Computer Algebra Independent Integration Tests

Summer 2024

1-Algebraic-functions/1.1-Binomial/1.1.2-Quadratic-binomial/34-1.1.2.6

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Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**396**]. This is test number [34].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Mathematica	76.77 (304)	23.23 (92)
Maple	66.67 (264)	33.33 (132)
Rubi	46.97 (186)	53.03 (210)
Fricas	29.04 (115)	70.96 (281)
Sympy	7.32 (29)	92.68 (367)
Reduce	4.29 (17)	95.71 (379)
Mupad	3.54 (14)	96.46 (382)
Giac	3.54 (14)	96.46 (382)
Maxima	3.54 (14)	96.46 (382)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

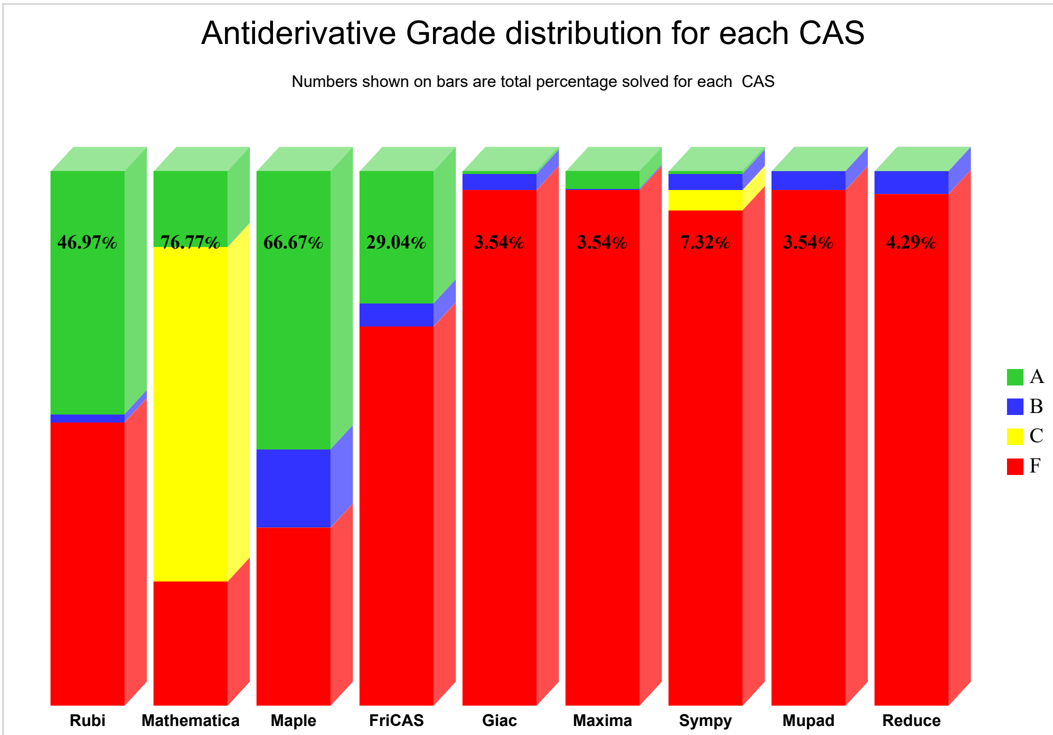
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

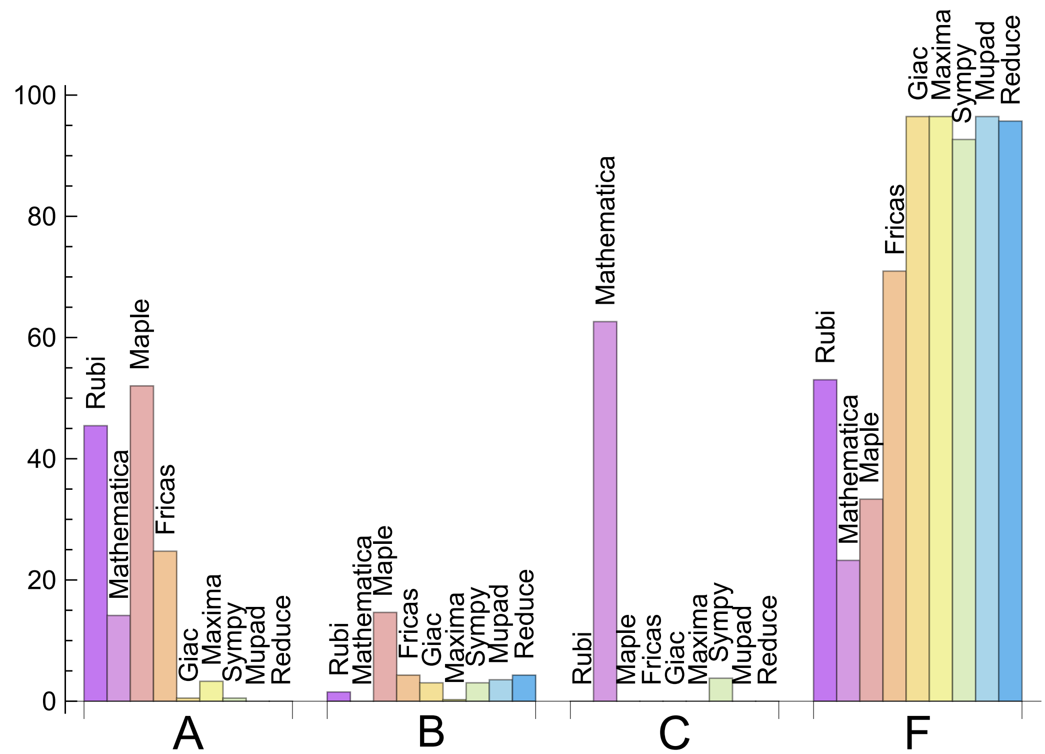
System	% A grade	% B grade	% C grade	% F grade
Maple	52.020	14.646	0.000	33.333
Rubi	45.455	1.515	0.000	53.030
Fricas	24.747	4.293	0.000	70.960
Mathematica	14.141	0.000	62.626	23.232
Maxima	3.283	0.253	0.000	96.465
Giac	0.505	3.030	0.000	96.465
Sympy	0.505	3.030	3.788	92.677
Mupad	0.000	3.535	0.000	96.465
Reduce	0.000	4.293	0.000	95.707

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Mathematica	92	100.00	0.00	0.00
Maple	132	100.00	0.00	0.00
Rubi	210	100.00	0.00	0.00
Fricas	281	43.77	56.23	0.00
Sympy	367	89.37	10.63	0.00
Reduce	379	100.00	0.00	0.00
Mupad	382	0.00	100.00	0.00
Giac	382	100.00	0.00	0.00
Maxima	382	100.00	0.00	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.07
Fricas	0.11
Giac	0.15
Reduce	0.22
Mupad	0.43
Rubi	0.87
Mathematica	4.29
Maple	11.82
Sympy	20.09

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Maxima	293.71	1.57	242.00	1.68
Mupad	379.36	2.05	305.00	2.21
Mathematica	425.81	0.89	353.00	0.80
Rubi	498.69	1.21	414.50	1.07
Fricas	555.34	1.43	434.00	1.03
Reduce	646.00	3.19	475.00	3.30
Maple	886.36	1.70	630.50	1.29
Giac	1512.00	6.99	1009.00	7.01
Sympy	3492.59	16.87	1460.00	13.52

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

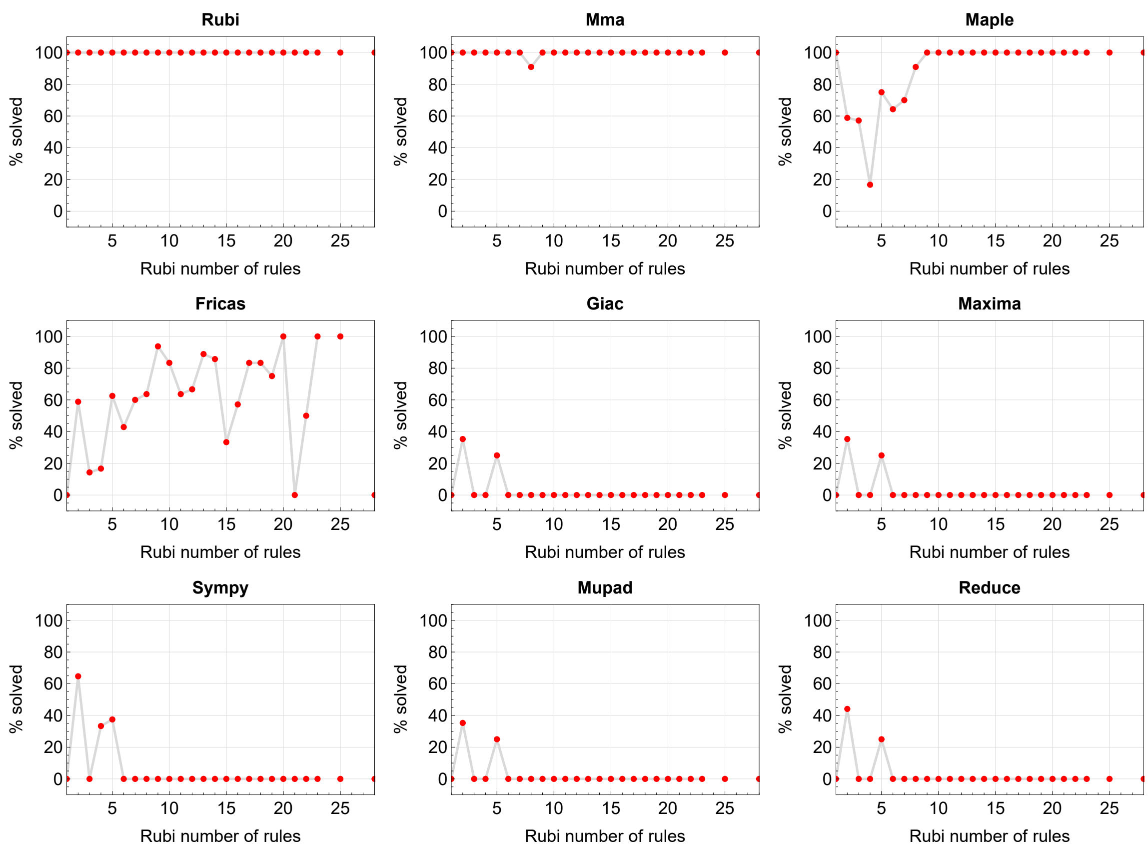


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

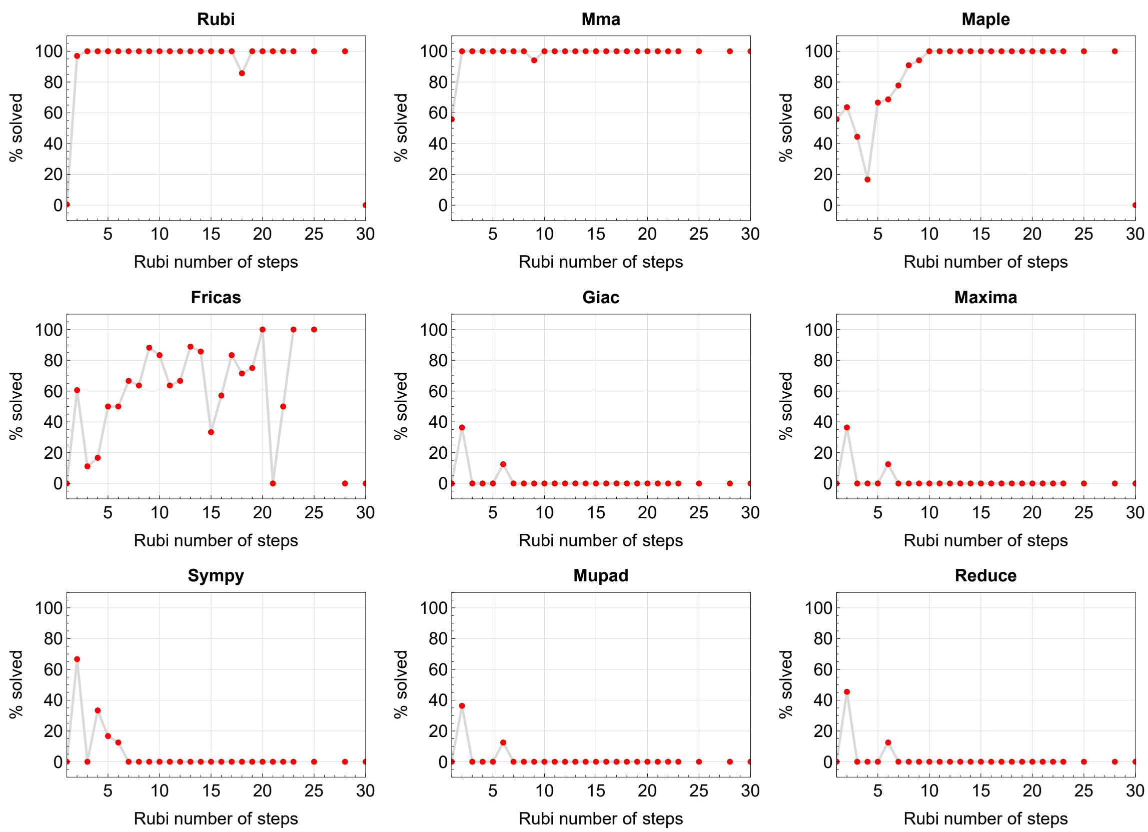


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

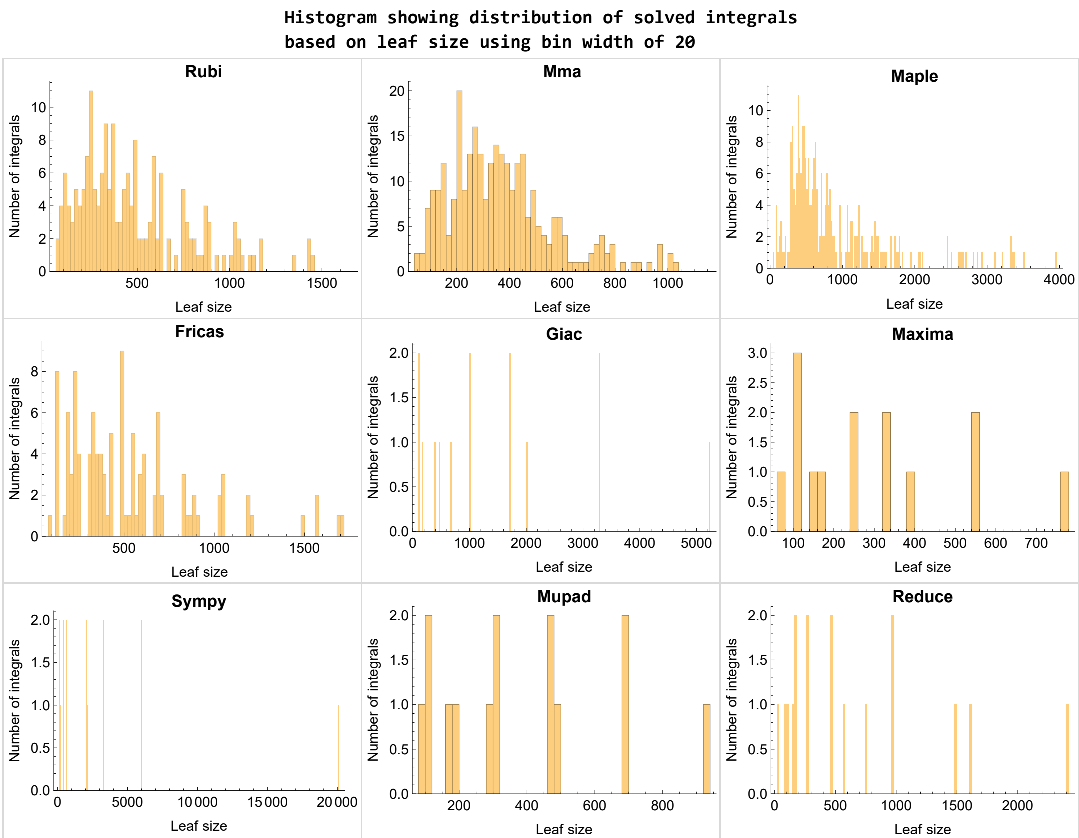


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

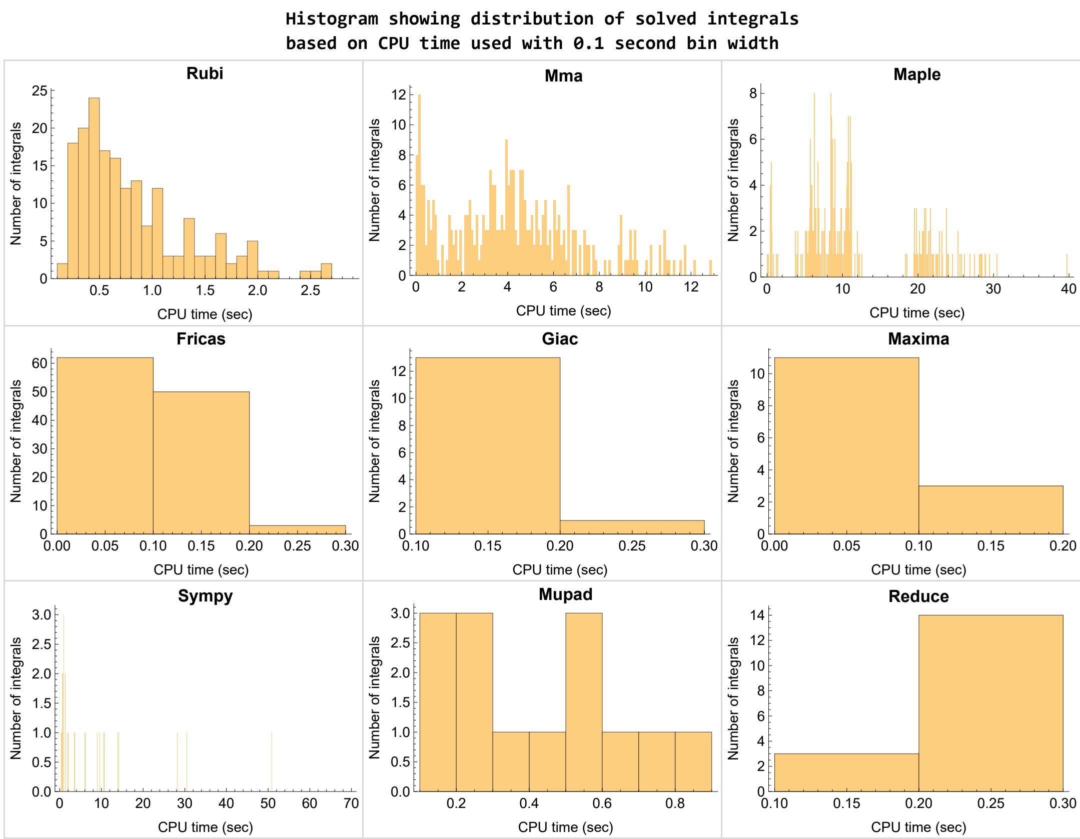


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

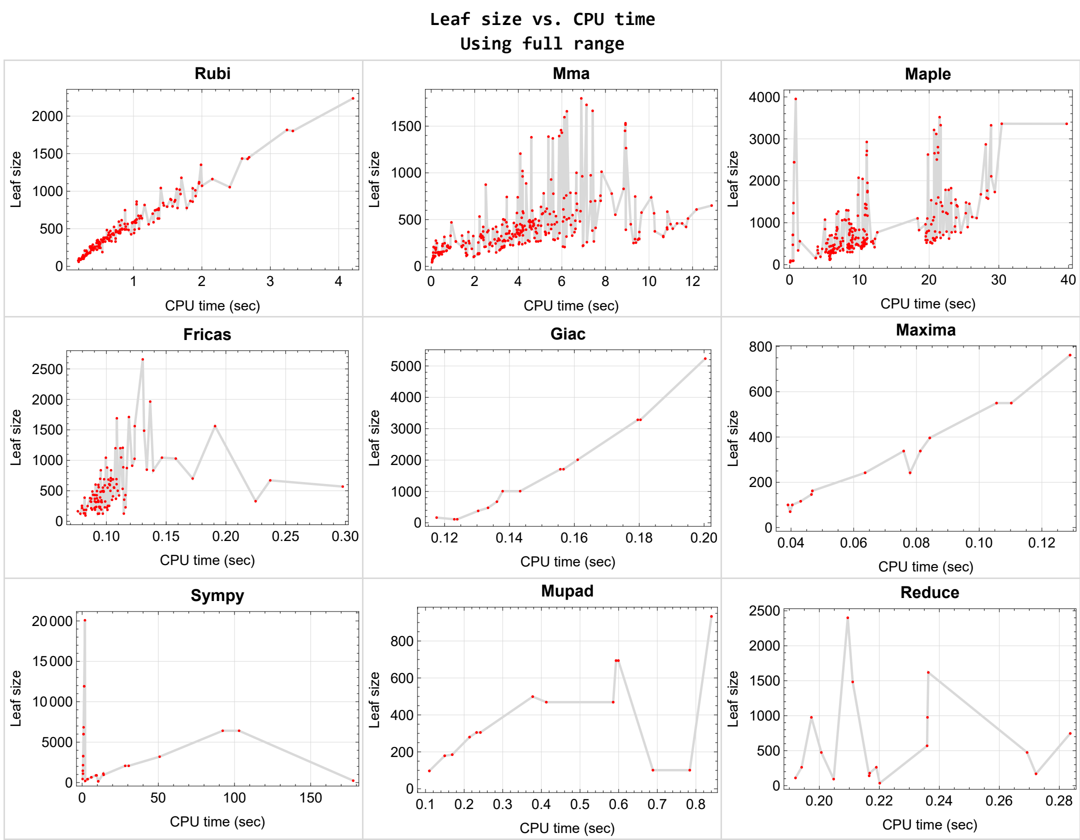


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {52, 53, 54, 56, 57, 58, 59, 83, 84, 85, 87, 88, 89, 110, 111, 112, 114, 115, 137, 138, 140, 141, 142, 167, 168, 170, 171, 172, 173, 204, 205, 207, 208, 209, 210, 233, 234, 235, 237, 238, 239, 270, 271, 273, 274}

Mathematica {}

Maple {}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

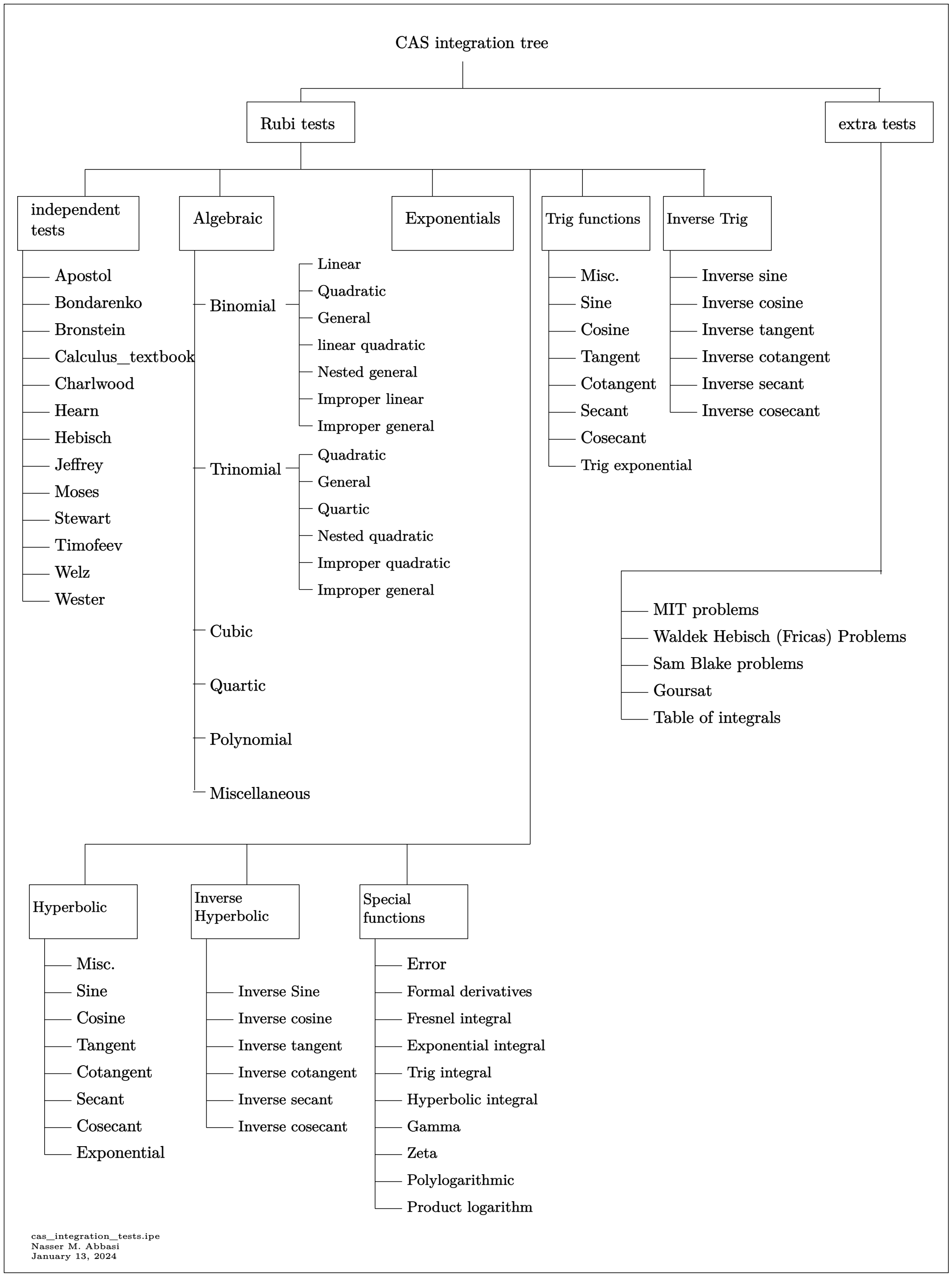
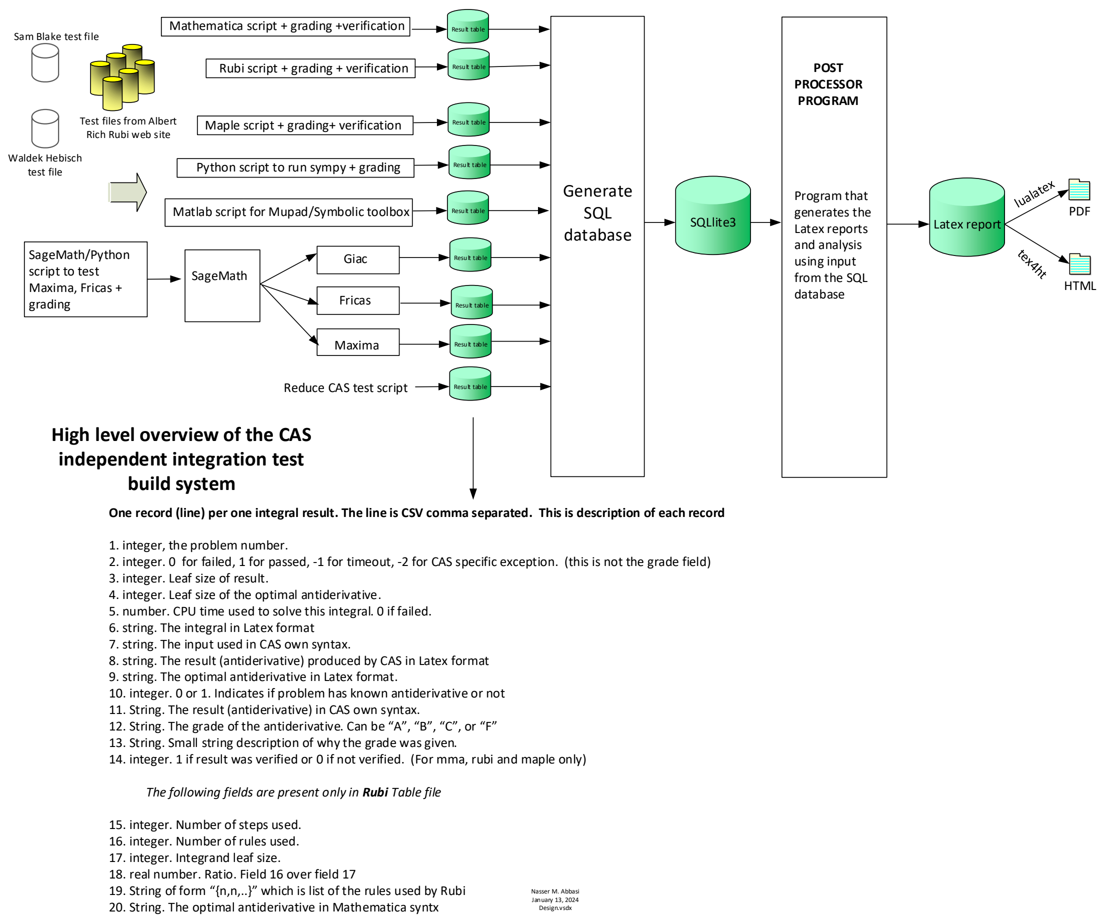


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 30, 31, 32, 33, 34, 35, 37, 38, 39, 40, 41, 42, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 72, 79, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 99, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 126, 133, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 151, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 185, 193, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 222, 230, 233, 234, 235, 237, 240, 241, 242, 243, 244, 245, 246, 252, 260, 270, 271, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 291, 299, 319, 326, 340, 362 }

B grade { 105, 157, 236, 238, 239, 272 }

C grade { }

F normal fail { 29, 36, 43, 68, 69, 70, 71, 73, 74, 75, 76, 77, 78, 80, 81, 82, 97, 98, 100, 101, 102, 103, 104, 106, 107, 108, 109, 123, 124, 125, 127, 128, 129, 130, 131, 132, 134, 135, 136, 149, 150, 152, 153, 154, 155, 156, 158, 159, 160, 161, 162, 181, 182, 183, 184, 186, 187, 188, 189, 190, 191, 192, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 218, 219, 220, 221, 223, 224, 225, 226, 227, 228, 229, 231, 232, 247, 248, 249, 250, 251, 253, 254, 255, 256, 257, 258, 259, 261, 262, 263, 264, 265, 266, 267, 268, 269, 287, 288, 289, 290, 292, 293, 294, 295, 296, 297, 298, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 320, 321, 322, 323, 324, 325, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396

}

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 283, 284, 319, 340, 362 }

B grade { }

C grade { 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301 }

F normal fail { 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 3, 4, 11, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 100, 101, 102, 103, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 192, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 262, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 289, 290, 291, 292, 293, 294, 301 }

B grade { 1, 2, 8, 9, 10, 15, 16, 17, 18, 76, 77, 99, 104, 105, 130, 131, 157, 189, 190, 191, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 226, 227, 228, 229, 230, 231, 232, 256, 257, 258, 259, 260, 261, 263, 264, 265, 266, 267, 268, 269, 288, 295, 296, 297, 298, 299, 300 }

C grade { }

F normal fail { 5, 6, 7, 12, 13, 14, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 4, 50, 51, 52, 53, 54, 55, 59, 60, 61, 62, 65, 66, 67, 83, 84, 85, 86, 90, 91, 95, 96, 110, 111, 112, 113, 116, 117, 118, 120, 121, 122, 137, 138, 139, 141, 142, 143, 144, 145, 146, 147, 148, 163, 164, 165, 166, 167, 168, 169, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 204, 205, 206, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 235, 236, 237, 240, 241, 242, 243, 244, 245, 246, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286 }

B grade { 1, 2, 3, 8, 9, 10, 11, 15, 16, 17, 18, 233, 234, 238, 239, 270, 271 }

C grade { }

F normal fail { 5, 6, 7, 12, 13, 14, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 56, 57, 58, 63, 64, 87, 88, 89, 92, 93, 94, 114, 115, 119, 140, 170, 207, 305, 306, 307, 308, 309, 312, 313, 314, 315, 316, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 333, 334, 335, 336, 337, 339, 341, 342, 343, 344, 345, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 362, 363, 364, 365, 366, 369, 370, 371, 372, 373, 375, 376, 377, 378, 379, 380, 389, 390, 391, 392, 393, 394, 395, 396 }

F(-1) timedout fail { 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 310, 311, 317, 318, 331, 332, 338, 340, 346, 359, 360, 361, 367, 368, 374, 381, 382, 383, 384, 385, 386, 387, 388 }

F(-2) exception fail { }

Maxima

A grade { 1, 2, 3, 4, 8, 9, 10, 11, 16, 17, 18, 50, 51 }

B grade { 15 }

C grade { }

F normal fail { 5, 6, 7, 12, 13, 14, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396 }

F(-1) timedout fail { }

F(-2) exception fail { }

Giac

A grade { 50, 51 }

B grade { 1, 2, 3, 4, 8, 9, 10, 11, 15, 16, 17, 18 }

C grade { }

F normal fail { 5, 6, 7, 12, 13, 14, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 8, 9, 10, 11, 15, 16, 17, 18, 50, 51 }

C grade { }

F normal fail { }

F(-1) timedout fail { 5, 6, 7, 12, 13, 14, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396 }

F(-2) exception fail { }

Sympy

A grade { 50, 51 }

B grade { 1, 2, 3, 4, 8, 9, 10, 11, 15, 16, 17, 18 }

C grade { 5, 6, 7, 12, 19, 22, 23, 24, 25, 26, 32, 33, 39, 40, 46 }

F normal fail { 13, 14, 20, 21, 27, 28, 30, 31, 34, 37, 38, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 104, 105, 106, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396 }

F(-1) timedout fail { 29, 35, 36, 41, 42, 43, 44, 45, 47, 48, 49, 103, 107, 108, 196, 197, 198, 199, 200, 201, 202, 203, 263, 264, 265, 266, 267, 268, 269, 295, 296, 297, 298, 299, 300, 301, 337, 350, 380 }

F(-2) exception fail { }

Reduce

A grade { }

B grade { 1, 2, 3, 4, 5, 8, 9, 10, 11, 12, 15, 16, 17, 18, 19, 50, 51 }

C grade { }

F normal fail { 6, 7, 13, 14, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	151	1229	338	911	5992	1708	747	469
N.S.	1	1.00	0.80	6.50	1.79	4.82	31.70	9.04	3.95	2.48
time (sec)	N/A	0.546	0.429	0.514	0.076	0.122	0.924	0.156	0.284	0.586

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	113	711	242	532	3271	1009	475	305
N.S.	1	1.00	0.78	4.94	1.68	3.69	22.72	7.01	3.30	2.12
time (sec)	N/A	0.300	0.103	0.457	0.064	0.114	0.702	0.138	0.201	0.232

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	73	96	146	235	1460	478	263	185
N.S.	1	1.00	0.75	0.99	1.51	2.42	15.05	4.93	2.71	1.91
time (sec)	N/A	0.230	0.055	0.105	0.047	0.092	0.459	0.133	0.219	0.169

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	43	59	70	94	439	167	111	97
N.S.	1	1.00	0.72	0.98	1.17	1.57	7.32	2.78	1.85	1.62
time (sec)	N/A	0.200	0.038	0.066	0.040	0.083	0.309	0.118	0.192	0.110

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	93	0	0	0	418	0	36	0
N.S.	1	1.00	0.79	0.00	0.00	0.00	3.54	0.00	0.31	0.00
time (sec)	N/A	0.253	0.123	0.000	0.000	0.000	3.449	0.000	0.220	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	153	174	108	0	0	0	2069	0	95	0
N.S.	1	1.14	0.71	0.00	0.00	0.00	13.52	0.00	0.62	0.00
time (sec)	N/A	0.342	0.177	0.000	0.000	0.000	30.583	0.000	0.198	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	206	220	133	0	0	0	6411	0	613	0
N.S.	1	1.07	0.65	0.00	0.00	0.00	31.12	0.00	2.98	0.00
time (sec)	N/A	0.384	0.381	0.000	0.000	0.000	103.008	0.000	0.228	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	292	292	247	2443	550	1711	11914	3283	1484	694
N.S.	1	1.00	0.85	8.37	1.88	5.86	40.80	11.24	5.08	2.38
time (sec)	N/A	0.499	0.620	0.638	0.110	0.119	1.354	0.180	0.211	0.599

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	178	1471	396	1043	6836	2010	977	499
N.S.	1	1.00	0.82	6.81	1.83	4.83	31.65	9.31	4.52	2.31
time (sec)	N/A	0.415	0.209	0.546	0.084	0.100	0.972	0.161	0.197	0.377

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	113	711	242	495	3271	1009	570	305
N.S.	1	1.00	0.78	4.94	1.68	3.44	22.72	7.01	3.96	2.12
time (sec)	N/A	0.310	0.106	0.453	0.078	0.092	0.710	0.143	0.236	0.242

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	67	90	116	217	1096	380	263	179
N.S.	1	1.00	0.74	0.99	1.27	2.38	12.04	4.18	2.89	1.97
time (sec)	N/A	0.224	0.050	0.416	0.043	0.080	0.572	0.130	0.194	0.150

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	146	0	0	0	649	0	94	0
N.S.	1	1.00	0.82	0.00	0.00	0.00	3.65	0.00	0.53	0.00
time (sec)	N/A	0.365	0.194	0.000	0.000	0.000	6.004	0.000	0.205	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	200	245	156	0	0	0	0	0	292	0
N.S.	1	1.22	0.78	0.00	0.00	0.00	0.00	0.00	1.46	0.00
time (sec)	N/A	0.517	0.511	0.000	0.000	0.000	0.000	0.000	0.214	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	290	305	165	0	0	0	0	0	1569	0
N.S.	1	1.05	0.57	0.00	0.00	0.00	0.00	0.00	5.41	0.00
time (sec)	N/A	0.510	0.697	0.000	0.000	0.000	0.000	0.000	0.238	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	379	379	327	3953	762	2657	20086	5234	2401	933
N.S.	1	1.00	0.86	10.43	2.01	7.01	53.00	13.81	6.34	2.46
time (sec)	N/A	0.612	0.891	0.867	0.129	0.130	1.854	0.200	0.209	0.840

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	284	239	2443	550	1690	11914	3283	1619	694
N.S.	1	1.00	0.84	8.60	1.94	5.95	41.95	11.56	5.70	2.44
time (sec)	N/A	0.505	0.528	0.630	0.106	0.109	1.331	0.180	0.236	0.593

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	151	1229	338	837	5992	1708	977	469
N.S.	1	1.00	0.80	6.50	1.79	4.43	31.70	9.04	5.17	2.48
time (sec)	N/A	0.368	0.354	0.507	0.081	0.095	0.923	0.157	0.236	0.413

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	90	475	162	381	2152	673	475	280
N.S.	1	1.00	0.74	3.93	1.34	3.15	17.79	5.56	3.93	2.31
time (sec)	N/A	0.262	0.084	0.450	0.047	0.086	0.592	0.136	0.269	0.214

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	258	258	217	0	0	0	887	0	179	0
N.S.	1	1.00	0.84	0.00	0.00	0.00	3.44	0.00	0.69	0.00
time (sec)	N/A	0.505	0.523	0.000	0.000	0.000	9.074	0.000	0.217	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	260	340	209	0	0	0	0	0	693	0
N.S.	1	1.31	0.80	0.00	0.00	0.00	0.00	0.00	2.67	0.00
time (sec)	N/A	0.646	0.745	0.000	0.000	0.000	0.000	0.000	0.218	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	340	489	218	0	0	0	0	0	0	0
N.S.	1	1.44	0.64	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.017	0.737	0.000	0.000	0.000	0.000	0.000	0.250	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	363	363	315	0	0	0	1102	0	0	0
N.S.	1	1.00	0.87	0.00	0.00	0.00	3.04	0.00	0.00	0.00
time (sec)	N/A	0.645	0.606	0.000	0.000	0.000	13.934	0.000	0.238	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	260	260	219	0	0	0	887	0	1357	0
N.S.	1	1.00	0.84	0.00	0.00	0.00	3.41	0.00	5.22	0.00
time (sec)	N/A	0.496	0.510	0.000	0.000	0.000	9.509	0.000	0.231	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	147	0	0	0	649	0	693	0
N.S.	1	1.00	0.82	0.00	0.00	0.00	3.61	0.00	3.85	0.00
time (sec)	N/A	0.367	0.219	0.000	0.000	0.000	6.130	0.000	0.207	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	93	0	0	0	418	0	292	0
N.S.	1	1.00	0.78	0.00	0.00	0.00	3.48	0.00	2.43	0.00
time (sec)	N/A	0.250	0.104	0.000	0.000	0.000	3.672	0.000	0.212	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	56	0	0	0	201	0	95	0
N.S.	1	1.00	0.73	0.00	0.00	0.00	2.61	0.00	1.23	0.00
time (sec)	N/A	0.193	0.055	0.000	0.000	0.000	2.019	0.000	0.224	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	100	0	0	0	0	0	19	0
N.S.	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	0.282	0.138	0.000	0.000	0.000	0.000	0.000	0.216	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	206	223	149	0	0	0	0	0	34	0
N.S.	1	1.08	0.72	0.00	0.00	0.00	0.00	0.00	0.17	0.00
time (sec)	N/A	0.462	0.316	0.000	0.000	0.000	0.000	0.000	0.182	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	342	0	195	0	0	0	0	0	58	0
N.S.	1	0.00	0.57	0.00	0.00	0.00	0.00	0.00	0.17	0.00
time (sec)	N/A	0.000	0.853	0.000	0.000	0.000	0.000	0.000	0.199	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	264	333	212	0	0	0	0	0	0	0
N.S.	1	1.26	0.80	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.680	0.755	0.000	0.000	0.000	0.000	0.000	0.268	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	202	245	158	0	0	0	0	0	0	0
N.S.	1	1.21	0.78	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.500	0.578	0.000	0.000	0.000	0.000	0.000	0.231	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	153	174	108	0	0	0	2069	0	1569	0
N.S.	1	1.14	0.71	0.00	0.00	0.00	13.52	0.00	10.25	0.00
time (sec)	N/A	0.323	0.180	0.000	0.000	0.000	28.269	0.000	0.214	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	81	0	0	0	954	0	613	0
N.S.	1	1.00	0.79	0.00	0.00	0.00	9.26	0.00	5.95	0.00
time (sec)	N/A	0.206	0.067	0.000	0.000	0.000	14.120	0.000	0.220	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	205	222	147	0	0	0	0	0	30	0
N.S.	1	1.08	0.72	0.00	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	0.468	0.295	0.000	0.000	0.000	0.000	0.000	0.228	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	304	329	207	0	0	0	0	0	58	0
N.S.	1	1.08	0.68	0.00	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	0.738	0.884	0.000	0.000	0.000	0.000	0.000	0.196	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	491	0	265	0	0	0	0	0	99	0
N.S.	1	0.00	0.54	0.00	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	0.000	1.133	0.000	0.000	0.000	0.000	0.000	0.267	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	326	443	222	0	0	0	0	0	0	0
N.S.	1	1.36	0.68	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.008	0.764	0.000	0.000	0.000	0.000	0.000	0.303	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	270	304	169	0	0	0	0	0	0	0
N.S.	1	1.13	0.63	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.514	0.712	0.000	0.000	0.000	0.000	0.000	0.236	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	206	219	133	0	0	0	6411	0	0	0
N.S.	1	1.06	0.65	0.00	0.00	0.00	31.12	0.00	0.00	0.00
time (sec)	N/A	0.387	0.390	0.000	0.000	0.000	92.256	0.000	0.231	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	81	0	0	0	3199	0	1550	0
N.S.	1	1.00	0.79	0.00	0.00	0.00	31.06	0.00	15.05	0.00
time (sec)	N/A	0.204	0.075	0.000	0.000	0.000	50.867	0.000	0.246	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	333	370	197	0	0	0	0	0	41	0
N.S.	1	1.11	0.59	0.00	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.739	0.814	0.000	0.000	0.000	0.000	0.000	0.184	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	452	500	266	0	0	0	0	0	82	0
N.S.	1	1.11	0.59	0.00	0.00	0.00	0.00	0.00	0.18	0.00
time (sec)	N/A	1.081	1.129	0.000	0.000	0.000	0.000	0.000	0.236	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	665	0	329	0	0	0	0	0	140	0
N.S.	1	0.00	0.49	0.00	0.00	0.00	0.00	0.00	0.21	0.00
time (sec)	N/A	0.000	1.417	0.000	0.000	0.000	0.000	0.000	0.260	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	688	1020	248	0	0	0	0	0	0	0
N.S.	1	1.48	0.36	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.675	0.265	0.000	0.000	0.000	0.000	0.000	0.418	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	399	455	198	0	0	0	0	0	0	0
N.S.	1	1.14	0.50	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.807	0.163	0.000	0.000	0.000	0.000	0.000	0.282	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	217	241	147	0	0	0	236	0	0	0
N.S.	1	1.11	0.68	0.00	0.00	0.00	1.09	0.00	0.00	0.00
time (sec)	N/A	0.433	0.094	0.000	0.000	0.000	177.905	0.000	0.235	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	118	0	0	0	0	0	0	0
N.S.	1	1.00	0.73	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.326	0.362	0.000	0.000	0.000	0.000	0.000	0.302	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	295	286	128	0	0	0	0	0	0	0
N.S.	1	0.97	0.43	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.497	0.340	0.000	0.000	0.000	0.000	0.000	0.727	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	483	491	128	0	0	0	0	0	0	0
N.S.	1	1.02	0.27	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.896	0.492	0.000	0.000	0.000	0.000	0.000	1.696	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	89	91	95	100	234	151	113	141	101
N.S.	1	0.95	0.97	1.01	1.06	2.49	1.61	1.20	1.50	1.07
time (sec)	N/A	0.229	0.101	0.525	0.040	0.095	10.611	0.124	0.217	0.784

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	89	91	95	100	220	151	113	170	101
N.S.	1	0.94	0.96	1.00	1.05	2.32	1.59	1.19	1.79	1.06
time (sec)	N/A	0.221	0.109	0.503	0.039	0.094	10.522	0.123	0.272	0.689

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1522	1802	1001	2627	0	1561	0	0	0	0
N.S.	1	1.18	0.66	1.73	0.00	1.03	0.00	0.00	0.00	0.00
time (sec)	N/A	3.329	6.671	19.832	0.000	0.191	0.000	0.000	2.052	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1130	1430	777	1668	0	1199	0	0	0	0
N.S.	1	1.27	0.69	1.48	0.00	1.06	0.00	0.00	0.00	0.00
time (sec)	N/A	2.668	5.609	9.787	0.000	0.112	0.000	0.000	1.626	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	841	1103	570	1075	0	873	0	0	0	0
N.S.	1	1.31	0.68	1.28	0.00	1.04	0.00	0.00	0.00	0.00
time (sec)	N/A	1.957	4.231	7.883	0.000	0.117	0.000	0.000	1.295	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	609	1043	422	704	0	593	0	0	1140	0
N.S.	1	1.71	0.69	1.16	0.00	0.97	0.00	0.00	1.87	0.00
time (sec)	N/A	1.400	2.390	6.022	0.000	0.103	0.000	0.000	0.984	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	692	670	324	516	0	0	0	0	921	0
N.S.	1	0.97	0.47	0.75	0.00	0.00	0.00	0.00	1.33	0.00
time (sec)	N/A	1.107	2.350	5.377	0.000	0.000	0.000	0.000	1.166	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	694	639	308	444	0	0	0	0	0	0
N.S.	1	0.92	0.44	0.64	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.124	2.397	5.918	0.000	0.000	0.000	0.000	1.992	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1067	751	383	547	0	0	0	0	0	0
N.S.	1	0.70	0.36	0.51	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.368	2.628	7.292	0.000	0.000	0.000	0.000	4.370	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1641	1012	520	755	0	589	0	0	0	0
N.S.	1	0.62	0.32	0.46	0.00	0.36	0.00	0.00	0.00	0.00
time (sec)	N/A	1.904	4.156	8.572	0.000	0.099	0.000	0.000	7.782	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	689	623	471	893	0	691	0	0	1358	0
N.S.	1	0.90	0.68	1.30	0.00	1.00	0.00	0.00	1.97	0.00
time (sec)	N/A	1.046	3.454	6.783	0.000	0.108	0.000	0.000	1.031	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	530	472	367	604	0	485	0	0	906	0
N.S.	1	0.89	0.69	1.14	0.00	0.92	0.00	0.00	1.71	0.00
time (sec)	N/A	0.794	1.709	5.480	0.000	0.098	0.000	0.000	0.820	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	381	350	269	431	0	313	0	0	520	0
N.S.	1	0.92	0.71	1.13	0.00	0.82	0.00	0.00	1.36	0.00
time (sec)	N/A	0.478	1.480	4.023	0.000	0.096	0.000	0.000	0.658	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	324	312	225	345	0	0	0	0	413	0
N.S.	1	0.96	0.69	1.06	0.00	0.00	0.00	0.00	1.27	0.00
time (sec)	N/A	0.461	1.465	3.998	0.000	0.000	0.000	0.000	0.874	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	332	319	251	369	0	0	0	0	1284	0
N.S.	1	0.96	0.76	1.11	0.00	0.00	0.00	0.00	3.87	0.00
time (sec)	N/A	0.483	1.589	5.265	0.000	0.000	0.000	0.000	1.682	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	391	424	324	477	0	314	0	0	1114	0
N.S.	1	1.08	0.83	1.22	0.00	0.80	0.00	0.00	2.85	0.00
time (sec)	N/A	0.695	1.933	6.417	0.000	0.091	0.000	0.000	3.268	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	536	580	446	636	0	483	0	0	1029	0
N.S.	1	1.08	0.83	1.19	0.00	0.90	0.00	0.00	1.92	0.00
time (sec)	N/A	0.984	3.260	7.563	0.000	0.095	0.000	0.000	4.641	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	695	752	571	833	0	686	0	0	0	0
N.S.	1	1.08	0.82	1.20	0.00	0.99	0.00	0.00	0.00	0.00
time (sec)	N/A	1.338	3.938	8.628	0.000	0.105	0.000	0.000	5.281	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1113	0	829	1105	0	0	0	0	33	0
N.S.	1	0.00	0.74	0.99	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	0.000	8.833	18.342	0.000	0.000	0.000	0.000	200.027	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	787	0	596	785	0	0	0	0	33	0
N.S.	1	0.00	0.76	1.00	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	0.000	4.642	8.973	0.000	0.000	0.000	0.000	200.018	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	558	0	426	565	0	0	0	0	1494	0
N.S.	1	0.00	0.76	1.01	0.00	0.00	0.00	0.00	2.68	0.00
time (sec)	N/A	0.000	3.495	7.647	0.000	0.000	0.000	0.000	12.271	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	401	0	320	699	0	0	0	0	600	0
N.S.	1	0.00	0.80	1.74	0.00	0.00	0.00	0.00	1.50	0.00
time (sec)	N/A	0.000	2.249	9.052	0.000	0.000	0.000	0.000	4.999	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	322	326	185	340	0	0	0	0	28	0
N.S.	1	1.01	0.57	1.06	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	0.473	1.584	1.203	0.000	0.000	0.000	0.000	0.528	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	415	0	533	412	0	0	0	0	32	0
N.S.	1	0.00	1.28	0.99	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	0.000	2.219	8.295	0.000	0.000	0.000	0.000	8.013	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	528	0	348	461	0	0	0	0	32	0
N.S.	1	0.00	0.66	0.87	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	0.000	3.440	8.494	0.000	0.000	0.000	0.000	27.332	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	741	0	1206	813	0	0	0	0	32	0
N.S.	1	0.00	1.63	1.10	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	0.000	4.095	9.385	0.000	0.000	0.000	0.000	52.806	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	771	0	505	1538	0	0	0	0	33	0
N.S.	1	0.00	0.65	1.99	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	0.000	4.787	20.293	0.000	0.000	0.000	0.000	200.029	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	587	0	378	1265	0	0	0	0	0	0
N.S.	1	0.00	0.64	2.16	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	3.598	21.960	0.000	0.000	0.000	0.000	49.287	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	531	0	874	849	0	0	0	0	0	0
N.S.	1	0.00	1.65	1.60	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	2.505	5.072	0.000	0.000	0.000	0.000	26.179	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	413	378	401	559	0	0	0	0	39	0
N.S.	1	0.92	0.97	1.35	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	0.563	2.834	1.458	0.000	0.000	0.000	0.000	9.241	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	651	0	244	791	0	0	0	0	43	0
N.S.	1	0.00	0.37	1.22	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	0.000	2.958	19.706	0.000	0.000	0.000	0.000	27.952	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	771	0	416	1135	0	0	0	0	33	0
N.S.	1	0.00	0.54	1.47	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	0.000	4.573	20.697	0.000	0.000	0.000	0.000	200.030	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1095	0	470	1785	0	0	0	0	33	0
N.S.	1	0.00	0.43	1.63	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	0.000	4.532	22.803	0.000	0.000	0.000	0.000	200.030	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1902	2236	1265	3327	0	1964	0	0	0	0
N.S.	1	1.18	0.67	1.75	0.00	1.03	0.00	0.00	0.00	0.00
time (sec)	N/A	4.208	8.944	21.655	0.000	0.137	0.000	0.000	2.643	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1435	1816	1012	2657	0	1561	0	0	0	0
N.S.	1	1.27	0.71	1.85	0.00	1.09	0.00	0.00	0.00	0.00
time (sec)	N/A	3.243	7.822	20.766	0.000	0.124	0.000	0.000	2.160	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1120	1435	782	2046	0	1199	0	0	0	0
N.S.	1	1.28	0.70	1.83	0.00	1.07	0.00	0.00	0.00	0.00
time (sec)	N/A	2.587	6.646	10.452	0.000	0.108	0.000	0.000	1.729	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	849	1352	584	1300	0	881	0	0	0	0
N.S.	1	1.59	0.69	1.53	0.00	1.04	0.00	0.00	0.00	0.00
time (sec)	N/A	1.987	5.630	8.128	0.000	0.101	0.000	0.000	1.355	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	927	894	451	854	0	0	0	0	1520	0
N.S.	1	0.96	0.49	0.92	0.00	0.00	0.00	0.00	1.64	0.00
time (sec)	N/A	1.541	4.086	6.832	0.000	0.000	0.000	0.000	1.660	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1035	794	392	644	0	0	0	0	0	0
N.S.	1	0.77	0.38	0.62	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.492	4.000	6.460	0.000	0.000	0.000	0.000	2.505	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1343	798	415	629	0	0	0	0	0	0
N.S.	1	0.59	0.31	0.47	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.571	4.205	6.770	0.000	0.000	0.000	0.000	5.691	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	684	624	480	1060	0	688	0	0	1358	0
N.S.	1	0.91	0.70	1.55	0.00	1.01	0.00	0.00	1.99	0.00
time (sec)	N/A	1.044	4.484	7.523	0.000	0.106	0.000	0.000	1.047	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	526	471	368	695	0	494	0	0	906	0
N.S.	1	0.90	0.70	1.32	0.00	0.94	0.00	0.00	1.72	0.00
time (sec)	N/A	0.663	3.034	5.756	0.000	0.097	0.000	0.000	0.814	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	430	415	288	509	0	0	0	0	749	0
N.S.	1	0.97	0.67	1.18	0.00	0.00	0.00	0.00	1.74	0.00
time (sec)	N/A	0.632	2.911	5.246	0.000	0.000	0.000	0.000	0.965	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	396	371	272	433	0	0	0	0	0	0
N.S.	1	0.94	0.69	1.09	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.638	3.044	5.710	0.000	0.000	0.000	0.000	1.641	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	453	419	349	502	0	0	0	0	1555	0
N.S.	1	0.92	0.77	1.11	0.00	0.00	0.00	0.00	3.43	0.00
time (sec)	N/A	0.712	3.477	7.315	0.000	0.000	0.000	0.000	3.447	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	526	580	429	643	0	489	0	0	1405	0
N.S.	1	1.10	0.82	1.22	0.00	0.93	0.00	0.00	2.67	0.00
time (sec)	N/A	1.025	4.680	9.002	0.000	0.088	0.000	0.000	5.791	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	690	755	560	832	0	684	0	0	0	0
N.S.	1	1.09	0.81	1.21	0.00	0.99	0.00	0.00	0.00	0.00
time (sec)	N/A	1.346	5.448	9.965	0.000	0.102	0.000	0.000	8.121	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	790	0	609	826	0	0	0	0	33	0
N.S.	1	0.00	0.77	1.05	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	0.000	6.068	18.555	0.000	0.000	0.000	0.000	200.020	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	547	0	437	601	0	0	0	0	1492	0
N.S.	1	0.00	0.80	1.10	0.00	0.00	0.00	0.00	2.73	0.00
time (sec)	N/A	0.000	4.542	8.976	0.000	0.000	0.000	0.000	12.967	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	402	382	742	831	0	0	0	0	779	0
N.S.	1	0.95	1.85	2.07	0.00	0.00	0.00	0.00	1.94	0.00
time (sec)	N/A	0.585	3.472	9.448	0.000	0.000	0.000	0.000	6.884	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	516	0	739	636	0	0	0	0	818	0
N.S.	1	0.00	1.43	1.23	0.00	0.00	0.00	0.00	1.59	0.00
time (sec)	N/A	0.000	3.974	9.846	0.000	0.000	0.000	0.000	12.451	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	627	0	369	498	0	0	0	0	0	0
N.S.	1	0.00	0.59	0.79	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	5.230	8.483	0.000	0.000	0.000	0.000	41.807	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	867	0	1369	841	0	0	0	0	0	0
N.S.	1	0.00	1.58	0.97	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	5.585	10.557	0.000	0.000	0.000	0.000	102.558	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	879	0	514	1603	0	0	0	0	33	0
N.S.	1	0.00	0.58	1.82	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	0.000	6.327	21.300	0.000	0.000	0.000	0.000	200.032	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	717	0	1381	1488	0	0	0	0	0	0
N.S.	1	0.00	1.93	2.08	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	4.602	21.676	0.000	0.000	0.000	0.000	67.714	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	482	1030	352	1075	0	0	0	0	0	0
N.S.	1	2.14	0.73	2.23	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.632	4.071	5.063	0.000	0.000	0.000	0.000	31.295	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	881	0	266	1102	0	0	0	0	0	0
N.S.	1	0.00	0.30	1.25	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	4.912	19.741	0.000	0.000	0.000	0.000	44.505	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1105	0	443	1458	0	0	0	0	33	0
N.S.	1	0.00	0.40	1.32	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	0.000	6.413	21.021	0.000	0.000	0.000	0.000	200.035	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1412	0	480	1826	0	0	0	0	33	0
N.S.	1	0.00	0.34	1.29	0.00	0.00	0.00	0.00	0.02	0.00
time (sec)	N/A	0.000	6.830	23.148	0.000	0.000	0.000	0.000	200.031	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	529	0	450	610	0	0	0	0	1575	0
N.S.	1	0.00	0.85	1.15	0.00	0.00	0.00	0.00	2.98	0.00
time (sec)	N/A	0.000	5.668	9.385	0.000	0.000	0.000	0.000	11.471	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1181	1451	787	1468	0	1206	0	0	0	0
N.S.	1	1.23	0.67	1.24	0.00	1.02	0.00	0.00	0.00	0.00
time (sec)	N/A	2.690	6.392	23.169	0.000	0.114	0.000	0.000	1.645	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	862	1120	586	963	0	886	0	0	0	0
N.S.	1	1.30	0.68	1.12	0.00	1.03	0.00	0.00	0.00	0.00
time (sec)	N/A	1.959	5.240	19.481	0.000	0.114	0.000	0.000	1.279	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	621	843	423	646	0	608	0	0	1140	0
N.S.	1	1.36	0.68	1.04	0.00	0.98	0.00	0.00	1.84	0.00
time (sec)	N/A	1.433	2.978	9.514	0.000	0.109	0.000	0.000	0.966	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	445	826	303	453	0	409	0	0	651	0
N.S.	1	1.86	0.68	1.02	0.00	0.92	0.00	0.00	1.46	0.00
time (sec)	N/A	1.045	2.308	7.339	0.000	0.106	0.000	0.000	0.665	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	532	529	255	373	0	0	0	0	490	0
N.S.	1	0.99	0.48	0.70	0.00	0.00	0.00	0.00	0.92	0.00
time (sec)	N/A	0.887	2.164	6.833	0.000	0.000	0.000	0.000	0.783	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	651	590	282	386	0	0	0	0	1617	0
N.S.	1	0.91	0.43	0.59	0.00	0.00	0.00	0.00	2.48	0.00
time (sec)	N/A	0.999	2.402	6.849	0.000	0.000	0.000	0.000	1.462	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	532	479	366	582	0	499	0	0	906	0
N.S.	1	0.90	0.69	1.09	0.00	0.94	0.00	0.00	1.70	0.00
time (sec)	N/A	0.744	2.577	8.799	0.000	0.103	0.000	0.000	0.792	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	388	356	270	417	0	324	0	0	521	0
N.S.	1	0.92	0.70	1.07	0.00	0.84	0.00	0.00	1.34	0.00
time (sec)	N/A	0.528	1.731	7.071	0.000	0.099	0.000	0.000	0.600	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	265	215	312	0	214	0	0	273	0
N.S.	1	0.93	0.76	1.10	0.00	0.75	0.00	0.00	0.96	0.00
time (sec)	N/A	0.365	1.396	5.178	0.000	0.090	0.000	0.000	0.421	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	256	206	298	0	0	0	0	224	0
N.S.	1	0.98	0.79	1.14	0.00	0.00	0.00	0.00	0.85	0.00
time (sec)	N/A	0.383	1.409	5.129	0.000	0.000	0.000	0.000	0.504	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	326	247	360	0	192	0	0	1015	0
N.S.	1	1.15	0.87	1.27	0.00	0.68	0.00	0.00	3.57	0.00
time (sec)	N/A	0.491	1.757	6.205	0.000	0.096	0.000	0.000	1.307	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	391	430	342	476	0	320	0	0	970	0
N.S.	1	1.10	0.87	1.22	0.00	0.82	0.00	0.00	2.48	0.00
time (sec)	N/A	0.687	2.279	8.316	0.000	0.096	0.000	0.000	1.880	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	531	583	464	640	0	488	0	0	910	0
N.S.	1	1.10	0.87	1.21	0.00	0.92	0.00	0.00	1.71	0.00
time (sec)	N/A	0.979	3.796	10.562	0.000	0.097	0.000	0.000	2.913	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	555	0	433	543	0	0	0	0	1495	0
N.S.	1	0.00	0.78	0.98	0.00	0.00	0.00	0.00	2.69	0.00
time (sec)	N/A	0.000	4.288	10.615	0.000	0.000	0.000	0.000	11.696	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	399	0	323	600	0	0	0	0	604	0
N.S.	1	0.00	0.81	1.50	0.00	0.00	0.00	0.00	1.51	0.00
time (sec)	N/A	0.000	3.217	10.948	0.000	0.000	0.000	0.000	5.040	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	316	0	187	274	0	0	0	0	46	0
N.S.	1	0.00	0.59	0.87	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	0.000	1.868	4.086	0.000	0.000	0.000	0.000	0.590	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	143	191	0	0	0	0	43	0
N.S.	1	1.00	1.40	1.87	0.00	0.00	0.00	0.00	0.42	0.00
time (sec)	N/A	0.207	1.596	4.412	0.000	0.000	0.000	0.000	0.350	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	387	0	373	302	0	0	0	0	46	0
N.S.	1	0.00	0.96	0.78	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.000	2.387	8.276	0.000	0.000	0.000	0.000	7.643	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	498	0	345	448	0	0	0	0	46	0
N.S.	1	0.00	0.69	0.90	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	0.000	3.722	8.648	0.000	0.000	0.000	0.000	25.131	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	702	0	1020	799	0	0	0	0	46	0
N.S.	1	0.00	1.45	1.14	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	0.000	4.193	10.756	0.000	0.000	0.000	0.000	36.996	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	643	0	438	1554	0	0	0	0	0	0
N.S.	1	0.00	0.68	2.42	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	4.909	23.783	0.000	0.000	0.000	0.000	89.808	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	538	0	359	1214	0	0	0	0	0	0
N.S.	1	0.00	0.67	2.26	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	3.967	6.740	0.000	0.000	0.000	0.000	19.004	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	496	0	253	812	0	0	0	0	70	0
N.S.	1	0.00	0.51	1.64	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	0.000	2.720	6.746	0.000	0.000	0.000	0.000	9.669	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	440	565	496	725	0	0	0	0	67	0
N.S.	1	1.28	1.13	1.65	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	0.871	3.377	6.612	0.000	0.000	0.000	0.000	9.938	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	648	0	293	1058	0	0	0	0	70	0
N.S.	1	0.00	0.45	1.63	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.000	3.690	20.564	0.000	0.000	0.000	0.000	21.409	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	840	0	392	1401	0	0	0	0	33	0
N.S.	1	0.00	0.47	1.67	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	0.000	4.213	21.854	0.000	0.000	0.000	0.000	200.027	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1095	0	470	1785	0	0	0	0	33	0
N.S.	1	0.00	0.43	1.63	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	0.000	0.930	22.353	0.000	0.000	0.000	0.000	200.029	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	630	962	488	964	0	1047	0	0	0	0
N.S.	1	1.53	0.77	1.53	0.00	1.66	0.00	0.00	0.00	0.00
time (sec)	N/A	1.688	4.946	20.312	0.000	0.112	0.000	0.000	2.224	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	443	745	363	718	0	708	0	0	0	0
N.S.	1	1.68	0.82	1.62	0.00	1.60	0.00	0.00	0.00	0.00
time (sec)	N/A	1.314	3.941	11.047	0.000	0.104	0.000	0.000	1.641	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	306	595	281	532	0	423	0	0	1420	0
N.S.	1	1.94	0.92	1.74	0.00	1.38	0.00	0.00	4.64	0.00
time (sec)	N/A	0.786	3.348	8.972	0.000	0.111	0.000	0.000	1.192	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	429	576	270	478	0	0	0	0	1342	0
N.S.	1	1.34	0.63	1.11	0.00	0.00	0.00	0.00	3.13	0.00
time (sec)	N/A	0.983	3.404	9.516	0.000	0.000	0.000	0.000	1.812	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	569	701	331	530	0	429	0	0	0	0
N.S.	1	1.23	0.58	0.93	0.00	0.75	0.00	0.00	0.00	0.00
time (sec)	N/A	1.281	4.210	10.405	0.000	0.116	0.000	0.000	2.948	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	879	881	441	675	0	664	0	0	0	0
N.S.	1	1.00	0.50	0.77	0.00	0.76	0.00	0.00	0.00	0.00
time (sec)	N/A	1.609	4.714	19.520	0.000	0.104	0.000	0.000	5.650	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	392	414	305	609	0	556	0	0	1483	0
N.S.	1	1.06	0.78	1.55	0.00	1.42	0.00	0.00	3.78	0.00
time (sec)	N/A	0.656	3.607	11.028	0.000	0.106	0.000	0.000	1.232	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	283	323	232	444	0	354	0	0	1048	0
N.S.	1	1.14	0.82	1.57	0.00	1.25	0.00	0.00	3.70	0.00
time (sec)	N/A	0.505	2.689	8.954	0.000	0.115	0.000	0.000	0.961	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	259	208	378	0	237	0	0	840	0
N.S.	1	1.26	1.01	1.83	0.00	1.15	0.00	0.00	4.08	0.00
time (sec)	N/A	0.364	2.450	4.665	0.000	0.094	0.000	0.000	0.741	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	309	217	382	0	214	0	0	797	0
N.S.	1	1.44	1.01	1.78	0.00	1.00	0.00	0.00	3.71	0.00
time (sec)	N/A	0.483	2.806	10.286	0.000	0.087	0.000	0.000	1.092	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	289	384	274	460	0	326	0	0	0	0
N.S.	1	1.33	0.95	1.59	0.00	1.13	0.00	0.00	0.00	0.00
time (sec)	N/A	0.612	3.358	9.864	0.000	0.090	0.000	0.000	1.910	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	391	489	383	587	0	552	0	0	0	0
N.S.	1	1.25	0.98	1.50	0.00	1.41	0.00	0.00	0.00	0.00
time (sec)	N/A	0.808	4.223	19.500	0.000	0.097	0.000	0.000	3.051	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	379	0	303	637	0	0	0	0	0	0
N.S.	1	0.00	0.80	1.68	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	4.745	6.762	0.000	0.000	0.000	0.000	17.509	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	253	0	275	390	0	0	0	0	70	0
N.S.	1	0.00	1.09	1.54	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.000	3.527	6.339	0.000	0.000	0.000	0.000	9.632	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	207	286	0	0	0	0	67	0
N.S.	1	1.00	0.99	1.37	0.00	0.00	0.00	0.00	0.32	0.00
time (sec)	N/A	0.312	3.274	6.588	0.000	0.000	0.000	0.000	8.918	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	463	0	493	586	0	0	0	0	70	0
N.S.	1	0.00	1.06	1.27	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	0.000	4.522	19.754	0.000	0.000	0.000	0.000	22.446	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	626	0	446	772	0	0	0	0	70	0
N.S.	1	0.00	0.71	1.23	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.000	5.533	21.519	0.000	0.000	0.000	0.000	36.646	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	887	0	1430	1123	0	0	0	0	70	0
N.S.	1	0.00	1.61	1.27	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	0.000	6.000	23.886	0.000	0.000	0.000	0.000	52.986	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	542	0	359	972	0	0	0	0	0	0
N.S.	1	0.00	0.66	1.79	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	5.258	8.820	0.000	0.000	0.000	0.000	63.864	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	514	0	266	866	0	0	0	0	111	0
N.S.	1	0.00	0.52	1.68	0.00	0.00	0.00	0.00	0.22	0.00
time (sec)	N/A	0.000	4.512	9.209	0.000	0.000	0.000	0.000	35.527	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	415	1179	277	873	0	0	0	0	108	0
N.S.	1	2.84	0.67	2.10	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	1.700	4.446	8.895	0.000	0.000	0.000	0.000	33.431	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	766	0	355	1228	0	0	0	0	33	0
N.S.	1	0.00	0.46	1.60	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	0.000	5.401	25.255	0.000	0.000	0.000	0.000	200.033	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	974	0	472	1678	0	0	0	0	33	0
N.S.	1	0.00	0.48	1.72	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	0.000	5.771	27.420	0.000	0.000	0.000	0.000	200.030	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1300	0	669	2108	0	0	0	0	33	0
N.S.	1	0.00	0.51	1.62	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	0.000	6.693	28.908	0.000	0.000	0.000	0.000	200.032	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	253	0	275	390	0	0	0	0	70	0
N.S.	1	0.00	1.09	1.54	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.000	0.217	6.448	0.000	0.000	0.000	0.000	9.595	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	357	0	354	382	0	0	0	0	71	0
N.S.	1	0.00	0.99	1.07	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	0.000	3.571	6.378	0.000	0.000	0.000	0.000	9.160	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	248	198	283	0	125	0	0	94	0
N.S.	1	1.19	0.95	1.35	0.00	0.60	0.00	0.00	0.45	0.00
time (sec)	N/A	0.370	3.271	5.505	0.000	0.115	0.000	0.000	0.299	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	228	207	282	0	124	0	0	100	0
N.S.	1	1.03	0.93	1.27	0.00	0.56	0.00	0.00	0.45	0.00
time (sec)	N/A	0.404	3.238	6.224	0.000	0.097	0.000	0.000	0.300	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	228	204	282	0	124	0	0	100	0
N.S.	1	1.03	0.92	1.27	0.00	0.56	0.00	0.00	0.45	0.00
time (sec)	N/A	0.401	3.352	6.323	0.000	0.093	0.000	0.000	0.304	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	231	209	283	0	123	0	0	102	0
N.S.	1	1.03	0.93	1.26	0.00	0.55	0.00	0.00	0.45	0.00
time (sec)	N/A	0.406	3.286	5.955	0.000	0.093	0.000	0.000	0.319	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	587	862	450	617	0	601	0	0	1140	0
N.S.	1	1.47	0.77	1.05	0.00	1.02	0.00	0.00	1.94	0.00
time (sec)	N/A	1.601	9.162	12.234	0.000	0.103	0.000	0.000	0.981	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	411	638	318	436	0	368	0	0	599	0
N.S.	1	1.55	0.77	1.06	0.00	0.90	0.00	0.00	1.46	0.00
time (sec)	N/A	1.089	6.210	8.532	0.000	0.108	0.000	0.000	0.670	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	302	483	236	325	0	224	0	0	288	0
N.S.	1	1.60	0.78	1.08	0.00	0.74	0.00	0.00	0.95	0.00
time (sec)	N/A	0.612	4.182	6.154	0.000	0.100	0.000	0.000	0.464	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	274	463	225	304	0	0	0	0	187	0
N.S.	1	1.69	0.82	1.11	0.00	0.00	0.00	0.00	0.68	0.00
time (sec)	N/A	0.761	3.756	5.978	0.000	0.000	0.000	0.000	0.557	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	581	272	374	0	228	0	0	998	0
N.S.	1	1.95	0.91	1.26	0.00	0.77	0.00	0.00	3.35	0.00
time (sec)	N/A	1.038	5.022	7.625	0.000	0.116	0.000	0.000	1.334	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	423	771	401	523	0	369	0	0	994	0
N.S.	1	1.82	0.95	1.24	0.00	0.87	0.00	0.00	2.35	0.00
time (sec)	N/A	1.369	6.624	10.708	0.000	0.090	0.000	0.000	1.930	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	608	1038	575	755	0	598	0	0	932	0
N.S.	1	1.71	0.95	1.24	0.00	0.98	0.00	0.00	1.53	0.00
time (sec)	N/A	1.866	9.656	19.594	0.000	0.093	0.000	0.000	3.097	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	524	489	382	575	0	494	0	0	906	0
N.S.	1	0.93	0.73	1.10	0.00	0.94	0.00	0.00	1.73	0.00
time (sec)	N/A	0.832	7.400	10.667	0.000	0.097	0.000	0.000	0.783	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	381	365	287	414	0	319	0	0	522	0
N.S.	1	0.96	0.75	1.09	0.00	0.84	0.00	0.00	1.37	0.00
time (sec)	N/A	0.574	5.612	8.375	0.000	0.093	0.000	0.000	0.616	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	265	218	302	0	186	0	0	229	0
N.S.	1	0.94	0.78	1.07	0.00	0.66	0.00	0.00	0.81	0.00
time (sec)	N/A	0.383	3.362	5.608	0.000	0.101	0.000	0.000	0.430	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	207	131	158	0	130	0	0	94	0
N.S.	1	1.00	0.64	0.77	0.00	0.63	0.00	0.00	0.46	0.00
time (sec)	N/A	0.297	2.192	3.725	0.000	0.082	0.000	0.000	0.237	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	248	198	283	0	125	0	0	94	0
N.S.	1	1.19	0.95	1.35	0.00	0.60	0.00	0.00	0.45	0.00
time (sec)	N/A	0.373	0.221	5.530	0.000	0.094	0.000	0.000	0.291	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	286	325	256	333	0	188	0	0	132	0
N.S.	1	1.14	0.90	1.16	0.00	0.66	0.00	0.00	0.46	0.00
time (sec)	N/A	0.486	4.045	7.229	0.000	0.091	0.000	0.000	0.851	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	391	438	355	481	0	319	0	0	287	0
N.S.	1	1.12	0.91	1.23	0.00	0.82	0.00	0.00	0.73	0.00
time (sec)	N/A	0.702	5.892	9.748	0.000	0.093	0.000	0.000	1.648	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	597	0	419	528	0	0	0	0	1496	0
N.S.	1	0.00	0.70	0.88	0.00	0.00	0.00	0.00	2.51	0.00
time (sec)	N/A	0.000	4.533	20.188	0.000	0.000	0.000	0.000	11.647	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	438	0	315	510	0	0	0	0	608	0
N.S.	1	0.00	0.72	1.16	0.00	0.00	0.00	0.00	1.39	0.00
time (sec)	N/A	0.000	3.898	20.434	0.000	0.000	0.000	0.000	5.098	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	334	0	172	202	0	0	0	0	78	0
N.S.	1	0.00	0.51	0.60	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	0.000	3.138	5.747	0.000	0.000	0.000	0.000	0.681	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	0	130	143	0	0	0	0	78	0
N.S.	1	0.00	0.63	0.70	0.00	0.00	0.00	0.00	0.38	0.00
time (sec)	N/A	0.000	2.144	5.819	0.000	0.000	0.000	0.000	0.474	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	100	101	118	0	0	0	0	75	0
N.S.	1	0.47	0.48	0.56	0.00	0.00	0.00	0.00	0.35	0.00
time (sec)	N/A	0.272	1.952	5.770	0.000	0.000	0.000	0.000	0.366	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	379	0	297	302	0	0	0	0	78	0
N.S.	1	0.00	0.78	0.80	0.00	0.00	0.00	0.00	0.21	0.00
time (sec)	N/A	0.000	3.095	8.981	0.000	0.000	0.000	0.000	1.066	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	441	0	341	445	0	0	0	0	626	0
N.S.	1	0.00	0.77	1.01	0.00	0.00	0.00	0.00	1.42	0.00
time (sec)	N/A	0.000	3.854	10.641	0.000	0.000	0.000	0.000	12.609	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	601	0	962	792	0	0	0	0	1538	0
N.S.	1	0.00	1.60	1.32	0.00	0.00	0.00	0.00	2.56	0.00
time (sec)	N/A	0.000	4.196	20.853	0.000	0.000	0.000	0.000	56.550	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	719	0	537	1454	0	0	0	0	0	0
N.S.	1	0.00	0.75	2.02	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	5.728	25.770	0.000	0.000	0.000	0.000	127.903	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	544	0	397	1269	0	0	0	0	130	0
N.S.	1	0.00	0.73	2.33	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	0.000	4.612	6.961	0.000	0.000	0.000	0.000	9.580	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	482	0	887	1061	0	0	0	0	130	0
N.S.	1	0.00	1.84	2.20	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	0.000	4.359	8.063	0.000	0.000	0.000	0.000	9.925	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	452	0	425	710	0	0	0	0	130	0
N.S.	1	0.00	0.94	1.57	0.00	0.00	0.00	0.00	0.29	0.00
time (sec)	N/A	0.000	3.892	8.552	0.000	0.000	0.000	0.000	8.551	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	426	448	587	973	0	0	0	0	127	0
N.S.	1	1.05	1.38	2.28	0.00	0.00	0.00	0.00	0.30	0.00
time (sec)	N/A	0.639	4.208	8.598	0.000	0.000	0.000	0.000	0.791	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	531	0	434	1214	0	0	0	0	1418	0
N.S.	1	0.00	0.82	2.29	0.00	0.00	0.00	0.00	2.67	0.00
time (sec)	N/A	0.000	5.044	22.639	0.000	0.000	0.000	0.000	18.644	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	705	0	476	1391	0	0	0	0	0	0
N.S.	1	0.00	0.68	1.97	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	4.621	24.068	0.000	0.000	0.000	0.000	55.807	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1080	0	730	3361	0	0	0	0	33	0
N.S.	1	0.00	0.68	3.11	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	0.000	6.100	30.414	0.000	0.000	0.000	0.000	200.025	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	852	0	559	2927	0	0	0	0	0	0
N.S.	1	0.00	0.66	3.44	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	5.089	11.052	0.000	0.000	0.000	0.000	108.548	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	666	0	458	2712	0	0	0	0	182	0
N.S.	1	0.00	0.69	4.07	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	0.000	4.704	11.073	0.000	0.000	0.000	0.000	18.430	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	582	0	392	2074	0	0	0	0	182	0
N.S.	1	0.00	0.67	3.56	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.000	4.354	9.879	0.000	0.000	0.000	0.000	21.109	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	583	0	391	1952	0	0	0	0	182	0
N.S.	1	0.00	0.67	3.35	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.000	3.933	11.151	0.000	0.000	0.000	0.000	19.051	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	669	0	438	2616	0	0	0	0	179	0
N.S.	1	0.00	0.65	3.91	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	0.000	4.168	11.024	0.000	0.000	0.000	0.000	55.324	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	859	0	567	2869	0	0	0	0	0	0
N.S.	1	0.00	0.66	3.34	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	5.129	28.131	0.000	0.000	0.000	0.000	53.006	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1129	0	758	3324	0	0	0	0	33	0
N.S.	1	0.00	0.67	2.94	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	0.000	6.202	28.886	0.000	0.000	0.000	0.000	200.030	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	529	870	459	632	0	608	0	0	1195	0
N.S.	1	1.64	0.87	1.19	0.00	1.15	0.00	0.00	2.26	0.00
time (sec)	N/A	1.831	11.520	11.125	0.000	0.102	0.000	0.000	1.029	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	378	636	328	449	0	376	0	0	630	0
N.S.	1	1.68	0.87	1.19	0.00	0.99	0.00	0.00	1.67	0.00
time (sec)	N/A	1.355	6.474	8.891	0.000	0.092	0.000	0.000	0.722	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	274	525	248	336	0	228	0	0	304	0
N.S.	1	1.92	0.91	1.23	0.00	0.83	0.00	0.00	1.11	0.00
time (sec)	N/A	0.715	4.318	6.222	0.000	0.099	0.000	0.000	0.468	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	426	237	315	0	0	0	0	199	0
N.S.	1	1.67	0.93	1.24	0.00	0.00	0.00	0.00	0.78	0.00
time (sec)	N/A	0.966	4.025	6.056	0.000	0.000	0.000	0.000	0.545	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	326	559	282	385	0	225	0	0	1029	0
N.S.	1	1.71	0.87	1.18	0.00	0.69	0.00	0.00	3.16	0.00
time (sec)	N/A	1.222	5.672	8.587	0.000	0.091	0.000	0.000	1.340	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	463	776	411	538	0	370	0	0	1024	0
N.S.	1	1.68	0.89	1.16	0.00	0.80	0.00	0.00	2.21	0.00
time (sec)	N/A	1.644	7.548	10.765	0.000	0.087	0.000	0.000	2.006	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	646	1054	585	772	0	602	0	0	960	0
N.S.	1	1.63	0.91	1.20	0.00	0.93	0.00	0.00	1.49	0.00
time (sec)	N/A	2.407	10.839	12.565	0.000	0.091	0.000	0.000	3.203	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	468	494	391	590	0	497	0	0	951	0
N.S.	1	1.06	0.84	1.26	0.00	1.06	0.00	0.00	2.03	0.00
time (sec)	N/A	0.868	8.936	10.773	0.000	0.098	0.000	0.000	0.850	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	347	368	297	427	0	327	0	0	550	0
N.S.	1	1.06	0.86	1.23	0.00	0.94	0.00	0.00	1.59	0.00
time (sec)	N/A	0.616	5.816	8.600	0.000	0.087	0.000	0.000	0.624	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	260	229	312	0	190	0	0	242	0
N.S.	1	1.02	0.90	1.23	0.00	0.75	0.00	0.00	0.95	0.00
time (sec)	N/A	0.436	3.632	5.635	0.000	0.088	0.000	0.000	0.422	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	138	161	0	130	0	0	100	0
N.S.	1	1.00	0.73	0.85	0.00	0.68	0.00	0.00	0.53	0.00
time (sec)	N/A	0.320	2.211	3.749	0.000	0.081	0.000	0.000	0.233	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	228	207	282	0	124	0	0	100	0
N.S.	1	1.03	0.93	1.27	0.00	0.56	0.00	0.00	0.45	0.00
time (sec)	N/A	0.395	0.111	6.261	0.000	0.078	0.000	0.000	0.289	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	308	323	266	343	0	187	0	0	140	0
N.S.	1	1.05	0.86	1.11	0.00	0.61	0.00	0.00	0.45	0.00
time (sec)	N/A	0.554	4.821	7.674	0.000	0.083	0.000	0.000	0.878	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	418	445	365	496	0	320	0	0	301	0
N.S.	1	1.06	0.87	1.19	0.00	0.77	0.00	0.00	0.72	0.00
time (sec)	N/A	0.781	6.580	9.603	0.000	0.087	0.000	0.000	1.785	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	514	0	433	539	0	0	0	0	1580	0
N.S.	1	0.00	0.84	1.05	0.00	0.00	0.00	0.00	3.07	0.00
time (sec)	N/A	0.000	4.684	20.186	0.000	0.000	0.000	0.000	10.642	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	384	0	331	524	0	0	0	0	643	0
N.S.	1	0.00	0.86	1.36	0.00	0.00	0.00	0.00	1.67	0.00
time (sec)	N/A	0.000	4.058	19.632	0.000	0.000	0.000	0.000	4.880	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	296	0	182	204	0	0	0	0	83	0
N.S.	1	0.00	0.61	0.69	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.000	3.181	5.674	0.000	0.000	0.000	0.000	0.679	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	0	138	145	0	0	0	0	83	0
N.S.	1	0.00	0.72	0.76	0.00	0.00	0.00	0.00	0.43	0.00
time (sec)	N/A	0.000	2.152	5.742	0.000	0.000	0.000	0.000	0.492	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	107	120	0	0	0	0	80	0
N.S.	1	1.00	1.07	1.20	0.00	0.00	0.00	0.00	0.80	0.00
time (sec)	N/A	0.253	1.982	5.728	0.000	0.000	0.000	0.000	0.358	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	324	0	280	300	0	0	0	0	83	0
N.S.	1	0.00	0.86	0.93	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.000	3.142	8.659	0.000	0.000	0.000	0.000	1.041	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	422	0	488	456	0	0	0	0	660	0
N.S.	1	0.00	1.16	1.08	0.00	0.00	0.00	0.00	1.56	0.00
time (sec)	N/A	0.000	4.524	10.365	0.000	0.000	0.000	0.000	12.144	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	584	0	781	809	0	0	0	0	1622	0
N.S.	1	0.00	1.34	1.39	0.00	0.00	0.00	0.00	2.78	0.00
time (sec)	N/A	0.000	5.165	19.956	0.000	0.000	0.000	0.000	51.535	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	613	0	552	1483	0	0	0	0	0	0
N.S.	1	0.00	0.90	2.42	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	8.453	25.293	0.000	0.000	0.000	0.000	120.533	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	481	0	406	1295	0	0	0	0	135	0
N.S.	1	0.00	0.84	2.69	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.000	4.800	8.405	0.000	0.000	0.000	0.000	9.469	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	427	0	929	1082	0	0	0	0	135	0
N.S.	1	0.00	2.18	2.53	0.00	0.00	0.00	0.00	0.32	0.00
time (sec)	N/A	0.000	5.507	8.475	0.000	0.000	0.000	0.000	9.788	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	407	0	441	725	0	0	0	0	135	0
N.S.	1	0.00	1.08	1.78	0.00	0.00	0.00	0.00	0.33	0.00
time (sec)	N/A	0.000	3.932	8.525	0.000	0.000	0.000	0.000	8.533	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	425	428	617	989	0	0	0	0	132	0
N.S.	1	1.01	1.45	2.33	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.709	5.308	8.543	0.000	0.000	0.000	0.000	0.764	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	537	0	447	1239	0	0	0	0	1469	0
N.S.	1	0.00	0.83	2.31	0.00	0.00	0.00	0.00	2.74	0.00
time (sec)	N/A	0.000	5.364	22.450	0.000	0.000	0.000	0.000	17.704	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	733	0	488	1417	0	0	0	0	0	0
N.S.	1	0.00	0.67	1.93	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	4.679	23.950	0.000	0.000	0.000	0.000	53.096	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	717	1162	650	913	0	1486	0	0	0	0
N.S.	1	1.62	0.91	1.27	0.00	2.07	0.00	0.00	0.00	0.00
time (sec)	N/A	2.153	12.874	23.700	0.000	0.132	0.000	0.000	2.293	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	520	885	451	634	0	1044	0	0	0	0
N.S.	1	1.70	0.87	1.22	0.00	2.01	0.00	0.00	0.00	0.00
time (sec)	N/A	1.551	10.989	20.566	0.000	0.146	0.000	0.000	1.506	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	374	673	314	474	0	671	0	0	1076	0
N.S.	1	1.80	0.84	1.27	0.00	1.79	0.00	0.00	2.88	0.00
time (sec)	N/A	1.072	10.657	10.555	0.000	0.237	0.000	0.000	1.069	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	278	862	249	448	0	500	0	0	657	0
N.S.	1	3.10	0.90	1.61	0.00	1.80	0.00	0.00	2.36	0.00
time (sec)	N/A	1.046	9.289	6.225	0.000	0.110	0.000	0.000	0.705	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	283	564	291	480	0	434	0	0	669	0
N.S.	1	1.99	1.03	1.70	0.00	1.53	0.00	0.00	2.36	0.00
time (sec)	N/A	0.915	9.358	10.751	0.000	0.089	0.000	0.000	1.135	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	A	F	B	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	377	810	415	547	0	701	0	0	0	0
N.S.	1	2.15	1.10	1.45	0.00	1.86	0.00	0.00	0.00	0.00
time (sec)	N/A	1.443	10.826	11.696	0.000	0.172	0.000	0.000	2.091	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	A	F	B	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	516	1072	608	709	0	1029	0	0	0	0
N.S.	1	2.08	1.18	1.37	0.00	1.99	0.00	0.00	0.00	0.00
time (sec)	N/A	2.001	12.181	22.026	0.000	0.158	0.000	0.000	3.394	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	459	500	396	576	0	846	0	0	1483	0
N.S.	1	1.09	0.86	1.25	0.00	1.84	0.00	0.00	3.23	0.00
time (sec)	N/A	0.832	10.839	20.852	0.000	0.134	0.000	0.000	1.254	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	338	377	285	431	0	569	0	0	694	0
N.S.	1	1.12	0.84	1.28	0.00	1.68	0.00	0.00	2.05	0.00
time (sec)	N/A	0.563	9.494	10.193	0.000	0.298	0.000	0.000	0.876	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	288	219	354	0	329	0	0	499	0
N.S.	1	1.17	0.89	1.44	0.00	1.34	0.00	0.00	2.03	0.00
time (sec)	N/A	0.406	6.966	6.144	0.000	0.225	0.000	0.000	0.615	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	206	334	0	253	0	0	142	0
N.S.	1	1.00	0.99	1.60	0.00	1.21	0.00	0.00	0.68	0.00
time (sec)	N/A	0.288	6.127	5.891	0.000	0.101	0.000	0.000	0.480	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	347	258	411	0	351	0	0	496	0
N.S.	1	1.36	1.01	1.61	0.00	1.38	0.00	0.00	1.95	0.00
time (sec)	N/A	0.553	7.148	10.287	0.000	0.105	0.000	0.000	0.852	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	343	455	367	497	0	558	0	0	845	0
N.S.	1	1.33	1.07	1.45	0.00	1.63	0.00	0.00	2.46	0.00
time (sec)	N/A	0.742	9.557	10.708	0.000	0.094	0.000	0.000	1.588	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	458	609	511	617	0	833	0	0	1010	0
N.S.	1	1.33	1.12	1.35	0.00	1.82	0.00	0.00	2.21	0.00
time (sec)	N/A	1.025	11.788	20.815	0.000	0.139	0.000	0.000	2.352	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	788	0	701	897	0	0	0	0	33	0
N.S.	1	0.00	0.89	1.14	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	0.000	7.781	25.525	0.000	0.000	0.000	0.000	200.042	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	576	0	480	772	0	0	0	0	0	0
N.S.	1	0.00	0.83	1.34	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	6.049	23.234	0.000	0.000	0.000	0.000	39.054	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	427	0	337	806	0	0	0	0	130	0
N.S.	1	0.00	0.79	1.89	0.00	0.00	0.00	0.00	0.30	0.00
time (sec)	N/A	0.000	5.261	8.685	0.000	0.000	0.000	0.000	8.159	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	339	0	354	397	0	0	0	0	130	0
N.S.	1	0.00	1.04	1.17	0.00	0.00	0.00	0.00	0.38	0.00
time (sec)	N/A	0.000	4.853	8.427	0.000	0.000	0.000	0.000	8.170	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	338	0	347	388	0	0	0	0	130	0
N.S.	1	0.00	1.03	1.15	0.00	0.00	0.00	0.00	0.38	0.00
time (sec)	N/A	0.000	3.983	9.197	0.000	0.000	0.000	0.000	8.534	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	342	343	365	412	0	0	0	0	127	0
N.S.	1	1.00	1.07	1.20	0.00	0.00	0.00	0.00	0.37	0.00
time (sec)	N/A	0.467	4.149	8.444	0.000	0.000	0.000	0.000	0.660	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	430	0	636	622	0	0	0	0	1414	0
N.S.	1	0.00	1.48	1.45	0.00	0.00	0.00	0.00	3.29	0.00
time (sec)	N/A	0.000	5.041	22.858	0.000	0.000	0.000	0.000	16.706	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	575	0	963	766	0	0	0	0	0	0
N.S.	1	0.00	1.67	1.33	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	6.936	24.540	0.000	0.000	0.000	0.000	39.971	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	787	0	1515	1113	0	0	0	0	0	0
N.S.	1	0.00	1.93	1.41	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	8.921	26.814	0.000	0.000	0.000	0.000	108.661	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	634	0	1797	1730	0	0	0	0	33	0
N.S.	1	0.00	2.83	2.73	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	0.000	6.891	11.147	0.000	0.000	0.000	0.000	200.030	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	514	0	1596	1431	0	0	0	0	218	0
N.S.	1	0.00	3.11	2.78	0.00	0.00	0.00	0.00	0.42	0.00
time (sec)	N/A	0.000	6.117	10.989	0.000	0.000	0.000	0.000	38.189	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	462	0	1394	1170	0	0	0	0	218	0
N.S.	1	0.00	3.02	2.53	0.00	0.00	0.00	0.00	0.47	0.00
time (sec)	N/A	0.000	5.876	11.161	0.000	0.000	0.000	0.000	29.828	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	466	0	1388	1168	0	0	0	0	218	0
N.S.	1	0.00	2.98	2.51	0.00	0.00	0.00	0.00	0.47	0.00
time (sec)	N/A	0.000	5.384	9.776	0.000	0.000	0.000	0.000	29.829	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	517	817	1659	1339	0	0	0	0	30	0
N.S.	1	1.58	3.21	2.59	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	1.162	6.230	11.020	0.000	0.000	0.000	0.000	200.033	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	672	0	737	1575	0	0	0	0	0	0
N.S.	1	0.00	1.10	2.34	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	10.098	28.460	0.000	0.000	0.000	0.000	61.750	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	917	0	693	1733	0	0	0	0	33	0
N.S.	1	0.00	0.76	1.89	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	0.000	7.318	29.421	0.000	0.000	0.000	0.000	200.029	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	951	0	756	3518	0	0	0	0	33	0
N.S.	1	0.00	0.79	3.70	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	0.000	7.771	21.501	0.000	0.000	0.000	0.000	200.027	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	808	0	590	3213	0	0	0	0	33	0
N.S.	1	0.00	0.73	3.98	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	0.000	6.857	20.704	0.000	0.000	0.000	0.000	200.033	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	728	0	548	2803	0	0	0	0	306	0
N.S.	1	0.00	0.75	3.85	0.00	0.00	0.00	0.00	0.42	0.00
time (sec)	N/A	0.000	6.491	21.260	0.000	0.000	0.000	0.000	23.636	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	663	0	502	2507	0	0	0	0	306	0
N.S.	1	0.00	0.76	3.78	0.00	0.00	0.00	0.00	0.46	0.00
time (sec)	N/A	0.000	6.221	21.105	0.000	0.000	0.000	0.000	22.567	0.000

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	727	0	526	2669	0	0	0	0	306	0
N.S.	1	0.00	0.72	3.67	0.00	0.00	0.00	0.00	0.42	0.00
time (sec)	N/A	0.000	6.107	21.300	0.000	0.000	0.000	0.000	24.816	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	810	0	600	3117	0	0	0	0	30	0
N.S.	1	0.00	0.74	3.85	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	0.000	6.678	21.025	0.000	0.000	0.000	0.000	200.032	0.000

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1047	0	778	3359	0	0	0	0	0	0
N.S.	1	0.00	0.74	3.21	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	8.288	39.729	0.000	0.000	0.000	0.000	90.385	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	546	861	459	652	0	1025	0	0	0	0
N.S.	1	1.58	0.84	1.19	0.00	1.88	0.00	0.00	0.00	0.00
time (sec)	N/A	1.865	11.285	21.079	0.000	0.124	0.000	0.000	1.574	0.000

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	380	636	323	488	0	697	0	0	1091	0
N.S.	1	1.67	0.85	1.28	0.00	1.83	0.00	0.00	2.87	0.00
time (sec)	N/A	1.374	10.653	10.667	0.000	0.095	0.000	0.000	1.103	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	283	614	253	462	0	430	0	0	667	0
N.S.	1	2.17	0.89	1.63	0.00	1.52	0.00	0.00	2.36	0.00
time (sec)	N/A	0.827	9.399	6.208	0.000	0.090	0.000	0.000	0.732	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	334	586	296	495	0	427	0	0	679	0
N.S.	1	1.75	0.89	1.48	0.00	1.28	0.00	0.00	2.03	0.00
time (sec)	N/A	1.293	9.537	11.017	0.000	0.092	0.000	0.000	1.173	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	472	777	422	564	0	690	0	0	0	0
N.S.	1	1.65	0.89	1.19	0.00	1.46	0.00	0.00	0.00	0.00
time (sec)	N/A	1.778	11.700	11.918	0.000	0.098	0.000	0.000	2.113	0.000

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	483	489	403	587	0	837	0	0	1507	0
N.S.	1	1.01	0.83	1.22	0.00	1.73	0.00	0.00	3.12	0.00
time (sec)	N/A	0.875	11.090	20.690	0.000	0.104	0.000	0.000	1.293	0.000

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	354	364	293	441	0	551	0	0	704	0
N.S.	1	1.03	0.83	1.25	0.00	1.56	0.00	0.00	1.99	0.00
time (sec)	N/A	0.626	9.555	10.675	0.000	0.101	0.000	0.000	0.866	0.000

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	264	223	365	0	344	0	0	505	0
N.S.	1	1.05	0.89	1.45	0.00	1.37	0.00	0.00	2.01	0.00
time (sec)	N/A	0.468	6.980	6.234	0.000	0.096	0.000	0.000	0.635	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	249	211	333	0	252	0	0	144	0
N.S.	1	1.05	0.89	1.41	0.00	1.06	0.00	0.00	0.61	0.00
time (sec)	N/A	0.410	6.049	5.918	0.000	0.102	0.000	0.000	0.501	0.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	300	329	266	423	0	340	0	0	504	0
N.S.	1	1.10	0.89	1.41	0.00	1.13	0.00	0.00	1.68	0.00
time (sec)	N/A	0.587	7.469	10.419	0.000	0.093	0.000	0.000	0.851	0.000

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	421	440	374	508	0	548	0	0	858	0
N.S.	1	1.05	0.89	1.21	0.00	1.30	0.00	0.00	2.04	0.00
time (sec)	N/A	0.805	10.267	11.990	0.000	0.106	0.000	0.000	1.658	0.000

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	319	330	269	417	0	390	0	0	534	0
N.S.	1	1.03	0.84	1.31	0.00	1.22	0.00	0.00	1.67	0.00
time (sec)	N/A	0.546	3.284	12.177	0.000	0.095	0.000	0.000	0.545	0.000

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	251	210	313	0	254	0	0	71	0
N.S.	1	1.05	0.88	1.32	0.00	1.07	0.00	0.00	0.30	0.00
time (sec)	N/A	0.410	2.499	6.228	0.000	0.080	0.000	0.000	0.334	0.000

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	122	138	0	164	0	0	71	0
N.S.	1	1.00	0.92	1.05	0.00	1.24	0.00	0.00	0.54	0.00
time (sec)	N/A	0.259	1.651	5.855	0.000	0.076	0.000	0.000	0.323	0.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	242	113	248	0	186	0	0	68	0
N.S.	1	1.08	0.50	1.11	0.00	0.83	0.00	0.00	0.30	0.00
time (sec)	N/A	0.384	1.620	5.822	0.000	0.081	0.000	0.000	0.179	0.000

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	280	305	241	398	0	249	0	0	347	0
N.S.	1	1.09	0.86	1.42	0.00	0.89	0.00	0.00	1.24	0.00
time (sec)	N/A	0.503	2.682	10.349	0.000	0.084	0.000	0.000	0.431	0.000

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	376	392	320	468	0	387	0	0	736	0
N.S.	1	1.04	0.85	1.24	0.00	1.03	0.00	0.00	1.96	0.00
time (sec)	N/A	0.665	3.775	10.596	0.000	0.082	0.000	0.000	0.788	0.000

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	540	0	488	789	0	0	0	0	0	0
N.S.	1	0.00	0.90	1.46	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	6.618	22.500	0.000	0.000	0.000	0.000	32.114	0.000

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	418	0	344	826	0	0	0	0	131	0
N.S.	1	0.00	0.82	1.98	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.000	5.261	8.660	0.000	0.000	0.000	0.000	7.637	0.000

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	362	0	369	398	0	0	0	0	131	0
N.S.	1	0.00	1.02	1.10	0.00	0.00	0.00	0.00	0.36	0.00
time (sec)	N/A	0.000	4.807	8.410	0.000	0.000	0.000	0.000	7.805	0.000

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	355	0	360	388	0	0	0	0	131	0
N.S.	1	0.00	1.01	1.09	0.00	0.00	0.00	0.00	0.37	0.00
time (sec)	N/A	0.000	3.983	8.479	0.000	0.000	0.000	0.000	7.985	0.000

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	363	484	382	411	0	0	0	0	128	0
N.S.	1	1.33	1.05	1.13	0.00	0.00	0.00	0.00	0.35	0.00
time (sec)	N/A	0.928	4.102	8.904	0.000	0.000	0.000	0.000	0.630	0.000

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	471	0	399	638	0	0	0	0	1423	0
N.S.	1	0.00	0.85	1.35	0.00	0.00	0.00	0.00	3.02	0.00
time (sec)	N/A	0.000	5.505	21.649	0.000	0.000	0.000	0.000	14.929	0.000

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	653	0	974	785	0	0	0	0	0	0
N.S.	1	0.00	1.49	1.20	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	7.322	23.753	0.000	0.000	0.000	0.000	36.068	0.000

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	905	0	1531	1138	0	0	0	0	0	0
N.S.	1	0.00	1.69	1.26	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	8.921	26.169	0.000	0.000	0.000	0.000	95.214	0.000

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	771	0	567	1758	0	0	0	0	34	0
N.S.	1	0.00	0.74	2.28	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	0.000	10.233	11.161	0.000	0.000	0.000	0.000	200.030	0.000

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	561	0	1664	1449	0	0	0	0	219	0
N.S.	1	0.00	2.97	2.58	0.00	0.00	0.00	0.00	0.39	0.00
time (sec)	N/A	0.000	7.411	10.830	0.000	0.000	0.000	0.000	37.707	0.000

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	515	0	1457	1186	0	0	0	0	219	0
N.S.	1	0.00	2.83	2.30	0.00	0.00	0.00	0.00	0.43	0.00
time (sec)	N/A	0.000	5.961	10.888	0.000	0.000	0.000	0.000	29.385	0.000

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	505	0	1450	1184	0	0	0	0	219	0
N.S.	1	0.00	2.87	2.34	0.00	0.00	0.00	0.00	0.43	0.00
time (sec)	N/A	0.000	8.899	10.790	0.000	0.000	0.000	0.000	28.966	0.000

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	566	935	1728	1355	0	0	0	0	31	0
N.S.	1	1.65	3.05	2.39	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	1.908	7.132	10.856	0.000	0.000	0.000	0.000	200.030	0.000

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	787	0	747	1598	0	0	0	0	0	0
N.S.	1	0.00	0.95	2.03	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	9.365	28.236	0.000	0.000	0.000	0.000	54.595	0.000

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1102	0	700	1765	0	0	0	0	34	0
N.S.	1	0.00	0.64	1.60	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	0.000	7.528	28.328	0.000	0.000	0.000	0.000	200.031	0.000

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1152	0	0	0	0	0	0	0	33	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.025	0.000

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	819	0	0	0	0	0	0	0	33	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.030	0.000

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	646	0	0	0	0	0	0	0	662	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.02	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	6.560	0.000

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	565	0	0	0	0	0	0	0	33	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.027	0.000

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	583	0	0	0	0	0	0	0	33	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.033	0.000

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	574	0	0	0	0	0	0	0	33	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.030	0.000

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	821	0	0	0	0	0	0	0	33	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.029	0.000

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1126	0	0	0	0	0	0	0	33	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.031	0.000

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1544	0	0	0	0	0	0	0	33	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.021	0.000

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1159	0	0	0	0	0	0	0	33	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.024	0.000

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	850	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.034	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	709	0	0	0	0	0	0	0	33	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.030	0.000

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	651	0	0	0	0	0	0	0	33	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.031	0.000

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	760	0	0	0	0	0	0	0	33	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.035	0.000

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	803	0	0	0	0	0	0	0	33	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.030	0.000

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	884	0	0	0	0	0	0	0	0	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	111.771	0.000

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	672	0	0	0	0	0	0	0	664	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.99	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	7.325	0.000

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	530	545	503	0	0	0	0	0	36	0
N.S.	1	1.03	0.95	0.00	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	0.806	3.308	0.000	0.000	0.000	0.000	0.000	0.587	0.000

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	503	0	0	0	0	0	0	0	33	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.030	0.000

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	425	0	0	0	0	0	0	0	33	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.027	0.000

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	593	0	0	0	0	0	0	0	33	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.030	0.000

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	828	0	0	0	0	0	0	0	33	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.027	0.000

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	718	0	0	0	0	0	0	0	33	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.026	0.000

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	573	0	0	0	0	0	0	0	33	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.031	0.000

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	481	748	0	0	0	0	0	0	30	0
N.S.	1	1.56	0.00	0.00	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	0.876	0.000	0.000	0.000	0.000	0.000	0.000	200.029	0.000

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	349	0	0	0	0	0	0	0	33	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.027	0.000

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	436	0	0	0	0	0	0	0	33	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.030	0.000

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	602	0	0	0	0	0	0	0	33	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.029	0.000

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	855	0	0	0	0	0	0	0	33	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.028	0.000

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	858	0	0	0	0	0	0	0	33	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.027	0.000

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	669	0	0	0	0	0	0	0	33	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.029	0.000

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	592	0	0	0	0	0	0	0	33	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.027	0.000

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	388	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.027	0.000

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	450	0	0	0	0	0	0	0	33	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.030	0.000

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	592	0	0	0	0	0	0	0	33	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.029	0.000

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	845	0	0	0	0	0	0	0	33	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.030	0.000

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	697	0	0	0	0	0	0	0	668	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.96	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	7.474	0.000

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	543	0	0	0	0	0	0	0	54	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.718	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	159	0	0	0	0	0	51	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	0.32	0.00
time (sec)	N/A	0.305	2.650	0.000	0.000	0.000	0.000	0.000	0.489	0.000

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	296	0	0	0	0	0	0	0	54	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.18	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.318	0.000

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	438	0	0	0	0	0	0	0	33	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.028	0.000

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	608	0	0	0	0	0	0	0	33	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.029	0.000

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	908	0	0	0	0	0	0	0	0	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	124.618	0.000

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	696	0	0	0	0	0	0	0	773	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.11	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	10.233	0.000

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	541	0	0	0	0	0	0	0	110	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.924	0.000

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	527	0	0	0	0	0	0	0	33	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.028	0.000

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	440	0	0	0	0	0	0	0	33	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.026	0.000

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	605	0	0	0	0	0	0	0	33	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.030	0.000

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	861	0	0	0	0	0	0	0	33	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.027	0.000

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	194	0	0	0	0	0	0	0	54	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.329	0.000

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	296	0	0	0	0	0	0	0	57	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.196	0.000

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	196	0	0	0	0	0	0	0	57	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.29	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.194	0.000

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	354	0	0	0	0	0	0	0	58	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.176	0.000

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	194	0	0	0	0	0	0	0	55	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.181	0.000

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	200	0	0	0	0	0	0	0	58	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.29	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.182	0.000

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	196	0	0	0	0	0	0	0	58	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.30	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.181	0.000

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	204	0	0	0	0	0	0	0	59	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.29	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.315	0.000

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	696	0	0	0	0	0	0	0	672	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.97	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	7.637	0.000

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	550	0	0	0	0	0	0	0	86	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.897	0.000

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	313	0	0	0	0	0	0	0	86	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.653	0.000

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	145	148	148	0	0	0	0	0	83	0
N.S.	1	1.02	1.02	0.00	0.00	0.00	0.00	0.00	0.57	0.00
time (sec)	N/A	0.264	2.984	0.000	0.000	0.000	0.000	0.000	0.427	0.000

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	341	0	0	0	0	0	0	0	86	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.310	0.000

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	458	0	0	0	0	0	0	0	86	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	2.326	0.000

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	638	0	0	0	0	0	0	0	33	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.027	0.000

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	995	0	0	0	0	0	0	0	33	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.029	0.000

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	616	0	0	0	0	0	0	0	138	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.22	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	14.978	0.000

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	493	0	0	0	0	0	0	0	138	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	15.227	0.000

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	314	0	0	0	0	0	0	0	138	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.44	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	15.073	0.000

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	320	0	0	0	0	0	0	0	135	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.42	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.945	0.000

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	404	0	0	0	0	0	0	0	1213	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	3.00	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	42.756	0.000

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	570	0	0	0	0	0	0	0	33	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.027	0.000

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1042	0	0	0	0	0	0	0	33	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.024	0.000

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	819	0	0	0	0	0	0	0	33	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.029	0.000

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	633	0	0	0	0	0	0	0	33	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.028	0.000

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	407	0	0	0	0	0	0	0	33	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.036	0.000

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	421	0	0	0	0	0	0	0	33	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.035	0.000

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	449	0	0	0	0	0	0	0	30	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.030	0.000

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	575	0	0	0	0	0	0	0	33	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.033	0.000

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	757	0	0	0	0	0	0	0	33	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.030	0.000

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	313	0	0	0	0	0	0	0	86	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.621	0.000

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	412	0	0	0	0	0	0	0	91	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.22	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.600	0.000

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	310	0	0	0	0	0	0	0	91	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.29	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.602	0.000

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	321	0	0	0	0	0	0	0	92	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.29	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.590	0.000

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	316	0	0	0	0	0	0	0	91	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.29	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.604	0.000

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	324	0	0	0	0	0	0	0	92	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.586	0.000

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	312	0	0	0	0	0	0	0	92	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.29	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.586	0.000

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	327	0	0	0	0	0	0	0	93	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.620	0.000

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	314	0	0	0	0	0	0	0	138	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.44	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	15.164	0.000

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	314	0	0	0	0	0	0	0	143	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.46	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	14.740	0.000

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	312	0	0	0	0	0	0	0	143	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.46	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	14.804	0.000

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	312	0	0	0	0	0	0	0	144	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.46	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	14.558	0.000

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	174	0	0	0	0	0	0	0	139	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.80	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	14.399	0.000

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	322	0	0	0	0	0	0	0	144	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.45	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	14.295	0.000

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	320	0	0	0	0	0	0	0	144	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.45	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	14.305	0.000

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	328	0	0	0	0	0	0	0	145	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.44	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	14.443	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [299] had the largest ratio of [.848485000000000045]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	2	1.00	29	0.069
2	A	2	2	1.00	29	0.069
3	A	2	2	1.00	27	0.074
4	A	2	2	1.00	20	0.100
5	A	2	2	1.00	29	0.069
6	A	4	4	1.14	29	0.138
7	A	4	4	1.07	29	0.138
8	A	2	2	1.00	31	0.065
9	A	2	2	1.00	31	0.065
10	A	2	2	1.00	29	0.069
11	A	2	2	1.00	22	0.091
12	A	2	2	1.00	31	0.065
13	A	4	4	1.22	31	0.129
14	A	6	6	1.05	31	0.194
15	A	2	2	1.00	31	0.065
16	A	2	2	1.00	31	0.065
17	A	2	2	1.00	29	0.069
18	A	2	2	1.00	22	0.091
19	A	2	2	1.00	31	0.065
20	A	4	4	1.31	31	0.129
21	A	6	6	1.44	31	0.194

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	2	2	1.00	31	0.065
23	A	2	2	1.00	31	0.065
24	A	2	2	1.00	31	0.065
25	A	2	2	1.00	29	0.069
26	A	2	2	1.00	22	0.091
27	A	2	2	1.00	31	0.065
28	A	3	3	1.08	31	0.097
29	F	0	0	N/A	0.000	N/A
30	A	4	4	1.26	31	0.129
31	A	4	4	1.21	31	0.129
32	A	4	4	1.14	29	0.138
33	A	2	2	1.00	22	0.091
34	A	3	3	1.08	31	0.097
35	A	5	5	1.08	31	0.161
36	F	0	0	N/A	0.000	N/A
37	A	6	6	1.36	31	0.194
38	A	6	6	1.13	31	0.194
39	A	4	4	1.06	29	0.138
40	A	2	2	1.00	22	0.091
41	A	4	4	1.11	31	0.129
42	A	7	7	1.11	31	0.226
43	F	0	0	N/A	0.000	N/A
44	A	7	7	1.48	31	0.226
45	A	6	6	1.14	31	0.194
46	A	5	5	1.11	29	0.172
47	A	2	2	1.00	31	0.065
48	A	3	3	0.97	31	0.097
49	A	4	4	1.02	31	0.129
50	A	6	5	0.95	29	0.172
51	A	6	5	0.94	29	0.172
52	A	18	18	1.18	35	0.514
53	A	17	17	1.27	35	0.486

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	15	15	1.31	35	0.429
55	A	2	2	1.71	32	0.062
56	A	14	14	0.97	35	0.400
57	A	13	13	0.92	35	0.371
58	A	15	15	0.70	35	0.429
59	A	18	18	0.62	35	0.514
60	A	9	9	0.90	33	0.273
61	A	8	8	0.89	33	0.242
62	A	6	6	0.92	30	0.200
63	A	7	7	0.96	33	0.212
64	A	6	6	0.96	33	0.182
65	A	10	10	1.08	33	0.303
66	A	11	11	1.08	33	0.333
67	A	13	13	1.08	33	0.394
68	F	0	0	N/A	0.000	N/A
69	F	0	0	N/A	0.000	N/A
70	F	0	0	N/A	0.000	N/A
71	F	0	0	N/A	0.000	N/A
72	A	6	6	1.01	32	0.188
73	F	0	0	N/A	0.000	N/A
74	F	0	0	N/A	0.000	N/A
75	F	0	0	N/A	0.000	N/A
76	F	0	0	N/A	0.000	N/A
77	F	0	0	N/A	0.000	N/A
78	F	0	0	N/A	0.000	N/A
79	A	8	8	0.92	32	0.250
80	F	0	0	N/A	0.000	N/A
81	F	0	0	N/A	0.000	N/A
82	F	0	0	N/A	0.000	N/A
83	A	19	19	1.18	35	0.543
84	A	17	17	1.27	35	0.486

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
85	A	16	16	1.28	35	0.457
86	A	2	2	1.59	32	0.062
87	A	16	16	0.96	35	0.457
88	A	18	18	0.77	35	0.514
89	A	19	19	0.59	35	0.543
90	A	9	9	0.91	33	0.273
91	A	7	7	0.90	30	0.233
92	A	8	8	0.97	33	0.242
93	A	10	10	0.94	33	0.303
94	A	11	11	0.92	33	0.333
95	A	12	12	1.10	33	0.364
96	A	13	13	1.09	33	0.394
97	F	0	0	N/A	0.000	N/A
98	F	0	0	N/A	0.000	N/A
99	A	9	9	0.95	32	0.281
100	F	0	0	N/A	0.000	N/A
101	F	0	0	N/A	0.000	N/A
102	F	0	0	N/A	0.000	N/A
103	F	0	0	N/A	0.000	N/A
104	F	0	0	N/A	0.000	N/A
105	B	21	21	2.14	32	0.656
106	F	0	0	N/A	0.000	N/A
107	F	0	0	N/A	0.000	N/A
108	F	0	0	N/A	0.000	N/A
109	F	0	0	N/A	0.000	N/A
110	A	16	16	1.23	35	0.457
111	A	14	14	1.30	35	0.400
112	A	12	12	1.36	35	0.343
113	A	2	2	1.86	32	0.062
114	A	11	11	0.99	35	0.314
115	A	12	12	0.91	35	0.343

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
116	A	7	7	0.90	33	0.212
117	A	6	6	0.92	33	0.182
118	A	5	5	0.93	30	0.167
119	A	5	5	0.98	33	0.152
120	A	7	7	1.15	33	0.212
121	A	9	9	1.10	33	0.273
122	A	10	10	1.10	33	0.303
123	F	0	0	N/A	0.000	N/A
124	F	0	0	N/A	0.000	N/A
125	F	0	0	N/A	0.000	N/A
126	A	1	1	1.00	32	0.031
127	F	0	0	N/A	0.000	N/A
128	F	0	0	N/A	0.000	N/A
129	F	0	0	N/A	0.000	N/A
130	F	0	0	N/A	0.000	N/A
131	F	0	0	N/A	0.000	N/A
132	F	0	0	N/A	0.000	N/A
133	A	12	12	1.28	32	0.375
134	F	0	0	N/A	0.000	N/A
135	F	0	0	N/A	0.000	N/A
136	F	0	0	N/A	0.000	N/A
137	A	14	14	1.53	35	0.400
138	A	13	13	1.68	35	0.371
139	A	2	2	1.94	32	0.062
140	A	16	16	1.34	35	0.457
141	A	16	16	1.23	35	0.457
142	A	19	19	1.00	35	0.543
143	A	8	8	1.06	33	0.242
144	A	7	7	1.14	33	0.212
145	A	6	6	1.26	30	0.200
146	A	9	9	1.44	33	0.273

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
147	A	11	11	1.33	33	0.333
148	A	13	13	1.25	33	0.394
149	F	0	0	N/A	0.000	N/A
150	F	0	0	N/A	0.000	N/A
151	A	3	3	1.00	32	0.094
152	F	0	0	N/A	0.000	N/A
153	F	0	0	N/A	0.000	N/A
154	F	0	0	N/A	0.000	N/A
155	F	0	0	N/A	0.000	N/A
156	F	0	0	N/A	0.000	N/A
157	B	22	22	2.84	32	0.688
158	F	0	0	N/A	0.000	N/A
159	F	0	0	N/A	0.000	N/A
160	F	0	0	N/A	0.000	N/A
161	F	0	0	N/A	0.000	N/A
162	F	0	0	N/A	0.000	N/A
163	A	6	6	1.19	33	0.182
164	A	9	9	1.03	34	0.265
165	A	9	9	1.03	34	0.265
166	A	9	9	1.03	35	0.257
167	A	14	14	1.47	35	0.400
168	A	12	12	1.55	35	0.343
169	A	2	2	1.60	32	0.062
170	A	11	11	1.69	35	0.314
171	A	14	14	1.95	35	0.400
172	A	16	16	1.82	35	0.457
173	A	17	17	1.71	35	0.486
174	A	9	9	0.93	33	0.273
175	A	7	7	0.96	33	0.212
176	A	5	5	0.94	33	0.152
177	A	4	4	1.00	30	0.133

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
178	A	6	6	1.19	33	0.182
179	A	8	8	1.14	33	0.242
180	A	9	9	1.12	33	0.273
181	F	0	0	N/A	0.000	N/A
182	F	0	0	N/A	0.000	N/A
183	F	0	0	N/A	0.000	N/A
184	F	0	0	N/A	0.000	N/A
185	A	3	3	0.47	32	0.094
186	F	0	0	N/A	0.000	N/A
187	F	0	0	N/A	0.000	N/A
188	F	0	0	N/A	0.000	N/A
189	F	0	0	N/A	0.000	N/A
190	F	0	0	N/A	0.000	N/A
191	F	0	0	N/A	0.000	N/A
192	F	0	0	N/A	0.000	N/A
193	A	8	8	1.05	32	0.250
194	F	0	0	N/A	0.000	N/A
195	F	0	0	N/A	0.000	N/A
196	F	0	0	N/A	0.000	N/A
197	F	0	0	N/A	0.000	N/A
198	F	0	0	N/A	0.000	N/A
199	F	0	0	N/A	0.000	N/A
200	F	0	0	N/A	0.000	N/A
201	F	0	0	N/A	0.000	N/A
202	F	0	0	N/A	0.000	N/A
203	F	0	0	N/A	0.000	N/A
204	A	18	18	1.64	36	0.500
205	A	17	17	1.68	36	0.472
206	A	2	2	1.92	33	0.061
207	A	17	17	1.67	36	0.472

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
208	A	20	20	1.71	36	0.556
209	A	23	23	1.68	36	0.639
210	A	25	25	1.63	36	0.694
211	A	10	10	1.06	34	0.294
212	A	9	9	1.06	34	0.265
213	A	8	8	1.02	34	0.235
214	A	7	7	1.00	31	0.226
215	A	9	9	1.03	34	0.265
216	A	12	12	1.05	34	0.353
217	A	14	14	1.06	34	0.412
218	F	0	0	N/A	0.000	N/A
219	F	0	0	N/A	0.000	N/A
220	F	0	0	N/A	0.000	N/A
221	F	0	0	N/A	0.000	N/A
222	A	3	3	1.00	33	0.091
223	F	0	0	N/A	0.000	N/A
224	F	0	0	N/A	0.000	N/A
225	F	0	0	N/A	0.000	N/A
226	F	0	0	N/A	0.000	N/A
227	F	0	0	N/A	0.000	N/A
228	F	0	0	N/A	0.000	N/A
229	F	0	0	N/A	0.000	N/A
230	A	11	11	1.01	33	0.333
231	F	0	0	N/A	0.000	N/A
232	F	0	0	N/A	0.000	N/A
233	A	14	14	1.62	35	0.400
234	A	13	13	1.70	35	0.371
235	A	11	11	1.80	35	0.314
236	B	2	2	3.10	32	0.062
237	A	13	13	1.99	35	0.371
238	B	17	17	2.15	35	0.486

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
239	B	18	18	2.08	35	0.514
240	A	8	8	1.09	33	0.242
241	A	6	6	1.12	33	0.182
242	A	5	5	1.17	33	0.152
243	A	3	3	1.00	30	0.100
244	A	9	9	1.36	33	0.273
245	A	10	10	1.33	33	0.303
246	A	11	11	1.33	33	0.333
247	F	0	0	N/A	0.000	N/A
248	F	0	0	N/A	0.000	N/A
249	F	0	0	N/A	0.000	N/A
250	F	0	0	N/A	0.000	N/A
251	F	0	0	N/A	0.000	N/A
252	A	6	6	1.00	32	0.188
253	F	0	0	N/A	0.000	N/A
254	F	0	0	N/A	0.000	N/A
255	F	0	0	N/A	0.000	N/A
256	F	0	0	N/A	0.000	N/A
257	F	0	0	N/A	0.000	N/A
258	F	0	0	N/A	0.000	N/A
259	F	0	0	N/A	0.000	N/A
260	A	15	15	1.58	32	0.469
261	F	0	0	N/A	0.000	N/A
262	F	0	0	N/A	0.000	N/A
263	F	0	0	N/A	0.000	N/A
264	F	0	0	N/A	0.000	N/A
265	F	0	0	N/A	0.000	N/A
266	F	0	0	N/A	0.000	N/A
267	F	0	0	N/A	0.000	N/A
268	F	0	0	N/A	0.000	N/A

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
269	F	0	0	N/A	0.000	N/A
270	A	19	19	1.58	36	0.528
271	A	18	18	1.67	36	0.500
272	B	2	2	2.17	33	0.061
273	A	20	20	1.75	36	0.556
274	A	22	22	1.65	36	0.611
275	A	11	11	1.01	34	0.324
276	A	10	10	1.03	34	0.294
277	A	9	9	1.05	34	0.265
278	A	8	8	1.05	31	0.258
279	A	11	11	1.10	34	0.324
280	A	13	13	1.05	34	0.382
281	A	9	9	1.03	27	0.333
282	A	8	8	1.05	27	0.296
283	A	4	4	1.00	27	0.148
284	A	9	9	1.08	24	0.375
285	A	11	11	1.09	27	0.407
286	A	13	13	1.04	27	0.481
287	F	0	0	N/A	0.000	N/A
288	F	0	0	N/A	0.000	N/A
289	F	0	0	N/A	0.000	N/A
290	F	0	0	N/A	0.000	N/A
291	A	16	16	1.33	33	0.485
292	F	0	0	N/A	0.000	N/A
293	F	0	0	N/A	0.000	N/A
294	F	0	0	N/A	0.000	N/A
295	F	0	0	N/A	0.000	N/A
296	F	0	0	N/A	0.000	N/A
297	F	0	0	N/A	0.000	N/A
298	F	0	0	N/A	0.000	N/A
299	A	28	28	1.65	33	0.848

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
300	F	0	0	N/A	0.000	N/A
301	F	0	0	N/A	0.000	N/A
302	F	0	0	N/A	0.000	N/A
303	F	0	0	N/A	0.000	N/A
304	F	0	0	N/A	0.000	N/A
305	F	0	0	N/A	0.000	N/A
306	F	0	0	N/A	0.000	N/A
307	F	0	0	N/A	0.000	N/A
308	F	0	0	N/A	0.000	N/A
309	F	0	0	N/A	0.000	N/A
310	F	0	0	N/A	0.000	N/A
311	F	0	0	N/A	0.000	N/A
312	F	0	0	N/A	0.000	N/A
313	F	0	0	N/A	0.000	N/A
314	F	0	0	N/A	0.000	N/A
315	F	0	0	N/A	0.000	N/A
316	F	0	0	N/A	0.000	N/A
317	F	0	0	N/A	0.000	N/A
318	F	0	0	N/A	0.000	N/A
319	A	8	7	1.03	34	0.206
320	F	0	0	N/A	0.000	N/A
321	F	0	0	N/A	0.000	N/A
322	F	0	0	N/A	0.000	N/A
323	F	0	0	N/A	0.000	N/A
324	F	0	0	N/A	0.000	N/A
325	F	0	0	N/A	0.000	N/A
326	A	9	8	1.56	34	0.235
327	F	0	0	N/A	0.000	N/A
328	F	0	0	N/A	0.000	N/A

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
329	F	0	0	N/A	0.000	N/A
330	F	0	0	N/A	0.000	N/A
331	F	0	0	N/A	0.000	N/A
332	F	0	0	N/A	0.000	N/A
333	F	0	0	N/A	0.000	N/A
334	F	0	0	N/A	0.000	N/A
335	F	0	0	N/A	0.000	N/A
336	F	0	0	N/A	0.000	N/A
337	F	0	0	N/A	0.000	N/A
338	F	0	0	N/A	0.000	N/A
339	F	0	0	N/A	0.000	N/A
340	A	3	2	1.00	34	0.059
341	F	0	0	N/A	0.000	N/A
342	F	0	0	N/A	0.000	N/A
343	F	0	0	N/A	0.000	N/A
344	F	0	0	N/A	0.000	N/A
345	F	0	0	N/A	0.000	N/A
346	F	0	0	N/A	0.000	N/A
347	F	0	0	N/A	0.000	N/A
348	F	0	0	N/A	0.000	N/A
349	F	0	0	N/A	0.000	N/A
350	F	0	0	N/A	0.000	N/A
351	F	0	0	N/A	0.000	N/A
352	F	0	0	N/A	0.000	N/A
353	F	0	0	N/A	0.000	N/A
354	F	0	0	N/A	0.000	N/A
355	F	0	0	N/A	0.000	N/A
356	F	0	0	N/A	0.000	N/A
357	F	0	0	N/A	0.000	N/A

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
358	F	0	0	N/A	0.000	N/A
359	F	0	0	N/A	0.000	N/A
360	F	0	0	N/A	0.000	N/A
361	F	0	0	N/A	0.000	N/A
362	A	3	2	1.02	34	0.059
363	F	0	0	N/A	0.000	N/A
364	F	0	0	N/A	0.000	N/A
365	F	0	0	N/A	0.000	N/A
366	F	0	0	N/A	0.000	N/A
367	F	0	0	N/A	0.000	N/A
368	F	0	0	N/A	0.000	N/A
369	F	0	0	N/A	0.000	N/A
370	F	0	0	N/A	0.000	N/A
371	F	0	0	N/A	0.000	N/A
372	F	0	0	N/A	0.000	N/A
373	F	0	0	N/A	0.000	N/A
374	F	0	0	N/A	0.000	N/A
375	F	0	0	N/A	0.000	N/A
376	F	0	0	N/A	0.000	N/A
377	F	0	0	N/A	0.000	N/A
378	F	0	0	N/A	0.000	N/A
379	F	0	0	N/A	0.000	N/A
380	F	0	0	N/A	0.000	N/A
381	F	0	0	N/A	0.000	N/A
382	F	0	0	N/A	0.000	N/A
383	F	0	0	N/A	0.000	N/A
384	F	0	0	N/A	0.000	N/A
385	F	0	0	N/A	0.000	N/A
386	F	0	0	N/A	0.000	N/A

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
387	F	0	0	N/A	0.000	N/A
388	F	0	0	N/A	0.000	N/A
389	F	0	0	N/A	0.000	N/A
390	F	0	0	N/A	0.000	N/A
391	F	0	0	N/A	0.000	N/A
392	F	0	0	N/A	0.000	N/A
393	F	0	0	N/A	0.000	N/A
394	F	0	0	N/A	0.000	N/A
395	F	0	0	N/A	0.000	N/A
396	F	0	0	N/A	0.000	N/A

CHAPTER 3

Listing of integrals

3.1 $\int (ex)^m \left(a+bx^2\right)^3 \left(A+Bx^2\right) \left(c+dx^2\right) \, dx$

Optimal result

Integrand size = 29, antiderivative size = 189

$$\begin{aligned}\int (ex)^m \left(a+bx^2\right)^3 \left(A+Bx^2\right) \left(c+dx^2\right) \, dx &= \frac{a^3Ac(ex)^{1+m}}{e(1+m)} + \frac{a^2(3Abc+aBc+aAd)(ex)^{3+m}}{e^3(3+m)} \\ &\quad + \frac{a(3Ab(bc+ad)+aB(3bc+ad))(ex)^{5+m}}{e^5(5+m)} \\ &\quad + \frac{b(3aB(bc+ad)+Ab(bc+3ad))(ex)^{7+m}}{e^7(7+m)} \\ &\quad + \frac{b^2(bBc+Abd+3aBd)(ex)^{9+m}}{e^9(9+m)} + \frac{b^3Bd(ex)^{11+m}}{e^{11}(11+m)}\end{aligned}$$

output
```
a^3*A*c*(e*x)^(1+m)/e/(1+m)+a^2*(A*a*d+3*A*b*c+B*a*c)*(e*x)^(3+m)/e^3/(3+m
)+a*(3*A*b*(a*d+b*c)+a*B*(a*d+3*b*c))*(e*x)^(5+m)/e^5/(5+m)+b*(3*a*B*(a*d+
b*c)+A*b*(3*a*d+b*c))*(e*x)^(7+m)/e^7/(7+m)+b^2*(A*b*d+3*B*a*d+B*b*c)*(e*x
)^(9+m)/e^9/(9+m)+b^3*B*d*(e*x)^(11+m)/e^11/(11+m)
```

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.80

$$\int (ex)^m \left(a+bx^2\right)^3 \left(A+Bx^2\right) \left(c+dx^2\right) \, dx = x(ex)^m \left(\frac{a^3Ac}{1+m}+\frac{a^2(3Abc+aBc+aAd)x^2}{3+m}+\frac{a(3Ab(bc+ad)+aB(3bc+ad))x^4}{5+m}+\frac{b(3aB(bc+ad)+Ab(bc+3ad))x^6}{7+m}+\frac{b^2(bBc+Abd+3aBd)x^8}{9+m}+\frac{b^3Bdx^{10}}{11+m}\right)$$

input
```
Integrate[(e*x)^m*(a + b*x^2)^3*(A + B*x^2)*(c + d*x^2),x]
```

output
```
x*(e*x)^m*((a^3*A*c)/(1 + m) + (a^2*(3*A*b*c + a*B*c + a*A*d)*x^2)/(3 + m)
 + (a*(3*A*b*(b*c + a*d) + a*B*(3*b*c + a*d))*x^4)/(5 + m) + (b*(3*a*B*(b*
c + a*d) + A*b*(b*c + 3*a*d))*x^6)/(7 + m) + (b^2*(b*B*c + A*b*d + 3*a*B*d
)*x^8)/(9 + m) + (b^3*B*d*x^10)/(11 + m))
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {437, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^3 \left(A+Bx^2\right) \left(c+dx^2\right) (ex)^m \, dx$$

$$\Big\downarrow \text{437}$$

$$\int \left(a^3Ac(ex)^m+\frac{a^2(ex)^{m+2}(aAd+aBc+3Abc)}{e^2}+\frac{b^2(ex)^{m+8}(3aBd+Abd+bBc)}{e^8}+\frac{b(ex)^{m+6}(Ab(3ad+bc)}{e^6}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3Ac(ex)^{m+1}}{e(m+1)}+\frac{a^2(ex)^{m+3}(aAd+aBc+3Abc)}{e^3(m+3)}+\frac{b^2(ex)^{m+9}(3aBd+Abd+bBc)}{e^9(m+9)}+\frac{b(ex)^{m+7}(Ab(3ad+bc)+3aB(ad+bc))}{e^7(m+7)}+\frac{a(ex)^{m+5}(3Ab(ad+bc)+aB(ad+3bc))}{e^5(m+5)}+\frac{b^3Bd(ex)^{m+11}}{e^{11}(m+11)}$$

input
```
Int[(e*x)^m*(a + b*x^2)^3*(A + B*x^2)*(c + d*x^2),x]
```

output
```
(a^3*A*c*(e*x)^(1 + m))/(e*(1 + m)) + (a^2*(3*A*b*c + a*B*c + a*A*d)*(e*x)
^(3 + m))/(e^3*(3 + m)) + (a*(3*A*b*(b*c + a*d) + a*B*(3*b*c + a*d))*(e*x)
^(5 + m))/(e^5*(5 + m)) + (b*(3*a*B*(b*c + a*d) + A*b*(b*c + 3*a*d))*(e*x)
^(7 + m))/(e^7*(7 + m)) + (b^2*(b*B*c + A*b*d + 3*a*B*d)*(e*x)^(9 + m))/(e
^9*(9 + m)) + (b^3*B*d*(e*x)^(11 + m))/(e^11*(11 + m))
```

Defintions of rubi rules used

rule 437
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(
a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f
, g, m}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1228 vs. $2(189) = 378$.

Time = 0.51 (sec) , antiderivative size = 1229, normalized size of antiderivative = 6.50

method	result	size
gosper	Expression too large to display	1229
risch	Expression too large to display	1229
orering	Expression too large to display	1229
parallelrisch	Expression too large to display	1709

input
```
int((e*x)^m*(b*x^2+a)^3*(B*x^2+A)*(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
x*(B*b^3*d*m^5*x^10+25*B*b^3*d*m^4*x^10+A*b^3*d*m^5*x^8+3*B*a*b^2*d*m^5*x^
8+B*b^3*c*m^5*x^8+230*B*b^3*d*m^3*x^10+27*A*b^3*d*m^4*x^8+81*B*a*b^2*d*m^4
*x^8+27*B*b^3*c*m^4*x^8+950*B*b^3*d*m^2*x^10+3*A*a*b^2*d*m^5*x^6+A*b^3*c*m
^5*x^6+262*A*b^3*d*m^3*x^8+3*B*a^2*b*d*m^5*x^6+3*B*a*b^2*c*m^5*x^6+786*B*a
*b^2*d*m^3*x^8+262*B*b^3*c*m^3*x^8+1689*B*b^3*d*m*x^10+87*A*a*b^2*d*m^4*x^
6+29*A*b^3*c*m^4*x^6+1122*A*b^3*d*m^2*x^8+87*B*a^2*b*d*m^4*x^6+87*B*a*b^2*
c*m^4*x^6+3366*B*a*b^2*d*m^2*x^8+1122*B*b^3*c*m^2*x^8+945*B*b^3*d*x^10+3*A
*a^2*b*d*m^5*x^4+3*A*a*b^2*c*m^5*x^4+906*A*a*b^2*d*m^3*x^6+302*A*b^3*c*m^3
*x^6+2041*A*b^3*d*m*x^8+B*a^3*d*m^5*x^4+3*B*a^2*b*c*m^5*x^4+906*B*a^2*b*d*
m^3*x^6+906*B*a*b^2*c*m^3*x^6+6123*B*a*b^2*d*m*x^8+2041*B*b^3*c*m*x^8+93*A
*a^2*b*d*m^4*x^4+93*A*a*b^2*c*m^4*x^4+4098*A*a*b^2*d*m^2*x^6+1366*A*b^3*c*
m^2*x^6+1155*A*b^3*d*x^8+31*B*a^3*d*m^4*x^4+93*B*a^2*b*c*m^4*x^4+4098*B*a^
2*b*d*m^2*x^6+4098*B*a*b^2*c*m^2*x^6+3465*B*a*b^2*d*x^8+1155*B*b^3*c*x^8+A
*a^3*d*m^5*x^2+3*A*a^2*b*c*m^5*x^2+1050*A*a^2*b*d*m^3*x^4+1050*A*a*b^2*c*m
^3*x^4+7731*A*a*b^2*d*m*x^6+2577*A*b^3*c*m*x^6+B*a^3*c*m^5*x^2+350*B*a^3*d
*m^3*x^4+1050*B*a^2*b*c*m^3*x^4+7731*B*a^2*b*d*m*x^6+7731*B*a*b^2*c*m*x^6+
33*A*a^3*d*m^4*x^2+99*A*a^2*b*c*m^4*x^2+5190*A*a^2*b*d*m^2*x^4+5190*A*a*b^
2*c*m^2*x^4+4455*A*a*b^2*d*x^6+1485*A*b^3*c*x^6+33*B*a^3*c*m^4*x^2+1730*B*
a^3*d*m^2*x^4+5190*B*a^2*b*c*m^2*x^4+4455*B*a^2*b*d*x^6+4455*B*a*b^2*c*x^6
+A*a^3*c*m^5+406*A*a^3*d*m^3*x^2+1218*A*a^2*b*c*m^3*x^2+10467*A*a^2*b*d...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 911 vs. $2(189) = 378$.

Time = 0.12 (sec) , antiderivative size = 911, normalized size of antiderivative = 4.82

$$\int (ex)^m \left(a + bx^2\right)^3 \left(A + Bx^2\right) \left(c + dx^2\right) \, dx = \text{Too large to display}$$

input
```
integrate((e*x)^m*(b*x^2+a)^3*(B*x^2+A)*(d*x^2+c),x, algorithm="fricas")
```

output
```
((B*b^3*d*m^5 + 25*B*b^3*d*m^4 + 230*B*b^3*d*m^3 + 950*B*b^3*d*m^2 + 1689*
B*b^3*d*m + 945*B*b^3*d)*x^11 + ((B*b^3*c + (3*B*a*b^2 + A*b^3)*d)*m^5 + 1
155*B*b^3*c + 27*(B*b^3*c + (3*B*a*b^2 + A*b^3)*d)*m^4 + 262*(B*b^3*c + (3
*B*a*b^2 + A*b^3)*d)*m^3 + 1122*(B*b^3*c + (3*B*a*b^2 + A*b^3)*d)*m^2 + 11
55*(3*B*a*b^2 + A*b^3)*d + 2041*(B*b^3*c + (3*B*a*b^2 + A*b^3)*d)*m)*x^9 +
 (((3*B*a*b^2 + A*b^3)*c + 3*(B*a^2*b + A*a*b^2)*d)*m^5 + 29*((3*B*a*b^2 +
 A*b^3)*c + 3*(B*a^2*b + A*a*b^2)*d)*m^4 + 302*((3*B*a*b^2 + A*b^3)*c + 3*
(B*a^2*b + A*a*b^2)*d)*m^3 + 1366*((3*B*a*b^2 + A*b^3)*c + 3*(B*a^2*b + A*
a*b^2)*d)*m^2 + 1485*(3*B*a*b^2 + A*b^3)*c + 4455*(B*a^2*b + A*a*b^2)*d +
2577*((3*B*a*b^2 + A*b^3)*c + 3*(B*a^2*b + A*a*b^2)*d)*m)*x^7 + ((3*(B*a^2
*b + A*a*b^2)*c + (B*a^3 + 3*A*a^2*b)*d)*m^5 + 31*(3*(B*a^2*b + A*a*b^2)*c
 + (B*a^3 + 3*A*a^2*b)*d)*m^4 + 350*(3*(B*a^2*b + A*a*b^2)*c + (B*a^3 + 3*
A*a^2*b)*d)*m^3 + 1730*(3*(B*a^2*b + A*a*b^2)*c + (B*a^3 + 3*A*a^2*b)*d)*m
^2 + 6237*(B*a^2*b + A*a*b^2)*c + 2079*(B*a^3 + 3*A*a^2*b)*d + 3489*(3*(B*
a^2*b + A*a*b^2)*c + (B*a^3 + 3*A*a^2*b)*d)*m)*x^5 + ((A*a^3*d + (B*a^3 +
3*A*a^2*b)*c)*m^5 + 3465*A*a^3*d + 33*(A*a^3*d + (B*a^3 + 3*A*a^2*b)*c)*m^
4 + 406*(A*a^3*d + (B*a^3 + 3*A*a^2*b)*c)*m^3 + 2262*(A*a^3*d + (B*a^3 + 3
*A*a^2*b)*c)*m^2 + 3465*(B*a^3 + 3*A*a^2*b)*c + 5353*(A*a^3*d + (B*a^3 + 3
*A*a^2*b)*c)*m)*x^3 + (A*a^3*c*m^5 + 35*A*a^3*c*m^4 + 470*A*a^3*c*m^3 + 30
10*A*a^3*c*m^2 + 9129*A*a^3*c*m + 10395*A*a^3*c)*x)*(e*x)^m/(m^6 + 36*m...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5992 vs. $2(184) = 368$.

Time = 0.92 (sec) , antiderivative size = 5992, normalized size of antiderivative = 31.70

$$\int (ex)^m \left(a+bx^2\right)^3 \left(A+Bx^2\right) \left(c+dx^2\right) \, dx = \text{Too large to display}$$

input
```
integrate((e*x)**m*(b*x**2+a)**3*(B*x**2+A)*(d*x**2+c),x)
```

output
```
Piecewise(((-A*a**3*c/(10*x**10) - A*a**3*d/(8*x**8) - 3*A*a**2*b*c/(8*x**
8) - A*a**2*b*d/(2*x**6) - A*a*b**2*c/(2*x**6) - 3*A*a*b**2*d/(4*x**4) - A
*b**3*c/(4*x**4) - A*b**3*d/(2*x**2) - B*a**3*c/(8*x**8) - B*a**3*d/(6*x**
6) - B*a**2*b*c/(2*x**6) - 3*B*a**2*b*d/(4*x**4) - 3*B*a*b**2*c/(4*x**4) -
 3*B*a*b**2*d/(2*x**2) - B*b**3*c/(2*x**2) + B*b**3*d*log(x))/e**11, Eq(m,
 -11)), ((-A*a**3*c/(8*x**8) - A*a**3*d/(6*x**6) - A*a**2*b*c/(2*x**6) - 3
*A*a**2*b*d/(4*x**4) - 3*A*a*b**2*c/(4*x**4) - 3*A*a*b**2*d/(2*x**2) - A*b
**3*c/(2*x**2) + A*b**3*d*log(x) - B*a**3*c/(6*x**6) - B*a**3*d/(4*x**4) -
 3*B*a**2*b*c/(4*x**4) - 3*B*a**2*b*d/(2*x**2) - 3*B*a*b**2*c/(2*x**2) + 3
*B*a*b**2*d*log(x) + B*b**3*c*log(x) + B*b**3*d*x**2/2)/e**9, Eq(m, -9)),
((-A*a**3*c/(6*x**6) - A*a**3*d/(4*x**4) - 3*A*a**2*b*c/(4*x**4) - 3*A*a**
2*b*d/(2*x**2) - 3*A*a*b**2*c/(2*x**2) + 3*A*a*b**2*d*log(x) + A*b**3*c*lo
g(x) + A*b**3*d*x**2/2 - B*a**3*c/(4*x**4) - B*a**3*d/(2*x**2) - 3*B*a**2*
b*c/(2*x**2) + 3*B*a**2*b*d*log(x) + 3*B*a*b**2*c*log(x) + 3*B*a*b**2*d*x*
*2/2 + B*b**3*c*x**2/2 + B*b**3*d*x**4/4)/e**7, Eq(m, -7)), ((-A*a**3*c/(4
*x**4) - A*a**3*d/(2*x**2) - 3*A*a**2*b*c/(2*x**2) + 3*A*a**2*b*d*log(x) +
 3*A*a*b**2*c*log(x) + 3*A*a*b**2*d*x**2/2 + A*b**3*c*x**2/2 + A*b**3*d*x*
*4/4 - B*a**3*c/(2*x**2) + B*a**3*d*log(x) + 3*B*a**2*b*c*log(x) + 3*B*a**
2*b*d*x**2/2 + 3*B*a*b**2*c*x**2/2 + 3*B*a*b**2*d*x**4/4 + B*b**3*c*x**4/4
 + B*b**3*d*x**6/6)/e**5, Eq(m, -5)), ((-A*a**3*c/(2*x**2) + A*a**3*d*l...
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.79

$$\begin{aligned}
&\int (ex)^m \left(a+bx^2\right)^3 \left(A+Bx^2\right) \left(c+dx^2\right)\,dx \\
&= \frac{Bb^3de^mx^{11}x^m}{m+11} + \frac{Bb^3ce^mx^9x^m}{m+9} + \frac{3\,Bab^2de^mx^9x^m}{m+9} + \frac{Ab^3de^mx^9x^m}{m+9} \\
&\quad + \frac{3\,Bab^2ce^mx^7x^m}{m+7} + \frac{Ab^3ce^mx^7x^m}{m+7} + \frac{3\,Ba^2bde^mx^7x^m}{m+7} + \frac{3\,Aab^2de^mx^7x^m}{m+7} \\
&\quad + \frac{3\,Ba^2bce^mx^5x^m}{m+5} + \frac{3\,Aab^2ce^mx^5x^m}{m+5} + \frac{Ba^3de^mx^5x^m}{m+5} + \frac{3\,Aa^2bde^mx^5x^m}{m+5} \\
&\quad + \frac{Ba^3ce^mx^3x^m}{m+3} + \frac{3\,Aa^2bce^mx^3x^m}{m+3} + \frac{Aa^3de^mx^3x^m}{m+3} + \frac{(ex)^{m+1}\,Aa^3c}{e(m+1)}
\end{aligned}$$

input
```
integrate((e*x)^m*(b*x^2+a)^3*(B*x^2+A)*(d*x^2+c),x, algorithm="maxima")
```

output
```
B*b^3*d*e^m*x^11*x^m/(m + 11) + B*b^3*c*e^m*x^9*x^m/(m + 9) + 3*B*a*b^2*d*
e^m*x^9*x^m/(m + 9) + A*b^3*d*e^m*x^9*x^m/(m + 9) + 3*B*a*b^2*c*e^m*x^7*x^
m/(m + 7) + A*b^3*c*e^m*x^7*x^m/(m + 7) + 3*B*a^2*b*d*e^m*x^7*x^m/(m + 7)
+ 3*A*a*b^2*d*e^m*x^7*x^m/(m + 7) + 3*B*a^2*b*c*e^m*x^5*x^m/(m + 5) + 3*A*
a*b^2*c*e^m*x^5*x^m/(m + 5) + B*a^3*d*e^m*x^5*x^m/(m + 5) + 3*A*a^2*b*d*e^
m*x^5*x^m/(m + 5) + B*a^3*c*e^m*x^3*x^m/(m + 3) + 3*A*a^2*b*c*e^m*x^3*x^m/
(m + 3) + A*a^3*d*e^m*x^3*x^m/(m + 3) + (e*x)^(m + 1)*A*a^3*c/(e*(m + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1708 vs. $2(189) = 378$.

Time = 0.16 (sec) , antiderivative size = 1708, normalized size of antiderivative = 9.04

$$\int (ex)^m \left(a+bx^2\right)^3 \left(A+Bx^2\right) \left(c+dx^2\right) \, dx = \text{Too large to display}$$

input
```
integrate((e*x)^m*(b*x^2+a)^3*(B*x^2+A)*(d*x^2+c),x, algorithm="giac")
```

output
```
((e*x)^m*B*b^3*d*m^5*x^11 + 25*(e*x)^m*B*b^3*d*m^4*x^11 + (e*x)^m*B*b^3*c*
m^5*x^9 + 3*(e*x)^m*B*a*b^2*d*m^5*x^9 + (e*x)^m*A*b^3*d*m^5*x^9 + 230*(e*x
)^m*B*b^3*d*m^3*x^11 + 27*(e*x)^m*B*b^3*c*m^4*x^9 + 81*(e*x)^m*B*a*b^2*d*m
^4*x^9 + 27*(e*x)^m*A*b^3*d*m^4*x^9 + 950*(e*x)^m*B*b^3*d*m^2*x^11 + 3*(e*
x)^m*B*a*b^2*c*m^5*x^7 + (e*x)^m*A*b^3*c*m^5*x^7 + 3*(e*x)^m*B*a^2*b*d*m^5
*x^7 + 3*(e*x)^m*A*a*b^2*d*m^5*x^7 + 262*(e*x)^m*B*b^3*c*m^3*x^9 + 786*(e*
x)^m*B*a*b^2*d*m^3*x^9 + 262*(e*x)^m*A*b^3*d*m^3*x^9 + 1689*(e*x)^m*B*b^3*
d*m*x^11 + 87*(e*x)^m*B*a*b^2*c*m^4*x^7 + 29*(e*x)^m*A*b^3*c*m^4*x^7 + 87*
(e*x)^m*B*a^2*b*d*m^4*x^7 + 87*(e*x)^m*A*a*b^2*d*m^4*x^7 + 1122*(e*x)^m*B*
b^3*c*m^2*x^9 + 3366*(e*x)^m*B*a*b^2*d*m^2*x^9 + 1122*(e*x)^m*A*b^3*d*m^2*
x^9 + 945*(e*x)^m*B*b^3*d*x^11 + 3*(e*x)^m*B*a^2*b*c*m^5*x^5 + 3*(e*x)^m*A
*a*b^2*c*m^5*x^5 + (e*x)^m*B*a^3*d*m^5*x^5 + 3*(e*x)^m*A*a^2*b*d*m^5*x^5 +
 906*(e*x)^m*B*a*b^2*c*m^3*x^7 + 302*(e*x)^m*A*b^3*c*m^3*x^7 + 906*(e*x)^m
*B*a^2*b*d*m^3*x^7 + 906*(e*x)^m*A*a*b^2*d*m^3*x^7 + 2041*(e*x)^m*B*b^3*c*
m*x^9 + 6123*(e*x)^m*B*a*b^2*d*m*x^9 + 2041*(e*x)^m*A*b^3*d*m*x^9 + 93*(e*
x)^m*B*a^2*b*c*m^4*x^5 + 93*(e*x)^m*A*a*b^2*c*m^4*x^5 + 31*(e*x)^m*B*a^3*d
*m^4*x^5 + 93*(e*x)^m*A*a^2*b*d*m^4*x^5 + 4098*(e*x)^m*B*a*b^2*c*m^2*x^7 +
 1366*(e*x)^m*A*b^3*c*m^2*x^7 + 4098*(e*x)^m*B*a^2*b*d*m^2*x^7 + 4098*(e*x
)^m*A*a*b^2*d*m^2*x^7 + 1155*(e*x)^m*B*b^3*c*x^9 + 3465*(e*x)^m*B*a*b^2*d*
x^9 + 1155*(e*x)^m*A*b^3*d*x^9 + (e*x)^m*B*a^3*c*m^5*x^3 + 3*(e*x)^m*A*...
```

Mupad [B] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 469, normalized size of antiderivative = 2.48

$$\int (ex)^m \left(a+bx^2\right)^3 \left(A+Bx^2\right) \left(c+dx^2\right) \, dx$$
$$= \frac{a^2\,x^3\,{\left(e\,x\right)}^m\,\left(A\,a\,d+3\,A\,b\,c+B\,a\,c\right)\,\left(m^5+33\,m^4+406\,m^3+2262\,m^2+5353\,m+3465\right)}{m^6+36\,m^5+505\,m^4+3480\,m^3+12139\,m^2+19524\,m+10395}$$
$$+\frac{b^2\,x^9\,{\left(e\,x\right)}^m\,\left(A\,b\,d+3\,B\,a\,d+B\,b\,c\right)\,\left(m^5+27\,m^4+262\,m^3+1122\,m^2+2041\,m+1155\right)}{m^6+36\,m^5+505\,m^4+3480\,m^3+12139\,m^2+19524\,m+10395}$$
$$+\frac{a\,x^5\,{\left(e\,x\right)}^m\,\left(3\,A\,b^2\,c+B\,a^2\,d+3\,A\,a\,b\,d+3\,B\,a\,b\,c\right)\,\left(m^5+31\,m^4+350\,m^3+1730\,m^2+3489\,m+207\right.}{m^6+36\,m^5+505\,m^4+3480\,m^3+12139\,m^2+19524\,m+10395}$$
$$+\frac{b\,x^7\,{\left(e\,x\right)}^m\,\left(A\,b^2\,c+3\,B\,a^2\,d+3\,A\,a\,b\,d+3\,B\,a\,b\,c\right)\,\left(m^5+29\,m^4+302\,m^3+1366\,m^2+2577\,m+148\right.}{m^6+36\,m^5+505\,m^4+3480\,m^3+12139\,m^2+19524\,m+10395}$$
$$+\frac{B\,b^3\,d\,x^{11}\,{\left(e\,x\right)}^m\,\left(m^5+25\,m^4+230\,m^3+950\,m^2+1689\,m+945\right)}{m^6+36\,m^5+505\,m^4+3480\,m^3+12139\,m^2+19524\,m+10395}$$
$$+\frac{A\,a^3\,c\,x\,{\left(e\,x\right)}^m\,\left(m^5+35\,m^4+470\,m^3+3010\,m^2+9129\,m+10395\right)}{m^6+36\,m^5+505\,m^4+3480\,m^3+12139\,m^2+19524\,m+10395}$$

input
```
int((A + B*x^2)*(e*x)^m*(a + b*x^2)^3*(c + d*x^2),x)
```

output
```
(a^2*x^3*(e*x)^m*(A*a*d + 3*A*b*c + B*a*c)*(5353*m + 2262*m^2 + 406*m^3 +
33*m^4 + m^5 + 3465))/(19524*m + 12139*m^2 + 3480*m^3 + 505*m^4 + 36*m^5 +
 m^6 + 10395) + (b^2*x^9*(e*x)^m*(A*b*d + 3*B*a*d + B*b*c)*(2041*m + 1122*
m^2 + 262*m^3 + 27*m^4 + m^5 + 1155))/(19524*m + 12139*m^2 + 3480*m^3 + 50
5*m^4 + 36*m^5 + m^6 + 10395) + (a*x^5*(e*x)^m*(3*A*b^2*c + B*a^2*d + 3*A*
a*b*d + 3*B*a*b*c)*(3489*m + 1730*m^2 + 350*m^3 + 31*m^4 + m^5 + 2079))/(1
9524*m + 12139*m^2 + 3480*m^3 + 505*m^4 + 36*m^5 + m^6 + 10395) + (b*x^7*(
e*x)^m*(A*b^2*c + 3*B*a^2*d + 3*A*a*b*d + 3*B*a*b*c)*(2577*m + 1366*m^2 +
302*m^3 + 29*m^4 + m^5 + 1485))/(19524*m + 12139*m^2 + 3480*m^3 + 505*m^4
+ 36*m^5 + m^6 + 10395) + (B*b^3*d*x^11*(e*x)^m*(1689*m + 950*m^2 + 230*m^
3 + 25*m^4 + m^5 + 945))/(19524*m + 12139*m^2 + 3480*m^3 + 505*m^4 + 36*m^
5 + m^6 + 10395) + (A*a^3*c*x*(e*x)^m*(9129*m + 3010*m^2 + 470*m^3 + 35*m^
4 + m^5 + 10395))/(19524*m + 12139*m^2 + 3480*m^3 + 505*m^4 + 36*m^5 + m^6
 + 10395)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 747, normalized size of antiderivative = 3.95

$$\int (ex)^m \left(a+bx^2\right)^3 \left(A+Bx^2\right) \left(c+dx^2\right) \, dx = \frac{x^m e^m x(b^4d\,m^5x^{10}+25b^4d\,m^4x^{10}+4a\,b^3d\,m^5x^8+b^4c\,m^5x^8+230b^4d\,m^3x^{10}+108a\,b^3d\,m^4x^8+27b^4c\,m^4}{}$$

input
```
int((e*x)^m*(b*x^2+a)^3*(B*x^2+A)*(d*x^2+c),x)
```

output

```
(x**m*e**m*x*(a**4*c*m**5 + 35*a**4*c*m**4 + 470*a**4*c*m**3 + 3010*a**4*c
*m**2 + 9129*a**4*c*m + 10395*a**4*c + a**4*d*m**5*x**2 + 33*a**4*d*m**4*x
**2 + 406*a**4*d*m**3*x**2 + 2262*a**4*d*m**2*x**2 + 5353*a**4*d*m*x**2 +
3465*a**4*d*x**2 + 4*a**3*b*c*m**5*x**2 + 132*a**3*b*c*m**4*x**2 + 1624*a*
*3*b*c*m**3*x**2 + 9048*a**3*b*c*m**2*x**2 + 21412*a**3*b*c*m*x**2 + 13860
*a**3*b*c*x**2 + 4*a**3*b*d*m**5*x**4 + 124*a**3*b*d*m**4*x**4 + 1400*a**3
*b*d*m**3*x**4 + 6920*a**3*b*d*m**2*x**4 + 13956*a**3*b*d*m*x**4 + 8316*a*
*3*b*d*x**4 + 6*a**2*b**2*c*m**5*x**4 + 186*a**2*b**2*c*m**4*x**4 + 2100*a
**2*b**2*c*m**3*x**4 + 10380*a**2*b**2*c*m**2*x**4 + 20934*a**2*b**2*c*m*x
**4 + 12474*a**2*b**2*c*x**4 + 6*a**2*b**2*d*m**5*x**6 + 174*a**2*b**2*d*m
**4*x**6 + 1812*a**2*b**2*d*m**3*x**6 + 8196*a**2*b**2*d*m**2*x**6 + 15462
*a**2*b**2*d*m*x**6 + 8910*a**2*b**2*d*x**6 + 4*a*b**3*c*m**5*x**6 + 116*a
*b**3*c*m**4*x**6 + 1208*a*b**3*c*m**3*x**6 + 5464*a*b**3*c*m**2*x**6 + 10
308*a*b**3*c*m*x**6 + 5940*a*b**3*c*x**6 + 4*a*b**3*d*m**5*x**8 + 108*a*b*
*3*d*m**4*x**8 + 1048*a*b**3*d*m**3*x**8 + 4488*a*b**3*d*m**2*x**8 + 8164*
a*b**3*d*m*x**8 + 4620*a*b**3*d*x**8 + b**4*c*m**5*x**8 + 27*b**4*c*m**4*x
**8 + 262*b**4*c*m**3*x**8 + 1122*b**4*c*m**2*x**8 + 2041*b**4*c*m*x**8 +
1155*b**4*c*x**8 + b**4*d*m**5*x**10 + 25*b**4*d*m**4*x**10 + 230*b**4*d*m
**3*x**10 + 950*b**4*d*m**2*x**10 + 1689*b**4*d*m*x**10 + 945*b**4*d*x**10
))/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 1039...
```

3.2 $\int (ex)^m (a+bx^2)^2 (A+Bx^2)(c+dx^2)\,dx$

Optimal result

Integrand size = 29, antiderivative size = 144

$$\begin{aligned}\int (ex)^m (a+bx^2)^2 (A+Bx^2)(c+dx^2)\,dx &= \frac{a^2Ac(ex)^{1+m}}{e(1+m)}+\frac{a(2Abc+aBc+aAd)(ex)^{3+m}}{e^3(3+m)} \\ &+\frac{(aB(2bc+ad)+Ab(bc+2ad))(ex)^{5+m}}{e^5(5+m)} \\ &+\frac{b(bBc+Abd+2aBd)(ex)^{7+m}}{e^7(7+m)}+\frac{b^2Bd(ex)^{9+m}}{e^9(9+m)}\end{aligned}$$

output
```
a^2*A*c*(e*x)^(1+m)/e/(1+m)+a*(A*a*d+2*A*b*c+B*a*c)*(e*x)^(3+m)/e^3/(3+m)+
(a*B*(a*d+2*b*c)+A*b*(2*a*d+b*c))*(e*x)^(5+m)/e^5/(5+m)+b*(A*b*d+2*B*a*d+B
*b*c)*(e*x)^(7+m)/e^7/(7+m)+b^2*B*d*(e*x)^(9+m)/e^9/(9+m)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.78

$$\int (ex)^m \left(a+bx^2\right)^2 \left(A+Bx^2\right) \left(c+dx^2\right) \, dx = x(ex)^m \left(\frac{a^2Ac}{1+m}+\frac{a(2Abc+aBc+aAd)x^2}{3+m}+\frac{(aB(2bc+ad)+Ab(bc+2ad))x^4}{5+m}+\frac{b(bBc+Abd+2aBd)x^6}{7+m}+\frac{b^2Bdx^8}{9+m}\right)$$

input
```
Integrate[(e*x)^m*(a + b*x^2)^2*(A + B*x^2)*(c + d*x^2),x]
```

output
```
x*(e*x)^m*((a^2*A*c)/(1 + m) + (a*(2*A*b*c + a*B*c + a*A*d)*x^2)/(3 + m) +
 ((a*B*(2*b*c + a*d) + A*b*(b*c + 2*a*d))*x^4)/(5 + m) + (b*(b*B*c + A*b*d
 + 2*a*B*d)*x^6)/(7 + m) + (b^2*B*d*x^8)/(9 + m))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {437, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^2 \left(A+Bx^2\right) \left(c+dx^2\right) (ex)^m \, dx$$

$$\Big\downarrow \text{437}$$

$$\int \left(a^2Ac(ex)^m+\frac{b(ex)^{m+6}(2aBd+Abd+bBc)}{e^6}+\frac{(ex)^{m+4}(Ab(2ad+bc)+aB(ad+2bc))}{e^4}+\frac{a(ex)^{m+2}(aAd+}{e^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2Ac(ex)^{m+1}}{e(m+1)} + \frac{b(ex)^{m+7}(2aBd + Abd + bBc)}{e^7(m+7)} + \frac{(ex)^{m+5}(Ab(2ad+bc) + aB(ad+2bc))}{e^5(m+5)} + \frac{a(ex)^{m+3}(aAd + aBc + 2Abc)}{e^3(m+3)} + \frac{b^2Bd(ex)^{m+9}}{e^9(m+9)}$$

input
```
Int[(e*x)^m*(a + b*x^2)^2*(A + B*x^2)*(c + d*x^2),x]
```

output
```
(a^2*A*c*(e*x)^(1 + m))/(e*(1 + m)) + (a*(2*A*b*c + a*B*c + a*A*d)*(e*x)^(
3 + m))/(e^3*(3 + m)) + ((a*B*(2*b*c + a*d) + A*b*(b*c + 2*a*d))*(e*x)^(5
+ m))/(e^5*(5 + m)) + (b*(b*B*c + A*b*d + 2*a*B*d)*(e*x)^(7 + m))/(e^7*(7
+ m)) + (b^2*B*d*(e*x)^(9 + m))/(e^9*(9 + m))
```

Defintions of rubi rules used

rule 437
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(
a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f
, g, m}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 710 vs. 2(144) = 288.

Time = 0.46 (sec) , antiderivative size = 711, normalized size of antiderivative = 4.94

method	result
gosper	$\frac{x(B b^2 d m^4 x^8+16 B b^2 d m^3 x^8+A b^2 d m^4 x^6+2 B a b d m^4 x^6+B b^2 c m^4 x^6+86 B b^2 d m^2 x^8+18 A b^2 d m^3 x^6+36 B a b d m^3 x^6+18}{}$
risch	$\frac{x(B b^2 d m^4 x^8+16 B b^2 d m^3 x^8+A b^2 d m^4 x^6+2 B a b d m^4 x^6+B b^2 c m^4 x^6+86 B b^2 d m^2 x^8+18 A b^2 d m^3 x^6+36 B a b d m^3 x^6+18}{}$
orering	$\frac{x(B b^2 d m^4 x^8+16 B b^2 d m^3 x^8+A b^2 d m^4 x^6+2 B a b d m^4 x^6+B b^2 c m^4 x^6+86 B b^2 d m^2 x^8+18 A b^2 d m^3 x^6+36 B a b d m^3 x^6+18}{}$
parallelrisch	Expression too large to display

input
```
int((e*x)^m*(b*x^2+a)^2*(B*x^2+A)*(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
x*(B*b^2*d*m^4*x^8+16*B*b^2*d*m^3*x^8+A*b^2*d*m^4*x^6+2*B*a*b*d*m^4*x^6+B*
b^2*c*m^4*x^6+86*B*b^2*d*m^2*x^8+18*A*b^2*d*m^3*x^6+36*B*a*b*d*m^3*x^6+18*
B*b^2*c*m^3*x^6+176*B*b^2*d*m*x^8+2*A*a*b*d*m^4*x^4+A*b^2*c*m^4*x^4+104*A*
b^2*d*m^2*x^6+B*a^2*d*m^4*x^4+2*B*a*b*c*m^4*x^4+208*B*a*b*d*m^2*x^6+104*B*
b^2*c*m^2*x^6+105*B*b^2*d*x^8+40*A*a*b*d*m^3*x^4+20*A*b^2*c*m^3*x^4+222*A*
b^2*d*m*x^6+20*B*a^2*d*m^3*x^4+40*B*a*b*c*m^3*x^4+444*B*a*b*d*m*x^6+222*B*
b^2*c*m*x^6+A*a^2*d*m^4*x^2+2*A*a*b*c*m^4*x^2+260*A*a*b*d*m^2*x^4+130*A*b^
2*c*m^2*x^4+135*A*b^2*d*x^6+B*a^2*c*m^4*x^2+130*B*a^2*d*m^2*x^4+260*B*a*b*
c*m^2*x^4+270*B*a*b*d*x^6+135*B*b^2*c*x^6+22*A*a^2*d*m^3*x^2+44*A*a*b*c*m^
3*x^2+600*A*a*b*d*m*x^4+300*A*b^2*c*m*x^4+22*B*a^2*c*m^3*x^2+300*B*a^2*d*m
*x^4+600*B*a*b*c*m*x^4+A*a^2*c*m^4+164*A*a^2*d*m^2*x^2+328*A*a*b*c*m^2*x^2
+378*A*a*b*d*x^4+189*A*b^2*c*x^4+164*B*a^2*c*m^2*x^2+189*B*a^2*d*x^4+378*B
*a*b*c*x^4+24*A*a^2*c*m^3+458*A*a^2*d*m*x^2+916*A*a*b*c*m*x^2+458*B*a^2*c*
m*x^2+206*A*a^2*c*m^2+315*A*a^2*d*x^2+630*A*a*b*c*x^2+315*B*a^2*c*x^2+744*
A*a^2*c*m+945*A*a^2*c)*(e*x)^m/(9+m)/(7+m)/(5+m)/(3+m)/(1+m)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 532 vs. $2(144) = 288$.

Time = 0.11 (sec) , antiderivative size = 532, normalized size of antiderivative = 3.69

$$\int (ex)^m (a+bx^2)^2 (A+Bx^2)(c+dx^2)\,dx$$
$$= \frac{((Bb^2dm^4 + 16\,Bb^2dm^3 + 86\,Bb^2dm^2 + 176\,Bb^2dm + 105\,Bb^2d)x^9 + ((Bb^2c + (2\,Bab + Ab^2)d)m^4 + 13}{}$$

input
```
integrate((e*x)^m*(b*x^2+a)^2*(B*x^2+A)*(d*x^2+c),x, algorithm="fricas")
```

output
```
((B*b^2*d*m^4 + 16*B*b^2*d*m^3 + 86*B*b^2*d*m^2 + 176*B*b^2*d*m + 105*B*b^
2*d)*x^9 + ((B*b^2*c + (2*B*a*b + A*b^2)*d)*m^4 + 135*B*b^2*c + 18*(B*b^2*
c + (2*B*a*b + A*b^2)*d)*m^3 + 104*(B*b^2*c + (2*B*a*b + A*b^2)*d)*m^2 + 1
35*(2*B*a*b + A*b^2)*d + 222*(B*b^2*c + (2*B*a*b + A*b^2)*d)*m)*x^7 + (((2
*B*a*b + A*b^2)*c + (B*a^2 + 2*A*a*b)*d)*m^4 + 20*((2*B*a*b + A*b^2)*c + (
B*a^2 + 2*A*a*b)*d)*m^3 + 130*((2*B*a*b + A*b^2)*c + (B*a^2 + 2*A*a*b)*d)*
m^2 + 189*(2*B*a*b + A*b^2)*c + 189*(B*a^2 + 2*A*a*b)*d + 300*((2*B*a*b +
A*b^2)*c + (B*a^2 + 2*A*a*b)*d)*m)*x^5 + ((A*a^2*d + (B*a^2 + 2*A*a*b)*c)*
m^4 + 315*A*a^2*d + 22*(A*a^2*d + (B*a^2 + 2*A*a*b)*c)*m^3 + 164*(A*a^2*d
+ (B*a^2 + 2*A*a*b)*c)*m^2 + 315*(B*a^2 + 2*A*a*b)*c + 458*(A*a^2*d + (B*a
^2 + 2*A*a*b)*c)*m)*x^3 + (A*a^2*c*m^4 + 24*A*a^2*c*m^3 + 206*A*a^2*c*m^2
+ 744*A*a^2*c*m + 945*A*a^2*c)*x)*(e*x)^m/(m^5 + 25*m^4 + 230*m^3 + 950*m^
2 + 1689*m + 945)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3271 vs. $2(139) = 278$.

Time = 0.70 (sec) , antiderivative size = 3271, normalized size of antiderivative = 22.72

$$\int (ex)^m \left(a+bx^2\right)^2 \left(A+Bx^2\right) \left(c+dx^2\right) \, dx = \text{Too large to display}$$

input
```
integrate((e*x)**m*(b*x**2+a)**2*(B*x**2+A)*(d*x**2+c),x)
```

output
```
Piecewise(((-A*a**2*c/(8*x**8) - A*a**2*d/(6*x**6) - A*a*b*c/(3*x**6) - A*
a*b*d/(2*x**4) - A*b**2*c/(4*x**4) - A*b**2*d/(2*x**2) - B*a**2*c/(6*x**6)
 - B*a**2*d/(4*x**4) - B*a*b*c/(2*x**4) - B*a*b*d/x**2 - B*b**2*c/(2*x**2)
 + B*b**2*d*log(x))/e**9, Eq(m, -9)), ((-A*a**2*c/(6*x**6) - A*a**2*d/(4*x
**4) - A*a*b*c/(2*x**4) - A*a*b*d/x**2 - A*b**2*c/(2*x**2) + A*b**2*d*log(
x) - B*a**2*c/(4*x**4) - B*a**2*d/(2*x**2) - B*a*b*c/x**2 + 2*B*a*b*d*log(
x) + B*b**2*c*log(x) + B*b**2*d*x**2/2)/e**7, Eq(m, -7)), ((-A*a**2*c/(4*x
**4) - A*a**2*d/(2*x**2) - A*a*b*c/x**2 + 2*A*a*b*d*log(x) + A*b**2*c*log(
x) + A*b**2*d*x**2/2 - B*a**2*c/(2*x**2) + B*a**2*d*log(x) + 2*B*a*b*c*log
(x) + B*a*b*d*x**2 + B*b**2*c*x**2/2 + B*b**2*d*x**4/4)/e**5, Eq(m, -5)),
((-A*a**2*c/(2*x**2) + A*a**2*d*log(x) + 2*A*a*b*c*log(x) + A*a*b*d*x**2 +
 A*b**2*c*x**2/2 + A*b**2*d*x**4/4 + B*a**2*c*log(x) + B*a**2*d*x**2/2 + B
*a*b*c*x**2 + B*a*b*d*x**4/2 + B*b**2*c*x**4/4 + B*b**2*d*x**6/6)/e**3, Eq
(m, -3)), ((A*a**2*c*log(x) + A*a**2*d*x**2/2 + A*a*b*c*x**2 + A*a*b*d*x**
4/2 + A*b**2*c*x**4/4 + A*b**2*d*x**6/6 + B*a**2*c*x**2/2 + B*a**2*d*x**4/
4 + B*a*b*c*x**4/2 + B*a*b*d*x**6/3 + B*b**2*c*x**6/6 + B*b**2*d*x**8/8)/e
, Eq(m, -1)), (A*a**2*c*m**4*x*(e*x)**m/(m**5 + 25*m**4 + 230*m**3 + 950*m
**2 + 1689*m + 945) + 24*A*a**2*c*m**3*x*(e*x)**m/(m**5 + 25*m**4 + 230*m*
*3 + 950*m**2 + 1689*m + 945) + 206*A*a**2*c*m**2*x*(e*x)**m/(m**5 + 25*m*
*4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 744*A*a**2*c*m*x*(e*x)**m/(m...
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.68

$$\int (ex)^m \left(a+bx^2\right)^2 \left(A+Bx^2\right) \left(c+dx^2\right) dx = \frac{Bb^2de^m x^9 x^m}{m+9} + \frac{Bb^2ce^m x^7 x^m}{m+7} + \frac{2\,Babde^m x^7 x^m}{m+7} + \frac{Ab^2de^m x^7 x^m}{m+7} + \frac{2\,Babce^m x^5 x^m}{m+5} + \frac{Ab^2ce^m x^5 x^m}{m+5} + \frac{Ba^2de^m x^5 x^m}{m+5} + \frac{2\,Aabde^m x^5 x^m}{m+5} + \frac{Ba^2ce^m x^3 x^m}{m+3} + \frac{2\,Aabce^m x^3 x^m}{m+3} + \frac{Aa^2de^m x^3 x^m}{m+3} + \frac{(ex)^{m+1} Aa^2c}{e(m+1)}$$

input
```
integrate((e*x)^m*(b*x^2+a)^2*(B*x^2+A)*(d*x^2+c),x, algorithm="maxima")
```

output
```
B*b^2*d*e^m*x^9*x^m/(m + 9) + B*b^2*c*e^m*x^7*x^m/(m + 7) + 2*B*a*b*d*e^m*
x^7*x^m/(m + 7) + A*b^2*d*e^m*x^7*x^m/(m + 7) + 2*B*a*b*c*e^m*x^5*x^m/(m +
 5) + A*b^2*c*e^m*x^5*x^m/(m + 5) + B*a^2*d*e^m*x^5*x^m/(m + 5) + 2*A*a*b*
d*e^m*x^5*x^m/(m + 5) + B*a^2*c*e^m*x^3*x^m/(m + 3) + 2*A*a*b*c*e^m*x^3*x^
m/(m + 3) + A*a^2*d*e^m*x^3*x^m/(m + 3) + (e*x)^(m + 1)*A*a^2*c/(e*(m + 1)
)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1009 vs. $2(144) = 288$.

Time = 0.14 (sec) , antiderivative size = 1009, normalized size of antiderivative = 7.01

$$\int (ex)^m \left(a+bx^2\right)^2 \left(A+Bx^2\right) \left(c+dx^2\right)\,dx = \text{Too large to display}$$

input
```
integrate((e*x)^m*(b*x^2+a)^2*(B*x^2+A)*(d*x^2+c),x, algorithm="giac")
```

output
```
((e*x)^m*B*b^2*d*m^4*x^9 + 16*(e*x)^m*B*b^2*d*m^3*x^9 + (e*x)^m*B*b^2*c*m^
4*x^7 + 2*(e*x)^m*B*a*b*d*m^4*x^7 + (e*x)^m*A*b^2*d*m^4*x^7 + 86*(e*x)^m*B
*b^2*d*m^2*x^9 + 18*(e*x)^m*B*b^2*c*m^3*x^7 + 36*(e*x)^m*B*a*b*d*m^3*x^7 +
 18*(e*x)^m*A*b^2*d*m^3*x^7 + 176*(e*x)^m*B*b^2*d*m*x^9 + 2*(e*x)^m*B*a*b*
c*m^4*x^5 + (e*x)^m*A*b^2*c*m^4*x^5 + (e*x)^m*B*a^2*d*m^4*x^5 + 2*(e*x)^m*
A*a*b*d*m^4*x^5 + 104*(e*x)^m*B*b^2*c*m^2*x^7 + 208*(e*x)^m*B*a*b*d*m^2*x^
7 + 104*(e*x)^m*A*b^2*d*m^2*x^7 + 105*(e*x)^m*B*b^2*d*x^9 + 40*(e*x)^m*B*a
*b*c*m^3*x^5 + 20*(e*x)^m*A*b^2*c*m^3*x^5 + 20*(e*x)^m*B*a^2*d*m^3*x^5 + 4
0*(e*x)^m*A*a*b*d*m^3*x^5 + 222*(e*x)^m*B*b^2*c*m*x^7 + 444*(e*x)^m*B*a*b*
d*m*x^7 + 222*(e*x)^m*A*b^2*d*m*x^7 + (e*x)^m*B*a^2*c*m^4*x^3 + 2*(e*x)^m*
A*a*b*c*m^4*x^3 + (e*x)^m*A*a^2*d*m^4*x^3 + 260*(e*x)^m*B*a*b*c*m^2*x^5 +
130*(e*x)^m*A*b^2*c*m^2*x^5 + 130*(e*x)^m*B*a^2*d*m^2*x^5 + 260*(e*x)^m*A*
a*b*d*m^2*x^5 + 135*(e*x)^m*B*b^2*c*x^7 + 270*(e*x)^m*B*a*b*d*x^7 + 135*(e
*x)^m*A*b^2*d*x^7 + 22*(e*x)^m*B*a^2*c*m^3*x^3 + 44*(e*x)^m*A*a*b*c*m^3*x^
3 + 22*(e*x)^m*A*a^2*d*m^3*x^3 + 600*(e*x)^m*B*a*b*c*m*x^5 + 300*(e*x)^m*A
*b^2*c*m*x^5 + 300*(e*x)^m*B*a^2*d*m*x^5 + 600*(e*x)^m*A*a*b*d*m*x^5 + (e*
x)^m*A*a^2*c*m^4*x + 164*(e*x)^m*B*a^2*c*m^2*x^3 + 328*(e*x)^m*A*a*b*c*m^2
*x^3 + 164*(e*x)^m*A*a^2*d*m^2*x^3 + 378*(e*x)^m*B*a*b*c*x^5 + 189*(e*x)^m
*A*b^2*c*x^5 + 189*(e*x)^m*B*a^2*d*x^5 + 378*(e*x)^m*A*a*b*d*x^5 + 24*(e*x
)^m*A*a^2*c*m^3*x + 458*(e*x)^m*B*a^2*c*m*x^3 + 916*(e*x)^m*A*a*b*c*m*x...
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 305, normalized size of antiderivative = 2.12

$$\int (ex)^m \left(a+bx^2\right)^2 \left(A+Bx^2\right) \left(c+dx^2\right) \, dx$$
$$= (e\,x)^m \left(\frac{x^5\,(A\,b^2\,c+B\,a^2\,d+2\,A\,a\,b\,d+2\,B\,a\,b\,c)\,(m^4+20\,m^3+130\,m^2+300\,m+189)}{m^5+25\,m^4+230\,m^3+950\,m^2+1689\,m+945} + \frac{a\,x^3\,(A\,a\,d+2\,A\,b\,c+B\,a\,c)\,(m^4+22\,m^3+164\,m^2+458\,m+315)}{m^5+25\,m^4+230\,m^3+950\,m^2+1689\,m+945} + \frac{b\,x^7\,(A\,b\,d+2\,B\,a\,d+B\,b\,c)\,(m^4+18\,m^3+104\,m^2+222\,m+135)}{m^5+25\,m^4+230\,m^3+950\,m^2+1689\,m+945} + \frac{A\,a^2\,c\,x\,(m^4+24\,m^3+206\,m^2+744\,m+945)}{m^5+25\,m^4+230\,m^3+950\,m^2+1689\,m+945} + \frac{B\,b^2\,d\,x^9\,(m^4+16\,m^3+86\,m^2+176\,m+105)}{m^5+25\,m^4+230\,m^3+950\,m^2+1689\,m+945}\right)$$

input
```
int((A + B*x^2)*(e*x)^m*(a + b*x^2)^2*(c + d*x^2),x)
```

output
```
(e*x)^m*((x^5*(A*b^2*c + B*a^2*d + 2*A*a*b*d + 2*B*a*b*c)*(300*m + 130*m^2
 + 20*m^3 + m^4 + 189))/(1689*m + 950*m^2 + 230*m^3 + 25*m^4 + m^5 + 945)
+ (a*x^3*(A*a*d + 2*A*b*c + B*a*c)*(458*m + 164*m^2 + 22*m^3 + m^4 + 315))
/(1689*m + 950*m^2 + 230*m^3 + 25*m^4 + m^5 + 945) + (b*x^7*(A*b*d + 2*B*a
*d + B*b*c)*(222*m + 104*m^2 + 18*m^3 + m^4 + 135))/(1689*m + 950*m^2 + 23
0*m^3 + 25*m^4 + m^5 + 945) + (A*a^2*c*x*(744*m + 206*m^2 + 24*m^3 + m^4 +
 945))/(1689*m + 950*m^2 + 230*m^3 + 25*m^4 + m^5 + 945) + (B*b^2*d*x^9*(1
76*m + 86*m^2 + 16*m^3 + m^4 + 105))/(1689*m + 950*m^2 + 230*m^3 + 25*m^4
+ m^5 + 945))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 475, normalized size of antiderivative = 3.30

$$\int (ex)^m \left(a+bx^2\right)^2 \left(A+Bx^2\right) \left(c+dx^2\right) \, dx$$
$$= \frac{x^m e^m x(b^3 d\,m^4 x^8 + 16 b^3 d\,m^3 x^8 + 3a\,b^2 d\,m^4 x^6 + b^3 c\,m^4 x^6 + 86 b^3 d\,m^2 x^8 + 54 a\,b^2 d\,m^3 x^6 + 18 b^3 c\,m^3 x^6 +}{}$$

input
```
int((e*x)^m*(b*x^2+a)^2*(B*x^2+A)*(d*x^2+c),x)
```

output
```
(x**m*e**m*x*(a**3*c*m**4 + 24*a**3*c*m**3 + 206*a**3*c*m**2 + 744*a**3*c*
m + 945*a**3*c + a**3*d*m**4*x**2 + 22*a**3*d*m**3*x**2 + 164*a**3*d*m**2*
x**2 + 458*a**3*d*m*x**2 + 315*a**3*d*x**2 + 3*a**2*b*c*m**4*x**2 + 66*a**
2*b*c*m**3*x**2 + 492*a**2*b*c*m**2*x**2 + 1374*a**2*b*c*m*x**2 + 945*a**2
*b*c*x**2 + 3*a**2*b*d*m**4*x**4 + 60*a**2*b*d*m**3*x**4 + 390*a**2*b*d*m*
*2*x**4 + 900*a**2*b*d*m*x**4 + 567*a**2*b*d*x**4 + 3*a*b**2*c*m**4*x**4 +
 60*a*b**2*c*m**3*x**4 + 390*a*b**2*c*m**2*x**4 + 900*a*b**2*c*m*x**4 + 56
7*a*b**2*c*x**4 + 3*a*b**2*d*m**4*x**6 + 54*a*b**2*d*m**3*x**6 + 312*a*b**
2*d*m**2*x**6 + 666*a*b**2*d*m*x**6 + 405*a*b**2*d*x**6 + b**3*c*m**4*x**6
 + 18*b**3*c*m**3*x**6 + 104*b**3*c*m**2*x**6 + 222*b**3*c*m*x**6 + 135*b*
*3*c*x**6 + b**3*d*m**4*x**8 + 16*b**3*d*m**3*x**8 + 86*b**3*d*m**2*x**8 +
 176*b**3*d*m*x**8 + 105*b**3*d*x**8))/(m**5 + 25*m**4 + 230*m**3 + 950*m*
*2 + 1689*m + 945)
```

3.3 $\int (ex)^m (a+bx^2) (A+Bx^2) (c+dx^2) \, dx$

Optimal result

Integrand size = 27, antiderivative size = 97

$$\int (ex)^m (a+bx^2) (A+Bx^2) (c+dx^2) \, dx = \frac{aAc(ex)^{1+m}}{e(1+m)} + \frac{(Abc+aBc+aAd)(ex)^{3+m}}{e^3(3+m)} + \frac{(bBc+Abd+aBd)(ex)^{5+m}}{e^5(5+m)} + \frac{bBd(ex)^{7+m}}{e^7(7+m)}$$

output
```
a*A*c*(e*x)^(1+m)/e/(1+m)+(A*a*d+A*b*c+B*a*c)*(e*x)^(3+m)/e^3/(3+m)+(A*b*d
+B*a*d+B*b*c)*(e*x)^(5+m)/e^5/(5+m)+b*B*d*(e*x)^(7+m)/e^7/(7+m)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.75

$$\int (ex)^m (a+bx^2) (A+Bx^2) (c+dx^2) \, dx = x(ex)^m \left(\frac{aAc}{1+m} + \frac{(Abc+aBc+aAd)x^2}{3+m} + \frac{(bBc+Abd+aBd)x^4}{5+m} + \frac{bBdx^6}{7+m}\right)$$

input
```
Integrate[(e*x)^m*(a + b*x^2)*(A + B*x^2)*(c + d*x^2),x]
```

output
```
x*(e*x)^m*((a*A*c)/(1 + m) + ((A*b*c + a*B*c + a*A*d)*x^2)/(3 + m) + ((b*B
*c + A*b*d + a*B*d)*x^4)/(5 + m) + (b*B*d*x^6)/(7 + m))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {437, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^2)(A+Bx^2)(c+dx^2)(ex)^m\,dx$$

$$\Big\downarrow \text{437}$$

$$\int \left(\frac{(ex)^{m+4}(aBd+Abd+bBc)}{e^4}+\frac{(ex)^{m+2}(aAd+aBc+Abc)}{e^2}+aAc(ex)^m+\frac{bBd(ex)^{m+6}}{e^6}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{(ex)^{m+5}(aBd+Abd+bBc)}{e^5(m+5)}+\frac{(ex)^{m+3}(aAd+aBc+Abc)}{e^3(m+3)}+\frac{aAc(ex)^{m+1}}{e(m+1)}+\frac{bBd(ex)^{m+7}}{e^7(m+7)}$$

input
```
Int[(e*x)^m*(a + b*x^2)*(A + B*x^2)*(c + d*x^2),x]
```

output
```
(a*A*c*(e*x)^(1 + m))/(e*(1 + m)) + ((A*b*c + a*B*c + a*A*d)*(e*x)^(3 + m)
)/(e^3*(3 + m)) + ((b*B*c + A*b*d + a*B*d)*(e*x)^(5 + m))/(e^5*(5 + m)) +
(b*B*d*(e*x)^(7 + m))/(e^7*(7 + m))
```

Defintions of rubi rules used

rule 437
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(
a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f
, g, m}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.99

method	result
norman	$\frac{(Abd+Bad+Bbc)x^5 e^{m\ln(ex)}}{5+m}+\frac{(Aad+Abc+Bac)x^3 e^{m\ln(ex)}}{3+m}+\frac{Aacx\,e^{m\ln(ex)}}{1+m}+\frac{Bbd\,x^7 e^{m\ln(ex)}}{7+m}$
gosper	$x(Bbd\,m^3x^6+9Bbd\,m^2x^6+Abd\,m^3x^4+Bad\,m^3x^4+Bbc\,m^3x^4+23Bbdm\,x^6+11Abd\,m^2x^4+11Bad\,m^2x^4+11Bbc\,m^2x^4+15B$
risch	$x(Bbd\,m^3x^6+9Bbd\,m^2x^6+Abd\,m^3x^4+Bad\,m^3x^4+Bbc\,m^3x^4+23Bbdm\,x^6+11Abd\,m^2x^4+11Bad\,m^2x^4+11Bbc\,m^2x^4+15B$
orering	$x(Bbd\,m^3x^6+9Bbd\,m^2x^6+Abd\,m^3x^4+Bad\,m^3x^4+Bbc\,m^3x^4+23Bbdm\,x^6+11Abd\,m^2x^4+11Bad\,m^2x^4+11Bbc\,m^2x^4+15B$
parallelrisch	$A\,x^3(ex)^m bc\,m^3+23B\,x^7(ex)^m bdm+B\,x^5(ex)^m ad\,m^3+B\,x^5(ex)^m bc\,m^3+11A\,x^5(ex)^m bd\,m^2+B\,x^7(ex)^m bd\,m^3+9B\,x^7(ex$

input
```
int((e*x)^m*(b*x^2+a)*(B*x^2+A)*(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
(A*b*d+B*a*d+B*b*c)/(5+m)*x^5*exp(m*ln(e*x))+(A*a*d+A*b*c+B*a*c)/(3+m)*x^3
*exp(m*ln(e*x))+A*a*c/(1+m)*x*exp(m*ln(e*x))+B*b*d/(7+m)*x^7*exp(m*ln(e*x)
)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 235 vs. $2(97) = 194$.

Time = 0.09 (sec) , antiderivative size = 235, normalized size of antiderivative = 2.42

$$\int (ex)^m (a+bx^2) (A+Bx^2) (c+dx^2)\,dx = \frac{((Bbdm^3 + 9\,Bbdm^2 + 23\,Bbdm + 15\,Bbd)x^7 + ((Bbc + (Ba + Ab)d)m^3 + 21\,Bbc + 11\,(Bbc + (Ba + A}{}$$

input
```
integrate((e*x)^m*(b*x^2+a)*(B*x^2+A)*(d*x^2+c),x, algorithm="fricas")
```

output
```
((B*b*d*m^3 + 9*B*b*d*m^2 + 23*B*b*d*m + 15*B*b*d)*x^7 + ((B*b*c + (B*a +
A*b)*d)*m^3 + 21*B*b*c + 11*(B*b*c + (B*a + A*b)*d)*m^2 + 21*(B*a + A*b)*d
 + 31*(B*b*c + (B*a + A*b)*d)*m)*x^5 + ((A*a*d + (B*a + A*b)*c)*m^3 + 35*A
*a*d + 13*(A*a*d + (B*a + A*b)*c)*m^2 + 35*(B*a + A*b)*c + 47*(A*a*d + (B*
a + A*b)*c)*m)*x^3 + (A*a*c*m^3 + 15*A*a*c*m^2 + 71*A*a*c*m + 105*A*a*c)*x
)*(e*x)^m/(m^4 + 16*m^3 + 86*m^2 + 176*m + 105)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1460 vs. $2(92) = 184$.

Time = 0.46 (sec) , antiderivative size = 1460, normalized size of antiderivative = 15.05

$$\int (ex)^m (a+bx^2) (A+Bx^2) (c+dx^2)\,dx = \text{Too large to display}$$

input
```
integrate((e*x)**m*(b*x**2+a)*(B*x**2+A)*(d*x**2+c),x)
```

output
```
Piecewise(((-A*a*c/(6*x**6) - A*a*d/(4*x**4) - A*b*c/(4*x**4) - A*b*d/(2*x
**2) - B*a*c/(4*x**4) - B*a*d/(2*x**2) - B*b*c/(2*x**2) + B*b*d*log(x))/e*
*7, Eq(m, -7)), ((-A*a*c/(4*x**4) - A*a*d/(2*x**2) - A*b*c/(2*x**2) + A*b*
d*log(x) - B*a*c/(2*x**2) + B*a*d*log(x) + B*b*c*log(x) + B*b*d*x**2/2)/e*
*5, Eq(m, -5)), ((-A*a*c/(2*x**2) + A*a*d*log(x) + A*b*c*log(x) + A*b*d*x*
*2/2 + B*a*c*log(x) + B*a*d*x**2/2 + B*b*c*x**2/2 + B*b*d*x**4/4)/e**3, Eq
(m, -3)), ((A*a*c*log(x) + A*a*d*x**2/2 + A*b*c*x**2/2 + A*b*d*x**4/4 + B*
a*c*x**2/2 + B*a*d*x**4/4 + B*b*c*x**4/4 + B*b*d*x**6/6)/e, Eq(m, -1)), (A
*a*c*m**3*x*(e*x)**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 15*A*a*c*m
**2*x*(e*x)**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 71*A*a*c*m*x*(e*
x)**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 105*A*a*c*x*(e*x)**m/(m**
4 + 16*m**3 + 86*m**2 + 176*m + 105) + A*a*d*m**3*x**3*(e*x)**m/(m**4 + 16
*m**3 + 86*m**2 + 176*m + 105) + 13*A*a*d*m**2*x**3*(e*x)**m/(m**4 + 16*m*
*3 + 86*m**2 + 176*m + 105) + 47*A*a*d*m*x**3*(e*x)**m/(m**4 + 16*m**3 + 8
6*m**2 + 176*m + 105) + 35*A*a*d*x**3*(e*x)**m/(m**4 + 16*m**3 + 86*m**2 +
 176*m + 105) + A*b*c*m**3*x**3*(e*x)**m/(m**4 + 16*m**3 + 86*m**2 + 176*m
 + 105) + 13*A*b*c*m**2*x**3*(e*x)**m/(m**4 + 16*m**3 + 86*m**2 + 176*m +
105) + 47*A*b*c*m*x**3*(e*x)**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) +
 35*A*b*c*x**3*(e*x)**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + A*b*d*m
**3*x**5*(e*x)**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 11*A*b*d*m...
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.51

$$\int (ex)^m \left(a+bx^2\right) \left(A+Bx^2\right) \left(c+dx^2\right) \, dx = \frac{Bbde^m x^7 x^m}{m+7} + \frac{Bbce^m x^5 x^m}{m+5} + \frac{Bade^m x^5 x^m}{m+5} + \frac{Abde^m x^5 x^m}{m+5} + \frac{Bace^m x^3 x^m}{m+3} + \frac{Abce^m x^3 x^m}{m+3} + \frac{Aade^m x^3 x^m}{m+3} + \frac{(ex)^{m+1} Aac}{e(m+1)}$$

input
```
integrate((e*x)^m*(b*x^2+a)*(B*x^2+A)*(d*x^2+c),x, algorithm="maxima")
```

output
```
B*b*d*e^m*x^7*x^m/(m + 7) + B*b*c*e^m*x^5*x^m/(m + 5) + B*a*d*e^m*x^5*x^m/
(m + 5) + A*b*d*e^m*x^5*x^m/(m + 5) + B*a*c*e^m*x^3*x^m/(m + 3) + A*b*c*e^
m*x^3*x^m/(m + 3) + A*a*d*e^m*x^3*x^m/(m + 3) + (e*x)^(m + 1)*A*a*c/(e*(m
+ 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 478 vs. $2(97) = 194$.

Time = 0.13 (sec) , antiderivative size = 478, normalized size of antiderivative = 4.93

$$\int (ex)^m (a+bx^2) (A+Bx^2) (c+dx^2)\, dx$$
$$= \frac{(ex)^m Bbdm^3x^7 + 9\,(ex)^m Bbdm^2x^7 + (ex)^m Bbcm^3x^5 + (ex)^m Badm^3x^5 + (ex)^m Abdm^3x^5 + 23\,(ex)^m}{}$$

input
```
integrate((e*x)^m*(b*x^2+a)*(B*x^2+A)*(d*x^2+c),x, algorithm="giac")
```

output
```
((e*x)^m*B*b*d*m^3*x^7 + 9*(e*x)^m*B*b*d*m^2*x^7 + (e*x)^m*B*b*c*m^3*x^5 +
 (e*x)^m*B*a*d*m^3*x^5 + (e*x)^m*A*b*d*m^3*x^5 + 23*(e*x)^m*B*b*d*m*x^7 +
11*(e*x)^m*B*b*c*m^2*x^5 + 11*(e*x)^m*B*a*d*m^2*x^5 + 11*(e*x)^m*A*b*d*m^2
*x^5 + 15*(e*x)^m*B*b*d*x^7 + (e*x)^m*B*a*c*m^3*x^3 + (e*x)^m*A*b*c*m^3*x^
3 + (e*x)^m*A*a*d*m^3*x^3 + 31*(e*x)^m*B*b*c*m*x^5 + 31*(e*x)^m*B*a*d*m*x^
5 + 31*(e*x)^m*A*b*d*m*x^5 + 13*(e*x)^m*B*a*c*m^2*x^3 + 13*(e*x)^m*A*b*c*m
^2*x^3 + 13*(e*x)^m*A*a*d*m^2*x^3 + 21*(e*x)^m*B*b*c*x^5 + 21*(e*x)^m*B*a*
d*x^5 + 21*(e*x)^m*A*b*d*x^5 + (e*x)^m*A*a*c*m^3*x + 47*(e*x)^m*B*a*c*m*x^
3 + 47*(e*x)^m*A*b*c*m*x^3 + 47*(e*x)^m*A*a*d*m*x^3 + 15*(e*x)^m*A*a*c*m^2
*x + 35*(e*x)^m*B*a*c*x^3 + 35*(e*x)^m*A*b*c*x^3 + 35*(e*x)^m*A*a*d*x^3 +
71*(e*x)^m*A*a*c*m*x + 105*(e*x)^m*A*a*c*x)/(m^4 + 16*m^3 + 86*m^2 + 176*m
 + 105)
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.91

$$\int (ex)^m \left(a+bx^2\right) \left(A+Bx^2\right) \left(c+dx^2\right) \, dx = (e\,x)^m \left(\frac{x^3\,(A\,a\,d+A\,b\,c+B\,a\,c)\,\left(m^3+13\,m^2+47\,m+35\right)}{m^4+16\,m^3+86\,m^2+176\,m+105} + \frac{x^5\,(A\,b\,d+B\,a\,d+B\,b\,c)\,\left(m^3+11\,m^2+31\,m+21\right)}{m^4+16\,m^3+86\,m^2+176\,m+105} + \frac{B\,b\,d\,x^7\,\left(m^3+9\,m^2+23\,m+15\right)}{m^4+16\,m^3+86\,m^2+176\,m+105} + \frac{A\,a\,c\,x\,\left(m^3+15\,m^2+71\,m+105\right)}{m^4+16\,m^3+86\,m^2+176\,m+105}\right)$$

input
```
int((A + B*x^2)*(e*x)^m*(a + b*x^2)*(c + d*x^2),x)
```

output
```
(e*x)^m*((x^3*(A*a*d + A*b*c + B*a*c)*(47*m + 13*m^2 + m^3 + 35))/(176*m +
 86*m^2 + 16*m^3 + m^4 + 105) + (x^5*(A*b*d + B*a*d + B*b*c)*(31*m + 11*m^
2 + m^3 + 21))/(176*m + 86*m^2 + 16*m^3 + m^4 + 105) + (B*b*d*x^7*(23*m +
9*m^2 + m^3 + 15))/(176*m + 86*m^2 + 16*m^3 + m^4 + 105) + (A*a*c*x*(71*m
+ 15*m^2 + m^3 + 105))/(176*m + 86*m^2 + 16*m^3 + m^4 + 105))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 263, normalized size of antiderivative = 2.71

$$\int (ex)^m \left(a+bx^2\right) \left(A+Bx^2\right) \left(c+dx^2\right) \, dx = \frac{x^m e^m x(b^2d\,m^3x^6+9b^2d\,m^2x^6+2abd\,m^3x^4+b^2c\,m^3x^4+23b^2dm\,x^6+22abd\,m^2x^4+11b^2c\,m^2x^4+15b^2}{}$$

input
```
int((e*x)^m*(b*x^2+a)*(B*x^2+A)*(d*x^2+c),x)
```

output

```
(x**m*e**m*x*(a**2*c*m**3 + 15*a**2*c*m**2 + 71*a**2*c*m + 105*a**2*c + a*
*2*d*m**3*x**2 + 13*a**2*d*m**2*x**2 + 47*a**2*d*m*x**2 + 35*a**2*d*x**2 +
 2*a*b*c*m**3*x**2 + 26*a*b*c*m**2*x**2 + 94*a*b*c*m*x**2 + 70*a*b*c*x**2
+ 2*a*b*d*m**3*x**4 + 22*a*b*d*m**2*x**4 + 62*a*b*d*m*x**4 + 42*a*b*d*x**4
 + b**2*c*m**3*x**4 + 11*b**2*c*m**2*x**4 + 31*b**2*c*m*x**4 + 21*b**2*c*x
**4 + b**2*d*m**3*x**6 + 9*b**2*d*m**2*x**6 + 23*b**2*d*m*x**6 + 15*b**2*d
*x**6))/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105)
```

3.4 $\int (ex)^m (A+Bx^2) (c+dx^2) \, dx$

Optimal result

Integrand size = 20, antiderivative size = 60

$$\int (ex)^m \left(A+Bx^2\right) \left(c+dx^2\right) \, dx = \frac{Ac(ex)^{1+m}}{e(1+m)} + \frac{(Bc+Ad)(ex)^{3+m}}{e^3(3+m)} + \frac{Bd(ex)^{5+m}}{e^5(5+m)}$$

output
```
A*c*(e*x)^(1+m)/e/(1+m)+(A*d+B*c)*(e*x)^(3+m)/e^3/(3+m)+B*d*(e*x)^(5+m)/e^
5/(5+m)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.72

$$\int (ex)^m \left(A+Bx^2\right) \left(c+dx^2\right) \, dx = x(ex)^m \left(\frac{Ac}{1+m} + \frac{(Bc+Ad)x^2}{3+m} + \frac{Bdx^4}{5+m}\right)$$

input
```
Integrate[(e*x)^m*(A + B*x^2)*(c + d*x^2),x]
```

output
```
x*(e*x)^m*((A*c)/(1 + m) + ((B*c + A*d)*x^2)/(3 + m) + (B*d*x^4)/(5 + m))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(A+Bx^2\right) \left(c+dx^2\right) (ex)^m \, dx$$

$$\Big\downarrow \text{355}$$

$$\int \left(\frac{(ex)^{m+2}(Ad+Bc)}{e^2}+Ac(ex)^m+\frac{Bd(ex)^{m+4}}{e^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{(ex)^{m+3}(Ad+Bc)}{e^3(m+3)}+\frac{Ac(ex)^{m+1}}{e(m+1)}+\frac{Bd(ex)^{m+5}}{e^5(m+5)}$$

input
```
Int[(e*x)^m*(A + B*x^2)*(c + d*x^2),x]
```

output
```
(A*c*(e*x)^(1 + m))/(e*(1 + m)) + ((B*c + A*d)*(e*x)^(3 + m))/(e^3*(3 + m)
) + (B*d*(e*x)^(5 + m))/(e^5*(5 + m))
```

Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.98

method	result
norman	$\frac{(Ad+Bc)x^3 e^{m\ln(ex)}}{3+m}+\frac{Acx\,e^{m\ln(ex)}}{1+m}+\frac{Bd\,x^5 e^{m\ln(ex)}}{5+m}$
gosper	$\frac{x(Bd\,m^2x^4+4Bdm\,x^4+Ad\,m^2x^2+Bc\,m^2x^2+3Bd\,x^4+6Adm\,x^2+6Bcm\,x^2+Ac\,m^2+5Ad\,x^2+5Bc\,x^2+8Acm+15Ac)(ex)^m}{(5+m)(3+m)(1+m)}$
risch	$\frac{x(Bd\,m^2x^4+4Bdm\,x^4+Ad\,m^2x^2+Bc\,m^2x^2+3Bd\,x^4+6Adm\,x^2+6Bcm\,x^2+Ac\,m^2+5Ad\,x^2+5Bc\,x^2+8Acm+15Ac)(ex)^m}{(5+m)(3+m)(1+m)}$
orering	$\frac{x(Bd\,m^2x^4+4Bdm\,x^4+Ad\,m^2x^2+Bc\,m^2x^2+3Bd\,x^4+6Adm\,x^2+6Bcm\,x^2+Ac\,m^2+5Ad\,x^2+5Bc\,x^2+8Acm+15Ac)(ex)^m}{(5+m)(3+m)(1+m)}$
parallelrisch	$\frac{B\,x^5(ex)^m d\,m^2+4B\,x^5(ex)^m dm+A\,x^3(ex)^m d\,m^2+3B\,x^5(ex)^m d+B\,x^3(ex)^m c\,m^2+6A\,x^3(ex)^m dm+6B\,x^3(ex)^m cm+5A\,x}{(5+m)(3+m)(1+m)}$

input
```
int((e*x)^m*(B*x^2+A)*(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
(A*d+B*c)/(3+m)*x^3*exp(m*ln(e*x))+A*c/(1+m)*x*exp(m*ln(e*x))+B*d/(5+m)*x^
5*exp(m*ln(e*x))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.57

$$\int (ex)^m\left(A+Bx^2\right)\left(c+dx^2\right)\,dx$$
$$=\frac{((Bdm^2+4\,Bdm+3\,Bd)x^5+((Bc+Ad)m^2+5\,Bc+5\,Ad+6\,(Bc+Ad)m)x^3+(Acm^2+8\,Acm+}{m^3+9\,m^2+23\,m+15}$$

input
```
integrate((e*x)^m*(B*x^2+A)*(d*x^2+c),x, algorithm="fricas")
```

output
```
((B*d*m^2 + 4*B*d*m + 3*B*d)*x^5 + ((B*c + A*d)*m^2 + 5*B*c + 5*A*d + 6*(B
*c + A*d)*m)*x^3 + (A*c*m^2 + 8*A*c*m + 15*A*c)*x)*(e*x)^m/(m^3 + 9*m^2 +
23*m + 15)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 439 vs. $2(51) = 102$.

Time = 0.31 (sec) , antiderivative size = 439, normalized size of antiderivative = 7.32

$$\int (ex)^m \left(A+Bx^2\right)\left(c+dx^2\right)\,dx = \begin{cases} \frac{-\frac{Ac}{4x^4}-\frac{Ad}{2x^2}-\frac{Bc}{2x^2}+Bd\log{(x)}}{e^5} \\ \frac{-\frac{Ac}{2x^2}+Ad\log{(x)}+Bc\log{(x)}+\frac{Bdx^2}{2}}{e^3} \\ \frac{Ac\log{(x)}+\frac{Adx^2}{2}+\frac{Bcx^2}{2}+\frac{Bdx^4}{4}}{e} \\ \frac{Acm^2x(ex)^m}{m^3+9m^2+23m+15}+\frac{8Acmx(ex)^m}{m^3+9m^2+23m+15}+\frac{15Acx(ex)^m}{m^3+9m^2+23m+15}+\frac{Adm^2x^3(ex)^m}{m^3+9m^2+23m+15}+\frac{6Admx^3(ex)^m}{m^3+9m^2+23m+15}+\frac{5Adx^3(ex)^m}{m^3+9m^2+23m+\cdots} \end{cases}$$

input
```
integrate((e*x)**m*(B*x**2+A)*(d*x**2+c),x)
```

output
```
Piecewise(((-A*c/(4*x**4) - A*d/(2*x**2) - B*c/(2*x**2) + B*d*log(x))/e**5
, Eq(m, -5)), ((-A*c/(2*x**2) + A*d*log(x) + B*c*log(x) + B*d*x**2/2)/e**3
, Eq(m, -3)), ((A*c*log(x) + A*d*x**2/2 + B*c*x**2/2 + B*d*x**4/4)/e, Eq(m
, -1)), (A*c*m**2*x*(e*x)**m/(m**3 + 9*m**2 + 23*m + 15) + 8*A*c*m*x*(e*x)
**m/(m**3 + 9*m**2 + 23*m + 15) + 15*A*c*x*(e*x)**m/(m**3 + 9*m**2 + 23*m
+ 15) + A*d*m**2*x**3*(e*x)**m/(m**3 + 9*m**2 + 23*m + 15) + 6*A*d*m*x**3*
(e*x)**m/(m**3 + 9*m**2 + 23*m + 15) + 5*A*d*x**3*(e*x)**m/(m**3 + 9*m**2
+ 23*m + 15) + B*c*m**2*x**3*(e*x)**m/(m**3 + 9*m**2 + 23*m + 15) + 6*B*c*
m*x**3*(e*x)**m/(m**3 + 9*m**2 + 23*m + 15) + 5*B*c*x**3*(e*x)**m/(m**3 +
9*m**2 + 23*m + 15) + B*d*m**2*x**5*(e*x)**m/(m**3 + 9*m**2 + 23*m + 15) +
 4*B*d*m*x**5*(e*x)**m/(m**3 + 9*m**2 + 23*m + 15) + 3*B*d*x**5*(e*x)**m/(
m**3 + 9*m**2 + 23*m + 15), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.17

$$\int (ex)^m \left(A+Bx^2\right)\left(c+dx^2\right)\,dx = \frac{Bde^mx^5x^m}{m+5}+\frac{Bce^mx^3x^m}{m+3}+\frac{Ade^mx^3x^m}{m+3}+\frac{(ex)^{m+1}Ac}{e(m+1)}$$

input
```
integrate((e*x)^m*(B*x^2+A)*(d*x^2+c),x, algorithm="maxima")
```

output
```
B*d*e^m*x^5*x^m/(m + 5) + B*c*e^m*x^3*x^m/(m + 3) + A*d*e^m*x^3*x^m/(m + 3
) + (e*x)^(m + 1)*A*c/(e*(m + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 167 vs. $2(60) = 120$.

Time = 0.12 (sec) , antiderivative size = 167, normalized size of antiderivative = 2.78

$$\int (ex)^m \left(A+Bx^2\right)\left(c+dx^2\right)\,dx$$
$$= \frac{(ex)^m Bdm^2x^5 + 4\,(ex)^m Bdmx^5 + (ex)^m Bcm^2x^3 + (ex)^m Adm^2x^3 + 3\,(ex)^m Bdx^5 + 6\,(ex)^m Bcmx^3 \cdots}{m^3 + 9\,m^2 + 2\cdots}$$

input
```
integrate((e*x)^m*(B*x^2+A)*(d*x^2+c),x, algorithm="giac")
```

output
```
((e*x)^m*B*d*m^2*x^5 + 4*(e*x)^m*B*d*m*x^5 + (e*x)^m*B*c*m^2*x^3 + (e*x)^m
*A*d*m^2*x^3 + 3*(e*x)^m*B*d*x^5 + 6*(e*x)^m*B*c*m*x^3 + 6*(e*x)^m*A*d*m*x
^3 + (e*x)^m*A*c*m^2*x + 5*(e*x)^m*B*c*x^3 + 5*(e*x)^m*A*d*x^3 + 8*(e*x)^m
*A*c*m*x + 15*(e*x)^m*A*c*x)/(m^3 + 9*m^2 + 23*m + 15)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.62

$$\int (ex)^m \left(A+Bx^2\right)\left(c+dx^2\right)\,dx = (e\,x)^m \left(\frac{x^3\,(A\,d+B\,c)\,\left(m^2+6\,m+5\right)}{m^3+9\,m^2+23\,m+15} + \frac{B\,d\,x^5\,\left(m^2+4\,m+3\right)}{m^3+9\,m^2+23\,m+15} + \frac{A\,c\,x\,\left(m^2+8\,m+15\right)}{m^3+9\,m^2+23\,m+15}\right)$$

input
```
int((A + B*x^2)*(e*x)^m*(c + d*x^2),x)
```

output
```
(e*x)^m*((x^3*(A*d + B*c)*(6*m + m^2 + 5))/(23*m + 9*m^2 + m^3 + 15) + (B*
d*x^5*(4*m + m^2 + 3))/(23*m + 9*m^2 + m^3 + 15) + (A*c*x*(8*m + m^2 + 15)
)/(23*m + 9*m^2 + m^3 + 15))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.85

$$\int (ex)^m (A + Bx^2) (c + dx^2) \, dx$$
$$= \frac{x^m e^m x(bd\,m^2x^4 + 4bdm\,x^4 + ad\,m^2x^2 + bc\,m^2x^2 + 3bd\,x^4 + 6adm\,x^2 + 6bcm\,x^2 + ac\,m^2 + 5ad\,x^2 + 5bc}{m^3 + 9m^2 + 23m + 15}$$

input
```
int((e*x)^m*(B*x^2+A)*(d*x^2+c),x)
```

output
```
(x**m*e**m*x*(a*c*m**2 + 8*a*c*m + 15*a*c + a*d*m**2*x**2 + 6*a*d*m*x**2 +
 5*a*d*x**2 + b*c*m**2*x**2 + 6*b*c*m*x**2 + 5*b*c*x**2 + b*d*m**2*x**4 +
4*b*d*m*x**4 + 3*b*d*x**4))/(m**3 + 9*m**2 + 23*m + 15)
```

3.5 $\int \frac{(ex)^m(A+Bx^2)(c+dx^2)}{a+bx^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 118

$$\int \frac{(ex)^m\left(A+Bx^2\right)\left(c+dx^2\right)}{a+bx^2}\,dx = \frac{(bBc+Abd-aBd)(ex)^{1+m}}{b^2e(1+m)}+\frac{Bd(ex)^{3+m}}{be^3(3+m)} + \frac{(Ab-aB)(bc-ad)(ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{bx^2}{a}\right)}{ab^2e(1+m)}$$

output
```
(A*b*d-B*a*d+B*b*c)*(e*x)^(1+m)/b^2/e/(1+m)+B*d*(e*x)^(3+m)/b/e^3/(3+m)+(A
*b-B*a)*(-a*d+b*c)*(e*x)^(1+m)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-b*x^2
/a)/a/b^2/e/(1+m)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.79

$$\int \frac{(ex)^m\left(A+Bx^2\right)\left(c+dx^2\right)}{a+bx^2}\,dx = \frac{x(ex)^m\left(\frac{bBc+Abd-aBd}{1+m}+\frac{bBdx^2}{3+m}+\frac{(-Ab+aB)(-bc+ad)\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{bx^2}{a}\right)}{a(1+m)}\right)}{b^2}$$

input
```
Integrate[((e*x)^m*(A + B*x^2)*(c + d*x^2))/(a + b*x^2),x]
```

output
```
(x*(e*x)^m*((b*B*c + A*b*d - a*B*d)/(1 + m) + (b*B*d*x^2)/(3 + m) + ((-(A*
b) + a*B)*(-(b*c) + a*d)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x
^2)/a)])/(a*(1 + m))))/b^2
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {437, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)(c+dx^2)(ex)^m}{a+bx^2}dx$$

$$\Big\downarrow \text{437}$$

$$\int \left(\frac{(ex)^m(a^2Bd-aAbd-abBc+Ab^2c)}{b^2(a+bx^2)}+\frac{(ex)^m(-aBd+Abd+bBc)}{b^2}+\frac{Bd(ex)^{m+2}}{be^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{(ex)^{m+1}(Ab-aB)(bc-ad)\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\frac{bx^2}{a}\right)}{ab^2e(m+1)}+\frac{(ex)^{m+1}(-aBd+Abd+bBc)}{b^2e(m+1)}+\frac{Bd(ex)^{m+3}}{be^3(m+3)}$$

input
```
Int[((e*x)^m*(A + B*x^2)*(c + d*x^2))/(a + b*x^2),x]
```

output
```
((b*B*c + A*b*d - a*B*d)*(e*x)^(1 + m))/(b^2*e*(1 + m)) + (B*d*(e*x)^(3 +
m))/(b*e^3*(3 + m)) + ((A*b - a*B)*(b*c - a*d)*(e*x)^(1 + m)*Hypergeometri
c2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(a*b^2*e*(1 + m))
```

Defintions of rubi rules used

rule 437
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(
a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f
, g, m}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{(ex)^m (x^2B + A) (x^2d + c)}{b\,x^2 + a}dx$$

input
```
int((e*x)^m*(B*x^2+A)*(d*x^2+c)/(b*x^2+a),x)
```

output
```
int((e*x)^m*(B*x^2+A)*(d*x^2+c)/(b*x^2+a),x)
```

Fricas [F]

$$\int \frac{(ex)^m (A + Bx^2) (c + dx^2)}{a + bx^2}\,dx = \int \frac{(Bx^2 + A)(dx^2 + c)(ex)^m}{bx^2 + a}\,dx$$

input
```
integrate((e*x)^m*(B*x^2+A)*(d*x^2+c)/(b*x^2+a),x, algorithm="fricas")
```

output
```
integral((B*d*x^4 + (B*c + A*d)*x^2 + A*c)*(e*x)^m/(b*x^2 + a), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.45 (sec) , antiderivative size = 418, normalized size of antiderivative = 3.54

$$\int \frac{(ex)^m (A+Bx^2)(c+dx^2)}{a+bx^2}\,dx = \frac{Ace^m m x^{m+1}\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)} + \frac{Ace^m x^{m+1}\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)} + \frac{Ade^m m x^{m+3}\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{3}{2}\right)\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)} + \frac{3Ade^m x^{m+3}\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{3}{2}\right)\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)} + \frac{Bce^m m x^{m+3}\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{3}{2}\right)\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)} + \frac{3Bce^m x^{m+3}\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{3}{2}\right)\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)} + \frac{Bde^m m x^{m+5}\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{5}{2}\right)\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{7}{2}\right)} + \frac{5Bde^m x^{m+5}\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{5}{2}\right)\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{7}{2}\right)}$$

input
```
integrate((e*x)**m*(B*x**2+A)*(d*x**2+c)/(b*x**2+a),x)
```

output
```
A*c*e**m*m*x**(m + 1)*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gam
ma(m/2 + 1/2)/(4*a*gamma(m/2 + 3/2)) + A*c*e**m*x**(m + 1)*lerchphi(b*x**2
*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(4*a*gamma(m/2 + 3/2))
+ A*d*e**m*m*x**(m + 3)*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 3/2)*g
amma(m/2 + 3/2)/(4*a*gamma(m/2 + 5/2)) + 3*A*d*e**m*x**(m + 3)*lerchphi(b*
x**2*exp_polar(I*pi)/a, 1, m/2 + 3/2)*gamma(m/2 + 3/2)/(4*a*gamma(m/2 + 5/
2)) + B*c*e**m*m*x**(m + 3)*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 3/
2)*gamma(m/2 + 3/2)/(4*a*gamma(m/2 + 5/2)) + 3*B*c*e**m*x**(m + 3)*lerchph
i(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 3/2)*gamma(m/2 + 3/2)/(4*a*gamma(m/2
+ 5/2)) + B*d*e**m*m*x**(m + 5)*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2
+ 5/2)*gamma(m/2 + 5/2)/(4*a*gamma(m/2 + 7/2)) + 5*B*d*e**m*x**(m + 5)*ler
chphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 5/2)*gamma(m/2 + 5/2)/(4*a*gamma(
m/2 + 7/2))
```

Maxima [F]

$$\int \frac{(ex)^m (A+Bx^2)(c+dx^2)}{a+bx^2}\,dx = \int \frac{(Bx^2+A)(dx^2+c)(ex)^m}{bx^2+a}\,dx$$

input
```
integrate((e*x)^m*(B*x^2+A)*(d*x^2+c)/(b*x^2+a),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*(d*x^2 + c)*(e*x)^m/(b*x^2 + a), x)
```

Giac [F]

$$\int \frac{(ex)^m (A+Bx^2)(c+dx^2)}{a+bx^2}\,dx = \int \frac{(Bx^2+A)(dx^2+c)(ex)^m}{bx^2+a}\,dx$$

input
```
integrate((e*x)^m*(B*x^2+A)*(d*x^2+c)/(b*x^2+a),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*(d*x^2 + c)*(e*x)^m/(b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m (A+Bx^2)(c+dx^2)}{a+bx^2}\,dx = \int \frac{(B\,x^2+A)\,(e\,x)^m\,(d\,x^2+c)}{b\,x^2+a}\,dx$$

input
```
int(((A + B*x^2)*(e*x)^m*(c + d*x^2))/(a + b*x^2),x)
```

output
```
int(((A + B*x^2)*(e*x)^m*(c + d*x^2))/(a + b*x^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.31

$$\int \frac{(ex)^m (A+Bx^2)(c+dx^2)}{a+bx^2}\,dx = \frac{x^m e^m x(dm\,x^2+d\,x^2+cm+3c)}{m^2+4m+3}$$

input
```
int((e*x)^m*(B*x^2+A)*(d*x^2+c)/(b*x^2+a),x)
```

output
```
(x**m*e**m*x*(c*m + 3*c + d*m*x**2 + d*x**2))/(m**2 + 4*m + 3)
```

3.6 $\int \frac{(ex)^m(A+Bx^2)(c+dx^2)}{(a+bx^2)^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 153

$$\int \frac{(ex)^m (A+Bx^2)(c+dx^2)}{(a+bx^2)^2}\,dx = \frac{Bd(ex)^{1+m}}{b^2e(1+m)} + \frac{(Ab-aB)(bc-ad)(ex)^{1+m}}{2ab^2e(a+bx^2)} + \frac{(aB(bc(1+m)-ad(3+m))+Ab(ad(1+m)+b(c-cm)))(ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+$$

$$2a^2b^2e(1+m)$$

output
```
B*d*(e*x)^(1+m)/b^2/e/(1+m)+1/2*(A*b-B*a)*(-a*d+b*c)*(e*x)^(1+m)/a/b^2/e/(
b*x^2+a)+1/2*(a*B*(b*c*(1+m)-a*d*(3+m))+A*b*(a*d*(1+m)+b*(-c*m+c)))*(e*x)^
(1+m)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-b*x^2/a)/a^2/b^2/e/(1+m)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.71

$$\int \frac{(ex)^m (A+Bx^2)(c+dx^2)}{(a+bx^2)^2}\,dx$$

$$= \frac{x(ex)^m \left(a^2Bd + a(bBc+Abd-2aBd)\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{bx^2}{a}\right) + (Ab-aB)(bc-ad\right.}{a^2b^2(1+m)}$$

input
```
Integrate[((e*x)^m*(A + B*x^2)*(c + d*x^2))/(a + b*x^2)^2,x]
```

output
```
(x*(e*x)^m*(a^2*B*d + a*(b*B*c + A*b*d - 2*a*B*d)*Hypergeometric2F1[1, (1
+ m)/2, (3 + m)/2, -((b*x^2)/a)] + (A*b - a*B)*(b*c - a*d)*Hypergeometric2
F1[2, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)]))/(a^2*b^2*(1 + m))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.14, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {439, 25, 363, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)(c+dx^2)(ex)^m}{(a+bx^2)^2}dx$$

$$\Big\downarrow \text{439}$$

$$\frac{(c+dx^2)(ex)^{m+1}(Ab-aB)}{2abe(a+bx^2)}-\frac{\int -\frac{(ex)^m(c(aB(m+1)+A(b-bm))-d(Ab(m+1)-aB(m+3))x^2)}{bx^2+a}dx}{2ab}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{(ex)^m(c(aB(m+1)+A(b-bm))-d(Ab(m+1)-aB(m+3))x^2)}{bx^2+a}dx}{2ab}+\frac{(c+dx^2)(ex)^{m+1}(Ab-aB)}{2abe(a+bx^2)}$$

$$\Big\downarrow \text{363}$$

$$\frac{\frac{(Ab(ad(m+1)+bc(1-m))+aB(bc(m+1)-ad(m+3)))\int \frac{(ex)^m}{bx^2+a}dx}{b}-\frac{d(ex)^{m+1}(Ab(m+1)-aB(m+3))}{be(m+1)}}{2ab}+\frac{(c+dx^2)(ex)^{m+1}(Ab-aB)}{2abe(a+bx^2)}$$

$$\Big\downarrow \text{278}$$

$$\frac{\frac{(ex)^{m+1}\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\frac{bx^2}{a}\right)(Ab(ad(m+1)+bc(1-m))+aB(bc(m+1)-ad(m+3)))}{abe(m+1)}-\frac{d(ex)^{m+1}(Ab(m+1)-aB(m+3))}{be(m+1)}}{2ab}$$
$$\frac{(c+dx^2)\,(ex)^{m+1}(Ab-aB)}{2abe\,(a+bx^2)}$$

input
```
Int[((e*x)^m*(A + B*x^2)*(c + d*x^2))/(a + b*x^2)^2,x]
```

output
```
((A*b - a*B)*(e*x)^(1 + m)*(c + d*x^2))/(2*a*b*e*(a + b*x^2)) + (-((d*(A*b
*(1 + m) - a*B*(3 + m))*(e*x)^(1 + m))/(b*e*(1 + m))) + ((A*b*(b*c*(1 - m)
 + a*d*(1 + m)) + a*B*(b*c*(1 + m) - a*d*(3 + m)))*(e*x)^(1 + m)*Hypergeom
etric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(a*b*e*(1 + m)))/(2*a*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 439
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
 + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*b*g*(p + 1))), x] + Simp[1/(2*a*b*(p
+ 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1)*Simp[c*(2*b*e*(
p + 1) + (b*e - a*f)*(m + 1)) + d*(2*b*e*(p + 1) + (b*e - a*f)*(m + 2*q + 1
))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && LtQ[p, -1] && G
tQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e - a*f])
```

Maple [F]

$$\int \frac{(ex)^m (x^2B + A) (x^2d + c)}{(b\,x^2 + a)^2}dx$$

input
```
int((e*x)^m*(B*x^2+A)*(d*x^2+c)/(b*x^2+a)^2,x)
```

output
```
int((e*x)^m*(B*x^2+A)*(d*x^2+c)/(b*x^2+a)^2,x)
```

Fricas [F]

$$\int \frac{(ex)^m (A + Bx^2) (c + dx^2)}{(a + bx^2)^2}\,dx = \int \frac{(Bx^2 + A)(dx^2 + c)(ex)^m}{(bx^2 + a)^2}\,dx$$

input
```
integrate((e*x)^m*(B*x^2+A)*(d*x^2+c)/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
integral((B*d*x^4 + (B*c + A*d)*x^2 + A*c)*(e*x)^m/(b^2*x^4 + 2*a*b*x^2 +
a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 30.58 (sec) , antiderivative size = 2069, normalized size of antiderivative = 13.52

$$\int \frac{(ex)^m (A + Bx^2) (c + dx^2)}{(a + bx^2)^2} dx = \text{Too large to display}$$

input
```
integrate((e*x)**m*(B*x**2+A)*(d*x**2+c)/(b*x**2+a)**2,x)
```

output
```
A*c*(-a*e**m*m**2*x**(m + 1)*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1
/2)*gamma(m/2 + 1/2)/(8*a**3*gamma(m/2 + 3/2) + 8*a**2*b*x**2*gamma(m/2 +
3/2)) + 2*a*e**m*m*x**(m + 1)*gamma(m/2 + 1/2)/(8*a**3*gamma(m/2 + 3/2) +
8*a**2*b*x**2*gamma(m/2 + 3/2)) + a*e**m*x**(m + 1)*lerchphi(b*x**2*exp_po
lar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(8*a**3*gamma(m/2 + 3/2) + 8*a
**2*b*x**2*gamma(m/2 + 3/2)) + 2*a*e**m*x**(m + 1)*gamma(m/2 + 1/2)/(8*a**
3*gamma(m/2 + 3/2) + 8*a**2*b*x**2*gamma(m/2 + 3/2)) - b*e**m*m**2*x**2*x*
*(m + 1)*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)
/(8*a**3*gamma(m/2 + 3/2) + 8*a**2*b*x**2*gamma(m/2 + 3/2)) + b*e**m*x**2*
x**(m + 1)*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/
2)/(8*a**3*gamma(m/2 + 3/2) + 8*a**2*b*x**2*gamma(m/2 + 3/2))) + A*d*(-a*e
**m*m**2*x**(m + 3)*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 3/2)*gamma
(m/2 + 3/2)/(8*a**3*gamma(m/2 + 5/2) + 8*a**2*b*x**2*gamma(m/2 + 5/2)) - 4
*a*e**m*m*x**(m + 3)*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 3/2)*gamm
a(m/2 + 3/2)/(8*a**3*gamma(m/2 + 5/2) + 8*a**2*b*x**2*gamma(m/2 + 5/2)) +
2*a*e**m*m*x**(m + 3)*gamma(m/2 + 3/2)/(8*a**3*gamma(m/2 + 5/2) + 8*a**2*b
*x**2*gamma(m/2 + 5/2)) - 3*a*e**m*x**(m + 3)*lerchphi(b*x**2*exp_polar(I*
pi)/a, 1, m/2 + 3/2)*gamma(m/2 + 3/2)/(8*a**3*gamma(m/2 + 5/2) + 8*a**2*b*
x**2*gamma(m/2 + 5/2)) + 6*a*e**m*x**(m + 3)*gamma(m/2 + 3/2)/(8*a**3*gamm
a(m/2 + 5/2) + 8*a**2*b*x**2*gamma(m/2 + 5/2)) - b*e**m*m**2*x**2*x**(m...
```

Maxima [F]

$$\int \frac{(ex)^m (A+Bx^2) (c+dx^2)}{(a+bx^2)^2} \, dx = \int \frac{(Bx^2+A)(dx^2+c)(ex)^m}{(bx^2+a)^2} \, dx$$

input
```
integrate((e*x)^m*(B*x^2+A)*(d*x^2+c)/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*(d*x^2 + c)*(e*x)^m/(b*x^2 + a)^2, x)
```

Giac [F]

$$\int \frac{(ex)^m (A+Bx^2) (c+dx^2)}{(a+bx^2)^2} \, dx = \int \frac{(Bx^2+A)(dx^2+c)(ex)^m}{(bx^2+a)^2} \, dx$$

input
```
integrate((e*x)^m*(B*x^2+A)*(d*x^2+c)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*(d*x^2 + c)*(e*x)^m/(b*x^2 + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m (A+Bx^2) (c+dx^2)}{(a+bx^2)^2} \, dx = \int \frac{(B\,x^2+A)\,(e\,x)^m\,(d\,x^2+c)}{(b\,x^2+a)^2} \, dx$$

input
```
int(((A + B*x^2)*(e*x)^m*(c + d*x^2))/(a + b*x^2)^2,x)
```

output
```
int(((A + B*x^2)*(e*x)^m*(c + d*x^2))/(a + b*x^2)^2, x)
```

Reduce [F]

$$\int \frac{(ex)^m (A+Bx^2)(c+dx^2)}{(a+bx^2)^2}\,dx$$
$$= \frac{e^m \left(x^m dx - \left(\int \frac{x^m}{b\,x^2+a}dx\right) adm - \left(\int \frac{x^m}{b\,x^2+a}dx\right) ad + \left(\int \frac{x^m}{b\,x^2+a}dx\right) bcm + \left(\int \frac{x^m}{b\,x^2+a}dx\right) bc\right)}{b\,(m+1)}$$

input
```
int((e*x)^m*(B*x^2+A)*(d*x^2+c)/(b*x^2+a)^2,x)
```

output
```
(e**m*(x**m*d*x - int(x**m/(a + b*x**2),x)*a*d*m - int(x**m/(a + b*x**2),x
)*a*d + int(x**m/(a + b*x**2),x)*b*c*m + int(x**m/(a + b*x**2),x)*b*c))/(b
*(m + 1))
```

3.7 $\int \frac{(ex)^m (A+Bx^2)(c+dx^2)}{(a+bx^2)^3} dx$

Optimal result

Integrand size = 29, antiderivative size = 206

$$\int \frac{(ex)^m (A+Bx^2)(c+dx^2)}{(a+bx^2)^3} dx = \frac{(Ab-aB)(bc-ad)(ex)^{1+m}}{4ab^2e(a+bx^2)^2} + \frac{(Ab(bc(3-m)+ad(1+m))+aB(bc(1+m)-ad(5+m)))(ex)^{1+m}}{8a^2b^2e(a+bx^2)} + \frac{(Ab(1-m)(bc(3-m)+ad(1+m))+aB(1+m)(ad(3+m)+b(c-cm)))(ex)^{1+m} \text{Hypergeometric}}{8a^3b^2e(1+m)}$$

output

```
1/4*(A*b-B*a)*(-a*d+b*c)*(e*x)^(1+m)/a/b^2/e/(b*x^2+a)^2+1/8*(A*b*(b*c*(3-
m)+a*d*(1+m))+a*B*(b*c*(1+m)-a*d*(5+m)))*(e*x)^(1+m)/a^2/b^2/e/(b*x^2+a)+1
/8*(A*b*(1-m)*(b*c*(3-m)+a*d*(1+m))+a*B*(1+m)*(a*d*(3+m)+b*(-c*m+c)))*(e*x
)^(1+m)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-b*x^2/a)/a^3/b^2/e/(1+m)
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.65

$$\int \frac{(ex)^m (A+Bx^2)(c+dx^2)}{(a+bx^2)^3}\,dx$$
$$= \frac{x(ex)^m \left(a^2Bd\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{bx^2}{a}\right)+a(bBc+Abd-2aBd)\,\text{Hypergeometric2F1}\left(\right.\right.}{a^3b^2(1+m)}$$

input
```
Integrate[((e*x)^m*(A + B*x^2)*(c + d*x^2))/(a + b*x^2)^3,x]
```

output
```
(x*(e*x)^m*(a^2*B*d*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a
)] + a*(b*B*c + A*b*d - 2*a*B*d)*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2
, -((b*x^2)/a)] + (A*b - a*B)*(b*c - a*d)*Hypergeometric2F1[3, (1 + m)/2,
(3 + m)/2, -((b*x^2)/a)]))/(a^3*b^2*(1 + m))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {439, 25, 362, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)(c+dx^2)(ex)^m}{(a+bx^2)^3}\,dx$$

$$\Big\downarrow \text{439}$$

$$\frac{(c+dx^2)(ex)^{m+1}(Ab-aB)}{4abe(a+bx^2)^2}-\frac{\int -\frac{(ex)^m(d(aB(m+3)+A(b-bm))x^2+c(Ab(3-m)+aB(m+1)))}{(bx^2+a)^2}dx}{4ab}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{(ex)^m(d(aB(m+3)+A(b-bm))x^2+c(Ab(3-m)+aB(m+1)))}{(bx^2+a)^2}dx}{4ab}+\frac{(c+dx^2)(ex)^{m+1}(Ab-aB)}{4abe(a+bx^2)^2}$$

$$\Big\downarrow \ 362$$

$$\frac{\frac{(Ab(1-m)(ad(m+1)+bc(3-m))+aB(m+1)(ad(m+3)+b(c-cm)))\int \frac{(ex)^m}{bx^2+a}dx}{2ab}-\frac{(ex)^{m+1}(Ab(ad(1-m)-bc(3-m))-aB(bc(m+1)-ad(m+3))}{2abe(a+bx^2)}}{4ab}$$
$$\frac{(c+dx^2)(ex)^{m+1}(Ab-aB)}{4abe(a+bx^2)^2}$$

$$\Big\downarrow \ 278$$

$$\frac{\frac{(ex)^{m+1}\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\frac{bx^2}{a}\right)(Ab(1-m)(ad(m+1)+bc(3-m))+aB(m+1)(ad(m+3)+b(c-cm)))}{2a^2be(m+1)}-\frac{(ex)^{m+1}(Ab(ad(1-m}{}}{4ab}$$
$$\frac{(c+dx^2)(ex)^{m+1}(Ab-aB)}{4abe(a+bx^2)^2}$$

input
```
Int[((e*x)^m*(A + B*x^2)*(c + d*x^2))/(a + b*x^2)^3,x]
```

output
```
((A*b - a*B)*(e*x)^(1 + m)*(c + d*x^2))/(4*a*b*e*(a + b*x^2)^2) + (-1/2*((
A*b*(a*d*(1 - m) - b*c*(3 - m)) - a*B*(b*c*(1 + m) - a*d*(3 + m)))*(e*x)^(
1 + m))/(a*b*e*(a + b*x^2)) + ((A*b*(1 - m)*(b*c*(3 - m) + a*d*(1 + m)) +
a*B*(1 + m)*(a*d*(3 + m) + b*(c - c*m)))*(e*x)^(1 + m)*Hypergeometric2F1[1
, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(2*a^2*b*e*(1 + m)))/(4*a*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

rule 439
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
 + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*b*g*(p + 1))), x] + Simp[1/(2*a*b*(p
+ 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1)*Simp[c*(2*b*e*(
p + 1) + (b*e - a*f)*(m + 1)) + d*(2*b*e*(p + 1) + (b*e - a*f)*(m + 2*q + 1
))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && LtQ[p, -1] && G
tQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e - a*f])
```

Maple [F]

$$\int \frac{(ex)^m (x^2B + A) (x^2d + c)}{(b\,x^2 + a)^3}dx$$

input
```
int((e*x)^m*(B*x^2+A)*(d*x^2+c)/(b*x^2+a)^3,x)
```

output
```
int((e*x)^m*(B*x^2+A)*(d*x^2+c)/(b*x^2+a)^3,x)
```

Fricas [F]

$$\int \frac{(ex)^m (A + Bx^2) (c + dx^2)}{(a + bx^2)^3}\,dx = \int \frac{(Bx^2 + A)(dx^2 + c)(ex)^m}{(bx^2 + a)^3}\,dx$$

input
```
integrate((e*x)^m*(B*x^2+A)*(d*x^2+c)/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
integral((B*d*x^4 + (B*c + A*d)*x^2 + A*c)*(e*x)^m/(b^3*x^6 + 3*a*b^2*x^4
+ 3*a^2*b*x^2 + a^3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 103.01 (sec) , antiderivative size = 6411, normalized size of antiderivative = 31.12

$$\int \frac{(ex)^m (A + Bx^2) (c + dx^2)}{(a + bx^2)^3} \, dx = \text{Too large to display}$$

input
```
integrate((e*x)**m*(B*x**2+A)*(d*x**2+c)/(b*x**2+a)**3,x)
```

output
```
A*c*(a**2*e**m*m**3*x**(m + 1)*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 +
 1/2)*gamma(m/2 + 1/2)/(32*a**5*gamma(m/2 + 3/2) + 64*a**4*b*x**2*gamma(m/
2 + 3/2) + 32*a**3*b**2*x**4*gamma(m/2 + 3/2)) - 3*a**2*e**m*m**2*x**(m +
1)*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(32*a
**5*gamma(m/2 + 3/2) + 64*a**4*b*x**2*gamma(m/2 + 3/2) + 32*a**3*b**2*x**4
*gamma(m/2 + 3/2)) - 2*a**2*e**m*m**2*x**(m + 1)*gamma(m/2 + 1/2)/(32*a**5
*gamma(m/2 + 3/2) + 64*a**4*b*x**2*gamma(m/2 + 3/2) + 32*a**3*b**2*x**4*ga
mma(m/2 + 3/2)) - a**2*e**m*m*x**(m + 1)*lerchphi(b*x**2*exp_polar(I*pi)/a
, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(32*a**5*gamma(m/2 + 3/2) + 64*a**4*b*x**
2*gamma(m/2 + 3/2) + 32*a**3*b**2*x**4*gamma(m/2 + 3/2)) + 8*a**2*e**m*m*x
**(m + 1)*gamma(m/2 + 1/2)/(32*a**5*gamma(m/2 + 3/2) + 64*a**4*b*x**2*gamm
a(m/2 + 3/2) + 32*a**3*b**2*x**4*gamma(m/2 + 3/2)) + 3*a**2*e**m*x**(m + 1
)*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(32*a*
*5*gamma(m/2 + 3/2) + 64*a**4*b*x**2*gamma(m/2 + 3/2) + 32*a**3*b**2*x**4*
gamma(m/2 + 3/2)) + 10*a**2*e**m*x**(m + 1)*gamma(m/2 + 1/2)/(32*a**5*gamm
a(m/2 + 3/2) + 64*a**4*b*x**2*gamma(m/2 + 3/2) + 32*a**3*b**2*x**4*gamma(m
/2 + 3/2)) + 2*a*b*e**m*m**3*x**2*x**(m + 1)*lerchphi(b*x**2*exp_polar(I*p
i)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(32*a**5*gamma(m/2 + 3/2) + 64*a**4*b
*x**2*gamma(m/2 + 3/2) + 32*a**3*b**2*x**4*gamma(m/2 + 3/2)) - 6*a*b*e**m*
m**2*x**2*x**(m + 1)*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*g...
```

Maxima [F]

$$\int \frac{(ex)^m (A+Bx^2) (c+dx^2)}{(a+bx^2)^3} \, dx = \int \frac{(Bx^2+A)(dx^2+c)(ex)^m}{(bx^2+a)^3} \, dx$$

input
```
integrate((e*x)^m*(B*x^2+A)*(d*x^2+c)/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*(d*x^2 + c)*(e*x)^m/(b*x^2 + a)^3, x)
```

Giac [F]

$$\int \frac{(ex)^m (A+Bx^2) (c+dx^2)}{(a+bx^2)^3} \, dx = \int \frac{(Bx^2+A)(dx^2+c)(ex)^m}{(bx^2+a)^3} \, dx$$

input
```
integrate((e*x)^m*(B*x^2+A)*(d*x^2+c)/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*(d*x^2 + c)*(e*x)^m/(b*x^2 + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m (A+Bx^2) (c+dx^2)}{(a+bx^2)^3} \, dx = \int \frac{(B\,x^2+A)\,(e\,x)^m\,(d\,x^2+c)}{(b\,x^2+a)^3} \, dx$$

input
```
int(((A + B*x^2)*(e*x)^m*(c + d*x^2))/(a + b*x^2)^3,x)
```

output
```
int(((A + B*x^2)*(e*x)^m*(c + d*x^2))/(a + b*x^2)^3, x)
```

Reduce [F]

$$\int \frac{(ex)^m (A+Bx^2)(c+dx^2)}{(a+bx^2)^3}\,dx$$
$$= \frac{e^m \left(x^m dx - \left(\int \frac{x^m}{b^2m\,x^4-b^2x^4+2abm\,x^2-2ab\,x^2+a^2m-a^2}dx\right) a^2d\,m^2 + \left(\int \frac{x^m}{b^2m\,x^4-b^2x^4+2abm\,x^2-2ab\,x^2+a^2m-a^2}dx\right) a^2d\right.}{}$$

input
```
int((e*x)^m*(B*x^2+A)*(d*x^2+c)/(b*x^2+a)^3,x)
```

output
```
(e**m*(x**m*d*x - int(x**m/(a**2*m - a**2 + 2*a*b*m*x**2 - 2*a*b*x**2 + b*
*2*m*x**4 - b**2*x**4),x)*a**2*d*m**2 + int(x**m/(a**2*m - a**2 + 2*a*b*m*
x**2 - 2*a*b*x**2 + b**2*m*x**4 - b**2*x**4),x)*a**2*d + int(x**m/(a**2*m
- a**2 + 2*a*b*m*x**2 - 2*a*b*x**2 + b**2*m*x**4 - b**2*x**4),x)*a*b*c*m**
2 - 2*int(x**m/(a**2*m - a**2 + 2*a*b*m*x**2 - 2*a*b*x**2 + b**2*m*x**4 -
b**2*x**4),x)*a*b*c*m + int(x**m/(a**2*m - a**2 + 2*a*b*m*x**2 - 2*a*b*x**
2 + b**2*m*x**4 - b**2*x**4),x)*a*b*c - int(x**m/(a**2*m - a**2 + 2*a*b*m*
x**2 - 2*a*b*x**2 + b**2*m*x**4 - b**2*x**4),x)*a*b*d*m**2*x**2 + int(x**m
/(a**2*m - a**2 + 2*a*b*m*x**2 - 2*a*b*x**2 + b**2*m*x**4 - b**2*x**4),x)*
a*b*d*x**2 + int(x**m/(a**2*m - a**2 + 2*a*b*m*x**2 - 2*a*b*x**2 + b**2*m*
x**4 - b**2*x**4),x)*b**2*c*m**2*x**2 - 2*int(x**m/(a**2*m - a**2 + 2*a*b*
m*x**2 - 2*a*b*x**2 + b**2*m*x**4 - b**2*x**4),x)*b**2*c*m*x**2 + int(x**m
/(a**2*m - a**2 + 2*a*b*m*x**2 - 2*a*b*x**2 + b**2*m*x**4 - b**2*x**4),x)*
b**2*c*x**2))/(b*(a*m - a + b*m*x**2 - b*x**2))
```

3.8 $\int (ex)^m \left(a+bx^2\right)^3 \left(A+Bx^2\right) \left(c+dx^2\right)^2 \, dx$

Optimal result

Integrand size = 31, antiderivative size = 292

$$\begin{aligned}\int (ex)^m \left(a+bx^2\right)^3 \left(A+Bx^2\right) \left(c+dx^2\right)^2 \, dx &= \frac{a^3Ac^2(ex)^{1+m}}{e(1+m)} + \frac{a^2c(3Abc+aBc+2aAd)(ex)^{3+m}}{e^3(3+m)} \\ &+ \frac{a\left(aBc(3bc+2ad)+A\left(3b^2c^2+6abcd+a^2d^2\right)\right)(ex)^{5+m}}{e^5(5+m)} \\ &+ \frac{\left(aB\left(3b^2c^2+6abcd+a^2d^2\right)+Ab\left(b^2c^2+6abcd+3a^2d^2\right)\right)(ex)^{7+m}}{e^7(7+m)} \\ &+ \frac{b\left(3a^2Bd^2+3abd(2Bc+Ad)+b^2c(Bc+2Ad)\right)(ex)^{9+m}}{e^9(9+m)} \\ &+ \frac{b^2d(2bBc+Abd+3aBd)(ex)^{11+m}}{e^{11}(11+m)} + \frac{b^3Bd^2(ex)^{13+m}}{e^{13}(13+m)}\end{aligned}$$

output
```
a^3*A*c^2*(e*x)^(1+m)/e/(1+m)+a^2*c*(2*A*a*d+3*A*b*c+B*a*c)*(e*x)^(3+m)/e^
3/(3+m)+a*(a*B*c*(2*a*d+3*b*c)+A*(a^2*d^2+6*a*b*c*d+3*b^2*c^2))*(e*x)^(5+m
)/e^5/(5+m)+(a*B*(a^2*d^2+6*a*b*c*d+3*b^2*c^2)+A*b*(3*a^2*d^2+6*a*b*c*d+b^
2*c^2))*(e*x)^(7+m)/e^7/(7+m)+b*(3*a^2*B*d^2+3*a*b*d*(A*d+2*B*c)+b^2*c*(2*
A*d+B*c))*(e*x)^(9+m)/e^9/(9+m)+b^2*d*(A*b*d+3*B*a*d+2*B*b*c)*(e*x)^(11+m)
/e^11/(11+m)+b^3*B*d^2*(e*x)^(13+m)/e^13/(13+m)
```

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.85

$$\int (ex)^m \left(a+bx^2\right)^3 \left(A+Bx^2\right) \left(c+dx^2\right)^2 \, dx = x(ex)^m \left(\frac{a^3Ac^2}{1+m}+\frac{a^2c(3Abc+aBc+2aAd)x^2}{3+m} + \frac{a(aBc(3bc+2ad)+A(3b^2c^2+6abcd+a^2d^2))\,x^4}{5+m} + \frac{(aB(3b^2c^2+6abcd+a^2d^2)+Ab(b^2c^2+6abcd+3a^2d^2))\,x^6}{7+m} + \frac{b(3a^2Bd^2+3abd(2Bc+Ad)+b^2c(Bc+2Ad))\,x^8}{9+m} + \frac{b^2d(2bBc+Abd+3aBd)x^{10}}{11+m}+\frac{b^3Bd^2x^{12}}{13+m}\right)$$

input
```
Integrate[(e*x)^m*(a + b*x^2)^3*(A + B*x^2)*(c + d*x^2)^2,x]
```

output
```
x*(e*x)^m*((a^3*A*c^2)/(1 + m) + (a^2*c*(3*A*b*c + a*B*c + 2*a*A*d)*x^2)/(
3 + m) + (a*(a*B*c*(3*b*c + 2*a*d) + A*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2))*
x^4)/(5 + m) + ((a*B*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2) + A*b*(b^2*c^2 + 6*
a*b*c*d + 3*a^2*d^2))*x^6)/(7 + m) + (b*(3*a^2*B*d^2 + 3*a*b*d*(2*B*c + A*
d) + b^2*c*(B*c + 2*A*d))*x^8)/(9 + m) + (b^2*d*(2*b*B*c + A*b*d + 3*a*B*d
)*x^10)/(11 + m) + (b^3*B*d^2*x^12)/(13 + m))
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {437, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^3 \left(A+Bx^2\right) \left(c+dx^2\right)^2 (ex)^m \, dx$$

$$\Big\downarrow \text{437}$$

$$\int \left(a^3Ac^2(ex)^m + \frac{(ex)^{m+6}\left(Ab(3a^2d^2+6abcd+b^2c^2)+aB(a^2d^2+6abcd+3b^2c^2)\right)}{e^6} + \frac{a(ex)^{m+4}\left(A(a^2d^2+6\right.}{}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3Ac^2(ex)^{m+1}}{e(m+1)} + \frac{(ex)^{m+7}\left(Ab(3a^2d^2+6abcd+b^2c^2)+aB(a^2d^2+6abcd+3b^2c^2)\right)}{e^7(m+7)} + \frac{a(ex)^{m+5}\left(A(a^2d^2+6abcd+3b^2c^2)+aBc(2ad+3bc)\right)}{e^5(m+5)} + \frac{b(ex)^{m+9}\left(3a^2Bd^2+3abd(Ad+2Bc)+b^2c(2Ad+Bc)\right)}{e^9(m+9)} + \frac{a^2c(ex)^{m+3}(2aAd+aBc+3Abc)}{e^3(m+3)} + \frac{b^2d(ex)^{m+11}(3aBd+Abd+2bBc)}{e^{11}(m+11)} + \frac{b^3Bd^2(ex)^{m+13}}{e^{13}(m+13)}$$

input
```
Int[(e*x)^m*(a + b*x^2)^3*(A + B*x^2)*(c + d*x^2)^2,x]
```

output
```
(a^3*A*c^2*(e*x)^(1 + m))/(e*(1 + m)) + (a^2*c*(3*A*b*c + a*B*c + 2*a*A*d)
*(e*x)^(3 + m))/(e^3*(3 + m)) + (a*(a*B*c*(3*b*c + 2*a*d) + A*(3*b^2*c^2 +
 6*a*b*c*d + a^2*d^2))*(e*x)^(5 + m))/(e^5*(5 + m)) + ((a*B*(3*b^2*c^2 + 6
*a*b*c*d + a^2*d^2) + A*b*(b^2*c^2 + 6*a*b*c*d + 3*a^2*d^2))*(e*x)^(7 + m)
)/(e^7*(7 + m)) + (b*(3*a^2*B*d^2 + 3*a*b*d*(2*B*c + A*d) + b^2*c*(B*c + 2
*A*d))*(e*x)^(9 + m))/(e^9*(9 + m)) + (b^2*d*(2*b*B*c + A*b*d + 3*a*B*d)*(
e*x)^(11 + m))/(e^11*(11 + m)) + (b^3*B*d^2*(e*x)^(13 + m))/(e^13*(13 + m)
)
```

Defintions of rubi rules used

rule 437
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(
a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f
, g, m}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2442 vs. $2(292) = 584$.

Time = 0.64 (sec) , antiderivative size = 2443, normalized size of antiderivative = 8.37

method	result	size
gosper	Expression too large to display	2443
risch	Expression too large to display	2443
orering	Expression too large to display	2443
parallelrisch	Expression too large to display	3284

input
```
int((e*x)^m*(b*x^2+a)^3*(B*x^2+A)*(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
x*(B*b^3*d^2*m^6*x^12+36*B*b^3*d^2*m^5*x^12+A*b^3*d^2*m^6*x^10+3*B*a*b^2*d
^2*m^6*x^10+2*B*b^3*c*d*m^6*x^10+505*B*b^3*d^2*m^4*x^12+38*A*b^3*d^2*m^5*x
^10+114*B*a*b^2*d^2*m^5*x^10+76*B*b^3*c*d*m^5*x^10+3480*B*b^3*d^2*m^3*x^12
+3*A*a*b^2*d^2*m^6*x^8+2*A*b^3*c*d*m^6*x^8+555*A*b^3*d^2*m^4*x^10+3*B*a^2*
b*d^2*m^6*x^8+6*B*a*b^2*c*d*m^6*x^8+1665*B*a*b^2*d^2*m^4*x^10+B*b^3*c^2*m^
6*x^8+1110*B*b^3*c*d*m^4*x^10+12139*B*b^3*d^2*m^2*x^12+120*A*a*b^2*d^2*m^5
*x^8+80*A*b^3*c*d*m^5*x^8+3940*A*b^3*d^2*m^3*x^10+120*B*a^2*b*d^2*m^5*x^8+
240*B*a*b^2*c*d*m^5*x^8+11820*B*a*b^2*d^2*m^3*x^10+40*B*b^3*c^2*m^5*x^8+78
80*B*b^3*c*d*m^3*x^10+19524*B*b^3*d^2*m*x^12+3*A*a^2*b*d^2*m^6*x^6+6*A*a*b
^2*c*d*m^6*x^6+1839*A*a*b^2*d^2*m^4*x^8+A*b^3*c^2*m^6*x^6+1226*A*b^3*c*d*m
^4*x^8+14039*A*b^3*d^2*m^2*x^10+B*a^3*d^2*m^6*x^6+6*B*a^2*b*c*d*m^6*x^6+18
39*B*a^2*b*d^2*m^4*x^8+3*B*a*b^2*c^2*m^6*x^6+3678*B*a*b^2*c*d*m^4*x^8+4211
7*B*a*b^2*d^2*m^2*x^10+613*B*b^3*c^2*m^4*x^8+28078*B*b^3*c*d*m^2*x^10+1039
5*B*b^3*d^2*x^12+126*A*a^2*b*d^2*m^5*x^6+252*A*a*b^2*c*d*m^5*x^6+13584*A*a
*b^2*d^2*m^3*x^8+42*A*b^3*c^2*m^5*x^6+9056*A*b^3*c*d*m^3*x^8+22902*A*b^3*d
^2*m*x^10+42*B*a^3*d^2*m^5*x^6+252*B*a^2*b*c*d*m^5*x^6+13584*B*a^2*b*d^2*m
^3*x^8+126*B*a*b^2*c^2*m^5*x^6+27168*B*a*b^2*c*d*m^3*x^8+68706*B*a*b^2*d^2
*m*x^10+4528*B*b^3*c^2*m^3*x^8+45804*B*b^3*c*d*m*x^10+A*a^3*d^2*m^6*x^4+6*
A*a^2*b*c*d*m^6*x^4+2037*A*a^2*b*d^2*m^4*x^6+3*A*a*b^2*c^2*m^6*x^4+4074*A*
a*b^2*c*d*m^4*x^6+49881*A*a*b^2*d^2*m^2*x^8+679*A*b^3*c^2*m^4*x^6+33254...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1711 vs. $2(292) = 584$.

Time = 0.12 (sec) , antiderivative size = 1711, normalized size of antiderivative = 5.86

$$\int (ex)^m (a+bx^2)^3 (A+Bx^2) (c+dx^2)^2 \, dx = \text{Too large to display}$$

input
```
integrate((e*x)^m*(b*x^2+a)^3*(B*x^2+A)*(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
((B*b^3*d^2*m^6 + 36*B*b^3*d^2*m^5 + 505*B*b^3*d^2*m^4 + 3480*B*b^3*d^2*m^
3 + 12139*B*b^3*d^2*m^2 + 19524*B*b^3*d^2*m + 10395*B*b^3*d^2)*x^13 + ((2*
B*b^3*c*d + (3*B*a*b^2 + A*b^3)*d^2)*m^6 + 24570*B*b^3*c*d + 38*(2*B*b^3*c
*d + (3*B*a*b^2 + A*b^3)*d^2)*m^5 + 555*(2*B*b^3*c*d + (3*B*a*b^2 + A*b^3)
*d^2)*m^4 + 3940*(2*B*b^3*c*d + (3*B*a*b^2 + A*b^3)*d^2)*m^3 + 12285*(3*B*
a*b^2 + A*b^3)*d^2 + 14039*(2*B*b^3*c*d + (3*B*a*b^2 + A*b^3)*d^2)*m^2 + 2
2902*(2*B*b^3*c*d + (3*B*a*b^2 + A*b^3)*d^2)*m)*x^11 + ((B*b^3*c^2 + 2*(3*
B*a*b^2 + A*b^3)*c*d + 3*(B*a^2*b + A*a*b^2)*d^2)*m^6 + 15015*B*b^3*c^2 +
40*(B*b^3*c^2 + 2*(3*B*a*b^2 + A*b^3)*c*d + 3*(B*a^2*b + A*a*b^2)*d^2)*m^5
 + 613*(B*b^3*c^2 + 2*(3*B*a*b^2 + A*b^3)*c*d + 3*(B*a^2*b + A*a*b^2)*d^2)
*m^4 + 4528*(B*b^3*c^2 + 2*(3*B*a*b^2 + A*b^3)*c*d + 3*(B*a^2*b + A*a*b^2)
*d^2)*m^3 + 30030*(3*B*a*b^2 + A*b^3)*c*d + 45045*(B*a^2*b + A*a*b^2)*d^2
+ 16627*(B*b^3*c^2 + 2*(3*B*a*b^2 + A*b^3)*c*d + 3*(B*a^2*b + A*a*b^2)*d^2
)*m^2 + 27688*(B*b^3*c^2 + 2*(3*B*a*b^2 + A*b^3)*c*d + 3*(B*a^2*b + A*a*b^
2)*d^2)*m)*x^9 + (((3*B*a*b^2 + A*b^3)*c^2 + 6*(B*a^2*b + A*a*b^2)*c*d + (
B*a^3 + 3*A*a^2*b)*d^2)*m^6 + 42*((3*B*a*b^2 + A*b^3)*c^2 + 6*(B*a^2*b + A
*a*b^2)*c*d + (B*a^3 + 3*A*a^2*b)*d^2)*m^5 + 679*((3*B*a*b^2 + A*b^3)*c^2
+ 6*(B*a^2*b + A*a*b^2)*c*d + (B*a^3 + 3*A*a^2*b)*d^2)*m^4 + 5292*((3*B*a*
b^2 + A*b^3)*c^2 + 6*(B*a^2*b + A*a*b^2)*c*d + (B*a^3 + 3*A*a^2*b)*d^2)*m^
3 + 19305*(3*B*a*b^2 + A*b^3)*c^2 + 115830*(B*a^2*b + A*a*b^2)*c*d + 19...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11914 vs. $2(294) = 588$.

Time = 1.35 (sec) , antiderivative size = 11914, normalized size of antiderivative = 40.80

$$\int (ex)^m \left(a+bx^2\right)^3 \left(A+Bx^2\right) \left(c+dx^2\right)^2 \, dx = \text{Too large to display}$$

input
```
integrate((e*x)**m*(b*x**2+a)**3*(B*x**2+A)*(d*x**2+c)**2,x)
```

output
```
Piecewise(((-A*a**3*c**2/(12*x**12) - A*a**3*c*d/(5*x**10) - A*a**3*d**2/(
8*x**8) - 3*A*a**2*b*c**2/(10*x**10) - 3*A*a**2*b*c*d/(4*x**8) - A*a**2*b*
d**2/(2*x**6) - 3*A*a*b**2*c**2/(8*x**8) - A*a*b**2*c*d/x**6 - 3*A*a*b**2*
d**2/(4*x**4) - A*b**3*c**2/(6*x**6) - A*b**3*c*d/(2*x**4) - A*b**3*d**2/(
2*x**2) - B*a**3*c**2/(10*x**10) - B*a**3*c*d/(4*x**8) - B*a**3*d**2/(6*x*
*6) - 3*B*a**2*b*c**2/(8*x**8) - B*a**2*b*c*d/x**6 - 3*B*a**2*b*d**2/(4*x*
*4) - B*a*b**2*c**2/(2*x**6) - 3*B*a*b**2*c*d/(2*x**4) - 3*B*a*b**2*d**2/(
2*x**2) - B*b**3*c**2/(4*x**4) - B*b**3*c*d/x**2 + B*b**3*d**2*log(x))/e**
13, Eq(m, -13)), ((-A*a**3*c**2/(10*x**10) - A*a**3*c*d/(4*x**8) - A*a**3*
d**2/(6*x**6) - 3*A*a**2*b*c**2/(8*x**8) - A*a**2*b*c*d/x**6 - 3*A*a**2*b*
d**2/(4*x**4) - A*a*b**2*c**2/(2*x**6) - 3*A*a*b**2*c*d/(2*x**4) - 3*A*a*b
**2*d**2/(2*x**2) - A*b**3*c**2/(4*x**4) - A*b**3*c*d/x**2 + A*b**3*d**2*l
og(x) - B*a**3*c**2/(8*x**8) - B*a**3*c*d/(3*x**6) - B*a**3*d**2/(4*x**4)
- B*a**2*b*c**2/(2*x**6) - 3*B*a**2*b*c*d/(2*x**4) - 3*B*a**2*b*d**2/(2*x*
*2) - 3*B*a*b**2*c**2/(4*x**4) - 3*B*a*b**2*c*d/x**2 + 3*B*a*b**2*d**2*log
(x) - B*b**3*c**2/(2*x**2) + 2*B*b**3*c*d*log(x) + B*b**3*d**2*x**2/2)/e**
11, Eq(m, -11)), ((-A*a**3*c**2/(8*x**8) - A*a**3*c*d/(3*x**6) - A*a**3*d*
*2/(4*x**4) - A*a**2*b*c**2/(2*x**6) - 3*A*a**2*b*c*d/(2*x**4) - 3*A*a**2*
b*d**2/(2*x**2) - 3*A*a*b**2*c**2/(4*x**4) - 3*A*a*b**2*c*d/x**2 + 3*A*a*b
**2*d**2*log(x) - A*b**3*c**2/(2*x**2) + 2*A*b**3*c*d*log(x) + A*b**3*d...
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 550, normalized size of antiderivative = 1.88

$$\int (ex)^m \left(a+bx^2\right)^3 \left(A+Bx^2\right) \left(c+dx^2\right)^2 \, dx = \frac{Bb^3d^2e^mx^{13}x^m}{m+13} + \frac{2\,Bb^3cde^mx^{11}x^m}{m+11} + \frac{3\,Bab^2d^2e^mx^{11}x^m}{m+11} + \frac{Ab^3d^2e^mx^{11}x^m}{m+11} + \frac{Bb^3c^2e^mx^9x^m}{m+9} + \frac{6\,Bab^2cde^mx^9x^m}{m+9} + \frac{2\,Ab^3cde^mx^9x^m}{m+9} + \frac{3\,Ba^2bd^2e^mx^9x^m}{m+9} + \frac{3\,Aab^2d^2e^mx^9x^m}{m+9} + \frac{3\,Bab^2c^2e^mx^7x^m}{m+7} + \frac{Ab^3c^2e^mx^7x^m}{m+7} + \frac{6\,Ba^2bcde^mx^7x^m}{m+7} + \frac{6\,Aab^2cde^mx^7x^m}{m+7} + \frac{Ba^3d^2e^mx^7x^m}{m+7} + \frac{3\,Aa^2bd^2e^mx^7x^m}{m+7} + \frac{3\,Ba^2bc^2e^mx^5x^m}{m+5} + \frac{3\,Aab^2c^2e^mx^5x^m}{m+5} + \frac{2\,Ba^3cde^mx^5x^m}{m+5} + \frac{6\,Aa^2bcde^mx^5x^m}{m+5} + \frac{Aa^3d^2e^mx^5x^m}{m+5} + \frac{Ba^3c^2e^mx^3x^m}{m+3} + \frac{3\,Aa^2bc^2e^mx^3x^m}{m+3} + \frac{2\,Aa^3cde^mx^3x^m}{m+3} + \frac{(ex)^{m+1}\,Aa^3c^2}{e(m+1)}$$

input
```
integrate((e*x)^m*(b*x^2+a)^3*(B*x^2+A)*(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
B*b^3*d^2*e^m*x^13*x^m/(m + 13) + 2*B*b^3*c*d*e^m*x^11*x^m/(m + 11) + 3*B*
a*b^2*d^2*e^m*x^11*x^m/(m + 11) + A*b^3*d^2*e^m*x^11*x^m/(m + 11) + B*b^3*
c^2*e^m*x^9*x^m/(m + 9) + 6*B*a*b^2*c*d*e^m*x^9*x^m/(m + 9) + 2*A*b^3*c*d*
e^m*x^9*x^m/(m + 9) + 3*B*a^2*b*d^2*e^m*x^9*x^m/(m + 9) + 3*A*a*b^2*d^2*e^
m*x^9*x^m/(m + 9) + 3*B*a*b^2*c^2*e^m*x^7*x^m/(m + 7) + A*b^3*c^2*e^m*x^7*
x^m/(m + 7) + 6*B*a^2*b*c*d*e^m*x^7*x^m/(m + 7) + 6*A*a*b^2*c*d*e^m*x^7*x^
m/(m + 7) + B*a^3*d^2*e^m*x^7*x^m/(m + 7) + 3*A*a^2*b*d^2*e^m*x^7*x^m/(m +
 7) + 3*B*a^2*b*c^2*e^m*x^5*x^m/(m + 5) + 3*A*a*b^2*c^2*e^m*x^5*x^m/(m + 5
) + 2*B*a^3*c*d*e^m*x^5*x^m/(m + 5) + 6*A*a^2*b*c*d*e^m*x^5*x^m/(m + 5) +
A*a^3*d^2*e^m*x^5*x^m/(m + 5) + B*a^3*c^2*e^m*x^3*x^m/(m + 3) + 3*A*a^2*b*
c^2*e^m*x^3*x^m/(m + 3) + 2*A*a^3*c*d*e^m*x^3*x^m/(m + 3) + (e*x)^(m + 1)*
A*a^3*c^2/(e*(m + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3283 vs. $2(292) = 584$.

Time = 0.18 (sec) , antiderivative size = 3283, normalized size of antiderivative = 11.24

$$\int (ex)^m \left(a+bx^2\right)^3 \left(A+Bx^2\right) \left(c+dx^2\right)^2 \, dx = \text{Too large to display}$$

input
```
integrate((e*x)^m*(b*x^2+a)^3*(B*x^2+A)*(d*x^2+c)^2,x, algorithm="giac")
```

output
```
((e*x)^m*B*b^3*d^2*m^6*x^13 + 36*(e*x)^m*B*b^3*d^2*m^5*x^13 + 2*(e*x)^m*B*
b^3*c*d*m^6*x^11 + 3*(e*x)^m*B*a*b^2*d^2*m^6*x^11 + (e*x)^m*A*b^3*d^2*m^6*
x^11 + 505*(e*x)^m*B*b^3*d^2*m^4*x^13 + 76*(e*x)^m*B*b^3*c*d*m^5*x^11 + 11
4*(e*x)^m*B*a*b^2*d^2*m^5*x^11 + 38*(e*x)^m*A*b^3*d^2*m^5*x^11 + 3480*(e*x
)^m*B*b^3*d^2*m^3*x^13 + (e*x)^m*B*b^3*c^2*m^6*x^9 + 6*(e*x)^m*B*a*b^2*c*d
*m^6*x^9 + 2*(e*x)^m*A*b^3*c*d*m^6*x^9 + 3*(e*x)^m*B*a^2*b*d^2*m^6*x^9 + 3
*(e*x)^m*A*a*b^2*d^2*m^6*x^9 + 1110*(e*x)^m*B*b^3*c*d*m^4*x^11 + 1665*(e*x
)^m*B*a*b^2*d^2*m^4*x^11 + 555*(e*x)^m*A*b^3*d^2*m^4*x^11 + 12139*(e*x)^m*
B*b^3*d^2*m^2*x^13 + 40*(e*x)^m*B*b^3*c^2*m^5*x^9 + 240*(e*x)^m*B*a*b^2*c*
d*m^5*x^9 + 80*(e*x)^m*A*b^3*c*d*m^5*x^9 + 120*(e*x)^m*B*a^2*b*d^2*m^5*x^9
 + 120*(e*x)^m*A*a*b^2*d^2*m^5*x^9 + 7880*(e*x)^m*B*b^3*c*d*m^3*x^11 + 118
20*(e*x)^m*B*a*b^2*d^2*m^3*x^11 + 3940*(e*x)^m*A*b^3*d^2*m^3*x^11 + 19524*
(e*x)^m*B*b^3*d^2*m*x^13 + 3*(e*x)^m*B*a*b^2*c^2*m^6*x^7 + (e*x)^m*A*b^3*c
^2*m^6*x^7 + 6*(e*x)^m*B*a^2*b*c*d*m^6*x^7 + 6*(e*x)^m*A*a*b^2*c*d*m^6*x^7
 + (e*x)^m*B*a^3*d^2*m^6*x^7 + 3*(e*x)^m*A*a^2*b*d^2*m^6*x^7 + 613*(e*x)^m
*B*b^3*c^2*m^4*x^9 + 3678*(e*x)^m*B*a*b^2*c*d*m^4*x^9 + 1226*(e*x)^m*A*b^3
*c*d*m^4*x^9 + 1839*(e*x)^m*B*a^2*b*d^2*m^4*x^9 + 1839*(e*x)^m*A*a*b^2*d^2
*m^4*x^9 + 28078*(e*x)^m*B*b^3*c*d*m^2*x^11 + 42117*(e*x)^m*B*a*b^2*d^2*m^
2*x^11 + 14039*(e*x)^m*A*b^3*d^2*m^2*x^11 + 10395*(e*x)^m*B*b^3*d^2*x^13 +
 126*(e*x)^m*B*a*b^2*c^2*m^5*x^7 + 42*(e*x)^m*A*b^3*c^2*m^5*x^7 + 252*(...
```

Mupad [B] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 694, normalized size of antiderivative = 2.38

$$\int (ex)^m \left(a+bx^2\right)^3 \left(A+Bx^2\right) \left(c+dx^2\right)^2 \, dx$$
$$= \frac{x^7\,(e\,x)^m\,(B\,a^3\,d^2+6\,B\,a^2\,b\,c\,d+3\,A\,a^2\,b\,d^2+3\,B\,a\,b^2\,c^2+6\,A\,a\,b^2\,c\,d+A\,b^3\,c^2)\,(m^6+42\,m^5+679\,m^4}{m^7+49\,m^6+973\,m^5+10045\,m^4+57379\,m^3+177331\,m^2+264207\,m}$$
$$+\frac{a\,x^5\,(e\,x)^m\,(2\,B\,a^2\,c\,d+A\,a^2\,d^2+3\,B\,a\,b\,c^2+6\,A\,a\,b\,c\,d+3\,A\,b^2\,c^2)\,(m^6+44\,m^5+753\,m^4+6280\,m^3}{m^7+49\,m^6+973\,m^5+10045\,m^4+57379\,m^3+177331\,m^2+264207\,m+}$$
$$+\frac{b\,x^9\,(e\,x)^m\,(3\,B\,a^2\,d^2+6\,B\,a\,b\,c\,d+3\,A\,a\,b\,d^2+B\,b^2\,c^2+2\,A\,b^2\,c\,d)\,(m^6+40\,m^5+613\,m^4+4528\,m^3}{m^7+49\,m^6+973\,m^5+10045\,m^4+57379\,m^3+177331\,m^2+264207\,m+}$$
$$+\frac{A\,a^3\,c^2\,x\,(e\,x)^m\,(m^6+48\,m^5+925\,m^4+9120\,m^3+48259\,m^2+129072\,m+135135)}{m^7+49\,m^6+973\,m^5+10045\,m^4+57379\,m^3+177331\,m^2+264207\,m+135135}$$
$$+\frac{a^2\,c\,x^3\,(e\,x)^m\,(2\,A\,a\,d+3\,A\,b\,c+B\,a\,c)\,(m^6+46\,m^5+835\,m^4+7540\,m^3+34759\,m^2+73054\,m+45}{m^7+49\,m^6+973\,m^5+10045\,m^4+57379\,m^3+177331\,m^2+264207\,m+135135}$$
$$+\frac{b^2\,d\,x^{11}\,(e\,x)^m\,(A\,b\,d+3\,B\,a\,d+2\,B\,b\,c)\,(m^6+38\,m^5+555\,m^4+3940\,m^3+14039\,m^2+22902\,m+1}{m^7+49\,m^6+973\,m^5+10045\,m^4+57379\,m^3+177331\,m^2+264207\,m+135135}$$
$$+\frac{B\,b^3\,d^2\,x^{13}\,(e\,x)^m\,(m^6+36\,m^5+505\,m^4+3480\,m^3+12139\,m^2+19524\,m+10395)}{m^7+49\,m^6+973\,m^5+10045\,m^4+57379\,m^3+177331\,m^2+264207\,m+135135}$$

input
```
int((A + B*x^2)*(e*x)^m*(a + b*x^2)^3*(c + d*x^2)^2,x)
```

output
```
(x^7*(e*x)^m*(A*b^3*c^2 + B*a^3*d^2 + 3*A*a^2*b*d^2 + 3*B*a*b^2*c^2 + 6*A*
a*b^2*c*d + 6*B*a^2*b*c*d)*(34986*m + 20335*m^2 + 5292*m^3 + 679*m^4 + 42*
m^5 + m^6 + 19305))/(264207*m + 177331*m^2 + 57379*m^3 + 10045*m^4 + 973*m
^5 + 49*m^6 + m^7 + 135135) + (a*x^5*(e*x)^m*(A*a^2*d^2 + 3*A*b^2*c^2 + 3*
B*a*b*c^2 + 2*B*a^2*c*d + 6*A*a*b*c*d)*(47436*m + 25979*m^2 + 6280*m^3 + 7
53*m^4 + 44*m^5 + m^6 + 27027))/(264207*m + 177331*m^2 + 57379*m^3 + 10045
*m^4 + 973*m^5 + 49*m^6 + m^7 + 135135) + (b*x^9*(e*x)^m*(3*B*a^2*d^2 + B*
b^2*c^2 + 3*A*a*b*d^2 + 2*A*b^2*c*d + 6*B*a*b*c*d)*(27688*m + 16627*m^2 +
4528*m^3 + 613*m^4 + 40*m^5 + m^6 + 15015))/(264207*m + 177331*m^2 + 57379
*m^3 + 10045*m^4 + 973*m^5 + 49*m^6 + m^7 + 135135) + (A*a^3*c^2*x*(e*x)^m
*(129072*m + 48259*m^2 + 9120*m^3 + 925*m^4 + 48*m^5 + m^6 + 135135))/(264
207*m + 177331*m^2 + 57379*m^3 + 10045*m^4 + 973*m^5 + 49*m^6 + m^7 + 1351
35) + (a^2*c*x^3*(e*x)^m*(2*A*a*d + 3*A*b*c + B*a*c)*(73054*m + 34759*m^2
+ 7540*m^3 + 835*m^4 + 46*m^5 + m^6 + 45045))/(264207*m + 177331*m^2 + 573
79*m^3 + 10045*m^4 + 973*m^5 + 49*m^6 + m^7 + 135135) + (b^2*d*x^11*(e*x)^
m*(A*b*d + 3*B*a*d + 2*B*b*c)*(22902*m + 14039*m^2 + 3940*m^3 + 555*m^4 +
38*m^5 + m^6 + 12285))/(264207*m + 177331*m^2 + 57379*m^3 + 10045*m^4 + 97
3*m^5 + 49*m^6 + m^7 + 135135) + (B*b^3*d^2*x^13*(e*x)^m*(19524*m + 12139*
m^2 + 3480*m^3 + 505*m^4 + 36*m^5 + m^6 + 10395))/(264207*m + 177331*m^2 +
 57379*m^3 + 10045*m^4 + 973*m^5 + 49*m^6 + m^7 + 135135)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 1484, normalized size of antiderivative = 5.08

$$\int (ex)^m \left(a+bx^2\right)^3 \left(A+Bx^2\right) \left(c+dx^2\right)^2 \, dx = \text{Too large to display}$$

input
```
int((e*x)^m*(b*x^2+a)^3*(B*x^2+A)*(d*x^2+c)^2,x)
```

output

```
(x**m*e**m*x*(a**4*c**2*m**6 + 48*a**4*c**2*m**5 + 925*a**4*c**2*m**4 + 91
20*a**4*c**2*m**3 + 48259*a**4*c**2*m**2 + 129072*a**4*c**2*m + 135135*a**
4*c**2 + 2*a**4*c*d*m**6*x**2 + 92*a**4*c*d*m**5*x**2 + 1670*a**4*c*d*m**4
*x**2 + 15080*a**4*c*d*m**3*x**2 + 69518*a**4*c*d*m**2*x**2 + 146108*a**4*
c*d*m*x**2 + 90090*a**4*c*d*x**2 + a**4*d**2*m**6*x**4 + 44*a**4*d**2*m**5
*x**4 + 753*a**4*d**2*m**4*x**4 + 6280*a**4*d**2*m**3*x**4 + 25979*a**4*d*
*2*m**2*x**4 + 47436*a**4*d**2*m*x**4 + 27027*a**4*d**2*x**4 + 4*a**3*b*c*
*2*m**6*x**2 + 184*a**3*b*c**2*m**5*x**2 + 3340*a**3*b*c**2*m**4*x**2 + 30
160*a**3*b*c**2*m**3*x**2 + 139036*a**3*b*c**2*m**2*x**2 + 292216*a**3*b*c
**2*m*x**2 + 180180*a**3*b*c**2*x**2 + 8*a**3*b*c*d*m**6*x**4 + 352*a**3*b
*c*d*m**5*x**4 + 6024*a**3*b*c*d*m**4*x**4 + 50240*a**3*b*c*d*m**3*x**4 +
207832*a**3*b*c*d*m**2*x**4 + 379488*a**3*b*c*d*m*x**4 + 216216*a**3*b*c*d
*x**4 + 4*a**3*b*d**2*m**6*x**6 + 168*a**3*b*d**2*m**5*x**6 + 2716*a**3*b*
d**2*m**4*x**6 + 21168*a**3*b*d**2*m**3*x**6 + 81340*a**3*b*d**2*m**2*x**6
 + 139944*a**3*b*d**2*m*x**6 + 77220*a**3*b*d**2*x**6 + 6*a**2*b**2*c**2*m
**6*x**4 + 264*a**2*b**2*c**2*m**5*x**4 + 4518*a**2*b**2*c**2*m**4*x**4 +
37680*a**2*b**2*c**2*m**3*x**4 + 155874*a**2*b**2*c**2*m**2*x**4 + 284616*
a**2*b**2*c**2*m*x**4 + 162162*a**2*b**2*c**2*x**4 + 12*a**2*b**2*c*d*m**6
*x**6 + 504*a**2*b**2*c*d*m**5*x**6 + 8148*a**2*b**2*c*d*m**4*x**6 + 63504
*a**2*b**2*c*d*m**3*x**6 + 244020*a**2*b**2*c*d*m**2*x**6 + 419832*a**2...
```

3.9 $\int (ex)^m (a+bx^2)^2 (A+Bx^2) (c+dx^2)^2 \, dx$

Optimal result

Integrand size = 31, antiderivative size = 216

$$\begin{aligned}\int (ex)^m \left(a+bx^2\right)^2 \left(A+Bx^2\right) \left(c+dx^2\right)^2 \, dx &= \frac{a^2Ac^2(ex)^{1+m}}{e(1+m)} + \frac{ac(aBc+2A(bc+ad))(ex)^{3+m}}{e^3(3+m)} \\ &+ \frac{(2aBc(bc+ad)+A(b^2c^2+4abcd+a^2d^2))\,(ex)^{5+m}}{e^5(5+m)} \\ &+ \frac{(a^2Bd^2+2abd(2Bc+Ad)+b^2c(Bc+2Ad))\,(ex)^{7+m}}{e^7(7+m)} \\ &+ \frac{bd(2bBc+Abd+2aBd)(ex)^{9+m}}{e^9(9+m)} + \frac{b^2Bd^2(ex)^{11+m}}{e^{11}(11+m)}\end{aligned}$$

output
```
a^2*A*c^2*(e*x)^(1+m)/e/(1+m)+a*c*(B*a*c+2*A*(a*d+b*c))*(e*x)^(3+m)/e^3/(3
+m)+(2*a*B*c*(a*d+b*c)+A*(a^2*d^2+4*a*b*c*d+b^2*c^2))*(e*x)^(5+m)/e^5/(5+m
)+(a^2*B*d^2+2*a*b*d*(A*d+2*B*c)+b^2*c*(2*A*d+B*c))*(e*x)^(7+m)/e^7/(7+m)+
b*d*(A*b*d+2*B*a*d+2*B*b*c)*(e*x)^(9+m)/e^9/(9+m)+b^2*B*d^2*(e*x)^(11+m)/e
^11/(11+m)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.82

$$\int (ex)^m \left(a+bx^2\right)^2 \left(A+Bx^2\right) \left(c+dx^2\right)^2 \, dx = x(ex)^m \left(\frac{a^2Ac^2}{1+m}+\frac{ac(aBc+2A(bc+ad))x^2}{3+m} + \frac{(2aBc(bc+ad)+A(b^2c^2+4abcd+a^2d^2))\,x^4}{5+m} + \frac{(a^2Bd^2+2abd(2Bc+Ad)+b^2c(Bc+2Ad))\,x^6}{7+m}+\frac{bd(2bBc+Abd+2aBd)x^8}{9+m} + \frac{b^2Bd^2x^{10}}{11+m}\right)$$

input
```
Integrate[(e*x)^m*(a + b*x^2)^2*(A + B*x^2)*(c + d*x^2)^2,x]
```

output
```
x*(e*x)^m*((a^2*A*c^2)/(1 + m) + (a*c*(a*B*c + 2*A*(b*c + a*d))*x^2)/(3 +
m) + ((2*a*B*c*(b*c + a*d) + A*(b^2*c^2 + 4*a*b*c*d + a^2*d^2))*x^4)/(5 +
m) + ((a^2*B*d^2 + 2*a*b*d*(2*B*c + A*d) + b^2*c*(B*c + 2*A*d))*x^6)/(7 +
m) + (b*d*(2*b*B*c + A*b*d + 2*a*B*d)*x^8)/(9 + m) + (b^2*B*d^2*x^10)/(11
+ m))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {437, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^2 \left(A+Bx^2\right) \left(c+dx^2\right)^2 (ex)^m \, dx$$

$$\Big\downarrow \text{ 437}$$

$$\int \left(\frac{(ex)^{m+4} \left(A\left(a^2d^2+4abcd+b^2c^2\right)+2aBc(ad+bc)\right)}{e^4} + \frac{(ex)^{m+6} \left(a^2Bd^2+2abd(Ad+2Bc)+b^2c(2Ad+B\right.}{e^6}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{(ex)^{m+5} \left(A\left(a^2d^2+4abcd+b^2c^2\right)+2aBc(ad+bc)\right)}{e^5(m+5)} + \frac{(ex)^{m+7} \left(a^2Bd^2+2abd(Ad+2Bc)+b^2c(2Ad+Bc)\right)}{e^7(m+7)} + \frac{a^2Ac^2(ex)^{m+1}}{e(m+1)} + \frac{bd(ex)^{m+9}(2aBd+Abd+2bBc)}{e^9(m+9)} + \frac{ac(ex)^{m+3}(2A(ad+bc)+aBc)}{e^3(m+3)} + \frac{b^2Bd^2(ex)^{m+11}}{e^{11}(m+11)}$$

input
```
Int[(e*x)^m*(a + b*x^2)^2*(A + B*x^2)*(c + d*x^2)^2,x]
```

output
```
(a^2*A*c^2*(e*x)^(1 + m))/(e*(1 + m)) + (a*c*(a*B*c + 2*A*(b*c + a*d))*(e*
x)^(3 + m))/(e^3*(3 + m)) + ((2*a*B*c*(b*c + a*d) + A*(b^2*c^2 + 4*a*b*c*d
 + a^2*d^2))*(e*x)^(5 + m))/(e^5*(5 + m)) + ((a^2*B*d^2 + 2*a*b*d*(2*B*c +
 A*d) + b^2*c*(B*c + 2*A*d))*(e*x)^(7 + m))/(e^7*(7 + m)) + (b*d*(2*b*B*c
+ A*b*d + 2*a*B*d)*(e*x)^(9 + m))/(e^9*(9 + m)) + (b^2*B*d^2*(e*x)^(11 + m
))/(e^11*(11 + m))
```

Defintions of rubi rules used

rule 437
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(
a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f
, g, m}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1470 vs. 2(216) = 432.

Time = 0.55 (sec) , antiderivative size = 1471, normalized size of antiderivative = 6.81

method	result	size
gosper	Expression too large to display	1471
risch	Expression too large to display	1471
orering	Expression too large to display	1471
parallelrisch	Expression too large to display	2011

input
```
int((e*x)^m*(b*x^2+a)^2*(B*x^2+A)*(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
x*(B*b^2*d^2*m^5*x^10+25*B*b^2*d^2*m^4*x^10+A*b^2*d^2*m^5*x^8+2*B*a*b*d^2*
m^5*x^8+2*B*b^2*c*d*m^5*x^8+230*B*b^2*d^2*m^3*x^10+27*A*b^2*d^2*m^4*x^8+54
*B*a*b*d^2*m^4*x^8+54*B*b^2*c*d*m^4*x^8+950*B*b^2*d^2*m^2*x^10+2*A*a*b*d^2
*m^5*x^6+2*A*b^2*c*d*m^5*x^6+262*A*b^2*d^2*m^3*x^8+B*a^2*d^2*m^5*x^6+4*B*a
*b*c*d*m^5*x^6+524*B*a*b*d^2*m^3*x^8+B*b^2*c^2*m^5*x^6+524*B*b^2*c*d*m^3*x
^8+1689*B*b^2*d^2*m*x^10+58*A*a*b*d^2*m^4*x^6+58*A*b^2*c*d*m^4*x^6+1122*A*
b^2*d^2*m^2*x^8+29*B*a^2*d^2*m^4*x^6+116*B*a*b*c*d*m^4*x^6+2244*B*a*b*d^2*
m^2*x^8+29*B*b^2*c^2*m^4*x^6+2244*B*b^2*c*d*m^2*x^8+945*B*b^2*d^2*x^10+A*a
^2*d^2*m^5*x^4+4*A*a*b*c*d*m^5*x^4+604*A*a*b*d^2*m^3*x^6+A*b^2*c^2*m^5*x^4
+604*A*b^2*c*d*m^3*x^6+2041*A*b^2*d^2*m*x^8+2*B*a^2*c*d*m^5*x^4+302*B*a^2*
d^2*m^3*x^6+2*B*a*b*c^2*m^5*x^4+1208*B*a*b*c*d*m^3*x^6+4082*B*a*b*d^2*m*x^
8+302*B*b^2*c^2*m^3*x^6+4082*B*b^2*c*d*m*x^8+31*A*a^2*d^2*m^4*x^4+124*A*a*
b*c*d*m^4*x^4+2732*A*a*b*d^2*m^2*x^6+31*A*b^2*c^2*m^4*x^4+2732*A*b^2*c*d*m
^2*x^6+1155*A*b^2*d^2*x^8+62*B*a^2*c*d*m^4*x^4+1366*B*a^2*d^2*m^2*x^6+62*B
*a*b*c^2*m^4*x^4+5464*B*a*b*c*d*m^2*x^6+2310*B*a*b*d^2*x^8+1366*B*b^2*c^2*
m^2*x^6+2310*B*b^2*c*d*x^8+2*A*a^2*c*d*m^5*x^2+350*A*a^2*d^2*m^3*x^4+2*A*a
*b*c^2*m^5*x^2+1400*A*a*b*c*d*m^3*x^4+5154*A*a*b*d^2*m*x^6+350*A*b^2*c^2*m
^3*x^4+5154*A*b^2*c*d*m*x^6+B*a^2*c^2*m^5*x^2+700*B*a^2*c*d*m^3*x^4+2577*B
*a^2*d^2*m*x^6+700*B*a*b*c^2*m^3*x^4+10308*B*a*b*c*d*m*x^6+2577*B*b^2*c^2*
m*x^6+66*A*a^2*c*d*m^4*x^2+1730*A*a^2*d^2*m^2*x^4+66*A*a*b*c^2*m^4*x^2+...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1043 vs. $2(216) = 432$.

Time = 0.10 (sec) , antiderivative size = 1043, normalized size of antiderivative = 4.83

$$\int (ex)^m \left(a + bx^2\right)^2 \left(A + Bx^2\right) \left(c + dx^2\right)^2 \, dx = \text{Too large to display}$$

input
```
integrate((e*x)^m*(b*x^2+a)^2*(B*x^2+A)*(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
((B*b^2*d^2*m^5 + 25*B*b^2*d^2*m^4 + 230*B*b^2*d^2*m^3 + 950*B*b^2*d^2*m^2
 + 1689*B*b^2*d^2*m + 945*B*b^2*d^2)*x^11 + ((2*B*b^2*c*d + (2*B*a*b + A*b
^2)*d^2)*m^5 + 2310*B*b^2*c*d + 27*(2*B*b^2*c*d + (2*B*a*b + A*b^2)*d^2)*m
^4 + 262*(2*B*b^2*c*d + (2*B*a*b + A*b^2)*d^2)*m^3 + 1155*(2*B*a*b + A*b^2
)*d^2 + 1122*(2*B*b^2*c*d + (2*B*a*b + A*b^2)*d^2)*m^2 + 2041*(2*B*b^2*c*d
 + (2*B*a*b + A*b^2)*d^2)*m)*x^9 + ((B*b^2*c^2 + 2*(2*B*a*b + A*b^2)*c*d +
 (B*a^2 + 2*A*a*b)*d^2)*m^5 + 1485*B*b^2*c^2 + 29*(B*b^2*c^2 + 2*(2*B*a*b
+ A*b^2)*c*d + (B*a^2 + 2*A*a*b)*d^2)*m^4 + 302*(B*b^2*c^2 + 2*(2*B*a*b +
A*b^2)*c*d + (B*a^2 + 2*A*a*b)*d^2)*m^3 + 2970*(2*B*a*b + A*b^2)*c*d + 148
5*(B*a^2 + 2*A*a*b)*d^2 + 1366*(B*b^2*c^2 + 2*(2*B*a*b + A*b^2)*c*d + (B*a
^2 + 2*A*a*b)*d^2)*m^2 + 2577*(B*b^2*c^2 + 2*(2*B*a*b + A*b^2)*c*d + (B*a^
2 + 2*A*a*b)*d^2)*m)*x^7 + ((A*a^2*d^2 + (2*B*a*b + A*b^2)*c^2 + 2*(B*a^2
+ 2*A*a*b)*c*d)*m^5 + 2079*A*a^2*d^2 + 31*(A*a^2*d^2 + (2*B*a*b + A*b^2)*c
^2 + 2*(B*a^2 + 2*A*a*b)*c*d)*m^4 + 350*(A*a^2*d^2 + (2*B*a*b + A*b^2)*c^2
 + 2*(B*a^2 + 2*A*a*b)*c*d)*m^3 + 2079*(2*B*a*b + A*b^2)*c^2 + 4158*(B*a^2
 + 2*A*a*b)*c*d + 1730*(A*a^2*d^2 + (2*B*a*b + A*b^2)*c^2 + 2*(B*a^2 + 2*A
*a*b)*c*d)*m^2 + 3489*(A*a^2*d^2 + (2*B*a*b + A*b^2)*c^2 + 2*(B*a^2 + 2*A*
a*b)*c*d)*m)*x^5 + ((2*A*a^2*c*d + (B*a^2 + 2*A*a*b)*c^2)*m^5 + 6930*A*a^2
*c*d + 33*(2*A*a^2*c*d + (B*a^2 + 2*A*a*b)*c^2)*m^4 + 406*(2*A*a^2*c*d + (
B*a^2 + 2*A*a*b)*c^2)*m^3 + 3465*(B*a^2 + 2*A*a*b)*c^2 + 2262*(2*A*a^2*...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6836 vs. $2(211) = 422$.

Time = 0.97 (sec) , antiderivative size = 6836, normalized size of antiderivative = 31.65

$$\int (ex)^m \left(a+bx^2\right)^2 \left(A+Bx^2\right) \left(c+dx^2\right)^2 \, dx = \text{Too large to display}$$

input
```
integrate((e*x)**m*(b*x**2+a)**2*(B*x**2+A)*(d*x**2+c)**2,x)
```

output
```
Piecewise(((-A*a**2*c**2/(10*x**10) - A*a**2*c*d/(4*x**8) - A*a**2*d**2/(6
*x**6) - A*a*b*c**2/(4*x**8) - 2*A*a*b*c*d/(3*x**6) - A*a*b*d**2/(2*x**4)
- A*b**2*c**2/(6*x**6) - A*b**2*c*d/(2*x**4) - A*b**2*d**2/(2*x**2) - B*a*
*2*c**2/(8*x**8) - B*a**2*c*d/(3*x**6) - B*a**2*d**2/(4*x**4) - B*a*b*c**2
/(3*x**6) - B*a*b*c*d/x**4 - B*a*b*d**2/x**2 - B*b**2*c**2/(4*x**4) - B*b*
*2*c*d/x**2 + B*b**2*d**2*log(x))/e**11, Eq(m, -11)), ((-A*a**2*c**2/(8*x*
*8) - A*a**2*c*d/(3*x**6) - A*a**2*d**2/(4*x**4) - A*a*b*c**2/(3*x**6) - A
*a*b*c*d/x**4 - A*a*b*d**2/x**2 - A*b**2*c**2/(4*x**4) - A*b**2*c*d/x**2 +
 A*b**2*d**2*log(x) - B*a**2*c**2/(6*x**6) - B*a**2*c*d/(2*x**4) - B*a**2*
d**2/(2*x**2) - B*a*b*c**2/(2*x**4) - 2*B*a*b*c*d/x**2 + 2*B*a*b*d**2*log(
x) - B*b**2*c**2/(2*x**2) + 2*B*b**2*c*d*log(x) + B*b**2*d**2*x**2/2)/e**9
, Eq(m, -9)), ((-A*a**2*c**2/(6*x**6) - A*a**2*c*d/(2*x**4) - A*a**2*d**2/
(2*x**2) - A*a*b*c**2/(2*x**4) - 2*A*a*b*c*d/x**2 + 2*A*a*b*d**2*log(x) -
A*b**2*c**2/(2*x**2) + 2*A*b**2*c*d*log(x) + A*b**2*d**2*x**2/2 - B*a**2*c
**2/(4*x**4) - B*a**2*c*d/x**2 + B*a**2*d**2*log(x) - B*a*b*c**2/x**2 + 4*
B*a*b*c*d*log(x) + B*a*b*d**2*x**2 + B*b**2*c**2*log(x) + B*b**2*c*d*x**2
+ B*b**2*d**2*x**4/4)/e**7, Eq(m, -7)), ((-A*a**2*c**2/(4*x**4) - A*a**2*c
*d/x**2 + A*a**2*d**2*log(x) - A*a*b*c**2/x**2 + 4*A*a*b*c*d*log(x) + A*a*
b*d**2*x**2 + A*b**2*c**2*log(x) + A*b**2*c*d*x**2 + A*b**2*d**2*x**4/4 -
B*a**2*c**2/(2*x**2) + 2*B*a**2*c*d*log(x) + B*a**2*d**2*x**2/2 + 2*B*a...
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.83

$$\int (ex)^m \left(a+bx^2\right)^2 \left(A+Bx^2\right) \left(c+dx^2\right)^2 dx = \frac{Bb^2d^2e^mx^{11}x^m}{m+11} + \frac{2\,Bb^2cde^mx^9x^m}{m+9} + \frac{2\,Babd^2e^mx^9x^m}{m+9} + \frac{Ab^2d^2e^mx^9x^m}{m+9} + \frac{Bb^2c^2e^mx^7x^m}{m+7} + \frac{4\,Babcde^mx^7x^m}{m+7} + \frac{2\,Ab^2cde^mx^7x^m}{m+7} + \frac{Ba^2d^2e^mx^7x^m}{m+7} + \frac{2\,Aabd^2e^mx^7x^m}{m+7} + \frac{2\,Babc^2e^mx^5x^m}{m+5} + \frac{Ab^2c^2e^mx^5x^m}{m+5} + \frac{2\,Ba^2cde^mx^5x^m}{m+5} + \frac{4\,Aabcde^mx^5x^m}{m+5} + \frac{Aa^2d^2e^mx^5x^m}{m+5} + \frac{Ba^2c^2e^mx^3x^m}{m+3} + \frac{2\,Aabc^2e^mx^3x^m}{m+3} + \frac{2\,Aa^2cde^mx^3x^m}{m+3} + \frac{(ex)^{m+1}\,Aa^2c^2}{e(m+1)}$$

input
```
integrate((e*x)^m*(b*x^2+a)^2*(B*x^2+A)*(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
B*b^2*d^2*e^m*x^11*x^m/(m + 11) + 2*B*b^2*c*d*e^m*x^9*x^m/(m + 9) + 2*B*a*
b*d^2*e^m*x^9*x^m/(m + 9) + A*b^2*d^2*e^m*x^9*x^m/(m + 9) + B*b^2*c^2*e^m*
x^7*x^m/(m + 7) + 4*B*a*b*c*d*e^m*x^7*x^m/(m + 7) + 2*A*b^2*c*d*e^m*x^7*x^
m/(m + 7) + B*a^2*d^2*e^m*x^7*x^m/(m + 7) + 2*A*a*b*d^2*e^m*x^7*x^m/(m + 7
) + 2*B*a*b*c^2*e^m*x^5*x^m/(m + 5) + A*b^2*c^2*e^m*x^5*x^m/(m + 5) + 2*B*
a^2*c*d*e^m*x^5*x^m/(m + 5) + 4*A*a*b*c*d*e^m*x^5*x^m/(m + 5) + A*a^2*d^2*
e^m*x^5*x^m/(m + 5) + B*a^2*c^2*e^m*x^3*x^m/(m + 3) + 2*A*a*b*c^2*e^m*x^3*
x^m/(m + 3) + 2*A*a^2*c*d*e^m*x^3*x^m/(m + 3) + (e*x)^(m + 1)*A*a^2*c^2/(e
*(m + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2010 vs. 2(216) = 432.

Time = 0.16 (sec) , antiderivative size = 2010, normalized size of antiderivative = 9.31

$$\int (ex)^m \left(a+bx^2\right)^2 \left(A+Bx^2\right) \left(c+dx^2\right)^2 dx = \text{Too large to display}$$

input
```
integrate((e*x)^m*(b*x^2+a)^2*(B*x^2+A)*(d*x^2+c)^2,x, algorithm="giac")
```

output
```
((e*x)^m*B*b^2*d^2*m^5*x^11 + 25*(e*x)^m*B*b^2*d^2*m^4*x^11 + 2*(e*x)^m*B*
b^2*c*d*m^5*x^9 + 2*(e*x)^m*B*a*b*d^2*m^5*x^9 + (e*x)^m*A*b^2*d^2*m^5*x^9
+ 230*(e*x)^m*B*b^2*d^2*m^3*x^11 + 54*(e*x)^m*B*b^2*c*d*m^4*x^9 + 54*(e*x)
^m*B*a*b*d^2*m^4*x^9 + 27*(e*x)^m*A*b^2*d^2*m^4*x^9 + 950*(e*x)^m*B*b^2*d^
2*m^2*x^11 + (e*x)^m*B*b^2*c^2*m^5*x^7 + 4*(e*x)^m*B*a*b*c*d*m^5*x^7 + 2*(
e*x)^m*A*b^2*c*d*m^5*x^7 + (e*x)^m*B*a^2*d^2*m^5*x^7 + 2*(e*x)^m*A*a*b*d^2
*m^5*x^7 + 524*(e*x)^m*B*b^2*c*d*m^3*x^9 + 524*(e*x)^m*B*a*b*d^2*m^3*x^9 +
 262*(e*x)^m*A*b^2*d^2*m^3*x^9 + 1689*(e*x)^m*B*b^2*d^2*m*x^11 + 29*(e*x)^
m*B*b^2*c^2*m^4*x^7 + 116*(e*x)^m*B*a*b*c*d*m^4*x^7 + 58*(e*x)^m*A*b^2*c*d
*m^4*x^7 + 29*(e*x)^m*B*a^2*d^2*m^4*x^7 + 58*(e*x)^m*A*a*b*d^2*m^4*x^7 + 2
244*(e*x)^m*B*b^2*c*d*m^2*x^9 + 2244*(e*x)^m*B*a*b*d^2*m^2*x^9 + 1122*(e*x
)^m*A*b^2*d^2*m^2*x^9 + 945*(e*x)^m*B*b^2*d^2*x^11 + 2*(e*x)^m*B*a*b*c^2*m
^5*x^5 + (e*x)^m*A*b^2*c^2*m^5*x^5 + 2*(e*x)^m*B*a^2*c*d*m^5*x^5 + 4*(e*x)
^m*A*a*b*c*d*m^5*x^5 + (e*x)^m*A*a^2*d^2*m^5*x^5 + 302*(e*x)^m*B*b^2*c^2*m
^3*x^7 + 1208*(e*x)^m*B*a*b*c*d*m^3*x^7 + 604*(e*x)^m*A*b^2*c*d*m^3*x^7 +
302*(e*x)^m*B*a^2*d^2*m^3*x^7 + 604*(e*x)^m*A*a*b*d^2*m^3*x^7 + 4082*(e*x)
^m*B*b^2*c*d*m*x^9 + 4082*(e*x)^m*B*a*b*d^2*m*x^9 + 2041*(e*x)^m*A*b^2*d^2
*m*x^9 + 62*(e*x)^m*B*a*b*c^2*m^4*x^5 + 31*(e*x)^m*A*b^2*c^2*m^4*x^5 + 62*
(e*x)^m*B*a^2*c*d*m^4*x^5 + 124*(e*x)^m*A*a*b*c*d*m^4*x^5 + 31*(e*x)^m*A*a
^2*d^2*m^4*x^5 + 1366*(e*x)^m*B*b^2*c^2*m^2*x^7 + 5464*(e*x)^m*B*a*b*c*...
```

Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 499, normalized size of antiderivative = 2.31

$$\int (ex)^m \left(a+bx^2\right)^2 \left(A+Bx^2\right) \left(c+dx^2\right)^2 \, dx$$
$$= \frac{x^5\,{\left(e\,x\right)}^m\,\left(2\,B\,a^2\,c\,d+A\,a^2\,d^2+2\,B\,a\,b\,c^2+4\,A\,a\,b\,c\,d+A\,b^2\,c^2\right)\,\left(m^5+31\,m^4+350\,m^3+1730\,m^2+34\right.}{m^6+36\,m^5+505\,m^4+3480\,m^3+12139\,m^2+19524\,m+10395}$$
$$+ \frac{x^7\,{\left(e\,x\right)}^m\,\left(B\,a^2\,d^2+4\,B\,a\,b\,c\,d+2\,A\,a\,b\,d^2+B\,b^2\,c^2+2\,A\,b^2\,c\,d\right)\,\left(m^5+29\,m^4+302\,m^3+1366\,m^2+\right.}{m^6+36\,m^5+505\,m^4+3480\,m^3+12139\,m^2+19524\,m+10395}$$
$$+ \frac{a\,c\,x^3\,{\left(e\,x\right)}^m\,\left(2\,A\,a\,d+2\,A\,b\,c+B\,a\,c\right)\,\left(m^5+33\,m^4+406\,m^3+2262\,m^2+5353\,m+3465\right)}{m^6+36\,m^5+505\,m^4+3480\,m^3+12139\,m^2+19524\,m+10395}$$
$$+ \frac{b\,d\,x^9\,{\left(e\,x\right)}^m\,\left(A\,b\,d+2\,B\,a\,d+2\,B\,b\,c\right)\,\left(m^5+27\,m^4+262\,m^3+1122\,m^2+2041\,m+1155\right)}{m^6+36\,m^5+505\,m^4+3480\,m^3+12139\,m^2+19524\,m+10395}$$
$$+ \frac{A\,a^2\,c^2\,x\,{\left(e\,x\right)}^m\,\left(m^5+35\,m^4+470\,m^3+3010\,m^2+9129\,m+10395\right)}{m^6+36\,m^5+505\,m^4+3480\,m^3+12139\,m^2+19524\,m+10395}$$
$$+ \frac{B\,b^2\,d^2\,x^{11}\,{\left(e\,x\right)}^m\,\left(m^5+25\,m^4+230\,m^3+950\,m^2+1689\,m+945\right)}{m^6+36\,m^5+505\,m^4+3480\,m^3+12139\,m^2+19524\,m+10395}$$

input
```
int((A + B*x^2)*(e*x)^m*(a + b*x^2)^2*(c + d*x^2)^2,x)
```

output
```
(x^5*(e*x)^m*(A*a^2*d^2 + A*b^2*c^2 + 2*B*a*b*c^2 + 2*B*a^2*c*d + 4*A*a*b*
c*d)*(3489*m + 1730*m^2 + 350*m^3 + 31*m^4 + m^5 + 2079))/(19524*m + 12139
*m^2 + 3480*m^3 + 505*m^4 + 36*m^5 + m^6 + 10395) + (x^7*(e*x)^m*(B*a^2*d^
2 + B*b^2*c^2 + 2*A*a*b*d^2 + 2*A*b^2*c*d + 4*B*a*b*c*d)*(2577*m + 1366*m^
2 + 302*m^3 + 29*m^4 + m^5 + 1485))/(19524*m + 12139*m^2 + 3480*m^3 + 505*
m^4 + 36*m^5 + m^6 + 10395) + (a*c*x^3*(e*x)^m*(2*A*a*d + 2*A*b*c + B*a*c)
*(5353*m + 2262*m^2 + 406*m^3 + 33*m^4 + m^5 + 3465))/(19524*m + 12139*m^2
 + 3480*m^3 + 505*m^4 + 36*m^5 + m^6 + 10395) + (b*d*x^9*(e*x)^m*(A*b*d +
2*B*a*d + 2*B*b*c)*(2041*m + 1122*m^2 + 262*m^3 + 27*m^4 + m^5 + 1155))/(1
9524*m + 12139*m^2 + 3480*m^3 + 505*m^4 + 36*m^5 + m^6 + 10395) + (A*a^2*c
^2*x*(e*x)^m*(9129*m + 3010*m^2 + 470*m^3 + 35*m^4 + m^5 + 10395))/(19524*
m + 12139*m^2 + 3480*m^3 + 505*m^4 + 36*m^5 + m^6 + 10395) + (B*b^2*d^2*x^
11*(e*x)^m*(1689*m + 950*m^2 + 230*m^3 + 25*m^4 + m^5 + 945))/(19524*m + 1
2139*m^2 + 3480*m^3 + 505*m^4 + 36*m^5 + m^6 + 10395)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 977, normalized size of antiderivative = 4.52

$$\int (ex)^m \left(a+bx^2\right)^2 \left(A+Bx^2\right) \left(c+dx^2\right)^2 \, dx = \text{Too large to display}$$

input
```
int((e*x)^m*(b*x^2+a)^2*(B*x^2+A)*(d*x^2+c)^2,x)
```

output

```
(x**m*e**m*x*(a**3*c**2*m**5 + 35*a**3*c**2*m**4 + 470*a**3*c**2*m**3 + 30
10*a**3*c**2*m**2 + 9129*a**3*c**2*m + 10395*a**3*c**2 + 2*a**3*c*d*m**5*x
**2 + 66*a**3*c*d*m**4*x**2 + 812*a**3*c*d*m**3*x**2 + 4524*a**3*c*d*m**2*
x**2 + 10706*a**3*c*d*m*x**2 + 6930*a**3*c*d*x**2 + a**3*d**2*m**5*x**4 +
31*a**3*d**2*m**4*x**4 + 350*a**3*d**2*m**3*x**4 + 1730*a**3*d**2*m**2*x**
4 + 3489*a**3*d**2*m*x**4 + 2079*a**3*d**2*x**4 + 3*a**2*b*c**2*m**5*x**2
+ 99*a**2*b*c**2*m**4*x**2 + 1218*a**2*b*c**2*m**3*x**2 + 6786*a**2*b*c**2
*m**2*x**2 + 16059*a**2*b*c**2*m*x**2 + 10395*a**2*b*c**2*x**2 + 6*a**2*b*
c*d*m**5*x**4 + 186*a**2*b*c*d*m**4*x**4 + 2100*a**2*b*c*d*m**3*x**4 + 103
80*a**2*b*c*d*m**2*x**4 + 20934*a**2*b*c*d*m*x**4 + 12474*a**2*b*c*d*x**4
+ 3*a**2*b*d**2*m**5*x**6 + 87*a**2*b*d**2*m**4*x**6 + 906*a**2*b*d**2*m**
3*x**6 + 4098*a**2*b*d**2*m**2*x**6 + 7731*a**2*b*d**2*m*x**6 + 4455*a**2*
b*d**2*x**6 + 3*a*b**2*c**2*m**5*x**4 + 93*a*b**2*c**2*m**4*x**4 + 1050*a*
b**2*c**2*m**3*x**4 + 5190*a*b**2*c**2*m**2*x**4 + 10467*a*b**2*c**2*m*x**
4 + 6237*a*b**2*c**2*x**4 + 6*a*b**2*c*d*m**5*x**6 + 174*a*b**2*c*d*m**4*x
**6 + 1812*a*b**2*c*d*m**3*x**6 + 8196*a*b**2*c*d*m**2*x**6 + 15462*a*b**2
*c*d*m*x**6 + 8910*a*b**2*c*d*x**6 + 3*a*b**2*d**2*m**5*x**8 + 81*a*b**2*d
**2*m**4*x**8 + 786*a*b**2*d**2*m**3*x**8 + 3366*a*b**2*d**2*m**2*x**8 + 6
123*a*b**2*d**2*m*x**8 + 3465*a*b**2*d**2*x**8 + b**3*c**2*m**5*x**6 + 29*
b**3*c**2*m**4*x**6 + 302*b**3*c**2*m**3*x**6 + 1366*b**3*c**2*m**2*x**...
```

3.10 $\int (ex)^m \left(a+bx^2\right) \left(A+Bx^2\right) \left(c+dx^2\right)^2 \, dx$

Optimal result

Integrand size = 29, antiderivative size = 144

$$\begin{aligned}
&\int (ex)^m \left(a+bx^2\right) \left(A+Bx^2\right) \left(c+dx^2\right)^2 \, dx \\
&= \frac{aAc^2(ex)^{1+m}}{e(1+m)} + \frac{c(Abc+aBc+2aAd)(ex)^{3+m}}{e^3(3+m)} \\
&\quad + \frac{(ad(2Bc+Ad)+bc(Bc+2Ad))(ex)^{5+m}}{e^5(5+m)} \\
&\quad + \frac{d(2bBc+Abd+aBd)(ex)^{7+m}}{e^7(7+m)} + \frac{bBd^2(ex)^{9+m}}{e^9(9+m)}
\end{aligned}$$

output
```
a*A*c^2*(e*x)^(1+m)/e/(1+m)+c*(2*A*a*d+A*b*c+B*a*c)*(e*x)^(3+m)/e^3/(3+m)+
(a*d*(A*d+2*B*c)+b*c*(2*A*d+B*c))*(e*x)^(5+m)/e^5/(5+m)+d*(A*b*d+B*a*d+2*B
*b*c)*(e*x)^(7+m)/e^7/(7+m)+b*B*d^2*(e*x)^(9+m)/e^9/(9+m)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.78

$$\int (ex)^m \left(a+bx^2\right) \left(A+Bx^2\right) \left(c+dx^2\right)^2 dx$$
$$= x(ex)^m \left(\frac{aAc^2}{1+m} + \frac{c(Abc+aBc+2aAd)x^2}{3+m} + \frac{(ad(2Bc+Ad)+bc(Bc+2Ad))x^4}{5+m} + \frac{d(2bBc+Abd+aBd)x^6}{7+m} + \frac{bBd^2x^8}{9+m}\right)$$

input
```
Integrate[(e*x)^m*(a + b*x^2)*(A + B*x^2)*(c + d*x^2)^2,x]
```

output
```
x*(e*x)^m*((a*A*c^2)/(1 + m) + (c*(A*b*c + a*B*c + 2*a*A*d)*x^2)/(3 + m) +
 ((a*d*(2*B*c + A*d) + b*c*(B*c + 2*A*d))*x^4)/(5 + m) + (d*(2*b*B*c + A*b
*d + a*B*d)*x^6)/(7 + m) + (b*B*d^2*x^8)/(9 + m))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {437, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right) \left(A+Bx^2\right) \left(c+dx^2\right)^2 (ex)^m dx$$

$$\Big\downarrow \text{ 437}$$

$$\int \left(\frac{d(ex)^{m+6}(aBd+Abd+2bBc)}{e^6} + \frac{(ex)^{m+4}(ad(Ad+2Bc)+bc(2Ad+Bc))}{e^4} + \frac{c(ex)^{m+2}(2aAd+aBc+Abc$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{d(ex)^{m+7}(aBd+Abd+2bBc)}{e^7(m+7)}+\frac{(ex)^{m+5}(ad(Ad+2Bc)+bc(2Ad+Bc))}{e^5(m+5)}+\frac{c(ex)^{m+3}(2aAd+aBc+Abc)}{e^3(m+3)}+\frac{aAc^2(ex)^{m+1}}{e(m+1)}+\frac{bBd^2(ex)^{m+9}}{e^9(m+9)}$$

input
```
Int[(e*x)^m*(a + b*x^2)*(A + B*x^2)*(c + d*x^2)^2,x]
```

output
```
(a*A*c^2*(e*x)^(1 + m))/(e*(1 + m)) + (c*(A*b*c + a*B*c + 2*a*A*d)*(e*x)^(
3 + m))/(e^3*(3 + m)) + ((a*d*(2*B*c + A*d) + b*c*(B*c + 2*A*d))*(e*x)^(5
+ m))/(e^5*(5 + m)) + (d*(2*b*B*c + A*b*d + a*B*d)*(e*x)^(7 + m))/(e^7*(7
+ m)) + (b*B*d^2*(e*x)^(9 + m))/(e^9*(9 + m))
```

Defintions of rubi rules used

rule 437
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(
a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f
, g, m}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 710 vs. $2(144) = 288$.

Time = 0.45 (sec) , antiderivative size = 711, normalized size of antiderivative = 4.94

method	result
gosper	$x\left(Bb\,d^2m^4x^8+16Bb\,d^2m^3x^8+Ab\,d^2m^4x^6+Ba\,d^2m^4x^6+2Bbcd\,m^4x^6+86Bb\,d^2m^2x^8+18Ab\,d^2m^3x^6+18Ba\,d^2m^3x^6+36Bb\right.$
risch	$x\left(Bb\,d^2m^4x^8+16Bb\,d^2m^3x^8+Ab\,d^2m^4x^6+Ba\,d^2m^4x^6+2Bbcd\,m^4x^6+86Bb\,d^2m^2x^8+18Ab\,d^2m^3x^6+18Ba\,d^2m^3x^6+36Bb\right.$
orering	$x\left(Bb\,d^2m^4x^8+16Bb\,d^2m^3x^8+Ab\,d^2m^4x^6+Ba\,d^2m^4x^6+2Bbcd\,m^4x^6+86Bb\,d^2m^2x^8+18Ab\,d^2m^3x^6+18Ba\,d^2m^3x^6+36Bb\right.$
parallelrisch	Expression too large to display

input
```
int((e*x)^m*(b*x^2+a)*(B*x^2+A)*(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
x*(B*b*d^2*m^4*x^8+16*B*b*d^2*m^3*x^8+A*b*d^2*m^4*x^6+B*a*d^2*m^4*x^6+2*B*
b*c*d*m^4*x^6+86*B*b*d^2*m^2*x^8+18*A*b*d^2*m^3*x^6+18*B*a*d^2*m^3*x^6+36*
B*b*c*d*m^3*x^6+176*B*b*d^2*m*x^8+A*a*d^2*m^4*x^4+2*A*b*c*d*m^4*x^4+104*A*
b*d^2*m^2*x^6+2*B*a*c*d*m^4*x^4+104*B*a*d^2*m^2*x^6+B*b*c^2*m^4*x^4+208*B*
b*c*d*m^2*x^6+105*B*b*d^2*x^8+20*A*a*d^2*m^3*x^4+40*A*b*c*d*m^3*x^4+222*A*
b*d^2*m*x^6+40*B*a*c*d*m^3*x^4+222*B*a*d^2*m*x^6+20*B*b*c^2*m^3*x^4+444*B*
b*c*d*m*x^6+2*A*a*c*d*m^4*x^2+130*A*a*d^2*m^2*x^4+A*b*c^2*m^4*x^2+260*A*b*
c*d*m^2*x^4+135*A*b*d^2*x^6+B*a*c^2*m^4*x^2+260*B*a*c*d*m^2*x^4+135*B*a*d^
2*x^6+130*B*b*c^2*m^2*x^4+270*B*b*c*d*x^6+44*A*a*c*d*m^3*x^2+300*A*a*d^2*m
*x^4+22*A*b*c^2*m^3*x^2+600*A*b*c*d*m*x^4+22*B*a*c^2*m^3*x^2+600*B*a*c*d*m
*x^4+300*B*b*c^2*m*x^4+A*a*c^2*m^4+328*A*a*c*d*m^2*x^2+189*A*a*d^2*x^4+164
*A*b*c^2*m^2*x^2+378*A*b*c*d*x^4+164*B*a*c^2*m^2*x^2+378*B*a*c*d*x^4+189*B
*b*c^2*x^4+24*A*a*c^2*m^3+916*A*a*c*d*m*x^2+458*A*b*c^2*m*x^2+458*B*a*c^2*
m*x^2+206*A*a*c^2*m^2+630*A*a*c*d*x^2+315*A*b*c^2*x^2+315*B*a*c^2*x^2+744*
A*a*c^2*m+945*A*a*c^2)*(e*x)^m/(9+m)/(7+m)/(5+m)/(3+m)/(1+m)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 495 vs. $2(144) = 288$.

Time = 0.09 (sec) , antiderivative size = 495, normalized size of antiderivative = 3.44

$$\int (ex)^m (a + bx^2) (A + Bx^2) (c + dx^2)^2 dx = \frac{((Bbd^2m^4 + 16\,Bbd^2m^3 + 86\,Bbd^2m^2 + 176\,Bbd^2m + 105\,Bbd^2)x^9 + ((2\,Bbcd + (Ba + Ab)d^2)m^4 + 27\ldots}{}$$

input
```
integrate((e*x)^m*(b*x^2+a)*(B*x^2+A)*(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
((B*b*d^2*m^4 + 16*B*b*d^2*m^3 + 86*B*b*d^2*m^2 + 176*B*b*d^2*m + 105*B*b*
d^2)*x^9 + ((2*B*b*c*d + (B*a + A*b)*d^2)*m^4 + 270*B*b*c*d + 18*(2*B*b*c*
d + (B*a + A*b)*d^2)*m^3 + 135*(B*a + A*b)*d^2 + 104*(2*B*b*c*d + (B*a + A
*b)*d^2)*m^2 + 222*(2*B*b*c*d + (B*a + A*b)*d^2)*m)*x^7 + ((B*b*c^2 + A*a*
d^2 + 2*(B*a + A*b)*c*d)*m^4 + 189*B*b*c^2 + 189*A*a*d^2 + 20*(B*b*c^2 + A
*a*d^2 + 2*(B*a + A*b)*c*d)*m^3 + 378*(B*a + A*b)*c*d + 130*(B*b*c^2 + A*a
*d^2 + 2*(B*a + A*b)*c*d)*m^2 + 300*(B*b*c^2 + A*a*d^2 + 2*(B*a + A*b)*c*d
)*m)*x^5 + ((2*A*a*c*d + (B*a + A*b)*c^2)*m^4 + 630*A*a*c*d + 22*(2*A*a*c*
d + (B*a + A*b)*c^2)*m^3 + 315*(B*a + A*b)*c^2 + 164*(2*A*a*c*d + (B*a + A
*b)*c^2)*m^2 + 458*(2*A*a*c*d + (B*a + A*b)*c^2)*m)*x^3 + (A*a*c^2*m^4 + 2
4*A*a*c^2*m^3 + 206*A*a*c^2*m^2 + 744*A*a*c^2*m + 945*A*a*c^2)*x)*(e*x)^m/
(m^5 + 25*m^4 + 230*m^3 + 950*m^2 + 1689*m + 945)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3271 vs. $2(139) = 278$.

Time = 0.71 (sec) , antiderivative size = 3271, normalized size of antiderivative = 22.72

$$\int (ex)^m \left(a+bx^2\right) \left(A+Bx^2\right) \left(c+dx^2\right)^2 dx = \text{Too large to display}$$

input
```
integrate((e*x)**m*(b*x**2+a)*(B*x**2+A)*(d*x**2+c)**2,x)
```

output
```
Piecewise(((-A*a*c**2/(8*x**8) - A*a*c*d/(3*x**6) - A*a*d**2/(4*x**4) - A*
b*c**2/(6*x**6) - A*b*c*d/(2*x**4) - A*b*d**2/(2*x**2) - B*a*c**2/(6*x**6)
 - B*a*c*d/(2*x**4) - B*a*d**2/(2*x**2) - B*b*c**2/(4*x**4) - B*b*c*d/x**2
 + B*b*d**2*log(x))/e**9, Eq(m, -9)), ((-A*a*c**2/(6*x**6) - A*a*c*d/(2*x*
*4) - A*a*d**2/(2*x**2) - A*b*c**2/(4*x**4) - A*b*c*d/x**2 + A*b*d**2*log(
x) - B*a*c**2/(4*x**4) - B*a*c*d/x**2 + B*a*d**2*log(x) - B*b*c**2/(2*x**2
) + 2*B*b*c*d*log(x) + B*b*d**2*x**2/2)/e**7, Eq(m, -7)), ((-A*a*c**2/(4*x
**4) - A*a*c*d/x**2 + A*a*d**2*log(x) - A*b*c**2/(2*x**2) + 2*A*b*c*d*log(
x) + A*b*d**2*x**2/2 - B*a*c**2/(2*x**2) + 2*B*a*c*d*log(x) + B*a*d**2*x**
2/2 + B*b*c**2*log(x) + B*b*c*d*x**2 + B*b*d**2*x**4/4)/e**5, Eq(m, -5)),
((-A*a*c**2/(2*x**2) + 2*A*a*c*d*log(x) + A*a*d**2*x**2/2 + A*b*c**2*log(x
) + A*b*c*d*x**2 + A*b*d**2*x**4/4 + B*a*c**2*log(x) + B*a*c*d*x**2 + B*a*
d**2*x**4/4 + B*b*c**2*x**2/2 + B*b*c*d*x**4/2 + B*b*d**2*x**6/6)/e**3, Eq
(m, -3)), ((A*a*c**2*log(x) + A*a*c*d*x**2 + A*a*d**2*x**4/4 + A*b*c**2*x*
*2/2 + A*b*c*d*x**4/2 + A*b*d**2*x**6/6 + B*a*c**2*x**2/2 + B*a*c*d*x**4/2
 + B*a*d**2*x**6/6 + B*b*c**2*x**4/4 + B*b*c*d*x**6/3 + B*b*d**2*x**8/8)/e
, Eq(m, -1)), (A*a*c**2*m**4*x*(e*x)**m/(m**5 + 25*m**4 + 230*m**3 + 950*m
**2 + 1689*m + 945) + 24*A*a*c**2*m**3*x*(e*x)**m/(m**5 + 25*m**4 + 230*m*
*3 + 950*m**2 + 1689*m + 945) + 206*A*a*c**2*m**2*x*(e*x)**m/(m**5 + 25*m*
*4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 744*A*a*c**2*m*x*(e*x)**m/(m...
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.68

$$\int (ex)^m (a+bx^2) (A+Bx^2) (c+dx^2)^2 dx = \frac{Bbd^2e^mx^9x^m}{m+9} + \frac{2\,Bbcde^mx^7x^m}{m+7} + \frac{Bad^2e^mx^7x^m}{m+7} + \frac{Abd^2e^mx^7x^m}{m+7} + \frac{Bbc^2e^mx^5x^m}{m+5} + \frac{2\,Bacde^mx^5x^m}{m+5} + \frac{2\,Abcde^mx^5x^m}{m+5} + \frac{Aad^2e^mx^5x^m}{m+5} + \frac{Bac^2e^mx^3x^m}{m+3} + \frac{Abc^2e^mx^3x^m}{m+3} + \frac{2\,Aacde^mx^3x^m}{m+3} + \frac{(ex)^{m+1}Aac^2}{e(m+1)}$$

input
```
integrate((e*x)^m*(b*x^2+a)*(B*x^2+A)*(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
B*b*d^2*e^m*x^9*x^m/(m + 9) + 2*B*b*c*d*e^m*x^7*x^m/(m + 7) + B*a*d^2*e^m*
x^7*x^m/(m + 7) + A*b*d^2*e^m*x^7*x^m/(m + 7) + B*b*c^2*e^m*x^5*x^m/(m + 5
) + 2*B*a*c*d*e^m*x^5*x^m/(m + 5) + 2*A*b*c*d*e^m*x^5*x^m/(m + 5) + A*a*d^
2*e^m*x^5*x^m/(m + 5) + B*a*c^2*e^m*x^3*x^m/(m + 3) + A*b*c^2*e^m*x^3*x^m/
(m + 3) + 2*A*a*c*d*e^m*x^3*x^m/(m + 3) + (e*x)^(m + 1)*A*a*c^2/(e*(m + 1)
)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1009 vs. $2(144) = 288$.

Time = 0.14 (sec) , antiderivative size = 1009, normalized size of antiderivative = 7.01

$$\int (ex)^m (a + bx^2) (A + Bx^2) (c + dx^2)^2 \, dx = \text{Too large to display}$$

input
```
integrate((e*x)^m*(b*x^2+a)*(B*x^2+A)*(d*x^2+c)^2,x, algorithm="giac")
```

output
```
((e*x)^m*B*b*d^2*m^4*x^9 + 16*(e*x)^m*B*b*d^2*m^3*x^9 + 2*(e*x)^m*B*b*c*d*
m^4*x^7 + (e*x)^m*B*a*d^2*m^4*x^7 + (e*x)^m*A*b*d^2*m^4*x^7 + 86*(e*x)^m*B
*b*d^2*m^2*x^9 + 36*(e*x)^m*B*b*c*d*m^3*x^7 + 18*(e*x)^m*B*a*d^2*m^3*x^7 +
 18*(e*x)^m*A*b*d^2*m^3*x^7 + 176*(e*x)^m*B*b*d^2*m*x^9 + (e*x)^m*B*b*c^2*
m^4*x^5 + 2*(e*x)^m*B*a*c*d*m^4*x^5 + 2*(e*x)^m*A*b*c*d*m^4*x^5 + (e*x)^m*
A*a*d^2*m^4*x^5 + 208*(e*x)^m*B*b*c*d*m^2*x^7 + 104*(e*x)^m*B*a*d^2*m^2*x^
7 + 104*(e*x)^m*A*b*d^2*m^2*x^7 + 105*(e*x)^m*B*b*d^2*x^9 + 20*(e*x)^m*B*b
*c^2*m^3*x^5 + 40*(e*x)^m*B*a*c*d*m^3*x^5 + 40*(e*x)^m*A*b*c*d*m^3*x^5 + 2
0*(e*x)^m*A*a*d^2*m^3*x^5 + 444*(e*x)^m*B*b*c*d*m*x^7 + 222*(e*x)^m*B*a*d^
2*m*x^7 + 222*(e*x)^m*A*b*d^2*m*x^7 + (e*x)^m*B*a*c^2*m^4*x^3 + (e*x)^m*A*
b*c^2*m^4*x^3 + 2*(e*x)^m*A*a*c*d*m^4*x^3 + 130*(e*x)^m*B*b*c^2*m^2*x^5 +
260*(e*x)^m*B*a*c*d*m^2*x^5 + 260*(e*x)^m*A*b*c*d*m^2*x^5 + 130*(e*x)^m*A*
a*d^2*m^2*x^5 + 270*(e*x)^m*B*b*c*d*x^7 + 135*(e*x)^m*B*a*d^2*x^7 + 135*(e
*x)^m*A*b*d^2*x^7 + 22*(e*x)^m*B*a*c^2*m^3*x^3 + 22*(e*x)^m*A*b*c^2*m^3*x^
3 + 44*(e*x)^m*A*a*c*d*m^3*x^3 + 300*(e*x)^m*B*b*c^2*m*x^5 + 600*(e*x)^m*B
*a*c*d*m*x^5 + 600*(e*x)^m*A*b*c*d*m*x^5 + 300*(e*x)^m*A*a*d^2*m*x^5 + (e*
x)^m*A*a*c^2*m^4*x + 164*(e*x)^m*B*a*c^2*m^2*x^3 + 164*(e*x)^m*A*b*c^2*m^2
*x^3 + 328*(e*x)^m*A*a*c*d*m^2*x^3 + 189*(e*x)^m*B*b*c^2*x^5 + 378*(e*x)^m
*B*a*c*d*x^5 + 378*(e*x)^m*A*b*c*d*x^5 + 189*(e*x)^m*A*a*d^2*x^5 + 24*(e*x
)^m*A*a*c^2*m^3*x + 458*(e*x)^m*B*a*c^2*m*x^3 + 458*(e*x)^m*A*b*c^2*m*x...
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 305, normalized size of antiderivative = 2.12

$$\int (ex)^m \left(a+bx^2\right) \left(A+Bx^2\right) \left(c+dx^2\right)^2 \, dx = (e\,x)^m \left(\frac{x^5\,(A\,a\,d^2+B\,b\,c^2+2\,A\,b\,c\,d+2\,B\,a\,c\,d)\,(m^4+20\,m^3+130\,m^2+300\,m+189)}{m^5+25\,m^4+230\,m^3+950\,m^2+1689\,m+945} + \frac{c\,x^3\,(2\,A\,a\,d+A\,b\,c+B\,a\,c)\,(m^4+22\,m^3+164\,m^2+458\,m+315)}{m^5+25\,m^4+230\,m^3+950\,m^2+1689\,m+945} + \frac{d\,x^7\,(A\,b\,d+B\,a\,d+2\,B\,b\,c)\,(m^4+18\,m^3+104\,m^2+222\,m+135)}{m^5+25\,m^4+230\,m^3+950\,m^2+1689\,m+945} + \frac{A\,a\,c^2\,x\,(m^4+24\,m^3+206\,m^2+744\,m+945)}{m^5+25\,m^4+230\,m^3+950\,m^2+1689\,m+945} + \frac{B\,b\,d^2\,x^9\,(m^4+16\,m^3+86\,m^2+176\,m+105)}{m^5+25\,m^4+230\,m^3+950\,m^2+1689\,m+945}\right)$$

input
```
int((A + B*x^2)*(e*x)^m*(a + b*x^2)*(c + d*x^2)^2,x)
```

output
```
(e*x)^m*((x^5*(A*a*d^2 + B*b*c^2 + 2*A*b*c*d + 2*B*a*c*d)*(300*m + 130*m^2
 + 20*m^3 + m^4 + 189))/(1689*m + 950*m^2 + 230*m^3 + 25*m^4 + m^5 + 945)
+ (c*x^3*(2*A*a*d + A*b*c + B*a*c)*(458*m + 164*m^2 + 22*m^3 + m^4 + 315))
/(1689*m + 950*m^2 + 230*m^3 + 25*m^4 + m^5 + 945) + (d*x^7*(A*b*d + B*a*d
 + 2*B*b*c)*(222*m + 104*m^2 + 18*m^3 + m^4 + 135))/(1689*m + 950*m^2 + 23
0*m^3 + 25*m^4 + m^5 + 945) + (A*a*c^2*x*(744*m + 206*m^2 + 24*m^3 + m^4 +
 945))/(1689*m + 950*m^2 + 230*m^3 + 25*m^4 + m^5 + 945) + (B*b*d^2*x^9*(1
76*m + 86*m^2 + 16*m^3 + m^4 + 105))/(1689*m + 950*m^2 + 230*m^3 + 25*m^4
+ m^5 + 945))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 570, normalized size of antiderivative = 3.96

$$\int (ex)^m \left(a+bx^2\right) \left(A+Bx^2\right) \left(c+dx^2\right)^2 \, dx = \frac{x^m e^m x(b^2 d^2 m^4 x^8 + 16b^2 d^2 m^3 x^8 + 2ab\,d^2 m^4 x^6 + 2b^2 cd\,m^4 x^6 + 86b^2 d^2 m^2 x^8 + 36ab\,d^2 m^3 x^6 + 36b^2 cd\,m^3}{}$$

input
```
int((e*x)^m*(b*x^2+a)*(B*x^2+A)*(d*x^2+c)^2,x)
```

output
```
(x**m*e**m*x*(a**2*c**2*m**4 + 24*a**2*c**2*m**3 + 206*a**2*c**2*m**2 + 74
4*a**2*c**2*m + 945*a**2*c**2 + 2*a**2*c*d*m**4*x**2 + 44*a**2*c*d*m**3*x*
*2 + 328*a**2*c*d*m**2*x**2 + 916*a**2*c*d*m*x**2 + 630*a**2*c*d*x**2 + a*
*2*d**2*m**4*x**4 + 20*a**2*d**2*m**3*x**4 + 130*a**2*d**2*m**2*x**4 + 300
*a**2*d**2*m*x**4 + 189*a**2*d**2*x**4 + 2*a*b*c**2*m**4*x**2 + 44*a*b*c**
2*m**3*x**2 + 328*a*b*c**2*m**2*x**2 + 916*a*b*c**2*m*x**2 + 630*a*b*c**2*
x**2 + 4*a*b*c*d*m**4*x**4 + 80*a*b*c*d*m**3*x**4 + 520*a*b*c*d*m**2*x**4
+ 1200*a*b*c*d*m*x**4 + 756*a*b*c*d*x**4 + 2*a*b*d**2*m**4*x**6 + 36*a*b*d
**2*m**3*x**6 + 208*a*b*d**2*m**2*x**6 + 444*a*b*d**2*m*x**6 + 270*a*b*d**
2*x**6 + b**2*c**2*m**4*x**4 + 20*b**2*c**2*m**3*x**4 + 130*b**2*c**2*m**2
*x**4 + 300*b**2*c**2*m*x**4 + 189*b**2*c**2*x**4 + 2*b**2*c*d*m**4*x**6 +
 36*b**2*c*d*m**3*x**6 + 208*b**2*c*d*m**2*x**6 + 444*b**2*c*d*m*x**6 + 27
0*b**2*c*d*x**6 + b**2*d**2*m**4*x**8 + 16*b**2*d**2*m**3*x**8 + 86*b**2*d
**2*m**2*x**8 + 176*b**2*d**2*m*x**8 + 105*b**2*d**2*x**8))/(m**5 + 25*m**
4 + 230*m**3 + 950*m**2 + 1689*m + 945)
```

3.11 $\int (ex)^m (A+Bx^2) (c+dx^2)^2 \, dx$

Optimal result

Integrand size = 22, antiderivative size = 91

$$\int (ex)^m (A+Bx^2) (c+dx^2)^2 \, dx = \frac{Ac^2(ex)^{1+m}}{e(1+m)} + \frac{c(Bc+2Ad)(ex)^{3+m}}{e^3(3+m)} + \frac{d(2Bc+Ad)(ex)^{5+m}}{e^5(5+m)} + \frac{Bd^2(ex)^{7+m}}{e^7(7+m)}$$

output
```
A*c^2*(e*x)^(1+m)/e/(1+m)+c*(2*A*d+B*c)*(e*x)^(3+m)/e^3/(3+m)+d*(A*d+2*B*c
)*(e*x)^(5+m)/e^5/(5+m)+B*d^2*(e*x)^(7+m)/e^7/(7+m)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.74

$$\int (ex)^m (A+Bx^2) (c+dx^2)^2 \, dx = x(ex)^m \left(\frac{Ac^2}{1+m} + \frac{c(Bc+2Ad)x^2}{3+m} + \frac{d(2Bc+Ad)x^4}{5+m} + \frac{Bd^2x^6}{7+m}\right)$$

input
```
Integrate[(e*x)^m*(A + B*x^2)*(c + d*x^2)^2,x]
```

output
```
x*(e*x)^m*((A*c^2)/(1 + m) + (c*(B*c + 2*A*d)*x^2)/(3 + m) + (d*(2*B*c + A
*d)*x^4)/(5 + m) + (B*d^2*x^6)/(7 + m))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (A+Bx^2)\left(c+dx^2\right)^2(ex)^m\,dx$$

$$\Big\downarrow \text{355}$$

$$\int \left(\frac{d(ex)^{m+4}(Ad+2Bc)}{e^4}+\frac{c(ex)^{m+2}(2Ad+Bc)}{e^2}+Ac^2(ex)^m+\frac{Bd^2(ex)^{m+6}}{e^6}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{d(ex)^{m+5}(Ad+2Bc)}{e^5(m+5)}+\frac{c(ex)^{m+3}(2Ad+Bc)}{e^3(m+3)}+\frac{Ac^2(ex)^{m+1}}{e(m+1)}+\frac{Bd^2(ex)^{m+7}}{e^7(m+7)}$$

input
```
Int[(e*x)^m*(A + B*x^2)*(c + d*x^2)^2,x]
```

output
```
(A*c^2*(e*x)^(1 + m))/(e*(1 + m)) + (c*(B*c + 2*A*d)*(e*x)^(3 + m))/(e^3*(
3 + m)) + (d*(2*B*c + A*d)*(e*x)^(5 + m))/(e^5*(5 + m)) + (B*d^2*(e*x)^(7
+ m))/(e^7*(7 + m))
```

Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.99

method	result
norman	$\frac{A\,c^2 x\,\mathrm{e}^{m\ln(ex)}}{1+m}+\frac{B\,d^2 x^7 \mathrm{e}^{m\ln(ex)}}{7+m}+\frac{c(2Ad+Bc)x^3\mathrm{e}^{m\ln(ex)}}{3+m}+\frac{d(Ad+2Bc)x^5\mathrm{e}^{m\ln(ex)}}{5+m}$
gosper	$x(B\,d^2m^3x^6+9B\,d^2m^2x^6+A\,d^2m^3x^4+2Bcd\,m^3x^4+23m\,x^6B\,d^2+11A\,d^2m^2x^4+22Bcd\,m^2x^4+15B\,d^2x^6+2Acd\,m^3x^2+31A$
risch	$x(B\,d^2m^3x^6+9B\,d^2m^2x^6+A\,d^2m^3x^4+2Bcd\,m^3x^4+23m\,x^6B\,d^2+11A\,d^2m^2x^4+22Bcd\,m^2x^4+15B\,d^2x^6+2Acd\,m^3x^2+31A$
orering	$x(B\,d^2m^3x^6+9B\,d^2m^2x^6+A\,d^2m^3x^4+2Bcd\,m^3x^4+23m\,x^6B\,d^2+11A\,d^2m^2x^4+22Bcd\,m^2x^4+15B\,d^2x^6+2Acd\,m^3x^2+31A$
parallelrisch	$62B\,x^5(ex)^m cdm+26A\,x^3(ex)^m cd\,m^2+94A\,x^3(ex)^m cdm+22B\,x^5(ex)^m cd\,m^2+2A\,x^3(ex)^m cd\,m^3+11A\,x^5(ex)^m d^2m^2+31$

input
```
int((e*x)^m*(B*x^2+A)*(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
A*c^2/(1+m)*x*exp(m*ln(e*x))+B*d^2/(7+m)*x^7*exp(m*ln(e*x))+c*(2*A*d+B*c)/
(3+m)*x^3*exp(m*ln(e*x))+d*(A*d+2*B*c)/(5+m)*x^5*exp(m*ln(e*x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 217 vs. $2(91) = 182$.

Time = 0.08 (sec) , antiderivative size = 217, normalized size of antiderivative = 2.38

$$\int (ex)^m (A+Bx^2) (c+dx^2)^2 \, dx = \frac{((Bd^2m^3 + 9\,Bd^2m^2 + 23\,Bd^2m + 15\,Bd^2)x^7 + ((2\,Bcd + Ad^2)m^3 + 42\,Bcd + 21\,Ad^2 + 11\,(2\,Bcd + Ad^2)m^2 + 31\,(2\,Bcd + Ad^2)m)x^5 + ((Bc^2 + 2\,Acd)m^3 + 35\,Bc^2 + 70\,Acd + 13\,(Bc^2 + 2\,Acd)m^2 + 47\,(Bc^2 + 2\,Acd)m)x^3 + (Ac^2m^3 + 15\,Ac^2m^2 + 71\,Ac^2m + 105\,Ac^2)x)(ex)^m}{m^4 + 16\,m^3 + 86\,m^2 + 176\,m + 105}$$

input
```
integrate((e*x)^m*(B*x^2+A)*(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
((B*d^2*m^3 + 9*B*d^2*m^2 + 23*B*d^2*m + 15*B*d^2)*x^7 + ((2*B*c*d + A*d^2
)*m^3 + 42*B*c*d + 21*A*d^2 + 11*(2*B*c*d + A*d^2)*m^2 + 31*(2*B*c*d + A*d
^2)*m)*x^5 + ((B*c^2 + 2*A*c*d)*m^3 + 35*B*c^2 + 70*A*c*d + 13*(B*c^2 + 2*
A*c*d)*m^2 + 47*(B*c^2 + 2*A*c*d)*m)*x^3 + (A*c^2*m^3 + 15*A*c^2*m^2 + 71*
A*c^2*m + 105*A*c^2)*x)*(e*x)^m/(m^4 + 16*m^3 + 86*m^2 + 176*m + 105)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1096 vs. $2(82) = 164$.

Time = 0.57 (sec) , antiderivative size = 1096, normalized size of antiderivative = 12.04

$$\int (ex)^m (A+Bx^2) (c+dx^2)^2 \, dx = \text{Too large to display}$$

input
```
integrate((e*x)**m*(B*x**2+A)*(d*x**2+c)**2,x)
```

output
```
Piecewise(((-A*c**2/(6*x**6) - A*c*d/(2*x**4) - A*d**2/(2*x**2) - B*c**2/(
4*x**4) - B*c*d/x**2 + B*d**2*log(x))/e**7, Eq(m, -7)), ((-A*c**2/(4*x**4)
 - A*c*d/x**2 + A*d**2*log(x) - B*c**2/(2*x**2) + 2*B*c*d*log(x) + B*d**2*
x**2/2)/e**5, Eq(m, -5)), ((-A*c**2/(2*x**2) + 2*A*c*d*log(x) + A*d**2*x**
2/2 + B*c**2*log(x) + B*c*d*x**2 + B*d**2*x**4/4)/e**3, Eq(m, -3)), ((A*c*
*2*log(x) + A*c*d*x**2 + A*d**2*x**4/4 + B*c**2*x**2/2 + B*c*d*x**4/2 + B*
d**2*x**6/6)/e, Eq(m, -1)), (A*c**2*m**3*x*(e*x)**m/(m**4 + 16*m**3 + 86*m
**2 + 176*m + 105) + 15*A*c**2*m**2*x*(e*x)**m/(m**4 + 16*m**3 + 86*m**2 +
 176*m + 105) + 71*A*c**2*m*x*(e*x)**m/(m**4 + 16*m**3 + 86*m**2 + 176*m +
 105) + 105*A*c**2*x*(e*x)**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 2
*A*c*d*m**3*x**3*(e*x)**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 26*A*
c*d*m**2*x**3*(e*x)**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 94*A*c*d
*m*x**3*(e*x)**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 70*A*c*d*x**3*
(e*x)**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + A*d**2*m**3*x**5*(e*x)
**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 11*A*d**2*m**2*x**5*(e*x)**
m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 31*A*d**2*m*x**5*(e*x)**m/(m*
*4 + 16*m**3 + 86*m**2 + 176*m + 105) + 21*A*d**2*x**5*(e*x)**m/(m**4 + 16
*m**3 + 86*m**2 + 176*m + 105) + B*c**2*m**3*x**3*(e*x)**m/(m**4 + 16*m**3
 + 86*m**2 + 176*m + 105) + 13*B*c**2*m**2*x**3*(e*x)**m/(m**4 + 16*m**3 +
 86*m**2 + 176*m + 105) + 47*B*c**2*m*x**3*(e*x)**m/(m**4 + 16*m**3 + 8...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.27

$$\int (ex)^m \left(A+Bx^2\right) \left(c+dx^2\right)^2 \, dx = \frac{Bd^2e^m x^7 x^m}{m+7} + \frac{2\,Bcde^m x^5 x^m}{m+5} + \frac{Ad^2e^m x^5 x^m}{m+5} + \frac{Bc^2e^m x^3 x^m}{m+3} + \frac{2\,Acde^m x^3 x^m}{m+3} + \frac{(ex)^{m+1}\,Ac^2}{e(m+1)}$$

input
```
integrate((e*x)^m*(B*x^2+A)*(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
B*d^2*e^m*x^7*x^m/(m + 7) + 2*B*c*d*e^m*x^5*x^m/(m + 5) + A*d^2*e^m*x^5*x^
m/(m + 5) + B*c^2*e^m*x^3*x^m/(m + 3) + 2*A*c*d*e^m*x^3*x^m/(m + 3) + (e*x
)^(m + 1)*A*c^2/(e*(m + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 380 vs. $2(91) = 182$.

Time = 0.13 (sec) , antiderivative size = 380, normalized size of antiderivative = 4.18

$$\int (ex)^m (A + Bx^2) (c + dx^2)^2 \, dx = \frac{(ex)^m Bd^2m^3x^7 + 9(ex)^m Bd^2m^2x^7 + 2(ex)^m Bcdm^3x^5 + (ex)^m Ad^2m^3x^5 + 23(ex)^m Bd^2mx^7 + 22(ex}{}$$

input
```
integrate((e*x)^m*(B*x^2+A)*(d*x^2+c)^2,x, algorithm="giac")
```

output
```
((e*x)^m*B*d^2*m^3*x^7 + 9*(e*x)^m*B*d^2*m^2*x^7 + 2*(e*x)^m*B*c*d*m^3*x^5
 + (e*x)^m*A*d^2*m^3*x^5 + 23*(e*x)^m*B*d^2*m*x^7 + 22*(e*x)^m*B*c*d*m^2*x
^5 + 11*(e*x)^m*A*d^2*m^2*x^5 + 15*(e*x)^m*B*d^2*x^7 + (e*x)^m*B*c^2*m^3*x
^3 + 2*(e*x)^m*A*c*d*m^3*x^3 + 62*(e*x)^m*B*c*d*m*x^5 + 31*(e*x)^m*A*d^2*m
*x^5 + 13*(e*x)^m*B*c^2*m^2*x^3 + 26*(e*x)^m*A*c*d*m^2*x^3 + 42*(e*x)^m*B*
c*d*x^5 + 21*(e*x)^m*A*d^2*x^5 + (e*x)^m*A*c^2*m^3*x + 47*(e*x)^m*B*c^2*m*
x^3 + 94*(e*x)^m*A*c*d*m*x^3 + 15*(e*x)^m*A*c^2*m^2*x + 35*(e*x)^m*B*c^2*x
^3 + 70*(e*x)^m*A*c*d*x^3 + 71*(e*x)^m*A*c^2*m*x + 105*(e*x)^m*A*c^2*x)/(m
^4 + 16*m^3 + 86*m^2 + 176*m + 105)
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.97

$$\int (ex)^m (A + Bx^2) (c + dx^2)^2 \, dx = (e\,x)^m \left(\frac{B\,d^2\,x^7\,(m^3 + 9\,m^2 + 23\,m + 15)}{m^4 + 16\,m^3 + 86\,m^2 + 176\,m + 105} + \frac{A\,c^2\,x\,(m^3 + 15\,m^2 + 71\,m + 105)}{m^4 + 16\,m^3 + 86\,m^2 + 176\,m + 105} + \frac{c\,x^3\,(2\,A\,d + B\,c)\,(m^3 + 13\,m^2 + 47\,m + 35)}{m^4 + 16\,m^3 + 86\,m^2 + 176\,m + 105} + \frac{d\,x^5\,(A\,d + 2\,B\,c)\,(m^3 + 11\,m^2 + 31\,m + 21)}{m^4 + 16\,m^3 + 86\,m^2 + 176\,m + 105}\right)$$

input
```
int((A + B*x^2)*(e*x)^m*(c + d*x^2)^2,x)
```

output
```
(e*x)^m*((B*d^2*x^7*(23*m + 9*m^2 + m^3 + 15))/(176*m + 86*m^2 + 16*m^3 +
m^4 + 105) + (A*c^2*x*(71*m + 15*m^2 + m^3 + 105))/(176*m + 86*m^2 + 16*m^
3 + m^4 + 105) + (c*x^3*(2*A*d + B*c)*(47*m + 13*m^2 + m^3 + 35))/(176*m +
 86*m^2 + 16*m^3 + m^4 + 105) + (d*x^5*(A*d + 2*B*c)*(31*m + 11*m^2 + m^3
+ 21))/(176*m + 86*m^2 + 16*m^3 + m^4 + 105))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 263, normalized size of antiderivative = 2.89

$$\int (ex)^m (A + Bx^2) (c + dx^2)^2 \, dx$$
$$= \frac{x^m e^m x(b\, d^2 m^3 x^6 + 9b\, d^2 m^2 x^6 + a\, d^2 m^3 x^4 + 2bcd\, m^3 x^4 + 23b\, d^2 m\, x^6 + 11a\, d^2 m^2 x^4 + 22bcd\, m^2 x^4 + 15b}{}$$

input
```
int((e*x)^m*(B*x^2+A)*(d*x^2+c)^2,x)
```

output
```
(x**m*e**m*x*(a*c**2*m**3 + 15*a*c**2*m**2 + 71*a*c**2*m + 105*a*c**2 + 2*
a*c*d*m**3*x**2 + 26*a*c*d*m**2*x**2 + 94*a*c*d*m*x**2 + 70*a*c*d*x**2 + a
*d**2*m**3*x**4 + 11*a*d**2*m**2*x**4 + 31*a*d**2*m*x**4 + 21*a*d**2*x**4
+ b*c**2*m**3*x**2 + 13*b*c**2*m**2*x**2 + 47*b*c**2*m*x**2 + 35*b*c**2*x*
*2 + 2*b*c*d*m**3*x**4 + 22*b*c*d*m**2*x**4 + 62*b*c*d*m*x**4 + 42*b*c*d*x
**4 + b*d**2*m**3*x**6 + 9*b*d**2*m**2*x**6 + 23*b*d**2*m*x**6 + 15*b*d**2
*x**6))/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105)
```

3.12 $\int \frac{(ex)^m(A+Bx^2)(c+dx^2)^2}{a+bx^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 178

$$\int \frac{(ex)^m(A+Bx^2)(c+dx^2)^2}{a+bx^2}\,dx = \frac{(a^2Bd^2-abd(2Bc+Ad)+b^2c(Bc+2Ad))(ex)^{1+m}}{b^3e(1+m)} + \frac{d(2bBc+Abd-aBd)(ex)^{3+m}}{b^2e^3(3+m)} + \frac{Bd^2(ex)^{5+m}}{be^5(5+m)} + \frac{(Ab-aB)(bc-ad)^2(ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{bx^2}{a}\right)}{ab^3e(1+m)}$$

output
```
(a^2*B*d^2-a*b*d*(A*d+2*B*c)+b^2*c*(2*A*d+B*c))*(e*x)^(1+m)/b^3/e/(1+m)+d*
(A*b*d-B*a*d+2*B*b*c)*(e*x)^(3+m)/b^2/e^3/(3+m)+B*d^2*(e*x)^(5+m)/b/e^5/(5
+m)+(A*b-B*a)*(-a*d+b*c)^2*(e*x)^(1+m)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m
],-b*x^2/a)/a/b^3/e/(1+m)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.82

$$\int \frac{(ex)^m (A+Bx^2) (c+dx^2)^2}{a+bx^2} dx$$

$$= \frac{x(ex)^m \left(\frac{a^2Bd^2-abd(2Bc+Ad)+b^2c(Bc+2Ad)}{1+m} + \frac{bd(2bBc+Abd-aBd)x^2}{3+m} + \frac{b^2Bd^2x^4}{5+m} + \frac{(Ab-aB)(bc-ad)^2 \text{Hypergeometric2F1}\left(1,\frac{1+}{}\right.}{a(1+m)}\right.}{b^3}$$

input
```
Integrate[((e*x)^m*(A + B*x^2)*(c + d*x^2)^2)/(a + b*x^2),x]
```

output
```
(x*(e*x)^m*((a^2*B*d^2 - a*b*d*(2*B*c + A*d) + b^2*c*(B*c + 2*A*d))/(1 + m
) + (b*d*(2*b*B*c + A*b*d - a*B*d)*x^2)/(3 + m) + (b^2*B*d^2*x^4)/(5 + m)
+ ((A*b - a*B)*(b*c - a*d)^2*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -(
(b*x^2)/a)])/(a*(1 + m))))/b^3
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {437, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2) (c+dx^2)^2 (ex)^m}{a+bx^2} dx$$

$$\Big\downarrow \text{437}$$

$$\int \left(\frac{(ex)^m (a^2Bd^2 - abd(Ad+2Bc) + b^2c(2Ad+Bc))}{b^3} + \frac{(ex)^m (a^3(-B)d^2 + a^2Abd^2 + 2a^2bBcd - 2aAb^2cd -}{b^3 (a+bx^2)}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{(ex)^{m+1}\left(a^2Bd^2-abd(Ad+2Bc)+b^2c(2Ad+Bc)\right)}{b^3e(m+1)}+\frac{(ex)^{m+1}(Ab-aB)(bc-ad)^2\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\frac{bx^2}{a}\right)}{ab^3e(m+1)}+\frac{d(ex)^{m+3}(-aBd+Abd+2bBc)}{b^2e^3(m+3)}+\frac{Bd^2(ex)^{m+5}}{be^5(m+5)}$$

input
```
Int[((e*x)^m*(A + B*x^2)*(c + d*x^2)^2)/(a + b*x^2),x]
```

output
```
((a^2*B*d^2 - a*b*d*(2*B*c + A*d) + b^2*c*(B*c + 2*A*d))*(e*x)^(1 + m))/(b
^3*e*(1 + m)) + (d*(2*b*B*c + A*b*d - a*B*d)*(e*x)^(3 + m))/(b^2*e^3*(3 +
m)) + (B*d^2*(e*x)^(5 + m))/(b*e^5*(5 + m)) + ((A*b - a*B)*(b*c - a*d)^2*(
e*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(a*
b^3*e*(1 + m))
```

Defintions of rubi rules used

rule 437
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(
a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f
, g, m}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{(ex)^m\,(x^2B+A)\,(x^2d+c)^2}{b\,x^2+a}dx$$

input
```
int((e*x)^m*(B*x^2+A)*(d*x^2+c)^2/(b*x^2+a),x)
```

output
```
int((e*x)^m*(B*x^2+A)*(d*x^2+c)^2/(b*x^2+a),x)
```

Fricas [F]

$$\int \frac{(ex)^m (A + Bx^2) (c + dx^2)^2}{a + bx^2} dx = \int \frac{(Bx^2 + A)(dx^2 + c)^2 (ex)^m}{bx^2 + a} dx$$

input
```
integrate((e*x)^m*(B*x^2+A)*(d*x^2+c)^2/(b*x^2+a),x, algorithm="fricas")
```

output
```
integral((B*d^2*x^6 + (2*B*c*d + A*d^2)*x^4 + A*c^2 + (B*c^2 + 2*A*c*d)*x^
2)*(e*x)^m/(b*x^2 + a), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.00 (sec) , antiderivative size = 649, normalized size of antiderivative = 3.65

$$\int \frac{(ex)^m (A + Bx^2) (c + dx^2)^2}{a + bx^2} dx = \text{Too large to display}$$

input
```
integrate((e*x)**m*(B*x**2+A)*(d*x**2+c)**2/(b*x**2+a),x)
```

output
```
A*c**2*e**m*m*x**(m + 1)*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*
gamma(m/2 + 1/2)/(4*a*gamma(m/2 + 3/2)) + A*c**2*e**m*x**(m + 1)*lerchphi(
b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(4*a*gamma(m/2 +
3/2)) + A*c*d*e**m*m*x**(m + 3)*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2
+ 3/2)*gamma(m/2 + 3/2)/(2*a*gamma(m/2 + 5/2)) + 3*A*c*d*e**m*x**(m + 3)*l
erchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 3/2)*gamma(m/2 + 3/2)/(2*a*gamm
a(m/2 + 5/2)) + A*d**2*e**m*m*x**(m + 5)*lerchphi(b*x**2*exp_polar(I*pi)/a
, 1, m/2 + 5/2)*gamma(m/2 + 5/2)/(4*a*gamma(m/2 + 7/2)) + 5*A*d**2*e**m*x*
*(m + 5)*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 5/2)*gamma(m/2 + 5/2)
/(4*a*gamma(m/2 + 7/2)) + B*c**2*e**m*m*x**(m + 3)*lerchphi(b*x**2*exp_pol
ar(I*pi)/a, 1, m/2 + 3/2)*gamma(m/2 + 3/2)/(4*a*gamma(m/2 + 5/2)) + 3*B*c*
*2*e**m*x**(m + 3)*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 3/2)*gamma(
m/2 + 3/2)/(4*a*gamma(m/2 + 5/2)) + B*c*d*e**m*m*x**(m + 5)*lerchphi(b*x**
2*exp_polar(I*pi)/a, 1, m/2 + 5/2)*gamma(m/2 + 5/2)/(2*a*gamma(m/2 + 7/2))
 + 5*B*c*d*e**m*x**(m + 5)*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 5/2
)*gamma(m/2 + 5/2)/(2*a*gamma(m/2 + 7/2)) + B*d**2*e**m*m*x**(m + 7)*lerch
phi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 7/2)*gamma(m/2 + 7/2)/(4*a*gamma(m/
2 + 9/2)) + 7*B*d**2*e**m*x**(m + 7)*lerchphi(b*x**2*exp_polar(I*pi)/a, 1,
 m/2 + 7/2)*gamma(m/2 + 7/2)/(4*a*gamma(m/2 + 9/2))
```

Maxima [F]

$$\int \frac{(ex)^m (A+Bx^2) (c+dx^2)^2}{a+bx^2} \, dx = \int \frac{(Bx^2+A)(dx^2+c)^2(ex)^m}{bx^2+a} \, dx$$

input
```
integrate((e*x)^m*(B*x^2+A)*(d*x^2+c)^2/(b*x^2+a),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*(d*x^2 + c)^2*(e*x)^m/(b*x^2 + a), x)
```

Giac [F]

$$\int \frac{(ex)^m (A+Bx^2)(c+dx^2)^2}{a+bx^2}\,dx = \int \frac{(Bx^2+A)(dx^2+c)^2(ex)^m}{bx^2+a}\,dx$$

input
```
integrate((e*x)^m*(B*x^2+A)*(d*x^2+c)^2/(b*x^2+a),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*(d*x^2 + c)^2*(e*x)^m/(b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m (A+Bx^2)(c+dx^2)^2}{a+bx^2}\,dx = \int \frac{(B\,x^2+A)\,(e\,x)^m\,(d\,x^2+c)^2}{b\,x^2+a}\,dx$$

input
```
int(((A + B*x^2)*(e*x)^m*(c + d*x^2)^2)/(a + b*x^2),x)
```

output
```
int(((A + B*x^2)*(e*x)^m*(c + d*x^2)^2)/(a + b*x^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.53

$$\int \frac{(ex)^m (A+Bx^2)(c+dx^2)^2}{a+bx^2}\,dx = \frac{x^m e^m x(d^2m^2x^4+4d^2m\,x^4+2cd\,m^2x^2+3d^2x^4+12cdm\,x^2+c^2m^2+10cd\,x^2+8c^2m+15c^2)}{m^3+9m^2+23m+15}$$

input
```
int((e*x)^m*(B*x^2+A)*(d*x^2+c)^2/(b*x^2+a),x)
```

output
```
(x**m*e**m*x*(c**2*m**2 + 8*c**2*m + 15*c**2 + 2*c*d*m**2*x**2 + 12*c*d*m*
x**2 + 10*c*d*x**2 + d**2*m**2*x**4 + 4*d**2*m*x**4 + 3*d**2*x**4))/(m**3
+ 9*m**2 + 23*m + 15)
```

3.13 $\int \frac{(ex)^m (A+Bx^2)(c+dx^2)^2}{(a+bx^2)^2} \, dx$

Optimal result

Integrand size = 31, antiderivative size = 200

$$\int \frac{(ex)^m (A+Bx^2)(c+dx^2)^2}{(a+bx^2)^2} \, dx = \frac{d(2bBc+Abd-2aBd)(ex)^{1+m}}{b^3e(1+m)} + \frac{Bd^2(ex)^{3+m}}{b^2e^3(3+m)} + \frac{(Ab-aB)(bc-ad)^2(ex)^{1+m}}{2ab^3e\,(a+bx^2)} + \frac{(bc-ad)(aB(bc(1+m)-ad(5+m))+Ab(ad(3+m)+b(c-cm)))(ex)^{1+m}\,\text{Hypergeometric2F1}\Big(}{2a^2b^3e(1+m)}$$

output
```
d*(A*b*d-2*B*a*d+2*B*b*c)*(e*x)^(1+m)/b^3/e/(1+m)+B*d^2*(e*x)^(3+m)/b^2/e^
3/(3+m)+1/2*(A*b-B*a)*(-a*d+b*c)^2*(e*x)^(1+m)/a/b^3/e/(b*x^2+a)+1/2*(-a*d
+b*c)*(a*B*(b*c*(1+m)-a*d*(5+m))+A*b*(a*d*(3+m)+b*(-c*m+c)))*(e*x)^(1+m)*h
ypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-b*x^2/a)/a^2/b^3/e/(1+m)
```

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.78

$$\int \frac{(ex)^m (A+Bx^2) (c+dx^2)^2}{(a+bx^2)^2} dx$$
$$= \frac{x(ex)^m \left(\frac{d(2bBc+Abd-2aBd)}{1+m} + \frac{bBd^2x^2}{3+m} + \frac{(bc-ad)(bBc+2Abd-3aBd)\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{bx^2}{a}\right)}{a(1+m)} + \frac{(Ab-aB)(bc-a\ldots}{\ldots}\right.}{b^3}$$

input
```
Integrate[((e*x)^m*(A + B*x^2)*(c + d*x^2)^2)/(a + b*x^2)^2,x]
```

output
```
(x*(e*x)^m*((d*(2*b*B*c + A*b*d - 2*a*B*d))/(1 + m) + (b*B*d^2*x^2)/(3 + m
) + ((b*c - a*d)*(b*B*c + 2*A*b*d - 3*a*B*d)*Hypergeometric2F1[1, (1 + m)/
2, (3 + m)/2, -((b*x^2)/a)])/(a*(1 + m)) + ((A*b - a*B)*(b*c - a*d)^2*Hype
rgeometric2F1[2, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(a^2*(1 + m))))/b^3
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.22, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {439, 25, 437, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)(c+dx^2)^2 (ex)^m}{(a+bx^2)^2} dx$$

$$\Big\downarrow \; 439$$

$$\frac{(c+dx^2)^2 (ex)^{m+1}(Ab-aB)}{2abe(a+bx^2)} - \frac{\int -\frac{(ex)^m (dx^2+c)(c(aB(m+1)+A(b-bm))-d(Ab(m+3)-aB(m+5))x^2)}{bx^2+a}dx}{2ab}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{(ex)^m (dx^2+c)(c(aB(m+1)+A(b-bm))-d(Ab(m+3)-aB(m+5))x^2)}{bx^2+a}dx}{2ab} + \frac{(c+dx^2)^2 (ex)^{m+1}(Ab-aB)}{2abe(a+bx^2)}$$

$\Big\downarrow$ 437

$$\frac{\int \left(-\frac{d(Ab(2bc(m+1)-ad(m+3))-aB(2bc(m+3)-ad(m+5)))(ex)^m}{b^2}+\frac{(5Bd^2a^3+Bd^2ma^3-3Abd^2a^2-6bBcda^2-Abd^2ma^2-2bBcdma^2+b^2 \cdots}{b^2(bx^2+\cdots}\right.}{2ab}$$

$$\frac{\left(c+dx^2\right)^2(ex)^{m+1}(Ab-aB)}{2abe\left(a+bx^2\right)}$$

$\Big\downarrow$ 2009

$$\frac{\frac{(ex)^{m+1}(bc-ad)\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\frac{bx^2}{a}\right)(Ab(ad(m+3)+bc(1-m))+aB(bc(m+1)-ad(m+5)))}{ab^2e(m+1)}-\frac{d(ex)^{m+1}(Ab(2bc(m+1)-\cdots}{\cdots}}{2ab}$$

$$\frac{\left(c+dx^2\right)^2(ex)^{m+1}(Ab-aB)}{2abe\left(a+bx^2\right)}$$

input
```
Int[((e*x)^m*(A + B*x^2)*(c + d*x^2)^2)/(a + b*x^2)^2,x]
```

output
```
((A*b - a*B)*(e*x)^(1 + m)*(c + d*x^2)^2)/(2*a*b*e*(a + b*x^2)) + (-((d*(A
*b*(2*b*c*(1 + m) - a*d*(3 + m)) - a*B*(2*b*c*(3 + m) - a*d*(5 + m)))*(e*x
)^(1 + m))/(b^2*e*(1 + m))) - (d^2*(A*b*(3 + m) - a*B*(5 + m))*(e*x)^(3 +
m))/(b*e^3*(3 + m)) + ((b*c - a*d)*(A*b*(b*c*(1 - m) + a*d*(3 + m)) + a*B*
(b*c*(1 + m) - a*d*(5 + m)))*(e*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/2,
 (3 + m)/2, -((b*x^2)/a)])/(a*b^2*e*(1 + m)))/(2*a*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 437
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(
a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f
, g, m}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

rule 439
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
 + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*b*g*(p + 1))), x] + Simp[1/(2*a*b*(p
+ 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1)*Simp[c*(2*b*e*(
p + 1) + (b*e - a*f)*(m + 1)) + d*(2*b*e*(p + 1) + (b*e - a*f)*(m + 2*q + 1
))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && LtQ[p, -1] && G
tQ[q, 0] && !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e - a*f])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{(ex)^m (x^2B + A) (x^2d + c)^2}{(b\,x^2 + a)^2}dx$$

input
```
int((e*x)^m*(B*x^2+A)*(d*x^2+c)^2/(b*x^2+a)^2,x)
```

output
```
int((e*x)^m*(B*x^2+A)*(d*x^2+c)^2/(b*x^2+a)^2,x)
```

Fricas [F]

$$\int \frac{(ex)^m (A + Bx^2) (c + dx^2)^2}{(a + bx^2)^2} \, dx = \int \frac{(Bx^2 + A)(dx^2 + c)^2(ex)^m}{(bx^2 + a)^2} \, dx$$

input
```
integrate((e*x)^m*(B*x^2+A)*(d*x^2+c)^2/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
integral((B*d^2*x^6 + (2*B*c*d + A*d^2)*x^4 + A*c^2 + (B*c^2 + 2*A*c*d)*x^
2)*(e*x)^m/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [F]

$$\int \frac{(ex)^m\,(A+Bx^2)\,(c+dx^2)^2}{(a+bx^2)^2}\,dx = \int \frac{(ex)^m\,(A+Bx^2)\,(c+dx^2)^2}{(a+bx^2)^2}\,dx$$

input
```
integrate((e*x)**m*(B*x**2+A)*(d*x**2+c)**2/(b*x**2+a)**2,x)
```

output
```
Integral((e*x)**m*(A + B*x**2)*(c + d*x**2)**2/(a + b*x**2)**2, x)
```

Maxima [F]

$$\int \frac{(ex)^m\,(A+Bx^2)\,(c+dx^2)^2}{(a+bx^2)^2}\,dx = \int \frac{(Bx^2+A)(dx^2+c)^2(ex)^m}{(bx^2+a)^2}\,dx$$

input
```
integrate((e*x)^m*(B*x^2+A)*(d*x^2+c)^2/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*(d*x^2 + c)^2*(e*x)^m/(b*x^2 + a)^2, x)
```

Giac [F]

$$\int \frac{(ex)^m\,(A+Bx^2)\,(c+dx^2)^2}{(a+bx^2)^2}\,dx = \int \frac{(Bx^2+A)(dx^2+c)^2(ex)^m}{(bx^2+a)^2}\,dx$$

input
```
integrate((e*x)^m*(B*x^2+A)*(d*x^2+c)^2/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*(d*x^2 + c)^2*(e*x)^m/(b*x^2 + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m (A+Bx^2) (c+dx^2)^2}{(a+bx^2)^2}\,dx = \int \frac{(B\,x^2+A)\,(e\,x)^m\,(d\,x^2+c)^2}{(b\,x^2+a)^2}\,dx$$

input
```
int(((A + B*x^2)*(e*x)^m*(c + d*x^2)^2)/(a + b*x^2)^2,x)
```

output
```
int(((A + B*x^2)*(e*x)^m*(c + d*x^2)^2)/(a + b*x^2)^2, x)
```

Reduce [F]

$$\int \frac{(ex)^m (A+Bx^2) (c+dx^2)^2}{(a+bx^2)^2}\,dx = \frac{e^m\left(-x^m a\,d^2mx - 3x^m a\,d^2x + 2x^m bcdmx + 6x^m bcdx + x^m b\,d^2m\,x^3 + x^m b\,d^2x^3 + \left(\int \frac{x^m}{b\,x^2+a}dx\right) a^2d^2m^2 \right.}{}$$

input
```
int((e*x)^m*(B*x^2+A)*(d*x^2+c)^2/(b*x^2+a)^2,x)
```

output
```
(e**m*( - x**m*a*d**2*m*x - 3*x**m*a*d**2*x + 2*x**m*b*c*d*m*x + 6*x**m*b*
c*d*x + x**m*b*d**2*m*x**3 + x**m*b*d**2*x**3 + int(x**m/(a + b*x**2),x)*a
**2*d**2*m**2 + 4*int(x**m/(a + b*x**2),x)*a**2*d**2*m + 3*int(x**m/(a + b
*x**2),x)*a**2*d**2 - 2*int(x**m/(a + b*x**2),x)*a*b*c*d*m**2 - 8*int(x**m
/(a + b*x**2),x)*a*b*c*d*m - 6*int(x**m/(a + b*x**2),x)*a*b*c*d + int(x**m
/(a + b*x**2),x)*b**2*c**2*m**2 + 4*int(x**m/(a + b*x**2),x)*b**2*c**2*m +
 3*int(x**m/(a + b*x**2),x)*b**2*c**2))/(b**2*(m**2 + 4*m + 3))
```

3.14 $\int \frac{(ex)^m (A+Bx^2) (c+dx^2)^2}{(a+bx^2)^3} dx$

Optimal result

Integrand size = 31, antiderivative size = 290

$$\int \frac{(ex)^m (A+Bx^2) (c+dx^2)^2}{(a+bx^2)^3} dx = \frac{Bd^2(ex)^{1+m}}{b^3e(1+m)} + \frac{(Ab-aB)(bc-ad)^2(ex)^{1+m}}{4ab^3e(a+bx^2)^2} + \frac{(bc-ad)(Ab(bc(3-m)+ad(5+m))+aB(bc(1+m)-ad(9+m)))(ex)^{1+m}}{8a^2b^3e(a+bx^2)} + \frac{(Ab(2abcd(1-m^2)+b^2c^2(3-4m+m^2)+a^2d^2(3+4m+m^2))+aB(b^2c^2(1-m^2)+2abcd(3+4m}{8a^3b^3e(1+m)}$$

output

```
B*d^2*(e*x)^(1+m)/b^3/e/(1+m)+1/4*(A*b-B*a)*(-a*d+b*c)^2*(e*x)^(1+m)/a/b^3
/e/(b*x^2+a)^2+1/8*(-a*d+b*c)*(A*b*(b*c*(3-m)+a*d*(5+m))+a*B*(b*c*(1+m)-a*
d*(9+m)))*(e*x)^(1+m)/a^2/b^3/e/(b*x^2+a)+1/8*(A*b*(2*a*b*c*d*(-m^2+1)+b^2
*c^2*(m^2-4*m+3)+a^2*d^2*(m^2+4*m+3))+a*B*(b^2*c^2*(-m^2+1)+2*a*b*c*d*(m^2
+4*m+3)-a^2*d^2*(m^2+8*m+15)))*(e*x)^(1+m)*hypergeom([1, 1/2+1/2*m],[3/2+1
/2*m],-b*x^2/a)/a^3/b^3/e/(1+m)
```

Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.57

$$\int \frac{(ex)^m (A+Bx^2) (c+dx^2)^2}{(a+bx^2)^3} dx$$
$$= \frac{x(ex)^m \left(Bd^2 + \frac{d(2bBc+Abd-3aBd) \text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{bx^2}{a}\right)}{a} + \frac{(bc-ad)(bBc+2Abd-3aBd) \text{Hypergeometric2F1}\left(2\right.}{a^2}\right.}{b^3(1+m)}$$

input
```
Integrate[((e*x)^m*(A + B*x^2)*(c + d*x^2)^2)/(a + b*x^2)^3,x]
```

output
```
(x*(e*x)^m*(B*d^2 + (d*(2*b*B*c + A*b*d - 3*a*B*d)*Hypergeometric2F1[1, (1
 + m)/2, (3 + m)/2, -((b*x^2)/a)])/a + ((b*c - a*d)*(b*B*c + 2*A*b*d - 3*a
*B*d)*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/a^2 + ((A*
b - a*B)*(b*c - a*d)^2*Hypergeometric2F1[3, (1 + m)/2, (3 + m)/2, -((b*x^2
)/a)])/a^3))/(b^3*(1 + m))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {439, 25, 439, 25, 363, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2) (c+dx^2)^2 (ex)^m}{(a+bx^2)^3} dx$$

$$\Big\downarrow \text{ 439}$$

$$\frac{(c+dx^2)^2 (ex)^{m+1}(Ab-aB)}{4abe (a+bx^2)^2} - \frac{\int -\frac{(ex)^m (dx^2+c) (c(Ab(3-m)+aB(m+1))-d(Ab(m+1)-aB(m+5))x^2)}{(bx^2+a)^2} dx}{4ab}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{(ex)^m (dx^2+c)\left(c(Ab(3-m)+aB(m+1))-d(Ab(m+1)-aB(m+5))x^2\right)}{(bx^2+a)^2}dx}{4ab}+\frac{\left(c+dx^2\right)^2(ex)^{m+1}(Ab-aB)}{4abe\left(a+bx^2\right)^2}$$

$\Big\downarrow$ 439

$$\frac{\frac{(ex)^{m+1}(bc-ad)\left(c(aB(m+1)+Ab(3-m))-dx^2(Ab(m+1)-aB(m+5))\right)}{2abe(a+bx^2)}-\frac{\int -\frac{(ex)^m\left(d(bc(m+1)-ad(m+3))(Ab(m+1)-aB(m+5))x^2+c(Ab(3-m)+\right.}{bx^2+a}}{2ab}}{4ab}$$

$$\frac{\left(c+dx^2\right)^2(ex)^{m+1}(Ab-aB)}{4abe\left(a+bx^2\right)^2}$$

$\Big\downarrow$ 25

$$\frac{\frac{\int \frac{(ex)^m\left(d(bc(m+1)-ad(m+3))(Ab(m+1)-aB(m+5))x^2+c(Ab(3-m)+aB(m+1))(ad(m+1)+b(c-cm))\right)}{bx^2+a}dx}{2ab}+\frac{(ex)^{m+1}(bc-ad)\left(c(aB(m+1)+Ab(3-m)\right.}{2abe(a+bx^2}}{4ab}$$

$$\frac{\left(c+dx^2\right)^2(ex)^{m+1}(Ab-aB)}{4abe\left(a+bx^2\right)^2}$$

$\Big\downarrow$ 363

$$\frac{\frac{\frac{d(ex)^{m+1}(Ab(m+1)-aB(m+5))(bc(m+1)-ad(m+3))}{be(m+1)}-\left(\frac{ad(Ab(m+1)-aB(m+5))(bc(m+1)-ad(m+3))}{b}-c(aB(m+1)+Ab(3-m))(ad(m+1)+b(c-cm))\right)}{2ab}}{4ab}$$

$$\frac{\left(c+dx^2\right)^2(ex)^{m+1}(Ab-aB)}{4abe\left(a+bx^2\right)^2}$$

$\Big\downarrow$ 278

$$\frac{\frac{\frac{d(ex)^{m+1}(Ab(m+1)-aB(m+5))(bc(m+1)-ad(m+3))}{be(m+1)}-\frac{(ex)^{m+1}\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\frac{bx^2}{a}\right)\left(\frac{ad(Ab(m+1)-aB(m+5))(bc(m+1)-ad(m+3))}{b}-\right.}{ae(m+1)}}{2ab}}{4ab}$$

$$\frac{\left(c+dx^2\right)^2(ex)^{m+1}(Ab-aB)}{4abe\left(a+bx^2\right)^2}$$

input
```
Int[((e*x)^m*(A + B*x^2)*(c + d*x^2)^2)/(a + b*x^2)^3,x]
```

output
```
((A*b - a*B)*(e*x)^(1 + m)*(c + d*x^2)^2)/(4*a*b*e*(a + b*x^2)^2) + (((b*c
 - a*d)*(e*x)^(1 + m)*(c*(A*b*(3 - m) + a*B*(1 + m)) - d*(A*b*(1 + m) - a*
B*(5 + m))*x^2))/(2*a*b*e*(a + b*x^2)) + ((d*(b*c*(1 + m) - a*d*(3 + m))*(
A*b*(1 + m) - a*B*(5 + m))*(e*x)^(1 + m))/(b*e*(1 + m)) - (((a*d*(b*c*(1 +
 m) - a*d*(3 + m))*(A*b*(1 + m) - a*B*(5 + m)))/b - c*(A*b*(3 - m) + a*B*(
1 + m))*(a*d*(1 + m) + b*(c - c*m)))*(e*x)^(1 + m)*Hypergeometric2F1[1, (1
 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(a*e*(1 + m)))/(2*a*b))/(4*a*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 439
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
 + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*b*g*(p + 1))), x] + Simp[1/(2*a*b*(p
+ 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1)*Simp[c*(2*b*e*(
p + 1) + (b*e - a*f)*(m + 1)) + d*(2*b*e*(p + 1) + (b*e - a*f)*(m + 2*q + 1
))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && LtQ[p, -1] && G
tQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e - a*f])
```

Maple [F]

$$\int \frac{(ex)^m (x^2B + A) (x^2d + c)^2}{(b\,x^2 + a)^3}dx$$

input
```
int((e*x)^m*(B*x^2+A)*(d*x^2+c)^2/(b*x^2+a)^3,x)
```

output
```
int((e*x)^m*(B*x^2+A)*(d*x^2+c)^2/(b*x^2+a)^3,x)
```

Fricas [F]

$$\int \frac{(ex)^m (A + Bx^2) (c + dx^2)^2}{(a + bx^2)^3} dx = \int \frac{(Bx^2 + A)(dx^2 + c)^2(ex)^m}{(bx^2 + a)^3} dx$$

input
```
integrate((e*x)^m*(B*x^2+A)*(d*x^2+c)^2/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
integral((B*d^2*x^6 + (2*B*c*d + A*d^2)*x^4 + A*c^2 + (B*c^2 + 2*A*c*d)*x^
2)*(e*x)^m/(b^3*x^6 + 3*a*b^2*x^4 + 3*a^2*b*x^2 + a^3), x)
```

Sympy [F]

$$\int \frac{(ex)^m (A + Bx^2) (c + dx^2)^2}{(a + bx^2)^3} dx = \int \frac{(ex)^m (A + Bx^2) (c + dx^2)^2}{(a + bx^2)^3} dx$$

input
```
integrate((e*x)**m*(B*x**2+A)*(d*x**2+c)**2/(b*x**2+a)**3,x)
```

output
```
Integral((e*x)**m*(A + B*x**2)*(c + d*x**2)**2/(a + b*x**2)**3, x)
```

Maxima [F]

$$\int \frac{(ex)^m (A+Bx^2) (c+dx^2)^2}{(a+bx^2)^3} dx = \int \frac{(Bx^2+A)(dx^2+c)^2(ex)^m}{(bx^2+a)^3} dx$$

input
```
integrate((e*x)^m*(B*x^2+A)*(d*x^2+c)^2/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*(d*x^2 + c)^2*(e*x)^m/(b*x^2 + a)^3, x)
```

Giac [F]

$$\int \frac{(ex)^m (A+Bx^2) (c+dx^2)^2}{(a+bx^2)^3} dx = \int \frac{(Bx^2+A)(dx^2+c)^2(ex)^m}{(bx^2+a)^3} dx$$

input
```
integrate((e*x)^m*(B*x^2+A)*(d*x^2+c)^2/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*(d*x^2 + c)^2*(e*x)^m/(b*x^2 + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m (A+Bx^2) (c+dx^2)^2}{(a+bx^2)^3} dx = \int \frac{(B\,x^2+A)\,(e\,x)^m\,(d\,x^2+c)^2}{(b\,x^2+a)^3} dx$$

input
```
int(((A + B*x^2)*(e*x)^m*(c + d*x^2)^2)/(a + b*x^2)^3,x)
```

output
```
int(((A + B*x^2)*(e*x)^m*(c + d*x^2)^2)/(a + b*x^2)^3, x)
```

Reduce [F]

$$\int \frac{(ex)^m (A + Bx^2) (c + dx^2)^2}{(a + bx^2)^3} \, dx = \text{Too large to display}$$

input
```
int((e*x)^m*(B*x^2+A)*(d*x^2+c)^2/(b*x^2+a)^3,x)
```

output
```
(e**m*( - x**m*a*d**2*m*x - 3*x**m*a*d**2*x + 2*x**m*b*c*d*m*x + 2*x**m*b*
c*d*x + x**m*b*d**2*m*x**3 - x**m*b*d**2*x**3 + int(x**m/(a**2*m - a**2 +
2*a*b*m*x**2 - 2*a*b*x**2 + b**2*m*x**4 - b**2*x**4),x)*a**3*d**2*m**3 + 3
*int(x**m/(a**2*m - a**2 + 2*a*b*m*x**2 - 2*a*b*x**2 + b**2*m*x**4 - b**2*
x**4),x)*a**3*d**2*m**2 - int(x**m/(a**2*m - a**2 + 2*a*b*m*x**2 - 2*a*b*x
**2 + b**2*m*x**4 - b**2*x**4),x)*a**3*d**2*m - 3*int(x**m/(a**2*m - a**2
+ 2*a*b*m*x**2 - 2*a*b*x**2 + b**2*m*x**4 - b**2*x**4),x)*a**3*d**2 - 2*in
t(x**m/(a**2*m - a**2 + 2*a*b*m*x**2 - 2*a*b*x**2 + b**2*m*x**4 - b**2*x**
4),x)*a**2*b*c*d*m**3 - 2*int(x**m/(a**2*m - a**2 + 2*a*b*m*x**2 - 2*a*b*x
**2 + b**2*m*x**4 - b**2*x**4),x)*a**2*b*c*d*m**2 + 2*int(x**m/(a**2*m - a
**2 + 2*a*b*m*x**2 - 2*a*b*x**2 + b**2*m*x**4 - b**2*x**4),x)*a**2*b*c*d*m
 + 2*int(x**m/(a**2*m - a**2 + 2*a*b*m*x**2 - 2*a*b*x**2 + b**2*m*x**4 - b
**2*x**4),x)*a**2*b*c*d + int(x**m/(a**2*m - a**2 + 2*a*b*m*x**2 - 2*a*b*x
**2 + b**2*m*x**4 - b**2*x**4),x)*a**2*b*d**2*m**3*x**2 + 3*int(x**m/(a**2
*m - a**2 + 2*a*b*m*x**2 - 2*a*b*x**2 + b**2*m*x**4 - b**2*x**4),x)*a**2*b
*d**2*m**2*x**2 - int(x**m/(a**2*m - a**2 + 2*a*b*m*x**2 - 2*a*b*x**2 + b*
*2*m*x**4 - b**2*x**4),x)*a**2*b*d**2*m*x**2 - 3*int(x**m/(a**2*m - a**2 +
 2*a*b*m*x**2 - 2*a*b*x**2 + b**2*m*x**4 - b**2*x**4),x)*a**2*b*d**2*x**2
+ int(x**m/(a**2*m - a**2 + 2*a*b*m*x**2 - 2*a*b*x**2 + b**2*m*x**4 - b**2
*x**4),x)*a*b**2*c**2*m**3 - int(x**m/(a**2*m - a**2 + 2*a*b*m*x**2 - 2...
```

3.15 $\int (ex)^m \left(a+bx^2\right)^3 \left(A+Bx^2\right) \left(c+dx^2\right)^3 \, dx$

Optimal result

Integrand size = 31, antiderivative size = 379

$$
\begin{aligned}
&\int (ex)^m \left(a+bx^2\right)^3 \left(A+Bx^2\right) \left(c+dx^2\right)^3 \, dx \\
&= \frac{a^3Ac^3(ex)^{1+m}}{e(1+m)} + \frac{a^2c^2(aBc+3A(bc+ad))(ex)^{3+m}}{e^3(3+m)} \\
&\quad + \frac{3ac\left(aBc(bc+ad)+A\left(b^2c^2+3abcd+a^2d^2\right)\right)(ex)^{5+m}}{e^5(5+m)} \\
&\quad + \frac{\left(3aBc\left(b^2c^2+3abcd+a^2d^2\right)+A\left(b^3c^3+9ab^2c^2d+9a^2bcd^2+a^3d^3\right)\right)(ex)^{7+m}}{e^7(7+m)} \\
&\quad + \frac{\left(a^3Bd^3+9ab^2cd(Bc+Ad)+3a^2bd^2(3Bc+Ad)+b^3c^2(Bc+3Ad)\right)(ex)^{9+m}}{e^9(9+m)} \\
&\quad + \frac{3bd\left(a^2Bd^2+b^2c(Bc+Ad)+abd(3Bc+Ad)\right)(ex)^{11+m}}{e^{11}(11+m)} \\
&\quad + \frac{b^2d^2(3bBc+Abd+3aBd)(ex)^{13+m}}{e^{13}(13+m)} + \frac{b^3Bd^3(ex)^{15+m}}{e^{15}(15+m)}
\end{aligned}
$$

output
```
a^3*A*c^3*(e*x)^(1+m)/e/(1+m)+a^2*c^2*(B*a*c+3*A*(a*d+b*c))*(e*x)^(3+m)/e^
3/(3+m)+3*a*c*(a*B*c*(a*d+b*c)+A*(a^2*d^2+3*a*b*c*d+b^2*c^2))*(e*x)^(5+m)/
e^5/(5+m)+(3*a*B*c*(a^2*d^2+3*a*b*c*d+b^2*c^2)+A*(a^3*d^3+9*a^2*b*c*d^2+9*
a*b^2*c^2*d+b^3*c^3))*(e*x)^(7+m)/e^7/(7+m)+(a^3*B*d^3+9*a*b^2*c*d*(A*d+B*
c)+3*a^2*b*d^2*(A*d+3*B*c)+b^3*c^2*(3*A*d+B*c))*(e*x)^(9+m)/e^9/(9+m)+3*b*
d*(a^2*B*d^2+b^2*c*(A*d+B*c)+a*b*d*(A*d+3*B*c))*(e*x)^(11+m)/e^11/(11+m)+b
^2*d^2*(A*b*d+3*B*a*d+3*B*b*c)*(e*x)^(13+m)/e^13/(13+m)+b^3*B*d^3*(e*x)^(1
5+m)/e^15/(15+m)
```

Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 327, normalized size of antiderivative = 0.86

$$\int (ex)^m \left(a+bx^2\right)^3 \left(A+Bx^2\right) \left(c+dx^2\right)^3 \,dx = x(ex)^m \left(\frac{a^3Ac^3}{1+m}+\frac{a^2c^2(aBc+3A(bc+ad))x^2}{3+m}+\frac{3ac(aBc(bc+ad)+A(b^2c^2+3abcd+a^2d^2))\,x^4}{5+m}+\frac{(3aBc(b^2c^2+3abcd+a^2d^2)+A(b^3c^3+9ab^2c^2d+9a^2bcd^2+a^3d^3))\,x^6}{7+m}+\frac{(a^3Bd^3+9ab^2cd(Bc+Ad)+3a^2bd^2(3Bc+Ad)+b^3c^2(Bc+3Ad))\,x^8}{9+m}+\frac{3bd(a^2Bd^2+b^2c(Bc+Ad)+abd(3Bc+Ad))\,x^{10}}{11+m}+\frac{b^2d^2(3bBc+Abd+3aBd)x^{12}}{13+m}+\frac{b^3Bd^3x^{14}}{15+m}\right)$$

input
```
Integrate[(e*x)^m*(a + b*x^2)^3*(A + B*x^2)*(c + d*x^2)^3,x]
```

output
```
x*(e*x)^m*((a^3*A*c^3)/(1 + m) + (a^2*c^2*(a*B*c + 3*A*(b*c + a*d))*x^2)/(
3 + m) + (3*a*c*(a*B*c*(b*c + a*d) + A*(b^2*c^2 + 3*a*b*c*d + a^2*d^2))*x^
4)/(5 + m) + ((3*a*B*c*(b^2*c^2 + 3*a*b*c*d + a^2*d^2) + A*(b^3*c^3 + 9*a*
b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3))*x^6)/(7 + m) + ((a^3*B*d^3 + 9*a*b^2
*c*d*(B*c + A*d) + 3*a^2*b*d^2*(3*B*c + A*d) + b^3*c^2*(B*c + 3*A*d))*x^8)
/(9 + m) + (3*b*d*(a^2*B*d^2 + b^2*c*(B*c + A*d) + a*b*d*(3*B*c + A*d))*x^
10)/(11 + m) + (b^2*d^2*(3*b*B*c + A*b*d + 3*a*B*d)*x^12)/(13 + m) + (b^3*
B*d^3*x^14)/(15 + m))
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 379, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {437, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^2)^3 (A+Bx^2) (c+dx^2)^3 (ex)^m \, dx$$

$$\Big\downarrow \text{437}$$

$$\int \left(a^3Ac^3(ex)^m + \frac{3ac(ex)^{m+4}\left(A\left(a^2d^2+3abcd+b^2c^2\right)+aBc(ad+bc)\right)}{e^4} + \frac{3bd(ex)^{m+10}\left(a^2Bd^2+abd(Ad+3\right.}{e^{10}}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3Ac^3(ex)^{m+1}}{e(m+1)} + \frac{3ac(ex)^{m+5}\left(A\left(a^2d^2+3abcd+b^2c^2\right)+aBc(ad+bc)\right)}{e^5(m+5)} + \frac{3bd(ex)^{m+11}\left(a^2Bd^2+abd(Ad+3Bc)+b^2c(Ad+Bc)\right)}{e^{11}(m+11)} + \frac{a^2c^2(ex)^{m+3}(3A(ad+bc)+aBc)}{e^3(m+3)} + \frac{(ex)^{m+9}\left(a^3Bd^3+3a^2bd^2(Ad+3Bc)+9ab^2cd(Ad+Bc)+b^3c^2(3Ad+Bc)\right)}{e^9(m+9)} + \frac{(ex)^{m+7}\left(3aBc\left(a^2d^2+3abcd+b^2c^2\right)+A\left(a^3d^3+9a^2bcd^2+9ab^2c^2d+b^3c^3\right)\right)}{e^7(m+7)} + \frac{b^2d^2(ex)^{m+13}(3aBd+Abd+3bBc)}{e^{13}(m+13)} + \frac{b^3Bd^3(ex)^{m+15}}{e^{15}(m+15)}$$

input
```
Int[(e*x)^m*(a + b*x^2)^3*(A + B*x^2)*(c + d*x^2)^3,x]
```

output
```
(a^3*A*c^3*(e*x)^(1 + m))/(e*(1 + m)) + (a^2*c^2*(a*B*c + 3*A*(b*c + a*d))
*(e*x)^(3 + m))/(e^3*(3 + m)) + (3*a*c*(a*B*c*(b*c + a*d) + A*(b^2*c^2 + 3
*a*b*c*d + a^2*d^2))*(e*x)^(5 + m))/(e^5*(5 + m)) + ((3*a*B*c*(b^2*c^2 + 3
*a*b*c*d + a^2*d^2) + A*(b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3
))*(e*x)^(7 + m))/(e^7*(7 + m)) + ((a^3*B*d^3 + 9*a*b^2*c*d*(B*c + A*d) +
3*a^2*b*d^2*(3*B*c + A*d) + b^3*c^2*(B*c + 3*A*d))*(e*x)^(9 + m))/(e^9*(9
+ m)) + (3*b*d*(a^2*B*d^2 + b^2*c*(B*c + A*d) + a*b*d*(3*B*c + A*d))*(e*x)
^(11 + m))/(e^11*(11 + m)) + (b^2*d^2*(3*b*B*c + A*b*d + 3*a*B*d)*(e*x)^(1
3 + m))/(e^13*(13 + m)) + (b^3*B*d^3*(e*x)^(15 + m))/(e^15*(15 + m))
```

Defintions of rubi rules used

rule 437
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(
a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f
, g, m}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3952 vs. $2(379) = 758$.

Time = 0.87 (sec) , antiderivative size = 3953, normalized size of antiderivative = 10.43

method	result	size
gosper	Expression too large to display	3953
risch	Expression too large to display	3953
orering	Expression too large to display	3953
parallelrisch	Expression too large to display	5235

input
```
int((e*x)^m*(b*x^2+a)^3*(B*x^2+A)*(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
x*(B*b^3*d^3*m^7*x^14+49*B*b^3*d^3*m^6*x^14+A*b^3*d^3*m^7*x^12+3*B*a*b^2*d
^3*m^7*x^12+3*B*b^3*c*d^2*m^7*x^12+973*B*b^3*d^3*m^5*x^14+51*A*b^3*d^3*m^6
*x^12+153*B*a*b^2*d^3*m^6*x^12+153*B*b^3*c*d^2*m^6*x^12+10045*B*b^3*d^3*m^
4*x^14+3*A*a*b^2*d^3*m^7*x^10+3*A*b^3*c*d^2*m^7*x^10+1045*A*b^3*d^3*m^5*x^
12+3*B*a^2*b*d^3*m^7*x^10+9*B*a*b^2*c*d^2*m^7*x^10+3135*B*a*b^2*d^3*m^5*x^
12+3*B*b^3*c^2*d*m^7*x^10+3135*B*b^3*c*d^2*m^5*x^12+57379*B*b^3*d^3*m^3*x^
14+159*A*a*b^2*d^3*m^6*x^10+159*A*b^3*c*d^2*m^6*x^10+11055*A*b^3*d^3*m^4*x
^12+159*B*a^2*b*d^3*m^6*x^10+477*B*a*b^2*c*d^2*m^6*x^10+33165*B*a*b^2*d^3*
m^4*x^12+159*B*b^3*c^2*d*m^6*x^10+33165*B*b^3*c*d^2*m^4*x^12+177331*B*b^3*
d^3*m^2*x^14+3*A*a^2*b*d^3*m^7*x^8+9*A*a*b^2*c*d^2*m^7*x^8+3375*A*a*b^2*d^
3*m^5*x^10+3*A*b^3*c^2*d*m^7*x^8+3375*A*b^3*c*d^2*m^5*x^10+64339*A*b^3*d^3
*m^3*x^12+B*a^3*d^3*m^7*x^8+9*B*a^2*b*c*d^2*m^7*x^8+3375*B*a^2*b*d^3*m^5*x
^10+9*B*a*b^2*c^2*d*m^7*x^8+10125*B*a*b^2*c*d^2*m^5*x^10+193017*B*a*b^2*d^
3*m^3*x^12+B*b^3*c^3*m^7*x^8+3375*B*b^3*c^2*d*m^5*x^10+193017*B*b^3*c*d^2*
m^3*x^12+264207*B*b^3*d^3*m*x^14+165*A*a^2*b*d^3*m^6*x^8+495*A*a*b^2*c*d^2
*m^6*x^8+36795*A*a*b^2*d^3*m^4*x^10+165*A*b^3*c^2*d*m^6*x^8+36795*A*b^3*c*
d^2*m^4*x^10+201609*A*b^3*d^3*m^2*x^12+55*B*a^3*d^3*m^6*x^8+495*B*a^2*b*c*
d^2*m^6*x^8+36795*B*a^2*b*d^3*m^4*x^10+495*B*a*b^2*c^2*d*m^6*x^8+110385*B*
a*b^2*c*d^2*m^4*x^10+604827*B*a*b^2*d^3*m^2*x^12+55*B*b^3*c^3*m^6*x^8+3679
5*B*b^3*c^2*d*m^4*x^10+604827*B*b^3*c*d^2*m^2*x^12+135135*B*b^3*d^3*x^1...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2657 vs. $2(379) = 758$.

Time = 0.13 (sec) , antiderivative size = 2657, normalized size of antiderivative = 7.01

$$\int (ex)^m \left(a+bx^2\right)^3 \left(A+Bx^2\right) \left(c+dx^2\right)^3 \, dx = \text{Too large to display}$$

input
```
integrate((e*x)^m*(b*x^2+a)^3*(B*x^2+A)*(d*x^2+c)^3,x, algorithm="fricas")
```

output
```
((B*b^3*d^3*m^7 + 49*B*b^3*d^3*m^6 + 973*B*b^3*d^3*m^5 + 10045*B*b^3*d^3*m
^4 + 57379*B*b^3*d^3*m^3 + 177331*B*b^3*d^3*m^2 + 264207*B*b^3*d^3*m + 135
135*B*b^3*d^3)*x^15 + ((3*B*b^3*c*d^2 + (3*B*a*b^2 + A*b^3)*d^3)*m^7 + 467
775*B*b^3*c*d^2 + 51*(3*B*b^3*c*d^2 + (3*B*a*b^2 + A*b^3)*d^3)*m^6 + 1045*
(3*B*b^3*c*d^2 + (3*B*a*b^2 + A*b^3)*d^3)*m^5 + 11055*(3*B*b^3*c*d^2 + (3*
B*a*b^2 + A*b^3)*d^3)*m^4 + 155925*(3*B*a*b^2 + A*b^3)*d^3 + 64339*(3*B*b^
3*c*d^2 + (3*B*a*b^2 + A*b^3)*d^3)*m^3 + 201609*(3*B*b^3*c*d^2 + (3*B*a*b^
2 + A*b^3)*d^3)*m^2 + 303255*(3*B*b^3*c*d^2 + (3*B*a*b^2 + A*b^3)*d^3)*m)*
x^13 + 3*((B*b^3*c^2*d + (3*B*a*b^2 + A*b^3)*c*d^2 + (B*a^2*b + A*a*b^2)*d
^3)*m^7 + 184275*B*b^3*c^2*d + 53*(B*b^3*c^2*d + (3*B*a*b^2 + A*b^3)*c*d^2
 + (B*a^2*b + A*a*b^2)*d^3)*m^6 + 1125*(B*b^3*c^2*d + (3*B*a*b^2 + A*b^3)*
c*d^2 + (B*a^2*b + A*a*b^2)*d^3)*m^5 + 12265*(B*b^3*c^2*d + (3*B*a*b^2 + A
*b^3)*c*d^2 + (B*a^2*b + A*a*b^2)*d^3)*m^4 + 184275*(3*B*a*b^2 + A*b^3)*c*
d^2 + 184275*(B*a^2*b + A*a*b^2)*d^3 + 73139*(B*b^3*c^2*d + (3*B*a*b^2 + A
*b^3)*c*d^2 + (B*a^2*b + A*a*b^2)*d^3)*m^3 + 233487*(B*b^3*c^2*d + (3*B*a*
b^2 + A*b^3)*c*d^2 + (B*a^2*b + A*a*b^2)*d^3)*m^2 + 355815*(B*b^3*c^2*d +
(3*B*a*b^2 + A*b^3)*c*d^2 + (B*a^2*b + A*a*b^2)*d^3)*m)*x^11 + ((B*b^3*c^3
 + 3*(3*B*a*b^2 + A*b^3)*c^2*d + 9*(B*a^2*b + A*a*b^2)*c*d^2 + (B*a^3 + 3*
A*a^2*b)*d^3)*m^7 + 225225*B*b^3*c^3 + 55*(B*b^3*c^3 + 3*(3*B*a*b^2 + A*b^
3)*c^2*d + 9*(B*a^2*b + A*a*b^2)*c*d^2 + (B*a^3 + 3*A*a^2*b)*d^3)*m^6 +...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 20086 vs. 2(377) = 754.

Time = 1.85 (sec) , antiderivative size = 20086, normalized size of antiderivative = 53.00

$$\int (ex)^m \left(a+bx^2\right)^3 \left(A+Bx^2\right) \left(c+dx^2\right)^3 \, dx = \text{Too large to display}$$

input
```
integrate((e*x)**m*(b*x**2+a)**3*(B*x**2+A)*(d*x**2+c)**3,x)
```

output
```
Piecewise(((-A*a**3*c**3/(14*x**14) - A*a**3*c**2*d/(4*x**12) - 3*A*a**3*c
*d**2/(10*x**10) - A*a**3*d**3/(8*x**8) - A*a**2*b*c**3/(4*x**12) - 9*A*a*
*2*b*c**2*d/(10*x**10) - 9*A*a**2*b*c*d**2/(8*x**8) - A*a**2*b*d**3/(2*x**
6) - 3*A*a*b**2*c**3/(10*x**10) - 9*A*a*b**2*c**2*d/(8*x**8) - 3*A*a*b**2*
c*d**2/(2*x**6) - 3*A*a*b**2*d**3/(4*x**4) - A*b**3*c**3/(8*x**8) - A*b**3
*c**2*d/(2*x**6) - 3*A*b**3*c*d**2/(4*x**4) - A*b**3*d**3/(2*x**2) - B*a**
3*c**3/(12*x**12) - 3*B*a**3*c**2*d/(10*x**10) - 3*B*a**3*c*d**2/(8*x**8)
- B*a**3*d**3/(6*x**6) - 3*B*a**2*b*c**3/(10*x**10) - 9*B*a**2*b*c**2*d/(8
*x**8) - 3*B*a**2*b*c*d**2/(2*x**6) - 3*B*a**2*b*d**3/(4*x**4) - 3*B*a*b**
2*c**3/(8*x**8) - 3*B*a*b**2*c**2*d/(2*x**6) - 9*B*a*b**2*c*d**2/(4*x**4)
- 3*B*a*b**2*d**3/(2*x**2) - B*b**3*c**3/(6*x**6) - 3*B*b**3*c**2*d/(4*x**
4) - 3*B*b**3*c*d**2/(2*x**2) + B*b**3*d**3*log(x))/e**15, Eq(m, -15)), ((
-A*a**3*c**3/(12*x**12) - 3*A*a**3*c**2*d/(10*x**10) - 3*A*a**3*c*d**2/(8*
x**8) - A*a**3*d**3/(6*x**6) - 3*A*a**2*b*c**3/(10*x**10) - 9*A*a**2*b*c**
2*d/(8*x**8) - 3*A*a**2*b*c*d**2/(2*x**6) - 3*A*a**2*b*d**3/(4*x**4) - 3*A
*a*b**2*c**3/(8*x**8) - 3*A*a*b**2*c**2*d/(2*x**6) - 9*A*a*b**2*c*d**2/(4*
x**4) - 3*A*a*b**2*d**3/(2*x**2) - A*b**3*c**3/(6*x**6) - 3*A*b**3*c**2*d/
(4*x**4) - 3*A*b**3*c*d**2/(2*x**2) + A*b**3*d**3*log(x) - B*a**3*c**3/(10
*x**10) - 3*B*a**3*c**2*d/(8*x**8) - B*a**3*c*d**2/(2*x**6) - B*a**3*d**3/
(4*x**4) - 3*B*a**2*b*c**3/(8*x**8) - 3*B*a**2*b*c**2*d/(2*x**6) - 9*B*...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 762 vs. $2(379) = 758$.

Time = 0.13 (sec) , antiderivative size = 762, normalized size of antiderivative = 2.01

$$\int (ex)^m (a+bx^2)^3 (A+Bx^2) (c+dx^2)^3 \, dx$$
$$= \frac{Bb^3d^3e^mx^{15}x^m}{m+15} + \frac{3\,Bb^3cd^2e^mx^{13}x^m}{m+13} + \frac{3\,Bab^2d^3e^mx^{13}x^m}{m+13} + \frac{Ab^3d^3e^mx^{13}x^m}{m+13} + \frac{3\,Bb^3c^2de^mx^{11}x^m}{m+11} + \frac{9\,Bab^2cd^2e^mx^{11}x^m}{m+11} + \frac{3\,Ab^3cd^2e^mx^{11}x^m}{m+11} + \frac{3\,Ba^2bd^3e^mx^{11}x^m}{m+11} + \frac{3\,Aab^2d^3e^mx^{11}x^m}{m+11} + \frac{Bb^3c^3e^mx^9x^m}{m+9} + \frac{9\,Bab^2c^2de^mx^9x^m}{m+9} + \frac{3\,Ab^3c^2de^mx^9x^m}{m+9} + \frac{9\,Ba^2bcd^2e^mx^9x^m}{m+9} + \frac{9\,Aab^2cd^2e^mx^9x^m}{m+9} + \frac{Ba^3d^3e^mx^9x^m}{m+9} + \frac{3\,Aa^2bd^3e^mx^9x^m}{m+9} + \frac{3\,Bab^2c^3e^mx^7x^m}{m+7} + \frac{Ab^3c^3e^mx^7x^m}{m+7} + \frac{9\,Ba^2bc^2de^mx^7x^m}{m+7} + \frac{9\,Aab^2c^2de^mx^7x^m}{m+7} + \frac{3\,Ba^3cd^2e^mx^7x^m}{m+7} + \frac{9\,Aa^2bcd^2e^mx^7x^m}{m+7} + \frac{Aa^3d^3e^mx^7x^m}{m+7} + \frac{3\,Ba^2bc^3e^mx^5x^m}{m+5} + \frac{3\,Aab^2c^3e^mx^5x^m}{m+5} + \frac{3\,Ba^3c^2de^mx^5x^m}{m+5} + \frac{9\,Aa^2bc^2de^mx^5x^m}{m+5} + \frac{3\,Aa^3cd^2e^mx^5x^m}{m+5} + \frac{Ba^3c^3e^mx^3x^m}{m+3} + \frac{3\,Aa^2bc^3e^mx^3x^m}{m+3} + \frac{3\,Aa^3c^2de^mx^3x^m}{m+3} + \frac{(ex)^{m+1}Aa^3c^3}{e(m+1)}$$

input
```
integrate((e*x)^m*(b*x^2+a)^3*(B*x^2+A)*(d*x^2+c)^3,x, algorithm="maxima")
```

output
```
B*b^3*d^3*e^m*x^15*x^m/(m + 15) + 3*B*b^3*c*d^2*e^m*x^13*x^m/(m + 13) + 3*
B*a*b^2*d^3*e^m*x^13*x^m/(m + 13) + A*b^3*d^3*e^m*x^13*x^m/(m + 13) + 3*B*
b^3*c^2*d*e^m*x^11*x^m/(m + 11) + 9*B*a*b^2*c*d^2*e^m*x^11*x^m/(m + 11) +
3*A*b^3*c*d^2*e^m*x^11*x^m/(m + 11) + 3*B*a^2*b*d^3*e^m*x^11*x^m/(m + 11)
+ 3*A*a*b^2*d^3*e^m*x^11*x^m/(m + 11) + B*b^3*c^3*e^m*x^9*x^m/(m + 9) + 9*
B*a*b^2*c^2*d*e^m*x^9*x^m/(m + 9) + 3*A*b^3*c^2*d*e^m*x^9*x^m/(m + 9) + 9*
B*a^2*b*c*d^2*e^m*x^9*x^m/(m + 9) + 9*A*a*b^2*c*d^2*e^m*x^9*x^m/(m + 9) +
B*a^3*d^3*e^m*x^9*x^m/(m + 9) + 3*A*a^2*b*d^3*e^m*x^9*x^m/(m + 9) + 3*B*a*
b^2*c^3*e^m*x^7*x^m/(m + 7) + A*b^3*c^3*e^m*x^7*x^m/(m + 7) + 9*B*a^2*b*c^
2*d*e^m*x^7*x^m/(m + 7) + 9*A*a*b^2*c^2*d*e^m*x^7*x^m/(m + 7) + 3*B*a^3*c*
d^2*e^m*x^7*x^m/(m + 7) + 9*A*a^2*b*c*d^2*e^m*x^7*x^m/(m + 7) + A*a^3*d^3*
e^m*x^7*x^m/(m + 7) + 3*B*a^2*b*c^3*e^m*x^5*x^m/(m + 5) + 3*A*a*b^2*c^3*e^
m*x^5*x^m/(m + 5) + 3*B*a^3*c^2*d*e^m*x^5*x^m/(m + 5) + 9*A*a^2*b*c^2*d*e^
m*x^5*x^m/(m + 5) + 3*A*a^3*c*d^2*e^m*x^5*x^m/(m + 5) + B*a^3*c^3*e^m*x^3*
x^m/(m + 3) + 3*A*a^2*b*c^3*e^m*x^3*x^m/(m + 3) + 3*A*a^3*c^2*d*e^m*x^3*x^
m/(m + 3) + (e*x)^(m + 1)*A*a^3*c^3/(e*(m + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5234 vs. $2(379) = 758$.

Time = 0.20 (sec) , antiderivative size = 5234, normalized size of antiderivative = 13.81

$$\int (ex)^m \left(a+bx^2\right)^3 \left(A+Bx^2\right) \left(c+dx^2\right)^3 \, dx = \text{Too large to display}$$

input
```
integrate((e*x)^m*(b*x^2+a)^3*(B*x^2+A)*(d*x^2+c)^3,x, algorithm="giac")
```

output
```
((e*x)^m*B*b^3*d^3*m^7*x^15 + 49*(e*x)^m*B*b^3*d^3*m^6*x^15 + 3*(e*x)^m*B*
b^3*c*d^2*m^7*x^13 + 3*(e*x)^m*B*a*b^2*d^3*m^7*x^13 + (e*x)^m*A*b^3*d^3*m^
7*x^13 + 973*(e*x)^m*B*b^3*d^3*m^5*x^15 + 153*(e*x)^m*B*b^3*c*d^2*m^6*x^13
 + 153*(e*x)^m*B*a*b^2*d^3*m^6*x^13 + 51*(e*x)^m*A*b^3*d^3*m^6*x^13 + 1004
5*(e*x)^m*B*b^3*d^3*m^4*x^15 + 3*(e*x)^m*B*b^3*c^2*d*m^7*x^11 + 9*(e*x)^m*
B*a*b^2*c*d^2*m^7*x^11 + 3*(e*x)^m*A*b^3*c*d^2*m^7*x^11 + 3*(e*x)^m*B*a^2*
b*d^3*m^7*x^11 + 3*(e*x)^m*A*a*b^2*d^3*m^7*x^11 + 3135*(e*x)^m*B*b^3*c*d^2
*m^5*x^13 + 3135*(e*x)^m*B*a*b^2*d^3*m^5*x^13 + 1045*(e*x)^m*A*b^3*d^3*m^5
*x^13 + 57379*(e*x)^m*B*b^3*d^3*m^3*x^15 + 159*(e*x)^m*B*b^3*c^2*d*m^6*x^1
1 + 477*(e*x)^m*B*a*b^2*c*d^2*m^6*x^11 + 159*(e*x)^m*A*b^3*c*d^2*m^6*x^11
+ 159*(e*x)^m*B*a^2*b*d^3*m^6*x^11 + 159*(e*x)^m*A*a*b^2*d^3*m^6*x^11 + 33
165*(e*x)^m*B*b^3*c*d^2*m^4*x^13 + 33165*(e*x)^m*B*a*b^2*d^3*m^4*x^13 + 11
055*(e*x)^m*A*b^3*d^3*m^4*x^13 + 177331*(e*x)^m*B*b^3*d^3*m^2*x^15 + (e*x)
^m*B*b^3*c^3*m^7*x^9 + 9*(e*x)^m*B*a*b^2*c^2*d*m^7*x^9 + 3*(e*x)^m*A*b^3*c
^2*d*m^7*x^9 + 9*(e*x)^m*B*a^2*b*c*d^2*m^7*x^9 + 9*(e*x)^m*A*a*b^2*c*d^2*m
^7*x^9 + (e*x)^m*B*a^3*d^3*m^7*x^9 + 3*(e*x)^m*A*a^2*b*d^3*m^7*x^9 + 3375*
(e*x)^m*B*b^3*c^2*d*m^5*x^11 + 10125*(e*x)^m*B*a*b^2*c*d^2*m^5*x^11 + 3375
*(e*x)^m*A*b^3*c*d^2*m^5*x^11 + 3375*(e*x)^m*B*a^2*b*d^3*m^5*x^11 + 3375*(
e*x)^m*A*a*b^2*d^3*m^5*x^11 + 193017*(e*x)^m*B*b^3*c*d^2*m^3*x^13 + 193017
*(e*x)^m*B*a*b^2*d^3*m^3*x^13 + 64339*(e*x)^m*A*b^3*d^3*m^3*x^13 + 2642...
```

Mupad [B] (verification not implemented)

Time = 0.84 (sec) , antiderivative size = 933, normalized size of antiderivative = 2.46

$$\int (ex)^m \left(a+bx^2\right)^3 \left(A+Bx^2\right) \left(c+dx^2\right)^3 \, dx = \text{Too large to display}$$

input
```
int((A + B*x^2)*(e*x)^m*(a + b*x^2)^3*(c + d*x^2)^3,x)
```

output
```
(x^7*(e*x)^m*(A*a^3*d^3 + A*b^3*c^3 + 3*B*a*b^2*c^3 + 3*B*a^3*c*d^2 + 9*A*
a*b^2*c^2*d + 9*A*a^2*b*c*d^2 + 9*B*a^2*b*c^2*d)*(544095*m + 340011*m^2 +
99715*m^3 + 15477*m^4 + 1309*m^5 + 57*m^6 + m^7 + 289575))/(4098240*m + 29
24172*m^2 + 1038016*m^3 + 208054*m^4 + 24640*m^5 + 1708*m^6 + 64*m^7 + m^8
 + 2027025) + (x^9*(e*x)^m*(B*a^3*d^3 + B*b^3*c^3 + 3*A*a^2*b*d^3 + 3*A*b^
3*c^2*d + 9*A*a*b^2*c*d^2 + 9*B*a*b^2*c^2*d + 9*B*a^2*b*c*d^2)*(430335*m +
 277093*m^2 + 84547*m^3 + 13723*m^4 + 1213*m^5 + 55*m^6 + m^7 + 225225))/(
4098240*m + 2924172*m^2 + 1038016*m^3 + 208054*m^4 + 24640*m^5 + 1708*m^6
+ 64*m^7 + m^8 + 2027025) + (B*b^3*d^3*x^15*(e*x)^m*(264207*m + 177331*m^2
 + 57379*m^3 + 10045*m^4 + 973*m^5 + 49*m^6 + m^7 + 135135))/(4098240*m +
2924172*m^2 + 1038016*m^3 + 208054*m^4 + 24640*m^5 + 1708*m^6 + 64*m^7 + m
^8 + 2027025) + (3*a*c*x^5*(e*x)^m*(A*a^2*d^2 + A*b^2*c^2 + B*a*b*c^2 + B*
a^2*c*d + 3*A*a*b*c*d)*(738567*m + 437121*m^2 + 120179*m^3 + 17575*m^4 + 1
413*m^5 + 59*m^6 + m^7 + 405405))/(4098240*m + 2924172*m^2 + 1038016*m^3 +
 208054*m^4 + 24640*m^5 + 1708*m^6 + 64*m^7 + m^8 + 2027025) + (3*b*d*x^11
*(e*x)^m*(B*a^2*d^2 + B*b^2*c^2 + A*a*b*d^2 + A*b^2*c*d + 3*B*a*b*c*d)*(35
5815*m + 233487*m^2 + 73139*m^3 + 12265*m^4 + 1125*m^5 + 53*m^6 + m^7 + 18
4275))/(4098240*m + 2924172*m^2 + 1038016*m^3 + 208054*m^4 + 24640*m^5 + 1
708*m^6 + 64*m^7 + m^8 + 2027025) + (a^2*c^2*x^3*(e*x)^m*(3*A*a*d + 3*A*b*
c + B*a*c)*(1140855*m + 594439*m^2 + 147859*m^3 + 20065*m^4 + 1525*m^5 ...
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 2401, normalized size of antiderivative = 6.34

$$\int (ex)^m \left(a+bx^2\right)^3 \left(A+Bx^2\right) \left(c+dx^2\right)^3 \, dx = \text{Too large to display}$$

input
```
int((e*x)^m*(b*x^2+a)^3*(B*x^2+A)*(d*x^2+c)^3,x)
```

output

```
(x**m*e**m*x*(a**4*c**3*m**7 + 63*a**4*c**3*m**6 + 1645*a**4*c**3*m**5 + 2
2995*a**4*c**3*m**4 + 185059*a**4*c**3*m**3 + 852957*a**4*c**3*m**2 + 2071
215*a**4*c**3*m + 2027025*a**4*c**3 + 3*a**4*c**2*d*m**7*x**2 + 183*a**4*c
**2*d*m**6*x**2 + 4575*a**4*c**2*d*m**5*x**2 + 60195*a**4*c**2*d*m**4*x**2
 + 443577*a**4*c**2*d*m**3*x**2 + 1783317*a**4*c**2*d*m**2*x**2 + 3422565*
a**4*c**2*d*m*x**2 + 2027025*a**4*c**2*d*x**2 + 3*a**4*c*d**2*m**7*x**4 +
177*a**4*c*d**2*m**6*x**4 + 4239*a**4*c*d**2*m**5*x**4 + 52725*a**4*c*d**2
*m**4*x**4 + 360537*a**4*c*d**2*m**3*x**4 + 1311363*a**4*c*d**2*m**2*x**4
+ 2215701*a**4*c*d**2*m*x**4 + 1216215*a**4*c*d**2*x**4 + a**4*d**3*m**7*x
**6 + 57*a**4*d**3*m**6*x**6 + 1309*a**4*d**3*m**5*x**6 + 15477*a**4*d**3*
m**4*x**6 + 99715*a**4*d**3*m**3*x**6 + 340011*a**4*d**3*m**2*x**6 + 54409
5*a**4*d**3*m*x**6 + 289575*a**4*d**3*x**6 + 4*a**3*b*c**3*m**7*x**2 + 244
*a**3*b*c**3*m**6*x**2 + 6100*a**3*b*c**3*m**5*x**2 + 80260*a**3*b*c**3*m*
*4*x**2 + 591436*a**3*b*c**3*m**3*x**2 + 2377756*a**3*b*c**3*m**2*x**2 + 4
563420*a**3*b*c**3*m*x**2 + 2702700*a**3*b*c**3*x**2 + 12*a**3*b*c**2*d*m*
*7*x**4 + 708*a**3*b*c**2*d*m**6*x**4 + 16956*a**3*b*c**2*d*m**5*x**4 + 21
0900*a**3*b*c**2*d*m**4*x**4 + 1442148*a**3*b*c**2*d*m**3*x**4 + 5245452*a
**3*b*c**2*d*m**2*x**4 + 8862804*a**3*b*c**2*d*m*x**4 + 4864860*a**3*b*c**
2*d*x**4 + 12*a**3*b*c*d**2*m**7*x**6 + 684*a**3*b*c*d**2*m**6*x**6 + 1570
8*a**3*b*c*d**2*m**5*x**6 + 185724*a**3*b*c*d**2*m**4*x**6 + 1196580*a*...
```

3.16 $\int (ex)^m \left(a+bx^2\right)^2 \left(A+Bx^2\right) \left(c+dx^2\right)^3 dx$

Optimal result

Integrand size = 31, antiderivative size = 284

$$\begin{aligned}
&\int (ex)^m \left(a+bx^2\right)^2 \left(A+Bx^2\right) \left(c+dx^2\right)^3 dx \\
&= \frac{a^2Ac^3(ex)^{1+m}}{e(1+m)} + \frac{ac^2(2Abc+aBc+3aAd)(ex)^{3+m}}{e^3(3+m)} \\
&\quad + \frac{c(aBc(2bc+3ad)+A(b^2c^2+6abcd+3a^2d^2))\,(ex)^{5+m}}{e^5(5+m)} \\
&\quad + \frac{(6abcd(Bc+Ad)+a^2d^2(3Bc+Ad)+b^2c^2(Bc+3Ad))\,(ex)^{7+m}}{e^7(7+m)} \\
&\quad + \frac{d(a^2Bd^2+3b^2c(Bc+Ad)+2abd(3Bc+Ad))\,(ex)^{9+m}}{e^9(9+m)} \\
&\quad + \frac{bd^2(3bBc+Abd+2aBd)(ex)^{11+m}}{e^{11}(11+m)} + \frac{b^2Bd^3(ex)^{13+m}}{e^{13}(13+m)}
\end{aligned}$$

output
```
a^2*A*c^3*(e*x)^(1+m)/e/(1+m)+a*c^2*(3*A*a*d+2*A*b*c+B*a*c)*(e*x)^(3+m)/e^
3/(3+m)+c*(a*B*c*(3*a*d+2*b*c)+A*(3*a^2*d^2+6*a*b*c*d+b^2*c^2))*(e*x)^(5+m
)/e^5/(5+m)+(6*a*b*c*d*(A*d+B*c)+a^2*d^2*(A*d+3*B*c)+b^2*c^2*(3*A*d+B*c))*
(e*x)^(7+m)/e^7/(7+m)+d*(a^2*B*d^2+3*b^2*c*(A*d+B*c)+2*a*b*d*(A*d+3*B*c))*
(e*x)^(9+m)/e^9/(9+m)+b*d^2*(A*b*d+2*B*a*d+3*B*b*c)*(e*x)^(11+m)/e^11/(11+
m)+b^2*B*d^3*(e*x)^(13+m)/e^13/(13+m)
```

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.84

$$\int (ex)^m \left(a+bx^2\right)^2 \left(A+Bx^2\right) \left(c+dx^2\right)^3 \, dx$$
$$= x(ex)^m \left(\frac{a^2Ac^3}{1+m} + \frac{ac^2(2Abc+aBc+3aAd)x^2}{3+m} + \frac{c(aBc(2bc+3ad)+A(b^2c^2+6abcd+3a^2d^2))\,x^4}{5+m} + \frac{(6abcd(Bc+Ad)+a^2d^2(3Bc+Ad)+b^2c^2(Bc+3Ad))\,x^6}{7+m} + \frac{d(a^2Bd^2+3b^2c(Bc+Ad)+2abd(3Bc+Ad))\,x^8}{9+m} + \frac{bd^2(3bBc+Abd+2aBd)x^{10}}{11+m} + \frac{b^2Bd^3x^{12}}{13+m}\right)$$

input
```
Integrate[(e*x)^m*(a + b*x^2)^2*(A + B*x^2)*(c + d*x^2)^3,x]
```

output
```
x*(e*x)^m*((a^2*A*c^3)/(1 + m) + (a*c^2*(2*A*b*c + a*B*c + 3*a*A*d)*x^2)/(
3 + m) + (c*(a*B*c*(2*b*c + 3*a*d) + A*(b^2*c^2 + 6*a*b*c*d + 3*a^2*d^2))*
x^4)/(5 + m) + ((6*a*b*c*d*(B*c + A*d) + a^2*d^2*(3*B*c + A*d) + b^2*c^2*(
B*c + 3*A*d))*x^6)/(7 + m) + (d*(a^2*B*d^2 + 3*b^2*c*(B*c + A*d) + 2*a*b*d
*(3*B*c + A*d))*x^8)/(9 + m) + (b*d^2*(3*b*B*c + A*b*d + 2*a*B*d)*x^10)/(1
1 + m) + (b^2*B*d^3*x^12)/(13 + m))
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {437, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^2 \left(A+Bx^2\right) \left(c+dx^2\right)^3 (ex)^m \, dx$$

$$\Big\downarrow \text{437}$$

$$\int \left(\frac{(ex)^{m+6}\left(a^2d^2(Ad+3Bc)+6abcd(Ad+Bc)+b^2c^2(3Ad+Bc)\right)}{e^6}+\frac{c(ex)^{m+4}\left(A\left(3a^2d^2+6abcd+b^2c^2\right)+\right.}{e^4}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{(ex)^{m+7}\left(a^2d^2(Ad+3Bc)+6abcd(Ad+Bc)+b^2c^2(3Ad+Bc)\right)}{e^7(m+7)}+\frac{c(ex)^{m+5}\left(A\left(3a^2d^2+6abcd+b^2c^2\right)+aBc(3ad+2bc)\right)}{e^5(m+5)}+\frac{d(ex)^{m+9}\left(a^2Bd^2+2abd(Ad+3Bc)+3b^2c(Ad+Bc)\right)}{e^9(m+9)}+\frac{a^2Ac^3(ex)^{m+1}}{e(m+1)}+\frac{ac^2(ex)^{m+3}(3aAd+aBc+2Abc)}{e^3(m+3)}+\frac{bd^2(ex)^{m+11}(2aBd+Abd+3bBc)}{e^{11}(m+11)}+\frac{b^2Bd^3(ex)^{m+13}}{e^{13}(m+13)}$$

input
```
Int[(e*x)^m*(a + b*x^2)^2*(A + B*x^2)*(c + d*x^2)^3,x]
```

output
```
(a^2*A*c^3*(e*x)^(1 + m))/(e*(1 + m)) + (a*c^2*(2*A*b*c + a*B*c + 3*a*A*d)
*(e*x)^(3 + m))/(e^3*(3 + m)) + (c*(a*B*c*(2*b*c + 3*a*d) + A*(b^2*c^2 + 6
*a*b*c*d + 3*a^2*d^2))*(e*x)^(5 + m))/(e^5*(5 + m)) + ((6*a*b*c*d*(B*c + A
*d) + a^2*d^2*(3*B*c + A*d) + b^2*c^2*(B*c + 3*A*d))*(e*x)^(7 + m))/(e^7*(
7 + m)) + (d*(a^2*B*d^2 + 3*b^2*c*(B*c + A*d) + 2*a*b*d*(3*B*c + A*d))*(e*
x)^(9 + m))/(e^9*(9 + m)) + (b*d^2*(3*b*B*c + A*b*d + 2*a*B*d)*(e*x)^(11 +
 m))/(e^11*(11 + m)) + (b^2*B*d^3*(e*x)^(13 + m))/(e^13*(13 + m))
```

Defintions of rubi rules used

rule 437
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(
a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f
, g, m}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2442 vs. $2(284) = 568$.

Time = 0.63 (sec) , antiderivative size = 2443, normalized size of antiderivative = 8.60

method	result	size
gosper	Expression too large to display	2443
risch	Expression too large to display	2443
orering	Expression too large to display	2443
parallelrisch	Expression too large to display	3284

input
```
int((e*x)^m*(b*x^2+a)^2*(B*x^2+A)*(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
x*(B*b^2*d^3*m^6*x^12+36*B*b^2*d^3*m^5*x^12+A*b^2*d^3*m^6*x^10+2*B*a*b*d^3
*m^6*x^10+3*B*b^2*c*d^2*m^6*x^10+505*B*b^2*d^3*m^4*x^12+38*A*b^2*d^3*m^5*x
^10+76*B*a*b*d^3*m^5*x^10+114*B*b^2*c*d^2*m^5*x^10+3480*B*b^2*d^3*m^3*x^12
+2*A*a*b*d^3*m^6*x^8+3*A*b^2*c*d^2*m^6*x^8+555*A*b^2*d^3*m^4*x^10+B*a^2*d^
3*m^6*x^8+6*B*a*b*c*d^2*m^6*x^8+1110*B*a*b*d^3*m^4*x^10+3*B*b^2*c^2*d*m^6*
x^8+1665*B*b^2*c*d^2*m^4*x^10+12139*B*b^2*d^3*m^2*x^12+80*A*a*b*d^3*m^5*x^
8+120*A*b^2*c*d^2*m^5*x^8+3940*A*b^2*d^3*m^3*x^10+40*B*a^2*d^3*m^5*x^8+240
*B*a*b*c*d^2*m^5*x^8+7880*B*a*b*d^3*m^3*x^10+120*B*b^2*c^2*d*m^5*x^8+11820
*B*b^2*c*d^2*m^3*x^10+19524*B*b^2*d^3*m*x^12+A*a^2*d^3*m^6*x^6+6*A*a*b*c*d
^2*m^6*x^6+1226*A*a*b*d^3*m^4*x^8+3*A*b^2*c^2*d*m^6*x^6+1839*A*b^2*c*d^2*m
^4*x^8+14039*A*b^2*d^3*m^2*x^10+3*B*a^2*c*d^2*m^6*x^6+613*B*a^2*d^3*m^4*x^
8+6*B*a*b*c^2*d*m^6*x^6+3678*B*a*b*c*d^2*m^4*x^8+28078*B*a*b*d^3*m^2*x^10+
B*b^2*c^3*m^6*x^6+1839*B*b^2*c^2*d*m^4*x^8+42117*B*b^2*c*d^2*m^2*x^10+1039
5*B*b^2*d^3*x^12+42*A*a^2*d^3*m^5*x^6+252*A*a*b*c*d^2*m^5*x^6+9056*A*a*b*d
^3*m^3*x^8+126*A*b^2*c^2*d*m^5*x^6+13584*A*b^2*c*d^2*m^3*x^8+22902*A*b^2*d
^3*m*x^10+126*B*a^2*c*d^2*m^5*x^6+4528*B*a^2*d^3*m^3*x^8+252*B*a*b*c^2*d*m
^5*x^6+27168*B*a*b*c*d^2*m^3*x^8+45804*B*a*b*d^3*m*x^10+42*B*b^2*c^3*m^5*x
^6+13584*B*b^2*c^2*d*m^3*x^8+68706*B*b^2*c*d^2*m*x^10+3*A*a^2*c*d^2*m^6*x^
4+679*A*a^2*d^3*m^4*x^6+6*A*a*b*c^2*d*m^6*x^4+4074*A*a*b*c*d^2*m^4*x^6+332
54*A*a*b*d^3*m^2*x^8+A*b^2*c^3*m^6*x^4+2037*A*b^2*c^2*d*m^4*x^6+49881*A...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1690 vs. $2(284) = 568$.

Time = 0.11 (sec) , antiderivative size = 1690, normalized size of antiderivative = 5.95

$$\int (ex)^m (a + bx^2)^2 (A + Bx^2) (c + dx^2)^3 \, dx = \text{Too large to display}$$

input
```
integrate((e*x)^m*(b*x^2+a)^2*(B*x^2+A)*(d*x^2+c)^3,x, algorithm="fricas")
```

output
```
((B*b^2*d^3*m^6 + 36*B*b^2*d^3*m^5 + 505*B*b^2*d^3*m^4 + 3480*B*b^2*d^3*m^
3 + 12139*B*b^2*d^3*m^2 + 19524*B*b^2*d^3*m + 10395*B*b^2*d^3)*x^13 + ((3*
B*b^2*c*d^2 + (2*B*a*b + A*b^2)*d^3)*m^6 + 36855*B*b^2*c*d^2 + 38*(3*B*b^2
*c*d^2 + (2*B*a*b + A*b^2)*d^3)*m^5 + 555*(3*B*b^2*c*d^2 + (2*B*a*b + A*b^
2)*d^3)*m^4 + 12285*(2*B*a*b + A*b^2)*d^3 + 3940*(3*B*b^2*c*d^2 + (2*B*a*b
 + A*b^2)*d^3)*m^3 + 14039*(3*B*b^2*c*d^2 + (2*B*a*b + A*b^2)*d^3)*m^2 + 2
2902*(3*B*b^2*c*d^2 + (2*B*a*b + A*b^2)*d^3)*m)*x^11 + ((3*B*b^2*c^2*d + 3
*(2*B*a*b + A*b^2)*c*d^2 + (B*a^2 + 2*A*a*b)*d^3)*m^6 + 45045*B*b^2*c^2*d
+ 40*(3*B*b^2*c^2*d + 3*(2*B*a*b + A*b^2)*c*d^2 + (B*a^2 + 2*A*a*b)*d^3)*m
^5 + 613*(3*B*b^2*c^2*d + 3*(2*B*a*b + A*b^2)*c*d^2 + (B*a^2 + 2*A*a*b)*d^
3)*m^4 + 45045*(2*B*a*b + A*b^2)*c*d^2 + 15015*(B*a^2 + 2*A*a*b)*d^3 + 452
8*(3*B*b^2*c^2*d + 3*(2*B*a*b + A*b^2)*c*d^2 + (B*a^2 + 2*A*a*b)*d^3)*m^3
+ 16627*(3*B*b^2*c^2*d + 3*(2*B*a*b + A*b^2)*c*d^2 + (B*a^2 + 2*A*a*b)*d^3
)*m^2 + 27688*(3*B*b^2*c^2*d + 3*(2*B*a*b + A*b^2)*c*d^2 + (B*a^2 + 2*A*a*
b)*d^3)*m)*x^9 + ((B*b^2*c^3 + A*a^2*d^3 + 3*(2*B*a*b + A*b^2)*c^2*d + 3*(
B*a^2 + 2*A*a*b)*c*d^2)*m^6 + 19305*B*b^2*c^3 + 19305*A*a^2*d^3 + 42*(B*b^
2*c^3 + A*a^2*d^3 + 3*(2*B*a*b + A*b^2)*c^2*d + 3*(B*a^2 + 2*A*a*b)*c*d^2)
*m^5 + 679*(B*b^2*c^3 + A*a^2*d^3 + 3*(2*B*a*b + A*b^2)*c^2*d + 3*(B*a^2 +
 2*A*a*b)*c*d^2)*m^4 + 57915*(2*B*a*b + A*b^2)*c^2*d + 57915*(B*a^2 + 2*A*
a*b)*c*d^2 + 5292*(B*b^2*c^3 + A*a^2*d^3 + 3*(2*B*a*b + A*b^2)*c^2*d + ...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11914 vs. $2(284) = 568$.

Time = 1.33 (sec) , antiderivative size = 11914, normalized size of antiderivative = 41.95

$$\int (ex)^m \left(a+bx^2\right)^2 \left(A+Bx^2\right) \left(c+dx^2\right)^3 \, dx = \text{Too large to display}$$

input
```
integrate((e*x)**m*(b*x**2+a)**2*(B*x**2+A)*(d*x**2+c)**3,x)
```

output
```
Piecewise(((-A*a**2*c**3/(12*x**12) - 3*A*a**2*c**2*d/(10*x**10) - 3*A*a**
2*c*d**2/(8*x**8) - A*a**2*d**3/(6*x**6) - A*a*b*c**3/(5*x**10) - 3*A*a*b*
c**2*d/(4*x**8) - A*a*b*c*d**2/x**6 - A*a*b*d**3/(2*x**4) - A*b**2*c**3/(8
*x**8) - A*b**2*c**2*d/(2*x**6) - 3*A*b**2*c*d**2/(4*x**4) - A*b**2*d**3/(
2*x**2) - B*a**2*c**3/(10*x**10) - 3*B*a**2*c**2*d/(8*x**8) - B*a**2*c*d**
2/(2*x**6) - B*a**2*d**3/(4*x**4) - B*a*b*c**3/(4*x**8) - B*a*b*c**2*d/x**
6 - 3*B*a*b*c*d**2/(2*x**4) - B*a*b*d**3/x**2 - B*b**2*c**3/(6*x**6) - 3*B
*b**2*c**2*d/(4*x**4) - 3*B*b**2*c*d**2/(2*x**2) + B*b**2*d**3*log(x))/e**
13, Eq(m, -13)), ((-A*a**2*c**3/(10*x**10) - 3*A*a**2*c**2*d/(8*x**8) - A*
a**2*c*d**2/(2*x**6) - A*a**2*d**3/(4*x**4) - A*a*b*c**3/(4*x**8) - A*a*b*
c**2*d/x**6 - 3*A*a*b*c*d**2/(2*x**4) - A*a*b*d**3/x**2 - A*b**2*c**3/(6*x
**6) - 3*A*b**2*c**2*d/(4*x**4) - 3*A*b**2*c*d**2/(2*x**2) + A*b**2*d**3*l
og(x) - B*a**2*c**3/(8*x**8) - B*a**2*c**2*d/(2*x**6) - 3*B*a**2*c*d**2/(4
*x**4) - B*a**2*d**3/(2*x**2) - B*a*b*c**3/(3*x**6) - 3*B*a*b*c**2*d/(2*x*
*4) - 3*B*a*b*c*d**2/x**2 + 2*B*a*b*d**3*log(x) - B*b**2*c**3/(4*x**4) - 3
*B*b**2*c**2*d/(2*x**2) + 3*B*b**2*c*d**2*log(x) + B*b**2*d**3*x**2/2)/e**
11, Eq(m, -11)), ((-A*a**2*c**3/(8*x**8) - A*a**2*c**2*d/(2*x**6) - 3*A*a*
*2*c*d**2/(4*x**4) - A*a**2*d**3/(2*x**2) - A*a*b*c**3/(3*x**6) - 3*A*a*b*
c**2*d/(2*x**4) - 3*A*a*b*c*d**2/x**2 + 2*A*a*b*d**3*log(x) - A*b**2*c**3/
(4*x**4) - 3*A*b**2*c**2*d/(2*x**2) + 3*A*b**2*c*d**2*log(x) + A*b**2*d...
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 550, normalized size of antiderivative = 1.94

$$\int (ex)^m \left(a+bx^2\right)^2 \left(A+Bx^2\right) \left(c+dx^2\right)^3 \, dx = \frac{Bb^2d^3e^m x^{13}x^m}{m+13} + \frac{3\,Bb^2cd^2e^m x^{11}x^m}{m+11} + \frac{2\,Babd^3e^m x^{11}x^m}{m+11} + \frac{Ab^2d^3e^m x^{11}x^m}{m+11} + \frac{3\,Bb^2c^2de^m x^9x^m}{m+9} + \frac{6\,Babcd^2e^m x^9x^m}{m+9} + \frac{3\,Ab^2cd^2e^m x^9x^m}{m+9} + \frac{Ba^2d^3e^m x^9x^m}{m+9} + \frac{2\,Aabd^3e^m x^9x^m}{m+9} + \frac{Bb^2c^3e^m x^7x^m}{m+7} + \frac{6\,Babc^2de^m x^7x^m}{m+7} + \frac{3\,Ab^2c^2de^m x^7x^m}{m+7} + \frac{3\,Ba^2cd^2e^m x^7x^m}{m+7} + \frac{6\,Aabcd^2e^m x^7x^m}{m+7} + \frac{Aa^2d^3e^m x^7x^m}{m+7} + \frac{2\,Babc^3e^m x^5x^m}{m+5} + \frac{Ab^2c^3e^m x^5x^m}{m+5} + \frac{3\,Ba^2c^2de^m x^5x^m}{m+5} + \frac{6\,Aabc^2de^m x^5x^m}{m+5} + \frac{3\,Aa^2cd^2e^m x^5x^m}{m+5} + \frac{Ba^2c^3e^m x^3x^m}{m+3} + \frac{2\,Aabc^3e^m x^3x^m}{m+3} + \frac{3\,Aa^2c^2de^m x^3x^m}{m+3} + \frac{(ex)^{m+1}Aa^2c^3}{e(m+1)}$$

input
```
integrate((e*x)^m*(b*x^2+a)^2*(B*x^2+A)*(d*x^2+c)^3,x, algorithm="maxima")
```

output
```
B*b^2*d^3*e^m*x^13*x^m/(m + 13) + 3*B*b^2*c*d^2*e^m*x^11*x^m/(m + 11) + 2*
B*a*b*d^3*e^m*x^11*x^m/(m + 11) + A*b^2*d^3*e^m*x^11*x^m/(m + 11) + 3*B*b^
2*c^2*d*e^m*x^9*x^m/(m + 9) + 6*B*a*b*c*d^2*e^m*x^9*x^m/(m + 9) + 3*A*b^2*
c*d^2*e^m*x^9*x^m/(m + 9) + B*a^2*d^3*e^m*x^9*x^m/(m + 9) + 2*A*a*b*d^3*e^
m*x^9*x^m/(m + 9) + B*b^2*c^3*e^m*x^7*x^m/(m + 7) + 6*B*a*b*c^2*d*e^m*x^7*
x^m/(m + 7) + 3*A*b^2*c^2*d*e^m*x^7*x^m/(m + 7) + 3*B*a^2*c*d^2*e^m*x^7*x^
m/(m + 7) + 6*A*a*b*c*d^2*e^m*x^7*x^m/(m + 7) + A*a^2*d^3*e^m*x^7*x^m/(m +
 7) + 2*B*a*b*c^3*e^m*x^5*x^m/(m + 5) + A*b^2*c^3*e^m*x^5*x^m/(m + 5) + 3*
B*a^2*c^2*d*e^m*x^5*x^m/(m + 5) + 6*A*a*b*c^2*d*e^m*x^5*x^m/(m + 5) + 3*A*
a^2*c*d^2*e^m*x^5*x^m/(m + 5) + B*a^2*c^3*e^m*x^3*x^m/(m + 3) + 2*A*a*b*c^
3*e^m*x^3*x^m/(m + 3) + 3*A*a^2*c^2*d*e^m*x^3*x^m/(m + 3) + (e*x)^(m + 1)*
A*a^2*c^3/(e*(m + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3283 vs. $2(284) = 568$.

Time = 0.18 (sec) , antiderivative size = 3283, normalized size of antiderivative = 11.56

$$\int (ex)^m \left(a+bx^2\right)^2 \left(A+Bx^2\right) \left(c+dx^2\right)^3 \, dx = \text{Too large to display}$$

input
```
integrate((e*x)^m*(b*x^2+a)^2*(B*x^2+A)*(d*x^2+c)^3,x, algorithm="giac")
```

output
```
((e*x)^m*B*b^2*d^3*m^6*x^13 + 36*(e*x)^m*B*b^2*d^3*m^5*x^13 + 3*(e*x)^m*B*
b^2*c*d^2*m^6*x^11 + 2*(e*x)^m*B*a*b*d^3*m^6*x^11 + (e*x)^m*A*b^2*d^3*m^6*
x^11 + 505*(e*x)^m*B*b^2*d^3*m^4*x^13 + 114*(e*x)^m*B*b^2*c*d^2*m^5*x^11 +
 76*(e*x)^m*B*a*b*d^3*m^5*x^11 + 38*(e*x)^m*A*b^2*d^3*m^5*x^11 + 3480*(e*x
)^m*B*b^2*d^3*m^3*x^13 + 3*(e*x)^m*B*b^2*c^2*d*m^6*x^9 + 6*(e*x)^m*B*a*b*c
*d^2*m^6*x^9 + 3*(e*x)^m*A*b^2*c*d^2*m^6*x^9 + (e*x)^m*B*a^2*d^3*m^6*x^9 +
 2*(e*x)^m*A*a*b*d^3*m^6*x^9 + 1665*(e*x)^m*B*b^2*c*d^2*m^4*x^11 + 1110*(e
*x)^m*B*a*b*d^3*m^4*x^11 + 555*(e*x)^m*A*b^2*d^3*m^4*x^11 + 12139*(e*x)^m*
B*b^2*d^3*m^2*x^13 + 120*(e*x)^m*B*b^2*c^2*d*m^5*x^9 + 240*(e*x)^m*B*a*b*c
*d^2*m^5*x^9 + 120*(e*x)^m*A*b^2*c*d^2*m^5*x^9 + 40*(e*x)^m*B*a^2*d^3*m^5*
x^9 + 80*(e*x)^m*A*a*b*d^3*m^5*x^9 + 11820*(e*x)^m*B*b^2*c*d^2*m^3*x^11 +
7880*(e*x)^m*B*a*b*d^3*m^3*x^11 + 3940*(e*x)^m*A*b^2*d^3*m^3*x^11 + 19524*
(e*x)^m*B*b^2*d^3*m*x^13 + (e*x)^m*B*b^2*c^3*m^6*x^7 + 6*(e*x)^m*B*a*b*c^2
*d*m^6*x^7 + 3*(e*x)^m*A*b^2*c^2*d*m^6*x^7 + 3*(e*x)^m*B*a^2*c*d^2*m^6*x^7
 + 6*(e*x)^m*A*a*b*c*d^2*m^6*x^7 + (e*x)^m*A*a^2*d^3*m^6*x^7 + 1839*(e*x)^
m*B*b^2*c^2*d*m^4*x^9 + 3678*(e*x)^m*B*a*b*c*d^2*m^4*x^9 + 1839*(e*x)^m*A*
b^2*c*d^2*m^4*x^9 + 613*(e*x)^m*B*a^2*d^3*m^4*x^9 + 1226*(e*x)^m*A*a*b*d^3
*m^4*x^9 + 42117*(e*x)^m*B*b^2*c*d^2*m^2*x^11 + 28078*(e*x)^m*B*a*b*d^3*m^
2*x^11 + 14039*(e*x)^m*A*b^2*d^3*m^2*x^11 + 10395*(e*x)^m*B*b^2*d^3*x^13 +
 42*(e*x)^m*B*b^2*c^3*m^5*x^7 + 252*(e*x)^m*B*a*b*c^2*d*m^5*x^7 + 126*(...
```

Mupad [B] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 694, normalized size of antiderivative = 2.44

$$\int (ex)^m \left(a+bx^2\right)^2 \left(A+Bx^2\right) \left(c+dx^2\right)^3 \, dx$$
$$= \frac{x^7\,(e\,x)^m\,(3\,B\,a^2\,c\,d^2+A\,a^2\,d^3+6\,B\,a\,b\,c^2\,d+6\,A\,a\,b\,c\,d^2+B\,b^2\,c^3+3\,A\,b^2\,c^2\,d)\,(m^6+42\,m^5+679\,m^4}{m^7+49\,m^6+973\,m^5+10045\,m^4+57379\,m^3+177331\,m^2+264207\,r}$$
$$+\frac{c\,x^5\,(e\,x)^m\,(3\,B\,a^2\,c\,d+3\,A\,a^2\,d^2+2\,B\,a\,b\,c^2+6\,A\,a\,b\,c\,d+A\,b^2\,c^2)\,(m^6+44\,m^5+753\,m^4+6280\,m^3}{m^7+49\,m^6+973\,m^5+10045\,m^4+57379\,m^3+177331\,m^2+264207\,m+}$$
$$+\frac{d\,x^9\,(e\,x)^m\,(B\,a^2\,d^2+6\,B\,a\,b\,c\,d+2\,A\,a\,b\,d^2+3\,B\,b^2\,c^2+3\,A\,b^2\,c\,d)\,(m^6+40\,m^5+613\,m^4+4528\,m^3}{m^7+49\,m^6+973\,m^5+10045\,m^4+57379\,m^3+177331\,m^2+264207\,m+}$$
$$+\frac{A\,a^2\,c^3\,x\,(e\,x)^m\,(m^6+48\,m^5+925\,m^4+9120\,m^3+48259\,m^2+129072\,m+135135)}{m^7+49\,m^6+973\,m^5+10045\,m^4+57379\,m^3+177331\,m^2+264207\,m+135135}$$
$$+\frac{a\,c^2\,x^3\,(e\,x)^m\,(3\,A\,a\,d+2\,A\,b\,c+B\,a\,c)\,(m^6+46\,m^5+835\,m^4+7540\,m^3+34759\,m^2+73054\,m+45}{m^7+49\,m^6+973\,m^5+10045\,m^4+57379\,m^3+177331\,m^2+264207\,m+135135}$$
$$+\frac{b\,d^2\,x^{11}\,(e\,x)^m\,(A\,b\,d+2\,B\,a\,d+3\,B\,b\,c)\,(m^6+38\,m^5+555\,m^4+3940\,m^3+14039\,m^2+22902\,m+1}{m^7+49\,m^6+973\,m^5+10045\,m^4+57379\,m^3+177331\,m^2+264207\,m+135135}$$
$$+\frac{B\,b^2\,d^3\,x^{13}\,(e\,x)^m\,(m^6+36\,m^5+505\,m^4+3480\,m^3+12139\,m^2+19524\,m+10395)}{m^7+49\,m^6+973\,m^5+10045\,m^4+57379\,m^3+177331\,m^2+264207\,m+135135}$$

input
```
int((A + B*x^2)*(e*x)^m*(a + b*x^2)^2*(c + d*x^2)^3,x)
```

output
```
(x^7*(e*x)^m*(A*a^2*d^3 + B*b^2*c^3 + 3*A*b^2*c^2*d + 3*B*a^2*c*d^2 + 6*A*
a*b*c*d^2 + 6*B*a*b*c^2*d)*(34986*m + 20335*m^2 + 5292*m^3 + 679*m^4 + 42*
m^5 + m^6 + 19305))/(264207*m + 177331*m^2 + 57379*m^3 + 10045*m^4 + 973*m
^5 + 49*m^6 + m^7 + 135135) + (c*x^5*(e*x)^m*(3*A*a^2*d^2 + A*b^2*c^2 + 2*
B*a*b*c^2 + 3*B*a^2*c*d + 6*A*a*b*c*d)*(47436*m + 25979*m^2 + 6280*m^3 + 7
53*m^4 + 44*m^5 + m^6 + 27027))/(264207*m + 177331*m^2 + 57379*m^3 + 10045
*m^4 + 973*m^5 + 49*m^6 + m^7 + 135135) + (d*x^9*(e*x)^m*(B*a^2*d^2 + 3*B*
b^2*c^2 + 2*A*a*b*d^2 + 3*A*b^2*c*d + 6*B*a*b*c*d)*(27688*m + 16627*m^2 +
4528*m^3 + 613*m^4 + 40*m^5 + m^6 + 15015))/(264207*m + 177331*m^2 + 57379
*m^3 + 10045*m^4 + 973*m^5 + 49*m^6 + m^7 + 135135) + (A*a^2*c^3*x*(e*x)^m
*(129072*m + 48259*m^2 + 9120*m^3 + 925*m^4 + 48*m^5 + m^6 + 135135))/(264
207*m + 177331*m^2 + 57379*m^3 + 10045*m^4 + 973*m^5 + 49*m^6 + m^7 + 1351
35) + (a*c^2*x^3*(e*x)^m*(3*A*a*d + 2*A*b*c + B*a*c)*(73054*m + 34759*m^2
+ 7540*m^3 + 835*m^4 + 46*m^5 + m^6 + 45045))/(264207*m + 177331*m^2 + 573
79*m^3 + 10045*m^4 + 973*m^5 + 49*m^6 + m^7 + 135135) + (b*d^2*x^11*(e*x)^
m*(A*b*d + 2*B*a*d + 3*B*b*c)*(22902*m + 14039*m^2 + 3940*m^3 + 555*m^4 +
38*m^5 + m^6 + 12285))/(264207*m + 177331*m^2 + 57379*m^3 + 10045*m^4 + 97
3*m^5 + 49*m^6 + m^7 + 135135) + (B*b^2*d^3*x^13*(e*x)^m*(19524*m + 12139*
m^2 + 3480*m^3 + 505*m^4 + 36*m^5 + m^6 + 10395))/(264207*m + 177331*m^2 +
 57379*m^3 + 10045*m^4 + 973*m^5 + 49*m^6 + m^7 + 135135)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 1619, normalized size of antiderivative = 5.70

$$\int (ex)^m \left(a+bx^2\right)^2 \left(A+Bx^2\right) \left(c+dx^2\right)^3 \, dx = \text{Too large to display}$$

input
```
int((e*x)^m*(b*x^2+a)^2*(B*x^2+A)*(d*x^2+c)^3,x)
```

output

```
(x**m*e**m*x*(a**3*c**3*m**6 + 48*a**3*c**3*m**5 + 925*a**3*c**3*m**4 + 91
20*a**3*c**3*m**3 + 48259*a**3*c**3*m**2 + 129072*a**3*c**3*m + 135135*a**
3*c**3 + 3*a**3*c**2*d*m**6*x**2 + 138*a**3*c**2*d*m**5*x**2 + 2505*a**3*c
**2*d*m**4*x**2 + 22620*a**3*c**2*d*m**3*x**2 + 104277*a**3*c**2*d*m**2*x*
*2 + 219162*a**3*c**2*d*m*x**2 + 135135*a**3*c**2*d*x**2 + 3*a**3*c*d**2*m
**6*x**4 + 132*a**3*c*d**2*m**5*x**4 + 2259*a**3*c*d**2*m**4*x**4 + 18840*
a**3*c*d**2*m**3*x**4 + 77937*a**3*c*d**2*m**2*x**4 + 142308*a**3*c*d**2*m
*x**4 + 81081*a**3*c*d**2*x**4 + a**3*d**3*m**6*x**6 + 42*a**3*d**3*m**5*x
**6 + 679*a**3*d**3*m**4*x**6 + 5292*a**3*d**3*m**3*x**6 + 20335*a**3*d**3
*m**2*x**6 + 34986*a**3*d**3*m*x**6 + 19305*a**3*d**3*x**6 + 3*a**2*b*c**3
*m**6*x**2 + 138*a**2*b*c**3*m**5*x**2 + 2505*a**2*b*c**3*m**4*x**2 + 2262
0*a**2*b*c**3*m**3*x**2 + 104277*a**2*b*c**3*m**2*x**2 + 219162*a**2*b*c**
3*m*x**2 + 135135*a**2*b*c**3*x**2 + 9*a**2*b*c**2*d*m**6*x**4 + 396*a**2*
b*c**2*d*m**5*x**4 + 6777*a**2*b*c**2*d*m**4*x**4 + 56520*a**2*b*c**2*d*m*
*3*x**4 + 233811*a**2*b*c**2*d*m**2*x**4 + 426924*a**2*b*c**2*d*m*x**4 + 2
43243*a**2*b*c**2*d*x**4 + 9*a**2*b*c*d**2*m**6*x**6 + 378*a**2*b*c*d**2*m
**5*x**6 + 6111*a**2*b*c*d**2*m**4*x**6 + 47628*a**2*b*c*d**2*m**3*x**6 +
183015*a**2*b*c*d**2*m**2*x**6 + 314874*a**2*b*c*d**2*m*x**6 + 173745*a**2
*b*c*d**2*x**6 + 3*a**2*b*d**3*m**6*x**8 + 120*a**2*b*d**3*m**5*x**8 + 183
9*a**2*b*d**3*m**4*x**8 + 13584*a**2*b*d**3*m**3*x**8 + 49881*a**2*b*d*...
```

3.17 $\int (ex)^m (a+bx^2) (A+Bx^2) (c+dx^2)^3 \, dx$

Optimal result

Integrand size = 29, antiderivative size = 189

$$\begin{aligned}\int (ex)^m (a+bx^2) (A+Bx^2) (c+dx^2)^3 \, dx &= \frac{aAc^3(ex)^{1+m}}{e(1+m)} + \frac{c^2(Abc+aBc+3aAd)(ex)^{3+m}}{e^3(3+m)} \\ &\quad + \frac{c(3ad(Bc+Ad)+bc(Bc+3Ad))(ex)^{5+m}}{e^5(5+m)} \\ &\quad + \frac{d(3bc(Bc+Ad)+ad(3Bc+Ad))(ex)^{7+m}}{e^7(7+m)} \\ &\quad + \frac{d^2(3bBc+Abd+aBd)(ex)^{9+m}}{e^9(9+m)} + \frac{bBd^3(ex)^{11+m}}{e^{11}(11+m)}\end{aligned}$$

output
```
a*A*c^3*(e*x)^(1+m)/e/(1+m)+c^2*(3*A*a*d+A*b*c+B*a*c)*(e*x)^(3+m)/e^3/(3+m
)+c*(3*a*d*(A*d+B*c)+b*c*(3*A*d+B*c))*(e*x)^(5+m)/e^5/(5+m)+d*(3*b*c*(A*d+
B*c)+a*d*(A*d+3*B*c))*(e*x)^(7+m)/e^7/(7+m)+d^2*(A*b*d+B*a*d+3*B*b*c)*(e*x
)^(9+m)/e^9/(9+m)+b*B*d^3*(e*x)^(11+m)/e^11/(11+m)
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.80

$$\int (ex)^m \left(a+bx^2\right) \left(A+Bx^2\right) \left(c+dx^2\right)^3 \, dx = x(ex)^m \left(\frac{aAc^3}{1+m}+\frac{c^2(Abc+aBc+3aAd)x^2}{3+m}+\frac{c(3ad(Bc+Ad)+bc(Bc+3Ad))x^4}{5+m} + \frac{d(3bc(Bc+Ad)+ad(3Bc+Ad))x^6}{7+m}+\frac{d^2(3bBc+Abd+aBd)x^8}{9+m}+\frac{bBd^3x^{10}}{11+m}\right)$$

input
```
Integrate[(e*x)^m*(a + b*x^2)*(A + B*x^2)*(c + d*x^2)^3,x]
```

output
```
x*(e*x)^m*((a*A*c^3)/(1 + m) + (c^2*(A*b*c + a*B*c + 3*a*A*d)*x^2)/(3 + m)
 + (c*(3*a*d*(B*c + A*d) + b*c*(B*c + 3*A*d))*x^4)/(5 + m) + (d*(3*b*c*(B*
c + A*d) + a*d*(3*B*c + A*d))*x^6)/(7 + m) + (d^2*(3*b*B*c + A*b*d + a*B*d
)*x^8)/(9 + m) + (b*B*d^3*x^10)/(11 + m))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {437, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right) \left(A+Bx^2\right) \left(c+dx^2\right)^3 (ex)^m \, dx$$

$$\Big\downarrow \text{437}$$

$$\int \left(\frac{c^2(ex)^{m+2}(3aAd+aBc+Abc)}{e^2}+\frac{d^2(ex)^{m+8}(aBd+Abd+3bBc)}{e^8}+\frac{d(ex)^{m+6}(ad(Ad+3Bc)+3bc(Ad+}{e^6}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{c^2(ex)^{m+3}(3aAd+aBc+Abc)}{e^3(m+3)}+\frac{d^2(ex)^{m+9}(aBd+Abd+3bBc)}{e^9(m+9)}+\frac{d(ex)^{m+7}(ad(Ad+3Bc)+3bc(Ad+Bc))}{e^7(m+7)}+\frac{c(ex)^{m+5}(3ad(Ad+Bc)+bc(3Ad+Bc))}{e^5(m+5)}+\frac{aAc^3(ex)^{m+1}}{e(m+1)}+\frac{bBd^3(ex)^{m+11}}{e^{11}(m+11)}$$

input
```
Int[(e*x)^m*(a + b*x^2)*(A + B*x^2)*(c + d*x^2)^3,x]
```

output
```
(a*A*c^3*(e*x)^(1 + m))/(e*(1 + m)) + (c^2*(A*b*c + a*B*c + 3*a*A*d)*(e*x)
^(3 + m))/(e^3*(3 + m)) + (c*(3*a*d*(B*c + A*d) + b*c*(B*c + 3*A*d))*(e*x)
^(5 + m))/(e^5*(5 + m)) + (d*(3*b*c*(B*c + A*d) + a*d*(3*B*c + A*d))*(e*x)
^(7 + m))/(e^7*(7 + m)) + (d^2*(3*b*B*c + A*b*d + a*B*d)*(e*x)^(9 + m))/(e
^9*(9 + m)) + (b*B*d^3*(e*x)^(11 + m))/(e^11*(11 + m))
```

Defintions of rubi rules used

rule 437
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(
a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f
, g, m}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1228 vs. $2(189) = 378$.

Time = 0.51 (sec) , antiderivative size = 1229, normalized size of antiderivative = 6.50

method	result	size
gosper	Expression too large to display	1229
risch	Expression too large to display	1229
orering	Expression too large to display	1229
parallelrisch	Expression too large to display	1709

input
```
int((e*x)^m*(b*x^2+a)*(B*x^2+A)*(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
x*(B*b*d^3*m^5*x^10+25*B*b*d^3*m^4*x^10+A*b*d^3*m^5*x^8+B*a*d^3*m^5*x^8+3*
B*b*c*d^2*m^5*x^8+230*B*b*d^3*m^3*x^10+27*A*b*d^3*m^4*x^8+27*B*a*d^3*m^4*x
^8+81*B*b*c*d^2*m^4*x^8+950*B*b*d^3*m^2*x^10+A*a*d^3*m^5*x^6+3*A*b*c*d^2*m
^5*x^6+262*A*b*d^3*m^3*x^8+3*B*a*c*d^2*m^5*x^6+262*B*a*d^3*m^3*x^8+3*B*b*c
^2*d*m^5*x^6+786*B*b*c*d^2*m^3*x^8+1689*B*b*d^3*m*x^10+29*A*a*d^3*m^4*x^6+
87*A*b*c*d^2*m^4*x^6+1122*A*b*d^3*m^2*x^8+87*B*a*c*d^2*m^4*x^6+1122*B*a*d^
3*m^2*x^8+87*B*b*c^2*d*m^4*x^6+3366*B*b*c*d^2*m^2*x^8+945*B*b*d^3*x^10+3*A
*a*c*d^2*m^5*x^4+302*A*a*d^3*m^3*x^6+3*A*b*c^2*d*m^5*x^4+906*A*b*c*d^2*m^3
*x^6+2041*A*b*d^3*m*x^8+3*B*a*c^2*d*m^5*x^4+906*B*a*c*d^2*m^3*x^6+2041*B*a
*d^3*m*x^8+B*b*c^3*m^5*x^4+906*B*b*c^2*d*m^3*x^6+6123*B*b*c*d^2*m*x^8+93*A
*a*c*d^2*m^4*x^4+1366*A*a*d^3*m^2*x^6+93*A*b*c^2*d*m^4*x^4+4098*A*b*c*d^2*
m^2*x^6+1155*A*b*d^3*x^8+93*B*a*c^2*d*m^4*x^4+4098*B*a*c*d^2*m^2*x^6+1155*
B*a*d^3*x^8+31*B*b*c^3*m^4*x^4+4098*B*b*c^2*d*m^2*x^6+3465*B*b*c*d^2*x^8+3
*A*a*c^2*d*m^5*x^2+1050*A*a*c*d^2*m^3*x^4+2577*A*a*d^3*m*x^6+A*b*c^3*m^5*x
^2+1050*A*b*c^2*d*m^3*x^4+7731*A*b*c*d^2*m*x^6+B*a*c^3*m^5*x^2+1050*B*a*c^
2*d*m^3*x^4+7731*B*a*c*d^2*m*x^6+350*B*b*c^3*m^3*x^4+7731*B*b*c^2*d*m*x^6+
99*A*a*c^2*d*m^4*x^2+5190*A*a*c*d^2*m^2*x^4+1485*A*a*d^3*x^6+33*A*b*c^3*m^
4*x^2+5190*A*b*c^2*d*m^2*x^4+4455*A*b*c*d^2*x^6+33*B*a*c^3*m^4*x^2+5190*B*
a*c^2*d*m^2*x^4+4455*B*a*c*d^2*x^6+1730*B*b*c^3*m^2*x^4+4455*B*b*c^2*d*x^6
+A*a*c^3*m^5+1218*A*a*c^2*d*m^3*x^2+10467*A*a*c*d^2*m*x^4+406*A*b*c^3*m...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 837 vs. $2(189) = 378$.

Time = 0.10 (sec) , antiderivative size = 837, normalized size of antiderivative = 4.43

$$\int (ex)^m \left(a+bx^2\right) \left(A+Bx^2\right) \left(c+dx^2\right)^3 \, dx = \text{Too large to display}$$

input
```
integrate((e*x)^m*(b*x^2+a)*(B*x^2+A)*(d*x^2+c)^3,x, algorithm="fricas")
```

output
```
((B*b*d^3*m^5 + 25*B*b*d^3*m^4 + 230*B*b*d^3*m^3 + 950*B*b*d^3*m^2 + 1689*
B*b*d^3*m + 945*B*b*d^3)*x^11 + ((3*B*b*c*d^2 + (B*a + A*b)*d^3)*m^5 + 346
5*B*b*c*d^2 + 27*(3*B*b*c*d^2 + (B*a + A*b)*d^3)*m^4 + 1155*(B*a + A*b)*d^
3 + 262*(3*B*b*c*d^2 + (B*a + A*b)*d^3)*m^3 + 1122*(3*B*b*c*d^2 + (B*a + A
*b)*d^3)*m^2 + 2041*(3*B*b*c*d^2 + (B*a + A*b)*d^3)*m)*x^9 + ((3*B*b*c^2*d
 + A*a*d^3 + 3*(B*a + A*b)*c*d^2)*m^5 + 4455*B*b*c^2*d + 1485*A*a*d^3 + 29
*(3*B*b*c^2*d + A*a*d^3 + 3*(B*a + A*b)*c*d^2)*m^4 + 4455*(B*a + A*b)*c*d^
2 + 302*(3*B*b*c^2*d + A*a*d^3 + 3*(B*a + A*b)*c*d^2)*m^3 + 1366*(3*B*b*c^
2*d + A*a*d^3 + 3*(B*a + A*b)*c*d^2)*m^2 + 2577*(3*B*b*c^2*d + A*a*d^3 + 3
*(B*a + A*b)*c*d^2)*m)*x^7 + ((B*b*c^3 + 3*A*a*c*d^2 + 3*(B*a + A*b)*c^2*d
)*m^5 + 2079*B*b*c^3 + 6237*A*a*c*d^2 + 31*(B*b*c^3 + 3*A*a*c*d^2 + 3*(B*a
 + A*b)*c^2*d)*m^4 + 6237*(B*a + A*b)*c^2*d + 350*(B*b*c^3 + 3*A*a*c*d^2 +
 3*(B*a + A*b)*c^2*d)*m^3 + 1730*(B*b*c^3 + 3*A*a*c*d^2 + 3*(B*a + A*b)*c^
2*d)*m^2 + 3489*(B*b*c^3 + 3*A*a*c*d^2 + 3*(B*a + A*b)*c^2*d)*m)*x^5 + ((3
*A*a*c^2*d + (B*a + A*b)*c^3)*m^5 + 10395*A*a*c^2*d + 33*(3*A*a*c^2*d + (B
*a + A*b)*c^3)*m^4 + 3465*(B*a + A*b)*c^3 + 406*(3*A*a*c^2*d + (B*a + A*b)
*c^3)*m^3 + 2262*(3*A*a*c^2*d + (B*a + A*b)*c^3)*m^2 + 5353*(3*A*a*c^2*d +
 (B*a + A*b)*c^3)*m)*x^3 + (A*a*c^3*m^5 + 35*A*a*c^3*m^4 + 470*A*a*c^3*m^3
 + 3010*A*a*c^3*m^2 + 9129*A*a*c^3*m + 10395*A*a*c^3)*x)*(e*x)^m/(m^6 + 36
*m^5 + 505*m^4 + 3480*m^3 + 12139*m^2 + 19524*m + 10395)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5992 vs. $2(184) = 368$.

Time = 0.92 (sec) , antiderivative size = 5992, normalized size of antiderivative = 31.70

$$\int (ex)^m \left(a+bx^2\right) \left(A+Bx^2\right) \left(c+dx^2\right)^3 \, dx = \text{Too large to display}$$

input
```
integrate((e*x)**m*(b*x**2+a)*(B*x**2+A)*(d*x**2+c)**3,x)
```

output
```
Piecewise(((-A*a*c**3/(10*x**10) - 3*A*a*c**2*d/(8*x**8) - A*a*c*d**2/(2*x
**6) - A*a*d**3/(4*x**4) - A*b*c**3/(8*x**8) - A*b*c**2*d/(2*x**6) - 3*A*b
*c*d**2/(4*x**4) - A*b*d**3/(2*x**2) - B*a*c**3/(8*x**8) - B*a*c**2*d/(2*x
**6) - 3*B*a*c*d**2/(4*x**4) - B*a*d**3/(2*x**2) - B*b*c**3/(6*x**6) - 3*B
*b*c**2*d/(4*x**4) - 3*B*b*c*d**2/(2*x**2) + B*b*d**3*log(x))/e**11, Eq(m,
 -11)), ((-A*a*c**3/(8*x**8) - A*a*c**2*d/(2*x**6) - 3*A*a*c*d**2/(4*x**4)
 - A*a*d**3/(2*x**2) - A*b*c**3/(6*x**6) - 3*A*b*c**2*d/(4*x**4) - 3*A*b*c
*d**2/(2*x**2) + A*b*d**3*log(x) - B*a*c**3/(6*x**6) - 3*B*a*c**2*d/(4*x**
4) - 3*B*a*c*d**2/(2*x**2) + B*a*d**3*log(x) - B*b*c**3/(4*x**4) - 3*B*b*c
**2*d/(2*x**2) + 3*B*b*c*d**2*log(x) + B*b*d**3*x**2/2)/e**9, Eq(m, -9)),
((-A*a*c**3/(6*x**6) - 3*A*a*c**2*d/(4*x**4) - 3*A*a*c*d**2/(2*x**2) + A*a
*d**3*log(x) - A*b*c**3/(4*x**4) - 3*A*b*c**2*d/(2*x**2) + 3*A*b*c*d**2*lo
g(x) + A*b*d**3*x**2/2 - B*a*c**3/(4*x**4) - 3*B*a*c**2*d/(2*x**2) + 3*B*a
*c*d**2*log(x) + B*a*d**3*x**2/2 - B*b*c**3/(2*x**2) + 3*B*b*c**2*d*log(x)
 + 3*B*b*c*d**2*x**2/2 + B*b*d**3*x**4/4)/e**7, Eq(m, -7)), ((-A*a*c**3/(4
*x**4) - 3*A*a*c**2*d/(2*x**2) + 3*A*a*c*d**2*log(x) + A*a*d**3*x**2/2 - A
*b*c**3/(2*x**2) + 3*A*b*c**2*d*log(x) + 3*A*b*c*d**2*x**2/2 + A*b*d**3*x*
*4/4 - B*a*c**3/(2*x**2) + 3*B*a*c**2*d*log(x) + 3*B*a*c*d**2*x**2/2 + B*a
*d**3*x**4/4 + B*b*c**3*log(x) + 3*B*b*c**2*d*x**2/2 + 3*B*b*c*d**2*x**4/4
 + B*b*d**3*x**6/6)/e**5, Eq(m, -5)), ((-A*a*c**3/(2*x**2) + 3*A*a*c**2...
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.79

$$\int (ex)^m (a+bx^2) (A+Bx^2) (c+dx^2)^3 \, dx = \frac{Bbd^3e^mx^{11}x^m}{m+11} + \frac{3\,Bbcd^2e^mx^9x^m}{m+9} + \frac{Bad^3e^mx^9x^m}{m+9} + \frac{Abd^3e^mx^9x^m}{m+9} + \frac{3\,Bbc^2de^mx^7x^m}{m+7} + \frac{3\,Bacd^2e^mx^7x^m}{m+7} + \frac{3\,Abcd^2e^mx^7x^m}{m+7} + \frac{Aad^3e^mx^7x^m}{m+7} + \frac{Bbc^3e^mx^5x^m}{m+5} + \frac{3\,Bac^2de^mx^5x^m}{m+5} + \frac{3\,Abc^2de^mx^5x^m}{m+5} + \frac{3\,Aacd^2e^mx^5x^m}{m+5} + \frac{Bac^3e^mx^3x^m}{m+3} + \frac{Abc^3e^mx^3x^m}{m+3} + \frac{3\,Aac^2de^mx^3x^m}{m+3} + \frac{(ex)^{m+1}\,Aac^3}{e(m+1)}$$

input
```
integrate((e*x)^m*(b*x^2+a)*(B*x^2+A)*(d*x^2+c)^3,x, algorithm="maxima")
```

output
```
B*b*d^3*e^m*x^11*x^m/(m + 11) + 3*B*b*c*d^2*e^m*x^9*x^m/(m + 9) + B*a*d^3*
e^m*x^9*x^m/(m + 9) + A*b*d^3*e^m*x^9*x^m/(m + 9) + 3*B*b*c^2*d*e^m*x^7*x^
m/(m + 7) + 3*B*a*c*d^2*e^m*x^7*x^m/(m + 7) + 3*A*b*c*d^2*e^m*x^7*x^m/(m +
 7) + A*a*d^3*e^m*x^7*x^m/(m + 7) + B*b*c^3*e^m*x^5*x^m/(m + 5) + 3*B*a*c^
2*d*e^m*x^5*x^m/(m + 5) + 3*A*b*c^2*d*e^m*x^5*x^m/(m + 5) + 3*A*a*c*d^2*e^
m*x^5*x^m/(m + 5) + B*a*c^3*e^m*x^3*x^m/(m + 3) + A*b*c^3*e^m*x^3*x^m/(m +
 3) + 3*A*a*c^2*d*e^m*x^3*x^m/(m + 3) + (e*x)^(m + 1)*A*a*c^3/(e*(m + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1708 vs. $2(189) = 378$.

Time = 0.16 (sec) , antiderivative size = 1708, normalized size of antiderivative = 9.04

$$\int (ex)^m \left(a+bx^2\right) \left(A+Bx^2\right) \left(c+dx^2\right)^3 \, dx = \text{Too large to display}$$

input
```
integrate((e*x)^m*(b*x^2+a)*(B*x^2+A)*(d*x^2+c)^3,x, algorithm="giac")
```

output
```
((e*x)^m*B*b*d^3*m^5*x^11 + 25*(e*x)^m*B*b*d^3*m^4*x^11 + 3*(e*x)^m*B*b*c*
d^2*m^5*x^9 + (e*x)^m*B*a*d^3*m^5*x^9 + (e*x)^m*A*b*d^3*m^5*x^9 + 230*(e*x
)^m*B*b*d^3*m^3*x^11 + 81*(e*x)^m*B*b*c*d^2*m^4*x^9 + 27*(e*x)^m*B*a*d^3*m
^4*x^9 + 27*(e*x)^m*A*b*d^3*m^4*x^9 + 950*(e*x)^m*B*b*d^3*m^2*x^11 + 3*(e*
x)^m*B*b*c^2*d*m^5*x^7 + 3*(e*x)^m*B*a*c*d^2*m^5*x^7 + 3*(e*x)^m*A*b*c*d^2
*m^5*x^7 + (e*x)^m*A*a*d^3*m^5*x^7 + 786*(e*x)^m*B*b*c*d^2*m^3*x^9 + 262*(
e*x)^m*B*a*d^3*m^3*x^9 + 262*(e*x)^m*A*b*d^3*m^3*x^9 + 1689*(e*x)^m*B*b*d^
3*m*x^11 + 87*(e*x)^m*B*b*c^2*d*m^4*x^7 + 87*(e*x)^m*B*a*c*d^2*m^4*x^7 + 8
7*(e*x)^m*A*b*c*d^2*m^4*x^7 + 29*(e*x)^m*A*a*d^3*m^4*x^7 + 3366*(e*x)^m*B*
b*c*d^2*m^2*x^9 + 1122*(e*x)^m*B*a*d^3*m^2*x^9 + 1122*(e*x)^m*A*b*d^3*m^2*
x^9 + 945*(e*x)^m*B*b*d^3*x^11 + (e*x)^m*B*b*c^3*m^5*x^5 + 3*(e*x)^m*B*a*c
^2*d*m^5*x^5 + 3*(e*x)^m*A*b*c^2*d*m^5*x^5 + 3*(e*x)^m*A*a*c*d^2*m^5*x^5 +
 906*(e*x)^m*B*b*c^2*d*m^3*x^7 + 906*(e*x)^m*B*a*c*d^2*m^3*x^7 + 906*(e*x)
^m*A*b*c*d^2*m^3*x^7 + 302*(e*x)^m*A*a*d^3*m^3*x^7 + 6123*(e*x)^m*B*b*c*d^
2*m*x^9 + 2041*(e*x)^m*B*a*d^3*m*x^9 + 2041*(e*x)^m*A*b*d^3*m*x^9 + 31*(e*
x)^m*B*b*c^3*m^4*x^5 + 93*(e*x)^m*B*a*c^2*d*m^4*x^5 + 93*(e*x)^m*A*b*c^2*d
*m^4*x^5 + 93*(e*x)^m*A*a*c*d^2*m^4*x^5 + 4098*(e*x)^m*B*b*c^2*d*m^2*x^7 +
 4098*(e*x)^m*B*a*c*d^2*m^2*x^7 + 4098*(e*x)^m*A*b*c*d^2*m^2*x^7 + 1366*(e
*x)^m*A*a*d^3*m^2*x^7 + 3465*(e*x)^m*B*b*c*d^2*x^9 + 1155*(e*x)^m*B*a*d^3*
x^9 + 1155*(e*x)^m*A*b*d^3*x^9 + (e*x)^m*B*a*c^3*m^5*x^3 + (e*x)^m*A*b*...
```

Mupad [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 469, normalized size of antiderivative = 2.48

$$\int (ex)^m \left(a+bx^2\right) \left(A+Bx^2\right) \left(c+dx^2\right)^3 \, dx$$
$$= \frac{c^2\,x^3\,{\left(e\,x\right)}^m\,\left(3\,A\,a\,d+A\,b\,c+B\,a\,c\right)\,\left(m^5+33\,m^4+406\,m^3+2262\,m^2+5353\,m+3465\right)}{m^6+36\,m^5+505\,m^4+3480\,m^3+12139\,m^2+19524\,m+10395}$$
$$+\frac{d^2\,x^9\,{\left(e\,x\right)}^m\,\left(A\,b\,d+B\,a\,d+3\,B\,b\,c\right)\,\left(m^5+27\,m^4+262\,m^3+1122\,m^2+2041\,m+1155\right)}{m^6+36\,m^5+505\,m^4+3480\,m^3+12139\,m^2+19524\,m+10395}$$
$$+\frac{c\,x^5\,{\left(e\,x\right)}^m\,\left(3\,A\,a\,d^2+B\,b\,c^2+3\,A\,b\,c\,d+3\,B\,a\,c\,d\right)\,\left(m^5+31\,m^4+350\,m^3+1730\,m^2+3489\,m+207\right.}{m^6+36\,m^5+505\,m^4+3480\,m^3+12139\,m^2+19524\,m+10395}$$
$$+\frac{d\,x^7\,{\left(e\,x\right)}^m\,\left(A\,a\,d^2+3\,B\,b\,c^2+3\,A\,b\,c\,d+3\,B\,a\,c\,d\right)\,\left(m^5+29\,m^4+302\,m^3+1366\,m^2+2577\,m+148\right.}{m^6+36\,m^5+505\,m^4+3480\,m^3+12139\,m^2+19524\,m+10395}$$
$$+\frac{B\,b\,d^3\,x^{11}\,{\left(e\,x\right)}^m\,\left(m^5+25\,m^4+230\,m^3+950\,m^2+1689\,m+945\right)}{m^6+36\,m^5+505\,m^4+3480\,m^3+12139\,m^2+19524\,m+10395}$$
$$+\frac{A\,a\,c^3\,x\,{\left(e\,x\right)}^m\,\left(m^5+35\,m^4+470\,m^3+3010\,m^2+9129\,m+10395\right)}{m^6+36\,m^5+505\,m^4+3480\,m^3+12139\,m^2+19524\,m+10395}$$

input
```
int((A + B*x^2)*(e*x)^m*(a + b*x^2)*(c + d*x^2)^3,x)
```

output
```
(c^2*x^3*(e*x)^m*(3*A*a*d + A*b*c + B*a*c)*(5353*m + 2262*m^2 + 406*m^3 +
33*m^4 + m^5 + 3465))/(19524*m + 12139*m^2 + 3480*m^3 + 505*m^4 + 36*m^5 +
 m^6 + 10395) + (d^2*x^9*(e*x)^m*(A*b*d + B*a*d + 3*B*b*c)*(2041*m + 1122*
m^2 + 262*m^3 + 27*m^4 + m^5 + 1155))/(19524*m + 12139*m^2 + 3480*m^3 + 50
5*m^4 + 36*m^5 + m^6 + 10395) + (c*x^5*(e*x)^m*(3*A*a*d^2 + B*b*c^2 + 3*A*
b*c*d + 3*B*a*c*d)*(3489*m + 1730*m^2 + 350*m^3 + 31*m^4 + m^5 + 2079))/(1
9524*m + 12139*m^2 + 3480*m^3 + 505*m^4 + 36*m^5 + m^6 + 10395) + (d*x^7*(
e*x)^m*(A*a*d^2 + 3*B*b*c^2 + 3*A*b*c*d + 3*B*a*c*d)*(2577*m + 1366*m^2 +
302*m^3 + 29*m^4 + m^5 + 1485))/(19524*m + 12139*m^2 + 3480*m^3 + 505*m^4
+ 36*m^5 + m^6 + 10395) + (B*b*d^3*x^11*(e*x)^m*(1689*m + 950*m^2 + 230*m^
3 + 25*m^4 + m^5 + 945))/(19524*m + 12139*m^2 + 3480*m^3 + 505*m^4 + 36*m^
5 + m^6 + 10395) + (A*a*c^3*x*(e*x)^m*(9129*m + 3010*m^2 + 470*m^3 + 35*m^
4 + m^5 + 10395))/(19524*m + 12139*m^2 + 3480*m^3 + 505*m^4 + 36*m^5 + m^6
 + 10395)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 977, normalized size of antiderivative = 5.17

$$\int (ex)^m (a+bx^2) (A+Bx^2) (c+dx^2)^3 \, dx = \text{Too large to display}$$

input
```
int((e*x)^m*(b*x^2+a)*(B*x^2+A)*(d*x^2+c)^3,x)
```

output

```
(x**m*e**m*x*(a**2*c**3*m**5 + 35*a**2*c**3*m**4 + 470*a**2*c**3*m**3 + 30
10*a**2*c**3*m**2 + 9129*a**2*c**3*m + 10395*a**2*c**3 + 3*a**2*c**2*d*m**
5*x**2 + 99*a**2*c**2*d*m**4*x**2 + 1218*a**2*c**2*d*m**3*x**2 + 6786*a**2
*c**2*d*m**2*x**2 + 16059*a**2*c**2*d*m*x**2 + 10395*a**2*c**2*d*x**2 + 3*
a**2*c*d**2*m**5*x**4 + 93*a**2*c*d**2*m**4*x**4 + 1050*a**2*c*d**2*m**3*x
**4 + 5190*a**2*c*d**2*m**2*x**4 + 10467*a**2*c*d**2*m*x**4 + 6237*a**2*c*
d**2*x**4 + a**2*d**3*m**5*x**6 + 29*a**2*d**3*m**4*x**6 + 302*a**2*d**3*m
**3*x**6 + 1366*a**2*d**3*m**2*x**6 + 2577*a**2*d**3*m*x**6 + 1485*a**2*d*
*3*x**6 + 2*a*b*c**3*m**5*x**2 + 66*a*b*c**3*m**4*x**2 + 812*a*b*c**3*m**3
*x**2 + 4524*a*b*c**3*m**2*x**2 + 10706*a*b*c**3*m*x**2 + 6930*a*b*c**3*x*
*2 + 6*a*b*c**2*d*m**5*x**4 + 186*a*b*c**2*d*m**4*x**4 + 2100*a*b*c**2*d*m
**3*x**4 + 10380*a*b*c**2*d*m**2*x**4 + 20934*a*b*c**2*d*m*x**4 + 12474*a*
b*c**2*d*x**4 + 6*a*b*c*d**2*m**5*x**6 + 174*a*b*c*d**2*m**4*x**6 + 1812*a
*b*c*d**2*m**3*x**6 + 8196*a*b*c*d**2*m**2*x**6 + 15462*a*b*c*d**2*m*x**6
+ 8910*a*b*c*d**2*x**6 + 2*a*b*d**3*m**5*x**8 + 54*a*b*d**3*m**4*x**8 + 52
4*a*b*d**3*m**3*x**8 + 2244*a*b*d**3*m**2*x**8 + 4082*a*b*d**3*m*x**8 + 23
10*a*b*d**3*x**8 + b**2*c**3*m**5*x**4 + 31*b**2*c**3*m**4*x**4 + 350*b**2
*c**3*m**3*x**4 + 1730*b**2*c**3*m**2*x**4 + 3489*b**2*c**3*m*x**4 + 2079*
b**2*c**3*x**4 + 3*b**2*c**2*d*m**5*x**6 + 87*b**2*c**2*d*m**4*x**6 + 906*
b**2*c**2*d*m**3*x**6 + 4098*b**2*c**2*d*m**2*x**6 + 7731*b**2*c**2*d*m...
```

3.18 $\int (ex)^m (A+Bx^2) (c+dx^2)^3 \, dx$

Optimal result

Integrand size = 22, antiderivative size = 121

$$\int (ex)^m (A+Bx^2) (c+dx^2)^3 \, dx = \frac{Ac^3(ex)^{1+m}}{e(1+m)} + \frac{c^2(Bc+3Ad)(ex)^{3+m}}{e^3(3+m)} + \frac{3cd(Bc+Ad)(ex)^{5+m}}{e^5(5+m)} + \frac{d^2(3Bc+Ad)(ex)^{7+m}}{e^7(7+m)} + \frac{Bd^3(ex)^{9+m}}{e^9(9+m)}$$

output
```
A*c^3*(e*x)^(1+m)/e/(1+m)+c^2*(3*A*d+B*c)*(e*x)^(3+m)/e^3/(3+m)+3*c*d*(A*d
+B*c)*(e*x)^(5+m)/e^5/(5+m)+d^2*(A*d+3*B*c)*(e*x)^(7+m)/e^7/(7+m)+B*d^3*(e
*x)^(9+m)/e^9/(9+m)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.74

$$\int (ex)^m (A+Bx^2) (c+dx^2)^3 \, dx = x(ex)^m \left(\frac{Ac^3}{1+m} + \frac{c^2(Bc+3Ad)x^2}{3+m} + \frac{3cd(Bc+Ad)x^4}{5+m} + \frac{d^2(3Bc+Ad)x^6}{7+m} + \frac{Bd^3x^8}{9+m}\right)$$

input
```
Integrate[(e*x)^m*(A + B*x^2)*(c + d*x^2)^3,x]
```

output
```
x*(e*x)^m*((A*c^3)/(1 + m) + (c^2*(B*c + 3*A*d)*x^2)/(3 + m) + (3*c*d*(B*c
 + A*d)*x^4)/(5 + m) + (d^2*(3*B*c + A*d)*x^6)/(7 + m) + (B*d^3*x^8)/(9 +
m))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(A+Bx^2\right)\left(c+dx^2\right)^3(ex)^m\,dx$$

$$\Big\downarrow \text{355}$$

$$\int \left(\frac{c^2(ex)^{m+2}(3Ad+Bc)}{e^2}+\frac{d^2(ex)^{m+6}(Ad+3Bc)}{e^6}+\frac{3cd(ex)^{m+4}(Ad+Bc)}{e^4}+Ac^3(ex)^m+\frac{Bd^3(ex)^{m+8}}{e^8}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{c^2(ex)^{m+3}(3Ad+Bc)}{e^3(m+3)}+\frac{d^2(ex)^{m+7}(Ad+3Bc)}{e^7(m+7)}+\frac{3cd(ex)^{m+5}(Ad+Bc)}{e^5(m+5)}+\frac{Ac^3(ex)^{m+1}}{e(m+1)}+\frac{Bd^3(ex)^{m+9}}{e^9(m+9)}$$

input
```
Int[(e*x)^m*(A + B*x^2)*(c + d*x^2)^3,x]
```

output
```
(A*c^3*(e*x)^(1 + m))/(e*(1 + m)) + (c^2*(B*c + 3*A*d)*(e*x)^(3 + m))/(e^3
*(3 + m)) + (3*c*d*(B*c + A*d)*(e*x)^(5 + m))/(e^5*(5 + m)) + (d^2*(3*B*c
+ A*d)*(e*x)^(7 + m))/(e^7*(7 + m)) + (B*d^3*(e*x)^(9 + m))/(e^9*(9 + m))
```

Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 474 vs. $2(121) = 242$.

Time = 0.45 (sec) , antiderivative size = 475, normalized size of antiderivative = 3.93

method	result
gosper	$x\left(B\,d^3m^4x^8+16B\,d^3m^3x^8+A\,d^3m^4x^6+3Bc\,d^2m^4x^6+86B\,d^3m^2x^8+18A\,d^3m^3x^6+54Bc\,d^2m^3x^6+176m\,x^8B\,d^3+3Ac\,d^2m\right.$
risch	$x\left(B\,d^3m^4x^8+16B\,d^3m^3x^8+A\,d^3m^4x^6+3Bc\,d^2m^4x^6+86B\,d^3m^2x^8+18A\,d^3m^3x^6+54Bc\,d^2m^3x^6+176m\,x^8B\,d^3+3Ac\,d^2m\right.$
orering	$x\left(B\,d^3m^4x^8+16B\,d^3m^3x^8+A\,d^3m^4x^6+3Bc\,d^2m^4x^6+86B\,d^3m^2x^8+18A\,d^3m^3x^6+54Bc\,d^2m^3x^6+176m\,x^8B\,d^3+3Ac\,d^2m\right.$
parallelrisch	$60B\,x^5(ex)^m c^2d\,m^3+390A\,x^5(ex)^m c\,d^2m^2+66A\,x^3(ex)^m c^2d\,m^3+900B\,x^5(ex)^m c^2dm+492A\,x^3(ex)^m c^2d\,m^2+1374A\,x^3(e$

input
```
int((e*x)^m*(B*x^2+A)*(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
x*(B*d^3*m^4*x^8+16*B*d^3*m^3*x^8+A*d^3*m^4*x^6+3*B*c*d^2*m^4*x^6+86*B*d^3
*m^2*x^8+18*A*d^3*m^3*x^6+54*B*c*d^2*m^3*x^6+176*B*d^3*m*x^8+3*A*c*d^2*m^4
*x^4+104*A*d^3*m^2*x^6+3*B*c^2*d*m^4*x^4+312*B*c*d^2*m^2*x^6+105*B*d^3*x^8
+60*A*c*d^2*m^3*x^4+222*A*d^3*m*x^6+60*B*c^2*d*m^3*x^4+666*B*c*d^2*m*x^6+3
*A*c^2*d*m^4*x^2+390*A*c*d^2*m^2*x^4+135*A*d^3*x^6+B*c^3*m^4*x^2+390*B*c^2
*d*m^2*x^4+405*B*c*d^2*x^6+66*A*c^2*d*m^3*x^2+900*A*c*d^2*m*x^4+22*B*c^3*m
^3*x^2+900*B*c^2*d*m*x^4+A*c^3*m^4+492*A*c^2*d*m^2*x^2+567*A*c*d^2*x^4+164
*B*c^3*m^2*x^2+567*B*c^2*d*x^4+24*A*c^3*m^3+1374*A*c^2*d*m*x^2+458*B*c^3*m
*x^2+206*A*c^3*m^2+945*A*c^2*d*x^2+315*B*c^3*x^2+744*A*c^3*m+945*A*c^3)*(e
*x)^m/(9+m)/(7+m)/(5+m)/(3+m)/(1+m)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 381 vs. 2(121) = 242.

Time = 0.09 (sec) , antiderivative size = 381, normalized size of antiderivative = 3.15

$$\int (ex)^m (A + Bx^2) (c + dx^2)^3 \, dx$$
$$= \frac{((Bd^3m^4 + 16\,Bd^3m^3 + 86\,Bd^3m^2 + 176\,Bd^3m + 105\,Bd^3)x^9 + ((3\,Bcd^2 + Ad^3)m^4 + 405\,Bcd^2 + 135\,A}{}$$

input
```
integrate((e*x)^m*(B*x^2+A)*(d*x^2+c)^3,x, algorithm="fricas")
```

output
```
((B*d^3*m^4 + 16*B*d^3*m^3 + 86*B*d^3*m^2 + 176*B*d^3*m + 105*B*d^3)*x^9 +
 ((3*B*c*d^2 + A*d^3)*m^4 + 405*B*c*d^2 + 135*A*d^3 + 18*(3*B*c*d^2 + A*d^
3)*m^3 + 104*(3*B*c*d^2 + A*d^3)*m^2 + 222*(3*B*c*d^2 + A*d^3)*m)*x^7 + 3*
((B*c^2*d + A*c*d^2)*m^4 + 189*B*c^2*d + 189*A*c*d^2 + 20*(B*c^2*d + A*c*d
^2)*m^3 + 130*(B*c^2*d + A*c*d^2)*m^2 + 300*(B*c^2*d + A*c*d^2)*m)*x^5 + (
(B*c^3 + 3*A*c^2*d)*m^4 + 315*B*c^3 + 945*A*c^2*d + 22*(B*c^3 + 3*A*c^2*d)
*m^3 + 164*(B*c^3 + 3*A*c^2*d)*m^2 + 458*(B*c^3 + 3*A*c^2*d)*m)*x^3 + (A*c
^3*m^4 + 24*A*c^3*m^3 + 206*A*c^3*m^2 + 744*A*c^3*m + 945*A*c^3)*x)*(e*x)^
m/(m^5 + 25*m^4 + 230*m^3 + 950*m^2 + 1689*m + 945)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2152 vs. 2(110) = 220.

Time = 0.59 (sec) , antiderivative size = 2152, normalized size of antiderivative = 17.79

$$\int (ex)^m (A + Bx^2) (c + dx^2)^3 \, dx = \text{Too large to display}$$

input
```
integrate((e*x)**m*(B*x**2+A)*(d*x**2+c)**3,x)
```

output
```
Piecewise(((-A*c**3/(8*x**8) - A*c**2*d/(2*x**6) - 3*A*c*d**2/(4*x**4) - A
*d**3/(2*x**2) - B*c**3/(6*x**6) - 3*B*c**2*d/(4*x**4) - 3*B*c*d**2/(2*x**
2) + B*d**3*log(x))/e**9, Eq(m, -9)), ((-A*c**3/(6*x**6) - 3*A*c**2*d/(4*x
**4) - 3*A*c*d**2/(2*x**2) + A*d**3*log(x) - B*c**3/(4*x**4) - 3*B*c**2*d/
(2*x**2) + 3*B*c*d**2*log(x) + B*d**3*x**2/2)/e**7, Eq(m, -7)), ((-A*c**3/
(4*x**4) - 3*A*c**2*d/(2*x**2) + 3*A*c*d**2*log(x) + A*d**3*x**2/2 - B*c**
3/(2*x**2) + 3*B*c**2*d*log(x) + 3*B*c*d**2*x**2/2 + B*d**3*x**4/4)/e**5,
Eq(m, -5)), ((-A*c**3/(2*x**2) + 3*A*c**2*d*log(x) + 3*A*c*d**2*x**2/2 + A
*d**3*x**4/4 + B*c**3*log(x) + 3*B*c**2*d*x**2/2 + 3*B*c*d**2*x**4/4 + B*d
**3*x**6/6)/e**3, Eq(m, -3)), ((A*c**3*log(x) + 3*A*c**2*d*x**2/2 + 3*A*c*
d**2*x**4/4 + A*d**3*x**6/6 + B*c**3*x**2/2 + 3*B*c**2*d*x**4/4 + B*c*d**2
*x**6/2 + B*d**3*x**8/8)/e, Eq(m, -1)), (A*c**3*m**4*x*(e*x)**m/(m**5 + 25
*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 24*A*c**3*m**3*x*(e*x)**m/(m
**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 206*A*c**3*m**2*x*(e
*x)**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 744*A*c**3*
m*x*(e*x)**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 945*A
*c**3*x*(e*x)**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 3
*A*c**2*d*m**4*x**3*(e*x)**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*
m + 945) + 66*A*c**2*d*m**3*x**3*(e*x)**m/(m**5 + 25*m**4 + 230*m**3 + 950
*m**2 + 1689*m + 945) + 492*A*c**2*d*m**2*x**3*(e*x)**m/(m**5 + 25*m**4...
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.34

$$\int (ex)^m \left(A+Bx^2\right) \left(c+dx^2\right)^3 \, dx = \frac{Bd^3e^m x^9 x^m}{m+9} + \frac{3\,Bcd^2e^m x^7 x^m}{m+7} + \frac{Ad^3e^m x^7 x^m}{m+7} + \frac{3\,Bc^2de^m x^5 x^m}{m+5} + \frac{3\,Acd^2e^m x^5 x^m}{m+5} + \frac{Bc^3e^m x^3 x^m}{m+3} + \frac{3\,Ac^2de^m x^3 x^m}{m+3} + \frac{(ex)^{m+1}\,Ac^3}{e(m+1)}$$

input
```
integrate((e*x)^m*(B*x^2+A)*(d*x^2+c)^3,x, algorithm="maxima")
```

output
```
B*d^3*e^m*x^9*x^m/(m + 9) + 3*B*c*d^2*e^m*x^7*x^m/(m + 7) + A*d^3*e^m*x^7*
x^m/(m + 7) + 3*B*c^2*d*e^m*x^5*x^m/(m + 5) + 3*A*c*d^2*e^m*x^5*x^m/(m + 5
) + B*c^3*e^m*x^3*x^m/(m + 3) + 3*A*c^2*d*e^m*x^3*x^m/(m + 3) + (e*x)^(m +
 1)*A*c^3/(e*(m + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 673 vs. $2(121) = 242$.

Time = 0.14 (sec) , antiderivative size = 673, normalized size of antiderivative = 5.56

$$\int (ex)^m (A + Bx^2) (c + dx^2)^3 \, dx$$
$$= \frac{(ex)^m Bd^3m^4x^9 + 16 (ex)^m Bd^3m^3x^9 + 3 (ex)^m Bcd^2m^4x^7 + (ex)^m Ad^3m^4x^7 + 86 (ex)^m Bd^3m^2x^9 + 54}{}$$

input
```
integrate((e*x)^m*(B*x^2+A)*(d*x^2+c)^3,x, algorithm="giac")
```

output
```
((e*x)^m*B*d^3*m^4*x^9 + 16*(e*x)^m*B*d^3*m^3*x^9 + 3*(e*x)^m*B*c*d^2*m^4*
x^7 + (e*x)^m*A*d^3*m^4*x^7 + 86*(e*x)^m*B*d^3*m^2*x^9 + 54*(e*x)^m*B*c*d^
2*m^3*x^7 + 18*(e*x)^m*A*d^3*m^3*x^7 + 176*(e*x)^m*B*d^3*m*x^9 + 3*(e*x)^m
*B*c^2*d*m^4*x^5 + 3*(e*x)^m*A*c*d^2*m^4*x^5 + 312*(e*x)^m*B*c*d^2*m^2*x^7
 + 104*(e*x)^m*A*d^3*m^2*x^7 + 105*(e*x)^m*B*d^3*x^9 + 60*(e*x)^m*B*c^2*d*
m^3*x^5 + 60*(e*x)^m*A*c*d^2*m^3*x^5 + 666*(e*x)^m*B*c*d^2*m*x^7 + 222*(e*
x)^m*A*d^3*m*x^7 + (e*x)^m*B*c^3*m^4*x^3 + 3*(e*x)^m*A*c^2*d*m^4*x^3 + 390
*(e*x)^m*B*c^2*d*m^2*x^5 + 390*(e*x)^m*A*c*d^2*m^2*x^5 + 405*(e*x)^m*B*c*d
^2*x^7 + 135*(e*x)^m*A*d^3*x^7 + 22*(e*x)^m*B*c^3*m^3*x^3 + 66*(e*x)^m*A*c
^2*d*m^3*x^3 + 900*(e*x)^m*B*c^2*d*m*x^5 + 900*(e*x)^m*A*c*d^2*m*x^5 + (e*
x)^m*A*c^3*m^4*x + 164*(e*x)^m*B*c^3*m^2*x^3 + 492*(e*x)^m*A*c^2*d*m^2*x^3
 + 567*(e*x)^m*B*c^2*d*x^5 + 567*(e*x)^m*A*c*d^2*x^5 + 24*(e*x)^m*A*c^3*m^
3*x + 458*(e*x)^m*B*c^3*m*x^3 + 1374*(e*x)^m*A*c^2*d*m*x^3 + 206*(e*x)^m*A
*c^3*m^2*x + 315*(e*x)^m*B*c^3*x^3 + 945*(e*x)^m*A*c^2*d*x^3 + 744*(e*x)^m
*A*c^3*m*x + 945*(e*x)^m*A*c^3*x)/(m^5 + 25*m^4 + 230*m^3 + 950*m^2 + 1689
*m + 945)
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 280, normalized size of antiderivative = 2.31

$$\int (ex)^m (A+Bx^2) (c+dx^2)^3 \, dx = (e\,x)^m \left(\frac{A\,c^3\,x\,(m^4+24\,m^3+206\,m^2+744\,m+945)}{m^5+25\,m^4+230\,m^3+950\,m^2+1689\,m+945} + \frac{B\,d^3\,x^9\,(m^4+16\,m^3+86\,m^2+176\,m+105)}{m^5+25\,m^4+230\,m^3+950\,m^2+1689\,m+945} + \frac{c^2\,x^3\,(3\,A\,d+B\,c)\,(m^4+22\,m^3+164\,m^2+458\,m+315)}{m^5+25\,m^4+230\,m^3+950\,m^2+1689\,m+945} + \frac{d^2\,x^7\,(A\,d+3\,B\,c)\,(m^4+18\,m^3+104\,m^2+222\,m+135)}{m^5+25\,m^4+230\,m^3+950\,m^2+1689\,m+945} + \frac{3\,c\,d\,x^5\,(A\,d+B\,c)\,(m^4+20\,m^3+130\,m^2+300\,m+189)}{m^5+25\,m^4+230\,m^3+950\,m^2+1689\,m+945}\right)$$

input
```
int((A + B*x^2)*(e*x)^m*(c + d*x^2)^3,x)
```

output
```
(e*x)^m*((A*c^3*x*(744*m + 206*m^2 + 24*m^3 + m^4 + 945))/(1689*m + 950*m^
2 + 230*m^3 + 25*m^4 + m^5 + 945) + (B*d^3*x^9*(176*m + 86*m^2 + 16*m^3 +
m^4 + 105))/(1689*m + 950*m^2 + 230*m^3 + 25*m^4 + m^5 + 945) + (c^2*x^3*(
3*A*d + B*c)*(458*m + 164*m^2 + 22*m^3 + m^4 + 315))/(1689*m + 950*m^2 + 2
30*m^3 + 25*m^4 + m^5 + 945) + (d^2*x^7*(A*d + 3*B*c)*(222*m + 104*m^2 + 1
8*m^3 + m^4 + 135))/(1689*m + 950*m^2 + 230*m^3 + 25*m^4 + m^5 + 945) + (3
*c*d*x^5*(A*d + B*c)*(300*m + 130*m^2 + 20*m^3 + m^4 + 189))/(1689*m + 950
*m^2 + 230*m^3 + 25*m^4 + m^5 + 945))
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 475, normalized size of antiderivative = 3.93

$$\int (ex)^m (A+Bx^2) (c+dx^2)^3 \, dx = \frac{x^m e^m x (b\,d^3 m^4 x^8 + 16b\,d^3 m^3 x^8 + a\,d^3 m^4 x^6 + 3bc\,d^2 m^4 x^6 + 86b\,d^3 m^2 x^8 + 18a\,d^3 m^3 x^6 + 54bc\,d^2 m^3 x^6 +}{}$$

input
```
int((e*x)^m*(B*x^2+A)*(d*x^2+c)^3,x)
```

output
```
(x**m*e**m*x*(a*c**3*m**4 + 24*a*c**3*m**3 + 206*a*c**3*m**2 + 744*a*c**3*
m + 945*a*c**3 + 3*a*c**2*d*m**4*x**2 + 66*a*c**2*d*m**3*x**2 + 492*a*c**2
*d*m**2*x**2 + 1374*a*c**2*d*m*x**2 + 945*a*c**2*d*x**2 + 3*a*c*d**2*m**4*
x**4 + 60*a*c*d**2*m**3*x**4 + 390*a*c*d**2*m**2*x**4 + 900*a*c*d**2*m*x**
4 + 567*a*c*d**2*x**4 + a*d**3*m**4*x**6 + 18*a*d**3*m**3*x**6 + 104*a*d**
3*m**2*x**6 + 222*a*d**3*m*x**6 + 135*a*d**3*x**6 + b*c**3*m**4*x**2 + 22*
b*c**3*m**3*x**2 + 164*b*c**3*m**2*x**2 + 458*b*c**3*m*x**2 + 315*b*c**3*x
**2 + 3*b*c**2*d*m**4*x**4 + 60*b*c**2*d*m**3*x**4 + 390*b*c**2*d*m**2*x**
4 + 900*b*c**2*d*m*x**4 + 567*b*c**2*d*x**4 + 3*b*c*d**2*m**4*x**6 + 54*b*
c*d**2*m**3*x**6 + 312*b*c*d**2*m**2*x**6 + 666*b*c*d**2*m*x**6 + 405*b*c*
d**2*x**6 + b*d**3*m**4*x**8 + 16*b*d**3*m**3*x**8 + 86*b*d**3*m**2*x**8 +
 176*b*d**3*m*x**8 + 105*b*d**3*x**8))/(m**5 + 25*m**4 + 230*m**3 + 950*m*
*2 + 1689*m + 945)
```

3.19 $\int \frac{(ex)^m(A+Bx^2)(c+dx^2)^3}{a+bx^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 258

$$\begin{aligned}\int \frac{(ex)^m(A+Bx^2)(c+dx^2)^3}{a+bx^2}\,dx &= -\frac{(a^3Bd^3+3ab^2cd(Bc+Ad)-a^2bd^2(3Bc+Ad)-b^3c^2(Bc+3Ad))(ex)^{1+m}}{b^4e(1+m)}\\ &\quad+\frac{d(a^2Bd^2+3b^2c(Bc+Ad)-abd(3Bc+Ad))(ex)^{3+m}}{b^3e^3(3+m)}\\ &\quad+\frac{d^2(3bBc+Abd-aBd)(ex)^{5+m}}{b^2e^5(5+m)}+\frac{Bd^3(ex)^{7+m}}{be^7(7+m)}\\ &\quad+\frac{(Ab-aB)(bc-ad)^3(ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{bx^2}{a}\right)}{ab^4e(1+m)}\end{aligned}$$

output
```
-(a^3*B*d^3+3*a*b^2*c*d*(A*d+B*c)-a^2*b*d^2*(A*d+3*B*c)-b^3*c^2*(3*A*d+B*c
))*(e*x)^(1+m)/b^4/e/(1+m)+d*(a^2*B*d^2+3*b^2*c*(A*d+B*c)-a*b*d*(A*d+3*B*c
))*(e*x)^(3+m)/b^3/e^3/(3+m)+d^2*(A*b*d-B*a*d+3*B*b*c)*(e*x)^(5+m)/b^2/e^5
/(5+m)+B*d^3*(e*x)^(7+m)/b/e^7/(7+m)+(A*b-B*a)*(-a*d+b*c)^3*(e*x)^(1+m)*hy
pergeom([1, 1/2+1/2*m],[3/2+1/2*m],-b*x^2/a)/a/b^4/e/(1+m)
```

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.84

$$\int \frac{(ex)^m (A+Bx^2)(c+dx^2)^3}{a+bx^2}\,dx$$
$$= \frac{x(ex)^m \left(\frac{-a^3Bd^3-3ab^2cd(Bc+Ad)+a^2bd^2(3Bc+Ad)+b^3c^2(Bc+3Ad)}{1+m}+\frac{bd(a^2Bd^2+3b^2c(Bc+Ad)-abd(3Bc+Ad))x^2}{3+m}+\frac{b^2d^2(3bBc+\ldots}{5+\ldots}\right.}{b^4}$$

input
```
Integrate[((e*x)^m*(A + B*x^2)*(c + d*x^2)^3)/(a + b*x^2),x]
```

output
```
(x*(e*x)^m*((-(a^3*B*d^3) - 3*a*b^2*c*d*(B*c + A*d) + a^2*b*d^2*(3*B*c + A
*d) + b^3*c^2*(B*c + 3*A*d))/(1 + m) + (b*d*(a^2*B*d^2 + 3*b^2*c*(B*c + A*
d) - a*b*d*(3*B*c + A*d))*x^2)/(3 + m) + (b^2*d^2*(3*b*B*c + A*b*d - a*B*d
)*x^4)/(5 + m) + (b^3*B*d^3*x^6)/(7 + m) + ((-(A*b) + a*B)*(-(b*c) + a*d)^
3*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(a*(1 + m))))/
b^4
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {437, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)(c+dx^2)^3 (ex)^m}{a+bx^2}\,dx$$
$$\Big\downarrow \text{437}$$
$$\int \left(\frac{d(ex)^{m+2}(a^2Bd^2-abd(Ad+3Bc)+3b^2c(Ad+Bc))}{b^3e^2}-\frac{(ex)^m(a^3Bd^3-a^2bd^2(Ad+3Bc)+3ab^2cd(Ad+\ldots}{b^4}\right.$$
$$\Big\downarrow \text{2009}$$

$$\frac{d(ex)^{m+3}\left(a^2Bd^2-abd(Ad+3Bc)+3b^2c(Ad+Bc)\right)}{b^3e^3(m+3)}-\frac{(ex)^{m+1}\left(a^3Bd^3-a^2bd^2(Ad+3Bc)+3ab^2cd(Ad+Bc)+b^3\left(-c^2\right)(3Ad+Bc)\right)}{b^4e(m+1)}+\frac{(ex)^{m+1}(Ab-aB)(bc-ad)^3\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\frac{bx^2}{a}\right)}{ab^4e(m+1)}+\frac{d^2(ex)^{m+5}(-aBd+Abd+3bBc)}{b^2e^5(m+5)}+\frac{Bd^3(ex)^{m+7}}{be^7(m+7)}$$

input
```
Int[((e*x)^m*(A + B*x^2)*(c + d*x^2)^3)/(a + b*x^2),x]
```

output
```
-(((a^3*B*d^3 + 3*a*b^2*c*d*(B*c + A*d) - a^2*b*d^2*(3*B*c + A*d) - b^3*c^
2*(B*c + 3*A*d))*(e*x)^(1 + m))/(b^4*e*(1 + m))) + (d*(a^2*B*d^2 + 3*b^2*c
*(B*c + A*d) - a*b*d*(3*B*c + A*d))*(e*x)^(3 + m))/(b^3*e^3*(3 + m)) + (d^
2*(3*b*B*c + A*b*d - a*B*d)*(e*x)^(5 + m))/(b^2*e^5*(5 + m)) + (B*d^3*(e*x
)^(7 + m))/(b*e^7*(7 + m)) + ((A*b - a*B)*(b*c - a*d)^3*(e*x)^(1 + m)*Hype
rgeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(a*b^4*e*(1 + m))
```

Defintions of rubi rules used

rule 437
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(
a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f
, g, m}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{(ex)^m\left(x^2B+A\right)\left(x^2d+c\right)^3}{b\,x^2+a}dx$$

input
```
int((e*x)^m*(B*x^2+A)*(d*x^2+c)^3/(b*x^2+a),x)
```

output
```
int((e*x)^m*(B*x^2+A)*(d*x^2+c)^3/(b*x^2+a),x)
```

Fricas [F]

$$\int \frac{(ex)^m (A+Bx^2)(c+dx^2)^3}{a+bx^2}\,dx=\int \frac{(Bx^2+A)(dx^2+c)^3(ex)^m}{bx^2+a}\,dx$$

input
```
integrate((e*x)^m*(B*x^2+A)*(d*x^2+c)^3/(b*x^2+a),x, algorithm="fricas")
```

output
```
integral((B*d^3*x^8 + (3*B*c*d^2 + A*d^3)*x^6 + 3*(B*c^2*d + A*c*d^2)*x^4
+ A*c^3 + (B*c^3 + 3*A*c^2*d)*x^2)*(e*x)^m/(b*x^2 + a), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 9.07 (sec) , antiderivative size = 887, normalized size of antiderivative = 3.44

$$\int \frac{(ex)^m (A+Bx^2)(c+dx^2)^3}{a+bx^2}\,dx=\text{Too large to display}$$

input
```
integrate((e*x)**m*(B*x**2+A)*(d*x**2+c)**3/(b*x**2+a),x)
```

output
```
A*c**3*e**m*m*x**(m + 1)*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*
gamma(m/2 + 1/2)/(4*a*gamma(m/2 + 3/2)) + A*c**3*e**m*x**(m + 1)*lerchphi(
b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(4*a*gamma(m/2 +
3/2)) + 3*A*c**2*d*e**m*m*x**(m + 3)*lerchphi(b*x**2*exp_polar(I*pi)/a, 1,
 m/2 + 3/2)*gamma(m/2 + 3/2)/(4*a*gamma(m/2 + 5/2)) + 9*A*c**2*d*e**m*x**(
m + 3)*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 3/2)*gamma(m/2 + 3/2)/(
4*a*gamma(m/2 + 5/2)) + 3*A*c*d**2*e**m*m*x**(m + 5)*lerchphi(b*x**2*exp_p
olar(I*pi)/a, 1, m/2 + 5/2)*gamma(m/2 + 5/2)/(4*a*gamma(m/2 + 7/2)) + 15*A
*c*d**2*e**m*x**(m + 5)*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 5/2)*g
amma(m/2 + 5/2)/(4*a*gamma(m/2 + 7/2)) + A*d**3*e**m*m*x**(m + 7)*lerchphi
(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 7/2)*gamma(m/2 + 7/2)/(4*a*gamma(m/2 +
 9/2)) + 7*A*d**3*e**m*x**(m + 7)*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/
2 + 7/2)*gamma(m/2 + 7/2)/(4*a*gamma(m/2 + 9/2)) + B*c**3*e**m*m*x**(m + 3
)*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 3/2)*gamma(m/2 + 3/2)/(4*a*g
amma(m/2 + 5/2)) + 3*B*c**3*e**m*x**(m + 3)*lerchphi(b*x**2*exp_polar(I*pi
)/a, 1, m/2 + 3/2)*gamma(m/2 + 3/2)/(4*a*gamma(m/2 + 5/2)) + 3*B*c**2*d*e*
*m*m*x**(m + 5)*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 5/2)*gamma(m/2
 + 5/2)/(4*a*gamma(m/2 + 7/2)) + 15*B*c**2*d*e**m*x**(m + 5)*lerchphi(b*x*
*2*exp_polar(I*pi)/a, 1, m/2 + 5/2)*gamma(m/2 + 5/2)/(4*a*gamma(m/2 + 7/2)
) + 3*B*c*d**2*e**m*m*x**(m + 7)*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, ...
```

Maxima [F]

$$\int \frac{(ex)^m (A+Bx^2) (c+dx^2)^3}{a+bx^2} \, dx = \int \frac{(Bx^2+A)(dx^2+c)^3(ex)^m}{bx^2+a} \, dx$$

input
```
integrate((e*x)^m*(B*x^2+A)*(d*x^2+c)^3/(b*x^2+a),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*(d*x^2 + c)^3*(e*x)^m/(b*x^2 + a), x)
```

Giac [F]

$$\int \frac{(ex)^m (A+Bx^2) (c+dx^2)^3}{a+bx^2} \, dx = \int \frac{(Bx^2+A)(dx^2+c)^3(ex)^m}{bx^2+a} \, dx$$

input
```
integrate((e*x)^m*(B*x^2+A)*(d*x^2+c)^3/(b*x^2+a),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*(d*x^2 + c)^3*(e*x)^m/(b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m (A+Bx^2) (c+dx^2)^3}{a+bx^2} \, dx = \int \frac{(B\,x^2+A)\,(e\,x)^m\,(d\,x^2+c)^3}{b\,x^2+a} \, dx$$

input
```
int(((A + B*x^2)*(e*x)^m*(c + d*x^2)^3)/(a + b*x^2),x)
```

output
```
int(((A + B*x^2)*(e*x)^m*(c + d*x^2)^3)/(a + b*x^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.69

$$\int \frac{(ex)^m (A+Bx^2) (c+dx^2)^3}{a+bx^2} \, dx = \frac{x^m e^m x(d^3m^3x^6+9d^3m^2x^6+3c\,d^2m^3x^4+23d^3m\,x^6+33c\,d^2m^2x^4+15d^3x^6+3c^2d\,m^3x^2+93c\,d^2m\,x^4}{m^4+16m^3+86m^2+17}$$

input
```
int((e*x)^m*(B*x^2+A)*(d*x^2+c)^3/(b*x^2+a),x)
```

output
```
(x**m*e**m*x*(c**3*m**3 + 15*c**3*m**2 + 71*c**3*m + 105*c**3 + 3*c**2*d*m
**3*x**2 + 39*c**2*d*m**2*x**2 + 141*c**2*d*m*x**2 + 105*c**2*d*x**2 + 3*c
*d**2*m**3*x**4 + 33*c*d**2*m**2*x**4 + 93*c*d**2*m*x**4 + 63*c*d**2*x**4
+ d**3*m**3*x**6 + 9*d**3*m**2*x**6 + 23*d**3*m*x**6 + 15*d**3*x**6))/(m**
4 + 16*m**3 + 86*m**2 + 176*m + 105)
```

3.20 $\int \frac{(ex)^m (A+Bx^2) (c+dx^2)^3}{(a+bx^2)^2} dx$

Optimal result

Integrand size = 31, antiderivative size = 260

$$\int \frac{(ex)^m (A+Bx^2) (c+dx^2)^3}{(a+bx^2)^2} dx = \frac{d(3a^2Bd^2+3b^2c(Bc+Ad)-2abd(3Bc+Ad))(ex)^{1+m}}{b^4e(1+m)} + \frac{d^2(3bBc+Abd-2aBd)(ex)^{3+m}}{b^3e^3(3+m)} + \frac{Bd^3(ex)^{5+m}}{b^2e^5(5+m)} + \frac{(Ab-aB)(bc-ad)^3(ex)^{1+m}}{2ab^4e(a+bx^2)} + \frac{(bc-ad)^2(aB(bc(1+m)-ad(7+m))+Ab(ad(5+m)+b(c-cm)))(ex)^{1+m}\text{Hypergeometric2F1}\left(\right.}{2a^2b^4e(1+m)}$$

output

```
d*(3*a^2*B*d^2+3*b^2*c*(A*d+B*c)-2*a*b*d*(A*d+3*B*c))*(e*x)^(1+m)/b^4/e/(1
+m)+d^2*(A*b*d-2*B*a*d+3*B*b*c)*(e*x)^(3+m)/b^3/e^3/(3+m)+B*d^3*(e*x)^(5+m
)/b^2/e^5/(5+m)+1/2*(A*b-B*a)*(-a*d+b*c)^3*(e*x)^(1+m)/a/b^4/e/(b*x^2+a)+1
/2*(-a*d+b*c)^2*(a*B*(b*c*(1+m)-a*d*(7+m))+A*b*(a*d*(5+m)+b*(-c*m+c)))*(e*
x)^(1+m)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-b*x^2/a)/a^2/b^4/e/(1+m)
```

Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.80

$$\int \frac{(ex)^m (A+Bx^2) (c+dx^2)^3}{(a+bx^2)^2} \, dx$$

$$= \frac{x(ex)^m \left(\frac{d(3a^2Bd^2+3b^2c(Bc+Ad)-2abd(3Bc+Ad))}{1+m} + \frac{bd^2(3bBc+Abd-2aBd)x^2}{3+m} + \frac{b^2Bd^3x^4}{5+m} + \frac{(bc-ad)^2(bBc+3Abd-4aBd)\,\text{Hyperg}}{a(1+} \right.}{b^4}$$

input
```
Integrate[((e*x)^m*(A + B*x^2)*(c + d*x^2)^3)/(a + b*x^2)^2,x]
```

output
```
(x*(e*x)^m*((d*(3*a^2*B*d^2 + 3*b^2*c*(B*c + A*d) - 2*a*b*d*(3*B*c + A*d))
)/(1 + m) + (b*d^2*(3*b*B*c + A*b*d - 2*a*B*d)*x^2)/(3 + m) + (b^2*B*d^3*x
^4)/(5 + m) + ((b*c - a*d)^2*(b*B*c + 3*A*b*d - 4*a*B*d)*Hypergeometric2F1
[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(a*(1 + m)) + ((-(A*b) + a*B)*(-(
b*c) + a*d)^3*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(a
^2*(1 + m))))/b^4
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.31, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {439, 25, 437, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2) (c+dx^2)^3 (ex)^m}{(a+bx^2)^2} \, dx$$

$$\Big\downarrow \; 439$$

$$\frac{(c+dx^2)^3 (ex)^{m+1}(Ab-aB)}{2abe\,(a+bx^2)} - \frac{\int -\frac{(ex)^m (dx^2+c)^2 (c(aB(m+1)+A(b-bm))-d(Ab(m+5)-aB(m+7))x^2)}{bx^2+a}dx}{2ab}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{(ex)^m (dx^2+c)^2 (c(aB(m+1)+A(b-bm))-d(Ab(m+5)-aB(m+7))x^2)}{bx^2+a}dx}{2ab}+\frac{(c+dx^2)^3 (ex)^{m+1}(Ab-aB)}{2abe\,(a+bx^2)}$$

$$\Big\downarrow \ 437$$

$$\frac{\int \left(-\frac{d(Ab(3b^2(m+1)c^2-3abd(m+3)c+a^2d^2(m+5))-aB(3b^2(m+3)c^2-3abd(m+5)c+a^2d^2(m+7)))(ex)^m}{b^3}+\frac{(-7Bd^3a^4-Bd^3ma^4+5Abd^3a\cdots}{\cdots}\right.}{\cdots}$$

$$\frac{(c+dx^2)^3 (ex)^{m+1}(Ab-aB)}{2abe\,(a+bx^2)}$$

$$\Big\downarrow \ 2009$$

$$\frac{-\frac{d(ex)^{m+1}(Ab(a^2d^2(m+5)-3abcd(m+3)+3b^2c^2(m+1))-aB(a^2d^2(m+7)-3abcd(m+5)+3b^2c^2(m+3)))}{b^3e(m+1)}+\frac{(ex)^{m+1}(bc-ad)^2\ \text{Hypergeomet}\cdots}{\cdots}}{\cdots}$$

$$\frac{(c+dx^2)^3 (ex)^{m+1}(Ab-aB)}{2abe\,(a+bx^2)}$$

input
```
Int[((e*x)^m*(A + B*x^2)*(c + d*x^2)^3)/(a + b*x^2)^2,x]
```

output
```
((A*b - a*B)*(e*x)^(1 + m)*(c + d*x^2)^3)/(2*a*b*e*(a + b*x^2)) + (-((d*(A
*b*(3*b^2*c^2*(1 + m) - 3*a*b*c*d*(3 + m) + a^2*d^2*(5 + m)) - a*B*(3*b^2*
c^2*(3 + m) - 3*a*b*c*d*(5 + m) + a^2*d^2*(7 + m)))*(e*x)^(1 + m))/(b^3*e*
(1 + m))) - (d^2*(A*b*(3*b*c*(3 + m) - a*d*(5 + m)) - a*B*(3*b*c*(5 + m) -
 a*d*(7 + m)))*(e*x)^(3 + m))/(b^2*e^3*(3 + m)) - (d^3*(A*b*(5 + m) - a*B*
(7 + m))*(e*x)^(5 + m))/(b*e^5*(5 + m)) + ((b*c - a*d)^2*(A*b*(b*c*(1 - m)
 + a*d*(5 + m)) + a*B*(b*c*(1 + m) - a*d*(7 + m)))*(e*x)^(1 + m)*Hypergeom
etric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(a*b^3*e*(1 + m)))/(2*a*b
)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 437
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(
a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f
, g, m}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

rule 439
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
 + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*b*g*(p + 1))), x] + Simp[1/(2*a*b*(p
+ 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1)*Simp[c*(2*b*e*(
p + 1) + (b*e - a*f)*(m + 1)) + d*(2*b*e*(p + 1) + (b*e - a*f)*(m + 2*q + 1
))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && LtQ[p, -1] && G
tQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e - a*f])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{(ex)^m (x^2B + A) (x^2d + c)^3}{(b\,x^2 + a)^2} dx$$

input
```
int((e*x)^m*(B*x^2+A)*(d*x^2+c)^3/(b*x^2+a)^2,x)
```

output
```
int((e*x)^m*(B*x^2+A)*(d*x^2+c)^3/(b*x^2+a)^2,x)
```

Fricas [F]

$$\int \frac{(ex)^m (A + Bx^2) (c + dx^2)^3}{(a + bx^2)^2} \, dx = \int \frac{(Bx^2 + A)(dx^2 + c)^3 (ex)^m}{(bx^2 + a)^2} \, dx$$

input
```
integrate((e*x)^m*(B*x^2+A)*(d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
integral((B*d^3*x^8 + (3*B*c*d^2 + A*d^3)*x^6 + 3*(B*c^2*d + A*c*d^2)*x^4
+ A*c^3 + (B*c^3 + 3*A*c^2*d)*x^2)*(e*x)^m/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [F]

$$\int \frac{(ex)^m (A + Bx^2) (c + dx^2)^3}{(a + bx^2)^2} \, dx = \int \frac{(ex)^m (A + Bx^2) (c + dx^2)^3}{(a + bx^2)^2} \, dx$$

input
```
integrate((e*x)**m*(B*x**2+A)*(d*x**2+c)**3/(b*x**2+a)**2,x)
```

output
```
Integral((e*x)**m*(A + B*x**2)*(c + d*x**2)**3/(a + b*x**2)**2, x)
```

Maxima [F]

$$\int \frac{(ex)^m (A + Bx^2) (c + dx^2)^3}{(a + bx^2)^2} \, dx = \int \frac{(Bx^2 + A)(dx^2 + c)^3 (ex)^m}{(bx^2 + a)^2} \, dx$$

input
```
integrate((e*x)^m*(B*x^2+A)*(d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*(d*x^2 + c)^3*(e*x)^m/(b*x^2 + a)^2, x)
```

Giac [F]

$$\int \frac{(ex)^m (A+Bx^2) (c+dx^2)^3}{(a+bx^2)^2} \, dx = \int \frac{(Bx^2+A)(dx^2+c)^3(ex)^m}{(bx^2+a)^2} \, dx$$

input
```
integrate((e*x)^m*(B*x^2+A)*(d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*(d*x^2 + c)^3*(e*x)^m/(b*x^2 + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m (A+Bx^2) (c+dx^2)^3}{(a+bx^2)^2} \, dx = \int \frac{(B\,x^2+A)\,(e\,x)^m\,(d\,x^2+c)^3}{(b\,x^2+a)^2} \, dx$$

input
```
int(((A + B*x^2)*(e*x)^m*(c + d*x^2)^3)/(a + b*x^2)^2,x)
```

output
```
int(((A + B*x^2)*(e*x)^m*(c + d*x^2)^3)/(a + b*x^2)^2, x)
```

Reduce [F]

$$\int \frac{(ex)^m (A+Bx^2) (c+dx^2)^3}{(a+bx^2)^2} \, dx$$
$$= \frac{e^m \left(3\left(\int \frac{x^m}{b\,x^2+a}dx\right) a^2bc\,d^2m^3 + 27\left(\int \frac{x^m}{b\,x^2+a}dx\right) a^2bc\,d^2m^2 + 69\left(\int \frac{x^m}{b\,x^2+a}dx\right) a^2bc\,d^2m - 3\left(\int \frac{x^m}{b\,x^2+a}dx\right) a\,b^2c\right.}{}$$

input
```
int((e*x)^m*(B*x^2+A)*(d*x^2+c)^3/(b*x^2+a)^2,x)
```

output
```
(e**m*(x**m*a**2*d**3*m**2*x + 8*x**m*a**2*d**3*m*x + 15*x**m*a**2*d**3*x
- 3*x**m*a*b*c*d**2*m**2*x - 24*x**m*a*b*c*d**2*m*x - 45*x**m*a*b*c*d**2*x
 - x**m*a*b*d**3*m**2*x**3 - 6*x**m*a*b*d**3*m*x**3 - 5*x**m*a*b*d**3*x**3
 + 3*x**m*b**2*c**2*d*m**2*x + 24*x**m*b**2*c**2*d*m*x + 45*x**m*b**2*c**2
*d*x + 3*x**m*b**2*c*d**2*m**2*x**3 + 18*x**m*b**2*c*d**2*m*x**3 + 15*x**m
*b**2*c*d**2*x**3 + x**m*b**2*d**3*m**2*x**5 + 4*x**m*b**2*d**3*m*x**5 + 3
*x**m*b**2*d**3*x**5 - int(x**m/(a + b*x**2),x)*a**3*d**3*m**3 - 9*int(x**
m/(a + b*x**2),x)*a**3*d**3*m**2 - 23*int(x**m/(a + b*x**2),x)*a**3*d**3*m
 - 15*int(x**m/(a + b*x**2),x)*a**3*d**3 + 3*int(x**m/(a + b*x**2),x)*a**2
*b*c*d**2*m**3 + 27*int(x**m/(a + b*x**2),x)*a**2*b*c*d**2*m**2 + 69*int(x
**m/(a + b*x**2),x)*a**2*b*c*d**2*m + 45*int(x**m/(a + b*x**2),x)*a**2*b*c
*d**2 - 3*int(x**m/(a + b*x**2),x)*a*b**2*c**2*d*m**3 - 27*int(x**m/(a + b
*x**2),x)*a*b**2*c**2*d*m**2 - 69*int(x**m/(a + b*x**2),x)*a*b**2*c**2*d*m
 - 45*int(x**m/(a + b*x**2),x)*a*b**2*c**2*d + int(x**m/(a + b*x**2),x)*b*
*3*c**3*m**3 + 9*int(x**m/(a + b*x**2),x)*b**3*c**3*m**2 + 23*int(x**m/(a
+ b*x**2),x)*b**3*c**3*m + 15*int(x**m/(a + b*x**2),x)*b**3*c**3))/(b**3*(
m**3 + 9*m**2 + 23*m + 15))
```

3.21 $\int \frac{(ex)^m (A+Bx^2) (c+dx^2)^3}{(a+bx^2)^3} \, dx$

Optimal result

Integrand size = 31, antiderivative size = 340

$$\int \frac{(ex)^m (A+Bx^2) (c+dx^2)^3}{(a+bx^2)^3} \, dx = \frac{d^2(3bBc+Abd-3aBd)(ex)^{1+m}}{b^4e(1+m)} + \frac{Bd^3(ex)^{3+m}}{b^3e^3(3+m)} + \frac{(Ab-aB)(bc-ad)^3(ex)^{1+m}}{4ab^4e\,(a+bx^2)^2} + \frac{(bc-ad)^2(Ab(bc(3-m)+ad(9+m))+aB(bc(1+m)-ad(13+m)))(ex)^{1+m}}{8a^2b^4e\,(a+bx^2)} + \frac{(bc-ad)\,(Ab(2abcd(3-2m-m^2)+b^2c^2(3-4m+m^2)+a^2d^2(15+8m+m^2))+aB(b^2c^2(1-m^2)}{8a^3b^4e(}$$

output
```
d^2*(A*b*d-3*B*a*d+3*B*b*c)*(e*x)^(1+m)/b^4/e/(1+m)+B*d^3*(e*x)^(3+m)/b^3/
e^3/(3+m)+1/4*(A*b-B*a)*(-a*d+b*c)^3*(e*x)^(1+m)/a/b^4/e/(b*x^2+a)^2+1/8*(
-a*d+b*c)^2*(A*b*(b*c*(3-m)+a*d*(9+m))+a*B*(b*c*(1+m)-a*d*(13+m)))*(e*x)^(
1+m)/a^2/b^4/e/(b*x^2+a)+1/8*(-a*d+b*c)*(A*b*(2*a*b*c*d*(-m^2-2*m+3)+b^2*c
^2*(m^2-4*m+3)+a^2*d^2*(m^2+8*m+15))+a*B*(b^2*c^2*(-m^2+1)+2*a*b*c*d*(m^2+
6*m+5)-a^2*d^2*(m^2+12*m+35)))*(e*x)^(1+m)*hypergeom([1, 1/2+1/2*m],[3/2+1
/2*m],-b*x^2/a)/a^3/b^4/e/(1+m)
```

Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.64

$$\int \frac{(ex)^m (A+Bx^2) (c+dx^2)^3}{(a+bx^2)^3} \, dx$$
$$= \frac{x(ex)^m \left(\frac{d^2(3bBc+Abd-3aBd)}{1+m} + \frac{bBd^3x^2}{3+m} + \frac{3d(bc-ad)(bBc+Abd-2aBd) \text{ Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{bx^2}{a}\right)}{a(1+m)} + \frac{(bc-ad)^2(bB\cdots}{\cdots}\right.}{b^4}$$

input
```
Integrate[((e*x)^m*(A + B*x^2)*(c + d*x^2)^3)/(a + b*x^2)^3,x]
```

output
```
(x*(e*x)^m*((d^2*(3*b*B*c + A*b*d - 3*a*B*d))/(1 + m) + (b*B*d^3*x^2)/(3 +
 m) + (3*d*(b*c - a*d)*(b*B*c + A*b*d - 2*a*B*d)*Hypergeometric2F1[1, (1 +
 m)/2, (3 + m)/2, -((b*x^2)/a)])/(a*(1 + m)) + ((b*c - a*d)^2*(b*B*c + 3*A
*b*d - 4*a*B*d)*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/
(a^2*(1 + m)) + ((-(A*b) + a*B)*(-(b*c) + a*d)^3*Hypergeometric2F1[3, (1 +
 m)/2, (3 + m)/2, -((b*x^2)/a)])/(a^3*(1 + m))))/b^4
```

Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 489, normalized size of antiderivative = 1.44, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {439, 25, 439, 25, 437, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2) (c+dx^2)^3 (ex)^m}{(a+bx^2)^3} \, dx$$
$$\Big\downarrow \; 439$$
$$\frac{(c+dx^2)^3 (ex)^{m+1}(Ab-aB)}{4abe\,(a+bx^2)^2} - \frac{\int -\frac{(ex)^m (dx^2+c)^2 (c(Ab(3-m)+aB(m+1))-d(Ab(m+3)-aB(m+7))x^2)}{(bx^2+a)^2}dx}{4ab}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{(ex)^m (dx^2+c)^2 (c(Ab(3-m)+aB(m+1))-d(Ab(m+3)-aB(m+7))x^2)}{(bx^2+a)^2}dx}{4ab}+\frac{(c+dx^2)^3 (ex)^{m+1}(Ab-aB)}{4abe\,(a+bx^2)^2}$$

$$\Big\downarrow \ 439$$

$$\frac{\frac{(c+dx^2)^2 (ex)^{m+1}(Ab(ad(m+3)+bc(3-m))+aB(bc(m+1)-ad(m+7)))}{2abe(a+bx^2)}-\int -\frac{(ex)^m \left(dx^2+c\right)\left(c\left(aB(m+1)(ad(m+7)+b(c-cm))+Ab\left(bc\left(m^2-4m+3\cdots\right.\right.\right.\right.}{\cdots}}{4ab}+$$

$$\frac{(c+dx^2)^3 (ex)^{m+1}(Ab-aB)}{4abe\,(a+bx^2)^2}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{\int \frac{(ex)^m \left(dx^2+c\right)\left(c\left(aB(m+1)(ad(m+7)+b(c-cm))+Ab\left(bc\left(m^2-4m+3\right)-ad\left(m^2+4m+3\right)\right)\right)-d\left(Ab(m+3)(bc(3-m)+ad(m+5))+aB\left(bc\left(m^2+4m+3\right)-ad\left(m^2\cdots\right.\right.\right.\right.}{bx^2+a}\cdots}{2ab}\cdots}{4ab}+$$

$$\frac{(c+dx^2)^3 (ex)^{m+1}(Ab-aB)}{4abe\,(a+bx^2)^2}$$

$$\Big\downarrow \ 437$$

$$\frac{\frac{\int \left(-\frac{d\left(Ab\left(2b^2\left(-m^2+2m+3\right)c^2+3abd\left(m^2+4m+3\right)c-a^2d^2\left(m^2+8m+15\right)\right)+aB\left(a^2\left(m^2+12m+35\right)d^2-3abc\left(m^2+8m+15\right)d+2b^2c^2(m+1)^2\right)\right)(ex)^m}{b^2}+\frac{\left(35Bd\cdots\right.}{\cdots}\right.\cdots}{\cdots}}{\cdots}+$$

$$\frac{(c+dx^2)^3 (ex)^{m+1}(Ab-aB)}{4abe\,(a+bx^2)^2}$$

$$\Big\downarrow \ 2009$$

$$\frac{\frac{\frac{(ex)^{m+1}(bc-ad)\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\frac{bx^2}{a}\right)\left(Ab\left(a^2d^2\left(m^2+8m+15\right)+2abcd\left(-m^2-2m+3\right)+b^2c^2\left(m^2-4m+3\right)\right)+aB\left(-a^2d^2\left(m^2+12m+35\right)+\cdots\right.\right.}{ab^2e(m+1)}\cdots}{\cdots}}{\cdots}+$$

$$\frac{(c+dx^2)^3 (ex)^{m+1}(Ab-aB)}{4abe\,(a+bx^2)^2}$$

input

```
Int[((e*x)^m*(A + B*x^2)*(c + d*x^2)^3)/(a + b*x^2)^3,x]
```

output
```
((A*b - a*B)*(e*x)^(1 + m)*(c + d*x^2)^3)/(4*a*b*e*(a + b*x^2)^2) + (((A*b
*(b*c*(3 - m) + a*d*(3 + m)) + a*B*(b*c*(1 + m) - a*d*(7 + m)))*(e*x)^(1 +
 m)*(c + d*x^2)^2)/(2*a*b*e*(a + b*x^2)) + (-((d*(A*b*(2*b^2*c^2*(3 + 2*m
- m^2) + 3*a*b*c*d*(3 + 4*m + m^2) - a^2*d^2*(15 + 8*m + m^2)) + a*B*(2*b^
2*c^2*(1 + m)^2 - 3*a*b*c*d*(15 + 8*m + m^2) + a^2*d^2*(35 + 12*m + m^2)))
*(e*x)^(1 + m))/(b^2*e*(1 + m))) - (d^2*(A*b*(3 + m)*(b*c*(3 - m) + a*d*(5
 + m)) + a*B*(b*c*(3 + 4*m + m^2) - a*d*(35 + 12*m + m^2)))*(e*x)^(3 + m))
/(b*e^3*(3 + m)) + ((b*c - a*d)*(A*b*(2*a*b*c*d*(3 - 2*m - m^2) + b^2*c^2*
(3 - 4*m + m^2) + a^2*d^2*(15 + 8*m + m^2)) + a*B*(b^2*c^2*(1 - m^2) + 2*a
*b*c*d*(5 + 6*m + m^2) - a^2*d^2*(35 + 12*m + m^2)))*(e*x)^(1 + m)*Hyperge
ometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(a*b^2*e*(1 + m)))/(2*a
*b))/(4*a*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 437
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(
a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f
, g, m}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

rule 439
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
 + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*b*g*(p + 1))), x] + Simp[1/(2*a*b*(p
+ 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1)*Simp[c*(2*b*e*(
p + 1) + (b*e - a*f)*(m + 1)) + d*(2*b*e*(p + 1) + (b*e - a*f)*(m + 2*q + 1
))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && LtQ[p, -1] && G
tQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e - a*f])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{(ex)^m (x^2B + A) (x^2d + c)^3}{(b\,x^2 + a)^3}dx$$

input
```
int((e*x)^m*(B*x^2+A)*(d*x^2+c)^3/(b*x^2+a)^3,x)
```

output
```
int((e*x)^m*(B*x^2+A)*(d*x^2+c)^3/(b*x^2+a)^3,x)
```

Fricas [F]

$$\int \frac{(ex)^m (A + Bx^2) (c + dx^2)^3}{(a + bx^2)^3} \, dx = \int \frac{(Bx^2 + A)(dx^2 + c)^3(ex)^m}{(bx^2 + a)^3} \, dx$$

input
```
integrate((e*x)^m*(B*x^2+A)*(d*x^2+c)^3/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
integral((B*d^3*x^8 + (3*B*c*d^2 + A*d^3)*x^6 + 3*(B*c^2*d + A*c*d^2)*x^4
+ A*c^3 + (B*c^3 + 3*A*c^2*d)*x^2)*(e*x)^m/(b^3*x^6 + 3*a*b^2*x^4 + 3*a^2*
b*x^2 + a^3), x)
```

Sympy [F]

$$\int \frac{(ex)^m (A + Bx^2) (c + dx^2)^3}{(a + bx^2)^3} \, dx = \int \frac{(ex)^m (A + Bx^2) (c + dx^2)^3}{(a + bx^2)^3} \, dx$$

input
```
integrate((e*x)**m*(B*x**2+A)*(d*x**2+c)**3/(b*x**2+a)**3,x)
```

output
```
Integral((e*x)**m*(A + B*x**2)*(c + d*x**2)**3/(a + b*x**2)**3, x)
```

Maxima [F]

$$\int \frac{(ex)^m (A+Bx^2) (c+dx^2)^3}{(a+bx^2)^3}\,dx = \int \frac{(Bx^2+A)(dx^2+c)^3(ex)^m}{(bx^2+a)^3}\,dx$$

input
```
integrate((e*x)^m*(B*x^2+A)*(d*x^2+c)^3/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*(d*x^2 + c)^3*(e*x)^m/(b*x^2 + a)^3, x)
```

Giac [F]

$$\int \frac{(ex)^m (A+Bx^2) (c+dx^2)^3}{(a+bx^2)^3}\,dx = \int \frac{(Bx^2+A)(dx^2+c)^3(ex)^m}{(bx^2+a)^3}\,dx$$

input
```
integrate((e*x)^m*(B*x^2+A)*(d*x^2+c)^3/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*(d*x^2 + c)^3*(e*x)^m/(b*x^2 + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m (A+Bx^2) (c+dx^2)^3}{(a+bx^2)^3}\,dx = \int \frac{(B\,x^2+A)\,(e\,x)^m\,(d\,x^2+c)^3}{(b\,x^2+a)^3}\,dx$$

input
```
int(((A + B*x^2)*(e*x)^m*(c + d*x^2)^3)/(a + b*x^2)^3,x)
```

output
```
int(((A + B*x^2)*(e*x)^m*(c + d*x^2)^3)/(a + b*x^2)^3, x)
```

Reduce [F]

$$\int \frac{(ex)^m (A + Bx^2) (c + dx^2)^3}{(a + bx^2)^3} \, dx = \text{too large to display}$$

input
```
int((e*x)^m*(B*x^2+A)*(d*x^2+c)^3/(b*x^2+a)^3,x)
```

output
```
(e**m*(x**m*a**2*d**3*m**2*x + 8*x**m*a**2*d**3*m*x + 15*x**m*a**2*d**3*x
- 3*x**m*a*b*c*d**2*m**2*x - 18*x**m*a*b*c*d**2*m*x - 27*x**m*a*b*c*d**2*x
 - x**m*a*b*d**3*m**2*x**3 - 4*x**m*a*b*d**3*m*x**3 + 5*x**m*a*b*d**3*x**3
 + 3*x**m*b**2*c**2*d*m**2*x + 12*x**m*b**2*c**2*d*m*x + 9*x**m*b**2*c**2*
d*x + 3*x**m*b**2*c*d**2*m**2*x**3 + 6*x**m*b**2*c*d**2*m*x**3 - 9*x**m*b*
*2*c*d**2*x**3 + x**m*b**2*d**3*m**2*x**5 - x**m*b**2*d**3*x**5 - int(x**m
/(a**2*m - a**2 + 2*a*b*m*x**2 - 2*a*b*x**2 + b**2*m*x**4 - b**2*x**4),x)*
a**4*d**3*m**4 - 8*int(x**m/(a**2*m - a**2 + 2*a*b*m*x**2 - 2*a*b*x**2 + b
**2*m*x**4 - b**2*x**4),x)*a**4*d**3*m**3 - 14*int(x**m/(a**2*m - a**2 + 2
*a*b*m*x**2 - 2*a*b*x**2 + b**2*m*x**4 - b**2*x**4),x)*a**4*d**3*m**2 + 8*
int(x**m/(a**2*m - a**2 + 2*a*b*m*x**2 - 2*a*b*x**2 + b**2*m*x**4 - b**2*x
**4),x)*a**4*d**3*m + 15*int(x**m/(a**2*m - a**2 + 2*a*b*m*x**2 - 2*a*b*x*
*2 + b**2*m*x**4 - b**2*x**4),x)*a**4*d**3 + 3*int(x**m/(a**2*m - a**2 + 2
*a*b*m*x**2 - 2*a*b*x**2 + b**2*m*x**4 - b**2*x**4),x)*a**3*b*c*d**2*m**4
+ 18*int(x**m/(a**2*m - a**2 + 2*a*b*m*x**2 - 2*a*b*x**2 + b**2*m*x**4 - b
**2*x**4),x)*a**3*b*c*d**2*m**3 + 24*int(x**m/(a**2*m - a**2 + 2*a*b*m*x**
2 - 2*a*b*x**2 + b**2*m*x**4 - b**2*x**4),x)*a**3*b*c*d**2*m**2 - 18*int(x
**m/(a**2*m - a**2 + 2*a*b*m*x**2 - 2*a*b*x**2 + b**2*m*x**4 - b**2*x**4),
x)*a**3*b*c*d**2*m - 27*int(x**m/(a**2*m - a**2 + 2*a*b*m*x**2 - 2*a*b*x**
2 + b**2*m*x**4 - b**2*x**4),x)*a**3*b*c*d**2 - int(x**m/(a**2*m - a**2...
```

3.22 $\int \frac{(ex)^m(a+bx^2)^4(A+Bx^2)}{c+dx^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 363

$$\int \frac{(ex)^m (a+bx^2)^4 (A+Bx^2)}{c+dx^2}\,dx = \frac{(a^4Bd^4+b^4c^3(Bc-Ad)-4ab^3c^2d(Bc-Ad)+6a^2b^2cd^2(Bc-Ad)-4a^3bd^3(Bc-Ad))\,(ex)^{1+m}}{d^5e(1+m)} + \frac{b(4a^3Bd^3-b^3c^2(Bc-Ad)+4ab^2cd(Bc-Ad)-6a^2bd^2(Bc-Ad))\,(ex)^{3+m}}{d^4e^3(3+m)} + \frac{b^2(6a^2Bd^2+b^2c(Bc-Ad)-4abd(Bc-Ad))\,(ex)^{5+m}}{d^3e^5(5+m)} - \frac{b^3(bBc-Abd-4aBd)(ex)^{7+m}}{d^2e^7(7+m)} + \frac{b^4B(ex)^{9+m}}{de^9(9+m)} - \frac{(bc-ad)^4(Bc-Ad)(ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{dx^2}{c}\right)}{cd^5e(1+m)}$$

output
```
(a^4*B*d^4+b^4*c^3*(-A*d+B*c)-4*a*b^3*c^2*d*(-A*d+B*c)+6*a^2*b^2*c*d^2*(-A
*d+B*c)-4*a^3*b*d^3*(-A*d+B*c))*(e*x)^(1+m)/d^5/e/(1+m)+b*(4*a^3*B*d^3-b^3
*c^2*(-A*d+B*c)+4*a*b^2*c*d*(-A*d+B*c)-6*a^2*b*d^2*(-A*d+B*c))*(e*x)^(3+m)
/d^4/e^3/(3+m)+b^2*(6*a^2*B*d^2+b^2*c*(-A*d+B*c)-4*a*b*d*(-A*d+B*c))*(e*x)
^(5+m)/d^3/e^5/(5+m)-b^3*(-A*b*d-4*B*a*d+B*b*c)*(e*x)^(7+m)/d^2/e^7/(7+m)+
b^4*B*(e*x)^(9+m)/d/e^9/(9+m)-(-a*d+b*c)^4*(-A*d+B*c)*(e*x)^(1+m)*hypergeo
m([1, 1/2+1/2*m],[3/2+1/2*m],-d*x^2/c)/c/d^5/e/(1+m)
```

Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 315, normalized size of antiderivative = 0.87

$$\int \frac{(ex)^m (a+bx^2)^4 (A+Bx^2)}{c+dx^2} dx$$

$$= \frac{x(ex)^m \left(\frac{a^4Bd^4+b^4c^3(Bc-Ad)+6a^2b^2cd^2(Bc-Ad)+4ab^3c^2d(-Bc+Ad)+4a^3bd^3(-Bc+Ad)}{1+m} + \frac{bd(4a^3Bd^3+4ab^2cd(Bc-Ad)+b^3c^2(-Bc\right.}{3+m}}{}$$

input
```
Integrate[((e*x)^m*(a + b*x^2)^4*(A + B*x^2))/(c + d*x^2),x]
```

output
```
(x*(e*x)^m*((a^4*B*d^4 + b^4*c^3*(B*c - A*d) + 6*a^2*b^2*c*d^2*(B*c - A*d)
 + 4*a*b^3*c^2*d*(-(B*c) + A*d) + 4*a^3*b*d^3*(-(B*c) + A*d))/(1 + m) + (b
*d*(4*a^3*B*d^3 + 4*a*b^2*c*d*(B*c - A*d) + b^3*c^2*(-(B*c) + A*d) + 6*a^2
*b*d^2*(-(B*c) + A*d))*x^2)/(3 + m) + (b^2*d^2*(6*a^2*B*d^2 + b^2*c*(B*c -
 A*d) + 4*a*b*d*(-(B*c) + A*d))*x^4)/(5 + m) + (b^3*d^3*(-(b*B*c) + A*b*d
+ 4*a*B*d)*x^6)/(7 + m) + (b^4*B*d^4*x^8)/(9 + m) - ((b*c - a*d)^4*(B*c -
A*d)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((d*x^2)/c)])/(c*(1 + m))
))/d^5
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {437, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^4 (A+Bx^2) (ex)^m}{c+dx^2} dx$$

$$\Big\downarrow \text{437}$$

$$\int \left(\frac{b^2(ex)^{m+4} (6a^2Bd^2-4abd(Bc-Ad)+b^2c(Bc-Ad))}{d^3e^4} + \frac{b(ex)^{m+2} (4a^3Bd^3-6a^2bd^2(Bc-Ad)+4ab^2cd(}{d^4e^2}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{b^2(ex)^{m+5}\left(6a^2Bd^2-4abd(Bc-Ad)+b^2c(Bc-Ad)\right)}{d^3e^5(m+5)}+\frac{b(ex)^{m+3}\left(4a^3Bd^3-6a^2bd^2(Bc-Ad)+4ab^2cd(Bc-Ad)+b^3\left(-c^2\right)(Bc-Ad)\right)}{d^4e^3(m+3)}+\frac{(ex)^{m+1}\left(a^4Bd^4-4a^3bd^3(Bc-Ad)+6a^2b^2cd^2(Bc-Ad)-4ab^3c^2d(Bc-Ad)+b^4c^3(Bc-Ad)\right)}{d^5e(m+1)}-\frac{b^3(ex)^{m+7}(-4aBd-Abd+bBc)}{d^2e^7(m+7)}-\frac{(ex)^{m+1}(bc-ad)^4(Bc-Ad)\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\frac{dx^2}{c}\right)}{cd^5e(m+1)}+\frac{b^4B(ex)^{m+9}}{de^9(m+9)}$$

input
```
Int[((e*x)^m*(a + b*x^2)^4*(A + B*x^2))/(c + d*x^2),x]
```

output
```
((a^4*B*d^4 + b^4*c^3*(B*c - A*d) - 4*a*b^3*c^2*d*(B*c - A*d) + 6*a^2*b^2*
c*d^2*(B*c - A*d) - 4*a^3*b*d^3*(B*c - A*d))*(e*x)^(1 + m))/(d^5*e*(1 + m)
) + (b*(4*a^3*B*d^3 - b^3*c^2*(B*c - A*d) + 4*a*b^2*c*d*(B*c - A*d) - 6*a^
2*b*d^2*(B*c - A*d))*(e*x)^(3 + m))/(d^4*e^3*(3 + m)) + (b^2*(6*a^2*B*d^2
+ b^2*c*(B*c - A*d) - 4*a*b*d*(B*c - A*d))*(e*x)^(5 + m))/(d^3*e^5*(5 + m)
) - (b^3*(b*B*c - A*b*d - 4*a*B*d)*(e*x)^(7 + m))/(d^2*e^7*(7 + m)) + (b^4
*B*(e*x)^(9 + m))/(d*e^9*(9 + m)) - ((b*c - a*d)^4*(B*c - A*d)*(e*x)^(1 +
m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((d*x^2)/c)])/(c*d^5*e*(1 +
 m))
```

Defintions of rubi rules used

rule 437
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(
a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f
, g, m}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{(ex)^m (b\,x^2+a)^4 (x^2B+A)}{x^2d+c}dx$$

input
```
int((e*x)^m*(b*x^2+a)^4*(B*x^2+A)/(d*x^2+c),x)
```

output
```
int((e*x)^m*(b*x^2+a)^4*(B*x^2+A)/(d*x^2+c),x)
```

Fricas [F]

$$\int \frac{(ex)^m (a+bx^2)^4 (A+Bx^2)}{c+dx^2}\,dx = \int \frac{(Bx^2+A)(bx^2+a)^4(ex)^m}{dx^2+c}\,dx$$

input
```
integrate((e*x)^m*(b*x^2+a)^4*(B*x^2+A)/(d*x^2+c),x, algorithm="fricas")
```

output
```
integral((B*b^4*x^10 + (4*B*a*b^3 + A*b^4)*x^8 + 2*(3*B*a^2*b^2 + 2*A*a*b^
3)*x^6 + A*a^4 + 2*(2*B*a^3*b + 3*A*a^2*b^2)*x^4 + (B*a^4 + 4*A*a^3*b)*x^2
)*(e*x)^m/(d*x^2 + c), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 13.93 (sec) , antiderivative size = 1102, normalized size of antiderivative = 3.04

$$\int \frac{(ex)^m (a+bx^2)^4 (A+Bx^2)}{c+dx^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*x)**m*(b*x**2+a)**4*(B*x**2+A)/(d*x**2+c),x)
```

output
```
A*a**4*e**m*m*x**(m + 1)*lerchphi(d*x**2*exp_polar(I*pi)/c, 1, m/2 + 1/2)*
gamma(m/2 + 1/2)/(4*c*gamma(m/2 + 3/2)) + A*a**4*e**m*x**(m + 1)*lerchphi(
d*x**2*exp_polar(I*pi)/c, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(4*c*gamma(m/2 +
3/2)) + A*a**3*b*e**m*m*x**(m + 3)*lerchphi(d*x**2*exp_polar(I*pi)/c, 1, m
/2 + 3/2)*gamma(m/2 + 3/2)/(c*gamma(m/2 + 5/2)) + 3*A*a**3*b*e**m*x**(m +
3)*lerchphi(d*x**2*exp_polar(I*pi)/c, 1, m/2 + 3/2)*gamma(m/2 + 3/2)/(c*ga
mma(m/2 + 5/2)) + 3*A*a**2*b**2*e**m*m*x**(m + 5)*lerchphi(d*x**2*exp_pola
r(I*pi)/c, 1, m/2 + 5/2)*gamma(m/2 + 5/2)/(2*c*gamma(m/2 + 7/2)) + 15*A*a*
*2*b**2*e**m*x**(m + 5)*lerchphi(d*x**2*exp_polar(I*pi)/c, 1, m/2 + 5/2)*g
amma(m/2 + 5/2)/(2*c*gamma(m/2 + 7/2)) + A*a*b**3*e**m*m*x**(m + 7)*lerchp
hi(d*x**2*exp_polar(I*pi)/c, 1, m/2 + 7/2)*gamma(m/2 + 7/2)/(c*gamma(m/2 +
 9/2)) + 7*A*a*b**3*e**m*x**(m + 7)*lerchphi(d*x**2*exp_polar(I*pi)/c, 1,
m/2 + 7/2)*gamma(m/2 + 7/2)/(c*gamma(m/2 + 9/2)) + A*b**4*e**m*m*x**(m + 9
)*lerchphi(d*x**2*exp_polar(I*pi)/c, 1, m/2 + 9/2)*gamma(m/2 + 9/2)/(4*c*g
amma(m/2 + 11/2)) + 9*A*b**4*e**m*x**(m + 9)*lerchphi(d*x**2*exp_polar(I*p
i)/c, 1, m/2 + 9/2)*gamma(m/2 + 9/2)/(4*c*gamma(m/2 + 11/2)) + B*a**4*e**m
*m*x**(m + 3)*lerchphi(d*x**2*exp_polar(I*pi)/c, 1, m/2 + 3/2)*gamma(m/2 +
 3/2)/(4*c*gamma(m/2 + 5/2)) + 3*B*a**4*e**m*x**(m + 3)*lerchphi(d*x**2*ex
p_polar(I*pi)/c, 1, m/2 + 3/2)*gamma(m/2 + 3/2)/(4*c*gamma(m/2 + 5/2)) + B
*a**3*b*e**m*m*x**(m + 5)*lerchphi(d*x**2*exp_polar(I*pi)/c, 1, m/2 + 5...
```

Maxima [F]

$$\int \frac{(ex)^m (a+bx^2)^4 (A+Bx^2)}{c+dx^2}\,dx = \int \frac{(Bx^2+A)(bx^2+a)^4(ex)^m}{dx^2+c}\,dx$$

input
```
integrate((e*x)^m*(b*x^2+a)^4*(B*x^2+A)/(d*x^2+c),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*(b*x^2 + a)^4*(e*x)^m/(d*x^2 + c), x)
```

Giac [F]

$$\int \frac{(ex)^m (a+bx^2)^4 (A+Bx^2)}{c+dx^2}\,dx = \int \frac{(Bx^2+A)(bx^2+a)^4(ex)^m}{dx^2+c}\,dx$$

input
```
integrate((e*x)^m*(b*x^2+a)^4*(B*x^2+A)/(d*x^2+c),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*(b*x^2 + a)^4*(e*x)^m/(d*x^2 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m (a+bx^2)^4 (A+Bx^2)}{c+dx^2}\,dx = \int \frac{(B\,x^2+A)\,(e\,x)^m\,(b\,x^2+a)^4}{d\,x^2+c}\,dx$$

input
```
int(((A + B*x^2)*(e*x)^m*(a + b*x^2)^4)/(c + d*x^2),x)
```

output
```
int(((A + B*x^2)*(e*x)^m*(a + b*x^2)^4)/(c + d*x^2), x)
```

Reduce [F]

$$\int \frac{(ex)^m (a+bx^2)^4 (A+Bx^2)}{c+dx^2}\,dx = \text{too large to display}$$

input
```
int((e*x)^m*(b*x^2+a)^4*(B*x^2+A)/(d*x^2+c),x)
```

output

```
(e**m*(5*x**m*a**4*b*d**4*m**4*x + 120*x**m*a**4*b*d**4*m**3*x + 1030*x**m
*a**4*b*d**4*m**2*x + 3720*x**m*a**4*b*d**4*m*x + 4725*x**m*a**4*b*d**4*x
- 10*x**m*a**3*b**2*c*d**3*m**4*x - 240*x**m*a**3*b**2*c*d**3*m**3*x - 206
0*x**m*a**3*b**2*c*d**3*m**2*x - 7440*x**m*a**3*b**2*c*d**3*m*x - 9450*x**
m*a**3*b**2*c*d**3*x + 10*x**m*a**3*b**2*d**4*m**4*x**3 + 220*x**m*a**3*b*
*2*d**4*m**3*x**3 + 1640*x**m*a**3*b**2*d**4*m**2*x**3 + 4580*x**m*a**3*b*
*2*d**4*m*x**3 + 3150*x**m*a**3*b**2*d**4*x**3 + 10*x**m*a**2*b**3*c**2*d*
*2*m**4*x + 240*x**m*a**2*b**3*c**2*d**2*m**3*x + 2060*x**m*a**2*b**3*c**2
*d**2*m**2*x + 7440*x**m*a**2*b**3*c**2*d**2*m*x + 9450*x**m*a**2*b**3*c**
2*d**2*x - 10*x**m*a**2*b**3*c*d**3*m**4*x**3 - 220*x**m*a**2*b**3*c*d**3*
m**3*x**3 - 1640*x**m*a**2*b**3*c*d**3*m**2*x**3 - 4580*x**m*a**2*b**3*c*d
**3*m*x**3 - 3150*x**m*a**2*b**3*c*d**3*x**3 + 10*x**m*a**2*b**3*d**4*m**4
*x**5 + 200*x**m*a**2*b**3*d**4*m**3*x**5 + 1300*x**m*a**2*b**3*d**4*m**2*
x**5 + 3000*x**m*a**2*b**3*d**4*m*x**5 + 1890*x**m*a**2*b**3*d**4*x**5 - 5
*x**m*a*b**4*c**3*d*m**4*x - 120*x**m*a*b**4*c**3*d*m**3*x - 1030*x**m*a*b
**4*c**3*d*m**2*x - 3720*x**m*a*b**4*c**3*d*m*x - 4725*x**m*a*b**4*c**3*d*
x + 5*x**m*a*b**4*c**2*d**2*m**4*x**3 + 110*x**m*a*b**4*c**2*d**2*m**3*x**
3 + 820*x**m*a*b**4*c**2*d**2*m**2*x**3 + 2290*x**m*a*b**4*c**2*d**2*m*x**
3 + 1575*x**m*a*b**4*c**2*d**2*x**3 - 5*x**m*a*b**4*c*d**3*m**4*x**5 - 100
*x**m*a*b**4*c*d**3*m**3*x**5 - 650*x**m*a*b**4*c*d**3*m**2*x**5 - 1500...
```

3.23 $\int \frac{(ex)^m(a+bx^2)^3(A+Bx^2)}{c+dx^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 260

$$\begin{aligned}\int \frac{(ex)^m\left(a+bx^2\right)^3\left(A+Bx^2\right)}{c+dx^2}\,dx &= \frac{(a^3Bd^3-b^3c^2(Bc-Ad)+3ab^2cd(Bc-Ad)-3a^2bd^2(Bc-Ad))\,(ex)^{1+m}}{d^4e(1+m)} \\ &\quad+\frac{b(3a^2Bd^2+b^2c(Bc-Ad)-3abd(Bc-Ad))\,(ex)^{3+m}}{d^3e^3(3+m)} \\ &\quad-\frac{b^2(bBc-Abd-3aBd)(ex)^{5+m}}{d^2e^5(5+m)}+\frac{b^3B(ex)^{7+m}}{de^7(7+m)} \\ &\quad+\frac{(bc-ad)^3(Bc-Ad)(ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{dx^2}{c}\right)}{cd^4e(1+m)}\end{aligned}$$

output
```
(a^3*B*d^3-b^3*c^2*(-A*d+B*c)+3*a*b^2*c*d*(-A*d+B*c)-3*a^2*b*d^2*(-A*d+B*c
))*(e*x)^(1+m)/d^4/e/(1+m)+b*(3*a^2*B*d^2+b^2*c*(-A*d+B*c)-3*a*b*d*(-A*d+B
*c))*(e*x)^(3+m)/d^3/e^3/(3+m)-b^2*(-A*b*d-3*B*a*d+B*b*c)*(e*x)^(5+m)/d^2/
e^5/(5+m)+b^3*B*(e*x)^(7+m)/d/e^7/(7+m)+(-a*d+b*c)^3*(-A*d+B*c)*(e*x)^(1+m
)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-d*x^2/c)/c/d^4/e/(1+m)
```

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.84

$$\int \frac{(ex)^m (a+bx^2)^3 (A+Bx^2)}{c+dx^2} dx$$
$$= \frac{x(ex)^m \left(\frac{a^3Bd^3+3ab^2cd(Bc-Ad)+b^3c^2(-Bc+Ad)+3a^2bd^2(-Bc+Ad)}{1+m} + \frac{bd(3a^2Bd^2+b^2c(Bc-Ad)+3abd(-Bc+Ad))x^2}{3+m} + \frac{b^2d^2(-bBc\right.}{d^4}$$

input
```
Integrate[((e*x)^m*(a + b*x^2)^3*(A + B*x^2))/(c + d*x^2),x]
```

output
```
(x*(e*x)^m*((a^3*B*d^3 + 3*a*b^2*c*d*(B*c - A*d) + b^3*c^2*(-(B*c) + A*d)
+ 3*a^2*b*d^2*(-(B*c) + A*d))/(1 + m) + (b*d*(3*a^2*B*d^2 + b^2*c*(B*c - A
*d) + 3*a*b*d*(-(B*c) + A*d))*x^2)/(3 + m) + (b^2*d^2*(-(b*B*c) + A*b*d +
3*a*B*d)*x^4)/(5 + m) + (b^3*B*d^3*x^6)/(7 + m) + ((b*c - a*d)^3*(B*c - A*
d)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((d*x^2)/c)])/(c*(1 + m)))
/d^4
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {437, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^3 (A+Bx^2) (ex)^m}{c+dx^2} dx$$

$$\Big\downarrow \text{437}$$

$$\int \left(\frac{b(ex)^{m+2} (3a^2Bd^2 - 3abd(Bc-Ad) + b^2c(Bc-Ad))}{d^3e^2} + \frac{(ex)^m (a^3Bd^3 - 3a^2bd^2(Bc-Ad) + 3ab^2cd(Bc-\right.}{d^4}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b(ex)^{m+3}\left(3a^2Bd^2-3abd(Bc-Ad)+b^2c(Bc-Ad)\right)}{d^3e^3(m+3)}+\frac{(ex)^{m+1}\left(a^3Bd^3-3a^2bd^2(Bc-Ad)+3ab^2cd(Bc-Ad)+b^3\left(-c^2\right)(Bc-Ad)\right)}{d^4e(m+1)}-\frac{b^2(ex)^{m+5}(-3aBd-Abd+bBc)}{d^2e^5(m+5)}+\frac{(ex)^{m+1}(bc-ad)^3(Bc-Ad)\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\frac{dx^2}{c}\right)}{cd^4e(m+1)}+\frac{b^3B(ex)^{m+7}}{de^7(m+7)}$$

input
```
Int[((e*x)^m*(a + b*x^2)^3*(A + B*x^2))/(c + d*x^2),x]
```

output
```
((a^3*B*d^3 - b^3*c^2*(B*c - A*d) + 3*a*b^2*c*d*(B*c - A*d) - 3*a^2*b*d^2*
(B*c - A*d))*(e*x)^(1 + m))/(d^4*e*(1 + m)) + (b*(3*a^2*B*d^2 + b^2*c*(B*c
 - A*d) - 3*a*b*d*(B*c - A*d))*(e*x)^(3 + m))/(d^3*e^3*(3 + m)) - (b^2*(b*
B*c - A*b*d - 3*a*B*d)*(e*x)^(5 + m))/(d^2*e^5*(5 + m)) + (b^3*B*(e*x)^(7
+ m))/(d*e^7*(7 + m)) + ((b*c - a*d)^3*(B*c - A*d)*(e*x)^(1 + m)*Hypergeom
etric2F1[1, (1 + m)/2, (3 + m)/2, -((d*x^2)/c)])/(c*d^4*e*(1 + m))
```

Defintions of rubi rules used

rule 437
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(
a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f
, g, m}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{(ex)^m\left(b\,x^2+a\right)^3\left(x^2B+A\right)}{x^2d+c}dx$$

input
```
int((e*x)^m*(b*x^2+a)^3*(B*x^2+A)/(d*x^2+c),x)
```

output
```
int((e*x)^m*(b*x^2+a)^3*(B*x^2+A)/(d*x^2+c),x)
```

Fricas [F]

$$\int \frac{(ex)^m (a+bx^2)^3 (A+Bx^2)}{c+dx^2} \, dx = \int \frac{(Bx^2+A)(bx^2+a)^3(ex)^m}{dx^2+c} \, dx$$

input
```
integrate((e*x)^m*(b*x^2+a)^3*(B*x^2+A)/(d*x^2+c),x, algorithm="fricas")
```

output
```
integral((B*b^3*x^8 + (3*B*a*b^2 + A*b^3)*x^6 + 3*(B*a^2*b + A*a*b^2)*x^4
+ A*a^3 + (B*a^3 + 3*A*a^2*b)*x^2)*(e*x)^m/(d*x^2 + c), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 9.51 (sec) , antiderivative size = 887, normalized size of antiderivative = 3.41

$$\int \frac{(ex)^m (a+bx^2)^3 (A+Bx^2)}{c+dx^2} \, dx = \text{Too large to display}$$

input
```
integrate((e*x)**m*(b*x**2+a)**3*(B*x**2+A)/(d*x**2+c),x)
```

output
```
A*a**3*e**m*m*x**(m + 1)*lerchphi(d*x**2*exp_polar(I*pi)/c, 1, m/2 + 1/2)*
gamma(m/2 + 1/2)/(4*c*gamma(m/2 + 3/2)) + A*a**3*e**m*x**(m + 1)*lerchphi(
d*x**2*exp_polar(I*pi)/c, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(4*c*gamma(m/2 +
3/2)) + 3*A*a**2*b*e**m*m*x**(m + 3)*lerchphi(d*x**2*exp_polar(I*pi)/c, 1,
 m/2 + 3/2)*gamma(m/2 + 3/2)/(4*c*gamma(m/2 + 5/2)) + 9*A*a**2*b*e**m*x**(
m + 3)*lerchphi(d*x**2*exp_polar(I*pi)/c, 1, m/2 + 3/2)*gamma(m/2 + 3/2)/(
4*c*gamma(m/2 + 5/2)) + 3*A*a*b**2*e**m*m*x**(m + 5)*lerchphi(d*x**2*exp_p
olar(I*pi)/c, 1, m/2 + 5/2)*gamma(m/2 + 5/2)/(4*c*gamma(m/2 + 7/2)) + 15*A
*a*b**2*e**m*x**(m + 5)*lerchphi(d*x**2*exp_polar(I*pi)/c, 1, m/2 + 5/2)*g
amma(m/2 + 5/2)/(4*c*gamma(m/2 + 7/2)) + A*b**3*e**m*m*x**(m + 7)*lerchphi
(d*x**2*exp_polar(I*pi)/c, 1, m/2 + 7/2)*gamma(m/2 + 7/2)/(4*c*gamma(m/2 +
 9/2)) + 7*A*b**3*e**m*x**(m + 7)*lerchphi(d*x**2*exp_polar(I*pi)/c, 1, m/
2 + 7/2)*gamma(m/2 + 7/2)/(4*c*gamma(m/2 + 9/2)) + B*a**3*e**m*m*x**(m + 3
)*lerchphi(d*x**2*exp_polar(I*pi)/c, 1, m/2 + 3/2)*gamma(m/2 + 3/2)/(4*c*g
amma(m/2 + 5/2)) + 3*B*a**3*e**m*x**(m + 3)*lerchphi(d*x**2*exp_polar(I*pi
)/c, 1, m/2 + 3/2)*gamma(m/2 + 3/2)/(4*c*gamma(m/2 + 5/2)) + 3*B*a**2*b*e*
*m*m*x**(m + 5)*lerchphi(d*x**2*exp_polar(I*pi)/c, 1, m/2 + 5/2)*gamma(m/2
 + 5/2)/(4*c*gamma(m/2 + 7/2)) + 15*B*a**2*b*e**m*x**(m + 5)*lerchphi(d*x*
*2*exp_polar(I*pi)/c, 1, m/2 + 5/2)*gamma(m/2 + 5/2)/(4*c*gamma(m/2 + 7/2)
) + 3*B*a*b**2*e**m*m*x**(m + 7)*lerchphi(d*x**2*exp_polar(I*pi)/c, 1, ...
```

Maxima [F]

$$\int \frac{(ex)^m\left(a+bx^2\right)^3\left(A+Bx^2\right)}{c+dx^2}\,dx=\int \frac{(Bx^2+A)(bx^2+a)^3(ex)^m}{dx^2+c}\,dx$$

input
```
integrate((e*x)^m*(b*x^2+a)^3*(B*x^2+A)/(d*x^2+c),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*(b*x^2 + a)^3*(e*x)^m/(d*x^2 + c), x)
```

Giac [F]

$$\int \frac{(ex)^m (a+bx^2)^3 (A+Bx^2)}{c+dx^2}\,dx = \int \frac{(Bx^2+A)(bx^2+a)^3(ex)^m}{dx^2+c}\,dx$$

input
```
integrate((e*x)^m*(b*x^2+a)^3*(B*x^2+A)/(d*x^2+c),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*(b*x^2 + a)^3*(e*x)^m/(d*x^2 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m (a+bx^2)^3 (A+Bx^2)}{c+dx^2}\,dx = \int \frac{(B\,x^2+A)\,(e\,x)^m\,(b\,x^2+a)^3}{d\,x^2+c}\,dx$$

input
```
int(((A + B*x^2)*(e*x)^m*(a + b*x^2)^3)/(c + d*x^2),x)
```

output
```
int(((A + B*x^2)*(e*x)^m*(a + b*x^2)^3)/(c + d*x^2), x)
```

Reduce [F]

$$\int \frac{(ex)^m (a+bx^2)^3 (A+Bx^2)}{c+dx^2}\,dx = \text{Too large to display}$$

input
```
int((e*x)^m*(b*x^2+a)^3*(B*x^2+A)/(d*x^2+c),x)
```

output
```
(e**m*(4*x**m*a**3*b*d**3*m**3*x + 60*x**m*a**3*b*d**3*m**2*x + 284*x**m*a
**3*b*d**3*m*x + 420*x**m*a**3*b*d**3*x - 6*x**m*a**2*b**2*c*d**2*m**3*x -
 90*x**m*a**2*b**2*c*d**2*m**2*x - 426*x**m*a**2*b**2*c*d**2*m*x - 630*x**
m*a**2*b**2*c*d**2*x + 6*x**m*a**2*b**2*d**3*m**3*x**3 + 78*x**m*a**2*b**2
*d**3*m**2*x**3 + 282*x**m*a**2*b**2*d**3*m*x**3 + 210*x**m*a**2*b**2*d**3
*x**3 + 4*x**m*a*b**3*c**2*d*m**3*x + 60*x**m*a*b**3*c**2*d*m**2*x + 284*x
**m*a*b**3*c**2*d*m*x + 420*x**m*a*b**3*c**2*d*x - 4*x**m*a*b**3*c*d**2*m*
*3*x**3 - 52*x**m*a*b**3*c*d**2*m**2*x**3 - 188*x**m*a*b**3*c*d**2*m*x**3
- 140*x**m*a*b**3*c*d**2*x**3 + 4*x**m*a*b**3*d**3*m**3*x**5 + 44*x**m*a*b
**3*d**3*m**2*x**5 + 124*x**m*a*b**3*d**3*m*x**5 + 84*x**m*a*b**3*d**3*x**
5 - x**m*b**4*c**3*m**3*x - 15*x**m*b**4*c**3*m**2*x - 71*x**m*b**4*c**3*m
*x - 105*x**m*b**4*c**3*x + x**m*b**4*c**2*d*m**3*x**3 + 13*x**m*b**4*c**2
*d*m**2*x**3 + 47*x**m*b**4*c**2*d*m*x**3 + 35*x**m*b**4*c**2*d*x**3 - x**
m*b**4*c*d**2*m**3*x**5 - 11*x**m*b**4*c*d**2*m**2*x**5 - 31*x**m*b**4*c*d
**2*m*x**5 - 21*x**m*b**4*c*d**2*x**5 + x**m*b**4*d**3*m**3*x**7 + 9*x**m*
b**4*d**3*m**2*x**7 + 23*x**m*b**4*d**3*m*x**7 + 15*x**m*b**4*d**3*x**7 +
int(x**m/(c + d*x**2),x)*a**4*d**4*m**4 + 16*int(x**m/(c + d*x**2),x)*a**4
*d**4*m**3 + 86*int(x**m/(c + d*x**2),x)*a**4*d**4*m**2 + 176*int(x**m/(c
+ d*x**2),x)*a**4*d**4*m + 105*int(x**m/(c + d*x**2),x)*a**4*d**4 - 4*int(
x**m/(c + d*x**2),x)*a**3*b*c*d**3*m**4 - 64*int(x**m/(c + d*x**2),x)*a...
```

3.24 $\int \frac{(ex)^m(a+bx^2)^2(A+Bx^2)}{c+dx^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 180

$$\int \frac{(ex)^m (a+bx^2)^2 (A+Bx^2)}{c+dx^2}\,dx = \frac{(a^2Bd^2+b^2c(Bc-Ad)-2abd(Bc-Ad))\,(ex)^{1+m}}{d^3e(1+m)} - \frac{b(bBc-Abd-2aBd)(ex)^{3+m}}{d^2e^3(3+m)} + \frac{b^2B(ex)^{5+m}}{de^5(5+m)} - \frac{(bc-ad)^2(Bc-Ad)(ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{dx^2}{c}\right)}{cd^3e(1+m)}$$

output

```
(a^2*B*d^2+b^2*c*(-A*d+B*c)-2*a*b*d*(-A*d+B*c))*(e*x)^(1+m)/d^3/e/(1+m)-b*
(-A*b*d-2*B*a*d+B*b*c)*(e*x)^(3+m)/d^2/e^3/(3+m)+b^2*B*(e*x)^(5+m)/d/e^5/(
5+m)-(-a*d+b*c)^2*(-A*d+B*c)*(e*x)^(1+m)*hypergeom([1, 1/2+1/2*m],[3/2+1/2
*m],-d*x^2/c)/c/d^3/e/(1+m)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.82

$$\int \frac{(ex)^m (a+bx^2)^2 (A+Bx^2)}{c+dx^2} dx$$
$$= \frac{x(ex)^m \left(\frac{a^2Bd^2+b^2c(Bc-Ad)+2abd(-Bc+Ad)}{1+m} + \frac{bd(-bBc+Abd+2aBd)x^2}{3+m} + \frac{b^2Bd^2x^4}{5+m} - \frac{(bc-ad)^2(Bc-Ad)\,\text{Hypergeometric2F1}\left(1\right.}{c(1+m)}\right.}{d^3}$$

input
```
Integrate[((e*x)^m*(a + b*x^2)^2*(A + B*x^2))/(c + d*x^2),x]
```

output
```
(x*(e*x)^m*((a^2*B*d^2 + b^2*c*(B*c - A*d) + 2*a*b*d*(-(B*c) + A*d))/(1 +
m) + (b*d*(-(b*B*c) + A*b*d + 2*a*B*d)*x^2)/(3 + m) + (b^2*B*d^2*x^4)/(5 +
 m) - ((b*c - a*d)^2*(B*c - A*d)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2
, -((d*x^2)/c)])/(c*(1 + m))))/d^3
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {437, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2 (A+Bx^2) (ex)^m}{c+dx^2} dx$$
$$\Big\downarrow \text{437}$$
$$\int \left(\frac{(ex)^m (a^2Ad^3 - a^2Bcd^2 - 2aAbcd^2 + 2abBc^2d + Ab^2c^2d - b^2Bc^3)}{d^3 (c+dx^2)} + \frac{(ex)^m (a^2Bd^2 - 2abd(Bc-Ad) + b^2}{d^3}\right.$$
$$\Big\downarrow \text{2009}$$

$$\frac{(ex)^{m+1}\left(a^2Bd^2-2abd(Bc-Ad)+b^2c(Bc-Ad)\right)}{d^3e(m+1)}-\frac{(ex)^{m+1}(bc-ad)^2(Bc-Ad)\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\frac{dx^2}{c}\right)}{cd^3e(m+1)}-\frac{b(ex)^{m+3}(-2aBd-Abd+bBc)}{d^2e^3(m+3)}+\frac{b^2B(ex)^{m+5}}{de^5(m+5)}$$

input
```
Int[((e*x)^m*(a + b*x^2)^2*(A + B*x^2))/(c + d*x^2),x]
```

output
```
((a^2*B*d^2 + b^2*c*(B*c - A*d) - 2*a*b*d*(B*c - A*d))*(e*x)^(1 + m))/(d^3
*e*(1 + m)) - (b*(b*B*c - A*b*d - 2*a*B*d)*(e*x)^(3 + m))/(d^2*e^3*(3 + m)
) + (b^2*B*(e*x)^(5 + m))/(d*e^5*(5 + m)) - ((b*c - a*d)^2*(B*c - A*d)*(e*
x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((d*x^2)/c)])/(c*d^
3*e*(1 + m))
```

Defintions of rubi rules used

rule 437
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(
a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f
, g, m}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{(ex)^m\left(b\,x^2+a\right)^2\left(x^2B+A\right)}{x^2d+c}dx$$

input
```
int((e*x)^m*(b*x^2+a)^2*(B*x^2+A)/(d*x^2+c),x)
```

output
```
int((e*x)^m*(b*x^2+a)^2*(B*x^2+A)/(d*x^2+c),x)
```

Fricas [F]

$$\int \frac{(ex)^m \left(a+bx^2\right)^2 \left(A+Bx^2\right)}{c+dx^2}\,dx=\int \frac{(Bx^2+A)(bx^2+a)^2(ex)^m}{dx^2+c}\,dx$$

input
```
integrate((e*x)^m*(b*x^2+a)^2*(B*x^2+A)/(d*x^2+c),x, algorithm="fricas")
```

output
```
integral((B*b^2*x^6 + (2*B*a*b + A*b^2)*x^4 + A*a^2 + (B*a^2 + 2*A*a*b)*x^
2)*(e*x)^m/(d*x^2 + c), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.13 (sec) , antiderivative size = 649, normalized size of antiderivative = 3.61

$$\int \frac{(ex)^m \left(a+bx^2\right)^2 \left(A+Bx^2\right)}{c+dx^2}\,dx=\text{Too large to display}$$

input
```
integrate((e*x)**m*(b*x**2+a)**2*(B*x**2+A)/(d*x**2+c),x)
```

output
```
A*a**2*e**m*m*x**(m + 1)*lerchphi(d*x**2*exp_polar(I*pi)/c, 1, m/2 + 1/2)*
gamma(m/2 + 1/2)/(4*c*gamma(m/2 + 3/2)) + A*a**2*e**m*x**(m + 1)*lerchphi(
d*x**2*exp_polar(I*pi)/c, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(4*c*gamma(m/2 +
3/2)) + A*a*b*e**m*m*x**(m + 3)*lerchphi(d*x**2*exp_polar(I*pi)/c, 1, m/2
+ 3/2)*gamma(m/2 + 3/2)/(2*c*gamma(m/2 + 5/2)) + 3*A*a*b*e**m*x**(m + 3)*l
erchphi(d*x**2*exp_polar(I*pi)/c, 1, m/2 + 3/2)*gamma(m/2 + 3/2)/(2*c*gamm
a(m/2 + 5/2)) + A*b**2*e**m*m*x**(m + 5)*lerchphi(d*x**2*exp_polar(I*pi)/c
, 1, m/2 + 5/2)*gamma(m/2 + 5/2)/(4*c*gamma(m/2 + 7/2)) + 5*A*b**2*e**m*x*
*(m + 5)*lerchphi(d*x**2*exp_polar(I*pi)/c, 1, m/2 + 5/2)*gamma(m/2 + 5/2)
/(4*c*gamma(m/2 + 7/2)) + B*a**2*e**m*m*x**(m + 3)*lerchphi(d*x**2*exp_pol
ar(I*pi)/c, 1, m/2 + 3/2)*gamma(m/2 + 3/2)/(4*c*gamma(m/2 + 5/2)) + 3*B*a*
*2*e**m*x**(m + 3)*lerchphi(d*x**2*exp_polar(I*pi)/c, 1, m/2 + 3/2)*gamma(
m/2 + 3/2)/(4*c*gamma(m/2 + 5/2)) + B*a*b*e**m*m*x**(m + 5)*lerchphi(d*x**
2*exp_polar(I*pi)/c, 1, m/2 + 5/2)*gamma(m/2 + 5/2)/(2*c*gamma(m/2 + 7/2))
 + 5*B*a*b*e**m*x**(m + 5)*lerchphi(d*x**2*exp_polar(I*pi)/c, 1, m/2 + 5/2
)*gamma(m/2 + 5/2)/(2*c*gamma(m/2 + 7/2)) + B*b**2*e**m*m*x**(m + 7)*lerch
phi(d*x**2*exp_polar(I*pi)/c, 1, m/2 + 7/2)*gamma(m/2 + 7/2)/(4*c*gamma(m/
2 + 9/2)) + 7*B*b**2*e**m*x**(m + 7)*lerchphi(d*x**2*exp_polar(I*pi)/c, 1,
 m/2 + 7/2)*gamma(m/2 + 7/2)/(4*c*gamma(m/2 + 9/2))
```

Maxima [F]

$$\int \frac{(ex)^m (a+bx^2)^2 (A+Bx^2)}{c+dx^2} \, dx = \int \frac{(Bx^2+A)(bx^2+a)^2(ex)^m}{dx^2+c} \, dx$$

input
```
integrate((e*x)^m*(b*x^2+a)^2*(B*x^2+A)/(d*x^2+c),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*(b*x^2 + a)^2*(e*x)^m/(d*x^2 + c), x)
```

Giac [F]

$$\int \frac{(ex)^m (a+bx^2)^2 (A+Bx^2)}{c+dx^2} \, dx = \int \frac{(Bx^2+A)(bx^2+a)^2(ex)^m}{dx^2+c} \, dx$$

input
```
integrate((e*x)^m*(b*x^2+a)^2*(B*x^2+A)/(d*x^2+c),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*(b*x^2 + a)^2*(e*x)^m/(d*x^2 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m (a+bx^2)^2 (A+Bx^2)}{c+dx^2} \, dx = \int \frac{(B\,x^2+A)\,(e\,x)^m\,(b\,x^2+a)^2}{d\,x^2+c} \, dx$$

input
```
int(((A + B*x^2)*(e*x)^m*(a + b*x^2)^2)/(c + d*x^2),x)
```

output
```
int(((A + B*x^2)*(e*x)^m*(a + b*x^2)^2)/(c + d*x^2), x)
```

Reduce [F]

$$\int \frac{(ex)^m (a+bx^2)^2 (A+Bx^2)}{c+dx^2} \, dx = \frac{e^m \left(15x^m b^3 c^2 x + 3x^m b^3 d^2 x^5 - 45\left(\int \frac{x^m}{d\,x^2+c} dx\right) a^2 bc\,d^2 + 45\left(\int \frac{x^m}{d\,x^2+c} dx\right) a\,b^2 c^2 d + 15\left(\int \frac{x^m}{d\,x^2+c} dx\right) a^3 d^3 - 1\right.}{}$$

input
```
int((e*x)^m*(b*x^2+a)^2*(B*x^2+A)/(d*x^2+c),x)
```

output
```
(e**m*(3*x**m*a**2*b*d**2*m**2*x + 24*x**m*a**2*b*d**2*m*x + 45*x**m*a**2*
b*d**2*x - 3*x**m*a*b**2*c*d*m**2*x - 24*x**m*a*b**2*c*d*m*x - 45*x**m*a*b
**2*c*d*x + 3*x**m*a*b**2*d**2*m**2*x**3 + 18*x**m*a*b**2*d**2*m*x**3 + 15
*x**m*a*b**2*d**2*x**3 + x**m*b**3*c**2*m**2*x + 8*x**m*b**3*c**2*m*x + 15
*x**m*b**3*c**2*x - x**m*b**3*c*d*m**2*x**3 - 6*x**m*b**3*c*d*m*x**3 - 5*x
**m*b**3*c*d*x**3 + x**m*b**3*d**2*m**2*x**5 + 4*x**m*b**3*d**2*m*x**5 + 3
*x**m*b**3*d**2*x**5 + int(x**m/(c + d*x**2),x)*a**3*d**3*m**3 + 9*int(x**
m/(c + d*x**2),x)*a**3*d**3*m**2 + 23*int(x**m/(c + d*x**2),x)*a**3*d**3*m
 + 15*int(x**m/(c + d*x**2),x)*a**3*d**3 - 3*int(x**m/(c + d*x**2),x)*a**2
*b*c*d**2*m**3 - 27*int(x**m/(c + d*x**2),x)*a**2*b*c*d**2*m**2 - 69*int(x
**m/(c + d*x**2),x)*a**2*b*c*d**2*m - 45*int(x**m/(c + d*x**2),x)*a**2*b*c
*d**2 + 3*int(x**m/(c + d*x**2),x)*a*b**2*c**2*d*m**3 + 27*int(x**m/(c + d
*x**2),x)*a*b**2*c**2*d*m**2 + 69*int(x**m/(c + d*x**2),x)*a*b**2*c**2*d*m
 + 45*int(x**m/(c + d*x**2),x)*a*b**2*c**2*d - int(x**m/(c + d*x**2),x)*b*
*3*c**3*m**3 - 9*int(x**m/(c + d*x**2),x)*b**3*c**3*m**2 - 23*int(x**m/(c
+ d*x**2),x)*b**3*c**3*m - 15*int(x**m/(c + d*x**2),x)*b**3*c**3))/(d**3*(
m**3 + 9*m**2 + 23*m + 15))
```

3.25 $\int \frac{(ex)^m(a+bx^2)(A+Bx^2)}{c+dx^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 120

$$\int \frac{(ex)^m(a+bx^2)(A+Bx^2)}{c+dx^2}\,dx = -\frac{(bBc-Abd-aBd)(ex)^{1+m}}{d^2e(1+m)}+\frac{bB(ex)^{3+m}}{de^3(3+m)} + \frac{(bc-ad)(Bc-Ad)(ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{dx^2}{c}\right)}{cd^2e(1+m)}$$

output
```
-(-A*b*d-B*a*d+B*b*c)*(e*x)^(1+m)/d^2/e/(1+m)+b*B*(e*x)^(3+m)/d/e^3/(3+m)+
(-a*d+b*c)*(-A*d+B*c)*(e*x)^(1+m)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-d*
x^2/c)/c/d^2/e/(1+m)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.78

$$\int \frac{(ex)^m(a+bx^2)(A+Bx^2)}{c+dx^2}\,dx = \frac{x(ex)^m\left(\frac{-bBc+Abd+aBd}{1+m}+\frac{bBdx^2}{3+m}+\frac{(bc-ad)(Bc-Ad)\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{dx^2}{c}\right)}{c(1+m)}\right)}{d^2}$$

input
```
Integrate[((e*x)^m*(a + b*x^2)*(A + B*x^2))/(c + d*x^2),x]
```

output
```
(x*(e*x)^m*((-(b*B*c) + A*b*d + a*B*d)/(1 + m) + (b*B*d*x^2)/(3 + m) + ((b
*c - a*d)*(B*c - A*d)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((d*x^2)
/c)])/(c*(1 + m))))/d^2
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {437, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)(A+Bx^2)(ex)^m}{c+dx^2}\,dx$$

$$\Big\downarrow \text{437}$$

$$\int \left(\frac{(ex)^m(aAd^2-aBcd-Abcd+bBc^2)}{d^2(c+dx^2)}-\frac{(ex)^m(-aBd-Abd+bBc)}{d^2}+\frac{bB(ex)^{m+2}}{de^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{(ex)^{m+1}(bc-ad)(Bc-Ad)\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\frac{dx^2}{c}\right)}{cd^2e(m+1)}-\frac{(ex)^{m+1}(-aBd-Abd+bBc)}{d^2e(m+1)}+\frac{bB(ex)^{m+3}}{de^3(m+3)}$$

input
```
Int[((e*x)^m*(a + b*x^2)*(A + B*x^2))/(c + d*x^2),x]
```

output
```
-(((b*B*c - A*b*d - a*B*d)*(e*x)^(1 + m))/(d^2*e*(1 + m))) + (b*B*(e*x)^(3
 + m))/(d*e^3*(3 + m)) + ((b*c - a*d)*(B*c - A*d)*(e*x)^(1 + m)*Hypergeome
tric2F1[1, (1 + m)/2, (3 + m)/2, -((d*x^2)/c)])/(c*d^2*e*(1 + m))
```

Defintions of rubi rules used

rule 437
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(
a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f
, g, m}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{(ex)^m (b\,x^2+a)\,(x^2B+A)}{x^2d+c}dx$$

input
```
int((e*x)^m*(b*x^2+a)*(B*x^2+A)/(d*x^2+c),x)
```

output
```
int((e*x)^m*(b*x^2+a)*(B*x^2+A)/(d*x^2+c),x)
```

Fricas [F]

$$\int \frac{(ex)^m\,(a+bx^2)\,(A+Bx^2)}{c+dx^2}\,dx = \int \frac{(Bx^2+A)(bx^2+a)(ex)^m}{dx^2+c}\,dx$$

input
```
integrate((e*x)^m*(b*x^2+a)*(B*x^2+A)/(d*x^2+c),x, algorithm="fricas")
```

output
```
integral((B*b*x^4 + (B*a + A*b)*x^2 + A*a)*(e*x)^m/(d*x^2 + c), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.67 (sec) , antiderivative size = 418, normalized size of antiderivative = 3.48

$$\int \frac{(ex)^m (a+bx^2)(A+Bx^2)}{c+dx^2}\,dx = \frac{Aae^m m x^{m+1}\Phi\left(\frac{dx^2e^{i\pi}}{c},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{4c\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)} + \frac{Aae^m x^{m+1}\Phi\left(\frac{dx^2e^{i\pi}}{c},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{4c\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)} + \frac{Abe^m m x^{m+3}\Phi\left(\frac{dx^2e^{i\pi}}{c},1,\frac{m}{2}+\frac{3}{2}\right)\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}{4c\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)} + \frac{3Abe^m x^{m+3}\Phi\left(\frac{dx^2e^{i\pi}}{c},1,\frac{m}{2}+\frac{3}{2}\right)\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}{4c\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)} + \frac{Bae^m m x^{m+3}\Phi\left(\frac{dx^2e^{i\pi}}{c},1,\frac{m}{2}+\frac{3}{2}\right)\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}{4c\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)} + \frac{3Bae^m x^{m+3}\Phi\left(\frac{dx^2e^{i\pi}}{c},1,\frac{m}{2}+\frac{3}{2}\right)\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}{4c\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)} + \frac{Bbe^m m x^{m+5}\Phi\left(\frac{dx^2e^{i\pi}}{c},1,\frac{m}{2}+\frac{5}{2}\right)\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)}{4c\Gamma\left(\frac{m}{2}+\frac{7}{2}\right)} + \frac{5Bbe^m x^{m+5}\Phi\left(\frac{dx^2e^{i\pi}}{c},1,\frac{m}{2}+\frac{5}{2}\right)\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)}{4c\Gamma\left(\frac{m}{2}+\frac{7}{2}\right)}$$

input
```
integrate((e*x)**m*(b*x**2+a)*(B*x**2+A)/(d*x**2+c),x)
```

output
```
A*a*e**m*m*x**(m + 1)*lerchphi(d*x**2*exp_polar(I*pi)/c, 1, m/2 + 1/2)*gam
ma(m/2 + 1/2)/(4*c*gamma(m/2 + 3/2)) + A*a*e**m*x**(m + 1)*lerchphi(d*x**2
*exp_polar(I*pi)/c, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(4*c*gamma(m/2 + 3/2))
 + A*b*e**m*m*x**(m + 3)*lerchphi(d*x**2*exp_polar(I*pi)/c, 1, m/2 + 3/2)*g
amma(m/2 + 3/2)/(4*c*gamma(m/2 + 5/2)) + 3*A*b*e**m*x**(m + 3)*lerchphi(d*
x**2*exp_polar(I*pi)/c, 1, m/2 + 3/2)*gamma(m/2 + 3/2)/(4*c*gamma(m/2 + 5/
2)) + B*a*e**m*m*x**(m + 3)*lerchphi(d*x**2*exp_polar(I*pi)/c, 1, m/2 + 3/
2)*gamma(m/2 + 3/2)/(4*c*gamma(m/2 + 5/2)) + 3*B*a*e**m*x**(m + 3)*lerchph
i(d*x**2*exp_polar(I*pi)/c, 1, m/2 + 3/2)*gamma(m/2 + 3/2)/(4*c*gamma(m/2
 + 5/2)) + B*b*e**m*m*x**(m + 5)*lerchphi(d*x**2*exp_polar(I*pi)/c, 1, m/2
 + 5/2)*gamma(m/2 + 5/2)/(4*c*gamma(m/2 + 7/2)) + 5*B*b*e**m*x**(m + 5)*ler
chphi(d*x**2*exp_polar(I*pi)/c, 1, m/2 + 5/2)*gamma(m/2 + 5/2)/(4*c*gamma(
m/2 + 7/2))
```

Maxima [F]

$$\int \frac{(ex)^m (a+bx^2) (A+Bx^2)}{c+dx^2} dx = \int \frac{(Bx^2+A)(bx^2+a)(ex)^m}{dx^2+c} dx$$

input
```
integrate((e*x)^m*(b*x^2+a)*(B*x^2+A)/(d*x^2+c),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*(b*x^2 + a)*(e*x)^m/(d*x^2 + c), x)
```

Giac [F]

$$\int \frac{(ex)^m (a+bx^2) (A+Bx^2)}{c+dx^2} dx = \int \frac{(Bx^2+A)(bx^2+a)(ex)^m}{dx^2+c} dx$$

input
```
integrate((e*x)^m*(b*x^2+a)*(B*x^2+A)/(d*x^2+c),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*(b*x^2 + a)*(e*x)^m/(d*x^2 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m (a+bx^2)(A+Bx^2)}{c+dx^2}\,dx = \int \frac{(B\,x^2+A)\,(e\,x)^m\,(b\,x^2+a)}{d\,x^2+c}\,dx$$

input
```
int(((A + B*x^2)*(e*x)^m*(a + b*x^2))/(c + d*x^2),x)
```

output
```
int(((A + B*x^2)*(e*x)^m*(a + b*x^2))/(c + d*x^2), x)
```

Reduce [F]

$$\int \frac{(ex)^m (a+bx^2)(A+Bx^2)}{c+dx^2}\,dx$$
$$= \frac{e^m\left(2x^m abdmx+6x^m abdx-x^m b^2cmx-3x^m b^2cx+x^m b^2dm\,x^3+x^m b^2d\,x^3+\left(\int \frac{x^m}{d\,x^2+c}dx\right)a^2d^2m^2+4\left(\right.}{}$$

input
```
int((e*x)^m*(b*x^2+a)*(B*x^2+A)/(d*x^2+c),x)
```

output
```
(e**m*(2*x**m*a*b*d*m*x + 6*x**m*a*b*d*x - x**m*b**2*c*m*x - 3*x**m*b**2*c
*x + x**m*b**2*d*m*x**3 + x**m*b**2*d*x**3 + int(x**m/(c + d*x**2),x)*a**2
*d**2*m**2 + 4*int(x**m/(c + d*x**2),x)*a**2*d**2*m + 3*int(x**m/(c + d*x*
*2),x)*a**2*d**2 - 2*int(x**m/(c + d*x**2),x)*a*b*c*d*m**2 - 8*int(x**m/(c
 + d*x**2),x)*a*b*c*d*m - 6*int(x**m/(c + d*x**2),x)*a*b*c*d + int(x**m/(c
 + d*x**2),x)*b**2*c**2*m**2 + 4*int(x**m/(c + d*x**2),x)*b**2*c**2*m + 3*
int(x**m/(c + d*x**2),x)*b**2*c**2))/(d**2*(m**2 + 4*m + 3))
```

3.26 $\int \frac{(ex)^m(A+Bx^2)}{c+dx^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 77

$$\int \frac{(ex)^m (A+Bx^2)}{c+dx^2}\,dx = \frac{B(ex)^{1+m}}{de(1+m)} - \frac{(Bc-Ad)(ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{dx^2}{c}\right)}{cde(1+m)}$$

output
```
B*(e*x)^(1+m)/d/e/(1+m)-(-A*d+B*c)*(e*x)^(1+m)*hypergeom([1, 1/2+1/2*m],[3
/2+1/2*m],-d*x^2/c)/c/d/e/(1+m)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.73

$$\int \frac{(ex)^m (A+Bx^2)}{c+dx^2}\,dx = \frac{x(ex)^m \left(Bc+(-Bc+Ad)\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{dx^2}{c}\right)\right)}{cd(1+m)}$$

input
```
Integrate[((e*x)^m*(A + B*x^2))/(c + d*x^2),x]
```

output
```
(x*(e*x)^m*(B*c + (-(B*c) + A*d)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2
, -((d*x^2)/c)]))/(c*d*(1 + m))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {363, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)(ex)^m}{c+dx^2}dx$$

$$\Big\downarrow \text{ 363}$$

$$\frac{B(ex)^{m+1}}{de(m+1)}-\frac{(Bc-Ad)\int \frac{(ex)^m}{dx^2+c}dx}{d}$$

$$\Big\downarrow \text{ 278}$$

$$\frac{B(ex)^{m+1}}{de(m+1)}-\frac{(ex)^{m+1}(Bc-Ad)\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\frac{dx^2}{c}\right)}{cde(m+1)}$$

input
```
Int[((e*x)^m*(A + B*x^2))/(c + d*x^2),x]
```

output
```
(B*(e*x)^(1 + m))/(d*e*(1 + m)) - ((B*c - A*d)*(e*x)^(1 + m)*Hypergeometri
c2F1[1, (1 + m)/2, (3 + m)/2, -((d*x^2)/c)])/(c*d*e*(1 + m))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

Maple [F]

$$\int \frac{(ex)^m (x^2B + A)}{x^2d + c}dx$$

input
```
int((e*x)^m*(B*x^2+A)/(d*x^2+c),x)
```

output
```
int((e*x)^m*(B*x^2+A)/(d*x^2+c),x)
```

Fricas [F]

$$\int \frac{(ex)^m (A + Bx^2)}{c + dx^2}\, dx = \int \frac{(Bx^2 + A)(ex)^m}{dx^2 + c}\, dx$$

input
```
integrate((e*x)^m*(B*x^2+A)/(d*x^2+c),x, algorithm="fricas")
```

output
```
integral((B*x^2 + A)*(e*x)^m/(d*x^2 + c), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.02 (sec) , antiderivative size = 201, normalized size of antiderivative = 2.61

$$\int \frac{(ex)^m (A+Bx^2)}{c+dx^2}\,dx = \frac{Ae^m m x^{m+1}\Phi\left(\frac{dx^2e^{i\pi}}{c},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{4c\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)} + \frac{Ae^m x^{m+1}\Phi\left(\frac{dx^2e^{i\pi}}{c},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{4c\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)} + \frac{Be^m m x^{m+3}\Phi\left(\frac{dx^2e^{i\pi}}{c},1,\frac{m}{2}+\frac{3}{2}\right)\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}{4c\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)} + \frac{3Be^m x^{m+3}\Phi\left(\frac{dx^2e^{i\pi}}{c},1,\frac{m}{2}+\frac{3}{2}\right)\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}{4c\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)}$$

input
```
integrate((e*x)**m*(B*x**2+A)/(d*x**2+c),x)
```

output
```
A*e**m*m*x**(m + 1)*lerchphi(d*x**2*exp_polar(I*pi)/c, 1, m/2 + 1/2)*gamma
(m/2 + 1/2)/(4*c*gamma(m/2 + 3/2)) + A*e**m*x**(m + 1)*lerchphi(d*x**2*exp
_polar(I*pi)/c, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(4*c*gamma(m/2 + 3/2)) + B*
e**m*m*x**(m + 3)*lerchphi(d*x**2*exp_polar(I*pi)/c, 1, m/2 + 3/2)*gamma(m
/2 + 3/2)/(4*c*gamma(m/2 + 5/2)) + 3*B*e**m*x**(m + 3)*lerchphi(d*x**2*exp
_polar(I*pi)/c, 1, m/2 + 3/2)*gamma(m/2 + 3/2)/(4*c*gamma(m/2 + 5/2))
```

Maxima [F]

$$\int \frac{(ex)^m (A+Bx^2)}{c+dx^2}\,dx = \int \frac{(Bx^2+A)(ex)^m}{dx^2+c}\,dx$$

input
```
integrate((e*x)^m*(B*x^2+A)/(d*x^2+c),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*(e*x)^m/(d*x^2 + c), x)
```

Giac [F]

$$\int \frac{(ex)^m (A + Bx^2)}{c + dx^2} \, dx = \int \frac{(Bx^2 + A)(ex)^m}{dx^2 + c} \, dx$$

input
```
integrate((e*x)^m*(B*x^2+A)/(d*x^2+c),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*(e*x)^m/(d*x^2 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m (A + Bx^2)}{c + dx^2} \, dx = \int \frac{(B\,x^2 + A)\,(e\,x)^m}{d\,x^2 + c} \, dx$$

input
```
int(((A + B*x^2)*(e*x)^m)/(c + d*x^2),x)
```

output
```
int(((A + B*x^2)*(e*x)^m)/(c + d*x^2), x)
```

Reduce [F]

$$\int \frac{(ex)^m (A + Bx^2)}{c + dx^2} \, dx = \frac{e^m \left(x^m bx + \left(\int \frac{x^m}{d\,x^2+c} dx\right) adm + \left(\int \frac{x^m}{d\,x^2+c} dx\right) ad - \left(\int \frac{x^m}{d\,x^2+c} dx\right) bcm - \left(\int \frac{x^m}{d\,x^2+c} dx\right) bc\right)}{d\,(m+1)}$$

input
```
int((e*x)^m*(B*x^2+A)/(d*x^2+c),x)
```

output
```
(e**m*(x**m*b*x + int(x**m/(c + d*x**2),x)*a*d*m + int(x**m/(c + d*x**2),x
)*a*d - int(x**m/(c + d*x**2),x)*b*c*m - int(x**m/(c + d*x**2),x)*b*c))/(d
*(m + 1))
```

3.27 $\int \frac{(ex)^m(A+Bx^2)}{(a+bx^2)(c+dx^2)}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 125

$$\int \frac{(ex)^m(A+Bx^2)}{(a+bx^2)(c+dx^2)}\,dx = \frac{(Ab-aB)(ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{bx^2}{a}\right)}{a(bc-ad)e(1+m)} + \frac{(Bc-Ad)(ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{dx^2}{c}\right)}{c(bc-ad)e(1+m)}$$

output
```
(A*b-B*a)*(e*x)^(1+m)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-b*x^2/a)/a/(-a
*d+b*c)/e/(1+m)+(-A*d+B*c)*(e*x)^(1+m)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m
],-d*x^2/c)/c/(-a*d+b*c)/e/(1+m)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.80

$$\int \frac{(ex)^m (A+Bx^2)}{(a+bx^2)(c+dx^2)} dx = \frac{x(ex)^m \left((-Abc+aBc) \text{Hypergeometric2F1}\left(1, \frac{1+m}{2}, \frac{3+m}{2}, -\frac{bx^2}{a}\right) + a(-Bc+Ad) \text{Hypergeometric2F1}\left(\right.\right.}{ac(-bc+ad)(1+m)}$$

input
```
Integrate[((e*x)^m*(A + B*x^2))/((a + b*x^2)*(c + d*x^2)),x]
```

output
```
(x*(e*x)^m*((-(A*b*c) + a*B*c)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2,
-((b*x^2)/a)] + a*(-(B*c) + A*d)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2
, -((d*x^2)/c)]))/(a*c*(-(b*c) + a*d)*(1 + m))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {446, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)(ex)^m}{(a+bx^2)(c+dx^2)} dx$$

$$\Big\downarrow \text{ 446}$$

$$\int \left(\frac{(ex)^m(Ab-aB)}{(a+bx^2)(bc-ad)} + \frac{(ex)^m(Bc-Ad)}{(c+dx^2)(bc-ad)}\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{(ex)^{m+1}(Ab-aB) \text{Hypergeometric2F1}\left(1, \frac{m+1}{2}, \frac{m+3}{2}, -\frac{bx^2}{a}\right)}{ae(m+1)(bc-ad)} + \frac{(ex)^{m+1}(Bc-Ad) \text{Hypergeometric2F1}\left(1, \frac{m+1}{2}, \frac{m+3}{2}, -\frac{dx^2}{c}\right)}{ce(m+1)(bc-ad)}$$

input
```
Int[((e*x)^m*(A + B*x^2))/((a + b*x^2)*(c + d*x^2)),x]
```

output
```
((A*b - a*B)*(e*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b
*x^2)/a)])/(a*(b*c - a*d)*e*(1 + m)) + ((B*c - A*d)*(e*x)^(1 + m)*Hypergeo
metric2F1[1, (1 + m)/2, (3 + m)/2, -((d*x^2)/c)])/(c*(b*c - a*d)*e*(1 + m)
)
```

Defintions of rubi rules used

rule 446
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((e_) + (f_.)*(x_)^2))/(
(c_) + (d_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a + b*x^2)^
p*((e + f*x^2)/(c + d*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{(ex)^m (x^2 B + A)}{(b\,x^2 + a)\,(x^2 d + c)}dx$$

input
```
int((e*x)^m*(B*x^2+A)/(b*x^2+a)/(d*x^2+c),x)
```

output
```
int((e*x)^m*(B*x^2+A)/(b*x^2+a)/(d*x^2+c),x)
```

Fricas [F]

$$\int \frac{(ex)^m (A + Bx^2)}{(a + bx^2)(c + dx^2)}\,dx = \int \frac{(Bx^2 + A)(ex)^m}{(bx^2 + a)(dx^2 + c)}\,dx$$

input
```
integrate((e*x)^m*(B*x^2+A)/(b*x^2+a)/(d*x^2+c),x, algorithm="fricas")
```

output
```
integral((B*x^2 + A)*(e*x)^m/(b*d*x^4 + (b*c + a*d)*x^2 + a*c), x)
```

Sympy [F]

$$\int \frac{(ex)^m (A + Bx^2)}{(a + bx^2)(c + dx^2)}\, dx = \int \frac{(ex)^m (A + Bx^2)}{(a + bx^2)(c + dx^2)}\, dx$$

input
```
integrate((e*x)**m*(B*x**2+A)/(b*x**2+a)/(d*x**2+c),x)
```

output
```
Integral((e*x)**m*(A + B*x**2)/((a + b*x**2)*(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{(ex)^m (A + Bx^2)}{(a + bx^2)(c + dx^2)}\, dx = \int \frac{(Bx^2 + A)(ex)^m}{(bx^2 + a)(dx^2 + c)}\, dx$$

input
```
integrate((e*x)^m*(B*x^2+A)/(b*x^2+a)/(d*x^2+c),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*(e*x)^m/((b*x^2 + a)*(d*x^2 + c)), x)
```

Giac [F]

$$\int \frac{(ex)^m (A + Bx^2)}{(a + bx^2)(c + dx^2)}\, dx = \int \frac{(Bx^2 + A)(ex)^m}{(bx^2 + a)(dx^2 + c)}\, dx$$

input
```
integrate((e*x)^m*(B*x^2+A)/(b*x^2+a)/(d*x^2+c),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*(e*x)^m/((b*x^2 + a)*(d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m (A+Bx^2)}{(a+bx^2)(c+dx^2)}\,dx = \int \frac{(B\,x^2+A)\,(e\,x)^m}{(b\,x^2+a)\,(d\,x^2+c)}\,dx$$

input
```
int(((A + B*x^2)*(e*x)^m)/((a + b*x^2)*(c + d*x^2)),x)
```

output
```
int(((A + B*x^2)*(e*x)^m)/((a + b*x^2)*(c + d*x^2)), x)
```

Reduce [F]

$$\int \frac{(ex)^m (A+Bx^2)}{(a+bx^2)(c+dx^2)}\,dx = e^m\left(\int \frac{x^m}{d\,x^2+c}dx\right)$$

input
```
int((e*x)^m*(B*x^2+A)/(b*x^2+a)/(d*x^2+c),x)
```

output
```
e**m*int(x**m/(c + d*x**2),x)
```

3.28 $\int \frac{(ex)^m(A+Bx^2)}{(a+bx^2)^2(c+dx^2)}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 206

$$\int \frac{(ex)^m(A+Bx^2)}{(a+bx^2)^2(c+dx^2)}\,dx = \frac{(Ab-aB)(ex)^{1+m}}{2a(bc-ad)e(a+bx^2)} + \frac{(Ab(bc(1-m)-ad(3-m))+aB(ad(1-m)+bc(1+m)))(ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+\cdots}{2}\cdots\right.}{2a^2(bc-ad)^2e(1+m)} - \frac{d(Bc-Ad)(ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{dx^2}{c}\right)}{c(bc-ad)^2e(1+m)}$$

output

```
1/2*(A*b-B*a)*(e*x)^(1+m)/a/(-a*d+b*c)/e/(b*x^2+a)+1/2*(A*b*(b*c*(1-m)-a*d
*(3-m))+a*B*(a*d*(1-m)+b*c*(1+m)))*(e*x)^(1+m)*hypergeom([1, 1/2+1/2*m],[3
/2+1/2*m],-b*x^2/a)/a^2/(-a*d+b*c)^2/e/(1+m)-d*(-A*d+B*c)*(e*x)^(1+m)*hype
rgeom([1, 1/2+1/2*m],[3/2+1/2*m],-d*x^2/c)/c/(-a*d+b*c)^2/e/(1+m)
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.72

$$\int \frac{(ex)^m (A+Bx^2)}{(a+bx^2)^2 (c+dx^2)} dx =$$
$$-\frac{x(ex)^m \left(abc(-Bc+Ad) \text{Hypergeometric2F1}\left(1, \frac{1+m}{2}, \frac{3+m}{2}, -\frac{bx^2}{a}\right) + a^2 d(Bc-Ad) \text{Hypergeometric2}\right.}{a^2 c(bc-ad)^2 (1}$$

input
```
Integrate[((e*x)^m*(A + B*x^2))/((a + b*x^2)^2*(c + d*x^2)),x]
```

output
```
-((x*(e*x)^m*(a*b*c*(-(B*c) + A*d)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)
/2, -((b*x^2)/a)] + a^2*d*(B*c - A*d)*Hypergeometric2F1[1, (1 + m)/2, (3 +
 m)/2, -((d*x^2)/c)] - (A*b - a*B)*c*(b*c - a*d)*Hypergeometric2F1[2, (1 +
 m)/2, (3 + m)/2, -((b*x^2)/a)]))/(a^2*c*(b*c - a*d)^2*(1 + m)))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {441, 446, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)(ex)^m}{(a+bx^2)^2 (c+dx^2)} dx$$

$$\Big\downarrow \text{441}$$

$$\frac{(ex)^{m+1}(Ab-aB)}{2ae(a+bx^2)(bc-ad)} - \frac{\int \frac{(ex)^m(-((Ab-aB)d(1-m)x^2)+2aAd-Abc(1-m)-aBc(m+1))}{(bx^2+a)(dx^2+c)}dx}{2a(bc-ad)}$$

$$\Big\downarrow \text{446}$$

$$\frac{(ex)^{m+1}(Ab-aB)}{2ae\left(a+bx^2\right)(bc-ad)}-\frac{\int\left(\frac{(-Ab(bc(1-m)-ad(3-m))-aB(ad(1-m)+bc(m+1)))(ex)^m}{(bc-ad)(bx^2+a)}+\frac{2ad(Ad-Bc)(ex)^m}{(ad-bc)(dx^2+c)}\right)dx}{2a(bc-ad)}$$

$$\Big\downarrow \; 2009$$

$$\frac{(ex)^{m+1}(Ab-aB)}{2ae\left(a+bx^2\right)(bc-ad)}-\frac{\frac{2ad(ex)^{m+1}(Bc-Ad)\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\frac{dx^2}{c}\right)}{ce(m+1)(bc-ad)}-\frac{(ex)^{m+1}\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\frac{bx^2}{a}\right)(Ab(bc(1-m)-ad(3-m}{ae(m+1)(bc-ad)}}{2a(bc-ad)}$$

input
```
Int[((e*x)^m*(A + B*x^2))/((a + b*x^2)^2*(c + d*x^2)),x]
```

output
```
((A*b - a*B)*(e*x)^(1 + m))/(2*a*(b*c - a*d)*e*(a + b*x^2)) - (-(((A*b*(b*
c*(1 - m) - a*d*(3 - m)) + a*B*(a*d*(1 - m) + b*c*(1 + m)))*(e*x)^(1 + m)*
Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(a*(b*c - a*d)*e
*(1 + m))) + (2*a*d*(B*c - A*d)*(e*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)
/2, (3 + m)/2, -((d*x^2)/c)])/(c*(b*c - a*d)*e*(1 + m)))/(2*a*(b*c - a*d))
```

Defintions of rubi rules used

rule 441
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
 + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(a*g*2*(b*c - a*d)*(p + 1))), x] + Si
mp[1/(a*2*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2
)^q*Simp[c*(b*e - a*f)*(m + 1) + e*2*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(m
 + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, q},
 x] && LtQ[p, -1]
```

rule 446
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((e_) + (f_.)*(x_)^2))/(
(c_) + (d_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a + b*x^2)^
p*((e + f*x^2)/(c + d*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{(ex)^m (x^2B + A)}{(b\,x^2 + a)^2 (x^2d + c)}dx$$

input
```
int((e*x)^m*(B*x^2+A)/(b*x^2+a)^2/(d*x^2+c),x)
```

output
```
int((e*x)^m*(B*x^2+A)/(b*x^2+a)^2/(d*x^2+c),x)
```

Fricas [F]

$$\int \frac{(ex)^m (A + Bx^2)}{(a + bx^2)^2 (c + dx^2)}\,dx = \int \frac{(Bx^2 + A)(ex)^m}{(bx^2 + a)^2(dx^2 + c)}\,dx$$

input
```
integrate((e*x)^m*(B*x^2+A)/(b*x^2+a)^2/(d*x^2+c),x, algorithm="fricas")
```

output
```
integral((B*x^2 + A)*(e*x)^m/(b^2*d*x^6 + (b^2*c + 2*a*b*d)*x^4 + a^2*c +
(2*a*b*c + a^2*d)*x^2), x)
```

Sympy [F]

$$\int \frac{(ex)^m (A + Bx^2)}{(a + bx^2)^2 (c + dx^2)}\,dx = \int \frac{(ex)^m (A + Bx^2)}{(a + bx^2)^2 (c + dx^2)}\,dx$$

input
```
integrate((e*x)**m*(B*x**2+A)/(b*x**2+a)**2/(d*x**2+c),x)
```

output
```
Integral((e*x)**m*(A + B*x**2)/((a + b*x**2)**2*(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{(ex)^m (A+Bx^2)}{(a+bx^2)^2 (c+dx^2)} \, dx = \int \frac{(Bx^2+A)(ex)^m}{(bx^2+a)^2(dx^2+c)} \, dx$$

input
```
integrate((e*x)^m*(B*x^2+A)/(b*x^2+a)^2/(d*x^2+c),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*(e*x)^m/((b*x^2 + a)^2*(d*x^2 + c)), x)
```

Giac [F]

$$\int \frac{(ex)^m (A+Bx^2)}{(a+bx^2)^2 (c+dx^2)} \, dx = \int \frac{(Bx^2+A)(ex)^m}{(bx^2+a)^2(dx^2+c)} \, dx$$

input
```
integrate((e*x)^m*(B*x^2+A)/(b*x^2+a)^2/(d*x^2+c),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*(e*x)^m/((b*x^2 + a)^2*(d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m (A+Bx^2)}{(a+bx^2)^2 (c+dx^2)} \, dx = \int \frac{(B\,x^2+A)\,(e\,x)^m}{(b\,x^2+a)^2\,(d\,x^2+c)} \, dx$$

input
```
int(((A + B*x^2)*(e*x)^m)/((a + b*x^2)^2*(c + d*x^2)),x)
```

output
```
int(((A + B*x^2)*(e*x)^m)/((a + b*x^2)^2*(c + d*x^2)), x)
```

Reduce [F]

$$\int \frac{(ex)^m (A + Bx^2)}{(a + bx^2)^2 (c + dx^2)}\, dx = e^m \left(\int \frac{x^m}{bd\, x^4 + ad\, x^2 + bc\, x^2 + ac} dx \right)$$

input
```
int((e*x)^m*(B*x^2+A)/(b*x^2+a)^2/(d*x^2+c),x)
```

output
```
e**m*int(x**m/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)
```

3.29 $\int \frac{(ex)^m(A+Bx^2)}{(a+bx^2)^3(c+dx^2)} \, dx$

Optimal result

Integrand size = 31, antiderivative size = 342

$$\int \frac{(ex)^m (A+Bx^2)}{(a+bx^2)^3 (c+dx^2)} \, dx = \frac{(Ab-aB)(ex)^{1+m}}{4a(bc-ad)e\,(a+bx^2)^2}$$
$$+ \frac{(Ab(bc(3-m)-ad(7-m))+aB(ad(3-m)+bc(1+m)))(ex)^{1+m}}{8a^2(bc-ad)^2e\,(a+bx^2)}$$
$$+ \frac{(Ab(a^2d^2(15-8m+m^2)-2abcd(5-6m+m^2)+b^2c^2(3-4m+m^2))+aB(b^2c^2(1-m^2)-2abcd(}{8a^3(bc-ad)^3e(1+}$$
$$+ \frac{d^2(Bc-Ad)(ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{dx^2}{c}\right)}{c(bc-ad)^3e(1+m)}$$

output
```
1/4*(A*b-B*a)*(e*x)^(1+m)/a/(-a*d+b*c)/e/(b*x^2+a)^2+1/8*(A*b*(b*c*(3-m)-a
*d*(7-m))+a*B*(a*d*(3-m)+b*c*(1+m)))*(e*x)^(1+m)/a^2/(-a*d+b*c)^2/e/(b*x^2
+a)+1/8*(A*b*(a^2*d^2*(m^2-8*m+15)-2*a*b*c*d*(m^2-6*m+5)+b^2*c^2*(m^2-4*m+
3))+a*B*(b^2*c^2*(-m^2+1)-2*a*b*c*d*(-m^2+2*m+3)-a^2*d^2*(m^2-4*m+3)))*(e*
x)^(1+m)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-b*x^2/a)/a^3/(-a*d+b*c)^3/e
/(1+m)+d^2*(-A*d+B*c)*(e*x)^(1+m)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-d*
x^2/c)/c/(-a*d+b*c)^3/e/(1+m)
```

Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.57

$$\int \frac{(ex)^m (A+Bx^2)}{(a+bx^2)^3 (c+dx^2)} \, dx$$
$$= \frac{x(ex)^m \left(\frac{bd(-Bc+Ad) \operatorname{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{bx^2}{a}\right)}{a} + \frac{d^2(Bc-Ad) \operatorname{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{dx^2}{c}\right)}{c} + \frac{b(bc-ad)(}{} \right.}{(bc-ad)^3(1+m}$$

input
```
Integrate[((e*x)^m*(A + B*x^2))/((a + b*x^2)^3*(c + d*x^2)),x]
```

output
```
(x*(e*x)^m*((b*d*(-(B*c) + A*d)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2,
 -((b*x^2)/a)])/a + (d^2*(B*c - A*d)*Hypergeometric2F1[1, (1 + m)/2, (3 +
m)/2, -((d*x^2)/c)])/c + (b*(b*c - a*d)*(B*c - A*d)*Hypergeometric2F1[2, (
1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/a^2 + ((A*b - a*B)*(b*c - a*d)^2*Hyper
geometric2F1[3, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/a^3))/((b*c - a*d)^3*
(1 + m))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)(ex)^m}{(a+bx^2)^3 (c+dx^2)} \, dx$$

$$\Big\downarrow \text{ 441}$$

$$\frac{(ex)^{m+1}(Ab-aB)}{4ae(a+bx^2)^2(bc-ad)} - \frac{\int \frac{(ex)^m(-((Ab-aB)d(3-m)x^2)+4aAd-Abc(3-m)-aBc(m+1))}{(bx^2+a)^2(dx^2+c)}dx}{4a(bc-ad)}$$

$$\Big\downarrow \text{ 441}$$

$$\frac{(ex)^{m+1}(Ab-aB)}{4ae\left(a+bx^2\right)^2(bc-ad)}-\frac{-\frac{\int -\frac{(ex)^m\left(-d(1-m)(Ab(bc(3-m)-ad(7-m))+aB(ad(3-m)+bc(m+1)))x^2+aBc(m+1)(ad(5-m)-b(c-cm))-A\left(b^2\left(m^2-4m+3\right)c^2-abd\left(m^2-8m+7\right)c+8a\ldots\right.\right.}{\left(bx^2+a\right)\left(dx^2+c\right)}}{2a(bc-ad)}}{4a(bc-ad)}$$

$$\Big\downarrow \; 25$$

$$\frac{(ex)^{m+1}(Ab-aB)}{4ae\left(a+bx^2\right)^2(bc-ad)}-\frac{\frac{\int -\frac{(ex)^m\left(d(1-m)(Ab(bc(3-m)-ad(7-m))+aB(ad(3-m)+bc(m+1)))x^2+aBc(bc(1-m)-ad(5-m))(m+1)+A\left(b^2\left(m^2-4m+3\right)c^2-abd\left(m^2-8m+7\right)c+8a^2d^2\ldots\right.\right.}{\left(bx^2+a\right)\left(dx^2+c\right)}}{2a(bc-ad)}}{4a(bc-ad)}$$

$$\Big\downarrow \; 25$$

$$\frac{(ex)^{m+1}(Ab-aB)}{4ae\left(a+bx^2\right)^2(bc-ad)}-\frac{-\frac{\int -\frac{(ex)^m\left(-d(1-m)(Ab(bc(3-m)-ad(7-m))+aB(ad(3-m)+bc(m+1)))x^2+aBc(m+1)(ad(5-m)-b(c-cm))-A\left(b^2\left(m^2-4m+3\right)c^2-abd\left(m^2-8m+7\right)c+8a\ldots\right.\right.}{\left(bx^2+a\right)\left(dx^2+c\right)}}{2a(bc-ad)}}{4a(bc-ad)}$$

$$\Big\downarrow \; 25$$

$$\frac{(ex)^{m+1}(Ab-aB)}{4ae\left(a+bx^2\right)^2(bc-ad)}-\frac{\frac{\int -\frac{(ex)^m\left(d(1-m)(Ab(bc(3-m)-ad(7-m))+aB(ad(3-m)+bc(m+1)))x^2+aBc(bc(1-m)-ad(5-m))(m+1)+A\left(b^2\left(m^2-4m+3\right)c^2-abd\left(m^2-8m+7\right)c+8a^2d^2\ldots\right.\right.}{\left(bx^2+a\right)\left(dx^2+c\right)}}{2a(bc-ad)}}{4a(bc-ad)}$$

$$\Big\downarrow \; 25$$

$$\frac{(ex)^{m+1}(Ab-aB)}{4ae\left(a+bx^2\right)^2(bc-ad)}-\frac{-\frac{\int -\frac{(ex)^m\left(-d(1-m)(Ab(bc(3-m)-ad(7-m))+aB(ad(3-m)+bc(m+1)))x^2+aBc(m+1)(ad(5-m)-b(c-cm))-A\left(b^2\left(m^2-4m+3\right)c^2-abd\left(m^2-8m+7\right)c+8a\ldots\right.\right.}{\left(bx^2+a\right)\left(dx^2+c\right)}}{2a(bc-ad)}}{4a(bc-ad)}$$

$$\Big\downarrow \; 25$$

$$\frac{(ex)^{m+1}(Ab-aB)}{4ae\left(a+bx^2\right)^2(bc-ad)}-\frac{\frac{\int -\frac{(ex)^m\left(d(1-m)(Ab(bc(3-m)-ad(7-m))+aB(ad(3-m)+bc(m+1)))x^2+aBc(bc(1-m)-ad(5-m))(m+1)+A\left(b^2\left(m^2-4m+3\right)c^2-abd\left(m^2-8m+7\right)c+8a^2d^2\ldots\right.\right.}{\left(bx^2+a\right)\left(dx^2+c\right)}}{2a(bc-ad)}}{4a(bc-ad)}$$

$\Big\downarrow$ 25

$$\frac{(ex)^{m+1}(Ab-aB)}{4ae\left(a+bx^2\right)^2(bc-ad)}-\frac{-\frac{\int -\frac{(ex)^m\left(-d(1-m)(Ab(bc(3-m)-ad(7-m))+aB(ad(3-m)+bc(m+1)))x^2+aBc(m+1)(ad(5-m)-b(c-cm))-A\left(b^2\left(m^2-4m+3\right)c^2-abd\left(m^2-8m+7\right)c+8a\cdots\right)\right)}{\left(bx^2+a\right)\left(dx^2+c\right)}}{2a(bc-ad)}}{4a(bc-ad)}$$

$\Big\downarrow$ 25

$$\frac{(ex)^{m+1}(Ab-aB)}{4ae\left(a+bx^2\right)^2(bc-ad)}-\frac{\frac{\int -\frac{(ex)^m\left(d(1-m)(Ab(bc(3-m)-ad(7-m))+aB(ad(3-m)+bc(m+1)))x^2+aBc(bc(1-m)-ad(5-m))(m+1)+A\left(b^2\left(m^2-4m+3\right)c^2-abd\left(m^2-8m+7\right)c+8a^2d^2\cdots\right)\right)}{\left(bx^2+a\right)\left(dx^2+c\right)}}{2a(bc-ad)}}{4a(bc-ad)}$$

$\Big\downarrow$ 25

$$\frac{(ex)^{m+1}(Ab-aB)}{4ae\left(a+bx^2\right)^2(bc-ad)}-\frac{-\frac{\int -\frac{(ex)^m\left(-d(1-m)(Ab(bc(3-m)-ad(7-m))+aB(ad(3-m)+bc(m+1)))x^2+aBc(m+1)(ad(5-m)-b(c-cm))-A\left(b^2\left(m^2-4m+3\right)c^2-abd\left(m^2-8m+7\right)c+8a\cdots\right)\right)}{\left(bx^2+a\right)\left(dx^2+c\right)}}{2a(bc-ad)}}{4a(bc-ad)}$$

$\Big\downarrow$ 25

$$\frac{(ex)^{m+1}(Ab-aB)}{4ae\left(a+bx^2\right)^2(bc-ad)}-\frac{\frac{\int -\frac{(ex)^m\left(d(1-m)(Ab(bc(3-m)-ad(7-m))+aB(ad(3-m)+bc(m+1)))x^2+aBc(bc(1-m)-ad(5-m))(m+1)+A\left(b^2\left(m^2-4m+3\right)c^2-abd\left(m^2-8m+7\right)c+8a^2d^2\cdots\right)\right)}{\left(bx^2+a\right)\left(dx^2+c\right)}}{2a(bc-ad)}}{4a(bc-ad)}$$

$\Big\downarrow$ 25

$$\frac{(ex)^{m+1}(Ab-aB)}{4ae\left(a+bx^2\right)^2(bc-ad)}-\frac{-\frac{\int -\frac{(ex)^m\left(-d(1-m)(Ab(bc(3-m)-ad(7-m))+aB(ad(3-m)+bc(m+1)))x^2+aBc(m+1)(ad(5-m)-b(c-cm))-A\left(b^2\left(m^2-4m+3\right)c^2-abd\left(m^2-8m+7\right)c+8a\cdots\right)\right)}{\left(bx^2+a\right)\left(dx^2+c\right)}}{2a(bc-ad)}}{4a(bc-ad)}$$

$\Big\downarrow$ 25

$$\frac{(ex)^{m+1}(Ab-aB)}{4ae\left(a+bx^2\right)^2(bc-ad)}-\frac{\frac{\int -\frac{(ex)^m\left(d(1-m)(Ab(bc(3-m)-ad(7-m))+aB(ad(3-m)+bc(m+1)))x^2+aBc(bc(1-m)-ad(5-m))(m+1)+A\left(b^2\left(m^2-4m+3\right)c^2-abd\left(m^2-8m+7\right)c+8a^2d^2\right.\cdots}{\left(bx^2+a\right)\left(dx^2+c\right)}}{2a(bc-ad)}\cdots}{4a(bc-ad)}$$

$$\Big\downarrow \; 25$$

$$\frac{(ex)^{m+1}(Ab-aB)}{4ae\left(a+bx^2\right)^2(bc-ad)}-\frac{-\frac{\int -\frac{(ex)^m\left(-d(1-m)(Ab(bc(3-m)-ad(7-m))+aB(ad(3-m)+bc(m+1)))x^2+aBc(m+1)(ad(5-m)-b(c-cm))-A\left(b^2\left(m^2-4m+3\right)c^2-abd\left(m^2-8m+7\right)c+8a\right.\cdots}{\left(bx^2+a\right)\left(dx^2+c\right)}}{2a(bc-ad)}\cdots}{4a(bc-ad)}$$

$$\Big\downarrow \; 25$$

$$\frac{(ex)^{m+1}(Ab-aB)}{4ae\left(a+bx^2\right)^2(bc-ad)}-\frac{\frac{\int -\frac{(ex)^m\left(d(1-m)(Ab(bc(3-m)-ad(7-m))+aB(ad(3-m)+bc(m+1)))x^2+aBc(bc(1-m)-ad(5-m))(m+1)+A\left(b^2\left(m^2-4m+3\right)c^2-abd\left(m^2-8m+7\right)c+8a^2d^2\right.\cdots}{\left(bx^2+a\right)\left(dx^2+c\right)}}{2a(bc-ad)}\cdots}{4a(bc-ad)}$$

$$\Big\downarrow \; 25$$

$$\frac{(ex)^{m+1}(Ab-aB)}{4ae\left(a+bx^2\right)^2(bc-ad)}-\frac{-\frac{\int -\frac{(ex)^m\left(-d(1-m)(Ab(bc(3-m)-ad(7-m))+aB(ad(3-m)+bc(m+1)))x^2+aBc(m+1)(ad(5-m)-b(c-cm))-A\left(b^2\left(m^2-4m+3\right)c^2-abd\left(m^2-8m+7\right)c+8a\right.\cdots}{\left(bx^2+a\right)\left(dx^2+c\right)}}{2a(bc-ad)}\cdots}{4a(bc-ad)}$$

$$\Big\downarrow \; 25$$

$$\frac{(ex)^{m+1}(Ab-aB)}{4ae\left(a+bx^2\right)^2(bc-ad)}-\frac{\frac{\int -\frac{(ex)^m\left(d(1-m)(Ab(bc(3-m)-ad(7-m))+aB(ad(3-m)+bc(m+1)))x^2+aBc(bc(1-m)-ad(5-m))(m+1)+A\left(b^2\left(m^2-4m+3\right)c^2-abd\left(m^2-8m+7\right)c+8a^2d^2\right.\cdots}{\left(bx^2+a\right)\left(dx^2+c\right)}}{2a(bc-ad)}\cdots}{4a(bc-ad)}$$

$$\Big\downarrow \; 25$$

$$\frac{(ex)^{m+1}(Ab-aB)}{4ae\left(a+bx^2\right)^2(bc-ad)}-\frac{-\frac{\int -\frac{(ex)^m\left(-d(1-m)(Ab(bc(3-m)-ad(7-m))+aB(ad(3-m)+bc(m+1)))x^2+aBc(m+1)(ad(5-m)-b(c-cm))-A\left(b^2\left(m^2-4m+3\right)c^2-abd\left(m^2-8m+7\right)c+8a\right.\cdots}{\left(bx^2+a\right)\left(dx^2+c\right)}}{2a(bc-ad)}\cdots}{4a(bc-ad)}$$

$$\Big\downarrow \color{blue}{25}$$

$$\frac{(ex)^{m+1}(Ab-aB)}{4ae\left(a+bx^2\right)^2(bc-ad)}-\frac{\frac{\int -\frac{(ex)^m\left(d(1-m)(Ab(bc(3-m)-ad(7-m))+aB(ad(3-m)+bc(m+1)))x^2+aBc(bc(1-m)-ad(5-m))(m+1)+A\left(b^2\left(m^2-4m+3\right)c^2-abd\left(m^2-8m+7\right)c+8a^2d^2\right.}{\left(bx^2+a\right)\left(dx^2+c\right)}}{2a(bc-ad)}}{4a(bc-ad)}$$

$$\Big\downarrow \color{blue}{25}$$

$$\frac{(ex)^{m+1}(Ab-aB)}{4ae\left(a+bx^2\right)^2(bc-ad)}-\frac{-\frac{\int -\frac{(ex)^m\left(-d(1-m)(Ab(bc(3-m)-ad(7-m))+aB(ad(3-m)+bc(m+1)))x^2+aBc(m+1)(ad(5-m)-b(c-cm))-A\left(b^2\left(m^2-4m+3\right)c^2-abd\left(m^2-8m+7\right)c+8a\right.}{\left(bx^2+a\right)\left(dx^2+c\right)}}{2a(bc-ad)}}{4a(bc-ad)}$$

$$\Big\downarrow \color{blue}{25}$$

$$\frac{(ex)^{m+1}(Ab-aB)}{4ae\left(a+bx^2\right)^2(bc-ad)}-\frac{\frac{\int -\frac{(ex)^m\left(d(1-m)(Ab(bc(3-m)-ad(7-m))+aB(ad(3-m)+bc(m+1)))x^2+aBc(bc(1-m)-ad(5-m))(m+1)+A\left(b^2\left(m^2-4m+3\right)c^2-abd\left(m^2-8m+7\right)c+8a^2d^2\right.}{\left(bx^2+a\right)\left(dx^2+c\right)}}{2a(bc-ad)}}{4a(bc-ad)}$$

$$\Big\downarrow \color{blue}{25}$$

$$\frac{(ex)^{m+1}(Ab-aB)}{4ae\left(a+bx^2\right)^2(bc-ad)}-\frac{-\frac{\int -\frac{(ex)^m\left(-d(1-m)(Ab(bc(3-m)-ad(7-m))+aB(ad(3-m)+bc(m+1)))x^2+aBc(m+1)(ad(5-m)-b(c-cm))-A\left(b^2\left(m^2-4m+3\right)c^2-abd\left(m^2-8m+7\right)c+8a\right.}{\left(bx^2+a\right)\left(dx^2+c\right)}}{2a(bc-ad)}}{4a(bc-ad)}$$

$$\Big\downarrow \color{blue}{25}$$

$$\frac{(ex)^{m+1}(Ab-aB)}{4ae\left(a+bx^2\right)^2(bc-ad)}-\frac{\frac{\int -\frac{(ex)^m\left(d(1-m)(Ab(bc(3-m)-ad(7-m))+aB(ad(3-m)+bc(m+1)))x^2+aBc(bc(1-m)-ad(5-m))(m+1)+A\left(b^2\left(m^2-4m+3\right)c^2-abd\left(m^2-8m+7\right)c+8a^2d^2\right.}{\left(bx^2+a\right)\left(dx^2+c\right)}}{2a(bc-ad)}}{4a(bc-ad)}$$

$$\Big\downarrow \color{blue}{25}$$

$$\frac{(ex)^{m+1}(Ab-aB)}{4ae\left(a+bx^2\right)^2(bc-ad)}-\frac{-\frac{\int -\frac{(ex)^m\left(-d(1-m)(Ab(bc(3-m)-ad(7-m))+aB(ad(3-m)+bc(m+1)))x^2+aBc(m+1)(ad(5-m)-b(c-cm))-A\left(b^2\left(m^2-4m+3\right)c^2-abd\left(m^2-8m+7\right)c+8a\cdots\right.\right.}{\left(bx^2+a\right)\left(dx^2+c\right)}\cdots}{2a(bc-ad)}\cdots}{4a(bc-ad)}$$

$\downarrow$ 25

$$\frac{(ex)^{m+1}(Ab-aB)}{4ae\left(a+bx^2\right)^2(bc-ad)}-\frac{\frac{\int -\frac{(ex)^m\left(d(1-m)(Ab(bc(3-m)-ad(7-m))+aB(ad(3-m)+bc(m+1)))x^2+aBc(bc(1-m)-ad(5-m))(m+1)+A\left(b^2\left(m^2-4m+3\right)c^2-abd\left(m^2-8m+7\right)c+8a^2d^2\cdots\right.\right.}{\left(bx^2+a\right)\left(dx^2+c\right)}\cdots}{2a(bc-ad)}\cdots}{4a(bc-ad)}$$

$\downarrow$ 25

$$\frac{(ex)^{m+1}(Ab-aB)}{4ae\left(a+bx^2\right)^2(bc-ad)}-\frac{-\frac{\int -\frac{(ex)^m\left(-d(1-m)(Ab(bc(3-m)-ad(7-m))+aB(ad(3-m)+bc(m+1)))x^2+aBc(m+1)(ad(5-m)-b(c-cm))-A\left(b^2\left(m^2-4m+3\right)c^2-abd\left(m^2-8m+7\right)c+8a\cdots\right.\right.}{\left(bx^2+a\right)\left(dx^2+c\right)}\cdots}{2a(bc-ad)}\cdots}{4a(bc-ad)}$$

$\downarrow$ 25

$$\frac{(ex)^{m+1}(Ab-aB)}{4ae\left(a+bx^2\right)^2(bc-ad)}-\frac{\frac{\int -\frac{(ex)^m\left(d(1-m)(Ab(bc(3-m)-ad(7-m))+aB(ad(3-m)+bc(m+1)))x^2+aBc(bc(1-m)-ad(5-m))(m+1)+A\left(b^2\left(m^2-4m+3\right)c^2-abd\left(m^2-8m+7\right)c+8a^2d^2\cdots\right.\right.}{\left(bx^2+a\right)\left(dx^2+c\right)}\cdots}{2a(bc-ad)}\cdots}{4a(bc-ad)}$$

$\downarrow$ 25

$$\frac{(ex)^{m+1}(Ab-aB)}{4ae\left(a+bx^2\right)^2(bc-ad)}-\frac{-\frac{\int -\frac{(ex)^m\left(-d(1-m)(Ab(bc(3-m)-ad(7-m))+aB(ad(3-m)+bc(m+1)))x^2+aBc(m+1)(ad(5-m)-b(c-cm))-A\left(b^2\left(m^2-4m+3\right)c^2-abd\left(m^2-8m+7\right)c+8a\cdots\right.\right.}{\left(bx^2+a\right)\left(dx^2+c\right)}\cdots}{2a(bc-ad)}\cdots}{4a(bc-ad)}$$

$\downarrow$ 25

$$\frac{(ex)^{m+1}(Ab-aB)}{4ae\left(a+bx^2\right)^2(bc-ad)}-\frac{\frac{\int -\frac{(ex)^m\left(d(1-m)(Ab(bc(3-m)-ad(7-m))+aB(ad(3-m)+bc(m+1)))x^2+aBc(bc(1-m)-ad(5-m))(m+1)+A\left(b^2\left(m^2-4m+3\right)c^2-abd\left(m^2-8m+7\right)c+8a^2d^2\cdots\right.\right.}{\left(bx^2+a\right)\left(dx^2+c\right)}\cdots}{2a(bc-ad)}\cdots}{4a(bc-ad)}$$

$$\Big\downarrow \text{25}$$

$$\frac{(ex)^{m+1}(Ab-aB)}{4ae\left(a+bx^2\right)^2(bc-ad)}-\frac{-\frac{\int -\frac{(ex)^m\left(-d(1-m)(Ab(bc(3-m)-ad(7-m))+aB(ad(3-m)+bc(m+1)))x^2+aBc(m+1)(ad(5-m)-b(c-cm))-A\left(b^2\left(m^2-4m+3\right)c^2-abd\left(m^2-8m+7\right)c+8a\right.}{\left(bx^2+a\right)\left(dx^2+c\right)}}{2a(bc-ad)}}{4a(bc-ad)}$$

input
```
Int[((e*x)^m*(A + B*x^2))/((a + b*x^2)^3*(c + d*x^2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 441
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
+ b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(a*g*2*(b*c - a*d)*(p + 1))), x] + Si
mp[1/(a*2*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2
)^q*Simp[c*(b*e - a*f)*(m + 1) + e*2*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(m
 + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, q},
 x] && LtQ[p, -1]
```

Maple [F]

$$\int \frac{(ex)^m\left(x^2B+A\right)}{\left(b\,x^2+a\right)^3\left(x^2d+c\right)}dx$$

input
```
int((e*x)^m*(B*x^2+A)/(b*x^2+a)^3/(d*x^2+c),x)
```

output
```
int((e*x)^m*(B*x^2+A)/(b*x^2+a)^3/(d*x^2+c),x)
```

Fricas [F]

$$\int \frac{(ex)^m \left(A+Bx^2\right)}{\left(a+bx^2\right)^3 \left(c+dx^2\right)}\,dx = \int \frac{(Bx^2+A)(ex)^m}{(bx^2+a)^3(dx^2+c)}\,dx$$

input
```
integrate((e*x)^m*(B*x^2+A)/(b*x^2+a)^3/(d*x^2+c),x, algorithm="fricas")
```

output
```
integral((B*x^2 + A)*(e*x)^m/(b^3*d*x^8 + (b^3*c + 3*a*b^2*d)*x^6 + 3*(a*b
^2*c + a^2*b*d)*x^4 + a^3*c + (3*a^2*b*c + a^3*d)*x^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ex)^m \left(A+Bx^2\right)}{\left(a+bx^2\right)^3 \left(c+dx^2\right)}\,dx = \text{Timed out}$$

input
```
integrate((e*x)**m*(B*x**2+A)/(b*x**2+a)**3/(d*x**2+c),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(ex)^m \left(A+Bx^2\right)}{\left(a+bx^2\right)^3 \left(c+dx^2\right)}\,dx = \int \frac{(Bx^2+A)(ex)^m}{(bx^2+a)^3(dx^2+c)}\,dx$$

input
```
integrate((e*x)^m*(B*x^2+A)/(b*x^2+a)^3/(d*x^2+c),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*(e*x)^m/((b*x^2 + a)^3*(d*x^2 + c)), x)
```

Giac [F]

$$\int \frac{(ex)^m (A+Bx^2)}{(a+bx^2)^3 (c+dx^2)} \, dx = \int \frac{(Bx^2+A)(ex)^m}{(bx^2+a)^3(dx^2+c)} \, dx$$

input
```
integrate((e*x)^m*(B*x^2+A)/(b*x^2+a)^3/(d*x^2+c),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*(e*x)^m/((b*x^2 + a)^3*(d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m (A+Bx^2)}{(a+bx^2)^3 (c+dx^2)} \, dx = \int \frac{(B\,x^2+A)\,(e\,x)^m}{(b\,x^2+a)^3\,(d\,x^2+c)} \, dx$$

input
```
int(((A + B*x^2)*(e*x)^m)/((a + b*x^2)^3*(c + d*x^2)),x)
```

output
```
int(((A + B*x^2)*(e*x)^m)/((a + b*x^2)^3*(c + d*x^2)), x)
```

Reduce [F]

$$\int \frac{(ex)^m (A+Bx^2)}{(a+bx^2)^3 (c+dx^2)} \, dx = e^m \left(\int \frac{x^m}{b^2 d\,x^6 + 2abd\,x^4 + b^2 c\,x^4 + a^2 d\,x^2 + 2abc\,x^2 + a^2 c} dx \right)$$

input
```
int((e*x)^m*(B*x^2+A)/(b*x^2+a)^3/(d*x^2+c),x)
```

output
```
e**m*int(x**m/(a**2*c + a**2*d*x**2 + 2*a*b*c*x**2 + 2*a*b*d*x**4 + b**2*c
*x**4 + b**2*d*x**6),x)
```

3.30 $\int \frac{(ex)^m(a+bx^2)^3(A+Bx^2)}{(c+dx^2)^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 264

$$\int \frac{(ex)^m(a+bx^2)^3(A+Bx^2)}{(c+dx^2)^2}\,dx = \frac{b(3a^2Bd^2+b^2c(3Bc-2Ad)-3abd(2Bc-Ad))(ex)^{1+m}}{d^4e(1+m)} - \frac{b^2(2bBc-Abd-3aBd)(ex)^{3+m}}{d^3e^3(3+m)} + \frac{b^3B(ex)^{5+m}}{d^2e^5(5+m)} + \frac{(bc-ad)^3(Bc-Ad)(ex)^{1+m}}{2cd^4e(c+dx^2)} + \frac{(bc-ad)^2(ad(Ad(1-m)+Bc(1+m))+bc(Ad(5+m)-Bc(7+m)))(ex)^{1+m}\,\text{Hypergeometric2F1}}{2c^2d^4e(1+m)}$$

output
```
b*(3*a^2*B*d^2+b^2*c*(-2*A*d+3*B*c)-3*a*b*d*(-A*d+2*B*c))*(e*x)^(1+m)/d^4/
e/(1+m)-b^2*(-A*b*d-3*B*a*d+2*B*b*c)*(e*x)^(3+m)/d^3/e^3/(3+m)+b^3*B*(e*x)
^(5+m)/d^2/e^5/(5+m)+1/2*(-a*d+b*c)^3*(-A*d+B*c)*(e*x)^(1+m)/c/d^4/e/(d*x^
2+c)+1/2*(-a*d+b*c)^2*(a*d*(A*d*(1-m)+B*c*(1+m))+b*c*(A*d*(5+m)-B*c*(7+m))
)*(e*x)^(1+m)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-d*x^2/c)/c^2/d^4/e/(1+
m)
```

Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.80

$$\int \frac{(ex)^m (a+bx^2)^3 (A+Bx^2)}{(c+dx^2)^2}\,dx$$
$$= \frac{x(ex)^m \left(\frac{b(3a^2Bd^2+b^2c(3Bc-2Ad)+3abd(-2Bc+Ad))}{1+m} + \frac{b^2d(-2bBc+Abd+3aBd)x^2}{3+m} + \frac{b^3Bd^2x^4}{5+m} - \frac{(bc-ad)^2(4bBc-3Abd-aBd)\,\mathrm{H}}{}\right.}{d^4}$$

input
```
Integrate[((e*x)^m*(a + b*x^2)^3*(A + B*x^2))/(c + d*x^2)^2,x]
```

output
```
(x*(e*x)^m*((b*(3*a^2*B*d^2 + b^2*c*(3*B*c - 2*A*d) + 3*a*b*d*(-2*B*c + A*
d)))/(1 + m) + (b^2*d*(-2*b*B*c + A*b*d + 3*a*B*d)*x^2)/(3 + m) + (b^3*B*d
^2*x^4)/(5 + m) - ((b*c - a*d)^2*(4*b*B*c - 3*A*b*d - a*B*d)*Hypergeometri
c2F1[1, (1 + m)/2, (3 + m)/2, -((d*x^2)/c)])/(c*(1 + m)) + ((b*c - a*d)^3*
(B*c - A*d)*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, -((d*x^2)/c)])/(c^2
*(1 + m))))/d^4
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.26, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {439, 25, 437, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^3 (A+Bx^2) (ex)^m}{(c+dx^2)^2}\,dx$$

$$\Big\downarrow \text{439}$$

$$-\frac{\int -\frac{(ex)^m (bx^2+a)^2 (a(Ad(1-m)+Bc(m+1))-b(Ad(m+5)-Bc(m+7))x^2)}{dx^2+c}dx}{2cd} - \frac{(a+bx^2)^3 (ex)^{m+1}(Bc-Ad)}{2cde\,(c+dx^2)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{(ex)^m\left(bx^2+a\right)^2\left(a(Ad(1-m)+Bc(m+1))-b(Ad(m+5)-Bc(m+7))x^2\right)}{dx^2+c}dx}{2cd}-\frac{\left(a+bx^2\right)^3(ex)^{m+1}(Bc-Ad)}{2cde\left(c+dx^2\right)}$$

$$\Big\downarrow \; 437$$

$$\frac{\int \left(-\frac{b\left(b^2(Ad(m+5)-Bc(m+7))c^2-3abd(Ad(m+3)-Bc(m+5))c+3a^2d^2(Ad(m+1)-Bc(m+3))\right)(ex)^m}{d^3}+\frac{\left(-7b^3Bc^4-b^3Bmc^4+5Ab^3dc^3+1\right.}{}\right.}{}$$

$$\frac{\left(a+bx^2\right)^3(ex)^{m+1}(Bc-Ad)}{2cde\left(c+dx^2\right)}$$

$$\Big\downarrow \; 2009$$

$$\frac{-\frac{b(ex)^{m+1}\left(3a^2d^2(Ad(m+1)-Bc(m+3))-3abcd(Ad(m+3)-Bc(m+5))+b^2c^2(Ad(m+5)-Bc(m+7))\right)}{d^3e(m+1)}-\frac{b^2(ex)^{m+3}(3ad(Ad(m+3)-Bc(m+}{d^2e^3(m}}{}$$

$$\frac{\left(a+bx^2\right)^3(ex)^{m+1}(Bc-Ad)}{2cde\left(c+dx^2\right)}$$

input
```
Int[((e*x)^m*(a + b*x^2)^3*(A + B*x^2))/(c + d*x^2)^2,x]
```

output
```
-1/2*((B*c - A*d)*(e*x)^(1 + m)*(a + b*x^2)^3)/(c*d*e*(c + d*x^2)) + (-((b
*(3*a^2*d^2*(A*d*(1 + m) - B*c*(3 + m)) - 3*a*b*c*d*(A*d*(3 + m) - B*c*(5
+ m)) + b^2*c^2*(A*d*(5 + m) - B*c*(7 + m)))*(e*x)^(1 + m))/(d^3*e*(1 + m)
)) - (b^2*(3*a*d*(A*d*(3 + m) - B*c*(5 + m)) - b*c*(A*d*(5 + m) - B*c*(7 +
 m)))*(e*x)^(3 + m))/(d^2*e^3*(3 + m)) - (b^3*(A*d*(5 + m) - B*c*(7 + m))*
(e*x)^(5 + m))/(d*e^5*(5 + m)) + ((b*c - a*d)^2*(a*d*(A*d*(1 - m) + B*c*(1
 + m)) + b*c*(A*d*(5 + m) - B*c*(7 + m)))*(e*x)^(1 + m)*Hypergeometric2F1[
1, (1 + m)/2, (3 + m)/2, -((d*x^2)/c)])/(c*d^3*e*(1 + m)))/(2*c*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 437
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(
a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f
, g, m}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

rule 439
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
 + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*b*g*(p + 1))), x] + Simp[1/(2*a*b*(p
+ 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1)*Simp[c*(2*b*e*(
p + 1) + (b*e - a*f)*(m + 1)) + d*(2*b*e*(p + 1) + (b*e - a*f)*(m + 2*q + 1
))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && LtQ[p, -1] && G
tQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e - a*f])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{(ex)^m (b\,x^2+a)^3 (x^2B+A)}{(x^2d+c)^2}dx$$

input
```
int((e*x)^m*(b*x^2+a)^3*(B*x^2+A)/(d*x^2+c)^2,x)
```

output
```
int((e*x)^m*(b*x^2+a)^3*(B*x^2+A)/(d*x^2+c)^2,x)
```

Fricas [F]

$$\int \frac{(ex)^m (a+bx^2)^3 (A+Bx^2)}{(c+dx^2)^2} \, dx = \int \frac{(Bx^2+A)(bx^2+a)^3 (ex)^m}{(dx^2+c)^2} \, dx$$

input
```
integrate((e*x)^m*(b*x^2+a)^3*(B*x^2+A)/(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
integral((B*b^3*x^8 + (3*B*a*b^2 + A*b^3)*x^6 + 3*(B*a^2*b + A*a*b^2)*x^4
+ A*a^3 + (B*a^3 + 3*A*a^2*b)*x^2)*(e*x)^m/(d^2*x^4 + 2*c*d*x^2 + c^2), x)
```

Sympy [F]

$$\int \frac{(ex)^m (a+bx^2)^3 (A+Bx^2)}{(c+dx^2)^2} \, dx = \int \frac{(ex)^m (A+Bx^2) (a+bx^2)^3}{(c+dx^2)^2} \, dx$$

input
```
integrate((e*x)**m*(b*x**2+a)**3*(B*x**2+A)/(d*x**2+c)**2,x)
```

output
```
Integral((e*x)**m*(A + B*x**2)*(a + b*x**2)**3/(c + d*x**2)**2, x)
```

Maxima [F]

$$\int \frac{(ex)^m (a+bx^2)^3 (A+Bx^2)}{(c+dx^2)^2} \, dx = \int \frac{(Bx^2+A)(bx^2+a)^3 (ex)^m}{(dx^2+c)^2} \, dx$$

input
```
integrate((e*x)^m*(b*x^2+a)^3*(B*x^2+A)/(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*(b*x^2 + a)^3*(e*x)^m/(d*x^2 + c)^2, x)
```

Giac [F]

$$\int \frac{(ex)^m (a+bx^2)^3 (A+Bx^2)}{(c+dx^2)^2}\, dx = \int \frac{(Bx^2+A)(bx^2+a)^3(ex)^m}{(dx^2+c)^2}\, dx$$

input
```
integrate((e*x)^m*(b*x^2+a)^3*(B*x^2+A)/(d*x^2+c)^2,x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*(b*x^2 + a)^3*(e*x)^m/(d*x^2 + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m (a+bx^2)^3 (A+Bx^2)}{(c+dx^2)^2}\, dx = \int \frac{(B\,x^2+A)\,(e\,x)^m\,(b\,x^2+a)^3}{(d\,x^2+c)^2}\, dx$$

input
```
int(((A + B*x^2)*(e*x)^m*(a + b*x^2)^3)/(c + d*x^2)^2,x)
```

output
```
int(((A + B*x^2)*(e*x)^m*(a + b*x^2)^3)/(c + d*x^2)^2, x)
```

Reduce [F]

$$\int \frac{(ex)^m (a+bx^2)^3 (A+Bx^2)}{(c+dx^2)^2}\, dx = \text{too large to display}$$

input
```
int((e*x)^m*(b*x^2+a)^3*(B*x^2+A)/(d*x^2+c)^2,x)
```

output

```
(e**m*(4*x**m*a**3*b*d**3*m**3*x + 36*x**m*a**3*b*d**3*m**2*x + 92*x**m*a*
*3*b*d**3*m*x + 60*x**m*a**3*b*d**3*x - 6*x**m*a**2*b**2*c*d**2*m**3*x - 6
6*x**m*a**2*b**2*c*d**2*m**2*x - 234*x**m*a**2*b**2*c*d**2*m*x - 270*x**m*
a**2*b**2*c*d**2*x + 6*x**m*a**2*b**2*d**3*m**3*x**3 + 42*x**m*a**2*b**2*d
**3*m**2*x**3 + 42*x**m*a**2*b**2*d**3*m*x**3 - 90*x**m*a**2*b**2*d**3*x**
3 + 4*x**m*a*b**3*c**2*d*m**3*x + 52*x**m*a*b**3*c**2*d*m**2*x + 220*x**m*
a*b**3*c**2*d*m*x + 300*x**m*a*b**3*c**2*d*x - 4*x**m*a*b**3*c*d**2*m**3*x
**3 - 36*x**m*a*b**3*c*d**2*m**2*x**3 - 60*x**m*a*b**3*c*d**2*m*x**3 + 100
*x**m*a*b**3*c*d**2*x**3 + 4*x**m*a*b**3*d**3*m**3*x**5 + 20*x**m*a*b**3*d
**3*m**2*x**5 - 4*x**m*a*b**3*d**3*m*x**5 - 20*x**m*a*b**3*d**3*x**5 - x**
m*b**4*c**3*m**3*x - 15*x**m*b**4*c**3*m**2*x - 71*x**m*b**4*c**3*m*x - 10
5*x**m*b**4*c**3*x + x**m*b**4*c**2*d*m**3*x**3 + 11*x**m*b**4*c**2*d*m**2
*x**3 + 23*x**m*b**4*c**2*d*m*x**3 - 35*x**m*b**4*c**2*d*x**3 - x**m*b**4*
c*d**2*m**3*x**5 - 7*x**m*b**4*c*d**2*m**2*x**5 + x**m*b**4*c*d**2*m*x**5
 + 7*x**m*b**4*c*d**2*x**5 + x**m*b**4*d**3*m**3*x**7 + 3*x**m*b**4*d**3*m*
*2*x**7 - x**m*b**4*d**3*m*x**7 - 3*x**m*b**4*d**3*x**7 + int(x**m/(c**2*m
 - c**2 + 2*c*d*m*x**2 - 2*c*d*x**2 + d**2*m*x**4 - d**2*x**4),x)*a**4*c*d
**4*m**5 + 7*int(x**m/(c**2*m - c**2 + 2*c*d*m*x**2 - 2*c*d*x**2 + d**2*m*
x**4 - d**2*x**4),x)*a**4*c*d**4*m**4 + 6*int(x**m/(c**2*m - c**2 + 2*c*d*
m*x**2 - 2*c*d*x**2 + d**2*m*x**4 - d**2*x**4),x)*a**4*c*d**4*m**3 - 22...
```

3.31 $\int \frac{(ex)^m(a+bx^2)^2(A+Bx^2)}{(c+dx^2)^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 202

$$\int \frac{(ex)^m(a+bx^2)^2(A+Bx^2)}{(c+dx^2)^2}\,dx = -\frac{b(2bBc-Abd-2aBd)(ex)^{1+m}}{d^3e(1+m)}+\frac{b^2B(ex)^{3+m}}{d^2e^3(3+m)}-\frac{(bc-ad)^2(Bc-Ad)(ex)^{1+m}}{2cd^3e\,(c+dx^2)} - \frac{(bc-ad)(ad(Ad(1-m)+Bc(1+m))+bc(Ad(3+m)-Bc(5+m)))(ex)^{1+m}\,\text{Hypergeometric2F1}}{2c^2d^3e(1+m)}$$

output
```
-b*(-A*b*d-2*B*a*d+2*B*b*c)*(e*x)^(1+m)/d^3/e/(1+m)+b^2*B*(e*x)^(3+m)/d^2/
e^3/(3+m)-1/2*(-a*d+b*c)^2*(-A*d+B*c)*(e*x)^(1+m)/c/d^3/e/(d*x^2+c)-1/2*(-
a*d+b*c)*(a*d*(A*d*(1-m)+B*c*(1+m))+b*c*(A*d*(3+m)-B*c*(5+m)))*(e*x)^(1+m)
*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-d*x^2/c)/c^2/d^3/e/(1+m)
```

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.78

$$\int \frac{(ex)^m (a+bx^2)^2 (A+Bx^2)}{(c+dx^2)^2} dx$$
$$= \frac{x(ex)^m \left(\frac{b(-2bBc+Abd+2aBd)}{1+m}+\frac{b^2Bdx^2}{3+m}+\frac{(bc-ad)(3bBc-2Abd-aBd)\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{dx^2}{c}\right)}{c(1+m)}-\frac{(bc-ad)^2(Bc-}{}\right.}{d^3}$$

input
```
Integrate[((e*x)^m*(a + b*x^2)^2*(A + B*x^2))/(c + d*x^2)^2,x]
```

output
```
(x*(e*x)^m*((b*(-2*b*B*c + A*b*d + 2*a*B*d))/(1 + m) + (b^2*B*d*x^2)/(3 +
m) + ((b*c - a*d)*(3*b*B*c - 2*A*b*d - a*B*d)*Hypergeometric2F1[1, (1 + m)
/2, (3 + m)/2, -((d*x^2)/c)])/(c*(1 + m)) - ((b*c - a*d)^2*(B*c - A*d)*Hyp
ergeometric2F1[2, (1 + m)/2, (3 + m)/2, -((d*x^2)/c)])/(c^2*(1 + m))))/d^3
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.21, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {439, 25, 437, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2 (A+Bx^2)(ex)^m}{(c+dx^2)^2} dx$$
$$\Big\downarrow \text{439}$$
$$-\frac{\int -\frac{(ex)^m(bx^2+a)(a(Ad(1-m)+Bc(m+1))-b(Ad(m+3)-Bc(m+5))x^2)}{dx^2+c}dx}{2cd} - \frac{(a+bx^2)^2 (ex)^{m+1}(Bc-Ad)}{2cde(c+dx^2)}$$
$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{(ex)^m(bx^2+a)(a(Ad(1-m)+Bc(m+1))-b(Ad(m+3)-Bc(m+5))x^2)}{dx^2+c}dx}{2cd}-\frac{(a+bx^2)^2(ex)^{m+1}(Bc-Ad)}{2cde(c+dx^2)}$$

$$\Big\downarrow \text{437}$$

$$\frac{\int \left(-\frac{b(2ad(Ad(m+1)-Bc(m+3))-bc(Ad(m+3)-Bc(m+5)))(ex)^m}{d^2}+\frac{(5b^2Bc^3+b^2Bmc^3-3Ab^2dc^2-6abBdc^2-Ab^2dmc^2-2abBdmc^2+2aA}{d^2(dx^2+}\right.}{2cd}$$

$$\frac{(a+bx^2)^2(ex)^{m+1}(Bc-Ad)}{2cde(c+dx^2)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{(ex)^{m+1}(bc-ad)\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\frac{dx^2}{c}\right)(ad(Ad(1-m)+Bc(m+1))+bc(Ad(m+3)-Bc(m+5)))}{cd^2e(m+1)}-\frac{b(ex)^{m+1}(2ad(Ad(m+}{}}{2cd}$$

$$\frac{(a+bx^2)^2(ex)^{m+1}(Bc-Ad)}{2cde(c+dx^2)}$$

input
```
Int[((e*x)^m*(a + b*x^2)^2*(A + B*x^2))/(c + d*x^2)^2,x]
```

output
```
-1/2*((B*c - A*d)*(e*x)^(1 + m)*(a + b*x^2)^2)/(c*d*e*(c + d*x^2)) + (-((b
*(2*a*d*(A*d*(1 + m) - B*c*(3 + m)) - b*c*(A*d*(3 + m) - B*c*(5 + m)))*(e*
x)^(1 + m))/(d^2*e*(1 + m))) - (b^2*(A*d*(3 + m) - B*c*(5 + m))*(e*x)^(3 +
 m))/(d*e^3*(3 + m)) - ((b*c - a*d)*(a*d*(A*d*(1 - m) + B*c*(1 + m)) + b*c
*(A*d*(3 + m) - B*c*(5 + m)))*(e*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/2
, (3 + m)/2, -((d*x^2)/c)])/(c*d^2*e*(1 + m)))/(2*c*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 437
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(
a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f
, g, m}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

rule 439
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
 + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*b*g*(p + 1))), x] + Simp[1/(2*a*b*(p
+ 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1)*Simp[c*(2*b*e*(
p + 1) + (b*e - a*f)*(m + 1)) + d*(2*b*e*(p + 1) + (b*e - a*f)*(m + 2*q + 1
))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && LtQ[p, -1] && G
tQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e - a*f])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{(ex)^m (b\,x^2+a)^2 (x^2B+A)}{(x^2d+c)^2}dx$$

input
```
int((e*x)^m*(b*x^2+a)^2*(B*x^2+A)/(d*x^2+c)^2,x)
```

output
```
int((e*x)^m*(b*x^2+a)^2*(B*x^2+A)/(d*x^2+c)^2,x)
```

Fricas [F]

$$\int \frac{(ex)^m (a+bx^2)^2 (A+Bx^2)}{(c+dx^2)^2}\,dx = \int \frac{(Bx^2+A)(bx^2+a)^2(ex)^m}{(dx^2+c)^2}\,dx$$

input
```
integrate((e*x)^m*(b*x^2+a)^2*(B*x^2+A)/(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
integral((B*b^2*x^6 + (2*B*a*b + A*b^2)*x^4 + A*a^2 + (B*a^2 + 2*A*a*b)*x^
2)*(e*x)^m/(d^2*x^4 + 2*c*d*x^2 + c^2), x)
```

Sympy [F]

$$\int \frac{(ex)^m \left(a+bx^2\right)^2 \left(A+Bx^2\right)}{\left(c+dx^2\right)^2} \, dx = \int \frac{(ex)^m \left(A+Bx^2\right) \left(a+bx^2\right)^2}{\left(c+dx^2\right)^2} \, dx$$

input
```
integrate((e*x)**m*(b*x**2+a)**2*(B*x**2+A)/(d*x**2+c)**2,x)
```

output
```
Integral((e*x)**m*(A + B*x**2)*(a + b*x**2)**2/(c + d*x**2)**2, x)
```

Maxima [F]

$$\int \frac{(ex)^m \left(a+bx^2\right)^2 \left(A+Bx^2\right)}{\left(c+dx^2\right)^2} \, dx = \int \frac{(Bx^2+A)(bx^2+a)^2(ex)^m}{(dx^2+c)^2} \, dx$$

input
```
integrate((e*x)^m*(b*x^2+a)^2*(B*x^2+A)/(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*(b*x^2 + a)^2*(e*x)^m/(d*x^2 + c)^2, x)
```

Giac [F]

$$\int \frac{(ex)^m \left(a+bx^2\right)^2 \left(A+Bx^2\right)}{\left(c+dx^2\right)^2} \, dx = \int \frac{(Bx^2+A)(bx^2+a)^2(ex)^m}{(dx^2+c)^2} \, dx$$

input
```
integrate((e*x)^m*(b*x^2+a)^2*(B*x^2+A)/(d*x^2+c)^2,x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*(b*x^2 + a)^2*(e*x)^m/(d*x^2 + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m (a+bx^2)^2 (A+Bx^2)}{(c+dx^2)^2}\,dx = \int \frac{(B\,x^2+A)\,(e\,x)^m\,(b\,x^2+a)^2}{(d\,x^2+c)^2}\,dx$$

input
```
int(((A + B*x^2)*(e*x)^m*(a + b*x^2)^2)/(c + d*x^2)^2,x)
```

output
```
int(((A + B*x^2)*(e*x)^m*(a + b*x^2)^2)/(c + d*x^2)^2, x)
```

Reduce [F]

$$\int \frac{(ex)^m (a+bx^2)^2 (A+Bx^2)}{(c+dx^2)^2}\,dx = \text{too large to display}$$

input
```
int((e*x)^m*(b*x^2+a)^2*(B*x^2+A)/(d*x^2+c)^2,x)
```

output
```
(e**m*(3*x**m*a**2*b*d**2*m**2*x + 12*x**m*a**2*b*d**2*m*x + 9*x**m*a**2*b
*d**2*x - 3*x**m*a*b**2*c*d*m**2*x - 18*x**m*a*b**2*c*d*m*x - 27*x**m*a*b*
*2*c*d*x + 3*x**m*a*b**2*d**2*m**2*x**3 + 6*x**m*a*b**2*d**2*m*x**3 - 9*x*
*m*a*b**2*d**2*x**3 + x**m*b**3*c**2*m**2*x + 8*x**m*b**3*c**2*m*x + 15*x*
*m*b**3*c**2*x - x**m*b**3*c*d*m**2*x**3 - 4*x**m*b**3*c*d*m*x**3 + 5*x**m
*b**3*c*d*x**3 + x**m*b**3*d**2*m**2*x**5 - x**m*b**3*d**2*x**5 + int(x**m
/(c**2*m - c**2 + 2*c*d*m*x**2 - 2*c*d*x**2 + d**2*m*x**4 - d**2*x**4),x)*
a**3*c*d**3*m**4 + 2*int(x**m/(c**2*m - c**2 + 2*c*d*m*x**2 - 2*c*d*x**2 +
 d**2*m*x**4 - d**2*x**4),x)*a**3*c*d**3*m**3 - 4*int(x**m/(c**2*m - c**2
+ 2*c*d*m*x**2 - 2*c*d*x**2 + d**2*m*x**4 - d**2*x**4),x)*a**3*c*d**3*m**2
 - 2*int(x**m/(c**2*m - c**2 + 2*c*d*m*x**2 - 2*c*d*x**2 + d**2*m*x**4 - d
**2*x**4),x)*a**3*c*d**3*m + 3*int(x**m/(c**2*m - c**2 + 2*c*d*m*x**2 - 2*
c*d*x**2 + d**2*m*x**4 - d**2*x**4),x)*a**3*c*d**3 + int(x**m/(c**2*m - c*
*2 + 2*c*d*m*x**2 - 2*c*d*x**2 + d**2*m*x**4 - d**2*x**4),x)*a**3*d**4*m**
4*x**2 + 2*int(x**m/(c**2*m - c**2 + 2*c*d*m*x**2 - 2*c*d*x**2 + d**2*m*x*
*4 - d**2*x**4),x)*a**3*d**4*m**3*x**2 - 4*int(x**m/(c**2*m - c**2 + 2*c*d
*m*x**2 - 2*c*d*x**2 + d**2*m*x**4 - d**2*x**4),x)*a**3*d**4*m**2*x**2 - 2
*int(x**m/(c**2*m - c**2 + 2*c*d*m*x**2 - 2*c*d*x**2 + d**2*m*x**4 - d**2*
x**4),x)*a**3*d**4*m*x**2 + 3*int(x**m/(c**2*m - c**2 + 2*c*d*m*x**2 - 2*c
*d*x**2 + d**2*m*x**4 - d**2*x**4),x)*a**3*d**4*x**2 - 3*int(x**m/(c**2...
```

3.32 $\int \frac{(ex)^m(a+bx^2)(A+Bx^2)}{(c+dx^2)^2} dx$

Optimal result

Integrand size = 29, antiderivative size = 153

$$\int \frac{(ex)^m(a+bx^2)(A+Bx^2)}{(c+dx^2)^2} dx = \frac{bB(ex)^{1+m}}{d^2e(1+m)} + \frac{(bc-ad)(Bc-Ad)(ex)^{1+m}}{2cd^2e(c+dx^2)} + \frac{(ad(Ad(1-m)+Bc(1+m))+bc(Ad(1+m)-Bc(3+m)))(ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\right.}{2c^2d^2e(1+m)}$$

output
```
b*B*(e*x)^(1+m)/d^2/e/(1+m)+1/2*(-a*d+b*c)*(-A*d+B*c)*(e*x)^(1+m)/c/d^2/e/
(d*x^2+c)+1/2*(a*d*(A*d*(1-m)+B*c*(1+m))+b*c*(A*d*(1+m)-B*c*(3+m)))*(e*x)^
(1+m)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-d*x^2/c)/c^2/d^2/e/(1+m)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.71

$$\int \frac{(ex)^m(a+bx^2)(A+Bx^2)}{(c+dx^2)^2} dx$$
$$= \frac{x(ex)^m\left(bBc^2 + c(-2bBc + Abd + aBd)\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{dx^2}{c}\right) + (bc-ad)(Bc-A\right.}{c^2d^2(1+m)}$$

input
```
Integrate[((e*x)^m*(a + b*x^2)*(A + B*x^2))/(c + d*x^2)^2,x]
```

output
```
(x*(e*x)^m*(b*B*c^2 + c*(-2*b*B*c + A*b*d + a*B*d)*Hypergeometric2F1[1, (1
 + m)/2, (3 + m)/2, -((d*x^2)/c)] + (b*c - a*d)*(B*c - A*d)*Hypergeometric
2F1[2, (1 + m)/2, (3 + m)/2, -((d*x^2)/c)]))/(c^2*d^2*(1 + m))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.14, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {439, 25, 363, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)(A+Bx^2)(ex)^m}{(c+dx^2)^2}dx$$

$$\Big\downarrow \text{439}$$

$$-\frac{\int -\frac{(ex)^m(A(ad(1-m)+bc(m+1))-B(ad(m+1)-bc(m+3))x^2)}{dx^2+c}dx}{2cd}-\frac{(A+Bx^2)(ex)^{m+1}(bc-ad)}{2cde(c+dx^2)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{(ex)^m(A(ad(1-m)+bc(m+1))-B(ad(m+1)-bc(m+3))x^2)}{dx^2+c}dx}{2cd}-\frac{(A+Bx^2)(ex)^{m+1}(bc-ad)}{2cde(c+dx^2)}$$

$$\Big\downarrow \text{363}$$

$$\frac{\frac{(ad(Ad(1-m)+Bc(m+1))+bc(Ad(m+1)-Bc(m+3)))\int \frac{(ex)^m}{dx^2+c}dx}{d}-\frac{B(ex)^{m+1}(ad(m+1)-bc(m+3))}{de(m+1)}}{2cd}-\frac{(A+Bx^2)(ex)^{m+1}(bc-ad)}{2cde(c+dx^2)}$$

$$\Big\downarrow \text{278}$$

$$\frac{\frac{(ex)^{m+1}\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\frac{dx^2}{c}\right)(ad(Ad(1-m)+Bc(m+1))+bc(Ad(m+1)-Bc(m+3)))}{cde(m+1)}-\frac{B(ex)^{m+1}(ad(m+1)-bc(m+3))}{de(m+1)}}{2cd}$$
$$\frac{(A+Bx^2)(ex)^{m+1}(bc-ad)}{2cde(c+dx^2)}$$

input
```
Int[((e*x)^m*(a + b*x^2)*(A + B*x^2))/(c + d*x^2)^2,x]
```

output
```
-1/2*((b*c - a*d)*(e*x)^(1 + m)*(A + B*x^2))/(c*d*e*(c + d*x^2)) + (-((B*(
a*d*(1 + m) - b*c*(3 + m))*(e*x)^(1 + m))/(d*e*(1 + m))) + ((a*d*(A*d*(1 -
 m) + B*c*(1 + m)) + b*c*(A*d*(1 + m) - B*c*(3 + m)))*(e*x)^(1 + m)*Hyperg
eometric2F1[1, (1 + m)/2, (3 + m)/2, -((d*x^2)/c)])/(c*d*e*(1 + m)))/(2*c*
d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 439
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
 + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*b*g*(p + 1))), x] + Simp[1/(2*a*b*(p
+ 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1)*Simp[c*(2*b*e*(
p + 1) + (b*e - a*f)*(m + 1)) + d*(2*b*e*(p + 1) + (b*e - a*f)*(m + 2*q + 1
))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && LtQ[p, -1] && G
tQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e - a*f])
```

Maple [F]

$$\int \frac{(ex)^m (b\,x^2+a)\,(x^2B+A)}{(x^2d+c)^2}dx$$

input
```
int((e*x)^m*(b*x^2+a)*(B*x^2+A)/(d*x^2+c)^2,x)
```

output
```
int((e*x)^m*(b*x^2+a)*(B*x^2+A)/(d*x^2+c)^2,x)
```

Fricas [F]

$$\int \frac{(ex)^m\,(a+bx^2)\,(A+Bx^2)}{(c+dx^2)^2}\,dx = \int \frac{(Bx^2+A)(bx^2+a)(ex)^m}{(dx^2+c)^2}\,dx$$

input
```
integrate((e*x)^m*(b*x^2+a)*(B*x^2+A)/(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
integral((B*b*x^4 + (B*a + A*b)*x^2 + A*a)*(e*x)^m/(d^2*x^4 + 2*c*d*x^2 +
c^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 28.27 (sec) , antiderivative size = 2069, normalized size of antiderivative = 13.52

$$\int \frac{(ex)^m (a+bx^2) (A+Bx^2)}{(c+dx^2)^2} \, dx = \text{Too large to display}$$

input
```
integrate((e*x)**m*(b*x**2+a)*(B*x**2+A)/(d*x**2+c)**2,x)
```

output
```
A*a*(-c*e**m*m**2*x**(m + 1)*lerchphi(d*x**2*exp_polar(I*pi)/c, 1, m/2 + 1
/2)*gamma(m/2 + 1/2)/(8*c**3*gamma(m/2 + 3/2) + 8*c**2*d*x**2*gamma(m/2 +
3/2)) + 2*c*e**m*m*x**(m + 1)*gamma(m/2 + 1/2)/(8*c**3*gamma(m/2 + 3/2) +
8*c**2*d*x**2*gamma(m/2 + 3/2)) + c*e**m*x**(m + 1)*lerchphi(d*x**2*exp_po
lar(I*pi)/c, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(8*c**3*gamma(m/2 + 3/2) + 8*c
**2*d*x**2*gamma(m/2 + 3/2)) + 2*c*e**m*x**(m + 1)*gamma(m/2 + 1/2)/(8*c**
3*gamma(m/2 + 3/2) + 8*c**2*d*x**2*gamma(m/2 + 3/2)) - d*e**m*m**2*x**2*x*
*(m + 1)*lerchphi(d*x**2*exp_polar(I*pi)/c, 1, m/2 + 1/2)*gamma(m/2 + 1/2)
/(8*c**3*gamma(m/2 + 3/2) + 8*c**2*d*x**2*gamma(m/2 + 3/2)) + d*e**m*x**2*
x**(m + 1)*lerchphi(d*x**2*exp_polar(I*pi)/c, 1, m/2 + 1/2)*gamma(m/2 + 1/
2)/(8*c**3*gamma(m/2 + 3/2) + 8*c**2*d*x**2*gamma(m/2 + 3/2))) + A*b*(-c*e
**m*m**2*x**(m + 3)*lerchphi(d*x**2*exp_polar(I*pi)/c, 1, m/2 + 3/2)*gamma
(m/2 + 3/2)/(8*c**3*gamma(m/2 + 5/2) + 8*c**2*d*x**2*gamma(m/2 + 5/2)) - 4
*c*e**m*m*x**(m + 3)*lerchphi(d*x**2*exp_polar(I*pi)/c, 1, m/2 + 3/2)*gamm
a(m/2 + 3/2)/(8*c**3*gamma(m/2 + 5/2) + 8*c**2*d*x**2*gamma(m/2 + 5/2)) +
2*c*e**m*m*x**(m + 3)*gamma(m/2 + 3/2)/(8*c**3*gamma(m/2 + 5/2) + 8*c**2*d
*x**2*gamma(m/2 + 5/2)) - 3*c*e**m*x**(m + 3)*lerchphi(d*x**2*exp_polar(I*
pi)/c, 1, m/2 + 3/2)*gamma(m/2 + 3/2)/(8*c**3*gamma(m/2 + 5/2) + 8*c**2*d*
x**2*gamma(m/2 + 5/2)) + 6*c*e**m*x**(m + 3)*gamma(m/2 + 3/2)/(8*c**3*gamm
a(m/2 + 5/2) + 8*c**2*d*x**2*gamma(m/2 + 5/2)) - d*e**m*m**2*x**2*x**(m...
```

Maxima [F]

$$\int \frac{(ex)^m (a+bx^2) (A+Bx^2)}{(c+dx^2)^2} \, dx = \int \frac{(Bx^2+A)(bx^2+a)(ex)^m}{(dx^2+c)^2} \, dx$$

input
```
integrate((e*x)^m*(b*x^2+a)*(B*x^2+A)/(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*(b*x^2 + a)*(e*x)^m/(d*x^2 + c)^2, x)
```

Giac [F]

$$\int \frac{(ex)^m (a+bx^2) (A+Bx^2)}{(c+dx^2)^2} \, dx = \int \frac{(Bx^2+A)(bx^2+a)(ex)^m}{(dx^2+c)^2} \, dx$$

input
```
integrate((e*x)^m*(b*x^2+a)*(B*x^2+A)/(d*x^2+c)^2,x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*(b*x^2 + a)*(e*x)^m/(d*x^2 + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m (a+bx^2) (A+Bx^2)}{(c+dx^2)^2} \, dx = \int \frac{(B\,x^2+A)\,(e\,x)^m\,(b\,x^2+a)}{(d\,x^2+c)^2} \, dx$$

input
```
int(((A + B*x^2)*(e*x)^m*(a + b*x^2))/(c + d*x^2)^2,x)
```

output
```
int(((A + B*x^2)*(e*x)^m*(a + b*x^2))/(c + d*x^2)^2, x)
```

Reduce [F]

$$\int \frac{(ex)^m (a+bx^2)(A+Bx^2)}{(c+dx^2)^2}\,dx = \text{Too large to display}$$

input

```
int((e*x)^m*(b*x^2+a)*(B*x^2+A)/(d*x^2+c)^2,x)
```

output

```
(e**m*(2*x**m*a*b*d*m*x + 2*x**m*a*b*d*x - x**m*b**2*c*m*x - 3*x**m*b**2*c
*x + x**m*b**2*d*m*x**3 - x**m*b**2*d*x**3 + int(x**m/(c**2*m - c**2 + 2*c
*d*m*x**2 - 2*c*d*x**2 + d**2*m*x**4 - d**2*x**4),x)*a**2*c*d**2*m**3 - in
t(x**m/(c**2*m - c**2 + 2*c*d*m*x**2 - 2*c*d*x**2 + d**2*m*x**4 - d**2*x**
4),x)*a**2*c*d**2*m**2 - int(x**m/(c**2*m - c**2 + 2*c*d*m*x**2 - 2*c*d*x*
*2 + d**2*m*x**4 - d**2*x**4),x)*a**2*c*d**2*m + int(x**m/(c**2*m - c**2 +
 2*c*d*m*x**2 - 2*c*d*x**2 + d**2*m*x**4 - d**2*x**4),x)*a**2*c*d**2 + int
(x**m/(c**2*m - c**2 + 2*c*d*m*x**2 - 2*c*d*x**2 + d**2*m*x**4 - d**2*x**4
),x)*a**2*d**3*m**3*x**2 - int(x**m/(c**2*m - c**2 + 2*c*d*m*x**2 - 2*c*d*
x**2 + d**2*m*x**4 - d**2*x**4),x)*a**2*d**3*m**2*x**2 - int(x**m/(c**2*m
- c**2 + 2*c*d*m*x**2 - 2*c*d*x**2 + d**2*m*x**4 - d**2*x**4),x)*a**2*d**3
*m*x**2 + int(x**m/(c**2*m - c**2 + 2*c*d*m*x**2 - 2*c*d*x**2 + d**2*m*x**
4 - d**2*x**4),x)*a**2*d**3*x**2 - 2*int(x**m/(c**2*m - c**2 + 2*c*d*m*x**
2 - 2*c*d*x**2 + d**2*m*x**4 - d**2*x**4),x)*a*b*c**2*d*m**3 - 2*int(x**m/
(c**2*m - c**2 + 2*c*d*m*x**2 - 2*c*d*x**2 + d**2*m*x**4 - d**2*x**4),x)*a
*b*c**2*d*m**2 + 2*int(x**m/(c**2*m - c**2 + 2*c*d*m*x**2 - 2*c*d*x**2 + d
**2*m*x**4 - d**2*x**4),x)*a*b*c**2*d*m + 2*int(x**m/(c**2*m - c**2 + 2*c*
d*m*x**2 - 2*c*d*x**2 + d**2*m*x**4 - d**2*x**4),x)*a*b*c**2*d - 2*int(x**
m/(c**2*m - c**2 + 2*c*d*m*x**2 - 2*c*d*x**2 + d**2*m*x**4 - d**2*x**4),x)
*a*b*c*d**2*m**3*x**2 - 2*int(x**m/(c**2*m - c**2 + 2*c*d*m*x**2 - 2*c*...
```

3.33 $\int \frac{(ex)^m(A+Bx^2)}{(c+dx^2)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 103

$$\int \frac{(ex)^m(A+Bx^2)}{(c+dx^2)^2}\,dx = -\frac{(Bc-Ad)(ex)^{1+m}}{2cde(c+dx^2)} + \frac{(Ad(1-m)+Bc(1+m))(ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{dx^2}{c}\right)}{2c^2de(1+m)}$$

output
```
-1/2*(-A*d+B*c)*(e*x)^(1+m)/c/d/e/(d*x^2+c)+1/2*(A*d*(1-m)+B*c*(1+m))*(e*x
)^(1+m)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-d*x^2/c)/c^2/d/e/(1+m)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.79

$$\int \frac{(ex)^m(A+Bx^2)}{(c+dx^2)^2}\,dx = \frac{x(ex)^m\left(Bc\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{dx^2}{c}\right)+(-Bc+Ad)\,\text{Hypergeometric2F1}\left(2,\frac{1+m}{2},\frac{3+m}{2},\right.\right.}{c^2d(1+m)}$$

input
```
Integrate[((e*x)^m*(A + B*x^2))/(c + d*x^2)^2,x]
```

output
```
(x*(e*x)^m*(B*c*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((d*x^2)/c)] +
 (-(B*c) + A*d)*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, -((d*x^2)/c)]))
/(c^2*d*(1 + m))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {362, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)(ex)^m}{(c+dx^2)^2}dx$$

$$\Big\downarrow \text{362}$$

$$\frac{(Ad(1-m)+Bc(m+1))\int \frac{(ex)^m}{dx^2+c}dx}{2cd}-\frac{(ex)^{m+1}(Bc-Ad)}{2cde(c+dx^2)}$$

$$\Big\downarrow \text{278}$$

$$\frac{(ex)^{m+1}(Ad(1-m)+Bc(m+1))\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\frac{dx^2}{c}\right)}{2c^2de(m+1)}-\frac{(ex)^{m+1}(Bc-Ad)}{2cde(c+dx^2)}$$

input
```
Int[((e*x)^m*(A + B*x^2))/(c + d*x^2)^2,x]
```

output
```
-1/2*((B*c - A*d)*(e*x)^(1 + m))/(c*d*e*(c + d*x^2)) + ((A*d*(1 - m) + B*c
*(1 + m))*(e*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((d*x^
2)/c)])/(2*c^2*d*e*(1 + m))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

Maple [F]

$$\int \frac{(ex)^m (x^2B + A)}{(x^2d + c)^2}dx$$

input
```
int((e*x)^m*(B*x^2+A)/(d*x^2+c)^2,x)
```

output
```
int((e*x)^m*(B*x^2+A)/(d*x^2+c)^2,x)
```

Fricas [F]

$$\int \frac{(ex)^m (A + Bx^2)}{(c + dx^2)^2} dx = \int \frac{(Bx^2 + A)(ex)^m}{(dx^2 + c)^2} dx$$

input
```
integrate((e*x)^m*(B*x^2+A)/(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
integral((B*x^2 + A)*(e*x)^m/(d^2*x^4 + 2*c*d*x^2 + c^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 14.12 (sec) , antiderivative size = 954, normalized size of antiderivative = 9.26

$$\int \frac{(ex)^m (A + Bx^2)}{(c + dx^2)^2} \, dx = \text{Too large to display}$$

input
```
integrate((e*x)**m*(B*x**2+A)/(d*x**2+c)**2,x)
```

output
```
A*(-c*e**m*m**2*x**(m + 1)*lerchphi(d*x**2*exp_polar(I*pi)/c, 1, m/2 + 1/2
)*gamma(m/2 + 1/2)/(8*c**3*gamma(m/2 + 3/2) + 8*c**2*d*x**2*gamma(m/2 + 3/
2)) + 2*c*e**m*m*x**(m + 1)*gamma(m/2 + 1/2)/(8*c**3*gamma(m/2 + 3/2) + 8*
c**2*d*x**2*gamma(m/2 + 3/2)) + c*e**m*x**(m + 1)*lerchphi(d*x**2*exp_pola
r(I*pi)/c, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(8*c**3*gamma(m/2 + 3/2) + 8*c**
2*d*x**2*gamma(m/2 + 3/2)) + 2*c*e**m*x**(m + 1)*gamma(m/2 + 1/2)/(8*c**3*
gamma(m/2 + 3/2) + 8*c**2*d*x**2*gamma(m/2 + 3/2)) - d*e**m*m**2*x**2*x**(
m + 1)*lerchphi(d*x**2*exp_polar(I*pi)/c, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(
8*c**3*gamma(m/2 + 3/2) + 8*c**2*d*x**2*gamma(m/2 + 3/2)) + d*e**m*x**2*x*
*(m + 1)*lerchphi(d*x**2*exp_polar(I*pi)/c, 1, m/2 + 1/2)*gamma(m/2 + 1/2)
/(8*c**3*gamma(m/2 + 3/2) + 8*c**2*d*x**2*gamma(m/2 + 3/2))) + B*(-c*e**m*
m**2*x**(m + 3)*lerchphi(d*x**2*exp_polar(I*pi)/c, 1, m/2 + 3/2)*gamma(m/2
 + 3/2)/(8*c**3*gamma(m/2 + 5/2) + 8*c**2*d*x**2*gamma(m/2 + 5/2)) - 4*c*e
**m*m*x**(m + 3)*lerchphi(d*x**2*exp_polar(I*pi)/c, 1, m/2 + 3/2)*gamma(m/
2 + 3/2)/(8*c**3*gamma(m/2 + 5/2) + 8*c**2*d*x**2*gamma(m/2 + 5/2)) + 2*c*
e**m*m*x**(m + 3)*gamma(m/2 + 3/2)/(8*c**3*gamma(m/2 + 5/2) + 8*c**2*d*x**
2*gamma(m/2 + 5/2)) - 3*c*e**m*x**(m + 3)*lerchphi(d*x**2*exp_polar(I*pi)/
c, 1, m/2 + 3/2)*gamma(m/2 + 3/2)/(8*c**3*gamma(m/2 + 5/2) + 8*c**2*d*x**2
*gamma(m/2 + 5/2)) + 6*c*e**m*x**(m + 3)*gamma(m/2 + 3/2)/(8*c**3*gamma(m/
2 + 5/2) + 8*c**2*d*x**2*gamma(m/2 + 5/2)) - d*e**m*m**2*x**2*x**(m + 3...
```

Maxima [F]

$$\int \frac{(ex)^m (A+Bx^2)}{(c+dx^2)^2} \, dx = \int \frac{(Bx^2+A)(ex)^m}{(dx^2+c)^2} \, dx$$

input
```
integrate((e*x)^m*(B*x^2+A)/(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*(e*x)^m/(d*x^2 + c)^2, x)
```

Giac [F]

$$\int \frac{(ex)^m (A+Bx^2)}{(c+dx^2)^2} \, dx = \int \frac{(Bx^2+A)(ex)^m}{(dx^2+c)^2} \, dx$$

input
```
integrate((e*x)^m*(B*x^2+A)/(d*x^2+c)^2,x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*(e*x)^m/(d*x^2 + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m (A+Bx^2)}{(c+dx^2)^2} \, dx = \int \frac{(B\,x^2+A)\,(e\,x)^m}{(d\,x^2+c)^2} \, dx$$

input
```
int(((A + B*x^2)*(e*x)^m)/(c + d*x^2)^2,x)
```

output
```
int(((A + B*x^2)*(e*x)^m)/(c + d*x^2)^2, x)
```

Reduce [F]

$$\int \frac{(ex)^m (A + Bx^2)}{(c + dx^2)^2} \, dx$$
$$= \frac{e^m \left(x^m bx + \left(\int \frac{x^m}{d^2m\, x^4 - d^2x^4 + 2cdm\, x^2 - 2cd\, x^2 + c^2m - c^2} dx\right) acd\, m^2 - 2\left(\int \frac{x^m}{d^2m\, x^4 - d^2x^4 + 2cdm\, x^2 - 2cd\, x^2 + c^2m - c^2} dx\right) acd\right.}{}$$

input
```
int((e*x)^m*(B*x^2+A)/(d*x^2+c)^2,x)
```

output
```
(e**m*(x**m*b*x + int(x**m/(c**2*m - c**2 + 2*c*d*m*x**2 - 2*c*d*x**2 + d*
*2*m*x**4 - d**2*x**4),x)*a*c*d*m**2 - 2*int(x**m/(c**2*m - c**2 + 2*c*d*m
*x**2 - 2*c*d*x**2 + d**2*m*x**4 - d**2*x**4),x)*a*c*d*m + int(x**m/(c**2*
m - c**2 + 2*c*d*m*x**2 - 2*c*d*x**2 + d**2*m*x**4 - d**2*x**4),x)*a*c*d +
 int(x**m/(c**2*m - c**2 + 2*c*d*m*x**2 - 2*c*d*x**2 + d**2*m*x**4 - d**2*
x**4),x)*a*d**2*m**2*x**2 - 2*int(x**m/(c**2*m - c**2 + 2*c*d*m*x**2 - 2*c
*d*x**2 + d**2*m*x**4 - d**2*x**4),x)*a*d**2*m*x**2 + int(x**m/(c**2*m - c
**2 + 2*c*d*m*x**2 - 2*c*d*x**2 + d**2*m*x**4 - d**2*x**4),x)*a*d**2*x**2
- int(x**m/(c**2*m - c**2 + 2*c*d*m*x**2 - 2*c*d*x**2 + d**2*m*x**4 - d**2
*x**4),x)*b*c**2*m**2 + int(x**m/(c**2*m - c**2 + 2*c*d*m*x**2 - 2*c*d*x**
2 + d**2*m*x**4 - d**2*x**4),x)*b*c**2 - int(x**m/(c**2*m - c**2 + 2*c*d*m
*x**2 - 2*c*d*x**2 + d**2*m*x**4 - d**2*x**4),x)*b*c*d*m**2*x**2 + int(x**
m/(c**2*m - c**2 + 2*c*d*m*x**2 - 2*c*d*x**2 + d**2*m*x**4 - d**2*x**4),x)
*b*c*d*x**2))/(d*(c*m - c + d*m*x**2 - d*x**2))
```

3.34 $\int \frac{(ex)^m(A+Bx^2)}{(a+bx^2)(c+dx^2)^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 205

$$\int \frac{(ex)^m(A+Bx^2)}{(a+bx^2)(c+dx^2)^2}\,dx = \frac{(Bc-Ad)(ex)^{1+m}}{2c(bc-ad)e(c+dx^2)} + \frac{b(Ab-aB)(ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{bx^2}{a}\right)}{a(bc-ad)^2e(1+m)} + \frac{(bc(Bc(1-m)-Ad(3-m))+ad(Ad(1-m)+Bc(1+m)))(ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3}{2}\right.}{2c^2(bc-ad)^2e(1+m)}$$

output
```
1/2*(-A*d+B*c)*(e*x)^(1+m)/c/(-a*d+b*c)/e/(d*x^2+c)+b*(A*b-B*a)*(e*x)^(1+m
)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-b*x^2/a)/a/(-a*d+b*c)^2/e/(1+m)+1/
2*(b*c*(B*c*(1-m)-A*d*(3-m))+a*d*(A*d*(1-m)+B*c*(1+m)))*(e*x)^(1+m)*hyperg
eom([1, 1/2+1/2*m],[3/2+1/2*m],-d*x^2/c)/c^2/(-a*d+b*c)^2/e/(1+m)
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.72

$$\int \frac{(ex)^m (A+Bx^2)}{(a+bx^2)(c+dx^2)^2}\,dx = \frac{x(ex)^m \left(b(Ab-aB)c^2 \,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{bx^2}{a}\right)+a(-Ab+aB)cd\,\text{Hypergeometric2F1}\right.}{ac^2(bc-ad)^2(1+}$$

input
```
Integrate[((e*x)^m*(A + B*x^2))/((a + b*x^2)*(c + d*x^2)^2),x]
```

output
```
(x*(e*x)^m*(b*(A*b - a*B)*c^2*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -
((b*x^2)/a)] + a*(-(A*b) + a*B)*c*d*Hypergeometric2F1[1, (1 + m)/2, (3 + m
)/2, -((d*x^2)/c)] + a*(b*c - a*d)*(B*c - A*d)*Hypergeometric2F1[2, (1 + m
)/2, (3 + m)/2, -((d*x^2)/c)]))/(a*c^2*(b*c - a*d)^2*(1 + m))
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {441, 446, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)(ex)^m}{(a+bx^2)(c+dx^2)^2}\,dx$$

$$\Big\downarrow \; 441$$

$$\frac{\int \frac{(ex)^m(b(Bc-Ad)(1-m)x^2+2Abc-aAd(1-m)-aBc(m+1))}{(bx^2+a)(dx^2+c)}dx}{2c(bc-ad)}+\frac{(ex)^{m+1}(Bc-Ad)}{2ce(c+dx^2)(bc-ad)}$$

$$\Big\downarrow \; 446$$

$$\frac{\int \left(\frac{2b(Ab-aB)c(ex)^m}{(bc-ad)(bx^2+a)}+\frac{(ad(Ad(1-m)+Bc(m+1))-bc(Ad(3-m)-B(c-cm)))(ex)^m}{(bc-ad)(dx^2+c)}\right)dx}{2c(bc-ad)}+\frac{(ex)^{m+1}(Bc-Ad)}{2ce\left(c+dx^2\right)(bc-ad)}$$

$\Big\downarrow$ 2009

$$\frac{\frac{2bc(ex)^{m+1}(Ab-aB)\text{ Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\frac{bx^2}{a}\right)}{ae(m+1)(bc-ad)}+\frac{(ex)^{m+1}\text{ Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\frac{dx^2}{c}\right)(ad(Ad(1-m)+Bc(m+1}{ce(m+1)(bc-ad)}}{2c(bc-ad)}$$

$$\frac{(ex)^{m+1}(Bc-Ad)}{2ce\left(c+dx^2\right)(bc-ad)}$$

input
```
Int[((e*x)^m*(A + B*x^2))/((a + b*x^2)*(c + d*x^2)^2),x]
```

output
```
((B*c - A*d)*(e*x)^(1 + m))/(2*c*(b*c - a*d)*e*(c + d*x^2)) + ((2*b*(A*b -
 a*B)*c*(e*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)
/a)])/(a*(b*c - a*d)*e*(1 + m)) + ((b*c*(B*c*(1 - m) - A*d*(3 - m)) + a*d*
(A*d*(1 - m) + B*c*(1 + m)))*(e*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/2,
 (3 + m)/2, -((d*x^2)/c)])/(c*(b*c - a*d)*e*(1 + m)))/(2*c*(b*c - a*d))
```

Defintions of rubi rules used

rule 441
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
+ b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(a*g*2*(b*c - a*d)*(p + 1))), x] + Si
mp[1/(a*2*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2
)^q*Simp[c*(b*e - a*f)*(m + 1) + e*2*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(m
 + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, q},
 x] && LtQ[p, -1]
```

rule 446
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((e_) + (f_.)*(x_)^2))/(
(c_) + (d_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a + b*x^2)^
p*((e + f*x^2)/(c + d*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{(ex)^m (x^2B + A)}{(b\,x^2 + a)\,(x^2d + c)^2}dx$$

input
```
int((e*x)^m*(B*x^2+A)/(b*x^2+a)/(d*x^2+c)^2,x)
```

output
```
int((e*x)^m*(B*x^2+A)/(b*x^2+a)/(d*x^2+c)^2,x)
```

Fricas [F]

$$\int \frac{(ex)^m (A + Bx^2)}{(a + bx^2)\,(c + dx^2)^2}\,dx = \int \frac{(Bx^2 + A)(ex)^m}{(bx^2 + a)(dx^2 + c)^2}\,dx$$

input
```
integrate((e*x)^m*(B*x^2+A)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
integral((B*x^2 + A)*(e*x)^m/(b*d^2*x^6 + (2*b*c*d + a*d^2)*x^4 + a*c^2 +
(b*c^2 + 2*a*c*d)*x^2), x)
```

Sympy [F]

$$\int \frac{(ex)^m (A + Bx^2)}{(a + bx^2)\,(c + dx^2)^2}\,dx = \int \frac{(ex)^m (A + Bx^2)}{(a + bx^2)\,(c + dx^2)^2}\,dx$$

input
```
integrate((e*x)**m*(B*x**2+A)/(b*x**2+a)/(d*x**2+c)**2,x)
```

output
```
Integral((e*x)**m*(A + B*x**2)/((a + b*x**2)*(c + d*x**2)**2), x)
```

Maxima [F]

$$\int \frac{(ex)^m (A+Bx^2)}{(a+bx^2)(c+dx^2)^2} \, dx = \int \frac{(Bx^2+A)(ex)^m}{(bx^2+a)(dx^2+c)^2} \, dx$$

input
```
integrate((e*x)^m*(B*x^2+A)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*(e*x)^m/((b*x^2 + a)*(d*x^2 + c)^2), x)
```

Giac [F]

$$\int \frac{(ex)^m (A+Bx^2)}{(a+bx^2)(c+dx^2)^2} \, dx = \int \frac{(Bx^2+A)(ex)^m}{(bx^2+a)(dx^2+c)^2} \, dx$$

input
```
integrate((e*x)^m*(B*x^2+A)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*(e*x)^m/((b*x^2 + a)*(d*x^2 + c)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m (A+Bx^2)}{(a+bx^2)(c+dx^2)^2} \, dx = \int \frac{(B\,x^2+A)\,(e\,x)^m}{(b\,x^2+a)\,(d\,x^2+c)^2} \, dx$$

input
```
int(((A + B*x^2)*(e*x)^m)/((a + b*x^2)*(c + d*x^2)^2),x)
```

output
```
int(((A + B*x^2)*(e*x)^m)/((a + b*x^2)*(c + d*x^2)^2), x)
```

Reduce [F]

$$\int \frac{(ex)^m (A+Bx^2)}{(a+bx^2)(c+dx^2)^2}\,dx = e^m\left(\int \frac{x^m}{d^2x^4+2cd\,x^2+c^2}dx\right)$$

input
```
int((e*x)^m*(B*x^2+A)/(b*x^2+a)/(d*x^2+c)^2,x)
```

output
```
e**m*int(x**m/(c**2 + 2*c*d*x**2 + d**2*x**4),x)
```

3.35 $\int \frac{(ex)^m(A+Bx^2)}{(a+bx^2)^2(c+dx^2)^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 304

$$
\begin{aligned}
&\int \frac{(ex)^m(A+Bx^2)}{(a+bx^2)^2(c+dx^2)^2}\,dx \\
&= \frac{d(Abc-2aBc+aAd)(ex)^{1+m}}{2ac(bc-ad)^2e(c+dx^2)} + \frac{(Ab-aB)(ex)^{1+m}}{2a(bc-ad)e(a+bx^2)(c+dx^2)} \\
&\quad + \frac{b(Ab(bc(1-m)-ad(5-m))+aB(ad(3-m)+bc(1+m)))(ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},3\right.}{2a^2(bc-ad)^3e(1+m)} \\
&\quad - \frac{d(bc(Bc(3-m)-Ad(5-m))+ad(Ad(1-m)+Bc(1+m)))(ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\right.}{2c^2(bc-ad)^3e(1+m)}
\end{aligned}
$$

output
```
1/2*d*(A*a*d+A*b*c-2*B*a*c)*(e*x)^(1+m)/a/c/(-a*d+b*c)^2/e/(d*x^2+c)+1/2*(
A*b-B*a)*(e*x)^(1+m)/a/(-a*d+b*c)/e/(b*x^2+a)/(d*x^2+c)+1/2*b*(A*b*(b*c*(1
-m)-a*d*(5-m))+a*B*(a*d*(3-m)+b*c*(1+m)))*(e*x)^(1+m)*hypergeom([1, 1/2+1/
2*m],[3/2+1/2*m],-b*x^2/a)/a^2/(-a*d+b*c)^3/e/(1+m)-1/2*d*(b*c*(B*c*(3-m)-
A*d*(5-m))+a*d*(A*d*(1-m)+B*c*(1+m)))*(e*x)^(1+m)*hypergeom([1, 1/2+1/2*m]
,[3/2+1/2*m],-d*x^2/c)/c^2/(-a*d+b*c)^3/e/(1+m)
```

Mathematica [A] (verified)

Time = 0.88 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.68

$$\int \frac{(ex)^m (A+Bx^2)}{(a+bx^2)^2 (c+dx^2)^2} \, dx$$
$$= \frac{x(ex)^m \left(-abc^2(bBc-2Abd+aBd) \text{Hypergeometric2F1}\left(1, \frac{1+m}{2}, \frac{3+m}{2}, -\frac{bx^2}{a}\right) + a^2cd(bBc-2Abd+aB\right.}{}$$

input
```
Integrate[((e*x)^m*(A + B*x^2))/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

output
```
(x*(e*x)^m*(-(a*b*c^2*(b*B*c - 2*A*b*d + a*B*d)*Hypergeometric2F1[1, (1 +
m)/2, (3 + m)/2, -((b*x^2)/a)]) + a^2*c*d*(b*B*c - 2*A*b*d + a*B*d)*Hyperg
eometric2F1[1, (1 + m)/2, (3 + m)/2, -((d*x^2)/c)] - (b*c - a*d)*(b*(A*b -
 a*B)*c^2*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)] + a^2*d
*(-(B*c) + A*d)*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, -((d*x^2)/c)]))
)/(a^2*c^2*(-(b*c) + a*d)^3*(1 + m))
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {441, 441, 27, 446, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)(ex)^m}{(a+bx^2)^2 (c+dx^2)^2} \, dx$$

$$\Big\downarrow \text{ 441}$$

$$\frac{(ex)^{m+1}(Ab-aB)}{2ae(a+bx^2)(c+dx^2)(bc-ad)} - \frac{\int \frac{(ex)^m(-((Ab-aB)d(3-m)x^2)+2aAd-Abc(1-m)-aBc(m+1))}{(bx^2+a)(dx^2+c)^2}dx}{2a(bc-ad)}$$

$$\Big\downarrow \text{ 441}$$

$$\frac{(ex)^{m+1}(Ab-aB)}{2ae\left(a+bx^2\right)\left(c+dx^2\right)(bc-ad)}-\frac{\frac{\int\frac{2(ex)^m\left(-bd(Abc-2aBc+aAd)(1-m)x^2+A\left(-b^2(1-m)c^2+4abdc-a^2d^2(1-m)\right)-aBc(bc+ad)(m+1)\right)}{\left(bx^2+a\right)\left(dx^2+c\right)}dx}{2c(bc-ad)}-\frac{d(ex)^{m+1}(aAd-2aBc+Abc)}{ce(c+dx^2)(bc-ad)}}{2a(bc-ad)}$$

$\Big\downarrow$ 27

$$\frac{(ex)^{m+1}(Ab-aB)}{2ae\left(a+bx^2\right)\left(c+dx^2\right)(bc-ad)}-\frac{\frac{\int\frac{(ex)^m\left(-bd(Abc-2aBc+aAd)(1-m)x^2+A\left(-b^2(1-m)c^2+4abdc-a^2d^2(1-m)\right)-aBc(bc+ad)(m+1)\right)}{\left(bx^2+a\right)\left(dx^2+c\right)}dx}{c(bc-ad)}-\frac{d(ex)^{m+1}(aAd-2aBc+Abc)}{ce(c+dx^2)(bc-ad)}}{2a(bc-ad)}$$

$\Big\downarrow$ 446

$$\frac{(ex)^{m+1}(Ab-aB)}{2ae\left(a+bx^2\right)\left(c+dx^2\right)(bc-ad)}-\frac{\frac{\int\left(\frac{bc(-Ab(bc(1-m)-ad(5-m))-aB(ad(3-m)+bc(m+1)))(ex)^m}{(bc-ad)\left(bx^2+a\right)}+\frac{ad(bc(Bc(3-m)-Ad(5-m))+ad(Ad(1-m)+Bc(m+1)))(ex)^m}{(bc-ad)\left(dx^2+c\right)}\right)dx}{c(bc-ad)}-\frac{d(ex)^{m+1}(aAd-}{ce(c+dx^2)(}}{2a(bc-ad)}$$

$\Big\downarrow$ 2009

$$\frac{(ex)^{m+1}(Ab-aB)}{2ae\left(a+bx^2\right)\left(c+dx^2\right)(bc-ad)}-\frac{\frac{\frac{ad(ex)^{m+1}\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\frac{dx^2}{c}\right)(ad(Ad(1-m)+Bc(m+1))+bc(Bc(3-m)-Ad(5-m)))}{ce(m+1)(bc-ad)}-\frac{bc(ex)^{m+1}\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\right.}{ae(}}{c(bc-ad)}}{2a(bc-ad)}$$

input

```
Int[((e*x)^m*(A + B*x^2))/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

output

```
((A*b - a*B)*(e*x)^(1 + m))/(2*a*(b*c - a*d)*e*(a + b*x^2)*(c + d*x^2)) -
(-((d*(A*b*c - 2*a*B*c + a*A*d)*(e*x)^(1 + m))/(c*(b*c - a*d)*e*(c + d*x^2
))) + (-((b*c*(A*b*(b*c*(1 - m) - a*d*(5 - m)) + a*B*(a*d*(3 - m) + b*c*(1
 + m)))*(e*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)
/a)])/(a*(b*c - a*d)*e*(1 + m))) + (a*d*(b*c*(B*c*(3 - m) - A*d*(5 - m)) +
 a*d*(A*d*(1 - m) + B*c*(1 + m)))*(e*x)^(1 + m)*Hypergeometric2F1[1, (1 +
m)/2, (3 + m)/2, -((d*x^2)/c)])/(c*(b*c - a*d)*e*(1 + m)))/(c*(b*c - a*d))
)/(2*a*(b*c - a*d))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 441
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
+ b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(a*g*2*(b*c - a*d)*(p + 1))), x] + Si
mp[1/(a*2*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2
)^q*Simp[c*(b*e - a*f)*(m + 1) + e*2*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(m
 + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, q},
 x] && LtQ[p, -1]
```

rule 446
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((e_) + (f_.)*(x_)^2))/(
(c_) + (d_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a + b*x^2)^
p*((e + f*x^2)/(c + d*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{(ex)^m (x^2B + A)}{(b\,x^2 + a)^2 (x^2d + c)^2}dx$$

input
```
int((e*x)^m*(B*x^2+A)/(b*x^2+a)^2/(d*x^2+c)^2,x)
```

output
```
int((e*x)^m*(B*x^2+A)/(b*x^2+a)^2/(d*x^2+c)^2,x)
```

Fricas [F]

$$\int \frac{(ex)^m (A + Bx^2)}{(a + bx^2)^2 (c + dx^2)^2} dx = \int \frac{(Bx^2 + A)(ex)^m}{(bx^2 + a)^2 (dx^2 + c)^2} dx$$

input
```
integrate((e*x)^m*(B*x^2+A)/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
integral((B*x^2 + A)*(e*x)^m/(b^2*d^2*x^8 + 2*(b^2*c*d + a*b*d^2)*x^6 + (b
^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(a*b*c^2 + a^2*c*d)*x^2),
x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ex)^m (A + Bx^2)}{(a + bx^2)^2 (c + dx^2)^2} dx = \text{Timed out}$$

input
```
integrate((e*x)**m*(B*x**2+A)/(b*x**2+a)**2/(d*x**2+c)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(ex)^m (A + Bx^2)}{(a + bx^2)^2 (c + dx^2)^2} dx = \int \frac{(Bx^2 + A)(ex)^m}{(bx^2 + a)^2 (dx^2 + c)^2} dx$$

input
```
integrate((e*x)^m*(B*x^2+A)/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*(e*x)^m/((b*x^2 + a)^2*(d*x^2 + c)^2), x)
```

Giac [F]

$$\int \frac{(ex)^m (A + Bx^2)}{(a + bx^2)^2 (c + dx^2)^2} \, dx = \int \frac{(Bx^2 + A)(ex)^m}{(bx^2 + a)^2 (dx^2 + c)^2} \, dx$$

input
```
integrate((e*x)^m*(B*x^2+A)/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*(e*x)^m/((b*x^2 + a)^2*(d*x^2 + c)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m (A + Bx^2)}{(a + bx^2)^2 (c + dx^2)^2} \, dx = \int \frac{(B\,x^2 + A)\,(e\,x)^m}{(b\,x^2 + a)^2\,(d\,x^2 + c)^2} \, dx$$

input
```
int(((A + B*x^2)*(e*x)^m)/((a + b*x^2)^2*(c + d*x^2)^2),x)
```

output
```
int(((A + B*x^2)*(e*x)^m)/((a + b*x^2)^2*(c + d*x^2)^2), x)
```

Reduce [F]

$$\int \frac{(ex)^m (A + Bx^2)}{(a + bx^2)^2 (c + dx^2)^2} \, dx = e^m \left(\int \frac{x^m}{b\,d^2x^6 + a\,d^2x^4 + 2bcd\,x^4 + 2acd\,x^2 + b\,c^2x^2 + a\,c^2} dx \right)$$

input
```
int((e*x)^m*(B*x^2+A)/(b*x^2+a)^2/(d*x^2+c)^2,x)
```

output
```
e**m*int(x**m/(a*c**2 + 2*a*c*d*x**2 + a*d**2*x**4 + b*c**2*x**2 + 2*b*c*d
*x**4 + b*d**2*x**6),x)
```

3.36 $\int \frac{(ex)^m(A+Bx^2)}{(a+bx^2)^3(c+dx^2)^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 491

$$\int \frac{(ex)^m(A+Bx^2)}{(a+bx^2)^3(c+dx^2)^2}\,dx =$$

$$-\frac{d(A(4a^2d^2-b^2c^2(3-m)+abcd(11-m))-aBc(ad(11-m)+bc(1+m)))(ex)^{1+m}}{8a^2c(bc-ad)^3e(c+dx^2)}$$

$$+\frac{(Ab-aB)(ex)^{1+m}}{4a(bc-ad)e(a+bx^2)^2(c+dx^2)}$$

$$+\frac{(Ab(bc(3-m)-ad(9-m))+aB(ad(5-m)+bc(1+m)))(ex)^{1+m}}{8a^2(bc-ad)^2e(a+bx^2)(c+dx^2)}$$

$$+\frac{b(aB(b^2c^2(1-m^2)-2abcd(5+4m-m^2)-a^2d^2(15-8m+m^2))+Ab(a^2d^2(35-12m+m^2)-2a\ldots}{8a^3(bc-ad)^4e(\ldots}$$

$$+\frac{d^2(bc(Bc(5-m)-Ad(7-m))+ad(Ad(1-m)+Bc(1+m)))(ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2}\ldots\right.}{2c^2(bc-ad)^4e(1+m)}$$

output
```
-1/8*d*(A*(4*a^2*d^2-b^2*c^2*(3-m)+a*b*c*d*(11-m))-a*B*c*(a*d*(11-m)+b*c*(
1+m)))*(e*x)^(1+m)/a^2/c/(-a*d+b*c)^3/e/(d*x^2+c)+1/4*(A*b-B*a)*(e*x)^(1+m
)/a/(-a*d+b*c)/e/(b*x^2+a)^2/(d*x^2+c)+1/8*(A*b*(b*c*(3-m)-a*d*(9-m))+a*B*
(a*d*(5-m)+b*c*(1+m)))*(e*x)^(1+m)/a^2/(-a*d+b*c)^2/e/(b*x^2+a)/(d*x^2+c)+
1/8*b*(a*B*(b^2*c^2*(-m^2+1)-2*a*b*c*d*(-m^2+4*m+5)-a^2*d^2*(m^2-8*m+15))+
A*b*(a^2*d^2*(m^2-12*m+35)-2*a*b*c*d*(m^2-8*m+7)+b^2*c^2*(m^2-4*m+3)))*(e*
x)^(1+m)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-b*x^2/a)/a^3/(-a*d+b*c)^4/e
/(1+m)+1/2*d^2*(b*c*(B*c*(5-m)-A*d*(7-m))+a*d*(A*d*(1-m)+B*c*(1+m)))*(e*x)
^(1+m)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-d*x^2/c)/c^2/(-a*d+b*c)^4/e/(
1+m)
```

Mathematica [A] (verified)

Time = 1.13 (sec) , antiderivative size = 265, normalized size of antiderivative = 0.54

$$\int \frac{(ex)^m (A+Bx^2)}{(a+bx^2)^3 (c+dx^2)^2} dx = \frac{x(ex)^m \left(-\frac{bd(2bBc-3Abd+aBd) \text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{bx^2}{a}\right)}{a} + \frac{d^2(2bBc-3Abd+aBd) \text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},\right.}{c}\right.}{}$$

input
```
Integrate[((e*x)^m*(A + B*x^2))/((a + b*x^2)^3*(c + d*x^2)^2),x]
```

output
```
(x*(e*x)^m*(-((b*d*(2*b*B*c - 3*A*b*d + a*B*d)*Hypergeometric2F1[1, (1 + m
)/2, (3 + m)/2, -((b*x^2)/a)])/a) + (d^2*(2*b*B*c - 3*A*b*d + a*B*d)*Hyper
geometric2F1[1, (1 + m)/2, (3 + m)/2, -((d*x^2)/c)])/c + (b*(b*c - a*d)*(b
*B*c - 2*A*b*d + a*B*d)*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, -((b*x^
2)/a)])/a^2 + (d^2*(b*c - a*d)*(B*c - A*d)*Hypergeometric2F1[2, (1 + m)/2,
 (3 + m)/2, -((d*x^2)/c)])/c^2 + (b*(A*b - a*B)*(b*c - a*d)^2*Hypergeometr
ic2F1[3, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/a^3))/((b*c - a*d)^4*(1 + m)
)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)(ex)^m}{(a+bx^2)^3(c+dx^2)^2}dx$$

$$\Big\downarrow \ 441$$

$$\frac{(ex)^{m+1}(Ab-aB)}{4ae(a+bx^2)^2(c+dx^2)(bc-ad)}-\frac{\int \frac{(ex)^m(-((Ab-aB)d(5-m)x^2)+4aAd-Abc(3-m)-aBc(m+1))}{(bx^2+a)^2(dx^2+c)^2}dx}{4a(bc-ad)}$$

$$\Big\downarrow \ 441$$

$$\frac{(ex)^{m+1}(Ab-aB)}{4ae(a+bx^2)^2(c+dx^2)(bc-ad)}-$$
$$-\frac{\int -\frac{(ex)^m (-d(3-m)(Ab(bc(3-m)-ad(9-m))+aB(ad(5-m)+bc(m+1)))x^2+aBc(m+1)(ad(7-m)-b(c-cm))-A(b^2(m^2-4m+3)c^2-abd(m^2-10m+5)c+8}{(bx^2+a)(dx^2+c)^2}}{2a(bc-ad)}$$
$$4a(bc-ad)$$

$$\Big\downarrow \ 25$$

$$\frac{(ex)^{m+1}(Ab-aB)}{4ae(a+bx^2)^2(c+dx^2)(bc-ad)}-$$
$$\frac{\int -\frac{(ex)^m (d(3-m)(Ab(bc(3-m)-ad(9-m))+aB(ad(5-m)+bc(m+1)))x^2+aBc(bc(1-m)-ad(7-m))(m+1)+A(b^2(m^2-4m+3)c^2-abd(m^2-10m+5)c+8a^2d^2}{(bx^2+a)(dx^2+c)^2}}{2a(bc-ad)}$$
$$4a(bc-ad)$$

$$\Big\downarrow \ 25$$

$$\frac{(ex)^{m+1}(Ab-aB)}{4ae(a+bx^2)^2(c+dx^2)(bc-ad)}-$$
$$-\frac{\int -\frac{(ex)^m (-d(3-m)(Ab(bc(3-m)-ad(9-m))+aB(ad(5-m)+bc(m+1)))x^2+aBc(m+1)(ad(7-m)-b(c-cm))-A(b^2(m^2-4m+3)c^2-abd(m^2-10m+5)c+8}{(bx^2+a)(dx^2+c)^2}}{2a(bc-ad)}$$
$$4a(bc-ad)$$

$$\Big\downarrow \ 25$$

$$\frac{(ex)^{m+1}(Ab-aB)}{4ae(a+bx^2)^2(c+dx^2)(bc-ad)}-$$
$$\frac{\int -\frac{(ex)^m (d(3-m)(Ab(bc(3-m)-ad(9-m))+aB(ad(5-m)+bc(m+1)))x^2+aBc(bc(1-m)-ad(7-m))(m+1)+A(b^2(m^2-4m+3)c^2-abd(m^2-10m+5)c+8a^2d^2}{(bx^2+a)(dx^2+c)^2}}{2a(bc-ad)}$$
$$4a(bc-ad)$$

$\Big\downarrow$ 25

$$\frac{(ex)^{m+1}(Ab-aB)}{4ae\left(a+bx^2\right)^2\left(c+dx^2\right)(bc-ad)}-\frac{-\frac{\int -\frac{(ex)^m\left(-d(3-m)(Ab(bc(3-m)-ad(9-m))+aB(ad(5-m)+bc(m+1)))x^2+aBc(m+1)(ad(7-m)-b(c-cm))-A\left(b^2\left(m^2-4m+3\right)c^2-abd\left(m^2-10m+5\right)c+8\cdots\right.\right.}{\left(bx^2+a\right)\left(dx^2+c\right)^2}}{2a(bc-ad)}\cdots}{4a(bc-ad)}$$

$\Big\downarrow$ 25

$$\frac{(ex)^{m+1}(Ab-aB)}{4ae\left(a+bx^2\right)^2\left(c+dx^2\right)(bc-ad)}-\frac{\frac{\int -\frac{(ex)^m\left(d(3-m)(Ab(bc(3-m)-ad(9-m))+aB(ad(5-m)+bc(m+1)))x^2+aBc(bc(1-m)-ad(7-m))(m+1)+A\left(b^2\left(m^2-4m+3\right)c^2-abd\left(m^2-10m+5\right)c+8a^2d^2\cdots\right.\right.}{\left(bx^2+a\right)\left(dx^2+c\right)^2}}{2a(bc-ad)}\cdots}{4a(bc-ad)}$$

$\Big\downarrow$ 25

$$\frac{(ex)^{m+1}(Ab-aB)}{4ae\left(a+bx^2\right)^2\left(c+dx^2\right)(bc-ad)}-\frac{-\frac{\int -\frac{(ex)^m\left(-d(3-m)(Ab(bc(3-m)-ad(9-m))+aB(ad(5-m)+bc(m+1)))x^2+aBc(m+1)(ad(7-m)-b(c-cm))-A\left(b^2\left(m^2-4m+3\right)c^2-abd\left(m^2-10m+5\right)c+8\cdots\right.\right.}{\left(bx^2+a\right)\left(dx^2+c\right)^2}}{2a(bc-ad)}\cdots}{4a(bc-ad)}$$

$\Big\downarrow$ 25

$$\frac{(ex)^{m+1}(Ab-aB)}{4ae\left(a+bx^2\right)^2\left(c+dx^2\right)(bc-ad)}-\frac{\frac{\int -\frac{(ex)^m\left(d(3-m)(Ab(bc(3-m)-ad(9-m))+aB(ad(5-m)+bc(m+1)))x^2+aBc(bc(1-m)-ad(7-m))(m+1)+A\left(b^2\left(m^2-4m+3\right)c^2-abd\left(m^2-10m+5\right)c+8a^2d^2\cdots\right.\right.}{\left(bx^2+a\right)\left(dx^2+c\right)^2}}{2a(bc-ad)}\cdots}{4a(bc-ad)}$$

$\Big\downarrow$ 25

$$\frac{(ex)^{m+1}(Ab-aB)}{4ae\left(a+bx^2\right)^2\left(c+dx^2\right)(bc-ad)}-\frac{-\frac{\int -\frac{(ex)^m\left(-d(3-m)(Ab(bc(3-m)-ad(9-m))+aB(ad(5-m)+bc(m+1)))x^2+aBc(m+1)(ad(7-m)-b(c-cm))-A\left(b^2\left(m^2-4m+3\right)c^2-abd\left(m^2-10m+5\right)c+8\cdots\right.\right.}{\left(bx^2+a\right)\left(dx^2+c\right)^2}}{2a(bc-ad)}\cdots}{4a(bc-ad)}$$

$\Big\downarrow$ 25

$$\frac{(ex)^{m+1}(Ab-aB)}{4ae\left(a+bx^2\right)^2\left(c+dx^2\right)(bc-ad)}-\frac{\frac{\int -\frac{(ex)^m\left(d(3-m)(Ab(bc(3-m)-ad(9-m))+aB(ad(5-m)+bc(m+1)))x^2+aBc(bc(1-m)-ad(7-m))(m+1)+A\left(b^2\left(m^2-4m+3\right)c^2-abd\left(m^2-10m+5\right)c+8a^2d^2\right.\right.}{\left(bx^2+a\right)\left(dx^2+c\right)^2}}{2a(bc-ad)}}{4a(bc-ad)}$$

$$\Big\downarrow \text{25}$$

$$\frac{(ex)^{m+1}(Ab-aB)}{4ae\left(a+bx^2\right)^2\left(c+dx^2\right)(bc-ad)}-\frac{-\frac{\int -\frac{(ex)^m\left(-d(3-m)(Ab(bc(3-m)-ad(9-m))+aB(ad(5-m)+bc(m+1)))x^2+aBc(m+1)(ad(7-m)-b(c-cm))-A\left(b^2\left(m^2-4m+3\right)c^2-abd\left(m^2-10m+5\right)c+8\right.\right.}{\left(bx^2+a\right)\left(dx^2+c\right)^2}}{2a(bc-ad)}}{4a(bc-ad)}$$

$$\Big\downarrow \text{25}$$

$$\frac{(ex)^{m+1}(Ab-aB)}{4ae\left(a+bx^2\right)^2\left(c+dx^2\right)(bc-ad)}-\frac{\frac{\int -\frac{(ex)^m\left(d(3-m)(Ab(bc(3-m)-ad(9-m))+aB(ad(5-m)+bc(m+1)))x^2+aBc(bc(1-m)-ad(7-m))(m+1)+A\left(b^2\left(m^2-4m+3\right)c^2-abd\left(m^2-10m+5\right)c+8a^2d^2\right.\right.}{\left(bx^2+a\right)\left(dx^2+c\right)^2}}{2a(bc-ad)}}{4a(bc-ad)}$$

$$\Big\downarrow \text{25}$$

$$\frac{(ex)^{m+1}(Ab-aB)}{4ae\left(a+bx^2\right)^2\left(c+dx^2\right)(bc-ad)}-\frac{-\frac{\int -\frac{(ex)^m\left(-d(3-m)(Ab(bc(3-m)-ad(9-m))+aB(ad(5-m)+bc(m+1)))x^2+aBc(m+1)(ad(7-m)-b(c-cm))-A\left(b^2\left(m^2-4m+3\right)c^2-abd\left(m^2-10m+5\right)c+8\right.\right.}{\left(bx^2+a\right)\left(dx^2+c\right)^2}}{2a(bc-ad)}}{4a(bc-ad)}$$

$$\Big\downarrow \text{25}$$

$$\frac{(ex)^{m+1}(Ab-aB)}{4ae\left(a+bx^2\right)^2\left(c+dx^2\right)(bc-ad)}-\frac{\frac{\int -\frac{(ex)^m\left(d(3-m)(Ab(bc(3-m)-ad(9-m))+aB(ad(5-m)+bc(m+1)))x^2+aBc(bc(1-m)-ad(7-m))(m+1)+A\left(b^2\left(m^2-4m+3\right)c^2-abd\left(m^2-10m+5\right)c+8a^2d^2\right.\right.}{\left(bx^2+a\right)\left(dx^2+c\right)^2}}{2a(bc-ad)}}{4a(bc-ad)}$$

$$\Big\downarrow \text{25}$$

$$\frac{(ex)^{m+1}(Ab-aB)}{4ae\left(a+bx^2\right)^2\left(c+dx^2\right)(bc-ad)}-\frac{-\frac{\int -\frac{(ex)^m\left(-d(3-m)(Ab(bc(3-m)-ad(9-m))+aB(ad(5-m)+bc(m+1)))x^2+aBc(m+1)(ad(7-m)-b(c-cm))-A\left(b^2\left(m^2-4m+3\right)c^2-abd\left(m^2-10m+5\right)c+8\right.\right.}{\left(bx^2+a\right)\left(dx^2+c\right)^2}}{2a(bc-ad)}}{4a(bc-ad)}$$

$$\Big\downarrow \; 25$$

$$\frac{(ex)^{m+1}(Ab-aB)}{4ae\left(a+bx^2\right)^2\left(c+dx^2\right)(bc-ad)}-\frac{\frac{\int -\frac{(ex)^m\left(d(3-m)(Ab(bc(3-m)-ad(9-m))+aB(ad(5-m)+bc(m+1)))x^2+aBc(bc(1-m)-ad(7-m))(m+1)+A\left(b^2\left(m^2-4m+3\right)c^2-abd\left(m^2-10m+5\right)c+8a^2d^2\right.\right.}{\left(bx^2+a\right)\left(dx^2+c\right)^2}}{2a(bc-ad)}}{4a(bc-ad)}$$

$$\Big\downarrow \; 25$$

$$\frac{(ex)^{m+1}(Ab-aB)}{4ae\left(a+bx^2\right)^2\left(c+dx^2\right)(bc-ad)}-\frac{-\frac{\int -\frac{(ex)^m\left(-d(3-m)(Ab(bc(3-m)-ad(9-m))+aB(ad(5-m)+bc(m+1)))x^2+aBc(m+1)(ad(7-m)-b(c-cm))-A\left(b^2\left(m^2-4m+3\right)c^2-abd\left(m^2-10m+5\right)c+8\right.\right.}{\left(bx^2+a\right)\left(dx^2+c\right)^2}}{2a(bc-ad)}}{4a(bc-ad)}$$

$$\Big\downarrow \; 25$$

$$\frac{(ex)^{m+1}(Ab-aB)}{4ae\left(a+bx^2\right)^2\left(c+dx^2\right)(bc-ad)}-\frac{\frac{\int -\frac{(ex)^m\left(d(3-m)(Ab(bc(3-m)-ad(9-m))+aB(ad(5-m)+bc(m+1)))x^2+aBc(bc(1-m)-ad(7-m))(m+1)+A\left(b^2\left(m^2-4m+3\right)c^2-abd\left(m^2-10m+5\right)c+8a^2d^2\right.\right.}{\left(bx^2+a\right)\left(dx^2+c\right)^2}}{2a(bc-ad)}}{4a(bc-ad)}$$

$$\Big\downarrow \; 25$$

$$\frac{(ex)^{m+1}(Ab-aB)}{4ae\left(a+bx^2\right)^2\left(c+dx^2\right)(bc-ad)}-\frac{-\frac{\int -\frac{(ex)^m\left(-d(3-m)(Ab(bc(3-m)-ad(9-m))+aB(ad(5-m)+bc(m+1)))x^2+aBc(m+1)(ad(7-m)-b(c-cm))-A\left(b^2\left(m^2-4m+3\right)c^2-abd\left(m^2-10m+5\right)c+8\right.\right.}{\left(bx^2+a\right)\left(dx^2+c\right)^2}}{2a(bc-ad)}}{4a(bc-ad)}$$

$$\Big\downarrow \; 25$$

$$\frac{(ex)^{m+1}(Ab-aB)}{4ae\left(a+bx^2\right)^2\left(c+dx^2\right)(bc-ad)}-\frac{\frac{\int -\frac{(ex)^m\left(d(3-m)(Ab(bc(3-m)-ad(9-m))+aB(ad(5-m)+bc(m+1)))x^2+aBc(bc(1-m)-ad(7-m))(m+1)+A\left(b^2\left(m^2-4m+3\right)c^2-abd\left(m^2-10m+5\right)c+8a^2d^2\cdots\right.\right.}{\left(bx^2+a\right)\left(dx^2+c\right)^2}}{2a(bc-ad)}}{4a(bc-ad)}$$

$$\Big\downarrow \; 25$$

$$\frac{(ex)^{m+1}(Ab-aB)}{4ae\left(a+bx^2\right)^2\left(c+dx^2\right)(bc-ad)}-\frac{-\frac{\int -\frac{(ex)^m\left(-d(3-m)(Ab(bc(3-m)-ad(9-m))+aB(ad(5-m)+bc(m+1)))x^2+aBc(m+1)(ad(7-m)-b(c-cm))-A\left(b^2\left(m^2-4m+3\right)c^2-abd\left(m^2-10m+5\right)c+8\cdots\right.\right.}{\left(bx^2+a\right)\left(dx^2+c\right)^2}}{2a(bc-ad)}}{4a(bc-ad)}$$

$$\Big\downarrow \; 25$$

$$\frac{(ex)^{m+1}(Ab-aB)}{4ae\left(a+bx^2\right)^2\left(c+dx^2\right)(bc-ad)}-\frac{\frac{\int -\frac{(ex)^m\left(d(3-m)(Ab(bc(3-m)-ad(9-m))+aB(ad(5-m)+bc(m+1)))x^2+aBc(bc(1-m)-ad(7-m))(m+1)+A\left(b^2\left(m^2-4m+3\right)c^2-abd\left(m^2-10m+5\right)c+8a^2d^2\cdots\right.\right.}{\left(bx^2+a\right)\left(dx^2+c\right)^2}}{2a(bc-ad)}}{4a(bc-ad)}$$

$$\Big\downarrow \; 25$$

$$\frac{(ex)^{m+1}(Ab-aB)}{4ae\left(a+bx^2\right)^2\left(c+dx^2\right)(bc-ad)}-\frac{-\frac{\int -\frac{(ex)^m\left(-d(3-m)(Ab(bc(3-m)-ad(9-m))+aB(ad(5-m)+bc(m+1)))x^2+aBc(m+1)(ad(7-m)-b(c-cm))-A\left(b^2\left(m^2-4m+3\right)c^2-abd\left(m^2-10m+5\right)c+8\cdots\right.\right.}{\left(bx^2+a\right)\left(dx^2+c\right)^2}}{2a(bc-ad)}}{4a(bc-ad)}$$

$$\Big\downarrow \; 25$$

$$\frac{(ex)^{m+1}(Ab-aB)}{4ae\left(a+bx^2\right)^2\left(c+dx^2\right)(bc-ad)}-\frac{\frac{\int -\frac{(ex)^m\left(d(3-m)(Ab(bc(3-m)-ad(9-m))+aB(ad(5-m)+bc(m+1)))x^2+aBc(bc(1-m)-ad(7-m))(m+1)+A\left(b^2\left(m^2-4m+3\right)c^2-abd\left(m^2-10m+5\right)c+8a^2d^2\cdots\right.\right.}{\left(bx^2+a\right)\left(dx^2+c\right)^2}}{2a(bc-ad)}}{4a(bc-ad)}$$

$$\Big\downarrow \; 25$$

$$\frac{(ex)^{m+1}(Ab-aB)}{4ae\left(a+bx^2\right)^2\left(c+dx^2\right)(bc-ad)}-\frac{-\frac{\int -\frac{(ex)^m\left(-d(3-m)(Ab(bc(3-m)-ad(9-m))+aB(ad(5-m)+bc(m+1)))x^2+aBc(m+1)(ad(7-m)-b(c-cm))-A\left(b^2\left(m^2-4m+3\right)c^2-abd\left(m^2-10m+5\right)c+8\right.\right.}{\left(bx^2+a\right)\left(dx^2+c\right)^2}}{2a(bc-ad)}}{4a(bc-ad)}$$

$\Big\downarrow$ 25

$$\frac{(ex)^{m+1}(Ab-aB)}{4ae\left(a+bx^2\right)^2\left(c+dx^2\right)(bc-ad)}-\frac{\frac{\int -\frac{(ex)^m\left(d(3-m)(Ab(bc(3-m)-ad(9-m))+aB(ad(5-m)+bc(m+1)))x^2+aBc(bc(1-m)-ad(7-m))(m+1)+A\left(b^2\left(m^2-4m+3\right)c^2-abd\left(m^2-10m+5\right)c+8a^2d^2\right.\right.}{\left(bx^2+a\right)\left(dx^2+c\right)^2}}{2a(bc-ad)}}{4a(bc-ad)}$$

$\Big\downarrow$ 25

$$\frac{(ex)^{m+1}(Ab-aB)}{4ae\left(a+bx^2\right)^2\left(c+dx^2\right)(bc-ad)}-\frac{-\frac{\int -\frac{(ex)^m\left(-d(3-m)(Ab(bc(3-m)-ad(9-m))+aB(ad(5-m)+bc(m+1)))x^2+aBc(m+1)(ad(7-m)-b(c-cm))-A\left(b^2\left(m^2-4m+3\right)c^2-abd\left(m^2-10m+5\right)c+8\right.\right.}{\left(bx^2+a\right)\left(dx^2+c\right)^2}}{2a(bc-ad)}}{4a(bc-ad)}$$

$\Big\downarrow$ 25

$$\frac{(ex)^{m+1}(Ab-aB)}{4ae\left(a+bx^2\right)^2\left(c+dx^2\right)(bc-ad)}-\frac{\frac{\int -\frac{(ex)^m\left(d(3-m)(Ab(bc(3-m)-ad(9-m))+aB(ad(5-m)+bc(m+1)))x^2+aBc(bc(1-m)-ad(7-m))(m+1)+A\left(b^2\left(m^2-4m+3\right)c^2-abd\left(m^2-10m+5\right)c+8a^2d^2\right.\right.}{\left(bx^2+a\right)\left(dx^2+c\right)^2}}{2a(bc-ad)}}{4a(bc-ad)}$$

$\Big\downarrow$ 25

$$\frac{(ex)^{m+1}(Ab-aB)}{4ae\left(a+bx^2\right)^2\left(c+dx^2\right)(bc-ad)}-\frac{-\frac{\int -\frac{(ex)^m\left(-d(3-m)(Ab(bc(3-m)-ad(9-m))+aB(ad(5-m)+bc(m+1)))x^2+aBc(m+1)(ad(7-m)-b(c-cm))-A\left(b^2\left(m^2-4m+3\right)c^2-abd\left(m^2-10m+5\right)c+8\right.\right.}{\left(bx^2+a\right)\left(dx^2+c\right)^2}}{2a(bc-ad)}}{4a(bc-ad)}$$

input
```
Int[((e*x)^m*(A + B*x^2))/((a + b*x^2)^3*(c + d*x^2)^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 441
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
+ b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(a*g*2*(b*c - a*d)*(p + 1))), x] + Si
mp[1/(a*2*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2
)^q*Simp[c*(b*e - a*f)*(m + 1) + e*2*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(m
 + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, q},
 x] && LtQ[p, -1]
```

Maple [F]

$$\int \frac{(ex)^m (x^2B + A)}{(b\,x^2 + a)^3 (x^2d + c)^2} dx$$

input
```
int((e*x)^m*(B*x^2+A)/(b*x^2+a)^3/(d*x^2+c)^2,x)
```

output
```
int((e*x)^m*(B*x^2+A)/(b*x^2+a)^3/(d*x^2+c)^2,x)
```

Fricas [F]

$$\int \frac{(ex)^m (A + Bx^2)}{(a + bx^2)^3 (c + dx^2)^2} \, dx = \int \frac{(Bx^2 + A)(ex)^m}{(bx^2 + a)^3 (dx^2 + c)^2} \, dx$$

input
```
integrate((e*x)^m*(B*x^2+A)/(b*x^2+a)^3/(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
integral((B*x^2 + A)*(e*x)^m/(b^3*d^2*x^10 + (2*b^3*c*d + 3*a*b^2*d^2)*x^8
 + (b^3*c^2 + 6*a*b^2*c*d + 3*a^2*b*d^2)*x^6 + a^3*c^2 + (3*a*b^2*c^2 + 6*
a^2*b*c*d + a^3*d^2)*x^4 + (3*a^2*b*c^2 + 2*a^3*c*d)*x^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ex)^m (A+Bx^2)}{(a+bx^2)^3 (c+dx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((e*x)**m*(B*x**2+A)/(b*x**2+a)**3/(d*x**2+c)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(ex)^m (A+Bx^2)}{(a+bx^2)^3 (c+dx^2)^2}\,dx = \int \frac{(Bx^2+A)(ex)^m}{(bx^2+a)^3(dx^2+c)^2}\,dx$$

input
```
integrate((e*x)^m*(B*x^2+A)/(b*x^2+a)^3/(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*(e*x)^m/((b*x^2 + a)^3*(d*x^2 + c)^2), x)
```

Giac [F]

$$\int \frac{(ex)^m (A+Bx^2)}{(a+bx^2)^3 (c+dx^2)^2}\,dx = \int \frac{(Bx^2+A)(ex)^m}{(bx^2+a)^3(dx^2+c)^2}\,dx$$

input
```
integrate((e*x)^m*(B*x^2+A)/(b*x^2+a)^3/(d*x^2+c)^2,x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*(e*x)^m/((b*x^2 + a)^3*(d*x^2 + c)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m (A+Bx^2)}{(a+bx^2)^3 (c+dx^2)^2}\,dx = \int \frac{(B\,x^2+A)\,(e\,x)^m}{(b\,x^2+a)^3\,(d\,x^2+c)^2}\,dx$$

input
```
int(((A + B*x^2)*(e*x)^m)/((a + b*x^2)^3*(c + d*x^2)^2),x)
```

output
```
int(((A + B*x^2)*(e*x)^m)/((a + b*x^2)^3*(c + d*x^2)^2), x)
```

Reduce [F]

$$\int \frac{(ex)^m (A+Bx^2)}{(a+bx^2)^3 (c+dx^2)^2}\,dx$$
$$= e^m \left(\int \frac{x^m}{b^2d^2x^8+2ab\,d^2x^6+2b^2cd\,x^6+a^2d^2x^4+4abcd\,x^4+b^2c^2x^4+2a^2cd\,x^2+2ab\,c^2x^2+a^2c^2}dx \right)$$

input
```
int((e*x)^m*(B*x^2+A)/(b*x^2+a)^3/(d*x^2+c)^2,x)
```

output
```
e**m*int(x**m/(a**2*c**2 + 2*a**2*c*d*x**2 + a**2*d**2*x**4 + 2*a*b*c**2*x
**2 + 4*a*b*c*d*x**4 + 2*a*b*d**2*x**6 + b**2*c**2*x**4 + 2*b**2*c*d*x**6
+ b**2*d**2*x**8),x)
```

3.37 $\int \frac{(ex)^m(a+bx^2)^3(A+Bx^2)}{(c+dx^2)^3}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 326

$$\int \frac{(ex)^m(a+bx^2)^3(A+Bx^2)}{(c+dx^2)^3}\,dx = -\frac{b^2(3bBc-Abd-3aBd)(ex)^{1+m}}{d^4e(1+m)}+\frac{b^3B(ex)^{3+m}}{d^3e^3(3+m)}+\frac{(bc-ad)^3(Bc-Ad)(ex)^{1+m}}{4cd^4e(c+dx^2)^2}+\frac{(bc-ad)^2(ad(Ad(3-m)+Bc(1+m))+bc(Ad(9+m)-Bc(13+m)))(ex)^{1+m}}{8c^2d^4e(c+dx^2)}-\frac{(bc-ad)(a^2d^2(1-m)(Ad(3-m)+Bc(1+m))+b^2c^2(5+m)(Ad(3+m)-Bc(7+m))+2abcd(\ldots}{8c^3d^4e(1+\ldots}$$

output
```
-b^2*(-A*b*d-3*B*a*d+3*B*b*c)*(e*x)^(1+m)/d^4/e/(1+m)+b^3*B*(e*x)^(3+m)/d^
3/e^3/(3+m)+1/4*(-a*d+b*c)^3*(-A*d+B*c)*(e*x)^(1+m)/c/d^4/e/(d*x^2+c)^2+1/
8*(-a*d+b*c)^2*(a*d*(A*d*(3-m)+B*c*(1+m))+b*c*(A*d*(9+m)-B*c*(13+m)))*(e*x
)^(1+m)/c^2/d^4/e/(d*x^2+c)-1/8*(-a*d+b*c)*(a^2*d^2*(1-m)*(A*d*(3-m)+B*c*(
1+m))+b^2*c^2*(5+m)*(A*d*(3+m)-B*c*(7+m))+2*a*b*c*d*(A*d*(-m^2-2*m+3)+B*c*
(m^2+6*m+5)))*(e*x)^(1+m)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-d*x^2/c)/c
^3/d^4/e/(1+m)
```

Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.68

$$\int \frac{(ex)^m (a+bx^2)^3 (A+Bx^2)}{(c+dx^2)^3}\,dx$$
$$= \frac{x(ex)^m\left(\frac{b^2(-3bBc+Abd+3aBd)}{1+m}+\frac{b^3Bdx^2}{3+m}+\frac{3b(bc-ad)(2bBc-Abd-aBd)\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{dx^2}{c}\right)}{c(1+m)}-\frac{(bc-ad)^2(4b}{}\right.}{d^4}$$

input
```
Integrate[((e*x)^m*(a + b*x^2)^3*(A + B*x^2))/(c + d*x^2)^3,x]
```

output
```
(x*(e*x)^m*((b^2*(-3*b*B*c + A*b*d + 3*a*B*d))/(1 + m) + (b^3*B*d*x^2)/(3
+ m) + (3*b*(b*c - a*d)*(2*b*B*c - A*b*d - a*B*d)*Hypergeometric2F1[1, (1
+ m)/2, (3 + m)/2, -((d*x^2)/c)])/(c*(1 + m)) - ((b*c - a*d)^2*(4*b*B*c -
3*A*b*d - a*B*d)*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, -((d*x^2)/c)])
/(c^2*(1 + m)) + ((b*c - a*d)^3*(B*c - A*d)*Hypergeometric2F1[3, (1 + m)/2
, (3 + m)/2, -((d*x^2)/c)])/(c^3*(1 + m))))/d^4
```

Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 443, normalized size of antiderivative = 1.36, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {439, 25, 439, 25, 437, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^3 (A+Bx^2)(ex)^m}{(c+dx^2)^3}\,dx$$
$$\Big\downarrow \ 439$$
$$-\frac{\int -\frac{(ex)^m(bx^2+a)^2\left(a(Ad(3-m)+Bc(m+1))-b(Ad(m+3)-Bc(m+7))x^2\right)}{(dx^2+c)^2}dx}{4cd}-\frac{(a+bx^2)^3(ex)^{m+1}(Bc-Ad)}{4cde(c+dx^2)^2}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{(ex)^m (bx^2+a)^2 (a(Ad(3-m)+Bc(m+1))-b(Ad(m+3)-Bc(m+7))x^2)}{(dx^2+c)^2}dx}{4cd} - \frac{(a+bx^2)^3 (ex)^{m+1}(Bc-Ad)}{4cde\,(c+dx^2)^2}$$

$$\Big\downarrow \ 439$$

$$\frac{\frac{(a+bx^2)^2 (ex)^{m+1}(ad(Ad(3-m)+Bc(m+1))+bc(Ad(m+3)-Bc(m+7)))}{2cde(c+dx^2)} - \frac{\int -\frac{(ex)^m \left(bx^2+a\right)\left(a(ad(1-m)(Ad(3-m)+Bc(m+1))-bc(m+1)(Ad(m+\right.}{}}{}}{4cd}$$

$$\frac{(a+bx^2)^3 (ex)^{m+1}(Bc-Ad)}{4cde\,(c+dx^2)^2}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{\int \frac{(ex)^m \left(bx^2+a\right)\left(a(ad(1-m)(Ad(3-m)+Bc(m+1))-bc(m+1)(Ad(m+3)-Bc(m+7)))-b(ad(m+3)(Ad(3-m)+Bc(m+1))+bc(m+5)(Ad(m+3)-Bc(m+7)))x^2\right)}{dx^2+c}}{2cd}}{4cd}$$

$$\frac{(a+bx^2)^3 (ex)^{m+1}(Bc-Ad)}{4cde\,(c+dx^2)^2}$$

$$\Big\downarrow \ 437$$

$$\int \left(-\frac{b\left(-b^2(m+5)(Ad(m+3)-Bc(m+7))c^2+3abd(m+3)(Ad(m+1)-Bc(m+5))c+2a^2d^2(m+1)(Ad(3-m)+Bc(m+1))\right)(ex)^m}{d^2}+\frac{\left(b^3Bm^2c^4+35b^3Bc^4+12b^3Bm\right.}{}\right.$$

$$\frac{(a+bx^2)^3 (ex)^{m+1}(Bc-Ad)}{4cde\,(c+dx^2)^2}$$

$$\Big\downarrow \ 2009$$

$$-\frac{(ex)^{m+1}(bc-ad)\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\frac{dx^2}{c}\right)\left(a^2d^2(1-m)(Ad(3-m)+Bc(m+1))+2abcd\left(Ad\left(-m^2-2m+3\right)+Bc\left(m^2+6m+5\right)\right)+b^2c^2(m+5)(\right.}{cd^2e(m+1)}$$

$$\frac{(a+bx^2)^3 (ex)^{m+1}(Bc-Ad)}{4cde\,(c+dx^2)^2}$$

input

```
Int[((e*x)^m*(a + b*x^2)^3*(A + B*x^2))/(c + d*x^2)^3,x]
```

output
```
-1/4*((B*c - A*d)*(e*x)^(1 + m)*(a + b*x^2)^3)/(c*d*e*(c + d*x^2)^2) + (((
a*d*(A*d*(3 - m) + B*c*(1 + m)) + b*c*(A*d*(3 + m) - B*c*(7 + m)))*(e*x)^(
1 + m)*(a + b*x^2)^2)/(2*c*d*e*(c + d*x^2)) + (-((b*(2*a^2*d^2*(1 + m)*(A*
d*(3 - m) + B*c*(1 + m)) + 3*a*b*c*d*(3 + m)*(A*d*(1 + m) - B*c*(5 + m)) -
 b^2*c^2*(5 + m)*(A*d*(3 + m) - B*c*(7 + m)))*(e*x)^(1 + m))/(d^2*e*(1 + m
))) - (b^2*(a*d*(3 + m)*(A*d*(3 - m) + B*c*(1 + m)) + b*c*(5 + m)*(A*d*(3
+ m) - B*c*(7 + m)))*(e*x)^(3 + m))/(d*e^3*(3 + m)) - ((b*c - a*d)*(a^2*d^
2*(1 - m)*(A*d*(3 - m) + B*c*(1 + m)) + b^2*c^2*(5 + m)*(A*d*(3 + m) - B*c
*(7 + m)) + 2*a*b*c*d*(A*d*(3 - 2*m - m^2) + B*c*(5 + 6*m + m^2)))*(e*x)^(
1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((d*x^2)/c)])/(c*d^2*e*
(1 + m)))/(2*c*d))/(4*c*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 437
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(
a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f
, g, m}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

rule 439
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
 + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*b*g*(p + 1))), x] + Simp[1/(2*a*b*(p
+ 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1)*Simp[c*(2*b*e*(
p + 1) + (b*e - a*f)*(m + 1)) + d*(2*b*e*(p + 1) + (b*e - a*f)*(m + 2*q + 1
))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && LtQ[p, -1] && G
tQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e - a*f])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{(ex)^m (b\,x^2+a)^3 (x^2B+A)}{(x^2d+c)^3}dx$$

input
```
int((e*x)^m*(b*x^2+a)^3*(B*x^2+A)/(d*x^2+c)^3,x)
```

output
```
int((e*x)^m*(b*x^2+a)^3*(B*x^2+A)/(d*x^2+c)^3,x)
```

Fricas [F]

$$\int \frac{(ex)^m (a+bx^2)^3 (A+Bx^2)}{(c+dx^2)^3}\,dx = \int \frac{(Bx^2+A)(bx^2+a)^3(ex)^m}{(dx^2+c)^3}\,dx$$

input
```
integrate((e*x)^m*(b*x^2+a)^3*(B*x^2+A)/(d*x^2+c)^3,x, algorithm="fricas")
```

output
```
integral((B*b^3*x^8 + (3*B*a*b^2 + A*b^3)*x^6 + 3*(B*a^2*b + A*a*b^2)*x^4
+ A*a^3 + (B*a^3 + 3*A*a^2*b)*x^2)*(e*x)^m/(d^3*x^6 + 3*c*d^2*x^4 + 3*c^2*
d*x^2 + c^3), x)
```

Sympy [F]

$$\int \frac{(ex)^m (a+bx^2)^3 (A+Bx^2)}{(c+dx^2)^3}\,dx = \int \frac{(ex)^m (A+Bx^2) (a+bx^2)^3}{(c+dx^2)^3}\,dx$$

input
```
integrate((e*x)**m*(b*x**2+a)**3*(B*x**2+A)/(d*x**2+c)**3,x)
```

output
```
Integral((e*x)**m*(A + B*x**2)*(a + b*x**2)**3/(c + d*x**2)**3, x)
```

Maxima [F]

$$\int \frac{(ex)^m (a+bx^2)^3 (A+Bx^2)}{(c+dx^2)^3} \, dx = \int \frac{(Bx^2+A)(bx^2+a)^3(ex)^m}{(dx^2+c)^3} \, dx$$

input
```
integrate((e*x)^m*(b*x^2+a)^3*(B*x^2+A)/(d*x^2+c)^3,x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*(b*x^2 + a)^3*(e*x)^m/(d*x^2 + c)^3, x)
```

Giac [F]

$$\int \frac{(ex)^m (a+bx^2)^3 (A+Bx^2)}{(c+dx^2)^3} \, dx = \int \frac{(Bx^2+A)(bx^2+a)^3(ex)^m}{(dx^2+c)^3} \, dx$$

input
```
integrate((e*x)^m*(b*x^2+a)^3*(B*x^2+A)/(d*x^2+c)^3,x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*(b*x^2 + a)^3*(e*x)^m/(d*x^2 + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m (a+bx^2)^3 (A+Bx^2)}{(c+dx^2)^3} \, dx = \int \frac{(B\,x^2+A)\,(e\,x)^m\,(b\,x^2+a)^3}{(d\,x^2+c)^3} \, dx$$

input
```
int(((A + B*x^2)*(e*x)^m*(a + b*x^2)^3)/(c + d*x^2)^3,x)
```

output
```
int(((A + B*x^2)*(e*x)^m*(a + b*x^2)^3)/(c + d*x^2)^3, x)
```

Reduce [F]

$$\int \frac{(ex)^m (a+bx^2)^3 (A+Bx^2)}{(c+dx^2)^3} \, dx = \text{too large to display}$$

input
```
int((e*x)^m*(b*x^2+a)^3*(B*x^2+A)/(d*x^2+c)^3,x)
```

output
```
(e**m*(4*x**m*a**3*b*d**3*m**3*x + 12*x**m*a**3*b*d**3*m**2*x - 4*x**m*a**
3*b*d**3*m*x - 12*x**m*a**3*b*d**3*x - 6*x**m*a**2*b**2*c*d**2*m**3*x - 42
*x**m*a**2*b**2*c*d**2*m**2*x - 90*x**m*a**2*b**2*c*d**2*m*x - 54*x**m*a**
2*b**2*c*d**2*x + 6*x**m*a**2*b**2*d**3*m**3*x**3 + 6*x**m*a**2*b**2*d**3*
m**2*x**3 - 54*x**m*a**2*b**2*d**3*m*x**3 - 54*x**m*a**2*b**2*d**3*x**3 +
4*x**m*a*b**3*c**2*d*m**3*x + 44*x**m*a*b**3*c**2*d*m**2*x + 156*x**m*a*b*
*3*c**2*d*m*x + 180*x**m*a*b**3*c**2*d*x - 4*x**m*a*b**3*c*d**2*m**3*x**3
- 20*x**m*a*b**3*c*d**2*m**2*x**3 + 36*x**m*a*b**3*c*d**2*m*x**3 + 180*x**
m*a*b**3*c*d**2*x**3 + 4*x**m*a*b**3*d**3*m**3*x**5 - 4*x**m*a*b**3*d**3*m
**2*x**5 - 36*x**m*a*b**3*d**3*m*x**5 + 36*x**m*a*b**3*d**3*x**5 - x**m*b*
*4*c**3*m**3*x - 15*x**m*b**4*c**3*m**2*x - 71*x**m*b**4*c**3*m*x - 105*x*
*m*b**4*c**3*x + x**m*b**4*c**2*d*m**3*x**3 + 9*x**m*b**4*c**2*d*m**2*x**3
 - x**m*b**4*c**2*d*m*x**3 - 105*x**m*b**4*c**2*d*x**3 - x**m*b**4*c*d**2*
m**3*x**5 - 3*x**m*b**4*c*d**2*m**2*x**5 + 25*x**m*b**4*c*d**2*m*x**5 - 21
*x**m*b**4*c*d**2*x**5 + x**m*b**4*d**3*m**3*x**7 - 3*x**m*b**4*d**3*m**2*
x**7 - x**m*b**4*d**3*m*x**7 + 3*x**m*b**4*d**3*x**7 + int(x**m/(c**3*m**2
 - 4*c**3*m + 3*c**3 + 3*c**2*d*m**2*x**2 - 12*c**2*d*m*x**2 + 9*c**2*d*x*
*2 + 3*c*d**2*m**2*x**4 - 12*c*d**2*m*x**4 + 9*c*d**2*x**4 + d**3*m**2*x**
6 - 4*d**3*m*x**6 + 3*d**3*x**6),x)*a**4*c**2*d**4*m**6 - 4*int(x**m/(c**3
*m**2 - 4*c**3*m + 3*c**3 + 3*c**2*d*m**2*x**2 - 12*c**2*d*m*x**2 + 9*c...
```

3.38 $\int \frac{(ex)^m (a+bx^2)^2 (A+Bx^2)}{(c+dx^2)^3} \, dx$

Optimal result

Integrand size = 31, antiderivative size = 270

$$\int \frac{(ex)^m (a+bx^2)^2 (A+Bx^2)}{(c+dx^2)^3} \, dx = \frac{b^2 B (ex)^{1+m}}{d^3 e (1+m)} - \frac{(bc-ad)^2 (Bc-Ad)(ex)^{1+m}}{4cd^3 e (c+dx^2)^2} - \frac{(bc-ad)(ad(Ad(3-m)+Bc(1+m))+bc(Ad(5+m)-Bc(9+m)))(ex)^{1+m}}{8c^2 d^3 e (c+dx^2)} + \frac{(a^2d^2(1-m)(Ad(3-m)+Bc(1+m))+2abcd(1+m)(Ad(1-m)+Bc(3+m))+b^2c^2(3+m)(Ac}{8c^3 d^3 e (1+m)}$$

output
```
b^2*B*(e*x)^(1+m)/d^3/e/(1+m)-1/4*(-a*d+b*c)^2*(-A*d+B*c)*(e*x)^(1+m)/c/d^
3/e/(d*x^2+c)^2-1/8*(-a*d+b*c)*(a*d*(A*d*(3-m)+B*c*(1+m))+b*c*(A*d*(5+m)-B
*c*(9+m)))*(e*x)^(1+m)/c^2/d^3/e/(d*x^2+c)+1/8*(a^2*d^2*(1-m)*(A*d*(3-m)+B
*c*(1+m))+2*a*b*c*d*(1+m)*(A*d*(1-m)+B*c*(3+m))+b^2*c^2*(3+m)*(A*d*(1+m)-B
*c*(5+m)))*(e*x)^(1+m)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-d*x^2/c)/c^3/
d^3/e/(1+m)
```

Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.63

$$\int \frac{(ex)^m (a+bx^2)^2 (A+Bx^2)}{(c+dx^2)^3}\,dx$$
$$= \frac{x(ex)^m \left(b^2B - \frac{b(3bBc-Abd-2aBd)\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{dx^2}{c}\right)}{c} + \frac{(bc-ad)(3bBc-2Abd-aBd)\,\text{Hypergeometric2F1}\left(2,\right.}{c^2}\right.}{d^3(1+m)}$$

input
```
Integrate[((e*x)^m*(a + b*x^2)^2*(A + B*x^2))/(c + d*x^2)^3,x]
```

output
```
(x*(e*x)^m*(b^2*B - (b*(3*b*B*c - A*b*d - 2*a*B*d)*Hypergeometric2F1[1, (1
 + m)/2, (3 + m)/2, -((d*x^2)/c)])/c + ((b*c - a*d)*(3*b*B*c - 2*A*b*d - a
*B*d)*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, -((d*x^2)/c)])/c^2 - ((b*
c - a*d)^2*(B*c - A*d)*Hypergeometric2F1[3, (1 + m)/2, (3 + m)/2, -((d*x^2
)/c)])/c^3))/(d^3*(1 + m))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.13, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {439, 25, 439, 25, 363, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2 (A+Bx^2) (ex)^m}{(c+dx^2)^3}\,dx$$

$$\Big\downarrow \; 439$$

$$-\frac{\int -\frac{(ex)^m (bx^2+a)(a(Ad(3-m)+Bc(m+1))-b(Ad(m+1)-Bc(m+5))x^2)}{(dx^2+c)^2}dx}{4cd} - \frac{(a+bx^2)^2 (ex)^{m+1}(Bc-Ad)}{4cde\,(c+dx^2)^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{(ex)^m (bx^2+a)\left(a(Ad(3-m)+Bc(m+1))-b(Ad(m+1)-Bc(m+5))x^2\right)}{(dx^2+c)^2}dx}{4cd} - \frac{\left(a+bx^2\right)^2 (ex)^{m+1}(Bc-Ad)}{4cde\left(c+dx^2\right)^2}$$

$\Big\downarrow$ 439

$$-\frac{\frac{\int -\frac{(ex)^m \left(b(ad(m+1)-bc(m+3))(Ad(m+1)-Bc(m+5))x^2+a(ad(1-m)+bc(m+1))(Ad(3-m)+Bc(m+1))\right)}{dx^2+c}dx}{2cd} - \frac{(ex)^{m+1}(bc-ad)\left(a(Ad(3-m)+Bc(m\right.}{2cde(c+}}{4cd}$$

$$\frac{\left(a+bx^2\right)^2 (ex)^{m+1}(Bc-Ad)}{4cde\left(c+dx^2\right)^2}$$

$\Big\downarrow$ 25

$$\frac{\frac{\int \frac{(ex)^m \left(b(ad(m+1)-bc(m+3))(Ad(m+1)-Bc(m+5))x^2+a(ad(1-m)+bc(m+1))(Ad(3-m)+Bc(m+1))\right)}{dx^2+c}dx}{2cd} - \frac{(ex)^{m+1}(bc-ad)\left(a(Ad(3-m)+Bc(m+1)\right.}{2cde(c+dx^2}}{4cd}$$

$$\frac{\left(a+bx^2\right)^2 (ex)^{m+1}(Bc-Ad)}{4cde\left(c+dx^2\right)^2}$$

$\Big\downarrow$ 363

$$\frac{\frac{\left(a(ad(1-m)+bc(m+1))(Ad(3-m)+Bc(m+1))-\frac{bc(ad(m+1)-bc(m+3))(Ad(m+1)-Bc(m+5))}{d}\right)\int \frac{(ex)^m}{dx^2+c}dx+\frac{b(ex)^{m+1}(ad(m+1)-bc(m+3))(Ad(m+1}{de(m+1)}}{2cd}}{4cd}$$

$$\frac{\left(a+bx^2\right)^2 (ex)^{m+1}(Bc-Ad)}{4cde\left(c+dx^2\right)^2}$$

$\Big\downarrow$ 278

$$\frac{\frac{\frac{(ex)^{m+1}\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\frac{dx^2}{c}\right)\left(a(ad(1-m)+bc(m+1))(Ad(3-m)+Bc(m+1))-\frac{bc(ad(m+1)-bc(m+3))(Ad(m+1)-Bc(m+5))}{d}\right)}{ce(m+1)}+\frac{b(ex)^m}{}}{2cd}}{4cd}$$

$$\frac{\left(a+bx^2\right)^2 (ex)^{m+1}(Bc-Ad)}{4cde\left(c+dx^2\right)^2}$$

input
```
Int[((e*x)^m*(a + b*x^2)^2*(A + B*x^2))/(c + d*x^2)^3,x]
```

output
```
-1/4*((B*c - A*d)*(e*x)^(1 + m)*(a + b*x^2)^2)/(c*d*e*(c + d*x^2)^2) + (-1
/2*((b*c - a*d)*(e*x)^(1 + m)*(a*(A*d*(3 - m) + B*c*(1 + m)) - b*(A*d*(1 +
 m) - B*c*(5 + m))*x^2))/(c*d*e*(c + d*x^2)) + ((b*(a*d*(1 + m) - b*c*(3 +
 m))*(A*d*(1 + m) - B*c*(5 + m))*(e*x)^(1 + m))/(d*e*(1 + m)) + ((a*(a*d*(
1 - m) + b*c*(1 + m))*(A*d*(3 - m) + B*c*(1 + m)) - (b*c*(a*d*(1 + m) - b*
c*(3 + m))*(A*d*(1 + m) - B*c*(5 + m)))/d)*(e*x)^(1 + m)*Hypergeometric2F1
[1, (1 + m)/2, (3 + m)/2, -((d*x^2)/c)])/(c*e*(1 + m)))/(2*c*d))/(4*c*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 439
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
 + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*b*g*(p + 1))), x] + Simp[1/(2*a*b*(p
+ 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1)*Simp[c*(2*b*e*(
p + 1) + (b*e - a*f)*(m + 1)) + d*(2*b*e*(p + 1) + (b*e - a*f)*(m + 2*q + 1
))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && LtQ[p, -1] && G
tQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e - a*f])
```

Maple [F]

$$\int \frac{(ex)^m (b\,x^2+a)^2 (x^2 B+A)}{(x^2 d+c)^3}dx$$

input
```
int((e*x)^m*(b*x^2+a)^2*(B*x^2+A)/(d*x^2+c)^3,x)
```

output
```
int((e*x)^m*(b*x^2+a)^2*(B*x^2+A)/(d*x^2+c)^3,x)
```

Fricas [F]

$$\int \frac{(ex)^m (a+bx^2)^2 (A+Bx^2)}{(c+dx^2)^3} \, dx = \int \frac{(Bx^2+A)(bx^2+a)^2(ex)^m}{(dx^2+c)^3} \, dx$$

input
```
integrate((e*x)^m*(b*x^2+a)^2*(B*x^2+A)/(d*x^2+c)^3,x, algorithm="fricas")
```

output
```
integral((B*b^2*x^6 + (2*B*a*b + A*b^2)*x^4 + A*a^2 + (B*a^2 + 2*A*a*b)*x^
2)*(e*x)^m/(d^3*x^6 + 3*c*d^2*x^4 + 3*c^2*d*x^2 + c^3), x)
```

Sympy [F]

$$\int \frac{(ex)^m (a+bx^2)^2 (A+Bx^2)}{(c+dx^2)^3} \, dx = \int \frac{(ex)^m (A+Bx^2) (a+bx^2)^2}{(c+dx^2)^3} \, dx$$

input
```
integrate((e*x)**m*(b*x**2+a)**2*(B*x**2+A)/(d*x**2+c)**3,x)
```

output
```
Integral((e*x)**m*(A + B*x**2)*(a + b*x**2)**2/(c + d*x**2)**3, x)
```

Maxima [F]

$$\int \frac{(ex)^m (a+bx^2)^2 (A+Bx^2)}{(c+dx^2)^3}\,dx = \int \frac{(Bx^2+A)(bx^2+a)^2(ex)^m}{(dx^2+c)^3}\,dx$$

input
```
integrate((e*x)^m*(b*x^2+a)^2*(B*x^2+A)/(d*x^2+c)^3,x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*(b*x^2 + a)^2*(e*x)^m/(d*x^2 + c)^3, x)
```

Giac [F]

$$\int \frac{(ex)^m (a+bx^2)^2 (A+Bx^2)}{(c+dx^2)^3}\,dx = \int \frac{(Bx^2+A)(bx^2+a)^2(ex)^m}{(dx^2+c)^3}\,dx$$

input
```
integrate((e*x)^m*(b*x^2+a)^2*(B*x^2+A)/(d*x^2+c)^3,x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*(b*x^2 + a)^2*(e*x)^m/(d*x^2 + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m (a+bx^2)^2 (A+Bx^2)}{(c+dx^2)^3}\,dx = \int \frac{(B\,x^2+A)\,(e\,x)^m\,(b\,x^2+a)^2}{(d\,x^2+c)^3}\,dx$$

input
```
int(((A + B*x^2)*(e*x)^m*(a + b*x^2)^2)/(c + d*x^2)^3,x)
```

output
```
int(((A + B*x^2)*(e*x)^m*(a + b*x^2)^2)/(c + d*x^2)^3, x)
```

Reduce [F]

$$\int \frac{(ex)^m (a+bx^2)^2 (A+Bx^2)}{(c+dx^2)^3} \, dx = \text{too large to display}$$

input
```
int((e*x)^m*(b*x^2+a)^2*(B*x^2+A)/(d*x^2+c)^3,x)
```

output
```
(e**m*(3*x**m*a**2*b*d**2*m**2*x - 3*x**m*a**2*b*d**2*x - 3*x**m*a*b**2*c*
d*m**2*x - 12*x**m*a*b**2*c*d*m*x - 9*x**m*a*b**2*c*d*x + 3*x**m*a*b**2*d*
*2*m**2*x**3 - 6*x**m*a*b**2*d**2*m*x**3 - 9*x**m*a*b**2*d**2*x**3 + x**m*
b**3*c**2*m**2*x + 8*x**m*b**3*c**2*m*x + 15*x**m*b**3*c**2*x - x**m*b**3*
c*d*m**2*x**3 - 2*x**m*b**3*c*d*m*x**3 + 15*x**m*b**3*c*d*x**3 + x**m*b**3
*d**2*m**2*x**5 - 4*x**m*b**3*d**2*m*x**5 + 3*x**m*b**3*d**2*x**5 + int(x*
*m/(c**3*m**2 - 4*c**3*m + 3*c**3 + 3*c**2*d*m**2*x**2 - 12*c**2*d*m*x**2
+ 9*c**2*d*x**2 + 3*c*d**2*m**2*x**4 - 12*c*d**2*m*x**4 + 9*c*d**2*x**4 +
d**3*m**2*x**6 - 4*d**3*m*x**6 + 3*d**3*x**6),x)*a**3*c**2*d**3*m**5 - 7*i
nt(x**m/(c**3*m**2 - 4*c**3*m + 3*c**3 + 3*c**2*d*m**2*x**2 - 12*c**2*d*m*
x**2 + 9*c**2*d*x**2 + 3*c*d**2*m**2*x**4 - 12*c*d**2*m*x**4 + 9*c*d**2*x*
*4 + d**3*m**2*x**6 - 4*d**3*m*x**6 + 3*d**3*x**6),x)*a**3*c**2*d**3*m**4
+ 14*int(x**m/(c**3*m**2 - 4*c**3*m + 3*c**3 + 3*c**2*d*m**2*x**2 - 12*c**
2*d*m*x**2 + 9*c**2*d*x**2 + 3*c*d**2*m**2*x**4 - 12*c*d**2*m*x**4 + 9*c*d
**2*x**4 + d**3*m**2*x**6 - 4*d**3*m*x**6 + 3*d**3*x**6),x)*a**3*c**2*d**3
*m**3 - 2*int(x**m/(c**3*m**2 - 4*c**3*m + 3*c**3 + 3*c**2*d*m**2*x**2 - 1
2*c**2*d*m*x**2 + 9*c**2*d*x**2 + 3*c*d**2*m**2*x**4 - 12*c*d**2*m*x**4 +
9*c*d**2*x**4 + d**3*m**2*x**6 - 4*d**3*m*x**6 + 3*d**3*x**6),x)*a**3*c**2
*d**3*m**2 - 15*int(x**m/(c**3*m**2 - 4*c**3*m + 3*c**3 + 3*c**2*d*m**2*x*
*2 - 12*c**2*d*m*x**2 + 9*c**2*d*x**2 + 3*c*d**2*m**2*x**4 - 12*c*d**2*...
```

3.39 $\int \frac{(ex)^m(a+bx^2)(A+Bx^2)}{(c+dx^2)^3}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 206

$$\int \frac{(ex)^m(a+bx^2)(A+Bx^2)}{(c+dx^2)^3}\,dx = \frac{(bc-ad)(Bc-Ad)(ex)^{1+m}}{4cd^2e(c+dx^2)^2} + \frac{(ad(Ad(3-m)+Bc(1+m))+bc(Ad(1+m)-Bc(5+m)))(ex)^{1+m}}{8c^2d^2e(c+dx^2)} + \frac{(ad(1-m)(Ad(3-m)+Bc(1+m))+bc(1+m)(Ad(1-m)+Bc(3+m)))(ex)^{1+m}\,\text{Hypergeometr}}{8c^3d^2e(1+m)}$$

output
```
1/4*(-a*d+b*c)*(-A*d+B*c)*(e*x)^(1+m)/c/d^2/e/(d*x^2+c)^2+1/8*(a*d*(A*d*(3
-m)+B*c*(1+m))+b*c*(A*d*(1+m)-B*c*(5+m)))*(e*x)^(1+m)/c^2/d^2/e/(d*x^2+c)+
1/8*(a*d*(1-m)*(A*d*(3-m)+B*c*(1+m))+b*c*(1+m)*(A*d*(1-m)+B*c*(3+m)))*(e*x
)^(1+m)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-d*x^2/c)/c^3/d^2/e/(1+m)
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.65

$$\int \frac{(ex)^m (a+bx^2) (A+Bx^2)}{(c+dx^2)^3} \, dx$$
$$= \frac{x(ex)^m \left(bBc^2 \text{Hypergeometric2F1}\left(1, \frac{1+m}{2}, \frac{3+m}{2}, -\frac{dx^2}{c}\right) + c(-2bBc+Abd+aBd) \text{Hypergeometric2F1}\right.}{c^3d^2(1+m)}$$

input
```
Integrate[((e*x)^m*(a + b*x^2)*(A + B*x^2))/(c + d*x^2)^3,x]
```

output
```
(x*(e*x)^m*(b*B*c^2*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((d*x^2)/c
)] + c*(-2*b*B*c + A*b*d + a*B*d)*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/
2, -((d*x^2)/c)] + (b*c - a*d)*(B*c - A*d)*Hypergeometric2F1[3, (1 + m)/2,
 (3 + m)/2, -((d*x^2)/c)]))/(c^3*d^2*(1 + m))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {439, 25, 362, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2) (A+Bx^2) (ex)^m}{(c+dx^2)^3} \, dx$$
$$\Big\downarrow \text{439}$$
$$-\frac{\int -\frac{(ex)^m (B(ad(1-m)+bc(m+3))x^2+A(ad(3-m)+bc(m+1)))}{(dx^2+c)^2}dx}{4cd} - \frac{(A+Bx^2)(ex)^{m+1}(bc-ad)}{4cde(c+dx^2)^2}$$
$$\Big\downarrow \text{25}$$
$$\frac{\int \frac{(ex)^m (B(ad(1-m)+bc(m+3))x^2+A(ad(3-m)+bc(m+1)))}{(dx^2+c)^2}dx}{4cd} - \frac{(A+Bx^2)(ex)^{m+1}(bc-ad)}{4cde(c+dx^2)^2}$$

$$\big\downarrow 362$$

$$\frac{\frac{(ad(1-m)(Ad(3-m)+Bc(m+1))+bc(m+1)(Ad(1-m)+Bc(m+3)))\int \frac{(ex)^m}{dx^2+c}dx}{2cd}+\frac{(ex)^{m+1}(ad(Ad(3-m)-B(c-cm))+bc(Ad(m+1)-Bc(m+}{2cde(c+dx^2)}}{4cd}$$

$$\frac{(A+Bx^2)(ex)^{m+1}(bc-ad)}{4cde(c+dx^2)^2}$$

$$\big\downarrow 278$$

$$\frac{\frac{(ex)^{m+1}\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\frac{dx^2}{c}\right)(ad(1-m)(Ad(3-m)+Bc(m+1))+bc(m+1)(Ad(1-m)+Bc(m+3)))}{2c^2de(m+1)}+\frac{(ex)^{m+1}(ad(Ad(3-}{}}{4cd}$$

$$\frac{(A+Bx^2)(ex)^{m+1}(bc-ad)}{4cde(c+dx^2)^2}$$

input
```
Int[((e*x)^m*(a + b*x^2)*(A + B*x^2))/(c + d*x^2)^3,x]
```

output
```
-1/4*((b*c - a*d)*(e*x)^(1 + m)*(A + B*x^2))/(c*d*e*(c + d*x^2)^2) + (((b*
c*(A*d*(1 + m) - B*c*(3 + m)) + a*d*(A*d*(3 - m) - B*(c - c*m)))*(e*x)^(1
+ m))/(2*c*d*e*(c + d*x^2)) + ((a*d*(1 - m)*(A*d*(3 - m) + B*c*(1 + m)) +
b*c*(1 + m)*(A*d*(1 - m) + B*c*(3 + m)))*(e*x)^(1 + m)*Hypergeometric2F1[1
, (1 + m)/2, (3 + m)/2, -((d*x^2)/c)])/(2*c^2*d*e*(1 + m)))/(4*c*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

rule 439
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
 + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*b*g*(p + 1))), x] + Simp[1/(2*a*b*(p
+ 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1)*Simp[c*(2*b*e*(
p + 1) + (b*e - a*f)*(m + 1)) + d*(2*b*e*(p + 1) + (b*e - a*f)*(m + 2*q + 1
))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && LtQ[p, -1] && G
tQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e - a*f])
```

Maple [F]

$$\int \frac{(ex)^m (b\,x^2+a)\,(x^2B+A)}{(x^2d+c)^3}dx$$

input
```
int((e*x)^m*(b*x^2+a)*(B*x^2+A)/(d*x^2+c)^3,x)
```

output
```
int((e*x)^m*(b*x^2+a)*(B*x^2+A)/(d*x^2+c)^3,x)
```

Fricas [F]

$$\int \frac{(ex)^m\,(a+bx^2)\,(A+Bx^2)}{(c+dx^2)^3}\,dx = \int \frac{(Bx^2+A)(bx^2+a)(ex)^m}{(dx^2+c)^3}\,dx$$

input
```
integrate((e*x)^m*(b*x^2+a)*(B*x^2+A)/(d*x^2+c)^3,x, algorithm="fricas")
```

output
```
integral((B*b*x^4 + (B*a + A*b)*x^2 + A*a)*(e*x)^m/(d^3*x^6 + 3*c*d^2*x^4
+ 3*c^2*d*x^2 + c^3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 92.26 (sec) , antiderivative size = 6411, normalized size of antiderivative = 31.12

$$\int \frac{(ex)^m (a+bx^2) (A+Bx^2)}{(c+dx^2)^3} dx = \text{Too large to display}$$

input
```
integrate((e*x)**m*(b*x**2+a)*(B*x**2+A)/(d*x**2+c)**3,x)
```

output
```
A*a*(c**2*e**m*m**3*x**(m + 1)*lerchphi(d*x**2*exp_polar(I*pi)/c, 1, m/2 +
 1/2)*gamma(m/2 + 1/2)/(32*c**5*gamma(m/2 + 3/2) + 64*c**4*d*x**2*gamma(m/
2 + 3/2) + 32*c**3*d**2*x**4*gamma(m/2 + 3/2)) - 3*c**2*e**m*m**2*x**(m +
1)*lerchphi(d*x**2*exp_polar(I*pi)/c, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(32*c
**5*gamma(m/2 + 3/2) + 64*c**4*d*x**2*gamma(m/2 + 3/2) + 32*c**3*d**2*x**4
*gamma(m/2 + 3/2)) - 2*c**2*e**m*m**2*x**(m + 1)*gamma(m/2 + 1/2)/(32*c**5
*gamma(m/2 + 3/2) + 64*c**4*d*x**2*gamma(m/2 + 3/2) + 32*c**3*d**2*x**4*ga
mma(m/2 + 3/2)) - c**2*e**m*m*x**(m + 1)*lerchphi(d*x**2*exp_polar(I*pi)/c
, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(32*c**5*gamma(m/2 + 3/2) + 64*c**4*d*x**
2*gamma(m/2 + 3/2) + 32*c**3*d**2*x**4*gamma(m/2 + 3/2)) + 8*c**2*e**m*m*x
**(m + 1)*gamma(m/2 + 1/2)/(32*c**5*gamma(m/2 + 3/2) + 64*c**4*d*x**2*gamm
a(m/2 + 3/2) + 32*c**3*d**2*x**4*gamma(m/2 + 3/2)) + 3*c**2*e**m*x**(m + 1
)*lerchphi(d*x**2*exp_polar(I*pi)/c, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(32*c*
*5*gamma(m/2 + 3/2) + 64*c**4*d*x**2*gamma(m/2 + 3/2) + 32*c**3*d**2*x**4*
gamma(m/2 + 3/2)) + 10*c**2*e**m*x**(m + 1)*gamma(m/2 + 1/2)/(32*c**5*gamm
a(m/2 + 3/2) + 64*c**4*d*x**2*gamma(m/2 + 3/2) + 32*c**3*d**2*x**4*gamma(m
/2 + 3/2)) + 2*c*d*e**m*m**3*x**2*x**(m + 1)*lerchphi(d*x**2*exp_polar(I*p
i)/c, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(32*c**5*gamma(m/2 + 3/2) + 64*c**4*d
*x**2*gamma(m/2 + 3/2) + 32*c**3*d**2*x**4*gamma(m/2 + 3/2)) - 6*c*d*e**m*
m**2*x**2*x**(m + 1)*lerchphi(d*x**2*exp_polar(I*pi)/c, 1, m/2 + 1/2)*g...
```

Maxima [F]

$$\int \frac{(ex)^m (a+bx^2) (A+Bx^2)}{(c+dx^2)^3} dx = \int \frac{(Bx^2+A)(bx^2+a)(ex)^m}{(dx^2+c)^3} dx$$

input
```
integrate((e*x)^m*(b*x^2+a)*(B*x^2+A)/(d*x^2+c)^3,x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*(b*x^2 + a)*(e*x)^m/(d*x^2 + c)^3, x)
```

Giac [F]

$$\int \frac{(ex)^m (a+bx^2) (A+Bx^2)}{(c+dx^2)^3} dx = \int \frac{(Bx^2+A)(bx^2+a)(ex)^m}{(dx^2+c)^3} dx$$

input
```
integrate((e*x)^m*(b*x^2+a)*(B*x^2+A)/(d*x^2+c)^3,x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*(b*x^2 + a)*(e*x)^m/(d*x^2 + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m (a+bx^2) (A+Bx^2)}{(c+dx^2)^3} dx = \int \frac{(B\,x^2+A)\,(e\,x)^m\,(b\,x^2+a)}{(d\,x^2+c)^3} dx$$

input
```
int(((A + B*x^2)*(e*x)^m*(a + b*x^2))/(c + d*x^2)^3,x)
```

output
```
int(((A + B*x^2)*(e*x)^m*(a + b*x^2))/(c + d*x^2)^3, x)
```

Reduce [F]

$$\int \frac{(ex)^m (a+bx^2)(A+Bx^2)}{(c+dx^2)^3}\,dx = \text{too large to display}$$

input
```
int((e*x)^m*(b*x^2+a)*(B*x^2+A)/(d*x^2+c)^3,x)
```

output
```
(e**m*(2*x**m*a*b*d*m*x - 2*x**m*a*b*d*x - x**m*b**2*c*m*x - 3*x**m*b**2*c
*x + x**m*b**2*d*m*x**3 - 3*x**m*b**2*d*x**3 + int(x**m/(c**3*m**2 - 4*c**
3*m + 3*c**3 + 3*c**2*d*m**2*x**2 - 12*c**2*d*m*x**2 + 9*c**2*d*x**2 + 3*c
*d**2*m**2*x**4 - 12*c*d**2*m*x**4 + 9*c*d**2*x**4 + d**3*m**2*x**6 - 4*d*
*3*m*x**6 + 3*d**3*x**6),x)*a**2*c**2*d**2*m**4 - 8*int(x**m/(c**3*m**2 -
4*c**3*m + 3*c**3 + 3*c**2*d*m**2*x**2 - 12*c**2*d*m*x**2 + 9*c**2*d*x**2
+ 3*c*d**2*m**2*x**4 - 12*c*d**2*m*x**4 + 9*c*d**2*x**4 + d**3*m**2*x**6 -
 4*d**3*m*x**6 + 3*d**3*x**6),x)*a**2*c**2*d**2*m**3 + 22*int(x**m/(c**3*m
**2 - 4*c**3*m + 3*c**3 + 3*c**2*d*m**2*x**2 - 12*c**2*d*m*x**2 + 9*c**2*d
*x**2 + 3*c*d**2*m**2*x**4 - 12*c*d**2*m*x**4 + 9*c*d**2*x**4 + d**3*m**2*
x**6 - 4*d**3*m*x**6 + 3*d**3*x**6),x)*a**2*c**2*d**2*m**2 - 24*int(x**m/(
c**3*m**2 - 4*c**3*m + 3*c**3 + 3*c**2*d*m**2*x**2 - 12*c**2*d*m*x**2 + 9*
c**2*d*x**2 + 3*c*d**2*m**2*x**4 - 12*c*d**2*m*x**4 + 9*c*d**2*x**4 + d**3
*m**2*x**6 - 4*d**3*m*x**6 + 3*d**3*x**6),x)*a**2*c**2*d**2*m + 9*int(x**m
/(c**3*m**2 - 4*c**3*m + 3*c**3 + 3*c**2*d*m**2*x**2 - 12*c**2*d*m*x**2 +
9*c**2*d*x**2 + 3*c*d**2*m**2*x**4 - 12*c*d**2*m*x**4 + 9*c*d**2*x**4 + d*
*3*m**2*x**6 - 4*d**3*m*x**6 + 3*d**3*x**6),x)*a**2*c**2*d**2 + 2*int(x**m
/(c**3*m**2 - 4*c**3*m + 3*c**3 + 3*c**2*d*m**2*x**2 - 12*c**2*d*m*x**2 +
9*c**2*d*x**2 + 3*c*d**2*m**2*x**4 - 12*c*d**2*m*x**4 + 9*c*d**2*x**4 + d*
*3*m**2*x**6 - 4*d**3*m*x**6 + 3*d**3*x**6),x)*a**2*c*d**3*m**4*x**2 - ...
```

3.40 $\int \frac{(ex)^m(A+Bx^2)}{(c+dx^2)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 103

$$\int \frac{(ex)^m(A+Bx^2)}{(c+dx^2)^3}\,dx = -\frac{(Bc-Ad)(ex)^{1+m}}{4cde(c+dx^2)^2} + \frac{(Ad(3-m)+Bc(1+m))(ex)^{1+m}\,\text{Hypergeometric2F1}\left(2,\frac{1+m}{2},\frac{3+m}{2},-\frac{dx^2}{c}\right)}{4c^3de(1+m)}$$

output

```
-1/4*(-A*d+B*c)*(e*x)^(1+m)/c/d/e/(d*x^2+c)^2+1/4*(A*d*(3-m)+B*c*(1+m))*(e
*x)^(1+m)*hypergeom([2, 1/2+1/2*m],[3/2+1/2*m],-d*x^2/c)/c^3/d/e/(1+m)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.79

$$\int \frac{(ex)^m(A+Bx^2)}{(c+dx^2)^3}\,dx = \frac{x(ex)^m\left(Bc\,\text{Hypergeometric2F1}\left(2,\frac{1+m}{2},\frac{3+m}{2},-\frac{dx^2}{c}\right)+(-Bc+Ad)\,\text{Hypergeometric2F1}\left(3,\frac{1+m}{2},\frac{3+m}{2},\right.\right.}{c^3d(1+m)}$$

input
```
Integrate[((e*x)^m*(A + B*x^2))/(c + d*x^2)^3,x]
```

output
```
(x*(e*x)^m*(B*c*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, -((d*x^2)/c)] +
 (-(B*c) + A*d)*Hypergeometric2F1[3, (1 + m)/2, (3 + m)/2, -((d*x^2)/c)]))
/(c^3*d*(1 + m))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {362, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)(ex)^m}{(c+dx^2)^3}dx$$

$$\Big\downarrow \text{362}$$

$$\frac{(Ad(3-m)+Bc(m+1))\int \frac{(ex)^m}{(dx^2+c)^2}dx}{4cd}-\frac{(ex)^{m+1}(Bc-Ad)}{4cde(c+dx^2)^2}$$

$$\Big\downarrow \text{278}$$

$$\frac{(ex)^{m+1}(Ad(3-m)+Bc(m+1))\,\text{Hypergeometric2F1}\left(2,\frac{m+1}{2},\frac{m+3}{2},-\frac{dx^2}{c}\right)}{4c^3de(m+1)}-\frac{(ex)^{m+1}(Bc-Ad)}{4cde(c+dx^2)^2}$$

input
```
Int[((e*x)^m*(A + B*x^2))/(c + d*x^2)^3,x]
```

output
```
-1/4*((B*c - A*d)*(e*x)^(1 + m))/(c*d*e*(c + d*x^2)^2) + ((A*d*(3 - m) + B
*c*(1 + m))*(e*x)^(1 + m)*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, -((d*
x^2)/c)])/(4*c^3*d*e*(1 + m))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

Maple [F]

$$\int \frac{(ex)^m (x^2B + A)}{(x^2d + c)^3}dx$$

input
```
int((e*x)^m*(B*x^2+A)/(d*x^2+c)^3,x)
```

output
```
int((e*x)^m*(B*x^2+A)/(d*x^2+c)^3,x)
```

Fricas [F]

$$\int \frac{(ex)^m (A + Bx^2)}{(c + dx^2)^3}\,dx = \int \frac{(Bx^2 + A)(ex)^m}{(dx^2 + c)^3}\,dx$$

input
```
integrate((e*x)^m*(B*x^2+A)/(d*x^2+c)^3,x, algorithm="fricas")
```

output
```
integral((B*x^2 + A)*(e*x)^m/(d^3*x^6 + 3*c*d^2*x^4 + 3*c^2*d*x^2 + c^3),
x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 50.87 (sec) , antiderivative size = 3199, normalized size of antiderivative = 31.06

$$\int \frac{(ex)^m (A + Bx^2)}{(c + dx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate((e*x)**m*(B*x**2+A)/(d*x**2+c)**3,x)
```

output
```
A*(c**2*e**m*m**3*x**(m + 1)*lerchphi(d*x**2*exp_polar(I*pi)/c, 1, m/2 + 1
/2)*gamma(m/2 + 1/2)/(32*c**5*gamma(m/2 + 3/2) + 64*c**4*d*x**2*gamma(m/2
+ 3/2) + 32*c**3*d**2*x**4*gamma(m/2 + 3/2)) - 3*c**2*e**m*m**2*x**(m + 1)
*lerchphi(d*x**2*exp_polar(I*pi)/c, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(32*c**
5*gamma(m/2 + 3/2) + 64*c**4*d*x**2*gamma(m/2 + 3/2) + 32*c**3*d**2*x**4*g
amma(m/2 + 3/2)) - 2*c**2*e**m*m**2*x**(m + 1)*gamma(m/2 + 1/2)/(32*c**5*g
amma(m/2 + 3/2) + 64*c**4*d*x**2*gamma(m/2 + 3/2) + 32*c**3*d**2*x**4*gamm
a(m/2 + 3/2)) - c**2*e**m*m*x**(m + 1)*lerchphi(d*x**2*exp_polar(I*pi)/c,
1, m/2 + 1/2)*gamma(m/2 + 1/2)/(32*c**5*gamma(m/2 + 3/2) + 64*c**4*d*x**2*
gamma(m/2 + 3/2) + 32*c**3*d**2*x**4*gamma(m/2 + 3/2)) + 8*c**2*e**m*m*x**
(m + 1)*gamma(m/2 + 1/2)/(32*c**5*gamma(m/2 + 3/2) + 64*c**4*d*x**2*gamma(
m/2 + 3/2) + 32*c**3*d**2*x**4*gamma(m/2 + 3/2)) + 3*c**2*e**m*x**(m + 1)*
lerchphi(d*x**2*exp_polar(I*pi)/c, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(32*c**5
*gamma(m/2 + 3/2) + 64*c**4*d*x**2*gamma(m/2 + 3/2) + 32*c**3*d**2*x**4*ga
mma(m/2 + 3/2)) + 10*c**2*e**m*x**(m + 1)*gamma(m/2 + 1/2)/(32*c**5*gamma(
m/2 + 3/2) + 64*c**4*d*x**2*gamma(m/2 + 3/2) + 32*c**3*d**2*x**4*gamma(m/2
 + 3/2)) + 2*c*d*e**m*m**3*x**2*x**(m + 1)*lerchphi(d*x**2*exp_polar(I*pi)
/c, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(32*c**5*gamma(m/2 + 3/2) + 64*c**4*d*x
**2*gamma(m/2 + 3/2) + 32*c**3*d**2*x**4*gamma(m/2 + 3/2)) - 6*c*d*e**m*m*
*2*x**2*x**(m + 1)*lerchphi(d*x**2*exp_polar(I*pi)/c, 1, m/2 + 1/2)*gam...
```

Maxima [F]

$$\int \frac{(ex)^m (A+Bx^2)}{(c+dx^2)^3}\,dx = \int \frac{(Bx^2+A)(ex)^m}{(dx^2+c)^3}\,dx$$

input
```
integrate((e*x)^m*(B*x^2+A)/(d*x^2+c)^3,x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*(e*x)^m/(d*x^2 + c)^3, x)
```

Giac [F]

$$\int \frac{(ex)^m (A+Bx^2)}{(c+dx^2)^3}\,dx = \int \frac{(Bx^2+A)(ex)^m}{(dx^2+c)^3}\,dx$$

input
```
integrate((e*x)^m*(B*x^2+A)/(d*x^2+c)^3,x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*(e*x)^m/(d*x^2 + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m (A+Bx^2)}{(c+dx^2)^3}\,dx = \int \frac{(B\,x^2+A)\,(e\,x)^m}{(d\,x^2+c)^3}\,dx$$

input
```
int(((A + B*x^2)*(e*x)^m)/(c + d*x^2)^3,x)
```

output
```
int(((A + B*x^2)*(e*x)^m)/(c + d*x^2)^3, x)
```

Reduce [F]

$$\int \frac{(ex)^m (A+Bx^2)}{(c+dx^2)^3}\,dx = \text{Too large to display}$$

input
```
int((e*x)^m*(B*x^2+A)/(d*x^2+c)^3,x)
```

output
```
(e**m*(x**m*b*x + int(x**m/(c**3*m - 3*c**3 + 3*c**2*d*m*x**2 - 9*c**2*d*x
**2 + 3*c*d**2*m*x**4 - 9*c*d**2*x**4 + d**3*m*x**6 - 3*d**3*x**6),x)*a*c*
*2*d*m**2 - 6*int(x**m/(c**3*m - 3*c**3 + 3*c**2*d*m*x**2 - 9*c**2*d*x**2
+ 3*c*d**2*m*x**4 - 9*c*d**2*x**4 + d**3*m*x**6 - 3*d**3*x**6),x)*a*c**2*d
*m + 9*int(x**m/(c**3*m - 3*c**3 + 3*c**2*d*m*x**2 - 9*c**2*d*x**2 + 3*c*d
**2*m*x**4 - 9*c*d**2*x**4 + d**3*m*x**6 - 3*d**3*x**6),x)*a*c**2*d + 2*in
t(x**m/(c**3*m - 3*c**3 + 3*c**2*d*m*x**2 - 9*c**2*d*x**2 + 3*c*d**2*m*x**
4 - 9*c*d**2*x**4 + d**3*m*x**6 - 3*d**3*x**6),x)*a*c*d**2*m**2*x**2 - 12*
int(x**m/(c**3*m - 3*c**3 + 3*c**2*d*m*x**2 - 9*c**2*d*x**2 + 3*c*d**2*m*x
**4 - 9*c*d**2*x**4 + d**3*m*x**6 - 3*d**3*x**6),x)*a*c*d**2*m*x**2 + 18*i
nt(x**m/(c**3*m - 3*c**3 + 3*c**2*d*m*x**2 - 9*c**2*d*x**2 + 3*c*d**2*m*x*
*4 - 9*c*d**2*x**4 + d**3*m*x**6 - 3*d**3*x**6),x)*a*c*d**2*x**2 + int(x**
m/(c**3*m - 3*c**3 + 3*c**2*d*m*x**2 - 9*c**2*d*x**2 + 3*c*d**2*m*x**4 - 9
*c*d**2*x**4 + d**3*m*x**6 - 3*d**3*x**6),x)*a*d**3*m**2*x**4 - 6*int(x**m
/(c**3*m - 3*c**3 + 3*c**2*d*m*x**2 - 9*c**2*d*x**2 + 3*c*d**2*m*x**4 - 9*
c*d**2*x**4 + d**3*m*x**6 - 3*d**3*x**6),x)*a*d**3*m*x**4 + 9*int(x**m/(c*
*3*m - 3*c**3 + 3*c**2*d*m*x**2 - 9*c**2*d*x**2 + 3*c*d**2*m*x**4 - 9*c*d*
*2*x**4 + d**3*m*x**6 - 3*d**3*x**6),x)*a*d**3*x**4 - int(x**m/(c**3*m - 3
*c**3 + 3*c**2*d*m*x**2 - 9*c**2*d*x**2 + 3*c*d**2*m*x**4 - 9*c*d**2*x**4
+ d**3*m*x**6 - 3*d**3*x**6),x)*b*c**3*m**2 + 2*int(x**m/(c**3*m - 3*c*...
```

3.41 $\int \frac{(ex)^m(A+Bx^2)}{(a+bx^2)(c+dx^2)^3}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 333

$$\int \frac{(ex)^m(A+Bx^2)}{(a+bx^2)(c+dx^2)^3}\,dx = \frac{(Bc-Ad)(ex)^{1+m}}{4c(bc-ad)e(c+dx^2)^2} + \frac{(bc(Bc(3-m)-Ad(7-m))+ad(Ad(3-m)+Bc(1+m)))(ex)^{1+m}}{8c^2(bc-ad)^2e(c+dx^2)} + \frac{b^2(Ab-aB)(ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{bx^2}{a}\right)}{a(bc-ad)^3e(1+m)} + \frac{(b^2c^2(Bc(1-m)-Ad(5-m))(3-m)-a^2d^2(1-m)(Ad(3-m)+Bc(1+m))+2abcd(Bc(3+2m\ldots}{8c^3(bc-ad)^3e(1+m\ldots}$$

output
```
1/4*(-A*d+B*c)*(e*x)^(1+m)/c/(-a*d+b*c)/e/(d*x^2+c)^2+1/8*(b*c*(B*c*(3-m)-
A*d*(7-m))+a*d*(A*d*(3-m)+B*c*(1+m)))*(e*x)^(1+m)/c^2/(-a*d+b*c)^2/e/(d*x^
2+c)+b^2*(A*b-B*a)*(e*x)^(1+m)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-b*x^2
/a)/a/(-a*d+b*c)^3/e/(1+m)+1/8*(b^2*c^2*(B*c*(1-m)-A*d*(5-m))*(3-m)-a^2*d^
2*(1-m)*(A*d*(3-m)+B*c*(1+m))+2*a*b*c*d*(B*c*(-m^2+2*m+3)+A*d*(m^2-6*m+5))
)*(e*x)^(1+m)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-d*x^2/c)/c^3/(-a*d+b*c
)^3/e/(1+m)
```

Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.59

$$\int \frac{(ex)^m (A+Bx^2)}{(a+bx^2)(c+dx^2)^3}\,dx = \frac{x(ex)^m\left(\frac{b^2(Ab-aB)\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{bx^2}{a}\right)}{a}-\frac{b(Ab-aB)d\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{dx^2}{c}\right)}{c}-\frac{(Ab-aB)d(}{}\right.}{(bc-ad)^3(1+m)}$$

input
```
Integrate[((e*x)^m*(A + B*x^2))/((a + b*x^2)*(c + d*x^2)^3),x]
```

output
```
(x*(e*x)^m*((b^2*(A*b - a*B)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -(
(b*x^2)/a)])/a - (b*(A*b - a*B)*d*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/
2, -((d*x^2)/c)])/c - ((A*b - a*B)*d*(b*c - a*d)*Hypergeometric2F1[2, (1 +
 m)/2, (3 + m)/2, -((d*x^2)/c)])/c^2 + ((b*c - a*d)^2*(B*c - A*d)*Hypergeo
metric2F1[3, (1 + m)/2, (3 + m)/2, -((d*x^2)/c)])/c^3))/((b*c - a*d)^3*(1
+ m))
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {441, 441, 446, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)(ex)^m}{(a+bx^2)(c+dx^2)^3}\,dx$$

$$\Big\downarrow \text{ 441}$$

$$\frac{\int \frac{(ex)^m(b(Bc-Ad)(3-m)x^2+4Abc-aAd(3-m)-aBc(m+1))}{(bx^2+a)(dx^2+c)^2}dx}{4c(bc-ad)}+\frac{(ex)^{m+1}(Bc-Ad)}{4ce(c+dx^2)^2(bc-ad)}$$

$$\Big\downarrow \text{ 441}$$

$$\frac{\frac{\int \frac{(ex)^m\left(b(1-m)(bc(Bc(3-m)-Ad(7-m))+ad(Ad(3-m)+Bc(m+1)))x^2+aBc(ad(1-m)-bc(5-m))(m+1)+A\left(8b^2c^2-abd\left(m^2-8m+7\right)c+a^2d^2\left(m^2-4m+3\right)\right)}{\left(bx^2+a\right)\left(dx^2+c\right)}}{2c(bc-ad)}}{4c(bc-ad)}$$

$$\frac{(ex)^{m+1}(Bc-Ad)}{4ce\left(c+dx^2\right)^2(bc-ad)}$$

$\Big\downarrow$ 446

$$\frac{\frac{\int \left(\frac{8b^2(Ab-aB)c^2(ex)^m}{(bc-ad)\left(bx^2+a\right)}+\frac{\left(b^2(Bc(1-m)-Ad(5-m))(3-m)c^2+2abd\left(Bc\left(-m^2+2m+3\right)+Ad\left(m^2-6m+5\right)\right)c-a^2d^2(1-m)(Ad(3-m)+Bc(m+1))\right)(ex)^m}{(bc-ad)\left(dx^2+c\right)}\right)dx}{2c(bc-ad)}}{4c(bc-ad)}$$

$$\frac{(ex)^{m+1}(Bc-Ad)}{4ce\left(c+dx^2\right)^2(bc-ad)}$$

$\Big\downarrow$ 2009

$$\frac{\frac{\frac{(ex)^{m+1}\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\frac{dx^2}{c}\right)\left(-a^2d^2(1-m)(Ad(3-m)+Bc(m+1))+2abcd\left(Ad\left(m^2-6m+5\right)+Bc\left(-m^2+2m+3\right)\right)+b^2c^2(3-m)(Bc(1-m)\right.}{ce(m+1)(bc-ad)}}{2c(bc-ad)}}{4c(bc}$$

$$\frac{(ex)^{m+1}(Bc-Ad)}{4ce\left(c+dx^2\right)^2(bc-ad)}$$

input

```
Int[((e*x)^m*(A + B*x^2))/((a + b*x^2)*(c + d*x^2)^3),x]
```

output

```
((B*c - A*d)*(e*x)^(1 + m))/(4*c*(b*c - a*d)*e*(c + d*x^2)^2) + (((b*c*(B*
c*(3 - m) - A*d*(7 - m)) + a*d*(A*d*(3 - m) + B*c*(1 + m)))*(e*x)^(1 + m))
/(2*c*(b*c - a*d)*e*(c + d*x^2)) + ((8*b^2*(A*b - a*B)*c^2*(e*x)^(1 + m)*H
ypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(a*(b*c - a*d)*e*
(1 + m)) + ((b^2*c^2*(B*c*(1 - m) - A*d*(5 - m))*(3 - m) - a^2*d^2*(1 - m)
*(A*d*(3 - m) + B*c*(1 + m)) + 2*a*b*c*d*(B*c*(3 + 2*m - m^2) + A*d*(5 - 6
*m + m^2)))*(e*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((d*
x^2)/c)])/(c*(b*c - a*d)*e*(1 + m)))/(2*c*(b*c - a*d)))/(4*c*(b*c - a*d))
```

Defintions of rubi rules used

rule 441
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
+ b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(a*g*2*(b*c - a*d)*(p + 1))), x] + Si
mp[1/(a*2*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2
)^q*Simp[c*(b*e - a*f)*(m + 1) + e*2*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(m
 + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, q},
 x] && LtQ[p, -1]
```

rule 446
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((e_) + (f_.)*(x_)^2))/(
(c_) + (d_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a + b*x^2)^
p*((e + f*x^2)/(c + d*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{(ex)^m (x^2B + A)}{(b\,x^2 + a)\,(x^2d + c)^3}dx$$

input
```
int((e*x)^m*(B*x^2+A)/(b*x^2+a)/(d*x^2+c)^3,x)
```

output
```
int((e*x)^m*(B*x^2+A)/(b*x^2+a)/(d*x^2+c)^3,x)
```

Fricas [F]

$$\int \frac{(ex)^m (A + Bx^2)}{(a + bx^2)(c + dx^2)^3}\,dx = \int \frac{(Bx^2 + A)(ex)^m}{(bx^2 + a)(dx^2 + c)^3}\,dx$$

input
```
integrate((e*x)^m*(B*x^2+A)/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="fricas")
```

output
```
integral((B*x^2 + A)*(e*x)^m/(b*d^3*x^8 + (3*b*c*d^2 + a*d^3)*x^6 + 3*(b*c
^2*d + a*c*d^2)*x^4 + a*c^3 + (b*c^3 + 3*a*c^2*d)*x^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ex)^m (A+Bx^2)}{(a+bx^2)(c+dx^2)^3} dx = \text{Timed out}$$

input
```
integrate((e*x)**m*(B*x**2+A)/(b*x**2+a)/(d*x**2+c)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(ex)^m (A+Bx^2)}{(a+bx^2)(c+dx^2)^3} dx = \int \frac{(Bx^2+A)(ex)^m}{(bx^2+a)(dx^2+c)^3} dx$$

input
```
integrate((e*x)^m*(B*x^2+A)/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*(e*x)^m/((b*x^2 + a)*(d*x^2 + c)^3), x)
```

Giac [F]

$$\int \frac{(ex)^m (A+Bx^2)}{(a+bx^2)(c+dx^2)^3} dx = \int \frac{(Bx^2+A)(ex)^m}{(bx^2+a)(dx^2+c)^3} dx$$

input
```
integrate((e*x)^m*(B*x^2+A)/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*(e*x)^m/((b*x^2 + a)*(d*x^2 + c)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m (A+Bx^2)}{(a+bx^2)(c+dx^2)^3}\,dx = \int \frac{(B\,x^2+A)\,(e\,x)^m}{(b\,x^2+a)\,(d\,x^2+c)^3}\,dx$$

input
```
int(((A + B*x^2)*(e*x)^m)/((a + b*x^2)*(c + d*x^2)^3),x)
```

output
```
int(((A + B*x^2)*(e*x)^m)/((a + b*x^2)*(c + d*x^2)^3), x)
```

Reduce [F]

$$\int \frac{(ex)^m (A+Bx^2)}{(a+bx^2)(c+dx^2)^3}\,dx = e^m\left(\int \frac{x^m}{d^3x^6+3c\,d^2x^4+3c^2d\,x^2+c^3}dx\right)$$

input
```
int((e*x)^m*(B*x^2+A)/(b*x^2+a)/(d*x^2+c)^3,x)
```

output
```
e**m*int(x**m/(c**3 + 3*c**2*d*x**2 + 3*c*d**2*x**4 + d**3*x**6),x)
```

3.42 $\int \frac{(ex)^m(A+Bx^2)}{(a+bx^2)^2(c+dx^2)^3}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 452

$$\int \frac{(ex)^m(A+Bx^2)}{(a+bx^2)^2(c+dx^2)^3}\,dx$$
$$= \frac{d(2Abc-3aBc+aAd)(ex)^{1+m}}{4ac(bc-ad)^2e(c+dx^2)^2} + \frac{(Ab-aB)(ex)^{1+m}}{2a(bc-ad)e(a+bx^2)(c+dx^2)^2}$$
$$+ \frac{d(A(4b^2c^2-a^2d^2(3-m)+abcd(11-m))-aBc(bc(11-m)+ad(1+m)))(ex)^{1+m}}{8ac^2(bc-ad)^3e(c+dx^2)}$$
$$+ \frac{b^2(Ab(bc(1-m)-ad(7-m))+aB(ad(5-m)+bc(1+m)))(ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\right.}{2a^2(bc-ad)^4e(1+m)}$$
$$- \frac{d(b^2c^2(Bc(3-m)-Ad(7-m))(5-m)-a^2d^2(1-m)(Ad(3-m)+Bc(1+m))+2abcd(Bc(5+4}{8c^3(bc-ad)^4e(1+n}$$

output
```
1/4*d*(A*a*d+2*A*b*c-3*B*a*c)*(e*x)^(1+m)/a/c/(-a*d+b*c)^2/e/(d*x^2+c)^2+1
/2*(A*b-B*a)*(e*x)^(1+m)/a/(-a*d+b*c)/e/(b*x^2+a)/(d*x^2+c)^2+1/8*d*(A*(4*
b^2*c^2-a^2*d^2*(3-m)+a*b*c*d*(11-m))-a*B*c*(b*c*(11-m)+a*d*(1+m)))*(e*x)^
(1+m)/a/c^2/(-a*d+b*c)^3/e/(d*x^2+c)+1/2*b^2*(A*b*(b*c*(1-m)-a*d*(7-m))+a*
B*(a*d*(5-m)+b*c*(1+m)))*(e*x)^(1+m)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],
-b*x^2/a)/a^2/(-a*d+b*c)^4/e/(1+m)-1/8*d*(b^2*c^2*(B*c*(3-m)-A*d*(7-m))*(5
-m)-a^2*d^2*(1-m)*(A*d*(3-m)+B*c*(1+m))+2*a*b*c*d*(B*c*(-m^2+4*m+5)+A*d*(m
^2-8*m+7)))*(e*x)^(1+m)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-d*x^2/c)/c^3
/(-a*d+b*c)^4/e/(1+m)
```

Mathematica [A] (verified)

Time = 1.13 (sec) , antiderivative size = 266, normalized size of antiderivative = 0.59

$$\int \frac{(ex)^m (A+Bx^2)}{(a+bx^2)^2 (c+dx^2)^3}\,dx$$
$$= \frac{x(ex)^m \left(\frac{b^2(bBc-3Abd+2aBd)\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{bx^2}{a}\right)}{a} - \frac{bd(bBc-3Abd+2aBd)\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\right.}{c}\right.}{}$$

input
```
Integrate[((e*x)^m*(A + B*x^2))/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

output
```
(x*(e*x)^m*((b^2*(b*B*c - 3*A*b*d + 2*a*B*d)*Hypergeometric2F1[1, (1 + m)/
2, (3 + m)/2, -((b*x^2)/a)])/a - (b*d*(b*B*c - 3*A*b*d + 2*a*B*d)*Hypergeo
metric2F1[1, (1 + m)/2, (3 + m)/2, -((d*x^2)/c)])/c + (b^2*(-(A*b) + a*B)*
(-(b*c) + a*d)*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/a
^2 - (d*(b*c - a*d)*(b*B*c - 2*A*b*d + a*B*d)*Hypergeometric2F1[2, (1 + m)
/2, (3 + m)/2, -((d*x^2)/c)])/c^2 + (d*(b*c - a*d)^2*(-(B*c) + A*d)*Hyperg
eometric2F1[3, (1 + m)/2, (3 + m)/2, -((d*x^2)/c)])/c^3))/((b*c - a*d)^4*(
1 + m))
```

Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 500, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {441, 441, 27, 441, 25, 446, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)(ex)^m}{(a+bx^2)^2(c+dx^2)^3}dx$$

$$\Big\downarrow \text{441}$$

$$\frac{(ex)^{m+1}(Ab-aB)}{2ae(a+bx^2)(c+dx^2)^2(bc-ad)}-\frac{\int \frac{(ex)^m(-((Ab-aB)d(5-m)x^2)+2aAd-Abc(1-m)-aBc(m+1))}{(bx^2+a)(dx^2+c)^3}dx}{2a(bc-ad)}$$

$$\Big\downarrow \text{441}$$

$$\frac{(ex)^{m+1}(Ab-aB)}{2ae(a+bx^2)(c+dx^2)^2(bc-ad)}-\frac{\frac{\int \frac{2(ex)^m\left(-bd(2Abc-3aBc+aAd)(3-m)x^2+A\left(-2b^2(1-m)c^2+8abdc-a^2d^2(3-m)\right)-aBc(2bc+ad)(m+1)\right)}{\left(bx^2+a\right)\left(dx^2+c\right)^2}dx}{4c(bc-ad)}-\frac{d(ex)^{m+1}(aAd-3aBc+2Abc)}{2ce(c+dx^2)^2(bc-ad)}}{2a(bc-ad)}$$

$$\Big\downarrow \text{27}$$

$$\frac{(ex)^{m+1}(Ab-aB)}{2ae(a+bx^2)(c+dx^2)^2(bc-ad)}-\frac{\frac{\int \frac{(ex)^m\left(-bd(2Abc-3aBc+aAd)(3-m)x^2+A\left(-2b^2(1-m)c^2+8abdc-a^2d^2(3-m)\right)-aBc(2bc+ad)(m+1)\right)}{\left(bx^2+a\right)\left(dx^2+c\right)^2}dx}{2c(bc-ad)}-\frac{d(ex)^{m+1}(aAd-3aBc+2Abc)}{2ce(c+dx^2)^2(bc-ad)}}{2a(bc-ad)}$$

$$\Big\downarrow \text{441}$$

$$\frac{(ex)^{m+1}(Ab-aB)}{2ae(a+bx^2)(c+dx^2)^2(bc-ad)}-\frac{\frac{\frac{\int -\frac{(ex)^m\left(bd(1-m)\left(A\left(4b^2c^2+abd(11-m)c-a^2d^2(3-m)\right)-aBc(bc(11-m)+ad(m+1))\right)x^2+aBc\left(4b^2c^2+abd(9-m)c-a^2d^2(1-m)\right)(m+1)-A\left(-4b^3(1-m)c^3+\cdots\right.}{\left(bx^2+a\right)\left(dx^2+c\right)}}{2c(bc-ad)}\cdots}{2c(bc-ad)}\cdots}{\cdots}$$

$$\Big\downarrow \text{25}$$

$$\frac{(ex)^{m+1}(Ab-aB)}{2ae\left(a+bx^2\right)\left(c+dx^2\right)^2(bc-ad)}-$$
$$-\frac{\int\frac{(ex)^m\left(bd(1-m)\left(A\left(4b^2c^2+abd(11-m)c-a^2d^2(3-m)\right)-aBc(bc(11-m)+ad(m+1))\right)x^2+aBc\left(4b^2c^2+abd(9-m)c-a^2d^2(1-m)\right)(m+1)-A\left(-4b^3(1-m)c^3+\right.}{\left(bx^2+a\right)\left(dx^2+c\right)}}{2c(bc-ad)}$$
$$2c(bc-ad)$$

$$\Big\downarrow \text{446}$$

$$\frac{(ex)^{m+1}(Ab-aB)}{2ae\left(a+bx^2\right)\left(c+dx^2\right)^2(bc-ad)}-$$
$$-\frac{\int\left(\frac{4b^2c^2(Ab(bc(1-m)-ad(7-m))+aB(ad(5-m)+bc(m+1)))(ex)^m}{(bc-ad)\left(bx^2+a\right)}+\frac{ad\left(-b^2(Bc(3-m)-Ad(7-m))(5-m)c^2-2abd\left(Bc\left(-m^2+4m+5\right)+Ad\left(m^2-8m+7\right)\right)c+\right.}{(bc-ad)\left(dx^2+c\right)}\right.}{2c(bc-ad)}$$
$$2c(bc-ad)$$
$$2a$$

$$\Big\downarrow \text{2009}$$

$$\frac{(ex)^{m+1}(Ab-aB)}{2ae\left(a+bx^2\right)\left(c+dx^2\right)^2(bc-ad)}-$$
$$-\frac{\frac{4b^2c^2(ex)^{m+1}\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\frac{bx^2}{a}\right)(Ab(bc(1-m)-ad(7-m))+aB(ad(5-m)+bc(m+1)))}{ae(m+1)(bc-ad)}-\frac{ad(ex)^{m+1}\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m}{2}\right.}{}}{2c(bc-ad)}$$

input

```
Int[((e*x)^m*(A + B*x^2))/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

output

```
((A*b - a*B)*(e*x)^(1 + m))/(2*a*(b*c - a*d)*e*(a + b*x^2)*(c + d*x^2)^2)
- (-1/2*(d*(2*A*b*c - 3*a*B*c + a*A*d)*(e*x)^(1 + m))/(c*(b*c - a*d)*e*(c
+ d*x^2)^2) + (-1/2*(d*(A*(4*b^2*c^2 - a^2*d^2*(3 - m) + a*b*c*d*(11 - m))
 - a*B*c*(b*c*(11 - m) + a*d*(1 + m)))*(e*x)^(1 + m))/(c*(b*c - a*d)*e*(c
+ d*x^2)) - ((4*b^2*c^2*(A*b*(b*c*(1 - m) - a*d*(7 - m)) + a*B*(a*d*(5 - m
) + b*c*(1 + m)))*(e*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2,
 -((b*x^2)/a)])/(a*(b*c - a*d)*e*(1 + m)) - (a*d*(b^2*c^2*(B*c*(3 - m) - A
*d*(7 - m))*(5 - m) - a^2*d^2*(1 - m)*(A*d*(3 - m) + B*c*(1 + m)) + 2*a*b*
c*d*(B*c*(5 + 4*m - m^2) + A*d*(7 - 8*m + m^2)))*(e*x)^(1 + m)*Hypergeomet
ric2F1[1, (1 + m)/2, (3 + m)/2, -((d*x^2)/c)])/(c*(b*c - a*d)*e*(1 + m)))/
(2*c*(b*c - a*d)))/(2*c*(b*c - a*d)))/(2*a*(b*c - a*d))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 441
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
+ b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(a*g*2*(b*c - a*d)*(p + 1))), x] + Si
mp[1/(a*2*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2
)^q*Simp[c*(b*e - a*f)*(m + 1) + e*2*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(m
 + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, q},
 x] && LtQ[p, -1]
```

rule 446
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((e_) + (f_.)*(x_)^2))/(
(c_) + (d_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a + b*x^2)^
p*((e + f*x^2)/(c + d*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{(ex)^m (x^2B + A)}{(b\,x^2 + a)^2 (x^2d + c)^3} dx$$

input
```
int((e*x)^m*(B*x^2+A)/(b*x^2+a)^2/(d*x^2+c)^3,x)
```

output
```
int((e*x)^m*(B*x^2+A)/(b*x^2+a)^2/(d*x^2+c)^3,x)
```

Fricas [F]

$$\int \frac{(ex)^m (A+Bx^2)}{(a+bx^2)^2 (c+dx^2)^3}\,dx = \int \frac{(Bx^2+A)(ex)^m}{(bx^2+a)^2(dx^2+c)^3}\,dx$$

input
```
integrate((e*x)^m*(B*x^2+A)/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
```

output
```
integral((B*x^2 + A)*(e*x)^m/(b^2*d^3*x^10 + (3*b^2*c*d^2 + 2*a*b*d^3)*x^8
 + (3*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^6 + a^2*c^3 + (b^2*c^3 + 6*a*b*
c^2*d + 3*a^2*c*d^2)*x^4 + (2*a*b*c^3 + 3*a^2*c^2*d)*x^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ex)^m (A+Bx^2)}{(a+bx^2)^2 (c+dx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((e*x)**m*(B*x**2+A)/(b*x**2+a)**2/(d*x**2+c)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(ex)^m (A+Bx^2)}{(a+bx^2)^2 (c+dx^2)^3}\,dx = \int \frac{(Bx^2+A)(ex)^m}{(bx^2+a)^2(dx^2+c)^3}\,dx$$

input
```
integrate((e*x)^m*(B*x^2+A)/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*(e*x)^m/((b*x^2 + a)^2*(d*x^2 + c)^3), x)
```

Giac [F]

$$\int \frac{(ex)^m (A+Bx^2)}{(a+bx^2)^2 (c+dx^2)^3}\,dx = \int \frac{(Bx^2+A)(ex)^m}{(bx^2+a)^2(dx^2+c)^3}\,dx$$

input
```
integrate((e*x)^m*(B*x^2+A)/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*(e*x)^m/((b*x^2 + a)^2*(d*x^2 + c)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m (A+Bx^2)}{(a+bx^2)^2 (c+dx^2)^3}\,dx = \int \frac{(B\,x^2+A)\,(e\,x)^m}{(b\,x^2+a)^2\,(d\,x^2+c)^3}\,dx$$

input
```
int(((A + B*x^2)*(e*x)^m)/((a + b*x^2)^2*(c + d*x^2)^3),x)
```

output
```
int(((A + B*x^2)*(e*x)^m)/((a + b*x^2)^2*(c + d*x^2)^3), x)
```

Reduce [F]

$$\int \frac{(ex)^m (A+Bx^2)}{(a+bx^2)^2 (c+dx^2)^3}\,dx$$
$$= e^m \left(\int \frac{x^m}{b\,d^3x^8 + a\,d^3x^6 + 3bc\,d^2x^6 + 3ac\,d^2x^4 + 3b\,c^2d\,x^4 + 3a\,c^2d\,x^2 + b\,c^3x^2 + a\,c^3}dx\right)$$

input
```
int((e*x)^m*(B*x^2+A)/(b*x^2+a)^2/(d*x^2+c)^3,x)
```

output
```
e**m*int(x**m/(a*c**3 + 3*a*c**2*d*x**2 + 3*a*c*d**2*x**4 + a*d**3*x**6 +
b*c**3*x**2 + 3*b*c**2*d*x**4 + 3*b*c*d**2*x**6 + b*d**3*x**8),x)
```

3.43 $\int \frac{(ex)^m(A+Bx^2)}{(a+bx^2)^3(c+dx^2)^3}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 665

$$\int \frac{(ex)^m (A+Bx^2)}{(a+bx^2)^3 (c+dx^2)^3}\,dx =$$
$$-\frac{d(A(2a^2d^2-b^2c^2(3-m)+abcd(13-m))-aBc(ad(11-m)+bc(1+m)))(ex)^{1+m}}{8a^2c(bc-ad)^3e(c+dx^2)^2}$$
$$+\frac{(Ab-aB)(ex)^{1+m}}{4a(bc-ad)e(a+bx^2)^2(c+dx^2)^2}$$
$$+\frac{(Ab(bc(3-m)-ad(11-m))+aB(ad(7-m)+bc(1+m)))(ex)^{1+m}}{8a^2(bc-ad)^2e(a+bx^2)(c+dx^2)^2}$$
$$+\frac{d(A(bc+ad)(b^2c^2(3-m)+a^2d^2(3-m)-2abcd(9-m))+aBc(2abcd(11-m)+b^2c^2(1+m)+a}{8a^2c^2(bc-ad)^4e(c+dx^2)}$$
$$+\frac{b^2(aB(b^2c^2(1-m^2)-2abcd(7+6m-m^2)-a^2d^2(35-12m+m^2))+Ab(a^2d^2(63-16m+m^2)-2}{8a^3(bc-ad)^5e}$$
$$+\frac{d^2(b^2c^2(Bc(5-m)-Ad(9-m))(7-m)-a^2d^2(1-m)(Ad(3-m)+Bc(1+m))+2abcd(Bc(7+}{8c^3(bc-ad)^5e(1+}$$

output
```
-1/8*d*(A*(2*a^2*d^2-b^2*c^2*(3-m)+a*b*c*d*(13-m))-a*B*c*(a*d*(11-m)+b*c*(
1+m)))*(e*x)^(1+m)/a^2/c/(-a*d+b*c)^3/e/(d*x^2+c)^2+1/4*(A*b-B*a)*(e*x)^(1
+m)/a/(-a*d+b*c)/e/(b*x^2+a)^2/(d*x^2+c)^2+1/8*(A*b*(b*c*(3-m)-a*d*(11-m))
+a*B*(a*d*(7-m)+b*c*(1+m)))*(e*x)^(1+m)/a^2/(-a*d+b*c)^2/e/(b*x^2+a)/(d*x^
2+c)^2+1/8*d*(A*(a*d+b*c)*(b^2*c^2*(3-m)+a^2*d^2*(3-m)-2*a*b*c*d*(9-m))+a*
B*c*(2*a*b*c*d*(11-m)+b^2*c^2*(1+m)+a^2*d^2*(1+m)))*(e*x)^(1+m)/a^2/c^2/(-
a*d+b*c)^4/e/(d*x^2+c)+1/8*b^2*(a*B*(b^2*c^2*(-m^2+1)-2*a*b*c*d*(-m^2+6*m+
7)-a^2*d^2*(m^2-12*m+35))+A*b*(a^2*d^2*(m^2-16*m+63)-2*a*b*c*d*(m^2-10*m+9
)+b^2*c^2*(m^2-4*m+3)))*(e*x)^(1+m)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-
b*x^2/a)/a^3/(-a*d+b*c)^5/e/(1+m)+1/8*d^2*(b^2*c^2*(B*c*(5-m)-A*d*(9-m))*(
7-m)-a^2*d^2*(1-m)*(A*d*(3-m)+B*c*(1+m))+2*a*b*c*d*(B*c*(-m^2+6*m+7)+A*d*(
m^2-10*m+9)))*(e*x)^(1+m)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-d*x^2/c)/c
^3/(-a*d+b*c)^5/e/(1+m)
```

Mathematica [A] (verified)

Time = 1.42 (sec) , antiderivative size = 329, normalized size of antiderivative = 0.49

$$\int \frac{(ex)^m (A+Bx^2)}{(a+bx^2)^3 (c+dx^2)^3} \, dx = \frac{x(ex)^m \left(-\frac{3b^2d(bBc-2Abd+aBd)\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{bx^2}{a}\right)}{a} + \frac{3bd^2(bBc-2Abd+aBd)\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2}\cdots\right.}{c}\cdots\right.}{\cdots}$$

input
```
Integrate[((e*x)^m*(A + B*x^2))/((a + b*x^2)^3*(c + d*x^2)^3),x]
```

output
```
(x*(e*x)^m*((-3*b^2*d*(b*B*c - 2*A*b*d + a*B*d)*Hypergeometric2F1[1, (1 +
m)/2, (3 + m)/2, -((b*x^2)/a)])/a + (3*b*d^2*(b*B*c - 2*A*b*d + a*B*d)*Hyp
ergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((d*x^2)/c)])/c + (b^2*(b*c - a*d
)*(b*B*c - 3*A*b*d + 2*a*B*d)*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, -
((b*x^2)/a)])/a^2 + (d^2*(b*c - a*d)*(2*b*B*c - 3*A*b*d + a*B*d)*Hypergeom
etric2F1[2, (1 + m)/2, (3 + m)/2, -((d*x^2)/c)])/c^2 + (b^2*(A*b - a*B)*(b
*c - a*d)^2*Hypergeometric2F1[3, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/a^3
+ (d^2*(b*c - a*d)^2*(B*c - A*d)*Hypergeometric2F1[3, (1 + m)/2, (3 + m)/2
, -((d*x^2)/c)])/c^3))/((b*c - a*d)^5*(1 + m))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)(ex)^m}{(a+bx^2)^3(c+dx^2)^3}dx$$

$$\Big\downarrow \ 441$$

$$\frac{(ex)^{m+1}(Ab-aB)}{4ae(a+bx^2)^2(c+dx^2)^2(bc-ad)}-\frac{\int \frac{(ex)^m(-((Ab-aB)d(7-m)x^2)+4aAd-Abc(3-m)-aBc(m+1))}{(bx^2+a)^2(dx^2+c)^3}dx}{4a(bc-ad)}$$

$$\Big\downarrow \ 441$$

$$\frac{(ex)^{m+1}(Ab-aB)}{4ae(a+bx^2)^2(c+dx^2)^2(bc-ad)}-$$

$$-\frac{\int -\frac{(ex)^m\left(-d(5-m)(Ab(bc(3-m)-ad(11-m))+aB(ad(7-m)+bc(m+1)))x^2+aBc(m+1)(ad(9-m)-b(c-cm))-A\left(b^2\left(m^2-4m+3\right)c^2-abd\left(m^2-12m+3\right)c+\right.\right.}{\left(bx^2+a\right)\left(dx^2+c\right)^3}}{2a(bc-ad)}$$

$$4a(bc-ad)$$

$$\Big\downarrow \ 25$$

$$\frac{(ex)^{m+1}(Ab-aB)}{4ae(a+bx^2)^2(c+dx^2)^2(bc-ad)}-$$

$$\frac{\int -\frac{(ex)^m\left(d(5-m)(Ab(bc(3-m)-ad(11-m))+aB(ad(7-m)+bc(m+1)))x^2+aBc(bc(1-m)-ad(9-m))(m+1)+A\left(b^2\left(m^2-4m+3\right)c^2-abd\left(m^2-12m+3\right)c+8a^2\right.\right.}{\left(bx^2+a\right)\left(dx^2+c\right)^3}}{2a(bc-ad)}$$

$$4a(bc-ad)$$

$$\Big\downarrow \ 25$$

$$\frac{(ex)^{m+1}(Ab-aB)}{4ae(a+bx^2)^2(c+dx^2)^2(bc-ad)}-$$

$$-\frac{\int -\frac{(ex)^m\left(-d(5-m)(Ab(bc(3-m)-ad(11-m))+aB(ad(7-m)+bc(m+1)))x^2+aBc(m+1)(ad(9-m)-b(c-cm))-A\left(b^2\left(m^2-4m+3\right)c^2-abd\left(m^2-12m+3\right)c+\right.\right.}{\left(bx^2+a\right)\left(dx^2+c\right)^3}}{2a(bc-ad)}$$

$$4a(bc-ad)$$

$$\Big\downarrow \ 25$$

$$\frac{(ex)^{m+1}(Ab-aB)}{4ae(a+bx^2)^2(c+dx^2)^2(bc-ad)}-$$

$$\frac{\int -\frac{(ex)^m\left(d(5-m)(Ab(bc(3-m)-ad(11-m))+aB(ad(7-m)+bc(m+1)))x^2+aBc(bc(1-m)-ad(9-m))(m+1)+A\left(b^2\left(m^2-4m+3\right)c^2-abd\left(m^2-12m+3\right)c+8a^2\right.\right.}{\left(bx^2+a\right)\left(dx^2+c\right)^3}}{2a(bc-ad)}$$

$$4a(bc-ad)$$

$\Big\downarrow$ 25

$$\frac{(ex)^{m+1}(Ab-aB)}{4ae\,(a+bx^2)^2\,(c+dx^2)^2\,(bc-ad)}-\frac{-\frac{\int -\frac{(ex)^m(-d(5-m)(Ab(bc(3-m)-ad(11-m))+aB(ad(7-m)+bc(m+1)))x^2+aBc(m+1)(ad(9-m)-b(c-cm))-A(b^2(m^2-4m+3)c^2-abd(m^2-12m+3)c+}{(bx^2+a)(dx^2+c)^3}}{2a(bc-ad)}}{4a(bc-ad)}$$

$\Big\downarrow$ 25

$$\frac{(ex)^{m+1}(Ab-aB)}{4ae\,(a+bx^2)^2\,(c+dx^2)^2\,(bc-ad)}-\frac{\frac{\int -\frac{(ex)^m(d(5-m)(Ab(bc(3-m)-ad(11-m))+aB(ad(7-m)+bc(m+1)))x^2+aBc(bc(1-m)-ad(9-m))(m+1)+A(b^2(m^2-4m+3)c^2-abd(m^2-12m+3)c+8a^2}{(bx^2+a)(dx^2+c)^3}}{2a(bc-ad)}}{4a(bc-ad)}$$

$\Big\downarrow$ 25

$$\frac{(ex)^{m+1}(Ab-aB)}{4ae\,(a+bx^2)^2\,(c+dx^2)^2\,(bc-ad)}-\frac{-\frac{\int -\frac{(ex)^m(-d(5-m)(Ab(bc(3-m)-ad(11-m))+aB(ad(7-m)+bc(m+1)))x^2+aBc(m+1)(ad(9-m)-b(c-cm))-A(b^2(m^2-4m+3)c^2-abd(m^2-12m+3)c+}{(bx^2+a)(dx^2+c)^3}}{2a(bc-ad)}}{4a(bc-ad)}$$

$\Big\downarrow$ 25

$$\frac{(ex)^{m+1}(Ab-aB)}{4ae\,(a+bx^2)^2\,(c+dx^2)^2\,(bc-ad)}-\frac{\frac{\int -\frac{(ex)^m(d(5-m)(Ab(bc(3-m)-ad(11-m))+aB(ad(7-m)+bc(m+1)))x^2+aBc(bc(1-m)-ad(9-m))(m+1)+A(b^2(m^2-4m+3)c^2-abd(m^2-12m+3)c+8a^2}{(bx^2+a)(dx^2+c)^3}}{2a(bc-ad)}}{4a(bc-ad)}$$

$\Big\downarrow$ 25

$$\frac{(ex)^{m+1}(Ab-aB)}{4ae\,(a+bx^2)^2\,(c+dx^2)^2\,(bc-ad)}-\frac{-\frac{\int -\frac{(ex)^m(-d(5-m)(Ab(bc(3-m)-ad(11-m))+aB(ad(7-m)+bc(m+1)))x^2+aBc(m+1)(ad(9-m)-b(c-cm))-A(b^2(m^2-4m+3)c^2-abd(m^2-12m+3)c+}{(bx^2+a)(dx^2+c)^3}}{2a(bc-ad)}}{4a(bc-ad)}$$

$\Big\downarrow$ 25

$$\frac{(ex)^{m+1}(Ab-aB)}{4ae\left(a+bx^2\right)^2\left(c+dx^2\right)^2(bc-ad)}-$$
$$\int -\frac{(ex)^m\left(d(5-m)(Ab(bc(3-m)-ad(11-m))+aB(ad(7-m)+bc(m+1)))x^2+aBc(bc(1-m)-ad(9-m))(m+1)+A\left(b^2\left(m^2-4m+3\right)c^2-abd\left(m^2-12m+3\right)c+8a^2 \right.\right.}{\left(bx^2+a\right)\left(dx^2+c\right)^3}$$
$$2a(bc-ad)$$
$$4a(bc-ad)$$

$\downarrow$ 25

$$\frac{(ex)^{m+1}(Ab-aB)}{4ae\left(a+bx^2\right)^2\left(c+dx^2\right)^2(bc-ad)}-$$
$$-\int -\frac{(ex)^m\left(-d(5-m)(Ab(bc(3-m)-ad(11-m))+aB(ad(7-m)+bc(m+1)))x^2+aBc(m+1)(ad(9-m)-b(c-cm))-A\left(b^2\left(m^2-4m+3\right)c^2-abd\left(m^2-12m+3\right)c+ \right.\right.}{\left(bx^2+a\right)\left(dx^2+c\right)^3}$$
$$2a(bc-ad)$$
$$4a(bc-ad)$$

$\downarrow$ 25

$$\frac{(ex)^{m+1}(Ab-aB)}{4ae\left(a+bx^2\right)^2\left(c+dx^2\right)^2(bc-ad)}-$$
$$\int -\frac{(ex)^m\left(d(5-m)(Ab(bc(3-m)-ad(11-m))+aB(ad(7-m)+bc(m+1)))x^2+aBc(bc(1-m)-ad(9-m))(m+1)+A\left(b^2\left(m^2-4m+3\right)c^2-abd\left(m^2-12m+3\right)c+8a^2 \right.\right.}{\left(bx^2+a\right)\left(dx^2+c\right)^3}$$
$$2a(bc-ad)$$
$$4a(bc-ad)$$

$\downarrow$ 25

$$\frac{(ex)^{m+1}(Ab-aB)}{4ae\left(a+bx^2\right)^2\left(c+dx^2\right)^2(bc-ad)}-$$
$$-\int -\frac{(ex)^m\left(-d(5-m)(Ab(bc(3-m)-ad(11-m))+aB(ad(7-m)+bc(m+1)))x^2+aBc(m+1)(ad(9-m)-b(c-cm))-A\left(b^2\left(m^2-4m+3\right)c^2-abd\left(m^2-12m+3\right)c+ \right.\right.}{\left(bx^2+a\right)\left(dx^2+c\right)^3}$$
$$2a(bc-ad)$$
$$4a(bc-ad)$$

$\downarrow$ 25

$$\frac{(ex)^{m+1}(Ab-aB)}{4ae\left(a+bx^2\right)^2\left(c+dx^2\right)^2(bc-ad)}-$$
$$\int -\frac{(ex)^m\left(d(5-m)(Ab(bc(3-m)-ad(11-m))+aB(ad(7-m)+bc(m+1)))x^2+aBc(bc(1-m)-ad(9-m))(m+1)+A\left(b^2\left(m^2-4m+3\right)c^2-abd\left(m^2-12m+3\right)c+8a^2 \right.\right.}{\left(bx^2+a\right)\left(dx^2+c\right)^3}$$
$$2a(bc-ad)$$
$$4a(bc-ad)$$

$\downarrow$ 25

$$\frac{(ex)^{m+1}(Ab-aB)}{4ae\left(a+bx^2\right)^2\left(c+dx^2\right)^2(bc-ad)}-\frac{-\frac{\int -\frac{(ex)^m\left(-d(5-m)(Ab(bc(3-m)-ad(11-m))+aB(ad(7-m)+bc(m+1)))x^2+aBc(m+1)(ad(9-m)-b(c-cm))-A\left(b^2\left(m^2-4m+3\right)c^2-abd\left(m^2-12m+3\right)c+\cdots\right.\right.}{\left(bx^2+a\right)\left(dx^2+c\right)^3}}{2a(bc-ad)}}{4a(bc-ad)}$$

$$\Big\downarrow \; 25$$

$$\frac{(ex)^{m+1}(Ab-aB)}{4ae\left(a+bx^2\right)^2\left(c+dx^2\right)^2(bc-ad)}-\frac{\frac{\int -\frac{(ex)^m\left(d(5-m)(Ab(bc(3-m)-ad(11-m))+aB(ad(7-m)+bc(m+1)))x^2+aBc(bc(1-m)-ad(9-m))(m+1)+A\left(b^2\left(m^2-4m+3\right)c^2-abd\left(m^2-12m+3\right)c+8a^2c\cdots\right.\right.}{\left(bx^2+a\right)\left(dx^2+c\right)^3}}{2a(bc-ad)}}{4a(bc-ad)}$$

$$\Big\downarrow \; 25$$

$$\frac{(ex)^{m+1}(Ab-aB)}{4ae\left(a+bx^2\right)^2\left(c+dx^2\right)^2(bc-ad)}-\frac{-\frac{\int -\frac{(ex)^m\left(-d(5-m)(Ab(bc(3-m)-ad(11-m))+aB(ad(7-m)+bc(m+1)))x^2+aBc(m+1)(ad(9-m)-b(c-cm))-A\left(b^2\left(m^2-4m+3\right)c^2-abd\left(m^2-12m+3\right)c+\cdots\right.\right.}{\left(bx^2+a\right)\left(dx^2+c\right)^3}}{2a(bc-ad)}}{4a(bc-ad)}$$

$$\Big\downarrow \; 25$$

$$\frac{(ex)^{m+1}(Ab-aB)}{4ae\left(a+bx^2\right)^2\left(c+dx^2\right)^2(bc-ad)}-\frac{\frac{\int -\frac{(ex)^m\left(d(5-m)(Ab(bc(3-m)-ad(11-m))+aB(ad(7-m)+bc(m+1)))x^2+aBc(bc(1-m)-ad(9-m))(m+1)+A\left(b^2\left(m^2-4m+3\right)c^2-abd\left(m^2-12m+3\right)c+8a^2c\cdots\right.\right.}{\left(bx^2+a\right)\left(dx^2+c\right)^3}}{2a(bc-ad)}}{4a(bc-ad)}$$

$$\Big\downarrow \; 25$$

$$\frac{(ex)^{m+1}(Ab-aB)}{4ae\left(a+bx^2\right)^2\left(c+dx^2\right)^2(bc-ad)}-\frac{-\frac{\int -\frac{(ex)^m\left(-d(5-m)(Ab(bc(3-m)-ad(11-m))+aB(ad(7-m)+bc(m+1)))x^2+aBc(m+1)(ad(9-m)-b(c-cm))-A\left(b^2\left(m^2-4m+3\right)c^2-abd\left(m^2-12m+3\right)c+\cdots\right.\right.}{\left(bx^2+a\right)\left(dx^2+c\right)^3}}{2a(bc-ad)}}{4a(bc-ad)}$$

$$\Big\downarrow \; 25$$

$$\frac{(ex)^{m+1}(Ab-aB)}{4ae\left(a+bx^2\right)^2\left(c+dx^2\right)^2(bc-ad)} - \frac{\frac{\int -\frac{(ex)^m\left(d(5-m)(Ab(bc(3-m)-ad(11-m))+aB(ad(7-m)+bc(m+1)))x^2+aBc(bc(1-m)-ad(9-m))(m+1)+A\left(b^2\left(m^2-4m+3\right)c^2-abd\left(m^2-12m+3\right)c+8a^2 \cdots}{\left(bx^2+a\right)\left(dx^2+c\right)^3}}{2a(bc-ad)} \cdots}{4a(bc-ad)}$$

$\downarrow$ 25

$$\frac{(ex)^{m+1}(Ab-aB)}{4ae\left(a+bx^2\right)^2\left(c+dx^2\right)^2(bc-ad)} - \frac{-\frac{\int -\frac{(ex)^m\left(-d(5-m)(Ab(bc(3-m)-ad(11-m))+aB(ad(7-m)+bc(m+1)))x^2+aBc(m+1)(ad(9-m)-b(c-cm))-A\left(b^2\left(m^2-4m+3\right)c^2-abd\left(m^2-12m+3\right)c+ \cdots}{\left(bx^2+a\right)\left(dx^2+c\right)^3}}{2a(bc-ad)} \cdots}{4a(bc-ad)}$$

$\downarrow$ 25

$$\frac{(ex)^{m+1}(Ab-aB)}{4ae\left(a+bx^2\right)^2\left(c+dx^2\right)^2(bc-ad)} - \frac{\frac{\int -\frac{(ex)^m\left(d(5-m)(Ab(bc(3-m)-ad(11-m))+aB(ad(7-m)+bc(m+1)))x^2+aBc(bc(1-m)-ad(9-m))(m+1)+A\left(b^2\left(m^2-4m+3\right)c^2-abd\left(m^2-12m+3\right)c+8a^2 \cdots}{\left(bx^2+a\right)\left(dx^2+c\right)^3}}{2a(bc-ad)} \cdots}{4a(bc-ad)}$$

$\downarrow$ 25

$$\frac{(ex)^{m+1}(Ab-aB)}{4ae\left(a+bx^2\right)^2\left(c+dx^2\right)^2(bc-ad)} - \frac{-\frac{\int -\frac{(ex)^m\left(-d(5-m)(Ab(bc(3-m)-ad(11-m))+aB(ad(7-m)+bc(m+1)))x^2+aBc(m+1)(ad(9-m)-b(c-cm))-A\left(b^2\left(m^2-4m+3\right)c^2-abd\left(m^2-12m+3\right)c+ \cdots}{\left(bx^2+a\right)\left(dx^2+c\right)^3}}{2a(bc-ad)} \cdots}{4a(bc-ad)}$$

$\downarrow$ 25

$$\frac{(ex)^{m+1}(Ab-aB)}{4ae\left(a+bx^2\right)^2\left(c+dx^2\right)^2(bc-ad)} - \frac{\frac{\int -\frac{(ex)^m\left(d(5-m)(Ab(bc(3-m)-ad(11-m))+aB(ad(7-m)+bc(m+1)))x^2+aBc(bc(1-m)-ad(9-m))(m+1)+A\left(b^2\left(m^2-4m+3\right)c^2-abd\left(m^2-12m+3\right)c+8a^2 \cdots}{\left(bx^2+a\right)\left(dx^2+c\right)^3}}{2a(bc-ad)} \cdots}{4a(bc-ad)}$$

$\downarrow$ 25

$$\frac{(ex)^{m+1}(Ab-aB)}{4ae\left(a+bx^2\right)^2\left(c+dx^2\right)^2(bc-ad)}-\frac{-\frac{\int -\frac{(ex)^m\left(-d(5-m)(Ab(bc(3-m)-ad(11-m))+aB(ad(7-m)+bc(m+1)))x^2+aBc(m+1)(ad(9-m)-b(c-cm))-A\left(b^2\left(m^2-4m+3\right)c^2-abd\left(m^2-12m+3\right)c+\right.\right.}{\left(bx^2+a\right)\left(dx^2+c\right)^3}}{2a(bc-ad)}}{4a(bc-ad)}$$

$\downarrow$ 25

$$\frac{(ex)^{m+1}(Ab-aB)}{4ae\left(a+bx^2\right)^2\left(c+dx^2\right)^2(bc-ad)}-\frac{\frac{\int -\frac{(ex)^m\left(d(5-m)(Ab(bc(3-m)-ad(11-m))+aB(ad(7-m)+bc(m+1)))x^2+aBc(bc(1-m)-ad(9-m))(m+1)+A\left(b^2\left(m^2-4m+3\right)c^2-abd\left(m^2-12m+3\right)c+8a^2c\right.\right.}{\left(bx^2+a\right)\left(dx^2+c\right)^3}}{2a(bc-ad)}}{4a(bc-ad)}$$

$\downarrow$ 25

$$\frac{(ex)^{m+1}(Ab-aB)}{4ae\left(a+bx^2\right)^2\left(c+dx^2\right)^2(bc-ad)}-\frac{-\frac{\int -\frac{(ex)^m\left(-d(5-m)(Ab(bc(3-m)-ad(11-m))+aB(ad(7-m)+bc(m+1)))x^2+aBc(m+1)(ad(9-m)-b(c-cm))-A\left(b^2\left(m^2-4m+3\right)c^2-abd\left(m^2-12m+3\right)c+\right.\right.}{\left(bx^2+a\right)\left(dx^2+c\right)^3}}{2a(bc-ad)}}{4a(bc-ad)}$$

$\downarrow$ 25

$$\frac{(ex)^{m+1}(Ab-aB)}{4ae\left(a+bx^2\right)^2\left(c+dx^2\right)^2(bc-ad)}-\frac{\frac{\int -\frac{(ex)^m\left(d(5-m)(Ab(bc(3-m)-ad(11-m))+aB(ad(7-m)+bc(m+1)))x^2+aBc(bc(1-m)-ad(9-m))(m+1)+A\left(b^2\left(m^2-4m+3\right)c^2-abd\left(m^2-12m+3\right)c+8a^2c\right.\right.}{\left(bx^2+a\right)\left(dx^2+c\right)^3}}{2a(bc-ad)}}{4a(bc-ad)}$$

$\downarrow$ 25

$$\frac{(ex)^{m+1}(Ab-aB)}{4ae\left(a+bx^2\right)^2\left(c+dx^2\right)^2(bc-ad)}-\frac{-\frac{\int -\frac{(ex)^m\left(-d(5-m)(Ab(bc(3-m)-ad(11-m))+aB(ad(7-m)+bc(m+1)))x^2+aBc(m+1)(ad(9-m)-b(c-cm))-A\left(b^2\left(m^2-4m+3\right)c^2-abd\left(m^2-12m+3\right)c+\right.\right.}{\left(bx^2+a\right)\left(dx^2+c\right)^3}}{2a(bc-ad)}}{4a(bc-ad)}$$

input
```
Int[((e*x)^m*(A + B*x^2))/((a + b*x^2)^3*(c + d*x^2)^3),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 441
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
+ b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(a*g*2*(b*c - a*d)*(p + 1))), x] + Si
mp[1/(a*2*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2
)^q*Simp[c*(b*e - a*f)*(m + 1) + e*2*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(m
 + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, q},
 x] && LtQ[p, -1]
```

Maple [F]

$$\int \frac{(ex)^m (x^2B + A)}{(b\,x^2 + a)^3 (x^2d + c)^3} dx$$

input
```
int((e*x)^m*(B*x^2+A)/(b*x^2+a)^3/(d*x^2+c)^3,x)
```

output
```
int((e*x)^m*(B*x^2+A)/(b*x^2+a)^3/(d*x^2+c)^3,x)
```

Fricas [F]

$$\int \frac{(ex)^m (A + Bx^2)}{(a + bx^2)^3 (c + dx^2)^3} \, dx = \int \frac{(Bx^2 + A)(ex)^m}{(bx^2 + a)^3 (dx^2 + c)^3} \, dx$$

input
```
integrate((e*x)^m*(B*x^2+A)/(b*x^2+a)^3/(d*x^2+c)^3,x, algorithm="fricas")
```

output
```
integral((B*x^2 + A)*(e*x)^m/(b^3*d^3*x^12 + 3*(b^3*c*d^2 + a*b^2*d^3)*x^1
0 + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*x^8 + (b^3*c^3 + 9*a*b^2*c^2
*d + 9*a^2*b*c*d^2 + a^3*d^3)*x^6 + a^3*c^3 + 3*(a*b^2*c^3 + 3*a^2*b*c^2*d
 + a^3*c*d^2)*x^4 + 3*(a^2*b*c^3 + a^3*c^2*d)*x^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ex)^m (A + Bx^2)}{(a + bx^2)^3 (c + dx^2)^3} \, dx = \text{Timed out}$$

input
```
integrate((e*x)**m*(B*x**2+A)/(b*x**2+a)**3/(d*x**2+c)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(ex)^m (A + Bx^2)}{(a + bx^2)^3 (c + dx^2)^3} \, dx = \int \frac{(Bx^2 + A)(ex)^m}{(bx^2 + a)^3 (dx^2 + c)^3} \, dx$$

input
```
integrate((e*x)^m*(B*x^2+A)/(b*x^2+a)^3/(d*x^2+c)^3,x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*(e*x)^m/((b*x^2 + a)^3*(d*x^2 + c)^3), x)
```

Giac [F]

$$\int \frac{(ex)^m (A + Bx^2)}{(a + bx^2)^3 (c + dx^2)^3} \, dx = \int \frac{(Bx^2 + A)(ex)^m}{(bx^2 + a)^3 (dx^2 + c)^3} \, dx$$

input
```
integrate((e*x)^m*(B*x^2+A)/(b*x^2+a)^3/(d*x^2+c)^3,x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*(e*x)^m/((b*x^2 + a)^3*(d*x^2 + c)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m (A+Bx^2)}{(a+bx^2)^3 (c+dx^2)^3} dx = \int \frac{(B x^2+A) (e x)^m}{(b x^2+a)^3 (d x^2+c)^3} dx$$

input
```
int(((A + B*x^2)*(e*x)^m)/((a + b*x^2)^3*(c + d*x^2)^3),x)
```

output
```
int(((A + B*x^2)*(e*x)^m)/((a + b*x^2)^3*(c + d*x^2)^3), x)
```

Reduce [F]

$$\int \frac{(ex)^m (A+Bx^2)}{(a+bx^2)^3 (c+dx^2)^3} dx$$
$$= e^m \left(\int \frac{x^m}{b^2d^3x^{10} + 2ab\,d^3x^8 + 3b^2c\,d^2x^8 + a^2d^3x^6 + 6abc\,d^2x^6 + 3b^2c^2d\,x^6 + 3a^2c\,d^2x^4 + 6ab\,c^2d\,x^4 + b^2c^3"$$

input
```
int((e*x)^m*(B*x^2+A)/(b*x^2+a)^3/(d*x^2+c)^3,x)
```

output
```
e**m*int(x**m/(a**2*c**3 + 3*a**2*c**2*d*x**2 + 3*a**2*c*d**2*x**4 + a**2*
d**3*x**6 + 2*a*b*c**3*x**2 + 6*a*b*c**2*d*x**4 + 6*a*b*c*d**2*x**6 + 2*a*
b*d**3*x**8 + b**2*c**3*x**4 + 3*b**2*c**2*d*x**6 + 3*b**2*c*d**2*x**8 + b
**2*d**3*x**10),x)
```

3.44 $\int (ex)^m (a+bx^2)^p (A+Bx^2) (c+dx^2)^3 \, dx$

Optimal result

Integrand size = 31, antiderivative size = 688

$$\int (ex)^m (a+bx^2)^p (A+Bx^2) (c+dx^2)^3 \, dx =$$

$$-\frac{(a^3Bd^3(105+71m+15m^2+m^3) - a^2bd^2(3Bc+Ad)(15+8m+m^2)(9+m+2p) + 3ab^2cd(Bc+ \ldots}{\ldots}$$

$$+\frac{d(a^2Bd^2(35+12m+m^2) - abd(3Bc+Ad)(5+m)(9+m+2p) + 3b^2c(Bc+Ad)(63+m^2+32p+ \ldots}{b^3e^3(5+m+2p)(7+m+2p)(9+m+2p)}$$

$$-\frac{d^2(aBd(7+m) - b(3Bc+Ad)(9+m+2p))(ex)^{5+m}(a+bx^2)^{1+p}}{b^2e^5(7+m+2p)(9+m+2p)}$$

$$+\frac{Bd^3(ex)^{7+m}(a+bx^2)^{1+p}}{be^7(9+m+2p)}$$

$$+\frac{\left(\frac{Ac^3}{1+m} + \frac{a(a^3Bd^3(105+71m+15m^2+m^3) - a^2bd^2(3Bc+Ad)(15+8m+m^2)(9+m+2p) + 3ab^2cd(Bc+Ad)(3+m)(63+m^2+32p+4p^2+4m(\ldots}{b^4(3+m+2p)(5+m+2p)(7+m+2p)(9+m+\ldots}\ldots}{\ldots}$$

output
```
-(a^3*B*d^3*(m^3+15*m^2+71*m+105)-a^2*b*d^2*(A*d+3*B*c)*(m^2+8*m+15)*(9+m+
2*p)+3*a*b^2*c*d*(A*d+B*c)*(3+m)*(63+m^2+32*p+4*p^2+4*m*(4+p))-b^3*c^2*(3*
A*d+B*c)*(315+m^3+286*p+84*p^2+8*p^3+3*m^2*(7+2*p)+m*(12*p^2+84*p+143)))*(
e*x)^(1+m)*(b*x^2+a)^(p+1)/b^4/e/(3+m+2*p)/(5+m+2*p)/(7+m+2*p)/(9+m+2*p)+d
*(a^2*B*d^2*(m^2+12*m+35)-a*b*d*(A*d+3*B*c)*(5+m)*(9+m+2*p)+3*b^2*c*(A*d+B
*c)*(63+m^2+32*p+4*p^2+4*m*(4+p)))*(e*x)^(3+m)*(b*x^2+a)^(p+1)/b^3/e^3/(5+
m+2*p)/(7+m+2*p)/(9+m+2*p)-d^2*(a*B*d*(7+m)-b*(A*d+3*B*c)*(9+m+2*p))*(e*x)
^(5+m)*(b*x^2+a)^(p+1)/b^2/e^5/(7+m+2*p)/(9+m+2*p)+B*d^3*(e*x)^(7+m)*(b*x^
2+a)^(p+1)/b/e^7/(9+m+2*p)+(A*c^3/(1+m)+a*(a^3*B*d^3*(m^3+15*m^2+71*m+105)
-a^2*b*d^2*(A*d+3*B*c)*(m^2+8*m+15)*(9+m+2*p)+3*a*b^2*c*d*(A*d+B*c)*(3+m)*
(63+m^2+32*p+4*p^2+4*m*(4+p))-b^3*c^2*(3*A*d+B*c)*(315+m^3+286*p+84*p^2+8*
p^3+3*m^2*(7+2*p)+m*(12*p^2+84*p+143)))/b^4/(3+m+2*p)/(5+m+2*p)/(7+m+2*p)/
(9+m+2*p))*(e*x)^(1+m)*(b*x^2+a)^p*hypergeom([-p, 1/2+1/2*m],[3/2+1/2*m],-
b*x^2/a)/e/((1+b*x^2/a)^p)
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.36

$$\int (ex)^m \left(a+bx^2\right)^p \left(A+Bx^2\right) \left(c+dx^2\right)^3 \, dx$$

$$= x(ex)^m \left(a+bx^2\right)^p \left(1+\frac{bx^2}{a}\right)^{-p} \left(\frac{Ac^3 \,\text{Hypergeometric2F1}\left(\frac{1+m}{2},-p,\frac{3+m}{2},-\frac{bx^2}{a}\right)}{1+m} + \frac{c^2(Bc+3Ad)x^2 \,\text{Hypergeometric2F1}\left(\frac{3+m}{2},-p,\frac{5+m}{2},-\frac{bx^2}{a}\right)}{3+m} + dx^4 \left(\frac{3c(Bc+Ad) \,\text{Hypergeometric2F1}\left(\frac{5+m}{2},-p,\frac{7+m}{2},-\frac{bx^2}{a}\right)}{5+m} + dx^2 \left(\frac{(3Bc+Ad) \,\text{Hypergeometric2F1}\left(\frac{7+m}{2},-p,\frac{9+m}{2},-\frac{bx^2}{a}\right)}{7+m} + \frac{Bdx^2 \,\text{Hypergeometric2F1}\left(\frac{9+m}{2},-p,\right.}{9+m}\right.\right.\right.$$

input
```
Integrate[(e*x)^m*(a + b*x^2)^p*(A + B*x^2)*(c + d*x^2)^3,x]
```

output
```
(x*(e*x)^m*(a + b*x^2)^p*((A*c^3*Hypergeometric2F1[(1 + m)/2, -p, (3 + m)/
2, -((b*x^2)/a)])/(1 + m) + (c^2*(B*c + 3*A*d)*x^2*Hypergeometric2F1[(3 +
m)/2, -p, (5 + m)/2, -((b*x^2)/a)])/(3 + m) + d*x^4*((3*c*(B*c + A*d)*Hype
rgeometric2F1[(5 + m)/2, -p, (7 + m)/2, -((b*x^2)/a)])/(5 + m) + d*x^2*(((
3*B*c + A*d)*Hypergeometric2F1[(7 + m)/2, -p, (9 + m)/2, -((b*x^2)/a)])/(7
 + m) + (B*d*x^2*Hypergeometric2F1[(9 + m)/2, -p, (11 + m)/2, -((b*x^2)/a)
])/(9 + m)))))/(1 + (b*x^2)/a)^p
```

Rubi [A] (verified)

Time = 1.68 (sec) , antiderivative size = 1020, normalized size of antiderivative = 1.48, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {443, 25, 443, 443, 363, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(A+Bx^2\right)\left(c+dx^2\right)^3(ex)^m\left(a+bx^2\right)^p\,dx$$

$$\Big\downarrow \text{443}$$

$$\frac{\int -(ex)^m\left(bx^2+a\right)^p\left(dx^2+c\right)^2\left(c(aB(m+1)-Ab(m+2p+9))-(6bBc-aBd(m+7)+Abd(m+2p+9))x\right.}{b(m+2p+9)}$$
$$\frac{B\left(c+dx^2\right)^3(ex)^{m+1}\left(a+bx^2\right)^{p+1}}{be(m+2p+9)}$$

$$\Big\downarrow \text{25}$$

$$\frac{B\left(c+dx^2\right)^3(ex)^{m+1}\left(a+bx^2\right)^{p+1}}{be(m+2p+9)}-$$
$$\frac{\int (ex)^m\left(bx^2+a\right)^p\left(dx^2+c\right)^2\left((aBd(m+7)-b(6Bc+Ad(m+2p+9)))x^2+c(aB(m+1)-Ab(m+2p+9))\right.}{b(m+2p+9)}$$

$$\Big\downarrow \text{443}$$

$$\frac{B\left(c+dx^2\right)^3(ex)^{m+1}\left(a+bx^2\right)^{p+1}}{be(m+2p+9)}-$$
$$\frac{\int (ex)^m\left(bx^2+a\right)^p\left(dx^2+c\right)\left((2bcd(p+3)(aB(m+1)-Ab(m+2p+9))+d(bc-ad)(m+1)(aB(m+7)-Ab(m+2p+9))+4(bc-ad)(aBd(m+7)-b(6Bc\right.}{b(m+2p+7)}$$

$\big\downarrow$ 443

$$\frac{B(c+dx^2)^3(ex)^{m+1}(a+bx^2)^{p+1}}{be(m+2p+9)}-$$

$\int (ex)^m \left(bx^2+a\right)^p \Big(\Big(-c^2\Big(48Bc+Ad\Big(m^3+(6p+23)m^2+\left(12p^2+92p+183\right)m+8p^3+92p^2+366p+513\Big)\Big)b^3+acd\Big(2Ad\Big(m^3+4(p+5)m^2+\left(4p^2+44p+123\right)m+8p^2+84$

$\big\downarrow$ 363

$$\frac{B(ex)^{m+1}(bx^2+a)^{p+1}(dx^2+c)^3}{be(m+2p+9)}-$$

$\Big(-c^2\Big(48Bc+Ad\Big(m^3+(6p+23)m^2+\left(12p^2+92p+183\right)m+8p^3+92p^2+366p+513\Big)\Big)b^3+acd\Big(2Ad\Big(m^3+4(p+5)m^2+\left(4p^2+44p+123\right)m+8p^2+84p+216\Big)+Bc\Big(m^3+($

$\big\downarrow$ 279

$$\frac{B(ex)^{m+1}(bx^2+a)^{p+1}(dx^2+c)^3}{be(m+2p+9)}-$$

$\Big(-c^2\Big(48Bc+Ad\Big(m^3+(6p+23)m^2+\left(12p^2+92p+183\right)m+8p^3+92p^2+366p+513\Big)\Big)b^3+acd\Big(2Ad\Big(m^3+4(p+5)m^2+\left(4p^2+44p+123\right)m+8p^2+84p+216\Big)+Bc\Big(m^3+($

$\big\downarrow$ 278

$$\frac{B(ex)^{m+1}(bx^2+a)^{p+1}(dx^2+c)^3}{be(m+2p+9)}-$$

$\Big(-c^2\Big(48Bc+Ad\Big(m^3+(6p+23)m^2+\left(12p^2+92p+183\right)m+8p^3+92p^2+366p+513\Big)\Big)b^3+acd\Big(2Ad\Big(m^3+4(p+5)m^2+\left(4p^2+44p+123\right)m+8p^2+84p+216\Big)+Bc\Big(m^3+($

input
```
Int[(e*x)^m*(a + b*x^2)^p*(A + B*x^2)*(c + d*x^2)^3,x]
```

output
```
(B*(e*x)^(1 + m)*(a + b*x^2)^(1 + p)*(c + d*x^2)^3)/(b*e*(9 + m + 2*p)) -
(-(((6*b*B*c - a*B*d*(7 + m) + A*b*d*(9 + m + 2*p))*(e*x)^(1 + m)*(a + b*x
^2)^(1 + p)*(c + d*x^2)^2)/(b*e*(7 + m + 2*p))) + (-(((a^2*B*d^2*(35 + 12*
m + m^2) + b^2*c*(24*B*c + A*d*(99 + m^2 + 40*p + 4*p^2 + 4*m*(5 + p))) -
a*b*d*(A*d*(5 + m)*(9 + m + 2*p) + B*c*(65 + m^2 + 2*p + 2*m*(9 + p))))*(e
*x)^(1 + m)*(a + b*x^2)^(1 + p)*(c + d*x^2))/(b*e*(5 + m + 2*p))) + (((a^3
*B*d^3*(105 + 71*m + 15*m^2 + m^3) - a^2*b*d^2*(5 + m)*(A*d*(3 + m)*(9 + m
 + 2*p) + 2*B*c*(30 + 13*m + m^2 + 2*p + 2*m*p)) + a*b^2*c*d*(2*A*d*(216 +
 m^3 + 84*p + 8*p^2 + 4*m^2*(5 + p) + m*(123 + 44*p + 4*p^2)) + B*c*(267 +
 m^3 + 40*p + 4*p^2 + m^2*(21 + 4*p) + m*(143 + 44*p + 4*p^2))) - b^3*c^2*
(48*B*c + A*d*(513 + m^3 + 366*p + 92*p^2 + 8*p^3 + m^2*(23 + 6*p) + m*(18
3 + 92*p + 12*p^2))))*(e*x)^(1 + m)*(a + b*x^2)^(1 + p))/(b*e*(3 + m + 2*p
)) - (((a*(1 + m)*(a^3*B*d^3*(105 + 71*m + 15*m^2 + m^3) - a^2*b*d^2*(5 +
m)*(A*d*(3 + m)*(9 + m + 2*p) + 2*B*c*(30 + 13*m + m^2 + 2*p + 2*m*p)) + a
*b^2*c*d*(2*A*d*(216 + m^3 + 84*p + 8*p^2 + 4*m^2*(5 + p) + m*(123 + 44*p
+ 4*p^2)) + B*c*(267 + m^3 + 40*p + 4*p^2 + m^2*(21 + 4*p) + m*(143 + 44*p
 + 4*p^2))) - b^3*c^2*(48*B*c + A*d*(513 + m^3 + 366*p + 92*p^2 + 8*p^3 +
m^2*(23 + 6*p) + m*(183 + 92*p + 12*p^2)))))/(b*(3 + m + 2*p)) - c*(2*b*c*
(2 + p)*(2*b*c*(3 + p)*(a*B*(1 + m) - A*b*(9 + m + 2*p)) + (b*c - a*d)*(1
+ m)*(a*B*(7 + m) - A*b*(9 + m + 2*p))) + (1 + m)*(b*c*(2*b*c*(3 + p)*(...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 443
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*(g*x)^(m + 1)*(a + b*x^2)^(p
 + 1)*((c + d*x^2)^q/(b*g*(m + 2*(p + q + 1) + 1))), x] + Simp[1/(b*(m + 2*
(p + q + 1) + 1))   Int[(g*x)^m*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[c*((
b*e - a*f)*(m + 1) + b*e*2*(p + q + 1)) + (d*(b*e - a*f)*(m + 1) + f*2*q*(b
*c - a*d) + b*e*d*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f
, g, m, p}, x] && GtQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^2, c + d*x^
2])
```

Maple [F]

$$\int (ex)^m \left(b\,x^2+a\right)^p \left(x^2B+A\right) \left(x^2d+c\right)^3dx$$

input
```
int((e*x)^m*(b*x^2+a)^p*(B*x^2+A)*(d*x^2+c)^3,x)
```

output
```
int((e*x)^m*(b*x^2+a)^p*(B*x^2+A)*(d*x^2+c)^3,x)
```

Fricas [F]

$$\int (ex)^m \left(a+bx^2\right)^p \left(A+Bx^2\right) \left(c+dx^2\right)^3 \,dx = \int \left(Bx^2+A\right)\left(dx^2+c\right)^3\left(bx^2+a\right)^p(ex)^m \,dx$$

input
```
integrate((e*x)^m*(b*x^2+a)^p*(B*x^2+A)*(d*x^2+c)^3,x, algorithm="fricas")
```

output
```
integral((B*d^3*x^8 + (3*B*c*d^2 + A*d^3)*x^6 + 3*(B*c^2*d + A*c*d^2)*x^4
+ A*c^3 + (B*c^3 + 3*A*c^2*d)*x^2)*(b*x^2 + a)^p*(e*x)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (ex)^m \left(a+bx^2\right)^p \left(A+Bx^2\right) \left(c+dx^2\right)^3 \, dx = \text{Timed out}$$

input
```
integrate((e*x)**m*(b*x**2+a)**p*(B*x**2+A)*(d*x**2+c)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (ex)^m \left(a+bx^2\right)^p \left(A+Bx^2\right) \left(c+dx^2\right)^3 \, dx = \int \left(Bx^2+A\right)\left(dx^2+c\right)^3 \left(bx^2+a\right)^p (ex)^m \, dx$$

input
```
integrate((e*x)^m*(b*x^2+a)^p*(B*x^2+A)*(d*x^2+c)^3,x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*(d*x^2 + c)^3*(b*x^2 + a)^p*(e*x)^m, x)
```

Giac [F]

$$\int (ex)^m \left(a+bx^2\right)^p \left(A+Bx^2\right) \left(c+dx^2\right)^3 \, dx = \int \left(Bx^2+A\right)\left(dx^2+c\right)^3 \left(bx^2+a\right)^p (ex)^m \, dx$$

input
```
integrate((e*x)^m*(b*x^2+a)^p*(B*x^2+A)*(d*x^2+c)^3,x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*(d*x^2 + c)^3*(b*x^2 + a)^p*(e*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m \left(a+bx^2\right)^p \left(A+Bx^2\right) \left(c+dx^2\right)^3 \, dx = \int \left(B\,x^2+A\right)\,{\left(e\,x\right)}^m\,{\left(b\,x^2+a\right)}^p\,{\left(d\,x^2+c\right)}^3 \,d x$$

input
```
int((A + B*x^2)*(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^3,x)
```

output
```
int((A + B*x^2)*(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^3, x)
```

Reduce [F]

$$\int (ex)^m \left(a+bx^2\right)^p \left(A+Bx^2\right) \left(c+dx^2\right)^3 \, dx = \text{too large to display}$$

input
```
int((e*x)^m*(b*x^2+a)^p*(B*x^2+A)*(d*x^2+c)^3,x)
```

output

```
(e**m*(4*x**m*(a + b*x**2)**p*a**4*d**3*m**2*p**2*x + 4*x**m*(a + b*x**2)*
*p*a**4*d**3*m**2*p*x + 32*x**m*(a + b*x**2)**p*a**4*d**3*m*p**2*x + 32*x*
*m*(a + b*x**2)**p*a**4*d**3*m*p*x + 60*x**m*(a + b*x**2)**p*a**4*d**3*p**
2*x + 60*x**m*(a + b*x**2)**p*a**4*d**3*p*x - 12*x**m*(a + b*x**2)**p*a**3
*b*c*d**2*m**2*p**2*x - 12*x**m*(a + b*x**2)**p*a**3*b*c*d**2*m**2*p*x - 2
4*x**m*(a + b*x**2)**p*a**3*b*c*d**2*m*p**3*x - 168*x**m*(a + b*x**2)**p*a
**3*b*c*d**2*m*p**2*x - 144*x**m*(a + b*x**2)**p*a**3*b*c*d**2*m*p*x - 72*
x**m*(a + b*x**2)**p*a**3*b*c*d**2*p**3*x - 396*x**m*(a + b*x**2)**p*a**3*
b*c*d**2*p**2*x - 324*x**m*(a + b*x**2)**p*a**3*b*c*d**2*p*x - 4*x**m*(a +
 b*x**2)**p*a**3*b*d**3*m**2*p**2*x**3 - 4*x**m*(a + b*x**2)**p*a**3*b*d**
3*m**2*p*x**3 - 8*x**m*(a + b*x**2)**p*a**3*b*d**3*m*p**3*x**3 - 32*x**m*(
a + b*x**2)**p*a**3*b*d**3*m*p**2*x**3 - 24*x**m*(a + b*x**2)**p*a**3*b*d*
*3*m*p*x**3 - 40*x**m*(a + b*x**2)**p*a**3*b*d**3*p**3*x**3 - 60*x**m*(a +
 b*x**2)**p*a**3*b*d**3*p**2*x**3 - 20*x**m*(a + b*x**2)**p*a**3*b*d**3*p*
x**3 + 12*x**m*(a + b*x**2)**p*a**2*b**2*c**2*d*m**2*p**2*x + 12*x**m*(a +
 b*x**2)**p*a**2*b**2*c**2*d*m**2*p*x + 48*x**m*(a + b*x**2)**p*a**2*b**2*
c**2*d*m*p**3*x + 240*x**m*(a + b*x**2)**p*a**2*b**2*c**2*d*m*p**2*x + 192
*x**m*(a + b*x**2)**p*a**2*b**2*c**2*d*m*p*x + 48*x**m*(a + b*x**2)**p*a**
2*b**2*c**2*d*p**4*x + 432*x**m*(a + b*x**2)**p*a**2*b**2*c**2*d*p**3*x +
1140*x**m*(a + b*x**2)**p*a**2*b**2*c**2*d*p**2*x + 756*x**m*(a + b*x**...
```

3.45 $\int (ex)^m (a+bx^2)^p (A+Bx^2) (c+dx^2)^2 \, dx$

Optimal result

Integrand size = 31, antiderivative size = 399

$$\int (ex)^m (a+bx^2)^p (A+Bx^2) (c+dx^2)^2 \, dx$$
$$= \frac{(a^2Bd^2(15+8m+m^2) - abd(2Bc+Ad)(3+m)(7+m+2p) + b^2c(Bc+2Ad)(35+m^2+24p+4p^2}{b^3e(3+m+2p)(5+m+2p)(7+m+2p)}$$
$$- \frac{d(aBd(5+m) - b(2Bc+Ad)(7+m+2p))(ex)^{3+m}(a+bx^2)^{1+p}}{b^2e^3(5+m+2p)(7+m+2p)}$$
$$+ \frac{Bd^2(ex)^{5+m}(a+bx^2)^{1+p}}{be^5(7+m+2p)}$$
$$+ \frac{\left(\frac{Ac^2}{1+m} - \frac{a(a^2Bd^2(15+8m+m^2) - abd(2Bc+Ad)(3+m)(7+m+2p) + b^2c(Bc+2Ad)(35+m^2+24p+4p^2+4m(3+p)))}{b^3(3+m+2p)(5+m+2p)(7+m+2p)}\right)(ex)^{1+m}(a+}{e}$$

output

```
(a^2*B*d^2*(m^2+8*m+15)-a*b*d*(A*d+2*B*c)*(3+m)*(7+m+2*p)+b^2*c*(2*A*d+B*c
)*(35+m^2+24*p+4*p^2+4*m*(3+p)))*(e*x)^(1+m)*(b*x^2+a)^(p+1)/b^3/e/(3+m+2*
p)/(5+m+2*p)/(7+m+2*p)-d*(a*B*d*(5+m)-b*(A*d+2*B*c)*(7+m+2*p))*(e*x)^(3+m)
*(b*x^2+a)^(p+1)/b^2/e^3/(5+m+2*p)/(7+m+2*p)+B*d^2*(e*x)^(5+m)*(b*x^2+a)^(
p+1)/b/e^5/(7+m+2*p)+(A*c^2/(1+m)-a*(a^2*B*d^2*(m^2+8*m+15)-a*b*d*(A*d+2*B
*c)*(3+m)*(7+m+2*p)+b^2*c*(2*A*d+B*c)*(35+m^2+24*p+4*p^2+4*m*(3+p)))/b^3/(
3+m+2*p)/(5+m+2*p)/(7+m+2*p))*(e*x)^(1+m)*(b*x^2+a)^p*hypergeom([-p, 1/2+1
/2*m],[3/2+1/2*m],-b*x^2/a)/e/((1+b*x^2/a)^p)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.50

$$\int (ex)^m \left(a+bx^2\right)^p \left(A+Bx^2\right) \left(c+dx^2\right)^2 dx$$
$$= x(ex)^m \left(a+bx^2\right)^p \left(1+\frac{bx^2}{a}\right)^{-p} \left(\frac{Ac^2 \operatorname{Hypergeometric2F1}\left(\frac{1+m}{2},-p,\frac{3+m}{2},-\frac{bx^2}{a}\right)}{1+m} + \frac{c(Bc+2Ad)x^2 \operatorname{Hypergeometric2F1}\left(\frac{3+m}{2},-p,\frac{5+m}{2},-\frac{bx^2}{a}\right)}{3+m} + dx^4 \left(\frac{(2Bc+Ad) \operatorname{Hypergeometric2F1}\left(\frac{5+m}{2},-p,\frac{7+m}{2},-\frac{bx^2}{a}\right)}{5+m} + \frac{Bdx^2 \operatorname{Hypergeometric2F1}\left(\frac{7+m}{2},-p,\frac{9+m}{2},-\frac{bx^2}{a}\right)}{7+m}\right)\right)$$

input
```
Integrate[(e*x)^m*(a + b*x^2)^p*(A + B*x^2)*(c + d*x^2)^2,x]
```

output
```
(x*(e*x)^m*(a + b*x^2)^p*((A*c^2*Hypergeometric2F1[(1 + m)/2, -p, (3 + m)/
2, -((b*x^2)/a)])/(1 + m) + (c*(B*c + 2*A*d)*x^2*Hypergeometric2F1[(3 + m)
/2, -p, (5 + m)/2, -((b*x^2)/a)])/(3 + m) + d*x^4*(((2*B*c + A*d)*Hypergeo
metric2F1[(5 + m)/2, -p, (7 + m)/2, -((b*x^2)/a)])/(5 + m) + (B*d*x^2*Hype
rgeometric2F1[(7 + m)/2, -p, (9 + m)/2, -((b*x^2)/a)])/(7 + m))))/(1 + (b*
x^2)/a)^p
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 455, normalized size of antiderivative = 1.14, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {443, 25, 443, 363, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(A+Bx^2\right) \left(c+dx^2\right)^2 (ex)^m \left(a+bx^2\right)^p \, dx$$

$$\Big\downarrow \text{443}$$

$$\frac{\int -(ex)^m \left(bx^2+a\right)^p \left(dx^2+c\right) \left(c(aB(m+1)-Ab(m+2p+7))-(4bBc-aBd(m+5)+Abd(m+2p+7))x^2 \cdots\right.}{b(m+2p+7)}$$

$$\frac{B\left(c+dx^2\right)^2 (ex)^{m+1} \left(a+bx^2\right)^{p+1}}{be(m+2p+7)}$$

$$\Big\downarrow \text{25}$$

$$\frac{B\left(c+dx^2\right)^2 (ex)^{m+1} \left(a+bx^2\right)^{p+1}}{be(m+2p+7)} -$$

$$\frac{\int (ex)^m \left(bx^2+a\right)^p \left(dx^2+c\right) \left((aBd(m+5)-b(4Bc+Ad(m+2p+7)))x^2+c(aB(m+1)-Ab(m+2p+7))\right) \cdots}{b(m+2p+7)}$$

$$\Big\downarrow \text{443}$$

$$\frac{B\left(c+dx^2\right)^2 (ex)^{m+1} \left(a+bx^2\right)^{p+1}}{be(m+2p+7)} -$$

$$\frac{\frac{\int (ex)^m \left(bx^2+a\right)^p \left((2bcd(p+2)(aB(m+1)-Ab(m+2p+7))+d(bc-ad)(m+1)(aB(m+5)-Ab(m+2p+7))+2(bc-ad)(aBd(m+5)-b(4Bc+Ad(m+ \cdots\right.}{b(m+2p+5)}}{\cdots}$$

$$\Big\downarrow \text{363}$$

$$\frac{B\left(c+dx^2\right)^2 (ex)^{m+1} \left(a+bx^2\right)^{p+1}}{be(m+2p+7)} -$$

$$\frac{\frac{\left(\frac{a(m+1)\left(a^2Bd^2\left(m^2+8m+15\right)-abd\left(Ad(m+3)(m+2p+7)+Bc\left(m^2+2m(p+6)+2p+27\right)\right)+b^2c\left(Ad(m+2p+7)^2+8Bc\right)\right)}{b(m+2p+3)}+c((m+1)(bc-ad)(aB(m+5)-Ab(\cdots\right.}{\cdots}}{\cdots}$$

$$\Big\downarrow \text{279}$$

$$\frac{B\left(c+dx^2\right)^2 (ex)^{m+1} \left(a+bx^2\right)^{p+1}}{be(m+2p+7)} -$$

$$\frac{\frac{\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(\frac{a(m+1)\left(a^2Bd^2\left(m^2+8m+15\right)-abd\left(Ad(m+3)(m+2p+7)+Bc\left(m^2+2m(p+6)+2p+27\right)\right)+b^2c\left(Ad(m+2p+7)^2+8Bc\right)\right)}{b(m+2p+3)}+c((m+1)(bc \cdots\right.}{\cdots}}{\cdots}$$

$$\Big\downarrow \text{278}$$

$$\frac{B\left(c+dx^2\right)^2(ex)^{m+1}\left(a+bx^2\right)^{p+1}}{be(m+2p+7)}-$$

$$(ex)^{m+1}\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{m+1}{2},-p,\frac{m+3}{2},-\frac{bx^2}{a}\right)\left(\frac{a(m+1)\left(a^2Bd^2\left(m^2+8m+15\right)-abd\left(Ad(m+3)(m+2p+7)+Bc\left(m^2+2m(p+6)+\right.\right.\right.}{b(m+2p+3)}\right.$$

$$e(m+1)$$

input
```
Int[(e*x)^m*(a + b*x^2)^p*(A + B*x^2)*(c + d*x^2)^2,x]
```

output
```
(B*(e*x)^(1 + m)*(a + b*x^2)^(1 + p)*(c + d*x^2)^2)/(b*e*(7 + m + 2*p)) -
(-(((4*b*B*c - a*B*d*(5 + m) + A*b*d*(7 + m + 2*p))*(e*x)^(1 + m)*(a + b*x
^2)^(1 + p)*(c + d*x^2))/(b*e*(5 + m + 2*p))) + (-(((a^2*B*d^2*(15 + 8*m +
 m^2) + b^2*c*(8*B*c + A*d*(7 + m + 2*p)^2) - a*b*d*(A*d*(3 + m)*(7 + m +
2*p) + B*c*(27 + m^2 + 2*p + 2*m*(6 + p))))*(e*x)^(1 + m)*(a + b*x^2)^(1 +
 p))/(b*e*(3 + m + 2*p))) + ((c*(2*b*c*(2 + p)*(a*B*(1 + m) - A*b*(7 + m +
 2*p)) + (b*c - a*d)*(1 + m)*(a*B*(5 + m) - A*b*(7 + m + 2*p))) + (a*(1 +
m)*(a^2*B*d^2*(15 + 8*m + m^2) + b^2*c*(8*B*c + A*d*(7 + m + 2*p)^2) - a*b
*d*(A*d*(3 + m)*(7 + m + 2*p) + B*c*(27 + m^2 + 2*p + 2*m*(6 + p)))))/(b*(
3 + m + 2*p)))*(e*x)^(1 + m)*(a + b*x^2)^p*Hypergeometric2F1[(1 + m)/2, -p
, (3 + m)/2, -((b*x^2)/a)])/(e*(1 + m)*(1 + (b*x^2)/a)^p))/(b*(5 + m + 2*p
)))/(b*(7 + m + 2*p))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 443
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*(g*x)^(m + 1)*(a + b*x^2)^(p
 + 1)*((c + d*x^2)^q/(b*g*(m + 2*(p + q + 1) + 1))), x] + Simp[1/(b*(m + 2*
(p + q + 1) + 1))   Int[(g*x)^m*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[c*((
b*e - a*f)*(m + 1) + b*e*2*(p + q + 1)) + (d*(b*e - a*f)*(m + 1) + f*2*q*(b
*c - a*d) + b*e*d*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f
, g, m, p}, x] && GtQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^2, c + d*x^
2])
```

Maple [F]

$$\int (ex)^m \left(b\,x^2+a\right)^p \left(x^2B+A\right) \left(x^2d+c\right)^2 dx$$

input
```
int((e*x)^m*(b*x^2+a)^p*(B*x^2+A)*(d*x^2+c)^2,x)
```

output
```
int((e*x)^m*(b*x^2+a)^p*(B*x^2+A)*(d*x^2+c)^2,x)
```

Fricas [F]

$$\int (ex)^m \left(a+bx^2\right)^p \left(A+Bx^2\right) \left(c+dx^2\right)^2 \, dx = \int \left(Bx^2+A\right)\left(dx^2+c\right)^2\left(bx^2+a\right)^p(ex)^m \, dx$$

input
```
integrate((e*x)^m*(b*x^2+a)^p*(B*x^2+A)*(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
integral((B*d^2*x^6 + (2*B*c*d + A*d^2)*x^4 + A*c^2 + (B*c^2 + 2*A*c*d)*x^
2)*(b*x^2 + a)^p*(e*x)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (ex)^m \left(a+bx^2\right)^p \left(A+Bx^2\right) \left(c+dx^2\right)^2 \, dx = \text{Timed out}$$

input
```
integrate((e*x)**m*(b*x**2+a)**p*(B*x**2+A)*(d*x**2+c)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (ex)^m \left(a+bx^2\right)^p \left(A+Bx^2\right) \left(c+dx^2\right)^2 \, dx = \int \left(Bx^2+A\right) \left(dx^2+c\right)^2 \left(bx^2+a\right)^p (ex)^m \, dx$$

input
```
integrate((e*x)^m*(b*x^2+a)^p*(B*x^2+A)*(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*(d*x^2 + c)^2*(b*x^2 + a)^p*(e*x)^m, x)
```

Giac [F]

$$\int (ex)^m \left(a+bx^2\right)^p \left(A+Bx^2\right) \left(c+dx^2\right)^2 \, dx = \int (Bx^2+A)(dx^2+c)^2(bx^2+a)^p(ex)^m \, dx$$

input
```
integrate((e*x)^m*(b*x^2+a)^p*(B*x^2+A)*(d*x^2+c)^2,x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*(d*x^2 + c)^2*(b*x^2 + a)^p*(e*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m \left(a+bx^2\right)^p \left(A+Bx^2\right) \left(c+dx^2\right)^2 \, dx = \int \left(B\,x^2+A\right)\,{\left(e\,x\right)}^m\,{\left(b\,x^2+a\right)}^p\,{\left(d\,x^2+c\right)}^2 \,d x$$

input
```
int((A + B*x^2)*(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^2,x)
```

output
```
int((A + B*x^2)*(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^2, x)
```

Reduce [F]

$$\int (ex)^m \left(a+bx^2\right)^p \left(A+Bx^2\right) \left(c+dx^2\right)^2 \, dx = \text{too large to display}$$

input
```
int((e*x)^m*(b*x^2+a)^p*(B*x^2+A)*(d*x^2+c)^2,x)
```

output
```
(e**m*( - 4*x**m*(a + b*x**2)**p*a**3*d**2*m*p**2*x - 4*x**m*(a + b*x**2)*
*p*a**3*d**2*m*p*x - 12*x**m*(a + b*x**2)**p*a**3*d**2*p**2*x - 12*x**m*(a
 + b*x**2)**p*a**3*d**2*p*x + 8*x**m*(a + b*x**2)**p*a**2*b*c*d*m*p**2*x +
 8*x**m*(a + b*x**2)**p*a**2*b*c*d*m*p*x + 16*x**m*(a + b*x**2)**p*a**2*b*
c*d*p**3*x + 72*x**m*(a + b*x**2)**p*a**2*b*c*d*p**2*x + 56*x**m*(a + b*x*
*2)**p*a**2*b*c*d*p*x + 4*x**m*(a + b*x**2)**p*a**2*b*d**2*m*p**2*x**3 + 4
*x**m*(a + b*x**2)**p*a**2*b*d**2*m*p*x**3 + 8*x**m*(a + b*x**2)**p*a**2*b
*d**2*p**3*x**3 + 12*x**m*(a + b*x**2)**p*a**2*b*d**2*p**2*x**3 + 4*x**m*(
a + b*x**2)**p*a**2*b*d**2*p*x**3 + x**m*(a + b*x**2)**p*a*b**2*c**2*m**3*
x + 8*x**m*(a + b*x**2)**p*a*b**2*c**2*m**2*p*x + 15*x**m*(a + b*x**2)**p*
a*b**2*c**2*m**2*x + 20*x**m*(a + b*x**2)**p*a*b**2*c**2*m*p**2*x + 84*x**
m*(a + b*x**2)**p*a*b**2*c**2*m*p*x + 71*x**m*(a + b*x**2)**p*a*b**2*c**2*
m*x + 16*x**m*(a + b*x**2)**p*a*b**2*c**2*p**3*x + 108*x**m*(a + b*x**2)**
p*a*b**2*c**2*p**2*x + 212*x**m*(a + b*x**2)**p*a*b**2*c**2*p*x + 105*x**m
*(a + b*x**2)**p*a*b**2*c**2*x + 2*x**m*(a + b*x**2)**p*a*b**2*c*d*m**3*x*
*3 + 16*x**m*(a + b*x**2)**p*a*b**2*c*d*m**2*p*x**3 + 26*x**m*(a + b*x**2)
**p*a*b**2*c*d*m**2*x**3 + 40*x**m*(a + b*x**2)**p*a*b**2*c*d*m*p**2*x**3
+ 136*x**m*(a + b*x**2)**p*a*b**2*c*d*m*p*x**3 + 94*x**m*(a + b*x**2)**p*a
*b**2*c*d*m*x**3 + 32*x**m*(a + b*x**2)**p*a*b**2*c*d*p**3*x**3 + 168*x**m
*(a + b*x**2)**p*a*b**2*c*d*p**2*x**3 + 216*x**m*(a + b*x**2)**p*a*b**2...
```

3.46 $\int (ex)^m \left(a+bx^2\right)^p \left(A+Bx^2\right) \left(c+dx^2\right) dx$

Optimal result

Integrand size = 29, antiderivative size = 217

$$\int (ex)^m \left(a+bx^2\right)^p \left(A+Bx^2\right) \left(c+dx^2\right) dx$$
$$= -\frac{(aBd(3+m)-b(Bc+Ad)(5+m+2p))(ex)^{1+m}\left(a+bx^2\right)^{1+p}}{b^2e(3+m+2p)(5+m+2p)}$$
$$+ \frac{Bd(ex)^{3+m}\left(a+bx^2\right)^{1+p}}{be^3(5+m+2p)}$$
$$+ \frac{\left(\frac{Ac}{1+m}+\frac{a(aBd(3+m)-b(Bc+Ad)(5+m+2p))}{b^2(3+m+2p)(5+m+2p)}\right)(ex)^{1+m}\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1+m}{2},-p,\right.}{e}$$

output
```
-(a*B*d*(3+m)-b*(A*d+B*c)*(5+m+2*p))*(e*x)^(1+m)*(b*x^2+a)^(p+1)/b^2/e/(3+
m+2*p)/(5+m+2*p)+B*d*(e*x)^(3+m)*(b*x^2+a)^(p+1)/b/e^3/(5+m+2*p)+(A*c/(1+m
)+a*(a*B*d*(3+m)-b*(A*d+B*c)*(5+m+2*p))/b^2/(3+m+2*p)/(5+m+2*p))*(e*x)^(1+
m)*(b*x^2+a)^p*hypergeom([-p, 1/2+1/2*m],[3/2+1/2*m],-b*x^2/a)/e/((1+b*x^2
/a)^p)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.68

$$\int (ex)^m (a+bx^2)^p (A+Bx^2)(c+dx^2)\,dx$$
$$= x(ex)^m (a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} \left(\frac{Ac\,\text{Hypergeometric2F1}\left(\frac{1+m}{2},-p,\frac{3+m}{2},-\frac{bx^2}{a}\right)}{1+m} + \frac{(Bc+Ad)x^2\,\text{Hypergeometric2F1}\left(\frac{3+m}{2},-p,\frac{5+m}{2},-\frac{bx^2}{a}\right)}{3+m} + \frac{Bdx^4\,\text{Hypergeometric2F1}\left(\frac{5+m}{2},-p,\frac{7+m}{2},-\frac{bx^2}{a}\right)}{5+m}\right)$$

input
```
Integrate[(e*x)^m*(a + b*x^2)^p*(A + B*x^2)*(c + d*x^2),x]
```

output
```
(x*(e*x)^m*(a + b*x^2)^p*((A*c*Hypergeometric2F1[(1 + m)/2, -p, (3 + m)/2,
 -((b*x^2)/a)])/(1 + m) + ((B*c + A*d)*x^2*Hypergeometric2F1[(3 + m)/2, -p
, (5 + m)/2, -((b*x^2)/a)])/(3 + m) + (B*d*x^4*Hypergeometric2F1[(5 + m)/2
, -p, (7 + m)/2, -((b*x^2)/a)])/(5 + m)))/(1 + (b*x^2)/a)^p
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {443, 25, 363, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (A+Bx^2)(c+dx^2)(ex)^m (a+bx^2)^p\,dx$$
$$\Big\downarrow \; 443$$

$$\frac{\int -(ex)^m \left(bx^2+a\right)^p \left(A(ad(m+1)-bc(m+2p+5))-(2Abd-aB(m+3)d+bBc(m+2p+5))x^2\right)dx}{b(m+2p+5)}+\frac{d\left(A+Bx^2\right)(ex)^{m+1}\left(a+bx^2\right)^{p+1}}{be(m+2p+5)}$$

$\downarrow$ 25

$$\frac{d\left(A+Bx^2\right)(ex)^{m+1}\left(a+bx^2\right)^{p+1}}{be(m+2p+5)}-\frac{\int (ex)^m \left(bx^2+a\right)^p \left((aBd(m+3)-b(2Ad+Bc(m+2p+5)))x^2+A(ad(m+1)-bc(m+2p+5))\right)dx}{b(m+2p+5)}$$

$\downarrow$ 363

$$\frac{d\left(A+Bx^2\right)(ex)^{m+1}\left(a+bx^2\right)^{p+1}}{be(m+2p+5)}-\frac{\left(-\frac{a(m+1)(aBd(m+3)-b(2Ad+Bc(m+2p+5)))}{b(m+2p+3)}+aAd(m+1)-Abc(m+2p+5)\right)\int (ex)^m \left(bx^2+a\right)^p dx+\frac{(ex)^{m+1}(a+b}{}}{b(m+2p+5)}$$

$\downarrow$ 279

$$\frac{d\left(A+Bx^2\right)(ex)^{m+1}\left(a+bx^2\right)^{p+1}}{be(m+2p+5)}-\frac{\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(-\frac{a(m+1)(aBd(m+3)-b(2Ad+Bc(m+2p+5)))}{b(m+2p+3)}+aAd(m+1)-Abc(m+2p+5)\right)\int (ex)^m \left(\frac{bx^2}{a}\right.}{b(m+2p+5)}$$

$\downarrow$ 278

$$\frac{d\left(A+Bx^2\right)(ex)^{m+1}\left(a+bx^2\right)^{p+1}}{be(m+2p+5)}-\frac{\frac{(ex)^{m+1}\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{m+1}{2},-p,\frac{m+3}{2},-\frac{bx^2}{a}\right)\left(-\frac{a(m+1)(aBd(m+3)-b(2Ad+Bc(m+2p+5)))}{b(m+2p+3)}+aAd(m+1)-Abc(m+2\right.}{e(m+1)}}{b(m+2p+5)}$$

input
```
Int[(e*x)^m*(a + b*x^2)^p*(A + B*x^2)*(c + d*x^2),x]
```

output
```
(d*(e*x)^(1 + m)*(a + b*x^2)^(1 + p)*(A + B*x^2))/(b*e*(5 + m + 2*p)) - ((
(a*B*d*(3 + m) - b*(2*A*d + B*c*(5 + m + 2*p)))*(e*x)^(1 + m)*(a + b*x^2)^
(1 + p))/(b*e*(3 + m + 2*p)) + ((a*A*d*(1 + m) - A*b*c*(5 + m + 2*p) - (a*
(1 + m)*(a*B*d*(3 + m) - b*(2*A*d + B*c*(5 + m + 2*p))))/(b*(3 + m + 2*p))
)*(e*x)^(1 + m)*(a + b*x^2)^p*Hypergeometric2F1[(1 + m)/2, -p, (3 + m)/2,
-((b*x^2)/a)])/(e*(1 + m)*(1 + (b*x^2)/a)^p))/(b*(5 + m + 2*p))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 443
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*(g*x)^(m + 1)*(a + b*x^2)^(p
 + 1)*((c + d*x^2)^q/(b*g*(m + 2*(p + q + 1) + 1))), x] + Simp[1/(b*(m + 2*
(p + q + 1) + 1))   Int[(g*x)^m*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[c*((
b*e - a*f)*(m + 1) + b*e*2*(p + q + 1)) + (d*(b*e - a*f)*(m + 1) + f*2*q*(b
*c - a*d) + b*e*d*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f
, g, m, p}, x] && GtQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^2, c + d*x^
2])
```

Maple [F]

$$\int (ex)^m \left(b\,x^2+a\right)^p \left(x^2B+A\right) \left(x^2d+c\right) dx$$

input
```
int((e*x)^m*(b*x^2+a)^p*(B*x^2+A)*(d*x^2+c),x)
```

output
```
int((e*x)^m*(b*x^2+a)^p*(B*x^2+A)*(d*x^2+c),x)
```

Fricas [F]

$$\int (ex)^m \left(a+bx^2\right)^p \left(A+Bx^2\right) \left(c+dx^2\right) \, dx = \int \left(Bx^2+A\right)\left(dx^2+c\right)\left(bx^2+a\right)^p (ex)^m \, dx$$

input
```
integrate((e*x)^m*(b*x^2+a)^p*(B*x^2+A)*(d*x^2+c),x, algorithm="fricas")
```

output
```
integral((B*d*x^4 + (B*c + A*d)*x^2 + A*c)*(b*x^2 + a)^p*(e*x)^m, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 177.90 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.09

$$\int (ex)^m \left(a+bx^2\right)^p \left(A+Bx^2\right) \left(c+dx^2\right) \, dx$$
$$= \frac{Aa^p ce^m x^{m+1}\Gamma\left(\frac{m}{2}+\frac{1}{2}\right) {}_2F_1\left(\begin{matrix} -p, \frac{m}{2}+\frac{1}{2} \\ \frac{m}{2}+\frac{3}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)} + \frac{Aa^p de^m x^{m+3}\Gamma\left(\frac{m}{2}+\frac{3}{2}\right) {}_2F_1\left(\begin{matrix} -p, \frac{m}{2}+\frac{3}{2} \\ \frac{m}{2}+\frac{5}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)} + \frac{Ba^p ce^m x^{m+3}\Gamma\left(\frac{m}{2}+\frac{3}{2}\right) {}_2F_1\left(\begin{matrix} -p, \frac{m}{2}+\frac{3}{2} \\ \frac{m}{2}+\frac{5}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)} + \frac{Ba^p de^m x^{m+5}\Gamma\left(\frac{m}{2}+\frac{5}{2}\right) {}_2F_1\left(\begin{matrix} -p, \frac{m}{2}+\frac{5}{2} \\ \frac{m}{2}+\frac{7}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{m}{2}+\frac{7}{2}\right)}$$

input
```
integrate((e*x)**m*(b*x**2+a)**p*(B*x**2+A)*(d*x**2+c),x)
```

output
```
A*a**p*c*e**m*x**(m + 1)*gamma(m/2 + 1/2)*hyper((-p, m/2 + 1/2), (m/2 + 3/
2,), b*x**2*exp_polar(I*pi)/a)/(2*gamma(m/2 + 3/2)) + A*a**p*d*e**m*x**(m
+ 3)*gamma(m/2 + 3/2)*hyper((-p, m/2 + 3/2), (m/2 + 5/2,), b*x**2*exp_pola
r(I*pi)/a)/(2*gamma(m/2 + 5/2)) + B*a**p*c*e**m*x**(m + 3)*gamma(m/2 + 3/2
)*hyper((-p, m/2 + 3/2), (m/2 + 5/2,), b*x**2*exp_polar(I*pi)/a)/(2*gamma(
m/2 + 5/2)) + B*a**p*d*e**m*x**(m + 5)*gamma(m/2 + 5/2)*hyper((-p, m/2 + 5
/2), (m/2 + 7/2,), b*x**2*exp_polar(I*pi)/a)/(2*gamma(m/2 + 7/2))
```

Maxima [F]

$$\int (ex)^m \left(a+bx^2\right)^p \left(A+Bx^2\right) \left(c+dx^2\right) \, dx = \int \left(Bx^2+A\right)\left(dx^2+c\right)\left(bx^2+a\right)^p (ex)^m \, dx$$

input
```
integrate((e*x)^m*(b*x^2+a)^p*(B*x^2+A)*(d*x^2+c),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*(d*x^2 + c)*(b*x^2 + a)^p*(e*x)^m, x)
```

Giac [F]

$$\int (ex)^m \left(a+bx^2\right)^p \left(A+Bx^2\right) \left(c+dx^2\right) \, dx = \int \left(Bx^2+A\right)\left(dx^2+c\right)\left(bx^2+a\right)^p (ex)^m \, dx$$

input
```
integrate((e*x)^m*(b*x^2+a)^p*(B*x^2+A)*(d*x^2+c),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*(d*x^2 + c)*(b*x^2 + a)^p*(e*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m \left(a+bx^2\right)^p \left(A+Bx^2\right) \left(c+dx^2\right) \, dx$$
$$= \int \left(B\,x^2+A\right)\,{\left(e\,x\right)}^m\,{\left(b\,x^2+a\right)}^p\,\left(d\,x^2+c\right) \,dx$$

input
```
int((A + B*x^2)*(e*x)^m*(a + b*x^2)^p*(c + d*x^2),x)
```

output
```
int((A + B*x^2)*(e*x)^m*(a + b*x^2)^p*(c + d*x^2), x)
```

Reduce [F]

$$\int (ex)^m \left(a+bx^2\right)^p \left(A+Bx^2\right)\left(c+dx^2\right)\,dx = \text{too large to display}$$

input
```
int((e*x)^m*(b*x^2+a)^p*(B*x^2+A)*(d*x^2+c),x)
```

output
```
(e**m*(4*x**m*(a + b*x**2)**p*a**2*d*p**2*x + 4*x**m*(a + b*x**2)**p*a**2*
d*p*x + x**m*(a + b*x**2)**p*a*b*c*m**2*x + 6*x**m*(a + b*x**2)**p*a*b*c*m
*p*x + 8*x**m*(a + b*x**2)**p*a*b*c*m*x + 8*x**m*(a + b*x**2)**p*a*b*c*p**
2*x + 26*x**m*(a + b*x**2)**p*a*b*c*p*x + 15*x**m*(a + b*x**2)**p*a*b*c*x
+ x**m*(a + b*x**2)**p*a*b*d*m**2*x**3 + 6*x**m*(a + b*x**2)**p*a*b*d*m*p*
x**3 + 6*x**m*(a + b*x**2)**p*a*b*d*m*x**3 + 8*x**m*(a + b*x**2)**p*a*b*d*
p**2*x**3 + 14*x**m*(a + b*x**2)**p*a*b*d*p*x**3 + 5*x**m*(a + b*x**2)**p*
a*b*d*x**3 + x**m*(a + b*x**2)**p*b**2*c*m**2*x**3 + 4*x**m*(a + b*x**2)**
p*b**2*c*m*p*x**3 + 6*x**m*(a + b*x**2)**p*b**2*c*m*x**3 + 4*x**m*(a + b*x
**2)**p*b**2*c*p**2*x**3 + 12*x**m*(a + b*x**2)**p*b**2*c*p*x**3 + 5*x**m*
(a + b*x**2)**p*b**2*c*x**3 + x**m*(a + b*x**2)**p*b**2*d*m**2*x**5 + 4*x*
*m*(a + b*x**2)**p*b**2*d*m*p*x**5 + 4*x**m*(a + b*x**2)**p*b**2*d*m*x**5
+ 4*x**m*(a + b*x**2)**p*b**2*d*p**2*x**5 + 8*x**m*(a + b*x**2)**p*b**2*d*
p*x**5 + 3*x**m*(a + b*x**2)**p*b**2*d*x**5 - 4*int((x**m*(a + b*x**2)**p)
/(a*m**3 + 6*a*m**2*p + 9*a*m**2 + 12*a*m*p**2 + 36*a*m*p + 23*a*m + 8*a*p
**3 + 36*a*p**2 + 46*a*p + 15*a + b*m**3*x**2 + 6*b*m**2*p*x**2 + 9*b*m**2
*x**2 + 12*b*m*p**2*x**2 + 36*b*m*p*x**2 + 23*b*m*x**2 + 8*b*p**3*x**2 + 3
6*b*p**2*x**2 + 46*b*p*x**2 + 15*b*x**2),x)*a**3*d*m**4*p**2 - 4*int((x**m
*(a + b*x**2)**p)/(a*m**3 + 6*a*m**2*p + 9*a*m**2 + 12*a*m*p**2 + 36*a*m*p
 + 23*a*m + 8*a*p**3 + 36*a*p**2 + 46*a*p + 15*a + b*m**3*x**2 + 6*b*m*...
```

3.47 $\int \frac{(ex)^m(a+bx^2)^p(A+Bx^2)}{c+dx^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 162

$$\int \frac{(ex)^m (a+bx^2)^p (A+Bx^2)}{c+dx^2}\,dx = -\frac{(Bc-Ad)(ex)^{1+m}(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\operatorname{AppellF1}\left(\frac{1+m}{2},-p,1,\frac{3+m}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{cde(1+m)} + \frac{B(ex)^{1+m}(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\operatorname{Hypergeometric2F1}\left(\frac{1+m}{2},-p,\frac{3+m}{2},-\frac{bx^2}{a}\right)}{de(1+m)}$$

output
```
-(-A*d+B*c)*(e*x)^(1+m)*(b*x^2+a)^p*AppellF1(1/2+1/2*m,-p,1,3/2+1/2*m,-b*x
^2/a,-d*x^2/c)/c/d/e/(1+m)/((1+b*x^2/a)^p)+B*(e*x)^(1+m)*(b*x^2+a)^p*hyper
geom([-p, 1/2+1/2*m],[3/2+1/2*m],-b*x^2/a)/d/e/(1+m)/((1+b*x^2/a)^p)
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.73

$$\int \frac{(ex)^m (a+bx^2)^p (A+Bx^2)}{c+dx^2}\,dx$$
$$= \frac{x(ex)^m (a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} \left((-Bc+Ad)\operatorname{AppellF1}\left(\frac{1+m}{2},-p,1,\frac{3+m}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+Bc\,\text{Hypergeom}\right.}{cd(1+m)}$$

input
```
Integrate[((e*x)^m*(a + b*x^2)^p*(A + B*x^2))/(c + d*x^2),x]
```

output
```
(x*(e*x)^m*(a + b*x^2)^p*((-(B*c) + A*d)*AppellF1[(1 + m)/2, -p, 1, (3 + m
)/2, -((b*x^2)/a), -((d*x^2)/c)] + B*c*Hypergeometric2F1[(1 + m)/2, -p, (3
 + m)/2, -((b*x^2)/a)]))/(c*d*(1 + m)*(1 + (b*x^2)/a)^p)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {446, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)(ex)^m (a+bx^2)^p}{c+dx^2}\,dx$$
$$\Big\downarrow \text{446}$$
$$\int \left(\frac{(ex)^m (a+bx^2)^p (Ad-Bc)}{d(c+dx^2)}+\frac{B(ex)^m (a+bx^2)^p}{d}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{B(ex)^{m+1}(a+bx^2)^p \left(\frac{bx^2}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{m+1}{2},-p,\frac{m+3}{2},-\frac{bx^2}{a}\right)}{de(m+1)}-\frac{(ex)^{m+1}(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(Bc-Ad)\operatorname{AppellF1}\left(\frac{m+1}{2},-p,1,\frac{m+3}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{cde(m+1)}$$

input
```
Int[((e*x)^m*(a + b*x^2)^p*(A + B*x^2))/(c + d*x^2),x]
```

output
```
-(((B*c - A*d)*(e*x)^(1 + m)*(a + b*x^2)^p*AppellF1[(1 + m)/2, -p, 1, (3 +
 m)/2, -((b*x^2)/a), -((d*x^2)/c)])/(c*d*e*(1 + m)*(1 + (b*x^2)/a)^p)) + (
B*(e*x)^(1 + m)*(a + b*x^2)^p*Hypergeometric2F1[(1 + m)/2, -p, (3 + m)/2,
-((b*x^2)/a)])/(d*e*(1 + m)*(1 + (b*x^2)/a)^p)
```

Defintions of rubi rules used

rule 446
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((e_) + (f_.)*(x_)^2))/(
(c_) + (d_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a + b*x^2)^
p*((e + f*x^2)/(c + d*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{(ex)^m (b\,x^2+a)^p (x^2B+A)}{x^2d+c}dx$$

input
```
int((e*x)^m*(b*x^2+a)^p*(B*x^2+A)/(d*x^2+c),x)
```

output
```
int((e*x)^m*(b*x^2+a)^p*(B*x^2+A)/(d*x^2+c),x)
```

Fricas [F]

$$\int \frac{(ex)^m (a+bx^2)^p (A+Bx^2)}{c+dx^2}\,dx = \int \frac{(Bx^2+A)(bx^2+a)^p(ex)^m}{dx^2+c}\,dx$$

input
```
integrate((e*x)^m*(b*x^2+a)^p*(B*x^2+A)/(d*x^2+c),x, algorithm="fricas")
```

output
```
integral((B*x^2 + A)*(b*x^2 + a)^p*(e*x)^m/(d*x^2 + c), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ex)^m (a+bx^2)^p (A+Bx^2)}{c+dx^2} \, dx = \text{Timed out}$$

input
```
integrate((e*x)**m*(b*x**2+a)**p*(B*x**2+A)/(d*x**2+c),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(ex)^m (a+bx^2)^p (A+Bx^2)}{c+dx^2} \, dx = \int \frac{(Bx^2+A)(bx^2+a)^p (ex)^m}{dx^2+c} \, dx$$

input
```
integrate((e*x)^m*(b*x^2+a)^p*(B*x^2+A)/(d*x^2+c),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*(b*x^2 + a)^p*(e*x)^m/(d*x^2 + c), x)
```

Giac [F]

$$\int \frac{(ex)^m (a+bx^2)^p (A+Bx^2)}{c+dx^2} \, dx = \int \frac{(Bx^2+A)(bx^2+a)^p (ex)^m}{dx^2+c} \, dx$$

input
```
integrate((e*x)^m*(b*x^2+a)^p*(B*x^2+A)/(d*x^2+c),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*(b*x^2 + a)^p*(e*x)^m/(d*x^2 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m (a+bx^2)^p (A+Bx^2)}{c+dx^2}\,dx = \int \frac{(B\,x^2+A)\,(e\,x)^m\,(b\,x^2+a)^p}{d\,x^2+c}\,dx$$

input
```
int(((A + B*x^2)*(e*x)^m*(a + b*x^2)^p)/(c + d*x^2),x)
```

output
```
int(((A + B*x^2)*(e*x)^m*(a + b*x^2)^p)/(c + d*x^2), x)
```

Reduce [F]

$$\int \frac{(ex)^m (a+bx^2)^p (A+Bx^2)}{c+dx^2}\,dx = \text{too large to display}$$

input
```
int((e*x)^m*(b*x^2+a)^p*(B*x^2+A)/(d*x^2+c),x)
```

output
```
(e**m*(x**m*(a + b*x**2)**p*b*x + int((x**m*(a + b*x**2)**p*x**2)/(a*c*m +
 2*a*c*p + a*c + a*d*m*x**2 + 2*a*d*p*x**2 + a*d*x**2 + b*c*m*x**2 + 2*b*c
*p*x**2 + b*c*x**2 + b*d*m*x**4 + 2*b*d*p*x**4 + b*d*x**4),x)*a*b*d*m**2 +
 6*int((x**m*(a + b*x**2)**p*x**2)/(a*c*m + 2*a*c*p + a*c + a*d*m*x**2 + 2
*a*d*p*x**2 + a*d*x**2 + b*c*m*x**2 + 2*b*c*p*x**2 + b*c*x**2 + b*d*m*x**4
 + 2*b*d*p*x**4 + b*d*x**4),x)*a*b*d*m*p + 2*int((x**m*(a + b*x**2)**p*x**
2)/(a*c*m + 2*a*c*p + a*c + a*d*m*x**2 + 2*a*d*p*x**2 + a*d*x**2 + b*c*m*x
**2 + 2*b*c*p*x**2 + b*c*x**2 + b*d*m*x**4 + 2*b*d*p*x**4 + b*d*x**4),x)*a
*b*d*m + 8*int((x**m*(a + b*x**2)**p*x**2)/(a*c*m + 2*a*c*p + a*c + a*d*m*
x**2 + 2*a*d*p*x**2 + a*d*x**2 + b*c*m*x**2 + 2*b*c*p*x**2 + b*c*x**2 + b*
d*m*x**4 + 2*b*d*p*x**4 + b*d*x**4),x)*a*b*d*p**2 + 6*int((x**m*(a + b*x**
2)**p*x**2)/(a*c*m + 2*a*c*p + a*c + a*d*m*x**2 + 2*a*d*p*x**2 + a*d*x**2
+ b*c*m*x**2 + 2*b*c*p*x**2 + b*c*x**2 + b*d*m*x**4 + 2*b*d*p*x**4 + b*d*x
**4),x)*a*b*d*p + int((x**m*(a + b*x**2)**p*x**2)/(a*c*m + 2*a*c*p + a*c +
 a*d*m*x**2 + 2*a*d*p*x**2 + a*d*x**2 + b*c*m*x**2 + 2*b*c*p*x**2 + b*c*x*
*2 + b*d*m*x**4 + 2*b*d*p*x**4 + b*d*x**4),x)*a*b*d - int((x**m*(a + b*x**
2)**p*x**2)/(a*c*m + 2*a*c*p + a*c + a*d*m*x**2 + 2*a*d*p*x**2 + a*d*x**2
+ b*c*m*x**2 + 2*b*c*p*x**2 + b*c*x**2 + b*d*m*x**4 + 2*b*d*p*x**4 + b*d*x
**4),x)*b**2*c*m**2 - 4*int((x**m*(a + b*x**2)**p*x**2)/(a*c*m + 2*a*c*p +
 a*c + a*d*m*x**2 + 2*a*d*p*x**2 + a*d*x**2 + b*c*m*x**2 + 2*b*c*p*x**2...
```

3.48 $\int \frac{(ex)^m(a+bx^2)^p(A+Bx^2)}{(c+dx^2)^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 295

$$\int \frac{(ex)^m(a+bx^2)^p(A+Bx^2)}{(c+dx^2)^2}\,dx = \frac{(Bc-Ad)(ex)^{1+m}(a+bx^2)^{1+p}}{2c(bc-ad)e(c+dx^2)} - \frac{(ad(Ad(1-m)+Bc(1+m))-bc(Ad(1-m-2p)+Bc(1+m+2p)))(ex)^{1+m}(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)}{2c^2d(bc-ad)e(1+m)} - \frac{b(Bc-Ad)(1+m+2p)(ex)^{1+m}(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1+m}{2},-p,\frac{3+m}{2},-\frac{bx^2}{a}\right)}{2cd(bc-ad)e(1+m)}$$

output
```
1/2*(-A*d+B*c)*(e*x)^(1+m)*(b*x^2+a)^(p+1)/c/(-a*d+b*c)/e/(d*x^2+c)-1/2*(a
*d*(A*d*(1-m)+B*c*(1+m))-b*c*(A*d*(1-m-2*p)+B*c*(1+m+2*p)))*(e*x)^(1+m)*(b
*x^2+a)^p*AppellF1(1/2+1/2*m,-p,1,3/2+1/2*m,-b*x^2/a,-d*x^2/c)/c^2/d/(-a*d
+b*c)/e/(1+m)/((1+b*x^2/a)^p)-1/2*b*(-A*d+B*c)*(1+m+2*p)*(e*x)^(1+m)*(b*x^
2+a)^p*hypergeom([-p, 1/2+1/2*m],[3/2+1/2*m],-b*x^2/a)/c/d/(-a*d+b*c)/e/(1
+m)/((1+b*x^2/a)^p)
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.43

$$\int \frac{(ex)^m (a+bx^2)^p (A+Bx^2)}{(c+dx^2)^2} dx$$
$$= \frac{x(ex)^m (a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} \left(Bc \operatorname{AppellF1}\left(\frac{1+m}{2},-p,1,\frac{3+m}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+(-Bc+Ad) \operatorname{AppellF1}\left(\right.\right.}{c^2d(1+m)}$$

input
```
Integrate[((e*x)^m*(a + b*x^2)^p*(A + B*x^2))/(c + d*x^2)^2,x]
```

output
```
(x*(e*x)^m*(a + b*x^2)^p*(B*c*AppellF1[(1 + m)/2, -p, 1, (3 + m)/2, -((b*x
^2)/a), -((d*x^2)/c)] + (-(B*c) + A*d)*AppellF1[(1 + m)/2, -p, 2, (3 + m)/
2, -((b*x^2)/a), -((d*x^2)/c)]))/(c^2*d*(1 + m)*(1 + (b*x^2)/a)^p)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 286, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {441, 446, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)(ex)^m (a+bx^2)^p}{(c+dx^2)^2} dx$$
$$\Big\downarrow \text{441}$$
$$\frac{\int \frac{(ex)^m (bx^2+a)^p (-b(Bc-Ad)(m+2p+1)x^2+2Abc-aAd(1-m)-aBc(m+1))}{dx^2+c}dx}{2c(bc-ad)}+\frac{(ex)^{m+1}(a+bx^2)^{p+1}(Bc-Ad)}{2ce(c+dx^2)(bc-ad)}$$
$$\Big\downarrow \text{446}$$

$$\frac{\int \left(\frac{(d(2Abc-aB(m+1)c-aAd(1-m))+bc(Bc-Ad)(m+2p+1))(ex)^m(bx^2+a)^p}{d(dx^2+c)}-\frac{b(Bc-Ad)(m+2p+1)(ex)^m(bx^2+a)^p}{d}\right)dx}{2c(bc-ad)}+\frac{(ex)^{m+1}\left(a+bx^2\right)^{p+1}(Bc-Ad)}{2ce\left(c+dx^2\right)(bc-ad)}$$

$$\Big\downarrow \ 2009$$

$$\frac{-\frac{(ex)^{m+1}(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(ad(Ad(1-m)+Bc(m+1))-bc(Ad(-m-2p+1)+Bc(m+2p+1)))\operatorname{AppellF1}\left(\frac{m+1}{2},-p,1,\frac{m+3}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{cde(m+1)}-}{2c(bc-ad)}$$

$$\frac{(ex)^{m+1}\left(a+bx^2\right)^{p+1}(Bc-Ad)}{2ce\left(c+dx^2\right)(bc-ad)}$$

input
```
Int[((e*x)^m*(a + b*x^2)^p*(A + B*x^2))/(c + d*x^2)^2,x]
```

output
```
((B*c - A*d)*(e*x)^(1 + m)*(a + b*x^2)^(1 + p))/(2*c*(b*c - a*d)*e*(c + d*
x^2)) + (-(((a*d*(A*d*(1 - m) + B*c*(1 + m)) - b*c*(A*d*(1 - m - 2*p) + B*
c*(1 + m + 2*p)))*(e*x)^(1 + m)*(a + b*x^2)^p*AppellF1[(1 + m)/2, -p, 1, (
3 + m)/2, -((b*x^2)/a), -((d*x^2)/c)])/(c*d*e*(1 + m)*(1 + (b*x^2)/a)^p))
- (b*(B*c - A*d)*(1 + m + 2*p)*(e*x)^(1 + m)*(a + b*x^2)^p*Hypergeometric2
F1[(1 + m)/2, -p, (3 + m)/2, -((b*x^2)/a)])/(d*e*(1 + m)*(1 + (b*x^2)/a)^p
))/(2*c*(b*c - a*d))
```

Defintions of rubi rules used

rule 441
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
 + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(a*g*2*(b*c - a*d)*(p + 1))), x] + Si
mp[1/(a*2*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2
)^q*Simp[c*(b*e - a*f)*(m + 1) + e*2*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(m
 + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, q},
 x] && LtQ[p, -1]
```

rule 446
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((e_) + (f_.)*(x_)^2))/(
(c_) + (d_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a + b*x^2)^
p*((e + f*x^2)/(c + d*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{(ex)^m\,(b\,x^2+a)^p\,(x^2B+A)}{(x^2d+c)^2}dx$$

input
```
int((e*x)^m*(b*x^2+a)^p*(B*x^2+A)/(d*x^2+c)^2,x)
```

output
```
int((e*x)^m*(b*x^2+a)^p*(B*x^2+A)/(d*x^2+c)^2,x)
```

Fricas [F]

$$\int \frac{(ex)^m\,(a+bx^2)^p\,(A+Bx^2)}{(c+dx^2)^2}\,dx = \int \frac{(Bx^2+A)(bx^2+a)^p(ex)^m}{(dx^2+c)^2}\,dx$$

input
```
integrate((e*x)^m*(b*x^2+a)^p*(B*x^2+A)/(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
integral((B*x^2 + A)*(b*x^2 + a)^p*(e*x)^m/(d^2*x^4 + 2*c*d*x^2 + c^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ex)^m (a+bx^2)^p (A+Bx^2)}{(c+dx^2)^2} \, dx = \text{Timed out}$$

input
```
integrate((e*x)**m*(b*x**2+a)**p*(B*x**2+A)/(d*x**2+c)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(ex)^m (a+bx^2)^p (A+Bx^2)}{(c+dx^2)^2} \, dx = \int \frac{(Bx^2+A)(bx^2+a)^p(ex)^m}{(dx^2+c)^2} \, dx$$

input
```
integrate((e*x)^m*(b*x^2+a)^p*(B*x^2+A)/(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*(b*x^2 + a)^p*(e*x)^m/(d*x^2 + c)^2, x)
```

Giac [F]

$$\int \frac{(ex)^m (a+bx^2)^p (A+Bx^2)}{(c+dx^2)^2} \, dx = \int \frac{(Bx^2+A)(bx^2+a)^p(ex)^m}{(dx^2+c)^2} \, dx$$

input
```
integrate((e*x)^m*(b*x^2+a)^p*(B*x^2+A)/(d*x^2+c)^2,x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*(b*x^2 + a)^p*(e*x)^m/(d*x^2 + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m (a+bx^2)^p (A+Bx^2)}{(c+dx^2)^2}\,dx=\int \frac{(B\,x^2+A)\,(e\,x)^m\,(b\,x^2+a)^p}{(d\,x^2+c)^2}\,dx$$

input
```
int(((A + B*x^2)*(e*x)^m*(a + b*x^2)^p)/(c + d*x^2)^2,x)
```

output
```
int(((A + B*x^2)*(e*x)^m*(a + b*x^2)^p)/(c + d*x^2)^2, x)
```

Reduce [F]

$$\int \frac{(ex)^m (a+bx^2)^p (A+Bx^2)}{(c+dx^2)^2}\,dx=\text{too large to display}$$

input
```
int((e*x)^m*(b*x^2+a)^p*(B*x^2+A)/(d*x^2+c)^2,x)
```

output

```
(e**m*(2*x**m*(a + b*x**2)**p*a*b*x - int((x**m*(a + b*x**2)**p*x**4)/(a**
2*c**2*d*m - a**2*c**2*d + 2*a**2*c*d**2*m*x**2 - 2*a**2*c*d**2*x**2 + a**
2*d**3*m*x**4 - a**2*d**3*x**4 + a*b*c**3*m + 2*a*b*c**3*p + a*b*c**3 + 3*
a*b*c**2*d*m*x**2 + 4*a*b*c**2*d*p*x**2 + a*b*c**2*d*x**2 + 3*a*b*c*d**2*m
*x**4 + 2*a*b*c*d**2*p*x**4 - a*b*c*d**2*x**4 + a*b*d**3*m*x**6 - a*b*d**3
*x**6 + b**2*c**3*m*x**2 + 2*b**2*c**3*p*x**2 + b**2*c**3*x**2 + 2*b**2*c*
*2*d*m*x**4 + 4*b**2*c**2*d*p*x**4 + 2*b**2*c**2*d*x**4 + b**2*c*d**2*m*x*
*6 + 2*b**2*c*d**2*p*x**6 + b**2*c*d**2*x**6),x)*a**2*b**2*c*d**2*m**2 - 4
*int((x**m*(a + b*x**2)**p*x**4)/(a**2*c**2*d*m - a**2*c**2*d + 2*a**2*c*d
**2*m*x**2 - 2*a**2*c*d**2*x**2 + a**2*d**3*m*x**4 - a**2*d**3*x**4 + a*b*
c**3*m + 2*a*b*c**3*p + a*b*c**3 + 3*a*b*c**2*d*m*x**2 + 4*a*b*c**2*d*p*x*
*2 + a*b*c**2*d*x**2 + 3*a*b*c*d**2*m*x**4 + 2*a*b*c*d**2*p*x**4 - a*b*c*d
**2*x**4 + a*b*d**3*m*x**6 - a*b*d**3*x**6 + b**2*c**3*m*x**2 + 2*b**2*c**
3*p*x**2 + b**2*c**3*x**2 + 2*b**2*c**2*d*m*x**4 + 4*b**2*c**2*d*p*x**4 +
2*b**2*c**2*d*x**4 + b**2*c*d**2*m*x**6 + 2*b**2*c*d**2*p*x**6 + b**2*c*d*
*2*x**6),x)*a**2*b**2*c*d**2*m*p + 2*int((x**m*(a + b*x**2)**p*x**4)/(a**2
*c**2*d*m - a**2*c**2*d + 2*a**2*c*d**2*m*x**2 - 2*a**2*c*d**2*x**2 + a**2
*d**3*m*x**4 - a**2*d**3*x**4 + a*b*c**3*m + 2*a*b*c**3*p + a*b*c**3 + 3*a
*b*c**2*d*m*x**2 + 4*a*b*c**2*d*p*x**2 + a*b*c**2*d*x**2 + 3*a*b*c*d**2*m*
x**4 + 2*a*b*c*d**2*p*x**4 - a*b*c*d**2*x**4 + a*b*d**3*m*x**6 - a*b*d*...
```

3.49 $\int \frac{(ex)^m (a+bx^2)^p (A+Bx^2)}{(c+dx^2)^3} \, dx$

Optimal result

Integrand size = 31, antiderivative size = 483

$$\int \frac{(ex)^m (a+bx^2)^p (A+Bx^2)}{(c+dx^2)^3} \, dx = \frac{(Bc-Ad)(ex)^{1+m} (a+bx^2)^{1+p}}{4c(bc-ad)e (c+dx^2)^2}$$

$$+ \frac{(ad(Ad(3-m)+Bc(1+m))+bc(Bc(1-m-2p)-Ad(5-m-2p)))(ex)^{1+m} (a+bx^2)^{1+p}}{8c^2(bc-ad)^2 e (c+dx^2)}$$

$$+ \frac{(a^2d^2(1-m)(Ad(3-m)+Bc(1+m))-2abcd(Bc(1+m)(1-m-2p)+Ad(1-m)(3-m-2p)}{}$$

$$- \frac{b(ad(Ad(3-m)+Bc(1+m))+bc(Bc(1-m-2p)-Ad(5-m-2p)))(1+m+2p)(ex)^{1+m} (a+}{8c^2d(bc-ad)^2 e(1+m)}$$

output
```
1/4*(-A*d+B*c)*(e*x)^(1+m)*(b*x^2+a)^(p+1)/c/(-a*d+b*c)/e/(d*x^2+c)^2+1/8*
(a*d*(A*d*(3-m)+B*c*(1+m))+b*c*(B*c*(1-m-2*p)-A*d*(5-m-2*p)))*(e*x)^(1+m)*
(b*x^2+a)^(p+1)/c^2/(-a*d+b*c)^2/e/(d*x^2+c)+1/8*(a^2*d^2*(1-m)*(A*d*(3-m)
+B*c*(1+m))-2*a*b*c*d*(B*c*(1+m)*(1-m-2*p)+A*d*(1-m)*(3-m-2*p))+b^2*c^2*(1
-m-2*p)*(A*d*(3-m-2*p)+B*c*(1+m+2*p)))*(e*x)^(1+m)*(b*x^2+a)^p*AppellF1(1/
2+1/2*m,-p,1,3/2+1/2*m,-b*x^2/a,-d*x^2/c)/c^3/d/(-a*d+b*c)^2/e/(1+m)/((1+b
*x^2/a)^p)-1/8*b*(a*d*(A*d*(3-m)+B*c*(1+m))+b*c*(B*c*(1-m-2*p)-A*d*(5-m-2*
p)))*(1+m+2*p)*(e*x)^(1+m)*(b*x^2+a)^p*hypergeom([-p, 1/2+1/2*m],[3/2+1/2*
m],-b*x^2/a)/c^2/d/(-a*d+b*c)^2/e/(1+m)/((1+b*x^2/a)^p)
```

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.27

$$\int \frac{(ex)^m (a+bx^2)^p (A+Bx^2)}{(c+dx^2)^3} dx$$
$$= \frac{x(ex)^m (a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} \left(Bc\,\text{AppellF1}\left(\frac{1+m}{2},-p,2,\frac{3+m}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+(-Bc+Ad)\,\text{AppellF1}\left(\right.\right.}{c^3d(1+m)}$$

input
```
Integrate[((e*x)^m*(a + b*x^2)^p*(A + B*x^2))/(c + d*x^2)^3,x]
```

output
```
(x*(e*x)^m*(a + b*x^2)^p*(B*c*AppellF1[(1 + m)/2, -p, 2, (3 + m)/2, -((b*x
^2)/a), -((d*x^2)/c)] + (-(B*c) + A*d)*AppellF1[(1 + m)/2, -p, 3, (3 + m)/
2, -((b*x^2)/a), -((d*x^2)/c)]))/(c^3*d*(1 + m)*(1 + (b*x^2)/a)^p)
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 491, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {441, 441, 446, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)(ex)^m (a+bx^2)^p}{(c+dx^2)^3} dx$$

$$\Big\downarrow \; 441$$

$$\frac{\int \frac{(ex)^m(bx^2+a)^p(b(Bc-Ad)(-m-2p+1)x^2+4Abc-aAd(3-m)-aBc(m+1))}{(dx^2+c)^2}dx}{4c(bc-ad)}+\frac{(ex)^{m+1}(a+bx^2)^{p+1}(Bc-Ad)}{4ce(c+dx^2)^2(bc-ad)}$$

$$\Big\downarrow \; 441$$

$$\int \frac{(ex)^m \left(bx^2+a\right)^p \left(b(d(4Abc-aB(m+1)c-aAd(3-m))-bc(Bc-Ad)(-m-2p+1))(m+2p+1)x^2+aBc(m+1)(ad(1-m)-bc(-m-2p+3))+A\left(8b^2c^2-abd\left(m^2-2(3\right.\right.\right.}{dx^2+c} \Big/ 2c(bc-ad)$$

$$4c(bc-ad$$

$$\frac{(ex)^{m+1}\left(a+bx^2\right)^{p+1}(Bc-Ad)}{4ce\left(c+dx^2\right)^2(bc-ad)}$$

$\big\downarrow$ 446

$$\int \left(\frac{b(d(4Abc-aB(m+1)c-aAd(3-m))-bc(Bc-Ad)(-m-2p+1))(m+2p+1)\left(bx^2+a\right)^p(ex)^m}{d}+\frac{\left(d\left(aBc(m+1)(ad(1-m)-bc(-m-2p+3))+A\left(8b^2c^2-abd\left(m^2-\right.\right.\right.\right.}{}\right. \Big/ 2c(bc-ad)$$

$$\frac{(ex)^{m+1}\left(a+bx^2\right)^{p+1}(Bc-Ad)}{4ce\left(c+dx^2\right)^2(bc-ad)}$$

$\big\downarrow$ 2009

$$\frac{(ex)^{m+1}\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(a^2d^2(1-m)(Ad(3-m)+Bc(m+1))-2abcd(Ad(1-m)(-m-2p+3)+Bc(m+1)(-m-2p+1))+b^2c^2(-m-2p+1)(Ad(-m-2p+3)-\right.}{cde(m+1)}$$

$$\frac{(ex)^{m+1}\left(a+bx^2\right)^{p+1}(Bc-Ad)}{4ce\left(c+dx^2\right)^2(bc-ad)}$$

input
```
Int[((e*x)^m*(a + b*x^2)^p*(A + B*x^2))/(c + d*x^2)^3,x]
```

output
```
((B*c - A*d)*(e*x)^(1 + m)*(a + b*x^2)^(1 + p))/(4*c*(b*c - a*d)*e*(c + d*
x^2)^2) + (((a*d*(A*d*(3 - m) + B*c*(1 + m)) + b*c*(B*c*(1 - m - 2*p) - A*
d*(5 - m - 2*p)))*(e*x)^(1 + m)*(a + b*x^2)^(1 + p))/(2*c*(b*c - a*d)*e*(c
 + d*x^2)) + (((a^2*d^2*(1 - m)*(A*d*(3 - m) + B*c*(1 + m)) - 2*a*b*c*d*(B
*c*(1 + m)*(1 - m - 2*p) + A*d*(1 - m)*(3 - m - 2*p)) + b^2*c^2*(1 - m - 2
*p)*(A*d*(3 - m - 2*p) + B*c*(1 + m + 2*p)))*(e*x)^(1 + m)*(a + b*x^2)^p*A
ppellF1[(1 + m)/2, -p, 1, (3 + m)/2, -((b*x^2)/a), -((d*x^2)/c)])/(c*d*e*(
1 + m)*(1 + (b*x^2)/a)^p) - (b*(a*d*(A*d*(3 - m) + B*c*(1 + m)) + b*c*(B*c
*(1 - m - 2*p) - A*d*(5 - m - 2*p)))*(1 + m + 2*p)*(e*x)^(1 + m)*(a + b*x^
2)^p*Hypergeometric2F1[(1 + m)/2, -p, (3 + m)/2, -((b*x^2)/a)])/(d*e*(1 +
m)*(1 + (b*x^2)/a)^p))/(2*c*(b*c - a*d)))/(4*c*(b*c - a*d))
```

Defintions of rubi rules used

rule 441
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
+ b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(a*g*2*(b*c - a*d)*(p + 1))), x] + Si
mp[1/(a*2*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2
)^q*Simp[c*(b*e - a*f)*(m + 1) + e*2*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(m
 + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, q},
 x] && LtQ[p, -1]
```

rule 446
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((e_) + (f_.)*(x_)^2))/(
(c_) + (d_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a + b*x^2)^
p*((e + f*x^2)/(c + d*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{(ex)^m (b\,x^2+a)^p (x^2B+A)}{(x^2d+c)^3}dx$$

input
```
int((e*x)^m*(b*x^2+a)^p*(B*x^2+A)/(d*x^2+c)^3,x)
```

output
```
int((e*x)^m*(b*x^2+a)^p*(B*x^2+A)/(d*x^2+c)^3,x)
```

Fricas [F]

$$\int \frac{(ex)^m (a+bx^2)^p (A+Bx^2)}{(c+dx^2)^3}\,dx = \int \frac{(Bx^2+A)(bx^2+a)^p(ex)^m}{(dx^2+c)^3}\,dx$$

input
```
integrate((e*x)^m*(b*x^2+a)^p*(B*x^2+A)/(d*x^2+c)^3,x, algorithm="fricas")
```

output
```
integral((B*x^2 + A)*(b*x^2 + a)^p*(e*x)^m/(d^3*x^6 + 3*c*d^2*x^4 + 3*c^2*
d*x^2 + c^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ex)^m (a+bx^2)^p (A+Bx^2)}{(c+dx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((e*x)**m*(b*x**2+a)**p*(B*x**2+A)/(d*x**2+c)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(ex)^m (a+bx^2)^p (A+Bx^2)}{(c+dx^2)^3}\,dx = \int \frac{(Bx^2+A)(bx^2+a)^p(ex)^m}{(dx^2+c)^3}\,dx$$

input
```
integrate((e*x)^m*(b*x^2+a)^p*(B*x^2+A)/(d*x^2+c)^3,x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*(b*x^2 + a)^p*(e*x)^m/(d*x^2 + c)^3, x)
```

Giac [F]

$$\int \frac{(ex)^m (a+bx^2)^p (A+Bx^2)}{(c+dx^2)^3}\,dx = \int \frac{(Bx^2+A)(bx^2+a)^p(ex)^m}{(dx^2+c)^3}\,dx$$

input
```
integrate((e*x)^m*(b*x^2+a)^p*(B*x^2+A)/(d*x^2+c)^3,x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*(b*x^2 + a)^p*(e*x)^m/(d*x^2 + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m (a+bx^2)^p (A+Bx^2)}{(c+dx^2)^3} \, dx = \int \frac{(B\,x^2+A)\,(e\,x)^m\,(b\,x^2+a)^p}{(d\,x^2+c)^3} \, dx$$

input
```
int(((A + B*x^2)*(e*x)^m*(a + b*x^2)^p)/(c + d*x^2)^3,x)
```

output
```
int(((A + B*x^2)*(e*x)^m*(a + b*x^2)^p)/(c + d*x^2)^3, x)
```

Reduce [F]

$$\int \frac{(ex)^m (a+bx^2)^p (A+Bx^2)}{(c+dx^2)^3} \, dx = \text{too large to display}$$

input
```
int((e*x)^m*(b*x^2+a)^p*(B*x^2+A)/(d*x^2+c)^3,x)
```

output
```
(e**m*(2*x**m*(a + b*x**2)**p*a*b*x - int((x**m*(a + b*x**2)**p*x**4)/(a**
2*c**3*d*m - 3*a**2*c**3*d + 3*a**2*c**2*d**2*m*x**2 - 9*a**2*c**2*d**2*x*
*2 + 3*a**2*c*d**3*m*x**4 - 9*a**2*c*d**3*x**4 + a**2*d**4*m*x**6 - 3*a**2
*d**4*x**6 + a*b*c**4*m + 2*a*b*c**4*p + a*b*c**4 + 4*a*b*c**3*d*m*x**2 +
6*a*b*c**3*d*p*x**2 + 6*a*b*c**2*d**2*m*x**4 + 6*a*b*c**2*d**2*p*x**4 - 6*
a*b*c**2*d**2*x**4 + 4*a*b*c*d**3*m*x**6 + 2*a*b*c*d**3*p*x**6 - 8*a*b*c*d
**3*x**6 + a*b*d**4*m*x**8 - 3*a*b*d**4*x**8 + b**2*c**4*m*x**2 + 2*b**2*c
**4*p*x**2 + b**2*c**4*x**2 + 3*b**2*c**3*d*m*x**4 + 6*b**2*c**3*d*p*x**4
+ 3*b**2*c**3*d*x**4 + 3*b**2*c**2*d**2*m*x**6 + 6*b**2*c**2*d**2*p*x**6 +
 3*b**2*c**2*d**2*x**6 + b**2*c*d**3*m*x**8 + 2*b**2*c*d**3*p*x**8 + b**2*
c*d**3*x**8),x)*a**2*b**2*c**2*d**2*m**2 - 4*int((x**m*(a + b*x**2)**p*x**
4)/(a**2*c**3*d*m - 3*a**2*c**3*d + 3*a**2*c**2*d**2*m*x**2 - 9*a**2*c**2*
d**2*x**2 + 3*a**2*c*d**3*m*x**4 - 9*a**2*c*d**3*x**4 + a**2*d**4*m*x**6 -
 3*a**2*d**4*x**6 + a*b*c**4*m + 2*a*b*c**4*p + a*b*c**4 + 4*a*b*c**3*d*m*
x**2 + 6*a*b*c**3*d*p*x**2 + 6*a*b*c**2*d**2*m*x**4 + 6*a*b*c**2*d**2*p*x*
*4 - 6*a*b*c**2*d**2*x**4 + 4*a*b*c*d**3*m*x**6 + 2*a*b*c*d**3*p*x**6 - 8*
a*b*c*d**3*x**6 + a*b*d**4*m*x**8 - 3*a*b*d**4*x**8 + b**2*c**4*m*x**2 + 2
*b**2*c**4*p*x**2 + b**2*c**4*x**2 + 3*b**2*c**3*d*m*x**4 + 6*b**2*c**3*d*
p*x**4 + 3*b**2*c**3*d*x**4 + 3*b**2*c**2*d**2*m*x**6 + 6*b**2*c**2*d**2*p
*x**6 + 3*b**2*c**2*d**2*x**6 + b**2*c*d**3*m*x**8 + 2*b**2*c*d**3*p*x*...
```

3.50 $\int \frac{\sqrt{a+bx^2}(A+Bx^2)(c+dx^2)}{x}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 94

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)(c+dx^2)}{x}\,dx = Ac\sqrt{a+bx^2}+\frac{(bBc+Abd-aBd)(a+bx^2)^{3/2}}{3b^2}+\frac{Bd(a+bx^2)^{5/2}}{5b^2}-\sqrt{a}A\text{carctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

output
```
A*c*(b*x^2+a)^(1/2)+1/3*(A*b*d-B*a*d+B*b*c)*(b*x^2+a)^(3/2)/b^2+1/5*B*d*(b
*x^2+a)^(5/2)/b^2-a^(1/2)*A*c*arctanh((b*x^2+a)^(1/2)/a^(1/2))
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.97

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)(c+dx^2)}{x}\,dx = \frac{\sqrt{a+bx^2}(-B(a+bx^2)(-5bc+2ad-3bdx^2)+5Ab(3bc+ad+bdx^2))}{15b^2}-\sqrt{a}A\text{carctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

input
```
Integrate[(Sqrt[a + b*x^2]*(A + B*x^2)*(c + d*x^2))/x,x]
```

output
```
(Sqrt[a + b*x^2]*(-(B*(a + b*x^2)*(-5*b*c + 2*a*d - 3*b*d*x^2)) + 5*A*b*(3
*b*c + a*d + b*d*x^2)))/(15*b^2) - Sqrt[a]*A*c*ArcTanh[Sqrt[a + b*x^2]/Sqr
t[a]]
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {435, 164, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\left(A+Bx^2\right)\left(c+dx^2\right)}{x}dx$$

$$\Big\downarrow \text{ 435}$$

$$\frac{1}{2}\int \frac{\sqrt{bx^2+a}\left(Bx^2+A\right)\left(dx^2+c\right)}{x^2}dx^2$$

$$\Big\downarrow \text{ 164}$$

$$\frac{1}{2}\left(Ac\int \frac{\sqrt{bx^2+a}}{x^2}dx^2-\frac{2\left(a+bx^2\right)^{3/2}\left(2aBd-5b(Ad+Bc)-3bBdx^2\right)}{15b^2}\right)$$

$$\Big\downarrow \text{ 60}$$

$$\frac{1}{2}\left(Ac\left(a\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2+2\sqrt{a+bx^2}\right)-\frac{2\left(a+bx^2\right)^{3/2}\left(2aBd-5b(Ad+Bc)-3bBdx^2\right)}{15b^2}\right)$$

$$\Big\downarrow \text{ 73}$$

$$\frac{1}{2}\left(Ac\left(\frac{2a\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{b}+2\sqrt{a+bx^2}\right)-\frac{2\left(a+bx^2\right)^{3/2}\left(2aBd-5b(Ad+Bc)-3bBdx^2\right)}{15b^2}\right)$$

$$\Big\downarrow \text{ 221}$$

$$\frac{1}{2}\left(Ac\left(2\sqrt{a+bx^2}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\right)-\frac{2(a+bx^2)^{3/2}\left(2aBd-5b(Ad+Bc)-3bBdx^2\right)}{15b^2}\right)$$

input
```
Int[(Sqrt[a + b*x^2]*(A + B*x^2)*(c + d*x^2))/x,x]
```

output
```
((-2*(a + b*x^2)^(3/2)*(2*a*B*d - 5*b*(B*c + A*d) - 3*b*B*d*x^2))/(15*b^2)
 + A*c*(2*Sqrt[a + b*x^2] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]))/2
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 435
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^(q_.)*((
e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)
*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^2], x] /; FreeQ[{a, b, c, d,
 e, f, p, q, r}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.01

method	result
pseudoelliptic	$\frac{-3A\sqrt{a}\,b^2 c\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)+\sqrt{b\,x^2+a}\left(\left(x^2\left(\frac{3x^2 d}{5}+c\right)B+3\left(\frac{x^2 d}{3}+c\right)A\right)b^2+a\left(\left(\frac{x^2 d}{5}+c\right)B+Ad\right)b-\frac{2B\,a^2 d}{5}\right)}{3b^2}$
default	$Ac\left(\sqrt{b\,x^2+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)\right)+\frac{Bc(b\,x^2+a)^{\frac{3}{2}}}{3b}+Bd\left(\frac{x^2(b\,x^2+a)^{\frac{3}{2}}}{5b}-\frac{2a(b\,x^2+a)^{\frac{3}{2}}}{15b^2}\right)$

input
```
int((b*x^2+a)^(1/2)*(B*x^2+A)*(d*x^2+c)/x,x,method=_RETURNVERBOSE)
```

output
```
1/3*(-3*A*a^(1/2)*b^2*c*arctanh((b*x^2+a)^(1/2)/a^(1/2))+(b*x^2+a)^(1/2)*(
(x^2*(3/5*x^2*d+c)*B+3*(1/3*x^2*d+c)*A)*b^2+a*((1/5*x^2*d+c)*B+A*d)*b-2/5*
B*a^2*d))/b^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 234, normalized size of antiderivative = 2.49

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)\,(c+dx^2)}{x}\,dx$$
$$=\left[\frac{15\,A\sqrt{a}b^2c\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\,(3\,Bb^2dx^4+(5\,Bb^2c+(Bab+5\,Ab^2)d)x^2+5\,(Bab+3\,Ab^2)c}{30\,b^2}\right.$$

input
```
integrate((b*x^2+a)^(1/2)*(B*x^2+A)*(d*x^2+c)/x,x, algorithm="fricas")
```

output
```
[1/30*(15*A*sqrt(a)*b^2*c*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x
^2) + 2*(3*B*b^2*d*x^4 + (5*B*b^2*c + (B*a*b + 5*A*b^2)*d)*x^2 + 5*(B*a*b
+ 3*A*b^2)*c - (2*B*a^2 - 5*A*a*b)*d)*sqrt(b*x^2 + a))/b^2, 1/15*(15*A*sqr
t(-a)*b^2*c*arctan(sqrt(b*x^2 + a)*sqrt(-a)/a) + (3*B*b^2*d*x^4 + (5*B*b^2
*c + (B*a*b + 5*A*b^2)*d)*x^2 + 5*(B*a*b + 3*A*b^2)*c - (2*B*a^2 - 5*A*a*b
)*d)*sqrt(b*x^2 + a))/b^2]
```

Sympy [A] (verification not implemented)

Time = 10.61 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.61

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)(c+dx^2)}{x}\,dx = \frac{\begin{cases} \frac{2Aac\operatorname{atan}\left(\frac{\sqrt{a+bx^2}}{\sqrt{-a}}\right)}{\sqrt{-a}} + 2Ac\sqrt{a+bx^2} + \frac{2Bd(a+bx^2)^{\frac{5}{2}}}{5b^2} + \frac{2(a+bx^2)^{\frac{3}{2}}(Abd-Bad+Bbc)}{3b^2} & \text{for } b \neq 0 \\ A\sqrt{a}c\log(x^2) + A\sqrt{a}dx^2 + B\sqrt{a}cx^2 + \frac{B\sqrt{a}dx^4}{2} & \text{otherwise} \end{cases}}{2}$$

input
```
integrate((b*x**2+a)**(1/2)*(B*x**2+A)*(d*x**2+c)/x,x)
```

output
```
Piecewise((2*A*a*c*atan(sqrt(a + b*x**2)/sqrt(-a))/sqrt(-a) + 2*A*c*sqrt(a
 + b*x**2) + 2*B*d*(a + b*x**2)**(5/2)/(5*b**2) + 2*(a + b*x**2)**(3/2)*(A
*b*d - B*a*d + B*b*c)/(3*b**2), Ne(b, 0)), (A*sqrt(a)*c*log(x**2) + A*sqrt
(a)*d*x**2 + B*sqrt(a)*c*x**2 + B*sqrt(a)*d*x**4/2, True))/2
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.06

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)(c+dx^2)}{x}\,dx = \frac{(bx^2+a)^{\frac{3}{2}}Bdx^2}{5\,b} - A\sqrt{a}c\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right) + \sqrt{bx^2+a}Ac + \frac{(bx^2+a)^{\frac{3}{2}}Bc}{3\,b} - \frac{2\,(bx^2+a)^{\frac{3}{2}}Bad}{15\,b^2} + \frac{(bx^2+a)^{\frac{3}{2}}Ad}{3\,b}$$

input
```
integrate((b*x^2+a)^(1/2)*(B*x^2+A)*(d*x^2+c)/x,x, algorithm="maxima")
```

output
```
1/5*(b*x^2 + a)^(3/2)*B*d*x^2/b - A*sqrt(a)*c*arcsinh(a/(sqrt(a*b)*abs(x))
) + sqrt(b*x^2 + a)*A*c + 1/3*(b*x^2 + a)^(3/2)*B*c/b - 2/15*(b*x^2 + a)^(
3/2)*B*a*d/b^2 + 1/3*(b*x^2 + a)^(3/2)*A*d/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.20

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)\left(c+dx^2\right)}{x}\,dx = \frac{Aac\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} + \frac{5\left(bx^2+a\right)^{\frac{3}{2}}Bb^9c+15\sqrt{bx^2+a}Ab^{10}c+3\left(bx^2+a\right)^{\frac{5}{2}}Bb^8d-5\left(bx^2+a\right)^{\frac{3}{2}}Bab^8d+5\left(bx^2+a\right)^{\frac{3}{2}}Ab^9d}{15\,b^{10}}$$

input
```
integrate((b*x^2+a)^(1/2)*(B*x^2+A)*(d*x^2+c)/x,x, algorithm="giac")
```

output
```
A*a*c*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) + 1/15*(5*(b*x^2 + a)^(3/2
)*B*b^9*c + 15*sqrt(b*x^2 + a)*A*b^10*c + 3*(b*x^2 + a)^(5/2)*B*b^8*d - 5*
(b*x^2 + a)^(3/2)*B*a*b^8*d + 5*(b*x^2 + a)^(3/2)*A*b^9*d)/b^10
```

Mupad [B] (verification not implemented)

Time = 0.78 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)\left(c+dx^2\right)}{x}\,dx = \sqrt{b\,x^2+a}\left(\frac{B\,d\,x^4}{5}-\frac{B\,a\,\left(2\,a\,d-5\,b\,c\right)}{15\,b^2}+\frac{B\,x^2\,\left(a\,d+5\,b\,c\right)}{15\,b}\right)+A\,c\,\sqrt{b\,x^2+a}+\frac{A\,d\,{\left(b\,x^2+a\right)}^{3/2}}{3\,b}-A\,\sqrt{a}\,c\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)$$

input
```
int(((A + B*x^2)*(a + b*x^2)^(1/2)*(c + d*x^2))/x,x)
```

output
```
(a + b*x^2)^(1/2)*((B*d*x^4)/5 - (B*a*(2*a*d - 5*b*c))/(15*b^2) + (B*x^2*(
a*d + 5*b*c))/(15*b)) + A*c*(a + b*x^2)^(1/2) + (A*d*(a + b*x^2)^(3/2))/(3
*b) - A*a^(1/2)*c*atanh((a + b*x^2)^(1/2)/a^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.50

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)(c+dx^2)}{x}\,dx$$
$$= \frac{3\sqrt{b\,x^2+a}\,a^2d+20\sqrt{b\,x^2+a}\,abc+6\sqrt{b\,x^2+a}\,abd\,x^2+5\sqrt{b\,x^2+a}\,b^2c\,x^2+3\sqrt{b\,x^2+a}\,b^2d\,x^4+15\sqrt{a}\,\ldots}{15b}$$

input
```
int((b*x^2+a)^(1/2)*(B*x^2+A)*(d*x^2+c)/x,x)
```

output
```
(3*sqrt(a + b*x**2)*a**2*d + 20*sqrt(a + b*x**2)*a*b*c + 6*sqrt(a + b*x**2
)*a*b*d*x**2 + 5*sqrt(a + b*x**2)*b**2*c*x**2 + 3*sqrt(a + b*x**2)*b**2*d*
x**4 + 15*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*a*
b*c - 15*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b
*c)/(15*b)
```

3.51 $\int \frac{(a+bx^2)(A+Bx^2)\sqrt{c+dx^2}}{x}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 95

$$\int \frac{(a+bx^2)(A+Bx^2)\sqrt{c+dx^2}}{x}\,dx = aA\sqrt{c+dx^2} - \frac{(bBc-Abd-aBd)(c+dx^2)^{3/2}}{3d^2} + \frac{bB(c+dx^2)^{5/2}}{5d^2} - aA\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)$$

output
```
a*A*(d*x^2+c)^(1/2)-1/3*(-A*b*d-B*a*d+B*b*c)*(d*x^2+c)^(3/2)/d^2+1/5*b*B*(
d*x^2+c)^(5/2)/d^2-a*A*c^(1/2)*arctanh((d*x^2+c)^(1/2)/c^(1/2))
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^2)(A+Bx^2)\sqrt{c+dx^2}}{x}\,dx = \frac{\sqrt{c+dx^2}(-b(c+dx^2)(2Bc-5Ad-3Bdx^2)+5ad(3Ad+B(c+dx^2)))}{15d^2} - aA\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)$$

input
```
Integrate[((a + b*x^2)*(A + B*x^2)*Sqrt[c + d*x^2])/x,x]
```

output
```
(Sqrt[c + d*x^2]*(-(b*(c + d*x^2)*(2*B*c - 5*A*d - 3*B*d*x^2)) + 5*a*d*(3*
A*d + B*(c + d*x^2))))/(15*d^2) - a*A*Sqrt[c]*ArcTanh[Sqrt[c + d*x^2]/Sqrt
[c]]
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {435, 164, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)(A+Bx^2)\sqrt{c+dx^2}}{x}dx$$

$$\Big\downarrow \text{435}$$

$$\frac{1}{2}\int \frac{(bx^2+a)(Bx^2+A)\sqrt{dx^2+c}}{x^2}dx^2$$

$$\Big\downarrow \text{164}$$

$$\frac{1}{2}\left(aA\int \frac{\sqrt{dx^2+c}}{x^2}dx^2-\frac{2(c+dx^2)^{3/2}(-5d(aB+Ab)+2bBc-3bBdx^2)}{15d^2}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(aA\left(c\int \frac{1}{x^2\sqrt{dx^2+c}}dx^2+2\sqrt{c+dx^2}\right)-\frac{2(c+dx^2)^{3/2}(-5d(aB+Ab)+2bBc-3bBdx^2)}{15d^2}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(aA\left(\frac{2c\int \frac{1}{\frac{x^4}{d}-\frac{c}{d}}d\sqrt{dx^2+c}}{d}+2\sqrt{c+dx^2}\right)-\frac{2(c+dx^2)^{3/2}(-5d(aB+Ab)+2bBc-3bBdx^2)}{15d^2}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(aA\left(2\sqrt{c+dx^2}-2\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)\right)-\frac{2(c+dx^2)^{3/2}\left(-5d(aB+Ab)+2bBc-3bBdx^2\right)}{15d^2}\right)$$

input
```
Int[((a + b*x^2)*(A + B*x^2)*Sqrt[c + d*x^2])/x,x]
```

output
```
((-2*(c + d*x^2)^(3/2)*(2*b*B*c - 5*(A*b + a*B)*d - 3*b*B*d*x^2))/(15*d^2)
 + a*A*(2*Sqrt[c + d*x^2] - 2*Sqrt[c]*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]]))/2
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 435
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^(q_.)*((
e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)
*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^2], x] /; FreeQ[{a, b, c, d,
 e, f, p, q, r}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00

method	result
pseudoelliptic	$\frac{-Aa\sqrt{c}\,d^2\operatorname{arctanh}\left(\frac{\sqrt{x^2d+c}}{\sqrt{c}}\right)+\sqrt{x^2d+c}\left(\left(\frac{x^2\left(\frac{3b\,x^2}{5}+a\right)B}{3}+A\left(\frac{b\,x^2}{3}+a\right)\right)d^2+\frac{\left(\left(\frac{b\,x^2}{5}+a\right)B+Ab\right)cd}{3}-\frac{2Bb\,c^2}{15}\right)}{d^2}$
default	$\frac{Ab(x^2d+c)^{\frac{3}{2}}}{3d}+\frac{Ba(x^2d+c)^{\frac{3}{2}}}{3d}+Aa\left(\sqrt{x^2d+c}-\sqrt{c}\,\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{x^2d+c}}{x}\right)\right)+Bb\left(\frac{x^2(x^2d+c)^{\frac{3}{2}}}{5d}-\frac{2}{}\right.$

input
```
int((b*x^2+a)*(B*x^2+A)*(d*x^2+c)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
(-A*a*c^(1/2)*d^2*arctanh((d*x^2+c)^(1/2)/c^(1/2))+(d*x^2+c)^(1/2)*((1/3*x
^2*(3/5*b*x^2+a)*B+A*(1/3*b*x^2+a))*d^2+1/3*((1/5*b*x^2+a)*B+A*b)*c*d-2/15
*B*b*c^2))/d^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 220, normalized size of antiderivative = 2.32

$$\int \frac{(a+bx^2)\left(A+Bx^2\right)\sqrt{c+dx^2}}{x}\,dx$$
$$=\left[\frac{15\,Aa\sqrt{c}d^2\log\left(-\frac{dx^2-2\,\sqrt{dx^2+c}\sqrt{c}+2\,c}{x^2}\right)+2\,(3\,Bbd^2x^4-2\,Bbc^2+15\,Aad^2+5\,(Ba+Ab)cd+(Bbcd+5}{30\,d^2}\right.$$

input
```
integrate((b*x^2+a)*(B*x^2+A)*(d*x^2+c)^(1/2)/x,x, algorithm="fricas")
```

output
```
[1/30*(15*A*a*sqrt(c)*d^2*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x
^2) + 2*(3*B*b*d^2*x^4 - 2*B*b*c^2 + 15*A*a*d^2 + 5*(B*a + A*b)*c*d + (B*b
*c*d + 5*(B*a + A*b)*d^2)*x^2)*sqrt(d*x^2 + c))/d^2, 1/15*(15*A*a*sqrt(-c)
*d^2*arctan(sqrt(d*x^2 + c)*sqrt(-c)/c) + (3*B*b*d^2*x^4 - 2*B*b*c^2 + 15*
A*a*d^2 + 5*(B*a + A*b)*c*d + (B*b*c*d + 5*(B*a + A*b)*d^2)*x^2)*sqrt(d*x^
2 + c))/d^2]
```

Sympy [A] (verification not implemented)

Time = 10.52 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.59

$$\int \frac{(a+bx^2)(A+Bx^2)\sqrt{c+dx^2}}{x}\,dx = \frac{\begin{cases} \frac{2Aac\,\mathrm{atan}\left(\frac{\sqrt{c+dx^2}}{\sqrt{-c}}\right)}{\sqrt{-c}} + 2Aa\sqrt{c+dx^2} + \frac{2Bb(c+dx^2)^{\frac{5}{2}}}{5d^2} + \frac{2(c+dx^2)^{\frac{3}{2}}(Abd+Bad-Bbc)}{3d^2} & \text{for } d\neq 0 \\ Aa\sqrt{c}\log(x^2) + Ab\sqrt{c}x^2 + Ba\sqrt{c}x^2 + \frac{Bb\sqrt{c}x^4}{2} & \text{otherwise} \end{cases}}{2}$$

input
```
integrate((b*x**2+a)*(B*x**2+A)*(d*x**2+c)**(1/2)/x,x)
```

output
```
Piecewise((2*A*a*c*atan(sqrt(c + d*x**2)/sqrt(-c))/sqrt(-c) + 2*A*a*sqrt(c
 + d*x**2) + 2*B*b*(c + d*x**2)**(5/2)/(5*d**2) + 2*(c + d*x**2)**(3/2)*(A
*b*d + B*a*d - B*b*c)/(3*d**2), Ne(d, 0)), (A*a*sqrt(c)*log(x**2) + A*b*sq
rt(c)*x**2 + B*a*sqrt(c)*x**2 + B*b*sqrt(c)*x**4/2, True))/2
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.05

$$\int \frac{(a+bx^2)(A+Bx^2)\sqrt{c+dx^2}}{x}\,dx = \frac{(dx^2+c)^{\frac{3}{2}}Bbx^2}{5\,d} - Aa\sqrt{c}\,\mathrm{arsinh}\left(\frac{c}{\sqrt{cd}|x|}\right) + \sqrt{dx^2+c}Aa - \frac{2\,(dx^2+c)^{\frac{3}{2}}Bbc}{15\,d^2} + \frac{(dx^2+c)^{\frac{3}{2}}Ba}{3\,d} + \frac{(dx^2+c)^{\frac{3}{2}}Ab}{3\,d}$$

input
```
integrate((b*x^2+a)*(B*x^2+A)*(d*x^2+c)^(1/2)/x,x, algorithm="maxima")
```

output
```
1/5*(d*x^2 + c)^(3/2)*B*b*x^2/d - A*a*sqrt(c)*arcsinh(c/(sqrt(c*d)*abs(x))
) + sqrt(d*x^2 + c)*A*a - 2/15*(d*x^2 + c)^(3/2)*B*b*c/d^2 + 1/3*(d*x^2 +
c)^(3/2)*B*a/d + 1/3*(d*x^2 + c)^(3/2)*A*b/d
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.19

$$\int \frac{(a+bx^2)(A+Bx^2)\sqrt{c+dx^2}}{x}\,dx = \frac{Aac\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}} + \frac{3\,(dx^2+c)^{\frac{5}{2}}Bbd^8 - 5\,(dx^2+c)^{\frac{3}{2}}Bbcd^8 + 5\,(dx^2+c)^{\frac{3}{2}}Bad^9 + 5\,(dx^2+c)^{\frac{3}{2}}Abd^9 + 15\sqrt{dx^2+c}Aad^{10}}{15\,d^{10}}$$

input
```
integrate((b*x^2+a)*(B*x^2+A)*(d*x^2+c)^(1/2)/x,x, algorithm="giac")
```

output
```
A*a*c*arctan(sqrt(d*x^2 + c)/sqrt(-c))/sqrt(-c) + 1/15*(3*(d*x^2 + c)^(5/2
)*B*b*d^8 - 5*(d*x^2 + c)^(3/2)*B*b*c*d^8 + 5*(d*x^2 + c)^(3/2)*B*a*d^9 +
5*(d*x^2 + c)^(3/2)*A*b*d^9 + 15*sqrt(d*x^2 + c)*A*a*d^10)/d^10
```

Mupad [B] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx^2)(A+Bx^2)\sqrt{c+dx^2}}{x}\,dx = \sqrt{d\,x^2+c}\left(\frac{B\,b\,x^4}{5}+\frac{B\,c\,(5\,a\,d-2\,b\,c)}{15\,d^2}+\frac{B\,x^2\,(5\,a\,d+b\,c)}{15\,d}\right)+A\,a\,\sqrt{d\,x^2+c}-A\,a\,\sqrt{c}\,\mathrm{atanh}\left(\frac{\sqrt{d\,x^2+c}}{\sqrt{c}}\right)+\frac{A\,b\,{\left(d\,x^2+c\right)}^{3/2}}{3\,d}$$

input
```
int(((A + B*x^2)*(a + b*x^2)*(c + d*x^2)^(1/2))/x,x)
```

output
```
(c + d*x^2)^(1/2)*((B*b*x^4)/5 + (B*c*(5*a*d - 2*b*c))/(15*d^2) + (B*x^2*(
5*a*d + b*c))/(15*d)) + A*a*(c + d*x^2)^(1/2) - A*a*c^(1/2)*atanh((c + d*x
^2)^(1/2)/c^(1/2)) + (A*b*(c + d*x^2)^(3/2))/(3*d)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.79

$$\int \frac{(a+bx^2)(A+Bx^2)\sqrt{c+dx^2}}{x}\,dx$$

$$= \frac{15\sqrt{d\,x^2+c}\,a^2d^2+10\sqrt{d\,x^2+c}\,abcd+10\sqrt{d\,x^2+c}\,ab\,d^2x^2-2\sqrt{d\,x^2+c}\,b^2c^2+\sqrt{d\,x^2+c}\,b^2cd\,x^2+3\sqrt{}}{15d^2}$$

input
```
int((b*x^2+a)*(B*x^2+A)*(d*x^2+c)^(1/2)/x,x)
```

output
```
(15*sqrt(c + d*x**2)*a**2*d**2 + 10*sqrt(c + d*x**2)*a*b*c*d + 10*sqrt(c +
 d*x**2)*a*b*d**2*x**2 - 2*sqrt(c + d*x**2)*b**2*c**2 + sqrt(c + d*x**2)*b
**2*c*d*x**2 + 3*sqrt(c + d*x**2)*b**2*d**2*x**4 + 15*sqrt(c)*log((sqrt(c
+ d*x**2) - sqrt(c) + sqrt(d)*x)/sqrt(c))*a**2*d**2 - 15*sqrt(c)*log((sqrt
(c + d*x**2) + sqrt(c) + sqrt(d)*x)/sqrt(c))*a**2*d**2)/(15*d**2)
```

3.52 $\int x^6\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^2\,dx$

Optimal result

Integrand size = 35, antiderivative size = 1522

$$\int x^6\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^2\,dx=\text{Too large to display}$$

output
```
-2/45045*(640*a^6*d^6*f^2-128*a^5*b*d^5*f*(2*c*f+13*d*e)+a^4*b^2*d^4*(-157
*c^2*f^2+728*c*d*e*f+1144*d^2*e^2)-4*a*b^5*c^3*d*(64*c^2*f^2-182*c*d*e*f+1
43*d^2*e^2)+8*b^6*c^4*(80*c^2*f^2-208*c*d*e*f+143*d^2*e^2)-a^3*b^3*c*d^3*(
139*c^2*f^2-481*c*d*e*f+572*d^2*e^2)-a^2*b^4*c^2*d^2*(157*c^2*f^2-481*c*d*
e*f+429*d^2*e^2))*x*(d*x^2+c)^(1/2)/b^5/d^6/(b*x^2+a)^(1/2)+1/45045*(640*a
^5*d^5*f^3-16*a^4*b*d^4*f^2*(11*c*f+104*d*e)+a^2*b^3*c*d^2*f*(-149*c^2*f^2
+468*c*d*e*f+2721*d^2*e^2)+a^3*b^2*d^3*f*(-149*c^2*f^2+520*c*d*e*f+1144*d^
2*e^2)+8*b^5*c^3*f*(80*c^2*f^2-208*c*d*e*f+143*d^2*e^2)-a*b^4*c*d*(176*c^3
*f^3-520*c^2*d*e*f^2-2721*c*d^2*e^2*f+1260*d^3*e^3))*x*(b*x^2+a)^(1/2)*(d*
x^2+c)^(1/2)/b^5/d^5/f-1/45045*(480*a^4*d^4*f^3-16*a^3*b*d^3*f^2*(7*c*f+78
*d*e)-2*a^2*b^2*d^2*f*(53*c^2*f^2+2981*c*d*e*f+1146*d^2*e^2)+a*b^3*d*(-112
*c^3*f^3-5962*c^2*d*e*f^2-601*c*d^2*e^2*f+1260*d^3*e^3)+12*b^4*c*(40*c^3*f
^3-104*c^2*d*e*f^2-191*c*d^2*e^2*f+105*d^3*e^3))*x^3*(b*x^2+a)^(1/2)*(d*x^
2+c)^(1/2)/b^4/d^4/f+1/9009*(80*a^3*d^3*f^3+a^2*b*d^2*f^2*(613*c*f+1052*d*
e)-a*b^2*d*f*(-613*c^2*f^2-934*c*d*e*f+424*d^2*e^2)-4*b^3*(-20*c^3*f^3-263
*c^2*d*e*f^2+106*c*d^2*e^2*f+63*d^3*e^3))*x^5*(b*x^2+a)^(1/2)*(d*x^2+c)^(1
/2)/b^3/d^3/f+1/1287*(80*a^2*d^2*f^2-a*b*d*f*(-83*c*f+28*d*e)-4*b^2*(-20*c
^2*f^2+7*c*d*e*f+7*d^2*e^2))*x^7*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b^2/d^2-2
/143*(5*a/b+5*c/d-2*e/f)*x*(b*x^2+a)^(3/2)*(d*x^2+c)^(3/2)*(f*x^2+e)^2/b/d
+1/13*x^3*(b*x^2+a)^(3/2)*(d*x^2+c)^(3/2)*(f*x^2+e)^2/b/d+2/45045*a^(1/...
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.67 (sec) , antiderivative size = 1001, normalized size of antiderivative = 0.66

$$\int x^6\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^2\,dx$$
$$=\frac{\sqrt{\frac{b}{a}}dx(a+bx^2)\left(c+dx^2\right)\left(640a^5d^5f^2-16a^4bd^4f(104de+11cf+30dfx^2)+a^3b^2d^3(-149c^2f^2+8cdf(6\cdots\right.}{}$$

input
```
Integrate[x^6*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^2,x]
```

output
```
(Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2)*(640*a^5*d^5*f^2 - 16*a^4*b*d^4*f*(
104*d*e + 11*c*f + 30*d*f*x^2) + a^3*b^2*d^3*(-149*c^2*f^2 + 8*c*d*f*(65*e
 + 14*f*x^2) + 8*d^2*(143*e^2 + 156*e*f*x^2 + 50*f^2*x^4)) + a*b^4*d*(-176
*c^4*f^2 + 8*c^3*d*f*(65*e + 14*f*x^2) + 2*c*d^3*x^2*(143*e^2 + 130*e*f*x^
2 + 35*f^2*x^4) + 5*d^4*x^4*(143*e^2 + 182*e*f*x^2 + 63*f^2*x^4) - c^2*d^2
*(429*e^2 + 338*e*f*x^2 + 85*f^2*x^4)) - a^2*b^3*d^2*(149*c^3*f^2 - 2*c^2*
d*f*(234*e + 53*f*x^2) + c*d^2*(429*e^2 + 338*e*f*x^2 + 85*f^2*x^4) + 2*d^
3*x^2*(429*e^2 + 520*e*f*x^2 + 175*f^2*x^4)) + b^5*(640*c^5*f^2 - 32*c^4*d
*f*(52*e + 15*f*x^2) + 8*c^3*d^2*(143*e^2 + 156*e*f*x^2 + 50*f^2*x^4) + 5*
c*d^4*x^4*(143*e^2 + 182*e*f*x^2 + 63*f^2*x^4) + 35*d^5*x^6*(143*e^2 + 234
*e*f*x^2 + 99*f^2*x^4) - 2*c^2*d^3*x^2*(429*e^2 + 520*e*f*x^2 + 175*f^2*x^
4))) + (2*I)*c*(640*a^6*d^6*f^2 - 128*a^5*b*d^5*f*(13*d*e + 2*c*f) + a^2*b
^4*c^2*d^2*(-429*d^2*e^2 + 481*c*d*e*f - 157*c^2*f^2) + a^4*b^2*d^4*(1144*
d^2*e^2 + 728*c*d*e*f - 157*c^2*f^2) + a^3*b^3*c*d^3*(-572*d^2*e^2 + 481*c
*d*e*f - 139*c^2*f^2) - 4*a*b^5*c^3*d*(143*d^2*e^2 - 182*c*d*e*f + 64*c^2*
f^2) + 8*b^6*c^4*(143*d^2*e^2 - 208*c*d*e*f + 80*c^2*f^2))*Sqrt[1 + (b*x^2
)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] -
I*c*(-(b*c) + a*d)*(640*a^5*d^5*f^2 - 16*a*b^4*c^3*d*f*(-13*d*e + 8*c*f) +
 16*a^4*b*d^4*f*(-104*d*e + 19*c*f) + a^2*b^3*c*d^2*(429*d^2*e^2 - 234*c*d
*e*f + 10*c^2*f^2) - 16*b^5*c^3*(143*d^2*e^2 - 208*c*d*e*f + 80*c^2*f^2...
```

Rubi [A] (warning: unable to verify)

Time = 3.33 (sec) , antiderivative size = 1802, normalized size of antiderivative = 1.18, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.514$, Rules used = {448, 443, 443, 444, 25, 444, 27, 444, 27, 406, 320, 388, 313, 444, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^6\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^2\,dx$$

$$\Big\downarrow \text{448}$$

$$\frac{f\int x^8\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)\,dx}{e^2}+e\int x^6\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)\,dx$$

$$\Big\downarrow \text{443}$$

$$\frac{f\left(\frac{\int \frac{x^8\sqrt{bx^2+a}\left((13bde+bcf-10adf)x^2+c(13be-9af)\right)}{\sqrt{dx^2+c}}dx}{13b}+\frac{fx^9\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{13b}\right)}{e^2}+$$
$$e\left(\frac{\int \frac{x^6\sqrt{bx^2+a}((11bde+bcf-8adf)x^2+c(11be-7af))}{\sqrt{dx^2+c}}dx}{11b}+\frac{fx^7\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{11b}\right)$$

$\Big\downarrow$ 443

$$\frac{f\left(\frac{\frac{\int \frac{x^8\left(ac(26bde-9bcf-9adf)-\left(-c(13de-10cf)b^2-ad(13de+2cf)b+10a^2d^2f\right)x^2\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{11d}+\frac{x^9\sqrt{a+bx^2}\sqrt{c+dx^2}(-10adf+bcf+13bde)}{11d}}{13b}+\frac{fx^9\left(a+bx^2\right)^{3/2}\sqrt{c}}{13b}\right.}{e^2}$$
$$e\left(\frac{\frac{\int \frac{x^6\left(ac(22bde-7bcf-7adf)-\left(-c(11de-8cf)b^2-ad(11de+2cf)b+8a^2d^2f\right)x^2\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{9d}+\frac{x^7\sqrt{a+bx^2}\sqrt{c+dx^2}(-8adf+bcf+11bde)}{9d}}{11b}+\frac{fx^7\left(a+bx^2\right.}{1}\right.$$

$\Big\downarrow$ 444

$$\frac{f\left(\frac{\frac{\frac{1}{9}x^7\sqrt{a+bx^2}\sqrt{c+dx^2}\left(-\frac{10a^2df}{b}+2acf+13ade-\frac{10bc^2f}{d}+13bce\right)-\frac{\int -\frac{x^6\left(\left(-8c^2(13de-10cf)b^3+acd(26de-17cf)b^2-a^2d^2(104de+17cf)b+80a^3d^3f\right)x^2+7\right.}{\sqrt{bx^2+a}\sqrt{dx^2+c}}}{9bd}}{11d}}{13b}\right.}{e^2}$$
$$e\left(\frac{\frac{\frac{1}{7}x^5\sqrt{a+bx^2}\sqrt{c+dx^2}\left(-\frac{8a^2df}{b}+2acf+11ade-\frac{8bc^2f}{d}+11bce\right)-\frac{\int -\frac{x^4\left(\left(-6c^2(11de-8cf)b^3+acd(22de-13cf)b^2-a^2d^2(66de+13cf)b+48a^3d^3f\right)x^2+\right.}{\sqrt{bx^2+a}\sqrt{dx^2+c}}}{7bd}}{9d}}{11b}\right.$$

$\Big\downarrow$ 25

$$f\left(\frac{\frac{\frac{\int \frac{x^6\left(\left(-8c^2(13de-10cf)b^3+acd(26de-17cf)b^2-a^2d^2(104de+17cf)b+80a^3d^3f\right)x^2+7ac\left(-c(13de-10cf)b^2-ad(13de+2cf)b+10a^2d^2f\right)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{9bd}+\frac{1}{9}x^7\sqrt{a+bx}\cdots}{11d}}{13b}\cdots\right.$$

$$e^2$$

$$e\left(\frac{\frac{\frac{\int \frac{x^4\left(\left(-6c^2(11de-8cf)b^3+acd(22de-13cf)b^2-a^2d^2(66de+13cf)b+48a^3d^3f\right)x^2+5ac\left(-c(11de-8cf)b^2-ad(11de+2cf)b+8a^2d^2f\right)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{7bd}+\frac{1}{7}x^5\sqrt{a+bx^2}\sqrt{c}\cdots}{9d}}{11b}\cdots\right.$$

$$\Big\downarrow \; 444$$

$$e\left(\frac{f\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}x^7}{11b}+\frac{(11bde+bcf-8adf)\sqrt{bx^2+a}\sqrt{dx^2+c}x^7}{9d}+\frac{\frac{1}{7}\left(-\frac{8dfa^2}{b}+11dea+2cfa+11bce-\frac{8bc^2f}{d}\right)\sqrt{bx^2+a}\sqrt{dx^2+c}\cdots}{\cdots}\right.$$

$$f\left(\frac{f\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}x^9}{13b}+\frac{(13bde+bcf-10adf)\sqrt{bx^2+a}\sqrt{dx^2+c}x^9}{11d}+\frac{\frac{1}{9}\left(-\frac{10dfa^2}{b}+13dea+2cfa+13bce-\frac{10bc^2f}{d}\right)\sqrt{bx^2+a}\sqrt{dx^2+c}x^7+\frac{\left(-8c^2(13de-\cdots\right.}{\cdots}}{\cdots}\right.$$

$$\Big\downarrow \; 27$$

$$e\left(\frac{f\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}x^7}{11b}+\frac{(11bde+bcf-8adf)\sqrt{bx^2+a}\sqrt{dx^2+c}x^7}{9d}+\frac{\frac{1}{7}\left(-\frac{8dfa^2}{b}+11dea+2cfa+11bce-\frac{8bc^2f}{d}\right)\sqrt{bx^2+a}\sqrt{dx^2+c}\cdots}{\cdots}\right.$$

$$f\left(\frac{f\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}x^9}{13b}+\frac{(13bde+bcf-10adf)\sqrt{bx^2+a}\sqrt{dx^2+c}x^9}{11d}+\frac{\frac{1}{9}\left(-\frac{10dfa^2}{b}+13dea+2cfa+13bce-\frac{10bc^2f}{d}\right)\sqrt{bx^2+a}\sqrt{dx^2+c}x^7+\frac{\left(-8c^2(13de-\cdots\right.}{\cdots}}{\cdots}\right.$$

$$\Big\downarrow \; 444$$

$$e\left(\frac{f\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}\,x^7}{11b}+\frac{(11bde+bcf-8adf)\sqrt{bx^2+a}\sqrt{dx^2+c}\,x^7}{9d}+\frac{\frac{1}{7}\left(-\frac{8dfa^2}{b}+11dea+2cfa+11bce-\frac{8bc^2f}{d}\right)\sqrt{bx^2+a}\sqrt{dx^2+c\cdots}}{\cdots}\right.$$

$$\left.f\left(\frac{f\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}\,x^9}{13b}+\frac{(13bde+bcf-10adf)\sqrt{bx^2+a}\sqrt{dx^2+c}\,x^9}{11d}+\frac{\frac{1}{9}\left(-\frac{10dfa^2}{b}+13dea+2cfa+13bce-\frac{10bc^2f}{d}\right)\sqrt{bx^2+a}\sqrt{dx^2+c}\,x^7+\frac{\left(-8c^2(13de-\cdots\right.}{\cdots}}{\cdots}\right.\right.$$

$$\Big\downarrow \; 27$$

$$e\left(\frac{f\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}\,x^7}{11b}+\frac{(11bde+bcf-8adf)\sqrt{bx^2+a}\sqrt{dx^2+c}\,x^7}{9d}+\frac{\frac{1}{7}\left(-\frac{8dfa^2}{b}+11dea+2cfa+11bce-\frac{8bc^2f}{d}\right)\sqrt{bx^2+a}\sqrt{dx^2+\cdots}}{\cdots}\right.$$

$$\left.f\left(\frac{f\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}\,x^9}{13b}+\frac{(13bde+bcf-10adf)\sqrt{bx^2+a}\sqrt{dx^2+c}\,x^9}{11d}+\frac{\frac{1}{9}\left(-\frac{10dfa^2}{b}+13dea+2cfa+13bce-\frac{10bc^2f}{d}\right)\sqrt{bx^2+a}\sqrt{dx^2+c}\,x^7+\frac{\left(-8c^2(13de-\cdots\right.}{\cdots}}{\cdots}\right.\right.$$

$$\Big\downarrow \; 406$$

$$e\left(\frac{f\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}x^7}{11b}+\frac{(11bde+bcf-8adf)\sqrt{bx^2+a}\sqrt{dx^2+c}x^7}{9d}+\frac{\frac{1}{7}\left(-\frac{8dfa^2}{b}+11dea+2cfa+11bce-\frac{8bc^2f}{d}\right)\sqrt{bx^2+a}\sqrt{dx^2+c}\ldots}{\ldots}\right.$$

$$f\left(\frac{f\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}x^9}{13b}+\frac{(13bde+bcf-10adf)\sqrt{bx^2+a}\sqrt{dx^2+c}x^9}{11d}+\frac{\frac{1}{9}\left(-\frac{10dfa^2}{b}+13dea+2cfa+13bce-\frac{10bc^2f}{d}\right)\sqrt{bx^2+a}\sqrt{dx^2+c}x^7+\frac{\left(-8c^2(13de-\ldots\right.}{\ldots}}{\ldots}\right.$$

$\downarrow$ 320

$$e\left(\frac{f\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}x^7}{11b}+\frac{(11bde+bcf-8adf)\sqrt{bx^2+a}\sqrt{dx^2+c}x^7}{9d}+\frac{\frac{1}{7}\left(-\frac{8dfa^2}{b}+11dea+2cfa+11bce-\frac{8bc^2f}{d}\right)\sqrt{bx^2+a}\sqrt{dx^2+c}\ldots}{\ldots}\right.$$

$$f\left(\frac{f\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}x^9}{13b}+\frac{(13bde+bcf-10adf)\sqrt{bx^2+a}\sqrt{dx^2+c}x^9}{11d}+\frac{\frac{1}{9}\left(-\frac{10dfa^2}{b}+13dea+2cfa+13bce-\frac{10bc^2f}{d}\right)\sqrt{bx^2+a}\sqrt{dx^2+c}x^7+\frac{\left(-8c^2(13de-\ldots\right.}{\ldots}}{\ldots}\right.$$

$\downarrow$ 388

$$e\left(\frac{f\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}x^7}{11b}+\frac{(11bde+bcf-8adf)\sqrt{bx^2+a}\sqrt{dx^2+c}x^7}{9d}+\frac{\frac{1}{7}\left(-\frac{8dfa^2}{b}+11dea+2cfa+11bce-\frac{8bc^2f}{d}\right)\sqrt{bx^2+a}\sqrt{dx^2+c}\ldots}{\ldots}\right.$$

$$f\left(\frac{f\left(bx^2+a\right)^{3/2}\sqrt{dx^2+cx^9}}{13b}+\frac{(13bde+bcf-10adf)\sqrt{bx^2+a}\sqrt{dx^2+cx^9}}{11d}+\frac{\frac{1}{9}\left(-\frac{10dfa^2}{b}+13dea+2cfa+13bce-\frac{10bc^2f}{d}\right)\sqrt{bx^2+a}\sqrt{dx^2+cx^7}+\frac{\left(-8c^2(13de-\ldots\right.}{\ldots}}{\ldots}\right.$$

$\downarrow$ 313

$$e\left(\frac{f\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}\,x^7}{11b}+\frac{(11bde+bcf-8adf)\sqrt{bx^2+a}\sqrt{dx^2+c}\,x^7}{9d}+\frac{\frac{1}{7}\left(-\frac{8dfa^2}{b}+11dea+2cfa+11bce-\frac{8bc^2f}{d}\right)\sqrt{bx^2+a}\sqrt{dx^2+c}\cdots}{\cdots}\right.$$

$$f\left(\frac{f\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}\,x^9}{13b}+\frac{(13bde+bcf-10adf)\sqrt{bx^2+a}\sqrt{dx^2+c}\,x^9}{11d}+\frac{\frac{1}{9}\left(-\frac{10dfa^2}{b}+13dea+2cfa+13bce-\frac{10bc^2f}{d}\right)\sqrt{bx^2+a}\sqrt{dx^2+c}\,x^7+\frac{\left(-8c^2(13de-\cdots\right.}{\cdots}}{\cdots}\right.$$

↓ 444

$$e\left(\frac{f\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}\,x^7}{11b}+\frac{(11bde+bcf-8adf)\sqrt{bx^2+a}\sqrt{dx^2+c}\,x^7}{9d}+\frac{\frac{1}{7}\left(-\frac{8dfa^2}{b}+11dea+2cfa+11bce-\frac{8bc^2f}{d}\right)\sqrt{bx^2+a}\sqrt{dx^2+c}\right.$$

$$f\left(\frac{f\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}\,x^9}{13b}+\frac{(13bde+bcf-10adf)\sqrt{bx^2+a}\sqrt{dx^2+c}\,x^9}{11d}+\frac{\frac{1}{9}\left(-\frac{10dfa^2}{b}+13dea+2cfa+13bce-\frac{10bc^2f}{d}\right)\sqrt{bx^2+a}\sqrt{dx^2+c}\,x^7+\frac{\left(-8c^2(13de-\right.}{}}{}\right.$$

↓ 406

$$e\left(\frac{f\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}x^7}{11b}+\frac{(11bde+bcf-8adf)\sqrt{bx^2+a}\sqrt{dx^2+c}x^7}{9d}+\frac{\frac{1}{7}\left(-\frac{8dfa^2}{b}+11dea+2cfa+11bce-\frac{8bc^2f}{d}\right)\sqrt{bx^2+a}\sqrt{dx^2+c}\right.$$

$$f\left(\frac{f\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}x^9}{13b}+\frac{(13bde+bcf-10adf)\sqrt{bx^2+a}\sqrt{dx^2+c}x^9}{11d}+\frac{\frac{1}{9}\left(-\frac{10dfa^2}{b}+13dea+2cfa+13bce-\frac{10bc^2f}{d}\right)\sqrt{bx^2+a}\sqrt{dx^2+c}x^7+\frac{\left(-8c^2(13de-\right.}{}}{}\right.$$

$\downarrow$ 320

$$e\left(\frac{f\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}\,x^7}{11b}+\frac{(11bde+bcf-8adf)\sqrt{bx^2+a}\sqrt{dx^2+c}\,x^7}{9d}+\frac{\frac{1}{7}\left(-\frac{8dfa^2}{b}+11dea+2cfa+11bce-\frac{8bc^2f}{d}\right)\sqrt{bx^2+a}\sqrt{dx^2+c}\right.$$

$$f\left(\frac{f\left(bx^2+a\right)^{3/2}\sqrt{dx^2+cx^9}}{13b}+\frac{(13bde+bcf-10adf)\sqrt{bx^2+a}\sqrt{dx^2+cx^9}}{11d}+\frac{\frac{1}{9}\left(-\frac{10dfa^2}{b}+13dea+2cfa+13bce-\frac{10bc^2f}{d}\right)\sqrt{bx^2+a}\sqrt{dx^2+cx^7}+\frac{\left(-8c^2(13de-\right.}{}}{}\right.$$

↓ 388

$$e\left(\frac{f\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}x^7}{11b}+\frac{(11bde+bcf-8adf)\sqrt{bx^2+a}\sqrt{dx^2+c}x^7}{9d}+\frac{\frac{1}{7}\left(-\frac{8dfa^2}{b}+11dea+2cfa+11bce-\frac{8bc^2f}{d}\right)\sqrt{bx^2+a}\sqrt{dx^2+c}\right.$$

$$f\left(\frac{f\left(bx^2+a\right)^{3/2}\sqrt{dx^2+cx^9}}{13b}+\frac{(13bde+bcf-10adf)\sqrt{bx^2+a}\sqrt{dx^2+cx^9}}{11d}+\frac{\frac{1}{9}\left(-\frac{10dfa^2}{b}+13dea+2cfa+13bce-\frac{10bc^2f}{d}\right)\sqrt{bx^2+a}\sqrt{dx^2+cx^7}+\frac{\left(-8c^2(13de-\right.}{}}{}\right.$$

↓ 313

$$e\left(\frac{f\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}x^7}{11b}+\frac{(11bde+bcf-8adf)\sqrt{bx^2+a}\sqrt{dx^2+c}x^7}{9d}+\frac{\frac{1}{7}\left(-\frac{8dfa^2}{b}+11dea+2cfa+11bce-\frac{8bc^2f}{d}\right)\sqrt{bx^2+a}\sqrt{dx^2+c}\right.$$

$$f\left(\frac{f(bx^2+a)^{3/2}\sqrt{dx^2+c}x^9}{13b}+\frac{(13bde+bcf-10adf)\sqrt{bx^2+a}\sqrt{dx^2+c}x^9}{11d}+\frac{\frac{1}{9}\left(-\frac{10dfa^2}{b}+13dea+2cfa+13bce-\frac{10bc^2f}{d}\right)\sqrt{bx^2+a}\sqrt{dx^2+c}x^7+\frac{\left(-8c^2(13de-\right.}{}}{}\right.$$

input
```
Int[x^6*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^2,x]
```

output
```
e*((f*x^7*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(11*b) + (((11*b*d*e + b*c*f
- 8*a*d*f)*x^7*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(9*d) + (((11*b*c*e + 11*a
*d*e + 2*a*c*f - (8*b*c^2*f)/d - (8*a^2*d*f)/b)*x^5*Sqrt[a + b*x^2]*Sqrt[c
 + d*x^2])/7 + (((48*a^3*d^3*f + a*b^2*c*d*(22*d*e - 13*c*f) - 6*b^3*c^2*(
11*d*e - 8*c*f) - a^2*b*d^2*(66*d*e + 13*c*f))*x^3*Sqrt[a + b*x^2]*Sqrt[c
+ d*x^2])/(5*b*d) - (3*(((64*a^4*d^4*f + a*b^3*c^2*d*(33*d*e - 20*c*f) - 8
*b^4*c^3*(11*d*e - 8*c*f) + 3*a^2*b^2*c*d^2*(11*d*e - 6*c*f) - 4*a^3*b*d^3
*(22*d*e + 5*c*f))*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(3*b*d) - ((128*a^5*
d^5*f + a^2*b^3*c^2*d^2*(66*d*e - 37*c*f) + a^3*b^2*c*d^3*(88*d*e - 37*c*f
) - 16*b^5*c^4*(11*d*e - 8*c*f) + 8*a*b^4*c^3*d*(11*d*e - 7*c*f) - 8*a^4*b
*d^4*(22*d*e + 7*c*f))*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]
*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/
(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (c^(3
/2)*(64*a^4*d^4*f + a*b^3*c^2*d*(33*d*e - 20*c*f) - 8*b^4*c^3*(11*d*e - 8*
c*f) + 3*a^2*b^2*c*d^2*(11*d*e - 6*c*f) - 4*a^3*b*d^3*(22*d*e + 5*c*f))*Sq
rt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sq
rt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))/(3*b*d)))/(5
*b*d))/(7*b*d))/(9*d))/(11*b)) + (f*((f*x^9*(a + b*x^2)^(3/2)*Sqrt[c + d*x
^2])/(13*b) + (((13*b*d*e + b*c*f - 10*a*d*f)*x^9*Sqrt[a + b*x^2]*Sqrt[c +
 d*x^2])/(11*d) + (((13*b*c*e + 13*a*d*e + 2*a*c*f - (10*b*c^2*f)/d - (...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 443
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*(g*x)^(m + 1)*(a + b*x^2)^(p
 + 1)*((c + d*x^2)^q/(b*g*(m + 2*(p + q + 1) + 1))), x] + Simp[1/(b*(m + 2*
(p + q + 1) + 1))   Int[(g*x)^m*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[c*((
b*e - a*f)*(m + 1) + b*e*2*(p + q + 1)) + (d*(b*e - a*f)*(m + 1) + f*2*q*(b
*c - a*d) + b*e*d*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f
, g, m, p}, x] && GtQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^2, c + d*x^
2])
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

rule 448
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[e   Int[(g*x)^m*(a + b*x
^2)^p*(c + d*x^2)^q*(e + f*x^2)^(r - 1), x], x] + Simp[f/e^2   Int[(g*x)^(m
 + 2)*(a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^(r - 1), x], x] /; FreeQ[{a,
b, c, d, e, f, g, m, p, q}, x] && IGtQ[r, 0]
```

Maple [A] (verified)

Time = 19.83 (sec) , antiderivative size = 2627, normalized size of antiderivative = 1.73

method	result	size
elliptic	Expression too large to display	2627
risch	Expression too large to display	2657
default	Expression too large to display	4274

input
```
int(x^6*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^2,x,method=_RETURNVERBOS
E)
```

output

```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/13*f^2*x^11
*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+1/11*(a*d*f^2+b*c*f^2+2*d*b*e*f-1/13*
f^2*(12*a*d+12*b*c))/b/d*x^9*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+1/9*(2/13
*a*c*f^2+2*a*d*e*f+2*b*c*e*f+b*d*e^2-1/11*(a*d*f^2+b*c*f^2+2*d*b*e*f-1/13*
f^2*(12*a*d+12*b*c))/b/d*(10*a*d+10*b*c))/b/d*x^7*(b*d*x^4+a*d*x^2+b*c*x^2
+a*c)^(1/2)+1/7*(2*a*c*e*f+a*d*e^2+b*c*e^2-9/11*(a*d*f^2+b*c*f^2+2*d*b*e*f
-1/13*f^2*(12*a*d+12*b*c))/b/d*a*c-1/9*(2/13*a*c*f^2+2*a*d*e*f+2*b*c*e*f+b
*d*e^2-1/11*(a*d*f^2+b*c*f^2+2*d*b*e*f-1/13*f^2*(12*a*d+12*b*c))/b/d*(10*a
*d+10*b*c))/b/d*(8*a*d+8*b*c))/b/d*x^5*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
+1/5*(a*c*e^2-7/9*(2/13*a*c*f^2+2*a*d*e*f+2*b*c*e*f+b*d*e^2-1/11*(a*d*f^2+
b*c*f^2+2*d*b*e*f-1/13*f^2*(12*a*d+12*b*c))/b/d*(10*a*d+10*b*c))/b/d*a*c-1
/7*(2*a*c*e*f+a*d*e^2+b*c*e^2-9/11*(a*d*f^2+b*c*f^2+2*d*b*e*f-1/13*f^2*(12
*a*d+12*b*c))/b/d*a*c-1/9*(2/13*a*c*f^2+2*a*d*e*f+2*b*c*e*f+b*d*e^2-1/11*(
a*d*f^2+b*c*f^2+2*d*b*e*f-1/13*f^2*(12*a*d+12*b*c))/b/d*(10*a*d+10*b*c))/b
/d*(8*a*d+8*b*c))/b/d*(6*a*d+6*b*c))/b/d*x^3*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)
^(1/2)+1/3*(-5/7*(2*a*c*e*f+a*d*e^2+b*c*e^2-9/11*(a*d*f^2+b*c*f^2+2*d*b*e*
f-1/13*f^2*(12*a*d+12*b*c))/b/d*a*c-1/9*(2/13*a*c*f^2+2*a*d*e*f+2*b*c*e*f+
b*d*e^2-1/11*(a*d*f^2+b*c*f^2+2*d*b*e*f-1/13*f^2*(12*a*d+12*b*c))/b/d*(10*
a*d+10*b*c))/b/d*(8*a*d+8*b*c))/b/d*a*c-1/5*(a*c*e^2-7/9*(2/13*a*c*f^2+2*a
*d*e*f+2*b*c*e*f+b*d*e^2-1/11*(a*d*f^2+b*c*f^2+2*d*b*e*f-1/13*f^2*(12*a...
```

Fricas [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 1561, normalized size of antiderivative = 1.03

$$\int x^6\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^2\,dx = \text{Too large to display}$$

input

```
integrate(x^6*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^2,x, algorithm="fr
icas")
```

output
```
1/45045*(2*(143*(8*b^6*c^5*d^2 - 4*a*b^5*c^4*d^3 - 3*a^2*b^4*c^3*d^4 - 4*a
^3*b^3*c^2*d^5 + 8*a^4*b^2*c*d^6)*e^2 - 13*(128*b^6*c^6*d - 56*a*b^5*c^5*d
^2 - 37*a^2*b^4*c^4*d^3 - 37*a^3*b^3*c^3*d^4 - 56*a^4*b^2*c^2*d^5 + 128*a^
5*b*c*d^6)*e*f + (640*b^6*c^7 - 256*a*b^5*c^6*d - 157*a^2*b^4*c^5*d^2 - 13
9*a^3*b^3*c^4*d^3 - 157*a^4*b^2*c^3*d^4 - 256*a^5*b*c^2*d^5 + 640*a^6*c*d^
6)*f^2)*sqrt(b*d)*x*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c))
 - (143*(16*b^6*c^5*d^2 - 8*a*b^5*c^4*d^3 + 8*a^4*b^2*d^7 - 2*(3*a^2*b^4 -
 4*a*b^5)*c^3*d^4 - (8*a^3*b^3 + 3*a^2*b^4)*c^2*d^5 + (16*a^4*b^2 - 3*a^3*
b^3)*c*d^6)*e^2 - 26*(128*b^6*c^6*d - 56*a*b^5*c^5*d^2 + 64*a^5*b*d^7 - (3
7*a^2*b^4 - 64*a*b^5)*c^4*d^3 - (37*a^3*b^3 + 20*a^2*b^4)*c^3*d^4 - 2*(28*
a^4*b^2 + 9*a^3*b^3)*c^2*d^5 + 4*(32*a^5*b - 5*a^4*b^2)*c*d^6)*e*f + (1280
*b^6*c^7 - 512*a*b^5*c^6*d + 640*a^6*d^7 - 2*(157*a^2*b^4 - 320*a*b^5)*c^5
*d^2 - 2*(139*a^3*b^3 + 88*a^2*b^4)*c^4*d^3 - (314*a^4*b^2 + 149*a^3*b^3)*
c^3*d^4 - (512*a^5*b + 149*a^4*b^2)*c^2*d^5 + 16*(80*a^6 - 11*a^5*b)*c*d^6
)*f^2)*sqrt(b*d)*x*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)/x), a*d/(b*c))
+ (3465*b^6*d^7*f^2*x^12 + 315*(26*b^6*d^7*e*f + (b^6*c*d^6 + a*b^5*d^7)*f
^2)*x^10 + 35*(143*b^6*d^7*e^2 + 26*(b^6*c*d^6 + a*b^5*d^7)*e*f - 2*(5*b^6
*c^2*d^5 - a*b^5*c*d^6 + 5*a^2*b^4*d^7)*f^2)*x^8 + 5*(143*(b^6*c*d^6 + a*b
^5*d^7)*e^2 - 52*(4*b^6*c^2*d^5 - a*b^5*c*d^6 + 4*a^2*b^4*d^7)*e*f + (80*b
^6*c^3*d^4 - 17*a*b^5*c^2*d^5 - 17*a^2*b^4*c*d^6 + 80*a^3*b^3*d^7)*f^2)...
```

Sympy [F]

$$\int x^6\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^2\,dx=\int x^6\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^2\,dx$$

input
```
integrate(x**6*(b*x**2+a)**(1/2)*(d*x**2+c)**(1/2)*(f*x**2+e)**2,x)
```

output
```
Integral(x**6*sqrt(a + b*x**2)*sqrt(c + d*x**2)*(e + f*x**2)**2, x)
```

Maxima [F]

$$\int x^6\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^2\,dx=\int\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^2x^6\,dx$$

input
```
integrate(x^6*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^2,x, algorithm="ma
xima")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^2*x^6, x)
```

Giac [F]

$$\int x^6\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^2\,dx=\int\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^2x^6\,dx$$

input
```
integrate(x^6*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^2,x, algorithm="gi
ac")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^2*x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^6\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^2\,dx=\int x^6\,\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,\left(f\,x^2+e\right)^2\,dx$$

input
```
int(x^6*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^2,x)
```

output
```
int(x^6*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^2, x)
```

Reduce [F]

$$\int x^6\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^2\,dx=\text{too large to display}$$

input
```
int(x^6*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^2,x)
```

output
```
(640*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**5*d**5*f**2*x - 176*sqrt(c + d*x
**2)*sqrt(a + b*x**2)*a**4*b*c*d**4*f**2*x - 1664*sqrt(c + d*x**2)*sqrt(a
+ b*x**2)*a**4*b*d**5*e*f*x - 480*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**4*b
*d**5*f**2*x**3 - 149*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**3*b**2*c**2*d**
3*f**2*x + 520*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**3*b**2*c*d**4*e*f*x +
112*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**3*b**2*c*d**4*f**2*x**3 + 1144*sq
rt(c + d*x**2)*sqrt(a + b*x**2)*a**3*b**2*d**5*e**2*x + 1248*sqrt(c + d*x*
*2)*sqrt(a + b*x**2)*a**3*b**2*d**5*e*f*x**3 + 400*sqrt(c + d*x**2)*sqrt(a
 + b*x**2)*a**3*b**2*d**5*f**2*x**5 - 149*sqrt(c + d*x**2)*sqrt(a + b*x**2
)*a**2*b**3*c**3*d**2*f**2*x + 468*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*
b**3*c**2*d**3*e*f*x + 106*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*b**3*c**
2*d**3*f**2*x**3 - 429*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*b**3*c*d**4*
e**2*x - 338*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*b**3*c*d**4*e*f*x**3 -
 85*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*b**3*c*d**4*f**2*x**5 - 858*sqr
t(c + d*x**2)*sqrt(a + b*x**2)*a**2*b**3*d**5*e**2*x**3 - 1040*sqrt(c + d*
x**2)*sqrt(a + b*x**2)*a**2*b**3*d**5*e*f*x**5 - 350*sqrt(c + d*x**2)*sqrt
(a + b*x**2)*a**2*b**3*d**5*f**2*x**7 - 176*sqrt(c + d*x**2)*sqrt(a + b*x*
*2)*a*b**4*c**4*d*f**2*x + 520*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**4*c*
*3*d**2*e*f*x + 112*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**4*c**3*d**2*f**
2*x**3 - 429*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**4*c**2*d**3*e**2*x ...
```

3.53 $\int x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^2\,dx$

Optimal result

Integrand size = 35, antiderivative size = 1130

$$\int x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^2\,dx = \text{Too large to display}$$

output
```
1/3465*(128*a^5*d^5*f^2-8*a^4*b*d^4*f*(7*c*f+44*d*e)+a^3*b^2*d^3*(-37*c^2*
f^2+176*c*d*e*f+264*d^2*e^2)+8*b^5*c^3*(16*c^2*f^2-44*c*d*e*f+33*d^2*e^2)-
a^2*b^3*c*d^2*(37*c^2*f^2-132*c*d*e*f+165*d^2*e^2)-a*b^4*c^2*d*(56*c^2*f^2
-176*c*d*e*f+165*d^2*e^2))*x*(d*x^2+c)^(1/2)/b^4/d^5/(b*x^2+a)^(1/2)-1/346
5*(64*a^4*d^4*f^2-4*a^3*b*d^3*f*(5*c*f+44*d*e)+a*b^3*c*d*(-20*c^2*f^2+66*c
*d*e*f+249*d^2*e^2)+6*a^2*b^2*d^2*(-3*c^2*f^2+11*c*d*e*f+22*d^2*e^2)+4*b^4
*c^2*(16*c^2*f^2-44*c*d*e*f+33*d^2*e^2))*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)
/b^4/d^4+1/3465*(48*a^3*d^3*f^2-a^2*b*d^2*f*(13*c*f+132*d*e)-12*b^3*c*(-4*
c^2*f^2+11*c*d*e*f+18*d^2*e^2)-a*b^2*d*(13*c^2*f^2+586*c*d*e*f+216*d^2*e^2
))*x^3*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b^3/d^3-1/693*(8*a^2*d*f^2/b+a*f*(6
1*c*f+104*d*e)-b*(36*d*e^2-104*c*e*f-8*c^2*f^2/d))*x^5*(b*x^2+a)^(1/2)*(d*
x^2+c)^(1/2)/b/d+4/99*f*(-2*a*d*f-2*b*c*f+b*d*e)*x^7*(b*x^2+a)^(1/2)*(d*x^
2+c)^(1/2)/b/d+1/11*x*(b*x^2+a)^(3/2)*(d*x^2+c)^(3/2)*(f*x^2+e)^2/b/d-1/34
65*a^(1/2)*(128*a^5*d^5*f^2-8*a^4*b*d^4*f*(7*c*f+44*d*e)+a^3*b^2*d^3*(-37*
c^2*f^2+176*c*d*e*f+264*d^2*e^2)+8*b^5*c^3*(16*c^2*f^2-44*c*d*e*f+33*d^2*e
^2)-a^2*b^3*c*d^2*(37*c^2*f^2-132*c*d*e*f+165*d^2*e^2)-a*b^4*c^2*d*(56*c^2
*f^2-176*c*d*e*f+165*d^2*e^2))*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)
/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/b^(9/2)/d^5/(b*x^2+a)^(1/2)/(a*(d*x^
2+c)/c/(b*x^2+a))^(1/2)+2/3465*a^(3/2)*(32*a^4*d^4*f^2-2*a^3*b*d^3*f*(5*c*
f+44*d*e)+3*a^2*b^2*d^2*(-3*c^2*f^2+11*c*d*e*f+22*d^2*e^2)-a*b^3*c*d*(1...
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.61 (sec) , antiderivative size = 777, normalized size of antiderivative = 0.69

$$\int x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^2\,dx$$

$$=\frac{\sqrt{\frac{b}{a}}dx(a+bx^2)\left(c+dx^2\right)\left(-64a^4d^4f^2+4a^3bd^3f(44de+5cf+12dfx^2)-a^2b^2d^2(-18c^2f^2+cdf(66e+\right.}{}$$

input
```
Integrate[x^4*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^2,x]
```

output
```
(Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2)*(-64*a^4*d^4*f^2 + 4*a^3*b*d^3*f*(4
4*d*e + 5*c*f + 12*d*f*x^2) - a^2*b^2*d^2*(-18*c^2*f^2 + c*d*f*(66*e + 13*
f*x^2) + 4*d^2*(33*e^2 + 33*e*f*x^2 + 10*f^2*x^4)) + a*b^3*d*(20*c^3*f^2 -
 c^2*d*f*(66*e + 13*f*x^2) + 2*c*d^2*(33*e^2 + 22*e*f*x^2 + 5*f^2*x^4) + d
^3*x^2*(99*e^2 + 110*e*f*x^2 + 35*f^2*x^4)) + b^4*(-64*c^4*f^2 + 16*c^3*d*
f*(11*e + 3*f*x^2) - 4*c^2*d^2*(33*e^2 + 33*e*f*x^2 + 10*f^2*x^4) + c*d^3*
x^2*(99*e^2 + 110*e*f*x^2 + 35*f^2*x^4) + 5*d^4*x^4*(99*e^2 + 154*e*f*x^2
+ 63*f^2*x^4))) - I*c*(128*a^5*d^5*f^2 - 8*a^4*b*d^4*f*(44*d*e + 7*c*f) +
a*b^4*c^2*d*(-165*d^2*e^2 + 176*c*d*e*f - 56*c^2*f^2) + a^2*b^3*c*d^2*(-16
5*d^2*e^2 + 132*c*d*e*f - 37*c^2*f^2) + a^3*b^2*d^3*(264*d^2*e^2 + 176*c*d
*e*f - 37*c^2*f^2) + 8*b^5*c^3*(33*d^2*e^2 - 44*c*d*e*f + 16*c^2*f^2))*Sqr
t[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*
d)/(b*c)] + I*c*(-(b*c) + a*d)*(64*a^4*d^4*f^2 - 4*a^3*b*d^3*f*(44*d*e - 7
*c*f) + 3*a^2*b^2*d^2*(44*d^2*e^2 - 22*c*d*e*f + 3*c^2*f^2) - 8*b^4*c^2*(3
3*d^2*e^2 - 44*c*d*e*f + 16*c^2*f^2) + a*b^3*(33*c*d^3*e^2 - 8*c^3*d*f^2))
*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x],
 (a*d)/(b*c)])/(3465*b^4*Sqrt[b/a]*d^5*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (warning: unable to verify)

Time = 2.67 (sec) , antiderivative size = 1430, normalized size of antiderivative = 1.27, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.486$, Rules used = {448, 443, 443, 444, 25, 27, 444, 27, 406, 320, 388, 313, 444, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^2\,dx$$

$$\Big\downarrow \text{448}$$

$$\frac{f\int x^6\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)dx}{e^2}+e\int x^4\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)dx$$

$$\Big\downarrow \text{443}$$

$$\frac{f\left(\frac{\int \frac{x^6\sqrt{bx^2+a}\left((11bde+bcf-8adf)x^2+c(11be-7af)\right)}{\sqrt{dx^2+c}}dx}{11b}+\frac{fx^7\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{11b}\right)}{e^2}+e\left(\frac{\int \frac{x^4\sqrt{bx^2+a}((9bde+bcf-6adf)x^2+c(9be-5af))}{\sqrt{dx^2+c}}dx}{9b}+\frac{fx^5\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{9b}\right)$$

$\downarrow$ 443

$$\frac{f\left(\frac{\frac{\int \frac{x^6\left(ac(22bde-7bcf-7adf)-\left(-c(11de-8cf)b^2-ad(11de+2cf)b+8a^2d^2f\right)x^2\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{9d}+\frac{x^7\sqrt{a+bx^2}\sqrt{c+dx^2}(-8adf+bcf+11bde)}{9d}}{11b}+\frac{fx^7\left(a+bx^2\right)^{3/2}\sqrt{c+d\ldots}}{11b}\right.}{e^2}$$

$$e\left(\frac{\frac{\int \frac{x^4\left(ac(18bde-5bcf-5adf)-\left(-3c(3de-2cf)b^2-ad(9de+2cf)b+6a^2d^2f\right)x^2\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{7d}+\frac{x^5\sqrt{a+bx^2}\sqrt{c+dx^2}(-6adf+bcf+9bde)}{7d}}{9b}+\frac{fx^5\left(a+bx^2\right)^{3}\ldots}{9b\ldots}\right.$$

$\downarrow$ 444

$$\frac{f\left(\frac{\frac{\frac{1}{7}x^5\sqrt{a+bx^2}\sqrt{c+dx^2}\left(-\frac{8a^2df}{b}+2acf+11ade-\frac{8bc^2f}{d}+11bce\right)-\frac{\int -\frac{x^4\left(\left(-6c^2(11de-8cf)b^3+acd(22de-13cf)b^2-a^2d^2(66de+13cf)b+48a^3d^3f\right)x^2+5ac\left(\ldots\right.\right.}{\sqrt{bx^2+a}\sqrt{dx^2+c}}\ldots}{7bd}}{9d}}{11b}\right.}{e^2}$$

$$e\left(\frac{\frac{\frac{1}{5}x^3\sqrt{a+bx^2}\sqrt{c+dx^2}\left(-\frac{6a^2df}{b}+2acf+9ade-\frac{6bc^2f}{d}+9bce\right)-\frac{\int -\frac{3x^2\left(\left(-4c^2(3de-2cf)b^3+3acd(2de-cf)b^2-3a^2d^2(4de+cf)b+8a^3d^3f\right)x^2+ac\left(-3c\ldots\right.\right.}{\sqrt{bx^2+a}\sqrt{dx^2+c}}\ldots}{5bd}}{7d}}{9b}\right.$$

$\downarrow$ 25

$$f\left(\frac{\frac{\frac{\int \frac{x^4\left(\left(-6c^2(11de-8cf)b^3+acd(22de-13cf)b^2-a^2d^2(66de+13cf)b+48a^3d^3f\right)x^2+5ac\left(-c(11de-8cf)b^2-ad(11de+2cf)b+8a^2d^2f\right)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{7bd}+\frac{1}{7}x^5\sqrt{a+bx^2}\sqrt{c\cdots}}{9d}\cdots}{11b}\cdots\right.$$

$$e^2$$

$$e\left(\frac{\frac{\frac{1}{5}x^3\sqrt{a+bx^2}\sqrt{c+dx^2}\left(-\frac{6a^2df}{b}+2acf+9ade-\frac{6bc^2f}{d}+9bce\right)-\frac{\int -\frac{3x^2\left(\left(-4c^2(3de-2cf)b^3+3acd(2de-cf)b^2-3a^2d^2(4de+cf)b+8a^3d^3f\right)x^2+ac\left(-3c\cdots\right.\right.}{\sqrt{bx^2+a}\sqrt{dx^2+c}}\cdots}{5bd}\cdots}{7d}\cdots}{9b}\cdots\right.$$

$$\downarrow 27$$

$$f\left(\frac{\frac{\frac{\int \frac{x^4\left(\left(-6c^2(11de-8cf)b^3+acd(22de-13cf)b^2-a^2d^2(66de+13cf)b+48a^3d^3f\right)x^2+5ac\left(-c(11de-8cf)b^2-ad(11de+2cf)b+8a^2d^2f\right)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{7bd}+\frac{1}{7}x^5\sqrt{a+bx^2}\sqrt{c\cdots}}{9d}\cdots}{11b}\cdots\right.$$

$$e^2$$

$$e\left(\frac{\frac{\frac{3\int \frac{x^2\left(\left(-4c^2(3de-2cf)b^3+3acd(2de-cf)b^2-3a^2d^2(4de+cf)b+8a^3d^3f\right)x^2+ac\left(-3c(3de-2cf)b^2-ad(9de+2cf)b+6a^2d^2f\right)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{5bd}+\frac{1}{5}x^3\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\cdots\right.}{7d}\cdots}{9b}\cdots\right.$$

$$\downarrow 444$$

$$e\left(\frac{f\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}x^5}{9b}+\frac{(9bde+bcf-6adf)\sqrt{bx^2+a}\sqrt{dx^2+c}x^5}{7d}+\frac{\frac{1}{5}\left(-\frac{6dfa^2}{b}+9dea+2cfa+9bce-\frac{6bc^2f}{d}\right)\sqrt{bx^2+a}\sqrt{dx^2+c}x^3\cdots}{\cdots}\cdots\right.$$

$$f\left(\frac{f\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}x^7}{11b}+\frac{(11bde+bcf-8adf)\sqrt{bx^2+a}\sqrt{dx^2+c}x^7}{9d}+\frac{\frac{1}{7}\left(-\frac{8dfa^2}{b}+11dea+2cfa+11bce-\frac{8bc^2f}{d}\right)\sqrt{bx^2+a}\sqrt{dx^2+c}x^5+\frac{\left(-6c^2(11de-8c\cdots\right.}{\cdots}}{\cdots}\cdots\right.$$

$$\Big\downarrow \; 27$$

$$e\left(\frac{f\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}x^5}{9b}+\frac{(9bde+bcf-6adf)\sqrt{bx^2+a}\sqrt{dx^2+c}x^5}{7d}+\frac{\frac{1}{5}\left(-\frac{6dfa^2}{b}+9dea+2cfa+9bce-\frac{6bc^2f}{d}\right)\sqrt{bx^2+a}\sqrt{dx^2+c}x^3\cdots}{\cdots}\right.$$

$$f\left(\frac{f\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}x^7}{11b}+\frac{(11bde+bcf-8adf)\sqrt{bx^2+a}\sqrt{dx^2+c}x^7}{9d}+\frac{\frac{1}{7}\left(-\frac{8dfa^2}{b}+11dea+2cfa+11bce-\frac{8bc^2f}{d}\right)\sqrt{bx^2+a}\sqrt{dx^2+c}x^5+\frac{\left(-6c^2(11de-8c\cdots\right.}{\cdots}}{\cdots}\right.$$

$$\Big\downarrow \; 406$$

$$e\left(\frac{f\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}x^5}{9b}+\frac{(9bde+bcf-6adf)\sqrt{bx^2+a}\sqrt{dx^2+c}x^5}{7d}+\frac{\frac{1}{5}\left(-\frac{6dfa^2}{b}+9dea+2cfa+9bce-\frac{6bc^2f}{d}\right)\sqrt{bx^2+a}\sqrt{dx^2+c}x^3\cdots}{\cdots}\right.$$

$$f\left(\frac{f\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}x^7}{11b}+\frac{(11bde+bcf-8adf)\sqrt{bx^2+a}\sqrt{dx^2+c}x^7}{9d}+\frac{\frac{1}{7}\left(-\frac{8dfa^2}{b}+11dea+2cfa+11bce-\frac{8bc^2f}{d}\right)\sqrt{bx^2+a}\sqrt{dx^2+c}x^5+\frac{\left(-6c^2(11de-8c\cdots\right.}{\cdots}}{\cdots}\right.$$

$$\Big\downarrow \; 320$$

$$e\left(\frac{f\left(bx^2+a\right)^{3/2}\sqrt{dx^2+cx}x^5}{9b}+\frac{(9bde+bcf-6adf)\sqrt{bx^2+a}\sqrt{dx^2+cx}x^5}{7d}+\frac{\frac{1}{5}\left(-\frac{6dfa^2}{b}+9dea+2cfa+9bce-\frac{6bc^2f}{d}\right)\sqrt{bx^2+a}\sqrt{dx^2+cx}x^3\cdots}{\cdots}\right.$$

$$f\left(\frac{f(bx^2+a)^{3/2}\sqrt{dx^2+cx}x^7}{11b}+\frac{(11bde+bcf-8adf)\sqrt{bx^2+a}\sqrt{dx^2+cx}x^7}{9d}+\frac{\frac{1}{7}\left(-\frac{8dfa^2}{b}+11dea+2cfa+11bce-\frac{8bc^2f}{d}\right)\sqrt{bx^2+a}\sqrt{dx^2+cx}x^5+\frac{\left(-6c^2(11de-8c\cdots\right.}{\cdots}}{\cdots}\right.$$

$\downarrow$ 388

$$e\left(\frac{f\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}\,x^5}{9b}+\frac{(9bde+bcf-6adf)\sqrt{bx^2+a}\sqrt{dx^2+c}\,x^5}{7d}+\frac{\frac{1}{5}\left(-\frac{6dfa^2}{b}+9dea+2cfa+9bce-\frac{6bc^2f}{d}\right)\sqrt{bx^2+a}\sqrt{dx^2+c}\,x^3\cdots}{\cdots}\right.$$

$$f\left(\frac{f\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}\,x^7}{11b}+\frac{(11bde+bcf-8adf)\sqrt{bx^2+a}\sqrt{dx^2+c}\,x^7}{9d}+\frac{\frac{1}{7}\left(-\frac{8dfa^2}{b}+11dea+2cfa+11bce-\frac{8bc^2f}{d}\right)\sqrt{bx^2+a}\sqrt{dx^2+c}\,x^5+\frac{\left(-6c^2(11de-8c\cdots\right.}{\cdots}}{\cdots}\right.$$

$\downarrow$ 313

$$e\left(\frac{f\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}\,x^5}{9b}+\frac{(9bde+bcf-6adf)\sqrt{bx^2+a}\sqrt{dx^2+c}\,x^5}{7d}+\frac{\frac{1}{5}\left(-\frac{6dfa^2}{b}+9dea+2cfa+9bce-\frac{6bc^2f}{d}\right)\sqrt{bx^2+a}\sqrt{dx^2+c}\,x^3 \cdots}{\cdots}\right.$$

$$f\left(\frac{f\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}\,x^7}{11b}+\frac{(11bde+bcf-8adf)\sqrt{bx^2+a}\sqrt{dx^2+c}\,x^7}{9d}+\frac{\frac{1}{7}\left(-\frac{8dfa^2}{b}+11dea+2cfa+11bce-\frac{8bc^2f}{d}\right)\sqrt{bx^2+a}\sqrt{dx^2+c}\,x^5+\frac{\left(-6c^2(11de-8c \cdots\right.}{\cdots}}{\cdots}\right.$$

$$\Big\downarrow \text{444}$$

$$e\left(\frac{f\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}x^5}{9b}+\frac{(9bde+bcf-6adf)\sqrt{bx^2+a}\sqrt{dx^2+c}x^5}{7d}+\frac{\frac{1}{5}\left(-\frac{6dfa^2}{b}+9dea+2cfa+9bce-\frac{6bc^2f}{d}\right)\sqrt{bx^2+a}\sqrt{dx^2+c}x^3}{}\right.$$

$$f\left(\frac{f\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}x^7}{11b}+\frac{(11bde+bcf-8adf)\sqrt{bx^2+a}\sqrt{dx^2+c}x^7}{9d}+\frac{\frac{1}{7}\left(-\frac{8dfa^2}{b}+11dea+2cfa+11bce-\frac{8bc^2f}{d}\right)\sqrt{bx^2+a}\sqrt{dx^2+c}x^5+\frac{\left(-6c^2(11de-8c\right.}{}}{}\right.$$

$\downarrow$ 406

$$e\left(\frac{f\left(bx^2+a\right)^{3/2}\sqrt{dx^2+cx}x^5}{9b}+\frac{(9bde+bcf-6adf)\sqrt{bx^2+a}\sqrt{dx^2+cx}x^5}{7d}+\frac{\frac{1}{5}\left(-\frac{6dfa^2}{b}+9dea+2cfa+9bce-\frac{6bc^2f}{d}\right)\sqrt{bx^2+a}\sqrt{dx^2+cx}x^3}{}\right.$$

$$f\left(\frac{f(bx^2+a)^{3/2}\sqrt{dx^2+cx}x^7}{11b}+\frac{(11bde+bcf-8adf)\sqrt{bx^2+a}\sqrt{dx^2+cx}x^7}{9d}+\frac{\frac{1}{7}\left(-\frac{8dfa^2}{b}+11dea+2cfa+11bce-\frac{8bc^2f}{d}\right)\sqrt{bx^2+a}\sqrt{dx^2+cx}x^5+\frac{\left(-6c^2(11de-8c\right.}{}}{}\right.$$

↓ 320

$$e\left(\frac{f\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}x^5}{9b}+\frac{(9bde+bcf-6adf)\sqrt{bx^2+a}\sqrt{dx^2+c}x^5}{7d}+\frac{\frac{1}{5}\left(-\frac{6dfa^2}{b}+9dea+2cfa+9bce-\frac{6bc^2f}{d}\right)\sqrt{bx^2+a}\sqrt{dx^2+c}x^3 \cdots}{\cdots}\right.$$

$$f\left(\frac{f\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}x^7}{11b}+\frac{(11bde+bcf-8adf)\sqrt{bx^2+a}\sqrt{dx^2+c}x^7}{9d}+\frac{\frac{1}{7}\left(-\frac{8dfa^2}{b}+11dea+2cfa+11bce-\frac{8bc^2f}{d}\right)\sqrt{bx^2+a}\sqrt{dx^2+c}x^5+\frac{\left(-6c^2(11de-8c \cdots\right.}{\cdots}}{\cdots}\right.$$

$\downarrow$ 388

$$e\left(\frac{f\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}\,x^5}{9b}+\frac{(9bde+bcf-6adf)\sqrt{bx^2+a}\sqrt{dx^2+c}\,x^5}{7d}+\frac{\frac{1}{5}\left(-\frac{6dfa^2}{b}+9dea+2cfa+9bce-\frac{6bc^2f}{d}\right)\sqrt{bx^2+a}\sqrt{dx^2+c}\,x^3 \cdots}{\cdots}\right.$$

$$f\left(\frac{f\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}\,x^7}{11b}+\frac{(11bde+bcf-8adf)\sqrt{bx^2+a}\sqrt{dx^2+c}\,x^7}{9d}+\frac{\frac{1}{7}\left(-\frac{8dfa^2}{b}+11dea+2cfa+11bce-\frac{8bc^2f}{d}\right)\sqrt{bx^2+a}\sqrt{dx^2+c}\,x^5+\frac{\left(-6c^2(11de-8c \cdots\right.}{\cdots}}{\cdots}\right.$$

↓ 313

$$e\left(\frac{f(bx^2+a)^{3/2}\sqrt{dx^2+c}x^5}{9b}+\frac{(9bde+bcf-6adf)\sqrt{bx^2+a}\sqrt{dx^2+c}x^5}{7d}+\frac{\frac{1}{5}\left(-\frac{6dfa^2}{b}+9dea+2cfa+9bce-\frac{6bc^2f}{d}\right)\sqrt{bx^2+a}\sqrt{dx^2+c}x^3\cdots}{\cdots}\right.$$

$$f\left(\frac{f(bx^2+a)^{3/2}\sqrt{dx^2+c}x^7}{11b}+\frac{(11bde+bcf-8adf)\sqrt{bx^2+a}\sqrt{dx^2+c}x^7}{9d}+\frac{\frac{1}{7}\left(-\frac{8dfa^2}{b}+11dea+2cfa+11bce-\frac{8bc^2f}{d}\right)\sqrt{bx^2+a}\sqrt{dx^2+c}x^5+\frac{(-6c^2(11de-8c\cdots}{\cdots}}{\cdots}\right.$$

input
```
Int[x^4*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^2,x]
```

output
```
e*((f*x^5*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(9*b) + (((9*b*d*e + b*c*f -
6*a*d*f)*x^5*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(7*d) + (((9*b*c*e + 9*a*d*e
 + 2*a*c*f - (6*b*c^2*f)/d - (6*a^2*d*f)/b)*x^3*Sqrt[a + b*x^2]*Sqrt[c + d
*x^2])/5 + (3*(((8*a^3*d^3*f - 4*b^3*c^2*(3*d*e - 2*c*f) + 3*a*b^2*c*d*(2*
d*e - c*f) - 3*a^2*b*d^2*(4*d*e + c*f))*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
/(3*b*d) - ((16*a^4*d^4*f + a*b^3*c^2*d*(15*d*e - 8*c*f) - 8*b^4*c^3*(3*d*
e - 2*c*f) + 3*a^2*b^2*c*d^2*(5*d*e - 2*c*f) - 8*a^3*b*d^3*(3*d*e + c*f))*
((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*Ellipt
icE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a +
 b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (c^(3/2)*(8*a^3*d^3*f - 4*b^
3*c^2*(3*d*e - 2*c*f) + 3*a*b^2*c*d*(2*d*e - c*f) - 3*a^2*b*d^2*(4*d*e + c
*f))*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d
)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))/(3*b*
d)))/(5*b*d))/(7*d))/(9*b)) + (f*((f*x^7*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2]
)/(11*b) + (((11*b*d*e + b*c*f - 8*a*d*f)*x^7*Sqrt[a + b*x^2]*Sqrt[c + d*x
^2])/(9*d) + (((11*b*c*e + 11*a*d*e + 2*a*c*f - (8*b*c^2*f)/d - (8*a^2*d*f
)/b)*x^5*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/7 + (((48*a^3*d^3*f + a*b^2*c*d*
(22*d*e - 13*c*f) - 6*b^3*c^2*(11*d*e - 8*c*f) - a^2*b*d^2*(66*d*e + 13*c*
f))*x^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(5*b*d) - (3*(((64*a^4*d^4*f + a*
b^3*c^2*d*(33*d*e - 20*c*f) - 8*b^4*c^3*(11*d*e - 8*c*f) + 3*a^2*b^2*c*...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
 + d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 443
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*(g*x)^(m + 1)*(a + b*x^2)^(p
 + 1)*((c + d*x^2)^q/(b*g*(m + 2*(p + q + 1) + 1))), x] + Simp[1/(b*(m + 2*
(p + q + 1) + 1))   Int[(g*x)^m*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[c*((
b*e - a*f)*(m + 1) + b*e*2*(p + q + 1)) + (d*(b*e - a*f)*(m + 1) + f*2*q*(b
*c - a*d) + b*e*d*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f
, g, m, p}, x] && GtQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^2, c + d*x^
2])
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

rule 448
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[e   Int[(g*x)^m*(a + b*x
^2)^p*(c + d*x^2)^q*(e + f*x^2)^(r - 1), x], x] + Simp[f/e^2   Int[(g*x)^(m
 + 2)*(a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^(r - 1), x], x] /; FreeQ[{a,
b, c, d, e, f, g, m, p, q}, x] && IGtQ[r, 0]
```

Maple [A] (verified)

Time = 9.79 (sec) , antiderivative size = 1668, normalized size of antiderivative = 1.48

method	result	size
elliptic	Expression too large to display	1668
risch	Expression too large to display	2046
default	Expression too large to display	3341

input
```
int(x^4*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^2,x,method=_RETURNVERBOS
E)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/11*f^2*x^9*
(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+1/9*(a*d*f^2+b*c*f^2+2*d*b*e*f-1/11*f^
2*(10*a*d+10*b*c))/b/d*x^7*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+1/7*(2/11*a
*c*f^2+2*a*d*e*f+2*b*c*e*f+b*d*e^2-1/9*(a*d*f^2+b*c*f^2+2*d*b*e*f-1/11*f^2
*(10*a*d+10*b*c))/b/d*(8*a*d+8*b*c))/b/d*x^5*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)
^(1/2)+1/5*(2*a*c*e*f+a*d*e^2+b*c*e^2-7/9*(a*d*f^2+b*c*f^2+2*d*b*e*f-1/11*
f^2*(10*a*d+10*b*c))/b/d*a*c-1/7*(2/11*a*c*f^2+2*a*d*e*f+2*b*c*e*f+b*d*e^2
-1/9*(a*d*f^2+b*c*f^2+2*d*b*e*f-1/11*f^2*(10*a*d+10*b*c))/b/d*(8*a*d+8*b*c
))/b/d*(6*a*d+6*b*c))/b/d*x^3*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+1/3*(a*c
*e^2-5/7*(2/11*a*c*f^2+2*a*d*e*f+2*b*c*e*f+b*d*e^2-1/9*(a*d*f^2+b*c*f^2+2*
d*b*e*f-1/11*f^2*(10*a*d+10*b*c))/b/d*(8*a*d+8*b*c))/b/d*a*c-1/5*(2*a*c*e*
f+a*d*e^2+b*c*e^2-7/9*(a*d*f^2+b*c*f^2+2*d*b*e*f-1/11*f^2*(10*a*d+10*b*c))
/b/d*a*c-1/7*(2/11*a*c*f^2+2*a*d*e*f+2*b*c*e*f+b*d*e^2-1/9*(a*d*f^2+b*c*f^
2+2*d*b*e*f-1/11*f^2*(10*a*d+10*b*c))/b/d*(8*a*d+8*b*c))/b/d*(6*a*d+6*b*c)
)/b/d*(4*a*d+4*b*c))/b/d*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)-1/3*(a*c*e^
2-5/7*(2/11*a*c*f^2+2*a*d*e*f+2*b*c*e*f+b*d*e^2-1/9*(a*d*f^2+b*c*f^2+2*d*b
*e*f-1/11*f^2*(10*a*d+10*b*c))/b/d*(8*a*d+8*b*c))/b/d*a*c-1/5*(2*a*c*e*f+a
*d*e^2+b*c*e^2-7/9*(a*d*f^2+b*c*f^2+2*d*b*e*f-1/11*f^2*(10*a*d+10*b*c))/b/
d*a*c-1/7*(2/11*a*c*f^2+2*a*d*e*f+2*b*c*e*f+b*d*e^2-1/9*(a*d*f^2+b*c*f^2+2
*d*b*e*f-1/11*f^2*(10*a*d+10*b*c))/b/d*(8*a*d+8*b*c))/b/d*(6*a*d+6*b*c)...
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 1199, normalized size of antiderivative = 1.06

$$\int x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^2\,dx=\text{Too large to display}$$

input
```
integrate(x^4*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^2,x, algorithm="fr
icas")
```

output
```
-1/3465*((33*(8*b^5*c^4*d^2 - 5*a*b^4*c^3*d^3 - 5*a^2*b^3*c^2*d^4 + 8*a^3*
b^2*c*d^5)*e^2 - 44*(8*b^5*c^5*d - 4*a*b^4*c^4*d^2 - 3*a^2*b^3*c^3*d^3 - 4
*a^3*b^2*c^2*d^4 + 8*a^4*b*c*d^5)*e*f + (128*b^5*c^6 - 56*a*b^4*c^5*d - 37
*a^2*b^3*c^4*d^2 - 37*a^3*b^2*c^3*d^3 - 56*a^4*b*c^2*d^4 + 128*a^5*c*d^5)*
f^2)*sqrt(b*d)*x*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c)) -
(33*(8*b^5*c^4*d^2 - 5*a*b^4*c^3*d^3 + 4*a^3*b^2*d^6 - (5*a^2*b^3 - 4*a*b^
4)*c^2*d^4 + 2*(4*a^3*b^2 - a^2*b^3)*c*d^5)*e^2 - 22*(16*b^5*c^5*d - 8*a*b
^4*c^4*d^2 + 8*a^4*b*d^6 - 2*(3*a^2*b^3 - 4*a*b^4)*c^3*d^3 - (8*a^3*b^2 +
3*a^2*b^3)*c^2*d^4 + (16*a^4*b - 3*a^3*b^2)*c*d^5)*e*f + (128*b^5*c^6 - 56
*a*b^4*c^5*d + 64*a^5*d^6 - (37*a^2*b^3 - 64*a*b^4)*c^4*d^2 - (37*a^3*b^2
+ 20*a^2*b^3)*c^3*d^3 - 2*(28*a^4*b + 9*a^3*b^2)*c^2*d^4 + 4*(32*a^5 - 5*a
^4*b)*c*d^5)*f^2)*sqrt(b*d)*x*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)/x),
a*d/(b*c)) - (315*b^5*d^6*f^2*x^10 + 35*(22*b^5*d^6*e*f + (b^5*c*d^5 + a*b
^4*d^6)*f^2)*x^8 + 5*(99*b^5*d^6*e^2 + 22*(b^5*c*d^5 + a*b^4*d^6)*e*f - 2*
(4*b^5*c^2*d^4 - a*b^4*c*d^5 + 4*a^2*b^3*d^6)*f^2)*x^6 + (99*(b^5*c*d^5 +
a*b^4*d^6)*e^2 - 44*(3*b^5*c^2*d^4 - a*b^4*c*d^5 + 3*a^2*b^3*d^6)*e*f + (4
8*b^5*c^3*d^3 - 13*a*b^4*c^2*d^4 - 13*a^2*b^3*c*d^5 + 48*a^3*b^2*d^6)*f^2)
*x^4 + 33*(8*b^5*c^3*d^3 - 5*a*b^4*c^2*d^4 - 5*a^2*b^3*c*d^5 + 8*a^3*b^2*d
^6)*e^2 - 44*(8*b^5*c^4*d^2 - 4*a*b^4*c^3*d^3 - 3*a^2*b^3*c^2*d^4 - 4*a^3*
b^2*c*d^5 + 8*a^4*b*d^6)*e*f + (128*b^5*c^5*d - 56*a*b^4*c^4*d^2 - 37*a...
```

Sympy [F]

$$\int x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^2\,dx=\int x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^2\,dx$$

input
```
integrate(x**4*(b*x**2+a)**(1/2)*(d*x**2+c)**(1/2)*(f*x**2+e)**2,x)
```

output
```
Integral(x**4*sqrt(a + b*x**2)*sqrt(c + d*x**2)*(e + f*x**2)**2, x)
```

Maxima [F]

$$\int x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^2\,dx=\int \sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^2x^4\,dx$$

input
```
integrate(x^4*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^2,x, algorithm="ma
xima")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^2*x^4, x)
```

Giac [F]

$$\int x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^2\,dx=\int \sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^2x^4\,dx$$

input
```
integrate(x^4*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^2,x, algorithm="gi
ac")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^2*x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^2\,dx=\int x^4\,\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,\left(f\,x^2+e\right)^2\,dx$$

input
```
int(x^4*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^2,x)
```

output
```
int(x^4*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^2, x)
```

Reduce [F]

$$\int x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^2\,dx=\text{too large to display}$$

input
```
int(x^4*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^2,x)
```

output
```
( - 64*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**4*d**4*f**2*x + 20*sqrt(c + d*
x**2)*sqrt(a + b*x**2)*a**3*b*c*d**3*f**2*x + 176*sqrt(c + d*x**2)*sqrt(a
+ b*x**2)*a**3*b*d**4*e*f*x + 48*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**3*b*
d**4*f**2*x**3 + 18*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*b**2*c**2*d**2*
f**2*x - 66*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*b**2*c*d**3*e*f*x - 13*
sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*b**2*c*d**3*f**2*x**3 - 132*sqrt(c
+ d*x**2)*sqrt(a + b*x**2)*a**2*b**2*d**4*e**2*x - 132*sqrt(c + d*x**2)*sq
rt(a + b*x**2)*a**2*b**2*d**4*e*f*x**3 - 40*sqrt(c + d*x**2)*sqrt(a + b*x*
*2)*a**2*b**2*d**4*f**2*x**5 + 20*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**3
*c**3*d*f**2*x - 66*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**3*c**2*d**2*e*f
*x - 13*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**3*c**2*d**2*f**2*x**3 + 66*
sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**3*c*d**3*e**2*x + 44*sqrt(c + d*x**
2)*sqrt(a + b*x**2)*a*b**3*c*d**3*e*f*x**3 + 10*sqrt(c + d*x**2)*sqrt(a +
b*x**2)*a*b**3*c*d**3*f**2*x**5 + 99*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b
**3*d**4*e**2*x**3 + 110*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**3*d**4*e*f
*x**5 + 35*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**3*d**4*f**2*x**7 - 64*sq
rt(c + d*x**2)*sqrt(a + b*x**2)*b**4*c**4*f**2*x + 176*sqrt(c + d*x**2)*sq
rt(a + b*x**2)*b**4*c**3*d*e*f*x + 48*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b*
*4*c**3*d*f**2*x**3 - 132*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**4*c**2*d**2
*e**2*x - 132*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**4*c**2*d**2*e*f*x**3...
```

3.54 $\int x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^2\,dx$

Optimal result

Integrand size = 35, antiderivative size = 841

$$\int x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^2\,dx =$$

$$-\frac{2(8a^4d^4f^2-4a^3bd^3f(6de+cf)+3a^2b^2d^2(7d^2e^2+5cdef-c^2f^2)-ab^3cd(21d^2e^2-15cdef+4c^2f^2)}{315b^3d^4\sqrt{a+bx^2}}$$

$$+\frac{(8a^3d^3f^2-3a^2bd^2f(8de+cf)+3ab^2d(7d^2e^2+4cdef-c^2f^2)+b^3c(21d^2e^2-24cdef+8c^2f^2))\,x\sqrt{a}}{315b^3d^3}$$

$$-\frac{\left(\frac{6a^2df^2}{b}-2af(9de+cf)-b\left(63de^2+18cef-\frac{6c^2f^2}{d}\right)\right)x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{315bd}$$

$$+\frac{f(18bde+bcf+adf)x^5\sqrt{a+bx^2}\sqrt{c+dx^2}}{63bd}+\frac{1}{9}f^2x^7\sqrt{a+bx^2}\sqrt{c+dx^2}$$

$$+\frac{2\sqrt{a}(8a^4d^4f^2-4a^3bd^3f(6de+cf)+3a^2b^2d^2(7d^2e^2+5cdef-c^2f^2)-ab^3cd(21d^2e^2-15cdef+4c^2}{315b^{7/2}d^4\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

$$-\frac{a^{3/2}(8a^3d^3f^2-3a^2bd^2f(8de+cf)+3ab^2d(7d^2e^2+4cdef-c^2f^2)+b^3c(21d^2e^2-24cdef+8c^2f^2))}{315b^{7/2}d^3\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
-2/315*(8*a^4*d^4*f^2-4*a^3*b*d^3*f*(c*f+6*d*e)+3*a^2*b^2*d^2*(-c^2*f^2+5*
c*d*e*f+7*d^2*e^2)-a*b^3*c*d*(4*c^2*f^2-15*c*d*e*f+21*d^2*e^2)+b^4*c^2*(8*
c^2*f^2-24*c*d*e*f+21*d^2*e^2))*x*(d*x^2+c)^(1/2)/b^3/d^4/(b*x^2+a)^(1/2)+
1/315*(8*a^3*d^3*f^2-3*a^2*b*d^2*f*(c*f+8*d*e)+3*a*b^2*d*(-c^2*f^2+4*c*d*e
*f+7*d^2*e^2)+b^3*c*(8*c^2*f^2-24*c*d*e*f+21*d^2*e^2))*x*(b*x^2+a)^(1/2)*(
d*x^2+c)^(1/2)/b^3/d^3-1/315*(6*a^2*d*f^2/b-2*a*f*(c*f+9*d*e)-b*(63*d*e^2+
18*c*e*f-6*c^2*f^2/d))*x^3*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b/d+1/63*f*(a*d
*f+b*c*f+18*b*d*e)*x^5*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b/d+1/9*f^2*x^7*(b*
x^2+a)^(1/2)*(d*x^2+c)^(1/2)+2/315*a^(1/2)*(8*a^4*d^4*f^2-4*a^3*b*d^3*f*(c
*f+6*d*e)+3*a^2*b^2*d^2*(-c^2*f^2+5*c*d*e*f+7*d^2*e^2)-a*b^3*c*d*(4*c^2*f^
2-15*c*d*e*f+21*d^2*e^2)+b^4*c^2*(8*c^2*f^2-24*c*d*e*f+21*d^2*e^2))*(d*x^2
+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))
/b^(7/2)/d^4/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)-1/315*a^(3/2)
*(8*a^3*d^3*f^2-3*a^2*b*d^2*f*(c*f+8*d*e)+3*a*b^2*d*(-c^2*f^2+4*c*d*e*f+7*
d^2*e^2)+b^3*c*(8*c^2*f^2-24*c*d*e*f+21*d^2*e^2))*(d*x^2+c)^(1/2)*InverseJ
acobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/b^(7/2)/d^3/(b*x^2+a)
^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.23 (sec) , antiderivative size = 570, normalized size of antiderivative = 0.68

$$\int x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^2\,dx$$
$$= \frac{\sqrt{\frac{b}{a}}dx(a+bx^2)\left(c+dx^2\right)\left(8a^3d^3f^2-3a^2bd^2f(8de+cf+2dfx^2)+ab^2d(-3c^2f^2+2cdf(6e+fx^2)+d^2\right.}{}$$

input
```
Integrate[x^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^2,x]
```

output
```
(Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2)*(8*a^3*d^3*f^2 - 3*a^2*b*d^2*f*(8*d
*e + c*f + 2*d*f*x^2) + a*b^2*d*(-3*c^2*f^2 + 2*c*d*f*(6*e + f*x^2) + d^2*
(21*e^2 + 18*e*f*x^2 + 5*f^2*x^4)) + b^3*(8*c^3*f^2 - 6*c^2*d*f*(4*e + f*x
^2) + c*d^2*(21*e^2 + 18*e*f*x^2 + 5*f^2*x^4) + d^3*x^2*(63*e^2 + 90*e*f*x
^2 + 35*f^2*x^4))) + (2*I)*c*(8*a^4*d^4*f^2 - 4*a^3*b*d^3*f*(6*d*e + c*f)
+ a*b^3*c*d*(-21*d^2*e^2 + 15*c*d*e*f - 4*c^2*f^2) + 3*a^2*b^2*d^2*(7*d^2*
e^2 + 5*c*d*e*f - c^2*f^2) + b^4*c^2*(21*d^2*e^2 - 24*c*d*e*f + 8*c^2*f^2)
)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x]
, (a*d)/(b*c)] - I*c*(-(b*c) + a*d)*(8*a^3*d^3*f^2 + 3*a*b^2*d^2*e*(7*d*e
- 2*c*f) + 3*a^2*b*d^2*f*(-8*d*e + c*f) - 2*b^3*c*(21*d^2*e^2 - 24*c*d*e*f
 + 8*c^2*f^2))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh
[Sqrt[b/a]*x], (a*d)/(b*c)])/(315*b^3*Sqrt[b/a]*d^4*Sqrt[a + b*x^2]*Sqrt[c
 + d*x^2])
```

Rubi [A] (warning: unable to verify)

Time = 1.96 (sec) , antiderivative size = 1103, normalized size of antiderivative = 1.31, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {448, 443, 443, 444, 25, 27, 406, 320, 388, 313, 444, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^2 dx$$

$$\Big\downarrow \text{448}$$

$$\frac{f\int x^4\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)dx}{e^2}+e\int x^2\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)dx$$

$$\Big\downarrow \text{443}$$

$$\frac{f\left(\frac{\int \frac{x^4\sqrt{bx^2+a}\left((9bde+bcf-6adf)x^2+c(9be-5af)\right)}{\sqrt{dx^2+c}}dx}{9b}+\frac{fx^5\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{9b}\right)}{e^2}+e\left(\frac{\int \frac{x^2\sqrt{bx^2+a}\left((7bde+bcf-4adf)x^2+c(7be-3af)\right)}{\sqrt{dx^2+c}}dx}{7b}+\frac{fx^3\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{7b}\right)$$

$$\Big\downarrow \text{443}$$

$$f\left(\frac{\frac{\int \frac{x^4\left(ac(18bde-5bcf-5adf)-\left(-3c(3de-2cf)b^2-ad(9de+2cf)b+6a^2d^2f\right)x^2\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{7d}+\frac{x^5\sqrt{a+bx^2}\sqrt{c+dx^2}(-6adf+bcf+9bde)}{7d}}{9b}+\frac{fx^5\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{9b}\right.$$

$$\overline{\qquad\qquad e^2 \qquad\qquad}$$

$$e\left(\frac{\frac{\int \frac{x^2\left(ac(14bde-3bcf-3adf)-\left(-c(7de-4cf)b^2-ad(7de+2cf)b+4a^2d^2f\right)x^2\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{5d}+\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}(-4adf+bcf+7bde)}{5d}}{7b}+\frac{fx^3\left(a+bx^2\right)^{3/}}{7b}\right.$$

$\downarrow$ 444

$$f\left(\frac{\frac{\frac{1}{5}x^3\sqrt{a+bx^2}\sqrt{c+dx^2}\left(-\frac{6a^2df}{b}+2acf+9ade-\frac{6bc^2f}{d}+9bce\right)-\frac{\int -\frac{3x^2\left(\left(\left(-4c^2(3de-2cf)b^3+3acd(2de-cf)b^2-3a^2d^2(4de+cf)b+8a^3d^3f\right)x^2+ac\left(-3c(3de-\right.\right.\right.}{\sqrt{bx^2+a}\sqrt{dx^2+c}}}{5bd}}{7d}}{9b}\right.$$

$$\overline{\qquad\qquad e^2 \qquad\qquad}$$

$$e\left(\frac{\frac{\frac{1}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(-\frac{4a^2df}{b}+2acf+7ade-\frac{4bc^2f}{d}+7bce\right)-\frac{\int -\frac{\left(-2c^2(7de-4cf)b^3+acd(14de-5cf)b^2-a^2d^2(14de+5cf)b+8a^3d^3f\right)x^2+ac\left(-c(7de-\right.}{\sqrt{bx^2+a}\sqrt{dx^2+c}}}{3bd}}{5d}}{7b}\right.$$

$\downarrow$ 25

$$f\left(\frac{\frac{\frac{1}{5}x^3\sqrt{a+bx^2}\sqrt{c+dx^2}\left(-\frac{6a^2df}{b}+2acf+9ade-\frac{6bc^2f}{d}+9bce\right)-\frac{\int -\frac{3x^2\left(\left(\left(-4c^2(3de-2cf)b^3+3acd(2de-cf)b^2-3a^2d^2(4de+cf)b+8a^3d^3f\right)x^2+ac\left(-3c(3de-\right.\right.\right.}{\sqrt{bx^2+a}\sqrt{dx^2+c}}}{5bd}}{7d}}{9b}\right.$$

$$\overline{\qquad\qquad e^2 \qquad\qquad}$$

$$e\left(\frac{\frac{\frac{\int \frac{\left(-2c^2(7de-4cf)b^3+acd(14de-5cf)b^2-a^2d^2(14de+5cf)b+8a^3d^3f\right)x^2+ac\left(-c(7de-4cf)b^2-ad(7de+2cf)b+4a^2d^2f\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3bd}+\frac{1}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(-\frac{4a^2}{b}\right.}{5d}}{7b}\right.$$

$\downarrow$ 27

$$f\left(\frac{\frac{\frac{3\int \frac{x^2\left(\left(-4c^2(3de-2cf)b^3+3acd(2de-cf)b^2-3a^2d^2(4de+cf)b+8a^3d^3f\right)x^2+ac\left(-3c(3de-2cf)b^2-ad(9de+2cf)b+6a^2d^2f\right)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{5bd}+\frac{1}{5}x^3\sqrt{a+bx^2}\sqrt{c+dx^2}\left(-\cdots\right.}{7d}\cdots}{9b}\cdots\right)$$

$$\overline{\qquad\qquad\qquad e^2 \qquad\qquad}$$

$$e\left(\frac{\frac{\frac{\int \frac{\left(-2c^2(7de-4cf)b^3+acd(14de-5cf)b^2-a^2d^2(14de+5cf)b+8a^3d^3f\right)x^2+ac\left(-c(7de-4cf)b^2-ad(7de+2cf)b+4a^2d^2f\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3bd}+\frac{1}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(-\frac{4a^2\cdots}{b\cdots}\right.}{5d}\cdots}{7b}\cdots\right)$$

$$\Big\downarrow \; 406$$

$$f\left(\frac{\frac{\frac{3\int \frac{x^2\left(\left(-4c^2(3de-2cf)b^3+3acd(2de-cf)b^2-3a^2d^2(4de+cf)b+8a^3d^3f\right)x^2+ac\left(-3c(3de-2cf)b^2-ad(9de+2cf)b+6a^2d^2f\right)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{5bd}+\frac{1}{5}x^3\sqrt{a+bx^2}\sqrt{c+dx^2}\left(-\cdots\right.}{7d}\cdots}{9b}\cdots\right)$$

$$\overline{\qquad\qquad\qquad e^2 \qquad\qquad}$$

$$e\left(\frac{\frac{\frac{ac\left(4a^2d^2f-abd(2cf+7de)+b^2(-c)(7de-4cf)\right)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\left(8a^3d^3f-a^2bd^2(5cf+14de)+ab^2cd(14de-5cf)-2b^3c^2(7de-4cf)\right)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2\cdots}}\cdots}{3bd}\cdots}{5d}\cdots}{7b}\cdots\right)$$

$$\Big\downarrow \; 320$$

$$e\left(\frac{\frac{\frac{\left(8a^3d^3f-a^2bd^2(5cf+14de)+ab^2cd(14de-5cf)-2b^3c^2(7de-4cf)\right)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}\left(4a^2d^2f-abd(2cf+7de)+b^2(-c)(7de-4cf)\right)\text{Ellipt}\cdots}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}\cdots}}{3bd}\cdots}{5d}\cdots}{7b}\cdots\right)$$

$$f\left(\frac{\frac{\frac{3\int \frac{x^2\left(\left(-4c^2(3de-2cf)b^3+3acd(2de-cf)b^2-3a^2d^2(4de+cf)b+8a^3d^3f\right)x^2+ac\left(-3c(3de-2cf)b^2-ad(9de+2cf)b+6a^2d^2f\right)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{5bd}+\frac{1}{5}x^3\sqrt{a+bx^2}\sqrt{c+dx^2}\left(-\cdots\right.}{7d}\cdots}{9b}\cdots\right)$$

$$\overline{\qquad\qquad\qquad e^2 \qquad\qquad}$$

$$\Big\downarrow \; 388$$

$$e\left(\frac{\frac{\frac{\left(8a^3d^3f-a^2bd^2(5cf+14de)+ab^2cd(14de-5cf)-2b^3c^2(7de-4cf)\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}\left(4a^2d^2f-abd(2cf+7de)+b^2(-c)(7de\right.}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(c}{a(c}}}}{3bd}}{5d}}{7}\right.$$

$$\frac{f\left(\frac{\frac{3\int\frac{x^2\left(\left(-4c^2(3de-2cf)b^3+3acd(2de-cf)b^2-3a^2d^2(4de+cf)b+8a^3d^3f\right)x^2+ac\left(-3c(3de-2cf)b^2-ad(9de+2cf)b+6a^2d^2f\right)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{5bd}+\frac{1}{5}x^3\sqrt{a+bx^2}\sqrt{c+dx^2}\left(-\right.}{7d}}{9b}\right.}{e^2}$$

$\Big\downarrow$ 313

$$\frac{f\left(\frac{\frac{3\int\frac{x^2\left(\left(-4c^2(3de-2cf)b^3+3acd(2de-cf)b^2-3a^2d^2(4de+cf)b+8a^3d^3f\right)x^2+ac\left(-3c(3de-2cf)b^2-ad(9de+2cf)b+6a^2d^2f\right)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{5bd}+\frac{1}{5}x^3\sqrt{a+bx^2}\sqrt{c+dx^2}\left(-\right.}{7d}}{9b}\right.}{e^2}$$

$$e\left(\frac{\frac{1}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(-\frac{4a^2df}{b}+2acf+7ade-\frac{4bc^2f}{d}+7bce\right)+\frac{c^{3/2}\sqrt{a+bx^2}\left(4a^2d^2f-abd(2cf+7de)+b^2(-c)(7de-4cf)\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{b}{c}\right.}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{5d}\right.$$

$\Big\downarrow$ 444

$$e\left(\frac{f\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}\,x^3}{7b}+\frac{\frac{(7bde+bcf-4adf)\sqrt{bx^2+a}\sqrt{dx^2+c}\,x^3}{5d}+\frac{\frac{1}{3}\left(-\frac{4dfa^2}{b}+7dea+2cfa+7bce-\frac{4bc^2f}{d}\right)\sqrt{bx^2+a}\sqrt{dx^2+c}\,x+\cdots}{\cdots}}{\cdots}\right.$$

$$\frac{f\left(\frac{f\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}\,x^5}{9b}+\frac{\frac{(9bde+bcf-6adf)\sqrt{bx^2+a}\sqrt{dx^2+c}\,x^5}{7d}+\frac{\frac{1}{5}\left(-\frac{6dfa^2}{b}+9dea+2cfa+9bce-\frac{6bc^2f}{d}\right)\sqrt{bx^2+a}\sqrt{dx^2+c}\,x^3+\frac{3\left(\frac{\left(-4c^2(3de-2cf)\cdots\right.}{\cdots}\right.}{\cdots}}{\cdots}}{\cdots}\right.}{\cdots}$$

$\downarrow$ 406

$$e\left(\frac{f\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}\,x^3}{7b}+\frac{\frac{(7bde+bcf-4adf)\sqrt{bx^2+a}\sqrt{dx^2+c}\,x^3}{5d}+\frac{\frac{1}{3}\left(-\frac{4dfa^2}{b}+7dea+2cfa+7bce-\frac{4bc^2f}{d}\right)\sqrt{bx^2+a}\sqrt{dx^2+c}\,x+\cdots}{\cdots}}{\cdots}\right.$$

$$\frac{f\left(\frac{f\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}\,x^5}{9b}+\frac{\frac{(9bde+bcf-6adf)\sqrt{bx^2+a}\sqrt{dx^2+c}\,x^5}{7d}+\frac{\frac{1}{5}\left(-\frac{6dfa^2}{b}+9dea+2cfa+9bce-\frac{6bc^2f}{d}\right)\sqrt{bx^2+a}\sqrt{dx^2+c}\,x^3+\frac{3\left(\frac{\left(-4c^2(3de-2cf)\cdots\right.}{\cdots}\right.}{\cdots}}{\cdots}}{\cdots}\right.}{\cdots}$$

$\downarrow$ 320

$$e\left(\frac{f\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}x^3}{7b}+\frac{(7bde+bcf-4adf)\sqrt{bx^2+a}\sqrt{dx^2+c}x^3}{5d}+\frac{\frac{1}{3}\left(-\frac{4dfa^2}{b}+7dea+2cfa+7bce-\frac{4bc^2f}{d}\right)\sqrt{bx^2+a}\sqrt{dx^2+c}x+\right.$$

$$f\left(\frac{f\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}x^5}{9b}+\frac{(9bde+bcf-6adf)\sqrt{bx^2+a}\sqrt{dx^2+c}x^5}{7d}+\frac{\frac{1}{5}\left(-\frac{6dfa^2}{b}+9dea+2cfa+9bce-\frac{6bc^2f}{d}\right)\sqrt{bx^2+a}\sqrt{dx^2+c}x^3+\frac{3\left(\frac{\left(-4c^2(3de-2cf)\right.}{}\right.}{}}{}\right.$$

↓ 388

$$e\left(\frac{f\left(bx^2+a\right)^{3/2}\sqrt{dx^2+cx^3}}{7b}+\frac{(7bde+bcf-4adf)\sqrt{bx^2+a}\sqrt{dx^2+cx^3}}{5d}+\frac{\frac{1}{3}\left(-\frac{4dfa^2}{b}+7dea+2cfa+7bce-\frac{4bc^2f}{d}\right)\sqrt{bx^2+a}\sqrt{dx^2+cx+}}{}\right.$$

$$f\left(\frac{f\left(bx^2+a\right)^{3/2}\sqrt{dx^2+cx^5}}{9b}+\frac{(9bde+bcf-6adf)\sqrt{bx^2+a}\sqrt{dx^2+cx^5}}{7d}+\frac{\frac{1}{5}\left(-\frac{6dfa^2}{b}+9dea+2cfa+9bce-\frac{6bc^2f}{d}\right)\sqrt{bx^2+a}\sqrt{dx^2+cx^3}+\frac{3\left(\frac{\left(-4c^2(3de-2cf)\right.}{}\right.}{}}{}\right.$$

$\downarrow$ 313

$$e\left(\frac{f\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}x^3}{7b}+\frac{(7bde+bcf-4adf)\sqrt{bx^2+a}\sqrt{dx^2+c}x^3}{5d}+\frac{\frac{1}{3}\left(-\frac{4dfa^2}{b}+7dea+2cfa+7bce-\frac{4bc^2f}{d}\right)\sqrt{bx^2+a}\sqrt{dx^2+c}x+}{}\right.$$

$$f\left(\frac{f\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}x^5}{9b}+\frac{(9bde+bcf-6adf)\sqrt{bx^2+a}\sqrt{dx^2+c}x^5}{7d}+\frac{\frac{1}{5}\left(-\frac{6dfa^2}{b}+9dea+2cfa+9bce-\frac{6bc^2f}{d}\right)\sqrt{bx^2+a}\sqrt{dx^2+c}x^3+\frac{3\left(\frac{\left(-4c^2(3de-2cf)\right.}{}\right.}{}}{}\right.$$

input
```
Int[x^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^2,x]
```

output
```
e*((f*x^3*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(7*b) + (((7*b*d*e + b*c*f -
4*a*d*f)*x^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(5*d) + (((7*b*c*e + 7*a*d*e
 + 2*a*c*f - (4*b*c^2*f)/d - (4*a^2*d*f)/b)*x*Sqrt[a + b*x^2]*Sqrt[c + d*x
^2])/3 + ((8*a^3*d^3*f + a*b^2*c*d*(14*d*e - 5*c*f) - 2*b^3*c^2*(7*d*e - 4
*c*f) - a^2*b*d^2*(14*d*e + 5*c*f))*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2
]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (
b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x
^2])) + (c^(3/2)*(4*a^2*d^2*f - b^2*c*(7*d*e - 4*c*f) - a*b*d*(7*d*e + 2*c
*f))*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d
)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))/(3*b*
d))/(5*d))/(7*b)) + (f*((f*x^5*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(9*b) +
(((9*b*d*e + b*c*f - 6*a*d*f)*x^5*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(7*d) +
 (((9*b*c*e + 9*a*d*e + 2*a*c*f - (6*b*c^2*f)/d - (6*a^2*d*f)/b)*x^3*Sqrt[
a + b*x^2]*Sqrt[c + d*x^2])/5 + (3*(((8*a^3*d^3*f - 4*b^3*c^2*(3*d*e - 2*c
*f) + 3*a*b^2*c*d*(2*d*e - c*f) - 3*a^2*b*d^2*(4*d*e + c*f))*x*Sqrt[a + b*
x^2]*Sqrt[c + d*x^2])/(3*b*d) - ((16*a^4*d^4*f + a*b^3*c^2*d*(15*d*e - 8*c
*f) - 8*b^4*c^3*(3*d*e - 2*c*f) + 3*a^2*b^2*c*d^2*(5*d*e - 2*c*f) - 8*a^3*
b*d^3*(3*d*e + c*f))*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*S
qrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b
*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (c^(...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 443
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*(g*x)^(m + 1)*(a + b*x^2)^(p
 + 1)*((c + d*x^2)^q/(b*g*(m + 2*(p + q + 1) + 1))), x] + Simp[1/(b*(m + 2*
(p + q + 1) + 1))   Int[(g*x)^m*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[c*((
b*e - a*f)*(m + 1) + b*e*2*(p + q + 1)) + (d*(b*e - a*f)*(m + 1) + f*2*q*(b
*c - a*d) + b*e*d*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f
, g, m, p}, x] && GtQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^2, c + d*x^
2])
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

rule 448
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[e   Int[(g*x)^m*(a + b*x
^2)^p*(c + d*x^2)^q*(e + f*x^2)^(r - 1), x], x] + Simp[f/e^2   Int[(g*x)^(m
 + 2)*(a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^(r - 1), x], x] /; FreeQ[{a,
b, c, d, e, f, g, m, p, q}, x] && IGtQ[r, 0]
```

Maple [A] (verified)

Time = 7.88 (sec) , antiderivative size = 1075, normalized size of antiderivative = 1.28

method	result	size
elliptic	Expression too large to display	1075
risch	Expression too large to display	1494
default	Expression too large to display	2491

input
```
int(x^2*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^2,x,method=_RETURNVERBOS
E)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/9*f^2*x^7*(
b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+1/7*(a*d*f^2+b*c*f^2+2*d*b*e*f-1/9*f^2*
(8*a*d+8*b*c))/b/d*x^5*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+1/5*(2/9*a*c*f^
2+2*a*d*e*f+2*b*c*e*f+b*d*e^2-1/7*(a*d*f^2+b*c*f^2+2*d*b*e*f-1/9*f^2*(8*a*
d+8*b*c))/b/d*(6*a*d+6*b*c))/b/d*x^3*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+1
/3*(2*a*c*e*f+a*d*e^2+b*c*e^2-5/7*(a*d*f^2+b*c*f^2+2*d*b*e*f-1/9*f^2*(8*a*
d+8*b*c))/b/d*a*c-1/5*(2/9*a*c*f^2+2*a*d*e*f+2*b*c*e*f+b*d*e^2-1/7*(a*d*f^
2+b*c*f^2+2*d*b*e*f-1/9*f^2*(8*a*d+8*b*c))/b/d*(6*a*d+6*b*c))/b/d*(4*a*d+4
*b*c))/b/d*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)-1/3*(2*a*c*e*f+a*d*e^2+b*
c*e^2-5/7*(a*d*f^2+b*c*f^2+2*d*b*e*f-1/9*f^2*(8*a*d+8*b*c))/b/d*a*c-1/5*(2
/9*a*c*f^2+2*a*d*e*f+2*b*c*e*f+b*d*e^2-1/7*(a*d*f^2+b*c*f^2+2*d*b*e*f-1/9*
f^2*(8*a*d+8*b*c))/b/d*(6*a*d+6*b*c))/b/d*(4*a*d+4*b*c))/b/d*a*c/(-b/a)^(1
/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2
)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-(a*c*e^2-3/5*(2/9*a*c
*f^2+2*a*d*e*f+2*b*c*e*f+b*d*e^2-1/7*(a*d*f^2+b*c*f^2+2*d*b*e*f-1/9*f^2*(8
*a*d+8*b*c))/b/d*(6*a*d+6*b*c))/b/d*a*c-1/3*(2*a*c*e*f+a*d*e^2+b*c*e^2-5/7
*(a*d*f^2+b*c*f^2+2*d*b*e*f-1/9*f^2*(8*a*d+8*b*c))/b/d*a*c-1/5*(2/9*a*c*f^
2+2*a*d*e*f+2*b*c*e*f+b*d*e^2-1/7*(a*d*f^2+b*c*f^2+2*d*b*e*f-1/9*f^2*(8*a*
d+8*b*c))/b/d*(6*a*d+6*b*c))/b/d*(4*a*d+4*b*c))/b/d*(2*a*d+2*b*c))*c/(-b/a
)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*...
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 873, normalized size of antiderivative = 1.04

$$\int x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^2\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^2,x, algorithm="fr
icas")
```

output
```
1/315*(2*(21*(b^4*c^3*d^2 - a*b^3*c^2*d^3 + a^2*b^2*c*d^4)*e^2 - 3*(8*b^4*
c^4*d - 5*a*b^3*c^3*d^2 - 5*a^2*b^2*c^2*d^3 + 8*a^3*b*c*d^4)*e*f + (8*b^4*
c^5 - 4*a*b^3*c^4*d - 3*a^2*b^2*c^3*d^2 - 4*a^3*b*c^2*d^3 + 8*a^4*c*d^4)*f
^2)*sqrt(b*d)*x*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - (
21*(2*b^4*c^3*d^2 - 2*a*b^3*c^2*d^3 + a^2*b^2*d^5 + (2*a^2*b^2 + a*b^3)*c*
d^4)*e^2 - 6*(8*b^4*c^4*d - 5*a*b^3*c^3*d^2 + 4*a^3*b*d^5 - (5*a^2*b^2 - 4
*a*b^3)*c^2*d^3 + 2*(4*a^3*b - a^2*b^2)*c*d^4)*e*f + (16*b^4*c^5 - 8*a*b^3
*c^4*d + 8*a^4*d^5 - 2*(3*a^2*b^2 - 4*a*b^3)*c^3*d^2 - (8*a^3*b + 3*a^2*b^
2)*c^2*d^3 + (16*a^4 - 3*a^3*b)*c*d^4)*f^2)*sqrt(b*d)*x*sqrt(-c/d)*ellipti
c_f(arcsin(sqrt(-c/d)/x), a*d/(b*c)) + (35*b^4*d^5*f^2*x^8 + 5*(18*b^4*d^5
*e*f + (b^4*c*d^4 + a*b^3*d^5)*f^2)*x^6 + (63*b^4*d^5*e^2 + 18*(b^4*c*d^4
+ a*b^3*d^5)*e*f - 2*(3*b^4*c^2*d^3 - a*b^3*c*d^4 + 3*a^2*b^2*d^5)*f^2)*x^
4 - 42*(b^4*c^2*d^3 - a*b^3*c*d^4 + a^2*b^2*d^5)*e^2 + 6*(8*b^4*c^3*d^2 -
5*a*b^3*c^2*d^3 - 5*a^2*b^2*c*d^4 + 8*a^3*b*d^5)*e*f - 2*(8*b^4*c^4*d - 4*
a*b^3*c^3*d^2 - 3*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + 8*a^4*d^5)*f^2 + (21*(
b^4*c*d^4 + a*b^3*d^5)*e^2 - 12*(2*b^4*c^2*d^3 - a*b^3*c*d^4 + 2*a^2*b^2*d
^5)*e*f + (8*b^4*c^3*d^2 - 3*a*b^3*c^2*d^3 - 3*a^2*b^2*c*d^4 + 8*a^3*b*d^5
)*f^2)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(b^4*d^5*x)
```

Sympy [F]

$$\int x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^2\,dx=\int x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^2\,dx$$

input
```
integrate(x**2*(b*x**2+a)**(1/2)*(d*x**2+c)**(1/2)*(f*x**2+e)**2,x)
```

output
```
Integral(x**2*sqrt(a + b*x**2)*sqrt(c + d*x**2)*(e + f*x**2)**2, x)
```

Maxima [F]

$$\int x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^2\,dx=\int \sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^2x^2\,dx$$

input
```
integrate(x^2*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^2,x, algorithm="ma
xima")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^2*x^2, x)
```

Giac [F]

$$\int x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^2\,dx=\int \sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^2x^2\,dx$$

input
```
integrate(x^2*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^2,x, algorithm="gi
ac")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^2*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^2\,dx=\int x^2\,\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,\left(f\,x^2+e\right)^2\,dx$$

input
```
int(x^2*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^2,x)
```

output
```
int(x^2*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^2, x)
```

Reduce [F]

$$\int x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^2\,dx=\text{too large to display}$$

input
```
int(x^2*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^2,x)
```

output
```
(8*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**3*d**3*f**2*x - 3*sqrt(c + d*x**2)
*sqrt(a + b*x**2)*a**2*b*c*d**2*f**2*x - 24*sqrt(c + d*x**2)*sqrt(a + b*x*
*2)*a**2*b*d**3*e*f*x - 6*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*b*d**3*f*
*2*x**3 - 3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**2*c**2*d*f**2*x + 12*sq
rt(c + d*x**2)*sqrt(a + b*x**2)*a*b**2*c*d**2*e*f*x + 2*sqrt(c + d*x**2)*s
qrt(a + b*x**2)*a*b**2*c*d**2*f**2*x**3 + 21*sqrt(c + d*x**2)*sqrt(a + b*x
**2)*a*b**2*d**3*e**2*x + 18*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**2*d**3
*e*f*x**3 + 5*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**2*d**3*f**2*x**5 + 8*
sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**3*c**3*f**2*x - 24*sqrt(c + d*x**2)*s
qrt(a + b*x**2)*b**3*c**2*d*e*f*x - 6*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b*
*3*c**2*d*f**2*x**3 + 21*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**3*c*d**2*e**
2*x + 18*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**3*c*d**2*e*f*x**3 + 5*sqrt(c
 + d*x**2)*sqrt(a + b*x**2)*b**3*c*d**2*f**2*x**5 + 63*sqrt(c + d*x**2)*sq
rt(a + b*x**2)*b**3*d**3*e**2*x**3 + 90*sqrt(c + d*x**2)*sqrt(a + b*x**2)*
b**3*d**3*e*f*x**5 + 35*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**3*d**3*f**2*x
**7 - 16*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*
c*x**2 + b*d*x**4),x)*a**4*d**4*f**2 + 8*int((sqrt(c + d*x**2)*sqrt(a + b*
x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a**3*b*c*d**3*f**2 +
 48*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**
2 + b*d*x**4),x)*a**3*b*d**4*e*f + 6*int((sqrt(c + d*x**2)*sqrt(a + b*x...
```

3.55 $\int \sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^2 \, dx$

Optimal result

Integrand size = 32, antiderivative size = 609

$$\int \sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^2 \, dx$$
$$= \frac{(8a^3d^3f^2-a^2bd^2f(28de+5cf)+ab^2d(35d^2e^2+28cdef-5c^2f^2)+b^3c(35d^2e^2-28cdef+8c^2f^2))\,x\sqrt{}}{105b^2d^3\sqrt{a+bx^2}}$$
$$-\frac{\left(\frac{4a^2df^2}{b}-2af(7de+cf)-b\left(35de^2+14cef-\frac{4c^2f^2}{d}\right)\right)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{105bd}$$
$$+\frac{f(14bde+bcf+adf)x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{35bd}+\frac{1}{7}f^2x^5\sqrt{a+bx^2}\sqrt{c+dx^2}$$
$$-\frac{\sqrt{a}(8a^3d^3f^2-a^2bd^2f(28de+5cf)+ab^2d(35d^2e^2+28cdef-5c^2f^2)+b^3c(35d^2e^2-28cdef+8c^2f^2}{105b^{5/2}d^3\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$
$$+\frac{2a^{3/2}(2a^2d^2f^2-abdf(7de+cf)+b^2(35d^2e^2-7cdef+2c^2f^2))\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1\right.}{105b^{5/2}d^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
1/105*(8*a^3*d^3*f^2-a^2*b*d^2*f*(5*c*f+28*d*e)+a*b^2*d*(-5*c^2*f^2+28*c*d
*e*f+35*d^2*e^2)+b^3*c*(8*c^2*f^2-28*c*d*e*f+35*d^2*e^2))*x*(d*x^2+c)^(1/2
)/b^2/d^3/(b*x^2+a)^(1/2)-1/105*(4*a^2*d*f^2/b-2*a*f*(c*f+7*d*e)-b*(35*d*e
^2+14*c*e*f-4*c^2*f^2/d))*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b/d+1/35*f*(a*
d*f+b*c*f+14*b*d*e)*x^3*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b/d+1/7*f^2*x^5*(b
*x^2+a)^(1/2)*(d*x^2+c)^(1/2)-1/105*a^(1/2)*(8*a^3*d^3*f^2-a^2*b*d^2*f*(5*
c*f+28*d*e)+a*b^2*d*(-5*c^2*f^2+28*c*d*e*f+35*d^2*e^2)+b^3*c*(8*c^2*f^2-28
*c*d*e*f+35*d^2*e^2))*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2
/a)^(1/2),(1-a*d/b/c)^(1/2))/b^(5/2)/d^3/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b
*x^2+a))^(1/2)+2/105*a^(3/2)*(2*a^2*d^2*f^2-a*b*d*f*(c*f+7*d*e)+b^2*(2*c^2
*f^2-7*c*d*e*f+35*d^2*e^2))*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)
*x/a^(1/2)),(1-a*d/b/c)^(1/2))/b^(5/2)/d^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/
(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.39 (sec) , antiderivative size = 422, normalized size of antiderivative = 0.69

$$\int \sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^2\,dx = \frac{-\sqrt{\frac{b}{a}}dx(a+bx^2)\left(c+dx^2\right)\left(4a^2d^2f^2-abdf\left(14de+2cf+3dfx^2\right)-b^2\left(-4c^2f^2+cdf\left(14e+3fx^2\right)+d^2(\right.\right.}{}$$

input
```
Integrate[Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^2,x]
```

output
```
(-(Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2)*(4*a^2*d^2*f^2 - a*b*d*f*(14*d*e
+ 2*c*f + 3*d*f*x^2) - b^2*(-4*c^2*f^2 + c*d*f*(14*e + 3*f*x^2) + d^2*(35*
e^2 + 42*e*f*x^2 + 15*f^2*x^4)))) - I*c*(8*a^3*d^3*f^2 - a^2*b*d^2*f*(28*d
*e + 5*c*f) + a*b^2*d*(35*d^2*e^2 + 28*c*d*e*f - 5*c^2*f^2) + b^3*c*(35*d^
2*e^2 - 28*c*d*e*f + 8*c^2*f^2))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*E
llipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + I*c*(-(b*c) + a*d)*(4*a^2*
d^2*f^2 + a*b*d*f*(-14*d*e + c*f) + b^2*(-35*d^2*e^2 + 28*c*d*e*f - 8*c^2*
f^2))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a
]*x], (a*d)/(b*c)])/(105*a^2*(b/a)^(5/2)*d^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^
2])
```

Rubi [A] (verified)

Time = 1.40 (sec) , antiderivative size = 1043, normalized size of antiderivative = 1.71, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^2 dx$$

$$\Big\downarrow \ 433$$

$$\int \left(e^2\sqrt{a+bx^2}\sqrt{c+dx^2}+2efx^2\sqrt{a+bx^2}\sqrt{c+dx^2}+f^2x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{7}f^2\sqrt{bx^2+a}\sqrt{dx^2+c}x^5+\frac{(bc+ad)f^2\sqrt{bx^2+a}\sqrt{dx^2+c}x^3}{35bd}+\frac{2}{5}ef\sqrt{bx^2+a}\sqrt{dx^2+c}x^3+\frac{1}{3}e^2\sqrt{bx^2+a}\sqrt{dx^2+c}x-\frac{2(2b^2c^2-abdc+2a^2d^2)f^2\sqrt{bx^2+a}\sqrt{dx^2+c}x}{105b^2d^2}+\frac{2(bc+ad)ef\sqrt{bx^2+a}\sqrt{dx^2+c}x}{15bd}+\frac{(bc+ad)e^2\sqrt{bx^2+a}x}{3b\sqrt{dx^2+c}}+\frac{(bc+ad)(8b^2c^2-13abdc+8a^2d^2)f^2\sqrt{bx^2+a}x}{105b^3d^2\sqrt{dx^2+c}}-\frac{4(b^2c^2-abdc+a^2d^2)ef\sqrt{bx^2+a}x}{15b^2d\sqrt{dx^2+c}}-\frac{\sqrt{c}(bc+ad)e^2\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{\sqrt{c}(bc+ad)(8b^2c^2-13abdc+8a^2d^2)f^2\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{105b^3d^{5/2}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{4\sqrt{c}(b^2c^2-abdc+a^2d^2)ef\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15b^2d^{3/2}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{2c^{3/2}e^2\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{2c^{3/2}(2b^2c^2-abdc+2a^2d^2)f^2\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{105b^2d^{5/2}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{2c^{3/2}(bc+ad)ef\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{15bd^{3/2}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}$$

input
```
Int[Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^2,x]
```

output
```
((b*c + a*d)*e^2*x*Sqrt[a + b*x^2])/(3*b*Sqrt[c + d*x^2]) - (4*(b^2*c^2 -
a*b*c*d + a^2*d^2)*e*f*x*Sqrt[a + b*x^2])/(15*b^2*d*Sqrt[c + d*x^2]) + ((b
*c + a*d)*(8*b^2*c^2 - 13*a*b*c*d + 8*a^2*d^2)*f^2*x*Sqrt[a + b*x^2])/(105
*b^3*d^2*Sqrt[c + d*x^2]) + (e^2*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/3 + (2
*(b*c + a*d)*e*f*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(15*b*d) - (2*(2*b^2*c
^2 - a*b*c*d + 2*a^2*d^2)*f^2*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(105*b^2*
d^2) + (2*e*f*x^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/5 + ((b*c + a*d)*f^2*x^
3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(35*b*d) + (f^2*x^5*Sqrt[a + b*x^2]*Sqr
t[c + d*x^2])/7 - (Sqrt[c]*(b*c + a*d)*e^2*Sqrt[a + b*x^2]*EllipticE[ArcTa
n[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*b*Sqrt[d]*Sqrt[(c*(a + b*x^2)
)/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (4*Sqrt[c]*(b^2*c^2 - a*b*c*d + a^2*
d^2)*e*f*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/
(a*d)])/(15*b^2*d^(3/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x
^2]) - (Sqrt[c]*(b*c + a*d)*(8*b^2*c^2 - 13*a*b*c*d + 8*a^2*d^2)*f^2*Sqrt[
a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(105*b
^3*d^(5/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (2*c^(
3/2)*e^2*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/
(a*d)])/(3*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
- (2*c^(3/2)*(b*c + a*d)*e*f*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/
Sqrt[c]], 1 - (b*c)/(a*d)])/(15*b*d^(3/2)*Sqrt[(c*(a + b*x^2))/(a*(c + ...
```

Defintions of rubi rules used

rule 433
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)
^q*(e + f*x^2)^r, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, f, p,
 q, r}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 6.02 (sec) , antiderivative size = 704, normalized size of antiderivative = 1.16

method	result
elliptic	$\frac{\sqrt{(b\,x^2+a)(x^2d+c)}\left(\frac{f^2x^5\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{7}+\frac{\left(ad\,f^2+bc\,f^2+2dbef-\frac{f^2(6ad+6bc)}{7}\right)x^3\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{5bd}+\frac{\left(\frac{2ac\,f^2}{7}+2adef+\right.}{}\right.}{}$
risch	$-\frac{x\left(-15f^2x^4b^2d^2-3ab\,d^2f^2x^2-3b^2cd\,f^2x^2-42b^2d^2ef\,x^2+4a^2d^2f^2-2abcd\,f^2-14ab\,d^2ef+4b^2c^2f^2-14b^2cdef-35b^2d^2e^2\right)\sqrt{b\,x^2}}{105b^2d^2}$
default	Expression too large to display

input

```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^2,x,method=_RETURNVERBOSE)
```

output

```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/7*f^2*x^5*(
b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+1/5*(a*d*f^2+b*c*f^2+2*d*b*e*f-1/7*f^2*
(6*a*d+6*b*c))/b/d*x^3*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+1/3*(2/7*a*c*f^
2+2*a*d*e*f+2*b*c*e*f+b*d*e^2-1/5*(a*d*f^2+b*c*f^2+2*d*b*e*f-1/7*f^2*(6*a*
d+6*b*c))/b/d*(4*a*d+4*b*c))/b/d*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+(a*
c*e^2-1/3*(2/7*a*c*f^2+2*a*d*e*f+2*b*c*e*f+b*d*e^2-1/5*(a*d*f^2+b*c*f^2+2*
d*b*e*f-1/7*f^2*(6*a*d+6*b*c))/b/d*(4*a*d+4*b*c))/b/d*a*c)/(-b/a)^(1/2)*(1
+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*Elli
pticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-(2*a*c*e*f+a*d*e^2+b*c*e^2-
3/5*(a*d*f^2+b*c*f^2+2*d*b*e*f-1/7*f^2*(6*a*d+6*b*c))/b/d*a*c-1/3*(2/7*a*c
*f^2+2*a*d*e*f+2*b*c*e*f+b*d*e^2-1/5*(a*d*f^2+b*c*f^2+2*d*b*e*f-1/7*f^2*(6
*a*d+6*b*c))/b/d*(4*a*d+4*b*c))/b/d*(2*a*d+2*b*c))*c/(-b/a)^(1/2)*(1+b*x^2
/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*(Ellipti
cF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(
a*d+b*c)/c/b)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 593, normalized size of antiderivative = 0.97

$$\int \sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^2\,dx =$$

$$-\frac{(35\,(b^3c^2d^2+ab^2cd^3)e^2-28\,(b^3c^3d-ab^2c^2d^2+a^2bcd^3)ef+(8\,b^3c^4-5\,ab^2c^3d-5\,a^2bc^2d^2+8\,a^3cd^3)\ldots}{\ldots}$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^2,x, algorithm="fricas
")
```

output
```
-1/105*((35*(b^3*c^2*d^2 + a*b^2*c*d^3)*e^2 - 28*(b^3*c^3*d - a*b^2*c^2*d^
2 + a^2*b*c*d^3)*e*f + (8*b^3*c^4 - 5*a*b^2*c^3*d - 5*a^2*b*c^2*d^2 + 8*a^
3*c*d^3)*f^2)*sqrt(b*d)*x*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), a*d/
(b*c)) - (35*(b^3*c^2*d^2 + a*b^2*c*d^3 + 2*a*b^2*d^4)*e^2 - 14*(2*b^3*c^3
*d - 2*a*b^2*c^2*d^2 + a^2*b*d^4 + (2*a^2*b + a*b^2)*c*d^3)*e*f + (8*b^3*c
^4 - 5*a*b^2*c^3*d + 4*a^3*d^4 - (5*a^2*b - 4*a*b^2)*c^2*d^2 + 2*(4*a^3 -
a^2*b)*c*d^3)*f^2)*sqrt(b*d)*x*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)/x),
 a*d/(b*c)) - (15*b^3*d^4*f^2*x^6 + 3*(14*b^3*d^4*e*f + (b^3*c*d^3 + a*b^2
*d^4)*f^2)*x^4 + 35*(b^3*c*d^3 + a*b^2*d^4)*e^2 - 28*(b^3*c^2*d^2 - a*b^2*
c*d^3 + a^2*b*d^4)*e*f + (8*b^3*c^3*d - 5*a*b^2*c^2*d^2 - 5*a^2*b*c*d^3 +
8*a^3*d^4)*f^2 + (35*b^3*d^4*e^2 + 14*(b^3*c*d^3 + a*b^2*d^4)*e*f - 2*(2*b
^3*c^2*d^2 - a*b^2*c*d^3 + 2*a^2*b*d^4)*f^2)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x
^2 + c))/(b^3*d^4*x)
```

Sympy [F]

$$\int \sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^2\,dx=\int \sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^2\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(1/2)*(f*x**2+e)**2,x)
```

output
```
Integral(sqrt(a + b*x**2)*sqrt(c + d*x**2)*(e + f*x**2)**2, x)
```

Maxima [F]

$$\int \sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^2\,dx=\int \sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^2\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^2,x, algorithm="maxima
")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^2, x)
```

Giac [F]

$$\int \sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^2\,dx=\int \sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^2\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^2,x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^2\,dx=\int \sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,{\left(f\,x^2+e\right)}^2\,dx$$

input
```
int((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^2,x)
```

output
```
int((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^2, x)
```

Reduce [F]

$$\int \sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^2\,dx=\text{Too large to display}$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^2,x)
```

output
```
( - 4*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*d**2*f**2*x + 2*sqrt(c + d*x*
*2)*sqrt(a + b*x**2)*a*b*c*d*f**2*x + 14*sqrt(c + d*x**2)*sqrt(a + b*x**2)
*a*b*d**2*e*f*x + 3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b*d**2*f**2*x**3 -
 4*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**2*c**2*f**2*x + 14*sqrt(c + d*x**2
)*sqrt(a + b*x**2)*b**2*c*d*e*f*x + 3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b*
*2*c*d*f**2*x**3 + 35*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**2*d**2*e**2*x +
 42*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**2*d**2*e*f*x**3 + 15*sqrt(c + d*x
**2)*sqrt(a + b*x**2)*b**2*d**2*f**2*x**5 + 8*int((sqrt(c + d*x**2)*sqrt(a
 + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a**3*d**3*f**2
- 5*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**
2 + b*d*x**4),x)*a**2*b*c*d**2*f**2 - 28*int((sqrt(c + d*x**2)*sqrt(a + b*
x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a**2*b*d**3*e*f - 5*
int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 +
b*d*x**4),x)*a*b**2*c**2*d*f**2 + 28*int((sqrt(c + d*x**2)*sqrt(a + b*x**2
)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a*b**2*c*d**2*e*f + 35*i
nt((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b
*d*x**4),x)*a*b**2*d**3*e**2 + 8*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x*
*2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*b**3*c**3*f**2 - 28*int((sqr
t(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4
),x)*b**3*c**2*d*e*f + 35*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/...
```

3.56 $\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^2}{x^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 692

$$
\begin{aligned}
&\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^2}{x^2}\,dx \\
&= -\frac{2(a^2d^2f^2-abdf(5de+cf)-b^2(15d^2e^2+5cdef-c^2f^2))\,x\sqrt{c+dx^2}}{15bd^2\sqrt{a+bx^2}} \\
&\quad+\frac{1}{15}\left(\frac{15be^2}{a}+\frac{15de^2}{c}+55ef+\frac{af^2}{b}+\frac{cf^2}{d}\right)x\sqrt{a+bx^2}\sqrt{c+dx^2} \\
&\quad+\frac{(5be(de+3cf)+af(15de+16cf))x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5ac} \\
&\quad+\frac{f(3be(de+cf)+af(3de+cf))x^5\sqrt{a+bx^2}\sqrt{c+dx^2}}{ace} \\
&\quad+\frac{f^2(3bde+bcf+adf)x^7\sqrt{a+bx^2}\sqrt{c+dx^2}}{ace} \\
&\quad+\frac{bdf^3x^9\sqrt{a+bx^2}\sqrt{c+dx^2}}{ace}-\frac{\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^3}{acex} \\
&\quad+\frac{2\sqrt{a}(a^2d^2f^2-abdf(5de+cf)-b^2(15d^2e^2+5cdef-c^2f^2))\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{15b^{3/2}d^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} \\
&\quad+\frac{\sqrt{a}(15b^2cde^2-a^2cdf^2+ab(15d^2e^2+20cdef-c^2f^2))\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{15b^{3/2}cd\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}
\end{aligned}
$$

output
```
-2/15*(a^2*d^2*f^2-a*b*d*f*(c*f+5*d*e)-b^2*(-c^2*f^2+5*c*d*e*f+15*d^2*e^2)
)*x*(d*x^2+c)^(1/2)/b/d^2/(b*x^2+a)^(1/2)+1/15*(15*b*e^2/a+15*d*e^2/c+55*e
*f+a*f^2/b+c*f^2/d)*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)+1/5*(5*b*e*(3*c*f+d*
e)+a*f*(16*c*f+15*d*e))*x^3*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a/c+f*(3*b*e*(
c*f+d*e)+a*f*(c*f+3*d*e))*x^5*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a/c/e+f^2*(a
*d*f+b*c*f+3*b*d*e)*x^7*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a/c/e+b*d*f^3*x^9*
(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a/c/e-(b*x^2+a)^(3/2)*(d*x^2+c)^(3/2)*(f*x
^2+e)^3/a/c/e/x+2/15*a^(1/2)*(a^2*d^2*f^2-a*b*d*f*(c*f+5*d*e)-b^2*(-c^2*f^
2+5*c*d*e*f+15*d^2*e^2))*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*
x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/b^(3/2)/d^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c
/(b*x^2+a))^(1/2)+1/15*a^(1/2)*(15*b^2*c*d*e^2-a^2*c*d*f^2+a*b*(-c^2*f^2+2
0*c*d*e*f+15*d^2*e^2))*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^
(1/2)),(1-a*d/b/c)^(1/2))/b^(3/2)/c/d/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^
2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.35 (sec) , antiderivative size = 324, normalized size of antiderivative = 0.47

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)^2}{x^2}\,dx$$
$$= \frac{\sqrt{\frac{b}{a}}d(a+bx^2)\,(c+dx^2)\,(bcf^2x^2+adf^2x^2+bd(-15e^2+10efx^2+3f^2x^4))+2ic(a^2d^2f^2-abdf(5de+c}{}$$

input
```
Integrate[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^2)/x^2,x]
```

output
```
(Sqrt[b/a]*d*(a + b*x^2)*(c + d*x^2)*(b*c*f^2*x^2 + a*d*f^2*x^2 + b*d*(-15
*e^2 + 10*e*f*x^2 + 3*f^2*x^4)) + (2*I)*c*(a^2*d^2*f^2 - a*b*d*f*(5*d*e +
c*f) + b^2*(-15*d^2*e^2 - 5*c*d*e*f + c^2*f^2))*x*Sqrt[1 + (b*x^2)/a]*Sqrt
[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*(-(b*c)
 + a*d)*(a*c*d*f^2 + b*(15*d^2*e^2 + 10*c*d*e*f - 2*c^2*f^2))*x*Sqrt[1 + (
b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c
)])/(15*b*Sqrt[b/a]*d^2*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (warning: unable to verify)

Time = 1.11 (sec) , antiderivative size = 670, normalized size of antiderivative = 0.97, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {448, 403, 403, 406, 320, 388, 313, 442, 403, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^2}{x^2}dx$$

$$\Big\downarrow \ 448$$

$$\frac{f\int \sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)dx}{e^2}+e\int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)}{x^2}dx$$

$$\Big\downarrow \ 403$$

$$\frac{f\left(\frac{\int \frac{\sqrt{bx^2+a}\left((5bde+bcf-2adf)x^2+c(5be-af)\right)}{\sqrt{dx^2+c}}dx}{5b}+\frac{fx\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{5b}\right)}{e^2}+ e\int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)}{x^2}dx$$

$$\Big\downarrow \ 403$$

$$\frac{f\left(\frac{\frac{\int \frac{ac(10bde-bcf-adf)-\left(-c(5de-2cf)b^2-ad(5de+2cf)b+2a^2d^2f\right)x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3d}+\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(-2adf+bcf+5bde)}{3d}}{5b}+\frac{fx\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{5b}\right)}{e^2}+ e\int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)}{x^2}dx$$

$$\Big\downarrow \ 406$$

$$\frac{f\left(\frac{\frac{ac(-adf-bcf+10bde)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx-\left(2a^2d^2f-abd(2cf+5de)+b^2(-c)(5de-2cf)\right)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3d}+\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(-2adf+bcf+5bde)}{3d}}{5b}\right.}{e^2}$$

$$e\int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)}{x^2}dx$$

$$\Big\downarrow \; 320$$

$$\frac{f\left(\frac{\frac{\frac{c^{3/2}\sqrt{a+bx^2}(-adf-bcf+10bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\left(2a^2d^2f-abd(2cf+5de)+b^2(-c)(5de-2cf)\right)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3d}+\frac{x\sqrt{a+bx^2}\sqrt{c+d}}{}}{5b}\right)}{e^2}$$

$$e\int\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)}{x^2}dx$$

$$\Big\downarrow \; 388$$

$$\frac{f\left(\frac{\frac{\frac{c^{3/2}\sqrt{a+bx^2}(-adf-bcf+10bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\left(2a^2d^2f-abd(2cf+5de)+b^2(-c)(5de-2cf)\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)}{3d}+\frac{x\sqrt{}}{}}{5b}\right)}{e^2}$$

$$e\int\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)}{x^2}dx$$

$$\Big\downarrow \; 313$$

$$e\int\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)}{x^2}dx+$$

$$\frac{f\left(\frac{\frac{\frac{c^{3/2}\sqrt{a+bx^2}(-adf-bcf+10bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\left(2a^2d^2f-abd(2cf+5de)+b^2(-c)(5de-2cf)\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}}{\sqrt{c}}\right.\right.}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+}{a(c+}}}\right.}{3d}}{5b}\right)}{e^2}$$

$$\Big\downarrow \; 442$$

$$e\left(\frac{\int \frac{\sqrt{bx^2+a}\left(d(3be+af)x^2+2bce+ade+acf\right)}{\sqrt{dx^2+c}}dx}{a}-\frac{e\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{ax}\right)+$$

$$\frac{f\left(\frac{\frac{\frac{c^{3/2}\sqrt{a+bx^2}(-adf-bcf+10bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\left(2a^2d^2f-abd(2cf+5de)+b^2(-c)(5de-2cf)\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\right.}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+\ldots\right)}{a\left(c+\ldots\right)}}}\right.}{3d}}{5b}\right.}{e^2}$$

↓ 403

$$e\left(\frac{\frac{\int \frac{ad\left((6bde+bcf+adf)x^2+3bce+3ade+2acf\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3d}+\frac{1}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}(af+3be)}{a}-\frac{e\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{ax}\right)+$$

$$\frac{f\left(\frac{\frac{\frac{c^{3/2}\sqrt{a+bx^2}(-adf-bcf+10bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\left(2a^2d^2f-abd(2cf+5de)+b^2(-c)(5de-2cf)\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\right.}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+\ldots\right)}{a\left(c+\ldots\right)}}}\right.}{3d}}{5b}\right.}{e^2}$$

↓ 27

$$e\left(\frac{\frac{1}{3}a\int \frac{(6bde+bcf+adf)x^2+3bce+3ade+2acf}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{1}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}(af+3be)}{a}-\frac{e\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{ax}\right)+$$

$$\frac{f\left(\frac{\frac{\frac{c^{3/2}\sqrt{a+bx^2}(-adf-bcf+10bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\left(2a^2d^2f-abd(2cf+5de)+b^2(-c)(5de-2cf)\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\right.}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+\ldots\right)}{a\left(c+\ldots\right)}}}\right.}{3d}}{5b}\right.}{e^2}$$

$\downarrow$ 406

$$\frac{e\left(\frac{\frac{1}{3}a\left((2acf+3ade+3bce)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+(adf+bcf+6bde)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)+\frac{1}{3}x\sqrt{a+bx^2}\sqrt{c+d\cdots}}{a}\right.\quad f\left(\frac{\frac{\frac{c^{3/2}\sqrt{a+bx^2}(-adf-bcf+10bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\left(2a^2d^2f-abd(2cf+5de)+b^2(-c)(5de-2cf)\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}\cdots}{\sqrt{c}}\right)\cdots\right.}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+\cdots\right)}{a\left(c+\cdots\right)}}}\right.}{3d}}{5b}\cdots\right.}{e^2}\cdots$$

$\downarrow$ 320

$$\frac{e\left(\frac{\frac{1}{3}a\left((adf+bcf+6bde)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{\sqrt{c}\sqrt{a+bx^2}(2acf+3ade+3bce)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{1}{3}x\sqrt{a+\cdots}}{a}\right.\quad f\left(\frac{\frac{\frac{c^{3/2}\sqrt{a+bx^2}(-adf-bcf+10bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\left(2a^2d^2f-abd(2cf+5de)+b^2(-c)(5de-2cf)\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}\cdots}{\sqrt{c}}\right)\cdots\right.}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+\cdots\right)}{a\left(c+\cdots\right)}}}\right.}{3d}}{5b}\cdots\right.}{e^2}\cdots$$

$\downarrow$ 388

$$e\left(\frac{\frac{1}{3}a\left((adf+bcf+6bde)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}}dx}{b}\right)+\frac{\sqrt{c}\sqrt{a+bx^2}(2acf+3ade+3bce)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{a}\right.$$

$$\frac{f\left(\frac{\frac{\frac{c^{3/2}\sqrt{a+bx^2}(-adf-bcf+10bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\left(2a^2d^2f-abd(2cf+5de)+b^2(-c)(5de-2cf)\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\ldots\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right.\ldots}{3d}}{5b}\ldots\right)}{e^2}$$

$\downarrow$ 313

$$\frac{f\left(\frac{\frac{\frac{c^{3/2}\sqrt{a+bx^2}(-adf-bcf+10bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\left(2a^2d^2f-abd(2cf+5de)+b^2(-c)(5de-2cf)\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\ldots\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right.\ldots}{3d}}{5b}\ldots\right)}{e^2}$$

$$e\left(\frac{\frac{1}{3}a\left(\frac{\sqrt{c}\sqrt{a+bx^2}(2acf+3ade+3bce)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+(adf+bcf+6bde)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\ldots\right)\ldots\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\ldots}}\right.\right.\ldots}{a}\right.\ldots$$

input
```
Int[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^2)/x^2,x]
```

output
```
e*(-((e*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(a*x)) + (((3*b*e + a*f)*x*Sqrt
[a + b*x^2]*Sqrt[c + d*x^2])/3 + (a*((6*b*d*e + b*c*f + a*d*f)*((x*Sqrt[a
+ b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcTan[
(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a
*(c + d*x^2))]*Sqrt[c + d*x^2])) + (Sqrt[c]*(3*b*c*e + 3*a*d*e + 2*a*c*f)*
Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(
a*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])))/3)/a) +
 (f*((f*x*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(5*b) + (((5*b*d*e + b*c*f -
2*a*d*f)*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(3*d) + (-((2*a^2*d^2*f - b^2*
c*(5*d*e - 2*c*f) - a*b*d*(5*d*e + 2*c*f))*((x*Sqrt[a + b*x^2])/(b*Sqrt[c
+ d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]]
, 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[
c + d*x^2]))) + (c^(3/2)*(10*b*d*e - b*c*f - a*d*f)*Sqrt[a + b*x^2]*Ellipt
icF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b
*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))/(3*d))/(5*b)))/e^2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 442
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^q/(a*g*(m + 1))), x] - Simp[1/(a*g^2*(m + 1))   Int[(g*x)
^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[c*(b*e - a*f)*(m + 1) + e*2
*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*2*(p + q + 1))*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && GtQ[q, 0] && LtQ[m, -1]
&&  !(EqQ[q, 1] && SimplerQ[e + f*x^2, c + d*x^2])
```

rule 448
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[e   Int[(g*x)^m*(a + b*x
^2)^p*(c + d*x^2)^q*(e + f*x^2)^(r - 1), x], x] + Simp[f/e^2   Int[(g*x)^(m
 + 2)*(a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^(r - 1), x], x] /; FreeQ[{a,
b, c, d, e, f, g, m, p, q}, x] && IGtQ[r, 0]
```

Maple [A] (verified)

Time = 5.38 (sec) , antiderivative size = 516, normalized size of antiderivative = 0.75

method	result
elliptic	$\frac{\sqrt{(b x^2+a)(x^2 d+c)}\left(-\frac{e^2\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{x}+\frac{f^2x^3\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{5}+\frac{\left(ad\,f^2+bc\,f^2+2dbef-\frac{f^2(4ad+4bc)}{5}\right)x\sqrt{bd\,x^4+ad\,x^2+}}{3bd}\right.}{}$
risch	$\frac{\sqrt{b\,x^2+a}\,\sqrt{x^2d+c}\,(3bd\,f^2x^4+ad\,f^2x^2+bc\,f^2x^2+10ef\,x^2bd-15bd\,e^2)}{15bdx}-\frac{\left(-\frac{(2a^2d^2f^2-2abcd\,f^2-10ab\,d^2ef+2b^2c^2f^2-10b^2cdef-30b}{}\right.}{}$
default	Expression too large to display

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^2/x^2,x,method=_RETURNVERBOS
E)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-e^2*(b*d*x^4
+a*d*x^2+b*c*x^2+a*c)^(1/2)/x+1/5*f^2*x^3*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1
/2)+1/3*(a*d*f^2+b*c*f^2+2*d*b*e*f-1/5*f^2*(4*a*d+4*b*c))/b/d*x*(b*d*x^4+a
*d*x^2+b*c*x^2+a*c)^(1/2)+(2*a*c*e*f+a*d*e^2+b*c*e^2-1/3*(a*d*f^2+b*c*f^2+
2*d*b*e*f-1/5*f^2*(4*a*d+4*b*c))/b/d*a*c)/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(
1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1
/2),(-1+(a*d+b*c)/c/b)^(1/2))-(2/5*a*c*f^2+2*a*d*e*f+2*b*c*e*f+2*b*d*e^2-1
/3*(a*d*f^2+b*c*f^2+2*d*b*e*f-1/5*f^2*(4*a*d+4*b*c))/b/d*(2*a*d+2*b*c))*c/
(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+
a*c)^(1/2)/d*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE
(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))))
```

Fricas [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)^2}{x^2}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^2}{x^2}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^2/x^2,x, algorithm="fr
icas")
```

output
```
integral((f^2*x^4 + 2*e*f*x^2 + e^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/x^2,
x)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)^2}{x^2}\,dx = \int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)^2}{x^2}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(1/2)*(f*x**2+e)**2/x**2,x)
```

output
```
Integral(sqrt(a + b*x**2)*sqrt(c + d*x**2)*(e + f*x**2)**2/x**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)^2}{x^2}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^2}{x^2}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^2/x^2,x, algorithm="ma
xima")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^2/x^2, x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)^2}{x^2}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^2}{x^2}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^2/x^2,x, algorithm="gi
ac")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^2/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)^2}{x^2}\,dx = \int \frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,(f\,x^2+e)^2}{x^2}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^2)/x^2,x)
```

output
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^2)/x^2, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)^2}{x^2}\,dx = \text{Too large to display}$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^2/x^2,x)
```

output

```
( - 2*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*d**2*f**2 + 2*sqrt(c + d*x**2
)*sqrt(a + b*x**2)*a*b*c*d*f**2 + 10*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b
*d**2*e*f + sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b*d**2*f**2*x**2 - 2*sqrt(
c + d*x**2)*sqrt(a + b*x**2)*b**2*c**2*f**2 + 10*sqrt(c + d*x**2)*sqrt(a +
 b*x**2)*b**2*c*d*e*f + sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**2*c*d*f**2*x*
*2 + 15*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**2*d**2*e**2 + 10*sqrt(c + d*x
**2)*sqrt(a + b*x**2)*b**2*d**2*e*f*x**2 + 3*sqrt(c + d*x**2)*sqrt(a + b*x
**2)*b**2*d**2*f**2*x**4 - 2*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c*
x**2 + a*d*x**4 + b*c*x**4 + b*d*x**6),x)*a**3*c*d**2*f**2*x + 2*int((sqrt
(c + d*x**2)*sqrt(a + b*x**2))/(a*c*x**2 + a*d*x**4 + b*c*x**4 + b*d*x**6)
,x)*a**2*b*c**2*d*f**2*x + 10*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c
*x**2 + a*d*x**4 + b*c*x**4 + b*d*x**6),x)*a**2*b*c*d**2*e*f*x - 2*int((sq
rt(c + d*x**2)*sqrt(a + b*x**2))/(a*c*x**2 + a*d*x**4 + b*c*x**4 + b*d*x**
6),x)*a*b**2*c**3*f**2*x + 10*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c
*x**2 + a*d*x**4 + b*c*x**4 + b*d*x**6),x)*a*b**2*c**2*d*e*f*x + 30*int((s
qrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c*x**2 + a*d*x**4 + b*c*x**4 + b*d*x*
*6),x)*a*b**2*c*d**2*e**2*x - int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c
 + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a**2*b*c*d**2*f**2*x - int((sqrt(c +
 d*x**2)*sqrt(a + b*x**2))/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a*b**
2*c**2*d*f**2*x + 20*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c + a*d...
```

3.57 $\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)^2}{x^4}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 694

$$\begin{aligned}
&\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)^2}{x^4}\,dx \\
&= \frac{\left(\frac{b^2e^2}{a}+af^2+b\left(\frac{de^2}{c}+12ef+\frac{cf^2}{d}\right)\right)x\sqrt{c+dx^2}}{3\sqrt{a+bx^2}} \\
&+\frac{(be(de+6cf)+af(6de+13cf))x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3ac} \\
&+\frac{2f(3be(de+2cf)+af(6de+5cf))x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{3ace} \\
&+\frac{f^2(af(10de+3cf)+2be(6de+5cf))x^5\sqrt{a+bx^2}\sqrt{c+dx^2}}{3ace^2} \\
&+\frac{f^3(10bde+3bcf+3adf)x^7\sqrt{a+bx^2}\sqrt{c+dx^2}}{3ace^2}+\frac{bdf^4x^9\sqrt{a+bx^2}\sqrt{c+dx^2}}{ace^2} \\
&-\frac{(a+bx^2)^{3/2}(c+dx^2)^{3/2}(e+fx^2)^3}{3acex^3}-\frac{f(a+bx^2)^{3/2}(c+dx^2)^{3/2}(e+fx^2)^3}{ace^2x} \\
&-\frac{(b^2cde^2+a^2cdf^2+ab(d^2e^2+12cdef+c^2f^2))\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{3\sqrt{a}\sqrt{b}cd\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} \\
&+\frac{2\sqrt{a}(af(3de+cf)+be(de+3cf))\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{3\sqrt{b}c\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}
\end{aligned}$$

output
```
1/3*(b^2*e^2/a+a*f^2+b*(d*e^2/c+12*e*f+c*f^2/d))*x*(d*x^2+c)^(1/2)/(b*x^2+
a)^(1/2)+1/3*(b*e*(6*c*f+d*e)+a*f*(13*c*f+6*d*e))*x*(b*x^2+a)^(1/2)*(d*x^2
+c)^(1/2)/a/c+2/3*f*(3*b*e*(2*c*f+d*e)+a*f*(5*c*f+6*d*e))*x^3*(b*x^2+a)^(1
/2)*(d*x^2+c)^(1/2)/a/c/e+1/3*f^2*(a*f*(3*c*f+10*d*e)+2*b*e*(5*c*f+6*d*e))
*x^5*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a/c/e^2+1/3*f^3*(3*a*d*f+3*b*c*f+10*b
*d*e)*x^7*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a/c/e^2+b*d*f^4*x^9*(b*x^2+a)^(1
/2)*(d*x^2+c)^(1/2)/a/c/e^2-1/3*(b*x^2+a)^(3/2)*(d*x^2+c)^(3/2)*(f*x^2+e)^
3/a/c/e/x^3-f*(b*x^2+a)^(3/2)*(d*x^2+c)^(3/2)*(f*x^2+e)^3/a/c/e^2/x-1/3*(b
^2*c*d*e^2+a^2*c*d*f^2+a*b*(c^2*f^2+12*c*d*e*f+d^2*e^2))*(d*x^2+c)^(1/2)*E
llipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/a^(1/2)/b^
(1/2)/c/d/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)+2/3*a^(1/2)*(a*f
*(c*f+3*d*e)+b*e*(3*c*f+d*e))*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/
2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/b^(1/2)/c/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/
(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.40 (sec) , antiderivative size = 308, normalized size of antiderivative = 0.44

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)^2}{x^4}\,dx$$

$$= \frac{\sqrt{\frac{b}{a}}\left(-\sqrt{\frac{b}{a}}d(a+bx^2)(c+dx^2)(bce^2x^2+ade^2x^2+ac(e^2+6efx^2-f^2x^4))-ic(b^2cde^2+a^2cdf^2+ab(a\right.}{}$$

input
```
Integrate[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^2)/x^4,x]
```

output
```
(Sqrt[b/a]*(-(Sqrt[b/a]*d*(a + b*x^2)*(c + d*x^2)*(b*c*e^2*x^2 + a*d*e^2*x
^2 + a*c*(e^2 + 6*e*f*x^2 - f^2*x^4))) - I*c*(b^2*c*d*e^2 + a^2*c*d*f^2 +
a*b*(d^2*e^2 + 12*c*d*e*f + c^2*f^2))*x^3*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*
x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*c*(-(b*c) + a*d
)*(b*d*e^2 + a*f*(6*d*e + c*f))*x^3*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c
]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(3*b*c*d*x^3*Sqrt[a + b
*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (warning: unable to verify)

Time = 1.12 (sec) , antiderivative size = 639, normalized size of antiderivative = 0.92, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.371$, Rules used = {448, 442, 403, 27, 406, 320, 388, 313, 442, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^2}{x^4}dx$$

$$\Big\downarrow \text{448}$$

$$\frac{f\int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)}{x^2}dx}{e^2}+e\int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)}{x^4}dx$$

$$\Big\downarrow \text{442}$$

$$\frac{f\left(\frac{\int \frac{\sqrt{bx^2+a}\left(d(3be+af)x^2+2bce+ade+acf\right)}{\sqrt{dx^2+c}}dx}{a}-\frac{e\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{ax}\right)}{e^2}+e\left(\frac{\int \frac{\sqrt{bx^2+a}(d(be+3af)x^2+a(de+3cf))}{x^2\sqrt{dx^2+c}}dx}{3a}-\frac{e\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{3ax^3}\right)$$

$$\Big\downarrow \text{403}$$

$$\frac{f\left(\frac{\frac{\int \frac{ad\left((6bde+bcf+adf)x^2+3bce+3ade+2acf\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3d}+\frac{1}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}(af+3be)}{a}-\frac{e\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{ax}\right)}{e^2}+e\left(\frac{\int \frac{\sqrt{bx^2+a}(d(be+3af)x^2+a(de+3cf))}{x^2\sqrt{dx^2+c}}dx}{3a}-\frac{e\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{3ax^3}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{f\left(\frac{\frac{1}{3}a\int \frac{(6bde+bcf+adf)x^2+3bce+3ade+2acf}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{1}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}(af+3be)}{a}-\frac{e\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{ax}\right)}{e^2}+e\left(\frac{\int \frac{\sqrt{bx^2+a}(d(be+3af)x^2+a(de+3cf))}{x^2\sqrt{dx^2+c}}dx}{3a}-\frac{e\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{3ax^3}\right)$$

$\downarrow$ 406

$$f\left(\frac{\frac{1}{3}a\left((2acf+3ade+3bce)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+(adf+bcf+6bde)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)+\frac{1}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}(af+3be)}{a}-\frac{e\left(a+bx^2\right)^{3/2}\sqrt{c+\cdots}}{ax}\right.$$
$$\overline{e^2}$$
$$e\left(\frac{\int \frac{\sqrt{bx^2+a}\left(d(be+3af)x^2+a(de+3cf)\right)}{x^2\sqrt{dx^2+c}}dx}{3a}-\frac{e\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{3ax^3}\right)$$

$\downarrow$ 320

$$f\left(\frac{\frac{1}{3}a\left((adf+bcf+6bde)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{\sqrt{c}\sqrt{a+bx^2}(2acf+3ade+3bce)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{1}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}(af+3be)}{a}-\cdots\right.$$
$$\overline{e^2}$$
$$e\left(\frac{\int \frac{\sqrt{bx^2+a}\left(d(be+3af)x^2+a(de+3cf)\right)}{x^2\sqrt{dx^2+c}}dx}{3a}-\frac{e\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{3ax^3}\right)$$

$\downarrow$ 388

$$f\left(\frac{\frac{1}{3}a\left((adf+bcf+6bde)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{\sqrt{c}\sqrt{a+bx^2}(2acf+3ade+3bce)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{1}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}\cdots}{a}\right.$$
$$\overline{e^2}$$
$$e\left(\frac{\int \frac{\sqrt{bx^2+a}\left(d(be+3af)x^2+a(de+3cf)\right)}{x^2\sqrt{dx^2+c}}dx}{3a}-\frac{e\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{3ax^3}\right)$$

$\downarrow$ 313

$$e\left(\frac{\int \frac{\sqrt{bx^2+a}\left(d(be+3af)x^2+a(de+3cf)\right)}{x^2\sqrt{dx^2+c}}dx}{3a}-\frac{e\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{3ax^3}\right)+$$

$$\frac{f\left(\frac{\frac{1}{3}a\left(\frac{\sqrt{c}\sqrt{a+bx^2}(2acf+3ade+3bce)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+(adf+bcf+6bde)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)\right)+\frac{1}{3}x\cdots}{a}\right.}{e^2}$$

$\downarrow$ 442

$$e\left(\frac{\frac{\int \frac{bd(bce+ade+6acf)x^2+ac(2bde+3bcf+3adf)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{c}-\frac{a\sqrt{a+bx^2}\sqrt{c+dx^2}(3cf+de)}{cx}}{3a}-\frac{e\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{3ax^3}\right)+$$

$$\frac{f\left(\frac{\frac{1}{3}a\left(\frac{\sqrt{c}\sqrt{a+bx^2}(2acf+3ade+3bce)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+(adf+bcf+6bde)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)\right)+\frac{1}{3}x\cdots}{a}\right.}{e^2}$$

$\downarrow$ 406

$$e\left(\frac{\frac{ac(3adf+3bcf+2bde)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+bd(6acf+ade+bce)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{c}-\frac{a\sqrt{a+bx^2}\sqrt{c+dx^2}(3cf+de)}{cx}}{3a}-\frac{e\left(a+bx^2\right)^{3/2}}{3ax^3}\cdots\right.$$

$$\frac{f\left(\frac{\frac{1}{3}a\left(\frac{\sqrt{c}\sqrt{a+bx^2}(2acf+3ade+3bce)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+(adf+bcf+6bde)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)\right)+\frac{1}{3}x\cdots}{a}\right.}{e^2}$$

$\downarrow$ 320

$$\frac{e\left(\frac{\frac{bd(6acf+ade+bce)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}(3adf+3bcf+2bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{c}-\frac{a\sqrt{a+bx^2}\sqrt{c+dx^2}(3cf+de)}{cx}}{3a}-e\cdots\right)}{}$$

$$\frac{f\left(\frac{\frac{1}{3}a\left(\frac{\sqrt{c}\sqrt{a+bx^2}(2acf+3ade+3bce)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+(adf+bcf+6bde)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)\right)+\frac{1}{3}x\cdots}{a}\cdots\right)}{e^2}$$

$\downarrow$ 388

$$\frac{e\left(\frac{\frac{bd(6acf+ade+bce)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(3adf+3bcf+2bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{c}-\frac{a\sqrt{a+bx^2}\sqrt{c+dx^2}(3c\cdots}{cx}}{3a}\cdots\right)}{}$$

$$\frac{f\left(\frac{\frac{1}{3}a\left(\frac{\sqrt{c}\sqrt{a+bx^2}(2acf+3ade+3bce)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+(adf+bcf+6bde)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)\right)+\frac{1}{3}x\cdots}{a}\cdots\right)}{e^2}$$

$\downarrow$ 313

$$e\left(\frac{\frac{\frac{c^{3/2}\sqrt{a+bx^2}(3adf+3bcf+2bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+bd(6acf+ade+bce)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{c}-\frac{a\sqrt{a}}{}}{3a}\right.$$

$$\frac{f\left(\frac{\frac{1}{3}a\left(\frac{\sqrt{c}\sqrt{a+bx^2}(2acf+3ade+3bce)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+(adf+bcf+6bde)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)\right)+\frac{1}{3}x}{a}\right.}{e^2}$$

input
```
Int[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^2)/x^4,x]
```

output
```
(f*(-((e*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(a*x)) + (((3*b*e + a*f)*x*Sqr
t[a + b*x^2]*Sqrt[c + d*x^2])/3 + (a*((6*b*d*e + b*c*f + a*d*f)*((x*Sqrt[a
 + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcTan
[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(
a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (Sqrt[c]*(3*b*c*e + 3*a*d*e + 2*a*c*f)
*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/
(a*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])))/3)/a))
/e^2 + e*(-1/3*(e*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(a*x^3) + (-((a*(d*e
+ 3*c*f)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(c*x)) + (b*d*(b*c*e + a*d*e + 6
*a*c*f)*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2
]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt
[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (c^(3/2)*(2*b*d*e +
3*b*c*f + 3*a*d*f)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]],
1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c +
d*x^2]))/c)/(3*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 442
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^q/(a*g*(m + 1))), x] - Simp[1/(a*g^2*(m + 1))   Int[(g*x)
^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[c*(b*e - a*f)*(m + 1) + e*2
*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*2*(p + q + 1))*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && GtQ[q, 0] && LtQ[m, -1]
&&  !(EqQ[q, 1] && SimplerQ[e + f*x^2, c + d*x^2])
```

rule 448
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[e   Int[(g*x)^m*(a + b*x
^2)^p*(c + d*x^2)^q*(e + f*x^2)^(r - 1), x], x] + Simp[f/e^2   Int[(g*x)^(m
 + 2)*(a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^(r - 1), x], x] /; FreeQ[{a,
b, c, d, e, f, g, m, p, q}, x] && IGtQ[r, 0]
```

Maple [A] (verified)

Time = 5.92 (sec) , antiderivative size = 444, normalized size of antiderivative = 0.64

method	result
elliptic	$\frac{\sqrt{(b x^2+a)(x^2 d+c)}\left(-\frac{e^2\sqrt{bd x^4+ad x^2+x^2bc+ac}}{3x^3}-\frac{e(6acf+ade+bce)\sqrt{bd x^4+ad x^2+x^2bc+ac}}{3acx}+\frac{f^2x\sqrt{bd x^4+ad x^2+x^2bc+ac}}{3}+\frac{\left(\frac{2}{3}ac f^2+2\right.}{}\right.}{}$
risch	$-\frac{\sqrt{b x^2+a}\sqrt{x^2d+c}\,(-ac f^2x^4+6acef x^2+ad e^2x^2+bc e^2x^2+ac e^2)}{3x^3ac}+\frac{\left(-\frac{\left(a^2cd f^2+ab c^2f^2+12abcdef+ab d^2e^2+b^2d e^2c\right)c\sqrt{1+\frac{b x^2}{a}}}{\sqrt{-}}\right.}{}$
default	Expression too large to display

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^2/x^4,x,method=_RETURNVERBOS
E)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-1/3*e^2*(b*d
*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/x^3-1/3*e*(6*a*c*f+a*d*e+b*c*e)/a/c*(b*d*x
^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/x+1/3*f^2*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1
/2)+(2/3*a*c*f^2+2*a*d*e*f+2*b*c*e*f+2/3*b*d*e^2)/(-b/a)^(1/2)*(1+b*x^2/a)
^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(
-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-(a*d*f^2+b*c*f^2+2*d*b*e*f+1/3*b*d*e
*(6*a*c*f+a*d*e+b*c*e)/a/c-1/3*f^2*(2*a*d+2*b*c))*c/(-b/a)^(1/2)*(1+b*x^2/
a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*(Elliptic
F(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a
*d+b*c)/c/b)^(1/2))))
```

Fricas [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)^2}{x^4}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^2}{x^4}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^2/x^4,x, algorithm="fr
icas")
```

output
```
integral((f^2*x^4 + 2*e*f*x^2 + e^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/x^4,
x)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)^2}{x^4}\,dx = \int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)^2}{x^4}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(1/2)*(f*x**2+e)**2/x**4,x)
```

output
```
Integral(sqrt(a + b*x**2)*sqrt(c + d*x**2)*(e + f*x**2)**2/x**4, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)^2}{x^4}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^2}{x^4}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^2/x^4,x, algorithm="ma
xima")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^2/x^4, x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)^2}{x^4}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^2}{x^4}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^2/x^4,x, algorithm="gi
ac")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^2/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)^2}{x^4}\,dx = \int \frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,{\left(f\,x^2+e\right)}^2}{x^4}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^2)/x^4,x)
```

output
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^2)/x^4, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)^2}{x^4}\,dx = \text{too large to display}$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^2/x^4,x)
```

output
```
( - sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*c*d*f**2 + 2*sqrt(c + d*x**2)*s
qrt(a + b*x**2)*a**2*d**2*f**2*x**2 - sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*
b*c**2*f**2 - 12*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b*c*d*e*f + 4*sqrt(c
+ d*x**2)*sqrt(a + b*x**2)*a*b*c*d*f**2*x**2 - 3*sqrt(c + d*x**2)*sqrt(a +
 b*x**2)*a*b*d**2*e**2 + 12*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b*d**2*e*f
*x**2 + 2*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b*d**2*f**2*x**4 + 2*sqrt(c
+ d*x**2)*sqrt(a + b*x**2)*b**2*c**2*f**2*x**2 - 3*sqrt(c + d*x**2)*sqrt(a
 + b*x**2)*b**2*c*d*e**2 + 12*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**2*c*d*e
*f*x**2 + 2*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**2*c*d*f**2*x**4 - 3*int((
sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a**2*c*d*x**4 + a**2*d**2*x**6 + a*b*c
**2*x**4 + 2*a*b*c*d*x**6 + a*b*d**2*x**8 + b**2*c**2*x**6 + b**2*c*d*x**8
),x)*a**4*c**2*d**2*f**2*x**3 - 6*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/
(a**2*c*d*x**4 + a**2*d**2*x**6 + a*b*c**2*x**4 + 2*a*b*c*d*x**6 + a*b*d**
2*x**8 + b**2*c**2*x**6 + b**2*c*d*x**8),x)*a**3*b*c**3*d*f**2*x**3 - 36*i
nt((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a**2*c*d*x**4 + a**2*d**2*x**6 + a
*b*c**2*x**4 + 2*a*b*c*d*x**6 + a*b*d**2*x**8 + b**2*c**2*x**6 + b**2*c*d*
x**8),x)*a**3*b*c**2*d**2*e*f*x**3 - 3*int((sqrt(c + d*x**2)*sqrt(a + b*x*
*2))/(a**2*c*d*x**4 + a**2*d**2*x**6 + a*b*c**2*x**4 + 2*a*b*c*d*x**6 + a*
b*d**2*x**8 + b**2*c**2*x**6 + b**2*c*d*x**8),x)*a**3*b*c*d**3*e**2*x**3 -
 3*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a**2*c*d*x**4 + a**2*d**2*x...
```

3.58 $\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^2}{x^6}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 1067

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^2}{x^6}\,dx = \text{Too large to display}$$

output
```
-2/15*b*(b^2*c^2*e^2-a*b*c*e*(5*c*f+d*e)+a^2*(-15*c^2*f^2-5*c*d*e*f+d^2*e^
2))*x*(d*x^2+c)^(1/2)/a^2/c^2/(b*x^2+a)^(1/2)+1/15*(b^2*c*d*e^3+3*a^2*c*f^
2*(5*c*f+9*d*e)+a*b*e*(27*c^2*f^2+19*c*d*e*f+d^2*e^2))*x*(b*x^2+a)^(1/2)*(
d*x^2+c)^(1/2)/a^2/c^2/e+1/15*(3*b^2*e^2*(4*c^2*f^2+3*c*d*e*f+d^2*e^2)+a^2
*f^2*(10*c^2*f^2+31*c*d*e*f+12*d^2*e^2)+a*b*e*f*(31*c^2*f^2+48*c*d*e*f+9*d
^2*e^2))*x^3*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a^2/c^2/e^2+1/15*f*(a^2*f^2*(
3*c^2*f^2+16*c*d*e*f+16*d^2*e^2)+b^2*e^2*(16*c^2*f^2+21*c*d*e*f+9*d^2*e^2)
+a*b*e*f*(16*c^2*f^2+50*c*d*e*f+21*d^2*e^2))*x^5*(b*x^2+a)^(1/2)*(d*x^2+c)
^(1/2)/a^2/c^2/e^3+1/15*f^2*(3*a^2*d*f^2*(c*f+2*d*e)+a*b*f*(3*c^2*f^2+22*c
*d*e*f+19*d^2*e^2)+b^2*e*(6*c^2*f^2+19*c*d*e*f+9*d^2*e^2))*x^7*(b*x^2+a)^(
1/2)*(d*x^2+c)^(1/2)/a^2/c^2/e^3+1/5*b*d*f^3*(a*f*(c*f+2*d*e)+b*e*(2*c*f+d
*e))*x^9*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a^2/c^2/e^3-1/5*(b*x^2+a)^(3/2)*(
d*x^2+c)^(3/2)*(f*x^2+e)^3/a/c/e/x^5+1/15*(-a*c*f+2*a*d*e+2*b*c*e)*(b*x^2+
a)^(3/2)*(d*x^2+c)^(3/2)*(f*x^2+e)^3/a^2/c^2/e^2/x^3-1/5*(a*f*(c*f+2*d*e)+
b*e*(2*c*f+d*e))*(b*x^2+a)^(3/2)*(d*x^2+c)^(3/2)*(f*x^2+e)^3/a^2/c^2/e^3/x
+2/15*b^(1/2)*(b^2*c^2*e^2-a*b*c*e*(5*c*f+d*e)+a^2*(-15*c^2*f^2-5*c*d*e*f+
d^2*e^2))*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1
-a*d/b/c)^(1/2))/a^(3/2)/c^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/
2)-1/15*(b^2*c*d*e^2-15*a^2*c*d*f^2+a*b*(-15*c^2*f^2-20*c*d*e*f+d^2*e^2))*
(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(...
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.63 (sec) , antiderivative size = 383, normalized size of antiderivative = 0.36

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)^2}{x^6}\,dx$$

$$= \frac{-\sqrt{\frac{b}{a}}(a+bx^2)(c+dx^2)(-2b^2c^2e^2x^4+abcex^2(2dex^2+c(e+10fx^2))+a^2(-2d^2e^2x^4+cdex^2(e+10f\ldots}{}$$

input
```
Integrate[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^2)/x^6,x]
```

output
```
(-(Sqrt[b/a]*(a + b*x^2)*(c + d*x^2)*(-2*b^2*c^2*e^2*x^4 + a*b*c*e*x^2*(2*
d*e*x^2 + c*(e + 10*f*x^2)) + a^2*(-2*d^2*e^2*x^4 + c*d*e*x^2*(e + 10*f*x^
2) + c^2*(3*e^2 + 10*e*f*x^2 + 15*f^2*x^4)))) - (2*I)*b*c*(-(b^2*c^2*e^2)
+ a*b*c*e*(d*e + 5*c*f) + a^2*(-(d^2*e^2) + 5*c*d*e*f + 15*c^2*f^2))*x^5*S
qrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (
a*d)/(b*c)] - I*c*(-(b*c) + a*d)*(-2*b^2*c*e^2 + 15*a^2*c*f^2 + a*b*e*(d*e
 + 10*c*f))*x^5*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSin
h[Sqrt[b/a]*x], (a*d)/(b*c)])/(15*a^2*Sqrt[b/a]*c^2*x^5*Sqrt[a + b*x^2]*Sq
rt[c + d*x^2])
```

Rubi [A] (warning: unable to verify)

Time = 1.37 (sec) , antiderivative size = 751, normalized size of antiderivative = 0.70, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {448, 442, 25, 442, 406, 320, 388, 313, 445, 25, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^2}{x^6}\,dx$$

$$\Big\downarrow \text{448}$$

$$\frac{f\int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)}{x^4}dx}{e^2}+e\int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)}{x^6}dx$$

$$\Big\downarrow \text{442}$$

$$\frac{f\left(\frac{\int \frac{\sqrt{bx^2+a}\left(d(be+3af)x^2+a(de+3cf)\right)}{x^2\sqrt{dx^2+c}}dx}{3a}-\frac{e\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{3ax^3}\right)}{e^2}+e\left(\frac{\int -\frac{\sqrt{bx^2+a}\left(d(be-5af)x^2+2bce-ade-5acf\right)}{x^4\sqrt{dx^2+c}}dx}{5a}-\frac{e\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{5ax^5}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{f\left(\frac{\int \frac{\sqrt{bx^2+a}\left(d(be+3af)x^2+a(de+3cf)\right)}{x^2\sqrt{dx^2+c}}dx}{3a}-\frac{e(a+bx^2)^{3/2}\sqrt{c+dx^2}}{3ax^3}\right)}{e^2}+$$
$$e\left(-\frac{\int \frac{\sqrt{bx^2+a}(d(be-5af)x^2+2bce-ade-5acf)}{x^4\sqrt{dx^2+c}}dx}{5a}-\frac{e(a+bx^2)^{3/2}\sqrt{c+dx^2}}{5ax^5}\right)$$

$\downarrow$ 442

$$e\left(-\frac{\frac{\int \frac{d(2de-5cf)a^2-bc(2de+5cf)a+bd(bce+ade-10acf)x^2+2b^2c^2e}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(-5acf-ade+2bce)}{3cx^3}}{5a}-\frac{e(a+bx^2)^{3/2}\sqrt{c+dx^2}}{5ax^5}\right.$$
$$\frac{f\left(\frac{\frac{\int \frac{bd(bce+ade+6acf)x^2+ac(2bde+3bcf+3adf)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{c}-\frac{a\sqrt{a+bx^2}\sqrt{c+dx^2}(3cf+de)}{cx}}{3a}-\frac{e(a+bx^2)^{3/2}\sqrt{c+dx^2}}{3ax^3}\right)}{e^2}$$

$\downarrow$ 406

$$e\left(-\frac{\frac{\int \frac{d(2de-5cf)a^2-bc(2de+5cf)a+bd(bce+ade-10acf)x^2+2b^2c^2e}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(-5acf-ade+2bce)}{3cx^3}}{5a}-\frac{e(a+bx^2)^{3/2}\sqrt{c+dx^2}}{5ax^5}\right.$$
$$\frac{f\left(\frac{\frac{ac(3adf+3bcf+2bde)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+bd(6acf+ade+bce)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{c}-\frac{a\sqrt{a+bx^2}\sqrt{c+dx^2}(3cf+de)}{cx}}{3a}-\frac{e(a+bx^2)^{3/2}\sqrt{c+dx^2}}{3ax^3}\right)}{e^2}$$

$\downarrow$ 320

$$e\left(-\frac{\frac{\int \frac{d(2de-5cf)a^2-bc(2de+5cf)a+bd(bce+ade-10acf)x^2+2b^2c^2e}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(-5acf-ade+2bce)}{3cx^3}}{5a}-\frac{e(a+bx^2)^{3/2}\sqrt{c+dx^2}}{5ax^5}\right.$$
$$\frac{f\left(\frac{\frac{bd(6acf+ade+bce)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}(3adf+3bcf+2bde)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}}{c}-\frac{a\sqrt{a+bx^2}\sqrt{c+dx^2}(3cf+de)}{cx}}{3a}-\frac{e(a+bx^2 \ldots}{\ldots}\right.}{e^2}$$

$\downarrow$ 388

$$\frac{e\left(-\frac{\frac{\int \frac{d(2de-5cf)a^2-bc(2de+5cf)a+bd(bce+ade-10acf)x^2+2b^2c^2e}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(-5acf-ade+2bce)}{3cx^3}}{5a}-\frac{e\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{5ax^5}\right)}{e^2} \quad f\left(\frac{\frac{bd(6acf+ade+bce)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(3adf+3bcf+2bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{c}-\frac{a\sqrt{a+bx^2}\sqrt{c+dx^2}(3cf+de)}{cx}}{3a}\right.$$

$$\Big\downarrow \; 313$$

$$\frac{e\left(-\frac{\frac{\int \frac{d(2de-5cf)a^2-bc(2de+5cf)a+bd(bce+ade-10acf)x^2+2b^2c^2e}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(-5acf-ade+2bce)}{3cx^3}}{5a}-\frac{e\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{5ax^5}\right)}{e^2} \quad f\left(\frac{\frac{\frac{c^{3/2}\sqrt{a+bx^2}(3adf+3bcf+2bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+bd(6acf+ade+bce)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{c}-\frac{a\sqrt{a+bx^2}\cdots}{}}{3a}\right.$$

$$\Big\downarrow \; 445$$

$$\frac{e\left(-\frac{\frac{-\frac{\int -\frac{bd\left(\left(d(2de-5cf)a^2-bc(2de+5cf)a+2b^2c^2e\right)x^2+ac(bce+ade-10acf)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{2b^2ce}{a}+\frac{2ad^2e}{c}-5adf-5bcf-2bde\right)}{x}}{3c}}{5a}-\frac{\sqrt{a+bx^2}\sqrt{\cdots}}{\cdots}\right)}{} \qquad f\left(\frac{\frac{\frac{c^{3/2}\sqrt{a+bx^2}(3adf+3bcf+2bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+bd(6acf+ade+bce)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{c}}{3a}-\frac{a\sqrt{a+bx^2}\cdots}{\cdots}\right) \Bigg/ e^2$$

$\downarrow$ 25

$$\frac{e\left(-\frac{\frac{\frac{\int \frac{bd\left(\left(d(2de-5cf)a^2-bc(2de+5cf)a+2b^2c^2e\right)x^2+ac(bce+ade-10acf)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{2b^2ce}{a}+\frac{2ad^2e}{c}-5adf-5bcf-2bde\right)}{x}}{3c}}{5a}-\frac{\sqrt{a+bx^2}\sqrt{c+\cdots}}{\cdots}\right)}{} \qquad f\left(\frac{\frac{\frac{c^{3/2}\sqrt{a+bx^2}(3adf+3bcf+2bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+bd(6acf+ade+bce)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{c}}{3a}-\frac{a\sqrt{a+bx^2}\cdots}{\cdots}\right) \Bigg/ e^2$$

$\downarrow$ 27

$$\frac{e\left(-\frac{\frac{bd\int \frac{\left(d(2de-5cf)a^2-bc(2de+5cf)a+2b^2c^2e\right)x^2+ac(bce+ade-10acf)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{2b^2ce}{a}+\frac{2ad^2e}{c}-5adf-5bcf-2bde\right)}{x}}{3c}}{5a}-\frac{\sqrt{a+bx^2}\sqrt{c+dx}\cdots}{\cdots}\right)+f\left(\frac{\frac{\frac{c^{3/2}\sqrt{a+bx^2}(3adf+3bcf+2bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+bd(6acf+ade+bce)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{c}-\frac{a\sqrt{a+bx^2}\cdots}{\cdots}}{3a}\cdots\right)}{e^2}$$

$\Big\downarrow$ 406

$$\frac{e\left(-\frac{\frac{bd\left(\left(a^2d(2de-5cf)-abc(5cf+2de)+2b^2c^2e\right)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+ac(-10acf+ade+bce)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{2b^2ce}{a}+\frac{2ad^2e}{c}-\cdots\right)}{x}}{3c}}{5a}\cdots\right)+f\left(\frac{\frac{\frac{c^{3/2}\sqrt{a+bx^2}(3adf+3bcf+2bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+bd(6acf+ade+bce)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{c}-\frac{a\sqrt{a+bx^2}\cdots}{\cdots}}{3a}\cdots\right)}{e^2}$$

$\Big\downarrow$ 320

$$e\left(-\frac{\frac{bd\left(\left(a^2d(2de-5cf)-abc(5cf+2de)+2b^2c^2e\right)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}(-10acf+ade+bce)\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c\cdots}}{\cdots}}{3c}}{5a}\cdots\right.$$

$$f\left(\frac{\frac{\frac{c^{3/2}\sqrt{a+bx^2}(3adf+3bcf+2bde)\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+bd(6acf+ade+bce)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{c}-\frac{a\sqrt{a+bx^2}\cdots}{\cdots}}{3a}\cdots\right.$$

$$\overline{\qquad\qquad e^2\qquad\qquad}$$

$\downarrow$ 388

$$e\left(-\frac{\frac{bd\left(\left(a^2d(2de-5cf)-abc(5cf+2de)+2b^2c^2e\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(-10acf+ade+bce)\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{ac}\cdots}{3c}}{5a}\cdots\right.$$

$$f\left(\frac{\frac{\frac{c^{3/2}\sqrt{a+bx^2}(3adf+3bcf+2bde)\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+bd(6acf+ade+bce)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{c}-\frac{a\sqrt{a+bx^2}\cdots}{\cdots}}{3a}\cdots\right.$$

$$\overline{\qquad\qquad e^2\qquad\qquad}$$

$\downarrow$ 313

$$e\left(-\frac{\frac{\frac{bd\left(\left(a^2d(2de-5cf)-abc(5cf+2de)+2b^2c^2e\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(-10acf+ade+bce)\,\mathrm{EllipticF}\left(\arctan\left(\right.\right.}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{ac}}{3c}}{5a}\right.$$

$$\frac{f\left(\frac{\frac{\frac{c^{3/2}\sqrt{a+bx^2}(3adf+3bcf+2bde)\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+bd(6acf+ade+bce)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{c}-\frac{a\sqrt{a+bx^2}}{}}{3a}\right.}{e^2}$$

input
```
Int[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^2)/x^6,x]
```

output
```
(f*(-1/3*(e*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(a*x^3) + (-((a*(d*e + 3*c*
f)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(c*x)) + (b*d*(b*c*e + a*d*e + 6*a*c*f
)*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*Elli
pticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a
 + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (c^(3/2)*(2*b*d*e + 3*b*c*
f + 3*a*d*f)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b
*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]
))/c)/(3*a)))/e^2 + e*(-1/5*(e*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(a*x^5)
- (-1/3*((2*b*c*e - a*d*e - 5*a*c*f)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(c*x
^3) + (-((((2*b^2*c*e)/a - 2*b*d*e + (2*a*d^2*e)/c - 5*b*c*f - 5*a*d*f)*Sq
rt[a + b*x^2]*Sqrt[c + d*x^2])/x) + (b*d*((2*b^2*c^2*e + a^2*d*(2*d*e - 5*
c*f) - a*b*c*(2*d*e + 5*c*f))*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (
Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(
a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))
+ (c^(3/2)*(b*c*e + a*d*e - 10*a*c*f)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sq
rt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c +
 d*x^2))]*Sqrt[c + d*x^2])))/(a*c))/(3*c))/(5*a))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 442
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^q/(a*g*(m + 1))), x] - Simp[1/(a*g^2*(m + 1))   Int[(g*x)
^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[c*(b*e - a*f)*(m + 1) + e*2
*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*2*(p + q + 1))*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && GtQ[q, 0] && LtQ[m, -1]
&&  !(EqQ[q, 1] && SimplerQ[e + f*x^2, c + d*x^2])
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 448
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[e   Int[(g*x)^m*(a + b*x
^2)^p*(c + d*x^2)^q*(e + f*x^2)^(r - 1), x], x] + Simp[f/e^2   Int[(g*x)^(m
 + 2)*(a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^(r - 1), x], x] /; FreeQ[{a,
b, c, d, e, f, g, m, p, q}, x] && IGtQ[r, 0]
```

Maple [A] (verified)

Time = 7.29 (sec) , antiderivative size = 547, normalized size of antiderivative = 0.51

method	result
elliptic	$\frac{\sqrt{(b\,x^2+a)(x^2d+c)}\left(-\frac{e^2\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{5x^5}-\frac{e(10acf+ade+bce)\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{15ac\,x^3}-\frac{\left(15a^2c^2f^2+10a^2cdef-2a^2d^2e^2+10ab\,c^2\right.}{1}\right.}{}$
risch	$-\frac{\sqrt{b\,x^2+a}\,\sqrt{x^2d+c}\,(15f^2x^4a^2c^2+10a^2cdef\,x^4-2a^2d^2e^2x^4+10ab\,c^2ef\,x^4+2abcd\,e^2x^4-2b^2c^2e^2x^4+10a^2c^2ef\,x^2+a^2cd\,e^2x^2+ab}{15x^5a^2c^2}$
default	Expression too large to display

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^2/x^6,x,method=_RETURNVERBOS
E)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-1/5*e^2*(b*d
*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/x^5-1/15*e*(10*a*c*f+a*d*e+b*c*e)/a/c*(b*d
*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/x^3-1/15/a^2/c^2*(15*a^2*c^2*f^2+10*a^2*c*
d*e*f-2*a^2*d^2*e^2+10*a*b*c^2*e*f+2*a*b*c*d*e^2-2*b^2*c^2*e^2)*(b*d*x^4+a
*d*x^2+b*c*x^2+a*c)^(1/2)/x+(a*d*f^2+b*c*f^2+2*d*b*e*f-1/15*b*d*e*(10*a*c*
f+a*d*e+b*c*e)/a/c)/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*
x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)
^(1/2))-(b*d*f^2+1/15*b*d*(15*a^2*c^2*f^2+10*a^2*c*d*e*f-2*a^2*d^2*e^2+10*
a*b*c^2*e*f+2*a*b*c*d*e^2-2*b^2*c^2*e^2)/a^2/c^2)*c/(-b/a)^(1/2)*(1+b*x^2/
a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*(Elliptic
F(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a
*d+b*c)/c/b)^(1/2))))
```

Fricas [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)^2}{x^6}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^2}{x^6}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^2/x^6,x, algorithm="fr
icas")
```

output
```
integral((f^2*x^4 + 2*e*f*x^2 + e^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/x^6,
x)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)^2}{x^6}\,dx = \int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)^2}{x^6}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(1/2)*(f*x**2+e)**2/x**6,x)
```

output
```
Integral(sqrt(a + b*x**2)*sqrt(c + d*x**2)*(e + f*x**2)**2/x**6, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)^2}{x^6}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^2}{x^6}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^2/x^6,x, algorithm="ma
xima")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^2/x^6, x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)^2}{x^6}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^2}{x^6}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^2/x^6,x, algorithm="gi
ac")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^2/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)^2}{x^6}\,dx = \int \frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,{\left(f\,x^2+e\right)}^2}{x^6}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^2)/x^6,x)
```

output
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^2)/x^6, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)^2}{x^6}\,dx = \text{too large to display}$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^2/x^6,x)
```

output
```
( - 5*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*c*d*f**2*x**2 + 10*sqrt(c + d
*x**2)*sqrt(a + b*x**2)*a**2*d**2*f**2*x**4 - 5*sqrt(c + d*x**2)*sqrt(a +
b*x**2)*a*b*c**2*f**2*x**2 - sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b*c*d*e**
2 - 10*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b*c*d*e*f*x**2 + 15*sqrt(c + d*
x**2)*sqrt(a + b*x**2)*a*b*c*d*f**2*x**4 + 10*sqrt(c + d*x**2)*sqrt(a + b*
x**2)*a*b*d**2*e*f*x**4 + 10*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**2*c**2*f
**2*x**4 + 10*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**2*c*d*e*f*x**4 - 2*sqrt
(c + d*x**2)*sqrt(a + b*x**2)*b**2*d**2*e**2*x**4 - 10*int((sqrt(c + d*x**
2)*sqrt(a + b*x**2)*x**2)/(a**2*c*d + a**2*d**2*x**2 + a*b*c**2 + 2*a*b*c*
d*x**2 + a*b*d**2*x**4 + b**2*c**2*x**2 + b**2*c*d*x**4),x)*a**3*b*d**4*f*
*2*x**5 - 20*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a**2*c*d + a**2
*d**2*x**2 + a*b*c**2 + 2*a*b*c*d*x**2 + a*b*d**2*x**4 + b**2*c**2*x**2 +
b**2*c*d*x**4),x)*a**2*b**2*c*d**3*f**2*x**5 - 10*int((sqrt(c + d*x**2)*sq
rt(a + b*x**2)*x**2)/(a**2*c*d + a**2*d**2*x**2 + a*b*c**2 + 2*a*b*c*d*x**
2 + a*b*d**2*x**4 + b**2*c**2*x**2 + b**2*c*d*x**4),x)*a**2*b**2*d**4*e*f*
x**5 - 20*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a**2*c*d + a**2*d*
*2*x**2 + a*b*c**2 + 2*a*b*c*d*x**2 + a*b*d**2*x**4 + b**2*c**2*x**2 + b**
2*c*d*x**4),x)*a*b**3*c**2*d**2*f**2*x**5 - 20*int((sqrt(c + d*x**2)*sqrt(
a + b*x**2)*x**2)/(a**2*c*d + a**2*d**2*x**2 + a*b*c**2 + 2*a*b*c*d*x**2 +
 a*b*d**2*x**4 + b**2*c**2*x**2 + b**2*c*d*x**4),x)*a*b**3*c*d**3*e*f*x...
```

3.59 $\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)^2}{x^8}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 1641

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)^2}{x^8}\,dx = \text{Too large to display}$$

output
```
1/105*b*(8*b^3*c^3*e^2-a*b^2*c^2*e*(28*c*f+5*d*e)-a^2*b*c*(-35*c^2*f^2-28*
c*d*e*f+5*d^2*e^2)+a^3*d*(35*c^2*f^2-28*c*d*e*f+8*d^2*e^2))*x*(d*x^2+c)^(1
/2)/a^3/c^3/(b*x^2+a)^(1/2)-1/105*(4*b^3*c^2*d*e^4+3*a^3*c*f^2*(5*c^2*f^2-
16*c*d*e*f+16*d^2*e^2)+a*b^2*c*e^2*(48*c^2*f^2+22*c*d*e*f+13*d^2*e^2)+a^2*
b*e*(-48*c^3*f^3-95*c^2*d*e*f^2+22*c*d^2*e^2*f+4*d^3*e^3))*x*(b*x^2+a)^(1/
2)*(d*x^2+c)^(1/2)/a^3/c^3/e^2-1/105*(12*b^3*c*e^3*(4*c^2*f^2+3*c*d*e*f+d^
2*e^2)+a^2*b*e*f*(-29*c^3*f^3-134*c^2*d*e*f^2+24*c*d^2*e^2*f+36*d^3*e^3)+2
*a*b^2*e^2*(8*c^3*f^3+12*c^2*d*e*f^2+15*c*d^2*e^2*f+6*d^3*e^3)+a^3*f^2*(10
*c^3*f^3-29*c^2*d*e*f^2+16*c*d^2*e^2*f+48*d^3*e^3))*x^3*(b*x^2+a)^(1/2)*(d
*x^2+c)^(1/2)/a^3/c^3/e^3-1/105*f*(4*b^3*c*e^3*(16*c^2*f^2+21*c*d*e*f+9*d^
2*e^2)+2*a*b^2*e^2*(-10*c^3*f^3-5*c^2*d*e*f^2+6*c*d^2*e^2*f+18*d^3*e^3)+2*
a^2*b*e*f*(-c^3*f^3-41*c^2*d*e*f^2-5*c*d^2*e^2*f+42*d^3*e^3)+a^3*f^2*(3*c^
3*f^3-2*c^2*d*e*f^2-20*c*d^2*e^2*f+64*d^3*e^3))*x^5*(b*x^2+a)^(1/2)*(d*x^2
+c)^(1/2)/a^3/c^3/e^4-1/105*f^2*(3*a^3*d*f^2*(c^2*f^2-4*c*d*e*f+8*d^2*e^2)
+4*b^3*c*e^2*(6*c^2*f^2+19*c*d*e*f+9*d^2*e^2)+a*b^2*e*(-12*c^3*f^3-29*c^2*
d*e*f^2-14*c*d^2*e^2*f+36*d^3*e^3)+a^2*b*f*(3*c^3*f^3-14*c^2*d*e*f^2-29*c*
d^2*e^2*f+76*d^3*e^3))*x^7*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a^3/c^3/e^4-1/3
5*b*d*f^3*(4*b^2*c*e^2*(2*c*f+d*e)+a*b*e*(-4*c^2*f^2-3*c*d*e*f+4*d^2*e^2)+
a^2*f*(c^2*f^2-4*c*d*e*f+8*d^2*e^2))*x^9*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a
^3/c^3/e^4-1/7*(b*x^2+a)^(3/2)*(d*x^2+c)^(3/2)*(f*x^2+e)^3/a/c/e/x^7+1/...
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.16 (sec) , antiderivative size = 520, normalized size of antiderivative = 0.32

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)^2}{x^8}\,dx$$
$$= \frac{-\sqrt{\frac{b}{a}}(a+bx^2)\,(c+dx^2)\,(8b^3c^3e^2x^6 - ab^2c^2ex^4(5dex^2+4c(e+7fx^2)) + a^2bcx^2(-5d^2e^2x^4+2cdex^2(e+$$

input
```
Integrate[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^2)/x^8,x]
```

output
```
(-(Sqrt[b/a]*(a + b*x^2)*(c + d*x^2)*(8*b^3*c^3*e^2*x^6 - a*b^2*c^2*e*x^4*
(5*d*e*x^2 + 4*c*(e + 7*f*x^2)) + a^2*b*c*x^2*(-5*d^2*e^2*x^4 + 2*c*d*e*x^
2*(e + 14*f*x^2) + c^2*(3*e^2 + 14*e*f*x^2 + 35*f^2*x^4)) + a^3*(c + d*x^2
)*(8*d^2*e^2*x^4 - 4*c*d*e*x^2*(3*e + 7*f*x^2) + c^2*(15*e^2 + 42*e*f*x^2
+ 35*f^2*x^4)))) - I*b*c*(8*b^3*c^3*e^2 - a*b^2*c^2*e*(5*d*e + 28*c*f) + a
^3*d*(8*d^2*e^2 - 28*c*d*e*f + 35*c^2*f^2) + a^2*b*c*(-5*d^2*e^2 + 28*c*d*
e*f + 35*c^2*f^2))*x^7*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I
*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + I*b*c*(b*c - a*d)*(8*b^2*c^2*e^2 - a
*b*c*e*(d*e + 28*c*f) + a^2*(-4*d^2*e^2 + 14*c*d*e*f + 35*c^2*f^2))*x^7*Sq
rt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a
*d)/(b*c)])/(105*a^3*Sqrt[b/a]*c^3*x^7*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (warning: unable to verify)

Time = 1.90 (sec) , antiderivative size = 1012, normalized size of antiderivative = 0.62, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.514$, Rules used = {448, 442, 25, 442, 445, 25, 27, 406, 320, 388, 313, 445, 25, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^2}{x^8}dx$$

$$\Big\downarrow \ 448$$

$$\frac{f\int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)}{x^6}dx}{e^2}+e\int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)}{x^8}dx$$

$$\Big\downarrow \ 442$$

$$\frac{f\left(\frac{\int -\frac{\sqrt{bx^2+a}\left(d(be-5af)x^2+2bce-ade-5acf\right)}{x^4\sqrt{dx^2+c}}dx}{5a}-\frac{e\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{5ax^5}\right)}{e^2}+e\left(\frac{\int -\frac{\sqrt{bx^2+a}\left(d(3be-7af)x^2+4bce-ade-7acf\right)}{x^6\sqrt{dx^2+c}}dx}{7a}-\frac{e\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{7ax^7}\right)$$

$$\Big\downarrow \ 25$$

$$\frac{f\left(-\frac{\int \frac{\sqrt{bx^2+a}\left(d(be-5af)x^2+2bce-ade-5acf\right)}{x^4\sqrt{dx^2+c}}dx}{5a}-\frac{e\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{5ax^5}\right)}{e^2}+e\left(-\frac{\int \frac{\sqrt{bx^2+a}\left(d(3be-7af)x^2+4bce-ade-7acf\right)}{x^6\sqrt{dx^2+c}}dx}{7a}-\frac{e\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{7ax^7}\right)$$

$$\Big\downarrow \; 442$$

$$\frac{f\left(-\frac{\frac{\int \frac{d(2de-5cf)a^2-bc(2de+5cf)a+bd(bce+ade-10acf)x^2+2b^2c^2e}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(-5acf-ade+2bce)}{3cx^3}}{5a}-\frac{e\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{5ax^5}\right)}{e^2}+e\left(-\frac{\frac{\int \frac{d(4de-7cf)a^2-bc(2de+7cf)a+bd(3bce+3ade-14acf)x^2+4b^2c^2e}{x^4\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{5c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(-7acf-ade+4bce)}{5cx^5}}{7a}-\frac{e\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}\cdots}{7ax^7}\cdots\right.$$

$$\Big\downarrow \; 445$$

$$\frac{f\left(-\frac{\frac{-\frac{\int -\frac{bd\left(\left(d(2de-5cf)a^2-bc(2de+5cf)a+2b^2c^2e\right)x^2+ac(bce+ade-10acf)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{2b^2ce}{a}+\frac{2ad^2e}{c}-5adf-5bcf-2bde\right)}{x}}{3c}-\frac{\sqrt{a+bx^2}\sqrt{c+\cdots}}{\cdots}}{5a}\cdots\right.}{e^2}$$

$$e\left(-\frac{\frac{-\frac{\int \frac{2d^2(4de-7cf)a^3-bcd(5de-14cf)a^2-b^2c^2(5de+14cf)a+bd\left(d(4de-7cf)a^2-bc(2de+7cf)a+4b^2c^2e\right)x^2+8b^3c^3e}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{4b^2ce}{a}+\frac{4ad^2\cdots}{c}\cdots\right)}{3x^3}}{5c}\cdots}{7a}\cdots\right.$$

$$\Big\downarrow \; 25$$

$$\frac{f\left(-\frac{\frac{\frac{\int \frac{bd\left(\left(d(2de-5cf)a^2-bc(2de+5cf)a+2b^2c^2e\right)x^2+ac(bce+ade-10acf)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{2b^2ce}{a}+\frac{2ad^2e}{c}-5adf-5bcf-2bde\right)}{x}}{3c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\cdots}{3\cdots}}{5a}\cdots\right.}{e^2}$$

$$e\left(-\frac{\frac{-\frac{\int \frac{2d^2(4de-7cf)a^3-bcd(5de-14cf)a^2-b^2c^2(5de+14cf)a+bd\left(d(4de-7cf)a^2-bc(2de+7cf)a+4b^2c^2e\right)x^2+8b^3c^3e}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{4b^2ce}{a}+\frac{4ad^2\cdots}{c}\cdots\right)}{3x^3}}{5c}\cdots}{7a}\cdots\right.$$

$$\downarrow 27$$

$$f\left(-\frac{\frac{\frac{bd\int \frac{\left(d(2de-5cf)a^2-bc(2de+5cf)a+2b^2c^2e\right)x^2+ac(bce+ade-10acf)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{2b^2ce}{a}+\frac{2ad^2e}{c}-5adf-5bcf-2bde\right)}{x}}{3c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(-}{3cx}}{5a}\right.$$

$$\overline{e^2}$$

$$e\left(-\frac{-\frac{\int \frac{2d^2(4de-7cf)a^3-bcd(5de-14cf)a^2-b^2c^2(5de+14cf)a+bd\left(d(4de-7cf)a^2-bc(2de+7cf)a+4b^2c^2e\right)x^2+8b^3c^3e}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{4b^2ce}{a}+\frac{4ad^2}{c}\right.}{3x^3}}{5c}}{7a}\right.$$

$$\downarrow 406$$

$$f\left(-\frac{\frac{\frac{bd\left(\left(a^2d(2de-5cf)-abc(5cf+2de)+2b^2c^2e\right)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+ac(-10acf+ade+bce)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{2b^2ce}{a}+\frac{2ad^2e}{c}\right.}{x}}{3c}}{5a}\right.$$

$$\overline{e^2}$$

$$e\left(-\frac{-\frac{\int \frac{2d^2(4de-7cf)a^3-bcd(5de-14cf)a^2-b^2c^2(5de+14cf)a+bd\left(d(4de-7cf)a^2-bc(2de+7cf)a+4b^2c^2e\right)x^2+8b^3c^3e}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{4b^2ce}{a}+\frac{4ad^2}{c}\right.}{3x^3}}{5c}}{7a}\right.$$

$$\downarrow 320$$

$$f\left(-\frac{\frac{\frac{bd\left(\left(a^2d(2de-5cf)-abc(5cf+2de)+2b^2c^2e\right)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}(-10acf+ade+bce)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{ac}-\frac{\sqrt{a+bx^2}\sqrt{}}{}}{3c}}{5a}\right.$$

$$\overline{e^2}$$

$$e\left(-\frac{-\frac{\int \frac{2d^2(4de-7cf)a^3-bcd(5de-14cf)a^2-b^2c^2(5de+14cf)a+bd\left(d(4de-7cf)a^2-bc(2de+7cf)a+4b^2c^2e\right)x^2+8b^3c^3e}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{4b^2ce}{a}+\frac{4ad^2}{c}\right.}{3x^3}}{5c}}{7a}\right.$$

$$\Big\downarrow \; 388$$

$$f\left(-\frac{\frac{bd\left(\left(a^2d(2de-5cf)-abc(5cf+2de)+2b^2c^2e\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(-10acf+ade+bce)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{ac}}{3c}}{5a}\right.$$

$$\overline{\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad e^2}$$

$$e\left(-\frac{-\frac{-\frac{\int\frac{2d^2(4de-7cf)a^3-bcd(5de-14cf)a^2-b^2c^2(5de+14cf)a+bd\left(d(4de-7cf)a^2-bc(2de+7cf)a+4b^2c^2e\right)x^2+8b^3c^3e}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{4b^2ce}{a}+\frac{4ad^2}{c}\right.}{3x^3}}{5c}}{7a}\right.$$

$$\Big\downarrow \; 313$$

$$e\left(-\frac{-\frac{-\frac{\int\frac{2d^2(4de-7cf)a^3-bcd(5de-14cf)a^2-b^2c^2(5de+14cf)a+bd\left(d(4de-7cf)a^2-bc(2de+7cf)a+4b^2c^2e\right)x^2+8b^3c^3e}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{4b^2ce}{a}+\frac{4ad^2}{c}\right.}{3x^3}}{5c}}{7a}\right.$$

$$f\left(-\frac{\frac{bd\left(\left(a^2d(2de-5cf)-abc(5cf+2de)+2b^2c^2e\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(-10acf+ade+bce)\,\text{EllipticF}\left(\arctan\right.}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{ac}}{3c}}{5a}\right.$$

$$\overline{\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad e^2}$$

$$\Big\downarrow \; 445$$

$$f\left(-\frac{e\sqrt{dx^2+c}(bx^2+a)^{3/2}}{5ax^5}-\frac{\frac{bd\left(\frac{(bce+ade-10acf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\left(d(2de-5cf)a^2-bc(2de+5cf)a+2b^2c^2e\right)\left(\frac{x\sqrt{bx^2+}}{b\sqrt{dx^2+}}\cdots\right.\right.}{ac}}{3c}\cdots\right)$$

$$e^2$$

$$e\left(-\frac{e\sqrt{dx^2+c}(bx^2+a)^{3/2}}{7ax^7}-\frac{-\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(\frac{4ceb^2}{a}-2deb-7cfb+\frac{4ad^2e}{c}-7adf\right)}{3x^3}-\frac{-\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(2d^2(4de-7cf)a^3-bcd(5de-14cf)a^2\cdots\right.}{acx}\cdots}{}}{}\cdots\right)$$

$$\Big\downarrow \; 25$$

$$f\left(-\frac{e\sqrt{dx^2+c}(bx^2+a)^{3/2}}{5ax^5}-\frac{\frac{bd\left(\frac{(bce+ade-10acf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\left(d(2de-5cf)a^2-bc(2de+5cf)a+2b^2c^2e\right)\left(\frac{x\sqrt{bx^2+}}{b\sqrt{dx^2+}}\cdots\right.\right.}{ac}}{3c}\cdots\right)$$

$$e^2$$

$$e\left(-\frac{e\sqrt{dx^2+c}(bx^2+a)^{3/2}}{7ax^7}-\frac{-\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(\frac{4ceb^2}{a}-2deb-7cfb+\frac{4ad^2e}{c}-7adf\right)}{3x^3}-\frac{\int\frac{bd\left(\left(2d^2(4de-7cf)a^3-bcd(5de-14cf)a^2-b^2c^2(5de+\cdots\right.\right.}{\sqrt{b\cdots}}}{}}{}\cdots\right)$$

$$\Big\downarrow \; 27$$

$$f\left(-\frac{e\sqrt{dx^2+c}\left(bx^2+a\right)^{3/2}}{5ax^5}-\frac{\frac{bd\left(\frac{(bce+ade-10acf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\left(d(2de-5cf)a^2-bc(2de+5cf)a+2b^2c^2e\right)\left(\frac{x\sqrt{bx^2+}}{b\sqrt{dx^2+}}\right.\right.}{ac}}{3c}\right.$$

$$e^2$$

$$e\left(-\frac{e\sqrt{dx^2+c}\left(bx^2+a\right)^{3/2}}{7ax^7}-\frac{-\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(\frac{4ceb^2}{a}-2deb-7cfb+\frac{4ad^2e}{c}-7adf\right)}{3x^3}-\frac{bd\int\frac{\left(2d^2(4de-7cf)a^3-bcd(5de-14cf)a^2-b^2c^2(5de+1\right.}{\sqrt{b}}}{}}{}\right.$$

$$\Big\downarrow 406$$

$$f\left(-\frac{e\sqrt{dx^2+c}\left(bx^2+a\right)^{3/2}}{5ax^5}-\frac{\frac{bd\left(\frac{(bce+ade-10acf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\left(d(2de-5cf)a^2-bc(2de+5cf)a+2b^2c^2e\right)\left(\frac{x\sqrt{bx^2+}}{b\sqrt{dx^2+}}\right.\right.}{ac}}{3c}\right.$$

$$e^2$$

$$e\left(-\frac{e\sqrt{dx^2+c}\left(bx^2+a\right)^{3/2}}{7ax^7}-\frac{-\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(\frac{4ceb^2}{a}-2deb-7cfb+\frac{4ad^2e}{c}-7adf\right)}{3x^3}-\frac{bd\left(ac\left(d(4de-7cf)a^2-bc(2de+7cf)a+4b^2c^2e\right)\int\frac{}{\sqrt{bx^2}}\right.}{}}{}\right.$$

$$\Big\downarrow 320$$

$$f\left(-\frac{e\sqrt{dx^2+c}\left(bx^2+a\right)^{3/2}}{5ax^5}-\frac{\frac{bd\left(\frac{(bce+ade-10acf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\left(d(2de-5cf)a^2-bc(2de+5cf)a+2b^2c^2e\right)\left(\frac{x\sqrt{bx^2+\cdots}}{b\sqrt{dx^2+\cdots}}\cdots\right.\right.}{ac}\cdots}{3c}\cdots\right.$$

$$\overline{\qquad\qquad\qquad\qquad e^2\qquad\cdots}$$

$$e\left(-\frac{e\sqrt{dx^2+c}\left(bx^2+a\right)^{3/2}}{7ax^7}-\frac{-\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(\frac{4ceb^2}{a}-2deb-7cfb+\frac{4ad^2e}{c}-7adf\right)}{3x^3}-\frac{bd\left(\frac{\left(d(4de-7cf)a^2-bc(2de+7cf)a+4b^2c^2e\right)\sqrt{bx^2+a}\,\text{E}\cdots}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{d\cdots}}\right.}{\cdots}}{\cdots}\right.$$

$\downarrow$ 388

$$f\left(-\frac{e\sqrt{dx^2+c}\left(bx^2+a\right)^{3/2}}{5ax^5}-\frac{\frac{bd\left(\frac{(bce+ade-10acf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\left(d(2de-5cf)a^2-bc(2de+5cf)a+2b^2c^2e\right)\left(\frac{x\sqrt{bx^2+\cdots}}{b\sqrt{dx^2+\cdots}}\cdots\right.\right.}{ac}\cdots}{3c}\cdots\right.$$

$$\overline{\qquad\qquad\qquad\qquad e^2\qquad\cdots}$$

$$e\left(-\frac{e\sqrt{dx^2+c}\left(bx^2+a\right)^{3/2}}{7ax^7}-\frac{-\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(\frac{4ceb^2}{a}-2deb-7cfb+\frac{4ad^2e}{c}-7adf\right)}{3x^3}-\frac{bd\left(\frac{\left(d(4de-7cf)a^2-bc(2de+7cf)a+4b^2c^2e\right)\sqrt{bx^2+a}\,\text{E}\cdots}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{d\cdots}}\right.}{\cdots}}{\cdots}\right.$$

$\downarrow$ 313

$$f\left(-\frac{e\sqrt{dx^2+c}\left(bx^2+a\right)^{3/2}}{5ax^5}-\frac{\frac{bd\left(\frac{(bce+ade-10acf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\left(d(2de-5cf)a^2-bc(2de+5cf)a+2b^2c^2e\right)\left(\frac{x\sqrt{bx^2+}}{b\sqrt{dx^2+}}\right.\right.}{ac}}{3c}\right.$$

$$e^2$$

$$e\left(-\frac{e\sqrt{dx^2+c}\left(bx^2+a\right)^{3/2}}{7ax^7}-\frac{-\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(\frac{4ceb^2}{a}-2deb-7cfb+\frac{4ad^2e}{c}-7adf\right)}{3x^3}-\frac{bd\left(\frac{\left(d(4de-7cf)a^2-bc(2de+7cf)a+4b^2c^2e\right)\sqrt{bx^2+a}\,\text{E}}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{d}}\right.}{}}{}\right.$$

input
```
Int[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^2)/x^8,x]
```

output
```
(f*(-1/5*(e*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(a*x^5) - (-1/3*((2*b*c*e -
 a*d*e - 5*a*c*f)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(c*x^3) + (-((((2*b^2*c
*e)/a - 2*b*d*e + (2*a*d^2*e)/c - 5*b*c*f - 5*a*d*f)*Sqrt[a + b*x^2]*Sqrt[
c + d*x^2])/x) + (b*d*((2*b^2*c^2*e + a^2*d*(2*d*e - 5*c*f) - a*b*c*(2*d*e
 + 5*c*f))*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*
x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*S
qrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (c^(3/2)*(b*c*e +
 a*d*e - 10*a*c*f)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]],
1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c +
d*x^2])))/(a*c))/(3*c))/(5*a)))/e^2 + e*(-1/7*(e*(a + b*x^2)^(3/2)*Sqrt[c
+ d*x^2])/(a*x^7) - (-1/5*((4*b*c*e - a*d*e - 7*a*c*f)*Sqrt[a + b*x^2]*Sqr
t[c + d*x^2])/(c*x^5) + (-1/3*(((4*b^2*c*e)/a - 2*b*d*e + (4*a*d^2*e)/c -
7*b*c*f - 7*a*d*f)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/x^3 - (-(((8*b^3*c^3*e
 - a^2*b*c*d*(5*d*e - 14*c*f) + 2*a^3*d^2*(4*d*e - 7*c*f) - a*b^2*c^2*(5*d
*e + 14*c*f))*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(a*c*x)) + (b*d*((8*b^3*c^3
*e - a^2*b*c*d*(5*d*e - 14*c*f) + 2*a^3*d^2*(4*d*e - 7*c*f) - a*b^2*c^2*(5
*d*e + 14*c*f))*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a
 + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt
[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (c^(3/2)*(4*
b^2*c^2*e + a^2*d*(4*d*e - 7*c*f) - a*b*c*(2*d*e + 7*c*f))*Sqrt[a + b*x...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 442
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^q/(a*g*(m + 1))), x] - Simp[1/(a*g^2*(m + 1))   Int[(g*x)
^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[c*(b*e - a*f)*(m + 1) + e*2
*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*2*(p + q + 1))*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && GtQ[q, 0] && LtQ[m, -1]
&&  !(EqQ[q, 1] && SimplerQ[e + f*x^2, c + d*x^2])
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 448
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[e   Int[(g*x)^m*(a + b*x
^2)^p*(c + d*x^2)^q*(e + f*x^2)^(r - 1), x], x] + Simp[f/e^2   Int[(g*x)^(m
 + 2)*(a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^(r - 1), x], x] /; FreeQ[{a,
b, c, d, e, f, g, m, p, q}, x] && IGtQ[r, 0]
```

Maple [A] (verified)

Time = 8.57 (sec) , antiderivative size = 755, normalized size of antiderivative = 0.46

method	result
elliptic	$\sqrt{(b x^2+a)(x^2 d+c)}\left(-\frac{e^2\sqrt{bd x^4+ad x^2+x^2bc+ac}}{7x^7}-\frac{e(14acf+ade+bce)\sqrt{bd x^4+ad x^2+x^2bc+ac}}{35ac\, x^5}-\frac{\left(35a^2c^2f^2+14a^2cdef-4a^2d^2e^2+14abc^2\right.}{10}\right.$
risch	Expression too large to display
default	Expression too large to display

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^2/x^8,x,method=_RETURNVERBOS
E)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-1/7*e^2*(b*d
*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/x^7-1/35*e*(14*a*c*f+a*d*e+b*c*e)/a/c*(b*d
*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/x^5-1/105/a^2/c^2*(35*a^2*c^2*f^2+14*a^2*c
*d*e*f-4*a^2*d^2*e^2+14*a*b*c^2*e*f+2*a*b*c*d*e^2-4*b^2*c^2*e^2)*(b*d*x^4+
a*d*x^2+b*c*x^2+a*c)^(1/2)/x^3-1/105/a^3/c^3*(35*a^3*c^2*d*f^2-28*a^3*c*d^
2*e*f+8*a^3*d^3*e^2+35*a^2*b*c^3*f^2+28*a^2*b*c^2*d*e*f-5*a^2*b*c*d^2*e^2-
28*a*b^2*c^3*e*f-5*a*b^2*c^2*d*e^2+8*b^3*c^3*e^2)*(b*d*x^4+a*d*x^2+b*c*x^2
+a*c)^(1/2)/x+(b*d*f^2-1/105*b*d*(35*a^2*c^2*f^2+14*a^2*c*d*e*f-4*a^2*d^2*
e^2+14*a*b*c^2*e*f+2*a*b*c*d*e^2-4*b^2*c^2*e^2)/a^2/c^2)/(-b/a)^(1/2)*(1+b
*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*Ellipt
icF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-1/105*b*(35*a^3*c^2*d*f^2-28*
a^3*c*d^2*e*f+8*a^3*d^3*e^2+35*a^2*b*c^3*f^2+28*a^2*b*c^2*d*e*f-5*a^2*b*c*
d^2*e^2-28*a*b^2*c^3*e*f-5*a*b^2*c^2*d*e^2+8*b^3*c^3*e^2)/c^2/a^3/(-b/a)^(
1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/
2)*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^
(1/2),(-1+(a*d+b*c)/c/b)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 589, normalized size of antiderivative = 0.36

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)^2}{x^8}\,dx$$
$$= \frac{((8\,b^4c^3-5\,ab^3c^2d-5\,a^2b^2cd^2+8\,a^3bd^3)e^2-28\,(ab^3c^3-a^2b^2c^2d+a^3bcd^2)ef+35\,(a^2b^2c^3+a^3bc^2d)f^2)}{}$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^2/x^8,x, algorithm="fr
icas")
```

output
```
1/105*(((8*b^4*c^3 - 5*a*b^3*c^2*d - 5*a^2*b^2*c*d^2 + 8*a^3*b*d^3)*e^2 -
28*(a*b^3*c^3 - a^2*b^2*c^2*d + a^3*b*c*d^2)*e*f + 35*(a^2*b^2*c^3 + a^3*b
*c^2*d)*f^2)*sqrt(a*c)*x^7*sqrt(-b/a)*elliptic_e(arcsin(x*sqrt(-b/a)), a*d
/(b*c)) - ((8*b^4*c^3 + (4*a^2*b^2 - 5*a*b^3)*c^2*d - (2*a^3*b + 5*a^2*b^2
)*c*d^2 + 4*(a^4 + 2*a^3*b)*d^3)*e^2 - 14*(2*a*b^3*c^3 + (a^3*b - 2*a^2*b^
2)*c^2*d + (a^4 + 2*a^3*b)*c*d^2)*e*f + 35*(a^2*b^2*c^3 + (2*a^4 + a^3*b)*
c^2*d)*f^2)*sqrt(a*c)*x^7*sqrt(-b/a)*elliptic_f(arcsin(x*sqrt(-b/a)), a*d/
(b*c)) - (15*a^4*c^3*e^2 + ((8*a*b^3*c^3 - 5*a^2*b^2*c^2*d - 5*a^3*b*c*d^2
 + 8*a^4*d^3)*e^2 - 28*(a^2*b^2*c^3 - a^3*b*c^2*d + a^4*c*d^2)*e*f + 35*(a
^3*b*c^3 + a^4*c^2*d)*f^2)*x^6 + (35*a^4*c^3*f^2 - 2*(2*a^2*b^2*c^3 - a^3*
b*c^2*d + 2*a^4*c*d^2)*e^2 + 14*(a^3*b*c^3 + a^4*c^2*d)*e*f)*x^4 + 3*(14*a
^4*c^3*e*f + (a^3*b*c^3 + a^4*c^2*d)*e^2)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2
+ c))/(a^4*c^3*x^7)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)^2}{x^8}\,dx = \int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)^2}{x^8}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(1/2)*(f*x**2+e)**2/x**8,x)
```

output
```
Integral(sqrt(a + b*x**2)*sqrt(c + d*x**2)*(e + f*x**2)**2/x**8, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)^2}{x^8}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^2}{x^8}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^2/x^8,x, algorithm="ma
xima")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^2/x^8, x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)^2}{x^8}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^2}{x^8}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^2/x^8,x, algorithm="gi
ac")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^2/x^8, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)^2}{x^8}\,dx = \int \frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,{\left(f\,x^2+e\right)}^2}{x^8}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^2)/x^8,x)
```

output
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^2)/x^8, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}{x^8}\,dx = \text{too large to display}$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^2/x^8,x)
```

output
```
(7*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*d*f**2*x**2 + 7*sqrt(c + d*x**2)*sq
rt(a + b*x**2)*b*c*f**2*x**2 - 3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b*d*e**
2 - 14*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b*d*e*f*x**2 - 21*sqrt(c + d*x**2
)*sqrt(a + b*x**2)*b*d*f**2*x**4 + 35*int((sqrt(c + d*x**2)*sqrt(a + b*x**
2))/(a**2*c*d*x**6 + a**2*d**2*x**8 + a*b*c**2*x**6 + 2*a*b*c*d*x**8 + a*b
*d**2*x**10 + b**2*c**2*x**8 + b**2*c*d*x**10),x)*a**3*c*d**2*f**2*x**7 +
70*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a**2*c*d*x**6 + a**2*d**2*x**8
 + a*b*c**2*x**6 + 2*a*b*c*d*x**8 + a*b*d**2*x**10 + b**2*c**2*x**8 + b**2
*c*d*x**10),x)*a**2*b*c**2*d*f**2*x**7 - 28*int((sqrt(c + d*x**2)*sqrt(a +
 b*x**2))/(a**2*c*d*x**6 + a**2*d**2*x**8 + a*b*c**2*x**6 + 2*a*b*c*d*x**8
 + a*b*d**2*x**10 + b**2*c**2*x**8 + b**2*c*d*x**10),x)*a**2*b*c*d**2*e*f*
x**7 + 3*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a**2*c*d*x**6 + a**2*d**
2*x**8 + a*b*c**2*x**6 + 2*a*b*c*d*x**8 + a*b*d**2*x**10 + b**2*c**2*x**8
+ b**2*c*d*x**10),x)*a**2*b*d**3*e**2*x**7 + 35*int((sqrt(c + d*x**2)*sqrt
(a + b*x**2))/(a**2*c*d*x**6 + a**2*d**2*x**8 + a*b*c**2*x**6 + 2*a*b*c*d*
x**8 + a*b*d**2*x**10 + b**2*c**2*x**8 + b**2*c*d*x**10),x)*a*b**2*c**3*f*
*2*x**7 - 28*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a**2*c*d*x**6 + a**2
*d**2*x**8 + a*b*c**2*x**6 + 2*a*b*c*d*x**8 + a*b*d**2*x**10 + b**2*c**2*x
**8 + b**2*c*d*x**10),x)*a*b**2*c**2*d*e*f*x**7 + 6*int((sqrt(c + d*x**2)*
sqrt(a + b*x**2))/(a**2*c*d*x**6 + a**2*d**2*x**8 + a*b*c**2*x**6 + 2*a...
```

3.60 $\int x^4\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)\,dx$

Optimal result

Integrand size = 33, antiderivative size = 689

$$\int x^4\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)\,dx = -\frac{(16a^4d^4f+ab^3c^2d(15de-8cf)-8b^4c^3(3de-2cf)+3a^2b^2cd^2(5de-2cf)-8a^3bd^3(3de+cf))\,x\sqrt{c}}{315b^3d^4\sqrt{a+bx^2}} + \frac{(8a^3d^3f-4b^3c^2(3de-2cf)+3ab^2cd(2de-cf)-3a^2bd^2(4de+cf))\,x\sqrt{a+bx^2}\sqrt{c+dx^2}}{315b^3d^3} + \frac{\left(9bce+9ade+2acf-\frac{6bc^2f}{d}-\frac{6a^2df}{b}\right)x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{315bd} + \frac{(9bde+bcf-6adf)x^5\sqrt{a+bx^2}\sqrt{c+dx^2}}{63bd} + \frac{fx^5(a+bx^2)^{3/2}\sqrt{c+dx^2}}{9b} + \frac{\sqrt{a}(16a^4d^4f+ab^3c^2d(15de-8cf)-8b^4c^3(3de-2cf)+3a^2b^2cd^2(5de-2cf)-8a^3bd^3(3de+cf))\sqrt{}}{315b^{7/2}d^4\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} - \frac{a^{3/2}(8a^3d^3f-4b^3c^2(3de-2cf)+3ab^2cd(2de-cf)-3a^2bd^2(4de+cf))\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\right.}{315b^{7/2}d^3\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
-1/315*(16*a^4*d^4*f+a*b^3*c^2*d*(-8*c*f+15*d*e)-8*b^4*c^3*(-2*c*f+3*d*e)+
3*a^2*b^2*c*d^2*(-2*c*f+5*d*e)-8*a^3*b*d^3*(c*f+3*d*e))*x*(d*x^2+c)^(1/2)/
b^3/d^4/(b*x^2+a)^(1/2)+1/315*(8*a^3*d^3*f-4*b^3*c^2*(-2*c*f+3*d*e)+3*a*b^
2*c*d*(-c*f+2*d*e)-3*a^2*b*d^2*(c*f+4*d*e))*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1
/2)/b^3/d^3+1/315*(9*b*c*e+9*a*d*e+2*a*c*f-6*b*c^2*f/d-6*a^2*d*f/b)*x^3*(b
*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b/d+1/63*(-6*a*d*f+b*c*f+9*b*d*e)*x^5*(b*x^2
+a)^(1/2)*(d*x^2+c)^(1/2)/b/d+1/9*f*x^5*(b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/b+
1/315*a^(1/2)*(16*a^4*d^4*f+a*b^3*c^2*d*(-8*c*f+15*d*e)-8*b^4*c^3*(-2*c*f+
3*d*e)+3*a^2*b^2*c*d^2*(-2*c*f+5*d*e)-8*a^3*b*d^3*(c*f+3*d*e))*(d*x^2+c)^(
1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/b^(7
/2)/d^4/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)-1/315*a^(3/2)*(8*a
^3*d^3*f-4*b^3*c^2*(-2*c*f+3*d*e)+3*a*b^2*c*d*(-c*f+2*d*e)-3*a^2*b*d^2*(c*
f+4*d*e))*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d
/b/c)^(1/2))/b^(7/2)/d^3/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.45 (sec) , antiderivative size = 471, normalized size of antiderivative = 0.68

$$\int x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)\,dx$$

$$=\frac{\sqrt{\frac{b}{a}}dx(a+bx^2)\left(c+dx^2\right)\left(8a^3d^3f-3a^2bd^2(4de+cf+2dfx^2)+ab^2d(-3c^2f+2cd(3e+fx^2)+d^2x^2(9\right.}{}$$

input
```
Integrate[x^4*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2),x]
```

output
```
(Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2)*(8*a^3*d^3*f - 3*a^2*b*d^2*(4*d*e +
 c*f + 2*d*f*x^2) + a*b^2*d*(-3*c^2*f + 2*c*d*(3*e + f*x^2) + d^2*x^2*(9*e
 + 5*f*x^2)) + b^3*(8*c^3*f - 6*c^2*d*(2*e + f*x^2) + c*d^2*x^2*(9*e + 5*f
*x^2) + 5*d^3*x^4*(9*e + 7*f*x^2))) + I*c*(16*a^4*d^4*f + a*b^3*c^2*d*(15*
d*e - 8*c*f) + 3*a^2*b^2*c*d^2*(5*d*e - 2*c*f) - 8*a^3*b*d^3*(3*d*e + c*f)
 + 8*b^4*c^3*(-3*d*e + 2*c*f))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Ell
ipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*c*(-(b*c) + a*d)*(-3*a*b^2
*c*d^2*e + 8*a^3*d^3*f + 3*a^2*b*d^2*(-4*d*e + c*f) - 8*b^3*c^2*(-3*d*e +
2*c*f))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b
/a]*x], (a*d)/(b*c)])/(315*b^3*Sqrt[b/a]*d^4*Sqrt[a + b*x^2]*Sqrt[c + d*x^
2])
```

Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 623, normalized size of antiderivative = 0.90, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {443, 443, 444, 27, 444, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \sqrt{a+bx^2} \sqrt{c+dx^2} \left(e+fx^2\right) \, dx$$

$$\Big\downarrow \text{443}$$

$$\frac{\int \frac{x^4 \sqrt{bx^2+a} \left((9bde+bcf-6adf)x^2+c(9be-5af)\right)}{\sqrt{dx^2+c}}dx}{9b}+\frac{fx^5 \left(a+bx^2\right)^{3/2} \sqrt{c+dx^2}}{9b}$$

$$\Big\downarrow \text{443}$$

$$\frac{\frac{\int \frac{x^4 \left(ac(18bde-5bcf-5adf)-\left(-3c(3de-2cf)b^2-ad(9de+2cf)b+6a^2d^2f\right)x^2\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{7d}+\frac{x^5\sqrt{a+bx^2}\sqrt{c+dx^2}(-6adf+bcf+9bde)}{7d}}{9b}+\frac{fx^5 \left(a+bx^2\right)^{3/2} \sqrt{c+dx^2}}{9b}$$

$$\Big\downarrow \text{444}$$

$$\frac{\frac{\frac{1}{5}x^3\sqrt{a+bx^2}\sqrt{c+dx^2}\left(-\frac{6a^2df}{b}+2acf+9ade-\frac{6bc^2f}{d}+9bce\right)-\frac{\int -\frac{3x^2\left(\left(\left(-4c^2(3de-2cf)b^3+3acd(2de-cf)b^2-3a^2d^2(4de+cf)b+8a^3d^3f\right)x^2+ac\left(-3c(3de-\cdots\right)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}\cdots}{5bd}\cdots}{7d}\cdots}{9b}\cdots$$

$$\frac{fx^5\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{9b}$$

$\downarrow$ 27

$$\frac{\frac{\frac{3\int \frac{x^2\left(\left(\left(-4c^2(3de-2cf)b^3+3acd(2de-cf)b^2-3a^2d^2(4de+cf)b+8a^3d^3f\right)x^2+ac\left(-3c(3de-2cf)b^2-ad(9de+2cf)b+6a^2d^2f\right)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{5bd}+\frac{1}{5}x^3\sqrt{a+bx^2}\sqrt{c+dx^2}\left(-6\cdots\right)}{7d}\cdots}{9b}\cdots$$

$$\frac{fx^5\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{9b}$$

$\downarrow$ 444

$$\frac{\frac{3\left(\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(8a^3d^3f-3a^2bd^2(cf+4de)+3ab^2cd(2de-cf)-4b^3c^2(3de-2cf)\right)}{3bd}-\frac{\int \frac{\left(-8c^3(3de-2cf)b^4+ac^2d(15de-8cf)b^3+3a^2cd^2(5de-2cf)b^2-8a^3d^3\cdots\right)}{\cdots}}{\cdots}\cdots\right)}{5bd}\cdots}{\cdots}$$

$$\frac{fx^5\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{9b}$$

$\downarrow$ 406

$$\frac{\frac{3\left(\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(8a^3d^3f-3a^2bd^2(cf+4de)+3ab^2cd(2de-cf)-4b^3c^2(3de-2cf)\right)}{3bd}-\frac{ac\left(8a^3d^3f-3a^2bd^2(cf+4de)+3ab^2cd(2de-cf)-4b^3c^2(3de-2cf)\right)\int \frac{\cdots}{\sqrt{\cdots}}}{\cdots}\cdots\right)}{5bd}\cdots}{\cdots}$$

$$\frac{fx^5\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{9b}$$

$\downarrow$ 320

$$3\left(\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(8a^3d^3f-3a^2bd^2(cf+4de)+3ab^2cd(2de-cf)-4b^3c^2(3de-2cf)\right)}{3bd}-\frac{\left(16a^4d^4f-8a^3bd^3(cf+3de)+3a^2b^2cd^2(5de-2cf)+ab^3c^2d(15de-8cf)-\right.}{}\right.$$

$$5bd$$

$$\frac{fx^5\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{9b}$$

$\downarrow$ 388

$$3\left(\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(8a^3d^3f-3a^2bd^2(cf+4de)+3ab^2cd(2de-cf)-4b^3c^2(3de-2cf)\right)}{3bd}-\frac{\left(16a^4d^4f-8a^3bd^3(cf+3de)+3a^2b^2cd^2(5de-2cf)+ab^3c^2d(15de-8cf)-\right.}{}\right.$$

$$\frac{fx^5\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{9b}$$

$\downarrow$ 313

$$\frac{1}{5}x^3\sqrt{a+bx^2}\sqrt{c+dx^2}\left(-\frac{6a^2df}{b}+2acf+9ade-\frac{6bc^2f}{d}+9bce\right)+\frac{3\left(\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(8a^3d^3f-3a^2bd^2(cf+4de)+3ab^2cd(2de-cf)-4b^3c^2(3de-2cf)\right)}{3bd}-\frac{c^{3/}}{}\right.}{}$$

$$\frac{fx^5\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{9b}$$

input
```
Int[x^4*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2),x]
```

output
```
(f*x^5*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(9*b) + (((9*b*d*e + b*c*f - 6*a
*d*f)*x^5*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(7*d) + (((9*b*c*e + 9*a*d*e +
2*a*c*f - (6*b*c^2*f)/d - (6*a^2*d*f)/b)*x^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^
2])/5 + (3*(((8*a^3*d^3*f - 4*b^3*c^2*(3*d*e - 2*c*f) + 3*a*b^2*c*d*(2*d*e
 - c*f) - 3*a^2*b*d^2*(4*d*e + c*f))*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(3
*b*d) - ((16*a^4*d^4*f + a*b^3*c^2*d*(15*d*e - 8*c*f) - 8*b^4*c^3*(3*d*e -
 2*c*f) + 3*a^2*b^2*c*d^2*(5*d*e - 2*c*f) - 8*a^3*b*d^3*(3*d*e + c*f))*((x
*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE
[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*
x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (c^(3/2)*(8*a^3*d^3*f - 4*b^3*c
^2*(3*d*e - 2*c*f) + 3*a*b^2*c*d*(2*d*e - c*f) - 3*a^2*b*d^2*(4*d*e + c*f)
)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])
/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))/(3*b*d))
)/(5*b*d))/(7*d))/(9*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
 + d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 443
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*(g*x)^(m + 1)*(a + b*x^2)^(p
 + 1)*((c + d*x^2)^q/(b*g*(m + 2*(p + q + 1) + 1))), x] + Simp[1/(b*(m + 2*
(p + q + 1) + 1))   Int[(g*x)^m*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[c*((
b*e - a*f)*(m + 1) + b*e*2*(p + q + 1)) + (d*(b*e - a*f)*(m + 1) + f*2*q*(b
*c - a*d) + b*e*d*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f
, g, m, p}, x] && GtQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^2, c + d*x^
2])
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

Maple [A] (verified)

Time = 6.78 (sec) , antiderivative size = 893, normalized size of antiderivative = 1.30

method	result
elliptic	$\frac{\sqrt{(b x^2+a)(x^2 d+c)}\left(\frac{f x^7 \sqrt{b d x^4+a d x^2+x^2 b c+a c}}{9}+\frac{\left(a d f+b c f+b d e-\frac{f(8 a d+8 b c)}{9}\right) x^5 \sqrt{b d x^4+a d x^2+x^2 b c+a c}}{7 b d}+\frac{\left(\frac{2 a c f}{9}+a d e+b c e-\frac{\left(a d f+b\right.}{}\right.}{}\right.}{}$
risch	Expression too large to display
default	Expression too large to display

input
```
int(x^4*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/9*f*x^7*(b*
d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+1/7*(a*d*f+b*c*f+b*d*e-1/9*f*(8*a*d+8*b*c
))/b/d*x^5*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+1/5*(2/9*a*c*f+a*d*e+b*c*e-
1/7*(a*d*f+b*c*f+b*d*e-1/9*f*(8*a*d+8*b*c))/b/d*(6*a*d+6*b*c))/b/d*x^3*(b*
d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+1/3*(a*c*e-5/7*(a*d*f+b*c*f+b*d*e-1/9*f*(
8*a*d+8*b*c))/b/d*a*c-1/5*(2/9*a*c*f+a*d*e+b*c*e-1/7*(a*d*f+b*c*f+b*d*e-1/
9*f*(8*a*d+8*b*c))/b/d*(6*a*d+6*b*c))/b/d*(4*a*d+4*b*c))/b/d*x*(b*d*x^4+a*
d*x^2+b*c*x^2+a*c)^(1/2)-1/3*(a*c*e-5/7*(a*d*f+b*c*f+b*d*e-1/9*f*(8*a*d+8*
b*c))/b/d*a*c-1/5*(2/9*a*c*f+a*d*e+b*c*e-1/7*(a*d*f+b*c*f+b*d*e-1/9*f*(8*a
*d+8*b*c))/b/d*(6*a*d+6*b*c))/b/d*(4*a*d+4*b*c))/b/d*a*c/(-b/a)^(1/2)*(1+b
*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*Ellipt
icF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-(-3/5*(2/9*a*c*f+a*d*e+b*c*e-
1/7*(a*d*f+b*c*f+b*d*e-1/9*f*(8*a*d+8*b*c))/b/d*(6*a*d+6*b*c))/b/d*a*c-1/3
*(a*c*e-5/7*(a*d*f+b*c*f+b*d*e-1/9*f*(8*a*d+8*b*c))/b/d*a*c-1/5*(2/9*a*c*f
+a*d*e+b*c*e-1/7*(a*d*f+b*c*f+b*d*e-1/9*f*(8*a*d+8*b*c))/b/d*(6*a*d+6*b*c)
)/b/d*(4*a*d+4*b*c))/b/d*(2*a*d+2*b*c))*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(
1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(-b/a)
^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/
b)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 691, normalized size of antiderivative = 1.00

$$\int x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)\,dx =$$

$$-\frac{\sqrt{bd}(3\,(8\,b^4c^4d-5\,ab^3c^3d^2-5\,a^2b^2c^2d^3+8\,a^3bcd^4)e-2\,(8\,b^4c^5-4\,ab^3c^4d-3\,a^2b^2c^3d^2-4\,a^3bc^2d^3+}{}$$

input
```
integrate(x^4*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e),x, algorithm="fric
as")
```

output
```
-1/315*(sqrt(b*d)*(3*(8*b^4*c^4*d - 5*a*b^3*c^3*d^2 - 5*a^2*b^2*c^2*d^3 +
8*a^3*b*c*d^4)*e - 2*(8*b^4*c^5 - 4*a*b^3*c^4*d - 3*a^2*b^2*c^3*d^2 - 4*a^
3*b*c^2*d^3 + 8*a^4*c*d^4)*f)*x*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x)
, a*d/(b*c)) - sqrt(b*d)*(3*(8*b^4*c^4*d - 5*a*b^3*c^3*d^2 + 4*a^3*b*d^5 -
 (5*a^2*b^2 - 4*a*b^3)*c^2*d^3 + 2*(4*a^3*b - a^2*b^2)*c*d^4)*e - (16*b^4*
c^5 - 8*a*b^3*c^4*d + 8*a^4*d^5 - 2*(3*a^2*b^2 - 4*a*b^3)*c^3*d^2 - (8*a^3
*b + 3*a^2*b^2)*c^2*d^3 + (16*a^4 - 3*a^3*b)*c*d^4)*f)*x*sqrt(-c/d)*ellipt
ic_f(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - (35*b^4*d^5*f*x^8 + 5*(9*b^4*d^5*e
 + (b^4*c*d^4 + a*b^3*d^5)*f)*x^6 + (9*(b^4*c*d^4 + a*b^3*d^5)*e - 2*(3*b^
4*c^2*d^3 - a*b^3*c*d^4 + 3*a^2*b^2*d^5)*f)*x^4 - (6*(2*b^4*c^2*d^3 - a*b^
3*c*d^4 + 2*a^2*b^2*d^5)*e - (8*b^4*c^3*d^2 - 3*a*b^3*c^2*d^3 - 3*a^2*b^2*
c*d^4 + 8*a^3*b*d^5)*f)*x^2 + 3*(8*b^4*c^3*d^2 - 5*a*b^3*c^2*d^3 - 5*a^2*b
^2*c*d^4 + 8*a^3*b*d^5)*e - 2*(8*b^4*c^4*d - 4*a*b^3*c^3*d^2 - 3*a^2*b^2*c
^2*d^3 - 4*a^3*b*c*d^4 + 8*a^4*d^5)*f)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(b
^4*d^5*x)
```

Sympy [F]

$$\int x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)\,dx=\int x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)\,dx$$

input
```
integrate(x**4*(b*x**2+a)**(1/2)*(d*x**2+c)**(1/2)*(f*x**2+e),x)
```

output
```
Integral(x**4*sqrt(a + b*x**2)*sqrt(c + d*x**2)*(e + f*x**2), x)
```

Maxima [F]

$$\int x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)\,dx=\int\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)x^4\,dx$$

input
```
integrate(x^4*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e),x, algorithm="maxi
ma")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)*x^4, x)
```

Giac [F]

$$\int x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)\,dx=\int \sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)x^4\,dx$$

input
```
integrate(x^4*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e),x, algorithm="giac
")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)*x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)\,dx=\int x^4\,\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,\left(f\,x^2+e\right)\,dx$$

input
```
int(x^4*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2),x)
```

output
```
int(x^4*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2), x)
```

Reduce [F]

$$\int x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)\,dx=\text{Too large to display}$$

input
```
int(x^4*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e),x)
```

output

```
(8*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**3*d**3*f*x - 3*sqrt(c + d*x**2)*sq
rt(a + b*x**2)*a**2*b*c*d**2*f*x - 12*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*
*2*b*d**3*e*x - 6*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*b*d**3*f*x**3 - 3
*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**2*c**2*d*f*x + 6*sqrt(c + d*x**2)*
sqrt(a + b*x**2)*a*b**2*c*d**2*e*x + 2*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a
*b**2*c*d**2*f*x**3 + 9*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**2*d**3*e*x*
*3 + 5*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**2*d**3*f*x**5 + 8*sqrt(c + d
*x**2)*sqrt(a + b*x**2)*b**3*c**3*f*x - 12*sqrt(c + d*x**2)*sqrt(a + b*x**
2)*b**3*c**2*d*e*x - 6*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**3*c**2*d*f*x**
3 + 9*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**3*c*d**2*e*x**3 + 5*sqrt(c + d*
x**2)*sqrt(a + b*x**2)*b**3*c*d**2*f*x**5 + 45*sqrt(c + d*x**2)*sqrt(a + b
*x**2)*b**3*d**3*e*x**5 + 35*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**3*d**3*f
*x**7 - 16*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 +
b*c*x**2 + b*d*x**4),x)*a**4*d**4*f + 8*int((sqrt(c + d*x**2)*sqrt(a + b*x
**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a**3*b*c*d**3*f + 24*
int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 +
b*d*x**4),x)*a**3*b*d**4*e + 6*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2
)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a**2*b**2*c**2*d**2*f - 15*int
((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d
*x**4),x)*a**2*b**2*c*d**3*e + 8*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)...
```

3.61 $\int x^2\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)\,dx$

Optimal result

Integrand size = 33, antiderivative size = 530

$$\begin{aligned}\int x^2\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)\,dx &= \frac{(8a^3d^3f+ab^2cd(14de-5cf)-2b^3c^2(7de-4cf)-a^2bd^2(14de+5cf))\,x\sqrt{c+dx^2}}{105b^2d^3\sqrt{a+bx^2}} \\ &- \frac{(4a^2d^2f-b^2c(7de-4cf)-abd(7de+2cf))\,x\sqrt{a+bx^2}\sqrt{c+dx^2}}{105b^2d^2} \\ &+ \frac{(7bde+bcf-4adf)x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{35bd}+\frac{fx^3(a+bx^2)^{3/2}\sqrt{c+dx^2}}{7b} \\ &- \frac{\sqrt{a}(8a^3d^3f+ab^2cd(14de-5cf)-2b^3c^2(7de-4cf)-a^2bd^2(14de+5cf))\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\right.}{105b^{5/2}d^3\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} \\ &+ \frac{a^{3/2}(4a^2d^2f-b^2c(7de-4cf)-abd(7de+2cf))\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{105b^{5/2}d^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}\end{aligned}$$

output
```
1/105*(8*a^3*d^3*f+a*b^2*c*d*(-5*c*f+14*d*e)-2*b^3*c^2*(-4*c*f+7*d*e)-a^2*
b*d^2*(5*c*f+14*d*e))*x*(d*x^2+c)^(1/2)/b^2/d^3/(b*x^2+a)^(1/2)-1/105*(4*a
^2*d^2*f-b^2*c*(-4*c*f+7*d*e)-a*b*d*(2*c*f+7*d*e))*x*(b*x^2+a)^(1/2)*(d*x^
2+c)^(1/2)/b^2/d^2+1/35*(-4*a*d*f+b*c*f+7*b*d*e)*x^3*(b*x^2+a)^(1/2)*(d*x^
2+c)^(1/2)/b/d+1/7*f*x^3*(b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/b-1/105*a^(1/2)*(
8*a^3*d^3*f+a*b^2*c*d*(-5*c*f+14*d*e)-2*b^3*c^2*(-4*c*f+7*d*e)-a^2*b*d^2*(
5*c*f+14*d*e))*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/
2),(1-a*d/b/c)^(1/2))/b^(5/2)/d^3/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a)
)^(1/2)+1/105*a^(3/2)*(4*a^2*d^2*f-b^2*c*(-4*c*f+7*d*e)-a*b*d*(2*c*f+7*d*e
))*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(
1/2))/b^(5/2)/d^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.71 (sec) , antiderivative size = 367, normalized size of antiderivative = 0.69

$$\int x^2\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)\ dx$$
$$= \frac{-\sqrt{\frac{b}{a}}dx(a+bx^2)\left(c+dx^2\right)\left(4a^2d^2f-abd(7de+2cf+3dfx^2)+b^2(4c^2f-cd(7e+3fx^2)-3d^2x^2(7e+\right.}{}$$

input
```
Integrate[x^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2),x]
```

output
```
(-(Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2)*(4*a^2*d^2*f - a*b*d*(7*d*e + 2*c
*f + 3*d*f*x^2) + b^2*(4*c^2*f - c*d*(7*e + 3*f*x^2) - 3*d^2*x^2*(7*e + 5*
f*x^2)))) - I*c*(8*a^3*d^3*f + a*b^2*c*d*(14*d*e - 5*c*f) + 2*b^3*c^2*(-7*
d*e + 4*c*f) - a^2*b*d^2*(14*d*e + 5*c*f))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d
*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + I*c*(-(b*c) + a*
d)*(4*a^2*d^2*f + 2*b^2*c*(7*d*e - 4*c*f) + a*b*d*(-7*d*e + c*f))*Sqrt[1 +
 (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b
*c)])/(105*a^2*(b/a)^(5/2)*d^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 472, normalized size of antiderivative = 0.89, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {443, 443, 444, 25, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)\,dx$$

$$\Big\downarrow \; 443$$

$$\frac{\int \frac{x^2\sqrt{bx^2+a}\left((7bde+bcf-4adf)x^2+c(7be-3af)\right)}{\sqrt{dx^2+c}}dx}{7b}+\frac{fx^3\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{7b}$$

$$\Big\downarrow \; 443$$

$$\frac{\frac{\int \frac{x^2\left(ac(14bde-3bcf-3adf)-\left(-c(7de-4cf)b^2-ad(7de+2cf)b+4a^2d^2f\right)x^2\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{5d}+\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}(-4adf+bcf+7bde)}{5d}}{7b}+\frac{fx^3\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{7b}$$

$$\Big\downarrow \; 444$$

$$\frac{\frac{\frac{1}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(-\frac{4a^2df}{b}+2acf+7ade-\frac{4bc^2f}{d}+7bce\right)-\frac{\int -\frac{\left(-2c^2(7de-4cf)b^3+acd(14de-5cf)b^2-a^2d^2(14de+5cf)b+8a^3d^3f\right)x^2+ac\left(-c(7de-4cf)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}}{3bd}}{5d}}{7b}$$

$$\frac{fx^3\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{7b}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\frac{\int \frac{\left(-2c^2(7de-4cf)b^3+acd(14de-5cf)b^2-a^2d^2(14de+5cf)b+8a^3d^3f\right)x^2+ac\left(-c(7de-4cf)b^2-ad(7de+2cf)b+4a^2d^2f\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3bd}+\frac{1}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(-\frac{4a^2df}{b}+\right)}{5d}}{7b}$$

$$\frac{fx^3\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{7b}$$

$$\Big\downarrow \; 406$$

$$\frac{\frac{\frac{ac\left(4a^2d^2f-abd(2cf+7de)+b^2(-c)(7de-4cf)\right)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\left(8a^3d^3f-a^2bd^2(5cf+14de)+ab^2cd(14de-5cf)-2b^3c^2(7de-4cf)\right)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}d}{3bd}}{5d}}{7b}$$

$$\frac{fx^3\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{7b}$$

$\Big\downarrow$ 320

$$\frac{\frac{\frac{\left(8a^3d^3f-a^2bd^2(5cf+14de)+ab^2cd(14de-5cf)-2b^3c^2(7de-4cf)\right)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}\left(4a^2d^2f-abd(2cf+7de)+b^2(-c)(7de-4cf)\right)\text{EllipticF}\left(\right.}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{3bd}}{5d}}{7b}$$

$$\frac{fx^3\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{7b}$$

$\Big\downarrow$ 388

$$\frac{\frac{\frac{\left(8a^3d^3f-a^2bd^2(5cf+14de)+ab^2cd(14de-5cf)-2b^3c^2(7de-4cf)\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}\left(4a^2d^2f-abd(2cf+7de)+b^2(-c)(7de-4c\right.}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx\right.}{a\left(c+dx\right.}}}}{3bd}}{5d}}{7b}$$

$$\frac{fx^3\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{7b}$$

$\Big\downarrow$ 313

$$\frac{\frac{\frac{1}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(-\frac{4a^2df}{b}+2acf+7ade-\frac{4bc^2f}{d}+7bce\right)+\frac{\frac{c^{3/2}\sqrt{a+bx^2}\left(4a^2d^2f-abd(2cf+7de)+b^2(-c)(7de-4cf)\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{}}{5d}}{}$$

$$\frac{fx^3\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{7b}$$

input
```
Int[x^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2),x]
```

output
```
(f*x^3*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(7*b) + (((7*b*d*e + b*c*f - 4*a
*d*f)*x^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(5*d) + (((7*b*c*e + 7*a*d*e +
2*a*c*f - (4*b*c^2*f)/d - (4*a^2*d*f)/b)*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]
)/3 + ((8*a^3*d^3*f + a*b^2*c*d*(14*d*e - 5*c*f) - 2*b^3*c^2*(7*d*e - 4*c*
f) - a^2*b*d^2*(14*d*e + 5*c*f))*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2])
- (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c
)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]
)) + (c^(3/2)*(4*a^2*d^2*f - b^2*c*(7*d*e - 4*c*f) - a*b*d*(7*d*e + 2*c*f)
)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])
/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))/(3*b*d))
/(5*d))/(7*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 443
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*(g*x)^(m + 1)*(a + b*x^2)^(p
 + 1)*((c + d*x^2)^q/(b*g*(m + 2*(p + q + 1) + 1))), x] + Simp[1/(b*(m + 2*
(p + q + 1) + 1))   Int[(g*x)^m*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[c*((
b*e - a*f)*(m + 1) + b*e*2*(p + q + 1)) + (d*(b*e - a*f)*(m + 1) + f*2*q*(b
*c - a*d) + b*e*d*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f
, g, m, p}, x] && GtQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^2, c + d*x^
2])
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

Maple [A] (verified)

Time = 5.48 (sec) , antiderivative size = 604, normalized size of antiderivative = 1.14

method	result
elliptic	$\frac{\sqrt{(b\,x^2+a)(x^2d+c)}\left(\frac{f\,x^5\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{7}+\frac{\left(adf+bcf+bde-\frac{f(6ad+6bc)}{7}\right)x^3\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{5bd}+\frac{\left(\frac{2acf}{7}+ade+bce-\frac{\left(adf+b\right.}{}\right.}{}\right.}{}$
risch	$-\frac{x(-15f\,x^4b^2d^2-3ab\,d^2f\,x^2-3b^2cf\,x^2d-21b^2d^2e\,x^2+4f\,d^2a^2-2fdcba-7ab\,d^2e+4f\,c^2b^2-7d\,b^2ce)\sqrt{b\,x^2+a}\,\sqrt{x^2d+c}}{105b^2d^2}+\frac{\left(-\frac{(8}{}\right.}{}$
default	Expression too large to display

input

```
int(x^2*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e),x,method=_RETURNVERBOSE)
```

output

```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/7*f*x^5*(b*
d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+1/5*(a*d*f+b*c*f+b*d*e-1/7*f*(6*a*d+6*b*c
))/b/d*x^3*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+1/3*(2/7*a*c*f+a*d*e+b*c*e-
1/5*(a*d*f+b*c*f+b*d*e-1/7*f*(6*a*d+6*b*c))/b/d*(4*a*d+4*b*c))/b/d*x*(b*d*
x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)-1/3*(2/7*a*c*f+a*d*e+b*c*e-1/5*(a*d*f+b*c*f
+b*d*e-1/7*f*(6*a*d+6*b*c))/b/d*(4*a*d+4*b*c))/b/d*a*c/(-b/a)^(1/2)*(1+b*x
^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*Elliptic
F(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-(a*c*e-3/5*(a*d*f+b*c*f+b*d*e-1
/7*f*(6*a*d+6*b*c))/b/d*a*c-1/3*(2/7*a*c*f+a*d*e+b*c*e-1/5*(a*d*f+b*c*f+b*
d*e-1/7*f*(6*a*d+6*b*c))/b/d*(4*a*d+4*b*c))/b/d*(2*a*d+2*b*c))*c/(-b/a)^(1
/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2
)/d*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)
^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 485, normalized size of antiderivative = 0.92

$$\int x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)\,dx$$

$$= \frac{\sqrt{bd}(14\,(b^3c^3d-ab^2c^2d^2+a^2bcd^3)e-(8\,b^3c^4-5\,ab^2c^3d-5\,a^2bc^2d^2+8\,a^3cd^3)f)x\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right.}{}$$

input
```
integrate(x^2*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e),x, algorithm="fric
as")
```

output
```
1/105*(sqrt(b*d)*(14*(b^3*c^3*d - a*b^2*c^2*d^2 + a^2*b*c*d^3)*e - (8*b^3*
c^4 - 5*a*b^2*c^3*d - 5*a^2*b*c^2*d^2 + 8*a^3*c*d^3)*f)*x*sqrt(-c/d)*ellip
tic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - sqrt(b*d)*(7*(2*b^3*c^3*d - 2*a*b
^2*c^2*d^2 + a^2*b*d^4 + (2*a^2*b + a*b^2)*c*d^3)*e - (8*b^3*c^4 - 5*a*b^2
*c^3*d + 4*a^3*d^4 - (5*a^2*b - 4*a*b^2)*c^2*d^2 + 2*(4*a^3 - a^2*b)*c*d^3
)*f)*x*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)/x), a*d/(b*c)) + (15*b^3*d^
4*f*x^6 + 3*(7*b^3*d^4*e + (b^3*c*d^3 + a*b^2*d^4)*f)*x^4 + (7*(b^3*c*d^3
+ a*b^2*d^4)*e - 2*(2*b^3*c^2*d^2 - a*b^2*c*d^3 + 2*a^2*b*d^4)*f)*x^2 - 14
*(b^3*c^2*d^2 - a*b^2*c*d^3 + a^2*b*d^4)*e + (8*b^3*c^3*d - 5*a*b^2*c^2*d^
2 - 5*a^2*b*c*d^3 + 8*a^3*d^4)*f)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(b^3*d^
4*x)
```

Sympy [F]

$$\int x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)\,dx=\int x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)\,dx$$

input
```
integrate(x**2*(b*x**2+a)**(1/2)*(d*x**2+c)**(1/2)*(f*x**2+e),x)
```

output
```
Integral(x**2*sqrt(a + b*x**2)*sqrt(c + d*x**2)*(e + f*x**2), x)
```

Maxima [F]

$$\int x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)\,dx=\int \sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)x^2\,dx$$

input
```
integrate(x^2*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e),x, algorithm="maxi
ma")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)*x^2, x)
```

Giac [F]

$$\int x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)\,dx=\int \sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)x^2\,dx$$

input
```
integrate(x^2*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e),x, algorithm="giac
")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)\,dx=\int x^2\,\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,\left(f\,x^2+e\right)\,dx$$

input
```
int(x^2*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2),x)
```

output
```
int(x^2*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2), x)
```

Reduce [F]

$$\int x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)\,dx=\text{Too large to display}$$

input
```
int(x^2*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e),x)
```

output
```
( - 4*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*d**2*f*x + 2*sqrt(c + d*x**2)
*sqrt(a + b*x**2)*a*b*c*d*f*x + 7*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b*d*
*2*e*x + 3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b*d**2*f*x**3 - 4*sqrt(c +
d*x**2)*sqrt(a + b*x**2)*b**2*c**2*f*x + 7*sqrt(c + d*x**2)*sqrt(a + b*x**
2)*b**2*c*d*e*x + 3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**2*c*d*f*x**3 + 21
*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**2*d**2*e*x**3 + 15*sqrt(c + d*x**2)*
sqrt(a + b*x**2)*b**2*d**2*f*x**5 + 8*int((sqrt(c + d*x**2)*sqrt(a + b*x**
2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a**3*d**3*f - 5*int((sq
rt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**
4),x)*a**2*b*c*d**2*f - 14*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a
*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a**2*b*d**3*e - 5*int((sqrt(c + d*
x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a*b
**2*c**2*d*f + 14*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*
x**2 + b*c*x**2 + b*d*x**4),x)*a*b**2*c*d**2*e + 8*int((sqrt(c + d*x**2)*s
qrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*b**3*c**3*
f - 14*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*
x**2 + b*d*x**4),x)*b**3*c**2*d*e + 4*int((sqrt(c + d*x**2)*sqrt(a + b*x**
2))/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a**3*c*d**2*f - 2*int((sqrt(
c + d*x**2)*sqrt(a + b*x**2))/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a*
*2*b*c**2*d*f - 7*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c + a*d*x*...
```

3.62 $\int \sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)\,dx$

Optimal result

Integrand size = 30, antiderivative size = 381

$$\int \sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)\,dx = \frac{\left(5bce+5ade+2acf-\frac{2bc^2f}{d}-\frac{2a^2df}{b}\right)x\sqrt{c+dx^2}}{15d\sqrt{a+bx^2}} + \frac{(5bde-2bcf+adf)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{15bd} + \frac{fx\sqrt{a+bx^2}\,(c+dx^2)^{3/2}}{5d} + \frac{\sqrt{a}(2a^2d^2f-b^2c(5de-2cf)-abd(5de+2cf))\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{15b^{3/2}d^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} + \frac{a^{3/2}(10bde-bcf-adf)\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{15b^{3/2}d\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
1/15*(5*b*c*e+5*a*d*e+2*a*c*f-2*b*c^2*f/d-2*a^2*d*f/b)*x*(d*x^2+c)^(1/2)/d
/(b*x^2+a)^(1/2)+1/15*(a*d*f-2*b*c*f+5*b*d*e)*x*(b*x^2+a)^(1/2)*(d*x^2+c)^
(1/2)/b/d+1/5*f*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/d+1/15*a^(1/2)*(2*a^2*d^
2*f-b^2*c*(-2*c*f+5*d*e)-a*b*d*(2*c*f+5*d*e))*(d*x^2+c)^(1/2)*EllipticE(b^
(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/b^(3/2)/d^2/(b*x^2+a)
^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)+1/15*a^(3/2)*(-a*d*f-b*c*f+10*b*d*e
)*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1
/2))/b^(3/2)/d/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.48 (sec) , antiderivative size = 269, normalized size of antiderivative = 0.71

$$\int \sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)\,dx$$
$$= \frac{\sqrt{\frac{b}{a}}dx(a+bx^2)\left(c+dx^2\right)\left(adf+b(5de+cf+3dfx^2)\right)+ic(2a^2d^2f+b^2c(-5de+2cf)-abd(5de+2cf}{}$$

input
```
Integrate[Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2),x]
```

output
```
(Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2)*(a*d*f + b*(5*d*e + c*f + 3*d*f*x^2
)) + I*c*(2*a^2*d^2*f + b^2*c*(-5*d*e + 2*c*f) - a*b*d*(5*d*e + 2*c*f))*Sq
rt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a
*d)/(b*c)] - I*c*(-(b*c) + a*d)*(5*b*d*e - 2*b*c*f + a*d*f)*Sqrt[1 + (b*x^
2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/
(15*b*Sqrt[b/a]*d^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 350, normalized size of antiderivative = 0.92, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {403, 403, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)\,dx$$

$$\Big\downarrow \text{403}$$

$$\frac{\int \frac{\sqrt{bx^2+a}\left((5bde+bcf-2adf)x^2+c(5be-af)\right)}{\sqrt{dx^2+c}}dx}{5b}+\frac{fx\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{5b}$$

$$\Big\downarrow \text{403}$$

$$\frac{\frac{\int \frac{ac(10bde-bcf-adf)-\left(-c(5de-2cf)b^2-ad(5de+2cf)b+2a^2d^2f\right)x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3d}+\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(-2adf+bcf+5bde)}{3d}}{5b}+\frac{fx\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{5b}$$

$$\Big\downarrow \; 406$$

$$\frac{\frac{ac(-adf-bcf+10bde)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx-\left(2a^2d^2f-abd(2cf+5de)+b^2(-c)(5de-2cf)\right)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3d}+\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(-2adf+b\ldots}{3d}}{5b}+\frac{fx\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{5b}$$

$$\Big\downarrow \; 320$$

$$\frac{\frac{\frac{c^{3/2}\sqrt{a+bx^2}(-adf-bcf+10bde)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\left(2a^2d^2f-abd(2cf+5de)+b^2(-c)(5de-2cf)\right)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3d}+\frac{x\sqrt{a+bx^2}\sqrt{\ldots}}{}}{5b}+\frac{fx\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{5b}$$

$$\Big\downarrow \; 388$$

$$\frac{\frac{\frac{c^{3/2}\sqrt{a+bx^2}(-adf-bcf+10bde)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\left(2a^2d^2f-abd(2cf+5de)+b^2(-c)(5de-2cf)\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)}{3d}+\ldots}{5b}+\frac{fx\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{5b}$$

$$\Big\downarrow \; 313$$

$$\frac{\frac{\frac{c^{3/2}\sqrt{a+bx^2}(-adf-bcf+10bde)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\left(2a^2d^2f-abd(2cf+5de)+b^2(-c)(5de-2cf)\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{\ldots}}{\sqrt{\ldots}}\right.\right.}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+\ldots\right)}{a\left(c+\ldots\right)}}}\right.}{3d}}{5b}+\frac{fx\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{5b}$$

input
```
Int[Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2),x]
```

output
```
(f*x*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(5*b) + (((5*b*d*e + b*c*f - 2*a*d
*f)*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(3*d) + (-((2*a^2*d^2*f - b^2*c*(5*
d*e - 2*c*f) - a*b*d*(5*d*e + 2*c*f))*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x
^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 -
 (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d
*x^2]))) + (c^(3/2)*(10*b*d*e - b*c*f - a*d*f)*Sqrt[a + b*x^2]*EllipticF[A
rcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2)
)/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))/(3*d))/(5*b)
```

Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

Maple [A] (verified)

Time = 4.02 (sec) , antiderivative size = 431, normalized size of antiderivative = 1.13

method	result
elliptic	$\sqrt{(b x^2+a)(x^2 d+c)}\left(\frac{f x^3\sqrt{bd x^4+ad x^2+x^2bc+ac}}{5}+\frac{\left(adf+bcf+bde-\frac{f(4ad+4bc)}{5}\right)x\sqrt{bd x^4+ad x^2+x^2bc+ac}}{3bd}+\frac{\left(ace-\frac{\left(adf+bcf+bde-\frac{f(4a\ldots}{}\right)}{3bd}\ldots\right.}{\ldots}\right.$
risch	$\frac{x(3bdf x^2+adf+bcf+5bde)\sqrt{b x^2+a}\,\sqrt{x^2 d+c}}{15bd}-\frac{\left(-\frac{\left(2f d^2a^2-2fdcba-5ab d^2e+2f c^2b^2-5d b^2ce\right)c\sqrt{1+\frac{b x^2}{a}}\,\sqrt{1+\frac{d x^2}{c}}\,\left(\text{EllipticF}\left(x\sqrt{-\ldots}\right.\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{bd x^4+ad x^2+x^2bc+ac\ldots}}\right.}{\ldots}$
default	$\frac{\sqrt{b x^2+a}\,\sqrt{x^2 d+c}\left(3\sqrt{-\frac{b}{a}}\,b^2d^3f x^7+4\sqrt{-\frac{b}{a}}\,ab d^3f x^5+4\sqrt{-\frac{b}{a}}\,b^2c d^2f x^5+5\sqrt{-\frac{b}{a}}\,b^2d^3e x^5+\sqrt{-\frac{b}{a}}\,a^2d^3f x^3+5\sqrt{-\frac{b}{a}}\,abc d^2f x^3\ldots\right.}{\ldots}$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/5*f*x^3*(b*
d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+1/3*(a*d*f+b*c*f+b*d*e-1/5*f*(4*a*d+4*b*c
))/b/d*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+(a*c*e-1/3*(a*d*f+b*c*f+b*d*e
-1/5*f*(4*a*d+4*b*c))/b/d*a*c)/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^
(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*
d+b*c)/c/b)^(1/2))-(2/5*a*c*f+a*d*e+b*c*e-1/3*(a*d*f+b*c*f+b*d*e-1/5*f*(4*
a*d+4*b*c))/b/d*(2*a*d+2*b*c))*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c
)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(-b/a)^(1/2),(-
1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))
))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 313, normalized size of antiderivative = 0.82

$$\int \sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)\,dx =$$

$$-\frac{\sqrt{bd}(5\left(b^2c^2d+abcd^2\right)e-2\left(b^2c^3-abc^2d+a^2cd^2\right)f)x\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ad}{bc})-\sqrt{bd}(5\left(b^2c^2d+\right.}{}$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e),x, algorithm="fricas")
```

output
```
-1/15*(sqrt(b*d)*(5*(b^2*c^2*d + a*b*c*d^2)*e - 2*(b^2*c^3 - a*b*c^2*d + a
^2*c*d^2)*f)*x*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - sq
rt(b*d)*(5*(b^2*c^2*d + a*b*c*d^2 + 2*a*b*d^3)*e - (2*b^2*c^3 - 2*a*b*c^2*
d + a^2*d^3 + (2*a^2 + a*b)*c*d^2)*f)*x*sqrt(-c/d)*elliptic_f(arcsin(sqrt(
-c/d)/x), a*d/(b*c)) - (3*b^2*d^3*f*x^4 + (5*b^2*d^3*e + (b^2*c*d^2 + a*b*
d^3)*f)*x^2 + 5*(b^2*c*d^2 + a*b*d^3)*e - 2*(b^2*c^2*d - a*b*c*d^2 + a^2*d
^3)*f)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(b^2*d^3*x)
```

Sympy [F]

$$\int \sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)\,dx = \int \sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(1/2)*(f*x**2+e),x)
```

output
```
Integral(sqrt(a + b*x**2)*sqrt(c + d*x**2)*(e + f*x**2), x)
```

Maxima [F]

$$\int \sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)\,dx=\int \sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e), x)
```

Giac [F]

$$\int \sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)\,dx=\int \sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)\,dx=\int \sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,\left(f\,x^2+e\right)\,dx$$

input
```
int((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2),x)
```

output
```
int((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2), x)
```

Reduce [F]

$$\int \sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)\,dx$$
$$=\frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,adfx+\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,bcfx+5\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,bdex+3\sqrt{d\,x^2+c}\,\sqrt{b\,x^2}}{}$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e),x)
```

output
```
(sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*d*f*x + sqrt(c + d*x**2)*sqrt(a + b*x
**2)*b*c*f*x + 5*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b*d*e*x + 3*sqrt(c + d*
x**2)*sqrt(a + b*x**2)*b*d*f*x**3 - 2*int((sqrt(c + d*x**2)*sqrt(a + b*x**
2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a**2*d**2*f + 2*int((sq
rt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**
4),x)*a*b*c*d*f + 5*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*
d*x**2 + b*c*x**2 + b*d*x**4),x)*a*b*d**2*e - 2*int((sqrt(c + d*x**2)*sqrt
(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*b**2*c**2*f +
 5*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2
 + b*d*x**4),x)*b**2*c*d*e - int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c
+ a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a**2*c*d*f - int((sqrt(c + d*x**2)*sq
rt(a + b*x**2))/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a*b*c**2*f + 10*
int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c + a*d*x**2 + b*c*x**2 + b*d*x
**4),x)*a*b*c*d*e)/(15*b*d)
```

3.63 $\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)}{x^2}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 324

$$\begin{aligned}\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)}{x^2}\,dx &= \frac{(6bde+bcf+adf)x\sqrt{c+dx^2}}{3d\sqrt{a+bx^2}} \\ &+ \frac{(3be+af)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3a} - \frac{e(a+bx^2)^{3/2}\sqrt{c+dx^2}}{ax} \\ &- \frac{\sqrt{a}(6bde+bcf+adf)\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{3\sqrt{b}d\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} \\ &+ \frac{\sqrt{a}(3bce+3ade+2acf)\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{3\sqrt{b}c\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}\end{aligned}$$

output
```
1/3*(a*d*f+b*c*f+6*b*d*e)*x*(d*x^2+c)^(1/2)/d/(b*x^2+a)^(1/2)+1/3*(a*f+3*b
*e)*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a-e*(b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/
a/x-1/3*a^(1/2)*(a*d*f+b*c*f+6*b*d*e)*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/
a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/b^(1/2)/d/(b*x^2+a)^(1/2)/(a*
(d*x^2+c)/c/(b*x^2+a))^(1/2)+1/3*a^(1/2)*(2*a*c*f+3*a*d*e+3*b*c*e)*(d*x^2+
c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/b^(1
/2)/c/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.47 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.69

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)}{x^2}\,dx$$
$$= \frac{\sqrt{\frac{b}{a}}d(a+bx^2)(c+dx^2)(-3e+fx^2) - ic(6bde+bcf+adf)x\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(i\text{arcsinh}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right.}{3\sqrt{\frac{b}{a}}dx\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

input
```
Integrate[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2))/x^2,x]
```

output
```
(Sqrt[b/a]*d*(a + b*x^2)*(c + d*x^2)*(-3*e + f*x^2) - I*c*(6*b*d*e + b*c*f
 + a*d*f)*x*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sq
rt[b/a]*x], (a*d)/(b*c)] - I*(-(b*c) + a*d)*(3*d*e + c*f)*x*Sqrt[1 + (b*x^
2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/
(3*Sqrt[b/a]*d*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 312, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {442, 403, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)}{x^2}\,dx$$

$$\Big\downarrow \; 442$$

$$\frac{\int \frac{\sqrt{bx^2+a}\left(d(3be+af)x^2+2bce+ade+acf\right)}{\sqrt{dx^2+c}}dx}{a} - \frac{e\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{ax}$$

$$\Big\downarrow \; 403$$

$$\frac{\frac{\frac{\int \frac{ad\left((6bde+bcf+adf)x^2+3bce+3ade+2acf\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3d}+\frac{1}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}(af+3be)}{a}-\frac{e\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{ax}}{}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{1}{3}a\int \frac{(6bde+bcf+adf)x^2+3bce+3ade+2acf}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{1}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}(af+3be)}{a}-\frac{e\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{ax}$$

$$\Big\downarrow \; 406$$

$$\frac{\frac{1}{3}a\left((2acf+3ade+3bce)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+(adf+bcf+6bde)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)+\frac{1}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}(\ldots}{a}$$
$$\frac{e\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{ax}$$

$$\Big\downarrow \; 320$$

$$\frac{\frac{1}{3}a\left((adf+bcf+6bde)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{\sqrt{c}\sqrt{a+bx^2}(2acf+3ade+3bce)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{1}{3}x\sqrt{a+bx^2}\ldots}{a}$$
$$\frac{e\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{ax}$$

$$\Big\downarrow \; 388$$

$$\frac{\frac{1}{3}a\left((adf+bcf+6bde)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{\sqrt{c}\sqrt{a+bx^2}(2acf+3ade+3bce)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\ldots}{a}$$
$$\frac{e\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{ax}$$

$$\Big\downarrow \; 313$$

$$\frac{\frac{1}{3}a\left(\frac{\sqrt{c}\sqrt{a+bx^2}(2acf+3ade+3bce)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+(adf+bcf+6bde)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{dx}}{\sqrt{c}}\right.\right.}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx\right.}{a\left(c+dx\right.}}}\right.\right.}{a}$$

$$\frac{e\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{ax}$$

input
```
Int[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2))/x^2,x]
```

output
```
-((e*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(a*x)) + (((3*b*e + a*f)*x*Sqrt[a
+ b*x^2]*Sqrt[c + d*x^2])/3 + (a*((6*b*d*e + b*c*f + a*d*f)*((x*Sqrt[a + b
*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sq
rt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c
 + d*x^2))]*Sqrt[c + d*x^2])) + (Sqrt[c]*(3*b*c*e + 3*a*d*e + 2*a*c*f)*Sqr
t[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*S
qrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])))/3)/a
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 442
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^q/(a*g*(m + 1))), x] - Simp[1/(a*g^2*(m + 1))   Int[(g*x)
^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[c*(b*e - a*f)*(m + 1) + e*2
*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*2*(p + q + 1))*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && GtQ[q, 0] && LtQ[m, -1]
&&  !(EqQ[q, 1] && SimplerQ[e + f*x^2, c + d*x^2])
```

Maple [A] (verified)

Time = 4.00 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.06

method	result
elliptic	$\frac{\sqrt{(b x^2+a)(x^2 d+c)}\left(-\frac{e\sqrt{bd x^4+ad x^2+x^2bc+ac}}{x}+\frac{f x\sqrt{bd x^4+ad x^2+x^2bc+ac}}{3}+\frac{\left(\frac{2}{3}acf+ade+bce\right)\sqrt{1+\frac{b x^2}{a}}\sqrt{1+\frac{d x^2}{c}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\cdots}\right.}{\sqrt{-\frac{b}{a}}\sqrt{bd x^4+ad x^2+x^2bc+ac}}\right.}{\sqrt{b x^2+a}\cdots}$
risch	$-\frac{\sqrt{b x^2+a}\sqrt{x^2 d+c}\,(-f x^2+3e)}{3x}+\frac{\left(-\frac{(adf+bcf+6bde)c\sqrt{1+\frac{b x^2}{a}}\sqrt{1+\frac{d x^2}{c}}\left(\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)-\mathrm{EllipticE}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1\cdots}\right)\right)}{3\sqrt{-\frac{b}{a}}\sqrt{bd x^4+ad x^2+x^2bc+ac}\,d}\right.}{\cdots}$
default	$\frac{\sqrt{b x^2+a}\sqrt{x^2 d+c}\left(\sqrt{-\frac{b}{a}}\,b d^2 f x^6+\sqrt{-\frac{b}{a}}\,a d^2 f x^4+\sqrt{-\frac{b}{a}}\,bcdf x^4-3\sqrt{-\frac{b}{a}}\,b d^2 e x^4+\sqrt{\frac{b x^2+a}{a}}\sqrt{\frac{x^2 d+c}{c}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)\cdots\right.}{\cdots}$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)/x^2,x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-e*(b*d*x^4+a
*d*x^2+b*c*x^2+a*c)^(1/2)/x+1/3*f*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+(2
/3*a*c*f+a*d*e+b*c*e)/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*
d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/
b)^(1/2))-(a*d*f+b*c*f+2*b*d*e-1/3*f*(2*a*d+2*b*c))*c/(-b/a)^(1/2)*(1+b*x^
2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*(Ellipt
icF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+
(a*d+b*c)/c/b)^(1/2))))
```

Fricas [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)}{x^2}\,dx=\int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)}{x^2}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)/x^2,x, algorithm="fric
as")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)/x^2, x)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)}{x^2}\,dx = \int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)}{x^2}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(1/2)*(f*x**2+e)/x**2,x)
```

output
```
Integral(sqrt(a + b*x**2)*sqrt(c + d*x**2)*(e + f*x**2)/x**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)}{x^2}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)}{x^2}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)/x^2,x, algorithm="maxi
ma")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)/x^2, x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)}{x^2}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)}{x^2}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)/x^2,x, algorithm="giac
")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)}{x^2}\,dx=\int \frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,\left(f\,x^2+e\right)}{x^2}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2))/x^2,x)
```

output
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2))/x^2, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)}{x^2}\,dx$$
$$=\frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,adf+\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,bcf+3\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,bde+\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,b}{}$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)/x^2,x)
```

output
```
(sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*d*f + sqrt(c + d*x**2)*sqrt(a + b*x**
2)*b*c*f + 3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b*d*e + sqrt(c + d*x**2)*sq
rt(a + b*x**2)*b*d*f*x**2 + int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c*x
**2 + a*d*x**4 + b*c*x**4 + b*d*x**6),x)*a**2*c*d*f*x + int((sqrt(c + d*x*
*2)*sqrt(a + b*x**2))/(a*c*x**2 + a*d*x**4 + b*c*x**4 + b*d*x**6),x)*a*b*c
**2*f*x + 6*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c*x**2 + a*d*x**4 +
 b*c*x**4 + b*d*x**6),x)*a*b*c*d*e*x + 2*int((sqrt(c + d*x**2)*sqrt(a + b*
x**2))/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a*b*c*d*f*x + 3*int((sqrt
(c + d*x**2)*sqrt(a + b*x**2))/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a
*b*d**2*e*x + 3*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c + a*d*x**2 +
b*c*x**2 + b*d*x**4),x)*b**2*c*d*e*x)/(3*b*d*x)
```

3.64 $\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)}{x^4}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 332

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)}{x^4}\,dx = \frac{b(bce+ade+6acf)x\sqrt{c+dx^2}}{3ac\sqrt{a+bx^2}} - \frac{(de+3cf)\sqrt{a+bx^2}\sqrt{c+dx^2}}{3cx} - \frac{e(a+bx^2)^{3/2}\sqrt{c+dx^2}}{3ax^3} - \frac{\sqrt{b}(bce+ade+6acf)\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{3\sqrt{a}c\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} + \frac{\sqrt{a}(2bde+3bcf+3adf)\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{3\sqrt{b}c\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
1/3*b*(6*a*c*f+a*d*e+b*c*e)*x*(d*x^2+c)^(1/2)/a/c/(b*x^2+a)^(1/2)-1/3*(3*c
*f+d*e)*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/c/x-1/3*e*(b*x^2+a)^(3/2)*(d*x^2+c
)^(1/2)/a/x^3-1/3*b^(1/2)*(6*a*c*f+a*d*e+b*c*e)*(d*x^2+c)^(1/2)*EllipticE(
b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/a^(1/2)/c/(b*x^2+a)
^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)+1/3*a^(1/2)*(3*a*d*f+3*b*c*f+2*b*d*
e)*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(
1/2))/b^(1/2)/c/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.59 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.76

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)}{x^4}\,dx$$
$$= \frac{\sqrt{\frac{b}{a}}\left(-\sqrt{\frac{b}{a}}(a+bx^2)\,(c+dx^2)\,(bcex^2+adex^2+ac(e+3fx^2))-ibc(bce+ade+6acf)x^3\sqrt{1+\frac{bx^2}{a}}\sqrt{1\right.}{3bcx^3\sqrt{c}}$$

input
```
Integrate[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2))/x^4,x]
```

output
```
(Sqrt[b/a]*(-(Sqrt[b/a]*(a + b*x^2)*(c + d*x^2)*(b*c*e*x^2 + a*d*e*x^2 + a
*c*(e + 3*f*x^2))) - I*b*c*(b*c*e + a*d*e + 6*a*c*f)*x^3*Sqrt[1 + (b*x^2)/
a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*
c*(-(b*c) + a*d)*(b*e + 3*a*f)*x^3*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]
*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(3*b*c*x^3*Sqrt[a + b*x^
2]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 319, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {442, 442, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)}{x^4}dx$$

$$\Big\downarrow \; 442$$

$$\frac{\int \frac{\sqrt{bx^2+a}(d(be+3af)x^2+a(de+3cf))}{x^2\sqrt{dx^2+c}}dx}{3a}-\frac{e(a+bx^2)^{3/2}\sqrt{c+dx^2}}{3ax^3}$$

$$\Big\downarrow \; 442$$

$$\frac{\frac{\int \frac{bd(bce+ade+6acf)x^2+ac(2bde+3bcf+3adf)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{c}-\frac{a\sqrt{a+bx^2}\sqrt{c+dx^2}(3cf+de)}{cx}}{3a}-\frac{e\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{3ax^3}$$

$\Big\downarrow$ 406

$$\frac{\frac{ac(3adf+3bcf+2bde)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+bd(6acf+ade+bce)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{c}-\frac{a\sqrt{a+bx^2}\sqrt{c+dx^2}(3cf+de)}{cx}}{3a}-\frac{e\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{3ax^3}$$

$\Big\downarrow$ 320

$$\frac{\frac{bd(6acf+ade+bce)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}(3adf+3bcf+2bde)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{c}-\frac{a\sqrt{a+bx^2}\sqrt{c+dx^2}(3cf+de)}{cx}}{3a}-\frac{e\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{3ax^3}$$

$\Big\downarrow$ 388

$$\frac{\frac{bd(6acf+ade+bce)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(3adf+3bcf+2bde)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{c}-\frac{a\sqrt{a+bx^2}\sqrt{c+dx^2}(3cf+a}{cx}}{3a}-\frac{e\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{3ax^3}$$

$\Big\downarrow$ 313

$$\frac{\frac{\frac{c^{3/2}\sqrt{a+bx^2}(3adf+3bcf+2bde)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+bd(6acf+ade+bce)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{c}-a\sqrt{a+bx}}{3a}-\frac{e\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{3ax^3}$$

input
```
Int[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2))/x^4,x]
```

output
```
-1/3*(e*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(a*x^3) + (-((a*(d*e + 3*c*f)*S
qrt[a + b*x^2]*Sqrt[c + d*x^2])/(c*x)) + (b*d*(b*c*e + a*d*e + 6*a*c*f)*((
x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*Elliptic
E[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b
*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (c^(3/2)*(2*b*d*e + 3*b*c*f +
3*a*d*f)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/
(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))/c
)/(3*a)
```

Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 442
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^q/(a*g*(m + 1))), x] - Simp[1/(a*g^2*(m + 1))   Int[(g*x)
^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[c*(b*e - a*f)*(m + 1) + e*2
*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*2*(p + q + 1))*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && GtQ[q, 0] && LtQ[m, -1]
&&  !(EqQ[q, 1] && SimplerQ[e + f*x^2, c + d*x^2])
```

Maple [A] (verified)

Time = 5.26 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.11

method	result
elliptic	$\frac{\sqrt{(b x^2+a)(x^2d+c)}\,\Bigg(-\frac{e\sqrt{bd x^4+ad x^2+x^2bc+ac}}{3x^3}-\frac{(3acf+ade+bce)\sqrt{bd x^4+ad x^2+x^2bc+ac}}{3acx}+\frac{\left(adf+bcf+\frac{2}{3}bde\right)\sqrt{1+\frac{b x^2}{a}}\,\sqrt{1+\frac{d x^2}{c}}\,\text{Ellipti}}{\sqrt{-\frac{b}{a}}\,\sqrt{bd x^4+ad x^2+x^2}}}{\sqrt{b x^2}}$
risch	$-\frac{\sqrt{b x^2+a}\,\sqrt{x^2d+c}\,(3acf\, x^2+ade\, x^2+bce\, x^2+ace)}{3x^3ac}+\frac{\Bigg(-\frac{b(6acf+ade+bce)c\sqrt{1+\frac{b x^2}{a}}\,\sqrt{1+\frac{d x^2}{c}}\,\left(\text{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)-\text{E}\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{bd x^4+ad x^2+x^2bc+ac}}}{}$
default	$\frac{\sqrt{b x^2+a}\,\sqrt{x^2d+c}\,\Bigg(-3\sqrt{-\frac{b}{a}}\,abcdf\, x^6-\sqrt{-\frac{b}{a}}\,ab\, d^2e\, x^6-\sqrt{-\frac{b}{a}}\,b^2cde\, x^6+3\sqrt{\frac{b x^2+a}{a}}\,\sqrt{\frac{x^2d+c}{c}}\,\text{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)a^2cdf\, x^3-}{}$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)/x^4,x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-1/3*e*(b*d*x
^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/x^3-1/3*(3*a*c*f+a*d*e+b*c*e)/a/c*(b*d*x^4+a
*d*x^2+b*c*x^2+a*c)^(1/2)/x+(a*d*f+b*c*f+2/3*b*d*e)/(-b/a)^(1/2)*(1+b*x^2/
a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x
*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-(b*d*f+1/3*b*d*(3*a*c*f+a*d*e+b*c*
e)/a/c)*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^
2+b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))
-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))))
```

Fricas [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)}{x^4}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)}{x^4}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)/x^4,x, algorithm="fric
as")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)/x^4, x)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)}{x^4}\,dx = \int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)}{x^4}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(1/2)*(f*x**2+e)/x**4,x)
```

output
```
Integral(sqrt(a + b*x**2)*sqrt(c + d*x**2)*(e + f*x**2)/x**4, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)}{x^4}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)}{x^4}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)/x^4,x, algorithm="maxi
ma")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)/x^4, x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)}{x^4}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)}{x^4}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)/x^4,x, algorithm="giac
")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)}{x^4}\,dx = \int \frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,(f\,x^2+e)}{x^4}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2))/x^4,x)
```

output
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2))/x^4, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)}{x^4}\,dx = \text{Too large to display}$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)/x^4,x)
```

output

```
( - 2*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*c*f - sqrt(c + d*x**2)*sqrt(a +
b*x**2)*a*d*e + 2*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*d*f*x**2 - sqrt(c +
d*x**2)*sqrt(a + b*x**2)*b*c*e + 2*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b*c*f
*x**2 - 6*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a**2*c*d*x**4 + a**2*d*
*2*x**6 + a*b*c**2*x**4 + 2*a*b*c*d*x**6 + a*b*d**2*x**8 + b**2*c**2*x**6
+ b**2*c*d*x**8),x)*a**3*c**2*d*f*x**3 - int((sqrt(c + d*x**2)*sqrt(a + b*
x**2))/(a**2*c*d*x**4 + a**2*d**2*x**6 + a*b*c**2*x**4 + 2*a*b*c*d*x**6 +
a*b*d**2*x**8 + b**2*c**2*x**6 + b**2*c*d*x**8),x)*a**3*c*d**2*e*x**3 - 6*
int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a**2*c*d*x**4 + a**2*d**2*x**6 +
a*b*c**2*x**4 + 2*a*b*c*d*x**6 + a*b*d**2*x**8 + b**2*c**2*x**6 + b**2*c*d
*x**8),x)*a**2*b*c**3*f*x**3 - 2*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(
a**2*c*d*x**4 + a**2*d**2*x**6 + a*b*c**2*x**4 + 2*a*b*c*d*x**6 + a*b*d**2
*x**8 + b**2*c**2*x**6 + b**2*c*d*x**8),x)*a**2*b*c**2*d*e*x**3 - int((sqr
t(c + d*x**2)*sqrt(a + b*x**2))/(a**2*c*d*x**4 + a**2*d**2*x**6 + a*b*c**2
*x**4 + 2*a*b*c*d*x**6 + a*b*d**2*x**8 + b**2*c**2*x**6 + b**2*c*d*x**8),x
)*a*b**2*c**3*e*x**3 + 2*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a**2*c*d
 + a**2*d**2*x**2 + a*b*c**2 + 2*a*b*c*d*x**2 + a*b*d**2*x**4 + b**2*c**2*
x**2 + b**2*c*d*x**4),x)*a**3*d**3*f*x**3 + 4*int((sqrt(c + d*x**2)*sqrt(a
 + b*x**2))/(a**2*c*d + a**2*d**2*x**2 + a*b*c**2 + 2*a*b*c*d*x**2 + a*b*d
**2*x**4 + b**2*c**2*x**2 + b**2*c*d*x**4),x)*a**2*b*c*d**2*f*x**3 + in...
```

3.65 $\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)}{x^6}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 391

$$\begin{aligned}
&\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)}{x^6}\,dx \\
&= \frac{(2b^2c^2e+a^2d(2de-5cf)-abc(2de+5cf))\sqrt{c+dx^2}}{15ac^2x\sqrt{a+bx^2}} \\
&\quad + \frac{(2bce-ade-5acf)\sqrt{a+bx^2}\sqrt{c+dx^2}}{15acx^3} - \frac{e(a+bx^2)^{3/2}\sqrt{c+dx^2}}{5ax^5} \\
&\quad + \frac{\sqrt{b}(2b^2c^2e+a^2d(2de-5cf)-abc(2de+5cf))\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{15a^{3/2}c^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} \\
&\quad - \frac{\sqrt{b}d(bce+ade-10acf)\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{15\sqrt{a}c^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}
\end{aligned}$$

output
```
1/15*(2*b^2*c^2*e+a^2*d*(-5*c*f+2*d*e)-a*b*c*(5*c*f+2*d*e))*(d*x^2+c)^(1/2
)/a/c^2/x/(b*x^2+a)^(1/2)+1/15*(-5*a*c*f-a*d*e+2*b*c*e)*(b*x^2+a)^(1/2)*(d
*x^2+c)^(1/2)/a/c/x^3-1/5*e*(b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/a/x^5+1/15*b^(
1/2)*(2*b^2*c^2*e+a^2*d*(-5*c*f+2*d*e)-a*b*c*(5*c*f+2*d*e))*(d*x^2+c)^(1/2
)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/a^(3/2)
/c^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)-1/15*b^(1/2)*d*(-10*a
*c*f+a*d*e+b*c*e)*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)
),(1-a*d/b/c)^(1/2))/a^(1/2)/c^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))
^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.93 (sec) , antiderivative size = 324, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)}{x^6}\,dx = \frac{-\sqrt{\frac{b}{a}}(a+bx^2)(c+dx^2)(-2b^2c^2ex^4+a^2(c+dx^2)(3ce-2dex^2+5cfx^2)+abcx^2(2dex^2+c(e+5fx^2)$$

input
```
Integrate[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2))/x^6,x]
```

output
```
(-(Sqrt[b/a]*(a + b*x^2)*(c + d*x^2)*(-2*b^2*c^2*e*x^4 + a^2*(c + d*x^2)*(
3*c*e - 2*d*e*x^2 + 5*c*f*x^2) + a*b*c*x^2*(2*d*e*x^2 + c*(e + 5*f*x^2))))
 - I*b*c*(-2*b^2*c^2*e + a^2*d*(-2*d*e + 5*c*f) + a*b*c*(2*d*e + 5*c*f))*x
^5*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x
], (a*d)/(b*c)] - I*b*c*(-(b*c) + a*d)*(-2*b*c*e + a*d*e + 5*a*c*f)*x^5*Sq
rt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a
*d)/(b*c)])/(15*a^2*Sqrt[b/a]*c^2*x^5*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 424, normalized size of antiderivative = 1.08, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.303$, Rules used = {442, 25, 442, 445, 25, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)}{x^6}\,dx$$

$$\Big\downarrow \; 442$$

$$\frac{\int -\frac{\sqrt{bx^2+a}\left(d(be-5af)x^2+2bce-ade-5acf\right)}{x^4\sqrt{dx^2+c}}dx}{5a}-\frac{e\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{5ax^5}$$

$$\Big\downarrow \; 25$$

$$-\frac{\int \frac{\sqrt{bx^2+a}\left(d(be-5af)x^2+2bce-ade-5acf\right)}{x^4\sqrt{dx^2+c}}dx}{5a}-\frac{e\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{5ax^5}$$

$$\Big\downarrow \; 442$$

$$-\frac{\frac{\int \frac{d(2de-5cf)a^2-bc(2de+5cf)a+bd(bce+ade-10acf)x^2+2b^2c^2e}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(-5acf-ade+2bce)}{3cx^3}}{5a}-\frac{e\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{5ax^5}$$

$$\Big\downarrow \; 445$$

$$-\frac{\frac{-\frac{\int -\frac{bd\left(\left(d(2de-5cf)a^2-bc(2de+5cf)a+2b^2c^2e\right)x^2+ac(bce+ade-10acf)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{2b^2ce}{a}+\frac{2ad^2e}{c}-5adf-5bcf-2bde\right)}{x}}{3c}-\sqrt{a+bx^2}\sqrt{c+d\ldots}}{5a}$$

$$\frac{e\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{5ax^5}$$

$$\Big\downarrow \; 25$$

$$-\frac{\frac{\frac{\int \frac{bd\left(\left(d(2de-5cf)a^2-bc(2de+5cf)a+2b^2c^2e\right)x^2+ac(bce+ade-10acf)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{2b^2ce}{a}+\frac{2ad^2e}{c}-5adf-5bcf-2bde\right)}{x}}{3c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(}{3c}}{5a}$$

$$\frac{e\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{5ax^5}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{\frac{bd\int \frac{\left(d(2de-5cf)a^2-bc(2de+5cf)a+2b^2c^2e\right)x^2+ac(bce+ade-10acf)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{2b^2ce}{a}+\frac{2ad^2e}{c}-5adf-5bcf-2bde\right)}{x}}{3c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(-}{3cx}}{5a}$$

$$\frac{e\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{5ax^5}$$

$$\Big\downarrow \; 406$$

$$-\frac{\frac{\frac{bd\left(\left(a^2d(2de-5cf)-abc(5cf+2de)+2b^2c^2e\right)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+ac(-10acf+ade+bce)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{2b^2ce}{a}+\frac{2ad^2e}{c}-5adf\right.}{x}}{3c}}{5a}$$

$$\frac{e\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{5ax^5}$$

$$\Big\downarrow \; 320$$

$$-\frac{\frac{\frac{bd\left(\left(a^2d(2de-5cf)-abc(5cf+2de)+2b^2c^2e\right)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}(-10acf+ade+bce)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{ac}-\sqrt{a+bx^2}\sqrt{c+dx}}{3c}}{5a}$$

$$\frac{e\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{5ax^5}$$

$$\Big\downarrow \; 388$$

$$-\frac{\frac{\frac{bd\left(\left(a^2d(2de-5cf)-abc(5cf+2de)+2b^2c^2e\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(-10acf+ade+bce)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{ac}-\sqrt{}}{3c}}{5a}$$

$$\frac{e\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{5ax^5}$$

$\Big\downarrow$ 313

$$-\frac{\frac{\frac{bd\left(\left(a^2d(2de-5cf)-abc(5cf+2de)+2b^2c^2e\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(-10acf+ade+bce)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\ldots\right.}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{ac}}{3c}}{5a}$$

$$\frac{e\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{5ax^5}$$

input
```
Int[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2))/x^6,x]
```

output
```
-1/5*(e*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(a*x^5) - (-1/3*((2*b*c*e - a*d
*e - 5*a*c*f)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(c*x^3) + (-((((2*b^2*c*e)/
a - 2*b*d*e + (2*a*d^2*e)/c - 5*b*c*f - 5*a*d*f)*Sqrt[a + b*x^2]*Sqrt[c +
d*x^2])/x) + (b*d*((2*b^2*c^2*e + a^2*d*(2*d*e - 5*c*f) - a*b*c*(2*d*e + 5
*c*f))*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]
*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[
(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (c^(3/2)*(b*c*e + a*d
*e - 10*a*c*f)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 -
(b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^
2])))/(a*c))/(3*c))/(5*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
 + d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 442
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^q/(a*g*(m + 1))), x] - Simp[1/(a*g^2*(m + 1))   Int[(g*x)
^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[c*(b*e - a*f)*(m + 1) + e*2
*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*2*(p + q + 1))*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && GtQ[q, 0] && LtQ[m, -1]
&&  !(EqQ[q, 1] && SimplerQ[e + f*x^2, c + d*x^2])
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 6.42 (sec) , antiderivative size = 477, normalized size of antiderivative = 1.22

method	result
elliptic	$\frac{\sqrt{(b\,x^2+a)(x^2d+c)}\left(-\frac{e\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{5x^5}-\frac{(5acf+ade+bce)\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{15ac\,x^3}-\frac{\left(5a^2cfd-2a^2d^2e+5ab\,c^2f+2abcde-2b^2c^2e\right)}{15a^2c^2x}\right.}{}$
risch	$-\frac{\sqrt{b\,x^2+a}\,\sqrt{x^2d+c}\,\left(5a^2cdf\,x^4-2a^2d^2e\,x^4+5ab\,c^2f\,x^4+2abcde\,x^4-2b^2c^2e\,x^4+5a^2c^2f\,x^2+a^2cde\,x^2+ab\,c^2e\,x^2+3a^2c^2e\right)}{15x^5a^2c^2}+\frac{bd\left(\right.}{}$
default	Expression too large to display

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)/x^6,x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-1/5*e*(b*d*x
^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/x^5-1/15*(5*a*c*f+a*d*e+b*c*e)/a/c*(b*d*x^4+
a*d*x^2+b*c*x^2+a*c)^(1/2)/x^3-1/15/a^2/c^2*(5*a^2*c*d*f-2*a^2*d^2*e+5*a*b
*c^2*f+2*a*b*c*d*e-2*b^2*c^2*e)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/x+(b*d
*f-1/15*b*d*(5*a*c*f+a*d*e+b*c*e)/a/c)/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d
*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2)
,(-1+(a*d+b*c)/c/b)^(1/2))-1/15*b*(5*a^2*c*d*f-2*a^2*d^2*e+5*a*b*c^2*f+2*a
*b*c*d*e-2*b^2*c^2*e)/a^2/c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/
2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+
b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 314, normalized size of antiderivative = 0.80

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)}{x^6}\,dx =$$
$$-\frac{\sqrt{ac}(2\left(b^3c^2-ab^2cd+a^2bd^2\right)e-5\left(ab^2c^2+a^2bcd\right)f)x^5\sqrt{-\frac{b}{a}}E(\arcsin\left(x\sqrt{-\frac{b}{a}}\right)\mid\frac{ad}{bc})-\sqrt{ac}((2\,b^3c^2+}{}$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)/x^6,x, algorithm="fric
as")
```

output
```
-1/15*(sqrt(a*c)*(2*(b^3*c^2 - a*b^2*c*d + a^2*b*d^2)*e - 5*(a*b^2*c^2 + a
^2*b*c*d)*f)*x^5*sqrt(-b/a)*elliptic_e(arcsin(x*sqrt(-b/a)), a*d/(b*c)) -
sqrt(a*c)*((2*b^3*c^2 + (a^2*b - 2*a*b^2)*c*d + (a^3 + 2*a^2*b)*d^2)*e - 5
*(a*b^2*c^2 + (2*a^3 + a^2*b)*c*d)*f)*x^5*sqrt(-b/a)*elliptic_f(arcsin(x*s
qrt(-b/a)), a*d/(b*c)) + (3*a^3*c^2*e - (2*(a*b^2*c^2 - a^2*b*c*d + a^3*d^
2)*e - 5*(a^2*b*c^2 + a^3*c*d)*f)*x^4 + (5*a^3*c^2*f + (a^2*b*c^2 + a^3*c*
d)*e)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(a^3*c^2*x^5)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)}{x^6}\,dx = \int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)}{x^6}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(1/2)*(f*x**2+e)/x**6,x)
```

output
```
Integral(sqrt(a + b*x**2)*sqrt(c + d*x**2)*(e + f*x**2)/x**6, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)}{x^6}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)}{x^6}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)/x^6,x, algorithm="maxi
ma")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)/x^6, x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)}{x^6}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)}{x^6}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)/x^6,x, algorithm="giac
")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)}{x^6}\,dx = \int \frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,(f\,x^2+e)}{x^6}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2))/x^6,x)
```

output
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2))/x^6, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)}{x^6}\,dx = \text{Too large to display}$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)/x^6,x)
```

output

```
( - sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*c*e - 5*sqrt(c + d*x**2)*sqrt(a +
b*x**2)*a*c*f*x**2 + 5*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*d*f*x**4 + 5*sq
rt(c + d*x**2)*sqrt(a + b*x**2)*b*c*f*x**4 - 2*sqrt(c + d*x**2)*sqrt(a + b
*x**2)*b*d*e*x**4 - 5*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a**2*c
*d + a**2*d**2*x**2 + a*b*c**2 + 2*a*b*c*d*x**2 + a*b*d**2*x**4 + b**2*c**
2*x**2 + b**2*c*d*x**4),x)*a**2*b*d**3*f*x**5 - 10*int((sqrt(c + d*x**2)*s
qrt(a + b*x**2)*x**2)/(a**2*c*d + a**2*d**2*x**2 + a*b*c**2 + 2*a*b*c*d*x*
*2 + a*b*d**2*x**4 + b**2*c**2*x**2 + b**2*c*d*x**4),x)*a*b**2*c*d**2*f*x*
*5 + 2*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a**2*c*d + a**2*d**2*
x**2 + a*b*c**2 + 2*a*b*c*d*x**2 + a*b*d**2*x**4 + b**2*c**2*x**2 + b**2*c
*d*x**4),x)*a*b**2*d**3*e*x**5 - 5*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*
x**2)/(a**2*c*d + a**2*d**2*x**2 + a*b*c**2 + 2*a*b*c*d*x**2 + a*b*d**2*x*
*4 + b**2*c**2*x**2 + b**2*c*d*x**4),x)*b**3*c**2*d*f*x**5 + 2*int((sqrt(c
 + d*x**2)*sqrt(a + b*x**2)*x**2)/(a**2*c*d + a**2*d**2*x**2 + a*b*c**2 +
2*a*b*c*d*x**2 + a*b*d**2*x**4 + b**2*c**2*x**2 + b**2*c*d*x**4),x)*b**3*c
*d**2*e*x**5 - 10*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a**2*c*d*x**4 +
 a**2*d**2*x**6 + a*b*c**2*x**4 + 2*a*b*c*d*x**6 + a*b*d**2*x**8 + b**2*c*
*2*x**6 + b**2*c*d*x**8),x)*a**3*c**2*d*f*x**5 + int((sqrt(c + d*x**2)*sqr
t(a + b*x**2))/(a**2*c*d*x**4 + a**2*d**2*x**6 + a*b*c**2*x**4 + 2*a*b*c*d
*x**6 + a*b*d**2*x**8 + b**2*c**2*x**6 + b**2*c*d*x**8),x)*a**3*c*d**2*...
```

3.66 $\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)}{x^8}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 536

$$
\begin{aligned}
&\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)}{x^8}\,dx \\
&= -\frac{(8b^3c^3e-a^2bcd(5de-14cf)+2a^3d^2(4de-7cf)-ab^2c^2(5de+14cf))\sqrt{c+dx^2}}{105a^2c^3x\sqrt{a+bx^2}} \\
&\quad+\frac{(4bce-ade-7acf)\sqrt{a+bx^2}\sqrt{c+dx^2}}{35acx^5} \\
&\quad+\frac{(4b^2c^2e+a^2d(4de-7cf)-abc(2de+7cf))\sqrt{a+bx^2}\sqrt{c+dx^2}}{105a^2c^2x^3} \\
&\quad-\frac{e(a+bx^2)^{3/2}\sqrt{c+dx^2}}{7ax^7} \\
&\quad-\frac{\sqrt{b}(8b^3c^3e-a^2bcd(5de-14cf)+2a^3d^2(4de-7cf)-ab^2c^2(5de+14cf))\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\right.}{105a^{5/2}c^3\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} \\
&\quad+\frac{\sqrt{b}d(4b^2c^2e+a^2d(4de-7cf)-abc(2de+7cf))\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{105a^{3/2}c^3\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}
\end{aligned}
$$

output
```
-1/105*(8*b^3*c^3*e-a^2*b*c*d*(-14*c*f+5*d*e)+2*a^3*d^2*(-7*c*f+4*d*e)-a*b
^2*c^2*(14*c*f+5*d*e))*(d*x^2+c)^(1/2)/a^2/c^3/x/(b*x^2+a)^(1/2)+1/35*(-7*
a*c*f-a*d*e+4*b*c*e)*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a/c/x^5+1/105*(4*b^2*
c^2*e+a^2*d*(-7*c*f+4*d*e)-a*b*c*(7*c*f+2*d*e))*(b*x^2+a)^(1/2)*(d*x^2+c)^
(1/2)/a^2/c^2/x^3-1/7*e*(b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/a/x^7-1/105*b^(1/2
)*(8*b^3*c^3*e-a^2*b*c*d*(-14*c*f+5*d*e)+2*a^3*d^2*(-7*c*f+4*d*e)-a*b^2*c^
2*(14*c*f+5*d*e))*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^
(1/2),(1-a*d/b/c)^(1/2))/a^(5/2)/c^3/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2
+a))^(1/2)+1/105*b^(1/2)*d*(4*b^2*c^2*e+a^2*d*(-7*c*f+4*d*e)-a*b*c*(7*c*f+
2*d*e))*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b
/c)^(1/2))/a^(3/2)/c^3/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.26 (sec) , antiderivative size = 446, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)}{x^8}\,dx$$
$$= \frac{-\sqrt{\frac{b}{a}}(a+bx^2)(c+dx^2)(8b^3c^3ex^6-ab^2c^2x^4(4ce+5dex^2+14cfx^2)+a^2bcx^2(-5d^2ex^4+2cdx^2(e+7f}{}$$

input
```
Integrate[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2))/x^8,x]
```

output
```
(-(Sqrt[b/a]*(a + b*x^2)*(c + d*x^2)*(8*b^3*c^3*e*x^6 - a*b^2*c^2*x^4*(4*c
*e + 5*d*e*x^2 + 14*c*f*x^2) + a^2*b*c*x^2*(-5*d^2*e*x^4 + 2*c*d*x^2*(e +
7*f*x^2) + c^2*(3*e + 7*f*x^2)) + a^3*(c + d*x^2)*(8*d^2*e*x^4 + 3*c^2*(5*
e + 7*f*x^2) - 2*c*d*x^2*(6*e + 7*f*x^2)))) + I*b*c*(-8*b^3*c^3*e + a^2*b*
c*d*(5*d*e - 14*c*f) + 2*a^3*d^2*(-4*d*e + 7*c*f) + a*b^2*c^2*(5*d*e + 14*
c*f))*x^7*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt
[b/a]*x], (a*d)/(b*c)] + I*b*c*(b*c - a*d)*(8*b^2*c^2*e + a^2*d*(-4*d*e +
7*c*f) - a*b*c*(d*e + 14*c*f))*x^7*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]
*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(105*a^3*Sqrt[b/a]*c^3*x^
7*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 580, normalized size of antiderivative = 1.08, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {442, 25, 442, 445, 445, 25, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)}{x^8}dx$$

$$\Big\downarrow \text{ 442}$$

$$\frac{\int -\frac{\sqrt{bx^2+a}\left(d(3be-7af)x^2+4bce-ade-7acf\right)}{x^6\sqrt{dx^2+c}}dx}{7a}-\frac{e\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{7ax^7}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\int \frac{\sqrt{bx^2+a}\left(d(3be-7af)x^2+4bce-ade-7acf\right)}{x^6\sqrt{dx^2+c}}dx}{7a}-\frac{e\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{7ax^7}$$

$$\Big\downarrow \text{ 442}$$

$$-\frac{\frac{\int \frac{d(4de-7cf)a^2-bc(2de+7cf)a+bd(3bce+3ade-14acf)x^2+4b^2c^2e}{x^4\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{5c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(-7acf-ade+4bce)}{5cx^5}}{7a}-\frac{e\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{7ax^7}$$

$$\Big\downarrow \text{ 445}$$

$$-\frac{\frac{-\frac{\int \frac{2d^2(4de-7cf)a^3-bcd(5de-14cf)a^2-b^2c^2(5de+14cf)a+bd\left(d(4de-7cf)a^2-bc(2de+7cf)a+4b^2c^2e\right)x^2+8b^3c^3e}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{4b^2ce}{a}+\frac{4ad^2e}{c}-7\cdots\right)}{3x^3}}{5c}}{7a}-\frac{e\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{7ax^7}$$

$$\Big\downarrow \text{ 445}$$

$$-\frac{-\frac{-\frac{\int -\frac{bd\left(\left(2d^2(4de-7cf)a^3-bcd(5de-14cf)a^2-b^2c^2(5de+14cf)a+8b^3c^3e\right)x^2+ac\left(d(4de-7cf)a^2-bc(2de+7cf)a+4b^2c^2e\right)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\sqrt{a+bx^2}\sqrt{c+dx^2}\left(2a^3d^2(\ldots\right.}{3ac}}{5c}}{\ldots}$$

$$-\frac{e\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{7ax^7}$$

$\downarrow$ 25

$$-\frac{-\frac{\frac{\int \frac{bd\left(\left(2d^2(4de-7cf)a^3-bcd(5de-14cf)a^2-b^2c^2(5de+14cf)a+8b^3c^3e\right)x^2+ac\left(d(4de-7cf)a^2-bc(2de+7cf)a+4b^2c^2e\right)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\sqrt{a+bx^2}\sqrt{c+dx^2}\left(2a^3d^2(4de\ldots\right.}{3ac}}{5c}}{7\ldots}$$

$$-\frac{e\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{7ax^7}$$

$\downarrow$ 27

$$-\frac{-\frac{\frac{bd\int \frac{\left(2d^2(4de-7cf)a^3-bcd(5de-14cf)a^2-b^2c^2(5de+14cf)a+8b^3c^3e\right)x^2+ac\left(d(4de-7cf)a^2-bc(2de+7cf)a+4b^2c^2e\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\sqrt{a+bx^2}\sqrt{c+dx^2}\left(2a^3d^2(4de-\ldots\right.}{3ac}}{5c}}{7a\ldots}$$

$$-\frac{e\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{7ax^7}$$

$\downarrow$ 406

$$-\frac{-\frac{\frac{bd\left(ac\left(a^2d(4de-7cf)-abc(7cf+2de)+4b^2c^2e\right)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\left(2a^3d^2(4de-7cf)-a^2bcd(5de-14cf)-ab^2c^2(14cf+5de)+8b^3c^3e\right)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2\ldots}}\right.}{ac}}{3ac}}{5c}\ldots$$

$$-\frac{e\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{7ax^7}$$

$\downarrow$ 320

$$-\frac{-\frac{bd\left(\left(2a^3d^2(4de-7cf)-a^2bcd(5de-14cf)-ab^2c^2(14cf+5de)+8b^3c^3e\right)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}\left(a^2d(4de-7cf)-abc(7cf+2de)+4b^2c^2e\right)\text{Ellipt}\ldots}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right.}{ac}}{3ac}\ldots$$

$$-\frac{e\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{7ax^7}$$

$\downarrow$ 388

$$-\frac{-\frac{\frac{bd\left(\left(2a^3d^2(4de-7cf)-a^2bcd(5de-14cf)-ab^2c^2(14cf+5de)+8b^3c^3e\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}\left(a^2d(4de-7cf)-abc(7cf+2de)+4\cdots\right.}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\cdots}}\right)}{ac}}{3ac}}{}-\frac{e(a+bx^2)^{3/2}\sqrt{c+dx^2}}{7ax^7}$$

$\downarrow$ 313

$$-\frac{-\frac{\frac{bd\left(\frac{c^{3/2}\sqrt{a+bx^2}\left(a^2d(4de-7cf)-abc(7cf+2de)+4b^2c^2e\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\left(2a^3d^2(4de-7cf)-a^2bcd(5de-14cf)-ab^2c^2(14cf+5de)+8b^3\cdots\right.\right)}{ac}}{3ac}}{}-\frac{e(a+bx^2)^{3/2}\sqrt{c+dx^2}}{7ax^7}$$

input
```
Int[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2))/x^8,x]
```

output
```
-1/7*(e*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(a*x^7) - (-1/5*((4*b*c*e - a*d
*e - 7*a*c*f)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(c*x^5) + (-1/3*(((4*b^2*c*
e)/a - 2*b*d*e + (4*a*d^2*e)/c - 7*b*c*f - 7*a*d*f)*Sqrt[a + b*x^2]*Sqrt[c
 + d*x^2])/x^3 - (-(((8*b^3*c^3*e - a^2*b*c*d*(5*d*e - 14*c*f) + 2*a^3*d^2
*(4*d*e - 7*c*f) - a*b^2*c^2*(5*d*e + 14*c*f))*Sqrt[a + b*x^2]*Sqrt[c + d*
x^2])/(a*c*x)) + (b*d*((8*b^3*c^3*e - a^2*b*c*d*(5*d*e - 14*c*f) + 2*a^3*d
^2*(4*d*e - 7*c*f) - a*b^2*c^2*(5*d*e + 14*c*f))*((x*Sqrt[a + b*x^2])/(b*S
qrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sq
rt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]
*Sqrt[c + d*x^2])) + (c^(3/2)*(4*b^2*c^2*e + a^2*d*(4*d*e - 7*c*f) - a*b*c
*(2*d*e + 7*c*f))*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1
 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d
*x^2])))/(a*c))/(3*a*c))/(5*c))/(7*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 442
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^q/(a*g*(m + 1))), x] - Simp[1/(a*g^2*(m + 1))   Int[(g*x)
^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[c*(b*e - a*f)*(m + 1) + e*2
*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*2*(p + q + 1))*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && GtQ[q, 0] && LtQ[m, -1]
&&  !(EqQ[q, 1] && SimplerQ[e + f*x^2, c + d*x^2])
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 7.56 (sec) , antiderivative size = 636, normalized size of antiderivative = 1.19

method	result
elliptic	$\frac{\sqrt{(bx^2+a)(x^2d+c)}\left(-\frac{e\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{7x^7}-\frac{(7acf+ade+bce)\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{35ac\,x^5}-\frac{\left(7a^2cfd-4a^2d^2e+7ab\,c^2f+2abcde-4b^2c^2e\right)}{105a^2c^2x^3}\right.}{}$
risch	$-\frac{\sqrt{bx^2+a}\,\sqrt{x^2d+c}\,(-14a^3cd^2f\,x^6+8a^3d^3e\,x^6+14a^2b\,c^2df\,x^6-5a^2bc\,d^2e\,x^6-14a\,b^2c^3f\,x^6-5a\,b^2c^2de\,x^6+8b^3c^3e\,x^6+7a^3c^2df\,x^4}{105x^7a^3c^3}$
default	Expression too large to display

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)/x^8,x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-1/7*e*(b*d*x
^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/x^7-1/35*(7*a*c*f+a*d*e+b*c*e)/a/c*(b*d*x^4+
a*d*x^2+b*c*x^2+a*c)^(1/2)/x^5-1/105/a^2/c^2*(7*a^2*c*d*f-4*a^2*d^2*e+7*a*
b*c^2*f+2*a*b*c*d*e-4*b^2*c^2*e)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/x^3+1
/105/a^3/c^3*(14*a^3*c*d^2*f-8*a^3*d^3*e-14*a^2*b*c^2*d*f+5*a^2*b*c*d^2*e+
14*a*b^2*c^3*f+5*a*b^2*c^2*d*e-8*b^3*c^3*e)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^
(1/2)/x-1/105*(7*a^2*c*d*f-4*a^2*d^2*e+7*a*b*c^2*f+2*a*b*c*d*e-4*b^2*c^2*e
)*b*d/a^2/c^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*
d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2)
)+1/105*b*(14*a^3*c*d^2*f-8*a^3*d^3*e-14*a^2*b*c^2*d*f+5*a^2*b*c*d^2*e+14*
a*b^2*c^3*f+5*a*b^2*c^2*d*e-8*b^3*c^3*e)/c^2/a^3/(-b/a)^(1/2)*(1+b*x^2/a)^
(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(EllipticF(x*(
-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*
c)/c/b)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 483, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)}{x^8}\,dx$$
$$= \frac{\sqrt{ac}((8\,b^4c^3-5\,ab^3c^2d-5\,a^2b^2cd^2+8\,a^3bd^3)e-14\,(ab^3c^3-a^2b^2c^2d+a^3bcd^2)f)x^7\sqrt{-\frac{b}{a}}E(\arcsin\left(x\sqrt{\right.}\right.}{}$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)/x^8,x, algorithm="fric
as")
```

output
```
1/105*(sqrt(a*c)*((8*b^4*c^3 - 5*a*b^3*c^2*d - 5*a^2*b^2*c*d^2 + 8*a^3*b*d
^3)*e - 14*(a*b^3*c^3 - a^2*b^2*c^2*d + a^3*b*c*d^2)*f)*x^7*sqrt(-b/a)*ell
iptic_e(arcsin(x*sqrt(-b/a)), a*d/(b*c)) - sqrt(a*c)*((8*b^4*c^3 + (4*a^2*
b^2 - 5*a*b^3)*c^2*d - (2*a^3*b + 5*a^2*b^2)*c*d^2 + 4*(a^4 + 2*a^3*b)*d^3
)*e - 7*(2*a*b^3*c^3 + (a^3*b - 2*a^2*b^2)*c^2*d + (a^4 + 2*a^3*b)*c*d^2)*
f)*x^7*sqrt(-b/a)*elliptic_f(arcsin(x*sqrt(-b/a)), a*d/(b*c)) - (15*a^4*c^
3*e + ((8*a*b^3*c^3 - 5*a^2*b^2*c^2*d - 5*a^3*b*c*d^2 + 8*a^4*d^3)*e - 14*
(a^2*b^2*c^3 - a^3*b*c^2*d + a^4*c*d^2)*f)*x^6 - (2*(2*a^2*b^2*c^3 - a^3*b
*c^2*d + 2*a^4*c*d^2)*e - 7*(a^3*b*c^3 + a^4*c^2*d)*f)*x^4 + 3*(7*a^4*c^3*
f + (a^3*b*c^3 + a^4*c^2*d)*e)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(a^4*
c^3*x^7)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)}{x^8}\,dx = \int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)}{x^8}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(1/2)*(f*x**2+e)/x**8,x)
```

output
```
Integral(sqrt(a + b*x**2)*sqrt(c + d*x**2)*(e + f*x**2)/x**8, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)}{x^8}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)}{x^8}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)/x^8,x, algorithm="maxi
ma")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)/x^8, x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)}{x^8}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)}{x^8}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)/x^8,x, algorithm="giac
")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)/x^8, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)}{x^8}\,dx = \int \frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,\left(f\,x^2+e\right)}{x^8}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2))/x^8,x)
```

output
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2))/x^8, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)}{x^8}\,dx = \text{Too large to display}$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)/x^8,x)
```

output
```
( - 3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*e - 7*sqrt(c + d*x**2)*sqrt(a + b*
x**2)*f*x**2 - 14*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a**2*c*d*x**6 +
 a**2*d**2*x**8 + a*b*c**2*x**6 + 2*a*b*c*d*x**8 + a*b*d**2*x**10 + b**2*c
**2*x**8 + b**2*c*d*x**10),x)*a**2*c*d*f*x**7 + 3*int((sqrt(c + d*x**2)*sq
rt(a + b*x**2))/(a**2*c*d*x**6 + a**2*d**2*x**8 + a*b*c**2*x**6 + 2*a*b*c*
d*x**8 + a*b*d**2*x**10 + b**2*c**2*x**8 + b**2*c*d*x**10),x)*a**2*d**2*e*
x**7 - 14*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a**2*c*d*x**6 + a**2*d*
*2*x**8 + a*b*c**2*x**6 + 2*a*b*c*d*x**8 + a*b*d**2*x**10 + b**2*c**2*x**8
 + b**2*c*d*x**10),x)*a*b*c**2*f*x**7 + 6*int((sqrt(c + d*x**2)*sqrt(a + b
*x**2))/(a**2*c*d*x**6 + a**2*d**2*x**8 + a*b*c**2*x**6 + 2*a*b*c*d*x**8 +
 a*b*d**2*x**10 + b**2*c**2*x**8 + b**2*c*d*x**10),x)*a*b*c*d*e*x**7 + 3*i
nt((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a**2*c*d*x**6 + a**2*d**2*x**8 + a
*b*c**2*x**6 + 2*a*b*c*d*x**8 + a*b*d**2*x**10 + b**2*c**2*x**8 + b**2*c*d
*x**10),x)*b**2*c**2*e*x**7 - 7*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a
**2*c*d*x**4 + a**2*d**2*x**6 + a*b*c**2*x**4 + 2*a*b*c*d*x**6 + a*b*d**2*
x**8 + b**2*c**2*x**6 + b**2*c*d*x**8),x)*a**2*d**2*f*x**7 - 14*int((sqrt(
c + d*x**2)*sqrt(a + b*x**2))/(a**2*c*d*x**4 + a**2*d**2*x**6 + a*b*c**2*x
**4 + 2*a*b*c*d*x**6 + a*b*d**2*x**8 + b**2*c**2*x**6 + b**2*c*d*x**8),x)*
a*b*c*d*f*x**7 + 6*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a**2*c*d*x**4
+ a**2*d**2*x**6 + a*b*c**2*x**4 + 2*a*b*c*d*x**6 + a*b*d**2*x**8 + b**...
```

3.67 $\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)}{x^{10}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 695

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)}{x^{10}}\,dx$$
$$= \frac{(16b^4c^4e - a^3bcd^2(8de-15cf) - 3a^2b^2c^2d(2de-5cf) + 8a^4d^3(2de-3cf) - 8ab^3c^3(de+3cf))\sqrt{c+dx}}{315a^3c^4x\sqrt{a+bx^2}}$$
$$+ \frac{(6bce-ade-9acf)\sqrt{a+bx^2}\sqrt{c+dx^2}}{63acx^7}$$
$$+ \frac{\left(\frac{6b^2ce}{a} - 2bde + \frac{6ad^2e}{c} - 9bcf - 9adf\right)\sqrt{a+bx^2}\sqrt{c+dx^2}}{315acx^5}$$
$$- \frac{(8b^3c^3e + 4a^3d^2(2de-3cf) - 3a^2bcd(de-2cf) - 3ab^2c^2(de+4cf))\sqrt{a+bx^2}\sqrt{c+dx^2}}{315a^3c^3x^3}$$
$$- \frac{e(a+bx^2)^{3/2}\sqrt{c+dx^2}}{9ax^9}$$
$$+ \frac{\sqrt{b}(16b^4c^4e - a^3bcd^2(8de-15cf) - 3a^2b^2c^2d(2de-5cf) + 8a^4d^3(2de-3cf) - 8ab^3c^3(de+3cf))\sqrt{c}}{315a^{7/2}c^4\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$
$$- \frac{\sqrt{b}d(8b^3c^3e + 4a^3d^2(2de-3cf) - 3a^2bcd(de-2cf) - 3ab^2c^2(de+4cf))\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\right.}{315a^{5/2}c^4\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
1/315*(16*b^4*c^4*e-a^3*b*c*d^2*(-15*c*f+8*d*e)-3*a^2*b^2*c^2*d*(-5*c*f+2*
d*e)+8*a^4*d^3*(-3*c*f+2*d*e)-8*a*b^3*c^3*(3*c*f+d*e))*(d*x^2+c)^(1/2)/a^3
/c^4/x/(b*x^2+a)^(1/2)+1/63*(-9*a*c*f-a*d*e+6*b*c*e)*(b*x^2+a)^(1/2)*(d*x^
2+c)^(1/2)/a/c/x^7+1/315*(6*b^2*c*e/a-2*b*d*e+6*a*d^2*e/c-9*b*c*f-9*a*d*f)
*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a/c/x^5-1/315*(8*b^3*c^3*e+4*a^3*d^2*(-3*
c*f+2*d*e)-3*a^2*b*c*d*(-2*c*f+d*e)-3*a*b^2*c^2*(4*c*f+d*e))*(b*x^2+a)^(1/
2)*(d*x^2+c)^(1/2)/a^3/c^3/x^3-1/9*e*(b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/a/x^9
+1/315*b^(1/2)*(16*b^4*c^4*e-a^3*b*c*d^2*(-15*c*f+8*d*e)-3*a^2*b^2*c^2*d*(
-5*c*f+2*d*e)+8*a^4*d^3*(-3*c*f+2*d*e)-8*a*b^3*c^3*(3*c*f+d*e))*(d*x^2+c)^
(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/a^(
7/2)/c^4/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)-1/315*b^(1/2)*d*(
8*b^3*c^3*e+4*a^3*d^2*(-3*c*f+2*d*e)-3*a^2*b*c*d*(-2*c*f+d*e)-3*a*b^2*c^2*
(4*c*f+d*e))*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-
a*d/b/c)^(1/2))/a^(5/2)/c^4/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2
)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.94 (sec) , antiderivative size = 571, normalized size of antiderivative = 0.82

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)}{x^{10}}\,dx$$

$$= \frac{-\sqrt{\frac{b}{a}}(a+bx^2)\,(c+dx^2)\,(-16b^4c^4ex^8+8ab^3c^3x^6(dex^2+c(e+3fx^2))-3a^2b^2c^2x^4(-2d^2ex^4+2c^2(e+$$

input
```
Integrate[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2))/x^10,x]
```

output
```
(-(Sqrt[b/a]*(a + b*x^2)*(c + d*x^2)*(-16*b^4*c^4*e*x^8 + 8*a*b^3*c^3*x^6*
(d*e*x^2 + c*(e + 3*f*x^2)) - 3*a^2*b^2*c^2*x^4*(-2*d^2*e*x^4 + 2*c^2*(e +
 2*f*x^2) + c*d*x^2*(e + 5*f*x^2)) + a^3*b*c*x^2*(8*d^3*e*x^6 + 2*c^2*d*x^
2*(e + 3*f*x^2) - 3*c*d^2*x^4*(e + 5*f*x^2) + c^3*(5*e + 9*f*x^2)) + a^4*(
c + d*x^2)*(-16*d^3*e*x^6 + 24*c*d^2*x^4*(e + f*x^2) - 6*c^2*d*x^2*(5*e +
6*f*x^2) + 5*c^3*(7*e + 9*f*x^2)))) - I*b*c*(-16*b^4*c^4*e + a^3*b*c*d^2*(
8*d*e - 15*c*f) + 3*a^2*b^2*c^2*d*(2*d*e - 5*c*f) + 8*a^4*d^3*(-2*d*e + 3*
c*f) + 8*a*b^3*c^3*(d*e + 3*c*f))*x^9*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)
/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + I*b*c*(-(b*c) + a*d)*
(16*b^3*c^3*e - 24*a*b^2*c^3*f + 3*a^2*b*c*d*(-(d*e) + c*f) + 4*a^3*d^2*(-
2*d*e + 3*c*f))*x^9*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*Ar
cSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(315*a^4*Sqrt[b/a]*c^4*x^9*Sqrt[a + b*x^
2]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 1.34 (sec) , antiderivative size = 752, normalized size of antiderivative = 1.08, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.394$, Rules used = {442, 25, 442, 445, 27, 445, 445, 25, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)}{x^{10}}dx$$

$$\Big\downarrow \text{442}$$

$$\frac{\int -\frac{\sqrt{bx^2+a}\left(d(5be-9af)x^2+6bce-ade-9acf\right)}{x^8\sqrt{dx^2+c}}dx}{9a}-\frac{e\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{9ax^9}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{\sqrt{bx^2+a}\left(d(5be-9af)x^2+6bce-ade-9acf\right)}{x^8\sqrt{dx^2+c}}dx}{9a}-\frac{e\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{9ax^9}$$

$$\Big\downarrow \text{442}$$

$$-\frac{\frac{\int \frac{3d(2de-3cf)a^2-bc(2de+9cf)a+bd(5bce+5ade-18acf)x^2+6b^2c^2e}{x^6\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{7c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(-9acf-ade+6bce)}{7cx^7}}{9a}-\frac{e\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{9ax^9}$$

$\Big\downarrow$ 445

$$-\frac{-\frac{\int \frac{3\left(4d^2(2de-3cf)a^3-3bcd(de-2cf)a^2-3b^2c^2(de+4cf)a+bd\left(3d(2de-3cf)a^2-bc(2de+9cf)a+6b^2c^2e\right)x^2+8b^3c^3e\right)}{x^4\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{5ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{6b^2ce}{a}+\frac{6ad^2e}{c}\ldots\right.}{5x^5}}{7c}}{9a}\ldots$$

$$\frac{e\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{9ax^9}$$

$\Big\downarrow$ 27

$$-\frac{-\frac{3\int \frac{4d^2(2de-3cf)a^3-3bcd(de-2cf)a^2-3b^2c^2(de+4cf)a+bd\left(3d(2de-3cf)a^2-bc(2de+9cf)a+6b^2c^2e\right)x^2+8b^3c^3e}{x^4\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{5ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{6b^2ce}{a}+\frac{6ad^2e}{c}-9\ldots\right.}{5x^5}}{7c}}{9a}\ldots$$

$$\frac{e\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{9ax^9}$$

$\Big\downarrow$ 445

$$-\frac{-\frac{3\left(-\frac{\int \frac{8d^3(2de-3cf)a^4-bcd^2(8de-15cf)a^3-3b^2c^2d(2de-5cf)a^2-8b^3c^3(de+3cf)a+bd\left(4d^2(2de-3cf)a^3-3bcd(de-2cf)a^2-3b^2c^2(de+4cf)a+8b^3c^3e\right)x^2\ldots}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}}{3ac}\ldots\right.}{5ac}}{7c}\ldots$$

$$\frac{e\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{9ax^9}$$

$\Big\downarrow$ 445

$$-\frac{-\frac{3\left(-\frac{-\frac{\int -\frac{bd\left(\left(8d^3(2de-3cf)a^4-bcd^2(8de-15cf)a^3-3b^2c^2d(2de-5cf)a^2-8b^3c^3(de+3cf)a+16b^4c^4e\right)x^2+ac\left(4d^2(2de-3cf)a^3-3bcd(de-2cf)a^2-3b^2c^2(\ldots\right.\right.}{\sqrt{bx^2+a}\sqrt{dx^2+c}}}{ac}\ldots}{3ac}\ldots\right.}{\ldots}}{\ldots}\ldots$$

$$\frac{e\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{9ax^9}$$

$\Big\downarrow$ 25

$$-\frac{3\left(-\frac{\frac{\int \frac{bd\left(\left(8d^3(2de-3cf)a^4-bcd^2(8de-15cf)a^3-3b^2c^2d(2de-5cf)a^2-8b^3c^3(de+3cf)a+16b^4c^4e\right)x^2+ac\left(4d^2(2de-3cf)a^3-3bcd(de-2cf)a^2-3b^2c^2(de+\cdots}{\sqrt{bx^2+a}\sqrt{dx^2+c}}}{ac}}{3ac}\cdots\right)}{\cdots}$$

$$\frac{e\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{9ax^9}$$

$$\Big\downarrow \ 27$$

$$-\frac{3\left(-\frac{\frac{bd\int \frac{\left(8d^3(2de-3cf)a^4-bcd^2(8de-15cf)a^3-3b^2c^2d(2de-5cf)a^2-8b^3c^3(de+3cf)a+16b^4c^4e\right)x^2+ac\left(4d^2(2de-3cf)a^3-3bcd(de-2cf)a^2-3b^2c^2(de+4\cdots}{\sqrt{bx^2+a}\sqrt{dx^2+c}}}{ac}}{3ac}\cdots\right)}{\cdots}$$

$$\frac{e\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{9ax^9}$$

$$\Big\downarrow \ 406$$

$$-\frac{3\left(-\frac{bd\left(ac\left(4a^3d^2(2de-3cf)-3a^2bcd(de-2cf)-3ab^2c^2(4cf+de)+8b^3c^3e\right)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\left(8a^4d^3(2de-3cf)-a^3bcd^2(8de-15cf)-3a^2b^2c^2d(2de-\cdots\right)\right)}{ac}\cdots\right)}{\cdots}$$

$$\frac{e\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{9ax^9}$$

$$\Big\downarrow \ 320$$

$$-\frac{3\left(-\frac{bd\left(\left(8a^4d^3(2de-3cf)-a^3bcd^2(8de-15cf)-3a^2b^2c^2d(2de-5cf)-8ab^3c^3(3cf+de)+16b^4c^4e\right)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}\left(4a^3d^2(2de-3cf)\cdots\right)}{\cdots}\right)}{ac}\cdots\right)}{\cdots}$$

$$\frac{e\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{9ax^9}$$

$\Big\downarrow$ 388

$$-\frac{-\frac{3\left(-\frac{bd\left(\left(8a^4d^3(2de-3cf)-a^3bcd^2(8de-15cf)-3a^2b^2c^2d(2de-5cf)-8ab^3c^3(3cf+de)+16b^4c^4e\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}\left(4a^3 \cdots\right.}{\cdots}\right)}{ac}\cdots\right)}{\cdots}}{\cdots}-\frac{e\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{9ax^9}$$

$\Big\downarrow$ 313

$$-\frac{-\frac{3\left(-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(4a^3d^2(2de-3cf)-3a^2bcd(de-2cf)-3ab^2c^2(4cf+de)+8b^3c^3e\right)}{3acx^3}-\frac{bd\left(\frac{c^{3/2}\sqrt{a+bx^2}\left(4a^3d^2(2de-3cf)-3a^2bcd(de-2cf)-3ab^2c^2(4c\cdots\right.}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+\cdots\right.}{a\left(c+\cdots\right.}}}\cdots\right)}{\cdots}\cdots\right)}{\cdots}}{\cdots}-\frac{e\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{9ax^9}$$

input
```
Int[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2))/x^10,x]
```

output
```
-1/9*(e*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(a*x^9) - (-1/7*((6*b*c*e - a*d
*e - 9*a*c*f)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(c*x^7) + (-1/5*(((6*b^2*c*
e)/a - 2*b*d*e + (6*a*d^2*e)/c - 9*b*c*f - 9*a*d*f)*Sqrt[a + b*x^2]*Sqrt[c
 + d*x^2])/x^5 - (3*(-1/3*((8*b^3*c^3*e + 4*a^3*d^2*(2*d*e - 3*c*f) - 3*a^
2*b*c*d*(d*e - 2*c*f) - 3*a*b^2*c^2*(d*e + 4*c*f))*Sqrt[a + b*x^2]*Sqrt[c
+ d*x^2])/(a*c*x^3) - (-(((16*b^4*c^4*e - a^3*b*c*d^2*(8*d*e - 15*c*f) - 3
*a^2*b^2*c^2*d*(2*d*e - 5*c*f) + 8*a^4*d^3*(2*d*e - 3*c*f) - 8*a*b^3*c^3*(
d*e + 3*c*f))*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(a*c*x)) + (b*d*((16*b^4*c^
4*e - a^3*b*c*d^2*(8*d*e - 15*c*f) - 3*a^2*b^2*c^2*d*(2*d*e - 5*c*f) + 8*a
^4*d^3*(2*d*e - 3*c*f) - 8*a*b^3*c^3*(d*e + 3*c*f))*((x*Sqrt[a + b*x^2])/(
b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)
/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2
))]*Sqrt[c + d*x^2])) + (c^(3/2)*(8*b^3*c^3*e + 4*a^3*d^2*(2*d*e - 3*c*f)
- 3*a^2*b*c*d*(d*e - 2*c*f) - 3*a*b^2*c^2*(d*e + 4*c*f))*Sqrt[a + b*x^2]*E
llipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(
a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))/(a*c))/(3*a*c)))/(5*a*c))/
(7*c))/(9*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 442
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^q/(a*g*(m + 1))), x] - Simp[1/(a*g^2*(m + 1))   Int[(g*x)
^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[c*(b*e - a*f)*(m + 1) + e*2
*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*2*(p + q + 1))*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && GtQ[q, 0] && LtQ[m, -1]
&&  !(EqQ[q, 1] && SimplerQ[e + f*x^2, c + d*x^2])
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 8.63 (sec) , antiderivative size = 833, normalized size of antiderivative = 1.20

method	result
elliptic	$\frac{\sqrt{(b\,x^2+a)(x^2d+c)}\left(-\frac{e\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{9x^9}-\frac{(9acf+ade+bce)\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{63ac\,x^7}-\frac{\left(9a^2cfd-6a^2d^2e+9ab\,c^2f+2abcde-6b^2c^2e\right)}{315a^2c^2x^5}\right.}{}$
risch	Expression too large to display
default	Expression too large to display

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)/x^10,x,method=_RETURNVERBOSE
)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-1/9*e*(b*d*x
^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/x^9-1/63*(9*a*c*f+a*d*e+b*c*e)/a/c*(b*d*x^4+
a*d*x^2+b*c*x^2+a*c)^(1/2)/x^7-1/315/a^2/c^2*(9*a^2*c*d*f-6*a^2*d^2*e+9*a*
b*c^2*f+2*a*b*c*d*e-6*b^2*c^2*e)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/x^5+1
/315/a^3/c^3*(12*a^3*c*d^2*f-8*a^3*d^3*e-6*a^2*b*c^2*d*f+3*a^2*b*c*d^2*e+1
2*a*b^2*c^3*f+3*a*b^2*c^2*d*e-8*b^3*c^3*e)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(
1/2)/x^3-1/315*(24*a^4*c*d^3*f-16*a^4*d^4*e-15*a^3*b*c^2*d^2*f+8*a^3*b*c*d
^3*e-15*a^2*b^2*c^3*d*f+6*a^2*b^2*c^2*d^2*e+24*a*b^3*c^4*f+8*a*b^3*c^3*d*e
-16*b^4*c^4*e)/a^4/c^4*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/x+1/315*(12*a^3
*c*d^2*f-8*a^3*d^3*e-6*a^2*b*c^2*d*f+3*a^2*b*c*d^2*e+12*a*b^2*c^3*f+3*a*b^
2*c^2*d*e-8*b^3*c^3*e)*b*d/c^3/a^3/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2
/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1
+(a*d+b*c)/c/b)^(1/2))-1/315*b*(24*a^4*c*d^3*f-16*a^4*d^4*e-15*a^3*b*c^2*d
^2*f+8*a^3*b*c*d^3*e-15*a^2*b^2*c^3*d*f+6*a^2*b^2*c^2*d^2*e+24*a*b^3*c^4*f
+8*a*b^3*c^3*d*e-16*b^4*c^4*e)/a^4/c^3/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d
*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(EllipticF(x*(-b/a)^(1/2
),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1
/2))))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 686, normalized size of antiderivative = 0.99

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)}{x^{10}}\,dx =$$

$$-\frac{\sqrt{ac}(2\,(8\,b^5c^4-4\,ab^4c^3d-3\,a^2b^3c^2d^2-4\,a^3b^2cd^3+8\,a^4bd^4)e-3\,(8\,ab^4c^4-5\,a^2b^3c^3d-5\,a^3b^2c^2d^2+8}{}$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)/x^10,x, algorithm="fri
cas")
```

output
```
-1/315*(sqrt(a*c)*(2*(8*b^5*c^4 - 4*a*b^4*c^3*d - 3*a^2*b^3*c^2*d^2 - 4*a^
3*b^2*c*d^3 + 8*a^4*b*d^4)*e - 3*(8*a*b^4*c^4 - 5*a^2*b^3*c^3*d - 5*a^3*b^
2*c^2*d^2 + 8*a^4*b*c*d^3)*f)*x^9*sqrt(-b/a)*elliptic_e(arcsin(x*sqrt(-b/a
)), a*d/(b*c)) - sqrt(a*c)*((16*b^5*c^4 + 8*(a^2*b^3 - a*b^4)*c^3*d - 3*(a
^3*b^2 + 2*a^2*b^3)*c^2*d^2 - (3*a^4*b + 8*a^3*b^2)*c*d^3 + 8*(a^5 + 2*a^4
*b)*d^4)*e - 3*(8*a*b^4*c^4 + (4*a^3*b^2 - 5*a^2*b^3)*c^3*d - (2*a^4*b + 5
*a^3*b^2)*c^2*d^2 + 4*(a^5 + 2*a^4*b)*c*d^3)*f)*x^9*sqrt(-b/a)*elliptic_f(
arcsin(x*sqrt(-b/a)), a*d/(b*c)) + (35*a^5*c^4*e - (2*(8*a*b^4*c^4 - 4*a^2
*b^3*c^3*d - 3*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 + 8*a^5*d^4)*e - 3*(8*a^2*b
^3*c^4 - 5*a^3*b^2*c^3*d - 5*a^4*b*c^2*d^2 + 8*a^5*c*d^3)*f)*x^8 + ((8*a^2
*b^3*c^4 - 3*a^3*b^2*c^3*d - 3*a^4*b*c^2*d^2 + 8*a^5*c*d^3)*e - 6*(2*a^3*b
^2*c^4 - a^4*b*c^3*d + 2*a^5*c^2*d^2)*f)*x^6 - (2*(3*a^3*b^2*c^4 - a^4*b*c
^3*d + 3*a^5*c^2*d^2)*e - 9*(a^4*b*c^4 + a^5*c^3*d)*f)*x^4 + 5*(9*a^5*c^4*
f + (a^4*b*c^4 + a^5*c^3*d)*e)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(a^5*
c^4*x^9)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)}{x^{10}}\,dx = \int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)}{x^{10}}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(1/2)*(f*x**2+e)/x**10,x)
```

output
```
Integral(sqrt(a + b*x**2)*sqrt(c + d*x**2)*(e + f*x**2)/x**10, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)}{x^{10}}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)}{x^{10}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)/x^10,x, algorithm="max
ima")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)/x^10, x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)}{x^{10}}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)}{x^{10}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)/x^10,x, algorithm="gia
c")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)/x^10, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)}{x^{10}}\,dx = \int \frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,(f\,x^2+e)}{x^{10}}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2))/x^10,x)
```

output
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2))/x^10, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)}{x^{10}}\,dx = \text{too large to display}$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)/x^10,x)
```

output
```
( - 200*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**3*c**2*d*e - 360*sqrt(c + d*x
**2)*sqrt(a + b*x**2)*a**3*c**2*d*f*x**2 + 72*sqrt(c + d*x**2)*sqrt(a + b*
x**2)*a**3*c*d**2*f*x**4 - 200*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*b*c*
*3*e - 360*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*b*c**3*f*x**2 + 18*sqrt(
c + d*x**2)*sqrt(a + b*x**2)*a**2*b*c**2*d*f*x**4 - 45*sqrt(c + d*x**2)*sq
rt(a + b*x**2)*a**2*b*c*d**2*e*x**4 - 216*sqrt(c + d*x**2)*sqrt(a + b*x**2
)*a**2*b*d**3*f*x**8 + 72*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**2*c**3*f*
x**4 - 45*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**2*c**2*d*e*x**4 - 54*sqrt
(c + d*x**2)*sqrt(a + b*x**2)*a*b**2*c*d**2*f*x**8 + 135*sqrt(c + d*x**2)*
sqrt(a + b*x**2)*a*b**2*d**3*e*x**8 - 216*sqrt(c + d*x**2)*sqrt(a + b*x**2
)*b**3*c**2*d*f*x**8 + 135*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**3*c*d**2*e
*x**8 + 216*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a**2*c*d + a**2*
d**2*x**2 + a*b*c**2 + 2*a*b*c*d*x**2 + a*b*d**2*x**4 + b**2*c**2*x**2 + b
**2*c*d*x**4),x)*a**3*b**2*d**5*f*x**9 + 270*int((sqrt(c + d*x**2)*sqrt(a
+ b*x**2)*x**2)/(a**2*c*d + a**2*d**2*x**2 + a*b*c**2 + 2*a*b*c*d*x**2 + a
*b*d**2*x**4 + b**2*c**2*x**2 + b**2*c*d*x**4),x)*a**2*b**3*c*d**4*f*x**9
- 135*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a**2*c*d + a**2*d**2*x
**2 + a*b*c**2 + 2*a*b*c*d*x**2 + a*b*d**2*x**4 + b**2*c**2*x**2 + b**2*c*
d*x**4),x)*a**2*b**3*d**5*e*x**9 + 270*int((sqrt(c + d*x**2)*sqrt(a + b*x*
*2)*x**2)/(a**2*c*d + a**2*d**2*x**2 + a*b*c**2 + 2*a*b*c*d*x**2 + a*b*...
```

3.68 $\int \frac{x^8\sqrt{a+bx^2}\sqrt{c+dx^2}}{e+fx^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 1113

$$\int \frac{x^8\sqrt{a+bx^2}\sqrt{c+dx^2}}{e+fx^2}\,dx = \text{Too large to display}$$

output
```
-1/63*(-a*d*f+6*b*c*f+9*b*d*e)*x^5*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b/d/f^2
+1/9*x^5*(b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/d/f-1/315*((16*a^4*d^4*f^4+8*a^3*
b*d^3*f^3*(-c*f+3*d*e)+3*a^2*b^2*d^2*f^2*(-2*c^2*f^2-5*c*d*e*f+14*d^2*e^2)
+a*b^3*d*f*(-8*c^3*f^3-15*c^2*d*e*f^2-42*c*d^2*e^2*f+105*d^3*e^3)-b^4*(-16
*c^4*f^4-24*c^3*d*e*f^3-42*c^2*d^2*e^2*f^2-105*c*d^3*e^3*f+315*d^4*e^4))*x
*(d*x^2+c)^(1/2)/b^2/d^3/f^3/(b*x^2+a)^(1/2)-(8*a^3*d^3*f^3+3*a^2*b*d^2*f^
2*(-c*f+4*d*e)+3*a*b^2*d*f*(-c^2*f^2-2*c*d*e*f+7*d^2*e^2)-b^3*(-8*c^3*f^3-
12*c^2*d*e*f^2-21*c*d^2*e^2*f+105*d^3*e^3))*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1
/2)/b^2/d^2/f^2+(9*b*c*e+9*a*d*e-63*b*d*e^2/f-2*a*c*f+6*b*c^2*f/d+6*a^2*d*
f/b)*x^3*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)-a^(1/2)*(16*a^4*d^4*f^4+8*a^3*b*d
^3*f^3*(-c*f+3*d*e)+3*a^2*b^2*d^2*f^2*(-2*c^2*f^2-5*c*d*e*f+14*d^2*e^2)+a*
b^3*d*f*(-8*c^3*f^3-15*c^2*d*e*f^2-42*c*d^2*e^2*f+105*d^3*e^3)-b^4*(-16*c^
4*f^4-24*c^3*d*e*f^3-42*c^2*d^2*e^2*f^2-105*c*d^3*e^3*f+315*d^4*e^4))*(d*x
^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2
))/b^(5/2)/d^3/f^3/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)+a^(3/2)
*(8*a^3*c*d^3*f^4+3*a^2*b*c*d^2*f^3*(-c*f+4*d*e)+3*a*b^2*c*d*f^2*(-c^2*f^2
-2*c*d*e*f+7*d^2*e^2)-b^3*(-8*c^4*f^4-12*c^3*d*e*f^3-21*c^2*d^2*e^2*f^2-21
0*c*d^3*e^3*f+315*d^4*e^4))*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)
*x/a^(1/2)),(1-a*d/b/c)^(1/2))/b^(5/2)/c/d^2/f^3/(b*x^2+a)^(1/2)/(a*(d*x^2
+c)/c/(b*x^2+a))^(1/2)+315*a^(3/2)*b^(1/2)*d*e^3*(-c*f+d*e)*(d*x^2+c)^(...
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 8.83 (sec) , antiderivative size = 829, normalized size of antiderivative = 0.74

$$\int \frac{x^8\sqrt{a+bx^2}\sqrt{c+dx^2}}{e+fx^2}\,dx$$
$$= \frac{\sqrt{\frac{b}{a}}df^2x(a+bx^2)\,(c+dx^2)\,(8a^3d^3f^3-3a^2bd^2f^2(-4de+cf+2dfx^2)+ab^2df(-3c^2f^2+2cdf(-3e+fx$$

input
```
Integrate[(x^8*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(e + f*x^2),x]
```

output
```
(Sqrt[b/a]*d*f^2*x*(a + b*x^2)*(c + d*x^2)*(8*a^3*d^3*f^3 - 3*a^2*b*d^2*f^
2*(-4*d*e + c*f + 2*d*f*x^2) + a*b^2*d*f*(-3*c^2*f^2 + 2*c*d*f*(-3*e + f*x
^2) + d^2*(21*e^2 - 9*e*f*x^2 + 5*f^2*x^4)) + b^3*(8*c^3*f^3 - 6*c^2*d*f^2
*(-2*e + f*x^2) + c*d^2*f*(21*e^2 - 9*e*f*x^2 + 5*f^2*x^4) + d^3*(-105*e^3
 + 63*e^2*f*x^2 - 45*e*f^2*x^4 + 35*f^3*x^6))) + I*c*f*(16*a^4*d^4*f^4 + 8
*a^3*b*d^3*f^3*(3*d*e - c*f) + 3*a^2*b^2*d^2*f^2*(14*d^2*e^2 - 5*c*d*e*f -
 2*c^2*f^2) + a*b^3*d*f*(105*d^3*e^3 - 42*c*d^2*e^2*f - 15*c^2*d*e*f^2 - 8
*c^3*f^3) + b^4*(-315*d^4*e^4 + 105*c*d^3*e^3*f + 42*c^2*d^2*e^2*f^2 + 24*
c^3*d*e*f^3 + 16*c^4*f^4))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Ellipti
cE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*(8*a^4*c*d^4*f^5 + a^3*b*c*d^3
*f^4*(12*d*e - 5*c*f) - 3*a^2*b^2*c*d^2*f^3*(-7*d^2*e^2 + 3*c*d*e*f + c^2*
f^2) + a*b^3*d*f*(315*d^4*e^4 - 105*c*d^3*e^3*f - 63*c^2*d^2*e^2*f^2 - 27*
c^3*d*e*f^3 - 16*c^4*f^4) + b^4*(-315*d^5*e^5 + 105*c^2*d^3*e^3*f^2 + 42*c
^3*d^2*e^2*f^3 + 24*c^4*d*e*f^4 + 16*c^5*f^5))*Sqrt[1 + (b*x^2)/a]*Sqrt[1
+ (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (315*I)*b^3*
d^4*e^3*(b*e - a*f)*(-(d*e) + c*f)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]
*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(315*b^3*Sq
rt[b/a]*d^4*f^6*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8\sqrt{a+bx^2}\sqrt{c+dx^2}}{e+fx^2}\,dx$$
$$\Big\downarrow \; 450$$
$$\int \frac{x^8\sqrt{a+bx^2}\sqrt{c+dx^2}}{e+fx^2}dx$$

input
```
Int[(x^8*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(e + f*x^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [A] (verified)

Time = 18.34 (sec) , antiderivative size = 1105, normalized size of antiderivative = 0.99

method	result	size
risch	Expression too large to display	1105
default	Expression too large to display	3424
elliptic	Expression too large to display	3939

input
```
int(x^8*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
1/315*x*(35*b^3*d^3*f^3*x^6+5*a*b^2*d^3*f^3*x^4+5*b^3*c*d^2*f^3*x^4-45*b^3
*d^3*e*f^2*x^4-6*a^2*b*d^3*f^3*x^2+2*a*b^2*c*d^2*f^3*x^2-9*a*b^2*d^3*e*f^2
*x^2-6*b^3*c^2*d*f^3*x^2-9*b^3*c*d^2*e*f^2*x^2+63*b^3*d^3*e^2*f*x^2+8*a^3*
d^3*f^3-3*a^2*b*c*d^2*f^3+12*a^2*b*d^3*e*f^2-3*a*b^2*c^2*d*f^3-6*a*b^2*c*d
^2*e*f^2+21*a*b^2*d^3*e^2*f+8*b^3*c^3*f^3+12*b^3*c^2*d*e*f^2+21*b^3*c*d^2*
e^2*f-105*b^3*d^3*e^3)*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b^3/d^3/f^4-1/315/d
^3/b^3/f^4*((8*a^4*c*d^3*f^5-3*a^3*b*c^2*d^2*f^5+12*a^3*b*c*d^3*e*f^4-3*a^
2*b^2*c^3*d*f^5-6*a^2*b^2*c^2*d^2*e*f^4+21*a^2*b^2*c*d^3*e^2*f^3+8*a*b^3*c
^4*f^5+12*a*b^3*c^3*d*e*f^4+21*a*b^3*c^2*d^2*e^2*f^3+210*a*b^3*c*d^3*e^3*f
^2-315*a*b^3*d^4*e^4*f-315*b^4*c*d^3*e^4*f+315*b^4*d^4*e^5)/f^2/(-b/a)^(1/
2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-1/f*(16*a^4*d^4*f^4-8*
a^3*b*c*d^3*f^4+24*a^3*b*d^4*e*f^3-6*a^2*b^2*c^2*d^2*f^4-15*a^2*b^2*c*d^3*
e*f^3+42*a^2*b^2*d^4*e^2*f^2-8*a*b^3*c^3*d*f^4-15*a*b^3*c^2*d^2*e*f^3-42*a
*b^3*c*d^3*e^2*f^2+105*a*b^3*d^4*e^3*f+16*b^4*c^4*f^4+24*b^4*c^3*d*e*f^3+4
2*b^4*c^2*d^2*e^2*f^2+105*b^4*c*d^3*e^3*f-315*b^4*d^4*e^4)*c/(-b/a)^(1/2)*
(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*
(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/
2),(-1+(a*d+b*c)/c/b)^(1/2)))-315*e^3*(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)*b^
3*d^3/f^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^8\sqrt{a+bx^2}\sqrt{c+dx^2}}{e+fx^2}\,dx = \text{Timed out}$$

input
```
integrate(x^8*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^8\sqrt{a+bx^2}\sqrt{c+dx^2}}{e+fx^2}\,dx=\int \frac{x^8\sqrt{a+bx^2}\sqrt{c+dx^2}}{e+fx^2}\,dx$$

input
```
integrate(x**8*(b*x**2+a)**(1/2)*(d*x**2+c)**(1/2)/(f*x**2+e),x)
```

output
```
Integral(x**8*sqrt(a + b*x**2)*sqrt(c + d*x**2)/(e + f*x**2), x)
```

Maxima [F]

$$\int \frac{x^8\sqrt{a+bx^2}\sqrt{c+dx^2}}{e+fx^2}\,dx=\int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}x^8}{fx^2+e}\,dx$$

input
```
integrate(x^8*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="maxi
ma")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x^8/(f*x^2 + e), x)
```

Giac [F]

$$\int \frac{x^8\sqrt{a+bx^2}\sqrt{c+dx^2}}{e+fx^2}\,dx=\int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}x^8}{fx^2+e}\,dx$$

input
```
integrate(x^8*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="giac
")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x^8/(f*x^2 + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^8\sqrt{a+bx^2}\sqrt{c+dx^2}}{e+fx^2}\,dx = \int \frac{x^8\,\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{f\,x^2+e}\,dx$$

input
```
int((x^8*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2))/(e + f*x^2),x)
```

output
```
int((x^8*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2))/(e + f*x^2), x)
```

Reduce [F]

$$\int \frac{x^8\sqrt{a+bx^2}\sqrt{c+dx^2}}{e+fx^2}\,dx = \int \frac{x^8\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{f\,x^2+e}\,dx$$

input
```
int(x^8*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e),x)
```

output
```
int(x^8*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e),x)
```

3.69 $\int \frac{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}}{e+fx^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 787

$$\int \frac{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}}{e+fx^2}\,dx$$

$$= -\frac{(7bde+4bcf-adf)x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{35bdf^2} + \frac{x^3\sqrt{a+bx^2}(c+dx^2)^{3/2}}{7df}$$

$$+ \frac{\frac{(8a^3d^3f^3+a^2bd^2f^2(14de-5cf)+ab^2df(35d^2e^2-14cdef-5c^2f^2)-b^3(105d^3e^3-35cd^2e^2f-14c^2def^2-8c^3f^3))x\sqrt{c+dx^2}}{bd^2f^2\sqrt{a+bx^2}} - \Big(7bce+7a}{}$$

output
```
-1/35*(-a*d*f+4*b*c*f+7*b*d*e)*x^3*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b/d/f^2
+1/7*x^3*(b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/d/f+1/105*((8*a^3*d^3*f^3+a^2*b*d
^2*f^2*(-5*c*f+14*d*e)+a*b^2*d*f*(-5*c^2*f^2-14*c*d*e*f+35*d^2*e^2)-b^3*(-
8*c^3*f^3-14*c^2*d*e*f^2-35*c*d^2*e^2*f+105*d^3*e^3))*x*(d*x^2+c)^(1/2)/b/
d^2/f^2/(b*x^2+a)^(1/2)-(7*b*c*e+7*a*d*e-35*b*d*e^2/f-2*a*c*f+4*b*c^2*f/d+
4*a^2*d*f/b)*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)-a^(1/2)*(8*a^3*d^3*f^3+a^2*
b*d^2*f^2*(-5*c*f+14*d*e)+a*b^2*d*f*(-5*c^2*f^2-14*c*d*e*f+35*d^2*e^2)-b^3
*(-8*c^3*f^3-14*c^2*d*e*f^2-35*c*d^2*e^2*f+105*d^3*e^3))*(d*x^2+c)^(1/2)*E
llipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/b^(3/2)/d^
2/f^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)+a^(3/2)*(4*a^2*c*d^2
*f^3+a*b*c*d*f^2*(-2*c*f+7*d*e)-b^2*(-4*c^3*f^3-7*c^2*d*e*f^2-70*c*d^2*e^2
*f+105*d^3*e^3))*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2))
,(1-a*d/b/c)^(1/2))/b^(3/2)/c/d/f^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+
a))^(1/2)+105*a^(3/2)*b^(1/2)*d*e^2*(-c*f+d*e)*(d*x^2+c)^(1/2)*EllipticPi(
b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),1-a*f/b/e,(1-a*d/b/c)^(1/2))/c/f^2/(b*
x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2))/b/d/f^2
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.64 (sec) , antiderivative size = 596, normalized size of antiderivative = 0.76

$$\int \frac{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}}{e+fx^2}\,dx$$
$$= \frac{-\sqrt{\frac{b}{a}}df^2x(a+bx^2)\left(c+dx^2\right)\left(4a^2d^2f^2+abdf(7de-2cf-3dfx^2)+b^2(4c^2f^2+cdf(7e-3fx^2)+d^2(-\right.}{}$$

input
```
Integrate[(x^6*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(e + f*x^2),x]
```

output
```
(-(Sqrt[b/a]*d*f^2*x*(a + b*x^2)*(c + d*x^2)*(4*a^2*d^2*f^2 + a*b*d*f*(7*d
*e - 2*c*f - 3*d*f*x^2) + b^2*(4*c^2*f^2 + c*d*f*(7*e - 3*f*x^2) + d^2*(-3
5*e^2 + 21*e*f*x^2 - 15*f^2*x^4)))) - I*c*f*(8*a^3*d^3*f^3 + a^2*b*d^2*f^2
*(14*d*e - 5*c*f) + a*b^2*d*f*(35*d^2*e^2 - 14*c*d*e*f - 5*c^2*f^2) + b^3*
(-105*d^3*e^3 + 35*c*d^2*e^2*f + 14*c^2*d*e*f^2 + 8*c^3*f^3))*Sqrt[1 + (b*
x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]
 + I*(4*a^3*c*d^3*f^4 + a^2*b*c*d^2*f^3*(7*d*e - 3*c*f) + a*b^2*d*f*(105*d
^3*e^3 - 35*c*d^2*e^2*f - 21*c^2*d*e*f^2 - 9*c^3*f^3) + b^3*(-105*d^4*e^4
+ 35*c^2*d^2*e^2*f^2 + 14*c^3*d*e*f^3 + 8*c^4*f^4))*Sqrt[1 + (b*x^2)/a]*Sq
rt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (105*I)
*b^2*d^3*e^2*(-(b*e) + a*f)*(-(d*e) + c*f)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d
*x^2)/c]*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(10
5*a^2*(b/a)^(5/2)*d^3*f^5*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}}{e+fx^2}\,dx$$

$$\Big\downarrow \text{ 450}$$

$$\int \frac{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}}{e+fx^2}dx$$

input
```
Int[(x^6*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(e + f*x^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [A] (verified)

Time = 8.97 (sec) , antiderivative size = 785, normalized size of antiderivative = 1.00

method	result
risch	$-\frac{x\left(-15f^2x^4b^2d^2-3ab\,d^2f^2x^2-3b^2cd\,f^2x^2+21b^2d^2ef\,x^2+4a^2d^2f^2-2abcd\,f^2+7ab\,d^2ef+4b^2c^2f^2+7b^2cdef-35b^2d^2e^2\right)\sqrt{b\,x^2+\cdots}}{105b^2d^2f^3}$
default	Expression too large to display
elliptic	Expression too large to display

input
```
int(x^6*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
-1/105*x*(-15*b^2*d^2*f^2*x^4-3*a*b*d^2*f^2*x^2-3*b^2*c*d*f^2*x^2+21*b^2*d
^2*e*f*x^2+4*a^2*d^2*f^2-2*a*b*c*d*f^2+7*a*b*d^2*e*f+4*b^2*c^2*f^2+7*b^2*c
*d*e*f-35*b^2*d^2*e^2)*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b^2/d^2/f^3+1/105/d
^2/b^2/f^3*((4*a^3*c*d^2*f^4-2*a^2*b*c^2*d*f^4+7*a^2*b*c*d^2*e*f^3+4*a*b^2
*c^3*f^4+7*a*b^2*c^2*d*e*f^3+70*a*b^2*c*d^2*e^2*f^2-105*a*b^2*d^3*e^3*f-10
5*b^3*c*d^2*e^3*f+105*b^3*d^3*e^4)/f^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d
*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2)
,(-1+(a*d+b*c)/c/b)^(1/2))-1/f*(8*a^3*d^3*f^3-5*a^2*b*c*d^2*f^3+14*a^2*b*d
^3*e*f^2-5*a*b^2*c^2*d*f^3-14*a*b^2*c*d^2*e*f^2+35*a*b^2*d^3*e^2*f+8*b^3*c
^3*f^3+14*b^3*c^2*d*e*f^2+35*b^3*c*d^2*e^2*f-105*b^3*d^3*e^3)*c/(-b/a)^(1/
2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
/d*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^
(1/2),(-1+(a*d+b*c)/c/b)^(1/2)))-105*e^2*(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)
*b^2*d^2/f^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d
*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(
-b/a)^(1/2)))*((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}}{e+fx^2}\,dx = \text{Timed out}$$

input
```
integrate(x^6*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}}{e+fx^2}\,dx = \int \frac{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}}{e+fx^2}\,dx$$

input
```
integrate(x**6*(b*x**2+a)**(1/2)*(d*x**2+c)**(1/2)/(f*x**2+e),x)
```

output
```
Integral(x**6*sqrt(a + b*x**2)*sqrt(c + d*x**2)/(e + f*x**2), x)
```

Maxima [F]

$$\int \frac{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}}{e+fx^2}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}x^6}{fx^2+e}\,dx$$

input
```
integrate(x^6*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="maxi
ma")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x^6/(f*x^2 + e), x)
```

Giac [F]

$$\int \frac{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}}{e+fx^2}\,dx=\int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}x^6}{fx^2+e}\,dx$$

input
```
integrate(x^6*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="giac
")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x^6/(f*x^2 + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}}{e+fx^2}\,dx=\int \frac{x^6\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{f\,x^2+e}\,dx$$

input
```
int((x^6*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2))/(e + f*x^2),x)
```

output
```
int((x^6*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2))/(e + f*x^2), x)
```

Reduce [F]

$$\int \frac{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}}{e+fx^2}\,dx=\int \frac{x^6\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{f\,x^2+e}dx$$

input
```
int(x^6*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e),x)
```

output
```
int(x^6*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e),x)
```

3.70 $\int \frac{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}{e+fx^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 558

$$\int \frac{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}{e+fx^2}\,dx = -\frac{\left(2a^2df+ab(5de-2cf)+b^2\left(5ce-\frac{15de^2}{f}+\frac{2c^2f}{d}\right)\right)x\sqrt{c+dx^2}}{15bdf^2\sqrt{a+bx^2}} - \frac{(5bde+2bcf-adf)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{15bdf^2} + \frac{x\sqrt{a+bx^2}(c+dx^2)^{3/2}}{5df} + \frac{\sqrt{a}(2a^2d^2f^2+abdf(5de-2cf)-b^2(15d^2e^2-5cdef-2c^2f^2))\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{15b^{3/2}d^2f^3\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} - \frac{a^{3/2}(acdf^2-b(15d^2e^2-10cdef-c^2f^2))\sqrt{c+dx^2}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{15b^{3/2}cdf^3\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} - \frac{a^{3/2}e(de-cf)\sqrt{c+dx^2}\,\mathrm{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}cf^3\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
-1/15*(2*a^2*d*f+a*b*(-2*c*f+5*d*e)+b^2*(5*c*e-15*d*e^2/f+2*c^2*f/d))*x*(d
*x^2+c)^(1/2)/b/d/f^2/(b*x^2+a)^(1/2)-1/15*(-a*d*f+2*b*c*f+5*b*d*e)*x*(b*x
^2+a)^(1/2)*(d*x^2+c)^(1/2)/b/d/f^2+1/5*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/
d/f+1/15*a^(1/2)*(2*a^2*d^2*f^2+a*b*d*f*(-2*c*f+5*d*e)-b^2*(-2*c^2*f^2-5*c
*d*e*f+15*d^2*e^2))*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a
)^(1/2),(1-a*d/b/c)^(1/2))/b^(3/2)/d^2/f^3/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/
(b*x^2+a))^(1/2)-1/15*a^(3/2)*(a*c*d*f^2-b*(-c^2*f^2-10*c*d*e*f+15*d^2*e^2
))*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(
1/2))/b^(3/2)/c/d/f^3/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)-a^(3
/2)*e*(-c*f+d*e)*(d*x^2+c)^(1/2)*EllipticPi(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^
(1/2),1-a*f/b/e,(1-a*d/b/c)^(1/2))/b^(1/2)/c/f^3/(b*x^2+a)^(1/2)/(a*(d*x^2
+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.49 (sec) , antiderivative size = 426, normalized size of antiderivative = 0.76

$$\int \frac{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}{e+fx^2}\,dx$$

$$= \frac{icf(2a^2d^2f^2+abdf(5de-2cf)+b^2(-15d^2e^2+5cdef+2c^2f^2))\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(i\text{arcsinh}\left(\sqrt{\frac{b}{a}}x\right)\right.}{}$$

input
```
Integrate[(x^4*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(e + f*x^2),x]
```

output
```
(I*c*f*(2*a^2*d^2*f^2 + a*b*d*f*(5*d*e - 2*c*f) + b^2*(-15*d^2*e^2 + 5*c*d
*e*f + 2*c^2*f^2))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*Arc
Sinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*(a^2*c*d^2*f^3 + a*b*d*f*(15*d^2*e^2 -
 5*c*d*e*f - 3*c^2*f^2) + b^2*(-15*d^3*e^3 + 5*c^2*d*e*f^2 + 2*c^3*f^3))*S
qrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (
a*d)/(b*c)] + d*(Sqrt[b/a]*f^2*x*(a + b*x^2)*(c + d*x^2)*(a*d*f + b*(-5*d*
e + c*f + 3*d*f*x^2)) - (15*I)*b*d*e*(-(b*e) + a*f)*(-(d*e) + c*f)*Sqrt[1
+ (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/
a]*x], (a*d)/(b*c)]))/(15*b*Sqrt[b/a]*d^2*f^4*Sqrt[a + b*x^2]*Sqrt[c + d*x
^2])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}{e+fx^2}\,dx$$
$$\Big\downarrow \; 450$$
$$\int \frac{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}{e+fx^2}\,dx$$

input
```
Int[(x^4*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(e + f*x^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [A] (verified)

Time = 7.65 (sec) , antiderivative size = 565, normalized size of antiderivative = 1.01

method	result
risch	$\frac{x\left(3bdf\,x^2+adf+bcf-5bde\right)\sqrt{b\,x^2+a}\,\sqrt{x^2d+c}}{15bd\,f^2}-\frac{\left(\frac{\left(a^2cd\,f^3+ab\,c^2f^3+10abcde\,f^2-15ab\,d^2e^2f-15b^2cd\,e^2f+15b^2d^2e^3\right)\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d}{}}}{f^2\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}\right.}{}$
default	Expression too large to display
elliptic	Expression too large to display

input
```
int(x^4*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
1/15*x*(3*b*d*f*x^2+a*d*f+b*c*f-5*b*d*e)*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b
/d/f^2-1/15/f^2/b/d*((a^2*c*d*f^3+a*b*c^2*f^3+10*a*b*c*d*e*f^2-15*a*b*d^2*
e^2*f-15*b^2*c*d*e^2*f+15*b^2*d^2*e^3)/f^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*
(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(
1/2),(-1+(a*d+b*c)/c/b)^(1/2))-1/f*(2*a^2*d^2*f^2-2*a*b*c*d*f^2+5*a*b*d^2*
e*f+2*b^2*c^2*f^2+5*b^2*c*d*e*f-15*b^2*d^2*e^2)*c/(-b/a)^(1/2)*(1+b*x^2/a)
^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*(EllipticF(
x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d
+b*c)/c/b)^(1/2)))-15*e*(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)*b*d/f^2/(-b/a)^(
1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/
2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2)))*((b*x^2
+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}{e+fx^2}\,dx = \text{Timed out}$$

input
```
integrate(x^4*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}{e+fx^2}\,dx = \int \frac{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}{e+fx^2}\,dx$$

input
```
integrate(x**4*(b*x**2+a)**(1/2)*(d*x**2+c)**(1/2)/(f*x**2+e),x)
```

output
```
Integral(x**4*sqrt(a + b*x**2)*sqrt(c + d*x**2)/(e + f*x**2), x)
```

Maxima [F]

$$\int \frac{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}{e+fx^2}\,dx=\int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}x^4}{fx^2+e}\,dx$$

input
```
integrate(x^4*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="maxi
ma")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x^4/(f*x^2 + e), x)
```

Giac [F]

$$\int \frac{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}{e+fx^2}\,dx=\int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}x^4}{fx^2+e}\,dx$$

input
```
integrate(x^4*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="giac
")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x^4/(f*x^2 + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}{e+fx^2}\,dx=\int \frac{x^4\,\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{f\,x^2+e}\,dx$$

input
```
int((x^4*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2))/(e + f*x^2),x)
```

output
```
int((x^4*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2))/(e + f*x^2), x)
```

Reduce [F]

$$\int \frac{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}{e+fx^2}\,dx = \text{Too large to display}$$

input
```
int(x^4*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e),x)
```

output
```
(sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*d*f*x + sqrt(c + d*x**2)*sqrt(a + b*x
**2)*b*c*f*x - 5*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b*d*e*x + 3*sqrt(c + d*
x**2)*sqrt(a + b*x**2)*b*d*f*x**3 - 2*int((sqrt(c + d*x**2)*sqrt(a + b*x**
2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*
f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*a**2*d**2*f**2 + 2*int((sqrt(c + d*x*
*2)*sqrt(a + b*x**2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 +
 b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*a*b*c*d*f**2 - 5*in
t((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*e*x**
2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*a*b
*d**2*e*f - 2*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c*e + a*c*f*
x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*
d*f*x**6),x)*b**2*c**2*f**2 - 5*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**
4)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4
 + b*d*e*x**4 + b*d*f*x**6),x)*b**2*c*d*e*f + 15*int((sqrt(c + d*x**2)*sqr
t(a + b*x**2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*
x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*b**2*d**2*e**2 - int((sqrt
(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d
*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*a**2*c*d*f
**2 - 2*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c*e + a*c*f*x**2 +
 a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f...
```

3.71 $\int \frac{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}{e+fx^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 401

$$\begin{aligned}\int \frac{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}{e+fx^2}\,dx &= -\frac{(3bde-bcf-adf)x\sqrt{c+dx^2}}{3df^2\sqrt{a+bx^2}}+\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3f}\\
&+\frac{\sqrt{a}(3bde-bcf-adf)\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{3\sqrt{b}df^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}\\
&-\frac{a^{3/2}(3de-2cf)\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{3\sqrt{b}cf^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}\\
&+\frac{a^{3/2}(de-cf)\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}cf^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}\end{aligned}$$

output
```
-1/3*(-a*d*f-b*c*f+3*b*d*e)*x*(d*x^2+c)^(1/2)/d/f^2/(b*x^2+a)^(1/2)+1/3*x*
(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/f+1/3*a^(1/2)*(-a*d*f-b*c*f+3*b*d*e)*(d*x^
2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2)
)/b^(1/2)/d/f^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)-1/3*a^(3/2
)*(-2*c*f+3*d*e)*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2))
,(1-a*d/b/c)^(1/2))/b^(1/2)/c/f^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a)
)^(1/2)+a^(3/2)*(-c*f+d*e)*(d*x^2+c)^(1/2)*EllipticPi(b^(1/2)*x/a^(1/2)/(1
+b*x^2/a)^(1/2),1-a*f/b/e,(1-a*d/b/c)^(1/2))/b^(1/2)/c/f^2/(b*x^2+a)^(1/2)
/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.25 (sec) , antiderivative size = 320, normalized size of antiderivative = 0.80

$$\int \frac{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}{e+fx^2}\,dx$$
$$= \frac{-icf(-3bde+bcf+adf)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(i\text{arcsinh}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)-i(adf(-3de+cf)+b(3d^2e^2-\cdots}{\cdots}$$

input
```
Integrate[(x^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(e + f*x^2),x]
```

output
```
((-I)*c*f*(-3*b*d*e + b*c*f + a*d*f)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/
c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*(a*d*f*(-3*d*e + c*f
) + b*(3*d^2*e^2 - c^2*f^2))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Ellip
ticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + d*(Sqrt[b/a]*f^2*x*(a + b*x^2)
*(c + d*x^2) + (3*I)*(-(b*e) + a*f)*(-(d*e) + c*f)*Sqrt[1 + (b*x^2)/a]*Sqr
t[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*
c)]))/(3*Sqrt[b/a]*d*f^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}{e+fx^2}\,dx$$

$$\Big\downarrow \ 450$$

$$\int \frac{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}{e+fx^2}dx$$

input

```
Int[(x^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(e + f*x^2),x]
```

output

```
$Aborted
```

Defintions of rubi rules used

rule 450

```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [A] (verified)

Time = 9.05 (sec) , antiderivative size = 699, normalized size of antiderivative = 1.74

method	result
risch	$\frac{x\sqrt{b\,x^{2}+a}\,\sqrt{x^{2}d+c}}{3f}+\frac{\left(-\frac{f(adf+bcf-3bde)c\sqrt{1+\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,\left(\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)-\operatorname{EllipticE}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^{4}+ad\,x^{2}+x^{2}bc+ac}\,d}+ \right.}{}$
default	$\frac{\sqrt{b\,x^{2}+a}\,\sqrt{x^{2}d+c}\,\left(\sqrt{-\frac{b}{a}}\,b\,d^{2}f^{2}x^{5}+\sqrt{-\frac{b}{a}}\,a\,d^{2}f^{2}x^{3}+\sqrt{-\frac{b}{a}}\,bcd\,f^{2}x^{3}+\sqrt{\frac{b\,x^{2}+a}{a}}\,\sqrt{\frac{x^{2}d+c}{c}}\,\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)acd\,f^{2}-3\sqrt{\frac{b\,x^{2}}{}} \right.}{}$
elliptic	Expression too large to display

input
```
int(x^2*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
1/3*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/f+1/3/f*(1/f^2*(-f*(a*d*f+b*c*f-3*b*
d*e)*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b
*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-El
lipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2)))+2*a*c*f^2/(-b/a)^(1/2)*(
1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*Ell
ipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+3*b*d*e^2/(-b/a)^(1/2)*(1+
b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*Ellip
ticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-3*a*d*e*f/(-b/a)^(1/2)*(1+b*
x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*Ellipti
cF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-3*b*c*e*f/(-b/a)^(1/2)*(1+b*x^
2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF
(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2)))-3*(a*c*f^2-a*d*e*f-b*c*e*f+b*d*
e^2)/f^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2
+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a
)^(1/2)))*((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}{e+fx^2}\,dx = \text{Timed out}$$

input
```
integrate(x^2*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}{e+fx^2}\,dx = \int \frac{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}{e+fx^2}\,dx$$

input
```
integrate(x**2*(b*x**2+a)**(1/2)*(d*x**2+c)**(1/2)/(f*x**2+e),x)
```

output
```
Integral(x**2*sqrt(a + b*x**2)*sqrt(c + d*x**2)/(e + f*x**2), x)
```

Maxima [F]

$$\int \frac{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}{e+fx^2}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}x^2}{fx^2+e}\,dx$$

input
```
integrate(x^2*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="maxi
ma")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x^2/(f*x^2 + e), x)
```

Giac [F]

$$\int \frac{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}{e+fx^2}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}x^2}{fx^2+e}\,dx$$

input
```
integrate(x^2*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="giac
")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x^2/(f*x^2 + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}{e+fx^2}\,dx = \int \frac{x^2\,\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{f\,x^2+e}\,dx$$

input
```
int((x^2*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2))/(e + f*x^2),x)
```

output
```
int((x^2*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2))/(e + f*x^2), x)
```

Reduce [F]

$$\int \frac{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}{e+fx^2}\,dx = \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x+\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^4}{bdf\,x^6+adf\,x^4+bcf\,x^4+bde\,x^4+acf\,x^2+ade\,x^2+bce\,x^2+ace}dx\right)adf+\left(\int \frac{\sqrt{d}}{bdf\,x^6+adf\,x^4+bcf\,x^4+}\right.}{}$$

input
```
int(x^2*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e),x)
```

output
```
(sqrt(c + d*x**2)*sqrt(a + b*x**2)*x + int((sqrt(c + d*x**2)*sqrt(a + b*x*
*2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c
*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*a*d*f + int((sqrt(c + d*x**2)*sqrt(a
 + b*x**2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**
2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*b*c*f - 3*int((sqrt(c + d*x**
2)*sqrt(a + b*x**2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 +
b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*b*d*e + 2*int((sqrt(
c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*
f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*a*c*f - 2*i
nt((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c*e + a*c*f*x**2 + a*d*e*x*
*2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*a*
d*e - 2*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c*e + a*c*f*x**2 +
 a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x*
*6),x)*b*c*e - int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c*e + a*c*f*x**2
 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*
x**6),x)*a*c*e)/(3*f)
```

3.72 $\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{e+fx^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 322

$$\begin{aligned}\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{e+fx^2}\,dx &= \frac{bx\sqrt{c+dx^2}}{f\sqrt{a+bx^2}}-\frac{\sqrt{a}\sqrt{b}\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{f\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}\\&+\frac{a^{3/2}d\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}cf\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}\\&-\frac{a^{3/2}(de-cf)\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}cef\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}\end{aligned}$$

output

```
b*x*(d*x^2+c)^(1/2)/f/(b*x^2+a)^(1/2)-a^(1/2)*b^(1/2)*(d*x^2+c)^(1/2)*Elli
pticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/f/(b*x^2+a)^(
1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)+a^(3/2)*d*(d*x^2+c)^(1/2)*InverseJaco
biAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/b^(1/2)/c/f/(b*x^2+a)^(1
/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)-a^(3/2)*(-c*f+d*e)*(d*x^2+c)^(1/2)*Ell
ipticPi(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),1-a*f/b/e,(1-a*d/b/c)^(1/2))/b
^(1/2)/c/e/f/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.58 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.57

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{e+fx^2}\,dx = -\frac{i\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\left(bcefE\left(i\text{arcsinh}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)-(be-af)\left(de\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{\frac{b}{a}}x\right),\frac{ad}{bc}\right)+\right.\right.}{\sqrt{\frac{b}{a}}ef^2\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

input
```
Integrate[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(e + f*x^2),x]
```

output
```
((-I)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(b*c*e*f*EllipticE[I*ArcSinh
[Sqrt[b/a]*x], (a*d)/(b*c)] - (b*e - a*f)*(d*e*EllipticF[I*ArcSinh[Sqrt[b/
a]*x], (a*d)/(b*c)] + (-(d*e) + c*f)*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqr
t[b/a]*x], (a*d)/(b*c)])))/(Sqrt[b/a]*e*f^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2
])
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {410, 324, 320, 388, 313, 414}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{e+fx^2}\,dx$$

$$\Big\downarrow \; 410$$

$$\frac{b\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}}dx}{f}-\frac{(be-af)\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}(fx^2+e)}dx}{f}$$

$$\Big\downarrow \; 324$$

$$\frac{b\left(c\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+d\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{f}-\frac{(be-af)\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}(fx^2+e)}dx}{f}$$

$$\Big\downarrow \text{320}$$

$$\frac{b\left(d\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{f}-\frac{(be-af)\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}(fx^2+e)}dx}{f}$$

$$\Big\downarrow \text{388}$$

$$\frac{b\left(d\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{f}-\frac{(be-af)\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}(fx^2+e)}dx}{f}$$

$$\Big\downarrow \text{313}$$

$$\frac{b\left(\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+d\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)\right)}{f}-\frac{(be-af)\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}(fx^2+e)}dx}{f}$$

$$\Big\downarrow \text{414}$$

$$\frac{b\left(\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+d\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)\right)}{f}-\frac{c^{3/2}\sqrt{a+bx^2}(be-af)\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}ef\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

input
```
Int[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(e + f*x^2),x]
```

output
```
(b*(d*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*
EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(
c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (c^(3/2)*Sqrt[a + b*x^
2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*Sqr
t[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])))/f - (c^(3/2)*(b*e -
a*f)*Sqrt[a + b*x^2]*EllipticPi[1 - (c*f)/(d*e), ArcTan[(Sqrt[d]*x)/Sqrt[c
]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*e*f*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]
*Sqrt[c + d*x^2])
```

Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 324
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Simp[b   Int[x^2/(Sqr
t[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c
] && PosQ[b/a]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 410
```
Int[(Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)^2])/((a_) + (b_.)*(x_
)^2), x_Symbol] :> Simp[d/b   Int[Sqrt[e + f*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*c - a*d)/b   Int[Sqrt[e + f*x^2]/((a + b*x^2)*Sqrt[c + d*x^2]), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !SimplerSqrtQ[-f/e, -d/c]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Maple [A] (verified)

Time = 1.20 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.06

method	result
default	$\frac{\sqrt{b x^2+a}\,\sqrt{x^2 d+c}\,\sqrt{\frac{b x^2+a}{a}}\,\sqrt{\frac{x^2 d+c}{c}}\left(\text{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)adef-\text{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)bde^2+\text{EllipticE}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)b\right.}{(bd}$
elliptic	$\frac{\sqrt{(b x^2+a)(x^2 d+c)}\left(\frac{\sqrt{1+\frac{b x^2}{a}}\,\sqrt{1+\frac{d x^2}{c}}\,\text{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)ad}{f\sqrt{-\frac{b}{a}}\,\sqrt{bd x^4+ad x^2+x^2bc+ac}}-\frac{\sqrt{1+\frac{b x^2}{a}}\,\sqrt{1+\frac{d x^2}{c}}\,\text{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)bde}{f^2\sqrt{-\frac{b}{a}}\,\sqrt{bd x^4+ad x^2+x^2bc+ac}}+\right.}{}$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*(E
llipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*d*e*f-EllipticF(x*(-b/a)^(1/2),
(a*d/b/c)^(1/2))*b*d*e^2+EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b*c*e*f
+EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*a*c*f^2-El
lipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*a*d*e*f-Ellip
ticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*b*c*e*f+Elliptic
Pi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*b*d*e^2)/(b*d*x^4+a
*d*x^2+b*c*x^2+a*c)/f^2/(-b/a)^(1/2)/e
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{e+fx^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{e+fx^2}\,dx = \int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{e+fx^2}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(1/2)/(f*x**2+e),x)
```

output
```
Integral(sqrt(a + b*x**2)*sqrt(c + d*x**2)/(e + f*x**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{e+fx^2}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{fx^2+e}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/(f*x^2 + e), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{e+fx^2}\,dx=\int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{fx^2+e}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/(f*x^2 + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{e+fx^2}\,dx=\int \frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{f\,x^2+e}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2))/(e + f*x^2),x)
```

output
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2))/(e + f*x^2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{e+fx^2}\,dx=\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{f\,x^2+e}dx$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e),x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(e + f*x**2),x)
```

3.73 $\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^2(e+fx^2)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 415

$$\begin{aligned}\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^2\left(e+fx^2\right)}\,dx &= \frac{bx\sqrt{c+dx^2}}{e\sqrt{a+bx^2}}+\frac{dx\sqrt{a+bx^2}\sqrt{c+dx^2}}{ce}+\frac{bx\sqrt{a+bx^2}(c+dx^2)^{3/2}}{ace}\\ &\quad-\frac{\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{acex}-\frac{\sqrt{a}\sqrt{b}\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{e\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}\\ &\quad+\frac{\sqrt{a}\sqrt{b}\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{e\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}\\ &\quad+\frac{a^{3/2}(de-cf)\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}\end{aligned}$$

output
```
b*x*(d*x^2+c)^(1/2)/e/(b*x^2+a)^(1/2)+d*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/
c/e+b*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/a/c/e-(b*x^2+a)^(3/2)*(d*x^2+c)^(3
/2)/a/c/e/x-a^(1/2)*b^(1/2)*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1
+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/e/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2
+a))^(1/2)+a^(1/2)*b^(1/2)*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*
x/a^(1/2)),(1-a*d/b/c)^(1/2))/e/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^
(1/2)+a^(3/2)*(-c*f+d*e)*(d*x^2+c)^(1/2)*EllipticPi(b^(1/2)*x/a^(1/2)/(1+b
*x^2/a)^(1/2),1-a*f/b/e,(1-a*d/b/c)^(1/2))/b^(1/2)/c/e^2/(b*x^2+a)^(1/2)/(
a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.22 (sec) , antiderivative size = 533, normalized size of antiderivative = 1.28

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^2(e+fx^2)}\,dx$$
$$= \frac{-a\sqrt{\frac{b}{a}}cef - b\sqrt{\frac{b}{a}}cefx^2 - a\sqrt{\frac{b}{a}}defx^2 - b\sqrt{\frac{b}{a}}defx^4 - ibcefx\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(i\text{arcsinh}\left(\sqrt{\frac{b}{a}}x\right)\mid\frac{ad}{bc}\right.}{}$$

input
```
Integrate[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(x^2*(e + f*x^2)),x]
```

output
```
(-(a*Sqrt[b/a]*c*e*f) - b*Sqrt[b/a]*c*e*f*x^2 - a*Sqrt[b/a]*d*e*f*x^2 - b*
Sqrt[b/a]*d*e*f*x^4 - I*b*c*e*f*x*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*
EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + I*b*e*(-(d*e) + c*f)*x*Sq
rt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a
*d)/(b*c)] + I*b*d*e^2*x*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticP
i[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*b*c*e*f*x*Sqrt[1 +
 (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a
]*x], (a*d)/(b*c)] - I*a*d*e*f*x*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*E
llipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + I*a*c*f^2*x*
Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e), I*ArcSinh[
Sqrt[b/a]*x], (a*d)/(b*c)])/(Sqrt[b/a]*e^2*f*x*Sqrt[a + b*x^2]*Sqrt[c + d*
x^2])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^2\left(e+fx^2\right)}dx$$
$$\Big\downarrow \; 450$$
$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^2\left(e+fx^2\right)}dx$$

input
```
Int[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(x^2*(e + f*x^2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [A] (verified)

Time = 8.30 (sec) , antiderivative size = 412, normalized size of antiderivative = 0.99

method	result
risch	$-\frac{\sqrt{b\,x^{2}+a}\,\sqrt{x^{2}d+c}}{ex}+\frac{\left(-\frac{\left(ac\,f^{2}-adef-bcef+bd\,e^{2}\right)\sqrt{1+\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,\operatorname{EllipticPi}\left(x\sqrt{-\frac{b}{a}},\frac{af}{be},\frac{\sqrt{-\frac{d}{c}}}{\sqrt{-\frac{b}{a}}}\right)}{fe\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^{4}+ad\,x^{2}+x^{2}bc+ac}}+\frac{bd\left(\frac{e\sqrt{1+\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,\operatorname{Elli}}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^{4}}}\right.}{}\right.}{}$
default	$\frac{\left(-\sqrt{-\frac{b}{a}}\,bdef\,x^{4}-\sqrt{\frac{b\,x^{2}+a}{a}}\,\sqrt{\frac{x^{2}d+c}{c}}\,\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)bcefx+\sqrt{\frac{b\,x^{2}+a}{a}}\,\sqrt{\frac{x^{2}d+c}{c}}\,\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)bd\,e^{2}x+\sqrt{\frac{b\,x}{}}\right.}{}$
elliptic	$\frac{\sqrt{\left(b\,x^{2}+a\right)\left(x^{2}d+c\right)}\left(-\frac{\sqrt{bd\,x^{4}+ad\,x^{2}+x^{2}bc+ac}}{ex}+\frac{bd\sqrt{1+\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{f\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^{4}+ad\,x^{2}+x^{2}bc+ac}}-\frac{bc\sqrt{1+\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,\operatorname{EllipticF}}{e\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^{4}+ad}}\right.}{}$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^2/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
-1/e*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x+1/e*(-(a*c*f^2-a*d*e*f-b*c*e*f+b*d*
e^2)/f/e/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2
+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a
)^(1/2))+b*d/f*(e/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^
4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(
1/2))-f*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^
2+b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))
-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2)))))*((b*x^2+a)*(d*x^2+c
))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^2\left(e+fx^2\right)}\,dx=\text{Timed out}$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^2/(f*x^2+e),x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^2\left(e+fx^2\right)}\,dx=\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^2\left(e+fx^2\right)}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(1/2)/x**2/(f*x**2+e),x)
```

output
```
Integral(sqrt(a + b*x**2)*sqrt(c + d*x**2)/(x**2*(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^2\left(e+fx^2\right)}\,dx=\int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{(fx^2+e)x^2}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^2/(f*x^2+e),x, algorithm="maxi
ma")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/((f*x^2 + e)*x^2), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^2\left(e+fx^2\right)}\,dx=\int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{(fx^2+e)x^2}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^2/(f*x^2+e),x, algorithm="giac
")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/((f*x^2 + e)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^2\left(e+fx^2\right)}\,dx=\int \frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{x^2\,\left(f\,x^2+e\right)}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2))/(x^2*(e + f*x^2)),x)
```

output
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2))/(x^2*(e + f*x^2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^2\left(e+fx^2\right)}\,dx=\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{f\,x^4+e\,x^2}dx$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^2/(f*x^2+e),x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(e*x**2 + f*x**4),x)
```

3.74 $\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^4(e+fx^2)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 528

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^4(e+fx^2)}\,dx = \frac{b(bce+ade-3acf)x\sqrt{c+dx^2}}{3ace^2\sqrt{a+bx^2}} + \frac{d(be-3af)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3ace^2} - \frac{bfx\sqrt{a+bx^2}(c+dx^2)^{3/2}}{ace^2} - \frac{(a+bx^2)^{3/2}(c+dx^2)^{3/2}}{3acex^3} + \frac{f(a+bx^2)^{3/2}(c+dx^2)^{3/2}}{ace^2x} - \frac{\sqrt{b}(bce+ade-3acf)\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{3\sqrt{a}ce^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} + \frac{\sqrt{a}\sqrt{b}(2de-3cf)\sqrt{c+dx^2}\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{3ce^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} - \frac{a^{3/2}f(de-cf)\sqrt{c+dx^2}\operatorname{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce^3\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
1/3*b*(-3*a*c*f+a*d*e+b*c*e)*x*(d*x^2+c)^(1/2)/a/c/e^2/(b*x^2+a)^(1/2)+1/3
*d*(-3*a*f+b*e)*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a/c/e^2-b*f*x*(b*x^2+a)^
(1/2)*(d*x^2+c)^(3/2)/a/c/e^2-1/3*(b*x^2+a)^(3/2)*(d*x^2+c)^(3/2)/a/c/e/x^
3+f*(b*x^2+a)^(3/2)*(d*x^2+c)^(3/2)/a/c/e^2/x-1/3*b^(1/2)*(-3*a*c*f+a*d*e+
b*c*e)*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*
d/b/c)^(1/2))/a^(1/2)/c/e^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2
)+1/3*a^(1/2)*b^(1/2)*(-3*c*f+2*d*e)*(d*x^2+c)^(1/2)*InverseJacobiAM(arcta
n(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/c/e^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)
/c/(b*x^2+a))^(1/2)-a^(3/2)*f*(-c*f+d*e)*(d*x^2+c)^(1/2)*EllipticPi(b^(1/2
)*x/a^(1/2)/(1+b*x^2/a)^(1/2),1-a*f/b/e,(1-a*d/b/c)^(1/2))/b^(1/2)/c/e^3/(
b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.44 (sec) , antiderivative size = 348, normalized size of antiderivative = 0.66

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^4(e+fx^2)}\,dx = \frac{\sqrt{\frac{b}{a}}\left(-\sqrt{\frac{b}{a}}e(a+bx^2)(c+dx^2)(bcex^2+adex^2+ac(e-3fx^2))+ibce(-bce-ade+3acf)x^3\sqrt{1+\frac{bx^2}{a}}\right.}{}$$

input
```
Integrate[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(x^4*(e + f*x^2)),x]
```

output
```
(Sqrt[b/a]*(-(Sqrt[b/a]*e*(a + b*x^2)*(c + d*x^2)*(b*c*e*x^2 + a*d*e*x^2 +
 a*c*(e - 3*f*x^2))) + I*b*c*e*(-(b*c*e) - a*d*e + 3*a*c*f)*x^3*Sqrt[1 + (
b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c
)] - I*b*c*e*(-(b*c*e) - 2*a*d*e + 3*a*c*f)*x^3*Sqrt[1 + (b*x^2)/a]*Sqrt[1
 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - (3*I)*a*c*(
-(b*e) + a*f)*(-(d*e) + c*f)*x^3*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*E
llipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(3*b*c*e^3*x
^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^4\left(e+fx^2\right)}\,dx$$

$$\Big\downarrow \ 450$$

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^4\left(e+fx^2\right)}dx$$

input
```
Int[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(x^4*(e + f*x^2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [A] (verified)

Time = 8.49 (sec) , antiderivative size = 461, normalized size of antiderivative = 0.87

method	result
risch	$-\frac{\sqrt{b\,x^{2}+a}\,\sqrt{x^{2}d+c}\,\left(-3acf\,x^{2}+ade\,x^{2}+bce\,x^{2}+ace\right)}{3ac\,e^{2}x^{3}}-\frac{\left(-\frac{b(3acf-ade-bce)c\sqrt{1+\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,\left(\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)-\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^{4}+ad\,x^{2}+x^{2}bc+ac}}\right.}{}$
default	$\frac{\sqrt{b\,x^{2}+a}\,\sqrt{x^{2}d+c}\,\left(3\sqrt{-\frac{b}{a}}\,abcdef\,x^{6}-\sqrt{-\frac{b}{a}}\,ab\,d^{2}e^{2}x^{6}-\sqrt{-\frac{b}{a}}\,b^{2}cd\,e^{2}x^{6}+3\sqrt{\frac{b\,x^{2}+a}{a}}\,\sqrt{\frac{x^{2}d+c}{c}}\,\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)ab\,c^{2}ef\,x\right.}{}$
elliptic	Expression too large to display

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^4/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(-3*a*c*f*x^2+a*d*e*x^2+b*c*e*x^2+a*c
*e)/a/c/e^2/x^3-1/3/a/c/e^2*(-b*(3*a*c*f-a*d*e-b*c*e)*c/(-b/a)^(1/2)*(1+b*
x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(Ellipt
icF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+
(a*d+b*c)/c/b)^(1/2)))+a*c*d*e*b/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c
)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(
a*d+b*c)/c/b)^(1/2))-3*(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)*a*c/e/(-b/a)^(1/2
)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*
EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2)))*((b*x^2+a)
*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^4\left(e+fx^2\right)}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^4/(f*x^2+e),x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^4\left(e+fx^2\right)}\,dx = \int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^4\left(e+fx^2\right)}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(1/2)/x**4/(f*x**2+e),x)
```

output
```
Integral(sqrt(a + b*x**2)*sqrt(c + d*x**2)/(x**4*(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^4(e+fx^2)}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{(fx^2+e)x^4}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^4/(f*x^2+e),x, algorithm="maxi
ma")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/((f*x^2 + e)*x^4), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^4(e+fx^2)}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{(fx^2+e)x^4}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^4/(f*x^2+e),x, algorithm="giac
")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/((f*x^2 + e)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^4(e+fx^2)}\,dx = \int \frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{x^4\,\left(f\,x^2+e\right)}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2))/(x^4*(e + f*x^2)),x)
```

output
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2))/(x^4*(e + f*x^2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^4(e+fx^2)}\,dx = \int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{f\,x^6+e\,x^4}dx$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^4/(f*x^2+e),x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(e*x**4 + f*x**6),x)
```

3.75 $\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^6(e+fx^2)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 741

$$\begin{aligned}
&\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^6\,(e+fx^2)}\,dx \\
&= -\frac{b(2b^2c^2e^2-abce(2de-5cf)+a^2(2d^2e^2+5cdef-15c^2f^2))\,x\sqrt{c+dx^2}}{15a^2c^2e^3\sqrt{a+bx^2}} \\
&\quad -\frac{d(2b^2ce^2-15a^2cf^2-abe(de-5cf))\,x\sqrt{a+bx^2}\sqrt{c+dx^2}}{15a^2c^2e^3} \\
&\quad +\frac{b(bde^2+5acf^2)\,x\sqrt{a+bx^2}(c+dx^2)^{3/2}}{5a^2c^2e^3}-\frac{(a+bx^2)^{3/2}\,(c+dx^2)^{3/2}}{5acex^5} \\
&\quad +\frac{(2bce+2ade+5acf)\,(a+bx^2)^{3/2}\,(c+dx^2)^{3/2}}{15a^2c^2e^2x^3} \\
&\quad -\frac{(bde^2+5acf^2)\,(a+bx^2)^{3/2}\,(c+dx^2)^{3/2}}{5a^2c^2e^3x} \\
&\quad +\frac{\sqrt{b}(2b^2c^2e^2-abce(2de-5cf)+a^2(2d^2e^2+5cdef-15c^2f^2))\,\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{15a^{3/2}c^2e^3\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} \\
&\quad -\frac{\sqrt{b}(bcde^2+a(d^2e^2+10cdef-15c^2f^2))\,\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{15\sqrt{a}c^2e^3\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} \\
&\quad +\frac{a^{3/2}f^2(de-cf)\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce^4\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}
\end{aligned}$$

output
```
-1/15*b*(2*b^2*c^2*e^2-a*b*c*e*(-5*c*f+2*d*e)+a^2*(-15*c^2*f^2+5*c*d*e*f+2
*d^2*e^2))*x*(d*x^2+c)^(1/2)/a^2/c^2/e^3/(b*x^2+a)^(1/2)-1/15*d*(2*b^2*c*e
^2-15*a^2*c*f^2-a*b*e*(-5*c*f+d*e))*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a^2/
c^2/e^3+1/5*b*(5*a*c*f^2+b*d*e^2)*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/a^2/c^
2/e^3-1/5*(b*x^2+a)^(3/2)*(d*x^2+c)^(3/2)/a/c/e/x^5+1/15*(5*a*c*f+2*a*d*e+
2*b*c*e)*(b*x^2+a)^(3/2)*(d*x^2+c)^(3/2)/a^2/c^2/e^2/x^3-1/5*(5*a*c*f^2+b*
d*e^2)*(b*x^2+a)^(3/2)*(d*x^2+c)^(3/2)/a^2/c^2/e^3/x+1/15*b^(1/2)*(2*b^2*c
^2*e^2-a*b*c*e*(-5*c*f+2*d*e)+a^2*(-15*c^2*f^2+5*c*d*e*f+2*d^2*e^2))*(d*x^
2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2)
)/a^(3/2)/c^2/e^3/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)-1/15*b^(
1/2)*(b*c*d*e^2+a*(-15*c^2*f^2+10*c*d*e*f+d^2*e^2))*(d*x^2+c)^(1/2)*Invers
eJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/a^(1/2)/c^2/e^3/(b*
x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)+a^(3/2)*f^2*(-c*f+d*e)*(d*x^2
+c)^(1/2)*EllipticPi(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),1-a*f/b/e,(1-a*d/
b/c)^(1/2))/b^(1/2)/c/e^4/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.10 (sec) , antiderivative size = 1206, normalized size of antiderivative = 1.63

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^6\left(e+fx^2\right)}\,dx = \text{Too large to display}$$

input
```
Integrate[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(x^6*(e + f*x^2)),x]
```

output
```
(-3*a^3*Sqrt[b/a]*c^3*e^3 - 4*a^3*(b/a)^(3/2)*c^3*e^3*x^2 - 4*a^3*Sqrt[b/a
]*c^2*d*e^3*x^2 + 5*a^3*Sqrt[b/a]*c^3*e^2*f*x^2 + a*b^2*Sqrt[b/a]*c^3*e^3*
x^4 - 7*a^3*(b/a)^(3/2)*c^2*d*e^3*x^4 + a^3*Sqrt[b/a]*c*d^2*e^3*x^4 + 10*a
^3*(b/a)^(3/2)*c^3*e^2*f*x^4 + 10*a^3*Sqrt[b/a]*c^2*d*e^2*f*x^4 - 15*a^3*S
qrt[b/a]*c^3*e*f^2*x^4 + 2*b^3*Sqrt[b/a]*c^3*e^3*x^6 - a*b^2*Sqrt[b/a]*c^2
*d*e^3*x^6 - a^3*(b/a)^(3/2)*c*d^2*e^3*x^6 + 2*a^3*Sqrt[b/a]*d^3*e^3*x^6 +
 5*a*b^2*Sqrt[b/a]*c^3*e^2*f*x^6 + 15*a^3*(b/a)^(3/2)*c^2*d*e^2*f*x^6 + 5*
a^3*Sqrt[b/a]*c*d^2*e^2*f*x^6 - 15*a^3*(b/a)^(3/2)*c^3*e*f^2*x^6 - 15*a^3*
Sqrt[b/a]*c^2*d*e*f^2*x^6 + 2*b^3*Sqrt[b/a]*c^2*d*e^3*x^8 - 2*a*b^2*Sqrt[b
/a]*c*d^2*e^3*x^8 + 2*a^3*(b/a)^(3/2)*d^3*e^3*x^8 + 5*a*b^2*Sqrt[b/a]*c^2*
d*e^2*f*x^8 + 5*a^3*(b/a)^(3/2)*c*d^2*e^2*f*x^8 - 15*a^3*(b/a)^(3/2)*c^2*d
*e*f^2*x^8 - I*b*c*e*(-2*b^2*c^2*e^2 + a*b*c*e*(2*d*e - 5*c*f) + a^2*(-2*d
^2*e^2 - 5*c*d*e*f + 15*c^2*f^2))*x^5*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)
/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + I*b*c*e*(-2*b^2*c^2*e
^2 + a*b*c*e*(3*d*e - 5*c*f) - a^2*(d^2*e^2 + 10*c*d*e*f - 15*c^2*f^2))*x^
5*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x]
, (a*d)/(b*c)] + (15*I)*a^2*b*c^2*d*e^2*f*x^5*Sqrt[1 + (b*x^2)/a]*Sqrt[1 +
 (d*x^2)/c]*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] -
 (15*I)*a^2*b*c^3*e*f^2*x^5*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Ellipt
icPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - (15*I)*a^3*c^2...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^6\left(e+fx^2\right)}\,dx$$

$$\Big\downarrow \; 450$$

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^6\left(e+fx^2\right)}dx$$

input
```
Int[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(x^6*(e + f*x^2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [A] (verified)

Time = 9.38 (sec) , antiderivative size = 813, normalized size of antiderivative = 1.10

method	result
risch	$-\frac{\sqrt{b\,x^2+a}\,\sqrt{x^2 d+c}\,(15 f^2 x^4 a^2 c^2-5 a^2 c d e f\,x^4-2 a^2 d^2 e^2 x^4-5 a b\,c^2 e f\,x^4+2 a b c d\,e^2 x^4-2 b^2 c^2 e^2 x^4-5 a^2 c^2 e f\,x^2+a^2 c d\,e^2 x^2+a b\,c^2 e^{2}}{15 a^2 c^2 e^3 x^5}$
default	Expression too large to display
elliptic	Expression too large to display

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^6/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output

```
-1/15*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(15*a^2*c^2*f^2*x^4-5*a^2*c*d*e*f*x^
4-2*a^2*d^2*e^2*x^4-5*a*b*c^2*e*f*x^4+2*a*b*c*d*e^2*x^4-2*b^2*c^2*e^2*x^4-
5*a^2*c^2*e*f*x^2+a^2*c*d*e^2*x^2+a*b*c^2*e^2*x^2+3*a^2*c^2*e^2)/a^2/c^2/e
^3/x^5+1/15/a^2/c^2/e^3*(-b*(15*a^2*c^2*f^2-5*a^2*c*d*e*f-2*a^2*d^2*e^2-5*
a*b*c^2*e*f+2*a*b*c*d*e^2-2*b^2*c^2*e^2)*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*
(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(EllipticF(x*(-b/a)^
(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b
)^(1/2)))-a*b^2*c^2*d*e^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)
/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c
)/c/b)^(1/2))-a^2*b*c*d^2*e^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(
1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d
+b*c)/c/b)^(1/2))-15*a^2*c^2*f*(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/e/(-b/a)^
(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1
/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))+5*a^2*b
*c^2*d*e*f/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x
^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2)))*
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^6\,(e+fx^2)}\,dx = \text{Timed out}$$

input

```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^6/(f*x^2+e),x, algorithm="fric
as")
```

output

```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^6\left(e+fx^2\right)}\,dx=\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^6\left(e+fx^2\right)}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(1/2)/x**6/(f*x**2+e),x)
```

output
```
Integral(sqrt(a + b*x**2)*sqrt(c + d*x**2)/(x**6*(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^6\left(e+fx^2\right)}\,dx=\int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{(fx^2+e)x^6}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^6/(f*x^2+e),x, algorithm="maxi
ma")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/((f*x^2 + e)*x^6), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^6\left(e+fx^2\right)}\,dx=\int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{(fx^2+e)x^6}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^6/(f*x^2+e),x, algorithm="giac
")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/((f*x^2 + e)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^6\,(e+fx^2)}\,dx = \int \frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{x^6\,\left(f\,x^2+e\right)}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2))/(x^6*(e + f*x^2)),x)
```

output
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2))/(x^6*(e + f*x^2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^6\,(e+fx^2)}\,dx = \int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{f\,x^8+e\,x^6}dx$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^6/(f*x^2+e),x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(e*x**6 + f*x**8),x)
```

3.76 $\int \frac{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 771

$$\int \frac{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^2}\,dx = -\frac{(4a^2d^2f^2+4abdf(5de-cf)-b^2(105d^2e^2-20cdef-4c^2f^2))\,x\sqrt{c+dx^2}}{30bd^2f^4\sqrt{a+bx^2}} + \frac{\left(\frac{2af}{b}-\frac{35d^2e^2-16cdef-4c^2f^2}{d(de-cf)}\right)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{30f^3} + \frac{(be(7de-2cf)-2af(de-cf))x\sqrt{a+bx^2}(c+dx^2)^{3/2}}{10df^2(be-af)(de-cf)} - \frac{e^2x(a+bx^2)^{3/2}(c+dx^2)^{3/2}}{2f(be-af)(de-cf)(e+fx^2)} + \frac{\sqrt{a}(4a^2d^2f^2+4abdf(5de-cf)-b^2(105d^2e^2-20cdef-4c^2f^2))\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{30b^{3/2}d^2f^4\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} + \frac{a^{3/2}(2a^2cdf^3+b^2e(105d^2e^2-55cdef-2c^2f^2)-2abf(45d^2e^2-19cdef-c^2f^2))\sqrt{c+dx^2}\,\text{EllipticF}(}{30b^{3/2}cdf^4(be-af)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} - \frac{a^{3/2}e(be(7de-6cf)-af(6de-5cf))\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{2\sqrt{b}cf^4(be-af)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
-1/30*(4*a^2*d^2*f^2+4*a*b*d*f*(-c*f+5*d*e)-b^2*(-4*c^2*f^2-20*c*d*e*f+105
*d^2*e^2))*x*(d*x^2+c)^(1/2)/b/d^2/f^4/(b*x^2+a)^(1/2)+1/30*(2*a*f/b-(-4*c
^2*f^2-16*c*d*e*f+35*d^2*e^2)/d/(-c*f+d*e))*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1
/2)/f^3+1/10*(b*e*(-2*c*f+7*d*e)-2*a*f*(-c*f+d*e))*x*(b*x^2+a)^(1/2)*(d*x^
2+c)^(3/2)/d/f^2/(-a*f+b*e)/(-c*f+d*e)-1/2*e^2*x*(b*x^2+a)^(3/2)*(d*x^2+c)
^(3/2)/f/(-a*f+b*e)/(-c*f+d*e)/(f*x^2+e)+1/30*a^(1/2)*(4*a^2*d^2*f^2+4*a*b
*d*f*(-c*f+5*d*e)-b^2*(-4*c^2*f^2-20*c*d*e*f+105*d^2*e^2))*(d*x^2+c)^(1/2)
*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/b^(3/2)/
d^2/f^4/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)+1/30*a^(3/2)*(2*a^
2*c*d*f^3+b^2*e*(-2*c^2*f^2-55*c*d*e*f+105*d^2*e^2)-2*a*b*f*(-c^2*f^2-19*c
*d*e*f+45*d^2*e^2))*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/
2)),(1-a*d/b/c)^(1/2))/b^(3/2)/c/d/f^4/(-a*f+b*e)/(b*x^2+a)^(1/2)/(a*(d*x^
2+c)/c/(b*x^2+a))^(1/2)-1/2*a^(3/2)*e*(b*e*(-6*c*f+7*d*e)-a*f*(-5*c*f+6*d*
e))*(d*x^2+c)^(1/2)*EllipticPi(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),1-a*f/b
/e,(1-a*d/b/c)^(1/2))/b^(1/2)/c/f^4/(-a*f+b*e)/(b*x^2+a)^(1/2)/(a*(d*x^2+c
)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.79 (sec) , antiderivative size = 505, normalized size of antiderivative = 0.65

$$\int \frac{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^2}\,dx$$

$$= \frac{icf(4a^2d^2f^2+4abdf(5de-cf)+b^2(-105d^2e^2+20cdef+4c^2f^2))\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}(e+fx^2)\,E\Big(i\mathrm{arc}}{}$$

input
```
Integrate[(x^6*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(e + f*x^2)^2,x]
```

output
```
(I*c*f*(4*a^2*d^2*f^2 + 4*a*b*d*f*(5*d*e - c*f) + b^2*(-105*d^2*e^2 + 20*c
*d*e*f + 4*c^2*f^2))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(e + f*x^2)*E
llipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*(2*a^2*c*d^2*f^3 + 2*a*b
*d*f*(45*d^2*e^2 - 10*c*d*e*f - 3*c^2*f^2) + b^2*(-105*d^3*e^3 - 15*c*d^2*
e^2*f + 20*c^2*d*e*f^2 + 4*c^3*f^3))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/
c]*(e + f*x^2)*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + d*(Sqrt[b/
a]*f^2*x*(a + b*x^2)*(c + d*x^2)*(2*b*c*f*(e + f*x^2) + 2*a*d*f*(e + f*x^2
) + b*d*(-35*e^2 - 14*e*f*x^2 + 6*f^2*x^4)) - (15*I)*b*d*e*(b*e*(7*d*e - 6
*c*f) + a*f*(-6*d*e + 5*c*f))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(e +
 f*x^2)*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(30
*b*Sqrt[b/a]*d^2*f^5*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^2}\,dx$$

$$\Big\downarrow \text{ 450}$$

$$\int \frac{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^2}dx$$

input
```
Int[(x^6*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(e + f*x^2)^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1537 vs. $2(727) = 1454$.

Time = 20.29 (sec) , antiderivative size = 1538, normalized size of antiderivative = 1.99

method	result	size
risch	Expression too large to display	1538
elliptic	Expression too large to display	1932
default	Expression too large to display	2886

input
```
int(x^6*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^2,x,method=_RETURNVERBOS
E)
```

output
```
1/15*x*(3*b*d*f*x^2+a*d*f+b*c*f-10*b*d*e)*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/
b/d/f^3-1/15/b/d/f^3*((a^2*c*d*f^3+a*b*c^2*f^3+20*a*b*c*d*e*f^2-45*a*b*d^2
*e^2*f-45*b^2*c*d*e^2*f+60*b^2*d^2*e^3)/f^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)
*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^
(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-1/f*(2*a^2*d^2*f^2-2*a*b*c*d*f^2+10*a*b*d^
2*e*f+2*b^2*c^2*f^2+10*b^2*c*d*e*f-45*b^2*d^2*e^2)*c/(-b/a)^(1/2)*(1+b*x^2
/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*(Ellipti
cF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(
a*d+b*c)/c/b)^(1/2)))-15*b*d*e/f^2*(3*a*c*f^2-4*a*d*e*f-4*b*c*e*f+5*b*d*e^
2)/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x
^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2
))+15*b*d*e^3*(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/f^2*(1/2*f^2/(a*c*f^2-a*d*
e*f-b*c*e*f+b*d*e^2)/e*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(f*x^2+e)-1/2
*d*b/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d
*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2)
,(-1+(a*d+b*c)/c/b)^(1/2))+1/2*f*b/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/e*c/(
-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a
*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-1/2*f*b/(a*c*
f^2-a*d*e*f-b*c*e*f+b*d*e^2)/e*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c
)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(-...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(x^6*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^2}\,dx=\int \frac{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^2}\,dx$$

input
```
integrate(x**6*(b*x**2+a)**(1/2)*(d*x**2+c)**(1/2)/(f*x**2+e)**2,x)
```

output
```
Integral(x**6*sqrt(a + b*x**2)*sqrt(c + d*x**2)/(e + f*x**2)**2, x)
```

Maxima [F]

$$\int \frac{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^2}\,dx=\int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}x^6}{(fx^2+e)^2}\,dx$$

input
```
integrate(x^6*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="ma
xima")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x^6/(f*x^2 + e)^2, x)
```

Giac [F]

$$\int \frac{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^2}\,dx=\int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}x^6}{(fx^2+e)^2}\,dx$$

input
```
integrate(x^6*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="gi
ac")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x^6/(f*x^2 + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}}{\left(e+fx^2\right)^2}\,dx=\int \frac{x^6\,\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{{\left(f\,x^2+e\right)}^2}\,dx$$

input
```
int((x^6*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2))/(e + f*x^2)^2,x)
```

output
```
int((x^6*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2))/(e + f*x^2)^2, x)
```

Reduce [F]

$$\int \frac{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}}{\left(e+fx^2\right)^2}\,dx=\int \frac{x^6\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{\left(f\,x^2+e\right)^2}dx$$

input
```
int(x^6*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^2,x)
```

output
```
int(x^6*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^2,x)
```

3.77 $\int \frac{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^2} \, dx$

Optimal result

Integrand size = 35, antiderivative size = 587

$$\begin{aligned}
&\int \frac{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^2} \, dx \\
&= -\frac{(15bde-2bcf-2adf)x\sqrt{c+dx^2}}{6df^3\sqrt{a+bx^2}} + \frac{(5de-2cf)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{6f^2(de-cf)} \\
&\quad - \frac{bex\sqrt{a+bx^2}(c+dx^2)^{3/2}}{2f(be-af)(de-cf)} + \frac{ex(a+bx^2)^{3/2}(c+dx^2)^{3/2}}{2(be-af)(de-cf)(e+fx^2)} \\
&\quad + \frac{\sqrt{a}(15bde-2bcf-2adf)\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{6\sqrt{b}df^3\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} \\
&\quad - \frac{a^{3/2}(be(15de-7cf)-4af(3de-cf))\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{6\sqrt{b}cf^3(be-af)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} \\
&\quad + \frac{a^{3/2}(be(5de-4cf)-af(4de-3cf))\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{2\sqrt{b}cf^3(be-af)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}
\end{aligned}$$

output
```
-1/6*(-2*a*d*f-2*b*c*f+15*b*d*e)*x*(d*x^2+c)^(1/2)/d/f^3/(b*x^2+a)^(1/2)+1
/6*(-2*c*f+5*d*e)*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/f^2/(-c*f+d*e)-1/2*b*e
*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/f/(-a*f+b*e)/(-c*f+d*e)+1/2*e*x*(b*x^2+
a)^(3/2)*(d*x^2+c)^(3/2)/(-a*f+b*e)/(-c*f+d*e)/(f*x^2+e)+1/6*a^(1/2)*(-2*a
*d*f-2*b*c*f+15*b*d*e)*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^
2/a)^(1/2),(1-a*d/b/c)^(1/2))/b^(1/2)/d/f^3/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c
/(b*x^2+a))^(1/2)-1/6*a^(3/2)*(b*e*(-7*c*f+15*d*e)-4*a*f*(-c*f+3*d*e))*(d*
x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/
b^(1/2)/c/f^3/(-a*f+b*e)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)+1
/2*a^(3/2)*(b*e*(-4*c*f+5*d*e)-a*f*(-3*c*f+4*d*e))*(d*x^2+c)^(1/2)*Ellipti
cPi(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),1-a*f/b/e,(1-a*d/b/c)^(1/2))/b^(1/
2)/c/f^3/(-a*f+b*e)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.60 (sec) , antiderivative size = 378, normalized size of antiderivative = 0.64

$$\int \frac{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^2}\,dx$$
$$= \frac{-icf(-15bde+2bcf+2adf)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}(e+fx^2)\,E\left(i\text{arcsinh}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)-i(2adf(-6de+cf}{}$$

input
```
Integrate[(x^4*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(e + f*x^2)^2,x]
```

output
```
((-I)*c*f*(-15*b*d*e + 2*b*c*f + 2*a*d*f)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*
x^2)/c]*(e + f*x^2)*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*(2*
a*d*f*(-6*d*e + c*f) + b*(15*d^2*e^2 + 3*c*d*e*f - 2*c^2*f^2))*Sqrt[1 + (b
*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(e + f*x^2)*EllipticF[I*ArcSinh[Sqrt[b/a]*x],
 (a*d)/(b*c)] + d*(Sqrt[b/a]*f^2*x*(a + b*x^2)*(c + d*x^2)*(5*e + 2*f*x^2)
 + (3*I)*(b*e*(5*d*e - 4*c*f) + a*f*(-4*d*e + 3*c*f))*Sqrt[1 + (b*x^2)/a]*
Sqrt[1 + (d*x^2)/c]*(e + f*x^2)*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a
]*x], (a*d)/(b*c)]))/(6*Sqrt[b/a]*d*f^4*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e
 + f*x^2))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^2}\,dx$$
$$\Big\downarrow \; 450$$
$$\int \frac{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^2}dx$$

input
```
Int[(x^4*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(e + f*x^2)^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1264 vs. 2(543) = 1086.

Time = 21.96 (sec) , antiderivative size = 1265, normalized size of antiderivative = 2.16

method	result	size
elliptic	Expression too large to display	1265
risch	Expression too large to display	1670
default	Expression too large to display	1756

input
```
int(x^4*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^2,x,method=_RETURNVERBOS
E)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/2/f^2*e*x*(
b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(f*x^2+e)+1/3/f^2*x*(b*d*x^4+a*d*x^2+b*
c*x^2+a*c)^(1/2)-2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x
^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^
(1/2))/f^3*a*d*e+1/2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d
*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b
)^(1/2))/f^3*b*c*e+5/2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b
*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c
/b)^(1/2))*b*d*e^2/f^4+1/3*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1
/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/f^2*a*EllipticE(x*(-b/a)^(1/2),(-1
+(a*d+b*c)/c/b)^(1/2))-1/3*c^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^
(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*b/f^2*EllipticF(x*(-b/a)^(1/2)
,(-1+(a*d+b*c)/c/b)^(1/2))+1/3*c^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2
/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*b/f^2*EllipticE(x*(-b/a)^(
1/2),(-1+(a*d+b*c)/c/b)^(1/2))+2*e/f^3/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d
*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2
),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*a*d+2*e/f^3/(-b/a)^(1/2)*(1+b*x^2/a
)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x
*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*b*c-5/2*e^2/f^4/(-b/a)^
(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(x^4*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}{\left(e+fx^2\right)^2}\,dx=\int \frac{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}{\left(e+fx^2\right)^2}\,dx$$

input
```
integrate(x**4*(b*x**2+a)**(1/2)*(d*x**2+c)**(1/2)/(f*x**2+e)**2,x)
```

output
```
Integral(x**4*sqrt(a + b*x**2)*sqrt(c + d*x**2)/(e + f*x**2)**2, x)
```

Maxima [F]

$$\int \frac{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}{\left(e+fx^2\right)^2}\,dx=\int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}x^4}{\left(fx^2+e\right)^2}\,dx$$

input
```
integrate(x^4*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="ma
xima")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x^4/(f*x^2 + e)^2, x)
```

Giac [F]

$$\int \frac{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}{\left(e+fx^2\right)^2}\,dx=\int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}x^4}{\left(fx^2+e\right)^2}\,dx$$

input
```
integrate(x^4*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="gi
ac")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x^4/(f*x^2 + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^2}\,dx = \int \frac{x^4\,\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{{\left(f\,x^2+e\right)}^2}\,dx$$

input
```
int((x^4*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2))/(e + f*x^2)^2,x)
```

output
```
int((x^4*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2))/(e + f*x^2)^2, x)
```

Reduce [F]

$$\int \frac{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^2}\,dx = \text{too large to display}$$

input
```
int(x^4*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^2,x)
```

output
```
(2*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*c*f*x - 4*sqrt(c + d*x**2)*sqrt(a +
 b*x**2)*a*d*e*x - 4*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b*c*e*x + 3*sqrt(c
+ d*x**2)*sqrt(a + b*x**2)*b*d*e*x**3 - 2*int((sqrt(c + d*x**2)*sqrt(a + b
*x**2)*x**6)/(a*c*e**2 + 2*a*c*e*f*x**2 + a*c*f**2*x**4 + a*d*e**2*x**2 +
2*a*d*e*f*x**4 + a*d*f**2*x**6 + b*c*e**2*x**2 + 2*b*c*e*f*x**4 + b*c*f**2
*x**6 + b*d*e**2*x**4 + 2*b*d*e*f*x**6 + b*d*f**2*x**8),x)*a*b*c*d*e*f**2
- 2*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**6)/(a*c*e**2 + 2*a*c*e*f*x**
2 + a*c*f**2*x**4 + a*d*e**2*x**2 + 2*a*d*e*f*x**4 + a*d*f**2*x**6 + b*c*e
**2*x**2 + 2*b*c*e*f*x**4 + b*c*f**2*x**6 + b*d*e**2*x**4 + 2*b*d*e*f*x**6
 + b*d*f**2*x**8),x)*a*b*c*d*f**3*x**2 + 7*int((sqrt(c + d*x**2)*sqrt(a +
b*x**2)*x**6)/(a*c*e**2 + 2*a*c*e*f*x**2 + a*c*f**2*x**4 + a*d*e**2*x**2 +
 2*a*d*e*f*x**4 + a*d*f**2*x**6 + b*c*e**2*x**2 + 2*b*c*e*f*x**4 + b*c*f**
2*x**6 + b*d*e**2*x**4 + 2*b*d*e*f*x**6 + b*d*f**2*x**8),x)*a*b*d**2*e**2*
f + 7*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**6)/(a*c*e**2 + 2*a*c*e*f*x
**2 + a*c*f**2*x**4 + a*d*e**2*x**2 + 2*a*d*e*f*x**4 + a*d*f**2*x**6 + b*c
*e**2*x**2 + 2*b*c*e*f*x**4 + b*c*f**2*x**6 + b*d*e**2*x**4 + 2*b*d*e*f*x*
*6 + b*d*f**2*x**8),x)*a*b*d**2*e*f**2*x**2 + 7*int((sqrt(c + d*x**2)*sqrt
(a + b*x**2)*x**6)/(a*c*e**2 + 2*a*c*e*f*x**2 + a*c*f**2*x**4 + a*d*e**2*x
**2 + 2*a*d*e*f*x**4 + a*d*f**2*x**6 + b*c*e**2*x**2 + 2*b*c*e*f*x**4 + b*
c*f**2*x**6 + b*d*e**2*x**4 + 2*b*d*e*f*x**6 + b*d*f**2*x**8),x)*b**2*c...
```

3.78 $\int \frac{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}{\left(e+fx^2\right)^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 531

$$\int \frac{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}{\left(e+fx^2\right)^2}\,dx = \frac{3bx\sqrt{c+dx^2}}{2f^2\sqrt{a+bx^2}}-\frac{dx\sqrt{a+bx^2}\sqrt{c+dx^2}}{2f(de-cf)}+\frac{bx\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}}{2(be-af)(de-cf)} -\frac{fx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{2(be-af)(de-cf)\left(e+fx^2\right)}-\frac{3\sqrt{a}\sqrt{b}\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{2f^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} +\frac{a^{3/2}(3bde-bcf-2adf)\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{2\sqrt{b}cf^2(be-af)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} -\frac{a^{3/2}(be(3de-2cf)-af(2de-cf))\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{2\sqrt{b}cef^2(be-af)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
3/2*b*x*(d*x^2+c)^(1/2)/f^2/(b*x^2+a)^(1/2)-1/2*d*x*(b*x^2+a)^(1/2)*(d*x^2
+c)^(1/2)/f/(-c*f+d*e)+1/2*b*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/(-a*f+b*e)/
(-c*f+d*e)-1/2*f*x*(b*x^2+a)^(3/2)*(d*x^2+c)^(3/2)/(-a*f+b*e)/(-c*f+d*e)/(
f*x^2+e)-3/2*a^(1/2)*b^(1/2)*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(
1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/f^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*
x^2+a))^(1/2)+1/2*a^(3/2)*(-2*a*d*f-b*c*f+3*b*d*e)*(d*x^2+c)^(1/2)*Inverse
JacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/b^(1/2)/c/f^2/(-a*f+
b*e)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)-1/2*a^(3/2)*(b*e*(-2*
c*f+3*d*e)-a*f*(-c*f+2*d*e))*(d*x^2+c)^(1/2)*EllipticPi(b^(1/2)*x/a^(1/2)/
(1+b*x^2/a)^(1/2),1-a*f/b/e,(1-a*d/b/c)^(1/2))/b^(1/2)/c/e/f^2/(-a*f+b*e)/
(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.51 (sec) , antiderivative size = 874, normalized size of antiderivative = 1.65

$$\int \frac{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^2}\,dx = \text{Too large to display}$$

input
```
Integrate[(x^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(e + f*x^2)^2,x]
```

output
```
(-(a*Sqrt[b/a]*c*e*f^2*x) - b*Sqrt[b/a]*c*e*f^2*x^3 - a*Sqrt[b/a]*d*e*f^2*
x^3 - b*Sqrt[b/a]*d*e*f^2*x^5 - (3*I)*b*c*e*f*Sqrt[1 + (b*x^2)/a]*Sqrt[1 +
 (d*x^2)/c]*(e + f*x^2)*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I
*e*(-3*b*d*e - b*c*f + 2*a*d*f)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(e
 + f*x^2)*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - (3*I)*b*d*e^3*S
qrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e), I*ArcSinh[S
qrt[b/a]*x], (a*d)/(b*c)] + (2*I)*b*c*e^2*f*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (
d*x^2)/c]*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (
2*I)*a*d*e^2*f*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b
*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*a*c*e*f^2*Sqrt[1 + (b*x^2)/a
]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d
)/(b*c)] - (3*I)*b*d*e^2*f*x^2*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Ell
ipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (2*I)*b*c*e*f^
2*x^2*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e), I*Ar
cSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (2*I)*a*d*e*f^2*x^2*Sqrt[1 + (b*x^2)/a]
*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)
/(b*c)] - I*a*c*f^3*x^2*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi
[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(2*Sqrt[b/a]*e*f^3*Sqr
t[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^2}\,dx$$
$$\Big\downarrow \; 450$$
$$\int \frac{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^2}dx$$

input
```
Int[(x^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(e + f*x^2)^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [A] (verified)

Time = 5.07 (sec) , antiderivative size = 849, normalized size of antiderivative = 1.60

method	result
elliptic	$\frac{\sqrt{(b\,x^2+a)(x^2d+c)}\left(-\frac{x\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{2f\left(f\,x^2+e\right)}+\frac{\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)ad}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}\,f^2}-\frac{\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\operatorname{EllipticF}}{2\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad}}\right.}{}$
default	Expression too large to display

input
```
int(x^2*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^2,x,method=_RETURNVERBOS
E)
```

output

```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-1/2/f*x*(b*d
*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(f*x^2+e)+1/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)
*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^
(1/2),(-1+(a*d+b*c)/c/b)^(1/2))/f^2*a*d-1/2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)
*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^
(1/2),(-1+(a*d+b*c)/c/b)^(1/2))*b*c/f^2-3/2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)
*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^
(1/2),(-1+(a*d+b*c)/c/b)^(1/2))*b*d*e/f^3+3/2*b/f^2*c/(-b/a)^(1/2)*(1+b*x^
2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticE
(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+1/2/e/f/(-b/a)^(1/2)*(1+b*x^2/a)
^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*
(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*a*c-1/f^2/(-b/a)^(1/2)*(
1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*Ell
ipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*a*d-1/f^2/(-b/
a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)
^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*b*c+
3/2/f^3*e/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^
2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/
a)^(1/2))*b*d)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}{\left(e+fx^2\right)^2}\,dx = \text{Timed out}$$

input

```
integrate(x^2*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="fr
icas")
```

output

```
Timed out
```

Sympy [F]

$$\int \frac{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^2}\,dx=\int \frac{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^2}\,dx$$

input
```
integrate(x**2*(b*x**2+a)**(1/2)*(d*x**2+c)**(1/2)/(f*x**2+e)**2,x)
```

output
```
Integral(x**2*sqrt(a + b*x**2)*sqrt(c + d*x**2)/(e + f*x**2)**2, x)
```

Maxima [F]

$$\int \frac{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^2}\,dx=\int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}x^2}{(fx^2+e)^2}\,dx$$

input
```
integrate(x^2*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="ma
xima")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x^2/(f*x^2 + e)^2, x)
```

Giac [F]

$$\int \frac{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^2}\,dx=\int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}x^2}{(fx^2+e)^2}\,dx$$

input
```
integrate(x^2*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="gi
ac")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x^2/(f*x^2 + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^2}\,dx = \int \frac{x^2\,\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{{\left(f\,x^2+e\right)}^2}\,dx$$

input
```
int((x^2*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2))/(e + f*x^2)^2,x)
```

output
```
int((x^2*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2))/(e + f*x^2)^2, x)
```

Reduce [F]

$$\int \frac{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^2}\,dx = \text{too large to display}$$

input
```
int(x^2*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^2,x)
```

output
```
(sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*d*x + sqrt(c + d*x**2)*sqrt(a + b*x**
2)*b*c*x - int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**6)/(a*c*e**2 + 2*a*c*
e*f*x**2 + a*c*f**2*x**4 + a*d*e**2*x**2 + 2*a*d*e*f*x**4 + a*d*f**2*x**6
+ b*c*e**2*x**2 + 2*b*c*e*f*x**4 + b*c*f**2*x**6 + b*d*e**2*x**4 + 2*b*d*e
*f*x**6 + b*d*f**2*x**8),x)*a*b*d**2*e*f - int((sqrt(c + d*x**2)*sqrt(a +
b*x**2)*x**6)/(a*c*e**2 + 2*a*c*e*f*x**2 + a*c*f**2*x**4 + a*d*e**2*x**2 +
 2*a*d*e*f*x**4 + a*d*f**2*x**6 + b*c*e**2*x**2 + 2*b*c*e*f*x**4 + b*c*f**
2*x**6 + b*d*e**2*x**4 + 2*b*d*e*f*x**6 + b*d*f**2*x**8),x)*a*b*d**2*f**2*
x**2 - int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**6)/(a*c*e**2 + 2*a*c*e*f*
x**2 + a*c*f**2*x**4 + a*d*e**2*x**2 + 2*a*d*e*f*x**4 + a*d*f**2*x**6 + b*
c*e**2*x**2 + 2*b*c*e*f*x**4 + b*c*f**2*x**6 + b*d*e**2*x**4 + 2*b*d*e*f*x
**6 + b*d*f**2*x**8),x)*b**2*c*d*e*f - int((sqrt(c + d*x**2)*sqrt(a + b*x*
*2)*x**6)/(a*c*e**2 + 2*a*c*e*f*x**2 + a*c*f**2*x**4 + a*d*e**2*x**2 + 2*a
*d*e*f*x**4 + a*d*f**2*x**6 + b*c*e**2*x**2 + 2*b*c*e*f*x**4 + b*c*f**2*x*
*6 + b*d*e**2*x**4 + 2*b*d*e*f*x**6 + b*d*f**2*x**8),x)*b**2*c*d*f**2*x**2
 + 3*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**6)/(a*c*e**2 + 2*a*c*e*f*x*
*2 + a*c*f**2*x**4 + a*d*e**2*x**2 + 2*a*d*e*f*x**4 + a*d*f**2*x**6 + b*c*
e**2*x**2 + 2*b*c*e*f*x**4 + b*c*f**2*x**6 + b*d*e**2*x**4 + 2*b*d*e*f*x**
6 + b*d*f**2*x**8),x)*b**2*d**2*e**2 + 3*int((sqrt(c + d*x**2)*sqrt(a + b*
x**2)*x**6)/(a*c*e**2 + 2*a*c*e*f*x**2 + a*c*f**2*x**4 + a*d*e**2*x**2 ...
```

3.79 $\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 413

$$\begin{aligned}
\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^2}\,dx &= -\frac{bx\sqrt{c+dx^2}}{2ef\sqrt{a+bx^2}}+\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}}{2e\,(e+fx^2)}+\frac{\sqrt{a}\sqrt{b}\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{2ef\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} \\
&\quad -\frac{a^{3/2}\sqrt{b}(de-cf)\sqrt{c+dx^2}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{2cef(be-af)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} \\
&\quad +\frac{a^{3/2}(bde^2-acf^2)\sqrt{c+dx^2}\,\mathrm{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{2\sqrt{b}ce^2f(be-af)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}
\end{aligned}$$

output
```
-1/2*b*x*(d*x^2+c)^(1/2)/e/f/(b*x^2+a)^(1/2)+1/2*x*(b*x^2+a)^(1/2)*(d*x^2+
c)^(1/2)/e/(f*x^2+e)+1/2*a^(1/2)*b^(1/2)*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)
*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/e/f/(b*x^2+a)^(1/2)/(a*(d*
x^2+c)/c/(b*x^2+a))^(1/2)-1/2*a^(3/2)*b^(1/2)*(-c*f+d*e)*(d*x^2+c)^(1/2)*I
nverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/c/e/f/(-a*f+b*
e)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)+1/2*a^(3/2)*(-a*c*f^2+b
*d*e^2)*(d*x^2+c)^(1/2)*EllipticPi(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),1-a
*f/b/e,(1-a*d/b/c)^(1/2))/b^(1/2)/c/e^2/f/(-a*f+b*e)/(b*x^2+a)^(1/2)/(a*(d
*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.83 (sec) , antiderivative size = 401, normalized size of antiderivative = 0.97

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^2}\,dx$$

$$= \frac{\frac{acx}{e+fx^2}+\frac{bcx^3}{e+fx^2}+\frac{adx^3}{e+fx^2}+\frac{bdx^5}{e+fx^2}+\frac{ia\sqrt{\frac{b}{a}}c\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(i\text{arcsinh}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)}{f}-\frac{ia\sqrt{\frac{b}{a}}(de+cf)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\text{Elli}}{f^2}}{}$$

input
```
Integrate[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(e + f*x^2)^2,x]
```

output
```
((a*c*x)/(e + f*x^2) + (b*c*x^3)/(e + f*x^2) + (a*d*x^3)/(e + f*x^2) + (b*
d*x^5)/(e + f*x^2) + (I*a*Sqrt[b/a]*c*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)
/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/f - (I*a*Sqrt[b/a]*(d*
e + c*f)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[
b/a]*x], (a*d)/(b*c)])/f^2 - (I*a*c*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c
]*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(Sqrt[b/a]
*e) + (I*a*Sqrt[b/a]*d*e*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticP
i[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/f^2)/(2*e*Sqrt[a + b*
x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 378, normalized size of antiderivative = 0.92, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {423, 406, 320, 388, 313, 413, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^2}dx$$

$$\Big\downarrow \text{ 423}$$

$$\frac{bd\int \frac{e-fx^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{2ef^2}+\frac{1}{2}\left(\frac{ac}{e}-\frac{bde}{f^2}\right)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\,(fx^2+e)}dx+\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}}{2e\,(e+fx^2)}$$

$$\Big\downarrow \text{ 406}$$

$$\frac{bd\left(e\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx-f\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{2ef^2}+\frac{1}{2}\left(\frac{ac}{e}-\frac{bde}{f^2}\right)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\,(fx^2+e)}dx+\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}}{2e\,(e+fx^2)}$$

$$\Big\downarrow \text{ 320}$$

$$\frac{bd\left(\frac{\sqrt{c}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-f\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{2ef^2}+\frac{1}{2}\left(\frac{ac}{e}-\frac{bde}{f^2}\right)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\,(fx^2+e)}dx+\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}}{2e\,(e+fx^2)}$$

$$\Big\downarrow \text{ 388}$$

$$\frac{bd\left(\frac{\sqrt{c}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-f\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}}dx}{b}\right)\right)}{2ef^2}+\frac{1}{2}\left(\frac{ac}{e}-\frac{bde}{f^2}\right)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\,(fx^2+e)}dx+\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}}{2e\,(e+fx^2)}$$

$$\Big\downarrow \color{blue}{313}$$

$$\frac{1}{2}\left(\frac{ac}{e}-\frac{bde}{f^2}\right)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\,(fx^2+e)}dx+\frac{bd\left(\frac{\sqrt{c}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-f\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)\right)}{2ef^2}+\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}}{2e\,(e+fx^2)}$$

$$\Big\downarrow \color{blue}{413}$$

$$\frac{\sqrt{\frac{bx^2}{a}+1}\left(\frac{ac}{e}-\frac{bde}{f^2}\right)\int \frac{1}{\sqrt{\frac{bx^2}{a}+1}\sqrt{dx^2+c}(fx^2+e)}dx}{2\sqrt{a+bx^2}}+\frac{bd\left(\frac{\sqrt{c}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-f\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)\right)}{2ef^2}+\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}}{2e\,(e+fx^2)}$$

$$\Big\downarrow \color{blue}{413}$$

$$\frac{\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\left(\frac{ac}{e}-\frac{bde}{f^2}\right)\int \frac{1}{\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}(fx^2+e)}dx}{2\sqrt{a+bx^2}\sqrt{c+dx^2}}+\frac{bd\left(\frac{\sqrt{c}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-f\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)\right)}{2ef^2}+\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}}{2e\,(e+fx^2)}$$

$$\Big\downarrow \color{blue}{412}$$

$$\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\left(\frac{ac}{e}-\frac{bde}{f^2}\right)\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{2\sqrt{b}e\sqrt{a+bx^2}\sqrt{c+dx^2}}+\frac{bd\left(\frac{\sqrt{c}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-f\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)\right)}{2ef^2}+\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}}{2e(e+fx^2)}$$

input

```
Int[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(e + f*x^2)^2,x]
```

output

```
(x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(2*e*(e + f*x^2)) + (b*d*(-(f*((x*Sqrt
[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcT
an[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))
/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))) + (Sqrt[c]*e*Sqrt[a + b*x^2]*Elliptic
F[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*Sqrt[(c*(a + b
*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])))/(2*e*f^2) + (Sqrt[-a]*((a*c)/e
- (b*d*e)/f^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b
*e), ArcSin[(Sqrt[b]*x)/Sqrt[-a]], (a*d)/(b*c)])/(2*Sqrt[b]*e*Sqrt[a + b*x
^2]*Sqrt[c + d*x^2])
```

Defintions of rubi rules used

rule 313

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 423
```
Int[(Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)^2])/((a_) + (b_.)*(x_
)^2)^2, x_Symbol] :> Simp[x*Sqrt[c + d*x^2]*(Sqrt[e + f*x^2]/(2*a*(a + b*x^
2))), x] + (Simp[(b^2*c*e - a^2*d*f)/(2*a*b^2)   Int[1/((a + b*x^2)*Sqrt[c
+ d*x^2]*Sqrt[e + f*x^2]), x], x] + Simp[d*(f/(2*a*b^2))   Int[(a - b*x^2)/
(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
```

Maple [A] (verified)

Time = 1.46 (sec) , antiderivative size = 559, normalized size of antiderivative = 1.35

method	result
elliptic	$\frac{\sqrt{(b x^2+a)(x^2 d+c)}\left(\frac{x\sqrt{bd x^4+ad x^2+x^2 bc+ac}}{2e\left(f x^2+e\right)}+\frac{bd\sqrt{1+\frac{b x^2}{a}}\sqrt{1+\frac{d x^2}{c}}\,\text{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{2f^2\sqrt{-\frac{b}{a}}\sqrt{bd x^4+ad x^2+x^2 bc+ac}}+\frac{bc\sqrt{1+\frac{b x^2}{a}}\sqrt{1+\frac{d x^2}{c}}\,\text{EllipticF}}{2ef\sqrt{-\frac{b}{a}}\sqrt{bd x^4+ad}}\right.}{}$
default	$\frac{\sqrt{b x^2+a}\sqrt{x^2 d+c}\left(\sqrt{-\frac{b}{a}}\,bde f^2 x^5+\sqrt{\frac{b x^2+a}{a}}\sqrt{\frac{x^2 d+c}{c}}\,\text{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)bce f^2 x^2+\sqrt{\frac{b x^2+a}{a}}\sqrt{\frac{x^2 d+c}{c}}\,\text{EllipticF}\left(x\sqrt{-\frac{b}{a}}\right.\right.}{}$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^2,x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/2*x/e*(b*d*
x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(f*x^2+e)+1/2*b*d/f^2/(-b/a)^(1/2)*(1+b*x^2
/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(
x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+1/2*b/e/f*c/(-b/a)^(1/2)*(1+b*x^2
/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(
x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-1/2*b/e/f*c/(-b/a)^(1/2)*(1+b*x^2
/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticE(
x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+1/2/e^2/(-b/a)^(1/2)*(1+b*x^2/a)^
(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(
-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*a*c-1/2/f^2/(-b/a)^(1/2)*
(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*El
lipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*b*d)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^2}\,dx=\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^2}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(1/2)/(f*x**2+e)**2,x)
```

output
```
Integral(sqrt(a + b*x**2)*sqrt(c + d*x**2)/(e + f*x**2)**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^2}\,dx=\int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{(fx^2+e)^2}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="maxima
")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/(f*x^2 + e)^2, x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^2}\,dx=\int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{(fx^2+e)^2}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/(f*x^2 + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^2}\,dx = \int \frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{{(f\,x^2+e)}^2}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2))/(e + f*x^2)^2,x)
```

output
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2))/(e + f*x^2)^2, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^2}\,dx = \int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{f^2x^4+2ef\,x^2+e^2}dx$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^2,x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(e**2 + 2*e*f*x**2 + f**2*x**4),x)
```

3.80 $\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^2(e+fx^2)^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 651

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^2(e+fx^2)^2}\,dx = \frac{3bx\sqrt{c+dx^2}}{2e^2\sqrt{a+bx^2}}+\frac{d(2de-3cf)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{2ce^2(de-cf)}$$
$$-\frac{b(af(2de-3cf)-2be(de-cf))x\sqrt{a+bx^2}(c+dx^2)^{3/2}}{2ace^2(be-af)(de-cf)}-\frac{(a+bx^2)^{3/2}(c+dx^2)^{3/2}}{acex(e+fx^2)}$$
$$+\frac{f(af(2de-3cf)-2be(de-cf))x(a+bx^2)^{3/2}(c+dx^2)^{3/2}}{2ace^2(be-af)(de-cf)(e+fx^2)}$$
$$-\frac{3\sqrt{a}\sqrt{b}\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{2e^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$
$$+\frac{\sqrt{a}\sqrt{b}(2bce+ade-3acf)\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{2ce^2(be-af)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$
$$-\frac{a^{3/2}(af(2de-3cf)-be(de-2cf))\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{2\sqrt{b}ce^3(be-af)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
3/2*b*x*(d*x^2+c)^(1/2)/e^2/(b*x^2+a)^(1/2)+1/2*d*(-3*c*f+2*d*e)*x*(b*x^2+
a)^(1/2)*(d*x^2+c)^(1/2)/c/e^2/(-c*f+d*e)-1/2*b*(a*f*(-3*c*f+2*d*e)-2*b*e*
(-c*f+d*e))*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/a/c/e^2/(-a*f+b*e)/(-c*f+d*e
)-(b*x^2+a)^(3/2)*(d*x^2+c)^(3/2)/a/c/e/x/(f*x^2+e)+1/2*f*(a*f*(-3*c*f+2*d
*e)-2*b*e*(-c*f+d*e))*x*(b*x^2+a)^(3/2)*(d*x^2+c)^(3/2)/a/c/e^2/(-a*f+b*e)
/(-c*f+d*e)/(f*x^2+e)-3/2*a^(1/2)*b^(1/2)*(d*x^2+c)^(1/2)*EllipticE(b^(1/2
)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/e^2/(b*x^2+a)^(1/2)/(a*(d
*x^2+c)/c/(b*x^2+a))^(1/2)+1/2*a^(1/2)*b^(1/2)*(-3*a*c*f+a*d*e+2*b*c*e)*(d
*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))
/c/e^2/(-a*f+b*e)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)-1/2*a^(3
/2)*(a*f*(-3*c*f+2*d*e)-b*e*(-2*c*f+d*e))*(d*x^2+c)^(1/2)*EllipticPi(b^(1/
2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),1-a*f/b/e,(1-a*d/b/c)^(1/2))/b^(1/2)/c/e^3/
(-a*f+b*e)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.96 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.37

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^2\left(e+fx^2\right)^2}\,dx = \frac{-\frac{e(a+bx^2)(c+dx^2)(2e+3fx^2)}{x(e+fx^2)} + \frac{i\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\left(-3bcefE\left(i\operatorname{arcsinh}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)-be(de-3cf)\operatorname{EllipticF}\left(i\operatorname{arcsinh}\left(\sqrt{\frac{b}{a}}x\right),\frac{ad}{bc}\right.\right.}{\sqrt{\frac{b}{a}}f}}{2e^3\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

input
```
Integrate[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(x^2*(e + f*x^2)^2),x]
```

output
```
(-((e*(a + b*x^2)*(c + d*x^2)*(2*e + 3*f*x^2))/(x*(e + f*x^2))) + (I*Sqrt[
1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(-3*b*c*e*f*EllipticE[I*ArcSinh[Sqrt[b/
a]*x], (a*d)/(b*c)] - b*e*(d*e - 3*c*f)*EllipticF[I*ArcSinh[Sqrt[b/a]*x],
(a*d)/(b*c)] + (b*e*(d*e - 2*c*f) + a*f*(-2*d*e + 3*c*f))*EllipticPi[(a*f)
/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(Sqrt[b/a]*f))/(2*e^3*Sqrt[
a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^2\left(e+fx^2\right)^2}\,dx$$

$$\Big\downarrow \; 450$$

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^2\left(e+fx^2\right)^2}dx$$

input
```
Int[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(x^2*(e + f*x^2)^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [A] (verified)

Time = 19.71 (sec) , antiderivative size = 791, normalized size of antiderivative = 1.22

method	result
elliptic	$\sqrt{\left(b\,x^2+a\right)\left(x^2 d+c\right)}\left(-\frac{f x\sqrt{bd\,x^4+ad\,x^2+x^2 bc+ac}}{2e^2\left(f\,x^2+e\right)}-\frac{\sqrt{bd\,x^4+ad\,x^2+x^2 bc+ac}}{e^2 x}+\frac{bd\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\;\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{2ef\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+x^2 bc+ac}}-\frac{3}{\ldots}\right.$
default	Expression too large to display
risch	Expression too large to display

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^2/(f*x^2+e)^2,x,method=_RETURNVERBOS
E)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-1/2*f/e^2*x*
(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(f*x^2+e)-1/e^2*(b*d*x^4+a*d*x^2+b*c*x
^2+a*c)^(1/2)/x+1/2*b*d/e/f/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/
2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b
*c)/c/b)^(1/2))-3/2*b/e^2*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/
2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b
*c)/c/b)^(1/2))+3/2*b/e^2*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/
2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b
*c)/c/b)^(1/2))-3/2/e^3*f/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)
/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1
/c*d)^(1/2)/(-b/a)^(1/2))*a*c+1/e^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^
2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a
*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*a*d+1/e^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/
2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/
a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*b*c-1/2/e/f/(-b/a)^(1/2)*(1+
b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*Ellip
ticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*b*d)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^2\left(e+fx^2\right)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^2/(f*x^2+e)^2,x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^2\left(e+fx^2\right)^2}\,dx = \int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^2\left(e+fx^2\right)^2}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(1/2)/x**2/(f*x**2+e)**2,x)
```

output
```
Integral(sqrt(a + b*x**2)*sqrt(c + d*x**2)/(x**2*(e + f*x**2)**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^2\left(e+fx^2\right)^2}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{\left(fx^2+e\right)^2x^2}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^2/(f*x^2+e)^2,x, algorithm="ma
xima")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/((f*x^2 + e)^2*x^2), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^2\left(e+fx^2\right)^2}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{\left(fx^2+e\right)^2x^2}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^2/(f*x^2+e)^2,x, algorithm="gi
ac")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/((f*x^2 + e)^2*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^2\left(e+fx^2\right)^2}\,dx=\int \frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{x^2\,\left(f\,x^2+e\right)^2}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2))/(x^2*(e + f*x^2)^2),x)
```

output
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2))/(x^2*(e + f*x^2)^2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^2\left(e+fx^2\right)^2}\,dx=\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{f^2x^6+2ef\,x^4+e^2x^2}dx$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^2/(f*x^2+e)^2,x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(e**2*x**2 + 2*e*f*x**4 + f**2*x**
6),x)
```

3.81 $\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^4(e+fx^2)^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 771

$$
\begin{aligned}
&\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^4\left(e+fx^2\right)^2}\,dx \\
&= \frac{b(2bce+2ade-15acf)x\sqrt{c+dx^2}}{6ace^3\sqrt{a+bx^2}}+\frac{d\left(2b-\frac{3af(4de-5cf)}{e(de-cf)}\right)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{6ace^2} \\
&+\frac{bf(af(4de-5cf)-4be(de-cf))x\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}}{2ace^3(be-af)(de-cf)} \\
&-\frac{\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{3acex^3\left(e+fx^2\right)}+\frac{5f\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{3ace^2x\left(e+fx^2\right)} \\
&-\frac{f^2(af(4de-5cf)-4be(de-cf))x\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{2ace^3(be-af)(de-cf)\left(e+fx^2\right)} \\
&-\frac{\sqrt{b}(2bce+2ade-15acf)\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{6\sqrt{a}ce^3\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} \\
&-\frac{\sqrt{a}\sqrt{b}(af(7de-15cf)-4be(de-3cf))\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{6ce^3(be-af)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} \\
&+\frac{a^{3/2}f(af(4de-5cf)-be(3de-4cf))\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{2\sqrt{b}ce^4(be-af)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}
\end{aligned}
$$

output
```
1/6*b*(-15*a*c*f+2*a*d*e+2*b*c*e)*x*(d*x^2+c)^(1/2)/a/c/e^3/(b*x^2+a)^(1/2
)+1/6*d*(2*b-3*a*f*(-5*c*f+4*d*e)/e/(-c*f+d*e))*x*(b*x^2+a)^(1/2)*(d*x^2+c
)^(1/2)/a/c/e^2+1/2*b*f*(a*f*(-5*c*f+4*d*e)-4*b*e*(-c*f+d*e))*x*(b*x^2+a)^
(1/2)*(d*x^2+c)^(3/2)/a/c/e^3/(-a*f+b*e)/(-c*f+d*e)-1/3*(b*x^2+a)^(3/2)*(d
*x^2+c)^(3/2)/a/c/e/x^3/(f*x^2+e)+5/3*f*(b*x^2+a)^(3/2)*(d*x^2+c)^(3/2)/a/
c/e^2/x/(f*x^2+e)-1/2*f^2*(a*f*(-5*c*f+4*d*e)-4*b*e*(-c*f+d*e))*x*(b*x^2+a
)^(3/2)*(d*x^2+c)^(3/2)/a/c/e^3/(-a*f+b*e)/(-c*f+d*e)/(f*x^2+e)-1/6*b^(1/2
)*(-15*a*c*f+2*a*d*e+2*b*c*e)*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/
(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/a^(1/2)/c/e^3/(b*x^2+a)^(1/2)/(a*(d*x
^2+c)/c/(b*x^2+a))^(1/2)-1/6*a^(1/2)*b^(1/2)*(a*f*(-15*c*f+7*d*e)-4*b*e*(-
3*c*f+d*e))*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a
*d/b/c)^(1/2))/c/e^3/(-a*f+b*e)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^
(1/2)+1/2*a^(3/2)*f*(a*f*(-5*c*f+4*d*e)-b*e*(-4*c*f+3*d*e))*(d*x^2+c)^(1/2
)*EllipticPi(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),1-a*f/b/e,(1-a*d/b/c)^(1/
2))/b^(1/2)/c/e^4/(-a*f+b*e)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/
2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.57 (sec) , antiderivative size = 416, normalized size of antiderivative = 0.54

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^4\,(e+fx^2)^2}\,dx$$

$$= \frac{\sqrt{\frac{b}{a}}\left(-\sqrt{\frac{b}{a}}e(a+bx^2)\,(c+dx^2)\,(2bcex^2(e+fx^2)+2adex^2(e+fx^2)+ac(2e^2-10efx^2-15f^2x^4))+i\right.}{}$$

input
```
Integrate[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(x^4*(e + f*x^2)^2),x]
```

output
```
(Sqrt[b/a]*(-(Sqrt[b/a]*e*(a + b*x^2)*(c + d*x^2)*(2*b*c*e*x^2*(e + f*x^2)
 + 2*a*d*e*x^2*(e + f*x^2) + a*c*(2*e^2 - 10*e*f*x^2 - 15*f^2*x^4))) + I*b
*c*e*(-2*b*c*e - 2*a*d*e + 15*a*c*f)*x^3*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x
^2)/c]*(e + f*x^2)*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*b*c*
e*(-2*b*c*e - 7*a*d*e + 15*a*c*f)*x^3*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)
/c]*(e + f*x^2)*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - (3*I)*a*c
*(b*e*(3*d*e - 4*c*f) + a*f*(-4*d*e + 5*c*f))*x^3*Sqrt[1 + (b*x^2)/a]*Sqrt
[1 + (d*x^2)/c]*(e + f*x^2)*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x]
, (a*d)/(b*c)]))/(6*b*c*e^4*x^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2
))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^4\left(e+fx^2\right)^2}\,dx$$

$$\Big\downarrow \; 450$$

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^4\left(e+fx^2\right)^2}dx$$

input
```
Int[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(x^4*(e + f*x^2)^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [A] (verified)

Time = 20.70 (sec) , antiderivative size = 1135, normalized size of antiderivative = 1.47

method	result	size
elliptic	Expression too large to display	1135
risch	Expression too large to display	1419
default	Expression too large to display	1862

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^4/(f*x^2+e)^2,x,method=_RETURNVERBOS
E)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-1/3/e^2*(b*d
*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/x^3+1/3/a/c*(6*a*c*f-a*d*e-b*c*e)/e^3*(b*d
*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/x+1/2*f^2/e^3*x*(b*d*x^4+a*d*x^2+b*c*x^2+a
*c)^(1/2)/(f*x^2+e)-7/6*b*d/e^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)
^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a
*d+b*c)/c/b)^(1/2))+5/2*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)
/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*f*b/e^3*EllipticF(x*(-b/a)^(1/2),(-1+
(a*d+b*c)/c/b)^(1/2))-5/2*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/
2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*f*b/e^3*EllipticE(x*(-b/a)^(1/2),(-
1+(a*d+b*c)/c/b)^(1/2))+1/3/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/
2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*d*b/e^2*EllipticE(x*(-b/a)^(1/2),(-
1+(a*d+b*c)/c/b)^(1/2))-1/3*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(
1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*b^2/a/e^2*EllipticF(x*(-b/a)^(1/2
),(-1+(a*d+b*c)/c/b)^(1/2))+1/3*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/
c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*b^2/a/e^2*EllipticE(x*(-b/a)^
(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+5/2/e^4*f^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)
*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)
^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*a*c-2/e^3*f/(-b/a)^(1/2)*(1+b*
x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*Ellipti
cPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*a*d-2/e^3*f/(-b...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^4\left(e+fx^2\right)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^4/(f*x^2+e)^2,x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^4\left(e+fx^2\right)^2}\,dx = \int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^4\left(e+fx^2\right)^2}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(1/2)/x**4/(f*x**2+e)**2,x)
```

output
```
Integral(sqrt(a + b*x**2)*sqrt(c + d*x**2)/(x**4*(e + f*x**2)**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^4\left(e+fx^2\right)^2}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{\left(fx^2+e\right)^2x^4}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^4/(f*x^2+e)^2,x, algorithm="ma
xima")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/((f*x^2 + e)^2*x^4), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^4\,(e+fx^2)^2}\,dx=\int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{(fx^2+e)^2x^4}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^4/(f*x^2+e)^2,x, algorithm="gi
ac")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/((f*x^2 + e)^2*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^4\,(e+fx^2)^2}\,dx=\int \frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{x^4\,(f\,x^2+e)^2}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2))/(x^4*(e + f*x^2)^2),x)
```

output
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2))/(x^4*(e + f*x^2)^2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^4\,(e+fx^2)^2}\,dx=\int \frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{x^4\,(f\,x^2+e)^2}dx$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^4/(f*x^2+e)^2,x)
```

output
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^4/(f*x^2+e)^2,x)
```

3.82 $\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^6\left(e+fx^2\right)^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 1095

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^6\left(e+fx^2\right)^2}\,dx = \text{Too large to display}$$

output
```
-1/30*b*(4*b^2*c^2*e^2-4*a*b*c*e*(-5*c*f+d*e)+a^2*(-105*c^2*f^2+20*c*d*e*f
+4*d^2*e^2))*x*(d*x^2+c)^(1/2)/a^2/c^2/e^4/(b*x^2+a)^(1/2)+1/30*d*(15*a^2*
c*f^2*(-7*c*f+6*d*e)-4*b^2*c*e^2*(-c*f+d*e)+2*a*b*e*(10*c^2*f^2-11*c*d*e*f
+d^2*e^2))*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a^2/c^2/e^4/(-c*f+d*e)-1/10*b
*(5*a^2*c*f^3*(-7*c*f+6*d*e)-2*b^2*d*e^3*(-c*f+d*e)+2*a*b*e*f*(15*c^2*f^2-
16*c*d*e*f+d^2*e^2))*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/a^2/c^2/e^4/(-a*f+b
*e)/(-c*f+d*e)-1/5*(b*x^2+a)^(3/2)*(d*x^2+c)^(3/2)/a/c/e/x^5/(f*x^2+e)+1/1
5*(7*a*c*f+2*a*d*e+2*b*c*e)*(b*x^2+a)^(3/2)*(d*x^2+c)^(3/2)/a^2/c^2/e^2/x^
3/(f*x^2+e)+1/15*(a*f*(-35*c*f+2*d*e)-b*e*(-2*c*f+3*d*e))*(b*x^2+a)^(3/2)*
(d*x^2+c)^(3/2)/a^2/c^2/e^3/x/(f*x^2+e)+1/10*f*(5*a^2*c*f^3*(-7*c*f+6*d*e)
-2*b^2*d*e^3*(-c*f+d*e)+2*a*b*e*f*(15*c^2*f^2-16*c*d*e*f+d^2*e^2))*x*(b*x^
2+a)^(3/2)*(d*x^2+c)^(3/2)/a^2/c^2/e^4/(-a*f+b*e)/(-c*f+d*e)/(f*x^2+e)+1/3
0*b^(1/2)*(4*b^2*c^2*e^2-4*a*b*c*e*(-5*c*f+d*e)+a^2*(-105*c^2*f^2+20*c*d*e
*f+4*d^2*e^2))*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/
2),(1-a*d/b/c)^(1/2))/a^(3/2)/c^2/e^4/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^
2+a))^(1/2)-1/30*b^(1/2)*(2*b^2*c*d*e^3-a^2*f*(-105*c^2*f^2+55*c*d*e*f+2*d
^2*e^2)+2*a*b*e*(-45*c^2*f^2+19*c*d*e*f+d^2*e^2))*(d*x^2+c)^(1/2)*InverseJ
acobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/a^(1/2)/c^2/e^4/(-a*f
+b*e)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)-1/2*a^(3/2)*f^2*(a*f
*(-7*c*f+6*d*e)-b*e*(-6*c*f+5*d*e))*(d*x^2+c)^(1/2)*EllipticPi(b^(1/2)*...
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.53 (sec) , antiderivative size = 470, normalized size of antiderivative = 0.43

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^6\left(e+fx^2\right)^2}\,dx$$

$$= \frac{-\frac{e(a+bx^2)(c+dx^2)\left(15a^2c^2f^3x^6+6a^2c^2e^2(e+fx^2)+2ace(bce+ade-10acf)x^2(e+fx^2)-2\left(2b^2c^2e^2+2abce(-de+5cf)+a^2\left(2d^2e^2+10cdef-4\right.\right.\right.}{x^5(e+fx^2)}}{}$$

input
```
Integrate[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(x^6*(e + f*x^2)^2),x]
```

output
```
(-((e*(a + b*x^2)*(c + d*x^2)*(15*a^2*c^2*f^3*x^6 + 6*a^2*c^2*e^2*(e + f*x
^2) + 2*a*c*e*(b*c*e + a*d*e - 10*a*c*f)*x^2*(e + f*x^2) - 2*(2*b^2*c^2*e^
2 + 2*a*b*c*e*(-(d*e) + 5*c*f) + a^2*(2*d^2*e^2 + 10*c*d*e*f - 45*c^2*f^2)
)*x^4*(e + f*x^2)))/(x^5*(e + f*x^2))) + (I*c*Sqrt[1 + (b*x^2)/a]*Sqrt[1 +
 (d*x^2)/c]*(b*e*(4*b^2*c^2*e^2 + 4*a*b*c*e*(-(d*e) + 5*c*f) + a^2*(4*d^2*
e^2 + 20*c*d*e*f - 105*c^2*f^2))*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(
b*c)] - b*e*(4*b^2*c^2*e^2 + 2*a*b*c*e*(-3*d*e + 10*c*f) + a^2*(2*d^2*e^2
+ 55*c*d*e*f - 105*c^2*f^2))*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)
] + 15*a^2*c*f*(b*e*(5*d*e - 6*c*f) + a*f*(-6*d*e + 7*c*f))*EllipticPi[(a*
f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/Sqrt[b/a])/(30*a^2*c^2*e^
5*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^6\left(e+fx^2\right)^2}\,dx$$

$$\Big\downarrow \; 450$$

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^6\left(e+fx^2\right)^2}dx$$

input
```
Int[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(x^6*(e + f*x^2)^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [A] (verified)

Time = 22.80 (sec) , antiderivative size = 1785, normalized size of antiderivative = 1.63

method	result	size
risch	Expression too large to display	1785
elliptic	Expression too large to display	1829
default	Expression too large to display	3178

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^6/(f*x^2+e)^2,x,method=_RETURNVERBOS
E)
```

output
```
-1/15*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(45*a^2*c^2*f^2*x^4-10*a^2*c*d*e*f*x
^4-2*a^2*d^2*e^2*x^4-10*a*b*c^2*e*f*x^4+2*a*b*c*d*e^2*x^4-2*b^2*c^2*e^2*x^
4-10*a^2*c^2*e*f*x^2+a^2*c*d*e^2*x^2+a*b*c^2*e^2*x^2+3*a^2*c^2*e^2)/a^2/c^
2/e^4/x^5+1/15/a^2/c^2/e^4*(-b*(45*a^2*c^2*f^2-10*a^2*c*d*e*f-2*a^2*d^2*e^
2-10*a*b*c^2*e*f+2*a*b*c*d*e^2-2*b^2*c^2*e^2)*c/(-b/a)^(1/2)*(1+b*x^2/a)^(
1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(EllipticF(x*(-
b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c
)/c/b)^(1/2)))-a*b^2*c^2*d*e^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^
(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*
d+b*c)/c/b)^(1/2))-a^2*b*c*d^2*e^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2
/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1
+(a*d+b*c)/c/b)^(1/2))-15*a^2*c^2*f*(3*a*c*f^2-2*a*d*e*f-2*b*c*e*f+b*d*e^2
)/e/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*
x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/
2))+10*a^2*b*c^2*d*e*f/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b
*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c
/b)^(1/2))-15*a^2*c^2*e*f*(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)*(1/2*f^2/(a*c*
f^2-a*d*e*f-b*c*e*f+b*d*e^2)/e*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(f*x^
2+e)-1/2*d*b/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/(-b/a)^(1/2)*(1+b*x^2/a)^(1
/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^6\left(e+fx^2\right)^2}\,dx=\text{Timed out}$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^6/(f*x^2+e)^2,x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^6\left(e+fx^2\right)^2}\,dx=\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^6\left(e+fx^2\right)^2}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(1/2)/x**6/(f*x**2+e)**2,x)
```

output
```
Integral(sqrt(a + b*x**2)*sqrt(c + d*x**2)/(x**6*(e + f*x**2)**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^6\left(e+fx^2\right)^2}\,dx=\int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{\left(fx^2+e\right)^2x^6}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^6/(f*x^2+e)^2,x, algorithm="ma
xima")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/((f*x^2 + e)^2*x^6), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^6\left(e+fx^2\right)^2}\,dx=\int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{\left(fx^2+e\right)^2x^6}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^6/(f*x^2+e)^2,x, algorithm="gi
ac")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/((f*x^2 + e)^2*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^6\left(e+fx^2\right)^2}\,dx=\int \frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{x^6\,{\left(f\,x^2+e\right)}^2}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2))/(x^6*(e + f*x^2)^2),x)
```

output
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2))/(x^6*(e + f*x^2)^2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^6\left(e+fx^2\right)^2}\,dx=\int \frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{x^6\,{\left(f\,x^2+e\right)}^2}dx$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^6/(f*x^2+e)^2,x)
```

output
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^6/(f*x^2+e)^2,x)
```

3.83 $\int x^6\sqrt{a+bx^2}(c+dx^2)^{3/2}\left(e+fx^2\right)^2\,dx$

Optimal result

Integrand size = 35, antiderivative size = 1902

$$\int x^6\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^2\,dx=\text{Too large to display}$$

output
```
1/45045*(1024*a^7*d^7*f^2-128*a^6*b*d^6*f*(13*c*f+20*d*e)-a^4*b^3*c*d^4*(-
129*c^2*f^2+828*c*d*e*f+3016*d^2*e^2)+16*a*b^6*c^4*d*(8*c^2*f^2-27*c*d*e*f
+26*d^2*e^2)+32*a^5*b^2*d^5*(9*c^2*f^2+136*c*d*e*f+52*d^2*e^2)-16*b^7*c^5*
(16*c^2*f^2-48*c*d*e*f+39*d^2*e^2)+a^2*b^5*c^3*d^2*(90*c^2*f^2-334*c*d*e*f
+377*d^2*e^2)+a^3*b^4*c^2*d^3*(93*c^2*f^2-406*c*d*e*f+663*d^2*e^2))*x*(d*x
^2+c)^(1/2)/b^6/d^6/(b*x^2+a)^(1/2)-1/45045*(512*a^6*d^6*f^3-256*a^5*b*d^5
*f^2*(3*c*f+5*d*e)-a^2*b^4*c^2*d^2*f*(-45*c^2*f^2+170*c*d*e*f+2577*d^2*e^2
)-3*a^3*b^3*c*d^3*f*(-17*c^2*f^2+74*c*d*e*f+468*d^2*e^2)+8*a^4*b^2*d^4*f*(
9*c^2*f^2+252*c*d*e*f+104*d^2*e^2)-8*b^6*c^4*f*(16*c^2*f^2-48*c*d*e*f+39*d
^2*e^2)+a*b^5*c^2*d*(48*c^3*f^3-168*c^2*d*e*f^2-2141*c*d^2*e^2*f+924*d^3*e
^3))*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b^6/d^5/f+1/45045*(384*a^5*d^5*f^3-
80*a^4*b*d^4*f^2*(7*c*f+12*d*e)+a^2*b^3*c*d^2*f*(33*c^2*f^2+5418*c*d*e*f+4
517*d^2*e^2)+a^3*b^2*d^3*f*(39*c^2*f^2+1472*c*d*e*f+624*d^2*e^2)-2*a*b^4*c
*d*(-16*c^3*f^3-2253*c^2*d*e*f^2-1329*c*d^2*e^2*f+924*d^3*e^3)-12*b^5*c^2*
(8*c^3*f^3-24*c^2*d*e*f^2-173*c*d^2*e^2*f+77*d^3*e^3))*x^3*(b*x^2+a)^(1/2)
*(d*x^2+c)^(1/2)/b^5/d^4/f-1/45045*(320*a^4*d^4*f^3-20*a^3*b*d^3*f^2*(23*c
*f+40*d*e)-a^2*b^2*d^2*f*(2745*c^2*f^2+9878*c*d*e*f+2252*d^2*e^2)+a*b^3*d*
(-2285*c^3*f^3-7944*c^2*d*e*f^2+1003*c*d^2*e^2*f+924*d^3*e^3)+4*b^4*c*(-20
*c^3*f^3-1095*c^2*d*e*f^2+9*c*d^2*e^2*f+462*d^3*e^3))*x^5*(b*x^2+a)^(1/2)*
(d*x^2+c)^(1/2)/b^4/d^3/f+1/6435*(40*a^3*d^3*f^3+a^2*b*d^2*f^2*(735*c*f...
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 8.94 (sec) , antiderivative size = 1265, normalized size of antiderivative = 0.67

$$\int x^6\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^2\,dx = \text{Too large to display}$$

input
```
Integrate[x^6*Sqrt[a + b*x^2]*(c + d*x^2)^(3/2)*(e + f*x^2)^2,x]
```

output
```
(Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2)*(-512*a^6*d^6*f^2 + 128*a^5*b*d^5*f
*(10*d*e + 6*c*f + 3*d*f*x^2) - 8*a^4*b^2*d^4*(9*c^2*f^2 + 14*c*d*f*(18*e
+ 5*f*x^2) + 8*d^2*(13*e^2 + 15*e*f*x^2 + 5*f^2*x^4)) + a^3*b^3*d^3*(-51*c
^3*f^2 + 3*c^2*d*f*(74*e + 13*f*x^2) + 8*d^3*x^2*(78*e^2 + 100*e*f*x^2 + 3
5*f^2*x^4) + 4*c*d^2*(351*e^2 + 368*e*f*x^2 + 115*f^2*x^4)) + b^6*(128*c^6
*f^2 - 96*c^5*d*f*(4*e + f*x^2) + 8*c^4*d^2*(39*e^2 + 36*e*f*x^2 + 10*f^2*
x^4) + 3*c^2*d^4*x^4*(65*e^2 + 70*e*f*x^2 + 21*f^2*x^4) - 2*c^3*d^3*x^2*(1
17*e^2 + 120*e*f*x^2 + 35*f^2*x^4) + 84*c*d^5*x^6*(65*e^2 + 105*e*f*x^2 +
44*f^2*x^4) + 21*d^6*x^8*(195*e^2 + 330*e*f*x^2 + 143*f^2*x^4)) + a*b^5*d*
(-48*c^5*f^2 + 8*c^4*d*f*(21*e + 4*f*x^2) + 3*c^2*d^3*x^2*(39*e^2 + 30*e*f
*x^2 + 7*f^2*x^4) - c^3*d^2*(169*e^2 + 114*e*f*x^2 + 25*f^2*x^4) + 7*d^5*x
^6*(65*e^2 + 90*e*f*x^2 + 33*f^2*x^4) + c*d^4*x^4*(845*e^2 + 1050*e*f*x^2
+ 357*f^2*x^4)) - a^2*b^4*d^2*(45*c^4*f^2 - c^3*d*f*(170*e + 33*f*x^2) + 3
*c^2*d^2*(65*e^2 + 42*e*f*x^2 + 9*f^2*x^4) + 4*d^4*x^4*(130*e^2 + 175*e*f*
x^2 + 63*f^2*x^4) + c*d^3*x^2*(1027*e^2 + 1210*e*f*x^2 + 399*f^2*x^4))) -
I*c*(1024*a^7*d^7*f^2 - 128*a^6*b*d^6*f*(20*d*e + 13*c*f) + 16*a*b^6*c^4*d
*(26*d^2*e^2 - 27*c*d*e*f + 8*c^2*f^2) + 32*a^5*b^2*d^5*(52*d^2*e^2 + 136*
c*d*e*f + 9*c^2*f^2) - 16*b^7*c^5*(39*d^2*e^2 - 48*c*d*e*f + 16*c^2*f^2) +
 a^2*b^5*c^3*d^2*(377*d^2*e^2 - 334*c*d*e*f + 90*c^2*f^2) + a^3*b^4*c^2*d^
3*(663*d^2*e^2 - 406*c*d*e*f + 93*c^2*f^2) + a^4*b^3*c*d^4*(-3016*d^2*e...
```

Rubi [A] (warning: unable to verify)

Time = 4.21 (sec) , antiderivative size = 2236, normalized size of antiderivative = 1.18, number of steps used = 19, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.543$, Rules used = {448, 443, 27, 443, 443, 444, 444, 27, 444, 27, 406, 320, 388, 313, 444, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^6\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^2\,dx$$

$$\Big\downarrow \text{448}$$

$$\frac{f\int x^8\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)dx}{e^2}+e\int x^6\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)dx$$

$$\Big\downarrow \text{443}$$

$$\frac{f\left(\frac{\int 3x^8\sqrt{bx^2+a}\sqrt{dx^2+c}\left((5bde+bcf-4adf)x^2+c(5be-3af)\right)dx}{15b}+\frac{fx^9\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{15b}\right)}{e^2}+$$
$$e\left(\frac{\int x^6\sqrt{bx^2+a}\sqrt{dx^2+c}\left((13bde+3bcf-10adf)x^2+c(13be-7af)\right)dx}{13b}+\frac{fx^7\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{13b}\right)$$

$\downarrow$ 27

$$\frac{f\left(\frac{\int x^8\sqrt{bx^2+a}\sqrt{dx^2+c}\left((5bde+bcf-4adf)x^2+c(5be-3af)\right)dx}{5b}+\frac{fx^9\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{15b}\right)}{e^2}+$$
$$e\left(\frac{\int x^6\sqrt{bx^2+a}\sqrt{dx^2+c}\left((13bde+3bcf-10adf)x^2+c(13be-7af)\right)dx}{13b}+\frac{fx^7\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{13b}\right)$$

$\downarrow$ 443

$$\frac{f\left(\frac{\frac{\int \frac{x^8\sqrt{bx^2+a}\left(\left(c(70de+cf)b^2-ad(50de+53cf)b+40a^2d^2f\right)x^2+c\left(36dfa^2-45bdea-48bcfa+65b^2ce\right)\right)}{\sqrt{dx^2+c}}dx}{13b}+\frac{x^9\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}(-4adf+bcf+5bde)}{13b}}{5b}+\right.}{e^2}$$
$$e\left(\frac{\frac{\int \frac{x^6\sqrt{bx^2+a}\left(\left(3c(52de+cf)b^2-ad(104de+111cf)b+80a^2d^2f\right)x^2+c\left(70dfa^2-91bdea-98bcfa+143b^2ce\right)\right)}{\sqrt{dx^2+c}}dx}{11b}+\frac{x^7\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}(-10adf+3bcf}{11b}}{13b}\right.$$

$\downarrow$ 443

$$\frac{f\left(\frac{\frac{\frac{\int \frac{x^8\left(\left(5c^2(3de-2cf)b^3+3acd(25de+cf)b^2-a^2d^2(50de+57cf)b+40a^3d^3f\right)x^2+ac\left(c(85de-9cf)b^2-3ad(15de+17cf)b+36a^2d^2f\right)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{11d}+x^9\sqrt{a+bx^2}\sqrt{c+dx^2}}{13b}}{5b}\right.}{e^2}$$
$$e\left(\frac{\frac{\frac{\int \frac{x^6\left(\left(3c^2(13de-8cf)b^3+acd(169de+9cf)b^2-a^2d^2(104de+121cf)b+80a^3d^3f\right)x^2+ac\left(3c(65de-7cf)b^2-7ad(13de+15cf)b+70a^2d^2f\right)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{9d}+x^7\sqrt{a+bx^2}\sqrt{c}}{11b}}{13b}\right.$$

$\downarrow$ 444

$$\frac{f\left(\frac{f(bx^2+a)^{3/2}(dx^2+c)^{3/2}x^9}{15b}+\frac{(5bde+bcf-4adf)\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}\,x^9}{13b}+\frac{\left(c(70de+cf)b^2-ad(50de+53cf)b+40a^2d^2f\right)\sqrt{bx^2+a}\sqrt{dx^2+c}\,x^9}{11d}+\frac{\left(5c^2(3\cdots\right.}{\cdots}\cdots\right.}{e\left(\frac{f\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}x^7}{13b}+\frac{(13bde+3bcf-10adf)(bx^2+a)^{3/2}\sqrt{dx^2+c}\,x^7}{11b}+\frac{\left(3c(52de+cf)b^2-ad(104de+111cf)b+80a^2d^2f\right)\sqrt{bx^2\cdots}}{9d}\cdots\right.}$$

$\Big\downarrow$ 444

$$\frac{f\left(\frac{f(bx^2+a)^{3/2}(dx^2+c)^{3/2}x^9}{15b}+\frac{(5bde+bcf-4adf)\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}\,x^9}{13b}+\frac{\left(c(70de+cf)b^2-ad(50de+53cf)b+40a^2d^2f\right)\sqrt{bx^2+a}\sqrt{dx^2+c}\,x^9}{11d}+\frac{\left(5c^2(3\cdots\right.}{\cdots}\cdots\right.}{e\left(\frac{f\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}x^7}{13b}+\frac{(13bde+3bcf-10adf)(bx^2+a)^{3/2}\sqrt{dx^2+c}\,x^7}{11b}+\frac{\left(3c(52de+cf)b^2-ad(104de+111cf)b+80a^2d^2f\right)\sqrt{bx^2\cdots}}{9d}\cdots\right.}$$

$\Big\downarrow$ 27

$$\frac{f\left(\frac{f(bx^2+a)^{3/2}(dx^2+c)^{3/2}x^9}{15b}+\frac{(5bde+bcf-4adf)\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}\,x^9}{13b}+\frac{\left(c(70de+cf)b^2-ad(50de+53cf)b+40a^2d^2f\right)\sqrt{bx^2+a}\sqrt{dx^2+c}\,x^9}{11d}+\frac{\left(5c^2(3\cdots\right.}{\cdots}\cdots\right.}{e\left(\frac{f\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}x^7}{13b}+\frac{(13bde+3bcf-10adf)(bx^2+a)^{3/2}\sqrt{dx^2+c}\,x^7}{11b}+\frac{\left(3c(52de+cf)b^2-ad(104de+111cf)b+80a^2d^2f\right)\sqrt{bx^2\cdots}}{9d}\cdots\right.}$$

$$\Big\downarrow \; 444$$

$$\frac{f\left(\frac{f(bx^2+a)^{3/2}(dx^2+c)^{3/2}x^9}{15b}+\frac{\frac{(5bde+bcf-4adf)\left(bx^2+a\right)^{3/2}\sqrt{dx^2+cx}^9}{13b}+\frac{\frac{\left(c(70de+cf)b^2-ad(50de+53cf)b+40a^2d^2f\right)\sqrt{bx^2+a}\sqrt{dx^2+cx}^9}{11d}+\frac{\left(5c^2(3e\right.}{}}{}}{}\right.}{e\left(\frac{f(bx^2+a)^{3/2}(dx^2+c)^{3/2}x^7}{13b}+\frac{\frac{(13bde+3bcf-10adf)(bx^2+a)^{3/2}\sqrt{dx^2+cx}^7}{11b}+\frac{\frac{\left(3c(52de+cf)b^2-ad(104de+111cf)b+80a^2d^2f\right)\sqrt{bx^2}}{9d}}{}}{}\right.}$$

$$\Big\downarrow \; 27$$

$$\frac{f\left(\frac{f(bx^2+a)^{3/2}(dx^2+c)^{3/2}x^9}{15b}+\frac{\frac{(5bde+bcf-4adf)\left(bx^2+a\right)^{3/2}\sqrt{dx^2+cx}^9}{13b}+\frac{\frac{\left(c(70de+cf)b^2-ad(50de+53cf)b+40a^2d^2f\right)\sqrt{bx^2+a}\sqrt{dx^2+cx}^9}{11d}+\frac{\left(5c^2(3e\right.}{}}{}}{}\right.}{e\left(\frac{f(bx^2+a)^{3/2}(dx^2+c)^{3/2}x^7}{13b}+\frac{\frac{(13bde+3bcf-10adf)(bx^2+a)^{3/2}\sqrt{dx^2+cx}^7}{11b}+\frac{\frac{\left(3c(52de+cf)b^2-ad(104de+111cf)b+80a^2d^2f\right)\sqrt{bx^2}}{9d}}{}}{}\right.}$$

$$\Big\downarrow \; 406$$

$$e\left(\frac{f\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}x^7}{13b}+\frac{(13bde+3bcf-10adf)\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}x^7}{11b}+\frac{\frac{\left(3c(52de+cf)b^2-ad(104de+111cf)b+80a^2d^2f\right)\sqrt{bx^2}}{9d}}{}\right.$$

$$f\left(\frac{f\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}x^9}{15b}+\frac{\frac{(5bde+bcf-4adf)\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}x^9}{13b}+\frac{\frac{\left(c(70de+cf)b^2-ad(50de+53cf)b+40a^2d^2f\right)\sqrt{bx^2+a}\sqrt{dx^2+c}x^9}{11d}+\frac{\left(5c^2(3c\right.}{}}{}}{}\right.$$

$\downarrow$ 320

$$e\left(\frac{f\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}x^7}{13b}+\frac{\frac{(13bde+3bcf-10adf)\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}x^7}{11b}+\frac{\left(3c(52de+cf)b^2-ad(104de+111cf)b+80a^2d^2f\right)\sqrt{bx^2}}{9d}}{}\right.$$

$$f\left(\frac{f\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}x^9}{15b}+\frac{\frac{(5bde+bcf-4adf)\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}x^9}{13b}+\frac{\left(c(70de+cf)b^2-ad(50de+53cf)b+40a^2d^2f\right)\sqrt{bx^2+a}\sqrt{dx^2+c}x^9}{11d}+\frac{\left(5c^2(3\right.}{}}{}\right.$$

$\downarrow$ 388

$$e\left(\frac{f\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}x^7}{13b}+\frac{(13bde+3bcf-10adf)\left(bx^2+a\right)^{3/2}\sqrt{dx^2+cx^7}}{11b}+\frac{\left(3c(52de+cf)b^2-ad(104de+111cf)b+80a^2d^2f\right)\sqrt{bx^2}}{9d}\right.$$

$$f\left(\frac{f\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}x^9}{15b}+\frac{(5bde+bcf-4adf)\left(bx^2+a\right)^{3/2}\sqrt{dx^2+cx^9}}{13b}+\frac{\left(c(70de+cf)b^2-ad(50de+53cf)b+40a^2d^2f\right)\sqrt{bx^2+a}\sqrt{dx^2+cx^9}}{11d}+\frac{\left(5c^2(3\right.}{}\right.$$

$\downarrow$ 313

$$e\left(\frac{f\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}x^7}{13b}+\frac{\frac{(13bde+3bcf-10adf)\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}x^7}{11b}+\frac{\frac{\left(3c(52de+cf)b^2-ad(104de+111cf)b+80a^2d^2f\right)\sqrt{bx^2}}{9d}}{}}{}\right.$$

$$f\left(\frac{f\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}x^9}{15b}+\frac{\frac{(5bde+bcf-4adf)\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}x^9}{13b}+\frac{\frac{\left(c(70de+cf)b^2-ad(50de+53cf)b+40a^2d^2f\right)\sqrt{bx^2+a}\sqrt{dx^2+c}x^9}{11d}+\frac{\left(5c^2(3\right.}{}}{}}{}\right.$$

$$\downarrow 444$$

$$e\left(\frac{f\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}x^7}{13b}+\frac{\frac{(13bde+3bcf-10adf)\left(bx^2+a\right)^{3/2}\sqrt{dx^2+cx^7}}{11b}+\frac{\frac{\left(3c(52de+cf)b^2-ad(104de+111cf)b+80a^2d^2f\right)\sqrt{bx^2}\cdots}{9d}\cdots}{\cdots}}{\cdots}\right.$$

$$f\left(\frac{f\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}x^9}{15b}+\frac{\frac{(5bde+bcf-4adf)\left(bx^2+a\right)^{3/2}\sqrt{dx^2+cx^9}}{13b}+\frac{\frac{\left(c(70de+cf)b^2-ad(50de+53cf)b+40a^2d^2f\right)\sqrt{bx^2+a}\sqrt{dx^2+cx^9}}{11d}+\frac{\left(5c^2(3\cdots\right.}{\cdots}}{\cdots}}{\cdots}\right.$$

↓ 406

$$e\left(\frac{f\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}x^7}{13b}+\frac{\frac{(13bde+3bcf-10adf)\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}x^7}{11b}+\frac{\left(3c(52de+cf)b^2-ad(104de+111cf)b+80a^2d^2f\right)\sqrt{bx^2}}{9d}\cdots}{\cdots}\right.$$

$$f\left(\frac{f\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}x^9}{15b}+\frac{\frac{(5bde+bcf-4adf)\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}x^9}{13b}+\frac{\frac{\left(c(70de+cf)b^2-ad(50de+53cf)b+40a^2d^2f\right)\sqrt{bx^2+a}\sqrt{dx^2+c}x^9}{11d}+\frac{\left(5c^2(3\cdots\right.}{\cdots}}{\cdots}}{\cdots}\right.$$

$\downarrow$ 320

$$e\left(\frac{f\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}x^7}{13b}+\frac{\frac{(13bde+3bcf-10adf)\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}x^7}{11b}+\frac{\left(3c(52de+cf)b^2-ad(104de+111cf)b+80a^2d^2f\right)\sqrt{bx^2}}{9d}}{}\right.$$

$$f\left(\frac{f\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}x^9}{15b}+\frac{\frac{(5bde+bcf-4adf)\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}x^9}{13b}+\frac{\frac{\left(c(70de+cf)b^2-ad(50de+53cf)b+40a^2d^2f\right)\sqrt{bx^2+a}\sqrt{dx^2+c}x^9}{11d}+\frac{\left(5c^2(3\right.}{}}{}}{}\right.$$

↓ 388

$$e\left(\frac{f\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}x^7}{13b}+\frac{\frac{(13bde+3bcf-10adf)\left(bx^2+a\right)^{3/2}\sqrt{dx^2+cx^7}}{11b}+\frac{\frac{\left(3c(52de+cf)b^2-ad(104de+111cf)b+80a^2d^2f\right)\sqrt{bx^2}}{9d}}{}}{}\right.$$

$$f\left(\frac{f\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}x^9}{15b}+\frac{\frac{(5bde+bcf-4adf)\left(bx^2+a\right)^{3/2}\sqrt{dx^2+cx^9}}{13b}+\frac{\frac{\left(c(70de+cf)b^2-ad(50de+53cf)b+40a^2d^2f\right)\sqrt{bx^2+a}\sqrt{dx^2+cx^9}}{11d}+\frac{\left(5c^2(3\right.}{}}{}}{}\right.$$

↓ 313

$$e\left(\frac{f\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}x^7}{13b}+\frac{\frac{(13bde+3bcf-10adf)\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}x^7}{11b}+\frac{\left(3c(52de+cf)b^2-ad(104de+111cf)b+80a^2d^2f\right)\sqrt{bx^2}}{9d}}{}\right.$$

$$f\left(\frac{f\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}x^9}{15b}+\frac{\frac{(5bde+bcf-4adf)\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}x^9}{13b}+\frac{\frac{\left(c(70de+cf)b^2-ad(50de+53cf)b+40a^2d^2f\right)\sqrt{bx^2+a}\sqrt{dx^2+c}x^9}{11d}+\frac{\left(5c^2(3\right.}{}}{}}{}\right.$$

input
```
Int[x^6*Sqrt[a + b*x^2]*(c + d*x^2)^(3/2)*(e + f*x^2)^2,x]
```

output
```
e*((f*x^7*(a + b*x^2)^(3/2)*(c + d*x^2)^(3/2))/(13*b) + (((13*b*d*e + 3*b*
c*f - 10*a*d*f)*x^7*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(11*b) + (((80*a^2*
d^2*f + 3*b^2*c*(52*d*e + c*f) - a*b*d*(104*d*e + 111*c*f))*x^7*Sqrt[a + b
*x^2]*Sqrt[c + d*x^2])/(9*d) + (((80*a^3*d^3*f + 3*b^3*c^2*(13*d*e - 8*c*f
) + a*b^2*c*d*(169*d*e + 9*c*f) - a^2*b*d^2*(104*d*e + 121*c*f))*x^5*Sqrt[
a + b*x^2]*Sqrt[c + d*x^2])/(7*b*d) - (((480*a^4*d^4*f - 3*a*b^3*c^2*d*(39
*d*e - 19*c*f) + 18*b^4*c^3*(13*d*e - 8*c*f) - 16*a^3*b*d^3*(39*d*e + 46*c
*f) + a^2*b^2*c*d^2*(1027*d*e + 63*c*f))*x^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^
2])/(5*b*d) - (3*(((640*a^5*d^5*f - a*b^4*c^3*d*(169*d*e - 84*c*f) - 5*a^2
*b^3*c^2*d^2*(39*d*e - 17*c*f) + 24*b^5*c^4*(13*d*e - 8*c*f) + 3*a^3*b^2*c
*d^3*(468*d*e + 37*c*f) - 16*a^4*b*d^4*(52*d*e + 63*c*f))*x*Sqrt[a + b*x^2
]*Sqrt[c + d*x^2])/(3*b*d) - ((1280*a^6*d^6*f - a^3*b^3*c^2*d^3*(663*d*e -
 203*c*f) - a^2*b^4*c^3*d^2*(377*d*e - 167*c*f) - 8*a*b^5*c^4*d*(52*d*e -
27*c*f) + 48*b^6*c^5*(13*d*e - 8*c*f) - 128*a^5*b*d^5*(13*d*e + 17*c*f) +
2*a^4*b^2*c*d^4*(1508*d*e + 207*c*f))*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x
^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 -
 (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d
*x^2])) + (c^(3/2)*(640*a^5*d^5*f - a*b^4*c^3*d*(169*d*e - 84*c*f) - 5*a^2
*b^3*c^2*d^2*(39*d*e - 17*c*f) + 24*b^5*c^4*(13*d*e - 8*c*f) + 3*a^3*b^2*c
*d^3*(468*d*e + 37*c*f) - 16*a^4*b*d^4*(52*d*e + 63*c*f))*Sqrt[a + b*x^...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 443
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*(g*x)^(m + 1)*(a + b*x^2)^(p
 + 1)*((c + d*x^2)^q/(b*g*(m + 2*(p + q + 1) + 1))), x] + Simp[1/(b*(m + 2*
(p + q + 1) + 1))   Int[(g*x)^m*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[c*((
b*e - a*f)*(m + 1) + b*e*2*(p + q + 1)) + (d*(b*e - a*f)*(m + 1) + f*2*q*(b
*c - a*d) + b*e*d*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f
, g, m, p}, x] && GtQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^2, c + d*x^
2])
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

rule 448
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[e   Int[(g*x)^m*(a + b*x
^2)^p*(c + d*x^2)^q*(e + f*x^2)^(r - 1), x], x] + Simp[f/e^2   Int[(g*x)^(m
 + 2)*(a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^(r - 1), x], x] /; FreeQ[{a,
b, c, d, e, f, g, m, p, q}, x] && IGtQ[r, 0]
```

Maple [A] (verified)

Time = 21.66 (sec) , antiderivative size = 3327, normalized size of antiderivative = 1.75

method	result	size
risch	Expression too large to display	3327
default	Expression too large to display	5290
elliptic	Expression too large to display	5327

input
```
int(x^6*(b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)*(f*x^2+e)^2,x,method=_RETURNVERBOS
E)
```

output
```
-1/45045*x/d^5*(-3003*b^6*d^6*f^2*x^12-231*a*b^5*d^6*f^2*x^10-3696*b^6*c*d
^5*f^2*x^10-6930*b^6*d^6*e*f*x^10+252*a^2*b^4*d^6*f^2*x^8-357*a*b^5*c*d^5*
f^2*x^8-630*a*b^5*d^6*e*f*x^8-63*b^6*c^2*d^4*f^2*x^8-8820*b^6*c*d^5*e*f*x^
8-4095*b^6*d^6*e^2*x^8-280*a^3*b^3*d^6*f^2*x^6+399*a^2*b^4*c*d^5*f^2*x^6+7
00*a^2*b^4*d^6*e*f*x^6-21*a*b^5*c^2*d^4*f^2*x^6-1050*a*b^5*c*d^5*e*f*x^6-4
55*a*b^5*d^6*e^2*x^6+70*b^6*c^3*d^3*f^2*x^6-210*b^6*c^2*d^4*e*f*x^6-5460*b
^6*c*d^5*e^2*x^6+320*a^4*b^2*d^6*f^2*x^4-460*a^3*b^3*c*d^5*f^2*x^4-800*a^3
*b^3*d^6*e*f*x^4+27*a^2*b^4*c^2*d^4*f^2*x^4+1210*a^2*b^4*c*d^5*e*f*x^4+520
*a^2*b^4*d^6*e^2*x^4+25*a*b^5*c^3*d^3*f^2*x^4-90*a*b^5*c^2*d^4*e*f*x^4-845
*a*b^5*c*d^5*e^2*x^4-80*b^6*c^4*d^2*f^2*x^4+240*b^6*c^3*d^3*e*f*x^4-195*b^
6*c^2*d^4*e^2*x^4-384*a^5*b*d^6*f^2*x^2+560*a^4*b^2*c*d^5*f^2*x^2+960*a^4*
b^2*d^6*e*f*x^2-39*a^3*b^3*c^2*d^4*f^2*x^2-1472*a^3*b^3*c*d^5*e*f*x^2-624*
a^3*b^3*d^6*e^2*x^2-33*a^2*b^4*c^3*d^3*f^2*x^2+126*a^2*b^4*c^2*d^4*e*f*x^2
+1027*a^2*b^4*c*d^5*e^2*x^2-32*a*b^5*c^4*d^2*f^2*x^2+114*a*b^5*c^3*d^3*e*f
*x^2-117*a*b^5*c^2*d^4*e^2*x^2+96*b^6*c^5*d*f^2*x^2-288*b^6*c^4*d^2*e*f*x^
2+234*b^6*c^3*d^3*e^2*x^2+512*a^6*d^6*f^2-768*a^5*b*c*d^5*f^2-1280*a^5*b*d
^6*e*f+72*a^4*b^2*c^2*d^4*f^2+2016*a^4*b^2*c*d^5*e*f+832*a^4*b^2*d^6*e^2+5
1*a^3*b^3*c^3*d^3*f^2-222*a^3*b^3*c^2*d^4*e*f-1404*a^3*b^3*c*d^5*e^2+45*a^
2*b^4*c^4*d^2*f^2-170*a^2*b^4*c^3*d^3*e*f+195*a^2*b^4*c^2*d^4*e^2+48*a*b^5
*c^5*d*f^2-168*a*b^5*c^4*d^2*e*f+169*a*b^5*c^3*d^3*e^2-128*b^6*c^6*f^2+...
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 1964, normalized size of antiderivative = 1.03

$$\int x^6\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^2\,dx=\text{Too large to display}$$

input
```
integrate(x^6*(b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)*(f*x^2+e)^2,x, algorithm="fr
icas")
```

output
```
1/45045*((13*(48*b^7*c^6*d^2 - 32*a*b^6*c^5*d^3 - 29*a^2*b^5*c^4*d^4 - 51*
a^3*b^4*c^3*d^5 + 232*a^4*b^3*c^2*d^6 - 128*a^5*b^2*c*d^7)*e^2 - 2*(384*b^
7*c^7*d - 216*a*b^6*c^6*d^2 - 167*a^2*b^5*c^5*d^3 - 203*a^3*b^4*c^4*d^4 -
414*a^4*b^3*c^3*d^5 + 2176*a^5*b^2*c^2*d^6 - 1280*a^6*b*c*d^7)*e*f + (256*
b^7*c^8 - 128*a*b^6*c^7*d - 90*a^2*b^5*c^6*d^2 - 93*a^3*b^4*c^5*d^3 - 129*
a^4*b^3*c^4*d^4 - 288*a^5*b^2*c^3*d^5 + 1664*a^6*b*c^2*d^6 - 1024*a^7*c*d^
7)*f^2)*sqrt(b*d)*x*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c))
 - (13*(48*b^7*c^6*d^2 - 32*a*b^6*c^5*d^3 - 64*a^5*b^2*d^8 - (29*a^2*b^5 -
 24*a*b^6)*c^4*d^4 - (51*a^3*b^4 + 13*a^2*b^5)*c^3*d^5 + (232*a^4*b^3 - 15
*a^3*b^4)*c^2*d^6 - 4*(32*a^5*b^2 - 27*a^4*b^3)*c*d^7)*e^2 - 2*(384*b^7*c^
7*d - 216*a*b^6*c^6*d^2 - 640*a^6*b*d^8 - (167*a^2*b^5 - 192*a*b^6)*c^5*d^
3 - 7*(29*a^3*b^4 + 12*a^2*b^5)*c^4*d^4 - (414*a^4*b^3 + 85*a^3*b^4)*c^3*d
^5 + (2176*a^5*b^2 - 111*a^4*b^3)*c^2*d^6 - 16*(80*a^6*b - 63*a^5*b^2)*c*d
^7)*e*f + (256*b^7*c^8 - 128*a*b^6*c^7*d - 512*a^7*d^8 - 2*(45*a^2*b^5 - 6
4*a*b^6)*c^6*d^2 - 3*(31*a^3*b^4 + 16*a^2*b^5)*c^5*d^3 - 3*(43*a^4*b^3 + 1
5*a^3*b^4)*c^4*d^4 - 3*(96*a^5*b^2 + 17*a^4*b^3)*c^3*d^5 + 8*(208*a^6*b -
9*a^5*b^2)*c^2*d^6 - 256*(4*a^7 - 3*a^6*b)*c*d^7)*f^2)*sqrt(b*d)*x*sqrt(-c
/d)*elliptic_f(arcsin(sqrt(-c/d)/x), a*d/(b*c)) + (3003*b^7*d^8*f^2*x^14 +
 231*(30*b^7*d^8*e*f + (16*b^7*c*d^7 + a*b^6*d^8)*f^2)*x^12 + 21*(195*b^7*
d^8*e^2 + 30*(14*b^7*c*d^7 + a*b^6*d^8)*e*f + (3*b^7*c^2*d^6 + 17*a*b^6...
```

Sympy [F]

$$\int x^6\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^2\,dx=\int x^6\sqrt{a+bx^2}\left(c+dx^2\right)^{\frac{3}{2}}\left(e+fx^2\right)^2\,dx$$

input
```
integrate(x**6*(b*x**2+a)**(1/2)*(d*x**2+c)**(3/2)*(f*x**2+e)**2,x)
```

output
```
Integral(x**6*sqrt(a + b*x**2)*(c + d*x**2)**(3/2)*(e + f*x**2)**2, x)
```

Maxima [F]

$$\int x^6\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^2\,dx=\int \sqrt{bx^2+a}\left(dx^2+c\right)^{\frac{3}{2}}\left(fx^2+e\right)^2x^6\,dx$$

input
```
integrate(x^6*(b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)*(f*x^2+e)^2,x, algorithm="ma
xima")
```

output
```
integrate(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)*(f*x^2 + e)^2*x^6, x)
```

Giac [F]

$$\int x^6\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^2\,dx=\int \sqrt{bx^2+a}\left(dx^2+c\right)^{\frac{3}{2}}\left(fx^2+e\right)^2x^6\,dx$$

input
```
integrate(x^6*(b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)*(f*x^2+e)^2,x, algorithm="gi
ac")
```

output
```
integrate(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)*(f*x^2 + e)^2*x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^6\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^2\,dx=\int x^6\,\sqrt{b\,x^2+a}\,{\left(d\,x^2+c\right)}^{3/2}\,{\left(f\,x^2+e\right)}^2\,dx$$

input
```
int(x^6*(a + b*x^2)^(1/2)*(c + d*x^2)^(3/2)*(e + f*x^2)^2,x)
```

output
```
int(x^6*(a + b*x^2)^(1/2)*(c + d*x^2)^(3/2)*(e + f*x^2)^2, x)
```

Reduce [F]

$$\int x^6\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^2\,dx=\text{too large to display}$$

input
```
int(x^6*(b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)*(f*x^2+e)^2,x)
```

output
```
( - 512*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**6*d**6*f**2*x + 768*sqrt(c +
d*x**2)*sqrt(a + b*x**2)*a**5*b*c*d**5*f**2*x + 1280*sqrt(c + d*x**2)*sqrt
(a + b*x**2)*a**5*b*d**6*e*f*x + 384*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**
5*b*d**6*f**2*x**3 - 72*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**4*b**2*c**2*d
**4*f**2*x - 2016*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**4*b**2*c*d**5*e*f*x
 - 560*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**4*b**2*c*d**5*f**2*x**3 - 832*
sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**4*b**2*d**6*e**2*x - 960*sqrt(c + d*x
**2)*sqrt(a + b*x**2)*a**4*b**2*d**6*e*f*x**3 - 320*sqrt(c + d*x**2)*sqrt(
a + b*x**2)*a**4*b**2*d**6*f**2*x**5 - 51*sqrt(c + d*x**2)*sqrt(a + b*x**2
)*a**3*b**3*c**3*d**3*f**2*x + 222*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**3*
b**3*c**2*d**4*e*f*x + 39*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**3*b**3*c**2
*d**4*f**2*x**3 + 1404*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**3*b**3*c*d**5*
e**2*x + 1472*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**3*b**3*c*d**5*e*f*x**3
+ 460*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**3*b**3*c*d**5*f**2*x**5 + 624*s
qrt(c + d*x**2)*sqrt(a + b*x**2)*a**3*b**3*d**6*e**2*x**3 + 800*sqrt(c + d
*x**2)*sqrt(a + b*x**2)*a**3*b**3*d**6*e*f*x**5 + 280*sqrt(c + d*x**2)*sqr
t(a + b*x**2)*a**3*b**3*d**6*f**2*x**7 - 45*sqrt(c + d*x**2)*sqrt(a + b*x*
*2)*a**2*b**4*c**4*d**2*f**2*x + 170*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**
2*b**4*c**3*d**3*e*f*x + 33*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*b**4*c*
*3*d**3*f**2*x**3 - 195*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*b**4*c**...
```

3.84 $\int x^4\sqrt{a+bx^2}(c+dx^2)^{3/2}(e+fx^2)^2\,dx$

Optimal result

Integrand size = 35, antiderivative size = 1435

$$\int x^4\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^2\,dx=\text{Too large to display}$$

output
```
-1/45045*(1280*a^6*d^6*f^2-128*a^5*b*d^5*f*(17*c*f+26*d*e)-a^3*b^3*c*d^3*(
-203*c^2*f^2+1326*c*d*e*f+4576*d^2*e^2)-8*b^6*c^4*(48*c^2*f^2-156*c*d*e*f+
143*d^2*e^2)+a^2*b^4*c^2*d^2*(167*c^2*f^2-754*c*d*e*f+1287*d^2*e^2)+2*a^4*
b^2*d^4*(207*c^2*f^2+3016*c*d*e*f+1144*d^2*e^2)+a*b^5*c^3*d*(216*c^2*f^2-8
32*c*d*e*f+1001*d^2*e^2))*x*(d*x^2+c)^(1/2)/b^5/d^5/(b*x^2+a)^(1/2)+1/4504
5*(640*a^5*d^5*f^2-16*a^4*b*d^4*f*(63*c*f+104*d*e)-2*a*b^4*c^2*d*(-42*c^2*
f^2+169*c*d*e*f+1518*d^2*e^2)-5*a^2*b^3*c*d^2*(-17*c^2*f^2+78*c*d*e*f+429*
d^2*e^2)-4*b^5*c^3*(48*c^2*f^2-156*c*d*e*f+143*d^2*e^2)+a^3*b^2*d^3*(111*c
^2*f^2+2808*c*d*e*f+1144*d^2*e^2))*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b^5/d
^4-1/45045*(480*a^4*d^4*f^2-32*a^3*b*d^3*f*(23*c*f+39*d*e)+12*b^4*c^2*(-12
*c^2*f^2+39*c*d*e*f+253*d^2*e^2)+a*b^3*c*d*(57*c^2*f^2+6696*c*d*e*f+5357*d
^2*e^2)+a^2*b^2*d^2*(63*c^2*f^2+2054*c*d*e*f+858*d^2*e^2))*x^3*(b*x^2+a)^(
1/2)*(d*x^2+c)^(1/2)/b^4/d^3+1/9009*(80*a^3*d^3*f^2-a^2*b*d^2*f*(121*c*f+2
08*d*e)+4*b^3*c*(-6*c^2*f^2-327*c*d*e*f+11*d^2*e^2)-2*a*b^2*d*(342*c^2*f^2
+1217*c*d*e*f+275*d^2*e^2))*x^5*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b^3/d^2-1/
1287*(10*a^2*d^2*f^2+a*b*d*f*(183*c*f+172*d*e)-b^2*(-96*c^2*f^2-84*c*d*e*f
+44*d^2*e^2))*x^7*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b^2/d+2/143*f*(-5*a*d*f-
4*b*c*f+2*b*d*e)*x^9*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b+1/13*x*(b*x^2+a)^(3
/2)*(d*x^2+c)^(5/2)*(f*x^2+e)^2/b/d+1/45045*a^(1/2)*(1280*a^6*d^6*f^2-128*
a^5*b*d^5*f*(17*c*f+26*d*e)-a^3*b^3*c*d^3*(-203*c^2*f^2+1326*c*d*e*f+45...
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 7.82 (sec) , antiderivative size = 1012, normalized size of antiderivative = 0.71

$$\int x^4\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^2\,dx=\text{Too large to display}$$

input
```
Integrate[x^4*Sqrt[a + b*x^2]*(c + d*x^2)^(3/2)*(e + f*x^2)^2,x]
```

output
```
(Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2)*(640*a^5*d^5*f^2 - 16*a^4*b*d^4*f*(
104*d*e + 63*c*f + 30*d*f*x^2) + a^3*b^2*d^3*(111*c^2*f^2 + 8*c*d*f*(351*e
 + 92*f*x^2) + 8*d^2*(143*e^2 + 156*e*f*x^2 + 50*f^2*x^4)) + b^5*(-192*c^5
*f^2 + 48*c^4*d*f*(13*e + 3*f*x^2) - 4*c^3*d^2*(143*e^2 + 117*e*f*x^2 + 30
*f^2*x^4) + 3*c^2*d^3*x^2*(143*e^2 + 130*e*f*x^2 + 35*f^2*x^4) + 35*d^5*x^
6*(143*e^2 + 234*e*f*x^2 + 99*f^2*x^4) + 10*c*d^4*x^4*(715*e^2 + 1092*e*f*
x^2 + 441*f^2*x^4)) + a*b^4*d*(84*c^4*f^2 - c^3*d*f*(338*e + 57*f*x^2) + 3
*c^2*d^2*(143*e^2 + 78*e*f*x^2 + 15*f^2*x^4) + 5*d^4*x^4*(143*e^2 + 182*e*
f*x^2 + 63*f^2*x^4) + c*d^3*x^2*(1573*e^2 + 1690*e*f*x^2 + 525*f^2*x^4)) -
 a^2*b^3*d^2*(-85*c^3*f^2 + 3*c^2*d*f*(130*e + 21*f*x^2) + 2*d^3*x^2*(429*
e^2 + 520*e*f*x^2 + 175*f^2*x^4) + c*d^2*(2145*e^2 + 2054*e*f*x^2 + 605*f^
2*x^4))) + I*c*(1280*a^6*d^6*f^2 - 128*a^5*b*d^5*f*(26*d*e + 17*c*f) - 8*b
^6*c^4*(143*d^2*e^2 - 156*c*d*e*f + 48*c^2*f^2) + a^2*b^4*c^2*d^2*(1287*d^
2*e^2 - 754*c*d*e*f + 167*c^2*f^2) + a^3*b^3*c*d^3*(-4576*d^2*e^2 - 1326*c
*d*e*f + 203*c^2*f^2) + 2*a^4*b^2*d^4*(1144*d^2*e^2 + 3016*c*d*e*f + 207*c
^2*f^2) + a*b^5*c^3*d*(1001*d^2*e^2 - 832*c*d*e*f + 216*c^2*f^2))*Sqrt[1 +
 (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b
*c)] + I*c*(b*c - a*d)*(640*a^5*d^5*f^2 - 16*a^4*b*d^4*f*(104*d*e + 33*c*f
) + a^3*b^2*d^3*(1144*d^2*e^2 + 1560*c*d*e*f - 225*c^2*f^2) + a^2*b^3*c*d^
2*(-1287*d^2*e^2 + 624*c*d*e*f - 107*c^2*f^2) + a*b^4*c^2*d*(-429*d^2*e...
```

Rubi [A] (warning: unable to verify)

Time = 3.24 (sec) , antiderivative size = 1816, normalized size of antiderivative = 1.27, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.486$, Rules used = {448, 443, 443, 443, 444, 27, 444, 27, 406, 320, 388, 313, 444, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^2\,dx$$

$$\Big\downarrow \text{448}$$

$$\frac{f\int x^6\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)dx}{e^2}+e\int x^4\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)dx$$

$$\Big\downarrow \text{443}$$

$$\frac{f\left(\frac{\int x^6\sqrt{bx^2+a}\sqrt{dx^2+c}((13bde+3bcf-10adf)x^2+c(13be-7af))dx}{13b}+\frac{fx^7(a+bx^2)^{3/2}(c+dx^2)^{3/2}}{13b}\right)}{e^2}+$$
$$e\left(\frac{\int x^4\sqrt{bx^2+a}\sqrt{dx^2+c}((11bde+3bcf-8adf)x^2+c(11be-5af))\,dx}{11b}+\frac{fx^5(a+bx^2)^{3/2}(c+dx^2)^{3/2}}{11b}\right)$$

$\downarrow$ 443

$$\frac{f\left(\frac{\frac{\int\frac{x^6\sqrt{bx^2+a}\left(\left(3c(52de+cf)b^2-ad(104de+111cf)b+80a^2d^2f\right)x^2+c\left(70dfa^2-91bdea-98bcfa+143b^2ce\right)\right)}{\sqrt{dx^2+c}}dx}{11b}+\frac{x^7(a+bx^2)^{3/2}\sqrt{c+dx^2}(-10adf+3bcf+13b}{11b}}{13b}\right.}{e^2}$$
$$e\left(\frac{\frac{\int\frac{x^4\sqrt{bx^2+a}\left(\left(c(110de+3cf)b^2-ad(66de+71cf)b+48a^2d^2f\right)x^2+c\left(40dfa^2-55bdea-60bcfa+99b^2ce\right)\right)}{\sqrt{dx^2+c}}dx}{9b}+\frac{x^5(a+bx^2)^{3/2}\sqrt{c+dx^2}(-8adf+3bcf+1}{9b}}{11b}\right.$$

$\downarrow$ 443

$$\frac{f\left(\frac{\frac{\frac{\int\frac{x^6\left(\left(3c^2(13de-8cf)b^3+acd(169de+9cf)b^2-a^2d^2(104de+121cf)b+80a^3d^3f\right)x^2+ac\left(3c(65de-7cf)b^2-7ad(13de+15cf)b+70a^2d^2f\right)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{9d}+\frac{x^7\sqrt{a+bx^2}\sqrt{c}}{}}{11b}}{13b}\right.}{e^2}$$
$$e\left(\frac{\frac{\frac{\int\frac{x^4\left(\left(3c^2(11de-6cf)b^3+acd(121de+9cf)b^2-a^2d^2(66de+79cf)b+48a^3d^3f\right)x^2+ac\left(c(143de-15cf)b^2-5ad(11de+13cf)b+40a^2d^2f\right)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{7d}+\frac{x^5\sqrt{a+bx^2}\sqrt{c+}}{}}{9b}}{11b}\right.$$

$\downarrow$ 444

$$e\left(\frac{f\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}x^5}{11b}+\frac{\frac{(11bde+3bcf-8adf)\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}x^5}{9b}+\frac{\left(c(110de+3cf)b^2-ad(66de+71cf)b+48a^2d^2f\right)\sqrt{bx^2+a}}{7d}}{}\right.$$
$$f\left(\frac{f(bx^2+a)^{3/2}(dx^2+c)^{3/2}x^7}{13b}+\frac{\frac{(13bde+3bcf-10adf)\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}x^7}{11b}+\frac{\left(3c(52de+cf)b^2-ad(104de+111cf)b+80a^2d^2f\right)\sqrt{bx^2+a}\sqrt{dx^2+c}x^7}{9d}+}{}\right.$$

$\big\downarrow$ 27

$$e\left(\frac{f\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}x^5}{11b}+\frac{(11bde+3bcf-8adf)\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}\,x^5}{9b}+\frac{\frac{\left(c(110de+3cf)b^2-ad(66de+71cf)b+48a^2d^2f\right)\sqrt{bx^2+a}}{7d}}{\cdots}\right.$$

$$f\left(\frac{f\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}x^7}{13b}+\frac{\frac{(13bde+3bcf-10adf)\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}\,x^7}{11b}+\frac{\left(3c(52de+cf)b^2-ad(104de+111cf)b+80a^2d^2f\right)\sqrt{bx^2+a}\sqrt{dx^2+c}\,x^7}{9d}+\cdots}{\cdots}\right.$$

$\big\downarrow$ 444

$$e\left(\frac{f\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}x^5}{11b}+\frac{(11bde+3bcf-8adf)\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}\,x^5}{9b}+\frac{\frac{\left(c(110de+3cf)b^2-ad(66de+71cf)b+48a^2d^2f\right)\sqrt{bx^2+a}}{7d}}{\cdots}\right.$$

$$f\left(\frac{f\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}x^7}{13b}+\frac{\frac{(13bde+3bcf-10adf)\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}\,x^7}{11b}+\frac{\left(3c(52de+cf)b^2-ad(104de+111cf)b+80a^2d^2f\right)\sqrt{bx^2+a}\sqrt{dx^2+c}\,x^7}{9d}+\cdots}{\cdots}\right.$$

$\big\downarrow$ 27

$$e\left(\frac{f\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}x^5}{11b}+\frac{(11bde+3bcf-8adf)\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}x^5}{9b}+\frac{\frac{\left(c(110de+3cf)b^2-ad(66de+71cf)b+48a^2d^2f\right)\sqrt{bx^2+a}\cdots}{7d}}{\cdots}\right.$$

$$f\left(\frac{f\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}x^7}{13b}+\frac{(13bde+3bcf-10adf)\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}x^7}{11b}+\frac{\frac{\left(3c(52de+cf)b^2-ad(104de+111cf)b+80a^2d^2f\right)\sqrt{bx^2+a}\sqrt{dx^2+c}x^7}{9d}+\cdots}{\cdots}\right.$$

$\downarrow$ 406

$$e\left(\frac{f\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}x^5}{11b}+\frac{(11bde+3bcf-8adf)\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}x^5}{9b}+\frac{\frac{\left(c(110de+3cf)b^2-ad(66de+71cf)b+48a^2d^2f\right)\sqrt{bx^2+a}\cdots}{7d}}{\cdots}\right.$$

$$f\left(\frac{f\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}x^7}{13b}+\frac{(13bde+3bcf-10adf)\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}x^7}{11b}+\frac{\frac{\left(3c(52de+cf)b^2-ad(104de+111cf)b+80a^2d^2f\right)\sqrt{bx^2+a}\sqrt{dx^2+c}x^7}{9d}+\cdots}{\cdots}\right.$$

$\downarrow$ 320

$$e\left(\frac{f\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}x^5}{11b}+\frac{\frac{(11bde+3bcf-8adf)\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}x^5}{9b}+\frac{\left(c(110de+3cf)b^2-ad(66de+71cf)b+48a^2d^2f\right)\sqrt{bx^2+a}\ldots}{7d}\ldots}{\ldots}\right.$$

$$f\left(\frac{f\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}x^7}{13b}+\frac{\frac{(13bde+3bcf-10adf)\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}x^7}{11b}+\frac{\left(3c(52de+cf)b^2-ad(104de+111cf)b+80a^2d^2f\right)\sqrt{bx^2+a}\sqrt{dx^2+c}x^7}{9d}+\ldots}{\ldots}\right.$$

$\downarrow$ 388

$$e\left(\frac{f\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}x^5}{11b}+\frac{\frac{(11bde+3bcf-8adf)\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}x^5}{9b}+\frac{\frac{\left(c(110de+3cf)b^2-ad(66de+71cf)b+48a^2d^2f\right)\sqrt{bx^2+c}}{7d}}{}}{}\right.$$

$$f\left(\frac{f\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}x^7}{13b}+\frac{\frac{(13bde+3bcf-10adf)\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}x^7}{11b}+\frac{\left(3c(52de+cf)b^2-ad(104de+111cf)b+80a^2d^2f\right)\sqrt{bx^2+a}\sqrt{dx^2+c}x^7}{9d}+}{}\right.$$

$\downarrow$ 313

$$e\left(\frac{f\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}x^5}{11b}+\frac{\frac{(11bde+3bcf-8adf)\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}x^5}{9b}+\frac{\left(c(110de+3cf)b^2-ad(66de+71cf)b+48a^2d^2f\right)\sqrt{bx^2+a}}{7d}}{}\right.$$

$$f\left(\frac{f\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}x^7}{13b}+\frac{\frac{(13bde+3bcf-10adf)\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}x^7}{11b}+\frac{\left(3c(52de+cf)b^2-ad(104de+111cf)b+80a^2d^2f\right)\sqrt{bx^2+a}\sqrt{dx^2+c}x^7}{9d}+}{}\right.$$

$\downarrow$ 444

$$e\left(\frac{f\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}x^5}{11b}+\frac{\frac{(11bde+3bcf-8adf)\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}x^5}{9b}+\frac{\left(c(110de+3cf)b^2-ad(66de+71cf)b+48a^2d^2f\right)\sqrt{bx^2+a}\cdots}{7d}\cdots}{\cdots}\right.$$

$$f\left(\frac{f\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}x^7}{13b}+\frac{\frac{(13bde+3bcf-10adf)\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}x^7}{11b}+\frac{\left(3c(52de+cf)b^2-ad(104de+111cf)b+80a^2d^2f\right)\sqrt{bx^2+a}\sqrt{dx^2+c}x^7}{9d}+\cdots}{\cdots}\right.$$

$$\Big\downarrow\ 406$$

$$e\left(\frac{f(bx^2+a)^{3/2}(dx^2+c)^{3/2}x^5}{11b}+\frac{\frac{(11bde+3bcf-8adf)(bx^2+a)^{3/2}\sqrt{dx^2+c}\,x^5}{9b}+\frac{\left(c(110de+3cf)b^2-ad(66de+71cf)b+48a^2d^2f\right)\sqrt{bx^2+a}}{7d}}{}\right.$$

$$f\left(\frac{f(bx^2+a)^{3/2}(dx^2+c)^{3/2}x^7}{13b}+\frac{\frac{(13bde+3bcf-10adf)(bx^2+a)^{3/2}\sqrt{dx^2+c}\,x^7}{11b}+\frac{\left(3c(52de+cf)b^2-ad(104de+111cf)b+80a^2d^2f\right)\sqrt{bx^2+a}\sqrt{dx^2+c}\,x^7}{9d}+}{}\right.$$

$\downarrow$ 320

$$e\left(\frac{f\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}x^5}{11b}+\frac{\frac{(11bde+3bcf-8adf)\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}\,x^5}{9b}+\frac{\frac{\left(c(110de+3cf)b^2-ad(66de+71cf)b+48a^2d^2f\right)\sqrt{bx^2+a}\cdots}{7d}\cdots}{\cdots}}{\cdots}\right.$$

$$f\left(\frac{f\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}x^7}{13b}+\frac{\frac{(13bde+3bcf-10adf)\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}\,x^7}{11b}+\frac{\frac{\left(3c(52de+cf)b^2-ad(104de+111cf)b+80a^2d^2f\right)\sqrt{bx^2+a}\sqrt{dx^2+c}\,x^7}{9d}+\cdots}{\cdots}}{\cdots}\right.$$

$\downarrow$ 388

$$e\left(\frac{f\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}x^5}{11b}+\frac{\frac{(11bde+3bcf-8adf)\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}x^5}{9b}+\frac{\frac{\left(c(110de+3cf)b^2-ad(66de+71cf)b+48a^2d^2f\right)\sqrt{bx^2+a}\dots}{7d}}{}}{}\right.$$

$$f\left(\frac{f(bx^2+a)^{3/2}(dx^2+c)^{3/2}x^7}{13b}+\frac{\frac{(13bde+3bcf-10adf)\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}x^7}{11b}+\frac{\frac{\left(3c(52de+cf)b^2-ad(104de+111cf)b+80a^2d^2f\right)\sqrt{bx^2+a}\sqrt{dx^2+c}x^7}{9d}+\dots}{}}{}\right.$$

$\downarrow$ 313

$$e\left(\frac{f\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}x^5}{11b}+\frac{\frac{(11bde+3bcf-8adf)\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}x^5}{9b}+\frac{\frac{\left(c(110de+3cf)b^2-ad(66de+71cf)b+48a^2d^2f\right)\sqrt{bx^2+a}}{7d}}{}}{}\right.$$

$$f\left(\frac{f\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}x^7}{13b}+\frac{\frac{(13bde+3bcf-10adf)\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}x^7}{11b}+\frac{\frac{\left(3c(52de+cf)b^2-ad(104de+111cf)b+80a^2d^2f\right)\sqrt{bx^2+a}\sqrt{dx^2+c}x^7}{9d}+}{}}{}\right.$$

input
```
Int[x^4*Sqrt[a + b*x^2]*(c + d*x^2)^(3/2)*(e + f*x^2)^2,x]
```

output
```
e*((f*x^5*(a + b*x^2)^(3/2)*(c + d*x^2)^(3/2))/(11*b) + (((11*b*d*e + 3*b*
c*f - 8*a*d*f)*x^5*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(9*b) + (((48*a^2*d^
2*f + b^2*c*(110*d*e + 3*c*f) - a*b*d*(66*d*e + 71*c*f))*x^5*Sqrt[a + b*x^
2]*Sqrt[c + d*x^2])/(7*d) + (((48*a^3*d^3*f + 3*b^3*c^2*(11*d*e - 6*c*f) +
 a*b^2*c*d*(121*d*e + 9*c*f) - a^2*b*d^2*(66*d*e + 79*c*f))*x^3*Sqrt[a + b
*x^2]*Sqrt[c + d*x^2])/(5*b*d) - (3*(((64*a^4*d^4*f - a*b^3*c^2*d*(33*d*e
- 13*c*f) + 4*b^4*c^3*(11*d*e - 6*c*f) + 15*a^2*b^2*c*d^2*(11*d*e + c*f) -
 4*a^3*b*d^3*(22*d*e + 27*c*f))*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(3*b*d)
 - ((128*a^5*d^5*f - a*b^4*c^3*d*(77*d*e - 32*c*f) - a^2*b^3*c^2*d^2*(99*d
*e - 29*c*f) + 8*b^5*c^4*(11*d*e - 6*c*f) - 8*a^4*b*d^4*(22*d*e + 29*c*f)
+ a^3*b^2*c*d^3*(352*d*e + 51*c*f))*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2
]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (
b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x
^2])) + (c^(3/2)*(64*a^4*d^4*f - a*b^3*c^2*d*(33*d*e - 13*c*f) + 4*b^4*c^3
*(11*d*e - 6*c*f) + 15*a^2*b^2*c*d^2*(11*d*e + c*f) - 4*a^3*b*d^3*(22*d*e
+ 27*c*f))*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c
)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))
/(3*b*d)))/(5*b*d))/(7*d))/(9*b))/(11*b)) + (f*((f*x^7*(a + b*x^2)^(3/2)*(
c + d*x^2)^(3/2))/(13*b) + (((13*b*d*e + 3*b*c*f - 10*a*d*f)*x^7*(a + b*x^
2)^(3/2)*Sqrt[c + d*x^2])/(11*b) + (((80*a^2*d^2*f + 3*b^2*c*(52*d*e + ...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 443
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*(g*x)^(m + 1)*(a + b*x^2)^(p
 + 1)*((c + d*x^2)^q/(b*g*(m + 2*(p + q + 1) + 1))), x] + Simp[1/(b*(m + 2*
(p + q + 1) + 1))   Int[(g*x)^m*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[c*((
b*e - a*f)*(m + 1) + b*e*2*(p + q + 1)) + (d*(b*e - a*f)*(m + 1) + f*2*q*(b
*c - a*d) + b*e*d*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f
, g, m, p}, x] && GtQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^2, c + d*x^
2])
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

rule 448
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[e   Int[(g*x)^m*(a + b*x
^2)^p*(c + d*x^2)^q*(e + f*x^2)^(r - 1), x], x] + Simp[f/e^2   Int[(g*x)^(m
 + 2)*(a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^(r - 1), x], x] /; FreeQ[{a,
b, c, d, e, f, g, m, p, q}, x] && IGtQ[r, 0]
```

Maple [A] (verified)

Time = 20.77 (sec) , antiderivative size = 2657, normalized size of antiderivative = 1.85

method	result	size
risch	Expression too large to display	2657
elliptic	Expression too large to display	3316
default	Expression too large to display	4274

input

```
int(x^4*(b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)*(f*x^2+e)^2,x,method=_RETURNVERBOS
E)
```

output

```
1/45045*x/d^4*(3465*b^5*d^5*f^2*x^10+315*a*b^4*d^5*f^2*x^8+4410*b^5*c*d^4*
f^2*x^8+8190*b^5*d^5*e*f*x^8-350*a^2*b^3*d^5*f^2*x^6+525*a*b^4*c*d^4*f^2*x
^6+910*a*b^4*d^5*e*f*x^6+105*b^5*c^2*d^3*f^2*x^6+10920*b^5*c*d^4*e*f*x^6+5
005*b^5*d^5*e^2*x^6+400*a^3*b^2*d^5*f^2*x^4-605*a^2*b^3*c*d^4*f^2*x^4-1040
*a^2*b^3*d^5*e*f*x^4+45*a*b^4*c^2*d^3*f^2*x^4+1690*a*b^4*c*d^4*e*f*x^4+715
*a*b^4*d^5*e^2*x^4-120*b^5*c^3*d^2*f^2*x^4+390*b^5*c^2*d^3*e*f*x^4+7150*b^
5*c*d^4*e^2*x^4-480*a^4*b*d^5*f^2*x^2+736*a^3*b^2*c*d^4*f^2*x^2+1248*a^3*b
^2*d^5*e*f*x^2-63*a^2*b^3*c^2*d^3*f^2*x^2-2054*a^2*b^3*c*d^4*e*f*x^2-858*a
^2*b^3*d^5*e^2*x^2-57*a*b^4*c^3*d^2*f^2*x^2+234*a*b^4*c^2*d^3*e*f*x^2+1573
*a*b^4*c*d^4*e^2*x^2+144*b^5*c^4*d*f^2*x^2-468*b^5*c^3*d^2*e*f*x^2+429*b^5
*c^2*d^3*e^2*x^2+640*a^5*d^5*f^2-1008*a^4*b*c*d^4*f^2-1664*a^4*b*d^5*e*f+1
11*a^3*b^2*c^2*d^3*f^2+2808*a^3*b^2*c*d^4*e*f+1144*a^3*b^2*d^5*e^2+85*a^2*
b^3*c^3*d^2*f^2-390*a^2*b^3*c^2*d^3*e*f-2145*a^2*b^3*c*d^4*e^2+84*a*b^4*c^
4*d*f^2-338*a*b^4*c^3*d^2*e*f+429*a*b^4*c^2*d^3*e^2-192*b^5*c^5*f^2+624*b^
5*c^4*d*e*f-572*b^5*c^3*d^2*e^2)*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b^5-1/450
45/d^4/b^5*(-(1280*a^6*d^6*f^2-2176*a^5*b*c*d^5*f^2-3328*a^5*b*d^6*e*f+414
*a^4*b^2*c^2*d^4*f^2+6032*a^4*b^2*c*d^5*e*f+2288*a^4*b^2*d^6*e^2+203*a^3*b
^3*c^3*d^3*f^2-1326*a^3*b^3*c^2*d^4*e*f-4576*a^3*b^3*c*d^5*e^2+167*a^2*b^4
*c^4*d^2*f^2-754*a^2*b^4*c^3*d^3*e*f+1287*a^2*b^4*c^2*d^4*e^2+216*a*b^5*c^
5*d*f^2-832*a*b^5*c^4*d^2*e*f+1001*a*b^5*c^3*d^3*e^2-384*b^6*c^6*f^2+12...
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 1561, normalized size of antiderivative = 1.09

$$\int x^4\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^2\,dx=\text{Too large to display}$$

input
```
integrate(x^4*(b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)*(f*x^2+e)^2,x, algorithm="fr
icas")
```

output
```
-1/45045*((143*(8*b^6*c^5*d^2 - 7*a*b^5*c^4*d^3 - 9*a^2*b^4*c^3*d^4 + 32*a
^3*b^3*c^2*d^5 - 16*a^4*b^2*c*d^6)*e^2 - 26*(48*b^6*c^6*d - 32*a*b^5*c^5*d
^2 - 29*a^2*b^4*c^4*d^3 - 51*a^3*b^3*c^3*d^4 + 232*a^4*b^2*c^2*d^5 - 128*a
^5*b*c*d^6)*e*f + (384*b^6*c^7 - 216*a*b^5*c^6*d - 167*a^2*b^4*c^5*d^2 - 2
03*a^3*b^3*c^4*d^3 - 414*a^4*b^2*c^3*d^4 + 2176*a^5*b*c^2*d^5 - 1280*a^6*c
*d^6)*f^2)*sqrt(b*d)*x*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*
c)) - (143*(8*b^6*c^5*d^2 - 7*a*b^5*c^4*d^3 - 8*a^4*b^2*d^7 - (9*a^2*b^4 -
 4*a*b^5)*c^3*d^4 + (32*a^3*b^3 - 3*a^2*b^4)*c^2*d^5 - (16*a^4*b^2 - 15*a^
3*b^3)*c*d^6)*e^2 - 26*(48*b^6*c^6*d - 32*a*b^5*c^5*d^2 - 64*a^5*b*d^7 - (
29*a^2*b^4 - 24*a*b^5)*c^4*d^3 - (51*a^3*b^3 + 13*a^2*b^4)*c^3*d^4 + (232*
a^4*b^2 - 15*a^3*b^3)*c^2*d^5 - 4*(32*a^5*b - 27*a^4*b^2)*c*d^6)*e*f + (38
4*b^6*c^7 - 216*a*b^5*c^6*d - 640*a^6*d^7 - (167*a^2*b^4 - 192*a*b^5)*c^5*
d^2 - 7*(29*a^3*b^3 + 12*a^2*b^4)*c^4*d^3 - (414*a^4*b^2 + 85*a^3*b^3)*c^3
*d^4 + (2176*a^5*b - 111*a^4*b^2)*c^2*d^5 - 16*(80*a^6 - 63*a^5*b)*c*d^6)*
f^2)*sqrt(b*d)*x*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)/x), a*d/(b*c)) -
(3465*b^6*d^7*f^2*x^12 + 315*(26*b^6*d^7*e*f + (14*b^6*c*d^6 + a*b^5*d^7)*
f^2)*x^10 + 35*(143*b^6*d^7*e^2 + 26*(12*b^6*c*d^6 + a*b^5*d^7)*e*f + (3*b
^6*c^2*d^5 + 15*a*b^5*c*d^6 - 10*a^2*b^4*d^7)*f^2)*x^8 + 5*(143*(10*b^6*c*
d^6 + a*b^5*d^7)*e^2 + 26*(3*b^6*c^2*d^5 + 13*a*b^5*c*d^6 - 8*a^2*b^4*d^7)
*e*f - (24*b^6*c^3*d^4 - 9*a*b^5*c^2*d^5 + 121*a^2*b^4*c*d^6 - 80*a^3*b...
```

Sympy [F]

$$\int x^4\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^2\,dx=\int x^4\sqrt{a+bx^2}\left(c+dx^2\right)^{\frac{3}{2}}\left(e+fx^2\right)^2\,dx$$

input
```
integrate(x**4*(b*x**2+a)**(1/2)*(d*x**2+c)**(3/2)*(f*x**2+e)**2,x)
```

output
```
Integral(x**4*sqrt(a + b*x**2)*(c + d*x**2)**(3/2)*(e + f*x**2)**2, x)
```

Maxima [F]

$$\int x^4\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^2\,dx=\int\sqrt{bx^2+a}\left(dx^2+c\right)^{\frac{3}{2}}\left(fx^2+e\right)^2x^4\,dx$$

input
```
integrate(x^4*(b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)*(f*x^2+e)^2,x, algorithm="ma
xima")
```

output
```
integrate(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)*(f*x^2 + e)^2*x^4, x)
```

Giac [F]

$$\int x^4\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^2\,dx=\int\sqrt{bx^2+a}\left(dx^2+c\right)^{\frac{3}{2}}\left(fx^2+e\right)^2x^4\,dx$$

input
```
integrate(x^4*(b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)*(f*x^2+e)^2,x, algorithm="gi
ac")
```

output
```
integrate(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)*(f*x^2 + e)^2*x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^2\,dx=\int x^4\,\sqrt{b\,x^2+a}\,{\left(d\,x^2+c\right)}^{3/2}\,{\left(f\,x^2+e\right)}^2\,dx$$

input
```
int(x^4*(a + b*x^2)^(1/2)*(c + d*x^2)^(3/2)*(e + f*x^2)^2,x)
```

output
```
int(x^4*(a + b*x^2)^(1/2)*(c + d*x^2)^(3/2)*(e + f*x^2)^2, x)
```

Reduce [F]

$$\int x^4\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^2\,dx=\text{too large to display}$$

input
```
int(x^4*(b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)*(f*x^2+e)^2,x)
```

output
```
(640*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**5*d**5*f**2*x - 1008*sqrt(c + d*
x**2)*sqrt(a + b*x**2)*a**4*b*c*d**4*f**2*x - 1664*sqrt(c + d*x**2)*sqrt(a
 + b*x**2)*a**4*b*d**5*e*f*x - 480*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**4*
b*d**5*f**2*x**3 + 111*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**3*b**2*c**2*d*
*3*f**2*x + 2808*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**3*b**2*c*d**4*e*f*x
+ 736*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**3*b**2*c*d**4*f**2*x**3 + 1144*
sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**3*b**2*d**5*e**2*x + 1248*sqrt(c + d*
x**2)*sqrt(a + b*x**2)*a**3*b**2*d**5*e*f*x**3 + 400*sqrt(c + d*x**2)*sqrt
(a + b*x**2)*a**3*b**2*d**5*f**2*x**5 + 85*sqrt(c + d*x**2)*sqrt(a + b*x**
2)*a**2*b**3*c**3*d**2*f**2*x - 390*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2
*b**3*c**2*d**3*e*f*x - 63*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*b**3*c**
2*d**3*f**2*x**3 - 2145*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*b**3*c*d**4
*e**2*x - 2054*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*b**3*c*d**4*e*f*x**3
 - 605*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*b**3*c*d**4*f**2*x**5 - 858*
sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*b**3*d**5*e**2*x**3 - 1040*sqrt(c +
 d*x**2)*sqrt(a + b*x**2)*a**2*b**3*d**5*e*f*x**5 - 350*sqrt(c + d*x**2)*s
qrt(a + b*x**2)*a**2*b**3*d**5*f**2*x**7 + 84*sqrt(c + d*x**2)*sqrt(a + b*
x**2)*a*b**4*c**4*d*f**2*x - 338*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**4*
c**3*d**2*e*f*x - 57*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**4*c**3*d**2*f*
*2*x**3 + 429*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**4*c**2*d**3*e**2*x...
```

3.85 $\int x^2\sqrt{a+bx^2}(c+dx^2)^{3/2}\left(e+fx^2\right)^2\,dx$

Optimal result

Integrand size = 35, antiderivative size = 1120

$$\int x^2\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^2\,dx=\text{Too large to display}$$

output
```
1/3465*(128*a^5*d^5*f^2-8*a^4*b*d^4*f*(29*c*f+44*d*e)-a^2*b^3*c*d^2*(-29*c
^2*f^2+198*c*d*e*f+627*d^2*e^2)-2*b^5*c^3*(24*c^2*f^2-88*c*d*e*f+99*d^2*e^
2)+a*b^4*c^2*d*(32*c^2*f^2-154*c*d*e*f+297*d^2*e^2)+a^3*b^2*d^3*(51*c^2*f^
2+704*c*d*e*f+264*d^2*e^2))*x*(d*x^2+c)^(1/2)/b^4/d^4/(b*x^2+a)^(1/2)-1/34
65*(64*a^4*d^4*f^2-4*a^3*b*d^3*f*(27*c*f+44*d*e)-a*b^3*c*d*(-13*c^2*f^2+66
*c*d*e*f+297*d^2*e^2)+3*a^2*b^2*d^2*(5*c^2*f^2+110*c*d*e*f+44*d^2*e^2)-b^4
*c^2*(24*c^2*f^2-88*c*d*e*f+99*d^2*e^2))*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)
/b^4/d^3+1/3465*(48*a^3*d^3*f^2-a^2*b*d^2*f*(79*c*f+132*d*e)+6*b^3*c*(-3*c
^2*f^2+11*c*d*e*f+132*d^2*e^2)+a*b^2*d*(9*c^2*f^2+242*c*d*e*f+99*d^2*e^2))
*x^3*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b^3/d^2-1/693*(8*a^2*d^2*f^2-a*b*d*f*
(13*c*f+22*d*e)-b^2*(3*c^2*f^2+220*c*d*e*f+99*d^2*e^2))*x^5*(b*x^2+a)^(1/2
)*(d*x^2+c)^(1/2)/b^2/d+1/99*f*(a*d*f+12*b*c*f+22*b*d*e)*x^7*(b*x^2+a)^(1/
2)*(d*x^2+c)^(1/2)/b+1/11*d*f^2*x^9*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)-1/3465
*a^(1/2)*(128*a^5*d^5*f^2-8*a^4*b*d^4*f*(29*c*f+44*d*e)-a^2*b^3*c*d^2*(-29
*c^2*f^2+198*c*d*e*f+627*d^2*e^2)-2*b^5*c^3*(24*c^2*f^2-88*c*d*e*f+99*d^2*
e^2)+a*b^4*c^2*d*(32*c^2*f^2-154*c*d*e*f+297*d^2*e^2)+a^3*b^2*d^3*(51*c^2*
f^2+704*c*d*e*f+264*d^2*e^2))*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/
(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/b^(9/2)/d^4/(b*x^2+a)^(1/2)/(a*(d*x^2
+c)/c/(b*x^2+a))^(1/2)+1/3465*a^(3/2)*(64*a^4*d^4*f^2-4*a^3*b*d^3*f*(27*c*
f+44*d*e)-a*b^3*c*d*(-13*c^2*f^2+66*c*d*e*f+297*d^2*e^2)+3*a^2*b^2*d^2*...
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.65 (sec) , antiderivative size = 782, normalized size of antiderivative = 0.70

$$\int x^2\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^2\,dx=\frac{\sqrt{\frac{b}{a}}dx(a+bx^2)\left(c+dx^2\right)\left(-64a^4d^4f^2+4a^3bd^3f(44de+27cf+12dfx^2)-a^2b^2d^2(15c^2f^2+\cdots\right.}{}$$

input
```
Integrate[x^2*Sqrt[a + b*x^2]*(c + d*x^2)^(3/2)*(e + f*x^2)^2,x]
```

output
```
(Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2)*(-64*a^4*d^4*f^2 + 4*a^3*b*d^3*f*(4
4*d*e + 27*c*f + 12*d*f*x^2) - a^2*b^2*d^2*(15*c^2*f^2 + c*d*f*(330*e + 79
*f*x^2) + 4*d^2*(33*e^2 + 33*e*f*x^2 + 10*f^2*x^4)) + a*b^3*d*(-13*c^3*f^2
 + 3*c^2*d*f*(22*e + 3*f*x^2) + d^3*x^2*(99*e^2 + 110*e*f*x^2 + 35*f^2*x^4
) + c*d^2*(297*e^2 + 242*e*f*x^2 + 65*f^2*x^4)) + b^4*(24*c^4*f^2 - 2*c^3*
d*f*(44*e + 9*f*x^2) + 3*c^2*d^2*(33*e^2 + 22*e*f*x^2 + 5*f^2*x^4) + 5*d^4
*x^4*(99*e^2 + 154*e*f*x^2 + 63*f^2*x^4) + 4*c*d^3*x^2*(198*e^2 + 275*e*f*
x^2 + 105*f^2*x^4))) - I*c*(128*a^5*d^5*f^2 - 8*a^4*b*d^4*f*(44*d*e + 29*c
*f) - 2*b^5*c^3*(99*d^2*e^2 - 88*c*d*e*f + 24*c^2*f^2) + a^2*b^3*c*d^2*(-6
27*d^2*e^2 - 198*c*d*e*f + 29*c^2*f^2) + a*b^4*c^2*d*(297*d^2*e^2 - 154*c*
d*e*f + 32*c^2*f^2) + a^3*b^2*d^3*(264*d^2*e^2 + 704*c*d*e*f + 51*c^2*f^2)
)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x]
, (a*d)/(b*c)] + (2*I)*c*(-(b*c) + a*d)*(32*a^4*d^4*f^2 - 2*a^3*b*d^3*f*(4
4*d*e + 15*c*f) + a*b^3*c*d*(-99*d^2*e^2 + 33*c*d*e*f - 4*c^2*f^2) + 3*a^2
*b^2*d^2*(22*d^2*e^2 + 33*c*d*e*f - 4*c^2*f^2) + b^4*c^2*(99*d^2*e^2 - 88*
c*d*e*f + 24*c^2*f^2))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I
*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(3465*b^4*Sqrt[b/a]*d^4*Sqrt[a + b*x^
2]*Sqrt[c + d*x^2])
```

Rubi [A] (warning: unable to verify)

Time = 2.59 (sec) , antiderivative size = 1435, normalized size of antiderivative = 1.28, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.457$, Rules used = {448, 443, 27, 443, 443, 444, 27, 406, 320, 388, 313, 444, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \sqrt{a+bx^2} \left(c+dx^2\right)^{3/2} \left(e+fx^2\right)^2 \, dx$$

$$\Big\downarrow \text{448}$$

$$\frac{f \int x^4 \sqrt{bx^2+a} \left(dx^2+c\right)^{3/2} \left(fx^2+e\right) dx}{e^2} + e \int x^2 \sqrt{bx^2+a} \left(dx^2+c\right)^{3/2} \left(fx^2+e\right) dx$$

$$\Big\downarrow \text{443}$$

$$\frac{f\left(\frac{\int x^4\sqrt{bx^2+a}\sqrt{dx^2+c}\left((11bde+3bcf-8adf)x^2+c(11be-5af)\right)dx}{11b}+\frac{fx^5\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{11b}\right)}{e^2}+e\left(\frac{\int 3x^2\sqrt{bx^2+a}\sqrt{dx^2+c}\left((3bde+bcf-2adf)x^2+c(3be-af)\right)dx}{9b}+\frac{fx^3\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{9b}\right)$$

$\downarrow$ 27

$$\frac{f\left(\frac{\int x^4\sqrt{bx^2+a}\sqrt{dx^2+c}\left((11bde+3bcf-8adf)x^2+c(11be-5af)\right)dx}{11b}+\frac{fx^5\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{11b}\right)}{e^2}+e\left(\frac{\int x^2\sqrt{bx^2+a}\sqrt{dx^2+c}\left((3bde+bcf-2adf)x^2+c(3be-af)\right)dx}{3b}+\frac{fx^3\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{9b}\right)$$

$\downarrow$ 443

$$\frac{f\left(\frac{\frac{\int\frac{x^4\sqrt{bx^2+a}\left(\left(c(110de+3cf)b^2-ad(66de+71cf)b+48a^2d^2f\right)x^2+c\left(40dfa^2-55bdea-60bcfa+99b^2ce\right)\right)}{\sqrt{dx^2+c}}dx}{9b}+\frac{x^5\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}(-8adf+3bcf+11bde)}{9b}}{11b}\right.}{e^2}$$

$$e\left(\frac{\frac{\int\frac{x^2\sqrt{bx^2+a}\left(\left(c(24de+cf)b^2-ad(12de+13cf)b+8a^2d^2f\right)x^2+c\left(6dfa^2-9bdea-10bcfa+21b^2ce\right)\right)}{\sqrt{dx^2+c}}dx}{7b}+\frac{x^3\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}(-2adf+bcf+3bde)}{7b}}{3b}\right.$$

$\downarrow$ 443

$$\frac{f\left(\frac{\frac{\frac{\int\frac{x^4\left(\left(3c^2(11de-6cf)b^3+acd(121de+9cf)b^2-a^2d^2(66de+79cf)b+48a^3d^3f\right)x^2+ac\left(c(143de-15cf)b^2-5ad(11de+13cf)b+40a^2d^2f\right)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{7d}+\frac{x^5\sqrt{a+bx^2}\sqrt{c+\cdots}}{\cdots}}{9b}}{11b}\right.}{e^2}$$

$$e\left(\frac{\frac{\frac{\int\frac{x^2\left(\left(c^2(9de-4cf)b^3+3acd(9de+cf)b^2-3a^2d^2(4de+5cf)b+8a^3d^3f\right)x^2+ac\left(3c(11de-cf)b^2-ad(9de+11cf)b+6a^2d^2f\right)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{5d}+\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}\left(8a^2d^2f\cdots\right.}{\cdots}}{7b}}{3b}\right.$$

$\downarrow$ 444

$$e\left(\frac{f\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}x^3}{9b}+\frac{\frac{(3bde+bcf-2adf)\left(bx^2+a\right)^{3/2}\sqrt{dx^2+cx^3}}{7b}+\frac{\left(c(24de+cf)b^2-ad(12de+13cf)b+8a^2d^2f\right)\sqrt{bx^2+a}\sqrt{dx^2}}{5d}}{}\right.$$

$$f\left(\frac{f\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}x^5}{11b}+\frac{\frac{(11bde+3bcf-8adf)\left(bx^2+a\right)^{3/2}\sqrt{dx^2+cx^5}}{9b}+\frac{\left(c(110de+3cf)b^2-ad(66de+71cf)b+48a^2d^2f\right)\sqrt{bx^2+a}\sqrt{dx^2+cx^5}}{7d}+}{}\right.$$

$\downarrow$ 27

$$e\left(\frac{f\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}x^3}{9b}+\frac{\frac{(3bde+bcf-2adf)\left(bx^2+a\right)^{3/2}\sqrt{dx^2+cx^3}}{7b}+\frac{\left(c(24de+cf)b^2-ad(12de+13cf)b+8a^2d^2f\right)\sqrt{bx^2+a}\sqrt{dx^2}}{5d}}{}\right.$$

$$f\left(\frac{f\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}x^5}{11b}+\frac{\frac{(11bde+3bcf-8adf)\left(bx^2+a\right)^{3/2}\sqrt{dx^2+cx^5}}{9b}+\frac{\left(c(110de+3cf)b^2-ad(66de+71cf)b+48a^2d^2f\right)\sqrt{bx^2+a}\sqrt{dx^2+cx^5}}{7d}+}{}\right.$$

$\downarrow$ 406

$$e\left(\frac{f\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}x^3}{9b}+\frac{\frac{(3bde+bcf-2adf)\left(bx^2+a\right)^{3/2}\sqrt{dx^2+cx^3}}{7b}+\frac{\left(c(24de+cf)b^2-ad(12de+13cf)b+8a^2d^2f\right)\sqrt{bx^2+a}\sqrt{dx^2}}{5d}}{}\right.$$

$$f\left(\frac{f\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}x^5}{11b}+\frac{\frac{(11bde+3bcf-8adf)\left(bx^2+a\right)^{3/2}\sqrt{dx^2+cx^5}}{9b}+\frac{\left(c(110de+3cf)b^2-ad(66de+71cf)b+48a^2d^2f\right)\sqrt{bx^2+a}\sqrt{dx^2+cx^5}}{7d}+}{}\right.$$

$\downarrow$ 320

$$e\left(\frac{f\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}x^3}{9b}+\frac{\frac{(3bde+bcf-2adf)\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}x^3}{7b}+\frac{\left(c(24de+cf)b^2-ad(12de+13cf)b+8a^2d^2f\right)\sqrt{bx^2+a}\sqrt{dx^2 \cdots}}{5d}\cdots}{\cdots}\right.$$

$$f\left(\frac{f\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}x^5}{11b}+\frac{\frac{(11bde+3bcf-8adf)\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}x^5}{9b}+\frac{\frac{\left(c(110de+3cf)b^2-ad(66de+71cf)b+48a^2d^2f\right)\sqrt{bx^2+a}\sqrt{dx^2+c}x^5}{7d}+\frac{(3\cdots}{\cdots}}{\cdots}}{\cdots}\right.$$

$$\Big\downarrow \text{ 388}$$

$$e\left(\frac{f\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}x^3}{9b}+\frac{\frac{(3bde+bcf-2adf)\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}x^3}{7b}+\frac{\left(c(24de+cf)b^2-ad(12de+13cf)b+8a^2d^2f\right)\sqrt{bx^2+a}\sqrt{dx^2 \cdots}}{5d}\cdots}{\cdots}\right.$$

$$f\left(\frac{f\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}x^5}{11b}+\frac{\frac{(11bde+3bcf-8adf)\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}x^5}{9b}+\frac{\frac{\left(c(110de+3cf)b^2-ad(66de+71cf)b+48a^2d^2f\right)\sqrt{bx^2+a}\sqrt{dx^2+c}x^5}{7d}+\frac{(3\cdots}{\cdots}}{\cdots}}{\cdots}\right.$$

$$\Big\downarrow \text{ 313}$$

$$e\left(\frac{f\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}x^3}{9b}+\frac{(3bde+bcf-2adf)\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}x^3}{7b}+\frac{\frac{\left(c(24de+cf)b^2-ad(12de+13cf)b+8a^2d^2f\right)\sqrt{bx^2+a}\sqrt{dx^2 \cdots}}{5d}}{\cdots}\right.$$

$$f\left(\frac{f\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}x^5}{11b}+\frac{\frac{(11bde+3bcf-8adf)\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}x^5}{9b}+\frac{\frac{\left(c(110de+3cf)b^2-ad(66de+71cf)b+48a^2d^2f\right)\sqrt{bx^2+a}\sqrt{dx^2+c}x^5}{7d}+\frac{(3\cdots}{\cdots}}{\cdots}}{\cdots}\right.$$

$\downarrow$ 444

$$e\left(\frac{f\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}x^3}{9b}+\frac{(3bde+bcf-2adf)\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}x^3}{7b}+\frac{\frac{\left(c(24de+cf)b^2-ad(12de+13cf)b+8a^2d^2f\right)\sqrt{bx^2+a}\sqrt{dx^2 \cdots}}{5d}}{\cdots}\right.$$

$$f\left(\frac{f\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}x^5}{11b}+\frac{\frac{(11bde+3bcf-8adf)\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}x^5}{9b}+\frac{\frac{\left(c(110de+3cf)b^2-ad(66de+71cf)b+48a^2d^2f\right)\sqrt{bx^2+a}\sqrt{dx^2+c}x^5}{7d}+\frac{(3\cdots}{\cdots}}{\cdots}}{\cdots}\right.$$

$\downarrow$ 406

$$e\left(\frac{f\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}x^3}{9b}+\frac{\frac{(3bde+bcf-2adf)\left(bx^2+a\right)^{3/2}\sqrt{dx^2+cx}^3}{7b}+\frac{\frac{\left(c(24de+cf)b^2-ad(12de+13cf)b+8a^2d^2f\right)\sqrt{bx^2+a}\sqrt{dx^2}}{5d}}{}}{}\right.$$

$$f\left(\frac{f\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}x^5}{11b}+\frac{\frac{(11bde+3bcf-8adf)\left(bx^2+a\right)^{3/2}\sqrt{dx^2+cx}^5}{9b}+\frac{\left(c(110de+3cf)b^2-ad(66de+71cf)b+48a^2d^2f\right)\sqrt{bx^2+a}\sqrt{dx^2+cx}^5}{7d}+\frac{\left(3\right.}{}}{}\right.$$

$$\Big\downarrow \; 320$$

$$e\left(\frac{f\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}x^3}{9b}+\frac{\frac{(3bde+bcf-2adf)\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}\,x^3}{7b}+\frac{\frac{\left(c(24de+cf)b^2-ad(12de+13cf)b+8a^2d^2f\right)\sqrt{bx^2+a}\sqrt{dx^2 \cdots}}{5d}\cdots}{\cdots}}{\cdots}\right.$$

$$f\left(\frac{f\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}x^5}{11b}+\frac{\frac{(11bde+3bcf-8adf)\left(bx^2+a\right)^{3/2}\sqrt{dx^2+cx^5}}{9b}+\frac{\frac{\left(c(110de+3cf)b^2-ad(66de+71cf)b+48a^2d^2f\right)\sqrt{bx^2+a}\sqrt{dx^2+cx^5}}{7d}+\frac{\left(3\cdots\right.}{\cdots}}{\cdots}}{\cdots}\right.$$

$\downarrow$ 388

$$e\left(\frac{f\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}x^3}{9b}+\frac{\frac{(3bde+bcf-2adf)\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}x^3}{7b}+\frac{\frac{\left(c(24de+cf)b^2-ad(12de+13cf)b+8a^2d^2f\right)\sqrt{bx^2+a}\sqrt{dx^2}\cdots}{5d}\cdots}{\cdots}}{\cdots}\right.$$

$$f\left(\frac{f\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}x^5}{11b}+\frac{\frac{(11bde+3bcf-8adf)\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}x^5}{9b}+\frac{\left(c(110de+3cf)b^2-ad(66de+71cf)b+48a^2d^2f\right)\sqrt{bx^2+a}\sqrt{dx^2+c}x^5}{7d}+\frac{\left(3\cdots\right.}{\cdots}}{\cdots}\right.$$

$\downarrow$ 313

$$e\left(\frac{f\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}x^3}{9b}+\frac{\frac{(3bde+bcf-2adf)\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}x^3}{7b}+\frac{\frac{\left(c(24de+cf)b^2-ad(12de+13cf)b+8a^2d^2f\right)\sqrt{bx^2+a}\sqrt{dx^2}}{5d}}{}}{}\right.$$

$$f\left(\frac{f\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}x^5}{11b}+\frac{\frac{(11bde+3bcf-8adf)\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}x^5}{9b}+\frac{\frac{\left(c(110de+3cf)b^2-ad(66de+71cf)b+48a^2d^2f\right)\sqrt{bx^2+a}\sqrt{dx^2+c}x^5}{7d}+\frac{(3}{}}{}}{}\right.$$

input
```
Int[x^2*Sqrt[a + b*x^2]*(c + d*x^2)^(3/2)*(e + f*x^2)^2,x]
```

output
```
e*((f*x^3*(a + b*x^2)^(3/2)*(c + d*x^2)^(3/2))/(9*b) + (((3*b*d*e + b*c*f
- 2*a*d*f)*x^3*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(7*b) + (((8*a^2*d^2*f +
 b^2*c*(24*d*e + c*f) - a*b*d*(12*d*e + 13*c*f))*x^3*Sqrt[a + b*x^2]*Sqrt[
c + d*x^2])/(5*d) + (((8*a^3*d^3*f + b^3*c^2*(9*d*e - 4*c*f) + 3*a*b^2*c*d
*(9*d*e + c*f) - 3*a^2*b*d^2*(4*d*e + 5*c*f))*x*Sqrt[a + b*x^2]*Sqrt[c + d
*x^2])/(3*b*d) - ((16*a^4*d^4*f - a*b^3*c^2*d*(27*d*e - 7*c*f) + 2*b^4*c^3
*(9*d*e - 4*c*f) + 3*a^2*b^2*c*d^2*(19*d*e + 3*c*f) - 8*a^3*b*d^3*(3*d*e +
 4*c*f))*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^
2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqr
t[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (c^(3/2)*(8*a^3*d^3
*f + b^3*c^2*(9*d*e - 4*c*f) + 3*a*b^2*c*d*(9*d*e + c*f) - 3*a^2*b*d^2*(4*
d*e + 5*c*f))*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (
b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2
]))/(3*b*d))/(5*d))/(7*b))/(3*b)) + (f*((f*x^5*(a + b*x^2)^(3/2)*(c + d*x^
2)^(3/2))/(11*b) + (((11*b*d*e + 3*b*c*f - 8*a*d*f)*x^5*(a + b*x^2)^(3/2)*
Sqrt[c + d*x^2])/(9*b) + (((48*a^2*d^2*f + b^2*c*(110*d*e + 3*c*f) - a*b*d
*(66*d*e + 71*c*f))*x^5*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(7*d) + (((48*a^3
*d^3*f + 3*b^3*c^2*(11*d*e - 6*c*f) + a*b^2*c*d*(121*d*e + 9*c*f) - a^2*b*
d^2*(66*d*e + 79*c*f))*x^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(5*b*d) - (3*(
((64*a^4*d^4*f - a*b^3*c^2*d*(33*d*e - 13*c*f) + 4*b^4*c^3*(11*d*e - 6*...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 443
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*(g*x)^(m + 1)*(a + b*x^2)^(p
 + 1)*((c + d*x^2)^q/(b*g*(m + 2*(p + q + 1) + 1))), x] + Simp[1/(b*(m + 2*
(p + q + 1) + 1))   Int[(g*x)^m*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[c*((
b*e - a*f)*(m + 1) + b*e*2*(p + q + 1)) + (d*(b*e - a*f)*(m + 1) + f*2*q*(b
*c - a*d) + b*e*d*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f
, g, m, p}, x] && GtQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^2, c + d*x^
2])
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

rule 448
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[e   Int[(g*x)^m*(a + b*x
^2)^p*(c + d*x^2)^q*(e + f*x^2)^(r - 1), x], x] + Simp[f/e^2   Int[(g*x)^(m
 + 2)*(a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^(r - 1), x], x] /; FreeQ[{a,
b, c, d, e, f, g, m, p, q}, x] && IGtQ[r, 0]
```

Maple [A] (verified)

Time = 10.45 (sec) , antiderivative size = 2046, normalized size of antiderivative = 1.83

method	result	size
risch	Expression too large to display	2046
elliptic	Expression too large to display	2067
default	Expression too large to display	3341

input
```
int(x^2*(b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)*(f*x^2+e)^2,x,method=_RETURNVERBOS
E)
```

output
```
-1/3465*x/d^3*(-315*b^4*d^4*f^2*x^8-35*a*b^3*d^4*f^2*x^6-420*b^4*c*d^3*f^2
*x^6-770*b^4*d^4*e*f*x^6+40*a^2*b^2*d^4*f^2*x^4-65*a*b^3*c*d^3*f^2*x^4-110
*a*b^3*d^4*e*f*x^4-15*b^4*c^2*d^2*f^2*x^4-1100*b^4*c*d^3*e*f*x^4-495*b^4*d
^4*e^2*x^4-48*a^3*b*d^4*f^2*x^2+79*a^2*b^2*c*d^3*f^2*x^2+132*a^2*b^2*d^4*e
*f*x^2-9*a*b^3*c^2*d^2*f^2*x^2-242*a*b^3*c*d^3*e*f*x^2-99*a*b^3*d^4*e^2*x^
2+18*b^4*c^3*d*f^2*x^2-66*b^4*c^2*d^2*e*f*x^2-792*b^4*c*d^3*e^2*x^2+64*a^4
*d^4*f^2-108*a^3*b*c*d^3*f^2-176*a^3*b*d^4*e*f+15*a^2*b^2*c^2*d^2*f^2+330*
a^2*b^2*c*d^3*e*f+132*a^2*b^2*d^4*e^2+13*a*b^3*c^3*d*f^2-66*a*b^3*c^2*d^2*
e*f-297*a*b^3*c*d^3*e^2-24*b^4*c^4*f^2+88*b^4*c^3*d*e*f-99*b^4*c^2*d^2*e^2
)*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b^4+1/3465/b^4/d^3*(-(128*a^5*d^5*f^2-23
2*a^4*b*c*d^4*f^2-352*a^4*b*d^5*e*f+51*a^3*b^2*c^2*d^3*f^2+704*a^3*b^2*c*d
^4*e*f+264*a^3*b^2*d^5*e^2+29*a^2*b^3*c^3*d^2*f^2-198*a^2*b^3*c^2*d^3*e*f-
627*a^2*b^3*c*d^4*e^2+32*a*b^4*c^4*d*f^2-154*a*b^4*c^3*d^2*e*f+297*a*b^4*c
^2*d^3*e^2-48*b^5*c^5*f^2+176*b^5*c^4*d*e*f-198*b^5*c^3*d^2*e^2)*c/(-b/a)^
(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1
/2)/d*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/
a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2)))-24*a*b^4*c^5*f^2/(-b/a)^(1/2)*(1+b*x^2
/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(
x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+64*a^5*c*d^4*f^2/(-b/a)^(1/2)*(1+
b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*El...
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 1199, normalized size of antiderivative = 1.07

$$\int x^2\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^2\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)*(f*x^2+e)^2,x, algorithm="fr
icas")
```

output
```
1/3465*((33*(6*b^5*c^4*d^2 - 9*a*b^4*c^3*d^3 + 19*a^2*b^3*c^2*d^4 - 8*a^3*
b^2*c*d^5)*e^2 - 22*(8*b^5*c^5*d - 7*a*b^4*c^4*d^2 - 9*a^2*b^3*c^3*d^3 + 3
2*a^3*b^2*c^2*d^4 - 16*a^4*b*c*d^5)*e*f + (48*b^5*c^6 - 32*a*b^4*c^5*d - 2
9*a^2*b^3*c^4*d^2 - 51*a^3*b^2*c^3*d^3 + 232*a^4*b*c^2*d^4 - 128*a^5*c*d^5
)*f^2)*sqrt(b*d)*x*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c))
- (33*(6*b^5*c^4*d^2 - 9*a*b^4*c^3*d^3 - 4*a^3*b^2*d^6 + (19*a^2*b^3 + 3*a
*b^4)*c^2*d^4 - (8*a^3*b^2 - 9*a^2*b^3)*c*d^5)*e^2 - 22*(8*b^5*c^5*d - 7*a
*b^4*c^4*d^2 - 8*a^4*b*d^6 - (9*a^2*b^3 - 4*a*b^4)*c^3*d^3 + (32*a^3*b^2 -
 3*a^2*b^3)*c^2*d^4 - (16*a^4*b - 15*a^3*b^2)*c*d^5)*e*f + (48*b^5*c^6 - 3
2*a*b^4*c^5*d - 64*a^5*d^6 - (29*a^2*b^3 - 24*a*b^4)*c^4*d^2 - (51*a^3*b^2
 + 13*a^2*b^3)*c^3*d^3 + (232*a^4*b - 15*a^3*b^2)*c^2*d^4 - 4*(32*a^5 - 27
*a^4*b)*c*d^5)*f^2)*sqrt(b*d)*x*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)/x)
, a*d/(b*c)) + (315*b^5*d^6*f^2*x^10 + 35*(22*b^5*d^6*e*f + (12*b^5*c*d^5
+ a*b^4*d^6)*f^2)*x^8 + 5*(99*b^5*d^6*e^2 + 22*(10*b^5*c*d^5 + a*b^4*d^6)*
e*f + (3*b^5*c^2*d^4 + 13*a*b^4*c*d^5 - 8*a^2*b^3*d^6)*f^2)*x^6 + (99*(8*b
^5*c*d^5 + a*b^4*d^6)*e^2 + 22*(3*b^5*c^2*d^4 + 11*a*b^4*c*d^5 - 6*a^2*b^3
*d^6)*e*f - (18*b^5*c^3*d^3 - 9*a*b^4*c^2*d^4 + 79*a^2*b^3*c*d^5 - 48*a^3*
b^2*d^6)*f^2)*x^4 - 33*(6*b^5*c^3*d^3 - 9*a*b^4*c^2*d^4 + 19*a^2*b^3*c*d^5
 - 8*a^3*b^2*d^6)*e^2 + 22*(8*b^5*c^4*d^2 - 7*a*b^4*c^3*d^3 - 9*a^2*b^3*c^
2*d^4 + 32*a^3*b^2*c*d^5 - 16*a^4*b*d^6)*e*f - (48*b^5*c^5*d - 32*a*b^4...
```

Sympy [F]

$$\int x^2\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^2\,dx = \int x^2\sqrt{a+bx^2}\left(c+dx^2\right)^{\frac{3}{2}}\left(e+fx^2\right)^2\,dx$$

input
```
integrate(x**2*(b*x**2+a)**(1/2)*(d*x**2+c)**(3/2)*(f*x**2+e)**2,x)
```

output
```
Integral(x**2*sqrt(a + b*x**2)*(c + d*x**2)**(3/2)*(e + f*x**2)**2, x)
```

Maxima [F]

$$\int x^2\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^2\,dx=\int\sqrt{bx^2+a}\left(dx^2+c\right)^{\frac{3}{2}}\left(fx^2+e\right)^2x^2\,dx$$

input
```
integrate(x^2*(b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)*(f*x^2+e)^2,x, algorithm="ma
xima")
```

output
```
integrate(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)*(f*x^2 + e)^2*x^2, x)
```

Giac [F]

$$\int x^2\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^2\,dx=\int\sqrt{bx^2+a}\left(dx^2+c\right)^{\frac{3}{2}}\left(fx^2+e\right)^2x^2\,dx$$

input
```
integrate(x^2*(b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)*(f*x^2+e)^2,x, algorithm="gi
ac")
```

output
```
integrate(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)*(f*x^2 + e)^2*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^2\,dx=\int x^2\,\sqrt{b\,x^2+a}\,\left(d\,x^2+c\right)^{3/2}\,\left(f\,x^2+e\right)^2\,dx$$

input
```
int(x^2*(a + b*x^2)^(1/2)*(c + d*x^2)^(3/2)*(e + f*x^2)^2,x)
```

output
```
int(x^2*(a + b*x^2)^(1/2)*(c + d*x^2)^(3/2)*(e + f*x^2)^2, x)
```

Reduce [F]

$$\int x^2\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^2\,dx=\text{too large to display}$$

input
```
int(x^2*(b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)*(f*x^2+e)^2,x)
```

output
```
( - 64*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**4*d**4*f**2*x + 108*sqrt(c + d
*x**2)*sqrt(a + b*x**2)*a**3*b*c*d**3*f**2*x + 176*sqrt(c + d*x**2)*sqrt(a
 + b*x**2)*a**3*b*d**4*e*f*x + 48*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**3*b
*d**4*f**2*x**3 - 15*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*b**2*c**2*d**2
*f**2*x - 330*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*b**2*c*d**3*e*f*x - 7
9*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*b**2*c*d**3*f**2*x**3 - 132*sqrt(
c + d*x**2)*sqrt(a + b*x**2)*a**2*b**2*d**4*e**2*x - 132*sqrt(c + d*x**2)*
sqrt(a + b*x**2)*a**2*b**2*d**4*e*f*x**3 - 40*sqrt(c + d*x**2)*sqrt(a + b*
x**2)*a**2*b**2*d**4*f**2*x**5 - 13*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b*
*3*c**3*d*f**2*x + 66*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**3*c**2*d**2*e
*f*x + 9*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**3*c**2*d**2*f**2*x**3 + 29
7*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**3*c*d**3*e**2*x + 242*sqrt(c + d*
x**2)*sqrt(a + b*x**2)*a*b**3*c*d**3*e*f*x**3 + 65*sqrt(c + d*x**2)*sqrt(a
 + b*x**2)*a*b**3*c*d**3*f**2*x**5 + 99*sqrt(c + d*x**2)*sqrt(a + b*x**2)*
a*b**3*d**4*e**2*x**3 + 110*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**3*d**4*
e*f*x**5 + 35*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**3*d**4*f**2*x**7 + 24
*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**4*c**4*f**2*x - 88*sqrt(c + d*x**2)*
sqrt(a + b*x**2)*b**4*c**3*d*e*f*x - 18*sqrt(c + d*x**2)*sqrt(a + b*x**2)*
b**4*c**3*d*f**2*x**3 + 99*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**4*c**2*d**
2*e**2*x + 66*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**4*c**2*d**2*e*f*x**3...
```

3.86 $\int \sqrt{a+bx^2}(c+dx^2)^{3/2}(e+fx^2)^2\,dx$

Optimal result

Integrand size = 32, antiderivative size = 849

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^2\,dx =$$

$$-\frac{(16a^4d^4f^2-16a^3bd^3f(3de+2cf)-ab^3cd(147d^2e^2+54cdef-7c^2f^2)+3a^2b^2d^2(14d^2e^2+38cdef+3c^2}{315b^3d^3\sqrt{a+bx^2}}$$

$$+\frac{(8a^3d^3f^2-3a^2bd^2f(8de+5cf)+2b^3c(63d^2e^2+9cdef-2c^2f^2)+3ab^2d(7d^2e^2+18cdef+c^2f^2))\,x\sqrt{a}}{315b^3d^2}$$

$$-\frac{(6a^2d^2f^2-abdf(18de+11cf)-3b^2(21d^2e^2+48cdef+c^2f^2))\,x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{315b^2d}$$

$$+\frac{f(18bde+10bcf+adf)x^5\sqrt{a+bx^2}\sqrt{c+dx^2}}{63b}+\frac{1}{9}df^2x^7\sqrt{a+bx^2}\sqrt{c+dx^2}$$

$$+\frac{\sqrt{a}(16a^4d^4f^2-16a^3bd^3f(3de+2cf)-ab^3cd(147d^2e^2+54cdef-7c^2f^2)+3a^2b^2d^2(14d^2e^2+38cdef+}{315b^{7/2}d^3\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

$$-\frac{a^{3/2}(8a^3d^3f^2-3a^2bd^2f(8de+5cf)+3ab^2d(7d^2e^2+18cdef+c^2f^2)-b^3c(189d^2e^2-18cdef+4c^2f^2))}{315b^{7/2}d^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
-1/315*(16*a^4*d^4*f^2-16*a^3*b*d^3*f*(2*c*f+3*d*e)-a*b^3*c*d*(-7*c^2*f^2+
54*c*d*e*f+147*d^2*e^2)+3*a^2*b^2*d^2*(3*c^2*f^2+38*c*d*e*f+14*d^2*e^2)-b^
4*c^2*(8*c^2*f^2-36*c*d*e*f+63*d^2*e^2))*x*(d*x^2+c)^(1/2)/b^3/d^3/(b*x^2+
a)^(1/2)+1/315*(8*a^3*d^3*f^2-3*a^2*b*d^2*f*(5*c*f+8*d*e)+2*b^3*c*(-2*c^2*
f^2+9*c*d*e*f+63*d^2*e^2)+3*a*b^2*d*(c^2*f^2+18*c*d*e*f+7*d^2*e^2))*x*(b*x
^2+a)^(1/2)*(d*x^2+c)^(1/2)/b^3/d^2-1/315*(6*a^2*d^2*f^2-a*b*d*f*(11*c*f+1
8*d*e)-3*b^2*(c^2*f^2+48*c*d*e*f+21*d^2*e^2))*x^3*(b*x^2+a)^(1/2)*(d*x^2+c
)^(1/2)/b^2/d+1/63*f*(a*d*f+10*b*c*f+18*b*d*e)*x^5*(b*x^2+a)^(1/2)*(d*x^2+
c)^(1/2)/b+1/9*d*f^2*x^7*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)+1/315*a^(1/2)*(16
*a^4*d^4*f^2-16*a^3*b*d^3*f*(2*c*f+3*d*e)-a*b^3*c*d*(-7*c^2*f^2+54*c*d*e*f
+147*d^2*e^2)+3*a^2*b^2*d^2*(3*c^2*f^2+38*c*d*e*f+14*d^2*e^2)-b^4*c^2*(8*c
^2*f^2-36*c*d*e*f+63*d^2*e^2))*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)
/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/b^(7/2)/d^3/(b*x^2+a)^(1/2)/(a*(d*x^
2+c)/c/(b*x^2+a))^(1/2)-1/315*a^(3/2)*(8*a^3*d^3*f^2-3*a^2*b*d^2*f*(5*c*f+
8*d*e)+3*a*b^2*d*(c^2*f^2+18*c*d*e*f+7*d^2*e^2)-b^3*c*(4*c^2*f^2-18*c*d*e*
f+189*d^2*e^2))*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),
(1-a*d/b/c)^(1/2))/b^(7/2)/d^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(
1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.63 (sec) , antiderivative size = 584, normalized size of antiderivative = 0.69

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^2\,dx = \frac{\sqrt{\frac{b}{a}}dx(a+bx^2)\left(c+dx^2\right)\left(8a^3d^3f^2-3a^2bd^2f(8de+5cf+2dfx^2)+ab^2d(3c^2f^2+cdf(54e+\right.}{}$$

input
```
Integrate[Sqrt[a + b*x^2]*(c + d*x^2)^(3/2)*(e + f*x^2)^2,x]
```

output
```
(Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2)*(8*a^3*d^3*f^2 - 3*a^2*b*d^2*f*(8*d
*e + 5*c*f + 2*d*f*x^2) + a*b^2*d*(3*c^2*f^2 + c*d*f*(54*e + 11*f*x^2) + d
^2*(21*e^2 + 18*e*f*x^2 + 5*f^2*x^4)) + b^3*(-4*c^3*f^2 + 3*c^2*d*f*(6*e +
 f*x^2) + 2*c*d^2*(63*e^2 + 72*e*f*x^2 + 25*f^2*x^4) + d^3*x^2*(63*e^2 + 9
0*e*f*x^2 + 35*f^2*x^4))) + I*c*(16*a^4*d^4*f^2 - 16*a^3*b*d^3*f*(3*d*e +
2*c*f) + b^4*c^2*(-63*d^2*e^2 + 36*c*d*e*f - 8*c^2*f^2) + 3*a^2*b^2*d^2*(1
4*d^2*e^2 + 38*c*d*e*f + 3*c^2*f^2) + a*b^3*c*d*(-147*d^2*e^2 - 54*c*d*e*f
 + 7*c^2*f^2))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh
[Sqrt[b/a]*x], (a*d)/(b*c)] + I*c*(b*c - a*d)*(8*a^3*d^3*f^2 - 3*a^2*b*d^2
*f*(8*d*e + 3*c*f) + 3*a*b^2*d*(7*d^2*e^2 + 12*c*d*e*f - c^2*f^2) + b^3*c*
(63*d^2*e^2 - 36*c*d*e*f + 8*c^2*f^2))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2
)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(315*b^3*Sqrt[b/a]*d^
3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 1.99 (sec) , antiderivative size = 1352, normalized size of antiderivative = 1.59, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^2dx$$

$$\Big\downarrow \ 433$$

$$\int \left(e^2\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}+2efx^2\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}+f^2x^4\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{9}f^2\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}x^5+\frac{(3bc+ad)f^2\sqrt{bx^2+a}\sqrt{dx^2+c}x^5}{63b}+\frac{2}{7}ef\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}x^3+\frac{\left(-\frac{6da^2}{b}+11ca+\frac{3bc^2}{d}\right)f^2\sqrt{bx^2+a}\sqrt{dx^2+c}x^3}{315b}+\frac{2(3bc+ad)ef\sqrt{bx^2+a}\sqrt{dx^2+c}x^3}{35b}+\frac{de^2\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}x}{5b}+\frac{2(3bc-ad)e^2\sqrt{bx^2+a}\sqrt{dx^2+c}x}{15b}-\frac{\left(4b^3c^3-3ab^2dc^2+15a^2bd^2c-8a^3d^3\right)f^2\sqrt{bx^2+a}\sqrt{dx^2+c}x}{315b^3d^2}+\frac{2\left(-\frac{4da^2}{b}+9ca+\frac{3bc^2}{d}\right)ef\sqrt{bx^2+a}\sqrt{dx^2+c}x}{105b}+\frac{\left(3b^2c^2+7abdc-2a^2d^2\right)e^2\sqrt{bx^2+a}x}{15b^2\sqrt{dx^2+c}}+\frac{\left(8b^4c^4-7ab^3dc^3-9a^2b^2d^2c^2+32a^3bd^3c-16a^4d^4\right)f^2\sqrt{bx^2+a}x}{315b^4d^2\sqrt{dx^2+c}}-\frac{2(2bc-ad)\left(3b^2c^2-3abdc+8a^2d^2\right)ef\sqrt{bx^2+a}x}{105b^3d\sqrt{dx^2+c}}-\frac{\sqrt{c}\left(3b^2c^2+7abdc-2a^2d^2\right)e^2\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15b^2\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{\sqrt{c}\left(8b^4c^4-7ab^3dc^3-9a^2b^2d^2c^2+32a^3bd^3c-16a^4d^4\right)f^2\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{315b^4d^{5/2}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{2\sqrt{c}(2bc-ad)\left(3b^2c^2-3abdc+8a^2d^2\right)ef\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{105b^3d^{3/2}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{c^{3/2}(9bc-ad)e^2\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{15b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{c^{3/2}\left(4b^3c^3-3ab^2dc^2+15a^2bd^2c-8a^3d^3\right)f^2\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{315b^3d^{5/2}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{2c^{3/2}\left(3b^2c^2+9abdc-4a^2d^2\right)ef\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{105b^2d^{3/2}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}$$

input
```
Int[Sqrt[a + b*x^2]*(c + d*x^2)^(3/2)*(e + f*x^2)^2,x]
```

output
```
((3*b^2*c^2 + 7*a*b*c*d - 2*a^2*d^2)*e^2*x*Sqrt[a + b*x^2])/(15*b^2*Sqrt[c
 + d*x^2]) - (2*(2*b*c - a*d)*(3*b^2*c^2 - 3*a*b*c*d + 8*a^2*d^2)*e*f*x*Sq
rt[a + b*x^2])/(105*b^3*d*Sqrt[c + d*x^2]) + ((8*b^4*c^4 - 7*a*b^3*c^3*d -
 9*a^2*b^2*c^2*d^2 + 32*a^3*b*c*d^3 - 16*a^4*d^4)*f^2*x*Sqrt[a + b*x^2])/(
315*b^4*d^2*Sqrt[c + d*x^2]) + (2*(3*b*c - a*d)*e^2*x*Sqrt[a + b*x^2]*Sqrt
[c + d*x^2])/(15*b) + (2*(9*a*c + (3*b*c^2)/d - (4*a^2*d)/b)*e*f*x*Sqrt[a
+ b*x^2]*Sqrt[c + d*x^2])/(105*b) - ((4*b^3*c^3 - 3*a*b^2*c^2*d + 15*a^2*b
*c*d^2 - 8*a^3*d^3)*f^2*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(315*b^3*d^2) +
 (2*(3*b*c + a*d)*e*f*x^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(35*b) + ((11*a
*c + (3*b*c^2)/d - (6*a^2*d)/b)*f^2*x^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(
315*b) + ((3*b*c + a*d)*f^2*x^5*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(63*b) +
(d*e^2*x*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(5*b) + (2*e*f*x^3*Sqrt[a + b*
x^2]*(c + d*x^2)^(3/2))/7 + (f^2*x^5*Sqrt[a + b*x^2]*(c + d*x^2)^(3/2))/9
- (Sqrt[c]*(3*b^2*c^2 + 7*a*b*c*d - 2*a^2*d^2)*e^2*Sqrt[a + b*x^2]*Ellipti
cE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(15*b^2*Sqrt[d]*Sqrt[(c*
(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (2*Sqrt[c]*(2*b*c - a*d)*
(3*b^2*c^2 - 3*a*b*c*d + 8*a^2*d^2)*e*f*Sqrt[a + b*x^2]*EllipticE[ArcTan[(
Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(105*b^3*d^(3/2)*Sqrt[(c*(a + b*x^2
))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) - (Sqrt[c]*(8*b^4*c^4 - 7*a*b^3*c^3*d
 - 9*a^2*b^2*c^2*d^2 + 32*a^3*b*c*d^3 - 16*a^4*d^4)*f^2*Sqrt[a + b*x^2]...
```

Defintions of rubi rules used

rule 433
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)
^q*(e + f*x^2)^r, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, f, p,
 q, r}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 8.13 (sec) , antiderivative size = 1300, normalized size of antiderivative = 1.53

method	result	size
elliptic	Expression too large to display	1300
risch	Expression too large to display	1494
default	Expression too large to display	2491

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)*(f*x^2+e)^2,x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/9*f^2*d*x^7
*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+1/7*(a*d^2*f^2+2*b*c*d*f^2+2*b*d^2*e*
f-1/9*f^2*d*(8*a*d+8*b*c))/b/d*x^5*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+1/5
*(11/9*a*c*d*f^2+2*a*d^2*e*f+b*c^2*f^2+4*b*c*d*e*f+b*d^2*e^2-1/7*(a*d^2*f^
2+2*b*c*d*f^2+2*b*d^2*e*f-1/9*f^2*d*(8*a*d+8*b*c))/b/d*(6*a*d+6*b*c))/b/d*
x^3*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+1/3*(a*c^2*f^2+4*a*c*e*f*d+a*d^2*e
^2+2*b*c^2*e*f+2*b*c*d*e^2-5/7*(a*d^2*f^2+2*b*c*d*f^2+2*b*d^2*e*f-1/9*f^2*
d*(8*a*d+8*b*c))/b/d*a*c-1/5*(11/9*a*c*d*f^2+2*a*d^2*e*f+b*c^2*f^2+4*b*c*d
*e*f+b*d^2*e^2-1/7*(a*d^2*f^2+2*b*c*d*f^2+2*b*d^2*e*f-1/9*f^2*d*(8*a*d+8*b
*c))/b/d*(6*a*d+6*b*c))/b/d*(4*a*d+4*b*c))/b/d*x*(b*d*x^4+a*d*x^2+b*c*x^2+
a*c)^(1/2)+(a*c^2*e^2-1/3*(a*c^2*f^2+4*a*c*e*f*d+a*d^2*e^2+2*b*c^2*e*f+2*b
*c*d*e^2-5/7*(a*d^2*f^2+2*b*c*d*f^2+2*b*d^2*e*f-1/9*f^2*d*(8*a*d+8*b*c))/b
/d*a*c-1/5*(11/9*a*c*d*f^2+2*a*d^2*e*f+b*c^2*f^2+4*b*c*d*e*f+b*d^2*e^2-1/7
*(a*d^2*f^2+2*b*c*d*f^2+2*b*d^2*e*f-1/9*f^2*d*(8*a*d+8*b*c))/b/d*(6*a*d+6*
b*c))/b/d*(4*a*d+4*b*c))/b/d*a*c)/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/
c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+
(a*d+b*c)/c/b)^(1/2))-(2*a*c^2*e*f+2*a*c*e^2*d+b*c^2*e^2-3/5*(11/9*a*c*d*f
^2+2*a*d^2*e*f+b*c^2*f^2+4*b*c*d*e*f+b*d^2*e^2-1/7*(a*d^2*f^2+2*b*c*d*f^2+
2*b*d^2*e*f-1/9*f^2*d*(8*a*d+8*b*c))/b/d*(6*a*d+6*b*c))/b/d*a*c-1/3*(a*c^2
*f^2+4*a*c*e*f*d+a*d^2*e^2+2*b*c^2*e*f+2*b*c*d*e^2-5/7*(a*d^2*f^2+2*b*c...
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 881, normalized size of antiderivative = 1.04

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^2\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)*(f*x^2+e)^2,x, algorithm="fricas
")
```

output
```
-1/315*((21*(3*b^4*c^3*d^2 + 7*a*b^3*c^2*d^3 - 2*a^2*b^2*c*d^4)*e^2 - 6*(6
*b^4*c^4*d - 9*a*b^3*c^3*d^2 + 19*a^2*b^2*c^2*d^3 - 8*a^3*b*c*d^4)*e*f + (
8*b^4*c^5 - 7*a*b^3*c^4*d - 9*a^2*b^2*c^3*d^2 + 32*a^3*b*c^2*d^3 - 16*a^4*
c*d^4)*f^2)*sqrt(b*d)*x*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), a*d/(b
*c)) - (21*(3*b^4*c^3*d^2 + 7*a*b^3*c^2*d^3 - a^2*b^2*d^5 - (2*a^2*b^2 - 9
*a*b^3)*c*d^4)*e^2 - 6*(6*b^4*c^4*d - 9*a*b^3*c^3*d^2 - 4*a^3*b*d^5 + (19*
a^2*b^2 + 3*a*b^3)*c^2*d^3 - (8*a^3*b - 9*a^2*b^2)*c*d^4)*e*f + (8*b^4*c^5
 - 7*a*b^3*c^4*d - 8*a^4*d^5 - (9*a^2*b^2 - 4*a*b^3)*c^3*d^2 + (32*a^3*b -
 3*a^2*b^2)*c^2*d^3 - (16*a^4 - 15*a^3*b)*c*d^4)*f^2)*sqrt(b*d)*x*sqrt(-c/
d)*elliptic_f(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - (35*b^4*d^5*f^2*x^8 + 5*(
18*b^4*d^5*e*f + (10*b^4*c*d^4 + a*b^3*d^5)*f^2)*x^6 + (63*b^4*d^5*e^2 + 1
8*(8*b^4*c*d^4 + a*b^3*d^5)*e*f + (3*b^4*c^2*d^3 + 11*a*b^3*c*d^4 - 6*a^2*
b^2*d^5)*f^2)*x^4 + 21*(3*b^4*c^2*d^3 + 7*a*b^3*c*d^4 - 2*a^2*b^2*d^5)*e^2
 - 6*(6*b^4*c^3*d^2 - 9*a*b^3*c^2*d^3 + 19*a^2*b^2*c*d^4 - 8*a^3*b*d^5)*e*
f + (8*b^4*c^4*d - 7*a*b^3*c^3*d^2 - 9*a^2*b^2*c^2*d^3 + 32*a^3*b*c*d^4 -
16*a^4*d^5)*f^2 + (21*(6*b^4*c*d^4 + a*b^3*d^5)*e^2 + 6*(3*b^4*c^2*d^3 + 9
*a*b^3*c*d^4 - 4*a^2*b^2*d^5)*e*f - (4*b^4*c^3*d^2 - 3*a*b^3*c^2*d^3 + 15*
a^2*b^2*c*d^4 - 8*a^3*b*d^5)*f^2)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(b
^4*d^4*x)
```

Sympy [F]

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^2\,dx = \int \sqrt{a+bx^2}\left(c+dx^2\right)^{\frac{3}{2}}\left(e+fx^2\right)^2\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(3/2)*(f*x**2+e)**2,x)
```

output
```
Integral(sqrt(a + b*x**2)*(c + d*x**2)**(3/2)*(e + f*x**2)**2, x)
```

Maxima [F]

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^2\,dx=\int \sqrt{bx^2+a}\left(dx^2+c\right)^{\frac{3}{2}}\left(fx^2+e\right)^2\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)*(f*x^2+e)^2,x, algorithm="maxima
")
```

output
```
integrate(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)*(f*x^2 + e)^2, x)
```

Giac [F]

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^2\,dx=\int \sqrt{bx^2+a}\left(dx^2+c\right)^{\frac{3}{2}}\left(fx^2+e\right)^2\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)*(f*x^2+e)^2,x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)*(f*x^2 + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^2\,dx=\int \sqrt{b\,x^2+a}\,\left(d\,x^2+c\right)^{3/2}\,\left(f\,x^2+e\right)^2\,dx$$

input
```
int((a + b*x^2)^(1/2)*(c + d*x^2)^(3/2)*(e + f*x^2)^2,x)
```

output
```
int((a + b*x^2)^(1/2)*(c + d*x^2)^(3/2)*(e + f*x^2)^2, x)
```

Reduce [F]

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^2\,dx = \text{too large to display}$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)*(f*x^2+e)^2,x)
```

output
```
(8*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**3*d**3*f**2*x - 15*sqrt(c + d*x**2
)*sqrt(a + b*x**2)*a**2*b*c*d**2*f**2*x - 24*sqrt(c + d*x**2)*sqrt(a + b*x
**2)*a**2*b*d**3*e*f*x - 6*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*b*d**3*f
**2*x**3 + 3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**2*c**2*d*f**2*x + 54*s
qrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**2*c*d**2*e*f*x + 11*sqrt(c + d*x**2)
*sqrt(a + b*x**2)*a*b**2*c*d**2*f**2*x**3 + 21*sqrt(c + d*x**2)*sqrt(a + b
*x**2)*a*b**2*d**3*e**2*x + 18*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**2*d*
*3*e*f*x**3 + 5*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**2*d**3*f**2*x**5 -
4*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**3*c**3*f**2*x + 18*sqrt(c + d*x**2)
*sqrt(a + b*x**2)*b**3*c**2*d*e*f*x + 3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*
b**3*c**2*d*f**2*x**3 + 126*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**3*c*d**2*
e**2*x + 144*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**3*c*d**2*e*f*x**3 + 50*s
qrt(c + d*x**2)*sqrt(a + b*x**2)*b**3*c*d**2*f**2*x**5 + 63*sqrt(c + d*x**
2)*sqrt(a + b*x**2)*b**3*d**3*e**2*x**3 + 90*sqrt(c + d*x**2)*sqrt(a + b*x
**2)*b**3*d**3*e*f*x**5 + 35*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**3*d**3*f
**2*x**7 - 16*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2
 + b*c*x**2 + b*d*x**4),x)*a**4*d**4*f**2 + 32*int((sqrt(c + d*x**2)*sqrt(
a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a**3*b*c*d**3*
f**2 + 48*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b
*c*x**2 + b*d*x**4),x)*a**3*b*d**4*e*f - 9*int((sqrt(c + d*x**2)*sqrt(a...
```

3.87 $\int \frac{\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^2}{x^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 927

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}(e+fx^2)^2}{x^2}\,dx = \frac{(8a^3d^3f^2-a^2bd^2f(28de+19cf)+b^3c(245d^2e^2+42cdef-6c^2f}{105b^2d^2\sqrt{a+bx^2}}$$

$$+\frac{1}{105}\left(\frac{105bce^2}{a}+245de^2+399cef+\frac{3c^2f^2}{d}\right.$$

$$\left.-\frac{4a^2df^2}{b^2}+\frac{af(14de+9cf)}{b}\right)x\sqrt{a+bx^2}\sqrt{c+dx^2}$$

$$+\frac{1}{35}\left(\frac{35d^2e^2}{c}+224def+113cf^2+\frac{adf^2}{b}+\frac{35be(2de+3cf)}{a}\right)x^3\sqrt{a+bx^2}\sqrt{c+dx^2}$$

$$+\frac{(7be(d^2e^2+6cdef+3c^2f^2)+af(21d^2e^2+43cdef+7c^2f^2))\,x^5\sqrt{a+bx^2}\sqrt{c+dx^2}}{7ace}$$

$$+\frac{f(adf(3de+2cf)+b(3d^2e^2+6cdef+c^2f^2))\,x^7\sqrt{a+bx^2}\sqrt{c+dx^2}}{ace}$$

$$+\frac{df^2(3bde+2bcf+adf)x^9\sqrt{a+bx^2}\sqrt{c+dx^2}}{ace}$$

$$+\frac{bd^2f^3x^{11}\sqrt{a+bx^2}\sqrt{c+dx^2}}{ace}-\frac{(a+bx^2)^{3/2}(c+dx^2)^{5/2}(e+fx^2)^3}{acex}$$

$$-\frac{\sqrt{a}(8a^3d^3f^2-a^2bd^2f(28de+19cf)+b^3c(245d^2e^2+42cdef-6c^2f^2)+ab^2d(35d^2e^2+98cdef+9c^2f^2)}{105b^{5/2}d^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

$$+\frac{\sqrt{a}(105b^3cde^2+4a^3d^2f^2-a^2bdf(14de+9cf)+ab^2(175d^2e^2+126cdef-3c^2f^2))\sqrt{c+dx^2}\,\text{EllipticF}\left(a\right.}{105b^{5/2}d\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
1/105*(8*a^3*d^3*f^2-a^2*b*d^2*f*(19*c*f+28*d*e)+b^3*c*(-6*c^2*f^2+42*c*d*
e*f+245*d^2*e^2)+a*b^2*d*(9*c^2*f^2+98*c*d*e*f+35*d^2*e^2))*x*(d*x^2+c)^(1
/2)/b^2/d^2/(b*x^2+a)^(1/2)+1/105*(105*b*c*e^2/a+245*d*e^2+399*c*e*f+3*c^2
*f^2/d-4*a^2*d*f^2/b^2+a*f*(9*c*f+14*d*e)/b)*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(
1/2)+1/35*(35*d^2*e^2/c+224*d*e*f+113*c*f^2+a*d*f^2/b+35*b*e*(3*c*f+2*d*e)
/a)*x^3*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)+1/7*(7*b*e*(3*c^2*f^2+6*c*d*e*f+d^
2*e^2)+a*f*(7*c^2*f^2+43*c*d*e*f+21*d^2*e^2))*x^5*(b*x^2+a)^(1/2)*(d*x^2+c
)^(1/2)/a/c/e+f*(a*d*f*(2*c*f+3*d*e)+b*(c^2*f^2+6*c*d*e*f+3*d^2*e^2))*x^7*
(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a/c/e+d*f^2*(a*d*f+2*b*c*f+3*b*d*e)*x^9*(b
*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a/c/e+b*d^2*f^3*x^11*(b*x^2+a)^(1/2)*(d*x^2+
c)^(1/2)/a/c/e-(b*x^2+a)^(3/2)*(d*x^2+c)^(5/2)*(f*x^2+e)^3/a/c/e/x-1/105*a
^(1/2)*(8*a^3*d^3*f^2-a^2*b*d^2*f*(19*c*f+28*d*e)+b^3*c*(-6*c^2*f^2+42*c*d
*e*f+245*d^2*e^2)+a*b^2*d*(9*c^2*f^2+98*c*d*e*f+35*d^2*e^2))*(d*x^2+c)^(1/
2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/b^(5/2
)/d^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)+1/105*a^(1/2)*(105*b
^3*c*d*e^2+4*a^3*d^2*f^2-a^2*b*d*f*(9*c*f+14*d*e)+a*b^2*(-3*c^2*f^2+126*c*
d*e*f+175*d^2*e^2))*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/
2)),(1-a*d/b/c)^(1/2))/b^(5/2)/d/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))
^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.09 (sec) , antiderivative size = 451, normalized size of antiderivative = 0.49

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}\left(e+fx^2\right)^2}{x^2}\,dx = \frac{-\sqrt{\frac{b}{a}}d(a+bx^2)\left(c+dx^2\right)\left(4a^2d^2f^2x^2-abdfx^2(14de+9cf+3d$$

input
```
Integrate[(Sqrt[a + b*x^2]*(c + d*x^2)^(3/2)*(e + f*x^2)^2)/x^2,x]
```

output
```
(-(Sqrt[b/a]*d*(a + b*x^2)*(c + d*x^2)*(4*a^2*d^2*f^2*x^2 - a*b*d*f*x^2*(1
4*d*e + 9*c*f + 3*d*f*x^2) - b^2*(3*c^2*f^2*x^2 + 3*c*d*(-35*e^2 + 28*e*f*
x^2 + 8*f^2*x^4) + d^2*x^2*(35*e^2 + 42*e*f*x^2 + 15*f^2*x^4)))) - I*c*(8*
a^3*d^3*f^2 - a^2*b*d^2*f*(28*d*e + 19*c*f) + b^3*c*(245*d^2*e^2 + 42*c*d*
e*f - 6*c^2*f^2) + a*b^2*d*(35*d^2*e^2 + 98*c*d*e*f + 9*c^2*f^2))*x*Sqrt[1
 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/
(b*c)] + (2*I)*c*(-(b*c) + a*d)*(2*a^2*d^2*f^2 - a*b*d*f*(7*d*e + 3*c*f) +
 b^2*(-70*d^2*e^2 - 21*c*d*e*f + 3*c^2*f^2))*x*Sqrt[1 + (b*x^2)/a]*Sqrt[1
+ (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(105*a^2*(b/a
)^(5/2)*d^2*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (warning: unable to verify)

Time = 1.54 (sec) , antiderivative size = 894, normalized size of antiderivative = 0.96, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.457$, Rules used = {448, 403, 403, 403, 406, 320, 388, 313, 442, 403, 403, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^2}{x^2}dx$$

$$\Big\downarrow \text{448}$$

$$\frac{f\int \sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)dx}{e^2}+e\int \frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)}{x^2}dx$$

$$\Big\downarrow \text{403}$$

$$\frac{f\left(\frac{\int \sqrt{bx^2+a}\sqrt{dx^2+c}\left((7bde+3bcf-4adf)x^2+c(7be-af)\right)dx}{7b}+\frac{fx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{7b}\right)}{e^2}+ e\int \frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)}{x^2}dx$$

$$\Big\downarrow \text{403}$$

$$f\left(\frac{\frac{\int \frac{\sqrt{bx^2+a}\left(\left(3c(14de+cf)b^2-ad(14de+15cf)b+8a^2d^2f\right)x^2+c\left(4dfa^2-7bdea-8bcfa+35b^2ce\right)\right)}{\sqrt{dx^2+c}}dx}{5b}+\frac{x\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}(-4adf+3bcf+7bde)}{5b}}{7b}+\frac{fx(a \cdots}{\cdots}\right. \cdots$$

$$\overline{\qquad e^2 \qquad}$$

$$e\int \frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)}{x^2}dx$$

$$\Big\downarrow \; 403$$

$$f\left(\frac{\frac{\frac{\int \frac{\left(3c^2(7de-2cf)b^3+acd(49de+9cf)b^2-a^2d^2(14de+19cf)b+8a^3d^3f\right)x^2+ac\left(3c(21de-cf)b^2-ad(7de+9cf)b+4a^2d^2f\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3d}+\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(8a^2d^2f-a \cdots\right.}{\cdots}}{5b}}{7b}\right. \cdots$$

$$\overline{\qquad e^2 \qquad}$$

$$e\int \frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)}{x^2}dx$$

$$\Big\downarrow \; 406$$

$$f\left(\frac{\frac{\frac{ac\left(4a^2d^2f-abd(9cf+7de)+3b^2c(21de-cf)\right)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\left(8a^3d^3f-a^2bd^2(19cf+14de)+ab^2cd(9cf+49de)+3b^3c^2(7de-2cf)\right)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+ \cdots}} \cdots}{3d}}{5b}}{7b}\right. \cdots$$

$$\overline{\qquad e^2 \qquad}$$

$$e\int \frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)}{x^2}dx$$

$$\Big\downarrow \; 320$$

$$f\left(\frac{\frac{\frac{\left(8a^3d^3f-a^2bd^2(19cf+14de)+ab^2cd(9cf+49de)+3b^3c^2(7de-2cf)\right)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}\left(4a^2d^2f-abd(9cf+7de)+3b^2c(21de-cf)\right)\text{Ellipti}\cdots}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{3d}}{5b}}{7b}\right. \cdots$$

$$e\int \frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)}{x^2}dx$$

$$\Big\downarrow \; 388$$

$$f\left(\frac{\frac{\frac{\left(8a^3d^3f-a^2bd^2(19cf+14de)+ab^2cd(9cf+49de)+3b^3c^2(7de-2cf)\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}\left(4a^2d^2f-abd(9cf+7de)+3b^2c(21de\ldots\right.}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(\ldots}{a(\ldots}}}}{3d}}{5b}}{7b\ldots}\right.$$

$$e\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)}{x^2}dx$$

$\downarrow$ 313

$$e\int\frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)}{x^2}dx+$$

$$f\left(\frac{\frac{\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(8a^2d^2f-abd(15cf+14de)+3b^2c(cf+14de)\right)}{3d}+\frac{c^{3/2}\sqrt{a+bx^2}\left(4a^2d^2f-abd(9cf+7de)+3b^2c(21de-cf)\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{5b}}{\ldots}\right.$$

$\downarrow$ 442

$$e\left(\frac{\int\sqrt{bx^2+a}\sqrt{dx^2+c}\left(d(5be+af)x^2+2bce+3ade+acf\right)dx}{a}-\frac{e\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{ax}\right)+$$

$$f\left(\frac{\frac{\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(8a^2d^2f-abd(15cf+14de)+3b^2c(cf+14de)\right)}{3d}+\frac{c^{3/2}\sqrt{a+bx^2}\left(4a^2d^2f-abd(9cf+7de)+3b^2c(21de-cf)\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{5b}}{\ldots}\right.$$

$\downarrow$ 403

$$e\left(\frac{\frac{\int \frac{\sqrt{bx^2+a}\left(d\left(-2dfa^2+b(5de+6cf)a+15b^2ce\right)x^2+c\left(-dfa^2+5b(2de+cf)a+10b^2ce\right)\right)}{\sqrt{dx^2+c}}dx}{5b}+\frac{dx\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}(af+5be)}{5b}}{a}-\frac{e\left(a+bx^2\right)^{3/2}\left(\right.}{ax}\right.$$

$$f\left(\frac{\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(8a^2d^2f-abd(15cf+14de)+3b^2c(cf+14de)\right)}{3d}+\frac{c^{3/2}\sqrt{a+bx^2}\left(4a^2d^2f-abd(9cf+7de)+3b^2c(21de-cf)\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{5b}\right.$$

$\downarrow$ 403

$$e\left(\frac{\frac{\frac{\int \frac{ad\left(c\left(-dfa^2+b(25de+9cf)a+15b^2ce\right)-\left(-c(35de+3cf)b^2-ad(5de+7cf)b+2a^2d^2f\right)x^2\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3d}+\frac{1}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(-2a^2df+ab(6cf+5de)+15b^2ce\right)}{5b}}{a}\right.$$

$$f\left(\frac{\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(8a^2d^2f-abd(15cf+14de)+3b^2c(cf+14de)\right)}{3d}+\frac{c^{3/2}\sqrt{a+bx^2}\left(4a^2d^2f-abd(9cf+7de)+3b^2c(21de-cf)\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{5b}\right.$$

$\downarrow$ 27

$$e\left(\frac{\frac{\frac{1}{3}a\int \frac{c\left(-dfa^2+b(25de+9cf)a+15b^2ce\right)-\left(-c(35de+3cf)b^2-ad(5de+7cf)b+2a^2d^2f\right)x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{1}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(-2a^2df+ab(6cf+5de)+15b^2ce\right)}{5b}+}{a}\right.$$

$$f\left(\frac{\frac{\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(8a^2d^2f-abd(15cf+14de)+3b^2c(cf+14de)\right)}{3d}+\frac{c^{3/2}\sqrt{a+bx^2}\left(4a^2d^2f-abd(9cf+7de)+3b^2c(21de-cf)\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{5b}}{}\right.$$

$$\Big\downarrow \text{406}$$

$$e\left(\frac{\frac{\frac{1}{3}a\left(c\left(a^2(-d)f+ab(9cf+25de)+15b^2ce\right)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx-\left(2a^2d^2f-abd(7cf+5de)+b^2(-c)(3cf+35de)\right)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)+\frac{1}{3}x\sqrt{}}{5b}}{a}\right.$$

$$f\left(\frac{\frac{\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(8a^2d^2f-abd(15cf+14de)+3b^2c(cf+14de)\right)}{3d}+\frac{c^{3/2}\sqrt{a+bx^2}\left(4a^2d^2f-abd(9cf+7de)+3b^2c(21de-cf)\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{5b}}{}\right.$$

$$\Big\downarrow \text{320}$$

$$\frac{f\left(\frac{fx(bx^2+a)^{3/2}(dx^2+c)^{3/2}}{7b}+\frac{\frac{(7bde+3bcf-4adf)x\sqrt{dx^2+c}\left(bx^2+a\right)^{3/2}}{5b}+\frac{\left(3c(14de+cf)b^2-ad(14de+15cf)b+8a^2d^2f\right)\sqrt{bx^2+a}\sqrt{dx^2+cx}}{3d}+\frac{\left(3c(21de-\right.}{}}{}\right.}{e\left(\frac{d(5be+af)x\sqrt{dx^2+c}(bx^2+a)^{3/2}}{5b}+\frac{\frac{1}{3}\left(-2dfa^2+b(5de+6cf)a+15b^2ce\right)\sqrt{bx^2+a}\sqrt{dx^2+cx}+\frac{1}{3}a\left(\frac{c^{3/2}\left(-dfa^2+b(25de+9cf)a+15b^2ce\right)\sqrt{bx^2+a}\,\mathrm{E}}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{d}}\right.}{5b}\right.}$$

$\downarrow$ 388

$$\frac{f\left(\frac{fx(bx^2+a)^{3/2}(dx^2+c)^{3/2}}{7b}+\frac{\frac{(7bde+3bcf-4adf)x\sqrt{dx^2+c}\left(bx^2+a\right)^{3/2}}{5b}+\frac{\left(3c(14de+cf)b^2-ad(14de+15cf)b+8a^2d^2f\right)\sqrt{bx^2+a}\sqrt{dx^2+cx}}{3d}+\frac{\left(3c(21de-\right.}{}}{}\right.}{e\left(\frac{d(5be+af)x\sqrt{dx^2+c}(bx^2+a)^{3/2}}{5b}+\frac{\frac{1}{3}\left(-2dfa^2+b(5de+6cf)a+15b^2ce\right)\sqrt{bx^2+a}\sqrt{dx^2+cx}+\frac{1}{3}a\left(\frac{c^{3/2}\left(-dfa^2+b(25de+9cf)a+15b^2ce\right)\sqrt{bx^2+a}\,\mathrm{E}}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{d}}\right.}{a}\right.}$$

$\downarrow$ 313

$$\left(3c(21de-\right.$$

$$f\left(\frac{fx(bx^2+a)^{3/2}(dx^2+c)^{3/2}}{7b}+\frac{\frac{(7bde+3bcf-4adf)x\sqrt{dx^2+c}\left(bx^2+a\right)^{3/2}}{5b}+\frac{\left(3c(14de+cf)b^2-ad(14de+15cf)b+8a^2d^2f\right)\sqrt{bx^2+a}\sqrt{dx^2+c}x}{3d}+\right.}{}\right.$$

$$e\left(\frac{d(5be+af)x\sqrt{dx^2+c}(bx^2+a)^{3/2}}{5b}+\frac{\frac{1}{3}\left(-2dfa^2+b(5de+6cf)a+15b^2ce\right)\sqrt{bx^2+a}\sqrt{dx^2+c}x+\frac{1}{3}a\left(\frac{c^{3/2}\left(-dfa^2+b(25de+9cf)a+15b^2ce\right)\sqrt{bx^2+a}\,\mathrm{E}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{d}}\right.}{a}\right.$$

input
```
Int[(Sqrt[a + b*x^2]*(c + d*x^2)^(3/2)*(e + f*x^2)^2)/x^2,x]
```

output
```
(f*((f*x*(a + b*x^2)^(3/2)*(c + d*x^2)^(3/2))/(7*b) + (((7*b*d*e + 3*b*c*f
 - 4*a*d*f)*x*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(5*b) + (((8*a^2*d^2*f +
3*b^2*c*(14*d*e + c*f) - a*b*d*(14*d*e + 15*c*f))*x*Sqrt[a + b*x^2]*Sqrt[c
 + d*x^2])/(3*d) + ((8*a^3*d^3*f + 3*b^3*c^2*(7*d*e - 2*c*f) + a*b^2*c*d*(
49*d*e + 9*c*f) - a^2*b*d^2*(14*d*e + 19*c*f))*((x*Sqrt[a + b*x^2])/(b*Sqr
t[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt
[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*S
qrt[c + d*x^2])) + (c^(3/2)*(4*a^2*d^2*f + 3*b^2*c*(21*d*e - c*f) - a*b*d*
(7*d*e + 9*c*f))*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1
- (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*
x^2]))/(3*d))/(5*b))/(7*b)))/e^2 + e*(-((e*(a + b*x^2)^(3/2)*(c + d*x^2)^(
3/2))/(a*x)) + ((d*(5*b*e + a*f)*x*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(5*b
) + (((15*b^2*c*e - 2*a^2*d*f + a*b*(5*d*e + 6*c*f))*x*Sqrt[a + b*x^2]*Sqr
t[c + d*x^2])/3 + (a*(-((2*a^2*d^2*f - b^2*c*(35*d*e + 3*c*f) - a*b*d*(5*d
*e + 7*c*f))*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a +
b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]
*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))) + (c^(3/2)*(15*b
^2*c*e - a^2*d*f + a*b*(25*d*e + 9*c*f))*Sqrt[a + b*x^2]*EllipticF[ArcTan[
(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a
*(c + d*x^2))]*Sqrt[c + d*x^2])))/3)/(5*b))/a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 442
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^q/(a*g*(m + 1))), x] - Simp[1/(a*g^2*(m + 1))   Int[(g*x)
^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[c*(b*e - a*f)*(m + 1) + e*2
*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*2*(p + q + 1))*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && GtQ[q, 0] && LtQ[m, -1]
&&  !(EqQ[q, 1] && SimplerQ[e + f*x^2, c + d*x^2])
```

rule 448
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[e   Int[(g*x)^m*(a + b*x
^2)^p*(c + d*x^2)^q*(e + f*x^2)^(r - 1), x], x] + Simp[f/e^2   Int[(g*x)^(m
 + 2)*(a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^(r - 1), x], x] /; FreeQ[{a,
b, c, d, e, f, g, m, p, q}, x] && IGtQ[r, 0]
```

Maple [A] (verified)

Time = 6.83 (sec) , antiderivative size = 854, normalized size of antiderivative = 0.92

method	result
elliptic	$\frac{\sqrt{(b x^2+a)(x^2 d+c)}\left(-\frac{c e^2 \sqrt{bd x^4+ad x^2+x^2 bc+ac}}{x}+\frac{f^2 d x^5 \sqrt{bd x^4+ad x^2+x^2 bc+ac}}{7}+\frac{\left(a d^2 f^2+2bcd f^2+2b d^2 e f-\frac{f^2 d(6ad+6bc)}{7}\right) x^3 \sqrt{bd}}{5bd}\right.}{}$
risch	Expression too large to display
default	Expression too large to display

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)*(f*x^2+e)^2/x^2,x,method=_RETURNVERBOS
E)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-c*e^2*(b*d*x
^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/x+1/7*f^2*d*x^5*(b*d*x^4+a*d*x^2+b*c*x^2+a*c
)^(1/2)+1/5*(a*d^2*f^2+2*b*c*d*f^2+2*b*d^2*e*f-1/7*f^2*d*(6*a*d+6*b*c))/b/
d*x^3*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+1/3*(9/7*a*c*d*f^2+2*a*d^2*e*f+b
*c^2*f^2+4*b*c*d*e*f+b*d^2*e^2-1/5*(a*d^2*f^2+2*b*c*d*f^2+2*b*d^2*e*f-1/7*
f^2*d*(6*a*d+6*b*c))/b/d*(4*a*d+4*b*c))/b/d*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c
)^(1/2)+(2*a*c^2*e*f+2*a*c*e^2*d+b*c^2*e^2-1/3*(9/7*a*c*d*f^2+2*a*d^2*e*f+
b*c^2*f^2+4*b*c*d*e*f+b*d^2*e^2-1/5*(a*d^2*f^2+2*b*c*d*f^2+2*b*d^2*e*f-1/7
*f^2*d*(6*a*d+6*b*c))/b/d*(4*a*d+4*b*c))/b/d*a*c)/(-b/a)^(1/2)*(1+b*x^2/a)
^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(
-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-(a*c^2*f^2+4*a*c*e*f*d+a*d^2*e^2+2*b
*c^2*e*f+3*b*c*d*e^2-3/5*(a*d^2*f^2+2*b*c*d*f^2+2*b*d^2*e*f-1/7*f^2*d*(6*a
*d+6*b*c))/b/d*a*c-1/3*(9/7*a*c*d*f^2+2*a*d^2*e*f+b*c^2*f^2+4*b*c*d*e*f+b*
d^2*e^2-1/5*(a*d^2*f^2+2*b*c*d*f^2+2*b*d^2*e*f-1/7*f^2*d*(6*a*d+6*b*c))/b/
d*(4*a*d+4*b*c))/b/d*(2*a*d+2*b*c))*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*
x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(-b/a)^(1/
2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(
1/2))))
```

Fricas [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}\left(e+fx^2\right)^2}{x^2}\,dx=\int \frac{\sqrt{bx^2+a}(dx^2+c)^{\frac{3}{2}}(fx^2+e)^2}{x^2}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)*(f*x^2+e)^2/x^2,x, algorithm="fr
icas")
```

output
```
integral((d*f^2*x^6 + (2*d*e*f + c*f^2)*x^4 + c*e^2 + (d*e^2 + 2*c*e*f)*x^
2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/x^2, x)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}(e+fx^2)^2}{x^2}\,dx = \int \frac{\sqrt{a+bx^2}(c+dx^2)^{\frac{3}{2}}(e+fx^2)^2}{x^2}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(3/2)*(f*x**2+e)**2/x**2,x)
```

output
```
Integral(sqrt(a + b*x**2)*(c + d*x**2)**(3/2)*(e + f*x**2)**2/x**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}(e+fx^2)^2}{x^2}\,dx = \int \frac{\sqrt{bx^2+a}(dx^2+c)^{\frac{3}{2}}(fx^2+e)^2}{x^2}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)*(f*x^2+e)^2/x^2,x, algorithm="ma
xima")
```

output
```
integrate(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)*(f*x^2 + e)^2/x^2, x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}(e+fx^2)^2}{x^2}\,dx = \int \frac{\sqrt{bx^2+a}(dx^2+c)^{\frac{3}{2}}(fx^2+e)^2}{x^2}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)*(f*x^2+e)^2/x^2,x, algorithm="gi
ac")
```

output
```
integrate(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)*(f*x^2 + e)^2/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}\left(e+fx^2\right)^2}{x^2}\,dx = \int \frac{\sqrt{b\,x^2+a}\,\left(d\,x^2+c\right)^{3/2}\,\left(f\,x^2+e\right)^2}{x^2}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(3/2)*(e + f*x^2)^2)/x^2,x)
```

output
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(3/2)*(e + f*x^2)^2)/x^2, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}\left(e+fx^2\right)^2}{x^2}\,dx = \text{Too large to display}$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)*(f*x^2+e)^2/x^2,x)
```

output
```
(8*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**3*d**3*f**2 - 19*sqrt(c + d*x**2)*
sqrt(a + b*x**2)*a**2*b*c*d**2*f**2 - 28*sqrt(c + d*x**2)*sqrt(a + b*x**2)
*a**2*b*d**3*e*f - 4*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*b*d**3*f**2*x*
*2 + 9*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**2*c**2*d*f**2 + 98*sqrt(c +
d*x**2)*sqrt(a + b*x**2)*a*b**2*c*d**2*e*f + 9*sqrt(c + d*x**2)*sqrt(a + b
*x**2)*a*b**2*c*d**2*f**2*x**2 + 35*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b*
*2*d**3*e**2 + 14*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**2*d**3*e*f*x**2 +
 3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**2*d**3*f**2*x**4 - 6*sqrt(c + d*
x**2)*sqrt(a + b*x**2)*b**3*c**3*f**2 + 42*sqrt(c + d*x**2)*sqrt(a + b*x**
2)*b**3*c**2*d*e*f + 3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**3*c**2*d*f**2*
x**2 + 140*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**3*c*d**2*e**2 + 84*sqrt(c
+ d*x**2)*sqrt(a + b*x**2)*b**3*c*d**2*e*f*x**2 + 24*sqrt(c + d*x**2)*sqrt
(a + b*x**2)*b**3*c*d**2*f**2*x**4 + 35*sqrt(c + d*x**2)*sqrt(a + b*x**2)*
b**3*d**3*e**2*x**2 + 42*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**3*d**3*e*f*x
**4 + 15*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**3*d**3*f**2*x**6 + 8*int((sq
rt(c + d*x**2)*sqrt(a + b*x**2))/(a*c*x**2 + a*d*x**4 + b*c*x**4 + b*d*x**
6),x)*a**4*c*d**3*f**2*x - 19*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c
*x**2 + a*d*x**4 + b*c*x**4 + b*d*x**6),x)*a**3*b*c**2*d**2*f**2*x - 28*in
t((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c*x**2 + a*d*x**4 + b*c*x**4 + b*
d*x**6),x)*a**3*b*c*d**3*e*f*x + 9*int((sqrt(c + d*x**2)*sqrt(a + b*x**...
```

3.88 $\int \frac{\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^2}{x^4} \, dx$

Optimal result

Integrand size = 35, antiderivative size = 1035

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}\left(e+fx^2\right)^2}{x^4}\,dx = \frac{\left(\frac{5b^2ce^2}{a}-\frac{2a^2df^2}{b}+af(10de+7cf)+b\left(35de^2+70cef+\frac{3c^2f^2}{d}\right)\right)}{15\sqrt{a+bx^2}}$$

$$+\frac{1}{15}\left(\frac{25d^2e^2}{c}+100def+66cf^2+\frac{adf^2}{b}+\frac{10be(2de+3cf)}{a}\right)x\sqrt{a+bx^2}\sqrt{c+dx^2}$$

$$+\frac{(5bce(5d^2e^2+18cdef+12c^2f^2)+a(10d^3e^3+90cd^2e^2f+153c^2def^2+50c^3f^3))\,x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{15ac^2e}$$

$$+\frac{(af(6d^3e^3+24cd^2e^2f+22c^2def^2+3c^3f^3)+2be(d^3e^3+9cd^2e^2f+15c^2def^2+5c^3f^3))\,x^5\sqrt{a+bx^2}\sqrt{c+}}{3ac^2e^2}$$

$$+\frac{f(2adf(3d^2e^2+7cdef+3c^2f^2)+b(6d^3e^3+24cd^2e^2f+22c^2def^2+3c^3f^3))\,x^7\sqrt{a+bx^2}\sqrt{c+dx^2}}{3ac^2e^2}$$

$$+\frac{df^2(adf(2de+3cf)+2b(3d^2e^2+7cdef+3c^2f^2))\,x^9\sqrt{a+bx^2}\sqrt{c+dx^2}}{3ac^2e^2}$$

$$+\frac{bd^2f^3(2de+3cf)x^{11}\sqrt{a+bx^2}\sqrt{c+dx^2}}{3ac^2e^2}-\frac{\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{5/2}\left(e+fx^2\right)^3}{3acex^3}$$

$$-\frac{(2de+3cf)\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{5/2}\left(e+fx^2\right)^3}{3ac^2e^2x}$$

$$-\frac{(5b^3cde^2-2a^3d^2f^2+a^2bdf(10de+7cf)+ab^2(35d^2e^2+70cdef+3c^2f^2))\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1\right.}{15\sqrt{a}b^{3/2}d\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

$$-\frac{\sqrt{a}(a^2cdf^2-5b^2ce(5de+6cf)-ab(15d^2e^2+50cdef+9c^2f^2))\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\right.}{15b^{3/2}c\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
1/15*(5*b^2*c*e^2/a-2*a^2*d*f^2/b+a*f*(7*c*f+10*d*e)+b*(35*d*e^2+70*c*e*f+
3*c^2*f^2/d))*x*(d*x^2+c)^(1/2)/(b*x^2+a)^(1/2)+1/15*(25*d^2*e^2/c+100*d*e
*f+66*c*f^2+a*d*f^2/b+10*b*e*(3*c*f+2*d*e)/a)*x*(b*x^2+a)^(1/2)*(d*x^2+c)^
(1/2)+1/15*(5*b*c*e*(12*c^2*f^2+18*c*d*e*f+5*d^2*e^2)+a*(50*c^3*f^3+153*c^
2*d*e*f^2+90*c*d^2*e^2*f+10*d^3*e^3))*x^3*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/
a/c^2/e+1/3*(a*f*(3*c^3*f^3+22*c^2*d*e*f^2+24*c*d^2*e^2*f+6*d^3*e^3)+2*b*e
*(5*c^3*f^3+15*c^2*d*e*f^2+9*c*d^2*e^2*f+d^3*e^3))*x^5*(b*x^2+a)^(1/2)*(d*
x^2+c)^(1/2)/a/c^2/e^2+1/3*f*(2*a*d*f*(3*c^2*f^2+7*c*d*e*f+3*d^2*e^2)+b*(3
*c^3*f^3+22*c^2*d*e*f^2+24*c*d^2*e^2*f+6*d^3*e^3))*x^7*(b*x^2+a)^(1/2)*(d*
x^2+c)^(1/2)/a/c^2/e^2+1/3*d*f^2*(a*d*f*(3*c*f+2*d*e)+2*b*(3*c^2*f^2+7*c*d
*e*f+3*d^2*e^2))*x^9*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a/c^2/e^2+1/3*b*d^2*f
^3*(3*c*f+2*d*e)*x^11*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a/c^2/e^2-1/3*(b*x^2
+a)^(3/2)*(d*x^2+c)^(5/2)*(f*x^2+e)^3/a/c/e/x^3-1/3*(3*c*f+2*d*e)*(b*x^2+a
)^(3/2)*(d*x^2+c)^(5/2)*(f*x^2+e)^3/a/c^2/e^2/x-1/15*(5*b^3*c*d*e^2-2*a^3*
d^2*f^2+a^2*b*d*f*(7*c*f+10*d*e)+a*b^2*(3*c^2*f^2+70*c*d*e*f+35*d^2*e^2))*
(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^
(1/2))/a^(1/2)/b^(3/2)/d/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)-1
/15*a^(1/2)*(a^2*c*d*f^2-5*b^2*c*e*(6*c*f+5*d*e)-a*b*(9*c^2*f^2+50*c*d*e*f
+15*d^2*e^2))*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1
-a*d/b/c)^(1/2))/b^(3/2)/c/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1...
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.00 (sec) , antiderivative size = 392, normalized size of antiderivative = 0.38

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}\left(e+fx^2\right)^2}{x^4}\,dx = \frac{\sqrt{\frac{b}{a}}d(a+bx^2)\left(c+dx^2\right)\left(-5b^2ce^2x^2+a^2df^2x^4+ab(dx^2(-20e^2+$$

input
```
Integrate[(Sqrt[a + b*x^2]*(c + d*x^2)^(3/2)*(e + f*x^2)^2)/x^4,x]
```

output
```
(Sqrt[b/a]*d*(a + b*x^2)*(c + d*x^2)*(-5*b^2*c*e^2*x^2 + a^2*d*f^2*x^4 + a
*b*(d*x^2*(-20*e^2 + 10*e*f*x^2 + 3*f^2*x^4) + c*(-5*e^2 - 30*e*f*x^2 + 6*
f^2*x^4))) + I*c*(-5*b^3*c*d*e^2 + 2*a^3*d^2*f^2 - a^2*b*d*f*(10*d*e + 7*c
*f) - a*b^2*(35*d^2*e^2 + 70*c*d*e*f + 3*c^2*f^2))*x^3*Sqrt[1 + (b*x^2)/a]
*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*(-
(b*c) + a*d)*(5*b^2*c*d*e^2 + a^2*c*d*f^2 + a*b*(15*d^2*e^2 + 40*c*d*e*f +
 3*c^2*f^2))*x^3*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSi
nh[Sqrt[b/a]*x], (a*d)/(b*c)])/(15*a^2*(b/a)^(3/2)*d*x^3*Sqrt[a + b*x^2]*S
qrt[c + d*x^2])
```

Rubi [A] (warning: unable to verify)

Time = 1.49 (sec) , antiderivative size = 794, normalized size of antiderivative = 0.77, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.514$, Rules used = {448, 442, 27, 403, 403, 27, 406, 320, 388, 313, 442, 27, 403, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^2}{x^4}dx$$

$$\Big\downarrow \text{448}$$

$$\frac{f\int \frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)}{x^2}dx}{e^2}+e\int \frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)}{x^4}dx$$

$$\Big\downarrow \text{442}$$

$$\frac{f\left(\frac{\int \sqrt{bx^2+a}\sqrt{dx^2+c}\left(d(5be+af)x^2+2bce+3ade+acf\right)dx}{a}-\frac{e\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{ax}\right)}{e^2}+e\left(\frac{\int \frac{3\sqrt{bx^2+a}\sqrt{dx^2+c}\left(d(be+af)x^2+a(de+cf)\right)}{x^2}dx}{3a}-\frac{e\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{3ax^3}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{f\left(\frac{\int \sqrt{bx^2+a}\sqrt{dx^2+c}\left(d(5be+af)x^2+2bce+3ade+acf\right)dx}{a}-\frac{e\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{ax}\right)}{e^2}+e\left(\frac{\int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(d(be+af)x^2+a(de+cf)\right)}{x^2}dx}{a}-\frac{e\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{3ax^3}\right)$$

$\downarrow$ 403

$$\frac{f\left(\frac{\frac{\int \frac{\sqrt{bx^2+a}\left(d\left(-2dfa^2+b(5de+6cf)a+15b^2ce\right)x^2+c\left(-dfa^2+5b(2de+cf)a+10b^2ce\right)\right)}{\sqrt{dx^2+c}}dx}{5b}+\frac{dx\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}(af+5be)}{5b}}{a}-\frac{e\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/}}{ax}\right.}{e^2}$$
$$e\left(\frac{\int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(d(be+af)x^2+a(de+cf)\right)}{x^2}dx}{a}-\frac{e\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{3ax^3}\right)$$

$\downarrow$ 403

$$\frac{f\left(\frac{\frac{\frac{\int \frac{ad\left(c\left(-dfa^2+b(25de+9cf)a+15b^2ce\right)-\left(-c(35de+3cf)b^2-ad(5de+7cf)b+2a^2d^2f\right)x^2\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3d}+\frac{1}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(-2a^2df+ab(6cf+5de)+15b^2ce\right)}{5b}+\frac{dx\left(\right.}{}}{a}\right.}{e^2}$$
$$e\left(\frac{\int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(d(be+af)x^2+a(de+cf)\right)}{x^2}dx}{a}-\frac{e\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{3ax^3}\right)$$

$\downarrow$ 27

$$\frac{f\left(\frac{\frac{\frac{1}{3}a\int \frac{c\left(-dfa^2+b(25de+9cf)a+15b^2ce\right)-\left(-c(35de+3cf)b^2-ad(5de+7cf)b+2a^2d^2f\right)x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{1}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(-2a^2df+ab(6cf+5de)+15b^2ce\right)}{5b}+\frac{dx\left(a+\right.}{}}{a}\right.}{e^2}$$
$$e\left(\frac{\int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(d(be+af)x^2+a(de+cf)\right)}{x^2}dx}{a}-\frac{e\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{3ax^3}\right)$$

$\downarrow$ 406

$$f\left(\frac{\frac{\frac{1}{3}a\left(c\left(a^2(-d)f+ab(9cf+25de)+15b^2ce\right)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx-\left(2a^2d^2f-abd(7cf+5de)+b^2(-c)(3cf+35de)\right)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)+\frac{1}{3}x\sqrt{a+bx^2}\sqrt{c+d}}{5b}}{a}\right.$$

$$\frac{}{e^2}$$

$$e\left(\frac{\int\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(d(be+af)x^2+a(de+cf)\right)}{x^2}dx}{a}-\frac{e\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{3ax^3}\right)$$

$\big\downarrow$ 320

$$f\left(\frac{\frac{\frac{1}{3}a\left(\frac{c^{3/2}\sqrt{a+bx^2}\left(a^2(-d)f+ab(9cf+25de)+15b^2ce\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\left(2a^2d^2f-abd(7cf+5de)+b^2(-c)(3cf+35de)\right)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}\right.}{5b}}{a}\right.$$

$$\frac{}{e^2}$$

$$e\left(\frac{\int\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(d(be+af)x^2+a(de+cf)\right)}{x^2}dx}{a}-\frac{e\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{3ax^3}\right)$$

$\big\downarrow$ 388

$$f\left(\frac{\frac{\frac{1}{3}a\left(\frac{c^{3/2}\sqrt{a+bx^2}\left(a^2(-d)f+ab(9cf+25de)+15b^2ce\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\left(2a^2d^2f-abd(7cf+5de)+b^2(-c)(3cf+35de)\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int -}{\left(}\right.\right.\right.}{5b}}{a}\right.$$

$$\frac{}{e^2}$$

$$e\left(\frac{\int\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(d(be+af)x^2+a(de+cf)\right)}{x^2}dx}{a}-\frac{e\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{3ax^3}\right)$$

$\big\downarrow$ 313

$$e\left(\frac{\int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(d(be+af)x^2+a(de+cf)\right)}{x^2}dx}{a}-\frac{e\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{3ax^3}\right)+$$

$$f\left(\frac{\frac{\frac{1}{3}a\left(\frac{c^{3/2}\sqrt{a+bx^2}\left(a^2(-d)f+ab(9cf+25de)+15b^2ce\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\left(2a^2d^2f-abd(7cf+5de)+b^2(-c)(3cf+35de)\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{\cdots}}{\cdots}\right.\right.}{5b}}{a}\right.$$

$\Big\downarrow$ 442

$$e\left(\frac{\frac{\int \frac{a\sqrt{bx^2+a}\left(d(4bde+3bcf+adf)x^2+ad(de+2cf)+bc(3de+2cf)\right)}{\sqrt{dx^2+c}}dx}{a}-\frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}(cf+de)}{x}}{a}-\frac{e\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{3ax^3}\right)+$$

$$f\left(\frac{\frac{\frac{1}{3}a\left(\frac{c^{3/2}\sqrt{a+bx^2}\left(a^2(-d)f+ab(9cf+25de)+15b^2ce\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\left(2a^2d^2f-abd(7cf+5de)+b^2(-c)(3cf+35de)\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{\cdots}}{\cdots}\right.\right.}{5b}}{a}\right.$$

$\Big\downarrow$ 27

$$e\left(\frac{\int \frac{\sqrt{bx^2+a}\left(d(4bde+3bcf+adf)x^2+ad(de+2cf)+bc(3de+2cf)\right)}{\sqrt{dx^2+c}}dx-\frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}(cf+de)}{x}}{a}-\frac{e\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{3ax^3}\right.$$

$$f\left(\frac{\frac{\frac{1}{3}a\left(\frac{c^{3/2}\sqrt{a+bx^2}\left(a^2(-d)f+ab(9cf+25de)+15b^2ce\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\left(2a^2d^2f-abd(7cf+5de)+b^2(-c)(3cf+35de)\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{\cdots}}{\cdots}\right.\right.}{5b}}{a}\right.$$

$\downarrow$ 403

$$e\left(\frac{\frac{\int \frac{d\left(d\left(dfa^2+7b(de+cf)a+b^2ce\right)x^2+a(bc(5de+3cf)+ad(3de+5cf))\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3d}-\frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}(cf+de)}{x}+\frac{1}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}(adf+3}{a}\right.$$

$$f\left(\frac{\frac{\frac{1}{3}a\left(\frac{c^{3/2}\sqrt{a+bx^2}\left(a^2(-d)f+ab(9cf+25de)+15b^2ce\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\left(2a^2d^2f-abd(7cf+5de)+b^2(-c)(3cf+35de)\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{}}{}\right.\right.}{5b}}{a}\right.$$

$\downarrow$ 27

$$e\left(\frac{\frac{1}{3}\int \frac{d\left(dfa^2+7b(de+cf)a+b^2ce\right)x^2+a(bc(5de+3cf)+ad(3de+5cf))}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx-\frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}(cf+de)}{x}+\frac{1}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}(ad}{a}\right.$$

$$f\left(\frac{\frac{\frac{1}{3}a\left(\frac{c^{3/2}\sqrt{a+bx^2}\left(a^2(-d)f+ab(9cf+25de)+15b^2ce\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\left(2a^2d^2f-abd(7cf+5de)+b^2(-c)(3cf+35de)\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{}}{}\right.\right.}{5b}}{a}\right.$$

$\downarrow$ 406

$$e\left(\frac{\frac{1}{3}\left(d\left(a^2df+7ab(cf+de)+b^2ce\right)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+a(ad(5cf+3de)+bc(3cf+5de))\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{a}\right.$$

$$f\left(\frac{\frac{\frac{1}{3}a\left(\frac{c^{3/2}\sqrt{a+bx^2}\left(a^2(-d)f+ab(9cf+25de)+15b^2ce\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\left(2a^2d^2f-abd(7cf+5de)+b^2(-c)(3cf+35de)\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{}}{}\right.\right.}{5b}}{a}\right.$$

$\downarrow$ 320

$$e\left(\frac{\frac{1}{3}\left(\left(d\left(a^2df+7ab(cf+de)+b^2ce\right)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{\sqrt{c}\sqrt{a+bx^2}(ad(5cf+3de)+bc(3cf+5de))\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\right.}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right.\right.}{a}\right.$$

$$f\left(\frac{\frac{\frac{1}{3}a\left(\frac{c^{3/2}\sqrt{a+bx^2}\left(a^2(-d)f+ab(9cf+25de)+15b^2ce\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\left(2a^2d^2f-abd(7cf+5de)+b^2(-c)(3cf+35de)\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{}}{}\right.\right.}{5b}}{a}\right.$$

$\downarrow$ 388

$$e\left(\frac{\frac{1}{3}\left(d\left(a^2df+7ab(cf+de)+b^2ce\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{\sqrt{c}\sqrt{a+bx^2}(ad(5cf+3de)+bc(3cf+5de))\,\text{EllipticF}\left(\text{ar}\right.}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right.}{a}\right.$$

$$f\left(\frac{\frac{\frac{1}{3}a\left(\frac{c^{3/2}\sqrt{a+bx^2}\left(a^2(-d)f+ab(9cf+25de)+15b^2ce\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\left(2a^2d^2f-abd(7cf+5de)+b^2(-c)(3cf+35de)\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{}}{}\right.\right.}{5b}}{a}\right.$$

$$\Big\downarrow \textbf{313}$$

$$f\left(\frac{\frac{\frac{1}{3}a\left(\frac{c^{3/2}\sqrt{a+bx^2}\left(a^2(-d)f+ab(9cf+25de)+15b^2ce\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\left(2a^2d^2f-abd(7cf+5de)+b^2(-c)(3cf+35de)\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{}}{}\right.\right.}{5b}}{a}\right.$$

$$e\left(\frac{\frac{1}{3}\left(d\left(a^2df+7ab(cf+de)+b^2ce\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{\sqrt{c}\sqrt{a+bx^2}(ad(5cf+3de)+bc(3cf-}{\sqrt{d}\sqrt{c+dx}}\right.}{a}\right.$$

input
```
Int[(Sqrt[a + b*x^2]*(c + d*x^2)^(3/2)*(e + f*x^2)^2)/x^4,x]
```

output
```
e*(-1/3*(e*(a + b*x^2)^(3/2)*(c + d*x^2)^(3/2))/(a*x^3) + (((4*b*d*e + 3*b
*c*f + a*d*f)*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/3 - ((d*e + c*f)*(a + b*x
^2)^(3/2)*Sqrt[c + d*x^2])/x + (d*(b^2*c*e + a^2*d*f + 7*a*b*(d*e + c*f))*
((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*Ellipt
icE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a +
 b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (Sqrt[c]*(b*c*(5*d*e + 3*c*f
) + a*d*(3*d*e + 5*c*f))*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt
[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqr
t[c + d*x^2]))/3)/a) + (f*(-((e*(a + b*x^2)^(3/2)*(c + d*x^2)^(3/2))/(a*x)
) + ((d*(5*b*e + a*f)*x*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(5*b) + (((15*b
^2*c*e - 2*a^2*d*f + a*b*(5*d*e + 6*c*f))*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2
])/3 + (a*(-((2*a^2*d^2*f - b^2*c*(35*d*e + 3*c*f) - a*b*d*(5*d*e + 7*c*f)
)*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*Elli
pticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a
 + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))) + (c^(3/2)*(15*b^2*c*e - a^
2*d*f + a*b*(25*d*e + 9*c*f))*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)
/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2
))]*Sqrt[c + d*x^2])))/3)/(5*b))/a))/e^2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 442
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^q/(a*g*(m + 1))), x] - Simp[1/(a*g^2*(m + 1))   Int[(g*x)
^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[c*(b*e - a*f)*(m + 1) + e*2
*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*2*(p + q + 1))*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && GtQ[q, 0] && LtQ[m, -1]
&&  !(EqQ[q, 1] && SimplerQ[e + f*x^2, c + d*x^2])
```

rule 448
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[e   Int[(g*x)^m*(a + b*x
^2)^p*(c + d*x^2)^q*(e + f*x^2)^(r - 1), x], x] + Simp[f/e^2   Int[(g*x)^(m
 + 2)*(a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^(r - 1), x], x] /; FreeQ[{a,
b, c, d, e, f, g, m, p, q}, x] && IGtQ[r, 0]
```

Maple [A] (verified)

Time = 6.46 (sec) , antiderivative size = 644, normalized size of antiderivative = 0.62

method	result
elliptic	$\frac{\sqrt{(b x^2+a)(x^2 d+c)}\left(-\frac{c e^2 \sqrt{b d x^4+a d x^2+x^2 b c+a c}}{3 x^3}-\frac{e(6 a c f+4 a d e+b c e)\sqrt{b d x^4+a d x^2+x^2 b c+a c}}{3 a x}+\frac{f^2 d x^3 \sqrt{b d x^4+a d x^2+x^2 b c+a c}}{5}+\frac{\left(a d^2\right.}{}\right.}{}$
risch	$\frac{\sqrt{b x^2+a}\,\sqrt{x^2 d+c}\,(3 a b d f^2 x^6+a^2 d f^2 x^4+6 a b c f^2 x^4+10 a b d e f x^4-30 a b c e f x^2-20 a b d e^2 x^2-5 b^2 c e^2 x^2-5 a c e^2 b)}{15 b a x^3}-\frac{\left(-\frac{\left(2 a^3 d^2 f^2\right.}{}\right.}{}$
default	Expression too large to display

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)*(f*x^2+e)^2/x^4,x,method=_RETURNVERBOS
E)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-1/3*c*e^2*(b
*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/x^3-1/3*e*(6*a*c*f+4*a*d*e+b*c*e)/a*(b*d
*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/x+1/5*f^2*d*x^3*(b*d*x^4+a*d*x^2+b*c*x^2+a
*c)^(1/2)+1/3*(a*d^2*f^2+2*b*c*d*f^2+2*b*d^2*e*f-1/5*f^2*d*(4*a*d+4*b*c))/
b/d*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+(a*c^2*f^2+4*a*c*e*f*d+a*d^2*e^2
+2*b*c^2*e*f+5/3*b*c*d*e^2-1/3*(a*d^2*f^2+2*b*c*d*f^2+2*b*d^2*e*f-1/5*f^2*
d*(4*a*d+4*b*c))/b/d*a*c)/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)
/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c
)/c/b)^(1/2))-(7/5*a*c*d*f^2+2*a*d^2*e*f+b*c^2*f^2+4*b*c*d*e*f+b*d^2*e^2+1
/3*b*d*e*(6*a*c*f+4*a*d*e+b*c*e)/a-1/3*(a*d^2*f^2+2*b*c*d*f^2+2*b*d^2*e*f-
1/5*f^2*d*(4*a*d+4*b*c))/b/d*(2*a*d+2*b*c))*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/
2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(-
b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c
)/c/b)^(1/2))))
```

Fricas [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}\left(e+fx^2\right)^2}{x^4}\,dx=\int \frac{\sqrt{bx^2+a}(dx^2+c)^{\frac{3}{2}}(fx^2+e)^2}{x^4}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)*(f*x^2+e)^2/x^4,x, algorithm="fr
icas")
```

output
```
integral((d*f^2*x^6 + (2*d*e*f + c*f^2)*x^4 + c*e^2 + (d*e^2 + 2*c*e*f)*x^
2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/x^4, x)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}\left(e+fx^2\right)^2}{x^4}\,dx=\int \frac{\sqrt{a+bx^2}(c+dx^2)^{\frac{3}{2}}\left(e+fx^2\right)^2}{x^4}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(3/2)*(f*x**2+e)**2/x**4,x)
```

output
```
Integral(sqrt(a + b*x**2)*(c + d*x**2)**(3/2)*(e + f*x**2)**2/x**4, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}\left(e+fx^2\right)^2}{x^4}\,dx=\int \frac{\sqrt{bx^2+a}(dx^2+c)^{\frac{3}{2}}(fx^2+e)^2}{x^4}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)*(f*x^2+e)^2/x^4,x, algorithm="ma
xima")
```

output
```
integrate(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)*(f*x^2 + e)^2/x^4, x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}\,(e+fx^2)^2}{x^4}\,dx = \int \frac{\sqrt{bx^2+a}(dx^2+c)^{\frac{3}{2}}(fx^2+e)^2}{x^4}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)*(f*x^2+e)^2/x^4,x, algorithm="gi
ac")
```

output
```
integrate(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)*(f*x^2 + e)^2/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}\,(e+fx^2)^2}{x^4}\,dx = \int \frac{\sqrt{b\,x^2+a}\,(d\,x^2+c)^{3/2}\,(f\,x^2+e)^2}{x^4}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(3/2)*(e + f*x^2)^2)/x^4,x)
```

output
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(3/2)*(e + f*x^2)^2)/x^4, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}\,(e+fx^2)^2}{x^4}\,dx = \text{too large to display}$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)*(f*x^2+e)^2/x^4,x)
```

output

```
(2*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**3*c*d**2*f**2 - 4*sqrt(c + d*x**2)
*sqrt(a + b*x**2)*a**3*d**3*f**2*x**2 - 7*sqrt(c + d*x**2)*sqrt(a + b*x**2
)*a**2*b*c**2*d*f**2 - 10*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*b*c*d**2*
e*f + 10*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*b*c*d**2*f**2*x**2 + 20*sq
rt(c + d*x**2)*sqrt(a + b*x**2)*a**2*b*d**3*e*f*x**2 + 2*sqrt(c + d*x**2)*
sqrt(a + b*x**2)*a**2*b*d**3*f**2*x**4 - 3*sqrt(c + d*x**2)*sqrt(a + b*x**
2)*a*b**2*c**3*f**2 - 70*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**2*c**2*d*e
*f + 20*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**2*c**2*d*f**2*x**2 - 45*sqr
t(c + d*x**2)*sqrt(a + b*x**2)*a*b**2*c*d**2*e**2 + 100*sqrt(c + d*x**2)*s
qrt(a + b*x**2)*a*b**2*c*d**2*e*f*x**2 + 14*sqrt(c + d*x**2)*sqrt(a + b*x*
*2)*a*b**2*c*d**2*f**2*x**4 + 30*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**2*
d**3*e**2*x**2 + 20*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**2*d**3*e*f*x**4
 + 6*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**2*d**3*f**2*x**6 + 6*sqrt(c +
d*x**2)*sqrt(a + b*x**2)*b**3*c**3*f**2*x**2 - 15*sqrt(c + d*x**2)*sqrt(a
+ b*x**2)*b**3*c**2*d*e**2 + 80*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**3*c**
2*d*e*f*x**2 + 12*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**3*c**2*d*f**2*x**4
+ 30*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**3*c*d**2*e**2*x**2 + 20*sqrt(c +
 d*x**2)*sqrt(a + b*x**2)*b**3*c*d**2*e*f*x**4 + 6*sqrt(c + d*x**2)*sqrt(a
 + b*x**2)*b**3*c*d**2*f**2*x**6 + 6*int((sqrt(c + d*x**2)*sqrt(a + b*x**2
))/(a**2*c*d*x**4 + a**2*d**2*x**6 + a*b*c**2*x**4 + 2*a*b*c*d*x**6 + a...
```

3.89 $\int \frac{\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^2}{x^6}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 1343

$$\int \frac{\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^2}{x^6}\,dx = \text{Too large to display}$$

output
```
-1/15*(2*b^3*c^2*e^2-5*a^3*c*d*f^2-a*b^2*c*e*(10*c*f+7*d*e)-a^2*b*(35*c^2*
f^2+70*c*d*e*f+3*d^2*e^2))*x*(d*x^2+c)^(1/2)/a^2/c/(b*x^2+a)^(1/2)+1/15*(b
^2*c*d*e^3+5*a^2*f*(3*c^2*f^2+19*c*d*e*f+10*d^2*e^2)+a*b*e*(27*c^2*f^2+55*
c*d*e*f+11*d^2*e^2))*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a^2/c/e+1/15*(b^2*c
*e^2*(12*c^2*f^2+15*c*d*e*f+8*d^2*e^2)+5*a^2*f*(2*c^3*f^3+18*c^2*d*e*f^2+2
7*c*d^2*e^2*f+4*d^3*e^3)+a*b*e*(31*c^3*f^3+129*c^2*d*e*f^2+80*c*d^2*e^2*f+
5*d^3*e^3))*x^3*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a^2/c^2/e^2+1/15*(a^2*f^2*
(3*c^3*f^3+40*c^2*d*e*f^2+135*c*d^2*e^2*f+60*d^3*e^3)+b^2*e^2*(16*c^3*f^3+
39*c^2*d*e*f^2+30*c*d^2*e^2*f+5*d^3*e^3)+a*b*e*f*(16*c^3*f^3+127*c^2*d*e*f
^2+177*c*d^2*e^2*f+35*d^3*e^3))*x^5*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a^2/c^
2/e^3+1/15*f*(2*a^2*d*f^2*(3*c^2*f^2+25*c*d*e*f+30*d^2*e^2)+a*b*f*(3*c^3*f
^3+52*c^2*d*e*f^2+161*c*d^2*e^2*f+75*d^3*e^3)+b^2*e*(6*c^3*f^3+37*c^2*d*e*
f^2+42*c*d^2*e^2*f+15*d^3*e^3))*x^7*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a^2/c^
2/e^3+1/15*d*f^2*(a^2*d*f^2*(3*c*f+20*d*e)+a*b*f*(6*c^2*f^2+56*c*d*e*f+65*
d^2*e^2)+b^2*e*(12*c^2*f^2+26*c*d*e*f+15*d^2*e^2))*x^9*(b*x^2+a)^(1/2)*(d*
x^2+c)^(1/2)/a^2/c^2/e^3+1/15*b*d^2*f^3*(a*f*(3*c*f+20*d*e)+b*e*(6*c*f+5*d
*e))*x^11*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a^2/c^2/e^3-1/5*(b*x^2+a)^(3/2)*
(d*x^2+c)^(5/2)*(f*x^2+e)^3/a/c/e/x^5+1/15*(-a*f+2*b*e)*(b*x^2+a)^(3/2)*(d
*x^2+c)^(5/2)*(f*x^2+e)^3/a^2/c/e^2/x^3-1/15*(a*f*(3*c*f+20*d*e)+b*e*(6*c*
f+5*d*e))*(b*x^2+a)^(3/2)*(d*x^2+c)^(5/2)*(f*x^2+e)^3/a^2/c^2/e^3/x+1/1...
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.21 (sec) , antiderivative size = 415, normalized size of antiderivative = 0.31

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}\left(e+fx^2\right)^2}{x^6}\,dx = \frac{-\sqrt{\frac{b}{a}}(a+bx^2)\left(c+dx^2\right)\left(-2b^2c^2e^2x^4+abcex^2(7dex^2+c(e+10\right.}{}$$

input
```
Integrate[(Sqrt[a + b*x^2]*(c + d*x^2)^(3/2)*(e + f*x^2)^2)/x^6,x]
```

output
```
(-(Sqrt[b/a]*(a + b*x^2)*(c + d*x^2)*(-2*b^2*c^2*e^2*x^4 + a*b*c*e*x^2*(7*
d*e*x^2 + c*(e + 10*f*x^2)) + a^2*(3*d^2*e^2*x^4 + c*d*x^2*(6*e^2 + 40*e*f
*x^2 - 5*f^2*x^4) + c^2*(3*e^2 + 10*e*f*x^2 + 15*f^2*x^4)))) - I*c*(-2*b^3
*c^2*e^2 + 5*a^3*c*d*f^2 + a*b^2*c*e*(7*d*e + 10*c*f) + a^2*b*(3*d^2*e^2 +
 70*c*d*e*f + 35*c^2*f^2))*x^5*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Ell
ipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - (2*I)*c*(-(b*c) + a*d)*(-(b^
2*c*e^2) + 5*a^2*f*(3*d*e + 2*c*f) + a*b*e*(3*d*e + 5*c*f))*x^5*Sqrt[1 + (
b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c
)])/(15*a^2*Sqrt[b/a]*c*x^5*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (warning: unable to verify)

Time = 1.57 (sec) , antiderivative size = 798, normalized size of antiderivative = 0.59, number of steps used = 19, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.543$, Rules used = {448, 442, 25, 27, 442, 25, 27, 403, 27, 406, 320, 388, 313, 442, 25, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^2}{x^6}dx$$

$$\Big\downarrow \text{448}$$

$$\frac{f\int \frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)}{x^4}dx}{e^2}+e\int \frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)}{x^6}dx$$

$$\Big\downarrow \text{442}$$

$$\frac{f\left(\frac{\int \frac{3\sqrt{bx^2+a}\sqrt{dx^2+c}\left(d(be+af)x^2+a(de+cf)\right)}{x^2}dx}{3a}-\frac{e\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{3ax^3}\right)}{e^2}+e\left(\frac{\int -\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(-d(be+5af)x^2+2bce-3ade-5acf\right)}{x^4}dx}{5a}-\frac{e\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{5ax^5}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{f\left(\frac{\int \frac{3\sqrt{bx^2+a}\sqrt{dx^2+c}\left(d(be+af)x^2+a(de+cf)\right)}{x^2}dx}{3a}-\frac{e(a+bx^2)^{3/2}(c+dx^2)^{3/2}}{3ax^3}\right)}{e^2}+e\left(-\frac{\int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}(-d(be+5af)x^2+2bce-3ade-5acf)}{x^4}dx}{5a}-\frac{e\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{5ax^5}\right)$$

$\downarrow$ 27

$$\frac{f\left(\frac{\int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(d(be+af)x^2+a(de+cf)\right)}{x^2}dx}{a}-\frac{e(a+bx^2)^{3/2}(c+dx^2)^{3/2}}{3ax^3}\right)}{e^2}+e\left(-\frac{\int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}(-d(be+5af)x^2+2bce-3ade-5acf)}{x^4}dx}{5a}-\frac{e\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{5ax^5}\right)$$

$\downarrow$ 442

$$e\left(-\frac{\frac{\int -\frac{d\sqrt{bx^2+a}\left(a(bce+3ade+20acf)-\left(-15dfa^2-b(6de+5cf)a+2b^2ce\right)x^2\right)}{x^2\sqrt{dx^2+c}}dx}{3a}-\frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}(-5acf-3ade+2bce)}{3ax^3}}{5a}-\frac{e\left(a+bx^2\right)^{3/2}}{5a\ldots}\right.$$

$$\frac{f\left(\frac{\frac{\int \frac{a\sqrt{bx^2+a}\left(d(4bde+3bcf+adf)x^2+ad(de+2cf)+bc(3de+2cf)\right)}{\sqrt{dx^2+c}}dx}{a}-\frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}(cf+de)}{x}}{a}-\frac{e(a+bx^2)^{3/2}(c+dx^2)^{3/2}}{3ax^3}\right)}{e^2}$$

$\downarrow$ 25

$$e\left(-\frac{\frac{-\frac{\int \frac{d\sqrt{bx^2+a}\left(a(bce+3ade+20acf)-\left(-15dfa^2-b(6de+5cf)a+2b^2ce\right)x^2\right)}{x^2\sqrt{dx^2+c}}dx}{3a}-\frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}(-5acf-3ade+2bce)}{3ax^3}}{5a}-\frac{e\left(a+bx^2\right)^{3/\ldots}}{5a\ldots}\right.$$

$$\frac{f\left(\frac{\frac{\int \frac{a\sqrt{bx^2+a}\left(d(4bde+3bcf+adf)x^2+ad(de+2cf)+bc(3de+2cf)\right)}{\sqrt{dx^2+c}}dx}{a}-\frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}(cf+de)}{x}}{a}-\frac{e(a+bx^2)^{3/2}(c+dx^2)^{3/2}}{3ax^3}\right)}{e^2}$$

$\downarrow$ 27

$$\frac{e\left(-\frac{-\frac{d\int \frac{\sqrt{bx^2+a}\left(a(bce+3ade+20acf)-\left(-15dfa^2-b(6de+5cf)a+2b^2ce\right)x^2\right)}{x^2\sqrt{dx^2+c}}dx}{3a}-\frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}(-5acf-3ade+2bce)}{3ax^3}}{5a}-\frac{e\left(a+bx^2\right)^{3/}}{5a}\right.}{}$$

$$\frac{f\left(\frac{\int \frac{\sqrt{bx^2+a}\left(d(4bde+3bcf+adf)x^2+ad(de+2cf)+bc(3de+2cf)\right)}{\sqrt{dx^2+c}}dx-\frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}(cf+de)}{x}}{a}-\frac{e\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{3ax^3}\right)}{e^2}$$

$$\Big\downarrow \text{403}$$

$$\frac{f\left(\frac{\frac{\int \frac{d\left(d\left(dfa^2+7b(de+cf)a+b^2ce\right)x^2+a(bc(5de+3cf)+ad(3de+5cf))\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3d}-\frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}(cf+de)}{x}+\frac{1}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}(adf+3bcf+4bde)}{a}-\frac{e(}{}\right.}{e^2}$$

$$e\left(-\frac{-\frac{d\int \frac{\sqrt{bx^2+a}\left(a(bce+3ade+20acf)-\left(-15dfa^2-b(6de+5cf)a+2b^2ce\right)x^2\right)}{x^2\sqrt{dx^2+c}}dx}{3a}-\frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}(-5acf-3ade+2bce)}{3ax^3}}{5a}-\frac{e\left(a+bx^2\right)^{3/}}{5a}\right.$$

$$\Big\downarrow \text{27}$$

$$\frac{f\left(\frac{\frac{1}{3}\int \frac{d\left(dfa^2+7b(de+cf)a+b^2ce\right)x^2+a(bc(5de+3cf)+ad(3de+5cf))}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx-\frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}(cf+de)}{x}+\frac{1}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}(adf+3bcf+4bde)}{a}-\frac{e(a}{}\right.}{e^2}$$

$$e\left(-\frac{-\frac{d\int \frac{\sqrt{bx^2+a}\left(a(bce+3ade+20acf)-\left(-15dfa^2-b(6de+5cf)a+2b^2ce\right)x^2\right)}{x^2\sqrt{dx^2+c}}dx}{3a}-\frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}(-5acf-3ade+2bce)}{3ax^3}}{5a}-\frac{e\left(a+bx^2\right)^{3/}}{5a}\right.$$

$$\Big\downarrow \text{406}$$

$$\frac{f\left(\frac{\frac{1}{3}\left(d\left(a^2df+7ab(cf+de)+b^2ce\right)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+a(ad(5cf+3de)+bc(3cf+5de))\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)-\frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}(cf+de)}{x}+\frac{1}{3}}{a}\right.}{e^2}$$

$$e\left(-\frac{-\frac{d\int \frac{\sqrt{bx^2+a}\left(a(bce+3ade+20acf)-\left(-15dfa^2-b(6de+5cf)a+2b^2ce\right)x^2\right)}{x^2\sqrt{dx^2+c}}dx}{3a}-\frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}(-5acf-3ade+2bce)}{3ax^3}}{5a}-\frac{e\left(a+bx^2\right)^{3/}}{5a}\right.$$

$$\Big\downarrow \; 320$$

$$f\left(\frac{\frac{1}{3}\left(d\left(a^2df+7ab(cf+de)+b^2ce\right)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{\sqrt{c}\sqrt{a+bx^2}(ad(5cf+3de)+bc(3cf+5de))\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{a}-\frac{\left(a+bx^2\right)^{3/2}\sqrt{c}}{x}\right.$$

$$\overline{\qquad\qquad e^2 \qquad\qquad}$$

$$e\left(-\frac{-\frac{d\int \frac{\sqrt{bx^2+a}\left(a(bce+3ade+20acf)-\left(-15dfa^2-b(6de+5cf)a+2b^2ce\right)x^2\right)}{x^2\sqrt{dx^2+c}}dx}{3a}-\frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}(-5acf-3ade+2bce)}{3ax^3}}{5a}-\frac{e\left(a+bx^2\right)^{3/}}{5}\right.$$

$$\Big\downarrow \; 388$$

$$f\left(\frac{\frac{1}{3}\left(d\left(a^2df+7ab(cf+de)+b^2ce\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{\sqrt{c}\sqrt{a+bx^2}(ad(5cf+3de)+bc(3cf+5de))\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{a}-\frac{(}{}\right.$$

$$\overline{\qquad\qquad e^2 \qquad\qquad}$$

$$e\left(-\frac{-\frac{d\int \frac{\sqrt{bx^2+a}\left(a(bce+3ade+20acf)-\left(-15dfa^2-b(6de+5cf)a+2b^2ce\right)x^2\right)}{x^2\sqrt{dx^2+c}}dx}{3a}-\frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}(-5acf-3ade+2bce)}{3ax^3}}{5a}-\frac{e\left(a+bx^2\right)^{3/}}{5}\right.$$

$$\Big\downarrow \; 313$$

$$e\left(-\frac{-\frac{d\int \frac{\sqrt{bx^2+a}\left(a(bce+3ade+20acf)-\left(-15dfa^2-b(6de+5cf)a+2b^2ce\right)x^2\right)}{x^2\sqrt{dx^2+c}}dx}{3a}-\frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}(-5acf-3ade+2bce)}{3ax^3}}{5a}-\frac{e\left(a+bx^2\right)^{3/}}{5}\right.$$

$$\frac{f\left(\frac{\frac{1}{3}\left(d\left(a^2df+7ab(cf+de)+b^2ce\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)+\frac{\sqrt{c}\sqrt{a+bx^2}(ad(5cf+3de)+bc(3cf+5de))\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}}{\sqrt{}}\right.\right.}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right.}{a}\right.}{e^2}$$

$\downarrow$ 442

$$e\left(-\frac{-\frac{d\left(\frac{\int -\frac{b\left(-d(3de+35cf)a^2-bc(7de+5cf)a+2b^2c^2e\right)x^2+ac\left(-15dfa^2-b(9de+25cf)a+b^2ce\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{c}-\frac{a\sqrt{a+bx^2}\sqrt{c+dx^2}(20acf+3ade+bce)}{cx}\right)}{3a}-\frac{(a+bx^{}}{}}{5a}\right.$$

$$\frac{f\left(\frac{\frac{1}{3}\left(d\left(a^2df+7ab(cf+de)+b^2ce\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)+\frac{\sqrt{c}\sqrt{a+bx^2}(ad(5cf+3de)+bc(3cf+5de))\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}}{\sqrt{}}\right.\right.}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right.}{a}\right.}{e^2}$$

$\downarrow$ 25

$$e\left(-\frac{-\frac{d\left(-\frac{\int \frac{b\left(-d(3de+35cf)a^2-bc(7de+5cf)a+2b^2c^2e\right)x^2+ac\left(-15dfa^2-b(9de+25cf)a+b^2ce\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{c}-\frac{a\sqrt{a+bx^2}\sqrt{c+dx^2}(20acf+3ade+bce)}{cx}\right)}{3a}-\frac{(a+bx\cdots}{}}{5a}\cdots\right.$$

$$\frac{f\left(\frac{\frac{1}{3}\left(d\left(a^2df+7ab(cf+de)+b^2ce\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{\sqrt{c}\sqrt{a+bx^2}(ad(5cf+3de)+bc(3cf+5de))\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}\cdots}{\sqrt{\cdots}}\right.\right.}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\cdots}{a}\cdots\right.}{e^2\cdots}$$

$\Big\downarrow$ 406

$$e\left(-\frac{-\frac{d\left(-\frac{b\left(a^2(-d)(35cf+3de)-abc(5cf+7de)+2b^2c^2e\right)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+ac\left(-15a^2df-ab(25cf+9de)+b^2ce\right)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{c}-\frac{a\sqrt{a+bx^2}\sqrt{c\cdots}}{}\cdots\right.}{3a}\cdots}{5a}\cdots\right.$$

$$\frac{f\left(\frac{\frac{1}{3}\left(d\left(a^2df+7ab(cf+de)+b^2ce\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{\sqrt{c}\sqrt{a+bx^2}(ad(5cf+3de)+bc(3cf+5de))\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}\cdots}{\sqrt{\cdots}}\right.\right.}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\cdots}{a}\cdots\right.}{e^2\cdots}$$

$\Big\downarrow$ 320

$$\frac{e\left(-\frac{-\frac{d\left(-\frac{b\left(a^2(-d)(35cf+3de)-abc(5cf+7de)+2b^2c^2e\right)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}\left(-15a^2df-ab(25cf+9de)+b^2ce\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1\ldots}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{c}\ldots\right)}{3a}\ldots}{5a}\ldots\right)\ldots f\left(\frac{\frac{1}{3}\left(d\left(a^2df+7ab(cf+de)+b^2ce\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{\sqrt{c}\sqrt{a+bx^2}(ad(5cf+3de)+bc(3cf+5de))\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}\ldots}{\sqrt{\ldots}}\right)\ldots}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\ldots\right)}{a}\ldots\right)}{e^2\ldots}$$

$$\Big\downarrow \; 388$$

$$e\left(-\frac{-\frac{d\left(-\frac{b\left(a^2(-d)(35cf+3de)-abc(5cf+7de)+2b^2c^2e\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}\left(-15a^2df-ab(25cf+9de)+b^2ce\right)\text{EllipticF}\left(\text{arc}\right.}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{c}\right.}{3a}}{5a}\right.$$

$$\frac{f\left(\frac{\frac{1}{3}\left(d\left(a^2df+7ab(cf+de)+b^2ce\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{\sqrt{c}\sqrt{a+bx^2}(ad(5cf+3de)+bc(3cf+5de))\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}}{\sqrt{}}\right.\right.}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right.}{a}\right.}{e^2}$$

$\downarrow$ 313

$$f\left(\frac{-\frac{(de+cf)\sqrt{dx^2+c}\left(bx^2+a\right)^{3/2}}{x}+\frac{1}{3}(4bde+3bcf+adf)x\sqrt{dx^2+c}\sqrt{bx^2+a}+\frac{1}{3}\left(d\left(dfa^2+7b(de+cf)a+b^2ce\right)\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}}{\sqrt{c}}\right.\right.}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{d}}\right.\right.}{a}\right.$$

$$\overline{\qquad e^2 \qquad}$$

$$e\left(-\frac{e\left(bx^2+a\right)^{3/2}\left(dx^2+c\right)^{3/2}}{5ax^5}-\frac{-\frac{(2bce-3ade-5acf)\sqrt{dx^2+c}\left(bx^2+a\right)^{3/2}}{3ax^3}-\frac{d\left(-\frac{a\sqrt{bx^2+a}\sqrt{dx^2+c}(bce+3ade+20acf)}{cx}-\frac{\left(-15dfa^2-\right.}{}\right.}{}}{}\right.$$

input
```
Int[(Sqrt[a + b*x^2]*(c + d*x^2)^(3/2)*(e + f*x^2)^2)/x^6,x]
```

output
```
(f*(-1/3*(e*(a + b*x^2)^(3/2)*(c + d*x^2)^(3/2))/(a*x^3) + (((4*b*d*e + 3*
b*c*f + a*d*f)*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/3 - ((d*e + c*f)*(a + b*
x^2)^(3/2)*Sqrt[c + d*x^2])/x + (d*(b^2*c*e + a^2*d*f + 7*a*b*(d*e + c*f))
*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*Ellip
ticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a
+ b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (Sqrt[c]*(b*c*(5*d*e + 3*c*
f) + a*d*(3*d*e + 5*c*f))*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqr
t[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sq
rt[c + d*x^2]))/3)/a))/e^2 + e*(-1/5*(e*(a + b*x^2)^(3/2)*(c + d*x^2)^(3/2
))/(a*x^5) - (-1/3*((2*b*c*e - 3*a*d*e - 5*a*c*f)*(a + b*x^2)^(3/2)*Sqrt[c
 + d*x^2])/(a*x^3) - (d*(-((a*(b*c*e + 3*a*d*e + 20*a*c*f)*Sqrt[a + b*x^2]
*Sqrt[c + d*x^2])/(c*x)) - (b*(2*b^2*c^2*e - a*b*c*(7*d*e + 5*c*f) - a^2*d
*(3*d*e + 35*c*f))*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqr
t[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*S
qrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (c^(3/2)*
(b^2*c*e - 15*a^2*d*f - a*b*(9*d*e + 25*c*f))*Sqrt[a + b*x^2]*EllipticF[Ar
cTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))
/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))/c))/(3*a))/(5*a))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 442
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^q/(a*g*(m + 1))), x] - Simp[1/(a*g^2*(m + 1))   Int[(g*x)
^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[c*(b*e - a*f)*(m + 1) + e*2
*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*2*(p + q + 1))*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && GtQ[q, 0] && LtQ[m, -1]
&&  !(EqQ[q, 1] && SimplerQ[e + f*x^2, c + d*x^2])
```

rule 448
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[e   Int[(g*x)^m*(a + b*x
^2)^p*(c + d*x^2)^q*(e + f*x^2)^(r - 1), x], x] + Simp[f/e^2   Int[(g*x)^(m
 + 2)*(a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^(r - 1), x], x] /; FreeQ[{a,
b, c, d, e, f, g, m, p, q}, x] && IGtQ[r, 0]
```

Maple [A] (verified)

Time = 6.77 (sec) , antiderivative size = 629, normalized size of antiderivative = 0.47

method	result
elliptic	$\frac{\sqrt{(b\,x^{2}+a)(x^{2}d+c)}\left(-\frac{c\,e^{2}\sqrt{bd\,x^{4}+ad\,x^{2}+x^{2}bc+ac}}{5x^{5}}-\frac{e(10acf+6ade+bce)\sqrt{bd\,x^{4}+ad\,x^{2}+x^{2}bc+ac}}{15a\,x^{3}}-\frac{(15a^{2}c^{2}f^{2}+40a^{2}cdef+3a^{2}d^{2}e^{2}+10ab}{}\right.}{}$
risch	$-\frac{\sqrt{b\,x^{2}+a}\,\sqrt{x^{2}d+c}\,(-5a^{2}cd\,f^{2}x^{6}+15f^{2}x^{4}a^{2}c^{2}+40a^{2}cdef\,x^{4}+3a^{2}d^{2}e^{2}x^{4}+10ab\,c^{2}ef\,x^{4}+7abcd\,e^{2}x^{4}-2b^{2}c^{2}e^{2}x^{4}+10a^{2}c^{2}ef\,x^{2}+}{15x^{5}a^{2}c}$
default	Expression too large to display

input

```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)*(f*x^2+e)^2/x^6,x,method=_RETURNVERBOS
E)
```

output

```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-1/5*c*e^2*(b
*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/x^5-1/15*e*(10*a*c*f+6*a*d*e+b*c*e)/a*(b
*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/x^3-1/15/a^2/c*(15*a^2*c^2*f^2+40*a^2*c*
d*e*f+3*a^2*d^2*e^2+10*a*b*c^2*e*f+7*a*b*c*d*e^2-2*b^2*c^2*e^2)*(b*d*x^4+a
*d*x^2+b*c*x^2+a*c)^(1/2)/x+1/3*f^2*d*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2
)+(5/3*a*c*d*f^2+2*a*d^2*e*f+b*c^2*f^2+4*b*c*d*e*f+b*d^2*e^2-1/15*b*d*e*(1
0*a*c*f+6*a*d*e+b*c*e)/a)/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)
/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c
)/c/b)^(1/2))-(a*d^2*f^2+2*b*c*d*f^2+2*b*d^2*e*f+1/15*b*d*(15*a^2*c^2*f^2+
40*a^2*c*d*e*f+3*a^2*d^2*e^2+10*a*b*c^2*e*f+7*a*b*c*d*e^2-2*b^2*c^2*e^2)/a
^2/c-1/3*f^2*d*(2*a*d+2*b*c))*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)
^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(-b/a)^(1/2),(-1
+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2)))
)
```

Fricas [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}\left(e+fx^2\right)^2}{x^6}\,dx = \int \frac{\sqrt{bx^2+a}(dx^2+c)^{\frac{3}{2}}(fx^2+e)^2}{x^6}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)*(f*x^2+e)^2/x^6,x, algorithm="fr
icas")
```

output
```
integral((d*f^2*x^6 + (2*d*e*f + c*f^2)*x^4 + c*e^2 + (d*e^2 + 2*c*e*f)*x^
2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/x^6, x)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}\left(e+fx^2\right)^2}{x^6}\,dx = \int \frac{\sqrt{a+bx^2}(c+dx^2)^{\frac{3}{2}}\left(e+fx^2\right)^2}{x^6}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(3/2)*(f*x**2+e)**2/x**6,x)
```

output
```
Integral(sqrt(a + b*x**2)*(c + d*x**2)**(3/2)*(e + f*x**2)**2/x**6, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}\left(e+fx^2\right)^2}{x^6}\,dx = \int \frac{\sqrt{bx^2+a}(dx^2+c)^{\frac{3}{2}}(fx^2+e)^2}{x^6}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)*(f*x^2+e)^2/x^6,x, algorithm="ma
xima")
```

output
```
integrate(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)*(f*x^2 + e)^2/x^6, x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}\left(e+fx^2\right)^2}{x^6}\,dx = \int \frac{\sqrt{bx^2+a}(dx^2+c)^{\frac{3}{2}}(fx^2+e)^2}{x^6}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)*(f*x^2+e)^2/x^6,x, algorithm="gi
ac")
```

output
```
integrate(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)*(f*x^2 + e)^2/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}\left(e+fx^2\right)^2}{x^6}\,dx = \int \frac{\sqrt{b\,x^2+a}\,(d\,x^2+c)^{3/2}\,(f\,x^2+e)^2}{x^6}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(3/2)*(e + f*x^2)^2)/x^6,x)
```

output
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(3/2)*(e + f*x^2)^2)/x^6, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}\left(e+fx^2\right)^2}{x^6}\,dx = \text{too large to display}$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)*(f*x^2+e)^2/x^6,x)
```

output

```
( - 25*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*c**2*d*f**2*x**2 - 30*sqrt(c
 + d*x**2)*sqrt(a + b*x**2)*a**2*c*d**2*e*f*x**2 + 50*sqrt(c + d*x**2)*sqr
t(a + b*x**2)*a**2*c*d**2*f**2*x**4 + 60*sqrt(c + d*x**2)*sqrt(a + b*x**2)
*a**2*d**3*e*f*x**4 - 15*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b*c**3*f**2*x
**2 - 3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b*c**2*d*e**2 - 60*sqrt(c + d*
x**2)*sqrt(a + b*x**2)*a*b*c**2*d*e*f*x**2 + 65*sqrt(c + d*x**2)*sqrt(a +
b*x**2)*a*b*c**2*d*f**2*x**4 - 15*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b*c*
d**2*e**2*x**2 + 120*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b*c*d**2*e*f*x**4
 + 5*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b*c*d**2*f**2*x**6 + 15*sqrt(c +
d*x**2)*sqrt(a + b*x**2)*a*b*d**3*e**2*x**4 + 30*sqrt(c + d*x**2)*sqrt(a +
 b*x**2)*b**2*c**3*f**2*x**4 + 90*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**2*c
**2*d*e*f*x**4 + 9*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**2*c*d**2*e**2*x**4
 - 45*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a**2*c*d + a**2*d**2*x
**2 + a*b*c**2 + 2*a*b*c*d*x**2 + a*b*d**2*x**4 + b**2*c**2*x**2 + b**2*c*
d*x**4),x)*a**3*b*c*d**4*f**2*x**5 - 60*int((sqrt(c + d*x**2)*sqrt(a + b*x
**2)*x**2)/(a**2*c*d + a**2*d**2*x**2 + a*b*c**2 + 2*a*b*c*d*x**2 + a*b*d*
*2*x**4 + b**2*c**2*x**2 + b**2*c*d*x**4),x)*a**3*b*d**5*e*f*x**5 - 90*int
((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a**2*c*d + a**2*d**2*x**2 + a*b
*c**2 + 2*a*b*c*d*x**2 + a*b*d**2*x**4 + b**2*c**2*x**2 + b**2*c*d*x**4),x
)*a**2*b**2*c**2*d**3*f**2*x**5 - 150*int((sqrt(c + d*x**2)*sqrt(a + b*...
```

3.90 $\int x^2\sqrt{a+bx^2}(c+dx^2)^{3/2}\left(e+fx^2\right)\,dx$

Optimal result

Integrand size = 33, antiderivative size = 684

$$\int x^2\sqrt{a+bx^2}(c+dx^2)^{3/2}\left(e+fx^2\right)\,dx =$$

$$-\frac{(16a^4d^4f-ab^3c^2d(27de-7cf)+2b^4c^3(9de-4cf)+3a^2b^2cd^2(19de+3cf)-8a^3bd^3(3de+4cf))\,x\sqrt{c}}{315b^3d^3\sqrt{a+bx^2}}$$

$$+\frac{(8a^3d^3f+b^3c^2(9de-4cf)+3ab^2cd(9de+cf)-3a^2bd^2(4de+5cf))\,x\sqrt{a+bx^2}\sqrt{c+dx^2}}{315b^3d^2}$$

$$+\frac{\left(24bce-12ade-13acf+\frac{bc^2f}{d}+\frac{8a^2df}{b}\right)x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{105b}$$

$$+\frac{(3bde+bcf-2adf)x^3(a+bx^2)^{3/2}\sqrt{c+dx^2}}{21b^2}+\frac{fx^3(a+bx^2)^{3/2}(c+dx^2)^{3/2}}{9b}$$

$$+\frac{\sqrt{a}(16a^4d^4f-ab^3c^2d(27de-7cf)+2b^4c^3(9de-4cf)+3a^2b^2cd^2(19de+3cf)-8a^3bd^3(3de+4cf))\,\sqrt{c}}{315b^{7/2}d^3\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

$$-\frac{a^{3/2}(8a^3d^3f+b^3c^2(9de-4cf)+3ab^2cd(9de+cf)-3a^2bd^2(4de+5cf))\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\right.\right.}{315b^{7/2}d^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
-1/315*(16*a^4*d^4*f-a*b^3*c^2*d*(-7*c*f+27*d*e)+2*b^4*c^3*(-4*c*f+9*d*e)+
3*a^2*b^2*c*d^2*(3*c*f+19*d*e)-8*a^3*b*d^3*(4*c*f+3*d*e))*x*(d*x^2+c)^(1/2
)/b^3/d^3/(b*x^2+a)^(1/2)+1/315*(8*a^3*d^3*f+b^3*c^2*(-4*c*f+9*d*e)+3*a*b^
2*c*d*(c*f+9*d*e)-3*a^2*b*d^2*(5*c*f+4*d*e))*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(
1/2)/b^3/d^2+1/105*(24*b*c*e-12*a*d*e-13*a*c*f+b*c^2*f/d+8*a^2*d*f/b)*x^3*
(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b+1/21*(-2*a*d*f+b*c*f+3*b*d*e)*x^3*(b*x^2
+a)^(3/2)*(d*x^2+c)^(1/2)/b^2+1/9*f*x^3*(b*x^2+a)^(3/2)*(d*x^2+c)^(3/2)/b+
1/315*a^(1/2)*(16*a^4*d^4*f-a*b^3*c^2*d*(-7*c*f+27*d*e)+2*b^4*c^3*(-4*c*f+
9*d*e)+3*a^2*b^2*c*d^2*(3*c*f+19*d*e)-8*a^3*b*d^3*(4*c*f+3*d*e))*(d*x^2+c)
^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/b^
(7/2)/d^3/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)-1/315*a^(3/2)*(8
*a^3*d^3*f+b^3*c^2*(-4*c*f+9*d*e)+3*a*b^2*c*d*(c*f+9*d*e)-3*a^2*b*d^2*(5*c
*f+4*d*e))*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*
d/b/c)^(1/2))/b^(7/2)/d^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.48 (sec) , antiderivative size = 480, normalized size of antiderivative = 0.70

$$\int x^2\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)\,dx = \frac{\sqrt{\frac{b}{a}}dx(a+bx^2)\left(c+dx^2\right)\left(8a^3d^3f-3a^2bd^2(4de+5cf+2dfx^2)+ab^2d(3c^2f+d^2x^2(9e+5fx\cdots\right.}{\cdots}$$

input
```
Integrate[x^2*Sqrt[a + b*x^2]*(c + d*x^2)^(3/2)*(e + f*x^2),x]
```

output
```
(Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2)*(8*a^3*d^3*f - 3*a^2*b*d^2*(4*d*e +
 5*c*f + 2*d*f*x^2) + a*b^2*d*(3*c^2*f + d^2*x^2*(9*e + 5*f*x^2) + c*d*(27
*e + 11*f*x^2)) + b^3*(-4*c^3*f + 3*c^2*d*(3*e + f*x^2) + 5*d^3*x^4*(9*e +
 7*f*x^2) + 2*c*d^2*x^2*(36*e + 25*f*x^2))) + I*c*(16*a^4*d^4*f + 2*b^4*c^
3*(9*d*e - 4*c*f) + 3*a^2*b^2*c*d^2*(19*d*e + 3*c*f) - 8*a^3*b*d^3*(3*d*e
+ 4*c*f) + a*b^3*c^2*d*(-27*d*e + 7*c*f))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*
x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + I*c*(b*c - a*d)*(
8*a^3*d^3*f - 3*a*b^2*c*d*(-6*d*e + c*f) - 3*a^2*b*d^2*(4*d*e + 3*c*f) + 2
*b^3*c^2*(-9*d*e + 4*c*f))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Ellipti
cF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(315*b^3*Sqrt[b/a]*d^3*Sqrt[a + b
*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 624, normalized size of antiderivative = 0.91, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {443, 27, 443, 443, 444, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)\,dx$$

$$\Big\downarrow \text{443}$$

$$\frac{\int 3x^2\sqrt{bx^2+a}\sqrt{dx^2+c}\left((3bde+bcf-2adf)x^2+c(3be-af)\right)dx}{9b}+\frac{fx^3\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{9b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int x^2\sqrt{bx^2+a}\sqrt{dx^2+c}\left((3bde+bcf-2adf)x^2+c(3be-af)\right)dx}{3b}+\frac{fx^3\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{9b}$$

$$\Big\downarrow \text{443}$$

$$\frac{\frac{\int \frac{x^2\sqrt{bx^2+a}\left(\left(c(24de+cf)b^2-ad(12de+13cf)b+8a^2d^2f\right)x^2+c\left(6dfa^2-9bdea-10bcfa+21b^2ce\right)\right)}{\sqrt{dx^2+c}}dx}{7b}+\frac{x^3\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}(-2adf+bcf+3bde)}{7b}}{3b}+$$

$$\frac{fx^3\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{9b}$$

$\downarrow$ 443

$$\frac{\frac{\frac{\int \frac{x^2\left(\left(c^2(9de-4cf)b^3+3acd(9de+cf)b^2-3a^2d^2(4de+5cf)b+8a^3d^3f\right)x^2+ac\left(3c(11de-cf)b^2-ad(9de+11cf)b+6a^2d^2f\right)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{5d}+\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}\left(8a^2d^2f-ab\right.}{5d}}{7b}}{3b}$$

$$\frac{fx^3\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{9b}$$

$\downarrow$ 444

$$\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(8a^3d^3f-3a^2bd^2(5cf+4de)+3ab^2cd(cf+9de)+b^3c^2(9de-4cf)\right)}{3bd}-\frac{\int \frac{\left(2c^3(9de-4cf)b^4-ac^2d(27de-7cf)b^3+3a^2cd^2(19de+3cf)b^2-8a^3d^3(3d\right.}{}}{5d}$$

$$\frac{fx^3\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{9b}$$

$\downarrow$ 406

$$\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(8a^3d^3f-3a^2bd^2(5cf+4de)+3ab^2cd(cf+9de)+b^3c^2(9de-4cf)\right)}{3bd}-\frac{ac\left(8a^3d^3f-3a^2bd^2(5cf+4de)+3ab^2cd(cf+9de)+b^3c^2(9de-4cf)\right)\int \frac{}{\sqrt{bx^2}}}{5d}$$

$$\frac{fx^3\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{9b}$$

$\downarrow$ 320

$$\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(8a^3d^3f-3a^2bd^2(5cf+4de)+3ab^2cd(cf+9de)+b^3c^2(9de-4cf)\right)}{3bd}-\frac{\left(16a^4d^4f-8a^3bd^3(4cf+3de)+3a^2b^2cd^2(3cf+19de)-ab^3c^2d(27de-7cf)+\right.}{5d}$$

$$\frac{fx^3\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{9b}$$

$\downarrow$ 388

$$\frac{\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(8a^3d^3f-3a^2bd^2(5cf+4de)+3ab^2cd(cf+9de)+b^3c^2(9de-4cf)\right)}{3bd}-\frac{\left(16a^4d^4f-8a^3bd^3(4cf+3de)+3a^2b^2cd^2(3cf+19de)-ab^3c^2d(27de-7cf)+\right.}{}}{}$$

$$\frac{fx^3\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{9b}$$

$\big\downarrow$ 313

$$\frac{\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}\left(8a^2d^2f-abd(13cf+12de)+b^2c(cf+24de)\right)}{5d}+\frac{\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(8a^3d^3f-3a^2bd^2(5cf+4de)+3ab^2cd(cf+9de)+b^3c^2(9de-4cf)\right)}{3bd}-\frac{c^{3/2}\sqrt{a}}{}}{}}{}$$

$$\frac{fx^3\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{9b}$$

input
```
Int[x^2*Sqrt[a + b*x^2]*(c + d*x^2)^(3/2)*(e + f*x^2),x]
```

output
```
(f*x^3*(a + b*x^2)^(3/2)*(c + d*x^2)^(3/2))/(9*b) + (((3*b*d*e + b*c*f - 2
*a*d*f)*x^3*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(7*b) + (((8*a^2*d^2*f + b^
2*c*(24*d*e + c*f) - a*b*d*(12*d*e + 13*c*f))*x^3*Sqrt[a + b*x^2]*Sqrt[c +
 d*x^2])/(5*d) + (((8*a^3*d^3*f + b^3*c^2*(9*d*e - 4*c*f) + 3*a*b^2*c*d*(9
*d*e + c*f) - 3*a^2*b*d^2*(4*d*e + 5*c*f))*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^
2])/(3*b*d) - ((16*a^4*d^4*f - a*b^3*c^2*d*(27*d*e - 7*c*f) + 2*b^4*c^3*(9
*d*e - 4*c*f) + 3*a^2*b^2*c*d^2*(19*d*e + 3*c*f) - 8*a^3*b*d^3*(3*d*e + 4*
c*f))*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*
EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(
c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (c^(3/2)*(8*a^3*d^3*f
+ b^3*c^2*(9*d*e - 4*c*f) + 3*a*b^2*c*d*(9*d*e + c*f) - 3*a^2*b*d^2*(4*d*e
 + 5*c*f))*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c
)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))
/(3*b*d))/(5*d))/(7*b))/(3*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 443
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*(g*x)^(m + 1)*(a + b*x^2)^(p
 + 1)*((c + d*x^2)^q/(b*g*(m + 2*(p + q + 1) + 1))), x] + Simp[1/(b*(m + 2*
(p + q + 1) + 1))   Int[(g*x)^m*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[c*((
b*e - a*f)*(m + 1) + b*e*2*(p + q + 1)) + (d*(b*e - a*f)*(m + 1) + f*2*q*(b
*c - a*d) + b*e*d*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f
, g, m, p}, x] && GtQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^2, c + d*x^
2])
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

Maple [A] (verified)

Time = 7.52 (sec) , antiderivative size = 1060, normalized size of antiderivative = 1.55

method	result	size
elliptic	Expression too large to display	1060
risch	Expression too large to display	1164
default	Expression too large to display	1846

input
```
int(x^2*(b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)*(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/9*d*f*x^7*(
b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+1/7*(a*d^2*f+2*b*c*d*f+b*d^2*e-1/9*d*f*
(8*a*d+8*b*c))/b/d*x^5*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+1/5*(11/9*a*c*d
*f+a*d^2*e+b*c^2*f+2*b*c*d*e-1/7*(a*d^2*f+2*b*c*d*f+b*d^2*e-1/9*d*f*(8*a*d
+8*b*c))/b/d*(6*a*d+6*b*c))/b/d*x^3*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+1/
3*(a*c^2*f+2*a*c*d*e+b*c^2*e-5/7*(a*d^2*f+2*b*c*d*f+b*d^2*e-1/9*d*f*(8*a*d
+8*b*c))/b/d*a*c-1/5*(11/9*a*c*d*f+a*d^2*e+b*c^2*f+2*b*c*d*e-1/7*(a*d^2*f+
2*b*c*d*f+b*d^2*e-1/9*d*f*(8*a*d+8*b*c))/b/d*(6*a*d+6*b*c))/b/d*(4*a*d+4*b
*c))/b/d*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)-1/3*(a*c^2*f+2*a*c*d*e+b*c^
2*e-5/7*(a*d^2*f+2*b*c*d*f+b*d^2*e-1/9*d*f*(8*a*d+8*b*c))/b/d*a*c-1/5*(11/
9*a*c*d*f+a*d^2*e+b*c^2*f+2*b*c*d*e-1/7*(a*d^2*f+2*b*c*d*f+b*d^2*e-1/9*d*f
*(8*a*d+8*b*c))/b/d*(6*a*d+6*b*c))/b/d*(4*a*d+4*b*c))/b/d*a*c/(-b/a)^(1/2)
*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*E
llipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-(a*c^2*e-3/5*(11/9*a*c*d
*f+a*d^2*e+b*c^2*f+2*b*c*d*e-1/7*(a*d^2*f+2*b*c*d*f+b*d^2*e-1/9*d*f*(8*a*d
+8*b*c))/b/d*(6*a*d+6*b*c))/b/d*a*c-1/3*(a*c^2*f+2*a*c*d*e+b*c^2*e-5/7*(a*
d^2*f+2*b*c*d*f+b*d^2*e-1/9*d*f*(8*a*d+8*b*c))/b/d*a*c-1/5*(11/9*a*c*d*f+a
*d^2*e+b*c^2*f+2*b*c*d*e-1/7*(a*d^2*f+2*b*c*d*f+b*d^2*e-1/9*d*f*(8*a*d+8*b
*c))/b/d*(6*a*d+6*b*c))/b/d*(4*a*d+4*b*c))/b/d*(2*a*d+2*b*c))*c/(-b/a)^(1/
2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1...
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 688, normalized size of antiderivative = 1.01

$$\int x^2\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)\,dx = \frac{\sqrt{bd}(3\left(6\,b^4c^4d-9\,ab^3c^3d^2+19\,a^2b^2c^2d^3-8\,a^3bcd^4\right)e-\left(8\,b^4c^5-7\,ab^3c^4d-9\,a^2b^2c^3d^2+32\,a^3\cdots\right.}{\cdots}$$

input
```
integrate(x^2*(b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)*(f*x^2+e),x, algorithm="fric
as")
```

output
```
1/315*(sqrt(b*d)*(3*(6*b^4*c^4*d - 9*a*b^3*c^3*d^2 + 19*a^2*b^2*c^2*d^3 -
8*a^3*b*c*d^4)*e - (8*b^4*c^5 - 7*a*b^3*c^4*d - 9*a^2*b^2*c^3*d^2 + 32*a^3
*b*c^2*d^3 - 16*a^4*c*d^4)*f)*x*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x)
, a*d/(b*c)) - sqrt(b*d)*(3*(6*b^4*c^4*d - 9*a*b^3*c^3*d^2 - 4*a^3*b*d^5 +
 (19*a^2*b^2 + 3*a*b^3)*c^2*d^3 - (8*a^3*b - 9*a^2*b^2)*c*d^4)*e - (8*b^4*
c^5 - 7*a*b^3*c^4*d - 8*a^4*d^5 - (9*a^2*b^2 - 4*a*b^3)*c^3*d^2 + (32*a^3*
b - 3*a^2*b^2)*c^2*d^3 - (16*a^4 - 15*a^3*b)*c*d^4)*f)*x*sqrt(-c/d)*ellipt
ic_f(arcsin(sqrt(-c/d)/x), a*d/(b*c)) + (35*b^4*d^5*f*x^8 + 5*(9*b^4*d^5*e
 + (10*b^4*c*d^4 + a*b^3*d^5)*f)*x^6 + (9*(8*b^4*c*d^4 + a*b^3*d^5)*e + (3
*b^4*c^2*d^3 + 11*a*b^3*c*d^4 - 6*a^2*b^2*d^5)*f)*x^4 + (3*(3*b^4*c^2*d^3
+ 9*a*b^3*c*d^4 - 4*a^2*b^2*d^5)*e - (4*b^4*c^3*d^2 - 3*a*b^3*c^2*d^3 + 15
*a^2*b^2*c*d^4 - 8*a^3*b*d^5)*f)*x^2 - 3*(6*b^4*c^3*d^2 - 9*a*b^3*c^2*d^3
+ 19*a^2*b^2*c*d^4 - 8*a^3*b*d^5)*e + (8*b^4*c^4*d - 7*a*b^3*c^3*d^2 - 9*a
^2*b^2*c^2*d^3 + 32*a^3*b*c*d^4 - 16*a^4*d^5)*f)*sqrt(b*x^2 + a)*sqrt(d*x^
2 + c))/(b^4*d^4*x)
```

Sympy [F]

$$\int x^2\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)\,dx=\int x^2\sqrt{a+bx^2}\left(c+dx^2\right)^{\frac{3}{2}}\left(e+fx^2\right)\,dx$$

input
```
integrate(x**2*(b*x**2+a)**(1/2)*(d*x**2+c)**(3/2)*(f*x**2+e),x)
```

output
```
Integral(x**2*sqrt(a + b*x**2)*(c + d*x**2)**(3/2)*(e + f*x**2), x)
```

Maxima [F]

$$\int x^2\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)\,dx=\int\sqrt{bx^2+a}\left(dx^2+c\right)^{\frac{3}{2}}\left(fx^2+e\right)x^2\,dx$$

input
```
integrate(x^2*(b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)*(f*x^2+e),x, algorithm="maxi
ma")
```

output
```
integrate(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)*(f*x^2 + e)*x^2, x)
```

Giac [F]

$$\int x^2\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)\,dx=\int \sqrt{bx^2+a}(dx^2+c)^{\frac{3}{2}}(fx^2+e)x^2\,dx$$

input
```
integrate(x^2*(b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)*(f*x^2+e),x, algorithm="giac
")
```

output
```
integrate(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)*(f*x^2 + e)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)\,dx=\int x^2\,\sqrt{b\,x^2+a}\,{\left(d\,x^2+c\right)}^{3/2}\,\left(f\,x^2+e\right)\,dx$$

input
```
int(x^2*(a + b*x^2)^(1/2)*(c + d*x^2)^(3/2)*(e + f*x^2),x)
```

output
```
int(x^2*(a + b*x^2)^(1/2)*(c + d*x^2)^(3/2)*(e + f*x^2), x)
```

Reduce [F]

$$\int x^2\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)\,dx=\text{Too large to display}$$

input
```
int(x^2*(b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)*(f*x^2+e),x)
```

output

```
(8*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**3*d**3*f*x - 15*sqrt(c + d*x**2)*s
qrt(a + b*x**2)*a**2*b*c*d**2*f*x - 12*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a
**2*b*d**3*e*x - 6*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*b*d**3*f*x**3 +
3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**2*c**2*d*f*x + 27*sqrt(c + d*x**2
)*sqrt(a + b*x**2)*a*b**2*c*d**2*e*x + 11*sqrt(c + d*x**2)*sqrt(a + b*x**2
)*a*b**2*c*d**2*f*x**3 + 9*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**2*d**3*e
*x**3 + 5*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**2*d**3*f*x**5 - 4*sqrt(c
+ d*x**2)*sqrt(a + b*x**2)*b**3*c**3*f*x + 9*sqrt(c + d*x**2)*sqrt(a + b*x
**2)*b**3*c**2*d*e*x + 3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**3*c**2*d*f*x
**3 + 72*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**3*c*d**2*e*x**3 + 50*sqrt(c
+ d*x**2)*sqrt(a + b*x**2)*b**3*c*d**2*f*x**5 + 45*sqrt(c + d*x**2)*sqrt(a
 + b*x**2)*b**3*d**3*e*x**5 + 35*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**3*d*
*3*f*x**7 - 16*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**
2 + b*c*x**2 + b*d*x**4),x)*a**4*d**4*f + 32*int((sqrt(c + d*x**2)*sqrt(a
+ b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a**3*b*c*d**3*f
+ 24*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x*
*2 + b*d*x**4),x)*a**3*b*d**4*e - 9*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)
*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a**2*b**2*c**2*d**2*f - 5
7*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2
+ b*d*x**4),x)*a**2*b**2*c*d**3*e - 7*int((sqrt(c + d*x**2)*sqrt(a + b*...
```

3.91 $\int \sqrt{a+bx^2}(c+dx^2)^{3/2}(e+fx^2)\,dx$

Optimal result

Integrand size = 30, antiderivative size = 526

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)\,dx=\frac{\left(8a^3d^3f+3b^3c^2(7de-2cf)+ab^2cd(49de+9cf)-a^2bd^2(14de+19cf)\right)x\sqrt{c+dx^2}}{105b^2d^2\sqrt{a+bx^2}}-\frac{\left(4a^2d^2f-3b^2c(7de-2cf)-abd(7de+6cf)\right)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{105b^2d}+\frac{(7bde-2bcf+adf)x\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}}{35bd}+\frac{fx\sqrt{a+bx^2}\left(c+dx^2\right)^{5/2}}{7d}-\frac{\sqrt{a}\left(8a^3d^3f+3b^3c^2(7de-2cf)+ab^2cd(49de+9cf)-a^2bd^2(14de+19cf)\right)\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|\right.}{105b^{5/2}d^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}+\frac{a^{3/2}\left(4a^2d^2f+3b^2c(21de-cf)-abd(7de+9cf)\right)\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{105b^{5/2}d\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
1/105*(8*a^3*d^3*f+3*b^3*c^2*(-2*c*f+7*d*e)+a*b^2*c*d*(9*c*f+49*d*e)-a^2*b
*d^2*(19*c*f+14*d*e))*x*(d*x^2+c)^(1/2)/b^2/d^2/(b*x^2+a)^(1/2)-1/105*(4*a
^2*d^2*f-3*b^2*c*(-2*c*f+7*d*e)-a*b*d*(6*c*f+7*d*e))*x*(b*x^2+a)^(1/2)*(d*
x^2+c)^(1/2)/b^2/d+1/35*(a*d*f-2*b*c*f+7*b*d*e)*x*(b*x^2+a)^(1/2)*(d*x^2+c
)^(3/2)/b/d+1/7*f*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(5/2)/d-1/105*a^(1/2)*(8*a^3
*d^3*f+3*b^3*c^2*(-2*c*f+7*d*e)+a*b^2*c*d*(9*c*f+49*d*e)-a^2*b*d^2*(19*c*f
+14*d*e))*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1
-a*d/b/c)^(1/2))/b^(5/2)/d^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/
2)+1/105*a^(3/2)*(4*a^2*d^2*f+3*b^2*c*(-c*f+21*d*e)-a*b*d*(9*c*f+7*d*e))*(
d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2)
)/b^(5/2)/d/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.03 (sec) , antiderivative size = 368, normalized size of antiderivative = 0.70

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)\,dx = \frac{-\sqrt{\frac{b}{a}}dx(a+bx^2)\left(c+dx^2\right)\left(4a^2d^2f-abd(7de+9cf+3dfx^2)-3b^2(c^2f+2cd(7e+4fx^2)+\right.}{}$$

input
```
Integrate[Sqrt[a + b*x^2]*(c + d*x^2)^(3/2)*(e + f*x^2),x]
```

output
```
(-(Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2)*(4*a^2*d^2*f - a*b*d*(7*d*e + 9*c
*f + 3*d*f*x^2) - 3*b^2*(c^2*f + 2*c*d*(7*e + 4*f*x^2) + d^2*x^2*(7*e + 5*
f*x^2)))) - I*c*(8*a^3*d^3*f + 3*b^3*c^2*(7*d*e - 2*c*f) + a*b^2*c*d*(49*d
*e + 9*c*f) - a^2*b*d^2*(14*d*e + 19*c*f))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d
*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + I*c*(-(b*c) + a*
d)*(4*a^2*d^2*f + 3*b^2*c*(-7*d*e + 2*c*f) - a*b*d*(7*d*e + 6*c*f))*Sqrt[1
 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/
(b*c)])/(105*a^2*(b/a)^(5/2)*d^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 471, normalized size of antiderivative = 0.90, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {403, 403, 403, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)\,dx$$

$$\Big\downarrow \text{403}$$

$$\frac{\int \sqrt{bx^2+a}\sqrt{dx^2+c}\left((7bde+3bcf-4adf)x^2+c(7be-af)\right)dx}{7b}+\frac{fx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{7b}$$

$$\Big\downarrow \text{403}$$

$$\frac{\frac{\int \frac{\sqrt{bx^2+a}\left(\left(3c(14de+cf)b^2-ad(14de+15cf)b+8a^2d^2f\right)x^2+c\left(4dfa^2-7bdea-8bcfa+35b^2ce\right)\right)}{\sqrt{dx^2+c}}dx}{5b}+\frac{x\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}(-4adf+3bcf+7bde)}{5b}}{7b}+\frac{fx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{7b}$$

$$\Big\downarrow \text{403}$$

$$\frac{\frac{\frac{\int \frac{\left(3c^2(7de-2cf)b^3+acd(49de+9cf)b^2-a^2d^2(14de+19cf)b+8a^3d^3f\right)x^2+ac\left(3c(21de-cf)b^2-ad(7de+9cf)b+4a^2d^2f\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3d}+\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(8a^2d^2f-abd(15\cdots\right.}{3d}\cdots}{5b}\cdots}{7b}$$

$$\frac{fx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{7b}$$

$$\Big\downarrow \text{406}$$

$$\frac{\frac{\frac{ac\left(4a^2d^2f-abd(9cf+7de)+3b^2c(21de-cf)\right)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\left(8a^3d^3f-a^2bd^2(19cf+14de)+ab^2cd(9cf+49de)+3b^3c^2(7de-2cf)\right)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\cdots}{3d}\cdots}{5b}\cdots}{7b}$$

$$\frac{fx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{7b}$$

$$\Big\downarrow \text{320}$$

$$\frac{\frac{\frac{\left(8a^3d^3f-a^2bd^2(19cf+14de)+ab^2cd(9cf+49de)+3b^3c^2(7de-2cf)\right)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}\left(4a^2d^2f-abd(9cf+7de)+3b^2c(21de-cf)\right)\text{EllipticF}\left(a\right.}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{3d}}{5b}}{7b}$$

$$\frac{fx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{7b}$$

$\downarrow$ 388

$$\frac{\frac{\frac{\left(8a^3d^3f-a^2bd^2(19cf+14de)+ab^2cd(9cf+49de)+3b^3c^2(7de-2cf)\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}\left(4a^2d^2f-abd(9cf+7de)+3b^2c(21de-cf)\right.}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx\right.}{a\left(c+dx\right.}}}}{3d}}{5b}}{7b}$$

$$\frac{fx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{7b}$$

$\downarrow$ 313

$$\frac{\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(8a^2d^2f-abd(15cf+14de)+3b^2c(cf+14de)\right)}{3d}+\frac{\frac{c^{3/2}\sqrt{a+bx^2}\left(4a^2d^2f-abd(9cf+7de)+3b^2c(21de-cf)\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+\left(8\right.}{5b}$$

$$\frac{fx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{7b}$$

input

```
Int[Sqrt[a + b*x^2]*(c + d*x^2)^(3/2)*(e + f*x^2),x]
```

output
```
(f*x*(a + b*x^2)^(3/2)*(c + d*x^2)^(3/2))/(7*b) + (((7*b*d*e + 3*b*c*f - 4
*a*d*f)*x*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(5*b) + (((8*a^2*d^2*f + 3*b^
2*c*(14*d*e + c*f) - a*b*d*(14*d*e + 15*c*f))*x*Sqrt[a + b*x^2]*Sqrt[c + d
*x^2])/(3*d) + ((8*a^3*d^3*f + 3*b^3*c^2*(7*d*e - 2*c*f) + a*b^2*c*d*(49*d
*e + 9*c*f) - a^2*b*d^2*(14*d*e + 19*c*f))*((x*Sqrt[a + b*x^2])/(b*Sqrt[c
+ d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]]
, 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[
c + d*x^2])) + (c^(3/2)*(4*a^2*d^2*f + 3*b^2*c*(21*d*e - c*f) - a*b*d*(7*d
*e + 9*c*f))*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b
*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]
))/(3*d))/(5*b))/(7*b)
```

Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

Maple [A] (verified)

Time = 5.76 (sec) , antiderivative size = 695, normalized size of antiderivative = 1.32

method	result
elliptic	$\frac{\sqrt{(b\,x^2+a)(x^2d+c)}\left(\frac{df\,x^5\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{7}+\frac{\left(a\,d^2f+2bcdf+b\,d^2e-\frac{df(6ad+6bc)}{7}\right)x^3\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{5bd}+\frac{\left(\frac{9acdf}{7}+a\,d^2e+b\,c^{2}\right.}{}\right.}{}$
risch	$-\frac{x\left(-15f\,x^4b^2d^2-3ab\,d^2f\,x^2-24b^2cf\,x^2d-21b^2d^2e\,x^2+4f\,d^2a^2-9fdcba-7ab\,d^2e-3f\,c^2b^2-42d\,b^2ce\right)\sqrt{b\,x^2+a}\,\sqrt{x^2d+c}}{105d\,b^2}+\frac{\left(-\right.}{}$
default	Expression too large to display

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)*(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/7*d*f*x^5*(
b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+1/5*(a*d^2*f+2*b*c*d*f+b*d^2*e-1/7*d*f*
(6*a*d+6*b*c))/b/d*x^3*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+1/3*(9/7*a*c*d*
f+a*d^2*e+b*c^2*f+2*b*c*d*e-1/5*(a*d^2*f+2*b*c*d*f+b*d^2*e-1/7*d*f*(6*a*d+
6*b*c))/b/d*(4*a*d+4*b*c))/b/d*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+(a*c^
2*e-1/3*(9/7*a*c*d*f+a*d^2*e+b*c^2*f+2*b*c*d*e-1/5*(a*d^2*f+2*b*c*d*f+b*d^
2*e-1/7*d*f*(6*a*d+6*b*c))/b/d*(4*a*d+4*b*c))/b/d*a*c)/(-b/a)^(1/2)*(1+b*x
^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*Elliptic
F(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-(a*c^2*f+2*a*c*d*e+b*c^2*e-3/5*
(a*d^2*f+2*b*c*d*f+b*d^2*e-1/7*d*f*(6*a*d+6*b*c))/b/d*a*c-1/3*(9/7*a*c*d*f
+a*d^2*e+b*c^2*f+2*b*c*d*e-1/5*(a*d^2*f+2*b*c*d*f+b*d^2*e-1/7*d*f*(6*a*d+6
*b*c))/b/d*(4*a*d+4*b*c))/b/d*(2*a*d+2*b*c))*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1
/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(
-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*
c)/c/b)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 494, normalized size of antiderivative = 0.94

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)\,dx =$$

$$-\frac{\sqrt{bd}(7\,(3\,b^3c^3d+7\,ab^2c^2d^2-2\,a^2bcd^3)e-(6\,b^3c^4-9\,ab^2c^3d+19\,a^2bc^2d^2-8\,a^3cd^3)f)x\sqrt{-\frac{c}{d}}E(\arcsin\Big(}{}$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)*(f*x^2+e),x, algorithm="fricas")
```

output
```
-1/105*(sqrt(b*d)*(7*(3*b^3*c^3*d + 7*a*b^2*c^2*d^2 - 2*a^2*b*c*d^3)*e - (
6*b^3*c^4 - 9*a*b^2*c^3*d + 19*a^2*b*c^2*d^2 - 8*a^3*c*d^3)*f)*x*sqrt(-c/d
)*elliptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - sqrt(b*d)*(7*(3*b^3*c^3*d
+ 7*a*b^2*c^2*d^2 - a^2*b*d^4 - (2*a^2*b - 9*a*b^2)*c*d^3)*e - (6*b^3*c^4
- 9*a*b^2*c^3*d - 4*a^3*d^4 + (19*a^2*b + 3*a*b^2)*c^2*d^2 - (8*a^3 - 9*a^
2*b)*c*d^3)*f)*x*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)/x), a*d/(b*c)) -
(15*b^3*d^4*f*x^6 + 3*(7*b^3*d^4*e + (8*b^3*c*d^3 + a*b^2*d^4)*f)*x^4 + (7
*(6*b^3*c*d^3 + a*b^2*d^4)*e + (3*b^3*c^2*d^2 + 9*a*b^2*c*d^3 - 4*a^2*b*d^
4)*f)*x^2 + 7*(3*b^3*c^2*d^2 + 7*a*b^2*c*d^3 - 2*a^2*b*d^4)*e - (6*b^3*c^3
*d - 9*a*b^2*c^2*d^2 + 19*a^2*b*c*d^3 - 8*a^3*d^4)*f)*sqrt(b*x^2 + a)*sqrt
(d*x^2 + c))/(b^3*d^3*x)
```

Sympy [F]

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)\,dx=\int \sqrt{a+bx^2}\left(c+dx^2\right)^{\frac{3}{2}}\left(e+fx^2\right)\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(3/2)*(f*x**2+e),x)
```

output
```
Integral(sqrt(a + b*x**2)*(c + d*x**2)**(3/2)*(e + f*x**2), x)
```

Maxima [F]

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)\,dx=\int \sqrt{bx^2+a}\left(dx^2+c\right)^{\frac{3}{2}}\left(fx^2+e\right)\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)*(f*x^2+e),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)*(f*x^2 + e), x)
```

Giac [F]

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)\,dx=\int \sqrt{bx^2+a}\left(dx^2+c\right)^{\frac{3}{2}}\left(fx^2+e\right)dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)*(f*x^2+e),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)*(f*x^2 + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)\,dx=\int \sqrt{b\,x^2+a}\,\left(d\,x^2+c\right)^{3/2}\,\left(f\,x^2+e\right)\,dx$$

input
```
int((a + b*x^2)^(1/2)*(c + d*x^2)^(3/2)*(e + f*x^2),x)
```

output
```
int((a + b*x^2)^(1/2)*(c + d*x^2)^(3/2)*(e + f*x^2), x)
```

Reduce [F]

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)\,dx=\text{Too large to display}$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)*(f*x^2+e),x)
```

output
```
( - 4*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*d**2*f*x + 9*sqrt(c + d*x**2)
*sqrt(a + b*x**2)*a*b*c*d*f*x + 7*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b*d*
*2*e*x + 3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b*d**2*f*x**3 + 3*sqrt(c +
d*x**2)*sqrt(a + b*x**2)*b**2*c**2*f*x + 42*sqrt(c + d*x**2)*sqrt(a + b*x*
*2)*b**2*c*d*e*x + 24*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**2*c*d*f*x**3 +
21*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**2*d**2*e*x**3 + 15*sqrt(c + d*x**2
)*sqrt(a + b*x**2)*b**2*d**2*f*x**5 + 8*int((sqrt(c + d*x**2)*sqrt(a + b*x
**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a**3*d**3*f - 19*int(
(sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*
x**4),x)*a**2*b*c*d**2*f - 14*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)
/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a**2*b*d**3*e + 9*int((sqrt(c +
 d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*
a*b**2*c**2*d*f + 49*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a
*d*x**2 + b*c*x**2 + b*d*x**4),x)*a*b**2*c*d**2*e - 6*int((sqrt(c + d*x**2
)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*b**3*c*
*3*f + 21*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b
*c*x**2 + b*d*x**4),x)*b**3*c**2*d*e + 4*int((sqrt(c + d*x**2)*sqrt(a + b*
x**2))/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a**3*c*d**2*f - 9*int((sq
rt(c + d*x**2)*sqrt(a + b*x**2))/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)
*a**2*b*c**2*d*f - 7*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c + a*d...
```

3.92 $\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}(e+fx^2)}{x^2} dx$

Optimal result

Integrand size = 33, antiderivative size = 430

$$\begin{aligned}
&\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}(e+fx^2)}{x^2} dx = \\
&-\frac{(2a^2d^2f-b^2c(35de+3cf)-abd(5de+7cf))x\sqrt{c+dx^2}}{15bd\sqrt{a+bx^2}} \\
&+\frac{(20bde+3bcf+adf)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{15b} \\
&+\frac{(5be+af)x\sqrt{a+bx^2}(c+dx^2)^{3/2}}{5a}-\frac{e(a+bx^2)^{3/2}(c+dx^2)^{3/2}}{ax} \\
&+\frac{\sqrt{a}(2a^2d^2f-b^2c(35de+3cf)-abd(5de+7cf))\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{15b^{3/2}d\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} \\
&+\frac{\sqrt{a}(15b^2ce-a^2df+ab(25de+9cf))\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{15b^{3/2}\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}
\end{aligned}$$

output
```
-1/15*(2*a^2*d^2*f-b^2*c*(3*c*f+35*d*e)-a*b*d*(7*c*f+5*d*e))*x*(d*x^2+c)^(
1/2)/b/d/(b*x^2+a)^(1/2)+1/15*(a*d*f+3*b*c*f+20*b*d*e)*x*(b*x^2+a)^(1/2)*(
d*x^2+c)^(1/2)/b+1/5*(a*f+5*b*e)*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/a-e*(b*
x^2+a)^(3/2)*(d*x^2+c)^(3/2)/a/x+1/15*a^(1/2)*(2*a^2*d^2*f-b^2*c*(3*c*f+35
*d*e)-a*b*d*(7*c*f+5*d*e))*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+
b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/b^(3/2)/d/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c
/(b*x^2+a))^(1/2)+1/15*a^(1/2)*(15*b^2*c*e-a^2*d*f+a*b*(9*c*f+25*d*e))*(d*
x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/
b^(3/2)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.91 (sec) , antiderivative size = 288, normalized size of antiderivative = 0.67

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}\,(e+fx^2)}{x^2}\,dx = \frac{\sqrt{\frac{b}{a}}d(a+bx^2)\,(c+dx^2)\,(adfx^2+b(-15ce+5dex^2+6cfx^2+3df$$

input
```
Integrate[(Sqrt[a + b*x^2]*(c + d*x^2)^(3/2)*(e + f*x^2))/x^2,x]
```

output
```
(Sqrt[b/a]*d*(a + b*x^2)*(c + d*x^2)*(a*d*f*x^2 + b*(-15*c*e + 5*d*e*x^2 +
 6*c*f*x^2 + 3*d*f*x^4)) + I*c*(2*a^2*d^2*f - b^2*c*(35*d*e + 3*c*f) - a*b
*d*(5*d*e + 7*c*f))*x*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*
ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*c*(-(b*c) + a*d)*(20*b*d*e + 3*b*c*
f + a*d*f)*x*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[S
qrt[b/a]*x], (a*d)/(b*c)])/(15*b*Sqrt[b/a]*d*x*Sqrt[a + b*x^2]*Sqrt[c + d*
x^2])
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 415, normalized size of antiderivative = 0.97, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {442, 403, 403, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)}{x^2}\,dx$$

$$\Big\downarrow \; 442$$

$$\frac{\int \sqrt{bx^2+a}\sqrt{dx^2+c}\left(d(5be+af)x^2+2bce+3ade+acf\right)dx}{a}-\frac{e\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{ax}$$

$$\Big\downarrow \; 403$$

$$\frac{\frac{\int \frac{\sqrt{bx^2+a}\left(d\left(-2dfa^2+b(5de+6cf)a+15b^2ce\right)x^2+c\left(-dfa^2+5b(2de+cf)a+10b^2ce\right)\right)}{\sqrt{dx^2+c}}dx}{5b}+\frac{dx\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}(af+5be)}{5b}}{a}-\frac{e\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{ax}$$

$$\Big\downarrow \; 403$$

$$\frac{\frac{\frac{\int \frac{ad\left(c\left(-dfa^2+b(25de+9cf)a+15b^2ce\right)-\left(-c(35de+3cf)b^2-ad(5de+7cf)b+2a^2d^2f\right)x^2\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3d}+\frac{1}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(-2a^2df+ab(6cf+5de)+15b^2ce\right)}{5b}+}{a}$$

$$\frac{e\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{ax}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{1}{3}a\int \frac{c\left(-dfa^2+b(25de+9cf)a+15b^2ce\right)-\left(-c(35de+3cf)b^2-ad(5de+7cf)b+2a^2d^2f\right)x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{1}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(-2a^2df+ab(6cf+5de)+15b^2ce\right)}{5b}+}{a}$$

$$\frac{e\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{ax}$$

$$\Big\downarrow \; 406$$

$$\frac{\frac{\frac{1}{3}a\left(c\left(a^2(-d)f+ab(9cf+25de)+15b^2ce\right)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx-\left(2a^2d^2f-abd(7cf+5de)+b^2(-c)(3cf+35de)\right)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)+\frac{1}{3}x\sqrt{a+b}}{5b}}{a} \quad \frac{e\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{ax}$$

$\downarrow$ 320

$$\frac{\frac{\frac{1}{3}a\left(\frac{c^{3/2}\sqrt{a+bx^2}\left(a^2(-d)f+ab(9cf+25de)+15b^2ce\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\left(2a^2d^2f-abd(7cf+5de)+b^2(-c)(3cf+35de)\right)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+}}\right.}{5b}}{a} \quad \frac{e\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{ax}$$

$\downarrow$ 388

$$\frac{\frac{\frac{1}{3}a\left(\frac{c^{3/2}\sqrt{a+bx^2}\left(a^2(-d)f+ab(9cf+25de)+15b^2ce\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\left(2a^2d^2f-abd(7cf+5de)+b^2(-c)(3cf+35de)\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int}{}\right.\right.}{5b}}{a} \quad \frac{e\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{ax}$$

$\downarrow$ 313

$$\frac{\frac{\frac{1}{3}a\left(\frac{c^{3/2}\sqrt{a+bx^2}\left(a^2(-d)f+ab(9cf+25de)+15b^2ce\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\left(2a^2d^2f-abd(7cf+5de)+b^2(-c)(3cf+35de)\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}}{}\right.\right.}{5b}}{a} \quad \frac{e\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{ax}$$

input

```
Int[(Sqrt[a + b*x^2]*(c + d*x^2)^(3/2)*(e + f*x^2))/x^2,x]
```

output
```
-((e*(a + b*x^2)^(3/2)*(c + d*x^2)^(3/2))/(a*x)) + ((d*(5*b*e + a*f)*x*(a
+ b*x^2)^(3/2)*Sqrt[c + d*x^2])/(5*b) + (((15*b^2*c*e - 2*a^2*d*f + a*b*(5
*d*e + 6*c*f))*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/3 + (a*(-((2*a^2*d^2*f -
 b^2*c*(35*d*e + 3*c*f) - a*b*d*(5*d*e + 7*c*f))*((x*Sqrt[a + b*x^2])/(b*S
qrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sq
rt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]
*Sqrt[c + d*x^2]))) + (c^(3/2)*(15*b^2*c*e - a^2*d*f + a*b*(25*d*e + 9*c*f
))*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)]
)/(a*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])))/3)/(
5*b))/a
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 442
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^q/(a*g*(m + 1))), x] - Simp[1/(a*g^2*(m + 1))   Int[(g*x)
^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[c*(b*e - a*f)*(m + 1) + e*2
*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*2*(p + q + 1))*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && GtQ[q, 0] && LtQ[m, -1]
&&  !(EqQ[q, 1] && SimplerQ[e + f*x^2, c + d*x^2])
```

Maple [A] (verified)

Time = 5.25 (sec) , antiderivative size = 509, normalized size of antiderivative = 1.18

method	result
elliptic	$\frac{\sqrt{(b x^2+a)(x^2 d+c)}\left(-\frac{ce\sqrt{bd x^4+ad x^2+x^2bc+ac}}{x}+\frac{df x^3\sqrt{bd x^4+ad x^2+x^2bc+ac}}{5}+\frac{\left(a d^2 f+2bcdf+b d^2 e-\frac{df(4ad+4bc)}{5}\right)x\sqrt{bd x^4+ad x^2+x}}{3bd}\right.}{}$
risch	$\frac{\sqrt{b x^2+a}\,\sqrt{x^2 d+c}\,(3bdf x^4+adf x^2+6bcf x^2+5bde x^2-15bce)}{15bx}-\frac{\left(-\frac{\left(2f d^2a^2-7fdcba-5ab d^2e-3f c^2b^2-35d b^2ce\right)c\sqrt{1+\frac{b x^2}{a}}\sqrt{1+\frac{d}{c}}}{\sqrt{-\frac{b}{a}}\sqrt{bd x}}\right.}{}$
default	$\frac{\sqrt{b x^2+a}\,\sqrt{x^2 d+c}\left(3\sqrt{-\frac{b}{a}}\,b^2d^3f x^8+4\sqrt{-\frac{b}{a}}\,ab d^3 f x^6+9\sqrt{-\frac{b}{a}}\,b^2c d^2 f x^6+5\sqrt{-\frac{b}{a}}\,b^2d^3e x^6+\sqrt{-\frac{b}{a}}\,a^2d^3f x^4+10\sqrt{-\frac{b}{a}}\,abc d^2 f x^4\right.}{}$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)*(f*x^2+e)/x^2,x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-c*e*(b*d*x^4
+a*d*x^2+b*c*x^2+a*c)^(1/2)/x+1/5*d*f*x^3*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1
/2)+1/3*(a*d^2*f+2*b*c*d*f+b*d^2*e-1/5*d*f*(4*a*d+4*b*c))/b/d*x*(b*d*x^4+a
*d*x^2+b*c*x^2+a*c)^(1/2)+(a*c^2*f+2*a*c*d*e+b*c^2*e-1/3*(a*d^2*f+2*b*c*d*
f+b*d^2*e-1/5*d*f*(4*a*d+4*b*c))/b/d*a*c)/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(
1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1
/2),(-1+(a*d+b*c)/c/b)^(1/2))-(7/5*a*c*d*f+a*d^2*e+b*c^2*f+3*b*c*d*e-1/3*(
a*d^2*f+2*b*c*d*f+b*d^2*e-1/5*d*f*(4*a*d+4*b*c))/b/d*(2*a*d+2*b*c))*c/(-b/
a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)
^(1/2)/d*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(
-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))))
```

Fricas [F]

$$\int \frac{\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)}{x^2}\,dx=\int \frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{\frac{3}{2}}\left(fx^2+e\right)}{x^2}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)*(f*x^2+e)/x^2,x, algorithm="fric
as")
```

output
```
integral((d*f*x^4 + (d*e + c*f)*x^2 + c*e)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)
/x^2, x)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)}{x^2}\,dx=\int \frac{\sqrt{a+bx^2}\left(c+dx^2\right)^{\frac{3}{2}}\left(e+fx^2\right)}{x^2}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(3/2)*(f*x**2+e)/x**2,x)
```

output
```
Integral(sqrt(a + b*x**2)*(c + d*x**2)**(3/2)*(e + f*x**2)/x**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}\left(e+fx^2\right)}{x^2}\,dx=\int \frac{\sqrt{bx^2+a}(dx^2+c)^{\frac{3}{2}}(fx^2+e)}{x^2}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)*(f*x^2+e)/x^2,x, algorithm="maxi
ma")
```

output
```
integrate(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)*(f*x^2 + e)/x^2, x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}\left(e+fx^2\right)}{x^2}\,dx=\int \frac{\sqrt{bx^2+a}(dx^2+c)^{\frac{3}{2}}(fx^2+e)}{x^2}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)*(f*x^2+e)/x^2,x, algorithm="giac
")
```

output
```
integrate(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)*(f*x^2 + e)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}\left(e+fx^2\right)}{x^2}\,dx=\int \frac{\sqrt{b\,x^2+a}\,{\left(d\,x^2+c\right)}^{3/2}\,\left(f\,x^2+e\right)}{x^2}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(3/2)*(e + f*x^2))/x^2,x)
```

output
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(3/2)*(e + f*x^2))/x^2, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}\,(e+fx^2)}{x^2}\,dx = \frac{-2\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,a^2d^2f+7\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,abcdf+5\sqrt{}}{}$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)*(f*x^2+e)/x^2,x)
```

output
```
( - 2*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*d**2*f + 7*sqrt(c + d*x**2)*s
qrt(a + b*x**2)*a*b*c*d*f + 5*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b*d**2*e
 + sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b*d**2*f*x**2 + 3*sqrt(c + d*x**2)*
sqrt(a + b*x**2)*b**2*c**2*f + 20*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**2*c
*d*e + 6*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**2*c*d*f*x**2 + 5*sqrt(c + d*
x**2)*sqrt(a + b*x**2)*b**2*d**2*e*x**2 + 3*sqrt(c + d*x**2)*sqrt(a + b*x*
*2)*b**2*d**2*f*x**4 - 2*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c*x**2
 + a*d*x**4 + b*c*x**4 + b*d*x**6),x)*a**3*c*d**2*f*x + 7*int((sqrt(c + d*
x**2)*sqrt(a + b*x**2))/(a*c*x**2 + a*d*x**4 + b*c*x**4 + b*d*x**6),x)*a**
2*b*c**2*d*f*x + 5*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c*x**2 + a*d
*x**4 + b*c*x**4 + b*d*x**6),x)*a**2*b*c*d**2*e*x + 3*int((sqrt(c + d*x**2
)*sqrt(a + b*x**2))/(a*c*x**2 + a*d*x**4 + b*c*x**4 + b*d*x**6),x)*a*b**2*
c**3*f*x + 35*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c*x**2 + a*d*x**4
 + b*c*x**4 + b*d*x**6),x)*a*b**2*c**2*d*e*x - int((sqrt(c + d*x**2)*sqrt(
a + b*x**2))/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a**2*b*c*d**2*f*x +
 9*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c + a*d*x**2 + b*c*x**2 + b*
d*x**4),x)*a*b**2*c**2*d*f*x + 25*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/
(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a*b**2*c*d**2*e*x + 15*int((sqrt
(c + d*x**2)*sqrt(a + b*x**2))/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*b
**3*c**2*d*e*x)/(15*b**2*d*x)
```

3.93 $\int \frac{\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)}{x^4}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 396

$$\begin{aligned}
\int \frac{\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)}{x^4}\,dx &= \frac{\left(b^2ce+a^2df+7ab(de+cf)\right)x\sqrt{c+dx^2}}{3a\sqrt{a+bx^2}} \\
&+ \frac{(4bde+3bcf+adf)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3a} \\
&- \frac{(de+cf)\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{ax} - \frac{e\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{3ax^3} \\
&- \frac{\left(b^2ce+a^2df+7ab(de+cf)\right)\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{3\sqrt{a}\sqrt{b}\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} \\
&+ \frac{\sqrt{a}(bc(5de+3cf)+ad(3de+5cf))\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{3\sqrt{b}c\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}
\end{aligned}$$

output
```
1/3*(b^2*c*e+a^2*d*f+7*a*b*(c*f+d*e))*x*(d*x^2+c)^(1/2)/a/(b*x^2+a)^(1/2)+
1/3*(a*d*f+3*b*c*f+4*b*d*e)*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a-(c*f+d*e)*
(b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/a/x-1/3*e*(b*x^2+a)^(3/2)*(d*x^2+c)^(3/2)/
a/x^3-1/3*(b^2*c*e+a^2*d*f+7*a*b*(c*f+d*e))*(d*x^2+c)^(1/2)*EllipticE(b^(1
/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/a^(1/2)/b^(1/2)/(b*x^2+
a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)+1/3*a^(1/2)*(b*c*(3*c*f+5*d*e)+a*
d*(5*c*f+3*d*e))*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2))
,(1-a*d/b/c)^(1/2))/b^(1/2)/c/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1
/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.04 (sec) , antiderivative size = 272, normalized size of antiderivative = 0.69

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}(e+fx^2)}{x^4}\,dx = \frac{\sqrt{\frac{b}{a}}\left(-\sqrt{\frac{b}{a}}(a+bx^2)(c+dx^2)(bcex^2+a(ce+4dex^2+3cfx^2-d\right.}{}$$

input
```
Integrate[(Sqrt[a + b*x^2]*(c + d*x^2)^(3/2)*(e + f*x^2))/x^4,x]
```

output
```
(Sqrt[b/a]*(-(Sqrt[b/a]*(a + b*x^2)*(c + d*x^2)*(b*c*e*x^2 + a*(c*e + 4*d*
e*x^2 + 3*c*f*x^2 - d*f*x^4))) - I*c*(b^2*c*e + a^2*d*f + 7*a*b*(d*e + c*f
))*x^3*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/
a]*x], (a*d)/(b*c)] - I*(-(b*c) + a*d)*(b*c*e + 3*a*d*e + 4*a*c*f)*x^3*Sqr
t[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*
d)/(b*c)]))/(3*b*x^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 371, normalized size of antiderivative = 0.94, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.303$, Rules used = {442, 27, 442, 27, 403, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)}{x^4}dx$$

$$\Big\downarrow \text{442}$$

$$\frac{\int \frac{3\sqrt{bx^2+a}\sqrt{dx^2+c}(d(be+af)x^2+a(de+cf))}{x^2}dx}{3a}-\frac{e\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{3ax^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}(d(be+af)x^2+a(de+cf))}{x^2}dx}{a}-\frac{e\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{3ax^3}$$

$$\Big\downarrow \text{442}$$

$$\frac{\frac{\int \frac{a\sqrt{bx^2+a}\left(d(4bde+3bcf+adf)x^2+ad(de+2cf)+bc(3de+2cf)\right)}{\sqrt{dx^2+c}}dx}{a}-\frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}(cf+de)}{x}}{a}-\frac{e\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{3ax^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sqrt{bx^2+a}(d(4bde+3bcf+adf)x^2+ad(de+2cf)+bc(3de+2cf))}{\sqrt{dx^2+c}}dx-\frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}(cf+de)}{x}}{a}-\frac{e\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{3ax^3}$$

$$\Big\downarrow \text{403}$$

$$\frac{\frac{\int \frac{d\left(d\left(dfa^2+7b(de+cf)a+b^2ce\right)x^2+a(bc(5de+3cf)+ad(3de+5cf))\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3d}-\frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}(cf+de)}{x}+\frac{1}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}(adf+3bc}{a}-\frac{e\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{3ax^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{3}\int \frac{d(dfa^2+7b(de+cf)a+b^2ce)x^2+a(bc(5de+3cf)+ad(3de+5cf))}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx-\frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}(cf+de)}{x}+\frac{1}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}(adf+}{a}-\frac{e\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{3ax^3}$$

$$\Big\downarrow \text{406}$$

$$\frac{\frac{\frac{1}{3}\left(d\left(a^2df+7ab(cf+de)+b^2ce\right)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+a(ad(5cf+3de)+bc(3cf+5de))\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)-}{a}}{\frac{e\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{3ax^3}}$$

$\downarrow$ 320

$$\frac{\frac{\frac{1}{3}\left(d\left(a^2df+7ab(cf+de)+b^2ce\right)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{\sqrt{c}\sqrt{a+bx^2}(ad(5cf+3de)+bc(3cf+5de))\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{a}}{\frac{e\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{3ax^3}}$$

$\downarrow$ 388

$$\frac{\frac{\frac{1}{3}\left(d\left(a^2df+7ab(cf+de)+b^2ce\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}}dx}{b}\right)+\frac{\sqrt{c}\sqrt{a+bx^2}(ad(5cf+3de)+bc(3cf+5de))\text{EllipticF}\left(\text{arctan}\right.}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right.}{a}}{\frac{e\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{3ax^3}}$$

$\downarrow$ 313

$$\frac{\frac{\frac{1}{3}\left(d\left(a^2df+7ab(cf+de)+b^2ce\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)+\frac{\sqrt{c}\sqrt{a+bx^2}(ad(5cf+3de)+bc(3cf+5de}{\sqrt{d}\sqrt{c+dx^2}\sqrt{}}\right.}{a}}{\frac{e\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{3ax^3}}$$

input
```
Int[(Sqrt[a + b*x^2]*(c + d*x^2)^(3/2)*(e + f*x^2))/x^4,x]
```

output
```
-1/3*(e*(a + b*x^2)^(3/2)*(c + d*x^2)^(3/2))/(a*x^3) + (((4*b*d*e + 3*b*c*
f + a*d*f)*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/3 - ((d*e + c*f)*(a + b*x^2)
^(3/2)*Sqrt[c + d*x^2])/x + (d*(b^2*c*e + a^2*d*f + 7*a*b*(d*e + c*f))*((x
*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE
[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*
x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (Sqrt[c]*(b*c*(5*d*e + 3*c*f) +
 a*d*(3*d*e + 5*c*f))*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]
], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c
 + d*x^2]))/3)/a
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 442
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^q/(a*g*(m + 1))), x] - Simp[1/(a*g^2*(m + 1))   Int[(g*x)
^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[c*(b*e - a*f)*(m + 1) + e*2
*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*2*(p + q + 1))*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && GtQ[q, 0] && LtQ[m, -1]
&&  !(EqQ[q, 1] && SimplerQ[e + f*x^2, c + d*x^2])
```

Maple [A] (verified)

Time = 5.71 (sec) , antiderivative size = 433, normalized size of antiderivative = 1.09

method	result
elliptic	$\frac{\sqrt{(bx^2+a)(x^2d+c)}\left(-\frac{ce\sqrt{bdx^4+adx^2+x^2bc+ac}}{3x^3}-\frac{(3acf+4ade+bce)\sqrt{bdx^4+adx^2+x^2bc+ac}}{3ax}+\frac{dfx\sqrt{bdx^4+adx^2+x^2bc+ac}}{3}+\frac{\left(\frac{5}{3}acdf+a\,c\cdots\right.}{\cdots}\right.}{\cdots}$
risch	$-\frac{\sqrt{bx^2+a}\sqrt{x^2d+c}\,(-adfx^4+3acfx^2+4adex^2+bcex^2+ace)}{3x^3a}+\frac{\left(-\frac{(a^2df+7abcf+7abde+ceb^2)c\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\left(\text{EllipticF}\left(x\sqrt{\cdots}\right.\right.}{\sqrt{-\frac{b}{a}}\sqrt{bdx^4+adx^2\cdots}}\right.}{\cdots}$
default	$\frac{\sqrt{bx^2+a}\sqrt{x^2d+c}\left(\sqrt{-\frac{b}{a}}\,abd^2fx^8+\sqrt{-\frac{b}{a}}\,a^2d^2fx^6-2\sqrt{-\frac{b}{a}}\,abcdfx^6-4\sqrt{-\frac{b}{a}}\,abd^2ex^6-\sqrt{-\frac{b}{a}}\,b^2cdex^6+4\sqrt{\frac{bx^2+a}{a}}\sqrt{\frac{x^2d+c}{c}}\,\text{El}\cdots\right.}{\cdots}$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)*(f*x^2+e)/x^4,x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-1/3*c*e*(b*d
*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/x^3-1/3*(3*a*c*f+4*a*d*e+b*c*e)/a*(b*d*x^4
+a*d*x^2+b*c*x^2+a*c)^(1/2)/x+1/3*d*f*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2
)+(5/3*a*c*d*f+a*d^2*e+b*c^2*f+5/3*b*c*d*e)/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)
*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^
(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-(a*d^2*f+2*b*c*d*f+b*d^2*e+1/3*b*d*(3*a*c*
f+4*a*d*e+b*c*e)/a-1/3*d*f*(2*a*d+2*b*c))*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)
*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(-b/
a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/
c/b)^(1/2))))
```

Fricas [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}\left(e+fx^2\right)}{x^4}\,dx=\int \frac{\sqrt{bx^2+a}(dx^2+c)^{\frac{3}{2}}(fx^2+e)}{x^4}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)*(f*x^2+e)/x^4,x, algorithm="fric
as")
```

output
```
integral((d*f*x^4 + (d*e + c*f)*x^2 + c*e)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)
/x^4, x)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}\left(e+fx^2\right)}{x^4}\,dx=\int \frac{\sqrt{a+bx^2}\left(c+dx^2\right)^{\frac{3}{2}}\left(e+fx^2\right)}{x^4}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(3/2)*(f*x**2+e)/x**4,x)
```

output
```
Integral(sqrt(a + b*x**2)*(c + d*x**2)**(3/2)*(e + f*x**2)/x**4, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}\left(e+fx^2\right)}{x^4}\,dx = \int \frac{\sqrt{bx^2+a}(dx^2+c)^{\frac{3}{2}}(fx^2+e)}{x^4}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)*(f*x^2+e)/x^4,x, algorithm="maxi
ma")
```

output
```
integrate(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)*(f*x^2 + e)/x^4, x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}\left(e+fx^2\right)}{x^4}\,dx = \int \frac{\sqrt{bx^2+a}(dx^2+c)^{\frac{3}{2}}(fx^2+e)}{x^4}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)*(f*x^2+e)/x^4,x, algorithm="giac
")
```

output
```
integrate(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)*(f*x^2 + e)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}\left(e+fx^2\right)}{x^4}\,dx = \int \frac{\sqrt{b\,x^2+a}\,{\left(d\,x^2+c\right)}^{3/2}\,\left(f\,x^2+e\right)}{x^4}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(3/2)*(e + f*x^2))/x^4,x)
```

output
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(3/2)*(e + f*x^2))/x^4, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}(e+fx^2)}{x^4}\,dx = \text{too large to display}$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)*(f*x^2+e)/x^4,x)
```

output
```
( - sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*c*d*f + 2*sqrt(c + d*x**2)*sqrt
(a + b*x**2)*a**2*d**2*f*x**2 - 7*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b*c*
*2*f - 9*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b*c*d*e + 10*sqrt(c + d*x**2)
*sqrt(a + b*x**2)*a*b*c*d*f*x**2 + 6*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b
*d**2*e*x**2 + 2*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b*d**2*f*x**4 - 3*sqr
t(c + d*x**2)*sqrt(a + b*x**2)*b**2*c**2*e + 8*sqrt(c + d*x**2)*sqrt(a + b
*x**2)*b**2*c**2*f*x**2 + 6*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**2*c*d*e*x
**2 + 2*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**2*c*d*f*x**4 - 3*int((sqrt(c
+ d*x**2)*sqrt(a + b*x**2))/(a**2*c*d*x**4 + a**2*d**2*x**6 + a*b*c**2*x**
4 + 2*a*b*c*d*x**6 + a*b*d**2*x**8 + b**2*c**2*x**6 + b**2*c*d*x**8),x)*a*
*4*c**2*d**2*f*x**3 - 24*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a**2*c*d
*x**4 + a**2*d**2*x**6 + a*b*c**2*x**4 + 2*a*b*c*d*x**6 + a*b*d**2*x**8 +
b**2*c**2*x**6 + b**2*c*d*x**8),x)*a**3*b*c**3*d*f*x**3 - 21*int((sqrt(c +
 d*x**2)*sqrt(a + b*x**2))/(a**2*c*d*x**4 + a**2*d**2*x**6 + a*b*c**2*x**4
 + 2*a*b*c*d*x**6 + a*b*d**2*x**8 + b**2*c**2*x**6 + b**2*c*d*x**8),x)*a**
3*b*c**2*d**2*e*x**3 - 21*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a**2*c*
d*x**4 + a**2*d**2*x**6 + a*b*c**2*x**4 + 2*a*b*c*d*x**6 + a*b*d**2*x**8 +
 b**2*c**2*x**6 + b**2*c*d*x**8),x)*a**2*b**2*c**4*f*x**3 - 24*int((sqrt(c
 + d*x**2)*sqrt(a + b*x**2))/(a**2*c*d*x**4 + a**2*d**2*x**6 + a*b*c**2*x*
*4 + 2*a*b*c*d*x**6 + a*b*d**2*x**8 + b**2*c**2*x**6 + b**2*c*d*x**8),x...
```

3.94 $\int \frac{\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)}{x^6}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 453

$$\int \frac{\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)}{x^6}\,dx = -\frac{b(2b^2c^2e-abc(7de+5cf)-a^2d(3de+35cf))\,x\sqrt{c+dx^2}}{15a^2c\sqrt{a+bx^2}} - \frac{d(bce+3ade+20acf)\sqrt{a+bx^2}\sqrt{c+dx^2}}{15acx} + \frac{(2bce-3ade-5acf)\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{15a^2x^3} - \frac{e\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{5ax^5} + \frac{\sqrt{b}(2b^2c^2e-abc(7de+5cf)-a^2d(3de+35cf))\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{15a^{3/2}c\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} - \frac{d(b^2ce-15a^2df-ab(9de+25cf))\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{15\sqrt{a}\sqrt{b}c\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
-1/15*b*(2*b^2*c^2*e-a*b*c*(5*c*f+7*d*e)-a^2*d*(35*c*f+3*d*e))*x*(d*x^2+c)
^(1/2)/a^2/c/(b*x^2+a)^(1/2)-1/15*d*(20*a*c*f+3*a*d*e+b*c*e)*(b*x^2+a)^(1/
2)*(d*x^2+c)^(1/2)/a/c/x+1/15*(-5*a*c*f-3*a*d*e+2*b*c*e)*(b*x^2+a)^(3/2)*(
d*x^2+c)^(1/2)/a^2/x^3-1/5*e*(b*x^2+a)^(3/2)*(d*x^2+c)^(3/2)/a/x^5+1/15*b^
(1/2)*(2*b^2*c^2*e-a*b*c*(5*c*f+7*d*e)-a^2*d*(35*c*f+3*d*e))*(d*x^2+c)^(1/
2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/a^(3/2
)/c/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)-1/15*d*(b^2*c*e-15*a^2
*d*f-a*b*(25*c*f+9*d*e))*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/
a^(1/2)),(1-a*d/b/c)^(1/2))/a^(1/2)/b^(1/2)/c/(b*x^2+a)^(1/2)/(a*(d*x^2+c)
/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.48 (sec) , antiderivative size = 349, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}\left(e+fx^2\right)}{x^6}\,dx = \frac{-\sqrt{\frac{b}{a}}(a+bx^2)\left(c+dx^2\right)\left(-2b^2c^2ex^4+abcx^2(7dex^2+c(e+5fx^2)\right.}{}$$

input
```
Integrate[(Sqrt[a + b*x^2]*(c + d*x^2)^(3/2)*(e + f*x^2))/x^6,x]
```

output
```
(-(Sqrt[b/a]*(a + b*x^2)*(c + d*x^2)*(-2*b^2*c^2*e*x^4 + a*b*c*x^2*(7*d*e*
x^2 + c*(e + 5*f*x^2)) + a^2*(3*d^2*e*x^4 + c^2*(3*e + 5*f*x^2) + c*d*(6*e
*x^2 + 20*f*x^4)))) - I*b*c*(-2*b^2*c^2*e + a*b*c*(7*d*e + 5*c*f) + a^2*d*
(3*d*e + 35*c*f))*x^5*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*
ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*c*(-(b*c) + a*d)*(-2*b^2*c*e + 15*a
^2*d*f + a*b*(6*d*e + 5*c*f))*x^5*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*
EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(15*a^2*Sqrt[b/a]*c*x^5*Sq
rt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 419, normalized size of antiderivative = 0.92, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {442, 25, 442, 25, 27, 442, 25, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)}{x^6}dx$$

$$\Big\downarrow \text{442}$$

$$\frac{\int -\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(-d(be+5af)x^2+2bce-3ade-5acf\right)}{x^4}dx}{5a}-\frac{e\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{5ax^5}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(-d(be+5af)x^2+2bce-3ade-5acf\right)}{x^4}dx}{5a}-\frac{e\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{5ax^5}$$

$$\Big\downarrow \text{442}$$

$$-\frac{\frac{\int -\frac{d\sqrt{bx^2+a}\left(a(bce+3ade+20acf)-\left(-15dfa^2-b(6de+5cf)a+2b^2ce\right)x^2\right)}{x^2\sqrt{dx^2+c}}dx}{3a}-\frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}(-5acf-3ade+2bce)}{3ax^3}}{5a}-\frac{e\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{5ax^5}$$

$$\Big\downarrow \text{25}$$

$$-\frac{-\frac{\int \frac{d\sqrt{bx^2+a}\left(a(bce+3ade+20acf)-\left(-15dfa^2-b(6de+5cf)a+2b^2ce\right)x^2\right)}{x^2\sqrt{dx^2+c}}dx}{3a}-\frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}(-5acf-3ade+2bce)}{3ax^3}}{5a}-\frac{e\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{5ax^5}$$

$$\Big\downarrow \text{27}$$

$$-\frac{-\frac{d\int \frac{\sqrt{bx^2+a}\left(a(bce+3ade+20acf)-\left(-15dfa^2-b(6de+5cf)a+2b^2ce\right)x^2\right)}{x^2\sqrt{dx^2+c}}dx}{3a}-\frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}(-5acf-3ade+2bce)}{3ax^3}}{5a}-\frac{e\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{5ax^5}$$

$\Big\downarrow$ 442

$$-\frac{-\frac{d\left(\frac{\int -\frac{b\left(-d(3de+35cf)a^2-bc(7de+5cf)a+2b^2c^2e\right)x^2+ac\left(-15dfa^2-b(9de+25cf)a+b^2ce\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{c}-\frac{a\sqrt{a+bx^2}\sqrt{c+dx^2}(20acf+3ade+bce)}{cx}\right)}{3a}-\frac{(a+bx^2)^{3/}}{}}{5a}$$

$$\frac{e\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{5ax^5}$$

$\Big\downarrow$ 25

$$-\frac{-\frac{d\left(-\frac{\int \frac{b\left(-d(3de+35cf)a^2-bc(7de+5cf)a+2b^2c^2e\right)x^2+ac\left(-15dfa^2-b(9de+25cf)a+b^2ce\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{c}-\frac{a\sqrt{a+bx^2}\sqrt{c+dx^2}(20acf+3ade+bce)}{cx}\right)}{3a}-\frac{(a+bx^2)^{3/}}{}}{5a}$$

$$\frac{e\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{5ax^5}$$

$\Big\downarrow$ 406

$$-\frac{-\frac{d\left(-\frac{b\left(a^2(-d)(35cf+3de)-abc(5cf+7de)+2b^2c^2e\right)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+ac\left(-15a^2df-ab(25cf+9de)+b^2ce\right)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{c}-\frac{a\sqrt{a+bx^2}\sqrt{c+dx}}{}\right.}{3a}}{5a}$$

$$\frac{e\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{5ax^5}$$

$\Big\downarrow$ 320

$$-\frac{-\frac{d\left(-\frac{b\left(a^2(-d)(35cf+3de)-abc(5cf+7de)+2b^2c^2e\right)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}\left(-15a^2df-ab(25cf+9de)+b^2ce\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{c}\right.}{3a}}{5a}$$

$$\frac{e\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{5ax^5}$$

$\Big\downarrow$ 388

$$-\frac{-\frac{d\left(-\frac{b\left(a^2(-d)(35cf+3de)-abc(5cf+7de)+2b^2c^2e\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}\left(-15a^2df-ab(25cf+9de)+b^2ce\right)\text{EllipticF}\left(\arctan\left(\right.\right.}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{c}\right.}{3a}}{5a}-\frac{e\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{5ax^5}$$

$\Big\downarrow$ 313

$$-\frac{-\frac{d\left(-\frac{\frac{c^{3/2}\sqrt{a+bx^2}\left(-15a^2df-ab(25cf+9de)+b^2ce\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+b\left(a^2(-d)(35cf+3de)-abc(5cf+7de)+2b^2c^2e\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{}}{}\right.}{c}\right.}{3a}}{5a}-\frac{e\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{5ax^5}$$

input
```
Int[(Sqrt[a + b*x^2]*(c + d*x^2)^(3/2)*(e + f*x^2))/x^6,x]
```

output
```
-1/5*(e*(a + b*x^2)^(3/2)*(c + d*x^2)^(3/2))/(a*x^5) - (-1/3*((2*b*c*e - 3
*a*d*e - 5*a*c*f)*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(a*x^3) - (d*(-((a*(b
*c*e + 3*a*d*e + 20*a*c*f)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(c*x)) - (b*(2
*b^2*c^2*e - a*b*c*(7*d*e + 5*c*f) - a^2*d*(3*d*e + 35*c*f))*((x*Sqrt[a +
b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(S
qrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(
c + d*x^2))]*Sqrt[c + d*x^2])) + (c^(3/2)*(b^2*c*e - 15*a^2*d*f - a*b*(9*d
*e + 25*c*f))*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (
b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2
]))/c))/(3*a))/(5*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 442
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^q/(a*g*(m + 1))), x] - Simp[1/(a*g^2*(m + 1))   Int[(g*x)
^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[c*(b*e - a*f)*(m + 1) + e*2
*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*2*(p + q + 1))*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && GtQ[q, 0] && LtQ[m, -1]
&&  !(EqQ[q, 1] && SimplerQ[e + f*x^2, c + d*x^2])
```

Maple [A] (verified)

Time = 7.32 (sec) , antiderivative size = 502, normalized size of antiderivative = 1.11

method	result
elliptic	$\frac{\sqrt{(b\,x^2+a)(x^2d+c)}\left(-\frac{ce\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{5x^5}-\frac{(5acf+6ade+bce)\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{15a\,x^3}-\frac{\left(20a^2cfd+3a^2d^2e+5ab\,c^2f+7abcde-2b^2c^2\right.}{15a^2cx}\right.}{}$
risch	$-\frac{\sqrt{b\,x^2+a}\,\sqrt{x^2d+c}\,(20df\,x^4a^2c+3a^2d^2e\,x^4+5ab\,c^2f\,x^4+7abcde\,x^4-2b^2c^2e\,x^4+5a^2c^2f\,x^2+6a^2cde\,x^2+ab\,c^2e\,x^2+3a^2c^2e)}{15x^5a^2c}+\frac{d}{}$
default	Expression too large to display

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)*(f*x^2+e)/x^6,x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-1/5*c*e*(b*d
*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/x^5-1/15*(5*a*c*f+6*a*d*e+b*c*e)/a*(b*d*x^
4+a*d*x^2+b*c*x^2+a*c)^(1/2)/x^3-1/15/a^2/c*(20*a^2*c*d*f+3*a^2*d^2*e+5*a*
b*c^2*f+7*a*b*c*d*e-2*b^2*c^2*e)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/x+(a*
d^2*f+2*b*c*d*f+b*d^2*e-1/15*b*d*(5*a*c*f+6*a*d*e+b*c*e)/a)/(-b/a)^(1/2)*(
1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*Ell
ipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-(f*b*d^2+1/15*b*d*(20*a^2*
c*d*f+3*a^2*d^2*e+5*a*b*c^2*f+7*a*b*c*d*e-2*b^2*c^2*e)/a^2/c)*c/(-b/a)^(1/
2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
/d*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^
(1/2),(-1+(a*d+b*c)/c/b)^(1/2))))
```

Fricas [F]

$$\int \frac{\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)}{x^6}\,dx=\int \frac{\sqrt{bx^2+a}(dx^2+c)^{\frac{3}{2}}(fx^2+e)}{x^6}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)*(f*x^2+e)/x^6,x, algorithm="fric
as")
```

output
```
integral((d*f*x^4 + (d*e + c*f)*x^2 + c*e)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)
/x^6, x)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}\,(e+fx^2)}{x^6}\,dx = \int \frac{\sqrt{a+bx^2}(c+dx^2)^{\frac{3}{2}}\,(e+fx^2)}{x^6}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(3/2)*(f*x**2+e)/x**6,x)
```

output
```
Integral(sqrt(a + b*x**2)*(c + d*x**2)**(3/2)*(e + f*x**2)/x**6, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}\,(e+fx^2)}{x^6}\,dx = \int \frac{\sqrt{bx^2+a}(dx^2+c)^{\frac{3}{2}}(fx^2+e)}{x^6}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)*(f*x^2+e)/x^6,x, algorithm="maxi
ma")
```

output
```
integrate(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)*(f*x^2 + e)/x^6, x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}\,(e+fx^2)}{x^6}\,dx = \int \frac{\sqrt{bx^2+a}(dx^2+c)^{\frac{3}{2}}(fx^2+e)}{x^6}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)*(f*x^2+e)/x^6,x, algorithm="giac
")
```

output
```
integrate(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)*(f*x^2 + e)/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}\,(e+fx^2)}{x^6}\,dx = \int \frac{\sqrt{b\,x^2+a}\,(d\,x^2+c)^{3/2}\,(f\,x^2+e)}{x^6}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(3/2)*(e + f*x^2))/x^6,x)
```

output
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(3/2)*(e + f*x^2))/x^6, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}\,(e+fx^2)}{x^6}\,dx = \text{Too large to display}$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)*(f*x^2+e)/x^6,x)
```

output
```
( - 5*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*c*d*f*x**2 + 10*sqrt(c + d*x*
*2)*sqrt(a + b*x**2)*a**2*d**2*f*x**4 - sqrt(c + d*x**2)*sqrt(a + b*x**2)*
a*b*c**2*e - 10*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b*c**2*f*x**2 - 5*sqrt
(c + d*x**2)*sqrt(a + b*x**2)*a*b*c*d*e*x**2 + 20*sqrt(c + d*x**2)*sqrt(a
+ b*x**2)*a*b*c*d*f*x**4 + 5*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b*d**2*e*
x**4 + 15*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**2*c**2*f*x**4 + 3*sqrt(c +
d*x**2)*sqrt(a + b*x**2)*b**2*c*d*e*x**4 - 10*int((sqrt(c + d*x**2)*sqrt(a
 + b*x**2)*x**2)/(a**2*c*d + a**2*d**2*x**2 + a*b*c**2 + 2*a*b*c*d*x**2 +
a*b*d**2*x**4 + b**2*c**2*x**2 + b**2*c*d*x**4),x)*a**3*b*d**4*f*x**5 - 25
*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a**2*c*d + a**2*d**2*x**2 +
 a*b*c**2 + 2*a*b*c*d*x**2 + a*b*d**2*x**4 + b**2*c**2*x**2 + b**2*c*d*x**
4),x)*a**2*b**2*c*d**3*f*x**5 - 5*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x
**2)/(a**2*c*d + a**2*d**2*x**2 + a*b*c**2 + 2*a*b*c*d*x**2 + a*b*d**2*x**
4 + b**2*c**2*x**2 + b**2*c*d*x**4),x)*a**2*b**2*d**4*e*x**5 - 30*int((sqr
t(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a**2*c*d + a**2*d**2*x**2 + a*b*c**2
 + 2*a*b*c*d*x**2 + a*b*d**2*x**4 + b**2*c**2*x**2 + b**2*c*d*x**4),x)*a*b
**3*c**2*d**2*f*x**5 - 8*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a**
2*c*d + a**2*d**2*x**2 + a*b*c**2 + 2*a*b*c*d*x**2 + a*b*d**2*x**4 + b**2*
c**2*x**2 + b**2*c*d*x**4),x)*a*b**3*c*d**3*e*x**5 - 15*int((sqrt(c + d*x*
*2)*sqrt(a + b*x**2)*x**2)/(a**2*c*d + a**2*d**2*x**2 + a*b*c**2 + 2*a*...
```

3.95 $\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}(e+fx^2)}{x^8}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 526

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}\,(e+fx^2)}{x^8}\,dx =$$

$$-\frac{(8b^3c^3e-3a^3d^2(2de-7cf)-ab^2c^2(19de+14cf)+a^2bcd(9de+49cf))\sqrt{c+dx^2}}{105a^2c^2x\sqrt{a+bx^2}}$$

$$-\frac{\left(\frac{8b^2ce}{a}-15bde+\frac{3ad^2e}{c}-14bcf+42adf\right)\sqrt{a+bx^2}\sqrt{c+dx^2}}{105ax^3}$$

$$+\frac{(4bce-3ade-7acf)\,(a+bx^2)^{3/2}\sqrt{c+dx^2}}{35a^2x^5}-\frac{e(a+bx^2)^{3/2}\,(c+dx^2)^{3/2}}{7ax^7}$$

$$-\frac{\sqrt{b}(8b^3c^3e-3a^3d^2(2de-7cf)-ab^2c^2(19de+14cf)+a^2bcd(9de+49cf))\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1}{105a^{5/2}c^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

$$+\frac{\sqrt{b}d(4b^2c^2e-3a^2d(de-21cf)-abc(9de+7cf))\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{105a^{3/2}c^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
-1/105*(8*b^3*c^3*e-3*a^3*d^2*(-7*c*f+2*d*e)-a*b^2*c^2*(14*c*f+19*d*e)+a^2
*b*c*d*(49*c*f+9*d*e))*(d*x^2+c)^(1/2)/a^2/c^2/x/(b*x^2+a)^(1/2)-1/105*(8*
b^2*c*e/a-15*b*d*e+3*a*d^2*e/c-14*b*c*f+42*a*d*f)*(b*x^2+a)^(1/2)*(d*x^2+c
)^(1/2)/a/x^3+1/35*(-7*a*c*f-3*a*d*e+4*b*c*e)*(b*x^2+a)^(3/2)*(d*x^2+c)^(1
/2)/a^2/x^5-1/7*e*(b*x^2+a)^(3/2)*(d*x^2+c)^(3/2)/a/x^7-1/105*b^(1/2)*(8*b
^3*c^3*e-3*a^3*d^2*(-7*c*f+2*d*e)-a*b^2*c^2*(14*c*f+19*d*e)+a^2*b*c*d*(49*
c*f+9*d*e))*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),
(1-a*d/b/c)^(1/2))/a^(5/2)/c^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(
1/2)+1/105*b^(1/2)*d*(4*b^2*c^2*e-3*a^2*d*(-21*c*f+d*e)-a*b*c*(7*c*f+9*d*e
))*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(
1/2))/a^(3/2)/c^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.68 (sec) , antiderivative size = 429, normalized size of antiderivative = 0.82

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}\,(e+fx^2)}{x^8}\,dx = \frac{-\sqrt{\frac{b}{a}}(a+bx^2)\,(c+dx^2)\left(8b^3c^3ex^6+3a^3(c+dx^2)^2\,(5ce-2dex^2\right.}{}$$

input
```
Integrate[(Sqrt[a + b*x^2]*(c + d*x^2)^(3/2)*(e + f*x^2))/x^8,x]
```

output
```
(-(Sqrt[b/a]*(a + b*x^2)*(c + d*x^2)*(8*b^3*c^3*e*x^6 + 3*a^3*(c + d*x^2)^
2*(5*c*e - 2*d*e*x^2 + 7*c*f*x^2) - a*b^2*c^2*x^4*(4*c*e + 19*d*e*x^2 + 14
*c*f*x^2) + a^2*b*c*x^2*(9*d^2*e*x^4 + c^2*(3*e + 7*f*x^2) + c*d*x^2*(9*e
+ 49*f*x^2)))) - I*b*c*(8*b^3*c^3*e + 3*a^3*d^2*(-2*d*e + 7*c*f) - a*b^2*c
^2*(19*d*e + 14*c*f) + a^2*b*c*d*(9*d*e + 49*c*f))*x^7*Sqrt[1 + (b*x^2)/a]
*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + I*b*
c*(b*c - a*d)*(8*b^2*c^2*e + 3*a^2*d*(d*e + 14*c*f) - a*b*c*(15*d*e + 14*c
*f))*x^7*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[
b/a]*x], (a*d)/(b*c)])/(105*a^3*Sqrt[b/a]*c^2*x^7*Sqrt[a + b*x^2]*Sqrt[c +
 d*x^2])
```

Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 580, normalized size of antiderivative = 1.10, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {442, 25, 442, 25, 442, 445, 25, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)}{x^8}dx$$

$$\Big\downarrow \text{ 442}$$

$$\frac{\int -\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(d(be-7af)x^2+4bce-3ade-7acf\right)}{x^6}dx}{7a}-\frac{e\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{7ax^7}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(d(be-7af)x^2+4bce-3ade-7acf\right)}{x^6}dx}{7a}-\frac{e\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{7ax^7}$$

$$\Big\downarrow \text{ 442}$$

$$-\frac{\frac{\int -\frac{\sqrt{bx^2+a}\left(3d(de+14cf)a^2-bc(15de+14cf)a+d\left(35dfa^2-b(8de+7cf)a+4b^2ce\right)x^2+8b^2c^2e\right)}{x^4\sqrt{dx^2+c}}dx}{5a}-\frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}(-7acf-3ade+4bce)}{5ax^5}}{7a}-\frac{e\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{7ax^7}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{-\frac{\int \frac{\sqrt{bx^2+a}\left(3d(de+14cf)a^2-bc(15de+14cf)a+d\left(35dfa^2-b(8de+7cf)a+4b^2ce\right)x^2+8b^2c^2e\right)}{x^4\sqrt{dx^2+c}}dx}{5a}-\frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}(-7acf-3ade+4bce)}{5ax^5}}{7a}-\frac{e\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{7ax^7}$$

$$\Big\downarrow \text{ 442}$$

$$-\frac{-\frac{\frac{\int \frac{-3d^2(2de-7cf)a^3+bcd(9de+49cf)a^2-b^2c^2(19de+14cf)a+bd\left(-3d(de-21cf)a^2-bc(9de+7cf)a+4b^2c^2e\right)x^2+8b^3c^3e}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(3a^2d(14cf+\cdots\right.}{3c\cdots}}{5a}\cdots}{7a}-\frac{e\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{7ax^7}$$

$\downarrow$ 445

$$-\frac{-\frac{\frac{-\frac{\int -\frac{bd\left(\left(-3d^2(2de-7cf)a^3+bcd(9de+49cf)a^2-b^2c^2(19de+14cf)a+8b^3c^3e\right)x^2+ac\left(-3d(de-21cf)a^2-bc(9de+7cf)a+4b^2c^2e\right)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}}{3c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\cdots\right.}{\cdots}}{5a}\cdots}{\cdots}-\frac{e\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{7ax^7}$$

$\downarrow$ 25

$$-\frac{-\frac{\frac{\frac{\int \frac{bd\left(\left(-3d^2(2de-7cf)a^3+bcd(9de+49cf)a^2-b^2c^2(19de+14cf)a+8b^3c^3e\right)x^2+ac\left(-3d(de-21cf)a^2-bc(9de+7cf)a+4b^2c^2e\right)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}}{3c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(-3a\cdots\right.}{\cdots}}{5a}\cdots}{\cdots}-\frac{e\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{7ax^7}$$

$\downarrow$ 27

$$-\frac{-\frac{\frac{\frac{bd\int \frac{\left(-3d^2(2de-7cf)a^3+bcd(9de+49cf)a^2-b^2c^2(19de+14cf)a+8b^3c^3e\right)x^2+ac\left(-3d(de-21cf)a^2-bc(9de+7cf)a+4b^2c^2e\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}}{3c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(-3a^3\cdots\right.}{\cdots}}{5a}\cdots}{\cdots}-\frac{e\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{7ax^7}$$

$\downarrow$ 406

$$-\frac{-\frac{\frac{bd\left(ac\left(-3a^2d(de-21cf)-abc(7cf+9de)+4b^2c^2e\right)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\left(-3a^3d^2(2de-7cf)+a^2bcd(49cf+9de)-ab^2c^2(14cf+19de)+8b^3c^3e\right)\int \frac{x\cdots}{\sqrt{bx^2+a}\cdots}\cdots\right.}{ac}}{3c}\cdots}{\cdots}-\frac{e\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{7ax^7}$$

$\downarrow$ 320

$$-\frac{-\frac{\frac{bd\left(\left(-3a^3d^2(2de-7cf)+a^2bcd(49cf+9de)-ab^2c^2(14cf+19de)+8b^3c^3e\right)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}\left(-3a^2d(de-21cf)-abc(7cf+9de)+4b^2c^2e\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right.}{ac}}{3c}}{}$$

$$\frac{e\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{7ax^7}$$

$$\Big\downarrow \; 388$$

$$-\frac{-\frac{\frac{bd\left(\left(-3a^3d^2(2de-7cf)+a^2bcd(49cf+9de)-ab^2c^2(14cf+19de)+8b^3c^3e\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}\left(-3a^2d(de-21cf)-abc(7cf+\right.}{\sqrt{d}\sqrt{c+dx}}\right.}{ac}}{3c}}{}$$

$$\frac{e\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{7ax^7}$$

$$\Big\downarrow \; 313$$

$$-\frac{-\frac{\frac{bd\left(\frac{c^{3/2}\sqrt{a+bx^2}\left(-3a^2d(de-21cf)-abc(7cf+9de)+4b^2c^2e\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\left(-3a^3d^2(2de-7cf)+a^2bcd(49cf+9de)-ab^2c^2(14cf+19d\right.\right.}{ac}}{}}{}$$

$$\frac{e\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{7ax^7}$$

input

```
Int[(Sqrt[a + b*x^2]*(c + d*x^2)^(3/2)*(e + f*x^2))/x^8,x]
```

output
```
-1/7*(e*(a + b*x^2)^(3/2)*(c + d*x^2)^(3/2))/(a*x^7) - (-1/5*((4*b*c*e - 3
*a*d*e - 7*a*c*f)*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(a*x^5) - (-1/3*((8*b
^2*c^2*e + 3*a^2*d*(d*e + 14*c*f) - a*b*c*(15*d*e + 14*c*f))*Sqrt[a + b*x^
2]*Sqrt[c + d*x^2])/(c*x^3) + (-(((8*b^3*c^3*e - 3*a^3*d^2*(2*d*e - 7*c*f)
 - a*b^2*c^2*(19*d*e + 14*c*f) + a^2*b*c*d*(9*d*e + 49*c*f))*Sqrt[a + b*x^
2]*Sqrt[c + d*x^2])/(a*c*x)) + (b*d*((8*b^3*c^3*e - 3*a^3*d^2*(2*d*e - 7*c
*f) - a*b^2*c^2*(19*d*e + 14*c*f) + a^2*b*c*d*(9*d*e + 49*c*f))*((x*Sqrt[a
 + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcTan
[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(
a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (c^(3/2)*(4*b^2*c^2*e - 3*a^2*d*(d*e -
 21*c*f) - a*b*c*(9*d*e + 7*c*f))*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d
]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x
^2))]*Sqrt[c + d*x^2]))/(a*c))/(3*c))/(5*a))/(7*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 442
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^q/(a*g*(m + 1))), x] - Simp[1/(a*g^2*(m + 1))   Int[(g*x)
^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[c*(b*e - a*f)*(m + 1) + e*2
*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*2*(p + q + 1))*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && GtQ[q, 0] && LtQ[m, -1]
&&  !(EqQ[q, 1] && SimplerQ[e + f*x^2, c + d*x^2])
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 9.00 (sec) , antiderivative size = 643, normalized size of antiderivative = 1.22

method	result
elliptic	$\frac{\sqrt{(b\,x^2+a)(x^2d+c)}\left(-\frac{ce\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{7x^7}-\frac{(7acf+8ade+bce)\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{35a\,x^5}-\frac{\left(42a^2cfd+3a^2d^2e+7ab\,c^2f+9abcde-4b^2c^{\cdots}\right.}{105a^2c\,x^3}\right.}{}$
risch	$-\frac{\sqrt{b\,x^2+a}\,\sqrt{x^2d+c}\,(21a^3c\,d^2f\,x^6-6a^3d^3e\,x^6+49a^2b\,c^2df\,x^6+9a^2bc\,d^2e\,x^6-14a\,b^2c^3f\,x^6-19a\,b^2c^2de\,x^6+8b^3c^3e\,x^6+42a^3c^2df\,x^{\cdots}}{105x^7a^3c^2}$
default	Expression too large to display

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)*(f*x^2+e)/x^8,x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-1/7*c*e*(b*d
*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/x^7-1/35*(7*a*c*f+8*a*d*e+b*c*e)/a*(b*d*x^
4+a*d*x^2+b*c*x^2+a*c)^(1/2)/x^5-1/105/a^2/c*(42*a^2*c*d*f+3*a^2*d^2*e+7*a
*b*c^2*f+9*a*b*c*d*e-4*b^2*c^2*e)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/x^3-
1/105/a^3/c^2*(21*a^3*c*d^2*f-6*a^3*d^3*e+49*a^2*b*c^2*d*f+9*a^2*b*c*d^2*e
-14*a*b^2*c^3*f-19*a*b^2*c^2*d*e+8*b^3*c^3*e)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c
)^(1/2)/x+(f*b*d^2-1/105*b*d*(42*a^2*c*d*f+3*a^2*d^2*e+7*a*b*c^2*f+9*a*b*c
*d*e-4*b^2*c^2*e)/a^2/c)/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/
(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)
/c/b)^(1/2))-1/105*b*(21*a^3*c*d^2*f-6*a^3*d^3*e+49*a^2*b*c^2*d*f+9*a^2*b*
c*d^2*e-14*a*b^2*c^3*f-19*a*b^2*c^2*d*e+8*b^3*c^3*e)/a^3/c/(-b/a)^(1/2)*(1
+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(Ell
ipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(
-1+(a*d+b*c)/c/b)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 489, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}\,(e+fx^2)}{x^8}\,dx = \frac{\sqrt{ac}((8\,b^4c^3-19\,ab^3c^2d+9\,a^2b^2cd^2-6\,a^3bd^3)e-7\,(2\,ab^3c^3-7\,a\cdots}{}$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)*(f*x^2+e)/x^8,x, algorithm="fric
as")
```

output
```
1/105*(sqrt(a*c)*((8*b^4*c^3 - 19*a*b^3*c^2*d + 9*a^2*b^2*c*d^2 - 6*a^3*b*
d^3)*e - 7*(2*a*b^3*c^3 - 7*a^2*b^2*c^2*d - 3*a^3*b*c*d^2)*f)*x^7*sqrt(-b/
a)*elliptic_e(arcsin(x*sqrt(-b/a)), a*d/(b*c)) - sqrt(a*c)*((8*b^4*c^3 + (
4*a^2*b^2 - 19*a*b^3)*c^2*d - 9*(a^3*b - a^2*b^2)*c*d^2 - 3*(a^4 + 2*a^3*b
)*d^3)*e - 7*(2*a*b^3*c^3 + (a^3*b - 7*a^2*b^2)*c^2*d - 3*(3*a^4 + a^3*b)*
c*d^2)*f)*x^7*sqrt(-b/a)*elliptic_f(arcsin(x*sqrt(-b/a)), a*d/(b*c)) - (15
*a^4*c^3*e + ((8*a*b^3*c^3 - 19*a^2*b^2*c^2*d + 9*a^3*b*c*d^2 - 6*a^4*d^3)
*e - 7*(2*a^2*b^2*c^3 - 7*a^3*b*c^2*d - 3*a^4*c*d^2)*f)*x^6 - ((4*a^2*b^2*
c^3 - 9*a^3*b*c^2*d - 3*a^4*c*d^2)*e - 7*(a^3*b*c^3 + 6*a^4*c^2*d)*f)*x^4
+ 3*(7*a^4*c^3*f + (a^3*b*c^3 + 8*a^4*c^2*d)*e)*x^2)*sqrt(b*x^2 + a)*sqrt(
d*x^2 + c))/(a^4*c^2*x^7)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}(e+fx^2)}{x^8}\,dx = \int \frac{\sqrt{a+bx^2}(c+dx^2)^{\frac{3}{2}}(e+fx^2)}{x^8}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(3/2)*(f*x**2+e)/x**8,x)
```

output
```
Integral(sqrt(a + b*x**2)*(c + d*x**2)**(3/2)*(e + f*x**2)/x**8, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}(e+fx^2)}{x^8}\,dx = \int \frac{\sqrt{bx^2+a}(dx^2+c)^{\frac{3}{2}}(fx^2+e)}{x^8}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)*(f*x^2+e)/x^8,x, algorithm="maxi
ma")
```

output
```
integrate(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)*(f*x^2 + e)/x^8, x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}\,(e+fx^2)}{x^8}\,dx = \int \frac{\sqrt{bx^2+a}(dx^2+c)^{\frac{3}{2}}(fx^2+e)}{x^8}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)*(f*x^2+e)/x^8,x, algorithm="giac
")
```

output
```
integrate(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)*(f*x^2 + e)/x^8, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}\,(e+fx^2)}{x^8}\,dx = \int \frac{\sqrt{b\,x^2+a}\,(d\,x^2+c)^{3/2}\,(f\,x^2+e)}{x^8}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(3/2)*(e + f*x^2))/x^8,x)
```

output
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(3/2)*(e + f*x^2))/x^8, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}\,(e+fx^2)}{x^8}\,dx = \text{Too large to display}$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)*(f*x^2+e)/x^8,x)
```

output
```
(7*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*d*f*x**2 - 3*sqrt(c + d*x**2)*sqrt(
a + b*x**2)*b*c*e - 7*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b*d*e*x**2 - 21*sq
rt(c + d*x**2)*sqrt(a + b*x**2)*b*d*f*x**4 + 35*int((sqrt(c + d*x**2)*sqrt
(a + b*x**2))/(a**2*c*d*x**6 + a**2*d**2*x**8 + a*b*c**2*x**6 + 2*a*b*c*d*
x**8 + a*b*d**2*x**10 + b**2*c**2*x**8 + b**2*c*d*x**10),x)*a**3*c*d**2*f*
x**7 + 56*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a**2*c*d*x**6 + a**2*d*
*2*x**8 + a*b*c**2*x**6 + 2*a*b*c*d*x**8 + a*b*d**2*x**10 + b**2*c**2*x**8
 + b**2*c*d*x**10),x)*a**2*b*c**2*d*f*x**7 - 11*int((sqrt(c + d*x**2)*sqrt
(a + b*x**2))/(a**2*c*d*x**6 + a**2*d**2*x**8 + a*b*c**2*x**6 + 2*a*b*c*d*
x**8 + a*b*d**2*x**10 + b**2*c**2*x**8 + b**2*c*d*x**10),x)*a**2*b*c*d**2*
e*x**7 + 21*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a**2*c*d*x**6 + a**2*
d**2*x**8 + a*b*c**2*x**6 + 2*a*b*c*d*x**8 + a*b*d**2*x**10 + b**2*c**2*x*
*8 + b**2*c*d*x**10),x)*a*b**2*c**3*f*x**7 - 8*int((sqrt(c + d*x**2)*sqrt(
a + b*x**2))/(a**2*c*d*x**6 + a**2*d**2*x**8 + a*b*c**2*x**6 + 2*a*b*c*d*x
**8 + a*b*d**2*x**10 + b**2*c**2*x**8 + b**2*c*d*x**10),x)*a*b**2*c**2*d*e
*x**7 + 3*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a**2*c*d*x**6 + a**2*d*
*2*x**8 + a*b*c**2*x**6 + 2*a*b*c*d*x**8 + a*b*d**2*x**10 + b**2*c**2*x**8
 + b**2*c*d*x**10),x)*b**3*c**3*e*x**7 + 28*int((sqrt(c + d*x**2)*sqrt(a +
 b*x**2))/(a**2*c*d*x**4 + a**2*d**2*x**6 + a*b*c**2*x**4 + 2*a*b*c*d*x**6
 + a*b*d**2*x**8 + b**2*c**2*x**6 + b**2*c*d*x**8),x)*a**3*d**3*f*x**7 ...
```

3.96 $\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}(e+fx^2)}{x^{10}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 690

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}\left(e+fx^2\right)}{x^{10}}\,dx = \frac{(16b^4c^4e+a^3bcd^2(7de-27cf)-2a^4d^3(4de-9cf)-8ab^3c^3(4de}{315a^3c^3x\sqrt{a+bx^2}}$$

$$-\frac{\left(\frac{8b^2ce}{a}-13bde+\frac{ad^2e}{c}-12bcf+24adf\right)\sqrt{a+bx^2}\sqrt{c+dx^2}}{105ax^5}$$

$$-\frac{(8b^3c^3e-a^3d^2(4de-9cf)-3ab^2c^2(5de+4cf)+3a^2bcd(de+9cf))\sqrt{a+bx^2}\sqrt{c+dx^2}}{315a^3c^2x^3}$$

$$+\frac{(2bce-ade-3acf)\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{21a^2x^7}-\frac{e(a+bx^2)^{3/2}\left(c+dx^2\right)^{3/2}}{9ax^9}$$

$$+\frac{\sqrt{b}(16b^4c^4e+a^3bcd^2(7de-27cf)-2a^4d^3(4de-9cf)-8ab^3c^3(4de+3cf)+3a^2b^2c^2d(3de+19cf))\sqrt{c}}{315a^{7/2}c^3\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

$$-\frac{\sqrt{b}d(8b^3c^3e-a^3d^2(4de-9cf)-3ab^2c^2(5de+4cf)+3a^2bcd(de+9cf))\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{}}{\sqrt{}}\right.\right.}{315a^{5/2}c^3\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
1/315*(16*b^4*c^4*e+a^3*b*c*d^2*(-27*c*f+7*d*e)-2*a^4*d^3*(-9*c*f+4*d*e)-8
*a*b^3*c^3*(3*c*f+4*d*e)+3*a^2*b^2*c^2*d*(19*c*f+3*d*e))*(d*x^2+c)^(1/2)/a
^3/c^3/x/(b*x^2+a)^(1/2)-1/105*(8*b^2*c*e/a-13*b*d*e+a*d^2*e/c-12*b*c*f+24
*a*d*f)*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a/x^5-1/315*(8*b^3*c^3*e-a^3*d^2*(
-9*c*f+4*d*e)-3*a*b^2*c^2*(4*c*f+5*d*e)+3*a^2*b*c*d*(9*c*f+d*e))*(b*x^2+a)
^(1/2)*(d*x^2+c)^(1/2)/a^3/c^2/x^3+1/21*(-3*a*c*f-a*d*e+2*b*c*e)*(b*x^2+a)
^(3/2)*(d*x^2+c)^(1/2)/a^2/x^7-1/9*e*(b*x^2+a)^(3/2)*(d*x^2+c)^(3/2)/a/x^9
+1/315*b^(1/2)*(16*b^4*c^4*e+a^3*b*c*d^2*(-27*c*f+7*d*e)-2*a^4*d^3*(-9*c*f
+4*d*e)-8*a*b^3*c^3*(3*c*f+4*d*e)+3*a^2*b^2*c^2*d*(19*c*f+3*d*e))*(d*x^2+c
)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/a
^(7/2)/c^3/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)-1/315*b^(1/2)*d
*(8*b^3*c^3*e-a^3*d^2*(-9*c*f+4*d*e)-3*a*b^2*c^2*(4*c*f+5*d*e)+3*a^2*b*c*d
*(9*c*f+d*e))*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1
-a*d/b/c)^(1/2))/a^(5/2)/c^3/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/
2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.45 (sec) , antiderivative size = 560, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}\,(e+fx^2)}{x^{10}}\,dx = \frac{-\sqrt{\frac{b}{a}}(a+bx^2)\,(c+dx^2)\left(-16b^4c^4ex^8+8ab^3c^3x^6(4dex^2+c(e+3\right.}{}$$

input
```
Integrate[(Sqrt[a + b*x^2]*(c + d*x^2)^(3/2)*(e + f*x^2))/x^10,x]
```

output
```
(-(Sqrt[b/a]*(a + b*x^2)*(c + d*x^2)*(-16*b^4*c^4*e*x^8 + 8*a*b^3*c^3*x^6*
(4*d*e*x^2 + c*(e + 3*f*x^2)) + a^4*(c + d*x^2)^2*(8*d^2*e*x^4 + 5*c^2*(7*
e + 9*f*x^2) - 2*c*d*x^2*(10*e + 9*f*x^2)) - 3*a^2*b^2*c^2*x^4*(3*d^2*e*x^
4 + 2*c^2*(e + 2*f*x^2) + c*d*x^2*(5*e + 19*f*x^2)) + a^3*b*c*x^2*(-7*d^3*
e*x^6 + 3*c*d^2*x^4*(e + 9*f*x^2) + c^3*(5*e + 9*f*x^2) + c^2*d*x^2*(11*e
+ 27*f*x^2)))) + I*b*c*(16*b^4*c^4*e + a^3*b*c*d^2*(7*d*e - 27*c*f) - 8*a*
b^3*c^3*(4*d*e + 3*c*f) + 2*a^4*d^3*(-4*d*e + 9*c*f) + 3*a^2*b^2*c^2*d*(3*
d*e + 19*c*f))*x^9*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*Arc
Sinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*b*c*(-(b*c) + a*d)*(-16*b^3*c^3*e - 45
*a^2*b*c^2*d*f + 24*a*b^2*c^2*(d*e + c*f) + a^3*d^2*(-4*d*e + 9*c*f))*x^9*
Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x],
(a*d)/(b*c)])/(315*a^4*Sqrt[b/a]*c^3*x^9*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 1.35 (sec) , antiderivative size = 755, normalized size of antiderivative = 1.09, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.394$, Rules used = {442, 27, 442, 25, 442, 445, 445, 25, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)}{x^{10}}dx$$

$$\Big\downarrow \textbf{442}$$

$$\frac{\int -\frac{3\sqrt{bx^2+a}\sqrt{dx^2+c}\left(d(be-3af)x^2+2bce-ade-3acf\right)}{x^8}dx}{9a}-\frac{e\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{9ax^9}$$

$$\Big\downarrow \textbf{27}$$

$$-\frac{\int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(d(be-3af)x^2+2bce-ade-3acf\right)}{x^8}dx}{3a}-\frac{e\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{9ax^9}$$

$$\Big\downarrow \textbf{442}$$

$$-\frac{\frac{\int -\frac{\sqrt{bx^2+a}\left(d(de+24cf)a^2-bc(13de+12cf)a+d\left(21dfa^2-b(10de+9cf)a+6b^2ce\right)x^2+8b^2c^2e\right)}{x^6\sqrt{dx^2+c}}dx}{7a}-\frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}(-3acf-ade+2bce)}{7ax^7}}{3a}-\frac{e\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{9ax^9}$$

$\downarrow$ 25

$$-\frac{-\frac{\int \frac{\sqrt{bx^2+a}\left(d(de+24cf)a^2-bc(13de+12cf)a+d\left(21dfa^2-b(10de+9cf)a+6b^2ce\right)x^2+8b^2c^2e\right)}{x^6\sqrt{dx^2+c}}dx}{7a}-\frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}(-3acf-ade+2bce)}{7ax^7}}{3a}-\frac{e\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{9ax^9}$$

$\downarrow$ 442

$$-\frac{-\frac{\frac{\int \frac{-d^2(4de-9cf)a^3+3bcd(de+9cf)a^2-3b^2c^2(5de+4cf)a+bd\left(-3d(de-11cf)a^2-bc(11de+9cf)a+6b^2c^2e\right)x^2+8b^3c^3e}{x^4\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{5c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(a^2d(24cf+de)\cdots\right)}{5cx^5}}{7a}\cdots}{3a}-\frac{e\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{9ax^9}$$

$\downarrow$ 445

$$-\frac{-\frac{-\frac{\int \frac{-2d^3(4de-9cf)a^4+bcd^2(7de-27cf)a^3+3b^2c^2d(3de+19cf)a^2-8b^3c^3(4de+3cf)a+bd\left(-d^2(4de-9cf)a^3+3bcd(de+9cf)a^2-3b^2c^2(5de+4cf)a+8b^3c^3e\right)\cdots}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}\cdots}{3ac}\cdots}{5c}\cdots}{\cdots}-\frac{e\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{9ax^9}$$

$\downarrow$ 445

$$-\frac{-\frac{-\frac{-\frac{\int -\frac{bd\left(\left(-2d^3(4de-9cf)a^4+bcd^2(7de-27cf)a^3+3b^2c^2d(3de+19cf)a^2-8b^3c^3(4de+3cf)a+16b^4c^4e\right)x^2+ac\left(-d^2(4de-9cf)a^3+3bcd(de+9cf)a^2-3b^2c^2\cdots\right)\cdots\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}\cdots}{ac}\cdots}{3ac}\cdots}{\cdots}\cdots}{\cdots}-\frac{e\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{9ax^9}$$

$\downarrow$ 25

$$-\frac{\int \frac{bd\left(\left(-2d^3(4de-9cf)a^4+bcd^2(7de-27cf)a^3+3b^2c^2d(3de+19cf)a^2-8b^3c^3(4de+3cf)a+16b^4c^4e\right)x^2+ac\left(-d^2(4de-9cf)a^3+3bcd(de+9cf)a^2-3b^2c^2(5d\cdots\right.\right.}{\sqrt{bx^2+a}\sqrt{dx^2+c}}}{ac} \cdots 3ac \cdots}{\cdots} - \frac{e\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{9ax^9}$$

$\downarrow$ 27

$$-\frac{\frac{bd\int \frac{\left(-2d^3(4de-9cf)a^4+bcd^2(7de-27cf)a^3+3b^2c^2d(3de+19cf)a^2-8b^3c^3(4de+3cf)a+16b^4c^4e\right)x^2+ac\left(-d^2(4de-9cf)a^3+3bcd(de+9cf)a^2-3b^2c^2(5d\cdots\right.}{\sqrt{bx^2+a}\sqrt{dx^2+c}}}{ac} \cdots 3ac \cdots}{\cdots} - \frac{e\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{9ax^9}$$

$\downarrow$ 406

$$-\frac{\frac{bd\left(ac\left(a^3\left(-d^2\right)(4de-9cf)+3a^2bcd(9cf+de)-3ab^2c^2(4cf+5de)+8b^3c^3e\right)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\left(-2a^4d^3(4de-9cf)+a^3bcd^2(7de-27cf)+3a^2b^2c^2d(1\cdots\right.\right.}{ac} \cdots}{\cdots} - \frac{e\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{9ax^9}$$

$\downarrow$ 320

$$-\frac{\frac{bd\left(\left(-2a^4d^3(4de-9cf)+a^3bcd^2(7de-27cf)+3a^2b^2c^2d(19cf+3de)-8ab^3c^3(3cf+4de)+16b^4c^4e\right)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}\left(a^3\left(-d^2\right)(4de-\cdots\right.}{\cdots}\right.}{ac} \cdots}{\cdots} - \frac{e\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{9ax^9}$$

$\downarrow$ 388

$$-\frac{-\frac{-\frac{bd\left(\left(-2a^4d^3(4de-9cf)+a^3bcd^2(7de-27cf)+3a^2b^2c^2d(19cf+3de)-8ab^3c^3(3cf+4de)+16b^4c^4e\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}\left(a^3\cdots\right.}{\cdots}\cdots\right)}{ac}\cdots}{\cdots}}{\cdots}-\frac{e\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{9ax^9}$$

$$\Big\downarrow \; 313$$

$$-\frac{-\frac{-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(a^3\left(-d^2\right)(4de-9cf)+3a^2bcd(9cf+de)-3ab^2c^2(4cf+5de)+8b^3c^3e\right)}{3acx^3}-\frac{bd\left(\frac{c^{3/2}\sqrt{a+bx^2}\left(a^3\left(-d^2\right)(4de-9cf)+3a^2bcd(9cf+de)-3ab^2c\cdots\right.}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\cdots}}\cdots\right)}{\cdots}}{\cdots}}{\cdots}-\frac{e\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}{9ax^9}$$

input
```
Int[(Sqrt[a + b*x^2]*(c + d*x^2)^(3/2)*(e + f*x^2))/x^10,x]
```

output
```
-1/9*(e*(a + b*x^2)^(3/2)*(c + d*x^2)^(3/2))/(a*x^9) - (-1/7*((2*b*c*e - a
*d*e - 3*a*c*f)*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(a*x^7) - (-1/5*((8*b^2
*c^2*e - a*b*c*(13*d*e + 12*c*f) + a^2*d*(d*e + 24*c*f))*Sqrt[a + b*x^2]*S
qrt[c + d*x^2])/(c*x^5) + (-1/3*((8*b^3*c^3*e - a^3*d^2*(4*d*e - 9*c*f) -
3*a*b^2*c^2*(5*d*e + 4*c*f) + 3*a^2*b*c*d*(d*e + 9*c*f))*Sqrt[a + b*x^2]*S
qrt[c + d*x^2])/(a*c*x^3) - (-(((16*b^4*c^4*e + a^3*b*c*d^2*(7*d*e - 27*c*
f) - 2*a^4*d^3*(4*d*e - 9*c*f) - 8*a*b^3*c^3*(4*d*e + 3*c*f) + 3*a^2*b^2*c
^2*d*(3*d*e + 19*c*f))*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(a*c*x)) + (b*d*((
16*b^4*c^4*e + a^3*b*c*d^2*(7*d*e - 27*c*f) - 2*a^4*d^3*(4*d*e - 9*c*f) -
8*a*b^3*c^3*(4*d*e + 3*c*f) + 3*a^2*b^2*c^2*d*(3*d*e + 19*c*f))*((x*Sqrt[a
 + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcTan
[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(
a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (c^(3/2)*(8*b^3*c^3*e - a^3*d^2*(4*d*e
 - 9*c*f) - 3*a*b^2*c^2*(5*d*e + 4*c*f) + 3*a^2*b*c*d*(d*e + 9*c*f))*Sqrt[
a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[
d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))/(a*c))/(3*a*c)
)/(5*c))/(7*a))/(3*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 442
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^q/(a*g*(m + 1))), x] - Simp[1/(a*g^2*(m + 1))   Int[(g*x)
^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[c*(b*e - a*f)*(m + 1) + e*2
*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*2*(p + q + 1))*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && GtQ[q, 0] && LtQ[m, -1]
&&  !(EqQ[q, 1] && SimplerQ[e + f*x^2, c + d*x^2])
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 9.96 (sec) , antiderivative size = 832, normalized size of antiderivative = 1.21

method	result
elliptic	$\frac{\sqrt{(b\,x^2+a)(x^2d+c)}\left(-\frac{ce\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{9x^9}-\frac{(9acf+10ade+bce)\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{63a\,x^7}-\frac{\left(72a^2cfd+3a^2d^2e+9ab\,c^2f+11abcde-6b\right.}{315a^2c\,x^5}\right.}{}$
risch	Expression too large to display
default	Expression too large to display

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)*(f*x^2+e)/x^10,x,method=_RETURNVERBOSE
)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-1/9*c*e*(b*d
*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/x^9-1/63*(9*a*c*f+10*a*d*e+b*c*e)/a*(b*d*x
^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/x^7-1/315/a^2/c*(72*a^2*c*d*f+3*a^2*d^2*e+9*
a*b*c^2*f+11*a*b*c*d*e-6*b^2*c^2*e)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/x^
5-1/315/a^3/c^2*(9*a^3*c*d^2*f-4*a^3*d^3*e+27*a^2*b*c^2*d*f+3*a^2*b*c*d^2*
e-12*a*b^2*c^3*f-15*a*b^2*c^2*d*e+8*b^3*c^3*e)*(b*d*x^4+a*d*x^2+b*c*x^2+a*
c)^(1/2)/x^3+1/315/a^4/c^3*(18*a^4*c*d^3*f-8*a^4*d^4*e-27*a^3*b*c^2*d^2*f+
7*a^3*b*c*d^3*e+57*a^2*b^2*c^3*d*f+9*a^2*b^2*c^2*d^2*e-24*a*b^3*c^4*f-32*a
*b^3*c^3*d*e+16*b^4*c^4*e)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/x-1/315*(9*
a^3*c*d^2*f-4*a^3*d^3*e+27*a^2*b*c^2*d*f+3*a^2*b*c*d^2*e-12*a*b^2*c^3*f-15
*a*b^2*c^2*d*e+8*b^3*c^3*e)*b*d/a^3/c^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+
d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2
),(-1+(a*d+b*c)/c/b)^(1/2))+1/315*b*(18*a^4*c*d^3*f-8*a^4*d^4*e-27*a^3*b*c
^2*d^2*f+7*a^3*b*c*d^3*e+57*a^2*b^2*c^3*d*f+9*a^2*b^2*c^2*d^2*e-24*a*b^3*c
^4*f-32*a*b^3*c^3*d*e+16*b^4*c^4*e)/a^4/c^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)
*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(EllipticF(x*(-b/a)
^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/
b)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 684, normalized size of antiderivative = 0.99

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}(e+fx^2)}{x^{10}}\,dx =$$

$$-\frac{\sqrt{ac}((16\,b^5c^4-32\,ab^4c^3d+9\,a^2b^3c^2d^2+7\,a^3b^2cd^3-8\,a^4bd^4)e-3\,(8\,ab^4c^4-19\,a^2b^3c^3d+9\,a^3b^2c^2d^2-6}{}$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)*(f*x^2+e)/x^10,x, algorithm="fri
cas")
```

output
```
-1/315*(sqrt(a*c)*((16*b^5*c^4 - 32*a*b^4*c^3*d + 9*a^2*b^3*c^2*d^2 + 7*a^
3*b^2*c*d^3 - 8*a^4*b*d^4)*e - 3*(8*a*b^4*c^4 - 19*a^2*b^3*c^3*d + 9*a^3*b
^2*c^2*d^2 - 6*a^4*b*c*d^3)*f)*x^9*sqrt(-b/a)*elliptic_e(arcsin(x*sqrt(-b/
a)), a*d/(b*c)) - sqrt(a*c)*((16*b^5*c^4 + 8*(a^2*b^3 - 4*a*b^4)*c^3*d - 3
*(5*a^3*b^2 - 3*a^2*b^3)*c^2*d^2 + (3*a^4*b + 7*a^3*b^2)*c*d^3 - 4*(a^5 +
2*a^4*b)*d^4)*e - 3*(8*a*b^4*c^4 + (4*a^3*b^2 - 19*a^2*b^3)*c^3*d - 9*(a^4
*b - a^3*b^2)*c^2*d^2 - 3*(a^5 + 2*a^4*b)*c*d^3)*f)*x^9*sqrt(-b/a)*ellipti
c_f(arcsin(x*sqrt(-b/a)), a*d/(b*c)) + (35*a^5*c^4*e - ((16*a*b^4*c^4 - 32
*a^2*b^3*c^3*d + 9*a^3*b^2*c^2*d^2 + 7*a^4*b*c*d^3 - 8*a^5*d^4)*e - 3*(8*a
^2*b^3*c^4 - 19*a^3*b^2*c^3*d + 9*a^4*b*c^2*d^2 - 6*a^5*c*d^3)*f)*x^8 + ((
8*a^2*b^3*c^4 - 15*a^3*b^2*c^3*d + 3*a^4*b*c^2*d^2 - 4*a^5*c*d^3)*e - 3*(4
*a^3*b^2*c^4 - 9*a^4*b*c^3*d - 3*a^5*c^2*d^2)*f)*x^6 - ((6*a^3*b^2*c^4 - 1
1*a^4*b*c^3*d - 3*a^5*c^2*d^2)*e - 9*(a^4*b*c^4 + 8*a^5*c^3*d)*f)*x^4 + 5*
(9*a^5*c^4*f + (a^4*b*c^4 + 10*a^5*c^3*d)*e)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x
^2 + c))/(a^5*c^3*x^9)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)}{x^{10}}\,dx=\int \frac{\sqrt{a+bx^2}\left(c+dx^2\right)^{\frac{3}{2}}\left(e+fx^2\right)}{x^{10}}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(3/2)*(f*x**2+e)/x**10,x)
```

output
```
Integral(sqrt(a + b*x**2)*(c + d*x**2)**(3/2)*(e + f*x**2)/x**10, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)}{x^{10}}\,dx=\int \frac{\sqrt{bx^2+a}\left(dx^2+c\right)^{\frac{3}{2}}\left(fx^2+e\right)}{x^{10}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)*(f*x^2+e)/x^10,x, algorithm="max
ima")
```

output
```
integrate(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)*(f*x^2 + e)/x^10, x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}(e+fx^2)}{x^{10}}\,dx=\int \frac{\sqrt{bx^2+a}(dx^2+c)^{\frac{3}{2}}(fx^2+e)}{x^{10}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)*(f*x^2+e)/x^10,x, algorithm="gia
c")
```

output
```
integrate(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)*(f*x^2 + e)/x^10, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}(e+fx^2)}{x^{10}}\,dx=\int \frac{\sqrt{b\,x^2+a}\,(d\,x^2+c)^{3/2}\,(f\,x^2+e)}{x^{10}}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(3/2)*(e + f*x^2))/x^10,x)
```

output
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(3/2)*(e + f*x^2))/x^10, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}(e+fx^2)}{x^{10}}\,dx=\text{too large to display}$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)*(f*x^2+e)/x^10,x)
```

output

```
(120*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**4*c**2*d**2*f*x**2 - 144*sqrt(c
+ d*x**2)*sqrt(a + b*x**2)*a**4*c*d**3*f*x**4 - 200*sqrt(c + d*x**2)*sqrt(
a + b*x**2)*a**3*b*c**3*d*e - 120*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**3*b
*c**3*d*f*x**2 - 360*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**3*b*c**2*d**2*e*
x**2 - 573*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**3*b*c**2*d**2*f*x**4 + 72*
sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**3*b*c*d**3*e*x**4 + 432*sqrt(c + d*x*
*2)*sqrt(a + b*x**2)*a**3*b*d**4*f*x**8 - 200*sqrt(c + d*x**2)*sqrt(a + b*
x**2)*a**2*b**2*c**4*e - 240*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*b**2*c
**4*f*x**2 - 360*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*b**2*c**3*d*e*x**2
 - 627*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*b**2*c**3*d*f*x**4 - 27*sqrt
(c + d*x**2)*sqrt(a + b*x**2)*a**2*b**2*c**2*d**2*e*x**4 - 81*sqrt(c + d*x
**2)*sqrt(a + b*x**2)*a**2*b**2*c*d**3*f*x**8 - 216*sqrt(c + d*x**2)*sqrt(
a + b*x**2)*a**2*b**2*d**4*e*x**8 - 72*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a
*b**3*c**4*f*x**4 + 27*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**3*c**3*d*e*x
**4 + 81*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**3*c**2*d**2*f*x**8 + 81*sq
rt(c + d*x**2)*sqrt(a + b*x**2)*a*b**3*c*d**3*e*x**8 + 216*sqrt(c + d*x**2
)*sqrt(a + b*x**2)*b**4*c**3*d*f*x**8 - 81*sqrt(c + d*x**2)*sqrt(a + b*x**
2)*b**4*c**2*d**2*e*x**8 - 432*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2
)/(a**2*c*d + a**2*d**2*x**2 + a*b*c**2 + 2*a*b*c*d*x**2 + a*b*d**2*x**4 +
 b**2*c**2*x**2 + b**2*c*d*x**4),x)*a**4*b**2*d**6*f*x**9 - 351*int((sq...
```

3.97 $\int \frac{x^4\sqrt{a+bx^2}(c+dx^2)^{3/2}}{e+fx^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 790

$$\int \frac{x^4\sqrt{a+bx^2}(c+dx^2)^{3/2}}{e+fx^2}\,dx = \frac{(8a^3d^3f^3+a^2bd^2f^2(14de-19cf)+ab^2df(35d^2e^2-49cdef+9c^2f^2)-b^3}{105b^2d^2f^4\sqrt{a+bx}}$$

$$-\frac{\left(\frac{4a^2d^2f}{b}+ad(7de-6cf)+b\left(21cde-\frac{35d^2e^2}{f}+6c^2f\right)\right)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{105bdf^2}$$

$$-\frac{(7bde+2bcf-adf)x\sqrt{a+bx^2}(c+dx^2)^{3/2}}{35bdf^2}+\frac{x\sqrt{a+bx^2}(c+dx^2)^{5/2}}{7df}$$

$$-\frac{\sqrt{a}(8a^3d^3f^3+a^2bd^2f^2(14de-19cf)+ab^2df(35d^2e^2-49cdef+9c^2f^2)-b^3(105d^3e^3-140cd^2e^2f+21}{105b^{5/2}d^2f^4\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

$$+\frac{a^{3/2}(4a^2cd^2f^3+abcdf^2(7de-9cf)-b^2(105d^3e^3-175cd^2e^2f+63c^2def^2+3c^3f^3))\sqrt{c+dx^2}\,\mathrm{EllipticF}(}{105b^{5/2}cdf^4\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

$$+\frac{a^{3/2}e(de-cf)^2\sqrt{c+dx^2}\,\mathrm{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}cf^4\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
1/105*(8*a^3*d^3*f^3+a^2*b*d^2*f^2*(-19*c*f+14*d*e)+a*b^2*d*f*(9*c^2*f^2-4
9*c*d*e*f+35*d^2*e^2)-b^3*(6*c^3*f^3+21*c^2*d*e*f^2-140*c*d^2*e^2*f+105*d^
3*e^3))*x*(d*x^2+c)^(1/2)/b^2/d^2/f^4/(b*x^2+a)^(1/2)-1/105*(4*a^2*d^2*f/b
+a*d*(-6*c*f+7*d*e)+b*(21*c*d*e-35*d^2*e^2/f+6*c^2*f))*x*(b*x^2+a)^(1/2)*(
d*x^2+c)^(1/2)/b/d/f^2-1/35*(-a*d*f+2*b*c*f+7*b*d*e)*x*(b*x^2+a)^(1/2)*(d*
x^2+c)^(3/2)/b/d/f^2+1/7*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(5/2)/d/f-1/105*a^(1/
2)*(8*a^3*d^3*f^3+a^2*b*d^2*f^2*(-19*c*f+14*d*e)+a*b^2*d*f*(9*c^2*f^2-49*c
*d*e*f+35*d^2*e^2)-b^3*(6*c^3*f^3+21*c^2*d*e*f^2-140*c*d^2*e^2*f+105*d^3*e
^3))*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/
b/c)^(1/2))/b^(5/2)/d^2/f^4/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2
)+1/105*a^(3/2)*(4*a^2*c*d^2*f^3+a*b*c*d*f^2*(-9*c*f+7*d*e)-b^2*(3*c^3*f^3
+63*c^2*d*e*f^2-175*c*d^2*e^2*f+105*d^3*e^3))*(d*x^2+c)^(1/2)*InverseJacob
iAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/b^(5/2)/c/d/f^4/(b*x^2+a)
^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)+a^(3/2)*e*(-c*f+d*e)^2*(d*x^2+c)^(1
/2)*EllipticPi(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),1-a*f/b/e,(1-a*d/b/c)^(
1/2))/b^(1/2)/c/f^4/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.07 (sec) , antiderivative size = 609, normalized size of antiderivative = 0.77

$$\int \frac{x^4\sqrt{a+bx^2}(c+dx^2)^{3/2}}{e+fx^2}\,dx = \frac{-icf(8a^3d^3f^3+a^2bd^2f^2(14de-19cf)+ab^2df(35d^2e^2-49cdef+9c^2f^2}{}$$

input
```
Integrate[(x^4*Sqrt[a + b*x^2]*(c + d*x^2)^(3/2))/(e + f*x^2),x]
```

output
```
((-I)*c*f*(8*a^3*d^3*f^3 + a^2*b*d^2*f^2*(14*d*e - 19*c*f) + a*b^2*d*f*(35
*d^2*e^2 - 49*c*d*e*f + 9*c^2*f^2) - b^3*(105*d^3*e^3 - 140*c*d^2*e^2*f +
21*c^2*d*e*f^2 + 6*c^3*f^3))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Ellip
ticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + I*(4*a^3*c*d^3*f^4 + a^2*b*c*d
^2*f^3*(7*d*e - 10*c*f) + a*b^2*d*f*(105*d^3*e^3 - 140*c*d^2*e^2*f + 14*c^
2*d*e*f^2 + 12*c^3*f^3) + b^3*(-105*d^4*e^4 + 105*c*d^3*e^3*f + 35*c^2*d^2
*e^2*f^2 - 21*c^3*d*e*f^3 - 6*c^4*f^4))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^
2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + d*(-(Sqrt[b/a]*f^2*
x*(a + b*x^2)*(c + d*x^2)*(4*a^2*d^2*f^2 + a*b*d*f*(7*d*e - 9*c*f - 3*d*f*
x^2) - b^2*(3*c^2*f^2 + 6*c*d*f*(-7*e + 4*f*x^2) + d^2*(35*e^2 - 21*e*f*x^
2 + 15*f^2*x^4)))) - (105*I)*b^2*d*e*(-(b*e) + a*f)*(d*e - c*f)^2*Sqrt[1 +
 (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a
]*x], (a*d)/(b*c)]))/(105*a^2*(b/a)^(5/2)*d^2*f^5*Sqrt[a + b*x^2]*Sqrt[c +
 d*x^2])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}}{e+fx^2}\,dx$$

$$\Big\downarrow \; 450$$

$$\int \frac{x^4\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}}{e+fx^2}dx$$

input
```
Int[(x^4*Sqrt[a + b*x^2]*(c + d*x^2)^(3/2))/(e + f*x^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [A] (verified)

Time = 18.56 (sec) , antiderivative size = 826, normalized size of antiderivative = 1.05

method	result
risch	$-\frac{x\left(-15f^2x^4b^2d^2-3ab\,d^2f^2x^2-24b^2cd\,f^2x^2+21b^2d^2ef\,x^2+4a^2d^2f^2-9abcd\,f^2+7ab\,d^2ef-3b^2c^2f^2+42b^2cdef-35b^2d^2e^2\right)\sqrt{b\,x^2}}{105d\,b^2f^3}$
default	Expression too large to display
elliptic	Expression too large to display

input
```
int(x^4*(b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
-1/105*x/d*(-15*b^2*d^2*f^2*x^4-3*a*b*d^2*f^2*x^2-24*b^2*c*d*f^2*x^2+21*b^
2*d^2*e*f*x^2+4*a^2*d^2*f^2-9*a*b*c*d*f^2+7*a*b*d^2*e*f-3*b^2*c^2*f^2+42*b
^2*c*d*e*f-35*b^2*d^2*e^2)*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b^2/f^3+1/105/b
^2/f^3/d*((4*a^3*c*d^2*f^4-9*a^2*b*c^2*d*f^4+7*a^2*b*c*d^2*e*f^3-3*a*b^2*c
^3*f^4-63*a*b^2*c^2*d*e*f^3+175*a*b^2*c*d^2*e^2*f^2-105*a*b^2*d^3*e^3*f+10
5*b^3*c^2*d*e^2*f^2-210*b^3*c*d^2*e^3*f+105*b^3*d^3*e^4)/f^2/(-b/a)^(1/2)*
(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*El
lipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-1/f*(8*a^3*d^3*f^3-19*a^2
*b*c*d^2*f^3+14*a^2*b*d^3*e*f^2+9*a*b^2*c^2*d*f^3-49*a*b^2*c*d^2*e*f^2+35*
a*b^2*d^3*e^2*f-6*b^3*c^3*f^3-21*b^3*c^2*d*e*f^2+140*b^3*c*d^2*e^2*f-105*b
^3*d^3*e^3)*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*
d*x^2+b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1
/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2)))+105*e*(a*c^2*f^3-
2*a*c*d*e*f^2+a*d^2*e^2*f-b*c^2*e*f^2+2*b*c*d*e^2*f-b*d^2*e^3)*d*b^2/f^2/(
-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a
*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2)))*
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^4\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}}{e+fx^2}\,dx=\text{Timed out}$$

input
```
integrate(x^4*(b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/(f*x^2+e),x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^4\sqrt{a+bx^2}\,(c+dx^2)^{3/2}}{e+fx^2}\,dx = \int \frac{x^4\sqrt{a+bx^2}\,(c+dx^2)^{\frac{3}{2}}}{e+fx^2}\,dx$$

input
```
integrate(x**4*(b*x**2+a)**(1/2)*(d*x**2+c)**(3/2)/(f*x**2+e),x)
```

output
```
Integral(x**4*sqrt(a + b*x**2)*(c + d*x**2)**(3/2)/(e + f*x**2), x)
```

Maxima [F]

$$\int \frac{x^4\sqrt{a+bx^2}\,(c+dx^2)^{3/2}}{e+fx^2}\,dx = \int \frac{\sqrt{bx^2+a}\,(dx^2+c)^{\frac{3}{2}}x^4}{fx^2+e}\,dx$$

input
```
integrate(x^4*(b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/(f*x^2+e),x, algorithm="maxi
ma")
```

output
```
integrate(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)*x^4/(f*x^2 + e), x)
```

Giac [F]

$$\int \frac{x^4\sqrt{a+bx^2}\,(c+dx^2)^{3/2}}{e+fx^2}\,dx = \int \frac{\sqrt{bx^2+a}\,(dx^2+c)^{\frac{3}{2}}x^4}{fx^2+e}\,dx$$

input
```
integrate(x^4*(b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/(f*x^2+e),x, algorithm="giac
")
```

output
```
integrate(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)*x^4/(f*x^2 + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4\sqrt{a+bx^2}(c+dx^2)^{3/2}}{e+fx^2}\,dx = \int \frac{x^4\,\sqrt{b\,x^2+a}\,{\left(d\,x^2+c\right)}^{3/2}}{f\,x^2+e}\,dx$$

input
```
int((x^4*(a + b*x^2)^(1/2)*(c + d*x^2)^(3/2))/(e + f*x^2),x)
```

output
```
int((x^4*(a + b*x^2)^(1/2)*(c + d*x^2)^(3/2))/(e + f*x^2), x)
```

Reduce [F]

$$\int \frac{x^4\sqrt{a+bx^2}(c+dx^2)^{3/2}}{e+fx^2}\,dx = \int \frac{x^4\sqrt{b\,x^2+a}\,(d\,x^2+c)^{\frac{3}{2}}}{f\,x^2+e}dx$$

input
```
int(x^4*(b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/(f*x^2+e),x)
```

output
```
int(x^4*(b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/(f*x^2+e),x)
```

3.98 $\int \frac{x^2\sqrt{a+bx^2}\,(c+dx^2)^{3/2}}{e+fx^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 547

$$\int \frac{x^2\sqrt{a+bx^2}(c+dx^2)^{3/2}}{e+fx^2}\,dx = -\frac{\left(2a^2df+ab(5de-7cf)+b^2\left(20ce-\frac{15de^2}{f}-\frac{3c^2f}{d}\right)\right)x\sqrt{c+dx^2}}{15bf^2\sqrt{a+bx^2}} - \frac{(5bde-3bcf-adf)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{15bf^2} + \frac{x\sqrt{a+bx^2}(c+dx^2)^{3/2}}{5f} + \frac{\sqrt{a}(2a^2d^2f^2+abdf(5de-7cf)-b^2(15d^2e^2-20cdef+3c^2f^2))\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{15b^{3/2}df^3\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} - \frac{a^{3/2}(acdf^2-b(15d^2e^2-25cdef+9c^2f^2))\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{15b^{3/2}cf^3\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} - \frac{a^{3/2}(de-cf)^2\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}cf^3\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
-1/15*(2*a^2*d*f+a*b*(-7*c*f+5*d*e)+b^2*(20*c*e-15*d*e^2/f-3*c^2*f/d))*x*(
d*x^2+c)^(1/2)/b/f^2/(b*x^2+a)^(1/2)-1/15*(-a*d*f-3*b*c*f+5*b*d*e)*x*(b*x^
2+a)^(1/2)*(d*x^2+c)^(1/2)/b/f^2+1/5*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/f+1
/15*a^(1/2)*(2*a^2*d^2*f^2+a*b*d*f*(-7*c*f+5*d*e)-b^2*(3*c^2*f^2-20*c*d*e*
f+15*d^2*e^2))*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/
2),(1-a*d/b/c)^(1/2))/b^(3/2)/d/f^3/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+
a))^(1/2)-1/15*a^(3/2)*(a*c*d*f^2-b*(9*c^2*f^2-25*c*d*e*f+15*d^2*e^2))*(d*
x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/
b^(3/2)/c/f^3/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)-a^(3/2)*(-c*
f+d*e)^2*(d*x^2+c)^(1/2)*EllipticPi(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),1-
a*f/b/e,(1-a*d/b/c)^(1/2))/b^(1/2)/c/f^3/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b
*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.54 (sec) , antiderivative size = 437, normalized size of antiderivative = 0.80

$$\int \frac{x^2\sqrt{a+bx^2}(c+dx^2)^{3/2}}{e+fx^2}\,dx = \frac{icf(2a^2d^2f^2+abdf(5de-7cf)+b^2(-15d^2e^2+20cdef-3c^2f^2))\sqrt{1+}}{}$$

input
```
Integrate[(x^2*Sqrt[a + b*x^2]*(c + d*x^2)^(3/2))/(e + f*x^2),x]
```

output
```
(I*c*f*(2*a^2*d^2*f^2 + a*b*d*f*(5*d*e - 7*c*f) + b^2*(-15*d^2*e^2 + 20*c*
d*e*f - 3*c^2*f^2))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*Ar
cSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*(a^2*c*d^2*f^3 + a*b*d*f*(15*d^2*e^2
- 20*c*d*e*f + 2*c^2*f^2) + b^2*(-15*d^3*e^3 + 15*c*d^2*e^2*f + 5*c^2*d*e*
f^2 - 3*c^3*f^3))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcS
inh[Sqrt[b/a]*x], (a*d)/(b*c)] + d*(Sqrt[b/a]*f^2*x*(a + b*x^2)*(c + d*x^2
)*(a*d*f + b*(-5*d*e + 6*c*f + 3*d*f*x^2)) + (15*I)*b*(-(b*e) + a*f)*(d*e
- c*f)^2*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e), I
*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(15*b*Sqrt[b/a]*d*f^4*Sqrt[a + b*x^2
]*Sqrt[c + d*x^2])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}}{e+fx^2}\,dx$$

$$\Big\downarrow \; 450$$

$$\int \frac{x^2\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}}{e+fx^2}dx$$

input
```
Int[(x^2*Sqrt[a + b*x^2]*(c + d*x^2)^(3/2))/(e + f*x^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [A] (verified)

Time = 8.98 (sec) , antiderivative size = 601, normalized size of antiderivative = 1.10

method	result
risch	$\frac{x\left(3bdf\,x^{2}+adf+6bcf-5bde\right)\sqrt{b\,x^{2}+a}\,\sqrt{x^{2}d+c}}{15b\,f^{2}}-\frac{\left(\frac{\left(a^{2}cd\,f^{3}-9ab\,c^{2}f^{3}+25abcde\,f^{2}-15ab\,d^{2}e^{2}f+15b^{2}c^{2}e\,f^{2}-30b^{2}cd\,e^{2}f+15b^{2}d^{2}e^{3}\right)\sqrt{}}{f^{2}\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^{4}+ad\,x^{2}+x^{2}bc+ac}}\right.}{}$
default	Expression too large to display
elliptic	Expression too large to display

input
```
int(x^2*(b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
1/15*x*(3*b*d*f*x^2+a*d*f+6*b*c*f-5*b*d*e)*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)
/b/f^2-1/15/f^2/b*((a^2*c*d*f^3-9*a*b*c^2*f^3+25*a*b*c*d*e*f^2-15*a*b*d^2*
e^2*f+15*b^2*c^2*e*f^2-30*b^2*c*d*e^2*f+15*b^2*d^2*e^3)/f^2/(-b/a)^(1/2)*(
1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*Ell
ipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-1/f*(2*a^2*d^2*f^2-7*a*b*c
*d*f^2+5*a*b*d^2*e*f-3*b^2*c^2*f^2+20*b^2*c*d*e*f-15*b^2*d^2*e^2)*c/(-b/a)
^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(
1/2)/d*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b
/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2)))+15*(a*c^2*f^3-2*a*c*d*e*f^2+a*d^2*e^2
*f-b*c^2*e*f^2+2*b*c*d*e^2*f-b*d^2*e^3)*b/f^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/
2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/
a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2)))*((b*x^2+a)*(d*x^2+c))^(1/2)
/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^2\sqrt{a+bx^2}(c+dx^2)^{3/2}}{e+fx^2}\,dx = \text{Timed out}$$

input
```
integrate(x^2*(b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/(f*x^2+e),x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^2\sqrt{a+bx^2}(c+dx^2)^{3/2}}{e+fx^2}\,dx = \int \frac{x^2\sqrt{a+bx^2}\,(c+dx^2)^{\frac{3}{2}}}{e+fx^2}\,dx$$

input
```
integrate(x**2*(b*x**2+a)**(1/2)*(d*x**2+c)**(3/2)/(f*x**2+e),x)
```

output
```
Integral(x**2*sqrt(a + b*x**2)*(c + d*x**2)**(3/2)/(e + f*x**2), x)
```

Maxima [F]

$$\int \frac{x^2\sqrt{a+bx^2}(c+dx^2)^{3/2}}{e+fx^2}\,dx = \int \frac{\sqrt{bx^2+a}(dx^2+c)^{\frac{3}{2}}x^2}{fx^2+e}\,dx$$

input
```
integrate(x^2*(b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/(f*x^2+e),x, algorithm="maxi
ma")
```

output
```
integrate(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)*x^2/(f*x^2 + e), x)
```

Giac [F]

$$\int \frac{x^2\sqrt{a+bx^2}(c+dx^2)^{3/2}}{e+fx^2}\,dx = \int \frac{\sqrt{bx^2+a}(dx^2+c)^{\frac{3}{2}}x^2}{fx^2+e}\,dx$$

input
```
integrate(x^2*(b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/(f*x^2+e),x, algorithm="giac
")
```

output
```
integrate(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)*x^2/(f*x^2 + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2\sqrt{a+bx^2}(c+dx^2)^{3/2}}{e+fx^2}\,dx = \int \frac{x^2\,\sqrt{b\,x^2+a}\,{\left(d\,x^2+c\right)}^{3/2}}{f\,x^2+e}\,dx$$

input
```
int((x^2*(a + b*x^2)^(1/2)*(c + d*x^2)^(3/2))/(e + f*x^2),x)
```

output
```
int((x^2*(a + b*x^2)^(1/2)*(c + d*x^2)^(3/2))/(e + f*x^2), x)
```

Reduce [F]

$$\int \frac{x^2\sqrt{a+bx^2}(c+dx^2)^{3/2}}{e+fx^2}\,dx = \text{Too large to display}$$

input
```
int(x^2*(b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/(f*x^2+e),x)
```

output
```
(sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*d*f*x + 6*sqrt(c + d*x**2)*sqrt(a + b
*x**2)*b*c*f*x - 5*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b*d*e*x + 3*sqrt(c +
d*x**2)*sqrt(a + b*x**2)*b*d*f*x**3 - 2*int((sqrt(c + d*x**2)*sqrt(a + b*x
**2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*
c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*a**2*d**2*f**2 + 7*int((sqrt(c + d*
x**2)*sqrt(a + b*x**2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4
 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*a*b*c*d*f**2 - 5*
int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*e*x
**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*a
*b*d**2*e*f + 3*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c*e + a*c*
f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 +
b*d*f*x**6),x)*b**2*c**2*f**2 - 20*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*
x**4)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x
**4 + b*d*e*x**4 + b*d*f*x**6),x)*b**2*c*d*e*f + 15*int((sqrt(c + d*x**2)*
sqrt(a + b*x**2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c
*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*b**2*d**2*e**2 - int((s
qrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 +
a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*a**2*c*
d*f**2 - 2*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c*e + a*c*f*x**
2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*...
```

3.99 $\int \frac{\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}}{e+fx^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 402

$$\begin{aligned}\int \frac{\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}}{e+fx^2}\,dx =& -\frac{(3bde-4bcf-adf)x\sqrt{c+dx^2}}{3f^2\sqrt{a+bx^2}}+\frac{dx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3f}\\ &+\frac{\sqrt{a}(3bde-4bcf-adf)\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{3\sqrt{b}f^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}\\ &-\frac{a^{3/2}d(3de-5cf)\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{3\sqrt{b}cf^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}\\ &+\frac{a^{3/2}(de-cf)^2\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}cef^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}\end{aligned}$$

output
```
-1/3*(-a*d*f-4*b*c*f+3*b*d*e)*x*(d*x^2+c)^(1/2)/f^2/(b*x^2+a)^(1/2)+1/3*d*
x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/f+1/3*a^(1/2)*(-a*d*f-4*b*c*f+3*b*d*e)*(
d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(
1/2))/b^(1/2)/f^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)-1/3*a^(3
/2)*d*(-5*c*f+3*d*e)*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1
/2)),(1-a*d/b/c)^(1/2))/b^(1/2)/c/f^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^
2+a))^(1/2)+a^(3/2)*(-c*f+d*e)^2*(d*x^2+c)^(1/2)*EllipticPi(b^(1/2)*x/a^(1
/2)/(1+b*x^2/a)^(1/2),1-a*f/b/e,(1-a*d/b/c)^(1/2))/b^(1/2)/c/e/f^2/(b*x^2+
a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.47 (sec) , antiderivative size = 742, normalized size of antiderivative = 1.85

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}}{e+fx^2}\,dx = \frac{a\sqrt{\frac{b}{a}}cdef^2x+b\sqrt{\frac{b}{a}}cdef^2x^3+a\sqrt{\frac{b}{a}}d^2ef^2x^3+b\sqrt{\frac{b}{a}}d^2ef^2x^5-icef(-3bde+$$

input
```
Integrate[(Sqrt[a + b*x^2]*(c + d*x^2)^(3/2))/(e + f*x^2),x]
```

output
```
(a*Sqrt[b/a]*c*d*e*f^2*x + b*Sqrt[b/a]*c*d*e*f^2*x^3 + a*Sqrt[b/a]*d^2*e*f
^2*x^3 + b*Sqrt[b/a]*d^2*e*f^2*x^5 - I*c*e*f*(-3*b*d*e + 4*b*c*f + a*d*f)*
Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x],
(a*d)/(b*c)] - I*e*(a*d*f*(-3*d*e + 4*c*f) + b*(3*d^2*e^2 - 3*c*d*e*f - c^
2*f^2))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b
/a]*x], (a*d)/(b*c)] + (3*I)*b*d^2*e^3*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2
)/c]*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - (6*I)*
b*c*d*e^2*f*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e)
, I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - (3*I)*a*d^2*e^2*f*Sqrt[1 + (b*x^2
)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (
a*d)/(b*c)] + (3*I)*b*c^2*e*f^2*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*El
lipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (6*I)*a*c*d*e
*f^2*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e), I*Arc
Sinh[Sqrt[b/a]*x], (a*d)/(b*c)] - (3*I)*a*c^2*f^3*Sqrt[1 + (b*x^2)/a]*Sqrt
[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c
)])/(3*Sqrt[b/a]*e*f^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 382, normalized size of antiderivative = 0.95, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.281$, Rules used = {418, 25, 403, 27, 406, 320, 388, 313, 414}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}}{e+fx^2}dx$$

$$\Big\downarrow \text{418}$$

$$\frac{(de-cf)^2\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)}dx}{f^2}+\frac{d\int -\frac{\sqrt{bx^2+a}(-dfx^2+de-2cf)}{\sqrt{dx^2+c}}dx}{f^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{(de-cf)^2\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)}dx}{f^2}-\frac{d\int \frac{\sqrt{bx^2+a}(-dfx^2+de-2cf)}{\sqrt{dx^2+c}}dx}{f^2}$$

$$\Big\downarrow \text{403}$$

$$\frac{(de-cf)^2\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)}dx}{f^2}-\frac{d\left(\frac{\int \frac{d\left((3bde-4bcf-adf)x^2+a(3de-5cf)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3d}-\frac{1}{3}fx\sqrt{a+bx^2}\sqrt{c+dx^2}\right)}{f^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{(de-cf)^2\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)}dx}{f^2}-\frac{d\left(\frac{1}{3}\int \frac{(3bde-4bcf-adf)x^2+a(3de-5cf)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx-\frac{1}{3}fx\sqrt{a+bx^2}\sqrt{c+dx^2}\right)}{f^2}$$

$$\Big\downarrow \text{406}$$

$$\frac{(de-cf)^2\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)}dx}{f^2}-\frac{d\left(\frac{1}{3}\left(a(3de-5cf)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+(-adf-4bcf+3bde)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)-\frac{1}{3}fx\sqrt{a+bx^2}\sqrt{c+dx^2}\right)}{f^2}$$

$$\Big\downarrow \text{320}$$

$$\frac{(de-cf)^2\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)}dx}{f^2}-$$

$$\frac{d\left(\frac{1}{3}\left((-adf-4bcf+3bde)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{\sqrt{c}\sqrt{a+bx^2}(3de-5cf)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)-\frac{1}{3}fx\sqrt{a+bx^2}\right.}{f^2}$$

$\downarrow$ 388

$$\frac{(de-cf)^2\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)}dx}{f^2}-$$

$$\frac{d\left(\frac{1}{3}\left((-adf-4bcf+3bde)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{\sqrt{c}\sqrt{a+bx^2}(3de-5cf)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)-\frac{1}{3}\right.}{f^2}$$

$\downarrow$ 313

$$\frac{(de-cf)^2\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)}dx}{f^2}-$$

$$\frac{d\left(\frac{1}{3}\left(\frac{\sqrt{c}\sqrt{a+bx^2}(3de-5cf)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+(-adf-4bcf+3bde)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1\right.}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right.\right.\right.}{f^2}$$

$\downarrow$ 414

$$\frac{a^{3/2}\sqrt{c+dx^2}(de-cf)^2\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}cef^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}-$$

$$\frac{d\left(\frac{1}{3}\left(\frac{\sqrt{c}\sqrt{a+bx^2}(3de-5cf)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+(-adf-4bcf+3bde)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1\right.}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right.\right.\right.}{f^2}$$

input
```
Int[(Sqrt[a + b*x^2]*(c + d*x^2)^(3/2))/(e + f*x^2),x]
```

output
```
-((d*(-1/3*(f*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]) + ((3*b*d*e - 4*b*c*f - a
*d*f)*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*
EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(
c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (Sqrt[c]*(3*d*e - 5*c*
f)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)]
)/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))/3))/f^2
) + (a^(3/2)*(d*e - c*f)^2*Sqrt[c + d*x^2]*EllipticPi[1 - (a*f)/(b*e), Arc
Tan[(Sqrt[b]*x)/Sqrt[a]], 1 - (a*d)/(b*c)])/(Sqrt[b]*c*e*f^2*Sqrt[a + b*x^
2]*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 418
```
Int[(((c_) + (d_.)*(x_)^2)^(3/2)*Sqrt[(e_) + (f_.)*(x_)^2])/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[(b*c - a*d)^2/b^2   Int[Sqrt[e + f*x^2]/((a + b*x
^2)*Sqrt[c + d*x^2]), x], x] + Simp[d/b^2   Int[(2*b*c - a*d + b*d*x^2)*(Sq
rt[e + f*x^2]/Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && P
osQ[d/c] && PosQ[f/e]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 830 vs. $2(372) = 744$.

Time = 9.45 (sec) , antiderivative size = 831, normalized size of antiderivative = 2.07

method	result
risch	$\frac{dx\sqrt{b\,x^2+a}\,\sqrt{x^2d+c}}{3f} + \frac{\left(-\frac{f(adf+4bcf-3bde)c\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\left(\text{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)-\text{EllipticE}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}\right.}{}$
default	Expression too large to display
elliptic	Expression too large to display

input

```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output

```
1/3*d*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/f+1/3/f*(1/f^2*(-f*(a*d*f+4*b*c*f-
3*b*d*e)*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x
^2+b*c*x^2+a*c)^(1/2)*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-
EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2)))+3*b*c^2*f^2/(-b/a)^(1/
2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+3*b*d^2*e^2/(-b/a)^(1/
2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+5*a*c*d*f^2/(-b/a)^(1/
2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-3*a*d^2*e*f/(-b/a)^(1/
2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-6*b*c*d*e*f/(-b/a)^(1/
2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2)))+3*(a*c^2*f^3-2*a*c*d*
e*f^2+a*d^2*e^2*f-b*c^2*e*f^2+2*b*c*d*e^2*f-b*d^2*e^3)/f^2/e/(-b/a)^(1/2)*
(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*El
lipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2)))*((b*x^2+a)*(
d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}}{e+fx^2}\,dx = \text{Timed out}$$

input

```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/(f*x^2+e),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}}{e+fx^2}\,dx = \int \frac{\sqrt{a+bx^2}(c+dx^2)^{\frac{3}{2}}}{e+fx^2}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(3/2)/(f*x**2+e),x)
```

output
```
Integral(sqrt(a + b*x**2)*(c + d*x**2)**(3/2)/(e + f*x**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}}{e+fx^2}\,dx = \int \frac{\sqrt{bx^2+a}(dx^2+c)^{\frac{3}{2}}}{fx^2+e}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/(f*x^2+e),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)/(f*x^2 + e), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}}{e+fx^2}\,dx = \int \frac{\sqrt{bx^2+a}(dx^2+c)^{\frac{3}{2}}}{fx^2+e}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/(f*x^2+e),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)/(f*x^2 + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}}{e+fx^2}\,dx = \int \frac{\sqrt{b\,x^2+a}\,{\left(d\,x^2+c\right)}^{3/2}}{f\,x^2+e}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(3/2))/(e + f*x^2),x)
```

output
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(3/2))/(e + f*x^2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}}{e+fx^2}\,dx = \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,dx+\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^4}{bdf\,x^6+adf\,x^4+bcf\,x^4+bde\,x^4+acf\,x^2+ade\,x^2+bce\,x^2+ace}dx\right.}{}$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/(f*x^2+e),x)
```

output
```
(sqrt(c + d*x**2)*sqrt(a + b*x**2)*d*x + int((sqrt(c + d*x**2)*sqrt(a + b*
x**2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b
*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*a*d**2*f + 4*int((sqrt(c + d*x**2)
*sqrt(a + b*x**2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*
c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*b*c*d*f - 3*int((sqrt(
c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*
f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*b*d**2*e +
5*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c*e + a*c*f*x**2 + a*d*e
*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)
*a*c*d*f - 2*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c*e + a*c*f*x
**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d
*f*x**6),x)*a*d**2*e + 3*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c
*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*
e*x**4 + b*d*f*x**6),x)*b*c**2*f - 2*int((sqrt(c + d*x**2)*sqrt(a + b*x**2
)*x**2)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f
*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*b*c*d*e + 3*int((sqrt(c + d*x**2)*sqrt
(a + b*x**2))/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 +
 b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*a*c**2*f - int((sqrt(c + d*x**2)
*sqrt(a + b*x**2))/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x
**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*a*c*d*e)/(3*f)
```

3.100 $\int \frac{\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}}{x^2\left(e+fx^2\right)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 516

$$\begin{aligned}\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}}{x^2\left(e+fx^2\right)}\,dx &= \frac{b(de+cf)x\sqrt{c+dx^2}}{ef\sqrt{a+bx^2}} \\ &+ \frac{(bc+2ad)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{ae} + \frac{d(2bc+ad)x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{ace} \\ &+ \frac{bd^2x^5\sqrt{a+bx^2}\sqrt{c+dx^2}}{ace} - \frac{\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{5/2}}{acex} \\ &- \frac{\sqrt{a}\sqrt{b}(de+cf)\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{ef\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} \\ &+ \frac{\sqrt{a}\left(ad^2e+bc^2f\right)\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}cef\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} \\ &- \frac{a^{3/2}(de-cf)^2\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce^2f\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}\end{aligned}$$

output
```
b*(c*f+d*e)*x*(d*x^2+c)^(1/2)/e/f/(b*x^2+a)^(1/2)+(2*a*d+b*c)*x*(b*x^2+a)^
(1/2)*(d*x^2+c)^(1/2)/a/e+d*(a*d+2*b*c)*x^3*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2
)/a/c/e+b*d^2*x^5*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a/c/e-(b*x^2+a)^(3/2)*(d
*x^2+c)^(5/2)/a/c/e/x-a^(1/2)*b^(1/2)*(c*f+d*e)*(d*x^2+c)^(1/2)*EllipticE(
b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/e/f/(b*x^2+a)^(1/2)
/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)+a^(1/2)*(a*d^2*e+b*c^2*f)*(d*x^2+c)^(1/2)
*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/b^(1/2)/c/e/
f/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)-a^(3/2)*(-c*f+d*e)^2*(d*
x^2+c)^(1/2)*EllipticPi(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),1-a*f/b/e,(1-a
*d/b/c)^(1/2))/b^(1/2)/c/e^2/f/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(
1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.97 (sec) , antiderivative size = 739, normalized size of antiderivative = 1.43

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}}{x^2(e+fx^2)}\,dx = \frac{-a\sqrt{\frac{b}{a}}c^2ef^2-b\sqrt{\frac{b}{a}}c^2ef^2x^2-a\sqrt{\frac{b}{a}}cdef^2x^2-b\sqrt{\frac{b}{a}}cdef^2x^4-ibcef(de+c$$

input
```
Integrate[(Sqrt[a + b*x^2]*(c + d*x^2)^(3/2))/(x^2*(e + f*x^2)),x]
```

output
```
(-(a*Sqrt[b/a]*c^2*e*f^2) - b*Sqrt[b/a]*c^2*e*f^2*x^2 - a*Sqrt[b/a]*c*d*e*
f^2*x^2 - b*Sqrt[b/a]*c*d*e*f^2*x^4 - I*b*c*e*f*(d*e + c*f)*x*Sqrt[1 + (b*
x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]
 - I*e*(-(b*d^2*e^2) + b*c*d*e*f + a*d^2*e*f - b*c^2*f^2)*x*Sqrt[1 + (b*x^
2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] -
 I*b*d^2*e^3*x*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b
*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (2*I)*b*c*d*e^2*f*x*Sqrt[1 + (
b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*
x], (a*d)/(b*c)] + I*a*d^2*e^2*f*x*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]
*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*b*c^2*e*
f^2*x*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e), I*Ar
cSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - (2*I)*a*c*d*e*f^2*x*Sqrt[1 + (b*x^2)/a]
*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)
/(b*c)] + I*a*c^2*f^3*x*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi
[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(Sqrt[b/a]*e^2*f^2*x*S
qrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}}{x^2\left(e+fx^2\right)}\,dx$$

$$\Big\downarrow \; 450$$

$$\int \frac{\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}}{x^2\left(e+fx^2\right)}dx$$

input
```
Int[(Sqrt[a + b*x^2]*(c + d*x^2)^(3/2))/(x^2*(e + f*x^2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [A] (verified)

Time = 9.85 (sec) , antiderivative size = 636, normalized size of antiderivative = 1.23

method	result
risch	$-\frac{c\sqrt{b\,x^2+a}\,\sqrt{x^2d+c}}{ex}+\frac{\left(d\left(-\frac{fb(cf+de)c\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\left(\text{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)-\text{EllipticE}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}\,d}+\frac{ac}{}\right.\right.}{}$
default	$\frac{\left(-\sqrt{-\frac{b}{a}}\,bcde\,f^2x^4+\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{x^2d+c}{c}}\,\text{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)a\,d^2e^2fx-\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{x^2d+c}{c}}\,\text{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)b\,c^2e\,f^2\right.}{}$
elliptic	Expression too large to display

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/x^2/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
-c/e*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x+1/e*(1/f^2*d*(-f*b*(c*f+d*e)*c/(-b/
a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)
^(1/2)/d*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(
-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2)))+a*d*e*f/(-b/a)^(1/2)*(1+b*x^2/a)^(1
/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/
a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-b*d*e^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*
(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(
1/2),(-1+(a*d+b*c)/c/b)^(1/2))+2*b*c*e*f/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1
+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/
2),(-1+(a*d+b*c)/c/b)^(1/2)))-(a*c^2*f^3-2*a*c*d*e*f^2+a*d^2*e^2*f-b*c^2*e
*f^2+2*b*c*d*e^2*f-b*d^2*e^3)/f^2/e/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^
2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a
*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2)))*((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)
^(1/2)/(d*x^2+c)^(1/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}}{x^2\left(e+fx^2\right)}\,dx=\text{Timed out}$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/x^2/(f*x^2+e),x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}}{x^2\left(e+fx^2\right)}\,dx=\int \frac{\sqrt{a+bx^2}\left(c+dx^2\right)^{\frac{3}{2}}}{x^2\left(e+fx^2\right)}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(3/2)/x**2/(f*x**2+e),x)
```

output
```
Integral(sqrt(a + b*x**2)*(c + d*x**2)**(3/2)/(x**2*(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}}{x^2(e+fx^2)}\,dx=\int \frac{\sqrt{bx^2+a}(dx^2+c)^{\frac{3}{2}}}{(fx^2+e)x^2}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/x^2/(f*x^2+e),x, algorithm="maxi
ma")
```

output
```
integrate(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)/((f*x^2 + e)*x^2), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}}{x^2(e+fx^2)}\,dx=\int \frac{\sqrt{bx^2+a}(dx^2+c)^{\frac{3}{2}}}{(fx^2+e)x^2}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/x^2/(f*x^2+e),x, algorithm="giac
")
```

output
```
integrate(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)/((f*x^2 + e)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}}{x^2(e+fx^2)}\,dx=\int \frac{\sqrt{b\,x^2+a}\,{(d\,x^2+c)}^{3/2}}{x^2\,(f\,x^2+e)}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(3/2))/(x^2*(e + f*x^2)),x)
```

output
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(3/2))/(x^2*(e + f*x^2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}}{x^2\left(e+fx^2\right)}\,dx = \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,ad+2\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,bc-\left(\int \frac{\sqrt{d\,x^2+\cdots}}{bdf\,x^6+adf\,x^4+bcf\,x^4+bd\cdots}\right.}{\cdots}$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/x^2/(f*x^2+e),x)
```

output
```
(sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*d + 2*sqrt(c + d*x**2)*sqrt(a + b*x**
2)*b*c - int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c*e + a*c*f*x**2
+ a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x
**6),x)*a*b*d**2*f*x - 2*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c
*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*
e*x**4 + b*d*f*x**6),x)*b**2*c*d*f*x + int((sqrt(c + d*x**2)*sqrt(a + b*x*
*2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c
*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*b**2*d**2*e*x + int((sqrt(c + d*x**2
)*sqrt(a + b*x**2))/(a*c*e*x**2 + a*c*f*x**4 + a*d*e*x**4 + a*d*f*x**6 + b
*c*e*x**4 + b*c*f*x**6 + b*d*e*x**6 + b*d*f*x**8),x)*a**2*c*d*e*x + 3*int(
(sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c*e*x**2 + a*c*f*x**4 + a*d*e*x**4
+ a*d*f*x**6 + b*c*e*x**4 + b*c*f*x**6 + b*d*e*x**6 + b*d*f*x**8),x)*a*b*c
**2*e*x + int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c*e + a*c*f*x**2 + a*
d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6)
,x)*a**2*c*d*f*x + 2*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c*e + a*c*
f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 +
b*d*f*x**6),x)*a*b*c**2*f*x + 2*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a
*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*
d*e*x**4 + b*d*f*x**6),x)*a*b*c*d*e*x + int((sqrt(c + d*x**2)*sqrt(a + b*x
**2))/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*...
```

3.101 $\int \frac{\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}}{x^4\left(e+fx^2\right)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 627

$$\begin{aligned}
\int \frac{\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}}{x^4\left(e+fx^2\right)}\,dx &= \frac{b(bce+4ade-3acf)x\sqrt{c+dx^2}}{3ae^2\sqrt{a+bx^2}} \\
&+ \frac{(ad(5de-6cf)+bc(4de-3cf))x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3ace^2} \\
&+ \frac{d(bc(5de-6cf)+ad(2de-3cf))x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{3ac^2e^2} \\
&+ \frac{bd^2(2de-3cf)x^5\sqrt{a+bx^2}\sqrt{c+dx^2}}{3ac^2e^2} - \frac{\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{5/2}}{3acex^3} \\
&- \frac{(2de-3cf)\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{5/2}}{3ac^2e^2x} \\
&- \frac{\sqrt{b}(bce+4ade-3acf)\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{3\sqrt{a}e^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} \\
&+ \frac{\sqrt{a}\sqrt{b}(5de-3cf)\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{3e^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} \\
&+ \frac{a^{3/2}(de-cf)^2\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce^3\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}
\end{aligned}$$

output
```
1/3*b*(-3*a*c*f+4*a*d*e+b*c*e)*x*(d*x^2+c)^(1/2)/a/e^2/(b*x^2+a)^(1/2)+1/3
*(a*d*(-6*c*f+5*d*e)+b*c*(-3*c*f+4*d*e))*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)
/a/c/e^2+1/3*d*(b*c*(-6*c*f+5*d*e)+a*d*(-3*c*f+2*d*e))*x^3*(b*x^2+a)^(1/2)
*(d*x^2+c)^(1/2)/a/c^2/e^2+1/3*b*d^2*(-3*c*f+2*d*e)*x^5*(b*x^2+a)^(1/2)*(d
*x^2+c)^(1/2)/a/c^2/e^2-1/3*(b*x^2+a)^(3/2)*(d*x^2+c)^(5/2)/a/c/e/x^3-1/3*
(-3*c*f+2*d*e)*(b*x^2+a)^(3/2)*(d*x^2+c)^(5/2)/a/c^2/e^2/x-1/3*b^(1/2)*(-3
*a*c*f+4*a*d*e+b*c*e)*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2
/a)^(1/2),(1-a*d/b/c)^(1/2))/a^(1/2)/e^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b
*x^2+a))^(1/2)+1/3*a^(1/2)*b^(1/2)*(-3*c*f+5*d*e)*(d*x^2+c)^(1/2)*InverseJ
acobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/e^2/(b*x^2+a)^(1/2)/(
a*(d*x^2+c)/c/(b*x^2+a))^(1/2)+a^(3/2)*(-c*f+d*e)^2*(d*x^2+c)^(1/2)*Ellipt
icPi(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),1-a*f/b/e,(1-a*d/b/c)^(1/2))/b^(1
/2)/c/e^3/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.23 (sec) , antiderivative size = 369, normalized size of antiderivative = 0.59

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}}{x^4(e+fx^2)}\,dx = \frac{\sqrt{\frac{b}{a}}\left(-\sqrt{\frac{b}{a}}ef(a+bx^2)(c+dx^2)(bcex^2+4adex^2+ac(e-3fx^2))+ibcef(\right.}{}$$

input
```
Integrate[(Sqrt[a + b*x^2]*(c + d*x^2)^(3/2))/(x^4*(e + f*x^2)),x]
```

output
```
(Sqrt[b/a]*(-(Sqrt[b/a]*e*f*(a + b*x^2)*(c + d*x^2)*(b*c*e*x^2 + 4*a*d*e*x
^2 + a*c*(e - 3*f*x^2))) + I*b*c*e*f*(-(b*c*e) - 4*a*d*e + 3*a*c*f)*x^3*Sq
rt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a
*d)/(b*c)] - I*b*e*(-(b*c^2*e*f) + a*(3*d^2*e^2 - 5*c*d*e*f + 3*c^2*f^2))*
x^3*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*
x], (a*d)/(b*c)] - (3*I)*a*(-(b*e) + a*f)*(d*e - c*f)^2*x^3*Sqrt[1 + (b*x^
2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x],
(a*d)/(b*c)]))/(3*b*e^3*f*x^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}}{x^4\left(e+fx^2\right)}dx$$

$$\Big\downarrow \text{450}$$

$$\int \frac{\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}}{x^4\left(e+fx^2\right)}dx$$

input
```
Int[(Sqrt[a + b*x^2]*(c + d*x^2)^(3/2))/(x^4*(e + f*x^2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [A] (verified)

Time = 8.48 (sec) , antiderivative size = 498, normalized size of antiderivative = 0.79

method	result
risch	$-\frac{\sqrt{b\,x^{2}+a}\,\sqrt{x^{2} d+c}\,\left(-3acf\,x^{2}+4ade\,x^{2}+bce\,x^{2}+ace\right)}{3a\,e^{2} x^{3}}-\frac{\left(-\frac{b(3acf-4ade-bce)c\sqrt{1+\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,\left(\text{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right.\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^{4}+ad\,x^{2}+x^{2}bc+ac}}\right.}{}$
default	Expression too large to display
elliptic	Expression too large to display

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/x^4/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(-3*a*c*f*x^2+4*a*d*e*x^2+b*c*e*x^2+a
*c*e)/a/e^2/x^3-1/3/e^2/a*(-b*(3*a*c*f-4*a*d*e-b*c*e)*c/(-b/a)^(1/2)*(1+b*
x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(Ellipt
icF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+
(a*d+b*c)/c/b)^(1/2)))+b*d*(c*f-3*d*e)*a*e/f/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2
)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)
^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-3*(a*c^2*f^3-2*a*c*d*e*f^2+a*d^2*e^2*f-b*
c^2*e*f^2+2*b*c*d*e^2*f-b*d^2*e^3)*a/f/e/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1
+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1
/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2)))*((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x
^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}}{x^4(e+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/x^4/(f*x^2+e),x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}}{x^4(e+fx^2)}\,dx = \int \frac{\sqrt{a+bx^2}(c+dx^2)^{\frac{3}{2}}}{x^4(e+fx^2)}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(3/2)/x**4/(f*x**2+e),x)
```

output
```
Integral(sqrt(a + b*x**2)*(c + d*x**2)**(3/2)/(x**4*(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}}{x^4(e+fx^2)}\,dx = \int \frac{\sqrt{bx^2+a}(dx^2+c)^{\frac{3}{2}}}{(fx^2+e)x^4}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/x^4/(f*x^2+e),x, algorithm="maxi
ma")
```

output
```
integrate(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)/((f*x^2 + e)*x^4), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}}{x^4(e+fx^2)}\,dx = \int \frac{\sqrt{bx^2+a}(dx^2+c)^{\frac{3}{2}}}{(fx^2+e)x^4}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/x^4/(f*x^2+e),x, algorithm="giac
")
```

output
```
integrate(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)/((f*x^2 + e)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}}{x^4(e+fx^2)}\,dx = \int \frac{\sqrt{b\,x^2+a}\,(d\,x^2+c)^{3/2}}{x^4\,(f\,x^2+e)}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(3/2))/(x^4*(e + f*x^2)),x)
```

output
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(3/2))/(x^4*(e + f*x^2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}}{x^4(e+fx^2)}\,dx = \text{too large to display}$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/x^4/(f*x^2+e),x)
```

output
```
( - sqrt(c + d*x**2)*sqrt(a + b*x**2)*c - 2*int((sqrt(c + d*x**2)*sqrt(a +
 b*x**2)*x**2)/(2*a**2*c*d*e*f + 2*a**2*c*d*f**2*x**2 + 2*a**2*d**2*e*f*x*
*2 + 2*a**2*d**2*f**2*x**4 + 2*a*b*c**2*e*f + 2*a*b*c**2*f**2*x**2 + a*b*c
*d*e**2 + 5*a*b*c*d*e*f*x**2 + 4*a*b*c*d*f**2*x**4 + a*b*d**2*e**2*x**2 +
3*a*b*d**2*e*f*x**4 + 2*a*b*d**2*f**2*x**6 + 2*b**2*c**2*e*f*x**2 + 2*b**2
*c**2*f**2*x**4 + b**2*c*d*e**2*x**2 + 3*b**2*c*d*e*f*x**4 + 2*b**2*c*d*f*
*2*x**6 + b**2*d**2*e**2*x**4 + b**2*d**2*e*f*x**6),x)*a*b*c*d**2*f**2*x**
3 + 6*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(2*a**2*c*d*e*f + 2*a**
2*c*d*f**2*x**2 + 2*a**2*d**2*e*f*x**2 + 2*a**2*d**2*f**2*x**4 + 2*a*b*c**
2*e*f + 2*a*b*c**2*f**2*x**2 + a*b*c*d*e**2 + 5*a*b*c*d*e*f*x**2 + 4*a*b*c
*d*f**2*x**4 + a*b*d**2*e**2*x**2 + 3*a*b*d**2*e*f*x**4 + 2*a*b*d**2*f**2*
x**6 + 2*b**2*c**2*e*f*x**2 + 2*b**2*c**2*f**2*x**4 + b**2*c*d*e**2*x**2 +
 3*b**2*c*d*e*f*x**4 + 2*b**2*c*d*f**2*x**6 + b**2*d**2*e**2*x**4 + b**2*d
**2*e*f*x**6),x)*a*b*d**3*e*f*x**3 - 2*int((sqrt(c + d*x**2)*sqrt(a + b*x*
*2)*x**2)/(2*a**2*c*d*e*f + 2*a**2*c*d*f**2*x**2 + 2*a**2*d**2*e*f*x**2 +
2*a**2*d**2*f**2*x**4 + 2*a*b*c**2*e*f + 2*a*b*c**2*f**2*x**2 + a*b*c*d*e*
*2 + 5*a*b*c*d*e*f*x**2 + 4*a*b*c*d*f**2*x**4 + a*b*d**2*e**2*x**2 + 3*a*b
*d**2*e*f*x**4 + 2*a*b*d**2*f**2*x**6 + 2*b**2*c**2*e*f*x**2 + 2*b**2*c**2
*f**2*x**4 + b**2*c*d*e**2*x**2 + 3*b**2*c*d*e*f*x**4 + 2*b**2*c*d*f**2*x*
*6 + b**2*d**2*e**2*x**4 + b**2*d**2*e*f*x**6),x)*b**2*c**2*d*f**2*x**3...
```

3.102 $\int \frac{\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}}{x^6\left(e+fx^2\right)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 867

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}}{x^6(e+fx^2)}\,dx = -\frac{b(2b^2c^2e^2-abce(7de-5cf)-a^2(3d^2e^2-20cdef+15c^2f^2))\,x\sqrt{c+dx^2}}{15a^2ce^3\sqrt{a+bx^2}} + \frac{(b^2cde^2-5a^2df(5de-6cf)+ab(11d^2e^2-20cdef+15c^2f^2))\,x\sqrt{a+bx^2}\sqrt{c+dx^2}}{15a^2ce^3} + \frac{d(8b^2cde^2-5a^2df(2de-3cf)+5ab(d^2e^2-5cdef+6c^2f^2))\,x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{15a^2c^2e^3} + \frac{bd^2(bde^2-af(2de-3cf))\,x^5\sqrt{a+bx^2}\sqrt{c+dx^2}}{3a^2c^2e^3} - \frac{(a+bx^2)^{3/2}(c+dx^2)^{5/2}}{5acex^5} + \frac{(2be+5af)(a+bx^2)^{3/2}(c+dx^2)^{5/2}}{15a^2ce^2x^3} - \frac{(bde^2-af(2de-3cf))(a+bx^2)^{3/2}(c+dx^2)^{5/2}}{3a^2c^2e^3x} + \frac{\sqrt{b}(2b^2c^2e^2-abce(7de-5cf)-a^2(3d^2e^2-20cdef+15c^2f^2))\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{15a^{3/2}ce^3\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} - \frac{\sqrt{b}(bcde^2-a(9d^2e^2-25cdef+15c^2f^2))\sqrt{c+dx^2}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{15\sqrt{a}ce^3\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} - \frac{a^{3/2}f(de-cf)^2\sqrt{c+dx^2}\,\mathrm{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce^4\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
-1/15*b*(2*b^2*c^2*e^2-a*b*c*e*(-5*c*f+7*d*e)-a^2*(15*c^2*f^2-20*c*d*e*f+3
*d^2*e^2))*x*(d*x^2+c)^(1/2)/a^2/c/e^3/(b*x^2+a)^(1/2)+1/15*(b^2*c*d*e^2-5
*a^2*d*f*(-6*c*f+5*d*e)+a*b*(15*c^2*f^2-20*c*d*e*f+11*d^2*e^2))*x*(b*x^2+a
)^(1/2)*(d*x^2+c)^(1/2)/a^2/c/e^3+1/15*d*(8*b^2*c*d*e^2-5*a^2*d*f*(-3*c*f+
2*d*e)+5*a*b*(6*c^2*f^2-5*c*d*e*f+d^2*e^2))*x^3*(b*x^2+a)^(1/2)*(d*x^2+c)^
(1/2)/a^2/c^2/e^3+1/3*b*d^2*(b*d*e^2-a*f*(-3*c*f+2*d*e))*x^5*(b*x^2+a)^(1/
2)*(d*x^2+c)^(1/2)/a^2/c^2/e^3-1/5*(b*x^2+a)^(3/2)*(d*x^2+c)^(5/2)/a/c/e/x
^5+1/15*(5*a*f+2*b*e)*(b*x^2+a)^(3/2)*(d*x^2+c)^(5/2)/a^2/c/e^2/x^3-1/3*(b
*d*e^2-a*f*(-3*c*f+2*d*e))*(b*x^2+a)^(3/2)*(d*x^2+c)^(5/2)/a^2/c^2/e^3/x+1
/15*b^(1/2)*(2*b^2*c^2*e^2-a*b*c*e*(-5*c*f+7*d*e)-a^2*(15*c^2*f^2-20*c*d*e
*f+3*d^2*e^2))*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/
2),(1-a*d/b/c)^(1/2))/a^(3/2)/c/e^3/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+
a))^(1/2)-1/15*b^(1/2)*(b*c*d*e^2-a*(15*c^2*f^2-25*c*d*e*f+9*d^2*e^2))*(d*
x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/
a^(1/2)/c/e^3/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)-a^(3/2)*f*(-
c*f+d*e)^2*(d*x^2+c)^(1/2)*EllipticPi(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),
1-a*f/b/e,(1-a*d/b/c)^(1/2))/b^(1/2)/c/e^4/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/
(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.59 (sec) , antiderivative size = 1369, normalized size of antiderivative = 1.58

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}}{x^6\,(e+fx^2)}\,dx = \text{Too large to display}$$

input
```
Integrate[(Sqrt[a + b*x^2]*(c + d*x^2)^(3/2))/(x^6*(e + f*x^2)),x]
```

output
```
(-3*a^3*Sqrt[b/a]*c^3*e^3 - 4*a^3*(b/a)^(3/2)*c^3*e^3*x^2 - 9*a^3*Sqrt[b/a
]*c^2*d*e^3*x^2 + 5*a^3*Sqrt[b/a]*c^3*e^2*f*x^2 + a*b^2*Sqrt[b/a]*c^3*e^3*
x^4 - 17*a^3*(b/a)^(3/2)*c^2*d*e^3*x^4 - 9*a^3*Sqrt[b/a]*c*d^2*e^3*x^4 + 1
0*a^3*(b/a)^(3/2)*c^3*e^2*f*x^4 + 25*a^3*Sqrt[b/a]*c^2*d*e^2*f*x^4 - 15*a^
3*Sqrt[b/a]*c^3*e*f^2*x^4 + 2*b^3*Sqrt[b/a]*c^3*e^3*x^6 - 6*a*b^2*Sqrt[b/a
]*c^2*d*e^3*x^6 - 16*a^3*(b/a)^(3/2)*c*d^2*e^3*x^6 - 3*a^3*Sqrt[b/a]*d^3*e
^3*x^6 + 5*a*b^2*Sqrt[b/a]*c^3*e^2*f*x^6 + 30*a^3*(b/a)^(3/2)*c^2*d*e^2*f*
x^6 + 20*a^3*Sqrt[b/a]*c*d^2*e^2*f*x^6 - 15*a^3*(b/a)^(3/2)*c^3*e*f^2*x^6
- 15*a^3*Sqrt[b/a]*c^2*d*e*f^2*x^6 + 2*b^3*Sqrt[b/a]*c^2*d*e^3*x^8 - 7*a*b
^2*Sqrt[b/a]*c*d^2*e^3*x^8 - 3*a^3*(b/a)^(3/2)*d^3*e^3*x^8 + 5*a*b^2*Sqrt[
b/a]*c^2*d*e^2*f*x^8 + 20*a^3*(b/a)^(3/2)*c*d^2*e^2*f*x^8 - 15*a^3*(b/a)^(
3/2)*c^2*d*e*f^2*x^8 - I*b*c*e*(-2*b^2*c^2*e^2 + a*b*c*e*(7*d*e - 5*c*f) +
 a^2*(3*d^2*e^2 - 20*c*d*e*f + 15*c^2*f^2))*x^5*Sqrt[1 + (b*x^2)/a]*Sqrt[1
 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + I*b*c*e*(-2
*b^2*c^2*e^2 + a*b*c*e*(8*d*e - 5*c*f) + a^2*(9*d^2*e^2 - 25*c*d*e*f + 15*
c^2*f^2))*x^5*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[
Sqrt[b/a]*x], (a*d)/(b*c)] - (15*I)*a^2*b*c*d^2*e^3*x^5*Sqrt[1 + (b*x^2)/a
]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d
)/(b*c)] + (30*I)*a^2*b*c^2*d*e^2*f*x^5*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^
2)/c]*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (1...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}}{x^6\left(e+fx^2\right)}\,dx$$

$$\Big\downarrow \text{ 450}$$

$$\int \frac{\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}}{x^6\left(e+fx^2\right)}dx$$

input
```
Int[(Sqrt[a + b*x^2]*(c + d*x^2)^(3/2))/(x^6*(e + f*x^2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [A] (verified)

Time = 10.56 (sec) , antiderivative size = 841, normalized size of antiderivative = 0.97

method	result
risch	$-\frac{\sqrt{b\,x^{2}+a}\,\sqrt{x^{2}d+c}\,(15f^{2}x^{4}a^{2}c^{2}-20a^{2}cdef\,x^{4}+3a^{2}d^{2}e^{2}x^{4}-5ab\,c^{2}ef\,x^{4}+7abcd\,e^{2}x^{4}-2b^{2}c^{2}e^{2}x^{4}-5a^{2}c^{2}ef\,x^{2}+6a^{2}cd\,e^{2}x^{2}+ab\,c}{15a^{2}c\,e^{3}x^{5}}$
default	Expression too large to display
elliptic	Expression too large to display

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/x^6/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
-1/15*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(15*a^2*c^2*f^2*x^4-20*a^2*c*d*e*f*x
^4+3*a^2*d^2*e^2*x^4-5*a*b*c^2*e*f*x^4+7*a*b*c*d*e^2*x^4-2*b^2*c^2*e^2*x^4
-5*a^2*c^2*e*f*x^2+6*a^2*c*d*e^2*x^2+a*b*c^2*e^2*x^2+3*a^2*c^2*e^2)/a^2/c/
e^3/x^5+1/15/e^3/a^2/c*(-b*(15*a^2*c^2*f^2-20*a^2*c*d*e*f+3*a^2*d^2*e^2-5*
a*b*c^2*e*f+7*a*b*c*d*e^2-2*b^2*c^2*e^2)*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*
(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(EllipticF(x*(-b/a)^
(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b
)^(1/2)))-15*(a*c^2*f^3-2*a*c*d*e*f^2+a*d^2*e^2*f-b*c^2*e*f^2+2*b*c*d*e^2*
f-b*d^2*e^3)*a^2*c/e/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d
*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)
^(1/2)/(-b/a)^(1/2))-a*b^2*c^2*d*e^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x
^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(
-1+(a*d+b*c)/c/b)^(1/2))-6*a^2*b*c*d^2*e^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*
(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(
1/2),(-1+(a*d+b*c)/c/b)^(1/2))+5*a^2*b*c^2*d*e*f/(-b/a)^(1/2)*(1+b*x^2/a)^
(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-
b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2)))*((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a
)^(1/2)/(d*x^2+c)^(1/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}}{x^6\,(e+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/x^6/(f*x^2+e),x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}}{x^6\,(e+fx^2)}\,dx=\int \frac{\sqrt{a+bx^2}(c+dx^2)^{\frac{3}{2}}}{x^6\,(e+fx^2)}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(3/2)/x**6/(f*x**2+e),x)
```

output
```
Integral(sqrt(a + b*x**2)*(c + d*x**2)**(3/2)/(x**6*(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}}{x^6\,(e+fx^2)}\,dx=\int \frac{\sqrt{bx^2+a}(dx^2+c)^{\frac{3}{2}}}{(fx^2+e)x^6}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/x^6/(f*x^2+e),x, algorithm="maxi
ma")
```

output
```
integrate(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)/((f*x^2 + e)*x^6), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}}{x^6\,(e+fx^2)}\,dx=\int \frac{\sqrt{bx^2+a}(dx^2+c)^{\frac{3}{2}}}{(fx^2+e)x^6}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/x^6/(f*x^2+e),x, algorithm="giac
")
```

output
```
integrate(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)/((f*x^2 + e)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}}{x^6(e+fx^2)}\,dx = \int \frac{\sqrt{b\,x^2+a}\,\left(d\,x^2+c\right)^{3/2}}{x^6\,\left(f\,x^2+e\right)}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(3/2))/(x^6*(e + f*x^2)),x)
```

output
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(3/2))/(x^6*(e + f*x^2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}}{x^6(e+fx^2)}\,dx = \text{too large to display}$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/x^6/(f*x^2+e),x)
```

output

```
( - 12*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*c**2*d*f - 15*sqrt(c + d*x**
2)*sqrt(a + b*x**2)*a**2*c*d**2*f*x**2 + 20*sqrt(c + d*x**2)*sqrt(a + b*x*
*2)*a**2*d**3*e*x**2 - 12*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b*c**3*f - 9
*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b*c**2*d*e - 10*sqrt(c + d*x**2)*sqrt
(a + b*x**2)*a*b*c**2*d*f*x**2 + 30*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b*
c*d**2*e*x**2 - 20*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**2*c**3*f*x**2 + 25
*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**2*c**2*d*e*x**2 - 60*int((sqrt(c + d
*x**2)*sqrt(a + b*x**2)*x**2)/(4*a**2*c*d*e*f + 4*a**2*c*d*f**2*x**2 + 4*a
**2*d**2*e*f*x**2 + 4*a**2*d**2*f**2*x**4 + 4*a*b*c**2*e*f + 4*a*b*c**2*f*
*2*x**2 + 3*a*b*c*d*e**2 + 11*a*b*c*d*e*f*x**2 + 8*a*b*c*d*f**2*x**4 + 3*a
*b*d**2*e**2*x**2 + 7*a*b*d**2*e*f*x**4 + 4*a*b*d**2*f**2*x**6 + 4*b**2*c*
*2*e*f*x**2 + 4*b**2*c**2*f**2*x**4 + 3*b**2*c*d*e**2*x**2 + 7*b**2*c*d*e*
f*x**4 + 4*b**2*c*d*f**2*x**6 + 3*b**2*d**2*e**2*x**4 + 3*b**2*d**2*e*f*x*
*6),x)*a**3*b*c*d**4*f**3*x**5 + 80*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)
*x**2)/(4*a**2*c*d*e*f + 4*a**2*c*d*f**2*x**2 + 4*a**2*d**2*e*f*x**2 + 4*a
**2*d**2*f**2*x**4 + 4*a*b*c**2*e*f + 4*a*b*c**2*f**2*x**2 + 3*a*b*c*d*e**
2 + 11*a*b*c*d*e*f*x**2 + 8*a*b*c*d*f**2*x**4 + 3*a*b*d**2*e**2*x**2 + 7*a
*b*d**2*e*f*x**4 + 4*a*b*d**2*f**2*x**6 + 4*b**2*c**2*e*f*x**2 + 4*b**2*c*
*2*f**2*x**4 + 3*b**2*c*d*e**2*x**2 + 7*b**2*c*d*e*f*x**4 + 4*b**2*c*d*f**
2*x**6 + 3*b**2*d**2*e**2*x**4 + 3*b**2*d**2*e*f*x**6),x)*a**3*b*d**5*e...
```

3.103 $\int \frac{x^4\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}}{\left(e+fx^2\right)^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 879

$$\int \frac{x^4\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}}{\left(e+fx^2\right)^2}\,dx =$$

$$-\frac{(4a^2d^2f^2+2abdf(10de-7cf)-b^2(105d^2e^2-95cdef+6c^2f^2))\,x\sqrt{c+dx^2}}{30bdf^4\sqrt{a+bx^2}}$$

$$-\frac{(2a^2df^2(de-cf)-abf(37d^2e^2-64cdef+12c^2f^2)+b^2e(35d^2e^2-62cdef+27c^2f^2))\,x\sqrt{a+bx^2}\sqrt{c+}}{30bf^3(be-af)(de-cf)}$$

$$+\frac{d(be(7de-12cf)-af(7de-2cf))x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{10f^2(be-af)(de-cf)}$$

$$-\frac{bd^2ex^5\sqrt{a+bx^2}\sqrt{c+dx^2}}{2f(be-af)(de-cf)}+\frac{ex\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{5/2}}{2(be-af)(de-cf)\left(e+fx^2\right)}$$

$$+\frac{\sqrt{a}(4a^2d^2f^2+2abdf(10de-7cf)-b^2(105d^2e^2-95cdef+6c^2f^2))\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{30b^{3/2}df^4\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

$$+\frac{a^{3/2}(2a^2cdf^3-2abf(45d^2e^2-49cdef+9c^2f^2)+b^2e(105d^2e^2-130cdef+33c^2f^2))\sqrt{c+dx^2}\,\text{EllipticF}}{30b^{3/2}cf^4(be-af)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

$$-\frac{a^{3/2}(de-cf)(be(7de-4cf)-3af(2de-cf))\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{2\sqrt{b}cf^4(be-af)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
-1/30*(4*a^2*d^2*f^2+2*a*b*d*f*(-7*c*f+10*d*e)-b^2*(6*c^2*f^2-95*c*d*e*f+1
05*d^2*e^2))*x*(d*x^2+c)^(1/2)/b/d/f^4/(b*x^2+a)^(1/2)-1/30*(2*a^2*d*f^2*(
-c*f+d*e)-a*b*f*(12*c^2*f^2-64*c*d*e*f+37*d^2*e^2)+b^2*e*(27*c^2*f^2-62*c*
d*e*f+35*d^2*e^2))*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b/f^3/(-a*f+b*e)/(-c*
f+d*e)+1/10*d*(b*e*(-12*c*f+7*d*e)-a*f*(-2*c*f+7*d*e))*x^3*(b*x^2+a)^(1/2)
*(d*x^2+c)^(1/2)/f^2/(-a*f+b*e)/(-c*f+d*e)-1/2*b*d^2*e*x^5*(b*x^2+a)^(1/2)
*(d*x^2+c)^(1/2)/f/(-a*f+b*e)/(-c*f+d*e)+1/2*e*x*(b*x^2+a)^(3/2)*(d*x^2+c)
^(5/2)/(-a*f+b*e)/(-c*f+d*e)/(f*x^2+e)+1/30*a^(1/2)*(4*a^2*d^2*f^2+2*a*b*d
*f*(-7*c*f+10*d*e)-b^2*(6*c^2*f^2-95*c*d*e*f+105*d^2*e^2))*(d*x^2+c)^(1/2)
*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/b^(3/2)/
d/f^4/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)+1/30*a^(3/2)*(2*a^2*
c*d*f^3-2*a*b*f*(9*c^2*f^2-49*c*d*e*f+45*d^2*e^2)+b^2*e*(33*c^2*f^2-130*c*
d*e*f+105*d^2*e^2))*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/
2)),(1-a*d/b/c)^(1/2))/b^(3/2)/c/f^4/(-a*f+b*e)/(b*x^2+a)^(1/2)/(a*(d*x^2+
c)/c/(b*x^2+a))^(1/2)-1/2*a^(3/2)*(-c*f+d*e)*(b*e*(-4*c*f+7*d*e)-3*a*f*(-c
*f+2*d*e))*(d*x^2+c)^(1/2)*EllipticPi(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),
1-a*f/b/e,(1-a*d/b/c)^(1/2))/b^(1/2)/c/f^4/(-a*f+b*e)/(b*x^2+a)^(1/2)/(a*(
d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.33 (sec) , antiderivative size = 514, normalized size of antiderivative = 0.58

$$\int \frac{x^4\sqrt{a+bx^2}(c+dx^2)^{3/2}}{(e+fx^2)^2}\,dx = \frac{icf(4a^2d^2f^2+2abdf(10de-7cf)+b^2(-105d^2e^2+95cdef-6c^2f^2))\sqrt{1}}{}$$

input
```
Integrate[(x^4*Sqrt[a + b*x^2]*(c + d*x^2)^(3/2))/(e + f*x^2)^2,x]
```

output
```
(I*c*f*(4*a^2*d^2*f^2 + 2*a*b*d*f*(10*d*e - 7*c*f) + b^2*(-105*d^2*e^2 + 9
5*c*d*e*f - 6*c^2*f^2))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(e + f*x^2
)*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*(2*a^2*c*d^2*f^3 + 2*
a*b*d*f*(45*d^2*e^2 - 40*c*d*e*f + 2*c^2*f^2) + b^2*(-105*d^3*e^3 + 60*c*d
^2*e^2*f + 35*c^2*d*e*f^2 - 6*c^3*f^3))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^
2)/c]*(e + f*x^2)*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + d*(Sqrt
[b/a]*f^2*x*(a + b*x^2)*(c + d*x^2)*(2*a*d*f*(e + f*x^2) + 3*b*c*f*(9*e +
4*f*x^2) + b*d*(-35*e^2 - 14*e*f*x^2 + 6*f^2*x^4)) + (15*I)*b*(-(d*e) + c*
f)*(b*e*(7*d*e - 4*c*f) + 3*a*f*(-2*d*e + c*f))*Sqrt[1 + (b*x^2)/a]*Sqrt[1
 + (d*x^2)/c]*(e + f*x^2)*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x],
(a*d)/(b*c)]))/(30*b*Sqrt[b/a]*d*f^5*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e +
f*x^2))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}}{\left(e+fx^2\right)^2}dx$$
$$\Big\downarrow \text{450}$$
$$\int \frac{x^4\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}}{\left(e+fx^2\right)^2}dx$$

input
```
Int[(x^4*Sqrt[a + b*x^2]*(c + d*x^2)^(3/2))/(e + f*x^2)^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [A] (verified)

Time = 21.30 (sec) , antiderivative size = 1603, normalized size of antiderivative = 1.82

method	result	size
risch	Expression too large to display	1603
elliptic	Expression too large to display	2151
default	Expression too large to display	3195

input
```
int(x^4*(b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/(f*x^2+e)^2,x,method=_RETURNVERBOS
E)
```

output
```
1/15*x*(3*b*d*f*x^2+a*d*f+6*b*c*f-10*b*d*e)*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2
)/b/f^3-1/15/b/f^3*((a^2*c*d*f^3-9*a*b*c^2*f^3+50*a*b*c*d*e*f^2-45*a*b*d^2
*e^2*f+30*b^2*c^2*e*f^2-90*b^2*c*d*e^2*f+60*b^2*d^2*e^3)/f^2/(-b/a)^(1/2)*
(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*El
lipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-1/f*(2*a^2*d^2*f^2-7*a*b*
c*d*f^2+10*a*b*d^2*e*f-3*b^2*c^2*f^2+40*b^2*c*d*e*f-45*b^2*d^2*e^2)*c/(-b/
a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)
^(1/2)/d*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(
-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2)))+15*b/f^2*(2*a*c^2*f^3-6*a*c*d*e*f^2
+4*a*d^2*e^2*f-3*b*c^2*e*f^2+8*b*c*d*e^2*f-5*b*d^2*e^3)/(-b/a)^(1/2)*(1+b*
x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*Ellipti
cPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))-15*b*e^2*(a*c^2*f^
3-2*a*c*d*e*f^2+a*d^2*e^2*f-b*c^2*e*f^2+2*b*c*d*e^2*f-b*d^2*e^3)/f^2*(1/2*
f^2/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/e*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1
/2)/(f*x^2+e)-1/2*d*b/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/(-b/a)^(1/2)*(1+b*
x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*Ellipti
cF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+1/2*f*b/(a*c*f^2-a*d*e*f-b*c*e
*f+b*d*e^2)/e*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+
a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/
2))-1/2*f*b/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/e*c/(-b/a)^(1/2)*(1+b*x^2...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^4\sqrt{a+bx^2}(c+dx^2)^{3/2}}{(e+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(x^4*(b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/(f*x^2+e)^2,x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^4\sqrt{a+bx^2}(c+dx^2)^{3/2}}{(e+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**4*(b*x**2+a)**(1/2)*(d*x**2+c)**(3/2)/(f*x**2+e)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^4\sqrt{a+bx^2}(c+dx^2)^{3/2}}{(e+fx^2)^2}\,dx = \int \frac{\sqrt{bx^2+a}(dx^2+c)^{\frac{3}{2}}x^4}{(fx^2+e)^2}\,dx$$

input
```
integrate(x^4*(b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/(f*x^2+e)^2,x, algorithm="ma
xima")
```

output
```
integrate(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)*x^4/(f*x^2 + e)^2, x)
```

Giac [F]

$$\int \frac{x^4\sqrt{a+bx^2}(c+dx^2)^{3/2}}{(e+fx^2)^2}\,dx = \int \frac{\sqrt{bx^2+a}(dx^2+c)^{\frac{3}{2}}x^4}{(fx^2+e)^2}\,dx$$

input
```
integrate(x^4*(b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/(f*x^2+e)^2,x, algorithm="gi
ac")
```

output
```
integrate(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)*x^4/(f*x^2 + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4\sqrt{a+bx^2}(c+dx^2)^{3/2}}{(e+fx^2)^2}\,dx = \int \frac{x^4\,\sqrt{b\,x^2+a}\,(d\,x^2+c)^{3/2}}{(f\,x^2+e)^2}\,dx$$

input
```
int((x^4*(a + b*x^2)^(1/2)*(c + d*x^2)^(3/2))/(e + f*x^2)^2,x)
```

output
```
int((x^4*(a + b*x^2)^(1/2)*(c + d*x^2)^(3/2))/(e + f*x^2)^2, x)
```

Reduce [F]

$$\int \frac{x^4\sqrt{a+bx^2}(c+dx^2)^{3/2}}{(e+fx^2)^2}\,dx = \int \frac{x^4\sqrt{b\,x^2+a}\,(d\,x^2+c)^{\frac{3}{2}}}{(f\,x^2+e)^2}dx$$

input
```
int(x^4*(b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/(f*x^2+e)^2,x)
```

output
```
int(x^4*(b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/(f*x^2+e)^2,x)
```

3.104 $\int \frac{x^2\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}}{\left(e+fx^2\right)^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 717

$$\begin{aligned}\int \frac{x^2\sqrt{a+bx^2}(c+dx^2)^{3/2}}{(e+fx^2)^2}\,dx &= -\frac{(15bde-11bcf-2adf)x\sqrt{c+dx^2}}{6f^3\sqrt{a+bx^2}}\\
&-\frac{(adf(5de-8cf)-b(5d^2e^2-8cdef+3c^2f^2))\,x\sqrt{a+bx^2}\sqrt{c+dx^2}}{6f^2(be-af)(de-cf)}\\
&-\frac{d(bde-2bcf-adf)x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{2f(be-af)(de-cf)}\\
&+\frac{bd^2x^5\sqrt{a+bx^2}\sqrt{c+dx^2}}{2(be-af)(de-cf)}-\frac{fx(a+bx^2)^{3/2}\,(c+dx^2)^{5/2}}{2(be-af)(de-cf)\,(e+fx^2)}\\
&+\frac{\sqrt{a}(15bde-11bcf-2adf)\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{6\sqrt{b}f^3\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}\\
&+\frac{a^{3/2}(2adf(6de-5cf)-b(15d^2e^2-16cdef+3c^2f^2))\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{6\sqrt{b}cf^3(be-af)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}\\
&+\frac{a^{3/2}(de-cf)(be(5de-2cf)-af(4de-cf))\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{2\sqrt{b}cef^3(be-af)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}\end{aligned}$$

output

```
-1/6*(-2*a*d*f-11*b*c*f+15*b*d*e)*x*(d*x^2+c)^(1/2)/f^3/(b*x^2+a)^(1/2)-1/
6*(a*d*f*(-8*c*f+5*d*e)-b*(3*c^2*f^2-8*c*d*e*f+5*d^2*e^2))*x*(b*x^2+a)^(1/
2)*(d*x^2+c)^(1/2)/f^2/(-a*f+b*e)/(-c*f+d*e)-1/2*d*(-a*d*f-2*b*c*f+b*d*e)*
x^3*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/f/(-a*f+b*e)/(-c*f+d*e)+1/2*b*d^2*x^5*
(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(-a*f+b*e)/(-c*f+d*e)-1/2*f*x*(b*x^2+a)^(3
/2)*(d*x^2+c)^(5/2)/(-a*f+b*e)/(-c*f+d*e)/(f*x^2+e)+1/6*a^(1/2)*(-2*a*d*f-
11*b*c*f+15*b*d*e)*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)
^(1/2),(1-a*d/b/c)^(1/2))/b^(1/2)/f^3/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^
2+a))^(1/2)+1/6*a^(3/2)*(2*a*d*f*(-5*c*f+6*d*e)-b*(3*c^2*f^2-16*c*d*e*f+15
*d^2*e^2))*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*
d/b/c)^(1/2))/b^(1/2)/c/f^3/(-a*f+b*e)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x
^2+a))^(1/2)+1/2*a^(3/2)*(-c*f+d*e)*(b*e*(-2*c*f+5*d*e)-a*f*(-c*f+4*d*e))*
(d*x^2+c)^(1/2)*EllipticPi(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),1-a*f/b/e,(
1-a*d/b/c)^(1/2))/b^(1/2)/c/e/f^3/(-a*f+b*e)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/
c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.60 (sec) , antiderivative size = 1381, normalized size of antiderivative = 1.93

$$\int \frac{x^2\sqrt{a+bx^2}(c+dx^2)^{3/2}}{(e+fx^2)^2}\,dx = \text{Too large to display}$$

input

```
Integrate[(x^2*Sqrt[a + b*x^2]*(c + d*x^2)^(3/2))/(e + f*x^2)^2,x]
```

output
```
(5*a*Sqrt[b/a]*c*d*e^2*f^2*x - 3*a*Sqrt[b/a]*c^2*e*f^3*x + 5*b*Sqrt[b/a]*c
*d*e^2*f^2*x^3 + 5*a*Sqrt[b/a]*d^2*e^2*f^2*x^3 - 3*b*Sqrt[b/a]*c^2*e*f^3*x
^3 - a*Sqrt[b/a]*c*d*e*f^3*x^3 + 5*b*Sqrt[b/a]*d^2*e^2*f^2*x^5 - b*Sqrt[b/
a]*c*d*e*f^3*x^5 + 2*a*Sqrt[b/a]*d^2*e*f^3*x^5 + 2*b*Sqrt[b/a]*d^2*e*f^3*x
^7 - I*c*e*f*(-15*b*d*e + 11*b*c*f + 2*a*d*f)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 +
 (d*x^2)/c]*(e + f*x^2)*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I
*e*(4*a*d*f*(-3*d*e + 2*c*f) + b*(15*d^2*e^2 - 6*c*d*e*f - 5*c^2*f^2))*Sqr
t[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(e + f*x^2)*EllipticF[I*ArcSinh[Sqrt[
b/a]*x], (a*d)/(b*c)] + (15*I)*b*d^2*e^4*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x
^2)/c]*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - (21*
I)*b*c*d*e^3*f*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b
*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - (12*I)*a*d^2*e^3*f*Sqrt[1 + (b
*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x
], (a*d)/(b*c)] + (6*I)*b*c^2*e^2*f^2*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)
/c]*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (15*I)*
a*c*d*e^2*f^2*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*
e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - (3*I)*a*c^2*e*f^3*Sqrt[1 + (b*x
^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x],
 (a*d)/(b*c)] + (15*I)*b*d^2*e^3*f*x^2*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2
)/c]*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - (21...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\sqrt{a+bx^2}(c+dx^2)^{3/2}}{(e+fx^2)^2}\,dx$$
$$\Big\downarrow \ 450$$
$$\int \frac{x^2\sqrt{a+bx^2}(c+dx^2)^{3/2}}{(e+fx^2)^2}dx$$

input
```
Int[(x^2*Sqrt[a + b*x^2]*(c + d*x^2)^(3/2))/(e + f*x^2)^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1487 vs. $2(667) = 1334$.

Time = 21.68 (sec) , antiderivative size = 1488, normalized size of antiderivative = 2.08

method	result	size
elliptic	Expression too large to display	1488
risch	Expression too large to display	1829
default	Expression too large to display	2132

input
```
int(x^2*(b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/(f*x^2+e)^2,x,method=_RETURNVERBOS
E)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-1/2*(c*f-d*e
)/f^2*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(f*x^2+e)+1/3/f^2*d*x*(b*d*x^4
+a*d*x^2+b*c*x^2+a*c)^(1/2)-5/6/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)
^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a
*d+b*c)/c/b)^(1/2))/f^2*b*c^2-1/f^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^
2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a
*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*b*c^2+4/3/(-b/a)^(1/2)*(1+b*x^2/a)^(1/
2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a
)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))/f^2*d*a*c-2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/
2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a
)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))/f^3*a*d^2*e+5/2/(-b/a)^(1/2)*(1+b*x^2/a)
^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(
-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))/f^4*b*d^2*e^2+1/3*c/(-b/a)^(1/2)*(1+
b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*d/f^2
*a*EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+11/6*c^2/(-b/a)^(1/2
)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/
f^2*b*EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+1/2/e/f/(-b/a)^(1
/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2
)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*a*c^2-5/2
/f^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^2\sqrt{a+bx^2}(c+dx^2)^{3/2}}{(e+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(x^2*(b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/(f*x^2+e)^2,x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^2\sqrt{a+bx^2}(c+dx^2)^{3/2}}{(e+fx^2)^2}\,dx = \int \frac{x^2\sqrt{a+bx^2}\left(c+dx^2\right)^{\frac{3}{2}}}{\left(e+fx^2\right)^2}\,dx$$

input
```
integrate(x**2*(b*x**2+a)**(1/2)*(d*x**2+c)**(3/2)/(f*x**2+e)**2,x)
```

output
```
Integral(x**2*sqrt(a + b*x**2)*(c + d*x**2)**(3/2)/(e + f*x**2)**2, x)
```

Maxima [F]

$$\int \frac{x^2\sqrt{a+bx^2}(c+dx^2)^{3/2}}{(e+fx^2)^2}\,dx = \int \frac{\sqrt{bx^2+a}(dx^2+c)^{\frac{3}{2}}x^2}{(fx^2+e)^2}\,dx$$

input
```
integrate(x^2*(b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/(f*x^2+e)^2,x, algorithm="ma
xima")
```

output
```
integrate(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)*x^2/(f*x^2 + e)^2, x)
```

Giac [F]

$$\int \frac{x^2\sqrt{a+bx^2}(c+dx^2)^{3/2}}{(e+fx^2)^2}\,dx = \int \frac{\sqrt{bx^2+a}(dx^2+c)^{\frac{3}{2}}x^2}{(fx^2+e)^2}\,dx$$

input
```
integrate(x^2*(b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/(f*x^2+e)^2,x, algorithm="gi
ac")
```

output
```
integrate(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)*x^2/(f*x^2 + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2\sqrt{a+bx^2}(c+dx^2)^{3/2}}{(e+fx^2)^2}\,dx=\int \frac{x^2\,\sqrt{b\,x^2+a}\,(d\,x^2+c)^{3/2}}{(f\,x^2+e)^2}\,dx$$

input
```
int((x^2*(a + b*x^2)^(1/2)*(c + d*x^2)^(3/2))/(e + f*x^2)^2,x)
```

output
```
int((x^2*(a + b*x^2)^(1/2)*(c + d*x^2)^(3/2))/(e + f*x^2)^2, x)
```

Reduce [F]

$$\int \frac{x^2\sqrt{a+bx^2}(c+dx^2)^{3/2}}{(e+fx^2)^2}\,dx=\text{too large to display}$$

input
```
int(x^2*(b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/(f*x^2+e)^2,x)
```

output

```
(5*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*c*d*f*x - 4*sqrt(c + d*x**2)*sqrt(a
 + b*x**2)*a*d**2*e*x + 3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b*c**2*f*x - 4
*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b*c*d*e*x + 3*sqrt(c + d*x**2)*sqrt(a +
 b*x**2)*b*d**2*e*x**3 - 5*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**6)/(a
*c*e**2 + 2*a*c*e*f*x**2 + a*c*f**2*x**4 + a*d*e**2*x**2 + 2*a*d*e*f*x**4
+ a*d*f**2*x**6 + b*c*e**2*x**2 + 2*b*c*e*f*x**4 + b*c*f**2*x**6 + b*d*e**
2*x**4 + 2*b*d*e*f*x**6 + b*d*f**2*x**8),x)*a*b*c*d**2*e*f**2 - 5*int((sqr
t(c + d*x**2)*sqrt(a + b*x**2)*x**6)/(a*c*e**2 + 2*a*c*e*f*x**2 + a*c*f**2
*x**4 + a*d*e**2*x**2 + 2*a*d*e*f*x**4 + a*d*f**2*x**6 + b*c*e**2*x**2 + 2
*b*c*e*f*x**4 + b*c*f**2*x**6 + b*d*e**2*x**4 + 2*b*d*e*f*x**6 + b*d*f**2*
x**8),x)*a*b*c*d**2*f**3*x**2 + 7*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x
**6)/(a*c*e**2 + 2*a*c*e*f*x**2 + a*c*f**2*x**4 + a*d*e**2*x**2 + 2*a*d*e*
f*x**4 + a*d*f**2*x**6 + b*c*e**2*x**2 + 2*b*c*e*f*x**4 + b*c*f**2*x**6 +
b*d*e**2*x**4 + 2*b*d*e*f*x**6 + b*d*f**2*x**8),x)*a*b*d**3*e**2*f + 7*int
((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**6)/(a*c*e**2 + 2*a*c*e*f*x**2 + a*c
*f**2*x**4 + a*d*e**2*x**2 + 2*a*d*e*f*x**4 + a*d*f**2*x**6 + b*c*e**2*x**
2 + 2*b*c*e*f*x**4 + b*c*f**2*x**6 + b*d*e**2*x**4 + 2*b*d*e*f*x**6 + b*d*
f**2*x**8),x)*a*b*d**3*e*f**2*x**2 - 3*int((sqrt(c + d*x**2)*sqrt(a + b*x*
*2)*x**6)/(a*c*e**2 + 2*a*c*e*f*x**2 + a*c*f**2*x**4 + a*d*e**2*x**2 + 2*a
*d*e*f*x**4 + a*d*f**2*x**6 + b*c*e**2*x**2 + 2*b*c*e*f*x**4 + b*c*f**2...
```

3.105 $\int \frac{\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}}{\left(e+fx^2\right)^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 482

$$\begin{aligned}\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}}{(e+fx^2)^2}\,dx &= \frac{b(3de-cf)x\sqrt{c+dx^2}}{2ef^2\sqrt{a+bx^2}}-\frac{(de-cf)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{2ef\left(e+fx^2\right)}\\
&-\frac{\sqrt{a}\sqrt{b}(3de-cf)\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{2ef^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}\\
&-\frac{a^{3/2}(2ad^2ef-b(3d^2e^2-2cdef+c^2f^2))\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{2\sqrt{b}cef^2(be-af)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}\\
&-\frac{a^{3/2}(de-cf)\left(3bde^2-af(2de+cf)\right)\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{2\sqrt{b}ce^2f^2(be-af)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}\end{aligned}$$

output
```
1/2*b*(-c*f+3*d*e)*x*(d*x^2+c)^(1/2)/e/f^2/(b*x^2+a)^(1/2)-1/2*(-c*f+d*e)*
x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/e/f/(f*x^2+e)-1/2*a^(1/2)*b^(1/2)*(-c*f+
3*d*e)*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*
d/b/c)^(1/2))/e/f^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)-1/2*a^
(3/2)*(2*a*d^2*e*f-b*(c^2*f^2-2*c*d*e*f+3*d^2*e^2))*(d*x^2+c)^(1/2)*Invers
eJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/b^(1/2)/c/e/f^2/(-a
*f+b*e)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)-1/2*a^(3/2)*(-c*f+
d*e)*(3*b*d*e^2-a*f*(c*f+2*d*e))*(d*x^2+c)^(1/2)*EllipticPi(b^(1/2)*x/a^(1
/2)/(1+b*x^2/a)^(1/2),1-a*f/b/e,(1-a*d/b/c)^(1/2))/b^(1/2)/c/e^2/f^2/(-a*f
+b*e)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.07 (sec) , antiderivative size = 352, normalized size of antiderivative = 0.73

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}}{(e+fx^2)^2}\,dx = \frac{ibcef(-3de+cf)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}(e+fx^2)\,E\left(i\operatorname{arcsinh}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)-ie}{}$$

input
```
Integrate[(Sqrt[a + b*x^2]*(c + d*x^2)^(3/2))/(e + f*x^2)^2,x]
```

output
```
(I*b*c*e*f*(-3*d*e + c*f)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(e + f*x
^2)*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*e*(-3*b*d^2*e^2 + 2
*a*d^2*e*f + b*c^2*f^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(e + f*x^2
)*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (-(d*e) + c*f)*(Sqrt[b/
a]*e*f^2*x*(a + b*x^2)*(c + d*x^2) - I*(-3*b*d*e^2 + a*f*(2*d*e + c*f))*Sq
rt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(e + f*x^2)*EllipticPi[(a*f)/(b*e),
I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(2*Sqrt[b/a]*e^2*f^3*Sqrt[a + b*x^2
]*Sqrt[c + d*x^2]*(e + f*x^2))
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1030 vs. $2(482) = 964$.

Time = 1.63 (sec) , antiderivative size = 1030, normalized size of antiderivative = 2.14, number of steps used = 21, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.656$, Rules used = {425, 420, 324, 320, 388, 313, 414, 425, 414, 425, 413, 413, 412, 424, 406, 320, 388, 313, 413, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}}{\left(e+fx^2\right)^2}dx$$

$$\Big\downarrow \text{ 425}$$

$$\frac{b\int \frac{\left(dx^2+c\right)^{3/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}-\frac{(be-af)\int \frac{\left(dx^2+c\right)^{3/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}$$

$$\Big\downarrow \text{ 420}$$

$$\frac{b\left(\frac{d\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}}dx}{f}-\frac{(de-cf)\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(be-af)\int \frac{\left(dx^2+c\right)^{3/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}$$

$$\Big\downarrow \text{ 324}$$

$$\frac{b\left(\frac{d\left(c\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+d\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{f}-\frac{(de-cf)\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}\right)}{f}-\frac{(be-af)\int \frac{\left(dx^2+c\right)^{3/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}$$

$$\Big\downarrow \text{ 320}$$

$$\frac{b\left(\frac{d\left(d\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{f}-\frac{(de-cf)\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}(fx^2+e)}dx}{f}\right)}{f}-\frac{(be-af)\int \frac{(dx^2+c)^{3/2}}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}$$

$$\Big\downarrow \text{388}$$

$$\frac{b\left(\frac{d\left(d\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{f}-\frac{(de-cf)\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}(fx^2+e)}dx}{f}\right)}{f}-\frac{(be-af)\int \frac{(dx^2+c)^{3/2}}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}$$

$$\Big\downarrow \text{313}$$

$$\frac{b\left(\frac{d\left(\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+d\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)\right)}{f}-\frac{(de-cf)\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}(fx^2+e)}dx}{f}\right)}{f}-\frac{(be-af)\int \frac{(dx^2+c)^{3/2}}{\sqrt{bx^2+a}(fx^2+e)^2}dx}{f}$$

$$\Big\downarrow \text{414}$$

$$\frac{b\left(\frac{d\left(\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+d\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)\right)}{f}-\frac{c^{3/2}\sqrt{a+bx^2}(de-cf)\,\text{EllipticPi}\left(1\ldots\right.}{a\sqrt{d}ef\sqrt{c+dx^2}\sqrt{\ldots}}\ldots\right.}{f}$$

$$\frac{(be-af)\int\frac{\left(dx^2+c\right)^{3/2}}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}$$

$\downarrow$ 425

$$\frac{b\left(\frac{d\left(\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+d\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)\right)}{f}-\frac{c^{3/2}\sqrt{a+bx^2}(de-cf)\,\text{EllipticPi}\left(1\ldots\right.}{a\sqrt{d}ef\sqrt{c+dx^2}\sqrt{\ldots}}\ldots\right.}{f}$$

$$\frac{(be-af)\left(\frac{d\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{f}-\frac{(de-cf)\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}\right)}{f}$$

$\downarrow$ 414

$$\frac{b\left(\frac{d\left(\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+d\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)\right)}{f}-\frac{c^{3/2}\sqrt{a+bx^2}(de-cf)\,\text{EllipticPi}\left(1\ldots\right.}{a\sqrt{d}ef\sqrt{c+dx^2}\sqrt{\ldots}}\ldots\right.}{f}$$

$$\frac{(be-af)\left(\frac{c^{3/2}\sqrt{d}\sqrt{a+bx^2}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{aef\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{(de-cf)\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)^2}dx}{f}\right)}{f}$$

$$\Big\downarrow \ 425$$

$$\frac{b\left(\frac{d\left(\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+d\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)\right)}{f}-\frac{c^{3/2}\sqrt{a+bx^2}(de-cf)\,\text{EllipticPi}\left(1\cdots\right.}{a\sqrt{d}ef\sqrt{c+dx^2}\sqrt{\cdots}}\cdots\right)}{f}$$

$$\frac{(be-af)\left(\frac{c^{3/2}\sqrt{d}\sqrt{a+bx^2}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{aef\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{(de-cf)\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{f}-\frac{(de-cf)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\cdots}}{f}\cdots\right.}{f}\cdots\right.}{f}$$

$$\Big\downarrow \ 413$$

$$\frac{b\left(\frac{d\left(\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+d\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)\right)}{f}-\frac{c^{3/2}\sqrt{a+bx^2}(de-cf)\,\text{EllipticPi}\left(1\cdots\right.}{a\sqrt{d}ef\sqrt{c+dx^2}\sqrt{\cdots}}\cdots\right)}{f}$$

$$\frac{(be-af)\left(\frac{c^{3/2}\sqrt{d}\sqrt{a+bx^2}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{aef\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{(de-cf)\left(\frac{d\sqrt{\frac{bx^2}{a}+1}\int\frac{1}{\sqrt{\frac{bx^2}{a}+1}\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{f\sqrt{a+bx^2}}-\frac{(de-cf)\int\frac{\cdots}{\sqrt{bx^2+\cdots}}}{\cdots}\cdots\right.}{f}\cdots\right.}{f}$$

$$\Big\downarrow \ 413$$

$$\frac{b\left(\frac{d\left(\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+d\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)\right)}{f}-\frac{c^{3/2}\sqrt{a+bx^2}(de-cf)\,\text{EllipticPi}\left(1\cdots\right.}{a\sqrt{d}ef\sqrt{c+dx^2}\sqrt{\cdots}}\right.}{f}$$

$$\frac{(be-af)\left(\frac{c^{3/2}\sqrt{d}\sqrt{a+bx^2}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{aef\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{(de-cf)\left(\frac{d\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\int\frac{1}{\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\left(fx^2+e\right)}dx}{f\sqrt{a+bx^2}\sqrt{c+dx^2}}-\frac{(de-cf)\cdots}{\cdots}\right.}{f}\right.}{f}$$

$\downarrow$ 412

$$\frac{b\left(\frac{d\left(\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+d\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)\right)}{f}-\frac{c^{3/2}\sqrt{a+bx^2}(de-cf)\,\text{EllipticPi}\left(1\cdots\right.}{a\sqrt{d}ef\sqrt{c+dx^2}\sqrt{\cdots}}\right.}{f}$$

$$\frac{(be-af)\left(\frac{c^{3/2}\sqrt{d}\sqrt{a+bx^2}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{aef\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{(de-cf)\left(\frac{\sqrt{-a}d\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{\sqrt{b}ef\sqrt{a+bx^2}\sqrt{c+dx^2}}-\cdots\right.}{f}\right.}{f}$$

$\downarrow$ 424

$$b\left(\frac{d\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(de-cf)\sqrt{bx^2+a}\,\text{EllipticPi}\left(1\cdots\right.}{a\sqrt{d}ef\sqrt{\frac{c\left(bx^2+a\right.}{a\left(dx^2+c\right.}}\cdots}\right.$$

$$\overline{\qquad f \qquad}$$

$$(be-af)\left(\frac{c^{3/2}\sqrt{d}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{aef\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{f}{(de-cf)\left(\frac{\sqrt{-a}d\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{\sqrt{b}ef\sqrt{bx^2+a}\sqrt{dx^2+c}}-\cdots\right.}\right.$$

$$\overline{\qquad f \qquad}$$

$\downarrow$ 406

$$b\left(\frac{d\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(de-cf)\sqrt{bx^2+a}\,\text{EllipticPi}\left(1\cdots\right.}{a\sqrt{d}ef\sqrt{\frac{c\left(bx^2+a\right.}{a\left(dx^2+c\right.}}\cdots}\right.$$

$$(be-af)\left(\frac{c^{3/2}\sqrt{d}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{aef\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{f}{(de-cf)\left(\frac{\sqrt{-a}d\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{\sqrt{b}ef\sqrt{bx^2+a}\sqrt{dx^2+c}}-\cdots\right.}\right.$$

$\downarrow$ 320

$$b\left(\frac{d\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(de-cf)\sqrt{bx^2+a}\,\text{EllipticPi}\left(1\cdots\right.}{a\sqrt{d}ef\sqrt{\frac{c\left(bx^2+a\cdots\right.}{a\left(dx^2+c\cdots\right.}}}\right.$$

$$\overline{\qquad\qquad f\qquad\qquad}$$

$$(de-cf)\left(\frac{\sqrt{-a}d\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{\sqrt{b}ef\sqrt{bx^2+a}\sqrt{dx^2+c}}-\cdots\right.$$

$$(be-af)\left(\frac{c^{3/2}\sqrt{d}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{aef\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\cdots}{\cdots}\right.$$

↓ 388

$$b\left(\frac{d\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(de-cf)\sqrt{bx^2+a}\,\text{EllipticPi}\left(1\cdots\right.}{a\sqrt{d}ef\sqrt{\frac{c\left(bx^2+a\cdots\right.}{a\left(dx^2+c\cdots\right.}}}\cdots\right.$$

$$\overline{\qquad f \qquad}$$

$$(be-af)\left(\frac{c^{3/2}\sqrt{d}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{aef\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{f\left(\cdots(de-cf)\left(\frac{\sqrt{-a}d\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{\sqrt{b}ef\sqrt{bx^2+a}\sqrt{dx^2+c}}-\cdots\right.\right.}{\cdots}\cdots\right.$$

↓ 313

$$\frac{b\left(\frac{d\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(de-cf)\sqrt{bx^2+a}\,\text{EllipticPi}\left(1\cdots\right.}{a\sqrt{d}ef\sqrt{\frac{c(bx^2+a}{a(dx^2+c}\cdots}}\right)}{f}$$

$$(be-af)\left(\frac{c^{3/2}\sqrt{d}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{aef\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{(de-cf)\left(\frac{\sqrt{-a}d\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{\sqrt{b}ef\sqrt{bx^2+a}\sqrt{dx^2+c}}-\cdots\right.}{\cdots}\right.$$

$\downarrow$ 413

$$\frac{b\left(\frac{d\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(de-cf)\sqrt{bx^2+a}\,\text{EllipticPi}\left(1\cdots\right.}{a\sqrt{d}ef\sqrt{\frac{c(bx^2+a\cdots}{a(dx^2+c\cdots}}}\right.}{f}$$

$$(be-af)\left(\frac{c^{3/2}\sqrt{d}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{aef\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{(de-cf)\left(\frac{\sqrt{-a}d\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{\sqrt{b}ef\sqrt{bx^2+a}\sqrt{dx^2+c}}-\cdots\right.}{\cdots}\right.$$

↓ 413

$$b\left(\frac{d\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(de-cf)\sqrt{bx^2+a}\,\text{EllipticPi}\Big(1\cdots}{a\sqrt{d}ef\sqrt{\frac{c(bx^2+a\cdots}{a(dx^2+c\cdots}}}\right)$$

$$f$$

$$(be-af)\left(\frac{c^{3/2}\sqrt{d}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{aef\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{(de-cf)\left(\frac{\sqrt{-a}d\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{\sqrt{b}ef\sqrt{bx^2+a}\sqrt{dx^2+c}}-\cdots\right.}{\cdots}\right.$$

↓ 412

$$b\left(\frac{d\left(\frac{\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+d\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)\right)}{f}-\frac{c^{3/2}(de-cf)\sqrt{bx^2+a}\,\text{EllipticPi}\Big(1\cdots}{a\sqrt{d}ef\sqrt{\frac{c\left(bx^2+a\cdots\right.}{a\left(dx^2+c\cdots\right.}}}\cdots\right.$$

$$\overline{\qquad f \qquad}$$

$$(be-af)\left(\frac{c^{3/2}\sqrt{d}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{aef\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{(de-cf)\left(\frac{\sqrt{-a}d\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{\sqrt{b}ef\sqrt{bx^2+a}\sqrt{dx^2+c}}-\cdots\right.}{\cdots}\right.$$

input
```
Int[(Sqrt[a + b*x^2]*(c + d*x^2)^(3/2))/(e + f*x^2)^2,x]
```

output
```
-(((b*e - a*f)*(-(((d*e - c*f)*((Sqrt[-a]*d*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (
d*x^2)/c]*EllipticPi[(a*f)/(b*e), ArcSin[(Sqrt[b]*x)/Sqrt[-a]], (a*d)/(b*c
)])/(Sqrt[b]*e*f*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]) - ((d*e - c*f)*((f^2*x*S
qrt[a + b*x^2]*Sqrt[c + d*x^2])/(2*e*(b*e - a*f)*(d*e - c*f)*(e + f*x^2))
- (b*d*(f*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x
^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sq
rt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (Sqrt[c]*e*Sqrt[a
+ b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[
d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])))/(2*e*(b*e - a*
f)*(d*e - c*f)) + (Sqrt[-a]*(b*e*(3*d*e - 2*c*f) - a*f*(2*d*e - c*f))*Sqrt
[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e), ArcSin[(Sqrt[b
]*x)/Sqrt[-a]], (a*d)/(b*c)])/(2*Sqrt[b]*e^2*(b*e - a*f)*(d*e - c*f)*Sqrt[
a + b*x^2]*Sqrt[c + d*x^2])))/f))/f) + (c^(3/2)*Sqrt[d]*Sqrt[a + b*x^2]*El
lipticPi[1 - (c*f)/(d*e), ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(
a*e*f*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])))/f) + (b*((d
*(d*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*El
lipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*
(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (c^(3/2)*Sqrt[a + b*x^2]
*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*Sqrt[
(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])))/f - (c^(3/2)*(d*e -...
```

Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 324
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Simp[b   Int[x^2/(Sqr
t[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c
] && PosQ[b/a]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 420
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[d/b   Int[(c + d*x^2)^(q - 1)*(e + f*x^2)^r, x],
x] + Simp[(b*c - a*d)/b   Int[(c + d*x^2)^(q - 1)*((e + f*x^2)^r/(a + b*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && GtQ[q, 1]
```

rule 424
```
Int[1/(((a_) + (b_.)*(x_)^2)^2*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*
(x_)^2]), x_Symbol] :> Simp[b^2*x*Sqrt[c + d*x^2]*(Sqrt[e + f*x^2]/(2*a*(b*
c - a*d)*(b*e - a*f)*(a + b*x^2))), x] + (Simp[(b^2*c*e + 3*a^2*d*f - 2*a*b
*(d*e + c*f))/(2*a*(b*c - a*d)*(b*e - a*f))   Int[1/((a + b*x^2)*Sqrt[c + d
*x^2]*Sqrt[e + f*x^2]), x], x] - Simp[d*(f/(2*a*(b*c - a*d)*(b*e - a*f)))
 Int[(a + b*x^2)/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x], x]) /; FreeQ[{a, b,
 c, d, e, f}, x]
```

rule 425
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[d/b   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q -
 1)*(e + f*x^2)^r, x], x] + Simp[(b*c - a*d)/b   Int[(a + b*x^2)^p*(c + d*x
^2)^(q - 1)*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && ILt
Q[p, 0] && GtQ[q, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1074 vs. $2(450) = 900$.

Time = 5.06 (sec) , antiderivative size = 1075, normalized size of antiderivative = 2.23

method	result	size
elliptic	Expression too large to display	1075
default	Expression too large to display	1576

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/(f*x^2+e)^2,x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/2*(c*f-d*e)
/e/f*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(f*x^2+e)+1/(-b/a)^(1/2)*(1+b*x
^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*Elliptic
F(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))*d^2/f^2*a+1/2/e^2/(-b/a)^(1/2)*
(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*El
lipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*a*c^2-3/2/(-b
/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c
)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))*b*d^2/f^3*e+3/2
*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x
^2+a*c)^(1/2)*d*b/f^2*EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-3
/2/f^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b
*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^
(1/2))*b*c*d+3/2*e/f^3/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b
*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*
d)^(1/2)/(-b/a)^(1/2))*b*d^2-1/f^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2
/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*
f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*a*d^2+1/2*c^2/(-b/a)^(1/2)*(1+b*x^2/a)^
(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/f*b/e*Elliptic
F(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-1/2*c^2/(-b/a)^(1/2)*(1+b*x^2/a
)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/f*b/e*Ell...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}}{(e+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/(f*x^2+e)^2,x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}}{(e+fx^2)^2}\,dx = \int \frac{\sqrt{a+bx^2}(c+dx^2)^{\frac{3}{2}}}{(e+fx^2)^2}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(3/2)/(f*x**2+e)**2,x)
```

output
```
Integral(sqrt(a + b*x**2)*(c + d*x**2)**(3/2)/(e + f*x**2)**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}}{(e+fx^2)^2}\,dx = \int \frac{\sqrt{bx^2+a}(dx^2+c)^{\frac{3}{2}}}{(fx^2+e)^2}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/(f*x^2+e)^2,x, algorithm="maxima
")
```

output
```
integrate(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)/(f*x^2 + e)^2, x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}}{(e+fx^2)^2}\,dx = \int \frac{\sqrt{bx^2+a}(dx^2+c)^{\frac{3}{2}}}{(fx^2+e)^2}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/(f*x^2+e)^2,x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)/(f*x^2 + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}}{(e+fx^2)^2}\,dx = \int \frac{\sqrt{b\,x^2+a}\,(d\,x^2+c)^{3/2}}{(f\,x^2+e)^2}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(3/2))/(e + f*x^2)^2,x)
```

output
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(3/2))/(e + f*x^2)^2, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}}{(e+fx^2)^2}\,dx = \text{too large to display}$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/(f*x^2+e)^2,x)
```

output

```
(sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*d*x + 2*sqrt(c + d*x**2)*sqrt(a + b*x
**2)*b*c*x - int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**6)/(a*c*e**2 + 2*a*
c*e*f*x**2 + a*c*f**2*x**4 + a*d*e**2*x**2 + 2*a*d*e*f*x**4 + a*d*f**2*x**
6 + b*c*e**2*x**2 + 2*b*c*e*f*x**4 + b*c*f**2*x**6 + b*d*e**2*x**4 + 2*b*d
*e*f*x**6 + b*d*f**2*x**8),x)*a*b*d**2*e*f - int((sqrt(c + d*x**2)*sqrt(a
+ b*x**2)*x**6)/(a*c*e**2 + 2*a*c*e*f*x**2 + a*c*f**2*x**4 + a*d*e**2*x**2
 + 2*a*d*e*f*x**4 + a*d*f**2*x**6 + b*c*e**2*x**2 + 2*b*c*e*f*x**4 + b*c*f
**2*x**6 + b*d*e**2*x**4 + 2*b*d*e*f*x**6 + b*d*f**2*x**8),x)*a*b*d**2*f**
2*x**2 - 2*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**6)/(a*c*e**2 + 2*a*c*
e*f*x**2 + a*c*f**2*x**4 + a*d*e**2*x**2 + 2*a*d*e*f*x**4 + a*d*f**2*x**6
+ b*c*e**2*x**2 + 2*b*c*e*f*x**4 + b*c*f**2*x**6 + b*d*e**2*x**4 + 2*b*d*e
*f*x**6 + b*d*f**2*x**8),x)*b**2*c*d*e*f - 2*int((sqrt(c + d*x**2)*sqrt(a
+ b*x**2)*x**6)/(a*c*e**2 + 2*a*c*e*f*x**2 + a*c*f**2*x**4 + a*d*e**2*x**2
 + 2*a*d*e*f*x**4 + a*d*f**2*x**6 + b*c*e**2*x**2 + 2*b*c*e*f*x**4 + b*c*f
**2*x**6 + b*d*e**2*x**4 + 2*b*d*e*f*x**6 + b*d*f**2*x**8),x)*b**2*c*d*f**
2*x**2 + 3*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**6)/(a*c*e**2 + 2*a*c*
e*f*x**2 + a*c*f**2*x**4 + a*d*e**2*x**2 + 2*a*d*e*f*x**4 + a*d*f**2*x**6
+ b*c*e**2*x**2 + 2*b*c*e*f*x**4 + b*c*f**2*x**6 + b*d*e**2*x**4 + 2*b*d*e
*f*x**6 + b*d*f**2*x**8),x)*b**2*d**2*e**2 + 3*int((sqrt(c + d*x**2)*sqrt(
a + b*x**2)*x**6)/(a*c*e**2 + 2*a*c*e*f*x**2 + a*c*f**2*x**4 + a*d*e**2...
```

3.106 $\int \frac{\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}}{x^2\left(e+fx^2\right)^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 881

$$\begin{aligned}\int \frac{\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}}{x^2\left(e+fx^2\right)^2}\,dx &= -\frac{b(de-3cf)x\sqrt{c+dx^2}}{2e^2f\sqrt{a+bx^2}} \\
&- \frac{\left(a^2df(5de-6cf)-2b^2ce(de-cf)-ab\left(5d^2e^2-8cdef+3c^2f^2\right)\right)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{2ae^2(be-af)(de-cf)} \\
&- \frac{d\left(a^2df(2de-3cf)-4b^2ce(de-cf)-ab\left(2d^2e^2-7cdef+6c^2f^2\right)\right)x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{2ace^2(be-af)(de-cf)} \\
&- \frac{bd^2(af(2de-3cf)-2be(de-cf))x^5\sqrt{a+bx^2}\sqrt{c+dx^2}}{2ace^2(be-af)(de-cf)} - \frac{\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{5/2}}{acex\left(e+fx^2\right)} \\
&+ \frac{f(af(2de-3cf)-2be(de-cf))x\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{5/2}}{2ace^2(be-af)(de-cf)\left(e+fx^2\right)} \\
&+ \frac{\sqrt{a}\sqrt{b}(de-3cf)\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{2e^2f\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} \\
&+ \frac{\sqrt{a}\sqrt{b}\left(2bc^2ef-a\left(d^2e^2-2cdef+3c^2f^2\right)\right)\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{2ce^2f(be-af)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} \\
&- \frac{a^{3/2}(de-cf)\left(3acf^2-be(de+2cf)\right)\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{2\sqrt{b}ce^3f(be-af)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}\end{aligned}$$

output
```
-1/2*b*(-3*c*f+d*e)*x*(d*x^2+c)^(1/2)/e^2/f/(b*x^2+a)^(1/2)-1/2*(a^2*d*f*(
-6*c*f+5*d*e)-2*b^2*c*e*(-c*f+d*e)-a*b*(3*c^2*f^2-8*c*d*e*f+5*d^2*e^2))*x*
(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a/e^2/(-a*f+b*e)/(-c*f+d*e)-1/2*d*(a^2*d*f
*(-3*c*f+2*d*e)-4*b^2*c*e*(-c*f+d*e)-a*b*(6*c^2*f^2-7*c*d*e*f+2*d^2*e^2))*
x^3*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a/c/e^2/(-a*f+b*e)/(-c*f+d*e)-1/2*b*d^
2*(a*f*(-3*c*f+2*d*e)-2*b*e*(-c*f+d*e))*x^5*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2
)/a/c/e^2/(-a*f+b*e)/(-c*f+d*e)-(b*x^2+a)^(3/2)*(d*x^2+c)^(5/2)/a/c/e/x/(f
*x^2+e)+1/2*f*(a*f*(-3*c*f+2*d*e)-2*b*e*(-c*f+d*e))*x*(b*x^2+a)^(3/2)*(d*x
^2+c)^(5/2)/a/c/e^2/(-a*f+b*e)/(-c*f+d*e)/(f*x^2+e)+1/2*a^(1/2)*b^(1/2)*(-
3*c*f+d*e)*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(
1-a*d/b/c)^(1/2))/e^2/f/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)+1/
2*a^(1/2)*b^(1/2)*(2*b*c^2*e*f-a*(3*c^2*f^2-2*c*d*e*f+d^2*e^2))*(d*x^2+c)^
(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/c/e^2/f
/(-a*f+b*e)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)-1/2*a^(3/2)*(-
c*f+d*e)*(3*a*c*f^2-b*e*(2*c*f+d*e))*(d*x^2+c)^(1/2)*EllipticPi(b^(1/2)*x/
a^(1/2)/(1+b*x^2/a)^(1/2),1-a*f/b/e,(1-a*d/b/c)^(1/2))/b^(1/2)/c/e^3/f/(-a
*f+b*e)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.91 (sec) , antiderivative size = 266, normalized size of antiderivative = 0.30

$$\int \frac{\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}}{x^2\left(e+fx^2\right)^2}\,dx = \frac{\frac{e(a+bx^2)(c+dx^2)(-2ce+dex^2-3cfx^2)}{x(e+fx^2)} + \frac{i\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\left(-bcef(-de+3cf)E\left(i\operatorname{arcsinh}\left(\sqrt{\frac{b}{a}}\right.\right.\right.}{}}{}$$

input
```
Integrate[(Sqrt[a + b*x^2]*(c + d*x^2)^(3/2))/(x^2*(e + f*x^2)^2),x]
```

output
```
((e*(a + b*x^2)*(c + d*x^2)*(-2*c*e + d*e*x^2 - 3*c*f*x^2))/(x*(e + f*x^2)
) + (I*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(-(b*c*e*f*(-(d*e) + 3*c*f)
*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]) - (d*e - c*f)*(b*e*(d*e +
 3*c*f)*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - (-3*a*c*f^2 + b*e
*(d*e + 2*c*f))*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c
)])))/(Sqrt[b/a]*f^2))/(2*e^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}}{x^2\left(e+fx^2\right)^2}dx$$
$$\Big\downarrow \text{450}$$
$$\int \frac{\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}}{x^2\left(e+fx^2\right)^2}dx$$

input
```
Int[(Sqrt[a + b*x^2]*(c + d*x^2)^(3/2))/(x^2*(e + f*x^2)^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [A] (verified)

Time = 19.74 (sec) , antiderivative size = 1102, normalized size of antiderivative = 1.25

method	result	size
elliptic	Expression too large to display	1102
risch	Expression too large to display	1423
default	Expression too large to display	1632

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/x^2/(f*x^2+e)^2,x,method=_RETURNVERBOS
E)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-1/2*(c*f-d*e
)/e^2*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(f*x^2+e)-c/e^2*(b*d*x^4+a*d*x
^2+b*c*x^2+a*c)^(1/2)/x-3/2*c^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)
^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*b/e^2*EllipticF(x*(-b/a)^(1/2),
(-1+(a*d+b*c)/c/b)^(1/2))+1/2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(
1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d
+b*c)/c/b)^(1/2))*b*d^2/f^2+1/e^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/
c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f
/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*b*c^2-1/2/f^2/(-b/a)^(1/2)*(1+b*x^2/a)^(
1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-
b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*b*d^2+3/2*c^2/(-b/a)^(1/2)
*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*b
/e^2*EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-3/2/e^3*f/(-b/a)^(
1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/
2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*a*c^2+3/
2/e^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*
c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(
1/2))*a*c*d+1/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*
d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2)
)/e/f*b*d*c-1/2*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}}{x^2\left(e+fx^2\right)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/x^2/(f*x^2+e)^2,x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}}{x^2(e+fx^2)^2}\,dx = \int \frac{\sqrt{a+bx^2}(c+dx^2)^{\frac{3}{2}}}{x^2(e+fx^2)^2}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(3/2)/x**2/(f*x**2+e)**2,x)
```

output
```
Integral(sqrt(a + b*x**2)*(c + d*x**2)**(3/2)/(x**2*(e + f*x**2)**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}}{x^2(e+fx^2)^2}\,dx = \int \frac{\sqrt{bx^2+a}(dx^2+c)^{\frac{3}{2}}}{(fx^2+e)^2x^2}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/x^2/(f*x^2+e)^2,x, algorithm="ma
xima")
```

output
```
integrate(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)/((f*x^2 + e)^2*x^2), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}}{x^2(e+fx^2)^2}\,dx = \int \frac{\sqrt{bx^2+a}(dx^2+c)^{\frac{3}{2}}}{(fx^2+e)^2x^2}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/x^2/(f*x^2+e)^2,x, algorithm="gi
ac")
```

output
```
integrate(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)/((f*x^2 + e)^2*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}}{x^2(e+fx^2)^2}\,dx=\int \frac{\sqrt{b\,x^2+a}\,\left(d\,x^2+c\right)^{3/2}}{x^2\,\left(f\,x^2+e\right)^2}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(3/2))/(x^2*(e + f*x^2)^2),x)
```

output
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(3/2))/(x^2*(e + f*x^2)^2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}}{x^2(e+fx^2)^2}\,dx=\text{too large to display}$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/x^2/(f*x^2+e)^2,x)
```

output

```
( - sqrt(c + d*x**2)*sqrt(a + b*x**2)*c - 2*int((sqrt(c + d*x**2)*sqrt(a +
 b*x**2)*x**4)/(2*a**2*c*d*e**2*f + 4*a**2*c*d*e*f**2*x**2 + 2*a**2*c*d*f*
*3*x**4 + 2*a**2*d**2*e**2*f*x**2 + 4*a**2*d**2*e*f**2*x**4 + 2*a**2*d**2*
f**3*x**6 + 2*a*b*c**2*e**2*f + 4*a*b*c**2*e*f**2*x**2 + 2*a*b*c**2*f**3*x
**4 - a*b*c*d*e**3 + 2*a*b*c*d*e**2*f*x**2 + 7*a*b*c*d*e*f**2*x**4 + 4*a*b
*c*d*f**3*x**6 - a*b*d**2*e**3*x**2 + 3*a*b*d**2*e*f**2*x**6 + 2*a*b*d**2*
f**3*x**8 + 2*b**2*c**2*e**2*f*x**2 + 4*b**2*c**2*e*f**2*x**4 + 2*b**2*c**
2*f**3*x**6 - b**2*c*d*e**3*x**2 + 3*b**2*c*d*e*f**2*x**6 + 2*b**2*c*d*f**
3*x**8 - b**2*d**2*e**3*x**4 - 2*b**2*d**2*e**2*f*x**6 - b**2*d**2*e*f**2*
x**8),x)*a*b*c*d**2*e*f**2*x - 2*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x*
*4)/(2*a**2*c*d*e**2*f + 4*a**2*c*d*e*f**2*x**2 + 2*a**2*c*d*f**3*x**4 + 2
*a**2*d**2*e**2*f*x**2 + 4*a**2*d**2*e*f**2*x**4 + 2*a**2*d**2*f**3*x**6 +
 2*a*b*c**2*e**2*f + 4*a*b*c**2*e*f**2*x**2 + 2*a*b*c**2*f**3*x**4 - a*b*c
*d*e**3 + 2*a*b*c*d*e**2*f*x**2 + 7*a*b*c*d*e*f**2*x**4 + 4*a*b*c*d*f**3*x
**6 - a*b*d**2*e**3*x**2 + 3*a*b*d**2*e*f**2*x**6 + 2*a*b*d**2*f**3*x**8 +
 2*b**2*c**2*e**2*f*x**2 + 4*b**2*c**2*e*f**2*x**4 + 2*b**2*c**2*f**3*x**6
 - b**2*c*d*e**3*x**2 + 3*b**2*c*d*e*f**2*x**6 + 2*b**2*c*d*f**3*x**8 - b*
*2*d**2*e**3*x**4 - 2*b**2*d**2*e**2*f*x**6 - b**2*d**2*e*f**2*x**8),x)*a*
b*c*d**2*f**3*x**3 + 2*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(2*a**
2*c*d*e**2*f + 4*a**2*c*d*e*f**2*x**2 + 2*a**2*c*d*f**3*x**4 + 2*a**2*d...
```

3.107 $\int \frac{\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}}{x^4\left(e+fx^2\right)^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 1105

$$\int \frac{\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}}{x^4\left(e+fx^2\right)^2}\,dx = \text{Too large to display}$$

output
```
1/6*b*(-15*a*c*f+11*a*d*e+2*b*c*e)*x*(d*x^2+c)^(1/2)/a/e^3/(b*x^2+a)^(1/2)
+1/6*(4*b^2*c*e*(3*c^2*f^2-5*c*d*e*f+2*d^2*e^2)-a^2*d*f*(30*c^2*f^2-37*c*d
*e*f+10*d^2*e^2)+5*a*b*(-3*c^3*f^3+10*c^2*d*e*f^2-9*c*d^2*e^2*f+2*d^3*e^3)
)*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a/c/e^3/(-a*f+b*e)/(-c*f+d*e)+1/6*d*(2
*b^2*c*e*(12*c^2*f^2-17*c*d*e*f+5*d^2*e^2)-a^2*d*f*(15*c^2*f^2-16*c*d*e*f+
4*d^2*e^2)+a*b*(-30*c^3*f^3+49*c^2*d*e*f^2-26*c*d^2*e^2*f+4*d^3*e^3))*x^3*
(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a/c^2/e^3/(-a*f+b*e)/(-c*f+d*e)+1/6*b*d^2*
(4*b*e*(3*c^2*f^2-4*c*d*e*f+d^2*e^2)-a*f*(15*c^2*f^2-16*c*d*e*f+4*d^2*e^2)
)*x^5*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a/c^2/e^3/(-a*f+b*e)/(-c*f+d*e)-1/3*
(b*x^2+a)^(3/2)*(d*x^2+c)^(5/2)/a/c/e/x^3/(f*x^2+e)-1/3*(-5*c*f+2*d*e)*(b*
x^2+a)^(3/2)*(d*x^2+c)^(5/2)/a/c^2/e^2/x/(f*x^2+e)-1/6*f*(4*b*e*(3*c^2*f^2
-4*c*d*e*f+d^2*e^2)-a*f*(15*c^2*f^2-16*c*d*e*f+4*d^2*e^2))*x*(b*x^2+a)^(3/
2)*(d*x^2+c)^(5/2)/a/c^2/e^3/(-a*f+b*e)/(-c*f+d*e)/(f*x^2+e)-1/6*b^(1/2)*(
-15*a*c*f+11*a*d*e+2*b*c*e)*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1
+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/a^(1/2)/e^3/(b*x^2+a)^(1/2)/(a*(d*x^2+c
)/c/(b*x^2+a))^(1/2)+1/6*a^(1/2)*b^(1/2)*(2*b*c*e*(-6*c*f+5*d*e)+a*(15*c^2
*f^2-16*c*d*e*f+3*d^2*e^2))*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)
*x/a^(1/2)),(1-a*d/b/c)^(1/2))/c/e^3/(-a*f+b*e)/(b*x^2+a)^(1/2)/(a*(d*x^2+
c)/c/(b*x^2+a))^(1/2)-1/2*a^(3/2)*(-c*f+d*e)*(a*f*(-5*c*f+2*d*e)-b*e*(-4*c
*f+d*e))*(d*x^2+c)^(1/2)*EllipticPi(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2)...
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.41 (sec) , antiderivative size = 443, normalized size of antiderivative = 0.40

$$\int \frac{\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}}{x^4\left(e+fx^2\right)^2}\,dx = \frac{\sqrt{\frac{b}{a}}\left(-\sqrt{\frac{b}{a}}ef\left(a+bx^2\right)\left(c+dx^2\right)\left(2bcex^2\left(e+fx^2\right)+adex^2\left(8e+11fx^2\right)+c\right.\right.}{}$$

input
```
Integrate[(Sqrt[a + b*x^2]*(c + d*x^2)^(3/2))/(x^4*(e + f*x^2)^2),x]
```

output
```
(Sqrt[b/a]*(-(Sqrt[b/a]*e*f*(a + b*x^2)*(c + d*x^2)*(2*b*c*e*x^2*(e + f*x^
2) + a*d*e*x^2*(8*e + 11*f*x^2) + a*c*(2*e^2 - 10*e*f*x^2 - 15*f^2*x^4)))
+ I*b*c*e*f*(-2*b*c*e - 11*a*d*e + 15*a*c*f)*x^3*Sqrt[1 + (b*x^2)/a]*Sqrt[
1 + (d*x^2)/c]*(e + f*x^2)*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]
- I*b*e*(-2*b*c^2*e*f + a*(3*d^2*e^2 - 16*c*d*e*f + 15*c^2*f^2))*x^3*Sqrt[
1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(e + f*x^2)*EllipticF[I*ArcSinh[Sqrt[b/
a]*x], (a*d)/(b*c)] - (3*I)*a*(-(d*e) + c*f)*(b*e*(d*e - 4*c*f) + a*f*(-2*
d*e + 5*c*f))*x^3*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(e + f*x^2)*Elli
pticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(6*b*e^4*f*x^3*
Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}}{x^4\left(e+fx^2\right)^2}\,dx$$

$$\Big\downarrow \ 450$$

$$\int \frac{\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}}{x^4\left(e+fx^2\right)^2}dx$$

input
```
Int[(Sqrt[a + b*x^2]*(c + d*x^2)^(3/2))/(x^4*(e + f*x^2)^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [A] (verified)

Time = 21.02 (sec) , antiderivative size = 1458, normalized size of antiderivative = 1.32

method	result	size
elliptic	Expression too large to display	1458
risch	Expression too large to display	1468
default	Expression too large to display	2343

input

```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/x^4/(f*x^2+e)^2,x,method=_RETURNVERBOS
E)
```

output

```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-1/3*c/e^2*(b
*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/x^3+1/3/a*(6*a*c*f-4*a*d*e-b*c*e)/e^3*(b
*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/x+1/2*(c*f-d*e)*f/e^3*x*(b*d*x^4+a*d*x^2
+b*c*x^2+a*c)^(1/2)/(f*x^2+e)-8/3/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/
c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+
(a*d+b*c)/c/b)^(1/2))*b*c*d/e^2+1/2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^
2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-
1+(a*d+b*c)/c/b)^(1/2))*b*d^2/e/f+11/6*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1
+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*d*b/e^2*EllipticE(x*(-
b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+5/2/e^4*f^2/(-b/a)^(1/2)*(1+b*x^2/a)^
(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(
-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*a*c^2-2/e^3*f/(-b/a)^(1/2
)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*
EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*b*c^2+5/2/e
^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x
^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2
))*b*c*d-1/2/e/f/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4
+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/
2)/(-b/a)^(1/2))*b*d^2+5/2*c^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^
(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*b/e^3*f*EllipticF(x*(-b/a)^(1...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}}{x^4\left(e+fx^2\right)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/x^4/(f*x^2+e)^2,x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}}{x^4\left(e+fx^2\right)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(3/2)/x**4/(f*x**2+e)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}}{x^4\left(e+fx^2\right)^2}\,dx = \int \frac{\sqrt{bx^2+a}(dx^2+c)^{\frac{3}{2}}}{(fx^2+e)^2x^4}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/x^4/(f*x^2+e)^2,x, algorithm="ma
xima")
```

output
```
integrate(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)/((f*x^2 + e)^2*x^4), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}}{x^4(e+fx^2)^2}\,dx=\int \frac{\sqrt{bx^2+a}(dx^2+c)^{\frac{3}{2}}}{(fx^2+e)^2x^4}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/x^4/(f*x^2+e)^2,x, algorithm="gi
ac")
```

output
```
integrate(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)/((f*x^2 + e)^2*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}}{x^4(e+fx^2)^2}\,dx=\int \frac{\sqrt{b\,x^2+a}\,(d\,x^2+c)^{3/2}}{x^4\,(f\,x^2+e)^2}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(3/2))/(x^4*(e + f*x^2)^2),x)
```

output
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(3/2))/(x^4*(e + f*x^2)^2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}}{x^4(e+fx^2)^2}\,dx=\int \frac{\sqrt{b\,x^2+a}\,(d\,x^2+c)^{\frac{3}{2}}}{x^4\,(f\,x^2+e)^2}dx$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/x^4/(f*x^2+e)^2,x)
```

output
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/x^4/(f*x^2+e)^2,x)
```

3.108 $\int \frac{\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}}{x^6\left(e+fx^2\right)^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 1412

$$\int \frac{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}}{x^6\,(e+fx^2)^2}\,dx = \text{Too large to display}$$

output
```
-1/30*b*(4*b^2*c^2*e^2-2*a*b*c*e*(-10*c*f+7*d*e)-a^2*(105*c^2*f^2-95*c*d*e
*f+6*d^2*e^2))*x*(d*x^2+c)^(1/2)/a^2/c/e^4/(b*x^2+a)^(1/2)+1/30*(2*b^3*c*d
*e^3*(-c*f+d*e)+5*a^3*d*f^2*(42*c^2*f^2-59*c*d*e*f+20*d^2*e^2)-a^2*b*f*(-1
05*c^3*f^3+380*c^2*d*e*f^2-397*c*d^2*e^2*f+122*d^3*e^3)+2*a*b^2*e*(-45*c^3
*f^3+86*c^2*d*e*f^2-52*c*d^2*e^2*f+11*d^3*e^3))*x*(b*x^2+a)^(1/2)*(d*x^2+c
)^(1/2)/a^2/c/e^4/(-a*f+b*e)/(-c*f+d*e)+1/30*d*(16*b^3*c*d*e^3*(-c*f+d*e)+
5*a^3*d*f^2*(21*c^2*f^2-26*c*d*e*f+8*d^2*e^2)+2*a*b^2*e*(-90*c^3*f^3+148*c
^2*d*e*f^2-63*c*d^2*e^2*f+5*d^3*e^3)-5*a^2*b*f*(-42*c^3*f^3+77*c^2*d*e*f^2
-48*c*d^2*e^2*f+10*d^3*e^3))*x^3*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a^2/c^2/e
^4/(-a*f+b*e)/(-c*f+d*e)+1/6*b*d^2*(2*b^2*d*e^3*(-c*f+d*e)-2*a*b*e*f*(9*c^
2*f^2-14*c*d*e*f+5*d^2*e^2)+a^2*f^2*(21*c^2*f^2-26*c*d*e*f+8*d^2*e^2))*x^5
*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a^2/c^2/e^4/(-a*f+b*e)/(-c*f+d*e)-1/5*(b*
x^2+a)^(3/2)*(d*x^2+c)^(5/2)/a/c/e/x^5/(f*x^2+e)+1/15*(7*a*f+2*b*e)*(b*x^2
+a)^(3/2)*(d*x^2+c)^(5/2)/a^2/c/e^2/x^3/(f*x^2+e)+1/15*(5*a*f*(-7*c*f+4*d*
e)-b*e*(-2*c*f+5*d*e))*(b*x^2+a)^(3/2)*(d*x^2+c)^(5/2)/a^2/c^2/e^3/x/(f*x^
2+e)-1/6*f*(2*b^2*d*e^3*(-c*f+d*e)-2*a*b*e*f*(9*c^2*f^2-14*c*d*e*f+5*d^2*e
^2)+a^2*f^2*(21*c^2*f^2-26*c*d*e*f+8*d^2*e^2))*x*(b*x^2+a)^(3/2)*(d*x^2+c)
^(5/2)/a^2/c^2/e^4/(-a*f+b*e)/(-c*f+d*e)/(f*x^2+e)+1/30*b^(1/2)*(4*b^2*c^2
*e^2-2*a*b*c*e*(-10*c*f+7*d*e)-a^2*(105*c^2*f^2-95*c*d*e*f+6*d^2*e^2))*(d*
x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^...
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.83 (sec) , antiderivative size = 480, normalized size of antiderivative = 0.34

$$\int \frac{\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}}{x^6\left(e+fx^2\right)^2}\,dx = \frac{\frac{e(a+bx^2)(c+dx^2)\left(15a^2cf^2(de-cf)x^6-6a^2c^2e^2(e+fx^2)-2ace(bce+6ade-10acf)x^2(e+fx^2)+2(2b^2c^2\cdots\right.}{cx^5(e+fx^2)}}{\cdots}$$

input
```
Integrate[(Sqrt[a + b*x^2]*(c + d*x^2)^(3/2))/(x^6*(e + f*x^2)^2),x]
```

output
```
((e*(a + b*x^2)*(c + d*x^2)*(15*a^2*c*f^2*(d*e - c*f)*x^6 - 6*a^2*c^2*e^2*
(e + f*x^2) - 2*a*c*e*(b*c*e + 6*a*d*e - 10*a*c*f)*x^2*(e + f*x^2) + 2*(2*
b^2*c^2*e^2 + a*b*c*e*(-7*d*e + 10*c*f) + a^2*(-3*d^2*e^2 + 40*c*d*e*f - 4
5*c^2*f^2))*x^4*(e + f*x^2)))/(c*x^5*(e + f*x^2)) + (I*Sqrt[1 + (b*x^2)/a]
*Sqrt[1 + (d*x^2)/c]*(b*e*(4*b^2*c^2*e^2 + 2*a*b*c*e*(-7*d*e + 10*c*f) + a
^2*(-6*d^2*e^2 + 95*c*d*e*f - 105*c^2*f^2))*EllipticE[I*ArcSinh[Sqrt[b/a]*
x], (a*d)/(b*c)] - b*e*(4*b^2*c^2*e^2 + 4*a*b*c*e*(-4*d*e + 5*c*f) + a^2*(
-33*d^2*e^2 + 130*c*d*e*f - 105*c^2*f^2))*EllipticF[I*ArcSinh[Sqrt[b/a]*x]
, (a*d)/(b*c)] + 15*a^2*(d*e - c*f)*(a*f*(4*d*e - 7*c*f) - 3*b*e*(d*e - 2*
c*f))*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/Sqrt[
b/a])/(30*a^2*e^5*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}}{x^6\left(e+fx^2\right)^2}\,dx$$

$$\Big\downarrow \; 450$$

$$\int \frac{\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}}{x^6\left(e+fx^2\right)^2}dx$$

input
```
Int[(Sqrt[a + b*x^2]*(c + d*x^2)^(3/2))/(x^6*(e + f*x^2)^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [A] (verified)

Time = 23.15 (sec) , antiderivative size = 1826, normalized size of antiderivative = 1.29

method	result	size
risch	Expression too large to display	1826
elliptic	Expression too large to display	2056
default	Expression too large to display	3497

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/x^6/(f*x^2+e)^2,x,method=_RETURNVERBOS
E)
```

output
```
-1/15*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(45*a^2*c^2*f^2*x^4-40*a^2*c*d*e*f*x
^4+3*a^2*d^2*e^2*x^4-10*a*b*c^2*e*f*x^4+7*a*b*c*d*e^2*x^4-2*b^2*c^2*e^2*x^
4-10*a^2*c^2*e*f*x^2+6*a^2*c*d*e^2*x^2+a*b*c^2*e^2*x^2+3*a^2*c^2*e^2)/a^2/
c/e^4/x^5+1/15/a^2/c/e^4*(-b*(45*a^2*c^2*f^2-40*a^2*c*d*e*f+3*a^2*d^2*e^2-
10*a*b*c^2*e*f+7*a*b*c*d*e^2-2*b^2*c^2*e^2)*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/
2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(EllipticF(x*(-b/
a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/
c/b)^(1/2)))-a*b^2*c^2*d*e^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1
/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+
b*c)/c/b)^(1/2))-6*a^2*b*c*d^2*e^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2
/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1
+(a*d+b*c)/c/b)^(1/2))-15*a^2*c*e*(a*c^2*f^3-2*a*c*d*e*f^2+a*d^2*e^2*f-b*c
^2*e*f^2+2*b*c*d*e^2*f-b*d^2*e^3)*(1/2*f^2/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^
2)/e*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(f*x^2+e)-1/2*d*b/(a*c*f^2-a*d*
e*f-b*c*e*f+b*d*e^2)/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d
*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b
)^(1/2))+1/2*f*b/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/e*c/(-b/a)^(1/2)*(1+b*x
^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*Elliptic
F(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-1/2*f*b/(a*c*f^2-a*d*e*f-b*c*e*
f+b*d*e^2)/e*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}}{x^6(e+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/x^6/(f*x^2+e)^2,x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}}{x^6(e+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(3/2)/x**6/(f*x**2+e)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}}{x^6(e+fx^2)^2}\,dx = \int \frac{\sqrt{bx^2+a}(dx^2+c)^{\frac{3}{2}}}{(fx^2+e)^2x^6}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/x^6/(f*x^2+e)^2,x, algorithm="ma
xima")
```

output
```
integrate(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)/((f*x^2 + e)^2*x^6), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}}{x^6(e+fx^2)^2}\,dx = \int \frac{\sqrt{bx^2+a}(dx^2+c)^{\frac{3}{2}}}{(fx^2+e)^2x^6}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/x^6/(f*x^2+e)^2,x, algorithm="gi
ac")
```

output
```
integrate(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)/((f*x^2 + e)^2*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}}{x^6(e+fx^2)^2}\,dx = \int \frac{\sqrt{b\,x^2+a}\,(d\,x^2+c)^{3/2}}{x^6\,(f\,x^2+e)^2}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(3/2))/(x^6*(e + f*x^2)^2),x)
```

output
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(3/2))/(x^6*(e + f*x^2)^2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}(c+dx^2)^{3/2}}{x^6(e+fx^2)^2}\,dx = \int \frac{\sqrt{b\,x^2+a}\,(d\,x^2+c)^{\frac{3}{2}}}{x^6\,(f\,x^2+e)^2}dx$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/x^6/(f*x^2+e)^2,x)
```

output
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/x^6/(f*x^2+e)^2,x)
```

3.109 $\int \frac{x^2\sqrt{a-bx^2}\left(c+dx^2\right)^{3/2}}{e+fx^2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 529

$$\int \frac{x^2\sqrt{a-bx^2}\left(c+dx^2\right)^{3/2}}{e+fx^2}\,dx = -\frac{(5bde-3bcf+adf)x\sqrt{a-bx^2}\sqrt{c+dx^2}}{15bf^2}+\frac{x\sqrt{a-bx^2}\left(c+dx^2\right)^{3/2}}{5f}$$
$$+\frac{\sqrt{a}\left(2a^2d^2f^2-abdf(5de-7cf)-b^2\left(15d^2e^2-20cdef+3c^2f^2\right)\right)\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\,E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\cdots\right.}{15b^{3/2}df^3\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}}$$
$$-\frac{\sqrt{a}\left(a^2cd^2f^3-abdf\left(15d^2e^2-20cdef+2c^2f^2\right)-b^2\left(15d^3e^3-15cd^2e^2f-5c^2def^2+3c^3f^3\right)\right)\sqrt{1-\frac{bx^2}{a}}\sqrt{\cdots}}{15b^{3/2}df^4\sqrt{a-bx^2}\sqrt{c+dx^2}}$$
$$-\frac{\sqrt{a}(be+af)(de-cf)^2\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}f^4\sqrt{a-bx^2}\sqrt{c+dx^2}}$$

output
```
-1/15*(a*d*f-3*b*c*f+5*b*d*e)*x*(-b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b/f^2+1/5
*x*(-b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/f+1/15*a^(1/2)*(2*a^2*d^2*f^2-a*b*d*f*
(-7*c*f+5*d*e)-b^2*(3*c^2*f^2-20*c*d*e*f+15*d^2*e^2))*(1-b*x^2/a)^(1/2)*(d
*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2),(-a*d/b/c)^(1/2))/b^(3/2)/d/f^3/
(-b*x^2+a)^(1/2)/(1+d*x^2/c)^(1/2)-1/15*a^(1/2)*(a^2*c*d^2*f^3-a*b*d*f*(2*
c^2*f^2-20*c*d*e*f+15*d^2*e^2)-b^2*(3*c^3*f^3-5*c^2*d*e*f^2-15*c*d^2*e^2*f
+15*d^3*e^3))*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(b^(1/2)*x/a^(1
/2),(-a*d/b/c)^(1/2))/b^(3/2)/d/f^4/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)-a^(1/
2)*(a*f+b*e)*(-c*f+d*e)^2*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)*EllipticPi(b
^(1/2)*x/a^(1/2),-a*f/b/e,(-a*d/b/c)^(1/2))/b^(1/2)/f^4/(-b*x^2+a)^(1/2)/(
d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.67 (sec) , antiderivative size = 450, normalized size of antiderivative = 0.85

$$\int \frac{x^2\sqrt{a-bx^2}(c+dx^2)^{3/2}}{e+fx^2}\,dx = \frac{-\sqrt{-\frac{b}{a}}df^2x(a-bx^2)\left(c+dx^2\right)\left(adf+b(5de-6cf-3dfx^2)\right)-icf(2a^2d$$

input
```
Integrate[(x^2*Sqrt[a - b*x^2]*(c + d*x^2)^(3/2))/(e + f*x^2),x]
```

output
```
(-(Sqrt[-(b/a)]*d*f^2*x*(a - b*x^2)*(c + d*x^2)*(a*d*f + b*(5*d*e - 6*c*f
- 3*d*f*x^2))) - I*c*f*(2*a^2*d^2*f^2 + a*b*d*f*(-5*d*e + 7*c*f) + b^2*(-1
5*d^2*e^2 + 20*c*d*e*f - 3*c^2*f^2))*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/
c]*EllipticE[I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))] + I*(a^2*c*d^2*f^3
 + a*b*d*f*(-15*d^2*e^2 + 20*c*d*e*f - 2*c^2*f^2) + b^2*(-15*d^3*e^3 + 15*
c*d^2*e^2*f + 5*c^2*d*e*f^2 - 3*c^3*f^3))*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*
x^2)/c]*EllipticF[I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))] + (15*I)*b*d*
(b*e + a*f)*(d*e - c*f)^2*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Elliptic
Pi[-((a*f)/(b*e)), I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))])/(15*b*Sqrt[
-(b/a)]*d*f^4*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\sqrt{a-bx^2}\left(c+dx^2\right)^{3/2}}{e+fx^2}\,dx$$
$$\Big\downarrow \ 450$$
$$\int \frac{x^2\sqrt{a-bx^2}\left(c+dx^2\right)^{3/2}}{e+fx^2}dx$$

input
```
Int[(x^2*Sqrt[a - b*x^2]*(c + d*x^2)^(3/2))/(e + f*x^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [A] (verified)

Time = 9.38 (sec) , antiderivative size = 610, normalized size of antiderivative = 1.15

method	result
risch	$-\frac{x\left(-3bdf\,x^{2}+adf-6bcf+5bde\right)\sqrt{-b\,x^{2}+a}\,\sqrt{x^{2}d+c}}{15b\,f^{2}}+\frac{\left(\frac{\left(a^{2}cd\,f^{3}+9ab\,c^{2}f^{3}-25abcde\,f^{2}+15ab\,d^{2}e^{2}f+15b^{2}c^{2}e\,f^{2}-30b^{2}cd\,e^{2}f+15b^{2}d\right.}{f^{2}\sqrt{\frac{b}{a}}\,\sqrt{-bd\,x^{4}+ad\,x^{2}-x^{2}}}\right.}{}$
default	Expression too large to display
elliptic	Expression too large to display

input
```
int(x^2*(-b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/(f*x^2+e),x,method=_RETURNVERBOSE
)
```

output
```
-1/15/b*x*(-3*b*d*f*x^2+a*d*f-6*b*c*f+5*b*d*e)*(-b*x^2+a)^(1/2)*(d*x^2+c)^
(1/2)/f^2+1/15/f^2/b*((a^2*c*d*f^3+9*a*b*c^2*f^3-25*a*b*c*d*e*f^2+15*a*b*d
^2*e^2*f+15*b^2*c^2*e*f^2-30*b^2*c*d*e^2*f+15*b^2*d^2*e^3)/f^2/(b/a)^(1/2)
*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)*
EllipticF(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2))-1/f*(2*a^2*d^2*f^2+7*a*b
*c*d*f^2-5*a*b*d^2*e*f-3*b^2*c^2*f^2+20*b^2*c*d*e*f-15*b^2*d^2*e^2)*c/(b/a
)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)
^(1/2)/d*(EllipticF(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2))-EllipticE(x*(b
/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2)))-15*(a*c^2*f^3-2*a*c*d*e*f^2+a*d^2*e^2
*f+b*c^2*e*f^2-2*b*c*d*e^2*f+b*d^2*e^3)*b/f^2/(b/a)^(1/2)*(1-b*x^2/a)^(1/2
)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)*EllipticPi(x*(b/a
)^(1/2),-a*f/b/e,(-1/c*d)^(1/2)/(b/a)^(1/2)))*((-b*x^2+a)*(d*x^2+c))^(1/2)
/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^2\sqrt{a-bx^2}(c+dx^2)^{3/2}}{e+fx^2}\,dx = \text{Timed out}$$

input
```
integrate(x^2*(-b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/(f*x^2+e),x, algorithm="fri
cas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^2\sqrt{a-bx^2}(c+dx^2)^{3/2}}{e+fx^2}\,dx = \int \frac{x^2\sqrt{a-bx^2}\,(c+dx^2)^{\frac{3}{2}}}{e+fx^2}\,dx$$

input
```
integrate(x**2*(-b*x**2+a)**(1/2)*(d*x**2+c)**(3/2)/(f*x**2+e),x)
```

output
```
Integral(x**2*sqrt(a - b*x**2)*(c + d*x**2)**(3/2)/(e + f*x**2), x)
```

Maxima [F]

$$\int \frac{x^2\sqrt{a-bx^2}(c+dx^2)^{3/2}}{e+fx^2}\,dx = \int \frac{\sqrt{-bx^2+a}\,(dx^2+c)^{\frac{3}{2}}x^2}{fx^2+e}\,dx$$

input
```
integrate(x^2*(-b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/(f*x^2+e),x, algorithm="max
ima")
```

output
```
integrate(sqrt(-b*x^2 + a)*(d*x^2 + c)^(3/2)*x^2/(f*x^2 + e), x)
```

Giac [F]

$$\int \frac{x^2\sqrt{a-bx^2}(c+dx^2)^{3/2}}{e+fx^2}\,dx = \int \frac{\sqrt{-bx^2+a}\,(dx^2+c)^{\frac{3}{2}}x^2}{fx^2+e}\,dx$$

input
```
integrate(x^2*(-b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/(f*x^2+e),x, algorithm="gia
c")
```

output
```
integrate(sqrt(-b*x^2 + a)*(d*x^2 + c)^(3/2)*x^2/(f*x^2 + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2\sqrt{a-bx^2}(c+dx^2)^{3/2}}{e+fx^2}\,dx = \int \frac{x^2\,\sqrt{a-b\,x^2}\,{\left(d\,x^2+c\right)}^{3/2}}{f\,x^2+e}\,dx$$

input
```
int((x^2*(a - b*x^2)^(1/2)*(c + d*x^2)^(3/2))/(e + f*x^2),x)
```

output
```
int((x^2*(a - b*x^2)^(1/2)*(c + d*x^2)^(3/2))/(e + f*x^2), x)
```

Reduce [F]

$$\int \frac{x^2\sqrt{a-bx^2}(c+dx^2)^{3/2}}{e+fx^2}\,dx = \text{Too large to display}$$

input
```
int(x^2*(-b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/(f*x^2+e),x)
```

output
```
( - sqrt(c + d*x**2)*sqrt(a - b*x**2)*a*d*f*x + 6*sqrt(c + d*x**2)*sqrt(a
- b*x**2)*b*c*f*x - 5*sqrt(c + d*x**2)*sqrt(a - b*x**2)*b*d*e*x + 3*sqrt(c
 + d*x**2)*sqrt(a - b*x**2)*b*d*f*x**3 + 2*int((sqrt(c + d*x**2)*sqrt(a -
b*x**2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 - b*c*e*x**2 -
 b*c*f*x**4 - b*d*e*x**4 - b*d*f*x**6),x)*a**2*d**2*f**2 + 7*int((sqrt(c +
 d*x**2)*sqrt(a - b*x**2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x
**4 - b*c*e*x**2 - b*c*f*x**4 - b*d*e*x**4 - b*d*f*x**6),x)*a*b*c*d*f**2 -
 5*int((sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*
e*x**2 + a*d*f*x**4 - b*c*e*x**2 - b*c*f*x**4 - b*d*e*x**4 - b*d*f*x**6),x
)*a*b*d**2*e*f - 3*int((sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**4)/(a*c*e + a
*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 - b*c*e*x**2 - b*c*f*x**4 - b*d*e*x**4
 - b*d*f*x**6),x)*b**2*c**2*f**2 + 20*int((sqrt(c + d*x**2)*sqrt(a - b*x**
2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 - b*c*e*x**2 - b*c*
f*x**4 - b*d*e*x**4 - b*d*f*x**6),x)*b**2*c*d*e*f - 15*int((sqrt(c + d*x**
2)*sqrt(a - b*x**2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 -
b*c*e*x**2 - b*c*f*x**4 - b*d*e*x**4 - b*d*f*x**6),x)*b**2*d**2*e**2 + int
((sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**2)/(a*c*e + a*c*f*x**2 + a*d*e*x**2
 + a*d*f*x**4 - b*c*e*x**2 - b*c*f*x**4 - b*d*e*x**4 - b*d*f*x**6),x)*a**2
*c*d*f**2 + 2*int((sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**2)/(a*c*e + a*c*f*
x**2 + a*d*e*x**2 + a*d*f*x**4 - b*c*e*x**2 - b*c*f*x**4 - b*d*e*x**4 -...
```

3.110 $\int \frac{x^6\sqrt{a+bx^2}(e+fx^2)^2}{\sqrt{c+dx^2}} dx$

Optimal result

Integrand size = 35, antiderivative size = 1181

$$\int \frac{x^6\sqrt{a+bx^2}(e+fx^2)^2}{\sqrt{c+dx^2}} dx = \text{Too large to display}$$

output
```
1/3465*(128*a^5*d^5*f^2-8*a^4*b*d^4*f*(-15*c*f+44*d*e)+16*a*b^4*c^2*d*(24*
c^2*f^2-55*c*d*e*f+33*d^2*e^2)-16*b^5*c^3*(80*c^2*f^2-176*c*d*e*f+99*d^2*e
^2)+a^3*b^2*d^3*(139*c^2*f^2-352*c*d*e*f+264*d^2*e^2)+a^2*b^3*c*d^2*(194*c
^2*f^2-462*c*d*e*f+297*d^2*e^2))*x*(d*x^2+c)^(1/2)/b^4/d^6/(b*x^2+a)^(1/2)
-1/3465*(64*a^4*d^4*f^3-4*a^3*b*d^3*f^2*(-17*c*f+44*d*e)-a^2*b^2*d^2*f*(-8
1*c^2*f^2+198*c*d*e*f+148*d^2*e^2)-8*b^4*c^2*f*(80*c^2*f^2-176*c*d*e*f+99*
d^2*e^2)+a*b^3*d*(112*c^3*f^3-264*c^2*d*e*f^2-185*c*d^2*e^2*f+140*d^3*e^3)
)*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b^4/d^5/f+1/3465*(48*a^3*d^3*f^3+a^2*b
*d^2*f^2*(53*c*f+428*d*e)-2*a*b^2*d*f*(-32*c^2*f^2-273*c*d*e*f+108*d^2*e^2
)-4*b^3*(120*c^3*f^3-264*c^2*d*e*f^2+61*c*d^2*e^2*f+35*d^3*e^3))*x^3*(b*x^
2+a)^(1/2)*(d*x^2+c)^(1/2)/b^3/d^4/f+1/693*(48*a^2*d^2*f^2-a*b*d*f*(-61*c*
f+20*d*e)-4*b^2*(-20*c^2*f^2+9*c*d*e*f+5*d^2*e^2))*x^5*(b*x^2+a)^(1/2)*(d*
x^2+c)^(1/2)/b^2/d^3-2/99*(4*a/b+5*c/d-2*e/f)*x*(b*x^2+a)^(3/2)*(d*x^2+c)^
(1/2)*(f*x^2+e)^2/b/d+1/11*x^3*(b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^2
/b/d-1/3465*a^(1/2)*(128*a^5*d^5*f^2-8*a^4*b*d^4*f*(-15*c*f+44*d*e)+16*a*b
^4*c^2*d*(24*c^2*f^2-55*c*d*e*f+33*d^2*e^2)-16*b^5*c^3*(80*c^2*f^2-176*c*d
*e*f+99*d^2*e^2)+a^3*b^2*d^3*(139*c^2*f^2-352*c*d*e*f+264*d^2*e^2)+a^2*b^3
*c*d^2*(194*c^2*f^2-462*c*d*e*f+297*d^2*e^2))*(d*x^2+c)^(1/2)*EllipticE(b^
(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/b^(9/2)/d^6/(b*x^2+a)
^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)+1/3465*a^(3/2)*(64*a^4*d^4*f^2-4...
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.39 (sec) , antiderivative size = 787, normalized size of antiderivative = 0.67

$$\int \frac{x^6\sqrt{a+bx^2}(e+fx^2)^2}{\sqrt{c+dx^2}}\,dx$$
$$= \frac{\sqrt{\frac{b}{a}}dx(a+bx^2)\left(c+dx^2\right)\left(-64a^4d^4f^2+4a^3bd^3f(44de-17cf+12dfx^2)-a^2b^2d^2(81c^2f^2-cdf(198e+\right.}{}$$

input
```
Integrate[(x^6*Sqrt[a + b*x^2]*(e + f*x^2)^2)/Sqrt[c + d*x^2],x]
```

output
```
(Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2)*(-64*a^4*d^4*f^2 + 4*a^3*b*d^3*f*(4
4*d*e - 17*c*f + 12*d*f*x^2) - a^2*b^2*d^2*(81*c^2*f^2 - c*d*f*(198*e + 53
*f*x^2) + 4*d^2*(33*e^2 + 33*e*f*x^2 + 10*f^2*x^4)) + a*b^3*d*(-112*c^3*f^
2 + 8*c^2*d*f*(33*e + 8*f*x^2) + d^3*x^2*(99*e^2 + 110*e*f*x^2 + 35*f^2*x^
4) - c*d^2*(165*e^2 + 154*e*f*x^2 + 45*f^2*x^4)) + b^4*(640*c^4*f^2 - 32*c
^3*d*f*(44*e + 15*f*x^2) + 8*c^2*d^2*(99*e^2 + 132*e*f*x^2 + 50*f^2*x^4) +
 5*d^4*x^4*(99*e^2 + 154*e*f*x^2 + 63*f^2*x^4) - 2*c*d^3*x^2*(297*e^2 + 44
0*e*f*x^2 + 175*f^2*x^4))) - I*c*(128*a^5*d^5*f^2 - 8*a^4*b*d^4*f*(44*d*e
- 15*c*f) + 16*a*b^4*c^2*d*(33*d^2*e^2 - 55*c*d*e*f + 24*c^2*f^2) - 16*b^5
*c^3*(99*d^2*e^2 - 176*c*d*e*f + 80*c^2*f^2) + a^3*b^2*d^3*(264*d^2*e^2 -
352*c*d*e*f + 139*c^2*f^2) + a^2*b^3*c*d^2*(297*d^2*e^2 - 462*c*d*e*f + 19
4*c^2*f^2))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sq
rt[b/a]*x], (a*d)/(b*c)] + (2*I)*c*(-(b*c) + a*d)*(32*a^4*d^4*f^2 + 2*a^3*
b*d^3*f*(-44*d*e + 29*c*f) + 3*a^2*b^2*d^2*(22*d^2*e^2 - 55*c*d*e*f + 29*c
^2*f^2) + 4*a*b^3*c*d*(33*d^2*e^2 - 66*c*d*e*f + 32*c^2*f^2) + 8*b^4*c^2*(
99*d^2*e^2 - 176*c*d*e*f + 80*c^2*f^2))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^
2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(3465*b^4*Sqrt[b/a]*
d^6*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (warning: unable to verify)

Time = 2.69 (sec) , antiderivative size = 1451, normalized size of antiderivative = 1.23, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.457$, Rules used = {448, 443, 444, 444, 27, 444, 27, 406, 320, 388, 313, 444, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6\sqrt{a+bx^2}\left(e+fx^2\right)^2}{\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \text{448}$$

$$\frac{f\int \frac{x^8\sqrt{bx^2+a}(fx^2+e)}{\sqrt{dx^2+c}}dx}{e^2}+e\int \frac{x^6\sqrt{bx^2+a}\left(fx^2+e\right)}{\sqrt{dx^2+c}}dx$$

$$\Big\downarrow \text{443}$$

$$\frac{f\left(\frac{\int \frac{x^8\left((11bde-10bcf+adf)x^2+a(11de-9cf)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{11d}+\frac{fx^9\sqrt{a+bx^2}\sqrt{c+dx^2}}{11d}\right)}{e^2}+$$
$$e\left(\frac{\int \frac{x^6\left((9bde-8bcf+adf)x^2+a(9de-7cf)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{9d}+\frac{fx^7\sqrt{a+bx^2}\sqrt{c+dx^2}}{9d}\right)$$

$\downarrow$ 444

$$\frac{f\left(\frac{\frac{x^7\sqrt{a+bx^2}\sqrt{c+dx^2}(adf-10bcf+11bde)}{9bd}-\frac{\int \frac{x^6\left(\left(\left(88cde-80c^2f\right)b^2-ad(11de-9cf)b+8a^2d^2f\right)x^2+7ac(11bde-10bcf+adf)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{9bd}}{11d}+\frac{fx^9\sqrt{a+bx^2}\sqrt{c+dx^2}}{11d}\right.}{e^2}$$
$$e\left(\frac{\frac{x^5\sqrt{a+bx^2}\sqrt{c+dx^2}(adf-8bcf+9bde)}{7bd}-\frac{\int \frac{x^4\left(\left(6c(9de-8cf)b^2-ad(9de-7cf)b+6a^2d^2f\right)x^2+5ac(9bde-8bcf+adf)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{7bd}}{9d}+\frac{fx^7\sqrt{a+bx^2}\sqrt{c-}}{9d}\right.$$

$\downarrow$ 444

$$\frac{f\left(\frac{\frac{x^7\sqrt{a+bx^2}\sqrt{c+dx^2}(adf-10bcf+11bde)}{9bd}-\frac{\frac{1}{7}x^5\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{8a^2df}{b}+9acf-11ade-\frac{80bc^2f}{d}+88bce\right)-\frac{\int \frac{x^4\left(\left(48c^2(11de-10cf)b^3-acd(77de-64cf)b^2-\right.\right.}{}}{}}{9bd}}{11d}\right.}{e^2}$$
$$e\left(\frac{\frac{x^5\sqrt{a+bx^2}\sqrt{c+dx^2}(adf-8bcf+9bde)}{7bd}-\frac{\frac{1}{5}x^3\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{6a^2df}{b}+7acf-9ade-\frac{48bc^2f}{d}+54bce\right)-\frac{\int \frac{3x^2\left(\left(8c^2(9de-8cf)b^3-3acd(5de-4c\right.\right.}{}}{}}{7bd}}{9d}\right.$$

$\downarrow$ 27

$$f\left(\frac{x^7\sqrt{a+bx^2}\sqrt{c+dx^2}(adf-10bcf+11bde)}{9bd}-\frac{\frac{1}{7}x^5\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{8a^2df}{b}+9acf-11ade-\frac{80bc^2f}{d}+88bce\right)-\frac{\int x^4\left(\left(48c^2(11de-10cf)b^3-acd(77de-64cf)b^2-\right.\right.}{9bd}}{11d}\right.$$

$$e^2$$

$$e\left(\frac{x^5\sqrt{a+bx^2}\sqrt{c+dx^2}(adf-8bcf+9bde)}{7bd}-\frac{\frac{1}{5}x^3\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{6a^2df}{b}+7acf-9ade-\frac{48bc^2f}{d}+54bce\right)-\frac{3\int x^2\left(\left(8c^2(9de-8cf)b^3-3acd(5de-4c\right.\right.}{7bd}}{9d}\right.$$

$\downarrow$ 444

$$f\left(\frac{f\sqrt{bx^2+a}\sqrt{dx^2+c}x^9}{11d}+\frac{(11bde-10bcf+adf)x^7\sqrt{bx^2+a}\sqrt{dx^2+c}}{9bd}-\frac{\frac{1}{7}\left(\frac{8dfa^2}{b}-11dea+9cfa+88bce-\frac{80bc^2f}{d}\right)x^5\sqrt{bx^2+a}\sqrt{dx^2+c}-\frac{\left(48c^2(11de-10cf)b\right.}{}}{}\right.$$

$$e\left(\frac{f\sqrt{bx^2+a}\sqrt{dx^2+c}x^7}{9d}+\frac{(9bde-8bcf+adf)x^5\sqrt{bx^2+a}\sqrt{dx^2+c}}{7bd}-\frac{\frac{1}{5}\left(\frac{6dfa^2}{b}-9dea+7cfa+54bce-\frac{48bc^2f}{d}\right)x^3\sqrt{bx^2+a}\sqrt{dx^2+c}-\frac{3\left(\right.}{}}{}\right.$$

$\downarrow$ 27

$$f\left(\frac{f\sqrt{bx^2+a}\sqrt{dx^2+c}\,x^9}{11d}+\frac{\frac{(11bde-10bcf+adf)x^7\sqrt{bx^2+a}\sqrt{dx^2+c}}{9bd}-\frac{\frac{1}{7}\left(\frac{8dfa^2}{b}-11dea+9cfa+88bce-\frac{80bc^2f}{d}\right)x^5\sqrt{bx^2+a}\sqrt{dx^2+c}-\frac{\left(48c^2(11de-10cf)b\right.}{}}{}}{}\right.$$

$$e\left(\frac{f\sqrt{bx^2+a}\sqrt{dx^2+c}\,x^7}{9d}+\frac{\frac{(9bde-8bcf+adf)x^5\sqrt{bx^2+a}\sqrt{dx^2+c}}{7bd}-\frac{\frac{1}{5}\left(\frac{6dfa^2}{b}-9dea+7cfa+54bce-\frac{48bc^2f}{d}\right)x^3\sqrt{bx^2+a}\sqrt{dx^2+c}-\frac{3\left(\right.}{}}{}}{}\right.$$

$\downarrow$ 406

$$e\left(\frac{f\sqrt{bx^2+a}\sqrt{dx^2+c}\,x^7}{9d}+\frac{\frac{(9bde-8bcf+adf)x^5\sqrt{bx^2+a}\sqrt{dx^2+c}}{7bd}-\frac{\frac{1}{5}\left(\frac{6dfa^2}{b}-9dea+7cfa+54bce-\frac{48bc^2f}{d}\right)x^3\sqrt{bx^2+a}\sqrt{dx^2+c}-\frac{3\left(\right.}{}}{}}{}\right.$$

$$f\left(\frac{f\sqrt{bx^2+a}\sqrt{dx^2+c}\,x^9}{11d}+\frac{\frac{(11bde-10bcf+adf)x^7\sqrt{bx^2+a}\sqrt{dx^2+c}}{9bd}-\frac{\frac{1}{7}\left(\frac{8dfa^2}{b}-11dea+9cfa+88bce-\frac{80bc^2f}{d}\right)x^5\sqrt{bx^2+a}\sqrt{dx^2+c}-\frac{\left(48c^2(11de-10cf)b\right.}{}}{}}{}\right.$$

$\downarrow$ 320

$$e\left(\frac{f\sqrt{bx^2+a}\sqrt{dx^2+c}x^7}{9d}+\frac{\frac{(9bde-8bcf+adf)x^5\sqrt{bx^2+a}\sqrt{dx^2+c}}{7bd}-\frac{\frac{1}{5}\left(\frac{6dfa^2}{b}-9dea+7cfa+54bce-\frac{48bc^2f}{d}\right)x^3\sqrt{bx^2+a}\sqrt{dx^2+c}-\frac{3\left(\cdots\right)}{\cdots}}{\cdots}}{\cdots}\right.$$

$$f\left(\frac{f\sqrt{bx^2+a}\sqrt{dx^2+c}x^9}{11d}+\frac{\frac{(11bde-10bcf+adf)x^7\sqrt{bx^2+a}\sqrt{dx^2+c}}{9bd}-\frac{\frac{1}{7}\left(\frac{8dfa^2}{b}-11dea+9cfa+88bce-\frac{80bc^2f}{d}\right)x^5\sqrt{bx^2+a}\sqrt{dx^2+c}-\frac{\left(48c^2(11de-10cf)b\cdots\right.}{\cdots}}{\cdots}}{\cdots}\right.$$

$$\Big\downarrow \; 388$$

$$e\left(\frac{f\sqrt{bx^2+a}\sqrt{dx^2+c}x^7}{9d}+\frac{(9bde-8bcf+adf)x^5\sqrt{bx^2+a}\sqrt{dx^2+c}}{7bd}-\frac{\frac{1}{5}\left(\frac{6dfa^2}{b}-9dea+7cfa+54bce-\frac{48bc^2f}{d}\right)x^3\sqrt{bx^2+a}\sqrt{dx^2+c}-\cdots 3\cdots}{\cdots}\right.$$

$$f\left(\frac{f\sqrt{bx^2+a}\sqrt{dx^2+c}x^9}{11d}+\frac{(11bde-10bcf+adf)x^7\sqrt{bx^2+a}\sqrt{dx^2+c}}{9bd}-\frac{\frac{1}{7}\left(\frac{8dfa^2}{b}-11dea+9cfa+88bce-\frac{80bc^2f}{d}\right)x^5\sqrt{bx^2+a}\sqrt{dx^2+c}-\frac{\left(48c^2(11de-10cf)b\cdots\right.}{\cdots}}{\cdots}\right.$$

$\downarrow$ 313

$$e\left(\frac{f\sqrt{bx^2+a}\sqrt{dx^2+c}x^7}{9d}+\frac{(9bde-8bcf+adf)x^5\sqrt{bx^2+a}\sqrt{dx^2+c}-\frac{\frac{1}{5}\left(\frac{6dfa^2}{b}-9dea+7cfa+54bce-\frac{48bc^2f}{d}\right)x^3\sqrt{bx^2+a}\sqrt{dx^2+c}-\cdots}{\cdots}}{7bd}\right.$$

$$f\left(\frac{f\sqrt{bx^2+a}\sqrt{dx^2+c}x^9}{11d}+\frac{(11bde-10bcf+adf)x^7\sqrt{bx^2+a}\sqrt{dx^2+c}-\frac{\frac{1}{7}\left(\frac{8dfa^2}{b}-11dea+9cfa+88bce-\frac{80bc^2f}{d}\right)x^5\sqrt{bx^2+a}\sqrt{dx^2+c}-\frac{\left(48c^2(11de-10cf)b\cdots\right.}{\cdots}}{\cdots}}{9bd}\right.$$

$\downarrow$ 444

$$e\left(\frac{f\sqrt{bx^2+a}\sqrt{dx^2+c}\,x^7}{9d}+\frac{\frac{(9bde-8bcf+adf)x^5\sqrt{bx^2+a}\sqrt{dx^2+c}}{7bd}-\frac{\frac{1}{5}\left(\frac{6dfa^2}{b}-9dea+7cfa+54bce-\frac{48bc^2f}{d}\right)x^3\sqrt{bx^2+a}\sqrt{dx^2+c}-\cdots}{\cdots}}{\cdots}\right.$$

3

$$f\left(\frac{f\sqrt{bx^2+a}\sqrt{dx^2+c}\,x^9}{11d}+\frac{\frac{(11bde-10bcf+adf)x^7\sqrt{bx^2+a}\sqrt{dx^2+c}}{9bd}-\frac{\frac{1}{7}\left(\frac{8dfa^2}{b}-11dea+9cfa+88bce-\frac{80bc^2f}{d}\right)x^5\sqrt{bx^2+a}\sqrt{dx^2+c}-\frac{\left(48c^2(11de-10cf)b\cdots\right.}{\cdots}}{\cdots}}{\cdots}\right.$$

↓ 406

$$e\left(\frac{f\sqrt{bx^2+a}\sqrt{dx^2+c}\,x^7}{9d}+\frac{(9bde-8bcf+adf)x^5\sqrt{bx^2+a}\sqrt{dx^2+c}}{7bd}-\frac{\frac{1}{5}\left(\frac{6dfa^2}{b}-9dea+7cfa+54bce-\frac{48bc^2f}{d}\right)x^3\sqrt{bx^2+a}\sqrt{dx^2+c}-\cdots}{\cdots}\right.$$

$$f\left(\frac{f\sqrt{bx^2+a}\sqrt{dx^2+c}\,x^9}{11d}+\frac{(11bde-10bcf+adf)x^7\sqrt{bx^2+a}\sqrt{dx^2+c}}{9bd}-\frac{\frac{1}{7}\left(\frac{8dfa^2}{b}-11dea+9cfa+88bce-\frac{80bc^2f}{d}\right)x^5\sqrt{bx^2+a}\sqrt{dx^2+c}-\frac{\left(48c^2(11de-10cf)b\cdots\right.}{\cdots}}{\cdots}\right.$$

↓ 320

$$e\left(\frac{f\sqrt{bx^2+a}\sqrt{dx^2+c}\,x^7}{9d}+\frac{(9bde-8bcf+adf)x^5\sqrt{bx^2+a}\sqrt{dx^2+c}}{7bd}-\frac{\frac{1}{5}\left(\frac{6dfa^2}{b}-9dea+7cfa+54bce-\frac{48bc^2f}{d}\right)x^3\sqrt{bx^2+a}\sqrt{dx^2+c}-\cdots 3\cdots}{\cdots}\right.$$

$$f\left(\frac{f\sqrt{bx^2+a}\sqrt{dx^2+c}\,x^9}{11d}+\frac{(11bde-10bcf+adf)x^7\sqrt{bx^2+a}\sqrt{dx^2+c}}{9bd}-\frac{\frac{1}{7}\left(\frac{8dfa^2}{b}-11dea+9cfa+88bce-\frac{80bc^2f}{d}\right)x^5\sqrt{bx^2+a}\sqrt{dx^2+c}-\frac{\left(48c^2(11de-10cf)b\cdots\right.}{\cdots}}{\cdots}\right.$$

$$\downarrow 388$$

$$e\left(\frac{f\sqrt{bx^2+a}\sqrt{dx^2+c}x^7}{9d}+\frac{(9bde-8bcf+adf)x^5\sqrt{bx^2+a}\sqrt{dx^2+c}}{7bd}-\frac{\frac{1}{5}\left(\frac{6dfa^2}{b}-9dea+7cfa+54bce-\frac{48bc^2f}{d}\right)x^3\sqrt{bx^2+a}\sqrt{dx^2+c}-\cdots}{\cdots}\right.$$

$$f\left(\frac{f\sqrt{bx^2+a}\sqrt{dx^2+c}x^9}{11d}+\frac{(11bde-10bcf+adf)x^7\sqrt{bx^2+a}\sqrt{dx^2+c}}{9bd}-\frac{\frac{1}{7}\left(\frac{8dfa^2}{b}-11dea+9cfa+88bce-\frac{80bc^2f}{d}\right)x^5\sqrt{bx^2+a}\sqrt{dx^2+c}-\frac{\left(48c^2(11de-10cf)b\cdots\right.}{\cdots}}{\cdots}\right.$$

↓ 313

$$e\left(\frac{f\sqrt{bx^2+a}\sqrt{dx^2+c}x^7}{9d}+\frac{(9bde-8bcf+adf)x^5\sqrt{bx^2+a}\sqrt{dx^2+c}}{7bd}-\frac{\frac{1}{5}\left(\frac{6dfa^2}{b}-9dea+7cfa+54bce-\frac{48bc^2f}{d}\right)x^3\sqrt{bx^2+a}\sqrt{dx^2+c}-\cdots}{\cdots}\right.$$

$$f\left(\frac{f\sqrt{bx^2+a}\sqrt{dx^2+c}x^9}{11d}+\frac{(11bde-10bcf+adf)x^7\sqrt{bx^2+a}\sqrt{dx^2+c}}{9bd}-\frac{\frac{1}{7}\left(\frac{8dfa^2}{b}-11dea+9cfa+88bce-\frac{80bc^2f}{d}\right)x^5\sqrt{bx^2+a}\sqrt{dx^2+c}-\frac{\left(48c^2(11de-10cf)b\cdots\right.}{\cdots}}{\cdots}\right.$$

input
```
Int[(x^6*Sqrt[a + b*x^2]*(e + f*x^2)^2)/Sqrt[c + d*x^2],x]
```

output
```
e*((f*x^7*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(9*d) + (((9*b*d*e - 8*b*c*f +
a*d*f)*x^5*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(7*b*d) - (((54*b*c*e - 9*a*d*
e + 7*a*c*f - (48*b*c^2*f)/d + (6*a^2*d*f)/b)*x^3*Sqrt[a + b*x^2]*Sqrt[c +
 d*x^2])/5 - (3*(((8*a^3*d^3*f + 8*b^3*c^2*(9*d*e - 8*c*f) - 3*a*b^2*c*d*(
5*d*e - 4*c*f) - 3*a^2*b*d^2*(4*d*e - 3*c*f))*x*Sqrt[a + b*x^2]*Sqrt[c + d
*x^2])/(3*b*d) - ((16*a^4*d^4*f + 16*b^4*c^3*(9*d*e - 8*c*f) - 3*a^2*b^2*c
*d^2*(9*d*e - 7*c*f) - 8*a*b^3*c^2*d*(6*d*e - 5*c*f) - 8*a^3*b*d^3*(3*d*e
- 2*c*f))*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x
^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sq
rt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (c^(3/2)*(8*a^3*d^
3*f + 8*b^3*c^2*(9*d*e - 8*c*f) - 3*a*b^2*c*d*(5*d*e - 4*c*f) - 3*a^2*b*d^
2*(4*d*e - 3*c*f))*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]],
1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c +
d*x^2]))/(3*b*d)))/(5*b*d))/(7*b*d))/(9*d)) + (f*((f*x^9*Sqrt[a + b*x^2]*S
qrt[c + d*x^2])/(11*d) + (((11*b*d*e - 10*b*c*f + a*d*f)*x^7*Sqrt[a + b*x^
2]*Sqrt[c + d*x^2])/(9*b*d) - (((88*b*c*e - 11*a*d*e + 9*a*c*f - (80*b*c^2
*f)/d + (8*a^2*d*f)/b)*x^5*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/7 - (((48*a^3*
d^3*f - a*b^2*c*d*(77*d*e - 64*c*f) - a^2*b*d^2*(66*d*e - 53*c*f) + 48*b^3
*c^2*(11*d*e - 10*c*f))*x^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(5*b*d) - (3*
(((64*a^4*d^4*f - 4*a*b^3*c^2*d*(33*d*e - 28*c*f) - 4*a^3*b*d^3*(22*d*e...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 443
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*(g*x)^(m + 1)*(a + b*x^2)^(p
 + 1)*((c + d*x^2)^q/(b*g*(m + 2*(p + q + 1) + 1))), x] + Simp[1/(b*(m + 2*
(p + q + 1) + 1))   Int[(g*x)^m*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[c*((
b*e - a*f)*(m + 1) + b*e*2*(p + q + 1)) + (d*(b*e - a*f)*(m + 1) + f*2*q*(b
*c - a*d) + b*e*d*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f
, g, m, p}, x] && GtQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^2, c + d*x^
2])
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

rule 448
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[e   Int[(g*x)^m*(a + b*x
^2)^p*(c + d*x^2)^q*(e + f*x^2)^(r - 1), x], x] + Simp[f/e^2   Int[(g*x)^(m
 + 2)*(a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^(r - 1), x], x] /; FreeQ[{a,
b, c, d, e, f, g, m, p, q}, x] && IGtQ[r, 0]
```

Maple [A] (verified)

Time = 23.17 (sec) , antiderivative size = 1468, normalized size of antiderivative = 1.24

method	result	size
elliptic	Expression too large to display	1468
risch	Expression too large to display	2046
default	Expression too large to display	3341

input
```
int(x^6*(b*x^2+a)^(1/2)*(f*x^2+e)^2/(d*x^2+c)^(1/2),x,method=_RETURNVERBOS
E)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/11*f^2/d*x^
9*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+1/9*(a*f^2+2*b*e*f-1/11*f^2/d*(10*a*
d+10*b*c))/b/d*x^7*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+1/7*(2*a*e*f+b*e^2-
9/11*a*c/d*f^2-1/9*(a*f^2+2*b*e*f-1/11*f^2/d*(10*a*d+10*b*c))/b/d*(8*a*d+8
*b*c))/b/d*x^5*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+1/5*(a*e^2-7/9*(a*f^2+2
*b*e*f-1/11*f^2/d*(10*a*d+10*b*c))/b/d*a*c-1/7*(2*a*e*f+b*e^2-9/11*a*c/d*f
^2-1/9*(a*f^2+2*b*e*f-1/11*f^2/d*(10*a*d+10*b*c))/b/d*(8*a*d+8*b*c))/b/d*(
6*a*d+6*b*c))/b/d*x^3*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+1/3*(-5/7*(2*a*e
*f+b*e^2-9/11*a*c/d*f^2-1/9*(a*f^2+2*b*e*f-1/11*f^2/d*(10*a*d+10*b*c))/b/d
*(8*a*d+8*b*c))/b/d*a*c-1/5*(a*e^2-7/9*(a*f^2+2*b*e*f-1/11*f^2/d*(10*a*d+1
0*b*c))/b/d*a*c-1/7*(2*a*e*f+b*e^2-9/11*a*c/d*f^2-1/9*(a*f^2+2*b*e*f-1/11*
f^2/d*(10*a*d+10*b*c))/b/d*(8*a*d+8*b*c))/b/d*(6*a*d+6*b*c))/b/d*(4*a*d+4*
b*c))/b/d*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)-1/3*(-5/7*(2*a*e*f+b*e^2-9
/11*a*c/d*f^2-1/9*(a*f^2+2*b*e*f-1/11*f^2/d*(10*a*d+10*b*c))/b/d*(8*a*d+8*
b*c))/b/d*a*c-1/5*(a*e^2-7/9*(a*f^2+2*b*e*f-1/11*f^2/d*(10*a*d+10*b*c))/b/
d*a*c-1/7*(2*a*e*f+b*e^2-9/11*a*c/d*f^2-1/9*(a*f^2+2*b*e*f-1/11*f^2/d*(10*
a*d+10*b*c))/b/d*(8*a*d+8*b*c))/b/d*(6*a*d+6*b*c))/b/d*(4*a*d+4*b*c))/b/d*
a*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*
x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-(-3/5*(a
*e^2-7/9*(a*f^2+2*b*e*f-1/11*f^2/d*(10*a*d+10*b*c))/b/d*a*c-1/7*(2*a*e*...
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 1206, normalized size of antiderivative = 1.02

$$\int \frac{x^6\sqrt{a+bx^2}(e+fx^2)^2}{\sqrt{c+dx^2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^6*(b*x^2+a)^(1/2)*(f*x^2+e)^2/(d*x^2+c)^(1/2),x, algorithm="fr
icas")
```

output
```
1/3465*((33*(48*b^5*c^4*d^2 - 16*a*b^4*c^3*d^3 - 9*a^2*b^3*c^2*d^4 - 8*a^3
*b^2*c*d^5)*e^2 - 22*(128*b^5*c^5*d - 40*a*b^4*c^4*d^2 - 21*a^2*b^3*c^3*d^
3 - 16*a^3*b^2*c^2*d^4 - 16*a^4*b*c*d^5)*e*f + (1280*b^5*c^6 - 384*a*b^4*c
^5*d - 194*a^2*b^3*c^4*d^2 - 139*a^3*b^2*c^3*d^3 - 120*a^4*b*c^2*d^4 - 128
*a^5*c*d^5)*f^2)*sqrt(b*d)*x*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), a
*d/(b*c)) - (33*(48*b^5*c^4*d^2 - 16*a*b^4*c^3*d^3 - 4*a^3*b^2*d^6 - 3*(3*
a^2*b^3 - 8*a*b^4)*c^2*d^4 - (8*a^3*b^2 + 5*a^2*b^3)*c*d^5)*e^2 - 22*(128*
b^5*c^5*d - 40*a*b^4*c^4*d^2 - 8*a^4*b*d^6 - (21*a^2*b^3 - 64*a*b^4)*c^3*d
^3 - 4*(4*a^3*b^2 + 3*a^2*b^3)*c^2*d^4 - (16*a^4*b + 9*a^3*b^2)*c*d^5)*e*f
 + (1280*b^5*c^6 - 384*a*b^4*c^5*d - 64*a^5*d^6 - 2*(97*a^2*b^3 - 320*a*b^
4)*c^4*d^2 - (139*a^3*b^2 + 112*a^2*b^3)*c^3*d^3 - 3*(40*a^4*b + 27*a^3*b^
2)*c^2*d^4 - 4*(32*a^5 + 17*a^4*b)*c*d^5)*f^2)*sqrt(b*d)*x*sqrt(-c/d)*elli
ptic_f(arcsin(sqrt(-c/d)/x), a*d/(b*c)) + (315*b^5*d^6*f^2*x^10 + 35*(22*b
^5*d^6*e*f - (10*b^5*c*d^5 - a*b^4*d^6)*f^2)*x^8 + 5*(99*b^5*d^6*e^2 - 22*
(8*b^5*c*d^5 - a*b^4*d^6)*e*f + (80*b^5*c^2*d^4 - 9*a*b^4*c*d^5 - 8*a^2*b^
3*d^6)*f^2)*x^6 - (99*(6*b^5*c*d^5 - a*b^4*d^6)*e^2 - 22*(48*b^5*c^2*d^4 -
 7*a*b^4*c*d^5 - 6*a^2*b^3*d^6)*e*f + (480*b^5*c^3*d^3 - 64*a*b^4*c^2*d^4
- 53*a^2*b^3*c*d^5 - 48*a^3*b^2*d^6)*f^2)*x^4 - 33*(48*b^5*c^3*d^3 - 16*a*
b^4*c^2*d^4 - 9*a^2*b^3*c*d^5 - 8*a^3*b^2*d^6)*e^2 + 22*(128*b^5*c^4*d^2 -
 40*a*b^4*c^3*d^3 - 21*a^2*b^3*c^2*d^4 - 16*a^3*b^2*c*d^5 - 16*a^4*b*d^...
```

Sympy [F]

$$\int \frac{x^6\sqrt{a+bx^2}(e+fx^2)^2}{\sqrt{c+dx^2}}\,dx = \int \frac{x^6\sqrt{a+bx^2}(e+fx^2)^2}{\sqrt{c+dx^2}}\,dx$$

input
```
integrate(x**6*(b*x**2+a)**(1/2)*(f*x**2+e)**2/(d*x**2+c)**(1/2),x)
```

output
```
Integral(x**6*sqrt(a + b*x**2)*(e + f*x**2)**2/sqrt(c + d*x**2), x)
```

Maxima [F]

$$\int \frac{x^6\sqrt{a+bx^2}(e+fx^2)^2}{\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{bx^2+a}(fx^2+e)^2x^6}{\sqrt{dx^2+c}}\,dx$$

input
```
integrate(x^6*(b*x^2+a)^(1/2)*(f*x^2+e)^2/(d*x^2+c)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate(sqrt(b*x^2 + a)*(f*x^2 + e)^2*x^6/sqrt(d*x^2 + c), x)
```

Giac [F]

$$\int \frac{x^6\sqrt{a+bx^2}(e+fx^2)^2}{\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{bx^2+a}(fx^2+e)^2x^6}{\sqrt{dx^2+c}}\,dx$$

input
```
integrate(x^6*(b*x^2+a)^(1/2)*(f*x^2+e)^2/(d*x^2+c)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate(sqrt(b*x^2 + a)*(f*x^2 + e)^2*x^6/sqrt(d*x^2 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6\sqrt{a+bx^2}(e+fx^2)^2}{\sqrt{c+dx^2}}\,dx = \int \frac{x^6\,\sqrt{b\,x^2+a}\,\left(f\,x^2+e\right)^2}{\sqrt{d\,x^2+c}}\,dx$$

input
```
int((x^6*(a + b*x^2)^(1/2)*(e + f*x^2)^2)/(c + d*x^2)^(1/2),x)
```

output
```
int((x^6*(a + b*x^2)^(1/2)*(e + f*x^2)^2)/(c + d*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^6\sqrt{a+bx^2}(e+fx^2)^2}{\sqrt{c+dx^2}}\,dx = \text{too large to display}$$

input
```
int(x^6*(b*x^2+a)^(1/2)*(f*x^2+e)^2/(d*x^2+c)^(1/2),x)
```

output
```
( - 64*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**4*d**4*f**2*x - 68*sqrt(c + d*
x**2)*sqrt(a + b*x**2)*a**3*b*c*d**3*f**2*x + 176*sqrt(c + d*x**2)*sqrt(a
+ b*x**2)*a**3*b*d**4*e*f*x + 48*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**3*b*
d**4*f**2*x**3 - 81*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*b**2*c**2*d**2*
f**2*x + 198*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*b**2*c*d**3*e*f*x + 53
*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*b**2*c*d**3*f**2*x**3 - 132*sqrt(c
 + d*x**2)*sqrt(a + b*x**2)*a**2*b**2*d**4*e**2*x - 132*sqrt(c + d*x**2)*s
qrt(a + b*x**2)*a**2*b**2*d**4*e*f*x**3 - 40*sqrt(c + d*x**2)*sqrt(a + b*x
**2)*a**2*b**2*d**4*f**2*x**5 - 112*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b*
*3*c**3*d*f**2*x + 264*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**3*c**2*d**2*
e*f*x + 64*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**3*c**2*d**2*f**2*x**3 -
165*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**3*c*d**3*e**2*x - 154*sqrt(c +
d*x**2)*sqrt(a + b*x**2)*a*b**3*c*d**3*e*f*x**3 - 45*sqrt(c + d*x**2)*sqrt
(a + b*x**2)*a*b**3*c*d**3*f**2*x**5 + 99*sqrt(c + d*x**2)*sqrt(a + b*x**2
)*a*b**3*d**4*e**2*x**3 + 110*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**3*d**
4*e*f*x**5 + 35*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**3*d**4*f**2*x**7 +
640*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**4*c**4*f**2*x - 1408*sqrt(c + d*x
**2)*sqrt(a + b*x**2)*b**4*c**3*d*e*f*x - 480*sqrt(c + d*x**2)*sqrt(a + b*
x**2)*b**4*c**3*d*f**2*x**3 + 792*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**4*c
**2*d**2*e**2*x + 1056*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**4*c**2*d**2...
```

3.111 $\int \frac{x^4\sqrt{a+bx^2}\left(e+fx^2\right)^2}{\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 862

$$\int \frac{x^4\sqrt{a+bx^2}\left(e+fx^2\right)^2}{\sqrt{c+dx^2}}\,dx =$$

$$-\frac{(16a^4d^4f^2-16a^3bd^3f(3de-cf)+3a^2b^2d^2(14d^2e^2-18cdef+7c^2f^2)-8b^4c^2(21d^2e^2-36cdef+16}{315b^3d^5\sqrt{a+bx^2}}$$

$$+\frac{(8a^3d^3f^2-3a^2bd^2f(8de-3cf)-2ab^2d(7d^2e^2+15cdef-6c^2f^2)-4b^3c(21d^2e^2-36cdef+16c^2f^2)}{315b^3d^4}$$

$$-\frac{(6a^2d^2f^2+abdf(52de+7cf)-4b^2(7d^2e^2-27cdef+12c^2f^2))\,x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{315b^2d^3}$$

$$+\frac{2f(2bde-4bcf-3adf)x^5\sqrt{a+bx^2}\sqrt{c+dx^2}}{63bd^2}+\frac{x\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}\left(e+fx^2\right)^2}{9bd}$$

$$+\frac{\sqrt{a}(16a^4d^4f^2-16a^3bd^3f(3de-cf)+3a^2b^2d^2(14d^2e^2-18cdef+7c^2f^2)-8b^4c^2(21d^2e^2-36cdef+}{315b^{7/2}d^5\sqrt{a+bx^2}\sqrt{\frac{a(c-}{c(a-}}}$$

$$-\frac{a^{3/2}(8a^3d^3f^2-3a^2bd^2f(8de-3cf)+3ab^2d(7d^2e^2-10cdef+4c^2f^2)-4b^3c(21d^2e^2-36cdef+16c^2}{315b^{7/2}d^4\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
-1/315*(16*a^4*d^4*f^2-16*a^3*b*d^3*f*(-c*f+3*d*e)+3*a^2*b^2*d^2*(7*c^2*f^
2-18*c*d*e*f+14*d^2*e^2)-8*b^4*c^2*(16*c^2*f^2-36*c*d*e*f+21*d^2*e^2)+a*b^
3*c*d*(40*c^2*f^2-96*c*d*e*f+63*d^2*e^2))*x*(d*x^2+c)^(1/2)/b^3/d^5/(b*x^2
+a)^(1/2)+1/315*(8*a^3*d^3*f^2-3*a^2*b*d^2*f*(-3*c*f+8*d*e)-2*a*b^2*d*(-6*
c^2*f^2+15*c*d*e*f+7*d^2*e^2)-4*b^3*c*(16*c^2*f^2-36*c*d*e*f+21*d^2*e^2))*
x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b^3/d^4-1/315*(6*a^2*d^2*f^2+a*b*d*f*(7*
c*f+52*d*e)-4*b^2*(12*c^2*f^2-27*c*d*e*f+7*d^2*e^2))*x^3*(b*x^2+a)^(1/2)*(
d*x^2+c)^(1/2)/b^2/d^3+2/63*f*(-3*a*d*f-4*b*c*f+2*b*d*e)*x^5*(b*x^2+a)^(1/
2)*(d*x^2+c)^(1/2)/b/d^2+1/9*x*(b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^2
/b/d+1/315*a^(1/2)*(16*a^4*d^4*f^2-16*a^3*b*d^3*f*(-c*f+3*d*e)+3*a^2*b^2*d
^2*(7*c^2*f^2-18*c*d*e*f+14*d^2*e^2)-8*b^4*c^2*(16*c^2*f^2-36*c*d*e*f+21*d
^2*e^2)+a*b^3*c*d*(40*c^2*f^2-96*c*d*e*f+63*d^2*e^2))*(d*x^2+c)^(1/2)*Elli
pticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/b^(7/2)/d^5/(
b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)-1/315*a^(3/2)*(8*a^3*d^3*f^
2-3*a^2*b*d^2*f*(-3*c*f+8*d*e)+3*a*b^2*d*(4*c^2*f^2-10*c*d*e*f+7*d^2*e^2)-
4*b^3*c*(16*c^2*f^2-36*c*d*e*f+21*d^2*e^2))*(d*x^2+c)^(1/2)*InverseJacobiA
M(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/b^(7/2)/d^4/(b*x^2+a)^(1/2)
/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.24 (sec) , antiderivative size = 586, normalized size of antiderivative = 0.68

$$\int \frac{x^4\sqrt{a+bx^2}(e+fx^2)^2}{\sqrt{c+dx^2}}\,dx$$
$$= \frac{\sqrt{\frac{b}{a}}dx(a+bx^2)\left(c+dx^2\right)\left(8a^3d^3f^2-3a^2bd^2f(8de-3cf+2dfx^2)+ab^2d(12c^2f^2-cdf(30e+7fx^2)+\right.}{}$$

input
```
Integrate[(x^4*Sqrt[a + b*x^2]*(e + f*x^2)^2)/Sqrt[c + d*x^2],x]
```

output
```
(Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2)*(8*a^3*d^3*f^2 - 3*a^2*b*d^2*f*(8*d
*e - 3*c*f + 2*d*f*x^2) + a*b^2*d*(12*c^2*f^2 - c*d*f*(30*e + 7*f*x^2) + d
^2*(21*e^2 + 18*e*f*x^2 + 5*f^2*x^4)) + b^3*(-64*c^3*f^2 + 48*c^2*d*f*(3*e
 + f*x^2) - 4*c*d^2*(21*e^2 + 27*e*f*x^2 + 10*f^2*x^4) + d^3*x^2*(63*e^2 +
 90*e*f*x^2 + 35*f^2*x^4))) + I*c*(16*a^4*d^4*f^2 + 16*a^3*b*d^3*f*(-3*d*e
 + c*f) + 3*a^2*b^2*d^2*(14*d^2*e^2 - 18*c*d*e*f + 7*c^2*f^2) - 8*b^4*c^2*
(21*d^2*e^2 - 36*c*d*e*f + 16*c^2*f^2) + a*b^3*c*d*(63*d^2*e^2 - 96*c*d*e*
f + 40*c^2*f^2))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSi
nh[Sqrt[b/a]*x], (a*d)/(b*c)] + I*c*(b*c - a*d)*(8*a^3*d^3*f^2 - 3*a^2*b*d
^2*f*(8*d*e - 5*c*f) + 3*a*b^2*d*(7*d^2*e^2 - 16*c*d*e*f + 8*c^2*f^2) + 8*
b^3*c*(21*d^2*e^2 - 36*c*d*e*f + 16*c^2*f^2))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 +
 (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(315*b^3*Sqrt[
b/a]*d^5*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (warning: unable to verify)

Time = 1.96 (sec) , antiderivative size = 1120, normalized size of antiderivative = 1.30, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {448, 443, 444, 444, 27, 406, 320, 388, 313, 444, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4\sqrt{a+bx^2}\left(e+fx^2\right)^2}{\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \ 448$$

$$\frac{f\int \frac{x^6\sqrt{bx^2+a}(fx^2+e)}{\sqrt{dx^2+c}}dx}{e^2}+e\int \frac{x^4\sqrt{bx^2+a}(fx^2+e)}{\sqrt{dx^2+c}}dx$$

$$\Big\downarrow \ 443$$

$$\frac{f\left(\frac{\int \frac{x^6\left((9bde-8bcf+adf)x^2+a(9de-7cf)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{9d}+\frac{fx^7\sqrt{a+bx^2}\sqrt{c+dx^2}}{9d}\right)}{e^2}+e\left(\frac{\int \frac{x^4((7bde-6bcf+adf)x^2+a(7de-5cf))}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{7d}+\frac{fx^5\sqrt{a+bx^2}\sqrt{c+dx^2}}{7d}\right)$$

$$\Big\downarrow \ 444$$

$$\frac{f\left(\frac{\frac{x^5\sqrt{a+bx^2}\sqrt{c+dx^2}(adf-8bcf+9bde)}{7bd}-\frac{\int \frac{x^4\left(\left(6c(9de-8cf)b^2-ad(9de-7cf)b+6a^2d^2f\right)x^2+5ac(9bde-8bcf+adf)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{7bd}}{9d}+\frac{fx^7\sqrt{a+bx^2}\sqrt{c+dx^2}}{9d}\right)}{e^2}+$$

$$e\left(\frac{\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}(adf-6bcf+7bde)}{5bd}-\frac{\int \frac{x^2\left(\left(4c(7de-6cf)b^2-ad(7de-5cf)b+4a^2d^2f\right)x^2+3ac(7bde-6bcf+adf)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{5bd}}{7d}+\frac{fx^5\sqrt{a+bx^2}\sqrt{c-\ldots}}{7d}\right.$$

$\Big\downarrow$ 444

$$\frac{f\left(\frac{\frac{x^5\sqrt{a+bx^2}\sqrt{c+dx^2}(adf-8bcf+9bde)}{7bd}-\frac{\frac{1}{5}x^3\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{6a^2df}{b}+7acf-9ade-\frac{48bc^2f}{d}+54bce\right)-\frac{\int\frac{3x^2\left(\left(8c^2(9de-8cf)b^3-3acd(5de-4cf)b^2-3a^2d^2\ldots\right.\right.}{\ldots}}{\ldots}}{7bd}}{9d}\ldots\right.}{e^2}$$

$$e\left(\frac{\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}(adf-6bcf+7bde)}{5bd}-\frac{\frac{1}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{4a^2df}{b}+5acf-7ade-\frac{24bc^2f}{d}+28bce\right)-\frac{\int\frac{\left(8c^2(7de-6cf)b^3-acd(21de-16cf)b^2-\ldots\right.}{\ldots}}{\ldots}}{5bd}}{7d}\ldots\right.$$

$\Big\downarrow$ 27

$$\frac{f\left(\frac{\frac{x^5\sqrt{a+bx^2}\sqrt{c+dx^2}(adf-8bcf+9bde)}{7bd}-\frac{\frac{1}{5}x^3\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{6a^2df}{b}+7acf-9ade-\frac{48bc^2f}{d}+54bce\right)-\frac{3\int\frac{x^2\left(\left(8c^2(9de-8cf)b^3-3acd(5de-4cf)b^2-3a^2d^2\ldots\right.\right.}{\ldots}}{\ldots}}{7bd}}{9d}\ldots\right.}{e^2}$$

$$e\left(\frac{\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}(adf-6bcf+7bde)}{5bd}-\frac{\frac{1}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{4a^2df}{b}+5acf-7ade-\frac{24bc^2f}{d}+28bce\right)-\frac{\int\frac{\left(8c^2(7de-6cf)b^3-acd(21de-16cf)b^2-\ldots\right.}{\ldots}}{\ldots}}{5bd}}{7d}\ldots\right.$$

$\Big\downarrow$ 406

$$\frac{f\left(\frac{\frac{x^5\sqrt{a+bx^2}\sqrt{c+dx^2}(adf-8bcf+9bde)}{7bd}-\frac{\frac{1}{5}x^3\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{6a^2df}{b}+7acf-9ade-\frac{48bc^2f}{d}+54bce\right)-\frac{3\int \frac{x^2\left(\left(8c^2(9de-8cf)b^3-3acd(5de-4cf)b^2-3a^2d^2\right.\right.}{}}{7bd}}{9d}\right.}{e^2}$$

$$e\left(\frac{\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}(adf-6bcf+7bde)}{5bd}-\frac{\frac{1}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{4a^2df}{b}+5acf-7ade-\frac{24bc^2f}{d}+28bce\right)-\frac{ac\left(4a^2d^2f-abd(7de-5cf)+4b^2c(7de-6c\right.}{}}{7d}\right.$$

$\downarrow$ 320

$$e\left(\frac{\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}(adf-6bcf+7bde)}{5bd}-\frac{\frac{1}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{4a^2df}{b}+5acf-7ade-\frac{24bc^2f}{d}+28bce\right)-\frac{\left(8a^3d^3f-a^2bd^2(14de-9cf)-ab^2cd(21de\right.}{}}{7d}\right.$$

$$\frac{f\left(\frac{\frac{x^5\sqrt{a+bx^2}\sqrt{c+dx^2}(adf-8bcf+9bde)}{7bd}-\frac{\frac{1}{5}x^3\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{6a^2df}{b}+7acf-9ade-\frac{48bc^2f}{d}+54bce\right)-\frac{3\int \frac{x^2\left(\left(8c^2(9de-8cf)b^3-3acd(5de-4cf)b^2-3a^2d^2\right.\right.}{}}{7bd}}{9d}\right.}{e^2}$$

$\downarrow$ 388

$$\left(8a^3d^3f-a^2bd^2(14de-9cf)-ab^2cd(21de \cdots\right.$$

$$e\left(\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}(adf-6bcf+7bde)}{5bd}-\frac{\frac{1}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{4a^2df}{b}+5acf-7ade-\frac{24bc^2f}{d}+28bce\right)-\cdots}{7\cdots}\right.$$

$$f\left(\frac{x^5\sqrt{a+bx^2}\sqrt{c+dx^2}(adf-8bcf+9bde)}{7bd}-\frac{\frac{1}{5}x^3\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{6a^2df}{b}+7acf-9ade-\frac{48bc^2f}{d}+54bce\right)-\frac{3\int\frac{x^2\left(\left(8c^2(9de-8cf)b^3-3acd(5de-4cf)b^2-3a^2d^2\cdots\right.\right.}{\cdots}}{7bd}}{9d}\right.$$

$$\overline{\qquad\qquad e^2 \qquad\qquad}$$

$\downarrow$ 313

$$\left(4c\cdots\right.$$

$$e\left(\frac{f\sqrt{bx^2+a}\sqrt{dx^2+c}x^5}{7d}+\frac{(7bde-6bcf+adf)x^3\sqrt{bx^2+a}\sqrt{dx^2+c}}{5bd}-\frac{\frac{1}{3}\left(\frac{4dfa^2}{b}-7dea+5cfa+28bce-\frac{24bc^2f}{d}\right)x\sqrt{bx^2+a}\sqrt{dx^2+c}-\cdots}{\cdots}\right.$$

$$f\left(\frac{f\sqrt{bx^2+a}\sqrt{dx^2+c}x^7}{9d}+\frac{(9bde-8bcf+adf)x^5\sqrt{bx^2+a}\sqrt{dx^2+c}}{7bd}-\frac{\frac{1}{5}\left(\frac{6dfa^2}{b}-9dea+7cfa+54bce-\frac{48bc^2f}{d}\right)x^3\sqrt{bx^2+a}\sqrt{dx^2+c}-\frac{3\int\frac{x^2\left(\left(8c^2(9de-8cf)b\cdots\right.\right.}{\cdots}}{\cdots}}{9d}\right.$$

$$\overline{\qquad\qquad e^2 \qquad\qquad}$$

$\downarrow$ 444

$$e\left(\frac{f\sqrt{bx^2+a}\sqrt{dx^2+cx^5}}{7d}+\frac{\frac{(7bde-6bcf+adf)x^3\sqrt{bx^2+a}\sqrt{dx^2+c}}{5bd}-\frac{\frac{1}{3}\left(\frac{4dfa^2}{b}-7dea+5cfa+28bce-\frac{24bc^2f}{d}\right)x\sqrt{bx^2+a}\sqrt{dx^2+c}-\frac{(4c\cdots}{\cdots}}{\cdots}}{\cdots}\right.$$

$$f\left(\frac{f\sqrt{bx^2+a}\sqrt{dx^2+cx^7}}{9d}+\frac{\frac{(9bde-8bcf+adf)x^5\sqrt{bx^2+a}\sqrt{dx^2+c}}{7bd}-\frac{\frac{1}{5}\left(\frac{6dfa^2}{b}-9dea+7cfa+54bce-\frac{48bc^2f}{d}\right)x^3\sqrt{bx^2+a}\sqrt{dx^2+c}-\frac{3\left(\frac{\left(8c^2(9de-8cf)b^3-3\cdots}{\cdots}\right.}{\cdots}}{\cdots}}{\cdots}\right.$$

$$\Big\downarrow \; 406$$

$$e\left(\frac{f\sqrt{bx^2+a}\sqrt{dx^2+cx^5}}{7d}+\frac{\frac{(7bde-6bcf+adf)x^3\sqrt{bx^2+a}\sqrt{dx^2+c}}{5bd}-\frac{\frac{1}{3}\left(\frac{4dfa^2}{b}-7dea+5cfa+28bce-\frac{24bc^2f}{d}\right)x\sqrt{bx^2+a}\sqrt{dx^2+c}-\frac{(4c\cdots}{\cdots}}{\cdots}}{\cdots}\right.$$

$$f\left(\frac{f\sqrt{bx^2+a}\sqrt{dx^2+cx^7}}{9d}+\frac{\frac{(9bde-8bcf+adf)x^5\sqrt{bx^2+a}\sqrt{dx^2+c}}{7bd}-\frac{\frac{1}{5}\left(\frac{6dfa^2}{b}-9dea+7cfa+54bce-\frac{48bc^2f}{d}\right)x^3\sqrt{bx^2+a}\sqrt{dx^2+c}-\frac{3\left(\frac{\left(8c^2(9de-8cf)b^3-3\cdots}{\cdots}\right.}{\cdots}}{\cdots}}{\cdots}\right.$$

$$\Big\downarrow \; 320$$

$$e\left(\frac{f\sqrt{bx^2+a}\sqrt{dx^2+c}\,x^5}{7d}+\frac{\frac{(7bde-6bcf+adf)x^3\sqrt{bx^2+a}\sqrt{dx^2+c}}{5bd}-\frac{\frac{1}{3}\left(\frac{4dfa^2}{b}-7dea+5cfa+28bce-\frac{24bc^2f}{d}\right)x\sqrt{bx^2+a}\sqrt{dx^2+c}-\frac{\left(4c\cdots\right.}{\cdots}}{\cdots}}{\cdots}\right.$$

$$f\left(\frac{f\sqrt{bx^2+a}\sqrt{dx^2+c}\,x^7}{9d}+\frac{\frac{(9bde-8bcf+adf)x^5\sqrt{bx^2+a}\sqrt{dx^2+c}}{7bd}-\frac{\frac{1}{5}\left(\frac{6dfa^2}{b}-9dea+7cfa+54bce-\frac{48bc^2f}{d}\right)x^3\sqrt{bx^2+a}\sqrt{dx^2+c}-\frac{3\left(\frac{\left(8c^2(9de-8cf)b^3-3\cdots\right.}{\cdots}\right.}{\cdots}}{\cdots}}{\cdots}\right.$$

↓ 388

$$e\left(\frac{f\sqrt{bx^2+a}\sqrt{dx^2+c}\,x^5}{7d}+\frac{(7bde-6bcf+adf)x^3\sqrt{bx^2+a}\sqrt{dx^2+c}}{5bd}-\frac{\frac{1}{3}\left(\frac{4dfa^2}{b}-7dea+5cfa+28bce-\frac{24bc^2f}{d}\right)x\sqrt{bx^2+a}\sqrt{dx^2+c}-\frac{\left(4c\ \cdots\right.}{\cdots}}{\cdots}\cdots\right.$$

$$f\left(\frac{f\sqrt{bx^2+a}\sqrt{dx^2+c}\,x^7}{9d}+\frac{(9bde-8bcf+adf)x^5\sqrt{bx^2+a}\sqrt{dx^2+c}}{7bd}-\frac{\frac{1}{5}\left(\frac{6dfa^2}{b}-9dea+7cfa+54bce-\frac{48bc^2f}{d}\right)x^3\sqrt{bx^2+a}\sqrt{dx^2+c}-\frac{3\left(\frac{\left(8c^2(9de-8cf)b^3-3\ \cdots\right.}{\cdots}\right.}{\cdots}}{\cdots}\cdots\right.$$

↓ 313

$$e\left(\frac{f\sqrt{bx^2+a}\sqrt{dx^2+c}\,x^5}{7d}+\frac{(7bde-6bcf+adf)x^3\sqrt{bx^2+a}\sqrt{dx^2+c}}{5bd}-\frac{\frac{1}{3}\left(\frac{4dfa^2}{b}-7dea+5cfa+28bce-\frac{24bc^2f}{d}\right)x\sqrt{bx^2+a}\sqrt{dx^2+c}-\frac{\left(4c\cdots\right.}{\cdots}}{\cdots}\right.$$

$$f\left(\frac{f\sqrt{bx^2+a}\sqrt{dx^2+c}\,x^7}{9d}+\frac{(9bde-8bcf+adf)x^5\sqrt{bx^2+a}\sqrt{dx^2+c}}{7bd}-\frac{\frac{1}{5}\left(\frac{6dfa^2}{b}-9dea+7cfa+54bce-\frac{48bc^2f}{d}\right)x^3\sqrt{bx^2+a}\sqrt{dx^2+c}-\frac{3\left(\frac{\left(8c^2(9de-8cf)b^3-3\cdots\right.}{\cdots}\right.}{\cdots}}{\cdots}\right.$$

input
```
Int[(x^4*Sqrt[a + b*x^2]*(e + f*x^2)^2)/Sqrt[c + d*x^2],x]
```

output
```
e*((f*x^5*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(7*d) + (((7*b*d*e - 6*b*c*f +
a*d*f)*x^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(5*b*d) - (((28*b*c*e - 7*a*d*
e + 5*a*c*f - (24*b*c^2*f)/d + (4*a^2*d*f)/b)*x*Sqrt[a + b*x^2]*Sqrt[c + d
*x^2])/3 - ((8*a^3*d^3*f - a*b^2*c*d*(21*d*e - 16*c*f) - a^2*b*d^2*(14*d*e
 - 9*c*f) + 8*b^3*c^2*(7*d*e - 6*c*f))*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*
x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1
- (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c +
d*x^2])) + (c^(3/2)*(4*a^2*d^2*f + 4*b^2*c*(7*d*e - 6*c*f) - a*b*d*(7*d*e
- 5*c*f))*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)
/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))/
(3*b*d))/(5*b*d))/(7*d)) + (f*((f*x^7*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(9*
d) + (((9*b*d*e - 8*b*c*f + a*d*f)*x^5*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(7
*b*d) - (((54*b*c*e - 9*a*d*e + 7*a*c*f - (48*b*c^2*f)/d + (6*a^2*d*f)/b)*
x^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/5 - (3*(((8*a^3*d^3*f + 8*b^3*c^2*(9*
d*e - 8*c*f) - 3*a*b^2*c*d*(5*d*e - 4*c*f) - 3*a^2*b*d^2*(4*d*e - 3*c*f))*
x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(3*b*d) - ((16*a^4*d^4*f + 16*b^4*c^3*(
9*d*e - 8*c*f) - 3*a^2*b^2*c*d^2*(9*d*e - 7*c*f) - 8*a*b^3*c^2*d*(6*d*e -
5*c*f) - 8*a^3*b*d^3*(3*d*e - 2*c*f))*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x
^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 -
 (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c ...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 443
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*(g*x)^(m + 1)*(a + b*x^2)^(p
 + 1)*((c + d*x^2)^q/(b*g*(m + 2*(p + q + 1) + 1))), x] + Simp[1/(b*(m + 2*
(p + q + 1) + 1))   Int[(g*x)^m*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[c*((
b*e - a*f)*(m + 1) + b*e*2*(p + q + 1)) + (d*(b*e - a*f)*(m + 1) + f*2*q*(b
*c - a*d) + b*e*d*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f
, g, m, p}, x] && GtQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^2, c + d*x^
2])
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

rule 448
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[e   Int[(g*x)^m*(a + b*x
^2)^p*(c + d*x^2)^q*(e + f*x^2)^(r - 1), x], x] + Simp[f/e^2   Int[(g*x)^(m
 + 2)*(a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^(r - 1), x], x] /; FreeQ[{a,
b, c, d, e, f, g, m, p, q}, x] && IGtQ[r, 0]
```

Maple [A] (verified)

Time = 19.48 (sec) , antiderivative size = 963, normalized size of antiderivative = 1.12

method	result
elliptic	$\frac{\sqrt{(b\,x^2+a)(x^2d+c)}\left(\frac{f^2x^7\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{9d}+\frac{\left(a\,f^2+2bef-\frac{f^2(8ad+8bc)}{9d}\right)x^5\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{7bd}+\frac{\left(2aef+b\,e^2-\frac{7ac\,f^2}{9d}-\frac{\left(a\,f \right.}{}\right.}{}\right.}{}$
risch	Expression too large to display
default	Expression too large to display

input
```
int(x^4*(b*x^2+a)^(1/2)*(f*x^2+e)^2/(d*x^2+c)^(1/2),x,method=_RETURNVERBOS
E)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/9*f^2/d*x^7
*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+1/7*(a*f^2+2*b*e*f-1/9*f^2/d*(8*a*d+8
*b*c))/b/d*x^5*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+1/5*(2*a*e*f+b*e^2-7/9*
a*c/d*f^2-1/7*(a*f^2+2*b*e*f-1/9*f^2/d*(8*a*d+8*b*c))/b/d*(6*a*d+6*b*c))/b
/d*x^3*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+1/3*(a*e^2-5/7*(a*f^2+2*b*e*f-1
/9*f^2/d*(8*a*d+8*b*c))/b/d*a*c-1/5*(2*a*e*f+b*e^2-7/9*a*c/d*f^2-1/7*(a*f^
2+2*b*e*f-1/9*f^2/d*(8*a*d+8*b*c))/b/d*(6*a*d+6*b*c))/b/d*(4*a*d+4*b*c))/b
/d*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)-1/3*(a*e^2-5/7*(a*f^2+2*b*e*f-1/9
*f^2/d*(8*a*d+8*b*c))/b/d*a*c-1/5*(2*a*e*f+b*e^2-7/9*a*c/d*f^2-1/7*(a*f^2+
2*b*e*f-1/9*f^2/d*(8*a*d+8*b*c))/b/d*(6*a*d+6*b*c))/b/d*(4*a*d+4*b*c))/b/d
*a*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c
*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-(-3/5*(
2*a*e*f+b*e^2-7/9*a*c/d*f^2-1/7*(a*f^2+2*b*e*f-1/9*f^2/d*(8*a*d+8*b*c))/b/
d*(6*a*d+6*b*c))/b/d*a*c-1/3*(a*e^2-5/7*(a*f^2+2*b*e*f-1/9*f^2/d*(8*a*d+8*
b*c))/b/d*a*c-1/5*(2*a*e*f+b*e^2-7/9*a*c/d*f^2-1/7*(a*f^2+2*b*e*f-1/9*f^2/
d*(8*a*d+8*b*c))/b/d*(6*a*d+6*b*c))/b/d*(4*a*d+4*b*c))/b/d*(2*a*d+2*b*c))*
c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^
2+a*c)^(1/2)/d*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-Ellipti
cE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 886, normalized size of antiderivative = 1.03

$$\int \frac{x^4\sqrt{a+bx^2}(e+fx^2)^2}{\sqrt{c+dx^2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^4*(b*x^2+a)^(1/2)*(f*x^2+e)^2/(d*x^2+c)^(1/2),x, algorithm="fr
icas")
```

output
```
-1/315*((21*(8*b^4*c^3*d^2 - 3*a*b^3*c^2*d^3 - 2*a^2*b^2*c*d^4)*e^2 - 6*(4
8*b^4*c^4*d - 16*a*b^3*c^3*d^2 - 9*a^2*b^2*c^2*d^3 - 8*a^3*b*c*d^4)*e*f +
(128*b^4*c^5 - 40*a*b^3*c^4*d - 21*a^2*b^2*c^3*d^2 - 16*a^3*b*c^2*d^3 - 16
*a^4*c*d^4)*f^2)*sqrt(b*d)*x*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), a
*d/(b*c)) - (21*(8*b^4*c^3*d^2 - 3*a*b^3*c^2*d^3 - a^2*b^2*d^5 - 2*(a^2*b^
2 - 2*a*b^3)*c*d^4)*e^2 - 6*(48*b^4*c^4*d - 16*a*b^3*c^3*d^2 - 4*a^3*b*d^5
 - 3*(3*a^2*b^2 - 8*a*b^3)*c^2*d^3 - (8*a^3*b + 5*a^2*b^2)*c*d^4)*e*f + (1
28*b^4*c^5 - 40*a*b^3*c^4*d - 8*a^4*d^5 - (21*a^2*b^2 - 64*a*b^3)*c^3*d^2
- 4*(4*a^3*b + 3*a^2*b^2)*c^2*d^3 - (16*a^4 + 9*a^3*b)*c*d^4)*f^2)*sqrt(b*
d)*x*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - (35*b^4*d^5*
f^2*x^8 + 5*(18*b^4*d^5*e*f - (8*b^4*c*d^4 - a*b^3*d^5)*f^2)*x^6 + (63*b^4
*d^5*e^2 - 18*(6*b^4*c*d^4 - a*b^3*d^5)*e*f + (48*b^4*c^2*d^3 - 7*a*b^3*c*
d^4 - 6*a^2*b^2*d^5)*f^2)*x^4 + 21*(8*b^4*c^2*d^3 - 3*a*b^3*c*d^4 - 2*a^2*
b^2*d^5)*e^2 - 6*(48*b^4*c^3*d^2 - 16*a*b^3*c^2*d^3 - 9*a^2*b^2*c*d^4 - 8*
a^3*b*d^5)*e*f + (128*b^4*c^4*d - 40*a*b^3*c^3*d^2 - 21*a^2*b^2*c^2*d^3 -
16*a^3*b*c*d^4 - 16*a^4*d^5)*f^2 - (21*(4*b^4*c*d^4 - a*b^3*d^5)*e^2 - 6*(
24*b^4*c^2*d^3 - 5*a*b^3*c*d^4 - 4*a^2*b^2*d^5)*e*f + (64*b^4*c^3*d^2 - 12
*a*b^3*c^2*d^3 - 9*a^2*b^2*c*d^4 - 8*a^3*b*d^5)*f^2)*x^2)*sqrt(b*x^2 + a)*
sqrt(d*x^2 + c))/(b^4*d^6*x)
```

Sympy [F]

$$\int \frac{x^4\sqrt{a+bx^2}(e+fx^2)^2}{\sqrt{c+dx^2}}\,dx=\int \frac{x^4\sqrt{a+bx^2}(e+fx^2)^2}{\sqrt{c+dx^2}}\,dx$$

input
```
integrate(x**4*(b*x**2+a)**(1/2)*(f*x**2+e)**2/(d*x**2+c)**(1/2),x)
```

output
```
Integral(x**4*sqrt(a + b*x**2)*(e + f*x**2)**2/sqrt(c + d*x**2), x)
```

Maxima [F]

$$\int \frac{x^4\sqrt{a+bx^2}(e+fx^2)^2}{\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{bx^2+a}(fx^2+e)^2x^4}{\sqrt{dx^2+c}}\,dx$$

input
```
integrate(x^4*(b*x^2+a)^(1/2)*(f*x^2+e)^2/(d*x^2+c)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate(sqrt(b*x^2 + a)*(f*x^2 + e)^2*x^4/sqrt(d*x^2 + c), x)
```

Giac [F]

$$\int \frac{x^4\sqrt{a+bx^2}(e+fx^2)^2}{\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{bx^2+a}(fx^2+e)^2x^4}{\sqrt{dx^2+c}}\,dx$$

input
```
integrate(x^4*(b*x^2+a)^(1/2)*(f*x^2+e)^2/(d*x^2+c)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate(sqrt(b*x^2 + a)*(f*x^2 + e)^2*x^4/sqrt(d*x^2 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4\sqrt{a+bx^2}(e+fx^2)^2}{\sqrt{c+dx^2}}\,dx = \int \frac{x^4\,\sqrt{b\,x^2+a}\,{\left(f\,x^2+e\right)}^2}{\sqrt{d\,x^2+c}}\,dx$$

input
```
int((x^4*(a + b*x^2)^(1/2)*(e + f*x^2)^2)/(c + d*x^2)^(1/2),x)
```

output
```
int((x^4*(a + b*x^2)^(1/2)*(e + f*x^2)^2)/(c + d*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^4\sqrt{a+bx^2}(e+fx^2)^2}{\sqrt{c+dx^2}}\,dx = \text{too large to display}$$

input
```
int(x^4*(b*x^2+a)^(1/2)*(f*x^2+e)^2/(d*x^2+c)^(1/2),x)
```

output
```
(8*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**3*d**3*f**2*x + 9*sqrt(c + d*x**2)
*sqrt(a + b*x**2)*a**2*b*c*d**2*f**2*x - 24*sqrt(c + d*x**2)*sqrt(a + b*x*
*2)*a**2*b*d**3*e*f*x - 6*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*b*d**3*f*
*2*x**3 + 12*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**2*c**2*d*f**2*x - 30*s
qrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**2*c*d**2*e*f*x - 7*sqrt(c + d*x**2)*
sqrt(a + b*x**2)*a*b**2*c*d**2*f**2*x**3 + 21*sqrt(c + d*x**2)*sqrt(a + b*
x**2)*a*b**2*d**3*e**2*x + 18*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**2*d**
3*e*f*x**3 + 5*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**2*d**3*f**2*x**5 - 6
4*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**3*c**3*f**2*x + 144*sqrt(c + d*x**2
)*sqrt(a + b*x**2)*b**3*c**2*d*e*f*x + 48*sqrt(c + d*x**2)*sqrt(a + b*x**2
)*b**3*c**2*d*f**2*x**3 - 84*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**3*c*d**2
*e**2*x - 108*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**3*c*d**2*e*f*x**3 - 40*
sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**3*c*d**2*f**2*x**5 + 63*sqrt(c + d*x*
*2)*sqrt(a + b*x**2)*b**3*d**3*e**2*x**3 + 90*sqrt(c + d*x**2)*sqrt(a + b*
x**2)*b**3*d**3*e*f*x**5 + 35*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**3*d**3*
f**2*x**7 - 16*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**
2 + b*c*x**2 + b*d*x**4),x)*a**4*d**4*f**2 - 16*int((sqrt(c + d*x**2)*sqrt
(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a**3*b*c*d**3
*f**2 + 48*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 +
b*c*x**2 + b*d*x**4),x)*a**3*b*d**4*e*f - 21*int((sqrt(c + d*x**2)*sqrt...
```

3.112 $\int \frac{x^2\sqrt{a+bx^2}\left(e+fx^2\right)^2}{\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 621

$$\int \frac{x^2\sqrt{a+bx^2}(e+fx^2)^2}{\sqrt{c+dx^2}}\,dx$$
$$= \frac{(8a^3d^3f^2 - a^2bd^2f(28de-9cf) + ab^2d(35d^2e^2-42cdef+16c^2f^2) - 2b^3c(35d^2e^2-56cdef+24c^2f^2))}{105b^2d^4\sqrt{a+bx^2}}$$
$$- \frac{(4a^2d^2f^2 - abdf(14de-5cf) - b^2(35d^2e^2-56cdef+24c^2f^2))\,x\sqrt{a+bx^2}\sqrt{c+dx^2}}{105b^2d^3}$$
$$+ \frac{f(14bde-6bcf+adf)x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{35bd^2} + \frac{f^2x^5\sqrt{a+bx^2}\sqrt{c+dx^2}}{7d}$$
$$- \frac{\sqrt{a}(8a^3d^3f^2 - a^2bd^2f(28de-9cf) + ab^2d(35d^2e^2-42cdef+16c^2f^2) - 2b^3c(35d^2e^2-56cdef+24c}{105b^{5/2}d^4\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$
$$+ \frac{a^{3/2}(4a^2d^2f^2 - abdf(14de-5cf) - b^2(35d^2e^2-56cdef+24c^2f^2))\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\right.}{105b^{5/2}d^3\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output

```
1/105*(8*a^3*d^3*f^2-a^2*b*d^2*f*(-9*c*f+28*d*e)+a*b^2*d*(16*c^2*f^2-42*c*
d*e*f+35*d^2*e^2)-2*b^3*c*(24*c^2*f^2-56*c*d*e*f+35*d^2*e^2))*x*(d*x^2+c)^
(1/2)/b^2/d^4/(b*x^2+a)^(1/2)-1/105*(4*a^2*d^2*f^2-a*b*d*f*(-5*c*f+14*d*e)
-b^2*(24*c^2*f^2-56*c*d*e*f+35*d^2*e^2))*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)
/b^2/d^3+1/35*f*(a*d*f-6*b*c*f+14*b*d*e)*x^3*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/
2)/b/d^2+1/7*f^2*x^5*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/d-1/105*a^(1/2)*(8*a^
3*d^3*f^2-a^2*b*d^2*f*(-9*c*f+28*d*e)+a*b^2*d*(16*c^2*f^2-42*c*d*e*f+35*d^
2*e^2)-2*b^3*c*(24*c^2*f^2-56*c*d*e*f+35*d^2*e^2))*(d*x^2+c)^(1/2)*Ellipti
cE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/b^(5/2)/d^4/(b*x
^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)+1/105*a^(3/2)*(4*a^2*d^2*f^2-a
*b*d*f*(-5*c*f+14*d*e)-b^2*(24*c^2*f^2-56*c*d*e*f+35*d^2*e^2))*(d*x^2+c)^(
1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/b^(5/2)/
d^3/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.98 (sec) , antiderivative size = 423, normalized size of antiderivative = 0.68

$$\int \frac{x^2\sqrt{a+bx^2}(e+fx^2)^2}{\sqrt{c+dx^2}}\,dx = \frac{-\sqrt{\frac{b}{a}}dx(a+bx^2)\left(c+dx^2\right)\left(4a^2d^2f^2+abdf(-14de+5cf-3dfx^2)-b^2(24c^2f^2-2cdf(28e+9fx^2)+\right.}{}$$

input

```
Integrate[(x^2*Sqrt[a + b*x^2]*(e + f*x^2)^2)/Sqrt[c + d*x^2],x]
```

output

```
(-(Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2)*(4*a^2*d^2*f^2 + a*b*d*f*(-14*d*e
 + 5*c*f - 3*d*f*x^2) - b^2*(24*c^2*f^2 - 2*c*d*f*(28*e + 9*f*x^2) + d^2*(
35*e^2 + 42*e*f*x^2 + 15*f^2*x^4)))) - I*c*(8*a^3*d^3*f^2 + a^2*b*d^2*f*(-
28*d*e + 9*c*f) + a*b^2*d*(35*d^2*e^2 - 42*c*d*e*f + 16*c^2*f^2) - 2*b^3*c
*(35*d^2*e^2 - 56*c*d*e*f + 24*c^2*f^2))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x
^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (2*I)*c*(-(b*c) +
a*d)*(2*a^2*d^2*f^2 + a*b*d*f*(-7*d*e + 4*c*f) + b^2*(35*d^2*e^2 - 56*c*d*
e*f + 24*c^2*f^2))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*Arc
Sinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(105*a^2*(b/a)^(5/2)*d^4*Sqrt[a + b*x^2]*
Sqrt[c + d*x^2])
```

Rubi [A] (warning: unable to verify)

Time = 1.43 (sec) , antiderivative size = 843, normalized size of antiderivative = 1.36, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.343, Rules used = {448, 443, 444, 406, 320, 388, 313, 444, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\sqrt{a+bx^2}\left(e+fx^2\right)^2}{\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \text{448}$$

$$\frac{f\int \frac{x^4\sqrt{bx^2+a}(fx^2+e)}{\sqrt{dx^2+c}}dx}{e^2}+e\int \frac{x^2\sqrt{bx^2+a}\left(fx^2+e\right)}{\sqrt{dx^2+c}}dx$$

$$\Big\downarrow \text{443}$$

$$\frac{f\left(\frac{\int \frac{x^4\left((7bde-6bcf+adf)x^2+a(7de-5cf)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{7d}+\frac{fx^5\sqrt{a+bx^2}\sqrt{c+dx^2}}{7d}\right)}{e^2}+e\left(\frac{\int \frac{x^2((5bde-4bcf+adf)x^2+a(5de-3cf))}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{5d}+\frac{fx^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5d}\right)$$

$$\Big\downarrow \text{444}$$

$$\frac{f\left(\frac{\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}(adf-6bcf+7bde)}{5bd}-\frac{\int \frac{x^2\left(\left(4c(7de-6cf)b^2-ad(7de-5cf)b+4a^2d^2f\right)x^2+3ac(7bde-6bcf+adf)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{5bd}}{7d}+\frac{fx^5\sqrt{a+bx^2}\sqrt{c+dx^2}}{7d}\right)}{e^2}+e\left(\frac{\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(adf-4bcf+5bde)}{3bd}-\frac{\int \frac{\left(2c(5de-4cf)b^2-ad(5de-3cf)b+2a^2d^2f\right)x^2+ac(5bde-4bcf+adf)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3bd}}{5d}+\frac{fx^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5d}\right.$$

$$\Big\downarrow \text{406}$$

$$\frac{f\left(\frac{\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}(adf-6bcf+7bde)}{5bd}-\frac{\int \frac{x^2\left(\left(4c(7de-6cf)b^2-ad(7de-5cf)b+4a^2d^2f\right)x^2+3ac(7bde-6bcf+adf)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{5bd}}{7d}+\frac{fx^5\sqrt{a+bx^2}\sqrt{c+dx^2}}{7d}\right)}{e^2}+$$

$$e\left(\frac{\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(adf-4bcf+5bde)}{3bd}-\frac{\left(2a^2d^2f-abd(5de-3cf)+2b^2c(5de-4cf)\right)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+ac(adf-4bcf+5bde)\int \frac{1}{\sqrt{bx^2+a}\sqrt{d}}}{3bd}}{5d}\right.$$

$\Big\downarrow$ 320

$$e\left(\frac{\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(adf-4bcf+5bde)}{3bd}-\frac{\left(2a^2d^2f-abd(5de-3cf)+2b^2c(5de-4cf)\right)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}(adf-4bcf+5bde)\text{Ellipti}}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a}{a(c}}}}{3bd}}{5d}\right.$$

$$\frac{f\left(\frac{\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}(adf-6bcf+7bde)}{5bd}-\frac{\int \frac{x^2\left(\left(4c(7de-6cf)b^2-ad(7de-5cf)b+4a^2d^2f\right)x^2+3ac(7bde-6bcf+adf)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{5bd}}{7d}+\frac{fx^5\sqrt{a+bx^2}\sqrt{c+dx^2}}{7d}\right)}{e^2}$$

$\Big\downarrow$ 388

$$e\left(\frac{\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(adf-4bcf+5bde)}{3bd}-\frac{\left(2a^2d^2f-abd(5de-3cf)+2b^2c(5de-4cf)\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(adf-4bcf}{\sqrt{d}}}{3bd}}{5d}\right.$$

$$\frac{f\left(\frac{\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}(adf-6bcf+7bde)}{5bd}-\frac{\int \frac{x^2\left(\left(4c(7de-6cf)b^2-ad(7de-5cf)b+4a^2d^2f\right)x^2+3ac(7bde-6bcf+adf)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{5bd}}{7d}+\frac{fx^5\sqrt{a+bx^2}\sqrt{c+dx^2}}{7d}\right)}{e^2}$$

$\Big\downarrow$ 313

$$\frac{f\left(\frac{\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}(adf-6bcf+7bde)}{5bd}-\frac{\int \frac{x^2\left(\left(4c(7de-6cf)b^2-ad(7de-5cf)b+4a^2d^2f\right)x^2+3ac(7bde-6bcf+adf)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{5bd}}{7d}+\frac{fx^5\sqrt{a+bx^2}\sqrt{c+dx^2}}{7d}\right)}{e^2}+$$

$$e\left(\frac{\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(adf-4bcf+5bde)}{3bd}-\frac{\left(2a^2d^2f-abd(5de-3cf)+2b^2c(5de-4cf)\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{c^{3/2}}{}}{3bd}}{5d}\right.$$

$\downarrow$ 444

$$\frac{f\left(\frac{\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}(adf-6bcf+7bde)}{5bd}-\frac{\frac{1}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{4a^2df}{b}+5acf-7ade-\frac{24bc^2f}{d}+28bce\right)-\int\frac{\left(8c^2(7de-6cf)b^3-acd(21de-16cf)b^2-a^2d^2(14de-\right.}{}}{5bd}}{7d}\right.}{e^2}$$

$$e\left(\frac{\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(adf-4bcf+5bde)}{3bd}-\frac{\left(2a^2d^2f-abd(5de-3cf)+2b^2c(5de-4cf)\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{c^{3/2}}{}}{3bd}}{5d}\right.$$

$\downarrow$ 406

$$\frac{f\left(\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}(adf-6bcf+7bde)}{5bd}-\frac{\frac{1}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{4a^2df}{b}+5acf-7ade-\frac{24bc^2f}{d}+28bce\right)-\frac{ac\left(4a^2d^2f-abd(7de-5cf)+4b^2c(7de-6cf)\right)\int\frac{}{\sqrt{bx^2}}}{5bd}}{7d}\right.}{e^2}$$

$$e\left(\frac{\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(adf-4bcf+5bde)}{3bd}-\frac{(2a^2d^2f-abd(5de-3cf)+2b^2c(5de-4cf))\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)+\frac{c^{3/2}}{}}{3bd}}{5d}\right.$$

$$\Big\downarrow \text{320}$$

$$\frac{f\left(\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}(adf-6bcf+7bde)}{5bd}-\frac{\frac{1}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{4a^2df}{b}+5acf-7ade-\frac{24bc^2f}{d}+28bce\right)-\frac{\left(8a^3d^3f-a^2bd^2(14de-9cf)-ab^2cd(21de-16cf)+8b^3\right.}{}}{7d}\right.}{e}$$

$$e\left(\frac{\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(adf-4bcf+5bde)}{3bd}-\frac{(2a^2d^2f-abd(5de-3cf)+2b^2c(5de-4cf))\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)+\frac{c^{3/2}}{}}{3bd}}{5d}\right.$$

$$\Big\downarrow \text{388}$$

$$f\left(\frac{\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}(adf-6bcf+7bde)}{5bd}-\frac{\frac{1}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{4a^2df}{b}+5acf-7ade-\frac{24bc^2f}{d}+28bce\right)-\frac{\left(8a^3d^3f-a^2bd^2(14de-9cf)-ab^2cd(21de-16cf)+8b^3\right.}{\cdots}}{\cdots}}{7d}\right.$$

$$e\left(\frac{\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(adf-4bcf+5bde)}{3bd}-\frac{(2a^2d^2f-abd(5de-3cf)+2b^2c(5de-4cf))\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{c^{3/2}}{\cdots}}{3bd}}{5d}\right.$$

$$\Big\downarrow 313$$

$$e\left(\frac{f\sqrt{bx^2+a}\sqrt{dx^2+c}x^3}{5d}+\frac{\frac{(5bde-4bcf+adf)x\sqrt{bx^2+a}\sqrt{dx^2+c}}{3bd}-\frac{\frac{(5bde-4bcf+adf)\sqrt{bx^2+a}\text{ EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+(2c(5\cdots}{\cdots}}{5d}\right.$$

$$f\left(\frac{f\sqrt{bx^2+a}\sqrt{dx^2+c}x^5}{7d}+\frac{\frac{(7bde-6bcf+adf)x^3\sqrt{bx^2+a}\sqrt{dx^2+c}}{5bd}-\frac{\frac{1}{3}\left(\frac{4dfa^2}{b}-7dea+5cfa+28bce-\frac{24bc^2f}{d}\right)x\sqrt{bx^2+a}\sqrt{dx^2+c}-\frac{\left(4c(7de-6cf)b^2-ad(7de\cdots\right.}{\cdots}}{\cdots}}{\cdots}\right.$$

input
```
Int[(x^2*Sqrt[a + b*x^2]*(e + f*x^2)^2)/Sqrt[c + d*x^2],x]
```

output
```
e*((f*x^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(5*d) + (((5*b*d*e - 4*b*c*f +
a*d*f)*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(3*b*d) - ((2*a^2*d^2*f + 2*b^2*
c*(5*d*e - 4*c*f) - a*b*d*(5*d*e - 3*c*f))*((x*Sqrt[a + b*x^2])/(b*Sqrt[c
+ d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]]
, 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[
c + d*x^2])) + (c^(3/2)*(5*b*d*e - 4*b*c*f + a*d*f)*Sqrt[a + b*x^2]*Ellipt
icF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b
*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))/(3*b*d))/(5*d)) + (f*((f*x^5*Sqr
t[a + b*x^2]*Sqrt[c + d*x^2])/(7*d) + (((7*b*d*e - 6*b*c*f + a*d*f)*x^3*Sq
rt[a + b*x^2]*Sqrt[c + d*x^2])/(5*b*d) - (((28*b*c*e - 7*a*d*e + 5*a*c*f -
 (24*b*c^2*f)/d + (4*a^2*d*f)/b)*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/3 - ((
8*a^3*d^3*f - a*b^2*c*d*(21*d*e - 16*c*f) - a^2*b*d^2*(14*d*e - 9*c*f) + 8
*b^3*c^2*(7*d*e - 6*c*f))*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt
[c]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)
])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (c
^(3/2)*(4*a^2*d^2*f + 4*b^2*c*(7*d*e - 6*c*f) - a*b*d*(7*d*e - 5*c*f))*Sqr
t[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqr
t[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))/(3*b*d))/(5*b
*d))/(7*d)))/e^2
```

Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 443
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*(g*x)^(m + 1)*(a + b*x^2)^(p
 + 1)*((c + d*x^2)^q/(b*g*(m + 2*(p + q + 1) + 1))), x] + Simp[1/(b*(m + 2*
(p + q + 1) + 1))   Int[(g*x)^m*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[c*((
b*e - a*f)*(m + 1) + b*e*2*(p + q + 1)) + (d*(b*e - a*f)*(m + 1) + f*2*q*(b
*c - a*d) + b*e*d*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f
, g, m, p}, x] && GtQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^2, c + d*x^
2])
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

rule 448
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[e   Int[(g*x)^m*(a + b*x
^2)^p*(c + d*x^2)^q*(e + f*x^2)^(r - 1), x], x] + Simp[f/e^2   Int[(g*x)^(m
 + 2)*(a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^(r - 1), x], x] /; FreeQ[{a,
b, c, d, e, f, g, m, p, q}, x] && IGtQ[r, 0]
```

Maple [A] (verified)

Time = 9.51 (sec) , antiderivative size = 646, normalized size of antiderivative = 1.04

method	result
elliptic	$\frac{\sqrt{(b\,x^2+a)(x^2d+c)}\left(\frac{f^2x^5\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{7d}+\frac{\left(a\,f^2+2bef-\frac{f^2(6ad+6bc)}{7d}\right)x^3\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{5bd}+\frac{\left(2aef+b\,e^2-\frac{5ac\,f^2}{7d}-\frac{\left(a\,f\right.}{}\right.}{}\right.}{}$
risch	$-\frac{x\left(-15f^2x^4b^2d^2-3ab\,d^2f^2x^2+18b^2cd\,f^2x^2-42b^2d^2ef\,x^2+4a^2d^2f^2+5abcd\,f^2-14ab\,d^2ef-24b^2c^2f^2+56b^2cdef-35b^2d^2e^2\right)\sqrt{b}}{105b^2d^3}$
default	Expression too large to display

input
```
int(x^2*(b*x^2+a)^(1/2)*(f*x^2+e)^2/(d*x^2+c)^(1/2),x,method=_RETURNVERBOS
E)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/7*f^2/d*x^5
*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+1/5*(a*f^2+2*b*e*f-1/7*f^2/d*(6*a*d+6
*b*c))/b/d*x^3*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+1/3*(2*a*e*f+b*e^2-5/7*
a*c/d*f^2-1/5*(a*f^2+2*b*e*f-1/7*f^2/d*(6*a*d+6*b*c))/b/d*(4*a*d+4*b*c))/b
/d*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)-1/3*(2*a*e*f+b*e^2-5/7*a*c/d*f^2-
1/5*(a*f^2+2*b*e*f-1/7*f^2/d*(6*a*d+6*b*c))/b/d*(4*a*d+4*b*c))/b/d*a*c/(-b
/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c
)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-(a*e^2-3/5*(a*f
^2+2*b*e*f-1/7*f^2/d*(6*a*d+6*b*c))/b/d*a*c-1/3*(2*a*e*f+b*e^2-5/7*a*c/d*f
^2-1/5*(a*f^2+2*b*e*f-1/7*f^2/d*(6*a*d+6*b*c))/b/d*(4*a*d+4*b*c))/b/d*(2*a
*d+2*b*c))*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d
*x^2+b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/
2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 608, normalized size of antiderivative = 0.98

$$\int \frac{x^2\sqrt{a+bx^2}(e+fx^2)^2}{\sqrt{c+dx^2}}\,dx$$
$$= \frac{(35\,(2\,b^3c^2d^2-ab^2cd^3)e^2-14\,(8\,b^3c^3d-3\,ab^2c^2d^2-2\,a^2bcd^3)ef+(48\,b^3c^4-16\,ab^2c^3d-9\,a^2bc^2d^2-8\,a}{}$$

input
```
integrate(x^2*(b*x^2+a)^(1/2)*(f*x^2+e)^2/(d*x^2+c)^(1/2),x, algorithm="fr
icas")
```

output
```
1/105*((35*(2*b^3*c^2*d^2 - a*b^2*c*d^3)*e^2 - 14*(8*b^3*c^3*d - 3*a*b^2*c
^2*d^2 - 2*a^2*b*c*d^3)*e*f + (48*b^3*c^4 - 16*a*b^2*c^3*d - 9*a^2*b*c^2*d
^2 - 8*a^3*c*d^3)*f^2)*sqrt(b*d)*x*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)
/x), a*d/(b*c)) - (35*(2*b^3*c^2*d^2 - a*b^2*c*d^3 + a*b^2*d^4)*e^2 - 14*(
8*b^3*c^3*d - 3*a*b^2*c^2*d^2 - a^2*b*d^4 - 2*(a^2*b - 2*a*b^2)*c*d^3)*e*f
 + (48*b^3*c^4 - 16*a*b^2*c^3*d - 4*a^3*d^4 - 3*(3*a^2*b - 8*a*b^2)*c^2*d^
2 - (8*a^3 + 5*a^2*b)*c*d^3)*f^2)*sqrt(b*d)*x*sqrt(-c/d)*elliptic_f(arcsin
(sqrt(-c/d)/x), a*d/(b*c)) + (15*b^3*d^4*f^2*x^6 + 3*(14*b^3*d^4*e*f - (6*
b^3*c*d^3 - a*b^2*d^4)*f^2)*x^4 - 35*(2*b^3*c*d^3 - a*b^2*d^4)*e^2 + 14*(8
*b^3*c^2*d^2 - 3*a*b^2*c*d^3 - 2*a^2*b*d^4)*e*f - (48*b^3*c^3*d - 16*a*b^2
*c^2*d^2 - 9*a^2*b*c*d^3 - 8*a^3*d^4)*f^2 + (35*b^3*d^4*e^2 - 14*(4*b^3*c*
d^3 - a*b^2*d^4)*e*f + (24*b^3*c^2*d^2 - 5*a*b^2*c*d^3 - 4*a^2*b*d^4)*f^2)
*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(b^3*d^5*x)
```

Sympy [F]

$$\int \frac{x^2\sqrt{a+bx^2}(e+fx^2)^2}{\sqrt{c+dx^2}}\,dx = \int \frac{x^2\sqrt{a+bx^2}(e+fx^2)^2}{\sqrt{c+dx^2}}\,dx$$

input
```
integrate(x**2*(b*x**2+a)**(1/2)*(f*x**2+e)**2/(d*x**2+c)**(1/2),x)
```

output
```
Integral(x**2*sqrt(a + b*x**2)*(e + f*x**2)**2/sqrt(c + d*x**2), x)
```

Maxima [F]

$$\int \frac{x^2\sqrt{a+bx^2}(e+fx^2)^2}{\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{bx^2+a}(fx^2+e)^2x^2}{\sqrt{dx^2+c}}\,dx$$

input
```
integrate(x^2*(b*x^2+a)^(1/2)*(f*x^2+e)^2/(d*x^2+c)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate(sqrt(b*x^2 + a)*(f*x^2 + e)^2*x^2/sqrt(d*x^2 + c), x)
```

Giac [F]

$$\int \frac{x^2\sqrt{a+bx^2}(e+fx^2)^2}{\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{bx^2+a}(fx^2+e)^2x^2}{\sqrt{dx^2+c}}\,dx$$

input
```
integrate(x^2*(b*x^2+a)^(1/2)*(f*x^2+e)^2/(d*x^2+c)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate(sqrt(b*x^2 + a)*(f*x^2 + e)^2*x^2/sqrt(d*x^2 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2\sqrt{a+bx^2}(e+fx^2)^2}{\sqrt{c+dx^2}}\,dx = \int \frac{x^2\,\sqrt{b\,x^2+a}\,{\left(f\,x^2+e\right)}^2}{\sqrt{d\,x^2+c}}\,dx$$

input
```
int((x^2*(a + b*x^2)^(1/2)*(e + f*x^2)^2)/(c + d*x^2)^(1/2),x)
```

output
```
int((x^2*(a + b*x^2)^(1/2)*(e + f*x^2)^2)/(c + d*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^2\sqrt{a+bx^2}(e+fx^2)^2}{\sqrt{c+dx^2}}\,dx = \text{Too large to display}$$

input
```
int(x^2*(b*x^2+a)^(1/2)*(f*x^2+e)^2/(d*x^2+c)^(1/2),x)
```

output
```
( - 4*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*d**2*f**2*x - 5*sqrt(c + d*x*
*2)*sqrt(a + b*x**2)*a*b*c*d*f**2*x + 14*sqrt(c + d*x**2)*sqrt(a + b*x**2)
*a*b*d**2*e*f*x + 3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b*d**2*f**2*x**3 +
 24*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**2*c**2*f**2*x - 56*sqrt(c + d*x**
2)*sqrt(a + b*x**2)*b**2*c*d*e*f*x - 18*sqrt(c + d*x**2)*sqrt(a + b*x**2)*
b**2*c*d*f**2*x**3 + 35*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**2*d**2*e**2*x
 + 42*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**2*d**2*e*f*x**3 + 15*sqrt(c + d
*x**2)*sqrt(a + b*x**2)*b**2*d**2*f**2*x**5 + 8*int((sqrt(c + d*x**2)*sqrt
(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a**3*d**3*f**
2 + 9*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x
**2 + b*d*x**4),x)*a**2*b*c*d**2*f**2 - 28*int((sqrt(c + d*x**2)*sqrt(a +
b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a**2*b*d**3*e*f +
16*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2
 + b*d*x**4),x)*a*b**2*c**2*d*f**2 - 42*int((sqrt(c + d*x**2)*sqrt(a + b*x
**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a*b**2*c*d**2*e*f + 3
5*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2
+ b*d*x**4),x)*a*b**2*d**3*e**2 - 48*int((sqrt(c + d*x**2)*sqrt(a + b*x**2
)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*b**3*c**3*f**2 + 112*int
((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d
*x**4),x)*b**3*c**2*d*e*f - 70*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x...
```

3.113 $\int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 445

$$\begin{aligned}
&\int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{\sqrt{c+dx^2}}\,dx \\
&= -\frac{(2a^2d^2f^2-abdf(10de-3cf)-b^2(15d^2e^2-20cdef+8c^2f^2))\,x\sqrt{c+dx^2}}{15bd^3\sqrt{a+bx^2}} \\
&\quad+\frac{f(10bde-4bcf+adf)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{15bd^2}+\frac{f^2x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5d} \\
&\quad+\frac{\sqrt{a}(2a^2d^2f^2-abdf(10de-3cf)-b^2(15d^2e^2-20cdef+8c^2f^2))\,\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{15b^{3/2}d^3\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} \\
&\quad-\frac{a^{3/2}(acdf^2-b(15d^2e^2-10cdef+4c^2f^2))\,\sqrt{c+dx^2}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{15b^{3/2}cd^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}
\end{aligned}$$

output
```
-1/15*(2*a^2*d^2*f^2-a*b*d*f*(-3*c*f+10*d*e)-b^2*(8*c^2*f^2-20*c*d*e*f+15*
d^2*e^2))*x*(d*x^2+c)^(1/2)/b/d^3/(b*x^2+a)^(1/2)+1/15*f*(a*d*f-4*b*c*f+10
*b*d*e)*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b/d^2+1/5*f^2*x^3*(b*x^2+a)^(1/2
)*(d*x^2+c)^(1/2)/d+1/15*a^(1/2)*(2*a^2*d^2*f^2-a*b*d*f*(-3*c*f+10*d*e)-b^
2*(8*c^2*f^2-20*c*d*e*f+15*d^2*e^2))*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a
^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/b^(3/2)/d^3/(b*x^2+a)^(1/2)/(a
*(d*x^2+c)/c/(b*x^2+a))^(1/2)-1/15*a^(3/2)*(a*c*d*f^2-b*(4*c^2*f^2-10*c*d*
e*f+15*d^2*e^2))*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2))
,(1-a*d/b/c)^(1/2))/b^(3/2)/c/d^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a)
)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.31 (sec) , antiderivative size = 303, normalized size of antiderivative = 0.68

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{\sqrt{c+dx^2}}\,dx$$

$$= \frac{\sqrt{\frac{b}{a}}dfx(a+bx^2)\left(c+dx^2\right)\left(adf+b(10de-4cf+3dfx^2)\right)+ic(2a^2d^2f^2+abdf(-10de+3cf)+b^2(-15}{}$$

input
```
Integrate[(Sqrt[a + b*x^2]*(e + f*x^2)^2)/Sqrt[c + d*x^2],x]
```

output
```
(Sqrt[b/a]*d*f*x*(a + b*x^2)*(c + d*x^2)*(a*d*f + b*(10*d*e - 4*c*f + 3*d*
f*x^2)) + I*c*(2*a^2*d^2*f^2 + a*b*d*f*(-10*d*e + 3*c*f) + b^2*(-15*d^2*e^
2 + 20*c*d*e*f - 8*c^2*f^2))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Ellip
ticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*(-(b*c) + a*d)*(a*c*d*f^2 +
b*(15*d^2*e^2 - 20*c*d*e*f + 8*c^2*f^2))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x
^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(15*b*Sqrt[b/a]*d^3
*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 826, normalized size of antiderivative = 1.86, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\left(e+fx^2\right)^2}{\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \text{433}$$

$$\int \left(\frac{e^2\sqrt{a+bx^2}}{\sqrt{c+dx^2}}+\frac{2efx^2\sqrt{a+bx^2}}{\sqrt{c+dx^2}}+\frac{f^2x^4\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{f^2\sqrt{bx^2+a}\sqrt{dx^2+c}x^3}{5d}-\frac{(4bc-ad)f^2\sqrt{bx^2+a}\sqrt{dx^2+c}x}{15bd^2}+\frac{2ef\sqrt{bx^2+a}\sqrt{dx^2+c}x}{3d}+\frac{e^2\sqrt{bx^2+a}x}{\sqrt{dx^2+c}}+\frac{\left(8b^2c^2-3abdc-2a^2d^2\right)f^2\sqrt{bx^2+a}x}{15b^2d^2\sqrt{dx^2+c}}-\frac{2(2bc-ad)ef\sqrt{bx^2+a}x}{3bd\sqrt{dx^2+c}}-\frac{\sqrt{c}e^2\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{\sqrt{c}\left(8b^2c^2-3abdc-2a^2d^2\right)f^2\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15b^2d^{5/2}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{2\sqrt{c}(2bc-ad)ef\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3bd^{3/2}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{\sqrt{c}e^2\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{c^{3/2}(4bc-ad)f^2\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{15bd^{5/2}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{2c^{3/2}ef\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3d^{3/2}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}$$

input
```
Int[(Sqrt[a + b*x^2]*(e + f*x^2)^2)/Sqrt[c + d*x^2],x]
```

output
```
(e^2*x*Sqrt[a + b*x^2])/Sqrt[c + d*x^2] - (2*(2*b*c - a*d)*e*f*x*Sqrt[a +
b*x^2])/(3*b*d*Sqrt[c + d*x^2]) + ((8*b^2*c^2 - 3*a*b*c*d - 2*a^2*d^2)*f^2
*x*Sqrt[a + b*x^2])/(15*b^2*d^2*Sqrt[c + d*x^2]) + (2*e*f*x*Sqrt[a + b*x^2
]*Sqrt[c + d*x^2])/(3*d) - ((4*b*c - a*d)*f^2*x*Sqrt[a + b*x^2]*Sqrt[c + d
*x^2])/(15*b*d^2) + (f^2*x^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(5*d) - (Sqr
t[c]*e^2*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/
(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) +
(2*Sqrt[c]*(2*b*c - a*d)*e*f*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/
Sqrt[c]], 1 - (b*c)/(a*d)])/(3*b*d^(3/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^
2))]*Sqrt[c + d*x^2]) - (Sqrt[c]*(8*b^2*c^2 - 3*a*b*c*d - 2*a^2*d^2)*f^2*S
qrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(1
5*b^2*d^(5/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (Sq
rt[c]*e^2*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)
/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) -
 (2*c^(3/2)*e*f*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 -
 (b*c)/(a*d)])/(3*d^(3/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d
*x^2]) + (c^(3/2)*(4*b*c - a*d)*f^2*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt
[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(15*b*d^(5/2)*Sqrt[(c*(a + b*x^2))/(a*(
c + d*x^2))]*Sqrt[c + d*x^2])
```

Defintions of rubi rules used

rule 433
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)
^q*(e + f*x^2)^r, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, f, p,
 q, r}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 7.34 (sec) , antiderivative size = 453, normalized size of antiderivative = 1.02

method	result
elliptic	$\frac{\sqrt{(b x^2+a)(x^2 d+c)}\left(\frac{f^2 x^3 \sqrt{bd x^4+ad x^2+x^2 bc+ac}}{5d}+\frac{\left(a f^2+2bef-\frac{f^2(4ad+4bc)}{5d}\right)x\sqrt{bd x^4+ad x^2+x^2 bc+ac}}{3bd}+\frac{\left(a e^2-\frac{\left(a f^2+2bef-\frac{f^2(4ac}{5}\right.}{3bd}\right.}{}\right.}{}$
risch	$\frac{fx(3bdf x^2+adf-4bcf+10bde)\sqrt{b x^2+a}\,\sqrt{x^2 d+c}}{15b d^2}-\frac{\left(-\frac{(2a^2d^2f^2+3abcd f^2-10ab d^2ef-8b^2c^2f^2+20b^2cdef-15b^2d^2e^2)c\sqrt{1+\frac{b x^2}{a}}\,\sqrt{1}}{\sqrt{-\frac{b}{a}}\,\sqrt{bd x^4+ac}}\right.}{}$
default	Expression too large to display

input

```
int((b*x^2+a)^(1/2)*(f*x^2+e)^2/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/5*f^2/d*x^3
*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+1/3*(a*f^2+2*b*e*f-1/5*f^2/d*(4*a*d+4
*b*c))/b/d*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+(a*e^2-1/3*(a*f^2+2*b*e*f
-1/5*f^2/d*(4*a*d+4*b*c))/b/d*a*c)/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2
/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1
+(a*d+b*c)/c/b)^(1/2))-(2*a*e*f+b*e^2-3/5*a*c/d*f^2-1/3*(a*f^2+2*b*e*f-1/5
*f^2/d*(4*a*d+4*b*c))/b/d*(2*a*d+2*b*c))*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*
(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(-b/a
)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c
/b)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 409, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{a+bx^2}\left(e+fx^2\right)^2}{\sqrt{c+dx^2}}\,dx =$$

$$-\frac{(15\,b^2c^2d^2e^2-10\,(2\,b^2c^3d-abc^2d^2)ef+(8\,b^2c^4-3\,abc^3d-2\,a^2c^2d^2)f^2)\sqrt{bd}x\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)}{}$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)^2/(d*x^2+c)^(1/2),x, algorithm="fricas
")
```

output
```
-1/15*((15*b^2*c^2*d^2*e^2 - 10*(2*b^2*c^3*d - a*b*c^2*d^2)*e*f + (8*b^2*c
^4 - 3*a*b*c^3*d - 2*a^2*c^2*d^2)*f^2)*sqrt(b*d)*x*sqrt(-c/d)*elliptic_e(a
rcsin(sqrt(-c/d)/x), a*d/(b*c)) - (15*(b^2*c^2*d^2 + a*b*d^4)*e^2 - 10*(2*
b^2*c^3*d - a*b*c^2*d^2 + a*b*c*d^3)*e*f + (8*b^2*c^4 - 3*a*b*c^3*d - a^2*
c*d^3 - 2*(a^2 - 2*a*b)*c^2*d^2)*f^2)*sqrt(b*d)*x*sqrt(-c/d)*elliptic_f(ar
csin(sqrt(-c/d)/x), a*d/(b*c)) - (3*b^2*c*d^3*f^2*x^4 + 15*b^2*c*d^3*e^2 -
 10*(2*b^2*c^2*d^2 - a*b*c*d^3)*e*f + (8*b^2*c^3*d - 3*a*b*c^2*d^2 - 2*a^2
*c*d^3)*f^2 + (10*b^2*c*d^3*e*f - (4*b^2*c^2*d^2 - a*b*c*d^3)*f^2)*x^2)*sq
rt(b*x^2 + a)*sqrt(d*x^2 + c))/(b^2*c*d^4*x)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{\sqrt{c+dx^2}}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(f*x**2+e)**2/(d*x**2+c)**(1/2),x)
```

output
```
Integral(sqrt(a + b*x**2)*(e + f*x**2)**2/sqrt(c + d*x**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{bx^2+a}(fx^2+e)^2}{\sqrt{dx^2+c}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)^2/(d*x^2+c)^(1/2),x, algorithm="maxima
")
```

output
```
integrate(sqrt(b*x^2 + a)*(f*x^2 + e)^2/sqrt(d*x^2 + c), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{\sqrt{c+dx^2}}\,dx=\int \frac{\sqrt{bx^2+a}(fx^2+e)^2}{\sqrt{dx^2+c}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)^2/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^2 + a)*(f*x^2 + e)^2/sqrt(d*x^2 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{\sqrt{c+dx^2}}\,dx=\int \frac{\sqrt{b\,x^2+a}\,(f\,x^2+e)^2}{\sqrt{d\,x^2+c}}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(e + f*x^2)^2)/(c + d*x^2)^(1/2),x)
```

output
```
int(((a + b*x^2)^(1/2)*(e + f*x^2)^2)/(c + d*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{\sqrt{c+dx^2}}\,dx$$
$$=\frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,ad\,f^2x-4\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,bc\,f^2x+10\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,bdefx+3\sqrt{d\,x^2+c}}{}$$

input
```
int((b*x^2+a)^(1/2)*(f*x^2+e)^2/(d*x^2+c)^(1/2),x)
```

output
```
(sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*d*f**2*x - 4*sqrt(c + d*x**2)*sqrt(a
+ b*x**2)*b*c*f**2*x + 10*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b*d*e*f*x + 3*
sqrt(c + d*x**2)*sqrt(a + b*x**2)*b*d*f**2*x**3 - 2*int((sqrt(c + d*x**2)*
sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a**2*d**2
*f**2 - 3*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b
*c*x**2 + b*d*x**4),x)*a*b*c*d*f**2 + 10*int((sqrt(c + d*x**2)*sqrt(a + b*
x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a*b*d**2*e*f + 8*int
((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d
*x**4),x)*b**2*c**2*f**2 - 20*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)
/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*b**2*c*d*e*f + 15*int((sqrt(c +
 d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*
b**2*d**2*e**2 - int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c + a*d*x**2 +
 b*c*x**2 + b*d*x**4),x)*a**2*c*d*f**2 + 4*int((sqrt(c + d*x**2)*sqrt(a +
b*x**2))/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a*b*c**2*f**2 - 10*int(
(sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4)
,x)*a*b*c*d*e*f + 15*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c + a*d*x*
*2 + b*c*x**2 + b*d*x**4),x)*a*b*d**2*e**2)/(15*b*d**2)
```

3.114 $\int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{x^2\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 532

$$\begin{aligned}
&\int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{x^2\sqrt{c+dx^2}}\,dx \\
&= \frac{(acdf^2+b(3d^2e^2+6cdef-2c^2f^2))\,x\sqrt{c+dx^2}}{3cd^2\sqrt{a+bx^2}} \\
&\quad+\frac{(3bde^2+af(9de+cf))\,x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3acd} \\
&\quad+\frac{3f(be+af)x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{ac}+\frac{f^2(3be+af)x^5\sqrt{a+bx^2}\sqrt{c+dx^2}}{ace} \\
&\quad+\frac{bf^3x^7\sqrt{a+bx^2}\sqrt{c+dx^2}}{ace}-\frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}(e+fx^2)^3}{acex} \\
&\quad-\frac{\sqrt{a}(acdf^2+b(3d^2e^2+6cdef-2c^2f^2))\,\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{3\sqrt{b}cd^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} \\
&\quad+\frac{\sqrt{a}(3bde^2+af(6de-cf))\,\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{3\sqrt{b}cd\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}
\end{aligned}$$

output
```
1/3*(a*c*d*f^2+b*(-2*c^2*f^2+6*c*d*e*f+3*d^2*e^2))*x*(d*x^2+c)^(1/2)/c/d^2
/(b*x^2+a)^(1/2)+1/3*(3*b*d*e^2+a*f*(c*f+9*d*e))*x*(b*x^2+a)^(1/2)*(d*x^2+
c)^(1/2)/a/c/d+3*f*(a*f+b*e)*x^3*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a/c+f^2*(
a*f+3*b*e)*x^5*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a/c/e+b*f^3*x^7*(b*x^2+a)^(
1/2)*(d*x^2+c)^(1/2)/a/c/e-(b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^3/a/c
/e/x-1/3*a^(1/2)*(a*c*d*f^2+b*(-2*c^2*f^2+6*c*d*e*f+3*d^2*e^2))*(d*x^2+c)^
(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/b^(
1/2)/c/d^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)+1/3*a^(1/2)*(3*
b*d*e^2+a*f*(-c*f+6*d*e))*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x
/a^(1/2)),(1-a*d/b/c)^(1/2))/b^(1/2)/c/d/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b
*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.16 (sec) , antiderivative size = 255, normalized size of antiderivative = 0.48

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{x^2\sqrt{c+dx^2}}\,dx$$
$$= \frac{\sqrt{\frac{b}{a}}d(a+bx^2)(c+dx^2)(-3de^2+cf^2x^2)-ic(acdf^2+b(3d^2e^2+6cdef-2c^2f^2))x\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}}{3\sqrt{\frac{b}{a}}cd^2x}$$

input
```
Integrate[(Sqrt[a + b*x^2]*(e + f*x^2)^2)/(x^2*Sqrt[c + d*x^2]),x]
```

output
```
(Sqrt[b/a]*d*(a + b*x^2)*(c + d*x^2)*(-3*d*e^2 + c*f^2*x^2) - I*c*(a*c*d*f
^2 + b*(3*d^2*e^2 + 6*c*d*e*f - 2*c^2*f^2))*x*Sqrt[1 + (b*x^2)/a]*Sqrt[1 +
 (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (2*I)*c*(-(b*
c) + a*d)*f*(-3*d*e + c*f)*x*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Ellip
ticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(3*Sqrt[b/a]*c*d^2*x*Sqrt[a + b
*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (warning: unable to verify)

Time = 0.89 (sec) , antiderivative size = 529, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.314, Rules used = {448, 403, 406, 320, 388, 313, 442, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\left(e+fx^2\right)^2}{x^2\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \ 448$$

$$\frac{f\int \frac{\sqrt{bx^2+a}\left(fx^2+e\right)}{\sqrt{dx^2+c}}dx}{e^2}+e\int \frac{\sqrt{bx^2+a}\left(fx^2+e\right)}{x^2\sqrt{dx^2+c}}dx$$

$$\Big\downarrow \ 403$$

$$\frac{f\left(\frac{\int \frac{(3bde-2bcf+adf)x^2+a(3de-cf)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3d}+\frac{fx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3d}\right)}{e^2}+e\int \frac{\sqrt{bx^2+a}\left(fx^2+e\right)}{x^2\sqrt{dx^2+c}}dx$$

$$\Big\downarrow \ 406$$

$$\frac{f\left(\frac{a(3de-cf)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+(adf-2bcf+3bde)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3d}+\frac{fx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3d}\right)}{e^2}+ e\int \frac{\sqrt{bx^2+a}\left(fx^2+e\right)}{x^2\sqrt{dx^2+c}}dx$$

$$\Big\downarrow \ 320$$

$$\frac{f\left(\frac{(adf-2bcf+3bde)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{\sqrt{c}\sqrt{a+bx^2}(3de-cf)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{3d}+\frac{fx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3d}\right)}{e^2}+ e\int \frac{\sqrt{bx^2+a}\left(fx^2+e\right)}{x^2\sqrt{dx^2+c}}dx$$

$$\Big\downarrow \ 388$$

$$\frac{f\left(\frac{(adf-2bcf+3bde)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}}dx}{b}\right)+\frac{\sqrt{c}\sqrt{a+bx^2}(3de-cf)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}}{3d}+\frac{fx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3d}\right)}{e^2}+$$

$$e\int\frac{\sqrt{bx^2+a}(fx^2+e)}{x^2\sqrt{dx^2+c}}dx$$

$\Big\downarrow$ 313

$$e\int\frac{\sqrt{bx^2+a}(fx^2+e)}{x^2\sqrt{dx^2+c}}dx+$$

$$\frac{f\left(\frac{\frac{\sqrt{c}\sqrt{a+bx^2}(3de-cf)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+(adf-2bcf+3bde)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{3d}+\frac{fx\sqrt{a+bx^2}\sqrt{c+}}{3d}\right.}{e^2}$$

$\Big\downarrow$ 442

$$e\left(\frac{\int\frac{b(de+cf)x^2+c(be+af)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{c}-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{cx}\right)+$$

$$\frac{f\left(\frac{\frac{\sqrt{c}\sqrt{a+bx^2}(3de-cf)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+(adf-2bcf+3bde)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{3d}+\frac{fx\sqrt{a+bx^2}\sqrt{c+}}{3d}\right.}{e^2}$$

$\Big\downarrow$ 406

$$\frac{e\left(\frac{c(af+be)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+b(cf+de)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{c}-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{cx}\right)+f\left(\frac{\frac{\sqrt{c}\sqrt{a+bx^2}(3de-cf)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+(adf-2bcf+3bde)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{3d}+\frac{fx\sqrt{a+bx^2}\sqrt{c+}}{3d}\right.}{e^2}$$

$\downarrow$ 320

$$\frac{e\left(\frac{b(cf+de)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}(af+be)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}}{c}-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{cx}\right)+f\left(\frac{\frac{\sqrt{c}\sqrt{a+bx^2}(3de-cf)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+(adf-2bcf+3bde)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{3d}+\frac{fx\sqrt{a+bx^2}\sqrt{c+}}{3d}\right.}{e^2}$$

$\downarrow$ 388

$$\frac{e\left(\frac{b(cf+de)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(af+be)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}}{c}-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{cx}\right)+f\left(\frac{\frac{\sqrt{c}\sqrt{a+bx^2}(3de-cf)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+(adf-2bcf+3bde)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{3d}+\frac{fx\sqrt{a+bx^2}\sqrt{c+\cdots}}{3d}\right)}{e^2}$$

$$\Big\downarrow \ 313$$

$$\frac{e\left(\frac{\frac{c^{3/2}\sqrt{a+bx^2}(af+be)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+b(cf+de)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{c}-\frac{e\sqrt{a\cdots}}{\cdots}\right)+f\left(\frac{\frac{\sqrt{c}\sqrt{a+bx^2}(3de-cf)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+(adf-2bcf+3bde)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{3d}+\frac{fx\sqrt{a+bx^2}\sqrt{c+\cdots}}{3d}\right)}{e^2}$$

input
```
Int[(Sqrt[a + b*x^2]*(e + f*x^2)^2)/(x^2*Sqrt[c + d*x^2]),x]
```

output
```
e*(-((e*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(c*x)) + (b*(d*e + c*f)*((x*Sqrt[
a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcTa
n[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/
(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (c^(3/2)*(b*e + a*f)*Sqrt[a + b*x^2]*
EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*Sqrt[(
c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))/c) + (f*((f*x*Sqrt[a + b
*x^2]*Sqrt[c + d*x^2])/(3*d) + ((3*b*d*e - 2*b*c*f + a*d*f)*((x*Sqrt[a + b
*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sq
rt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c
 + d*x^2))]*Sqrt[c + d*x^2])) + (Sqrt[c]*(3*d*e - c*f)*Sqrt[a + b*x^2]*Ell
ipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a
+ b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))/(3*d)))/e^2
```

Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 442
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^q/(a*g*(m + 1))), x] - Simp[1/(a*g^2*(m + 1))   Int[(g*x)
^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[c*(b*e - a*f)*(m + 1) + e*2
*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*2*(p + q + 1))*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && GtQ[q, 0] && LtQ[m, -1]
&&  !(EqQ[q, 1] && SimplerQ[e + f*x^2, c + d*x^2])
```

rule 448
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[e   Int[(g*x)^m*(a + b*x
^2)^p*(c + d*x^2)^q*(e + f*x^2)^(r - 1), x], x] + Simp[f/e^2   Int[(g*x)^(m
 + 2)*(a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^(r - 1), x], x] /; FreeQ[{a,
b, c, d, e, f, g, m, p, q}, x] && IGtQ[r, 0]
```

Maple [A] (verified)

Time = 6.83 (sec) , antiderivative size = 373, normalized size of antiderivative = 0.70

method	result
elliptic	$\frac{\sqrt{(b\,x^2+a)(x^2d+c)}\left(-\frac{e^2\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{cx}+\frac{f^2x\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{3d}+\frac{\left(2aef+b\,e^2-\frac{ac\,f^2}{3d}\right)\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\mathrm{EllipticF}\left(x\sqrt{-\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}}{\sqrt{b\,x}}$
risch	$\frac{\sqrt{b\,x^2+a}\,\sqrt{x^2d+c}\,(c\,f^2x^2-3d\,e^2)}{3dcx}-\frac{\left(\frac{\left(acd\,f^2-2b\,c^2f^2+6bcdef+3b\,d^2e^2\right)c\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\left(\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)-\mathrm{Ellipt}\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}\,d}\right.}{}$
default	$-\frac{\sqrt{b\,x^2+a}\,\sqrt{x^2d+c}\left(-\sqrt{-\frac{b}{a}}\,bc\,d^2f^2x^6-\sqrt{-\frac{b}{a}}\,ac\,d^2f^2x^4-\sqrt{-\frac{b}{a}}\,b\,c^2d\,f^2x^4+3\sqrt{-\frac{b}{a}}\,b\,d^3e^2x^4+2\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{x^2d+c}{c}}\,\mathrm{EllipticF}\left(x\right.\right.}{}$

input
```
int((b*x^2+a)^(1/2)*(f*x^2+e)^2/x^2/(d*x^2+c)^(1/2),x,method=_RETURNVERBOS
E)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-e^2/c*(b*d*x
^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/x+1/3*f^2/d*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^
(1/2)+(2*a*e*f+b*e^2-1/3*a*c/d*f^2)/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^
2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-
1+(a*d+b*c)/c/b)^(1/2))-(a*f^2+2*b*e*f+d*b/c*e^2-1/3*f^2/d*(2*a*d+2*b*c))*
c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^
2+a*c)^(1/2)/d*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-Ellipti
cE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))))
```

Fricas [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{x^2\sqrt{c+dx^2}}\,dx=\int \frac{\sqrt{bx^2+a}(fx^2+e)^2}{\sqrt{dx^2+cx^2}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)^2/x^2/(d*x^2+c)^(1/2),x, algorithm="fr
icas")
```

output
```
integral((f^2*x^4 + 2*e*f*x^2 + e^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/(d*x^
4 + c*x^2), x)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}\,(e+fx^2)^2}{x^2\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{a+bx^2}\,(e+fx^2)^2}{x^2\sqrt{c+dx^2}}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(f*x**2+e)**2/x**2/(d*x**2+c)**(1/2),x)
```

output
```
Integral(sqrt(a + b*x**2)*(e + f*x**2)**2/(x**2*sqrt(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}\,(e+fx^2)^2}{x^2\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{bx^2+a}\,(fx^2+e)^2}{\sqrt{dx^2+c}\,x^2}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)^2/x^2/(d*x^2+c)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate(sqrt(b*x^2 + a)*(f*x^2 + e)^2/(sqrt(d*x^2 + c)*x^2), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}\,(e+fx^2)^2}{x^2\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{bx^2+a}\,(fx^2+e)^2}{\sqrt{dx^2+c}\,x^2}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)^2/x^2/(d*x^2+c)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate(sqrt(b*x^2 + a)*(f*x^2 + e)^2/(sqrt(d*x^2 + c)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{x^2\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{b\,x^2+a}\,(f\,x^2+e)^2}{x^2\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(e + f*x^2)^2)/(x^2*(c + d*x^2)^(1/2)),x)
```

output
```
int(((a + b*x^2)^(1/2)*(e + f*x^2)^2)/(x^2*(c + d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{x^2\sqrt{c+dx^2}}\,dx = \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,ad\,f^2-2\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,bc\,f^2+6\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,bdef+\sqrt{d\,x^2+c}\,\sqrt{b\,x^2}}{}$$

input
```
int((b*x^2+a)^(1/2)*(f*x^2+e)^2/x^2/(d*x^2+c)^(1/2),x)
```

output

```
(sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*d*f**2 - 2*sqrt(c + d*x**2)*sqrt(a +
b*x**2)*b*c*f**2 + 6*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b*d*e*f + sqrt(c +
d*x**2)*sqrt(a + b*x**2)*b*d*f**2*x**2 + int((sqrt(c + d*x**2)*sqrt(a + b*
x**2))/(a*c*x**2 + a*d*x**4 + b*c*x**4 + b*d*x**6),x)*a**2*c*d*f**2*x - 2*
int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c*x**2 + a*d*x**4 + b*c*x**4 +
b*d*x**6),x)*a*b*c**2*f**2*x + 6*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(
a*c*x**2 + a*d*x**4 + b*c*x**4 + b*d*x**6),x)*a*b*c*d*e*f*x + 3*int((sqrt(
c + d*x**2)*sqrt(a + b*x**2))/(a*c*x**2 + a*d*x**4 + b*c*x**4 + b*d*x**6),
x)*a*b*d**2*e**2*x - int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c + a*d*x*
*2 + b*c*x**2 + b*d*x**4),x)*a*b*c*d*f**2*x + 6*int((sqrt(c + d*x**2)*sqrt
(a + b*x**2))/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a*b*d**2*e*f*x + 3
*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c + a*d*x**2 + b*c*x**2 + b*d*
x**4),x)*b**2*d**2*e**2*x)/(3*b*d**2*x)
```

3.115 $\int \frac{\sqrt{a+bx^2}\left(e+fx^2\right)^2}{x^4\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 651

$$\begin{aligned}
\int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{x^4\sqrt{c+dx^2}}\,dx
&= \frac{b(bcde^2-a(2d^2e^2-6cdef-3c^2f^2))\,x\sqrt{c+dx^2}}{3ac^2d\sqrt{a+bx^2}} \\
&- \frac{2(be(de-3cf)+3af(de-2cf))x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3ac^2} \\
&- \frac{2f(af(3de-5cf)+3be(de-2cf))x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{3ac^2e} \\
&- \frac{f^2(2be(3de-5cf)+af(2de-3cf))x^5\sqrt{a+bx^2}\sqrt{c+dx^2}}{3ac^2e^2} \\
&- \frac{bf^3(2de-3cf)x^7\sqrt{a+bx^2}\sqrt{c+dx^2}}{3ac^2e^2} - \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}(e+fx^2)^3}{3acex^3} \\
&+ \frac{(2de-3cf)\,(a+bx^2)^{3/2}\sqrt{c+dx^2}(e+fx^2)^3}{3ac^2e^2x} \\
&- \frac{\sqrt{b}(bcde^2-a(2d^2e^2-6cdef-3c^2f^2))\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{3\sqrt{a}c^2d\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} \\
&+ \frac{\sqrt{a}(3acf^2-be(de-6cf))\sqrt{c+dx^2}\,\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{3\sqrt{b}c^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}
\end{aligned}$$

output
```
1/3*b*(b*c*d*e^2-a*(-3*c^2*f^2-6*c*d*e*f+2*d^2*e^2))*x*(d*x^2+c)^(1/2)/a/c
^2/d/(b*x^2+a)^(1/2)-2/3*(b*e*(-3*c*f+d*e)+3*a*f*(-2*c*f+d*e))*x*(b*x^2+a)
^(1/2)*(d*x^2+c)^(1/2)/a/c^2-2/3*f*(a*f*(-5*c*f+3*d*e)+3*b*e*(-2*c*f+d*e))
*x^3*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a/c^2/e-1/3*f^2*(2*b*e*(-5*c*f+3*d*e)
+a*f*(-3*c*f+2*d*e))*x^5*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a/c^2/e^2-1/3*b*f
^3*(-3*c*f+2*d*e)*x^7*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a/c^2/e^2-1/3*(b*x^2
+a)^(3/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^3/a/c/e/x^3+1/3*(-3*c*f+2*d*e)*(b*x^2+
a)^(3/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^3/a/c^2/e^2/x-1/3*b^(1/2)*(b*c*d*e^2-a*
(-3*c^2*f^2-6*c*d*e*f+2*d^2*e^2))*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1
/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/a^(1/2)/c^2/d/(b*x^2+a)^(1/2)/(a*
(d*x^2+c)/c/(b*x^2+a))^(1/2)+1/3*a^(1/2)*(3*a*c*f^2-b*e*(-6*c*f+d*e))*(d*x
^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/b
^(1/2)/c^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.40 (sec) , antiderivative size = 282, normalized size of antiderivative = 0.43

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{x^4\sqrt{c+dx^2}}\,dx$$
$$= \frac{\sqrt{\frac{b}{a}}\left(-\sqrt{\frac{b}{a}}de(a+bx^2)\,(c+dx^2)\,(bcex^2-2adex^2+ac(e+6fx^2))-ibc(bcde^2+a(-2d^2e^2+6cdef+3c\right.}{}$$

input
```
Integrate[(Sqrt[a + b*x^2]*(e + f*x^2)^2)/(x^4*Sqrt[c + d*x^2]),x]
```

output
```
(Sqrt[b/a]*(-(Sqrt[b/a]*d*e*(a + b*x^2)*(c + d*x^2)*(b*c*e*x^2 - 2*a*d*e*x
^2 + a*c*(e + 6*f*x^2))) - I*b*c*(b*c*d*e^2 + a*(-2*d^2*e^2 + 6*c*d*e*f +
3*c^2*f^2))*x^3*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSin
h[Sqrt[b/a]*x], (a*d)/(b*c)] - I*c*(-(b*c) + a*d)*(b*d*e^2 + 3*a*c*f^2)*x^
3*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x]
, (a*d)/(b*c)]))/(3*b*c^2*d*x^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (warning: unable to verify)

Time = 1.00 (sec) , antiderivative size = 590, normalized size of antiderivative = 0.91, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.343$, Rules used = {448, 442, 406, 320, 388, 313, 445, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\left(e+fx^2\right)^2}{x^4\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \text{448}$$

$$\frac{f\int \frac{\sqrt{bx^2+a}(fx^2+e)}{x^2\sqrt{dx^2+c}}dx}{e^2}+e\int \frac{\sqrt{bx^2+a}(fx^2+e)}{x^4\sqrt{dx^2+c}}dx$$

$$\Big\downarrow \text{442}$$

$$\frac{f\left(\frac{\int \frac{b(de+cf)x^2+c(be+af)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{c}-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{cx}\right)}{e^2}+e\left(\frac{\int \frac{-b(de-3cf)x^2+bce-2ade+3acf}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3c}-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{3cx^3}\right)$$

$$\Big\downarrow \text{406}$$

$$\frac{f\left(\frac{c(af+be)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+b(cf+de)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{c}-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{cx}\right)}{e^2}+e\left(\frac{\int \frac{-b(de-3cf)x^2+bce-2ade+3acf}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3c}-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{3cx^3}\right)$$

$$\Big\downarrow \text{320}$$

$$\frac{f\left(\frac{b(cf+de)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}(af+be)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}}{c}-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{cx}\right)}{e^2}+e\left(\frac{\int \frac{-b(de-3cf)x^2+bce-2ade+3acf}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3c}-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{3cx^3}\right)$$

$$\Big\downarrow \text{388}$$

$$\frac{f\left(\frac{b(cf+de)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(af+be)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}}{c}-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{cx}\right)}{e^2}+e\left(\frac{\int \frac{-b(de-3cf)x^2+bce-2ade+3acf}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3c}-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{3cx^3}\right)$$

$$\Big\downarrow \text{313}$$

$$e\left(\frac{\int \frac{-b(de-3cf)x^2+bce-2ade+3acf}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3c}-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{3cx^3}\right)+\frac{f\left(\frac{\frac{c^{3/2}\sqrt{a+bx^2}(af+be)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+b(cf+de)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{c}-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{cx}\right)}{e^2}$$

$$\Big\downarrow \text{445}$$

$$\frac{e\left(\frac{-\frac{\int \frac{b\left(ac(de-3cf)-d(bce-2ade+3acf)x^2\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(3acf-2ade+bce)}{acx}}{3c}-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{3cx^3}\right)+f\left(\frac{\frac{c^{3/2}\sqrt{a+bx^2}(af+be)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+b(cf+de)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{c}-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{cx}\right)}{e^2}$$

$\downarrow$ 27

$$\frac{e\left(\frac{-\frac{b\int \frac{ac(de-3cf)-d(bce-2ade+3acf)x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(3acf-2ade+bce)}{acx}}{3c}-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{3cx^3}\right)+f\left(\frac{\frac{c^{3/2}\sqrt{a+bx^2}(af+be)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+b(cf+de)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{c}-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{cx}\right)}{e^2}$$

$\downarrow$ 406

$$\frac{e\left(\frac{-\frac{b\left(ac(de-3cf)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx-d(3acf-2ade+bce)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(3acf-2ade+bce)}{acx}}{3c}-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{3cx^3}\right)+f\left(\frac{\frac{c^{3/2}\sqrt{a+bx^2}(af+be)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+b(cf+de)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{c}-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{cx}\right)}{e^2}$$

$\downarrow$ 320

$$\frac{e\left(\frac{-\frac{b\left(\frac{c^{3/2}\sqrt{a+bx^2}(de-3cf)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-d(3acf-2ade+bce)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(3acf-2ade+bce)}{acx}}{3c}\right)+f\left(\frac{\frac{c^{3/2}\sqrt{a+bx^2}(af+be)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+b(cf+de)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{c}-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{cx}\right)}{e^2}$$

$\downarrow$ 388

$$\frac{e\left(\frac{-\frac{b\left(\frac{c^{3/2}\sqrt{a+bx^2}(de-3cf)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-d(3acf-2ade+bce)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}}dx}{b}\right)\right)}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(3acf}{acx}}{3c}\right)+f\left(\frac{\frac{c^{3/2}\sqrt{a+bx^2}(af+be)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+b(cf+de)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{c}-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{cx}\right)}{e^2}$$

$\downarrow$ 313

$$\frac{f\left(\frac{\frac{c^{3/2}\sqrt{a+bx^2}(af+be)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+b(cf+de)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{c}-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{cx}\right)}{e^2}+$$

$$e\left(\frac{-\frac{b\left(\frac{c^{3/2}\sqrt{a+bx^2}(de-3cf)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-d(3acf-2ade+bce)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)\right)}{ac}-\frac{\sqrt{a+b}}{}}{3c}\right.$$

input
```
Int[(Sqrt[a + b*x^2]*(e + f*x^2)^2)/(x^4*Sqrt[c + d*x^2]),x]
```

output
```
(f*(-((e*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(c*x)) + (b*(d*e + c*f)*((x*Sqrt
[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcT
an[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))
/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (c^(3/2)*(b*e + a*f)*Sqrt[a + b*x^2]
*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*Sqrt[
(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))/c))/e^2 + e*(-1/3*(e*Sq
rt[a + b*x^2]*Sqrt[c + d*x^2])/(c*x^3) + (-(((b*c*e - 2*a*d*e + 3*a*c*f)*S
qrt[a + b*x^2]*Sqrt[c + d*x^2])/(a*c*x)) - (b*(-(d*(b*c*e - 2*a*d*e + 3*a*
c*f)*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*E
llipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c
*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))) + (c^(3/2)*(d*e - 3*c*f)
*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/
(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])))/(a*c))/(
3*c))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 442
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^q/(a*g*(m + 1))), x] - Simp[1/(a*g^2*(m + 1))   Int[(g*x)
^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[c*(b*e - a*f)*(m + 1) + e*2
*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*2*(p + q + 1))*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && GtQ[q, 0] && LtQ[m, -1]
&&  !(EqQ[q, 1] && SimplerQ[e + f*x^2, c + d*x^2])
```

rule 445

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 448

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[e   Int[(g*x)^m*(a + b*x
^2)^p*(c + d*x^2)^q*(e + f*x^2)^(r - 1), x], x] + Simp[f/e^2   Int[(g*x)^(m
 + 2)*(a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^(r - 1), x], x] /; FreeQ[{a,
b, c, d, e, f, g, m, p, q}, x] && IGtQ[r, 0]
```

Maple [A] (verified)

Time = 6.85 (sec) , antiderivative size = 386, normalized size of antiderivative = 0.59

method	result
elliptic	$\frac{\sqrt{(b\,x^2+a)(x^2d+c)}\left(-\frac{e^2\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{3c\,x^3}-\frac{e(6acf-2ade+bce)\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{3a\,c^2x}+\frac{\left(a\,f^2+2bef-\frac{db\,e^2}{3c}\right)\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x}}\right.}{\sqrt{}}$
risch	$-\frac{\sqrt{b\,x^2+a}\,\sqrt{x^2d+c}\,e(6acf\,x^2-2ade\,x^2+bce\,x^2+ace)}{3c^2x^3a}+\frac{\left(-\frac{b\left(3a\,c^2f^2+6acefd-2a\,d^2e^2+bcd\,e^2\right)c\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\left(\text{EllipticF}\left(x\sqrt{-}\right.\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+x^2b}}\right.}{}$
default	$\frac{\sqrt{b\,x^2+a}\,\sqrt{x^2d+c}\left(-6\sqrt{-\frac{b}{a}}\,abc\,d^2ef\,x^6+2\sqrt{-\frac{b}{a}}\,ab\,d^3e^2x^6-\sqrt{-\frac{b}{a}}\,b^2c\,d^2e^2x^6+3\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{x^2d+c}{c}}\,\text{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)a^2c\right.}{}$

input

```
int((b*x^2+a)^(1/2)*(f*x^2+e)^2/x^4/(d*x^2+c)^(1/2),x,method=_RETURNVERBOS
E)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-1/3/c*e^2*(b
*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/x^3-1/3/a/c^2*e*(6*a*c*f-2*a*d*e+b*c*e)*
(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/x+(a*f^2+2*b*e*f-1/3*d*b/c*e^2)/(-b/a)
^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(
1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-(f^2*b+1/3*b*d*e*(
6*a*c*f-2*a*d*e+b*c*e)/a/c^2)*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)
^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(-b/a)^(1/2),(-1
+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2)))
)
```

Fricas [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{x^4\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{bx^2+a}(fx^2+e)^2}{\sqrt{dx^2+c}x^4}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)^2/x^4/(d*x^2+c)^(1/2),x, algorithm="fr
icas")
```

output
```
integral((f^2*x^4 + 2*e*f*x^2 + e^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/(d*x^
6 + c*x^4), x)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{x^4\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{x^4\sqrt{c+dx^2}}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(f*x**2+e)**2/x**4/(d*x**2+c)**(1/2),x)
```

output
```
Integral(sqrt(a + b*x**2)*(e + f*x**2)**2/(x**4*sqrt(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{x^4\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{bx^2+a}(fx^2+e)^2}{\sqrt{dx^2+cx^4}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)^2/x^4/(d*x^2+c)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate(sqrt(b*x^2 + a)*(f*x^2 + e)^2/(sqrt(d*x^2 + c)*x^4), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{x^4\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{bx^2+a}(fx^2+e)^2}{\sqrt{dx^2+cx^4}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)^2/x^4/(d*x^2+c)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate(sqrt(b*x^2 + a)*(f*x^2 + e)^2/(sqrt(d*x^2 + c)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{x^4\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{b\,x^2+a}\,(f\,x^2+e)^2}{x^4\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(e + f*x^2)^2)/(x^4*(c + d*x^2)^(1/2)),x)
```

output
```
int(((a + b*x^2)^(1/2)*(e + f*x^2)^2)/(x^4*(c + d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{x^4\sqrt{c+dx^2}}\,dx = \text{Too large to display}$$

input
```
int((b*x^2+a)^(1/2)*(f*x^2+e)^2/x^4/(d*x^2+c)^(1/2),x)
```

output
```
( - sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*c*f**2 - 2*sqrt(c + d*x**2)*sqrt(a
 + b*x**2)*a*d*e*f + 2*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*d*f**2*x**2 + 2
*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b*c*f**2*x**2 - sqrt(c + d*x**2)*sqrt(a
 + b*x**2)*b*d*e**2 - 3*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a**2*c*d*
x**4 + a**2*d**2*x**6 + a*b*c**2*x**4 + 2*a*b*c*d*x**6 + a*b*d**2*x**8 + b
**2*c**2*x**6 + b**2*c*d*x**8),x)*a**3*c**2*d*f**2*x**3 - 6*int((sqrt(c +
d*x**2)*sqrt(a + b*x**2))/(a**2*c*d*x**4 + a**2*d**2*x**6 + a*b*c**2*x**4
+ 2*a*b*c*d*x**6 + a*b*d**2*x**8 + b**2*c**2*x**6 + b**2*c*d*x**8),x)*a**3
*c*d**2*e*f*x**3 + 2*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a**2*c*d*x**
4 + a**2*d**2*x**6 + a*b*c**2*x**4 + 2*a*b*c*d*x**6 + a*b*d**2*x**8 + b**2
*c**2*x**6 + b**2*c*d*x**8),x)*a**3*d**3*e**2*x**3 - 3*int((sqrt(c + d*x**
2)*sqrt(a + b*x**2))/(a**2*c*d*x**4 + a**2*d**2*x**6 + a*b*c**2*x**4 + 2*a
*b*c*d*x**6 + a*b*d**2*x**8 + b**2*c**2*x**6 + b**2*c*d*x**8),x)*a**2*b*c*
*3*f**2*x**3 - 6*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a**2*c*d*x**4 +
a**2*d**2*x**6 + a*b*c**2*x**4 + 2*a*b*c*d*x**6 + a*b*d**2*x**8 + b**2*c**
2*x**6 + b**2*c*d*x**8),x)*a**2*b*c**2*d*e*f*x**3 + int((sqrt(c + d*x**2)*
sqrt(a + b*x**2))/(a**2*c*d*x**4 + a**2*d**2*x**6 + a*b*c**2*x**4 + 2*a*b*
c*d*x**6 + a*b*d**2*x**8 + b**2*c**2*x**6 + b**2*c*d*x**8),x)*a**2*b*c*d**
2*e**2*x**3 - int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a**2*c*d*x**4 + a**
2*d**2*x**6 + a*b*c**2*x**4 + 2*a*b*c*d*x**6 + a*b*d**2*x**8 + b**2*c**...
```

3.116 $\int \frac{x^4\sqrt{a+bx^2}(e+fx^2)}{\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 532

$$\begin{aligned}
\int \frac{x^4\sqrt{a+bx^2}(e+fx^2)}{\sqrt{c+dx^2}}\,dx
&= \frac{(8a^3d^3f - ab^2cd(21de-16cf) - a^2bd^2(14de-9cf) + 8b^3c^2(7de-6cf))\,x\sqrt{c+dx^2}}{105b^2d^4\sqrt{a+bx^2}} \\
&- \frac{(4a^2d^2f + 4b^2c(7de-6cf) - abd(7de-5cf))\,x\sqrt{a+bx^2}\sqrt{c+dx^2}}{105b^2d^3} \\
&+ \frac{(7bde - 6bcf + adf)x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{35bd^2} + \frac{fx^5\sqrt{a+bx^2}\sqrt{c+dx^2}}{7d} \\
&- \frac{\sqrt{a}(8a^3d^3f - ab^2cd(21de-16cf) - a^2bd^2(14de-9cf) + 8b^3c^2(7de-6cf))\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right.\right.}{105b^{5/2}d^4\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} \\
&+ \frac{a^{3/2}(4a^2d^2f + 4b^2c(7de-6cf) - abd(7de-5cf))\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right), 1-\frac{ad}{bc}\right)}{105b^{5/2}d^3\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}
\end{aligned}$$

output
```
1/105*(8*a^3*d^3*f-a*b^2*c*d*(-16*c*f+21*d*e)-a^2*b*d^2*(-9*c*f+14*d*e)+8*
b^3*c^2*(-6*c*f+7*d*e))*x*(d*x^2+c)^(1/2)/b^2/d^4/(b*x^2+a)^(1/2)-1/105*(4
*a^2*d^2*f+4*b^2*c*(-6*c*f+7*d*e)-a*b*d*(-5*c*f+7*d*e))*x*(b*x^2+a)^(1/2)*
(d*x^2+c)^(1/2)/b^2/d^3+1/35*(a*d*f-6*b*c*f+7*b*d*e)*x^3*(b*x^2+a)^(1/2)*(
d*x^2+c)^(1/2)/b/d^2+1/7*f*x^5*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/d-1/105*a^(
1/2)*(8*a^3*d^3*f-a*b^2*c*d*(-16*c*f+21*d*e)-a^2*b*d^2*(-9*c*f+14*d*e)+8*b
^3*c^2*(-6*c*f+7*d*e))*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^
2/a)^(1/2),(1-a*d/b/c)^(1/2))/b^(5/2)/d^4/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(
b*x^2+a))^(1/2)+1/105*a^(3/2)*(4*a^2*d^2*f+4*b^2*c*(-6*c*f+7*d*e)-a*b*d*(-
5*c*f+7*d*e))*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1
-a*d/b/c)^(1/2))/b^(5/2)/d^3/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/
2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.58 (sec) , antiderivative size = 366, normalized size of antiderivative = 0.69

$$\int \frac{x^4\sqrt{a+bx^2}(e+fx^2)}{\sqrt{c+dx^2}}\,dx$$
$$= \frac{-\sqrt{\frac{b}{a}}dx(a+bx^2)\left(c+dx^2\right)\left(4a^2d^2f+abd(-7de+5cf-3dfx^2)+b^2(-24c^2f-3d^2x^2(7e+5fx^2)+2c\cdots\right.}{\cdots}$$

input
```
Integrate[(x^4*Sqrt[a + b*x^2]*(e + f*x^2))/Sqrt[c + d*x^2],x]
```

output
```
(-(Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2)*(4*a^2*d^2*f + a*b*d*(-7*d*e + 5*
c*f - 3*d*f*x^2) + b^2*(-24*c^2*f - 3*d^2*x^2*(7*e + 5*f*x^2) + 2*c*d*(14*
e + 9*f*x^2)))) - I*c*(8*a^3*d^3*f - 8*b^3*c^2*(-7*d*e + 6*c*f) + a^2*b*d^
2*(-14*d*e + 9*c*f) + a*b^2*c*d*(-21*d*e + 16*c*f))*Sqrt[1 + (b*x^2)/a]*Sq
rt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + I*c*(-(
b*c) + a*d)*(4*a^2*d^2*f + 8*b^2*c*(-7*d*e + 6*c*f) + a*b*d*(-7*d*e + 8*c*
f))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*
x], (a*d)/(b*c)])/(105*a^2*(b/a)^(5/2)*d^4*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]
)
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 479, normalized size of antiderivative = 0.90, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {443, 444, 444, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4\sqrt{a+bx^2}(e+fx^2)}{\sqrt{c+dx^2}}\,dx$$

$$\Big\downarrow \text{443}$$

$$\frac{\int \frac{x^4((7bde-6bcf+adf)x^2+a(7de-5cf))}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{7d}+\frac{fx^5\sqrt{a+bx^2}\sqrt{c+dx^2}}{7d}$$

$$\Big\downarrow \text{444}$$

$$\frac{\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}(adf-6bcf+7bde)}{5bd}-\frac{\int \frac{x^2\left(\left(4c(7de-6cf)b^2-ad(7de-5cf)b+4a^2d^2f\right)x^2+3ac(7bde-6bcf+adf)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{5bd}}{7d}+\frac{fx^5\sqrt{a+bx^2}\sqrt{c+dx^2}}{7d}$$

$$\Big\downarrow \text{444}$$

$$\frac{\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}(adf-6bcf+7bde)}{5bd}-\frac{\frac{1}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{4a^2df}{b}+5acf-7ade-\frac{24bc^2f}{d}+28bce\right)-\frac{\int \left(8c^2(7de-6cf)b^3-acd(21de-16cf)b^2-a^2d^2 \cdots\right.}{\cdots}}{5bd}}{7d}$$
$$\frac{fx^5\sqrt{a+bx^2}\sqrt{c+dx^2}}{7d}$$

$$\Big\downarrow \text{406}$$

$$\frac{\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}(adf-6bcf+7bde)}{5bd}-\frac{\frac{1}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{4a^2df}{b}+5acf-7ade-\frac{24bc^2f}{d}+28bce\right)-\frac{ac\left(4a^2d^2f-abd(7de-5cf)+4b^2c(7de-6cf)\right)\int \cdots}{\cdots}}{5\cdots}}{7d}$$
$$\frac{fx^5\sqrt{a+bx^2}\sqrt{c+dx^2}}{7d}$$

$$\Big\downarrow \text{320}$$

$$\frac{\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}(adf-6bcf+7bde)}{5bd}-\frac{\frac{1}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{4a^2df}{b}+5acf-7ade-\frac{24bc^2f}{d}+28bce\right)-\frac{\left(8a^3d^3f-a^2bd^2(14de-9cf)-ab^2cd(21de-16c\right.}{}}{7d}}{}\ \frac{fx^5\sqrt{a+bx^2}\sqrt{c+dx^2}}{7d}$$

$\downarrow$ 388

$$\frac{\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}(adf-6bcf+7bde)}{5bd}-\frac{\frac{1}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{4a^2df}{b}+5acf-7ade-\frac{24bc^2f}{d}+28bce\right)-\frac{\left(8a^3d^3f-a^2bd^2(14de-9cf)-ab^2cd(21de-16c\right.}{}}{7d}}{}\ \frac{fx^5\sqrt{a+bx^2}\sqrt{c+dx^2}}{7d}$$

$\downarrow$ 313

$$\frac{\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}(adf-6bcf+7bde)}{5bd}-\frac{\frac{1}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{4a^2df}{b}+5acf-7ade-\frac{24bc^2f}{d}+28bce\right)-\frac{\frac{c^{3/2}\sqrt{a+bx^2}\left(4a^2d^2f-abd(7de-5cf)+4b^2c(7\right.}{\sqrt{d}\sqrt{c+dx^2}\sqrt{}}}{}}{}}{}\ \frac{fx^5\sqrt{a+bx^2}\sqrt{c+dx^2}}{7d}$$

input
```
Int[(x^4*Sqrt[a + b*x^2]*(e + f*x^2))/Sqrt[c + d*x^2],x]
```

output
```
(f*x^5*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(7*d) + (((7*b*d*e - 6*b*c*f + a*d
*f)*x^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(5*b*d) - (((28*b*c*e - 7*a*d*e +
 5*a*c*f - (24*b*c^2*f)/d + (4*a^2*d*f)/b)*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^
2])/3 - ((8*a^3*d^3*f - a*b^2*c*d*(21*d*e - 16*c*f) - a^2*b*d^2*(14*d*e -
9*c*f) + 8*b^3*c^2*(7*d*e - 6*c*f))*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2
]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (
b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x
^2])) + (c^(3/2)*(4*a^2*d^2*f + 4*b^2*c*(7*d*e - 6*c*f) - a*b*d*(7*d*e - 5
*c*f))*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a
*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))/(3*
b*d))/(5*b*d))/(7*d)
```

Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 443
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*(g*x)^(m + 1)*(a + b*x^2)^(p
 + 1)*((c + d*x^2)^q/(b*g*(m + 2*(p + q + 1) + 1))), x] + Simp[1/(b*(m + 2*
(p + q + 1) + 1))   Int[(g*x)^m*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[c*((
b*e - a*f)*(m + 1) + b*e*2*(p + q + 1)) + (d*(b*e - a*f)*(m + 1) + f*2*q*(b
*c - a*d) + b*e*d*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f
, g, m, p}, x] && GtQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^2, c + d*x^
2])
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

Maple [A] (verified)

Time = 8.80 (sec) , antiderivative size = 582, normalized size of antiderivative = 1.09

method	result
elliptic	$\frac{\sqrt{(bx^2+a)(x^2d+c)}\left(\frac{f x^5\sqrt{bdx^4+adx^2+x^2bc+ac}}{7d}+\frac{\left(af+be-\frac{f(6ad+6bc)}{7d}\right)x^3\sqrt{bdx^4+adx^2+x^2bc+ac}}{5bd}+\frac{\left(ae-\frac{5acf}{7d}-\frac{\left(af+be-\frac{f(6ad+6bc)}{7d}\right)\cdots}{5bd}\cdots\right)\cdots}{\cdots}\cdots\right)}{\cdots}$
risch	$-\frac{x\left(-15fx^4b^2d^2-3abd^2fx^2+18b^2cfx^2d-21b^2d^2ex^2+4fd^2a^2+5fdcba-7abd^2e-24fc^2b^2+28db^2ce\right)\sqrt{bx^2+a}\,\sqrt{x^2d+c}}{105b^2d^3}+\cdots$
default	Expression too large to display

input
```
int(x^4*(b*x^2+a)^(1/2)*(f*x^2+e)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/7*f/d*x^5*(
b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+1/5*(a*f+b*e-1/7*f/d*(6*a*d+6*b*c))/b/d
*x^3*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+1/3*(a*e-5/7*a*c*f/d-1/5*(a*f+b*e
-1/7*f/d*(6*a*d+6*b*c))/b/d*(4*a*d+4*b*c))/b/d*x*(b*d*x^4+a*d*x^2+b*c*x^2+
a*c)^(1/2)-1/3*(a*e-5/7*a*c*f/d-1/5*(a*f+b*e-1/7*f/d*(6*a*d+6*b*c))/b/d*(4
*a*d+4*b*c))/b/d*a*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d
*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b
)^(1/2))-(-3/5*(a*f+b*e-1/7*f/d*(6*a*d+6*b*c))/b/d*a*c-1/3*(a*e-5/7*a*c*f/
d-1/5*(a*f+b*e-1/7*f/d*(6*a*d+6*b*c))/b/d*(4*a*d+4*b*c))/b/d*(2*a*d+2*b*c)
)*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*
x^2+a*c)^(1/2)/d*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-Ellip
ticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 499, normalized size of antiderivative = 0.94

$$\int \frac{x^4\sqrt{a+bx^2}(e+fx^2)}{\sqrt{c+dx^2}}\,dx =$$

$$-\frac{\sqrt{bd}(7\,(8\,b^3c^3d-3\,ab^2c^2d^2-2\,a^2bcd^3)e-(48\,b^3c^4-16\,ab^2c^3d-9\,a^2bc^2d^2-8\,a^3cd^3)f)x\sqrt{-\frac{c}{d}}E(\arcsi}{}$$

input
```
integrate(x^4*(b*x^2+a)^(1/2)*(f*x^2+e)/(d*x^2+c)^(1/2),x, algorithm="fric
as")
```

output
```
-1/105*(sqrt(b*d)*(7*(8*b^3*c^3*d - 3*a*b^2*c^2*d^2 - 2*a^2*b*c*d^3)*e - (
48*b^3*c^4 - 16*a*b^2*c^3*d - 9*a^2*b*c^2*d^2 - 8*a^3*c*d^3)*f)*x*sqrt(-c/
d)*elliptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - sqrt(b*d)*(7*(8*b^3*c^3*d
 - 3*a*b^2*c^2*d^2 - a^2*b*d^4 - 2*(a^2*b - 2*a*b^2)*c*d^3)*e - (48*b^3*c^
4 - 16*a*b^2*c^3*d - 4*a^3*d^4 - 3*(3*a^2*b - 8*a*b^2)*c^2*d^2 - (8*a^3 +
5*a^2*b)*c*d^3)*f)*x*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)/x), a*d/(b*c)
) - (15*b^3*d^4*f*x^6 + 3*(7*b^3*d^4*e - (6*b^3*c*d^3 - a*b^2*d^4)*f)*x^4
 - (7*(4*b^3*c*d^3 - a*b^2*d^4)*e - (24*b^3*c^2*d^2 - 5*a*b^2*c*d^3 - 4*a^2
*b*d^4)*f)*x^2 + 7*(8*b^3*c^2*d^2 - 3*a*b^2*c*d^3 - 2*a^2*b*d^4)*e - (48*b
^3*c^3*d - 16*a*b^2*c^2*d^2 - 9*a^2*b*c*d^3 - 8*a^3*d^4)*f)*sqrt(b*x^2 + a
)*sqrt(d*x^2 + c))/(b^3*d^5*x)
```

Sympy [F]

$$\int \frac{x^4\sqrt{a+bx^2}(e+fx^2)}{\sqrt{c+dx^2}}\,dx = \int \frac{x^4\sqrt{a+bx^2}\,(e+fx^2)}{\sqrt{c+dx^2}}\,dx$$

input
```
integrate(x**4*(b*x**2+a)**(1/2)*(f*x**2+e)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(x**4*sqrt(a + b*x**2)*(e + f*x**2)/sqrt(c + d*x**2), x)
```

Maxima [F]

$$\int \frac{x^4\sqrt{a+bx^2}(e+fx^2)}{\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{bx^2+a}(fx^2+e)x^4}{\sqrt{dx^2+c}}\,dx$$

input
```
integrate(x^4*(b*x^2+a)^(1/2)*(f*x^2+e)/(d*x^2+c)^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate(sqrt(b*x^2 + a)*(f*x^2 + e)*x^4/sqrt(d*x^2 + c), x)
```

Giac [F]

$$\int \frac{x^4\sqrt{a+bx^2}(e+fx^2)}{\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{bx^2+a}(fx^2+e)x^4}{\sqrt{dx^2+c}}\,dx$$

input
```
integrate(x^4*(b*x^2+a)^(1/2)*(f*x^2+e)/(d*x^2+c)^(1/2),x, algorithm="giac
")
```

output
```
integrate(sqrt(b*x^2 + a)*(f*x^2 + e)*x^4/sqrt(d*x^2 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4\sqrt{a+bx^2}(e+fx^2)}{\sqrt{c+dx^2}}\,dx = \int \frac{x^4\,\sqrt{b\,x^2+a}\,(f\,x^2+e)}{\sqrt{d\,x^2+c}}\,dx$$

input
```
int((x^4*(a + b*x^2)^(1/2)*(e + f*x^2))/(c + d*x^2)^(1/2),x)
```

output
```
int((x^4*(a + b*x^2)^(1/2)*(e + f*x^2))/(c + d*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^4\sqrt{a+bx^2}(e+fx^2)}{\sqrt{c+dx^2}}\,dx = \text{Too large to display}$$

input
```
int(x^4*(b*x^2+a)^(1/2)*(f*x^2+e)/(d*x^2+c)^(1/2),x)
```

output

```
( - 4*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*d**2*f*x - 5*sqrt(c + d*x**2)
*sqrt(a + b*x**2)*a*b*c*d*f*x + 7*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b*d*
*2*e*x + 3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b*d**2*f*x**3 + 24*sqrt(c +
 d*x**2)*sqrt(a + b*x**2)*b**2*c**2*f*x - 28*sqrt(c + d*x**2)*sqrt(a + b*x
**2)*b**2*c*d*e*x - 18*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**2*c*d*f*x**3 +
 21*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**2*d**2*e*x**3 + 15*sqrt(c + d*x**
2)*sqrt(a + b*x**2)*b**2*d**2*f*x**5 + 8*int((sqrt(c + d*x**2)*sqrt(a + b*
x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a**3*d**3*f + 9*int(
(sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*
x**4),x)*a**2*b*c*d**2*f - 14*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)
/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a**2*b*d**3*e + 16*int((sqrt(c
+ d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)
*a*b**2*c**2*d*f - 21*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c +
a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a*b**2*c*d**2*e - 48*int((sqrt(c + d*x*
*2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*b**3*
c**3*f + 56*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 +
 b*c*x**2 + b*d*x**4),x)*b**3*c**2*d*e + 4*int((sqrt(c + d*x**2)*sqrt(a +
b*x**2))/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a**3*c*d**2*f + 5*int((
sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),
x)*a**2*b*c**2*d*f - 7*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c + a...
```

3.117 $\int \frac{x^2\sqrt{a+bx^2}(e+fx^2)}{\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 388

$$
\begin{aligned}
&\int \frac{x^2\sqrt{a+bx^2}(e+fx^2)}{\sqrt{c+dx^2}}\,dx \\
&= -\frac{(2a^2d^2f+2b^2c(5de-4cf)-abd(5de-3cf))\,x\sqrt{c+dx^2}}{15bd^3\sqrt{a+bx^2}} \\
&\quad+\frac{(5bde-4bcf+adf)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{15bd^2}+\frac{fx^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5d} \\
&\quad+\frac{\sqrt{a}(2a^2d^2f+2b^2c(5de-4cf)-abd(5de-3cf))\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{15b^{3/2}d^3\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} \\
&\quad-\frac{a^{3/2}(5bde-4bcf+adf)\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{15b^{3/2}d^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}
\end{aligned}
$$

output
```
-1/15*(2*a^2*d^2*f+2*b^2*c*(-4*c*f+5*d*e)-a*b*d*(-3*c*f+5*d*e))*x*(d*x^2+c
)^(1/2)/b/d^3/(b*x^2+a)^(1/2)+1/15*(a*d*f-4*b*c*f+5*b*d*e)*x*(b*x^2+a)^(1/
2)*(d*x^2+c)^(1/2)/b/d^2+1/5*f*x^3*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/d+1/15*
a^(1/2)*(2*a^2*d^2*f+2*b^2*c*(-4*c*f+5*d*e)-a*b*d*(-3*c*f+5*d*e))*(d*x^2+c
)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/b
^(3/2)/d^3/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)-1/15*a^(3/2)*(a
*d*f-4*b*c*f+5*b*d*e)*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(
1/2)),(1-a*d/b/c)^(1/2))/b^(3/2)/d^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2
+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.73 (sec) , antiderivative size = 270, normalized size of antiderivative = 0.70

$$\int \frac{x^2\sqrt{a+bx^2}(e+fx^2)}{\sqrt{c+dx^2}}\,dx$$
$$= \frac{\sqrt{\frac{b}{a}}dx(a+bx^2)\left(c+dx^2\right)\left(adf+b(5de-4cf+3dfx^2)\right)+ic(2a^2d^2f+2b^2c(5de-4cf)+abd(-5de+3}{}$$

input
```
Integrate[(x^2*Sqrt[a + b*x^2]*(e + f*x^2))/Sqrt[c + d*x^2],x]
```

output
```
(Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2)*(a*d*f + b*(5*d*e - 4*c*f + 3*d*f*x
^2)) + I*c*(2*a^2*d^2*f + 2*b^2*c*(5*d*e - 4*c*f) + a*b*d*(-5*d*e + 3*c*f)
)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x]
, (a*d)/(b*c)] - I*c*(-(b*c) + a*d)*(-10*b*d*e + 8*b*c*f + a*d*f)*Sqrt[1 +
 (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b
*c)])/(15*b*Sqrt[b/a]*d^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 356, normalized size of antiderivative = 0.92, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {443, 444, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\sqrt{a+bx^2}\left(e+fx^2\right)}{\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \text{443}$$

$$\frac{\int \frac{x^2\left((5bde-4bcf+adf)x^2+a(5de-3cf)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{5d}+\frac{fx^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5d}$$

$$\Big\downarrow \text{444}$$

$$\frac{\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(adf-4bcf+5bde)}{3bd}-\frac{\int \frac{\left(2c(5de-4cf)b^2-ad(5de-3cf)b+2a^2d^2f\right)x^2+ac(5bde-4bcf+adf)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3bd}}{5d}+\frac{fx^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5d}$$

$$\Big\downarrow \text{406}$$

$$\frac{\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(adf-4bcf+5bde)}{3bd}-\frac{\left(2a^2d^2f-abd(5de-3cf)+2b^2c(5de-4cf)\right)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+ac(adf-4bcf+5bde)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3bd}}{5d}+\frac{fx^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5d}$$

$$\Big\downarrow \text{320}$$

$$\frac{\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(adf-4bcf+5bde)}{3bd}-\frac{\left(2a^2d^2f-abd(5de-3cf)+2b^2c(5de-4cf)\right)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}(adf-4bcf+5bde)\text{EllipticF}\left(\ldots\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}\ldots}}{3bd}}{5d}+\frac{fx^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5d}$$

$$\Big\downarrow \text{388}$$

$$\frac{\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(adf-4bcf+5bde)}{3bd}-\frac{(2a^2d^2f-abd(5de-3cf)+2b^2c(5de-4cf))\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(adf-4bcf+5b\cdots}{\sqrt{d}\sqrt{c+\cdots}}}{3bd}}{5d}$$

$$\frac{fx^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5d}$$

$$\Big\downarrow \text{313}$$

$$\frac{\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(adf-4bcf+5bde)}{3bd}-\frac{(2a^2d^2f-abd(5de-3cf)+2b^2c(5de-4cf))\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)+\frac{c^{3/2}\sqrt{a+\cdots}}{\cdots}}{3bd}}{5d}$$

$$\frac{fx^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5d}$$

input
```
Int[(x^2*Sqrt[a + b*x^2]*(e + f*x^2))/Sqrt[c + d*x^2],x]
```

output
```
(f*x^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(5*d) + (((5*b*d*e - 4*b*c*f + a*d
*f)*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(3*b*d) - ((2*a^2*d^2*f + 2*b^2*c*(
5*d*e - 4*c*f) - a*b*d*(5*d*e - 3*c*f))*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d
*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1
 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c +
 d*x^2])) + (c^(3/2)*(5*b*d*e - 4*b*c*f + a*d*f)*Sqrt[a + b*x^2]*EllipticF
[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^
2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))/(3*b*d))/(5*d)
```

Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 443
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*(g*x)^(m + 1)*(a + b*x^2)^(p
 + 1)*((c + d*x^2)^q/(b*g*(m + 2*(p + q + 1) + 1))), x] + Simp[1/(b*(m + 2*
(p + q + 1) + 1))   Int[(g*x)^m*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[c*((
b*e - a*f)*(m + 1) + b*e*2*(p + q + 1)) + (d*(b*e - a*f)*(m + 1) + f*2*q*(b
*c - a*d) + b*e*d*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f
, g, m, p}, x] && GtQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^2, c + d*x^
2])
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

Maple [A] (verified)

Time = 7.07 (sec) , antiderivative size = 417, normalized size of antiderivative = 1.07

method	result
elliptic	$\frac{\sqrt{(b\,x^2+a)(x^2d+c)}\left(\frac{f\,x^3\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{5d}+\frac{\left(af+be-\frac{f(4ad+4bc)}{5d}\right)x\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{3bd}-\frac{\left(af+be-\frac{f(4ad+4bc)}{5d}\right)ac\sqrt{1+\frac{b\,x^2}{a}}}{3bd\sqrt{-\frac{b}{a}}\,\sqrt{b}}\right.}{}$
risch	$\frac{x\left(3bdf\,x^2+adf-4bcf+5bde\right)\sqrt{b\,x^2+a}\,\sqrt{x^2d+c}}{15b\,d^2}-\frac{\left(-\frac{\left(2f\,d^2a^2+3fdcba-5ab\,d^2e-8f\,c^2b^2+10d\,b^2ce\right)c\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\left(\operatorname{EllipticF}\left(x\sqrt{}\right.\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+x^2bc+}}\right.}{}$
default	$\frac{\sqrt{b\,x^2+a}\,\sqrt{x^2d+c}\,\left(3\sqrt{-\frac{b}{a}}\,b^2d^3f\,x^7+4\sqrt{-\frac{b}{a}}\,ab\,d^3f\,x^5-\sqrt{-\frac{b}{a}}\,b^2c\,d^2f\,x^5+5\sqrt{-\frac{b}{a}}\,b^2d^3e\,x^5+\sqrt{-\frac{b}{a}}\,a^2d^3f\,x^3+5\sqrt{-\frac{b}{a}}\,ab\,d^3e\,x^3-4\right.}{}$

input
```
int(x^2*(b*x^2+a)^(1/2)*(f*x^2+e)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/5*f/d*x^3*(
b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+1/3*(a*f+b*e-1/5*f/d*(4*a*d+4*b*c))/b/d
*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)-1/3*(a*f+b*e-1/5*f/d*(4*a*d+4*b*c))
/b/d*a*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2
+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-(a*
e-3/5*a*c*f/d-1/3*(a*f+b*e-1/5*f/d*(4*a*d+4*b*c))/b/d*(2*a*d+2*b*c))*c/(-b
/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c
)^(1/2)/d*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*
(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 324, normalized size of antiderivative = 0.84

$$\int \frac{x^2\sqrt{a+bx^2}(e+fx^2)}{\sqrt{c+dx^2}}\,dx$$

$$= \frac{\sqrt{bd}(5\,(2\,b^2c^2d-abcd^2)e-(8\,b^2c^3-3\,abc^2d-2\,a^2cd^2)f)x\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ad}{bc})-\sqrt{bd}(5\,(2\,b^2c^2}{}$$

input
```
integrate(x^2*(b*x^2+a)^(1/2)*(f*x^2+e)/(d*x^2+c)^(1/2),x, algorithm="fric
as")
```

output
```
1/15*(sqrt(b*d)*(5*(2*b^2*c^2*d - a*b*c*d^2)*e - (8*b^2*c^3 - 3*a*b*c^2*d
- 2*a^2*c*d^2)*f)*x*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c))
 - sqrt(b*d)*(5*(2*b^2*c^2*d - a*b*c*d^2 + a*b*d^3)*e - (8*b^2*c^3 - 3*a*b
*c^2*d - a^2*d^3 - 2*(a^2 - 2*a*b)*c*d^2)*f)*x*sqrt(-c/d)*elliptic_f(arcsi
n(sqrt(-c/d)/x), a*d/(b*c)) + (3*b^2*d^3*f*x^4 + (5*b^2*d^3*e - (4*b^2*c*d
^2 - a*b*d^3)*f)*x^2 - 5*(2*b^2*c*d^2 - a*b*d^3)*e + (8*b^2*c^2*d - 3*a*b*
c*d^2 - 2*a^2*d^3)*f)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(b^2*d^4*x)
```

Sympy [F]

$$\int \frac{x^2\sqrt{a+bx^2}(e+fx^2)}{\sqrt{c+dx^2}}\,dx=\int \frac{x^2\sqrt{a+bx^2}(e+fx^2)}{\sqrt{c+dx^2}}\,dx$$

input
```
integrate(x**2*(b*x**2+a)**(1/2)*(f*x**2+e)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(x**2*sqrt(a + b*x**2)*(e + f*x**2)/sqrt(c + d*x**2), x)
```

Maxima [F]

$$\int \frac{x^2\sqrt{a+bx^2}(e+fx^2)}{\sqrt{c+dx^2}}\,dx=\int \frac{\sqrt{bx^2+a}(fx^2+e)x^2}{\sqrt{dx^2+c}}\,dx$$

input
```
integrate(x^2*(b*x^2+a)^(1/2)*(f*x^2+e)/(d*x^2+c)^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate(sqrt(b*x^2 + a)*(f*x^2 + e)*x^2/sqrt(d*x^2 + c), x)
```

Giac [F]

$$\int \frac{x^2\sqrt{a+bx^2}(e+fx^2)}{\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{bx^2+a}(fx^2+e)x^2}{\sqrt{dx^2+c}}\,dx$$

input
```
integrate(x^2*(b*x^2+a)^(1/2)*(f*x^2+e)/(d*x^2+c)^(1/2),x, algorithm="giac
")
```

output
```
integrate(sqrt(b*x^2 + a)*(f*x^2 + e)*x^2/sqrt(d*x^2 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2\sqrt{a+bx^2}(e+fx^2)}{\sqrt{c+dx^2}}\,dx = \int \frac{x^2\,\sqrt{b\,x^2+a}\,\left(f\,x^2+e\right)}{\sqrt{d\,x^2+c}}\,dx$$

input
```
int((x^2*(a + b*x^2)^(1/2)*(e + f*x^2))/(c + d*x^2)^(1/2),x)
```

output
```
int((x^2*(a + b*x^2)^(1/2)*(e + f*x^2))/(c + d*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^2\sqrt{a+bx^2}(e+fx^2)}{\sqrt{c+dx^2}}\,dx$$
$$= \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,adfx-4\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,bcfx+5\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,bdex+3\sqrt{d\,x^2+c}\,\sqrt{b\,x^2}}{}$$

input
```
int(x^2*(b*x^2+a)^(1/2)*(f*x^2+e)/(d*x^2+c)^(1/2),x)
```

output
```
(sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*d*f*x - 4*sqrt(c + d*x**2)*sqrt(a + b
*x**2)*b*c*f*x + 5*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b*d*e*x + 3*sqrt(c +
d*x**2)*sqrt(a + b*x**2)*b*d*f*x**3 - 2*int((sqrt(c + d*x**2)*sqrt(a + b*x
**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a**2*d**2*f - 3*int((
sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x
**4),x)*a*b*c*d*f + 5*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c +
a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a*b*d**2*e + 8*int((sqrt(c + d*x**2)*sq
rt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*b**2*c**2*f
 - 10*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x
**2 + b*d*x**4),x)*b**2*c*d*e - int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a
*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a**2*c*d*f + 4*int((sqrt(c + d*x**
2)*sqrt(a + b*x**2))/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a*b*c**2*f
- 5*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c + a*d*x**2 + b*c*x**2 + b
*d*x**4),x)*a*b*c*d*e)/(15*b*d**2)
```

3.118 $\int \frac{\sqrt{a+bx^2}(e+fx^2)}{\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 284

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{\sqrt{c+dx^2}}\,dx = \frac{(3bde-2bcf+adf)x\sqrt{c+dx^2}}{3d^2\sqrt{a+bx^2}}+\frac{fx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3d} - \frac{\sqrt{a}(3bde-2bcf+adf)\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{3\sqrt{b}d^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} + \frac{a^{3/2}(3de-cf)\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{3\sqrt{b}cd\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
1/3*(a*d*f-2*b*c*f+3*b*d*e)*x*(d*x^2+c)^(1/2)/d^2/(b*x^2+a)^(1/2)+1/3*f*x*
(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/d-1/3*a^(1/2)*(a*d*f-2*b*c*f+3*b*d*e)*(d*x
^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2
))/b^(1/2)/d^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)+1/3*a^(3/2)
*(-c*f+3*d*e)*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1
-a*d/b/c)^(1/2))/b^(1/2)/c/d/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/
2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.40 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.76

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{\sqrt{c+dx^2}}\,dx = \frac{\sqrt{\frac{b}{a}}dfx(a+bx^2)\left(c+dx^2\right)-ic(3bde-2bcf+adf)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(i\text{arcsinh}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)+i(-bc}{3\sqrt{\frac{b}{a}}d^2\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

input
```
Integrate[(Sqrt[a + b*x^2]*(e + f*x^2))/Sqrt[c + d*x^2],x]
```

output
```
(Sqrt[b/a]*d*f*x*(a + b*x^2)*(c + d*x^2) - I*c*(3*b*d*e - 2*b*c*f + a*d*f)
*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x],
 (a*d)/(b*c)] + I*(-(b*c) + a*d)*(-3*d*e + 2*c*f)*Sqrt[1 + (b*x^2)/a]*Sqrt
[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(3*Sqrt[b/
a]*d^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 265, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {403, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{\sqrt{c+dx^2}}\,dx$$

$$\Big\downarrow \text{ 403}$$

$$\frac{\int \frac{(3bde-2bcf+adf)x^2+a(3de-cf)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3d}+\frac{fx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3d}$$

$$\Big\downarrow \text{ 406}$$

$$\frac{\frac{a(3de-cf)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+(adf-2bcf+3bde)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3d}+\frac{fx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3d}}{}$$

$$\Big\downarrow \; 320$$

$$\frac{(adf-2bcf+3bde)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{\sqrt{c}\sqrt{a+bx^2}(3de-cf)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}}{3d}+\frac{fx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3d}$$

$$\Big\downarrow \; 388$$

$$\frac{(adf-2bcf+3bde)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}}dx}{b}\right)+\frac{\sqrt{c}\sqrt{a+bx^2}(3de-cf)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}}{3d}+\frac{fx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3d}$$

$$\Big\downarrow \; 313$$

$$\frac{\frac{\sqrt{c}\sqrt{a+bx^2}(3de-cf)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+(adf-2bcf+3bde)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{3d}+\frac{fx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3d}$$

input
```
Int[(Sqrt[a + b*x^2]*(e + f*x^2))/Sqrt[c + d*x^2],x]
```

output
```
(f*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(3*d) + ((3*b*d*e - 2*b*c*f + a*d*f)
*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*Ellip
ticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a
+ b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (Sqrt[c]*(3*d*e - c*f)*Sqrt
[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt
[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))/(3*d)
```

Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

Maple [A] (verified)

Time = 5.18 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.10

method	result
elliptic	$\frac{\sqrt{(b\,x^2+a)(x^2d+c)}\left(\frac{fx\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{3d}+\frac{\left(ae-\frac{acf}{3d}\right)\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}-\frac{\left(af+be-\frac{f(2ad+2bc)}{3d}\right.}{}\right.}{\sqrt{b\,x^2+a}\,\sqrt{x^2d+c}}$
risch	$\frac{fx\sqrt{b\,x^2+a}\,\sqrt{x^2d+c}}{3d}-\frac{\left(\frac{(adf-2bcf+3bde)c\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\left(\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)-\operatorname{EllipticE}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}\,d}+\right.}{}$
default	$-\frac{\sqrt{b\,x^2+a}\,\sqrt{x^2d+c}\,\left(-\sqrt{-\frac{b}{a}}\,b\,d^2f\,x^5-\sqrt{-\frac{b}{a}}\,a\,d^2f\,x^3-\sqrt{-\frac{b}{a}}\,bcdf\,x^3+2\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{x^2d+c}{c}}\,\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)acdf-3\sqrt{\frac{b}{}}\right.}{}$

input
```
int((b*x^2+a)^(1/2)*(f*x^2+e)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/3*f/d*x*(b*
d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+(a*e-1/3*a*c*f/d)/(-b/a)^(1/2)*(1+b*x^2/a
)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*
(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-(a*f+b*e-1/3*f/d*(2*a*d+2*b*c))*c/(
-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a
*c)^(1/2)/d*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(
x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.75

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{\sqrt{c+dx^2}}\,dx = -\frac{(3\,bc^2de-(2\,bc^3-ac^2d)f)\sqrt{bd}x\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ad}{bc})-\sqrt{bd}(3\,(bc^2d+ad^3)e-(2\,bc^3-ac^2d-}{3\,bc}$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*((3*b*c^2*d*e - (2*b*c^3 - a*c^2*d)*f)*sqrt(b*d)*x*sqrt(-c/d)*ellipti
c_e(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - sqrt(b*d)*(3*(b*c^2*d + a*d^3)*e -
(2*b*c^3 - a*c^2*d + a*c*d^2)*f)*x*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)
/x), a*d/(b*c)) - (b*c*d^2*f*x^2 + 3*b*c*d^2*e - (2*b*c^2*d - a*c*d^2)*f)*
sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(b*c*d^3*x)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{a+bx^2}(e+fx^2)}{\sqrt{c+dx^2}}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(f*x**2+e)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(sqrt(a + b*x**2)*(e + f*x**2)/sqrt(c + d*x**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{bx^2+a}(fx^2+e)}{\sqrt{dx^2+c}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)*(f*x^2 + e)/sqrt(d*x^2 + c), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{bx^2+a}(fx^2+e)}{\sqrt{dx^2+c}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^2 + a)*(f*x^2 + e)/sqrt(d*x^2 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{b\,x^2+a}\,(f\,x^2+e)}{\sqrt{d\,x^2+c}}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(e + f*x^2))/(c + d*x^2)^(1/2),x)
```

output
```
int(((a + b*x^2)^(1/2)*(e + f*x^2))/(c + d*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{\sqrt{c+dx^2}}\,dx$$
$$= \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,fx+\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^2}{bd\,x^4+ad\,x^2+bc\,x^2+ac}dx\right)adf-2\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^2}{bd\,x^4+ad\,x^2+bc\,x^2+ac}dx\right)bcf+3\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{}}{bd\,x^4+ad\,x^2}\right.}{3d}$$

input
```
int((b*x^2+a)^(1/2)*(f*x^2+e)/(d*x^2+c)^(1/2),x)
```

output
```
(sqrt(c + d*x**2)*sqrt(a + b*x**2)*f*x + int((sqrt(c + d*x**2)*sqrt(a + b*
x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a*d*f - 2*int((sqrt(
c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),
x)*b*c*f + 3*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2
+ b*c*x**2 + b*d*x**4),x)*b*d*e - int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/
(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a*c*f + 3*int((sqrt(c + d*x**2)*
sqrt(a + b*x**2))/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a*d*e)/(3*d)
```

3.119 $\int \frac{\sqrt{a+bx^2}(e+fx^2)}{x^2\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 262

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{x^2\sqrt{c+dx^2}}\,dx = \frac{b(de+cf)x\sqrt{c+dx^2}}{cd\sqrt{a+bx^2}} - \frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{cx} - \frac{\sqrt{a}\sqrt{b}(de+cf)\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{cd\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} + \frac{\sqrt{a}(be+af)\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}c\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
b*(c*f+d*e)*x*(d*x^2+c)^(1/2)/c/d/(b*x^2+a)^(1/2)-e*(b*x^2+a)^(1/2)*(d*x^2
+c)^(1/2)/c/x-a^(1/2)*b^(1/2)*(c*f+d*e)*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*
x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/c/d/(b*x^2+a)^(1/2)/(a*(d*x
^2+c)/c/(b*x^2+a))^(1/2)+a^(1/2)*(a*f+b*e)*(d*x^2+c)^(1/2)*InverseJacobiAM
(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/b^(1/2)/c/(b*x^2+a)^(1/2)/(a
*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.41 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{x^2\sqrt{c+dx^2}}\,dx$$
$$= \frac{-\sqrt{\frac{b}{a}}de(a+bx^2)\,(c+dx^2)-ibc(de+cf)x\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(i\text{arcsinh}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)-ic(-bc+ad)}{\sqrt{\frac{b}{a}}cdx\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

input
```
Integrate[(Sqrt[a + b*x^2]*(e + f*x^2))/(x^2*Sqrt[c + d*x^2]),x]
```

output
```
(-(Sqrt[b/a]*d*e*(a + b*x^2)*(c + d*x^2)) - I*b*c*(d*e + c*f)*x*Sqrt[1 + (
b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c
)] - I*c*(-(b*c) + a*d)*f*x*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Ellipt
icF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(Sqrt[b/a]*c*d*x*Sqrt[a + b*x^2]
*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 256, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {442, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{x^2\sqrt{c+dx^2}}\,dx$$
$$\Big\downarrow \text{ 442}$$
$$\frac{\int \frac{b(de+cf)x^2+c(be+af)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{c}-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{cx}$$
$$\Big\downarrow \text{ 406}$$

$$\frac{c(af+be)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+b(cf+de)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{c}-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{cx}$$

$$\Big\downarrow \text{320}$$

$$\frac{b(cf+de)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}(af+be)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}}{c}-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{cx}$$

$$\Big\downarrow \text{388}$$

$$\frac{b(cf+de)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(af+be)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}}{c}-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{cx}$$

$$\Big\downarrow \text{313}$$

$$\frac{\frac{c^{3/2}\sqrt{a+bx^2}(af+be)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+b(cf+de)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{c}-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{cx}$$

input
```
Int[(Sqrt[a + b*x^2]*(e + f*x^2))/(x^2*Sqrt[c + d*x^2]),x]
```

output
```
-((e*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(c*x)) + (b*(d*e + c*f)*((x*Sqrt[a +
 b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(
Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*
(c + d*x^2))]*Sqrt[c + d*x^2])) + (c^(3/2)*(b*e + a*f)*Sqrt[a + b*x^2]*Ell
ipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*Sqrt[(c*(
a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))/c
```

Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 442
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^q/(a*g*(m + 1))), x] - Simp[1/(a*g^2*(m + 1))   Int[(g*x)
^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[c*(b*e - a*f)*(m + 1) + e*2
*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*2*(p + q + 1))*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && GtQ[q, 0] && LtQ[m, -1]
&&  !(EqQ[q, 1] && SimplerQ[e + f*x^2, c + d*x^2])
```

Maple [A] (verified)

Time = 5.13 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.14

method	result
elliptic	$\frac{\sqrt{(b\,x^{2}+a)(x^{2}d+c)}\,\left(-\frac{e\sqrt{bd\,x^{4}+ad\,x^{2}+x^{2}bc+ac}}{cx}+\frac{(af+be)\sqrt{1+\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^{4}+ad\,x^{2}+x^{2}bc+ac}}-\frac{\left(fb+\frac{dbe}{c}\right)c\sqrt{1+\frac{b\,x^{2}}{a}}\,\sqrt{\cdots}}{\cdots}\right.}{\sqrt{b\,x^{2}+a}\,\sqrt{x^{2}d+c}}$
default	$\frac{\sqrt{b\,x^{2}+a}\,\sqrt{x^{2}d+c}\,\left(-\sqrt{-\frac{b}{a}}\,b\,d^{2}e\,x^{4}+\sqrt{\frac{b\,x^{2}+a}{a}}\,\sqrt{\frac{x^{2}d+c}{c}}\,\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)acdfx-\sqrt{\frac{b\,x^{2}+a}{a}}\,\sqrt{\frac{x^{2}d+c}{c}}\,\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\cdots}\right.\right.}{\cdots}$
risch	$-\frac{e\sqrt{b\,x^{2}+a}\,\sqrt{x^{2}d+c}}{cx}+\frac{\left(-\frac{b(cf+de)c\sqrt{1+\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,\left(\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)-\operatorname{EllipticE}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^{4}+ad\,x^{2}+x^{2}bc+ac}\,d}+\frac{acf\sqrt{\cdots}}{\cdots}\right.}{\cdots}$

input

```
int((b*x^2+a)^(1/2)*(f*x^2+e)/x^2/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-1/c*e*(b*d*x
^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/x+(a*f+b*e)/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(
1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1
/2),(-1+(a*d+b*c)/c/b)^(1/2))-(f*b+d*b/c*e)*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/
2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(-
b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c
)/c/b)^(1/2))))
```

Fricas [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{x^2\sqrt{c+dx^2}}\,dx=\int \frac{\sqrt{bx^2+a}(fx^2+e)}{\sqrt{dx^2+c}x^2}\,dx$$

input

```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)/x^2/(d*x^2+c)^(1/2),x, algorithm="fric
as")
```

output

```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)/(d*x^4 + c*x^2), x)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{x^2\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{a+bx^2}(e+fx^2)}{x^2\sqrt{c+dx^2}}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(f*x**2+e)/x**2/(d*x**2+c)**(1/2),x)
```

output
```
Integral(sqrt(a + b*x**2)*(e + f*x**2)/(x**2*sqrt(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{x^2\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{bx^2+a}(fx^2+e)}{\sqrt{dx^2+c}x^2}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)/x^2/(d*x^2+c)^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate(sqrt(b*x^2 + a)*(f*x^2 + e)/(sqrt(d*x^2 + c)*x^2), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{x^2\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{bx^2+a}(fx^2+e)}{\sqrt{dx^2+c}x^2}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)/x^2/(d*x^2+c)^(1/2),x, algorithm="giac
")
```

output
```
integrate(sqrt(b*x^2 + a)*(f*x^2 + e)/(sqrt(d*x^2 + c)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{x^2\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{b\,x^2+a}\,(f\,x^2+e)}{x^2\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(e + f*x^2))/(x^2*(c + d*x^2)^(1/2)),x)
```

output
```
int(((a + b*x^2)^(1/2)*(e + f*x^2))/(x^2*(c + d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{x^2\sqrt{c+dx^2}}\,dx$$
$$= \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,f + \left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{bd\,x^6+ad\,x^4+bc\,x^4+ac\,x^2}dx\right)acfx + \left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{bd\,x^6+ad\,x^4+bc\,x^4+ac\,x^2}dx\right)adex + \left(\int \frac{\sqrt{d\,x}}{bd\,x^4+}\right.}{dx}$$

input
```
int((b*x^2+a)^(1/2)*(f*x^2+e)/x^2/(d*x^2+c)^(1/2),x)
```

output
```
(sqrt(c + d*x**2)*sqrt(a + b*x**2)*f + int((sqrt(c + d*x**2)*sqrt(a + b*x*
*2))/(a*c*x**2 + a*d*x**4 + b*c*x**4 + b*d*x**6),x)*a*c*f*x + int((sqrt(c
+ d*x**2)*sqrt(a + b*x**2))/(a*c*x**2 + a*d*x**4 + b*c*x**4 + b*d*x**6),x)
*a*d*e*x + int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c + a*d*x**2 + b*c*x
**2 + b*d*x**4),x)*a*d*f*x + int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c
+ a*d*x**2 + b*c*x**2 + b*d*x**4),x)*b*d*e*x)/(d*x)
```

3.120 $\int \frac{\sqrt{a+bx^2}(e+fx^2)}{x^4\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 284

$$\begin{aligned}\int \frac{\sqrt{a+bx^2}(e+fx^2)}{x^4\sqrt{c+dx^2}}\,dx &= -\frac{(bce-2ade+3acf)\sqrt{c+dx^2}}{3c^2x\sqrt{a+bx^2}} - \frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{3cx^3} \\ &\quad - \frac{\sqrt{b}(bce-2ade+3acf)\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{3\sqrt{a}c^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} \\ &\quad - \frac{\sqrt{a}\sqrt{b}(de-3cf)\sqrt{c+dx^2}\,\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{3c^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}\end{aligned}$$

output
```
-1/3*(3*a*c*f-2*a*d*e+b*c*e)*(d*x^2+c)^(1/2)/c^2/x/(b*x^2+a)^(1/2)-1/3*e*(
b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/c/x^3-1/3*b^(1/2)*(3*a*c*f-2*a*d*e+b*c*e)*(
d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(
1/2))/a^(1/2)/c^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)-1/3*a^(1
/2)*b^(1/2)*(-3*c*f+d*e)*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/
a^(1/2)),(1-a*d/b/c)^(1/2))/c^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^
(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.76 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.87

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{x^4\sqrt{c+dx^2}}\,dx$$
$$= \frac{\sqrt{\frac{b}{a}}\left(-\sqrt{\frac{b}{a}}(a+bx^2)(c+dx^2)(bcex^2-2adex^2+ac(e+3fx^2))-ibc(bce-2ade+3acf)x^3\sqrt{1+\frac{bx^2}{a}}\sqrt{\right.}{3bc^2x^3\sqrt{a+}}$$

input
```
Integrate[(Sqrt[a + b*x^2]*(e + f*x^2))/(x^4*Sqrt[c + d*x^2]),x]
```

output
```
(Sqrt[b/a]*(-(Sqrt[b/a]*(a + b*x^2)*(c + d*x^2)*(b*c*e*x^2 - 2*a*d*e*x^2 +
 a*c*(e + 3*f*x^2))) - I*b*c*(b*c*e - 2*a*d*e + 3*a*c*f)*x^3*Sqrt[1 + (b*x
^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]
- I*b*c*(-(b*c) + a*d)*e*x^3*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Ellip
ticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(3*b*c^2*x^3*Sqrt[a + b*x^2]*S
qrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.15, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {442, 445, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{x^4\sqrt{c+dx^2}}\,dx$$
$$\Big\downarrow \; 442$$
$$\frac{\int \frac{-b(de-3cf)x^2+bce-2ade+3acf}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3c}-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{3cx^3}$$
$$\Big\downarrow \; 445$$

$$\frac{-\frac{\int \frac{b\left(ac(de-3cf)-d(bce-2ade+3acf)x^2\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(3acf-2ade+bce)}{acx}}{3c}-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{3cx^3}$$

$\downarrow$ 27

$$\frac{-\frac{b\int \frac{ac(de-3cf)-d(bce-2ade+3acf)x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(3acf-2ade+bce)}{acx}}{3c}-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{3cx^3}$$

$\downarrow$ 406

$$\frac{-\frac{b\left(ac(de-3cf)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx-d(3acf-2ade+bce)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(3acf-2ade+bce)}{acx}}{3c}-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{3cx^3}$$

$\downarrow$ 320

$$\frac{-\frac{b\left(\frac{c^{3/2}\sqrt{a+bx^2}(de-3cf)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-d(3acf-2ade+bce)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(3acf-2ade+bce)}{acx}}{3c}-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{3cx^3}$$

$\downarrow$ 388

$$\frac{-\frac{b\left(\frac{c^{3/2}\sqrt{a+bx^2}(de-3cf)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-d(3acf-2ade+bce)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)\right)}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(3acf-2a}{acx}}{3c}$$

$$\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{3cx^3}$$

$\downarrow$ 313

$$-\frac{-\frac{b\left(\frac{c^{3/2}\sqrt{a+bx^2}(de-3cf)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-d(3acf-2ade+bce)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)\right)}{ac}-\frac{\sqrt{a+bx^2}\sqrt{}}{}}{3c}-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{3cx^3}$$

input
```
Int[(Sqrt[a + b*x^2]*(e + f*x^2))/(x^4*Sqrt[c + d*x^2]),x]
```

output
```
-1/3*(e*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(c*x^3) + (-(((b*c*e - 2*a*d*e +
3*a*c*f)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(a*c*x)) - (b*(-(d*(b*c*e - 2*a*
d*e + 3*a*c*f)*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a
+ b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[
d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))) + (c^(3/2)*(d*
e - 3*c*f)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c
)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))
)/(a*c))/(3*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 442
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^q/(a*g*(m + 1))), x] - Simp[1/(a*g^2*(m + 1))   Int[(g*x)
^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[c*(b*e - a*f)*(m + 1) + e*2
*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*2*(p + q + 1))*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && GtQ[q, 0] && LtQ[m, -1]
&&  !(EqQ[q, 1] && SimplerQ[e + f*x^2, c + d*x^2])
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 6.20 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.27

method	result
elliptic	$\frac{\sqrt{(bx^2+a)(x^2d+c)}\,\Big(-\frac{e\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{3c\,x^3}-\frac{(3acf-2ade+bce)\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{3a\,c^2x}+\frac{\left(fb-\frac{dbe}{3c}\right)\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\mathrm{EllipticF}\Big(x\sqrt{}}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+x^2bc}}}{\sqrt{b\,x^2+a}\,\sqrt{x}}$
risch	$-\frac{\sqrt{b\,x^2+a}\,\sqrt{x^2d+c}\,(3acf\,x^2-2ade\,x^2+bce\,x^2+ace)}{3c^2x^3a}+\frac{b\Big(-\frac{(3acf-2ade+bce)c\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\left(\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}}{}$
default	$\frac{\sqrt{b\,x^2+a}\,\sqrt{x^2d+c}\,\Big(-3\sqrt{-\frac{b}{a}}\,abcdf\,x^6+2\sqrt{-\frac{b}{a}}\,ab\,d^2e\,x^6-\sqrt{-\frac{b}{a}}\,b^2cde\,x^6+\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{x^2d+c}{c}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)abcde\,x^3-}{}$

input
```
int((b*x^2+a)^(1/2)*(f*x^2+e)/x^4/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-1/3/c*e*(b*d
*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/x^3-1/3/a/c^2*(3*a*c*f-2*a*d*e+b*c*e)*(b*d
*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/x+(f*b-1/3*d*b/c*e)/(-b/a)^(1/2)*(1+b*x^2/
a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x
*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-1/3*b*(3*a*c*f-2*a*d*e+b*c*e)/a/c/
(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+
a*c)^(1/2)*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x
*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.68

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{x^4\sqrt{c+dx^2}}\,dx$$
$$=\frac{(3\,abcf+(b^2c-2\,abd)e)\sqrt{ac}x^3\sqrt{-\frac{b}{a}}E(\arcsin\left(x\sqrt{-\frac{b}{a}}\right)\mid\frac{ad}{bc})-(3\,(a^2+ab)cf+(b^2c-(a^2+2\,ab)d)e)}{3\,a^2c^2x^3}$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)/x^4/(d*x^2+c)^(1/2),x, algorithm="fric
as")
```

output
```
1/3*((3*a*b*c*f + (b^2*c - 2*a*b*d)*e)*sqrt(a*c)*x^3*sqrt(-b/a)*elliptic_e
(arcsin(x*sqrt(-b/a)), a*d/(b*c)) - (3*(a^2 + a*b)*c*f + (b^2*c - (a^2 + 2
*a*b)*d)*e)*sqrt(a*c)*x^3*sqrt(-b/a)*elliptic_f(arcsin(x*sqrt(-b/a)), a*d/
(b*c)) - (a^2*c*e + (3*a^2*c*f + (a*b*c - 2*a^2*d)*e)*x^2)*sqrt(b*x^2 + a)
*sqrt(d*x^2 + c))/(a^2*c^2*x^3)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{x^4\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{a+bx^2}(e+fx^2)}{x^4\sqrt{c+dx^2}}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(f*x**2+e)/x**4/(d*x**2+c)**(1/2),x)
```

output
```
Integral(sqrt(a + b*x**2)*(e + f*x**2)/(x**4*sqrt(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{x^4\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{bx^2+a}(fx^2+e)}{\sqrt{dx^2+c}x^4}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)/x^4/(d*x^2+c)^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate(sqrt(b*x^2 + a)*(f*x^2 + e)/(sqrt(d*x^2 + c)*x^4), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{x^4\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{bx^2+a}(fx^2+e)}{\sqrt{dx^2+cx^4}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)/x^4/(d*x^2+c)^(1/2),x, algorithm="giac
")
```

output
```
integrate(sqrt(b*x^2 + a)*(f*x^2 + e)/(sqrt(d*x^2 + c)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{x^4\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{b\,x^2+a}\,\left(f\,x^2+e\right)}{x^4\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(e + f*x^2))/(x^4*(c + d*x^2)^(1/2)),x)
```

output
```
int(((a + b*x^2)^(1/2)*(e + f*x^2))/(x^4*(c + d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{x^4\sqrt{c+dx^2}}\,dx = \text{Too large to display}$$

input
```
int((b*x^2+a)^(1/2)*(f*x^2+e)/x^4/(d*x^2+c)^(1/2),x)
```

output

```
( - sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*f - sqrt(c + d*x**2)*sqrt(a + b*x*
*2)*b*e - 3*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a**2*c*d*x**4 + a**2*
d**2*x**6 + a*b*c**2*x**4 + 2*a*b*c*d*x**6 + a*b*d**2*x**8 + b**2*c**2*x**
6 + b**2*c*d*x**8),x)*a**3*c*d*f*x**3 + 2*int((sqrt(c + d*x**2)*sqrt(a + b
*x**2))/(a**2*c*d*x**4 + a**2*d**2*x**6 + a*b*c**2*x**4 + 2*a*b*c*d*x**6 +
 a*b*d**2*x**8 + b**2*c**2*x**6 + b**2*c*d*x**8),x)*a**3*d**2*e*x**3 - 3*i
nt((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a**2*c*d*x**4 + a**2*d**2*x**6 + a
*b*c**2*x**4 + 2*a*b*c*d*x**6 + a*b*d**2*x**8 + b**2*c**2*x**6 + b**2*c*d*
x**8),x)*a**2*b*c**2*f*x**3 + int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a**
2*c*d*x**4 + a**2*d**2*x**6 + a*b*c**2*x**4 + 2*a*b*c*d*x**6 + a*b*d**2*x*
*8 + b**2*c**2*x**6 + b**2*c*d*x**8),x)*a**2*b*c*d*e*x**3 - int((sqrt(c +
d*x**2)*sqrt(a + b*x**2))/(a**2*c*d*x**4 + a**2*d**2*x**6 + a*b*c**2*x**4
+ 2*a*b*c*d*x**6 + a*b*d**2*x**8 + b**2*c**2*x**6 + b**2*c*d*x**8),x)*a*b*
*2*c**2*e*x**3 + int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a**2*c*d + a**2*
d**2*x**2 + a*b*c**2 + 2*a*b*c*d*x**2 + a*b*d**2*x**4 + b**2*c**2*x**2 + b
**2*c*d*x**4),x)*a**2*b*d**2*f*x**3 + 3*int((sqrt(c + d*x**2)*sqrt(a + b*x
**2))/(a**2*c*d + a**2*d**2*x**2 + a*b*c**2 + 2*a*b*c*d*x**2 + a*b*d**2*x*
*4 + b**2*c**2*x**2 + b**2*c*d*x**4),x)*a*b**2*c*d*f*x**3 - int((sqrt(c +
d*x**2)*sqrt(a + b*x**2))/(a**2*c*d + a**2*d**2*x**2 + a*b*c**2 + 2*a*b*c*
d*x**2 + a*b*d**2*x**4 + b**2*c**2*x**2 + b**2*c*d*x**4),x)*a*b**2*d**2...
```

3.121 $\int \frac{\sqrt{a+bx^2}(e+fx^2)}{x^6\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 391

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{x^6\sqrt{c+dx^2}}\,dx = \frac{(2b^2c^2e+abc(3de-5cf)-2a^2d(4de-5cf))\sqrt{c+dx^2}}{15ac^3x\sqrt{a+bx^2}} - \frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{5cx^5} - \frac{(bce-4ade+5acf)\sqrt{a+bx^2}\sqrt{c+dx^2}}{15ac^2x^3} + \frac{\sqrt{b}(2b^2c^2e+abc(3de-5cf)-2a^2d(4de-5cf))\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{15a^{3/2}c^3\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} - \frac{\sqrt{b}d(bce-4ade+5acf)\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{15\sqrt{a}c^3\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
1/15*(2*b^2*c^2*e+a*b*c*(-5*c*f+3*d*e)-2*a^2*d*(-5*c*f+4*d*e))*(d*x^2+c)^(
1/2)/a/c^3/x/(b*x^2+a)^(1/2)-1/5*e*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/c/x^5-1
/15*(5*a*c*f-4*a*d*e+b*c*e)*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a/c^2/x^3+1/15
*b^(1/2)*(2*b^2*c^2*e+a*b*c*(-5*c*f+3*d*e)-2*a^2*d*(-5*c*f+4*d*e))*(d*x^2+
c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/
a^(3/2)/c^3/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)-1/15*b^(1/2)*d
*(5*a*c*f-4*a*d*e+b*c*e)*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/
a^(1/2)),(1-a*d/b/c)^(1/2))/a^(1/2)/c^3/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*
x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.28 (sec) , antiderivative size = 342, normalized size of antiderivative = 0.87

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{x^6\sqrt{c+dx^2}}\,dx$$
$$= \frac{\sqrt{\frac{b}{a}}(a+bx^2)\,(c+dx^2)\,(2b^2c^2ex^4-abcx^2(-3dex^2+c(e+5fx^2))+a^2(-8d^2ex^4+2cdx^2(2e+5fx^2)-}{}$$

input
```
Integrate[(Sqrt[a + b*x^2]*(e + f*x^2))/(x^6*Sqrt[c + d*x^2]),x]
```

output
```
(Sqrt[b/a]*(a + b*x^2)*(c + d*x^2)*(2*b^2*c^2*e*x^4 - a*b*c*x^2*(-3*d*e*x^
2 + c*(e + 5*f*x^2)) + a^2*(-8*d^2*e*x^4 + 2*c*d*x^2*(2*e + 5*f*x^2) - c^2
*(3*e + 5*f*x^2))) + I*b*c*(2*b^2*c^2*e + a*b*c*(3*d*e - 5*c*f) + 2*a^2*d*
(-4*d*e + 5*c*f))*x^5*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*
ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*b*c*(-(b*c) + a*d)*(-2*b*c*e - 4*a*
d*e + 5*a*c*f)*x^5*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*Arc
Sinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(15*a^2*Sqrt[b/a]*c^3*x^5*Sqrt[a + b*x^2]
*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 430, normalized size of antiderivative = 1.10, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {442, 445, 445, 25, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{x^6\sqrt{c+dx^2}}\,dx$$
$$\Big\downarrow \text{442}$$
$$\frac{\int \frac{-b(3de-5cf)x^2+bce-4ade+5acf}{x^4\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{5c}-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{5cx^5}$$

$\Big\downarrow$ 445

$$-\frac{-\frac{\int \frac{-2d(4de-5cf)a^2+bc(3de-5cf)a+bd(bce-4ade+5acf)x^2+2b^2c^2e}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(5acf-4ade+bce)}{3acx^3}}{5c}-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{5cx^5}$$

$\Big\downarrow$ 445

$$-\frac{-\frac{-\frac{\int -\frac{bd\left(\left(-2d(4de-5cf)a^2+bc(3de-5cf)a+2b^2c^2e\right)x^2+ac(bce-4ade+5acf)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{2b^2ce}{a}-\frac{8ad^2e}{c}+10adf-5bcf+3bde\right)}{x}}{3ac}-\sqrt{a+bx^2}\sqrt{c\cdots}}{5c}-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{5cx^5}$$

$\Big\downarrow$ 25

$$-\frac{-\frac{\frac{\int \frac{bd\left(\left(-2d(4de-5cf)a^2+bc(3de-5cf)a+2b^2c^2e\right)x^2+ac(bce-4ade+5acf)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{2b^2ce}{a}-\frac{8ad^2e}{c}+10adf-5bcf+3bde\right)}{x}}{3ac}-\sqrt{a+bx^2}\sqrt{c+d\cdots}}{5c}-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{5cx^5}$$

$\Big\downarrow$ 27

$$-\frac{-\frac{\frac{bd\int \frac{\left(-2d(4de-5cf)a^2+bc(3de-5cf)a+2b^2c^2e\right)x^2+ac(bce-4ade+5acf)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{2b^2ce}{a}-\frac{8ad^2e}{c}+10adf-5bcf+3bde\right)}{x}}{3ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\cdots}{3a\cdots}}{5c}-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{5cx^5}$$

$\Big\downarrow$ 406

$$-\frac{-\frac{\frac{bd\left(\left(-2a^2d(4de-5cf)+abc(3de-5cf)+2b^2c^2e\right)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+ac(5acf-4ade+bce)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{2b^2ce}{a}-\frac{8ad^2e}{c}+10a\cdots\right)}{x}}{3ac}\cdots}{5c}-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{5cx^5}$$

$\Big\downarrow$ 320

$$-\frac{\frac{bd\left(\left(-2a^2d(4de-5cf)+abc(3de-5cf)+2b^2c^2e\right)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}(5acf-4ade+bce)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+d\cdots}}{\cdots}}{3ac}}{5c}$$

$$\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{5cx^5}$$

$$\Big\downarrow \; 388$$

$$-\frac{\frac{bd\left(\left(-2a^2d(4de-5cf)+abc(3de-5cf)+2b^2c^2e\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(5acf-4ade+bce)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{ac}-\frac{\sqrt{\cdots}}{\cdots}}{3ac}}{5c}$$

$$\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{5cx^5}$$

$$\Big\downarrow \; 313$$

$$-\frac{\frac{bd\left(\left(-2a^2d(4de-5cf)+abc(3de-5cf)+2b^2c^2e\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(5acf-4ade+bce)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{c}\cdots}{\sqrt{\cdots}}\right.\cdots\right.}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\cdots\right)}{ac}\cdots}{3ac}}{5c}$$

$$\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{5cx^5}$$

input

```
Int[(Sqrt[a + b*x^2]*(e + f*x^2))/(x^6*Sqrt[c + d*x^2]),x]
```

output
```
-1/5*(e*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(c*x^5) + (-1/3*((b*c*e - 4*a*d*e
 + 5*a*c*f)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(a*c*x^3) - (-((((2*b^2*c*e)/
a + 3*b*d*e - (8*a*d^2*e)/c - 5*b*c*f + 10*a*d*f)*Sqrt[a + b*x^2]*Sqrt[c +
 d*x^2])/x) + (b*d*((2*b^2*c^2*e + a*b*c*(3*d*e - 5*c*f) - 2*a^2*d*(4*d*e
- 5*c*f))*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x
^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sq
rt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (c^(3/2)*(b*c*e -
4*a*d*e + 5*a*c*f)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]],
1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c +
d*x^2])))/(a*c))/(3*a*c))/(5*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 442
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^q/(a*g*(m + 1))), x] - Simp[1/(a*g^2*(m + 1))   Int[(g*x)
^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[c*(b*e - a*f)*(m + 1) + e*2
*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*2*(p + q + 1))*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && GtQ[q, 0] && LtQ[m, -1]
&&  !(EqQ[q, 1] && SimplerQ[e + f*x^2, c + d*x^2])
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 8.32 (sec) , antiderivative size = 476, normalized size of antiderivative = 1.22

method	result
elliptic	$\frac{\sqrt{(b\,x^2+a)(x^2d+c)}\left(-\frac{e\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{5c\,x^5}-\frac{(5acf-4ade+bce)\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{15a\,c^2x^3}+\frac{\left(10a^2cfd-8a^2d^2e-5ab\,c^2f+3abcde+2b^2c^2\right.}{15a^2c^3x}\right.}{}$
risch	$-\frac{\sqrt{b\,x^2+a}\,\sqrt{x^2d+c}\,\left(-10a^2cdf\,x^4+8a^2d^2e\,x^4+5ab\,c^2f\,x^4-3abcde\,x^4-2b^2c^2e\,x^4+5a^2c^2f\,x^2-4a^2cde\,x^2+ab\,c^2e\,x^2+3a^2c^2e\right)}{15c^3x^5a^2}-$
default	Expression too large to display

input
```
int((b*x^2+a)^(1/2)*(f*x^2+e)/x^6/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-1/5/c*e*(b*d
*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/x^5-1/15/a/c^2*(5*a*c*f-4*a*d*e+b*c*e)*(b*
d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/x^3+1/15/a^2/c^3*(10*a^2*c*d*f-8*a^2*d^2*
e-5*a*b*c^2*f+3*a*b*c*d*e+2*b^2*c^2*e)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
/x-1/15*(5*a*c*f-4*a*d*e+b*c*e)*b*d/a/c^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(
1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1
/2),(-1+(a*d+b*c)/c/b)^(1/2))+1/15*b*(10*a^2*c*d*f-8*a^2*d^2*e-5*a*b*c^2*f
+3*a*b*c*d*e+2*b^2*c^2*e)/a^2/c^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/
c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(EllipticF(x*(-b/a)^(1/2),(-1
+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2)))
)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 320, normalized size of antiderivative = 0.82

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{x^6\sqrt{c+dx^2}}\,dx =$$
$$-\frac{\sqrt{ac}((2\,b^3c^2+3\,ab^2cd-8\,a^2bd^2)e-5\,(ab^2c^2-2\,a^2bcd)f)x^5\sqrt{-\frac{b}{a}}E(\arcsin\left(x\sqrt{-\frac{b}{a}}\right)\mid\frac{ad}{bc})-\sqrt{ac}((2\,b$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)/x^6/(d*x^2+c)^(1/2),x, algorithm="fric
as")
```

output
```
-1/15*(sqrt(a*c)*((2*b^3*c^2 + 3*a*b^2*c*d - 8*a^2*b*d^2)*e - 5*(a*b^2*c^2
 - 2*a^2*b*c*d)*f)*x^5*sqrt(-b/a)*elliptic_e(arcsin(x*sqrt(-b/a)), a*d/(b*
c)) - sqrt(a*c)*((2*b^3*c^2 + (a^2*b + 3*a*b^2)*c*d - 4*(a^3 + 2*a^2*b)*d^
2)*e - 5*(a*b^2*c^2 - (a^3 + 2*a^2*b)*c*d)*f)*x^5*sqrt(-b/a)*elliptic_f(ar
csin(x*sqrt(-b/a)), a*d/(b*c)) + (3*a^3*c^2*e - ((2*a*b^2*c^2 + 3*a^2*b*c*
d - 8*a^3*d^2)*e - 5*(a^2*b*c^2 - 2*a^3*c*d)*f)*x^4 + (5*a^3*c^2*f + (a^2*
b*c^2 - 4*a^3*c*d)*e)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(a^3*c^3*x^5)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{x^6\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{a+bx^2}(e+fx^2)}{x^6\sqrt{c+dx^2}}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(f*x**2+e)/x**6/(d*x**2+c)**(1/2),x)
```

output
```
Integral(sqrt(a + b*x**2)*(e + f*x**2)/(x**6*sqrt(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{x^6\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{bx^2+a}(fx^2+e)}{\sqrt{dx^2+c}x^6}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)/x^6/(d*x^2+c)^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate(sqrt(b*x^2 + a)*(f*x^2 + e)/(sqrt(d*x^2 + c)*x^6), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{x^6\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{bx^2+a}(fx^2+e)}{\sqrt{dx^2+c}x^6}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)/x^6/(d*x^2+c)^(1/2),x, algorithm="giac
")
```

output
```
integrate(sqrt(b*x^2 + a)*(f*x^2 + e)/(sqrt(d*x^2 + c)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{x^6\sqrt{c+dx^2}}\,dx=\int \frac{\sqrt{b\,x^2+a}\,\left(f\,x^2+e\right)}{x^6\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(e + f*x^2))/(x^6*(c + d*x^2)^(1/2)),x)
```

output
```
int(((a + b*x^2)^(1/2)*(e + f*x^2))/(x^6*(c + d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{x^6\sqrt{c+dx^2}}\,dx=\text{Too large to display}$$

input
```
int((b*x^2+a)^(1/2)*(f*x^2+e)/x^6/(d*x^2+c)^(1/2),x)
```

output
```
( - sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*c*e - 5*sqrt(c + d*x**2)*sqrt(a +
b*x**2)*b*c*f*x**4 + 3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b*d*e*x**4 + 5*in
t((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a**2*c*d + a**2*d**2*x**2 + a*
b*c**2 + 2*a*b*c*d*x**2 + a*b*d**2*x**4 + b**2*c**2*x**2 + b**2*c*d*x**4),
x)*a*b**2*c*d**2*f*x**5 - 3*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(
a**2*c*d + a**2*d**2*x**2 + a*b*c**2 + 2*a*b*c*d*x**2 + a*b*d**2*x**4 + b*
*2*c**2*x**2 + b**2*c*d*x**4),x)*a*b**2*d**3*e*x**5 + 5*int((sqrt(c + d*x*
*2)*sqrt(a + b*x**2)*x**2)/(a**2*c*d + a**2*d**2*x**2 + a*b*c**2 + 2*a*b*c
*d*x**2 + a*b*d**2*x**4 + b**2*c**2*x**2 + b**2*c*d*x**4),x)*b**3*c**2*d*f
*x**5 - 3*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a**2*c*d + a**2*d*
*2*x**2 + a*b*c**2 + 2*a*b*c*d*x**2 + a*b*d**2*x**4 + b**2*c**2*x**2 + b**
2*c*d*x**4),x)*b**3*c*d**2*e*x**5 + 5*int((sqrt(c + d*x**2)*sqrt(a + b*x**
2))/(a**2*c*d*x**4 + a**2*d**2*x**6 + a*b*c**2*x**4 + 2*a*b*c*d*x**6 + a*b
*d**2*x**8 + b**2*c**2*x**6 + b**2*c*d*x**8),x)*a**3*c**2*d*f*x**5 - 4*int
((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a**2*c*d*x**4 + a**2*d**2*x**6 + a*b
*c**2*x**4 + 2*a*b*c*d*x**6 + a*b*d**2*x**8 + b**2*c**2*x**6 + b**2*c*d*x*
*8),x)*a**3*c*d**2*e*x**5 + 5*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a**
2*c*d*x**4 + a**2*d**2*x**6 + a*b*c**2*x**4 + 2*a*b*c*d*x**6 + a*b*d**2*x*
*8 + b**2*c**2*x**6 + b**2*c*d*x**8),x)*a**2*b*c**3*f*x**5 - 3*int((sqrt(c
 + d*x**2)*sqrt(a + b*x**2))/(a**2*c*d*x**4 + a**2*d**2*x**6 + a*b*c**2...
```

3.122 $\int \frac{\sqrt{a+bx^2}(e+fx^2)}{x^8\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 531

$$\begin{aligned}
&\int \frac{\sqrt{a+bx^2}(e+fx^2)}{x^8\sqrt{c+dx^2}}\,dx \\
&= -\frac{(8b^3c^3e+a^2bcd(16de-21cf)+ab^2c^2(9de-14cf)-8a^3d^2(6de-7cf))\sqrt{c+dx^2}}{105a^2c^4x\sqrt{a+bx^2}} \\
&\quad -\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{7cx^7}-\frac{(bce-6ade+7acf)\sqrt{a+bx^2}\sqrt{c+dx^2}}{35ac^2x^5} \\
&\quad +\frac{(4b^2c^2e+abc(5de-7cf)-4a^2d(6de-7cf))\sqrt{a+bx^2}\sqrt{c+dx^2}}{105a^2c^3x^3} \\
&\quad -\frac{\sqrt{b}(8b^3c^3e+a^2bcd(16de-21cf)+ab^2c^2(9de-14cf)-8a^3d^2(6de-7cf))\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\right.}{105a^{5/2}c^4\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} \\
&\quad +\frac{\sqrt{b}d(4b^2c^2e+abc(5de-7cf)-4a^2d(6de-7cf))\sqrt{c+dx^2}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{105a^{3/2}c^4\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}
\end{aligned}$$

output
```
-1/105*(8*b^3*c^3*e+a^2*b*c*d*(-21*c*f+16*d*e)+a*b^2*c^2*(-14*c*f+9*d*e)-8
*a^3*d^2*(-7*c*f+6*d*e))*(d*x^2+c)^(1/2)/a^2/c^4/x/(b*x^2+a)^(1/2)-1/7*e*(
b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/c/x^7-1/35*(7*a*c*f-6*a*d*e+b*c*e)*(b*x^2+a
)^(1/2)*(d*x^2+c)^(1/2)/a/c^2/x^5+1/105*(4*b^2*c^2*e+a*b*c*(-7*c*f+5*d*e)-
4*a^2*d*(-7*c*f+6*d*e))*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a^2/c^3/x^3-1/105*
b^(1/2)*(8*b^3*c^3*e+a^2*b*c*d*(-21*c*f+16*d*e)+a*b^2*c^2*(-14*c*f+9*d*e)-
8*a^3*d^2*(-7*c*f+6*d*e))*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b
*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/a^(5/2)/c^4/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/
c/(b*x^2+a))^(1/2)+1/105*b^(1/2)*d*(4*b^2*c^2*e+a*b*c*(-7*c*f+5*d*e)-4*a^2
*d*(-7*c*f+6*d*e))*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2
)),(1-a*d/b/c)^(1/2))/a^(3/2)/c^4/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a)
)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.80 (sec) , antiderivative size = 464, normalized size of antiderivative = 0.87

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{x^8\sqrt{c+dx^2}}\,dx$$
$$= \frac{-\sqrt{\frac{b}{a}}(a+bx^2)\,(c+dx^2)\,(8b^3c^3ex^6+ab^2c^2x^4(9dex^2-2c(2e+7fx^2))+a^3(-48d^3ex^6+8cd^2x^4(3e+7f}{}$$

input
```
Integrate[(Sqrt[a + b*x^2]*(e + f*x^2))/(x^8*Sqrt[c + d*x^2]),x]
```

output
```
(-(Sqrt[b/a]*(a + b*x^2)*(c + d*x^2)*(8*b^3*c^3*e*x^6 + a*b^2*c^2*x^4*(9*d
*e*x^2 - 2*c*(2*e + 7*f*x^2)) + a^3*(-48*d^3*e*x^6 + 8*c*d^2*x^4*(3*e + 7*
f*x^2) + 3*c^3*(5*e + 7*f*x^2) - 2*c^2*d*x^2*(9*e + 14*f*x^2)) + a^2*b*c*x
^2*(16*d^2*e*x^4 + c^2*(3*e + 7*f*x^2) - c*d*x^2*(5*e + 21*f*x^2)))) - I*b
*c*(8*b^3*c^3*e + a^2*b*c*d*(16*d*e - 21*c*f) + a*b^2*c^2*(9*d*e - 14*c*f)
 + 8*a^3*d^2*(-6*d*e + 7*c*f))*x^7*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]
*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + I*b*c*(b*c - a*d)*(8*b^2
*c^2*e + a*b*c*(13*d*e - 14*c*f) + 4*a^2*d*(6*d*e - 7*c*f))*x^7*Sqrt[1 + (
b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c
)])/(105*a^3*Sqrt[b/a]*c^4*x^7*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 583, normalized size of antiderivative = 1.10, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.303$, Rules used = {442, 445, 445, 445, 25, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{x^8\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \text{442}$$

$$\frac{\int \frac{-b(5de-7cf)x^2+bce-6ade+7acf}{x^6\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{7c}-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{7cx^7}$$

$$\Big\downarrow \text{445}$$

$$\frac{-\frac{\int \frac{-4d(6de-7cf)a^2+bc(5de-7cf)a+3bd(bce-6ade+7acf)x^2+4b^2c^2e}{x^4\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{5ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(7acf-6ade+bce)}{5acx^5}}{7c}-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{7cx^7}$$

$$\Big\downarrow \text{445}$$

$$\frac{-\frac{-\frac{\int \frac{-8d^2(6de-7cf)a^3+bcd(16de-21cf)a^2+b^2c^2(9de-14cf)a+bd\left(-4d(6de-7cf)a^2+bc(5de-7cf)a+4b^2c^2e\right)x^2+8b^3c^3e}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{4b^2ce}{a}-\frac{4ad}{}\right.}{3x^3}}{5ac}}{7c}-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{7cx^7}$$

$$\Big\downarrow \text{445}$$

$$\frac{-\frac{-\frac{-\frac{\int -\frac{bd\left(\left(-8d^2(6de-7cf)a^3+bcd(16de-21cf)a^2+b^2c^2(9de-14cf)a+8b^3c^3e\right)x^2+ac\left(-4d(6de-7cf)a^2+bc(5de-7cf)a+4b^2c^2e\right)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(-\right.}{}}{3ac}}{5ac}}{}-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{7cx^7}$$

$$\Big\downarrow \text{25}$$

$$-\frac{-\frac{-\frac{\int \frac{bd\left(\left(-8d^2(6de-7cf)a^3+bcd(16de-21cf)a^2+b^2c^2(9de-14cf)a+8b^3c^3e\right)x^2+ac\left(-4d(6de-7cf)a^2+bc(5de-7cf)a+4b^2c^2e\right)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(-8a^3 \cdots\right.}{\cdots}}{3ac}}{5ac}}{\cdots}$$

$$\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{7cx^7}$$

$\downarrow$ 27

$$-\frac{-\frac{-\frac{bd\int \frac{\left(-8d^2(6de-7cf)a^3+bcd(16de-21cf)a^2+b^2c^2(9de-14cf)a+8b^3c^3e\right)x^2+ac\left(-4d(6de-7cf)a^2+bc(5de-7cf)a+4b^2c^2e\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(-8a^3d \cdots\right.}{\cdots}}{3ac}}{5ac}}{\cdots}$$

$$\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{7cx^7}$$

$\downarrow$ 406

$$-\frac{-\frac{bd\left(ac\left(-4a^2d(6de-7cf)+abc(5de-7cf)+4b^2c^2e\right)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\left(-8a^3d^2(6de-7cf)+a^2bcd(16de-21cf)+ab^2c^2(9de-14cf)+8b^3c^3e\right)\int \frac{x^{2}}{\sqrt{bx^2+a \cdots}}\cdots\right.}{ac}\cdots}{3ac}\cdots$$

$$\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{7cx^7}$$

$\downarrow$ 320

$$-\frac{-\frac{bd\left(\left(-8a^3d^2(6de-7cf)+a^2bcd(16de-21cf)+ab^2c^2(9de-14cf)+8b^3c^3e\right)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}\left(-4a^2d(6de-7cf)+abc(5de-7cf)+4b^2c^2e\right)\cdots}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\cdots\right.}{ac}\cdots}{3ac}\cdots$$

$$\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{7cx^7}$$

$\downarrow$ 388

$$-\frac{-\frac{bd\left(\left(-8a^3d^2(6de-7cf)+a^2bcd(16de-21cf)+ab^2c^2(9de-14cf)+8b^3c^3e\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}\left(-4a^2d(6de-7cf)+abc(5de-7\cdots\right.}{\sqrt{d}\sqrt{c+dx^2}\cdots}\right.}{ac}}{3ac}}{\cdots}$$

$$\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{7cx^7}$$

$$\Big\downarrow \; 313$$

$$-\frac{-\frac{bd\left(\frac{c^{3/2}\sqrt{a+bx^2}\left(-4a^2d(6de-7cf)+abc(5de-7cf)+4b^2c^2e\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+\left(-8a^3d^2(6de-7cf)+a^2bcd(16de-21cf)+ab^2c^2(9de-14c\cdots\right.\right.}{ac}}{3\cdots}}{\cdots}$$

$$\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{7cx^7}$$

input
```
Int[(Sqrt[a + b*x^2]*(e + f*x^2))/(x^8*Sqrt[c + d*x^2]),x]
```

output
```
-1/7*(e*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(c*x^7) + (-1/5*((b*c*e - 6*a*d*e
 + 7*a*c*f)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(a*c*x^5) - (-1/3*(((4*b^2*c*
e)/a + b*(5*d*e - 7*c*f) - (4*a*d*(6*d*e - 7*c*f))/c)*Sqrt[a + b*x^2]*Sqrt
[c + d*x^2])/x^3 - (-(((8*b^3*c^3*e + a^2*b*c*d*(16*d*e - 21*c*f) + a*b^2*
c^2*(9*d*e - 14*c*f) - 8*a^3*d^2*(6*d*e - 7*c*f))*Sqrt[a + b*x^2]*Sqrt[c +
 d*x^2])/(a*c*x)) + (b*d*((8*b^3*c^3*e + a^2*b*c*d*(16*d*e - 21*c*f) + a*b
^2*c^2*(9*d*e - 14*c*f) - 8*a^3*d^2*(6*d*e - 7*c*f))*((x*Sqrt[a + b*x^2])/
(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x
)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^
2))]*Sqrt[c + d*x^2])) + (c^(3/2)*(4*b^2*c^2*e + a*b*c*(5*d*e - 7*c*f) - 4
*a^2*d*(6*d*e - 7*c*f))*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[
c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt
[c + d*x^2])))/(a*c))/(3*a*c))/(5*a*c))/(7*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 442
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^q/(a*g*(m + 1))), x] - Simp[1/(a*g^2*(m + 1))   Int[(g*x)
^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[c*(b*e - a*f)*(m + 1) + e*2
*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*2*(p + q + 1))*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && GtQ[q, 0] && LtQ[m, -1]
&&  !(EqQ[q, 1] && SimplerQ[e + f*x^2, c + d*x^2])
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 10.56 (sec) , antiderivative size = 640, normalized size of antiderivative = 1.21

method	result
elliptic	$\frac{\sqrt{(b x^2+a)(x^2 d+c)}\left(-\frac{e\sqrt{bd x^4+ad x^2+x^2bc+ac}}{7c x^7}-\frac{(7acf-6ade+bce)\sqrt{bd x^4+ad x^2+x^2bc+ac}}{35a c^2x^5}+\frac{(28a^2cfd-24a^2d^2e-7ab c^2f+5abcde+4b^2c}{105a^2c^3x^3}\right.}{}$
risch	$-\frac{\sqrt{b x^2+a}\sqrt{x^2d+c}\,(56a^3cd^2f x^6-48a^3d^3e x^6-21a^2bc^2df x^6+16a^2bcd^2e x^6-14ab^2c^3f x^6+9ab^2c^2de x^6+8b^3c^3e x^6-28a^3c^2df}{105c^4x^7a^3}$
default	Expression too large to display

input
```
int((b*x^2+a)^(1/2)*(f*x^2+e)/x^8/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-1/7/c*e*(b*d
*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/x^7-1/35/a/c^2*(7*a*c*f-6*a*d*e+b*c*e)*(b*
d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/x^5+1/105/a^2/c^3*(28*a^2*c*d*f-24*a^2*d^
2*e-7*a*b*c^2*f+5*a*b*c*d*e+4*b^2*c^2*e)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/
2)/x^3-1/105*(56*a^3*c*d^2*f-48*a^3*d^3*e-21*a^2*b*c^2*d*f+16*a^2*b*c*d^2*
e-14*a*b^2*c^3*f+9*a*b^2*c^2*d*e+8*b^3*c^3*e)/a^3/c^4*(b*d*x^4+a*d*x^2+b*c
*x^2+a*c)^(1/2)/x+1/105*(28*a^2*c*d*f-24*a^2*d^2*e-7*a*b*c^2*f+5*a*b*c*d*e
+4*b^2*c^2*e)*b*d/a^2/c^3/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)
/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c
)/c/b)^(1/2))-1/105*b*(56*a^3*c*d^2*f-48*a^3*d^3*e-21*a^2*b*c^2*d*f+16*a^2
*b*c*d^2*e-14*a*b^2*c^3*f+9*a*b^2*c^2*d*e+8*b^3*c^3*e)/a^3/c^3/(-b/a)^(1/2
)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*
(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/
2),(-1+(a*d+b*c)/c/b)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 488, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{x^8\sqrt{c+dx^2}}\,dx$$
$$= \frac{\sqrt{ac}((8\,b^4c^3+9\,ab^3c^2d+16\,a^2b^2cd^2-48\,a^3bd^3)e-7\,(2\,ab^3c^3+3\,a^2b^2c^2d-8\,a^3bcd^2)f)x^7\sqrt{-\frac{b}{a}}E(\arcsin}{}$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)/x^8/(d*x^2+c)^(1/2),x, algorithm="fric
as")
```

output
```
1/105*(sqrt(a*c)*((8*b^4*c^3 + 9*a*b^3*c^2*d + 16*a^2*b^2*c*d^2 - 48*a^3*b
*d^3)*e - 7*(2*a*b^3*c^3 + 3*a^2*b^2*c^2*d - 8*a^3*b*c*d^2)*f)*x^7*sqrt(-b
/a)*elliptic_e(arcsin(x*sqrt(-b/a)), a*d/(b*c)) - sqrt(a*c)*((8*b^4*c^3 +
(4*a^2*b^2 + 9*a*b^3)*c^2*d + (5*a^3*b + 16*a^2*b^2)*c*d^2 - 24*(a^4 + 2*a
^3*b)*d^3)*e - 7*(2*a*b^3*c^3 + (a^3*b + 3*a^2*b^2)*c^2*d - 4*(a^4 + 2*a^3
*b)*c*d^2)*f)*x^7*sqrt(-b/a)*elliptic_f(arcsin(x*sqrt(-b/a)), a*d/(b*c)) -
 (15*a^4*c^3*e + ((8*a*b^3*c^3 + 9*a^2*b^2*c^2*d + 16*a^3*b*c*d^2 - 48*a^4
*d^3)*e - 7*(2*a^2*b^2*c^3 + 3*a^3*b*c^2*d - 8*a^4*c*d^2)*f)*x^6 - ((4*a^2
*b^2*c^3 + 5*a^3*b*c^2*d - 24*a^4*c*d^2)*e - 7*(a^3*b*c^3 - 4*a^4*c^2*d)*f
)*x^4 + 3*(7*a^4*c^3*f + (a^3*b*c^3 - 6*a^4*c^2*d)*e)*x^2)*sqrt(b*x^2 + a)
*sqrt(d*x^2 + c))/(a^4*c^4*x^7)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{x^8\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{a+bx^2}\,(e+fx^2)}{x^8\sqrt{c+dx^2}}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(f*x**2+e)/x**8/(d*x**2+c)**(1/2),x)
```

output
```
Integral(sqrt(a + b*x**2)*(e + f*x**2)/(x**8*sqrt(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{x^8\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{bx^2+a}(fx^2+e)}{\sqrt{dx^2+c}x^8}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)/x^8/(d*x^2+c)^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate(sqrt(b*x^2 + a)*(f*x^2 + e)/(sqrt(d*x^2 + c)*x^8), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{x^8\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{bx^2+a}(fx^2+e)}{\sqrt{dx^2+c}x^8}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)/x^8/(d*x^2+c)^(1/2),x, algorithm="giac
")
```

output
```
integrate(sqrt(b*x^2 + a)*(f*x^2 + e)/(sqrt(d*x^2 + c)*x^8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{x^8\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{b\,x^2+a}\,\left(f\,x^2+e\right)}{x^8\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(e + f*x^2))/(x^8*(c + d*x^2)^(1/2)),x)
```

output
```
int(((a + b*x^2)^(1/2)*(e + f*x^2))/(x^8*(c + d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{x^8\sqrt{c+dx^2}}\,dx = \text{Too large to display}$$

input
```
int((b*x^2+a)^(1/2)*(f*x^2+e)/x^8/(d*x^2+c)^(1/2),x)
```

output
```
( - sqrt(c + d*x**2)*sqrt(a + b*x**2)*e + 7*int((sqrt(c + d*x**2)*sqrt(a +
 b*x**2))/(a**2*c*d*x**6 + a**2*d**2*x**8 + a*b*c**2*x**6 + 2*a*b*c*d*x**8
 + a*b*d**2*x**10 + b**2*c**2*x**8 + b**2*c*d*x**10),x)*a**2*c*d*f*x**7 -
6*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a**2*c*d*x**6 + a**2*d**2*x**8
+ a*b*c**2*x**6 + 2*a*b*c*d*x**8 + a*b*d**2*x**10 + b**2*c**2*x**8 + b**2*
c*d*x**10),x)*a**2*d**2*e*x**7 + 7*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))
/(a**2*c*d*x**6 + a**2*d**2*x**8 + a*b*c**2*x**6 + 2*a*b*c*d*x**8 + a*b*d*
*2*x**10 + b**2*c**2*x**8 + b**2*c*d*x**10),x)*a*b*c**2*f*x**7 - 5*int((sq
rt(c + d*x**2)*sqrt(a + b*x**2))/(a**2*c*d*x**6 + a**2*d**2*x**8 + a*b*c**
2*x**6 + 2*a*b*c*d*x**8 + a*b*d**2*x**10 + b**2*c**2*x**8 + b**2*c*d*x**10
),x)*a*b*c*d*e*x**7 + int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a**2*c*d*x*
*6 + a**2*d**2*x**8 + a*b*c**2*x**6 + 2*a*b*c*d*x**8 + a*b*d**2*x**10 + b*
*2*c**2*x**8 + b**2*c*d*x**10),x)*b**2*c**2*e*x**7 + 7*int((sqrt(c + d*x**
2)*sqrt(a + b*x**2))/(a**2*c*d*x**4 + a**2*d**2*x**6 + a*b*c**2*x**4 + 2*a
*b*c*d*x**6 + a*b*d**2*x**8 + b**2*c**2*x**6 + b**2*c*d*x**8),x)*a*b*c*d*f
*x**7 - 5*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a**2*c*d*x**4 + a**2*d*
*2*x**6 + a*b*c**2*x**4 + 2*a*b*c*d*x**6 + a*b*d**2*x**8 + b**2*c**2*x**6
+ b**2*c*d*x**8),x)*a*b*d**2*e*x**7 + 7*int((sqrt(c + d*x**2)*sqrt(a + b*x
**2))/(a**2*c*d*x**4 + a**2*d**2*x**6 + a*b*c**2*x**4 + 2*a*b*c*d*x**6 + a
*b*d**2*x**8 + b**2*c**2*x**6 + b**2*c*d*x**8),x)*b**2*c**2*f*x**7 - 5*...
```

3.123 $\int \frac{x^6\sqrt{a+bx^2}}{\sqrt{c+dx^2}(e+fx^2)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 555

$$
\begin{aligned}
&\int \frac{x^6\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)}\,dx \\
&= -\frac{\left(2a^2df+ab(5de+3cf)-b^2\left(10ce+\frac{15de^2}{f}+\frac{8c^2f}{d}\right)\right)x\sqrt{c+dx^2}}{15bd^2f^2\sqrt{a+bx^2}} \\
&\quad -\frac{(5bde+4bcf-adf)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{15bd^2f^2}+\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5df} \\
&\quad +\frac{\sqrt{a}(2a^2d^2f^2+abdf(5de+3cf)-b^2(15d^2e^2+10cdef+8c^2f^2))\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{15b^{3/2}d^3f^3\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} \\
&\quad -\frac{a^{3/2}(acdf^2-b(15d^2e^2+5cdef+4c^2f^2))\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{15b^{3/2}cd^2f^3\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} \\
&\quad -\frac{a^{3/2}e^2\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}cf^3\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}
\end{aligned}
$$

output
```
-1/15*(2*a^2*d*f+a*b*(3*c*f+5*d*e)-b^2*(10*c*e+15*d*e^2/f+8*c^2*f/d))*x*(d
*x^2+c)^(1/2)/b/d^2/f^2/(b*x^2+a)^(1/2)-1/15*(-a*d*f+4*b*c*f+5*b*d*e)*x*(b
*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b/d^2/f^2+1/5*x^3*(b*x^2+a)^(1/2)*(d*x^2+c)^
(1/2)/d/f+1/15*a^(1/2)*(2*a^2*d^2*f^2+a*b*d*f*(3*c*f+5*d*e)-b^2*(8*c^2*f^2
+10*c*d*e*f+15*d^2*e^2))*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*
x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/b^(3/2)/d^3/f^3/(b*x^2+a)^(1/2)/(a*(d*x^2+
c)/c/(b*x^2+a))^(1/2)-1/15*a^(3/2)*(a*c*d*f^2-b*(4*c^2*f^2+5*c*d*e*f+15*d^
2*e^2))*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b
/c)^(1/2))/b^(3/2)/c/d^2/f^3/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/
2)-a^(3/2)*e^2*(d*x^2+c)^(1/2)*EllipticPi(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1
/2),1-a*f/b/e,(1-a*d/b/c)^(1/2))/b^(1/2)/c/f^3/(b*x^2+a)^(1/2)/(a*(d*x^2+c
)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.29 (sec) , antiderivative size = 433, normalized size of antiderivative = 0.78

$$\int \frac{x^6\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)}\,dx$$
$$= \frac{\sqrt{\frac{b}{a}}df^2x(a+bx^2)\,(c+dx^2)\,(adf+b(-5de-4cf+3dfx^2))+icf(2a^2d^2f^2+abdf(5de+3cf)-b^2(15d^2}{}$$

input
```
Integrate[(x^6*Sqrt[a + b*x^2])/(Sqrt[c + d*x^2]*(e + f*x^2)),x]
```

output
```
(Sqrt[b/a]*d*f^2*x*(a + b*x^2)*(c + d*x^2)*(a*d*f + b*(-5*d*e - 4*c*f + 3*
d*f*x^2)) + I*c*f*(2*a^2*d^2*f^2 + a*b*d*f*(5*d*e + 3*c*f) - b^2*(15*d^2*e
^2 + 10*c*d*e*f + 8*c^2*f^2))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Elli
pticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*(a^2*c*d^2*f^3 + a*b*d*f*(1
5*d^2*e^2 + 10*c*d*e*f + 7*c^2*f^2) - b^2*(15*d^3*e^3 + 15*c*d^2*e^2*f + 1
0*c^2*d*e*f^2 + 8*c^3*f^3))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Ellipt
icF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (15*I)*b*d^3*e^2*(-(b*e) + a*f)
*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e), I*ArcSinh
[Sqrt[b/a]*x], (a*d)/(b*c)])/(15*b*Sqrt[b/a]*d^3*f^4*Sqrt[a + b*x^2]*Sqrt[
c + d*x^2])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)}\,dx$$
$$\Big\downarrow \; 450$$
$$\int \frac{x^6\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)}dx$$

input
```
Int[(x^6*Sqrt[a + b*x^2])/(Sqrt[c + d*x^2]*(e + f*x^2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [A] (verified)

Time = 10.62 (sec) , antiderivative size = 543, normalized size of antiderivative = 0.98

method	result
risch	$\frac{x(3bdf\,x^2+adf-4bcf-5bde)\sqrt{b\,x^2+a}\,\sqrt{x^2d+c}}{15b\,d^2f^2}-\frac{\left(\frac{(a^2cd\,f^3-4ab\,c^2f^3-5abcde\,f^2-15ab\,d^2e^2f+15b^2d^2e^3)\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\text{Elliptic}}{f^2\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}\right.}{}$
default	Expression too large to display
elliptic	Expression too large to display

input
```
int(x^6*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
1/15*x*(3*b*d*f*x^2+a*d*f-4*b*c*f-5*b*d*e)*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)
/b/d^2/f^2-1/15/d^2/b/f^2*((a^2*c*d*f^3-4*a*b*c^2*f^3-5*a*b*c*d*e*f^2-15*a
*b*d^2*e^2*f+15*b^2*d^2*e^3)/f^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c
)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(
a*d+b*c)/c/b)^(1/2))-1/f*(2*a^2*d^2*f^2+3*a*b*c*d*f^2+5*a*b*d^2*e*f-8*b^2*
c^2*f^2-10*b^2*c*d*e*f-15*b^2*d^2*e^2)*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1
+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(-b/a)^
(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b
)^(1/2)))+15*e^2*(a*f-b*e)*b*d^2/f^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x
^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),
a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2)))*((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a
)^(1/2)/(d*x^2+c)^(1/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^6\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate(x^6*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^6\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{x^6\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)}\,dx$$

input
```
integrate(x**6*(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2)/(f*x**2+e),x)
```

output
```
Integral(x**6*sqrt(a + b*x**2)/(sqrt(c + d*x**2)*(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{x^6\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{\sqrt{bx^2+a}x^6}{\sqrt{dx^2+c}(fx^2+e)}\,dx$$

input
```
integrate(x^6*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="maxi
ma")
```

output
```
integrate(sqrt(b*x^2 + a)*x^6/(sqrt(d*x^2 + c)*(f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{x^6\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{\sqrt{bx^2+a}x^6}{\sqrt{dx^2+c}(fx^2+e)}\,dx$$

input
```
integrate(x^6*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="giac
")
```

output
```
integrate(sqrt(b*x^2 + a)*x^6/(sqrt(d*x^2 + c)*(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{x^6\,\sqrt{b\,x^2+a}}{\sqrt{d\,x^2+c}\,\left(f\,x^2+e\right)}\,dx$$

input
```
int((x^6*(a + b*x^2)^(1/2))/((c + d*x^2)^(1/2)*(e + f*x^2)),x)
```

output
```
int((x^6*(a + b*x^2)^(1/2))/((c + d*x^2)^(1/2)*(e + f*x^2)), x)
```

Reduce [F]

$$\int \frac{x^6\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \text{Too large to display}$$

input
```
int(x^6*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x)
```

output
```
(sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*d*f*x - 4*sqrt(c + d*x**2)*sqrt(a + b
*x**2)*b*c*f*x - 5*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b*d*e*x + 3*sqrt(c +
d*x**2)*sqrt(a + b*x**2)*b*d*f*x**3 - 2*int((sqrt(c + d*x**2)*sqrt(a + b*x
**2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*
c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*a**2*d**2*f**2 - 3*int((sqrt(c + d*
x**2)*sqrt(a + b*x**2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4
 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*a*b*c*d*f**2 - 5*
int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*e*x
**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*a
*b*d**2*e*f + 8*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c*e + a*c*
f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 +
b*d*f*x**6),x)*b**2*c**2*f**2 + 10*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*
x**4)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x
**4 + b*d*e*x**4 + b*d*f*x**6),x)*b**2*c*d*e*f + 15*int((sqrt(c + d*x**2)*
sqrt(a + b*x**2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c
*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*b**2*d**2*e**2 - int((s
qrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 +
a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*a**2*c*
d*f**2 - 2*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c*e + a*c*f*x**
2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*...
```

3.124 $\int \frac{x^4\sqrt{a+bx^2}}{\sqrt{c+dx^2}(e+fx^2)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 399

$$
\begin{aligned}
&\int \frac{x^4\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)}\,dx\\
&= -\frac{(3bde+2bcf-adf)x\sqrt{c+dx^2}}{3d^2f^2\sqrt{a+bx^2}}+\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3df}\\
&\quad+\frac{\sqrt{a}(3bde+2bcf-adf)\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{3\sqrt{b}d^2f^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}\\
&\quad-\frac{a^{3/2}(3de+cf)\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{3\sqrt{b}cdf^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}\\
&\quad+\frac{a^{3/2}e\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}cf^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}
\end{aligned}
$$

output
```
-1/3*(-a*d*f+2*b*c*f+3*b*d*e)*x*(d*x^2+c)^(1/2)/d^2/f^2/(b*x^2+a)^(1/2)+1/
3*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/d/f+1/3*a^(1/2)*(-a*d*f+2*b*c*f+3*b*d*
e)*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/
c)^(1/2))/b^(1/2)/d^2/f^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)-
1/3*a^(3/2)*(c*f+3*d*e)*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a
^(1/2)),(1-a*d/b/c)^(1/2))/b^(1/2)/c/d/f^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/
(b*x^2+a))^(1/2)+a^(3/2)*e*(d*x^2+c)^(1/2)*EllipticPi(b^(1/2)*x/a^(1/2)/(1
+b*x^2/a)^(1/2),1-a*f/b/e,(1-a*d/b/c)^(1/2))/b^(1/2)/c/f^2/(b*x^2+a)^(1/2)
/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.22 (sec) , antiderivative size = 323, normalized size of antiderivative = 0.81

$$\int \frac{x^4\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)}\,dx$$
$$= \frac{-icf(-3bde-2bcf+adf)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(i\operatorname{arcsinh}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)+i(adf(3de+2cf)-b(3d^2e^2+}{}$$

input
```
Integrate[(x^4*Sqrt[a + b*x^2])/(Sqrt[c + d*x^2]*(e + f*x^2)),x]
```

output
```
((-I)*c*f*(-3*b*d*e - 2*b*c*f + a*d*f)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2
)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + I*(a*d*f*(3*d*e + 2*
c*f) - b*(3*d^2*e^2 + 3*c*d*e*f + 2*c^2*f^2))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 +
 (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + d*(Sqrt[b/a]*
f^2*x*(a + b*x^2)*(c + d*x^2) - (3*I)*d*e*(-(b*e) + a*f)*Sqrt[1 + (b*x^2)/
a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*
d)/(b*c)]))/(3*Sqrt[b/a]*d^2*f^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)}\,dx$$
$$\Big\downarrow \text{450}$$
$$\int \frac{x^4\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)}dx$$

input
```
Int[(x^4*Sqrt[a + b*x^2])/(Sqrt[c + d*x^2]*(e + f*x^2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [A] (verified)

Time = 10.95 (sec) , antiderivative size = 600, normalized size of antiderivative = 1.50

method	result
risch	$\frac{x\sqrt{b\,x^2+a}\,\sqrt{x^2d+c}}{3df}+\frac{\left(-\frac{f(adf-2bcf-3bde)c\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\left(\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)-\operatorname{EllipticE}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}\,d}\right.}{}$
default	$-\frac{\left(-\sqrt{-\frac{b}{a}}\,b\,d^2f^2x^5-\sqrt{-\frac{b}{a}}\,a\,d^2f^2x^3-\sqrt{-\frac{b}{a}}\,bcd\,f^2x^3+2\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{x^2d+c}{c}}\,\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)acd\,f^2+3\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{x^2d+c}{c}}\right.}{}$
elliptic	Expression too large to display

input
```
int(x^4*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
1/3*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/d/f+1/3/d/f*(1/f^2*(-f*(a*d*f-2*b*c*
f-3*b*d*e)*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d
*x^2+b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/
2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2)))-a*c*f^2/(-b/a)^(1/
2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+3*b*d*e^2/(-b/a)^(1/2)
*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*E
llipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-3*a*d*e*f/(-b/a)^(1/2)*(
1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*Ell
ipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2)))+3*e*(a*f-b*e)*d/f^2/(-b/a
)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^
(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2)))*((b*
x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^4\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate(x^4*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^4\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{x^4\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)}\,dx$$

input
```
integrate(x**4*(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2)/(f*x**2+e),x)
```

output
```
Integral(x**4*sqrt(a + b*x**2)/(sqrt(c + d*x**2)*(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{x^4\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{\sqrt{bx^2+a}x^4}{\sqrt{dx^2+c}(fx^2+e)}\,dx$$

input
```
integrate(x^4*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="maxi
ma")
```

output
```
integrate(sqrt(b*x^2 + a)*x^4/(sqrt(d*x^2 + c)*(f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{x^4\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{\sqrt{bx^2+a}x^4}{\sqrt{dx^2+c}(fx^2+e)}\,dx$$

input
```
integrate(x^4*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="giac
")
```

output
```
integrate(sqrt(b*x^2 + a)*x^4/(sqrt(d*x^2 + c)*(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{x^4\,\sqrt{b\,x^2+a}}{\sqrt{d\,x^2+c}\,\left(f\,x^2+e\right)}\,dx$$

input
```
int((x^4*(a + b*x^2)^(1/2))/((c + d*x^2)^(1/2)*(e + f*x^2)),x)
```

output
```
int((x^4*(a + b*x^2)^(1/2))/((c + d*x^2)^(1/2)*(e + f*x^2)), x)
```

Reduce [F]

$$\int \frac{x^4\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)}\,dx$$
$$= \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x + \left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^4}{bdf\,x^6+adf\,x^4+bcf\,x^4+bde\,x^4+acf\,x^2+ade\,x^2+bce\,x^2+ace}dx\right) adf - 2\left(\int \frac{\sqrt{}}{bdf\,x^6+adf\,x^4+bcf\,x}\right.}{}$$

input
```
int(x^4*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x)
```

output
```
(sqrt(c + d*x**2)*sqrt(a + b*x**2)*x + int((sqrt(c + d*x**2)*sqrt(a + b*x*
*2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c
*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*a*d*f - 2*int((sqrt(c + d*x**2)*sqrt
(a + b*x**2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x
**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*b*c*f - 3*int((sqrt(c + d*x
**2)*sqrt(a + b*x**2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4
+ b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*b*d*e - int((sqrt(
c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*
f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*a*c*f - 2*i
nt((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c*e + a*c*f*x**2 + a*d*e*x*
*2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*a*
d*e - 2*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c*e + a*c*f*x**2 +
 a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x*
*6),x)*b*c*e - int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c*e + a*c*f*x**2
 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*
x**6),x)*a*c*e)/(3*d*f)
```

3.125 $\int \frac{x^2\sqrt{a+bx^2}}{\sqrt{c+dx^2}(e+fx^2)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 316

$$\int \frac{x^2\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \frac{bx\sqrt{c+dx^2}}{df\sqrt{a+bx^2}} - \frac{\sqrt{a}\sqrt{b}\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{df\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} + \frac{a^{3/2}\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}cf\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} - \frac{a^{3/2}\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}cf\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
b*x*(d*x^2+c)^(1/2)/d/f/(b*x^2+a)^(1/2)-a^(1/2)*b^(1/2)*(d*x^2+c)^(1/2)*El
lipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/d/f/(b*x^2+
a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)+a^(3/2)*(d*x^2+c)^(1/2)*InverseJa
cobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/b^(1/2)/c/f/(b*x^2+a)^
(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)-a^(3/2)*(d*x^2+c)^(1/2)*EllipticPi(b
^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),1-a*f/b/e,(1-a*d/b/c)^(1/2))/b^(1/2)/c/
f/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.87 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.59

$$\int \frac{x^2\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)}\,dx =$$
$$-\frac{i\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\left(bcfE\left(i\text{arcsinh}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)-(bde+bcf-adf)\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{\frac{b}{a}}x\right),\frac{ad}{bc}\right.\right.}{\sqrt{\frac{b}{a}}df^2\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

input
```
Integrate[(x^2*Sqrt[a + b*x^2])/(Sqrt[c + d*x^2]*(e + f*x^2)),x]
```

output
```
((-I)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(b*c*f*EllipticE[I*ArcSinh[S
qrt[b/a]*x], (a*d)/(b*c)] - (b*d*e + b*c*f - a*d*f)*EllipticF[I*ArcSinh[Sq
rt[b/a]*x], (a*d)/(b*c)] + d*(b*e - a*f)*EllipticPi[(a*f)/(b*e), I*ArcSinh
[Sqrt[b/a]*x], (a*d)/(b*c)]))/(Sqrt[b/a]*d*f^2*Sqrt[a + b*x^2]*Sqrt[c + d*
x^2])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)}\,dx$$
$$\Big\downarrow \; 450$$
$$\int \frac{x^2\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)}dx$$

input
```
Int[(x^2*Sqrt[a + b*x^2])/(Sqrt[c + d*x^2]*(e + f*x^2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [A] (verified)

Time = 4.09 (sec) , antiderivative size = 274, normalized size of antiderivative = 0.87

method	result
default	$\frac{\left(\text{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)adf-\text{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)bcf-\text{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)bde+\text{EllipticE}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)bcf-\text{Ellip}}{f^2d\sqrt{-\frac{b}{a}}\,(bd\,x^4+ad\,x^2+x^2bc}$
elliptic	$\frac{\sqrt{(b\,x^2+a)(x^2d+c)}\left(\frac{\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\text{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)a}{f\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}-\frac{\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\text{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)be}{f^2\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}-\frac{bc}{}\right.}{}$

input
```
int(x^2*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
(EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*d*f-EllipticF(x*(-b/a)^(1/2),
(a*d/b/c)^(1/2))*b*c*f-EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b*d*e+Ell
ipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b*c*f-EllipticPi(x*(-b/a)^(1/2),a*f
/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*a*d*f+EllipticPi(x*(-b/a)^(1/2),a*f/b/e,
(-1/c*d)^(1/2)/(-b/a)^(1/2))*b*d*e)*((d*x^2+c)/c)^(1/2)*((b*x^2+a)/a)^(1/2
)*(d*x^2+c)^(1/2)*(b*x^2+a)^(1/2)/f^2/d/(-b/a)^(1/2)/(b*d*x^4+a*d*x^2+b*c*
x^2+a*c)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^2\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate(x^2*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^2\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{x^2\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)}\,dx$$

input
```
integrate(x**2*(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2)/(f*x**2+e),x)
```

output
```
Integral(x**2*sqrt(a + b*x**2)/(sqrt(c + d*x**2)*(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{x^2\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{\sqrt{bx^2+a}x^2}{\sqrt{dx^2+c}(fx^2+e)}\,dx$$

input
```
integrate(x^2*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="maxi
ma")
```

output
```
integrate(sqrt(b*x^2 + a)*x^2/(sqrt(d*x^2 + c)*(f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{x^2\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{\sqrt{bx^2+a}x^2}{\sqrt{dx^2+c}(fx^2+e)}\,dx$$

input
```
integrate(x^2*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="giac
")
```

output
```
integrate(sqrt(b*x^2 + a)*x^2/(sqrt(d*x^2 + c)*(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{x^2\,\sqrt{b\,x^2+a}}{\sqrt{d\,x^2+c}\,\left(f\,x^2+e\right)}\,dx$$

input
```
int((x^2*(a + b*x^2)^(1/2))/((c + d*x^2)^(1/2)*(e + f*x^2)),x)
```

output
```
int((x^2*(a + b*x^2)^(1/2))/((c + d*x^2)^(1/2)*(e + f*x^2)), x)
```

Reduce [F]

$$\int \frac{x^2\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^2}{df\,x^4+cf\,x^2+de\,x^2+ce}dx$$

input
```
int(x^2*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(c*e + c*f*x**2 + d*e*x**2 +
d*f*x**4),x)
```

3.126 $\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}(e+fx^2)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 102

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \frac{a^{3/2}\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
a^(3/2)*(d*x^2+c)^(1/2)*EllipticPi(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),1-a
*f/b/e,(1-a*d/b/c)^(1/2))/b^(1/2)/c/e/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^
2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.60 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.40

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)}\,dx = -\frac{i\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\left(be\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{\frac{b}{a}}x\right),\frac{ad}{bc}\right)+(-be+af)\,\text{EllipticPi}\left(\frac{af}{be},i\text{arcsinh}\left(\sqrt{\frac{b}{a}}x\right)\right.\right.}{\sqrt{\frac{b}{a}}ef\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

input
```
Integrate[Sqrt[a + b*x^2]/(Sqrt[c + d*x^2]*(e + f*x^2)),x]
```

output
```
((-I)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(b*e*EllipticF[I*ArcSinh[Sqr
t[b/a]*x], (a*d)/(b*c)] + (-(b*e) + a*f)*EllipticPi[(a*f)/(b*e), I*ArcSinh
[Sqrt[b/a]*x], (a*d)/(b*c)]))/(Sqrt[b/a]*e*f*Sqrt[a + b*x^2]*Sqrt[c + d*x^
2])
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.031$, Rules used = {414}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)}\,dx$$

$$\Big\downarrow \text{414}$$

$$\frac{a^{3/2}\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

input
```
Int[Sqrt[a + b*x^2]/(Sqrt[c + d*x^2]*(e + f*x^2)),x]
```

output
```
(a^(3/2)*Sqrt[c + d*x^2]*EllipticPi[1 - (a*f)/(b*e), ArcTan[(Sqrt[b]*x)/Sq
rt[a]], 1 - (a*d)/(b*c)])/(Sqrt[b]*c*e*Sqrt[a + b*x^2]*Sqrt[(a*(c + d*x^2)
)/(c*(a + b*x^2))])
```

Defintions of rubi rules used

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Maple [A] (verified)

Time = 4.41 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.87

method	result
default	$\frac{\left(\text{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)be+\text{EllipticPi}\left(x\sqrt{-\frac{b}{a}},\frac{af}{be},\frac{\sqrt{-\frac{d}{c}}}{\sqrt{-\frac{b}{a}}}\right)af-\text{EllipticPi}\left(x\sqrt{-\frac{b}{a}},\frac{af}{be},\frac{\sqrt{-\frac{d}{c}}}{\sqrt{-\frac{b}{a}}}\right)be\right)\sqrt{\frac{x^2d+c}{c}}\,\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{x^2d+c}}{fe\sqrt{-\frac{b}{a}}\,(bd\,x^4+ad\,x^2+x^2bc+ac)}$
elliptic	$\frac{\sqrt{(b\,x^2+a)(x^2d+c)}\left(\frac{b\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\text{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{f\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}+\frac{\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\text{EllipticPi}\left(x\sqrt{-\frac{b}{a}},\frac{af}{be},\frac{\sqrt{-\frac{d}{c}}}{\sqrt{-\frac{b}{a}}}\right)a}{e\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}-\right.}{\sqrt{b\,x^2+a}\,\sqrt{x^2d+c}}$

input
```
int((b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
(EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b*e+EllipticPi(x*(-b/a)^(1/2),a
*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*a*f-EllipticPi(x*(-b/a)^(1/2),a*f/b/e,
(-1/c*d)^(1/2)/(-b/a)^(1/2))*b*e)*((d*x^2+c)/c)^(1/2)*((b*x^2+a)/a)^(1/2)/
f*(d*x^2+c)^(1/2)*(b*x^2+a)^(1/2)/e/(-b/a)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+
a*c)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)/(d*x**2+c)**(1/2)/(f*x**2+e),x)
```

output
```
Integral(sqrt(a + b*x**2)/(sqrt(c + d*x**2)*(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)/(sqrt(d*x^2 + c)*(f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)}\,dx=\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^2 + a)/(sqrt(d*x^2 + c)*(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)}\,dx=\int \frac{\sqrt{b\,x^2+a}}{\sqrt{d\,x^2+c}\,\left(f\,x^2+e\right)}\,dx$$

input
```
int((a + b*x^2)^(1/2)/((c + d*x^2)^(1/2)*(e + f*x^2)),x)
```

output
```
int((a + b*x^2)^(1/2)/((c + d*x^2)^(1/2)*(e + f*x^2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)}\,dx=\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{df\,x^4+cf\,x^2+de\,x^2+ce}dx$$

input
```
int((b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(c*e + c*f*x**2 + d*e*x**2 + d*f*x
**4),x)
```

3.127 $\int \frac{\sqrt{a+bx^2}}{x^2\sqrt{c+dx^2}(e+fx^2)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 387

$$\int \frac{\sqrt{a+bx^2}}{x^2\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \frac{bx\sqrt{c+dx^2}}{ce\sqrt{a+bx^2}}+\frac{bx\sqrt{a+bx^2}\sqrt{c+dx^2}}{ace}-\frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{acex} - \frac{\sqrt{a}\sqrt{b}\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{ce\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} + \frac{\sqrt{a}\sqrt{b}\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{ce\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} - \frac{a^{3/2}f\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
b*x*(d*x^2+c)^(1/2)/c/e/(b*x^2+a)^(1/2)+b*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2
)/a/c/e-(b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/a/c/e/x-a^(1/2)*b^(1/2)*(d*x^2+c)^
(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/c/e
/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)+a^(1/2)*b^(1/2)*(d*x^2+c)
^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/c/e/(b
*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)-a^(3/2)*f*(d*x^2+c)^(1/2)*El
lipticPi(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),1-a*f/b/e,(1-a*d/b/c)^(1/2))/
b^(1/2)/c/e^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.39 (sec) , antiderivative size = 373, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{a+bx^2}}{x^2\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \frac{-a\sqrt{\frac{b}{a}}ce - b\sqrt{\frac{b}{a}}cex^2 - a\sqrt{\frac{b}{a}}dex^2 - b\sqrt{\frac{b}{a}}dex^4 - ibcex\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(i\text{arcsinh}\left(\sqrt{\frac{b}{a}}x\right)\mid\frac{ad}{bc}\right) + ibc}{}$$

input
```
Integrate[Sqrt[a + b*x^2]/(x^2*Sqrt[c + d*x^2]*(e + f*x^2)),x]
```

output
```
(-(a*Sqrt[b/a]*c*e) - b*Sqrt[b/a]*c*e*x^2 - a*Sqrt[b/a]*d*e*x^2 - b*Sqrt[b
/a]*d*e*x^4 - I*b*c*e*x*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[
I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + I*b*c*e*x*Sqrt[1 + (b*x^2)/a]*Sqrt[
1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*b*c*e*x*
Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e), I*ArcSinh[
Sqrt[b/a]*x], (a*d)/(b*c)] + I*a*c*f*x*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2
)/c]*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(Sqrt[b
/a]*c*e^2*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}}{x^2\sqrt{c+dx^2}\,(e+fx^2)}\,dx$$

$$\Big\downarrow \text{450}$$

$$\int \frac{\sqrt{a+bx^2}}{x^2\sqrt{c+dx^2}\,(e+fx^2)}dx$$

input
```
Int[Sqrt[a + b*x^2]/(x^2*Sqrt[c + d*x^2]*(e + f*x^2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [A] (verified)

Time = 8.28 (sec) , antiderivative size = 302, normalized size of antiderivative = 0.78

method	result
risch	$-\frac{\sqrt{b\,x^{2}+a}\,\sqrt{x^{2}d+c}}{cex}+\frac{\left(-\frac{bc\sqrt{1+\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,\left(\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)-\operatorname{EllipticE}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^{4}+ad\,x^{2}+x^{2}bc+ac}}-\frac{c(af-be)\sqrt{1+\frac{b}{}}}{}\right.}{ce\sqrt{b\,x^{2}+a}\,\sqrt{x^{2}d+c}}$
default	$\frac{\left(-\sqrt{-\frac{b}{a}}\,bde\,x^{4}-\sqrt{\frac{b\,x^{2}+a}{a}}\,\sqrt{\frac{x^{2}d+c}{c}}\,\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)bcex+\sqrt{\frac{b\,x^{2}+a}{a}}\,\sqrt{\frac{x^{2}d+c}{c}}\,\operatorname{EllipticE}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)bcex-\sqrt{\frac{b\,x^{2}+a}{a}}\right.}{\sqrt{}}$
elliptic	$\frac{\sqrt{(b\,x^{2}+a)(x^{2}d+c)}\left(-\frac{\sqrt{bd\,x^{4}+ad\,x^{2}+x^{2}bc+ac}}{cex}-\frac{b\sqrt{1+\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{e\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^{4}+ad\,x^{2}+x^{2}bc+ac}}+\frac{b\sqrt{1+\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,\operatorname{EllipticE}\left(\right.}{e\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^{4}+ad\,x^{2}}}\right.}{\sqrt{b\,x^{2}+a}\,\sqrt{}}$

input
```
int((b*x^2+a)^(1/2)/x^2/(d*x^2+c)^(1/2)/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
-1/c/e*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x+1/c/e*(-b*c/(-b/a)^(1/2)*(1+b*x^2
/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(EllipticF
(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*
d+b*c)/c/b)^(1/2)))-c*(a*f-b*e)/e/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/
c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f
/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2)))*((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(
1/2)/(d*x^2+c)^(1/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{x^2\sqrt{c+dx^2}\,(e+fx^2)}\,dx=\text{Timed out}$$

input
```
integrate((b*x^2+a)^(1/2)/x^2/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}}{x^2\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{\sqrt{a+bx^2}}{x^2\sqrt{c+dx^2}\,(e+fx^2)}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)/x**2/(d*x**2+c)**(1/2)/(f*x**2+e),x)
```

output
```
Integral(sqrt(a + b*x**2)/(x**2*sqrt(c + d*x**2)*(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}}{x^2\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)x^2}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/x^2/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="maxi
ma")
```

output
```
integrate(sqrt(b*x^2 + a)/(sqrt(d*x^2 + c)*(f*x^2 + e)*x^2), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}}{x^2\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)x^2}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/x^2/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="giac
")
```

output
```
integrate(sqrt(b*x^2 + a)/(sqrt(d*x^2 + c)*(f*x^2 + e)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{x^2\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{\sqrt{b\,x^2+a}}{x^2\,\sqrt{d\,x^2+c}\,\left(f\,x^2+e\right)}\,dx$$

input
```
int((a + b*x^2)^(1/2)/(x^2*(c + d*x^2)^(1/2)*(e + f*x^2)),x)
```

output
```
int((a + b*x^2)^(1/2)/(x^2*(c + d*x^2)^(1/2)*(e + f*x^2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}}{x^2\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{df\,x^6+cf\,x^4+de\,x^4+ce\,x^2}dx$$

input
```
int((b*x^2+a)^(1/2)/x^2/(d*x^2+c)^(1/2)/(f*x^2+e),x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(c*e*x**2 + c*f*x**4 + d*e*x**4 +
d*f*x**6),x)
```

3.128 $\int \frac{\sqrt{a+bx^2}}{x^4\sqrt{c+dx^2}(e+fx^2)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 498

$$\begin{aligned}\int \frac{\sqrt{a+bx^2}}{x^4\sqrt{c+dx^2}\,(e+fx^2)}\,dx &= \frac{b(bce-2ade-3acf)x\sqrt{c+dx^2}}{3ac^2e^2\sqrt{a+bx^2}}-\frac{b(2de+3cf)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3ac^2e^2} \\ &\quad -\frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{3acex^3}+\frac{(2de+3cf)\,(a+bx^2)^{3/2}\sqrt{c+dx^2}}{3ac^2e^2x} \\ &\quad -\frac{\sqrt{b}(bce-2ade-3acf)\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{3\sqrt{a}c^2e^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} \\ &\quad -\frac{\sqrt{a}\sqrt{b}(de+3cf)\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{3c^2e^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} \\ &\quad +\frac{a^{3/2}f^2\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce^3\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}\end{aligned}$$

output
```
1/3*b*(-3*a*c*f-2*a*d*e+b*c*e)*x*(d*x^2+c)^(1/2)/a/c^2/e^2/(b*x^2+a)^(1/2)
-1/3*b*(3*c*f+2*d*e)*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a/c^2/e^2-1/3*(b*x^
2+a)^(3/2)*(d*x^2+c)^(1/2)/a/c/e/x^3+1/3*(3*c*f+2*d*e)*(b*x^2+a)^(3/2)*(d*
x^2+c)^(1/2)/a/c^2/e^2/x-1/3*b^(1/2)*(-3*a*c*f-2*a*d*e+b*c*e)*(d*x^2+c)^(1
/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/a^(1/
2)/c^2/e^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)-1/3*a^(1/2)*b^(
1/2)*(3*c*f+d*e)*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2))
,(1-a*d/b/c)^(1/2))/c^2/e^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2
)+a^(3/2)*f^2*(d*x^2+c)^(1/2)*EllipticPi(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/
2),1-a*f/b/e,(1-a*d/b/c)^(1/2))/b^(1/2)/c/e^3/(b*x^2+a)^(1/2)/(a*(d*x^2+c)
/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.72 (sec) , antiderivative size = 345, normalized size of antiderivative = 0.69

$$\int \frac{\sqrt{a+bx^2}}{x^4\sqrt{c+dx^2}\,(e+fx^2)}\,dx$$
$$= \frac{\sqrt{\frac{b}{a}}\left(\sqrt{\frac{b}{a}}e(a+bx^2)\,(c+dx^2)\,(-bcex^2+a(-ce+2dex^2+3cfx^2))+ibce(-bce+2ade+3acf)x^3\sqrt{1+\right.}{}$$

input
```
Integrate[Sqrt[a + b*x^2]/(x^4*Sqrt[c + d*x^2]*(e + f*x^2)),x]
```

output
```
(Sqrt[b/a]*(Sqrt[b/a]*e*(a + b*x^2)*(c + d*x^2)*(-(b*c*e*x^2) + a*(-(c*e)
+ 2*d*e*x^2 + 3*c*f*x^2)) + I*b*c*e*(-(b*c*e) + 2*a*d*e + 3*a*c*f)*x^3*Sqr
t[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*
d)/(b*c)] - I*b*c*e*(-(b*c*e) + a*d*e + 3*a*c*f)*x^3*Sqrt[1 + (b*x^2)/a]*S
qrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - (3*I)*
a*c^2*f*(-(b*e) + a*f)*x^3*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Ellipti
cPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(3*b*c^2*e^3*x^3*S
qrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}}{x^4\sqrt{c+dx^2}\,(e+fx^2)}\,dx$$

$$\Big\downarrow \; 450$$

$$\int \frac{\sqrt{a+bx^2}}{x^4\sqrt{c+dx^2}\,(e+fx^2)}dx$$

input
```
Int[Sqrt[a + b*x^2]/(x^4*Sqrt[c + d*x^2]*(e + f*x^2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [A] (verified)

Time = 8.65 (sec) , antiderivative size = 448, normalized size of antiderivative = 0.90

method	result
risch	$-\frac{\sqrt{b x^2+a}\,\sqrt{x^2 d+c}\,(-3acf\,x^2-2ade\,x^2+bce\,x^2+ace)}{3a\,c^2e^2x^3}-\frac{\left(-\frac{b(3acf+2ade-bce)c\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\left(\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right.\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}\right.}{}$
default	$\frac{\left(3\sqrt{-\frac{b}{a}}\,abcdef\,x^6+2\sqrt{-\frac{b}{a}}\,ab\,d^2e^2x^6-\sqrt{-\frac{b}{a}}\,b^2cd\,e^2x^6+3\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{x^2d+c}{c}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)ab\,c^2ef\,x^3+\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{x^2}\right.}{}$
elliptic	$\frac{\sqrt{(b\,x^2+a)(x^2d+c)}\left(-\frac{\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{3ce\,x^3}+\frac{(3acf+2ade-bce)\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{3a\,c^2e^2x}+\frac{db\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-}\right.}{3ce\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}\right.}{}$

input
```
int((b*x^2+a)^(1/2)/x^4/(d*x^2+c)^(1/2)/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(-3*a*c*f*x^2-2*a*d*e*x^2+b*c*e*x^2+a
*c*e)/a/c^2/e^2/x^3-1/3/c^2/e^2/a*(-b*(3*a*c*f+2*a*d*e-b*c*e)*c/(-b/a)^(1/
2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1
/2),(-1+(a*d+b*c)/c/b)^(1/2)))+a*c*d*e*b/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1
+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/
2),(-1+(a*d+b*c)/c/b)^(1/2))-3*a*c^2*f*(a*f-b*e)/e/(-b/a)^(1/2)*(1+b*x^2/a
)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x
*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2)))*((b*x^2+a)*(d*x^2+c))^
(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{x^4\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(1/2)/x^4/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}}{x^4\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{\sqrt{a+bx^2}}{x^4\sqrt{c+dx^2}\,(e+fx^2)}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)/x**4/(d*x**2+c)**(1/2)/(f*x**2+e),x)
```

output
```
Integral(sqrt(a + b*x**2)/(x**4*sqrt(c + d*x**2)*(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}}{x^4\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)x^4}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/x^4/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="maxi
ma")
```

output
```
integrate(sqrt(b*x^2 + a)/(sqrt(d*x^2 + c)*(f*x^2 + e)*x^4), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}}{x^4\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)x^4}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/x^4/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="giac
")
```

output
```
integrate(sqrt(b*x^2 + a)/(sqrt(d*x^2 + c)*(f*x^2 + e)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{x^4\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{\sqrt{b\,x^2+a}}{x^4\,\sqrt{d\,x^2+c}\,\left(f\,x^2+e\right)}\,dx$$

input
```
int((a + b*x^2)^(1/2)/(x^4*(c + d*x^2)^(1/2)*(e + f*x^2)),x)
```

output
```
int((a + b*x^2)^(1/2)/(x^4*(c + d*x^2)^(1/2)*(e + f*x^2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}}{x^4\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{df\,x^8+cf\,x^6+de\,x^6+ce\,x^4}dx$$

input
```
int((b*x^2+a)^(1/2)/x^4/(d*x^2+c)^(1/2)/(f*x^2+e),x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(c*e*x**4 + c*f*x**6 + d*e*x**6 +
d*f*x**8),x)
```

3.129 $\int \frac{\sqrt{a+bx^2}}{x^6\sqrt{c+dx^2}(e+fx^2)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 702

$$\int \frac{\sqrt{a+bx^2}}{x^6\sqrt{c+dx^2}\,(e+fx^2)}\,dx = -\frac{b(2b^2c^2e^2+abce(3de+5cf)-a^2(8d^2e^2+10cdef+15c^2f^2))\,x\sqrt{c+dx^2}}{15a^2c^3e^3\sqrt{a+bx^2}} + \frac{b(bcde^2+a(8d^2e^2+10cdef+15c^2f^2))\,x\sqrt{a+bx^2}\sqrt{c+dx^2}}{15a^2c^3e^3} - \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{5acex^5} + \frac{(2bce+4ade+5acf)\,(a+bx^2)^{3/2}\sqrt{c+dx^2}}{15a^2c^2e^2x^3} - \frac{\left(bde+a\left(\frac{8d^2e}{c}+10df+\frac{15cf^2}{e}\right)\right)(a+bx^2)^{3/2}\sqrt{c+dx^2}}{15a^2c^2e^2x} + \frac{\sqrt{b}(2b^2c^2e^2+abce(3de+5cf)-a^2(8d^2e^2+10cdef+15c^2f^2))\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{15a^{3/2}c^3e^3\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} - \frac{\sqrt{b}(bcde^2-a(4d^2e^2+5cdef+15c^2f^2))\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{15\sqrt{a}c^3e^3\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} - \frac{a^{3/2}f^3\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce^4\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
-1/15*b*(2*b^2*c^2*e^2+a*b*c*e*(5*c*f+3*d*e)-a^2*(15*c^2*f^2+10*c*d*e*f+8*
d^2*e^2))*x*(d*x^2+c)^(1/2)/a^2/c^3/e^3/(b*x^2+a)^(1/2)+1/15*b*(b*c*d*e^2+
a*(15*c^2*f^2+10*c*d*e*f+8*d^2*e^2))*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a^2
/c^3/e^3-1/5*(b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/a/c/e/x^5+1/15*(5*a*c*f+4*a*d
*e+2*b*c*e)*(b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/a^2/c^2/e^2/x^3-1/15*(b*d*e+a*
(8*d^2*e/c+10*d*f+15*c*f^2/e))*(b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/a^2/c^2/e^2
/x+1/15*b^(1/2)*(2*b^2*c^2*e^2+a*b*c*e*(5*c*f+3*d*e)-a^2*(15*c^2*f^2+10*c*
d*e*f+8*d^2*e^2))*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^
(1/2),(1-a*d/b/c)^(1/2))/a^(3/2)/c^3/e^3/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b
*x^2+a))^(1/2)-1/15*b^(1/2)*(b*c*d*e^2-a*(15*c^2*f^2+5*c*d*e*f+4*d^2*e^2))
*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/
2))/a^(1/2)/c^3/e^3/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)-a^(3/2
)*f^3*(d*x^2+c)^(1/2)*EllipticPi(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),1-a*f
/b/e,(1-a*d/b/c)^(1/2))/b^(1/2)/c/e^4/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^
2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.19 (sec) , antiderivative size = 1020, normalized size of antiderivative = 1.45

$$\int \frac{\sqrt{a+bx^2}}{x^6\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \text{Too large to display}$$

input
```
Integrate[Sqrt[a + b*x^2]/(x^6*Sqrt[c + d*x^2]*(e + f*x^2)),x]
```

output
```
(-3*a^3*Sqrt[b/a]*c^3*e^3 - 4*a^3*(b/a)^(3/2)*c^3*e^3*x^2 + a^3*Sqrt[b/a]*
c^2*d*e^3*x^2 + 5*a^3*Sqrt[b/a]*c^3*e^2*f*x^2 + a*b^2*Sqrt[b/a]*c^3*e^3*x^
4 + 3*a^3*(b/a)^(3/2)*c^2*d*e^3*x^4 - 4*a^3*Sqrt[b/a]*c*d^2*e^3*x^4 + 10*a
^3*(b/a)^(3/2)*c^3*e^2*f*x^4 - 5*a^3*Sqrt[b/a]*c^2*d*e^2*f*x^4 - 15*a^3*Sq
rt[b/a]*c^3*e*f^2*x^4 + 2*b^3*Sqrt[b/a]*c^3*e^3*x^6 + 4*a*b^2*Sqrt[b/a]*c^
2*d*e^3*x^6 - a^3*(b/a)^(3/2)*c*d^2*e^3*x^6 - 8*a^3*Sqrt[b/a]*d^3*e^3*x^6
+ 5*a*b^2*Sqrt[b/a]*c^3*e^2*f*x^6 - 10*a^3*Sqrt[b/a]*c*d^2*e^2*f*x^6 - 15*
a^3*(b/a)^(3/2)*c^3*e*f^2*x^6 - 15*a^3*Sqrt[b/a]*c^2*d*e*f^2*x^6 + 2*b^3*S
qrt[b/a]*c^2*d*e^3*x^8 + 3*a*b^2*Sqrt[b/a]*c*d^2*e^3*x^8 - 8*a^3*(b/a)^(3/
2)*d^3*e^3*x^8 + 5*a*b^2*Sqrt[b/a]*c^2*d*e^2*f*x^8 - 10*a^3*(b/a)^(3/2)*c*
d^2*e^2*f*x^8 - 15*a^3*(b/a)^(3/2)*c^2*d*e*f^2*x^8 - I*b*c*e*(-2*b^2*c^2*e
^2 - a*b*c*e*(3*d*e + 5*c*f) + a^2*(8*d^2*e^2 + 10*c*d*e*f + 15*c^2*f^2))*
x^5*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*
x], (a*d)/(b*c)] + I*b*c*e*(-2*b^2*c^2*e^2 - a*b*c*e*(2*d*e + 5*c*f) + a^2
*(4*d^2*e^2 + 5*c*d*e*f + 15*c^2*f^2))*x^5*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d
*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - (15*I)*a^2*b*c^3
*e*f^2*x^5*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e),
 I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (15*I)*a^3*c^3*f^3*x^5*Sqrt[1 + (b
*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x
], (a*d)/(b*c)])/(15*a^2*Sqrt[b/a]*c^3*e^4*x^5*Sqrt[a + b*x^2]*Sqrt[c +...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}}{x^6\sqrt{c+dx^2}\,(e+fx^2)}\,dx$$

$$\Big\downarrow \ 450$$

$$\int \frac{\sqrt{a+bx^2}}{x^6\sqrt{c+dx^2}\,(e+fx^2)}dx$$

input
```
Int[Sqrt[a + b*x^2]/(x^6*Sqrt[c + d*x^2]*(e + f*x^2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [A] (verified)

Time = 10.76 (sec) , antiderivative size = 799, normalized size of antiderivative = 1.14

method	result
risch	$-\frac{\sqrt{b x^2+a}\,\sqrt{x^2 d+c}\,\left(15 f^2 x^4 a^2 c^2+10 a^2 c d e f\, x^4+8 a^2 d^2 e^2 x^4-5 a b\, c^2 e f\, x^4-3 a b c d\, e^2 x^4-2 b^2 c^2 e^2 x^4-5 a^2 c^2 e f\, x^2-4 a^2 c d\, e^2 x^2+a b\, c^{2}\right.}{15 a^2 c^3 e^3 x^5}$
default	Expression too large to display
elliptic	Expression too large to display

input
```
int((b*x^2+a)^(1/2)/x^6/(d*x^2+c)^(1/2)/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
-1/15*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(15*a^2*c^2*f^2*x^4+10*a^2*c*d*e*f*x
^4+8*a^2*d^2*e^2*x^4-5*a*b*c^2*e*f*x^4-3*a*b*c*d*e^2*x^4-2*b^2*c^2*e^2*x^4
-5*a^2*c^2*e*f*x^2-4*a^2*c*d*e^2*x^2+a*b*c^2*e^2*x^2+3*a^2*c^2*e^2)/a^2/c^
3/e^3/x^5+1/15/a^2/c^3/e^3*(-b*(15*a^2*c^2*f^2+10*a^2*c*d*e*f+8*a^2*d^2*e^
2-5*a*b*c^2*e*f-3*a*b*c*d*e^2-2*b^2*c^2*e^2)*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1
/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(EllipticF(x*(-b
/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)
/c/b)^(1/2)))-a*b^2*c^2*d*e^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(
1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d
+b*c)/c/b)^(1/2))+4*a^2*b*c*d^2*e^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^
2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-
1+(a*d+b*c)/c/b)^(1/2))-15*a^2*c^3*f^2*(a*f-b*e)/e/(-b/a)^(1/2)*(1+b*x^2/a
)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x
*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))+5*a^2*b*c^2*d*e*f/(-b/a
)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^
(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2)))*((b*x^2+a)*(d*x^
2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{x^6\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(1/2)/x^6/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}}{x^6\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{\sqrt{a+bx^2}}{x^6\sqrt{c+dx^2}\,(e+fx^2)}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)/x**6/(d*x**2+c)**(1/2)/(f*x**2+e),x)
```

output
```
Integral(sqrt(a + b*x**2)/(x**6*sqrt(c + d*x**2)*(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}}{x^6\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)x^6}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/x^6/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="maxi
ma")
```

output
```
integrate(sqrt(b*x^2 + a)/(sqrt(d*x^2 + c)*(f*x^2 + e)*x^6), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}}{x^6\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)x^6}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/x^6/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="giac
")
```

output
```
integrate(sqrt(b*x^2 + a)/(sqrt(d*x^2 + c)*(f*x^2 + e)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{x^6\sqrt{c+dx^2}\,(e+fx^2)}\,dx=\int \frac{\sqrt{b\,x^2+a}}{x^6\,\sqrt{d\,x^2+c}\,\left(f\,x^2+e\right)}\,dx$$

input
```
int((a + b*x^2)^(1/2)/(x^6*(c + d*x^2)^(1/2)*(e + f*x^2)),x)
```

output
```
int((a + b*x^2)^(1/2)/(x^6*(c + d*x^2)^(1/2)*(e + f*x^2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}}{x^6\sqrt{c+dx^2}\,(e+fx^2)}\,dx=\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{df\,x^{10}+cf\,x^8+de\,x^8+ce\,x^6}dx$$

input
```
int((b*x^2+a)^(1/2)/x^6/(d*x^2+c)^(1/2)/(f*x^2+e),x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(c*e*x**6 + c*f*x**8 + d*e*x**8 +
d*f*x**10),x)
```

3.130 $\int \frac{x^6\sqrt{a+bx^2}}{\sqrt{c+dx^2}(e+fx^2)^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 643

$$\begin{aligned}\int \frac{x^6\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx &= \frac{\left(2adf-\frac{b(15d^2e^2-8cdef-4c^2f^2)}{de-cf}\right)x\sqrt{c+dx^2}}{6d^2f^3\sqrt{a+bx^2}} \\ &+\frac{(be(5de-2cf)-2af(de-cf))x\sqrt{a+bx^2}\sqrt{c+dx^2}}{6df^2(be-af)(de-cf)} \\ &-\frac{e^2x(a+bx^2)^{3/2}\sqrt{c+dx^2}}{2f(be-af)(de-cf)\,(e+fx^2)} \\ &-\frac{\sqrt{a}(2adf(de-cf)-b(15d^2e^2-8cdef-4c^2f^2))\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{6\sqrt{b}d^2f^3(de-cf)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} \\ &+\frac{a^{3/2}(2af(6de+cf)-be(15de+2cf))\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{6\sqrt{b}cdf^3(be-af)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} \\ &+\frac{a^{3/2}e(be(5de-6cf)-af(4de-5cf))\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{2\sqrt{b}cf^3(be-af)(de-cf)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}\end{aligned}$$

output
```
1/6*(2*a*d*f-b*(-4*c^2*f^2-8*c*d*e*f+15*d^2*e^2)/(-c*f+d*e))*x*(d*x^2+c)^(
1/2)/d^2/f^3/(b*x^2+a)^(1/2)+1/6*(b*e*(-2*c*f+5*d*e)-2*a*f*(-c*f+d*e))*x*(
b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/d/f^2/(-a*f+b*e)/(-c*f+d*e)-1/2*e^2*x*(b*x^
2+a)^(3/2)*(d*x^2+c)^(1/2)/f/(-a*f+b*e)/(-c*f+d*e)/(f*x^2+e)-1/6*a^(1/2)*(
2*a*d*f*(-c*f+d*e)-b*(-4*c^2*f^2-8*c*d*e*f+15*d^2*e^2))*(d*x^2+c)^(1/2)*El
lipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/b^(1/2)/d^2
/f^3/(-c*f+d*e)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)+1/6*a^(3/2
)*(2*a*f*(c*f+6*d*e)-b*e*(2*c*f+15*d*e))*(d*x^2+c)^(1/2)*InverseJacobiAM(a
rctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/b^(1/2)/c/d/f^3/(-a*f+b*e)/(b*
x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)+1/2*a^(3/2)*e*(b*e*(-6*c*f+5*
d*e)-a*f*(-5*c*f+4*d*e))*(d*x^2+c)^(1/2)*EllipticPi(b^(1/2)*x/a^(1/2)/(1+b
*x^2/a)^(1/2),1-a*f/b/e,(1-a*d/b/c)^(1/2))/b^(1/2)/c/f^3/(-a*f+b*e)/(-c*f+
d*e)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.91 (sec) , antiderivative size = 438, normalized size of antiderivative = 0.68

$$\int \frac{x^6\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx$$
$$= \frac{-icf(2adf(-de+cf)+b(15d^2e^2-8cdef-4c^2f^2))\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}(e+fx^2)\,E\left(i\operatorname{arcsinh}\left(\sqrt{\frac{b}{a}}x\right)|\frac{a}{b}\right.}{}$$

input
```
Integrate[(x^6*Sqrt[a + b*x^2])/(Sqrt[c + d*x^2]*(e + f*x^2)^2),x]
```

output
```
((-I)*c*f*(2*a*d*f*(-(d*e) + c*f) + b*(15*d^2*e^2 - 8*c*d*e*f - 4*c^2*f^2)
)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(e + f*x^2)*EllipticE[I*ArcSinh[
Sqrt[b/a]*x], (a*d)/(b*c)] + I*(-(d*e) + c*f)*(4*a*d*f*(3*d*e + c*f) - b*(
15*d^2*e^2 + 12*c*d*e*f + 4*c^2*f^2))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)
/c]*(e + f*x^2)*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + d*(Sqrt[b
/a]*f^2*x*(a + b*x^2)*(c + d*x^2)*(2*c*f*(e + f*x^2) - d*e*(5*e + 2*f*x^2)
) - (3*I)*d*e*(b*e*(5*d*e - 6*c*f) + a*f*(-4*d*e + 5*c*f))*Sqrt[1 + (b*x^2
)/a]*Sqrt[1 + (d*x^2)/c]*(e + f*x^2)*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqr
t[b/a]*x], (a*d)/(b*c)]))/(6*Sqrt[b/a]*d^2*f^4*(-(d*e) + c*f)*Sqrt[a + b*x
^2]*Sqrt[c + d*x^2]*(e + f*x^2))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx$$

$$\Big\downarrow \; 450$$

$$\int \frac{x^6\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)^2}dx$$

input
```
Int[(x^6*Sqrt[a + b*x^2])/(Sqrt[c + d*x^2]*(e + f*x^2)^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1553 vs. $2(605) = 1210$.

Time = 23.78 (sec) , antiderivative size = 1554, normalized size of antiderivative = 2.42

method	result	size
risch	Expression too large to display	1554
elliptic	Expression too large to display	1650
default	Expression too large to display	2397

input
```
int(x^6*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x,method=_RETURNVERBOS
E)
```

output
```
1/3/f^2/d*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)+1/3/f^2/d*(1/f^2*(-f*(a*d*f-2*
b*c*f-6*b*d*e)*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4
+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)
^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2)))-a*c*f^2/(-b/a)
^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(
1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+9*b*d*e^2/(-b/a)^(
1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/
2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-6*a*d*e*f/(-b/a)^(1/
2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2)))-3*d*e^3*(a*f-b*e)/f^2
*(1/2*f^2/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/e*x*(b*d*x^4+a*d*x^2+b*c*x^2+a
*c)^(1/2)/(f*x^2+e)-1/2*d*b/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/(-b/a)^(1/2)
*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*E
llipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+1/2*f*b/(a*c*f^2-a*d*e*f
-b*c*e*f+b*d*e^2)/e*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*
d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/
b)^(1/2))-1/2*f*b/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/e*c/(-b/a)^(1/2)*(1+b*
x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*Ellipti
cE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+1/2/(a*c*f^2-a*d*e*f-b*c*e*f+b
*d*e^2)/e^2*f^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^6\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(x^6*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^6\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{x^6\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx$$

input
```
integrate(x**6*(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**2,x)
```

output
```
Integral(x**6*sqrt(a + b*x**2)/(sqrt(c + d*x**2)*(e + f*x**2)**2), x)
```

Maxima [F]

$$\int \frac{x^6\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{\sqrt{bx^2+a}x^6}{\sqrt{dx^2+c}(fx^2+e)^2}\,dx$$

input
```
integrate(x^6*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="ma
xima")
```

output
```
integrate(sqrt(b*x^2 + a)*x^6/(sqrt(d*x^2 + c)*(f*x^2 + e)^2), x)
```

Giac [F]

$$\int \frac{x^6\sqrt{a+bx^2}}{\sqrt{c+dx^2}\left(e+fx^2\right)^2}\,dx = \int \frac{\sqrt{bx^2+a}x^6}{\sqrt{dx^2+c}(fx^2+e)^2}\,dx$$

input
```
integrate(x^6*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="gi
ac")
```

output
```
integrate(sqrt(b*x^2 + a)*x^6/(sqrt(d*x^2 + c)*(f*x^2 + e)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6\sqrt{a+bx^2}}{\sqrt{c+dx^2}\left(e+fx^2\right)^2}\,dx = \int \frac{x^6\,\sqrt{b\,x^2+a}}{\sqrt{d\,x^2+c}\,{\left(f\,x^2+e\right)}^2}\,dx$$

input
```
int((x^6*(a + b*x^2)^(1/2))/((c + d*x^2)^(1/2)*(e + f*x^2)^2),x)
```

output
```
int((x^6*(a + b*x^2)^(1/2))/((c + d*x^2)^(1/2)*(e + f*x^2)^2), x)
```

Reduce [F]

$$\int \frac{x^6\sqrt{a+bx^2}}{\sqrt{c+dx^2}\left(e+fx^2\right)^2}\,dx = \text{too large to display}$$

input
```
int(x^6*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x)
```

output

```
( - sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*c*f*x - 4*sqrt(c + d*x**2)*sqrt(a
+ b*x**2)*a*d*e*x - 4*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b*c*e*x + 3*sqrt(c
 + d*x**2)*sqrt(a + b*x**2)*b*d*e*x**3 + int((sqrt(c + d*x**2)*sqrt(a + b*
x**2)*x**6)/(a*c*e**2 + 2*a*c*e*f*x**2 + a*c*f**2*x**4 + a*d*e**2*x**2 + 2
*a*d*e*f*x**4 + a*d*f**2*x**6 + b*c*e**2*x**2 + 2*b*c*e*f*x**4 + b*c*f**2*
x**6 + b*d*e**2*x**4 + 2*b*d*e*f*x**6 + b*d*f**2*x**8),x)*a*b*c*d*e*f**2 +
 int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**6)/(a*c*e**2 + 2*a*c*e*f*x**2 +
 a*c*f**2*x**4 + a*d*e**2*x**2 + 2*a*d*e*f*x**4 + a*d*f**2*x**6 + b*c*e**2
*x**2 + 2*b*c*e*f*x**4 + b*c*f**2*x**6 + b*d*e**2*x**4 + 2*b*d*e*f*x**6 +
b*d*f**2*x**8),x)*a*b*c*d*f**3*x**2 + 7*int((sqrt(c + d*x**2)*sqrt(a + b*x
**2)*x**6)/(a*c*e**2 + 2*a*c*e*f*x**2 + a*c*f**2*x**4 + a*d*e**2*x**2 + 2*
a*d*e*f*x**4 + a*d*f**2*x**6 + b*c*e**2*x**2 + 2*b*c*e*f*x**4 + b*c*f**2*x
**6 + b*d*e**2*x**4 + 2*b*d*e*f*x**6 + b*d*f**2*x**8),x)*a*b*d**2*e**2*f +
 7*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**6)/(a*c*e**2 + 2*a*c*e*f*x**2
 + a*c*f**2*x**4 + a*d*e**2*x**2 + 2*a*d*e*f*x**4 + a*d*f**2*x**6 + b*c*e*
*2*x**2 + 2*b*c*e*f*x**4 + b*c*f**2*x**6 + b*d*e**2*x**4 + 2*b*d*e*f*x**6
+ b*d*f**2*x**8),x)*a*b*d**2*e*f**2*x**2 - 2*int((sqrt(c + d*x**2)*sqrt(a
+ b*x**2)*x**6)/(a*c*e**2 + 2*a*c*e*f*x**2 + a*c*f**2*x**4 + a*d*e**2*x**2
 + 2*a*d*e*f*x**4 + a*d*f**2*x**6 + b*c*e**2*x**2 + 2*b*c*e*f*x**4 + b*c*f
**2*x**6 + b*d*e**2*x**4 + 2*b*d*e*f*x**6 + b*d*f**2*x**8),x)*b**2*c*d*...
```

3.131 $\int \frac{x^4\sqrt{a+bx^2}}{\sqrt{c+dx^2}(e+fx^2)^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 538

$$\int \frac{x^4\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \frac{b(3de-2cf)x\sqrt{c+dx^2}}{2df^2(de-cf)\sqrt{a+bx^2}} - \frac{bex\sqrt{a+bx^2}\sqrt{c+dx^2}}{2f(be-af)(de-cf)} + \frac{ex(a+bx^2)^{3/2}\sqrt{c+dx^2}}{2(be-af)(de-cf)(e+fx^2)} - \frac{\sqrt{a}\sqrt{b}(3de-2cf)\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{2df^2(de-cf)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} + \frac{a^{3/2}(3be-2af)\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{2\sqrt{b}cf^2(be-af)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} - \frac{a^{3/2}(be(3de-4cf)-af(2de-3cf))\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{2\sqrt{b}cf^2(be-af)(de-cf)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
1/2*b*(-2*c*f+3*d*e)*x*(d*x^2+c)^(1/2)/d/f^2/(-c*f+d*e)/(b*x^2+a)^(1/2)-1/
2*b*e*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/f/(-a*f+b*e)/(-c*f+d*e)+1/2*e*x*(b
*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/(-a*f+b*e)/(-c*f+d*e)/(f*x^2+e)-1/2*a^(1/2)*
b^(1/2)*(-2*c*f+3*d*e)*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^
2/a)^(1/2),(1-a*d/b/c)^(1/2))/d/f^2/(-c*f+d*e)/(b*x^2+a)^(1/2)/(a*(d*x^2+c
)/c/(b*x^2+a))^(1/2)+1/2*a^(3/2)*(-2*a*f+3*b*e)*(d*x^2+c)^(1/2)*InverseJac
obiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/b^(1/2)/c/f^2/(-a*f+b*e
)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)-1/2*a^(3/2)*(b*e*(-4*c*f
+3*d*e)-a*f*(-3*c*f+2*d*e))*(d*x^2+c)^(1/2)*EllipticPi(b^(1/2)*x/a^(1/2)/(
1+b*x^2/a)^(1/2),1-a*f/b/e,(1-a*d/b/c)^(1/2))/b^(1/2)/c/f^2/(-a*f+b*e)/(-c
*f+d*e)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.97 (sec) , antiderivative size = 359, normalized size of antiderivative = 0.67

$$\int \frac{x^4\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx$$
$$= \frac{ibcf(-3de+2cf)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}(e+fx^2)\,E\left(i\operatorname{arcsinh}\left(\sqrt{\frac{b}{a}}x\right)\middle|\frac{ad}{bc}\right)+i(-de+cf)(-3bde-2bcf+\cdots}{\cdots}$$

input
```
Integrate[(x^4*Sqrt[a + b*x^2])/(Sqrt[c + d*x^2]*(e + f*x^2)^2),x]
```

output
```
(I*b*c*f*(-3*d*e + 2*c*f)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(e + f*x
^2)*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + I*(-(d*e) + c*f)*(-3*
b*d*e - 2*b*c*f + 2*a*d*f)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(e + f*
x^2)*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - d*(Sqrt[b/a]*e*f^2*x
*(a + b*x^2)*(c + d*x^2) + I*(b*e*(3*d*e - 4*c*f) + a*f*(-2*d*e + 3*c*f))*
Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(e + f*x^2)*EllipticPi[(a*f)/(b*e)
, I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(2*Sqrt[b/a]*d*f^3*(d*e - c*f)*Sq
rt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx$$

$$\Big\downarrow \text{450}$$

$$\int \frac{x^4\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)^2}dx$$

input
```
Int[(x^4*Sqrt[a + b*x^2])/(Sqrt[c + d*x^2]*(e + f*x^2)^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1213 vs. $2(500) = 1000$.

Time = 6.74 (sec) , antiderivative size = 1214, normalized size of antiderivative = 2.26

method	result	size
elliptic	Expression too large to display	1214
default	Expression too large to display	1568

input
```
int(x^4*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x,method=_RETURNVERBOS
E)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/2*e/(c*f-d*
e)/f*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(f*x^2+e)+1/(-b/a)^(1/2)*(1+b*x
^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*Elliptic
F(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))*a/f^2-2/(-b/a)^(1/2)*(1+b*x^2/a
)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*
(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))*b/f^3*e-1/2/(-b/a)^(1/2)*(1+b*x^2/a
)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*
(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))*b*d*e^2/f^3/(c*f-d*e)-c/(-b/a)^(1/2
)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/
d*b/f^2*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+c/(-b/a)^(1/2)*
(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*
b/f^2*EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+1/2*c/(-b/a)^(1/2
)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*
b*e/f^2/(c*f-d*e)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-1/2*c
/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2
+a*c)^(1/2)*b*e/f^2/(c*f-d*e)*EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^
(1/2))-3/2/(c*f-d*e)/f/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b
*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*
d)^(1/2)/(-b/a)^(1/2))*a*c+e/(c*f-d*e)/f^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*
(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^4\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(x^4*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^4\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{x^4\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx$$

input
```
integrate(x**4*(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**2,x)
```

output
```
Integral(x**4*sqrt(a + b*x**2)/(sqrt(c + d*x**2)*(e + f*x**2)**2), x)
```

Maxima [F]

$$\int \frac{x^4\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{\sqrt{bx^2+a}x^4}{\sqrt{dx^2+c}(fx^2+e)^2}\,dx$$

input
```
integrate(x^4*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="ma
xima")
```

output
```
integrate(sqrt(b*x^2 + a)*x^4/(sqrt(d*x^2 + c)*(f*x^2 + e)^2), x)
```

Giac [F]

$$\int \frac{x^4\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{\sqrt{bx^2+a}x^4}{\sqrt{dx^2+c}(fx^2+e)^2}\,dx$$

input
```
integrate(x^4*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="gi
ac")
```

output
```
integrate(sqrt(b*x^2 + a)*x^4/(sqrt(d*x^2 + c)*(f*x^2 + e)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx=\int \frac{x^4\,\sqrt{b\,x^2+a}}{\sqrt{d\,x^2+c}\,(f\,x^2+e)^2}\,dx$$

input
```
int((x^4*(a + b*x^2)^(1/2))/((c + d*x^2)^(1/2)*(e + f*x^2)^2),x)
```

output
```
int((x^4*(a + b*x^2)^(1/2))/((c + d*x^2)^(1/2)*(e + f*x^2)^2), x)
```

Reduce [F]

$$\int \frac{x^4\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx=\text{too large to display}$$

input
```
int(x^4*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x)
```

output
```
(sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*x - int((sqrt(c + d*x**2)*sqrt(a + b*
x**2)*x**6)/(a*c*e**2 + 2*a*c*e*f*x**2 + a*c*f**2*x**4 + a*d*e**2*x**2 + 2
*a*d*e*f*x**4 + a*d*f**2*x**6 + b*c*e**2*x**2 + 2*b*c*e*f*x**4 + b*c*f**2*
x**6 + b*d*e**2*x**4 + 2*b*d*e*f*x**6 + b*d*f**2*x**8),x)*a*b*d*e*f - int(
(sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**6)/(a*c*e**2 + 2*a*c*e*f*x**2 + a*c*
f**2*x**4 + a*d*e**2*x**2 + 2*a*d*e*f*x**4 + a*d*f**2*x**6 + b*c*e**2*x**2
 + 2*b*c*e*f*x**4 + b*c*f**2*x**6 + b*d*e**2*x**4 + 2*b*d*e*f*x**6 + b*d*f
**2*x**8),x)*a*b*d*f**2*x**2 + 3*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x*
*6)/(a*c*e**2 + 2*a*c*e*f*x**2 + a*c*f**2*x**4 + a*d*e**2*x**2 + 2*a*d*e*f
*x**4 + a*d*f**2*x**6 + b*c*e**2*x**2 + 2*b*c*e*f*x**4 + b*c*f**2*x**6 + b
*d*e**2*x**4 + 2*b*d*e*f*x**6 + b*d*f**2*x**8),x)*b**2*d*e**2 + 3*int((sqr
t(c + d*x**2)*sqrt(a + b*x**2)*x**6)/(a*c*e**2 + 2*a*c*e*f*x**2 + a*c*f**2
*x**4 + a*d*e**2*x**2 + 2*a*d*e*f*x**4 + a*d*f**2*x**6 + b*c*e**2*x**2 + 2
*b*c*e*f*x**4 + b*c*f**2*x**6 + b*d*e**2*x**4 + 2*b*d*e*f*x**6 + b*d*f**2*
x**8),x)*b**2*d*e*f*x**2 + int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a
*c*e**2 + 2*a*c*e*f*x**2 + a*c*f**2*x**4 + a*d*e**2*x**2 + 2*a*d*e*f*x**4
+ a*d*f**2*x**6 + b*c*e**2*x**2 + 2*b*c*e*f*x**4 + b*c*f**2*x**6 + b*d*e**
2*x**4 + 2*b*d*e*f*x**6 + b*d*f**2*x**8),x)*a**2*c*e*f + int((sqrt(c + d*x
**2)*sqrt(a + b*x**2)*x**2)/(a*c*e**2 + 2*a*c*e*f*x**2 + a*c*f**2*x**4 + a
*d*e**2*x**2 + 2*a*d*e*f*x**4 + a*d*f**2*x**6 + b*c*e**2*x**2 + 2*b*c*e...
```

3.132 $\int \frac{x^2\sqrt{a+bx^2}}{\sqrt{c+dx^2}(e+fx^2)^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 496

$$\int \frac{x^2\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = -\frac{bx\sqrt{c+dx^2}}{2f(de-cf)\sqrt{a+bx^2}}+\frac{bx\sqrt{a+bx^2}\sqrt{c+dx^2}}{2(be-af)(de-cf)} - \frac{fx(a+bx^2)^{3/2}\sqrt{c+dx^2}}{2(be-af)(de-cf)\,(e+fx^2)}+\frac{\sqrt{a}\sqrt{b}\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{2f(de-cf)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} - \frac{a^{3/2}\sqrt{b}\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{2cf(be-af)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} + \frac{a^{3/2}(acf^2+be(de-2cf))\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{2\sqrt{b}cef(be-af)(de-cf)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
-1/2*b*x*(d*x^2+c)^(1/2)/f/(-c*f+d*e)/(b*x^2+a)^(1/2)+1/2*b*x*(b*x^2+a)^(1
/2)*(d*x^2+c)^(1/2)/(-a*f+b*e)/(-c*f+d*e)-1/2*f*x*(b*x^2+a)^(3/2)*(d*x^2+c
)^(1/2)/(-a*f+b*e)/(-c*f+d*e)/(f*x^2+e)+1/2*a^(1/2)*b^(1/2)*(d*x^2+c)^(1/2
)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/f/(-c*f
+d*e)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)-1/2*a^(3/2)*b^(1/2)*
(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2
))/c/f/(-a*f+b*e)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)+1/2*a^(3
/2)*(a*c*f^2+b*e*(-2*c*f+d*e))*(d*x^2+c)^(1/2)*EllipticPi(b^(1/2)*x/a^(1/2
)/(1+b*x^2/a)^(1/2),1-a*f/b/e,(1-a*d/b/c)^(1/2))/b^(1/2)/c/e/f/(-a*f+b*e)/
(-c*f+d*e)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.72 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.51

$$\int \frac{x^2\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \frac{\sqrt{\frac{b}{a}}ef^2x(a+bx^2)\,(c+dx^2)+i\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}(e+fx^2)\left(bcefE\left(i\mathrm{arcsinh}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)-be(de-c\right.}{2\sqrt{\frac{b}{a}}ef^2(de-cf)\sqrt{a+bx}}$$

input
```
Integrate[(x^2*Sqrt[a + b*x^2])/(Sqrt[c + d*x^2]*(e + f*x^2)^2),x]
```

output
```
(Sqrt[b/a]*e*f^2*x*(a + b*x^2)*(c + d*x^2) + I*Sqrt[1 + (b*x^2)/a]*Sqrt[1
+ (d*x^2)/c]*(e + f*x^2)*(b*c*e*f*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/
(b*c)] - b*e*(d*e - c*f)*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] +
(a*c*f^2 + b*e*(d*e - 2*c*f))*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*
x], (a*d)/(b*c)]))/(2*Sqrt[b/a]*e*f^2*(d*e - c*f)*Sqrt[a + b*x^2]*Sqrt[c +
 d*x^2]*(e + f*x^2))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx$$

$$\Big\downarrow \; 450$$

$$\int \frac{x^2\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)^2}dx$$

input
```
Int[(x^2*Sqrt[a + b*x^2])/(Sqrt[c + d*x^2]*(e + f*x^2)^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [A] (verified)

Time = 6.75 (sec) , antiderivative size = 812, normalized size of antiderivative = 1.64

method	result
elliptic	$\frac{\sqrt{(b x^2+a)(x^2 d+c)}\left(-\frac{x\sqrt{bd x^4+ad x^2+x^2 bc+ac}}{2(cf-de)\left(f x^2+e\right)}+\frac{\sqrt{1+\frac{b x^2}{a}}\sqrt{1+\frac{d x^2}{c}}\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)b}{\sqrt{-\frac{b}{a}}\sqrt{bd x^4+ad x^2+x^2 bc+ac}\,f^2}+\frac{\sqrt{1+\frac{b x^2}{a}}\sqrt{1+\frac{d x^2}{c}}\operatorname{EllipticF}\left(\right.}{2\sqrt{-\frac{b}{a}}\sqrt{bd x^4+ad x^2+}}$
default	$\frac{\left(-\sqrt{-\frac{b}{a}}\,bde\,f^2x^5+\sqrt{\frac{b x^2+a}{a}}\sqrt{\frac{x^2d+c}{c}}\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)bce\,f^2x^2-\sqrt{\frac{b x^2+a}{a}}\sqrt{\frac{x^2d+c}{c}}\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)bd\,e^2f\,x^2\right.}{}$

input
```
int(x^2*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x,method=_RETURNVERBOS
E)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-1/2/(c*f-d*e
)*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(f*x^2+e)+1/(-b/a)^(1/2)*(1+b*x^2/
a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x
*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))*b/f^2+1/2/(-b/a)^(1/2)*(1+b*x^2/a)
^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(
-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))*b*d*e/f^2/(c*f-d*e)-1/2*b/(c*f-d*e)/
f*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*
x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+1/2*b/(c
*f-d*e)/f*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*
x^2+b*c*x^2+a*c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+
1/2/(c*f-d*e)/e/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+
a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2
)/(-b/a)^(1/2))*a*c-1/(c*f-d*e)/f/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/
c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f
/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*b*c+1/2/f^2/(c*f-d*e)*e/(-b/a)^(1/2)*(1+
b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*Ellip
ticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*b*d)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^2\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(x^2*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^2\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{x^2\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx$$

input
```
integrate(x**2*(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**2,x)
```

output
```
Integral(x**2*sqrt(a + b*x**2)/(sqrt(c + d*x**2)*(e + f*x**2)**2), x)
```

Maxima [F]

$$\int \frac{x^2\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{\sqrt{bx^2+a}x^2}{\sqrt{dx^2+c}(fx^2+e)^2}\,dx$$

input
```
integrate(x^2*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="ma
xima")
```

output
```
integrate(sqrt(b*x^2 + a)*x^2/(sqrt(d*x^2 + c)*(f*x^2 + e)^2), x)
```

Giac [F]

$$\int \frac{x^2\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{\sqrt{bx^2+a}x^2}{\sqrt{dx^2+c}(fx^2+e)^2}\,dx$$

input
```
integrate(x^2*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="gi
ac")
```

output
```
integrate(sqrt(b*x^2 + a)*x^2/(sqrt(d*x^2 + c)*(f*x^2 + e)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx=\int \frac{x^2\,\sqrt{b\,x^2+a}}{\sqrt{d\,x^2+c}\,{\left(f\,x^2+e\right)}^2}\,dx$$

input
```
int((x^2*(a + b*x^2)^(1/2))/((c + d*x^2)^(1/2)*(e + f*x^2)^2),x)
```

output
```
int((x^2*(a + b*x^2)^(1/2))/((c + d*x^2)^(1/2)*(e + f*x^2)^2), x)
```

Reduce [F]

$$\int \frac{x^2\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx=\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^2}{d\,f^2x^6+c\,f^2x^4+2def\,x^4+2cef\,x^2+d\,e^2x^2+c\,e^2}dx$$

input
```
int(x^2*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(c*e**2 + 2*c*e*f*x**2 + c*f*
*2*x**4 + d*e**2*x**2 + 2*d*e*f*x**4 + d*f**2*x**6),x)
```

3.133 $\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\left(e+fx^2\right)^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 440

$$\begin{aligned}\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\left(e+fx^2\right)^2}\,dx &= \frac{bx\sqrt{c+dx^2}}{2e(de-cf)\sqrt{a+bx^2}}-\frac{fx\sqrt{a+bx^2}\sqrt{c+dx^2}}{2e(de-cf)\left(e+fx^2\right)}\\
&\quad-\frac{\sqrt{a}\sqrt{b}\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{2e(de-cf)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}\\
&\quad+\frac{a^{3/2}\sqrt{b}\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{2ce(be-af)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}\\
&\quad+\frac{a^{3/2}(bde^2-af(2de-cf))\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{2\sqrt{b}ce^2(be-af)(de-cf)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}\end{aligned}$$

output
```
1/2*b*x*(d*x^2+c)^(1/2)/e/(-c*f+d*e)/(b*x^2+a)^(1/2)-1/2*f*x*(b*x^2+a)^(1/
2)*(d*x^2+c)^(1/2)/e/(-c*f+d*e)/(f*x^2+e)-1/2*a^(1/2)*b^(1/2)*(d*x^2+c)^(1
/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/e/(-c
*f+d*e)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)+1/2*a^(3/2)*b^(1/2
)*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1
/2))/c/e/(-a*f+b*e)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)+1/2*a^
(3/2)*(b*d*e^2-a*f*(-c*f+2*d*e))*(d*x^2+c)^(1/2)*EllipticPi(b^(1/2)*x/a^(1
/2)/(1+b*x^2/a)^(1/2),1-a*f/b/e,(1-a*d/b/c)^(1/2))/b^(1/2)/c/e^2/(-a*f+b*e
)/(-c*f+d*e)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.38 (sec) , antiderivative size = 496, normalized size of antiderivative = 1.13

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx$$
$$= \frac{-\frac{acfx}{e+fx^2}-\frac{bcfx^3}{e+fx^2}-\frac{adfx^3}{e+fx^2}-\frac{bdfx^5}{e+fx^2}-ia\sqrt{\frac{b}{a}}c\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(i\text{arcsinh}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)+\frac{ia\sqrt{\frac{b}{a}}(-de+cf)\sqrt{1-}}{}}{}$$

input
```
Integrate[Sqrt[a + b*x^2]/(Sqrt[c + d*x^2]*(e + f*x^2)^2),x]
```

output
```
(-((a*c*f*x)/(e + f*x^2)) - (b*c*f*x^3)/(e + f*x^2) - (a*d*f*x^3)/(e + f*x
^2) - (b*d*f*x^5)/(e + f*x^2) - I*a*Sqrt[b/a]*c*Sqrt[1 + (b*x^2)/a]*Sqrt[1
 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (I*a*Sqrt[b
/a]*(-(d*e) + c*f)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*Arc
Sinh[Sqrt[b/a]*x], (a*d)/(b*c)])/f - ((2*I)*a*d*Sqrt[1 + (b*x^2)/a]*Sqrt[1
 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]
)/Sqrt[b/a] + (I*a*Sqrt[b/a]*d*e*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*E
llipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/f + (I*a*c*f*
Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e), I*ArcSinh[
Sqrt[b/a]*x], (a*d)/(b*c)])/(Sqrt[b/a]*e))/(2*e*(d*e - c*f)*Sqrt[a + b*x^2
]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 565, normalized size of antiderivative = 1.28, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {425, 413, 413, 412, 424, 406, 320, 388, 313, 413, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)^2}dx$$

$$\Big\downarrow \text{425}$$

$$\frac{b\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)}dx}{f}-\frac{(be-af)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^2}dx}{f}$$

$$\Big\downarrow \text{413}$$

$$\frac{b\sqrt{\frac{bx^2}{a}+1}\int \frac{1}{\sqrt{\frac{bx^2}{a}+1}\sqrt{dx^2+c}(fx^2+e)}dx}{f\sqrt{a+bx^2}}-\frac{(be-af)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^2}dx}{f}$$

$$\Big\downarrow \text{413}$$

$$\frac{b\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\int \frac{1}{\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}(fx^2+e)}dx}{f\sqrt{a+bx^2}\sqrt{c+dx^2}}-\frac{(be-af)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^2}dx}{f}$$

$$\Big\downarrow \text{412}$$

$$\frac{\sqrt{-a}\sqrt{b}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{ef\sqrt{a+bx^2}\sqrt{c+dx^2}}-\frac{(be-af)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^2}dx}{f}$$

$$\Big\downarrow \text{424}$$

$$\frac{\sqrt{-a}\sqrt{b}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{ef\sqrt{a+bx^2}\sqrt{c+dx^2}}-\frac{(be-af)\left(\frac{(be(3de-2cf)-af(2de-cf))\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{2e(be-af)(de-cf)}-\frac{bd\int \frac{fx^2+e}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{2e(be-af)(de-cf)}+\frac{f^2x\sqrt{a+bx^2}\sqrt{c+dx^2}}{2e(e+fx^2)(be-af)(de-cf)}\right)}{f}$$

$$\Big\downarrow \text{406}$$

$$\frac{\sqrt{-a}\sqrt{b}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{ef\sqrt{a+bx^2}\sqrt{c+dx^2}}-\frac{(be-af)\left(-\frac{bd\left(e\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+f\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{2e(be-af)(de-cf)}+\frac{(be(3de-2cf)-af(2de-cf))\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{2e(be-af)(de-cf)}+\frac{f^2x}{2e(e+f}\right.}{f}$$

$$\Big\downarrow \text{320}$$

$$\frac{\sqrt{-a}\sqrt{b}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{ef\sqrt{a+bx^2}\sqrt{c+dx^2}}-\frac{(be-af)\left(-\frac{bd\left(f\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{\sqrt{c}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{2e(be-af)(de-cf)}+\frac{(be(3de-2cf)-af(2de-cf))\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(f\right.}}{2e(be-af)(de-cf)}\right.}{f}$$

$$\Big\downarrow \text{388}$$

$$\frac{\sqrt{-a}\sqrt{b}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{ef\sqrt{a+bx^2}\sqrt{c+dx^2}}-\frac{(be-af)\left(-\frac{bd\left(f\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{\sqrt{c}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{2e(be-af)(de-cf)}+\frac{(be(3de-2cf)-af(2de-cf))\int\frac{}{\sqrt{bx^2}}}{2e(be-af)(de-c}\right.}{f}$$

$$\Big\downarrow \text{313}$$

$$\frac{\sqrt{-a}\sqrt{b}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{ef\sqrt{a+bx^2}\sqrt{c+dx^2}}-\frac{(be-af)\left(\frac{(be(3de-2cf)-af(2de-cf))\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)}dx}{2e(be-af)(de-cf)}-\frac{bd\left(\frac{\sqrt{c}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+f\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\cdots}{\cdots}\right.\right.}{2e(be-af)(de-cf)}\cdots\right)}{f}$$

$\downarrow$ 413

$$\frac{\sqrt{-a}\sqrt{b}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{ef\sqrt{a+bx^2}\sqrt{c+dx^2}}-\frac{(be-af)\left(\frac{\sqrt{\frac{bx^2}{a}+1}(be(3de-2cf)-af(2de-cf))\int\frac{1}{\sqrt{\frac{bx^2}{a}+1}\sqrt{dx^2+c}(fx^2+e)}dx}{2e\sqrt{a+bx^2}(be-af)(de-cf)}-\frac{bd\left(\frac{\sqrt{c}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+f\left(\frac{x\sqrt{a\cdots}}{b\sqrt{c\cdots}}\cdots\right.\right.}{2e(be-af)(de-\cdots}\cdots\right)}{f}$$

$\downarrow$ 413

$$\frac{\sqrt{-a}\sqrt{b}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{ef\sqrt{a+bx^2}\sqrt{c+dx^2}}-\frac{(be-af)\left(\frac{\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}(be(3de-2cf)-af(2de-cf))\int\frac{1}{\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}(fx^2+e)}dx}{2e\sqrt{a+bx^2}\sqrt{c+dx^2}(be-af)(de-cf)}-\frac{bd\left(\frac{\sqrt{c}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\cdots\right.}{2e(be\cdots}\cdots\right)}{f}$$

$\downarrow$ 412

$$\frac{\sqrt{-a}\sqrt{b}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{ef\sqrt{a+bx^2}\sqrt{c+dx^2}} - \frac{(be-af)\left(\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}(be(3de-2cf)-af(2de-cf))\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{2\sqrt{b}e^2\sqrt{a+bx^2}\sqrt{c+dx^2}(be-af)(de-cf)} - \frac{bd\left(\frac{\sqrt{c}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\right.}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right.}{}\right.}{f}$$

input
```
Int[Sqrt[a + b*x^2]/(Sqrt[c + d*x^2]*(e + f*x^2)^2),x]
```

output
```
(Sqrt[-a]*Sqrt[b]*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)
/(b*e), ArcSin[(Sqrt[b]*x)/Sqrt[-a]], (a*d)/(b*c)])/(e*f*Sqrt[a + b*x^2]*S
qrt[c + d*x^2]) - ((b*e - a*f)*((f^2*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(2
*e*(b*e - a*f)*(d*e - c*f)*(e + f*x^2)) - (b*d*(f*((x*Sqrt[a + b*x^2])/(b*
Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/S
qrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))
]*Sqrt[c + d*x^2])) + (Sqrt[c]*e*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]
*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*
x^2))]*Sqrt[c + d*x^2])))/(2*e*(b*e - a*f)*(d*e - c*f)) + (Sqrt[-a]*(b*e*(
3*d*e - 2*c*f) - a*f*(2*d*e - c*f))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c
]*EllipticPi[(a*f)/(b*e), ArcSin[(Sqrt[b]*x)/Sqrt[-a]], (a*d)/(b*c)])/(2*S
qrt[b]*e^2*(b*e - a*f)*(d*e - c*f)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])))/f
```

Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 424
```
Int[1/(((a_) + (b_.)*(x_)^2)^2*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*
(x_)^2]), x_Symbol] :> Simp[b^2*x*Sqrt[c + d*x^2]*(Sqrt[e + f*x^2]/(2*a*(b*
c - a*d)*(b*e - a*f)*(a + b*x^2))), x] + (Simp[(b^2*c*e + 3*a^2*d*f - 2*a*b
*(d*e + c*f))/(2*a*(b*c - a*d)*(b*e - a*f))   Int[1/((a + b*x^2)*Sqrt[c + d
*x^2]*Sqrt[e + f*x^2]), x], x] - Simp[d*(f/(2*a*(b*c - a*d)*(b*e - a*f)))
 Int[(a + b*x^2)/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x], x]) /; FreeQ[{a, b,
 c, d, e, f}, x]
```

rule 425
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[d/b   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q -
 1)*(e + f*x^2)^r, x], x] + Simp[(b*c - a*d)/b   Int[(a + b*x^2)^p*(c + d*x
^2)^(q - 1)*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && ILt
Q[p, 0] && GtQ[q, 0]
```

Maple [A] (verified)

Time = 6.61 (sec) , antiderivative size = 725, normalized size of antiderivative = 1.65

method	result
elliptic	$\frac{\sqrt{(bx^2+a)(x^2d+c)}\left(\frac{fx\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{2(cf-de)e\left(f\,x^2+e\right)}-\frac{bd\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\text{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{2(cf-de)f\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}+\frac{bc\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\text{Ellipti}}{2e(cf-de)\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4}}\right.}{}$
default	$\frac{\left(\sqrt{-\frac{b}{a}}\,bde\,f^2x^5+\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{x^2d+c}{c}}\,\text{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)bce\,f^2x^2-\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{x^2d+c}{c}}\,\text{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)bd\,e^2f\,x^2-\right.}{}$

input
```
int((b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/2*f/(c*f-d*
e)/e*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(f*x^2+e)-1/2*b*d/(c*f-d*e)/f/(
-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a
*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+1/2*b/e/(c*f-
d*e)*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b
*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-1/2*b
/e/(c*f-d*e)*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a
*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2
))+1/2/e^2*f/(c*f-d*e)/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b
*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*
d)^(1/2)/(-b/a)^(1/2))*a*c-1/(c*f-d*e)/e/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1
+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1
/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*a*d+1/2/(c*f-d*e)/f/(-b/a)^(1/2)*
(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*El
lipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*b*d)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**2,x)
```

output
```
Integral(sqrt(a + b*x**2)/(sqrt(c + d*x**2)*(e + f*x**2)**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)^2}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="maxima
")
```

output
```
integrate(sqrt(b*x^2 + a)/(sqrt(d*x^2 + c)*(f*x^2 + e)^2), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)^2}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^2 + a)/(sqrt(d*x^2 + c)*(f*x^2 + e)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{\sqrt{b\,x^2+a}}{\sqrt{d\,x^2+c}\,(f\,x^2+e)^2}\,dx$$

input
```
int((a + b*x^2)^(1/2)/((c + d*x^2)^(1/2)*(e + f*x^2)^2),x)
```

output
```
int((a + b*x^2)^(1/2)/((c + d*x^2)^(1/2)*(e + f*x^2)^2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{d\,f^2x^6+c\,f^2x^4+2def\,x^4+2cef\,x^2+d\,e^2x^2+c\,e^2}dx$$

input
```
int((b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(c*e**2 + 2*c*e*f*x**2 + c*f**2*x**
*4 + d*e**2*x**2 + 2*d*e*f*x**4 + d*f**2*x**6),x)
```

3.134 $\int \frac{\sqrt{a+bx^2}}{x^2\sqrt{c+dx^2}(e+fx^2)^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 648

$$\begin{aligned}
&\int \frac{\sqrt{a+bx^2}}{x^2\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx \\
&= \frac{b(2de-3cf)x\sqrt{c+dx^2}}{2ce^2(de-cf)\sqrt{a+bx^2}} - \frac{b(af(2de-3cf)-2be(de-cf))x\sqrt{a+bx^2}\sqrt{c+dx^2}}{2ace^2(be-af)(de-cf)} \\
&\quad - \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{acex\,(e+fx^2)} + \frac{f(af(2de-3cf)-2be(de-cf))x(a+bx^2)^{3/2}\sqrt{c+dx^2}}{2ace^2(be-af)(de-cf)\,(e+fx^2)} \\
&\quad - \frac{\sqrt{a}\sqrt{b}(2de-3cf)\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{2ce^2(de-cf)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} \\
&\quad + \frac{\sqrt{a}\sqrt{b}(2be-3af)\sqrt{c+dx^2}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{2ce^2(be-af)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} \\
&\quad + \frac{a^{3/2}f(af(4de-3cf)-be(3de-2cf))\sqrt{c+dx^2}\,\mathrm{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{2\sqrt{b}ce^3(be-af)(de-cf)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}
\end{aligned}$$

output
```
1/2*b*(-3*c*f+2*d*e)*x*(d*x^2+c)^(1/2)/c/e^2/(-c*f+d*e)/(b*x^2+a)^(1/2)-1/
2*b*(a*f*(-3*c*f+2*d*e)-2*b*e*(-c*f+d*e))*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2
)/a/c/e^2/(-a*f+b*e)/(-c*f+d*e)-(b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/a/c/e/x/(f
*x^2+e)+1/2*f*(a*f*(-3*c*f+2*d*e)-2*b*e*(-c*f+d*e))*x*(b*x^2+a)^(3/2)*(d*x
^2+c)^(1/2)/a/c/e^2/(-a*f+b*e)/(-c*f+d*e)/(f*x^2+e)-1/2*a^(1/2)*b^(1/2)*(-
3*c*f+2*d*e)*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2)
,(1-a*d/b/c)^(1/2))/c/e^2/(-c*f+d*e)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2
+a))^(1/2)+1/2*a^(1/2)*b^(1/2)*(-3*a*f+2*b*e)*(d*x^2+c)^(1/2)*InverseJacob
iAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/c/e^2/(-a*f+b*e)/(b*x^2+a
)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)+1/2*a^(3/2)*f*(a*f*(-3*c*f+4*d*e)-
b*e*(-2*c*f+3*d*e))*(d*x^2+c)^(1/2)*EllipticPi(b^(1/2)*x/a^(1/2)/(1+b*x^2/
a)^(1/2),1-a*f/b/e,(1-a*d/b/c)^(1/2))/b^(1/2)/c/e^3/(-a*f+b*e)/(-c*f+d*e)/
(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.69 (sec) , antiderivative size = 293, normalized size of antiderivative = 0.45

$$\int \frac{\sqrt{a+bx^2}}{x^2\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx$$
$$= \frac{\sqrt{\frac{b}{a}}e(a+bx^2)\,(c+dx^2)\,(-2de(e+fx^2)+cf(2e+3fx^2))-icx\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}(e+fx^2)\left(be(2de-\right.}{}$$

input
```
Integrate[Sqrt[a + b*x^2]/(x^2*Sqrt[c + d*x^2]*(e + f*x^2)^2),x]
```

output
```
(Sqrt[b/a]*e*(a + b*x^2)*(c + d*x^2)*(-2*d*e*(e + f*x^2) + c*f*(2*e + 3*f*
x^2)) - I*c*x*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(e + f*x^2)*(b*e*(2*
d*e - 3*c*f)*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - 3*b*e*(d*e -
 c*f)*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (b*e*(3*d*e - 2*c*f
) + a*f*(-4*d*e + 3*c*f))*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x],
(a*d)/(b*c)]))/(2*Sqrt[b/a]*c*e^3*(d*e - c*f)*x*Sqrt[a + b*x^2]*Sqrt[c + d
*x^2]*(e + f*x^2))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}}{x^2\sqrt{c+dx^2}\left(e+fx^2\right)^2}\,dx$$
$$\Big\downarrow \text{450}$$
$$\int \frac{\sqrt{a+bx^2}}{x^2\sqrt{c+dx^2}\left(e+fx^2\right)^2}dx$$

input
```
Int[Sqrt[a + b*x^2]/(x^2*Sqrt[c + d*x^2]*(e + f*x^2)^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [A] (verified)

Time = 20.56 (sec) , antiderivative size = 1058, normalized size of antiderivative = 1.63

method	result	size
elliptic	Expression too large to display	1058
risch	Expression too large to display	1244
default	Expression too large to display	1388

input
```
int((b*x^2+a)^(1/2)/x^2/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x,method=_RETURNVERBOS
E)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-1/2*f^2/e^2/
(c*f-d*e)*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(f*x^2+e)-1/c/e^2*(b*d*x^4
+a*d*x^2+b*c*x^2+a*c)^(1/2)/x+1/2*d*b/e/(c*f-d*e)/(-b/a)^(1/2)*(1+b*x^2/a)
^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(
-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-1/2*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)
*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*b*f/e^2/(c*f-d*e)*E
llipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+1/2*c/(-b/a)^(1/2)*(1+b*
x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*b*f/e^2
/(c*f-d*e)*EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-1/(-b/a)^(1/
2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
*b/e^2*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+1/(-b/a)^(1/2)*(
1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*b/e
^2*EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-3/2/e^3/(c*f-d*e)*f^
2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^
2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2)
)*a*c+2/e^2/(c*f-d*e)*f/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(
b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c
*d)^(1/2)/(-b/a)^(1/2))*a*d+1/e^2/(c*f-d*e)*f/(-b/a)^(1/2)*(1+b*x^2/a)^(1/
2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/
a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*b*c-3/2/(c*f-d*e)/e/(-b/a...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{x^2\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(1/2)/x^2/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}}{x^2\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{\sqrt{a+bx^2}}{x^2\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)/x**2/(d*x**2+c)**(1/2)/(f*x**2+e)**2,x)
```

output
```
Integral(sqrt(a + b*x**2)/(x**2*sqrt(c + d*x**2)*(e + f*x**2)**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}}{x^2\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)^2x^2}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/x^2/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="ma
xima")
```

output
```
integrate(sqrt(b*x^2 + a)/(sqrt(d*x^2 + c)*(f*x^2 + e)^2*x^2), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}}{x^2\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)^2x^2}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/x^2/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="gi
ac")
```

output
```
integrate(sqrt(b*x^2 + a)/(sqrt(d*x^2 + c)*(f*x^2 + e)^2*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{x^2\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{\sqrt{b\,x^2+a}}{x^2\,\sqrt{d\,x^2+c}\,(f\,x^2+e)^2}\,dx$$

input
```
int((a + b*x^2)^(1/2)/(x^2*(c + d*x^2)^(1/2)*(e + f*x^2)^2),x)
```

output
```
int((a + b*x^2)^(1/2)/(x^2*(c + d*x^2)^(1/2)*(e + f*x^2)^2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}}{x^2\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{d\,f^2x^8+c\,f^2x^6+2def\,x^6+2cef\,x^4+d\,e^2x^4+c\,e^2x^2}dx$$

input
```
int((b*x^2+a)^(1/2)/x^2/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(c*e**2*x**2 + 2*c*e*f*x**4 + c*f*
*2*x**6 + d*e**2*x**4 + 2*d*e*f*x**6 + d*f**2*x**8),x)
```

3.135 $\int \frac{\sqrt{a+bx^2}}{x^4\sqrt{c+dx^2}(e+fx^2)^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 840

$$
\begin{aligned}
\int \frac{\sqrt{a+bx^2}}{x^4\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx &= \frac{b\left(2bc-\frac{a(4d^2e^2+8cdef-15c^2f^2)}{e(de-cf)}\right)x\sqrt{c+dx^2}}{6ac^2e^2\sqrt{a+bx^2}} \\
&+ \frac{b(af(4d^2e^2+8cdef-15c^2f^2)-4be(d^2e^2+2cdef-3c^2f^2))\,x\sqrt{a+bx^2}\sqrt{c+dx^2}}{6ac^2e^3(be-af)(de-cf)} \\
&- \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{3acex^3\,(e+fx^2)} + \frac{(2de+5cf)\,(a+bx^2)^{3/2}\sqrt{c+dx^2}}{3ac^2e^2x\,(e+fx^2)} \\
&- \frac{f(af(4d^2e^2+8cdef-15c^2f^2)-4be(d^2e^2+2cdef-3c^2f^2))\,x(a+bx^2)^{3/2}\sqrt{c+dx^2}}{6ac^2e^3(be-af)(de-cf)\,(e+fx^2)} \\
&- \frac{\sqrt{b}(2bce(de-cf)-a(4d^2e^2+8cdef-15c^2f^2))\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{6\sqrt{a}c^2e^3(de-cf)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} \\
&- \frac{\sqrt{a}\sqrt{b}(2be(de+6cf)-af(2de+15cf))\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{6c^2e^3(be-af)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} \\
&- \frac{a^{3/2}f^2(af(6de-5cf)-be(5de-4cf))\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{2\sqrt{b}ce^4(be-af)(de-cf)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}
\end{aligned}
$$

output
```
1/6*b*(2*b*c-a*(-15*c^2*f^2+8*c*d*e*f+4*d^2*e^2)/e/(-c*f+d*e))*x*(d*x^2+c)
^(1/2)/a/c^2/e^2/(b*x^2+a)^(1/2)+1/6*b*(a*f*(-15*c^2*f^2+8*c*d*e*f+4*d^2*e
^2)-4*b*e*(-3*c^2*f^2+2*c*d*e*f+d^2*e^2))*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2
)/a/c^2/e^3/(-a*f+b*e)/(-c*f+d*e)-1/3*(b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/a/c/
e/x^3/(f*x^2+e)+1/3*(5*c*f+2*d*e)*(b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/a/c^2/e^
2/x/(f*x^2+e)-1/6*f*(a*f*(-15*c^2*f^2+8*c*d*e*f+4*d^2*e^2)-4*b*e*(-3*c^2*f
^2+2*c*d*e*f+d^2*e^2))*x*(b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/a/c^2/e^3/(-a*f+b
*e)/(-c*f+d*e)/(f*x^2+e)-1/6*b^(1/2)*(2*b*c*e*(-c*f+d*e)-a*(-15*c^2*f^2+8*
c*d*e*f+4*d^2*e^2))*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a
)^(1/2),(1-a*d/b/c)^(1/2))/a^(1/2)/c^2/e^3/(-c*f+d*e)/(b*x^2+a)^(1/2)/(a*(
d*x^2+c)/c/(b*x^2+a))^(1/2)-1/6*a^(1/2)*b^(1/2)*(2*b*e*(6*c*f+d*e)-a*f*(15
*c*f+2*d*e))*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-
a*d/b/c)^(1/2))/c^2/e^3/(-a*f+b*e)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a
))^(1/2)-1/2*a^(3/2)*f^2*(a*f*(-5*c*f+6*d*e)-b*e*(-4*c*f+5*d*e))*(d*x^2+c)
^(1/2)*EllipticPi(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),1-a*f/b/e,(1-a*d/b/c
)^(1/2))/b^(1/2)/c/e^4/(-a*f+b*e)/(-c*f+d*e)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/
c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.21 (sec) , antiderivative size = 392, normalized size of antiderivative = 0.47

$$\int \frac{\sqrt{a+bx^2}}{x^4\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx$$
$$= \frac{\sqrt{\frac{b}{a}}\left(-\sqrt{\frac{b}{a}}e(a+bx^2)\,(c+dx^2)\,(3ac^2f^3x^4+2ace(de-cf)\,(e+fx^2)+2(de-cf)(bce-2a(de+3cf))\right.}{}$$

input
```
Integrate[Sqrt[a + b*x^2]/(x^4*Sqrt[c + d*x^2]*(e + f*x^2)^2),x]
```

output
```
(Sqrt[b/a]*(-(Sqrt[b/a]*e*(a + b*x^2)*(c + d*x^2)*(3*a*c^2*f^3*x^4 + 2*a*c
*e*(d*e - c*f)*(e + f*x^2) + 2*(d*e - c*f)*(b*c*e - 2*a*(d*e + 3*c*f))*x^2
*(e + f*x^2))) + I*c*x^3*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(e + f*x^
2)*(b*e*(2*b*c*e*(-(d*e) + c*f) + a*(4*d^2*e^2 + 8*c*d*e*f - 15*c^2*f^2))*
EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + b*e*(d*e - c*f)*(2*b*c*e
- 2*a*d*e - 15*a*c*f)*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + 3*a
*c*f*(b*e*(5*d*e - 4*c*f) + a*f*(-6*d*e + 5*c*f))*EllipticPi[(a*f)/(b*e),
I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])))/(6*b*c^2*e^4*(d*e - c*f)*x^3*Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}}{x^4\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx$$

$$\Big\downarrow \text{ 450}$$

$$\int \frac{\sqrt{a+bx^2}}{x^4\sqrt{c+dx^2}\,(e+fx^2)^2}dx$$

input
```
Int[Sqrt[a + b*x^2]/(x^4*Sqrt[c + d*x^2]*(e + f*x^2)^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [A] (verified)

Time = 21.85 (sec) , antiderivative size = 1401, normalized size of antiderivative = 1.67

method	result	size
risch	Expression too large to display	1401
elliptic	Expression too large to display	1508
default	Expression too large to display	2633

input
```
int((b*x^2+a)^(1/2)/x^4/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x,method=_RETURNVERBOS
E)
```

output
```
-1/3*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(-6*a*c*f*x^2-2*a*d*e*x^2+b*c*e*x^2+a
*c*e)/a/c^2/e^3/x^3-1/3/a/c^2/e^3*(-b*(6*a*c*f+2*a*d*e-b*c*e)*c/(-b/a)^(1/
2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1
/2),(-1+(a*d+b*c)/c/b)^(1/2)))+a*c*d*e*b/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1
+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/
2),(-1+(a*d+b*c)/c/b)^(1/2))-3*a*c^2*f*(2*a*f-b*e)/e/(-b/a)^(1/2)*(1+b*x^2
/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi
(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))-3*a*c^2*e*f*(a*f-b*e)
*(1/2*f^2/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/e*x*(b*d*x^4+a*d*x^2+b*c*x^2+a
*c)^(1/2)/(f*x^2+e)-1/2*d*b/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/(-b/a)^(1/2)
*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*E
llipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+1/2*f*b/(a*c*f^2-a*d*e*f
-b*c*e*f+b*d*e^2)/e*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*
d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/
b)^(1/2))-1/2*f*b/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/e*c/(-b/a)^(1/2)*(1+b*
x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*Ellipti
cE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+1/2/(a*c*f^2-a*d*e*f-b*c*e*f+b
*d*e^2)/e^2*f^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+
a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{x^4\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(1/2)/x^4/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}}{x^4\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{\sqrt{a+bx^2}}{x^4\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)/x**4/(d*x**2+c)**(1/2)/(f*x**2+e)**2,x)
```

output
```
Integral(sqrt(a + b*x**2)/(x**4*sqrt(c + d*x**2)*(e + f*x**2)**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}}{x^4\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)^2x^4}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/x^4/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="ma
xima")
```

output
```
integrate(sqrt(b*x^2 + a)/(sqrt(d*x^2 + c)*(f*x^2 + e)^2*x^4), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}}{x^4\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)^2x^4}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/x^4/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="gi
ac")
```

output
```
integrate(sqrt(b*x^2 + a)/(sqrt(d*x^2 + c)*(f*x^2 + e)^2*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{x^4\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{\sqrt{b\,x^2+a}}{x^4\,\sqrt{d\,x^2+c}\,(f\,x^2+e)^2}\,dx$$

input
```
int((a + b*x^2)^(1/2)/(x^4*(c + d*x^2)^(1/2)*(e + f*x^2)^2),x)
```

output
```
int((a + b*x^2)^(1/2)/(x^4*(c + d*x^2)^(1/2)*(e + f*x^2)^2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}}{x^4\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{\sqrt{b\,x^2+a}}{x^4\sqrt{d\,x^2+c}\,(f\,x^2+e)^2}dx$$

input
```
int((b*x^2+a)^(1/2)/x^4/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x)
```

output
```
int((b*x^2+a)^(1/2)/x^4/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x)
```

3.136 $\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^6(e+fx^2)^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 1095

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^6\left(e+fx^2\right)^2}\,dx=\text{Too large to display}$$

output

```
-1/30*b*(4*b^2*c^2*e^2-4*a*b*c*e*(-5*c*f+d*e)+a^2*(-105*c^2*f^2+20*c*d*e*f
+4*d^2*e^2))*x*(d*x^2+c)^(1/2)/a^2/c^2/e^4/(b*x^2+a)^(1/2)+1/30*d*(15*a^2*
c*f^2*(-7*c*f+6*d*e)-4*b^2*c*e^2*(-c*f+d*e)+2*a*b*e*(10*c^2*f^2-11*c*d*e*f
+d^2*e^2))*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a^2/c^2/e^4/(-c*f+d*e)-1/10*b
*(5*a^2*c*f^3*(-7*c*f+6*d*e)-2*b^2*d*e^3*(-c*f+d*e)+2*a*b*e*f*(15*c^2*f^2-
16*c*d*e*f+d^2*e^2))*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(3/2)/a^2/c^2/e^4/(-a*f+b
*e)/(-c*f+d*e)-1/5*(b*x^2+a)^(3/2)*(d*x^2+c)^(3/2)/a/c/e/x^5/(f*x^2+e)+1/1
5*(7*a*c*f+2*a*d*e+2*b*c*e)*(b*x^2+a)^(3/2)*(d*x^2+c)^(3/2)/a^2/c^2/e^2/x^
3/(f*x^2+e)+1/15*(a*f*(-35*c*f+2*d*e)-b*e*(-2*c*f+3*d*e))*(b*x^2+a)^(3/2)*
(d*x^2+c)^(3/2)/a^2/c^2/e^3/x/(f*x^2+e)+1/10*f*(5*a^2*c*f^3*(-7*c*f+6*d*e)
-2*b^2*d*e^3*(-c*f+d*e)+2*a*b*e*f*(15*c^2*f^2-16*c*d*e*f+d^2*e^2))*x*(b*x^
2+a)^(3/2)*(d*x^2+c)^(3/2)/a^2/c^2/e^4/(-a*f+b*e)/(-c*f+d*e)/(f*x^2+e)+1/3
0*b^(1/2)*(4*b^2*c^2*e^2-4*a*b*c*e*(-5*c*f+d*e)+a^2*(-105*c^2*f^2+20*c*d*e
*f+4*d^2*e^2))*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/
2),(1-a*d/b/c)^(1/2))/a^(3/2)/c^2/e^4/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^
2+a))^(1/2)-1/30*b^(1/2)*(2*b^2*c*d*e^3-a^2*f*(-105*c^2*f^2+55*c*d*e*f+2*d
^2*e^2)+2*a*b*e*(-45*c^2*f^2+19*c*d*e*f+d^2*e^2))*(d*x^2+c)^(1/2)*InverseJ
acobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/a^(1/2)/c^2/e^4/(-a*f
+b*e)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)-1/2*a^(3/2)*f^2*(a*f
*(-7*c*f+6*d*e)-b*e*(-6*c*f+5*d*e))*(d*x^2+c)^(1/2)*EllipticPi(b^(1/2)*...
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.93 (sec) , antiderivative size = 470, normalized size of antiderivative = 0.43

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^6\left(e+fx^2\right)^2}\,dx$$

$$= \frac{-\frac{e(a+bx^2)(c+dx^2)\left(15a^2c^2f^3x^6+6a^2c^2e^2(e+fx^2)+2ace(bce+ade-10acf)x^2(e+fx^2)-2\left(2b^2c^2e^2+2abce(-de+5cf)+a^2\left(2d^2e^2+10cdef-4\right.\right.\right.}{x^5(e+fx^2)}}{}$$

input

```
Integrate[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(x^6*(e + f*x^2)^2),x]
```

output
```
(-((e*(a + b*x^2)*(c + d*x^2)*(15*a^2*c^2*f^3*x^6 + 6*a^2*c^2*e^2*(e + f*x
^2) + 2*a*c*e*(b*c*e + a*d*e - 10*a*c*f)*x^2*(e + f*x^2) - 2*(2*b^2*c^2*e^
2 + 2*a*b*c*e*(-(d*e) + 5*c*f) + a^2*(2*d^2*e^2 + 10*c*d*e*f - 45*c^2*f^2)
)*x^4*(e + f*x^2)))/(x^5*(e + f*x^2))) + (I*c*Sqrt[1 + (b*x^2)/a]*Sqrt[1 +
 (d*x^2)/c]*(b*e*(4*b^2*c^2*e^2 + 4*a*b*c*e*(-(d*e) + 5*c*f) + a^2*(4*d^2*
e^2 + 20*c*d*e*f - 105*c^2*f^2))*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(
b*c)] - b*e*(4*b^2*c^2*e^2 + 2*a*b*c*e*(-3*d*e + 10*c*f) + a^2*(2*d^2*e^2
+ 55*c*d*e*f - 105*c^2*f^2))*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)
] + 15*a^2*c*f*(b*e*(5*d*e - 6*c*f) + a*f*(-6*d*e + 7*c*f))*EllipticPi[(a*
f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/Sqrt[b/a])/(30*a^2*c^2*e^
5*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^6\left(e+fx^2\right)^2}\,dx$$

$$\Big\downarrow \; 450$$

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^6\left(e+fx^2\right)^2}dx$$

input
```
Int[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(x^6*(e + f*x^2)^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [A] (verified)

Time = 22.35 (sec) , antiderivative size = 1785, normalized size of antiderivative = 1.63

method	result	size
risch	Expression too large to display	1785
elliptic	Expression too large to display	1829
default	Expression too large to display	3178

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^6/(f*x^2+e)^2,x,method=_RETURNVERBOS
E)
```

output
```
-1/15*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(45*a^2*c^2*f^2*x^4-10*a^2*c*d*e*f*x
^4-2*a^2*d^2*e^2*x^4-10*a*b*c^2*e*f*x^4+2*a*b*c*d*e^2*x^4-2*b^2*c^2*e^2*x^
4-10*a^2*c^2*e*f*x^2+a^2*c*d*e^2*x^2+a*b*c^2*e^2*x^2+3*a^2*c^2*e^2)/a^2/c^
2/e^4/x^5+1/15/a^2/c^2/e^4*(-b*(45*a^2*c^2*f^2-10*a^2*c*d*e*f-2*a^2*d^2*e^
2-10*a*b*c^2*e*f+2*a*b*c*d*e^2-2*b^2*c^2*e^2)*c/(-b/a)^(1/2)*(1+b*x^2/a)^(
1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(EllipticF(x*(-
b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c
)/c/b)^(1/2)))-a*b^2*c^2*d*e^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^
(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*
d+b*c)/c/b)^(1/2))-a^2*b*c*d^2*e^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2
/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1
+(a*d+b*c)/c/b)^(1/2))-15*a^2*c^2*f*(3*a*c*f^2-2*a*d*e*f-2*b*c*e*f+b*d*e^2
)/e/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*
x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/
2))+10*a^2*b*c^2*d*e*f/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b
*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c
/b)^(1/2))-15*a^2*c^2*e*f*(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)*(1/2*f^2/(a*c*
f^2-a*d*e*f-b*c*e*f+b*d*e^2)/e*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(f*x^
2+e)-1/2*d*b/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/(-b/a)^(1/2)*(1+b*x^2/a)^(1
/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^6\left(e+fx^2\right)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^6/(f*x^2+e)^2,x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^6\left(e+fx^2\right)^2}\,dx=\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^6\left(e+fx^2\right)^2}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(1/2)/x**6/(f*x**2+e)**2,x)
```

output
```
Integral(sqrt(a + b*x**2)*sqrt(c + d*x**2)/(x**6*(e + f*x**2)**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^6\left(e+fx^2\right)^2}\,dx=\int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{\left(fx^2+e\right)^2x^6}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^6/(f*x^2+e)^2,x, algorithm="ma
xima")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/((f*x^2 + e)^2*x^6), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^6\left(e+fx^2\right)^2}\,dx=\int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{\left(fx^2+e\right)^2x^6}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^6/(f*x^2+e)^2,x, algorithm="gi
ac")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/((f*x^2 + e)^2*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^6\left(e+fx^2\right)^2}\,dx=\int \frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{x^6\,\left(f\,x^2+e\right)^2}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2))/(x^6*(e + f*x^2)^2),x)
```

output
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2))/(x^6*(e + f*x^2)^2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^6\left(e+fx^2\right)^2}\,dx=\int \frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{x^6\,\left(f\,x^2+e\right)^2}dx$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^6/(f*x^2+e)^2,x)
```

output
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^6/(f*x^2+e)^2,x)
```

3.137 $\int \frac{x^4\sqrt{a+bx^2}\left(e+fx^2\right)^2}{\left(c+dx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 630

$$\int \frac{x^4\sqrt{a+bx^2}(e+fx^2)^2}{(c+dx^2)^{3/2}}\,dx = \frac{(8a^3d^3f^2 - a^2bd^2f(28de-19cf) + 2ab^2d(10d^2e^2 - 49cdef + 30c^2f^2) - 4b}{105b^3d^4\sqrt{c+dx^2}}$$

$$- \frac{(4a^2d^2f^2 + abdf(16de+9cf) - 4b^2(5d^2e^2 - 21cdef + 12c^2f^2))\,x^3\sqrt{a+bx^2}}{105b^2d^3\sqrt{c+dx^2}}$$

$$+ \frac{4f(bde-2bcf-adf)x^5\sqrt{a+bx^2}}{35bd^2\sqrt{c+dx^2}} + \frac{x(a+bx^2)^{3/2}\,(e+fx^2)^2}{7bd\sqrt{c+dx^2}}$$

$$- \frac{\sqrt{c}(8a^3d^3f^2 - a^2bd^2f(28de-23cf) - 8b^3c(35d^2e^2 - 84cdef + 48c^2f^2) + ab^2d(35d^2e^2 - 112cdef + 72c}{105b^3d^{9/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

$$+ \frac{2c^{3/2}(2a^2d^2f^2 - abdf(7de-6cf) - 2b^2(35d^2e^2 - 84cdef + 48c^2f^2))\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\right.}{105b^2d^{9/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output
```
1/105*(8*a^3*d^3*f^2-a^2*b*d^2*f*(-19*c*f+28*d*e)+2*a*b^2*d*(30*c^2*f^2-49
*c*d*e*f+10*d^2*e^2)-4*b^3*c*(48*c^2*f^2-84*c*d*e*f+35*d^2*e^2))*x*(b*x^2+
a)^(1/2)/b^3/d^4/(d*x^2+c)^(1/2)-1/105*(4*a^2*d^2*f^2+a*b*d*f*(9*c*f+16*d*
e)-4*b^2*(12*c^2*f^2-21*c*d*e*f+5*d^2*e^2))*x^3*(b*x^2+a)^(1/2)/b^2/d^3/(d
*x^2+c)^(1/2)+4/35*f*(-a*d*f-2*b*c*f+b*d*e)*x^5*(b*x^2+a)^(1/2)/b/d^2/(d*x
^2+c)^(1/2)+1/7*x*(b*x^2+a)^(3/2)*(f*x^2+e)^2/b/d/(d*x^2+c)^(1/2)-1/105*c^
(1/2)*(8*a^3*d^3*f^2-a^2*b*d^2*f*(-23*c*f+28*d*e)-8*b^3*c*(48*c^2*f^2-84*c
*d*e*f+35*d^2*e^2)+a*b^2*d*(72*c^2*f^2-112*c*d*e*f+35*d^2*e^2))*(b*x^2+a)^
(1/2)*EllipticE(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))/b^3
/d^(9/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)+2/105*c^(3/2)*(2*
a^2*d^2*f^2-a*b*d*f*(-6*c*f+7*d*e)-2*b^2*(48*c^2*f^2-84*c*d*e*f+35*d^2*e^2
))*(b*x^2+a)^(1/2)*InverseJacobiAM(arctan(d^(1/2)*x/c^(1/2)),(1-b*c/a/d)^(
1/2))/b^2/d^(9/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.95 (sec) , antiderivative size = 488, normalized size of antiderivative = 0.77

$$\int \frac{x^4\sqrt{a+bx^2}(e+fx^2)^2}{(c+dx^2)^{3/2}}\,dx = \frac{-\sqrt{\frac{b}{a}}dx(a+bx^2)\left(4a^2d^2f^2(c+dx^2)+abdf(c+dx^2)\left(-14de+12cf-3df\right.\right.}{}$$

input
```
Integrate[(x^4*Sqrt[a + b*x^2]*(e + f*x^2)^2)/(c + d*x^2)^(3/2),x]
```

output
```
(-(Sqrt[b/a]*d*x*(a + b*x^2)*(4*a^2*d^2*f^2*(c + d*x^2) + a*b*d*f*(c + d*x
^2)*(-14*d*e + 12*c*f - 3*d*f*x^2) - b^2*(192*c^3*f^2 + 48*c^2*d*f*(-7*e +
 f*x^2) + 4*c*d^2*(35*e^2 - 21*e*f*x^2 - 6*f^2*x^4) + d^3*x^2*(35*e^2 + 42
*e*f*x^2 + 15*f^2*x^4)))) - I*c*(8*a^3*d^3*f^2 + a^2*b*d^2*f*(-28*d*e + 23
*c*f) - 8*b^3*c*(35*d^2*e^2 - 84*c*d*e*f + 48*c^2*f^2) + a*b^2*d*(35*d^2*e
^2 - 112*c*d*e*f + 72*c^2*f^2))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*El
lipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + I*c*(4*a^3*d^3*f^2 + a^2*b*
d^2*f*(-14*d*e + 11*c*f) - 8*b^3*c*(35*d^2*e^2 - 84*c*d*e*f + 48*c^2*f^2)
+ a*b^2*d*(175*d^2*e^2 - 448*c*d*e*f + 264*c^2*f^2))*Sqrt[1 + (b*x^2)/a]*S
qrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(105*a^
2*(b/a)^(5/2)*d^5*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (warning: unable to verify)

Time = 1.69 (sec) , antiderivative size = 962, normalized size of antiderivative = 1.53, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {448, 439, 444, 27, 444, 406, 320, 388, 313, 444, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4\sqrt{a+bx^2}\left(e+fx^2\right)^2}{\left(c+dx^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{448}$$

$$\frac{f\int \frac{x^6\sqrt{bx^2+a}\left(fx^2+e\right)}{\left(dx^2+c\right)^{3/2}}dx}{e^2}+e\int \frac{x^4\sqrt{bx^2+a}\left(fx^2+e\right)}{\left(dx^2+c\right)^{3/2}}dx$$

$$\Big\downarrow \text{439}$$

$$\frac{f\left(\frac{x^7\sqrt{a+bx^2}(de-cf)}{cd\sqrt{c+dx^2}}-\frac{\int \frac{x^6\left(b(7de-8cf)x^2+a(6de-7cf)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{cd}\right)}{e^2}+e\left(\frac{x^5\sqrt{a+bx^2}(de-cf)}{cd\sqrt{c+dx^2}}-\frac{\int \frac{x^4\left(b(5de-6cf)x^2+a(4de-5cf)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{cd}\right)$$

$$\Big\downarrow \text{444}$$

$$\frac{f\left(\frac{x^7\sqrt{a+bx^2}(de-cf)}{cd\sqrt{c+dx^2}}-\frac{\frac{x^5\sqrt{a+bx^2}\sqrt{c+dx^2}(7de-8cf)}{7d}-\frac{\int \frac{bcx^4\left((adf+6b(7de-8cf))x^2+5a(7de-8cf)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{7bd}}{cd}\right)}{e^2}+e\left(\frac{x^5\sqrt{a+bx^2}(de-cf)}{cd\sqrt{c+dx^2}}-\frac{\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}(5de-6cf)}{5d}-\frac{\int \frac{bcx^2\left((adf+4b(5de-6cf))x^2+3a(5de-6cf)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{5bd}}{cd}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{f\left(\frac{x^7\sqrt{a+bx^2}(de-cf)}{cd\sqrt{c+dx^2}}-\frac{\frac{x^5\sqrt{a+bx^2}\sqrt{c+dx^2}(7de-8cf)}{7d}-\frac{c\int\frac{x^4\left((42bde-48bcf+adf)x^2+5a(7de-8cf)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{7d}}{cd}\right)}{e^2}+e\left(\frac{x^5\sqrt{a+bx^2}(de-cf)}{cd\sqrt{c+dx^2}}-\frac{\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}(5de-6cf)}{5d}-\frac{c\int\frac{x^2\left((20bde-24bcf+adf)x^2+3a(5de-6cf)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{5d}}{cd}\right)$$

$\downarrow$ 444

$$\frac{f\left(\frac{x^7\sqrt{a+bx^2}(de-cf)}{cd\sqrt{c+dx^2}}-\frac{\frac{x^5\sqrt{a+bx^2}\sqrt{c+dx^2}(7de-8cf)}{7d}-\frac{c\left(\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}(adf-48bcf+42bde)}{5bd}-\frac{\int\frac{x^2\left(\left(24c(7de-8cf)b^2-ad(7de-12cf)b+4a^2d^2f\right)\cdots\right.}{\sqrt{bx^2+a}\sqrt{dx^2+c}}}{5bd}\cdots\right.}{7d}}{cd}\cdots\right.}{e^2}$$

$$e\left(\frac{x^5\sqrt{a+bx^2}(de-cf)}{cd\sqrt{c+dx^2}}-\frac{\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}(5de-6cf)}{5d}-\frac{c\left(\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(adf-24bcf+20bde)}{3bd}-\frac{\int\frac{\left(8c(5de-6cf)b^2-ad(5de-8cf)b+2\cdots\right.}{\sqrt{bx^2+a}\cdots}}{3\cdots}\cdots\right.}{5d}}{cd}\cdots\right.$$

$\downarrow$ 406

$$\frac{f\left(\frac{x^7\sqrt{a+bx^2}(de-cf)}{cd\sqrt{c+dx^2}}-\frac{\frac{x^5\sqrt{a+bx^2}\sqrt{c+dx^2}(7de-8cf)}{7d}-\frac{c\left(\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}(adf-48bcf+42bde)}{5bd}-\frac{\int\frac{x^2\left(\left(24c(7de-8cf)b^2-ad(7de-12cf)b+4a^2d^2f\right)\cdots\right.}{\sqrt{bx^2+a}\sqrt{dx^2+c}}}{5bd}\cdots\right.}{7d}}{cd}\cdots\right.}{e^2}$$

$$e\left(\frac{x^5\sqrt{a+bx^2}(de-cf)}{cd\sqrt{c+dx^2}}-\frac{\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}(5de-6cf)}{5d}-\frac{c\left(\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(adf-24bcf+20bde)}{3bd}-\frac{\left(2a^2d^2f-abd(5de-8cf)+8b^2c(5de-\cdots\right.}{\cdots}\right.}{5d}}{cd}\cdots\right.$$

$\downarrow$ 320

$$e\Bigg(\frac{x^5\sqrt{a+bx^2}(de-cf)}{cd\sqrt{c+dx^2}}-\frac{\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}(5de-6cf)}{5d}-\frac{c\Big(\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(adf-24bcf+20bde)}{3bd}-\frac{\big(2a^2d^2f-abd(5de-8cf)+8b^2c(5de-}{}}{}}{cd}$$

$$f\Bigg(\frac{x^7\sqrt{a+bx^2}(de-cf)}{cd\sqrt{c+dx^2}}-\frac{\frac{x^5\sqrt{a+bx^2}\sqrt{c+dx^2}(7de-8cf)}{7d}-\frac{c\Big(\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}(adf-48bcf+42bde)}{5bd}-\frac{\int\frac{x^2\big(\big(24c(7de-8cf)b^2-ad(7de-12cf)b+4a^2d^2f}{\sqrt{bx^2+a}\sqrt{dx^2+c}}}{5bd}}{7d}}{cd}$$

$$\overline{\qquad\qquad\qquad\qquad}$$
$$e^2$$

$\downarrow$ 388

$$\frac{e\left(\frac{x^5\sqrt{a+bx^2}(de-cf)}{cd\sqrt{c+dx^2}}-\frac{\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}(5de-6cf)}{5d}-\frac{c\left(\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(adf-24bcf+20bde)}{3bd}-\frac{\left(2a^2d^2f-abd(5de-8cf)+8b^2c(5de-\cdots\right.}{\cdots}\cdots\right.}{\cdots}}{cd}\cdots\right.\quad f\left(\frac{x^7\sqrt{a+bx^2}(de-cf)}{cd\sqrt{c+dx^2}}-\frac{\frac{x^5\sqrt{a+bx^2}\sqrt{c+dx^2}(7de-8cf)}{7d}-\frac{c\left(\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}(adf-48bcf+42bde)}{5bd}-\frac{\int\frac{x^2\left(\left(24c(7de-8cf)b^2-ad(7de-12cf)b+4a^2d^2f\cdots\right)\right.}{\sqrt{bx^2+a}\sqrt{dx^2+c}\cdots}}{5bd}\cdots\right.}{7d}}{cd}\cdots\right.}{e^2}$$

$\downarrow$ 313

$$\frac{f\left(\frac{x^7\sqrt{a+bx^2}(de-cf)}{cd\sqrt{c+dx^2}}-\frac{\frac{x^5\sqrt{a+bx^2}\sqrt{c+dx^2}(7de-8cf)}{7d}-\frac{c\left(\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}(adf-48bcf+42bde)}{5bd}-\frac{\int\frac{x^2\left(\left(24c(7de-8cf)b^2-ad(7de-12cf)b+4a^2d^2f\right.\right.}{\sqrt{bx^2+a}\sqrt{dx^2+c}}}{5bd}\right.}{7d}}{cd}\right.}{e^2}$$

$$e\left(\frac{x^5\sqrt{a+bx^2}(de-cf)}{cd\sqrt{c+dx^2}}-\frac{\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}(5de-6cf)}{5d}-\frac{c\left(\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(adf-24bcf+20bde)}{3bd}-\frac{\left(2a^2d^2f-abd(5de-8cf)+8b^2c(5de-\right.}{}\right.}{}}{}\right.$$

$\downarrow$ 444

$$\frac{f\left(\frac{x^7\sqrt{a+bx^2}(de-cf)}{cd\sqrt{c+dx^2}}-\frac{\frac{x^5\sqrt{a+bx^2}\sqrt{c+dx^2}(7de-8cf)}{7d}-\frac{c\left(\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}(adf-48bcf+42bde)}{5bd}-\frac{\frac{1}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{4a^2df}{b}-a(7de-12cf)+\frac{24bc}{}\right.}{}\right.}{}}{}\right.}{e^2}$$

$$e\left(\frac{x^5\sqrt{a+bx^2}(de-cf)}{cd\sqrt{c+dx^2}}-\frac{\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}(5de-6cf)}{5d}-\frac{c\left(\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(adf-24bcf+20bde)}{3bd}-\frac{\left(2a^2d^2f-abd(5de-8cf)+8b^2c(5de-\right.}{}\right.}{}}{}\right.$$

↓ 406

$$(20bde-24bcf+adf)\sqrt{bx^2+a}\,\text{Elliptic}$$

$$\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}$$

$$c\left(\frac{(20bde-24bcf+adf)x\sqrt{bx^2+a}\sqrt{dx^2+c}}{3bd}-\right.$$

$$e\left(\frac{(de-cf)x^5\sqrt{bx^2+a}}{cd\sqrt{dx^2+c}}-\frac{(5de-6cf)x^3\sqrt{bx^2+a}\sqrt{dx^2+c}}{5d}-\right.$$

$$c\left(\frac{(42bde-48bcf+adf)x^3\sqrt{bx^2+a}\sqrt{dx^2+c}}{5bd}-\frac{\frac{1}{3}\left(\frac{4dfa^2}{b}-(7de-12cf)a+\frac{24bc(7de-8cf)}{d}\right)x\sqrt{bx^2}}{}\right.$$

$$f\left(\frac{(de-cf)x^7\sqrt{bx^2+a}}{cd\sqrt{dx^2+c}}-\frac{(7de-8cf)x^5\sqrt{bx^2+a}\sqrt{dx^2+c}}{7d}-\right.$$

$\downarrow$ 320

$$e\left(\frac{(de-cf)x^5\sqrt{bx^2+a}}{cd\sqrt{dx^2+c}}-\frac{\frac{(5de-6cf)x^3\sqrt{bx^2+a}\sqrt{dx^2+c}}{5d}-\frac{c\left(\frac{(20bde-24bcf+adf)x\sqrt{bx^2+a}\sqrt{dx^2+c}}{3bd}-\frac{\frac{(20bde-24bcf+adf)\sqrt{bx^2+a}\,\mathrm{Elliptic}}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}}}{}\right.}{}}{}\right.$$

$$f\left(\frac{(de-cf)x^7\sqrt{bx^2+a}}{cd\sqrt{dx^2+c}}-\frac{\frac{(7de-8cf)x^5\sqrt{bx^2+a}\sqrt{dx^2+c}}{7d}-\frac{c\left(\frac{(42bde-48bcf+adf)x^3\sqrt{bx^2+a}\sqrt{dx^2+c}}{5bd}-\frac{\frac{1}{3}\left(\frac{4dfa^2}{b}-(7de-12cf)a+\frac{24bc(7de-8cf)}{d}\right)x\sqrt{bx^2}}{}\right.}{}}{}\right.$$

$\downarrow$ 388

$$e\left(\frac{(de-cf)x^5\sqrt{bx^2+a}}{cd\sqrt{dx^2+c}}-\frac{\frac{(5de-6cf)x^3\sqrt{bx^2+a}\sqrt{dx^2+c}}{5d}-\frac{c\left(\frac{(20bde-24bcf+adf)x\sqrt{bx^2+a}\sqrt{dx^2+c}}{3bd}-\frac{\frac{(20bde-24bcf+adf)\sqrt{bx^2+a}\,\text{Elliptic}\cdots}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\cdots}}{\cdots}\cdots\right)}{\cdots}}{\cdots}\cdots\right)$$

$$f\left(\frac{(de-cf)x^7\sqrt{bx^2+a}}{cd\sqrt{dx^2+c}}-\frac{\frac{(7de-8cf)x^5\sqrt{bx^2+a}\sqrt{dx^2+c}}{7d}-\frac{c\left(\frac{(42bde-48bcf+adf)x^3\sqrt{bx^2+a}\sqrt{dx^2+c}}{5bd}-\frac{\frac{1}{3}\left(\frac{4dfa^2}{b}-(7de-12cf)a+\frac{24bc(7de-8cf)}{d}\right)x\sqrt{bx^2\cdots}}{\cdots}\cdots\right)}{\cdots}}{\cdots}\cdots\right)$$

$\downarrow$ 313

$$e\left(\frac{(de-cf)x^5\sqrt{bx^2+a}}{cd\sqrt{dx^2+c}}-\frac{\frac{(5de-6cf)x^3\sqrt{bx^2+a}\sqrt{dx^2+c}}{5d}-\frac{c\left(\frac{(20bde-24bcf+adf)x\sqrt{bx^2+a}\sqrt{dx^2+c}}{3bd}-\frac{(20bde-24bcf+adf)\sqrt{bx^2+a}\,\text{Elliptic}}{\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}}\right.}{}}{}\right.$$

$$f\left(\frac{(de-cf)x^7\sqrt{bx^2+a}}{cd\sqrt{dx^2+c}}-\frac{\frac{(7de-8cf)x^5\sqrt{bx^2+a}\sqrt{dx^2+c}}{7d}-\frac{c\left(\frac{(42bde-48bcf+adf)x^3\sqrt{bx^2+a}\sqrt{dx^2+c}}{5bd}-\frac{\frac{1}{3}\left(\frac{4dfa^2}{b}-(7de-12cf)a+\frac{24bc(7de-8cf)}{d}\right)x\sqrt{bx^2}}{}\right.}{}}{}\right.$$

input
```
Int[(x^4*Sqrt[a + b*x^2]*(e + f*x^2)^2)/(c + d*x^2)^(3/2),x]
```

output
```
e*(((d*e - c*f)*x^5*Sqrt[a + b*x^2])/(c*d*Sqrt[c + d*x^2]) - (((5*d*e - 6*
c*f)*x^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(5*d) - (c*(((20*b*d*e - 24*b*c*
f + a*d*f)*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(3*b*d) - ((2*a^2*d^2*f - a*
b*d*(5*d*e - 8*c*f) + 8*b^2*c*(5*d*e - 6*c*f))*((x*Sqrt[a + b*x^2])/(b*Sqr
t[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt
[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*S
qrt[c + d*x^2])) + (c^(3/2)*(20*b*d*e - 24*b*c*f + a*d*f)*Sqrt[a + b*x^2]*
EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*
(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))/(3*b*d)))/(5*d))/(c*d)) +
(f*(((d*e - c*f)*x^7*Sqrt[a + b*x^2])/(c*d*Sqrt[c + d*x^2]) - (((7*d*e - 8
*c*f)*x^5*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(7*d) - (c*(((42*b*d*e - 48*b*c
*f + a*d*f)*x^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(5*b*d) - ((((4*a^2*d*f)/
b - a*(7*d*e - 12*c*f) + (24*b*c*(7*d*e - 8*c*f))/d)*x*Sqrt[a + b*x^2]*Sqr
t[c + d*x^2])/3 - ((8*a^3*d^3*f - a^2*b*d^2*(14*d*e - 23*c*f) - 8*a*b^2*c*
d*(7*d*e - 9*c*f) + 48*b^3*c^2*(7*d*e - 8*c*f))*((x*Sqrt[a + b*x^2])/(b*Sq
rt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqr
t[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*
Sqrt[c + d*x^2])) + (c^(3/2)*(4*a^2*d^2*f - a*b*d*(7*d*e - 12*c*f) + 24*b^
2*c*(7*d*e - 8*c*f))*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]]
, 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 439
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
 + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*b*g*(p + 1))), x] + Simp[1/(2*a*b*(p
+ 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1)*Simp[c*(2*b*e*(
p + 1) + (b*e - a*f)*(m + 1)) + d*(2*b*e*(p + 1) + (b*e - a*f)*(m + 2*q + 1
))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && LtQ[p, -1] && G
tQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e - a*f])
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

rule 448
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[e   Int[(g*x)^m*(a + b*x
^2)^p*(c + d*x^2)^q*(e + f*x^2)^(r - 1), x], x] + Simp[f/e^2   Int[(g*x)^(m
 + 2)*(a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^(r - 1), x], x] /; FreeQ[{a,
b, c, d, e, f, g, m, p, q}, x] && IGtQ[r, 0]
```

Maple [A] (verified)

Time = 20.31 (sec) , antiderivative size = 964, normalized size of antiderivative = 1.53

method	result
risch	$-\frac{x\left(-15f^2x^4b^2d^2-3ab\,d^2f^2x^2+39b^2cd\,f^2x^2-42b^2d^2ef\,x^2+4a^2d^2f^2+12abcd\,f^2-14ab\,d^2ef-87b^2c^2f^2+126b^2cdef-35b^2d^2e^2\right)\surd}{105b^2d^4}$
elliptic	Expression too large to display
default	Expression too large to display

input
```
int(x^4*(b*x^2+a)^(1/2)*(f*x^2+e)^2/(d*x^2+c)^(3/2),x,method=_RETURNVERBOS
E)
```

output
```
-1/105/b^2*x*(-15*b^2*d^2*f^2*x^4-3*a*b*d^2*f^2*x^2+39*b^2*c*d*f^2*x^2-42*
b^2*d^2*e*f*x^2+4*a^2*d^2*f^2+12*a*b*c*d*f^2-14*a*b*d^2*e*f-87*b^2*c^2*f^2
+126*b^2*c*d*e*f-35*b^2*d^2*e^2)*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/d^4+1/105
/b^2/d^4*(-(8*a^3*d^3*f^2+23*a^2*b*c*d^2*f^2-28*a^2*b*d^3*e*f+72*a*b^2*c^2
*d*f^2-112*a*b^2*c*d^2*e*f+35*a*b^2*d^3*e^2-279*b^3*c^3*f^2+462*b^3*c^2*d*
e*f-175*b^3*c*d^2*e^2)*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/
(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b
*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2)))+c*(4*a
^3*d^3*f^2+12*a^2*b*c*d^2*f^2-14*a^2*b*d^3*e*f-192*a*b^2*c^2*d*f^2+336*a*b
^2*c*d^2*e*f-140*a*b^2*d^3*e^2+105*b^3*c^3*f^2-210*b^3*c^2*d*e*f+105*b^3*c
*d^2*e^2)/d/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*
x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+
105*c^2*(a*c^2*d*f^2-2*a*c*d^2*e*f+a*d^3*e^2-b*c^3*f^2+2*b*c^2*d*e*f-b*c*d
^2*e^2)*b^2/d*((b*d*x^2+a*d)/c/(a*d-b*c)*x/((x^2+c/d)*(b*d*x^2+a*d))^(1/2)
+(1/c-1/(a*d-b*c)/c*a*d)/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/
(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)
/c/b)^(1/2))+b/(a*d-b*c)/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/
(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c
)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2)))))*((b*x^
2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 1047, normalized size of antiderivative = 1.66

$$\int \frac{x^4\sqrt{a+bx^2}(e+fx^2)^2}{(c+dx^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^4*(b*x^2+a)^(1/2)*(f*x^2+e)^2/(d*x^2+c)^(3/2),x, algorithm="fr
icas")
```

output
```
1/105*(((35*(8*b^3*c^2*d^3 - a*b^2*c*d^4)*e^2 - 28*(24*b^3*c^3*d^2 - 4*a*b
^2*c^2*d^3 - a^2*b*c*d^4)*e*f + (384*b^3*c^4*d - 72*a*b^2*c^3*d^2 - 23*a^2
*b*c^2*d^3 - 8*a^3*c*d^4)*f^2)*x^3 + (35*(8*b^3*c^3*d^2 - a*b^2*c^2*d^3)*e
^2 - 28*(24*b^3*c^4*d - 4*a*b^2*c^3*d^2 - a^2*b*c^2*d^3)*e*f + (384*b^3*c^
5 - 72*a*b^2*c^4*d - 23*a^2*b*c^3*d^2 - 8*a^3*c^2*d^3)*f^2)*x)*sqrt(b*d)*s
qrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - ((35*(8*b^3*c^2*d^
3 - a*b^2*c*d^4 + 4*a*b^2*d^5)*e^2 - 14*(48*b^3*c^3*d^2 - 8*a*b^2*c^2*d^3
- a^2*b*d^5 - 2*(a^2*b - 12*a*b^2)*c*d^4)*e*f + (384*b^3*c^4*d - 72*a*b^2*
c^3*d^2 - 4*a^3*d^5 - (23*a^2*b - 192*a*b^2)*c^2*d^3 - 4*(2*a^3 + 3*a^2*b)
*c*d^4)*f^2)*x^3 + (35*(8*b^3*c^3*d^2 - a*b^2*c^2*d^3 + 4*a*b^2*c*d^4)*e^2
 - 14*(48*b^3*c^4*d - 8*a*b^2*c^3*d^2 - a^2*b*c*d^4 - 2*(a^2*b - 12*a*b^2)
*c^2*d^3)*e*f + (384*b^3*c^5 - 72*a*b^2*c^4*d - 4*a^3*c*d^4 - (23*a^2*b -
192*a*b^2)*c^3*d^2 - 4*(2*a^3 + 3*a^2*b)*c^2*d^3)*f^2)*x)*sqrt(b*d)*sqrt(-
c/d)*elliptic_f(arcsin(sqrt(-c/d)/x), a*d/(b*c)) + (15*b^3*d^5*f^2*x^8 + 3
*(14*b^3*d^5*e*f - (8*b^3*c*d^4 - a*b^2*d^5)*f^2)*x^6 + (35*b^3*d^5*e^2 -
14*(6*b^3*c*d^4 - a*b^2*d^5)*e*f + (48*b^3*c^2*d^3 - 9*a*b^2*c*d^4 - 4*a^2
*b*d^5)*f^2)*x^4 - 35*(8*b^3*c^2*d^3 - a*b^2*c*d^4)*e^2 + 28*(24*b^3*c^3*d
^2 - 4*a*b^2*c^2*d^3 - a^2*b*c*d^4)*e*f - (384*b^3*c^4*d - 72*a*b^2*c^3*d^
2 - 23*a^2*b*c^2*d^3 - 8*a^3*c*d^4)*f^2 - (35*(4*b^3*c*d^4 - a*b^2*d^5)*e^
2 - 14*(24*b^3*c^2*d^3 - 7*a*b^2*c*d^4 - 2*a^2*b*d^5)*e*f + (192*b^3*c^...
```

Sympy [F]

$$\int \frac{x^4\sqrt{a+bx^2}(e+fx^2)^2}{(c+dx^2)^{3/2}}\,dx = \int \frac{x^4\sqrt{a+bx^2}(e+fx^2)^2}{(c+dx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**4*(b*x**2+a)**(1/2)*(f*x**2+e)**2/(d*x**2+c)**(3/2),x)
```

output
```
Integral(x**4*sqrt(a + b*x**2)*(e + f*x**2)**2/(c + d*x**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{x^4\sqrt{a+bx^2}(e+fx^2)^2}{(c+dx^2)^{3/2}}\,dx = \int \frac{\sqrt{bx^2+a}(fx^2+e)^2x^4}{(dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4*(b*x^2+a)^(1/2)*(f*x^2+e)^2/(d*x^2+c)^(3/2),x, algorithm="ma
xima")
```

output
```
integrate(sqrt(b*x^2 + a)*(f*x^2 + e)^2*x^4/(d*x^2 + c)^(3/2), x)
```

Giac [F]

$$\int \frac{x^4\sqrt{a+bx^2}(e+fx^2)^2}{(c+dx^2)^{3/2}}\,dx = \int \frac{\sqrt{bx^2+a}(fx^2+e)^2x^4}{(dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4*(b*x^2+a)^(1/2)*(f*x^2+e)^2/(d*x^2+c)^(3/2),x, algorithm="gi
ac")
```

output
```
integrate(sqrt(b*x^2 + a)*(f*x^2 + e)^2*x^4/(d*x^2 + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4\sqrt{a+bx^2}(e+fx^2)^2}{(c+dx^2)^{3/2}}\,dx=\int \frac{x^4\,\sqrt{b\,x^2+a}\,{\left(f\,x^2+e\right)}^2}{{\left(d\,x^2+c\right)}^{3/2}}\,dx$$

input
```
int((x^4*(a + b*x^2)^(1/2)*(e + f*x^2)^2)/(c + d*x^2)^(3/2),x)
```

output
```
int((x^4*(a + b*x^2)^(1/2)*(e + f*x^2)^2)/(c + d*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^4\sqrt{a+bx^2}(e+fx^2)^2}{(c+dx^2)^{3/2}}\,dx=\text{too large to display}$$

input
```
int(x^4*(b*x^2+a)^(1/2)*(f*x^2+e)^2/(d*x^2+c)^(3/2),x)
```

output

```
(12*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**3*d**2*f**2*x + 27*sqrt(c + d*x**
2)*sqrt(a + b*x**2)*a**2*b*c*d*f**2*x - 42*sqrt(c + d*x**2)*sqrt(a + b*x**
2)*a**2*b*d**2*e*f*x - 8*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*b*d**2*f**
2*x**3 - 144*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**2*c**2*f**2*x + 252*sq
rt(c + d*x**2)*sqrt(a + b*x**2)*a*b**2*c*d*e*f*x - 18*sqrt(c + d*x**2)*sqr
t(a + b*x**2)*a*b**2*c*d*f**2*x**3 - 105*sqrt(c + d*x**2)*sqrt(a + b*x**2)
*a*b**2*d**2*e**2*x + 28*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**2*d**2*e*f
*x**3 + 6*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**2*d**2*f**2*x**5 + 96*sqr
t(c + d*x**2)*sqrt(a + b*x**2)*b**3*c**2*f**2*x**3 - 168*sqrt(c + d*x**2)*
sqrt(a + b*x**2)*b**3*c*d*e*f*x**3 - 48*sqrt(c + d*x**2)*sqrt(a + b*x**2)*
b**3*c*d*f**2*x**5 + 70*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**3*d**2*e**2*x
**3 + 84*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**3*d**2*e*f*x**5 + 30*sqrt(c
+ d*x**2)*sqrt(a + b*x**2)*b**3*d**2*f**2*x**7 + 4*int((sqrt(c + d*x**2)*s
qrt(a + b*x**2)*x**4)/(a*c**2 + 2*a*c*d*x**2 + a*d**2*x**4 + b*c**2*x**2 +
 2*b*c*d*x**4 + b*d**2*x**6),x)*a**3*b*c*d**3*f**2 + 4*int((sqrt(c + d*x**
2)*sqrt(a + b*x**2)*x**4)/(a*c**2 + 2*a*c*d*x**2 + a*d**2*x**4 + b*c**2*x*
*2 + 2*b*c*d*x**4 + b*d**2*x**6),x)*a**3*b*d**4*f**2*x**2 + 11*int((sqrt(c
 + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c**2 + 2*a*c*d*x**2 + a*d**2*x**4 + b
*c**2*x**2 + 2*b*c*d*x**4 + b*d**2*x**6),x)*a**2*b**2*c**2*d**2*f**2 - 14*
int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c**2 + 2*a*c*d*x**2 + a...
```

3.138 $\int \frac{x^2\sqrt{a+bx^2}(e+fx^2)^2}{(c+dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 443

$$\begin{aligned}
\int \frac{x^2\sqrt{a+bx^2}(e+fx^2)^2}{(c+dx^2)^{3/2}}\,dx &= -\frac{(2a^2d^2f^2-abdf(10de-7cf)-b^2(15d^2e^2-40cdef+24c^2f^2))\,x\sqrt{a+bx^2}}{15b^2d^3\sqrt{c+dx^2}} \\
&+\frac{f(10bde-6bcf+adf)x^3\sqrt{a+bx^2}}{15bd^2\sqrt{c+dx^2}}+\frac{f^2x^5\sqrt{a+bx^2}}{5d\sqrt{c+dx^2}} \\
&+\frac{2\sqrt{c}(a^2d^2f^2-abdf(5de-4cf)-b^2(15d^2e^2-40cdef+24c^2f^2))\,\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15b^2d^{7/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} \\
&-\frac{\sqrt{c}(acdf^2-b(15d^2e^2-40cdef+24c^2f^2))\,\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{15bd^{7/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}
\end{aligned}$$

output
```
-1/15*(2*a^2*d^2*f^2-a*b*d*f*(-7*c*f+10*d*e)-b^2*(24*c^2*f^2-40*c*d*e*f+15
*d^2*e^2))*x*(b*x^2+a)^(1/2)/b^2/d^3/(d*x^2+c)^(1/2)+1/15*f*(a*d*f-6*b*c*f
+10*b*d*e)*x^3*(b*x^2+a)^(1/2)/b/d^2/(d*x^2+c)^(1/2)+1/5*f^2*x^5*(b*x^2+a)
^(1/2)/d/(d*x^2+c)^(1/2)+2/15*c^(1/2)*(a^2*d^2*f^2-a*b*d*f*(-4*c*f+5*d*e)-
b^2*(24*c^2*f^2-40*c*d*e*f+15*d^2*e^2))*(b*x^2+a)^(1/2)*EllipticE(d^(1/2)*
x/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))/b^2/d^(7/2)/(c*(b*x^2+a)/a/
(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-1/15*c^(1/2)*(a*c*d*f^2-b*(24*c^2*f^2-40*
c*d*e*f+15*d^2*e^2))*(b*x^2+a)^(1/2)*InverseJacobiAM(arctan(d^(1/2)*x/c^(1
/2)),(1-b*c/a/d)^(1/2))/b/d^(7/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c
)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.94 (sec) , antiderivative size = 363, normalized size of antiderivative = 0.82

$$\int \frac{x^2\sqrt{a+bx^2}(e+fx^2)^2}{(c+dx^2)^{3/2}}\,dx = \frac{\sqrt{\frac{b}{a}}dx(a+bx^2)\left(adf^2(c+dx^2)+b(-24c^2f^2+2cdf(20e-3fx^2)+d^2(-15\right.}{}$$

input
```
Integrate[(x^2*Sqrt[a + b*x^2]*(e + f*x^2)^2)/(c + d*x^2)^(3/2),x]
```

output
```
(Sqrt[b/a]*d*x*(a + b*x^2)*(a*d*f^2*(c + d*x^2) + b*(-24*c^2*f^2 + 2*c*d*f
*(20*e - 3*f*x^2) + d^2*(-15*e^2 + 10*e*f*x^2 + 3*f^2*x^4))) + (2*I)*c*(a^
2*d^2*f^2 + a*b*d*f*(-5*d*e + 4*c*f) + b^2*(-15*d^2*e^2 + 40*c*d*e*f - 24*
c^2*f^2))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt
[b/a]*x], (a*d)/(b*c)] - I*(a^2*c*d^2*f^2 - 2*b^2*c*(15*d^2*e^2 - 40*c*d*e
*f + 24*c^2*f^2) + a*b*d*(15*d^2*e^2 - 50*c*d*e*f + 32*c^2*f^2))*Sqrt[1 +
(b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*
c)])/(15*b*Sqrt[b/a]*d^4*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (warning: unable to verify)

Time = 1.31 (sec) , antiderivative size = 745, normalized size of antiderivative = 1.68, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.371, Rules used = {448, 439, 444, 27, 406, 320, 388, 313, 444, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\sqrt{a+bx^2}\left(e+fx^2\right)^2}{\left(c+dx^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{448}$$

$$\frac{f\int \frac{x^4\sqrt{bx^2+a}\left(fx^2+e\right)}{\left(dx^2+c\right)^{3/2}}dx}{e^2}+e\int \frac{x^2\sqrt{bx^2+a}\left(fx^2+e\right)}{\left(dx^2+c\right)^{3/2}}dx$$

$$\Big\downarrow \text{439}$$

$$\frac{f\left(\frac{x^5\sqrt{a+bx^2}(de-cf)}{cd\sqrt{c+dx^2}}-\frac{\int \frac{x^4\left(b(5de-6cf)x^2+a(4de-5cf)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{cd}\right)}{e^2}+e\left(\frac{x^3\sqrt{a+bx^2}(de-cf)}{cd\sqrt{c+dx^2}}-\frac{\int \frac{x^2\left(b(3de-4cf)x^2+a(2de-3cf)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{cd}\right)$$

$$\Big\downarrow \text{444}$$

$$\frac{f\left(\frac{x^5\sqrt{a+bx^2}(de-cf)}{cd\sqrt{c+dx^2}}-\frac{\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}(5de-6cf)}{5d}-\frac{\int \frac{bcx^2\left((adf+4b(5de-6cf))x^2+3a(5de-6cf)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{5bd}}{cd}\right)}{e^2}+e\left(\frac{x^3\sqrt{a+bx^2}(de-cf)}{cd\sqrt{c+dx^2}}-\frac{\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(3de-4cf)}{3d}-\frac{\int \frac{bc\left((6bde-8bcf+adf)x^2+a(3de-4cf)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3bd}}{cd}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{f\left(\frac{x^5\sqrt{a+bx^2}(de-cf)}{cd\sqrt{c+dx^2}}-\frac{\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}(5de-6cf)}{5d}-\frac{c\int\frac{x^2\left((20bde-24bcf+adf)x^2+3a(5de-6cf)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{5d}}{cd}\right)}{e^2}+$$

$$e\left(\frac{x^3\sqrt{a+bx^2}(de-cf)}{cd\sqrt{c+dx^2}}-\frac{\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(3de-4cf)}{3d}-\frac{c\int\frac{(6bde-8bcf+adf)x^2+a(3de-4cf)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3d}}{cd}\right)$$

$$\Big\downarrow \text{406}$$

$$\frac{f\left(\frac{x^5\sqrt{a+bx^2}(de-cf)}{cd\sqrt{c+dx^2}}-\frac{\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}(5de-6cf)}{5d}-\frac{c\int\frac{x^2\left((20bde-24bcf+adf)x^2+3a(5de-6cf)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{5d}}{cd}\right)}{e^2}+$$

$$e\left(\frac{x^3\sqrt{a+bx^2}(de-cf)}{cd\sqrt{c+dx^2}}-\frac{\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(3de-4cf)}{3d}-\frac{c\left(a(3de-4cf)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+(adf-8bcf+6bde)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}d\right.}{3d}}{cd}\right.$$

$$\Big\downarrow \text{320}$$

$$e\left(\frac{x^3\sqrt{a+bx^2}(de-cf)}{cd\sqrt{c+dx^2}}-\frac{\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(3de-4cf)}{3d}-\frac{c\left((adf-8bcf+6bde)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{\sqrt{c}\sqrt{a+bx^2}(3de-4cf)\,\text{EllipticF}\left(a\right.}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a-\right.}{a\left(c-\right.}}}\right.}{3d}}{cd}\right.$$

$$\frac{f\left(\frac{x^5\sqrt{a+bx^2}(de-cf)}{cd\sqrt{c+dx^2}}-\frac{\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}(5de-6cf)}{5d}-\frac{c\int\frac{x^2\left((20bde-24bcf+adf)x^2+3a(5de-6cf)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{5d}}{cd}\right)}{e^2}$$

$$\Big\downarrow \text{388}$$

$$e\left(\frac{x^3\sqrt{a+bx^2}(de-cf)}{cd\sqrt{c+dx^2}}-\frac{\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(3de-4cf)}{3d}-\frac{c\left((adf-8bcf+6bde)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{\sqrt{c}\sqrt{a+bx^2}(3de-4c\cdots}{\sqrt{d}\sqrt{\cdots}}\cdots\right.}{3d}\cdots}{cd}\cdots\right.$$

$$\frac{f\left(\frac{x^5\sqrt{a+bx^2}(de-cf)}{cd\sqrt{c+dx^2}}-\frac{\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}(5de-6cf)}{5d}-\frac{c\int\frac{x^2\left((20bde-24bcf+adf)x^2+3a(5de-6cf)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{5d}}{cd}\right)}{e^2}$$

$\downarrow$ 313

$$\frac{f\left(\frac{x^5\sqrt{a+bx^2}(de-cf)}{cd\sqrt{c+dx^2}}-\frac{\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}(5de-6cf)}{5d}-\frac{c\int\frac{x^2\left((20bde-24bcf+adf)x^2+3a(5de-6cf)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{5d}}{cd}\right)}{e^2}+$$

$$e\left(\frac{x^3\sqrt{a+bx^2}(de-cf)}{cd\sqrt{c+dx^2}}-\frac{\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(3de-4cf)}{3d}-\frac{c\left(\frac{\sqrt{c}\sqrt{a+bx^2}(3de-4cf)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+(adf-8bcf+6bde)\left(\cdots\right.\right.}{3d}\cdots}{cd}\cdots\right.$$

$\downarrow$ 444

$$\frac{f\left(\frac{x^5\sqrt{a+bx^2}(de-cf)}{cd\sqrt{c+dx^2}}-\frac{\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}(5de-6cf)}{5d}-\frac{c\left(\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(adf-24bcf+20bde)}{3bd}-\frac{\int\frac{\left(8c(5de-6cf)b^2-ad(5de-8cf)b+2a^2d^2f\right)x^2+ac\cdots}{\sqrt{bx^2+a}\sqrt{dx^2+c}}}{3bd}\cdots\right)}{5d}}{cd}\cdots\right)}{e^2}\cdots$$

$$e\left(\frac{x^3\sqrt{a+bx^2}(de-cf)}{cd\sqrt{c+dx^2}}-\frac{\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(3de-4cf)}{3d}-\frac{c\left(\frac{\sqrt{c}\sqrt{a+bx^2}(3de-4cf)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+(adf-8bcf+6bde)\cdots\right)}{3d}}{cd}\cdots\right)$$

$$\Big\downarrow \; 406$$

$$\frac{f\left(\frac{x^5\sqrt{a+bx^2}(de-cf)}{cd\sqrt{c+dx^2}}-\frac{\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}(5de-6cf)}{5d}-\frac{c\left(\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(adf-24bcf+20bde)}{3bd}-\frac{\left(2a^2d^2f-abd(5de-8cf)+8b^2c(5de-6cf)\right)\int\frac{\cdots}{\sqrt{bx^2\cdots}}\cdots}{\cdots}\right)}{5d}}{cd}\cdots\right)}{e^2}\cdots$$

$$e\left(\frac{x^3\sqrt{a+bx^2}(de-cf)}{cd\sqrt{c+dx^2}}-\frac{\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(3de-4cf)}{3d}-\frac{c\left(\frac{\sqrt{c}\sqrt{a+bx^2}(3de-4cf)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+(adf-8bcf+6bde)\cdots\right)}{3d}}{cd}\cdots\right)$$

$$\Big\downarrow \; 320$$

$$f\left(\frac{x^5\sqrt{a+bx^2}(de-cf)}{cd\sqrt{c+dx^2}}-\frac{\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}(5de-6cf)}{5d}-\frac{c\left(\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(adf-24bcf+20bde)}{3bd}-\frac{\left(2a^2d^2f-abd(5de-8cf)+8b^2c(5de-6cf)\right)\int\frac{}{\sqrt{bx^2\cdots}}}{\cdots}\right.}{5d\cdots}}{cd\cdots}\right.$$

$$\overline{\qquad e^2 \qquad}$$

$$e\left(\frac{x^3\sqrt{a+bx^2}(de-cf)}{cd\sqrt{c+dx^2}}-\frac{\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(3de-4cf)}{3d}-\frac{c\left(\frac{\sqrt{c}\sqrt{a+bx^2}(3de-4cf)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+(adf-8bcf+6bde)\left(\cdots\right.\right.}{3d\cdots}}{cd\cdots}\right.$$

$\downarrow$ 388

$$\frac{f\left(\frac{x^5\sqrt{a+bx^2}(de-cf)}{cd\sqrt{c+dx^2}}-\frac{\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}(5de-6cf)}{5d}-\frac{c\left(\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(adf-24bcf+20bde)}{3bd}-\frac{\left(2a^2d^2f-abd(5de-8cf)+8b^2c(5de-6cf)\right)\left(\frac{x\sqrt{a+}}{b\sqrt{c+}}\right.}{}\right.}{}}{cd}\right.}{e^2}$$

$$e\left(\frac{x^3\sqrt{a+bx^2}(de-cf)}{cd\sqrt{c+dx^2}}-\frac{\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(3de-4cf)}{3d}-\frac{c\left(\frac{\sqrt{c}\sqrt{a+bx^2}(3de-4cf)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+(adf-8bcf+6bde)\right.}{3d}}{cd}\right.$$

↓ 313

$$\frac{f\left(\frac{x^5\sqrt{a+bx^2}(de-cf)}{cd\sqrt{c+dx^2}}-\frac{\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}(5de-6cf)}{5d}-\frac{c\left(\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(adf-24bcf+20bde)}{3bd}-\frac{\left(2a^2d^2f-abd(5de-8cf)+8b^2c(5de-6cf)\right)\left(\frac{x\sqrt{a+\cdots}}{b\sqrt{c+\cdots}}\cdots\right.}{\cdots}\cdots\right.}{\cdots}}{cd}\cdots\right.}{e^2\cdots}$$

$$e\left(\frac{x^3\sqrt{a+bx^2}(de-cf)}{cd\sqrt{c+dx^2}}-\frac{\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(3de-4cf)}{3d}-\frac{c\left(\frac{\sqrt{c}\sqrt{a+bx^2}(3de-4cf)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+(adf-8bcf+6bde)\left(\cdots\right.\right.}{3d\cdots}}{cd}\cdots\right.$$

input
```
Int[(x^2*Sqrt[a + b*x^2]*(e + f*x^2)^2)/(c + d*x^2)^(3/2),x]
```

output
```
e*(((d*e - c*f)*x^3*Sqrt[a + b*x^2])/(c*d*Sqrt[c + d*x^2]) - (((3*d*e - 4*
c*f)*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(3*d) - (c*((6*b*d*e - 8*b*c*f + a
*d*f)*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*
EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(
c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (Sqrt[c]*(3*d*e - 4*c*
f)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)]
)/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])))/(3*d))
/(c*d)) + (f*(((d*e - c*f)*x^5*Sqrt[a + b*x^2])/(c*d*Sqrt[c + d*x^2]) - ((
(5*d*e - 6*c*f)*x^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(5*d) - (c*(((20*b*d*
e - 24*b*c*f + a*d*f)*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(3*b*d) - ((2*a^2
*d^2*f - a*b*d*(5*d*e - 8*c*f) + 8*b^2*c*(5*d*e - 6*c*f))*((x*Sqrt[a + b*x
^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt
[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c +
 d*x^2))]*Sqrt[c + d*x^2])) + (c^(3/2)*(20*b*d*e - 24*b*c*f + a*d*f)*Sqrt[
a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[
d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))/(3*b*d)))/(5*d)
)/(c*d)))/e^2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 439
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
 + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*b*g*(p + 1))), x] + Simp[1/(2*a*b*(p
+ 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1)*Simp[c*(2*b*e*(
p + 1) + (b*e - a*f)*(m + 1)) + d*(2*b*e*(p + 1) + (b*e - a*f)*(m + 2*q + 1
))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && LtQ[p, -1] && G
tQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e - a*f])
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

rule 448
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[e   Int[(g*x)^m*(a + b*x
^2)^p*(c + d*x^2)^q*(e + f*x^2)^(r - 1), x], x] + Simp[f/e^2   Int[(g*x)^(m
 + 2)*(a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^(r - 1), x], x] /; FreeQ[{a,
b, c, d, e, f, g, m, p, q}, x] && IGtQ[r, 0]
```

Maple [A] (verified)

Time = 11.05 (sec) , antiderivative size = 718, normalized size of antiderivative = 1.62

method	result
elliptic	$\frac{\sqrt{(b x^2+a)(x^2 d+c)}\left(-\frac{\left(bd\,x^2+ad\right)\left(c^2 f^2-2cdef+d^2 e^2\right)x}{d^4\sqrt{\left(x^2+\frac{c}{d}\right)\left(bd\,x^2+ad\right)}}+\frac{f^2 x^3\sqrt{bd\,x^4+ad\,x^2+x^2 bc+ac}}{5d^2}+\frac{\left(\frac{f(adf-bcf+2bde)}{d^2}-\frac{f^2(4ad+4bc)}{5d^2}\right)x\sqrt{bd\,x^4+}}{3bd}\right.}{}$
risch	$\frac{fx\left(3bdf\,x^2+adf-9bcf+10bde\right)\sqrt{b\,x^2+a}\,\sqrt{x^2 d+c}}{15b\,d^3}-\frac{\left(-\frac{\left(2a^2 d^2 f^2+8abcd\,f^2-10ab\,d^2 ef-33b^2 c^2 f^2+50b^2 cdef-15b^2 d^2 e^2\right)c\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{}}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+a}}\right.}{}$
default	Expression too large to display

input
```
int(x^2*(b*x^2+a)^(1/2)*(f*x^2+e)^2/(d*x^2+c)^(3/2),x,method=_RETURNVERBOS
E)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-(b*d*x^2+a*d
)*(c^2*f^2-2*c*d*e*f+d^2*e^2)/d^4*x/((x^2+c/d)*(b*d*x^2+a*d))^(1/2)+1/5*f^
2/d^2*x^3*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+1/3*(1/d^2*f*(a*d*f-b*c*f+2*
b*d*e)-1/5*f^2/d^2*(4*a*d+4*b*c))/b/d*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2
)+(1/d^4*(a*c^2*d*f^2-2*a*c*d^2*e*f+a*d^3*e^2-b*c^3*f^2+2*b*c^2*d*e*f-b*c*
d^2*e^2)-(c^2*f^2-2*c*d*e*f+d^2*e^2)/d^4*(a*d-b*c)+a/d^3*(c^2*f^2-2*c*d*e*
f+d^2*e^2)-1/3*(1/d^2*f*(a*d*f-b*c*f+2*b*d*e)-1/5*f^2/d^2*(4*a*d+4*b*c))/b
/d*a*c)/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+
b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-(-(a
*c*d*f^2-2*a*d^2*e*f-b*c^2*f^2+2*b*c*d*e*f-b*d^2*e^2)/d^3+(c^2*f^2-2*c*d*e
*f+d^2*e^2)/d^3*b-3/5*a*c/d^2*f^2-1/3*(1/d^2*f*(a*d*f-b*c*f+2*b*d*e)-1/5*f
^2/d^2*(4*a*d+4*b*c))/b/d*(2*a*d+2*b*c))*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*
(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(-b/a
)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c
/b)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 708, normalized size of antiderivative = 1.60

$$\int \frac{x^2\sqrt{a+bx^2}(e+fx^2)^2}{(c+dx^2)^{3/2}}\,dx =$$

$$-\frac{2\left(\left(15\,b^2c^2d^3e^2-5\,(8\,b^2c^3d^2-abc^2d^3)ef+(24\,b^2c^4d-4\,abc^3d^2-a^2c^2d^3)f^2\right)x^3+\left(15\,b^2c^3d^2e^2-5\,(8\,b^2c^4\right.\right.}{}$$

input
```
integrate(x^2*(b*x^2+a)^(1/2)*(f*x^2+e)^2/(d*x^2+c)^(3/2),x, algorithm="fr
icas")
```

output
```
-1/15*(2*((15*b^2*c^2*d^3*e^2 - 5*(8*b^2*c^3*d^2 - a*b*c^2*d^3)*e*f + (24*
b^2*c^4*d - 4*a*b*c^3*d^2 - a^2*c^2*d^3)*f^2)*x^3 + (15*b^2*c^3*d^2*e^2 -
5*(8*b^2*c^4*d - a*b*c^3*d^2)*e*f + (24*b^2*c^5 - 4*a*b*c^4*d - a^2*c^3*d^
2)*f^2)*x)*sqrt(b*d)*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c)
) - ((15*(2*b^2*c^2*d^3 + a*b*d^5)*e^2 - 10*(8*b^2*c^3*d^2 - a*b*c^2*d^3 +
 4*a*b*c*d^4)*e*f + (48*b^2*c^4*d - 8*a*b*c^3*d^2 - a^2*c*d^4 - 2*(a^2 - 1
2*a*b)*c^2*d^3)*f^2)*x^3 + (15*(2*b^2*c^3*d^2 + a*b*c*d^4)*e^2 - 10*(8*b^2
*c^4*d - a*b*c^3*d^2 + 4*a*b*c^2*d^3)*e*f + (48*b^2*c^5 - 8*a*b*c^4*d - a^
2*c^2*d^3 - 2*(a^2 - 12*a*b)*c^3*d^2)*f^2)*x)*sqrt(b*d)*sqrt(-c/d)*ellipti
c_f(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - (3*b^2*c*d^4*f^2*x^6 + 30*b^2*c^2*d
^3*e^2 + (10*b^2*c*d^4*e*f - (6*b^2*c^2*d^3 - a*b*c*d^4)*f^2)*x^4 - 10*(8*
b^2*c^3*d^2 - a*b*c^2*d^3)*e*f + 2*(24*b^2*c^4*d - 4*a*b*c^3*d^2 - a^2*c^2
*d^3)*f^2 + (15*b^2*c*d^4*e^2 - 10*(4*b^2*c^2*d^3 - a*b*c*d^4)*e*f + (24*b
^2*c^3*d^2 - 7*a*b*c^2*d^3 - 2*a^2*c*d^4)*f^2)*x^2)*sqrt(b*x^2 + a)*sqrt(d
*x^2 + c))/(b^2*c*d^6*x^3 + b^2*c^2*d^5*x)
```

Sympy [F]

$$\int \frac{x^2\sqrt{a+bx^2}(e+fx^2)^2}{(c+dx^2)^{3/2}}\,dx = \int \frac{x^2\sqrt{a+bx^2}\,(e+fx^2)^2}{(c+dx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2*(b*x**2+a)**(1/2)*(f*x**2+e)**2/(d*x**2+c)**(3/2),x)
```

output
```
Integral(x**2*sqrt(a + b*x**2)*(e + f*x**2)**2/(c + d*x**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{x^2\sqrt{a+bx^2}(e+fx^2)^2}{(c+dx^2)^{3/2}}\,dx = \int \frac{\sqrt{bx^2+a}(fx^2+e)^2x^2}{(dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*(b*x^2+a)^(1/2)*(f*x^2+e)^2/(d*x^2+c)^(3/2),x, algorithm="ma
xima")
```

output
```
integrate(sqrt(b*x^2 + a)*(f*x^2 + e)^2*x^2/(d*x^2 + c)^(3/2), x)
```

Giac [F]

$$\int \frac{x^2\sqrt{a+bx^2}(e+fx^2)^2}{(c+dx^2)^{3/2}}\,dx = \int \frac{\sqrt{bx^2+a}(fx^2+e)^2x^2}{(dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*(b*x^2+a)^(1/2)*(f*x^2+e)^2/(d*x^2+c)^(3/2),x, algorithm="gi
ac")
```

output
```
integrate(sqrt(b*x^2 + a)*(f*x^2 + e)^2*x^2/(d*x^2 + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2\sqrt{a+bx^2}(e+fx^2)^2}{(c+dx^2)^{3/2}}\,dx = \int \frac{x^2\,\sqrt{b\,x^2+a}\,{\left(f\,x^2+e\right)}^2}{{\left(d\,x^2+c\right)}^{3/2}} \,d x$$

input
```
int((x^2*(a + b*x^2)^(1/2)*(e + f*x^2)^2)/(c + d*x^2)^(3/2),x)
```

output
```
int((x^2*(a + b*x^2)^(1/2)*(e + f*x^2)^2)/(c + d*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^2\sqrt{a+bx^2}(e+fx^2)^2}{(c+dx^2)^{3/2}}\,dx = \text{too large to display}$$

input
```
int(x^2*(b*x^2+a)^(1/2)*(f*x^2+e)^2/(d*x^2+c)^(3/2),x)
```

output
```
( - 3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*c*d*f**2*x + 18*sqrt(c + d*x*
*2)*sqrt(a + b*x**2)*a*b*c**2*f**2*x - 30*sqrt(c + d*x**2)*sqrt(a + b*x**2
)*a*b*c*d*e*f*x + 2*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b*c*d*f**2*x**3 +
15*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b*d**2*e**2*x - 12*sqrt(c + d*x**2)
*sqrt(a + b*x**2)*b**2*c**2*f**2*x**3 + 20*sqrt(c + d*x**2)*sqrt(a + b*x**
2)*b**2*c*d*e*f*x**3 + 6*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**2*c*d*f**2*x
**5 - int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c**2 + 2*a*c*d*x**2
+ a*d**2*x**4 + b*c**2*x**2 + 2*b*c*d*x**4 + b*d**2*x**6),x)*a**2*b*c**2*d
**2*f**2 - int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c**2 + 2*a*c*d*
x**2 + a*d**2*x**4 + b*c**2*x**2 + 2*b*c*d*x**4 + b*d**2*x**6),x)*a**2*b*c
*d**3*f**2*x**2 - 32*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c**2
+ 2*a*c*d*x**2 + a*d**2*x**4 + b*c**2*x**2 + 2*b*c*d*x**4 + b*d**2*x**6),x
)*a*b**2*c**3*d*f**2 + 50*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*
c**2 + 2*a*c*d*x**2 + a*d**2*x**4 + b*c**2*x**2 + 2*b*c*d*x**4 + b*d**2*x*
*6),x)*a*b**2*c**2*d**2*e*f - 32*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x*
*4)/(a*c**2 + 2*a*c*d*x**2 + a*d**2*x**4 + b*c**2*x**2 + 2*b*c*d*x**4 + b*
d**2*x**6),x)*a*b**2*c**2*d**2*f**2*x**2 - 15*int((sqrt(c + d*x**2)*sqrt(a
 + b*x**2)*x**4)/(a*c**2 + 2*a*c*d*x**2 + a*d**2*x**4 + b*c**2*x**2 + 2*b*
c*d*x**4 + b*d**2*x**6),x)*a*b**2*c*d**3*e**2 + 50*int((sqrt(c + d*x**2)*s
qrt(a + b*x**2)*x**4)/(a*c**2 + 2*a*c*d*x**2 + a*d**2*x**4 + b*c**2*x**...
```

3.139 $\int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{(c+dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 306

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{(c+dx^2)^{3/2}}\,dx = \frac{f(6bde-4bcf+adf)x\sqrt{a+bx^2}}{3bd^2\sqrt{c+dx^2}} + \frac{f^2x^3\sqrt{a+bx^2}}{3d\sqrt{c+dx^2}} - \frac{(acdf^2-b(3d^2e^2-12cdef+8c^2f^2))\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3b\sqrt{c}d^{5/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} + \frac{2\sqrt{c}f(3de-2cf)\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3d^{5/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output
```
1/3*f*(a*d*f-4*b*c*f+6*b*d*e)*x*(b*x^2+a)^(1/2)/b/d^2/(d*x^2+c)^(1/2)+1/3*
f^2*x^3*(b*x^2+a)^(1/2)/d/(d*x^2+c)^(1/2)-1/3*(a*c*d*f^2-b*(8*c^2*f^2-12*c
*d*e*f+3*d^2*e^2))*(b*x^2+a)^(1/2)*EllipticE(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)
^(1/2),(1-b*c/a/d)^(1/2))/b/c^(1/2)/d^(5/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2
)/(d*x^2+c)^(1/2)+2/3*c^(1/2)*f*(-2*c*f+3*d*e)*(b*x^2+a)^(1/2)*InverseJaco
biAM(arctan(d^(1/2)*x/c^(1/2)),(1-b*c/a/d)^(1/2))/d^(5/2)/(c*(b*x^2+a)/a/(
d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.35 (sec) , antiderivative size = 281, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{a+bx^2}\left(e+fx^2\right)^2}{\left(c+dx^2\right)^{3/2}}\,dx = \frac{\sqrt{\frac{b}{a}}dx(a+bx^2)\left(3d^2e^2+4c^2f^2+cdf(-6e+fx^2)\right)-ic(acdf^2+b(-3d^2e^2+}{}$$

input
```
Integrate[(Sqrt[a + b*x^2]*(e + f*x^2)^2)/(c + d*x^2)^(3/2),x]
```

output
```
(Sqrt[b/a]*d*x*(a + b*x^2)*(3*d^2*e^2 + 4*c^2*f^2 + c*d*f*(-6*e + f*x^2))
- I*c*(a*c*d*f^2 + b*(-3*d^2*e^2 + 12*c*d*e*f - 8*c^2*f^2))*Sqrt[1 + (b*x^
2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] +
 I*c*(a*d*f*(-6*d*e + 5*c*f) + b*(-3*d^2*e^2 + 12*c*d*e*f - 8*c^2*f^2))*Sq
rt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a
*d)/(b*c)])/(3*Sqrt[b/a]*c*d^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 595, normalized size of antiderivative = 1.94, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = $\{433, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\left(e+fx^2\right)^2}{\left(c+dx^2\right)^{3/2}}\,dx$$

$$\Big\downarrow \ 433$$

$$\int \left(\frac{e^2\sqrt{a+bx^2}}{\left(c+dx^2\right)^{3/2}}+\frac{2efx^2\sqrt{a+bx^2}}{\left(c+dx^2\right)^{3/2}}+\frac{f^2x^4\sqrt{a+bx^2}}{\left(c+dx^2\right)^{3/2}}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{4c^{3/2}f^2\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3d^{5/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{2\sqrt{c}ef\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{d^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{4\sqrt{c}ef\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{d^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{\sqrt{c}f^2\sqrt{a+bx^2}(8bc-ad)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3bd^{5/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{e^2\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{4f^2x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3d^2}-\frac{f^2x\sqrt{a+bx^2}(8bc-ad)}{3bd^2\sqrt{c+dx^2}}+\frac{2efx\sqrt{a+bx^2}}{d\sqrt{c+dx^2}}-\frac{f^2x^3\sqrt{a+bx^2}}{d\sqrt{c+dx^2}}$$

input
```
Int[(Sqrt[a + b*x^2]*(e + f*x^2)^2)/(c + d*x^2)^(3/2),x]
```

output
```
(2*e*f*x*Sqrt[a + b*x^2])/(d*Sqrt[c + d*x^2]) - ((8*b*c - a*d)*f^2*x*Sqrt[
a + b*x^2])/(3*b*d^2*Sqrt[c + d*x^2]) - (f^2*x^3*Sqrt[a + b*x^2])/(d*Sqrt[
c + d*x^2]) + (4*f^2*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(3*d^2) + (e^2*Sqr
t[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqr
t[c]*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) - (4*S
qrt[c]*e*f*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c
)/(a*d)])/(d^(3/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
 + (Sqrt[c]*(8*b*c - a*d)*f^2*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/
Sqrt[c]], 1 - (b*c)/(a*d)])/(3*b*d^(5/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^
2))]*Sqrt[c + d*x^2]) + (2*Sqrt[c]*e*f*Sqrt[a + b*x^2]*EllipticF[ArcTan[(S
qrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(d^(3/2)*Sqrt[(c*(a + b*x^2))/(a*(c
 + d*x^2))]*Sqrt[c + d*x^2]) - (4*c^(3/2)*f^2*Sqrt[a + b*x^2]*EllipticF[Arc
Tan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*d^(5/2)*Sqrt[(c*(a + b*x^2)
)/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Defintions of rubi rules used

rule 433
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)
^q*(e + f*x^2)^r, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, f, p,
 q, r}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 8.97 (sec) , antiderivative size = 532, normalized size of antiderivative = 1.74

method	result
elliptic	$\frac{\sqrt{(b x^2+a)(x^2 d+c)}\left(\frac{\left(b d x^2+a d\right)\left(c^2 f^2-2 c d e f+d^2 e^2\right) x}{c d^3 \sqrt{\left(x^2+\frac{c}{d}\right)\left(b d x^2+a d\right)}}+\frac{f^2 x \sqrt{b d x^4+a d x^2+x^2 b c+a c}}{3 d^2}+\frac{\left(-\frac{a c d f^2-2 a d^2 e f-b c^2 f^2+2 b c d e f-b d^2 e^2}{d^3}+\frac{(c^2 \ldots}{\ldots}\right.}{\ldots}\right.}{\ldots}$
risch	$\frac{f^2 x \sqrt{b x^2+a}\,\sqrt{x^2 d+c}}{3 d^2}+\frac{\left(-\frac{f(a d f-5 b c f+6 b d e) c \sqrt{1+\frac{b x^2}{a}}\,\sqrt{1+\frac{d x^2}{c}}\left(\text{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{a d+b c}{c b}}\right)-\text{EllipticE}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{a d+b c}{c b}}\right)\ldots\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{b d x^4+a d x^2+x^2 b c+a c}\, d}\right.}{\ldots}$
default	$-\frac{\sqrt{b x^2+a}\,\sqrt{x^2 d+c}\left(-\sqrt{-\frac{b}{a}}\, b c d^2 f^2 x^5-\sqrt{-\frac{b}{a}}\, a c d^2 f^2 x^3-4\sqrt{-\frac{b}{a}}\, b c^2 d f^2 x^3+6\sqrt{-\frac{b}{a}}\, b c d^2 e f x^3-3\sqrt{-\frac{b}{a}}\, b d^3 e^2 x^3+5\sqrt{\frac{b x^2+a}{a}}\,\sqrt{\ldots}\right.}{\ldots}$

input
```
int((b*x^2+a)^(1/2)*(f*x^2+e)^2/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*((b*d*x^2+a*d)
*(c^2*f^2-2*c*d*e*f+d^2*e^2)/c/d^3*x/((x^2+c/d)*(b*d*x^2+a*d))^(1/2)+1/3*f
^2/d^2*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+(-(a*c*d*f^2-2*a*d^2*e*f-b*c^
2*f^2+2*b*c*d*e*f-b*d^2*e^2)/d^3+(c^2*f^2-2*c*d*e*f+d^2*e^2)/d^3*(a*d-b*c)
/c-a/d^2*(c^2*f^2-2*c*d*e*f+d^2*e^2)/c-1/3*a*c/d^2*f^2)/(-b/a)^(1/2)*(1+b*
x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*Ellipti
cF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-(1/d^2*f*(a*d*f-b*c*f+2*b*d*e)
-(c^2*f^2-2*c*d*e*f+d^2*e^2)/d^2*b/c-1/3*f^2/d^2*(2*a*d+2*b*c))*c/(-b/a)^(
1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/
2)/d*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a
)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 423, normalized size of antiderivative = 1.38

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{(c+dx^2)^{3/2}}\,dx = \frac{((3\,bcd^3e^2 - 12\,bc^2d^2ef + (8\,bc^3d - ac^2d^2)f^2)x^3 + (3\,bc^2d^2e^2 - 12\,bc^3def + (8}{}$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)^2/(d*x^2+c)^(3/2),x, algorithm="fricas
")
```

output
```
1/3*(((3*b*c*d^3*e^2 - 12*b*c^2*d^2*e*f + (8*b*c^3*d - a*c^2*d^2)*f^2)*x^3
 + (3*b*c^2*d^2*e^2 - 12*b*c^3*d*e*f + (8*b*c^4 - a*c^3*d)*f^2)*x)*sqrt(b*
d)*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - ((3*b*c*d^3*e^
2 - 6*(2*b*c^2*d^2 + a*d^4)*e*f + (8*b*c^3*d - a*c^2*d^2 + 4*a*c*d^3)*f^2)
*x^3 + (3*b*c^2*d^2*e^2 - 6*(2*b*c^3*d + a*c*d^3)*e*f + (8*b*c^4 - a*c^3*d
 + 4*a*c^2*d^2)*f^2)*x)*sqrt(b*d)*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)/
x), a*d/(b*c)) + (b*c*d^3*f^2*x^4 - 3*b*c*d^3*e^2 + 12*b*c^2*d^2*e*f - (8*
b*c^3*d - a*c^2*d^2)*f^2 + (6*b*c*d^3*e*f - (4*b*c^2*d^2 - a*c*d^3)*f^2)*x
^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(b*c*d^5*x^3 + b*c^2*d^4*x)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{(c+dx^2)^{3/2}}\,dx = \int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{(c+dx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(f*x**2+e)**2/(d*x**2+c)**(3/2),x)
```

output
```
Integral(sqrt(a + b*x**2)*(e + f*x**2)**2/(c + d*x**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{(c+dx^2)^{3/2}}\,dx = \int \frac{\sqrt{bx^2+a}(fx^2+e)^2}{(dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)^2/(d*x^2+c)^(3/2),x, algorithm="maxima
")
```

output
```
integrate(sqrt(b*x^2 + a)*(f*x^2 + e)^2/(d*x^2 + c)^(3/2), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{(c+dx^2)^{3/2}}\,dx = \int \frac{\sqrt{bx^2+a}(fx^2+e)^2}{(dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)^2/(d*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^2 + a)*(f*x^2 + e)^2/(d*x^2 + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{(c+dx^2)^{3/2}}\,dx = \int \frac{\sqrt{b\,x^2+a}\,(f\,x^2+e)^2}{(d\,x^2+c)^{3/2}}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(e + f*x^2)^2)/(c + d*x^2)^(3/2),x)
```

output
```
int(((a + b*x^2)^(1/2)*(e + f*x^2)^2)/(c + d*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{(c+dx^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((b*x^2+a)^(1/2)*(f*x^2+e)^2/(d*x^2+c)^(3/2),x)
```

output
```
( - 3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*c*f**2*x + 6*sqrt(c + d*x**2)*sq
rt(a + b*x**2)*a*d*e*f*x + 2*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b*c*f**2*x*
*3 + 3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b*d*e**2*x + 5*int((sqrt(c + d*x*
*2)*sqrt(a + b*x**2)*x**4)/(a*c**2 + 2*a*c*d*x**2 + a*d**2*x**4 + b*c**2*x
**2 + 2*b*c*d*x**4 + b*d**2*x**6),x)*a*b*c**2*d*f**2 - 6*int((sqrt(c + d*x
**2)*sqrt(a + b*x**2)*x**4)/(a*c**2 + 2*a*c*d*x**2 + a*d**2*x**4 + b*c**2*
x**2 + 2*b*c*d*x**4 + b*d**2*x**6),x)*a*b*c*d**2*e*f + 5*int((sqrt(c + d*x
**2)*sqrt(a + b*x**2)*x**4)/(a*c**2 + 2*a*c*d*x**2 + a*d**2*x**4 + b*c**2*
x**2 + 2*b*c*d*x**4 + b*d**2*x**6),x)*a*b*c*d**2*f**2*x**2 - 6*int((sqrt(c
 + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c**2 + 2*a*c*d*x**2 + a*d**2*x**4 + b
*c**2*x**2 + 2*b*c*d*x**4 + b*d**2*x**6),x)*a*b*d**3*e*f*x**2 - 8*int((sqr
t(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c**2 + 2*a*c*d*x**2 + a*d**2*x**4
 + b*c**2*x**2 + 2*b*c*d*x**4 + b*d**2*x**6),x)*b**2*c**3*f**2 + 12*int((sq
rt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c**2 + 2*a*c*d*x**2 + a*d**2*x**4
 + b*c**2*x**2 + 2*b*c*d*x**4 + b*d**2*x**6),x)*b**2*c**2*d*e*f - 8*int((s
qrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c**2 + 2*a*c*d*x**2 + a*d**2*x**
4 + b*c**2*x**2 + 2*b*c*d*x**4 + b*d**2*x**6),x)*b**2*c**2*d*f**2*x**2 - 3
*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c**2 + 2*a*c*d*x**2 + a*d
**2*x**4 + b*c**2*x**2 + 2*b*c*d*x**4 + b*d**2*x**6),x)*b**2*c*d**2*e**2 +
 12*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c**2 + 2*a*c*d*x**2...
```

3.140 $\int \frac{\sqrt{a+bx^2}\left(e+fx^2\right)^2}{x^2\left(c+dx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 429

$$\begin{aligned}\int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{x^2\,(c+dx^2)^{3/2}}\,dx &= \frac{(bde^2+af(3de+cf))\,x\sqrt{a+bx^2}}{acd\sqrt{c+dx^2}} \\ &+ \frac{3f(be+af)x^3\sqrt{a+bx^2}}{ac\sqrt{c+dx^2}} + \frac{f^2(3be+af)x^5\sqrt{a+bx^2}}{ace\sqrt{c+dx^2}} \\ &+ \frac{bf^3x^7\sqrt{a+bx^2}}{ace\sqrt{c+dx^2}} - \frac{(a+bx^2)^{3/2}\,(e+fx^2)^3}{acex\sqrt{c+dx^2}} \\ &- \frac{2(d^2e^2-cdef+c^2f^2)\,\sqrt{a+bx^2}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{c^{3/2}d^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} \\ &+ \frac{(bde^2+acf^2)\,\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{c}d^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}\end{aligned}$$

output
```
(b*d*e^2+a*f*(c*f+3*d*e))*x*(b*x^2+a)^(1/2)/a/c/d/(d*x^2+c)^(1/2)+3*f*(a*f
+b*e)*x^3*(b*x^2+a)^(1/2)/a/c/(d*x^2+c)^(1/2)+f^2*(a*f+3*b*e)*x^5*(b*x^2+a
)^(1/2)/a/c/e/(d*x^2+c)^(1/2)+b*f^3*x^7*(b*x^2+a)^(1/2)/a/c/e/(d*x^2+c)^(1
/2)-(b*x^2+a)^(3/2)*(f*x^2+e)^3/a/c/e/x/(d*x^2+c)^(1/2)-2*(c^2*f^2-c*d*e*f
+d^2*e^2)*(b*x^2+a)^(1/2)*EllipticE(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),(1
-b*c/a/d)^(1/2))/c^(3/2)/d^(3/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)
^(1/2)+(a*c*f^2+b*d*e^2)*(b*x^2+a)^(1/2)*InverseJacobiAM(arctan(d^(1/2)*x/
c^(1/2)),(1-b*c/a/d)^(1/2))/a/c^(1/2)/d^(3/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1
/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.40 (sec) , antiderivative size = 270, normalized size of antiderivative = 0.63

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{x^2\,(c+dx^2)^{3/2}}\,dx = \frac{-\sqrt{\frac{b}{a}}d(a+bx^2)\,(2d^2e^2x^2+c^2f^2x^2+cde(e-2fx^2))-2ibc(d^2e^2-cdef+c^2}{}$$

input
```
Integrate[(Sqrt[a + b*x^2]*(e + f*x^2)^2)/(x^2*(c + d*x^2)^(3/2)),x]
```

output
```
(-(Sqrt[b/a]*d*(a + b*x^2)*(2*d^2*e^2*x^2 + c^2*f^2*x^2 + c*d*e*(e - 2*f*x
^2))) - (2*I)*b*c*(d^2*e^2 - c*d*e*f + c^2*f^2)*x*Sqrt[1 + (b*x^2)/a]*Sqrt
[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*c*(a*c*
d*f^2 - b*(d^2*e^2 - 2*c*d*e*f + 2*c^2*f^2))*x*Sqrt[1 + (b*x^2)/a]*Sqrt[1
+ (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(Sqrt[b/a]*c^
2*d^2*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (warning: unable to verify)

Time = 0.98 (sec) , antiderivative size = 576, normalized size of antiderivative = 1.34, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.457, Rules used = {448, 401, 25, 406, 320, 388, 313, 439, 25, 445, 25, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\left(e+fx^2\right)^2}{x^2\left(c+dx^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 448}$$

$$\frac{f\int \frac{\sqrt{bx^2+a}\left(fx^2+e\right)}{\left(dx^2+c\right)^{3/2}}dx}{e^2}+e\int \frac{\sqrt{bx^2+a}\left(fx^2+e\right)}{x^2\left(dx^2+c\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 401}$$

$$\frac{f\left(\frac{x\sqrt{a+bx^2}(de-cf)}{cd\sqrt{c+dx^2}}-\frac{\int -\frac{acf-b(de-2cf)x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{cd}\right)}{e^2}+e\int \frac{\sqrt{bx^2+a}\left(fx^2+e\right)}{x^2\left(dx^2+c\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 25}$$

$$\frac{f\left(\frac{\int \frac{acf-b(de-2cf)x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{cd}+\frac{x\sqrt{a+bx^2}(de-cf)}{cd\sqrt{c+dx^2}}\right)}{e^2}+e\int \frac{\sqrt{bx^2+a}\left(fx^2+e\right)}{x^2\left(dx^2+c\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 406}$$

$$\frac{f\left(\frac{acf\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx-b(de-2cf)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{cd}+\frac{x\sqrt{a+bx^2}(de-cf)}{cd\sqrt{c+dx^2}}\right)}{e^2}+ e\int \frac{\sqrt{bx^2+a}\left(fx^2+e\right)}{x^2\left(dx^2+c\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 320}$$

$$\frac{f\left(\frac{\frac{c^{3/2}f\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-b(de-2cf)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{cd}+\frac{x\sqrt{a+bx^2}(de-cf)}{cd\sqrt{c+dx^2}}\right)}{e^2}+e\int\frac{\sqrt{bx^2+a}\left(fx^2+e\right)}{x^2\left(dx^2+c\right)^{3/2}}dx$$

$$\Big\downarrow \; 388$$

$$\frac{f\left(\frac{\frac{c^{3/2}f\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-b(de-2cf)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)}{cd}+\frac{x\sqrt{a+bx^2}(de-cf)}{cd\sqrt{c+dx^2}}\right)}{e^2}+e\int\frac{\sqrt{bx^2+a}\left(fx^2+e\right)}{x^2\left(dx^2+c\right)^{3/2}}dx$$

$$\Big\downarrow \; 313$$

$$e\int\frac{\sqrt{bx^2+a}\left(fx^2+e\right)}{x^2\left(dx^2+c\right)^{3/2}}dx+\frac{f\left(\frac{\frac{c^{3/2}f\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-b(de-2cf)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{cd}+\frac{x\sqrt{a+bx^2}(de-cf)}{cd\sqrt{c+dx^2}}\right)}{e^2}$$

$$\Big\downarrow \; 439$$

$$\frac{e\left(\frac{\sqrt{a+bx^2}(de-cf)}{cdx\sqrt{c+dx^2}}-\frac{\int -\frac{bdex^2+a(2de-cf)}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{cd}\right)+f\left(\frac{\frac{c^{3/2}f\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-b(de-2cf)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{cd}+\frac{x\sqrt{a+bx^2}(de-cf)}{cd\sqrt{c+dx^2}}\right)}{e^2}$$

$\downarrow$ 25

$$\frac{e\left(\frac{\int \frac{bdex^2+a(2de-cf)}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{cd}+\frac{\sqrt{a+bx^2}(de-cf)}{cdx\sqrt{c+dx^2}}\right)+f\left(\frac{\frac{c^{3/2}f\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-b(de-2cf)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{cd}+\frac{x\sqrt{a+bx^2}(de-cf)}{cd\sqrt{c+dx^2}}\right)}{e^2}$$

$\downarrow$ 445

$$\frac{e\left(\frac{-\frac{\int -\frac{abd\left((2de-cf)x^2+ce\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(2de-cf)}{cx}}{cd}+\frac{\sqrt{a+bx^2}(de-cf)}{cdx\sqrt{c+dx^2}}\right)+f\left(\frac{\frac{c^{3/2}f\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-b(de-2cf)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{cd}+\frac{x\sqrt{a+bx^2}(de-cf)}{cd\sqrt{c+dx^2}}\right)}{e^2}$$

$\downarrow$ 25

$$\frac{e\left(\frac{\frac{\int \frac{abd\left((2de-cf)x^2+ce\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(2de-cf)}{cx}}{cd}+\frac{\sqrt{a+bx^2}(de-cf)}{cdx\sqrt{c+dx^2}}\right)+f\left(\frac{\frac{c^{3/2}f\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-b(de-2cf)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{cd}+\frac{x\sqrt{a+bx^2}(de-cf)}{cd\sqrt{c+dx^2}}\right)}{e^2}$$

$$\Big\downarrow \ 27$$

$$\frac{e\left(\frac{\frac{bd\int \frac{(2de-cf)x^2+ce}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(2de-cf)}{cx}}{cd}+\frac{\sqrt{a+bx^2}(de-cf)}{cdx\sqrt{c+dx^2}}\right)+f\left(\frac{\frac{c^{3/2}f\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-b(de-2cf)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{cd}+\frac{x\sqrt{a+bx^2}(de-cf)}{cd\sqrt{c+dx^2}}\right)}{e^2}$$

$$\Big\downarrow \ 406$$

$$\frac{e\left(\frac{\frac{bd\left((2de-cf)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+ce\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(2de-cf)}{cx}}{cd}+\frac{\sqrt{a+bx^2}(de-cf)}{cdx\sqrt{c+dx^2}}\right)+f\left(\frac{\frac{c^{3/2}f\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-b(de-2cf)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{cd}+\frac{x\sqrt{a+bx^2}(de-cf)}{cd\sqrt{c+dx^2}}\right)}{e^2}$$

$$\Big\downarrow \ 320$$

$$\frac{e\left(\frac{\frac{bd\left((2de-cf)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(2de-cf)}{cx}}{cd}+\frac{\sqrt{a+bx^2}(de-c}{cdx\sqrt{c+dx^2}}\right)+f\left(\frac{\frac{c^{3/2}f\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-b(de-2cf)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{cd}+\frac{x\sqrt{a+bx^2}(de-cf)}{cd\sqrt{c+dx^2}}\right)}{e^2}$$

$$\Big\downarrow \; 388$$

$$\frac{e\left(\frac{\frac{bd\left((2de-cf)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{c^{3/2}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(2de-cf)}{cx}}{cd}+\frac{\sqrt{a+b}{cdx\sqrt{}}\right)+f\left(\frac{\frac{c^{3/2}f\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-b(de-2cf)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{cd}+\frac{x\sqrt{a+bx^2}(de-cf)}{cd\sqrt{c+dx^2}}\right)}{e^2}$$

$$\Big\downarrow \; 313$$

$$\frac{f\left(\frac{\frac{c^{3/2}f\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-b(de-2cf)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{cd}+\frac{x\sqrt{a+bx^2}(de-cf)}{cd\sqrt{c+dx^2}}\right)}{e^2}+$$

$$e\left(\frac{\frac{bd\left(\frac{c^{3/2}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+(2de-cf)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)\right)}{c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(2de-c}{cx}}{cd}\right.$$

input
```
Int[(Sqrt[a + b*x^2]*(e + f*x^2)^2)/(x^2*(c + d*x^2)^(3/2)),x]
```

output
```
(f*(((d*e - c*f)*x*Sqrt[a + b*x^2])/(c*d*Sqrt[c + d*x^2]) + (-(b*(d*e - 2*
c*f)*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*E
llipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c
*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))) + (c^(3/2)*f*Sqrt[a + b*
x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqr
t[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))/(c*d)))/e^2 + e*(((d*
e - c*f)*Sqrt[a + b*x^2])/(c*d*x*Sqrt[c + d*x^2]) + (-(((2*d*e - c*f)*Sqrt
[a + b*x^2]*Sqrt[c + d*x^2])/(c*x)) + (b*d*((2*d*e - c*f)*((x*Sqrt[a + b*x
^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt
[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c +
 d*x^2))]*Sqrt[c + d*x^2])) + (c^(3/2)*e*Sqrt[a + b*x^2]*EllipticF[ArcTan[
(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a
*(c + d*x^2))]*Sqrt[c + d*x^2])))/c)/(c*d))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 439
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
 + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*b*g*(p + 1))), x] + Simp[1/(2*a*b*(p
+ 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1)*Simp[c*(2*b*e*(
p + 1) + (b*e - a*f)*(m + 1)) + d*(2*b*e*(p + 1) + (b*e - a*f)*(m + 2*q + 1
))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && LtQ[p, -1] && G
tQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e - a*f])
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 448
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[e   Int[(g*x)^m*(a + b*x
^2)^p*(c + d*x^2)^q*(e + f*x^2)^(r - 1), x], x] + Simp[f/e^2   Int[(g*x)^(m
 + 2)*(a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^(r - 1), x], x] /; FreeQ[{a,
b, c, d, e, f, g, m, p, q}, x] && IGtQ[r, 0]
```

Maple [A] (verified)

Time = 9.52 (sec) , antiderivative size = 478, normalized size of antiderivative = 1.11

method	result
elliptic	$\frac{\sqrt{(b x^2+a)(x^2 d+c)}\left(-\frac{e^2\sqrt{bd x^4+ad x^2+x^2 bc+ac}}{c^2 x}-\frac{(bd x^2+ad)(c^2f^2-2cdef+d^2e^2)x}{c^2d^2\sqrt{(x^2+\frac{c}{d})(bd x^2+ad)}}+\frac{\left(\frac{f(adf-bcf+2bde)}{d^2}-\frac{(c^2f^2-2cdef+d^2e^2)(ad-b}{d^2c^2}\right.}{}\right.}{}$
default	$\frac{\sqrt{bx^2+a}\sqrt{x^2d+c}\left(-\sqrt{-\frac{b}{a}}\,bc^2df^2x^4+2\sqrt{-\frac{b}{a}}\,bcd^2efx^4-2\sqrt{-\frac{b}{a}}\,bd^3e^2x^4+\sqrt{\frac{bx^2+a}{a}}\sqrt{\frac{x^2d+c}{c}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)ac^2df^2\right.}{}$
risch	$-\frac{e^2\sqrt{bx^2+a}\sqrt{x^2d+c}}{c^2x}+\frac{\left(-\frac{b(c^2f^2+d^2e^2)c\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\left(\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)-\mathrm{EllipticE}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)\right)}{d^2\sqrt{-\frac{b}{a}}\sqrt{bdx^4+adx^2+x^2bc+ac}}-\right.}{}$

input
```
int((b*x^2+a)^(1/2)*(f*x^2+e)^2/x^2/(d*x^2+c)^(3/2),x,method=_RETURNVERBOS
E)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-e^2/c^2*(b*d
*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/x-(b*d*x^2+a*d)*(c^2*f^2-2*c*d*e*f+d^2*e^2
)/c^2/d^2*x/((x^2+c/d)*(b*d*x^2+a*d))^(1/2)+(1/d^2*f*(a*d*f-b*c*f+2*b*d*e)
-(c^2*f^2-2*c*d*e*f+d^2*e^2)/d^2*(a*d-b*c)/c^2+a/d*(c^2*f^2-2*c*d*e*f+d^2*
e^2)/c^2)/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^
2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-(f
^2*b/d+d*b/c^2*e^2+(c^2*f^2-2*c*d*e*f+d^2*e^2)/d*b/c^2)*c/(-b/a)^(1/2)*(1+
b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*(El
lipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),
(-1+(a*d+b*c)/c/b)^(1/2))))
```

Fricas [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{x^2\,(c+dx^2)^{3/2}}\,dx=\int \frac{\sqrt{bx^2+a}(fx^2+e)^2}{(dx^2+c)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)^2/x^2/(d*x^2+c)^(3/2),x, algorithm="fr
icas")
```

output
```
integral((f^2*x^4 + 2*e*f*x^2 + e^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/(d^2*
x^6 + 2*c*d*x^4 + c^2*x^2), x)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{x^2\left(c+dx^2\right)^{3/2}}\,dx=\int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{x^2\left(c+dx^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(f*x**2+e)**2/x**2/(d*x**2+c)**(3/2),x)
```

output
```
Integral(sqrt(a + b*x**2)*(e + f*x**2)**2/(x**2*(c + d*x**2)**(3/2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{x^2\left(c+dx^2\right)^{3/2}}\,dx=\int \frac{\sqrt{bx^2+a}(fx^2+e)^2}{(dx^2+c)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)^2/x^2/(d*x^2+c)^(3/2),x, algorithm="ma
xima")
```

output
```
integrate(sqrt(b*x^2 + a)*(f*x^2 + e)^2/((d*x^2 + c)^(3/2)*x^2), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{x^2\left(c+dx^2\right)^{3/2}}\,dx=\int \frac{\sqrt{bx^2+a}(fx^2+e)^2}{(dx^2+c)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)^2/x^2/(d*x^2+c)^(3/2),x, algorithm="gi
ac")
```

output
```
integrate(sqrt(b*x^2 + a)*(f*x^2 + e)^2/((d*x^2 + c)^(3/2)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{x^2\,(c+dx^2)^{3/2}}\,dx = \int \frac{\sqrt{b\,x^2+a}\,(f\,x^2+e)^2}{x^2\,(d\,x^2+c)^{3/2}}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(e + f*x^2)^2)/(x^2*(c + d*x^2)^(3/2)),x)
```

output
```
int(((a + b*x^2)^(1/2)*(e + f*x^2)^2)/(x^2*(c + d*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{x^2\,(c+dx^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((b*x^2+a)^(1/2)*(f*x^2+e)^2/x^2/(d*x^2+c)^(3/2),x)
```

output

```
(sqrt(c + d*x**2)*sqrt(a + b*x**2)*c*f**2*x**2 - sqrt(c + d*x**2)*sqrt(a +
 b*x**2)*d*e**2 + int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c**2 + 2
*a*c*d*x**2 + a*d**2*x**4 + b*c**2*x**2 + 2*b*c*d*x**4 + b*d**2*x**6),x)*a
*c**2*d*f**2*x + int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c**2 + 2*
a*c*d*x**2 + a*d**2*x**4 + b*c**2*x**2 + 2*b*c*d*x**4 + b*d**2*x**6),x)*a*
c*d**2*f**2*x**3 - 2*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c**2
+ 2*a*c*d*x**2 + a*d**2*x**4 + b*c**2*x**2 + 2*b*c*d*x**4 + b*d**2*x**6),x
)*b*c**3*f**2*x + 2*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c**2 +
 2*a*c*d*x**2 + a*d**2*x**4 + b*c**2*x**2 + 2*b*c*d*x**4 + b*d**2*x**6),x)
*b*c**2*d*e*f*x - 2*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c**2 +
 2*a*c*d*x**2 + a*d**2*x**4 + b*c**2*x**2 + 2*b*c*d*x**4 + b*d**2*x**6),x)
*b*c**2*d*f**2*x**3 - int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c**2
 + 2*a*c*d*x**2 + a*d**2*x**4 + b*c**2*x**2 + 2*b*c*d*x**4 + b*d**2*x**6),
x)*b*c*d**2*e**2*x + 2*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c**
2 + 2*a*c*d*x**2 + a*d**2*x**4 + b*c**2*x**2 + 2*b*c*d*x**4 + b*d**2*x**6)
,x)*b*c*d**2*e*f*x**3 - int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c*
*2 + 2*a*c*d*x**2 + a*d**2*x**4 + b*c**2*x**2 + 2*b*c*d*x**4 + b*d**2*x**6
),x)*b*d**3*e**2*x**3 - int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c**2 +
2*a*c*d*x**2 + a*d**2*x**4 + b*c**2*x**2 + 2*b*c*d*x**4 + b*d**2*x**6),x)*
a*c**3*f**2*x + 2*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c**2 + 2*a...
```

3.141 $\int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{x^4(c+dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 569

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{x^4(c+dx^2)^{3/2}}\,dx = -\frac{2(be(2de-3cf)+6af(de-cf))x\sqrt{a+bx^2}}{3ac^2\sqrt{c+dx^2}}$$
$$-\frac{2f(af(6de-5cf)+6be(de-cf))x^3\sqrt{a+bx^2}}{3ac^2e\sqrt{c+dx^2}}$$
$$-\frac{f^2(2be(6de-5cf)+af(4de-3cf))x^5\sqrt{a+bx^2}}{3ac^2e^2\sqrt{c+dx^2}}$$
$$-\frac{bf^3(4de-3cf)x^7\sqrt{a+bx^2}}{3ac^2e^2\sqrt{c+dx^2}}-\frac{(a+bx^2)^{3/2}(e+fx^2)^3}{3acex^3\sqrt{c+dx^2}}$$
$$+\frac{(4de-3cf)(a+bx^2)^{3/2}(e+fx^2)^3}{3ac^2e^2x\sqrt{c+dx^2}}$$
$$-\frac{(bcde^2-a(8d^2e^2-12cdef+3c^2f^2))\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3ac^{5/2}\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$
$$-\frac{2be(2de-3cf)\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3ac^{3/2}\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output
```
-2/3*(b*e*(-3*c*f+2*d*e)+6*a*f*(-c*f+d*e))*x*(b*x^2+a)^(1/2)/a/c^2/(d*x^2+
c)^(1/2)-2/3*f*(a*f*(-5*c*f+6*d*e)+6*b*e*(-c*f+d*e))*x^3*(b*x^2+a)^(1/2)/a
/c^2/e/(d*x^2+c)^(1/2)-1/3*f^2*(2*b*e*(-5*c*f+6*d*e)+a*f*(-3*c*f+4*d*e))*x
^5*(b*x^2+a)^(1/2)/a/c^2/e^2/(d*x^2+c)^(1/2)-1/3*b*f^3*(-3*c*f+4*d*e)*x^7*
(b*x^2+a)^(1/2)/a/c^2/e^2/(d*x^2+c)^(1/2)-1/3*(b*x^2+a)^(3/2)*(f*x^2+e)^3/
a/c/e/x^3/(d*x^2+c)^(1/2)+1/3*(-3*c*f+4*d*e)*(b*x^2+a)^(3/2)*(f*x^2+e)^3/a
/c^2/e^2/x/(d*x^2+c)^(1/2)-1/3*(b*c*d*e^2-a*(3*c^2*f^2-12*c*d*e*f+8*d^2*e^
2))*(b*x^2+a)^(1/2)*EllipticE(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a
/d)^(1/2))/a/c^(5/2)/d^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/
2)-2/3*b*e*(-3*c*f+2*d*e)*(b*x^2+a)^(1/2)*InverseJacobiAM(arctan(d^(1/2)*x
/c^(1/2)),(1-b*c/a/d)^(1/2))/a/c^(3/2)/d^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(
1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.21 (sec) , antiderivative size = 331, normalized size of antiderivative = 0.58

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{x^4\left(c+dx^2\right)^{3/2}}\,dx = \frac{\sqrt{\frac{b}{a}}\left(-\sqrt{\frac{b}{a}}d(a+bx^2)\left(bce^2x^2(c+dx^2)+a(-8d^2e^2x^4-4cdex^2(e-3fx^2)+\right.\right.}{}$$

input
```
Integrate[(Sqrt[a + b*x^2]*(e + f*x^2)^2)/(x^4*(c + d*x^2)^(3/2)),x]
```

output
```
(Sqrt[b/a]*(-(Sqrt[b/a]*d*(a + b*x^2)*(b*c*e^2*x^2*(c + d*x^2) + a*(-8*d^2
*e^2*x^4 - 4*c*d*e*x^2*(e - 3*f*x^2) + c^2*(e^2 + 6*e*f*x^2 - 3*f^2*x^4)))
) + I*b*c*(-(b*c*d*e^2) + a*(8*d^2*e^2 - 12*c*d*e*f + 3*c^2*f^2))*x^3*Sqrt
[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d
)/(b*c)] - I*b*c*(-(b*c*d*e^2) + a*(4*d^2*e^2 - 6*c*d*e*f + 3*c^2*f^2))*x^
3*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x]
, (a*d)/(b*c)]))/(3*b*c^3*d*x^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (warning: unable to verify)

Time = 1.28 (sec) , antiderivative size = 701, normalized size of antiderivative = 1.23, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.457$, Rules used = {448, 439, 25, 445, 25, 27, 406, 320, 388, 313, 445, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\left(e+fx^2\right)^2}{x^4\left(c+dx^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{448}$$

$$\frac{f\int \frac{\sqrt{bx^2+a}\left(fx^2+e\right)}{x^2\left(dx^2+c\right)^{3/2}}dx}{e^2}+e\int \frac{\sqrt{bx^2+a}\left(fx^2+e\right)}{x^4\left(dx^2+c\right)^{3/2}}dx$$

$$\Big\downarrow \text{439}$$

$$\frac{f\left(\frac{\sqrt{a+bx^2}(de-cf)}{cdx\sqrt{c+dx^2}}-\frac{\int -\frac{bdex^2+a(2de-cf)}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{cd}\right)}{e^2}+e\left(\frac{\sqrt{a+bx^2}(de-cf)}{cdx^3\sqrt{c+dx^2}}-\frac{\int -\frac{b(3de-2cf)x^2+a(4de-3cf)}{x^4\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{cd}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{f\left(\frac{\int \frac{bdex^2+a(2de-cf)}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{cd}+\frac{\sqrt{a+bx^2}(de-cf)}{cdx\sqrt{c+dx^2}}\right)}{e^2}+e\left(\frac{\int \frac{b(3de-2cf)x^2+a(4de-3cf)}{x^4\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{cd}+\frac{\sqrt{a+bx^2}(de-cf)}{cdx^3\sqrt{c+dx^2}}\right)$$

$$\Big\downarrow \text{445}$$

$$\frac{f\left(\frac{-\frac{\int -\frac{abd\left((2de-cf)x^2+ce\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(2de-cf)}{cx}}{cd}+\frac{\sqrt{a+bx^2}(de-cf)}{cdx\sqrt{c+dx^2}}\right)}{e^2}+e\left(\frac{-\frac{\int -\frac{ad\left(-b(4de-3cf)x^2+bce-8ade+6acf\right)}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(4de-3cf)}{3cx^3}}{cd}+\frac{\sqrt{a+bx^2}(de-cf)}{cdx^3\sqrt{c+dx^2}}\right)$$

$$\Big\downarrow \text{ 25}$$

$$\frac{f\left(\frac{\frac{\int \frac{abd\left((2de-cf)x^2+ce\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(2de-cf)}{cx}}{cd}+\frac{\sqrt{a+bx^2}(de-cf)}{cdx\sqrt{c+dx^2}}\right)}{e^2}+e\left(\frac{\frac{\int \frac{ad\left(-b(4de-3cf)x^2+bce-8ade+6acf\right)}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(4de-3cf)}{3cx^3}}{cd}+\frac{\sqrt{a+bx^2}(de-cf)}{cdx^3\sqrt{c+dx^2}}\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{f\left(\frac{\frac{bd\int \frac{(2de-cf)x^2+ce}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(2de-cf)}{cx}}{cd}+\frac{\sqrt{a+bx^2}(de-cf)}{cdx\sqrt{c+dx^2}}\right)}{e^2}+e\left(\frac{\frac{d\int \frac{-b(4de-3cf)x^2+bce-8ade+6acf}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(4de-3cf)}{3cx^3}}{cd}+\frac{\sqrt{a+bx^2}(de-cf)}{cdx^3\sqrt{c+dx^2}}\right)$$

$$\Big\downarrow \text{ 406}$$

$$\frac{f\left(\frac{\frac{bd\left((2de-cf)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+ce\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(2de-cf)}{cx}}{cd}+\frac{\sqrt{a+bx^2}(de-cf)}{cdx\sqrt{c+dx^2}}\right)}{e^2}+e\left(\frac{\frac{d\int \frac{-b(4de-3cf)x^2+bce-8ade+6acf}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(4de-3cf)}{3cx^3}}{cd}+\frac{\sqrt{a+bx^2}(de-cf)}{cdx^3\sqrt{c+dx^2}}\right)$$

$$\Big\downarrow \text{ 320}$$

$$\frac{f\left(\frac{\frac{bd\left((2de-cf)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(2de-cf)}{cx}}{cd}+\frac{\sqrt{a+bx^2}(de-cf)}{cdx\sqrt{c+dx^2}}\right)}{e^2}+$$

$$e\left(\frac{\frac{d\int\frac{-b(4de-3cf)x^2+bce-8ade+6acf}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(4de-3cf)}{3cx^3}}{cd}+\frac{\sqrt{a+bx^2}(de-cf)}{cdx^3\sqrt{c+dx^2}}\right)$$

$$\Big\downarrow \text{388}$$

$$\frac{f\left(\frac{\frac{bd\left((2de-cf)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}}dx}{b}\right)+\frac{c^{3/2}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(2de-cf)}{cx}}{cd}+\frac{\sqrt{a+bx^2}(de-cf)}{cdx\sqrt{c+dx^2}}\right)}{e^2}$$

$$e\left(\frac{\frac{d\int\frac{-b(4de-3cf)x^2+bce-8ade+6acf}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(4de-3cf)}{3cx^3}}{cd}+\frac{\sqrt{a+bx^2}(de-cf)}{cdx^3\sqrt{c+dx^2}}\right)$$

$$\Big\downarrow \text{313}$$

$$e\left(\frac{\frac{d\int\frac{-b(4de-3cf)x^2+bce-8ade+6acf}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(4de-3cf)}{3cx^3}}{cd}+\frac{\sqrt{a+bx^2}(de-cf)}{cdx^3\sqrt{c+dx^2}}\right)+$$

$$\frac{f\left(\frac{\frac{bd\left(\frac{c^{3/2}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+(2de-cf)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)\right)}{c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(2de-cf)}{cx}}{cd}+\right.}{e^2}$$

$\Big\downarrow$ 445

$$\frac{e\left(\frac{\frac{d\left(-\frac{\int \frac{b\left(ac(4de-3cf)-d(bce-8ade+6acf)x^2\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(6acf-8ade+bce)}{acx}\right)}{3c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(4de-3cf)}{3cx^3}}{cd}+\frac{\sqrt{a+bx^2}(de-c}{cdx^3\sqrt{c+dx^2}}\right.}{}$$

$$\frac{f\left(\frac{\frac{bd\left(\frac{c^{3/2}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+(2de-cf)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)\right)}{c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(2de-cf)}{cx}}{cd}+\right.}{e^2}$$

$\Big\downarrow$ 27

$$\frac{e\left(\frac{\frac{d\left(-\frac{b\int \frac{ac(4de-3cf)-d(bce-8ade+6acf)x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(6acf-8ade+bce)}{acx}\right)}{3c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(4de-3cf)}{3cx^3}}{cd}+\frac{\sqrt{a+bx^2}(de-cf}{cdx^3\sqrt{c+dx^2}}\right.}{}$$

$$\frac{f\left(\frac{\frac{bd\left(\frac{c^{3/2}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+(2de-cf)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)\right)}{c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(2de-cf)}{cx}}{cd}+\right.}{e^2}$$

$\Big\downarrow$ 406

$$\frac{e\left(\frac{\frac{d\left(-\frac{b\left(ac(4de-3cf)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx-d(6acf-8ade+bce)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(6acf-8ade+bce)}{acx}\right)}{3c}}{cd}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(4\cdots}{3cx^3}\cdots\right)+f\left(\frac{\frac{bd\left(\frac{c^{3/2}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+(2de-cf)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)\right)}{c}}{cd}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(2de-cf)}{cx}+\cdots\right)}{e^2}$$

$\Big\downarrow$ 320

$$\frac{e\left(\frac{\frac{d\left(-\frac{b\left(\frac{c^{3/2}\sqrt{a+bx^2}(4de-3cf)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-d(6acf-8ade+bce)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(6acf-8ade+bce)}{acx}\right)}{3c}}{cd}-\cdots\right)+f\left(\frac{\frac{bd\left(\frac{c^{3/2}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+(2de-cf)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)\right)}{c}}{cd}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(2de-cf)}{cx}+\cdots\right)}{e^2}$$

$\Big\downarrow$ 388

$$e\left(\frac{d\left(-\frac{b\left(\frac{c^{3/2}\sqrt{a+bx^2}(4de-3cf)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-d(6acf-8ade+bce)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)\right)}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(6acf-8a}{acx}\right.}{3c}}{cd}\right.$$

$$\left.f\left(\frac{\frac{bd\left(\frac{c^{3/2}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+(2de-cf)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)\right)}{c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(2de-cf)}{cx}}{cd}+\right.\right.}{e^2}$$

$\Big\downarrow$ 313

$$\frac{f\left(\frac{\frac{bd\left(\frac{c^{3/2}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+(2de-cf)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)\right)}{c}}{cd}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(2de-cf)}{cx}+\right.}{e^2}$$

$$e\left(\frac{d\left(-\frac{b\left(\frac{c^{3/2}\sqrt{a+bx^2}(4de-3cf)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-d(6acf-8ade+bce)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)\right)}{ac}-\frac{\sqrt{a+bx^2}}{\cdots}\right.}{3c}}{cd}\right.$$

input
```
Int[(Sqrt[a + b*x^2]*(e + f*x^2)^2)/(x^4*(c + d*x^2)^(3/2)),x]
```

output

```
(f*(((d*e - c*f)*Sqrt[a + b*x^2])/(c*d*x*Sqrt[c + d*x^2]) + (-(((2*d*e - c
*f)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(c*x)) + (b*d*((2*d*e - c*f)*((x*Sqrt
[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcT
an[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))
/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (c^(3/2)*e*Sqrt[a + b*x^2]*EllipticF
[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*Sqrt[(c*(a + b*
x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])))/c)/(c*d)))/e^2 + e*(((d*e - c*f)
*Sqrt[a + b*x^2])/(c*d*x^3*Sqrt[c + d*x^2]) + (-1/3*((4*d*e - 3*c*f)*Sqrt[
a + b*x^2]*Sqrt[c + d*x^2])/(c*x^3) + (d*(-(((b*c*e - 8*a*d*e + 6*a*c*f)*S
qrt[a + b*x^2]*Sqrt[c + d*x^2])/(a*c*x)) - (b*(-(d*(b*c*e - 8*a*d*e + 6*a*
c*f)*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*E
llipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c
*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))) + (c^(3/2)*(4*d*e - 3*c*
f)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)]
)/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])))/(a*c))
)/(3*c))/(c*d))
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 439
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
 + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*b*g*(p + 1))), x] + Simp[1/(2*a*b*(p
+ 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1)*Simp[c*(2*b*e*(
p + 1) + (b*e - a*f)*(m + 1)) + d*(2*b*e*(p + 1) + (b*e - a*f)*(m + 2*q + 1
))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && LtQ[p, -1] && G
tQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e - a*f])
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 448
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[e   Int[(g*x)^m*(a + b*x
^2)^p*(c + d*x^2)^q*(e + f*x^2)^(r - 1), x], x] + Simp[f/e^2   Int[(g*x)^(m
 + 2)*(a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^(r - 1), x], x] /; FreeQ[{a,
b, c, d, e, f, g, m, p, q}, x] && IGtQ[r, 0]
```

Maple [A] (verified)

Time = 10.40 (sec) , antiderivative size = 530, normalized size of antiderivative = 0.93

method	result
elliptic	$\frac{\sqrt{(b\,x^2+a)(x^2d+c)}\left(-\frac{e^2\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{3c^2x^3}-\frac{e(6acf-5ade+bce)\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{3a\,c^3x}+\frac{\left(bd\,x^2+ad\right)\left(c^2f^2-2cdef+d^2e^2\right)x}{c^3d\sqrt{\left(x^2+\frac{c}{d}\right)\left(bd\,x^2+ad\right)}}+\right.}{}$
risch	$-\frac{\sqrt{b\,x^2+a}\,\sqrt{x^2d+c}\,e\left(6acf\,x^2-5ade\,x^2+bce\,x^2+ace\right)}{3c^3x^3a}+\frac{\left(-\frac{be(6acf-5ade+bce)c\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\left(\text{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right.\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}\right.}{}$
default	$\frac{\sqrt{b\,x^2+a}\,\sqrt{x^2d+c}\left(3\sqrt{-\frac{b}{a}}\,ab\,c^2d\,f^2x^6-12\sqrt{-\frac{b}{a}}\,abc\,d^2ef\,x^6+8\sqrt{-\frac{b}{a}}\,ab\,d^3e^2x^6-\sqrt{-\frac{b}{a}}\,b^2c\,d^2e^2x^6+3\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{x^2d+c}{c}}\,\text{EllipticF}\right.}{}$

input
```
int((b*x^2+a)^(1/2)*(f*x^2+e)^2/x^4/(d*x^2+c)^(3/2),x,method=_RETURNVERBOS
E)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-1/3*e^2/c^2*
(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/x^3-1/3/a/c^3*e*(6*a*c*f-5*a*d*e+b*c*e
)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/x+(b*d*x^2+a*d)*(c^2*f^2-2*c*d*e*f+d
^2*e^2)/c^3/d*x/((x^2+c/d)*(b*d*x^2+a*d))^(1/2)+(f^2*b/d-1/3*d*b/c^2*e^2+(
c^2*f^2-2*c*d*e*f+d^2*e^2)/d*(a*d-b*c)/c^3-a*(c^2*f^2-2*c*d*e*f+d^2*e^2)/c
^3)/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*
x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-(1/3*e*b
*d*(6*a*c*f-5*a*d*e+b*c*e)/a/c^3-(c^2*f^2-2*c*d*e*f+d^2*e^2)*b/c^3)*c/(-b/
a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)
^(1/2)/d*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(
-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 429, normalized size of antiderivative = 0.75

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{x^4\,(c+dx^2)^{3/2}}\,dx = \frac{((12\,abcd^2ef-3\,abc^2df^2+(b^2cd^2-8\,abd^3)e^2)x^5+(12\,abc^2def-3\,abc^3f^2+$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)^2/x^4/(d*x^2+c)^(3/2),x, algorithm="fr
icas")
```

output
```
1/3*(((12*a*b*c*d^2*e*f - 3*a*b*c^2*d*f^2 + (b^2*c*d^2 - 8*a*b*d^3)*e^2)*x
^5 + (12*a*b*c^2*d*e*f - 3*a*b*c^3*f^2 + (b^2*c^2*d - 8*a*b*c*d^2)*e^2)*x^
3)*sqrt(a*c)*sqrt(-b/a)*elliptic_e(arcsin(x*sqrt(-b/a)), a*d/(b*c)) + ((3*
a*b*c^2*d*f^2 - 6*(a^2 + 2*a*b)*c*d^2*e*f - (b^2*c*d^2 - 4*(a^2 + 2*a*b)*d
^3)*e^2)*x^5 + (3*a*b*c^3*f^2 - 6*(a^2 + 2*a*b)*c^2*d*e*f - (b^2*c^2*d - 4
*(a^2 + 2*a*b)*c*d^2)*e^2)*x^3)*sqrt(a*c)*sqrt(-b/a)*elliptic_f(arcsin(x*s
qrt(-b/a)), a*d/(b*c)) - (a^2*c^2*d*e^2 + (12*a^2*c*d^2*e*f - 3*a^2*c^2*d*
f^2 + (a*b*c*d^2 - 8*a^2*d^3)*e^2)*x^4 + (6*a^2*c^2*d*e*f + (a*b*c^2*d - 4
*a^2*c*d^2)*e^2)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(a^2*c^3*d^2*x^5 +
a^2*c^4*d*x^3)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{x^4\,(c+dx^2)^{3/2}}\,dx = \int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{x^4\,(c+dx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(f*x**2+e)**2/x**4/(d*x**2+c)**(3/2),x)
```

output
```
Integral(sqrt(a + b*x**2)*(e + f*x**2)**2/(x**4*(c + d*x**2)**(3/2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{x^4(c+dx^2)^{3/2}}\,dx = \int \frac{\sqrt{bx^2+a}(fx^2+e)^2}{(dx^2+c)^{\frac{3}{2}}x^4}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)^2/x^4/(d*x^2+c)^(3/2),x, algorithm="ma
xima")
```

output
```
integrate(sqrt(b*x^2 + a)*(f*x^2 + e)^2/((d*x^2 + c)^(3/2)*x^4), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{x^4(c+dx^2)^{3/2}}\,dx = \int \frac{\sqrt{bx^2+a}(fx^2+e)^2}{(dx^2+c)^{\frac{3}{2}}x^4}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)^2/x^4/(d*x^2+c)^(3/2),x, algorithm="gi
ac")
```

output
```
integrate(sqrt(b*x^2 + a)*(f*x^2 + e)^2/((d*x^2 + c)^(3/2)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{x^4(c+dx^2)^{3/2}}\,dx = \int \frac{\sqrt{b\,x^2+a}\,(f\,x^2+e)^2}{x^4\,(d\,x^2+c)^{3/2}}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(e + f*x^2)^2)/(x^4*(c + d*x^2)^(3/2)),x)
```

output
```
int(((a + b*x^2)^(1/2)*(e + f*x^2)^2)/(x^4*(c + d*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{x^4(c+dx^2)^{3/2}}\,dx = \text{too large to display}$$

input
```
int((b*x^2+a)^(1/2)*(f*x^2+e)^2/x^4/(d*x^2+c)^(3/2),x)
```

output
```
( - 3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*c*f**2*x**2 - sqrt(c + d*x**2)*sqr
t(a + b*x**2)*d*e**2 - 6*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(2*a**2*c
**2*d*x**2 + 4*a**2*c*d**2*x**4 + 2*a**2*d**3*x**6 + a*b*c**3*x**2 + 4*a*b
*c**2*d*x**4 + 5*a*b*c*d**2*x**6 + 2*a*b*d**3*x**8 + b**2*c**3*x**4 + 2*b*
*2*c**2*d*x**6 + b**2*c*d**2*x**8),x)*a**2*c**3*d*f**2*x**3 + 12*int((sqrt
(c + d*x**2)*sqrt(a + b*x**2))/(2*a**2*c**2*d*x**2 + 4*a**2*c*d**2*x**4 +
2*a**2*d**3*x**6 + a*b*c**3*x**2 + 4*a*b*c**2*d*x**4 + 5*a*b*c*d**2*x**6 +
 2*a*b*d**3*x**8 + b**2*c**3*x**4 + 2*b**2*c**2*d*x**6 + b**2*c*d**2*x**8)
,x)*a**2*c**2*d**2*e*f*x**3 - 6*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(2
*a**2*c**2*d*x**2 + 4*a**2*c*d**2*x**4 + 2*a**2*d**3*x**6 + a*b*c**3*x**2
+ 4*a*b*c**2*d*x**4 + 5*a*b*c*d**2*x**6 + 2*a*b*d**3*x**8 + b**2*c**3*x**4
 + 2*b**2*c**2*d*x**6 + b**2*c*d**2*x**8),x)*a**2*c**2*d**2*f**2*x**5 - 8*
int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(2*a**2*c**2*d*x**2 + 4*a**2*c*d**
2*x**4 + 2*a**2*d**3*x**6 + a*b*c**3*x**2 + 4*a*b*c**2*d*x**4 + 5*a*b*c*d*
*2*x**6 + 2*a*b*d**3*x**8 + b**2*c**3*x**4 + 2*b**2*c**2*d*x**6 + b**2*c*d
**2*x**8),x)*a**2*c*d**3*e**2*x**3 + 12*int((sqrt(c + d*x**2)*sqrt(a + b*x
**2))/(2*a**2*c**2*d*x**2 + 4*a**2*c*d**2*x**4 + 2*a**2*d**3*x**6 + a*b*c*
*3*x**2 + 4*a*b*c**2*d*x**4 + 5*a*b*c*d**2*x**6 + 2*a*b*d**3*x**8 + b**2*c
**3*x**4 + 2*b**2*c**2*d*x**6 + b**2*c*d**2*x**8),x)*a**2*c*d**3*e*f*x**5
- 8*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(2*a**2*c**2*d*x**2 + 4*a**...
```

3.142 $\int \frac{\sqrt{a+bx^2}\left(e+fx^2\right)^2}{x^6\left(c+dx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 879

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{x^6(c+dx^2)^{3/2}}\,dx =$$

$$-\frac{(b^2cde^3-3a^2f(24d^2e^2-28cdef+5c^2f^2)-abe(24d^2e^2-43cdef+27c^2f^2))\,x\sqrt{a+bx^2}}{15a^2c^3e\sqrt{c+dx^2}}$$
$$-\frac{f(3b^2ce^2(de-4cf)-2a^2f(36d^2e^2-36cdef+5c^2f^2)-abe(72d^2e^2-87cdef+31c^2f^2))\,x^3\sqrt{a+bx^2}}{15a^2c^3e^2\sqrt{c+dx^2}}$$
$$-\frac{f^2(b^2ce^2(3de-16cf)-a^2f(24d^2e^2-22cdef+3c^2f^2)-abe(72d^2e^2-73cdef+16c^2f^2))\,x^5\sqrt{a+bx^2}}{15a^2c^3e^3\sqrt{c+dx^2}}$$
$$-\frac{bf^3(bce(de-6cf)-a(24d^2e^2-22cdef+3c^2f^2))\,x^7\sqrt{a+bx^2}}{15a^2c^3e^3\sqrt{c+dx^2}}$$
$$-\frac{(a+bx^2)^{3/2}(e+fx^2)^3}{5acex^5\sqrt{c+dx^2}}+\frac{(2bce+6ade-acf)(a+bx^2)^{3/2}(e+fx^2)^3}{15a^2c^2e^2x^3\sqrt{c+dx^2}}$$
$$+\frac{(bce(de-6cf)-a(24d^2e^2-22cdef+3c^2f^2))(a+bx^2)^{3/2}(e+fx^2)^3}{15a^2c^3e^3x\sqrt{c+dx^2}}$$
$$+\frac{2\sqrt{d}(b^2c^2e^2+abce(4de-5cf)-a^2(24d^2e^2-40cdef+15c^2f^2))\sqrt{a+bx^2}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15a^2c^{7/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$
$$-\frac{b(bcde^2-a(24d^2e^2-40cdef+15c^2f^2))\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{15a^2c^{5/2}\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output

```
-1/15*(b^2*c*d*e^3-3*a^2*f*(5*c^2*f^2-28*c*d*e*f+24*d^2*e^2)-a*b*e*(27*c^2
*f^2-43*c*d*e*f+24*d^2*e^2))*x*(b*x^2+a)^(1/2)/a^2/c^3/e/(d*x^2+c)^(1/2)-1
/15*f*(3*b^2*c*e^2*(-4*c*f+d*e)-2*a^2*f*(5*c^2*f^2-36*c*d*e*f+36*d^2*e^2)-
a*b*e*(31*c^2*f^2-87*c*d*e*f+72*d^2*e^2))*x^3*(b*x^2+a)^(1/2)/a^2/c^3/e^2/
(d*x^2+c)^(1/2)-1/15*f^2*(b^2*c*e^2*(-16*c*f+3*d*e)-a^2*f*(3*c^2*f^2-22*c*
d*e*f+24*d^2*e^2)-a*b*e*(16*c^2*f^2-73*c*d*e*f+72*d^2*e^2))*x^5*(b*x^2+a)^
(1/2)/a^2/c^3/e^3/(d*x^2+c)^(1/2)-1/15*b*f^3*(b*c*e*(-6*c*f+d*e)-a*(3*c^2*
f^2-22*c*d*e*f+24*d^2*e^2))*x^7*(b*x^2+a)^(1/2)/a^2/c^3/e^3/(d*x^2+c)^(1/2
)-1/5*(b*x^2+a)^(3/2)*(f*x^2+e)^3/a/c/e/x^5/(d*x^2+c)^(1/2)+1/15*(-a*c*f+6
*a*d*e+2*b*c*e)*(b*x^2+a)^(3/2)*(f*x^2+e)^3/a^2/c^2/e^2/x^3/(d*x^2+c)^(1/2
)+1/15*(b*c*e*(-6*c*f+d*e)-a*(3*c^2*f^2-22*c*d*e*f+24*d^2*e^2))*(b*x^2+a)^
(3/2)*(f*x^2+e)^3/a^2/c^3/e^3/x/(d*x^2+c)^(1/2)+2/15*d^(1/2)*(b^2*c^2*e^2+
a*b*c*e*(-5*c*f+4*d*e)-a^2*(15*c^2*f^2-40*c*d*e*f+24*d^2*e^2))*(b*x^2+a)^(
1/2)*EllipticE(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))/a^2/
c^(7/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-1/15*b*(b*c*d*e^2-
a*(15*c^2*f^2-40*c*d*e*f+24*d^2*e^2))*(b*x^2+a)^(1/2)*InverseJacobiAM(arct
an(d^(1/2)*x/c^(1/2)),(1-b*c/a/d)^(1/2))/a^2/c^(5/2)/d^(1/2)/(c*(b*x^2+a)/
a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.71 (sec) , antiderivative size = 441, normalized size of antiderivative = 0.50

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{x^6\,(c+dx^2)^{3/2}}\,dx = \frac{-\sqrt{\frac{b}{a}}(a+bx^2)\left(-2b^2c^2e^2x^4(c+dx^2)+abcex^2(c+dx^2)\left(-8dex^2+c(e+10f\right.\right.}{}$$

input

```
Integrate[(Sqrt[a + b*x^2]*(e + f*x^2)^2)/(x^6*(c + d*x^2)^(3/2)),x]
```

output
```
(-(Sqrt[b/a]*(a + b*x^2)*(-2*b^2*c^2*e^2*x^4*(c + d*x^2) + a*b*c*e*x^2*(c
+ d*x^2)*(-8*d*e*x^2 + c*(e + 10*f*x^2)) + a^2*(48*d^3*e^2*x^6 + 8*c*d^2*e
*x^4*(3*e - 10*f*x^2) + 2*c^2*d*x^2*(-3*e^2 - 20*e*f*x^2 + 15*f^2*x^4) + c
^3*(3*e^2 + 10*e*f*x^2 + 15*f^2*x^4)))) - (2*I)*b*c*(-(b^2*c^2*e^2) + a*b*
c*e*(-4*d*e + 5*c*f) + a^2*(24*d^2*e^2 - 40*c*d*e*f + 15*c^2*f^2))*x^5*Sqr
t[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*
d)/(b*c)] + I*b*c*(-2*b^2*c^2*e^2 + a*b*c*e*(-7*d*e + 10*c*f) + a^2*(24*d^
2*e^2 - 40*c*d*e*f + 15*c^2*f^2))*x^5*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)
/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(15*a^2*Sqrt[b/a]*c^4*
x^5*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (warning: unable to verify)

Time = 1.61 (sec) , antiderivative size = 881, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.543$, Rules used = {448, 439, 25, 445, 25, 27, 445, 27, 406, 320, 388, 313, 445, 25, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\left(e+fx^2\right)^2}{x^6\left(c+dx^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{448}$$

$$\frac{f\int \frac{\sqrt{bx^2+a}\left(fx^2+e\right)}{x^4\left(dx^2+c\right)^{3/2}}dx}{e^2}+e\int \frac{\sqrt{bx^2+a}\left(fx^2+e\right)}{x^6\left(dx^2+c\right)^{3/2}}dx$$

$$\Big\downarrow \text{439}$$

$$\frac{f\left(\frac{\sqrt{a+bx^2}(de-cf)}{cdx^3\sqrt{c+dx^2}}-\frac{\int -\frac{b(3de-2cf)x^2+a(4de-3cf)}{x^4\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{cd}\right)}{e^2}+e\left(\frac{\sqrt{a+bx^2}(de-cf)}{cdx^5\sqrt{c+dx^2}}-\frac{\int -\frac{b(5de-4cf)x^2+a(6de-5cf)}{x^6\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{cd}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{f\left(\frac{\int \frac{b(3de-2cf)x^2+a(4de-3cf)}{x^4\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{cd}+\frac{\sqrt{a+bx^2}(de-cf)}{cdx^3\sqrt{c+dx^2}}\right)}{e^2}+e\left(\frac{\int \frac{b(5de-4cf)x^2+a(6de-5cf)}{x^6\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{cd}+\frac{\sqrt{a+bx^2}(de-cf)}{cdx^5\sqrt{c+dx^2}}\right)$$

$$\Big\downarrow \text{ 445}$$

$$\frac{f\left(\frac{-\frac{\int -\frac{ad\left(-b(4de-3cf)x^2+bce-8ade+6acf\right)}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(4de-3cf)}{3cx^3}}{cd}+\frac{\sqrt{a+bx^2}(de-cf)}{cdx^3\sqrt{c+dx^2}}\right)}{e^2}+e\left(\frac{-\frac{\int -\frac{ad\left(-3b(6de-5cf)x^2+bce-24ade+20acf\right)}{x^4\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{5ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(6de-5cf)}{5cx^5}}{cd}+\frac{\sqrt{a+bx^2}(de-cf)}{cdx^5\sqrt{c+dx^2}}\right)$$

$$\Big\downarrow \text{ 25}$$

$$\frac{f\left(\frac{\frac{\int \frac{ad\left(-b(4de-3cf)x^2+bce-8ade+6acf\right)}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(4de-3cf)}{3cx^3}}{cd}+\frac{\sqrt{a+bx^2}(de-cf)}{cdx^3\sqrt{c+dx^2}}\right)}{e^2}+e\left(\frac{\frac{\int \frac{ad\left(-3b(6de-5cf)x^2+bce-24ade+20acf\right)}{x^4\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{5ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(6de-5cf)}{5cx^5}}{cd}+\frac{\sqrt{a+bx^2}(de-cf)}{cdx^5\sqrt{c+dx^2}}\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{f\left(\frac{\frac{d\int \frac{-b(4de-3cf)x^2+bce-8ade+6acf}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(4de-3cf)}{3cx^3}}{cd}+\frac{\sqrt{a+bx^2}(de-cf)}{cdx^3\sqrt{c+dx^2}}\right)}{e^2}+e\left(\frac{\frac{d\int \frac{-3b(6de-5cf)x^2+bce-24ade+20acf}{x^4\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{5c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(6de-5cf)}{5cx^5}}{cd}+\frac{\sqrt{a+bx^2}(de-cf)}{cdx^5\sqrt{c+dx^2}}\right)$$

$$\Big\downarrow \text{ 445}$$

$$\frac{e\left(\frac{\frac{d\left(-\frac{\int \frac{-8d(6de-5cf)a^2+bc(8de-5cf)a+bd(bce-24ade+20acf)x^2+2b^2c^2e}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(20acf-24ade+bce)}{3acx^3}\right)}{5c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(6de-5cj}{5cx^5}}{cd}\right.}{}$$

$$\frac{f\left(\frac{\frac{d\left(-\frac{\int \frac{b\left(ac(4de-3cf)-d(bce-8ade+6acf)x^2\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(6acf-8ade+bce)}{acx}\right)}{3c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(4de-3cf)}{3cx^3}}{cd}+\frac{\sqrt{a+bx^2}(de-cf)}{cdx^3\sqrt{c+dx^2}}\right)}{e^2}$$

$$\downarrow 27$$

$$\frac{e\left(\frac{\frac{d\left(-\frac{\int \frac{-8d(6de-5cf)a^2+bc(8de-5cf)a+bd(bce-24ade+20acf)x^2+2b^2c^2e}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(20acf-24ade+bce)}{3acx^3}\right)}{5c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(6de-5cj}{5cx^5}}{cd}\right.}{}$$

$$\frac{f\left(\frac{\frac{d\left(-\frac{b\int \frac{ac(4de-3cf)-d(bce-8ade+6acf)x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(6acf-8ade+bce)}{acx}\right)}{3c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(4de-3cf)}{3cx^3}}{cd}+\frac{\sqrt{a+bx^2}(de-cf)}{cdx^3\sqrt{c+dx^2}}\right)}{e^2}$$

$$\downarrow 406$$

$$\frac{e\left(\frac{\frac{d\left(-\frac{\int \frac{-8d(6de-5cf)a^2+bc(8de-5cf)a+bd(bce-24ade+20acf)x^2+2b^2c^2e}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(20acf-24ade+bce)}{3acx^3}\right)}{5c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(6de-5c}{5cx^5}}{cd}\right. \quad f\left(\frac{\frac{d\left(-\frac{b\left(ac(4de-3cf)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx-d(6acf-8ade+bce)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(6acf-8ade+bce)}{acx}\right)}{3c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(4de-3}{3cx^3}}{cd}\right.}{e^2}$$

$\downarrow$ 320

$$\frac{e\left(\frac{\frac{d\left(-\frac{\int \frac{-8d(6de-5cf)a^2+bc(8de-5cf)a+bd(bce-24ade+20acf)x^2+2b^2c^2e}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(20acf-24ade+bce)}{3acx^3}\right)}{5c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(6de-5c}{5cx^5}}{cd}\right. \quad f\left(\frac{\frac{d\left(-\frac{b\left(\frac{c^{3/2}\sqrt{a+bx^2}(4de-3cf)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-d(6acf-8ade+bce)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(6acf-8ade+bce)}{acx}\right)}{3c}-}{cd}\right.}{e^2}$$

$\downarrow$ 388

$$\frac{e\left(\frac{\frac{d\left(-\frac{\int \frac{-8d(6de-5cf)a^2+bc(8de-5cf)a+bd(bce-24ade+20acf)x^2+2b^2c^2e}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(20acf-24ade+bce)}{3acx^3}\right)}{5c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(6de-5c}{5cx^5}}{cd}\right.}{}$$

$$\frac{f\left(\frac{\frac{d\left(-\frac{b\left(\frac{c^{3/2}\sqrt{a+bx^2}(4de-3cf)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-d(6acf-8ade+bce)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}}dx}{b}\right)\right)}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(6acf-8a}{acx}}{3c}}{cd}\right.}{e^2}$$

$$\Big\downarrow \; 313$$

$$e\left(\frac{\frac{d\left(-\frac{\int \frac{-8d(6de-5cf)a^2+bc(8de-5cf)a+bd(bce-24ade+20acf)x^2+2b^2c^2e}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(20acf-24ade+bce)}{3acx^3}\right)}{5c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(6de-5c}{5cx^5}}{cd}\right.$$

$$f\left(\frac{\frac{d\left(-\frac{b\left(\frac{c^{3/2}\sqrt{a+bx^2}(4de-3cf)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-d(6acf-8ade+bce)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)\right)}{ac}-\frac{\sqrt{a+bx^2}}{}\right.}{3c}}{cd}\right.$$

$$\overline{\qquad e^2 \qquad}$$

$\downarrow$ 445

$$e\left(\frac{d\left(-\frac{-\frac{\int -\frac{bd\left(\left(-8d(6de-5cf)a^2+bc(8de-5cf)a+2b^2c^2e\right)x^2+ac(bce-24ade+20acf)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{2b^2ce}{a}-\frac{8ad(6de-5cf)}{c}+b(8de-5cf)\right)}{x}}{3ac}-\sqrt{a}\cdots\right.}{5c}}{cd}\right.$$

$$\frac{f\left(\frac{d\left(-\frac{b\left(\frac{c^{3/2}\sqrt{a+bx^2}(4de-3cf)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-d(6acf-8ade+bce)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)\right)}{ac}-\sqrt{a+bx^2}\cdots\right.}{3c}}{cd}\right.}{e^2}$$

$$\downarrow 25$$

$$e\left(\frac{\frac{d\left(-\frac{\frac{\int \frac{bd\left(\left(-8d(6de-5cf)a^2+bc(8de-5cf)a+2b^2c^2e\right)x^2+ac(bce-24ade+20acf)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{2b^2ce}{a}-\frac{8ad(6de-5cf)}{c}+b(8de-5cf)\right)}{x}}{3ac}-\frac{\sqrt{a+bx}}{}\right.}{5c}}{cd}\right.$$

$$\frac{f\left(\frac{\frac{d\left(-\frac{b\left(\frac{c^{3/2}\sqrt{a+bx^2}(4de-3cf)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-d(6acf-8ade+bce)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)\right)}{ac}-\frac{\sqrt{a+bx^2}}{}\right.}{3c}}{cd}\right.}{e^2}$$

$\downarrow$ 27

$$\frac{e\left(\frac{d\left(-\frac{\frac{bd\int\frac{\left(-8d(6de-5cf)a^2+bc(8de-5cf)a+2b^2c^2e\right)x^2+ac(bce-24ade+20acf)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}}{3ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{2b^2ce}{a}-\frac{8ad(6de-5cf)}{c}+b(8de-5cf)\right)}{x}-\frac{\sqrt{a+bx^2}\cdots}{\cdots}\right.}{5c}}{cd}\right.\cdots}{}$$

$$\frac{f\left(\frac{d\left(-\frac{b\left(\frac{c^{3/2}\sqrt{a+bx^2}(4de-3cf)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-d(6acf-8ade+bce)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)\right)}{ac}-\frac{\sqrt{a+bx^2}\cdots}{\cdots}\right.}{3c}}{cd}\right.\cdots}{e^2}$$

$\downarrow$ 406

$$\frac{e\left(\frac{d\left(-\frac{\frac{bd\left(\left(-8a^2d(6de-5cf)+abc(8de-5cf)+2b^2c^2e\right)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+ac(20acf-24ade+bce)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{ac}}{3ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{2b^2ce}{a}-8\cdots\right.}{3\cdots}\cdots\right)}{5c}\cdots}{cd}\cdots\right)\cdots f\left(\frac{d\left(-\frac{b\left(\frac{c^{3/2}\sqrt{a+bx^2}(4de-3cf)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-d(6acf-8ade+bce)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)\right)}{ac}-\frac{\sqrt{a+bx^2}\cdots}{\cdots}\cdots\right)}{3c}\cdots}{cd}\cdots\right)\cdots}{e^2}$$

$\downarrow$ 320

$$e\left(\frac{d\left(-\frac{\frac{bd\left(\left(-8a^2d(6de-5cf)+abc(8de-5cf)+2b^2c^2e\right)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}(20acf-24ade+bce)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{ac}}{3ac}-\frac{\sqrt{a+\cdots}}{\cdots}\cdots\right)}{5c}\cdots\right.$$

$$\frac{\cdots}{cd}$$

$$f\left(\frac{d\left(-\frac{b\left(\frac{c^{3/2}\sqrt{a+bx^2}(4de-3cf)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-d(6acf-8ade+bce)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)\right)}{ac}-\frac{\sqrt{a+bx^2}\cdots}{\cdots}\cdots\right)}{3c}\cdots\right.$$

$$\frac{\cdots}{cd}$$

$$\frac{\cdots}{e^2}$$

↓ 388

$$f\left(\frac{\sqrt{bx^2+a}(de-cf)}{cdx^3\sqrt{dx^2+c}}+\frac{\frac{d\left(-\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}(bce-8ade+6acf)}{acx}-\frac{b\left(\frac{c^{3/2}(4de-3cf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-d(bce-8ade+6acf)\left(\frac{x\sqrt{b}}{b\sqrt{d}}\cdots\right.\right.}{ac}\cdots}{3c}}{cd}\cdots\right.$$

$$\overline{\qquad\qquad e^2\qquad\qquad}$$

$$e\left(\frac{\sqrt{bx^2+a}(de-cf)}{cdx^5\sqrt{dx^2+c}}+\frac{d\left(-\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}(bce-24ade+20acf)}{3acx^3}-\frac{bd\left(\frac{(bce-24ade+20acf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\left(-8\cdots\right.\right.}{\cdots}\cdots\right.}{\cdots}\cdots\right.$$

↓ 313

$$\frac{f\left(\frac{\sqrt{bx^2+a}(de-cf)}{cdx^3\sqrt{dx^2+c}}+\frac{\frac{d\left(-\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}(bce-8ade+6acf)}{acx}-\frac{b\left(\frac{c^{3/2}(4de-3cf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-d(bce-8ade+6acf)\left(\frac{x\sqrt{b}}{b\sqrt{d}}\right.\right.}{ac}}{3c}}{cd}\right.}{e^2}$$

$$e\left(\frac{\sqrt{bx^2+a}(de-cf)}{cdx^5\sqrt{dx^2+c}}+\frac{d\left(-\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}(bce-24ade+20acf)}{3acx^3}-\frac{bd\left(\frac{(bce-24ade+20acf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+(-8\right.}{}\right.}{}\right.$$

input
```
Int[(Sqrt[a + b*x^2]*(e + f*x^2)^2)/(x^6*(c + d*x^2)^(3/2)),x]
```

output
```
(f*(((d*e - c*f)*Sqrt[a + b*x^2])/(c*d*x^3*Sqrt[c + d*x^2]) + (-1/3*((4*d*
e - 3*c*f)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(c*x^3) + (d*(-(((b*c*e - 8*a*
d*e + 6*a*c*f)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(a*c*x)) - (b*(-(d*(b*c*e
- 8*a*d*e + 6*a*c*f)*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*S
qrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b
*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))) + (c^(3/
2)*(4*d*e - 3*c*f)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]],
1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c +
d*x^2])))/(a*c)))/(3*c))/(c*d)))/e^2 + e*(((d*e - c*f)*Sqrt[a + b*x^2])/(c
*d*x^5*Sqrt[c + d*x^2]) + (-1/5*((6*d*e - 5*c*f)*Sqrt[a + b*x^2]*Sqrt[c +
d*x^2])/(c*x^5) + (d*(-1/3*((b*c*e - 24*a*d*e + 20*a*c*f)*Sqrt[a + b*x^2]*
Sqrt[c + d*x^2])/(a*c*x^3) - (-((((2*b^2*c*e)/a - (8*a*d*(6*d*e - 5*c*f))/
c + b*(8*d*e - 5*c*f))*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/x) + (b*d*((2*b^2*
c^2*e - 8*a^2*d*(6*d*e - 5*c*f) + a*b*c*(8*d*e - 5*c*f))*((x*Sqrt[a + b*x^
2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[
d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c +
d*x^2))]*Sqrt[c + d*x^2])) + (c^(3/2)*(b*c*e - 24*a*d*e + 20*a*c*f)*Sqrt[a
 + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d
]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])))/(a*c))/(3*a*c))
)/(5*c))/(c*d))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] && !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 439
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
 + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*b*g*(p + 1))), x] + Simp[1/(2*a*b*(p
+ 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1)*Simp[c*(2*b*e*(
p + 1) + (b*e - a*f)*(m + 1)) + d*(2*b*e*(p + 1) + (b*e - a*f)*(m + 2*q + 1
))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && LtQ[p, -1] && G
tQ[q, 0] && !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e - a*f])
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 448
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[e   Int[(g*x)^m*(a + b*x
^2)^p*(c + d*x^2)^q*(e + f*x^2)^(r - 1), x], x] + Simp[f/e^2   Int[(g*x)^(m
 + 2)*(a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^(r - 1), x], x] /; FreeQ[{a,
b, c, d, e, f, g, m, p, q}, x] && IGtQ[r, 0]
```

Maple [A] (verified)

Time = 19.52 (sec) , antiderivative size = 675, normalized size of antiderivative = 0.77

method	result
elliptic	$\frac{\sqrt{(b\,x^2+a)(x^2d+c)}\left(-\frac{e^2\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{5c^2x^5}-\frac{e(10acf-9ade+bce)\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{15a\,c^3x^3}-\frac{\left(15a^2c^2f^2-50a^2cdef+33a^2d^2e^2+10ab\right.}{}\right.}{}$
risch	Expression too large to display
default	Expression too large to display

input
```
int((b*x^2+a)^(1/2)*(f*x^2+e)^2/x^6/(d*x^2+c)^(3/2),x,method=_RETURNVERBOS
E)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-1/5/c^2*e^2*
(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/x^5-1/15/a/c^3*e*(10*a*c*f-9*a*d*e+b*c
*e)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/x^3-1/15/a^2/c^4*(15*a^2*c^2*f^2-5
0*a^2*c*d*e*f+33*a^2*d^2*e^2+10*a*b*c^2*e*f-8*a*b*c*d*e^2-2*b^2*c^2*e^2)*(
b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/x-(b*d*x^2+a*d)*(c^2*f^2-2*c*d*e*f+d^2*
e^2)/c^4*x/((x^2+c/d)*(b*d*x^2+a*d))^(1/2)+(-1/15*b*d*e*(10*a*c*f-9*a*d*e+
b*c*e)/c^3/a-(c^2*f^2-2*c*d*e*f+d^2*e^2)*(a*d-b*c)/c^4+a*d*(c^2*f^2-2*c*d*
e*f+d^2*e^2)/c^4)/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^
4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(
1/2))-(1/15*b*d*(15*a^2*c^2*f^2-50*a^2*c*d*e*f+33*a^2*d^2*e^2+10*a*b*c^2*e
*f-8*a*b*c*d*e^2-2*b^2*c^2*e^2)/a^2/c^4+(c^2*f^2-2*c*d*e*f+d^2*e^2)*b*d/c^
4)*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c
*x^2+a*c)^(1/2)/d*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-Elli
pticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 664, normalized size of antiderivative = 0.76

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{x^6\left(c+dx^2\right)^{3/2}}\,dx = \frac{2\left(\left(15\,a^2bc^2df^2-\left(b^3c^2d+4\,ab^2cd^2-24\,a^2bd^3\right)e^2+5\left(ab^2c^2d-8\,a^2bcd^2\right)ef\right)x\right.}{}$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)^2/x^6/(d*x^2+c)^(3/2),x, algorithm="fr
icas")
```

output
```
1/15*(2*((15*a^2*b*c^2*d*f^2 - (b^3*c^2*d + 4*a*b^2*c*d^2 - 24*a^2*b*d^3)*
e^2 + 5*(a*b^2*c^2*d - 8*a^2*b*c*d^2)*e*f)*x^7 + (15*a^2*b*c^3*f^2 - (b^3*
c^3 + 4*a*b^2*c^2*d - 24*a^2*b*c*d^2)*e^2 + 5*(a*b^2*c^3 - 8*a^2*b*c^2*d)*
e*f)*x^5)*sqrt(a*c)*sqrt(-b/a)*elliptic_e(arcsin(x*sqrt(-b/a)), a*d/(b*c))
 - ((15*(a^3 + 2*a^2*b)*c^2*d*f^2 - (2*b^3*c^2*d + (a^2*b + 8*a*b^2)*c*d^2
 - 24*(a^3 + 2*a^2*b)*d^3)*e^2 + 10*(a*b^2*c^2*d - 4*(a^3 + 2*a^2*b)*c*d^2
)*e*f)*x^7 + (15*(a^3 + 2*a^2*b)*c^3*f^2 - (2*b^3*c^3 + (a^2*b + 8*a*b^2)*
c^2*d - 24*(a^3 + 2*a^2*b)*c*d^2)*e^2 + 10*(a*b^2*c^3 - 4*(a^3 + 2*a^2*b)*
c^2*d)*e*f)*x^5)*sqrt(a*c)*sqrt(-b/a)*elliptic_f(arcsin(x*sqrt(-b/a)), a*d
/(b*c)) - (3*a^3*c^3*e^2 + 2*(15*a^3*c^2*d*f^2 - (a*b^2*c^2*d + 4*a^2*b*c*
d^2 - 24*a^3*d^3)*e^2 + 5*(a^2*b*c^2*d - 8*a^3*c*d^2)*e*f)*x^6 + (15*a^3*c
^3*f^2 - (2*a*b^2*c^3 + 7*a^2*b*c^2*d - 24*a^3*c*d^2)*e^2 + 10*(a^2*b*c^3
- 4*a^3*c^2*d)*e*f)*x^4 + (10*a^3*c^3*e*f + (a^2*b*c^3 - 6*a^3*c^2*d)*e^2)
*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(a^3*c^4*d*x^7 + a^3*c^5*x^5)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{x^6(c+dx^2)^{3/2}}\,dx = \int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{x^6(c+dx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(f*x**2+e)**2/x**6/(d*x**2+c)**(3/2),x)
```

output
```
Integral(sqrt(a + b*x**2)*(e + f*x**2)**2/(x**6*(c + d*x**2)**(3/2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{x^6\left(c+dx^2\right)^{3/2}}\,dx=\int \frac{\sqrt{bx^2+a}(fx^2+e)^2}{(dx^2+c)^{\frac{3}{2}}x^6}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)^2/x^6/(d*x^2+c)^(3/2),x, algorithm="ma
xima")
```

output
```
integrate(sqrt(b*x^2 + a)*(f*x^2 + e)^2/((d*x^2 + c)^(3/2)*x^6), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{x^6\left(c+dx^2\right)^{3/2}}\,dx=\int \frac{\sqrt{bx^2+a}(fx^2+e)^2}{(dx^2+c)^{\frac{3}{2}}x^6}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)^2/x^6/(d*x^2+c)^(3/2),x, algorithm="gi
ac")
```

output
```
integrate(sqrt(b*x^2 + a)*(f*x^2 + e)^2/((d*x^2 + c)^(3/2)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{x^6\left(c+dx^2\right)^{3/2}}\,dx=\int \frac{\sqrt{b\,x^2+a}\,\left(f\,x^2+e\right)^2}{x^6\,\left(d\,x^2+c\right)^{3/2}}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(e + f*x^2)^2)/(x^6*(c + d*x^2)^(3/2)),x)
```

output
```
int(((a + b*x^2)^(1/2)*(e + f*x^2)^2)/(x^6*(c + d*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)^2}{x^6 (c+dx^2)^{3/2}} \, dx = \text{too large to display}$$

input
```
int((b*x^2+a)^(1/2)*(f*x^2+e)^2/x^6/(d*x^2+c)^(3/2),x)
```

output
```
( - 30*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*c*d*f**2*x**2 - 18*sqrt(c +
d*x**2)*sqrt(a + b*x**2)*a**2*d**2*e**2 + 30*sqrt(c + d*x**2)*sqrt(a + b*x
**2)*a**2*d**2*f**2*x**4 - 20*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b*c**2*f
**2*x**2 - 12*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b*c*d*e**2 + 5*sqrt(c +
d*x**2)*sqrt(a + b*x**2)*a*b*c*d*f**2*x**4 - 30*sqrt(c + d*x**2)*sqrt(a +
b*x**2)*a*b*d**2*e*f*x**4 + 40*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**2*c**2
*f**2*x**4 - 120*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**2*c*d*e*f*x**4 + 75*
sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**2*d**2*e**2*x**4 + 90*int((sqrt(c + d
*x**2)*sqrt(a + b*x**2)*x**2)/(3*a**2*c**2*d + 6*a**2*c*d**2*x**2 + 3*a**2
*d**3*x**4 + 2*a*b*c**3 + 7*a*b*c**2*d*x**2 + 8*a*b*c*d**2*x**4 + 3*a*b*d*
*3*x**6 + 2*b**2*c**3*x**2 + 4*b**2*c**2*d*x**4 + 2*b**2*c*d**2*x**6),x)*a
**3*b*c*d**4*f**2*x**5 + 90*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(
3*a**2*c**2*d + 6*a**2*c*d**2*x**2 + 3*a**2*d**3*x**4 + 2*a*b*c**3 + 7*a*b
*c**2*d*x**2 + 8*a*b*c*d**2*x**4 + 3*a*b*d**3*x**6 + 2*b**2*c**3*x**2 + 4*
b**2*c**2*d*x**4 + 2*b**2*c*d**2*x**6),x)*a**3*b*d**5*f**2*x**7 + 75*int((
sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(3*a**2*c**2*d + 6*a**2*c*d**2*x**
2 + 3*a**2*d**3*x**4 + 2*a*b*c**3 + 7*a*b*c**2*d*x**2 + 8*a*b*c*d**2*x**4
+ 3*a*b*d**3*x**6 + 2*b**2*c**3*x**2 + 4*b**2*c**2*d*x**4 + 2*b**2*c*d**2*
x**6),x)*a**2*b**2*c**2*d**3*f**2*x**5 - 90*int((sqrt(c + d*x**2)*sqrt(a +
 b*x**2)*x**2)/(3*a**2*c**2*d + 6*a**2*c*d**2*x**2 + 3*a**2*d**3*x**4 +...
```

3.143 $\int \frac{x^4\sqrt{a+bx^2}(e+fx^2)}{(c+dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 392

$$\int \frac{x^4\sqrt{a+bx^2}(e+fx^2)}{(c+dx^2)^{3/2}}\,dx = -\frac{(2a^2d^2f-abd(5de-7cf)+4b^2c(5de-6cf))\,x\sqrt{a+bx^2}}{15b^2d^3\sqrt{c+dx^2}} + \frac{(5bde-6bcf+adf)x^3\sqrt{a+bx^2}}{15bd^2\sqrt{c+dx^2}} + \frac{fx^5\sqrt{a+bx^2}}{5d\sqrt{c+dx^2}} + \frac{\sqrt{c}(2a^2d^2f-abd(5de-8cf)+8b^2c(5de-6cf))\,\sqrt{a+bx^2}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15b^2d^{7/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} - \frac{c^{3/2}(adf+4b(5de-6cf))\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{15bd^{7/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output
```
-1/15*(2*a^2*d^2*f-a*b*d*(-7*c*f+5*d*e)+4*b^2*c*(-6*c*f+5*d*e))*x*(b*x^2+a
)^(1/2)/b^2/d^3/(d*x^2+c)^(1/2)+1/15*(a*d*f-6*b*c*f+5*b*d*e)*x^3*(b*x^2+a)
^(1/2)/b/d^2/(d*x^2+c)^(1/2)+1/5*f*x^5*(b*x^2+a)^(1/2)/d/(d*x^2+c)^(1/2)+1
/15*c^(1/2)*(2*a^2*d^2*f-a*b*d*(-8*c*f+5*d*e)+8*b^2*c*(-6*c*f+5*d*e))*(b*x
^2+a)^(1/2)*EllipticE(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2
))/b^2/d^(7/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-1/15*c^(3/2
)*(a*d*f+4*b*(-6*c*f+5*d*e))*(b*x^2+a)^(1/2)*InverseJacobiAM(arctan(d^(1/2
)*x/c^(1/2)),(1-b*c/a/d)^(1/2))/b/d^(7/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/
(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.61 (sec) , antiderivative size = 305, normalized size of antiderivative = 0.78

$$\int \frac{x^4\sqrt{a+bx^2}(e+fx^2)}{(c+dx^2)^{3/2}}\,dx = \frac{\sqrt{\frac{b}{a}}dx(a+bx^2)\left(adf(c+dx^2)+b(-24c^2f+cd(20e-6fx^2)+d^2x^2(5e+3\right.}{}$$

input
```
Integrate[(x^4*Sqrt[a + b*x^2]*(e + f*x^2))/(c + d*x^2)^(3/2),x]
```

output
```
(Sqrt[b/a]*d*x*(a + b*x^2)*(a*d*f*(c + d*x^2) + b*(-24*c^2*f + c*d*(20*e -
 6*f*x^2) + d^2*x^2*(5*e + 3*f*x^2))) + I*c*(2*a^2*d^2*f - 8*b^2*c*(-5*d*e
 + 6*c*f) + a*b*d*(-5*d*e + 8*c*f))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c
]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*c*(a^2*d^2*f - 8*b^2*
c*(-5*d*e + 6*c*f) + a*b*d*(-25*d*e + 32*c*f))*Sqrt[1 + (b*x^2)/a]*Sqrt[1
+ (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(15*b*Sqrt[b/
a]*d^4*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 414, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {439, 444, 27, 444, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4\sqrt{a+bx^2}\left(e+fx^2\right)}{\left(c+dx^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 439}$$

$$\frac{x^5\sqrt{a+bx^2}(de-cf)}{cd\sqrt{c+dx^2}}-\frac{\int \frac{x^4\left(b(5de-6cf)x^2+a(4de-5cf)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{cd}$$

$$\Big\downarrow \text{ 444}$$

$$\frac{x^5\sqrt{a+bx^2}(de-cf)}{cd\sqrt{c+dx^2}}-\frac{\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}(5de-6cf)}{5d}-\frac{\int \frac{bcx^2\left((adf+4b(5de-6cf))x^2+3a(5de-6cf)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{5bd}}{cd}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{x^5\sqrt{a+bx^2}(de-cf)}{cd\sqrt{c+dx^2}}-\frac{\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}(5de-6cf)}{5d}-\frac{c\int \frac{x^2\left((20bde-24bcf+adf)x^2+3a(5de-6cf)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{5d}}{cd}$$

$$\Big\downarrow \text{ 444}$$

$$\frac{x^5\sqrt{a+bx^2}(de-cf)}{cd\sqrt{c+dx^2}}-\frac{\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}(5de-6cf)}{5d}-\frac{c\left(\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(adf-24bcf+20bde)}{3bd}-\frac{\int \frac{\left(8c(5de-6cf)b^2-ad(5de-8cf)b+2a^2d^2f\right)x^2+ac(20bde-24bcf+adf)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3bd}\right)}{5d}}{cd}$$

$$\Big\downarrow \text{ 406}$$

$$\frac{x^5\sqrt{a+bx^2}(de-cf)}{cd\sqrt{c+dx^2}}-\frac{\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}(5de-6cf)}{5d}-\frac{c\left(\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(adf-24bcf+20bde)}{3bd}-\frac{\left(2a^2d^2f-abd(5de-8cf)+8b^2c(5de-6cf)\right)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+ac(adf-2}{3bd}\right.}{5d}}{cd}$$

$\big\downarrow$ 320

$$\frac{x^5\sqrt{a+bx^2}(de-cf)}{cd\sqrt{c+dx^2}}-\frac{\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}(5de-6cf)}{5d}-\frac{c\left(\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(adf-24bcf+20bde)}{3bd}-\frac{\left(2a^2d^2f-abd(5de-8cf)+8b^2c(5de-6cf)\right)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+\cdots}}{\cdots}}{3bd}\cdots\right)}{5d}}{cd}$$

$\big\downarrow$ 388

$$\frac{x^5\sqrt{a+bx^2}(de-cf)}{cd\sqrt{c+dx^2}}-\frac{\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}(5de-6cf)}{5d}-\frac{c\left(\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(adf-24bcf+20bde)}{3bd}-\frac{\left(2a^2d^2f-abd(5de-8cf)+8b^2c(5de-6cf)\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\cdots\right)\cdots}{3bd}\right)}{5d}}{cd}$$

$\big\downarrow$ 313

$$\frac{x^5\sqrt{a+bx^2}(de-cf)}{cd\sqrt{c+dx^2}}-\frac{\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}(5de-6cf)}{5d}-\frac{c\left(\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(adf-24bcf+20bde)}{3bd}-\frac{\left(2a^2d^2f-abd(5de-8cf)+8b^2c(5de-6cf)\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\cdots\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\cdots}}\cdots\right)}{\cdots}\right)}{5d}}{cd}$$

input
```
Int[(x^4*Sqrt[a + b*x^2]*(e + f*x^2))/(c + d*x^2)^(3/2),x]
```

output
```
((d*e - c*f)*x^5*Sqrt[a + b*x^2])/(c*d*Sqrt[c + d*x^2]) - (((5*d*e - 6*c*f
)*x^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(5*d) - (c*(((20*b*d*e - 24*b*c*f +
 a*d*f)*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(3*b*d) - ((2*a^2*d^2*f - a*b*d
*(5*d*e - 8*c*f) + 8*b^2*c*(5*d*e - 6*c*f))*((x*Sqrt[a + b*x^2])/(b*Sqrt[c
 + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]
], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt
[c + d*x^2])) + (c^(3/2)*(20*b*d*e - 24*b*c*f + a*d*f)*Sqrt[a + b*x^2]*Ell
ipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a
+ b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))/(3*b*d)))/(5*d))/(c*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 439
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
 + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*b*g*(p + 1))), x] + Simp[1/(2*a*b*(p
+ 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1)*Simp[c*(2*b*e*(
p + 1) + (b*e - a*f)*(m + 1)) + d*(2*b*e*(p + 1) + (b*e - a*f)*(m + 2*q + 1
))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && LtQ[p, -1] && G
tQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e - a*f])
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

Maple [A] (verified)

Time = 11.03 (sec) , antiderivative size = 609, normalized size of antiderivative = 1.55

method	result
elliptic	$\frac{\sqrt{(b x^2+a)(x^2d+c)}\left(-\frac{(bd x^2+ad)(cf-de)cx}{d^4\sqrt{\left(x^2+\frac{c}{d}\right)(bd x^2+ad)}}+\frac{f x^3\sqrt{bd x^4+ad x^2+x^2bc+ac}}{5d^2}+\frac{\left(\frac{adf-bcf+bde}{d^2}-\frac{f(4ad+4bc)}{5d^2}\right)x\sqrt{bd x^4+ad x^2+x^2bc+ac}}{3bd}\right.}{}$
risch	$\frac{x(3bdf x^2+adf-9bcf+5bde)\sqrt{b x^2+a}\sqrt{x^2d+c}}{15b d^3}-\frac{\left(-\frac{(2f d^2a^2+8fdcba-5ab d^2e-33f c^2b^2+25d b^2ce)c\sqrt{1+\frac{b x^2}{a}}\sqrt{1+\frac{d x^2}{c}}\left(\text{EllipticF}\left(x\right.\right.}{\sqrt{-\frac{b}{a}}\sqrt{bd x^4+ad x^2+x^2bc+}}\right.}{}$
default	$\frac{\sqrt{b x^2+a}\sqrt{x^2d+c}\left(3\sqrt{-\frac{b}{a}}b^2d^3f x^7+4\sqrt{-\frac{b}{a}}ab d^3f x^5-6\sqrt{-\frac{b}{a}}b^2c d^2f x^5+5\sqrt{-\frac{b}{a}}b^2d^3e x^5+\sqrt{-\frac{b}{a}}a^2d^3f x^3-5\sqrt{-\frac{b}{a}}abc d^2f x^3\right.}{}$

input
```
int(x^4*(b*x^2+a)^(1/2)*(f*x^2+e)/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-(b*d*x^2+a*d
)*(c*f-d*e)*c/d^4*x/((x^2+c/d)*(b*d*x^2+a*d))^(1/2)+1/5/d^2*f*x^3*(b*d*x^4
+a*d*x^2+b*c*x^2+a*c)^(1/2)+1/3*((a*d*f-b*c*f+b*d*e)/d^2-1/5/d^2*f*(4*a*d+
4*b*c))/b/d*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+(c*(a*c*d*f-a*d^2*e-b*c^
2*f+b*c*d*e)/d^4-(c*f-d*e)*c/d^4*(a*d-b*c)+a/d^3*(c*f-d*e)*c-1/3*((a*d*f-b
*c*f+b*d*e)/d^2-1/5/d^2*f*(4*a*d+4*b*c))/b/d*a*c)/(-b/a)^(1/2)*(1+b*x^2/a)
^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(
-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-(-1/d^3*(a*c*d*f-a*d^2*e-b*c^2*f+b*c
*d*e)+(c*f-d*e)*c/d^3*b-3/5*a*c/d^2*f-1/3*((a*d*f-b*c*f+b*d*e)/d^2-1/5/d^2
*f*(4*a*d+4*b*c))/b/d*(2*a*d+2*b*c))*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d
*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(-b/a)^(1
/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^
(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 556, normalized size of antiderivative = 1.42

$$\int \frac{x^4\sqrt{a+bx^2}(e+fx^2)}{(c+dx^2)^{3/2}}\,dx = \frac{((5\,(8\,b^2c^2d^2-abcd^3)e-2\,(24\,b^2c^3d-4\,abc^2d^2-a^2cd^3)f)x^3+(5\,(8\,b^2c^3d-$$

input
```
integrate(x^4*(b*x^2+a)^(1/2)*(f*x^2+e)/(d*x^2+c)^(3/2),x, algorithm="fric
as")
```

output
```
1/15*(((5*(8*b^2*c^2*d^2 - a*b*c*d^3)*e - 2*(24*b^2*c^3*d - 4*a*b*c^2*d^2
- a^2*c*d^3)*f)*x^3 + (5*(8*b^2*c^3*d - a*b*c^2*d^2)*e - 2*(24*b^2*c^4 - 4
*a*b*c^3*d - a^2*c^2*d^2)*f)*x)*sqrt(b*d)*sqrt(-c/d)*elliptic_e(arcsin(sqr
t(-c/d)/x), a*d/(b*c)) - ((5*(8*b^2*c^2*d^2 - a*b*c*d^3 + 4*a*b*d^4)*e - (
48*b^2*c^3*d - 8*a*b*c^2*d^2 - a^2*d^4 - 2*(a^2 - 12*a*b)*c*d^3)*f)*x^3 +
(5*(8*b^2*c^3*d - a*b*c^2*d^2 + 4*a*b*c*d^3)*e - (48*b^2*c^4 - 8*a*b*c^3*d
 - a^2*c*d^3 - 2*(a^2 - 12*a*b)*c^2*d^2)*f)*x)*sqrt(b*d)*sqrt(-c/d)*ellipt
ic_f(arcsin(sqrt(-c/d)/x), a*d/(b*c)) + (3*b^2*d^4*f*x^6 + (5*b^2*d^4*e -
(6*b^2*c*d^3 - a*b*d^4)*f)*x^4 - (5*(4*b^2*c*d^3 - a*b*d^4)*e - (24*b^2*c^
2*d^2 - 7*a*b*c*d^3 - 2*a^2*d^4)*f)*x^2 - 5*(8*b^2*c^2*d^2 - a*b*c*d^3)*e
+ 2*(24*b^2*c^3*d - 4*a*b*c^2*d^2 - a^2*c*d^3)*f)*sqrt(b*x^2 + a)*sqrt(d*x
^2 + c))/(b^2*d^6*x^3 + b^2*c*d^5*x)
```

Sympy [F]

$$\int \frac{x^4\sqrt{a+bx^2}(e+fx^2)}{(c+dx^2)^{3/2}}\,dx = \int \frac{x^4\sqrt{a+bx^2}(e+fx^2)}{(c+dx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**4*(b*x**2+a)**(1/2)*(f*x**2+e)/(d*x**2+c)**(3/2),x)
```

output
```
Integral(x**4*sqrt(a + b*x**2)*(e + f*x**2)/(c + d*x**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{x^4\sqrt{a+bx^2}(e+fx^2)}{(c+dx^2)^{3/2}}\,dx = \int \frac{\sqrt{bx^2+a}(fx^2+e)x^4}{(dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4*(b*x^2+a)^(1/2)*(f*x^2+e)/(d*x^2+c)^(3/2),x, algorithm="maxi
ma")
```

output
```
integrate(sqrt(b*x^2 + a)*(f*x^2 + e)*x^4/(d*x^2 + c)^(3/2), x)
```

Giac [F]

$$\int \frac{x^4\sqrt{a+bx^2}(e+fx^2)}{(c+dx^2)^{3/2}}\,dx = \int \frac{\sqrt{bx^2+a}(fx^2+e)x^4}{(dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4*(b*x^2+a)^(1/2)*(f*x^2+e)/(d*x^2+c)^(3/2),x, algorithm="giac
")
```

output
```
integrate(sqrt(b*x^2 + a)*(f*x^2 + e)*x^4/(d*x^2 + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4\sqrt{a+bx^2}(e+fx^2)}{(c+dx^2)^{3/2}}\,dx=\int \frac{x^4\,\sqrt{b\,x^2+a}\,(f\,x^2+e)}{{(d\,x^2+c)}^{3/2}} \,dx$$

input
```
int((x^4*(a + b*x^2)^(1/2)*(e + f*x^2))/(c + d*x^2)^(3/2),x)
```

output
```
int((x^4*(a + b*x^2)^(1/2)*(e + f*x^2))/(c + d*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^4\sqrt{a+bx^2}(e+fx^2)}{(c+dx^2)^{3/2}}\,dx=\text{Too large to display}$$

input
```
int(x^4*(b*x^2+a)^(1/2)*(f*x^2+e)/(d*x^2+c)^(3/2),x)
```

output
```
( - 3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*d*f*x + 18*sqrt(c + d*x**2)*s
qrt(a + b*x**2)*a*b*c*f*x - 15*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b*d*e*x
 + 2*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b*d*f*x**3 - 12*sqrt(c + d*x**2)*
sqrt(a + b*x**2)*b**2*c*f*x**3 + 10*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**2
*d*e*x**3 + 6*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**2*d*f*x**5 - int((sqrt(
c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c**2 + 2*a*c*d*x**2 + a*d**2*x**4 +
b*c**2*x**2 + 2*b*c*d*x**4 + b*d**2*x**6),x)*a**2*b*c*d**2*f - int((sqrt(c
 + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c**2 + 2*a*c*d*x**2 + a*d**2*x**4 + b
*c**2*x**2 + 2*b*c*d*x**4 + b*d**2*x**6),x)*a**2*b*d**3*f*x**2 - 32*int((s
qrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c**2 + 2*a*c*d*x**2 + a*d**2*x**
4 + b*c**2*x**2 + 2*b*c*d*x**4 + b*d**2*x**6),x)*a*b**2*c**2*d*f + 25*int(
(sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c**2 + 2*a*c*d*x**2 + a*d**2*x
**4 + b*c**2*x**2 + 2*b*c*d*x**4 + b*d**2*x**6),x)*a*b**2*c*d**2*e - 32*in
t((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c**2 + 2*a*c*d*x**2 + a*d**2
*x**4 + b*c**2*x**2 + 2*b*c*d*x**4 + b*d**2*x**6),x)*a*b**2*c*d**2*f*x**2
+ 25*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c**2 + 2*a*c*d*x**2 +
 a*d**2*x**4 + b*c**2*x**2 + 2*b*c*d*x**4 + b*d**2*x**6),x)*a*b**2*d**3*e*
x**2 + 48*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c**2 + 2*a*c*d*x
**2 + a*d**2*x**4 + b*c**2*x**2 + 2*b*c*d*x**4 + b*d**2*x**6),x)*b**3*c**3
*f - 40*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c**2 + 2*a*c*d*...
```

3.144 $\int \frac{x^2\sqrt{a+bx^2}(e+fx^2)}{(c+dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 283

$$\begin{aligned}\int \frac{x^2\sqrt{a+bx^2}(e+fx^2)}{(c+dx^2)^{3/2}}\,dx &= \frac{(3bde-4bcf+adf)x\sqrt{a+bx^2}}{3bd^2\sqrt{c+dx^2}}+\frac{fx^3\sqrt{a+bx^2}}{3d\sqrt{c+dx^2}}\\ &\quad-\frac{\sqrt{c}(6bde-8bcf+adf)\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3bd^{5/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}\\ &\quad+\frac{\sqrt{c}(3de-4cf)\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3d^{5/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}\end{aligned}$$

output
```
1/3*(a*d*f-4*b*c*f+3*b*d*e)*x*(b*x^2+a)^(1/2)/b/d^2/(d*x^2+c)^(1/2)+1/3*f*
x^3*(b*x^2+a)^(1/2)/d/(d*x^2+c)^(1/2)-1/3*c^(1/2)*(a*d*f-8*b*c*f+6*b*d*e)*
(b*x^2+a)^(1/2)*EllipticE(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^
(1/2))/b/d^(5/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)+1/3*c^(1/
2)*(-4*c*f+3*d*e)*(b*x^2+a)^(1/2)*InverseJacobiAM(arctan(d^(1/2)*x/c^(1/2)
),(1-b*c/a/d)^(1/2))/d^(5/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/
2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.69 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.82

$$\int \frac{x^2\sqrt{a+bx^2}(e+fx^2)}{(c+dx^2)^{3/2}}\,dx = \frac{\sqrt{\frac{b}{a}}dx(a+bx^2)\left(-3de+4cf+dfx^2\right)-ic(6bde-8bcf+adf)\sqrt{1+\frac{bx^2}{a}}\sqrt{1}$$

input
```
Integrate[(x^2*Sqrt[a + b*x^2]*(e + f*x^2))/(c + d*x^2)^(3/2),x]
```

output
```
(Sqrt[b/a]*d*x*(a + b*x^2)*(-3*d*e + 4*c*f + d*f*x^2) - I*c*(6*b*d*e - 8*b
*c*f + a*d*f)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[
Sqrt[b/a]*x], (a*d)/(b*c)] + I*(a*d*(-3*d*e + 5*c*f) + b*(6*c*d*e - 8*c^2*
f))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*
x], (a*d)/(b*c)])/(3*Sqrt[b/a]*d^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.14, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {439, 444, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\sqrt{a+bx^2}(e+fx^2)}{(c+dx^2)^{3/2}}dx$$

$$\Big\downarrow \text{ 439}$$

$$\frac{x^3\sqrt{a+bx^2}(de-cf)}{cd\sqrt{c+dx^2}}-\frac{\int \frac{x^2\left(b(3de-4cf)x^2+a(2de-3cf)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{cd}$$

$$\Big\downarrow \text{ 444}$$

$$\frac{x^3\sqrt{a+bx^2}(de-cf)}{cd\sqrt{c+dx^2}}-\frac{\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(3de-4cf)}{3d}-\frac{\int\frac{bc\left((6bde-8bcf+adf)x^2+a(3de-4cf)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3bd}}{cd}$$

$\downarrow$ 27

$$\frac{x^3\sqrt{a+bx^2}(de-cf)}{cd\sqrt{c+dx^2}}-\frac{\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(3de-4cf)}{3d}-\frac{c\int\frac{(6bde-8bcf+adf)x^2+a(3de-4cf)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3d}}{cd}$$

$\downarrow$ 406

$$\frac{x^3\sqrt{a+bx^2}(de-cf)}{cd\sqrt{c+dx^2}}-\frac{\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(3de-4cf)}{3d}-\frac{c\left(a(3de-4cf)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+(adf-8bcf+6bde)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{3d}}{cd}$$

$\downarrow$ 320

$$\frac{x^3\sqrt{a+bx^2}(de-cf)}{cd\sqrt{c+dx^2}}-\frac{\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(3de-4cf)}{3d}-\frac{c\left((adf-8bcf+6bde)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{\sqrt{c}\sqrt{a+bx^2}(3de-4cf)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{3d}}{cd}$$

$\downarrow$ 388

$$\frac{x^3\sqrt{a+bx^2}(de-cf)}{cd\sqrt{c+dx^2}}-\frac{\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(3de-4cf)}{3d}-\frac{c\left((adf-8bcf+6bde)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{\sqrt{c}\sqrt{a+bx^2}(3de-4cf)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{3d}}{cd}$$

$\downarrow$ 313

$$\frac{x^3\sqrt{a+bx^2}(de-cf)}{cd\sqrt{c+dx^2}}-\frac{\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(3de-4cf)}{3d}-\frac{c\left(\frac{\sqrt{c}\sqrt{a+bx^2}(3de-4cf)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+(adf-8bcf+6bde)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{c}}{\sqrt{}}\right.\right.}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+}{a(c+}}}\right.\right.}{3d}}{cd}$$

input
```
Int[(x^2*Sqrt[a + b*x^2]*(e + f*x^2))/(c + d*x^2)^(3/2),x]
```

output
```
((d*e - c*f)*x^3*Sqrt[a + b*x^2])/(c*d*Sqrt[c + d*x^2]) - (((3*d*e - 4*c*f
)*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(3*d) - (c*((6*b*d*e - 8*b*c*f + a*d*
f)*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*Ell
ipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(
a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (Sqrt[c]*(3*d*e - 4*c*f)*
Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(
Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])))/(3*d))/(c
*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 439
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
 + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*b*g*(p + 1))), x] + Simp[1/(2*a*b*(p
+ 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1)*Simp[c*(2*b*e*(
p + 1) + (b*e - a*f)*(m + 1)) + d*(2*b*e*(p + 1) + (b*e - a*f)*(m + 2*q + 1
))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && LtQ[p, -1] && G
tQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e - a*f])
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

Maple [A] (verified)

Time = 8.95 (sec) , antiderivative size = 444, normalized size of antiderivative = 1.57

method	result
elliptic	$\frac{\sqrt{(b x^2+a)(x^2 d+c)}\left(\frac{\left(bd x^2+ad\right)(cf-de)x}{d^3\sqrt{\left(x^2+\frac{c}{d}\right)\left(bd x^2+ad\right)}}+\frac{fx\sqrt{bd x^4+ad x^2+x^2bc+ac}}{3d^2}+\frac{\left(-\frac{acdf-a d^2e-b c^2f+bcde}{d^3}+\frac{(cf-de)(ad-bc)}{d^3}-\frac{a(cf-de)}{d^2}-\cdots\right.}{\sqrt{-\frac{b}{a}}\sqrt{bd x^4+a\cdots}}\right.}{\cdots}$
default	$-\frac{\sqrt{b x^2+a}\sqrt{x^2d+c}\left(-\sqrt{-\frac{b}{a}}\,b d^2 f x^5-\sqrt{-\frac{b}{a}}\,a d^2 f x^3-4\sqrt{-\frac{b}{a}}\,bcdf x^3+3\sqrt{-\frac{b}{a}}\,b d^2 e x^3+5\sqrt{\frac{b x^2+a}{a}}\sqrt{\frac{x^2d+c}{c}}\,\text{EllipticF}\left(x\sqrt{-\frac{b}{a}},\cdots\right.\right.}{\cdots}$
risch	$\frac{fx\sqrt{b x^2+a}\sqrt{x^2d+c}}{3d^2}+\frac{\left(-\frac{(adf-5bcf+3bde)c\sqrt{1+\frac{b x^2}{a}}\sqrt{1+\frac{d x^2}{c}}\left(\text{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)-\text{EllipticE}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)\right)}{\sqrt{-\frac{b}{a}}\sqrt{bd x^4+ad x^2+x^2bc+ac}\,d}-\cdots\right.}{\cdots}$

input
```
int(x^2*(b*x^2+a)^(1/2)*(f*x^2+e)/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*((b*d*x^2+a*d)
*(c*f-d*e)/d^3*x/((x^2+c/d)*(b*d*x^2+a*d))^(1/2)+1/3*f/d^2*x*(b*d*x^4+a*d*
x^2+b*c*x^2+a*c)^(1/2)+(-1/d^3*(a*c*d*f-a*d^2*e-b*c^2*f+b*c*d*e)+(c*f-d*e)
/d^3*(a*d-b*c)-a/d^2*(c*f-d*e)-1/3*a*c/d^2*f)/(-b/a)^(1/2)*(1+b*x^2/a)^(1/
2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a
)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-((a*d*f-b*c*f+b*d*e)/d^2-(c*f-d*e)/d^2*b
-1/3*f/d^2*(2*a*d+2*b*c))*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/
2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(-b/a)^(1/2),(-1+(a*
d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.25

$$\int \frac{x^2\sqrt{a+bx^2}(e+fx^2)}{(c+dx^2)^{3/2}}\,dx =$$

$$-\frac{((6\,bc^2d^2e-(8\,bc^3d-ac^2d^2)f)x^3+(6\,bc^3de-(8\,bc^4-ac^3d)f)x)\sqrt{bd}\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ad}{bc})-((3}{}$$

input
```
integrate(x^2*(b*x^2+a)^(1/2)*(f*x^2+e)/(d*x^2+c)^(3/2),x, algorithm="fric
as")
```

output
```
-1/3*(((6*b*c^2*d^2*e - (8*b*c^3*d - a*c^2*d^2)*f)*x^3 + (6*b*c^3*d*e - (8
*b*c^4 - a*c^3*d)*f)*x)*sqrt(b*d)*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/
x), a*d/(b*c)) - ((3*(2*b*c^2*d^2 + a*d^4)*e - (8*b*c^3*d - a*c^2*d^2 + 4*
a*c*d^3)*f)*x^3 + (3*(2*b*c^3*d + a*c*d^3)*e - (8*b*c^4 - a*c^3*d + 4*a*c^
2*d^2)*f)*x)*sqrt(b*d)*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)/x), a*d/(b*
c)) - (b*c*d^3*f*x^4 + 6*b*c^2*d^2*e + (3*b*c*d^3*e - (4*b*c^2*d^2 - a*c*d
^3)*f)*x^2 - (8*b*c^3*d - a*c^2*d^2)*f)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(
b*c*d^5*x^3 + b*c^2*d^4*x)
```

Sympy [F]

$$\int \frac{x^2\sqrt{a+bx^2}(e+fx^2)}{(c+dx^2)^{3/2}}\,dx = \int \frac{x^2\sqrt{a+bx^2}(e+fx^2)}{(c+dx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2*(b*x**2+a)**(1/2)*(f*x**2+e)/(d*x**2+c)**(3/2),x)
```

output
```
Integral(x**2*sqrt(a + b*x**2)*(e + f*x**2)/(c + d*x**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{x^2\sqrt{a+bx^2}(e+fx^2)}{(c+dx^2)^{3/2}}\,dx = \int \frac{\sqrt{bx^2+a}(fx^2+e)x^2}{(dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*(b*x^2+a)^(1/2)*(f*x^2+e)/(d*x^2+c)^(3/2),x, algorithm="maxi
ma")
```

output
```
integrate(sqrt(b*x^2 + a)*(f*x^2 + e)*x^2/(d*x^2 + c)^(3/2), x)
```

Giac [F]

$$\int \frac{x^2\sqrt{a+bx^2}(e+fx^2)}{(c+dx^2)^{3/2}}\,dx = \int \frac{\sqrt{bx^2+a}(fx^2+e)x^2}{(dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*(b*x^2+a)^(1/2)*(f*x^2+e)/(d*x^2+c)^(3/2),x, algorithm="giac
")
```

output
```
integrate(sqrt(b*x^2 + a)*(f*x^2 + e)*x^2/(d*x^2 + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2\sqrt{a+bx^2}(e+fx^2)}{(c+dx^2)^{3/2}}\,dx = \int \frac{x^2\,\sqrt{b\,x^2+a}\,(f\,x^2+e)}{(d\,x^2+c)^{3/2}}\,dx$$

input
```
int((x^2*(a + b*x^2)^(1/2)*(e + f*x^2))/(c + d*x^2)^(3/2),x)
```

output
```
int((x^2*(a + b*x^2)^(1/2)*(e + f*x^2))/(c + d*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^2\sqrt{a+bx^2}(e+fx^2)}{(c+dx^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int(x^2*(b*x^2+a)^(1/2)*(f*x^2+e)/(d*x^2+c)^(3/2),x)
```

output
```
( - 3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*c*f*x + 3*sqrt(c + d*x**2)*sqrt(
a + b*x**2)*a*d*e*x + 2*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b*c*f*x**3 + 5*i
nt((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c**2 + 2*a*c*d*x**2 + a*d**
2*x**4 + b*c**2*x**2 + 2*b*c*d*x**4 + b*d**2*x**6),x)*a*b*c**2*d*f - 3*int
((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c**2 + 2*a*c*d*x**2 + a*d**2*
x**4 + b*c**2*x**2 + 2*b*c*d*x**4 + b*d**2*x**6),x)*a*b*c*d**2*e + 5*int((
sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c**2 + 2*a*c*d*x**2 + a*d**2*x*
*4 + b*c**2*x**2 + 2*b*c*d*x**4 + b*d**2*x**6),x)*a*b*c*d**2*f*x**2 - 3*in
t((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c**2 + 2*a*c*d*x**2 + a*d**2
*x**4 + b*c**2*x**2 + 2*b*c*d*x**4 + b*d**2*x**6),x)*a*b*d**3*e*x**2 - 8*i
nt((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c**2 + 2*a*c*d*x**2 + a*d**
2*x**4 + b*c**2*x**2 + 2*b*c*d*x**4 + b*d**2*x**6),x)*b**2*c**3*f + 6*int(
(sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c**2 + 2*a*c*d*x**2 + a*d**2*x
**4 + b*c**2*x**2 + 2*b*c*d*x**4 + b*d**2*x**6),x)*b**2*c**2*d*e - 8*int((
sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c**2 + 2*a*c*d*x**2 + a*d**2*x*
*4 + b*c**2*x**2 + 2*b*c*d*x**4 + b*d**2*x**6),x)*b**2*c**2*d*f*x**2 + 6*i
nt((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c**2 + 2*a*c*d*x**2 + a*d**
2*x**4 + b*c**2*x**2 + 2*b*c*d*x**4 + b*d**2*x**6),x)*b**2*c*d**2*e*x**2 +
 3*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c**2 + 2*a*c*d*x**2 + a*d**2
*x**4 + b*c**2*x**2 + 2*b*c*d*x**4 + b*d**2*x**6),x)*a**2*c**3*f - 3*in...
```

3.145 $\int \frac{\sqrt{a+bx^2}(e+fx^2)}{(c+dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 206

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{(c+dx^2)^{3/2}}\,dx = \frac{fx\sqrt{a+bx^2}}{d\sqrt{c+dx^2}} + \frac{(de-2cf)\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}d^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} + \frac{\sqrt{c}f\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{d^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output
```
f*x*(b*x^2+a)^(1/2)/d/(d*x^2+c)^(1/2)+(-2*c*f+d*e)*(b*x^2+a)^(1/2)*Ellipti
cE(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))/c^(1/2)/d^(3/2)/
(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)+c^(1/2)*f*(b*x^2+a)^(1/2)*
InverseJacobiAM(arctan(d^(1/2)*x/c^(1/2)),(1-b*c/a/d)^(1/2))/d^(3/2)/(c*(b
*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.45 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.01

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{(c+dx^2)^{3/2}}\,dx = \frac{\sqrt{\frac{b}{a}}d(de-cf)x(a+bx^2)-ibc(-de+2cf)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\Big(i\text{arcsinh}\Big(\sqrt{}}{\sqrt{\frac{b}{a}}cd^2\sqrt{c}}$$

input
```
Integrate[(Sqrt[a + b*x^2]*(e + f*x^2))/(c + d*x^2)^(3/2),x]
```

output
```
(Sqrt[b/a]*d*(d*e - c*f)*x*(a + b*x^2) - I*b*c*(-(d*e) + 2*c*f)*Sqrt[1 + (
b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c
)] - I*c*(b*d*e - 2*b*c*f + a*d*f)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]
*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(Sqrt[b/a]*c*d^2*Sqrt[a +
 b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.26, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {401, 25, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{(c+dx^2)^{3/2}}dx$$

$$\Big\downarrow \text{401}$$

$$\frac{x\sqrt{a+bx^2}(de-cf)}{cd\sqrt{c+dx^2}}-\frac{\int -\frac{acf-b(de-2cf)x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{cd}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{acf-b(de-2cf)x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{cd}+\frac{x\sqrt{a+bx^2}(de-cf)}{cd\sqrt{c+dx^2}}$$

$$\Big\downarrow \text{406}$$

$$\frac{acf\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx-b(de-2cf)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{cd}+\frac{x\sqrt{a+bx^2}(de-cf)}{cd\sqrt{c+dx^2}}$$

$$\Big\downarrow \text{320}$$

$$\frac{\frac{c^{3/2}f\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-b(de-2cf)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{cd}+\frac{x\sqrt{a+bx^2}(de-cf)}{cd\sqrt{c+dx^2}}$$

$$\Big\downarrow \text{388}$$

$$\frac{\frac{c^{3/2}f\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-b(de-2cf)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)}{cd}+\frac{x\sqrt{a+bx^2}(de-cf)}{cd\sqrt{c+dx^2}}$$

$$\Big\downarrow \text{313}$$

$$\frac{\frac{c^{3/2}f\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-b(de-2cf)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{cd}+\frac{x\sqrt{a+bx^2}(de-cf)}{cd\sqrt{c+dx^2}}$$

input
```
Int[(Sqrt[a + b*x^2]*(e + f*x^2))/(c + d*x^2)^(3/2),x]
```

output
```
((d*e - c*f)*x*Sqrt[a + b*x^2])/(c*d*Sqrt[c + d*x^2]) + (-(b*(d*e - 2*c*f)
*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*Ellip
ticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a
+ b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))) + (c^(3/2)*f*Sqrt[a + b*x^2]
*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c
*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))/(c*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
 + d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

Maple [A] (verified)

Time = 4.66 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.83

method	result
elliptic	$\frac{\sqrt{(b\,x^2+a)(x^2d+c)}\left(-\frac{(bd\,x^2+ad)(cf-de)x}{c\,d^2\sqrt{\left(x^2+\frac{c}{d}\right)(bd\,x^2+ad)}}+\frac{\left(\frac{adf-bcf+bde}{d^2}-\frac{(cf-de)(ad-bc)}{d^2c}+\frac{a(cf-de)}{dc}\right)\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\right.}}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}\right.}{\sqrt{b\,x^2+a}\,\sqrt{x^2d}}$
default	$\frac{\sqrt{b\,x^2+a}\,\sqrt{x^2d+c}\left(-\sqrt{-\frac{b}{a}}\,bcdf\,x^3+\sqrt{-\frac{b}{a}}\,b\,d^2e\,x^3+\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{x^2d+c}{c}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)acdf-2\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{x^2d+c}{c}}\,\mathrm{Ellip}\right.}{}$

input
```
int((b*x^2+a)^(1/2)*(f*x^2+e)/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-(b*d*x^2+a*d
)*(c*f-d*e)/c/d^2*x/((x^2+c/d)*(b*d*x^2+a*d))^(1/2)+((a*d*f-b*c*f+b*d*e)/d
^2-(c*f-d*e)/d^2*(a*d-b*c)/c+a/d*(c*f-d*e)/c)/(-b/a)^(1/2)*(1+b*x^2/a)^(1/
2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a
)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-(f*b/d+(c*f-d*e)/d*b/c)*c/(-b/a)^(1/2)*(
1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*(
EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2
),(-1+(a*d+b*c)/c/b)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.15

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{(c+dx^2)^{3/2}}\,dx = \frac{((bcd^2e-2\,bc^2df)x^3+(bc^2de-2\,bc^3f)x)\sqrt{bd}\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ad}{bc})-(}{}$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
(((b*c*d^2*e - 2*b*c^2*d*f)*x^3 + (b*c^2*d*e - 2*b*c^3*f)*x)*sqrt(b*d)*sqr
t(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - ((b*c*d^2*e - (2*b*c
^2*d + a*d^3)*f)*x^3 + (b*c^2*d*e - (2*b*c^3 + a*c*d^2)*f)*x)*sqrt(b*d)*sq
rt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)/x), a*d/(b*c)) + (b*c*d^2*f*x^2 - b*
c*d^2*e + 2*b*c^2*d*f)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(b*c*d^4*x^3 + b*c
^2*d^3*x)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{(c+dx^2)^{3/2}}\,dx = \int \frac{\sqrt{a+bx^2}(e+fx^2)}{(c+dx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(f*x**2+e)/(d*x**2+c)**(3/2),x)
```

output
```
Integral(sqrt(a + b*x**2)*(e + f*x**2)/(c + d*x**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{(c+dx^2)^{3/2}}\,dx = \int \frac{\sqrt{bx^2+a}(fx^2+e)}{(dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)*(f*x^2 + e)/(d*x^2 + c)^(3/2), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{(c+dx^2)^{3/2}}\,dx = \int \frac{\sqrt{bx^2+a}(fx^2+e)}{(dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)/(d*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^2 + a)*(f*x^2 + e)/(d*x^2 + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{(c+dx^2)^{3/2}}\,dx = \int \frac{\sqrt{b\,x^2+a}\,(f\,x^2+e)}{(d\,x^2+c)^{3/2}}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(e + f*x^2))/(c + d*x^2)^(3/2),x)
```

output
```
int(((a + b*x^2)^(1/2)*(e + f*x^2))/(c + d*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{(c+dx^2)^{3/2}}\,dx = \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,afx+\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,bex-\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2}}{b\,d^2x^6+a\,d^2x^4+2bcd\,x^4+2c}\right.}{}$$

input
```
int((b*x^2+a)^(1/2)*(f*x^2+e)/(d*x^2+c)^(3/2),x)
```

output
```
(sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*f*x + sqrt(c + d*x**2)*sqrt(a + b*x**
2)*b*e*x - int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c**2 + 2*a*c*d*
x**2 + a*d**2*x**4 + b*c**2*x**2 + 2*b*c*d*x**4 + b*d**2*x**6),x)*a*b*c*d*
f - int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c**2 + 2*a*c*d*x**2 +
a*d**2*x**4 + b*c**2*x**2 + 2*b*c*d*x**4 + b*d**2*x**6),x)*a*b*d**2*f*x**2
 + 2*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c**2 + 2*a*c*d*x**2 +
 a*d**2*x**4 + b*c**2*x**2 + 2*b*c*d*x**4 + b*d**2*x**6),x)*b**2*c**2*f -
int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c**2 + 2*a*c*d*x**2 + a*d*
*2*x**4 + b*c**2*x**2 + 2*b*c*d*x**4 + b*d**2*x**6),x)*b**2*c*d*e + 2*int(
(sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c**2 + 2*a*c*d*x**2 + a*d**2*x
**4 + b*c**2*x**2 + 2*b*c*d*x**4 + b*d**2*x**6),x)*b**2*c*d*f*x**2 - int((
sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c**2 + 2*a*c*d*x**2 + a*d**2*x*
*4 + b*c**2*x**2 + 2*b*c*d*x**4 + b*d**2*x**6),x)*b**2*d**2*e*x**2 - int((
sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c**2 + 2*a*c*d*x**2 + a*d**2*x**4 +
b*c**2*x**2 + 2*b*c*d*x**4 + b*d**2*x**6),x)*a**2*c**2*f - int((sqrt(c + d
*x**2)*sqrt(a + b*x**2))/(a*c**2 + 2*a*c*d*x**2 + a*d**2*x**4 + b*c**2*x**
2 + 2*b*c*d*x**4 + b*d**2*x**6),x)*a**2*c*d*f*x**2 + int((sqrt(c + d*x**2)
*sqrt(a + b*x**2))/(a*c**2 + 2*a*c*d*x**2 + a*d**2*x**4 + b*c**2*x**2 + 2*
b*c*d*x**4 + b*d**2*x**6),x)*a*b*c**2*e + int((sqrt(c + d*x**2)*sqrt(a + b
*x**2))/(a*c**2 + 2*a*c*d*x**2 + a*d**2*x**4 + b*c**2*x**2 + 2*b*c*d*x*...
```

3.146 $\int \frac{\sqrt{a+bx^2}(e+fx^2)}{x^2(c+dx^2)^{3/2}} dx$

Optimal result

Integrand size = 33, antiderivative size = 215

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{x^2(c+dx^2)^{3/2}} dx = -\frac{e\sqrt{a+bx^2}}{cx\sqrt{c+dx^2}} - \frac{(2de-cf)\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{c^{3/2}\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} + \frac{be\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{c}\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output
```
-e*(b*x^2+a)^(1/2)/c/x/(d*x^2+c)^(1/2)-(-c*f+2*d*e)*(b*x^2+a)^(1/2)*Ellipt
icE(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))/c^(3/2)/d^(1/2)
/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)+b*e*(b*x^2+a)^(1/2)*Inver
seJacobiAM(arctan(d^(1/2)*x/c^(1/2)),(1-b*c/a/d)^(1/2))/a/c^(1/2)/d^(1/2)/
(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.81 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.01

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{x^2\left(c+dx^2\right)^{3/2}}\,dx = \frac{-\sqrt{\frac{b}{a}}d(a+bx^2)\left(2dex^2+c(e-fx^2)\right)+ibc(-2de+cf)x\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E}{\sqrt{\frac{b}{a}}c^2}$$

input
```
Integrate[(Sqrt[a + b*x^2]*(e + f*x^2))/(x^2*(c + d*x^2)^(3/2)),x]
```

output
```
(-(Sqrt[b/a]*d*(a + b*x^2)*(2*d*e*x^2 + c*(e - f*x^2))) + I*b*c*(-2*d*e +
c*f)*x*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/
a]*x], (a*d)/(b*c)] - I*b*c*(-(d*e) + c*f)*x*Sqrt[1 + (b*x^2)/a]*Sqrt[1 +
(d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(Sqrt[b/a]*c^2*
d*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.44, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {439, 25, 445, 25, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{x^2\left(c+dx^2\right)^{3/2}}dx$$

$$\Big\downarrow \; 439$$

$$\frac{\sqrt{a+bx^2}(de-cf)}{cdx\sqrt{c+dx^2}}-\frac{\int -\frac{bdex^2+a(2de-cf)}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{cd}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{bdex^2+a(2de-cf)}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{cd}+\frac{\sqrt{a+bx^2}(de-cf)}{cdx\sqrt{c+dx^2}}$$

$$\Big\downarrow \text{445}$$

$$\frac{-\frac{\int -\frac{abd\left((2de-cf)x^2+ce\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(2de-cf)}{cx}}{cd}+\frac{\sqrt{a+bx^2}(de-cf)}{cdx\sqrt{c+dx^2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{abd\left((2de-cf)x^2+ce\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(2de-cf)}{cx}}{cd}+\frac{\sqrt{a+bx^2}(de-cf)}{cdx\sqrt{c+dx^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{bd\int \frac{(2de-cf)x^2+ce}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(2de-cf)}{cx}}{cd}+\frac{\sqrt{a+bx^2}(de-cf)}{cdx\sqrt{c+dx^2}}$$

$$\Big\downarrow \text{406}$$

$$\frac{\frac{bd\left((2de-cf)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+ce\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(2de-cf)}{cx}}{cd}+\frac{\sqrt{a+bx^2}(de-cf)}{cdx\sqrt{c+dx^2}}$$

$$\Big\downarrow \text{320}$$

$$\frac{\frac{bd\left((2de-cf)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(2de-cf)}{cx}}{cd}+\frac{\sqrt{a+bx^2}(de-cf)}{cdx\sqrt{c+dx^2}}$$

$$\Big\downarrow \text{388}$$

$$\frac{\frac{bd\left((2de-cf)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{c^{3/2}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(2de-cf)}{cx}}{cd}+\frac{\sqrt{a+bx^2}(de-cf)}{cdx\sqrt{c+dx^2}}$$

$$\Big\downarrow \text{313}$$

$$\frac{\frac{bd\left(\frac{c^{3/2}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+(2de-cf)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)\right)}{c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(2de-cf)}{cx}}{cd} -$$

$$\frac{\sqrt{a+bx^2}(de-cf)}{cdx\sqrt{c+dx^2}}$$

input
```
Int[(Sqrt[a + b*x^2]*(e + f*x^2))/(x^2*(c + d*x^2)^(3/2)),x]
```

output
```
((d*e - c*f)*Sqrt[a + b*x^2])/(c*d*x*Sqrt[c + d*x^2]) + (-(((2*d*e - c*f)*
Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(c*x)) + (b*d*((2*d*e - c*f)*((x*Sqrt[a +
 b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(
Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*
(c + d*x^2))]*Sqrt[c + d*x^2])) + (c^(3/2)*e*Sqrt[a + b*x^2]*EllipticF[Arc
Tan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*Sqrt[(c*(a + b*x^2)
)/(a*(c + d*x^2))]*Sqrt[c + d*x^2])))/c)/(c*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
 + d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 439
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
 + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*b*g*(p + 1))), x] + Simp[1/(2*a*b*(p
+ 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1)*Simp[c*(2*b*e*(
p + 1) + (b*e - a*f)*(m + 1)) + d*(2*b*e*(p + 1) + (b*e - a*f)*(m + 2*q + 1
))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && LtQ[p, -1] && G
tQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e - a*f])
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 10.29 (sec) , antiderivative size = 382, normalized size of antiderivative = 1.78

method	result
default	$\frac{\sqrt{b x^2+a}\,\sqrt{x^2 d+c}\left(\sqrt{-\frac{b}{a}}\,bcdf\,x^4-2\sqrt{-\frac{b}{a}}\,b\,d^2 e\,x^4+\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{x^2 d+c}{c}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)b\,c^2 f x-\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{x^2 d+c}{c}}\,\mathrm{Ellip}\right.}{}$
elliptic	$\frac{\sqrt{(b\,x^2+a)(x^2 d+c)}\left(-\frac{e\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{c^2x}+\frac{\left(bd\,x^2+ad\right)(cf-de)x}{c^2d\sqrt{\left(x^2+\frac{c}{d}\right)\left(bd\,x^2+ad\right)}}+\frac{\left(\frac{fb}{d}+\frac{(cf-de)(ad-bc)}{d\,c^2}-\frac{a(cf-de)}{c^2}\right)\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\mathrm{Ell}}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+x^2bc+a}}\right.}{\sqrt{b\,x^2+}}$
risch	$-\frac{e\sqrt{b\,x^2+a}\,\sqrt{x^2 d+c}}{c^2x}+\frac{\left(\frac{b\left(\frac{c^2f\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}-\frac{dec\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\left(\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+}\right.\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ac}}\right.}{d}\right.}{}$

input
```
int((b*x^2+a)^(1/2)*(f*x^2+e)/x^2/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*((-b/a)^(1/2)*b*c*d*f*x^4-2*(-b/a)^(1/2)*b
*d^2*e*x^4+((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2
),(a*d/b/c)^(1/2))*b*c^2*f*x-((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*Ellip
ticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b*c*d*e*x-((b*x^2+a)/a)^(1/2)*((d*x^2
+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b*c^2*f*x+2*((b*x^2
+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))
*b*c*d*e*x+(-b/a)^(1/2)*a*c*d*f*x^2-2*(-b/a)^(1/2)*a*d^2*e*x^2-(-b/a)^(1/2
)*b*c*d*e*x^2-(-b/a)^(1/2)*a*c*d*e)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)/c^2/x/(-
b/a)^(1/2)/d
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{x^2\left(c+dx^2\right)^{3/2}}\,dx=\frac{((2\,bd^2e-bcdf)x^3+(2\,bcde-bc^2f)x)\sqrt{ac}\sqrt{-\frac{b}{a}}E(\arcsin\left(x\sqrt{-\frac{b}{a}}\right)\mid\frac{ad}{bc})-(((}{}$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)/x^2/(d*x^2+c)^(3/2),x, algorithm="fric
as")
```

output
```
(((2*b*d^2*e - b*c*d*f)*x^3 + (2*b*c*d*e - b*c^2*f)*x)*sqrt(a*c)*sqrt(-b/a
)*elliptic_e(arcsin(x*sqrt(-b/a)), a*d/(b*c)) - (((a + 2*b)*d^2*e - b*c*d*
f)*x^3 + ((a + 2*b)*c*d*e - b*c^2*f)*x)*sqrt(a*c)*sqrt(-b/a)*elliptic_f(ar
csin(x*sqrt(-b/a)), a*d/(b*c)) - (a*c*d*e + (2*a*d^2*e - a*c*d*f)*x^2)*sqr
t(b*x^2 + a)*sqrt(d*x^2 + c))/(a*c^2*d^2*x^3 + a*c^3*d*x)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{x^2\left(c+dx^2\right)^{3/2}}\,dx=\int \frac{\sqrt{a+bx^2}(e+fx^2)}{x^2\left(c+dx^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(f*x**2+e)/x**2/(d*x**2+c)**(3/2),x)
```

output
```
Integral(sqrt(a + b*x**2)*(e + f*x**2)/(x**2*(c + d*x**2)**(3/2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{x^2\left(c+dx^2\right)^{3/2}}\,dx=\int \frac{\sqrt{bx^2+a}(fx^2+e)}{(dx^2+c)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)/x^2/(d*x^2+c)^(3/2),x, algorithm="maxi
ma")
```

output
```
integrate(sqrt(b*x^2 + a)*(f*x^2 + e)/((d*x^2 + c)^(3/2)*x^2), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{x^2\left(c+dx^2\right)^{3/2}}\,dx = \int \frac{\sqrt{bx^2+a}(fx^2+e)}{(dx^2+c)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)/x^2/(d*x^2+c)^(3/2),x, algorithm="giac
")
```

output
```
integrate(sqrt(b*x^2 + a)*(f*x^2 + e)/((d*x^2 + c)^(3/2)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{x^2\left(c+dx^2\right)^{3/2}}\,dx = \int \frac{\sqrt{b\,x^2+a}\,(f\,x^2+e)}{x^2\,{(d\,x^2+c)}^{3/2}}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(e + f*x^2))/(x^2*(c + d*x^2)^(3/2)),x)
```

output
```
int(((a + b*x^2)^(1/2)*(e + f*x^2))/(x^2*(c + d*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{x^2\left(c+dx^2\right)^{3/2}}\,dx = \frac{-\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,e+\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^2}{b\,d^2x^6+a\,d^2x^4+2bcd\,x^4+2acd\,x^2+b\,c^2x^2+a\,c^2}dx\right)b\,c^2fx-}{}$$

input
```
int((b*x^2+a)^(1/2)*(f*x^2+e)/x^2/(d*x^2+c)^(3/2),x)
```

output
```
( - sqrt(c + d*x**2)*sqrt(a + b*x**2)*e + int((sqrt(c + d*x**2)*sqrt(a + b
*x**2)*x**2)/(a*c**2 + 2*a*c*d*x**2 + a*d**2*x**4 + b*c**2*x**2 + 2*b*c*d*
x**4 + b*d**2*x**6),x)*b*c**2*f*x - int((sqrt(c + d*x**2)*sqrt(a + b*x**2)
*x**2)/(a*c**2 + 2*a*c*d*x**2 + a*d**2*x**4 + b*c**2*x**2 + 2*b*c*d*x**4 +
 b*d**2*x**6),x)*b*c*d*e*x + int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/
(a*c**2 + 2*a*c*d*x**2 + a*d**2*x**4 + b*c**2*x**2 + 2*b*c*d*x**4 + b*d**2
*x**6),x)*b*c*d*f*x**3 - int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c
**2 + 2*a*c*d*x**2 + a*d**2*x**4 + b*c**2*x**2 + 2*b*c*d*x**4 + b*d**2*x**
6),x)*b*d**2*e*x**3 + int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c**2 + 2*
a*c*d*x**2 + a*d**2*x**4 + b*c**2*x**2 + 2*b*c*d*x**4 + b*d**2*x**6),x)*a*
c**2*f*x - 2*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c**2 + 2*a*c*d*x**
2 + a*d**2*x**4 + b*c**2*x**2 + 2*b*c*d*x**4 + b*d**2*x**6),x)*a*c*d*e*x +
 int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c**2 + 2*a*c*d*x**2 + a*d**2*x
**4 + b*c**2*x**2 + 2*b*c*d*x**4 + b*d**2*x**6),x)*a*c*d*f*x**3 - 2*int((s
qrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c**2 + 2*a*c*d*x**2 + a*d**2*x**4 + b
*c**2*x**2 + 2*b*c*d*x**4 + b*d**2*x**6),x)*a*d**2*e*x**3 + int((sqrt(c +
d*x**2)*sqrt(a + b*x**2))/(a*c**2 + 2*a*c*d*x**2 + a*d**2*x**4 + b*c**2*x*
*2 + 2*b*c*d*x**4 + b*d**2*x**6),x)*b*c**2*e*x + int((sqrt(c + d*x**2)*sqr
t(a + b*x**2))/(a*c**2 + 2*a*c*d*x**2 + a*d**2*x**4 + b*c**2*x**2 + 2*b*c*
d*x**4 + b*d**2*x**6),x)*b*c*d*e*x**3)/(c*x*(c + d*x**2))
```

3.147 $\int \frac{\sqrt{a+bx^2}(e+fx^2)}{x^4(c+dx^2)^{3/2}} \, dx$

Optimal result

Integrand size = 33, antiderivative size = 289

$$\begin{aligned}\int \frac{\sqrt{a+bx^2}(e+fx^2)}{x^4(c+dx^2)^{3/2}} \, dx &= -\frac{e\sqrt{a+bx^2}}{3cx^3\sqrt{c+dx^2}} - \frac{(bce-4ade+3acf)\sqrt{a+bx^2}}{3ac^2x\sqrt{c+dx^2}} \\ &\quad - \frac{\sqrt{d}(bce-8ade+6acf)\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3ac^{5/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} \\ &\quad - \frac{b(4de-3cf)\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3ac^{3/2}\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}\end{aligned}$$

output

```
-1/3*e*(b*x^2+a)^(1/2)/c/x^3/(d*x^2+c)^(1/2)-1/3*(3*a*c*f-4*a*d*e+b*c*e)*(
b*x^2+a)^(1/2)/a/c^2/x/(d*x^2+c)^(1/2)-1/3*d^(1/2)*(6*a*c*f-8*a*d*e+b*c*e)
*(b*x^2+a)^(1/2)*EllipticE(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)
^(1/2))/a/c^(5/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-1/3*b*(-
3*c*f+4*d*e)*(b*x^2+a)^(1/2)*InverseJacobiAM(arctan(d^(1/2)*x/c^(1/2)),(1-
b*c/a/d)^(1/2))/a/c^(3/2)/d^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c
)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.36 (sec) , antiderivative size = 274, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{x^4 (c+dx^2)^{3/2}}\,dx = \frac{\sqrt{\frac{b}{a}}\left(-\sqrt{\frac{b}{a}}(a+bx^2)\left(bcex^2(c+dx^2)+a(-8d^2ex^4+c^2(e+3fx^2)+cd(-4ex^2\right.\right.}{}$$

input
```
Integrate[(Sqrt[a + b*x^2]*(e + f*x^2))/(x^4*(c + d*x^2)^(3/2)),x]
```

output
```
(Sqrt[b/a]*(-(Sqrt[b/a]*(a + b*x^2)*(b*c*e*x^2*(c + d*x^2) + a*(-8*d^2*e*x
^4 + c^2*(e + 3*f*x^2) + c*d*(-4*e*x^2 + 6*f*x^4)))) - I*b*c*(b*c*e - 8*a*
d*e + 6*a*c*f)*x^3*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*Arc
Sinh[Sqrt[b/a]*x], (a*d)/(b*c)] + I*b*c*(b*c*e - 4*a*d*e + 3*a*c*f)*x^3*Sq
rt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a
*d)/(b*c)]))/(3*b*c^3*x^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.33, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {439, 25, 445, 25, 27, 445, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{x^4 (c+dx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{439}$$

$$\frac{\sqrt{a+bx^2}(de-cf)}{cdx^3\sqrt{c+dx^2}} - \frac{\int -\frac{b(3de-2cf)x^2+a(4de-3cf)}{x^4\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{cd}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{b(3de-2cf)x^2+a(4de-3cf)}{x^4\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{cd}+\frac{\sqrt{a+bx^2}(de-cf)}{cdx^3\sqrt{c+dx^2}}$$

$$\Big\downarrow \text{445}$$

$$\frac{-\frac{\int -\frac{ad\left(-b(4de-3cf)x^2+bce-8ade+6acf\right)}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(4de-3cf)}{3cx^3}}{cd}+\frac{\sqrt{a+bx^2}(de-cf)}{cdx^3\sqrt{c+dx^2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{ad\left(-b(4de-3cf)x^2+bce-8ade+6acf\right)}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(4de-3cf)}{3cx^3}}{cd}+\frac{\sqrt{a+bx^2}(de-cf)}{cdx^3\sqrt{c+dx^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{d\int \frac{-b(4de-3cf)x^2+bce-8ade+6acf}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(4de-3cf)}{3cx^3}}{cd}+\frac{\sqrt{a+bx^2}(de-cf)}{cdx^3\sqrt{c+dx^2}}$$

$$\Big\downarrow \text{445}$$

$$\frac{\frac{d\left(-\frac{\int \frac{b\left(ac(4de-3cf)-d(bce-8ade+6acf)x^2\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(6acf-8ade+bce)}{acx}\right)}{3c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(4de-3cf)}{3cx^3}}{cd}+\frac{\sqrt{a+bx^2}(de-cf)}{cdx^3\sqrt{c+dx^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{d\left(-\frac{b\int \frac{ac(4de-3cf)-d(bce-8ade+6acf)x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(6acf-8ade+bce)}{acx}\right)}{3c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(4de-3cf)}{3cx^3}}{cd}+\frac{\sqrt{a+bx^2}(de-cf)}{cdx^3\sqrt{c+dx^2}}$$

$$\Big\downarrow \text{406}$$

$$\frac{\frac{d\left(-\frac{b\left(ac(4de-3cf)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx-d(6acf-8ade+bce)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(6acf-8ade+bce)}{acx}\right)}{3c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(4de-}{3cx^3}}{cd}$$

$$\frac{\sqrt{a+bx^2}(de-cf)}{cdx^3\sqrt{c+dx^2}}$$

$$\Big\downarrow \; 320$$

$$\frac{\frac{d\left(-\frac{b\left(\frac{c^{3/2}\sqrt{a+bx^2}(4de-3cf)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-d(6acf-8ade+bce)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(6acf-8ade+bce)}{acx}\right)}{3c}-\sqrt{\cdots}}{cd}$$

$$\frac{\sqrt{a+bx^2}(de-cf)}{cdx^3\sqrt{c+dx^2}}$$

$$\Big\downarrow \; 388$$

$$\frac{\frac{d\left(-\frac{b\left(\frac{c^{3/2}\sqrt{a+bx^2}(4de-3cf)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-d(6acf-8ade+bce)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}}dx}{b}\right)\right)}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(6acf-8ade+b\cdots}{acx}\cdots\right)}{3c}\cdots}{cd}$$

$$\frac{\sqrt{a+bx^2}(de-cf)}{cdx^3\sqrt{c+dx^2}}$$

$$\Big\downarrow \; 313$$

$$\frac{\frac{d\left(-\frac{b\left(\frac{c^{3/2}\sqrt{a+bx^2}(4de-3cf)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-d(6acf-8ade+bce)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)\right)}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+\cdots}}{\cdots}\right)}{3c}\cdots}{cd}$$

$$\frac{\sqrt{a+bx^2}(de-cf)}{cdx^3\sqrt{c+dx^2}}$$

input
```
Int[(Sqrt[a + b*x^2]*(e + f*x^2))/(x^4*(c + d*x^2)^(3/2)),x]
```

output
```
((d*e - c*f)*Sqrt[a + b*x^2])/(c*d*x^3*Sqrt[c + d*x^2]) + (-1/3*((4*d*e -
3*c*f)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(c*x^3) + (d*(-(((b*c*e - 8*a*d*e
+ 6*a*c*f)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(a*c*x)) - (b*(-(d*(b*c*e - 8*
a*d*e + 6*a*c*f)*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[
a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqr
t[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))) + (c^(3/2)*(
4*d*e - 3*c*f)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 -
(b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^
2])))/(a*c)))/(3*c))/(c*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
 + d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 439
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
 + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*b*g*(p + 1))), x] + Simp[1/(2*a*b*(p
+ 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1)*Simp[c*(2*b*e*(
p + 1) + (b*e - a*f)*(m + 1)) + d*(2*b*e*(p + 1) + (b*e - a*f)*(m + 2*q + 1
))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && LtQ[p, -1] && G
tQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e - a*f])
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 9.86 (sec) , antiderivative size = 460, normalized size of antiderivative = 1.59

method	result
elliptic	$\frac{\sqrt{(b x^2+a)(x^2 d+c)}\left(-\frac{e\sqrt{bd x^4+ad x^2+x^2 bc+ac}}{3c^2 x^3}-\frac{(3acf-5ade+bce)\sqrt{bd x^4+ad x^2+x^2 bc+ac}}{3a c^3 x}-\frac{\left(bd x^2+ad\right)(cf-de)x}{c^3\sqrt{\left(x^2+\frac{c}{d}\right)\left(bd x^2+ad\right)}}+\frac{\left(-\frac{dbe}{3c^2}-\frac{(cf-d}{}\right.}{}\right.}{}$
default	$-\frac{\sqrt{b x^2+a}\sqrt{x^2 d+c}\left(6\sqrt{-\frac{b}{a}}\, abcdf\, x^6-8\sqrt{-\frac{b}{a}}\, ab\, d^2 e\, x^6+\sqrt{-\frac{b}{a}}\, b^2 cde\, x^6+3\sqrt{\frac{b x^2+a}{a}}\sqrt{\frac{x^2 d+c}{c}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)ab\, c^2 f\, x^3\right.}{}$
risch	$-\frac{\sqrt{b x^2+a}\sqrt{x^2 d+c}\left(3acf\, x^2-5ade\, x^2+bce\, x^2+ace\right)}{3c^3 x^3 a}+\frac{\left(-\frac{b(3acf-5ade+bce)c\sqrt{1+\frac{b x^2}{a}}\sqrt{1+\frac{d x^2}{c}}\left(\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)\right.}{\sqrt{-\frac{b}{a}}\sqrt{bd x^4+ad x^2+x^2 bc+ac}}\right.}{}$

input
```
int((b*x^2+a)^(1/2)*(f*x^2+e)/x^4/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-1/3/c^2*e*(b
*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/x^3-1/3/a/c^3*(3*a*c*f-5*a*d*e+b*c*e)*(b
*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/x-(b*d*x^2+a*d)*(c*f-d*e)/c^3*x/((x^2+c/
d)*(b*d*x^2+a*d))^(1/2)+(-1/3*d*b/c^2*e-(c*f-d*e)*(a*d-b*c)/c^3+a*d*(c*f-d
*e)/c^3)/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2
+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-(1/
3*b*d*(3*a*c*f-5*a*d*e+b*c*e)/a/c^3+b*d*(c*f-d*e)/c^3)*c/(-b/a)^(1/2)*(1+b
*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*(Ell
ipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(
-1+(a*d+b*c)/c/b)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.13

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{x^4\left(c+dx^2\right)^{3/2}}\,dx = \frac{((6\,abcdf+(b^2cd-8\,abd^2)e)x^5+(6\,abc^2f+(b^2c^2-8\,abcd)e)x^3)\sqrt{ac}\sqrt{-\frac{b}{a}}E(}{}$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)/x^4/(d*x^2+c)^(3/2),x, algorithm="fric
as")
```

output
```
1/3*(((6*a*b*c*d*f + (b^2*c*d - 8*a*b*d^2)*e)*x^5 + (6*a*b*c^2*f + (b^2*c^
2 - 8*a*b*c*d)*e)*x^3)*sqrt(a*c)*sqrt(-b/a)*elliptic_e(arcsin(x*sqrt(-b/a)
), a*d/(b*c)) - ((3*(a^2 + 2*a*b)*c*d*f + (b^2*c*d - 4*(a^2 + 2*a*b)*d^2)*
e)*x^5 + (3*(a^2 + 2*a*b)*c^2*f + (b^2*c^2 - 4*(a^2 + 2*a*b)*c*d)*e)*x^3)*
sqrt(a*c)*sqrt(-b/a)*elliptic_f(arcsin(x*sqrt(-b/a)), a*d/(b*c)) - (a^2*c^
2*e + (6*a^2*c*d*f + (a*b*c*d - 8*a^2*d^2)*e)*x^4 + (3*a^2*c^2*f + (a*b*c^
2 - 4*a^2*c*d)*e)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(a^2*c^3*d*x^5 + a
^2*c^4*x^3)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{x^4(c+dx^2)^{3/2}}\,dx = \int \frac{\sqrt{a+bx^2}(e+fx^2)}{x^4(c+dx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(f*x**2+e)/x**4/(d*x**2+c)**(3/2),x)
```

output
```
Integral(sqrt(a + b*x**2)*(e + f*x**2)/(x**4*(c + d*x**2)**(3/2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{x^4(c+dx^2)^{3/2}}\,dx = \int \frac{\sqrt{bx^2+a}(fx^2+e)}{(dx^2+c)^{\frac{3}{2}}x^4}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)/x^4/(d*x^2+c)^(3/2),x, algorithm="maxi
ma")
```

output
```
integrate(sqrt(b*x^2 + a)*(f*x^2 + e)/((d*x^2 + c)^(3/2)*x^4), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{x^4(c+dx^2)^{3/2}}\,dx = \int \frac{\sqrt{bx^2+a}(fx^2+e)}{(dx^2+c)^{\frac{3}{2}}x^4}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)/x^4/(d*x^2+c)^(3/2),x, algorithm="giac
")
```

output
```
integrate(sqrt(b*x^2 + a)*(f*x^2 + e)/((d*x^2 + c)^(3/2)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{x^4\left(c+dx^2\right)^{3/2}}\,dx=\int \frac{\sqrt{b\,x^2+a}\,\left(f\,x^2+e\right)}{x^4\,{\left(d\,x^2+c\right)}^{3/2}}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(e + f*x^2))/(x^4*(c + d*x^2)^(3/2)),x)
```

output
```
int(((a + b*x^2)^(1/2)*(e + f*x^2))/(x^4*(c + d*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{x^4\left(c+dx^2\right)^{3/2}}\,dx=\text{too large to display}$$

input
```
int((b*x^2+a)^(1/2)*(f*x^2+e)/x^4/(d*x^2+c)^(3/2),x)
```

output
```
( - sqrt(c + d*x**2)*sqrt(a + b*x**2)*e + 6*int((sqrt(c + d*x**2)*sqrt(a +
 b*x**2))/(2*a**2*c**2*d*x**2 + 4*a**2*c*d**2*x**4 + 2*a**2*d**3*x**6 + a*
b*c**3*x**2 + 4*a*b*c**2*d*x**4 + 5*a*b*c*d**2*x**6 + 2*a*b*d**3*x**8 + b*
*2*c**3*x**4 + 2*b**2*c**2*d*x**6 + b**2*c*d**2*x**8),x)*a**2*c**2*d*f*x**
3 - 8*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(2*a**2*c**2*d*x**2 + 4*a**2
*c*d**2*x**4 + 2*a**2*d**3*x**6 + a*b*c**3*x**2 + 4*a*b*c**2*d*x**4 + 5*a*
b*c*d**2*x**6 + 2*a*b*d**3*x**8 + b**2*c**3*x**4 + 2*b**2*c**2*d*x**6 + b*
*2*c*d**2*x**8),x)*a**2*c*d**2*e*x**3 + 6*int((sqrt(c + d*x**2)*sqrt(a + b
*x**2))/(2*a**2*c**2*d*x**2 + 4*a**2*c*d**2*x**4 + 2*a**2*d**3*x**6 + a*b*
c**3*x**2 + 4*a*b*c**2*d*x**4 + 5*a*b*c*d**2*x**6 + 2*a*b*d**3*x**8 + b**2
*c**3*x**4 + 2*b**2*c**2*d*x**6 + b**2*c*d**2*x**8),x)*a**2*c*d**2*f*x**5
- 8*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(2*a**2*c**2*d*x**2 + 4*a**2*c
*d**2*x**4 + 2*a**2*d**3*x**6 + a*b*c**3*x**2 + 4*a*b*c**2*d*x**4 + 5*a*b*
c*d**2*x**6 + 2*a*b*d**3*x**8 + b**2*c**3*x**4 + 2*b**2*c**2*d*x**6 + b**2
*c*d**2*x**8),x)*a**2*d**3*e*x**5 + 3*int((sqrt(c + d*x**2)*sqrt(a + b*x**
2))/(2*a**2*c**2*d*x**2 + 4*a**2*c*d**2*x**4 + 2*a**2*d**3*x**6 + a*b*c**3
*x**2 + 4*a*b*c**2*d*x**4 + 5*a*b*c*d**2*x**6 + 2*a*b*d**3*x**8 + b**2*c**
3*x**4 + 2*b**2*c**2*d*x**6 + b**2*c*d**2*x**8),x)*a*b*c**3*f*x**3 - 2*int
((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(2*a**2*c**2*d*x**2 + 4*a**2*c*d**2*x
**4 + 2*a**2*d**3*x**6 + a*b*c**3*x**2 + 4*a*b*c**2*d*x**4 + 5*a*b*c*d*...
```

3.148 $\int \frac{\sqrt{a+bx^2}(e+fx^2)}{x^6(c+dx^2)^{3/2}} \, dx$

Optimal result

Integrand size = 33, antiderivative size = 391

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{x^6\left(c+dx^2\right)^{3/2}} \, dx = -\frac{e\sqrt{a+bx^2}}{5cx^5\sqrt{c+dx^2}} - \frac{(bce-6ade+5acf)\sqrt{a+bx^2}}{15ac^2x^3\sqrt{c+dx^2}}$$
$$+ \frac{\left(2b^2c^2e-4a^2d(6de-5cf)+abc(7de-5cf)\right)\sqrt{a+bx^2}}{15a^2c^3x\sqrt{c+dx^2}}$$
$$+ \frac{\sqrt{d}\left(2b^2c^2e-8a^2d(6de-5cf)+abc(8de-5cf)\right)\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15a^2c^{7/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$
$$- \frac{b\sqrt{d}(bce-24ade+20acf)\sqrt{a+bx^2}\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{15a^2c^{5/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output
```
-1/5*e*(b*x^2+a)^(1/2)/c/x^5/(d*x^2+c)^(1/2)-1/15*(5*a*c*f-6*a*d*e+b*c*e)*
(b*x^2+a)^(1/2)/a/c^2/x^3/(d*x^2+c)^(1/2)+1/15*(2*b^2*c^2*e-4*a^2*d*(-5*c*
f+6*d*e)+a*b*c*(-5*c*f+7*d*e))*(b*x^2+a)^(1/2)/a^2/c^3/x/(d*x^2+c)^(1/2)+1
/15*d^(1/2)*(2*b^2*c^2*e-8*a^2*d*(-5*c*f+6*d*e)+a*b*c*(-5*c*f+8*d*e))*(b*x
^2+a)^(1/2)*EllipticE(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2
))/a^2/c^(7/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-1/15*b*d^(1
/2)*(20*a*c*f-24*a*d*e+b*c*e)*(b*x^2+a)^(1/2)*InverseJacobiAM(arctan(d^(1/
2)*x/c^(1/2)),(1-b*c/a/d)^(1/2))/a^2/c^(5/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/
2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.22 (sec) , antiderivative size = 383, normalized size of antiderivative = 0.98

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{x^6\,(c+dx^2)^{3/2}}\,dx = \frac{\sqrt{\frac{b}{a}}(a+bx^2)\left(2b^2c^2ex^4(c+dx^2)-abcx^2(c+dx^2)\left(-8dex^2+c(e+5fx^2)\right)+\cdots\right.}{\cdots}$$

input
```
Integrate[(Sqrt[a + b*x^2]*(e + f*x^2))/(x^6*(c + d*x^2)^(3/2)),x]
```

output
```
(Sqrt[b/a]*(a + b*x^2)*(2*b^2*c^2*e*x^4*(c + d*x^2) - a*b*c*x^2*(c + d*x^2
)*(-8*d*e*x^2 + c*(e + 5*f*x^2)) + a^2*(-48*d^3*e*x^6 + 8*c*d^2*x^4*(-3*e
+ 5*f*x^2) - c^3*(3*e + 5*f*x^2) + c^2*d*(6*e*x^2 + 20*f*x^4))) + I*b*c*(2
*b^2*c^2*e + a*b*c*(8*d*e - 5*c*f) + 8*a^2*d*(-6*d*e + 5*c*f))*x^5*Sqrt[1
+ (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(
b*c)] - I*b*c*(2*b^2*c^2*e + a*b*c*(7*d*e - 5*c*f) + 4*a^2*d*(-6*d*e + 5*c
*f))*x^5*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[
b/a]*x], (a*d)/(b*c)])/(15*a^2*Sqrt[b/a]*c^4*x^5*Sqrt[a + b*x^2]*Sqrt[c +
d*x^2])
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 489, normalized size of antiderivative = 1.25, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.394$, Rules used = {439, 25, 445, 25, 27, 445, 445, 25, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\,(e+fx^2)}{x^6\,(c+dx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{439}$$

$$\frac{\sqrt{a+bx^2}(de-cf)}{cdx^5\sqrt{c+dx^2}}-\frac{\int -\frac{b(5de-4cf)x^2+a(6de-5cf)}{x^6\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{cd}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{b(5de-4cf)x^2+a(6de-5cf)}{x^6\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{cd}+\frac{\sqrt{a+bx^2}(de-cf)}{cdx^5\sqrt{c+dx^2}}$$

$$\Big\downarrow \text{445}$$

$$\frac{-\frac{\int -\frac{ad\left(-3b(6de-5cf)x^2+bce-24ade+20acf\right)}{x^4\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{5ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(6de-5cf)}{5cx^5}}{cd}+\frac{\sqrt{a+bx^2}(de-cf)}{cdx^5\sqrt{c+dx^2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{ad\left(-3b(6de-5cf)x^2+bce-24ade+20acf\right)}{x^4\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{5ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(6de-5cf)}{5cx^5}}{cd}+\frac{\sqrt{a+bx^2}(de-cf)}{cdx^5\sqrt{c+dx^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{d\int \frac{-3b(6de-5cf)x^2+bce-24ade+20acf}{x^4\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{5c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(6de-5cf)}{5cx^5}}{cd}+\frac{\sqrt{a+bx^2}(de-cf)}{cdx^5\sqrt{c+dx^2}}$$

$$\Big\downarrow \text{445}$$

$$\frac{\frac{d\left(-\frac{\int \frac{-8d(6de-5cf)a^2+bc(8de-5cf)a+bd(bce-24ade+20acf)x^2+2b^2c^2e}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(20acf-24ade+bce)}{3acx^3}\right)}{5c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(6de-5cf)}{5cx^5}}{cd}+$$

$$\frac{\sqrt{a+bx^2}(de-cf)}{cdx^5\sqrt{c+dx^2}}$$

$\downarrow$ 445

$$\frac{\frac{d\left(-\frac{-\frac{\int -\frac{bd\left(\left(-8d(6de-5cf)a^2+bc(8de-5cf)a+2b^2c^2e\right)x^2+ac(bce-24ade+20acf)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{2b^2ce}{a}-\frac{8ad(6de-5cf)}{c}+b(8de-5cf)\right)}{x}}{3ac}-\frac{\sqrt{a+bx^2}\cdots}{\cdots}\right)}{5c}}{cd}$$

$$\frac{\sqrt{a+bx^2}(de-cf)}{cdx^5\sqrt{c+dx^2}}$$

$\downarrow$ 25

$$\frac{\frac{d\left(-\frac{-\frac{\int \frac{bd\left(\left(-8d(6de-5cf)a^2+bc(8de-5cf)a+2b^2c^2e\right)x^2+ac(bce-24ade+20acf)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{2b^2ce}{a}-\frac{8ad(6de-5cf)}{c}+b(8de-5cf)\right)}{x}}{3ac}-\frac{\sqrt{a+bx^2}\sqrt{c\cdots}}{\cdots}\right)}{5c}}{cd}$$

$$\frac{\sqrt{a+bx^2}(de-cf)}{cdx^5\sqrt{c+dx^2}}$$

$\downarrow$ 27

$$\frac{\frac{d\left(-\frac{-\frac{bd\int \frac{\left(-8d(6de-5cf)a^2+bc(8de-5cf)a+2b^2c^2e\right)x^2+ac(bce-24ade+20acf)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{2b^2ce}{a}-\frac{8ad(6de-5cf)}{c}+b(8de-5cf)\right)}{x}}{3ac}-\frac{\sqrt{a+bx^2}\sqrt{c+\cdots}}{\cdots}\right)}{5c}}{cd}$$

$$\frac{\sqrt{a+bx^2}(de-cf)}{cdx^5\sqrt{c+dx^2}}$$

$\downarrow$ 406

$$\frac{d\left(-\frac{\frac{bd\left(\left(-8a^2d(6de-5cf)+abc(8de-5cf)+2b^2c^2e\right)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+ac(20acf-24ade+bce)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{ac}}{3ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{2b^2ce}{a}-\frac{8ad(6\cdots}{}\right.}{x}\cdots\right)}{5c}}{cd}$$

$$\frac{\sqrt{a+bx^2}(de-cf)}{cdx^5\sqrt{c+dx^2}}$$

$\big\downarrow$ 320

$$\frac{d\left(-\frac{\frac{bd\left(\left(-8a^2d(6de-5cf)+abc(8de-5cf)+2b^2c^2e\right)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}(20acf-24ade+bce)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{ac}}{3ac}-\frac{\sqrt{a+bx^2}\cdots}{}\cdots\right)}{5c}}{cd}$$

$$\frac{\sqrt{a+bx^2}(de-cf)}{cdx^5\sqrt{c+dx^2}}$$

$\big\downarrow$ 388

$$\frac{d\left(-\frac{\frac{bd\left(\left(-8a^2d(6de-5cf)+abc(8de-5cf)+2b^2c^2e\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(20acf-24ade+bce)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\cdots\right)}{ac}}{3ac}\cdots\right)}{5c}}{cd}$$

$$\frac{\sqrt{a+bx^2}(de-cf)}{cdx^5\sqrt{c+dx^2}}$$

$\big\downarrow$ 313

$$d\left(-\frac{bd\left(\left(-8a^2d(6de-5cf)+abc(8de-5cf)+2b^2c^2e\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(20acf-24ade+bce)\,\text{EllipticF}\left(\text{arcta}\right.}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}}{ac} \over 3ac \right. \over 5c} \over cd$$

$$\frac{\sqrt{a+bx^2}(de-cf)}{cdx^5\sqrt{c+dx^2}}$$

input
```
Int[(Sqrt[a + b*x^2]*(e + f*x^2))/(x^6*(c + d*x^2)^(3/2)),x]
```

output
```
((d*e - c*f)*Sqrt[a + b*x^2])/(c*d*x^5*Sqrt[c + d*x^2]) + (-1/5*((6*d*e -
5*c*f)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(c*x^5) + (d*(-1/3*((b*c*e - 24*a*
d*e + 20*a*c*f)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(a*c*x^3) - (-((((2*b^2*c
*e)/a - (8*a*d*(6*d*e - 5*c*f))/c + b*(8*d*e - 5*c*f))*Sqrt[a + b*x^2]*Sqr
t[c + d*x^2])/x) + (b*d*((2*b^2*c^2*e - 8*a^2*d*(6*d*e - 5*c*f) + a*b*c*(8
*d*e - 5*c*f))*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a
+ b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[
d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (c^(3/2)*(b*c
*e - 24*a*d*e + 20*a*c*f)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqr
t[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sq
rt[c + d*x^2])))/(a*c))/(3*a*c)))/(5*c))/(c*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 439
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
 + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*b*g*(p + 1))), x] + Simp[1/(2*a*b*(p
+ 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1)*Simp[c*(2*b*e*(
p + 1) + (b*e - a*f)*(m + 1)) + d*(2*b*e*(p + 1) + (b*e - a*f)*(m + 2*q + 1
))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && LtQ[p, -1] && G
tQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e - a*f])
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 19.50 (sec) , antiderivative size = 587, normalized size of antiderivative = 1.50

method	result
elliptic	$\frac{\sqrt{(b\,x^2+a)(x^2d+c)}\left(-\frac{e\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{5c^2x^5}-\frac{(5acf-9ade+bce)\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{15a\,c^3x^3}+\frac{\left(25a^2cfd-33a^2d^2e-5ab\,c^2f+8abcde+2b^2c\right.}{15a^2c^4x}\right.}{}$
risch	$-\frac{\sqrt{b\,x^2+a}\,\sqrt{x^2d+c}\,(-25a^2cdf\,x^4+33a^2d^2e\,x^4+5ab\,c^2f\,x^4-8abcde\,x^4-2b^2c^2e\,x^4+5a^2c^2f\,x^2-9a^2cde\,x^2+ab\,c^2e\,x^2+3a^2c^2e)}{15c^4x^5a^2}-$
default	Expression too large to display

input
```
int((b*x^2+a)^(1/2)*(f*x^2+e)/x^6/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-1/5/c^2*e*(b
*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/x^5-1/15/a/c^3*(5*a*c*f-9*a*d*e+b*c*e)*(
b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/x^3+1/15/a^2/c^4*(25*a^2*c*d*f-33*a^2*d
^2*e-5*a*b*c^2*f+8*a*b*c*d*e+2*b^2*c^2*e)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1
/2)/x+(b*d*x^2+a*d)*(c*f-d*e)*d/c^4*x/((x^2+c/d)*(b*d*x^2+a*d))^(1/2)+(-1/
15*b*d*(5*a*c*f-9*a*d*e+b*c*e)/c^3/a+(c*f-d*e)*d*(a*d-b*c)/c^4-a*d^2*(c*f-
d*e)/c^4)/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^
2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-(-
1/15*b*d*(25*a^2*c*d*f-33*a^2*d^2*e-5*a*b*c^2*f+8*a*b*c*d*e+2*b^2*c^2*e)/a
^2/c^4-(c*f-d*e)*d^2*b/c^4)*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(
1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(-b/a)^(1/2),(-1+(
a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 552, normalized size of antiderivative = 1.41

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{x^6\,(c+dx^2)^{3/2}}\,dx =$$

$$-\frac{((2\,(b^3c^2d+4\,ab^2cd^2-24\,a^2bd^3)e-5\,(ab^2c^2d-8\,a^2bcd^2)f)x^7+(2\,(b^3c^3+4\,ab^2c^2d-24\,a^2bcd^2)e-5\,(a}{}$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)/x^6/(d*x^2+c)^(3/2),x, algorithm="fric
as")
```

output
```
-1/15*(((2*(b^3*c^2*d + 4*a*b^2*c*d^2 - 24*a^2*b*d^3)*e - 5*(a*b^2*c^2*d -
 8*a^2*b*c*d^2)*f)*x^7 + (2*(b^3*c^3 + 4*a*b^2*c^2*d - 24*a^2*b*c*d^2)*e -
 5*(a*b^2*c^3 - 8*a^2*b*c^2*d)*f)*x^5)*sqrt(a*c)*sqrt(-b/a)*elliptic_e(arc
sin(x*sqrt(-b/a)), a*d/(b*c)) - (((2*b^3*c^2*d + (a^2*b + 8*a*b^2)*c*d^2 -
 24*(a^3 + 2*a^2*b)*d^3)*e - 5*(a*b^2*c^2*d - 4*(a^3 + 2*a^2*b)*c*d^2)*f)*
x^7 + ((2*b^3*c^3 + (a^2*b + 8*a*b^2)*c^2*d - 24*(a^3 + 2*a^2*b)*c*d^2)*e
- 5*(a*b^2*c^3 - 4*(a^3 + 2*a^2*b)*c^2*d)*f)*x^5)*sqrt(a*c)*sqrt(-b/a)*ell
iptic_f(arcsin(x*sqrt(-b/a)), a*d/(b*c)) + (3*a^3*c^3*e - (2*(a*b^2*c^2*d
+ 4*a^2*b*c*d^2 - 24*a^3*d^3)*e - 5*(a^2*b*c^2*d - 8*a^3*c*d^2)*f)*x^6 - (
(2*a*b^2*c^3 + 7*a^2*b*c^2*d - 24*a^3*c*d^2)*e - 5*(a^2*b*c^3 - 4*a^3*c^2*
d)*f)*x^4 + (5*a^3*c^3*f + (a^2*b*c^3 - 6*a^3*c^2*d)*e)*x^2)*sqrt(b*x^2 +
a)*sqrt(d*x^2 + c))/(a^3*c^4*d*x^7 + a^3*c^5*x^5)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{x^6\,(c+dx^2)^{3/2}}\,dx = \int \frac{\sqrt{a+bx^2}(e+fx^2)}{x^6\,(c+dx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(f*x**2+e)/x**6/(d*x**2+c)**(3/2),x)
```

output
```
Integral(sqrt(a + b*x**2)*(e + f*x**2)/(x**6*(c + d*x**2)**(3/2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{x^6\,(c+dx^2)^{3/2}}\,dx = \int \frac{\sqrt{bx^2+a}(fx^2+e)}{(dx^2+c)^{\frac{3}{2}}x^6}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)/x^6/(d*x^2+c)^(3/2),x, algorithm="maxi
ma")
```

output
```
integrate(sqrt(b*x^2 + a)*(f*x^2 + e)/((d*x^2 + c)^(3/2)*x^6), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{x^6\,(c+dx^2)^{3/2}}\,dx = \int \frac{\sqrt{bx^2+a}(fx^2+e)}{(dx^2+c)^{\frac{3}{2}}x^6}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(f*x^2+e)/x^6/(d*x^2+c)^(3/2),x, algorithm="giac
")
```

output
```
integrate(sqrt(b*x^2 + a)*(f*x^2 + e)/((d*x^2 + c)^(3/2)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{x^6\,(c+dx^2)^{3/2}}\,dx = \int \frac{\sqrt{b\,x^2+a}\,(f\,x^2+e)}{x^6\,(d\,x^2+c)^{3/2}}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(e + f*x^2))/(x^6*(c + d*x^2)^(3/2)),x)
```

output
```
int(((a + b*x^2)^(1/2)*(e + f*x^2))/(x^6*(c + d*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}(e+fx^2)}{x^6\,(c+dx^2)^{3/2}}\,dx = \text{too large to display}$$

input
```
int((b*x^2+a)^(1/2)*(f*x^2+e)/x^6/(d*x^2+c)^(3/2),x)
```

output

```
( - 6*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*d*e - 4*sqrt(c + d*x**2)*sqrt
(a + b*x**2)*a*b*c*e - 5*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b*d*f*x**4 -
20*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**2*c*f*x**4 + 25*sqrt(c + d*x**2)*s
qrt(a + b*x**2)*b**2*d*e*x**4 - 15*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*
x**2)/(3*a**2*c**2*d + 6*a**2*c*d**2*x**2 + 3*a**2*d**3*x**4 + 2*a*b*c**3
+ 7*a*b*c**2*d*x**2 + 8*a*b*c*d**2*x**4 + 3*a*b*d**3*x**6 + 2*b**2*c**3*x*
*2 + 4*b**2*c**2*d*x**4 + 2*b**2*c*d**2*x**6),x)*a**2*b**2*c*d**3*f*x**5 -
 15*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(3*a**2*c**2*d + 6*a**2*c
*d**2*x**2 + 3*a**2*d**3*x**4 + 2*a*b*c**3 + 7*a*b*c**2*d*x**2 + 8*a*b*c*d
**2*x**4 + 3*a*b*d**3*x**6 + 2*b**2*c**3*x**2 + 4*b**2*c**2*d*x**4 + 2*b**
2*c*d**2*x**6),x)*a**2*b**2*d**4*f*x**7 - 70*int((sqrt(c + d*x**2)*sqrt(a
+ b*x**2)*x**2)/(3*a**2*c**2*d + 6*a**2*c*d**2*x**2 + 3*a**2*d**3*x**4 + 2
*a*b*c**3 + 7*a*b*c**2*d*x**2 + 8*a*b*c*d**2*x**4 + 3*a*b*d**3*x**6 + 2*b*
*2*c**3*x**2 + 4*b**2*c**2*d*x**4 + 2*b**2*c*d**2*x**6),x)*a*b**3*c**2*d**
2*f*x**5 + 75*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(3*a**2*c**2*d
+ 6*a**2*c*d**2*x**2 + 3*a**2*d**3*x**4 + 2*a*b*c**3 + 7*a*b*c**2*d*x**2 +
 8*a*b*c*d**2*x**4 + 3*a*b*d**3*x**6 + 2*b**2*c**3*x**2 + 4*b**2*c**2*d*x*
*4 + 2*b**2*c*d**2*x**6),x)*a*b**3*c*d**3*e*x**5 - 70*int((sqrt(c + d*x**2
)*sqrt(a + b*x**2)*x**2)/(3*a**2*c**2*d + 6*a**2*c*d**2*x**2 + 3*a**2*d**3
*x**4 + 2*a*b*c**3 + 7*a*b*c**2*d*x**2 + 8*a*b*c*d**2*x**4 + 3*a*b*d**3...
```

3.149 $\int \frac{x^4\sqrt{a+bx^2}}{(c+dx^2)^{3/2}(e+fx^2)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 379

$$\int \frac{x^4\sqrt{a+bx^2}}{(c+dx^2)^{3/2}(e+fx^2)}\,dx = \frac{x\sqrt{a+bx^2}}{df\sqrt{c+dx^2}} - \frac{\sqrt{c}(de-2cf)\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{d^{3/2}f(de-cf)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} + \frac{c^{3/2}(bde^2-af(2de-cf))\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{ad^{3/2}f(de-cf)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} - \frac{c^{3/2}e(be-af)\sqrt{a+bx^2}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}f(de-cf)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output
```
x*(b*x^2+a)^(1/2)/d/f/(d*x^2+c)^(1/2)-c^(1/2)*(-2*c*f+d*e)*(b*x^2+a)^(1/2)
*EllipticE(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))/d^(3/2)/
f/(-c*f+d*e)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)+c^(3/2)*(b*d*
e^2-a*f*(-c*f+2*d*e))*(b*x^2+a)^(1/2)*InverseJacobiAM(arctan(d^(1/2)*x/c^(
1/2)),(1-b*c/a/d)^(1/2))/a/d^(3/2)/f/(-c*f+d*e)^2/(c*(b*x^2+a)/a/(d*x^2+c)
)^(1/2)/(d*x^2+c)^(1/2)-c^(3/2)*e*(-a*f+b*e)*(b*x^2+a)^(1/2)*EllipticPi(d^
(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),1-c*f/d/e,(1-b*c/a/d)^(1/2))/a/d^(1/2)/f
/(-c*f+d*e)^2/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.75 (sec) , antiderivative size = 303, normalized size of antiderivative = 0.80

$$\int \frac{x^4\sqrt{a+bx^2}}{(c+dx^2)^{3/2}(e+fx^2)}\,dx = \frac{ibcf(-de+2cf)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(i\text{arcsinh}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)+i(-de+cf)}{}$$

input
```
Integrate[(x^4*Sqrt[a + b*x^2])/((c + d*x^2)^(3/2)*(e + f*x^2)),x]
```

output
```
(I*b*c*f*(-(d*e) + 2*c*f)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Elliptic
E[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + I*(-(d*e) + c*f)*(a*d*f - b*(d*e
+ 2*c*f))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt
[b/a]*x], (a*d)/(b*c)] + d*(Sqrt[b/a]*c*f^2*x*(a + b*x^2) + I*d*e*(-(b*e)
+ a*f)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e), I*A
rcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(Sqrt[b/a]*d^2*f^2*(d*e - c*f)*Sqrt[a
+ b*x^2]*Sqrt[c + d*x^2])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4\sqrt{a+bx^2}}{(c+dx^2)^{3/2}\,(e+fx^2)}\,dx$$

$$\Big\downarrow \; 450$$

$$\int \frac{x^4\sqrt{a+bx^2}}{(c+dx^2)^{3/2}\,(e+fx^2)}dx$$

input
```
Int[(x^4*Sqrt[a + b*x^2])/((c + d*x^2)^(3/2)*(e + f*x^2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [A] (verified)

Time = 6.76 (sec) , antiderivative size = 637, normalized size of antiderivative = 1.68

method	result
default	$\frac{\left(-\sqrt{-\frac{b}{a}}\,bcd\,f^2x^3+\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{x^2d+c}{c}}\,\text{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)acd\,f^2-\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{x^2d+c}{c}}\,\text{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)a\,d^2ef-2\sqrt{\right.}{}$
elliptic	$\frac{\sqrt{(b\,x^2+a)(x^2d+c)}\left(-\frac{\left(bd\,x^2+ad\right)cx}{d^2(cf-de)\sqrt{\left(x^2+\frac{c}{d}\right)\left(bd\,x^2+ad\right)}}+\frac{\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\text{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)a}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}\,df}-\frac{2\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\text{E}}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x}}\right.}{}$

input
```
int(x^4*(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
(-(-b/a)^(1/2)*b*c*d*f^2*x^3+((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*Ellip
ticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*c*d*f^2-((b*x^2+a)/a)^(1/2)*((d*x^2
+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*d^2*e*f-2*((b*x^2
+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))
*b*c^2*f^2+((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2
),(a*d/b/c)^(1/2))*b*c*d*e*f+((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*Ellip
ticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b*d^2*e^2+2*((b*x^2+a)/a)^(1/2)*((d*x
^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b*c^2*f^2-((b*x^2
+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))
*b*c*d*e*f+((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticPi(x*(-b/a)^(1/
2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*a*d^2*e*f-((b*x^2+a)/a)^(1/2)*((d*
x^2+c)/c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1
/2))*b*d^2*e^2-(-b/a)^(1/2)*a*c*d*f^2*x)*(d*x^2+c)^(1/2)*(b*x^2+a)^(1/2)/d
^2/(-b/a)^(1/2)/f^2/(c*f-d*e)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^4\sqrt{a+bx^2}}{(c+dx^2)^{3/2}(e+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate(x^4*(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)/(f*x^2+e),x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^4\sqrt{a+bx^2}}{(c+dx^2)^{3/2}\,(e+fx^2)}\,dx = \int \frac{x^4\sqrt{a+bx^2}}{(c+dx^2)^{\frac{3}{2}}\,(e+fx^2)}\,dx$$

input
```
integrate(x**4*(b*x**2+a)**(1/2)/(d*x**2+c)**(3/2)/(f*x**2+e),x)
```

output
```
Integral(x**4*sqrt(a + b*x**2)/((c + d*x**2)**(3/2)*(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{x^4\sqrt{a+bx^2}}{(c+dx^2)^{3/2}\,(e+fx^2)}\,dx = \int \frac{\sqrt{bx^2+a}x^4}{(dx^2+c)^{\frac{3}{2}}(fx^2+e)}\,dx$$

input
```
integrate(x^4*(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)/(f*x^2+e),x, algorithm="maxi
ma")
```

output
```
integrate(sqrt(b*x^2 + a)*x^4/((d*x^2 + c)^(3/2)*(f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{x^4\sqrt{a+bx^2}}{(c+dx^2)^{3/2}\,(e+fx^2)}\,dx = \int \frac{\sqrt{bx^2+a}x^4}{(dx^2+c)^{\frac{3}{2}}(fx^2+e)}\,dx$$

input
```
integrate(x^4*(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)/(f*x^2+e),x, algorithm="giac
")
```

output
```
integrate(sqrt(b*x^2 + a)*x^4/((d*x^2 + c)^(3/2)*(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4\sqrt{a+bx^2}}{(c+dx^2)^{3/2}(e+fx^2)}\,dx=\int \frac{x^4\,\sqrt{b\,x^2+a}}{{\left(d\,x^2+c\right)}^{3/2}\,\left(f\,x^2+e\right)}\,dx$$

input
```
int((x^4*(a + b*x^2)^(1/2))/((c + d*x^2)^(3/2)*(e + f*x^2)),x)
```

output
```
int((x^4*(a + b*x^2)^(1/2))/((c + d*x^2)^(3/2)*(e + f*x^2)), x)
```

Reduce [F]

$$\int \frac{x^4\sqrt{a+bx^2}}{(c+dx^2)^{3/2}(e+fx^2)}\,dx=\text{too large to display}$$

input
```
int(x^4*(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)/(f*x^2+e),x)
```

output

```
(sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*x - 2*int((sqrt(c + d*x**2)*sqrt(a +
b*x**2)*x**6)/(2*a*c**3*e*f + 2*a*c**3*f**2*x**2 + a*c**2*d*e**2 + 5*a*c**
2*d*e*f*x**2 + 4*a*c**2*d*f**2*x**4 + 2*a*c*d**2*e**2*x**2 + 4*a*c*d**2*e*
f*x**4 + 2*a*c*d**2*f**2*x**6 + a*d**3*e**2*x**4 + a*d**3*e*f*x**6 + 2*b*c
**3*e*f*x**2 + 2*b*c**3*f**2*x**4 + b*c**2*d*e**2*x**2 + 5*b*c**2*d*e*f*x*
*4 + 4*b*c**2*d*f**2*x**6 + 2*b*c*d**2*e**2*x**4 + 4*b*c*d**2*e*f*x**6 + 2
*b*c*d**2*f**2*x**8 + b*d**3*e**2*x**6 + b*d**3*e*f*x**8),x)*a*b*c**2*d*f*
*2 - int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**6)/(2*a*c**3*e*f + 2*a*c**3
*f**2*x**2 + a*c**2*d*e**2 + 5*a*c**2*d*e*f*x**2 + 4*a*c**2*d*f**2*x**4 +
2*a*c*d**2*e**2*x**2 + 4*a*c*d**2*e*f*x**4 + 2*a*c*d**2*f**2*x**6 + a*d**3
*e**2*x**4 + a*d**3*e*f*x**6 + 2*b*c**3*e*f*x**2 + 2*b*c**3*f**2*x**4 + b*
c**2*d*e**2*x**2 + 5*b*c**2*d*e*f*x**4 + 4*b*c**2*d*f**2*x**6 + 2*b*c*d**2
*e**2*x**4 + 4*b*c*d**2*e*f*x**6 + 2*b*c*d**2*f**2*x**8 + b*d**3*e**2*x**6
 + b*d**3*e*f*x**8),x)*a*b*c*d**2*e*f - 2*int((sqrt(c + d*x**2)*sqrt(a + b
*x**2)*x**6)/(2*a*c**3*e*f + 2*a*c**3*f**2*x**2 + a*c**2*d*e**2 + 5*a*c**2
*d*e*f*x**2 + 4*a*c**2*d*f**2*x**4 + 2*a*c*d**2*e**2*x**2 + 4*a*c*d**2*e*f
*x**4 + 2*a*c*d**2*f**2*x**6 + a*d**3*e**2*x**4 + a*d**3*e*f*x**6 + 2*b*c*
*3*e*f*x**2 + 2*b*c**3*f**2*x**4 + b*c**2*d*e**2*x**2 + 5*b*c**2*d*e*f*x**
4 + 4*b*c**2*d*f**2*x**6 + 2*b*c*d**2*e**2*x**4 + 4*b*c*d**2*e*f*x**6 + 2*
b*c*d**2*f**2*x**8 + b*d**3*e**2*x**6 + b*d**3*e*f*x**8),x)*a*b*c*d**2*...
```

3.150 $\int \frac{x^2\sqrt{a+bx^2}}{\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 253

$$\int \frac{x^2\sqrt{a+bx^2}}{\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)}\,dx = \frac{(bc-ad)x}{d(de-cf)\sqrt{a+bx^2}\sqrt{c+dx^2}} - \frac{\sqrt{a}\sqrt{b}\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{d(de-cf)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} + \frac{a^{3/2}\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}c(de-cf)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
(-a*d+b*c)*x/d/(-c*f+d*e)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)-a^(1/2)*b^(1/2)*
(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^
(1/2))/d/(-c*f+d*e)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)+a^(3/2
)*(d*x^2+c)^(1/2)*EllipticPi(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),1-a*f/b/e
,(1-a*d/b/c)^(1/2))/b^(1/2)/c/(-c*f+d*e)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b
*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.53 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.09

$$\int \frac{x^2\sqrt{a+bx^2}}{(c+dx^2)^{3/2}(e+fx^2)}\,dx = \frac{-ibcf\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(i\text{arcsinh}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)+ib(-de+cf)\sqrt{1+\frac{bx^2}{a}}}{}$$

input
```
Integrate[(x^2*Sqrt[a + b*x^2])/((c + d*x^2)^(3/2)*(e + f*x^2)),x]
```

output
```
((-I)*b*c*f*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sq
rt[b/a]*x], (a*d)/(b*c)] + I*b*(-(d*e) + c*f)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 +
 (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - d*(Sqrt[b/a]*
f*x*(a + b*x^2) + I*(-(b*e) + a*f)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]
*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(Sqrt[b/a]
*d*f*(d*e - c*f)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\sqrt{a+bx^2}}{(c+dx^2)^{3/2}(e+fx^2)}\,dx$$

$$\Big\downarrow \text{ 450}$$

$$\int \frac{x^2\sqrt{a+bx^2}}{(c+dx^2)^{3/2}(e+fx^2)}dx$$

input
```
Int[(x^2*Sqrt[a + b*x^2])/((c + d*x^2)^(3/2)*(e + f*x^2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [A] (verified)

Time = 6.34 (sec) , antiderivative size = 390, normalized size of antiderivative = 1.54

method	result
default	$\frac{\left(\sqrt{-\frac{b}{a}}\, b d f\, x^{3}+\sqrt{\frac{b\,x^{2}+a}{a}}\,\sqrt{\frac{x^{2} d+c}{c}}\, \operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}}, \sqrt{\frac{a d}{b c}}\right) b c f-\sqrt{\frac{b\,x^{2}+a}{a}}\,\sqrt{\frac{x^{2} d+c}{c}}\, \operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}}, \sqrt{\frac{a d}{b c}}\right) b d e-\sqrt{\frac{b\,x^{2}+a}{a}}\,\sqrt{x \cdots}\right.}{\cdots}$
elliptic	$\frac{\sqrt{\left(b\,x^{2}+a\right)\left(x^{2} d+c\right)}\left(\frac{\left(b d\,x^{2}+a d\right) x}{d\left(c f-d e\right)\sqrt{\left(x^{2}+\frac{c}{d}\right)\left(b d\,x^{2}+a d\right)}}+\frac{\sqrt{1+\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\, \operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}}, \sqrt{-1+\frac{a d+b c}{c b}}\right) b}{\sqrt{-\frac{b}{a}}\,\sqrt{b d\,x^{4}+a d\,x^{2}+x^{2} b c+a c}\, d f}-\frac{b c\sqrt{1+\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\, \operatorname{Ellip}\cdots}{\left(c f-d e\right)\sqrt{-\frac{b}{a}}\,\sqrt{b d \cdots}}\right.}{\sqrt{b\,x^{2}+a \cdots}}$

input
```
int(x^2*(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
((-b/a)^(1/2)*b*d*f*x^3+((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(
x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b*c*f-((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1
/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b*d*e-((b*x^2+a)/a)^(1/2)*((
d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b*c*f-((b*x^2+
a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)
^(1/2)/(-b/a)^(1/2))*a*d*f+e*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*Ellip
ticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*b*d+(-b/a)^(1/2)
*a*d*f*x)*(d*x^2+c)^(1/2)*(b*x^2+a)^(1/2)/f/(-b/a)^(1/2)/d/(c*f-d*e)/(b*d*
x^4+a*d*x^2+b*c*x^2+a*c)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^2\sqrt{a+bx^2}}{(c+dx^2)^{3/2}\,(e+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate(x^2*(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)/(f*x^2+e),x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^2\sqrt{a+bx^2}}{(c+dx^2)^{3/2}\,(e+fx^2)}\,dx = \int \frac{x^2\sqrt{a+bx^2}}{(c+dx^2)^{\frac{3}{2}}\,(e+fx^2)}\,dx$$

input
```
integrate(x**2*(b*x**2+a)**(1/2)/(d*x**2+c)**(3/2)/(f*x**2+e),x)
```

output
```
Integral(x**2*sqrt(a + b*x**2)/((c + d*x**2)**(3/2)*(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{x^2\sqrt{a+bx^2}}{(c+dx^2)^{3/2}\,(e+fx^2)}\,dx = \int \frac{\sqrt{bx^2+a}x^2}{(dx^2+c)^{\frac{3}{2}}(fx^2+e)}\,dx$$

input
```
integrate(x^2*(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)/(f*x^2+e),x, algorithm="maxi
ma")
```

output
```
integrate(sqrt(b*x^2 + a)*x^2/((d*x^2 + c)^(3/2)*(f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{x^2\sqrt{a+bx^2}}{(c+dx^2)^{3/2}\,(e+fx^2)}\,dx = \int \frac{\sqrt{bx^2+a}x^2}{(dx^2+c)^{\frac{3}{2}}(fx^2+e)}\,dx$$

input
```
integrate(x^2*(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)/(f*x^2+e),x, algorithm="giac
")
```

output
```
integrate(sqrt(b*x^2 + a)*x^2/((d*x^2 + c)^(3/2)*(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2\sqrt{a+bx^2}}{(c+dx^2)^{3/2}\,(e+fx^2)}\,dx = \int \frac{x^2\,\sqrt{b\,x^2+a}}{{\left(d\,x^2+c\right)}^{3/2}\,\left(f\,x^2+e\right)}\,dx$$

input
```
int((x^2*(a + b*x^2)^(1/2))/((c + d*x^2)^(3/2)*(e + f*x^2)),x)
```

output
```
int((x^2*(a + b*x^2)^(1/2))/((c + d*x^2)^(3/2)*(e + f*x^2)), x)
```

Reduce [F]

$$\int \frac{x^2\sqrt{a+bx^2}}{(c+dx^2)^{3/2}\,(e+fx^2)}\,dx = \int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^2}{d^2f\,x^6+2cdf\,x^4+d^2e\,x^4+c^2f\,x^2+2cde\,x^2+c^2e}dx$$

input
```
int(x^2*(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)/(f*x^2+e),x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(c**2*e + c**2*f*x**2 + 2*c*d
*e*x**2 + 2*c*d*f*x**4 + d**2*e*x**4 + d**2*f*x**6),x)
```

3.151 $\int \frac{\sqrt{a+bx^2}}{\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 209

$$\int \frac{\sqrt{a+bx^2}}{\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)}\,dx = \frac{\sqrt{d}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(de-cf)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} - \frac{a^{3/2}f\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(de-cf)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output

```
d^(1/2)*(b*x^2+a)^(1/2)*EllipticE(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b
*c/a/d)^(1/2))/c^(1/2)/(-c*f+d*e)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c
)^(1/2)-a^(3/2)*f*(d*x^2+c)^(1/2)*EllipticPi(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)
^(1/2),1-a*f/b/e,(1-a*d/b/c)^(1/2))/b^(1/2)/c/e/(-c*f+d*e)/(b*x^2+a)^(1/2)
/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.27 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.99

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}(e+fx^2)}\,dx = \frac{-\sqrt{\frac{b}{a}}dex(a+bx^2) - ibce\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(i\operatorname{arcsinh}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right) - ic(\cdots}{\sqrt{\frac{b}{a}}ce(-de+cf)\sqrt{a+\cdots}}$$

input
```
Integrate[Sqrt[a + b*x^2]/((c + d*x^2)^(3/2)*(e + f*x^2)),x]
```

output
```
(-(Sqrt[b/a]*d*e*x*(a + b*x^2)) - I*b*c*e*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*
x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*c*(-(b*e) + a*f
)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e), I*ArcSin
h[Sqrt[b/a]*x], (a*d)/(b*c)])/(Sqrt[b/a]*c*e*(-(d*e) + c*f)*Sqrt[a + b*x^2
]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {416, 313, 414}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}(e+fx^2)}\,dx$$

$$\Big\downarrow \text{416}$$

$$\frac{d\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}}dx}{de-cf} - \frac{f\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)}dx}{de-cf}$$

$$\Big\downarrow \text{313}$$

$$\frac{\sqrt{d}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{f\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)}dx}{de-cf}$$

$$\Big\downarrow \text{414}$$

$$\frac{\sqrt{d}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{a^{3/2}f\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce\sqrt{a+bx^2}(de-cf)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

input
```
Int[Sqrt[a + b*x^2]/((c + d*x^2)^(3/2)*(e + f*x^2)),x]
```

output
```
(Sqrt[d]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/
(a*d)])/(Sqrt[c]*(d*e - c*f)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c
+ d*x^2]) - (a^(3/2)*f*Sqrt[c + d*x^2]*EllipticPi[1 - (a*f)/(b*e), ArcTan[
(Sqrt[b]*x)/Sqrt[a]], 1 - (a*d)/(b*c)])/(Sqrt[b]*c*e*(d*e - c*f)*Sqrt[a +
b*x^2]*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))])
```

Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 416
```
Int[Sqrt[(e_) + (f_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[b/(b*c - a*d)   Int[Sqrt[e + f*x^2]/((a + b*x^2)*
Sqrt[c + d*x^2]), x], x] - Simp[d/(b*c - a*d)   Int[Sqrt[e + f*x^2]/(c + d*
x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[d/c] && PosQ[f/e
]
```

Maple [A] (verified)

Time = 6.59 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.37

method	result
default	$\frac{\left(-\sqrt{-\frac{b}{a}}\, bde\, x^{3}+\sqrt{\frac{b\, x^{2}+a}{a}}\, \sqrt{\frac{x^{2} d+c}{c}}\, \operatorname{EllipticE}\left(x \sqrt{-\frac{b}{a}}, \sqrt{\frac{a d}{b c}}\right) bce+\sqrt{\frac{b\, x^{2}+a}{a}}\, \sqrt{\frac{x^{2} d+c}{c}}\, \operatorname{EllipticPi}\left(x \sqrt{-\frac{b}{a}}, \frac{a f}{b e}, \frac{\sqrt{-\frac{d}{c}}}{\sqrt{-\frac{b}{a}}}\right) acf-\sqrt{\frac{b\, x^{2}}{a}}\right.}{ce \sqrt{-\frac{b}{a}}\, (cf-de)(bd\, x^{4}+ad\, x^{2}+x^{2} bc+ac)}$
elliptic	$\frac{\sqrt{(b\, x^{2}+a)(x^{2} d+c)}\, \left(-\frac{\left(bd\, x^{2}+ad\right) x}{c(cf-de)\sqrt{\left(x^{2}+\frac{c}{d}\right)\left(bd\, x^{2}+ad\right)}}+\frac{b\sqrt{1+\frac{b\, x^{2}}{a}}\, \sqrt{1+\frac{d\, x^{2}}{c}}\, \operatorname{EllipticE}\left(x \sqrt{-\frac{b}{a}}, \sqrt{-1+\frac{ad+bc}{cb}}\right)}{(cf-de)\sqrt{-\frac{b}{a}}\, \sqrt{bd\, x^{4}+ad\, x^{2}+x^{2} bc+ac}}+\frac{f\sqrt{1+\frac{b\, x^{2}}{a}}\, \sqrt{1+\frac{d\, x^{2}}{c}}\, \mathrm{Ell}}{(cf-de)e\sqrt{-\frac{b}{a}}\, \sqrt{}}\right.}{\sqrt{b\, x^{2}+a}\, \sqrt{x^{2} d+c}}$

input
```
int((b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
(-(-b/a)^(1/2)*b*d*e*x^3+((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE
(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b*c*e+((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(
1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*a*c*f-
((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,
(-1/c*d)^(1/2)/(-b/a)^(1/2))*b*c*e-(-b/a)^(1/2)*a*d*e*x)*(d*x^2+c)^(1/2)*(
b*x^2+a)^(1/2)/c/e/(-b/a)^(1/2)/(c*f-d*e)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)}\, dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)/(f*x^2+e),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}(e+fx^2)}\,dx = \int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{\frac{3}{2}}(e+fx^2)}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)/(d*x**2+c)**(3/2)/(f*x**2+e),x)
```

output
```
Integral(sqrt(a + b*x**2)/((c + d*x**2)**(3/2)*(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}(e+fx^2)}\,dx = \int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{\frac{3}{2}}(fx^2+e)}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)/(f*x^2+e),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)/((d*x^2 + c)^(3/2)*(f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}(e+fx^2)}\,dx = \int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{\frac{3}{2}}(fx^2+e)}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)/(f*x^2+e),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^2 + a)/((d*x^2 + c)^(3/2)*(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}(e+fx^2)}\,dx = \int \frac{\sqrt{b\,x^2+a}}{{\left(d\,x^2+c\right)}^{3/2}\,\left(f\,x^2+e\right)}\,dx$$

input
```
int((a + b*x^2)^(1/2)/((c + d*x^2)^(3/2)*(e + f*x^2)),x)
```

output
```
int((a + b*x^2)^(1/2)/((c + d*x^2)^(3/2)*(e + f*x^2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}(e+fx^2)}\,dx = \int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{d^2f\,x^6+2cdf\,x^4+d^2e\,x^4+c^2f\,x^2+2cde\,x^2+c^2e}dx$$

input
```
int((b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)/(f*x^2+e),x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(c**2*e + c**2*f*x**2 + 2*c*d*e*x*
*2 + 2*c*d*f*x**4 + d**2*e*x**4 + d**2*f*x**6),x)
```

3.152 $\int \frac{\sqrt{a+bx^2}}{x^2(c+dx^2)^{3/2}(e+fx^2)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 463

$$\begin{aligned}\int \frac{\sqrt{a+bx^2}}{x^2(c+dx^2)^{3/2}(e+fx^2)}\,dx &= \frac{\left(b-\frac{ad(2de-cf)}{c(de-cf)}\right)x\sqrt{a+bx^2}}{ace\sqrt{c+dx^2}}-\frac{(a+bx^2)^{3/2}}{acex\sqrt{c+dx^2}}\\
&+\frac{b(2de-cf)x\sqrt{c+dx^2}}{c^2e(de-cf)\sqrt{a+bx^2}}-\frac{\sqrt{a}\sqrt{b}(2de-cf)\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{c^2e(de-cf)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}\\
&+\frac{\sqrt{a}\sqrt{b}\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{c^2e\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}\\
&+\frac{a^{3/2}f^2\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce^2(de-cf)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}\end{aligned}$$

output
```
(b-a*d*(-c*f+2*d*e)/c/(-c*f+d*e))*x*(b*x^2+a)^(1/2)/a/c/e/(d*x^2+c)^(1/2)-
(b*x^2+a)^(3/2)/a/c/e/x/(d*x^2+c)^(1/2)+b*(-c*f+2*d*e)*x*(d*x^2+c)^(1/2)/c
^2/e/(-c*f+d*e)/(b*x^2+a)^(1/2)-a^(1/2)*b^(1/2)*(-c*f+2*d*e)*(d*x^2+c)^(1/
2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/c^2/e/
(-c*f+d*e)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)+a^(1/2)*b^(1/2)
*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/
2))/c^2/e/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)+a^(3/2)*f^2*(d*x
^2+c)^(1/2)*EllipticPi(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),1-a*f/b/e,(1-a*
d/b/c)^(1/2))/b^(1/2)/c/e^2/(-c*f+d*e)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x
^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.52 (sec) , antiderivative size = 493, normalized size of antiderivative = 1.06

$$\int \frac{\sqrt{a+bx^2}}{x^2\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)}\,dx = \frac{a\sqrt{\frac{b}{a}}cde^2 - a\sqrt{\frac{b}{a}}c^2ef + b\sqrt{\frac{b}{a}}cde^2x^2 + 2a\sqrt{\frac{b}{a}}d^2e^2x^2 - b\sqrt{\frac{b}{a}}c^2efx^2 - a\sqrt{}}{}$$

input
```
Integrate[Sqrt[a + b*x^2]/(x^2*(c + d*x^2)^(3/2)*(e + f*x^2)),x]
```

output
```
(a*Sqrt[b/a]*c*d*e^2 - a*Sqrt[b/a]*c^2*e*f + b*Sqrt[b/a]*c*d*e^2*x^2 + 2*a
*Sqrt[b/a]*d^2*e^2*x^2 - b*Sqrt[b/a]*c^2*e*f*x^2 - a*Sqrt[b/a]*c*d*e*f*x^2
 + 2*b*Sqrt[b/a]*d^2*e^2*x^4 - b*Sqrt[b/a]*c*d*e*f*x^4 - I*b*c*e*(-2*d*e +
 c*f)*x*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b
/a]*x], (a*d)/(b*c)] + I*b*c*e*(-(d*e) + c*f)*x*Sqrt[1 + (b*x^2)/a]*Sqrt[1
 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*b*c^2*e*f
*x*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e), I*ArcSi
nh[Sqrt[b/a]*x], (a*d)/(b*c)] + I*a*c^2*f^2*x*Sqrt[1 + (b*x^2)/a]*Sqrt[1 +
 (d*x^2)/c]*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/
(Sqrt[b/a]*c^2*e^2*(-(d*e) + c*f)*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}}{x^2 (c+dx^2)^{3/2} (e+fx^2)} \, dx$$

$$\Big\downarrow \; 450$$

$$\int \frac{\sqrt{a+bx^2}}{x^2 (c+dx^2)^{3/2} (e+fx^2)} dx$$

input
```
Int[Sqrt[a + b*x^2]/(x^2*(c + d*x^2)^(3/2)*(e + f*x^2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [A] (verified)

Time = 19.75 (sec) , antiderivative size = 586, normalized size of antiderivative = 1.27

method	result
default	$\frac{\left(-\sqrt{-\frac{b}{a}}\, bcdef\, x^{4}+2\sqrt{-\frac{b}{a}}\, b\, d^{2} e^{2} x^{4}-\sqrt{\frac{b\,x^{2}+a}{a}}\, \sqrt{\frac{x^{2} d+c}{c}}\, \operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}}, \sqrt{\frac{ad}{bc}}\right) b\, c^{2} e f x+\sqrt{\frac{b\,x^{2}+a}{a}}\, \sqrt{\frac{x^{2} d+c}{c}}\, \operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}}\right.\right.}{}$
risch	$-\frac{\sqrt{b\,x^{2}+a}\,\sqrt{x^{2} d+c}}{c^{2} e x}+\frac{\left(-\frac{bc\sqrt{1+\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,\left(\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}}, \sqrt{-1+\frac{ad+bc}{cb}}\right)-\operatorname{EllipticE}\left(x\sqrt{-\frac{b}{a}}, \sqrt{-1+\frac{ad+bc}{cb}}\right)\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^{4}+ad\,x^{2}+x^{2} bc+ac}}+\frac{ce(ad-bc)d\left(\right.}{}\right.}{}$
elliptic	$\frac{\sqrt{(b\,x^{2}+a)(x^{2} d+c)}\left(\frac{\left(bd\,x^{2}+ad\right) dx}{c^{2}(cf-de)\sqrt{\left(x^{2}+\frac{c}{d}\right)\left(bd\,x^{2}+ad\right)}}-\frac{\sqrt{bd\,x^{4}+ad\,x^{2}+x^{2} bc+ac}}{c^{2} e x}-\frac{\sqrt{1+\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\, db\,\operatorname{EllipticE}\left(x\sqrt{-\frac{b}{a}}, \sqrt{-1+\frac{ad+bc}{cb}}\right)}{c\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^{4}+ad\,x^{2}+x^{2} bc+ac}\,(cf-de)}\right.}{}$

input
```
int((b*x^2+a)^(1/2)/x^2/(d*x^2+c)^(3/2)/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
(-(-b/a)^(1/2)*b*c*d*e*f*x^4+2*(-b/a)^(1/2)*b*d^2*e^2*x^4-((b*x^2+a)/a)^(1
/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b*c^2*e*
f*x+((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/
b/c)^(1/2))*b*c*d*e^2*x+((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(
x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b*c^2*e*f*x-2*((b*x^2+a)/a)^(1/2)*((d*x^2+
c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b*c*d*e^2*x-((b*x^2+
a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)
^(1/2)/(-b/a)^(1/2))*a*c^2*f^2*x+((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*E
llipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*b*c^2*e*f*x-
(-b/a)^(1/2)*a*c*d*e*f*x^2+2*(-b/a)^(1/2)*a*d^2*e^2*x^2-(-b/a)^(1/2)*b*c^2
*e*f*x^2+(-b/a)^(1/2)*b*c*d*e^2*x^2-(-b/a)^(1/2)*a*c^2*e*f+(-b/a)^(1/2)*a*
c*d*e^2)*(d*x^2+c)^(1/2)*(b*x^2+a)^(1/2)/(c*f-d*e)/(-b/a)^(1/2)/x/e^2/c^2/
(b*d*x^4+a*d*x^2+b*c*x^2+a*c)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{x^2 (c+dx^2)^{3/2} (e+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(1/2)/x^2/(d*x^2+c)^(3/2)/(f*x^2+e),x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}}{x^2 (c+dx^2)^{3/2} (e+fx^2)}\,dx = \int \frac{\sqrt{a+bx^2}}{x^2 (c+dx^2)^{\frac{3}{2}} (e+fx^2)}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)/x**2/(d*x**2+c)**(3/2)/(f*x**2+e),x)
```

output
```
Integral(sqrt(a + b*x**2)/(x**2*(c + d*x**2)**(3/2)*(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}}{x^2 (c+dx^2)^{3/2} (e+fx^2)}\,dx = \int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{\frac{3}{2}}(fx^2+e)x^2}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/x^2/(d*x^2+c)^(3/2)/(f*x^2+e),x, algorithm="maxi
ma")
```

output
```
integrate(sqrt(b*x^2 + a)/((d*x^2 + c)^(3/2)*(f*x^2 + e)*x^2), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}}{x^2 (c+dx^2)^{3/2} (e+fx^2)} \, dx = \int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{\frac{3}{2}}(fx^2+e)x^2} \, dx$$

input
```
integrate((b*x^2+a)^(1/2)/x^2/(d*x^2+c)^(3/2)/(f*x^2+e),x, algorithm="giac
")
```

output
```
integrate(sqrt(b*x^2 + a)/((d*x^2 + c)^(3/2)*(f*x^2 + e)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{x^2 (c+dx^2)^{3/2} (e+fx^2)} \, dx = \int \frac{\sqrt{b\,x^2+a}}{x^2\,(d\,x^2+c)^{3/2}\,(f\,x^2+e)} \, dx$$

input
```
int((a + b*x^2)^(1/2)/(x^2*(c + d*x^2)^(3/2)*(e + f*x^2)),x)
```

output
```
int((a + b*x^2)^(1/2)/(x^2*(c + d*x^2)^(3/2)*(e + f*x^2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}}{x^2 (c+dx^2)^{3/2} (e+fx^2)} \, dx = \int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{d^2 f\,x^8+2cdf\,x^6+d^2 e\,x^6+c^2 f\,x^4+2cde\,x^4+c^2 e\,x^2} dx$$

input
```
int((b*x^2+a)^(1/2)/x^2/(d*x^2+c)^(3/2)/(f*x^2+e),x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(c**2*e*x**2 + c**2*f*x**4 + 2*c*d
*e*x**4 + 2*c*d*f*x**6 + d**2*e*x**6 + d**2*f*x**8),x)
```

3.153 $\int \frac{\sqrt{a+bx^2}}{x^4\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 626

$$\int \frac{\sqrt{a+bx^2}}{x^4\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)}\,dx = -\frac{\left(bc(5d^2e^2-2cdef-3c^2f^2)-ad(8d^2e^2-2cdef-3c^2f^2)\right)x\sqrt{a+bx^2}}{3ac^3e^2(de-cf)\sqrt{c+dx^2}}$$

$$-\frac{\left(a+bx^2\right)^{3/2}}{3acex^3\sqrt{c+dx^2}}+\frac{(4de+3cf)\left(a+bx^2\right)^{3/2}}{3ac^2e^2x\sqrt{c+dx^2}}+\frac{b\left(bc-\frac{a(8d^2e^2-2cdef-3c^2f^2)}{e(de-cf)}\right)x\sqrt{c+dx^2}}{3ac^3e\sqrt{a+bx^2}}$$

$$-\frac{\sqrt{b}\left(bce(de-cf)-a(8d^2e^2-2cdef-3c^2f^2)\right)\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{3\sqrt{a}c^3e^2(de-cf)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

$$-\frac{\sqrt{a}\sqrt{b}(4de+3cf)\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{3c^3e^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

$$-\frac{a^{3/2}f^3\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce^3(de-cf)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
-1/3*(b*c*(-3*c^2*f^2-2*c*d*e*f+5*d^2*e^2)-a*d*(-3*c^2*f^2-2*c*d*e*f+8*d^2
*e^2))*x*(b*x^2+a)^(1/2)/a/c^3/e^2/(-c*f+d*e)/(d*x^2+c)^(1/2)-1/3*(b*x^2+a
)^(3/2)/a/c/e/x^3/(d*x^2+c)^(1/2)+1/3*(3*c*f+4*d*e)*(b*x^2+a)^(3/2)/a/c^2/
e^2/x/(d*x^2+c)^(1/2)+1/3*b*(b*c-a*(-3*c^2*f^2-2*c*d*e*f+8*d^2*e^2)/e/(-c*
f+d*e))*x*(d*x^2+c)^(1/2)/a/c^3/e/(b*x^2+a)^(1/2)-1/3*b^(1/2)*(b*c*e*(-c*f
+d*e)-a*(-3*c^2*f^2-2*c*d*e*f+8*d^2*e^2))*(d*x^2+c)^(1/2)*EllipticE(b^(1/2
)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/a^(1/2)/c^3/e^2/(-c*f+d*e
)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)-1/3*a^(1/2)*b^(1/2)*(3*c
*f+4*d*e)*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d
/b/c)^(1/2))/c^3/e^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)-a^(3/
2)*f^3*(d*x^2+c)^(1/2)*EllipticPi(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),1-a*
f/b/e,(1-a*d/b/c)^(1/2))/b^(1/2)/c/e^3/(-c*f+d*e)/(b*x^2+a)^(1/2)/(a*(d*x^
2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.53 (sec) , antiderivative size = 446, normalized size of antiderivative = 0.71

$$\int \frac{\sqrt{a+bx^2}}{x^4\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)}\,dx = \frac{\sqrt{\frac{b}{a}}\left(-\sqrt{\frac{b}{a}}e(a+bx^2)\left(bce(-de+cf)x^2(c+dx^2)+a(8d^3e^2x^4+c^3f(e-\right.\right.}{}$$

input
```
Integrate[Sqrt[a + b*x^2]/(x^4*(c + d*x^2)^(3/2)*(e + f*x^2)),x]
```

output
```
(Sqrt[b/a]*(-(Sqrt[b/a]*e*(a + b*x^2)*(b*c*e*(-(d*e) + c*f)*x^2*(c + d*x^2
) + a*(8*d^3*e^2*x^4 + c^3*f*(e - 3*f*x^2) + 2*c*d^2*e*x^2*(2*e - f*x^2) -
 c^2*d*(e^2 + e*f*x^2 + 3*f^2*x^4)))) + I*b*c*e*(b*c*e*(d*e - c*f) + a*(-8
*d^2*e^2 + 2*c*d*e*f + 3*c^2*f^2))*x^3*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2
)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*b*c*e*(-(d*e) + c*
f)*(-(b*c*e) + 4*a*d*e + 3*a*c*f)*x^3*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)
/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - (3*I)*a*c^3*f^2*(-(b*
e) + a*f)*x^3*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*
e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(3*b*c^3*e^3*(-(d*e) + c*f)*x^3
*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}}{x^4 \left(c+dx^2\right)^{3/2} \left(e+fx^2\right)} \, dx$$

$$\Big\downarrow \ 450$$

$$\int \frac{\sqrt{a+bx^2}}{x^4 \left(c+dx^2\right)^{3/2} \left(e+fx^2\right)}dx$$

input
```
Int[Sqrt[a + b*x^2]/(x^4*(c + d*x^2)^(3/2)*(e + f*x^2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [A] (verified)

Time = 21.52 (sec) , antiderivative size = 772, normalized size of antiderivative = 1.23

method	result
risch	$-\frac{\sqrt{b\,x^2+a}\,\sqrt{x^2 d+c}\,\left(-3acf\,x^2-5ade\,x^2+bce\,x^2+ace\right)}{3a\,c^3 e^2 x^3}-\frac{\left(-\frac{b(3acf+5ade-bce)c\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\left(\text{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right.\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}\right.}{}$
elliptic	Expression too large to display
default	Expression too large to display

input
```
int((b*x^2+a)^(1/2)/x^4/(d*x^2+c)^(3/2)/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(-3*a*c*f*x^2-5*a*d*e*x^2+b*c*e*x^2+a
*c*e)/a/c^3/e^2/x^3-1/3/e^2/c^3/a*(-b*(3*a*c*f+5*a*d*e-b*c*e)*c/(-b/a)^(1/
2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1
/2),(-1+(a*d+b*c)/c/b)^(1/2)))+a*c*d*e*b/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1
+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/
2),(-1+(a*d+b*c)/c/b)^(1/2))-3*a*c^3*f^2*(a*f-b*e)/(c*f-d*e)/e/(-b/a)^(1/2
)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*
EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))+3*a*c*e^2*d
^2*(a*d-b*c)/(c*f-d*e)*((b*d*x^2+a*d)/c/(a*d-b*c)*x/((x^2+c/d)*(b*d*x^2+a*
d))^(1/2)+(1/c-1/(a*d-b*c)/c*a*d)/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/
c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+
(a*d+b*c)/c/b)^(1/2))+b/(a*d-b*c)/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/
c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(EllipticF(x*(-b/a)^(1/2),(-1
+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2)))
))*((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{x^4 (c+dx^2)^{3/2} (e+fx^2)} \, dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(1/2)/x^4/(d*x^2+c)^(3/2)/(f*x^2+e),x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}}{x^4\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)}\,dx=\int \frac{\sqrt{a+bx^2}}{x^4\left(c+dx^2\right)^{\frac{3}{2}}\left(e+fx^2\right)}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)/x**4/(d*x**2+c)**(3/2)/(f*x**2+e),x)
```

output
```
Integral(sqrt(a + b*x**2)/(x**4*(c + d*x**2)**(3/2)*(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}}{x^4\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)}\,dx=\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{\frac{3}{2}}(fx^2+e)x^4}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/x^4/(d*x^2+c)^(3/2)/(f*x^2+e),x, algorithm="maxi
ma")
```

output
```
integrate(sqrt(b*x^2 + a)/((d*x^2 + c)^(3/2)*(f*x^2 + e)*x^4), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}}{x^4\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)}\,dx=\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{\frac{3}{2}}(fx^2+e)x^4}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/x^4/(d*x^2+c)^(3/2)/(f*x^2+e),x, algorithm="giac
")
```

output
```
integrate(sqrt(b*x^2 + a)/((d*x^2 + c)^(3/2)*(f*x^2 + e)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{x^4\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)}\,dx=\int \frac{\sqrt{b\,x^2+a}}{x^4\,{\left(d\,x^2+c\right)}^{3/2}\,\left(f\,x^2+e\right)}\,dx$$

input
```
int((a + b*x^2)^(1/2)/(x^4*(c + d*x^2)^(3/2)*(e + f*x^2)),x)
```

output
```
int((a + b*x^2)^(1/2)/(x^4*(c + d*x^2)^(3/2)*(e + f*x^2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}}{x^4\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)}\,dx=\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{d^2 f\,x^{10}+2cdf\,x^8+d^2 e\,x^8+c^2 f\,x^6+2cde\,x^6+c^2 e\,x^4}dx$$

input
```
int((b*x^2+a)^(1/2)/x^4/(d*x^2+c)^(3/2)/(f*x^2+e),x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(c**2*e*x**4 + c**2*f*x**6 + 2*c*d
*e*x**6 + 2*c*d*f*x**8 + d**2*e*x**8 + d**2*f*x**10),x)
```

3.154 $\int \frac{\sqrt{a+bx^2}}{x^6 (c+dx^2)^{3/2} (e+fx^2)} \, dx$

Optimal result

Integrand size = 35, antiderivative size = 887

$$\int \frac{\sqrt{a+bx^2}}{x^6 (c+dx^2)^{3/2} (e+fx^2)} \, dx = \frac{(b^2c^2de^2(de-cf) - a^2d(48d^3e^3 - 8cd^2e^2f - 10c^2def^2 - 15c^3f^3) + abc(3}{15a^2c^4e^3(de-cf)\sqrt{c+}$$

$$- \frac{(a+bx^2)^{3/2}}{5acex^5\sqrt{c+dx^2}} + \frac{(2bce+6ade+5acf)(a+bx^2)^{3/2}}{15a^2c^2e^2x^3\sqrt{c+dx^2}}$$

$$+ \frac{(bcde^2 - a(24d^2e^2 + 20cdef + 15c^2f^2))(a+bx^2)^{3/2}}{15a^2c^3e^3x\sqrt{c+dx^2}}$$

$$- \frac{b(2b^2c^2e^2(de-cf) + abce(8d^2e^2 - 3cdef - 5c^2f^2) - a^2(48d^3e^3 - 8cd^2e^2f - 10c^2def^2 - 15c^3f^3))x\sqrt{c+}}{15a^2c^4e^3(de-cf)\sqrt{a+bx^2}}$$

$$+ \frac{\sqrt{b}(2b^2c^2e^2(de-cf) + abce(8d^2e^2 - 3cdef - 5c^2f^2) - a^2(48d^3e^3 - 8cd^2e^2f - 10c^2def^2 - 15c^3f^3))\sqrt{c+}}{15a^{3/2}c^4e^3(de-cf)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

$$- \frac{\sqrt{b}(bcde^2 - a(24d^2e^2 + 20cdef + 15c^2f^2))\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right), 1 - \frac{ad}{bc}\right)}{15\sqrt{a}c^4e^3\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

$$+ \frac{a^{3/2}f^4\sqrt{c+dx^2}\,\text{EllipticPi}\left(1 - \frac{af}{be}, \arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right), 1 - \frac{ad}{bc}\right)}{\sqrt{b}ce^4(de-cf)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
1/15*(b^2*c^2*d*e^2*(-c*f+d*e)-a^2*d*(-15*c^3*f^3-10*c^2*d*e*f^2-8*c*d^2*e
^2*f+48*d^3*e^3)+a*b*c*(-15*c^3*f^3-10*c^2*d*e*f^2-7*c*d^2*e^2*f+32*d^3*e^
3))*x*(b*x^2+a)^(1/2)/a^2/c^4/e^3/(-c*f+d*e)/(d*x^2+c)^(1/2)-1/5*(b*x^2+a)
^(3/2)/a/c/e/x^5/(d*x^2+c)^(1/2)+1/15*(5*a*c*f+6*a*d*e+2*b*c*e)*(b*x^2+a)^
(3/2)/a^2/c^2/e^2/x^3/(d*x^2+c)^(1/2)+1/15*(b*c*d*e^2-a*(15*c^2*f^2+20*c*d
*e*f+24*d^2*e^2))*(b*x^2+a)^(3/2)/a^2/c^3/e^3/x/(d*x^2+c)^(1/2)-1/15*b*(2*
b^2*c^2*e^2*(-c*f+d*e)+a*b*c*e*(-5*c^2*f^2-3*c*d*e*f+8*d^2*e^2)-a^2*(-15*c
^3*f^3-10*c^2*d*e*f^2-8*c*d^2*e^2*f+48*d^3*e^3))*x*(d*x^2+c)^(1/2)/a^2/c^4
/e^3/(-c*f+d*e)/(b*x^2+a)^(1/2)+1/15*b^(1/2)*(2*b^2*c^2*e^2*(-c*f+d*e)+a*b
*c*e*(-5*c^2*f^2-3*c*d*e*f+8*d^2*e^2)-a^2*(-15*c^3*f^3-10*c^2*d*e*f^2-8*c*
d^2*e^2*f+48*d^3*e^3))*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^
2/a)^(1/2),(1-a*d/b/c)^(1/2))/a^(3/2)/c^4/e^3/(-c*f+d*e)/(b*x^2+a)^(1/2)/(
a*(d*x^2+c)/c/(b*x^2+a))^(1/2)-1/15*b^(1/2)*(b*c*d*e^2-a*(15*c^2*f^2+20*c*
d*e*f+24*d^2*e^2))*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2
)),(1-a*d/b/c)^(1/2))/a^(1/2)/c^4/e^3/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^
2+a))^(1/2)+a^(3/2)*f^4*(d*x^2+c)^(1/2)*EllipticPi(b^(1/2)*x/a^(1/2)/(1+b*
x^2/a)^(1/2),1-a*f/b/e,(1-a*d/b/c)^(1/2))/b^(1/2)/c/e^4/(-c*f+d*e)/(b*x^2+
a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.00 (sec) , antiderivative size = 1430, normalized size of antiderivative = 1.61

$$\int \frac{\sqrt{a+bx^2}}{x^6\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)}\,dx = \text{Too large to display}$$

input
```
Integrate[Sqrt[a + b*x^2]/(x^6*(c + d*x^2)^(3/2)*(e + f*x^2)),x]
```

output
```
(3*a^3*Sqrt[b/a]*c^3*d*e^4 - 3*a^3*Sqrt[b/a]*c^4*e^3*f + 4*a^3*(b/a)^(3/2)
*c^3*d*e^4*x^2 - 6*a^3*Sqrt[b/a]*c^2*d^2*e^4*x^2 - 4*a^3*(b/a)^(3/2)*c^4*e
^3*f*x^2 + a^3*Sqrt[b/a]*c^3*d*e^3*f*x^2 + 5*a^3*Sqrt[b/a]*c^4*e^2*f^2*x^2
 - a*b^2*Sqrt[b/a]*c^3*d*e^4*x^4 - 13*a^3*(b/a)^(3/2)*c^2*d^2*e^4*x^4 + 24
*a^3*Sqrt[b/a]*c*d^3*e^4*x^4 + a*b^2*Sqrt[b/a]*c^4*e^3*f*x^4 + 3*a^3*(b/a)
^(3/2)*c^3*d*e^3*f*x^4 - 4*a^3*Sqrt[b/a]*c^2*d^2*e^3*f*x^4 + 10*a^3*(b/a)^
(3/2)*c^4*e^2*f^2*x^4 - 5*a^3*Sqrt[b/a]*c^3*d*e^2*f^2*x^4 - 15*a^3*Sqrt[b/
a]*c^4*e*f^3*x^4 - 2*b^3*Sqrt[b/a]*c^3*d*e^4*x^6 - 9*a*b^2*Sqrt[b/a]*c^2*d
^2*e^4*x^6 + 16*a^3*(b/a)^(3/2)*c*d^3*e^4*x^6 + 48*a^3*Sqrt[b/a]*d^4*e^4*x
^6 + 2*b^3*Sqrt[b/a]*c^4*e^3*f*x^6 + 4*a*b^2*Sqrt[b/a]*c^3*d*e^3*f*x^6 - a
^3*(b/a)^(3/2)*c^2*d^2*e^3*f*x^6 - 8*a^3*Sqrt[b/a]*c*d^3*e^3*f*x^6 + 5*a*b
^2*Sqrt[b/a]*c^4*e^2*f^2*x^6 - 10*a^3*Sqrt[b/a]*c^2*d^2*e^2*f^2*x^6 - 15*a
^3*(b/a)^(3/2)*c^4*e*f^3*x^6 - 15*a^3*Sqrt[b/a]*c^3*d*e*f^3*x^6 - 2*b^3*Sq
rt[b/a]*c^2*d^2*e^4*x^8 - 8*a*b^2*Sqrt[b/a]*c*d^3*e^4*x^8 + 48*a^3*(b/a)^(
3/2)*d^4*e^4*x^8 + 2*b^3*Sqrt[b/a]*c^3*d*e^3*f*x^8 + 3*a*b^2*Sqrt[b/a]*c^2
*d^2*e^3*f*x^8 - 8*a^3*(b/a)^(3/2)*c*d^3*e^3*f*x^8 + 5*a*b^2*Sqrt[b/a]*c^3
*d*e^2*f^2*x^8 - 10*a^3*(b/a)^(3/2)*c^2*d^2*e^2*f^2*x^8 - 15*a^3*(b/a)^(3/
2)*c^3*d*e*f^3*x^8 - I*b*c*e*(2*b^2*c^2*e^2*(d*e - c*f) + a*b*c*e*(8*d^2*e
^2 - 3*c*d*e*f - 5*c^2*f^2) + a^2*(-48*d^3*e^3 + 8*c*d^2*e^2*f + 10*c^2*d*
e*f^2 + 15*c^3*f^3))*x^5*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Ellipt...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}}{x^6 \left(c+dx^2\right)^{3/2} \left(e+fx^2\right)} \, dx$$
$$\Big\downarrow \text{450}$$
$$\int \frac{\sqrt{a+bx^2}}{x^6 \left(c+dx^2\right)^{3/2} \left(e+fx^2\right)}dx$$

input
```
Int[Sqrt[a + b*x^2]/(x^6*(c + d*x^2)^(3/2)*(e + f*x^2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [A] (verified)

Time = 23.89 (sec) , antiderivative size = 1123, normalized size of antiderivative = 1.27

method	result	size
risch	Expression too large to display	1123
elliptic	Expression too large to display	1780
default	Expression too large to display	2310

input
```
int((b*x^2+a)^(1/2)/x^6/(d*x^2+c)^(3/2)/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
-1/15*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(15*a^2*c^2*f^2*x^4+25*a^2*c*d*e*f*x
^4+33*a^2*d^2*e^2*x^4-5*a*b*c^2*e*f*x^4-8*a*b*c*d*e^2*x^4-2*b^2*c^2*e^2*x^
4-5*a^2*c^2*e*f*x^2-9*a^2*c*d*e^2*x^2+a*b*c^2*e^2*x^2+3*a^2*c^2*e^2)/a^2/c
^4/e^3/x^5+1/15/e^3/c^4/a^2*(-b*(15*a^2*c^2*f^2+25*a^2*c*d*e*f+33*a^2*d^2*
e^2-5*a*b*c^2*e*f-8*a*b*c*d*e^2-2*b^2*c^2*e^2)*c/(-b/a)^(1/2)*(1+b*x^2/a)^
(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(EllipticF(x*(
-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*
c)/c/b)^(1/2)))-a*b^2*c^2*d*e^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)
^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a
*d+b*c)/c/b)^(1/2))+9*a^2*b*c*d^2*e^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*
x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),
(-1+(a*d+b*c)/c/b)^(1/2))+5*a^2*b*c^2*d*e*f/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)
*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^
(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-15*a^2*c^4*f^3*(a*f-b*e)/(c*f-d*e)/e/(-b/a
)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^
(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))+15*a^
2*c*d^3*e^3*(a*d-b*c)/(c*f-d*e)*((b*d*x^2+a*d)/c/(a*d-b*c)*x/((x^2+c/d)*(b
*d*x^2+a*d))^(1/2)+(1/c-1/(a*d-b*c)/c*a*d)/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*
(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(
1/2),(-1+(a*d+b*c)/c/b)^(1/2))+b/(a*d-b*c)/(-b/a)^(1/2)*(1+b*x^2/a)^(1/...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{x^6\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)}\,dx=\text{Timed out}$$

input
```
integrate((b*x^2+a)^(1/2)/x^6/(d*x^2+c)^(3/2)/(f*x^2+e),x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}}{x^6 (c+dx^2)^{3/2} (e+fx^2)}\,dx = \int \frac{\sqrt{a+bx^2}}{x^6 (c+dx^2)^{\frac{3}{2}} (e+fx^2)}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)/x**6/(d*x**2+c)**(3/2)/(f*x**2+e),x)
```

output
```
Integral(sqrt(a + b*x**2)/(x**6*(c + d*x**2)**(3/2)*(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}}{x^6 (c+dx^2)^{3/2} (e+fx^2)}\,dx = \int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{\frac{3}{2}}(fx^2+e)x^6}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/x^6/(d*x^2+c)^(3/2)/(f*x^2+e),x, algorithm="maxi
ma")
```

output
```
integrate(sqrt(b*x^2 + a)/((d*x^2 + c)^(3/2)*(f*x^2 + e)*x^6), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}}{x^6 (c+dx^2)^{3/2} (e+fx^2)}\,dx = \int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{\frac{3}{2}}(fx^2+e)x^6}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/x^6/(d*x^2+c)^(3/2)/(f*x^2+e),x, algorithm="giac
")
```

output
```
integrate(sqrt(b*x^2 + a)/((d*x^2 + c)^(3/2)*(f*x^2 + e)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{x^6\,(c+dx^2)^{3/2}\,(e+fx^2)}\,dx=\int \frac{\sqrt{b\,x^2+a}}{x^6\,{\left(d\,x^2+c\right)}^{3/2}\,\left(f\,x^2+e\right)}\,dx$$

input
```
int((a + b*x^2)^(1/2)/(x^6*(c + d*x^2)^(3/2)*(e + f*x^2)),x)
```

output
```
int((a + b*x^2)^(1/2)/(x^6*(c + d*x^2)^(3/2)*(e + f*x^2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}}{x^6\,(c+dx^2)^{3/2}\,(e+fx^2)}\,dx=\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{d^2f\,x^{12}+2cdf\,x^{10}+d^2e\,x^{10}+c^2f\,x^8+2cde\,x^8+c^2e\,x^6}dx$$

input
```
int((b*x^2+a)^(1/2)/x^6/(d*x^2+c)^(3/2)/(f*x^2+e),x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(c**2*e*x**6 + c**2*f*x**8 + 2*c*d
*e*x**8 + 2*c*d*f*x**10 + d**2*e*x**10 + d**2*f*x**12),x)
```

3.155 $\int \frac{x^4\sqrt{a+bx^2}}{(c+dx^2)^{3/2}(e+fx^2)^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 542

$$\int \frac{x^4\sqrt{a+bx^2}}{(c+dx^2)^{3/2}(e+fx^2)^2}\,dx = \frac{(3bce-ade-2acf)x\sqrt{a+bx^2}}{2(be-af)(de-cf)^2\sqrt{c+dx^2}} - \frac{b(de+2cf)x\sqrt{c+dx^2}}{2df(de-cf)^2\sqrt{a+bx^2}} + \frac{ex(a+bx^2)^{3/2}}{2(be-af)(de-cf)\sqrt{c+dx^2}\,(e+fx^2)} + \frac{\sqrt{a}\sqrt{b}(de+2cf)\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{2df(de-cf)^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} - \frac{a^{3/2}\sqrt{b}e\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{2cf(be-af)(de-cf)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} + \frac{a^{3/2}(3acf^2+be(de-4cf))\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{2\sqrt{b}cf(be-af)(de-cf)^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
1/2*(-2*a*c*f-a*d*e+3*b*c*e)*x*(b*x^2+a)^(1/2)/(-a*f+b*e)/(-c*f+d*e)^2/(d*
x^2+c)^(1/2)-1/2*b*(2*c*f+d*e)*x*(d*x^2+c)^(1/2)/d/f/(-c*f+d*e)^2/(b*x^2+a
)^(1/2)+1/2*e*x*(b*x^2+a)^(3/2)/(-a*f+b*e)/(-c*f+d*e)/(d*x^2+c)^(1/2)/(f*x
^2+e)+1/2*a^(1/2)*b^(1/2)*(2*c*f+d*e)*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/
a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/d/f/(-c*f+d*e)^2/(b*x^2+a)^(1
/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)-1/2*a^(3/2)*b^(1/2)*e*(d*x^2+c)^(1/2)*
InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/c/f/(-a*f+b*e
)/(-c*f+d*e)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)+1/2*a^(3/2)*(
3*a*c*f^2+b*e*(-4*c*f+d*e))*(d*x^2+c)^(1/2)*EllipticPi(b^(1/2)*x/a^(1/2)/(
1+b*x^2/a)^(1/2),1-a*f/b/e,(1-a*d/b/c)^(1/2))/b^(1/2)/c/f/(-a*f+b*e)/(-c*f
+d*e)^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.26 (sec) , antiderivative size = 359, normalized size of antiderivative = 0.66

$$\int \frac{x^4\sqrt{a+bx^2}}{(c+dx^2)^{3/2}(e+fx^2)^2}\,dx = \frac{\sqrt{\frac{b}{a}}df^2x(a+bx^2)\left(3ce+dex^2+2cfx^2\right)+ibcf(de+2cf)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\ldots}}{\ldots}$$

input
```
Integrate[(x^4*Sqrt[a + b*x^2])/((c + d*x^2)^(3/2)*(e + f*x^2)^2),x]
```

output
```
(Sqrt[b/a]*d*f^2*x*(a + b*x^2)*(3*c*e + d*e*x^2 + 2*c*f*x^2) + I*b*c*f*(d*
e + 2*c*f)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(e + f*x^2)*EllipticE[I
*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*b*(d^2*e^2 - 3*c*d*e*f + 2*c^2*f^2
)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(e + f*x^2)*EllipticF[I*ArcSinh[
Sqrt[b/a]*x], (a*d)/(b*c)] + I*d*(3*a*c*f^2 + b*e*(d*e - 4*c*f))*Sqrt[1 +
(b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(e + f*x^2)*EllipticPi[(a*f)/(b*e), I*ArcSi
nh[Sqrt[b/a]*x], (a*d)/(b*c)])/(2*Sqrt[b/a]*d*f^2*(d*e - c*f)^2*Sqrt[a + b
*x^2]*Sqrt[c + d*x^2]*(e + f*x^2))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4\sqrt{a+bx^2}}{(c+dx^2)^{3/2}(e+fx^2)^2}\,dx$$
$$\Big\downarrow \text{ 450}$$
$$\int \frac{x^4\sqrt{a+bx^2}}{(c+dx^2)^{3/2}(e+fx^2)^2}dx$$

input
```
Int[(x^4*Sqrt[a + b*x^2])/((c + d*x^2)^(3/2)*(e + f*x^2)^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [A] (verified)

Time = 8.82 (sec) , antiderivative size = 972, normalized size of antiderivative = 1.79

method	result
elliptic	$\frac{\sqrt{(b\,x^2+a)(x^2d+c)}\left(\frac{e x\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{2(cf-de)^2\left(f\,x^2+e\right)}+\frac{\left(bd\,x^2+ad\right)cx}{d(cf-de)^2\sqrt{\left(x^2+\frac{c}{d}\right)\left(bd\,x^2+ad\right)}}+\frac{\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}\,f^2d}\right.}{}$
default	Expression too large to display

input
```
int(x^4*(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^2,x,method=_RETURNVERBOS
E)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/2*e/(c*f-d*
e)^2*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(f*x^2+e)+(b*d*x^2+a*d)*c/d*x/(
c*f-d*e)^2/((x^2+c/d)*(b*d*x^2+a*d))^(1/2)+1/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2
)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)
^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))*b/f^2/d-1/2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2
)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)
^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))*b*d*e^2/f^2/(c*f-d*e)^2+1/2*c/(-b/a)^(1/2
)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*
b*e/(c*f-d*e)^2/f*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-1/2*c
/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2
+a*c)^(1/2)*b*e/(c*f-d*e)^2/f*EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^
(1/2))-c^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x
^2+b*c*x^2+a*c)^(1/2)/d*b/(c*f-d*e)^2*EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*
c)/c/b)^(1/2))-3/2/(c*f-d*e)^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^
(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/
e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*a*c+2*e/(c*f-d*e)^2/f/(-b/a)^(1/2)*(1+b*x^2
/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi
(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*b*c-1/2*e^2/f^2/(c*f-
d*e)^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b
*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^4\sqrt{a+bx^2}}{(c+dx^2)^{3/2}(e+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(x^4*(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^2,x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^4\sqrt{a+bx^2}}{(c+dx^2)^{3/2}\,(e+fx^2)^2}\,dx = \int \frac{x^4\sqrt{a+bx^2}}{(c+dx^2)^{\frac{3}{2}}\,(e+fx^2)^2}\,dx$$

input
```
integrate(x**4*(b*x**2+a)**(1/2)/(d*x**2+c)**(3/2)/(f*x**2+e)**2,x)
```

output
```
Integral(x**4*sqrt(a + b*x**2)/((c + d*x**2)**(3/2)*(e + f*x**2)**2), x)
```

Maxima [F]

$$\int \frac{x^4\sqrt{a+bx^2}}{(c+dx^2)^{3/2}\,(e+fx^2)^2}\,dx = \int \frac{\sqrt{bx^2+a}\,x^4}{(dx^2+c)^{\frac{3}{2}}(fx^2+e)^2}\,dx$$

input
```
integrate(x^4*(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^2,x, algorithm="ma
xima")
```

output
```
integrate(sqrt(b*x^2 + a)*x^4/((d*x^2 + c)^(3/2)*(f*x^2 + e)^2), x)
```

Giac [F]

$$\int \frac{x^4\sqrt{a+bx^2}}{(c+dx^2)^{3/2}\,(e+fx^2)^2}\,dx = \int \frac{\sqrt{bx^2+a}\,x^4}{(dx^2+c)^{\frac{3}{2}}(fx^2+e)^2}\,dx$$

input
```
integrate(x^4*(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^2,x, algorithm="gi
ac")
```

output
```
integrate(sqrt(b*x^2 + a)*x^4/((d*x^2 + c)^(3/2)*(f*x^2 + e)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4\sqrt{a+bx^2}}{(c+dx^2)^{3/2}\,(e+fx^2)^2}\,dx = \int \frac{x^4\,\sqrt{b\,x^2+a}}{(d\,x^2+c)^{3/2}\,(f\,x^2+e)^2}\,dx$$

input
```
int((x^4*(a + b*x^2)^(1/2))/((c + d*x^2)^(3/2)*(e + f*x^2)^2),x)
```

output
```
int((x^4*(a + b*x^2)^(1/2))/((c + d*x^2)^(3/2)*(e + f*x^2)^2), x)
```

Reduce [F]

$$\int \frac{x^4\sqrt{a+bx^2}}{(c+dx^2)^{3/2}\,(e+fx^2)^2}\,dx = \text{too large to display}$$

input
```
int(x^4*(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^2,x)
```

output

```
( - sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*x + 2*int((sqrt(c + d*x**2)*sqrt(a
 + b*x**2)*x**6)/(2*a**2*c**2*e**2*f + 4*a**2*c**2*e*f**2*x**2 + 2*a**2*c*
*2*f**3*x**4 + 4*a**2*c*d*e**2*f*x**2 + 8*a**2*c*d*e*f**2*x**4 + 4*a**2*c*
d*f**3*x**6 + 2*a**2*d**2*e**2*f*x**4 + 4*a**2*d**2*e*f**2*x**6 + 2*a**2*d
**2*f**3*x**8 - a*b*c**2*e**3 + 3*a*b*c**2*e*f**2*x**4 + 2*a*b*c**2*f**3*x
**6 - 2*a*b*c*d*e**3*x**2 + 6*a*b*c*d*e*f**2*x**6 + 4*a*b*c*d*f**3*x**8 -
a*b*d**2*e**3*x**4 + 3*a*b*d**2*e*f**2*x**8 + 2*a*b*d**2*f**3*x**10 - b**2
*c**2*e**3*x**2 - 2*b**2*c**2*e**2*f*x**4 - b**2*c**2*e*f**2*x**6 - 2*b**2
*c*d*e**3*x**4 - 4*b**2*c*d*e**2*f*x**6 - 2*b**2*c*d*e*f**2*x**8 - b**2*d*
*2*e**3*x**6 - 2*b**2*d**2*e**2*f*x**8 - b**2*d**2*e*f**2*x**10),x)*a**2*b
*c*d*e*f**2 + 2*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**6)/(2*a**2*c**2*
e**2*f + 4*a**2*c**2*e*f**2*x**2 + 2*a**2*c**2*f**3*x**4 + 4*a**2*c*d*e**2
*f*x**2 + 8*a**2*c*d*e*f**2*x**4 + 4*a**2*c*d*f**3*x**6 + 2*a**2*d**2*e**2
*f*x**4 + 4*a**2*d**2*e*f**2*x**6 + 2*a**2*d**2*f**3*x**8 - a*b*c**2*e**3
+ 3*a*b*c**2*e*f**2*x**4 + 2*a*b*c**2*f**3*x**6 - 2*a*b*c*d*e**3*x**2 + 6*
a*b*c*d*e*f**2*x**6 + 4*a*b*c*d*f**3*x**8 - a*b*d**2*e**3*x**4 + 3*a*b*d**
2*e*f**2*x**8 + 2*a*b*d**2*f**3*x**10 - b**2*c**2*e**3*x**2 - 2*b**2*c**2*
e**2*f*x**4 - b**2*c**2*e*f**2*x**6 - 2*b**2*c*d*e**3*x**4 - 4*b**2*c*d*e*
*2*f*x**6 - 2*b**2*c*d*e*f**2*x**8 - b**2*d**2*e**3*x**6 - 2*b**2*d**2*e**
2*f*x**8 - b**2*d**2*e*f**2*x**10),x)*a**2*b*c*d*f**3*x**2 + 2*int((sqr...
```

3.156 $\int \frac{x^2\sqrt{a+bx^2}}{(c+dx^2)^{3/2}(e+fx^2)^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 514

$$\begin{aligned}\int \frac{x^2\sqrt{a+bx^2}}{(c+dx^2)^{3/2}(e+fx^2)^2}\,dx &= -\frac{(2bde+bcf-3adf)x\sqrt{a+bx^2}}{2(be-af)(de-cf)^2\sqrt{c+dx^2}} \\ &+ \frac{3bx\sqrt{c+dx^2}}{2(de-cf)^2\sqrt{a+bx^2}} - \frac{fx(a+bx^2)^{3/2}}{2(be-af)(de-cf)\sqrt{c+dx^2}\,(e+fx^2)} \\ &- \frac{3\sqrt{a}\sqrt{b}\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{2(de-cf)^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} \\ &+ \frac{a^{3/2}\sqrt{b}\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{2c(be-af)(de-cf)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} \\ &- \frac{a^{3/2}(af(2de+cf)-be(de+2cf))\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{2\sqrt{b}ce(be-af)(de-cf)^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}\end{aligned}$$

output
```
-1/2*(-3*a*d*f+b*c*f+2*b*d*e)*x*(b*x^2+a)^(1/2)/(-a*f+b*e)/(-c*f+d*e)^2/(d
*x^2+c)^(1/2)+3/2*b*x*(d*x^2+c)^(1/2)/(-c*f+d*e)^2/(b*x^2+a)^(1/2)-1/2*f*x
*(b*x^2+a)^(3/2)/(-a*f+b*e)/(-c*f+d*e)/(d*x^2+c)^(1/2)/(f*x^2+e)-3/2*a^(1/
2)*b^(1/2)*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(
1-a*d/b/c)^(1/2))/(-c*f+d*e)^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(
1/2)+1/2*a^(3/2)*b^(1/2)*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/
a^(1/2)),(1-a*d/b/c)^(1/2))/c/(-a*f+b*e)/(-c*f+d*e)/(b*x^2+a)^(1/2)/(a*(d*
x^2+c)/c/(b*x^2+a))^(1/2)-1/2*a^(3/2)*(a*f*(c*f+2*d*e)-b*e*(2*c*f+d*e))*(d
*x^2+c)^(1/2)*EllipticPi(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),1-a*f/b/e,(1-
a*d/b/c)^(1/2))/b^(1/2)/c/e/(-a*f+b*e)/(-c*f+d*e)^2/(b*x^2+a)^(1/2)/(a*(d*
x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.51 (sec) , antiderivative size = 266, normalized size of antiderivative = 0.52

$$\int \frac{x^2\sqrt{a+bx^2}}{(c+dx^2)^{3/2}(e+fx^2)^2}\,dx = \frac{-\sqrt{\frac{b}{a}}efx(a+bx^2)\left(2de+cf+3dfx^2\right)-i\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}(e+fx^2)\Big(3}{}$$

input
```
Integrate[(x^2*Sqrt[a + b*x^2])/((c + d*x^2)^(3/2)*(e + f*x^2)^2),x]
```

output
```
(-(Sqrt[b/a]*e*f*x*(a + b*x^2)*(2*d*e + c*f + 3*d*f*x^2)) - I*Sqrt[1 + (b*
x^2)/a]*Sqrt[1 + (d*x^2)/c]*(e + f*x^2)*(3*b*c*e*f*EllipticE[I*ArcSinh[Sqr
t[b/a]*x], (a*d)/(b*c)] + b*e*(d*e - c*f)*EllipticF[I*ArcSinh[Sqrt[b/a]*x]
, (a*d)/(b*c)] + (a*f*(2*d*e + c*f) - b*e*(d*e + 2*c*f))*EllipticPi[(a*f)/
(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(2*Sqrt[b/a]*e*f*(d*e - c*f)
^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\sqrt{a+bx^2}}{(c+dx^2)^{3/2}(e+fx^2)^2}\,dx$$

$$\Big\downarrow \; 450$$

$$\int \frac{x^2\sqrt{a+bx^2}}{(c+dx^2)^{3/2}(e+fx^2)^2}dx$$

input
```
Int[(x^2*Sqrt[a + b*x^2])/((c + d*x^2)^(3/2)*(e + f*x^2)^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [A] (verified)

Time = 9.21 (sec) , antiderivative size = 866, normalized size of antiderivative = 1.68

method	result
elliptic	$\frac{\sqrt{(b\,x^2+a)(x^2d+c)}\left(-\frac{fx\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{2(cf-de)^2\left(f\,x^2+e\right)}-\frac{\left(bd\,x^2+ad\right)x}{(cf-de)^2\sqrt{\left(x^2+\frac{c}{d}\right)\left(bd\,x^2+ad\right)}}+\frac{\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\ \text{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right.}{2\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}\,(cf-de)^2f}\right.}{}$
default	Expression too large to display

input
```
int(x^2*(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^2,x,method=_RETURNVERBOS
E)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-1/2*f/(c*f-d
*e)^2*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(f*x^2+e)-(b*d*x^2+a*d)*x/(c*f
-d*e)^2/((x^2+c/d)*(b*d*x^2+a*d))^(1/2)+1/2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)
*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^
(1/2),(-1+(a*d+b*c)/c/b)^(1/2))*b*d*e/(c*f-d*e)^2/f-1/2/(-b/a)^(1/2)*(1+b*
x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*Ellipti
cF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))*b*c/(c*f-d*e)^2+3/2*b/(c*f-d*e
)^2*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*
c*x^2+a*c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+1/2/(c
*f-d*e)^2*f/e/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*
d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/
(-b/a)^(1/2))*a*c+1/(c*f-d*e)^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)
^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b
/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*a*d-1/(c*f-d*e)^2/(-b/a)^(1/2)*(1+b*x^2/a)
^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*
(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*b*c-1/2/(c*f-d*e)^2*e/f/
(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+
a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*
b*d)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^2\sqrt{a+bx^2}}{(c+dx^2)^{3/2}\left(e+fx^2\right)^2}\,dx=\text{Timed out}$$

input
```
integrate(x^2*(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^2,x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^2\sqrt{a+bx^2}}{(c+dx^2)^{3/2}\left(e+fx^2\right)^2}\,dx=\int \frac{x^2\sqrt{a+bx^2}}{(c+dx^2)^{\frac{3}{2}}\left(e+fx^2\right)^2}\,dx$$

input
```
integrate(x**2*(b*x**2+a)**(1/2)/(d*x**2+c)**(3/2)/(f*x**2+e)**2,x)
```

output
```
Integral(x**2*sqrt(a + b*x**2)/((c + d*x**2)**(3/2)*(e + f*x**2)**2), x)
```

Maxima [F]

$$\int \frac{x^2\sqrt{a+bx^2}}{(c+dx^2)^{3/2}\left(e+fx^2\right)^2}\,dx=\int \frac{\sqrt{bx^2+a}x^2}{(dx^2+c)^{\frac{3}{2}}(fx^2+e)^2}\,dx$$

input
```
integrate(x^2*(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^2,x, algorithm="ma
xima")
```

output
```
integrate(sqrt(b*x^2 + a)*x^2/((d*x^2 + c)^(3/2)*(f*x^2 + e)^2), x)
```

Giac [F]

$$\int \frac{x^2\sqrt{a+bx^2}}{(c+dx^2)^{3/2}\left(e+fx^2\right)^2}\,dx=\int \frac{\sqrt{bx^2+a}x^2}{(dx^2+c)^{\frac{3}{2}}(fx^2+e)^2}\,dx$$

input
```
integrate(x^2*(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^2,x, algorithm="gi
ac")
```

output
```
integrate(sqrt(b*x^2 + a)*x^2/((d*x^2 + c)^(3/2)*(f*x^2 + e)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2\sqrt{a+bx^2}}{(c+dx^2)^{3/2}(e+fx^2)^2}\,dx = \int \frac{x^2\sqrt{b\,x^2+a}}{(d\,x^2+c)^{3/2}\,(f\,x^2+e)^2}\,dx$$

input
```
int((x^2*(a + b*x^2)^(1/2))/((c + d*x^2)^(3/2)*(e + f*x^2)^2),x)
```

output
```
int((x^2*(a + b*x^2)^(1/2))/((c + d*x^2)^(3/2)*(e + f*x^2)^2), x)
```

Reduce [F]

$$\int \frac{x^2\sqrt{a+bx^2}}{(c+dx^2)^{3/2}(e+fx^2)^2}\,dx = \int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^2}{d^2f^2x^8+2cd\,f^2x^6+2d^2ef\,x^6+c^2f^2x^4+4cdef\,x^4+d^2e^2x^4+2c^2ef\,x^2}$$

input
```
int(x^2*(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^2,x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(c**2*e**2 + 2*c**2*e*f*x**2
+ c**2*f**2*x**4 + 2*c*d*e**2*x**2 + 4*c*d*e*f*x**4 + 2*c*d*f**2*x**6 + d*
*2*e**2*x**4 + 2*d**2*e*f*x**6 + d**2*f**2*x**8),x)
```

3.157 $\int \frac{\sqrt{a+bx^2}}{\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 415

$$\int \frac{\sqrt{a+bx^2}}{\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^2}\,dx = -\frac{fx\sqrt{a+bx^2}}{2e(de-cf)\sqrt{c+dx^2}\,(e+fx^2)} + \frac{\sqrt{d}(2de+cf)\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{2\sqrt{c}e(de-cf)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} + \frac{\sqrt{c}\sqrt{d}f(3bce-4ade+acf)\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{2ae(de-cf)^3\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} - \frac{c^{3/2}f(3bde^2-af(4de-cf))\sqrt{a+bx^2}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{2a\sqrt{d}e^2(de-cf)^3\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output
```
-1/2*f*x*(b*x^2+a)^(1/2)/e/(-c*f+d*e)/(d*x^2+c)^(1/2)/(f*x^2+e)+1/2*d^(1/2
)*(c*f+2*d*e)*(b*x^2+a)^(1/2)*EllipticE(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2
),(1-b*c/a/d)^(1/2))/c^(1/2)/e/(-c*f+d*e)^2/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2
)/(d*x^2+c)^(1/2)+1/2*c^(1/2)*d^(1/2)*f*(a*c*f-4*a*d*e+3*b*c*e)*(b*x^2+a)^
(1/2)*InverseJacobiAM(arctan(d^(1/2)*x/c^(1/2)),(1-b*c/a/d)^(1/2))/a/e/(-c
*f+d*e)^3/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-1/2*c^(3/2)*f*(3
*b*d*e^2-a*f*(-c*f+4*d*e))*(b*x^2+a)^(1/2)*EllipticPi(d^(1/2)*x/c^(1/2)/(1
+d*x^2/c)^(1/2),1-c*f/d/e,(1-b*c/a/d)^(1/2))/a/d^(1/2)/e^2/(-c*f+d*e)^3/(c
*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.45 (sec) , antiderivative size = 277, normalized size of antiderivative = 0.67

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}(e+fx^2)^2}\,dx = \frac{\sqrt{\frac{b}{a}}ex(a+bx^2)\left(cf^2(c+dx^2)+2d^2e(e+fx^2)\right)+ic\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}(e}{}$$

input
```
Integrate[Sqrt[a + b*x^2]/((c + d*x^2)^(3/2)*(e + f*x^2)^2),x]
```

output
```
(Sqrt[b/a]*e*x*(a + b*x^2)*(c*f^2*(c + d*x^2) + 2*d^2*e*(e + f*x^2)) + I*c
*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(e + f*x^2)*(b*e*(2*d*e + c*f)*El
lipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + b*e*(d*e - c*f)*EllipticF[I
*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - (3*b*d*e^2 + a*f*(-4*d*e + c*f))*Ell
ipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(2*Sqrt[b/a]*c
*e^2*(d*e - c*f)^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2))
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1179 vs. 2(415) = 830.

Time = 1.70 (sec) , antiderivative size = 1179, normalized size of antiderivative = 2.84, number of steps used = 22, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.688$, Rules used = {425, 421, 25, 400, 313, 320, 414, 426, 421, 25, 400, 313, 320, 414, 424, 406, 320, 388, 313, 413, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}(e+fx^2)^2}dx$$

$$\Big\downarrow \text{425}$$

$$\frac{b\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^{3/2}(fx^2+e)}dx}{f}-\frac{(be-af)\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^{3/2}(fx^2+e)^2}dx}{f}$$

$$\Big\downarrow \text{421}$$

$$\frac{b\left(\frac{f^2\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{d\int -\frac{-dfx^2+de-2cf}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{(de-cf)^2}\right)}{f}-\frac{(be-af)\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^{3/2}(fx^2+e)^2}dx}{f}$$

$$\Big\downarrow \text{25}$$

$$\frac{b\left(\frac{f^2\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{(de-cf)^2}+\frac{d\int \frac{-dfx^2+de-2cf}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{(de-cf)^2}\right)}{f}-\frac{(be-af)\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^{3/2}(fx^2+e)^2}dx}{f}$$

$$\Big\downarrow \text{400}$$

$$\frac{b\left(\frac{f^2\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{(de-cf)^2}+\frac{d\left(\frac{(adf-2bcf+bde)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}-\frac{d(de-cf)\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{bc-ad}\right)}{(de-cf)^2}\right)}{f}-\frac{(be-af)\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^{3/2}(fx^2+e)^2}dx}{f}$$

$$\Big\downarrow \text{313}$$

$$\frac{b\left(\frac{d\left(\frac{(adf-2bcf+bde)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}-\frac{\sqrt{d}\sqrt{a+bx^2}(de-cf)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{(de-cf)^2}+\frac{f^2\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}(fx^2+e)}dx}{(de-cf)^2}\right)}{f}-\frac{(be-af)\int\frac{1}{\sqrt{bx^2+a}(dx^2+c)^{3/2}(fx^2+e)^2}dx}{f}$$

$$\Big\downarrow \text{320}$$

$$\frac{b\left(\frac{f^2\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}(fx^2+e)}dx}{(de-cf)^2}+\frac{d\left(\frac{\sqrt{c}\sqrt{a+bx^2}(adf-2bcf+bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{d}\sqrt{a+bx^2}(de-cf)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{(de-cf)^2}\right)}{f}-\frac{(be-af)\int\frac{1}{\sqrt{bx^2+a}(dx^2+c)^{3/2}(fx^2+e)^2}dx}{f}$$

$$\Big\downarrow \text{414}$$

$$\frac{b\left(\frac{c^{3/2}f^2\sqrt{a+bx^2}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}e\sqrt{c+dx^2}(de-cf)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{d\left(\frac{\sqrt{c}\sqrt{a+bx^2}(adf-2bcf+bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{d}\sqrt{a+bx^2}(de-cf)E}{\sqrt{c}\sqrt{c+dx^2}(bc}\right.}{(de-cf)^2}\right.}{f}$$

$$\frac{(be-af)\int\frac{1}{\sqrt{bx^2+a}(dx^2+c)^{3/2}(fx^2+e)^2}dx}{f}$$

$$\Big\downarrow \text{426}$$

$$\frac{b\left(\frac{c^{3/2}f^2\sqrt{a+bx^2}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}e\sqrt{c+dx^2}(de-cf)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{d\left(\frac{\sqrt{c}\sqrt{a+bx^2}(adf-2bcf+bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{d}\sqrt{a+bx^2}(de-cf)E}{\sqrt{c}\sqrt{c+dx^2}(bc}\right.}{(de-cf)^2}\right.}{f}$$

$$\frac{(be-af)\left(\frac{d\int\frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}\left(fx^2+e\right)}dx}{de-cf}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^2}dx}{de-cf}\right)}{f}$$

$\downarrow$ 421

$$\frac{b\left(\frac{c^{3/2}f^2\sqrt{a+bx^2}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}e\sqrt{c+dx^2}(de-cf)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{d\left(\frac{\sqrt{c}\sqrt{a+bx^2}(adf-2bcf+bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{d}\sqrt{a+bx^2}(de-cf)E}{\sqrt{c}\sqrt{c+dx^2}(bc}\right.}{(de-cf)^2}\right.}{f}$$

$$\frac{(be-af)\left(\frac{d\left(\frac{f^2\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{(de-cf)^2}-\frac{d\int-\frac{-dfx^2+de-2cf}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{(de-cf)^2}\right)}{de-cf}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^2}dx}{de-cf}\right)}{f}$$

$\downarrow$ 25

$$\frac{b\left(\frac{c^{3/2}f^2\sqrt{a+bx^2}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}e\sqrt{c+dx^2}(de-cf)^2\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+\frac{d\left(\frac{\sqrt{c}\sqrt{a+bx^2}(adf-2bcf+bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{\sqrt{d}\sqrt{a+bx^2}(de-cf)E\cdots}{\sqrt{c}\sqrt{c+dx^2}(bc\cdots}\right)}{(de-cf)^2}\cdots\right)}{f}$$

$$\frac{(be-af)\left(\frac{d\left(\frac{f^2\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{(de-cf)^2}+\frac{d\int\frac{-dfx^2+de-2cf}{\sqrt{bx^2+a}\left(dx^2+c\right)^{3/2}}dx}{(de-cf)^2}\right)}{de-cf}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^2}dx}{de-cf}\right)}{f}$$

$$\Big\downarrow \; 400$$

$$\frac{b\left(\frac{c^{3/2}f^2\sqrt{a+bx^2}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}e\sqrt{c+dx^2}(de-cf)^2\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+\frac{d\left(\frac{\sqrt{c}\sqrt{a+bx^2}(adf-2bcf+bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{\sqrt{d}\sqrt{a+bx^2}(de-cf)E\cdots}{\sqrt{c}\sqrt{c+dx^2}(bc\cdots}\right)}{(de-cf)^2}\cdots\right)}{f}$$

$$\frac{(be-af)\left(\frac{d\left(\frac{f^2\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{(de-cf)^2}+\frac{d\left(\frac{(adf-2bcf+bde)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}-\frac{d(de-cf)\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{bc-ad}\right)}{(de-cf)^2}\right)}{de-cf}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+\cdots\right.}}{de-cf}\cdots\right)}{f}$$

$$\Big\downarrow \; 313$$

$$\frac{b\left(\frac{c^{3/2}f^2\sqrt{a+bx^2}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}e\sqrt{c+dx^2}(de-cf)^2\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+\frac{d\left(\frac{\sqrt{c}\sqrt{a+bx^2}(adf-2bcf+bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{\sqrt{d}\sqrt{a+bx^2}(de-cf)E}{\sqrt{c}\sqrt{c+dx^2}(bc}\right.}{(de-cf)^2}\right.}{f}$$

$$\frac{(be-af)\left(\frac{d\left(\frac{d\left(\frac{(adf-2bcf+bde)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}-\frac{\sqrt{d}\sqrt{a+bx^2}(de-cf)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{(de-cf)^2}+\frac{f^2\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{(de-cf)^2}\right)}{de-cf}-\frac{f\int}{\sqrt{bx^2}}\right.}{f}$$

$\downarrow$ 320

$$b\left(\frac{c^{3/2}f^2\sqrt{a+bx^2}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}e\sqrt{c+dx^2}(de-cf)^2\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+\frac{d\left(\frac{\sqrt{c}\sqrt{a+bx^2}(adf-2bcf+bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{\sqrt{d}\sqrt{a+bx^2}(de-cf)E}{\sqrt{c}\sqrt{c+dx^2}(bc}\right.}{(de-cf)^2}\right.}{f}$$

$$\frac{(be-af)\left(\frac{d\left(\frac{f^2\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)}dx}{(de-cf)^2}+\frac{d\left(\frac{\sqrt{c}\sqrt{a+bx^2}(adf-2bcf+bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{\sqrt{d}\sqrt{a+bx^2}(de-cf)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{b}{a}\right.}{\sqrt{c}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right.}{(de-cf)^2}\right.}{de-cf}\right.}{f}$$

$\downarrow$ 414

$$\frac{b\left(\frac{c^{3/2}f^2\sqrt{a+bx^2}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}e\sqrt{c+dx^2}(de-cf)^2\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+\frac{d\left(\frac{\sqrt{c}\sqrt{a+bx^2}(adf-2bcf+bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{\sqrt{d}\sqrt{a+bx^2}(de-cf)E}{\sqrt{c}\sqrt{c+dx^2}(bc}\right.}{(de-cf)^2}\right.}{f}$$

$$\frac{(be-af)\left(\frac{d\left(\frac{c^{3/2}f^2\sqrt{a+bx^2}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}e\sqrt{c+dx^2}(de-cf)^2\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+\frac{d\left(\frac{\sqrt{c}\sqrt{a+bx^2}(adf-2bcf+bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{c}\sqrt{}}\right.}{(de-cf)^2}\right.}{de-cf}\right.}{f}$$

$\downarrow$ 424

$$\frac{b\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}E}{\sqrt{c}(bc-ad)\sqrt{\frac{c}{a}}}\right.}{(de-cf)^2}}{f}$$

$$(be-af)\left(\frac{d\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)}{\sqrt{c}(b}\right.}{(de-cf)^2}\right.}{de-cf}\right.$$

$$\downarrow 406$$

$$\frac{b\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}E}{\sqrt{c}(bc-ad)\sqrt{\frac{c}{a}}}\right.}{(de-cf)^2}}{f}$$

$$(be-af)\left(\frac{d\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)}{\sqrt{c}(bc}\right.}{(de-cf)^2}}{de-cf}\right.$$

↓ 320

$$\frac{b\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}E}{\sqrt{c}(bc-ad)\sqrt{\frac{c}{a}}}\right.}{(de-cf)^2}}{f}$$

$$(be-af)\left(\frac{d\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)}{\sqrt{c}(bc}\right.}{(de-cf)^2}\right)}{de-cf}\right.$$

$$\Big\downarrow \; 388$$

$$b\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}E\ldots}{\sqrt{c}(bc-ad)\sqrt{\ldots}}\ldots\right)}{(de-cf)^2}\ldots\right)$$

$$\overline{\qquad f \qquad}$$

$$(be-af)\left(\frac{d\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\ldots}{\sqrt{c}(b\ldots}\ldots\right)}{(de-cf)^2}\ldots\right)}{de-cf}\ldots\right)$$

$$\downarrow 313$$

$$\frac{b\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}E}{\sqrt{c}(bc-ad)\sqrt{\frac{c}{a}}}\right.}{(de-cf)^2}}{f}$$

$$\frac{(be-af)\left(\frac{d\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)}{\sqrt{c}(b}\right.}{(de-cf)^2}\right)}{de-cf}\right.}{}$$

$\downarrow$ 413

$$\frac{b\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}\,E\cdots}{\sqrt{c}(bc-ad)\sqrt{\frac{c\cdots}{a\cdots}}}\cdots\right)}{(de-cf)^2}\cdots\right)}{f}\cdots$$

$$\frac{(be-af)\left(\frac{d\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\cdots}{\sqrt{c}(b\cdots}\cdots\right)}{(de-cf)^2}\cdots\right)}{de-cf}\cdots\right)}{\cdots}$$

↓ 413

$$\frac{b\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}E}{\sqrt{c}(bc-ad)\sqrt{\frac{c}{a}}}\right.}{(de-cf)^2}}{f}$$

$$\frac{(be-af)\left(\frac{d\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)}{\sqrt{c}(bc}\right.}{(de-cf)^2}\right)}{de-cf}\right.}{}$$

$$\downarrow 412$$

$$b\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)\sqrt{bx^2+a}E}{\sqrt{c}(bc-ad)\sqrt{\frac{c}{a}}}\right.}{(de-cf)^2}\right.$$

$$\overline{\qquad\qquad\qquad\qquad f \qquad\qquad\qquad\qquad}$$

$$(be-af)\left(\frac{d\left(\frac{c^{3/2}\sqrt{bx^2+a}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)f^2}{a\sqrt{d}e(de-cf)^2\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\frac{d\left(\frac{\sqrt{c}(bde-2bcf+adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\frac{\sqrt{d}(de-cf)}{\sqrt{c}(bc}\right.}{(de-cf)^2}\right.}{de-cf}\right.$$

input
```
Int[Sqrt[a + b*x^2]/((c + d*x^2)^(3/2)*(e + f*x^2)^2),x]
```

output
```
(b*((d*(-((Sqrt[d]*(d*e - c*f)*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x
)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[c]*(b*c - a*d)*Sqrt[(c*(a + b*x^2))/(a
*(c + d*x^2))]*Sqrt[c + d*x^2])) + (Sqrt[c]*(b*d*e - 2*b*c*f + a*d*f)*Sqrt
[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sq
rt[d]*(b*c - a*d)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])))
/(d*e - c*f)^2 + (c^(3/2)*f^2*Sqrt[a + b*x^2]*EllipticPi[1 - (c*f)/(d*e),
ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*e*(d*e - c*f)^2*
Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])))/f - ((b*e - a*f)*
(-((f*((f^2*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(2*e*(b*e - a*f)*(d*e - c*f
)*(e + f*x^2)) - (b*d*(f*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[
c]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)]
)/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (Sq
rt[c]*e*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(
a*d)])/(a*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])))
/(2*e*(b*e - a*f)*(d*e - c*f)) + (Sqrt[-a]*(b*e*(3*d*e - 2*c*f) - a*f*(2*d
*e - c*f))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e),
 ArcSin[(Sqrt[b]*x)/Sqrt[-a]], (a*d)/(b*c)])/(2*Sqrt[b]*e^2*(b*e - a*f)*(d
*e - c*f)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])))/(d*e - c*f)) + (d*((d*(-((Sqr
t[d]*(d*e - c*f)*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1
- (b*c)/(a*d)])/(Sqrt[c]*(b*c - a*d)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 400
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 421
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b^2/(b*c - a*d)^2   Int[(c + d*x^2)^(q + 2)*((e +
 f*x^2)^r/(a + b*x^2)), x], x] - Simp[d/(b*c - a*d)^2   Int[(c + d*x^2)^q*(
e + f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r}
, x] && LtQ[q, -1]
```

rule 424
```
Int[1/(((a_) + (b_.)*(x_)^2)^2*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*
(x_)^2]), x_Symbol] :> Simp[b^2*x*Sqrt[c + d*x^2]*(Sqrt[e + f*x^2]/(2*a*(b*
c - a*d)*(b*e - a*f)*(a + b*x^2))), x] + (Simp[(b^2*c*e + 3*a^2*d*f - 2*a*b
*(d*e + c*f))/(2*a*(b*c - a*d)*(b*e - a*f))   Int[1/((a + b*x^2)*Sqrt[c + d
*x^2]*Sqrt[e + f*x^2]), x], x] - Simp[d*(f/(2*a*(b*c - a*d)*(b*e - a*f)))
 Int[(a + b*x^2)/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x], x]) /; FreeQ[{a, b,
 c, d, e, f}, x]
```

rule 425
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[d/b   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q -
 1)*(e + f*x^2)^r, x], x] + Simp[(b*c - a*d)/b   Int[(a + b*x^2)^p*(c + d*x
^2)^(q - 1)*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && ILt
Q[p, 0] && GtQ[q, 0]
```

rule 426
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[b/(b*c - a*d)   Int[(a + b*x^2)^p*(c + d*x^2)^
(q + 1)*(e + f*x^2)^r, x], x] - Simp[d/(b*c - a*d)   Int[(a + b*x^2)^(p + 1
)*(c + d*x^2)^q*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] &&
 ILtQ[p, 0] && LeQ[q, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 872 vs. $2(389) = 778$.

Time = 8.90 (sec) , antiderivative size = 873, normalized size of antiderivative = 2.10

method	result
elliptic	$\frac{\sqrt{(b\,x^2+a)(x^2d+c)}\left(\frac{f^2x\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{2(cf-de)^2e\left(f\,x^2+e\right)}+\frac{\left(bd\,x^2+ad\right)dx}{c(cf-de)^2\sqrt{\left(x^2+\frac{c}{d}\right)\left(bd\,x^2+ad\right)}}-\frac{\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right.}{2\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}\,(cf-de)^2}\right.}{}$
default	Expression too large to display

input
```
int((b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^2,x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/2*f^2/(c*f-
d*e)^2/e*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(f*x^2+e)+(b*d*x^2+a*d)/c*d
*x/(c*f-d*e)^2/((x^2+c/d)*(b*d*x^2+a*d))^(1/2)-1/2/(-b/a)^(1/2)*(1+b*x^2/a
)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*
(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))*d*b/(c*f-d*e)^2+1/2*c/(-b/a)^(1/2)*
(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*f*
b/(c*f-d*e)^2/e*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-1/2*c/(
-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a
*c)^(1/2)*f*b/(c*f-d*e)^2/e*EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1
/2))-1/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b
*c*x^2+a*c)^(1/2)*d*b/(c*f-d*e)^2*EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c
/b)^(1/2))+1/2/(c*f-d*e)^2/e^2*f^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2
/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*
f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*a*c-2/(c*f-d*e)^2/e*f/(-b/a)^(1/2)*(1+b
*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*Ellipt
icPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*a*d+3/2/(c*f-d*e)
^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x
^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2
))*b*d)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}(e+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^2,x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}(e+fx^2)^2}\,dx = \int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{\frac{3}{2}}(e+fx^2)^2}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)/(d*x**2+c)**(3/2)/(f*x**2+e)**2,x)
```

output
```
Integral(sqrt(a + b*x**2)/((c + d*x**2)**(3/2)*(e + f*x**2)**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}(e+fx^2)^2}\,dx = \int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{\frac{3}{2}}(fx^2+e)^2}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^2,x, algorithm="maxima
")
```

output
```
integrate(sqrt(b*x^2 + a)/((d*x^2 + c)^(3/2)*(f*x^2 + e)^2), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}(e+fx^2)^2}\,dx = \int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{\frac{3}{2}}(fx^2+e)^2}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^2,x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^2 + a)/((d*x^2 + c)^(3/2)*(f*x^2 + e)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}(e+fx^2)^2}\,dx = \int \frac{\sqrt{b\,x^2+a}}{(d\,x^2+c)^{3/2}\,(f\,x^2+e)^2}\,dx$$

input
```
int((a + b*x^2)^(1/2)/((c + d*x^2)^(3/2)*(e + f*x^2)^2),x)
```

output
```
int((a + b*x^2)^(1/2)/((c + d*x^2)^(3/2)*(e + f*x^2)^2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}(e+fx^2)^2}\,dx = \int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{d^2f^2x^8+2cd\,f^2x^6+2d^2ef\,x^6+c^2f^2x^4+4cdef\,x^4+d^2e^2x^4+2c^2ef\,x^2}$$

input
```
int((b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)/(f*x^2+e)^2,x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(c**2*e**2 + 2*c**2*e*f*x**2 + c**
2*f**2*x**4 + 2*c*d*e**2*x**2 + 4*c*d*e*f*x**4 + 2*c*d*f**2*x**6 + d**2*e*
*2*x**4 + 2*d**2*e*f*x**6 + d**2*f**2*x**8),x)
```

3.158 $\int \frac{\sqrt{a+bx^2}}{x^2(c+dx^2)^{3/2}(e+fx^2)^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 766

$$
\begin{aligned}
\int \frac{\sqrt{a+bx^2}}{x^2(c+dx^2)^{3/2}(e+fx^2)^2}\,dx &= \frac{(2b^2ce(de-cf)^2+a^2df(4d^2e^2-4cdef+3c^2f^2)-ab(4d^3e^3-2cd^2e^2f}{2ac^2e^2(be-af)(de-cf)^2\sqrt{c+dx^2}} \\
&+ \frac{b(4d^2e^2-4cdef+3c^2f^2)\,x\sqrt{c+dx^2}}{2c^2e^2(de-cf)^2\sqrt{a+bx^2}} - \frac{(a+bx^2)^{3/2}}{acex\sqrt{c+dx^2}\,(e+fx^2)} \\
&+ \frac{f(af(2de-3cf)-2be(de-cf))x(a+bx^2)^{3/2}}{2ace^2(be-af)(de-cf)\sqrt{c+dx^2}\,(e+fx^2)} \\
&- \frac{\sqrt{a}\sqrt{b}(4d^2e^2-4cdef+3c^2f^2)\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{2c^2e^2(de-cf)^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} \\
&- \frac{\sqrt{a}\sqrt{b}(af(2de-3cf)-2be(de-cf))\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{2c^2e^2(be-af)(de-cf)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} \\
&+ \frac{a^{3/2}f^2(be(5de-2cf)-3af(2de-cf))\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{2\sqrt{b}ce^3(be-af)(de-cf)^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}
\end{aligned}
$$

output
```
1/2*(2*b^2*c*e*(-c*f+d*e)^2+a^2*d*f*(3*c^2*f^2-4*c*d*e*f+4*d^2*e^2)-a*b*(3
*c^3*f^3-2*c^2*d*e*f^2-2*c*d^2*e^2*f+4*d^3*e^3))*x*(b*x^2+a)^(1/2)/a/c^2/e
^2/(-a*f+b*e)/(-c*f+d*e)^2/(d*x^2+c)^(1/2)+1/2*b*(3*c^2*f^2-4*c*d*e*f+4*d^
2*e^2)*x*(d*x^2+c)^(1/2)/c^2/e^2/(-c*f+d*e)^2/(b*x^2+a)^(1/2)-(b*x^2+a)^(3
/2)/a/c/e/x/(d*x^2+c)^(1/2)/(f*x^2+e)+1/2*f*(a*f*(-3*c*f+2*d*e)-2*b*e*(-c*
f+d*e))*x*(b*x^2+a)^(3/2)/a/c/e^2/(-a*f+b*e)/(-c*f+d*e)/(d*x^2+c)^(1/2)/(f
*x^2+e)-1/2*a^(1/2)*b^(1/2)*(3*c^2*f^2-4*c*d*e*f+4*d^2*e^2)*(d*x^2+c)^(1/2
)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/c^2/e^2
/(-c*f+d*e)^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)-1/2*a^(1/2)*
b^(1/2)*(a*f*(-3*c*f+2*d*e)-2*b*e*(-c*f+d*e))*(d*x^2+c)^(1/2)*InverseJacob
iAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/c^2/e^2/(-a*f+b*e)/(-c*f+
d*e)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)+1/2*a^(3/2)*f^2*(b*e*
(-2*c*f+5*d*e)-3*a*f*(-c*f+2*d*e))*(d*x^2+c)^(1/2)*EllipticPi(b^(1/2)*x/a^
(1/2)/(1+b*x^2/a)^(1/2),1-a*f/b/e,(1-a*d/b/c)^(1/2))/b^(1/2)/c/e^3/(-a*f+b
*e)/(-c*f+d*e)^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.40 (sec) , antiderivative size = 355, normalized size of antiderivative = 0.46

$$\int \frac{\sqrt{a+bx^2}}{x^2\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^2}\,dx = \frac{-\sqrt{\frac{b}{a}}e(a+bx^2)\left(c^2f^3x^2(c+dx^2)+2d^3e^2x^2(e+fx^2)+2(de-cf)^2\left(c\right.\right.}{}$$

input
```
Integrate[Sqrt[a + b*x^2]/(x^2*(c + d*x^2)^(3/2)*(e + f*x^2)^2),x]
```

output
```
(-(Sqrt[b/a]*e*(a + b*x^2)*(c^2*f^3*x^2*(c + d*x^2) + 2*d^3*e^2*x^2*(e + f
*x^2) + 2*(d*e - c*f)^2*(c + d*x^2)*(e + f*x^2))) + I*c*x*Sqrt[1 + (b*x^2)
/a]*Sqrt[1 + (d*x^2)/c]*(e + f*x^2)*(-(b*e*(4*d^2*e^2 - 4*c*d*e*f + 3*c^2*
f^2)*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]) + b*e*(2*d^2*e^2 - 5*
c*d*e*f + 3*c^2*f^2)*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + c*f*
(b*e*(5*d*e - 2*c*f) + 3*a*f*(-2*d*e + c*f))*EllipticPi[(a*f)/(b*e), I*Arc
Sinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(2*Sqrt[b/a]*c^2*e^3*(d*e - c*f)^2*x*Sqr
t[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}}{x^2\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^2}dx$$
$$\Big\downarrow \text{ 450}$$
$$\int \frac{\sqrt{a+bx^2}}{x^2\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^2}dx$$

input
```
Int[Sqrt[a + b*x^2]/(x^2*(c + d*x^2)^(3/2)*(e + f*x^2)^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [A] (verified)

Time = 25.26 (sec) , antiderivative size = 1228, normalized size of antiderivative = 1.60

method	result	size
elliptic	Expression too large to display	1228
risch	Expression too large to display	1598
default	Expression too large to display	1856

input
```
int((b*x^2+a)^(1/2)/x^2/(d*x^2+c)^(3/2)/(f*x^2+e)^2,x,method=_RETURNVERBOS
E)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-1/2*f^3/e^2/
(c*f-d*e)^2*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(f*x^2+e)-1/c^2/e^2*(b*d
*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/x-(b*d*x^2+a*d)/c^2*d^2*x/(c*f-d*e)^2/((x^
2+c/d)*(b*d*x^2+a*d))^(1/2)+1/2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)
^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a
*d+b*c)/c/b)^(1/2))*f*d*b/(c*f-d*e)^2/e-1/2*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/
2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*b*f^2/e^2/(c*f-d*
e)^2*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+1/2*c/(-b/a)^(1/2)
*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*b
*f^2/e^2/(c*f-d*e)^2*EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-1/
c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^
2+a*c)^(1/2)*b/e^2*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+1/c/
(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+
a*c)^(1/2)*b/e^2*EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+1/c/(-
b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*
c)^(1/2)*d^2*b/(c*f-d*e)^2*EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/
2))-3/2/e^3/(c*f-d*e)^2*f^3/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/
2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(
-1/c*d)^(1/2)/(-b/a)^(1/2))*a*c+3/e^2/(c*f-d*e)^2*f^2/(-b/a)^(1/2)*(1+b*x^
2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*Ellipt...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{x^2\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^2}\,dx=\text{Timed out}$$

input
```
integrate((b*x^2+a)^(1/2)/x^2/(d*x^2+c)^(3/2)/(f*x^2+e)^2,x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}}{x^2 (c+dx^2)^{3/2} (e+fx^2)^2} \, dx = \int \frac{\sqrt{a+bx^2}}{x^2 (c+dx^2)^{\frac{3}{2}} (e+fx^2)^2} \, dx$$

input
```
integrate((b*x**2+a)**(1/2)/x**2/(d*x**2+c)**(3/2)/(f*x**2+e)**2,x)
```

output
```
Integral(sqrt(a + b*x**2)/(x**2*(c + d*x**2)**(3/2)*(e + f*x**2)**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}}{x^2 (c+dx^2)^{3/2} (e+fx^2)^2} \, dx = \int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{\frac{3}{2}} (fx^2+e)^2 x^2} \, dx$$

input
```
integrate((b*x^2+a)^(1/2)/x^2/(d*x^2+c)^(3/2)/(f*x^2+e)^2,x, algorithm="ma
xima")
```

output
```
integrate(sqrt(b*x^2 + a)/((d*x^2 + c)^(3/2)*(f*x^2 + e)^2*x^2), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}}{x^2 (c+dx^2)^{3/2} (e+fx^2)^2} \, dx = \int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{\frac{3}{2}} (fx^2+e)^2 x^2} \, dx$$

input
```
integrate((b*x^2+a)^(1/2)/x^2/(d*x^2+c)^(3/2)/(f*x^2+e)^2,x, algorithm="gi
ac")
```

output
```
integrate(sqrt(b*x^2 + a)/((d*x^2 + c)^(3/2)*(f*x^2 + e)^2*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{x^2\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^2}\,dx=\int \frac{\sqrt{b\,x^2+a}}{x^2\,\left(d\,x^2+c\right)^{3/2}\,\left(f\,x^2+e\right)^2}\,dx$$

input
```
int((a + b*x^2)^(1/2)/(x^2*(c + d*x^2)^(3/2)*(e + f*x^2)^2),x)
```

output
```
int((a + b*x^2)^(1/2)/(x^2*(c + d*x^2)^(3/2)*(e + f*x^2)^2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}}{x^2\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^2}\,dx=\int \frac{\sqrt{b\,x^2+a}}{x^2\,\left(d\,x^2+c\right)^{\frac{3}{2}}\,\left(f\,x^2+e\right)^2}dx$$

input
```
int((b*x^2+a)^(1/2)/x^2/(d*x^2+c)^(3/2)/(f*x^2+e)^2,x)
```

output
```
int((b*x^2+a)^(1/2)/x^2/(d*x^2+c)^(3/2)/(f*x^2+e)^2,x)
```

3.159 $\int \frac{\sqrt{a+bx^2}}{x^4\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 974

$$\int \frac{\sqrt{a+bx^2}}{x^4\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^2}\,dx =$$

$$-\frac{(2b^2ce(de-cf)^2(5de+6cf)+a^2df(16d^3e^3-8cd^2e^2f-14c^2def^2+15c^3f^3)-ab(16d^4e^4+2cd^3e^3f-2}{6ac^3e^3(be-af)(de-cf)^2\sqrt{c+dx^2}}$$

$$+\frac{b\left(2b-\frac{a(16d^3e^3-8cd^2e^2f-14c^2def^2+15c^3f^3)}{ce(de-cf)^2}\right)x\sqrt{c+dx^2}}{6ac^2e^2\sqrt{a+bx^2}}$$

$$-\frac{\left(a+bx^2\right)^{3/2}}{3acex^3\sqrt{c+dx^2}\left(e+fx^2\right)}+\frac{(4de+5cf)\left(a+bx^2\right)^{3/2}}{3ac^2e^2x\sqrt{c+dx^2}\left(e+fx^2\right)}$$

$$-\frac{f(af(8d^2e^2+4cdef-15c^2f^2)-4be(2d^2e^2+cdef-3c^2f^2))\,x\left(a+bx^2\right)^{3/2}}{6ac^2e^3(be-af)(de-cf)\sqrt{c+dx^2}\left(e+fx^2\right)}$$

$$-\frac{\sqrt{b}\left(2b-\frac{a(16d^3e^3-8cd^2e^2f-14c^2def^2+15c^3f^3)}{ce(de-cf)^2}\right)\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{6\sqrt{a}c^2e^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

$$+\frac{\sqrt{a}\sqrt{b}(af(8d^2e^2+4cdef-15c^2f^2)-4be(2d^2e^2+cdef-3c^2f^2))\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\right.}{6c^3e^3(be-af)(de-cf)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

$$+\frac{a^{3/2}f^3(af(8de-5cf)-be(7de-4cf))\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{2\sqrt{b}ce^4(be-af)(de-cf)^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
-1/6*(2*b^2*c*e*(-c*f+d*e)^2*(6*c*f+5*d*e)+a^2*d*f*(15*c^3*f^3-14*c^2*d*e*
f^2-8*c*d^2*e^2*f+16*d^3*e^3)-a*b*(15*c^4*f^4-2*c^3*d*e*f^3-22*c^2*d^2*e^2
*f^2+2*c*d^3*e^3*f+16*d^4*e^4))*x*(b*x^2+a)^(1/2)/a/c^3/e^3/(-a*f+b*e)/(-c
*f+d*e)^2/(d*x^2+c)^(1/2)+1/6*b*(2*b-a*(15*c^3*f^3-14*c^2*d*e*f^2-8*c*d^2*
e^2*f+16*d^3*e^3)/c/e/(-c*f+d*e)^2)*x*(d*x^2+c)^(1/2)/a/c^2/e^2/(b*x^2+a)^
(1/2)-1/3*(b*x^2+a)^(3/2)/a/c/e/x^3/(d*x^2+c)^(1/2)/(f*x^2+e)+1/3*(5*c*f+4
*d*e)*(b*x^2+a)^(3/2)/a/c^2/e^2/x/(d*x^2+c)^(1/2)/(f*x^2+e)-1/6*f*(a*f*(-1
5*c^2*f^2+4*c*d*e*f+8*d^2*e^2)-4*b*e*(-3*c^2*f^2+c*d*e*f+2*d^2*e^2))*x*(b*
x^2+a)^(3/2)/a/c^2/e^3/(-a*f+b*e)/(-c*f+d*e)/(d*x^2+c)^(1/2)/(f*x^2+e)-1/6
*b^(1/2)*(2*b-a*(15*c^3*f^3-14*c^2*d*e*f^2-8*c*d^2*e^2*f+16*d^3*e^3)/c/e/(
-c*f+d*e)^2)*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2)
,(1-a*d/b/c)^(1/2))/a^(1/2)/c^2/e^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+
a))^(1/2)+1/6*a^(1/2)*b^(1/2)*(a*f*(-15*c^2*f^2+4*c*d*e*f+8*d^2*e^2)-4*b*e
*(-3*c^2*f^2+c*d*e*f+2*d^2*e^2))*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^
(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/c^3/e^3/(-a*f+b*e)/(-c*f+d*e)/(b*x^2+a
)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)+1/2*a^(3/2)*f^3*(a*f*(-5*c*f+8*d*e
)-b*e*(-4*c*f+7*d*e))*(d*x^2+c)^(1/2)*EllipticPi(b^(1/2)*x/a^(1/2)/(1+b*x^
2/a)^(1/2),1-a*f/b/e,(1-a*d/b/c)^(1/2))/b^(1/2)/c/e^4/(-a*f+b*e)/(-c*f+d*e
)^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.77 (sec) , antiderivative size = 472, normalized size of antiderivative = 0.48

$$\int \frac{\sqrt{a+bx^2}}{x^4 (c+dx^2)^{3/2} (e+fx^2)^2} \, dx = \frac{\sqrt{\frac{b}{a}} \left(\sqrt{\frac{b}{a}} e(a+bx^2) \left(3ac^3 f^4 x^4 (c+dx^2) + 6ad^4 e^3 x^4 (e+fx^2) - 2ace(de\right.\right.}{}$$

input
```
Integrate[Sqrt[a + b*x^2]/(x^4*(c + d*x^2)^(3/2)*(e + f*x^2)^2),x]
```

output
```
(Sqrt[b/a]*(Sqrt[b/a]*e*(a + b*x^2)*(3*a*c^3*f^4*x^4*(c + d*x^2) + 6*a*d^4
*e^3*x^4*(e + f*x^2) - 2*a*c*e*(d*e - c*f)^2*(c + d*x^2)*(e + f*x^2) + 2*(
d*e - c*f)^2*(-(b*c*e) + 5*a*d*e + 6*a*c*f)*x^2*(c + d*x^2)*(e + f*x^2)) +
 I*c*x^3*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(e + f*x^2)*(-(b*e*(2*b*c
*e*(d*e - c*f)^2 + a*(-16*d^3*e^3 + 8*c*d^2*e^2*f + 14*c^2*d*e*f^2 - 15*c^
3*f^3))*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]) + b*e*(-(d*e) + c*
f)*(2*b*c*e*(-(d*e) + c*f) + a*(8*d^2*e^2 + 4*c*d*e*f - 15*c^2*f^2))*Ellip
ticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - 3*a*c^2*f^2*(b*e*(7*d*e - 4*c*
f) + a*f*(-8*d*e + 5*c*f))*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x],
 (a*d)/(b*c)])))/(6*b*c^3*e^4*(d*e - c*f)^2*x^3*Sqrt[a + b*x^2]*Sqrt[c + d
*x^2]*(e + f*x^2))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}}{x^4 \left(c+dx^2\right)^{3/2} \left(e+fx^2\right)^2} \, dx$$

$$\Big\downarrow \ 450$$

$$\int \frac{\sqrt{a+bx^2}}{x^4 \left(c+dx^2\right)^{3/2} \left(e+fx^2\right)^2}dx$$

input
```
Int[Sqrt[a + b*x^2]/(x^4*(c + d*x^2)^(3/2)*(e + f*x^2)^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [A] (verified)

Time = 27.42 (sec) , antiderivative size = 1678, normalized size of antiderivative = 1.72

method	result	size
elliptic	Expression too large to display	1678
risch	Expression too large to display	1755
default	Expression too large to display	3523

input
```
int((b*x^2+a)^(1/2)/x^4/(d*x^2+c)^(3/2)/(f*x^2+e)^2,x,method=_RETURNVERBOS
E)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-1/3/c^2/e^2*
(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/x^3+1/3/a/c^3*(6*a*c*f+5*a*d*e-b*c*e)/
e^3*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/x+1/2*f^4/e^3/(c*f-d*e)^2*x*(b*d*x
^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(f*x^2+e)+(b*d*x^2+a*d)/c^3*d^3*x/(c*f-d*e)^
2/((x^2+c/d)*(b*d*x^2+a*d))^(1/2)+5/2*f^4/e^4/(c*f-d*e)^2/(-b/a)^(1/2)*(1+
b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*Ellip
ticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*a*c-4*f^3/e^3/(c
*f-d*e)^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^
2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/
a)^(1/2))*a*d-2*f^3/e^3/(c*f-d*e)^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^
2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a
*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*b*c+7/2*f^2/e^2/(c*f-d*e)^2/(-b/a)^(1/
2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*b*d-1/2/(-
b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*
c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))*b*d*f^2/e^2/(c
*f-d*e)^2+2/c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*
d*x^2+b*c*x^2+a*c)^(1/2)*b/e^3*f*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/
b)^(1/2))-2/c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*
d*x^2+b*c*x^2+a*c)^(1/2)*b/e^3*f*EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{x^4\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^2}\,dx=\text{Timed out}$$

input
```
integrate((b*x^2+a)^(1/2)/x^4/(d*x^2+c)^(3/2)/(f*x^2+e)^2,x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}}{x^4 (c+dx^2)^{3/2} (e+fx^2)^2} \, dx = \int \frac{\sqrt{a+bx^2}}{x^4 (c+dx^2)^{\frac{3}{2}} (e+fx^2)^2} \, dx$$

input
```
integrate((b*x**2+a)**(1/2)/x**4/(d*x**2+c)**(3/2)/(f*x**2+e)**2,x)
```

output
```
Integral(sqrt(a + b*x**2)/(x**4*(c + d*x**2)**(3/2)*(e + f*x**2)**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}}{x^4 (c+dx^2)^{3/2} (e+fx^2)^2} \, dx = \int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{\frac{3}{2}} (fx^2+e)^2 x^4} \, dx$$

input
```
integrate((b*x^2+a)^(1/2)/x^4/(d*x^2+c)^(3/2)/(f*x^2+e)^2,x, algorithm="ma
xima")
```

output
```
integrate(sqrt(b*x^2 + a)/((d*x^2 + c)^(3/2)*(f*x^2 + e)^2*x^4), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}}{x^4 (c+dx^2)^{3/2} (e+fx^2)^2} \, dx = \int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{\frac{3}{2}} (fx^2+e)^2 x^4} \, dx$$

input
```
integrate((b*x^2+a)^(1/2)/x^4/(d*x^2+c)^(3/2)/(f*x^2+e)^2,x, algorithm="gi
ac")
```

output
```
integrate(sqrt(b*x^2 + a)/((d*x^2 + c)^(3/2)*(f*x^2 + e)^2*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{x^4 (c+dx^2)^{3/2} (e+fx^2)^2} \, dx = \int \frac{\sqrt{b\,x^2+a}}{x^4\,\left(d\,x^2+c\right)^{3/2}\,\left(f\,x^2+e\right)^2} \,dx$$

input
```
int((a + b*x^2)^(1/2)/(x^4*(c + d*x^2)^(3/2)*(e + f*x^2)^2),x)
```

output
```
int((a + b*x^2)^(1/2)/(x^4*(c + d*x^2)^(3/2)*(e + f*x^2)^2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}}{x^4 (c+dx^2)^{3/2} (e+fx^2)^2} \, dx = \int \frac{\sqrt{b\,x^2+a}}{x^4\,\left(d\,x^2+c\right)^{\frac{3}{2}}\,\left(f\,x^2+e\right)^2}dx$$

input
```
int((b*x^2+a)^(1/2)/x^4/(d*x^2+c)^(3/2)/(f*x^2+e)^2,x)
```

output
```
int((b*x^2+a)^(1/2)/x^4/(d*x^2+c)^(3/2)/(f*x^2+e)^2,x)
```

3.160 $\int \frac{\sqrt{a+bx^2}}{x^6 \left(c+dx^2\right)^{3/2} \left(e+fx^2\right)^2} \, dx$

Optimal result

Integrand size = 35, antiderivative size = 1300

$$\int \frac{\sqrt{a+bx^2}}{x^6 \left(c+dx^2\right)^{3/2} \left(e+fx^2\right)^2} \, dx = \text{Too large to display}$$

output
```
1/30*(2*b^3*c^2*d*e^3*(-c*f+d*e)^2+2*a*b^2*c*e*(-c*f+d*e)^2*(45*c^2*f^2+49
*c*d*e*f+32*d^2*e^2)+a^3*d*f*(105*c^4*f^4-80*c^3*d*e*f^3-44*c^2*d^2*e^2*f^
2-32*c*d^3*e^3*f+96*d^4*e^4)-a^2*b*(105*c^5*f^5+10*c^4*d*e*f^4-126*c^3*d^2
*e^2*f^3-72*c^2*d^3*e^3*f^2+32*c*d^4*e^4*f+96*d^5*e^5))*x*(b*x^2+a)^(1/2)/
a^2/c^4/e^4/(-a*f+b*e)/(-c*f+d*e)^2/(d*x^2+c)^(1/2)-1/30*b*(4*b^2*c*e+4*a*
b*(5*c*f+4*d*e)-a^2*(105*c^4*f^4-80*c^3*d*e*f^3-44*c^2*d^2*e^2*f^2-32*c*d^
3*e^3*f+96*d^4*e^4)/c/e/(-c*f+d*e)^2)*x*(d*x^2+c)^(1/2)/a^2/c^3/e^3/(b*x^2
+a)^(1/2)-1/5*(b*x^2+a)^(3/2)/a/c/e/x^5/(d*x^2+c)^(1/2)/(f*x^2+e)+1/15*(7*
a*c*f+6*a*d*e+2*b*c*e)*(b*x^2+a)^(3/2)/a^2/c^2/e^2/x^3/(d*x^2+c)^(1/2)/(f*
x^2+e)+1/15*(b*c*e*(2*c*f+d*e)-a*(35*c^2*f^2+34*c*d*e*f+24*d^2*e^2))*(b*x^
2+a)^(3/2)/a^2/c^3/e^3/x/(d*x^2+c)^(1/2)/(f*x^2+e)+1/30*f*(2*b^2*c*d*e^3*(
-c*f+d*e)+a^2*f*(-105*c^3*f^3+10*c^2*d*e*f^2+32*c*d^2*e^2*f+48*d^3*e^3)-2*
a*b*e*(-45*c^3*f^3+4*c^2*d*e*f^2+17*c*d^2*e^2*f+24*d^3*e^3))*x*(b*x^2+a)^(
3/2)/a^2/c^3/e^4/(-a*f+b*e)/(-c*f+d*e)/(d*x^2+c)^(1/2)/(f*x^2+e)+1/30*b^(1
/2)*(4*b^2*c*e+4*a*b*(5*c*f+4*d*e)-a^2*(105*c^4*f^4-80*c^3*d*e*f^3-44*c^2*
d^2*e^2*f^2-32*c*d^3*e^3*f+96*d^4*e^4)/c/e/(-c*f+d*e)^2)*(d*x^2+c)^(1/2)*E
llipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/a^(3/2)/c^
3/e^3/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)-1/30*b^(1/2)*(2*b^2*
c*d*e^3*(-c*f+d*e)+a^2*f*(-105*c^3*f^3+10*c^2*d*e*f^2+32*c*d^2*e^2*f+48*d^
3*e^3)-2*a*b*e*(-45*c^3*f^3+4*c^2*d*e*f^2+17*c*d^2*e^2*f+24*d^3*e^3))*(...
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.69 (sec) , antiderivative size = 669, normalized size of antiderivative = 0.51

$$\int \frac{\sqrt{a+bx^2}}{x^6 (c+dx^2)^{3/2} (e+fx^2)^2} dx = \frac{-\sqrt{\frac{b}{a}}e(a+bx^2)\left(15a^2c^4f^5x^6(c+dx^2)+30a^2d^5e^4x^6(e+fx^2)+6a^2c^2e^2\cdots\right.}{\cdots}$$

input
```
Integrate[Sqrt[a + b*x^2]/(x^6*(c + d*x^2)^(3/2)*(e + f*x^2)^2),x]
```

output
```
(-(Sqrt[b/a]*e*(a + b*x^2)*(15*a^2*c^4*f^5*x^6*(c + d*x^2) + 30*a^2*d^5*e^
4*x^6*(e + f*x^2) + 6*a^2*c^2*e^2*(d*e - c*f)^2*(c + d*x^2)*(e + f*x^2) +
2*a*c*e*(d*e - c*f)^2*(b*c*e - 9*a*d*e - 10*a*c*f)*x^2*(c + d*x^2)*(e + f*
x^2) + 2*(d*e - c*f)^2*(-2*b^2*c^2*e^2 - 2*a*b*c*e*(4*d*e + 5*c*f) + a^2*(
33*d^2*e^2 + 50*c*d*e*f + 45*c^2*f^2))*x^4*(c + d*x^2)*(e + f*x^2))) + I*c
*x^5*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(e + f*x^2)*(b*e*(4*b^2*c^2*e
^2*(d*e - c*f)^2 + 4*a*b*c*e*(d*e - c*f)^2*(4*d*e + 5*c*f) + a^2*(-96*d^4*
e^4 + 32*c*d^3*e^3*f + 44*c^2*d^2*e^2*f^2 + 80*c^3*d*e*f^3 - 105*c^4*f^4))
*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - b*e*(-(d*e) + c*f)*(4*b^
2*c^2*e^2*(-(d*e) + c*f) + 2*a*b*c*e*(-7*d^2*e^2 - 3*c*d*e*f + 10*c^2*f^2)
 + a^2*(48*d^3*e^3 + 32*c*d^2*e^2*f + 10*c^2*d*e*f^2 - 105*c^3*f^3))*Ellip
ticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + 15*a^2*c^3*f^3*(3*b*e*(3*d*e -
 2*c*f) + a*f*(-10*d*e + 7*c*f))*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/
a]*x], (a*d)/(b*c)]))/(30*a^2*Sqrt[b/a]*c^4*e^5*(d*e - c*f)^2*x^5*Sqrt[a +
 b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}}{x^6 (c+dx^2)^{3/2} (e+fx^2)^2} \, dx$$

$$\Big\downarrow \; 450$$

$$\int \frac{\sqrt{a+bx^2}}{x^6 (c+dx^2)^{3/2} (e+fx^2)^2}dx$$

input
```
Int[Sqrt[a + b*x^2]/(x^6*(c + d*x^2)^(3/2)*(e + f*x^2)^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [A] (verified)

Time = 28.91 (sec) , antiderivative size = 2108, normalized size of antiderivative = 1.62

method	result	size
risch	Expression too large to display	2108
elliptic	Expression too large to display	2377
default	Expression too large to display	5749

input
```
int((b*x^2+a)^(1/2)/x^6/(d*x^2+c)^(3/2)/(f*x^2+e)^2,x,method=_RETURNVERBOS
E)
```

output
```
-1/15*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(45*a^2*c^2*f^2*x^4+50*a^2*c*d*e*f*x
^4+33*a^2*d^2*e^2*x^4-10*a*b*c^2*e*f*x^4-8*a*b*c*d*e^2*x^4-2*b^2*c^2*e^2*x
^4-10*a^2*c^2*e*f*x^2-9*a^2*c*d*e^2*x^2+a*b*c^2*e^2*x^2+3*a^2*c^2*e^2)/a^2
/c^4/e^4/x^5+1/15/e^4/a^2/c^4*(-b*(45*a^2*c^2*f^2+50*a^2*c*d*e*f+33*a^2*d^
2*e^2-10*a*b*c^2*e*f-8*a*b*c*d*e^2-2*b^2*c^2*e^2)*c/(-b/a)^(1/2)*(1+b*x^2/
a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(EllipticF(
x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d
+b*c)/c/b)^(1/2)))-a*b^2*c^2*d*e^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2
/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1
+(a*d+b*c)/c/b)^(1/2))+9*a^2*b*c*d^2*e^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1
+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/
2),(-1+(a*d+b*c)/c/b)^(1/2))+10*a^2*b*c^2*d*e*f/(-b/a)^(1/2)*(1+b*x^2/a)^(
1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b
/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-15*a^2*c^4*f^3*(3*a*c*f^2-4*a*d*e*f-2*
b*c*e*f+3*b*d*e^2)/(c*f-d*e)^2/e/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c
)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/
b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))-15*a^2*c*d^4*e^4*(a*d-b*c)/(c*f-d*e)^2*((
b*d*x^2+a*d)/c/(a*d-b*c)*x/((x^2+c/d)*(b*d*x^2+a*d))^(1/2)+(1/c-1/(a*d-b*c
)/c*a*d)/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2
+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{x^6 (c+dx^2)^{3/2} (e+fx^2)^2} \, dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(1/2)/x^6/(d*x^2+c)^(3/2)/(f*x^2+e)^2,x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}}{x^6\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^2}\,dx = \int \frac{\sqrt{a+bx^2}}{x^6\left(c+dx^2\right)^{\frac{3}{2}}\left(e+fx^2\right)^2}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)/x**6/(d*x**2+c)**(3/2)/(f*x**2+e)**2,x)
```

output
```
Integral(sqrt(a + b*x**2)/(x**6*(c + d*x**2)**(3/2)*(e + f*x**2)**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}}{x^6\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^2}\,dx = \int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{\frac{3}{2}}\left(fx^2+e\right)^2x^6}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/x^6/(d*x^2+c)^(3/2)/(f*x^2+e)^2,x, algorithm="ma
xima")
```

output
```
integrate(sqrt(b*x^2 + a)/((d*x^2 + c)^(3/2)*(f*x^2 + e)^2*x^6), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}}{x^6\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^2}\,dx = \int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{\frac{3}{2}}\left(fx^2+e\right)^2x^6}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/x^6/(d*x^2+c)^(3/2)/(f*x^2+e)^2,x, algorithm="gi
ac")
```

output
```
integrate(sqrt(b*x^2 + a)/((d*x^2 + c)^(3/2)*(f*x^2 + e)^2*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{x^6\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^2}\,dx=\int \frac{\sqrt{b\,x^2+a}}{x^6\,\left(d\,x^2+c\right)^{3/2}\,\left(f\,x^2+e\right)^2}\,dx$$

input
```
int((a + b*x^2)^(1/2)/(x^6*(c + d*x^2)^(3/2)*(e + f*x^2)^2),x)
```

output
```
int((a + b*x^2)^(1/2)/(x^6*(c + d*x^2)^(3/2)*(e + f*x^2)^2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}}{x^6\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^2}\,dx=\int \frac{\sqrt{b\,x^2+a}}{x^6\,\left(d\,x^2+c\right)^{\frac{3}{2}}\,\left(f\,x^2+e\right)^2}dx$$

input
```
int((b*x^2+a)^(1/2)/x^6/(d*x^2+c)^(3/2)/(f*x^2+e)^2,x)
```

output
```
int((b*x^2+a)^(1/2)/x^6/(d*x^2+c)^(3/2)/(f*x^2+e)^2,x)
```

3.161 $\int \frac{x^2\sqrt{a+bx^2}}{(c+dx^2)^{3/2}(e+fx^2)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 253

$$\int \frac{x^2\sqrt{a+bx^2}}{(c+dx^2)^{3/2}(e+fx^2)}\,dx = \frac{(bc-ad)x}{d(de-cf)\sqrt{a+bx^2}\sqrt{c+dx^2}} - \frac{\sqrt{a}\sqrt{b}\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{d(de-cf)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} + \frac{a^{3/2}\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}c(de-cf)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
(-a*d+b*c)*x/d/(-c*f+d*e)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)-a^(1/2)*b^(1/2)*
(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^
(1/2))/d/(-c*f+d*e)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)+a^(3/2
)*(d*x^2+c)^(1/2)*EllipticPi(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),1-a*f/b/e
,(1-a*d/b/c)^(1/2))/b^(1/2)/c/(-c*f+d*e)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b
*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.22 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.09

$$\int \frac{x^2\sqrt{a+bx^2}}{(c+dx^2)^{3/2}(e+fx^2)}\,dx = \frac{-ibcf\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(i\operatorname{arcsinh}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)+ib(-de+cf)\sqrt{1+\frac{bx^2}{a}}}{}$$

input
```
Integrate[(x^2*Sqrt[a + b*x^2])/((c + d*x^2)^(3/2)*(e + f*x^2)),x]
```

output
```
((-I)*b*c*f*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sq
rt[b/a]*x], (a*d)/(b*c)] + I*b*(-(d*e) + c*f)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 +
 (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - d*(Sqrt[b/a]*
f*x*(a + b*x^2) + I*(-(b*e) + a*f)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]
*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(Sqrt[b/a]
*d*f*(d*e - c*f)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\sqrt{a+bx^2}}{(c+dx^2)^{3/2}(e+fx^2)}\,dx$$

$$\Big\downarrow \; 450$$

$$\int \frac{x^2\sqrt{a+bx^2}}{(c+dx^2)^{3/2}(e+fx^2)}dx$$

input
```
Int[(x^2*Sqrt[a + b*x^2])/((c + d*x^2)^(3/2)*(e + f*x^2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [A] (verified)

Time = 6.45 (sec) , antiderivative size = 390, normalized size of antiderivative = 1.54

method	result
default	$\frac{\left(\sqrt{-\frac{b}{a}}\, bdf\, x^3+\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{x^2d+c}{c}}\,\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)bcf-\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{x^2d+c}{c}}\,\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)bde-\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{x}\right.}{}$
elliptic	$\frac{\sqrt{(b\,x^2+a)(x^2d+c)}\left(\frac{\left(bd\,x^2+ad\right)x}{d(cf-de)\sqrt{\left(x^2+\frac{c}{d}\right)\left(bd\,x^2+ad\right)}}+\frac{\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)b}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}\,df}-\frac{bc\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\operatorname{Ellip}}{(cf-de)\sqrt{-\frac{b}{a}}\,\sqrt{bd}}\right.}{\sqrt{b\,x^2+a}}$

input
```
int(x^2*(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
((-b/a)^(1/2)*b*d*f*x^3+((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(
x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b*c*f-((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1
/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b*d*e-((b*x^2+a)/a)^(1/2)*((
d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b*c*f-((b*x^2+
a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)
^(1/2)/(-b/a)^(1/2))*a*d*f+e*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*Ellip
ticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*b*d+(-b/a)^(1/2)
*a*d*f*x)*(d*x^2+c)^(1/2)*(b*x^2+a)^(1/2)/f/(-b/a)^(1/2)/d/(c*f-d*e)/(b*d*
x^4+a*d*x^2+b*c*x^2+a*c)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^2\sqrt{a+bx^2}}{(c+dx^2)^{3/2}(e+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate(x^2*(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)/(f*x^2+e),x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^2\sqrt{a+bx^2}}{(c+dx^2)^{3/2}(e+fx^2)}\,dx = \int \frac{x^2\sqrt{a+bx^2}}{(c+dx^2)^{\frac{3}{2}}(e+fx^2)}\,dx$$

input
```
integrate(x**2*(b*x**2+a)**(1/2)/(d*x**2+c)**(3/2)/(f*x**2+e),x)
```

output
```
Integral(x**2*sqrt(a + b*x**2)/((c + d*x**2)**(3/2)*(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{x^2\sqrt{a+bx^2}}{(c+dx^2)^{3/2}(e+fx^2)}\,dx = \int \frac{\sqrt{bx^2+a}x^2}{(dx^2+c)^{\frac{3}{2}}(fx^2+e)}\,dx$$

input
```
integrate(x^2*(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)/(f*x^2+e),x, algorithm="maxi
ma")
```

output
```
integrate(sqrt(b*x^2 + a)*x^2/((d*x^2 + c)^(3/2)*(f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{x^2\sqrt{a+bx^2}}{(c+dx^2)^{3/2}(e+fx^2)}\,dx = \int \frac{\sqrt{bx^2+a}\,x^2}{(dx^2+c)^{\frac{3}{2}}(fx^2+e)}\,dx$$

input
```
integrate(x^2*(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)/(f*x^2+e),x, algorithm="giac
")
```

output
```
integrate(sqrt(b*x^2 + a)*x^2/((d*x^2 + c)^(3/2)*(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2\sqrt{a+bx^2}}{(c+dx^2)^{3/2}(e+fx^2)}\,dx = \int \frac{x^2\,\sqrt{b\,x^2+a}}{{(d\,x^2+c)}^{3/2}\,(f\,x^2+e)}\,dx$$

input
```
int((x^2*(a + b*x^2)^(1/2))/((c + d*x^2)^(3/2)*(e + f*x^2)),x)
```

output
```
int((x^2*(a + b*x^2)^(1/2))/((c + d*x^2)^(3/2)*(e + f*x^2)), x)
```

Reduce [F]

$$\int \frac{x^2\sqrt{a+bx^2}}{(c+dx^2)^{3/2}(e+fx^2)}\,dx = \int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^2}{d^2f\,x^6+2cdf\,x^4+d^2e\,x^4+c^2f\,x^2+2cde\,x^2+c^2e}dx$$

input
```
int(x^2*(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2)/(f*x^2+e),x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(c**2*e + c**2*f*x**2 + 2*c*d
*e*x**2 + 2*c*d*f*x**4 + d**2*e*x**4 + d**2*f*x**6),x)
```

3.162 $\int \frac{x^2\sqrt{a+bx^2}}{(c-dx^2)^{3/2}(e+fx^2)}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 357

$$\int \frac{x^2\sqrt{a+bx^2}}{(c-dx^2)^{3/2}(e+fx^2)}\,dx = \frac{x\sqrt{a+bx^2}}{(de+cf)\sqrt{c-dx^2}} - \frac{\sqrt{c}\sqrt{a+bx^2}\sqrt{1-\frac{dx^2}{c}}E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{\sqrt{d}(de+cf)\sqrt{1+\frac{bx^2}{a}}\sqrt{c-dx^2}} - \frac{\sqrt{c}(be-af)\sqrt{1+\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{bc}{ad}\right)}{\sqrt{d}f(de+cf)\sqrt{a+bx^2}\sqrt{c-dx^2}} + \frac{\sqrt{c}(be-af)\sqrt{1+\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{cf}{de},\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{bc}{ad}\right)}{\sqrt{d}f(de+cf)\sqrt{a+bx^2}\sqrt{c-dx^2}}$$

output
```
x*(b*x^2+a)^(1/2)/(c*f+d*e)/(-d*x^2+c)^(1/2)-c^(1/2)*(b*x^2+a)^(1/2)*(1-d*
x^2/c)^(1/2)*EllipticE(d^(1/2)*x/c^(1/2),(-b*c/a/d)^(1/2))/d^(1/2)/(c*f+d*
e)/(1+b*x^2/a)^(1/2)/(-d*x^2+c)^(1/2)-c^(1/2)*(-a*f+b*e)*(1+b*x^2/a)^(1/2)
*(1-d*x^2/c)^(1/2)*EllipticF(d^(1/2)*x/c^(1/2),(-b*c/a/d)^(1/2))/d^(1/2)/f
/(c*f+d*e)/(b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2)+c^(1/2)*(-a*f+b*e)*(1+b*x^2/a)
^(1/2)*(1-d*x^2/c)^(1/2)*EllipticPi(d^(1/2)*x/c^(1/2),-c*f/d/e,(-b*c/a/d)^
(1/2))/d^(1/2)/f/(c*f+d*e)/(b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.57 (sec) , antiderivative size = 354, normalized size of antiderivative = 0.99

$$\int \frac{x^2\sqrt{a+bx^2}}{(c-dx^2)^{3/2}(e+fx^2)}\,dx = \frac{a\sqrt{\frac{b}{a}}dfx + b\sqrt{\frac{b}{a}}dfx^3 - ibcf\sqrt{1+\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}E\left(i\text{arcsinh}\left(\sqrt{\frac{b}{a}}x\right)|-\frac{ad}{bc}\right) +$$

input
```
Integrate[(x^2*Sqrt[a + b*x^2])/((c - d*x^2)^(3/2)*(e + f*x^2)),x]
```

output
```
(a*Sqrt[b/a]*d*f*x + b*Sqrt[b/a]*d*f*x^3 - I*b*c*f*Sqrt[1 + (b*x^2)/a]*Sqr
t[1 - (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], -((a*d)/(b*c))] + I*b*(
d*e + c*f)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 - (d*x^2)/c]*EllipticF[I*ArcSinh[Sqr
t[b/a]*x], -((a*d)/(b*c))] - I*b*d*e*Sqrt[1 + (b*x^2)/a]*Sqrt[1 - (d*x^2)/
c]*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], -((a*d)/(b*c))] + I*a*d
*f*Sqrt[1 + (b*x^2)/a]*Sqrt[1 - (d*x^2)/c]*EllipticPi[(a*f)/(b*e), I*ArcSi
nh[Sqrt[b/a]*x], -((a*d)/(b*c))])/(Sqrt[b/a]*d*f*(d*e + c*f)*Sqrt[a + b*x^
2]*Sqrt[c - d*x^2])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\sqrt{a+bx^2}}{(c-dx^2)^{3/2}(e+fx^2)}\,dx$$
$$\Big\downarrow \; 450$$
$$\int \frac{x^2\sqrt{a+bx^2}}{(c-dx^2)^{3/2}(e+fx^2)}dx$$

input
```
Int[(x^2*Sqrt[a + b*x^2])/((c - d*x^2)^(3/2)*(e + f*x^2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [A] (verified)

Time = 6.38 (sec) , antiderivative size = 382, normalized size of antiderivative = 1.07

method	result
default	$\frac{\left(\sqrt{\frac{d}{c}}\, b f\, x^{3}+\sqrt{\frac{-x^{2} d+c}{c}}\, \sqrt{\frac{b\, x^{2}+a}{a}}\, \operatorname{EllipticF}\left(x \sqrt{\frac{d}{c}}, \sqrt{-\frac{b c}{a d}}\right) a f-\sqrt{\frac{-x^{2} d+c}{c}}\, \sqrt{\frac{b\, x^{2}+a}{a}}\, \operatorname{EllipticF}\left(x \sqrt{\frac{d}{c}}, \sqrt{-\frac{b c}{a d}}\right) b e-\sqrt{\frac{-x^{2} d+c}{c}}\, \sqrt{b \ldots}\right.}{}$
elliptic	$\frac{\sqrt{\left(-x^{2} d+c\right)\left(b\, x^{2}+a\right)}\, \left(-\frac{\left(-b d\, x^{2}-a d\right) x}{d \left(c f+d e\right) \sqrt{\left(x^{2}-\frac{c}{d}\right)\left(-b d\, x^{2}-a d\right)}}-\frac{\sqrt{1-\frac{d\, x^{2}}{c}}\, \sqrt{1+\frac{b\, x^{2}}{a}}\, \operatorname{EllipticF}\left(x \sqrt{\frac{d}{c}}, \sqrt{-1-\frac{-a d+b c}{a d}}\right) b}{\sqrt{\frac{d}{c}}\, \sqrt{-b d\, x^{4}-a d\, x^{2}+x^{2} b c+a c}\, d f}+\frac{\sqrt{1-\frac{d\, x^{2}}{c}}\, \sqrt{1+\frac{b\, x^{2}}{a}} \ldots}{\sqrt{\frac{d}{c}}\, \sqrt{-b d\, x^{4} \ldots}}\right.}{}$

input
```
int(x^2*(b*x^2+a)^(1/2)/(-d*x^2+c)^(3/2)/(f*x^2+e),x,method=_RETURNVERBOSE
)
```

output
```
((1/c*d)^(1/2)*b*f*x^3+((-d*x^2+c)/c)^(1/2)*((b*x^2+a)/a)^(1/2)*EllipticF(
x*(1/c*d)^(1/2),(-b*c/a/d)^(1/2))*a*f-((-d*x^2+c)/c)^(1/2)*((b*x^2+a)/a)^(
1/2)*EllipticF(x*(1/c*d)^(1/2),(-b*c/a/d)^(1/2))*b*e-((-d*x^2+c)/c)^(1/2)*
((b*x^2+a)/a)^(1/2)*EllipticE(x*(1/c*d)^(1/2),(-b*c/a/d)^(1/2))*a*f-((-d*x
^2+c)/c)^(1/2)*((b*x^2+a)/a)^(1/2)*EllipticPi(x*(1/c*d)^(1/2),-c*f/d/e,(-b
/a)^(1/2)/(1/c*d)^(1/2))*a*f+((-d*x^2+c)/c)^(1/2)*((b*x^2+a)/a)^(1/2)*Elli
pticPi(x*(1/c*d)^(1/2),-c*f/d/e,(-b/a)^(1/2)/(1/c*d)^(1/2))*b*e+(1/c*d)^(1
/2)*a*f*x)*(-d*x^2+c)^(1/2)*(b*x^2+a)^(1/2)/f/(1/c*d)^(1/2)/(c*f+d*e)/(-b*
d*x^4-a*d*x^2+b*c*x^2+a*c)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^2\sqrt{a+bx^2}}{(c-dx^2)^{3/2}(e+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate(x^2*(b*x^2+a)^(1/2)/(-d*x^2+c)^(3/2)/(f*x^2+e),x, algorithm="fri
cas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^2\sqrt{a+bx^2}}{(c-dx^2)^{3/2}(e+fx^2)}\,dx = \int \frac{x^2\sqrt{a+bx^2}}{(c-dx^2)^{\frac{3}{2}}(e+fx^2)}\,dx$$

input
```
integrate(x**2*(b*x**2+a)**(1/2)/(-d*x**2+c)**(3/2)/(f*x**2+e),x)
```

output
```
Integral(x**2*sqrt(a + b*x**2)/((c - d*x**2)**(3/2)*(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{x^2\sqrt{a+bx^2}}{(c-dx^2)^{3/2}(e+fx^2)}\,dx = \int \frac{\sqrt{bx^2+a}x^2}{(-dx^2+c)^{\frac{3}{2}}(fx^2+e)}\,dx$$

input
```
integrate(x^2*(b*x^2+a)^(1/2)/(-d*x^2+c)^(3/2)/(f*x^2+e),x, algorithm="max
ima")
```

output
```
integrate(sqrt(b*x^2 + a)*x^2/((-d*x^2 + c)^(3/2)*(f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{x^2\sqrt{a+bx^2}}{(c-dx^2)^{3/2}(e+fx^2)}\,dx = \int \frac{\sqrt{bx^2+a}\,x^2}{(-dx^2+c)^{\frac{3}{2}}(fx^2+e)}\,dx$$

input
```
integrate(x^2*(b*x^2+a)^(1/2)/(-d*x^2+c)^(3/2)/(f*x^2+e),x, algorithm="gia
c")
```

output
```
integrate(sqrt(b*x^2 + a)*x^2/((-d*x^2 + c)^(3/2)*(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2\sqrt{a+bx^2}}{(c-dx^2)^{3/2}(e+fx^2)}\,dx = \int \frac{x^2\,\sqrt{b\,x^2+a}}{(c-d\,x^2)^{3/2}\,(f\,x^2+e)}\,dx$$

input
```
int((x^2*(a + b*x^2)^(1/2))/((c - d*x^2)^(3/2)*(e + f*x^2)),x)
```

output
```
int((x^2*(a + b*x^2)^(1/2))/((c - d*x^2)^(3/2)*(e + f*x^2)), x)
```

Reduce [F]

$$\int \frac{x^2\sqrt{a+bx^2}}{(c-dx^2)^{3/2}(e+fx^2)}\,dx = \int \frac{\sqrt{-d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^2}{d^2f\,x^6-2cdf\,x^4+d^2e\,x^4+c^2f\,x^2-2cde\,x^2+c^2e}dx$$

input
```
int(x^2*(b*x^2+a)^(1/2)/(-d*x^2+c)^(3/2)/(f*x^2+e),x)
```

output
```
int((sqrt(c - d*x**2)*sqrt(a + b*x**2)*x**2)/(c**2*e + c**2*f*x**2 - 2*c*d
*e*x**2 - 2*c*d*f*x**4 + d**2*e*x**4 + d**2*f*x**6),x)
```

3.163 $\int \frac{e+fx^2}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 209

$$\int \frac{e+fx^2}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx=-\frac{e\sqrt{c+dx^2}}{cx\sqrt{a+bx^2}}-\frac{\sqrt{b}e\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}c\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}+\frac{\sqrt{a}f\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}c\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output

```
-e*(d*x^2+c)^(1/2)/c/x/(b*x^2+a)^(1/2)-b^(1/2)*e*(d*x^2+c)^(1/2)*EllipticE
(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/a^(1/2)/c/(b*x^2+a
)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)+a^(1/2)*f*(d*x^2+c)^(1/2)*InverseJ
acobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/b^(1/2)/c/(b*x^2+a)^(
1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.27 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.95

$$\int \frac{e+fx^2}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$
$$= \frac{\sqrt{\frac{b}{a}}\left(-\sqrt{\frac{b}{a}}e(a+bx^2)\left(c+dx^2\right)-ibcex\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(i\text{arcsinh}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)-ic(-be+af)x\sqrt{1}\right.}{bcx\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

input
```
Integrate[(e + f*x^2)/(x^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
(Sqrt[b/a]*(-(Sqrt[b/a]*e*(a + b*x^2)*(c + d*x^2)) - I*b*c*e*x*Sqrt[1 + (b
*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)
] - I*c*(-(b*e) + a*f)*x*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF
[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(b*c*x*Sqrt[a + b*x^2]*Sqrt[c + d*
x^2])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.19, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {445, 25, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx^2}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$
$$\Big\downarrow \ 445$$
$$-\frac{\int -\frac{bdex^2+acf}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{acx}$$
$$\Big\downarrow \ 25$$

$$\frac{\int \frac{bdex^2+acf}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{acx}$$

$$\Big\downarrow \text{406}$$

$$\frac{bde\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+acf\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{acx}$$

$$\Big\downarrow \text{320}$$

$$\frac{bde\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}f\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{ac}-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{acx}$$

$$\Big\downarrow \text{388}$$

$$\frac{bde\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{c^{3/2}f\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{ac}-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{acx}$$

$$\Big\downarrow \text{313}$$

$$\frac{\frac{c^{3/2}f\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+bde\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{ac}-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{acx}$$

input
```
Int[(e + f*x^2)/(x^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
-((e*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(a*c*x)) + (b*d*e*((x*Sqrt[a + b*x^2
])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d
]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d
*x^2))]*Sqrt[c + d*x^2])) + (c^(3/2)*f*Sqrt[a + b*x^2]*EllipticF[ArcTan[(S
qrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c
+ d*x^2))]*Sqrt[c + d*x^2]))/(a*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 5.50 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.35

method	result
default	$\frac{\left(-\sqrt{-\frac{b}{a}}\, bde\, x^4+\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{x^2d+c}{c}}\,\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)acfx-\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{x^2d+c}{c}}\,\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)bcex+\sqrt{\frac{b\,x^2+a}{a}}}{\sqrt{-\frac{b}{a}}\,xca(bd\,x^4+ad\,x^2+x^2bc+c}$
elliptic	$\frac{\sqrt{(b\,x^2+a)(x^2d+c)}\left(-\frac{e\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{acx}+\frac{f\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}-\frac{be\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,(\operatorname{Elliptic}}{}\right.}{\sqrt{b\,x^2+a}\,\sqrt{x^2d+c}}$
risch	$-\frac{e\sqrt{b\,x^2+a}\,\sqrt{x^2d+c}}{acx}+\frac{\left(\frac{acf\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}-\frac{bec\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\left(\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad}{}}\right.\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2}}\right.}{ac\sqrt{b\,x^2+a}\,\sqrt{x^2d+c}}$

input
```
int((f*x^2+e)/x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-(-b/a)^(1/2)*b*d*e*x^4+((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF
(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*c*f*x-((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)
^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b*c*e*x+((b*x^2+a)/a)^(1/
2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b*c*e*x-(
-b/a)^(1/2)*a*d*e*x^2-(-b/a)^(1/2)*b*c*e*x^2-(-b/a)^(1/2)*a*c*e)*(d*x^2+c)
^(1/2)*(b*x^2+a)^(1/2)/(-b/a)^(1/2)/x/c/a/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.60

$$\int \frac{e+fx^2}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$
$$=\frac{\sqrt{ac}b^2ex\sqrt{-\frac{b}{a}}E(\arcsin\left(x\sqrt{-\frac{b}{a}}\right)\mid\frac{ad}{bc})-\sqrt{bx^2+a}\sqrt{dx^2+c}abe-(b^2e+a^2f)\sqrt{ac}x\sqrt{-\frac{b}{a}}F(\arcsin\left(x\sqrt{}\right.}{a^2bcx}$$

input
```
integrate((f*x^2+e)/x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fric
as")
```

output
```
(sqrt(a*c)*b^2*e*x*sqrt(-b/a)*elliptic_e(arcsin(x*sqrt(-b/a)), a*d/(b*c))
- sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*a*b*e - (b^2*e + a^2*f)*sqrt(a*c)*x*sqrt
(-b/a)*elliptic_f(arcsin(x*sqrt(-b/a)), a*d/(b*c)))/(a^2*b*c*x)
```

Sympy [F]

$$\int \frac{e+fx^2}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx=\int \frac{e+fx^2}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

input
```
integrate((f*x**2+e)/x**2/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral((e + f*x**2)/(x**2*sqrt(a + b*x**2)*sqrt(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{e+fx^2}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx=\int \frac{fx^2+e}{\sqrt{bx^2+a}\sqrt{dx^2+c}x^2}\,dx$$

input
```
integrate((f*x^2+e)/x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate((f*x^2 + e)/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x^2), x)
```

Giac [F]

$$\int \frac{e+fx^2}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx=\int \frac{fx^2+e}{\sqrt{bx^2+a}\sqrt{dx^2+c}x^2}\,dx$$

input
```
integrate((f*x^2+e)/x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac
")
```

output
```
integrate((f*x^2 + e)/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx^2}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx=\int \frac{f\,x^2+e}{x^2\,\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int((e + f*x^2)/(x^2*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int((e + f*x^2)/(x^2*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{e+fx^2}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx=\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{bd\,x^6+ad\,x^4+bc\,x^4+ac\,x^2}dx\right)e+\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{bd\,x^4+ad\,x^2+bc\,x^2+ac}dx\right)f$$

input
```
int((f*x^2+e)/x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c*x**2 + a*d*x**4 + b*c*x**4 +
b*d*x**6),x)*e + int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c + a*d*x**2 +
 b*c*x**2 + b*d*x**4),x)*f
```

3.164 $\int \frac{e+fx^2}{x^2\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 222

$$\int \frac{e+fx^2}{x^2\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx = -\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{acx} - \frac{\sqrt{b}e\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\,E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|-\frac{ad}{bc}\right)}{\sqrt{a}c\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}} + \frac{(be+af)\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}$$

output

```
-e*(-b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a/c/x-b^(1/2)*e*(1-b*x^2/a)^(1/2)*(d*x
^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2),(-a*d/b/c)^(1/2))/a^(1/2)/c/(-b*x^
2+a)^(1/2)/(1+d*x^2/c)^(1/2)+(a*f+b*e)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)
*EllipticF(b^(1/2)*x/a^(1/2),(-a*d/b/c)^(1/2))/a^(1/2)/b^(1/2)/(-b*x^2+a)^
(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.24 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.93

$$\int \frac{e+fx^2}{x^2\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx$$
$$= \frac{-\sqrt{-\frac{b}{a}}e(a-bx^2)(c+dx^2)+ibcex\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(i\text{arcsinh}\left(\sqrt{-\frac{b}{a}}x\right)|-\frac{ad}{bc}\right)-ic(be+af)x\sqrt{1-}}{a\sqrt{-\frac{b}{a}}cx\sqrt{a-bx^2}\sqrt{c+dx^2}}$$

input

```
Integrate[(e + f*x^2)/(x^2*Sqrt[a - b*x^2]*Sqrt[c + d*x^2]),x]
```

output

```
(-(Sqrt[-(b/a)]*e*(a - b*x^2)*(c + d*x^2)) + I*b*c*e*x*Sqrt[1 - (b*x^2)/a]
*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))]
- I*c*(b*e + a*f)*x*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*Ar
cSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))])/(a*Sqrt[-(b/a)]*c*x*Sqrt[a - b*x^2
]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.265$, Rules used = {445, 25, 399, 323, 323, 321, 331, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx^2}{x^2\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx$$
$$\Big\downarrow \; 445$$
$$-\frac{\int -\frac{acf-bdex^2}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{ac}-\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{acx}$$
$$\Big\downarrow \; 25$$

$$\frac{\int \frac{acf-bdex^2}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{ac}-\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{acx}$$

$$\Big\downarrow \ 399$$

$$\frac{c(af+be)\int \frac{1}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx-be\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{ac}-\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{acx}$$

$$\Big\downarrow \ 323$$

$$\frac{\frac{c\sqrt{\frac{dx^2}{c}+1}(af+be)\int \frac{1}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}dx}{\sqrt{c+dx^2}}-be\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{ac}-\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{acx}$$

$$\Big\downarrow \ 323$$

$$\frac{\frac{c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(af+be)\int \frac{1}{\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}}dx}{\sqrt{a-bx^2}\sqrt{c+dx^2}}-be\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{ac}-\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{acx}$$

$$\Big\downarrow \ 321$$

$$\frac{\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(af+be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-be\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{ac}-\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{acx}$$

$$\Big\downarrow \ 331$$

$$\frac{\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(af+be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{be\sqrt{1-\frac{bx^2}{a}}\int \frac{\sqrt{dx^2+c}}{\sqrt{1-\frac{bx^2}{a}}}dx}{\sqrt{a-bx^2}}}{ac}-\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{acx}$$

$$\Big\downarrow \ 330$$

$$\frac{\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(af+be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{be\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\int \frac{\sqrt{\frac{dx^2}{c}+1}}{\sqrt{1-\frac{bx^2}{a}}}dx}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}}{ac}-\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{acx}$$

$$\Big\downarrow \ 327$$

$$\frac{\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(af+be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{b}e\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}}{ac}-\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{acx}$$

input
```
Int[(e + f*x^2)/(x^2*Sqrt[a - b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
-((e*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])/(a*c*x)) + (-((Sqrt[a]*Sqrt[b]*e*Sqr
t[1 - (b*x^2)/a]*Sqrt[c + d*x^2]*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -(
(a*d)/(b*c))])/(Sqrt[a - b*x^2]*Sqrt[1 + (d*x^2)/c])) + (Sqrt[a]*c*(b*e +
a*f)*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[ArcSin[(Sqrt[b]*x)/
Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*Sqrt[a - b*x^2]*Sqrt[c + d*x^2]))/(a*c
)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 6.22 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.27

method	result
default	$\frac{\left(\sqrt{\frac{b}{a}}\, b d e\, x^{4}+\sqrt{\frac{-b\,x^{2}+a}{a}}\,\sqrt{\frac{x^{2} d+c}{c}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{b}{a}},\sqrt{-\frac{a d}{b c}}\right) a c f x+\sqrt{\frac{-b\,x^{2}+a}{a}}\,\sqrt{\frac{x^{2} d+c}{c}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{b}{a}},\sqrt{-\frac{a d}{b c}}\right) b c e x-\sqrt{\frac{-b\,x^{2}+}{a}}\right.}{\sqrt{\frac{b}{a}}\, x c a(-b d\,x^{4}+a d\,x^{2}-x^{2} b c+a c}$
risch	$-\frac{e\sqrt{-b\,x^{2}+a}\,\sqrt{x^{2} d+c}}{a c x}+\frac{\left(\frac{b e c\sqrt{1-\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{b}{a}},\sqrt{-1-\frac{a d-b c}{c b}}\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{b}{a}},\sqrt{-1-\frac{a d-b c}{c b}}\right)\right)}{\sqrt{\frac{b}{a}}\,\sqrt{-b d\,x^{4}+a d\,x^{2}-x^{2} b c+a c}}+\frac{a c f\sqrt{1-\frac{b\,x^{2}}{a}}\,\sqrt{}}{\sqrt{\frac{b}{a}}}\right.}{a c\sqrt{-b\,x^{2}+a}\,\sqrt{x^{2} d+c}}$
elliptic	$\frac{\sqrt{(-b\,x^{2}+a)(x^{2} d+c)}\,\left(-\frac{e\sqrt{-b d\,x^{4}+a d\,x^{2}-x^{2} b c+a c}}{a c x}+\frac{f\sqrt{1-\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{b}{a}},\sqrt{-1-\frac{a d-b c}{c b}}\right)}{\sqrt{\frac{b}{a}}\,\sqrt{-b d\,x^{4}+a d\,x^{2}-x^{2} b c+a c}}+\frac{b e\sqrt{1-\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,\left(\operatorname{Ellip}\right.}{}\right.}{\sqrt{-b\,x^{2}+a}\,\sqrt{x^{2} d+c}}$

input
```
int((f*x^2+e)/x^2/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE
)
```

output
```
((b/a)^(1/2)*b*d*e*x^4+((-b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(
x*(b/a)^(1/2),(-a*d/b/c)^(1/2))*a*c*f*x+((-b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)
^(1/2)*EllipticF(x*(b/a)^(1/2),(-a*d/b/c)^(1/2))*b*c*e*x-((-b*x^2+a)/a)^(1
/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(b/a)^(1/2),(-a*d/b/c)^(1/2))*b*c*e*x-
(b/a)^(1/2)*a*d*e*x^2+(b/a)^(1/2)*b*c*e*x^2-(b/a)^(1/2)*a*c*e)*(d*x^2+c)^(
1/2)*(-b*x^2+a)^(1/2)/(b/a)^(1/2)/x/c/a/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.56

$$\int \frac{e+fx^2}{x^2\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx = -\frac{\sqrt{ac}b^2ex\sqrt{\frac{b}{a}}E\left(\arcsin\left(x\sqrt{\frac{b}{a}}\right)|-\frac{ad}{bc}\right)+\sqrt{-bx^2+a}\sqrt{dx^2+c}\,cabe-(b^2e+a^2f)\sqrt{ac}x\sqrt{\frac{b}{a}}F\left(\arcsin\left(x\sqrt{\cdots}\right)\right)}{a^2bcx}$$

input
```
integrate((f*x^2+e)/x^2/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fri
cas")
```

output
```
-(sqrt(a*c)*b^2*e*x*sqrt(b/a)*elliptic_e(arcsin(x*sqrt(b/a)), -a*d/(b*c))
+ sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)*a*b*e - (b^2*e + a^2*f)*sqrt(a*c)*x*sqr
t(b/a)*elliptic_f(arcsin(x*sqrt(b/a)), -a*d/(b*c)))/(a^2*b*c*x)
```

Sympy [F]

$$\int \frac{e+fx^2}{x^2\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{e+fx^2}{x^2\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx$$

input
```
integrate((f*x**2+e)/x**2/(-b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral((e + f*x**2)/(x**2*sqrt(a - b*x**2)*sqrt(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{e+fx^2}{x^2\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx=\int \frac{fx^2+e}{\sqrt{-bx^2+a}\sqrt{dx^2+c}x^2}\,dx$$

input
```
integrate((f*x^2+e)/x^2/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="max
ima")
```

output
```
integrate((f*x^2 + e)/(sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)*x^2), x)
```

Giac [F]

$$\int \frac{e+fx^2}{x^2\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx=\int \frac{fx^2+e}{\sqrt{-bx^2+a}\sqrt{dx^2+c}x^2}\,dx$$

input
```
integrate((f*x^2+e)/x^2/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="gia
c")
```

output
```
integrate((f*x^2 + e)/(sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx^2}{x^2\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx=\int \frac{f\,x^2+e}{x^2\,\sqrt{a-b\,x^2}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int((e + f*x^2)/(x^2*(a - b*x^2)^(1/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int((e + f*x^2)/(x^2*(a - b*x^2)^(1/2)*(c + d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{e+fx^2}{x^2\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx = \left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{-b\,x^2+a}}{-bd\,x^6+ad\,x^4-bc\,x^4+ac\,x^2}dx\right)e + \left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{-b\,x^2+a}}{-bd\,x^4+ad\,x^2-bc\,x^2+ac}dx\right)f$$

input
```
int((f*x^2+e)/x^2/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a - b*x**2))/(a*c*x**2 + a*d*x**4 - b*c*x**4 -
b*d*x**6),x)*e + int((sqrt(c + d*x**2)*sqrt(a - b*x**2))/(a*c + a*d*x**2 -
 b*c*x**2 - b*d*x**4),x)*f
```

3.165 $\int \frac{e+fx^2}{x^2\sqrt{a+bx^2}\sqrt{c-dx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 222

$$\begin{aligned}\int \frac{e+fx^2}{x^2\sqrt{a+bx^2}\sqrt{c-dx^2}}\,dx &= -\frac{e\sqrt{a+bx^2}\sqrt{c-dx^2}}{acx} - \frac{\sqrt{d}e\sqrt{a+bx^2}\sqrt{1-\frac{dx^2}{c}}E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{a\sqrt{c}\sqrt{1+\frac{bx^2}{a}}\sqrt{c-dx^2}} \\ &\quad + \frac{(de+cf)\sqrt{1+\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}\sqrt{c-dx^2}}\end{aligned}$$

output
```
-e*(b*x^2+a)^(1/2)*(-d*x^2+c)^(1/2)/a/c/x-d^(1/2)*e*(b*x^2+a)^(1/2)*(1-d*x
^2/c)^(1/2)*EllipticE(d^(1/2)*x/c^(1/2),(-b*c/a/d)^(1/2))/a/c^(1/2)/(1+b*x
^2/a)^(1/2)/(-d*x^2+c)^(1/2)+(c*f+d*e)*(1+b*x^2/a)^(1/2)*(1-d*x^2/c)^(1/2)
*EllipticF(d^(1/2)*x/c^(1/2),(-b*c/a/d)^(1/2))/c^(1/2)/d^(1/2)/(b*x^2+a)^(
1/2)/(-d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.35 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.92

$$\int \frac{e+fx^2}{x^2\sqrt{a+bx^2}\sqrt{c-dx^2}}\,dx = \frac{\sqrt{\frac{b}{a}}\left(\sqrt{\frac{b}{a}}e(a+bx^2)\left(-c+dx^2\right)-ibcex\sqrt{1+\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}E\left(i\text{arcsinh}\left(\sqrt{\frac{b}{a}}x\right)|-\frac{ad}{bc}\right)-ic(-be+af)x\sqrt{\right.}{bcx\sqrt{a+bx^2}\sqrt{c-dx^2}}$$

input
```
Integrate[(e + f*x^2)/(x^2*Sqrt[a + b*x^2]*Sqrt[c - d*x^2]),x]
```

output
```
(Sqrt[b/a]*(Sqrt[b/a]*e*(a + b*x^2)*(-c + d*x^2) - I*b*c*e*x*Sqrt[1 + (b*x
^2)/a]*Sqrt[1 - (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], -((a*d)/(b*c)
)] - I*c*(-(b*e) + a*f)*x*Sqrt[1 + (b*x^2)/a]*Sqrt[1 - (d*x^2)/c]*Elliptic
F[I*ArcSinh[Sqrt[b/a]*x], -((a*d)/(b*c))]))/(b*c*x*Sqrt[a + b*x^2]*Sqrt[c
- d*x^2])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.265, Rules used = {445, 25, 399, 323, 323, 321, 331, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx^2}{x^2\sqrt{a+bx^2}\sqrt{c-dx^2}}\,dx$$

$$\Big\downarrow \text{445}$$

$$-\frac{\int -\frac{acf-bdex^2}{\sqrt{bx^2+a}\sqrt{c-dx^2}}dx}{ac}-\frac{e\sqrt{a+bx^2}\sqrt{c-dx^2}}{acx}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{acf-bdex^2}{\sqrt{bx^2+a}\sqrt{c-dx^2}}dx}{ac}-\frac{e\sqrt{a+bx^2}\sqrt{c-dx^2}}{acx}$$

$$\Big\downarrow \text{399}$$

$$\frac{a(cf+de)\int \frac{1}{\sqrt{bx^2+a}\sqrt{c-dx^2}}dx-de\int \frac{\sqrt{bx^2+a}}{\sqrt{c-dx^2}}dx}{ac}-\frac{e\sqrt{a+bx^2}\sqrt{c-dx^2}}{acx}$$

$$\Big\downarrow \text{323}$$

$$\frac{\frac{a\sqrt{1-\frac{dx^2}{c}}(cf+de)\int \frac{1}{\sqrt{bx^2+a}\sqrt{1-\frac{dx^2}{c}}}dx}{\sqrt{c-dx^2}}-de\int \frac{\sqrt{bx^2+a}}{\sqrt{c-dx^2}}dx}{ac}-\frac{e\sqrt{a+bx^2}\sqrt{c-dx^2}}{acx}$$

$$\Big\downarrow \text{323}$$

$$\frac{\frac{a\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}(cf+de)\int \frac{1}{\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}}dx}{\sqrt{a+bx^2}\sqrt{c-dx^2}}-de\int \frac{\sqrt{bx^2+a}}{\sqrt{c-dx^2}}dx}{ac}-\frac{e\sqrt{a+bx^2}\sqrt{c-dx^2}}{acx}$$

$$\Big\downarrow \text{321}$$

$$\frac{\frac{a\sqrt{c}\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}(cf+de)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{a+bx^2}\sqrt{c-dx^2}}-de\int \frac{\sqrt{bx^2+a}}{\sqrt{c-dx^2}}dx}{ac}-\frac{e\sqrt{a+bx^2}\sqrt{c-dx^2}}{acx}$$

$$\Big\downarrow \text{331}$$

$$\frac{\frac{a\sqrt{c}\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}(cf+de)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{a+bx^2}\sqrt{c-dx^2}}-\frac{de\sqrt{1-\frac{dx^2}{c}}\int \frac{\sqrt{bx^2+a}}{\sqrt{1-\frac{dx^2}{c}}}dx}{\sqrt{c-dx^2}}}{ac}-\frac{e\sqrt{a+bx^2}\sqrt{c-dx^2}}{acx}$$

$$\Big\downarrow \text{330}$$

$$\frac{\frac{a\sqrt{c}\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}(cf+de)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{a+bx^2}\sqrt{c-dx^2}}-\frac{de\sqrt{a+bx^2}\sqrt{1-\frac{dx^2}{c}}\int \frac{\sqrt{\frac{bx^2}{a}+1}}{\sqrt{1-\frac{dx^2}{c}}}dx}{\sqrt{\frac{bx^2}{a}+1}\sqrt{c-dx^2}}}{ac}-\frac{e\sqrt{a+bx^2}\sqrt{c-dx^2}}{acx}$$

$$\Big\downarrow \text{327}$$

$$\frac{\frac{\frac{a\sqrt{c}\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}(cf+de)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{a+bx^2}\sqrt{c-dx^2}}-\frac{\sqrt{c}\sqrt{d}e\sqrt{a+bx^2}\sqrt{1-\frac{dx^2}{c}}E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{\sqrt{\frac{bx^2}{a}+1}\sqrt{c-dx^2}}}{ac}-\frac{e\sqrt{a+bx^2}\sqrt{c-dx^2}}{acx}$$

input
```
Int[(e + f*x^2)/(x^2*Sqrt[a + b*x^2]*Sqrt[c - d*x^2]),x]
```

output
```
-((e*Sqrt[a + b*x^2]*Sqrt[c - d*x^2])/(a*c*x)) + (-((Sqrt[c]*Sqrt[d]*e*Sqr
t[a + b*x^2]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(Sqrt[d]*x)/Sqrt[c]], -(
(b*c)/(a*d))])/(Sqrt[1 + (b*x^2)/a]*Sqrt[c - d*x^2])) + (a*Sqrt[c]*(d*e +
c*f)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(Sqrt[d]*x)/
Sqrt[c]], -((b*c)/(a*d))])/(Sqrt[d]*Sqrt[a + b*x^2]*Sqrt[c - d*x^2]))/(a*c
)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 6.32 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.27

method	result
default	$\frac{\left(\sqrt{\frac{d}{c}}\, bde\, x^4+\sqrt{\frac{-x^2 d+c}{c}}\,\sqrt{\frac{b\,x^2+a}{a}}\ \text{EllipticF}\left(x\sqrt{\frac{d}{c}},\sqrt{-\frac{bc}{ad}}\right) acfx+\sqrt{\frac{-x^2 d+c}{c}}\,\sqrt{\frac{b\,x^2+a}{a}}\ \text{EllipticF}\left(x\sqrt{\frac{d}{c}},\sqrt{-\frac{bc}{ad}}\right) adex-\sqrt{\frac{-x^2 d+\cdots}{c}}\cdots\right.}{\sqrt{\frac{d}{c}}\, xca(-bd\,x^4-ad\,x^2+x^2bc+a\cdots}$
risch	$-\frac{e\sqrt{b\,x^2+a}\,\sqrt{-x^2 d+c}}{acx}+\frac{\left(\frac{dea\sqrt{1-\frac{d\,x^2}{c}}\,\sqrt{1+\frac{b\,x^2}{a}}\,\left(\text{EllipticF}\left(x\sqrt{\frac{d}{c}},\sqrt{-1-\frac{-ad+bc}{ad}}\right)-\text{EllipticE}\left(x\sqrt{\frac{d}{c}},\sqrt{-1-\frac{-ad+bc}{ad}}\right)\right)}{\sqrt{\frac{d}{c}}\,\sqrt{-bd\,x^4-ad\,x^2+x^2bc+ac}}+\frac{acf\sqrt{1-\frac{d\,x^2}{c}}\cdots}{\cdots}\right.}{ac\sqrt{-x^2 d+c}\,\sqrt{b\,x^2+a}\cdots}$
elliptic	$\frac{\sqrt{(-x^2 d+c)(b\,x^2+a)}\left(-\frac{e\sqrt{-bd\,x^4-ad\,x^2+x^2bc+ac}}{acx}+\frac{f\sqrt{1-\frac{d\,x^2}{c}}\,\sqrt{1+\frac{b\,x^2}{a}}\ \text{EllipticF}\left(x\sqrt{\frac{d}{c}},\sqrt{-1-\frac{-ad+bc}{ad}}\right)}{\sqrt{\frac{d}{c}}\,\sqrt{-bd\,x^4-ad\,x^2+x^2bc+ac}}+\frac{de\sqrt{1-\frac{d\,x^2}{c}}\,\sqrt{1+\frac{b\,x^2}{a}}\,\left(\text{Ell}\cdots\right.}{\cdots}\right.}{\sqrt{-x^2 d+c}\,\sqrt{b\,x^2+a}\cdots}$

input
```
int((f*x^2+e)/x^2/(b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x,method=_RETURNVERBOSE
)
```

output
```
((1/c*d)^(1/2)*b*d*e*x^4+((-d*x^2+c)/c)^(1/2)*((b*x^2+a)/a)^(1/2)*Elliptic
F(x*(1/c*d)^(1/2),(-b*c/a/d)^(1/2))*a*c*f*x+((-d*x^2+c)/c)^(1/2)*((b*x^2+a
)/a)^(1/2)*EllipticF(x*(1/c*d)^(1/2),(-b*c/a/d)^(1/2))*a*d*e*x-((-d*x^2+c)
/c)^(1/2)*((b*x^2+a)/a)^(1/2)*EllipticE(x*(1/c*d)^(1/2),(-b*c/a/d)^(1/2))*
a*d*e*x+(1/c*d)^(1/2)*a*d*e*x^2-(1/c*d)^(1/2)*b*c*e*x^2-(1/c*d)^(1/2)*a*c*
e)*(-d*x^2+c)^(1/2)*(b*x^2+a)^(1/2)/(1/c*d)^(1/2)/x/c/a/(-b*d*x^4-a*d*x^2+
b*c*x^2+a*c)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.56

$$\int \frac{e+fx^2}{x^2\sqrt{a+bx^2}\sqrt{c-dx^2}}\,dx = -\frac{\sqrt{ac}d^2ex\sqrt{\frac{d}{c}}E(\arcsin\left(x\sqrt{\frac{d}{c}}\right)|-\frac{bc}{ad})+\sqrt{bx^2+a}\sqrt{-dx^2+c}cde-(d^2e+c^2f)\sqrt{ac}x\sqrt{\frac{d}{c}}F(\arcsin\left(x\sqrt{}\right.}{ac^2dx}$$

input
```
integrate((f*x^2+e)/x^2/(b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x, algorithm="fri
cas")
```

output
```
-(sqrt(a*c)*d^2*e*x*sqrt(d/c)*elliptic_e(arcsin(x*sqrt(d/c)), -b*c/(a*d))
+ sqrt(b*x^2 + a)*sqrt(-d*x^2 + c)*c*d*e - (d^2*e + c^2*f)*sqrt(a*c)*x*sqr
t(d/c)*elliptic_f(arcsin(x*sqrt(d/c)), -b*c/(a*d)))/(a*c^2*d*x)
```

Sympy [F]

$$\int \frac{e+fx^2}{x^2\sqrt{a+bx^2}\sqrt{c-dx^2}}\,dx = \int \frac{e+fx^2}{x^2\sqrt{a+bx^2}\sqrt{c-dx^2}}\,dx$$

input
```
integrate((f*x**2+e)/x**2/(b*x**2+a)**(1/2)/(-d*x**2+c)**(1/2),x)
```

output
```
Integral((e + f*x**2)/(x**2*sqrt(a + b*x**2)*sqrt(c - d*x**2)), x)
```

Maxima [F]

$$\int \frac{e+fx^2}{x^2\sqrt{a+bx^2}\sqrt{c-dx^2}}\,dx = \int \frac{fx^2+e}{\sqrt{bx^2+a}\sqrt{-dx^2+c}x^2}\,dx$$

input
```
integrate((f*x^2+e)/x^2/(b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x, algorithm="max
ima")
```

output
```
integrate((f*x^2 + e)/(sqrt(b*x^2 + a)*sqrt(-d*x^2 + c)*x^2), x)
```

Giac [F]

$$\int \frac{e+fx^2}{x^2\sqrt{a+bx^2}\sqrt{c-dx^2}}\,dx = \int \frac{fx^2+e}{\sqrt{bx^2+a}\sqrt{-dx^2+c}x^2}\,dx$$

input
```
integrate((f*x^2+e)/x^2/(b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x, algorithm="gia
c")
```

output
```
integrate((f*x^2 + e)/(sqrt(b*x^2 + a)*sqrt(-d*x^2 + c)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx^2}{x^2\sqrt{a+bx^2}\sqrt{c-dx^2}}\,dx=\int \frac{f\,x^2+e}{x^2\,\sqrt{b\,x^2+a}\,\sqrt{c-d\,x^2}}\,dx$$

input
```
int((e + f*x^2)/(x^2*(a + b*x^2)^(1/2)*(c - d*x^2)^(1/2)),x)
```

output
```
int((e + f*x^2)/(x^2*(a + b*x^2)^(1/2)*(c - d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{e+fx^2}{x^2\sqrt{a+bx^2}\sqrt{c-dx^2}}\,dx=\left(\int \frac{\sqrt{-d\,x^2+c}\,\sqrt{b\,x^2+a}}{-bd\,x^6-ad\,x^4+bc\,x^4+ac\,x^2}dx\right)e+\left(\int \frac{\sqrt{-d\,x^2+c}\,\sqrt{b\,x^2+a}}{-bd\,x^4-ad\,x^2+bc\,x^2+ac}dx\right)f$$

input
```
int((f*x^2+e)/x^2/(b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x)
```

output
```
int((sqrt(c - d*x**2)*sqrt(a + b*x**2))/(a*c*x**2 - a*d*x**4 + b*c*x**4 -
b*d*x**6),x)*e + int((sqrt(c - d*x**2)*sqrt(a + b*x**2))/(a*c - a*d*x**2 +
 b*c*x**2 - b*d*x**4),x)*f
```

3.166 $\int \frac{e+fx^2}{x^2\sqrt{a-bx^2}\sqrt{c-dx^2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 225

$$\int \frac{e+fx^2}{x^2\sqrt{a-bx^2}\sqrt{c-dx^2}}\,dx = -\frac{e\sqrt{a-bx^2}\sqrt{c-dx^2}}{acx} - \frac{\sqrt{b}e\sqrt{1-\frac{bx^2}{a}}\sqrt{c-dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|\frac{ad}{bc}\right)}{\sqrt{a}c\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}} + \frac{(be+af)\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{b}\sqrt{a-bx^2}\sqrt{c-dx^2}}$$

output
```
-e*(-b*x^2+a)^(1/2)*(-d*x^2+c)^(1/2)/a/c/x-b^(1/2)*e*(1-b*x^2/a)^(1/2)*(-d
*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2),(a*d/b/c)^(1/2))/a^(1/2)/c/(-b*x
^2+a)^(1/2)/(1-d*x^2/c)^(1/2)+(a*f+b*e)*(1-b*x^2/a)^(1/2)*(1-d*x^2/c)^(1/2
)*EllipticF(b^(1/2)*x/a^(1/2),(a*d/b/c)^(1/2))/a^(1/2)/b^(1/2)/(-b*x^2+a)^
(1/2)/(-d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.29 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.93

$$\int \frac{e+fx^2}{x^2\sqrt{a-bx^2}\sqrt{c-dx^2}}\,dx$$
$$= \frac{\sqrt{-\frac{b}{a}}e(a-bx^2)\left(-c+dx^2\right)+ibcex\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}E\left(i\text{arcsinh}\left(\sqrt{-\frac{b}{a}}x\right)|\frac{ad}{bc}\right)-ic(be+af)x\sqrt{1-}}{a\sqrt{-\frac{b}{a}}cx\sqrt{a-bx^2}\sqrt{c-dx^2}}$$

input
```
Integrate[(e + f*x^2)/(x^2*Sqrt[a - b*x^2]*Sqrt[c - d*x^2]),x]
```

output
```
(Sqrt[-(b/a)]*e*(a - b*x^2)*(-c + d*x^2) + I*b*c*e*x*Sqrt[1 - (b*x^2)/a]*S
qrt[1 - (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[-(b/a)]*x], (a*d)/(b*c)] - I*c
*(b*e + a*f)*x*Sqrt[1 - (b*x^2)/a]*Sqrt[1 - (d*x^2)/c]*EllipticF[I*ArcSinh
[Sqrt[-(b/a)]*x], (a*d)/(b*c)])/(a*Sqrt[-(b/a)]*c*x*Sqrt[a - b*x^2]*Sqrt[c
 - d*x^2])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {445, 25, 399, 323, 323, 321, 331, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx^2}{x^2\sqrt{a-bx^2}\sqrt{c-dx^2}}\,dx$$
$$\Big\downarrow \text{445}$$
$$-\frac{\int -\frac{bdex^2+acf}{\sqrt{a-bx^2}\sqrt{c-dx^2}}dx}{ac}-\frac{e\sqrt{a-bx^2}\sqrt{c-dx^2}}{acx}$$
$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{bdex^2+acf}{\sqrt{a-bx^2}\sqrt{c-dx^2}}dx}{ac}-\frac{e\sqrt{a-bx^2}\sqrt{c-dx^2}}{acx}$$

$$\Big\downarrow \text{399}$$

$$\frac{a(cf+de)\int \frac{1}{\sqrt{a-bx^2}\sqrt{c-dx^2}}dx-de\int \frac{\sqrt{a-bx^2}}{\sqrt{c-dx^2}}dx}{ac}-\frac{e\sqrt{a-bx^2}\sqrt{c-dx^2}}{acx}$$

$$\Big\downarrow \text{323}$$

$$\frac{\frac{a\sqrt{1-\frac{dx^2}{c}}(cf+de)\int \frac{1}{\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}}dx}{\sqrt{c-dx^2}}-de\int \frac{\sqrt{a-bx^2}}{\sqrt{c-dx^2}}dx}{ac}-\frac{e\sqrt{a-bx^2}\sqrt{c-dx^2}}{acx}$$

$$\Big\downarrow \text{323}$$

$$\frac{\frac{a\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}(cf+de)\int \frac{1}{\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}}dx}{\sqrt{a-bx^2}\sqrt{c-dx^2}}-de\int \frac{\sqrt{a-bx^2}}{\sqrt{c-dx^2}}dx}{ac}-\frac{e\sqrt{a-bx^2}\sqrt{c-dx^2}}{acx}$$

$$\Big\downarrow \text{321}$$

$$\frac{\frac{a\sqrt{c}\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}(cf+de)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{a-bx^2}\sqrt{c-dx^2}}-de\int \frac{\sqrt{a-bx^2}}{\sqrt{c-dx^2}}dx}{ac}-\frac{e\sqrt{a-bx^2}\sqrt{c-dx^2}}{acx}$$

$$\Big\downarrow \text{331}$$

$$\frac{\frac{a\sqrt{c}\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}(cf+de)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{a-bx^2}\sqrt{c-dx^2}}-\frac{de\sqrt{1-\frac{dx^2}{c}}\int \frac{\sqrt{a-bx^2}}{\sqrt{1-\frac{dx^2}{c}}}dx}{\sqrt{c-dx^2}}}{ac}-\frac{e\sqrt{a-bx^2}\sqrt{c-dx^2}}{acx}$$

$$\Big\downarrow \text{330}$$

$$\frac{\frac{a\sqrt{c}\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}(cf+de)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{a-bx^2}\sqrt{c-dx^2}}-\frac{de\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}\int \frac{\sqrt{1-\frac{bx^2}{a}}}{\sqrt{1-\frac{dx^2}{c}}}dx}{\sqrt{1-\frac{bx^2}{a}}\sqrt{c-dx^2}}}{ac}-\frac{e\sqrt{a-bx^2}\sqrt{c-dx^2}}{acx}$$

$$\Big\downarrow \text{327}$$

$$\frac{\frac{a\sqrt{c}\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}(cf+de)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{a-bx^2}\sqrt{c-dx^2}}-\frac{\sqrt{c}\sqrt{d}e\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{bc}{ad}\right)}{\sqrt{1-\frac{bx^2}{a}}\sqrt{c-dx^2}}}{ac}-\frac{e\sqrt{a-bx^2}\sqrt{c-dx^2}}{acx}$$

input
```
Int[(e + f*x^2)/(x^2*Sqrt[a - b*x^2]*Sqrt[c - d*x^2]),x]
```

output
```
-((e*Sqrt[a - b*x^2]*Sqrt[c - d*x^2])/(a*c*x)) + (-((Sqrt[c]*Sqrt[d]*e*Sqr
t[a - b*x^2]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(Sqrt[d]*x)/Sqrt[c]], (b
*c)/(a*d)])/(Sqrt[1 - (b*x^2)/a]*Sqrt[c - d*x^2])) + (a*Sqrt[c]*(d*e + c*f
)*Sqrt[1 - (b*x^2)/a]*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(Sqrt[d]*x)/Sqr
t[c]], (b*c)/(a*d)])/(Sqrt[d]*Sqrt[a - b*x^2]*Sqrt[c - d*x^2]))/(a*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 5.96 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.26

method	result
default	$\frac{\left(-\sqrt{\frac{d}{c}}\, bde\, x^4+\sqrt{\frac{-x^2 d+c}{c}}\, \sqrt{\frac{-b\, x^2+a}{a}}\, \operatorname{EllipticF}\left(x\sqrt{\frac{d}{c}}, \sqrt{\frac{bc}{ad}}\right) acfx+\sqrt{\frac{-x^2 d+c}{c}}\, \sqrt{\frac{-b\, x^2+a}{a}}\, \operatorname{EllipticF}\left(x\sqrt{\frac{d}{c}}, \sqrt{\frac{bc}{ad}}\right) adex-\sqrt{\frac{-x^2 d}{c}}\right.}{\sqrt{\frac{d}{c}}\, xca(bd\, x^4-ad\, x^2-x^2 bc+a}$
elliptic	$\frac{\sqrt{(-x^2 d+c)(-b\, x^2+a)}\, \left(-\frac{e\sqrt{bd\, x^4-ad\, x^2-x^2 bc+ac}}{acx}+\frac{f\sqrt{1-\frac{d\, x^2}{c}}\, \sqrt{1-\frac{b\, x^2}{a}}\, \operatorname{EllipticF}\left(x\sqrt{\frac{d}{c}}, \sqrt{-1-\frac{-ad-bc}{ad}}\right)}{\sqrt{\frac{d}{c}}\, \sqrt{bd\, x^4-ad\, x^2-x^2 bc+ac}}+\frac{de\sqrt{1-\frac{d\, x^2}{c}}\, \sqrt{1-\frac{b\, x^2}{a}}\, (\mathrm{El}}{}\right.}{\sqrt{-x^2 d+c}\, \sqrt{-b\, x^2+a}}$
risch	$-\frac{e\sqrt{-b\, x^2+a}\, \sqrt{-x^2 d+c}}{acx}+\frac{\left(\frac{acf\sqrt{1-\frac{d\, x^2}{c}}\, \sqrt{1-\frac{b\, x^2}{a}}\, \operatorname{EllipticF}\left(x\sqrt{\frac{d}{c}}, \sqrt{-1-\frac{-ad-bc}{ad}}\right)}{\sqrt{\frac{d}{c}}\, \sqrt{bd\, x^4-ad\, x^2-x^2 bc+ac}}+\frac{dea\sqrt{1-\frac{d\, x^2}{c}}\, \sqrt{1-\frac{b\, x^2}{a}}\, \left(\operatorname{EllipticF}\left(x\sqrt{\frac{d}{c}}, \sqrt{-1-}\right.\right.}{\sqrt{\frac{d}{c}}\, \sqrt{bd\, x^4-a}}\right.}{ac\sqrt{-x^2 d+c}\, \sqrt{-b\, x^2+a}}$

input
```
int((f*x^2+e)/x^2/(-b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x,method=_RETURNVERBOS
E)
```

output
```
(-(1/c*d)^(1/2)*b*d*e*x^4+((-d*x^2+c)/c)^(1/2)*((-b*x^2+a)/a)^(1/2)*Ellipt
icF(x*(1/c*d)^(1/2),(b*c/a/d)^(1/2))*a*c*f*x+((-d*x^2+c)/c)^(1/2)*((-b*x^2
+a)/a)^(1/2)*EllipticF(x*(1/c*d)^(1/2),(b*c/a/d)^(1/2))*a*d*e*x-((-d*x^2+c
)/c)^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticE(x*(1/c*d)^(1/2),(b*c/a/d)^(1/2))
*a*d*e*x+(1/c*d)^(1/2)*a*d*e*x^2+(1/c*d)^(1/2)*b*c*e*x^2-(1/c*d)^(1/2)*a*c
*e)*(-d*x^2+c)^(1/2)*(-b*x^2+a)^(1/2)/(1/c*d)^(1/2)/x/c/a/(b*d*x^4-a*d*x^2
-b*c*x^2+a*c)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.55

$$\int \frac{e+fx^2}{x^2\sqrt{a-bx^2}\sqrt{c-dx^2}}\,dx = -\frac{\sqrt{ac}d^2ex\sqrt{\frac{d}{c}}E\left(\arcsin\left(x\sqrt{\frac{d}{c}}\right)\mid\frac{bc}{ad}\right)+\sqrt{-bx^2+a}\sqrt{-dx^2+c}cde-(d^2e+c^2f)\sqrt{ac}x\sqrt{\frac{d}{c}}F\left(\arcsin\left(x\sqrt{\frac{d}{c}}\right)\mid\frac{bc}{ad}\right)}{ac^2dx}$$

input
```
integrate((f*x^2+e)/x^2/(-b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x, algorithm="fr
icas")
```

output
```
-(sqrt(a*c)*d^2*e*x*sqrt(d/c)*elliptic_e(arcsin(x*sqrt(d/c)), b*c/(a*d)) +
 sqrt(-b*x^2 + a)*sqrt(-d*x^2 + c)*c*d*e - (d^2*e + c^2*f)*sqrt(a*c)*x*sqr
t(d/c)*elliptic_f(arcsin(x*sqrt(d/c)), b*c/(a*d)))/(a*c^2*d*x)
```

Sympy [F]

$$\int \frac{e+fx^2}{x^2\sqrt{a-bx^2}\sqrt{c-dx^2}}\,dx = \int \frac{e+fx^2}{x^2\sqrt{a-bx^2}\sqrt{c-dx^2}}\,dx$$

input
```
integrate((f*x**2+e)/x**2/(-b*x**2+a)**(1/2)/(-d*x**2+c)**(1/2),x)
```

output
```
Integral((e + f*x**2)/(x**2*sqrt(a - b*x**2)*sqrt(c - d*x**2)), x)
```

Maxima [F]

$$\int \frac{e+fx^2}{x^2\sqrt{a-bx^2}\sqrt{c-dx^2}}\,dx = \int \frac{fx^2+e}{\sqrt{-bx^2+a}\sqrt{-dx^2+cx^2}}\,dx$$

input
```
integrate((f*x^2+e)/x^2/(-b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate((f*x^2 + e)/(sqrt(-b*x^2 + a)*sqrt(-d*x^2 + c)*x^2), x)
```

Giac [F]

$$\int \frac{e+fx^2}{x^2\sqrt{a-bx^2}\sqrt{c-dx^2}}\,dx = \int \frac{fx^2+e}{\sqrt{-bx^2+a}\sqrt{-dx^2+cx^2}}\,dx$$

input
```
integrate((f*x^2+e)/x^2/(-b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate((f*x^2 + e)/(sqrt(-b*x^2 + a)*sqrt(-d*x^2 + c)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx^2}{x^2\sqrt{a-bx^2}\sqrt{c-dx^2}}\,dx = \int \frac{f\,x^2+e}{x^2\,\sqrt{a-b\,x^2}\,\sqrt{c-d\,x^2}}\,dx$$

input
```
int((e + f*x^2)/(x^2*(a - b*x^2)^(1/2)*(c - d*x^2)^(1/2)),x)
```

output
```
int((e + f*x^2)/(x^2*(a - b*x^2)^(1/2)*(c - d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{e+fx^2}{x^2\sqrt{a-bx^2}\sqrt{c-dx^2}}\,dx = \left(\int \frac{\sqrt{-d\,x^2+c}\,\sqrt{-b\,x^2+a}}{bd\,x^6-ad\,x^4-bc\,x^4+ac\,x^2}dx\right)e + \left(\int \frac{\sqrt{-d\,x^2+c}\,\sqrt{-b\,x^2+a}}{bd\,x^4-ad\,x^2-bc\,x^2+ac}dx\right)f$$

input
```
int((f*x^2+e)/x^2/(-b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x)
```

output
```
int((sqrt(c - d*x**2)*sqrt(a - b*x**2))/(a*c*x**2 - a*d*x**4 - b*c*x**4 +
b*d*x**6),x)*e + int((sqrt(c - d*x**2)*sqrt(a - b*x**2))/(a*c - a*d*x**2 -
 b*c*x**2 + b*d*x**4),x)*f
```

3.167 $\int \frac{x^4(e+fx^2)^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 587

$$\int \frac{x^4(e+fx^2)^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = -\frac{2(9abcdf(7bde-3bcf-3adf)-(bc+ad)\,(8(bc+ad)f(7bde-3bcf-3adf)-5bd(7bde^2-5acf^2)))}{105b^3d^4\sqrt{a+bx^2}}$$
$$-\frac{(8(bc+ad)f(7bde-3bcf-3adf)-5bd(7bde^2-5acf^2))\,x\sqrt{a+bx^2}\sqrt{c+dx^2}}{105b^3d^3}$$
$$+\frac{2f(7bde-3bcf-3adf)x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{35b^2d^2}+\frac{f^2x^5\sqrt{a+bx^2}\sqrt{c+dx^2}}{7bd}$$
$$+\frac{2\sqrt{a}(9abcdf(7bde-3bcf-3adf)-(bc+ad)\,(8(bc+ad)f(7bde-3bcf-3adf)-5bd(7bde^2-5acf}{105b^{7/2}d^4\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$
$$+\frac{a^{3/2}(8(bc+ad)f(7bde-3bcf-3adf)-5bd(7bde^2-5acf^2))\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\right.}{105b^{7/2}d^3\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
-2/105*(9*a*b*c*d*f*(-3*a*d*f-3*b*c*f+7*b*d*e)-(a*d+b*c)*(8*(a*d+b*c)*f*(-
3*a*d*f-3*b*c*f+7*b*d*e)-5*b*d*(-5*a*c*f^2+7*b*d*e^2)))*x*(d*x^2+c)^(1/2)/
b^3/d^4/(b*x^2+a)^(1/2)-1/105*(8*(a*d+b*c)*f*(-3*a*d*f-3*b*c*f+7*b*d*e)-5*
b*d*(-5*a*c*f^2+7*b*d*e^2))*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b^3/d^3+2/35
*f*(-3*a*d*f-3*b*c*f+7*b*d*e)*x^3*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b^2/d^2+
1/7*f^2*x^5*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b/d+2/105*a^(1/2)*(9*a*b*c*d*f
*(-3*a*d*f-3*b*c*f+7*b*d*e)-(a*d+b*c)*(8*(a*d+b*c)*f*(-3*a*d*f-3*b*c*f+7*b
*d*e)-5*b*d*(-5*a*c*f^2+7*b*d*e^2)))*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a
^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/b^(7/2)/d^4/(b*x^2+a)^(1/2)/(a
*(d*x^2+c)/c/(b*x^2+a))^(1/2)+1/105*a^(3/2)*(8*(a*d+b*c)*f*(-3*a*d*f-3*b*c
*f+7*b*d*e)-5*b*d*(-5*a*c*f^2+7*b*d*e^2))*(d*x^2+c)^(1/2)*InverseJacobiAM(
arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/b^(7/2)/d^3/(b*x^2+a)^(1/2)/(
a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 9.16 (sec) , antiderivative size = 450, normalized size of antiderivative = 0.77

$$\int \frac{x^4(e+fx^2)^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$
$$= \frac{\sqrt{\frac{b}{a}}dx(a+bx^2)\left(c+dx^2\right)\left(24a^2d^2f^2+abdf(23cf-2d(28e+9fx^2))+b^2(24c^2f^2-2cdf(28e+9fx^2)+\right.}{}$$

input
```
Integrate[(x^4*(e + f*x^2)^2)/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
(Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2)*(24*a^2*d^2*f^2 + a*b*d*f*(23*c*f -
 2*d*(28*e + 9*f*x^2)) + b^2*(24*c^2*f^2 - 2*c*d*f*(28*e + 9*f*x^2) + d^2*
(35*e^2 + 42*e*f*x^2 + 15*f^2*x^4))) + (2*I)*c*(24*a^3*d^3*f^2 + 4*a^2*b*d
^2*f*(-14*d*e + 5*c*f) + a*b^2*d*(35*d^2*e^2 - 49*c*d*e*f + 20*c^2*f^2) +
b^3*c*(35*d^2*e^2 - 56*c*d*e*f + 24*c^2*f^2))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 +
 (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*c*(24*a^3*d
^3*f^2 + a^2*b*d^2*f*(-56*d*e + 17*c*f) + a*b^2*d*(35*d^2*e^2 - 42*c*d*e*f
 + 16*c^2*f^2) + 2*b^3*c*(35*d^2*e^2 - 56*c*d*e*f + 24*c^2*f^2))*Sqrt[1 +
(b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*
c)])/(105*b^3*Sqrt[b/a]*d^4*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (warning: unable to verify)

Time = 1.60 (sec) , antiderivative size = 862, normalized size of antiderivative = 1.47, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {448, 444, 444, 25, 406, 320, 388, 313, 444, 25, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 (e+fx^2)^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \text{448}$$

$$\frac{f\int \frac{x^6(fx^2+e)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{e^2}+e\int \frac{x^4(fx^2+e)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx$$

$$\Big\downarrow \text{444}$$

$$\frac{f\left(\frac{fx^5\sqrt{a+bx^2}\sqrt{c+dx^2}}{7bd}-\frac{\int \frac{x^4\left(5acf-(7bde-6bcf-6adf)x^2\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{7bd}\right)}{e^2}+e\left(\frac{fx^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5bd}-\frac{\int \frac{x^2(3acf-(5bde-4bcf-4adf)x^2)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{5bd}\right)$$

$$\Big\downarrow \text{444}$$

$$\frac{f\left(\frac{fx^5\sqrt{a+bx^2}\sqrt{c+dx^2}}{7bd}-\frac{-\frac{\int -\frac{x^2\left(3ac(7bde-6bcf-6adf)-\left(-4c(7de-6cf)b^2-ad(28de-23cf)b+24a^2d^2f\right)x^2\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{5bd}-\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}(-6adf-6bcf+7}{5bd}}{7bd}\right.}{e^2}$$

$$e\left(\frac{fx^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5bd}-\frac{-\frac{\int -\frac{ac(5bde-4bcf-4adf)-\left(-2c(5de-4cf)b^2-ad(10de-7cf)b+8a^2d^2f\right)x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3bd}-\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(-4adf-4}{3bd}}{5bd}\right.$$

$$\Big\downarrow \text{25}$$

$$f\left(\frac{fx^5\sqrt{a+bx^2}\sqrt{c+dx^2}}{7bd}-\frac{\frac{\int\frac{x^2\left(3ac(7bde-6bcf-6adf)-\left(-4c(7de-6cf)b^2-ad(28de-23cf)b+24a^2d^2f\right)x^2\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{5bd}-\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}(-6adf-6bcf+7bde}{5bd}}{7bd}\right.$$

$$\overline{\qquad e^2 \qquad}$$

$$e\left(\frac{fx^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5bd}-\frac{\frac{\int\frac{ac(5bde-4bcf-4adf)-\left(-2c(5de-4cf)b^2-ad(10de-7cf)b+8a^2d^2f\right)x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3bd}-\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(-4adf-4bcf+}{3bd}}{5bd}\right.$$

$$\Big\downarrow \; 406$$

$$f\left(\frac{fx^5\sqrt{a+bx^2}\sqrt{c+dx^2}}{7bd}-\frac{\frac{\int\frac{x^2\left(3ac(7bde-6bcf-6adf)-\left(-4c(7de-6cf)b^2-ad(28de-23cf)b+24a^2d^2f\right)x^2\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{5bd}-\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}(-6adf-6bcf+7bde}{5bd}}{7bd}\right.$$

$$\overline{\qquad e^2 \qquad}$$

$$e\left(\frac{fx^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5bd}-\frac{\frac{ac(-4adf-4bcf+5bde)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx-\left(8a^2d^2f-abd(10de-7cf)-2b^2c(5de-4cf)\right)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2-}}}{3bd}}{5bd}\right.$$

$$\Big\downarrow \; 320$$

$$e\left(\frac{fx^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5bd}-\frac{\frac{\frac{c^{3/2}\sqrt{a+bx^2}(-4adf-4bcf+5bde)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\left(8a^2d^2f-abd(10de-7cf)-2b^2c(5de-4cf)\right)}{3bd}}{5bd}\right.$$

$$f\left(\frac{fx^5\sqrt{a+bx^2}\sqrt{c+dx^2}}{7bd}-\frac{\frac{\int\frac{x^2\left(3ac(7bde-6bcf-6adf)-\left(-4c(7de-6cf)b^2-ad(28de-23cf)b+24a^2d^2f\right)x^2\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{5bd}-\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}(-6adf-6bcf+7bde}{5bd}}{7bd}\right.$$

$$\overline{\qquad e^2 \qquad}$$

$$\Big\downarrow \; 388$$

$$e\left(\frac{fx^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5bd}-\frac{\frac{\frac{c^{3/2}\sqrt{a+bx^2}(-4adf-4bcf+5bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\left(8a^2d^2f-abd(10de-7cf)-2b^2c(5de-4cf)\right)}{3bd}}{5bd}\right.$$

$$\frac{f\left(\frac{fx^5\sqrt{a+bx^2}\sqrt{c+dx^2}}{7bd}-\frac{\frac{\int\frac{x^2\left(3ac(7bde-6bcf-6adf)-\left(-4c(7de-6cf)b^2-ad(28de-23cf)b+24a^2d^2f\right)x^2\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{5bd}-\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}(-6adf-6bcf+7bde}{5bd}}{7bd}\right.}{e^2}$$

$\downarrow$ 313

$$\frac{f\left(\frac{fx^5\sqrt{a+bx^2}\sqrt{c+dx^2}}{7bd}-\frac{\frac{\int\frac{x^2\left(3ac(7bde-6bcf-6adf)-\left(-4c(7de-6cf)b^2-ad(28de-23cf)b+24a^2d^2f\right)x^2\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{5bd}-\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}(-6adf-6bcf+7bde}{5bd}}{7bd}\right.}{e^2}$$

$$e\left(\frac{fx^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5bd}-\frac{\frac{\frac{c^{3/2}\sqrt{a+bx^2}(-4adf-4bcf+5bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\left(8a^2d^2f-abd(10de-7cf)-2b^2c(5de-4cf)\right)}{3bd}}{5bd}\right.$$

$\downarrow$ 444

$$\frac{f\left(\frac{fx^5\sqrt{a+bx^2}\sqrt{c+dx^2}}{7bd}-\frac{-\frac{\int -\frac{\left(-8c^2(7de-6cf)b^3-acd(49de-40cf)b^2-8a^2d^2(7de-5cf)b+48a^3d^3f\right)x^2+ac\left(-4c(7de-6cf)b^2-ad(28de-23cf)b+24a^2d\cdots\right.}{\sqrt{bx^2+a}\sqrt{dx^2+c}}\cdots}{3bd}\cdots}{5bd}\cdots}{7bd}\cdots\right)}{e^2\cdots}$$

$$e\left(\frac{fx^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5bd}-\frac{\frac{\frac{c^{3/2}\sqrt{a+bx^2}(-4adf-4bcf+5bde)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\left(8a^2d^2f-abd(10de-7cf)-2b^2c(5de-4cf)\right)\cdots}{3bd}\cdots}{5bd}\cdots\right)$$

$\downarrow$ 25

$$\frac{f\left(\frac{fx^5\sqrt{a+bx^2}\sqrt{c+dx^2}}{7bd}-\frac{\frac{\int \frac{\left(-8c^2(7de-6cf)b^3-acd(49de-40cf)b^2-8a^2d^2(7de-5cf)b+48a^3d^3f\right)x^2+ac\left(-4c(7de-6cf)b^2-ad(28de-23cf)b+24a^2d^2f\right)\cdots}{\sqrt{bx^2+a}\sqrt{dx^2+c}}\cdots}{3bd}\cdots}{5bd}\cdots}{7bd}\cdots\right)}{e^2\cdots}$$

$$e\left(\frac{fx^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5bd}-\frac{\frac{\frac{c^{3/2}\sqrt{a+bx^2}(-4adf-4bcf+5bde)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\left(8a^2d^2f-abd(10de-7cf)-2b^2c(5de-4cf)\right)\cdots}{3bd}\cdots}{5bd}\cdots\right)$$

$\downarrow$ 406

$$\frac{f\left(\frac{fx^5\sqrt{a+bx^2}\sqrt{c+dx^2}}{7bd}-\frac{\frac{ac\left(24a^2d^2f-abd(28de-23cf)-4b^2c(7de-6cf)\right)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\left(48a^3d^3f-8a^2bd^2(7de-5cf)-ab^2cd(49de-40cf)-8b^3\right.}{3bd}}{5bd}\right.}{e^2}$$

$$e\left(\frac{fx^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5bd}-\frac{\frac{\frac{c^{3/2}\sqrt{a+bx^2}(-4adf-4bcf+5bde)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-(8a^2d^2f-abd(10de-7cf)-2b^2c(5de-4cf))}{3bd}}{5bd}\right.$$

$$\downarrow 320$$

$$f\left(\frac{fx^5\sqrt{a+bx^2}\sqrt{c+dx^2}}{7bd}-\frac{\frac{\left(48a^3d^3f-8a^2bd^2(7de-5cf)-ab^2cd(49de-40cf)-8b^3c^2(7de-6cf)\right)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}\left(24a^2d^2f-abd(28d\right.}{}}{3bd}}{5b}\right.$$

$$e\left(\frac{fx^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5bd}-\frac{\frac{\frac{c^{3/2}\sqrt{a+bx^2}(-4adf-4bcf+5bde)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-(8a^2d^2f-abd(10de-7cf)-2b^2c(5de-4cf))}{3bd}}{5bd}\right.$$

$$\downarrow 388$$

$$e\left(\frac{fx^3\sqrt{bx^2+a}\sqrt{dx^2+c}}{5bd}-\frac{\frac{\frac{c^{3/2}(5bde-4bcf-4adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\left(-2c(5de-4cf)b^2-ad(10de-7cf)b+8a^2d^2f\right)\cdots}{3bd}}{5bd}\right.$$

$$f\left(\frac{fx^5\sqrt{bx^2+a}\sqrt{dx^2+c}}{7bd}-\frac{\frac{\frac{\left(-4c(7de-6cf)b^2-ad(28de-23cf)b+24a^2d^2f\right)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\left(-8c^2(7de-6cf)b^3-acd(49de\cdots\right.}{3bd}}{\cdots}\right.$$

$$\Big\downarrow \; 313$$

$$e\left(\frac{fx^3\sqrt{bx^2+a}\sqrt{dx^2+c}}{5bd}-\frac{\frac{\frac{c^{3/2}(5bde-4bcf-4adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\left(-2c(5de-4cf)b^2-ad(10de-7cf)b+8a^2d^2f\right)\cdots}{3bd}}{5bd}\right.$$

$$f\left(\frac{fx^5\sqrt{bx^2+a}\sqrt{dx^2+c}}{7bd}-\frac{\frac{\frac{\left(-4c(7de-6cf)b^2-ad(28de-23cf)b+24a^2d^2f\right)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\left(-8c^2(7de-6cf)b^3-acd(49de\cdots\right.}{3bd}}{\cdots}\right.$$

input
```
Int[(x^4*(e + f*x^2)^2)/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
e*((f*x^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(5*b*d) - (-1/3*((5*b*d*e - 4*b
*c*f - 4*a*d*f)*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(b*d) + (-((8*a^2*d^2*f
 - a*b*d*(10*d*e - 7*c*f) - 2*b^2*c*(5*d*e - 4*c*f))*((x*Sqrt[a + b*x^2])/
(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x
)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^
2))]*Sqrt[c + d*x^2]))) + (c^(3/2)*(5*b*d*e - 4*b*c*f - 4*a*d*f)*Sqrt[a +
b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*S
qrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))/(3*b*d))/(5*b*d)) +
 (f*((f*x^5*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(7*b*d) - (-1/5*((7*b*d*e - 6
*b*c*f - 6*a*d*f)*x^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(b*d) + (-1/3*(((24
*a^2*d*f)/b - a*(28*d*e - 23*c*f) - (4*b*c*(7*d*e - 6*c*f))/d)*x*Sqrt[a +
b*x^2]*Sqrt[c + d*x^2]) + ((48*a^3*d^3*f - a*b^2*c*d*(49*d*e - 40*c*f) - 8
*b^3*c^2*(7*d*e - 6*c*f) - 8*a^2*b*d^2*(7*d*e - 5*c*f))*((x*Sqrt[a + b*x^2
])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d
]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d
*x^2))]*Sqrt[c + d*x^2])) + (c^(3/2)*(24*a^2*d^2*f - a*b*d*(28*d*e - 23*c*
f) - 4*b^2*c*(7*d*e - 6*c*f))*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)
/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))
]*Sqrt[c + d*x^2]))/(3*b*d))/(5*b*d))/(7*b*d)))/e^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

rule 448
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[e   Int[(g*x)^m*(a + b*x
^2)^p*(c + d*x^2)^q*(e + f*x^2)^(r - 1), x], x] + Simp[f/e^2   Int[(g*x)^(m
 + 2)*(a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^(r - 1), x], x] /; FreeQ[{a,
b, c, d, e, f, g, m, p, q}, x] && IGtQ[r, 0]
```

Maple [A] (verified)

Time = 12.23 (sec) , antiderivative size = 617, normalized size of antiderivative = 1.05

method	result
elliptic	$\frac{\sqrt{(b\,x^2+a)(x^2d+c)}\left(\frac{f^2x^5\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{7bd}+\frac{\left(2ef-\frac{f^2(6ad+6bc)}{7bd}\right)x^3\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{5bd}+\frac{\left(e^2-\frac{5ac\,f^2}{7bd}-\frac{\left(2ef-\frac{f^2(6ad+6bc}{7bd}\right.}{5bd}\right.}{}\right.}{}$
risch	$\frac{x(15f^2x^4b^2d^2-18ab\,d^2f^2x^2-18b^2cd\,f^2x^2+42b^2d^2ef\,x^2+24a^2d^2f^2+23abcd\,f^2-56ab\,d^2ef+24b^2c^2f^2-56b^2cdef+35b^2d^2e^2)\sqrt{b\,x}}{105b^3d^3}$
default	Expression too large to display

input

```
int(x^4*(f*x^2+e)^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOS
E)
```

output

```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/7*f^2/b/d*x
^5*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+1/5*(2*e*f-1/7*f^2/b/d*(6*a*d+6*b*c
))/b/d*x^3*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+1/3*(e^2-5/7*a/b*c/d*f^2-1/
5*(2*e*f-1/7*f^2/b/d*(6*a*d+6*b*c))/b/d*(4*a*d+4*b*c))/b/d*x*(b*d*x^4+a*d*
x^2+b*c*x^2+a*c)^(1/2)-1/3*(e^2-5/7*a/b*c/d*f^2-1/5*(2*e*f-1/7*f^2/b/d*(6*
a*d+6*b*c))/b/d*(4*a*d+4*b*c))/b/d*a*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d
*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2)
,(-1+(a*d+b*c)/c/b)^(1/2))-(-3/5*(2*e*f-1/7*f^2/b/d*(6*a*d+6*b*c))/b/d*a*c
-1/3*(e^2-5/7*a/b*c/d*f^2-1/5*(2*e*f-1/7*f^2/b/d*(6*a*d+6*b*c))/b/d*(4*a*d
+4*b*c))/b/d*(2*a*d+2*b*c))*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(
1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(-b/a)^(1/2),(-1+(
a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 601, normalized size of antiderivative = 1.02

$$\int \frac{x^4(e+fx^2)^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$
$$= \frac{2\left(35\left(b^3c^2d^2+ab^2cd^3\right)e^2-7\left(8\,b^3c^3d+7\,ab^2c^2d^2+8\,a^2bcd^3\right)ef+4\left(6\,b^3c^4+5\,ab^2c^3d+5\,a^2bc^2d^2+6\,a^3c\right.}{}$$

input
```
integrate(x^4*(f*x^2+e)^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fr
icas")
```

output
```
1/105*(2*(35*(b^3*c^2*d^2 + a*b^2*c*d^3)*e^2 - 7*(8*b^3*c^3*d + 7*a*b^2*c^
2*d^2 + 8*a^2*b*c*d^3)*e*f + 4*(6*b^3*c^4 + 5*a*b^2*c^3*d + 5*a^2*b*c^2*d^
2 + 6*a^3*c*d^3)*f^2)*sqrt(b*d)*x*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/
x), a*d/(b*c)) - (35*(2*b^3*c^2*d^2 + 2*a*b^2*c*d^3 + a*b^2*d^4)*e^2 - 14*
(8*b^3*c^3*d + 7*a*b^2*c^2*d^2 + 4*a^2*b*d^4 + 4*(2*a^2*b + a*b^2)*c*d^3)*
e*f + (48*b^3*c^4 + 40*a*b^2*c^3*d + 24*a^3*d^4 + 8*(5*a^2*b + 3*a*b^2)*c^
2*d^2 + (48*a^3 + 23*a^2*b)*c*d^3)*f^2)*sqrt(b*d)*x*sqrt(-c/d)*elliptic_f(
arcsin(sqrt(-c/d)/x), a*d/(b*c)) + (15*b^3*d^4*f^2*x^6 + 6*(7*b^3*d^4*e*f
- 3*(b^3*c*d^3 + a*b^2*d^4)*f^2)*x^4 - 70*(b^3*c*d^3 + a*b^2*d^4)*e^2 + 14
*(8*b^3*c^2*d^2 + 7*a*b^2*c*d^3 + 8*a^2*b*d^4)*e*f - 8*(6*b^3*c^3*d + 5*a*
b^2*c^2*d^2 + 5*a^2*b*c*d^3 + 6*a^3*d^4)*f^2 + (35*b^3*d^4*e^2 - 56*(b^3*c
*d^3 + a*b^2*d^4)*e*f + (24*b^3*c^2*d^2 + 23*a*b^2*c*d^3 + 24*a^2*b*d^4)*f
^2)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(b^4*d^5*x)
```

Sympy [F]

$$\int \frac{x^4(e+fx^2)^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{x^4(e+fx^2)^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

input
```
integrate(x**4*(f*x**2+e)**2/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(x**4*(e + f*x**2)**2/(sqrt(a + b*x**2)*sqrt(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{x^4(e+fx^2)^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{(fx^2+e)^2x^4}{\sqrt{bx^2+a}\sqrt{dx^2+c}}\,dx$$

input
```
integrate(x^4*(f*x^2+e)^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate((f*x^2 + e)^2*x^4/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

Giac [F]

$$\int \frac{x^4(e+fx^2)^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{(fx^2+e)^2x^4}{\sqrt{bx^2+a}\sqrt{dx^2+c}}\,dx$$

input
```
integrate(x^4*(f*x^2+e)^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate((f*x^2 + e)^2*x^4/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(e+fx^2)^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{x^4\,(f\,x^2+e)^2}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int((x^4*(e + f*x^2)^2)/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int((x^4*(e + f*x^2)^2)/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^4(e + fx^2)^2}{\sqrt{a + bx^2}\sqrt{c + dx^2}}\,dx = \text{Too large to display}$$

input
```
int(x^4*(f*x^2+e)^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

output
```
(24*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*d**2*f**2*x + 23*sqrt(c + d*x**
2)*sqrt(a + b*x**2)*a*b*c*d*f**2*x - 56*sqrt(c + d*x**2)*sqrt(a + b*x**2)*
a*b*d**2*e*f*x - 18*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b*d**2*f**2*x**3 +
 24*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**2*c**2*f**2*x - 56*sqrt(c + d*x**
2)*sqrt(a + b*x**2)*b**2*c*d*e*f*x - 18*sqrt(c + d*x**2)*sqrt(a + b*x**2)*
b**2*c*d*f**2*x**3 + 35*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**2*d**2*e**2*x
 + 42*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**2*d**2*e*f*x**3 + 15*sqrt(c + d
*x**2)*sqrt(a + b*x**2)*b**2*d**2*f**2*x**5 - 48*int((sqrt(c + d*x**2)*sqr
t(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a**3*d**3*f*
*2 - 40*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c
*x**2 + b*d*x**4),x)*a**2*b*c*d**2*f**2 + 112*int((sqrt(c + d*x**2)*sqrt(a
 + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a**2*b*d**3*e*f
 - 40*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x
**2 + b*d*x**4),x)*a*b**2*c**2*d*f**2 + 98*int((sqrt(c + d*x**2)*sqrt(a +
b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a*b**2*c*d**2*e*f
- 70*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x*
*2 + b*d*x**4),x)*a*b**2*d**3*e**2 - 48*int((sqrt(c + d*x**2)*sqrt(a + b*x
**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*b**3*c**3*f**2 + 112*
int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 +
b*d*x**4),x)*b**3*c**2*d*e*f - 70*int((sqrt(c + d*x**2)*sqrt(a + b*x**2...
```

3.168 $\int \frac{x^2(e+fx^2)^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 411

$$\begin{aligned}
&\int \frac{x^2(e+fx^2)^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx \\
&= -\frac{(4(bc+ad)f(5bde-2bcf-2adf)-3bd(5bde^2-3acf^2))\,x\sqrt{c+dx^2}}{15b^2d^3\sqrt{a+bx^2}} \\
&\quad+\frac{2f(5bde-2bcf-2adf)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{15b^2d^2}+\frac{f^2x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5bd} \\
&\quad+\frac{\sqrt{a}(4(bc+ad)f(5bde-2bcf-2adf)-3bd(5bde^2-3acf^2))\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{15b^{5/2}d^3\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} \\
&\quad-\frac{2a^{3/2}f(5bde-2bcf-2adf)\sqrt{c+dx^2}\,\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{15b^{5/2}d^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}
\end{aligned}$$

output
```
-1/15*(4*(a*d+b*c)*f*(-2*a*d*f-2*b*c*f+5*b*d*e)-3*b*d*(-3*a*c*f^2+5*b*d*e^
2))*x*(d*x^2+c)^(1/2)/b^2/d^3/(b*x^2+a)^(1/2)+2/15*f*(-2*a*d*f-2*b*c*f+5*b
*d*e)*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b^2/d^2+1/5*f^2*x^3*(b*x^2+a)^(1/2
)*(d*x^2+c)^(1/2)/b/d+1/15*a^(1/2)*(4*(a*d+b*c)*f*(-2*a*d*f-2*b*c*f+5*b*d*
e)-3*b*d*(-3*a*c*f^2+5*b*d*e^2))*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/
2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/b^(5/2)/d^3/(b*x^2+a)^(1/2)/(a*(d*
x^2+c)/c/(b*x^2+a))^(1/2)-2/15*a^(3/2)*f*(-2*a*d*f-2*b*c*f+5*b*d*e)*(d*x^2
+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/b^(
5/2)/d^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.21 (sec) , antiderivative size = 318, normalized size of antiderivative = 0.77

$$\int \frac{x^2(e+fx^2)^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$
$$= \frac{-\sqrt{\frac{b}{a}}dfx(a+bx^2)(c+dx^2)(4adf+b(-10de+4cf-3dfx^2))-ic(8a^2d^2f^2+abdf(-20de+7cf)+b^2}{}$$

input
```
Integrate[(x^2*(e + f*x^2)^2)/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
(-(Sqrt[b/a]*d*f*x*(a + b*x^2)*(c + d*x^2)*(4*a*d*f + b*(-10*d*e + 4*c*f -
 3*d*f*x^2))) - I*c*(8*a^2*d^2*f^2 + a*b*d*f*(-20*d*e + 7*c*f) + b^2*(15*d
^2*e^2 - 20*c*d*e*f + 8*c^2*f^2))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*
EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + I*c*(4*a^2*d^2*f^2 + a*b*
d*f*(-10*d*e + 3*c*f) + b^2*(15*d^2*e^2 - 20*c*d*e*f + 8*c^2*f^2))*Sqrt[1
+ (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(
b*c)])/(15*a^2*(b/a)^(5/2)*d^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (warning: unable to verify)

Time = 1.09 (sec) , antiderivative size = 638, normalized size of antiderivative = 1.55, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.343, Rules used = {448, 444, 406, 320, 388, 313, 444, 25, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \left(e+f x^2\right)^2}{\sqrt{a+b x^2} \sqrt{c+d x^2}} \, dx$$

$$\Big\downarrow \text{ 448}$$

$$\frac{f \int \frac{x^4 \left(f x^2+e\right)}{\sqrt{b x^2+a} \sqrt{d x^2+c}}dx}{e^2}+e \int \frac{x^2 \left(f x^2+e\right)}{\sqrt{b x^2+a} \sqrt{d x^2+c}}dx$$

$$\Big\downarrow \text{ 444}$$

$$\frac{f \left(\frac{f x^3 \sqrt{a+b x^2} \sqrt{c+d x^2}}{5 b d}-\frac{\int \frac{x^2 \left(3 a c f-(5 b d e-4 b c f-4 a d f) x^2\right)}{\sqrt{b x^2+a} \sqrt{d x^2+c}}dx}{5 b d}\right)}{e^2}+e \left(\frac{f x \sqrt{a+b x^2} \sqrt{c+d x^2}}{3 b d}-\frac{\int \frac{a c f-(3 b d e-2 b c f-2 a d f) x^2}{\sqrt{b x^2+a} \sqrt{d x^2+c}}dx}{3 b d}\right)$$

$$\Big\downarrow \text{ 406}$$

$$\frac{f \left(\frac{f x^3 \sqrt{a+b x^2} \sqrt{c+d x^2}}{5 b d}-\frac{\int \frac{x^2 \left(3 a c f-(5 b d e-4 b c f-4 a d f) x^2\right)}{\sqrt{b x^2+a} \sqrt{d x^2+c}}dx}{5 b d}\right)}{e^2}+e \left(\frac{f x \sqrt{a+b x^2} \sqrt{c+d x^2}}{3 b d}-\frac{a c f \int \frac{1}{\sqrt{b x^2+a} \sqrt{d x^2+c}}dx-(-2 a d f-2 b c f+3 b d e) \int \frac{x^2}{\sqrt{b x^2+a} \sqrt{d x^2+c}}dx}{3 b d}\right)$$

$$\Big\downarrow \text{ 320}$$

$$e\left(\frac{fx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3bd}-\frac{\frac{c^{3/2}f\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-(-2adf-2bcf+3bde)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3bd}\right)$$

$$\frac{f\left(\frac{fx^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5bd}-\frac{\int\frac{x^2\left(3acf-(5bde-4bcf-4adf)x^2\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{5bd}\right)}{e^2}$$

$\downarrow$ 388

$$e\left(\frac{fx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3bd}-\frac{\frac{c^{3/2}f\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-(-2adf-2bcf+3bde)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{b\ldots}}{(dx^2+\ldots}}{b\ldots}\right.}{3bd}\right.$$

$$\frac{f\left(\frac{fx^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5bd}-\frac{\int\frac{x^2\left(3acf-(5bde-4bcf-4adf)x^2\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{5bd}\right)}{e^2}$$

$\downarrow$ 313

$$\frac{f\left(\frac{fx^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5bd}-\frac{\int\frac{x^2\left(3acf-(5bde-4bcf-4adf)x^2\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{5bd}\right)}{e^2}+$$

$$e\left(\frac{fx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3bd}-\frac{\frac{c^{3/2}f\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-(-2adf-2bcf+3bde)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+b\ldots}}{b\sqrt{\ldots}}\right.}{3bd}\right.$$

$\downarrow$ 444

$$\frac{f\left(\frac{fx^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5bd}-\frac{-\frac{\int -\frac{ac(5bde-4bcf-4adf)-\left(-2c(5de-4cf)b^2-ad(10de-7cf)b+8a^2d^2f\right)x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3bd}-\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(-4adf-4bcf+5bde)}{3bd}}{5bd}\right)}{e^2}-$$

$$e\left(\frac{fx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3bd}-\frac{\frac{c^{3/2}f\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-(-2adf-2bcf+3bde)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+b}}{b\sqrt{}}\right.}{3bd}\right.$$

$\downarrow$ 25

$$\frac{f\left(\frac{fx^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5bd}-\frac{\frac{\int \frac{ac(5bde-4bcf-4adf)-\left(-2c(5de-4cf)b^2-ad(10de-7cf)b+8a^2d^2f\right)x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3bd}-\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(-4adf-4bcf+5bde)}{3bd}}{5bd}\right)}{e^2}+$$

$$e\left(\frac{fx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3bd}-\frac{\frac{c^{3/2}f\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-(-2adf-2bcf+3bde)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+b}}{b\sqrt{}}\right.}{3bd}\right.$$

$\downarrow$ 406

$$\frac{f\left(\frac{fx^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5bd}-\frac{\frac{ac(-4adf-4bcf+5bde)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx-\left(8a^2d^2f-abd(10de-7cf)-2b^2c(5de-4cf)\right)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3bd}-\frac{x\sqrt{a+bx^2}}{}}{5bd}\right.}{e^2}$$

$$e\left(\frac{fx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3bd}-\frac{\frac{c^{3/2}f\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-(-2adf-2bcf+3bde)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+b}}{b\sqrt{}}\right.}{3bd}\right.$$

$\downarrow$ 320

$$\frac{f\left(\frac{fx^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5bd}-\frac{\frac{\frac{c^{3/2}\sqrt{a+bx^2}(-4adf-4bcf+5bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\left(8a^2d^2f-abd(10de-7cf)-2b^2c(5de-4cf)\right)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{\cdots}}\cdots}{3bd}}{5bd}\cdots\right)}{e^2}\cdots$$

$$e\left(\frac{fx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3bd}-\frac{\frac{c^{3/2}f\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-(-2adf-2bcf+3bde)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+b\cdots}}{b\sqrt{\cdots}}\cdots\right)}{3bd}\cdots\right)$$

$\downarrow$ 388

$$\frac{f\left(\frac{fx^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5bd}-\frac{\frac{\frac{c^{3/2}\sqrt{a+bx^2}(-4adf-4bcf+5bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\left(8a^2d^2f-abd(10de-7cf)-2b^2c(5de-4cf)\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\cdots\right)}{3bd}}{5bd}\cdots\right)}{e^2}\cdots$$

$$e\left(\frac{fx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3bd}-\frac{\frac{c^{3/2}f\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-(-2adf-2bcf+3bde)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+b\cdots}}{b\sqrt{\cdots}}\cdots\right)}{3bd}\cdots\right)$$

$\downarrow$ 313

$$f\left(\frac{fx^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5bd}-\frac{\frac{\frac{c^{3/2}\sqrt{a+bx^2}(-4adf-4bcf+5bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\left(8a^2d^2f-abd(10de-7cf)-2b^2c(5de-4cf)\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}\right.}{3bd}}{5bd}\right.$$

$$e^2$$

$$e\left(\frac{fx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3bd}-\frac{\frac{c^{3/2}f\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-(-2adf-2bcf+3bde)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+b}}{b\sqrt{}}\right.}{3bd}\right.$$

input

```
Int[(x^2*(e + f*x^2)^2)/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

output

```
e*((f*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(3*b*d) - (-((3*b*d*e - 2*b*c*f -
 2*a*d*f)*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x
^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sq
rt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))) + (c^(3/2)*f*Sqrt[a
 + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d
]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))/(3*b*d)) + (f*((
f*x^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(5*b*d) - (-1/3*((5*b*d*e - 4*b*c*f
 - 4*a*d*f)*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(b*d) + (-((8*a^2*d^2*f - a
*b*d*(10*d*e - 7*c*f) - 2*b^2*c*(5*d*e - 4*c*f))*((x*Sqrt[a + b*x^2])/(b*S
qrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sq
rt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]
*Sqrt[c + d*x^2]))) + (c^(3/2)*(5*b*d*e - 4*b*c*f - 4*a*d*f)*Sqrt[a + b*x^
2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[
(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))/(3*b*d))/(5*b*d)))/e^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

rule 448
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[e   Int[(g*x)^m*(a + b*x
^2)^p*(c + d*x^2)^q*(e + f*x^2)^(r - 1), x], x] + Simp[f/e^2   Int[(g*x)^(m
 + 2)*(a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^(r - 1), x], x] /; FreeQ[{a,
b, c, d, e, f, g, m, p, q}, x] && IGtQ[r, 0]
```

Maple [A] (verified)

Time = 8.53 (sec) , antiderivative size = 436, normalized size of antiderivative = 1.06

method	result
elliptic	$\frac{\sqrt{(b\,x^2+a)(x^2d+c)}\left(\frac{f^2x^3\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{5bd}+\frac{\left(2ef-\frac{f^2(4ad+4bc)}{5bd}\right)x\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{3bd}-\frac{\left(2ef-\frac{f^2(4ad+4bc)}{5bd}\right)ac\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{\cdots}}{3bd\sqrt{-\frac{b}{a}}\,\sqrt{bd\cdots}}\cdots\right.}{\cdots}$
risch	$-\frac{fx(-3bdf\,x^2+4adf+4bcf-10bde)\sqrt{b\,x^2+a}\,\sqrt{x^2d+c}}{15b^2d^2}+\frac{\left(-\frac{(8a^2d^2f^2+7abcd\,f^2-20ab\,d^2ef+8b^2c^2f^2-20b^2cdef+15b^2d^2e^2)c\sqrt{1+\frac{b\,x^2}{a}}\cdots}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\cdots}}\cdots\right.}{\cdots}$
default	Expression too large to display

input
```
int(x^2*(f*x^2+e)^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOS
E)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/5*f^2/b/d*x
^3*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+1/3*(2*e*f-1/5*f^2/b/d*(4*a*d+4*b*c
))/b/d*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)-1/3*(2*e*f-1/5*f^2/b/d*(4*a*d
+4*b*c))/b/d*a*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4
+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1
/2))-(e^2-3/5*a/b*c/d*f^2-1/3*(2*e*f-1/5*f^2/b/d*(4*a*d+4*b*c))/b/d*(2*a*d
+2*b*c))*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x
^2+b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2)
)-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 368, normalized size of antiderivative = 0.90

$$\int \frac{x^2(e+fx^2)^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = -\frac{(15\,b^2cd^2e^2-20\,(b^2c^2d+abcd^2)ef+(8\,b^2c^3+7\,abc^2d+8\,a^2cd^2)f^2)\sqrt{bd}x\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ad}{bc}}{}$$

input
```
integrate(x^2*(f*x^2+e)^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fr
icas")
```

output
```
-1/15*((15*b^2*c*d^2*e^2 - 20*(b^2*c^2*d + a*b*c*d^2)*e*f + (8*b^2*c^3 + 7
*a*b*c^2*d + 8*a^2*c*d^2)*f^2)*sqrt(b*d)*x*sqrt(-c/d)*elliptic_e(arcsin(sq
rt(-c/d)/x), a*d/(b*c)) - (15*b^2*c*d^2*e^2 - 10*(2*b^2*c^2*d + 2*a*b*c*d^
2 + a*b*d^3)*e*f + (8*b^2*c^3 + 7*a*b*c^2*d + 4*a^2*d^3 + 4*(2*a^2 + a*b)*
c*d^2)*f^2)*sqrt(b*d)*x*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)/x), a*d/(b
*c)) - (3*b^2*d^3*f^2*x^4 + 15*b^2*d^3*e^2 - 20*(b^2*c*d^2 + a*b*d^3)*e*f
+ (8*b^2*c^2*d + 7*a*b*c*d^2 + 8*a^2*d^3)*f^2 + 2*(5*b^2*d^3*e*f - 2*(b^2*
c*d^2 + a*b*d^3)*f^2)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(b^3*d^4*x)
```

Sympy [F]

$$\int \frac{x^2(e+fx^2)^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{x^2(e+fx^2)^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

input
```
integrate(x**2*(f*x**2+e)**2/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(x**2*(e + f*x**2)**2/(sqrt(a + b*x**2)*sqrt(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{x^2(e+fx^2)^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{(fx^2+e)^2x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}\,dx$$

input
```
integrate(x^2*(f*x^2+e)^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate((f*x^2 + e)^2*x^2/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

Giac [F]

$$\int \frac{x^2(e+fx^2)^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{(fx^2+e)^2x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}\,dx$$

input
```
integrate(x^2*(f*x^2+e)^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate((f*x^2 + e)^2*x^2/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(e+fx^2)^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{x^2\,(f\,x^2+e)^2}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int((x^2*(e + f*x^2)^2)/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int((x^2*(e + f*x^2)^2)/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^2(e+fx^2)^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$
$$= \frac{-4\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,ad\,f^2x-4\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,bc\,f^2x+10\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,bdefx+3\sqrt{d\,x^2}}{}$$

input
```
int(x^2*(f*x^2+e)^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

output
```
( - 4*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*d*f**2*x - 4*sqrt(c + d*x**2)*sq
rt(a + b*x**2)*b*c*f**2*x + 10*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b*d*e*f*x
 + 3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b*d*f**2*x**3 + 8*int((sqrt(c + d*x
**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a**2
*d**2*f**2 + 7*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**
2 + b*c*x**2 + b*d*x**4),x)*a*b*c*d*f**2 - 20*int((sqrt(c + d*x**2)*sqrt(a
 + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a*b*d**2*e*f +
8*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2
+ b*d*x**4),x)*b**2*c**2*f**2 - 20*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*
x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*b**2*c*d*e*f + 15*int((sqr
t(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4
),x)*b**2*d**2*e**2 + 4*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c + a*d
*x**2 + b*c*x**2 + b*d*x**4),x)*a**2*c*d*f**2 + 4*int((sqrt(c + d*x**2)*sq
rt(a + b*x**2))/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a*b*c**2*f**2 -
10*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c + a*d*x**2 + b*c*x**2 + b*
d*x**4),x)*a*b*c*d*e*f)/(15*b**2*d**2)
```

3.169 $\int \frac{(e+fx^2)^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 302

$$\begin{aligned}\int \frac{(e+fx^2)^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx &= \frac{2f(3bde-bcf-adf)x\sqrt{c+dx^2}}{3bd^2\sqrt{a+bx^2}}+\frac{f^2x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3bd}\\ &\quad-\frac{2\sqrt{a}f(3bde-bcf-adf)\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{3b^{3/2}d^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}\\ &\quad+\frac{\sqrt{a}(3bde^2-acf^2)\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{3b^{3/2}cd\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}\end{aligned}$$

output
```
2/3*f*(-a*d*f-b*c*f+3*b*d*e)*x*(d*x^2+c)^(1/2)/b/d^2/(b*x^2+a)^(1/2)+1/3*f
^2*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b/d-2/3*a^(1/2)*f*(-a*d*f-b*c*f+3*b*d
*e)*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b
/c)^(1/2))/b^(3/2)/d^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)+1/3
*a^(1/2)*(-a*c*f^2+3*b*d*e^2)*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/
2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/b^(3/2)/c/d/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/
c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.18 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.78

$$\int \frac{(e+fx^2)^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \frac{\sqrt{\frac{b}{a}}df^2x(a+bx^2)(c+dx^2)+2icf(-3bde+bcf+adf)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(i\operatorname{arcsinh}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)-i(}{3b\sqrt{\frac{b}{a}}d^2\sqrt{a+bx^2}\sqrt{c-}}$$

input
```
Integrate[(e + f*x^2)^2/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
(Sqrt[b/a]*d*f^2*x*(a + b*x^2)*(c + d*x^2) + (2*I)*c*f*(-3*b*d*e + b*c*f +
 a*d*f)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b
/a]*x], (a*d)/(b*c)] - I*(a*c*d*f^2 + b*(3*d^2*e^2 - 6*c*d*e*f + 2*c^2*f^2
))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x
], (a*d)/(b*c)])/(3*b*Sqrt[b/a]*d^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 483, normalized size of antiderivative = 1.60, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx^2)^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

$$\Big\downarrow \ 433$$

$$\int \left(\frac{e^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}+\frac{2efx^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}+\frac{f^2x^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{2\sqrt{c}f^2\sqrt{a+bx^2}(ad+bc)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3b^2d^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{c^{3/2}f^2\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3bd^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{\sqrt{c}e^2\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{2\sqrt{c}ef\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{2f^2x\sqrt{a+bx^2}(ad+bc)}{3b^2d\sqrt{c+dx^2}}+\frac{2efx\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}+\frac{f^2x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3bd}$$

input
```
Int[(e + f*x^2)^2/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
(2*e*f*x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (2*(b*c + a*d)*f^2*x*Sqrt[
a + b*x^2])/(3*b^2*d*Sqrt[c + d*x^2]) + (f^2*x*Sqrt[a + b*x^2]*Sqrt[c + d*
x^2])/(3*b*d) - (2*Sqrt[c]*e*f*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x
)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^
2))]*Sqrt[c + d*x^2]) + (2*Sqrt[c]*(b*c + a*d)*f^2*Sqrt[a + b*x^2]*Ellipti
cE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*b^2*d^(3/2)*Sqrt[(c*(
a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (Sqrt[c]*e^2*Sqrt[a + b*x^
2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*Sqr
t[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) - (c^(3/2)*f^2*Sqrt[a
+ b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*b*d^(
3/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Defintions of rubi rules used

rule 433
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)
^q*(e + f*x^2)^r, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, f, p,
 q, r}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 6.15 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.08

method	result
elliptic	$\frac{\sqrt{(b\,x^2+a)(x^2d+c)}\left(\frac{f^2x\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{3bd}+\frac{\left(e^2-\frac{ac\,f^2}{3bd}\right)\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}-\left(2ef-\frac{f^2(2ad+2b}{3bd}\right.\right.}{\sqrt{b\,x^2+a}\,\sqrt{x^2d+c}}$
risch	$\frac{f^2x\sqrt{b\,x^2+a}\,\sqrt{x^2d+c}}{3bd}-\frac{\left(-\frac{2f(adf+bcf-3bde)c\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\left(\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)-\mathrm{EllipticE}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}\,d}\right.}{}$
default	$\frac{\left(\sqrt{-\frac{b}{a}}\,b\,d^2f^2x^5+\sqrt{-\frac{b}{a}}\,a\,d^2f^2x^3+\sqrt{-\frac{b}{a}}\,bcd\,f^2x^3+\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{x^2d+c}{c}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)acd\,f^2+2\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{x^2d+c}{c}}\,\mathrm{Elli}\right.}{}$

input
```
int((f*x^2+e)^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/3*f^2/b/d*x
*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+(e^2-1/3*a/b*c/d*f^2)/(-b/a)^(1/2)*(1
+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*Elli
pticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-(2*e*f-1/3*f^2/b/d*(2*a*d+2
*b*c))*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2
+b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-
EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.74

$$\int \frac{(e+fx^2)^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = -\frac{2\,(3\,bc^2def-(bc^3+ac^2d)f^2)\sqrt{bd}x\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ad}{bc})-(3\,bd^3e^2+6\,bc^2def-(2\,bc^3+2\,ac^2}{}$$

input
```
integrate((f*x^2+e)^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas
")
```

output
```
-1/3*(2*(3*b*c^2*d*e*f - (b*c^3 + a*c^2*d)*f^2)*sqrt(b*d)*x*sqrt(-c/d)*ell
iptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - (3*b*d^3*e^2 + 6*b*c^2*d*e*f -
(2*b*c^3 + 2*a*c^2*d + a*c*d^2)*f^2)*sqrt(b*d)*x*sqrt(-c/d)*elliptic_f(arc
sin(sqrt(-c/d)/x), a*d/(b*c)) - (b*c*d^2*f^2*x^2 + 6*b*c*d^2*e*f - 2*(b*c^
2*d + a*c*d^2)*f^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(b^2*c*d^3*x)
```

Sympy [F]

$$\int \frac{(e+fx^2)^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{(e+fx^2)^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

input
```
integrate((f*x**2+e)**2/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral((e + f*x**2)**2/(sqrt(a + b*x**2)*sqrt(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{(e+fx^2)^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{(fx^2+e)^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}\,dx$$

input
```
integrate((f*x^2+e)^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima
")
```

output
```
integrate((f*x^2 + e)^2/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

Giac [F]

$$\int \frac{(e+fx^2)^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{(fx^2+e)^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}\,dx$$

input
```
integrate((f*x^2+e)^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((f*x^2 + e)^2/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx^2)^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{(f\,x^2+e)^2}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int((e + f*x^2)^2/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int((e + f*x^2)^2/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{(e+fx^2)^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,f^2x-2\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^2}{bd\,x^4+ad\,x^2+bc\,x^2+ac}dx\right)ad\,f^2-2\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^2}{bd\,x^4+ad\,x^2+bc\,x^2+ac}dx\right)bc\,f^2+6\left(\int \frac{\sqrt{d}}{bd\,x}\right.}{3bd}$$

input
```
int((f*x^2+e)^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

output
```
(sqrt(c + d*x**2)*sqrt(a + b*x**2)*f**2*x - 2*int((sqrt(c + d*x**2)*sqrt(a
 + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a*d*f**2 - 2*in
t((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*
d*x**4),x)*b*c*f**2 + 6*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c
+ a*d*x**2 + b*c*x**2 + b*d*x**4),x)*b*d*e*f - int((sqrt(c + d*x**2)*sqrt(
a + b*x**2))/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a*c*f**2 + 3*int((s
qrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x
)*b*d*e**2)/(3*b*d)
```

3.170 $\int \frac{(e+fx^2)^2}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 274

$$\int \frac{(e+fx^2)^2}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \frac{\left(\frac{be^2}{ac}+\frac{f^2}{d}\right)x\sqrt{c+dx^2}}{\sqrt{a+bx^2}} - \frac{e^2\sqrt{a+bx^2}\sqrt{c+dx^2}}{acx} - \frac{(bde^2+acf^2)\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{b}cd\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} + \frac{2\sqrt{a}ef\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}c\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
(b*e^2/a/c+f^2/d)*x*(d*x^2+c)^(1/2)/(b*x^2+a)^(1/2)-e^2*(b*x^2+a)^(1/2)*(d
*x^2+c)^(1/2)/a/c/x-(a*c*f^2+b*d*e^2)*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/
a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/a^(1/2)/b^(1/2)/c/d/(b*x^2+a)
^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)+2*a^(1/2)*e*f*(d*x^2+c)^(1/2)*Inver
seJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/b^(1/2)/c/(b*x^2+a
)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.76 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.82

$$\int \frac{(e+fx^2)^2}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \frac{\sqrt{\frac{b}{a}}\left(-\sqrt{\frac{b}{a}}de^2(a+bx^2)(c+dx^2)-ic(bde^2+acf^2)x\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(i\text{arcsinh}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)+ic(b\right.}{bcdx\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

input
```
Integrate[(e + f*x^2)^2/(x^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
(Sqrt[b/a]*(-(Sqrt[b/a]*d*e^2*(a + b*x^2)*(c + d*x^2)) - I*c*(b*d*e^2 + a*
c*f^2)*x*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[
b/a]*x], (a*d)/(b*c)] + I*c*(b*d*e^2 + a*f*(-2*d*e + c*f))*x*Sqrt[1 + (b*x
^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])
)/(b*c*d*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (warning: unable to verify)

Time = 0.76 (sec) , antiderivative size = 463, normalized size of antiderivative = 1.69, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.314$, Rules used = {448, 406, 320, 388, 313, 445, 25, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx^2)^2}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

$$\Big\downarrow \ 448$$

$$\frac{f\int \frac{fx^2+e}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{e^2}+e\int \frac{fx^2+e}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx$$

$$\Big\downarrow \ 406$$

$$\frac{f\left(e\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+f\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{e^2}+e\int \frac{fx^2+e}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx$$

$$\Big\downarrow \text{ 320}$$

$$\frac{f\left(f\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{\sqrt{c}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{e^2}+e\int \frac{fx^2+e}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx$$

$$\Big\downarrow \text{ 388}$$

$$\frac{f\left(f\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{\sqrt{c}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{e^2}+e\int \frac{fx^2+e}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx$$

$$\Big\downarrow \text{ 313}$$

$$e\int \frac{fx^2+e}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{f\left(\frac{\sqrt{c}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+f\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)\right)}{e^2}$$

$$\Big\downarrow \text{ 445}$$

$$e\left(-\frac{\int -\frac{bdex^2+acf}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{acx}\right)+\frac{f\left(\frac{\sqrt{c}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+f\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)\right)}{e^2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{e\left(\frac{\int \frac{bdex^2+acf}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{acx}\right)+f\left(\frac{\sqrt{c}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+f\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)\right)}{e^2}$$

$$\Big\downarrow \text{406}$$

$$\frac{e\left(\frac{bde\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+acf\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{acx}\right)+f\left(\frac{\sqrt{c}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+f\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)\right)}{e^2}$$

$$\Big\downarrow \text{320}$$

$$\frac{e\left(\frac{bde\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}f\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}}{ac}-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{acx}\right)+f\left(\frac{\sqrt{c}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+f\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)\right)}{e^2}$$

$$\Big\downarrow \text{388}$$

$$\frac{e\left(\frac{bde\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}}dx}{b}\right)+\frac{c^{3/2}f\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}}{ac}-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{acx}\right)+f\left(\frac{\sqrt{c}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+f\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)\right)}{e^2}$$

$$\Big\downarrow \text{313}$$

$$e\left(\frac{\frac{c^{3/2}f\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+bde\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{ac}-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{acx}\right)$$

$$\frac{f\left(\frac{\sqrt{c}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+f\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)\right)}{e^2}$$

input
```
Int[(e + f*x^2)^2/(x^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
(f*(f*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*
EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(
c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (Sqrt[c]*e*Sqrt[a + b*
x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*S
qrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])))/e^2 + e*(-((e*Sqrt
[a + b*x^2]*Sqrt[c + d*x^2])/(a*c*x)) + (b*d*e*((x*Sqrt[a + b*x^2])/(b*Sqr
t[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt
[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*S
qrt[c + d*x^2])) + (c^(3/2)*f*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)
/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))
]*Sqrt[c + d*x^2]))/(a*c))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 448
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[e   Int[(g*x)^m*(a + b*x
^2)^p*(c + d*x^2)^q*(e + f*x^2)^(r - 1), x], x] + Simp[f/e^2   Int[(g*x)^(m
 + 2)*(a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^(r - 1), x], x] /; FreeQ[{a,
b, c, d, e, f, g, m, p, q}, x] && IGtQ[r, 0]
```

Maple [A] (verified)

Time = 5.98 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.11

method	result
risch	$-\frac{e^2\sqrt{b\,x^2+a}\,\sqrt{x^2d+c}}{acx}+\frac{\Big(-\frac{(ac\,f^2+bd\,e^2)c\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,(\text{EllipticF}(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}})-\text{EllipticE}(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}))}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}\,d}+}{ac\sqrt{b\,x^2+a}\,\sqrt{x^2d+c}}$
elliptic	$\frac{\sqrt{(b\,x^2+a)(x^2d+c)}\,\Big(-\frac{e^2\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{acx}+\frac{2ef\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\text{EllipticF}(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}})}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}-\frac{(f^2+\frac{bd\,e^2}{ac})c\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1}}{}}{\sqrt{b\,x^2+a}\,\sqrt{x^2d+c}}$
default	$\frac{\Big(-\sqrt{-\frac{b}{a}}\,b\,d^2e^2x^4-\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{x^2d+c}{c}}\,\text{EllipticF}(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}})a\,c^2f^2x+2\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{x^2d+c}{c}}\,\text{EllipticF}(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}})acdefx-}{}$

input
```
int((f*x^2+e)^2/x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOS
E)
```

output
```
-e^2*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a/c/x+1/a/c*(-(a*c*f^2+b*d*e^2)*c/(-b
/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c
)^(1/2)/d*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*
(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2)))+2*a*c*e*f/(-b/a)^(1/2)*(1+b*x^2/a)
^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(
-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2)))*((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+
a)^(1/2)/(d*x^2+c)^(1/2)
```

Fricas [F]

$$\int \frac{(e+fx^2)^2}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx=\int \frac{(fx^2+e)^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}\,x^2}\,dx$$

input
```
integrate((f*x^2+e)^2/x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fr
icas")
```

output
```
integral((f^2*x^4 + 2*e*f*x^2 + e^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/(b*d*
x^6 + (b*c + a*d)*x^4 + a*c*x^2), x)
```

Sympy [F]

$$\int \frac{(e+fx^2)^2}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx=\int \frac{(e+fx^2)^2}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

input
```
integrate((f*x**2+e)**2/x**2/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral((e + f*x**2)**2/(x**2*sqrt(a + b*x**2)*sqrt(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{(e+fx^2)^2}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx=\int \frac{(fx^2+e)^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}x^2}\,dx$$

input
```
integrate((f*x^2+e)^2/x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate((f*x^2 + e)^2/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x^2), x)
```

Giac [F]

$$\int \frac{(e+fx^2)^2}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx=\int \frac{(fx^2+e)^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}x^2}\,dx$$

input
```
integrate((f*x^2+e)^2/x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate((f*x^2 + e)^2/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx^2)^2}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{(f\,x^2+e)^2}{x^2\,\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int((e + f*x^2)^2/(x^2*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int((e + f*x^2)^2/(x^2*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{(e+fx^2)^2}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,f^2+\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{bd\,x^6+ad\,x^4+bc\,x^4+ac\,x^2}dx\right)ac\,f^2x+\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{bd\,x^6+ad\,x^4+bc\,x^4+ac\,x^2}dx\right)bd\,e^2x+2\left(\int \cdots\right.}{bdx}$$

input
```
int((f*x^2+e)^2/x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

output
```
(sqrt(c + d*x**2)*sqrt(a + b*x**2)*f**2 + int((sqrt(c + d*x**2)*sqrt(a + b
*x**2))/(a*c*x**2 + a*d*x**4 + b*c*x**4 + b*d*x**6),x)*a*c*f**2*x + int((s
qrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c*x**2 + a*d*x**4 + b*c*x**4 + b*d*x*
*6),x)*b*d*e**2*x + 2*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c + a*d*x
**2 + b*c*x**2 + b*d*x**4),x)*b*d*e*f*x)/(b*d*x)
```

3.171 $\int \frac{(e+fx^2)^2}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 298

$$\begin{aligned}\int \frac{(e+fx^2)^2}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx &= \frac{2e(bce+ade-3acf)\sqrt{c+dx^2}}{3ac^2x\sqrt{a+bx^2}} - \frac{e^2\sqrt{a+bx^2}\sqrt{c+dx^2}}{3acx^3} \\ &\quad + \frac{2\sqrt{b}e(bce+ade-3acf)\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{3a^{3/2}c^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} \\ &\quad - \frac{(bde^2-3acf^2)\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{3\sqrt{a}\sqrt{b}c^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}\end{aligned}$$

output

```
2/3*e*(-3*a*c*f+a*d*e+b*c*e)*(d*x^2+c)^(1/2)/a/c^2/x/(b*x^2+a)^(1/2)-1/3*e
^2*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a/c/x^3+2/3*b^(1/2)*e*(-3*a*c*f+a*d*e+b
*c*e)*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d
/b/c)^(1/2))/a^(3/2)/c^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)-1
/3*(-3*a*c*f^2+b*d*e^2)*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a
^(1/2)),(1-a*d/b/c)^(1/2))/a^(1/2)/b^(1/2)/c^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c
)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.02 (sec) , antiderivative size = 272, normalized size of antiderivative = 0.91

$$\int \frac{(e+fx^2)^2}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$
$$= \frac{\sqrt{\frac{b}{a}}e(a+bx^2)(c+dx^2)(2bcex^2+2adex^2-ac(e+6fx^2))-2ibce(-bce-ade+3acf)x^3\sqrt{1+\frac{bx^2}{a}}\sqrt{1}}{3a^2\sqrt{}}$$

input
```
Integrate[(e + f*x^2)^2/(x^4*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
(Sqrt[b/a]*e*(a + b*x^2)*(c + d*x^2)*(2*b*c*e*x^2 + 2*a*d*e*x^2 - a*c*(e +
 6*f*x^2)) - (2*I)*b*c*e*(-(b*c*e) - a*d*e + 3*a*c*f)*x^3*Sqrt[1 + (b*x^2)
/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I
*c*(2*b^2*c*e^2 + 3*a^2*c*f^2 + a*b*e*(d*e - 6*c*f))*x^3*Sqrt[1 + (b*x^2)/
a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(3*
a^2*Sqrt[b/a]*c^2*x^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (warning: unable to verify)

Time = 1.04 (sec) , antiderivative size = 581, normalized size of antiderivative = 1.95, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {448, 445, 25, 406, 320, 388, 313, 445, 25, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx^2)^2}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

$$\Big\downarrow \text{448}$$

$$\frac{f\int \frac{fx^2+e}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{e^2}+e\int \frac{fx^2+e}{x^4\sqrt{bx^2+a}\sqrt{dx^2+c}}dx$$

$$\Big\downarrow \text{445}$$

$$\frac{f\left(-\frac{\int -\frac{bdex^2+acf}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{acx}\right)}{e^2}+$$
$$e\left(-\frac{\int \frac{bdex^2+2bce+2ade-3acf}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3ac}-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{3acx^3}\right)$$

$\downarrow$ 25

$$\frac{f\left(\frac{\int \frac{bdex^2+acf}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{acx}\right)}{e^2}+$$
$$e\left(-\frac{\int \frac{bdex^2+2bce+2ade-3acf}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3ac}-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{3acx^3}\right)$$

$\downarrow$ 406

$$\frac{f\left(\frac{bde\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+acf\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{acx}\right)}{e^2}+$$
$$e\left(-\frac{\int \frac{bdex^2+2bce+2ade-3acf}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3ac}-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{3acx^3}\right)$$

$\downarrow$ 320

$$\frac{f\left(\frac{bde\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}f\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{ac}-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{acx}\right)}{e^2}+$$
$$e\left(-\frac{\int \frac{bdex^2+2bce+2ade-3acf}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3ac}-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{3acx^3}\right)$$

$\downarrow$ 388

$$\frac{f\left(\frac{bde\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}}dx}{b}\right)+\frac{c^{3/2}f\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}}{ac}-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{acx}\right)}{e^2}+$$

$$e\left(-\frac{\int\frac{bdex^2+2bce+2ade-3acf}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3ac}-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{3acx^3}\right)$$

$\downarrow$ 313

$$e\left(-\frac{\int\frac{bdex^2+2bce+2ade-3acf}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3ac}-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{3acx^3}\right)+$$

$$\frac{f\left(\frac{\frac{c^{3/2}f\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+bde\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{ac}-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{acx}\right)}{e^2}$$

$\downarrow$ 445

$$e\left(-\frac{-\frac{\int-\frac{bd\left((2bce+2ade-3acf)x^2+ace\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(-3acf+2ade+2bce)}{acx}}{3ac}-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{3acx^3}\right)+$$

$$\frac{f\left(\frac{\frac{c^{3/2}f\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+bde\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{ac}-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{acx}\right)}{e^2}$$

$\downarrow$ 25

$$\frac{e\left(-\frac{\frac{\int \frac{bd\left((2bce+2ade-3acf)x^2+ace\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(-3acf+2ade+2bce)}{acx}}{3ac}-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{3acx^3}\right)+f\left(\frac{\frac{c^{3/2}f\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+bde\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{ac}-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{acx}\right)}{e^2}$$

$\Big\downarrow$ 27

$$\frac{e\left(-\frac{\frac{bd\int \frac{(2bce+2ade-3acf)x^2+ace}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(-3acf+2ade+2bce)}{acx}}{3ac}-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{3acx^3}\right)+f\left(\frac{\frac{c^{3/2}f\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+bde\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{ac}-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{acx}\right)}{e^2}$$

$\Big\downarrow$ 406

$$\frac{e\left(-\frac{\frac{bd\left((-3acf+2ade+2bce)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+ace\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(-3acf+2ade+2bce)}{acx}}{3ac}-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{3acx^3}\right)+f\left(\frac{\frac{c^{3/2}f\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+bde\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{ac}-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{acx}\right)}{e^2}$$

$\Big\downarrow$ 320

$$\frac{e\left(-\frac{\frac{bd\left((-3acf+2ade+2bce)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(-3acf+2ade+2bce)}{acx}}{3ac}\right.}{}$$

$$\frac{f\left(\frac{\frac{c^{3/2}f\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+bde\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{ac}-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{acx}\right)}{e^2}$$

$\downarrow$ 388

$$\frac{e\left(-\frac{\frac{bd\left((-3acf+2ade+2bce)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}}dx}{b}\right)+\frac{c^{3/2}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(-3acf+}{acx}}{3ac}\right.}{}$$

$$\frac{f\left(\frac{\frac{c^{3/2}f\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+bde\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{ac}-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{acx}\right)}{e^2}$$

$\downarrow$ 313

$$\frac{f\left(\frac{\frac{c^{3/2}f\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+bde\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{ac}-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{acx}\right)}{e^2}+$$

$$e\left(-\frac{\frac{bd\left(\frac{c^{3/2}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+(-3acf+2ade+2bce)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)\right)}{ac}-\frac{\sqrt{a+bx^2}\sqrt{\cdots}}{\cdots}}{3ac}\right.$$

input
```
Int[(e + f*x^2)^2/(x^4*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
(f*(-((e*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(a*c*x)) + (b*d*e*((x*Sqrt[a + b
*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sq
rt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c
 + d*x^2))]*Sqrt[c + d*x^2])) + (c^(3/2)*f*Sqrt[a + b*x^2]*EllipticF[ArcTa
n[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a
*(c + d*x^2))]*Sqrt[c + d*x^2]))/(a*c)))/e^2 + e*(-1/3*(e*Sqrt[a + b*x^2]*
Sqrt[c + d*x^2])/(a*c*x^3) - (-(((2*b*c*e + 2*a*d*e - 3*a*c*f)*Sqrt[a + b*
x^2]*Sqrt[c + d*x^2])/(a*c*x)) + (b*d*((2*b*c*e + 2*a*d*e - 3*a*c*f)*((x*S
qrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[A
rcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^
2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (c^(3/2)*e*Sqrt[a + b*x^2]*Ellipt
icF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b
*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])))/(a*c))/(3*a*c))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 448
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[e   Int[(g*x)^m*(a + b*x
^2)^p*(c + d*x^2)^q*(e + f*x^2)^(r - 1), x], x] + Simp[f/e^2   Int[(g*x)^(m
 + 2)*(a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^(r - 1), x], x] /; FreeQ[{a,
b, c, d, e, f, g, m, p, q}, x] && IGtQ[r, 0]
```

Maple [A] (verified)

Time = 7.62 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.26

method	result
elliptic	$\frac{\sqrt{(b\,x^2+a)(x^2d+c)}\left(-\frac{e^2\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{3ac\,x^3}-\frac{2e(3acf-ade-bce)\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{3a^2c^2x}+\frac{\left(f^2-\frac{bd\,e^2}{3ac}\right)\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\ \mathrm{Elliptic}}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+x}}\right.}{\sqrt{b\,x^2+a}}$
risch	$-\frac{\sqrt{b\,x^2+a}\,\sqrt{x^2d+c}\,e(6acf\,x^2-2ade\,x^2-2bce\,x^2+ace)}{3a^2c^2x^3}+\frac{\left(-\frac{2be(3acf-ade-bce)c\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\left(\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+b}{cb}}\right.\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}\right.}{}$
default	$\frac{\left(-6\sqrt{-\frac{b}{a}}\,abcdef\,x^6+2\sqrt{-\frac{b}{a}}\,ab\,d^2e^2x^6+2\sqrt{-\frac{b}{a}}\,b^2cd\,e^2x^6+3\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{x^2d+c}{c}}\ \mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)a^2c^2f^2x^3-6\sqrt{\frac{b\,x^2+a}{a}}\right.}{}$

input
```
int((f*x^2+e)^2/x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOS
E)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-1/3*e^2/a/c*
(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/x^3-2/3*e*(3*a*c*f-a*d*e-b*c*e)/a^2/c^
2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/x+(f^2-1/3*b*d*e^2/a/c)/(-b/a)^(1/2)
*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*E
llipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-2/3*b*e*(3*a*c*f-a*d*e-b
*c*e)/a^2/c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*
x^2+b*c*x^2+a*c)^(1/2)*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))
-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.77

$$\int \frac{(e+fx^2)^2}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$
$$= \frac{2\,(3\,ab^2cef-(b^3c+ab^2d)e^2)\sqrt{ac}x^3\sqrt{-\frac{b}{a}}E(\arcsin\left(x\sqrt{-\frac{b}{a}}\right)\mid\frac{ad}{bc})-(6\,ab^2cef+3\,a^3cf^2-(2\,b^3c+(a^2b}{}$$

input
```
integrate((f*x^2+e)^2/x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fr
icas")
```

output
```
1/3*(2*(3*a*b^2*c*e*f - (b^3*c + a*b^2*d)*e^2)*sqrt(a*c)*x^3*sqrt(-b/a)*el
liptic_e(arcsin(x*sqrt(-b/a)), a*d/(b*c)) - (6*a*b^2*c*e*f + 3*a^3*c*f^2 -
 (2*b^3*c + (a^2*b + 2*a*b^2)*d)*e^2)*sqrt(a*c)*x^3*sqrt(-b/a)*elliptic_f(
arcsin(x*sqrt(-b/a)), a*d/(b*c)) - (a^2*b*c*e^2 + 2*(3*a^2*b*c*e*f - (a*b^
2*c + a^2*b*d)*e^2)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(a^3*b*c^2*x^3)
```

Sympy [F]

$$\int \frac{(e+fx^2)^2}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{(e+fx^2)^2}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

input
```
integrate((f*x**2+e)**2/x**4/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral((e + f*x**2)**2/(x**4*sqrt(a + b*x**2)*sqrt(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{(e+fx^2)^2}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{(fx^2+e)^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}x^4}\,dx$$

input
```
integrate((f*x^2+e)^2/x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate((f*x^2 + e)^2/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x^4), x)
```

Giac [F]

$$\int \frac{(e+fx^2)^2}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{(fx^2+e)^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}x^4}\,dx$$

input
```
integrate((f*x^2+e)^2/x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate((f*x^2 + e)^2/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx^2)^2}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{(f\,x^2+e)^2}{x^4\,\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int((e + f*x^2)^2/(x^4*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int((e + f*x^2)^2/(x^4*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{(e+fx^2)^2}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \text{Too large to display}$$

input
```
int((f*x^2+e)^2/x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

output
```
( - sqrt(c + d*x**2)*sqrt(a + b*x**2)*e*f - 3*int((sqrt(c + d*x**2)*sqrt(a
 + b*x**2))/(a**2*c*d*x**4 + a**2*d**2*x**6 + a*b*c**2*x**4 + 2*a*b*c*d*x*
*6 + a*b*d**2*x**8 + b**2*c**2*x**6 + b**2*c*d*x**8),x)*a**2*c*d*e*f*x**3
+ int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a**2*c*d*x**4 + a**2*d**2*x**6
+ a*b*c**2*x**4 + 2*a*b*c*d*x**6 + a*b*d**2*x**8 + b**2*c**2*x**6 + b**2*c
*d*x**8),x)*a**2*d**2*e**2*x**3 - 3*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)
)/(a**2*c*d*x**4 + a**2*d**2*x**6 + a*b*c**2*x**4 + 2*a*b*c*d*x**6 + a*b*d
**2*x**8 + b**2*c**2*x**6 + b**2*c*d*x**8),x)*a*b*c**2*e*f*x**3 + 2*int((s
qrt(c + d*x**2)*sqrt(a + b*x**2))/(a**2*c*d*x**4 + a**2*d**2*x**6 + a*b*c*
*2*x**4 + 2*a*b*c*d*x**6 + a*b*d**2*x**8 + b**2*c**2*x**6 + b**2*c*d*x**8)
,x)*a*b*c*d*e**2*x**3 + int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a**2*c*d*
x**4 + a**2*d**2*x**6 + a*b*c**2*x**4 + 2*a*b*c*d*x**6 + a*b*d**2*x**8 + b
**2*c**2*x**6 + b**2*c*d*x**8),x)*b**2*c**2*e**2*x**3 + int((sqrt(c + d*x*
*2)*sqrt(a + b*x**2))/(a**2*c*d + a**2*d**2*x**2 + a*b*c**2 + 2*a*b*c*d*x*
*2 + a*b*d**2*x**4 + b**2*c**2*x**2 + b**2*c*d*x**4),x)*a**2*d**2*f**2*x**
3 + 2*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a**2*c*d + a**2*d**2*x**2 +
 a*b*c**2 + 2*a*b*c*d*x**2 + a*b*d**2*x**4 + b**2*c**2*x**2 + b**2*c*d*x**
4),x)*a*b*c*d*f**2*x**3 - int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a**2*c*
d + a**2*d**2*x**2 + a*b*c**2 + 2*a*b*c*d*x**2 + a*b*d**2*x**4 + b**2*c**2
*x**2 + b**2*c*d*x**4),x)*a*b*d**2*e*f*x**3 + int((sqrt(c + d*x**2)*sqr...
```

3.172 $\int \frac{(e+fx^2)^2}{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 423

$$\begin{aligned}
&\int \frac{(e+fx^2)^2}{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx \\
&= -\frac{(4(bc+ad)e(2bce+2ade-5acf)-3ac(3bde^2-5acf^2))\sqrt{c+dx^2}}{15a^2c^3x\sqrt{a+bx^2}} \\
&\quad -\frac{e^2\sqrt{a+bx^2}\sqrt{c+dx^2}}{5acx^5}+\frac{2e(2bce+2ade-5acf)\sqrt{a+bx^2}\sqrt{c+dx^2}}{15a^2c^2x^3} \\
&\quad -\frac{\sqrt{b}(8b^2c^2e^2+abce(7de-20cf)+a^2(8d^2e^2-20cdef+15c^2f^2))\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{15a^{5/2}c^3\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} \\
&\quad +\frac{2\sqrt{b}de(2bce+2ade-5acf)\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{15a^{3/2}c^3\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}
\end{aligned}$$

output
```
-1/15*(4*(a*d+b*c)*e*(-5*a*c*f+2*a*d*e+2*b*c*e)-3*a*c*(-5*a*c*f^2+3*b*d*e^
2))*(d*x^2+c)^(1/2)/a^2/c^3/x/(b*x^2+a)^(1/2)-1/5*e^2*(b*x^2+a)^(1/2)*(d*x
^2+c)^(1/2)/a/c/x^5+2/15*e*(-5*a*c*f+2*a*d*e+2*b*c*e)*(b*x^2+a)^(1/2)*(d*x
^2+c)^(1/2)/a^2/c^2/x^3-1/15*b^(1/2)*(8*b^2*c^2*e^2+a*b*c*e*(-20*c*f+7*d*e
)+a^2*(15*c^2*f^2-20*c*d*e*f+8*d^2*e^2))*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)
*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/a^(5/2)/c^3/(b*x^2+a)^(1/2
)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)+2/15*b^(1/2)*d*e*(-5*a*c*f+2*a*d*e+2*b*c
*e)*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^
(1/2))/a^(3/2)/c^3/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.62 (sec) , antiderivative size = 401, normalized size of antiderivative = 0.95

$$\int \frac{(e+fx^2)^2}{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \frac{-\sqrt{\frac{b}{a}}(a+bx^2)(c+dx^2)\left(8b^2c^2e^2x^4+abcex^2(7dex^2-4c(e+5fx^2))+a^2(8d^2e^2x^4-4cdex^2(e+5fx^2)\right.}{}$$

input
```
Integrate[(e + f*x^2)^2/(x^6*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
(-(Sqrt[b/a]*(a + b*x^2)*(c + d*x^2)*(8*b^2*c^2*e^2*x^4 + a*b*c*e*x^2*(7*d
*e*x^2 - 4*c*(e + 5*f*x^2)) + a^2*(8*d^2*e^2*x^4 - 4*c*d*e*x^2*(e + 5*f*x^
2) + c^2*(3*e^2 + 10*e*f*x^2 + 15*f^2*x^4)))) - I*b*c*(8*b^2*c^2*e^2 + a*b
*c*e*(7*d*e - 20*c*f) + a^2*(8*d^2*e^2 - 20*c*d*e*f + 15*c^2*f^2))*x^5*Sqr
t[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*
d)/(b*c)] + I*b*c*(8*b^2*c^2*e^2 + a*b*c*e*(3*d*e - 20*c*f) + a^2*(4*d^2*e
^2 - 10*c*d*e*f + 15*c^2*f^2))*x^5*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]
*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(15*a^3*Sqrt[b/a]*c^3*x^5
*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (warning: unable to verify)

Time = 1.37 (sec) , antiderivative size = 771, normalized size of antiderivative = 1.82, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.457$, Rules used = {448, 445, 445, 25, 27, 406, 320, 388, 313, 445, 25, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(e+fx^2\right)^2}{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \text{448}$$

$$\frac{f\int \frac{fx^2+e}{x^4\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{e^2}+e\int \frac{fx^2+e}{x^6\sqrt{bx^2+a}\sqrt{dx^2+c}}dx$$

$$\Big\downarrow \text{445}$$

$$\frac{f\left(-\frac{\int \frac{bdex^2+2bce+2ade-3acf}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3ac}-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{3acx^3}\right)}{e^2}+e\left(-\frac{\int \frac{3bdex^2+4bce+4ade-5acf}{x^4\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{5ac}-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{5acx^5}\right)$$

$$\Big\downarrow \text{445}$$

$$e\left(-\frac{-\frac{\int \frac{2d(4de-5cf)a^2+bc(7de-10cf)a+bd(4bce+4ade-5acf)x^2+8b^2c^2e}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(-5acf+4ade+4bce)}{3acx^3}}{5ac}-\frac{e\sqrt{a+bx^2}\sqrt{c+d\ldots}}{5acx^5}\right.$$
$$\frac{f\left(-\frac{-\frac{\int -\frac{bd\left((2bce+2ade-3acf)x^2+ace\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(-3acf+2ade+2bce)}{acx}}{3ac}-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{3acx^3}\right)}{e^2}$$

$$\Big\downarrow \text{25}$$

$$e\left(-\frac{-\frac{\int \frac{2d(4de-5cf)a^2+bc(7de-10cf)a+bd(4bce+4ade-5acf)x^2+8b^2c^2e}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(-5acf+4ade+4bce)}{3acx^3}}{5ac}-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{5acx^5}\right)$$

$$\frac{f\left(-\frac{\frac{\int \frac{bd\left((2bce+2ade-3acf)x^2+ace\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(-3acf+2ade+2bce)}{acx}}{3ac}-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{3acx^3}\right)}{e^2}$$

$\downarrow$ 27

$$e\left(-\frac{-\frac{\int \frac{2d(4de-5cf)a^2+bc(7de-10cf)a+bd(4bce+4ade-5acf)x^2+8b^2c^2e}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(-5acf+4ade+4bce)}{3acx^3}}{5ac}-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{5acx^5}\right)$$

$$\frac{f\left(-\frac{\frac{bd\int \frac{(2bce+2ade-3acf)x^2+ace}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(-3acf+2ade+2bce)}{acx}}{3ac}-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{3acx^3}\right)}{e^2}$$

$\downarrow$ 406

$$e\left(-\frac{-\frac{\int \frac{2d(4de-5cf)a^2+bc(7de-10cf)a+bd(4bce+4ade-5acf)x^2+8b^2c^2e}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(-5acf+4ade+4bce)}{3acx^3}}{5ac}-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{5acx^5}\right)$$

$$\frac{f\left(-\frac{\frac{bd\left((-3acf+2ade+2bce)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+ace\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(-3acf+2ade+2bce)}{acx}}{3ac}-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{3acx^3}\right)}{e^2}$$

$\downarrow$ 320

$$\frac{e\left(-\frac{-\frac{\int \frac{2d(4de-5cf)a^2+bc(7de-10cf)a+bd(4bce+4ade-5acf)x^2+8b^2c^2e}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(-5acf+4ade+4bce)}{3acx^3}}{5ac}-\frac{e\sqrt{a+bx^2}\sqrt{c+d}}{5acx^5}\right.}{} \\ \frac{f\left(-\frac{\frac{bd\left((-3acf+2ade+2bce)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}e\sqrt{a+bx^2}\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(-3acf+2ade+2bce)}{acx}}{3ac}-\frac{e\sqrt{a+}}{}\right.}{e^2}$$

$\big\downarrow$ 388

$$\frac{e\left(-\frac{-\frac{\int \frac{2d(4de-5cf)a^2+bc(7de-10cf)a+bd(4bce+4ade-5acf)x^2+8b^2c^2e}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(-5acf+4ade+4bce)}{3acx^3}}{5ac}-\frac{e\sqrt{a+bx^2}\sqrt{c+d}}{5acx^5}\right.}{} \\ \frac{f\left(-\frac{\frac{bd\left((-3acf+2ade+2bce)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}}dx}{b}\right)+\frac{c^{3/2}e\sqrt{a+bx^2}\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(-3acf+2ade+2bc}{acx}}{3ac}\right.}{e^2}$$

$\big\downarrow$ 313

$$\frac{e\left(-\frac{-\frac{\int \frac{2d(4de-5cf)a^2+bc(7de-10cf)a+bd(4bce+4ade-5acf)x^2+8b^2c^2e}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(-5acf+4ade+4bce)}{3acx^3}}{5ac}-\frac{e\sqrt{a+bx^2}\sqrt{c+d}}{5acx^5}\right.}{} \\ \frac{f\left(-\frac{\frac{bd\left(\frac{c^{3/2}e\sqrt{a+bx^2}\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+(-3acf+2ade+2bce)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)\right)}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(}{}}{3ac}\right.}{e^2}$$

$\Big\downarrow$ 445

$$\frac{e\left(-\frac{-\frac{-\frac{\int -\frac{bd\left(\left(2d(4de-5cf)a^2+bc(7de-10cf)a+8b^2c^2e\right)x^2+ac(4bce+4ade-5acf)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{8b^2ce}{a}+\frac{8ad^2e}{c}-10adf-10bcf+7bde\right)}{x}}{3ac}-\frac{\sqrt{a}\cdots}{}}{5ac}\right)}{e^2}$$

$$f\left(-\frac{\frac{bd\left(\frac{c^{3/2}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+(-3acf+2ade+2bce)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)\right)}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(\cdots}{c\cdots}}{3ac}\cdots\right)$$

$\Big\downarrow$ 25

$$\frac{e\left(-\frac{-\frac{\frac{\int \frac{bd\left(\left(2d(4de-5cf)a^2+bc(7de-10cf)a+8b^2c^2e\right)x^2+ac(4bce+4ade-5acf)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{8b^2ce}{a}+\frac{8ad^2e}{c}-10adf-10bcf+7bde\right)}{x}}{3ac}-\frac{\sqrt{a+bx^2}\cdots}{}}{5ac}\right)}{e^2}$$

$$f\left(-\frac{\frac{bd\left(\frac{c^{3/2}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+(-3acf+2ade+2bce)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)\right)}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(\cdots}{c\cdots}}{3ac}\cdots\right)$$

$\Big\downarrow$ 27

$$\frac{e\left(-\frac{-\frac{bd\int \frac{\left(2d(4de-5cf)a^2+bc(7de-10cf)a+8b^2c^2e\right)x^2+ac(4bce+4ade-5acf)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{8b^2ce}{a}+\frac{8ad^2e}{c}-10adf-10bcf+7bde\right)}{x}}{3ac}}{5ac}-\frac{\sqrt{a+bx^2}\cdots}{\cdots}\right)\;f\left(-\frac{\frac{bd\left(\frac{c^{3/2}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+(-3acf+2ade+2bce)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)\right)}{ac}}{3ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\cdots\right.}{c\cdots}\cdots\right)}{e^2}\cdots$$

$\downarrow$ 406

$$\frac{e\left(-\frac{-\frac{bd\left(\left(2a^2d(4de-5cf)+abc(7de-10cf)+8b^2c^2e\right)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+ac(-5acf+4ade+4bce)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{8b^2ce}{a}+\frac{8\cdots}{\cdots}\right.}{x\cdots}}{3ac}}{5ac}\cdots\right)\;f\left(-\frac{\frac{bd\left(\frac{c^{3/2}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+(-3acf+2ade+2bce)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)\right)}{ac}}{3ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\cdots\right.}{c\cdots}\cdots\right)}{e^2}\cdots$$

$\downarrow$ 320

$$\frac{e\left(-\frac{-\frac{bd\left(\left(2a^2d(4de-5cf)+abc(7de-10cf)+8b^2c^2e\right)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}(-5acf+4ade+4bce)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{ac}-\sqrt{a+b\cdots}}{3ac}}{5ac}\cdots\right)\quad f\left(-\frac{\frac{bd\left(\frac{c^{3/2}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+(-3acf+2ade+2bce)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)\right)}{ac}}{3ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(\cdots}{\cdots}\cdots\right)}{e^2}$$

$\downarrow$ 388

$$\frac{e\left(-\frac{-\frac{bd\left(\left(2a^2d(4de-5cf)+abc(7de-10cf)+8b^2c^2e\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(-5acf+4ade+4bce)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\cdots\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\cdots}{ac}\cdots}{3ac}}{5ac}\cdots\right)\quad f\left(-\frac{\frac{bd\left(\frac{c^{3/2}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+(-3acf+2ade+2bce)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)\right)}{ac}}{3ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(\cdots}{\cdots}\cdots\right)}{e^2}$$

$\downarrow$ 313

$$\frac{e\left(-\frac{-\frac{bd\left(\left(2a^2d(4de-5cf)+abc(7de-10cf)+8b^2c^2e\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(-5acf+4ade+4bce)\operatorname{EllipticF}\left(\operatorname{ar}\right.}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}}{ac}}{3ac}}{5ac}\right.}{}$$

$$\frac{f\left(-\frac{\frac{bd\left(\frac{c^{3/2}e\sqrt{a+bx^2}\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+(-3acf+2ade+2bce)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)\right)}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(\cdots}{}}{3ac}\right.}{e^2}$$

input
```
Int[(e + f*x^2)^2/(x^6*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
(f*(-1/3*(e*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(a*c*x^3) - (-(((2*b*c*e + 2*
a*d*e - 3*a*c*f)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(a*c*x)) + (b*d*((2*b*c*
e + 2*a*d*e - 3*a*c*f)*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]
*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/
(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (c^(3
/2)*e*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*
d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])))/(a*
c))/(3*a*c)))/e^2 + e*(-1/5*(e*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(a*c*x^5)
 - (-1/3*((4*b*c*e + 4*a*d*e - 5*a*c*f)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(a
*c*x^3) - (-((((8*b^2*c*e)/a + 7*b*d*e + (8*a*d^2*e)/c - 10*b*c*f - 10*a*d
*f)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/x) + (b*d*((8*b^2*c^2*e + a*b*c*(7*d*
e - 10*c*f) + 2*a^2*d*(4*d*e - 5*c*f))*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*
x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1
 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c +
d*x^2])) + (c^(3/2)*(4*b*c*e + 4*a*d*e - 5*a*c*f)*Sqrt[a + b*x^2]*Elliptic
F[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x
^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])))/(a*c))/(3*a*c))/(5*a*c))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 448
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[e   Int[(g*x)^m*(a + b*x
^2)^p*(c + d*x^2)^q*(e + f*x^2)^(r - 1), x], x] + Simp[f/e^2   Int[(g*x)^(m
 + 2)*(a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^(r - 1), x], x] /; FreeQ[{a,
b, c, d, e, f, g, m, p, q}, x] && IGtQ[r, 0]
```

Maple [A] (verified)

Time = 10.71 (sec) , antiderivative size = 523, normalized size of antiderivative = 1.24

method	result
elliptic	$\frac{\sqrt{(b\,x^2+a)(x^2d+c)}\left(-\frac{e^2\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{5ac\,x^5}-\frac{2(5acf-2ade-2bce)e\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{15a^2c^2x^3}-\frac{\left(15a^2c^2f^2-20a^2cdef+8a^2d^2e^2-20ab\right.}{}\right.}{}$
risch	$-\frac{\sqrt{b\,x^2+a}\,\sqrt{x^2d+c}\,(15f^2x^4a^2c^2-20a^2cdef\,x^4+8a^2d^2e^2x^4-20ab\,c^2ef\,x^4+7abcd\,e^2x^4+8b^2c^2e^2x^4+10a^2c^2ef\,x^2-4a^2cd\,e^2x^2-4a}{15a^3c^3x^5}$
default	Expression too large to display

input
```
int((f*x^2+e)^2/x^6/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOS
E)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-1/5*e^2/a/c*
(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/x^5-2/15*(5*a*c*f-2*a*d*e-2*b*c*e)*e/a
^2/c^2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/x^3-1/15/a^3/c^3*(15*a^2*c^2*f^
2-20*a^2*c*d*e*f+8*a^2*d^2*e^2-20*a*b*c^2*e*f+7*a*b*c*d*e^2+8*b^2*c^2*e^2)
*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/x-2/15*b*d*e*(5*a*c*f-2*a*d*e-2*b*c*e
)/a^2/c^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^
2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-1/
15*(15*a^2*c^2*f^2-20*a^2*c*d*e*f+8*a^2*d^2*e^2-20*a*b*c^2*e*f+7*a*b*c*d*e
^2+8*b^2*c^2*e^2)*b/c^2/a^3/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/
2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+
b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 369, normalized size of antiderivative = 0.87

$$\int \frac{(e+fx^2)^2}{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \frac{(15\,a^2bc^2f^2+(8\,b^3c^2+7\,ab^2cd+8\,a^2bd^2)e^2-20\,(ab^2c^2+a^2bcd)ef)\sqrt{ac}x^5\sqrt{-\frac{b}{a}}E(\arcsin\left(x\sqrt{-\frac{b}{a}}\right)\mid\frac{ad}{bc}}{}$$

input
```
integrate((f*x^2+e)^2/x^6/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fr
icas")
```

output
```
1/15*((15*a^2*b*c^2*f^2 + (8*b^3*c^2 + 7*a*b^2*c*d + 8*a^2*b*d^2)*e^2 - 20
*(a*b^2*c^2 + a^2*b*c*d)*e*f)*sqrt(a*c)*x^5*sqrt(-b/a)*elliptic_e(arcsin(x
*sqrt(-b/a)), a*d/(b*c)) - (15*a^2*b*c^2*f^2 + (8*b^3*c^2 + (4*a^2*b + 7*a
*b^2)*c*d + 4*(a^3 + 2*a^2*b)*d^2)*e^2 - 10*(2*a*b^2*c^2 + (a^3 + 2*a^2*b)
*c*d)*e*f)*sqrt(a*c)*x^5*sqrt(-b/a)*elliptic_f(arcsin(x*sqrt(-b/a)), a*d/(
b*c)) - (3*a^3*c^2*e^2 + (15*a^3*c^2*f^2 + (8*a*b^2*c^2 + 7*a^2*b*c*d + 8*
a^3*d^2)*e^2 - 20*(a^2*b*c^2 + a^3*c*d)*e*f)*x^4 + 2*(5*a^3*c^2*e*f - 2*(a
^2*b*c^2 + a^3*c*d)*e^2)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(a^4*c^3*x^
5)
```

Sympy [F]

$$\int \frac{(e+fx^2)^2}{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{(e+fx^2)^2}{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

input
```
integrate((f*x**2+e)**2/x**6/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral((e + f*x**2)**2/(x**6*sqrt(a + b*x**2)*sqrt(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{(e+fx^2)^2}{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{(fx^2+e)^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}x^6}\,dx$$

input
```
integrate((f*x^2+e)^2/x^6/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate((f*x^2 + e)^2/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x^6), x)
```

Giac [F]

$$\int \frac{(e+fx^2)^2}{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{(fx^2+e)^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}x^6}\,dx$$

input
```
integrate((f*x^2+e)^2/x^6/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate((f*x^2 + e)^2/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx^2)^2}{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{(f\,x^2+e)^2}{x^6\,\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int((e + f*x^2)^2/(x^6*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int((e + f*x^2)^2/(x^6*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{(e+fx^2)^2}{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \text{Too large to display}$$

input
```
int((f*x^2+e)^2/x^6/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

output
```
( - sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*c*e**2 - 5*sqrt(c + d*x**2)*sqrt(a
 + b*x**2)*a*c*f**2*x**4 + 3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b*d*e**2*x*
*4 + 5*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a**2*c*d + a**2*d**2*
x**2 + a*b*c**2 + 2*a*b*c*d*x**2 + a*b*d**2*x**4 + b**2*c**2*x**2 + b**2*c
*d*x**4),x)*a**2*b*c*d**2*f**2*x**5 + 5*int((sqrt(c + d*x**2)*sqrt(a + b*x
**2)*x**2)/(a**2*c*d + a**2*d**2*x**2 + a*b*c**2 + 2*a*b*c*d*x**2 + a*b*d*
*2*x**4 + b**2*c**2*x**2 + b**2*c*d*x**4),x)*a*b**2*c**2*d*f**2*x**5 - 3*i
nt((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a**2*c*d + a**2*d**2*x**2 + a
*b*c**2 + 2*a*b*c*d*x**2 + a*b*d**2*x**4 + b**2*c**2*x**2 + b**2*c*d*x**4)
,x)*a*b**2*d**3*e**2*x**5 - 3*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)
/(a**2*c*d + a**2*d**2*x**2 + a*b*c**2 + 2*a*b*c*d*x**2 + a*b*d**2*x**4 +
b**2*c**2*x**2 + b**2*c*d*x**4),x)*b**3*c*d**2*e**2*x**5 + 10*int((sqrt(c
+ d*x**2)*sqrt(a + b*x**2))/(a**2*c*d*x**4 + a**2*d**2*x**6 + a*b*c**2*x**
4 + 2*a*b*c*d*x**6 + a*b*d**2*x**8 + b**2*c**2*x**6 + b**2*c*d*x**8),x)*a*
*3*c**2*d*e*f*x**5 - 4*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a**2*c*d*x
**4 + a**2*d**2*x**6 + a*b*c**2*x**4 + 2*a*b*c*d*x**6 + a*b*d**2*x**8 + b*
*2*c**2*x**6 + b**2*c*d*x**8),x)*a**3*c*d**2*e**2*x**5 + 10*int((sqrt(c +
d*x**2)*sqrt(a + b*x**2))/(a**2*c*d*x**4 + a**2*d**2*x**6 + a*b*c**2*x**4
+ 2*a*b*c*d*x**6 + a*b*d**2*x**8 + b**2*c**2*x**6 + b**2*c*d*x**8),x)*a**2
*b*c**3*e*f*x**5 - 8*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a**2*c*d*...
```

3.173 $\int \frac{(e+fx^2)^2}{x^8\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 608

$$\int \frac{(e+fx^2)^2}{x^8\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = -\frac{2(9abcde(3bce+3ade-7acf)-(bc+ad)\left(8(bc+ad)e(3bce+3ade-7acf)-5ac(5bde^2-7acf^2)\right))}{105a^3c^4x\sqrt{a+bx^2}}$$

$$-\frac{e^2\sqrt{a+bx^2}\sqrt{c+dx^2}}{7acx^7}+\frac{2e(3bce+3ade-7acf)\sqrt{a+bx^2}\sqrt{c+dx^2}}{35a^2c^2x^5}$$

$$-\frac{\left(8(bc+ad)e(3bce+3ade-7acf)-5ac(5bde^2-7acf^2)\right)\sqrt{a+bx^2}\sqrt{c+dx^2}}{105a^3c^3x^3}$$

$$+\frac{2\sqrt{b}(24b^3c^3e^2+4ab^2c^2e(5de-14cf)+a^3d(24d^2e^2-56cdef+35c^2f^2)+a^2bc(20d^2e^2-49cdef+35}{105a^{7/2}c^4\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

$$-\frac{\sqrt{b}d(24b^2c^2e^2+abce(23de-56cf)+a^2(24d^2e^2-56cdef+35c^2f^2))\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}}{\sqrt{}}\right.\right.}{105a^{5/2}c^4\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
-2/105*(9*a*b*c*d*e*(-7*a*c*f+3*a*d*e+3*b*c*e)-(a*d+b*c)*(8*(a*d+b*c)*e*(-
7*a*c*f+3*a*d*e+3*b*c*e)-5*a*c*(-7*a*c*f^2+5*b*d*e^2)))*(d*x^2+c)^(1/2)/a^
3/c^4/x/(b*x^2+a)^(1/2)-1/7*e^2*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a/c/x^7+2/
35*e*(-7*a*c*f+3*a*d*e+3*b*c*e)*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a^2/c^2/x^
5-1/105*(8*(a*d+b*c)*e*(-7*a*c*f+3*a*d*e+3*b*c*e)-5*a*c*(-7*a*c*f^2+5*b*d*
e^2))*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a^3/c^3/x^3+2/105*b^(1/2)*(24*b^3*c^
3*e^2+4*a*b^2*c^2*e*(-14*c*f+5*d*e)+a^3*d*(35*c^2*f^2-56*c*d*e*f+24*d^2*e^
2)+a^2*b*c*(35*c^2*f^2-49*c*d*e*f+20*d^2*e^2))*(d*x^2+c)^(1/2)*EllipticE(b
^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/a^(7/2)/c^4/(b*x^2+a
)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)-1/105*b^(1/2)*d*(24*b^2*c^2*e^2+a*
b*c*e*(-56*c*f+23*d*e)+a^2*(35*c^2*f^2-56*c*d*e*f+24*d^2*e^2))*(d*x^2+c)^(
1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/a^(5/2)/
c^4/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 9.66 (sec) , antiderivative size = 575, normalized size of antiderivative = 0.95

$$\int \frac{(e+fx^2)^2}{x^8\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$
$$= \frac{\sqrt{\frac{b}{a}}(a+bx^2)(c+dx^2)(48b^3c^3e^2x^6-8ab^2c^2ex^4(3ce-5dex^2+14cfx^2)+a^2bcx^2(40d^2e^2x^4-cdex^2(23e}{}$$

input
```
Integrate[(e + f*x^2)^2/(x^8*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
(Sqrt[b/a]*(a + b*x^2)*(c + d*x^2)*(48*b^3*c^3*e^2*x^6 - 8*a*b^2*c^2*e*x^4
*(3*c*e - 5*d*e*x^2 + 14*c*f*x^2) + a^2*b*c*x^2*(40*d^2*e^2*x^4 - c*d*e*x^
2*(23*e + 98*f*x^2) + 2*c^2*(9*e^2 + 28*e*f*x^2 + 35*f^2*x^4)) + a^3*(48*d
^3*e^2*x^6 - 8*c*d^2*e*x^4*(3*e + 14*f*x^2) + 2*c^2*d*x^2*(9*e^2 + 28*e*f*
x^2 + 35*f^2*x^4) - c^3*(15*e^2 + 42*e*f*x^2 + 35*f^2*x^4))) + (2*I)*b*c*(
24*b^3*c^3*e^2 + 4*a*b^2*c^2*e*(5*d*e - 14*c*f) + a^3*d*(24*d^2*e^2 - 56*c
*d*e*f + 35*c^2*f^2) + a^2*b*c*(20*d^2*e^2 - 49*c*d*e*f + 35*c^2*f^2))*x^7
*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x],
 (a*d)/(b*c)] - I*b*c*(48*b^3*c^3*e^2 - 16*a*b^2*c^2*e*(-(d*e) + 7*c*f) +
a^3*d*(24*d^2*e^2 - 56*c*d*e*f + 35*c^2*f^2) + a^2*b*c*(17*d^2*e^2 - 42*c*
d*e*f + 70*c^2*f^2))*x^7*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF
[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(105*a^4*Sqrt[b/a]*c^4*x^7*Sqrt[a +
 b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (warning: unable to verify)

Time = 1.87 (sec) , antiderivative size = 1038, normalized size of antiderivative = 1.71, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.486$, Rules used = {448, 445, 445, 445, 25, 27, 406, 320, 388, 313, 445, 25, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx^2)^2}{x^8\sqrt{a+bx^2}\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \text{448}$$

$$\frac{f\int \frac{fx^2+e}{x^6\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{e^2}+e\int \frac{fx^2+e}{x^8\sqrt{bx^2+a}\sqrt{dx^2+c}}dx$$

$$\Big\downarrow \text{445}$$

$$\frac{f\left(-\frac{\int \frac{3bdex^2+4bce+4ade-5acf}{x^4\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{5ac}-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{5acx^5}\right)}{e^2}+e\left(-\frac{\int \frac{5bdex^2+6bce+6ade-7acf}{x^6\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{7ac}-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{7acx^7}\right)$$

$$\Big\downarrow \text{445}$$

$$\frac{f\left(-\frac{-\frac{\int \frac{2d(4de-5cf)a^2+bc(7de-10cf)a+bd(4bce+4ade-5acf)x^2+8b^2c^2e}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(-5acf+4ade+4bce)}{3acx^3}}{5ac}-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{5acx^5}\right)}{e^2}+$$

$$e\left(-\frac{-\frac{\int \frac{4d(6de-7cf)a^2+bc(23de-28cf)a+3bd(6bce+6ade-7acf)x^2+24b^2c^2e}{x^4\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{5ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(-7acf+6ade+6bce)}{5acx^5}}{7ac}-\frac{e\sqrt{a+bx^2}\sqrt{c+\cdots}}{7acx^7}\cdots\right.$$

$\Big\downarrow$ 445

$$\frac{f\left(-\frac{-\frac{-\frac{\int -\frac{bd\left(\left(2d(4de-5cf)a^2+bc(7de-10cf)a+8b^2c^2e\right)x^2+ac(4bce+4ade-5acf)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{8b^2ce}{a}+\frac{8ad^2e}{c}-10adf-10bcf+7bde\right)}{x}}{3ac}-\frac{\sqrt{a+b\cdots}}{\cdots}}{5ac}\cdots\right.}{e^2}\cdots$$

$$e\left(-\frac{-\frac{-\frac{\int \frac{8d^2(6de-7cf)a^3+bcd(40de-49cf)a^2+8b^2c^2(5de-7cf)a+bd\left(4d(6de-7cf)a^2+bc(23de-28cf)a+24b^2c^2e\right)x^2+48b^3c^3e}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{24b\cdots}{\cdots}\cdots\right)}{\cdots}}{5ac}\cdots}{7ac}\cdots\right.$$

$\Big\downarrow$ 25

$$\frac{f\left(-\frac{-\frac{-\frac{\int \frac{bd\left(\left(2d(4de-5cf)a^2+bc(7de-10cf)a+8b^2c^2e\right)x^2+ac(4bce+4ade-5acf)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{8b^2ce}{a}+\frac{8ad^2e}{c}-10adf-10bcf+7bde\right)}{x}}{3ac}-\frac{\sqrt{a+bx^2}\cdots}{\cdots}}{5ac}\cdots\right.}{e^2}\cdots$$

$$e\left(-\frac{-\frac{-\frac{\int \frac{8d^2(6de-7cf)a^3+bcd(40de-49cf)a^2+8b^2c^2(5de-7cf)a+bd\left(4d(6de-7cf)a^2+bc(23de-28cf)a+24b^2c^2e\right)x^2+48b^3c^3e}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{24b\cdots}{\cdots}\cdots\right)}{\cdots}}{5ac}\cdots}{7ac}\cdots\right.$$

$\Big\downarrow$ 27

$$\frac{f\left(-\frac{-\frac{-\frac{bd\int\frac{\left(2d(4de-5cf)a^2+bc(7de-10cf)a+8b^2c^2e\right)x^2+ac(4bce+4ade-5acf)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{8b^2ce}{a}+\frac{8ad^2e}{c}-10adf-10bcf+7bde\right)}{x}}{3ac}-\sqrt{a+bx^2}\sqrt{c\cdots}}{5ac}\cdots\right)}{e^2}$$

$$e\left(-\frac{-\frac{-\frac{\int\frac{8d^2(6de-7cf)a^3+bcd(40de-49cf)a^2+8b^2c^2(5de-7cf)a+bd\left(4d(6de-7cf)a^2+bc(23de-28cf)a+24b^2c^2e\right)x^2+48b^3c^3e}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3ac}-\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{24b\cdots}{c\cdots}\cdots\right)}{5ac}\cdots}{7ac}\cdots\right)$$

$\downarrow$ 406

$$\frac{f\left(-\frac{-\frac{-\frac{bd\left(\left(2a^2d(4de-5cf)+abc(7de-10cf)+8b^2c^2e\right)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+ac(-5acf+4ade+4bce)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{8b^2ce}{a}+\frac{8\cdots}{\cdots}\cdots\right)}{x}}{3ac}\cdots}{5ac}\cdots\right)}{e^2}$$

$$e\left(-\frac{-\frac{-\frac{\int\frac{8d^2(6de-7cf)a^3+bcd(40de-49cf)a^2+8b^2c^2(5de-7cf)a+bd\left(4d(6de-7cf)a^2+bc(23de-28cf)a+24b^2c^2e\right)x^2+48b^3c^3e}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3ac}-\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{24b\cdots}{c\cdots}\cdots\right)}{5ac}\cdots}{7ac}\cdots\right)$$

$\downarrow$ 320

$$\frac{f\left(-\frac{-\frac{-\frac{bd\left(\left(2a^2d(4de-5cf)+abc(7de-10cf)+8b^2c^2e\right)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}(-5acf+4ade+4bce)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{ac}-\sqrt{a+b\cdots}}{3ac}\cdots}{5ac}\cdots\right)}{e^2}$$

$$e\left(-\frac{-\frac{-\frac{\int\frac{8d^2(6de-7cf)a^3+bcd(40de-49cf)a^2+8b^2c^2(5de-7cf)a+bd\left(4d(6de-7cf)a^2+bc(23de-28cf)a+24b^2c^2e\right)x^2+48b^3c^3e}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3ac}-\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{24b\cdots}{c\cdots}\cdots\right)}{5ac}\cdots}{7ac}\cdots\right)$$

$\downarrow$ 388

$$\frac{f\left(-\frac{-\frac{bd\left(\left(2a^2d(4de-5cf)+abc(7de-10cf)+8b^2c^2e\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(-5acf+4ade+4bce)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{b}{a}\cdots\right.}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\cdots}{ac}\cdots}{3ac}\cdots}{5ac}\cdots\right)}{e^2}\cdots$$

$$e\left(-\frac{-\frac{-\frac{\int\frac{8d^2(6de-7cf)a^3+bcd(40de-49cf)a^2+8b^2c^2(5de-7cf)a+bd\left(4d(6de-7cf)a^2+bc(23de-28cf)a+24b^2c^2e\right)x^2+48b^3c^3e}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{24b\cdots}{c\cdots}\right.}{\cdots}}{5ac}\cdots}{7ac}\cdots\right.$$

$\Big\downarrow$ 313

$$e\left(-\frac{-\frac{-\frac{\int\frac{8d^2(6de-7cf)a^3+bcd(40de-49cf)a^2+8b^2c^2(5de-7cf)a+bd\left(4d(6de-7cf)a^2+bc(23de-28cf)a+24b^2c^2e\right)x^2+48b^3c^3e}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{24b\cdots}{c\cdots}\right.}{\cdots}}{5ac}\cdots}{7ac}\cdots\right.$$

$$\frac{f\left(-\frac{-\frac{bd\left(\left(2a^2d(4de-5cf)+abc(7de-10cf)+8b^2c^2e\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(-5acf+4ade+4bce)\,\text{EllipticF}\left(a\cdots\right.}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}\cdots}\cdots}{ac}\cdots}{3ac}\cdots}{5ac}\cdots\right)}{e^2}\cdots$$

$\Big\downarrow$ 445

$$\frac{f\left(-\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}e}{5acx^5}-\frac{-\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}(4bce+4ade-5acf)}{3acx^3}-\frac{bd\left(\frac{(4bce+4ade-5acf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\left(2d(4de-5cf\right.\cdots}{a\cdots}}{\cdots}\right.}{e^2\cdots}$$

$$e\left(-\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}e}{7acx^7}-\frac{-\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}(6bce+6ade-7acf)}{5acx^5}-\frac{-\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(\frac{24ceb^2}{a}+(23de-28cf)b+\frac{4ad(6de-7cf)}{c}\right)}{3x^3}-\frac{-\frac{\sqrt{bx\cdots}}{\cdots}}{\cdots}}{\cdots}}{\cdots}\right.$$

$\downarrow$ 25

$$\frac{f\left(-\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}e}{5acx^5}-\frac{-\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}(4bce+4ade-5acf)}{3acx^3}-\frac{bd\left(\frac{(4bce+4ade-5acf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\left(2d(4de-5cf\right.\cdots}{a\cdots}}{\cdots}\right.}{e^2\cdots}$$

$$e\left(-\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}e}{7acx^7}-\frac{-\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}(6bce+6ade-7acf)}{5acx^5}-\frac{-\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(\frac{24ceb^2}{a}+(23de-28cf)b+\frac{4ad(6de-7cf)}{c}\right)}{3x^3}-\frac{\int\frac{bd(\cdots}{\cdots}}{\cdots}}{\cdots}}{\cdots}\right.$$

$\downarrow$ 27

$$f\left(-\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}e}{5acx^5}-\frac{-\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}(4bce+4ade-5acf)}{3acx^3}-\frac{bd\left(\frac{(4bce+4ade-5acf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\left(2d(4de-5cf\right.\right.}{a}}{}\right.$$

$$e^2$$

$$e\left(-\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}e}{7acx^7}-\frac{-\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}(6bce+6ade-7acf)}{5acx^5}-\frac{-\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(\frac{24ceb^2}{a}+(23de-28cf)b+\frac{4ad(6de-7cf)}{c}\right)}{3x^3}-\frac{bd\int\left(\right.}{}}{}}{}\right.$$

$\downarrow$ 406

$$f\left(-\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}e}{5acx^5}-\frac{-\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}(4bce+4ade-5acf)}{3acx^3}-\frac{bd\left(\frac{(4bce+4ade-5acf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\left(2d(4de-5cf\right.\right.}{a}}{}\right.$$

$$e^2$$

$$e\left(-\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}e}{7acx^7}-\frac{-\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}(6bce+6ade-7acf)}{5acx^5}-\frac{-\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(\frac{24ceb^2}{a}+(23de-28cf)b+\frac{4ad(6de-7cf)}{c}\right)}{3x^3}-\frac{bd\left(ac\right.}{}}{}}{}\right.$$

$\downarrow$ 320

$$f\left(-\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}e}{5acx^5}-\frac{-\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}(4bce+4ade-5acf)}{3acx^3}-\frac{bd\left(\frac{(4bce+4ade-5acf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\left(2d(4de-5cf\right.\right.}{a}}{}\right.$$

$$e\left(-\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}e}{7acx^7}-\frac{-\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}(6bce+6ade-7acf)}{5acx^5}-\frac{-\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(\frac{24ceb^2}{a}+(23de-28cf)b+\frac{4ad(6de-7cf)}{c}\right)}{3x^3}-\frac{e^2\ bd\left(\frac{(4}{}\right.}{}}{}\right.$$

$\downarrow$ 388

$$f\left(-\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}e}{5acx^5}-\frac{-\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}(4bce+4ade-5acf)}{3acx^3}-\frac{bd\left(\frac{(4bce+4ade-5acf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\left(2d(4de-5cf\right.\right.}{a}}{}\right.$$

$$e\left(-\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}e}{7acx^7}-\frac{-\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}(6bce+6ade-7acf)}{5acx^5}-\frac{-\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(\frac{24ceb^2}{a}+(23de-28cf)b+\frac{4ad(6de-7cf)}{c}\right)}{3x^3}-\frac{e^2\ bd\left(\frac{(4}{}\right.}{}}{}\right.$$

$\downarrow$ 313

$$f\left(-\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}e}{5acx^5}-\frac{-\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}(4bce+4ade-5acf)}{3acx^3}-\frac{bd\left(\frac{(4bce+4ade-5acf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\left(2d(4de-5cf\right.\right.}{}}{}\right.$$

$$\overline{\qquad e^2}$$

$$e\left(-\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}e}{7acx^7}-\frac{-\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}(6bce+6ade-7acf)}{5acx^5}-\frac{-\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(\frac{24ceb^2}{a}+(23de-28cf)b+\frac{4ad(6de-7cf)}{c}\right)}{3x^3}-\frac{bd\left(\frac{(4}{}\right.}{}}{}}{}\right.$$

input
```
Int[(e + f*x^2)^2/(x^8*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
(f*(-1/5*(e*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(a*c*x^5) - (-1/3*((4*b*c*e +
 4*a*d*e - 5*a*c*f)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(a*c*x^3) - (-((((8*b
^2*c*e)/a + 7*b*d*e + (8*a*d^2*e)/c - 10*b*c*f - 10*a*d*f)*Sqrt[a + b*x^2]
*Sqrt[c + d*x^2])/x) + (b*d*((8*b^2*c^2*e + a*b*c*(7*d*e - 10*c*f) + 2*a^2
*d*(4*d*e - 5*c*f))*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sq
rt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*
Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (c^(3/2)
*(4*b*c*e + 4*a*d*e - 5*a*c*f)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x
)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2)
)]*Sqrt[c + d*x^2])))/(a*c))/(3*a*c))/(5*a*c)))/e^2 + e*(-1/7*(e*Sqrt[a +
b*x^2]*Sqrt[c + d*x^2])/(a*c*x^7) - (-1/5*((6*b*c*e + 6*a*d*e - 7*a*c*f)*S
qrt[a + b*x^2]*Sqrt[c + d*x^2])/(a*c*x^5) - (-1/3*(((24*b^2*c*e)/a + b*(23
*d*e - 28*c*f) + (4*a*d*(6*d*e - 7*c*f))/c)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2
])/x^3 - (-(((48*b^3*c^3*e + a^2*b*c*d*(40*d*e - 49*c*f) + 8*a*b^2*c^2*(5*
d*e - 7*c*f) + 8*a^3*d^2*(6*d*e - 7*c*f))*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
/(a*c*x)) + (b*d*((48*b^3*c^3*e + a^2*b*c*d*(40*d*e - 49*c*f) + 8*a*b^2*c^
2*(5*d*e - 7*c*f) + 8*a^3*d^2*(6*d*e - 7*c*f))*((x*Sqrt[a + b*x^2])/(b*Sqr
t[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt
[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*S
qrt[c + d*x^2])) + (c^(3/2)*(24*b^2*c^2*e + a*b*c*(23*d*e - 28*c*f) + 4...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 448
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[e   Int[(g*x)^m*(a + b*x
^2)^p*(c + d*x^2)^q*(e + f*x^2)^(r - 1), x], x] + Simp[f/e^2   Int[(g*x)^(m
 + 2)*(a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^(r - 1), x], x] /; FreeQ[{a,
b, c, d, e, f, g, m, p, q}, x] && IGtQ[r, 0]
```

Maple [A] (verified)

Time = 19.59 (sec) , antiderivative size = 755, normalized size of antiderivative = 1.24

method	result
elliptic	$\frac{\sqrt{(b x^2+a)(x^2 d+c)}\left(-\frac{e^2\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{7ac\,x^7}-\frac{2e(7acf-3ade-3bce)\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{35a^2c^2x^5}-\frac{(35a^2c^2f^2-56a^2cdef+24a^2d^2e^2-56a}{}\right)}{}$
risch	Expression too large to display
default	Expression too large to display

input
```
int((f*x^2+e)^2/x^8/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOS
E)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-1/7/a/c*e^2*
(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/x^7-2/35*e*(7*a*c*f-3*a*d*e-3*b*c*e)/a
^2/c^2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/x^5-1/105/a^3/c^3*(35*a^2*c^2*f
^2-56*a^2*c*d*e*f+24*a^2*d^2*e^2-56*a*b*c^2*e*f+23*a*b*c*d*e^2+24*b^2*c^2*
e^2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/x^3+2/105*(35*a^3*c^2*d*f^2-56*a^
3*c*d^2*e*f+24*a^3*d^3*e^2+35*a^2*b*c^3*f^2-49*a^2*b*c^2*d*e*f+20*a^2*b*c*
d^2*e^2-56*a*b^2*c^3*e*f+20*a*b^2*c^2*d*e^2+24*b^3*c^3*e^2)/c^4/a^4*(b*d*x
^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/x-1/105*b*d*(35*a^2*c^2*f^2-56*a^2*c*d*e*f+2
4*a^2*d^2*e^2-56*a*b*c^2*e*f+23*a*b*c*d*e^2+24*b^2*c^2*e^2)/c^3/a^3/(-b/a)
^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(
1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+2/105*b*(35*a^3*c^
2*d*f^2-56*a^3*c*d^2*e*f+24*a^3*d^3*e^2+35*a^2*b*c^3*f^2-49*a^2*b*c^2*d*e*
f+20*a^2*b*c*d^2*e^2-56*a*b^2*c^3*e*f+20*a*b^2*c^2*d*e^2+24*b^3*c^3*e^2)/c
^3/a^4/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b
*c*x^2+a*c)^(1/2)*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-Elli
pticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 598, normalized size of antiderivative = 0.98

$$\int \frac{(e+fx^2)^2}{x^8\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx =$$
$$-\frac{2\,(4\,(6\,b^4c^3+5\,ab^3c^2d+5\,a^2b^2cd^2+6\,a^3bd^3)e^2-7\,(8\,ab^3c^3+7\,a^2b^2c^2d+8\,a^3bcd^2)ef+35\,(a^2b^2c^3+a}{}$$

input
```
integrate((f*x^2+e)^2/x^8/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fr
icas")
```

output
```
-1/105*(2*(4*(6*b^4*c^3 + 5*a*b^3*c^2*d + 5*a^2*b^2*c*d^2 + 6*a^3*b*d^3)*e
^2 - 7*(8*a*b^3*c^3 + 7*a^2*b^2*c^2*d + 8*a^3*b*c*d^2)*e*f + 35*(a^2*b^2*c
^3 + a^3*b*c^2*d)*f^2)*sqrt(a*c)*x^7*sqrt(-b/a)*elliptic_e(arcsin(x*sqrt(-
b/a)), a*d/(b*c)) - ((48*b^4*c^3 + 8*(3*a^2*b^2 + 5*a*b^3)*c^2*d + (23*a^3
*b + 40*a^2*b^2)*c*d^2 + 24*(a^4 + 2*a^3*b)*d^3)*e^2 - 14*(8*a*b^3*c^3 + (
4*a^3*b + 7*a^2*b^2)*c^2*d + 4*(a^4 + 2*a^3*b)*c*d^2)*e*f + 35*(2*a^2*b^2*
c^3 + (a^4 + 2*a^3*b)*c^2*d)*f^2)*sqrt(a*c)*x^7*sqrt(-b/a)*elliptic_f(arcs
in(x*sqrt(-b/a)), a*d/(b*c)) + (15*a^4*c^3*e^2 - 2*(4*(6*a*b^3*c^3 + 5*a^2
*b^2*c^2*d + 5*a^3*b*c*d^2 + 6*a^4*d^3)*e^2 - 7*(8*a^2*b^2*c^3 + 7*a^3*b*c
^2*d + 8*a^4*c*d^2)*e*f + 35*(a^3*b*c^3 + a^4*c^2*d)*f^2)*x^6 + (35*a^4*c^
3*f^2 + (24*a^2*b^2*c^3 + 23*a^3*b*c^2*d + 24*a^4*c*d^2)*e^2 - 56*(a^3*b*c
^3 + a^4*c^2*d)*e*f)*x^4 + 6*(7*a^4*c^3*e*f - 3*(a^3*b*c^3 + a^4*c^2*d)*e^
2)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(a^5*c^4*x^7)
```

Sympy [F]

$$\int \frac{(e+fx^2)^2}{x^8\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{(e+fx^2)^2}{x^8\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

input
```
integrate((f*x**2+e)**2/x**8/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral((e + f*x**2)**2/(x**8*sqrt(a + b*x**2)*sqrt(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{(e+fx^2)^2}{x^8\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx=\int \frac{(fx^2+e)^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}x^8}\,dx$$

input
```
integrate((f*x^2+e)^2/x^8/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate((f*x^2 + e)^2/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x^8), x)
```

Giac [F]

$$\int \frac{(e+fx^2)^2}{x^8\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx=\int \frac{(fx^2+e)^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}x^8}\,dx$$

input
```
integrate((f*x^2+e)^2/x^8/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate((f*x^2 + e)^2/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x^8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx^2)^2}{x^8\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx=\int \frac{(f\,x^2+e)^2}{x^8\,\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int((e + f*x^2)^2/(x^8*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int((e + f*x^2)^2/(x^8*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{(e+fx^2)^2}{x^8\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \text{Too large to display}$$

input
```
int((f*x^2+e)^2/x^8/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

output
```
( - sqrt(c + d*x**2)*sqrt(a + b*x**2)*e**2 + 14*int((sqrt(c + d*x**2)*sqrt
(a + b*x**2))/(a**2*c*d*x**6 + a**2*d**2*x**8 + a*b*c**2*x**6 + 2*a*b*c*d*
x**8 + a*b*d**2*x**10 + b**2*c**2*x**8 + b**2*c*d*x**10),x)*a**2*c*d*e*f*x
**7 - 6*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a**2*c*d*x**6 + a**2*d**2
*x**8 + a*b*c**2*x**6 + 2*a*b*c*d*x**8 + a*b*d**2*x**10 + b**2*c**2*x**8 +
 b**2*c*d*x**10),x)*a**2*d**2*e**2*x**7 + 14*int((sqrt(c + d*x**2)*sqrt(a
+ b*x**2))/(a**2*c*d*x**6 + a**2*d**2*x**8 + a*b*c**2*x**6 + 2*a*b*c*d*x**
8 + a*b*d**2*x**10 + b**2*c**2*x**8 + b**2*c*d*x**10),x)*a*b*c**2*e*f*x**7
 - 12*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a**2*c*d*x**6 + a**2*d**2*x
**8 + a*b*c**2*x**6 + 2*a*b*c*d*x**8 + a*b*d**2*x**10 + b**2*c**2*x**8 + b
**2*c*d*x**10),x)*a*b*c*d*e**2*x**7 - 6*int((sqrt(c + d*x**2)*sqrt(a + b*x
**2))/(a**2*c*d*x**6 + a**2*d**2*x**8 + a*b*c**2*x**6 + 2*a*b*c*d*x**8 + a
*b*d**2*x**10 + b**2*c**2*x**8 + b**2*c*d*x**10),x)*b**2*c**2*e**2*x**7 +
7*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a**2*c*d*x**4 + a**2*d**2*x**6
+ a*b*c**2*x**4 + 2*a*b*c*d*x**6 + a*b*d**2*x**8 + b**2*c**2*x**6 + b**2*c
*d*x**8),x)*a**2*c*d*f**2*x**7 + 7*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))
/(a**2*c*d*x**4 + a**2*d**2*x**6 + a*b*c**2*x**4 + 2*a*b*c*d*x**6 + a*b*d*
*2*x**8 + b**2*c**2*x**6 + b**2*c*d*x**8),x)*a*b*c**2*f**2*x**7 - 5*int((s
qrt(c + d*x**2)*sqrt(a + b*x**2))/(a**2*c*d*x**4 + a**2*d**2*x**6 + a*b*c*
*2*x**4 + 2*a*b*c*d*x**6 + a*b*d**2*x**8 + b**2*c**2*x**6 + b**2*c*d*x*...
```

3.174 $\int \frac{x^6(e+fx^2)}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 524

$$\begin{aligned}
\int \frac{x^6(e+fx^2)}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx =
&-\frac{(48a^3d^3f-ab^2cd(49de-40cf)-8b^3c^2(7de-6cf)-8a^2bd^2(7de-5cf))\,x\sqrt{c+dx^2}}{105b^3d^4\sqrt{a+bx^2}} \\
&-\frac{(25abcdf+4(bc+ad)(7bde-6bcf-6adf))x\sqrt{a+bx^2}\sqrt{c+dx^2}}{105b^3d^3} \\
&+\frac{(7bde-6bcf-6adf)x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{35b^2d^2}+\frac{fx^5\sqrt{a+bx^2}\sqrt{c+dx^2}}{7bd} \\
&+\frac{\sqrt{a}(48a^3d^3f-ab^2cd(49de-40cf)-8b^3c^2(7de-6cf)-8a^2bd^2(7de-5cf))\,\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{b}}{\sqrt{c}}\right.\right.}{105b^{7/2}d^4\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} \\
&+\frac{a^{3/2}(25abcdf+4(bc+ad)(7bde-6bcf-6adf))\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{105b^{7/2}d^3\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}
\end{aligned}$$

output
```
-1/105*(48*a^3*d^3*f-a*b^2*c*d*(-40*c*f+49*d*e)-8*b^3*c^2*(-6*c*f+7*d*e)-8
*a^2*b*d^2*(-5*c*f+7*d*e))*x*(d*x^2+c)^(1/2)/b^3/d^4/(b*x^2+a)^(1/2)-1/105
*(25*a*b*c*d*f+4*(a*d+b*c)*(-6*a*d*f-6*b*c*f+7*b*d*e))*x*(b*x^2+a)^(1/2)*(
d*x^2+c)^(1/2)/b^3/d^3+1/35*(-6*a*d*f-6*b*c*f+7*b*d*e)*x^3*(b*x^2+a)^(1/2)
*(d*x^2+c)^(1/2)/b^2/d^2+1/7*f*x^5*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b/d+1/1
05*a^(1/2)*(48*a^3*d^3*f-a*b^2*c*d*(-40*c*f+49*d*e)-8*b^3*c^2*(-6*c*f+7*d*
e)-8*a^2*b*d^2*(-5*c*f+7*d*e))*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)
/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/b^(7/2)/d^4/(b*x^2+a)^(1/2)/(a*(d*x^
2+c)/c/(b*x^2+a))^(1/2)+1/105*a^(3/2)*(25*a*b*c*d*f+4*(a*d+b*c)*(-6*a*d*f-
6*b*c*f+7*b*d*e))*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)
),(1-a*d/b/c)^(1/2))/b^(7/2)/d^3/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))
^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 7.40 (sec) , antiderivative size = 382, normalized size of antiderivative = 0.73

$$\int \frac{x^6(e+fx^2)}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \frac{\sqrt{\frac{b}{a}}dx(a+bx^2)\left(c+dx^2\right)\left(24a^2d^2f+abd(23cf-2d(14e+9fx^2))+b^2(24c^2f+3d^2x^2(7e+5fx^2)-2c\right.}{}$$

input
```
Integrate[(x^6*(e + f*x^2))/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
(Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2)*(24*a^2*d^2*f + a*b*d*(23*c*f - 2*d
*(14*e + 9*f*x^2)) + b^2*(24*c^2*f + 3*d^2*x^2*(7*e + 5*f*x^2) - 2*c*d*(14
*e + 9*f*x^2))) + I*c*(48*a^3*d^3*f - 8*a^2*b*d^2*(7*d*e - 5*c*f) + 8*b^3*
c^2*(-7*d*e + 6*c*f) + a*b^2*c*d*(-49*d*e + 40*c*f))*Sqrt[1 + (b*x^2)/a]*S
qrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*c*(2
4*a^3*d^3*f + 8*b^3*c^2*(-7*d*e + 6*c*f) + a*b^2*c*d*(-21*d*e + 16*c*f) +
a^2*b*d^2*(-28*d*e + 17*c*f))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Elli
pticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(105*b^3*Sqrt[b/a]*d^4*Sqrt[a
+ b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 489, normalized size of antiderivative = 0.93, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {444, 444, 25, 444, 25, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6\left(e+fx^2\right)}{\sqrt{a+bx^2}\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \ \textbf{444}$$

$$\frac{fx^5\sqrt{a+bx^2}\sqrt{c+dx^2}}{7bd}-\frac{\int \frac{x^4\left(5acf-(7bde-6bcf-6adf)x^2\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{7bd}$$

$$\Big\downarrow \ \textbf{444}$$

$$\frac{fx^5\sqrt{a+bx^2}\sqrt{c+dx^2}}{7bd}-\frac{-\frac{\int -\frac{x^2\left(3ac(7bde-6bcf-6adf)-\left(-4c(7de-6cf)b^2-ad(28de-23cf)b+24a^2d^2f\right)x^2\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{5bd}-\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}(-6adf-6bcf+7bde)}{5bd}}{7bd}$$

$$\Big\downarrow \ \textbf{25}$$

$$\frac{fx^5\sqrt{a+bx^2}\sqrt{c+dx^2}}{7bd}-\frac{\frac{\int \frac{x^2\left(3ac(7bde-6bcf-6adf)-\left(-4c(7de-6cf)b^2-ad(28de-23cf)b+24a^2d^2f\right)x^2\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{5bd}-\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}(-6adf-6bcf+7bde)}{5bd}}{7bd}$$

$$\Big\downarrow \ \textbf{444}$$

$$\frac{fx^5\sqrt{a+bx^2}\sqrt{c+dx^2}}{7bd}-\frac{\frac{-\frac{\int -\frac{\left(-8c^2(7de-6cf)b^3-acd(49de-40cf)b^2-8a^2d^2(7de-5cf)b+48a^3d^3f\right)x^2+ac\left(-4c(7de-6cf)b^2-ad(28de-23cf)b+24a^2d^2f\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3bd}-\frac{1}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}(}{5bd}}{7bd}$$

$$\Big\downarrow \ \textbf{25}$$

$$\frac{fx^5\sqrt{a+bx^2}\sqrt{c+dx^2}}{7bd} - \frac{\frac{\frac{\int \frac{\left(-8c^2(7de-6cf)b^3-acd(49de-40cf)b^2-8a^2d^2(7de-5cf)b+48a^3d^3f\right)x^2+ac\left(-4c(7de-6cf)b^2-ad(28de-23cf)b+24a^2d^2f\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3bd}-\frac{1}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(24a\right.}{5bd}}{7bd}$$

$\big\downarrow$ 406

$$\frac{fx^5\sqrt{a+bx^2}\sqrt{c+dx^2}}{7bd} - \frac{\frac{\frac{ac\left(24a^2d^2f-abd(28de-23cf)-4b^2c(7de-6cf)\right)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\left(48a^3d^3f-8a^2bd^2(7de-5cf)-ab^2cd(49de-40cf)-8b^3c^2(7de-6cf)\right)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+}}}{3bd}}{5bd}}{7bd}$$

$\big\downarrow$ 320

$$\frac{fx^5\sqrt{a+bx^2}\sqrt{c+dx^2}}{7bd} - \frac{\frac{\frac{\left(48a^3d^3f-8a^2bd^2(7de-5cf)-ab^2cd(49de-40cf)-8b^3c^2(7de-6cf)\right)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}\left(24a^2d^2f-abd(28de-23cf)-4b^2c(7de-6cf)\right)\text{Ellipti}}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{3bd}}{5bd}}{7bd}$$

$\big\downarrow$ 388

$$\frac{fx^5\sqrt{a+bx^2}\sqrt{c+dx^2}}{7bd} - \frac{\frac{\frac{\left(48a^3d^3f-8a^2bd^2(7de-5cf)-ab^2cd(49de-40cf)-8b^3c^2(7de-6cf)\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}\left(24a^2d^2f-abd(28de-23cf)-4b^2c(7de-\right.}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+\right.}{a\left(c+\right.}}}}{3bd}}{5bd}}{7}$$

$\big\downarrow$ 313

$$\frac{fx^5\sqrt{a+bx^2}\sqrt{c+dx^2}}{7bd} - \frac{\frac{\frac{\frac{c^{3/2}\sqrt{a+bx^2}\left(24a^2d^2f-abd(28de-23cf)-4b^2c(7de-6cf)\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+\left(48a^3d^3f-8a^2bd^2(7de-5cf)-ab^2cd(49de-40cf)-8b^3c^2(7de-6\right.}{3bd}}{5bd}}{}$$

input
```
Int[(x^6*(e + f*x^2))/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
(f*x^5*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(7*b*d) - (-1/5*((7*b*d*e - 6*b*c*
f - 6*a*d*f)*x^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(b*d) + (-1/3*(((24*a^2*
d*f)/b - a*(28*d*e - 23*c*f) - (4*b*c*(7*d*e - 6*c*f))/d)*x*Sqrt[a + b*x^2
]*Sqrt[c + d*x^2]) + ((48*a^3*d^3*f - a*b^2*c*d*(49*d*e - 40*c*f) - 8*b^3*
c^2*(7*d*e - 6*c*f) - 8*a^2*b*d^2*(7*d*e - 5*c*f))*((x*Sqrt[a + b*x^2])/(b
*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/
Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2)
)]*Sqrt[c + d*x^2])) + (c^(3/2)*(24*a^2*d^2*f - a*b*d*(28*d*e - 23*c*f) -
4*b^2*c*(7*d*e - 6*c*f))*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt
[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqr
t[c + d*x^2]))/(3*b*d))/(5*b*d))/(7*b*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

Maple [A] (verified)

Time = 10.67 (sec) , antiderivative size = 575, normalized size of antiderivative = 1.10

method	result
elliptic	$\frac{\sqrt{(b\,x^2+a)(x^2d+c)}\left(\frac{f\,x^5\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{7bd}+\frac{\left(e-\frac{f(6ad+6bc)}{7bd}\right)x^3\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{5bd}+\frac{\left(-\frac{5acf}{7bd}-\frac{\left(e-\frac{f(6ad+6bc)}{7bd}\right)(4ad+4bc)}{5bd}\right.}{3bd}\right.}{}$
risch	$\frac{x(15f\,x^4b^2d^2-18ab\,d^2f\,x^2-18b^2cf\,x^2d+21b^2d^2e\,x^2+24f\,d^2a^2+23fdcba-28ab\,d^2e+24f\,c^2b^2-28d\,b^2ce)\sqrt{b\,x^2+a}\,\sqrt{x^2d+c}}{105b^3d^3}-$
default	Expression too large to display

input
```
int(x^6*(f*x^2+e)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/7/b/d*f*x^5
*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+1/5*(e-1/7/b/d*f*(6*a*d+6*b*c))/b/d*x
^3*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+1/3*(-5/7*a/b*c/d*f-1/5*(e-1/7/b/d*
f*(6*a*d+6*b*c))/b/d*(4*a*d+4*b*c))/b/d*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1
/2)-1/3*(-5/7*a/b*c/d*f-1/5*(e-1/7/b/d*f*(6*a*d+6*b*c))/b/d*(4*a*d+4*b*c))
/b/d*a*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2
+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-(-3
/5*(e-1/7/b/d*f*(6*a*d+6*b*c))/b/d*a*c-1/3*(-5/7*a/b*c/d*f-1/5*(e-1/7/b/d*
f*(6*a*d+6*b*c))/b/d*(4*a*d+4*b*c))/b/d*(2*a*d+2*b*c))*c/(-b/a)^(1/2)*(1+b
*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*(Ell
ipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(
-1+(a*d+b*c)/c/b)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 494, normalized size of antiderivative = 0.94

$$\int \frac{x^6(e+fx^2)}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx =$$

$$-\frac{\sqrt{bd}(7\,(8\,b^3c^3d+7\,ab^2c^2d^2+8\,a^2bcd^3)e-8\,(6\,b^3c^4+5\,ab^2c^3d+5\,a^2bc^2d^2+6\,a^3cd^3)\,f)x\sqrt{-\frac{c}{d}}E(\arcsin}{}$$

input
```
integrate(x^6*(f*x^2+e)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fric
as")
```

output
```
-1/105*(sqrt(b*d)*(7*(8*b^3*c^3*d + 7*a*b^2*c^2*d^2 + 8*a^2*b*c*d^3)*e - 8
*(6*b^3*c^4 + 5*a*b^2*c^3*d + 5*a^2*b*c^2*d^2 + 6*a^3*c*d^3)*f)*x*sqrt(-c/
d)*elliptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - sqrt(b*d)*(7*(8*b^3*c^3*d
 + 7*a*b^2*c^2*d^2 + 4*a^2*b*d^4 + 4*(2*a^2*b + a*b^2)*c*d^3)*e - (48*b^3*
c^4 + 40*a*b^2*c^3*d + 24*a^3*d^4 + 8*(5*a^2*b + 3*a*b^2)*c^2*d^2 + (48*a^
3 + 23*a^2*b)*c*d^3)*f)*x*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)/x), a*d/
(b*c)) - (15*b^3*d^4*f*x^6 + 3*(7*b^3*d^4*e - 6*(b^3*c*d^3 + a*b^2*d^4)*f)
*x^4 - (28*(b^3*c*d^3 + a*b^2*d^4)*e - (24*b^3*c^2*d^2 + 23*a*b^2*c*d^3 +
24*a^2*b*d^4)*f)*x^2 + 7*(8*b^3*c^2*d^2 + 7*a*b^2*c*d^3 + 8*a^2*b*d^4)*e -
 8*(6*b^3*c^3*d + 5*a*b^2*c^2*d^2 + 5*a^2*b*c*d^3 + 6*a^3*d^4)*f)*sqrt(b*x
^2 + a)*sqrt(d*x^2 + c))/(b^4*d^5*x)
```

Sympy [F]

$$\int \frac{x^6(e+fx^2)}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{x^6(e+fx^2)}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

input
```
integrate(x**6*(f*x**2+e)/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(x**6*(e + f*x**2)/(sqrt(a + b*x**2)*sqrt(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{x^6(e+fx^2)}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{(fx^2+e)x^6}{\sqrt{bx^2+a}\sqrt{dx^2+c}}\,dx$$

input
```
integrate(x^6*(f*x^2+e)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate((f*x^2 + e)*x^6/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

Giac [F]

$$\int \frac{x^6(e+fx^2)}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{(fx^2+e)x^6}{\sqrt{bx^2+a}\sqrt{dx^2+c}}\,dx$$

input
```
integrate(x^6*(f*x^2+e)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac
")
```

output
```
integrate((f*x^2 + e)*x^6/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6(e+fx^2)}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{x^6\,(f\,x^2+e)}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int((x^6*(e + f*x^2))/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int((x^6*(e + f*x^2))/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^6(e+fx^2)}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \text{Too large to display}$$

input
```
int(x^6*(f*x^2+e)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

output

```
(24*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*d**2*f*x + 23*sqrt(c + d*x**2)*
sqrt(a + b*x**2)*a*b*c*d*f*x - 28*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b*d*
*2*e*x - 18*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b*d**2*f*x**3 + 24*sqrt(c
+ d*x**2)*sqrt(a + b*x**2)*b**2*c**2*f*x - 28*sqrt(c + d*x**2)*sqrt(a + b*
x**2)*b**2*c*d*e*x - 18*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**2*c*d*f*x**3
+ 21*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**2*d**2*e*x**3 + 15*sqrt(c + d*x*
*2)*sqrt(a + b*x**2)*b**2*d**2*f*x**5 - 48*int((sqrt(c + d*x**2)*sqrt(a +
b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a**3*d**3*f - 40*i
nt((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b
*d*x**4),x)*a**2*b*c*d**2*f + 56*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x*
*2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a**2*b*d**3*e - 40*int((sqrt
(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4)
,x)*a*b**2*c**2*d*f + 49*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c
 + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a*b**2*c*d**2*e - 48*int((sqrt(c + d
*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*b*
*3*c**3*f + 56*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**
2 + b*c*x**2 + b*d*x**4),x)*b**3*c**2*d*e - 24*int((sqrt(c + d*x**2)*sqrt(
a + b*x**2))/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a**3*c*d**2*f - 23*
int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c + a*d*x**2 + b*c*x**2 + b*d*x
**4),x)*a**2*b*c**2*d*f + 28*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a...
```

3.175 $\int \frac{x^4(e+fx^2)}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 381

$$\begin{aligned}
&\int \frac{x^4(e+fx^2)}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx \\
&= -\frac{(9abcdf+2(bc+ad)(5bde-4bcf-4adf))x\sqrt{c+dx^2}}{15b^2d^3\sqrt{a+bx^2}} \\
&\quad+\frac{(5bde-4bcf-4adf)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{15b^2d^2}+\frac{fx^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5bd} \\
&\quad+\frac{\sqrt{a}(9abcdf+2(bc+ad)(5bde-4bcf-4adf))\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{15b^{5/2}d^3\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} \\
&\quad-\frac{a^{3/2}(5bde-4bcf-4adf)\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{15b^{5/2}d^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}
\end{aligned}$$

output
```
-1/15*(9*a*b*c*d*f+2*(a*d+b*c)*(-4*a*d*f-4*b*c*f+5*b*d*e))*x*(d*x^2+c)^(1/
2)/b^2/d^3/(b*x^2+a)^(1/2)+1/15*(-4*a*d*f-4*b*c*f+5*b*d*e)*x*(b*x^2+a)^(1/
2)*(d*x^2+c)^(1/2)/b^2/d^2+1/5*f*x^3*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b/d+1
/15*a^(1/2)*(9*a*b*c*d*f+2*(a*d+b*c)*(-4*a*d*f-4*b*c*f+5*b*d*e))*(d*x^2+c)
^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/b^
(5/2)/d^3/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)-1/15*a^(3/2)*(-4
*a*d*f-4*b*c*f+5*b*d*e)*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a
^(1/2)),(1-a*d/b/c)^(1/2))/b^(5/2)/d^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x
^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.61 (sec) , antiderivative size = 287, normalized size of antiderivative = 0.75

$$\int \frac{x^4(e+fx^2)}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$
$$= \frac{-\sqrt{\frac{b}{a}}dx(a+bx^2)\left(c+dx^2\right)\left(4adf+b(-5de+4cf-3dfx^2)\right)-ic(8a^2d^2f+2b^2c(-5de+4cf)+abd(-1}{}$$

input
```
Integrate[(x^4*(e + f*x^2))/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
(-(Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2)*(4*a*d*f + b*(-5*d*e + 4*c*f - 3*
d*f*x^2))) - I*c*(8*a^2*d^2*f + 2*b^2*c*(-5*d*e + 4*c*f) + a*b*d*(-10*d*e
+ 7*c*f))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt
[b/a]*x], (a*d)/(b*c)] + I*c*(4*a^2*d^2*f + a*b*d*(-5*d*e + 3*c*f) + 2*b^2
*c*(-5*d*e + 4*c*f))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*A
rcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(15*a^2*(b/a)^(5/2)*d^3*Sqrt[a + b*x^2]
*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 365, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {444, 444, 25, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(e+fx^2)}{\sqrt{a+bx^2}\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \text{444}$$

$$\frac{fx^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5bd}-\frac{\int \frac{x^2(3acf-(5bde-4bcf-4adf)x^2)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{5bd}$$

$$\Big\downarrow \text{444}$$

$$\frac{fx^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5bd}-\frac{-\frac{\int -\frac{ac(5bde-4bcf-4adf)-\left(-2c(5de-4cf)b^2-ad(10de-7cf)b+8a^2d^2f\right)x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3bd}-\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(-4adf-4bcf+5bde)}{3bd}}{5bd}$$

$$\Big\downarrow \text{25}$$

$$\frac{fx^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5bd}-\frac{\frac{\int \frac{ac(5bde-4bcf-4adf)-\left(-2c(5de-4cf)b^2-ad(10de-7cf)b+8a^2d^2f\right)x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3bd}-\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(-4adf-4bcf+5bde)}{3bd}}{5bd}$$

$$\Big\downarrow \text{406}$$

$$\frac{fx^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5bd}-\frac{\frac{ac(-4adf-4bcf+5bde)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx-\left(8a^2d^2f-abd(10de-7cf)-2b^2c(5de-4cf)\right)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3bd}-\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(-4adf-4}{3bd}}{5bd}$$

$$\Big\downarrow \text{320}$$

$$\frac{fx^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5bd}-\frac{\frac{\frac{c^{3/2}\sqrt{a+bx^2}(-4adf-4bcf+5bde)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-(8a^2d^2f-abd(10de-7cf)-2b^2c(5de-4cf))\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3bd}-\frac{x\sqrt{a+bx^2}\sqrt{}}{}}{5bd}$$

$\downarrow$ 388

$$\frac{fx^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5bd}-\frac{\frac{\frac{c^{3/2}\sqrt{a+bx^2}(-4adf-4bcf+5bde)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-(8a^2d^2f-abd(10de-7cf)-2b^2c(5de-4cf))\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}}dx}{b}\right)}{3bd}-}{5bd}$$

$\downarrow$ 313

$$\frac{fx^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5bd}-\frac{\frac{\frac{c^{3/2}\sqrt{a+bx^2}(-4adf-4bcf+5bde)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-(8a^2d^2f-abd(10de-7cf)-2b^2c(5de-4cf))\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}}{\sqrt{c}}\right.\right.}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+}{a(c+}}}\right.}{3bd}}{5bd}$$

input
```
Int[(x^4*(e + f*x^2))/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
(f*x^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(5*b*d) - (-1/3*((5*b*d*e - 4*b*c*
f - 4*a*d*f)*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(b*d) + (-((8*a^2*d^2*f -
a*b*d*(10*d*e - 7*c*f) - 2*b^2*c*(5*d*e - 4*c*f))*((x*Sqrt[a + b*x^2])/(b*
Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/S
qrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))
]*Sqrt[c + d*x^2]))) + (c^(3/2)*(5*b*d*e - 4*b*c*f - 4*a*d*f)*Sqrt[a + b*x
^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt
[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))/(3*b*d))/(5*b*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

Maple [A] (verified)

Time = 8.38 (sec) , antiderivative size = 414, normalized size of antiderivative = 1.09

method	result
elliptic	$\frac{\sqrt{(b\,x^2+a)(x^2d+c)}\left(\frac{f\,x^3\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{5bd}+\frac{\left(e-\frac{f(4ad+4bc)}{5bd}\right)x\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{3bd}-\frac{\left(e-\frac{f(4ad+4bc)}{5bd}\right)ac\sqrt{1+\frac{b\,x^2}{a}}\sqrt{1+\frac{d\,x^2}{c}}}{3bd\sqrt{-\frac{b}{a}}\sqrt{bd\,x^4+ac}}\right.}{}$
risch	$-\frac{x(-3bdf\,x^2+4adf+4bcf-5bde)\sqrt{b\,x^2+a}\,\sqrt{x^2d+c}}{15b^2d^2}+\frac{\left(-\frac{\left(8f\,d^2a^2+7fdcba-10ab\,d^2e+8f\,c^2b^2-10d\,b^2ce\right)c\sqrt{1+\frac{b\,x^2}{a}}\sqrt{1+\frac{d\,x^2}{c}}\left(\text{Ellipt}\right.}{\sqrt{-\frac{b}{a}}\sqrt{bd\,x^4+ad\,x^2+}}\right.}{}$
default	$-\frac{\left(-3\sqrt{-\frac{b}{a}}\,b^2d^3f\,x^7+\sqrt{-\frac{b}{a}}\,ab\,d^3f\,x^5+\sqrt{-\frac{b}{a}}\,b^2c\,d^2f\,x^5-5\sqrt{-\frac{b}{a}}\,b^2d^3e\,x^5+4\sqrt{-\frac{b}{a}}\,a^2d^3f\,x^3+5\sqrt{-\frac{b}{a}}\,abc\,d^2f\,x^3-5\sqrt{-\frac{b}{a}}\,ab\,d^3e\right.}{}$

input
```
int(x^4*(f*x^2+e)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/5/b/d*f*x^3
*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+1/3*(e-1/5/b/d*f*(4*a*d+4*b*c))/b/d*x
*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)-1/3*(e-1/5/b/d*f*(4*a*d+4*b*c))/b/d*a
*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x
^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-(-3/5*a/b
*c/d*f-1/3*(e-1/5/b/d*f*(4*a*d+4*b*c))/b/d*(2*a*d+2*b*c))*c/(-b/a)^(1/2)*(
1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*(
EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2
),(-1+(a*d+b*c)/c/b)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 319, normalized size of antiderivative = 0.84

$$\int \frac{x^4(e+fx^2)}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$
$$=\frac{\sqrt{bd}(10\,(b^2c^2d+abcd^2)e-(8\,b^2c^3+7\,abc^2d+8\,a^2cd^2)f)x\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ad}{bc})-\sqrt{bd}(5\,(2\,b^2c^2d}{}$$

input
```
integrate(x^4*(f*x^2+e)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fric
as")
```

output
```
1/15*(sqrt(b*d)*(10*(b^2*c^2*d + a*b*c*d^2)*e - (8*b^2*c^3 + 7*a*b*c^2*d +
 8*a^2*c*d^2)*f)*x*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c))
- sqrt(b*d)*(5*(2*b^2*c^2*d + 2*a*b*c*d^2 + a*b*d^3)*e - (8*b^2*c^3 + 7*a*
b*c^2*d + 4*a^2*d^3 + 4*(2*a^2 + a*b)*c*d^2)*f)*x*sqrt(-c/d)*elliptic_f(ar
csin(sqrt(-c/d)/x), a*d/(b*c)) + (3*b^2*d^3*f*x^4 + (5*b^2*d^3*e - 4*(b^2*
c*d^2 + a*b*d^3)*f)*x^2 - 10*(b^2*c*d^2 + a*b*d^3)*e + (8*b^2*c^2*d + 7*a*
b*c*d^2 + 8*a^2*d^3)*f)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(b^3*d^4*x)
```

Sympy [F]

$$\int \frac{x^4(e+fx^2)}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{x^4(e+fx^2)}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

input
```
integrate(x**4*(f*x**2+e)/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(x**4*(e + f*x**2)/(sqrt(a + b*x**2)*sqrt(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{x^4(e+fx^2)}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{(fx^2+e)x^4}{\sqrt{bx^2+a}\sqrt{dx^2+c}}\,dx$$

input
```
integrate(x^4*(f*x^2+e)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate((f*x^2 + e)*x^4/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

Giac [F]

$$\int \frac{x^4(e+fx^2)}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx=\int \frac{(fx^2+e)x^4}{\sqrt{bx^2+a}\sqrt{dx^2+c}}\,dx$$

input
```
integrate(x^4*(f*x^2+e)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac
")
```

output
```
integrate((f*x^2 + e)*x^4/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(e+fx^2)}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx=\int \frac{x^4\,(f\,x^2+e)}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int((x^4*(e + f*x^2))/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int((x^4*(e + f*x^2))/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^4(e+fx^2)}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$
$$=\frac{-4\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,adfx-4\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,bcfx+5\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,bdex+3\sqrt{d\,x^2+c}\,\sqrt{}$$

input
```
int(x^4*(f*x^2+e)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

output

```
( - 4*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*d*f*x - 4*sqrt(c + d*x**2)*sqrt(
a + b*x**2)*b*c*f*x + 5*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b*d*e*x + 3*sqrt
(c + d*x**2)*sqrt(a + b*x**2)*b*d*f*x**3 + 8*int((sqrt(c + d*x**2)*sqrt(a
+ b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a**2*d**2*f + 7*
int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 +
b*d*x**4),x)*a*b*c*d*f - 10*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(
a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a*b*d**2*e + 8*int((sqrt(c + d*x*
*2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*b**2*
c**2*f - 10*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 +
 b*c*x**2 + b*d*x**4),x)*b**2*c*d*e + 4*int((sqrt(c + d*x**2)*sqrt(a + b*x
**2))/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a**2*c*d*f + 4*int((sqrt(c
 + d*x**2)*sqrt(a + b*x**2))/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a*b
*c**2*f - 5*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c + a*d*x**2 + b*c*
x**2 + b*d*x**4),x)*a*b*c*d*e)/(15*b**2*d**2)
```

3.176 $\int \frac{x^2(e+fx^2)}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 281

$$\begin{aligned}\int \frac{x^2(e+fx^2)}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx &= \frac{(3bde-2bcf-2adf)x\sqrt{c+dx^2}}{3bd^2\sqrt{a+bx^2}}+\frac{fx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3bd}\\ &\quad-\frac{\sqrt{a}(3bde-2bcf-2adf)\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{3b^{3/2}d^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}\\ &\quad-\frac{a^{3/2}f\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{3b^{3/2}d\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}\end{aligned}$$

output

```
1/3*(-2*a*d*f-2*b*c*f+3*b*d*e)*x*(d*x^2+c)^(1/2)/b/d^2/(b*x^2+a)^(1/2)+1/3
*f*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b/d-1/3*a^(1/2)*(-2*a*d*f-2*b*c*f+3*b
*d*e)*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d
/b/c)^(1/2))/b^(3/2)/d^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)-1
/3*a^(3/2)*f*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-
a*d/b/c)^(1/2))/b^(3/2)/d/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.36 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.78

$$\int \frac{x^2(e+fx^2)}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \frac{\sqrt{\frac{b}{a}}dfx(a+bx^2)(c+dx^2)+ic(-3bde+2bcf+2adf)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(i\operatorname{arcsinh}\left(\sqrt{\frac{b}{a}}x\right)\mid\frac{ad}{bc}\right)-ic(}{3b\sqrt{\frac{b}{a}}d^2\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

input
```
Integrate[(x^2*(e + f*x^2))/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
(Sqrt[b/a]*d*f*x*(a + b*x^2)*(c + d*x^2) + I*c*(-3*b*d*e + 2*b*c*f + 2*a*d
*f)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*
x], (a*d)/(b*c)] - I*c*(-3*b*d*e + 2*b*c*f + a*d*f)*Sqrt[1 + (b*x^2)/a]*Sq
rt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(3*b*Sqr
t[b/a]*d^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 265, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {444, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(e+fx^2)}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

$$\Big\downarrow \ 444$$

$$\frac{fx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3bd}-\frac{\int \frac{acf-(3bde-2bcf-2adf)x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3bd}$$

$$\Big\downarrow \ 406$$

$$\frac{fx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3bd}-\frac{acf\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx-(-2adf-2bcf+3bde)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3bd}$$

$$\Big\downarrow \text{320}$$

$$\frac{fx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3bd}-\frac{\frac{c^{3/2}f\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-(-2adf-2bcf+3bde)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3bd}$$

$$\Big\downarrow \text{388}$$

$$\frac{fx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3bd}-\frac{\frac{c^{3/2}f\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-(-2adf-2bcf+3bde)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)}{3bd}$$

$$\Big\downarrow \text{313}$$

$$\frac{fx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3bd}-\frac{\frac{c^{3/2}f\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-(-2adf-2bcf+3bde)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{3bd}$$

input
```
Int[(x^2*(e + f*x^2))/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
(f*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(3*b*d) - (-((3*b*d*e - 2*b*c*f - 2*
a*d*f)*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]
*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[
(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))) + (c^(3/2)*f*Sqrt[a +
b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*S
qrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))/(3*b*d)
```

Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

Maple [A] (verified)

Time = 5.61 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.07

method	result
risch	$\frac{fx\sqrt{b\,x^2+a}\,\sqrt{x^2d+c}}{3bd}-\frac{\left(-\frac{(2adf+2bcf-3bde)c\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\left(\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)-\operatorname{EllipticE}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}\,d}\right.}{3bd\sqrt{b\,x^2+a}\,\sqrt{x^2d+c}}$
elliptic	$\frac{\sqrt{(b\,x^2+a)(x^2d+c)}\left(\frac{fx\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{3bd}-\frac{acf\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{3bd\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}-\frac{\left(e-\frac{f(2ad+2bc)}{3bd}\right)c\sqrt{1+\frac{b\,x^2}{a}}}{}\right.}{\sqrt{b\,x^2+a}\,\sqrt{x^2d+c}}$
default	$\frac{\left(\sqrt{-\frac{b}{a}}\,b\,d^2f\,x^5+\sqrt{-\frac{b}{a}}\,a\,d^2f\,x^3+\sqrt{-\frac{b}{a}}\,bcdf\,x^3+\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{x^2d+c}{c}}\,\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)acdf+2\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{x^2d+c}{c}}\,\operatorname{EllipticF}\right.}{}$

input
```
int(x^2*(f*x^2+e)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*f*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b/d-1/3/b/d*(-(2*a*d*f+2*b*c*f-3*b
*d*e)*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+
b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-E
llipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2)))+a*c*f/(-b/a)^(1/2)*(1+b
*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*Ellipt
icF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2)))*((b*x^2+a)*(d*x^2+c))^(1/2)/
(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.66

$$\int \frac{x^2(e+fx^2)}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = -\frac{(3\,bcde-2\,(bc^2+acd)f)\sqrt{bd}x\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ad}{bc})-(3\,bcde-(2\,bc^2+2\,acd+ad^2)f)\sqrt{bd}x\,\sqrt{}}{3\,b^2d^3x}$$

input
```
integrate(x^2*(f*x^2+e)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fric
as")
```

output
```
-1/3*((3*b*c*d*e - 2*(b*c^2 + a*c*d)*f)*sqrt(b*d)*x*sqrt(-c/d)*elliptic_e(
arcsin(sqrt(-c/d)/x), a*d/(b*c)) - (3*b*c*d*e - (2*b*c^2 + 2*a*c*d + a*d^2
)*f)*sqrt(b*d)*x*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)/x), a*d/(b*c)) -
(b*d^2*f*x^2 + 3*b*d^2*e - 2*(b*c*d + a*d^2)*f)*sqrt(b*x^2 + a)*sqrt(d*x^2
 + c))/(b^2*d^3*x)
```

Sympy [F]

$$\int \frac{x^2(e+fx^2)}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{x^2(e+fx^2)}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

input
```
integrate(x**2*(f*x**2+e)/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(x**2*(e + f*x**2)/(sqrt(a + b*x**2)*sqrt(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{x^2(e+fx^2)}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{(fx^2+e)x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}\,dx$$

input
```
integrate(x^2*(f*x^2+e)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate((f*x^2 + e)*x^2/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

Giac [F]

$$\int \frac{x^2(e+fx^2)}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx=\int \frac{(fx^2+e)x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}\,dx$$

input
```
integrate(x^2*(f*x^2+e)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac
")
```

output
```
integrate((f*x^2 + e)*x^2/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(e+fx^2)}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx=\int \frac{x^2\,(f\,x^2+e)}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int((x^2*(e + f*x^2))/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int((x^2*(e + f*x^2))/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^2(e+fx^2)}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$
$$=\frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,fx-2\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^2}{bd\,x^4+ad\,x^2+bc\,x^2+ac}dx\right)adf-2\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^2}{bd\,x^4+ad\,x^2+bc\,x^2+ac}dx\right)bcf+3\left(\int \frac{\sqrt{d\,x^2+c}}{bd\,x^4+ad}\right.}{3bd}$$

input
```
int(x^2*(f*x^2+e)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

output

```
(sqrt(c + d*x**2)*sqrt(a + b*x**2)*f*x - 2*int((sqrt(c + d*x**2)*sqrt(a +
b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a*d*f - 2*int((sqr
t(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4
),x)*b*c*f + 3*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**
2 + b*c*x**2 + b*d*x**4),x)*b*d*e - int((sqrt(c + d*x**2)*sqrt(a + b*x**2)
)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a*c*f)/(3*b*d)
```

3.177 $\int \frac{e+fx^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 206

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx=\frac{fx\sqrt{c+dx^2}}{d\sqrt{a+bx^2}}-\frac{\sqrt{a}f\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}+\frac{\sqrt{a}e\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}c\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
f*x*(d*x^2+c)^(1/2)/d/(b*x^2+a)^(1/2)-a^(1/2)*f*(d*x^2+c)^(1/2)*EllipticE(
b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/b^(1/2)/d/(b*x^2+a)
^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)+a^(1/2)*e*(d*x^2+c)^(1/2)*InverseJa
cobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/b^(1/2)/c/(b*x^2+a)^(1
/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.19 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.64

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = -\frac{i\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\left(cfE\left(i\operatorname{arcsinh}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)+(de-cf)\operatorname{EllipticF}\left(i\operatorname{arcsinh}\left(\sqrt{\frac{b}{a}}x\right),\frac{ad}{bc}\right)\right)}{\sqrt{\frac{b}{a}}d\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

input
```
Integrate[(e + f*x^2)/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
((-I)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(c*f*EllipticE[I*ArcSinh[Sqr
t[b/a]*x], (a*d)/(b*c)] + (d*e - c*f)*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a
*d)/(b*c)]))/(Sqrt[b/a]*d*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \text{406}$$

$$e\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+f\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx$$

$$\Big\downarrow \text{320}$$

$$f\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{\sqrt{c}e\sqrt{a+bx^2}\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

$$\Big\downarrow \text{388}$$

$$f\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}}dx}{b}\right)+\frac{\sqrt{c}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

$$\Big\downarrow \text{313}$$

$$\frac{\sqrt{c}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+f\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)$$

input
```
Int[(e + f*x^2)/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
f*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*Elli
pticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a
 + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (Sqrt[c]*e*Sqrt[a + b*x^2]
*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*Sqrt[
(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

Maple [A] (verified)

Time = 3.72 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.77

method	result
default	$\frac{\left(-\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)cf+\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)de+\operatorname{EllipticE}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)cf\right)\sqrt{\frac{x^2d+c}{c}}\sqrt{\frac{b\,x^2+a}{a}}\sqrt{b\,x^2+a}\sqrt{x^2d+c}}{d\sqrt{-\frac{b}{a}}\,(bd\,x^4+ad\,x^2+x^2bc+ac)}$
elliptic	$\frac{\sqrt{(b\,x^2+a)(x^2d+c)}\left(\frac{e\sqrt{1+\frac{b\,x^2}{a}}\sqrt{1+\frac{d\,x^2}{c}}\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{\sqrt{-\frac{b}{a}}\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}-\frac{fc\sqrt{1+\frac{b\,x^2}{a}}\sqrt{1+\frac{d\,x^2}{c}}\left(\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)-\mathrm{El}\right.}{\sqrt{-\frac{b}{a}}\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}\right.}{\sqrt{b\,x^2+a}\sqrt{x^2d+c}}$

input
```
int((f*x^2+e)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*c*f+EllipticF(x*(-b/a)^(1/2),(
a*d/b/c)^(1/2))*d*e+EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*c*f)*((d*x^2
+c)/c)^(1/2)*((b*x^2+a)/a)^(1/2)*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/d/(-b/a)^
(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.63

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = -\frac{\sqrt{bd}c^2fx\sqrt{-\frac{c}{d}}E\left(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ad}{bc}\right)-\sqrt{bx^2+a}\sqrt{dx^2+c}cdf-(d^2e+c^2f)\sqrt{bd}x\sqrt{-\frac{c}{d}}F\left(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ad}{bc}\right)}{bcd^2x}$$

input
```
integrate((f*x^2+e)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
-(sqrt(b*d)*c^2*f*x*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c))
 - sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*c*d*f - (d^2*e + c^2*f)*sqrt(b*d)*x*sqr
t(-c/d)*elliptic_f(arcsin(sqrt(-c/d)/x), a*d/(b*c)))/(b*c*d^2*x)
```

Sympy [F]

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{e+fx^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

input
```
integrate((f*x**2+e)/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral((e + f*x**2)/(sqrt(a + b*x**2)*sqrt(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{fx^2+e}{\sqrt{bx^2+a}\sqrt{dx^2+c}}\,dx$$

input
```
integrate((f*x^2+e)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((f*x^2 + e)/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

Giac [F]

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx=\int \frac{fx^2+e}{\sqrt{bx^2+a}\sqrt{dx^2+c}}\,dx$$

input
```
integrate((f*x^2+e)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((f*x^2 + e)/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx=\int \frac{f\,x^2+e}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int((e + f*x^2)/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int((e + f*x^2)/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx=\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^2}{bd\,x^4+ad\,x^2+bc\,x^2+ac}dx\right)f+\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{bd\,x^4+ad\,x^2+bc\,x^2+ac}dx\right)e$$

input
```
int((f*x^2+e)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 +
b*d*x**4),x)*f + int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c + a*d*x**2 +
 b*c*x**2 + b*d*x**4),x)*e
```

3.178 $\int \frac{e+fx^2}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 209

$$\int \frac{e+fx^2}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = -\frac{e\sqrt{c+dx^2}}{cx\sqrt{a+bx^2}} - \frac{\sqrt{b}e\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}c\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} + \frac{\sqrt{a}f\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}c\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
-e*(d*x^2+c)^(1/2)/c/x/(b*x^2+a)^(1/2)-b^(1/2)*e*(d*x^2+c)^(1/2)*EllipticE
(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/a^(1/2)/c/(b*x^2+a
)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)+a^(1/2)*f*(d*x^2+c)^(1/2)*InverseJ
acobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/b^(1/2)/c/(b*x^2+a)^(
1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.22 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.95

$$\int \frac{e+fx^2}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \frac{\sqrt{\frac{b}{a}}\left(-\sqrt{\frac{b}{a}}e(a+bx^2)\left(c+dx^2\right)-ibcex\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(i\operatorname{arcsinh}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)-ic(-be+af)x\sqrt{1}\right.}{bcx\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

input
```
Integrate[(e + f*x^2)/(x^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
(Sqrt[b/a]*(-(Sqrt[b/a]*e*(a + b*x^2)*(c + d*x^2)) - I*b*c*e*x*Sqrt[1 + (b
*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)
] - I*c*(-(b*e) + a*f)*x*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF
[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(b*c*x*Sqrt[a + b*x^2]*Sqrt[c + d*
x^2])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.19, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {445, 25, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx^2}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

$$\Big\downarrow \text{ 445}$$

$$-\frac{\int -\frac{bdex^2+acf}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{acx}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{bdex^2+acf}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac} - \frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{acx}$$

$\Big\downarrow$ 406

$$\frac{bde\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx + acf\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac} - \frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{acx}$$

$\Big\downarrow$ 320

$$\frac{bde\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx + \frac{c^{3/2}f\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{ac} - \frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{acx}$$

$\Big\downarrow$ 388

$$\frac{bde\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}} - \frac{c\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right) + \frac{c^{3/2}f\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{ac} - \frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{acx}$$

$\Big\downarrow$ 313

$$\frac{\frac{c^{3/2}f\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}} + bde\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}} - \frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{ac} - \frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{acx}$$

input

```
Int[(e + f*x^2)/(x^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

output

```
-((e*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(a*c*x)) + (b*d*e*((x*Sqrt[a + b*x^2
])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d
]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d
*x^2))]*Sqrt[c + d*x^2])) + (c^(3/2)*f*Sqrt[a + b*x^2]*EllipticF[ArcTan[(S
qrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c
+ d*x^2))]*Sqrt[c + d*x^2]))/(a*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 5.53 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.35

method	result
default	$\frac{\left(-\sqrt{-\frac{b}{a}}\, bde\, x^{4}+\sqrt{\frac{b\, x^{2}+a}{a}}\, \sqrt{\frac{x^{2} d+c}{c}}\, \operatorname{EllipticF}\left(x \sqrt{-\frac{b}{a}}, \sqrt{\frac{a d}{b c}}\right) acfx-\sqrt{\frac{b\, x^{2}+a}{a}}\, \sqrt{\frac{x^{2} d+c}{c}}\, \operatorname{EllipticF}\left(x \sqrt{-\frac{b}{a}}, \sqrt{\frac{a d}{b c}}\right) bcex+\sqrt{\frac{b\, x^{2}+a}{a}}\right.}{\sqrt{-\frac{b}{a}}\, xca\left(bd\, x^{4}+ad\, x^{2}+x^{2} bc+a\right.}$
elliptic	$\frac{\sqrt{\left(b\, x^{2}+a\right)\left(x^{2} d+c\right)}\left(-\frac{e \sqrt{bd\, x^{4}+ad\, x^{2}+x^{2} bc+ac}}{acx}+\frac{f \sqrt{1+\frac{b\, x^{2}}{a}}\, \sqrt{1+\frac{d\, x^{2}}{c}}\, \operatorname{EllipticF}\left(x \sqrt{-\frac{b}{a}}, \sqrt{-1+\frac{ad+bc}{cb}}\right)}{\sqrt{-\frac{b}{a}}\, \sqrt{bd\, x^{4}+ad\, x^{2}+x^{2} bc+ac}}-\frac{be \sqrt{1+\frac{b\, x^{2}}{a}}\, \sqrt{1+\frac{d\, x^{2}}{c}}\, \left(\operatorname{Elliptic}\right.}{}\right.}{\sqrt{b\, x^{2}+a}\, \sqrt{x^{2} d+c}}$
risch	$-\frac{e \sqrt{b\, x^{2}+a}\, \sqrt{x^{2} d+c}}{acx}+\frac{\left(\frac{acf \sqrt{1+\frac{b\, x^{2}}{a}}\, \sqrt{1+\frac{d\, x^{2}}{c}}\, \operatorname{EllipticF}\left(x \sqrt{-\frac{b}{a}}, \sqrt{-1+\frac{ad+bc}{cb}}\right)}{\sqrt{-\frac{b}{a}}\, \sqrt{bd\, x^{4}+ad\, x^{2}+x^{2} bc+ac}}-\frac{bec \sqrt{1+\frac{b\, x^{2}}{a}}\, \sqrt{1+\frac{d\, x^{2}}{c}}\, \left(\operatorname{EllipticF}\left(x \sqrt{-\frac{b}{a}}, \sqrt{-1+\frac{ad}{}}\right.\right.}{\sqrt{-\frac{b}{a}}\, \sqrt{bd\, x^{4}+ad\, x^{2}}}\right.}{ac \sqrt{b\, x^{2}+a}\, \sqrt{x^{2} d+c}}$

input
```
int((f*x^2+e)/x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-(-b/a)^(1/2)*b*d*e*x^4+((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF
(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*c*f*x-((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)
^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b*c*e*x+((b*x^2+a)/a)^(1/
2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b*c*e*x-(
-b/a)^(1/2)*a*d*e*x^2-(-b/a)^(1/2)*b*c*e*x^2-(-b/a)^(1/2)*a*c*e)*(d*x^2+c)
^(1/2)*(b*x^2+a)^(1/2)/(-b/a)^(1/2)/x/c/a/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.60

$$\int \frac{e+fx^2}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$
$$=\frac{\sqrt{ac}b^2ex\sqrt{-\frac{b}{a}}E(\arcsin\left(x\sqrt{-\frac{b}{a}}\right)\mid\frac{ad}{bc})-\sqrt{bx^2+a}\sqrt{dx^2+c}abe-(b^2e+a^2f)\sqrt{ac}x\sqrt{-\frac{b}{a}}F(\arcsin\left(x\sqrt{\right.\right.}{a^2bcx}$$

input
```
integrate((f*x^2+e)/x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fric
as")
```

output
```
(sqrt(a*c)*b^2*e*x*sqrt(-b/a)*elliptic_e(arcsin(x*sqrt(-b/a)), a*d/(b*c))
- sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*a*b*e - (b^2*e + a^2*f)*sqrt(a*c)*x*sqrt
(-b/a)*elliptic_f(arcsin(x*sqrt(-b/a)), a*d/(b*c)))/(a^2*b*c*x)
```

Sympy [F]

$$\int \frac{e+fx^2}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx=\int \frac{e+fx^2}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

input
```
integrate((f*x**2+e)/x**2/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral((e + f*x**2)/(x**2*sqrt(a + b*x**2)*sqrt(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{e+fx^2}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx=\int \frac{fx^2+e}{\sqrt{bx^2+a}\sqrt{dx^2+c}x^2}\,dx$$

input
```
integrate((f*x^2+e)/x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate((f*x^2 + e)/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x^2), x)
```

Giac [F]

$$\int \frac{e+fx^2}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx=\int \frac{fx^2+e}{\sqrt{bx^2+a}\sqrt{dx^2+c}x^2}\,dx$$

input
```
integrate((f*x^2+e)/x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac
")
```

output
```
integrate((f*x^2 + e)/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx^2}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx=\int \frac{f\,x^2+e}{x^2\,\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int((e + f*x^2)/(x^2*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int((e + f*x^2)/(x^2*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{e+fx^2}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx=\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{bd\,x^6+ad\,x^4+bc\,x^4+ac\,x^2}dx\right)e+\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{bd\,x^4+ad\,x^2+bc\,x^2+ac}dx\right)f$$

input
```
int((f*x^2+e)/x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c*x**2 + a*d*x**4 + b*c*x**4 +
b*d*x**6),x)*e + int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c + a*d*x**2 +
 b*c*x**2 + b*d*x**4),x)*f
```

3.179 $\int \frac{e+fx^2}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 286

$$\begin{aligned}\int \frac{e+fx^2}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx &= \frac{(2bce+2ade-3acf)\sqrt{c+dx^2}}{3ac^2x\sqrt{a+bx^2}}-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{3acx^3}\\ &\quad+\frac{\sqrt{b}(2bce+2ade-3acf)\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{3a^{3/2}c^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}\\ &\quad-\frac{\sqrt{b}de\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{3\sqrt{a}c^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}\end{aligned}$$

output
```
1/3*(-3*a*c*f+2*a*d*e+2*b*c*e)*(d*x^2+c)^(1/2)/a/c^2/x/(b*x^2+a)^(1/2)-1/3
*e*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a/c/x^3+1/3*b^(1/2)*(-3*a*c*f+2*a*d*e+2
*b*c*e)*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a
*d/b/c)^(1/2))/a^(3/2)/c^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
-1/3*b^(1/2)*d*e*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2))
,(1-a*d/b/c)^(1/2))/a^(1/2)/c^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^
(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.04 (sec) , antiderivative size = 256, normalized size of antiderivative = 0.90

$$\int \frac{e+fx^2}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \frac{-\sqrt{\frac{b}{a}}(a+bx^2)(c+dx^2)(-2bcex^2-2adex^2+ac(e+3fx^2))-ibc(-2bce-2ade+3acf)x^3\sqrt{1+\frac{bx^2}{a}}\sqrt{}}{3a^2\sqrt{\frac{b}{a}}c^2}$$

input
```
Integrate[(e + f*x^2)/(x^4*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
(-(Sqrt[b/a]*(a + b*x^2)*(c + d*x^2)*(-2*b*c*e*x^2 - 2*a*d*e*x^2 + a*c*(e
+ 3*f*x^2))) - I*b*c*(-2*b*c*e - 2*a*d*e + 3*a*c*f)*x^3*Sqrt[1 + (b*x^2)/a
]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + I*b
*c*(-2*b*c*e - a*d*e + 3*a*c*f)*x^3*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c
]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(3*a^2*Sqrt[b/a]*c^2*x^3
*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.14, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {445, 445, 25, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx^2}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

$$\Big\downarrow \text{445}$$

$$-\frac{\int \frac{bdex^2+2bce+2ade-3acf}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3ac}-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{3acx^3}$$

$$\Big\downarrow \text{445}$$

$$-\frac{-\frac{\int -\frac{bd\left((2bce+2ade-3acf)x^2+ace\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(-3acf+2ade+2bce)}{acx}}{3ac}-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{3acx^3}$$

$\big\downarrow$ 25

$$-\frac{\frac{\int \frac{bd\left((2bce+2ade-3acf)x^2+ace\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(-3acf+2ade+2bce)}{acx}}{3ac}-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{3acx^3}$$

$\big\downarrow$ 27

$$-\frac{\frac{bd\int \frac{(2bce+2ade-3acf)x^2+ace}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(-3acf+2ade+2bce)}{acx}}{3ac}-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{3acx^3}$$

$\big\downarrow$ 406

$$-\frac{\frac{bd\left((-3acf+2ade+2bce)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+ace\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(-3acf+2ade+2bce)}{acx}}{3ac}-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{3acx^3}$$

$\big\downarrow$ 320

$$-\frac{\frac{bd\left((-3acf+2ade+2bce)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}e\sqrt{a+bx^2}\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(-3acf+2ade+2bce)}{acx}}{3ac}-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{3acx^3}$$

$\big\downarrow$ 388

$$-\frac{\frac{bd\left((-3acf+2ade+2bce)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{c^{3/2}e\sqrt{a+bx^2}\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(-3acf+2ade}{acx}}{3ac}-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{3acx^3}$$

$\big\downarrow$ 313

$$-\frac{\frac{bd\left(\frac{c^{3/2}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+(-3acf+2ade+2bce)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)\right)}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+d}}{}}{3ac}$$

$$\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{3acx^3}$$

input
```
Int[(e + f*x^2)/(x^4*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
-1/3*(e*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(a*c*x^3) - (-(((2*b*c*e + 2*a*d*
e - 3*a*c*f)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(a*c*x)) + (b*d*((2*b*c*e +
2*a*d*e - 3*a*c*f)*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqr
t[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*S
qrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (c^(3/2)*
e*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])
/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])))/(a*c))/
(3*a*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
 + d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] && !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 7.23 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.16

method	result
risch	$-\frac{\sqrt{b x^2+a}\,\sqrt{x^2 d+c}\,\left(3acf\,x^2-2ade\,x^2-2bce\,x^2+ace\right)}{3a^2c^2x^3}-\frac{bd\left(\frac{(3acf-2ade-2bce)c\sqrt{1+\frac{b x^2}{a}}\,\sqrt{1+\frac{d x^2}{c}}\,\left(\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right.\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}\,d}\cdots\right.}{\cdots}$
elliptic	$\frac{\sqrt{(b x^2+a)(x^2 d+c)}\left(-\frac{e\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{3ac\,x^3}-\frac{(3acf-2ade-2bce)\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{3a^2c^2x}-\frac{bde\sqrt{1+\frac{b x^2}{a}}\,\sqrt{1+\frac{d x^2}{c}}\,\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\cdots\right)}{3ac\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+x^2bc+\cdots}}\cdots\right)}{\sqrt{b x^2+a}\,\sqrt{x^2 d+\cdots}}$
default	$-\frac{\left(3\sqrt{-\frac{b}{a}}\,abcdf\,x^6-2\sqrt{-\frac{b}{a}}\,ab\,d^2e\,x^6-2\sqrt{-\frac{b}{a}}\,b^2cde\,x^6+3\sqrt{\frac{b x^2+a}{a}}\,\sqrt{\frac{x^2d+c}{c}}\,\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)ab\,c^2f\,x^3-\sqrt{\frac{b x^2+a}{a}}\,\sqrt{\frac{x^2\cdots}{\cdots}}\cdots\right.}{\cdots}$

input
```
int((f*x^2+e)/x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(3*a*c*f*x^2-2*a*d*e*x^2-2*b*c*e*x^2+
a*c*e)/a^2/c^2/x^3-1/3/a^2/c^2*b*d*((3*a*c*f-2*a*d*e-2*b*c*e)*c/(-b/a)^(1/
2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
/d*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^
(1/2),(-1+(a*d+b*c)/c/b)^(1/2)))+a*c*e/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d
*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2)
,(-1+(a*d+b*c)/c/b)^(1/2)))*((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d
*x^2+c)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.66

$$\int \frac{e+fx^2}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \frac{(3\,abcf-2\,(b^2c+abd)e)\sqrt{ac}x^3\sqrt{-\frac{b}{a}}E\left(\arcsin\left(x\sqrt{-\frac{b}{a}}\right)\mid\frac{ad}{bc}\right)-(3\,abcf-(2\,b^2c+(a^2+2\,ab)d)e)\sqrt{ac}x^3}{3\,a^3c^2x^3}$$

input
```
integrate((f*x^2+e)/x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fric
as")
```

output
```
1/3*((3*a*b*c*f - 2*(b^2*c + a*b*d)*e)*sqrt(a*c)*x^3*sqrt(-b/a)*elliptic_e
(arcsin(x*sqrt(-b/a)), a*d/(b*c)) - (3*a*b*c*f - (2*b^2*c + (a^2 + 2*a*b)*
d)*e)*sqrt(a*c)*x^3*sqrt(-b/a)*elliptic_f(arcsin(x*sqrt(-b/a)), a*d/(b*c))
 - (a^2*c*e + (3*a^2*c*f - 2*(a*b*c + a^2*d)*e)*x^2)*sqrt(b*x^2 + a)*sqrt(
d*x^2 + c))/(a^3*c^2*x^3)
```

Sympy [F]

$$\int \frac{e+fx^2}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx=\int \frac{e+fx^2}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

input
```
integrate((f*x**2+e)/x**4/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral((e + f*x**2)/(x**4*sqrt(a + b*x**2)*sqrt(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{e+fx^2}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx=\int \frac{fx^2+e}{\sqrt{bx^2+a}\sqrt{dx^2+c}x^4}\,dx$$

input
```
integrate((f*x^2+e)/x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate((f*x^2 + e)/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x^4), x)
```

Giac [F]

$$\int \frac{e+fx^2}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx=\int \frac{fx^2+e}{\sqrt{bx^2+a}\sqrt{dx^2+c}x^4}\,dx$$

input
```
integrate((f*x^2+e)/x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac
")
```

output
```
integrate((f*x^2 + e)/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx^2}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{f\,x^2+e}{x^4\,\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int((e + f*x^2)/(x^4*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int((e + f*x^2)/(x^4*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{e+fx^2}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \frac{-\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,f+\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^2}{bd\,x^4+ad\,x^2+bc\,x^2+ac}dx\right)bdfx+\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{bd\,x^8+ad\,x^6+bc\,x^6+ac\,x^4}dx\right)acex}{acx}$$

input
```
int((f*x^2+e)/x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

output
```
( - sqrt(c + d*x**2)*sqrt(a + b*x**2)*f + int((sqrt(c + d*x**2)*sqrt(a + b
*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*b*d*f*x + int((sqrt
(c + d*x**2)*sqrt(a + b*x**2))/(a*c*x**4 + a*d*x**6 + b*c*x**6 + b*d*x**8)
,x)*a*c*e*x)/(a*c*x)
```

3.180 $\int \frac{e+fx^2}{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 391

$$\begin{aligned}\int \frac{e+fx^2}{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx &= \frac{(9abcde-2(bc+ad)(4bce+4ade-5acf))\sqrt{c+dx^2}}{15a^2c^3x\sqrt{a+bx^2}} \\ &- \frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{5acx^5}+\frac{(4bce+4ade-5acf)\sqrt{a+bx^2}\sqrt{c+dx^2}}{15a^2c^2x^3} \\ &- \frac{\sqrt{b}(8b^2c^2e+abc(7de-10cf)+2a^2d(4de-5cf))\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{15a^{5/2}c^3\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} \\ &+ \frac{\sqrt{b}d(4bce+4ade-5acf)\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{15a^{3/2}c^3\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}\end{aligned}$$

output

```
1/15*(9*a*b*c*d*e-2*(a*d+b*c)*(-5*a*c*f+4*a*d*e+4*b*c*e))*(d*x^2+c)^(1/2)/
a^2/c^3/x/(b*x^2+a)^(1/2)-1/5*e*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a/c/x^5+1/
15*(-5*a*c*f+4*a*d*e+4*b*c*e)*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a^2/c^2/x^3-
1/15*b^(1/2)*(8*b^2*c^2*e+a*b*c*(-10*c*f+7*d*e)+2*a^2*d*(-5*c*f+4*d*e))*(d
*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1
/2))/a^(5/2)/c^3/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)+1/15*b^(1
/2)*d*(-5*a*c*f+4*a*d*e+4*b*c*e)*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^
(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/a^(3/2)/c^3/(b*x^2+a)^(1/2)/(a*(d*x^2+
c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.89 (sec) , antiderivative size = 355, normalized size of antiderivative = 0.91

$$\int \frac{e+fx^2}{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$
$$= \frac{\sqrt{\frac{b}{a}}(a+bx^2)(c+dx^2)(-8b^2c^2ex^4+abcx^2(4ce-7dex^2+10cfx^2)+a^2(-8d^2ex^4+2cdx^2(2e+5fx^2)-$$

input
```
Integrate[(e + f*x^2)/(x^6*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
(Sqrt[b/a]*(a + b*x^2)*(c + d*x^2)*(-8*b^2*c^2*e*x^4 + a*b*c*x^2*(4*c*e -
7*d*e*x^2 + 10*c*f*x^2) + a^2*(-8*d^2*e*x^4 + 2*c*d*x^2*(2*e + 5*f*x^2) -
c^2*(3*e + 5*f*x^2))) + I*b*c*(-8*b^2*c^2*e + 2*a^2*d*(-4*d*e + 5*c*f) + a
*b*c*(-7*d*e + 10*c*f))*x^5*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Ellipt
icE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*b*c*(-8*b^2*c^2*e + a^2*d*(-4
*d*e + 5*c*f) + a*b*c*(-3*d*e + 10*c*f))*x^5*Sqrt[1 + (b*x^2)/a]*Sqrt[1 +
(d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(15*a^3*Sqrt[b/
a]*c^3*x^5*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 438, normalized size of antiderivative = 1.12, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {445, 445, 445, 25, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx^2}{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$
$$\Big\downarrow \; 445$$
$$-\frac{\int \frac{3bdex^2+4bce+4ade-5acf}{x^4\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{5ac}-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{5acx^5}$$

$\downarrow$ 445

$$-\frac{-\frac{\int \frac{2d(4de-5cf)a^2+bc(7de-10cf)a+bd(4bce+4ade-5acf)x^2+8b^2c^2e}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(-5acf+4ade+4bce)}{3acx^3}}{5ac}-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{5acx^5}$$

$\downarrow$ 445

$$-\frac{-\frac{-\frac{\int -\frac{bd\left(\left(2d(4de-5cf)a^2+bc(7de-10cf)a+8b^2c^2e\right)x^2+ac(4bce+4ade-5acf)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{8b^2ce}{a}+\frac{8ad^2e}{c}-10adf-10bcf+7bde\right)}{x}}{3ac}-\sqrt{a+bx\cdots}}{5ac}-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{5acx^5}$$

$\downarrow$ 25

$$-\frac{-\frac{\frac{\int \frac{bd\left(\left(2d(4de-5cf)a^2+bc(7de-10cf)a+8b^2c^2e\right)x^2+ac(4bce+4ade-5acf)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{8b^2ce}{a}+\frac{8ad^2e}{c}-10adf-10bcf+7bde\right)}{x}}{3ac}-\sqrt{a+bx^2}\sqrt{\cdots}}{5ac}-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{5acx^5}$$

$\downarrow$ 27

$$-\frac{-\frac{\frac{bd\int \frac{\left(2d(4de-5cf)a^2+bc(7de-10cf)a+8b^2c^2e\right)x^2+ac(4bce+4ade-5acf)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{8b^2ce}{a}+\frac{8ad^2e}{c}-10adf-10bcf+7bde\right)}{x}}{3ac}-\sqrt{a+bx^2}\sqrt{c+\cdots}}{5ac}-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{5acx^5}$$

$\downarrow$ 406

$$-\frac{-\frac{\frac{bd\left(\left(2a^2d(4de-5cf)+abc(7de-10cf)+8b^2c^2e\right)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+ac(-5acf+4ade+4bce)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{8b^2ce}{a}+\frac{8ad^2e}{c}\cdots\right)}{x}}{3ac}\cdots}{5ac}-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{5acx^5}$$

$\downarrow$ 320

$$-\frac{-\frac{\frac{bd\left(\left(2a^2d(4de-5cf)+abc(7de-10cf)+8b^2c^2e\right)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}(-5acf+4ade+4bce)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{ac}-\frac{\sqrt{a+bx^2}\sqrt{\cdots}}{\cdots}}{3ac}\cdots}{5ac}$$

$$\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{5acx^5}$$

$\downarrow$ 388

$$-\frac{-\frac{\frac{bd\left(\left(2a^2d(4de-5cf)+abc(7de-10cf)+8b^2c^2e\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(-5acf+4ade+4bce)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{ac}\cdots}{3ac}\cdots}{5ac}$$

$$\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{5acx^5}$$

$\downarrow$ 313

$$-\frac{-\frac{\frac{bd\left(\left(2a^2d(4de-5cf)+abc(7de-10cf)+8b^2c^2e\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(-5acf+4ade+4bce)\,\text{EllipticF}\left(\arctan\cdots\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\cdots\right)}{ac}\cdots}{3ac}\cdots}{5ac}$$

$$\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}}{5acx^5}$$

input
```
Int[(e + f*x^2)/(x^6*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
-1/5*(e*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(a*c*x^5) - (-1/3*((4*b*c*e + 4*a
*d*e - 5*a*c*f)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(a*c*x^3) - (-((((8*b^2*c
*e)/a + 7*b*d*e + (8*a*d^2*e)/c - 10*b*c*f - 10*a*d*f)*Sqrt[a + b*x^2]*Sqr
t[c + d*x^2])/x) + (b*d*((8*b^2*c^2*e + a*b*c*(7*d*e - 10*c*f) + 2*a^2*d*(
4*d*e - 5*c*f))*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a
 + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt
[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (c^(3/2)*(4*
b*c*e + 4*a*d*e - 5*a*c*f)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sq
rt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*S
qrt[c + d*x^2])))/(a*c))/(3*a*c))/(5*a*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 9.75 (sec) , antiderivative size = 481, normalized size of antiderivative = 1.23

method	result
elliptic	$\frac{\sqrt{(b\,x^2+a)(x^2d+c)}\left(-\frac{e\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{5ac\,x^5}-\frac{(5acf-4ade-4bce)\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{15a^2c^2x^3}+\frac{\left(10a^2cfd-8a^2d^2e+10ab\,c^2f-7abcde-8b^2\right.}{15c^3a^3x}\right.}{}$
risch	$-\frac{\sqrt{b\,x^2+a}\,\sqrt{x^2d+c}\,\left(-10a^2cdf\,x^4+8a^2d^2e\,x^4-10ab\,c^2f\,x^4+7abcde\,x^4+8b^2c^2e\,x^4+5a^2c^2f\,x^2-4a^2cde\,x^2-4ab\,c^2e\,x^2+3a^2c^2e\right)}{15a^3c^3x^5}-$
default	Expression too large to display

input
```
int((f*x^2+e)/x^6/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-1/5*e/a/c*(b
*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/x^5-1/15/a^2/c^2*(5*a*c*f-4*a*d*e-4*b*c*
e)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/x^3+1/15*(10*a^2*c*d*f-8*a^2*d^2*e+
10*a*b*c^2*f-7*a*b*c*d*e-8*b^2*c^2*e)/c^3/a^3*(b*d*x^4+a*d*x^2+b*c*x^2+a*c
)^(1/2)/x-1/15*(5*a*c*f-4*a*d*e-4*b*c*e)*b*d/a^2/c^2/(-b/a)^(1/2)*(1+b*x^2
/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(
x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+1/15*b*(10*a^2*c*d*f-8*a^2*d^2*e+
10*a*b*c^2*f-7*a*b*c*d*e-8*b^2*c^2*e)/c^2/a^3/(-b/a)^(1/2)*(1+b*x^2/a)^(1/
2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(EllipticF(x*(-b/
a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/
c/b)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 319, normalized size of antiderivative = 0.82

$$\int \frac{e+fx^2}{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \frac{\sqrt{ac}((8\,b^3c^2+7\,ab^2cd+8\,a^2bd^2)e-10\,(ab^2c^2+a^2bcd)f)x^5\sqrt{-\frac{b}{a}}E(\arcsin\left(x\sqrt{-\frac{b}{a}}\right)\mid\frac{ad}{bc})-\sqrt{ac}((8\,b^3c^2}{}$$

input
```
integrate((f*x^2+e)/x^6/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fric
as")
```

output
```
1/15*(sqrt(a*c)*((8*b^3*c^2 + 7*a*b^2*c*d + 8*a^2*b*d^2)*e - 10*(a*b^2*c^2
 + a^2*b*c*d)*f)*x^5*sqrt(-b/a)*elliptic_e(arcsin(x*sqrt(-b/a)), a*d/(b*c)
) - sqrt(a*c)*((8*b^3*c^2 + (4*a^2*b + 7*a*b^2)*c*d + 4*(a^3 + 2*a^2*b)*d^
2)*e - 5*(2*a*b^2*c^2 + (a^3 + 2*a^2*b)*c*d)*f)*x^5*sqrt(-b/a)*elliptic_f(
arcsin(x*sqrt(-b/a)), a*d/(b*c)) - (3*a^3*c^2*e + ((8*a*b^2*c^2 + 7*a^2*b*
c*d + 8*a^3*d^2)*e - 10*(a^2*b*c^2 + a^3*c*d)*f)*x^4 + (5*a^3*c^2*f - 4*(a
^2*b*c^2 + a^3*c*d)*e)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(a^4*c^3*x^5)
```

Sympy [F]

$$\int \frac{e+fx^2}{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx=\int \frac{e+fx^2}{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

input
```
integrate((f*x**2+e)/x**6/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral((e + f*x**2)/(x**6*sqrt(a + b*x**2)*sqrt(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{e+fx^2}{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx=\int \frac{fx^2+e}{\sqrt{bx^2+a}\sqrt{dx^2+c}x^6}\,dx$$

input
```
integrate((f*x^2+e)/x^6/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate((f*x^2 + e)/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x^6), x)
```

Giac [F]

$$\int \frac{e+fx^2}{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx=\int \frac{fx^2+e}{\sqrt{bx^2+a}\sqrt{dx^2+c}x^6}\,dx$$

input
```
integrate((f*x^2+e)/x^6/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac
")
```

output
```
integrate((f*x^2 + e)/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx^2}{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx=\int \frac{f\,x^2+e}{x^6\,\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int((e + f*x^2)/(x^6*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int((e + f*x^2)/(x^6*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{e+fx^2}{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \frac{-\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,ace+3\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,bde\,x^4-3\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^2}{bd\,x^4+ad\,x^2+bc\,x^2+ac}dx\right)b^2d^2e\,x^5+5\left(\int \frac{}{bd}\right.}{5}$$

input
```
int((f*x^2+e)/x^6/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

output
```
( - sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*c*e + 3*sqrt(c + d*x**2)*sqrt(a +
b*x**2)*b*d*e*x**4 - 3*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c +
 a*d*x**2 + b*c*x**2 + b*d*x**4),x)*b**2*d**2*e*x**5 + 5*int((sqrt(c + d*x
**2)*sqrt(a + b*x**2))/(a*c*x**4 + a*d*x**6 + b*c*x**6 + b*d*x**8),x)*a**2
*c**2*f*x**5 - 4*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c*x**4 + a*d*x
**6 + b*c*x**6 + b*d*x**8),x)*a**2*c*d*e*x**5 - 4*int((sqrt(c + d*x**2)*sq
rt(a + b*x**2))/(a*c*x**4 + a*d*x**6 + b*c*x**6 + b*d*x**8),x)*a*b*c**2*e*
x**5)/(5*a**2*c**2*x**5)
```

3.181 $\int \frac{x^8}{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 597

$$\int \frac{x^8}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \frac{\left(8a^2df+ab(10de+7cf)+b^2\left(10ce+\frac{15de^2}{f}+\frac{8c^2f}{d}\right)\right)x\sqrt{c+dx^2}}{15b^2d^2f^2\sqrt{a+bx^2}} - \frac{(5bde+4bcf+4adf)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{15b^2d^2f^2} + \frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5bdf} - \frac{\sqrt{a}(8a^2d^2f^2+abdf(10de+7cf)+b^2(15d^2e^2+10cdef+8c^2f^2))\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{15b^{5/2}d^3f^3\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} - \frac{a^{3/2}(4a^2cdf^3+abcf^2(de+4cf)-b^2e(15d^2e^2+5cdef+4c^2f^2))\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1\right.}{15b^{5/2}cd^2f^3(be-af)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} - \frac{a^{3/2}e^3\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}cf^3(be-af)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
1/15*(8*a^2*d*f+a*b*(7*c*f+10*d*e)+b^2*(10*c*e+15*d*e^2/f+8*c^2*f/d))*x*(d
*x^2+c)^(1/2)/b^2/d^2/f^2/(b*x^2+a)^(1/2)-1/15*(4*a*d*f+4*b*c*f+5*b*d*e)*x
*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b^2/d^2/f^2+1/5*x^3*(b*x^2+a)^(1/2)*(d*x^
2+c)^(1/2)/b/d/f-1/15*a^(1/2)*(8*a^2*d^2*f^2+a*b*d*f*(7*c*f+10*d*e)+b^2*(8
*c^2*f^2+10*c*d*e*f+15*d^2*e^2))*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/
2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/b^(5/2)/d^3/f^3/(b*x^2+a)^(1/2)/(a
*(d*x^2+c)/c/(b*x^2+a))^(1/2)-1/15*a^(3/2)*(4*a^2*c*d*f^3+a*b*c*f^2*(4*c*f
+d*e)-b^2*e*(4*c^2*f^2+5*c*d*e*f+15*d^2*e^2))*(d*x^2+c)^(1/2)*InverseJacob
iAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/b^(5/2)/c/d^2/f^3/(-a*f+b
*e)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)-a^(3/2)*e^3*(d*x^2+c)^
(1/2)*EllipticPi(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),1-a*f/b/e,(1-a*d/b/c)
^(1/2))/b^(1/2)/c/f^3/(-a*f+b*e)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))
^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.53 (sec) , antiderivative size = 419, normalized size of antiderivative = 0.70

$$\int \frac{x^8}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx$$
$$= \frac{-icf(8a^2d^2f^2+abdf(10de+7cf)+b^2(15d^2e^2+10cdef+8c^2f^2))\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,E\left(i\operatorname{arcsinh}\left(\sqrt{\frac{b}{a}}\right.\right.}{}$$

input
```
Integrate[x^8/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)),x]
```

output
```
((-I)*c*f*(8*a^2*d^2*f^2 + a*b*d*f*(10*d*e + 7*c*f) + b^2*(15*d^2*e^2 + 10
*c*d*e*f + 8*c^2*f^2))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I
*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + I*(4*a^2*c*d^2*f^3 + a*b*c*d*f^2*(5*
d*e + 3*c*f) + b^2*(15*d^3*e^3 + 15*c*d^2*e^2*f + 10*c^2*d*e*f^2 + 8*c^3*f
^3))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]
*x], (a*d)/(b*c)] - d*(Sqrt[b/a]*f^2*x*(a + b*x^2)*(c + d*x^2)*(4*a*d*f +
b*(5*d*e + 4*c*f - 3*d*f*x^2)) + (15*I)*b^2*d^2*e^3*Sqrt[1 + (b*x^2)/a]*Sq
rt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b
*c)]))/(15*a^2*(b/a)^(5/2)*d^3*f^4*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx$$
$$\Big\downarrow \ 450$$
$$\int \frac{x^8}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)}dx$$

input
```
Int[x^8/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [A] (verified)

Time = 20.19 (sec) , antiderivative size = 528, normalized size of antiderivative = 0.88

method	result
risch	$-\frac{x\left(-3bdf\,x^{2}+4adf+4bcf+5bde\right)\sqrt{b\,x^{2}+a}\,\sqrt{x^{2}d+c}}{15b^{2}d^{2}f^{2}}+\frac{\left(\frac{\left(4a^{2}cd\,f^{3}+4ab\,c^{2}f^{3}+5abcde\,f^{2}-15b^{2}d^{2}e^{3}\right)\sqrt{1+\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,\mathrm{EllipticF}\left(x\sqrt{\cdots}\right.}{f^{2}\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^{4}+ad\,x^{2}+x^{2}bc+ac}}\cdots\right.}{\cdots}$
default	Expression too large to display
elliptic	Expression too large to display

input
```
int(x^8/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
-1/15*x*(-3*b*d*f*x^2+4*a*d*f+4*b*c*f+5*b*d*e)*(b*x^2+a)^(1/2)*(d*x^2+c)^(
1/2)/b^2/d^2/f^2+1/15/f^2/b^2/d^2*((4*a^2*c*d*f^3+4*a*b*c^2*f^3+5*a*b*c*d*
e*f^2-15*b^2*d^2*e^3)/f^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)
/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c
)/c/b)^(1/2))-1/f*(8*a^2*d^2*f^2+7*a*b*c*d*f^2+10*a*b*d^2*e*f+8*b^2*c^2*f^
2+10*b^2*c*d*e*f+15*b^2*d^2*e^2)*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2
/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(-b/a)^(1/2),
(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2
)))+15*b^2*d^2*e^3/f^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b
*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*
d)^(1/2)/(-b/a)^(1/2)))*((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2
+c)^(1/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^8}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate(x^8/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^8}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{x^8}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx$$

input
```
integrate(x**8/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2)/(f*x**2+e),x)
```

output
```
Integral(x**8/(sqrt(a + b*x**2)*sqrt(c + d*x**2)*(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{x^8}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{x^8}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)}\,dx$$

input
```
integrate(x^8/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="maxi
ma")
```

output
```
integrate(x^8/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{x^8}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{x^8}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)}\,dx$$

input
```
integrate(x^8/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="giac
")
```

output
```
integrate(x^8/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^8}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{x^8}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,\left(f\,x^2+e\right)}\,dx$$

input
```
int(x^8/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)),x)
```

output
```
int(x^8/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)), x)
```

Reduce [F]

$$\int \frac{x^8}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \text{Too large to display}$$

input
```
int(x^8/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x)
```

output
```
( - 4*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*d*f*x - 4*sqrt(c + d*x**2)*sqrt(
a + b*x**2)*b*c*f*x - 5*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b*d*e*x + 3*sqrt
(c + d*x**2)*sqrt(a + b*x**2)*b*d*f*x**3 + 8*int((sqrt(c + d*x**2)*sqrt(a
+ b*x**2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2
 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*a**2*d**2*f**2 + 7*int((sqrt(c
 + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f
*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*a*b*c*d*f**2
 + 10*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c*e + a*c*f*x**2 + a
*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6
),x)*a*b*d**2*e*f + 8*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c*e
+ a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x
**4 + b*d*f*x**6),x)*b**2*c**2*f**2 + 10*int((sqrt(c + d*x**2)*sqrt(a + b*
x**2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b
*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*b**2*c*d*e*f + 15*int((sqrt(c + d*
x**2)*sqrt(a + b*x**2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4
 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*b**2*d**2*e**2 +
4*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c*e + a*c*f*x**2 + a*d*e
*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)
*a**2*c*d*f**2 + 8*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c*e + a
*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x...
```

3.182 $\int \frac{x^6}{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 438

$$\begin{aligned}
\int \frac{x^6}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx
&= -\frac{(3bde+2bcf+2adf)x\sqrt{c+dx^2}}{3bd^2f^2\sqrt{a+bx^2}}+\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3bdf} \\
&\quad+\frac{\sqrt{a}(3bde+2bcf+2adf)\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{3b^{3/2}d^2f^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} \\
&\quad+\frac{a^{3/2}(acf^2-be(3de+cf))\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{3b^{3/2}cdf^2(be-af)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} \\
&\quad+\frac{a^{3/2}e^2\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}cf^2(be-af)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}
\end{aligned}$$

output
```
-1/3*(2*a*d*f+2*b*c*f+3*b*d*e)*x*(d*x^2+c)^(1/2)/b/d^2/f^2/(b*x^2+a)^(1/2)
+1/3*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b/d/f+1/3*a^(1/2)*(2*a*d*f+2*b*c*f+
3*b*d*e)*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-
a*d/b/c)^(1/2))/b^(3/2)/d^2/f^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^
(1/2)+1/3*a^(3/2)*(a*c*f^2-b*e*(c*f+3*d*e))*(d*x^2+c)^(1/2)*InverseJacobiA
M(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/b^(3/2)/c/d/f^2/(-a*f+b*e)/
(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)+a^(3/2)*e^2*(d*x^2+c)^(1/2
)*EllipticPi(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),1-a*f/b/e,(1-a*d/b/c)^(1/
2))/b^(1/2)/c/f^2/(-a*f+b*e)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/
2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.90 (sec) , antiderivative size = 315, normalized size of antiderivative = 0.72

$$\int \frac{x^6}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx$$
$$= \frac{icf(3bde+2bcf+2adf)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(i\text{arcsinh}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)-i(acdf^2+b(3d^2e^2+3cdef+2c^2}{}$$

input
```
Integrate[x^6/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)),x]
```

output
```
(I*c*f*(3*b*d*e + 2*b*c*f + 2*a*d*f)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/
c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*(a*c*d*f^2 + b*(3*d^
2*e^2 + 3*c*d*e*f + 2*c^2*f^2))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*El
lipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + d*(Sqrt[b/a]*f^2*x*(a + b*x
^2)*(c + d*x^2) + (3*I)*b*d*e^2*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*El
lipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(3*b*Sqrt[b/a
]*d^2*f^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx$$
$$\Big\downarrow \text{ 450}$$
$$\int \frac{x^6}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)}dx$$

input
```
Int[x^6/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [A] (verified)

Time = 20.43 (sec) , antiderivative size = 510, normalized size of antiderivative = 1.16

method	result
risch	$\frac{x\sqrt{b\,x^2+a}\,\sqrt{x^2d+c}}{3bdf}-\frac{\left(-\frac{f(2adf+2bcf+3bde)c\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\left(\text{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)-\text{EllipticE}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}\,d}\right.}{}$
default	$\frac{\left(\sqrt{-\frac{b}{a}}\,b\,d^2f^2x^5+\sqrt{-\frac{b}{a}}\,a\,d^2f^2x^3+\sqrt{-\frac{b}{a}}\,bcd\,f^2x^3+\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{x^2d+c}{c}}\,\text{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)acd\,f^2+2\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{x^2d+c}{c}}\,\text{Elli}\right.}{}$
elliptic	$\frac{\sqrt{(b\,x^2+a)(x^2d+c)}\,\left(\frac{x\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{3fbd}+\frac{\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\text{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)e^2}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}\,f^3}+\frac{\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\text{EllipticF}\left(x\right.}{3\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2}}\right.}{}$

input
```
int(x^6/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
1/3*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b/d/f-1/3/f/b/d*(1/f^2*(-f*(2*a*d*f+
2*b*c*f+3*b*d*e)*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x
^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/
b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2)))+a*c*f^2/(-b/
a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)
^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-3*b*d*e^2/(-b/a)
^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(
1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2)))+3*b*d*e^2/f^2/(-b
/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c
)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2)))*((
b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^6}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate(x^6/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^6}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{x^6}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx$$

input
```
integrate(x**6/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2)/(f*x**2+e),x)
```

output
```
Integral(x**6/(sqrt(a + b*x**2)*sqrt(c + d*x**2)*(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{x^6}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{x^6}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)}\,dx$$

input
```
integrate(x^6/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="maxi
ma")
```

output
```
integrate(x^6/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{x^6}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{x^6}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)}\,dx$$

input
```
integrate(x^6/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="giac
")
```

output
```
integrate(x^6/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{x^6}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,\left(f\,x^2+e\right)}\,dx$$

input
```
int(x^6/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)),x)
```

output
```
int(x^6/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)), x)
```

Reduce [F]

$$\int \frac{x^6}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx$$
$$= \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x-2\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^4}{bdf\,x^6+adf\,x^4+bcf\,x^4+bde\,x^4+acf\,x^2+ade\,x^2+bce\,x^2+ace}dx\right)adf-2\left(\int \frac{}{bdf\,x^6+adf\,x^4+bcf}\right.}{}$$

input
```
int(x^6/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x)
```

output

```
(sqrt(c + d*x**2)*sqrt(a + b*x**2)*x - 2*int((sqrt(c + d*x**2)*sqrt(a + b*
x**2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b
*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*a*d*f - 2*int((sqrt(c + d*x**2)*sq
rt(a + b*x**2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e
*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*b*c*f - 3*int((sqrt(c + d
*x**2)*sqrt(a + b*x**2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**
4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*b*d*e - int((sqr
t(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*
d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*a*c*f - 2
*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c*e + a*c*f*x**2 + a*d*e*
x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*
a*d*e - 2*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c*e + a*c*f*x**2
 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*
x**6),x)*b*c*e - int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c*e + a*c*f*x*
*2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*
f*x**6),x)*a*c*e)/(3*b*d*f)
```

3.183 $\int \frac{x^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 334

$$\begin{aligned}\int \frac{x^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx &= \frac{x\sqrt{c+dx^2}}{df\sqrt{a+bx^2}} - \frac{\sqrt{a}\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{b}df\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} \\ &+ \frac{\sqrt{a}e\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}f(de-cf)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} \\ &- \frac{c^{3/2}e\sqrt{a+bx^2}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}f(de-cf)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}\end{aligned}$$

output

```
x*(d*x^2+c)^(1/2)/d/f/(b*x^2+a)^(1/2)-a^(1/2)*(d*x^2+c)^(1/2)*EllipticE(b^
(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/b^(1/2)/d/f/(b*x^2+a)
^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)+a^(1/2)*e*(d*x^2+c)^(1/2)*InverseJa
cobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/b^(1/2)/f/(-c*f+d*e)/(
b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)-c^(3/2)*e*(b*x^2+a)^(1/2)*E
llipticPi(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),1-c*f/d/e,(1-b*c/a/d)^(1/2))
/a/d^(1/2)/f/(-c*f+d*e)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.14 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.51

$$\int \frac{x^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx =$$
$$-\frac{i\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\left(cfE\left(i\text{arcsinh}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)-(de+cf)\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{\frac{b}{a}}x\right),\frac{ad}{bc}\right)+de\,\text{El}\right.}{\sqrt{\frac{b}{a}}df^2\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

input
```
Integrate[x^4/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)),x]
```

output
```
((-I)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(c*f*EllipticE[I*ArcSinh[Sqr
t[b/a]*x], (a*d)/(b*c)] - (d*e + c*f)*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a
*d)/(b*c)] + d*e*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*
c)]))/(Sqrt[b/a]*d*f^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx$$
$$\Big\downarrow \; 450$$
$$\int \frac{x^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)}dx$$

input
```
Int[x^4/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [A] (verified)

Time = 5.75 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.60

method	result
default	$\frac{\left(-\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)cf-\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)de+\operatorname{EllipticE}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)cf+\operatorname{EllipticPi}\left(x\sqrt{-\frac{b}{a}},\frac{af}{be},\frac{\sqrt{-\frac{d}{c}}}{\sqrt{-\frac{b}{a}}}\right)de\right)\sqrt{\cdots}}{d\,f^{2}\sqrt{-\frac{b}{a}}\,(bd\,x^{4}+ad\,x^{2}+x^{2}bc+ac)\cdots}$
elliptic	$\frac{\sqrt{(b\,x^{2}+a)(x^{2}d+c)}\left(-\frac{e\sqrt{1+\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{f^{2}\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^{4}+ad\,x^{2}+x^{2}bc+ac}}-\frac{c\sqrt{1+\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{f\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^{4}+ad\,x^{2}+x^{2}bc+ac}\,d}+\cdots\right.}{\sqrt{b\,x^{2}+a}\,\sqrt{x^{2}d+c}}$

input
```
int(x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
(-EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*c*f-EllipticF(x*(-b/a)^(1/2),(
a*d/b/c)^(1/2))*d*e+EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*c*f+Elliptic
Pi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*d*e)*((d*x^2+c)/c)^
(1/2)*((b*x^2+a)/a)^(1/2)*(d*x^2+c)^(1/2)*(b*x^2+a)^(1/2)/d/f^2/(-b/a)^(1/
2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx=\text{Timed out}$$

input
```
integrate(x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{x^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx$$

input
```
integrate(x**4/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2)/(f*x**2+e),x)
```

output
```
Integral(x**4/(sqrt(a + b*x**2)*sqrt(c + d*x**2)*(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{x^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{x^4}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)}\,dx$$

input
```
integrate(x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="maxi
ma")
```

output
```
integrate(x^4/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{x^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{x^4}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)}\,dx$$

input
```
integrate(x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="giac
")
```

output
```
integrate(x^4/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{x^4}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,(f\,x^2+e)}\,dx$$

input
```
int(x^4/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)),x)
```

output
```
int(x^4/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)), x)
```

Reduce [F]

$$\int \frac{x^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^4}{bdf\,x^6+adf\,x^4+bcf\,x^4+bde\,x^4+acf\,x^2+ade\,x^2+bce\,x^2+ace}dx$$

input
```
int(x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*e*x
**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)
```

3.184 $\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 205

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = -\frac{\sqrt{a}\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}(de-cf)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} + \frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(de-cf)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output
```
-a^(1/2)*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/
b/c)^(1/2))/b^(1/2)/(-c*f+d*e)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(
1/2)+c^(3/2)*(b*x^2+a)^(1/2)*EllipticPi(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2
),1-c*f/d/e,(1-b*c/a/d)^(1/2))/a/d^(1/2)/(-c*f+d*e)/(c*(b*x^2+a)/a/(d*x^2+
c))^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.14 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.63

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = -\frac{i\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\left(\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{\frac{b}{a}}x\right),\frac{ad}{bc}\right)-\text{EllipticPi}\left(\frac{af}{be},i\text{arcsinh}\left(\sqrt{\frac{b}{a}}x\right),\frac{ad}{bc}\right)\right)}{\sqrt{\frac{b}{a}}f\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

input
```
Integrate[x^2/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)),x]
```

output
```
((-I)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(EllipticF[I*ArcSinh[Sqrt[b/
a]*x], (a*d)/(b*c)] - EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d
)/(b*c)]))/(Sqrt[b/a]*f*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx$$
$$\Big\downarrow \text{ 450}$$
$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)}dx$$

input
```
Int[x^2/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [A] (verified)

Time = 5.82 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.70

method	result	siz
default	$\frac{\left(\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)-\operatorname{EllipticPi}\left(x\sqrt{-\frac{b}{a}},\frac{af}{be},\frac{\sqrt{-\frac{d}{c}}}{\sqrt{-\frac{b}{a}}}\right)\right)\sqrt{\frac{x^2d+c}{c}}\,\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{x^2d+c}\,\sqrt{b\,x^2+a}}{f\sqrt{-\frac{b}{a}}\,(bd\,x^4+ad\,x^2+x^2bc+ac)}$	14
elliptic	$\frac{\sqrt{(b\,x^2+a)(x^2d+c)}\left(\frac{\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{f\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}-\frac{\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\operatorname{EllipticPi}\left(x\sqrt{-\frac{b}{a}},\frac{af}{be},\frac{\sqrt{-\frac{d}{c}}}{\sqrt{-\frac{b}{a}}}\right)}{f\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}\right)}{\sqrt{b\,x^2+a}\,\sqrt{x^2d+c}}$	22

input
```
int(x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
(EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))-EllipticPi(x*(-b/a)^(1/2),a*f/b
/e,(-1/c*d)^(1/2)/(-b/a)^(1/2)))/f*((d*x^2+c)/c)^(1/2)*((b*x^2+a)/a)^(1/2)
*(d*x^2+c)^(1/2)*(b*x^2+a)^(1/2)/(-b/a)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c
)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate(x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx$$

input
```
integrate(x**2/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2)/(f*x**2+e),x)
```

output
```
Integral(x**2/(sqrt(a + b*x**2)*sqrt(c + d*x**2)*(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)}\,dx$$

input
```
integrate(x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="maxi
ma")
```

output
```
integrate(x^2/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)}\,dx$$

input
```
integrate(x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="giac
")
```

output
```
integrate(x^2/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{x^2}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,(f\,x^2+e)}\,dx$$

input
```
int(x^2/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)),x)
```

output
```
int(x^2/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)), x)
```

Reduce [F]

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^2}{bdf\,x^6+adf\,x^4+bcf\,x^4+bde\,x^4+acf\,x^2+ade\,x^2+bce\,x^2+ace}dx$$

input
```
int(x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c*e + a*c*f*x**2 + a*d*e*x
**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)
```

3.185 $\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 212

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \frac{\sqrt{a}\sqrt{b}\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{c(be-af)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} - \frac{a^{3/2}f\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(be-af)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
a^(1/2)*b^(1/2)*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),
(1-a*d/b/c)^(1/2))/c/(-a*f+b*e)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^
(1/2)-a^(3/2)*f*(d*x^2+c)^(1/2)*EllipticPi(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(
1/2),1-a*f/b/e,(1-a*d/b/c)^(1/2))/b^(1/2)/c/e/(-a*f+b*e)/(b*x^2+a)^(1/2)/(
a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.95 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.48

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = -\frac{i\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticPi}\left(\frac{af}{be},i\text{arcsinh}\left(\sqrt{\frac{b}{a}}x\right),\frac{ad}{bc}\right)}{\sqrt{\frac{b}{a}}e\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

input
```
Integrate[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)),x]
```

output
```
((-I)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e), I*Ar
cSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(Sqrt[b/a]*e*Sqrt[a + b*x^2]*Sqrt[c + d*
x^2])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.47, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {413, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx$$

$$\Big\downarrow \text{413}$$

$$\frac{\sqrt{\frac{bx^2}{a}+1}\int \frac{1}{\sqrt{\frac{bx^2}{a}+1}\sqrt{dx^2+c}(fx^2+e)}dx}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{413}$$

$$\frac{\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\int \frac{1}{\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}(fx^2+e)}dx}{\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

$$\Big\downarrow \text{412}$$

$$\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{\sqrt{b}e\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

input
```
Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)),x]
```

output
```
(Sqrt[-a]*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e),
ArcSin[(Sqrt[b]*x)/Sqrt[-a]], (a*d)/(b*c)])/(Sqrt[b]*e*Sqrt[a + b*x^2]*Sqr
t[c + d*x^2])
```

Defintions of rubi rules used

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

Maple [A] (verified)

Time = 5.77 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.56

method	result	size
default	$\frac{\operatorname{EllipticPi}\left(x\sqrt{-\frac{b}{a}}, \frac{af}{be}, \frac{\sqrt{-\frac{d}{c}}}{\sqrt{-\frac{b}{a}}}\right)\sqrt{\frac{x^2d+c}{c}}\,\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{x^2d+c}\,\sqrt{b\,x^2+a}}{e\sqrt{-\frac{b}{a}}\,(bd\,x^4+ad\,x^2+x^2bc+ac)}$	118
elliptic	$\frac{\sqrt{(b\,x^2+a)(x^2d+c)}\,\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\operatorname{EllipticPi}\left(x\sqrt{-\frac{b}{a}}, \frac{af}{be}, \frac{\sqrt{-\frac{d}{c}}}{\sqrt{-\frac{b}{a}}}\right)}{\sqrt{b\,x^2+a}\,\sqrt{x^2d+c}\,e\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}$	133

input
```
int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*((d*x^2+c)/
c)^(1/2)*((b*x^2+a)/a)^(1/2)*(d*x^2+c)^(1/2)*(b*x^2+a)^(1/2)/e/(-b/a)^(1/2
)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx$$

input
```
integrate(1/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2)/(f*x**2+e),x)
```

output
```
Integral(1/(sqrt(a + b*x**2)*sqrt(c + d*x**2)*(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="maxima
")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx=\int \frac{1}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,(f\,x^2+e)}\,dx$$

input
```
int(1/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)),x)
```

output
```
int(1/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx$$
$$=\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{bdf\,x^6+adf\,x^4+bcf\,x^4+bde\,x^4+acf\,x^2+ade\,x^2+bce\,x^2+ace}dx$$

input
```
int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c*e + a*c*f*x**2 + a*d*e*x**2 +
 a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)
```

3.186 $\int \frac{1}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 379

$$\int \frac{1}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \frac{bx\sqrt{c+dx^2}}{ace\sqrt{a+bx^2}}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{acex}-\frac{\sqrt{b}\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}ce\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}-\frac{\sqrt{a}\sqrt{b}f\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{ce(be-af)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}+\frac{a^{3/2}f^2\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce^2(be-af)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output

```
b*x*(d*x^2+c)^(1/2)/a/c/e/(b*x^2+a)^(1/2)-(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/
a/c/e/x-b^(1/2)*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1
/2),(1-a*d/b/c)^(1/2))/a^(1/2)/c/e/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a
))^(1/2)-a^(1/2)*b^(1/2)*f*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*
x/a^(1/2)),(1-a*d/b/c)^(1/2))/c/e/(-a*f+b*e)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/
c/(b*x^2+a))^(1/2)+a^(3/2)*f^2*(d*x^2+c)^(1/2)*EllipticPi(b^(1/2)*x/a^(1/2
)/(1+b*x^2/a)^(1/2),1-a*f/b/e,(1-a*d/b/c)^(1/2))/b^(1/2)/c/e^2/(-a*f+b*e)/
(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.10 (sec) , antiderivative size = 297, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx =$$
$$-\frac{abce+b^2cex^2+abdex^2+b^2dex^4+iab\sqrt{\frac{b}{a}}cex\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(i\text{arcsinh}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)-iab\sqrt{\frac{b}{a}}ce}{ab}$$

input
```
Integrate[1/(x^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)),x]
```

output
```
-((a*b*c*e + b^2*c*e*x^2 + a*b*d*e*x^2 + b^2*d*e*x^4 + I*a*b*Sqrt[b/a]*c*e
*x*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x
], (a*d)/(b*c)] - I*a*b*Sqrt[b/a]*c*e*x*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^
2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*a^2*Sqrt[b/a]*c*f
*x*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e), I*ArcSi
nh[Sqrt[b/a]*x], (a*d)/(b*c)])/(a*b*c*e^2*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2
]))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx$$
$$\Big\downarrow \ 450$$
$$\int \frac{1}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)}dx$$

input
```
Int[1/(x^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [A] (verified)

Time = 8.98 (sec) , antiderivative size = 302, normalized size of antiderivative = 0.80

method	result
default	$\frac{\left(-\sqrt{-\frac{b}{a}}\, bde\, x^4-\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{x^2 d+c}{c}}\,\text{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)bcex+\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{x^2 d+c}{c}}\,\text{EllipticE}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)bcex-\sqrt{\frac{b\,x^2+a}{a}} \cdots\right.}{\sqrt{-\frac{b}{a}}\,x\,e^2 ca(bd\,x^4+ad\,x^2+x^2 \cdots}$
risch	$-\frac{\sqrt{b\,x^2+a}\,\sqrt{x^2 d+c}}{acex}+\frac{\left(-\frac{bc\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\left(\text{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)-\text{EllipticE}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}-\frac{acf\sqrt{1+\frac{b\,x^2}{a}}\,\cdots}{e\sqrt{\cdots}}\cdots\right.}{ace\sqrt{b\,x^2+a}\,\sqrt{x^2 d+c}\cdots}$
elliptic	$\frac{\sqrt{(b\,x^2+a)(x^2 d+c)}\left(-\frac{\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{acex}-\frac{b\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\text{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{ae\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}+\frac{b\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\text{EllipticE}\left(\cdots\right.}{ae\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x\cdots}}\cdots\right.}{\sqrt{b\,x^2+a}\,\sqrt{x^2 d+c}\cdots}$

input
```
int(1/x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x,method=_RETURNVERBOS
E)
```

output
```
(-(-b/a)^(1/2)*b*d*e*x^4-((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF
(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b*c*e*x+((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)
^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b*c*e*x-((b*x^2+a)/a)^(1/
2)*((d*x^2+c)/c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-
b/a)^(1/2))*a*c*f*x-(-b/a)^(1/2)*a*d*e*x^2-(-b/a)^(1/2)*b*c*e*x^2-(-b/a)^(
1/2)*a*c*e)*(d*x^2+c)^(1/2)*(b*x^2+a)^(1/2)/(-b/a)^(1/2)/x/e^2/c/a/(b*d*x^
4+a*d*x^2+b*c*x^2+a*c)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate(1/x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{1}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx$$

input
```
integrate(1/x**2/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2)/(f*x**2+e),x)
```

output
```
Integral(1/(x**2*sqrt(a + b*x**2)*sqrt(c + d*x**2)*(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="ma
xima")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="gi
ac")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{1}{x^2\,\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,\left(f\,x^2+e\right)}\,dx$$

input
```
int(1/(x^2*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)),x)
```

output
```
int(1/(x^2*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)), x)
```

Reduce [F]

$$\int \frac{1}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx$$
$$= \int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{bdf\,x^8+adf\,x^6+bcf\,x^6+bde\,x^6+acf\,x^4+ade\,x^4+bce\,x^4+ace\,x^2}dx$$

input
```
int(1/x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c*e*x**2 + a*c*f*x**4 + a*d*e*x
**4 + a*d*f*x**6 + b*c*e*x**4 + b*c*f*x**6 + b*d*e*x**6 + b*d*f*x**8),x)
```

3.187 $\int \frac{1}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 441

$$\begin{aligned}
&\int \frac{1}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx \\
&= \frac{(2bce+2ade+3acf)\sqrt{a+bx^2}}{3a^2ce^2x\sqrt{c+dx^2}} - \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{3acex^3} \\
&+ \frac{\sqrt{d}(2bce+2ade+3acf)\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3a^2c^{3/2}e^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} \\
&+ \frac{\sqrt{d}(3acf^2-be(de-cf))\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3a^2\sqrt{c}e^2(de-cf)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} \\
&- \frac{c^{3/2}f^3\sqrt{a+bx^2}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}e^3(de-cf)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}
\end{aligned}$$

output
```
1/3*(3*a*c*f+2*a*d*e+2*b*c*e)*(b*x^2+a)^(1/2)/a^2/c/e^2/x/(d*x^2+c)^(1/2)-
1/3*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a/c/e/x^3+1/3*d^(1/2)*(3*a*c*f+2*a*d*e
+2*b*c*e)*(b*x^2+a)^(1/2)*EllipticE(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),(1
-b*c/a/d)^(1/2))/a^2/c^(3/2)/e^2/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)
^(1/2)+1/3*d^(1/2)*(3*a*c*f^2-b*e*(-c*f+d*e))*(b*x^2+a)^(1/2)*InverseJacob
iAM(arctan(d^(1/2)*x/c^(1/2)),(1-b*c/a/d)^(1/2))/a^2/c^(1/2)/e^2/(-c*f+d*e
)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-c^(3/2)*f^3*(b*x^2+a)^(1
/2)*EllipticPi(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),1-c*f/d/e,(1-b*c/a/d)^(
1/2))/a/d^(1/2)/e^3/(-c*f+d*e)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(
1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.85 (sec) , antiderivative size = 341, normalized size of antiderivative = 0.77

$$\int \frac{1}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx$$
$$= \frac{\sqrt{\frac{b}{a}}e(a+bx^2)\,(c+dx^2)\,(2bcex^2+a(-ce+2dex^2+3cfx^2))+ibce(2bce+2ade+3acf)x^3\sqrt{1+\frac{bx^2}{a}}\sqrt{1}}{}$$

input
```
Integrate[1/(x^4*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)),x]
```

output
```
(Sqrt[b/a]*e*(a + b*x^2)*(c + d*x^2)*(2*b*c*e*x^2 + a*(-(c*e) + 2*d*e*x^2
+ 3*c*f*x^2)) + I*b*c*e*(2*b*c*e + 2*a*d*e + 3*a*c*f)*x^3*Sqrt[1 + (b*x^2)
/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I
*b*c*e*(2*b*c*e + a*d*e + 3*a*c*f)*x^3*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2
)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - (3*I)*a^2*c^2*f^2*x^
3*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e), I*ArcSin
h[Sqrt[b/a]*x], (a*d)/(b*c)])/(3*a^2*Sqrt[b/a]*c^2*e^3*x^3*Sqrt[a + b*x^2]
*Sqrt[c + d*x^2])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx$$
$$\Big\downarrow \; 450$$
$$\int \frac{1}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)}dx$$

input
```
Int[1/(x^4*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [A] (verified)

Time = 10.64 (sec) , antiderivative size = 445, normalized size of antiderivative = 1.01

method	result
risch	$-\frac{\sqrt{b x^2+a}\,\sqrt{x^2 d+c}\,(-3acf\,x^2-2ade\,x^2-2bce\,x^2+ace)}{3a^2c^2e^2x^3}-\frac{\left(-\frac{b(3acf+2ade+2bce)c\sqrt{1+\frac{b x^2}{a}}\,\sqrt{1+\frac{d x^2}{c}}\,\left(\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+}{cb}}\right.\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}\right.}{}$
default	$\frac{\left(3\sqrt{-\frac{b}{a}}\,abcdef\,x^6+2\sqrt{-\frac{b}{a}}\,ab\,d^2e^2x^6+2\sqrt{-\frac{b}{a}}\,b^2cd\,e^2x^6+3\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{x^2d+c}{c}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)ab\,c^2ef\,x^3+\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\right.}{}$
elliptic	$\frac{\sqrt{(b\,x^2+a)(x^2d+c)}\left(-\frac{\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{3ace\,x^3}+\frac{(3acf+2ade+2bce)\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{3a^2c^2e^2x}+\frac{db\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-}\right.}{3ace\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+x^2bc+}}\right.}{}$

input
```
int(1/x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x,method=_RETURNVERBOS
E)
```

output
```
-1/3*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(-3*a*c*f*x^2-2*a*d*e*x^2-2*b*c*e*x^2
+a*c*e)/a^2/c^2/e^2/x^3-1/3/a^2/c^2/e^2*(-b*(3*a*c*f+2*a*d*e+2*b*c*e)*c/(-
b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*
c)^(1/2)*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(
-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2)))+a*c*d*e*b/(-b/a)^(1/2)*(1+b*x^2/a)^
(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-
b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-3*a^2*c^2*f^2/e/(-b/a)^(1/2)*(1+b*x^2
/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi
(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2)))*((b*x^2+a)*(d*x^2+c)
)^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx=\text{Timed out}$$

input
```
integrate(1/x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx=\int \frac{1}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx$$

input
```
integrate(1/x**4/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2)/(f*x**2+e),x)
```

output
```
Integral(1/(x**4*sqrt(a + b*x**2)*sqrt(c + d*x**2)*(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx=\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="ma
xima")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)*x^4), x)
```

Giac [F]

$$\int \frac{1}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx=\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="gi
ac")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx=\int \frac{1}{x^4\,\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,\left(f\,x^2+e\right)}\,dx$$

input
```
int(1/(x^4*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)),x)
```

output
```
int(1/(x^4*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)), x)
```

Reduce [F]

$$\int \frac{1}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx$$
$$=\frac{-\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}-\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^2}{bdf\,x^6+adf\,x^4+bcf\,x^4+bde\,x^4+acf\,x^2+ade\,x^2+bce\,x^2+ace}dx\right)bdf\,x^3-3\left(\int \frac{}{bdf\,x^8+adf\,x^6+bc}\right.}{}$$

input
```
int(1/x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x)
```

output
```
( - sqrt(c + d*x**2)*sqrt(a + b*x**2) - int((sqrt(c + d*x**2)*sqrt(a + b*x
**2)*x**2)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*
c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*b*d*f*x**3 - 3*int((sqrt(c + d*x**2
)*sqrt(a + b*x**2))/(a*c*e*x**2 + a*c*f*x**4 + a*d*e*x**4 + a*d*f*x**6 + b
*c*e*x**4 + b*c*f*x**6 + b*d*e*x**6 + b*d*f*x**8),x)*a*c*f*x**3 - 2*int((s
qrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c*e*x**2 + a*c*f*x**4 + a*d*e*x**4 +
a*d*f*x**6 + b*c*e*x**4 + b*c*f*x**6 + b*d*e*x**6 + b*d*f*x**8),x)*a*d*e*x
**3 - 2*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c*e*x**2 + a*c*f*x**4 +
 a*d*e*x**4 + a*d*f*x**6 + b*c*e*x**4 + b*c*f*x**6 + b*d*e*x**6 + b*d*f*x*
*8),x)*b*c*e*x**3 - 2*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c*e + a*c
*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 +
 b*d*f*x**6),x)*a*d*f*x**3 - 2*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*
c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d
*e*x**4 + b*d*f*x**6),x)*b*c*f*x**3 - int((sqrt(c + d*x**2)*sqrt(a + b*x**
2))/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**
4 + b*d*e*x**4 + b*d*f*x**6),x)*b*d*e*x**3)/(3*a*c*e*x**3)
```

3.188 $\int \frac{1}{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 601

$$\begin{aligned}
&\int \frac{1}{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx \\
&= -\frac{\left(\frac{8b^2c^2e}{a}+bc(7de+10cf)+a\left(8d^2e+10cdf+\frac{15c^2f^2}{e}\right)\right)\sqrt{a+bx^2}}{15a^2c^2e^2x\sqrt{c+dx^2}} \\
&\quad-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{5acex^5}+\frac{(4bce+4ade+5acf)\sqrt{a+bx^2}\sqrt{c+dx^2}}{15a^2c^2e^2x^3} \\
&\quad-\frac{\sqrt{d}(8b^2c^2e^2+abce(7de+10cf)+a^2(8d^2e^2+10cdef+15c^2f^2))\sqrt{a+bx^2}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15a^3c^{5/2}e^3\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} \\
&\quad-\frac{\sqrt{d}(15a^2c^2f^3-4b^2ce^2(de-cf)-abe(4d^2e^2+cdef-5c^2f^2))\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\right.}{15a^3c^{3/2}e^3(de-cf)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} \\
&\quad+\frac{c^{3/2}f^4\sqrt{a+bx^2}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}e^4(de-cf)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}
\end{aligned}$$

output
```
-1/15*(8*b^2*c^2*e/a+b*c*(10*c*f+7*d*e)+a*(8*d^2*e+10*c*d*f+15*c^2*f^2/e))
*(b*x^2+a)^(1/2)/a^2/c^2/e^2/x/(d*x^2+c)^(1/2)-1/5*(b*x^2+a)^(1/2)*(d*x^2+
c)^(1/2)/a/c/e/x^5+1/15*(5*a*c*f+4*a*d*e+4*b*c*e)*(b*x^2+a)^(1/2)*(d*x^2+c
)^(1/2)/a^2/c^2/e^2/x^3-1/15*d^(1/2)*(8*b^2*c^2*e^2+a*b*c*e*(10*c*f+7*d*e)
+a^2*(15*c^2*f^2+10*c*d*e*f+8*d^2*e^2))*(b*x^2+a)^(1/2)*EllipticE(d^(1/2)*
x/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))/a^3/c^(5/2)/e^3/(c*(b*x^2+a
)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-1/15*d^(1/2)*(15*a^2*c^2*f^3-4*b^2*c*
e^2*(-c*f+d*e)-a*b*e*(-5*c^2*f^2+c*d*e*f+4*d^2*e^2))*(b*x^2+a)^(1/2)*Inver
seJacobiAM(arctan(d^(1/2)*x/c^(1/2)),(1-b*c/a/d)^(1/2))/a^3/c^(3/2)/e^3/(-
c*f+d*e)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)+c^(3/2)*f^4*(b*x^
2+a)^(1/2)*EllipticPi(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),1-c*f/d/e,(1-b*c
/a/d)^(1/2))/a/d^(1/2)/e^4/(-c*f+d*e)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x
^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.20 (sec) , antiderivative size = 962, normalized size of antiderivative = 1.60

$$\int \frac{1}{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \frac{-3a^3\sqrt{\frac{b}{a}}c^3e^3+a^3\left(\frac{b}{a}\right)^{3/2}c^3e^3x^2+a^3\sqrt{\frac{b}{a}}c^2de^3x^2+5a^3\sqrt{\frac{b}{a}}c^3e^2fx^2-4ab^2\sqrt{\frac{b}{a}}c^3e^3x^4-2a^3\left(\frac{b}{a}\right)^{3/2}c^2de^3x^4}{}$$

input
```
Integrate[1/(x^6*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)),x]
```

output
```
(-3*a^3*Sqrt[b/a]*c^3*e^3 + a^3*(b/a)^(3/2)*c^3*e^3*x^2 + a^3*Sqrt[b/a]*c^
2*d*e^3*x^2 + 5*a^3*Sqrt[b/a]*c^3*e^2*f*x^2 - 4*a*b^2*Sqrt[b/a]*c^3*e^3*x^
4 - 2*a^3*(b/a)^(3/2)*c^2*d*e^3*x^4 - 4*a^3*Sqrt[b/a]*c*d^2*e^3*x^4 - 5*a^
3*(b/a)^(3/2)*c^3*e^2*f*x^4 - 5*a^3*Sqrt[b/a]*c^2*d*e^2*f*x^4 - 15*a^3*Sqr
t[b/a]*c^3*e*f^2*x^4 - 8*b^3*Sqrt[b/a]*c^3*e^3*x^6 - 11*a*b^2*Sqrt[b/a]*c^
2*d*e^3*x^6 - 11*a^3*(b/a)^(3/2)*c*d^2*e^3*x^6 - 8*a^3*Sqrt[b/a]*d^3*e^3*x
^6 - 10*a*b^2*Sqrt[b/a]*c^3*e^2*f*x^6 - 15*a^3*(b/a)^(3/2)*c^2*d*e^2*f*x^6
 - 10*a^3*Sqrt[b/a]*c*d^2*e^2*f*x^6 - 15*a^3*(b/a)^(3/2)*c^3*e*f^2*x^6 - 1
5*a^3*Sqrt[b/a]*c^2*d*e*f^2*x^6 - 8*b^3*Sqrt[b/a]*c^2*d*e^3*x^8 - 7*a*b^2*
Sqrt[b/a]*c*d^2*e^3*x^8 - 8*a^3*(b/a)^(3/2)*d^3*e^3*x^8 - 10*a*b^2*Sqrt[b/
a]*c^2*d*e^2*f*x^8 - 10*a^3*(b/a)^(3/2)*c*d^2*e^2*f*x^8 - 15*a^3*(b/a)^(3/
2)*c^2*d*e*f^2*x^8 - I*b*c*e*(8*b^2*c^2*e^2 + a*b*c*e*(7*d*e + 10*c*f) + a
^2*(8*d^2*e^2 + 10*c*d*e*f + 15*c^2*f^2))*x^5*Sqrt[1 + (b*x^2)/a]*Sqrt[1 +
 (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + I*b*c*e*(8*b^
2*c^2*e^2 + a*b*c*e*(3*d*e + 10*c*f) + a^2*(4*d^2*e^2 + 5*c*d*e*f + 15*c^2
*f^2))*x^5*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqr
t[b/a]*x], (a*d)/(b*c)] + (15*I)*a^3*c^3*f^3*x^5*Sqrt[1 + (b*x^2)/a]*Sqrt[
1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)
])/(15*a^3*Sqrt[b/a]*c^3*e^4*x^5*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx$$
$$\Big\downarrow \text{ 450}$$
$$\int \frac{1}{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)}dx$$

input
```
Int[1/(x^6*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [A] (verified)

Time = 20.85 (sec) , antiderivative size = 792, normalized size of antiderivative = 1.32

method	result
risch	$-\frac{\sqrt{b\,x^2+a}\,\sqrt{x^2d+c}\,(15f^2x^4a^2c^2+10a^2cdef\,x^4+8a^2d^2e^2x^4+10ab\,c^2ef\,x^4+7abcd\,e^2x^4+8b^2c^2e^2x^4-5a^2c^2ef\,x^2-4a^2cd\,e^2x^2-4ab}{15a^3c^3e^3x^5}$
default	Expression too large to display
elliptic	Expression too large to display

input
```
int(1/x^6/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x,method=_RETURNVERBOS
E)
```

output
```
-1/15*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(15*a^2*c^2*f^2*x^4+10*a^2*c*d*e*f*x
^4+8*a^2*d^2*e^2*x^4+10*a*b*c^2*e*f*x^4+7*a*b*c*d*e^2*x^4+8*b^2*c^2*e^2*x^
4-5*a^2*c^2*e*f*x^2-4*a^2*c*d*e^2*x^2-4*a*b*c^2*e^2*x^2+3*a^2*c^2*e^2)/a^3
/c^3/e^3/x^5+1/15/c^3/a^3/e^3*(-b*(15*a^2*c^2*f^2+10*a^2*c*d*e*f+8*a^2*d^2
*e^2+10*a*b*c^2*e*f+7*a*b*c*d*e^2+8*b^2*c^2*e^2)*c/(-b/a)^(1/2)*(1+b*x^2/a
)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(EllipticF(x
*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+
b*c)/c/b)^(1/2)))-15*c^3*a^3*f^3/e/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2
/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*
f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))+4*a*b^2*c^2*d*e^2/(-b/a)^(1/2)*(1+b*x^2
/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(
x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+4*a^2*b*c*d^2*e^2/(-b/a)^(1/2)*(1
+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*Elli
pticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+5*a^2*b*c^2*d*e*f/(-b/a)^(1
/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2
)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2)))*((b*x^2+a)*(d*x^2+c)
)^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate(1/x^6/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{1}{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx$$

input
```
integrate(1/x**6/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2)/(f*x**2+e),x)
```

output
```
Integral(1/(x**6*sqrt(a + b*x**2)*sqrt(c + d*x**2)*(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)x^6}\,dx$$

input
```
integrate(1/x^6/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="ma
xima")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)*x^6), x)
```

Giac [F]

$$\int \frac{1}{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)x^6}\,dx$$

input
```
integrate(1/x^6/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="gi
ac")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx=\int \frac{1}{x^6\,\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,\left(f\,x^2+e\right)}\,dx$$

input
```
int(1/(x^6*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)),x)
```

output
```
int(1/(x^6*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)), x)
```

Reduce [F]

$$\int \frac{1}{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx=\text{Too large to display}$$

input
```
int(1/x^6/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x)
```

output

```
( - 3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*c*e + 5*sqrt(c + d*x**2)*sqrt(a
+ b*x**2)*a*c*f*x**2 + 4*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*d*e*x**2 + 4*
sqrt(c + d*x**2)*sqrt(a + b*x**2)*b*c*e*x**2 + 5*int((sqrt(c + d*x**2)*sqr
t(a + b*x**2)*x**2)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*
x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*a*b*c*d*f**2*x**5 + 4*int(
(sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c*e + a*c*f*x**2 + a*d*e*x**2
+ a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*a*b*d
**2*e*f*x**5 + 4*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c*e + a*c
*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 +
 b*d*f*x**6),x)*b**2*c*d*e*f*x**5 + 15*int((sqrt(c + d*x**2)*sqrt(a + b*x*
*2))/(a*c*e*x**2 + a*c*f*x**4 + a*d*e*x**4 + a*d*f*x**6 + b*c*e*x**4 + b*c
*f*x**6 + b*d*e*x**6 + b*d*f*x**8),x)*a**2*c**2*f**2*x**5 + 10*int((sqrt(c
 + d*x**2)*sqrt(a + b*x**2))/(a*c*e*x**2 + a*c*f*x**4 + a*d*e*x**4 + a*d*f
*x**6 + b*c*e*x**4 + b*c*f*x**6 + b*d*e*x**6 + b*d*f*x**8),x)*a**2*c*d*e*f
*x**5 + 8*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c*e*x**2 + a*c*f*x**4
 + a*d*e*x**4 + a*d*f*x**6 + b*c*e*x**4 + b*c*f*x**6 + b*d*e*x**6 + b*d*f*
x**8),x)*a**2*d**2*e**2*x**5 + 10*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/
(a*c*e*x**2 + a*c*f*x**4 + a*d*e*x**4 + a*d*f*x**6 + b*c*e*x**4 + b*c*f*x*
*6 + b*d*e*x**6 + b*d*f*x**8),x)*a*b*c**2*e*f*x**5 + 7*int((sqrt(c + d*x**
2)*sqrt(a + b*x**2))/(a*c*e*x**2 + a*c*f*x**4 + a*d*e*x**4 + a*d*f*x**6...
```

3.189 $\int \frac{x^8}{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 719

$$\int \frac{x^8}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx$$
$$= \frac{(4a^2df^2(de-cf)-b^2e(15d^2e^2-8cdef-4c^2f^2)+4abf(2d^2e^2-cdef-c^2f^2))\,x\sqrt{c+dx^2}}{6bd^2f^3(be-af)(de-cf)\sqrt{a+bx^2}}$$
$$+\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3bdf^2}+\frac{e^3x\sqrt{a+bx^2}\sqrt{c+dx^2}}{2f^2(be-af)(de-cf)\,(e+fx^2)}$$
$$-\frac{\sqrt{a}(4a^2df^2(de-cf)-b^2e(15d^2e^2-8cdef-4c^2f^2)+4abf(2d^2e^2-cdef-c^2f^2))\,\sqrt{c+dx^2}E\Big(\text{arcta}}{6b^{3/2}d^2f^3(be-af)(de-cf)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$
$$-\frac{a^{3/2}(2a^2cf^3-2abef(9de+2cf)+b^2e^2(15de+2cf))\,\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{6b^{3/2}cdf^3(be-af)^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$
$$-\frac{a^{3/2}e^2(af(6de-7cf)-be(5de-6cf))\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{2\sqrt{b}cf^3(be-af)^2(de-cf)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
1/6*(4*a^2*d*f^2*(-c*f+d*e)-b^2*e*(-4*c^2*f^2-8*c*d*e*f+15*d^2*e^2)+4*a*b*
f*(-c^2*f^2-c*d*e*f+2*d^2*e^2))*x*(d*x^2+c)^(1/2)/b/d^2/f^3/(-a*f+b*e)/(-c
*f+d*e)/(b*x^2+a)^(1/2)+1/3*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b/d/f^2+1/2*
e^3*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/f^2/(-a*f+b*e)/(-c*f+d*e)/(f*x^2+e)-
1/6*a^(1/2)*(4*a^2*d*f^2*(-c*f+d*e)-b^2*e*(-4*c^2*f^2-8*c*d*e*f+15*d^2*e^2
)+4*a*b*f*(-c^2*f^2-c*d*e*f+2*d^2*e^2))*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*
x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/b^(3/2)/d^2/f^3/(-a*f+b*e)/
(-c*f+d*e)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)-1/6*a^(3/2)*(2*
a^2*c*f^3-2*a*b*e*f*(2*c*f+9*d*e)+b^2*e^2*(2*c*f+15*d*e))*(d*x^2+c)^(1/2)*
InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/b^(3/2)/c/d/f
^3/(-a*f+b*e)^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)-1/2*a^(3/2
)*e^2*(a*f*(-7*c*f+6*d*e)-b*e*(-6*c*f+5*d*e))*(d*x^2+c)^(1/2)*EllipticPi(b
^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),1-a*f/b/e,(1-a*d/b/c)^(1/2))/b^(1/2)/c/
f^3/(-a*f+b*e)^2/(-c*f+d*e)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2
)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.73 (sec) , antiderivative size = 537, normalized size of antiderivative = 0.75

$$\int \frac{x^8}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx$$

$$= \frac{icf(4a^2df^2(-de+cf)+b^2e(15d^2e^2-8cdef-4c^2f^2)+4abf(-2d^2e^2+cdef+c^2f^2))\sqrt{1+\frac{bx^2}{a}}\sqrt{1+ \ldots}}{\ldots}$$

input
```
Integrate[x^8/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^2),x]
```

output
```
(I*c*f*(4*a^2*d*f^2*(-(d*e) + c*f) + b^2*e*(15*d^2*e^2 - 8*c*d*e*f - 4*c^2
*f^2) + 4*a*b*f*(-2*d^2*e^2 + c*d*e*f + c^2*f^2))*Sqrt[1 + (b*x^2)/a]*Sqrt
[1 + (d*x^2)/c]*(e + f*x^2)*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]
 - I*(-(d*e) + c*f)*(2*a^2*c*d*f^3 + 2*a*b*f*(9*d^2*e^2 + 5*c*d*e*f + 2*c^
2*f^2) - b^2*e*(15*d^2*e^2 + 12*c*d*e*f + 4*c^2*f^2))*Sqrt[1 + (b*x^2)/a]*
Sqrt[1 + (d*x^2)/c]*(e + f*x^2)*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b
*c)] + d*(Sqrt[b/a]*f^2*x*(a + b*x^2)*(c + d*x^2)*(2*a*f*(-(d*e) + c*f)*(e
 + f*x^2) + b*e*(-2*c*f*(e + f*x^2) + d*e*(5*e + 2*f*x^2))) + (3*I)*b*d*e^
2*(b*e*(5*d*e - 6*c*f) + a*f*(-6*d*e + 7*c*f))*Sqrt[1 + (b*x^2)/a]*Sqrt[1
+ (d*x^2)/c]*(e + f*x^2)*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (
a*d)/(b*c)]))/(6*b*Sqrt[b/a]*d^2*f^4*(b*e - a*f)*(d*e - c*f)*Sqrt[a + b*x^
2]*Sqrt[c + d*x^2]*(e + f*x^2))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx$$

$$\Big\downarrow \; 450$$

$$\int \frac{x^8}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}dx$$

input
```
Int[x^8/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1453 vs. $2(681) = 1362$.

Time = 25.77 (sec) , antiderivative size = 1454, normalized size of antiderivative = 2.02

method	result	size
risch	Expression too large to display	1454
elliptic	Expression too large to display	1700
default	Expression too large to display	3438

input
```
int(x^8/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x,method=_RETURNVERBOS
E)
```

output
```
1/3*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b/d/f^2-1/3/f^2/b/d*(1/f^2*(-2*f*(a*
d*f+b*c*f+3*b*d*e)*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d
*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/
c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2)))+a*c*f^2/(-
b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*
c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-9*b*d*e^2/(-b/
a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)
^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2)))+12*b*d*e^2/f^2/
(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+
a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))-
3*b*d*e^4/f^2*(1/2*f^2/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/e*x*(b*d*x^4+a*d*
x^2+b*c*x^2+a*c)^(1/2)/(f*x^2+e)-1/2*d*b/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)
/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2
+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+1/2*f*b/(a*
c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/e*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2
/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1
+(a*d+b*c)/c/b)^(1/2))-1/2*f*b/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/e*c/(-b/a
)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^
(1/2)*EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+1/2/(a*c*f^2-a*d*
e*f-b*c*e*f+b*d*e^2)/e^2*f^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^8}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(x^8/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^8}{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^2}\,dx = \int \frac{x^8}{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^2}\,dx$$

input
```
integrate(x**8/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**2,x)
```

output
```
Integral(x**8/(sqrt(a + b*x**2)*sqrt(c + d*x**2)*(e + f*x**2)**2), x)
```

Maxima [F]

$$\int \frac{x^8}{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^2}\,dx = \int \frac{x^8}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^2}\,dx$$

input
```
integrate(x^8/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="ma
xima")
```

output
```
integrate(x^8/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^2), x)
```

Giac [F]

$$\int \frac{x^8}{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^2}\,dx = \int \frac{x^8}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^2}\,dx$$

input
```
integrate(x^8/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="gi
ac")
```

output
```
integrate(x^8/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^8}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{x^8}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,(f\,x^2+e)^2}\,dx$$

input
```
int(x^8/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^2),x)
```

output
```
int(x^8/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^2), x)
```

Reduce [F]

$$\int \frac{x^8}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \text{too large to display}$$

input
```
int(x^8/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x)
```

output

```
( - sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*c*f*x - 4*sqrt(c + d*x**2)*sqrt(a
+ b*x**2)*a*d*e*x - 4*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b*c*e*x + 3*sqrt(c
 + d*x**2)*sqrt(a + b*x**2)*b*d*e*x**3 + int((sqrt(c + d*x**2)*sqrt(a + b*
x**2)*x**6)/(a*c*e**2 + 2*a*c*e*f*x**2 + a*c*f**2*x**4 + a*d*e**2*x**2 + 2
*a*d*e*f*x**4 + a*d*f**2*x**6 + b*c*e**2*x**2 + 2*b*c*e*f*x**4 + b*c*f**2*
x**6 + b*d*e**2*x**4 + 2*b*d*e*f*x**6 + b*d*f**2*x**8),x)*a*b*c*d*e*f**2 +
 int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**6)/(a*c*e**2 + 2*a*c*e*f*x**2 +
 a*c*f**2*x**4 + a*d*e**2*x**2 + 2*a*d*e*f*x**4 + a*d*f**2*x**6 + b*c*e**2
*x**2 + 2*b*c*e*f*x**4 + b*c*f**2*x**6 + b*d*e**2*x**4 + 2*b*d*e*f*x**6 +
b*d*f**2*x**8),x)*a*b*c*d*f**3*x**2 - 2*int((sqrt(c + d*x**2)*sqrt(a + b*x
**2)*x**6)/(a*c*e**2 + 2*a*c*e*f*x**2 + a*c*f**2*x**4 + a*d*e**2*x**2 + 2*
a*d*e*f*x**4 + a*d*f**2*x**6 + b*c*e**2*x**2 + 2*b*c*e*f*x**4 + b*c*f**2*x
**6 + b*d*e**2*x**4 + 2*b*d*e*f*x**6 + b*d*f**2*x**8),x)*a*b*d**2*e**2*f -
 2*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**6)/(a*c*e**2 + 2*a*c*e*f*x**2
 + a*c*f**2*x**4 + a*d*e**2*x**2 + 2*a*d*e*f*x**4 + a*d*f**2*x**6 + b*c*e*
*2*x**2 + 2*b*c*e*f*x**4 + b*c*f**2*x**6 + b*d*e**2*x**4 + 2*b*d*e*f*x**6
+ b*d*f**2*x**8),x)*a*b*d**2*e*f**2*x**2 - 2*int((sqrt(c + d*x**2)*sqrt(a
+ b*x**2)*x**6)/(a*c*e**2 + 2*a*c*e*f*x**2 + a*c*f**2*x**4 + a*d*e**2*x**2
 + 2*a*d*e*f*x**4 + a*d*f**2*x**6 + b*c*e**2*x**2 + 2*b*c*e*f*x**4 + b*c*f
**2*x**6 + b*d*e**2*x**4 + 2*b*d*e*f*x**6 + b*d*f**2*x**8),x)*b**2*c*d*...
```

3.190 $\int \frac{x^6}{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 544

$$\begin{aligned}\int \frac{x^6}{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^2}\,dx &= \frac{(be(3de-2cf)-2af(de-cf))x\sqrt{c+dx^2}}{2df^2(be-af)(de-cf)\sqrt{a+bx^2}} - \frac{e^2x\sqrt{a+bx^2}\sqrt{c+dx^2}}{2f(be-af)(de-cf)\left(e+fx^2\right)} \\ &\quad - \frac{\sqrt{a}(be(3de-2cf)-2af(de-cf))\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{2\sqrt{b}df^2(be-af)(de-cf)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} \\ &\quad + \frac{a^{3/2}e(3be-4af)\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{2\sqrt{b}cf^2(be-af)^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} \\ &\quad + \frac{a^{3/2}e(af(4de-5cf)-be(3de-4cf))\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{2\sqrt{b}cf^2(be-af)^2(de-cf)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}\end{aligned}$$

output
```
1/2*(b*e*(-2*c*f+3*d*e)-2*a*f*(-c*f+d*e))*x*(d*x^2+c)^(1/2)/d/f^2/(-a*f+b*
e)/(-c*f+d*e)/(b*x^2+a)^(1/2)-1/2*e^2*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/f/
(-a*f+b*e)/(-c*f+d*e)/(f*x^2+e)-1/2*a^(1/2)*(b*e*(-2*c*f+3*d*e)-2*a*f*(-c*
f+d*e))*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a
*d/b/c)^(1/2))/b^(1/2)/d/f^2/(-a*f+b*e)/(-c*f+d*e)/(b*x^2+a)^(1/2)/(a*(d*x
^2+c)/c/(b*x^2+a))^(1/2)+1/2*a^(3/2)*e*(-4*a*f+3*b*e)*(d*x^2+c)^(1/2)*Inve
rseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/b^(1/2)/c/f^2/(-a
*f+b*e)^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)+1/2*a^(3/2)*e*(a
*f*(-5*c*f+4*d*e)-b*e*(-4*c*f+3*d*e))*(d*x^2+c)^(1/2)*EllipticPi(b^(1/2)*x
/a^(1/2)/(1+b*x^2/a)^(1/2),1-a*f/b/e,(1-a*d/b/c)^(1/2))/b^(1/2)/c/f^2/(-a*
f+b*e)^2/(-c*f+d*e)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.61 (sec) , antiderivative size = 397, normalized size of antiderivative = 0.73

$$\int \frac{x^6}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \frac{-icf(be(3de-2cf)+2af(-de+cf))\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}(e+fx^2)\,E\left(i\operatorname{arcsinh}\left(\sqrt{\frac{b}{a}}x\right)\,|\frac{ad}{bc}\right)+i(-de+}{}$$

input
```
Integrate[x^6/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^2),x]
```

output
```
((-I)*c*f*(b*e*(3*d*e - 2*c*f) + 2*a*f*(-(d*e) + c*f))*Sqrt[1 + (b*x^2)/a]
*Sqrt[1 + (d*x^2)/c]*(e + f*x^2)*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(
b*c)] + I*(-(d*e) + c*f)*(2*a*f*(2*d*e + c*f) - b*e*(3*d*e + 2*c*f))*Sqrt[
1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(e + f*x^2)*EllipticF[I*ArcSinh[Sqrt[b/
a]*x], (a*d)/(b*c)] - I*d*e*((-I)*Sqrt[b/a]*e*f^2*x*(a + b*x^2)*(c + d*x^2
) + (b*e*(3*d*e - 4*c*f) + a*f*(-4*d*e + 5*c*f))*Sqrt[1 + (b*x^2)/a]*Sqrt[
1 + (d*x^2)/c]*(e + f*x^2)*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x],
 (a*d)/(b*c)]))/(2*Sqrt[b/a]*d*f^3*(b*e - a*f)*(d*e - c*f)*Sqrt[a + b*x^2]
*Sqrt[c + d*x^2]*(e + f*x^2))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx$$
$$\Big\downarrow \text{ 450}$$
$$\int \frac{x^6}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}dx$$

input
```
Int[x^6/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1268 vs. 2(512) = 1024.

Time = 6.96 (sec) , antiderivative size = 1269, normalized size of antiderivative = 2.33

method	result	size
elliptic	Expression too large to display	1269
default	Expression too large to display	1965

input
```
int(x^6/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x,method=_RETURNVERBOS
E)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-1/2/f/(a*c*f
^2-a*d*e*f-b*c*e*f+b*d*e^2)*e^2*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(f*x
^2+e)-2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+
b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))*e/f^
3+1/2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*
c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))*b*d/f^
3*e^3/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)-c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(
1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d/f^2*EllipticF(x*(-b
/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*
x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d/f^2*EllipticE(x*(-b/a)^
(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-1/2*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*
x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*b*e^2/f^2/(a*c*f^2-a*d*e*
f-b*c*e*f+b*d*e^2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+1/2*
c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^
2+a*c)^(1/2)*b*e^2/f^2/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)*EllipticE(x*(-b/a
)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+5/2*e/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/
f/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^
2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2)
)*a*c-2*e^2/f^2/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/(-b/a)^(1/2)*(1+b*x^2/a)
^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^6}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(x^6/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^6}{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^2}\,dx=\int \frac{x^6}{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^2}\,dx$$

input
```
integrate(x**6/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**2,x)
```

output
```
Integral(x**6/(sqrt(a + b*x**2)*sqrt(c + d*x**2)*(e + f*x**2)**2), x)
```

Maxima [F]

$$\int \frac{x^6}{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^2}\,dx=\int \frac{x^6}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^2}\,dx$$

input
```
integrate(x^6/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="ma
xima")
```

output
```
integrate(x^6/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^2), x)
```

Giac [F]

$$\int \frac{x^6}{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^2}\,dx=\int \frac{x^6}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^2}\,dx$$

input
```
integrate(x^6/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="gi
ac")
```

output
```
integrate(x^6/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^2}\,dx=\int \frac{x^6}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,{\left(f\,x^2+e\right)}^2}\,dx$$

input
```
int(x^6/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^2),x)
```

output
```
int(x^6/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^2), x)
```

Reduce [F]

$$\int \frac{x^6}{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^2}\,dx$$
$$=\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^6}{bd\,f^2x^8+ad\,f^2x^6+bc\,f^2x^6+2bdef\,x^6+ac\,f^2x^4+2adef\,x^4+2bcef\,x^4+bd\,e^2x^4+2acef\,x^2+ad\,e}$$

input
```
int(x^6/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**6)/(a*c*e**2 + 2*a*c*e*f*x**2 +
a*c*f**2*x**4 + a*d*e**2*x**2 + 2*a*d*e*f*x**4 + a*d*f**2*x**6 + b*c*e**2*
x**2 + 2*b*c*e*f*x**4 + b*c*f**2*x**6 + b*d*e**2*x**4 + 2*b*d*e*f*x**6 + b
*d*f**2*x**8),x)
```

3.191 $\int \frac{x^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 482

$$\begin{aligned}\int \frac{x^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx &= -\frac{bex\sqrt{c+dx^2}}{2f(be-af)(de-cf)\sqrt{a+bx^2}} \\ &+ \frac{ex\sqrt{a+bx^2}\sqrt{c+dx^2}}{2(be-af)(de-cf)\,(e+fx^2)} + \frac{\sqrt{a}\sqrt{b}e\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{2f(be-af)(de-cf)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} \\ &- \frac{a^{3/2}(be-2af)\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{2\sqrt{b}cf(be-af)^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} \\ &- \frac{a^{3/2}(af(2de-3cf)-be(de-2cf))\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{2\sqrt{b}cf(be-af)^2(de-cf)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}\end{aligned}$$

output
```
-1/2*b*e*x*(d*x^2+c)^(1/2)/f/(-a*f+b*e)/(-c*f+d*e)/(b*x^2+a)^(1/2)+1/2*e*x
*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(-a*f+b*e)/(-c*f+d*e)/(f*x^2+e)+1/2*a^(1/
2)*b^(1/2)*e*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2)
,(1-a*d/b/c)^(1/2))/f/(-a*f+b*e)/(-c*f+d*e)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c
/(b*x^2+a))^(1/2)-1/2*a^(3/2)*(-2*a*f+b*e)*(d*x^2+c)^(1/2)*InverseJacobiAM
(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/b^(1/2)/c/f/(-a*f+b*e)^2/(b*
x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)-1/2*a^(3/2)*(a*f*(-3*c*f+2*d*
e)-b*e*(-2*c*f+d*e))*(d*x^2+c)^(1/2)*EllipticPi(b^(1/2)*x/a^(1/2)/(1+b*x^2
/a)^(1/2),1-a*f/b/e,(1-a*d/b/c)^(1/2))/b^(1/2)/c/f/(-a*f+b*e)^2/(-c*f+d*e)
/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.36 (sec) , antiderivative size = 887, normalized size of antiderivative = 1.84

$$\int \frac{x^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \text{Too large to display}$$

input
```
Integrate[x^4/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^2),x]
```

output
```
(a*Sqrt[b/a]*c*e*f^2*x + b*Sqrt[b/a]*c*e*f^2*x^3 + a*Sqrt[b/a]*d*e*f^2*x^3
 + b*Sqrt[b/a]*d*e*f^2*x^5 + I*b*c*e*f*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2
)/c]*(e + f*x^2)*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*(-(b*e
) + 2*a*f)*(-(d*e) + c*f)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(e + f*x
^2)*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + I*b*d*e^3*Sqrt[1 + (b
*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x
], (a*d)/(b*c)] - (2*I)*b*c*e^2*f*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*
EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - (2*I)*a*d*e
^2*f*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e), I*Arc
Sinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (3*I)*a*c*e*f^2*Sqrt[1 + (b*x^2)/a]*Sqrt
[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c
)] + I*b*d*e^2*f*x^2*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a
*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - (2*I)*b*c*e*f^2*x^2*Sqrt
[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt
[b/a]*x], (a*d)/(b*c)] - (2*I)*a*d*e*f^2*x^2*Sqrt[1 + (b*x^2)/a]*Sqrt[1 +
(d*x^2)/c]*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] +
(3*I)*a*c*f^3*x^2*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)
/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(2*Sqrt[b/a]*f^2*(b*e - a*f)
*(d*e - c*f)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx$$

$$\Big\downarrow \text{ 450}$$

$$\int \frac{x^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}dx$$

input
```
Int[x^4/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1060 vs. $2(450) = 900$.

Time = 8.06 (sec) , antiderivative size = 1061, normalized size of antiderivative = 2.20

method	result	size
elliptic	Expression too large to display	1061
default	Expression too large to display	1305

input
```
int(x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x,method=_RETURNVERBOS
E)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/2*e/(a*c*f^
2-a*d*e*f-b*c*e*f+b*d*e^2)*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(f*x^2+e)
+1/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x
^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))/f^2-1/2/(
-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a
*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))*b*d*e^2/f^2/(
a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)+1/2*e/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)*b
/f*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c
*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-1/2*e/(
a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)*b/f*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d
*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticE(x*(-b/a)^(1/2)
,(-1+(a*d+b*c)/c/b)^(1/2))-3/2/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/(-b/a)^(1
/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2
)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*a*c+e/(a*
c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/f/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c
)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/
b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*a*d+e/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/f
/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2
+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))
*b*c-1/2*e^2/f^2/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/(-b/a)^(1/2)*(1+b*x^...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{x^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx$$

input
```
integrate(x**4/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**2,x)
```

output
```
Integral(x**4/(sqrt(a + b*x**2)*sqrt(c + d*x**2)*(e + f*x**2)**2), x)
```

Maxima [F]

$$\int \frac{x^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{x^4}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^2}\,dx$$

input
```
integrate(x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="ma
xima")
```

output
```
integrate(x^4/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^2), x)
```

Giac [F]

$$\int \frac{x^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{x^4}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^2}\,dx$$

input
```
integrate(x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="gi
ac")
```

output
```
integrate(x^4/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx=\int \frac{x^4}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,(f\,x^2+e)^2}\,dx$$

input
```
int(x^4/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^2),x)
```

output
```
int(x^4/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^2), x)
```

Reduce [F]

$$\int \frac{x^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx$$
$$=\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^4}{bd\,f^2x^8+ad\,f^2x^6+bc\,f^2x^6+2bdef\,x^6+ac\,f^2x^4+2adef\,x^4+2bcef\,x^4+bd\,e^2x^4+2acef\,x^2+ad\,e}$$

input
```
int(x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c*e**2 + 2*a*c*e*f*x**2 +
a*c*f**2*x**4 + a*d*e**2*x**2 + 2*a*d*e*f*x**4 + a*d*f**2*x**6 + b*c*e**2*
x**2 + 2*b*c*e*f*x**4 + b*c*f**2*x**6 + b*d*e**2*x**4 + 2*b*d*e*f*x**6 + b
*d*f**2*x**8),x)
```

3.192 $\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 452

$$\begin{aligned}
&\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^2}\,dx \\
&= \frac{bx\sqrt{c+dx^2}}{2(be-af)(de-cf)\sqrt{a+bx^2}} - \frac{fx\sqrt{a+bx^2}\sqrt{c+dx^2}}{2(be-af)(de-cf)\left(e+fx^2\right)} \\
&\quad - \frac{\sqrt{a}\sqrt{b}\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{2(be-af)(de-cf)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} \\
&\quad - \frac{a^{3/2}\sqrt{b}\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{2c(be-af)^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} \\
&\quad + \frac{a^{3/2}(bde^2-acf^2)\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{2\sqrt{b}ce(be-af)^2(de-cf)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}
\end{aligned}$$

output

```
1/2*b*x*(d*x^2+c)^(1/2)/(-a*f+b*e)/(-c*f+d*e)/(b*x^2+a)^(1/2)-1/2*f*x*(b*x
^2+a)^(1/2)*(d*x^2+c)^(1/2)/(-a*f+b*e)/(-c*f+d*e)/(f*x^2+e)-1/2*a^(1/2)*b^
(1/2)*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d
/b/c)^(1/2))/(-a*f+b*e)/(-c*f+d*e)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a
))^(1/2)-1/2*a^(3/2)*b^(1/2)*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2
)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/c/(-a*f+b*e)^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c
)/c/(b*x^2+a))^(1/2)+1/2*a^(3/2)*(-a*c*f^2+b*d*e^2)*(d*x^2+c)^(1/2)*Ellipt
icPi(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),1-a*f/b/e,(1-a*d/b/c)^(1/2))/b^(1
/2)/c/e/(-a*f+b*e)^2/(-c*f+d*e)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^
(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.89 (sec) , antiderivative size = 425, normalized size of antiderivative = 0.94

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx$$
$$= \frac{-\frac{acfx}{e+fx^2}-\frac{bcfx^3}{e+fx^2}-\frac{adfx^3}{e+fx^2}-\frac{bdfx^5}{e+fx^2}-ia\sqrt{\frac{b}{a}}c\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(i\text{arcsinh}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)+\frac{ia\sqrt{\frac{b}{a}}(-de+cf)\sqrt{1-}}{}}{2(}$$

input

```
Integrate[x^2/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^2),x]
```

output

```
(-((a*c*f*x)/(e + f*x^2)) - (b*c*f*x^3)/(e + f*x^2) - (a*d*f*x^3)/(e + f*x
^2) - (b*d*f*x^5)/(e + f*x^2) - I*a*Sqrt[b/a]*c*Sqrt[1 + (b*x^2)/a]*Sqrt[1
 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (I*a*Sqrt[b
/a]*(-(d*e) + c*f)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*Arc
Sinh[Sqrt[b/a]*x], (a*d)/(b*c)])/f + (I*a*Sqrt[b/a]*d*e*Sqrt[1 + (b*x^2)/a
]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d
)/(b*c)])/f - (I*a*c*f*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[
(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(Sqrt[b/a]*e))/(2*(b*e
- a*f)*(d*e - c*f)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx$$
$$\Big\downarrow \; 450$$
$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}dx$$

input
```
Int[x^2/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [A] (verified)

Time = 8.55 (sec) , antiderivative size = 710, normalized size of antiderivative = 1.57

method	result
elliptic	$\frac{\sqrt{(b x^2+a)(x^2 d+c)}\left(-\frac{f x\sqrt{bd x^4+ad x^2+x^2 bc+ac}}{2\left(ac f^2-adef-bcef+bd e^2\right)\left(f x^2+e\right)}+\frac{ebd\sqrt{1+\frac{b x^2}{a}}\sqrt{1+\frac{d x^2}{c}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{2\left(ac f^2-adef-bcef+bd e^2\right)f\sqrt{-\frac{b}{a}}\sqrt{bd x^4+ad x^2+x^2 bc+ac}}-\frac{bc\sqrt{1+\frac{b x^2}{a}}}{2\left(ac f^2-ade\right.}\right.}{}$
default	$-\frac{\left(\sqrt{-\frac{b}{a}}\,bde f^2 x^5+\sqrt{\frac{b x^2+a}{a}}\sqrt{\frac{x^2 d+c}{c}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)bce f^2 x^2-\sqrt{\frac{b x^2+a}{a}}\sqrt{\frac{x^2 d+c}{c}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)bd e^2 f x\right.}{}$

input
```
int(x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x,method=_RETURNVERBOS
E)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-1/2*f/(a*c*f
^2-a*d*e*f-b*c*e*f+b*d*e^2)*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(f*x^2+e
)+1/2*e/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)*b*d/f/(-b/a)^(1/2)*(1+b*x^2/a)^(
1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b
/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-1/2*b/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2
)*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*
x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+1/2*b/(a
*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/
c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(-1+
(a*d+b*c)/c/b)^(1/2))+1/2/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)*f/e/(-b/a)^(1/
2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*a*c-1/2/(a
*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)*e/f/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^
2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a
*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*b*d)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx$$

input
```
integrate(x**2/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**2,x)
```

output
```
Integral(x**2/(sqrt(a + b*x**2)*sqrt(c + d*x**2)*(e + f*x**2)**2), x)
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^2}\,dx$$

input
```
integrate(x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="ma
xima")
```

output
```
integrate(x^2/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^2), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^2}\,dx$$

input
```
integrate(x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="gi
ac")
```

output
```
integrate(x^2/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{x^2}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,(f\,x^2+e)^2}\,dx$$

input
```
int(x^2/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^2),x)
```

output
```
int(x^2/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^2), x)
```

Reduce [F]

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx$$
$$= \int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^2}{bd\,f^2x^8+ad\,f^2x^6+bc\,f^2x^6+2bdef\,x^6+ac\,f^2x^4+2adef\,x^4+2bcef\,x^4+bd\,e^2x^4+2acef\,x^2+ad\,e}$$

input
```
int(x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c*e**2 + 2*a*c*e*f*x**2 +
a*c*f**2*x**4 + a*d*e**2*x**2 + 2*a*d*e*f*x**4 + a*d*f**2*x**6 + b*c*e**2*
x**2 + 2*b*c*e*f*x**4 + b*c*f**2*x**6 + b*d*e**2*x**4 + 2*b*d*e*f*x**6 + b
*d*f**2*x**8),x)
```

3.193 $\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 426

$$\begin{aligned}
&\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx \\
&= -\frac{fx\sqrt{a+bx^2}}{2e(be-af)\sqrt{c+dx^2}\,(e+fx^2)} + \frac{\sqrt{c}\sqrt{d}f\sqrt{a+bx^2}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{2e(be-af)(de-cf)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} \\
&\quad + \frac{\sqrt{c}\sqrt{d}(2de-cf)\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{2ae(de-cf)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} \\
&\quad - \frac{c^{3/2}f(be(3de-2cf)-af(2de-cf))\sqrt{a+bx^2}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{2a\sqrt{d}e^2(be-af)(de-cf)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}
\end{aligned}$$

output
```
-1/2*f*x*(b*x^2+a)^(1/2)/e/(-a*f+b*e)/(d*x^2+c)^(1/2)/(f*x^2+e)+1/2*c^(1/2
)*d^(1/2)*f*(b*x^2+a)^(1/2)*EllipticE(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),
(1-b*c/a/d)^(1/2))/e/(-a*f+b*e)/(-c*f+d*e)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)
/(d*x^2+c)^(1/2)+1/2*c^(1/2)*d^(1/2)*(-c*f+2*d*e)*(b*x^2+a)^(1/2)*InverseJ
acobiAM(arctan(d^(1/2)*x/c^(1/2)),(1-b*c/a/d)^(1/2))/a/e/(-c*f+d*e)^2/(c*(
b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-1/2*c^(3/2)*f*(b*e*(-2*c*f+3*d
*e)-a*f*(-c*f+2*d*e))*(b*x^2+a)^(1/2)*EllipticPi(d^(1/2)*x/c^(1/2)/(1+d*x^
2/c)^(1/2),1-c*f/d/e,(1-b*c/a/d)^(1/2))/a/d^(1/2)/e^2/(-a*f+b*e)/(-c*f+d*e
)^2/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.21 (sec) , antiderivative size = 587, normalized size of antiderivative = 1.38

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx$$

$$= \frac{\frac{acf^2x}{e+fx^2}+\frac{bcf^2x^3}{e+fx^2}+\frac{adf^2x^3}{e+fx^2}+\frac{bdf^2x^5}{e+fx^2}+ia\sqrt{\frac{b}{a}}cf\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(i\text{arcsinh}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)-ia\sqrt{\frac{b}{a}}(-de+c}{}$$

input
```
Integrate[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^2),x]
```

output
```
((a*c*f^2*x)/(e + f*x^2) + (b*c*f^2*x^3)/(e + f*x^2) + (a*d*f^2*x^3)/(e +
f*x^2) + (b*d*f^2*x^5)/(e + f*x^2) + I*a*Sqrt[b/a]*c*f*Sqrt[1 + (b*x^2)/a]
*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*a*
Sqrt[b/a]*(-(d*e) + c*f)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF
[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - (3*I)*a*Sqrt[b/a]*d*e*Sqrt[1 + (b*
x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x]
, (a*d)/(b*c)] + (2*I)*a*Sqrt[b/a]*c*f*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2
)/c]*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + ((2*I)
*a*d*f*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e), I*A
rcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/Sqrt[b/a] - (I*a*c*f^2*Sqrt[1 + (b*x^2)
/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a
*d)/(b*c)])/(Sqrt[b/a]*e))/(2*e*(b*e - a*f)*(d*e - c*f)*Sqrt[a + b*x^2]*Sq
rt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 448, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {424, 406, 320, 388, 313, 413, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}dx$$

$$\Big\downarrow \text{424}$$

$$\frac{(be(3de-2cf)-af(2de-cf))\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)}dx}{2e(be-af)(de-cf)}-\frac{bd\int \frac{fx^2+e}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{2e(be-af)(de-cf)}+\frac{f^2x\sqrt{a+bx^2}\sqrt{c+dx^2}}{2e\,(e+fx^2)\,(be-af)(de-cf)}$$

$$\Big\downarrow \text{406}$$

$$-\frac{bd\left(e\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+f\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{2e(be-af)(de-cf)}+\frac{(be(3de-2cf)-af(2de-cf))\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)}dx}{2e(be-af)(de-cf)}+\frac{f^2x\sqrt{a+bx^2}\sqrt{c+dx^2}}{2e\,(e+fx^2)\,(be-af)(de-cf)}$$

$$\Big\downarrow \text{320}$$

$$-\frac{bd\left(f\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{\sqrt{c}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{2e(be-af)(de-cf)}+\frac{(be(3de-2cf)-af(2de-cf))\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)}dx}{2e(be-af)(de-cf)}+\frac{f^2x\sqrt{a+bx^2}\sqrt{c+dx^2}}{2e\,(e+fx^2)\,(be-af)(de-cf)}$$

$$\Big\downarrow \text{388}$$

$$-\frac{bd\left(f\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{\sqrt{c}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{2e(be-af)(de-cf)}+\frac{(be(3de-2cf)-af(2de-cf))\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)}dx}{2e(be-af)(de-cf)}+\frac{f^2x\sqrt{a+bx^2}\sqrt{c+dx^2}}{2e\,(e+fx^2)\,(be-af)(de-cf)}$$

$$\Big\downarrow \; 313$$

$$\frac{(be(3de-2cf)-af(2de-cf))\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)}dx}{2e(be-af)(de-cf)}-\frac{bd\left(\frac{\sqrt{c}e\sqrt{a+bx^2}\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+f\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)\right)}{2e(be-af)(de-cf)}+\frac{f^2x\sqrt{a+bx^2}\sqrt{c+dx^2}}{2e(e+fx^2)(be-af)(de-cf)}$$

$$\Big\downarrow \; 413$$

$$\frac{\sqrt{\frac{bx^2}{a}+1}(be(3de-2cf)-af(2de-cf))\int \frac{1}{\sqrt{\frac{bx^2}{a}+1}\sqrt{dx^2+c}(fx^2+e)}dx}{2e\sqrt{a+bx^2}(be-af)(de-cf)}-\frac{bd\left(\frac{\sqrt{c}e\sqrt{a+bx^2}\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+f\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)\right)}{2e(be-af)(de-cf)}+\frac{f^2x\sqrt{a+bx^2}\sqrt{c+dx^2}}{2e(e+fx^2)(be-af)(de-cf)}$$

$$\Big\downarrow \; 413$$

$$\frac{\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}(be(3de-2cf)-af(2de-cf))\int \frac{1}{\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}(fx^2+e)}dx}{2e\sqrt{a+bx^2}\sqrt{c+dx^2}(be-af)(de-cf)}-\frac{bd\left(\frac{\sqrt{c}e\sqrt{a+bx^2}\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+f\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)\right)}{2e(be-af)(de-cf)}+\frac{f^2x\sqrt{a+bx^2}\sqrt{c+dx^2}}{2e(e+fx^2)(be-af)(de-cf)}$$

$$\Big\downarrow \; 412$$

$$\frac{\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}(be(3de-2cf)-af(2de-cf))\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{2\sqrt{b}e^2\sqrt{a+bx^2}\sqrt{c+dx^2}(be-af)(de-cf)} - \frac{bd\left(\frac{\sqrt{c}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+f\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)\right)}{2e(be-af)(de-cf)} + \frac{f^2x\sqrt{a+bx^2}\sqrt{c+dx^2}}{2e\left(e+fx^2\right)(be-af)(de-cf)}$$

input
```
Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^2),x]
```

output
```
(f^2*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(2*e*(b*e - a*f)*(d*e - c*f)*(e +
f*x^2)) - (b*d*(f*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt
[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sq
rt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (Sqrt[c]*e
*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/
(a*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])))/(2*e*(
b*e - a*f)*(d*e - c*f)) + (Sqrt[-a]*(b*e*(3*d*e - 2*c*f) - a*f*(2*d*e - c*
f))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e), ArcSin
[(Sqrt[b]*x)/Sqrt[-a]], (a*d)/(b*c)])/(2*Sqrt[b]*e^2*(b*e - a*f)*(d*e - c*
f)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
 + d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 424
```
Int[1/(((a_) + (b_.)*(x_)^2)^2*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*
(x_)^2]), x_Symbol] :> Simp[b^2*x*Sqrt[c + d*x^2]*(Sqrt[e + f*x^2]/(2*a*(b*
c - a*d)*(b*e - a*f)*(a + b*x^2))), x] + (Simp[(b^2*c*e + 3*a^2*d*f - 2*a*b
*(d*e + c*f))/(2*a*(b*c - a*d)*(b*e - a*f))   Int[1/((a + b*x^2)*Sqrt[c + d
*x^2]*Sqrt[e + f*x^2]), x], x] - Simp[d*(f/(2*a*(b*c - a*d)*(b*e - a*f)))
 Int[(a + b*x^2)/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x], x]) /; FreeQ[{a, b,
 c, d, e, f}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 972 vs. $2(400) = 800$.

Time = 8.60 (sec) , antiderivative size = 973, normalized size of antiderivative = 2.28

method	result
elliptic	$\frac{\sqrt{(b x^2+a)(x^2 d+c)}\left(\frac{f^2 x \sqrt{bd x^4+ad x^2+x^2 bc+ac}}{2\left(ac f^2-adef-bcef+bd e^2\right) e \left(f x^2+e\right)}-\frac{db\sqrt{1+\frac{b x^2}{a}}\sqrt{1+\frac{d x^2}{c}}\,\text{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{2\left(ac f^2-adef-bcef+bd e^2\right)\sqrt{-\frac{b}{a}}\sqrt{bd x^4+ad x^2+x^2 bc+ac}}+\frac{fbc\sqrt{1+\frac{b x^2}{a}}}{2\left(ac f^2-adef-\right.}\right.}{}$
default	Expression too large to display

input
```
int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/2*f^2/(a*c*
f^2-a*d*e*f-b*c*e*f+b*d*e^2)/e*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(f*x^
2+e)-1/2*d*b/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/(-b/a)^(1/2)*(1+b*x^2/a)^(1
/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/
a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+1/2*f*b/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^
2)/e*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b
*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-1/2*f
*b/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/e*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1
+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticE(x*(-b/a)^(1/
2),(-1+(a*d+b*c)/c/b)^(1/2))+1/2/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/e^2*f^2
/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2
+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))
*a*c-1/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/e*f/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2
)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a
)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*a*d-1/(a*c*f^2-a*d*e*f-b*c*e*
f+b*d*e^2)/e*f/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a
*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)
/(-b/a)^(1/2))*b*c+3/2/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/(-b/a)^(1/2)*(1+b
*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*Ellipt
icPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*b*d)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx$$

input
```
integrate(1/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**2,x)
```

output
```
Integral(1/(sqrt(a + b*x**2)*sqrt(c + d*x**2)*(e + f*x**2)**2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^2}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="maxi
ma")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^2}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="giac
")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{1}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,(f\,x^2+e)^2}\,dx$$

input
```
int(1/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^2),x)
```

output
```
int(1/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx$$
$$= \int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{bd\,f^2x^8+ad\,f^2x^6+bc\,f^2x^6+2bdef\,x^6+ac\,f^2x^4+2adef\,x^4+2bcef\,x^4+bd\,e^2x^4+2acef\,x^2+ad\,e}$$

input
```
int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x)
```

output

```
int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c*e**2 + 2*a*c*e*f*x**2 + a*c*f
**2*x**4 + a*d*e**2*x**2 + 2*a*d*e*f*x**4 + a*d*f**2*x**6 + b*c*e**2*x**2
+ 2*b*c*e*f*x**4 + b*c*f**2*x**6 + b*d*e**2*x**4 + 2*b*d*e*f*x**6 + b*d*f*
*2*x**8),x)
```

3.194 $\int \frac{1}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 531

$$\begin{aligned}
&\int \frac{1}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx \\
&= -\frac{(af(2de-3cf)-2be(de-cf))x\sqrt{a+bx^2}}{2ace^2(be-af)\sqrt{c+dx^2}\,(e+fx^2)} - \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{acex\,(e+fx^2)} \\
&\quad + \frac{\sqrt{d}(af(2de-3cf)-2be(de-cf))\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{2a\sqrt{c}e^2(be-af)(de-cf)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} \\
&\quad - \frac{\sqrt{c}\sqrt{d}f(4de-3cf)\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{2ae^2(de-cf)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} \\
&\quad + \frac{c^{3/2}f^2(be(5de-4cf)-af(4de-3cf))\sqrt{a+bx^2}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{2a\sqrt{d}e^3(be-af)(de-cf)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}
\end{aligned}$$

output
```
-1/2*(a*f*(-3*c*f+2*d*e)-2*b*e*(-c*f+d*e))*x*(b*x^2+a)^(1/2)/a/c/e^2/(-a*f
+b*e)/(d*x^2+c)^(1/2)/(f*x^2+e)-(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a/c/e/x/(f
*x^2+e)+1/2*d^(1/2)*(a*f*(-3*c*f+2*d*e)-2*b*e*(-c*f+d*e))*(b*x^2+a)^(1/2)*
EllipticE(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))/a/c^(1/2)
/e^2/(-a*f+b*e)/(-c*f+d*e)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)
-1/2*c^(1/2)*d^(1/2)*f*(-3*c*f+4*d*e)*(b*x^2+a)^(1/2)*InverseJacobiAM(arct
an(d^(1/2)*x/c^(1/2)),(1-b*c/a/d)^(1/2))/a/e^2/(-c*f+d*e)^2/(c*(b*x^2+a)/a
/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)+1/2*c^(3/2)*f^2*(b*e*(-4*c*f+5*d*e)-a*f*
(-3*c*f+4*d*e))*(b*x^2+a)^(1/2)*EllipticPi(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(
1/2),1-c*f/d/e,(1-b*c/a/d)^(1/2))/a/d^(1/2)/e^3/(-a*f+b*e)/(-c*f+d*e)^2/(c
*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.04 (sec) , antiderivative size = 434, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx$$
$$= \frac{\sqrt{\frac{b}{a}}\left(\sqrt{\frac{b}{a}}e(a+bx^2)\,(c+dx^2)\,(-2be(de-cf)\,(e+fx^2)+af(2de(e+fx^2)-cf(2e+3fx^2)))-ibce(2b\right.}{}$$

input
```
Integrate[1/(x^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^2),x]
```

output
```
(Sqrt[b/a]*(Sqrt[b/a]*e*(a + b*x^2)*(c + d*x^2)*(-2*b*e*(d*e - c*f)*(e + f
*x^2) + a*f*(2*d*e*(e + f*x^2) - c*f*(2*e + 3*f*x^2))) - I*b*c*e*(2*b*e*(d
*e - c*f) + a*f*(-2*d*e + 3*c*f))*x*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c
]*(e + f*x^2)*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + I*b*c*e*(-2
*b*e + 3*a*f)*(-(d*e) + c*f)*x*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(e
+ f*x^2)*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + I*a*c*f*(b*e*(5*
d*e - 4*c*f) + a*f*(-4*d*e + 3*c*f))*x*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2
)/c]*(e + f*x^2)*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*
c)]))/(2*b*c*e^3*(-(b*e) + a*f)*(-(d*e) + c*f)*x*Sqrt[a + b*x^2]*Sqrt[c +
d*x^2]*(e + f*x^2))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx$$
$$\Big\downarrow \text{ 450}$$
$$\int \frac{1}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}dx$$

input
```
Int[1/(x^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1213 vs. 2(501) = 1002.

Time = 22.64 (sec) , antiderivative size = 1214, normalized size of antiderivative = 2.29

method	result	size
elliptic	Expression too large to display	1214
risch	Expression too large to display	1244
default	Expression too large to display	2134

input
```
int(1/x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x,method=_RETURNVERB
OSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-1/2*f^3/(a*c
*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/e^2*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(f
*x^2+e)-1/a/c/e^2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/x+1/2*f*d*b/(a*c*f^2
-a*d*e*f-b*c*e*f+b*d*e^2)/e/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/
2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b
*c)/c/b)^(1/2))-1/2*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*
d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*b*f^2/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/e
^2*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+1/2*c/(-b/a)^(1/2)*(
1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*b*f
^2/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/e^2*EllipticE(x*(-b/a)^(1/2),(-1+(a*d
+b*c)/c/b)^(1/2))-1/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*
x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*b/a/e^2*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b
*c)/c/b)^(1/2))+1/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^
4+a*d*x^2+b*c*x^2+a*c)^(1/2)*b/a/e^2*EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c
)/c/b)^(1/2))-3/2/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/e^3*f^3/(-b/a)^(1/2)*(
1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*Ell
ipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*a*c+2/(a*c*f^2
-a*d*e*f-b*c*e*f+b*d*e^2)/e^2*f^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/
c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f
/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*a*d+2/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="
fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{1}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx$$

input
```
integrate(1/x**2/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**2,x)
```

output
```
Integral(1/(x**2*sqrt(a + b*x**2)*sqrt(c + d*x**2)*(e + f*x**2)**2), x)
```

Maxima [F]

$$\int \frac{1}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^2x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="
maxima")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^2*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^2x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="
giac")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^2*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{1}{x^2\,\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,(f\,x^2+e)^2}\,dx$$

input
```
int(1/(x^2*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^2),x)
```

output
```
int(1/(x^2*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^2), x)
```

Reduce [F]

$$\int \frac{1}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \text{Too large to display}$$

input
```
int(1/x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x)
```

output
```
( - sqrt(c + d*x**2)*sqrt(a + b*x**2) - int((sqrt(c + d*x**2)*sqrt(a + b*x
**2)*x**4)/(a*c*e**2 + 2*a*c*e*f*x**2 + a*c*f**2*x**4 + a*d*e**2*x**2 + 2*
a*d*e*f*x**4 + a*d*f**2*x**6 + b*c*e**2*x**2 + 2*b*c*e*f*x**4 + b*c*f**2*x
**6 + b*d*e**2*x**4 + 2*b*d*e*f*x**6 + b*d*f**2*x**8),x)*b*d*e*f*x - int((
sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c*e**2 + 2*a*c*e*f*x**2 + a*c*f
**2*x**4 + a*d*e**2*x**2 + 2*a*d*e*f*x**4 + a*d*f**2*x**6 + b*c*e**2*x**2
+ 2*b*c*e*f*x**4 + b*c*f**2*x**6 + b*d*e**2*x**4 + 2*b*d*e*f*x**6 + b*d*f*
*2*x**8),x)*b*d*f**2*x**3 - 2*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)
/(a*c*e**2 + 2*a*c*e*f*x**2 + a*c*f**2*x**4 + a*d*e**2*x**2 + 2*a*d*e*f*x*
*4 + a*d*f**2*x**6 + b*c*e**2*x**2 + 2*b*c*e*f*x**4 + b*c*f**2*x**6 + b*d*
e**2*x**4 + 2*b*d*e*f*x**6 + b*d*f**2*x**8),x)*a*d*e*f*x - 2*int((sqrt(c +
 d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c*e**2 + 2*a*c*e*f*x**2 + a*c*f**2*x**4
 + a*d*e**2*x**2 + 2*a*d*e*f*x**4 + a*d*f**2*x**6 + b*c*e**2*x**2 + 2*b*c*
e*f*x**4 + b*c*f**2*x**6 + b*d*e**2*x**4 + 2*b*d*e*f*x**6 + b*d*f**2*x**8)
,x)*a*d*f**2*x**3 - 2*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c*e*
*2 + 2*a*c*e*f*x**2 + a*c*f**2*x**4 + a*d*e**2*x**2 + 2*a*d*e*f*x**4 + a*d
*f**2*x**6 + b*c*e**2*x**2 + 2*b*c*e*f*x**4 + b*c*f**2*x**6 + b*d*e**2*x**
4 + 2*b*d*e*f*x**6 + b*d*f**2*x**8),x)*b*c*e*f*x - 2*int((sqrt(c + d*x**2)
*sqrt(a + b*x**2)*x**2)/(a*c*e**2 + 2*a*c*e*f*x**2 + a*c*f**2*x**4 + a*d*e
**2*x**2 + 2*a*d*e*f*x**4 + a*d*f**2*x**6 + b*c*e**2*x**2 + 2*b*c*e*f*x...
```

3.195 $\int \frac{1}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 705

$$\int \frac{1}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^2}\,dx =$$
$$-\frac{(4b^2ce^2(de-cf)-a^2f(4d^2e^2+8cdef-15c^2f^2)+4abe(d^2e^2+cdef-2c^2f^2))\,x\sqrt{a+bx^2}}{6a^2c^2e^3(be-af)\sqrt{c+dx^2}\,(e+fx^2)}$$
$$-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{3acex^3\,(e+fx^2)}+\frac{(2bce+2ade+5acf)\sqrt{a+bx^2}\sqrt{c+dx^2}}{3a^2c^2e^2x\,(e+fx^2)}$$
$$+\frac{\sqrt{d}(4b^2ce^2(de-cf)-a^2f(4d^2e^2+8cdef-15c^2f^2)+4abe(d^2e^2+cdef-2c^2f^2))\,\sqrt{a+bx^2}\,E\Big(\text{arcta}}{6a^2c^{3/2}e^3(be-af)(de-cf)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$
$$-\frac{\sqrt{d}\left(2b-\frac{3acf^2(6de-5cf)}{e(de-cf)^2}\right)\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{6a^2\sqrt{c}e^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$
$$-\frac{c^{3/2}f^3(be(7de-6cf)-af(6de-5cf))\sqrt{a+bx^2}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{2a\sqrt{d}e^4(be-af)(de-cf)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output
```
-1/6*(4*b^2*c*e^2*(-c*f+d*e)-a^2*f*(-15*c^2*f^2+8*c*d*e*f+4*d^2*e^2)+4*a*b
*e*(-2*c^2*f^2+c*d*e*f+d^2*e^2))*x*(b*x^2+a)^(1/2)/a^2/c^2/e^3/(-a*f+b*e)/
(d*x^2+c)^(1/2)/(f*x^2+e)-1/3*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a/c/e/x^3/(f
*x^2+e)+1/3*(5*a*c*f+2*a*d*e+2*b*c*e)*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a^2/
c^2/e^2/x/(f*x^2+e)+1/6*d^(1/2)*(4*b^2*c*e^2*(-c*f+d*e)-a^2*f*(-15*c^2*f^2
+8*c*d*e*f+4*d^2*e^2)+4*a*b*e*(-2*c^2*f^2+c*d*e*f+d^2*e^2))*(b*x^2+a)^(1/2
)*EllipticE(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))/a^2/c^(
3/2)/e^3/(-a*f+b*e)/(-c*f+d*e)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(
1/2)-1/6*d^(1/2)*(2*b-3*a*c*f^2*(-5*c*f+6*d*e)/e/(-c*f+d*e)^2)*(b*x^2+a)^(
1/2)*InverseJacobiAM(arctan(d^(1/2)*x/c^(1/2)),(1-b*c/a/d)^(1/2))/a^2/c^(1
/2)/e^2/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-1/2*c^(3/2)*f^3*(b
*e*(-6*c*f+7*d*e)-a*f*(-5*c*f+6*d*e))*(b*x^2+a)^(1/2)*EllipticPi(d^(1/2)*x
/c^(1/2)/(1+d*x^2/c)^(1/2),1-c*f/d/e,(1-b*c/a/d)^(1/2))/a/d^(1/2)/e^4/(-a*
f+b*e)/(-c*f+d*e)^2/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.62 (sec) , antiderivative size = 476, normalized size of antiderivative = 0.68

$$\int \frac{1}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \frac{\sqrt{\frac{b}{a}}e(a+bx^2)\,(c+dx^2)\,(3a^2c^2f^4x^4-2ace(-be+af)(-de+cf)\,(e+fx^2)+4(be-af)(de-cf)(bce+}{}$$

input
```
Integrate[1/(x^4*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^2),x]
```

output
```
(Sqrt[b/a]*e*(a + b*x^2)*(c + d*x^2)*(3*a^2*c^2*f^4*x^4 - 2*a*c*e*(-(b*e)
+ a*f)*(-(d*e) + c*f)*(e + f*x^2) + 4*(b*e - a*f)*(d*e - c*f)*(b*c*e + a*d
*e + 3*a*c*f)*x^2*(e + f*x^2)) + I*c*x^3*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x
^2)/c]*(e + f*x^2)*(b*e*(4*b^2*c*e^2*(d*e - c*f) + 4*a*b*e*(d^2*e^2 + c*d*
e*f - 2*c^2*f^2) + a^2*f*(-4*d^2*e^2 - 8*c*d*e*f + 15*c^2*f^2))*EllipticE[
I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + b*e*(-(d*e) + c*f)*(4*b^2*c*e^2 + 2
*a*b*e*(d*e + 4*c*f) - a^2*f*(2*d*e + 15*c*f))*EllipticF[I*ArcSinh[Sqrt[b/
a]*x], (a*d)/(b*c)] - 3*a^2*c*f^2*(b*e*(7*d*e - 6*c*f) + a*f*(-6*d*e + 5*c
*f))*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(6*a^2
*Sqrt[b/a]*c^2*e^4*(b*e - a*f)*(d*e - c*f)*x^3*Sqrt[a + b*x^2]*Sqrt[c + d*
x^2]*(e + f*x^2))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx$$
$$\Big\downarrow \; 450$$
$$\int \frac{1}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}dx$$

input
```
Int[1/(x^4*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1390 vs. $2(667) = 1334$.

Time = 24.07 (sec) , antiderivative size = 1391, normalized size of antiderivative = 1.97

method	result	size
risch	Expression too large to display	1391
elliptic	Expression too large to display	1661
default	Expression too large to display	3933

input
```
int(1/x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x,method=_RETURNVERB
OSE)
```

output
```
-1/3*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(-6*a*c*f*x^2-2*a*d*e*x^2-2*b*c*e*x^2
+a*c*e)/a^2/c^2/e^3/x^3-1/3/a^2/c^2/e^3*(-2*b*(3*a*c*f+a*d*e+b*c*e)*c/(-b/
a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)
^(1/2)*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b
/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2)))+a*c*d*e*b/(-b/a)^(1/2)*(1+b*x^2/a)^(1
/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/
a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-6*a^2*c^2*f^2/e/(-b/a)^(1/2)*(1+b*x^2/a
)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x
*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))-3*a^2*c^2*e*f^2*(1/2*f^
2/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/e*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2
)/(f*x^2+e)-1/2*d*b/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/(-b/a)^(1/2)*(1+b*x^
2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF
(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+1/2*f*b/(a*c*f^2-a*d*e*f-b*c*e*f
+b*d*e^2)/e*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*
d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2)
)-1/2*f*b/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/e*c/(-b/a)^(1/2)*(1+b*x^2/a)^(
1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticE(x*(-b
/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+1/2/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/
e^2*f^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+
b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="
fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{1}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx$$

input
```
integrate(1/x**4/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**2,x)
```

output
```
Integral(1/(x**4*sqrt(a + b*x**2)*sqrt(c + d*x**2)*(e + f*x**2)**2), x)
```

Maxima [F]

$$\int \frac{1}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^2x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="
maxima")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^2*x^4), x)
```

Giac [F]

$$\int \frac{1}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^2x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="
giac")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^2*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{1}{x^4\,\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,(f\,x^2+e)^2}\,dx$$

input
```
int(1/(x^4*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^2),x)
```

output
```
int(1/(x^4*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^2), x)
```

Reduce [F]

$$\int \frac{1}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \text{too large to display}$$

input
```
int(1/x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x)
```

output
```
( - sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*c*e + 5*sqrt(c + d*x**2)*sqrt(a +
b*x**2)*a*c*f*x**2 + 2*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*d*e*x**2 + 2*sq
rt(c + d*x**2)*sqrt(a + b*x**2)*b*c*e*x**2 + 5*int((sqrt(c + d*x**2)*sqrt(
a + b*x**2)*x**4)/(a*c*e**2 + 2*a*c*e*f*x**2 + a*c*f**2*x**4 + a*d*e**2*x*
*2 + 2*a*d*e*f*x**4 + a*d*f**2*x**6 + b*c*e**2*x**2 + 2*b*c*e*f*x**4 + b*c
*f**2*x**6 + b*d*e**2*x**4 + 2*b*d*e*f*x**6 + b*d*f**2*x**8),x)*a*b*c*d*e*
f**2*x**3 + 5*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c*e**2 + 2*a
*c*e*f*x**2 + a*c*f**2*x**4 + a*d*e**2*x**2 + 2*a*d*e*f*x**4 + a*d*f**2*x*
*6 + b*c*e**2*x**2 + 2*b*c*e*f*x**4 + b*c*f**2*x**6 + b*d*e**2*x**4 + 2*b*
d*e*f*x**6 + b*d*f**2*x**8),x)*a*b*c*d*f**3*x**5 + 2*int((sqrt(c + d*x**2)
*sqrt(a + b*x**2)*x**4)/(a*c*e**2 + 2*a*c*e*f*x**2 + a*c*f**2*x**4 + a*d*e
**2*x**2 + 2*a*d*e*f*x**4 + a*d*f**2*x**6 + b*c*e**2*x**2 + 2*b*c*e*f*x**4
 + b*c*f**2*x**6 + b*d*e**2*x**4 + 2*b*d*e*f*x**6 + b*d*f**2*x**8),x)*a*b*
d**2*e**2*f*x**3 + 2*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c*e**
2 + 2*a*c*e*f*x**2 + a*c*f**2*x**4 + a*d*e**2*x**2 + 2*a*d*e*f*x**4 + a*d*
f**2*x**6 + b*c*e**2*x**2 + 2*b*c*e*f*x**4 + b*c*f**2*x**6 + b*d*e**2*x**4
 + 2*b*d*e*f*x**6 + b*d*f**2*x**8),x)*a*b*d**2*e*f**2*x**5 + 2*int((sqrt(c
 + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c*e**2 + 2*a*c*e*f*x**2 + a*c*f**2*x*
*4 + a*d*e**2*x**2 + 2*a*d*e*f*x**4 + a*d*f**2*x**6 + b*c*e**2*x**2 + 2*b*
c*e*f*x**4 + b*c*f**2*x**6 + b*d*e**2*x**4 + 2*b*d*e*f*x**6 + b*d*f**2*...
```

3.196 $\int \frac{x^{10}}{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)^3}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 1080

$$\int \frac{x^{10}}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx =$$

$$-\frac{(16a^3df^3(de-cf)^2+8a^2bf^2(de-cf)^2(5de+2cf)+b^3e^2(105d^3e^3-170cd^2e^2f+40c^2def^2+16c^3f^3}{24bd^2f^4(be-af)^2(de-cf)^2\sqrt{a+b}}$$

$$+\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3bdf^3}-\frac{e^4x\sqrt{a+bx^2}\sqrt{c+dx^2}}{4f^3(be-af)(de-cf)\,(e+fx^2)^2}$$

$$-\frac{e^3(af(14de-17cf)-be(11de-14cf))x\sqrt{a+bx^2}\sqrt{c+dx^2}}{8f^3(be-af)^2(de-cf)^2\,(e+fx^2)}$$

$$+\frac{\sqrt{a}(16a^3df^3(de-cf)^2+8a^2bf^2(de-cf)^2(5de+2cf)+b^3e^2(105d^3e^3-170cd^2e^2f+40c^2def^2+16c}{24b^{3/2}d^2f^4(be-af)^2(de-cf}$$

$$+\frac{a^{3/2}(8a^3cf^4(de-cf)-b^3e^3(105d^2e^2-100cdef-8c^2f^2)+3ab^2e^2f(80d^2e^2-73cdef-8c^2f^2)-24a}{24b^{3/2}cdf^4(be-af)^3(de-cf)\sqrt{a+bx}}$$

$$+\frac{a^{3/2}e^2(3a^2f^2(16d^2e^2-36cdef+21c^2f^2)+b^2e^2(35d^2e^2-80cdef+48c^2f^2)-2abef(40d^2e^2-91cde}{8\sqrt{b}cf^4(be-af)^3(de-cf)^2\sqrt{a+bx^2}\sqrt{\frac{a(}{c(}}}$$

output
```
-1/24*(16*a^3*d*f^3*(-c*f+d*e)^2+8*a^2*b*f^2*(-c*f+d*e)^2*(2*c*f+5*d*e)+b^
3*e^2*(16*c^3*f^3+40*c^2*d*e*f^2-170*c*d^2*e^2*f+105*d^3*e^3)-a*b^2*e*f*(3
2*c^3*f^3+64*c^2*d*e*f^2-275*c*d^2*e^2*f+170*d^3*e^3))*x*(d*x^2+c)^(1/2)/b
/d^2/f^4/(-a*f+b*e)^2/(-c*f+d*e)^2/(b*x^2+a)^(1/2)+1/3*x*(b*x^2+a)^(1/2)*(
d*x^2+c)^(1/2)/b/d/f^3-1/4*e^4*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/f^3/(-a*f
+b*e)/(-c*f+d*e)/(f*x^2+e)^2-1/8*e^3*(a*f*(-17*c*f+14*d*e)-b*e*(-14*c*f+11
*d*e))*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/f^3/(-a*f+b*e)^2/(-c*f+d*e)^2/(f*
x^2+e)+1/24*a^(1/2)*(16*a^3*d*f^3*(-c*f+d*e)^2+8*a^2*b*f^2*(-c*f+d*e)^2*(2
*c*f+5*d*e)+b^3*e^2*(16*c^3*f^3+40*c^2*d*e*f^2-170*c*d^2*e^2*f+105*d^3*e^3
)-a*b^2*e*f*(32*c^3*f^3+64*c^2*d*e*f^2-275*c*d^2*e^2*f+170*d^3*e^3))*(d*x^
2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2)
)/b^(3/2)/d^2/f^4/(-a*f+b*e)^2/(-c*f+d*e)^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c
/(b*x^2+a))^(1/2)+1/24*a^(3/2)*(8*a^3*c*f^4*(-c*f+d*e)-b^3*e^3*(-8*c^2*f^2
-100*c*d*e*f+105*d^2*e^2)+3*a*b^2*e^2*f*(-8*c^2*f^2-73*c*d*e*f+80*d^2*e^2)
-24*a^2*b*e*f^2*(-c^2*f^2-5*c*d*e*f+6*d^2*e^2))*(d*x^2+c)^(1/2)*InverseJac
obiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/b^(3/2)/c/d/f^4/(-a*f+b
*e)^3/(-c*f+d*e)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)+1/8*a^(3/
2)*e^2*(3*a^2*f^2*(21*c^2*f^2-36*c*d*e*f+16*d^2*e^2)+b^2*e^2*(48*c^2*f^2-8
0*c*d*e*f+35*d^2*e^2)-2*a*b*e*f*(54*c^2*f^2-91*c*d*e*f+40*d^2*e^2))*(d*x^2
+c)^(1/2)*EllipticPi(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),1-a*f/b/e,(1-a...
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.10 (sec) , antiderivative size = 730, normalized size of antiderivative = 0.68

$$\int \frac{x^{10}}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx$$

$$= \frac{-\sqrt{\frac{b}{a}}df^2x(a+bx^2)\,(c+dx^2)\,\Big(6bde^4(be-af)(de-cf)-3bde^3(be(11de-14cf)+af(-14de+17cf))}{}$$

input
```
Integrate[x^10/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^3),x]
```

output
```
(-(Sqrt[b/a]*d*f^2*x*(a + b*x^2)*(c + d*x^2)*(6*b*d*e^4*(b*e - a*f)*(d*e -
 c*f) - 3*b*d*e^3*(b*e*(11*d*e - 14*c*f) + a*f*(-14*d*e + 17*c*f))*(e + f*
x^2) - 8*(b*e - a*f)^2*(d*e - c*f)^2*(e + f*x^2)^2)) + I*Sqrt[1 + (b*x^2)/
a]*Sqrt[1 + (d*x^2)/c]*(e + f*x^2)^2*(c*f*(16*a^3*d*f^3*(d*e - c*f)^2 + 8*
a^2*b*f^2*(d*e - c*f)^2*(5*d*e + 2*c*f) + b^3*e^2*(105*d^3*e^3 - 170*c*d^2
*e^2*f + 40*c^2*d*e*f^2 + 16*c^3*f^3) - a*b^2*e*f*(170*d^3*e^3 - 275*c*d^2
*e^2*f + 64*c^2*d*e*f^2 + 32*c^3*f^3))*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (
a*d)/(b*c)] - (d*e - c*f)*(8*a^3*c*d*f^4*(d*e - c*f) + b^3*e^2*(105*d^3*e^
3 - 30*c*d^2*e^2*f - 56*c^2*d*e*f^2 - 16*c^3*f^3) - 8*a^2*b*f^2*(-18*d^3*e
^3 + 11*c*d^2*e^2*f + 5*c^2*d*e*f^2 + 2*c^3*f^3) + a*b^2*e*f*(-240*d^3*e^3
 + 101*c*d^2*e^2*f + 104*c^2*d*e*f^2 + 32*c^3*f^3))*EllipticF[I*ArcSinh[Sq
rt[b/a]*x], (a*d)/(b*c)] + 3*b*d^2*e^2*(3*a^2*f^2*(16*d^2*e^2 - 36*c*d*e*f
 + 21*c^2*f^2) + b^2*e^2*(35*d^2*e^2 - 80*c*d*e*f + 48*c^2*f^2) - 2*a*b*e*
f*(40*d^2*e^2 - 91*c*d*e*f + 54*c^2*f^2))*EllipticPi[(a*f)/(b*e), I*ArcSin
h[Sqrt[b/a]*x], (a*d)/(b*c)]))/(24*b*Sqrt[b/a]*d^2*f^5*(b*e - a*f)^2*(d*e
- c*f)^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^2)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{10}}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx$$

$$\Big\downarrow \; 450$$

$$\int \frac{x^{10}}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^3}dx$$

input
```
Int[x^10/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^3),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3360 vs. 2(1036) = 2072.

Time = 30.41 (sec) , antiderivative size = 3361, normalized size of antiderivative = 3.11

method	result	size
elliptic	Expression too large to display	3361
risch	Expression too large to display	4045
default	Expression too large to display	10282

input
```
int(x^10/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x,method=_RETURNVERBO
SE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(3*c/(-b/a)^(1
/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2
)/d*e/f^4*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-3*c/(-b/a)^(1
/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2
)/d*e/f^4*EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-33/8/(-b/a)^(
1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/
2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))*b*d*e^4/f^3/(a*c*f^2
-a*d*e*f-b*c*e*f+b*d*e^2)^2*a*c-6*e^4/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)^2/
f^3/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*
x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/
2))*a^2*d^2-6*e^4/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)^2/f^3/(-b/a)^(1/2)*(1+
b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*Ellip
ticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*b^2*c^2-35/8*e^6
/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)^2/f^5/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1
+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1
/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*b^2*d^2+2/3*c^2/(-b/a)^(1/2)*(1+b
*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d^2/f^
3*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-2/3*c^2/(-b/a)^(1/2)*
(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d^
2/f^3*EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-1/4/f^3/(a*c*f...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^{10}}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(x^10/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x, algorithm="f
ricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{10}}{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**10/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{10}}{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^3}\,dx = \int \frac{x^{10}}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^3}\,dx$$

input
```
integrate(x^10/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x, algorithm="m
axima")
```

output
```
integrate(x^10/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^3), x)
```

Giac [F]

$$\int \frac{x^{10}}{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^3}\,dx = \int \frac{x^{10}}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^3}\,dx$$

input
```
integrate(x^10/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x, algorithm="g
iac")
```

output
```
integrate(x^10/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{10}}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \int \frac{x^{10}}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,(f\,x^2+e)^3}\,dx$$

input
```
int(x^10/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^3),x)
```

output
```
int(x^10/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^3), x)
```

Reduce [F]

$$\int \frac{x^{10}}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \int \frac{x^{10}}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,(f\,x^2+e)^3}dx$$

input
```
int(x^10/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x)
```

output
```
int(x^10/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x)
```

3.197 $\int \frac{x^8}{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)^3}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 852

$$\int \frac{x^8}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx$$
$$= \frac{(8a^2f^2(de-cf)^2+b^2e^2(15d^2e^2-26cdef+8c^2f^2)-abef(26d^2e^2-45cdef+16c^2f^2))\,x\sqrt{c+dx^2}}{8df^3(be-af)^2(de-cf)^2\sqrt{a+bx^2}}$$
$$+\frac{e^3x\sqrt{a+bx^2}\sqrt{c+dx^2}}{4f^2(be-af)(de-cf)\,(e+fx^2)^2}$$
$$+\frac{e^2(af(10de-13cf)-be(7de-10cf))x\sqrt{a+bx^2}\sqrt{c+dx^2}}{8f^2(be-af)^2(de-cf)^2\,(e+fx^2)}$$
$$-\frac{\sqrt{a}(8a^2f^2(de-cf)^2+b^2e^2(15d^2e^2-26cdef+8c^2f^2)-abef(26d^2e^2-45cdef+16c^2f^2))\,\sqrt{c+dx^2}}{8\sqrt{b}df^3(be-af)^2(de-cf)^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$
$$-\frac{a^{3/2}e(abef(36de-37cf)-b^2e^2(15de-16cf)-24a^2f^2(de-cf))\,\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\right.}{8\sqrt{b}cf^3(be-af)^3(de-cf)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$
$$-\frac{a^{3/2}e(3b^2e^2(5d^2e^2-12cdef+8c^2f^2)-2abef(18d^2e^2-43cdef+28c^2f^2)+a^2f^2(24d^2e^2-56cdef+}{8\sqrt{b}cf^3(be-af)^3(de-cf)^2\sqrt{a+bx^2}\sqrt{\frac{a(c+}{c(a+}}}$$

output
```
1/8*(8*a^2*f^2*(-c*f+d*e)^2+b^2*e^2*(8*c^2*f^2-26*c*d*e*f+15*d^2*e^2)-a*b*
e*f*(16*c^2*f^2-45*c*d*e*f+26*d^2*e^2))*x*(d*x^2+c)^(1/2)/d/f^3/(-a*f+b*e)
^2/(-c*f+d*e)^2/(b*x^2+a)^(1/2)+1/4*e^3*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/
f^2/(-a*f+b*e)/(-c*f+d*e)/(f*x^2+e)^2+1/8*e^2*(a*f*(-13*c*f+10*d*e)-b*e*(-
10*c*f+7*d*e))*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/f^2/(-a*f+b*e)^2/(-c*f+d*
e)^2/(f*x^2+e)-1/8*a^(1/2)*(8*a^2*f^2*(-c*f+d*e)^2+b^2*e^2*(8*c^2*f^2-26*c
*d*e*f+15*d^2*e^2)-a*b*e*f*(16*c^2*f^2-45*c*d*e*f+26*d^2*e^2))*(d*x^2+c)^(
1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/b^(1
/2)/d/f^3/(-a*f+b*e)^2/(-c*f+d*e)^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+
a))^(1/2)-1/8*a^(3/2)*e*(a*b*e*f*(-37*c*f+36*d*e)-b^2*e^2*(-16*c*f+15*d*e)
-24*a^2*f^2*(-c*f+d*e))*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a
^(1/2)),(1-a*d/b/c)^(1/2))/b^(1/2)/c/f^3/(-a*f+b*e)^3/(-c*f+d*e)/(b*x^2+a)
^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)-1/8*a^(3/2)*e*(3*b^2*e^2*(8*c^2*f^2
-12*c*d*e*f+5*d^2*e^2)-2*a*b*e*f*(28*c^2*f^2-43*c*d*e*f+18*d^2*e^2)+a^2*f^
2*(35*c^2*f^2-56*c*d*e*f+24*d^2*e^2))*(d*x^2+c)^(1/2)*EllipticPi(b^(1/2)*x
/a^(1/2)/(1+b*x^2/a)^(1/2),1-a*f/b/e,(1-a*d/b/c)^(1/2))/b^(1/2)/c/f^3/(-a*
f+b*e)^3/(-c*f+d*e)^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.09 (sec) , antiderivative size = 559, normalized size of antiderivative = 0.66

$$\int \frac{x^8}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx$$
$$= \frac{\sqrt{\frac{b}{a}}de^2f^2x(a+bx^2)\,(c+dx^2)\,(2e(be-af)(de-cf)-(be(7de-10cf)+af(-10de+13cf))\,(e+fx^2)}{}$$

input
```
Integrate[x^8/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^3),x]
```

output
```
(Sqrt[b/a]*d*e^2*f^2*x*(a + b*x^2)*(c + d*x^2)*(2*e*(b*e - a*f)*(d*e - c*f
) - (b*e*(7*d*e - 10*c*f) + a*f*(-10*d*e + 13*c*f))*(e + f*x^2)) - I*Sqrt[
1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(e + f*x^2)^2*(c*f*(8*a^2*f^2*(d*e - c*
f)^2 + a*b*e*f*(-26*d^2*e^2 + 45*c*d*e*f - 16*c^2*f^2) + b^2*e^2*(15*d^2*e
^2 - 26*c*d*e*f + 8*c^2*f^2))*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c
)] - (d*e - c*f)*(b^2*e^2*(15*d^2*e^2 - 6*c*d*e*f - 8*c^2*f^2) - 8*a^2*f^2
*(-3*d^2*e^2 + 2*c*d*e*f + c^2*f^2) + a*b*e*f*(-36*d^2*e^2 + 19*c*d*e*f +
16*c^2*f^2))*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + d*e*(3*b^2*e
^2*(5*d^2*e^2 - 12*c*d*e*f + 8*c^2*f^2) - 2*a*b*e*f*(18*d^2*e^2 - 43*c*d*e
*f + 28*c^2*f^2) + a^2*f^2*(24*d^2*e^2 - 56*c*d*e*f + 35*c^2*f^2))*Ellipti
cPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(8*Sqrt[b/a]*d*f^4
*(b*e - a*f)^2*(d*e - c*f)^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^2
)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx$$

$$\Big\downarrow \; 450$$

$$\int \frac{x^8}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^3}dx$$

input
```
Int[x^8/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^3),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2926 vs. $2(814) = 1628$.

Time = 11.05 (sec) , antiderivative size = 2927, normalized size of antiderivative = 3.44

method	result	size
elliptic	Expression too large to display	2927
default	Expression too large to display	6862

input
```
int(x^8/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x,method=_RETURNVERBOS
E)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(7/8*c/(-b/a)^
(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1
/2)*d*e^4*b^2/f^3/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)^2*EllipticE(x*(-b/a)^(
1/2),(-1+(a*d+b*c)/c/b)^(1/2))-7*e^2/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)^2/f
/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2
+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))
*a^2*c*d-7*e^2/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)^2/f/(-b/a)^(1/2)*(1+b*x^2
/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi
(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*a*b*c^2-3/(-b/a)^(1/2
)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*
EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))*e/f^4-1/8*e^2*(13*a*c*f
^2-10*a*d*e*f-10*b*c*e*f+7*b*d*e^2)/f^2/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)^
2*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(f*x^2+e)+9/8/(-b/a)^(1/2)*(1+b*x^
2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF
(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))*b^2*d^2*e^5/f^4/(a*c*f^2-a*d*e*f
-b*c*e*f+b*d*e^2)^2+43/4*e^3/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)^2/f^2/(-b/a
)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^
(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*a*b*c
*d-9/2*e^4/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)^2/f^3/(-b/a)^(1/2)*(1+b*x^2/a
)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticP...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^8}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(x^8/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^8}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**8/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^8}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \int \frac{x^8}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^3}\,dx$$

input
```
integrate(x^8/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x, algorithm="ma
xima")
```

output
```
integrate(x^8/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^3), x)
```

Giac [F]

$$\int \frac{x^8}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \int \frac{x^8}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^3}\,dx$$

input
```
integrate(x^8/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x, algorithm="gi
ac")
```

output
```
integrate(x^8/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^8}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \int \frac{x^8}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,(f\,x^2+e)^3}\,dx$$

input
```
int(x^8/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^3),x)
```

output
```
int(x^8/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^3), x)
```

Reduce [F]

$$\int \frac{x^8}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \text{too large to display}$$

input
```
int(x^8/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x)
```

output
```
( - sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*c*f*x + 4*sqrt(c + d*x**2)*sqrt(a
+ b*x**2)*a*d*e*x + 2*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*d*f*x**3 + 4*sqr
t(c + d*x**2)*sqrt(a + b*x**2)*b*c*e*x + 2*sqrt(c + d*x**2)*sqrt(a + b*x**
2)*b*c*f*x**3 - 3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b*d*e*x**3 - 2*int((sq
rt(c + d*x**2)*sqrt(a + b*x**2)*x**6)/(2*a**2*c*d*e**3*f + 6*a**2*c*d*e**2
*f**2*x**2 + 6*a**2*c*d*e*f**3*x**4 + 2*a**2*c*d*f**4*x**6 + 2*a**2*d**2*e
**3*f*x**2 + 6*a**2*d**2*e**2*f**2*x**4 + 6*a**2*d**2*e*f**3*x**6 + 2*a**2
*d**2*f**4*x**8 + 2*a*b*c**2*e**3*f + 6*a*b*c**2*e**2*f**2*x**2 + 6*a*b*c*
*2*e*f**3*x**4 + 2*a*b*c**2*f**4*x**6 - 3*a*b*c*d*e**4 - 5*a*b*c*d*e**3*f*
x**2 + 3*a*b*c*d*e**2*f**2*x**4 + 9*a*b*c*d*e*f**3*x**6 + 4*a*b*c*d*f**4*x
**8 - 3*a*b*d**2*e**4*x**2 - 7*a*b*d**2*e**3*f*x**4 - 3*a*b*d**2*e**2*f**2
*x**6 + 3*a*b*d**2*e*f**3*x**8 + 2*a*b*d**2*f**4*x**10 + 2*b**2*c**2*e**3*
f*x**2 + 6*b**2*c**2*e**2*f**2*x**4 + 6*b**2*c**2*e*f**3*x**6 + 2*b**2*c**
2*f**4*x**8 - 3*b**2*c*d*e**4*x**2 - 7*b**2*c*d*e**3*f*x**4 - 3*b**2*c*d*e
**2*f**2*x**6 + 3*b**2*c*d*e*f**3*x**8 + 2*b**2*c*d*f**4*x**10 - 3*b**2*d*
*2*e**4*x**4 - 9*b**2*d**2*e**3*f*x**6 - 9*b**2*d**2*e**2*f**2*x**8 - 3*b*
*2*d**2*e*f**3*x**10),x)*a**2*b*c*d**2*e**2*f**3 - 4*int((sqrt(c + d*x**2)
*sqrt(a + b*x**2)*x**6)/(2*a**2*c*d*e**3*f + 6*a**2*c*d*e**2*f**2*x**2 + 6
*a**2*c*d*e*f**3*x**4 + 2*a**2*c*d*f**4*x**6 + 2*a**2*d**2*e**3*f*x**2 + 6
*a**2*d**2*e**2*f**2*x**4 + 6*a**2*d**2*e*f**3*x**6 + 2*a**2*d**2*f**4*...
```

3.198 $\int \frac{x^6}{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)^3}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 666

$$\int \frac{x^6}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = -\frac{e^2x\sqrt{a+bx^2}\sqrt{c+dx^2}}{4f(be-af)(de-cf)\,(e+fx^2)^2}+\frac{3e(af(2de-3cf)-be(de-2cf))x\sqrt{a+bx^2}}{8f^2(be-af)^2(de-cf)\sqrt{c+dx^2}\,(e+fx^2)}$$
$$-\frac{3\sqrt{c}\sqrt{d}e(af(2de-3cf)-be(de-2cf))\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{8f^2(be-af)^2(de-cf)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$
$$+\frac{c^{3/2}(af(4d^2e^2-9cdef+8c^2f^2)-be(3d^2e^2-8cdef+8c^2f^2))\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\right.}{8a\sqrt{d}f^2(be-af)(de-cf)^3\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$
$$+\frac{c^{3/2}(b^2e^2(3d^2e^2-8cdef+8c^2f^2)-2abef(4d^2e^2-11cdef+10c^2f^2)+a^2f^2(8d^2e^2-20cdef+15c^2f}{8a\sqrt{d}f^2(be-af)^2(de-cf)^3\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx}}$$

output
```
-1/4*e^2*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/f/(-a*f+b*e)/(-c*f+d*e)/(f*x^2+
e)^2+3/8*e*(a*f*(-3*c*f+2*d*e)-b*e*(-2*c*f+d*e))*x*(b*x^2+a)^(1/2)/f^2/(-a
*f+b*e)^2/(-c*f+d*e)/(d*x^2+c)^(1/2)/(f*x^2+e)-3/8*c^(1/2)*d^(1/2)*e*(a*f*
(-3*c*f+2*d*e)-b*e*(-2*c*f+d*e))*(b*x^2+a)^(1/2)*EllipticE(d^(1/2)*x/c^(1/
2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))/f^2/(-a*f+b*e)^2/(-c*f+d*e)^2/(c*(
b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)+1/8*c^(3/2)*(a*f*(8*c^2*f^2-9*
c*d*e*f+4*d^2*e^2)-b*e*(8*c^2*f^2-8*c*d*e*f+3*d^2*e^2))*(b*x^2+a)^(1/2)*In
verseJacobiAM(arctan(d^(1/2)*x/c^(1/2)),(1-b*c/a/d)^(1/2))/a/d^(1/2)/f^2/(
-a*f+b*e)/(-c*f+d*e)^3/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)+1/8
*c^(3/2)*(b^2*e^2*(8*c^2*f^2-8*c*d*e*f+3*d^2*e^2)-2*a*b*e*f*(10*c^2*f^2-11
*c*d*e*f+4*d^2*e^2)+a^2*f^2*(15*c^2*f^2-20*c*d*e*f+8*d^2*e^2))*(b*x^2+a)^(
1/2)*EllipticPi(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),1-c*f/d/e,(1-b*c/a/d)^
(1/2))/a/d^(1/2)/f^2/(-a*f+b*e)^2/(-c*f+d*e)^3/(c*(b*x^2+a)/a/(d*x^2+c))^(
1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.70 (sec) , antiderivative size = 458, normalized size of antiderivative = 0.69

$$\int \frac{x^6}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx$$
$$= \frac{-\sqrt{\frac{b}{a}}ef^2x(a+bx^2)\,(c+dx^2)\,(2e(be-af)(de-cf)-3(be(de-2cf)+af(-2de+3cf))\,(e+fx^2))-}{}$$

input
```
Integrate[x^6/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^3),x]
```

output
```
(-(Sqrt[b/a]*e*f^2*x*(a + b*x^2)*(c + d*x^2)*(2*e*(b*e - a*f)*(d*e - c*f)
- 3*(b*e*(d*e - 2*c*f) + a*f*(-2*d*e + 3*c*f))*(e + f*x^2))) - I*Sqrt[1 +
(b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(e + f*x^2)^2*(3*b*c*e*f*(a*f*(2*d*e - 3*c*
f) + b*e*(-(d*e) + 2*c*f))*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]
+ (d*e - c*f)*(b^2*e^2*(3*d*e - 2*c*f) + 8*a^2*f^2*(d*e - c*f) + a*b*e*f*(
-8*d*e + 7*c*f))*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (a^2*f^2
*(-8*d^2*e^2 + 20*c*d*e*f - 15*c^2*f^2) + b^2*e^2*(-3*d^2*e^2 + 8*c*d*e*f
- 8*c^2*f^2) + 2*a*b*e*f*(4*d^2*e^2 - 11*c*d*e*f + 10*c^2*f^2))*EllipticPi
[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(8*Sqrt[b/a]*f^3*(b*e
 - a*f)^2*(d*e - c*f)^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^2)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx$$
$$\Big\downarrow \; 450$$
$$\int \frac{x^6}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^3}dx$$

input
```
Int[x^6/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^3),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2711 vs. $2(634) = 1268$.

Time = 11.07 (sec) , antiderivative size = 2712, normalized size of antiderivative = 4.07

method	result	size
elliptic	Expression too large to display	2712
default	Expression too large to display	5216

input
```
int(x^6/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x,method=_RETURNVERBOS
E)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-1/4/f/(a*c*f
^2-a*d*e*f-b*c*e*f+b*d*e^2)*e^2*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(f*x
^2+e)^2+3/8*(3*a*c*f^2-2*a*d*e*f-2*b*c*e*f+b*d*e^2)*e/f/(a*c*f^2-a*d*e*f-b
*c*e*f+b*d*e^2)^2*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(f*x^2+e)-5/8/(-b/
a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)
^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))*b^2*d^2*e^4/f^3/
(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)^2-11/4*e^2/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*
e^2)^2/f/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2
+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a
)^(1/2))*a*b*c*d-3/4*b^2*e^2/f/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)^2*c^2/(-b
/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c
)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+3/4*b^2*e^2/f/(
a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)^2*c^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d
*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticE(x*(-b/a)^(1/2)
,(-1+(a*d+b*c)/c/b)^(1/2))+5/2*e/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)^2/(-b/a
)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^
(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*a^2*c
*d-e^2/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)^2/f/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2
)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a
)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*a^2*d^2+5/2*e/(a*c*f^2-a*d...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^6}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(x^6/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^6}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**6/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^6}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \int \frac{x^6}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^3}\,dx$$

input
```
integrate(x^6/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x, algorithm="ma
xima")
```

output
```
integrate(x^6/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^3), x)
```

Giac [F]

$$\int \frac{x^6}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \int \frac{x^6}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^3}\,dx$$

input
```
integrate(x^6/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x, algorithm="gi
ac")
```

output
```
integrate(x^6/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \int \frac{x^6}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,(f\,x^2+e)^3}\,dx$$

input
```
int(x^6/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^3),x)
```

output
```
int(x^6/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^3), x)
```

Reduce [F]

$$\int \frac{x^6}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx$$
$$= \int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^6}{bd\,f^3x^{10}+ad\,f^3x^8+bc\,f^3x^8+3bde\,f^2x^8+ac\,f^3x^6+3ade\,f^2x^6+3bce\,f^2x^6+3bd\,e^2f\,x^6+3ace\,f^2x}$$

input
```
int(x^6/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x)
```

output

```
int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**6)/(a*c*e**3 + 3*a*c*e**2*f*x**2
 + 3*a*c*e*f**2*x**4 + a*c*f**3*x**6 + a*d*e**3*x**2 + 3*a*d*e**2*f*x**4 +
 3*a*d*e*f**2*x**6 + a*d*f**3*x**8 + b*c*e**3*x**2 + 3*b*c*e**2*f*x**4 + 3
*b*c*e*f**2*x**6 + b*c*f**3*x**8 + b*d*e**3*x**4 + 3*b*d*e**2*f*x**6 + 3*b
*d*e*f**2*x**8 + b*d*f**3*x**10),x)
```

3.199 $\int \frac{x^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^3}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 582

$$\begin{aligned}
&\int \frac{x^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^3}\,dx \\
&= \frac{ex\sqrt{a+bx^2}\sqrt{c+dx^2}}{4(be-af)(de-cf)\left(e+fx^2\right)^2}-\frac{(af(2de-5cf)+be(de+2cf))x\sqrt{a+bx^2}}{8f(be-af)^2(de-cf)\sqrt{c+dx^2}\left(e+fx^2\right)} \\
&\quad+\frac{\sqrt{c}\sqrt{d}(af(2de-5cf)+be(de+2cf))\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{8f(be-af)^2(de-cf)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} \\
&\quad-\frac{c^{3/2}\sqrt{d}\left(3acf^2+be(de-4cf)\right)\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{8af(be-af)(de-cf)^3\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} \\
&\quad-\frac{c^{3/2}\left(3a^2c^2f^4+2abde^2f(2de-5cf)-b^2de^3(de-4cf)\right)\sqrt{a+bx^2}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1\right.}{8a\sqrt{d}ef(be-af)^2(de-cf)^3\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}
\end{aligned}$$

output
```
1/4*e*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(-a*f+b*e)/(-c*f+d*e)/(f*x^2+e)^2-
1/8*(a*f*(-5*c*f+2*d*e)+b*e*(2*c*f+d*e))*x*(b*x^2+a)^(1/2)/f/(-a*f+b*e)^2/
(-c*f+d*e)/(d*x^2+c)^(1/2)/(f*x^2+e)+1/8*c^(1/2)*d^(1/2)*(a*f*(-5*c*f+2*d*
e)+b*e*(2*c*f+d*e))*(b*x^2+a)^(1/2)*EllipticE(d^(1/2)*x/c^(1/2)/(1+d*x^2/c
)^(1/2),(1-b*c/a/d)^(1/2))/f/(-a*f+b*e)^2/(-c*f+d*e)^2/(c*(b*x^2+a)/a/(d*x
^2+c))^(1/2)/(d*x^2+c)^(1/2)-1/8*c^(3/2)*d^(1/2)*(3*a*c*f^2+b*e*(-4*c*f+d*
e))*(b*x^2+a)^(1/2)*InverseJacobiAM(arctan(d^(1/2)*x/c^(1/2)),(1-b*c/a/d)^
(1/2))/a/f/(-a*f+b*e)/(-c*f+d*e)^3/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+
c)^(1/2)-1/8*c^(3/2)*(3*a^2*c^2*f^4+2*a*b*d*e^2*f*(-5*c*f+2*d*e)-b^2*d*e^3
*(-4*c*f+d*e))*(b*x^2+a)^(1/2)*EllipticPi(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1
/2),1-c*f/d/e,(1-b*c/a/d)^(1/2))/a/d^(1/2)/e/f/(-a*f+b*e)^2/(-c*f+d*e)^3/(
c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.35 (sec) , antiderivative size = 392, normalized size of antiderivative = 0.67

$$\int \frac{x^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \frac{\sqrt{\frac{b}{a}}ef^2x(a+bx^2)\,(c+dx^2)\,(af^2(-3ce+2dex^2-5cfx^2)+be(2cf^2x^2+de(3e+fx^2)))-i\sqrt{1+\frac{bx^2}{a}}\sqrt{}}{}$$

input
```
Integrate[x^4/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^3),x]
```

output
```
(Sqrt[b/a]*e*f^2*x*(a + b*x^2)*(c + d*x^2)*(a*f^2*(-3*c*e + 2*d*e*x^2 - 5*
c*f*x^2) + b*e*(2*c*f^2*x^2 + d*e*(3*e + f*x^2))) - I*Sqrt[1 + (b*x^2)/a]*
Sqrt[1 + (d*x^2)/c]*(e + f*x^2)^2*(-(b*c*e*f*(a*f*(2*d*e - 5*c*f) + b*e*(d
*e + 2*c*f))*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]) + b*e*(d*e -
c*f)*(b*e*(d*e - 2*c*f) + a*f*(-4*d*e + 5*c*f))*EllipticF[I*ArcSinh[Sqrt[b
/a]*x], (a*d)/(b*c)] + (3*a^2*c^2*f^4 + 2*a*b*d*e^2*f*(2*d*e - 5*c*f) + b^
2*d*e^3*(-(d*e) + 4*c*f))*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x],
(a*d)/(b*c)]))/(8*Sqrt[b/a]*e*f^2*(b*e - a*f)^2*(d*e - c*f)^2*Sqrt[a + b*x
^2]*Sqrt[c + d*x^2]*(e + f*x^2)^2)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx$$
$$\Big\downarrow \text{450}$$
$$\int \frac{x^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^3}dx$$

input
```
Int[x^4/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^3),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2073 vs. 2(550) = 1100.

Time = 9.88 (sec) , antiderivative size = 2074, normalized size of antiderivative = 3.56

method	result	size
elliptic	Expression too large to display	2074
default	Expression too large to display	3617

input
```
int(x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x,method=_RETURNVERBOS
E)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/4/(a*c*f^2-
a*d*e*f-b*c*e*f+b*d*e^2)*e*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(f*x^2+e)
^2-1/8*(5*a*c*f^2-2*a*d*e*f-2*b*c*e*f-b*d*e^2)/(a*c*f^2-a*d*e*f-b*c*e*f+b*
d*e^2)^2*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(f*x^2+e)-5/4/(a*c*f^2-a*d*
e*f-b*c*e*f+b*d*e^2)^2*e/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/
(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/
c*d)^(1/2)/(-b/a)^(1/2))*a*b*c*d+1/2/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)^2*e
^2/f/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c
*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1
/2))*a*b*d^2+1/2/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)^2*e^2/f/(-b/a)^(1/2)*(1
+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*Elli
pticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*b^2*c*d+1/8*b^2
*d^2*e^3/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)^2/f^2/(-b/a)^(1/2)*(1+b*x^2/a)^
(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-
b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-5/8*b/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^
2)^2*f*c^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x
^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))*a
+1/4*b^2/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)^2*c^2/(-b/a)^(1/2)*(1+b*x^2/a)^
(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-
b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))*e+5/8*b/(a*c*f^2-a*d*e*f-b*c*e*f+b...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**4/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \int \frac{x^4}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^3}\,dx$$

input
```
integrate(x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x, algorithm="ma
xima")
```

output
```
integrate(x^4/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^3), x)
```

Giac [F]

$$\int \frac{x^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \int \frac{x^4}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^3}\,dx$$

input
```
integrate(x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x, algorithm="gi
ac")
```

output
```
integrate(x^4/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \int \frac{x^4}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,(f\,x^2+e)^3}\,dx$$

input
```
int(x^4/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^3),x)
```

output
```
int(x^4/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^3), x)
```

Reduce [F]

$$\int \frac{x^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx$$
$$= \int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^4}{bd\,f^3x^{10}+ad\,f^3x^8+bc\,f^3x^8+3bde\,f^2x^8+ac\,f^3x^6+3ade\,f^2x^6+3bce\,f^2x^6+3bd\,e^2f\,x^6+3ace\,f^2x}$$

input
```
int(x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c*e**3 + 3*a*c*e**2*f*x**2
 + 3*a*c*e*f**2*x**4 + a*c*f**3*x**6 + a*d*e**3*x**2 + 3*a*d*e**2*f*x**4 +
 3*a*d*e*f**2*x**6 + a*d*f**3*x**8 + b*c*e**3*x**2 + 3*b*c*e**2*f*x**4 + 3
*b*c*e*f**2*x**6 + b*c*f**3*x**8 + b*d*e**3*x**4 + 3*b*d*e**2*f*x**6 + 3*b
*d*e*f**2*x**8 + b*d*f**3*x**10),x)
```

3.200 $\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^3}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 583

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^3}\,dx$$
$$= -\frac{fx\sqrt{a+bx^2}\sqrt{c+dx^2}}{4(be-af)(de-cf)\left(e+fx^2\right)^2} + \frac{(be(5de-2cf)-af(2de+cf))x\sqrt{a+bx^2}}{8e(be-af)^2(de-cf)\sqrt{c+dx^2}\left(e+fx^2\right)}$$
$$- \frac{\sqrt{c}\sqrt{d}(be(5de-2cf)-af(2de+cf))\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{8e(be-af)^2(de-cf)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$
$$- \frac{c^{3/2}\sqrt{d}\left(3bde^2-af(4de-cf)\right)\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{8ae(be-af)(de-cf)^3\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$
$$+ \frac{c^{3/2}\left(3b^2d^2e^4-2abcef^2(5de-2cf)+a^2cf^3(4de-cf)\right)\sqrt{a+bx^2}\,\text{EllipticPi}\left(1-\frac{cf}{de},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\right.}{8a\sqrt{d}e^2(be-af)^2(de-cf)^3\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output
```
-1/4*f*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(-a*f+b*e)/(-c*f+d*e)/(f*x^2+e)^2
+1/8*(b*e*(-2*c*f+5*d*e)-a*f*(c*f+2*d*e))*x*(b*x^2+a)^(1/2)/e/(-a*f+b*e)^2
/(-c*f+d*e)/(d*x^2+c)^(1/2)/(f*x^2+e)-1/8*c^(1/2)*d^(1/2)*(b*e*(-2*c*f+5*d
*e)-a*f*(c*f+2*d*e))*(b*x^2+a)^(1/2)*EllipticE(d^(1/2)*x/c^(1/2)/(1+d*x^2/
c)^(1/2),(1-b*c/a/d)^(1/2))/e/(-a*f+b*e)^2/(-c*f+d*e)^2/(c*(b*x^2+a)/a/(d*
x^2+c))^(1/2)/(d*x^2+c)^(1/2)-1/8*c^(3/2)*d^(1/2)*(3*b*d*e^2-a*f*(-c*f+4*d
*e))*(b*x^2+a)^(1/2)*InverseJacobiAM(arctan(d^(1/2)*x/c^(1/2)),(1-b*c/a/d)
^(1/2))/a/e/(-a*f+b*e)/(-c*f+d*e)^3/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2
+c)^(1/2)+1/8*c^(3/2)*(3*b^2*d^2*e^4-2*a*b*c*e*f^2*(-2*c*f+5*d*e)+a^2*c*f^
3*(-c*f+4*d*e))*(b*x^2+a)^(1/2)*EllipticPi(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(
1/2),1-c*f/d/e,(1-b*c/a/d)^(1/2))/a/d^(1/2)/e^2/(-a*f+b*e)^2/(-c*f+d*e)^3/
(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.93 (sec) , antiderivative size = 391, normalized size of antiderivative = 0.67

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx$$
$$= \frac{-\sqrt{\frac{b}{a}}ef^2x(a+bx^2)\,(c+dx^2)\,(2e(be-af)(de-cf)+(be(5de-2cf)-af(2de+cf))\,(e+fx^2))-i\sqrt{}}{}$$

input
```
Integrate[x^2/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^3),x]
```

output
```
(-(Sqrt[b/a]*e*f^2*x*(a + b*x^2)*(c + d*x^2)*(2*e*(b*e - a*f)*(d*e - c*f)
+ (b*e*(5*d*e - 2*c*f) - a*f*(2*d*e + c*f))*(e + f*x^2))) - I*Sqrt[1 + (b*
x^2)/a]*Sqrt[1 + (d*x^2)/c]*(e + f*x^2)^2*(-(b*c*e*f*(a*f*(2*d*e + c*f) +
b*e*(-5*d*e + 2*c*f))*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]) + b*
e*(d*e - c*f)*(-(a*c*f^2) + b*e*(3*d*e - 2*c*f))*EllipticF[I*ArcSinh[Sqrt[
b/a]*x], (a*d)/(b*c)] + (-3*b^2*d^2*e^4 + 2*a*b*c*e*f^2*(5*d*e - 2*c*f) +
a^2*c*f^3*(-4*d*e + c*f))*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x],
(a*d)/(b*c)]))/(8*Sqrt[b/a]*e^2*f*(b*e - a*f)^2*(d*e - c*f)^2*Sqrt[a + b*x
^2]*Sqrt[c + d*x^2]*(e + f*x^2)^2)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx$$
$$\Big\downarrow \ 450$$
$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^3}dx$$

input
```
Int[x^2/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^3),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1951 vs. 2(551) = 1102.

Time = 11.15 (sec) , antiderivative size = 1952, normalized size of antiderivative = 3.35

method	result	size
elliptic	Expression too large to display	1952
default	Expression too large to display	3520

input
```
int(x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x,method=_RETURNVERBOS
E)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-1/4*f/(a*c*f
^2-a*d*e*f-b*c*e*f+b*d*e^2)*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(f*x^2+e
)^2+1/8*f*(a*c*f^2+2*a*d*e*f+2*b*c*e*f-5*b*d*e^2)/(a*c*f^2-a*d*e*f-b*c*e*f
+b*d*e^2)^2/e*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(f*x^2+e)-1/4*b/(a*c*f
^2-a*d*e*f-b*c*e*f+b*d*e^2)^2*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)
^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(-1+(a
*d+b*c)/c/b)^(1/2))*a*d*f+1/8/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)^2*f^3/e^2/
(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+
a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*
a^2*c^2-3/8/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)^2*e^2/f/(-b/a)^(1/2)*(1+b*x^
2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticP
i(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*b^2*d^2+3/8*b^2*d^2/
(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)^2/f/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*
x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),
(-1+(a*d+b*c)/c/b)^(1/2))*e^2+1/4*b^2/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)^2*
c^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*
x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))*f-5/8*b^
2/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)^2*e*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(
1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1
/2),(-1+(a*d+b*c)/c/b)^(1/2))*d-1/4*b^2/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**2/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^3}\,dx$$

input
```
integrate(x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x, algorithm="ma
xima")
```

output
```
integrate(x^2/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^3), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^3}\,dx$$

input
```
integrate(x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x, algorithm="gi
ac")
```

output
```
integrate(x^2/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \int \frac{x^2}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,(f\,x^2+e)^3}\,dx$$

input
```
int(x^2/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^3),x)
```

output
```
int(x^2/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^3), x)
```

Reduce [F]

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx$$
$$= \int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^2}{bd\,f^3x^{10}+ad\,f^3x^8+bc\,f^3x^8+3bde\,f^2x^8+ac\,f^3x^6+3ade\,f^2x^6+3bce\,f^2x^6+3bd\,e^2f\,x^6+3ace\,f^2x}$$

input
```
int(x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c*e**3 + 3*a*c*e**2*f*x**2
 + 3*a*c*e*f**2*x**4 + a*c*f**3*x**6 + a*d*e**3*x**2 + 3*a*d*e**2*f*x**4 +
 3*a*d*e*f**2*x**6 + a*d*f**3*x**8 + b*c*e**3*x**2 + 3*b*c*e**2*f*x**4 + 3
*b*c*e*f**2*x**6 + b*c*f**3*x**8 + b*d*e**3*x**4 + 3*b*d*e**2*f*x**6 + 3*b
*d*e*f**2*x**8 + b*d*f**3*x**10),x)
```

3.201 $\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)^3}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 669

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx$$
$$= \frac{f^2x\sqrt{a+bx^2}\sqrt{c+dx^2}}{4e(be-af)(de-cf)\,(e+fx^2)^2} - \frac{3f(be(3de-2cf)-af(2de-cf))x\sqrt{a+bx^2}}{8e^2(be-af)^2(de-cf)\sqrt{c+dx^2}\,(e+fx^2)}$$
$$+ \frac{3\sqrt{c}\sqrt{d}f(be(3de-2cf)-af(2de-cf))\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{8e^2(be-af)^2(de-cf)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$
$$- \frac{\sqrt{c}\sqrt{d}(af(8d^2e^2-8cdef+3c^2f^2)-be(8d^2e^2-9cdef+4c^2f^2))\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\right.}{8ae^2(be-af)(de-cf)^3\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$
$$- \frac{c^{3/2}f(a^2f^2(8d^2e^2-8cdef+3c^2f^2)-2abef(10d^2e^2-11cdef+4c^2f^2)+b^2e^2(15d^2e^2-20cdef+8c^2}{8a\sqrt{d}e^3(be-af)^2(de-cf)^3\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx}}$$

output
```
1/4*f^2*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/e/(-a*f+b*e)/(-c*f+d*e)/(f*x^2+e
)^2-3/8*f*(b*e*(-2*c*f+3*d*e)-a*f*(-c*f+2*d*e))*x*(b*x^2+a)^(1/2)/e^2/(-a*
f+b*e)^2/(-c*f+d*e)/(d*x^2+c)^(1/2)/(f*x^2+e)+3/8*c^(1/2)*d^(1/2)*f*(b*e*(
-2*c*f+3*d*e)-a*f*(-c*f+2*d*e))*(b*x^2+a)^(1/2)*EllipticE(d^(1/2)*x/c^(1/2
)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))/e^2/(-a*f+b*e)^2/(-c*f+d*e)^2/(c*(b
*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-1/8*c^(1/2)*d^(1/2)*(a*f*(3*c^2
*f^2-8*c*d*e*f+8*d^2*e^2)-b*e*(4*c^2*f^2-9*c*d*e*f+8*d^2*e^2))*(b*x^2+a)^(
1/2)*InverseJacobiAM(arctan(d^(1/2)*x/c^(1/2)),(1-b*c/a/d)^(1/2))/a/e^2/(-
a*f+b*e)/(-c*f+d*e)^3/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-1/8*
c^(3/2)*f*(a^2*f^2*(3*c^2*f^2-8*c*d*e*f+8*d^2*e^2)-2*a*b*e*f*(4*c^2*f^2-11
*c*d*e*f+10*d^2*e^2)+b^2*e^2*(8*c^2*f^2-20*c*d*e*f+15*d^2*e^2))*(b*x^2+a)^
(1/2)*EllipticPi(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),1-c*f/d/e,(1-b*c/a/d)
^(1/2))/a/d^(1/2)/e^3/(-a*f+b*e)^2/(-c*f+d*e)^3/(c*(b*x^2+a)/a/(d*x^2+c))^
(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.17 (sec) , antiderivative size = 438, normalized size of antiderivative = 0.65

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx$$

$$= \frac{\sqrt{\frac{b}{a}}ef^2x(a+bx^2)\,(c+dx^2)\,(2e(be-af)(de-cf)+3(be(3de-2cf)+af(-2de+cf))\,(e+fx^2))-i\cdots}{}$$

input
```
Integrate[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^3),x]
```

output
```
(Sqrt[b/a]*e*f^2*x*(a + b*x^2)*(c + d*x^2)*(2*e*(b*e - a*f)*(d*e - c*f) +
3*(b*e*(3*d*e - 2*c*f) + a*f*(-2*d*e + c*f))*(e + f*x^2)) - I*Sqrt[1 + (b*
x^2)/a]*Sqrt[1 + (d*x^2)/c]*(e + f*x^2)^2*(3*b*c*e*f*(a*f*(2*d*e - c*f) +
b*e*(-3*d*e + 2*c*f))*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - b*e
*(d*e - c*f)*(b*e*(7*d*e - 6*c*f) + a*f*(-4*d*e + 3*c*f))*EllipticF[I*ArcS
inh[Sqrt[b/a]*x], (a*d)/(b*c)] + (a^2*f^2*(8*d^2*e^2 - 8*c*d*e*f + 3*c^2*f
^2) - 2*a*b*e*f*(10*d^2*e^2 - 11*c*d*e*f + 4*c^2*f^2) + b^2*e^2*(15*d^2*e^
2 - 20*c*d*e*f + 8*c^2*f^2))*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x
], (a*d)/(b*c)]))/(8*Sqrt[b/a]*e^3*(b*e - a*f)^2*(d*e - c*f)^2*Sqrt[a + b*
x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^2)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx$$

$$\Big\downarrow \; 433$$

$$\int \left(-\frac{3f}{8e^2\sqrt{a+bx^2}\sqrt{c+dx^2}\,(-ef-f^2x^2)}-\frac{3f}{16e^2\sqrt{a+bx^2}\sqrt{c+dx^2}\,\left(\sqrt{-e}\sqrt{f}-fx\right)^2}-\frac{3f}{16e^2\sqrt{a+bx^2}\sqrt{c+d\right.$$

$$\Big\downarrow \; 2009$$

$$-\frac{3f\int \frac{1}{\left(\sqrt{-e}\sqrt{f}-fx\right)^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{16e^2}-\frac{3f\int \frac{1}{\left(fx+\sqrt{-e}\sqrt{f}\right)^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{16e^2}-\frac{f^{3/2}\int \frac{1}{\left(\sqrt{-e}\sqrt{f}-fx\right)^3\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{8(-e)^{3/2}}-\frac{f^{3/2}\int \frac{1}{\left(fx+\sqrt{-e}\sqrt{f}\right)^3\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{8(-e)^{3/2}}+\frac{3\sqrt{-a}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{-a}}\right),\frac{ad}{bc}\right)}{8\sqrt{b}e^3\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

input
```
Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^3),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 433
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)
^q*(e + f*x^2)^r, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, f, p,
 q, r}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2615 vs. $2(637) = 1274$.

Time = 11.02 (sec) , antiderivative size = 2616, normalized size of antiderivative = 3.91

method	result	size
elliptic	Expression too large to display	2616
default	Expression too large to display	4650

input
```
int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/4*f^2/(a*c*
f^2-a*d*e*f-b*c*e*f+b*d*e^2)/e*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(f*x^
2+e)^2+3/8*f^2*(a*c*f^2-2*a*d*e*f-2*b*c*e*f+3*b*d*e^2)/(a*c*f^2-a*d*e*f-b*
c*e*f+b*d*e^2)^2/e^2*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(f*x^2+e)+11/4/
(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)^2/e*f^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(
1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(
1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*a*b*c*d-3/4*f^2*b^2/(a*c*f^2-a*d
*e*f-b*c*e*f+b*d*e^2)^2/e*c^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(
1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d
+b*c)/c/b)^(1/2))+3/4*f^2*b^2/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)^2/e*c^2/(-
b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*
c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+3/8/(a*c*f^2-a
*d*e*f-b*c*e*f+b*d*e^2)^2/e^3*f^4/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/
c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f
/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*a^2*c^2+1/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e
^2)^2/e*f^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*
x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-
b/a)^(1/2))*a^2*d^2-7/8*b^2*d^2/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)^2*e/(-b/
a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)
^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-1/(a*c*f^2-a*...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^3}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x, algorithm="maxi
ma")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^3}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x, algorithm="giac
")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \int \frac{1}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,(f\,x^2+e)^3}\,dx$$

input
```
int(1/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^3),x)
```

output
```
int(1/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^3), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx$$
$$= \int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{bd\,f^3x^{10}+ad\,f^3x^8+bc\,f^3x^8+3bde\,f^2x^8+ac\,f^3x^6+3ade\,f^2x^6+3bce\,f^2x^6+3bd\,e^2f\,x^6+3ace\,f^2x}$$

input
```
int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c*e**3 + 3*a*c*e**2*f*x**2 + 3*
a*c*e*f**2*x**4 + a*c*f**3*x**6 + a*d*e**3*x**2 + 3*a*d*e**2*f*x**4 + 3*a*
d*e*f**2*x**6 + a*d*f**3*x**8 + b*c*e**3*x**2 + 3*b*c*e**2*f*x**4 + 3*b*c*
e*f**2*x**6 + b*c*f**3*x**8 + b*d*e**3*x**4 + 3*b*d*e**2*f*x**6 + 3*b*d*e*
f**2*x**8 + b*d*f**3*x**10),x)
```

3.202 $\int \frac{1}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)^3} dx$

Optimal result

Integrand size = 35, antiderivative size = 859

$$\int \frac{1}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx$$
$$= -\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{acex\,(e+fx^2)^2} + \frac{f(af(4de-5cf)-4be(de-cf))x\sqrt{a+bx^2}\sqrt{c+dx^2}}{4ace^2(be-af)(de-cf)\,(e+fx^2)^2}$$
$$+ \frac{(8b^2e^2(de-cf)^2+a^2f^2(8d^2e^2-26cdef+15c^2f^2)-abef(16d^2e^2-45cdef+26c^2f^2))\,x\sqrt{a+bx^2}}{8ace^3(be-af)^2(de-cf)\sqrt{c+dx^2}\,(e+fx^2)}$$
$$- \frac{\sqrt{d}(8b^2e^2(de-cf)^2+a^2f^2(8d^2e^2-26cdef+15c^2f^2)-abef(16d^2e^2-45cdef+26c^2f^2))\,\sqrt{a+bx^2}}{8a\sqrt{c}e^3(be-af)^2(de-cf)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$
$$+ \frac{\sqrt{c}\sqrt{d}f(3af(8d^2e^2-12cdef+5c^2f^2)-be(24d^2e^2-37cdef+16c^2f^2))\,\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\right.\right.}{8ae^3(be-af)(de-cf)^3\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$
$$+ \frac{c^{3/2}f^2(3a^2f^2(8d^2e^2-12cdef+5c^2f^2)-2abef(28d^2e^2-43cdef+18c^2f^2)+b^2e^2(35d^2e^2-56cdef}{8a\sqrt{d}e^4(be-af)^2(de-cf)^3\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+}}$$

output
```
-(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a/c/e/x/(f*x^2+e)^2+1/4*f*(a*f*(-5*c*f+4*
d*e)-4*b*e*(-c*f+d*e))*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a/c/e^2/(-a*f+b*e
)/(-c*f+d*e)/(f*x^2+e)^2+1/8*(8*b^2*e^2*(-c*f+d*e)^2+a^2*f^2*(15*c^2*f^2-2
6*c*d*e*f+8*d^2*e^2)-a*b*e*f*(26*c^2*f^2-45*c*d*e*f+16*d^2*e^2))*x*(b*x^2+
a)^(1/2)/a/c/e^3/(-a*f+b*e)^2/(-c*f+d*e)/(d*x^2+c)^(1/2)/(f*x^2+e)-1/8*d^(
1/2)*(8*b^2*e^2*(-c*f+d*e)^2+a^2*f^2*(15*c^2*f^2-26*c*d*e*f+8*d^2*e^2)-a*b
*e*f*(26*c^2*f^2-45*c*d*e*f+16*d^2*e^2))*(b*x^2+a)^(1/2)*EllipticE(d^(1/2)
*x/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))/a/c^(1/2)/e^3/(-a*f+b*e)^2
/(-c*f+d*e)^2/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)+1/8*c^(1/2)*
d^(1/2)*f*(3*a*f*(5*c^2*f^2-12*c*d*e*f+8*d^2*e^2)-b*e*(16*c^2*f^2-37*c*d*e
*f+24*d^2*e^2))*(b*x^2+a)^(1/2)*InverseJacobiAM(arctan(d^(1/2)*x/c^(1/2)),
(1-b*c/a/d)^(1/2))/a/e^3/(-a*f+b*e)/(-c*f+d*e)^3/(c*(b*x^2+a)/a/(d*x^2+c))
^(1/2)/(d*x^2+c)^(1/2)+1/8*c^(3/2)*f^2*(3*a^2*f^2*(5*c^2*f^2-12*c*d*e*f+8*
d^2*e^2)-2*a*b*e*f*(18*c^2*f^2-43*c*d*e*f+28*d^2*e^2)+b^2*e^2*(24*c^2*f^2-
56*c*d*e*f+35*d^2*e^2))*(b*x^2+a)^(1/2)*EllipticPi(d^(1/2)*x/c^(1/2)/(1+d*
x^2/c)^(1/2),1-c*f/d/e,(1-b*c/a/d)^(1/2))/a/d^(1/2)/e^4/(-a*f+b*e)^2/(-c*f
+d*e)^3/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.13 (sec) , antiderivative size = 567, normalized size of antiderivative = 0.66

$$\int \frac{1}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx$$

$$= \frac{\sqrt{\frac{b}{a}}\left(-\sqrt{\frac{b}{a}}e(a+bx^2)\,(c+dx^2)\left(2acef^3(-be+af)(-de+cf)x^2+acf^3(be(13de-10cf)+af(-10de\right.\right.}{}$$

input
```
Integrate[1/(x^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^3),x]
```

output
```
(Sqrt[b/a]*(-(Sqrt[b/a]*e*(a + b*x^2)*(c + d*x^2)*(2*a*c*e*f^3*(-(b*e) + a
*f)*(-(d*e) + c*f)*x^2 + a*c*f^3*(b*e*(13*d*e - 10*c*f) + a*f*(-10*d*e + 7
*c*f))*x^2*(e + f*x^2) + 8*(b*e - a*f)^2*(d*e - c*f)^2*(e + f*x^2)^2)) + I
*c*x*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(e + f*x^2)^2*(-(b*e*(8*b^2*e
^2*(d*e - c*f)^2 + a*b*e*f*(-16*d^2*e^2 + 45*c*d*e*f - 26*c^2*f^2) + a^2*f
^2*(8*d^2*e^2 - 26*c*d*e*f + 15*c^2*f^2))*EllipticE[I*ArcSinh[Sqrt[b/a]*x]
, (a*d)/(b*c)]) + b*e*(d*e - c*f)*(a^2*f^2*(16*d*e - 15*c*f) + 8*b^2*e^2*(
d*e - c*f) + a*b*e*f*(-27*d*e + 26*c*f))*EllipticF[I*ArcSinh[Sqrt[b/a]*x],
 (a*d)/(b*c)] + a*f*(3*a^2*f^2*(8*d^2*e^2 - 12*c*d*e*f + 5*c^2*f^2) - 2*a*
b*e*f*(28*d^2*e^2 - 43*c*d*e*f + 18*c^2*f^2) + b^2*e^2*(35*d^2*e^2 - 56*c*
d*e*f + 24*c^2*f^2))*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)
/(b*c)])))/(8*b*c*e^4*(b*e - a*f)^2*(d*e - c*f)^2*x*Sqrt[a + b*x^2]*Sqrt[c
 + d*x^2]*(e + f*x^2)^2)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx$$
$$\Big\downarrow \; 450$$
$$\int \frac{1}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^3}dx$$

input
```
Int[1/(x^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^3),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2868 vs. $2(823) = 1646$.

Time = 28.13 (sec) , antiderivative size = 2869, normalized size of antiderivative = 3.34

method	result	size
elliptic	Expression too large to display	2869
risch	Expression too large to display	3831
default	Expression too large to display	7270

input
```
int(1/x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x,method=_RETURNVERB
OSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-15/8/(a*c*f^
2-a*d*e*f-b*c*e*f+b*d*e^2)^2/e^4*f^5/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x
^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),
a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*a^2*c^2-3/(a*c*f^2-a*d*e*f-b*c*e*f+b*
d*e^2)^2/e^2*f^3/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4
+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/
2)/(-b/a)^(1/2))*a^2*d^2-3/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)^2/e^2*f^3/(-b
/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c
)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*b^2
*c^2+9/2/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)^2/e^3*f^4/(-b/a)^(1/2)*(1+b*x^2
/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi
(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*a^2*c*d+9/2/(a*c*f^2-
a*d*e*f-b*c*e*f+b*d*e^2)^2/e^3*f^4/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2
/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*
f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*a*b*c^2+7/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*
e^2)^2*f^2/e/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d
*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(
-b/a)^(1/2))*a*b*d^2+11/8*b^2*d^2*f/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)^2/(-
b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*
c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-1/(-b/a)^(1...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x, algorithm="
fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/x**2/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^3x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x, algorithm="
maxima")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^3*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^3x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x, algorithm="
giac")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^3*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \int \frac{1}{x^2\,\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,{(f\,x^2+e)}^3}\,dx$$

input
```
int(1/(x^2*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^3),x)
```

output
```
int(1/(x^2*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^3), x)
```

Reduce [F]

$$\int \frac{1}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \text{too large to display}$$

input
```
int(1/x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x)
```

output

```
( - sqrt(c + d*x**2)*sqrt(a + b*x**2) - 3*int((sqrt(c + d*x**2)*sqrt(a + b
*x**2)*x**4)/(a*c*e**3 + 3*a*c*e**2*f*x**2 + 3*a*c*e*f**2*x**4 + a*c*f**3*
x**6 + a*d*e**3*x**2 + 3*a*d*e**2*f*x**4 + 3*a*d*e*f**2*x**6 + a*d*f**3*x*
*8 + b*c*e**3*x**2 + 3*b*c*e**2*f*x**4 + 3*b*c*e*f**2*x**6 + b*c*f**3*x**8
 + b*d*e**3*x**4 + 3*b*d*e**2*f*x**6 + 3*b*d*e*f**2*x**8 + b*d*f**3*x**10)
,x)*b*d*e**2*f*x - 6*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c*e**
3 + 3*a*c*e**2*f*x**2 + 3*a*c*e*f**2*x**4 + a*c*f**3*x**6 + a*d*e**3*x**2
+ 3*a*d*e**2*f*x**4 + 3*a*d*e*f**2*x**6 + a*d*f**3*x**8 + b*c*e**3*x**2 +
3*b*c*e**2*f*x**4 + 3*b*c*e*f**2*x**6 + b*c*f**3*x**8 + b*d*e**3*x**4 + 3*
b*d*e**2*f*x**6 + 3*b*d*e*f**2*x**8 + b*d*f**3*x**10),x)*b*d*e*f**2*x**3 -
 3*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c*e**3 + 3*a*c*e**2*f*x
**2 + 3*a*c*e*f**2*x**4 + a*c*f**3*x**6 + a*d*e**3*x**2 + 3*a*d*e**2*f*x**
4 + 3*a*d*e*f**2*x**6 + a*d*f**3*x**8 + b*c*e**3*x**2 + 3*b*c*e**2*f*x**4
+ 3*b*c*e*f**2*x**6 + b*c*f**3*x**8 + b*d*e**3*x**4 + 3*b*d*e**2*f*x**6 +
3*b*d*e*f**2*x**8 + b*d*f**3*x**10),x)*b*d*f**3*x**5 - 4*int((sqrt(c + d*x
**2)*sqrt(a + b*x**2)*x**2)/(a*c*e**3 + 3*a*c*e**2*f*x**2 + 3*a*c*e*f**2*x
**4 + a*c*f**3*x**6 + a*d*e**3*x**2 + 3*a*d*e**2*f*x**4 + 3*a*d*e*f**2*x**
6 + a*d*f**3*x**8 + b*c*e**3*x**2 + 3*b*c*e**2*f*x**4 + 3*b*c*e*f**2*x**6
+ b*c*f**3*x**8 + b*d*e**3*x**4 + 3*b*d*e**2*f*x**6 + 3*b*d*e*f**2*x**8 +
b*d*f**3*x**10),x)*a*d*e**2*f*x - 8*int((sqrt(c + d*x**2)*sqrt(a + b*x*...
```

3.203 $\int \frac{1}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)^3}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 1129

$$\int \frac{1}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \text{Too large to display}$$

output
```
-1/3*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a/c/e/x^3/(f*x^2+e)^2+1/3*(7*a*c*f+2*
a*d*e+2*b*c*e)*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a^2/c^2/e^2/x/(f*x^2+e)^2+1
/12*f*(8*b^2*c*e^2*(-c*f+d*e)-a^2*f*(-35*c^2*f^2+24*c*d*e*f+8*d^2*e^2)+8*a
*b*e*(-3*c^2*f^2+2*c*d*e*f+d^2*e^2))*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a^2
/c^2/e^3/(-a*f+b*e)/(-c*f+d*e)/(f*x^2+e)^2-1/24*(16*b^3*c*e^3*(-c*f+d*e)^2
+8*a*b^2*e^2*(-c*f+d*e)^2*(5*c*f+2*d*e)+a^3*f^2*(105*c^3*f^3-170*c^2*d*e*f
^2+40*c*d^2*e^2*f+16*d^3*e^3)-a^2*b*e*f*(170*c^3*f^3-275*c^2*d*e*f^2+64*c*
d^2*e^2*f+32*d^3*e^3))*x*(b*x^2+a)^(1/2)/a^2/c^2/e^4/(-a*f+b*e)^2/(-c*f+d*
e)/(d*x^2+c)^(1/2)/(f*x^2+e)+1/24*d^(1/2)*(16*b^3*c*e^3*(-c*f+d*e)^2+8*a*b
^2*e^2*(-c*f+d*e)^2*(5*c*f+2*d*e)+a^3*f^2*(105*c^3*f^3-170*c^2*d*e*f^2+40*
c*d^2*e^2*f+16*d^3*e^3)-a^2*b*e*f*(170*c^3*f^3-275*c^2*d*e*f^2+64*c*d^2*e^
2*f+32*d^3*e^3))*(b*x^2+a)^(1/2)*EllipticE(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(
1/2),(1-b*c/a/d)^(1/2))/a^2/c^(3/2)/e^4/(-a*f+b*e)^2/(-c*f+d*e)^2/(c*(b*x^
2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-1/24*d^(1/2)*(8*b^2*e^2*(-c*f+d*e)
^3+3*a^2*c*f^3*(35*c^2*f^2-80*c*d*e*f+48*d^2*e^2)-a*b*e*f*(100*c^3*f^3-219
*c^2*d*e*f^2+120*c*d^2*e^2*f+8*d^3*e^3))*(b*x^2+a)^(1/2)*InverseJacobiAM(a
rctan(d^(1/2)*x/c^(1/2)),(1-b*c/a/d)^(1/2))/a^2/c^(1/2)/e^4/(-a*f+b*e)/(-c
*f+d*e)^3/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-1/8*c^(3/2)*f^3*
(3*b^2*e^2*(16*c^2*f^2-36*c*d*e*f+21*d^2*e^2)+a^2*f^2*(35*c^2*f^2-80*c*d*e
*f+48*d^2*e^2)-2*a*b*e*f*(40*c^2*f^2-91*c*d*e*f+54*d^2*e^2))*(b*x^2+a)^...
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.20 (sec) , antiderivative size = 758, normalized size of antiderivative = 0.67

$$\int \frac{1}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx$$
$$= \frac{\sqrt{\frac{b}{a}}e(a+bx^2)(c+dx^2)\Big(6a^2c^2ef^4(-be+af)(-de+cf)x^4+3a^2c^2f^4(be(17de-14cf)+af(-14de+$$

input
```
Integrate[1/(x^4*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^3),x]
```

output
```
(Sqrt[b/a]*e*(a + b*x^2)*(c + d*x^2)*(6*a^2*c^2*e*f^4*(-(b*e) + a*f)*(-(d*
e) + c*f)*x^4 + 3*a^2*c^2*f^4*(b*e*(17*d*e - 14*c*f) + a*f*(-14*d*e + 11*c
*f))*x^4*(e + f*x^2) - 8*a*c*e*(b*e - a*f)^2*(d*e - c*f)^2*(e + f*x^2)^2 +
 8*(b*e - a*f)^2*(d*e - c*f)^2*(2*b*c*e + 2*a*d*e + 9*a*c*f)*x^2*(e + f*x^
2)^2) - I*c*x^3*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(e + f*x^2)^2*(-(b
*e*(16*b^3*c*e^3*(d*e - c*f)^2 + 8*a*b^2*e^2*(d*e - c*f)^2*(2*d*e + 5*c*f)
 + a^3*f^2*(16*d^3*e^3 + 40*c*d^2*e^2*f - 170*c^2*d*e*f^2 + 105*c^3*f^3) -
 a^2*b*e*f*(32*d^3*e^3 + 64*c*d^2*e^2*f - 275*c^2*d*e*f^2 + 170*c^3*f^3))*
EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]) + b*e*(d*e - c*f)*(-16*b^3
*c*e^3*(-(d*e) + c*f) + a^3*f^2*(8*d^2*e^2 + 100*c*d*e*f - 105*c^2*f^2) +
8*a*b^2*e^2*(d^2*e^2 + 4*c*d*e*f - 5*c^2*f^2) + a^2*b*e*f*(-16*d^2*e^2 - 1
57*c*d*e*f + 170*c^2*f^2))*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]
+ 3*a^2*c*f^2*(3*b^2*e^2*(21*d^2*e^2 - 36*c*d*e*f + 16*c^2*f^2) + a^2*f^2*
(48*d^2*e^2 - 80*c*d*e*f + 35*c^2*f^2) - 2*a*b*e*f*(54*d^2*e^2 - 91*c*d*e*
f + 40*c^2*f^2))*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*
c)]))/(24*a^2*Sqrt[b/a]*c^2*e^5*(b*e - a*f)^2*(d*e - c*f)^2*x^3*Sqrt[a + b
*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^2)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx$$

$$\Big\downarrow \text{ 450}$$

$$\int \frac{1}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^3}dx$$

input
```
Int[1/(x^4*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^3),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3323 vs. 2(1085) = 2170.

Time = 28.89 (sec) , antiderivative size = 3324, normalized size of antiderivative = 2.94

method	result	size
elliptic	Expression too large to display	3324
risch	Expression too large to display	3986
default	Expression too large to display	11152

input
```
int(1/x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x,method=_RETURNVERB
OSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(63/8*f^2/(a*c
*f^2-a*d*e*f-b*c*e*f+b*d*e^2)^2/e/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/
c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f
/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*b^2*d^2-1/3/a/c/e^3*(b*d*x^4+a*d*x^2+b*c
*x^2+a*c)^(1/2)/x^3+1/8*f^4*(11*a*c*f^2-14*a*d*e*f-14*b*c*e*f+17*b*d*e^2)/
(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)^2/e^4*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1
/2)/(f*x^2+e)-17/8*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d
*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*d*f^3*b^2/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2
)^2/e^2*EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-10*f^5/(a*c*f^2
-a*d*e*f-b*c*e*f+b*d*e^2)^2/e^4/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)
^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b
/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*a^2*c*d-10*f^5/(a*c*f^2-a*d*e*f-b*c*e*f+b*
d*e^2)^2/e^4/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d
*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(
-b/a)^(1/2))*a*b*c^2-27/2*f^3/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)^2/e^2/(-b/
a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)
^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*a*b*
d^2-27/2*f^3/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)^2/e^2/(-b/a)^(1/2)*(1+b*x^2
/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi
(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*b^2*c*d-23/8/(-b/a...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^3}\,dx=\text{Timed out}$$

input
```
integrate(1/x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x, algorithm="
fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/x**4/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^3x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x, algorithm="
maxima")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^3*x^4), x)
```

Giac [F]

$$\int \frac{1}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^3x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x, algorithm="
giac")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^3*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \int \frac{1}{x^4\,\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,(f\,x^2+e)^3}\,dx$$

input
```
int(1/(x^4*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^3),x)
```

output
```
int(1/(x^4*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^3), x)
```

Reduce [F]

$$\int \frac{1}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \int \frac{1}{x^4\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,(f\,x^2+e)^3}dx$$

input
```
int(1/x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x)
```

output
```
int(1/x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x)
```

3.204 $\int \frac{x^4(e+fx^2)^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 529

$$\int \frac{x^4(e+fx^2)^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx = \frac{(8(bc-ad)f(7bde-3bcf+3adf)-5bd(7bde^2+5acf^2))\,x\sqrt{a-bx^2}\sqrt{c+dx^2}}{105b^3d^3} - \frac{2f(7bde-3bcf+3adf)x^3\sqrt{a-bx^2}\sqrt{c+dx^2}}{35b^2d^2} - \frac{f^2x^5\sqrt{a-bx^2}\sqrt{c+dx^2}}{7bd} + \frac{2\sqrt{a}(9abcdf(7bde-3bcf+3adf)+(bc-ad)\,(8(bc-ad)f(7bde-3bcf+3adf)-5bd(7bde^2+5acf^2 \ldots}{105b^{7/2}d^4\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}} - \frac{\sqrt{a}c(24a^3d^3f^2+a^2bd^2f(56de-17cf)+ab^2d(35d^2e^2-42cdef+16c^2f^2)-2b^3c(35d^2e^2-56cdef+ \ldots}{105b^{7/2}d^4\sqrt{a-bx^2}\sqrt{c+dx^2}}$$

output
```
1/105*(8*(-a*d+b*c)*f*(3*a*d*f-3*b*c*f+7*b*d*e)-5*b*d*(5*a*c*f^2+7*b*d*e^2
))*x*(-b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b^3/d^3-2/35*f*(3*a*d*f-3*b*c*f+7*b*
d*e)*x^3*(-b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b^2/d^2-1/7*f^2*x^5*(-b*x^2+a)^(
1/2)*(d*x^2+c)^(1/2)/b/d+2/105*a^(1/2)*(9*a*b*c*d*f*(3*a*d*f-3*b*c*f+7*b*d
*e)+(-a*d+b*c)*(8*(-a*d+b*c)*f*(3*a*d*f-3*b*c*f+7*b*d*e)-5*b*d*(5*a*c*f^2+
7*b*d*e^2)))*(1-b*x^2/a)^(1/2)*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)
,(-a*d/b/c)^(1/2))/b^(7/2)/d^4/(-b*x^2+a)^(1/2)/(1+d*x^2/c)^(1/2)-1/105*a^
(1/2)*c*(24*a^3*d^3*f^2+a^2*b*d^2*f*(-17*c*f+56*d*e)+a*b^2*d*(16*c^2*f^2-4
2*c*d*e*f+35*d^2*e^2)-2*b^3*c*(24*c^2*f^2-56*c*d*e*f+35*d^2*e^2))*(1-b*x^2
/a)^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(b^(1/2)*x/a^(1/2),(-a*d/b/c)^(1/2))/
b^(7/2)/d^4/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.52 (sec) , antiderivative size = 459, normalized size of antiderivative = 0.87

$$\int \frac{x^4(e+fx^2)^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx = \frac{-\sqrt{-\frac{b}{a}}dx(a-bx^2)(c+dx^2)(24a^2d^2f^2+abdf(56de-23cf+18dfx^2)+b^2(24c^2f^2-2cdf(28e+9fx^2)}{}$$

input
```
Integrate[(x^4*(e + f*x^2)^2)/(Sqrt[a - b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
(-(Sqrt[-(b/a)]*d*x*(a - b*x^2)*(c + d*x^2)*(24*a^2*d^2*f^2 + a*b*d*f*(56*
d*e - 23*c*f + 18*d*f*x^2) + b^2*(24*c^2*f^2 - 2*c*d*f*(28*e + 9*f*x^2) +
d^2*(35*e^2 + 42*e*f*x^2 + 15*f^2*x^4)))) - (2*I)*c*(24*a^3*d^3*f^2 + 4*a^
2*b*d^2*f*(14*d*e - 5*c*f) + b^3*c*(-35*d^2*e^2 + 56*c*d*e*f - 24*c^2*f^2)
 + a*b^2*d*(35*d^2*e^2 - 49*c*d*e*f + 20*c^2*f^2))*Sqrt[1 - (b*x^2)/a]*Sqr
t[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))] + I*
c*(24*a^3*d^3*f^2 + a^2*b*d^2*f*(56*d*e - 17*c*f) + a*b^2*d*(35*d^2*e^2 -
42*c*d*e*f + 16*c^2*f^2) - 2*b^3*c*(35*d^2*e^2 - 56*c*d*e*f + 24*c^2*f^2))
*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[-(b/a)]*
x], -((a*d)/(b*c))])/(105*b^3*Sqrt[-(b/a)]*d^4*Sqrt[a - b*x^2]*Sqrt[c + d*
x^2])
```

Rubi [A] (warning: unable to verify)

Time = 1.83 (sec) , antiderivative size = 870, normalized size of antiderivative = 1.64, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {448, 444, 444, 399, 323, 323, 321, 331, 330, 327, 444, 399, 323, 323, 321, 331, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4\left(e+fx^2\right)^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \ 448$$

$$\frac{f\int \frac{x^6\left(fx^2+e\right)}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{e^2}+e\int \frac{x^4\left(fx^2+e\right)}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx$$

$$\Big\downarrow \ 444$$

$$\frac{f\left(\frac{\int \frac{x^4\left((7bde-6bcf+6adf)x^2+5acf\right)}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{7bd}-\frac{fx^5\sqrt{a-bx^2}\sqrt{c+dx^2}}{7bd}\right)}{e^2}+e\left(\frac{\int \frac{x^2\left((5bde-4bcf+4adf)x^2+3acf\right)}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{5bd}-\frac{fx^3\sqrt{a-bx^2}\sqrt{c+dx^2}}{5bd}\right)$$

$$\Big\downarrow \ 444$$

$$\frac{f\left(\frac{\frac{\int \frac{x^2\left(\left(-4c(7de-6cf)b^2+ad(28de-23cf)b+24a^2d^2f\right)x^2+3ac(7bde-6bcf+6adf)\right)}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{5bd}-\frac{x^3\sqrt{a-bx^2}\sqrt{c+dx^2}(6adf-6bcf+7bde)}{5bd}}{7bd}-\frac{fx^5\sqrt{a-bx^2}\sqrt{c+dx^2}}{7bd}\right.}{e^2}$$

$$e\left(\frac{\frac{\int \frac{\left(-2c(5de-4cf)b^2+ad(10de-7cf)b+8a^2d^2f\right)x^2+ac(5bde-4bcf+4adf)}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{3bd}-\frac{x\sqrt{a-bx^2}\sqrt{c+dx^2}(4adf-4bcf+5bde)}{3bd}}{5bd}-\frac{fx^3\sqrt{a-bx^2}\sqrt{c+}}{5bd}\right.$$

$$\Big\downarrow \ 399$$

$$\frac{f\left(\frac{\frac{\int \frac{x^2\left(\left(-4c(7de-6cf)b^2+ad(28de-23cf)b+24a^2d^2f\right)x^2+3ac(7bde-6bcf+6adf)\right)}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{5bd}-\frac{x^3\sqrt{a-bx^2}\sqrt{c+dx^2}(6adf-6bcf+7bde)}{5bd}}{7bd}-\frac{fx^5\sqrt{a-bx^2}\sqrt{c+dx^2}}{7bd}\right.}{e^2}$$

$$e\left(\frac{\frac{\frac{\left(8a^2d^2f+abd(10de-7cf)-2b^2c(5de-4cf)\right)\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}-\frac{c\left(4a^2d^2f+abd(5de-3cf)-2b^2c(5de-4cf)\right)\int \frac{1}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{d}}{3bd}-\frac{x\sqrt{a-bx^2}\sqrt{c+dx^2}(4c}{3bd}}{5bd}\right.$$

$\downarrow$ 323

$$\frac{f\left(\frac{\frac{\int \frac{x^2\left(\left(-4c(7de-6cf)b^2+ad(28de-23cf)b+24a^2d^2f\right)x^2+3ac(7bde-6bcf+6adf)\right)}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{5bd}-\frac{x^3\sqrt{a-bx^2}\sqrt{c+dx^2}(6adf-6bcf+7bde)}{5bd}}{7bd}-\frac{fx^5\sqrt{a-bx^2}\sqrt{c+dx^2}}{7bd}\right.}{e^2}$$

$$e\left(\frac{\frac{\frac{\left(8a^2d^2f+abd(10de-7cf)-2b^2c(5de-4cf)\right)\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}-\frac{c\sqrt{\frac{dx^2}{c}+1}\left(4a^2d^2f+abd(5de-3cf)-2b^2c(5de-4cf)\right)\int \frac{1}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}dx}{d\sqrt{c+dx^2}}}{3bd}-\frac{x\sqrt{a-bx^2}\sqrt{}}{}}{5bd}\right.$$

$\downarrow$ 323

$$\frac{f\left(\frac{\frac{\int \frac{x^2\left(\left(-4c(7de-6cf)b^2+ad(28de-23cf)b+24a^2d^2f\right)x^2+3ac(7bde-6bcf+6adf)\right)}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{5bd}-\frac{x^3\sqrt{a-bx^2}\sqrt{c+dx^2}(6adf-6bcf+7bde)}{5bd}}{7bd}-\frac{fx^5\sqrt{a-bx^2}\sqrt{c+dx^2}}{7bd}\right.}{e^2}$$

$$e\left(\frac{\frac{\frac{\left(8a^2d^2f+abd(10de-7cf)-2b^2c(5de-4cf)\right)\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}-\frac{c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\left(4a^2d^2f+abd(5de-3cf)-2b^2c(5de-4cf)\right)\int \frac{1}{\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}}dx}{d\sqrt{a-bx^2}\sqrt{c+dx^2}}}{3bd}-x\sqrt{}}{5bd}\right.$$

$\downarrow$ 321

$$\frac{e\left(\frac{\frac{\frac{\left(8a^2d^2f+abd(10de-7cf)-2b^2c(5de-4cf)\right)\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}-\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\left(4a^2d^2f+abd(5de-3cf)-2b^2c(5de-4cf)\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right.}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}}{3bd}}{5bd}\right.}{e^2}$$

$$f\left(\frac{\frac{\int \frac{x^2\left(\left(-4c(7de-6cf)b^2+ad(28de-23cf)b+24a^2d^2f\right)x^2+3ac(7bde-6bcf+6adf)\right)}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{5bd}-\frac{x^3\sqrt{a-bx^2}\sqrt{c+dx^2}(6adf-6bcf+7bde)}{5bd}}{7bd}-\frac{fx^5\sqrt{a-bx^2}\sqrt{c+dx^2}}{7bd}\right.$$

$\downarrow$ 331

$$\frac{e\left(\frac{\frac{\frac{\sqrt{1-\frac{bx^2}{a}}\left(8a^2d^2f+abd(10de-7cf)-2b^2c(5de-4cf)\right)\int \frac{\sqrt{dx^2+c}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\left(4a^2d^2f+abd(5de-3cf)-2b^2c(5de-4cf)\right)\text{EllipticF}\left(\arcsin\left(\right.\right.}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}}{3bd}}{5bd}\right.}{e^2}$$

$$f\left(\frac{\frac{\int \frac{x^2\left(\left(-4c(7de-6cf)b^2+ad(28de-23cf)b+24a^2d^2f\right)x^2+3ac(7bde-6bcf+6adf)\right)}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{5bd}-\frac{x^3\sqrt{a-bx^2}\sqrt{c+dx^2}(6adf-6bcf+7bde)}{5bd}}{7bd}-\frac{fx^5\sqrt{a-bx^2}\sqrt{c+dx^2}}{7bd}\right.$$

$\downarrow$ 330

$$\frac{e\left(\frac{\frac{\frac{\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\left(8a^2d^2f+abd(10de-7cf)-2b^2c(5de-4cf)\right)\int \frac{\sqrt{\frac{dx^2}{c}+1}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}-\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\left(4a^2d^2f+abd(5de-3cf)-2b^2c(5de-4cf)\right)\text{EllipticF}\left(\right.}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}}{3bd}}{5bd}\right.}{e^2}$$

$$f\left(\frac{\frac{\int \frac{x^2\left(\left(-4c(7de-6cf)b^2+ad(28de-23cf)b+24a^2d^2f\right)x^2+3ac(7bde-6bcf+6adf)\right)}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{5bd}-\frac{x^3\sqrt{a-bx^2}\sqrt{c+dx^2}(6adf-6bcf+7bde)}{5bd}}{7bd}-\frac{fx^5\sqrt{a-bx^2}\sqrt{c+dx^2}}{7bd}\right.$$

$\downarrow$ 327

$$f\left(\frac{\frac{\int \frac{x^2\left(\left(-4c(7de-6cf)b^2+ad(28de-23cf)b+24a^2d^2f\right)x^2+3ac(7bde-6bcf+6adf)\right)}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{5bd}-\frac{x^3\sqrt{a-bx^2}\sqrt{c+dx^2}(6adf-6bcf+7bde)}{5bd}}{7bd}-\frac{fx^5\sqrt{a-bx^2}\sqrt{c+dx^2}}{7bd}\right.$$

$$e^2$$

$$e\left(\frac{\frac{\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\left(8a^2d^2f+abd(10de-7cf)-2b^2c(5de-4cf)\right)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}-\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\left(4a^2d^2f+abd(5de-3cf)-2b^2c(5de-4cf\right.}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}}{3bd}}{5bd}\right.$$

$\downarrow$ 444

$$f\left(\frac{\frac{\frac{\int \frac{\left(8c^2(7de-6cf)b^3-acd(49de-40cf)b^2+8a^2d^2(7de-5cf)b+48a^3d^3f\right)x^2+ac\left(-4c(7de-6cf)b^2+ad(28de-23cf)b+24a^2d^2f\right)}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{3bd}-\frac{1}{3}x\sqrt{a-bx^2}\sqrt{c+dx^2}\left(24\right.}{5bd}}{7bd}\right.$$

$$e^2$$

$$e\left(\frac{\frac{\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\left(8a^2d^2f+abd(10de-7cf)-2b^2c(5de-4cf)\right)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}-\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\left(4a^2d^2f+abd(5de-3cf)-2b^2c(5de-4cf\right.}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}}{3bd}}{5bd}\right.$$

$\downarrow$ 399

$$f\left(\frac{\frac{\frac{\left(48a^3d^3f+8a^2bd^2(7de-5cf)-ab^2cd(49de-40cf)+8b^3c^2(7de-6cf)\right)\int\frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}-\frac{c\left(24a^3d^3f+a^2bd^2(28de-17cf)-ab^2cd(21de-16cf)+8b^3c^2(7de-6cf\right.}{d}}{3bd}}{5bd}}{7bd}\right.$$

$$e\left(\frac{\frac{\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\left(8a^2d^2f+abd(10de-7cf)-2b^2c(5de-4cf)\right)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}-\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\left(4a^2d^2f+abd(5de-3cf)-2b^2c(5de-4cf\right.}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}}{3bd}}{5bd}\right.$$

$\downarrow$ 323

$$f\left(\frac{\frac{\frac{\left(48a^3d^3f+8a^2bd^2(7de-5cf)-ab^2cd(49de-40cf)+8b^3c^2(7de-6cf)\right)\int\frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}-\frac{c\sqrt{\frac{dx^2}{c}+1}\left(24a^3d^3f+a^2bd^2(28de-17cf)-ab^2cd(21de-16cf)+8b^3c^2}{d\sqrt{c+dx^2}}}{3bd}}{5bd}}{7bd}\right.$$

$$e\left(\frac{\frac{\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\left(8a^2d^2f+abd(10de-7cf)-2b^2c(5de-4cf)\right)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}-\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\left(4a^2d^2f+abd(5de-3cf)-2b^2c(5de-4cf)\right.}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}}{3bd}}{5bd}\right.$$

$\downarrow$ 323

$$e\left(\frac{\frac{\frac{\sqrt{a}\left(-2c(5de-4cf)b^2+ad(10de-7cf)b+8a^2d^2f\right)\sqrt{1-\frac{bx^2}{a}}\sqrt{dx^2+c}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}-\frac{\sqrt{a}c\left(-2c(5de-4cf)b^2+ad(5de-3cf)b+4a^2d^2f\right)\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{d}{}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{dx^2+c}}}{3bd}}{5bd}\right.$$

$$f\left(\frac{\frac{\frac{\left(8c^2(7de-6cf)b^3-acd(49de-40cf)b^2+8a^2d^2(7de-5cf)b+48a^3d^3f\right)\int\frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}-\frac{c\left(8c^2(7de-6cf)b^3-acd(21de-16cf)b^2+a^2d^2(28de-17cf)b+24a^3d^3\right.}{d\sqrt{a-bx^2}\sqrt{dx^2+c}}}{3bd}}{5bd}}{7bd}\right.$$

$\downarrow$ 321

$$e\left(\frac{\frac{\frac{\sqrt{a}\left(-2c(5de-4cf)b^2+ad(10de-7cf)b+8a^2d^2f\right)\sqrt{1-\frac{bx^2}{a}}\sqrt{dx^2+c}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}-\frac{\sqrt{a}c\left(-2c(5de-4cf)b^2+ad(5de-3cf)b+4a^2d^2f\right)\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}\cdots}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{dx^2+c}}}{3bd}}{5bd}\right.$$

$$f\left(\frac{\frac{\frac{\left(8c^2(7de-6cf)b^3-acd(49de-40cf)b^2+8a^2d^2(7de-5cf)b+48a^3d^3f\right)\int\frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}-\frac{\sqrt{a}c\left(8c^2(7de-6cf)b^3-acd(21de-16cf)b^2+a^2d^2(28de-17cf)b+24a^3\cdots\right.}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{dx\cdots}}}{3bd}}{5bd}\right.$$

$\downarrow$ 331

$$e\left(\frac{\frac{\frac{\sqrt{a}\left(-2c(5de-4cf)b^2+ad(10de-7cf)b+8a^2d^2f\right)\sqrt{1-\frac{bx^2}{a}}\sqrt{dx^2+c}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}-\frac{\sqrt{a}c\left(-2c(5de-4cf)b^2+ad(5de-3cf)b+4a^2d^2f\right)\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}\cdots}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{dx^2+c}}}{3bd}}{5bd}\right.$$

$$f\left(\frac{\frac{\frac{\left(8c^2(7de-6cf)b^3-acd(49de-40cf)b^2+8a^2d^2(7de-5cf)b+48a^3d^3f\right)\sqrt{1-\frac{bx^2}{a}}\int\frac{\sqrt{dx^2+c}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{\sqrt{a}c\left(8c^2(7de-6cf)b^3-acd(21de-16cf)b^2+a^2d^2(28de-17c\cdots\right.}{\sqrt{b}d\sqrt{a\cdots}}}{3bd}}{5bd}\right.$$

$\downarrow$ 330

$$e\left(\frac{\frac{\frac{\sqrt{a}\left(-2c(5de-4cf)b^2+ad(10de-7cf)b+8a^2d^2f\right)\sqrt{1-\frac{bx^2}{a}}\sqrt{dx^2+c}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}-\frac{\sqrt{a}c\left(-2c(5de-4cf)b^2+ad(5de-3cf)b+4a^2d^2f\right)\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{dx^2+c}}}{3bd}}{5bd}\right.$$

$$f\left(\frac{\frac{\frac{\left(8c^2(7de-6cf)b^3-acd(49de-40cf)b^2+8a^2d^2(7de-5cf)b+48a^3d^3f\right)\sqrt{1-\frac{bx^2}{a}}\sqrt{dx^2+c}\int\frac{\sqrt{\frac{dx^2}{c}+1}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}-\sqrt{a}c\left(8c^2(7de-6cf)b^3-acd(21de-16cf)b^2+a^2d^2(2\right.}{3bd}}{5bd}\right.$$

$\big\downarrow$ 327

$$e\left(\frac{\frac{\frac{\sqrt{a}\left(-2c(5de-4cf)b^2+ad(10de-7cf)b+8a^2d^2f\right)\sqrt{1-\frac{bx^2}{a}}\sqrt{dx^2+c}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}-\frac{\sqrt{a}c\left(-2c(5de-4cf)b^2+ad(5de-3cf)b+4a^2d^2f\right)\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{dx^2+c}}}{3bd}}{5bd}\right.$$

$$f\left(\frac{\frac{\frac{\sqrt{a}\left(8c^2(7de-6cf)b^3-acd(49de-40cf)b^2+8a^2d^2(7de-5cf)b+48a^3d^3f\right)\sqrt{1-\frac{bx^2}{a}}\sqrt{dx^2+c}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}-\sqrt{a}c\left(8c^2(7de-6cf)b^3-acd(21de-16c\right.}{3bd}}{5bd}\right.$$

input
```
Int[(x^4*(e + f*x^2)^2)/(Sqrt[a - b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
e*(-1/5*(f*x^3*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])/(b*d) + (-1/3*((5*b*d*e -
4*b*c*f + 4*a*d*f)*x*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])/(b*d) + ((Sqrt[a]*(8
*a^2*d^2*f + a*b*d*(10*d*e - 7*c*f) - 2*b^2*c*(5*d*e - 4*c*f))*Sqrt[1 - (b
*x^2)/a]*Sqrt[c + d*x^2]*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b
*c))])/(Sqrt[b]*d*Sqrt[a - b*x^2]*Sqrt[1 + (d*x^2)/c]) - (Sqrt[a]*c*(4*a^2
*d^2*f - 2*b^2*c*(5*d*e - 4*c*f) + a*b*d*(5*d*e - 3*c*f))*Sqrt[1 - (b*x^2)
/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*
c))])/(Sqrt[b]*d*Sqrt[a - b*x^2]*Sqrt[c + d*x^2]))/(3*b*d))/(5*b*d)) + (f*
(-1/7*(f*x^5*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])/(b*d) + (-1/5*((7*b*d*e - 6*
b*c*f + 6*a*d*f)*x^3*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])/(b*d) + (-1/3*(((24*
a^2*d*f)/b + a*(28*d*e - 23*c*f) - (4*b*c*(7*d*e - 6*c*f))/d)*x*Sqrt[a - b
*x^2]*Sqrt[c + d*x^2]) + ((Sqrt[a]*(48*a^3*d^3*f - a*b^2*c*d*(49*d*e - 40*
c*f) + 8*b^3*c^2*(7*d*e - 6*c*f) + 8*a^2*b*d^2*(7*d*e - 5*c*f))*Sqrt[1 - (
b*x^2)/a]*Sqrt[c + d*x^2]*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(
b*c))])/(Sqrt[b]*d*Sqrt[a - b*x^2]*Sqrt[1 + (d*x^2)/c]) - (Sqrt[a]*c*(24*a
^3*d^3*f + a^2*b*d^2*(28*d*e - 17*c*f) - a*b^2*c*d*(21*d*e - 16*c*f) + 8*b
^3*c^2*(7*d*e - 6*c*f))*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[
ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*d*Sqrt[a - b*x^2]*S
qrt[c + d*x^2]))/(3*b*d))/(5*b*d))/(7*b*d)))/e^2
```

Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

rule 448
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[e   Int[(g*x)^m*(a + b*x
^2)^p*(c + d*x^2)^q*(e + f*x^2)^(r - 1), x], x] + Simp[f/e^2   Int[(g*x)^(m
 + 2)*(a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^(r - 1), x], x] /; FreeQ[{a,
b, c, d, e, f, g, m, p, q}, x] && IGtQ[r, 0]
```

Maple [A] (verified)

Time = 11.12 (sec) , antiderivative size = 632, normalized size of antiderivative = 1.19

method	result
elliptic	$\frac{\sqrt{(-b\,x^2+a)(x^2d+c)}\left(-\frac{f^2x^5\sqrt{-bd\,x^4+ad\,x^2-x^2bc+ac}}{7bd}-\frac{\left(2ef+\frac{f^2(6ad-6bc)}{7bd}\right)x^3\sqrt{-bd\,x^4+ad\,x^2-x^2bc+ac}}{5bd}-\frac{\left(e^2+\frac{5ac\,f^2}{7bd}+\frac{\left(2ef+\frac{f^2(6\right.}{}\right.}{}\right.}{}$
risch	Expression too large to display
default	Expression too large to display

input
```
int(x^4*(f*x^2+e)^2/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBO
SE)
```

output
```
((-b*x^2+a)*(d*x^2+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-1/7*f^2/b/
d*x^5*(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)-1/5*(2*e*f+1/7*f^2/b/d*(6*a*d-6
*b*c))/b/d*x^3*(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)-1/3*(e^2+5/7*a/b*c/d*f
^2+1/5*(2*e*f+1/7*f^2/b/d*(6*a*d-6*b*c))/b/d*(4*a*d-4*b*c))/b/d*x*(-b*d*x^
4+a*d*x^2-b*c*x^2+a*c)^(1/2)+1/3*(e^2+5/7*a/b*c/d*f^2+1/5*(2*e*f+1/7*f^2/b
/d*(6*a*d-6*b*c))/b/d*(4*a*d-4*b*c))/b/d*a*c/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)
*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)*EllipticF(x*(b/a)^
(1/2),(-1-(a*d-b*c)/c/b)^(1/2))-(3/5*(2*e*f+1/7*f^2/b/d*(6*a*d-6*b*c))/b/d
*a*c+1/3*(e^2+5/7*a/b*c/d*f^2+1/5*(2*e*f+1/7*f^2/b/d*(6*a*d-6*b*c))/b/d*(4
*a*d-4*b*c))/b/d*(2*a*d-2*b*c))*c/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c
)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(b/a)^(1/2),(-
1-(a*d-b*c)/c/b)^(1/2))-EllipticE(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2)))
)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 608, normalized size of antiderivative = 1.15

$$\int \frac{x^4 (e + f x^2)^2}{\sqrt{a - b x^2} \sqrt{c + d x^2}} \, dx$$
$$= \frac{2\,(35\,(ab^3cd^2 - a^2b^2d^3)e^2 - 7\,(8\,ab^3c^2d - 7\,a^2b^2cd^2 + 8\,a^3bd^3)ef + 4\,(6\,ab^3c^3 - 5\,a^2b^2c^2d + 5\,a^3bcd^2 - 6\,a}{}$$

input
```
integrate(x^4*(f*x^2+e)^2/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="f
ricas")
```

output
```
1/105*(2*(35*(a*b^3*c*d^2 - a^2*b^2*d^3)*e^2 - 7*(8*a*b^3*c^2*d - 7*a^2*b^
2*c*d^2 + 8*a^3*b*d^3)*e*f + 4*(6*a*b^3*c^3 - 5*a^2*b^2*c^2*d + 5*a^3*b*c*
d^2 - 6*a^4*d^3)*f^2)*sqrt(-b*d)*x*sqrt(a/b)*elliptic_e(arcsin(sqrt(a/b)/x
), -b*c/(a*d)) + (35*(2*a^2*b^2*d^3 - (2*a*b^3 - b^4)*c*d^2)*e^2 + 14*(8*a
^3*b*d^3 + 4*(2*a*b^3 - b^4)*c^2*d - (7*a^2*b^2 - 4*a*b^3)*c*d^2)*e*f + (4
8*a^4*d^3 - 24*(2*a*b^3 - b^4)*c^3 + (40*a^2*b^2 - 23*a*b^3)*c^2*d - 8*(5*
a^3*b - 3*a^2*b^2)*c*d^2)*f^2)*sqrt(-b*d)*x*sqrt(a/b)*elliptic_f(arcsin(sq
rt(a/b)/x), -b*c/(a*d)) - (15*b^4*d^3*f^2*x^6 + 6*(7*b^4*d^3*e*f - 3*(b^4*
c*d^2 - a*b^3*d^3)*f^2)*x^4 - 70*(b^4*c*d^2 - a*b^3*d^3)*e^2 + 14*(8*b^4*c
^2*d - 7*a*b^3*c*d^2 + 8*a^2*b^2*d^3)*e*f - 8*(6*b^4*c^3 - 5*a*b^3*c^2*d +
 5*a^2*b^2*c*d^2 - 6*a^3*b*d^3)*f^2 + (35*b^4*d^3*e^2 - 56*(b^4*c*d^2 - a*
b^3*d^3)*e*f + (24*b^4*c^2*d - 23*a*b^3*c*d^2 + 24*a^2*b^2*d^3)*f^2)*x^2)*
sqrt(-b*x^2 + a)*sqrt(d*x^2 + c))/(b^5*d^4*x)
```

Sympy [F]

$$\int \frac{x^4 (e + f x^2)^2}{\sqrt{a - b x^2} \sqrt{c + d x^2}} \, dx = \int \frac{x^4 (e + f x^2)^2}{\sqrt{a - b x^2} \sqrt{c + d x^2}} \, dx$$

input
```
integrate(x**4*(f*x**2+e)**2/(-b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(x**4*(e + f*x**2)**2/(sqrt(a - b*x**2)*sqrt(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{x^4(e+fx^2)^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{(fx^2+e)^2x^4}{\sqrt{-bx^2+a}\sqrt{dx^2+c}}\,dx$$

input
```
integrate(x^4*(f*x^2+e)^2/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="m
axima")
```

output
```
integrate((f*x^2 + e)^2*x^4/(sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

Giac [F]

$$\int \frac{x^4(e+fx^2)^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{(fx^2+e)^2x^4}{\sqrt{-bx^2+a}\sqrt{dx^2+c}}\,dx$$

input
```
integrate(x^4*(f*x^2+e)^2/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="g
iac")
```

output
```
integrate((f*x^2 + e)^2*x^4/(sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(e+fx^2)^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{x^4\,{\left(f\,x^2+e\right)}^2}{\sqrt{a-b\,x^2}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int((x^4*(e + f*x^2)^2)/((a - b*x^2)^(1/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int((x^4*(e + f*x^2)^2)/((a - b*x^2)^(1/2)*(c + d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^4(e+fx^2)^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx = \text{Too large to display}$$

input
```
int(x^4*(f*x^2+e)^2/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

output
```
( - 24*sqrt(c + d*x**2)*sqrt(a - b*x**2)*a**2*d**2*f**2*x + 23*sqrt(c + d*
x**2)*sqrt(a - b*x**2)*a*b*c*d*f**2*x - 56*sqrt(c + d*x**2)*sqrt(a - b*x**
2)*a*b*d**2*e*f*x - 18*sqrt(c + d*x**2)*sqrt(a - b*x**2)*a*b*d**2*f**2*x**
3 - 24*sqrt(c + d*x**2)*sqrt(a - b*x**2)*b**2*c**2*f**2*x + 56*sqrt(c + d*
x**2)*sqrt(a - b*x**2)*b**2*c*d*e*f*x + 18*sqrt(c + d*x**2)*sqrt(a - b*x**
2)*b**2*c*d*f**2*x**3 - 35*sqrt(c + d*x**2)*sqrt(a - b*x**2)*b**2*d**2*e**
2*x - 42*sqrt(c + d*x**2)*sqrt(a - b*x**2)*b**2*d**2*e*f*x**3 - 15*sqrt(c
+ d*x**2)*sqrt(a - b*x**2)*b**2*d**2*f**2*x**5 + 48*int((sqrt(c + d*x**2)*
sqrt(a - b*x**2)*x**2)/(a*c + a*d*x**2 - b*c*x**2 - b*d*x**4),x)*a**3*d**3
*f**2 - 40*int((sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**2)/(a*c + a*d*x**2 -
b*c*x**2 - b*d*x**4),x)*a**2*b*c*d**2*f**2 + 112*int((sqrt(c + d*x**2)*sqr
t(a - b*x**2)*x**2)/(a*c + a*d*x**2 - b*c*x**2 - b*d*x**4),x)*a**2*b*d**3*
e*f + 40*int((sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**2)/(a*c + a*d*x**2 - b*
c*x**2 - b*d*x**4),x)*a*b**2*c**2*d*f**2 - 98*int((sqrt(c + d*x**2)*sqrt(a
 - b*x**2)*x**2)/(a*c + a*d*x**2 - b*c*x**2 - b*d*x**4),x)*a*b**2*c*d**2*e
*f + 70*int((sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**2)/(a*c + a*d*x**2 - b*c
*x**2 - b*d*x**4),x)*a*b**2*d**3*e**2 - 48*int((sqrt(c + d*x**2)*sqrt(a -
b*x**2)*x**2)/(a*c + a*d*x**2 - b*c*x**2 - b*d*x**4),x)*b**3*c**3*f**2 + 1
12*int((sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**2)/(a*c + a*d*x**2 - b*c*x**2
 - b*d*x**4),x)*b**3*c**2*d*e*f - 70*int((sqrt(c + d*x**2)*sqrt(a - b*x...
```

3.205 $\int \frac{x^2(e+fx^2)^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 378

$$\int \frac{x^2(e+fx^2)^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx = -\frac{2f(5bde-2bcf+2adf)x\sqrt{a-bx^2}\sqrt{c+dx^2}}{15b^2d^2}-\frac{f^2x^3\sqrt{a-bx^2}\sqrt{c+dx^2}}{5bd}$$
$$-\frac{\sqrt{a}(4(bc-ad)f(5bde-2bcf+2adf)-3bd(5bde^2+3acf^2))\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right.}{15b^{5/2}d^3\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}}$$
$$-\frac{\sqrt{a}c(4a^2d^2f^2+abdf(10de-3cf)+b^2(15d^2e^2-20cdef+8c^2f^2))\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\mathrm{EllipticF}\left(\mathrm{arcs}\right.}{15b^{5/2}d^3\sqrt{a-bx^2}\sqrt{c+dx^2}}$$

output
```
-2/15*f*(2*a*d*f-2*b*c*f+5*b*d*e)*x*(-b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b^2/d
^2-1/5*f^2*x^3*(-b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b/d-1/15*a^(1/2)*(4*(-a*d+
b*c)*f*(2*a*d*f-2*b*c*f+5*b*d*e)-3*b*d*(3*a*c*f^2+5*b*d*e^2))*(1-b*x^2/a)^
(1/2)*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2),(-a*d/b/c)^(1/2))/b^(5/2
)/d^3/(-b*x^2+a)^(1/2)/(1+d*x^2/c)^(1/2)-1/15*a^(1/2)*c*(4*a^2*d^2*f^2+a*b
*d*f*(-3*c*f+10*d*e)+b^2*(8*c^2*f^2-20*c*d*e*f+15*d^2*e^2))*(1-b*x^2/a)^(1
/2)*(1+d*x^2/c)^(1/2)*EllipticF(b^(1/2)*x/a^(1/2),(-a*d/b/c)^(1/2))/b^(5/2
)/d^3/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.47 (sec) , antiderivative size = 328, normalized size of antiderivative = 0.87

$$\int \frac{x^2(e+fx^2)^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx$$
$$= \frac{-\sqrt{-\frac{b}{a}}dfx(a-bx^2)(c+dx^2)(4adf+b(10de-4cf+3dfx^2))-ic(8a^2d^2f^2+abdf(20de-7cf)+b^2(1}{}$$

input
```
Integrate[(x^2*(e + f*x^2)^2)/(Sqrt[a - b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
(-(Sqrt[-(b/a)]*d*f*x*(a - b*x^2)*(c + d*x^2)*(4*a*d*f + b*(10*d*e - 4*c*f
 + 3*d*f*x^2))) - I*c*(8*a^2*d^2*f^2 + a*b*d*f*(20*d*e - 7*c*f) + b^2*(15*
d^2*e^2 - 20*c*d*e*f + 8*c^2*f^2))*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]
*EllipticE[I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))] + I*c*(4*a^2*d^2*f^2
 + a*b*d*f*(10*d*e - 3*c*f) + b^2*(15*d^2*e^2 - 20*c*d*e*f + 8*c^2*f^2))*S
qrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[-(b/a)]*x]
, -((a*d)/(b*c))])/(15*b^2*Sqrt[-(b/a)]*d^3*Sqrt[a - b*x^2]*Sqrt[c + d*x^2
])
```

Rubi [A] (warning: unable to verify)

Time = 1.36 (sec) , antiderivative size = 636, normalized size of antiderivative = 1.68, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.472$, Rules used = {448, 444, 399, 323, 323, 321, 331, 330, 327, 444, 399, 323, 323, 321, 331, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(e+fx^2)^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx$$
$$\Big\downarrow \text{448}$$

$$\frac{f\int \frac{x^4(fx^2+e)}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{e^2}+e\int \frac{x^2(fx^2+e)}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx$$

$\Big\downarrow$ 444

$$\frac{f\left(\frac{\int \frac{x^2\left((5bde-4bcf+4adf)x^2+3acf\right)}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{5bd}-\frac{fx^3\sqrt{a-bx^2}\sqrt{c+dx^2}}{5bd}\right)}{e^2}+e\left(\frac{\int \frac{(3bde-2bcf+2adf)x^2+acf}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{3bd}-\frac{fx\sqrt{a-bx^2}\sqrt{c+dx^2}}{3bd}\right)$$

$\Big\downarrow$ 399

$$\frac{f\left(\frac{\int \frac{x^2\left((5bde-4bcf+4adf)x^2+3acf\right)}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{5bd}-\frac{fx^3\sqrt{a-bx^2}\sqrt{c+dx^2}}{5bd}\right)}{e^2}+e\left(\frac{\frac{(2adf-2bcf+3bde)\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}-\frac{c(adf-2bcf+3bde)\int \frac{1}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{d}}{3bd}-\frac{fx\sqrt{a-bx^2}\sqrt{c+dx^2}}{3bd}\right)$$

$\Big\downarrow$ 323

$$\frac{f\left(\frac{\int \frac{x^2\left((5bde-4bcf+4adf)x^2+3acf\right)}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{5bd}-\frac{fx^3\sqrt{a-bx^2}\sqrt{c+dx^2}}{5bd}\right)}{e^2}+e\left(\frac{\frac{(2adf-2bcf+3bde)\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}-\frac{c\sqrt{\frac{dx^2}{c}+1}(adf-2bcf+3bde)\int \frac{1}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}dx}{d\sqrt{c+dx^2}}}{3bd}-\frac{fx\sqrt{a-bx^2}\sqrt{c+dx^2}}{3bd}\right)$$

$\Big\downarrow$ 323

$$\frac{f\left(\frac{\int \frac{x^2\left((5bde-4bcf+4adf)x^2+3acf\right)}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{5bd}-\frac{fx^3\sqrt{a-bx^2}\sqrt{c+dx^2}}{5bd}\right)}{e^2}+e\left(\frac{\frac{(2adf-2bcf+3bde)\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}-\frac{c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(adf-2bcf+3bde)\int \frac{1}{\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}}dx}{d\sqrt{a-bx^2}\sqrt{c+dx^2}}}{3bd}-\frac{fx\sqrt{a-bx^2}\sqrt{c+dx^2}}{3bd}\right)$$

$\Big\downarrow$ 321

$$e\left(\frac{\frac{(2adf-2bcf+3bde)\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}-\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(adf-2bcf+3bde)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}}{3bd}-\frac{fx\sqrt{a-bx^2}\sqrt{c+dx}}{3bd}\right.$$

$$\frac{f\left(\frac{\int \frac{x^2\left((5bde-4bcf+4adf)x^2+3acf\right)}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{5bd}-\frac{fx^3\sqrt{a-bx^2}\sqrt{c+dx^2}}{5bd}\right)}{e^2}$$

$$\Big\downarrow \; 331$$

$$e\left(\frac{\frac{\sqrt{1-\frac{bx^2}{a}}(2adf-2bcf+3bde)\int \frac{\sqrt{dx^2+c}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(adf-2bcf+3bde)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}}{3bd}-\frac{fx\sqrt{a-bx^2}\sqrt{}}{3bd}\right.$$

$$\frac{f\left(\frac{\int \frac{x^2\left((5bde-4bcf+4adf)x^2+3acf\right)}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{5bd}-\frac{fx^3\sqrt{a-bx^2}\sqrt{c+dx^2}}{5bd}\right)}{e^2}$$

$$\Big\downarrow \; 330$$

$$e\left(\frac{\frac{\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(2adf-2bcf+3bde)\int \frac{\sqrt{\frac{dx^2}{c}+1}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}-\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(adf-2bcf+3bde)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}}{3bd}-\frac{fx\sqrt{a}}{}\right.$$

$$\frac{f\left(\frac{\int \frac{x^2\left((5bde-4bcf+4adf)x^2+3acf\right)}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{5bd}-\frac{fx^3\sqrt{a-bx^2}\sqrt{c+dx^2}}{5bd}\right)}{e^2}$$

$$\Big\downarrow \; 327$$

$$\frac{f\left(\frac{\int \frac{x^2\left((5bde-4bcf+4adf)x^2+3acf\right)}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{5bd}-\frac{fx^3\sqrt{a-bx^2}\sqrt{c+dx^2}}{5bd}\right)}{e^2}+e\left(\frac{\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(2adf-2bcf+3bde)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}-\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(adf-2bcf+3bde)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}}{3bd}\right.$$

$$\Big\downarrow \; 444$$

$$\frac{f\left(\frac{\frac{\int \frac{\left(-2c(5de-4cf)b^2+ad(10de-7cf)b+8a^2d^2f\right)x^2+ac(5bde-4bcf+4adf)}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{3bd}-\frac{x\sqrt{a-bx^2}\sqrt{c+dx^2}(4adf-4bcf+5bde)}{3bd}}{5bd}-\frac{fx^3\sqrt{a-bx^2}\sqrt{c+dx^2}}{5bd}\right)}{e^2}+e\left(\frac{\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(2adf-2bcf+3bde)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}-\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(adf-2bcf+3bde)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}}{3bd}\right.$$

$$\Big\downarrow \; 399$$

$$\frac{f\left(\frac{\frac{\frac{\left(8a^2d^2f+abd(10de-7cf)-2b^2c(5de-4cf)\right)\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}-\frac{c\left(4a^2d^2f+abd(5de-3cf)-2b^2c(5de-4cf)\right)\int \frac{1}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{d}}{3bd}-\frac{x\sqrt{a-bx^2}\sqrt{c+dx^2}(4adf-}{3bd}}{5bd}\right.}{e^2}$$

$$e\left(\frac{\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(2adf-2bcf+3bde)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}-\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(adf-2bcf+3bde)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}}{3bd}\right.$$

$$\Big\downarrow \; 323$$

$$\frac{f\left(\frac{\frac{\frac{\left(8a^2d^2f+abd(10de-7cf)-2b^2c(5de-4cf)\right)\int\frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}-\frac{c\sqrt{\frac{dx^2}{c}+1}\left(4a^2d^2f+abd(5de-3cf)-2b^2c(5de-4cf)\right)\int\frac{1}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}dx}{d\sqrt{c+dx^2}}}{3bd}-\frac{x\sqrt{a-bx^2}\sqrt{c+\cdots}}{\cdots}}{5bd}\cdots\right)}{e^2\cdots}$$

$$e\left(\frac{\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(2adf-2bcf+3bde)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}-\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(adf-2bcf+3bde)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}\cdots}{3bd}\cdots\right)$$

$$\Big\downarrow \; 323$$

$$\frac{f\left(\frac{\frac{\frac{\left(8a^2d^2f+abd(10de-7cf)-2b^2c(5de-4cf)\right)\int\frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}-\frac{c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\left(4a^2d^2f+abd(5de-3cf)-2b^2c(5de-4cf)\right)\int\frac{1}{\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}}dx}{d\sqrt{a-bx^2}\sqrt{c+dx^2}}}{3bd}-\frac{x\sqrt{a\cdots}}{\cdots}}{5bd}\cdots\right)}{e^2\cdots}$$

$$e\left(\frac{\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(2adf-2bcf+3bde)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}-\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(adf-2bcf+3bde)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}\cdots}{3bd}\cdots\right)$$

$$\Big\downarrow \; 321$$

$$\frac{f\left(\frac{\frac{\frac{\left(8a^2d^2f+abd(10de-7cf)-2b^2c(5de-4cf)\right)\int\frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}-\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\left(4a^2d^2f+abd(5de-3cf)-2b^2c(5de-4cf)\right)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{a\cdots}{b\cdots}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}}{3bd}\cdots}{5bd}\cdots\right)}{e^2\cdots}$$

$$e\left(\frac{\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(2adf-2bcf+3bde)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}-\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(adf-2bcf+3bde)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}\cdots}{3bd}\cdots\right)$$

$$\Big\downarrow \; 331$$

$$f\left(\frac{\frac{\frac{\sqrt{1-\frac{bx^2}{a}}\left(8a^2d^2f+abd(10de-7cf)-2b^2c(5de-4cf)\right)\int\frac{\sqrt{dx^2+c}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\left(4a^2d^2f+abd(5de-3cf)-2b^2c(5de-4cf)\right)\text{EllipticF}\left(\arcsin\left(\right.\right.}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}}{3bd}}{5bd}\right.$$

$$e^2$$

$$e\left(\frac{\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(2adf-2bcf+3bde)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}-\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(adf-2bcf+3bde)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}}{3bd}\right.$$

$$\Big\downarrow \; 330$$

$$f\left(\frac{\frac{\frac{\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\left(8a^2d^2f+abd(10de-7cf)-2b^2c(5de-4cf)\right)\int\frac{\sqrt{\frac{dx^2}{c}+1}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}-\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\left(4a^2d^2f+abd(5de-3cf)-2b^2c(5de-4cf)\right)\text{EllipticF}\left(\right.}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}}{3bd}}{5bd}\right.$$

$$e^2$$

$$e\left(\frac{\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(2adf-2bcf+3bde)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}-\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(adf-2bcf+3bde)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}}{3bd}\right.$$

$$\Big\downarrow \; 327$$

$$f\left(\frac{\frac{\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\left(8a^2d^2f+abd(10de-7cf)-2b^2c(5de-4cf)\right)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}-\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\left(4a^2d^2f+abd(5de-3cf)-2b^2c(5de-4cf\right.}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}}{3bd}}{5bd}\right.$$

$$e^2$$

$$e\left(\frac{\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(2adf-2bcf+3bde)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}-\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(adf-2bcf+3bde)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}}{3bd}\right.$$

input
```
Int[(x^2*(e + f*x^2)^2)/(Sqrt[a - b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
e*(-1/3*(f*x*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])/(b*d) + ((Sqrt[a]*(3*b*d*e -
 2*b*c*f + 2*a*d*f)*Sqrt[1 - (b*x^2)/a]*Sqrt[c + d*x^2]*EllipticE[ArcSin[(
Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*d*Sqrt[a - b*x^2]*Sqrt[1 +
(d*x^2)/c]) - (Sqrt[a]*c*(3*b*d*e - 2*b*c*f + a*d*f)*Sqrt[1 - (b*x^2)/a]*S
qrt[1 + (d*x^2)/c]*EllipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])
/(Sqrt[b]*d*Sqrt[a - b*x^2]*Sqrt[c + d*x^2]))/(3*b*d)) + (f*(-1/5*(f*x^3*S
qrt[a - b*x^2]*Sqrt[c + d*x^2])/(b*d) + (-1/3*((5*b*d*e - 4*b*c*f + 4*a*d*
f)*x*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])/(b*d) + ((Sqrt[a]*(8*a^2*d^2*f + a*b
*d*(10*d*e - 7*c*f) - 2*b^2*c*(5*d*e - 4*c*f))*Sqrt[1 - (b*x^2)/a]*Sqrt[c
+ d*x^2]*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*
d*Sqrt[a - b*x^2]*Sqrt[1 + (d*x^2)/c]) - (Sqrt[a]*c*(4*a^2*d^2*f - 2*b^2*c
*(5*d*e - 4*c*f) + a*b*d*(5*d*e - 3*c*f))*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*
x^2)/c]*EllipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*d
*Sqrt[a - b*x^2]*Sqrt[c + d*x^2]))/(3*b*d))/(5*b*d)))/e^2
```

Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

rule 448
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[e   Int[(g*x)^m*(a + b*x
^2)^p*(c + d*x^2)^q*(e + f*x^2)^(r - 1), x], x] + Simp[f/e^2   Int[(g*x)^(m
 + 2)*(a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^(r - 1), x], x] /; FreeQ[{a,
b, c, d, e, f, g, m, p, q}, x] && IGtQ[r, 0]
```

Maple [A] (verified)

Time = 8.89 (sec) , antiderivative size = 449, normalized size of antiderivative = 1.19

method	result
elliptic	$\frac{\sqrt{(-b x^2+a)(x^2d+c)}\left(-\frac{f^2x^3\sqrt{-bd x^4+ad x^2-x^2bc+ac}}{5bd}-\frac{\left(2ef+\frac{f^2(4ad-4bc)}{5bd}\right)x\sqrt{-bd x^4+ad x^2-x^2bc+ac}}{3bd}+\frac{\left(2ef+\frac{f^2(4ad-4bc)}{5bd}\right)ac\sqrt{1-}}{3bd\sqrt{\frac{b}{a}}}\right.}{}$
risch	$-\frac{fx(3bdf x^2+4adf-4bcf+10bde)\sqrt{-b x^2+a}\,\sqrt{x^2d+c}}{15b^2d^2}+\frac{\left(-\frac{(8a^2d^2f^2-7abcd f^2+20ab d^2ef+8b^2c^2f^2-20b^2cdef+15b^2d^2e^2)c\sqrt{1-\frac{b}{}}}{\sqrt{\frac{b}{a}}\sqrt{-bd x}}\right.}{}$
default	Expression too large to display

input
```
int(x^2*(f*x^2+e)^2/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBO
SE)
```

output
```
((-b*x^2+a)*(d*x^2+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-1/5*f^2/b/
d*x^3*(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)-1/3*(2*e*f+1/5*f^2/b/d*(4*a*d-4
*b*c))/b/d*x*(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)+1/3*(2*e*f+1/5*f^2/b/d*(
4*a*d-4*b*c))/b/d*a*c/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*
d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)*EllipticF(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b
)^(1/2))-(e^2+3/5*a/b*c/d*f^2+1/3*(2*e*f+1/5*f^2/b/d*(4*a*d-4*b*c))/b/d*(2
*a*d-2*b*c))*c/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a
*d*x^2-b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1
/2))-EllipticE(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 376, normalized size of antiderivative = 0.99

$$\int \frac{x^2(e+fx^2)^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx =$$

$$-\frac{(15\,ab^2d^2e^2-20\,(ab^2cd-a^2bd^2)ef+(8\,ab^2c^2-7\,a^2bcd+8\,a^3d^2)f^2)\sqrt{-bd}x\sqrt{\frac{a}{b}}E(\arcsin\left(\frac{\sqrt{\frac{a}{b}}}{x}\right)\mid-\frac{b}{a}}{}$$

input
```
integrate(x^2*(f*x^2+e)^2/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="f
ricas")
```

output
```
-1/15*((15*a*b^2*d^2*e^2 - 20*(a*b^2*c*d - a^2*b*d^2)*e*f + (8*a*b^2*c^2 -
 7*a^2*b*c*d + 8*a^3*d^2)*f^2)*sqrt(-b*d)*x*sqrt(a/b)*elliptic_e(arcsin(sq
rt(a/b)/x), -b*c/(a*d)) - (15*a*b^2*d^2*e^2 + 10*(2*a^2*b*d^2 - (2*a*b^2 -
 b^3)*c*d)*e*f + (8*a^3*d^2 + 4*(2*a*b^2 - b^3)*c^2 - (7*a^2*b - 4*a*b^2)*
c*d)*f^2)*sqrt(-b*d)*x*sqrt(a/b)*elliptic_f(arcsin(sqrt(a/b)/x), -b*c/(a*d
)) + (3*b^3*d^2*f^2*x^4 + 15*b^3*d^2*e^2 - 20*(b^3*c*d - a*b^2*d^2)*e*f +
(8*b^3*c^2 - 7*a*b^2*c*d + 8*a^2*b*d^2)*f^2 + 2*(5*b^3*d^2*e*f - 2*(b^3*c*
d - a*b^2*d^2)*f^2)*x^2)*sqrt(-b*x^2 + a)*sqrt(d*x^2 + c))/(b^4*d^3*x)
```

Sympy [F]

$$\int \frac{x^2(e+fx^2)^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{x^2(e+fx^2)^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx$$

input
```
integrate(x**2*(f*x**2+e)**2/(-b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(x**2*(e + f*x**2)**2/(sqrt(a - b*x**2)*sqrt(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{x^2(e+fx^2)^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{(fx^2+e)^2x^2}{\sqrt{-bx^2+a}\sqrt{dx^2+c}}\,dx$$

input
```
integrate(x^2*(f*x^2+e)^2/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="m
axima")
```

output
```
integrate((f*x^2 + e)^2*x^2/(sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

Giac [F]

$$\int \frac{x^2(e+fx^2)^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{(fx^2+e)^2x^2}{\sqrt{-bx^2+a}\sqrt{dx^2+c}}\,dx$$

input
```
integrate(x^2*(f*x^2+e)^2/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="g
iac")
```

output
```
integrate((f*x^2 + e)^2*x^2/(sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(e+fx^2)^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{x^2\,(f\,x^2+e)^2}{\sqrt{a-b\,x^2}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int((x^2*(e + f*x^2)^2)/((a - b*x^2)^(1/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int((x^2*(e + f*x^2)^2)/((a - b*x^2)^(1/2)*(c + d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^2(e+fx^2)^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx$$
$$= \frac{-4\sqrt{d\,x^2+c}\,\sqrt{-b\,x^2+a}\,ad\,f^2x+4\sqrt{d\,x^2+c}\,\sqrt{-b\,x^2+a}\,bc\,f^2x-10\sqrt{d\,x^2+c}\,\sqrt{-b\,x^2+a}\,bdefx-3\sqrt{}}{}$$

input
```
int(x^2*(f*x^2+e)^2/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

output
```
( - 4*sqrt(c + d*x**2)*sqrt(a - b*x**2)*a*d*f**2*x + 4*sqrt(c + d*x**2)*sq
rt(a - b*x**2)*b*c*f**2*x - 10*sqrt(c + d*x**2)*sqrt(a - b*x**2)*b*d*e*f*x
 - 3*sqrt(c + d*x**2)*sqrt(a - b*x**2)*b*d*f**2*x**3 + 8*int((sqrt(c + d*x
**2)*sqrt(a - b*x**2)*x**2)/(a*c + a*d*x**2 - b*c*x**2 - b*d*x**4),x)*a**2
*d**2*f**2 - 7*int((sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**2)/(a*c + a*d*x**
2 - b*c*x**2 - b*d*x**4),x)*a*b*c*d*f**2 + 20*int((sqrt(c + d*x**2)*sqrt(a
 - b*x**2)*x**2)/(a*c + a*d*x**2 - b*c*x**2 - b*d*x**4),x)*a*b*d**2*e*f +
8*int((sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**2)/(a*c + a*d*x**2 - b*c*x**2
- b*d*x**4),x)*b**2*c**2*f**2 - 20*int((sqrt(c + d*x**2)*sqrt(a - b*x**2)*
x**2)/(a*c + a*d*x**2 - b*c*x**2 - b*d*x**4),x)*b**2*c*d*e*f + 15*int((sqr
t(c + d*x**2)*sqrt(a - b*x**2)*x**2)/(a*c + a*d*x**2 - b*c*x**2 - b*d*x**4
),x)*b**2*d**2*e**2 + 4*int((sqrt(c + d*x**2)*sqrt(a - b*x**2))/(a*c + a*d
*x**2 - b*c*x**2 - b*d*x**4),x)*a**2*c*d*f**2 - 4*int((sqrt(c + d*x**2)*sq
rt(a - b*x**2))/(a*c + a*d*x**2 - b*c*x**2 - b*d*x**4),x)*a*b*c**2*f**2 +
10*int((sqrt(c + d*x**2)*sqrt(a - b*x**2))/(a*c + a*d*x**2 - b*c*x**2 - b*
d*x**4),x)*a*b*c*d*e*f)/(15*b**2*d**2)
```

3.206 $\int \frac{(e+fx^2)^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 274

$$\int \frac{(e+fx^2)^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx = -\frac{f^2x\sqrt{a-bx^2}\sqrt{c+dx^2}}{3bd} + \frac{2\sqrt{a}f(3bde-bcf+adf)\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\,E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{3b^{3/2}d^2\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}} - \frac{\sqrt{a}(acdf^2-b(3d^2e^2-6cdef+2c^2f^2))\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{3b^{3/2}d^2\sqrt{a-bx^2}\sqrt{c+dx^2}}$$

output
```
-1/3*f^2*x*(-b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b/d+2/3*a^(1/2)*f*(a*d*f-b*c*f
+3*b*d*e)*(1-b*x^2/a)^(1/2)*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2),(-
a*d/b/c)^(1/2))/b^(3/2)/d^2/(-b*x^2+a)^(1/2)/(1+d*x^2/c)^(1/2)-1/3*a^(1/2)
*(a*c*d*f^2-b*(2*c^2*f^2-6*c*d*e*f+3*d^2*e^2))*(1-b*x^2/a)^(1/2)*(1+d*x^2/
c)^(1/2)*EllipticF(b^(1/2)*x/a^(1/2),(-a*d/b/c)^(1/2))/b^(3/2)/d^2/(-b*x^2
+a)^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.32 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.91

$$\int \frac{(e+fx^2)^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx$$
$$= \frac{-\sqrt{-\frac{b}{a}}df^2x(a-bx^2)(c+dx^2) - 2icf(3bde-bcf+adf)\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(i\text{arcsinh}\left(\sqrt{-\frac{b}{a}}x\right)|-\frac{ad}{bc}\right.}{3b\sqrt{-\frac{b}{a}}d^2\sqrt{a-b}}$$

input
```
Integrate[(e + f*x^2)^2/(Sqrt[a - b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
(-(Sqrt[-(b/a)]*d*f^2*x*(a - b*x^2)*(c + d*x^2)) - (2*I)*c*f*(3*b*d*e - b*
c*f + a*d*f)*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[S
qrt[-(b/a)]*x], -((a*d)/(b*c))] + I*(a*c*d*f^2 + b*(-3*d^2*e^2 + 6*c*d*e*f
 - 2*c^2*f^2))*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh
[Sqrt[-(b/a)]*x], -((a*d)/(b*c))])/(3*b*Sqrt[-(b/a)]*d^2*Sqrt[a - b*x^2]*S
qrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 525, normalized size of antiderivative = 1.92, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.061$, Rules used = {433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx^2)^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx$$
$$\Big\downarrow \text{433}$$
$$\int \left(\frac{e^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}}+\frac{2efx^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}}+\frac{f^2x^4}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a}cf^2\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(2bc-ad)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{3b^{3/2}d^2\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{2\sqrt{a}f^2\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(bc-ad)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{3b^{3/2}d^2\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}+\frac{\sqrt{a}e^2\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{2\sqrt{a}cef\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}+\frac{2\sqrt{a}ef\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}-\frac{f^2x\sqrt{a-bx^2}\sqrt{c+dx^2}}{3bd}$$

input
```
Int[(e + f*x^2)^2/(Sqrt[a - b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
-1/3*(f^2*x*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])/(b*d) + (2*Sqrt[a]*e*f*Sqrt[1
 - (b*x^2)/a]*Sqrt[c + d*x^2]*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*
d)/(b*c))])/(Sqrt[b]*d*Sqrt[a - b*x^2]*Sqrt[1 + (d*x^2)/c]) - (2*Sqrt[a]*(
b*c - a*d)*f^2*Sqrt[1 - (b*x^2)/a]*Sqrt[c + d*x^2]*EllipticE[ArcSin[(Sqrt[
b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(3*b^(3/2)*d^2*Sqrt[a - b*x^2]*Sqrt[1 + (
d*x^2)/c]) + (Sqrt[a]*e^2*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Elliptic
F[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*Sqrt[a - b*x^2]*S
qrt[c + d*x^2]) - (2*Sqrt[a]*c*e*f*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]
*EllipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*d*Sqrt[a
 - b*x^2]*Sqrt[c + d*x^2]) + (Sqrt[a]*c*(2*b*c - a*d)*f^2*Sqrt[1 - (b*x^2)
/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*
c))])/(3*b^(3/2)*d^2*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])
```

Defintions of rubi rules used

rule 433
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)
^q*(e + f*x^2)^r, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, f, p,
 q, r}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 6.22 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.23

method	result
elliptic	$\frac{\sqrt{(-b x^2+a)(x^2 d+c)}\left(-\frac{f^2 x\sqrt{-bd x^4+ad x^2-x^2 bc+ac}}{3bd}+\frac{\left(e^2+\frac{ac f^2}{3bd}\right)\sqrt{1-\frac{b x^2}{a}}\sqrt{1+\frac{d x^2}{c}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{b}{a}},\sqrt{-1-\frac{ad-bc}{cb}}\right)}{\sqrt{\frac{b}{a}}\sqrt{-bd x^4+ad x^2-x^2 bc+ac}}-\frac{\left(2ef+\frac{f^2(2ac}{3b}\right.}{}\right.}{\sqrt{-b x^2+a}\sqrt{x^2 d+c}}$
risch	$-\frac{f^2 x\sqrt{-b x^2+a}\sqrt{x^2 d+c}}{3bd}+\frac{\left(-\frac{2f(adf-bcf+3bde)c\sqrt{1-\frac{b x^2}{a}}\sqrt{1+\frac{d x^2}{c}}\left(\mathrm{EllipticF}\left(x\sqrt{\frac{b}{a}},\sqrt{-1-\frac{ad-bc}{cb}}\right)-\mathrm{EllipticE}\left(x\sqrt{\frac{b}{a}},\sqrt{-1-\frac{ad-bc}{cb}}\right.\right.}{\sqrt{\frac{b}{a}}\sqrt{-bd x^4+ad x^2-x^2 bc+ac}\,d}\right.}{}$
default	$-\frac{\left(-\sqrt{\frac{b}{a}}\,b d^2 f^2 x^5+\sqrt{\frac{b}{a}}\,a d^2 f^2 x^3-\sqrt{\frac{b}{a}}\,bcd f^2 x^3+\sqrt{\frac{-b x^2+a}{a}}\sqrt{\frac{x^2 d+c}{c}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{b}{a}},\sqrt{-\frac{ad}{bc}}\right)acd f^2-2\sqrt{\frac{-b x^2+a}{a}}\sqrt{\frac{x^2 d+c}{c}}\,\mathrm{E}\right.}{}$

input
```
int((f*x^2+e)^2/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((-b*x^2+a)*(d*x^2+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-1/3*f^2/b/
d*x*(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)+(e^2+1/3*a/b*c/d*f^2)/(b/a)^(1/2)
*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)*
EllipticF(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2))-(2*e*f+1/3*f^2/b/d*(2*a*
d-2*b*c))*c/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*
x^2-b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2)
)-EllipticE(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.83

$$\int \frac{(e+fx^2)^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx =$$

$$-\frac{2\left(3\,a^2bdef-(a^2bc-a^3d)f^2\right)\sqrt{-bd}x\sqrt{\frac{a}{b}}E(\arcsin\left(\frac{\sqrt{\frac{a}{b}}}{x}\right)\mid-\frac{bc}{ad})-(3\,b^3de^2+6\,a^2bdef+(2\,a^3d-(2\,a}{}$$

input
```
integrate((f*x^2+e)^2/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="frica
s")
```

output
```
-1/3*(2*(3*a^2*b*d*e*f - (a^2*b*c - a^3*d)*f^2)*sqrt(-b*d)*x*sqrt(a/b)*ell
iptic_e(arcsin(sqrt(a/b)/x), -b*c/(a*d)) - (3*b^3*d*e^2 + 6*a^2*b*d*e*f +
(2*a^3*d - (2*a^2*b - a*b^2)*c)*f^2)*sqrt(-b*d)*x*sqrt(a/b)*elliptic_f(arc
sin(sqrt(a/b)/x), -b*c/(a*d)) + (a*b^2*d*f^2*x^2 + 6*a*b^2*d*e*f - 2*(a*b^
2*c - a^2*b*d)*f^2)*sqrt(-b*x^2 + a)*sqrt(d*x^2 + c))/(a*b^3*d^2*x)
```

Sympy [F]

$$\int \frac{(e+fx^2)^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{(e+fx^2)^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx$$

input
```
integrate((f*x**2+e)**2/(-b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral((e + f*x**2)**2/(sqrt(a - b*x**2)*sqrt(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{(e+fx^2)^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{(fx^2+e)^2}{\sqrt{-bx^2+a}\sqrt{dx^2+c}}\,dx$$

input
```
integrate((f*x^2+e)^2/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxim
a")
```

output
```
integrate((f*x^2 + e)^2/(sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

Giac [F]

$$\int \frac{(e+fx^2)^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{(fx^2+e)^2}{\sqrt{-bx^2+a}\sqrt{dx^2+c}}\,dx$$

input
```
integrate((f*x^2+e)^2/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac"
)
```

output
```
integrate((f*x^2 + e)^2/(sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx^2)^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{(f\,x^2+e)^2}{\sqrt{a-b\,x^2}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int((e + f*x^2)^2/((a - b*x^2)^(1/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int((e + f*x^2)^2/((a - b*x^2)^(1/2)*(c + d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{(e+fx^2)^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx = \frac{-\sqrt{d\,x^2+c}\,\sqrt{-b\,x^2+a}\,f^2x+2\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{-b\,x^2+a}\,x^2}{-bd\,x^4+ad\,x^2-bc\,x^2+ac}dx\right)ad\,f^2-2\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{-b\,x^2+a}\,x^2}{-bd\,x^4+ad\,x^2-bc\,x^2+ac}dx\right)bc\,f^2+6}{3bd}$$

input
```
int((f*x^2+e)^2/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

output
```
( - sqrt(c + d*x**2)*sqrt(a - b*x**2)*f**2*x + 2*int((sqrt(c + d*x**2)*sqr
t(a - b*x**2)*x**2)/(a*c + a*d*x**2 - b*c*x**2 - b*d*x**4),x)*a*d*f**2 - 2
*int((sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**2)/(a*c + a*d*x**2 - b*c*x**2 -
 b*d*x**4),x)*b*c*f**2 + 6*int((sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**2)/(a
*c + a*d*x**2 - b*c*x**2 - b*d*x**4),x)*b*d*e*f + int((sqrt(c + d*x**2)*sq
rt(a - b*x**2))/(a*c + a*d*x**2 - b*c*x**2 - b*d*x**4),x)*a*c*f**2 + 3*int
((sqrt(c + d*x**2)*sqrt(a - b*x**2))/(a*c + a*d*x**2 - b*c*x**2 - b*d*x**4
),x)*b*d*e**2)/(3*b*d)
```

3.207 $\int \frac{(e+fx^2)^2}{x^2\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 255

$$\int \frac{(e+fx^2)^2}{x^2\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx = -\frac{e^2\sqrt{a-bx^2}\sqrt{c+dx^2}}{acx} - \frac{(bde^2-acf^2)\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{b}cd\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}} + \frac{(bde^2+af(2de-cf))\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}$$

output

```
-e^2*(-b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a/c/x-(-a*c*f^2+b*d*e^2)*(1-b*x^2/a)
^(1/2)*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2),(-a*d/b/c)^(1/2))/a^(1/
2)/b^(1/2)/c/d/(-b*x^2+a)^(1/2)/(1+d*x^2/c)^(1/2)+(b*d*e^2+a*f*(-c*f+2*d*e
))*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(b^(1/2)*x/a^(1/2),(-a*d/b
/c)^(1/2))/a^(1/2)/b^(1/2)/d/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.03 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.93

$$\int \frac{(e+fx^2)^2}{x^2\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx$$
$$= \frac{-\sqrt{-\frac{b}{a}}de^2(a-bx^2)(c+dx^2) - ic(-bde^2+acf^2)x\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(i\operatorname{arcsinh}\left(\sqrt{-\frac{b}{a}}x\right)|-\frac{ad}{bc}\right)+ic}{a\sqrt{-\frac{b}{a}}cdx\sqrt{a-bx^2}\sqrt{c+}}$$

input
```
Integrate[(e + f*x^2)^2/(x^2*Sqrt[a - b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
(-(Sqrt[-(b/a)]*d*e^2*(a - b*x^2)*(c + d*x^2)) - I*c*(-(b*d*e^2) + a*c*f^2
)*x*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[-(b/a
)]*x], -((a*d)/(b*c))] + I*c*(-(b*d*e^2) + a*f*(-2*d*e + c*f))*x*Sqrt[1 -
(b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d
)/(b*c))])/(a*Sqrt[-(b/a)]*c*d*x*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (warning: unable to verify)

Time = 0.97 (sec) , antiderivative size = 426, normalized size of antiderivative = 1.67, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.472$, Rules used = {448, 399, 323, 323, 321, 331, 330, 327, 445, 25, 399, 323, 323, 321, 331, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx^2)^2}{x^2\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx$$
$$\Big\downarrow \; 448$$
$$\frac{f\int \frac{fx^2+e}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{e^2} + e\int \frac{fx^2+e}{x^2\sqrt{a-bx^2}\sqrt{dx^2+c}}dx$$

$$\Big\downarrow \text{399}$$

$$\frac{f\left(\frac{(de-cf)\int \frac{1}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{d}+\frac{f\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}\right)}{e^2}+e\int \frac{fx^2+e}{x^2\sqrt{a-bx^2}\sqrt{dx^2+c}}dx$$

$$\Big\downarrow \text{323}$$

$$\frac{f\left(\frac{\sqrt{\frac{dx^2}{c}+1}(de-cf)\int \frac{1}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}dx}{d\sqrt{c+dx^2}}+\frac{f\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}\right)}{e^2}+e\int \frac{fx^2+e}{x^2\sqrt{a-bx^2}\sqrt{dx^2+c}}dx$$

$$\Big\downarrow \text{323}$$

$$\frac{f\left(\frac{\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(de-cf)\int \frac{1}{\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}}dx}{d\sqrt{a-bx^2}\sqrt{c+dx^2}}+\frac{f\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}\right)}{e^2}+e\int \frac{fx^2+e}{x^2\sqrt{a-bx^2}\sqrt{dx^2+c}}dx$$

$$\Big\downarrow \text{321}$$

$$\frac{f\left(\frac{f\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}+\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(de-cf)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}\right)}{e^2}+ e\int \frac{fx^2+e}{x^2\sqrt{a-bx^2}\sqrt{dx^2+c}}dx$$

$$\Big\downarrow \text{331}$$

$$\frac{f\left(\frac{f\sqrt{1-\frac{bx^2}{a}}\int \frac{\sqrt{dx^2+c}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}+\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(de-cf)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}\right)}{e^2}+ e\int \frac{fx^2+e}{x^2\sqrt{a-bx^2}\sqrt{dx^2+c}}dx$$

$$\Big\downarrow \text{330}$$

$$\frac{f\left(\frac{f\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\int \frac{\sqrt{\frac{dx^2}{c}+1}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}+\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(de-cf)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}\right)}{e^2}+ e\int \frac{fx^2+e}{x^2\sqrt{a-bx^2}\sqrt{dx^2+c}}dx$$

$$\Big\downarrow \; 327$$

$$e\int \frac{fx^2+e}{x^2\sqrt{a-bx^2}\sqrt{dx^2+c}}dx+\frac{f\left(\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(de-cf)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}+\frac{\sqrt{a}f\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}\right)}{e^2}$$

$$\Big\downarrow \; 445$$

$$e\left(-\frac{\int -\frac{acf-bdex^2}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{ac}-\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{acx}\right)+\frac{f\left(\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(de-cf)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}+\frac{\sqrt{a}f\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}\right)}{e^2}$$

$$\Big\downarrow \; 25$$

$$e\left(\frac{\int \frac{acf-bdex^2}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{ac}-\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{acx}\right)+\frac{f\left(\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(de-cf)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}+\frac{\sqrt{a}f\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}\right)}{e^2}$$

$$\Big\downarrow \; 399$$

$$e\left(\frac{c(af+be)\int \frac{1}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx-be\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{ac}-\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{acx}\right)+\frac{f\left(\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(de-cf)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}+\frac{\sqrt{a}f\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}\right)}{e^2}$$

$$\Big\downarrow \; 323$$

$$e\left(\frac{\frac{c\sqrt{\frac{dx^2}{c}+1}(af+be)\int \frac{1}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}dx}{\sqrt{c+dx^2}}-be\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{ac}-\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{acx}\right)+\frac{f\left(\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(de-cf)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}+\frac{\sqrt{a}f\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}\right)}{e^2}$$

$$\Big\downarrow \; 323$$

$$\frac{e\left(\frac{\frac{c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(af+be)\int \frac{1}{\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}}dx}{\sqrt{a-bx^2}\sqrt{c+dx^2}}-be\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{ac}-\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{acx}\right)+f\left(\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(de-cf)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}+\frac{\sqrt{a}f\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}\right)}{e^2}$$

$$\Big\downarrow \; 321$$

$$\frac{e\left(\frac{\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(af+be)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-be\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{ac}-\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{acx}\right)+f\left(\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(de-cf)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}+\frac{\sqrt{a}f\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}\right)}{e^2}$$

$$\Big\downarrow \; 331$$

$$\frac{e\left(\frac{\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(af+be)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{be\sqrt{1-\frac{bx^2}{a}}\int \frac{\sqrt{dx^2+c}}{\sqrt{1-\frac{bx^2}{a}}}dx}{\sqrt{a-bx^2}}}{ac}-\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{acx}\right)+f\left(\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(de-cf)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}+\frac{\sqrt{a}f\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}\right)}{e^2}$$

$$\Big\downarrow \; 330$$

$$\frac{e\left(\frac{\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(af+be)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{be\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\int \frac{\sqrt{\frac{dx^2}{c}+1}}{\sqrt{1-\frac{bx^2}{a}}}dx}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}}{ac}-\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{acx}\right)+f\left(\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(de-cf)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}+\frac{\sqrt{a}f\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}\right)}{e^2}$$

$$\Big\downarrow 327$$

$$\frac{f\left(\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(de-cf)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}+\frac{\sqrt{a}f\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}\right)}{e^2}+ e\left(\frac{\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(af+be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{b}e\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}}{ac}-\frac{e\sqrt{a-bx^2}\sqrt{c+}}{acx}\right.$$

input
```
Int[(e + f*x^2)^2/(x^2*Sqrt[a - b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
(f*((Sqrt[a]*f*Sqrt[1 - (b*x^2)/a]*Sqrt[c + d*x^2]*EllipticE[ArcSin[(Sqrt[
b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*d*Sqrt[a - b*x^2]*Sqrt[1 + (d*x^
2)/c]) + (Sqrt[a]*(d*e - c*f)*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Elli
pticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*d*Sqrt[a - b*
x^2]*Sqrt[c + d*x^2])))/e^2 + e*(-((e*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])/(a*
c*x)) + (-((Sqrt[a]*Sqrt[b]*e*Sqrt[1 - (b*x^2)/a]*Sqrt[c + d*x^2]*Elliptic
E[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[a - b*x^2]*Sqrt[1 +
(d*x^2)/c])) + (Sqrt[a]*c*(b*e + a*f)*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)
/c]*EllipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*Sqrt[
a - b*x^2]*Sqrt[c + d*x^2]))/(a*c))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 448
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[e   Int[(g*x)^m*(a + b*x
^2)^p*(c + d*x^2)^q*(e + f*x^2)^(r - 1), x], x] + Simp[f/e^2   Int[(g*x)^(m
 + 2)*(a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^(r - 1), x], x] /; FreeQ[{a,
b, c, d, e, f, g, m, p, q}, x] && IGtQ[r, 0]
```

Maple [A] (verified)

Time = 6.06 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.24

method	result
risch	$-\frac{e^{2}\sqrt{-b\,x^{2}+a}\,\sqrt{x^{2}d+c}}{a c x}+\frac{\left(-\frac{\left(a c\,f^{2}-b d\,e^{2}\right) c\sqrt{1-\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,\left(\mathrm{EllipticF}\left(x\sqrt{\frac{b}{a}},\sqrt{-1-\frac{ad-bc}{cb}}\right)-\mathrm{EllipticE}\left(x\sqrt{\frac{b}{a}},\sqrt{-1-\frac{ad-bc}{cb}}\right)\right)}{\sqrt{\frac{b}{a}}\,\sqrt{-b d\,x^{4}+a d\,x^{2}-x^{2} b c+a c}\,d}+\cdots\right.}{a c\sqrt{-b\,x^{2}+a}\,\sqrt{x^{2}d+c}}$
elliptic	$\frac{\sqrt{\left(-b\,x^{2}+a\right)\left(x^{2}d+c\right)}\,\left(-\frac{e^{2}\sqrt{-b d\,x^{4}+a d\,x^{2}-x^{2} b c+a c}}{a c x}+\frac{2 e f\sqrt{1-\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{b}{a}},\sqrt{-1-\frac{ad-bc}{cb}}\right)}{\sqrt{\frac{b}{a}}\,\sqrt{-b d\,x^{4}+a d\,x^{2}-x^{2} b c+a c}}-\frac{\left(f^{2}-\frac{b d\,e^{2}}{a c}\right) c\sqrt{1-\frac{b\,x^{2}}{a}}\cdots}{\cdots}\right.}{\sqrt{-b\,x^{2}+a}\,\sqrt{x^{2}d+c}}$
default	$\frac{\left(\sqrt{\frac{b}{a}}\,b\,d^{2}e^{2}x^{4}-\sqrt{\frac{-b\,x^{2}+a}{a}}\,\sqrt{\frac{x^{2}d+c}{c}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{b}{a}},\sqrt{-\frac{ad}{bc}}\right) a\,c^{2} f^{2} x+2\sqrt{\frac{-b\,x^{2}+a}{a}}\,\sqrt{\frac{x^{2}d+c}{c}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{b}{a}},\sqrt{-\frac{ad}{bc}}\right) a c d e f x+\cdots\right.}{\cdots}$

input
```
int((f*x^2+e)^2/x^2/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBO
SE)
```

output
```
-e^2*(-b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a/c/x+1/a/c*(-(a*c*f^2-b*d*e^2)*c/(b
/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*
c)^(1/2)/d*(EllipticF(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2))-EllipticE(x*
(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2)))+2*a*c*e*f/(b/a)^(1/2)*(1-b*x^2/a)^(
1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)*EllipticF(x*(b
/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2)))*((-b*x^2+a)*(d*x^2+c))^(1/2)/(-b*x^2+
a)^(1/2)/(d*x^2+c)^(1/2)
```

Fricas [F]

$$\int \frac{(e+fx^2)^2}{x^2\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx=\int \frac{\left(fx^2+e\right)^2}{\sqrt{-bx^2+a}\sqrt{dx^2+c}\,x^2}\,dx$$

input
```
integrate((f*x^2+e)^2/x^2/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="f
ricas")
```

output
```
integral(-(f^2*x^4 + 2*e*f*x^2 + e^2)*sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)/(b*
d*x^6 + (b*c - a*d)*x^4 - a*c*x^2), x)
```

Sympy [F]

$$\int \frac{(e+fx^2)^2}{x^2\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx=\int \frac{(e+fx^2)^2}{x^2\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx$$

input
```
integrate((f*x**2+e)**2/x**2/(-b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral((e + f*x**2)**2/(x**2*sqrt(a - b*x**2)*sqrt(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{(e+fx^2)^2}{x^2\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx=\int \frac{(fx^2+e)^2}{\sqrt{-bx^2+a}\sqrt{dx^2+cx^2}}\,dx$$

input
```
integrate((f*x^2+e)^2/x^2/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="m
axima")
```

output
```
integrate((f*x^2 + e)^2/(sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)*x^2), x)
```

Giac [F]

$$\int \frac{(e+fx^2)^2}{x^2\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx=\int \frac{(fx^2+e)^2}{\sqrt{-bx^2+a}\sqrt{dx^2+cx^2}}\,dx$$

input
```
integrate((f*x^2+e)^2/x^2/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="g
iac")
```

output
```
integrate((f*x^2 + e)^2/(sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx^2)^2}{x^2\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{{\left(f\,x^2+e\right)}^2}{x^2\,\sqrt{a-b\,x^2}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int((e + f*x^2)^2/(x^2*(a - b*x^2)^(1/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int((e + f*x^2)^2/(x^2*(a - b*x^2)^(1/2)*(c + d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{(e+fx^2)^2}{x^2\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx = \frac{-\sqrt{d\,x^2+c}\,\sqrt{-b\,x^2+a}\,f^2-\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{-b\,x^2+a}}{-bd\,x^6+ad\,x^4-bc\,x^4+ac\,x^2}dx\right)ac\,f^2x+\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{-b\,x^2+a}}{-bd\,x^6+ad\,x^4-bc\,x^4+ac\,x^2}dx\right)bd\,e^2x +}{bdx}$$

input
```
int((f*x^2+e)^2/x^2/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

output
```
( - sqrt(c + d*x**2)*sqrt(a - b*x**2)*f**2 - int((sqrt(c + d*x**2)*sqrt(a
- b*x**2))/(a*c*x**2 + a*d*x**4 - b*c*x**4 - b*d*x**6),x)*a*c*f**2*x + int
((sqrt(c + d*x**2)*sqrt(a - b*x**2))/(a*c*x**2 + a*d*x**4 - b*c*x**4 - b*d
*x**6),x)*b*d*e**2*x + 2*int((sqrt(c + d*x**2)*sqrt(a - b*x**2))/(a*c + a*
d*x**2 - b*c*x**2 - b*d*x**4),x)*b*d*e*f*x)/(b*d*x)
```

3.208 $\int \frac{(e+fx^2)^2}{x^4\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 326

$$\begin{aligned}\int \frac{(e+fx^2)^2}{x^4\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx &= -\frac{e^2\sqrt{a-bx^2}\sqrt{c+dx^2}}{3acx^3}-\frac{2e(bce-ade+3acf)\sqrt{a-bx^2}\sqrt{c+dx^2}}{3a^2c^2x}\\ &\quad-\frac{2\sqrt{b}e(bce-ade+3acf)\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{3a^{3/2}c^2\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}}\\ &\quad+\frac{(2b^2ce^2+3a^2cf^2-abe(de-6cf))\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{3a^{3/2}\sqrt{b}c\sqrt{a-bx^2}\sqrt{c+dx^2}}\end{aligned}$$

output
```
-1/3*e^2*(-b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a/c/x^3-2/3*e*(3*a*c*f-a*d*e+b*c
*e)*(-b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a^2/c^2/x-2/3*b^(1/2)*e*(3*a*c*f-a*d*
e+b*c*e)*(1-b*x^2/a)^(1/2)*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2),(-a
*d/b/c)^(1/2))/a^(3/2)/c^2/(-b*x^2+a)^(1/2)/(1+d*x^2/c)^(1/2)+1/3*(2*b^2*c
*e^2+3*a^2*c*f^2-a*b*e*(-6*c*f+d*e))*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)*E
llipticF(b^(1/2)*x/a^(1/2),(-a*d/b/c)^(1/2))/a^(3/2)/b^(1/2)/c/(-b*x^2+a)^
(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.67 (sec) , antiderivative size = 282, normalized size of antiderivative = 0.87

$$\int \frac{(e+fx^2)^2}{x^4\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx$$
$$= \frac{\sqrt{-\frac{b}{a}}e(-a+bx^2)\,(c+dx^2)\,(2bcex^2-2adex^2+ac(e+6fx^2))+2ibce(bce-ade+3acf)x^3\sqrt{1-\frac{bx^2}{a}}\sqrt{}}{3}$$

input
```
Integrate[(e + f*x^2)^2/(x^4*Sqrt[a - b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
(Sqrt[-(b/a)]*e*(-a + b*x^2)*(c + d*x^2)*(2*b*c*e*x^2 - 2*a*d*e*x^2 + a*c*
(e + 6*f*x^2)) + (2*I)*b*c*e*(b*c*e - a*d*e + 3*a*c*f)*x^3*Sqrt[1 - (b*x^2
)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c
))] - I*c*(2*b^2*c*e^2 + 3*a^2*c*f^2 + a*b*e*(-(d*e) + 6*c*f))*x^3*Sqrt[1
- (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[-(b/a)]*x], -((a
*d)/(b*c))])/(3*a^2*Sqrt[-(b/a)]*c^2*x^3*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (warning: unable to verify)

Time = 1.22 (sec) , antiderivative size = 559, normalized size of antiderivative = 1.71, number of steps used = 20, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {448, 445, 25, 399, 323, 323, 321, 331, 330, 327, 445, 25, 27, 399, 323, 323, 321, 331, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx^2)^2}{x^4\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx$$
$$\Big\downarrow \text{448}$$
$$\frac{f\int \frac{fx^2+e}{x^2\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{e^2}+e\int \frac{fx^2+e}{x^4\sqrt{a-bx^2}\sqrt{dx^2+c}}dx$$

$$\Big\downarrow \ 445$$

$$\frac{f\left(-\frac{\int -\frac{acf-bdex^2}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{ac}-\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{acx}\right)}{e^2}+e\left(-\frac{\int -\frac{bdex^2+2bce-2ade+3acf}{x^2\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{3ac}-\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{3acx^3}\right)$$

$$\Big\downarrow \ 25$$

$$\frac{f\left(\frac{\int \frac{acf-bdex^2}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{ac}-\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{acx}\right)}{e^2}+e\left(\frac{\int \frac{bdex^2+2bce-2ade+3acf}{x^2\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{3ac}-\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{3acx^3}\right)$$

$$\Big\downarrow \ 399$$

$$\frac{f\left(\frac{c(af+be)\int \frac{1}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx-be\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{ac}-\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{acx}\right)}{e^2}+e\left(\frac{\int \frac{bdex^2+2bce-2ade+3acf}{x^2\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{3ac}-\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{3acx^3}\right)$$

$$\Big\downarrow \ 323$$

$$\frac{f\left(\frac{\frac{c\sqrt{\frac{dx^2}{c}+1}(af+be)\int \frac{1}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}dx}{\sqrt{c+dx^2}}-be\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{ac}-\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{acx}\right)}{e^2}+e\left(\frac{\int \frac{bdex^2+2bce-2ade+3acf}{x^2\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{3ac}-\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{3acx^3}\right)$$

$$\Big\downarrow \ 323$$

$$\frac{f\left(\frac{\frac{c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(af+be)\int\frac{1}{\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}}dx}{\sqrt{a-bx^2}\sqrt{c+dx^2}}-be\int\frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{ac}-\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{acx}\right)}{e^2}+e\left(\frac{\int\frac{bdex^2+2bce-2ade+3acf}{x^2\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{3ac}-\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{3acx^3}\right)$$

$\Big\downarrow$ 321

$$\frac{f\left(\frac{\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(af+be)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-be\int\frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{ac}-\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{acx}\right)}{e^2}+e\left(\frac{\int\frac{bdex^2+2bce-2ade+3acf}{x^2\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{3ac}-\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{3acx^3}\right)$$

$\Big\downarrow$ 331

$$\frac{f\left(\frac{\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(af+be)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{be\sqrt{1-\frac{bx^2}{a}}\int\frac{\sqrt{dx^2+c}}{\sqrt{1-\frac{bx^2}{a}}}dx}{\sqrt{a-bx^2}}}{ac}-\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{acx}\right)}{e^2}+e\left(\frac{\int\frac{bdex^2+2bce-2ade+3acf}{x^2\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{3ac}-\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{3acx^3}\right)$$

$\Big\downarrow$ 330

$$\frac{f\left(\frac{\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(af+be)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{be\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\int\frac{\sqrt{\frac{dx^2}{c}+1}}{\sqrt{1-\frac{bx^2}{a}}}dx}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}}{ac}-\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{acx}\right)}{e^2}+e\left(\frac{\int\frac{bdex^2+2bce-2ade+3acf}{x^2\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{3ac}-\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{3acx^3}\right)$$

$\Big\downarrow$ 327

$$\frac{e\left(\frac{\int \frac{bdex^2+2bce-2ade+3acf}{x^2\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{3ac}-\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{3acx^3}\right)+f\left(\frac{\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(af+be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{b}e\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}}{ac}-\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{acx}\right)}{e^2}$$

$$\Big\downarrow \; 445$$

$$\frac{e\left(\frac{-\frac{\int -\frac{bd\left(ace-(2bce-2ade+3acf)x^2\right)}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}(3acf-2ade+2bce)}{acx}}{3ac}-\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{3acx^3}\right)+f\left(\frac{\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(af+be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{b}e\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}}{ac}-\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{acx}\right)}{e^2}$$

$$\Big\downarrow \; 25$$

$$\frac{e\left(\frac{\frac{\int \frac{bd\left(ace-(2bce-2ade+3acf)x^2\right)}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}(3acf-2ade+2bce)}{acx}}{3ac}-\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{3acx^3}\right)+f\left(\frac{\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(af+be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{b}e\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}}{ac}-\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{acx}\right)}{e^2}$$

$$\Big\downarrow \; 27$$

$$\frac{e\left(\frac{\frac{bd\int \frac{ace-(2bce-2ade+3acf)x^2}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}(3acf-2ade+2bce)}{acx}}{3ac}-\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{3acx^3}\right)+f\left(\frac{\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(af+be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{b}e\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}}{ac}-\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{acx}\right)}{e^2}$$

$\downarrow$ 399

$$\frac{e\left(\frac{bd\left(\frac{c(3acf-ade+2bce)\int\frac{1}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{d}-\frac{(3acf-2ade+2bce)\int\frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}\right)}{ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}(3acf-2ade+2bce)}{acx}}{3ac}-\frac{e\sqrt{a-bx^2}\sqrt{c}}{3acx^3}\right.}{}$$

$$\frac{f\left(\frac{\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(af+be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{b}e\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}}{ac}-\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{acx}\right)}{e^2}$$

$\downarrow$ 323

$$\frac{e\left(\frac{bd\left(\frac{c\sqrt{\frac{dx^2}{c}+1}(3acf-ade+2bce)\int\frac{1}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}dx}{d\sqrt{c+dx^2}}-\frac{(3acf-2ade+2bce)\int\frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}\right)}{ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}(3acf-2ade+2bce)}{acx}}{3ac}-\frac{e\sqrt{a-}}{}\right.}{}$$

$$\frac{f\left(\frac{\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(af+be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{b}e\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}}{ac}-\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{acx}\right)}{e^2}$$

$\downarrow$ 323

$$\frac{e\left(\frac{\frac{bd\left(\frac{c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(3acf-ade+2bce)\int\frac{1}{\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}}dx}{d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{(3acf-2ade+2bce)\int\frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}\right)}{ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}(3acf-2ade+2bce)}{acx}}{3ac}-\right.}{} \quad f\left(\frac{\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(af+be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{b}e\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}}{ac}-\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{acx}\right) \Big/ e^2$$

$$\Big\downarrow \; 321$$

$$\frac{e\left(\frac{\frac{bd\left(\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(3acf-ade+2bce)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{(3acf-2ade+2bce)\int\frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}\right)}{ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}(3acf-2ade+}{acx}}{3ac}\right.}{} \quad f\left(\frac{\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(af+be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{b}e\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}}{ac}-\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{acx}\right) \Big/ e^2$$

$$\Big\downarrow \; 331$$

$$\frac{e\left(\frac{\frac{bd\left(\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(3acf-ade+2bce)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{1-\frac{bx^2}{a}}(3acf-2ade+2bce)\int\frac{\sqrt{dx^2+c}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}\right)}{ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}(3ac}{acx}}{3ac}\right)\ \ f\left(\frac{\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(af+be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{b}e\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}}{ac}-\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{acx}\right)}{e^2}$$

$\downarrow$ 330

$$\frac{e\left(\frac{\frac{bd\left(\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(3acf-ade+2bce)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(3acf-2ade+2bce)\int\frac{\sqrt{\frac{dx^2}{c}+1}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}\right)}{ac}-\frac{\sqrt{a-bx^2}\sqrt{c+d}}{}}{3ac}\right)\ \ f\left(\frac{\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(af+be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{b}e\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}}{ac}-\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{acx}\right)}{e^2}$$

$\downarrow$ 327

$$\frac{f\left(\frac{\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(af+be)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{b}e\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}}{ac}-\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{acx}\right)}{e^2}+$$

$$e\left(\frac{\frac{bd\left(\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(3acf-ade+2bce)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(3acf-2ade+2bce)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}\right)}{ac}-\frac{\sqrt{a}\cdots}{\cdots}}{3ac}\right.$$

input
```
Int[(e + f*x^2)^2/(x^4*Sqrt[a - b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
(f*(-((e*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])/(a*c*x)) + (-((Sqrt[a]*Sqrt[b]*e
*Sqrt[1 - (b*x^2)/a]*Sqrt[c + d*x^2]*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]]
, -((a*d)/(b*c))])/(Sqrt[a - b*x^2]*Sqrt[1 + (d*x^2)/c])) + (Sqrt[a]*c*(b*
e + a*f)*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[ArcSin[(Sqrt[b]
*x)/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*Sqrt[a - b*x^2]*Sqrt[c + d*x^2]))/
(a*c)))/e^2 + e*(-1/3*(e*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])/(a*c*x^3) + (-((
(2*b*c*e - 2*a*d*e + 3*a*c*f)*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])/(a*c*x)) +
(b*d*(-((Sqrt[a]*(2*b*c*e - 2*a*d*e + 3*a*c*f)*Sqrt[1 - (b*x^2)/a]*Sqrt[c
+ d*x^2]*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*
d*Sqrt[a - b*x^2]*Sqrt[1 + (d*x^2)/c])) + (Sqrt[a]*c*(2*b*c*e - a*d*e + 3*
a*c*f)*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[ArcSin[(Sqrt[b]*x
)/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*d*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])))
/(a*c))/(3*a*c))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 448
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[e   Int[(g*x)^m*(a + b*x
^2)^p*(c + d*x^2)^q*(e + f*x^2)^(r - 1), x], x] + Simp[f/e^2   Int[(g*x)^(m
 + 2)*(a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^(r - 1), x], x] /; FreeQ[{a,
b, c, d, e, f, g, m, p, q}, x] && IGtQ[r, 0]
```

Maple [A] (verified)

Time = 8.59 (sec) , antiderivative size = 385, normalized size of antiderivative = 1.18

method	result
elliptic	$\frac{\sqrt{(-b\,x^2+a)(x^2d+c)}\left(-\frac{e^2\sqrt{-bd\,x^4+ad\,x^2-x^2bc+ac}}{3ac\,x^3}-\frac{2e(3acf-ade+bce)\sqrt{-bd\,x^4+ad\,x^2-x^2bc+ac}}{3a^2c^2x}+\frac{\left(f^2+\frac{bd\,e^2}{3ac}\right)\sqrt{1-\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\ \mathrm{E}}{\sqrt{\frac{b}{a}}\,\sqrt{-bd\,x^4+ad}}\right.}{\sqrt{-b\,x^2+}}$
risch	$-\frac{\sqrt{-b\,x^2+a}\,\sqrt{x^2d+c}\,e\left(6acf\,x^2-2ade\,x^2+2bce\,x^2+ace\right)}{3a^2c^2x^3}+\frac{\left(\frac{2be(3acf-ade+bce)c\sqrt{1-\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\left(\mathrm{EllipticF}\left(x\sqrt{\frac{b}{a}},\sqrt{-1-\frac{ad-bc}{cb}}\right.\right.}{\sqrt{\frac{b}{a}}\,\sqrt{-bd\,x^4+ad\,x^2-x^2bc+ac}}\right.}{}$
default	$\frac{\left(6\sqrt{\frac{b}{a}}\,abcdef\,x^6-2\sqrt{\frac{b}{a}}\,ab\,d^2e^2x^6+2\sqrt{\frac{b}{a}}\,b^2cd\,e^2x^6+3\sqrt{\frac{-b\,x^2+a}{a}}\,\sqrt{\frac{x^2d+c}{c}}\ \mathrm{EllipticF}\left(x\sqrt{\frac{b}{a}},\sqrt{-\frac{ad}{bc}}\right)a^2c^2f^2x^3+6\sqrt{\frac{-b\,x^2+a}{a}}\,\sqrt{\frac{x^2}{}}\right.}{}$

input
```
int((f*x^2+e)^2/x^4/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBO
SE)
```

output
```
((-b*x^2+a)*(d*x^2+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-1/3*e^2/a/
c*(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)/x^3-2/3*e*(3*a*c*f-a*d*e+b*c*e)/a^2
/c^2*(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)/x+(f^2+1/3*b*d*e^2/a/c)/(b/a)^(1
/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/
2)*EllipticF(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2))+2/3*b*e*(3*a*c*f-a*d*
e+b*c*e)/a^2/c/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a
*d*x^2-b*c*x^2+a*c)^(1/2)*(EllipticF(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2
))-EllipticE(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.69

$$\int \frac{(e+fx^2)^2}{x^4\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx =$$
$$-\frac{2\left(3\,ab^2cef+(b^3c-ab^2d)e^2\right)\sqrt{ac}x^3\sqrt{\frac{b}{a}}E\left(\arcsin\left(x\sqrt{\frac{b}{a}}\right)\,|-\frac{ad}{bc}\right)-\left(6\,ab^2cef+3\,a^3cf^2+\left(2\,b^3c+(a^2b\right.\right.}{}$$

input
```
integrate((f*x^2+e)^2/x^4/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="f
ricas")
```

output
```
-1/3*(2*(3*a*b^2*c*e*f + (b^3*c - a*b^2*d)*e^2)*sqrt(a*c)*x^3*sqrt(b/a)*el
liptic_e(arcsin(x*sqrt(b/a)), -a*d/(b*c)) - (6*a*b^2*c*e*f + 3*a^3*c*f^2 +
 (2*b^3*c + (a^2*b - 2*a*b^2)*d)*e^2)*sqrt(a*c)*x^3*sqrt(b/a)*elliptic_f(a
rcsin(x*sqrt(b/a)), -a*d/(b*c)) + (a^2*b*c*e^2 + 2*(3*a^2*b*c*e*f + (a*b^2
*c - a^2*b*d)*e^2)*x^2)*sqrt(-b*x^2 + a)*sqrt(d*x^2 + c))/(a^3*b*c^2*x^3)
```

Sympy [F]

$$\int \frac{(e+fx^2)^2}{x^4\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{(e+fx^2)^2}{x^4\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx$$

input
```
integrate((f*x**2+e)**2/x**4/(-b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral((e + f*x**2)**2/(x**4*sqrt(a - b*x**2)*sqrt(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{(e+fx^2)^2}{x^4\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{(fx^2+e)^2}{\sqrt{-bx^2+a}\sqrt{dx^2+cx^4}}\,dx$$

input
```
integrate((f*x^2+e)^2/x^4/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="m
axima")
```

output
```
integrate((f*x^2 + e)^2/(sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)*x^4), x)
```

Giac [F]

$$\int \frac{(e+fx^2)^2}{x^4\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{(fx^2+e)^2}{\sqrt{-bx^2+a}\sqrt{dx^2+cx^4}}\,dx$$

input
```
integrate((f*x^2+e)^2/x^4/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="g
iac")
```

output
```
integrate((f*x^2 + e)^2/(sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx^2)^2}{x^4\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{(f\,x^2+e)^2}{x^4\,\sqrt{a-b\,x^2}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int((e + f*x^2)^2/(x^4*(a - b*x^2)^(1/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int((e + f*x^2)^2/(x^4*(a - b*x^2)^(1/2)*(c + d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{(e+fx^2)^2}{x^4\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx = \text{Too large to display}$$

input
```
int((f*x^2+e)^2/x^4/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

output
```
( - sqrt(c + d*x**2)*sqrt(a - b*x**2)*e*f - 3*int((sqrt(c + d*x**2)*sqrt(a
 - b*x**2))/(a**2*c*d*x**4 + a**2*d**2*x**6 - a*b*c**2*x**4 - 2*a*b*c*d*x*
*6 - a*b*d**2*x**8 + b**2*c**2*x**6 + b**2*c*d*x**8),x)*a**2*c*d*e*f*x**3
+ int((sqrt(c + d*x**2)*sqrt(a - b*x**2))/(a**2*c*d*x**4 + a**2*d**2*x**6
- a*b*c**2*x**4 - 2*a*b*c*d*x**6 - a*b*d**2*x**8 + b**2*c**2*x**6 + b**2*c
*d*x**8),x)*a**2*d**2*e**2*x**3 + 3*int((sqrt(c + d*x**2)*sqrt(a - b*x**2)
)/(a**2*c*d*x**4 + a**2*d**2*x**6 - a*b*c**2*x**4 - 2*a*b*c*d*x**6 - a*b*d
**2*x**8 + b**2*c**2*x**6 + b**2*c*d*x**8),x)*a*b*c**2*e*f*x**3 - 2*int((s
qrt(c + d*x**2)*sqrt(a - b*x**2))/(a**2*c*d*x**4 + a**2*d**2*x**6 - a*b*c*
*2*x**4 - 2*a*b*c*d*x**6 - a*b*d**2*x**8 + b**2*c**2*x**6 + b**2*c*d*x**8)
,x)*a*b*c*d*e**2*x**3 + int((sqrt(c + d*x**2)*sqrt(a - b*x**2))/(a**2*c*d*
x**4 + a**2*d**2*x**6 - a*b*c**2*x**4 - 2*a*b*c*d*x**6 - a*b*d**2*x**8 + b
**2*c**2*x**6 + b**2*c*d*x**8),x)*b**2*c**2*e**2*x**3 + int((sqrt(c + d*x*
*2)*sqrt(a - b*x**2))/(a**2*c*d + a**2*d**2*x**2 - a*b*c**2 - 2*a*b*c*d*x*
*2 - a*b*d**2*x**4 + b**2*c**2*x**2 + b**2*c*d*x**4),x)*a**2*d**2*f**2*x**
3 - 2*int((sqrt(c + d*x**2)*sqrt(a - b*x**2))/(a**2*c*d + a**2*d**2*x**2 -
 a*b*c**2 - 2*a*b*c*d*x**2 - a*b*d**2*x**4 + b**2*c**2*x**2 + b**2*c*d*x**
4),x)*a*b*c*d*f**2*x**3 + int((sqrt(c + d*x**2)*sqrt(a - b*x**2))/(a**2*c*
d + a**2*d**2*x**2 - a*b*c**2 - 2*a*b*c*d*x**2 - a*b*d**2*x**4 + b**2*c**2
*x**2 + b**2*c*d*x**4),x)*a*b*d**2*e*f*x**3 + int((sqrt(c + d*x**2)*sqr...
```

3.209 $\int \frac{(e+fx^2)^2}{x^6\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 463

$$\int \frac{(e+fx^2)^2}{x^6\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx$$

$$= -\frac{e^2\sqrt{a-bx^2}\sqrt{c+dx^2}}{5acx^5} - \frac{2e(2bce-2ade+5acf)\sqrt{a-bx^2}\sqrt{c+dx^2}}{15a^2c^2x^3}$$

$$- \frac{(4(bc-ad)e(2bce-2ade+5acf)+3ac(3bde^2+5acf^2))\sqrt{a-bx^2}\sqrt{c+dx^2}}{15a^3c^3x}$$

$$- \frac{\sqrt{b}(4(bc-ad)e(2bce-2ade+5acf)+3ac(3bde^2+5acf^2))\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\,E\big(\arcsin\big(\frac{\sqrt{b}x}{\sqrt{a}}\big)\,|-\frac{ad}{bc}}{15a^{5/2}c^3\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}}$$

$$+ \frac{\sqrt{b}(8b^2c^2e^2-abce(3de-20cf)+a^2(4d^2e^2-10cdef+15c^2f^2))\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\mathrm{EllipticF}\big(\arcsin}{15a^{5/2}c^2\sqrt{a-bx^2}\sqrt{c+dx^2}}$$

output
```
-1/5*e^2*(-b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a/c/x^5-2/15*e*(5*a*c*f-2*a*d*e+
2*b*c*e)*(-b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a^2/c^2/x^3-1/15*(4*(-a*d+b*c)*e
*(5*a*c*f-2*a*d*e+2*b*c*e)+3*a*c*(5*a*c*f^2+3*b*d*e^2))*(-b*x^2+a)^(1/2)*(
d*x^2+c)^(1/2)/a^3/c^3/x-1/15*b^(1/2)*(4*(-a*d+b*c)*e*(5*a*c*f-2*a*d*e+2*b
*c*e)+3*a*c*(5*a*c*f^2+3*b*d*e^2))*(1-b*x^2/a)^(1/2)*(d*x^2+c)^(1/2)*Ellip
ticE(b^(1/2)*x/a^(1/2),(-a*d/b/c)^(1/2))/a^(5/2)/c^3/(-b*x^2+a)^(1/2)/(1+d
*x^2/c)^(1/2)+1/15*b^(1/2)*(8*b^2*c^2*e^2-a*b*c*e*(-20*c*f+3*d*e)+a^2*(15*
c^2*f^2-10*c*d*e*f+4*d^2*e^2))*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)*Ellipti
cF(b^(1/2)*x/a^(1/2),(-a*d/b/c)^(1/2))/a^(5/2)/c^2/(-b*x^2+a)^(1/2)/(d*x^2
+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 7.55 (sec) , antiderivative size = 411, normalized size of antiderivative = 0.89

$$\int \frac{(e+fx^2)^2}{x^6\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx$$
$$= \frac{\sqrt{-\frac{b}{a}}(-a+bx^2)(c+dx^2)\left(8b^2c^2e^2x^4+abcex^2(-7dex^2+4c(e+5fx^2))+a^2(8d^2e^2x^4-4cdex^2(e+5f\cdots\right.}{\cdots}$$

input
```
Integrate[(e + f*x^2)^2/(x^6*Sqrt[a - b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
(Sqrt[-(b/a)]*(-a + b*x^2)*(c + d*x^2)*(8*b^2*c^2*e^2*x^4 + a*b*c*e*x^2*(-
7*d*e*x^2 + 4*c*(e + 5*f*x^2)) + a^2*(8*d^2*e^2*x^4 - 4*c*d*e*x^2*(e + 5*f
*x^2) + c^2*(3*e^2 + 10*e*f*x^2 + 15*f^2*x^4))) + I*b*c*(8*b^2*c^2*e^2 + a
*b*c*e*(-7*d*e + 20*c*f) + a^2*(8*d^2*e^2 - 20*c*d*e*f + 15*c^2*f^2))*x^5*
Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[-(b/a)]*x
], -((a*d)/(b*c))] - I*b*c*(8*b^2*c^2*e^2 + a*b*c*e*(-3*d*e + 20*c*f) + a^
2*(4*d^2*e^2 - 10*c*d*e*f + 15*c^2*f^2))*x^5*Sqrt[1 - (b*x^2)/a]*Sqrt[1 +
(d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))])/(15*a^3*S
qrt[-(b/a)]*c^3*x^5*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (warning: unable to verify)

Time = 1.64 (sec) , antiderivative size = 776, normalized size of antiderivative = 1.68, number of steps used = 23, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.639$, Rules used = {448, 445, 25, 445, 25, 27, 399, 323, 323, 321, 331, 330, 327, 445, 25, 27, 399, 323, 323, 321, 331, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(e+fx^2\right)^2}{x^6\sqrt{a-bx^2}\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \text{448}$$

$$\frac{f\int \frac{fx^2+e}{x^4\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{e^2}+e\int \frac{fx^2+e}{x^6\sqrt{a-bx^2}\sqrt{dx^2+c}}dx$$

$$\Big\downarrow \text{445}$$

$$\frac{f\left(-\frac{\int -\frac{bdex^2+2bce-2ade+3acf}{x^2\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{3ac}-\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{3acx^3}\right)}{e^2}+e\left(-\frac{\int -\frac{3bdex^2+4bce-4ade+5acf}{x^4\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{5ac}-\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{5acx^5}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{f\left(\frac{\int \frac{bdex^2+2bce-2ade+3acf}{x^2\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{3ac}-\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{3acx^3}\right)}{e^2}+e\left(\frac{\int \frac{3bdex^2+4bce-4ade+5acf}{x^4\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{5ac}-\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{5acx^5}\right)$$

$$\Big\downarrow \text{445}$$

$$e\left(\frac{-\frac{\int -\frac{2d(4de-5cf)a^2-bc(7de-10cf)a+bd(4bce-4ade+5acf)x^2+8b^2c^2e}{x^2\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{3ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}(5acf-4ade+4bce)}{3acx^3}}{5ac}-\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{5acx^5}\right.$$

$$\frac{f\left(\frac{-\frac{\int -\frac{bd\left(ace-(2bce-2ade+3acf)x^2\right)}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}(3acf-2ade+2bce)}{acx}}{3ac}-\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{3acx^3}\right)}{e^2}$$

$$\Big\downarrow \; 25$$

$$e\left(\frac{\frac{\int \frac{2d(4de-5cf)a^2-bc(7de-10cf)a+bd(4bce-4ade+5acf)x^2+8b^2c^2e}{x^2\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{3ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}(5acf-4ade+4bce)}{3acx^3}}{5ac}-\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{5acx^5}\right)-$$

$$\frac{f\left(\frac{\frac{\int \frac{bd\left(ace-(2bce-2ade+3acf)x^2\right)}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}(3acf-2ade+2bce)}{acx}}{3ac}-\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{3acx^3}\right)}{e^2}$$

$$\Big\downarrow \; 27$$

$$e\left(\frac{\frac{\int \frac{2d(4de-5cf)a^2-bc(7de-10cf)a+bd(4bce-4ade+5acf)x^2+8b^2c^2e}{x^2\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{3ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}(5acf-4ade+4bce)}{3acx^3}}{5ac}-\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{5acx^5}\right)-$$

$$\frac{f\left(\frac{\frac{bd\int \frac{ace-(2bce-2ade+3acf)x^2}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}(3acf-2ade+2bce)}{acx}}{3ac}-\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{3acx^3}\right)}{e^2}$$

$$\Big\downarrow \; 399$$

$$e\left(\frac{\frac{\int \frac{2d(4de-5cf)a^2-bc(7de-10cf)a+bd(4bce-4ade+5acf)x^2+8b^2c^2e}{x^2\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{3ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}(5acf-4ade+4bce)}{3acx^3}}{5ac}-\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{5acx^5}\right)-$$

$$\frac{f\left(\frac{\frac{bd\left(\frac{c(3acf-ade+2bce)\int \frac{1}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{d}-\frac{(3acf-2ade+2bce)\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}\right)}{ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}(3acf-2ade+2bce)}{acx}}{3ac}-\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{3acx^3}\right)}{e^2}$$

$\downarrow$ 323

$$\frac{e\left(\frac{\frac{\int \frac{2d(4de-5cf)a^2-bc(7de-10cf)a+bd(4bce-4ade+5acf)x^2+8b^2c^2e}{x^2\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{3ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}(5acf-4ade+4bce)}{3acx^3}}{5ac}-\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{5acx^5}\right)-\cdots f\left(\frac{\frac{bd\left(\frac{c\sqrt{\frac{dx^2}{c}+1}(3acf-ade+2bce)\int \frac{1}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}dx}{d\sqrt{c+dx^2}}-\frac{(3acf-2ade+2bce)\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}\right)}{ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}(3acf-2ade+2bce)}{acx}}{3ac}-\frac{e\sqrt{a-bx^2}\sqrt{c+\cdots}}{3acx^3}\cdots\right)}{e^2}$$

$\downarrow$ 323

$$\frac{e\left(\frac{\frac{\int \frac{2d(4de-5cf)a^2-bc(7de-10cf)a+bd(4bce-4ade+5acf)x^2+8b^2c^2e}{x^2\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{3ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}(5acf-4ade+4bce)}{3acx^3}}{5ac}-\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{5acx^5}\right)-\cdots f\left(\frac{\frac{bd\left(\frac{c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(3acf-ade+2bce)\int \frac{1}{\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}}dx}{d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{(3acf-2ade+2bce)\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}\right)}{ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}(3acf-2ade+2bce)}{acx}}{3ac}-e\sqrt{a-\cdots}\right)}{e^2}$$

$\downarrow$ 321

$$\frac{e\left(\frac{\frac{\int \frac{2d(4de-5cf)a^2-bc(7de-10cf)a+bd(4bce-4ade+5acf)x^2+8b^2c^2e}{x^2\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{3ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}(5acf-4ade+4bce)}{3acx^3}}{5ac}-\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{5acx^5}\right)-\cdots f\left(\frac{\frac{bd\left(\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(3acf-ade+2bce)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{(3acf-2ade+2bce)\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}\right)}{ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}(3acf-2ade+2bce)}{acx}}{3ac}\cdots\right)}{e^2}$$

$$\Big\downarrow \ 331$$

$$e\left(\frac{\frac{\int \frac{2d(4de-5cf)a^2-bc(7de-10cf)a+bd(4bce-4ade+5acf)x^2+8b^2c^2e}{x^2\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{3ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}(5acf-4ade+4bce)}{3acx^3}}{5ac}-\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{5acx^5}\right)-$$

$$\frac{f\left(\frac{\frac{bd\left(\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(3acf-ade+2bce)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{1-\frac{bx^2}{a}}(3acf-2ade+2bce)\int \frac{\sqrt{dx^2+c}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}\right)}{ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}(3acf-2a\ldots}{acx}}{3ac}\ldots\right)}{e^2}$$

$$\Big\downarrow \ 330$$

$$e\left(\frac{\frac{\int \frac{2d(4de-5cf)a^2-bc(7de-10cf)a+bd(4bce-4ade+5acf)x^2+8b^2c^2e}{x^2\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{3ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}(5acf-4ade+4bce)}{3acx^3}}{5ac}-\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{5acx^5}\right)-$$

$$\frac{f\left(\frac{\frac{bd\left(\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(3acf-ade+2bce)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(3acf-2ade+2bce)\int \frac{\sqrt{\frac{dx^2}{c}+1}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}\right)}{ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}(3\ldots}{ac\ldots}}{3ac}\ldots\right)}{e^2}$$

$$\Big\downarrow \ 327$$

$$\frac{e\left(\frac{\frac{\int \frac{2d(4de-5cf)a^2-bc(7de-10cf)a+bd(4bce-4ade+5acf)x^2+8b^2c^2e}{x^2\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{3ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}(5acf-4ade+4bce)}{3acx^3}}{5ac}-\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{5acx^5}\right)-\cdots \quad f\left(\frac{\frac{bd\left(\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(3acf-ade+2bce)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(3acf-2ade+2bce)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}\right)}{ac}-\frac{\sqrt{a-b\cdots}}{\cdots}}{3ac}\cdots\right.}{e^2}$$

$\downarrow$ 445

$$\frac{e\left(\frac{\frac{-\frac{\int -\frac{bd\left(ac(4bce-4ade+5acf)-\left(2d(4de-5cf)a^2-bc(7de-10cf)a+8b^2c^2e\right)x^2\right)}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}\left(\frac{8b^2ce}{a}+\frac{8ad^2e}{c}-10adf+10bcf-7bde\right)}{x}}{3ac}}{5ac}-\frac{\sqrt{a-bx^2}\cdots}{\cdots}\cdots\right) \quad f\left(\frac{\frac{bd\left(\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(3acf-ade+2bce)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(3acf-2ade+2bce)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}\right)}{ac}-\frac{\sqrt{a-b\cdots}}{\cdots}}{3ac}\cdots\right.}{e^2}$$

$\downarrow$ 25

$$\frac{e\left(\frac{\frac{\frac{\int \frac{bd\left(ac(4bce-4ade+5acf)-\left(2d(4de-5cf)a^2-bc(7de-10cf)a+8b^2c^2e\right)x^2\right)}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}\left(\frac{8b^2ce}{a}+\frac{8ad^2e}{c}-10adf+10bcf-7bde\right)}{x}}{3ac}}{5ac}-\frac{\sqrt{a-bx^2}\sqrt{c\cdots}}{\cdots}\cdots\right) \quad f\left(\frac{\frac{bd\left(\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(3acf-ade+2bce)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(3acf-2ade+2bce)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}\right)}{ac}-\frac{\sqrt{a-b\cdots}}{\cdots}}{3ac}\cdots\right.}{e^2}$$

$\downarrow$ 27

$$e\left(\frac{\frac{\frac{bd\int \frac{ac(4bce-4ade+5acf)-\left(2d(4de-5cf)a^2-bc(7de-10cf)a+8b^2c^2e\right)x^2}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}\left(\frac{8b^2ce}{a}+\frac{8ad^2e}{c}-10adf+10bcf-7bde\right)}{x}}{3ac}-\frac{\sqrt{a-bx^2}\sqrt{c+\cdots}}{\cdots}}{5ac}\right.$$

$$f\left(\frac{\frac{bd\left(\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(3acf-ade+2bce)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(3acf-2ade+2bce)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}\right)}{ac}}{3ac}-\frac{\sqrt{a-b\cdots}}{\cdots}\right.$$

$$\overline{\qquad\qquad e^2 \qquad\qquad}$$

$$\Big\downarrow \text{399}$$

$$e\left(\frac{\frac{\frac{bd\left(\frac{c\left(a^2d(4de-5cf)-abc(3de-10cf)+8b^2c^2e\right)\int \frac{1}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{d}-\frac{\left(2a^2d(4de-5cf)-abc(7de-10cf)+8b^2c^2e\right)\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}\right)}{ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}\left(\frac{8\cdots}{\cdots}\right.}{\cdots}}{3ac}}{5ac}\right.$$

$$f\left(\frac{\frac{bd\left(\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(3acf-ade+2bce)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(3acf-2ade+2bce)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}\right)}{ac}}{3ac}-\frac{\sqrt{a-b\cdots}}{\cdots}\right.$$

$$\overline{\qquad\qquad e^2 \qquad\qquad}$$

$$\Big\downarrow \text{323}$$

$$\frac{e\left(\frac{\frac{bd\left(\frac{c\sqrt{\frac{dx^2}{c}+1}\left(a^2d(4de-5cf)-abc(3de-10cf)+8b^2c^2e\right)\int\frac{1}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}dx}{d\sqrt{c+dx^2}}-\frac{\left(2a^2d(4de-5cf)-abc(7de-10cf)+8b^2c^2e\right)\int\frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}\right)}{ac}-\frac{\sqrt{a-bx^2}\sqrt{\cdots}}{\cdots}}{3ac}\cdots}{5ac}\cdots\right)\cdots f\left(\frac{\frac{bd\left(\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(3acf-ade+2bce)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(3acf-2ade+2bce)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}\right)}{ac}-\frac{\sqrt{a-b\cdots}}{\cdots}}{3ac}\cdots\right)}{e^2}$$

$\downarrow$ 323

$$\frac{e\left(\frac{\frac{bd\left(\frac{c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\left(a^2d(4de-5cf)-abc(3de-10cf)+8b^2c^2e\right)\int\frac{1}{\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}}dx}{d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\left(2a^2d(4de-5cf)-abc(7de-10cf)+8b^2c^2e\right)\int\frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}\right)}{ac}-\cdots}{3ac}\cdots}{5ac}\cdots\right)\cdots f\left(\frac{\frac{bd\left(\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(3acf-ade+2bce)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(3acf-2ade+2bce)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}\right)}{ac}-\frac{\sqrt{a-b\cdots}}{\cdots}}{3ac}\cdots\right)}{e^2}$$

$\downarrow$ 321

$$\frac{e\left(\frac{\frac{bd\left(\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\left(a^2d(4de-5cf)-abc(3de-10cf)+8b^2c^2e\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\left(2a^2d(4de-5cf)-abc(7de-10cf)+8b^2c^2e\right)\int\frac{\sqrt{dx^2}}{\sqrt{a-b}}}{d}\right.}{ac}}{3ac}}{5ac}\right.+f\left(\frac{\frac{bd\left(\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(3acf-ade+2bce)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(3acf-2ade+2bce)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}\right)}{ac}-\sqrt{a-b}}{3ac}\right.}{e^2}$$

$\downarrow$ 331

$$\frac{e\left(\frac{\frac{bd\left(\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\left(a^2d(4de-5cf)-abc(3de-10cf)+8b^2c^2e\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{1-\frac{bx^2}{a}}\left(2a^2d(4de-5cf)-abc(7de-10cf)+8b^2c^2e\right)}{d\sqrt{a-bx^2}}\right.}{ac}}{3ac}}{5ac}\right.+f\left(\frac{\frac{bd\left(\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(3acf-ade+2bce)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(3acf-2ade+2bce)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}\right)}{ac}-\sqrt{a-b}}{3ac}\right.}{e^2}$$

$\downarrow$ 330

$$\frac{e\left(\frac{\frac{bd\left(\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\left(a^2d(4de-5cf)-abc(3de-10cf)+8b^2c^2e\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\left(2a^2d(4de-5cf)-abc(7de-10cf)\cdots\right.}{d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}\right.}{ac}}{3ac}}{5ac}\cdots\right)$$

$$\frac{f\left(\frac{\frac{bd\left(\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(3acf-ade+2bce)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(3acf-2ade+2bce)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}\right)}{ac}}{3ac}-\frac{\sqrt{a-b\cdots}}{\cdots}\right)}{e^2}$$

$\downarrow$ 327

$$\frac{e\left(\frac{\frac{bd\left(\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\left(a^2d(4de-5cf)-abc(3de-10cf)+8b^2c^2e\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\left(2a^2d(4de-5cf)-abc(7de-10c\cdots\right.}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}\cdots}}\right.}{ac}}{3ac}}{5ac}\cdots\right)$$

$$\frac{f\left(\frac{\frac{bd\left(\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(3acf-ade+2bce)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(3acf-2ade+2bce)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}\right)}{ac}}{3ac}-\frac{\sqrt{a-b\cdots}}{\cdots}\right)}{e^2}$$

input

```
Int[(e + f*x^2)^2/(x^6*Sqrt[a - b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
(f*(-1/3*(e*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])/(a*c*x^3) + (-(((2*b*c*e - 2*
a*d*e + 3*a*c*f)*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])/(a*c*x)) + (b*d*(-((Sqrt
[a]*(2*b*c*e - 2*a*d*e + 3*a*c*f)*Sqrt[1 - (b*x^2)/a]*Sqrt[c + d*x^2]*Elli
pticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*d*Sqrt[a - b*
x^2]*Sqrt[1 + (d*x^2)/c])) + (Sqrt[a]*c*(2*b*c*e - a*d*e + 3*a*c*f)*Sqrt[1
 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -
((a*d)/(b*c))])/(Sqrt[b]*d*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])))/(a*c))/(3*a*
c)))/e^2 + e*(-1/5*(e*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])/(a*c*x^5) + (-1/3*(
(4*b*c*e - 4*a*d*e + 5*a*c*f)*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])/(a*c*x^3) +
 (-((((8*b^2*c*e)/a - 7*b*d*e + (8*a*d^2*e)/c + 10*b*c*f - 10*a*d*f)*Sqrt[
a - b*x^2]*Sqrt[c + d*x^2])/x) + (b*d*(-((Sqrt[a]*(8*b^2*c^2*e - a*b*c*(7*
d*e - 10*c*f) + 2*a^2*d*(4*d*e - 5*c*f))*Sqrt[1 - (b*x^2)/a]*Sqrt[c + d*x^
2]*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*d*Sqrt
[a - b*x^2]*Sqrt[1 + (d*x^2)/c])) + (Sqrt[a]*c*(8*b^2*c^2*e - a*b*c*(3*d*e
 - 10*c*f) + a^2*d*(4*d*e - 5*c*f))*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c
]*EllipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*d*Sqrt[
a - b*x^2]*Sqrt[c + d*x^2])))/(a*c))/(3*a*c))/(5*a*c))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 448
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[e   Int[(g*x)^m*(a + b*x
^2)^p*(c + d*x^2)^q*(e + f*x^2)^(r - 1), x], x] + Simp[f/e^2   Int[(g*x)^(m
 + 2)*(a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^(r - 1), x], x] /; FreeQ[{a,
b, c, d, e, f, g, m, p, q}, x] && IGtQ[r, 0]
```

Maple [A] (verified)

Time = 10.76 (sec) , antiderivative size = 538, normalized size of antiderivative = 1.16

method	result
elliptic	$\frac{\sqrt{(-b\,x^2+a)(x^2d+c)}\left(-\frac{e^2\sqrt{-bd\,x^4+ad\,x^2-x^2bc+ac}}{5ac\,x^5}-\frac{2(5acf-2ade+2bce)e\sqrt{-bd\,x^4+ad\,x^2-x^2bc+ac}}{15a^2c^2x^3}-\frac{(15a^2c^2f^2-20a^2cdef+8a^2d^2e^2+}{}\right.}{}$
risch	$-\frac{\sqrt{-b\,x^2+a}\,\sqrt{x^2d+c}\,(15f^2x^4a^2c^2-20a^2cdef\,x^4+8a^2d^2e^2x^4+20ab\,c^2ef\,x^4-7abcd\,e^2x^4+8b^2c^2e^2x^4+10a^2c^2ef\,x^2-4a^2cd\,e^2x^2+}{15a^3c^3x^5}$
default	Expression too large to display

input
```
int((f*x^2+e)^2/x^6/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBO
SE)
```

output
```
((-b*x^2+a)*(d*x^2+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-1/5*e^2/a/
c*(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)/x^5-2/15*(5*a*c*f-2*a*d*e+2*b*c*e)*
e/a^2/c^2*(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)/x^3-1/15/a^3/c^3*(15*a^2*c^
2*f^2-20*a^2*c*d*e*f+8*a^2*d^2*e^2+20*a*b*c^2*e*f-7*a*b*c*d*e^2+8*b^2*c^2*
e^2)*(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)/x+2/15*b*d*e*(5*a*c*f-2*a*d*e+2*
b*c*e)/a^2/c^2/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a
*d*x^2-b*c*x^2+a*c)^(1/2)*EllipticF(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2)
)+1/15*(15*a^2*c^2*f^2-20*a^2*c*d*e*f+8*a^2*d^2*e^2+20*a*b*c^2*e*f-7*a*b*c
*d*e^2+8*b^2*c^2*e^2)*b/c^2/a^3/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^
(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)*(EllipticF(x*(b/a)^(1/2),(-1-(a
*d-b*c)/c/b)^(1/2))-EllipticE(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 370, normalized size of antiderivative = 0.80

$$\int \frac{(e+fx^2)^2}{x^6\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx =$$

$$-\frac{(15\,a^2bc^2f^2+(8\,b^3c^2-7\,ab^2cd+8\,a^2bd^2)e^2+20\,(ab^2c^2-a^2bcd)ef)\sqrt{ac}x^5\sqrt{\frac{b}{a}}E\left(\arcsin\left(x\sqrt{\frac{b}{a}}\right)\,|-\frac{ad}{bc}\right.}{}$$

input
```
integrate((f*x^2+e)^2/x^6/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="f
ricas")
```

output
```
-1/15*((15*a^2*b*c^2*f^2 + (8*b^3*c^2 - 7*a*b^2*c*d + 8*a^2*b*d^2)*e^2 + 2
0*(a*b^2*c^2 - a^2*b*c*d)*e*f)*sqrt(a*c)*x^5*sqrt(b/a)*elliptic_e(arcsin(x
*sqrt(b/a)), -a*d/(b*c)) - (15*a^2*b*c^2*f^2 + (8*b^3*c^2 + (4*a^2*b - 7*a
*b^2)*c*d - 4*(a^3 - 2*a^2*b)*d^2)*e^2 + 10*(2*a*b^2*c^2 + (a^3 - 2*a^2*b)
*c*d)*e*f)*sqrt(a*c)*x^5*sqrt(b/a)*elliptic_f(arcsin(x*sqrt(b/a)), -a*d/(b
*c)) + (3*a^3*c^2*e^2 + (15*a^3*c^2*f^2 + (8*a*b^2*c^2 - 7*a^2*b*c*d + 8*a
^3*d^2)*e^2 + 20*(a^2*b*c^2 - a^3*c*d)*e*f)*x^4 + 2*(5*a^3*c^2*e*f + 2*(a^
2*b*c^2 - a^3*c*d)*e^2)*x^2)*sqrt(-b*x^2 + a)*sqrt(d*x^2 + c))/(a^4*c^3*x^
5)
```

Sympy [F]

$$\int \frac{(e+fx^2)^2}{x^6\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx=\int \frac{(e+fx^2)^2}{x^6\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx$$

input
```
integrate((f*x**2+e)**2/x**6/(-b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral((e + f*x**2)**2/(x**6*sqrt(a - b*x**2)*sqrt(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{(e+fx^2)^2}{x^6\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx=\int \frac{(fx^2+e)^2}{\sqrt{-bx^2+a}\sqrt{dx^2+cx^6}}\,dx$$

input
```
integrate((f*x^2+e)^2/x^6/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="m
axima")
```

output
```
integrate((f*x^2 + e)^2/(sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)*x^6), x)
```

Giac [F]

$$\int \frac{(e+fx^2)^2}{x^6\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{(fx^2+e)^2}{\sqrt{-bx^2+a}\sqrt{dx^2+c}x^6}\,dx$$

input
```
integrate((f*x^2+e)^2/x^6/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="g
iac")
```

output
```
integrate((f*x^2 + e)^2/(sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx^2)^2}{x^6\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{(f\,x^2+e)^2}{x^6\,\sqrt{a-b\,x^2}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int((e + f*x^2)^2/(x^6*(a - b*x^2)^(1/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int((e + f*x^2)^2/(x^6*(a - b*x^2)^(1/2)*(c + d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{(e+fx^2)^2}{x^6\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx = \text{Too large to display}$$

input
```
int((f*x^2+e)^2/x^6/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

output

```
( - sqrt(c + d*x**2)*sqrt(a - b*x**2)*a*c*e**2 - 5*sqrt(c + d*x**2)*sqrt(a
 - b*x**2)*a*c*f**2*x**4 - 3*sqrt(c + d*x**2)*sqrt(a - b*x**2)*b*d*e**2*x*
*4 - 5*int((sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**2)/(a**2*c*d + a**2*d**2*
x**2 - a*b*c**2 - 2*a*b*c*d*x**2 - a*b*d**2*x**4 + b**2*c**2*x**2 + b**2*c
*d*x**4),x)*a**2*b*c*d**2*f**2*x**5 + 5*int((sqrt(c + d*x**2)*sqrt(a - b*x
**2)*x**2)/(a**2*c*d + a**2*d**2*x**2 - a*b*c**2 - 2*a*b*c*d*x**2 - a*b*d*
*2*x**4 + b**2*c**2*x**2 + b**2*c*d*x**4),x)*a*b**2*c**2*d*f**2*x**5 - 3*i
nt((sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**2)/(a**2*c*d + a**2*d**2*x**2 - a
*b*c**2 - 2*a*b*c*d*x**2 - a*b*d**2*x**4 + b**2*c**2*x**2 + b**2*c*d*x**4)
,x)*a*b**2*d**3*e**2*x**5 + 3*int((sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**2)
/(a**2*c*d + a**2*d**2*x**2 - a*b*c**2 - 2*a*b*c*d*x**2 - a*b*d**2*x**4 +
b**2*c**2*x**2 + b**2*c*d*x**4),x)*b**3*c*d**2*e**2*x**5 + 10*int((sqrt(c
+ d*x**2)*sqrt(a - b*x**2))/(a**2*c*d*x**4 + a**2*d**2*x**6 - a*b*c**2*x**
4 - 2*a*b*c*d*x**6 - a*b*d**2*x**8 + b**2*c**2*x**6 + b**2*c*d*x**8),x)*a*
*3*c**2*d*e*f*x**5 - 4*int((sqrt(c + d*x**2)*sqrt(a - b*x**2))/(a**2*c*d*x
**4 + a**2*d**2*x**6 - a*b*c**2*x**4 - 2*a*b*c*d*x**6 - a*b*d**2*x**8 + b*
*2*c**2*x**6 + b**2*c*d*x**8),x)*a**3*c*d**2*e**2*x**5 - 10*int((sqrt(c +
d*x**2)*sqrt(a - b*x**2))/(a**2*c*d*x**4 + a**2*d**2*x**6 - a*b*c**2*x**4
- 2*a*b*c*d*x**6 - a*b*d**2*x**8 + b**2*c**2*x**6 + b**2*c*d*x**8),x)*a**2
*b*c**3*e*f*x**5 + 8*int((sqrt(c + d*x**2)*sqrt(a - b*x**2))/(a**2*c*d*...
```

3.210 $\int \frac{(e+fx^2)^2}{x^8\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 646

$$\int \frac{(e+fx^2)^2}{x^8\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx$$
$$= -\frac{e^2\sqrt{a-bx^2}\sqrt{c+dx^2}}{7acx^7} - \frac{2e(3bce-3ade+7acf)\sqrt{a-bx^2}\sqrt{c+dx^2}}{35a^2c^2x^5}$$
$$- \frac{(8(bc-ad)e(3bce-3ade+7acf)+5ac(5bde^2+7acf^2))\sqrt{a-bx^2}\sqrt{c+dx^2}}{105a^3c^3x^3}$$
$$- \frac{2(9abcde(3bce-3ade+7acf)+(bc-ad)(8(bc-ad)e(3bce-3ade+7acf)+5ac(5bde^2+7acf^2)))}{105a^4c^4x}$$
$$- \frac{2\sqrt{b}(9abcde(3bce-3ade+7acf)+(bc-ad)(8(bc-ad)e(3bce-3ade+7acf)+5ac(5bde^2+7acf^2}{105a^{7/2}c^4\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}}$$
$$+ \frac{\sqrt{b}(48b^3c^3e^2-16ab^2c^2e(de-7cf)-a^3d(24d^2e^2-56cdef+35c^2f^2)+a^2bc(17d^2e^2-42cdef+70c^2}{105a^{7/2}c^3\sqrt{a-bx^2}\sqrt{c+dx^2}}$$

output
```
-1/7*e^2*(-b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a/c/x^7-2/35*e*(7*a*c*f-3*a*d*e+
3*b*c*e)*(-b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a^2/c^2/x^5-1/105*(8*(-a*d+b*c)*
e*(7*a*c*f-3*a*d*e+3*b*c*e)+5*a*c*(7*a*c*f^2+5*b*d*e^2))*(-b*x^2+a)^(1/2)*
(d*x^2+c)^(1/2)/a^3/c^3/x^3-2/105*(9*a*b*c*d*e*(7*a*c*f-3*a*d*e+3*b*c*e)+(
-a*d+b*c)*(8*(-a*d+b*c)*e*(7*a*c*f-3*a*d*e+3*b*c*e)+5*a*c*(7*a*c*f^2+5*b*d
*e^2)))*(-b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a^4/c^4/x-2/105*b^(1/2)*(9*a*b*c*
d*e*(7*a*c*f-3*a*d*e+3*b*c*e)+(-a*d+b*c)*(8*(-a*d+b*c)*e*(7*a*c*f-3*a*d*e+
3*b*c*e)+5*a*c*(7*a*c*f^2+5*b*d*e^2)))*(1-b*x^2/a)^(1/2)*(d*x^2+c)^(1/2)*E
llipticE(b^(1/2)*x/a^(1/2),(-a*d/b/c)^(1/2))/a^(7/2)/c^4/(-b*x^2+a)^(1/2)/
(1+d*x^2/c)^(1/2)+1/105*b^(1/2)*(48*b^3*c^3*e^2-16*a*b^2*c^2*e*(-7*c*f+d*e
)-a^3*d*(35*c^2*f^2-56*c*d*e*f+24*d^2*e^2)+a^2*b*c*(70*c^2*f^2-42*c*d*e*f+
17*d^2*e^2))*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(b^(1/2)*x/a^(1/
2),(-a*d/b/c)^(1/2))/a^(7/2)/c^3/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.84 (sec) , antiderivative size = 585, normalized size of antiderivative = 0.91

$$\int \frac{(e+fx^2)^2}{x^8\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx$$
$$= \frac{\sqrt{-\frac{b}{a}}(-a+bx^2)\,(c+dx^2)\,(48b^3c^3e^2x^6+8ab^2c^2ex^4(3ce-5dex^2+14cfx^2)+a^2bcx^2(40d^2e^2x^4-cdex^2(\ldots}{\ldots}$$

input
```
Integrate[(e + f*x^2)^2/(x^8*Sqrt[a - b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
(Sqrt[-(b/a)]*(-a + b*x^2)*(c + d*x^2)*(48*b^3*c^3*e^2*x^6 + 8*a*b^2*c^2*e
*x^4*(3*c*e - 5*d*e*x^2 + 14*c*f*x^2) + a^2*b*c*x^2*(40*d^2*e^2*x^4 - c*d*
e*x^2*(23*e + 98*f*x^2) + 2*c^2*(9*e^2 + 28*e*f*x^2 + 35*f^2*x^4)) + a^3*(
-48*d^3*e^2*x^6 + 8*c*d^2*e*x^4*(3*e + 14*f*x^2) - 2*c^2*d*x^2*(9*e^2 + 28
*e*f*x^2 + 35*f^2*x^4) + c^3*(15*e^2 + 42*e*f*x^2 + 35*f^2*x^4))) - (2*I)*
b*c*(-24*b^3*c^3*e^2 + 4*a*b^2*c^2*e*(5*d*e - 14*c*f) + a^2*b*c*(-20*d^2*e
^2 + 49*c*d*e*f - 35*c^2*f^2) + a^3*d*(24*d^2*e^2 - 56*c*d*e*f + 35*c^2*f^
2))*x^7*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[-
(b/a)]*x], -((a*d)/(b*c))] - I*b*c*(48*b^3*c^3*e^2 + 16*a*b^2*c^2*e*(-(d*e
) + 7*c*f) + a^3*d*(-24*d^2*e^2 + 56*c*d*e*f - 35*c^2*f^2) + a^2*b*c*(17*d
^2*e^2 - 42*c*d*e*f + 70*c^2*f^2))*x^7*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2
)/c]*EllipticF[I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))])/(105*a^4*Sqrt[-
(b/a)]*c^4*x^7*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (warning: unable to verify)

Time = 2.41 (sec) , antiderivative size = 1054, normalized size of antiderivative = 1.63, number of steps used = 25, number of rules used = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.694$, Rules used = {448, 445, 25, 445, 25, 445, 25, 27, 399, 323, 323, 321, 331, 330, 327, 445, 25, 27, 399, 323, 323, 321, 331, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(e+fx^2\right)^2}{x^8\sqrt{a-bx^2}\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \text{448}$$

$$\frac{f\int \frac{fx^2+e}{x^6\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{e^2}+e\int \frac{fx^2+e}{x^8\sqrt{a-bx^2}\sqrt{dx^2+c}}dx$$

$$\Big\downarrow \text{445}$$

$$\frac{f\left(-\frac{\int -\frac{3bdex^2+4bce-4ade+5acf}{x^4\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{5ac}-\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{5acx^5}\right)}{e^2}+e\left(-\frac{\int -\frac{5bdex^2+6bce-6ade+7acf}{x^6\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{7ac}-\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{7acx^7}\right)$$

$$\Big\downarrow 25$$

$$\frac{f\left(\frac{\int \frac{3bdex^2+4bce-4ade+5acf}{x^4\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{5ac}-\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{5acx^5}\right)}{e^2}+e\left(\frac{\int \frac{5bdex^2+6bce-6ade+7acf}{x^6\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{7ac}-\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{7acx^7}\right)$$

$$\Big\downarrow 445$$

$$\frac{f\left(\frac{-\frac{\int -\frac{2d(4de-5cf)a^2-bc(7de-10cf)a+bd(4bce-4ade+5acf)x^2+8b^2c^2e}{x^2\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{3ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}(5acf-4ade+4bce)}{3acx^3}}{5ac}-\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{5acx^5}\right)}{e^2}+$$
$$e\left(\frac{-\frac{\int -\frac{4d(6de-7cf)a^2-bc(23de-28cf)a+3bd(6bce-6ade+7acf)x^2+24b^2c^2e}{x^4\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{5ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}(7acf-6ade+6bce)}{5acx^5}}{7ac}-\frac{e\sqrt{a-bx^2}\sqrt{c+d\cdots}}{7acx^7}\cdots\right.$$

$$\Big\downarrow 25$$

$$\frac{f\left(\frac{\frac{\int \frac{2d(4de-5cf)a^2-bc(7de-10cf)a+bd(4bce-4ade+5acf)x^2+8b^2c^2e}{x^2\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{3ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}(5acf-4ade+4bce)}{3acx^3}}{5ac}-\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{5acx^5}\right)}{e^2}+$$
$$e\left(\frac{\frac{\int \frac{4d(6de-7cf)a^2-bc(23de-28cf)a+3bd(6bce-6ade+7acf)x^2+24b^2c^2e}{x^4\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{5ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}(7acf-6ade+6bce)}{5acx^5}}{7ac}-\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{7acx^7}\cdots\right.$$

$$\Big\downarrow 445$$

$$\frac{f\left(\frac{\frac{-\frac{\int -\frac{bd\left(ac(4bce-4ade+5acf)-\left(2d(4de-5cf)a^2-bc(7de-10cf)a+8b^2c^2e\right)x^2\right)}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}\left(\frac{8b^2ce}{a}+\frac{8ad^2e}{c}-10adf+10bcf-7bde\right)}{x}}{3ac}-\sqrt{a-bx^2}\sqrt{\cdots}}{5ac}\cdots\right.}{e^2}$$
$$e\left(\frac{\frac{-\frac{\int -\frac{-8d^2(6de-7cf)a^3+bcd(40de-49cf)a^2-8b^2c^2(5de-7cf)a+bd\left(4d(6de-7cf)a^2-bc(23de-28cf)a+24b^2c^2e\right)x^2+48b^3c^3e}{x^2\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{3ac}-\sqrt{a-bx^2}\sqrt{c+dx^2}\left(\frac{24b^2\cdots}{a}\cdots\right.}{5ac}}{7ac}\cdots\right.$$

$$\Big\downarrow \ 25$$

$$f\left(\frac{\frac{\frac{\int \frac{bd\left(ac(4bce-4ade+5acf)-\left(2d(4de-5cf)a^2-bc(7de-10cf)a+8b^2c^2e\right)x^2\right)}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}\left(\frac{8b^2ce}{a}+\frac{8ad^2e}{c}-10adf+10bcf-7bde\right)}{x}}{3ac}-\frac{\sqrt{a-bx^2}\sqrt{c+\cdots}}{\cdots}}{5ac}\cdots\right)$$

$$\overline{\qquad e^2 \qquad}$$

$$e\left(\frac{\frac{\frac{\int \frac{-8d^2(6de-7cf)a^3+bcd(40de-49cf)a^2-8b^2c^2(5de-7cf)a+bd\left(4d(6de-7cf)a^2-bc(23de-28cf)a+24b^2c^2e\right)x^2+48b^3c^3e}{x^2\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{3ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}\left(\frac{24b^2ce}{a}+\cdots\right)}{\cdots}}{5ac}\cdots}{7ac}\cdots\right)$$

$$\Big\downarrow \ 27$$

$$f\left(\frac{\frac{\frac{bd\int \frac{ac(4bce-4ade+5acf)-\left(2d(4de-5cf)a^2-bc(7de-10cf)a+8b^2c^2e\right)x^2}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}\left(\frac{8b^2ce}{a}+\frac{8ad^2e}{c}-10adf+10bcf-7bde\right)}{x}}{3ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}\cdots}{3\cdots}}{5ac}\cdots\right)$$

$$\overline{\qquad e^2 \qquad}$$

$$e\left(\frac{\frac{\frac{\int \frac{-8d^2(6de-7cf)a^3+bcd(40de-49cf)a^2-8b^2c^2(5de-7cf)a+bd\left(4d(6de-7cf)a^2-bc(23de-28cf)a+24b^2c^2e\right)x^2+48b^3c^3e}{x^2\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{3ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}\left(\frac{24b^2ce}{a}+\cdots\right)}{\cdots}}{5ac}\cdots}{7ac}\cdots\right)$$

$$\Big\downarrow \ 399$$

$$f\left(\frac{\frac{\frac{bd\left(\frac{c\left(a^2d(4de-5cf)-abc(3de-10cf)+8b^2c^2e\right)\int \frac{1}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{d}-\frac{\left(2a^2d(4de-5cf)-abc(7de-10cf)+8b^2c^2e\right)\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}\right)}{ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}\left(\cdots\right)}{\cdots}}{3ac}\cdots}{5ac}\cdots\right)$$

$$\overline{\qquad e^2 \qquad}$$

$$e\left(\frac{\frac{\frac{\int \frac{-8d^2(6de-7cf)a^3+bcd(40de-49cf)a^2-8b^2c^2(5de-7cf)a+bd\left(4d(6de-7cf)a^2-bc(23de-28cf)a+24b^2c^2e\right)x^2+48b^3c^3e}{x^2\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{3ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}\left(\frac{24b^2ce}{a}+\cdots\right)}{\cdots}}{5ac}\cdots}{7ac}\cdots\right)$$

$$\Big\downarrow \ 323$$

$$
\frac{f\left(\frac{\frac{\frac{bd\left(\frac{c\sqrt{\frac{dx^2}{c}+1}\left(a^2d(4de-5cf)-abc(3de-10cf)+8b^2c^2e\right)\int\frac{1}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}dx}{d\sqrt{c+dx^2}}-\frac{\left(2a^2d(4de-5cf)-abc(7de-10cf)+8b^2c^2e\right)\int\frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}\right)}{ac}-\frac{\sqrt{a-bx^2}\cdots}{\cdots}}{3ac}\cdots}{5ac}\cdots\right)}{e^2}\cdots
$$

$$
e\left(\frac{\frac{\frac{\int\frac{-8d^2(6de-7cf)a^3+bcd(40de-49cf)a^2-8b^2c^2(5de-7cf)a+bd\left(4d(6de-7cf)a^2-bc(23de-28cf)a+24b^2c^2e\right)x^2+48b^3c^3e}{x^2\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{3ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}\left(\frac{24b^2ce}{a}+\cdots\right)}{\cdots}}{5ac}\cdots}{7ac}\cdots\right)
$$

$\downarrow$ 323

$$
\frac{f\left(\frac{\frac{\frac{bd\left(\frac{c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\left(a^2d(4de-5cf)-abc(3de-10cf)+8b^2c^2e\right)\int\frac{1}{\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}}dx}{d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\left(2a^2d(4de-5cf)-abc(7de-10cf)+8b^2c^2e\right)\int\frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}\right)}{ac}-\cdots}{3ac}\cdots}{5ac}\cdots\right)}{e^2}\cdots
$$

$$
e\left(\frac{\frac{\frac{\int\frac{-8d^2(6de-7cf)a^3+bcd(40de-49cf)a^2-8b^2c^2(5de-7cf)a+bd\left(4d(6de-7cf)a^2-bc(23de-28cf)a+24b^2c^2e\right)x^2+48b^3c^3e}{x^2\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{3ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}\left(\frac{24b^2ce}{a}+\cdots\right)}{\cdots}}{5ac}\cdots}{7ac}\cdots\right)
$$

$\downarrow$ 321

$$f\left(\frac{\frac{\frac{bd\left(\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\left(a^2d(4de-5cf)-abc(3de-10cf)+8b^2c^2e\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\left(2a^2d(4de-5cf)-abc(7de-10cf)+8b^2c^2e\right)\int\frac{\sqrt{dx^2}}{\sqrt{a-}}}{d}\right.}{ac}}{3ac}}{5ac}\right.$$

$$\overline{\qquad e^2 \qquad}$$

$$e\left(\frac{\frac{\frac{\int\frac{-8d^2(6de-7cf)a^3+bcd(40de-49cf)a^2-8b^2c^2(5de-7cf)a+bd\left(4d(6de-7cf)a^2-bc(23de-28cf)a+24b^2c^2e\right)x^2+48b^3c^3e}{x^2\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{3ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}\left(\frac{24b^2ce}{a}+\right.}{}}{5ac}}{7ac}\right.$$

$$\Big\downarrow \textbf{331}$$

$$f\left(\frac{\frac{\frac{bd\left(\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\left(a^2d(4de-5cf)-abc(3de-10cf)+8b^2c^2e\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{1-\frac{bx^2}{a}}\left(2a^2d(4de-5cf)-abc(7de-10cf)+8b^2c^2e\right.}{d\sqrt{a-bx^2}}\right.}{ac}}{3ac}}{5ac}\right.$$

$$\overline{\qquad e^2 \qquad}$$

$$e\left(\frac{\frac{\frac{\int\frac{-8d^2(6de-7cf)a^3+bcd(40de-49cf)a^2-8b^2c^2(5de-7cf)a+bd\left(4d(6de-7cf)a^2-bc(23de-28cf)a+24b^2c^2e\right)x^2+48b^3c^3e}{x^2\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{3ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}\left(\frac{24b^2ce}{a}+\right.}{}}{5ac}}{7ac}\right.$$

$$\Big\downarrow \textbf{330}$$

$$\frac{f\left(\frac{\frac{bd\left(\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\left(a^2d(4de-5cf)-abc(3de-10cf)+8b^2c^2e\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\left(2a^2d(4de-5cf)-abc(7de-10cf)\cdots\right.}{d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}\cdots}\right.}{ac}\cdots}{3ac}\cdots}{5ac}\cdots\right)\cdots}{e^2}\cdots$$

$$e\left(\frac{\frac{\frac{\int\frac{-8d^2(6de-7cf)a^3+bcd(40de-49cf)a^2-8b^2c^2(5de-7cf)a+bd\left(4d(6de-7cf)a^2-bc(23de-28cf)a+24b^2c^2e\right)x^2+48b^3c^3e}{x^2\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{3ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}\left(\frac{24b^2ce}{a}+\cdots\right.}{\cdots}}{5ac}\cdots}{7ac}\cdots\right.$$

$\downarrow$ 327

$$e\left(\frac{\frac{\frac{\int\frac{-8d^2(6de-7cf)a^3+bcd(40de-49cf)a^2-8b^2c^2(5de-7cf)a+bd\left(4d(6de-7cf)a^2-bc(23de-28cf)a+24b^2c^2e\right)x^2+48b^3c^3e}{x^2\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{3ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}\left(\frac{24b^2ce}{a}+\cdots\right.}{\cdots}}{5ac}\cdots}{7ac}\cdots\right.$$

$$f\left(\frac{\frac{bd\left(\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\left(a^2d(4de-5cf)-abc(3de-10cf)+8b^2c^2e\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\left(2a^2d(4de-5cf)-abc(7de-10\cdots\right.}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{dx}{c}\cdots}}\right.}{ac}\cdots}{3ac}\cdots}{5ac}\cdots\right.$$

$\downarrow$ 445

$$f\left(\frac{\frac{bd\left(\frac{\sqrt{a}c\left(d(4de-5cf)a^2-bc(3de-10cf)a+8b^2c^2e\right)\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{dx^2+c}}-\frac{\sqrt{a}\left(2d(4de-5cf)a^2-bc(7de-10cf)a+8b^2c^2e\right)\sqrt{1-}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{dx}{c}}}}{ac}}{3ac}}{5ac}\right.$$

$$e\left(\frac{\frac{-\frac{\sqrt{a-bx^2}\sqrt{dx^2+c}\left(-8d^2(6de-7cf)a^3+bcd(40de-49cf)a^2-8b^2c^2(5de-7cf)a+48b^3c^3e\right)}{acx}-\frac{\int-\frac{bd\left(ac\left(4d(6de-7cf)a^2-bc(23de-28cf)a+24b^2c^2e\right)-\left(-8d^2\right.\right.}{\sqrt{a-}}}{}}{3ac}}{5ac}\right.$$

$\downarrow$ 25

$$f\left(\frac{\frac{bd\left(\frac{\sqrt{a}c\left(d(4de-5cf)a^2-bc(3de-10cf)a+8b^2c^2e\right)\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{dx^2+c}}-\frac{\sqrt{a}\left(2d(4de-5cf)a^2-bc(7de-10cf)a+8b^2c^2e\right)\sqrt{1-}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{dx}{c}}}}{ac}}{3ac}}{5ac}\right.$$

$$e\left(\frac{\frac{\frac{\int\frac{bd\left(ac\left(4d(6de-7cf)a^2-bc(23de-28cf)a+24b^2c^2e\right)-\left(-8d^2(6de-7cf)a^3+bcd(40de-49cf)a^2-8b^2c^2(5de-7cf)a+48b^3c^3e\right)x^2\right)}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{ac}-\left(-8d^2(6de-7cf)a^3-\right.}{3ac}}{5ac}\right.$$

$\downarrow$ 27

$$f\left(\frac{\frac{bd\left(\frac{\sqrt{a}c\left(d(4de-5cf)a^2-bc(3de-10cf)a+8b^2c^2e\right)\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{dx^2+c}}-\frac{\sqrt{a}\left(2d(4de-5cf)a^2-bc(7de-10cf)a+8b^2c^2e\right)\sqrt{1-\cdots}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{dx}{c}\cdots}}\cdots}{ac}\cdots}{3ac}\cdots}{5ac}\cdots\right.$$

$$e\left(\frac{\frac{\frac{bd\int\frac{ac\left(4d(6de-7cf)a^2-bc(23de-28cf)a+24b^2c^2e\right)-\left(-8d^2(6de-7cf)a^3+bcd(40de-49cf)a^2-8b^2c^2(5de-7cf)a+48b^3c^3e\right)x^2}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{ac}-\frac{\left(-8d^2(6de-7cf)a^3+b\cdots\right.}{\cdots}}{3ac}\cdots}{5ac}\cdots\right.$$

$$\Big\downarrow\ 399$$

$$f\left(\frac{\frac{bd\left(\frac{\sqrt{a}c\left(d(4de-5cf)a^2-bc(3de-10cf)a+8b^2c^2e\right)\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{dx^2+c}}-\frac{\sqrt{a}\left(2d(4de-5cf)a^2-bc(7de-10cf)a+8b^2c^2e\right)\sqrt{1-\cdots}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{dx}{c}\cdots}}\cdots}{ac}\cdots}{3ac}\cdots}{5ac}\cdots\right.$$

$$e\left(\frac{\frac{bd\left(\frac{c\left(-4d^2(6de-7cf)a^3+bcd(17de-21cf)a^2-8b^2c^2(2de-7cf)a+48b^3c^3e\right)\int\frac{1}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{d}-\frac{\left(-8d^2(6de-7cf)a^3+bcd(40de-49cf)a^2-8b^2c^2(5de-\cdots\right.}{d}\cdots\right)}{ac}\cdots}{3ac}\cdots\right.$$

$$\Big\downarrow\ 323$$

$$f\left(\frac{\frac{bd\left(\frac{\sqrt{a}c\left(d(4de-5cf)a^2-bc(3de-10cf)a+8b^2c^2e\right)\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{dx^2+c}}-\frac{\sqrt{a}\left(2d(4de-5cf)a^2-bc(7de-10cf)a+8b^2c^2e\right)\sqrt{1-}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{dx}{c}}}\right.}{ac}}{3ac}}{5ac}\right.$$

$$e\left(\frac{\frac{bd\left(\frac{c\left(-4d^2(6de-7cf)a^3+bcd(17de-21cf)a^2-8b^2c^2(2de-7cf)a+48b^3c^3e\right)\sqrt{\frac{dx^2}{c}+1}\int\frac{1}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}dx}{d\sqrt{dx^2+c}}-\frac{\left(-8d^2(6de-7cf)a^3+bcd(40de-49cf)a^2-8b\right.}{d}\right.}{ac}}{3ac}\right.$$

$$\Big\downarrow \; 323$$

$$f\left(\frac{\frac{bd\left(\frac{\sqrt{a}c\left(d(4de-5cf)a^2-bc(3de-10cf)a+8b^2c^2e\right)\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{dx^2+c}}-\frac{\sqrt{a}\left(2d(4de-5cf)a^2-bc(7de-10cf)a+8b^2c^2e\right)\sqrt{1-}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{dx}{c}}}\right.}{ac}}{3ac}}{5ac}\right.$$

$$e\left(\frac{\frac{bd\left(\frac{c\left(-4d^2(6de-7cf)a^3+bcd(17de-21cf)a^2-8b^2c^2(2de-7cf)a+48b^3c^3e\right)\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\int\frac{1}{\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}}dx}{d\sqrt{a-bx^2}\sqrt{dx^2+c}}-\left(-8d^2(6de-7cf)a^3+bcd(40de-49\right.\right.}{ac}}{3ac}\right.$$

$$\Big\downarrow \; 321$$

$$\frac{f\left(\frac{\frac{bd\left(\frac{\sqrt{a}c\left(d(4de-5cf)a^2-bc(3de-10cf)a+8b^2c^2e\right)\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{dx^2+c}}-\frac{\sqrt{a}\left(2d(4de-5cf)a^2-bc(7de-10cf)a+8b^2c^2e\right)\sqrt{1-\cdots}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{dx\cdots}{c}}}\cdots\right)}{ac}\cdots}{3ac}\cdots}{5ac}\cdots\right)}{e\left(\frac{\frac{bd\left(\frac{\sqrt{a}c\left(-4d^2(6de-7cf)a^3+bcd(17de-21cf)a^2-8b^2c^2(2de-7cf)a+48b^3c^3e\right)\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{dx^2+c}}-\frac{\left(-8d^2(6de-7cf)a^3+bc\cdots\right.}{\cdots}\right)}{ac}\cdots}{3a\cdots}\cdots\right)}$$

$\downarrow$ 331

$$\frac{f\left(\frac{\frac{bd\left(\frac{\sqrt{a}c\left(d(4de-5cf)a^2-bc(3de-10cf)a+8b^2c^2e\right)\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{dx^2+c}}-\frac{\sqrt{a}\left(2d(4de-5cf)a^2-bc(7de-10cf)a+8b^2c^2e\right)\sqrt{1-\cdots}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{dx\cdots}{c}}}\cdots\right)}{ac}\cdots}{3ac}\cdots}{5ac}\cdots\right)}{e\left(\frac{\frac{bd\left(\frac{\sqrt{a}c\left(-4d^2(6de-7cf)a^3+bcd(17de-21cf)a^2-8b^2c^2(2de-7cf)a+48b^3c^3e\right)\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{dx^2+c}}-\frac{\left(-8d^2(6de-7cf)a^3+bc\cdots\right.}{\cdots}\right)}{ac}\cdots}{\cdots}\cdots\right)}$$

$\downarrow$ 330

$$f\left(\frac{\frac{\frac{bd\left(\frac{\sqrt{a}c\left(d(4de-5cf)a^2-bc(3de-10cf)a+8b^2c^2e\right)\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{dx^2+c}}-\frac{\sqrt{a}\left(2d(4de-5cf)a^2-bc(7de-10cf)a+8b^2c^2e\right)\sqrt{1-\cdots}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{dx\cdots}{c}}}\cdots}{ac}\cdots}{3ac}\cdots}{5ac}\cdots\right.$$

$$e\left(\frac{bd\left(\frac{\sqrt{a}c\left(-4d^2(6de-7cf)a^3+bcd(17de-21cf)a^2-8b^2c^2(2de-7cf)a+48b^3c^3e\right)\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{dx^2+c}}-\frac{\left(-8d^2(6de-7cf)a^3+bc\cdots\right.}{\cdots}\cdots\right)}{ac}\cdots\right.$$

$$\Big\downarrow \; 327$$

$$f\left(\frac{\frac{\frac{bd\left(\frac{\sqrt{a}c\left(d(4de-5cf)a^2-bc(3de-10cf)a+8b^2c^2e\right)\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{dx^2+c}}-\frac{\sqrt{a}\left(2d(4de-5cf)a^2-bc(7de-10cf)a+8b^2c^2e\right)\sqrt{1-\cdots}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{dx\cdots}{c}}}\cdots}{ac}\cdots}{3ac}\cdots}{5ac}\cdots\right.$$

$$e\left(\frac{bd\left(\frac{\sqrt{a}c\left(-4d^2(6de-7cf)a^3+bcd(17de-21cf)a^2-8b^2c^2(2de-7cf)a+48b^3c^3e\right)\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{dx^2+c}}-\frac{\sqrt{a}\left(-8d^2(6de-7cf)a^3\cdots\right.}{\cdots}\cdots\right)}{ac}\cdots\right.$$

input
```
Int[(e + f*x^2)^2/(x^8*Sqrt[a - b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
(f*(-1/5*(e*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])/(a*c*x^5) + (-1/3*((4*b*c*e -
 4*a*d*e + 5*a*c*f)*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])/(a*c*x^3) + (-((((8*b
^2*c*e)/a - 7*b*d*e + (8*a*d^2*e)/c + 10*b*c*f - 10*a*d*f)*Sqrt[a - b*x^2]
*Sqrt[c + d*x^2])/x) + (b*d*(-((Sqrt[a]*(8*b^2*c^2*e - a*b*c*(7*d*e - 10*c
*f) + 2*a^2*d*(4*d*e - 5*c*f))*Sqrt[1 - (b*x^2)/a]*Sqrt[c + d*x^2]*Ellipti
cE[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*d*Sqrt[a - b*x^2
]*Sqrt[1 + (d*x^2)/c])) + (Sqrt[a]*c*(8*b^2*c^2*e - a*b*c*(3*d*e - 10*c*f)
 + a^2*d*(4*d*e - 5*c*f))*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Elliptic
F[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*d*Sqrt[a - b*x^2]
*Sqrt[c + d*x^2])))/(a*c))/(3*a*c))/(5*a*c)))/e^2 + e*(-1/7*(e*Sqrt[a - b*
x^2]*Sqrt[c + d*x^2])/(a*c*x^7) + (-1/5*((6*b*c*e - 6*a*d*e + 7*a*c*f)*Sqr
t[a - b*x^2]*Sqrt[c + d*x^2])/(a*c*x^5) + (-1/3*(((24*b^2*c*e)/a - b*(23*d
*e - 28*c*f) + (4*a*d*(6*d*e - 7*c*f))/c)*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])
/x^3 + (-(((48*b^3*c^3*e + a^2*b*c*d*(40*d*e - 49*c*f) - 8*a*b^2*c^2*(5*d*
e - 7*c*f) - 8*a^3*d^2*(6*d*e - 7*c*f))*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])/(
a*c*x)) + (b*d*(-((Sqrt[a]*(48*b^3*c^3*e + a^2*b*c*d*(40*d*e - 49*c*f) - 8
*a*b^2*c^2*(5*d*e - 7*c*f) - 8*a^3*d^2*(6*d*e - 7*c*f))*Sqrt[1 - (b*x^2)/a
]*Sqrt[c + d*x^2]*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/
(Sqrt[b]*d*Sqrt[a - b*x^2]*Sqrt[1 + (d*x^2)/c])) + (Sqrt[a]*c*(48*b^3*c^3*
e + a^2*b*c*d*(17*d*e - 21*c*f) - 8*a*b^2*c^2*(2*d*e - 7*c*f) - 4*a^3*d...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 448
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[e   Int[(g*x)^m*(a + b*x
^2)^p*(c + d*x^2)^q*(e + f*x^2)^(r - 1), x], x] + Simp[f/e^2   Int[(g*x)^(m
 + 2)*(a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^(r - 1), x], x] /; FreeQ[{a,
b, c, d, e, f, g, m, p, q}, x] && IGtQ[r, 0]
```

Maple [A] (verified)

Time = 12.56 (sec) , antiderivative size = 772, normalized size of antiderivative = 1.20

method	result
elliptic	$\frac{\sqrt{(-b\,x^2+a)(x^2d+c)}\left(-\frac{e^2\sqrt{-bd\,x^4+ad\,x^2-x^2bc+ac}}{7ac\,x^7}-\frac{2e(7acf-3ade+3bce)\sqrt{-bd\,x^4+ad\,x^2-x^2bc+ac}}{35a^2c^2x^5}-\frac{(35a^2c^2f^2-56a^2cdef+24a^2d^2e^2}{}\right.}{}$
risch	Expression too large to display
default	Expression too large to display

input
```
int((f*x^2+e)^2/x^8/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBO
SE)
```

output
```
((-b*x^2+a)*(d*x^2+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-1/7*e^2/a/
c*(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)/x^7-2/35*e*(7*a*c*f-3*a*d*e+3*b*c*e
)/a^2/c^2*(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)/x^5-1/105/a^3/c^3*(35*a^2*c
^2*f^2-56*a^2*c*d*e*f+24*a^2*d^2*e^2+56*a*b*c^2*e*f-23*a*b*c*d*e^2+24*b^2*
c^2*e^2)*(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)/x^3+2/105*(35*a^3*c^2*d*f^2-
56*a^3*c*d^2*e*f+24*a^3*d^3*e^2-35*a^2*b*c^3*f^2+49*a^2*b*c^2*d*e*f-20*a^2
*b*c*d^2*e^2-56*a*b^2*c^3*e*f+20*a*b^2*c^2*d*e^2-24*b^3*c^3*e^2)/a^4/c^4*(
-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)/x+1/105*(35*a^2*c^2*f^2-56*a^2*c*d*e*f
+24*a^2*d^2*e^2+56*a*b*c^2*e*f-23*a*b*c*d*e^2+24*b^2*c^2*e^2)*b*d/c^3/a^3/
(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+
a*c)^(1/2)*EllipticF(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2))-2/105*(35*a^3
*c^2*d*f^2-56*a^3*c*d^2*e*f+24*a^3*d^3*e^2-35*a^2*b*c^3*f^2+49*a^2*b*c^2*d
*e*f-20*a^2*b*c*d^2*e^2-56*a*b^2*c^3*e*f+20*a*b^2*c^2*d*e^2-24*b^3*c^3*e^2
)*b/a^4/c^3/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*
x^2-b*c*x^2+a*c)^(1/2)*(EllipticF(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2))-
EllipticE(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 602, normalized size of antiderivative = 0.93

$$\int \frac{(e+fx^2)^2}{x^8\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx = -\frac{2\left(4\left(6b^4c^3-5ab^3c^2d+5a^2b^2cd^2-6a^3bd^3\right)e^2+7\left(8ab^3c^3-7a^2b^2c^2d+8a^3bcd^2\right)ef+35\left(a^2b^2c^3-a\right.\right.}{}$$

input
```
integrate((f*x^2+e)^2/x^8/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="f
ricas")
```

output
```
-1/105*(2*(4*(6*b^4*c^3 - 5*a*b^3*c^2*d + 5*a^2*b^2*c*d^2 - 6*a^3*b*d^3)*e
^2 + 7*(8*a*b^3*c^3 - 7*a^2*b^2*c^2*d + 8*a^3*b*c*d^2)*e*f + 35*(a^2*b^2*c
^3 - a^3*b*c^2*d)*f^2)*sqrt(a*c)*x^7*sqrt(b/a)*elliptic_e(arcsin(x*sqrt(b/
a)), -a*d/(b*c)) - ((48*b^4*c^3 + 8*(3*a^2*b^2 - 5*a*b^3)*c^2*d - (23*a^3*
b - 40*a^2*b^2)*c*d^2 + 24*(a^4 - 2*a^3*b)*d^3)*e^2 + 14*(8*a*b^3*c^3 + (4
*a^3*b - 7*a^2*b^2)*c^2*d - 4*(a^4 - 2*a^3*b)*c*d^2)*e*f + 35*(2*a^2*b^2*c
^3 + (a^4 - 2*a^3*b)*c^2*d)*f^2)*sqrt(a*c)*x^7*sqrt(b/a)*elliptic_f(arcsin
(x*sqrt(b/a)), -a*d/(b*c)) + (15*a^4*c^3*e^2 + 2*(4*(6*a*b^3*c^3 - 5*a^2*b
^2*c^2*d + 5*a^3*b*c*d^2 - 6*a^4*d^3)*e^2 + 7*(8*a^2*b^2*c^3 - 7*a^3*b*c^2
*d + 8*a^4*c*d^2)*e*f + 35*(a^3*b*c^3 - a^4*c^2*d)*f^2)*x^6 + (35*a^4*c^3*
f^2 + (24*a^2*b^2*c^3 - 23*a^3*b*c^2*d + 24*a^4*c*d^2)*e^2 + 56*(a^3*b*c^3
 - a^4*c^2*d)*e*f)*x^4 + 6*(7*a^4*c^3*e*f + 3*(a^3*b*c^3 - a^4*c^2*d)*e^2)
*x^2)*sqrt(-b*x^2 + a)*sqrt(d*x^2 + c))/(a^5*c^4*x^7)
```

Sympy [F]

$$\int \frac{(e+fx^2)^2}{x^8\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{(e+fx^2)^2}{x^8\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx$$

input
```
integrate((f*x**2+e)**2/x**8/(-b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral((e + f*x**2)**2/(x**8*sqrt(a - b*x**2)*sqrt(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{(e+fx^2)^2}{x^8\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx=\int \frac{(fx^2+e)^2}{\sqrt{-bx^2+a}\sqrt{dx^2+c}x^8}\,dx$$

input
```
integrate((f*x^2+e)^2/x^8/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="m
axima")
```

output
```
integrate((f*x^2 + e)^2/(sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)*x^8), x)
```

Giac [F]

$$\int \frac{(e+fx^2)^2}{x^8\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx=\int \frac{(fx^2+e)^2}{\sqrt{-bx^2+a}\sqrt{dx^2+c}x^8}\,dx$$

input
```
integrate((f*x^2+e)^2/x^8/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="g
iac")
```

output
```
integrate((f*x^2 + e)^2/(sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)*x^8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx^2)^2}{x^8\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx=\int \frac{(f\,x^2+e)^2}{x^8\,\sqrt{a-b\,x^2}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int((e + f*x^2)^2/(x^8*(a - b*x^2)^(1/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int((e + f*x^2)^2/(x^8*(a - b*x^2)^(1/2)*(c + d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{(e+fx^2)^2}{x^8\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx = \text{Too large to display}$$

input
```
int((f*x^2+e)^2/x^8/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

output
```
( - sqrt(c + d*x**2)*sqrt(a - b*x**2)*e**2 + 14*int((sqrt(c + d*x**2)*sqrt
(a - b*x**2))/(a**2*c*d*x**6 + a**2*d**2*x**8 - a*b*c**2*x**6 - 2*a*b*c*d*
x**8 - a*b*d**2*x**10 + b**2*c**2*x**8 + b**2*c*d*x**10),x)*a**2*c*d*e*f*x
**7 - 6*int((sqrt(c + d*x**2)*sqrt(a - b*x**2))/(a**2*c*d*x**6 + a**2*d**2
*x**8 - a*b*c**2*x**6 - 2*a*b*c*d*x**8 - a*b*d**2*x**10 + b**2*c**2*x**8 +
 b**2*c*d*x**10),x)*a**2*d**2*e**2*x**7 - 14*int((sqrt(c + d*x**2)*sqrt(a
- b*x**2))/(a**2*c*d*x**6 + a**2*d**2*x**8 - a*b*c**2*x**6 - 2*a*b*c*d*x**
8 - a*b*d**2*x**10 + b**2*c**2*x**8 + b**2*c*d*x**10),x)*a*b*c**2*e*f*x**7
 + 12*int((sqrt(c + d*x**2)*sqrt(a - b*x**2))/(a**2*c*d*x**6 + a**2*d**2*x
**8 - a*b*c**2*x**6 - 2*a*b*c*d*x**8 - a*b*d**2*x**10 + b**2*c**2*x**8 + b
**2*c*d*x**10),x)*a*b*c*d*e**2*x**7 - 6*int((sqrt(c + d*x**2)*sqrt(a - b*x
**2))/(a**2*c*d*x**6 + a**2*d**2*x**8 - a*b*c**2*x**6 - 2*a*b*c*d*x**8 - a
*b*d**2*x**10 + b**2*c**2*x**8 + b**2*c*d*x**10),x)*b**2*c**2*e**2*x**7 +
7*int((sqrt(c + d*x**2)*sqrt(a - b*x**2))/(a**2*c*d*x**4 + a**2*d**2*x**6
- a*b*c**2*x**4 - 2*a*b*c*d*x**6 - a*b*d**2*x**8 + b**2*c**2*x**6 + b**2*c
*d*x**8),x)*a**2*c*d*f**2*x**7 - 7*int((sqrt(c + d*x**2)*sqrt(a - b*x**2))
/(a**2*c*d*x**4 + a**2*d**2*x**6 - a*b*c**2*x**4 - 2*a*b*c*d*x**6 - a*b*d*
*2*x**8 + b**2*c**2*x**6 + b**2*c*d*x**8),x)*a*b*c**2*f**2*x**7 + 5*int((s
qrt(c + d*x**2)*sqrt(a - b*x**2))/(a**2*c*d*x**4 + a**2*d**2*x**6 - a*b*c*
*2*x**4 - 2*a*b*c*d*x**6 - a*b*d**2*x**8 + b**2*c**2*x**6 + b**2*c*d*x*...
```

3.211 $\int \frac{x^6(e+fx^2)}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 468

$$\int \frac{x^6(e+fx^2)}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx$$
$$= -\frac{(25abcdf-4(bc-ad)(7bde-6bcf+6adf))x\sqrt{a-bx^2}\sqrt{c+dx^2}}{105b^3d^3}$$
$$-\frac{(7bde-6bcf+6adf)x^3\sqrt{a-bx^2}\sqrt{c+dx^2}}{35b^2d^2}-\frac{fx^5\sqrt{a-bx^2}\sqrt{c+dx^2}}{7bd}$$
$$+\frac{\sqrt{a}(48a^3d^3f-ab^2cd(49de-40cf)+8b^3c^2(7de-6cf)+8a^2bd^2(7de-5cf))\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(a\right.}{105b^{7/2}d^4\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}}$$
$$-\frac{\sqrt{a}c(24a^3d^3f+a^2bd^2(28de-17cf)-ab^2cd(21de-16cf)+8b^3c^2(7de-6cf))\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\mathrm{E}}{105b^{7/2}d^4\sqrt{a-bx^2}\sqrt{c+dx^2}}$$

output
```
-1/105*(25*a*b*c*d*f-4*(-a*d+b*c)*(6*a*d*f-6*b*c*f+7*b*d*e))*x*(-b*x^2+a)^
(1/2)*(d*x^2+c)^(1/2)/b^3/d^3-1/35*(6*a*d*f-6*b*c*f+7*b*d*e)*x^3*(-b*x^2+a
)^(1/2)*(d*x^2+c)^(1/2)/b^2/d^2-1/7*f*x^5*(-b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)
/b/d+1/105*a^(1/2)*(48*a^3*d^3*f-a*b^2*c*d*(-40*c*f+49*d*e)+8*b^3*c^2*(-6*
c*f+7*d*e)+8*a^2*b*d^2*(-5*c*f+7*d*e))*(1-b*x^2/a)^(1/2)*(d*x^2+c)^(1/2)*E
llipticE(b^(1/2)*x/a^(1/2),(-a*d/b/c)^(1/2))/b^(7/2)/d^4/(-b*x^2+a)^(1/2)/
(1+d*x^2/c)^(1/2)-1/105*a^(1/2)*c*(24*a^3*d^3*f+a^2*b*d^2*(-17*c*f+28*d*e)
-a*b^2*c*d*(-16*c*f+21*d*e)+8*b^3*c^2*(-6*c*f+7*d*e))*(1-b*x^2/a)^(1/2)*(1
+d*x^2/c)^(1/2)*EllipticF(b^(1/2)*x/a^(1/2),(-a*d/b/c)^(1/2))/b^(7/2)/d^4/
(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 8.94 (sec) , antiderivative size = 391, normalized size of antiderivative = 0.84

$$\int \frac{x^6(e+fx^2)}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx$$
$$= \frac{\sqrt{-\frac{b}{a}}dx(-a+bx^2)\left(c+dx^2\right)\left(24a^2d^2f+abd(28de-23cf+18dfx^2)+b^2(24c^2f+3d^2x^2(7e+5fx^2)-\right.}{}$$

input
```
Integrate[(x^6*(e + f*x^2))/(Sqrt[a - b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
(Sqrt[-(b/a)]*d*x*(-a + b*x^2)*(c + d*x^2)*(24*a^2*d^2*f + a*b*d*(28*d*e -
 23*c*f + 18*d*f*x^2) + b^2*(24*c^2*f + 3*d^2*x^2*(7*e + 5*f*x^2) - 2*c*d*
(14*e + 9*f*x^2))) - I*c*(48*a^3*d^3*f + 8*a^2*b*d^2*(7*d*e - 5*c*f) - 8*b
^3*c^2*(-7*d*e + 6*c*f) + a*b^2*c*d*(-49*d*e + 40*c*f))*Sqrt[1 - (b*x^2)/a
]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))]
 + I*c*(24*a^3*d^3*f + a^2*b*d^2*(28*d*e - 17*c*f) - 8*b^3*c^2*(-7*d*e + 6
*c*f) + a*b^2*c*d*(-21*d*e + 16*c*f))*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)
/c]*EllipticF[I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))])/(105*b^3*Sqrt[-(
b/a)]*d^4*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 494, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {444, 444, 444, 399, 323, 323, 321, 331, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6\left(e+fx^2\right)}{\sqrt{a-bx^2}\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \text{444}$$

$$\frac{\int \frac{x^4\left((7bde-6bcf+6adf)x^2+5acf\right)}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{7bd}-\frac{fx^5\sqrt{a-bx^2}\sqrt{c+dx^2}}{7bd}$$

$$\Big\downarrow \text{444}$$

$$\frac{\frac{\int \frac{x^2\left(\left(-4c(7de-6cf)b^2+ad(28de-23cf)b+24a^2d^2f\right)x^2+3ac(7bde-6bcf+6adf)\right)}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{5bd}-\frac{x^3\sqrt{a-bx^2}\sqrt{c+dx^2}(6adf-6bcf+7bde)}{5bd}}{7bd}-\frac{fx^5\sqrt{a-bx^2}\sqrt{c+dx^2}}{7bd}$$

$$\Big\downarrow \text{444}$$

$$\frac{\frac{\frac{\int \frac{\left(8c^2(7de-6cf)b^3-acd(49de-40cf)b^2+8a^2d^2(7de-5cf)b+48a^3d^3f\right)x^2+ac\left(-4c(7de-6cf)b^2+ad(28de-23cf)b+24a^2d^2f\right)}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{3bd}-\frac{1}{3}x\sqrt{a-bx^2}\sqrt{c+dx^2}\left(\frac{24a^2}{b}\cdots\right.}{5bd}\cdots}{7bd}-\frac{fx^5\sqrt{a-bx^2}\sqrt{c+dx^2}}{7bd}$$

$$\Big\downarrow \text{399}$$

$$\frac{\frac{\frac{\left(48a^3d^3f+8a^2bd^2(7de-5cf)-ab^2cd(49de-40cf)+8b^3c^2(7de-6cf)\right)\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}-\frac{c\left(24a^3d^3f+a^2bd^2(28de-17cf)-ab^2cd(21de-16cf)+8b^3c^2(7de-6cf)\right)\int \cdots}{d}}{3bd}\cdots}{5bd}\cdots}{7bd}-\frac{fx^5\sqrt{a-bx^2}\sqrt{c+dx^2}}{7bd}$$

$$\Big\downarrow \text{323}$$

$$\frac{\frac{\frac{\left(48a^3d^3f+8a^2bd^2(7de-5cf)-ab^2cd(49de-40cf)+8b^3c^2(7de-6cf)\right)\int\frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}-\frac{c\sqrt{\frac{dx^2}{c}+1}\left(24a^3d^3f+a^2bd^2(28de-17cf)-ab^2cd(21de-16cf)+8b^3c^2(7de-\right.}{d\sqrt{c+dx^2}}}{3bd}}{5bd}}{7bd}$$

$$\frac{fx^5\sqrt{a-bx^2}\sqrt{c+dx^2}}{7bd}$$

$\downarrow$ 323

$$\frac{\frac{\frac{\left(48a^3d^3f+8a^2bd^2(7de-5cf)-ab^2cd(49de-40cf)+8b^3c^2(7de-6cf)\right)\int\frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}-\frac{c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\left(24a^3d^3f+a^2bd^2(28de-17cf)-ab^2cd(21de-16cf)+8b\right.}{d\sqrt{a-bx^2}\sqrt{c+dx^2}}}{3bd}}{5bd}}{7b}$$

$$\frac{fx^5\sqrt{a-bx^2}\sqrt{c+dx^2}}{7bd}$$

$\downarrow$ 321

$$\frac{\frac{\left(48a^3d^3f+8a^2bd^2(7de-5cf)-ab^2cd(49de-40cf)+8b^3c^2(7de-6cf)\right)\int\frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}-\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\left(24a^3d^3f+a^2bd^2(28de-17cf)-ab^2cd(21de-16cf)\right.}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}}{3bd}$$

$$5bd$$

$$\frac{fx^5\sqrt{a-bx^2}\sqrt{c+dx^2}}{7bd}$$

$\downarrow$ 331

$$\frac{\frac{\sqrt{1-\frac{bx^2}{a}}\left(48a^3d^3f+8a^2bd^2(7de-5cf)-ab^2cd(49de-40cf)+8b^3c^2(7de-6cf)\right)\int\frac{\sqrt{dx^2+c}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\left(24a^3d^3f+a^2bd^2(28de-17cf)-ab^2cd(21\right.}{\sqrt{b}d\sqrt{a-bx^2}}}{3bd}$$

$$5bd$$

$$\frac{fx^5\sqrt{a-bx^2}\sqrt{c+dx^2}}{7bd}$$

$\downarrow$ 330

$$\frac{\frac{\frac{\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\left(48a^3d^3f+8a^2bd^2(7de-5cf)-ab^2cd(49de-40cf)+8b^3c^2(7de-6cf)\right)\int\frac{\sqrt{\frac{dx^2}{c}+1}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}-\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\left(24a^3d^3f+a^2bd^2(28de-17cf)-\ldots\right.}{\sqrt{b}d\ldots}}{3bd}}{5bd}$$

$$\frac{fx^5\sqrt{a-bx^2}\sqrt{c+dx^2}}{7bd}$$

$$\Big\downarrow \ 327$$

$$\frac{\frac{\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\left(48a^3d^3f+8a^2bd^2(7de-5cf)-ab^2cd(49de-40cf)+8b^3c^2(7de-6cf)\right)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}-\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\left(24a^3d^3f+a^2bd^2(2\ldots\right.}{\ldots}}{3bd}}{5bd}$$

$$\frac{fx^5\sqrt{a-bx^2}\sqrt{c+dx^2}}{7bd}$$

input
```
Int[(x^6*(e + f*x^2))/(Sqrt[a - b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
-1/7*(f*x^5*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])/(b*d) + (-1/5*((7*b*d*e - 6*b
*c*f + 6*a*d*f)*x^3*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])/(b*d) + (-1/3*(((24*a
^2*d*f)/b + a*(28*d*e - 23*c*f) - (4*b*c*(7*d*e - 6*c*f))/d)*x*Sqrt[a - b*
x^2]*Sqrt[c + d*x^2]) + ((Sqrt[a]*(48*a^3*d^3*f - a*b^2*c*d*(49*d*e - 40*c
*f) + 8*b^3*c^2*(7*d*e - 6*c*f) + 8*a^2*b*d^2*(7*d*e - 5*c*f))*Sqrt[1 - (b
*x^2)/a]*Sqrt[c + d*x^2]*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b
*c))])/(Sqrt[b]*d*Sqrt[a - b*x^2]*Sqrt[1 + (d*x^2)/c]) - (Sqrt[a]*c*(24*a^
3*d^3*f + a^2*b*d^2*(28*d*e - 17*c*f) - a*b^2*c*d*(21*d*e - 16*c*f) + 8*b^
3*c^2*(7*d*e - 6*c*f))*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[A
rcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*d*Sqrt[a - b*x^2]*Sq
rt[c + d*x^2]))/(3*b*d))/(5*b*d))/(7*b*d)
```

Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

Maple [A] (verified)

Time = 10.77 (sec) , antiderivative size = 590, normalized size of antiderivative = 1.26

method	result
elliptic	$\frac{\sqrt{(-b x^2+a)(x^2 d+c)}\left(-\frac{f x^5\sqrt{-bd x^4+ad x^2-x^2bc+ac}}{7bd}-\frac{\left(e+\frac{f(6ad-6bc)}{7bd}\right)x^3\sqrt{-bd x^4+ad x^2-x^2bc+ac}}{5bd}-\frac{\left(\frac{5acf}{7bd}+\frac{\left(e+\frac{f(6ad-6bc)}{7bd}\right)(4ac}{5bd}\right.}{}\right.}{}$
risch	$-\frac{x(15f x^4b^2d^2+18ab d^2f x^2-18b^2cf x^2d+21b^2d^2e x^2+24f d^2a^2-23f dcba+28ab d^2e+24f c^2b^2-28d b^2ce)\sqrt{-b x^2+a}\,\sqrt{x^2d+c}}{105b^3d^3}+$
default	Expression too large to display

input
```
int(x^6*(f*x^2+e)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE
)
```

output
```
((-b*x^2+a)*(d*x^2+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-1/7/b/d*f*
x^5*(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)-1/5*(e+1/7/b/d*f*(6*a*d-6*b*c))/b
/d*x^3*(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)-1/3*(5/7*a/b*c/d*f+1/5*(e+1/7/
b/d*f*(6*a*d-6*b*c))/b/d*(4*a*d-4*b*c))/b/d*x*(-b*d*x^4+a*d*x^2-b*c*x^2+a*
c)^(1/2)+1/3*(5/7*a/b*c/d*f+1/5*(e+1/7/b/d*f*(6*a*d-6*b*c))/b/d*(4*a*d-4*b
*c))/b/d*a*c/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d
*x^2-b*c*x^2+a*c)^(1/2)*EllipticF(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2))-
(3/5*(e+1/7/b/d*f*(6*a*d-6*b*c))/b/d*a*c+1/3*(5/7*a/b*c/d*f+1/5*(e+1/7/b/d
*f*(6*a*d-6*b*c))/b/d*(4*a*d-4*b*c))/b/d*(2*a*d-2*b*c))*c/(b/a)^(1/2)*(1-b
*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)/d*(El
lipticF(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2))-EllipticE(x*(b/a)^(1/2),(-
1-(a*d-b*c)/c/b)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 497, normalized size of antiderivative = 1.06

$$\int \frac{x^6(e+fx^2)}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx =$$

$$-\frac{\sqrt{-bd}(7\,(8\,ab^3c^2d-7\,a^2b^2cd^2+8\,a^3bd^3)e-8\,(6\,ab^3c^3-5\,a^2b^2c^2d+5\,a^3bcd^2-6\,a^4d^3)f)x\sqrt{\frac{a}{b}}E(\text{arcsi}}{}$$

input
```
integrate(x^6*(f*x^2+e)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fri
cas")
```

output
```
-1/105*(sqrt(-b*d)*(7*(8*a*b^3*c^2*d - 7*a^2*b^2*c*d^2 + 8*a^3*b*d^3)*e -
8*(6*a*b^3*c^3 - 5*a^2*b^2*c^2*d + 5*a^3*b*c*d^2 - 6*a^4*d^3)*f)*x*sqrt(a/
b)*elliptic_e(arcsin(sqrt(a/b)/x), -b*c/(a*d)) - sqrt(-b*d)*(7*(8*a^3*b*d^
3 + 4*(2*a*b^3 - b^4)*c^2*d - (7*a^2*b^2 - 4*a*b^3)*c*d^2)*e + (48*a^4*d^3
 - 24*(2*a*b^3 - b^4)*c^3 + (40*a^2*b^2 - 23*a*b^3)*c^2*d - 8*(5*a^3*b - 3
*a^2*b^2)*c*d^2)*f)*x*sqrt(a/b)*elliptic_f(arcsin(sqrt(a/b)/x), -b*c/(a*d)
) + (15*b^4*d^3*f*x^6 + 3*(7*b^4*d^3*e - 6*(b^4*c*d^2 - a*b^3*d^3)*f)*x^4
- (28*(b^4*c*d^2 - a*b^3*d^3)*e - (24*b^4*c^2*d - 23*a*b^3*c*d^2 + 24*a^2*
b^2*d^3)*f)*x^2 + 7*(8*b^4*c^2*d - 7*a*b^3*c*d^2 + 8*a^2*b^2*d^3)*e - 8*(6
*b^4*c^3 - 5*a*b^3*c^2*d + 5*a^2*b^2*c*d^2 - 6*a^3*b*d^3)*f)*sqrt(-b*x^2 +
 a)*sqrt(d*x^2 + c))/(b^5*d^4*x)
```

Sympy [F]

$$\int \frac{x^6(e+fx^2)}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{x^6(e+fx^2)}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx$$

input
```
integrate(x**6*(f*x**2+e)/(-b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(x**6*(e + f*x**2)/(sqrt(a - b*x**2)*sqrt(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{x^6(e+fx^2)}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{(fx^2+e)x^6}{\sqrt{-bx^2+a}\sqrt{dx^2+c}}\,dx$$

input
```
integrate(x^6*(f*x^2+e)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="max
ima")
```

output
```
integrate((f*x^2 + e)*x^6/(sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

Giac [F]

$$\int \frac{x^6(e+fx^2)}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{(fx^2+e)x^6}{\sqrt{-bx^2+a}\sqrt{dx^2+c}}\,dx$$

input
```
integrate(x^6*(f*x^2+e)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="gia
c")
```

output
```
integrate((f*x^2 + e)*x^6/(sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6(e+fx^2)}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{x^6\,(f\,x^2+e)}{\sqrt{a-b\,x^2}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int((x^6*(e + f*x^2))/((a - b*x^2)^(1/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int((x^6*(e + f*x^2))/((a - b*x^2)^(1/2)*(c + d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^6(e+fx^2)}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx = \text{Too large to display}$$

input
```
int(x^6*(f*x^2+e)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

output

```
( - 24*sqrt(c + d*x**2)*sqrt(a - b*x**2)*a**2*d**2*f*x + 23*sqrt(c + d*x**
2)*sqrt(a - b*x**2)*a*b*c*d*f*x - 28*sqrt(c + d*x**2)*sqrt(a - b*x**2)*a*b
*d**2*e*x - 18*sqrt(c + d*x**2)*sqrt(a - b*x**2)*a*b*d**2*f*x**3 - 24*sqrt
(c + d*x**2)*sqrt(a - b*x**2)*b**2*c**2*f*x + 28*sqrt(c + d*x**2)*sqrt(a -
 b*x**2)*b**2*c*d*e*x + 18*sqrt(c + d*x**2)*sqrt(a - b*x**2)*b**2*c*d*f*x*
*3 - 21*sqrt(c + d*x**2)*sqrt(a - b*x**2)*b**2*d**2*e*x**3 - 15*sqrt(c + d
*x**2)*sqrt(a - b*x**2)*b**2*d**2*f*x**5 + 48*int((sqrt(c + d*x**2)*sqrt(a
 - b*x**2)*x**2)/(a*c + a*d*x**2 - b*c*x**2 - b*d*x**4),x)*a**3*d**3*f - 4
0*int((sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**2)/(a*c + a*d*x**2 - b*c*x**2
- b*d*x**4),x)*a**2*b*c*d**2*f + 56*int((sqrt(c + d*x**2)*sqrt(a - b*x**2)
*x**2)/(a*c + a*d*x**2 - b*c*x**2 - b*d*x**4),x)*a**2*b*d**3*e + 40*int((s
qrt(c + d*x**2)*sqrt(a - b*x**2)*x**2)/(a*c + a*d*x**2 - b*c*x**2 - b*d*x*
*4),x)*a*b**2*c**2*d*f - 49*int((sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**2)/(
a*c + a*d*x**2 - b*c*x**2 - b*d*x**4),x)*a*b**2*c*d**2*e - 48*int((sqrt(c
+ d*x**2)*sqrt(a - b*x**2)*x**2)/(a*c + a*d*x**2 - b*c*x**2 - b*d*x**4),x)
*b**3*c**3*f + 56*int((sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**2)/(a*c + a*d*
x**2 - b*c*x**2 - b*d*x**4),x)*b**3*c**2*d*e + 24*int((sqrt(c + d*x**2)*sq
rt(a - b*x**2))/(a*c + a*d*x**2 - b*c*x**2 - b*d*x**4),x)*a**3*c*d**2*f -
23*int((sqrt(c + d*x**2)*sqrt(a - b*x**2))/(a*c + a*d*x**2 - b*c*x**2 - b*
d*x**4),x)*a**2*b*c**2*d*f + 28*int((sqrt(c + d*x**2)*sqrt(a - b*x**2))...
```

3.212 $\int \frac{x^4(e+fx^2)}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 347

$$\int \frac{x^4(e+fx^2)}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx = -\frac{(5bde-4bcf+4adf)x\sqrt{a-bx^2}\sqrt{c+dx^2}}{15b^2d^2} - \frac{fx^3\sqrt{a-bx^2}\sqrt{c+dx^2}}{5bd} + \frac{\sqrt{a}(9abcdf-2(bc-ad)(5bde-4bcf+4adf))\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\,E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{15b^{5/2}d^3\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}} - \frac{\sqrt{a}c(4a^2d^2f-2b^2c(5de-4cf)+abd(5de-3cf))\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{15b^{5/2}d^3\sqrt{a-bx^2}\sqrt{c+dx^2}}$$

output
```
-1/15*(4*a*d*f-4*b*c*f+5*b*d*e)*x*(-b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b^2/d^2
-1/5*f*x^3*(-b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b/d+1/15*a^(1/2)*(9*a*b*c*d*f-
2*(-a*d+b*c)*(4*a*d*f-4*b*c*f+5*b*d*e))*(1-b*x^2/a)^(1/2)*(d*x^2+c)^(1/2)*
EllipticE(b^(1/2)*x/a^(1/2),(-a*d/b/c)^(1/2))/b^(5/2)/d^3/(-b*x^2+a)^(1/2)
/(1+d*x^2/c)^(1/2)-1/15*a^(1/2)*c*(4*a^2*d^2*f-2*b^2*c*(-4*c*f+5*d*e)+a*b*
d*(-3*c*f+5*d*e))*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(b^(1/2)*x/
a^(1/2),(-a*d/b/c)^(1/2))/b^(5/2)/d^3/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.82 (sec) , antiderivative size = 297, normalized size of antiderivative = 0.86

$$\int \frac{x^4(e+fx^2)}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx$$
$$= \frac{-\sqrt{-\frac{b}{a}}dx(a-bx^2)(c+dx^2)(4adf+b(5de-4cf+3dfx^2))-ic(8a^2d^2f+abd(10de-7cf)+2b^2c(-5}{}$$

input
```
Integrate[(x^4*(e + f*x^2))/(Sqrt[a - b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
(-(Sqrt[-(b/a)]*d*x*(a - b*x^2)*(c + d*x^2)*(4*a*d*f + b*(5*d*e - 4*c*f + 
3*d*f*x^2))) - I*c*(8*a^2*d^2*f + a*b*d*(10*d*e - 7*c*f) + 2*b^2*c*(-5*d*e
 + 4*c*f))*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqr
t[-(b/a)]*x], -((a*d)/(b*c))] + I*c*(4*a^2*d^2*f + a*b*d*(5*d*e - 3*c*f) +
 2*b^2*c*(-5*d*e + 4*c*f))*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Ellipti
cF[I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))])/(15*b^2*Sqrt[-(b/a)]*d^3*Sq
rt[a - b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.265$, Rules used = {444, 444, 399, 323, 323, 321, 331, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(e+fx^2)}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx$$
$$\Big\downarrow \text{444}$$
$$\frac{\int \frac{x^2((5bde-4bcf+4adf)x^2+3acf)}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{5bd}-\frac{fx^3\sqrt{a-bx^2}\sqrt{c+dx^2}}{5bd}$$

$\Big\downarrow$ 444

$$\frac{\frac{\int \frac{\left(-2c(5de-4cf)b^2+ad(10de-7cf)b+8a^2d^2f\right)x^2+ac(5bde-4bcf+4adf)}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{3bd}-\frac{x\sqrt{a-bx^2}\sqrt{c+dx^2}(4adf-4bcf+5bde)}{3bd}}{5bd}-\frac{fx^3\sqrt{a-bx^2}\sqrt{c+dx^2}}{5bd}$$

$\Big\downarrow$ 399

$$\frac{\frac{\frac{\left(8a^2d^2f+abd(10de-7cf)-2b^2c(5de-4cf)\right)\int\frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}-\frac{c\left(4a^2d^2f+abd(5de-3cf)-2b^2c(5de-4cf)\right)\int\frac{1}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{d}}{3bd}-\frac{x\sqrt{a-bx^2}\sqrt{c+dx^2}(4adf-}{3bd}}{5bd}$$
$$\frac{fx^3\sqrt{a-bx^2}\sqrt{c+dx^2}}{5bd}$$

$\Big\downarrow$ 323

$$\frac{\frac{\frac{\left(8a^2d^2f+abd(10de-7cf)-2b^2c(5de-4cf)\right)\int\frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}-\frac{c\sqrt{\frac{dx^2}{c}+1}\left(4a^2d^2f+abd(5de-3cf)-2b^2c(5de-4cf)\right)\int\frac{1}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}dx}{d\sqrt{c+dx^2}}}{3bd}-x\sqrt{a-bx^2}\sqrt{c+d}}{5bd}$$
$$\frac{fx^3\sqrt{a-bx^2}\sqrt{c+dx^2}}{5bd}$$

$\Big\downarrow$ 323

$$\frac{\frac{\frac{\left(8a^2d^2f+abd(10de-7cf)-2b^2c(5de-4cf)\right)\int\frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}-\frac{c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\left(4a^2d^2f+abd(5de-3cf)-2b^2c(5de-4cf)\right)\int\frac{1}{\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}}dx}{d\sqrt{a-bx^2}\sqrt{c+dx^2}}}{3bd}-x\sqrt{a-b}}{5bd}$$
$$\frac{fx^3\sqrt{a-bx^2}\sqrt{c+dx^2}}{5bd}$$

$\Big\downarrow$ 321

$$\frac{\frac{\frac{\left(8a^2d^2f+abd(10de-7cf)-2b^2c(5de-4cf)\right)\int\frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}-\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\left(4a^2d^2f+abd(5de-3cf)-2b^2c(5de-4cf)\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}}{3bd}}{5bd}$$
$$\frac{fx^3\sqrt{a-bx^2}\sqrt{c+dx^2}}{5bd}$$

$\Big\downarrow$ 331

$$\frac{\frac{\sqrt{1-\frac{bx^2}{a}}\left(8a^2d^2f+abd(10de-7cf)-2b^2c(5de-4cf)\right)\int \frac{\sqrt{dx^2+c}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\left(4a^2d^2f+abd(5de-3cf)-2b^2c(5de-4cf)\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\right.}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}}{3bd}$$

$$5bd$$

$$\frac{fx^3\sqrt{a-bx^2}\sqrt{c+dx^2}}{5bd}$$

$\downarrow$ 330

$$\frac{\frac{\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\left(8a^2d^2f+abd(10de-7cf)-2b^2c(5de-4cf)\right)\int \frac{\sqrt{\frac{dx^2}{c}+1}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}-\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\left(4a^2d^2f+abd(5de-3cf)-2b^2c(5de-4cf)\right)\text{EllipticF}\left(\text{arcsi}\right.}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}}{3bd}$$

$$5bd$$

$$\frac{fx^3\sqrt{a-bx^2}\sqrt{c+dx^2}}{5bd}$$

$\downarrow$ 327

$$\frac{\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\left(8a^2d^2f+abd(10de-7cf)-2b^2c(5de-4cf)\right)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}-\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\left(4a^2d^2f+abd(5de-3cf)-2b^2c(5de-4cf)\right)\text{El}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}}{3bd}$$

$$5bd$$

$$\frac{fx^3\sqrt{a-bx^2}\sqrt{c+dx^2}}{5bd}$$

input
```
Int[(x^4*(e + f*x^2))/(Sqrt[a - b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
-1/5*(f*x^3*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])/(b*d) + (-1/3*((5*b*d*e - 4*b
*c*f + 4*a*d*f)*x*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])/(b*d) + ((Sqrt[a]*(8*a^
2*d^2*f + a*b*d*(10*d*e - 7*c*f) - 2*b^2*c*(5*d*e - 4*c*f))*Sqrt[1 - (b*x^
2)/a]*Sqrt[c + d*x^2]*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c)
)])/(Sqrt[b]*d*Sqrt[a - b*x^2]*Sqrt[1 + (d*x^2)/c]) - (Sqrt[a]*c*(4*a^2*d^
2*f - 2*b^2*c*(5*d*e - 4*c*f) + a*b*d*(5*d*e - 3*c*f))*Sqrt[1 - (b*x^2)/a]
*Sqrt[1 + (d*x^2)/c]*EllipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))
])/(Sqrt[b]*d*Sqrt[a - b*x^2]*Sqrt[c + d*x^2]))/(3*b*d))/(5*b*d)
```

Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

Maple [A] (verified)

Time = 8.60 (sec) , antiderivative size = 427, normalized size of antiderivative = 1.23

method	result
elliptic	$\frac{\sqrt{(-b\,x^2+a)(x^2d+c)}\left(-\frac{f\,x^3\sqrt{-bd\,x^4+ad\,x^2-x^2bc+ac}}{5bd}-\frac{\left(e+\frac{f(4ad-4bc)}{5bd}\right)x\sqrt{-bd\,x^4+ad\,x^2-x^2bc+ac}}{3bd}+\frac{\left(e+\frac{f(4ad-4bc)}{5bd}\right)ac\sqrt{1-\frac{b\,x^2}{a}}\sqrt{}}{3bd\sqrt{\frac{b}{a}}\sqrt{-bd}}\right.}{}$
risch	$-\frac{x\left(3bdf\,x^2+4adf-4bcf+5bde\right)\sqrt{-b\,x^2+a}\,\sqrt{x^2d+c}}{15b^2d^2}+\frac{\left(-\frac{\left(8f\,d^2a^2-7fdcba+10ab\,d^2e+8f\,c^2b^2-10d\,b^2ce\right)c\sqrt{1-\frac{b\,x^2}{a}}\sqrt{1+\frac{d\,x^2}{c}}\left(\mathrm{Ellipt}\right.}{\sqrt{\frac{b}{a}}\sqrt{-bd\,x^4+ad\,x^2-x}}\right.}{}$
default	$-\frac{\left(-3\sqrt{\frac{b}{a}}\,b^2d^3f\,x^7-\sqrt{\frac{b}{a}}\,ab\,d^3f\,x^5+\sqrt{\frac{b}{a}}\,b^2c\,d^2f\,x^5-5\sqrt{\frac{b}{a}}\,b^2d^3e\,x^5+4\sqrt{\frac{b}{a}}\,a^2d^3f\,x^3-5\sqrt{\frac{b}{a}}\,abc\,d^2f\,x^3+5\sqrt{\frac{b}{a}}\,ab\,d^3e\,x^3+4\sqrt{\frac{b}{a}}\,b^2\right.}{}$

input
```
int(x^4*(f*x^2+e)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE
)
```

output
```
((-b*x^2+a)*(d*x^2+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-1/5/b/d*f*
x^3*(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)-1/3*(e+1/5/b/d*f*(4*a*d-4*b*c))/b
/d*x*(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)+1/3*(e+1/5/b/d*f*(4*a*d-4*b*c))/
b/d*a*c/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-
b*c*x^2+a*c)^(1/2)*EllipticF(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2))-(3/5*
a/b*c/d*f+1/3*(e+1/5/b/d*f*(4*a*d-4*b*c))/b/d*(2*a*d-2*b*c))*c/(b/a)^(1/2)
*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)/
d*(EllipticF(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2))-EllipticE(x*(b/a)^(1/
2),(-1-(a*d-b*c)/c/b)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 327, normalized size of antiderivative = 0.94

$$\int \frac{x^4(e+fx^2)}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx$$
$$= \frac{\sqrt{-bd}(10\,(ab^2cd-a^2bd^2)e-(8\,ab^2c^2-7\,a^2bcd+8\,a^3d^2)f)x\sqrt{\frac{a}{b}}E(\arcsin\left(\frac{\sqrt{\frac{a}{b}}}{x}\right)|-\frac{bc}{ad})+\sqrt{-bd}(5\,(2\,a^2}{}$$

input
```
integrate(x^4*(f*x^2+e)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fri
cas")
```

output
```
1/15*(sqrt(-b*d)*(10*(a*b^2*c*d - a^2*b*d^2)*e - (8*a*b^2*c^2 - 7*a^2*b*c*
d + 8*a^3*d^2)*f)*x*sqrt(a/b)*elliptic_e(arcsin(sqrt(a/b)/x), -b*c/(a*d))
+ sqrt(-b*d)*(5*(2*a^2*b*d^2 - (2*a*b^2 - b^3)*c*d)*e + (8*a^3*d^2 + 4*(2*
a*b^2 - b^3)*c^2 - (7*a^2*b - 4*a*b^2)*c*d)*f)*x*sqrt(a/b)*elliptic_f(arcs
in(sqrt(a/b)/x), -b*c/(a*d)) - (3*b^3*d^2*f*x^4 + (5*b^3*d^2*e - 4*(b^3*c*
d - a*b^2*d^2)*f)*x^2 - 10*(b^3*c*d - a*b^2*d^2)*e + (8*b^3*c^2 - 7*a*b^2*
c*d + 8*a^2*b*d^2)*f)*sqrt(-b*x^2 + a)*sqrt(d*x^2 + c))/(b^4*d^3*x)
```

Sympy [F]

$$\int \frac{x^4(e+fx^2)}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{x^4(e+fx^2)}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx$$

input
```
integrate(x**4*(f*x**2+e)/(-b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(x**4*(e + f*x**2)/(sqrt(a - b*x**2)*sqrt(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{x^4(e+fx^2)}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{(fx^2+e)x^4}{\sqrt{-bx^2+a}\sqrt{dx^2+c}}\,dx$$

input
```
integrate(x^4*(f*x^2+e)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="max
ima")
```

output
```
integrate((f*x^2 + e)*x^4/(sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

Giac [F]

$$\int \frac{x^4(e+fx^2)}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{(fx^2+e)x^4}{\sqrt{-bx^2+a}\sqrt{dx^2+c}}\,dx$$

input
```
integrate(x^4*(f*x^2+e)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="gia
c")
```

output
```
integrate((f*x^2 + e)*x^4/(sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(e+fx^2)}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{x^4\,(f\,x^2+e)}{\sqrt{a-b\,x^2}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int((x^4*(e + f*x^2))/((a - b*x^2)^(1/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int((x^4*(e + f*x^2))/((a - b*x^2)^(1/2)*(c + d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^4(e+fx^2)}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx$$
$$= \frac{-4\sqrt{d\,x^2+c}\,\sqrt{-b\,x^2+a}\,adfx+4\sqrt{d\,x^2+c}\,\sqrt{-b\,x^2+a}\,bcfx-5\sqrt{d\,x^2+c}\,\sqrt{-b\,x^2+a}\,bdex-3\sqrt{d\,x^2}}{}$$

input
```
int(x^4*(f*x^2+e)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

output
```
( - 4*sqrt(c + d*x**2)*sqrt(a - b*x**2)*a*d*f*x + 4*sqrt(c + d*x**2)*sqrt(
a - b*x**2)*b*c*f*x - 5*sqrt(c + d*x**2)*sqrt(a - b*x**2)*b*d*e*x - 3*sqrt
(c + d*x**2)*sqrt(a - b*x**2)*b*d*f*x**3 + 8*int((sqrt(c + d*x**2)*sqrt(a
- b*x**2)*x**2)/(a*c + a*d*x**2 - b*c*x**2 - b*d*x**4),x)*a**2*d**2*f - 7*
int((sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**2)/(a*c + a*d*x**2 - b*c*x**2 -
b*d*x**4),x)*a*b*c*d*f + 10*int((sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**2)/(
a*c + a*d*x**2 - b*c*x**2 - b*d*x**4),x)*a*b*d**2*e + 8*int((sqrt(c + d*x*
*2)*sqrt(a - b*x**2)*x**2)/(a*c + a*d*x**2 - b*c*x**2 - b*d*x**4),x)*b**2*
c**2*f - 10*int((sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**2)/(a*c + a*d*x**2 -
 b*c*x**2 - b*d*x**4),x)*b**2*c*d*e + 4*int((sqrt(c + d*x**2)*sqrt(a - b*x
**2))/(a*c + a*d*x**2 - b*c*x**2 - b*d*x**4),x)*a**2*c*d*f - 4*int((sqrt(c
 + d*x**2)*sqrt(a - b*x**2))/(a*c + a*d*x**2 - b*c*x**2 - b*d*x**4),x)*a*b
*c**2*f + 5*int((sqrt(c + d*x**2)*sqrt(a - b*x**2))/(a*c + a*d*x**2 - b*c*
x**2 - b*d*x**4),x)*a*b*c*d*e)/(15*b**2*d**2)
```

3.213 $\int \frac{x^2(e+fx^2)}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 254

$$\begin{aligned}\int \frac{x^2(e+fx^2)}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx &= -\frac{fx\sqrt{a-bx^2}\sqrt{c+dx^2}}{3bd} \\ &\quad + \frac{\sqrt{a}(3bde-2bcf+2adf)\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\,E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{3b^{3/2}d^2\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}} \\ &\quad - \frac{\sqrt{a}c(3bde-2bcf+adf)\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{3b^{3/2}d^2\sqrt{a-bx^2}\sqrt{c+dx^2}}\end{aligned}$$

output

```
-1/3*f*x*(-b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b/d+1/3*a^(1/2)*(2*a*d*f-2*b*c*f
+3*b*d*e)*(1-b*x^2/a)^(1/2)*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2),(-
a*d/b/c)^(1/2))/b^(3/2)/d^2/(-b*x^2+a)^(1/2)/(1+d*x^2/c)^(1/2)-1/3*a^(1/2)
*c*(a*d*f-2*b*c*f+3*b*d*e)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(b
^(1/2)*x/a^(1/2),(-a*d/b/c)^(1/2))/b^(3/2)/d^2/(-b*x^2+a)^(1/2)/(d*x^2+c)^
(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.63 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.90

$$\int \frac{x^2(e+fx^2)}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx = \frac{-\sqrt{-\frac{b}{a}}dfx(a-bx^2)(c+dx^2)-ic(3bde-2bcf+2adf)\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(i\operatorname{arcsinh}\left(\sqrt{-\frac{b}{a}}x\right)|-\frac{ad}{bc}\right)}{3b\sqrt{-\frac{b}{a}}d^2\sqrt{a-bx^2}\sqrt{c+}}$$

input
```
Integrate[(x^2*(e + f*x^2))/(Sqrt[a - b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
(-(Sqrt[-(b/a)]*d*f*x*(a - b*x^2)*(c + d*x^2)) - I*c*(3*b*d*e - 2*b*c*f +
2*a*d*f)*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[
-(b/a)]*x], -((a*d)/(b*c))] + I*c*(3*b*d*e - 2*b*c*f + a*d*f)*Sqrt[1 - (b*
x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(
b*c))])/(3*b*Sqrt[-(b/a)]*d^2*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {444, 399, 323, 323, 321, 331, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(e+fx^2)}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx$$

$$\Big\downarrow \ 444$$

$$\frac{\int \frac{(3bde-2bcf+2adf)x^2+acf}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{3bd}-\frac{fx\sqrt{a-bx^2}\sqrt{c+dx^2}}{3bd}$$

$$\Big\downarrow \ 399$$

$$\frac{\frac{(2adf-2bcf+3bde)\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}-\frac{c(adf-2bcf+3bde)\int \frac{1}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{d}}{3bd}-\frac{fx\sqrt{a-bx^2}\sqrt{c+dx^2}}{3bd}$$

$$\Big\downarrow \text{ 323}$$

$$\frac{\frac{(2adf-2bcf+3bde)\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}-\frac{c\sqrt{\frac{dx^2}{c}+1}(adf-2bcf+3bde)\int \frac{1}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}dx}{d\sqrt{c+dx^2}}}{3bd}-\frac{fx\sqrt{a-bx^2}\sqrt{c+dx^2}}{3bd}$$

$$\Big\downarrow \text{ 323}$$

$$\frac{\frac{(2adf-2bcf+3bde)\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}-\frac{c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(adf-2bcf+3bde)\int \frac{1}{\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}}dx}{d\sqrt{a-bx^2}\sqrt{c+dx^2}}}{3bd}-\frac{fx\sqrt{a-bx^2}\sqrt{c+dx^2}}{3bd}$$

$$\Big\downarrow \text{ 321}$$

$$\frac{\frac{(2adf-2bcf+3bde)\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}-\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(adf-2bcf+3bde)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}}{3bd}-\frac{fx\sqrt{a-bx^2}\sqrt{c+dx^2}}{3bd}$$

$$\Big\downarrow \text{ 331}$$

$$\frac{\frac{\sqrt{1-\frac{bx^2}{a}}(2adf-2bcf+3bde)\int \frac{\sqrt{dx^2+c}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(adf-2bcf+3bde)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}}{3bd}-\frac{fx\sqrt{a-bx^2}\sqrt{c+dx^2}}{3bd}$$

$$\Big\downarrow \text{ 330}$$

$$\frac{\frac{\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(2adf-2bcf+3bde)\int \frac{\sqrt{\frac{dx^2}{c}+1}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}-\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(adf-2bcf+3bde)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}}{3bd}-\frac{fx\sqrt{a-bx^2}\sqrt{c+dx^2}}{3bd}$$

$$\Big\downarrow \text{ 327}$$

$$\frac{\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(2adf-2bcf+3bde)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}-\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(adf-2bcf+3bde)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}}{3bd}-\frac{fx\sqrt{a-bx^2}\sqrt{c+dx^2}}{3bd}$$

input
```
Int[(x^2*(e + f*x^2))/(Sqrt[a - b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
-1/3*(f*x*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])/(b*d) + ((Sqrt[a]*(3*b*d*e - 2*
b*c*f + 2*a*d*f)*Sqrt[1 - (b*x^2)/a]*Sqrt[c + d*x^2]*EllipticE[ArcSin[(Sqr
t[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*d*Sqrt[a - b*x^2]*Sqrt[1 + (d*
x^2)/c]) - (Sqrt[a]*c*(3*b*d*e - 2*b*c*f + a*d*f)*Sqrt[1 - (b*x^2)/a]*Sqrt
[1 + (d*x^2)/c]*EllipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(S
qrt[b]*d*Sqrt[a - b*x^2]*Sqrt[c + d*x^2]))/(3*b*d)
```

Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

Maple [A] (verified)

Time = 5.64 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.23

method	result
risch	$-\frac{fx\sqrt{-b\,x^2+a}\,\sqrt{x^2d+c}}{3bd}+\frac{\left(-\frac{(2adf-2bcf+3bde)c\sqrt{1-\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\left(\text{EllipticF}\left(x\sqrt{\frac{b}{a}},\sqrt{-1-\frac{ad-bc}{cb}}\right)-\text{EllipticE}\left(x\sqrt{\frac{b}{a}},\sqrt{-1-\frac{ad-bc}{cb}}\right)\right.}{\sqrt{\frac{b}{a}}\,\sqrt{-bd\,x^4+ad\,x^2-x^2bc+ac}\,d}}{3bd\sqrt{-b\,x^2+a}\,\sqrt{x^2d+c}}$
elliptic	$\frac{\sqrt{(-b\,x^2+a)(x^2d+c)}\left(-\frac{fx\sqrt{-bd\,x^4+ad\,x^2-x^2bc+ac}}{3bd}+\frac{acf\sqrt{1-\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\text{EllipticF}\left(x\sqrt{\frac{b}{a}},\sqrt{-1-\frac{ad-bc}{cb}}\right)}{3bd\sqrt{\frac{b}{a}}\,\sqrt{-bd\,x^4+ad\,x^2-x^2bc+ac}}-\frac{\left(e+\frac{f(2ad-2bc)}{3bd}\right)c\sqrt{1-}}{}\right.}{\sqrt{-b\,x^2+a}\,\sqrt{x^2d+c}}$
default	$-\frac{\left(-\sqrt{\frac{b}{a}}\,b\,d^2f\,x^5+\sqrt{\frac{b}{a}}\,a\,d^2f\,x^3-\sqrt{\frac{b}{a}}\,bcdf\,x^3+\sqrt{\frac{-b\,x^2+a}{a}}\,\sqrt{\frac{x^2d+c}{c}}\,\text{EllipticF}\left(x\sqrt{\frac{b}{a}},\sqrt{-\frac{ad}{bc}}\right)acdf-2\sqrt{\frac{-b\,x^2+a}{a}}\,\sqrt{\frac{x^2d+c}{c}}\,\text{Ellipti}\right.}{}$

input
```
int(x^2*(f*x^2+e)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE
)
```

output
```
-1/3*f*x*(-b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b/d+1/3/b/d*(-(2*a*d*f-2*b*c*f+3
*b*d*e)*c/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^
2-b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2))-
EllipticE(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2)))+a*c*f/(b/a)^(1/2)*(1-b*
x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)*Ellipt
icF(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2)))*((-b*x^2+a)*(d*x^2+c))^(1/2)/
(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.75

$$\int \frac{x^2(e+fx^2)}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx =$$

$$-\frac{(3\,abde-2\,(abc-a^2d)\,f)\sqrt{-bd}x\sqrt{\frac{a}{b}}E(\arcsin\left(\frac{\sqrt{\frac{a}{b}}}{x}\right)\mid -\frac{bc}{ad})-(3\,abde+(2\,a^2d-(2\,ab-b^2)c)f)\sqrt{-b}}{3\,b^3d^2x}$$

input
```
integrate(x^2*(f*x^2+e)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fri
cas")
```

output
```
-1/3*((3*a*b*d*e - 2*(a*b*c - a^2*d)*f)*sqrt(-b*d)*x*sqrt(a/b)*elliptic_e(
arcsin(sqrt(a/b)/x), -b*c/(a*d)) - (3*a*b*d*e + (2*a^2*d - (2*a*b - b^2)*c
)*f)*sqrt(-b*d)*x*sqrt(a/b)*elliptic_f(arcsin(sqrt(a/b)/x), -b*c/(a*d)) +
(b^2*d*f*x^2 + 3*b^2*d*e - 2*(b^2*c - a*b*d)*f)*sqrt(-b*x^2 + a)*sqrt(d*x^
2 + c))/(b^3*d^2*x)
```

Sympy [F]

$$\int \frac{x^2(e+fx^2)}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{x^2(e+fx^2)}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx$$

input
```
integrate(x**2*(f*x**2+e)/(-b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(x**2*(e + f*x**2)/(sqrt(a - b*x**2)*sqrt(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{x^2(e+fx^2)}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{(fx^2+e)x^2}{\sqrt{-bx^2+a}\sqrt{dx^2+c}}\,dx$$

input
```
integrate(x^2*(f*x^2+e)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="max
ima")
```

output
```
integrate((f*x^2 + e)*x^2/(sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

Giac [F]

$$\int \frac{x^2(e+fx^2)}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{(fx^2+e)x^2}{\sqrt{-bx^2+a}\sqrt{dx^2+c}}\,dx$$

input
```
integrate(x^2*(f*x^2+e)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="gia
c")
```

output
```
integrate((f*x^2 + e)*x^2/(sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(e+fx^2)}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{x^2\,(f\,x^2+e)}{\sqrt{a-b\,x^2}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int((x^2*(e + f*x^2))/((a - b*x^2)^(1/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int((x^2*(e + f*x^2))/((a - b*x^2)^(1/2)*(c + d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^2(e+fx^2)}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx = \frac{-\sqrt{d\,x^2+c}\,\sqrt{-b\,x^2+a}\,fx+2\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{-b\,x^2+a}\,x^2}{-bd\,x^4+ad\,x^2-bc\,x^2+ac}dx\right)adf-2\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{-b\,x^2+a}\,x^2}{-bd\,x^4+ad\,x^2-bc\,x^2+ac}dx\right)bcf+3\left(\int \ldots\right.}{3bd}$$

input
```
int(x^2*(f*x^2+e)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

output

```
( - sqrt(c + d*x**2)*sqrt(a - b*x**2)*f*x + 2*int((sqrt(c + d*x**2)*sqrt(a
 - b*x**2)*x**2)/(a*c + a*d*x**2 - b*c*x**2 - b*d*x**4),x)*a*d*f - 2*int((
sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**2)/(a*c + a*d*x**2 - b*c*x**2 - b*d*x
**4),x)*b*c*f + 3*int((sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**2)/(a*c + a*d*
x**2 - b*c*x**2 - b*d*x**4),x)*b*d*e + int((sqrt(c + d*x**2)*sqrt(a - b*x*
*2))/(a*c + a*d*x**2 - b*c*x**2 - b*d*x**4),x)*a*c*f)/(3*b*d)
```

3.214 $\int \frac{e+fx^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 190

$$\begin{aligned}\int \frac{e+fx^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx &= \frac{\sqrt{a}f\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}}\\&\quad+\frac{\sqrt{a}(de-cf)\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}\end{aligned}$$

output
```
a^(1/2)*f*(1-b*x^2/a)^(1/2)*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2),(-
a*d/b/c)^(1/2))/b^(1/2)/d/(-b*x^2+a)^(1/2)/(1+d*x^2/c)^(1/2)+a^(1/2)*(-c*f
+d*e)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(b^(1/2)*x/a^(1/2),(-a*
d/b/c)^(1/2))/b^(1/2)/d/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.21 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.73

$$\int \frac{e+fx^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx = -\frac{i\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\left(cfE\left(i\text{arcsinh}\left(\sqrt{-\frac{b}{a}}x\right)|-\frac{ad}{bc}\right)+(de-cf)\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{-\frac{b}{a}}x\right),-\frac{ad}{bc}\right)\right)}{\sqrt{-\frac{b}{a}}d\sqrt{a-bx^2}\sqrt{c+dx^2}}$$

input
```
Integrate[(e + f*x^2)/(Sqrt[a - b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
((-I)*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(c*f*EllipticE[I*ArcSinh[Sqr
t[-(b/a)]*x], -((a*d)/(b*c))] + (d*e - c*f)*EllipticF[I*ArcSinh[Sqrt[-(b/a
)]*x], -((a*d)/(b*c))]))/(Sqrt[-(b/a)]*d*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {399, 323, 323, 321, 331, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \text{ 399}$$

$$\frac{(de-cf)\int \frac{1}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{d}+\frac{f\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}$$

$$\Big\downarrow \text{ 323}$$

$$\frac{\sqrt{\frac{dx^2}{c}+1}(de-cf)\int \frac{1}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}dx}{d\sqrt{c+dx^2}}+\frac{f\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}$$

$$\Big\downarrow \text{323}$$

$$\frac{\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(de-cf)\int \frac{1}{\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}}dx}{d\sqrt{a-bx^2}\sqrt{c+dx^2}}+\frac{f\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}$$

$$\Big\downarrow \text{321}$$

$$\frac{f\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}+\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(de-cf)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}$$

$$\Big\downarrow \text{331}$$

$$\frac{f\sqrt{1-\frac{bx^2}{a}}\int \frac{\sqrt{dx^2+c}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}+\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(de-cf)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}$$

$$\Big\downarrow \text{330}$$

$$\frac{f\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\int \frac{\sqrt{\frac{dx^2}{c}+1}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}+\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(de-cf)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}$$

$$\Big\downarrow \text{327}$$

$$\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(de-cf)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}+\frac{\sqrt{a}f\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}$$

input
```
Int[(e + f*x^2)/(Sqrt[a - b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
(Sqrt[a]*f*Sqrt[1 - (b*x^2)/a]*Sqrt[c + d*x^2]*EllipticE[ArcSin[(Sqrt[b]*x
)/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*d*Sqrt[a - b*x^2]*Sqrt[1 + (d*x^2)/c
]) + (Sqrt[a]*(d*e - c*f)*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Elliptic
F[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*d*Sqrt[a - b*x^2]
*Sqrt[c + d*x^2])
```

Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

Maple [A] (verified)

Time = 3.75 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.85

method	result
default	$\frac{(-\mathrm{EllipticF}(x\sqrt{\frac{b}{a}},\sqrt{-\frac{ad}{bc}})cf+\mathrm{EllipticF}(x\sqrt{\frac{b}{a}},\sqrt{-\frac{ad}{bc}})de+\mathrm{EllipticE}(x\sqrt{\frac{b}{a}},\sqrt{-\frac{ad}{bc}})cf)\sqrt{\frac{x^{2}d+c}{c}}\,\sqrt{\frac{-b\,x^{2}+a}{a}}\,\sqrt{-b\,x^{2}+a}\,\sqrt{x^{2}}}{d\sqrt{\frac{b}{a}}\,(-bd\,x^{4}+ad\,x^{2}-x^{2}bc+ac)}$
elliptic	$\frac{\sqrt{(-b\,x^{2}+a)(x^{2}d+c)}\,(\frac{e\sqrt{1-\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,\mathrm{EllipticF}(x\sqrt{\frac{b}{a}},\sqrt{-1-\frac{ad-bc}{cb}})}{\sqrt{\frac{b}{a}}\,\sqrt{-bd\,x^{4}+ad\,x^{2}-x^{2}bc+ac}}-\frac{fc\sqrt{1-\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,(\mathrm{EllipticF}(x\sqrt{\frac{b}{a}},\sqrt{-1-\frac{ad-bc}{cb}})-\mathrm{Elli}}{\sqrt{\frac{b}{a}}\,\sqrt{-bd\,x^{4}+ad\,x^{2}-x^{2}bc+ac}\,d}}{\sqrt{-b\,x^{2}+a}\,\sqrt{x^{2}d+c}}$

input
```
int((f*x^2+e)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-EllipticF(x*(b/a)^(1/2),(-a*d/b/c)^(1/2))*c*f+EllipticF(x*(b/a)^(1/2),(-
a*d/b/c)^(1/2))*d*e+EllipticE(x*(b/a)^(1/2),(-a*d/b/c)^(1/2))*c*f)*((d*x^2
+c)/c)^(1/2)*((-b*x^2+a)/a)^(1/2)*(-b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/d/(b/a)
^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.68

$$\int \frac{e+fx^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx = -\frac{\sqrt{-bd}a^2fx\sqrt{\frac{a}{b}}E(\arcsin\left(\frac{\sqrt{\frac{a}{b}}}{x}\right)\mid-\frac{bc}{ad})+\sqrt{-bx^2+a}\sqrt{dx^2+c}abf-(b^2e+a^2f)\sqrt{-bd}x\sqrt{\frac{a}{b}}F(\arcsin}{ab^2dx}$$

input
```
integrate((f*x^2+e)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas"
)
```

output
```
-(sqrt(-b*d)*a^2*f*x*sqrt(a/b)*elliptic_e(arcsin(sqrt(a/b)/x), -b*c/(a*d))
 + sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)*a*b*f - (b^2*e + a^2*f)*sqrt(-b*d)*x*s
qrt(a/b)*elliptic_f(arcsin(sqrt(a/b)/x), -b*c/(a*d)))/(a*b^2*d*x)
```

Sympy [F]

$$\int \frac{e+fx^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx=\int \frac{e+fx^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx$$

input
```
integrate((f*x**2+e)/(-b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral((e + f*x**2)/(sqrt(a - b*x**2)*sqrt(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{e+fx^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx=\int \frac{fx^2+e}{\sqrt{-bx^2+a}\sqrt{dx^2+c}}\,dx$$

input
```
integrate((f*x^2+e)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima"
)
```

output
```
integrate((f*x^2 + e)/(sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

Giac [F]

$$\int \frac{e+fx^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx=\int \frac{fx^2+e}{\sqrt{-bx^2+a}\sqrt{dx^2+c}}\,dx$$

input
```
integrate((f*x^2+e)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((f*x^2 + e)/(sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{f\,x^2+e}{\sqrt{a-b\,x^2}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int((e + f*x^2)/((a - b*x^2)^(1/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int((e + f*x^2)/((a - b*x^2)^(1/2)*(c + d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{e+fx^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx = \left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{-b\,x^2+a}\,x^2}{-bd\,x^4+ad\,x^2-bc\,x^2+ac}dx\right)f + \left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{-b\,x^2+a}}{-bd\,x^4+ad\,x^2-bc\,x^2+ac}dx\right)e$$

input
```
int((f*x^2+e)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**2)/(a*c + a*d*x**2 - b*c*x**2 -
b*d*x**4),x)*f + int((sqrt(c + d*x**2)*sqrt(a - b*x**2))/(a*c + a*d*x**2 -
 b*c*x**2 - b*d*x**4),x)*e
```

3.215 $\int \frac{e+fx^2}{x^2\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 222

$$\int \frac{e+fx^2}{x^2\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx = -\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{acx} - \frac{\sqrt{b}e\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\,E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{a}c\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}} + \frac{(be+af)\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}$$

output
```
-e*(-b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a/c/x-b^(1/2)*e*(1-b*x^2/a)^(1/2)*(d*x
^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2),(-a*d/b/c)^(1/2))/a^(1/2)/c/(-b*x^
2+a)^(1/2)/(1+d*x^2/c)^(1/2)+(a*f+b*e)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)
*EllipticF(b^(1/2)*x/a^(1/2),(-a*d/b/c)^(1/2))/a^(1/2)/b^(1/2)/(-b*x^2+a)^
(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.93

$$\int \frac{e+fx^2}{x^2\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx = \frac{-\sqrt{-\frac{b}{a}}e(a-bx^2)(c+dx^2)+ibcex\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(i\text{arcsinh}\left(\sqrt{-\frac{b}{a}}x\right)|-\frac{ad}{bc}\right)-ic(be+af)x\sqrt{1-}}{a\sqrt{-\frac{b}{a}}cx\sqrt{a-bx^2}\sqrt{c+dx^2}}$$

input
```
Integrate[(e + f*x^2)/(x^2*Sqrt[a - b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
(-(Sqrt[-(b/a)]*e*(a - b*x^2)*(c + d*x^2)) + I*b*c*e*x*Sqrt[1 - (b*x^2)/a]
*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))]
- I*c*(b*e + a*f)*x*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*Ar
cSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))])/(a*Sqrt[-(b/a)]*c*x*Sqrt[a - b*x^2
]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.265$, Rules used = {445, 25, 399, 323, 323, 321, 331, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx^2}{x^2\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx$$

$$\Big\downarrow \text{445}$$

$$-\frac{\int -\frac{acf-bdex^2}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{ac}-\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{acx}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{acf-bdex^2}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{ac}-\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{acx}$$

$\Big\downarrow$ 399

$$\frac{c(af+be)\int \frac{1}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx-be\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{ac}-\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{acx}$$

$\Big\downarrow$ 323

$$\frac{\frac{c\sqrt{\frac{dx^2}{c}+1}(af+be)\int \frac{1}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}dx}{\sqrt{c+dx^2}}-be\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{ac}-\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{acx}$$

$\Big\downarrow$ 323

$$\frac{\frac{c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(af+be)\int \frac{1}{\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}}dx}{\sqrt{a-bx^2}\sqrt{c+dx^2}}-be\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{ac}-\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{acx}$$

$\Big\downarrow$ 321

$$\frac{\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(af+be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-be\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{ac}-\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{acx}$$

$\Big\downarrow$ 331

$$\frac{\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(af+be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{be\sqrt{1-\frac{bx^2}{a}}\int \frac{\sqrt{dx^2+c}}{\sqrt{1-\frac{bx^2}{a}}}dx}{\sqrt{a-bx^2}}}{ac}-\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{acx}$$

$\Big\downarrow$ 330

$$\frac{\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(af+be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{be\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\int \frac{\sqrt{\frac{dx^2}{c}+1}}{\sqrt{1-\frac{bx^2}{a}}}dx}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}}{ac}-\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{acx}$$

$\Big\downarrow$ 327

$$\frac{\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(af+be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{b}e\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}}{ac}-\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{acx}$$

input
```
Int[(e + f*x^2)/(x^2*Sqrt[a - b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
-((e*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])/(a*c*x)) + (-((Sqrt[a]*Sqrt[b]*e*Sqr
t[1 - (b*x^2)/a]*Sqrt[c + d*x^2]*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -(
(a*d)/(b*c))])/(Sqrt[a - b*x^2]*Sqrt[1 + (d*x^2)/c])) + (Sqrt[a]*c*(b*e +
a*f)*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[ArcSin[(Sqrt[b]*x)/
Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*Sqrt[a - b*x^2]*Sqrt[c + d*x^2]))/(a*c
)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 6.26 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.27

method	result
default	$\frac{\left(\sqrt{\frac{b}{a}}\, b d e\, x^{4}+\sqrt{\frac{-b\,x^{2}+a}{a}}\,\sqrt{\frac{x^{2} d+c}{c}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{b}{a}},\sqrt{-\frac{a d}{b c}}\right) a c f x+\sqrt{\frac{-b\,x^{2}+a}{a}}\,\sqrt{\frac{x^{2} d+c}{c}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{b}{a}},\sqrt{-\frac{a d}{b c}}\right) b c e x-\sqrt{\frac{-b\,x^{2}+}{a}}\right.}{\sqrt{\frac{b}{a}}\, x c a\left(-b d\,x^{4}+a d\,x^{2}-x^{2} b c+a c\right.}$
risch	$-\frac{e\sqrt{-b\,x^{2}+a}\,\sqrt{x^{2} d+c}}{a c x}+\frac{\left(\frac{b e c\sqrt{1-\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{b}{a}},\sqrt{-1-\frac{a d-b c}{c b}}\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{b}{a}},\sqrt{-1-\frac{a d-b c}{c b}}\right)\right)}{\sqrt{\frac{b}{a}}\,\sqrt{-b d\,x^{4}+a d\,x^{2}-x^{2} b c+a c}}+\frac{a c f\sqrt{1-\frac{b\,x^{2}}{a}}}{\sqrt{\frac{b}{a}}}\right.}{a c\sqrt{-b\,x^{2}+a}\,\sqrt{x^{2} d+c}}$
elliptic	$\frac{\sqrt{\left(-b\,x^{2}+a\right)\left(x^{2} d+c\right)}\,\left(-\frac{e\sqrt{-b d\,x^{4}+a d\,x^{2}-x^{2} b c+a c}}{a c x}+\frac{f\sqrt{1-\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{b}{a}},\sqrt{-1-\frac{a d-b c}{c b}}\right)}{\sqrt{\frac{b}{a}}\,\sqrt{-b d\,x^{4}+a d\,x^{2}-x^{2} b c+a c}}+\frac{b e\sqrt{1-\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,\left(\operatorname{Ellip}\right.}{}\right.}{\sqrt{-b\,x^{2}+a}\,\sqrt{x^{2} d+c}}$

input
```
int((f*x^2+e)/x^2/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE
)
```

output
```
((b/a)^(1/2)*b*d*e*x^4+((-b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(
x*(b/a)^(1/2),(-a*d/b/c)^(1/2))*a*c*f*x+((-b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)
^(1/2)*EllipticF(x*(b/a)^(1/2),(-a*d/b/c)^(1/2))*b*c*e*x-((-b*x^2+a)/a)^(1
/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(b/a)^(1/2),(-a*d/b/c)^(1/2))*b*c*e*x-
(b/a)^(1/2)*a*d*e*x^2+(b/a)^(1/2)*b*c*e*x^2-(b/a)^(1/2)*a*c*e)*(d*x^2+c)^(
1/2)*(-b*x^2+a)^(1/2)/(b/a)^(1/2)/x/c/a/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.56

$$\int \frac{e+fx^2}{x^2\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx =$$
$$-\frac{\sqrt{ac}b^2ex\sqrt{\frac{b}{a}}E(\arcsin\left(x\sqrt{\frac{b}{a}}\right)|-\frac{ad}{bc})+\sqrt{-bx^2+a}\sqrt{dx^2+c}abe-(b^2e+a^2f)\sqrt{ac}x\sqrt{\frac{b}{a}}F(\arcsin\left(x\sqrt{\cdots}\right.}{a^2bcx}$$

input
```
integrate((f*x^2+e)/x^2/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fri
cas")
```

output
```
-(sqrt(a*c)*b^2*e*x*sqrt(b/a)*elliptic_e(arcsin(x*sqrt(b/a)), -a*d/(b*c))
+ sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)*a*b*e - (b^2*e + a^2*f)*sqrt(a*c)*x*sqr
t(b/a)*elliptic_f(arcsin(x*sqrt(b/a)), -a*d/(b*c)))/(a^2*b*c*x)
```

Sympy [F]

$$\int \frac{e+fx^2}{x^2\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{e+fx^2}{x^2\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx$$

input
```
integrate((f*x**2+e)/x**2/(-b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral((e + f*x**2)/(x**2*sqrt(a - b*x**2)*sqrt(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{e+fx^2}{x^2\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx=\int \frac{fx^2+e}{\sqrt{-bx^2+a}\sqrt{dx^2+c}x^2}\,dx$$

input
```
integrate((f*x^2+e)/x^2/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="max
ima")
```

output
```
integrate((f*x^2 + e)/(sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)*x^2), x)
```

Giac [F]

$$\int \frac{e+fx^2}{x^2\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx=\int \frac{fx^2+e}{\sqrt{-bx^2+a}\sqrt{dx^2+c}x^2}\,dx$$

input
```
integrate((f*x^2+e)/x^2/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="gia
c")
```

output
```
integrate((f*x^2 + e)/(sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx^2}{x^2\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx=\int \frac{f\,x^2+e}{x^2\,\sqrt{a-b\,x^2}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int((e + f*x^2)/(x^2*(a - b*x^2)^(1/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int((e + f*x^2)/(x^2*(a - b*x^2)^(1/2)*(c + d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{e+fx^2}{x^2\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx = \left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{-b\,x^2+a}}{-bd\,x^6+ad\,x^4-bc\,x^4+ac\,x^2}dx\right)e + \left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{-b\,x^2+a}}{-bd\,x^4+ad\,x^2-bc\,x^2+ac}dx\right)f$$

input
```
int((f*x^2+e)/x^2/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a - b*x**2))/(a*c*x**2 + a*d*x**4 - b*c*x**4 -
b*d*x**6),x)*e + int((sqrt(c + d*x**2)*sqrt(a - b*x**2))/(a*c + a*d*x**2 -
 b*c*x**2 - b*d*x**4),x)*f
```

3.216 $\int \frac{e+fx^2}{x^4\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 308

$$\begin{aligned}
&\int \frac{e+fx^2}{x^4\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx \\
&= -\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{3acx^3} - \frac{(2bce-2ade+3acf)\sqrt{a-bx^2}\sqrt{c+dx^2}}{3a^2c^2x} \\
&\quad - \frac{\sqrt{b}(2bce-2ade+3acf)\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\,E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|-\frac{ad}{bc}\right)}{3a^{3/2}c^2\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}} \\
&\quad + \frac{\sqrt{b}(2bce-ade+3acf)\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{3a^{3/2}c\sqrt{a-bx^2}\sqrt{c+dx^2}}
\end{aligned}$$

output

```
-1/3*e*(-b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a/c/x^3-1/3*(3*a*c*f-2*a*d*e+2*b*c
*e)*(-b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a^2/c^2/x-1/3*b^(1/2)*(3*a*c*f-2*a*d*
e+2*b*c*e)*(1-b*x^2/a)^(1/2)*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2),(
-a*d/b/c)^(1/2))/a^(3/2)/c^2/(-b*x^2+a)^(1/2)/(1+d*x^2/c)^(1/2)+1/3*b^(1/2
)*(3*a*c*f-a*d*e+2*b*c*e)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(b^
(1/2)*x/a^(1/2),(-a*d/b/c)^(1/2))/a^(3/2)/c/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/
2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.82 (sec) , antiderivative size = 266, normalized size of antiderivative = 0.86

$$\int \frac{e+fx^2}{x^4\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx$$
$$= \frac{-\sqrt{-\frac{b}{a}}(a-bx^2)(c+dx^2)(2bcex^2-2adex^2+ac(e+3fx^2))+ibc(2bce-2ade+3acf)x^3\sqrt{1-\frac{bx^2}{a}}\sqrt{1}}{3a^2\sqrt{-\frac{b}{a}}}$$

input
```
Integrate[(e + f*x^2)/(x^4*Sqrt[a - b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
(-(Sqrt[-(b/a)]*(a - b*x^2)*(c + d*x^2)*(2*b*c*e*x^2 - 2*a*d*e*x^2 + a*c*(
e + 3*f*x^2))) + I*b*c*(2*b*c*e - 2*a*d*e + 3*a*c*f)*x^3*Sqrt[1 - (b*x^2)/
a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))
] - I*b*c*(2*b*c*e - a*d*e + 3*a*c*f)*x^3*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*
x^2)/c]*EllipticF[I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))])/(3*a^2*Sqrt[
-(b/a)]*c^2*x^3*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.05, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {445, 25, 445, 25, 27, 399, 323, 323, 321, 331, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx^2}{x^4\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx$$
$$\Big\downarrow \; 445$$
$$-\frac{\int -\frac{bdex^2+2bce-2ade+3acf}{x^2\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{3ac}-\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{3acx^3}$$
$$\Big\downarrow \; 25$$

$$\frac{\int \frac{bdex^2+2bce-2ade+3acf}{x^2\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{3ac}-\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{3acx^3}$$

$\Big\downarrow$ 445

$$\frac{-\frac{\int -\frac{bd\left(ace-(2bce-2ade+3acf)x^2\right)}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}(3acf-2ade+2bce)}{acx}}{3ac}-\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{3acx^3}$$

$\Big\downarrow$ 25

$$\frac{\frac{\int \frac{bd\left(ace-(2bce-2ade+3acf)x^2\right)}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}(3acf-2ade+2bce)}{acx}}{3ac}-\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{3acx^3}$$

$\Big\downarrow$ 27

$$\frac{\frac{bd\int \frac{ace-(2bce-2ade+3acf)x^2}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}(3acf-2ade+2bce)}{acx}}{3ac}-\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{3acx^3}$$

$\Big\downarrow$ 399

$$\frac{\frac{bd\left(\frac{c(3acf-ade+2bce)\int \frac{1}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{d}-\frac{(3acf-2ade+2bce)\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}\right)}{ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}(3acf-2ade+2bce)}{acx}}{3ac}-\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{3acx^3}$$

$\Big\downarrow$ 323

$$\frac{\frac{bd\left(\frac{c\sqrt{\frac{dx^2}{c}+1}(3acf-ade+2bce)\int \frac{1}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}dx}{d\sqrt{c+dx^2}}-\frac{(3acf-2ade+2bce)\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}\right)}{ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}(3acf-2ade+2bce)}{acx}}{3ac}-\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{3acx^3}$$

$\Big\downarrow$ 323

$$\frac{\frac{bd\left(\frac{c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(3acf-ade+2bce)\int \frac{1}{\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}}dx}{d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{(3acf-2ade+2bce)\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}\right)}{ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}(3acf-2ade+2bce)}{acx}}{3ac}-\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{3acx^3}$$

$\Big\downarrow$ 321

$$\frac{\frac{bd\left(\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(3acf-ade+2bce)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{(3acf-2ade+2bce)\int\frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}\right)}{ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}(3acf-2ade+2bce}{acx}}{3ac}$$

$$\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{3acx^3}$$

$\Big\downarrow$ 331

$$\frac{\frac{bd\left(\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(3acf-ade+2bce)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{1-\frac{bx^2}{a}}(3acf-2ade+2bce)\int\frac{\sqrt{dx^2+c}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}\right)}{ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}(3acf-2a}{acx}}{3ac}$$

$$\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{3acx^3}$$

$\Big\downarrow$ 330

$$\frac{\frac{bd\left(\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(3acf-ade+2bce)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(3acf-2ade+2bce)\int\frac{\sqrt{\frac{dx^2}{c}+1}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}\right)}{ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}(3}{ac}}{3ac}$$

$$\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{3acx^3}$$

$\Big\downarrow$ 327

$$\frac{\frac{bd\left(\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(3acf-ade+2bce)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(3acf-2ade+2bce)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}\right)}{ac}-\sqrt{a-bx}}{3ac}$$

$$\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{3acx^3}$$

input
```
Int[(e + f*x^2)/(x^4*Sqrt[a - b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
-1/3*(e*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])/(a*c*x^3) + (-(((2*b*c*e - 2*a*d*
e + 3*a*c*f)*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])/(a*c*x)) + (b*d*(-((Sqrt[a]*
(2*b*c*e - 2*a*d*e + 3*a*c*f)*Sqrt[1 - (b*x^2)/a]*Sqrt[c + d*x^2]*Elliptic
E[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*d*Sqrt[a - b*x^2]
*Sqrt[1 + (d*x^2)/c])) + (Sqrt[a]*c*(2*b*c*e - a*d*e + 3*a*c*f)*Sqrt[1 - (
b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*
d)/(b*c))])/(Sqrt[b]*d*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])))/(a*c))/(3*a*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 7.67 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.11

method	result
risch	$-\frac{\sqrt{-b\,x^{2}+a}\,\sqrt{x^{2} d+c}\,\left(3acf\,x^{2}-2ade\,x^{2}+2bce\,x^{2}+ace\right)}{3a^{2}c^{2}x^{3}}+\frac{bd\left(\frac{\left(3acf-2ade+2bce\right)c\sqrt{1-\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{b}{a}},\sqrt{-1-\frac{ad-bc}{cb}}\cdots\right.\right.}{\sqrt{\frac{b}{a}}\,\sqrt{-bd\,x^{4}+ad\,x^{2}-x^{2}bc+ac}\,d\cdots}\cdots\right)}{\cdots}$
elliptic	$\frac{\sqrt{\left(-b\,x^{2}+a\right)\left(x^{2} d+c\right)}\,\left(-\frac{e\sqrt{-bd\,x^{4}+ad\,x^{2}-x^{2}bc+ac}}{3ac\,x^{3}}-\frac{\left(3acf-2ade+2bce\right)\sqrt{-bd\,x^{4}+ad\,x^{2}-x^{2}bc+ac}}{3a^{2}c^{2}x}+\frac{bde\sqrt{1-\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,\operatorname{EllipticF}\left(x\sqrt{\cdots}\right.}{3ac\sqrt{\frac{b}{a}}\,\sqrt{-bd\,x^{4}+ad\,x^{2}-\cdots}}\cdots\right)}{\sqrt{-b\,x^{2}+a}\,\sqrt{x^{2}\cdots}}$
default	$\frac{\left(3\sqrt{\frac{b}{a}}\,abcdf\,x^{6}-2\sqrt{\frac{b}{a}}\,ab\,d^{2}e\,x^{6}+2\sqrt{\frac{b}{a}}\,b^{2}cde\,x^{6}+3\sqrt{\frac{-b\,x^{2}+a}{a}}\,\sqrt{\frac{x^{2} d+c}{c}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{b}{a}},\sqrt{-\frac{ad}{bc}}\right)ab\,c^{2}f\,x^{3}-\sqrt{\frac{-b\,x^{2}+a}{a}}\,\sqrt{\frac{x^{2} d+c}{c}}\cdots\right.}{\cdots}$

input
```
int((f*x^2+e)/x^4/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE
)
```

output
```
-1/3*(-b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(3*a*c*f*x^2-2*a*d*e*x^2+2*b*c*e*x^2
+a*c*e)/a^2/c^2/x^3+1/3/a^2/c^2*b*d*((3*a*c*f-2*a*d*e+2*b*c*e)*c/(b/a)^(1/
2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2
)/d*(EllipticF(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2))-EllipticE(x*(b/a)^(
1/2),(-1-(a*d-b*c)/c/b)^(1/2)))+a*c*e/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x
^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)*EllipticF(x*(b/a)^(1/2),(
-1-(a*d-b*c)/c/b)^(1/2)))*((-b*x^2+a)*(d*x^2+c))^(1/2)/(-b*x^2+a)^(1/2)/(d
*x^2+c)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.61

$$\int \frac{e+fx^2}{x^4\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx = -\frac{(3\,abcf+2\,(b^2c-abd)e)\sqrt{ac}x^3\sqrt{\frac{b}{a}}E(\arcsin\left(x\sqrt{\frac{b}{a}}\right)\mid-\frac{ad}{bc})-(3\,abcf+(2\,b^2c+(a^2-2\,ab)d)e)\sqrt{ac}x}{3\,a^3c^2x^3}$$

input
```
integrate((f*x^2+e)/x^4/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fri
cas")
```

output
```
-1/3*((3*a*b*c*f + 2*(b^2*c - a*b*d)*e)*sqrt(a*c)*x^3*sqrt(b/a)*elliptic_e
(arcsin(x*sqrt(b/a)), -a*d/(b*c)) - (3*a*b*c*f + (2*b^2*c + (a^2 - 2*a*b)*
d)*e)*sqrt(a*c)*x^3*sqrt(b/a)*elliptic_f(arcsin(x*sqrt(b/a)), -a*d/(b*c))
 + (a^2*c*e + (3*a^2*c*f + 2*(a*b*c - a^2*d)*e)*x^2)*sqrt(-b*x^2 + a)*sqrt(
d*x^2 + c))/(a^3*c^2*x^3)
```

Sympy [F]

$$\int \frac{e+fx^2}{x^4\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{e+fx^2}{x^4\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx$$

input
```
integrate((f*x**2+e)/x**4/(-b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral((e + f*x**2)/(x**4*sqrt(a - b*x**2)*sqrt(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{e+fx^2}{x^4\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{fx^2+e}{\sqrt{-bx^2+a}\sqrt{dx^2+cx^4}}\,dx$$

input
```
integrate((f*x^2+e)/x^4/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="max
ima")
```

output
```
integrate((f*x^2 + e)/(sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)*x^4), x)
```

Giac [F]

$$\int \frac{e+fx^2}{x^4\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{fx^2+e}{\sqrt{-bx^2+a}\sqrt{dx^2+cx^4}}\,dx$$

input
```
integrate((f*x^2+e)/x^4/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="gia
c")
```

output
```
integrate((f*x^2 + e)/(sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx^2}{x^4\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{f\,x^2+e}{x^4\,\sqrt{a-b\,x^2}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int((e + f*x^2)/(x^4*(a - b*x^2)^(1/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int((e + f*x^2)/(x^4*(a - b*x^2)^(1/2)*(c + d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{e + f x^2}{x^4 \sqrt{a - b x^2} \sqrt{c + d x^2}} \, dx$$
$$= \frac{-\sqrt{d\,x^2 + c}\,\sqrt{-b\,x^2 + a}\,f - \left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{-b\,x^2+a}\,x^2}{-bd\,x^4+ad\,x^2-bc\,x^2+ac}dx\right) bdf x + \left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{-b\,x^2+a}}{-bd\,x^8+ad\,x^6-bc\,x^6+ac\,x^4}dx\right) acex}{acx}$$

input
```
int((f*x^2+e)/x^4/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

output
```
( - sqrt(c + d*x**2)*sqrt(a - b*x**2)*f - int((sqrt(c + d*x**2)*sqrt(a - b
*x**2)*x**2)/(a*c + a*d*x**2 - b*c*x**2 - b*d*x**4),x)*b*d*f*x + int((sqrt
(c + d*x**2)*sqrt(a - b*x**2))/(a*c*x**4 + a*d*x**6 - b*c*x**6 - b*d*x**8)
,x)*a*c*e*x)/(a*c*x)
```

3.217 $\int \frac{e+fx^2}{x^6\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 418

$$\begin{aligned}
&\int \frac{e+fx^2}{x^6\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx \\
&= -\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{5acx^5}-\frac{(4bce-4ade+5acf)\sqrt{a-bx^2}\sqrt{c+dx^2}}{15a^2c^2x^3} \\
&\quad-\frac{(9abcde+2(bc-ad)(4bce-4ade+5acf))\sqrt{a-bx^2}\sqrt{c+dx^2}}{15a^3c^3x} \\
&\quad-\frac{\sqrt{b}(9abcde+2(bc-ad)(4bce-4ade+5acf))\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{15a^{5/2}c^3\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}} \\
&\quad+\frac{\sqrt{b}(8b^2c^2e-abc(3de-10cf)+a^2d(4de-5cf))\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{15a^{5/2}c^2\sqrt{a-bx^2}\sqrt{c+dx^2}}
\end{aligned}$$

output
```
-1/5*e*(-b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a/c/x^5-1/15*(5*a*c*f-4*a*d*e+4*b*
c*e)*(-b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a^2/c^2/x^3-1/15*(9*a*b*c*d*e+2*(-a*
d+b*c)*(5*a*c*f-4*a*d*e+4*b*c*e))*(-b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a^3/c^3
/x-1/15*b^(1/2)*(9*a*b*c*d*e+2*(-a*d+b*c)*(5*a*c*f-4*a*d*e+4*b*c*e))*(1-b*
x^2/a)^(1/2)*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2),(-a*d/b/c)^(1/2))
/a^(5/2)/c^3/(-b*x^2+a)^(1/2)/(1+d*x^2/c)^(1/2)+1/15*b^(1/2)*(8*b^2*c^2*e-
a*b*c*(-10*c*f+3*d*e)+a^2*d*(-5*c*f+4*d*e))*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^
(1/2)*EllipticF(b^(1/2)*x/a^(1/2),(-a*d/b/c)^(1/2))/a^(5/2)/c^2/(-b*x^2+a)
^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.58 (sec) , antiderivative size = 365, normalized size of antiderivative = 0.87

$$\int \frac{e+fx^2}{x^6\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx = \frac{\sqrt{-\frac{b}{a}}(-a+bx^2)\,(c+dx^2)\,(8b^2c^2ex^4+abcx^2(4ce-7dex^2+10cfx^2)+a^2(8d^2ex^4-2cdx^2(2e+5fx^2)+\cdots}{}$$

input
```
Integrate[(e + f*x^2)/(x^6*Sqrt[a - b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
(Sqrt[-(b/a)]*(-a + b*x^2)*(c + d*x^2)*(8*b^2*c^2*e*x^4 + a*b*c*x^2*(4*c*e
 - 7*d*e*x^2 + 10*c*f*x^2) + a^2*(8*d^2*e*x^4 - 2*c*d*x^2*(2*e + 5*f*x^2)
+ c^2*(3*e + 5*f*x^2))) - I*b*c*(-8*b^2*c^2*e + a*b*c*(7*d*e - 10*c*f) + 2
*a^2*d*(-4*d*e + 5*c*f))*x^5*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Ellip
ticE[I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))] + I*b*c*(-8*b^2*c^2*e + a*
b*c*(3*d*e - 10*c*f) + a^2*d*(-4*d*e + 5*c*f))*x^5*Sqrt[1 - (b*x^2)/a]*Sqr
t[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))])/(15
*a^3*Sqrt[-(b/a)]*c^3*x^5*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 445, normalized size of antiderivative = 1.06, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {445, 25, 445, 25, 445, 25, 27, 399, 323, 323, 321, 331, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx^2}{x^6\sqrt{a-bx^2}\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \; 445$$

$$-\frac{\int -\frac{3bdex^2+4bce-4ade+5acf}{x^4\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{5ac}-\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{5acx^5}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{3bdex^2+4bce-4ade+5acf}{x^4\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{5ac}-\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{5acx^5}$$

$$\Big\downarrow \; 445$$

$$\frac{-\frac{\int -\frac{2d(4de-5cf)a^2-bc(7de-10cf)a+bd(4bce-4ade+5acf)x^2+8b^2c^2e}{x^2\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{3ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}(5acf-4ade+4bce)}{3acx^3}}{5ac}-\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{5acx^5}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{2d(4de-5cf)a^2-bc(7de-10cf)a+bd(4bce-4ade+5acf)x^2+8b^2c^2e}{x^2\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{3ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}(5acf-4ade+4bce)}{3acx^3}}{5ac}-\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{5acx^5}$$

$$\Big\downarrow \; 445$$

$$\frac{\frac{-\frac{\int -\frac{bd\left(ac(4bce-4ade+5acf)-\left(2d(4de-5cf)a^2-bc(7de-10cf)a+8b^2c^2e\right)x^2\right)}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}\left(\frac{8b^2ce}{a}+\frac{8ad^2e}{c}-10adf+10bcf-7bde\right)}{x}}{3ac}-\frac{\sqrt{a-bx^2}\sqrt{c\cdots}}{\cdots}}{5ac}-\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{5acx^5}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\frac{\int \frac{bd\left(ac(4bce-4ade+5acf)-\left(2d(4de-5cf)a^2-bc(7de-10cf)a+8b^2c^2e\right)x^2\right)}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}\left(\frac{8b^2ce}{a}+\frac{8ad^2e}{c}-10adf+10bcf-7bde\right)}{x}}{3ac}-\sqrt{a-bx^2}\sqrt{c+dx\ldots}}{5ac}$$

$$\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{5acx^5}$$

$\Big\downarrow$ 27

$$\frac{\frac{\frac{bd\int \frac{ac(4bce-4ade+5acf)-\left(2d(4de-5cf)a^2-bc(7de-10cf)a+8b^2c^2e\right)x^2}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}\left(\frac{8b^2ce}{a}+\frac{8ad^2e}{c}-10adf+10bcf-7bde\right)}{x}}{3ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}\ldots}{3a\ldots}}{5ac}$$

$$\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{5acx^5}$$

$\Big\downarrow$ 399

$$\frac{\frac{\frac{bd\left(\frac{c\left(a^2d(4de-5cf)-abc(3de-10cf)+8b^2c^2e\right)\int \frac{1}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{d}-\frac{\left(2a^2d(4de-5cf)-abc(7de-10cf)+8b^2c^2e\right)\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}\right)}{ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}\left(\frac{8b^2ce}{a}\ldots\right.}{\ldots}}{3ac}}{5ac}$$

$$\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{5acx^5}$$

$\Big\downarrow$ 323

$$\frac{\frac{\frac{bd\left(\frac{c\sqrt{\frac{dx^2}{c}+1}\left(a^2d(4de-5cf)-abc(3de-10cf)+8b^2c^2e\right)\int \frac{1}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}dx}{d\sqrt{c+dx^2}}-\frac{\left(2a^2d(4de-5cf)-abc(7de-10cf)+8b^2c^2e\right)\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}\right)}{ac}-\frac{\sqrt{a-bx^2}\sqrt{c+d\ldots}}{\ldots}}{3ac}}{5ac}$$

$$\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{5acx^5}$$

$\Big\downarrow$ 323

$$\frac{\frac{\frac{bd\left(\frac{c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\left(a^2d(4de-5cf)-abc(3de-10cf)+8b^2c^2e\right)\int \frac{1}{\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}}dx}{d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\left(2a^2d(4de-5cf)-abc(7de-10cf)+8b^2c^2e\right)\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}\right)}{ac}-\frac{\sqrt{a-\ldots}}{\ldots}}{3ac}}{5ac}$$

$$\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{5acx^5}$$

$\Big\downarrow$ 321

$$\frac{\frac{\frac{bd\left(\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\left(a^2d(4de-5cf)-abc(3de-10cf)+8b^2c^2e\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\left(2a^2d(4de-5cf)-abc(7de-10cf)+8b^2c^2e\right)\int\frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}d\cdots}{d}\right.}{ac}\cdots}{3ac}\cdots}{5ac}\cdots$$

$$\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{5acx^5}$$

$\Big\downarrow$ 331

$$\frac{\frac{\frac{bd\left(\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\left(a^2d(4de-5cf)-abc(3de-10cf)+8b^2c^2e\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{1-\frac{bx^2}{a}}\left(2a^2d(4de-5cf)-abc(7de-10cf)+8b^2c^2e\right)\int\cdots}{d\sqrt{a-bx^2}}\right.}{ac}\cdots}{3ac}\cdots}{5ac}\cdots$$

$$\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{5acx^5}$$

$\Big\downarrow$ 330

$$\frac{\frac{\frac{bd\left(\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\left(a^2d(4de-5cf)-abc(3de-10cf)+8b^2c^2e\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\left(2a^2d(4de-5cf)-abc(7de-10cf)+8b^2\cdots\right.}{d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}\right.}{ac}\cdots}{3ac}\cdots}{5ac}\cdots$$

$$\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{5acx^5}$$

$\Big\downarrow$ 327

$$\frac{\frac{\frac{bd\left(\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\left(a^2d(4de-5cf)-abc(3de-10cf)+8b^2c^2e\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\left(2a^2d(4de-5cf)-abc(7de-10cf)+\cdots\right.}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}\right.}{ac}\cdots}{3ac}\cdots}{5ac}\cdots$$

$$\frac{e\sqrt{a-bx^2}\sqrt{c+dx^2}}{5acx^5}$$

input

```
Int[(e + f*x^2)/(x^6*Sqrt[a - b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
-1/5*(e*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])/(a*c*x^5) + (-1/3*((4*b*c*e - 4*a
*d*e + 5*a*c*f)*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])/(a*c*x^3) + (-((((8*b^2*c
*e)/a - 7*b*d*e + (8*a*d^2*e)/c + 10*b*c*f - 10*a*d*f)*Sqrt[a - b*x^2]*Sqr
t[c + d*x^2])/x) + (b*d*(-((Sqrt[a]*(8*b^2*c^2*e - a*b*c*(7*d*e - 10*c*f)
+ 2*a^2*d*(4*d*e - 5*c*f))*Sqrt[1 - (b*x^2)/a]*Sqrt[c + d*x^2]*EllipticE[A
rcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*d*Sqrt[a - b*x^2]*Sq
rt[1 + (d*x^2)/c])) + (Sqrt[a]*c*(8*b^2*c^2*e - a*b*c*(3*d*e - 10*c*f) + a
^2*d*(4*d*e - 5*c*f))*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[Ar
cSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*d*Sqrt[a - b*x^2]*Sqr
t[c + d*x^2])))/(a*c))/(3*a*c))/(5*a*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 9.60 (sec) , antiderivative size = 496, normalized size of antiderivative = 1.19

method	result
elliptic	$\frac{\sqrt{(-b\,x^2+a)(x^2d+c)}\left(-\frac{e\sqrt{-bd\,x^4+ad\,x^2-x^2bc+ac}}{5ac\,x^5}-\frac{(5acf-4ade+4bce)\sqrt{-bd\,x^4+ad\,x^2-x^2bc+ac}}{15a^2c^2x^3}+\frac{\left(10a^2cfd-8a^2d^2e-10ab\,c^2f+7abcde\right.}{15c^3}\right.}{}$
risch	$-\frac{\sqrt{-b\,x^2+a}\,\sqrt{x^2d+c}\,(-10a^2cdf\,x^4+8a^2d^2e\,x^4+10ab\,c^2f\,x^4-7abcde\,x^4+8b^2c^2e\,x^4+5a^2c^2f\,x^2-4a^2cde\,x^2+4ab\,c^2e\,x^2+3a^2c^2e)}{15a^3c^3x^5}$
default	Expression too large to display

input
```
int((f*x^2+e)/x^6/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE
)
```

output
```
((-b*x^2+a)*(d*x^2+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-1/5*e/a/c*
(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)/x^5-1/15/a^2/c^2*(5*a*c*f-4*a*d*e+4*b
*c*e)*(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)/x^3+1/15*(10*a^2*c*d*f-8*a^2*d^
2*e-10*a*b*c^2*f+7*a*b*c*d*e-8*b^2*c^2*e)/c^3/a^3*(-b*d*x^4+a*d*x^2-b*c*x^
2+a*c)^(1/2)/x+1/15*(5*a*c*f-4*a*d*e+4*b*c*e)*b*d/a^2/c^2/(b/a)^(1/2)*(1-b
*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)*Ellip
ticF(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2))-1/15*b*(10*a^2*c*d*f-8*a^2*d^
2*e-10*a*b*c^2*f+7*a*b*c*d*e-8*b^2*c^2*e)/c^2/a^3/(b/a)^(1/2)*(1-b*x^2/a)^
(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)*(EllipticF(x*
(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2))-EllipticE(x*(b/a)^(1/2),(-1-(a*d-b*c
)/c/b)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 320, normalized size of antiderivative = 0.77

$$\int \frac{e+fx^2}{x^6\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx = -\frac{\sqrt{ac}((8\,b^3c^2-7\,ab^2cd+8\,a^2bd^2)e+10\,(ab^2c^2-a^2bcd)f)x^5\sqrt{\frac{b}{a}}E(\arcsin\left(x\sqrt{\frac{b}{a}}\right)|-\frac{ad}{bc})-\sqrt{ac}((8\,b^3c^2 \cdots}{\cdots}$$

input
```
integrate((f*x^2+e)/x^6/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fri
cas")
```

output
```
-1/15*(sqrt(a*c)*((8*b^3*c^2 - 7*a*b^2*c*d + 8*a^2*b*d^2)*e + 10*(a*b^2*c^
2 - a^2*b*c*d)*f)*x^5*sqrt(b/a)*elliptic_e(arcsin(x*sqrt(b/a)), -a*d/(b*c)
) - sqrt(a*c)*((8*b^3*c^2 + (4*a^2*b - 7*a*b^2)*c*d - 4*(a^3 - 2*a^2*b)*d^
2)*e + 5*(2*a*b^2*c^2 + (a^3 - 2*a^2*b)*c*d)*f)*x^5*sqrt(b/a)*elliptic_f(a
rcsin(x*sqrt(b/a)), -a*d/(b*c)) + (3*a^3*c^2*e + ((8*a*b^2*c^2 - 7*a^2*b*c
*d + 8*a^3*d^2)*e + 10*(a^2*b*c^2 - a^3*c*d)*f)*x^4 + (5*a^3*c^2*f + 4*(a^
2*b*c^2 - a^3*c*d)*e)*x^2)*sqrt(-b*x^2 + a)*sqrt(d*x^2 + c))/(a^4*c^3*x^5)
```

Sympy [F]

$$\int \frac{e+fx^2}{x^6\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{e+fx^2}{x^6\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx$$

input
```
integrate((f*x**2+e)/x**6/(-b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral((e + f*x**2)/(x**6*sqrt(a - b*x**2)*sqrt(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{e+fx^2}{x^6\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{fx^2+e}{\sqrt{-bx^2+a}\sqrt{dx^2+c}x^6}\,dx$$

input
```
integrate((f*x^2+e)/x^6/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="max
ima")
```

output
```
integrate((f*x^2 + e)/(sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)*x^6), x)
```

Giac [F]

$$\int \frac{e+fx^2}{x^6\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{fx^2+e}{\sqrt{-bx^2+a}\sqrt{dx^2+c}x^6}\,dx$$

input
```
integrate((f*x^2+e)/x^6/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="gia
c")
```

output
```
integrate((f*x^2 + e)/(sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx^2}{x^6\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{f\,x^2+e}{x^6\,\sqrt{a-b\,x^2}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int((e + f*x^2)/(x^6*(a - b*x^2)^(1/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int((e + f*x^2)/(x^6*(a - b*x^2)^(1/2)*(c + d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{e+fx^2}{x^6\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx = \frac{-\sqrt{d\,x^2+c}\,\sqrt{-b\,x^2+a}\,ace-3\sqrt{d\,x^2+c}\,\sqrt{-b\,x^2+a}\,bde\,x^4-3\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{-b\,x^2+a}\,x^2}{-bd\,x^4+ad\,x^2-bc\,x^2+ac}dx\right)b^2d^2e\,x^5+5\left(\right.}{}$$

input
```
int((f*x^2+e)/x^6/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

output
```
( - sqrt(c + d*x**2)*sqrt(a - b*x**2)*a*c*e - 3*sqrt(c + d*x**2)*sqrt(a -
b*x**2)*b*d*e*x**4 - 3*int((sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**2)/(a*c +
 a*d*x**2 - b*c*x**2 - b*d*x**4),x)*b**2*d**2*e*x**5 + 5*int((sqrt(c + d*x
**2)*sqrt(a - b*x**2))/(a*c*x**4 + a*d*x**6 - b*c*x**6 - b*d*x**8),x)*a**2
*c**2*f*x**5 - 4*int((sqrt(c + d*x**2)*sqrt(a - b*x**2))/(a*c*x**4 + a*d*x
**6 - b*c*x**6 - b*d*x**8),x)*a**2*c*d*e*x**5 + 4*int((sqrt(c + d*x**2)*sq
rt(a - b*x**2))/(a*c*x**4 + a*d*x**6 - b*c*x**6 - b*d*x**8),x)*a*b*c**2*e*
x**5)/(5*a**2*c**2*x**5)
```

3.218 $\int \frac{x^8}{\sqrt{a-bx^2}\sqrt{c+dx^2}(e+fx^2)}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 514

$$\int \frac{x^8}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx$$

$$= \frac{(5bde+4bcf-4adf)x\sqrt{a-bx^2}\sqrt{c+dx^2}}{15b^2d^2f^2} - \frac{x^3\sqrt{a-bx^2}\sqrt{c+dx^2}}{5bdf}$$

$$+ \frac{\sqrt{a}(8a^2d^2f^2-abdf(10de+7cf)+b^2(15d^2e^2+10cdef+8c^2f^2))\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\,E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\right.}{15b^{5/2}d^3f^3\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}}$$

$$- \frac{\sqrt{a}(4a^2cd^2f^3-abcdf^2(5de+3cf)+b^2(15d^3e^3+15cd^2e^2f+10c^2def^2+8c^3f^3))\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}}{15b^{5/2}d^3f^4\sqrt{a-bx^2}\sqrt{c+dx^2}}$$

$$+ \frac{\sqrt{a}e^3\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\mathrm{EllipticPi}\left(-\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}f^4\sqrt{a-bx^2}\sqrt{c+dx^2}}$$

output
```
1/15*(-4*a*d*f+4*b*c*f+5*b*d*e)*x*(-b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b^2/d^2
/f^2-1/5*x^3*(-b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b/d/f+1/15*a^(1/2)*(8*a^2*d^
2*f^2-a*b*d*f*(7*c*f+10*d*e)+b^2*(8*c^2*f^2+10*c*d*e*f+15*d^2*e^2))*(1-b*x
^2/a)^(1/2)*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2),(-a*d/b/c)^(1/2))/
b^(5/2)/d^3/f^3/(-b*x^2+a)^(1/2)/(1+d*x^2/c)^(1/2)-1/15*a^(1/2)*(4*a^2*c*d
^2*f^3-a*b*c*d*f^2*(3*c*f+5*d*e)+b^2*(8*c^3*f^3+10*c^2*d*e*f^2+15*c*d^2*e^
2*f+15*d^3*e^3))*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(b^(1/2)*x/a
^(1/2),(-a*d/b/c)^(1/2))/b^(5/2)/d^3/f^4/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)+
a^(1/2)*e^3*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)*EllipticPi(b^(1/2)*x/a^(1/
2),-a*f/b/e,(-a*d/b/c)^(1/2))/b^(1/2)/f^4/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.68 (sec) , antiderivative size = 433, normalized size of antiderivative = 0.84

$$\int \frac{x^8}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx$$
$$= \frac{-\sqrt{-\frac{b}{a}}df^2x(a-bx^2)\,(c+dx^2)\,(4adf+b(-5de-4cf+3dfx^2))-icf(8a^2d^2f^2-abdf(10de+7cf)+b}{}$$

input
```
Integrate[x^8/(Sqrt[a - b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)),x]
```

output
```
(-(Sqrt[-(b/a)]*d*f^2*x*(a - b*x^2)*(c + d*x^2)*(4*a*d*f + b*(-5*d*e - 4*c
*f + 3*d*f*x^2))) - I*c*f*(8*a^2*d^2*f^2 - a*b*d*f*(10*d*e + 7*c*f) + b^2*
(15*d^2*e^2 + 10*c*d*e*f + 8*c^2*f^2))*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2
)/c]*EllipticE[I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))] + I*(4*a^2*c*d^2
*f^3 - a*b*c*d*f^2*(5*d*e + 3*c*f) + b^2*(15*d^3*e^3 + 15*c*d^2*e^2*f + 10
*c^2*d*e*f^2 + 8*c^3*f^3))*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Ellipti
cF[I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))] - (15*I)*b^2*d^3*e^3*Sqrt[1
- (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[-((a*f)/(b*e)), I*ArcSinh[Sqrt
[-(b/a)]*x], -((a*d)/(b*c))])/(15*b^2*Sqrt[-(b/a)]*d^3*f^4*Sqrt[a - b*x^2]
*Sqrt[c + d*x^2])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx$$
$$\Big\downarrow \text{450}$$
$$\int \frac{x^8}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)}dx$$

input
```
Int[x^8/(Sqrt[a - b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [A] (verified)

Time = 20.19 (sec) , antiderivative size = 539, normalized size of antiderivative = 1.05

method	result
risch	$-\frac{x\left(3bdf\,x^{2}+4adf-4bcf-5bde\right)\sqrt{-b\,x^{2}+a}\,\sqrt{x^{2}d+c}}{15b^{2}d^{2}f^{2}}+\frac{\left(\frac{\left(4a^{2}cd\,f^{3}-4ab\,c^{2}f^{3}-5abcde\,f^{2}-15b^{2}d^{2}e^{3}\right)\sqrt{1-\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,\operatorname{EllipticF}\left(x\sqrt{\right.}{f^{2}\sqrt{\frac{b}{a}}\,\sqrt{-bd\,x^{4}+ad\,x^{2}-x^{2}bc+ac}}\right.}{}$
default	Expression too large to display
elliptic	Expression too large to display

input
```
int(x^8/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x,method=_RETURNVERBOSE
)
```

output
```
-1/15/b^2/d^2*x*(3*b*d*f*x^2+4*a*d*f-4*b*c*f-5*b*d*e)*(-b*x^2+a)^(1/2)*(d*
x^2+c)^(1/2)/f^2+1/15/b^2/d^2/f^2*((4*a^2*c*d*f^3-4*a*b*c^2*f^3-5*a*b*c*d*
e*f^2-15*b^2*d^2*e^3)/f^2/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/
(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)*EllipticF(x*(b/a)^(1/2),(-1-(a*d-b*c)
/c/b)^(1/2))-1/f*(8*a^2*d^2*f^2-7*a*b*c*d*f^2-10*a*b*d^2*e*f+8*b^2*c^2*f^2
+10*b^2*c*d*e*f+15*b^2*d^2*e^2)*c/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c
)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(b/a)^(1/2),(-
1-(a*d-b*c)/c/b)^(1/2))-EllipticE(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2)))
+15*b^2*d^2*e^3/f^2/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*
x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)*EllipticPi(x*(b/a)^(1/2),-a*f/b/e,(-1/c*d)^
(1/2)/(b/a)^(1/2)))*((-b*x^2+a)*(d*x^2+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x^2+c
)^(1/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^8}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate(x^8/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="fri
cas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^8}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{x^8}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx$$

input
```
integrate(x**8/(-b*x**2+a)**(1/2)/(d*x**2+c)**(1/2)/(f*x**2+e),x)
```

output
```
Integral(x**8/(sqrt(a - b*x**2)*sqrt(c + d*x**2)*(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{x^8}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx=\int \frac{x^8}{\sqrt{-bx^2+a}\sqrt{dx^2+c}(fx^2+e)}\,dx$$

input
```
integrate(x^8/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="max
ima")
```

output
```
integrate(x^8/(sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{x^8}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx=\int \frac{x^8}{\sqrt{-bx^2+a}\sqrt{dx^2+c}(fx^2+e)}\,dx$$

input
```
integrate(x^8/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="gia
c")
```

output
```
integrate(x^8/(sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^8}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx=\int \frac{x^8}{\sqrt{a-b\,x^2}\,\sqrt{d\,x^2+c}\,\left(f\,x^2+e\right)}\,dx$$

input
```
int(x^8/((a - b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)),x)
```

output
```
int(x^8/((a - b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)), x)
```

Reduce [F]

$$\int \frac{x^8}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \text{Too large to display}$$

input
```
int(x^8/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x)
```

output
```
( - 4*sqrt(c + d*x**2)*sqrt(a - b*x**2)*a*d*f*x + 4*sqrt(c + d*x**2)*sqrt(
a - b*x**2)*b*c*f*x + 5*sqrt(c + d*x**2)*sqrt(a - b*x**2)*b*d*e*x - 3*sqrt
(c + d*x**2)*sqrt(a - b*x**2)*b*d*f*x**3 + 8*int((sqrt(c + d*x**2)*sqrt(a
- b*x**2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 - b*c*e*x**2
 - b*c*f*x**4 - b*d*e*x**4 - b*d*f*x**6),x)*a**2*d**2*f**2 - 7*int((sqrt(c
 + d*x**2)*sqrt(a - b*x**2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f
*x**4 - b*c*e*x**2 - b*c*f*x**4 - b*d*e*x**4 - b*d*f*x**6),x)*a*b*c*d*f**2
 - 10*int((sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**4)/(a*c*e + a*c*f*x**2 + a
*d*e*x**2 + a*d*f*x**4 - b*c*e*x**2 - b*c*f*x**4 - b*d*e*x**4 - b*d*f*x**6
),x)*a*b*d**2*e*f + 8*int((sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**4)/(a*c*e
+ a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 - b*c*e*x**2 - b*c*f*x**4 - b*d*e*x
**4 - b*d*f*x**6),x)*b**2*c**2*f**2 + 10*int((sqrt(c + d*x**2)*sqrt(a - b*
x**2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 - b*c*e*x**2 - b
*c*f*x**4 - b*d*e*x**4 - b*d*f*x**6),x)*b**2*c*d*e*f + 15*int((sqrt(c + d*
x**2)*sqrt(a - b*x**2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4
 - b*c*e*x**2 - b*c*f*x**4 - b*d*e*x**4 - b*d*f*x**6),x)*b**2*d**2*e**2 +
4*int((sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**2)/(a*c*e + a*c*f*x**2 + a*d*e
*x**2 + a*d*f*x**4 - b*c*e*x**2 - b*c*f*x**4 - b*d*e*x**4 - b*d*f*x**6),x)
*a**2*c*d*f**2 + 8*int((sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**2)/(a*c*e + a
*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 - b*c*e*x**2 - b*c*f*x**4 - b*d*e*x...
```

3.219 $\int \frac{x^6}{\sqrt{a-bx^2}\sqrt{c+dx^2}(e+fx^2)}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 384

$$\int \frac{x^6}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = -\frac{x\sqrt{a-bx^2}\sqrt{c+dx^2}}{3bdf} - \frac{\sqrt{a}(3bde+2bcf-2adf)\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\,E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|-\frac{ad}{bc}\right)}{3b^{3/2}d^2f^2\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}} + \frac{\sqrt{a}(3bd^2e^2+3bcdef+2bc^2f^2-acdf^2)\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{3b^{3/2}d^2f^3\sqrt{a-bx^2}\sqrt{c+dx^2}} - \frac{\sqrt{a}e^2\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}f^3\sqrt{a-bx^2}\sqrt{c+dx^2}}$$

output
```
-1/3*x*(-b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b/d/f-1/3*a^(1/2)*(-2*a*d*f+2*b*c*
f+3*b*d*e)*(1-b*x^2/a)^(1/2)*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2),(
-a*d/b/c)^(1/2))/b^(3/2)/d^2/f^2/(-b*x^2+a)^(1/2)/(1+d*x^2/c)^(1/2)+1/3*a^
(1/2)*(-a*c*d*f^2+2*b*c^2*f^2+3*b*c*d*e*f+3*b*d^2*e^2)*(1-b*x^2/a)^(1/2)*(
1+d*x^2/c)^(1/2)*EllipticF(b^(1/2)*x/a^(1/2),(-a*d/b/c)^(1/2))/b^(3/2)/d^2
/f^3/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)-a^(1/2)*e^2*(1-b*x^2/a)^(1/2)*(1+d*x
^2/c)^(1/2)*EllipticPi(b^(1/2)*x/a^(1/2),-a*f/b/e,(-a*d/b/c)^(1/2))/b^(1/2
)/f^3/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.06 (sec) , antiderivative size = 331, normalized size of antiderivative = 0.86

$$\int \frac{x^6}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \frac{-\sqrt{-\frac{b}{a}}df^2x(a-bx^2)\,(c+dx^2)-icf(-3bde-2bcf+2adf)\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(i\text{arcsinh}\left(\sqrt{-\frac{b}{a}}x\right)|-\right.}{}$$

input
```
Integrate[x^6/(Sqrt[a - b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)),x]
```

output
```
(-(Sqrt[-(b/a)]*d*f^2*x*(a - b*x^2)*(c + d*x^2)) - I*c*f*(-3*b*d*e - 2*b*c
*f + 2*a*d*f)*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[
Sqrt[-(b/a)]*x], -((a*d)/(b*c))] + I*(a*c*d*f^2 - b*(3*d^2*e^2 + 3*c*d*e*f
 + 2*c^2*f^2))*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh
[Sqrt[-(b/a)]*x], -((a*d)/(b*c))] + (3*I)*b*d^2*e^2*Sqrt[1 - (b*x^2)/a]*Sq
rt[1 + (d*x^2)/c]*EllipticPi[-((a*f)/(b*e)), I*ArcSinh[Sqrt[-(b/a)]*x], -(
(a*d)/(b*c))])/(3*b*Sqrt[-(b/a)]*d^2*f^3*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx$$

$$\Big\downarrow \text{450}$$

$$\int \frac{x^6}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)}dx$$

input
```
Int[x^6/(Sqrt[a - b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [A] (verified)

Time = 19.63 (sec) , antiderivative size = 524, normalized size of antiderivative = 1.36

method	result
risch	$-\frac{x\sqrt{-b x^2+a}\,\sqrt{x^2 d+c}}{3bdf}+\frac{\left(-\frac{f(2adf-2bcf-3bde)c\sqrt{1-\frac{b x^2}{a}}\,\sqrt{1+\frac{d x^2}{c}}\left(\operatorname{EllipticF}\left(x\sqrt{\frac{b}{a}},\sqrt{-1-\frac{ad-bc}{cb}}\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{b}{a}},\sqrt{-1-\frac{ad-bc}{cb}}\right)\right.}{\sqrt{\frac{b}{a}}\,\sqrt{-bd x^4+ad x^2-x^2bc+ac}\,d}\right.}{}$
default	$-\frac{\left(-\sqrt{\frac{b}{a}}\,b d^2 f^2 x^5+\sqrt{\frac{b}{a}}\,a d^2 f^2 x^3-\sqrt{\frac{b}{a}}\,bcd f^2 x^3+\sqrt{\frac{-b x^2+a}{a}}\,\sqrt{\frac{x^2 d+c}{c}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{b}{a}},\sqrt{-\frac{ad}{bc}}\right)acd f^2-2\sqrt{\frac{-b x^2+a}{a}}\,\sqrt{\frac{x^2 d+c}{c}}\,\mathrm{E}\right.}{}$
elliptic	$\frac{\sqrt{(-b x^2+a)(x^2 d+c)}\left(-\frac{x\sqrt{-bd x^4+ad x^2-x^2bc+ac}}{3fbd}+\frac{\sqrt{1-\frac{b x^2}{a}}\,\sqrt{1+\frac{d x^2}{c}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{b}{a}},\sqrt{-1-\frac{ad-bc}{cb}}\right)e^2}{\sqrt{\frac{b}{a}}\,\sqrt{-bd x^4+ad x^2-x^2bc+ac}\,f^3}-\frac{\sqrt{1-\frac{b x^2}{a}}\,\sqrt{1+\frac{d x^2}{c}}\,\mathrm{Ellipti}}{3\sqrt{\frac{b}{a}}\,\sqrt{-bd x^4+ac}}\right.}{}$

input
```
int(x^6/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x,method=_RETURNVERBOSE
)
```

output
```
-1/3*x*(-b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b/d/f+1/3/b/d/f*(1/f^2*(-f*(2*a*d*
f-2*b*c*f-3*b*d*e)*c/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d
*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(b/a)^(1/2),(-1-(a*d-b*c)/c
/b)^(1/2))-EllipticE(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2)))+a*c*f^2/(b/a
)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)
^(1/2)*EllipticF(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2))+3*b*d*e^2/(b/a)^(
1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1
/2)*EllipticF(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2)))-3*b*d*e^2/f^2/(b/a)
^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^
(1/2)*EllipticPi(x*(b/a)^(1/2),-a*f/b/e,(-1/c*d)^(1/2)/(b/a)^(1/2)))*((-b*
x^2+a)*(d*x^2+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^6}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate(x^6/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="fri
cas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^6}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{x^6}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx$$

input
```
integrate(x**6/(-b*x**2+a)**(1/2)/(d*x**2+c)**(1/2)/(f*x**2+e),x)
```

output
```
Integral(x**6/(sqrt(a - b*x**2)*sqrt(c + d*x**2)*(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{x^6}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx=\int \frac{x^6}{\sqrt{-bx^2+a}\sqrt{dx^2+c}(fx^2+e)}\,dx$$

input
```
integrate(x^6/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="max
ima")
```

output
```
integrate(x^6/(sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{x^6}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx=\int \frac{x^6}{\sqrt{-bx^2+a}\sqrt{dx^2+c}(fx^2+e)}\,dx$$

input
```
integrate(x^6/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="gia
c")
```

output
```
integrate(x^6/(sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx=\int \frac{x^6}{\sqrt{a-b\,x^2}\,\sqrt{d\,x^2+c}\,\left(f\,x^2+e\right)}\,dx$$

input
```
int(x^6/((a - b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)),x)
```

output
```
int(x^6/((a - b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)), x)
```

Reduce [F]

$$\int \frac{x^6}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx$$
$$= \frac{-\sqrt{d\,x^2+c}\,\sqrt{-b\,x^2+a}\,x+2\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{-b\,x^2+a}\,x^4}{-bdf\,x^6+adf\,x^4-bcf\,x^4-bde\,x^4+acf\,x^2+ade\,x^2-bce\,x^2+ace}dx\right)adf-2\left(\int \frac{}{-bdf\,x^6+adf}\right.}{}$$

input
```
int(x^6/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x)
```

output
```
( - sqrt(c + d*x**2)*sqrt(a - b*x**2)*x + 2*int((sqrt(c + d*x**2)*sqrt(a -
 b*x**2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 - b*c*e*x**2
- b*c*f*x**4 - b*d*e*x**4 - b*d*f*x**6),x)*a*d*f - 2*int((sqrt(c + d*x**2)
*sqrt(a - b*x**2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 - b*
c*e*x**2 - b*c*f*x**4 - b*d*e*x**4 - b*d*f*x**6),x)*b*c*f - 3*int((sqrt(c
+ d*x**2)*sqrt(a - b*x**2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*
x**4 - b*c*e*x**2 - b*c*f*x**4 - b*d*e*x**4 - b*d*f*x**6),x)*b*d*e + int((
sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**2)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 +
 a*d*f*x**4 - b*c*e*x**2 - b*c*f*x**4 - b*d*e*x**4 - b*d*f*x**6),x)*a*c*f
+ 2*int((sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**2)/(a*c*e + a*c*f*x**2 + a*d
*e*x**2 + a*d*f*x**4 - b*c*e*x**2 - b*c*f*x**4 - b*d*e*x**4 - b*d*f*x**6),
x)*a*d*e - 2*int((sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**2)/(a*c*e + a*c*f*x
**2 + a*d*e*x**2 + a*d*f*x**4 - b*c*e*x**2 - b*c*f*x**4 - b*d*e*x**4 - b*d
*f*x**6),x)*b*c*e + int((sqrt(c + d*x**2)*sqrt(a - b*x**2))/(a*c*e + a*c*f
*x**2 + a*d*e*x**2 + a*d*f*x**4 - b*c*e*x**2 - b*c*f*x**4 - b*d*e*x**4 - b
*d*f*x**6),x)*a*c*e)/(3*b*d*f)
```

3.220 $\int \frac{x^4}{\sqrt{a-bx^2}\sqrt{c+dx^2}(e+fx^2)}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 296

$$\begin{aligned}\int \frac{x^4}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx &= \frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\,E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}df\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}} \\ &- \frac{\sqrt{a}(de+cf)\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}df^2\sqrt{a-bx^2}\sqrt{c+dx^2}} \\ &+ \frac{\sqrt{a}e\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}f^2\sqrt{a-bx^2}\sqrt{c+dx^2}}\end{aligned}$$

output
```
a^(1/2)*(1-b*x^2/a)^(1/2)*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2),(-a*
d/b/c)^(1/2))/b^(1/2)/d/f/(-b*x^2+a)^(1/2)/(1+d*x^2/c)^(1/2)-a^(1/2)*(c*f+
d*e)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(b^(1/2)*x/a^(1/2),(-a*d
/b/c)^(1/2))/b^(1/2)/d/f^2/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)+a^(1/2)*e*(1-b
*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)*EllipticPi(b^(1/2)*x/a^(1/2),-a*f/b/e,(-a*
d/b/c)^(1/2))/b^(1/2)/f^2/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.18 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.61

$$\int \frac{x^4}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx =$$
$$-\frac{i\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\left(cfE\left(i\,\mathrm{arcsinh}\left(\sqrt{-\frac{b}{a}}x\right)|-\frac{ad}{bc}\right)-(de+cf)\,\mathrm{EllipticF}\left(i\,\mathrm{arcsinh}\left(\sqrt{-\frac{b}{a}}x\right),-\frac{ad}{bc}\right)\right.}{\sqrt{-\frac{b}{a}}df^2\sqrt{a-bx^2}\sqrt{c+dx^2}}$$

input
```
Integrate[x^4/(Sqrt[a - b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)),x]
```

output
```
((-I)*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(c*f*EllipticE[I*ArcSinh[Sqr
t[-(b/a)]*x], -((a*d)/(b*c))] - (d*e + c*f)*EllipticF[I*ArcSinh[Sqrt[-(b/a
)]*x], -((a*d)/(b*c))] + d*e*EllipticPi[-((a*f)/(b*e)), I*ArcSinh[Sqrt[-(b
/a)]*x], -((a*d)/(b*c))]))/(Sqrt[-(b/a)]*d*f^2*Sqrt[a - b*x^2]*Sqrt[c + d*
x^2])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx$$
$$\Big\downarrow \; 450$$
$$\int \frac{x^4}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)}dx$$

input
```
Int[x^4/(Sqrt[a - b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [A] (verified)

Time = 5.67 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.69

method	result
default	$\frac{\left(-\operatorname{EllipticF}\left(x\sqrt{\frac{b}{a}},\sqrt{-\frac{ad}{bc}}\right)cf-\operatorname{EllipticF}\left(x\sqrt{\frac{b}{a}},\sqrt{-\frac{ad}{bc}}\right)de+\operatorname{EllipticE}\left(x\sqrt{\frac{b}{a}},\sqrt{-\frac{ad}{bc}}\right)cf+\operatorname{EllipticPi}\left(x\sqrt{\frac{b}{a}},-\frac{af}{be},\frac{\sqrt{-\frac{d}{c}}}{\sqrt{\frac{b}{a}}}\right)de\right)\sqrt{}}{d\sqrt{\frac{b}{a}}\,f^2\left(-bd\,x^4+ad\,x^2-x^2bc+ac\right)}$
elliptic	$\frac{\sqrt{(-b\,x^2+a)(x^2d+c)}\left(-\frac{e\sqrt{1-\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{b}{a}},\sqrt{-1-\frac{ad-bc}{cb}}\right)}{f^2\sqrt{\frac{b}{a}}\,\sqrt{-bd\,x^4+ad\,x^2-x^2bc+ac}}-\frac{c\sqrt{1-\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{b}{a}},\sqrt{-1-\frac{ad-bc}{cb}}\right)}{f\sqrt{\frac{b}{a}}\,\sqrt{-bd\,x^4+ad\,x^2-x^2bc+ac}\,d}+\frac{c\sqrt{}}{}\right.}{\sqrt{-b\,x^2+a}\,\sqrt{x^2d+c}}$

input
```
int(x^4/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x,method=_RETURNVERBOSE
)
```

output
```
(-EllipticF(x*(b/a)^(1/2),(-a*d/b/c)^(1/2))*c*f-EllipticF(x*(b/a)^(1/2),(-
a*d/b/c)^(1/2))*d*e+EllipticE(x*(b/a)^(1/2),(-a*d/b/c)^(1/2))*c*f+Elliptic
Pi(x*(b/a)^(1/2),-a*f/b/e,(-1/c*d)^(1/2)/(b/a)^(1/2))*d*e)*((d*x^2+c)/c)^(
1/2)*((-b*x^2+a)/a)^(1/2)*(d*x^2+c)^(1/2)*(-b*x^2+a)^(1/2)/d/(b/a)^(1/2)/f
^2/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx=\text{Timed out}$$

input
```
integrate(x^4/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="fri
cas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^4}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx=\int \frac{x^4}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx$$

input
```
integrate(x**4/(-b*x**2+a)**(1/2)/(d*x**2+c)**(1/2)/(f*x**2+e),x)
```

output
```
Integral(x**4/(sqrt(a - b*x**2)*sqrt(c + d*x**2)*(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{x^4}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx=\int \frac{x^4}{\sqrt{-bx^2+a}\sqrt{dx^2+c}(fx^2+e)}\,dx$$

input
```
integrate(x^4/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="max
ima")
```

output
```
integrate(x^4/(sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{x^4}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{x^4}{\sqrt{-bx^2+a}\sqrt{dx^2+c}(fx^2+e)}\,dx$$

input
```
integrate(x^4/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="gia
c")
```

output
```
integrate(x^4/(sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{x^4}{\sqrt{a-b\,x^2}\,\sqrt{d\,x^2+c}\,(f\,x^2+e)}\,dx$$

input
```
int(x^4/((a - b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)),x)
```

output
```
int(x^4/((a - b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)), x)
```

Reduce [F]

$$\int \frac{x^4}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx$$
$$= \int \frac{\sqrt{d\,x^2+c}\,\sqrt{-b\,x^2+a}\,x^4}{-bdf\,x^6+adf\,x^4-bcf\,x^4-bde\,x^4+acf\,x^2+ade\,x^2-bce\,x^2+ace}dx$$

input
```
int(x^4/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*e*x
**2 + a*d*f*x**4 - b*c*e*x**2 - b*c*f*x**4 - b*d*e*x**4 - b*d*f*x**6),x)
```

3.221 $\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}(e+fx^2)}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 192

$$\begin{aligned}\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx &= \frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}f\sqrt{a-bx^2}\sqrt{c+dx^2}} \\ &\quad - \frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}f\sqrt{a-bx^2}\sqrt{c+dx^2}}\end{aligned}$$

output
```
a^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(b^(1/2)*x/a^(1/2),(-
a*d/b/c)^(1/2))/b^(1/2)/f/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)-a^(1/2)*(1-b*x^
2/a)^(1/2)*(1+d*x^2/c)^(1/2)*EllipticPi(b^(1/2)*x/a^(1/2),-a*f/b/e,(-a*d/b
/c)^(1/2))/b^(1/2)/f/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.15 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.72

$$\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = -\frac{i\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\left(\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{-\frac{b}{a}}x\right),-\frac{ad}{bc}\right)-\text{EllipticPi}\left(-\frac{af}{be},i\text{arcsinh}\left(\sqrt{-\frac{b}{a}}x\right),-\frac{ad}{bc}\right)\right)}{\sqrt{-\frac{b}{a}}f\sqrt{a-bx^2}\sqrt{c+dx^2}}$$

input
```
Integrate[x^2/(Sqrt[a - b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)),x]
```

output
```
((-I)*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(EllipticF[I*ArcSinh[Sqrt[-(
b/a)]*x], -((a*d)/(b*c))] - EllipticPi[-((a*f)/(b*e)), I*ArcSinh[Sqrt[-(b/
a)]*x], -((a*d)/(b*c))]))/(Sqrt[-(b/a)]*f*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx$$
$$\Big\downarrow \ 450$$
$$\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)}dx$$

input
```
Int[x^2/(Sqrt[a - b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [A] (verified)

Time = 5.74 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.76

method	result	siz
default	$\frac{\left(\operatorname{EllipticF}\left(x\sqrt{\frac{b}{a}},\sqrt{-\frac{ad}{bc}}\right)-\operatorname{EllipticPi}\left(x\sqrt{\frac{b}{a}},-\frac{af}{be},\frac{\sqrt{-\frac{d}{c}}}{\sqrt{\frac{b}{a}}}\right)\right)\sqrt{\frac{x^{2}d+c}{c}}\,\sqrt{\frac{-b\,x^{2}+a}{a}}\,\sqrt{x^{2}d+c}\,\sqrt{-b\,x^{2}+a}}{f\sqrt{\frac{b}{a}}\,(-bd\,x^{4}+ad\,x^{2}-x^{2}bc+ac)}$	14
elliptic	$\frac{\sqrt{(-b\,x^{2}+a)(x^{2}d+c)}\,\left(\frac{\sqrt{1-\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{b}{a}},\sqrt{-1-\frac{ad-bc}{cb}}\right)}{f\sqrt{\frac{b}{a}}\,\sqrt{-bd\,x^{4}+ad\,x^{2}-x^{2}bc+ac}}-\frac{\sqrt{1-\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,\operatorname{EllipticPi}\left(x\sqrt{\frac{b}{a}},-\frac{af}{be},\frac{\sqrt{-\frac{d}{c}}}{\sqrt{\frac{b}{a}}}\right)}{f\sqrt{\frac{b}{a}}\,\sqrt{-bd\,x^{4}+ad\,x^{2}-x^{2}bc+ac}}\right)}{\sqrt{-b\,x^{2}+a}\,\sqrt{x^{2}d+c}}$	23

input
```
int(x^2/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x,method=_RETURNVERBOSE
)
```

output
```
(EllipticF(x*(b/a)^(1/2),(-a*d/b/c)^(1/2))-EllipticPi(x*(b/a)^(1/2),-a*f/b
/e,(-1/c*d)^(1/2)/(b/a)^(1/2)))/f*((d*x^2+c)/c)^(1/2)*((-b*x^2+a)/a)^(1/2)
*(d*x^2+c)^(1/2)*(-b*x^2+a)^(1/2)/(b/a)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*
c)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx=\text{Timed out}$$

input
```
integrate(x^2/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="fri
cas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx$$

input
```
integrate(x**2/(-b*x**2+a)**(1/2)/(d*x**2+c)**(1/2)/(f*x**2+e),x)
```

output
```
Integral(x**2/(sqrt(a - b*x**2)*sqrt(c + d*x**2)*(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{x^2}{\sqrt{-bx^2+a}\sqrt{dx^2+c}(fx^2+e)}\,dx$$

input
```
integrate(x^2/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="max
ima")
```

output
```
integrate(x^2/(sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{x^2}{\sqrt{-bx^2+a}\sqrt{dx^2+c}(fx^2+e)}\,dx$$

input
```
integrate(x^2/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="gia
c")
```

output
```
integrate(x^2/(sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{x^2}{\sqrt{a-b\,x^2}\,\sqrt{d\,x^2+c}\,\left(f\,x^2+e\right)}\,dx$$

input
```
int(x^2/((a - b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)),x)
```

output
```
int(x^2/((a - b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)), x)
```

Reduce [F]

$$\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{\sqrt{d\,x^2+c}\,\sqrt{-b\,x^2+a}\,x^2}{-bdf\,x^6+adf\,x^4-bcf\,x^4-bde\,x^4+acf\,x^2+ade\,x^2-bce\,x^2+ace}dx$$

input
```
int(x^2/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**2)/(a*c*e + a*c*f*x**2 + a*d*e*x
**2 + a*d*f*x**4 - b*c*e*x**2 - b*c*f*x**4 - b*d*e*x**4 - b*d*f*x**6),x)
```

3.222 $\int \frac{1}{\sqrt{a-bx^2}\sqrt{c+dx^2}(e+fx^2)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 100

$$\int \frac{1}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}e\sqrt{a-bx^2}\sqrt{c+dx^2}}$$

output
```
a^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)*EllipticPi(b^(1/2)*x/a^(1/2),-
a*f/b/e,(-a*d/b/c)^(1/2))/b^(1/2)/e/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.98 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.07

$$\int \frac{1}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = -\frac{i\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{af}{be},i\text{arcsinh}\left(\sqrt{-\frac{b}{a}}x\right),-\frac{ad}{bc}\right)}{\sqrt{-\frac{b}{a}}e\sqrt{a-bx^2}\sqrt{c+dx^2}}$$

input
```
Integrate[1/(Sqrt[a - b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)),x]
```

output
```
((-I)*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[-((a*f)/(b*e)), I
*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))])/(Sqrt[-(b/a)]*e*Sqrt[a - b*x^2]
*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {413, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)}dx$$

$$\Big\downarrow \ 413$$

$$\frac{\sqrt{1-\frac{bx^2}{a}}\int \frac{1}{\sqrt{1-\frac{bx^2}{a}}\sqrt{dx^2+c}(fx^2+e)}dx}{\sqrt{a-bx^2}}$$

$$\Big\downarrow \ 413$$

$$\frac{\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\int \frac{1}{\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(fx^2+e)}dx}{\sqrt{a-bx^2}\sqrt{c+dx^2}}$$

$$\Big\downarrow \ 412$$

$$\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(-\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}e\sqrt{a-bx^2}\sqrt{c+dx^2}}$$

input
```
Int[1/(Sqrt[a - b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)),x]
```

output
```
(Sqrt[a]*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[-((a*f)/(b*e))
, ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*e*Sqrt[a - b*x^2]
*Sqrt[c + d*x^2])
```

Defintions of rubi rules used

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

Maple [A] (verified)

Time = 5.73 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.20

method	result	size
default	$\frac{\operatorname{EllipticPi}\left(x\sqrt{\frac{b}{a}}, -\frac{af}{be}, \frac{\sqrt{-\frac{d}{c}}}{\sqrt{\frac{b}{a}}}\right)\sqrt{\frac{x^2 d+c}{c}}\,\sqrt{\frac{-b\,x^2+a}{a}}\,\sqrt{x^2 d+c}\,\sqrt{-b\,x^2+a}}{e\sqrt{\frac{b}{a}}\,(-bd\,x^4+ad\,x^2-x^2bc+ac)}$	120
elliptic	$\frac{\sqrt{(-b\,x^2+a)(x^2 d+c)}\,\sqrt{1-\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\operatorname{EllipticPi}\left(x\sqrt{\frac{b}{a}}, -\frac{af}{be}, \frac{\sqrt{-\frac{d}{c}}}{\sqrt{\frac{b}{a}}}\right)}{\sqrt{-b\,x^2+a}\,\sqrt{x^2 d+c}\,e\sqrt{\frac{b}{a}}\,\sqrt{-bd\,x^4+ad\,x^2-x^2bc+ac}}$	136

input
```
int(1/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
EllipticPi(x*(b/a)^(1/2),-a*f/b/e,(-1/c*d)^(1/2)/(b/a)^(1/2))*((d*x^2+c)/c
)^(1/2)*((-b*x^2+a)/a)^(1/2)*(d*x^2+c)^(1/2)*(-b*x^2+a)^(1/2)/e/(b/a)^(1/2
)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate(1/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="frica
s")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{1}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx$$

input
```
integrate(1/(-b*x**2+a)**(1/2)/(d*x**2+c)**(1/2)/(f*x**2+e),x)
```

output
```
Integral(1/(sqrt(a - b*x**2)*sqrt(c + d*x**2)*(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{1}{\sqrt{-bx^2+a}\sqrt{dx^2+c}(fx^2+e)}\,dx$$

input
```
integrate(1/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="maxim
a")
```

output
```
integrate(1/(sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{1}{\sqrt{-bx^2+a}\sqrt{dx^2+c}(fx^2+e)}\,dx$$

input
```
integrate(1/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="giac"
)
```

output
```
integrate(1/(sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{1}{\sqrt{a-b\,x^2}\,\sqrt{d\,x^2+c}\,(f\,x^2+e)}\,dx$$

input
```
int(1/((a - b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)),x)
```

output
```
int(1/((a - b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx$$
$$= \int \frac{\sqrt{d\,x^2+c}\,\sqrt{-b\,x^2+a}}{-bdf\,x^6+adf\,x^4-bcf\,x^4-bde\,x^4+acf\,x^2+ade\,x^2-bce\,x^2+ace}dx$$

input
```
int(1/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a - b*x**2))/(a*c*e + a*c*f*x**2 + a*d*e*x**2 +
 a*d*f*x**4 - b*c*e*x**2 - b*c*f*x**4 - b*d*e*x**4 - b*d*f*x**6),x)
```

3.223 $\int \frac{1}{x^2\sqrt{a-bx^2}\sqrt{c+dx^2}(e+fx^2)}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 324

$$\int \frac{1}{x^2\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = -\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}}{acex} - \frac{\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{a}ce\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}} + \frac{\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{a}e\sqrt{a-bx^2}\sqrt{c+dx^2}} - \frac{\sqrt{a}f\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}e^2\sqrt{a-bx^2}\sqrt{c+dx^2}}$$

output
```
-(-b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a/c/e/x-b^(1/2)*(1-b*x^2/a)^(1/2)*(d*x^2
+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2),(-a*d/b/c)^(1/2))/a^(1/2)/c/e/(-b*x^
2+a)^(1/2)/(1+d*x^2/c)^(1/2)+b^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)*E
llipticF(b^(1/2)*x/a^(1/2),(-a*d/b/c)^(1/2))/a^(1/2)/e/(-b*x^2+a)^(1/2)/(d
*x^2+c)^(1/2)-a^(1/2)*f*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)*EllipticPi(b^(
1/2)*x/a^(1/2),-a*f/b/e,(-a*d/b/c)^(1/2))/b^(1/2)/e^2/(-b*x^2+a)^(1/2)/(d*
x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.14 (sec) , antiderivative size = 280, normalized size of antiderivative = 0.86

$$\int \frac{1}{x^2\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx$$
$$= \frac{-\sqrt{-\frac{b}{a}}e(a-bx^2)\,(c+dx^2)+ibcex\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\Big(i\text{arcsinh}\Big(\sqrt{-\frac{b}{a}}x\Big)\,|-\frac{ad}{bc}\Big)-ibcex\sqrt{1-\frac{bx^2}{a}}\sqrt{}}{a\sqrt{-\frac{b}{a}}ce^2}$$

input
```
Integrate[1/(x^2*Sqrt[a - b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)),x]
```

output
```
(-(Sqrt[-(b/a)]*e*(a - b*x^2)*(c + d*x^2)) + I*b*c*e*x*Sqrt[1 - (b*x^2)/a]
*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))]
- I*b*c*e*x*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sq
rt[-(b/a)]*x], -((a*d)/(b*c))] + I*a*c*f*x*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d
*x^2)/c]*EllipticPi[-((a*f)/(b*e)), I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*
c))])/(a*Sqrt[-(b/a)]*c*e^2*x*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx$$
$$\Big\downarrow \; 450$$
$$\int \frac{1}{x^2\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)}dx$$

input
```
Int[1/(x^2*Sqrt[a - b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [A] (verified)

Time = 8.66 (sec) , antiderivative size = 300, normalized size of antiderivative = 0.93

method	result
default	$\frac{\left(\sqrt{\frac{b}{a}}\, bde\, x^4+\sqrt{\frac{-b\,x^2+a}{a}}\,\sqrt{\frac{x^2d+c}{c}}\ \text{EllipticF}\left(x\sqrt{\frac{b}{a}},\sqrt{-\frac{ad}{bc}}\right)bcex-\sqrt{\frac{-b\,x^2+a}{a}}\,\sqrt{\frac{x^2d+c}{c}}\ \text{EllipticE}\left(x\sqrt{\frac{b}{a}},\sqrt{-\frac{ad}{bc}}\right)bcex-\sqrt{\frac{-b\,x^2+}{a}}\right.}{\sqrt{\frac{b}{a}}\,x\,e^2ca(-bd\,x^4+ad\,x^2-x^2bc}$
risch	$-\frac{\sqrt{-b\,x^2+a}\,\sqrt{x^2d+c}}{acex}-\frac{\left(-\frac{bc\sqrt{1-\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\left(\text{EllipticF}\left(x\sqrt{\frac{b}{a}},\sqrt{-1-\frac{ad-bc}{cb}}\right)-\text{EllipticE}\left(x\sqrt{\frac{b}{a}},\sqrt{-1-\frac{ad-bc}{cb}}\right)\right)}{\sqrt{\frac{b}{a}}\,\sqrt{-bd\,x^4+ad\,x^2-x^2bc+ac}}+\frac{acf\sqrt{1-\frac{b\,x^2}{a}}\,\sqrt{1}}{e\sqrt{\frac{b}{a}}}\right.}{ace\sqrt{-b\,x^2+a}\,\sqrt{x^2d+c}}$
elliptic	$\frac{\sqrt{(-b\,x^2+a)(x^2d+c)}\left(-\frac{\sqrt{-bd\,x^4+ad\,x^2-x^2bc+ac}}{acex}+\frac{b\sqrt{1-\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\ \text{EllipticF}\left(x\sqrt{\frac{b}{a}},\sqrt{-1-\frac{ad-bc}{cb}}\right)}{ae\sqrt{\frac{b}{a}}\,\sqrt{-bd\,x^4+ad\,x^2-x^2bc+ac}}-\frac{b\sqrt{1-\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\ \text{EllipticE}}{ae\sqrt{\frac{b}{a}}\,\sqrt{-bd\,x^4+ad}}\right.}{\sqrt{-b\,x^2+a}\,\sqrt{x^2d+c}}$

input
```
int(1/x^2/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x,method=_RETURNVERBO
SE)
```

output
```
((b/a)^(1/2)*b*d*e*x^4+((-b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(
x*(b/a)^(1/2),(-a*d/b/c)^(1/2))*b*c*e*x-((-b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)
^(1/2)*EllipticE(x*(b/a)^(1/2),(-a*d/b/c)^(1/2))*b*c*e*x-((-b*x^2+a)/a)^(1
/2)*((d*x^2+c)/c)^(1/2)*EllipticPi(x*(b/a)^(1/2),-a*f/b/e,(-1/c*d)^(1/2)/(
b/a)^(1/2))*a*c*f*x-(b/a)^(1/2)*a*d*e*x^2+(b/a)^(1/2)*b*c*e*x^2-(b/a)^(1/2
)*a*c*e)*(d*x^2+c)^(1/2)*(-b*x^2+a)^(1/2)/(b/a)^(1/2)/x/e^2/c/a/(-b*d*x^4+
a*d*x^2-b*c*x^2+a*c)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate(1/x^2/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="f
ricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{x^2\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{1}{x^2\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx$$

input
```
integrate(1/x**2/(-b*x**2+a)**(1/2)/(d*x**2+c)**(1/2)/(f*x**2+e),x)
```

output
```
Integral(1/(x**2*sqrt(a - b*x**2)*sqrt(c + d*x**2)*(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{x^2\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{1}{\sqrt{-bx^2+a}\sqrt{dx^2+c}(fx^2+e)x^2}\,dx$$

input
```
integrate(1/x^2/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="m
axima")
```

output
```
integrate(1/(sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{1}{\sqrt{-bx^2+a}\sqrt{dx^2+c}(fx^2+e)x^2}\,dx$$

input
```
integrate(1/x^2/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="g
iac")
```

output
```
integrate(1/(sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{1}{x^2\,\sqrt{a-b\,x^2}\,\sqrt{d\,x^2+c}\,\left(f\,x^2+e\right)}\,dx$$

input
```
int(1/(x^2*(a - b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)),x)
```

output
```
int(1/(x^2*(a - b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)), x)
```

Reduce [F]

$$\int \frac{1}{x^2\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{\sqrt{d\,x^2+c}\,\sqrt{-b\,x^2+a}}{-bdf\,x^8+adf\,x^6-bcf\,x^6-bde\,x^6+acf\,x^4+ade\,x^4-bce\,x^4+ace\,x^2}dx$$

input
```
int(1/x^2/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a - b*x**2))/(a*c*e*x**2 + a*c*f*x**4 + a*d*e*x
**4 + a*d*f*x**6 - b*c*e*x**4 - b*c*f*x**6 - b*d*e*x**6 - b*d*f*x**8),x)
```

3.224 $\int \frac{1}{x^4\sqrt{a-bx^2}\sqrt{c+dx^2}(e+fx^2)}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 422

$$\begin{aligned}
&\int \frac{1}{x^4\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx \\
&= -\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}}{3acex^3} - \frac{(2bce-2ade-3acf)\sqrt{a-bx^2}\sqrt{c+dx^2}}{3a^2c^2e^2x} \\
&\quad - \frac{\sqrt{b}(2bce-2ade-3acf)\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\,E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{3a^{3/2}c^2e^2\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}} \\
&\quad + \frac{\sqrt{b}(2bce-ade-3acf)\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{3a^{3/2}ce^2\sqrt{a-bx^2}\sqrt{c+dx^2}} \\
&\quad + \frac{\sqrt{a}f^2\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\mathrm{EllipticPi}\left(-\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}e^3\sqrt{a-bx^2}\sqrt{c+dx^2}}
\end{aligned}$$

output
```
-1/3*(-b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a/c/e/x^3-1/3*(-3*a*c*f-2*a*d*e+2*b*
c*e)*(-b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a^2/c^2/e^2/x-1/3*b^(1/2)*(-3*a*c*f-
2*a*d*e+2*b*c*e)*(1-b*x^2/a)^(1/2)*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(
1/2),(-a*d/b/c)^(1/2))/a^(3/2)/c^2/e^2/(-b*x^2+a)^(1/2)/(1+d*x^2/c)^(1/2)+
1/3*b^(1/2)*(-3*a*c*f-a*d*e+2*b*c*e)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)*E
llipticF(b^(1/2)*x/a^(1/2),(-a*d/b/c)^(1/2))/a^(3/2)/c/e^2/(-b*x^2+a)^(1/2
)/(d*x^2+c)^(1/2)+a^(1/2)*f^2*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)*Elliptic
Pi(b^(1/2)*x/a^(1/2),-a*f/b/e,(-a*d/b/c)^(1/2))/b^(1/2)/e^3/(-b*x^2+a)^(1/
2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.52 (sec) , antiderivative size = 488, normalized size of antiderivative = 1.16

$$\int \frac{1}{x^4\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx$$
$$= \frac{-a^2bc^2e^2-ab^2c^2e^2x^2+a^2bcde^2x^2+3a^2bc^2efx^2+2b^3c^2e^2x^4-3ab^2cde^2x^4+2a^2bd^2e^2x^4-3ab^2c^2efx^4 \cdots}{\cdots}$$

input
```
Integrate[1/(x^4*Sqrt[a - b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)),x]
```

output
```
(-(a^2*b*c^2*e^2) - a*b^2*c^2*e^2*x^2 + a^2*b*c*d*e^2*x^2 + 3*a^2*b*c^2*e*
f*x^2 + 2*b^3*c^2*e^2*x^4 - 3*a*b^2*c*d*e^2*x^4 + 2*a^2*b*d^2*e^2*x^4 - 3*
a*b^2*c^2*e*f*x^4 + 3*a^2*b*c*d*e*f*x^4 + 2*b^3*c*d*e^2*x^6 - 2*a*b^2*d^2*
e^2*x^6 - 3*a*b^2*c*d*e*f*x^6 + I*a*b*Sqrt[-(b/a)]*c*e*(-2*b*c*e + 2*a*d*e
 + 3*a*c*f)*x^3*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSin
h[Sqrt[-(b/a)]*x], -((a*d)/(b*c))] - I*a*b*Sqrt[-(b/a)]*c*e*(-2*b*c*e + a*
d*e + 3*a*c*f)*x^3*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*Arc
Sinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))] + (3*I)*a^3*Sqrt[-(b/a)]*c^2*f^2*x^3
*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[-((a*f)/(b*e)), I*ArcS
inh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))])/(3*a^2*b*c^2*e^3*x^3*Sqrt[a - b*x^2]
*Sqrt[c + d*x^2])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx$$
$$\Big\downarrow \; 450$$
$$\int \frac{1}{x^4\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)}dx$$

input
```
Int[1/(x^4*Sqrt[a - b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [A] (verified)

Time = 10.36 (sec) , antiderivative size = 456, normalized size of antiderivative = 1.08

method	result
risch	$-\frac{\sqrt{-b\,x^2+a}\,\sqrt{x^2d+c}\,(-3acf\,x^2-2ade\,x^2+2bce\,x^2+ace)}{3a^2c^2e^2x^3}+\frac{\left(-\frac{b(3acf+2ade-2bce)c\sqrt{1-\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\left(\mathrm{EllipticF}\left(x\sqrt{\frac{b}{a}},\sqrt{-1-\frac{ad-\cdots}{cb}}\right)\cdots\right.}{\sqrt{\frac{b}{a}}\,\sqrt{-bd\,x^4+ad\,x^2-x^2bc+ac\cdots}}\right.\cdots}{\cdots}$
default	$-\frac{\left(3\sqrt{\frac{b}{a}}\,abcdef\,x^6+2\sqrt{\frac{b}{a}}\,ab\,d^2e^2x^6-2\sqrt{\frac{b}{a}}\,b^2cd\,e^2x^6+3\sqrt{\frac{-b\,x^2+a}{a}}\,\sqrt{\frac{x^2d+c}{c}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{b}{a}},\sqrt{-\frac{ad}{bc}}\right)ab\,c^2ef\,x^3+\sqrt{\frac{-b\,x^2+a}{a}}\,\sqrt{\cdots}\right.\cdots}{\cdots}$
elliptic	$\frac{\sqrt{(-b\,x^2+a)(x^2d+c)}\left(-\frac{\sqrt{-bd\,x^4+ad\,x^2-x^2bc+ac}}{3ace\,x^3}+\frac{(3acf+2ade-2bce)\sqrt{-bd\,x^4+ad\,x^2-x^2bc+ac}}{3a^2c^2e^2x}-\frac{db\sqrt{1-\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{b}{a}},\cdots\right)}{3ace\sqrt{\frac{b}{a}}\,\sqrt{-bd\,x^4+ad\,x^2-x^2\cdots}}\cdots\right)}{\cdots}$

input

```
int(1/x^4/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x,method=_RETURNVERBO
SE)
```

output

```
-1/3*(-b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(-3*a*c*f*x^2-2*a*d*e*x^2+2*b*c*e*x^
2+a*c*e)/a^2/c^2/e^2/x^3+1/3/a^2/c^2/e^2*(-b*(3*a*c*f+2*a*d*e-2*b*c*e)*c/(
b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a
*c)^(1/2)*(EllipticF(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2))-EllipticE(x*(
b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2)))+a*b*c*d*e/(b/a)^(1/2)*(1-b*x^2/a)^(1
/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)*EllipticF(x*(b/
a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2))+3*a^2*c^2*f^2/e/(b/a)^(1/2)*(1-b*x^2/a)
^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)*EllipticPi(x
*(b/a)^(1/2),-a*f/b/e,(-1/c*d)^(1/2)/(b/a)^(1/2)))*((-b*x^2+a)*(d*x^2+c))^
(1/2)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^4\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx=\text{Timed out}$$

input
```
integrate(1/x^4/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="f
ricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{x^4\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx=\int \frac{1}{x^4\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx$$

input
```
integrate(1/x**4/(-b*x**2+a)**(1/2)/(d*x**2+c)**(1/2)/(f*x**2+e),x)
```

output
```
Integral(1/(x**4*sqrt(a - b*x**2)*sqrt(c + d*x**2)*(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{x^4\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx=\int \frac{1}{\sqrt{-bx^2+a}\sqrt{dx^2+c}(fx^2+e)x^4}\,dx$$

input
```
integrate(1/x^4/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="m
axima")
```

output
```
integrate(1/(sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)*x^4), x)
```

Giac [F]

$$\int \frac{1}{x^4\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{1}{\sqrt{-bx^2+a}\sqrt{dx^2+c}(fx^2+e)x^4}\,dx$$

input
```
integrate(1/x^4/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="g
iac")
```

output
```
integrate(1/(sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{1}{x^4\,\sqrt{a-b\,x^2}\,\sqrt{d\,x^2+c}\,\left(f\,x^2+e\right)}\,dx$$

input
```
int(1/(x^4*(a - b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)),x)
```

output
```
int(1/(x^4*(a - b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)), x)
```

Reduce [F]

$$\int \frac{1}{x^4\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx$$
$$= \frac{-\sqrt{d\,x^2+c}\,\sqrt{-b\,x^2+a}+\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{-b\,x^2+a}\,x^2}{-bdf\,x^6+adf\,x^4-bcf\,x^4-bde\,x^4+acf\,x^2+ade\,x^2-bce\,x^2+ace}dx\right) bdf\,x^3-3\left(\int \frac{}{-bdf\,x^8+adf}\right.}{}$$

input
```
int(1/x^4/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x)
```

output
```
( - sqrt(c + d*x**2)*sqrt(a - b*x**2) + int((sqrt(c + d*x**2)*sqrt(a - b*x
**2)*x**2)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 - b*c*e*x**2 - b*
c*f*x**4 - b*d*e*x**4 - b*d*f*x**6),x)*b*d*f*x**3 - 3*int((sqrt(c + d*x**2
)*sqrt(a - b*x**2))/(a*c*e*x**2 + a*c*f*x**4 + a*d*e*x**4 + a*d*f*x**6 - b
*c*e*x**4 - b*c*f*x**6 - b*d*e*x**6 - b*d*f*x**8),x)*a*c*f*x**3 - 2*int((s
qrt(c + d*x**2)*sqrt(a - b*x**2))/(a*c*e*x**2 + a*c*f*x**4 + a*d*e*x**4 +
a*d*f*x**6 - b*c*e*x**4 - b*c*f*x**6 - b*d*e*x**6 - b*d*f*x**8),x)*a*d*e*x
**3 + 2*int((sqrt(c + d*x**2)*sqrt(a - b*x**2))/(a*c*e*x**2 + a*c*f*x**4 +
 a*d*e*x**4 + a*d*f*x**6 - b*c*e*x**4 - b*c*f*x**6 - b*d*e*x**6 - b*d*f*x*
*8),x)*b*c*e*x**3 - 2*int((sqrt(c + d*x**2)*sqrt(a - b*x**2))/(a*c*e + a*c
*f*x**2 + a*d*e*x**2 + a*d*f*x**4 - b*c*e*x**2 - b*c*f*x**4 - b*d*e*x**4 -
 b*d*f*x**6),x)*a*d*f*x**3 + 2*int((sqrt(c + d*x**2)*sqrt(a - b*x**2))/(a*
c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 - b*c*e*x**2 - b*c*f*x**4 - b*d
*e*x**4 - b*d*f*x**6),x)*b*c*f*x**3 + int((sqrt(c + d*x**2)*sqrt(a - b*x**
2))/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 - b*c*e*x**2 - b*c*f*x**
4 - b*d*e*x**4 - b*d*f*x**6),x)*b*d*e*x**3)/(3*a*c*e*x**3)
```

3.225 $\int \frac{1}{x^6\sqrt{a-bx^2}\sqrt{c+dx^2}(e+fx^2)}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 584

$$\int \frac{1}{x^6\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx$$
$$= -\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}}{5acex^5} - \frac{(4bce-4ade-5acf)\sqrt{a-bx^2}\sqrt{c+dx^2}}{15a^2c^2e^2x^3}$$
$$-\frac{\left(\frac{8b^2ce}{a}-7bde+\frac{8ad^2e}{c}-10bcf+10adf+\frac{15acf^2}{e}\right)\sqrt{a-bx^2}\sqrt{c+dx^2}}{15a^2c^2e^2x}$$
$$-\frac{\sqrt{b}(8b^2c^2e^2-abce(7de+10cf)+a^2(8d^2e^2+10cdef+15c^2f^2))\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|\right.}{15a^{5/2}c^3e^3\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}}$$
$$+\frac{\sqrt{b}(8b^2c^2e^2-abce(3de+10cf)+a^2(4d^2e^2+5cdef+15c^2f^2))\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\right.}{15a^{5/2}c^2e^3\sqrt{a-bx^2}\sqrt{c+dx^2}}$$
$$-\frac{\sqrt{a}f^3\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}e^4\sqrt{a-bx^2}\sqrt{c+dx^2}}$$

output
```
-1/5*(-b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a/c/e/x^5-1/15*(-5*a*c*f-4*a*d*e+4*b
*c*e)*(-b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a^2/c^2/e^2/x^3-1/15*(8*b^2*c*e/a-7
*b*d*e+8*a*d^2*e/c-10*b*c*f+10*a*d*f+15*a*c*f^2/e)*(-b*x^2+a)^(1/2)*(d*x^2
+c)^(1/2)/a^2/c^2/e^2/x-1/15*b^(1/2)*(8*b^2*c^2*e^2-a*b*c*e*(10*c*f+7*d*e)
+a^2*(15*c^2*f^2+10*c*d*e*f+8*d^2*e^2))*(1-b*x^2/a)^(1/2)*(d*x^2+c)^(1/2)*
EllipticE(b^(1/2)*x/a^(1/2),(-a*d/b/c)^(1/2))/a^(5/2)/c^3/e^3/(-b*x^2+a)^(
1/2)/(1+d*x^2/c)^(1/2)+1/15*b^(1/2)*(8*b^2*c^2*e^2-a*b*c*e*(10*c*f+3*d*e)+
a^2*(15*c^2*f^2+5*c*d*e*f+4*d^2*e^2))*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)*
EllipticF(b^(1/2)*x/a^(1/2),(-a*d/b/c)^(1/2))/a^(5/2)/c^2/e^3/(-b*x^2+a)^(
1/2)/(d*x^2+c)^(1/2)-a^(1/2)*f^3*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)*Ellip
ticPi(b^(1/2)*x/a^(1/2),-a*f/b/e,(-a*d/b/c)^(1/2))/b^(1/2)/e^4/(-b*x^2+a)^
(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.16 (sec) , antiderivative size = 781, normalized size of antiderivative = 1.34

$$\int \frac{1}{x^6\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx =$$

$$-\frac{3a^3c^3e^3 + a^2bc^3e^3x^2 - a^3c^2de^3x^2 - 5a^3c^3e^2fx^2 + 4ab^2c^3e^3x^4 - 2a^2bc^2de^3x^4 + 4a^3cd^2e^3x^4 - 5a^2bc^3e^2f}{}$$

input
```
Integrate[1/(x^6*Sqrt[a - b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)),x]
```

output
```
-1/15*(3*a^3*c^3*e^3 + a^2*b*c^3*e^3*x^2 - a^3*c^2*d*e^3*x^2 - 5*a^3*c^3*e
^2*f*x^2 + 4*a*b^2*c^3*e^3*x^4 - 2*a^2*b*c^2*d*e^3*x^4 + 4*a^3*c*d^2*e^3*x
^4 - 5*a^2*b*c^3*e^2*f*x^4 + 5*a^3*c^2*d*e^2*f*x^4 + 15*a^3*c^3*e*f^2*x^4
- 8*b^3*c^3*e^3*x^6 + 11*a*b^2*c^2*d*e^3*x^6 - 11*a^2*b*c*d^2*e^3*x^6 + 8*
a^3*d^3*e^3*x^6 + 10*a*b^2*c^3*e^2*f*x^6 - 15*a^2*b*c^2*d*e^2*f*x^6 + 10*a
^3*c*d^2*e^2*f*x^6 - 15*a^2*b*c^3*e*f^2*x^6 + 15*a^3*c^2*d*e*f^2*x^6 - 8*b
^3*c^2*d*e^3*x^8 + 7*a*b^2*c*d^2*e^3*x^8 - 8*a^2*b*d^3*e^3*x^8 + 10*a*b^2*
c^2*d*e^2*f*x^8 - 10*a^2*b*c*d^2*e^2*f*x^8 - 15*a^2*b*c^2*d*e*f^2*x^8 + I*
a*Sqrt[-(b/a)]*c*e*(8*b^2*c^2*e^2 - a*b*c*e*(7*d*e + 10*c*f) + a^2*(8*d^2*
e^2 + 10*c*d*e*f + 15*c^2*f^2))*x^5*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c
]*EllipticE[I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))] + (I*b*c*e*(8*b^2*c
^2*e^2 - a*b*c*e*(3*d*e + 10*c*f) + a^2*(4*d^2*e^2 + 5*c*d*e*f + 15*c^2*f^
2))*x^5*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[-
(b/a)]*x], -((a*d)/(b*c))])/Sqrt[-(b/a)] - ((15*I)*a^3*c^3*f^3*x^5*Sqrt[1
- (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[-((a*f)/(b*e)), I*ArcSinh[Sqrt
[-(b/a)]*x], -((a*d)/(b*c))])/Sqrt[-(b/a)])/(a^3*c^3*e^4*x^5*Sqrt[a - b*x^
2]*Sqrt[c + d*x^2])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx$$
$$\Big\downarrow \ 450$$
$$\int \frac{1}{x^6\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)}dx$$

input
```
Int[1/(x^6*Sqrt[a - b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [A] (verified)

Time = 19.96 (sec) , antiderivative size = 809, normalized size of antiderivative = 1.39

method	result
risch	$-\frac{\sqrt{-b x^2+a}\,\sqrt{x^2 d+c}\,(15 f^2 x^4 a^2 c^2+10 a^2 c d e f x^4+8 a^2 d^2 e^2 x^4-10 a b c^2 e f x^4-7 a b c d e^2 x^4+8 b^2 c^2 e^2 x^4-5 a^2 c^2 e f x^2-4 a^2 c d e^2 x^2+4}{15 a^3 c^3 e^3 x^5}$
default	Expression too large to display
elliptic	Expression too large to display

input
```
int(1/x^6/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x,method=_RETURNVERBO
SE)
```

output
```
-1/15*(-b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(15*a^2*c^2*f^2*x^4+10*a^2*c*d*e*f*
x^4+8*a^2*d^2*e^2*x^4-10*a*b*c^2*e*f*x^4-7*a*b*c*d*e^2*x^4+8*b^2*c^2*e^2*x
^4-5*a^2*c^2*e*f*x^2-4*a^2*c*d*e^2*x^2+4*a*b*c^2*e^2*x^2+3*a^2*c^2*e^2)/a^
3/c^3/e^3/x^5-1/15/c^3/a^3/e^3*(-b*(15*a^2*c^2*f^2+10*a^2*c*d*e*f+8*a^2*d^
2*e^2-10*a*b*c^2*e*f-7*a*b*c*d*e^2+8*b^2*c^2*e^2)*c/(b/a)^(1/2)*(1-b*x^2/a
)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)*(EllipticF(
x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2))-EllipticE(x*(b/a)^(1/2),(-1-(a*d-b
*c)/c/b)^(1/2)))+15*c^3*a^3*f^3/e/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c
)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)*EllipticPi(x*(b/a)^(1/2),-a*f
/b/e,(-1/c*d)^(1/2)/(b/a)^(1/2))-4*a*b^2*c^2*d*e^2/(b/a)^(1/2)*(1-b*x^2/a)
^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)*EllipticF(x*
(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2))+4*a^2*b*c*d^2*e^2/(b/a)^(1/2)*(1-b*x
^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)*Ellipti
cF(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2))+5*a^2*b*c^2*d*e*f/(b/a)^(1/2)*(
1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)*El
lipticF(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2)))*((-b*x^2+a)*(d*x^2+c))^(1
/2)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^6\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate(1/x^6/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="f
ricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{x^6\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{1}{x^6\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx$$

input
```
integrate(1/x**6/(-b*x**2+a)**(1/2)/(d*x**2+c)**(1/2)/(f*x**2+e),x)
```

output
```
Integral(1/(x**6*sqrt(a - b*x**2)*sqrt(c + d*x**2)*(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{x^6\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{1}{\sqrt{-bx^2+a}\sqrt{dx^2+c}(fx^2+e)x^6}\,dx$$

input
```
integrate(1/x^6/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="m
axima")
```

output
```
integrate(1/(sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)*x^6), x)
```

Giac [F]

$$\int \frac{1}{x^6\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{1}{\sqrt{-bx^2+a}\sqrt{dx^2+c}(fx^2+e)x^6}\,dx$$

input
```
integrate(1/x^6/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="g
iac")
```

output
```
integrate(1/(sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^6\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{1}{x^6\,\sqrt{a-b\,x^2}\,\sqrt{d\,x^2+c}\,(f\,x^2+e)}\,dx$$

input
```
int(1/(x^6*(a - b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)),x)
```

output
```
int(1/(x^6*(a - b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)), x)
```

Reduce [F]

$$\int \frac{1}{x^6\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \text{Too large to display}$$

input
```
int(1/x^6/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x)
```

output

```
( - 3*sqrt(c + d*x**2)*sqrt(a - b*x**2)*a*c*e + 5*sqrt(c + d*x**2)*sqrt(a
- b*x**2)*a*c*f*x**2 + 4*sqrt(c + d*x**2)*sqrt(a - b*x**2)*a*d*e*x**2 - 4*
sqrt(c + d*x**2)*sqrt(a - b*x**2)*b*c*e*x**2 - 5*int((sqrt(c + d*x**2)*sqr
t(a - b*x**2)*x**2)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 - b*c*e*
x**2 - b*c*f*x**4 - b*d*e*x**4 - b*d*f*x**6),x)*a*b*c*d*f**2*x**5 - 4*int(
(sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**2)/(a*c*e + a*c*f*x**2 + a*d*e*x**2
+ a*d*f*x**4 - b*c*e*x**2 - b*c*f*x**4 - b*d*e*x**4 - b*d*f*x**6),x)*a*b*d
**2*e*f*x**5 + 4*int((sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**2)/(a*c*e + a*c
*f*x**2 + a*d*e*x**2 + a*d*f*x**4 - b*c*e*x**2 - b*c*f*x**4 - b*d*e*x**4 -
 b*d*f*x**6),x)*b**2*c*d*e*f*x**5 + 15*int((sqrt(c + d*x**2)*sqrt(a - b*x*
*2))/(a*c*e*x**2 + a*c*f*x**4 + a*d*e*x**4 + a*d*f*x**6 - b*c*e*x**4 - b*c
*f*x**6 - b*d*e*x**6 - b*d*f*x**8),x)*a**2*c**2*f**2*x**5 + 10*int((sqrt(c
 + d*x**2)*sqrt(a - b*x**2))/(a*c*e*x**2 + a*c*f*x**4 + a*d*e*x**4 + a*d*f
*x**6 - b*c*e*x**4 - b*c*f*x**6 - b*d*e*x**6 - b*d*f*x**8),x)*a**2*c*d*e*f
*x**5 + 8*int((sqrt(c + d*x**2)*sqrt(a - b*x**2))/(a*c*e*x**2 + a*c*f*x**4
 + a*d*e*x**4 + a*d*f*x**6 - b*c*e*x**4 - b*c*f*x**6 - b*d*e*x**6 - b*d*f*
x**8),x)*a**2*d**2*e**2*x**5 - 10*int((sqrt(c + d*x**2)*sqrt(a - b*x**2))/
(a*c*e*x**2 + a*c*f*x**4 + a*d*e*x**4 + a*d*f*x**6 - b*c*e*x**4 - b*c*f*x*
*6 - b*d*e*x**6 - b*d*f*x**8),x)*a*b*c**2*e*f*x**5 - 7*int((sqrt(c + d*x**
2)*sqrt(a - b*x**2))/(a*c*e*x**2 + a*c*f*x**4 + a*d*e*x**4 + a*d*f*x**6...
```

3.226 $\int \frac{x^8}{\sqrt{a-bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 613

$$\int \frac{x^8}{\sqrt{a-bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^2}\,dx$$

$$= -\frac{x\sqrt{a-bx^2}\sqrt{c+dx^2}}{3bdf^2} - \frac{e^3x\sqrt{a-bx^2}\sqrt{c+dx^2}}{2f^2(be+af)(de-cf)\left(e+fx^2\right)}$$

$$+\frac{\sqrt{a}(4a^2df^2(de-cf)-b^2e(15d^2e^2-8cdef-4c^2f^2)-4abf(2d^2e^2-cdef-c^2f^2))\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}}{6b^{3/2}d^2f^3(be+af)(de-cf)\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}}$$

$$-\frac{\sqrt{a}(2a^2cdf^3-2abf(9d^2e^2+5cdef+2c^2f^2)-b^2e(15d^2e^2+12cdef+4c^2f^2))\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\text{Elli}}{6b^{3/2}d^2f^4(be+af)\sqrt{a-bx^2}\sqrt{c+dx^2}}$$

$$-\frac{\sqrt{a}e^2(af(6de-7cf)+be(5de-6cf))\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{2\sqrt{b}f^4(be+af)(de-cf)\sqrt{a-bx^2}\sqrt{c+dx^2}}$$

output
```
-1/3*x*(-b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b/d/f^2-1/2*e^3*x*(-b*x^2+a)^(1/2)
*(d*x^2+c)^(1/2)/f^2/(a*f+b*e)/(-c*f+d*e)/(f*x^2+e)+1/6*a^(1/2)*(4*a^2*d*f
^2*(-c*f+d*e)-b^2*e*(-4*c^2*f^2-8*c*d*e*f+15*d^2*e^2)-4*a*b*f*(-c^2*f^2-c*
d*e*f+2*d^2*e^2))*(1-b*x^2/a)^(1/2)*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^
(1/2),(-a*d/b/c)^(1/2))/b^(3/2)/d^2/f^3/(a*f+b*e)/(-c*f+d*e)/(-b*x^2+a)^(1
/2)/(1+d*x^2/c)^(1/2)-1/6*a^(1/2)*(2*a^2*c*d*f^3-2*a*b*f*(2*c^2*f^2+5*c*d*
e*f+9*d^2*e^2)-b^2*e*(4*c^2*f^2+12*c*d*e*f+15*d^2*e^2))*(1-b*x^2/a)^(1/2)*
(1+d*x^2/c)^(1/2)*EllipticF(b^(1/2)*x/a^(1/2),(-a*d/b/c)^(1/2))/b^(3/2)/d^
2/f^4/(a*f+b*e)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)-1/2*a^(1/2)*e^2*(a*f*(-7*
c*f+6*d*e)+b*e*(-6*c*f+5*d*e))*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)*Ellipti
cPi(b^(1/2)*x/a^(1/2),-a*f/b/e,(-a*d/b/c)^(1/2))/b^(1/2)/f^4/(a*f+b*e)/(-c
*f+d*e)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 8.45 (sec) , antiderivative size = 552, normalized size of antiderivative = 0.90

$$\int \frac{x^8}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx$$
$$= \frac{-\sqrt{-\frac{b}{a}}df^2x(a-bx^2)\,(c+dx^2)\,(2af(-de+cf)\,(e+fx^2)+be(2cf(e+fx^2)-de(5e+2fx^2)))-icf(4}{}$$

input
```
Integrate[x^8/(Sqrt[a - b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^2),x]
```

output
```
(-(Sqrt[-(b/a)]*d*f^2*x*(a - b*x^2)*(c + d*x^2)*(2*a*f*(-(d*e) + c*f)*(e +
 f*x^2) + b*e*(2*c*f*(e + f*x^2) - d*e*(5*e + 2*f*x^2)))) - I*c*f*(4*a^2*d
*f^2*(-(d*e) + c*f) + b^2*e*(15*d^2*e^2 - 8*c*d*e*f - 4*c^2*f^2) - 4*a*b*f
*(-2*d^2*e^2 + c*d*e*f + c^2*f^2))*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]
*(e + f*x^2)*EllipticE[I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))] + I*(-(d
*e) + c*f)*(2*a^2*c*d*f^3 - 2*a*b*f*(9*d^2*e^2 + 5*c*d*e*f + 2*c^2*f^2) -
b^2*e*(15*d^2*e^2 + 12*c*d*e*f + 4*c^2*f^2))*Sqrt[1 - (b*x^2)/a]*Sqrt[1 +
(d*x^2)/c]*(e + f*x^2)*EllipticF[I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))
] + (3*I)*b*d^2*e^2*(b*e*(-5*d*e + 6*c*f) + a*f*(-6*d*e + 7*c*f))*Sqrt[1 -
 (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(e + f*x^2)*EllipticPi[-((a*f)/(b*e)), I*A
rcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))])/(6*b*Sqrt[-(b/a)]*d^2*f^4*(b*e +
a*f)*(-(d*e) + c*f)*Sqrt[a - b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx$$

$$\Big\downarrow \; 450$$

$$\int \frac{x^8}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}dx$$

input
```
Int[x^8/(Sqrt[a - b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1482 vs. $2(550) = 1100$.

Time = 25.29 (sec) , antiderivative size = 1483, normalized size of antiderivative = 2.42

method	result	size
risch	Expression too large to display	1483
elliptic	Expression too large to display	1740
default	Expression too large to display	3455

input
```
int(x^8/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x,method=_RETURNVERBO
SE)
```

output
```
-1/3*x*(-b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b/d/f^2+1/3/f^2/b/d*(1/f^2*(-2*f*(
a*d*f-b*c*f-3*b*d*e)*c/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b
*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(b/a)^(1/2),(-1-(a*d-b*c)
/c/b)^(1/2))-EllipticE(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2)))+a*c*f^2/(b
/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*
c)^(1/2)*EllipticF(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2))+9*b*d*e^2/(b/a)
^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^
(1/2)*EllipticF(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2)))-12*b*d*e^2/f^2/(b
/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*
c)^(1/2)*EllipticPi(x*(b/a)^(1/2),-a*f/b/e,(-1/c*d)^(1/2)/(b/a)^(1/2))+3*b
*d*e^4/f^2*(1/2*f^2/(a*c*f^2-a*d*e*f+b*c*e*f-b*d*e^2)/e*x*(-b*d*x^4+a*d*x^
2-b*c*x^2+a*c)^(1/2)/(f*x^2+e)+1/2*d*b/(a*c*f^2-a*d*e*f+b*c*e*f-b*d*e^2)/(
b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a
*c)^(1/2)*EllipticF(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2))-1/2*b*f/(a*c*f
^2-a*d*e*f+b*c*e*f-b*d*e^2)/e*c/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^
(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)*EllipticF(x*(b/a)^(1/2),(-1-(a*
d-b*c)/c/b)^(1/2))+1/2*b*f/(a*c*f^2-a*d*e*f+b*c*e*f-b*d*e^2)/e*c/(b/a)^(1/
2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2
)*EllipticE(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2))+1/2/(a*c*f^2-a*d*e*f+b
*c*e*f-b*d*e^2)/e^2*f^2/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^8}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(x^8/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="f
ricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^8}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{x^8}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx$$

input
```
integrate(x**8/(-b*x**2+a)**(1/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**2,x)
```

output
```
Integral(x**8/(sqrt(a - b*x**2)*sqrt(c + d*x**2)*(e + f*x**2)**2), x)
```

Maxima [F]

$$\int \frac{x^8}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{x^8}{\sqrt{-bx^2+a}\sqrt{dx^2+c}(fx^2+e)^2}\,dx$$

input
```
integrate(x^8/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="m
axima")
```

output
```
integrate(x^8/(sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^2), x)
```

Giac [F]

$$\int \frac{x^8}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{x^8}{\sqrt{-bx^2+a}\sqrt{dx^2+c}(fx^2+e)^2}\,dx$$

input
```
integrate(x^8/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="g
iac")
```

output
```
integrate(x^8/(sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^8}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{x^8}{\sqrt{a-b\,x^2}\,\sqrt{d\,x^2+c}\,(f\,x^2+e)^2}\,dx$$

input
```
int(x^8/((a - b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^2),x)
```

output
```
int(x^8/((a - b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^2), x)
```

Reduce [F]

$$\int \frac{x^8}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \text{too large to display}$$

input
```
int(x^8/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x)
```

output

```
( - sqrt(c + d*x**2)*sqrt(a - b*x**2)*a*c*f*x - 4*sqrt(c + d*x**2)*sqrt(a
- b*x**2)*a*d*e*x + 4*sqrt(c + d*x**2)*sqrt(a - b*x**2)*b*c*e*x - 3*sqrt(c
 + d*x**2)*sqrt(a - b*x**2)*b*d*e*x**3 - int((sqrt(c + d*x**2)*sqrt(a - b*
x**2)*x**6)/(a*c*e**2 + 2*a*c*e*f*x**2 + a*c*f**2*x**4 + a*d*e**2*x**2 + 2
*a*d*e*f*x**4 + a*d*f**2*x**6 - b*c*e**2*x**2 - 2*b*c*e*f*x**4 - b*c*f**2*
x**6 - b*d*e**2*x**4 - 2*b*d*e*f*x**6 - b*d*f**2*x**8),x)*a*b*c*d*e*f**2 -
 int((sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**6)/(a*c*e**2 + 2*a*c*e*f*x**2 +
 a*c*f**2*x**4 + a*d*e**2*x**2 + 2*a*d*e*f*x**4 + a*d*f**2*x**6 - b*c*e**2
*x**2 - 2*b*c*e*f*x**4 - b*c*f**2*x**6 - b*d*e**2*x**4 - 2*b*d*e*f*x**6 -
b*d*f**2*x**8),x)*a*b*c*d*f**3*x**2 + 2*int((sqrt(c + d*x**2)*sqrt(a - b*x
**2)*x**6)/(a*c*e**2 + 2*a*c*e*f*x**2 + a*c*f**2*x**4 + a*d*e**2*x**2 + 2*
a*d*e*f*x**4 + a*d*f**2*x**6 - b*c*e**2*x**2 - 2*b*c*e*f*x**4 - b*c*f**2*x
**6 - b*d*e**2*x**4 - 2*b*d*e*f*x**6 - b*d*f**2*x**8),x)*a*b*d**2*e**2*f +
 2*int((sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**6)/(a*c*e**2 + 2*a*c*e*f*x**2
 + a*c*f**2*x**4 + a*d*e**2*x**2 + 2*a*d*e*f*x**4 + a*d*f**2*x**6 - b*c*e*
*2*x**2 - 2*b*c*e*f*x**4 - b*c*f**2*x**6 - b*d*e**2*x**4 - 2*b*d*e*f*x**6
- b*d*f**2*x**8),x)*a*b*d**2*e*f**2*x**2 - 2*int((sqrt(c + d*x**2)*sqrt(a
- b*x**2)*x**6)/(a*c*e**2 + 2*a*c*e*f*x**2 + a*c*f**2*x**4 + a*d*e**2*x**2
 + 2*a*d*e*f*x**4 + a*d*f**2*x**6 - b*c*e**2*x**2 - 2*b*c*e*f*x**4 - b*c*f
**2*x**6 - b*d*e**2*x**4 - 2*b*d*e*f*x**6 - b*d*f**2*x**8),x)*b**2*c*d*...
```

3.227 $\int \frac{x^6}{\sqrt{a-bx^2}\sqrt{c+dx^2}(e+fx^2)^2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 481

$$\begin{aligned}\int \frac{x^6}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx &= \frac{e^2x\sqrt{a-bx^2}\sqrt{c+dx^2}}{2f(be+af)(de-cf)\,(e+fx^2)} \\ &+ \frac{\sqrt{a}(be(3de-2cf)+2af(de-cf))\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{2\sqrt{b}df^2(be+af)(de-cf)\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}} \\ &- \frac{\sqrt{a}(2af(2de+cf)+be(3de+2cf))\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{2\sqrt{b}df^3(be+af)\sqrt{a-bx^2}\sqrt{c+dx^2}} \\ &+ \frac{\sqrt{a}e(af(4de-5cf)+be(3de-4cf))\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{2\sqrt{b}f^3(be+af)(de-cf)\sqrt{a-bx^2}\sqrt{c+dx^2}}\end{aligned}$$

output
```
1/2*e^2*x*(-b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/f/(a*f+b*e)/(-c*f+d*e)/(f*x^2+e
)+1/2*a^(1/2)*(b*e*(-2*c*f+3*d*e)+2*a*f*(-c*f+d*e))*(1-b*x^2/a)^(1/2)*(d*x
^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2),(-a*d/b/c)^(1/2))/b^(1/2)/d/f^2/(a
*f+b*e)/(-c*f+d*e)/(-b*x^2+a)^(1/2)/(1+d*x^2/c)^(1/2)-1/2*a^(1/2)*(2*a*f*(
c*f+2*d*e)+b*e*(2*c*f+3*d*e))*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)*Elliptic
F(b^(1/2)*x/a^(1/2),(-a*d/b/c)^(1/2))/b^(1/2)/d/f^3/(a*f+b*e)/(-b*x^2+a)^(
1/2)/(d*x^2+c)^(1/2)+1/2*a^(1/2)*e*(a*f*(-5*c*f+4*d*e)+b*e*(-4*c*f+3*d*e))
*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)*EllipticPi(b^(1/2)*x/a^(1/2),-a*f/b/e
,(-a*d/b/c)^(1/2))/b^(1/2)/f^3/(a*f+b*e)/(-c*f+d*e)/(-b*x^2+a)^(1/2)/(d*x^
2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.80 (sec) , antiderivative size = 406, normalized size of antiderivative = 0.84

$$\int \frac{x^6}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx$$
$$= \frac{icf(2af(-de+cf)+be(-3de+2cf))\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}(e+fx^2)\,E\left(i\text{arcsinh}\left(\sqrt{-\frac{b}{a}}x\right)|-\frac{ad}{bc}\right)-i(-c}{}$$

input
```
Integrate[x^6/(Sqrt[a - b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^2),x]
```

output
```
(I*c*f*(2*a*f*(-(d*e) + c*f) + b*e*(-3*d*e + 2*c*f))*Sqrt[1 - (b*x^2)/a]*S
qrt[1 + (d*x^2)/c]*(e + f*x^2)*EllipticE[I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d
)/(b*c))] - I*(-(d*e) + c*f)*(2*a*f*(2*d*e + c*f) + b*e*(3*d*e + 2*c*f))*S
qrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(e + f*x^2)*EllipticF[I*ArcSinh[Sqr
t[-(b/a)]*x], -((a*d)/(b*c))] + d*e*(Sqrt[-(b/a)]*e*f^2*x*(a - b*x^2)*(c +
 d*x^2) + I*(b*e*(-3*d*e + 4*c*f) + a*f*(-4*d*e + 5*c*f))*Sqrt[1 - (b*x^2)
/a]*Sqrt[1 + (d*x^2)/c]*(e + f*x^2)*EllipticPi[-((a*f)/(b*e)), I*ArcSinh[S
qrt[-(b/a)]*x], -((a*d)/(b*c))]))/(2*Sqrt[-(b/a)]*d*f^3*(b*e + a*f)*(d*e -
 c*f)*Sqrt[a - b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx$$

$$\Big\downarrow \text{450}$$

$$\int \frac{x^6}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}dx$$

input
```
Int[x^6/(Sqrt[a - b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1294 vs. $2(424) = 848$.

Time = 8.40 (sec) , antiderivative size = 1295, normalized size of antiderivative = 2.69

method	result	size
elliptic	Expression too large to display	1295
default	Expression too large to display	1982

input
```
int(x^6/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x,method=_RETURNVERBO
SE)
```

output
```
((-b*x^2+a)*(d*x^2+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-1/2/f/(a*c
*f^2-a*d*e*f+b*c*e*f-b*d*e^2)*e^2*x*(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)/(
f*x^2+e)-2/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x
^2-b*c*x^2+a*c)^(1/2)*EllipticF(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2))*e/
f^3-1/2/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-
b*c*x^2+a*c)^(1/2)*EllipticF(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2))*b*d/f
^3*e^3/(a*c*f^2-a*d*e*f+b*c*e*f-b*d*e^2)-c/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(
1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)/d/f^2*EllipticF(x*(b
/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2))+c/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x
^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)/d/f^2*EllipticE(x*(b/a)^(
1/2),(-1-(a*d-b*c)/c/b)^(1/2))+1/2*c/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^
2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)*b*e^2/f^2/(a*c*f^2-a*d*e*f
+b*c*e*f-b*d*e^2)*EllipticF(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2))-1/2*c/
(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+
a*c)^(1/2)*b*e^2/f^2/(a*c*f^2-a*d*e*f+b*c*e*f-b*d*e^2)*EllipticE(x*(b/a)^(
1/2),(-1-(a*d-b*c)/c/b)^(1/2))+5/2*e/(a*c*f^2-a*d*e*f+b*c*e*f-b*d*e^2)/f/(
b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a
*c)^(1/2)*EllipticPi(x*(b/a)^(1/2),-a*f/b/e,(-1/c*d)^(1/2)/(b/a)^(1/2))*a*
c-2*e^2/f^2/(a*c*f^2-a*d*e*f+b*c*e*f-b*d*e^2)/(b/a)^(1/2)*(1-b*x^2/a)^(1/2
)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)*EllipticPi(x*(...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^6}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(x^6/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="f
ricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^6}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{x^6}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx$$

input
```
integrate(x**6/(-b*x**2+a)**(1/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**2,x)
```

output
```
Integral(x**6/(sqrt(a - b*x**2)*sqrt(c + d*x**2)*(e + f*x**2)**2), x)
```

Maxima [F]

$$\int \frac{x^6}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{x^6}{\sqrt{-bx^2+a}\sqrt{dx^2+c}(fx^2+e)^2}\,dx$$

input
```
integrate(x^6/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="m
axima")
```

output
```
integrate(x^6/(sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^2), x)
```

Giac [F]

$$\int \frac{x^6}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{x^6}{\sqrt{-bx^2+a}\sqrt{dx^2+c}(fx^2+e)^2}\,dx$$

input
```
integrate(x^6/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="g
iac")
```

output
```
integrate(x^6/(sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx=\int \frac{x^6}{\sqrt{a-b\,x^2}\,\sqrt{d\,x^2+c}\,(f\,x^2+e)^2}\,dx$$

input
```
int(x^6/((a - b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^2),x)
```

output
```
int(x^6/((a - b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^2), x)
```

Reduce [F]

$$\int \frac{x^6}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{\sqrt{d\,x^2+c}\,\sqrt{-b\,x^2+a}\,x^6}{-bd\,f^2x^8+ad\,f^2x^6-bc\,f^2x^6-2bdef\,x^6+ac\,f^2x^4+2adef\,x^4-2bcef\,x^4-bd\,e^2x^4+2acef\,x^2+a}$$

input
```
int(x^6/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**6)/(a*c*e**2 + 2*a*c*e*f*x**2 +
a*c*f**2*x**4 + a*d*e**2*x**2 + 2*a*d*e*f*x**4 + a*d*f**2*x**6 - b*c*e**2*
x**2 - 2*b*c*e*f*x**4 - b*c*f**2*x**6 - b*d*e**2*x**4 - 2*b*d*e*f*x**6 - b
*d*f**2*x**8),x)
```

3.228 $\int \frac{x^4}{\sqrt{a-bx^2}\sqrt{c+dx^2}(e+fx^2)^2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 427

$$\int \frac{x^4}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = -\frac{ex\sqrt{a-bx^2}\sqrt{c+dx^2}}{2(be+af)(de-cf)\,(e+fx^2)} - \frac{\sqrt{a}\sqrt{b}e\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\,E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{2f(be+af)(de-cf)\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}} + \frac{\sqrt{a}(be+2af)\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{2\sqrt{b}f^2(be+af)\sqrt{a-bx^2}\sqrt{c+dx^2}} - \frac{\sqrt{a}(af(2de-3cf)+be(de-2cf))\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{2\sqrt{b}f^2(be+af)(de-cf)\sqrt{a-bx^2}\sqrt{c+dx^2}}$$

output
```
-1/2*e*x*(-b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(a*f+b*e)/(-c*f+d*e)/(f*x^2+e)-1
/2*a^(1/2)*b^(1/2)*e*(1-b*x^2/a)^(1/2)*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x
/a^(1/2),(-a*d/b/c)^(1/2))/f/(a*f+b*e)/(-c*f+d*e)/(-b*x^2+a)^(1/2)/(1+d*x^
2/c)^(1/2)+1/2*a^(1/2)*(2*a*f+b*e)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)*Ell
ipticF(b^(1/2)*x/a^(1/2),(-a*d/b/c)^(1/2))/b^(1/2)/f^2/(a*f+b*e)/(-b*x^2+a
)^(1/2)/(d*x^2+c)^(1/2)-1/2*a^(1/2)*(a*f*(-3*c*f+2*d*e)+b*e*(-2*c*f+d*e))*
(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)*EllipticPi(b^(1/2)*x/a^(1/2),-a*f/b/e,
(-a*d/b/c)^(1/2))/b^(1/2)/f^2/(a*f+b*e)/(-c*f+d*e)/(-b*x^2+a)^(1/2)/(d*x^2
+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.51 (sec) , antiderivative size = 929, normalized size of antiderivative = 2.18

$$\int \frac{x^4}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \text{Too large to display}$$

input
```
Integrate[x^4/(Sqrt[a - b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^2),x]
```

output
```
((b*c*e*f^2*x)/Sqrt[-(b/a)] + b*Sqrt[-(b/a)]*c*e*f^2*x^3 + (b*d*e*f^2*x^3)
/Sqrt[-(b/a)] + b*Sqrt[-(b/a)]*d*e*f^2*x^5 + I*b*c*e*f*Sqrt[1 - (b*x^2)/a]
*Sqrt[1 + (d*x^2)/c]*(e + f*x^2)*EllipticE[I*ArcSinh[Sqrt[-(b/a)]*x], -((a
*d)/(b*c))] + I*(b*e + 2*a*f)*(-(d*e) + c*f)*Sqrt[1 - (b*x^2)/a]*Sqrt[1 +
(d*x^2)/c]*(e + f*x^2)*EllipticF[I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))
] + I*b*d*e^3*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[-((a*f)/(
b*e)), I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))] - (2*I)*b*c*e^2*f*Sqrt[1
 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[-((a*f)/(b*e)), I*ArcSinh[Sqr
t[-(b/a)]*x], -((a*d)/(b*c))] + (2*I)*a*d*e^2*f*Sqrt[1 - (b*x^2)/a]*Sqrt[1
 + (d*x^2)/c]*EllipticPi[-((a*f)/(b*e)), I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d
)/(b*c))] - (3*I)*a*c*e*f^2*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Ellipt
icPi[-((a*f)/(b*e)), I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))] + I*b*d*e^
2*f*x^2*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[-((a*f)/(b*e)),
 I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))] - (2*I)*b*c*e*f^2*x^2*Sqrt[1 -
 (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[-((a*f)/(b*e)), I*ArcSinh[Sqrt[
-(b/a)]*x], -((a*d)/(b*c))] + (2*I)*a*d*e*f^2*x^2*Sqrt[1 - (b*x^2)/a]*Sqrt
[1 + (d*x^2)/c]*EllipticPi[-((a*f)/(b*e)), I*ArcSinh[Sqrt[-(b/a)]*x], -((a
*d)/(b*c))] - (3*I)*a*c*f^3*x^2*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*El
lipticPi[-((a*f)/(b*e)), I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))])/(2*Sq
rt[-(b/a)]*f^2*(b*e + a*f)*(d*e - c*f)*Sqrt[a - b*x^2]*Sqrt[c + d*x^2]*...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx$$
$$\Big\downarrow \text{ 450}$$
$$\int \frac{x^4}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}dx$$

input
```
Int[x^4/(Sqrt[a - b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1081 vs. $2(370) = 740$.

Time = 8.48 (sec) , antiderivative size = 1082, normalized size of antiderivative = 2.53

method	result	size
elliptic	Expression too large to display	1082
default	Expression too large to display	1310

input
```
int(x^4/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x,method=_RETURNVERBO
SE)
```

output
```
((-b*x^2+a)*(d*x^2+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/2*e/(a*c*
f^2-a*d*e*f+b*c*e*f-b*d*e^2)*x*(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)/(f*x^2
+e)+1/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*
c*x^2+a*c)^(1/2)*EllipticF(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2))/f^2+1/2
/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2
+a*c)^(1/2)*EllipticF(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2))*b*d*e^2/f^2/
(a*c*f^2-a*d*e*f+b*c*e*f-b*d*e^2)-1/2*b*e/(a*c*f^2-a*d*e*f+b*c*e*f-b*d*e^2
)/f*c/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*
c*x^2+a*c)^(1/2)*EllipticF(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2))+1/2*b*e
/(a*c*f^2-a*d*e*f+b*c*e*f-b*d*e^2)/f*c/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*
x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)*EllipticE(x*(b/a)^(1/2),
(-1-(a*d-b*c)/c/b)^(1/2))-3/2/(a*c*f^2-a*d*e*f+b*c*e*f-b*d*e^2)/(b/a)^(1/2
)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)
*EllipticPi(x*(b/a)^(1/2),-a*f/b/e,(-1/c*d)^(1/2)/(b/a)^(1/2))*a*c+e/(a*c*
f^2-a*d*e*f+b*c*e*f-b*d*e^2)/f/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(
1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)*EllipticPi(x*(b/a)^(1/2),-a*f/b/
e,(-1/c*d)^(1/2)/(b/a)^(1/2))*a*d-e/(a*c*f^2-a*d*e*f+b*c*e*f-b*d*e^2)/f/(b
/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*
c)^(1/2)*EllipticPi(x*(b/a)^(1/2),-a*f/b/e,(-1/c*d)^(1/2)/(b/a)^(1/2))*b*c
+1/2*e^2/f^2/(a*c*f^2-a*d*e*f+b*c*e*f-b*d*e^2)/(b/a)^(1/2)*(1-b*x^2/a)^...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{a-bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^2}\,dx=\text{Timed out}$$

input
```
integrate(x^4/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="f
ricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^4}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{x^4}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx$$

input
```
integrate(x**4/(-b*x**2+a)**(1/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**2,x)
```

output
```
Integral(x**4/(sqrt(a - b*x**2)*sqrt(c + d*x**2)*(e + f*x**2)**2), x)
```

Maxima [F]

$$\int \frac{x^4}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{x^4}{\sqrt{-bx^2+a}\sqrt{dx^2+c}(fx^2+e)^2}\,dx$$

input
```
integrate(x^4/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="m
axima")
```

output
```
integrate(x^4/(sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^2), x)
```

Giac [F]

$$\int \frac{x^4}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{x^4}{\sqrt{-bx^2+a}\sqrt{dx^2+c}(fx^2+e)^2}\,dx$$

input
```
integrate(x^4/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="g
iac")
```

output
```
integrate(x^4/(sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{x^4}{\sqrt{a-b\,x^2}\,\sqrt{d\,x^2+c}\,(f\,x^2+e)^2}\,dx$$

input
```
int(x^4/((a - b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^2),x)
```

output
```
int(x^4/((a - b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^2), x)
```

Reduce [F]

$$\int \frac{x^4}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx$$
$$= \int \frac{\sqrt{d\,x^2+c}\,\sqrt{-b\,x^2+a}\,x^4}{-bd\,f^2x^8+ad\,f^2x^6-bc\,f^2x^6-2bdef\,x^6+ac\,f^2x^4+2adef\,x^4-2bcef\,x^4-bd\,e^2x^4+2acef\,x^2+ac}$$

input
```
int(x^4/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**4)/(a*c*e**2 + 2*a*c*e*f*x**2 +
a*c*f**2*x**4 + a*d*e**2*x**2 + 2*a*d*e*f*x**4 + a*d*f**2*x**6 - b*c*e**2*
x**2 - 2*b*c*e*f*x**4 - b*c*f**2*x**6 - b*d*e**2*x**4 - 2*b*d*e*f*x**6 - b
*d*f**2*x**8),x)
```

3.229 $\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^2} \, dx$

Optimal result

Integrand size = 36, antiderivative size = 407

$$\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^2} \, dx = \frac{fx\sqrt{a-bx^2}\sqrt{c+dx^2}}{2(be+af)(de-cf)\left(e+fx^2\right)}+\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{2(be+af)(de-cf)\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}}+\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{2f(be+af)\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\left(bde^2+acf^2\right)\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{2\sqrt{b}ef(be+af)(de-cf)\sqrt{a-bx^2}\sqrt{c+dx^2}}$$

output
```
1/2*f*x*(-b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(a*f+b*e)/(-c*f+d*e)/(f*x^2+e)+1/
2*a^(1/2)*b^(1/2)*(1-b*x^2/a)^(1/2)*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^
(1/2),(-a*d/b/c)^(1/2))/(a*f+b*e)/(-c*f+d*e)/(-b*x^2+a)^(1/2)/(1+d*x^2/c)^
(1/2)+1/2*a^(1/2)*b^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(b^
(1/2)*x/a^(1/2),(-a*d/b/c)^(1/2))/f/(a*f+b*e)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(
1/2)-1/2*a^(1/2)*(a*c*f^2+b*d*e^2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)*Ell
ipticPi(b^(1/2)*x/a^(1/2),-a*f/b/e,(-a*d/b/c)^(1/2))/b^(1/2)/e/f/(a*f+b*e)
/(-c*f+d*e)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.93 (sec) , antiderivative size = 441, normalized size of antiderivative = 1.08

$$\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx$$
$$= \frac{\frac{acfx}{e+fx^2}-\frac{bcfx^3}{e+fx^2}+\frac{adfx^3}{e+fx^2}-\frac{bdfx^5}{e+fx^2}+ia\sqrt{-\frac{b}{a}}c\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(i\operatorname{arcsinh}\left(\sqrt{-\frac{b}{a}}x\right)|-\frac{ad}{bc}\right)-\frac{ia\sqrt{-\frac{b}{a}}(-de+}{}}{}$$

input
```
Integrate[x^2/(Sqrt[a - b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^2),x]
```

output
```
((a*c*f*x)/(e + f*x^2) - (b*c*f*x^3)/(e + f*x^2) + (a*d*f*x^3)/(e + f*x^2)
 - (b*d*f*x^5)/(e + f*x^2) + I*a*Sqrt[-(b/a)]*c*Sqrt[1 - (b*x^2)/a]*Sqrt[1
 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))] - (I*a*
Sqrt[-(b/a)]*(-(d*e) + c*f)*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Ellipt
icF[I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))])/f - (I*a*Sqrt[-(b/a)]*d*e*
Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[-((a*f)/(b*e)), I*ArcSi
nh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))])/f + (I*a*c*f*Sqrt[1 - (b*x^2)/a]*Sqrt
[1 + (d*x^2)/c]*EllipticPi[-((a*f)/(b*e)), I*ArcSinh[Sqrt[-(b/a)]*x], -((a
*d)/(b*c))])/(Sqrt[-(b/a)]*e))/(2*(b*e + a*f)*(d*e - c*f)*Sqrt[a - b*x^2]*
Sqrt[c + d*x^2])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx$$
$$\Big\downarrow \; 450$$
$$\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}dx$$

input
```
Int[x^2/(Sqrt[a - b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 724 vs. $2(350) = 700$.

Time = 8.52 (sec) , antiderivative size = 725, normalized size of antiderivative = 1.78

method	result
elliptic	$\frac{\sqrt{(-b\,x^2+a)(x^2d+c)}\left(-\frac{fx\sqrt{-bd\,x^4+ad\,x^2-x^2bc+ac}}{2\left(ac\,f^2-adef+bcef-bd\,e^2\right)\left(f\,x^2+e\right)}-\frac{bde\sqrt{1-\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\text{EllipticF}\left(x\sqrt{\frac{b}{a}},\sqrt{-1-\frac{ad-bc}{cb}}\right)}{2\left(ac\,f^2-adef+bcef-bd\,e^2\right)f\sqrt{\frac{b}{a}}\,\sqrt{-bd\,x^4+ad\,x^2-x^2bc+ac}}+\frac{bc\sqrt{1-b}}{2\left(ac\,f^2-a\right.}\right.}{}$
default	$\frac{\left(\sqrt{\frac{b}{a}}\,bde\,f^2x^5+\sqrt{\frac{-b\,x^2+a}{a}}\,\sqrt{\frac{x^2d+c}{c}}\,\text{EllipticF}\left(x\sqrt{\frac{b}{a}},\sqrt{-\frac{ad}{bc}}\right)bce\,f^2x^2-\sqrt{\frac{-b\,x^2+a}{a}}\,\sqrt{\frac{x^2d+c}{c}}\,\text{EllipticF}\left(x\sqrt{\frac{b}{a}},\sqrt{-\frac{ad}{bc}}\right)bd\,e^2f\,x^2\right.}{}$

input
```
int(x^2/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x,method=_RETURNVERBO
SE)
```

output
```
((-b*x^2+a)*(d*x^2+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-1/2*f/(a*c
*f^2-a*d*e*f+b*c*e*f-b*d*e^2)*x*(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)/(f*x^
2+e)-1/2*b*d*e/(a*c*f^2-a*d*e*f+b*c*e*f-b*d*e^2)/f/(b/a)^(1/2)*(1-b*x^2/a)
^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)*EllipticF(x*
(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2))+1/2*b/(a*c*f^2-a*d*e*f+b*c*e*f-b*d*e
^2)*c/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*
c*x^2+a*c)^(1/2)*EllipticF(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2))-1/2*b/(
a*c*f^2-a*d*e*f+b*c*e*f-b*d*e^2)*c/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/
c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)*EllipticE(x*(b/a)^(1/2),(-1-
(a*d-b*c)/c/b)^(1/2))+1/2/(a*c*f^2-a*d*e*f+b*c*e*f-b*d*e^2)*f/e/(b/a)^(1/2
)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)
*EllipticPi(x*(b/a)^(1/2),-a*f/b/e,(-1/c*d)^(1/2)/(b/a)^(1/2))*a*c+1/2/(a*
c*f^2-a*d*e*f+b*c*e*f-b*d*e^2)*e/f/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/
c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)*EllipticPi(x*(b/a)^(1/2),-a*
f/b/e,(-1/c*d)^(1/2)/(b/a)^(1/2))*b*d)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(x^2/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="f
ricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx$$

input
```
integrate(x**2/(-b*x**2+a)**(1/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**2,x)
```

output
```
Integral(x**2/(sqrt(a - b*x**2)*sqrt(c + d*x**2)*(e + f*x**2)**2), x)
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{x^2}{\sqrt{-bx^2+a}\sqrt{dx^2+c}(fx^2+e)^2}\,dx$$

input
```
integrate(x^2/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="m
axima")
```

output
```
integrate(x^2/(sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^2), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{x^2}{\sqrt{-bx^2+a}\sqrt{dx^2+c}(fx^2+e)^2}\,dx$$

input
```
integrate(x^2/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="g
iac")
```

output
```
integrate(x^2/(sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{x^2}{\sqrt{a-b\,x^2}\,\sqrt{d\,x^2+c}\,(f\,x^2+e)^2}\,dx$$

input
```
int(x^2/((a - b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^2),x)
```

output
```
int(x^2/((a - b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^2), x)
```

Reduce [F]

$$\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx$$
$$= \int \frac{\sqrt{d\,x^2+c}\,\sqrt{-b\,x^2+a}\,x^2}{-bd\,f^2x^8+ad\,f^2x^6-bc\,f^2x^6-2bdef\,x^6+ac\,f^2x^4+2adef\,x^4-2bcef\,x^4-bd\,e^2x^4+2acef\,x^2+ac}$$

input
```
int(x^2/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**2)/(a*c*e**2 + 2*a*c*e*f*x**2 +
a*c*f**2*x**4 + a*d*e**2*x**2 + 2*a*d*e*f*x**4 + a*d*f**2*x**6 - b*c*e**2*
x**2 - 2*b*c*e*f*x**4 - b*c*f**2*x**6 - b*d*e**2*x**4 - 2*b*d*e*f*x**6 - b
*d*f**2*x**8),x)
```

3.230 $\int \frac{1}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 425

$$\int \frac{1}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = -\frac{f^2x\sqrt{a-bx^2}\sqrt{c+dx^2}}{2e(be+af)(de-cf)\,(e+fx^2)} - \frac{\sqrt{a}\sqrt{b}f\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\,E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{2e(be+af)(de-cf)\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}} - \frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{2e(be+af)\sqrt{a-bx^2}\sqrt{c+dx^2}} + \frac{\sqrt{a}(be(3de-2cf)+af(2de-cf))\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{2\sqrt{b}e^2(be+af)(de-cf)\sqrt{a-bx^2}\sqrt{c+dx^2}}$$

output
```
-1/2*f^2*x*(-b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/e/(a*f+b*e)/(-c*f+d*e)/(f*x^2+
e)-1/2*a^(1/2)*b^(1/2)*f*(1-b*x^2/a)^(1/2)*(d*x^2+c)^(1/2)*EllipticE(b^(1/
2)*x/a^(1/2),(-a*d/b/c)^(1/2))/e/(a*f+b*e)/(-c*f+d*e)/(-b*x^2+a)^(1/2)/(1+
d*x^2/c)^(1/2)-1/2*a^(1/2)*b^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)*Ell
ipticF(b^(1/2)*x/a^(1/2),(-a*d/b/c)^(1/2))/e/(a*f+b*e)/(-b*x^2+a)^(1/2)/(d
*x^2+c)^(1/2)+1/2*a^(1/2)*(b*e*(-2*c*f+3*d*e)+a*f*(-c*f+2*d*e))*(1-b*x^2/a
)^(1/2)*(1+d*x^2/c)^(1/2)*EllipticPi(b^(1/2)*x/a^(1/2),-a*f/b/e,(-a*d/b/c)
^(1/2))/b^(1/2)/e^2/(a*f+b*e)/(-c*f+d*e)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.31 (sec) , antiderivative size = 617, normalized size of antiderivative = 1.45

$$\int \frac{1}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx$$
$$= \frac{-\frac{acf^2x}{e+fx^2}+\frac{bcf^2x^3}{e+fx^2}-\frac{adf^2x^3}{e+fx^2}+\frac{bdf^2x^5}{e+fx^2}-ia\sqrt{-\frac{b}{a}}cf\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(i\operatorname{arcsinh}\left(\sqrt{-\frac{b}{a}}x\right)|-\frac{ad}{bc}\right)+ia\sqrt{-\frac{b}{a}}}{}$$

input
```
Integrate[1/(Sqrt[a - b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^2),x]
```

output
```
(-((a*c*f^2*x)/(e + f*x^2)) + (b*c*f^2*x^3)/(e + f*x^2) - (a*d*f^2*x^3)/(e
 + f*x^2) + (b*d*f^2*x^5)/(e + f*x^2) - I*a*Sqrt[-(b/a)]*c*f*Sqrt[1 - (b*x
^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b
*c))] + I*a*Sqrt[-(b/a)]*(-(d*e) + c*f)*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^
2)/c]*EllipticF[I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))] + (3*I)*a*Sqrt[
-(b/a)]*d*e*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[-((a*f)/(b*
e)), I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))] - (2*I)*a*Sqrt[-(b/a)]*c*f
*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[-((a*f)/(b*e)), I*ArcS
inh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))] + ((2*I)*b*d*f*Sqrt[1 - (b*x^2)/a]*Sq
rt[1 + (d*x^2)/c]*EllipticPi[-((a*f)/(b*e)), I*ArcSinh[Sqrt[-(b/a)]*x], -(
(a*d)/(b*c))])/(-(b/a))^(3/2) + (I*a*c*f^2*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d
*x^2)/c]*EllipticPi[-((a*f)/(b*e)), I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*
c))])/(Sqrt[-(b/a)]*e))/(2*e*(b*e + a*f)*(d*e - c*f)*Sqrt[a - b*x^2]*Sqrt[
c + d*x^2])
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 428, normalized size of antiderivative = 1.01, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {424, 399, 323, 323, 321, 331, 330, 327, 413, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a-bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^2}dx$$

$$\Big\downarrow \ 424$$

$$\frac{(af(2de-cf)+be(3de-2cf))\int \frac{1}{\sqrt{a-bx^2}\sqrt{dx^2+c}(fx^2+e)}dx}{2e(af+be)(de-cf)}-\frac{bd\int \frac{fx^2+e}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{2e(af+be)(de-cf)}-\frac{f^2x\sqrt{a-bx^2}\sqrt{c+dx^2}}{2e\left(e+fx^2\right)(af+be)(de-cf)}$$

$$\Big\downarrow \ 399$$

$$-\frac{bd\left(\frac{(de-cf)\int \frac{1}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{d}+\frac{f\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}\right)}{2e(af+be)(de-cf)}+\frac{(af(2de-cf)+be(3de-2cf))\int \frac{1}{\sqrt{a-bx^2}\sqrt{dx^2+c}(fx^2+e)}dx}{2e(af+be)(de-cf)}-\frac{f^2x\sqrt{a-bx^2}\sqrt{c+dx^2}}{2e\left(e+fx^2\right)(af+be)(de-cf)}$$

$$\Big\downarrow \ 323$$

$$-\frac{bd\left(\frac{\sqrt{\frac{dx^2}{c}+1}(de-cf)\int \frac{1}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}dx}{d\sqrt{c+dx^2}}+\frac{f\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}\right)}{2e(af+be)(de-cf)}+\frac{(af(2de-cf)+be(3de-2cf))\int \frac{1}{\sqrt{a-bx^2}\sqrt{dx^2+c}(fx^2+e)}dx}{2e(af+be)(de-cf)}-\frac{f^2x\sqrt{a-bx^2}\sqrt{c+dx^2}}{2e\left(e+fx^2\right)(af+be)(de-cf)}$$

$$\Big\downarrow \ 323$$

$$-\frac{bd\left(\frac{\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(de-cf)\int \frac{1}{\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}}dx}{d\sqrt{a-bx^2}\sqrt{c+dx^2}}+\frac{f\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}\right)}{2e(af+be)(de-cf)}+\frac{(af(2de-cf)+be(3de-2cf))\int \frac{1}{\sqrt{a-bx^2}\sqrt{dx^2+c}(fx^2+e)}dx}{2e(af+be)(de-cf)}-\frac{f^2x\sqrt{a-bx^2}\sqrt{c+dx^2}}{2e\left(e+fx^2\right)(af+be)(de-cf)}$$

$$\Big\downarrow \ 321$$

$$-\frac{bd\left(\frac{f\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}+\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(de-cf)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}\right)}{2e(af+be)(de-cf)}+\frac{(af(2de-cf)+be(3de-2cf))\int \frac{1}{\sqrt{a-bx^2}\sqrt{dx^2+c}(fx^2+e)}dx}{2e(af+be)(de-cf)}-\frac{f^2x\sqrt{a-bx^2}\sqrt{c+dx^2}}{2e\left(e+fx^2\right)(af+be)(de-cf)}$$

$$\Big\downarrow \ 331$$

$$-\frac{bd\left(\frac{f\sqrt{1-\frac{bx^2}{a}}\int\frac{\sqrt{dx^2+c}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}+\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(de-cf)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}\right)}{2e(af+be)(de-cf)}+\frac{(af(2de-cf)+be(3de-2cf))\int\frac{1}{\sqrt{a-bx^2}\sqrt{dx^2+c}(fx^2+e)}dx}{2e(af+be)(de-cf)}-\frac{f^2x\sqrt{a-bx^2}\sqrt{c+dx^2}}{2e(e+fx^2)(af+be)(de-cf)}$$

$\big\downarrow$ 330

$$-\frac{bd\left(\frac{f\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\int\frac{\sqrt{\frac{dx^2}{c}+1}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}+\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(de-cf)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}\right)}{2e(af+be)(de-cf)}+\frac{(af(2de-cf)+be(3de-2cf))\int\frac{1}{\sqrt{a-bx^2}\sqrt{dx^2+c}(fx^2+e)}dx}{2e(af+be)(de-cf)}-\frac{f^2x\sqrt{a-bx^2}\sqrt{c+dx^2}}{2e(e+fx^2)(af+be)(de-cf)}$$

$\big\downarrow$ 327

$$\frac{(af(2de-cf)+be(3de-2cf))\int\frac{1}{\sqrt{a-bx^2}\sqrt{dx^2+c}(fx^2+e)}dx}{2e(af+be)(de-cf)}-\frac{bd\left(\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(de-cf)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}+\frac{\sqrt{a}f\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}\right)}{2e(af+be)(de-cf)}-\frac{f^2x\sqrt{a-bx^2}\sqrt{c+dx^2}}{2e(e+fx^2)(af+be)(de-cf)}$$

$\big\downarrow$ 413

$$\frac{\sqrt{1-\frac{bx^2}{a}}(af(2de-cf)+be(3de-2cf))\int\frac{1}{\sqrt{1-\frac{bx^2}{a}}\sqrt{dx^2+c}(fx^2+e)}dx}{2e\sqrt{a-bx^2}(af+be)(de-cf)}-\frac{bd\left(\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(de-cf)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}+\frac{\sqrt{a}f\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}\right)}{2e(af+be)(de-cf)}-\frac{f^2x\sqrt{a-bx^2}\sqrt{c+dx^2}}{2e(e+fx^2)(af+be)(de-cf)}$$

$\big\downarrow$ 413

$$\frac{\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(af(2de-cf)+be(3de-2cf))\int \frac{1}{\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(fx^2+e)}dx}{2e\sqrt{a-bx^2}\sqrt{c+dx^2}(af+be)(de-cf)}-\frac{bd\left(\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(de-cf)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}+\frac{\sqrt{a}f\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}\right)}{2e(af+be)(de-cf)}-\frac{f^2x\sqrt{a-bx^2}\sqrt{c+dx^2}}{2e\left(e+fx^2\right)(af+be)(de-cf)}$$

$$\Big\downarrow \text{412}$$

$$\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(af(2de-cf)+be(3de-2cf))\operatorname{EllipticPi}\left(-\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{2\sqrt{b}e^2\sqrt{a-bx^2}\sqrt{c+dx^2}(af+be)(de-cf)}-\frac{bd\left(\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(de-cf)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}+\frac{\sqrt{a}f\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}\right)}{2e(af+be)(de-cf)}-\frac{f^2x\sqrt{a-bx^2}\sqrt{c+dx^2}}{2e\left(e+fx^2\right)(af+be)(de-cf)}$$

input
```
Int[1/(Sqrt[a - b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^2),x]
```

output
```
-1/2*(f^2*x*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])/(e*(b*e + a*f)*(d*e - c*f)*(e
 + f*x^2)) - (b*d*((Sqrt[a]*f*Sqrt[1 - (b*x^2)/a]*Sqrt[c + d*x^2]*Elliptic
E[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*d*Sqrt[a - b*x^2]
*Sqrt[1 + (d*x^2)/c]) + (Sqrt[a]*(d*e - c*f)*Sqrt[1 - (b*x^2)/a]*Sqrt[1 +
(d*x^2)/c]*EllipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b
]*d*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])))/(2*e*(b*e + a*f)*(d*e - c*f)) + (Sq
rt[a]*(b*e*(3*d*e - 2*c*f) + a*f*(2*d*e - c*f))*Sqrt[1 - (b*x^2)/a]*Sqrt[1
 + (d*x^2)/c]*EllipticPi[-((a*f)/(b*e)), ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a
*d)/(b*c))])/(2*Sqrt[b]*e^2*(b*e + a*f)*(d*e - c*f)*Sqrt[a - b*x^2]*Sqrt[c
 + d*x^2])
```

Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 424
```
Int[1/(((a_) + (b_.)*(x_)^2)^2*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*
(x_)^2]), x_Symbol] :> Simp[b^2*x*Sqrt[c + d*x^2]*(Sqrt[e + f*x^2]/(2*a*(b*
c - a*d)*(b*e - a*f)*(a + b*x^2))), x] + (Simp[(b^2*c*e + 3*a^2*d*f - 2*a*b
*(d*e + c*f))/(2*a*(b*c - a*d)*(b*e - a*f))   Int[1/((a + b*x^2)*Sqrt[c + d
*x^2]*Sqrt[e + f*x^2]), x], x] - Simp[d*(f/(2*a*(b*c - a*d)*(b*e - a*f)))
 Int[(a + b*x^2)/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x], x]) /; FreeQ[{a, b,
 c, d, e, f}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 988 vs. $2(368) = 736$.

Time = 8.54 (sec) , antiderivative size = 989, normalized size of antiderivative = 2.33

method	result
elliptic	$\frac{\sqrt{(-b\,x^2+a)(x^2d+c)}\left(\frac{f^2x\sqrt{-bd\,x^4+ad\,x^2-x^2bc+ac}}{2\left(ac\,f^2-adef+bcef-bd\,e^2\right)e\left(f\,x^2+e\right)}+\frac{db\sqrt{1-\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\text{EllipticF}\left(x\sqrt{\frac{b}{a}},\sqrt{-1-\frac{ad-bc}{cb}}\right)}{2\left(ac\,f^2-adef+bcef-bd\,e^2\right)\sqrt{\frac{b}{a}}\,\sqrt{-bd\,x^4+ad\,x^2-x^2bc+ac}}-\frac{bfc\sqrt{1-\frac{b\,x}{c}}}{2\left(ac\,f^2-ade\right.}\right.}{}$
default	Expression too large to display

input
```
int(1/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x,method=_RETURNVERBOSE
)
```

output
```
((-b*x^2+a)*(d*x^2+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/2*f^2/(a*
c*f^2-a*d*e*f+b*c*e*f-b*d*e^2)/e*x*(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)/(f
*x^2+e)+1/2*d*b/(a*c*f^2-a*d*e*f+b*c*e*f-b*d*e^2)/(b/a)^(1/2)*(1-b*x^2/a)^
(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)*EllipticF(x*(
b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2))-1/2*b*f/(a*c*f^2-a*d*e*f+b*c*e*f-b*d*
e^2)/e*c/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2
-b*c*x^2+a*c)^(1/2)*EllipticF(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2))+1/2*
b*f/(a*c*f^2-a*d*e*f+b*c*e*f-b*d*e^2)/e*c/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1
+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)*EllipticE(x*(b/a)^(1/
2),(-1-(a*d-b*c)/c/b)^(1/2))+1/2/(a*c*f^2-a*d*e*f+b*c*e*f-b*d*e^2)/e^2*f^2
/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2
+a*c)^(1/2)*EllipticPi(x*(b/a)^(1/2),-a*f/b/e,(-1/c*d)^(1/2)/(b/a)^(1/2))*
a*c-1/(a*c*f^2-a*d*e*f+b*c*e*f-b*d*e^2)/e*f/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*
(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)*EllipticPi(x*(b/a)^
(1/2),-a*f/b/e,(-1/c*d)^(1/2)/(b/a)^(1/2))*a*d+1/(a*c*f^2-a*d*e*f+b*c*e*f-
b*d*e^2)/e*f/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d
*x^2-b*c*x^2+a*c)^(1/2)*EllipticPi(x*(b/a)^(1/2),-a*f/b/e,(-1/c*d)^(1/2)/(
b/a)^(1/2))*b*c-3/2/(a*c*f^2-a*d*e*f+b*c*e*f-b*d*e^2)/(b/a)^(1/2)*(1-b*x^2
/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)*EllipticP
i(x*(b/a)^(1/2),-a*f/b/e,(-1/c*d)^(1/2)/(b/a)^(1/2))*b*d)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="fri
cas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{1}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx$$

input
```
integrate(1/(-b*x**2+a)**(1/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**2,x)
```

output
```
Integral(1/(sqrt(a - b*x**2)*sqrt(c + d*x**2)*(e + f*x**2)**2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{1}{\sqrt{-bx^2+a}\sqrt{dx^2+c}(fx^2+e)^2}\,dx$$

input
```
integrate(1/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="max
ima")
```

output
```
integrate(1/(sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{1}{\sqrt{-bx^2+a}\sqrt{dx^2+c}(fx^2+e)^2}\,dx$$

input
```
integrate(1/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="gia
c")
```

output
```
integrate(1/(sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx=\int \frac{1}{\sqrt{a-b\,x^2}\,\sqrt{d\,x^2+c}\,{\left(f\,x^2+e\right)}^2}\,dx$$

input
```
int(1/((a - b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^2),x)
```

output
```
int(1/((a - b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx$$
$$=\int \frac{\sqrt{d\,x^2+c}\,\sqrt{-b\,x^2+a}}{-bd\,f^2x^8+ad\,f^2x^6-bc\,f^2x^6-2bdef\,x^6+ac\,f^2x^4+2adef\,x^4-2bcef\,x^4-bd\,e^2x^4+2acef\,x^2+a}$$

input
```
int(1/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a - b*x**2))/(a*c*e**2 + 2*a*c*e*f*x**2 + a*c*f
**2*x**4 + a*d*e**2*x**2 + 2*a*d*e*f*x**4 + a*d*f**2*x**6 - b*c*e**2*x**2
- 2*b*c*e*f*x**4 - b*c*f**2*x**6 - b*d*e**2*x**4 - 2*b*d*e*f*x**6 - b*d*f*
*2*x**8),x)
```

3.231 $\int \frac{1}{x^2\sqrt{a-bx^2}\sqrt{c+dx^2}(e+fx^2)^2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 537

$$\begin{aligned}
&\int \frac{1}{x^2\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx \\
&= -\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}}{acex\,(e+fx^2)} - \frac{f(af(2de-3cf)+2be(de-cf))x\sqrt{a-bx^2}\sqrt{c+dx^2}}{2ace^2(be+af)(de-cf)\,(e+fx^2)} \\
&\quad - \frac{\sqrt{b}(af(2de-3cf)+2be(de-cf))\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{2\sqrt{a}ce^2(be+af)(de-cf)\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}} \\
&\quad + \frac{\sqrt{b}(2be+3af)\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{2\sqrt{a}e^2(be+af)\sqrt{a-bx^2}\sqrt{c+dx^2}} \\
&\quad - \frac{\sqrt{a}f(be(5de-4cf)+af(4de-3cf))\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{2\sqrt{b}e^3(be+af)(de-cf)\sqrt{a-bx^2}\sqrt{c+dx^2}}
\end{aligned}$$

output
```
-(-b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a/c/e/x/(f*x^2+e)-1/2*f*(a*f*(-3*c*f+2*d
*e)+2*b*e*(-c*f+d*e))*x*(-b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a/c/e^2/(a*f+b*e)
/(-c*f+d*e)/(f*x^2+e)-1/2*b^(1/2)*(a*f*(-3*c*f+2*d*e)+2*b*e*(-c*f+d*e))*(1
-b*x^2/a)^(1/2)*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2),(-a*d/b/c)^(1/
2))/a^(1/2)/c/e^2/(a*f+b*e)/(-c*f+d*e)/(-b*x^2+a)^(1/2)/(1+d*x^2/c)^(1/2)+
1/2*b^(1/2)*(3*a*f+2*b*e)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(b^
(1/2)*x/a^(1/2),(-a*d/b/c)^(1/2))/a^(1/2)/e^2/(a*f+b*e)/(-b*x^2+a)^(1/2)/(
d*x^2+c)^(1/2)-1/2*a^(1/2)*f*(b*e*(-4*c*f+5*d*e)+a*f*(-3*c*f+4*d*e))*(1-b*
x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)*EllipticPi(b^(1/2)*x/a^(1/2),-a*f/b/e,(-a*d
/b/c)^(1/2))/b^(1/2)/e^3/(a*f+b*e)/(-c*f+d*e)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(
1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.36 (sec) , antiderivative size = 447, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^2\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx$$
$$= \frac{-\sqrt{-\frac{b}{a}}e(a-bx^2)\,(c+dx^2)\,(-2be(de-cf)\,(e+fx^2)+af(-2de(e+fx^2)+cf(2e+3fx^2)))+ibce(2b}{}$$

input
```
Integrate[1/(x^2*Sqrt[a - b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^2),x]
```

output
```
(-(Sqrt[-(b/a)]*e*(a - b*x^2)*(c + d*x^2)*(-2*b*e*(d*e - c*f)*(e + f*x^2)
+ a*f*(-2*d*e*(e + f*x^2) + c*f*(2*e + 3*f*x^2)))) + I*b*c*e*(2*b*e*(-(d*e
) + c*f) + a*f*(-2*d*e + 3*c*f))*x*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]
*(e + f*x^2)*EllipticE[I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))] - I*b*c*
e*(2*b*e + 3*a*f)*(-(d*e) + c*f)*x*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]
*(e + f*x^2)*EllipticF[I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))] + I*a*c*
f*(a*f*(-4*d*e + 3*c*f) + b*e*(-5*d*e + 4*c*f))*x*Sqrt[1 - (b*x^2)/a]*Sqrt
[1 + (d*x^2)/c]*(e + f*x^2)*EllipticPi[-((a*f)/(b*e)), I*ArcSinh[Sqrt[-(b/
a)]*x], -((a*d)/(b*c))])/(2*a*Sqrt[-(b/a)]*c*e^3*(b*e + a*f)*(-(d*e) + c*f
)*x*Sqrt[a - b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx$$
$$\Big\downarrow \ 450$$
$$\int \frac{1}{x^2\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}dx$$

input
```
Int[1/(x^2*Sqrt[a - b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1238 vs. 2(476) = 952.

Time = 22.45 (sec) , antiderivative size = 1239, normalized size of antiderivative = 2.31

method	result	size
elliptic	Expression too large to display	1239
risch	Expression too large to display	1265
default	Expression too large to display	2131

input
```
int(1/x^2/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x,method=_RETURNVER
BOSE)
```

output
```
((-b*x^2+a)*(d*x^2+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-1/2*f^3/(a
*c*f^2-a*d*e*f+b*c*e*f-b*d*e^2)/e^2*x*(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)
/(f*x^2+e)-1/a/c/e^2*(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)/x-1/2*b*d*f/(a*c
*f^2-a*d*e*f+b*c*e*f-b*d*e^2)/e/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^
(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)*EllipticF(x*(b/a)^(1/2),(-1-(a*
d-b*c)/c/b)^(1/2))+1/2*c/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(
-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)*b*f^2/(a*c*f^2-a*d*e*f+b*c*e*f-b*d*e^2
)/e^2*EllipticF(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2))-1/2*c/(b/a)^(1/2)*
(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)*b
*f^2/(a*c*f^2-a*d*e*f+b*c*e*f-b*d*e^2)/e^2*EllipticE(x*(b/a)^(1/2),(-1-(a*
d-b*c)/c/b)^(1/2))+1/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d
*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)*b/a/e^2*EllipticF(x*(b/a)^(1/2),(-1-(a*d-b
*c)/c/b)^(1/2))-1/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^
4+a*d*x^2-b*c*x^2+a*c)^(1/2)*b/a/e^2*EllipticE(x*(b/a)^(1/2),(-1-(a*d-b*c)
/c/b)^(1/2))-3/2/(a*c*f^2-a*d*e*f+b*c*e*f-b*d*e^2)/e^3*f^3/(b/a)^(1/2)*(1-
b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)*Elli
pticPi(x*(b/a)^(1/2),-a*f/b/e,(-1/c*d)^(1/2)/(b/a)^(1/2))*a*c+2/(a*c*f^2-a
*d*e*f+b*c*e*f-b*d*e^2)/e^2*f^2/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^
(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)*EllipticPi(x*(b/a)^(1/2),-a*f/b
/e,(-1/c*d)^(1/2)/(b/a)^(1/2))*a*d-2/(a*c*f^2-a*d*e*f+b*c*e*f-b*d*e^2)/...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/x^2/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm=
"fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{x^2\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx=\int \frac{1}{x^2\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx$$

input
```
integrate(1/x**2/(-b*x**2+a)**(1/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**2,x)
```

output
```
Integral(1/(x**2*sqrt(a - b*x**2)*sqrt(c + d*x**2)*(e + f*x**2)**2), x)
```

Maxima [F]

$$\int \frac{1}{x^2\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx=\int \frac{1}{\sqrt{-bx^2+a}\sqrt{dx^2+c}(fx^2+e)^2x^2}\,dx$$

input
```
integrate(1/x^2/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm=
"maxima")
```

output
```
integrate(1/(sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^2*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx=\int \frac{1}{\sqrt{-bx^2+a}\sqrt{dx^2+c}(fx^2+e)^2x^2}\,dx$$

input
```
integrate(1/x^2/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm=
"giac")
```

output
```
integrate(1/(sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^2*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{1}{x^2\,\sqrt{a-b\,x^2}\,\sqrt{d\,x^2+c}\,(f\,x^2+e)^2}\,dx$$

input
```
int(1/(x^2*(a - b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^2),x)
```

output
```
int(1/(x^2*(a - b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^2), x)
```

Reduce [F]

$$\int \frac{1}{x^2\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \text{Too large to display}$$

input
```
int(1/x^2/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x)
```

output

```
( - sqrt(c + d*x**2)*sqrt(a - b*x**2) + int((sqrt(c + d*x**2)*sqrt(a - b*x
**2)*x**4)/(a*c*e**2 + 2*a*c*e*f*x**2 + a*c*f**2*x**4 + a*d*e**2*x**2 + 2*
a*d*e*f*x**4 + a*d*f**2*x**6 - b*c*e**2*x**2 - 2*b*c*e*f*x**4 - b*c*f**2*x
**6 - b*d*e**2*x**4 - 2*b*d*e*f*x**6 - b*d*f**2*x**8),x)*b*d*e*f*x + int((
sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**4)/(a*c*e**2 + 2*a*c*e*f*x**2 + a*c*f
**2*x**4 + a*d*e**2*x**2 + 2*a*d*e*f*x**4 + a*d*f**2*x**6 - b*c*e**2*x**2
- 2*b*c*e*f*x**4 - b*c*f**2*x**6 - b*d*e**2*x**4 - 2*b*d*e*f*x**6 - b*d*f*
*2*x**8),x)*b*d*f**2*x**3 - 2*int((sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**2)
/(a*c*e**2 + 2*a*c*e*f*x**2 + a*c*f**2*x**4 + a*d*e**2*x**2 + 2*a*d*e*f*x*
*4 + a*d*f**2*x**6 - b*c*e**2*x**2 - 2*b*c*e*f*x**4 - b*c*f**2*x**6 - b*d*
e**2*x**4 - 2*b*d*e*f*x**6 - b*d*f**2*x**8),x)*a*d*e*f*x - 2*int((sqrt(c +
 d*x**2)*sqrt(a - b*x**2)*x**2)/(a*c*e**2 + 2*a*c*e*f*x**2 + a*c*f**2*x**4
 + a*d*e**2*x**2 + 2*a*d*e*f*x**4 + a*d*f**2*x**6 - b*c*e**2*x**2 - 2*b*c*
e*f*x**4 - b*c*f**2*x**6 - b*d*e**2*x**4 - 2*b*d*e*f*x**6 - b*d*f**2*x**8)
,x)*a*d*f**2*x**3 + 2*int((sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**2)/(a*c*e*
*2 + 2*a*c*e*f*x**2 + a*c*f**2*x**4 + a*d*e**2*x**2 + 2*a*d*e*f*x**4 + a*d
*f**2*x**6 - b*c*e**2*x**2 - 2*b*c*e*f*x**4 - b*c*f**2*x**6 - b*d*e**2*x**
4 - 2*b*d*e*f*x**6 - b*d*f**2*x**8),x)*b*c*e*f*x + 2*int((sqrt(c + d*x**2)
*sqrt(a - b*x**2)*x**2)/(a*c*e**2 + 2*a*c*e*f*x**2 + a*c*f**2*x**4 + a*d*e
**2*x**2 + 2*a*d*e*f*x**4 + a*d*f**2*x**6 - b*c*e**2*x**2 - 2*b*c*e*f*x...
```

3.232 $\int \frac{1}{x^4\sqrt{a-bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 733

$$\begin{aligned}
&\int \frac{1}{x^4\sqrt{a-bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^2}\,dx \\
&= -\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}}{3acex^3\left(e+fx^2\right)} - \frac{(2bce-2ade-5acf)\sqrt{a-bx^2}\sqrt{c+dx^2}}{3a^2c^2e^2x\left(e+fx^2\right)} \\
&\quad - \frac{f(4b^2ce^2(de-cf)-a^2f(4d^2e^2+8cdef-15c^2f^2)-4abe(d^2e^2+cdef-2c^2f^2))\,x\sqrt{a-bx^2}\sqrt{c+dx^2}}{6a^2c^2e^3(be+af)(de-cf)\left(e+fx^2\right)} \\
&\quad - \frac{\sqrt{b}(4b^2ce^2(de-cf)-a^2f(4d^2e^2+8cdef-15c^2f^2)-4abe(d^2e^2+cdef-2c^2f^2))\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}}{6a^{3/2}c^2e^3(be+af)(de-cf)\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}} \\
&\quad + \frac{\sqrt{b}(4b^2ce^2-2abe(de+4cf)-a^2f(2de+15cf))\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{6a^{3/2}ce^3(be+af)\sqrt{a-bx^2}\sqrt{c+dx^2}} \\
&\quad + \frac{\sqrt{a}f^2(be(7de-6cf)+af(6de-5cf))\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{2\sqrt{b}e^4(be+af)(de-cf)\sqrt{a-bx^2}\sqrt{c+dx^2}}
\end{aligned}$$

output
```
-1/3*(-b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a/c/e/x^3/(f*x^2+e)-1/3*(-5*a*c*f-2*
a*d*e+2*b*c*e)*(-b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a^2/c^2/e^2/x/(f*x^2+e)-1/
6*f*(4*b^2*c*e^2*(-c*f+d*e)-a^2*f*(-15*c^2*f^2+8*c*d*e*f+4*d^2*e^2)-4*a*b*
e*(-2*c^2*f^2+c*d*e*f+d^2*e^2))*x*(-b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a^2/c^2
/e^3/(a*f+b*e)/(-c*f+d*e)/(f*x^2+e)-1/6*b^(1/2)*(4*b^2*c*e^2*(-c*f+d*e)-a^
2*f*(-15*c^2*f^2+8*c*d*e*f+4*d^2*e^2)-4*a*b*e*(-2*c^2*f^2+c*d*e*f+d^2*e^2)
)*(1-b*x^2/a)^(1/2)*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2),(-a*d/b/c)
^(1/2))/a^(3/2)/c^2/e^3/(a*f+b*e)/(-c*f+d*e)/(-b*x^2+a)^(1/2)/(1+d*x^2/c)^
(1/2)+1/6*b^(1/2)*(4*b^2*c*e^2-2*a*b*e*(4*c*f+d*e)-a^2*f*(15*c*f+2*d*e))*(
1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(b^(1/2)*x/a^(1/2),(-a*d/b/c)^
(1/2))/a^(3/2)/c/e^3/(a*f+b*e)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)+1/2*a^(1/2
)*f^2*(b*e*(-6*c*f+7*d*e)+a*f*(-5*c*f+6*d*e))*(1-b*x^2/a)^(1/2)*(1+d*x^2/c
)^(1/2)*EllipticPi(b^(1/2)*x/a^(1/2),-a*f/b/e,(-a*d/b/c)^(1/2))/b^(1/2)/e^
4/(a*f+b*e)/(-c*f+d*e)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.68 (sec) , antiderivative size = 488, normalized size of antiderivative = 0.67

$$\int \frac{1}{x^4\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx$$
$$= \frac{-\sqrt{-\frac{b}{a}}e(a-bx^2)\,(c+dx^2)\,(3a^2c^2f^4x^4+2ace(be+af)(de-cf)\,(e+fx^2)+4(be+af)(de-cf)(bce\cdots}{}$$

input
```
Integrate[1/(x^4*Sqrt[a - b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^2),x]
```

output
```
(-(Sqrt[-(b/a)]*e*(a - b*x^2)*(c + d*x^2)*(3*a^2*c^2*f^4*x^4 + 2*a*c*e*(b*
e + a*f)*(d*e - c*f)*(e + f*x^2) + 4*(b*e + a*f)*(d*e - c*f)*(b*c*e - a*(d
*e + 3*c*f))*x^2*(e + f*x^2))) + I*c*x^3*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x
^2)/c]*(e + f*x^2)*(b*e*(4*b^2*c*e^2*(d*e - c*f) - 4*a*b*e*(d^2*e^2 + c*d*
e*f - 2*c^2*f^2) + a^2*f*(-4*d^2*e^2 - 8*c*d*e*f + 15*c^2*f^2))*EllipticE[
I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))] + b*e*(-(d*e) + c*f)*(4*b^2*c*e
^2 - 2*a*b*e*(d*e + 4*c*f) - a^2*f*(2*d*e + 15*c*f))*EllipticF[I*ArcSinh[S
qrt[-(b/a)]*x], -((a*d)/(b*c))] + 3*a^2*c*f^2*(a*f*(-6*d*e + 5*c*f) + b*e*
(-7*d*e + 6*c*f))*EllipticPi[-((a*f)/(b*e)), I*ArcSinh[Sqrt[-(b/a)]*x], -(
(a*d)/(b*c))]))/(6*a^2*Sqrt[-(b/a)]*c^2*e^4*(b*e + a*f)*(d*e - c*f)*x^3*Sq
rt[a - b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx$$

$$\Big\downarrow \; 450$$

$$\int \frac{1}{x^4\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}dx$$

input
```
Int[1/(x^4*Sqrt[a - b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1416 vs. $2(664) = 1328$.

Time = 23.95 (sec) , antiderivative size = 1417, normalized size of antiderivative = 1.93

method	result	size
risch	Expression too large to display	1417
elliptic	Expression too large to display	1701
default	Expression too large to display	3921

input
```
int(1/x^4/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x,method=_RETURNVER
BOSE)
```

output
```
-1/3*(-b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(-6*a*c*f*x^2-2*a*d*e*x^2+2*b*c*e*x^
2+a*c*e)/a^2/c^2/e^3/x^3+1/3/a^2/c^2/e^3*(-2*b*(3*a*c*f+a*d*e-b*c*e)*c/(b/
a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c
)^(1/2)*(EllipticF(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2))-EllipticE(x*(b/
a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2)))+a*b*c*d*e/(b/a)^(1/2)*(1-b*x^2/a)^(1/2
)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)*EllipticF(x*(b/a)
^(1/2),(-1-(a*d-b*c)/c/b)^(1/2))+6*a^2*c^2*f^2/e/(b/a)^(1/2)*(1-b*x^2/a)^(
1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)*EllipticPi(x*(
b/a)^(1/2),-a*f/b/e,(-1/c*d)^(1/2)/(b/a)^(1/2))+3*a^2*c^2*e*f^2*(1/2*f^2/(
a*c*f^2-a*d*e*f+b*c*e*f-b*d*e^2)/e*x*(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)/
(f*x^2+e)+1/2*d*b/(a*c*f^2-a*d*e*f+b*c*e*f-b*d*e^2)/(b/a)^(1/2)*(1-b*x^2/a
)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)*EllipticF(x
*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2))-1/2*b*f/(a*c*f^2-a*d*e*f+b*c*e*f-b*
d*e^2)/e*c/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x
^2-b*c*x^2+a*c)^(1/2)*EllipticF(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2))+1/
2*b*f/(a*c*f^2-a*d*e*f+b*c*e*f-b*d*e^2)/e*c/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*
(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)*EllipticE(x*(b/a)^(
1/2),(-1-(a*d-b*c)/c/b)^(1/2))+1/2/(a*c*f^2-a*d*e*f+b*c*e*f-b*d*e^2)/e^2*f
^2/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x
^2+a*c)^(1/2)*EllipticPi(x*(b/a)^(1/2),-a*f/b/e,(-1/c*d)^(1/2)/(b/a)^(1...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^4\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/x^4/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm=
"fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{x^4\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx=\int \frac{1}{x^4\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx$$

input
```
integrate(1/x**4/(-b*x**2+a)**(1/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**2,x)
```

output
```
Integral(1/(x**4*sqrt(a - b*x**2)*sqrt(c + d*x**2)*(e + f*x**2)**2), x)
```

Maxima [F]

$$\int \frac{1}{x^4\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx=\int \frac{1}{\sqrt{-bx^2+a}\sqrt{dx^2+c}(fx^2+e)^2x^4}\,dx$$

input
```
integrate(1/x^4/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm=
"maxima")
```

output
```
integrate(1/(sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^2*x^4), x)
```

Giac [F]

$$\int \frac{1}{x^4\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx=\int \frac{1}{\sqrt{-bx^2+a}\sqrt{dx^2+c}(fx^2+e)^2x^4}\,dx$$

input
```
integrate(1/x^4/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm=
"giac")
```

output
```
integrate(1/(sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^2*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{1}{x^4\,\sqrt{a-b\,x^2}\,\sqrt{d\,x^2+c}\,(f\,x^2+e)^2}\,dx$$

input
```
int(1/(x^4*(a - b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^2),x)
```

output
```
int(1/(x^4*(a - b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^2), x)
```

Reduce [F]

$$\int \frac{1}{x^4\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \text{too large to display}$$

input
```
int(1/x^4/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x)
```

output

```
( - sqrt(c + d*x**2)*sqrt(a - b*x**2)*a*c*e + 5*sqrt(c + d*x**2)*sqrt(a -
b*x**2)*a*c*f*x**2 + 2*sqrt(c + d*x**2)*sqrt(a - b*x**2)*a*d*e*x**2 - 2*sq
rt(c + d*x**2)*sqrt(a - b*x**2)*b*c*e*x**2 - 5*int((sqrt(c + d*x**2)*sqrt(
a - b*x**2)*x**4)/(a*c*e**2 + 2*a*c*e*f*x**2 + a*c*f**2*x**4 + a*d*e**2*x*
*2 + 2*a*d*e*f*x**4 + a*d*f**2*x**6 - b*c*e**2*x**2 - 2*b*c*e*f*x**4 - b*c
*f**2*x**6 - b*d*e**2*x**4 - 2*b*d*e*f*x**6 - b*d*f**2*x**8),x)*a*b*c*d*e*
f**2*x**3 - 5*int((sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**4)/(a*c*e**2 + 2*a
*c*e*f*x**2 + a*c*f**2*x**4 + a*d*e**2*x**2 + 2*a*d*e*f*x**4 + a*d*f**2*x*
*6 - b*c*e**2*x**2 - 2*b*c*e*f*x**4 - b*c*f**2*x**6 - b*d*e**2*x**4 - 2*b*
d*e*f*x**6 - b*d*f**2*x**8),x)*a*b*c*d*f**3*x**5 - 2*int((sqrt(c + d*x**2)
*sqrt(a - b*x**2)*x**4)/(a*c*e**2 + 2*a*c*e*f*x**2 + a*c*f**2*x**4 + a*d*e
**2*x**2 + 2*a*d*e*f*x**4 + a*d*f**2*x**6 - b*c*e**2*x**2 - 2*b*c*e*f*x**4
 - b*c*f**2*x**6 - b*d*e**2*x**4 - 2*b*d*e*f*x**6 - b*d*f**2*x**8),x)*a*b*
d**2*e**2*f*x**3 - 2*int((sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**4)/(a*c*e**
2 + 2*a*c*e*f*x**2 + a*c*f**2*x**4 + a*d*e**2*x**2 + 2*a*d*e*f*x**4 + a*d*
f**2*x**6 - b*c*e**2*x**2 - 2*b*c*e*f*x**4 - b*c*f**2*x**6 - b*d*e**2*x**4
 - 2*b*d*e*f*x**6 - b*d*f**2*x**8),x)*a*b*d**2*e*f**2*x**5 + 2*int((sqrt(c
 + d*x**2)*sqrt(a - b*x**2)*x**4)/(a*c*e**2 + 2*a*c*e*f*x**2 + a*c*f**2*x*
*4 + a*d*e**2*x**2 + 2*a*d*e*f*x**4 + a*d*f**2*x**6 - b*c*e**2*x**2 - 2*b*
c*e*f*x**4 - b*c*f**2*x**6 - b*d*e**2*x**4 - 2*b*d*e*f*x**6 - b*d*f**2*...
```

3.233 $\int \frac{x^6(e+fx^2)^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 717

$$\int \frac{x^6(e+fx^2)^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = -\frac{2(96a^3d^3f^2-6a^2bd^2f(28de-9cf)+b^3c(35d^2e^2-56cdef+24c^2f^2)+ab^2d(70d^2e^2-91cdef+36c^2f^2)}{105b^4d^4\sqrt{a+bx^2}}$$
$$+\frac{(48a^2d^2f^2-3abdf(28de-11cf)+b^2(35d^2e^2-56cdef+24c^2f^2))\,x^3\sqrt{c+dx^2}}{105b^3d^3\sqrt{a+bx^2}}$$
$$+\frac{2f(7bde-3bcf-4adf)x^5\sqrt{c+dx^2}}{35b^2d^2\sqrt{a+bx^2}}+\frac{f^2x^7\sqrt{c+dx^2}}{7bd\sqrt{a+bx^2}}$$
$$-\frac{\sqrt{a}(384a^4d^4f^2-24a^3bd^3f(28de+5cf)+7a^2b^2d^2(40d^2e^2+32cdef-9c^2f^2)-3ab^3cd(35d^2e^2-42cdef}{105b^{9/2}d^4(bc-ad)\sqrt{a+bx^2}}$$
$$+\frac{a^{3/2}(192a^3d^3f^2-12a^2bd^2f(28de+3cf)+ab^2d(140d^2e^2+70cdef-27c^2f^2)-b^3c(35d^2e^2-56cdef+2}{105b^{9/2}d^3(bc-ad)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
-2/105*(96*a^3*d^3*f^2-6*a^2*b*d^2*f*(-9*c*f+28*d*e)+b^3*c*(24*c^2*f^2-56*
c*d*e*f+35*d^2*e^2)+a*b^2*d*(36*c^2*f^2-91*c*d*e*f+70*d^2*e^2))*x*(d*x^2+c
)^(1/2)/b^4/d^4/(b*x^2+a)^(1/2)+1/105*(48*a^2*d^2*f^2-3*a*b*d*f*(-11*c*f+2
8*d*e)+b^2*(24*c^2*f^2-56*c*d*e*f+35*d^2*e^2))*x^3*(d*x^2+c)^(1/2)/b^3/d^3
/(b*x^2+a)^(1/2)+2/35*f*(-4*a*d*f-3*b*c*f+7*b*d*e)*x^5*(d*x^2+c)^(1/2)/b^2
/d^2/(b*x^2+a)^(1/2)+1/7*f^2*x^7*(d*x^2+c)^(1/2)/b/d/(b*x^2+a)^(1/2)-1/105
*a^(1/2)*(384*a^4*d^4*f^2-24*a^3*b*d^3*f*(5*c*f+28*d*e)+7*a^2*b^2*d^2*(-9*
c^2*f^2+32*c*d*e*f+40*d^2*e^2)-3*a*b^3*c*d*(16*c^2*f^2-42*c*d*e*f+35*d^2*e
^2)-2*b^4*c^2*(24*c^2*f^2-56*c*d*e*f+35*d^2*e^2))*(d*x^2+c)^(1/2)*Elliptic
E(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/b^(9/2)/d^4/(-a*d
+b*c)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)+1/105*a^(3/2)*(192*a
^3*d^3*f^2-12*a^2*b*d^2*f*(3*c*f+28*d*e)+a*b^2*d*(-27*c^2*f^2+70*c*d*e*f+1
40*d^2*e^2)-b^3*c*(24*c^2*f^2-56*c*d*e*f+35*d^2*e^2))*(d*x^2+c)^(1/2)*Inve
rseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/b^(9/2)/d^3/(-a*d
+b*c)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 12.87 (sec) , antiderivative size = 650, normalized size of antiderivative = 0.91

$$\int \frac{x^6(e+fx^2)^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \frac{\sqrt{\frac{b}{a}}dx(c+dx^2)\left(192a^4d^3f^2+12a^3bd^2f(-28de-3cf+4dfx^2)+a^2b^2d(-2\right.}{}$$

input
```
Integrate[(x^6*(e + f*x^2)^2)/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

output
```
(Sqrt[b/a]*d*x*(c + d*x^2)*(192*a^4*d^3*f^2 + 12*a^3*b*d^2*f*(-28*d*e - 3*
c*f + 4*d*f*x^2) + a^2*b^2*d*(-27*c^2*f^2 + 5*c*d*f*(14*e - 3*f*x^2) + 4*d
^2*(35*e^2 - 21*e*f*x^2 - 6*f^2*x^4)) - b^4*c*x^2*(24*c^2*f^2 - 2*c*d*f*(2
8*e + 9*f*x^2) + d^2*(35*e^2 + 42*e*f*x^2 + 15*f^2*x^4)) + a*b^3*(-24*c^3*
f^2 + c^2*d*f*(56*e - 9*f*x^2) + c*d^2*(-35*e^2 + 28*e*f*x^2 + 6*f^2*x^4)
+ d^3*x^2*(35*e^2 + 42*e*f*x^2 + 15*f^2*x^4))) + I*c*(384*a^4*d^4*f^2 - 24
*a^3*b*d^3*f*(28*d*e + 5*c*f) + 7*a^2*b^2*d^2*(40*d^2*e^2 + 32*c*d*e*f - 9
*c^2*f^2) - 3*a*b^3*c*d*(35*d^2*e^2 - 42*c*d*e*f + 16*c^2*f^2) - 2*b^4*c^2
*(35*d^2*e^2 - 56*c*d*e*f + 24*c^2*f^2))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x
^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (2*I)*c*(b*c - a*d
)*(96*a^3*d^3*f^2 - 6*a^2*b*d^2*f*(28*d*e - 9*c*f) + b^3*c*(35*d^2*e^2 - 5
6*c*d*e*f + 24*c^2*f^2) + a*b^2*d*(70*d^2*e^2 - 91*c*d*e*f + 36*c^2*f^2))*
Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x],
(a*d)/(b*c)])/(105*b^4*Sqrt[b/a]*d^4*(-(b*c) + a*d)*Sqrt[a + b*x^2]*Sqrt[c
 + d*x^2])
```

Rubi [A] (warning: unable to verify)

Time = 2.15 (sec) , antiderivative size = 1162, normalized size of antiderivative = 1.62, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {448, 440, 444, 444, 25, 406, 320, 388, 313, 444, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6 \left(e+f x^2\right)^2}{\left(a+b x^2\right)^{3/2} \sqrt{c+d x^2}} \, dx$$

$$\Big\downarrow \text{448}$$

$$\frac{f \int \frac{x^8 \left(f x^2+e\right)}{\left(b x^2+a\right)^{3/2} \sqrt{d x^2+c}}dx}{e^2}+e \int \frac{x^6 \left(f x^2+e\right)}{\left(b x^2+a\right)^{3/2} \sqrt{d x^2+c}}dx$$

$$\Big\downarrow \text{440}$$

$$\frac{f\left(\frac{\int \frac{x^6\left((7bde+bcf-8adf)x^2+7c(be-af)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{b(bc-ad)}-\frac{x^7\sqrt{c+dx^2}(be-af)}{b\sqrt{a+bx^2}(bc-ad)}\right)}{e^2}+$$

$$e\left(\frac{\int \frac{x^4\left((5bde+bcf-6adf)x^2+5c(be-af)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{b(bc-ad)}-\frac{x^5\sqrt{c+dx^2}(be-af)}{b\sqrt{a+bx^2}(bc-ad)}\right)$$

$\Big\downarrow$ 444

$$\frac{f\left(\frac{\frac{x^5\sqrt{a+bx^2}\sqrt{c+dx^2}(-8adf+bcf+7bde)}{7bd}-\frac{\int \frac{x^4\left(5ac(7bde+bcf-8adf)-\left(c(7de-6cf)b^2-7ad(6de+cf)b+48a^2d^2f\right)x^2\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{7bd}}{b(bc-ad)}-\frac{x^7\sqrt{c+dx^2}(be-af)}{b\sqrt{a+bx^2}(bc-ad)}\right)}{e^2}+$$

$$e\left(\frac{\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}(-6adf+bcf+5bde)}{5bd}-\frac{\int \frac{x^2\left(3ac(5bde+bcf-6adf)-\left(c(5de-4cf)b^2-5ad(4de+cf)b+24a^2d^2f\right)x^2\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{5bd}}{b(bc-ad)}-\frac{x^5\sqrt{c+dx^2}(be-\cdots}{b\sqrt{a+bx^2}(bc-\cdots}\right.$$

$\Big\downarrow$ 444

$$\frac{f\left(\frac{\frac{x^5\sqrt{a+bx^2}\sqrt{c+dx^2}(-8adf+bcf+7bde)}{7bd}-\frac{-\frac{\int -\frac{x^2\left(\left(4c^2(7de-6cf)b^3+acd(35de-27cf)b^2-12a^2d^2(14de+3cf)b+192a^3d^3f\right)x^2+3ac\left(c(7de-6cf)b^2-7ad(6\cdots\right.\right.}{\sqrt{bx^2+a}\sqrt{dx^2+c}}}{5bd}\cdots}{7bd}}{b(bc-ad)}\cdots\right.}{e^2}$$

$$e\left(\frac{\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}(-6adf+bcf+5bde)}{5bd}-\frac{-\frac{\int -\frac{\left(2c^2(5de-4cf)b^3+3acd(5de-3cf)b^2-8a^2d^2(5de+2cf)b+48a^3d^3f\right)x^2+ac\left(c(5de-4cf)b^2-5ad(4de+\cdots\right.}{\sqrt{bx^2+a}\sqrt{dx^2+c}}}{3bd}\cdots}{5bd}}{b(bc-ad)}\cdots\right.$$

$\Big\downarrow$ 25

$$f\left(\frac{\frac{x^5\sqrt{a+bx^2}\sqrt{c+dx^2}(-8adf+bcf+7bde)}{7bd}-\frac{\frac{\int\frac{x^2\left(\left(4c^2(7de-6cf)b^3+acd(35de-27cf)b^2-12a^2d^2(14de+3cf)b+192a^3d^3f\right)x^2+3ac\left(c(7de-6cf)b^2-7ad(6de+\right.}{\sqrt{bx^2+a}\sqrt{dx^2+c}}}{5bd}}{7bd}}{b(bc-ad)}\right.$$

$$\overline{\qquad e^2 \qquad}$$

$$e\left(\frac{\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}(-6adf+bcf+5bde)}{5bd}-\frac{\frac{\int\frac{\left(2c^2(5de-4cf)b^3+3acd(5de-3cf)b^2-8a^2d^2(5de+2cf)b+48a^3d^3f\right)x^2+ac\left(c(5de-4cf)b^2-5ad(4de+cf)b\right.}{\sqrt{bx^2+a}\sqrt{dx^2+c}}}{3bd}}{5bd}}{b(bc-ad)}\right.$$

$\downarrow$ 406

$$f\left(\frac{\frac{x^5\sqrt{a+bx^2}\sqrt{c+dx^2}(-8adf+bcf+7bde)}{7bd}-\frac{\frac{\int\frac{x^2\left(\left(4c^2(7de-6cf)b^3+acd(35de-27cf)b^2-12a^2d^2(14de+3cf)b+192a^3d^3f\right)x^2+3ac\left(c(7de-6cf)b^2-7ad(6de+\right.}{\sqrt{bx^2+a}\sqrt{dx^2+c}}}{5bd}}{7bd}}{b(bc-ad)}\right.$$

$$\overline{\qquad e^2 \qquad}$$

$$e\left(\frac{\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}(-6adf+bcf+5bde)}{5bd}-\frac{ac\left(24a^2d^2f-5abd(cf+4de)+b^2c(5de-4cf)\right)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\left(48a^3d^3f-8a^2bd^2(2cf+5de)+3ab^2cd\right.}{3bd}}{b(bc-ad)}\right.$$

$\downarrow$ 320

$$e\left(\frac{\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}(-6adf+bcf+5bde)}{5bd}-\frac{\left(48a^3d^3f-8a^2bd^2(2cf+5de)+3ab^2cd(5de-3cf)+2b^3c^2(5de-4cf)\right)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}\left(\right.}{}}{3bd}}{b(bc-ad)}\right.$$

$$f\left(\frac{\frac{x^5\sqrt{a+bx^2}\sqrt{c+dx^2}(-8adf+bcf+7bde)}{7bd}-\frac{\frac{\int\frac{x^2\left(\left(4c^2(7de-6cf)b^3+acd(35de-27cf)b^2-12a^2d^2(14de+3cf)b+192a^3d^3f\right)x^2+3ac\left(c(7de-6cf)b^2-7ad(6de+\right.}{\sqrt{bx^2+a}\sqrt{dx^2+c}}}{5bd}}{7bd}}{b(bc-ad)}\right.$$

$$\overline{\qquad e^2 \qquad}$$

$\downarrow$ 388

$$\frac{e\left(\frac{\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}(-6adf+bcf+5bde)}{5bd}-\frac{\frac{\left(48a^3d^3f-8a^2bd^2(2cf+5de)+3ab^2cd(5de-3cf)+2b^3c^2(5de-4cf)\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\cdots}{3bd}}{}}{b(bc-\cdots}\right)+f\left(\frac{\frac{x^5\sqrt{a+bx^2}\sqrt{c+dx^2}(-8adf+bcf+7bde)}{7bd}-\frac{\frac{\int\frac{x^2\left(\left(4c^2(7de-6cf)b^3+acd(35de-27cf)b^2-12a^2d^2(14de+3cf)b+192a^3d^3f\right)x^2+3ac\left(c(7de-6cf)b^2-7ad(6de+\cdots\right.\right.}{\sqrt{bx^2+a}\sqrt{dx^2+c}}}{5bd}\cdots}{7bd}}{b(bc-ad)}\right)}{e^2}$$

$\Big\downarrow$ 313

$$\frac{e\left(\frac{\frac{(5bde+bcf-6adf)x^3\sqrt{bx^2+a}\sqrt{dx^2+c}}{5bd}-\frac{\frac{\left(c(5de-4cf)b^2-5ad(4de+cf)b+24a^2d^2f\right)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\left(2c^2(5de-4cf)b^3\cdots\right.}{3bd}}{}\right)+f\left(\frac{\frac{(7bde+bcf-8adf)x^5\sqrt{bx^2+a}\sqrt{dx^2+c}}{7bd}-\frac{\frac{\int\frac{x^2\left(\left(4c^2(7de-6cf)b^3+acd(35de-27cf)b^2-12a^2d^2(14de+3cf)b+192a^3d^3f\right)x^2+3ac\left(c(7de-6cf)b^2-7ad(6de+cf\cdots\right.\right.}{\sqrt{bx^2+a}\sqrt{dx^2+c}}}{5bd}\cdots}{7bd}}{b(bc-ad)}\right)}{e^2}$$

$\Big\downarrow$ 444

$$e\left(\frac{(5bde+bcf-6adf)x^3\sqrt{bx^2+a}\sqrt{dx^2+c}}{5bd}-\frac{\frac{\frac{\left(c(5de-4cf)b^2-5ad(4de+cf)b+24a^2d^2f\right)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\left(2c^2(5de-4cf)b^3\cdots\right.}{3bd\cdots}}{\cdots}\right.$$

$$f\left(\frac{(7bde+bcf-8adf)x^5\sqrt{bx^2+a}\sqrt{dx^2+c}}{7bd}-\frac{\frac{\left(4c^2(7de-6cf)b^3+acd(35de-27cf)b^2-12a^2d^2(14de+3cf)b+192a^3d^3f\right)x\sqrt{bx^2+a}\sqrt{dx^2+c}}{3bd}-\int\frac{\left(8c^3(7de-6cf)b^4+\cdots\right.}{\cdots}}{\cdots}\right.$$

$\downarrow$ 406

$$e\left(\frac{(5bde+bcf-6adf)x^3\sqrt{bx^2+a}\sqrt{dx^2+c}}{5bd}-\frac{\frac{\frac{\left(c(5de-4cf)b^2-5ad(4de+cf)b+24a^2d^2f\right)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\left(2c^2(5de-4cf)b^3\cdots\right.}{3bd\cdots}}{\cdots}\right.$$

$$f\left(\frac{(7bde+bcf-8adf)x^5\sqrt{bx^2+a}\sqrt{dx^2+c}}{7bd}-\frac{\frac{\left(4c^2(7de-6cf)b^3+acd(35de-27cf)b^2-12a^2d^2(14de+3cf)b+192a^3d^3f\right)x\sqrt{bx^2+a}\sqrt{dx^2+c}}{3bd}-\frac{ac\left(4c^2(7de-6cf)b^3+\cdots\right.}{\cdots}}{\cdots}\right.$$

$\downarrow$ 320

$$e\left(\frac{(5bde+bcf-6adf)x^3\sqrt{bx^2+a}\sqrt{dx^2+c}}{5bd}-\frac{\frac{\frac{\left(c(5de-4cf)b^2-5ad(4de+cf)b+24a^2d^2f\right)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\left(2c^2(5de-4cf)b^3\cdots\right.}{3bd\cdots}}{\cdots}\right.$$

$$f\left(\frac{(7bde+bcf-8adf)x^5\sqrt{bx^2+a}\sqrt{dx^2+c}}{7bd}-\frac{\frac{\left(4c^2(7de-6cf)b^3+acd(35de-27cf)b^2-12a^2d^2(14de+3cf)b+192a^3d^3f\right)x\sqrt{bx^2+a}\sqrt{dx^2+c}}{3bd}-\frac{\left(4c^2(7de-6cf)b^3+a\cdots\right.}{\cdots}}{\cdots}\right.$$

$$\Big\downarrow \; 388$$

$$e\left(\frac{(5bde+bcf-6adf)x^3\sqrt{bx^2+a}\sqrt{dx^2+c}}{5bd}-\frac{\frac{\frac{\left(c(5de-4cf)b^2-5ad(4de+cf)b+24a^2d^2f\right)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\left(2c^2(5de-4cf)b^3\cdots\right.}{3bd\cdots}}{\cdots}\right.$$

$$f\left(\frac{(7bde+bcf-8adf)x^5\sqrt{bx^2+a}\sqrt{dx^2+c}}{7bd}-\frac{\frac{\left(4c^2(7de-6cf)b^3+acd(35de-27cf)b^2-12a^2d^2(14de+3cf)b+192a^3d^3f\right)x\sqrt{bx^2+a}\sqrt{dx^2+c}}{3bd}-\frac{\left(4c^2(7de-6cf)b^3+a\cdots\right.}{\cdots}}{\cdots}\right.$$

$\downarrow$ 313

$$e\left(\frac{\frac{(5bde+bcf-6adf)x^3\sqrt{bx^2+a}\sqrt{dx^2+c}}{5bd}-\frac{\frac{\left(c(5de-4cf)b^2-5ad(4de+cf)b+24a^2d^2f\right)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\left(2c^2(5de-4cf)b^3\right.}{3bd}}{}\right.$$

$$f\left(\frac{\frac{(7bde+bcf-8adf)x^5\sqrt{bx^2+a}\sqrt{dx^2+c}}{7bd}-\frac{\frac{\left(4c^2(7de-6cf)b^3+acd(35de-27cf)b^2-12a^2d^2(14de+3cf)b+192a^3d^3f\right)x\sqrt{bx^2+a}\sqrt{dx^2+c}}{3bd}-\frac{\left(4c^2(7de-6cf)b^3+a\right.}{}}{}}{}\right.$$

input
```
Int[(x^6*(e + f*x^2)^2)/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

output
```
e*(-(((b*e - a*f)*x^5*Sqrt[c + d*x^2])/(b*(b*c - a*d)*Sqrt[a + b*x^2])) +
(((5*b*d*e + b*c*f - 6*a*d*f)*x^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(5*b*d)
 - (-1/3*(((24*a^2*d*f)/b + (b*c*(5*d*e - 4*c*f))/d - 5*a*(4*d*e + c*f))*x
*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]) + ((48*a^3*d^3*f + 2*b^3*c^2*(5*d*e - 4*
c*f) + 3*a*b^2*c*d*(5*d*e - 3*c*f) - 8*a^2*b*d^2*(5*d*e + 2*c*f))*((x*Sqrt
[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcT
an[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))
/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (c^(3/2)*(24*a^2*d^2*f + b^2*c*(5*d*
e - 4*c*f) - 5*a*b*d*(4*d*e + c*f))*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt
[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d
*x^2))]*Sqrt[c + d*x^2]))/(3*b*d))/(5*b*d))/(b*(b*c - a*d))) + (f*(-(((b*e
 - a*f)*x^7*Sqrt[c + d*x^2])/(b*(b*c - a*d)*Sqrt[a + b*x^2])) + (((7*b*d*e
 + b*c*f - 8*a*d*f)*x^5*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(7*b*d) - (-1/5*(
((48*a^2*d*f)/b + (b*c*(7*d*e - 6*c*f))/d - 7*a*(6*d*e + c*f))*x^3*Sqrt[a
+ b*x^2]*Sqrt[c + d*x^2]) + (((192*a^3*d^3*f + a*b^2*c*d*(35*d*e - 27*c*f)
 + 4*b^3*c^2*(7*d*e - 6*c*f) - 12*a^2*b*d^2*(14*d*e + 3*c*f))*x*Sqrt[a + b
*x^2]*Sqrt[c + d*x^2])/(3*b*d) - ((384*a^4*d^4*f + 3*a*b^3*c^2*d*(21*d*e -
 16*c*f) + 7*a^2*b^2*c*d^2*(16*d*e - 9*c*f) + 8*b^4*c^3*(7*d*e - 6*c*f) -
24*a^3*b*d^3*(14*d*e + 5*c*f))*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) -
(Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 440
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[g*(b*e - a*f)*(g*x)^(m - 1)*(a +
 b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] - Simp[
g^2/(2*b*(b*c - a*d)*(p + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c +
d*x^2)^q*Simp[c*(b*e - a*f)*(m - 1) + (d*(b*e - a*f)*(m + 2*q + 1) - b*2*(c
*f - d*e)*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, q}, x] &&
 LtQ[p, -1] && GtQ[m, 1]
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

rule 448
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[e   Int[(g*x)^m*(a + b*x
^2)^p*(c + d*x^2)^q*(e + f*x^2)^(r - 1), x], x] + Simp[f/e^2   Int[(g*x)^(m
 + 2)*(a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^(r - 1), x], x] /; FreeQ[{a,
b, c, d, e, f, g, m, p, q}, x] && IGtQ[r, 0]
```

Maple [A] (verified)

Time = 23.70 (sec) , antiderivative size = 913, normalized size of antiderivative = 1.27

method	result
elliptic	$\frac{\sqrt{(b\,x^2+a)(x^2d+c)}\left(\frac{\left(bd\,x^2+bc\right)a^2x\left(a^2f^2-2abfe+b^2e^2\right)}{b^5(ad-bc)\sqrt{\left(x^2+\frac{a}{b}\right)\left(bd\,x^2+bc\right)}}+\frac{f^2x^5\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{7b^2d}+\frac{\left(-\frac{f(af-2be)}{b^2}-\frac{f^2(6ad+6bc)}{7b^2d}\right)x^3\sqrt{bd\,x^4+ad\,x}}{5bd}\right.}{}$
risch	$\frac{x\left(15f^2x^4b^2d^2-39ab\,d^2f^2x^2-18b^2cd\,f^2x^2+42b^2d^2ef\,x^2+87a^2d^2f^2+51abcd\,f^2-126ab\,d^2ef+24b^2c^2f^2-56b^2cdef+35b^2d^2e^2\right)\sqrt{b}}{105b^4d^3}$
default	Expression too large to display

input
```
int(x^6*(f*x^2+e)^2/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOS
E)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*((b*d*x^2+b*c)
/b^5*a^2/(a*d-b*c)*x*(a^2*f^2-2*a*b*e*f+b^2*e^2)/((x^2+a/b)*(b*d*x^2+b*c))
^(1/2)+1/7*f^2/b^2/d*x^5*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+1/5*(-f*(a*f-
2*b*e)/b^2-1/7*f^2/b^2/d*(6*a*d+6*b*c))/b/d*x^3*(b*d*x^4+a*d*x^2+b*c*x^2+a
*c)^(1/2)+1/3*(1/b^3*(a^2*f^2-2*a*b*e*f+b^2*e^2)-5/7*f^2/b^2/d*a*c-1/5*(-f
*(a*f-2*b*e)/b^2-1/7*f^2/b^2/d*(6*a*d+6*b*c))/b/d*(4*a*d+4*b*c))/b/d*x*(b*
d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+(-1/b^4*c*a^2/(a*d-b*c)*(a^2*f^2-2*a*b*e*
f+b^2*e^2)-1/3*(1/b^3*(a^2*f^2-2*a*b*e*f+b^2*e^2)-5/7*f^2/b^2/d*a*c-1/5*(-
f*(a*f-2*b*e)/b^2-1/7*f^2/b^2/d*(6*a*d+6*b*c))/b/d*(4*a*d+4*b*c))/b/d*a*c)
/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2
+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-(-a/b^4*(a^
2*f^2-2*a*b*e*f+b^2*e^2)-1/b^4*d*a^2*(a^2*f^2-2*a*b*e*f+b^2*e^2)/(a*d-b*c)
-3/5*(-f*(a*f-2*b*e)/b^2-1/7*f^2/b^2/d*(6*a*d+6*b*c))/b/d*a*c-1/3*(1/b^3*(
a^2*f^2-2*a*b*e*f+b^2*e^2)-5/7*f^2/b^2/d*a*c-1/5*(-f*(a*f-2*b*e)/b^2-1/7*f
^2/b^2/d*(6*a*d+6*b*c))/b/d*(4*a*d+4*b*c))/b/d*(2*a*d+2*b*c))*c/(-b/a)^(1/
2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
/d*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^
(1/2),(-1+(a*d+b*c)/c/b)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1486 vs. $2(676) = 1352$.

Time = 0.13 (sec) , antiderivative size = 1486, normalized size of antiderivative = 2.07

$$\int \frac{x^6(e+fx^2)^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^6*(f*x^2+e)^2/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="fr
icas")
```

output
```
1/105*(((35*(2*b^5*c^3*d^2 + 3*a*b^4*c^2*d^3 - 8*a^2*b^3*c*d^4)*e^2 - 14*(
8*b^5*c^4*d + 9*a*b^4*c^3*d^2 + 16*a^2*b^3*c^2*d^3 - 48*a^3*b^2*c*d^4)*e*f
 + 3*(16*b^5*c^5 + 16*a*b^4*c^4*d + 21*a^2*b^3*c^3*d^2 + 40*a^3*b^2*c^2*d^
3 - 128*a^4*b*c*d^4)*f^2)*x^3 + (35*(2*a*b^4*c^3*d^2 + 3*a^2*b^3*c^2*d^3 -
 8*a^3*b^2*c*d^4)*e^2 - 14*(8*a*b^4*c^4*d + 9*a^2*b^3*c^3*d^2 + 16*a^3*b^2
*c^2*d^3 - 48*a^4*b*c*d^4)*e*f + 3*(16*a*b^4*c^5 + 16*a^2*b^3*c^4*d + 21*a
^3*b^2*c^3*d^2 + 40*a^4*b*c^2*d^3 - 128*a^5*c*d^4)*f^2)*x)*sqrt(b*d)*sqrt(
-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - ((35*(2*b^5*c^3*d^2 +
3*a*b^4*c^2*d^3 - 4*a^2*b^3*d^5 - (8*a^2*b^3 - a*b^4)*c*d^4)*e^2 - 14*(8*b
^5*c^4*d + 9*a*b^4*c^3*d^2 - 24*a^3*b^2*d^5 + 4*(4*a^2*b^3 + a*b^4)*c^2*d^
3 - (48*a^3*b^2 - 5*a^2*b^3)*c*d^4)*e*f + 3*(16*b^5*c^5 + 16*a*b^4*c^4*d -
 64*a^4*b*d^5 + (21*a^2*b^3 + 8*a*b^4)*c^3*d^2 + (40*a^3*b^2 + 9*a^2*b^3)*
c^2*d^3 - 4*(32*a^4*b - 3*a^3*b^2)*c*d^4)*f^2)*x^3 + (35*(2*a*b^4*c^3*d^2
+ 3*a^2*b^3*c^2*d^3 - 4*a^3*b^2*d^5 - (8*a^3*b^2 - a^2*b^3)*c*d^4)*e^2 - 1
4*(8*a*b^4*c^4*d + 9*a^2*b^3*c^3*d^2 - 24*a^4*b*d^5 + 4*(4*a^3*b^2 + a^2*b
^3)*c^2*d^3 - (48*a^4*b - 5*a^3*b^2)*c*d^4)*e*f + 3*(16*a*b^4*c^5 + 16*a^2
*b^3*c^4*d - 64*a^5*d^5 + (21*a^3*b^2 + 8*a^2*b^3)*c^3*d^2 + (40*a^4*b + 9
*a^3*b^2)*c^2*d^3 - 4*(32*a^5 - 3*a^4*b)*c*d^4)*f^2)*x)*sqrt(b*d)*sqrt(-c/
d)*elliptic_f(arcsin(sqrt(-c/d)/x), a*d/(b*c)) + (15*(b^5*c*d^4 - a*b^4*d^
5)*f^2*x^8 + 6*(7*(b^5*c*d^4 - a*b^4*d^5)*e*f - (3*b^5*c^2*d^3 + a*b^4*...
```

Sympy [F]

$$\int \frac{x^6(e+fx^2)^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \int \frac{x^6(e+fx^2)^2}{(a+bx^2)^{\frac{3}{2}}\sqrt{c+dx^2}}\,dx$$

input
```
integrate(x**6*(f*x**2+e)**2/(b*x**2+a)**(3/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(x**6*(e + f*x**2)**2/((a + b*x**2)**(3/2)*sqrt(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{x^6(e+fx^2)^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx=\int \frac{(fx^2+e)^2x^6}{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}}\,dx$$

input
```
integrate(x^6*(f*x^2+e)^2/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate((f*x^2 + e)^2*x^6/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)), x)
```

Giac [F]

$$\int \frac{x^6(e+fx^2)^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx=\int \frac{(fx^2+e)^2x^6}{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}}\,dx$$

input
```
integrate(x^6*(f*x^2+e)^2/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate((f*x^2 + e)^2*x^6/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6(e+fx^2)^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx=\int \frac{x^6\,{\left(f\,x^2+e\right)}^2}{{\left(b\,x^2+a\right)}^{3/2}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int((x^6*(e + f*x^2)^2)/((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int((x^6*(e + f*x^2)^2)/((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^6 (e+fx^2)^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \text{too large to display}$$

input
```
int(x^6*(f*x^2+e)^2/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x)
```

output
```
( - 144*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*c*d**2*f**2*x + 96*sqrt(c +
 d*x**2)*sqrt(a + b*x**2)*a**2*d**3*f**2*x**3 - 99*sqrt(c + d*x**2)*sqrt(a
 + b*x**2)*a*b*c**2*d*f**2*x + 252*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b*c
*d**2*e*f*x + 66*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b*c*d**2*f**2*x**3 -
168*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b*d**3*e*f*x**3 - 48*sqrt(c + d*x*
*2)*sqrt(a + b*x**2)*a*b*d**3*f**2*x**5 - 72*sqrt(c + d*x**2)*sqrt(a + b*x
**2)*b**2*c**3*f**2*x + 168*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**2*c**2*d*
e*f*x + 48*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**2*c**2*d*f**2*x**3 - 105*s
qrt(c + d*x**2)*sqrt(a + b*x**2)*b**2*c*d**2*e**2*x - 112*sqrt(c + d*x**2)
*sqrt(a + b*x**2)*b**2*c*d**2*e*f*x**3 - 36*sqrt(c + d*x**2)*sqrt(a + b*x*
*2)*b**2*c*d**2*f**2*x**5 + 70*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**2*d**3
*e**2*x**3 + 84*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**2*d**3*e*f*x**5 + 30*
sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**2*d**3*f**2*x**7 - 384*int((sqrt(c +
d*x**2)*sqrt(a + b*x**2)*x**4)/(a**2*c + a**2*d*x**2 + 2*a*b*c*x**2 + 2*a*
b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a**4*d**4*f**2 - 72*int((sqrt(c +
 d*x**2)*sqrt(a + b*x**2)*x**4)/(a**2*c + a**2*d*x**2 + 2*a*b*c*x**2 + 2*a
*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a**3*b*c*d**3*f**2 + 672*int((sq
rt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a**2*c + a**2*d*x**2 + 2*a*b*c*x**2
 + 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a**3*b*d**4*e*f - 384*int(
(sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a**2*c + a**2*d*x**2 + 2*a*b*...
```

3.234 $\int \frac{x^4(e+fx^2)^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 520

$$\int \frac{x^4(e+fx^2)^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \frac{(24a^2d^2f^2-abdf(40de-13cf)+b^2(15d^2e^2-20cdef+8c^2f^2))\,x\sqrt{c+dx^2}}{15b^3d^3\sqrt{a+bx^2}}$$
$$+\frac{2f(5bde-2bcf-3adf)x^3\sqrt{c+dx^2}}{15b^2d^2\sqrt{a+bx^2}}+\frac{f^2x^5\sqrt{c+dx^2}}{5bd\sqrt{a+bx^2}}$$
$$+\frac{\sqrt{a}(48a^3d^3f^2-16a^2bd^2f(5de+cf)+3ab^2d(10d^2e^2+10cdef-3c^2f^2)-b^3c(15d^2e^2-20cdef+8c^2f^2)}{15b^{7/2}d^3(bc-ad)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$
$$-\frac{a^{3/2}(24a^2d^2f^2-5abdf(8de+cf)+b^2(15d^2e^2+10cdef-4c^2f^2))\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1\right.}{15b^{7/2}d^2(bc-ad)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
1/15*(24*a^2*d^2*f^2-a*b*d*f*(-13*c*f+40*d*e)+b^2*(8*c^2*f^2-20*c*d*e*f+15
*d^2*e^2))*x*(d*x^2+c)^(1/2)/b^3/d^3/(b*x^2+a)^(1/2)+2/15*f*(-3*a*d*f-2*b*
c*f+5*b*d*e)*x^3*(d*x^2+c)^(1/2)/b^2/d^2/(b*x^2+a)^(1/2)+1/5*f^2*x^5*(d*x^
2+c)^(1/2)/b/d/(b*x^2+a)^(1/2)+1/15*a^(1/2)*(48*a^3*d^3*f^2-16*a^2*b*d^2*f
*(c*f+5*d*e)+3*a*b^2*d*(-3*c^2*f^2+10*c*d*e*f+10*d^2*e^2)-b^3*c*(8*c^2*f^2
-20*c*d*e*f+15*d^2*e^2))*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*
x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/b^(7/2)/d^3/(-a*d+b*c)/(b*x^2+a)^(1/2)/(a*
(d*x^2+c)/c/(b*x^2+a))^(1/2)-1/15*a^(3/2)*(24*a^2*d^2*f^2-5*a*b*d*f*(c*f+8
*d*e)+b^2*(-4*c^2*f^2+10*c*d*e*f+15*d^2*e^2))*(d*x^2+c)^(1/2)*InverseJacob
iAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/b^(7/2)/d^2/(-a*d+b*c)/(b
*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.99 (sec) , antiderivative size = 451, normalized size of antiderivative = 0.87

$$\int \frac{x^4(e+fx^2)^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \frac{\sqrt{\frac{b}{a}}dx(c+dx^2)\left(24a^3d^2f^2+b^3cfx^2(10de-4cf+3dfx^2)+a^2bdf(-40de-\right.}{}$$

input
```
Integrate[(x^4*(e + f*x^2)^2)/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

output
```
(Sqrt[b/a]*d*x*(c + d*x^2)*(24*a^3*d^2*f^2 + b^3*c*f*x^2*(10*d*e - 4*c*f +
 3*d*f*x^2) + a^2*b*d*f*(-40*d*e - 5*c*f + 6*d*f*x^2) + a*b^2*(-4*c^2*f^2
- 2*c*d*f*(-5*e + f*x^2) + d^2*(15*e^2 - 10*e*f*x^2 - 3*f^2*x^4))) + I*c*(
48*a^3*d^3*f^2 - 16*a^2*b*d^2*f*(5*d*e + c*f) + b^3*c*(-15*d^2*e^2 + 20*c*
d*e*f - 8*c^2*f^2) + 3*a*b^2*d*(10*d^2*e^2 + 10*c*d*e*f - 3*c^2*f^2))*Sqrt
[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d
)/(b*c)] - I*c*(-(b*c) + a*d)*(24*a^2*d^2*f^2 + a*b*d*f*(-40*d*e + 13*c*f)
 + b^2*(15*d^2*e^2 - 20*c*d*e*f + 8*c^2*f^2))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 +
 (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(15*b^3*Sqrt[b
/a]*d^3*(b*c - a*d)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (warning: unable to verify)

Time = 1.55 (sec) , antiderivative size = 885, normalized size of antiderivative = 1.70, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.371$, Rules used = {448, 440, 444, 406, 320, 388, 313, 444, 25, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 (e+fx^2)^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \text{448}$$

$$\frac{f\int \frac{x^6(fx^2+e)}{(bx^2+a)^{3/2}\sqrt{dx^2+c}}dx}{e^2}+e\int \frac{x^4(fx^2+e)}{(bx^2+a)^{3/2}\sqrt{dx^2+c}}dx$$

$$\Big\downarrow \text{440}$$

$$\frac{f\left(\frac{\int \frac{x^4\left((5bde+bcf-6adf)x^2+5c(be-af)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{b(bc-ad)}-\frac{x^5\sqrt{c+dx^2}(be-af)}{b\sqrt{a+bx^2}(bc-ad)}\right)}{e^2}+e\left(\frac{\int \frac{x^2((3bde+bcf-4adf)x^2+3c(be-af))}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{b(bc-ad)}-\frac{x^3\sqrt{c+dx^2}(be-af)}{b\sqrt{a+bx^2}(bc-ad)}\right)$$

$$\Big\downarrow \text{444}$$

$$\frac{f\left(\frac{\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}(-6adf+bcf+5bde)}{5bd}-\frac{\int \frac{x^2\left(3ac(5bde+bcf-6adf)-\left(c(5de-4cf)b^2-5ad(4de+cf)b+24a^2d^2f\right)x^2\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{5bd}}{b(bc-ad)}-\frac{x^5\sqrt{c+dx^2}(be-af)}{b\sqrt{a+bx^2}(bc-ad)}\right)}{e^2}+e\left(\frac{\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(-4adf+bcf+3bde)}{3bd}-\frac{\int \frac{ac(3bde+bcf-4adf)-\left(c(3de-2cf)b^2-3ad(2de+cf)b+8a^2d^2f\right)x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3bd}}{b(bc-ad)}-\frac{x^3\sqrt{c+dx^2}(be-af)}{b\sqrt{a+bx^2}(bc-ad)}\right)$$

$$\Big\downarrow \text{406}$$

$$\frac{f\left(\frac{\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}(-6adf+bcf+5bde)}{5bd}-\frac{\int\frac{x^2\left(3ac(5bde+bcf-6adf)-\left(c(5de-4cf)b^2-5ad(4de+cf)b+24a^2d^2f\right)x^2\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{5bd}}{b(bc-ad)}-\frac{x^5\sqrt{c+dx^2}(be-af)}{b\sqrt{a+bx^2}(bc-ad)}\right)}{e^2}+$$

$$e\left(\frac{\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(-4adf+bcf+3bde)}{3bd}-\frac{ac(-4adf+bcf+3bde)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx-\left(8a^2d^2f-3abd(cf+2de)+b^2c(3de-2cf)\right)\int\frac{x^2}{\sqrt{bx^2+a}\cdots}}{3bd}}{b(bc-ad)}\right.$$

$\downarrow$ 320

$$e\left(\frac{\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(-4adf+bcf+3bde)}{3bd}-\frac{\frac{c^{3/2}\sqrt{a+bx^2}(-4adf+bcf+3bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\left(8a^2d^2f-3abd(cf+2de)+b^2c(3de-\cdots\right.}{3bd}}{b(bc-ad)}\right.$$

$$\frac{f\left(\frac{\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}(-6adf+bcf+5bde)}{5bd}-\frac{\int\frac{x^2\left(3ac(5bde+bcf-6adf)-\left(c(5de-4cf)b^2-5ad(4de+cf)b+24a^2d^2f\right)x^2\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{5bd}}{b(bc-ad)}-\frac{x^5\sqrt{c+dx^2}(be-af)}{b\sqrt{a+bx^2}(bc-ad)}\right)}{e^2}$$

$\downarrow$ 388

$$e\left(\frac{\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(-4adf+bcf+3bde)}{3bd}-\frac{\frac{c^{3/2}\sqrt{a+bx^2}(-4adf+bcf+3bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\left(8a^2d^2f-3abd(cf+2de)+b^2c(3de-\cdots\right.}{3bd}}{b(bc-ad)}\right.$$

$$\frac{f\left(\frac{\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}(-6adf+bcf+5bde)}{5bd}-\frac{\int\frac{x^2\left(3ac(5bde+bcf-6adf)-\left(c(5de-4cf)b^2-5ad(4de+cf)b+24a^2d^2f\right)x^2\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{5bd}}{b(bc-ad)}-\frac{x^5\sqrt{c+dx^2}(be-af)}{b\sqrt{a+bx^2}(bc-ad)}\right)}{e^2}$$

$\downarrow$ 313

$$\frac{f\left(\frac{\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}(-6adf+bcf+5bde)}{5bd}-\frac{\int\frac{x^2\left(3ac(5bde+bcf-6adf)-\left(c(5de-4cf)b^2-5ad(4de+cf)b+24a^2d^2f\right)x^2\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{5bd}}{b(bc-ad)}-\frac{x^5\sqrt{c+dx^2}(be-af)}{b\sqrt{a+bx^2}(bc-ad)}\right)}{e^2}+$$

$$e\left(\frac{\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(-4adf+bcf+3bde)}{3bd}-\frac{\frac{c^{3/2}\sqrt{a+bx^2}(-4adf+bcf+3bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\left(8a^2d^2f-3abd(cf+2de)+b^2c(3de-\right.}{3bd}}{b(bc-ad)}\right.$$

$$\Big\downarrow 444$$

$$\frac{f\left(\frac{\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}(-6adf+bcf+5bde)}{5bd}-\frac{-\frac{\int-\frac{\left(2c^2(5de-4cf)b^3+3acd(5de-3cf)b^2-8a^2d^2(5de+2cf)b+48a^3d^3f\right)x^2+ac\left(c(5de-4cf)b^2-5ad(4de+cf)b+\right.}{\sqrt{bx^2+a}\sqrt{dx^2+c}}}{3bd}}{5bd}}{b(bc-ad)}\right.}{e^2}$$

$$e\left(\frac{\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(-4adf+bcf+3bde)}{3bd}-\frac{\frac{c^{3/2}\sqrt{a+bx^2}(-4adf+bcf+3bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\left(8a^2d^2f-3abd(cf+2de)+b^2c(3de-\right.}{3bd}}{b(bc-ad)}\right.$$

$$\Big\downarrow 25$$

$$f\left(\frac{\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}(-6adf+bcf+5bde)}{5bd}-\frac{\frac{\int\frac{\left(2c^2(5de-4cf)b^3+3acd(5de-3cf)b^2-8a^2d^2(5de+2cf)b+48a^3d^3f\right)x^2+ac\left(c(5de-4cf)b^2-5ad(4de+cf)b+24a\right.}{\sqrt{bx^2+a}\sqrt{dx^2+c}}}{3bd}}{5bd}}{b(bc-ad)}\right.$$

$$\overline{\qquad\qquad e^2\qquad\qquad}$$

$$e\left(\frac{\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(-4adf+bcf+3bde)}{3bd}-\frac{\frac{c^{3/2}\sqrt{a+bx^2}(-4adf+bcf+3bde)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\left(8a^2d^2f-3abd(cf+2de)+b^2c(3de-\right.}{3bd}}{b(bc-ad)}\right.$$

$\downarrow$ 406

$$f\left(\frac{\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}(-6adf+bcf+5bde)}{5bd}-\frac{\frac{ac\left(24a^2d^2f-5abd(cf+4de)+b^2c(5de-4cf)\right)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\left(48a^3d^3f-8a^2bd^2(2cf+5de)+3ab^2cd(5de-\right.}{3bd}}{5bd}}{b(bc-ad)}\right.$$

$$\overline{\qquad\qquad e^2\qquad\qquad}$$

$$e\left(\frac{\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(-4adf+bcf+3bde)}{3bd}-\frac{\frac{c^{3/2}\sqrt{a+bx^2}(-4adf+bcf+3bde)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\left(8a^2d^2f-3abd(cf+2de)+b^2c(3de-\right.}{3bd}}{b(bc-ad)}\right.$$

$\downarrow$ 320

$$e\left(\frac{\frac{(3bde+bcf-4adf)x\sqrt{bx^2+a}\sqrt{dx^2+c}}{3bd}-\frac{\frac{c^{3/2}(3bde+bcf-4adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\left(c(3de-2cf)b^2-3ad(2de+cf)b+8a^2d^2\right.}{3bd}}{b(bc-ad)}\right.$$

$$f\left(\frac{\frac{(5bde+bcf-6adf)x^3\sqrt{bx^2+a}\sqrt{dx^2+c}}{5bd}-\frac{\frac{\left(c(5de-4cf)b^2-5ad(4de+cf)b+24a^2d^2f\right)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\left(2c^2(5de-4cf)b^3+3ac\right.}{3bd}}{b(bc-ad)}\right.$$

$$e^{\cdots}$$

$\downarrow$ 388

$$e\left(\frac{\frac{(3bde+bcf-4adf)x\sqrt{bx^2+a}\sqrt{dx^2+c}}{3bd}-\frac{\frac{c^{3/2}(3bde+bcf-4adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}-\left(c(3de-2cf)b^2-3ad(2de+cf)b+8a^2d^2\right.}{3bd}}{b(bc-ad)}\right.$$

$$f\left(\frac{\frac{(5bde+bcf-6adf)x^3\sqrt{bx^2+a}\sqrt{dx^2+c}}{5bd}-\frac{\frac{\left(c(5de-4cf)b^2-5ad(4de+cf)b+24a^2d^2f\right)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\left(2c^2(5de-4cf)b^3+3ac\right.}{3bd}}{b(bc-ad)}\right.$$

$\downarrow$ 313

$$e\left(\frac{\frac{(3bde+bcf-4adf)x\sqrt{bx^2+a}\sqrt{dx^2+c}}{3bd}-\frac{\frac{c^{3/2}(3bde+bcf-4adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}-\left(c(3de-2cf)b^2-3ad(2de+cf)b+8a^2d^2\right.}{3bd}}{b(bc-ad)}\right.$$

$$f\left(\frac{\frac{(5bde+bcf-6adf)x^3\sqrt{bx^2+a}\sqrt{dx^2+c}}{5bd}-\frac{\frac{\left(c(5de-4cf)b^2-5ad(4de+cf)b+24a^2d^2f\right)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\left(2c^2(5de-4cf)b^3+3ac\right.}{3bd}}{b(\right.$$

input
```
Int[(x^4*(e + f*x^2)^2)/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

output
```
e*(-(((b*e - a*f)*x^3*Sqrt[c + d*x^2])/(b*(b*c - a*d)*Sqrt[a + b*x^2])) +
(((3*b*d*e + b*c*f - 4*a*d*f)*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(3*b*d) -
 (-((8*a^2*d^2*f + b^2*c*(3*d*e - 2*c*f) - 3*a*b*d*(2*d*e + c*f))*((x*Sqrt
[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcT
an[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))
/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))) + (c^(3/2)*(3*b*d*e + b*c*f - 4*a*d*f
)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])
/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))/(3*b*d))
/(b*(b*c - a*d))) + (f*(-(((b*e - a*f)*x^5*Sqrt[c + d*x^2])/(b*(b*c - a*d)
*Sqrt[a + b*x^2])) + (((5*b*d*e + b*c*f - 6*a*d*f)*x^3*Sqrt[a + b*x^2]*Sqr
t[c + d*x^2])/(5*b*d) - (-1/3*(((24*a^2*d*f)/b + (b*c*(5*d*e - 4*c*f))/d -
 5*a*(4*d*e + c*f))*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]) + ((48*a^3*d^3*f +
2*b^3*c^2*(5*d*e - 4*c*f) + 3*a*b^2*c*d*(5*d*e - 3*c*f) - 8*a^2*b*d^2*(5*d
*e + 2*c*f))*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a +
b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]
*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (c^(3/2)*(24*a^
2*d^2*f + b^2*c*(5*d*e - 4*c*f) - 5*a*b*d*(4*d*e + c*f))*Sqrt[a + b*x^2]*E
llipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(
a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))/(3*b*d))/(5*b*d))/(b*(b*c -
 a*d))))/e^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 440
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[g*(b*e - a*f)*(g*x)^(m - 1)*(a +
 b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] - Simp[
g^2/(2*b*(b*c - a*d)*(p + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c +
d*x^2)^q*Simp[c*(b*e - a*f)*(m - 1) + (d*(b*e - a*f)*(m + 2*q + 1) - b*2*(c
*f - d*e)*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, q}, x] &&
 LtQ[p, -1] && GtQ[m, 1]
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

rule 448
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[e   Int[(g*x)^m*(a + b*x
^2)^p*(c + d*x^2)^q*(e + f*x^2)^(r - 1), x], x] + Simp[f/e^2   Int[(g*x)^(m
 + 2)*(a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^(r - 1), x], x] /; FreeQ[{a,
b, c, d, e, f, g, m, p, q}, x] && IGtQ[r, 0]
```

Maple [A] (verified)

Time = 20.57 (sec) , antiderivative size = 634, normalized size of antiderivative = 1.22

method	result
elliptic	$\frac{\sqrt{(b x^2+a)(x^2 d+c)}\left(-\frac{(bd x^2+bc) a x \left(a^2 f^2-2abfe+b^2e^2\right)}{b^4(ad-bc)\sqrt{\left(x^2+\frac{a}{b}\right)(bd x^2+bc)}}+\frac{f^2 x^3\sqrt{bd x^4+ad x^2+x^2bc+ac}}{5b^2 d}+\frac{\left(-\frac{f(af-2be)}{b^2}-\frac{f^2(4ad+4bc)}{5b^2 d}\right)x\sqrt{bd x^4+ad x}}{3bd}\right.}{}$
risch	$-\frac{fx(-3bdf x^2+9adf+4bcf-10bde)\sqrt{b x^2+a}\sqrt{x^2d+c}}{15b^3d^2}+\frac{\left(-\frac{(33a^2d^2f^2+17abcdf^2-50abd^2ef+8b^2c^2f^2-20b^2cdef+15b^2d^2e^2)c\sqrt{1+}}{\sqrt{-\frac{b}{a}}\sqrt{b}}\right.}{}$
default	Expression too large to display

input
```
int(x^4*(f*x^2+e)^2/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOS
E)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-(b*d*x^2+b*c
)/b^4*a/(a*d-b*c)*x*(a^2*f^2-2*a*b*e*f+b^2*e^2)/((x^2+a/b)*(b*d*x^2+b*c))^
(1/2)+1/5*f^2/b^2/d*x^3*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+1/3*(-f*(a*f-2
*b*e)/b^2-1/5*f^2/b^2/d*(4*a*d+4*b*c))/b/d*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)
^(1/2)+(1/b^3*c*a/(a*d-b*c)*(a^2*f^2-2*a*b*e*f+b^2*e^2)-1/3*(-f*(a*f-2*b*e
)/b^2-1/5*f^2/b^2/d*(4*a*d+4*b*c))/b/d*a*c)/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)
*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^
(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-(1/b^3*(a^2*f^2-2*a*b*e*f+b^2*e^2)+1/b^3*d
*a*(a^2*f^2-2*a*b*e*f+b^2*e^2)/(a*d-b*c)-3/5*f^2/b^2/d*a*c-1/3*(-f*(a*f-2*
b*e)/b^2-1/5*f^2/b^2/d*(4*a*d+4*b*c))/b/d*(2*a*d+2*b*c))*c/(-b/a)^(1/2)*(1
+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*(E
llipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2)
,(-1+(a*d+b*c)/c/b)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1044 vs. $2(485) = 970$.

Time = 0.15 (sec) , antiderivative size = 1044, normalized size of antiderivative = 2.01

$$\int \frac{x^4(e+fx^2)^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^4*(f*x^2+e)^2/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="fr
icas")
```

output
```
-1/15*(((15*(b^4*c^2*d^2 - 2*a*b^3*c*d^3)*e^2 - 10*(2*b^4*c^3*d + 3*a*b^3*
c^2*d^2 - 8*a^2*b^2*c*d^3)*e*f + (8*b^4*c^4 + 9*a*b^3*c^3*d + 16*a^2*b^2*c
^2*d^2 - 48*a^3*b*c*d^3)*f^2)*x^3 + (15*(a*b^3*c^2*d^2 - 2*a^2*b^2*c*d^3)*
e^2 - 10*(2*a*b^3*c^3*d + 3*a^2*b^2*c^2*d^2 - 8*a^3*b*c*d^3)*e*f + (8*a*b^
3*c^4 + 9*a^2*b^2*c^3*d + 16*a^3*b*c^2*d^2 - 48*a^4*c*d^3)*f^2)*x)*sqrt(b*
d)*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - ((15*(b^4*c^2*
d^2 - 2*a*b^3*c*d^3 - a*b^3*d^4)*e^2 - 10*(2*b^4*c^3*d + 3*a*b^3*c^2*d^2 -
 4*a^2*b^2*d^4 - (8*a^2*b^2 - a*b^3)*c*d^3)*e*f + (8*b^4*c^4 + 9*a*b^3*c^3
*d - 24*a^3*b*d^4 + 4*(4*a^2*b^2 + a*b^3)*c^2*d^2 - (48*a^3*b - 5*a^2*b^2)
*c*d^3)*f^2)*x^3 + (15*(a*b^3*c^2*d^2 - 2*a^2*b^2*c*d^3 - a^2*b^2*d^4)*e^2
 - 10*(2*a*b^3*c^3*d + 3*a^2*b^2*c^2*d^2 - 4*a^3*b*d^4 - (8*a^3*b - a^2*b^
2)*c*d^3)*e*f + (8*a*b^3*c^4 + 9*a^2*b^2*c^3*d - 24*a^4*d^4 + 4*(4*a^3*b +
 a^2*b^2)*c^2*d^2 - (48*a^4 - 5*a^3*b)*c*d^3)*f^2)*x)*sqrt(b*d)*sqrt(-c/d)
*elliptic_f(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - (3*(b^4*c*d^3 - a*b^3*d^4)*
f^2*x^6 + 2*(5*(b^4*c*d^3 - a*b^3*d^4)*e*f - (2*b^4*c^2*d^2 + a*b^3*c*d^3
- 3*a^2*b^2*d^4)*f^2)*x^4 + 15*(a*b^3*c*d^3 - 2*a^2*b^2*d^4)*e^2 - 10*(2*a
*b^3*c^2*d^2 + 3*a^2*b^2*c*d^3 - 8*a^3*b*d^4)*e*f + (8*a*b^3*c^3*d + 9*a^2
*b^2*c^2*d^2 + 16*a^3*b*c*d^3 - 48*a^4*d^4)*f^2 + (15*(b^4*c*d^3 - a*b^3*d
^4)*e^2 - 20*(b^4*c^2*d^2 + a*b^3*c*d^3 - 2*a^2*b^2*d^4)*e*f + (8*b^4*c^3*
d + 5*a*b^3*c^2*d^2 + 11*a^2*b^2*c*d^3 - 24*a^3*b*d^4)*f^2)*x^2)*sqrt(b...
```

Sympy [F]

$$\int \frac{x^4(e+fx^2)^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \int \frac{x^4(e+fx^2)^2}{(a+bx^2)^{\frac{3}{2}}\sqrt{c+dx^2}}\,dx$$

input
```
integrate(x**4*(f*x**2+e)**2/(b*x**2+a)**(3/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(x**4*(e + f*x**2)**2/((a + b*x**2)**(3/2)*sqrt(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{x^4(e+fx^2)^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \int \frac{(fx^2+e)^2x^4}{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}}\,dx$$

input
```
integrate(x^4*(f*x^2+e)^2/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate((f*x^2 + e)^2*x^4/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)), x)
```

Giac [F]

$$\int \frac{x^4(e+fx^2)^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \int \frac{(fx^2+e)^2x^4}{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}}\,dx$$

input
```
integrate(x^4*(f*x^2+e)^2/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate((f*x^2 + e)^2*x^4/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(e+fx^2)^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx=\int \frac{x^4\,(f\,x^2+e)^2}{(b\,x^2+a)^{3/2}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int((x^4*(e + f*x^2)^2)/((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int((x^4*(e + f*x^2)^2)/((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^4(e+fx^2)^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx=\text{too large to display}$$

input
```
int(x^4*(f*x^2+e)^2/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x)
```

output
```
(9*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*c*d*f**2*x - 6*sqrt(c + d*x**2)*sqr
t(a + b*x**2)*a*d**2*f**2*x**3 + 6*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b*c**
2*f**2*x - 15*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b*c*d*e*f*x - 4*sqrt(c + d
*x**2)*sqrt(a + b*x**2)*b*c*d*f**2*x**3 + 10*sqrt(c + d*x**2)*sqrt(a + b*x
**2)*b*d**2*e*f*x**3 + 3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b*d**2*f**2*x**
5 + 24*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a**2*c + a**2*d*x**2
+ 2*a*b*c*x**2 + 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a**3*d**3*f*
*2 + 4*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a**2*c + a**2*d*x**2
+ 2*a*b*c*x**2 + 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a**2*b*c*d**
2*f**2 - 40*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a**2*c + a**2*d*
x**2 + 2*a*b*c*x**2 + 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a**2*b*
d**3*e*f + 24*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a**2*c + a**2*
d*x**2 + 2*a*b*c*x**2 + 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a**2*
b*d**3*f**2*x**2 + 2*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a**2*c
+ a**2*d*x**2 + 2*a*b*c*x**2 + 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x
)*a*b**2*c**2*d*f**2 - 5*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a**
2*c + a**2*d*x**2 + 2*a*b*c*x**2 + 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**
6),x)*a*b**2*c*d**2*e*f + 4*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(
a**2*c + a**2*d*x**2 + 2*a*b*c*x**2 + 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*
x**6),x)*a*b**2*c*d**2*f**2*x**2 + 15*int((sqrt(c + d*x**2)*sqrt(a + b*...
```

3.235 $\int \frac{x^2(e+fx^2)^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 374

$$\int \frac{x^2(e+fx^2)^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \frac{2f(3bde-bcf-2adf)x\sqrt{c+dx^2}}{3b^2d^2\sqrt{a+bx^2}}+\frac{f^2x^3\sqrt{c+dx^2}}{3bd\sqrt{a+bx^2}}$$
$$-\frac{\sqrt{a}(8a^2d^2f^2-3abdf(4de+cf)+b^2(3d^2e^2+6cdef-2c^2f^2))\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{3b^{5/2}d^2(bc-ad)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$
$$+\frac{\sqrt{a}(3b^2de^2+4a^2df^2-abf(6de+cf))\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{3b^{5/2}d(bc-ad)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
2/3*f*(-2*a*d*f-b*c*f+3*b*d*e)*x*(d*x^2+c)^(1/2)/b^2/d^2/(b*x^2+a)^(1/2)+1
/3*f^2*x^3*(d*x^2+c)^(1/2)/b/d/(b*x^2+a)^(1/2)-1/3*a^(1/2)*(8*a^2*d^2*f^2-
3*a*b*d*f*(c*f+4*d*e)+b^2*(-2*c^2*f^2+6*c*d*e*f+3*d^2*e^2))*(d*x^2+c)^(1/2
)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/b^(5/2)
/d^2/(-a*d+b*c)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)+1/3*a^(1/2
)*(3*b^2*d*e^2+4*a^2*d*f^2-a*b*f*(c*f+6*d*e))*(d*x^2+c)^(1/2)*InverseJacob
iAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/b^(5/2)/d/(-a*d+b*c)/(b*x
^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.66 (sec) , antiderivative size = 314, normalized size of antiderivative = 0.84

$$\int \frac{x^2(e+fx^2)^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \frac{\sqrt{\frac{b}{a}}dx(c+dx^2)\left(-4a^2df^2+abf(6de+cf-dfx^2)+b^2(-3de^2+cf^2x^2)\right)-}{}$$

input
```
Integrate[(x^2*(e + f*x^2)^2)/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

output
```
(Sqrt[b/a]*d*x*(c + d*x^2)*(-4*a^2*d*f^2 + a*b*f*(6*d*e + c*f - d*f*x^2) +
 b^2*(-3*d*e^2 + c*f^2*x^2)) - I*c*(8*a^2*d^2*f^2 - 3*a*b*d*f*(4*d*e + c*f
) + b^2*(3*d^2*e^2 + 6*c*d*e*f - 2*c^2*f^2))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 +
(d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (2*I)*c*(-(b*c
) + a*d)*f*(-3*b*d*e + b*c*f + 2*a*d*f)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^
2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(3*a^2*(b/a)^(5/2)*d
^2*(b*c - a*d)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (warning: unable to verify)

Time = 1.07 (sec) , antiderivative size = 673, normalized size of antiderivative = 1.80, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.314$, Rules used = {448, 440, 406, 320, 388, 313, 444, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(e+fx^2)^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \text{448}$$

$$\frac{f\int \frac{x^4(fx^2+e)}{(bx^2+a)^{3/2}\sqrt{dx^2+c}}dx}{e^2}+e\int \frac{x^2(fx^2+e)}{(bx^2+a)^{3/2}\sqrt{dx^2+c}}dx$$

$$\Big\downarrow \text{440}$$

$$\frac{f\left(\frac{\int \frac{x^2\left((3bde+bcf-4adf)x^2+3c(be-af)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{b(bc-ad)}-\frac{x^3\sqrt{c+dx^2}(be-af)}{b\sqrt{a+bx^2}(bc-ad)}\right)}{e^2}+e\left(\frac{\int \frac{(bde+bcf-2adf)x^2+c(be-af)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{b(bc-ad)}-\frac{x\sqrt{c+dx^2}(be-af)}{b\sqrt{a+bx^2}(bc-ad)}\right)$$

$$\Big\downarrow \text{406}$$

$$\frac{f\left(\frac{\int \frac{x^2\left((3bde+bcf-4adf)x^2+3c(be-af)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{b(bc-ad)}-\frac{x^3\sqrt{c+dx^2}(be-af)}{b\sqrt{a+bx^2}(bc-ad)}\right)}{e^2}+e\left(\frac{c(be-af)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+(-2adf+bcf+bde)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{b(bc-ad)}-\frac{x\sqrt{c+dx^2}(be-af)}{b\sqrt{a+bx^2}(bc-ad)}\right)$$

$$\Big\downarrow \text{320}$$

$$e\left(\frac{(-2adf+bcf+bde)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}(be-af)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{b(bc-ad)}-\frac{x\sqrt{c+dx^2}(be-af)}{b\sqrt{a+bx^2}(bc-ad)}\right)+\frac{f\left(\frac{\int \frac{x^2\left((3bde+bcf-4adf)x^2+3c(be-af)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{b(bc-ad)}-\frac{x^3\sqrt{c+dx^2}(be-af)}{b\sqrt{a+bx^2}(bc-ad)}\right)}{e^2}$$

$$\Big\downarrow \text{388}$$

$$e\left(\frac{(-2adf+bcf+bde)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(be-af)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{b(bc-ad)}-\frac{x\sqrt{c+dx^2}(be-af)}{b\sqrt{a+bx^2}(bc-ad)}\right)+\frac{f\left(\frac{\int \frac{x^2\left((3bde+bcf-4adf)x^2+3c(be-af)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{b(bc-ad)}-\frac{x^3\sqrt{c+dx^2}(be-af)}{b\sqrt{a+bx^2}(bc-ad)}\right)}{e^2}$$

$$\Big\downarrow \text{313}$$

$$\frac{f\left(\frac{\int \frac{x^2\left((3bde+bcf-4adf)x^2+3c(be-af)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{b(bc-ad)}-\frac{x^3\sqrt{c+dx^2}(be-af)}{b\sqrt{a+bx^2}(bc-ad)}\right)}{e^2}+$$

$$e\left(\frac{\frac{c^{3/2}\sqrt{a+bx^2}(be-af)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+(-2adf+bcf+bde)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right.}{b(bc-ad)}\right.$$

$\downarrow$ 444

$$\frac{f\left(\frac{\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(-4adf+bcf+3bde)}{3bd}-\frac{\int \frac{ac(3bde+bcf-4adf)-\left(c(3de-2cf)b^2-3ad(2de+cf)b+8a^2d^2f\right)x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3bd}}{b(bc-ad)}-\frac{x^3\sqrt{c+dx^2}(be-af)}{b\sqrt{a+bx^2}(bc-ad)}\right)}{e^2}+$$

$$e\left(\frac{\frac{c^{3/2}\sqrt{a+bx^2}(be-af)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+(-2adf+bcf+bde)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right.}{b(bc-ad)}\right.$$

$\downarrow$ 406

$$\frac{f\left(\frac{\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(-4adf+bcf+3bde)}{3bd}-\frac{ac(-4adf+bcf+3bde)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx-\left(8a^2d^2f-3abd(cf+2de)+b^2c(3de-2cf)\right)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3bd}}{b(bc-ad)}-\right.}{e^2}$$

$$e\left(\frac{\frac{c^{3/2}\sqrt{a+bx^2}(be-af)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+(-2adf+bcf+bde)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right.}{b(bc-ad)}\right.$$

$\downarrow$ 320

$$\frac{f\left(\frac{\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(-4adf+bcf+3bde)}{3bd}-\frac{\frac{c^{3/2}\sqrt{a+bx^2}(-4adf+bcf+3bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\left(8a^2d^2f-3abd(cf+2de)+b^2c(3de-2cf)\right)\int\frac{}{\sqrt{}}}{3bd}}{b(bc-ad)}\right.}{e^2}$$

$$e\left(\frac{\frac{c^{3/2}\sqrt{a+bx^2}(be-af)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+(-2adf+bcf+bde)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right.}{b(bc-ad)}\right.$$

$$\downarrow 388$$

$$\frac{f\left(\frac{\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(-4adf+bcf+3bde)}{3bd}-\frac{\frac{c^{3/2}\sqrt{a+bx^2}(-4adf+bcf+3bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\left(8a^2d^2f-3abd(cf+2de)+b^2c(3de-2cf)\right)\left(\frac{x}{b}\right.}{3bd}}{b(bc-ad)}\right.}{e^2}$$

$$e\left(\frac{\frac{c^{3/2}\sqrt{a+bx^2}(be-af)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+(-2adf+bcf+bde)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right.}{b(bc-ad)}\right.$$

$$\downarrow 313$$

$$\frac{f\left(\frac{\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(-4adf+bcf+3bde)}{3bd}-\frac{\frac{c^{3/2}\sqrt{a+bx^2}(-4adf+bcf+3bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\left(8a^2d^2f-3abd(cf+2de)+b^2c(3de-2cf)\right)\left(\frac{x\sqrt{\cdots}}{b\sqrt{\cdots}}\cdots\right.}{3bd}}{b(bc-ad)}\cdots\right)}{e^2}$$

$$e\left(\frac{\frac{c^{3/2}\sqrt{a+bx^2}(be-af)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+(-2adf+bcf+bde)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right.\cdots}{b(bc-ad)}\cdots\right.$$

input
```
Int[(x^2*(e + f*x^2)^2)/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

output
```
e*(-(((b*e - a*f)*x*Sqrt[c + d*x^2])/(b*(b*c - a*d)*Sqrt[a + b*x^2])) + ((
b*d*e + b*c*f - 2*a*d*f)*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[
c]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)]
)/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (c^
(3/2)*(b*e - a*f)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1
 - (b*c)/(a*d)])/(a*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c +
 d*x^2]))/(b*(b*c - a*d))) + (f*(-(((b*e - a*f)*x^3*Sqrt[c + d*x^2])/(b*(b
*c - a*d)*Sqrt[a + b*x^2])) + (((3*b*d*e + b*c*f - 4*a*d*f)*x*Sqrt[a + b*x
^2]*Sqrt[c + d*x^2])/(3*b*d) - (-((8*a^2*d^2*f + b^2*c*(3*d*e - 2*c*f) - 3
*a*b*d*(2*d*e + c*f))*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*
Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(
b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))) + (c^(3
/2)*(3*b*d*e + b*c*f - 4*a*d*f)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*
x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2
))]*Sqrt[c + d*x^2]))/(3*b*d))/(b*(b*c - a*d))))/e^2
```

Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 440
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[g*(b*e - a*f)*(g*x)^(m - 1)*(a +
 b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] - Simp[
g^2/(2*b*(b*c - a*d)*(p + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c +
d*x^2)^q*Simp[c*(b*e - a*f)*(m - 1) + (d*(b*e - a*f)*(m + 2*q + 1) - b*2*(c
*f - d*e)*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, q}, x] &&
 LtQ[p, -1] && GtQ[m, 1]
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

rule 448
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[e   Int[(g*x)^m*(a + b*x
^2)^p*(c + d*x^2)^q*(e + f*x^2)^(r - 1), x], x] + Simp[f/e^2   Int[(g*x)^(m
 + 2)*(a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^(r - 1), x], x] /; FreeQ[{a,
b, c, d, e, f, g, m, p, q}, x] && IGtQ[r, 0]
```

Maple [A] (verified)

Time = 10.56 (sec) , antiderivative size = 474, normalized size of antiderivative = 1.27

method	result
elliptic	$\frac{\sqrt{(b x^2+a)(x^2 d+c)}\left(\frac{(bd x^2+bc)x(a^2 f^2-2abfe+b^2e^2)}{b^3(ad-bc)\sqrt{\left(x^2+\frac{a}{b}\right)(bd x^2+bc)}}+\frac{f^2 x\sqrt{bd x^4+ad x^2+x^2bc+ac}}{3d b^2}+\frac{\left(-\frac{c(a^2f^2-2abfe+b^2e^2)}{b^2(ad-bc)}-\frac{f^2ac}{3b^2d}\right)\sqrt{1+\frac{b x^2}{a}}\sqrt{1+}}{\sqrt{-\frac{b}{a}}\sqrt{bd x^4+ad x^2-}}\right.}{}$
risch	$\frac{f^2x\sqrt{b x^2+a}\sqrt{x^2d+c}}{3b^2d}-\frac{\left(-\frac{f(5adf+2bcf-6bde)c\sqrt{1+\frac{b x^2}{a}}\sqrt{1+\frac{d x^2}{c}}\left(\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)-\mathrm{EllipticE}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right.\right.}{\sqrt{-\frac{b}{a}}\sqrt{bd x^4+ad x^2+x^2bc+ac}\,d}\right.}{}$
default	$\frac{\left(\sqrt{-\frac{b}{a}}\,abd^3f^2x^5-\sqrt{-\frac{b}{a}}\,b^2cd^2f^2x^5+4\sqrt{-\frac{b}{a}}\,a^2d^3f^2x^3-6\sqrt{-\frac{b}{a}}\,abd^3efx^3-\sqrt{-\frac{b}{a}}\,b^2c^2df^2x^3+3\sqrt{-\frac{b}{a}}\,b^2d^3e^2x^3+4\sqrt{\frac{b x^2+a}{a}}\sqrt{\right.}{}$

input
```
int(x^2*(f*x^2+e)^2/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOS
E)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*((b*d*x^2+b*c)
/b^3/(a*d-b*c)*x*(a^2*f^2-2*a*b*e*f+b^2*e^2)/((x^2+a/b)*(b*d*x^2+b*c))^(1/
2)+1/3*f^2/d/b^2*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+(-1/b^2*c/(a*d-b*c)
*(a^2*f^2-2*a*b*e*f+b^2*e^2)-1/3*f^2/b^2/d*a*c)/(-b/a)^(1/2)*(1+b*x^2/a)^(
1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b
/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-(-f*(a*f-2*b*e)/b^2-1/b^2*d*(a^2*f^2-2
*a*b*e*f+b^2*e^2)/(a*d-b*c)-1/3*f^2/d/b^2*(2*a*d+2*b*c))*c/(-b/a)^(1/2)*(1
+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*(E
llipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2)
,(-1+(a*d+b*c)/c/b)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 671, normalized size of antiderivative = 1.79

$$\int \frac{x^2(e+fx^2)^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx =$$

$$-\frac{((3\,b^3cd^2e^2+6\,(b^3c^2d-2\,ab^2cd^2)ef-(2\,b^3c^3+3\,ab^2c^2d-8\,a^2bcd^2)f^2)x^3+(3\,ab^2cd^2e^2+6\,(ab^2c^2d-2\,a\cdots}{\cdots}$$

input
```
integrate(x^2*(f*x^2+e)^2/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="fr
icas")
```

output
```
-1/3*(((3*b^3*c*d^2*e^2 + 6*(b^3*c^2*d - 2*a*b^2*c*d^2)*e*f - (2*b^3*c^3 +
 3*a*b^2*c^2*d - 8*a^2*b*c*d^2)*f^2)*x^3 + (3*a*b^2*c*d^2*e^2 + 6*(a*b^2*c
^2*d - 2*a^2*b*c*d^2)*e*f - (2*a*b^2*c^3 + 3*a^2*b*c^2*d - 8*a^3*c*d^2)*f^
2)*x)*sqrt(b*d)*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - (
(3*(b^3*c*d^2 + b^3*d^3)*e^2 + 6*(b^3*c^2*d - 2*a*b^2*c*d^2 - a*b^2*d^3)*e
*f - (2*b^3*c^3 + 3*a*b^2*c^2*d - 4*a^2*b*d^3 - (8*a^2*b - a*b^2)*c*d^2)*f
^2)*x^3 + (3*(a*b^2*c*d^2 + a*b^2*d^3)*e^2 + 6*(a*b^2*c^2*d - 2*a^2*b*c*d^
2 - a^2*b*d^3)*e*f - (2*a*b^2*c^3 + 3*a^2*b*c^2*d - 4*a^3*d^3 - (8*a^3 - a
^2*b)*c*d^2)*f^2)*x)*sqrt(b*d)*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)/x),
 a*d/(b*c)) - (3*a*b^2*d^3*e^2 + (b^3*c*d^2 - a*b^2*d^3)*f^2*x^4 + 6*(a*b^
2*c*d^2 - 2*a^2*b*d^3)*e*f - (2*a*b^2*c^2*d + 3*a^2*b*c*d^2 - 8*a^3*d^3)*f
^2 + 2*(3*(b^3*c*d^2 - a*b^2*d^3)*e*f - (b^3*c^2*d + a*b^2*c*d^2 - 2*a^2*b
*d^3)*f^2)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/((b^5*c*d^3 - a*b^4*d^4)*
x^3 + (a*b^4*c*d^3 - a^2*b^3*d^4)*x)
```

Sympy [F]

$$\int \frac{x^2(e+fx^2)^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \int \frac{x^2(e+fx^2)^2}{(a+bx^2)^{\frac{3}{2}}\sqrt{c+dx^2}}\,dx$$

input
```
integrate(x**2*(f*x**2+e)**2/(b*x**2+a)**(3/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(x**2*(e + f*x**2)**2/((a + b*x**2)**(3/2)*sqrt(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{x^2(e+fx^2)^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \int \frac{(fx^2+e)^2x^2}{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}}\,dx$$

input
```
integrate(x^2*(f*x^2+e)^2/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate((f*x^2 + e)^2*x^2/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)), x)
```

Giac [F]

$$\int \frac{x^2(e+fx^2)^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \int \frac{(fx^2+e)^2x^2}{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}}\,dx$$

input
```
integrate(x^2*(f*x^2+e)^2/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate((f*x^2 + e)^2*x^2/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(e+fx^2)^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \int \frac{x^2\,(f\,x^2+e)^2}{(b\,x^2+a)^{3/2}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int((x^2*(e + f*x^2)^2)/((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int((x^2*(e + f*x^2)^2)/((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^2(e+fx^2)^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \text{Too large to display}$$

input
```
int(x^2*(f*x^2+e)^2/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x)
```

output

```
( - 3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*c*f**2*x + 2*sqrt(c + d*x**2)*sq
rt(a + b*x**2)*a*d*f**2*x**3 + 3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b*d*e**
2*x - 8*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a**2*c + a**2*d*x**2
 + 2*a*b*c*x**2 + 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a**3*d**2*f
**2 - int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a**2*c + a**2*d*x**2 +
 2*a*b*c*x**2 + 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a**2*b*c*d*f*
*2 + 12*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a**2*c + a**2*d*x**2
 + 2*a*b*c*x**2 + 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a**2*b*d**2
*e*f - 8*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a**2*c + a**2*d*x**
2 + 2*a*b*c*x**2 + 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a**2*b*d**
2*f**2*x**2 - int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a**2*c + a**2*
d*x**2 + 2*a*b*c*x**2 + 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a*b**
2*c*d*f**2*x**2 - 3*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a**2*c +
 a**2*d*x**2 + 2*a*b*c*x**2 + 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)
*a*b**2*d**2*e**2 + 12*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a**2*
c + a**2*d*x**2 + 2*a*b*c*x**2 + 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6)
,x)*a*b**2*d**2*e*f*x**2 - 3*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/
(a**2*c + a**2*d*x**2 + 2*a*b*c*x**2 + 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d
*x**6),x)*b**3*d**2*e**2*x**2 + 3*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/
(a**2*c + a**2*d*x**2 + 2*a*b*c*x**2 + 2*a*b*d*x**4 + b**2*c*x**4 + b**...
```

3.236 $\int \frac{(e+fx^2)^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 278

$$\int \frac{(e+fx^2)^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \frac{f^2x\sqrt{c+dx^2}}{bd\sqrt{a+bx^2}} + \frac{(b^2de^2+2a^2df^2-abf(2de+cf))\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}b^{3/2}d(bc-ad)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} - \frac{\sqrt{a}(acf^2+be(de-2cf))\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{b^{3/2}c(bc-ad)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
f^2*x*(d*x^2+c)^(1/2)/b/d/(b*x^2+a)^(1/2)+(b^2*d*e^2+2*a^2*d*f^2-a*b*f*(c*
f+2*d*e))*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1
-a*d/b/c)^(1/2))/a^(1/2)/b^(3/2)/d/(-a*d+b*c)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)
/c/(b*x^2+a))^(1/2)-a^(1/2)*(a*c*f^2+b*e*(-2*c*f+d*e))*(d*x^2+c)^(1/2)*Inv
erseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/b^(3/2)/c/(-a*d+
b*c)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 9.29 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.90

$$\int \frac{(e+fx^2)^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \frac{-\sqrt{\frac{b}{a}}d(be-af)^2x(c+dx^2)-ic(b^2de^2+2a^2df^2-abf(2de+cf))\sqrt{1+\frac{bx}{c}}}{}$$

input
```
Integrate[(e + f*x^2)^2/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

output
```
(-(Sqrt[b/a]*d*(b*e - a*f)^2*x*(c + d*x^2)) - I*c*(b^2*d*e^2 + 2*a^2*d*f^2
 - a*b*f*(2*d*e + c*f))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[
I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + I*(-(b*c) + a*d)*(-(b*d*e^2) + a*c*
f^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]
*x], (a*d)/(b*c)])/(a^2*(b/a)^(3/2)*d*(-(b*c) + a*d)*Sqrt[a + b*x^2]*Sqrt[
c + d*x^2])
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 862 vs. 2(278) = 556.

Time = 1.05 (sec) , antiderivative size = 862, normalized size of antiderivative = 3.10, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx^2)^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx$$

$$\Big\downarrow \; 433$$

$$\int \left(\frac{e^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}+\frac{2efx^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}+\frac{f^2x^4}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{\sqrt{c}\sqrt{d}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)e^2}{a(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}} - \frac{\sqrt{c}\sqrt{d}\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)e^2}{a(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}} + \frac{bx\sqrt{dx^2+c}e^2}{a(bc-ad)\sqrt{bx^2+a}} - \frac{dx\sqrt{bx^2+a}e^2}{a(bc-ad)\sqrt{dx^2+c}} - \frac{2\sqrt{c}\sqrt{d}f\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)e}{b(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}} + \frac{2c^{3/2}f\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)e}{a\sqrt{d}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}} - \frac{2fx\sqrt{dx^2+c}e}{(bc-ad)\sqrt{bx^2+a}} + \frac{2dfx\sqrt{bx^2+a}e}{b(bc-ad)\sqrt{dx^2+c}} - \frac{\sqrt{c}(bc-2ad)f^2\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b^2\sqrt{d}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}} - \frac{c^{3/2}f^2\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{b\sqrt{d}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}} + \frac{af^2x\sqrt{dx^2+c}}{b(bc-ad)\sqrt{bx^2+a}} + \frac{(bc-2ad)f^2x\sqrt{bx^2+a}}{b^2(bc-ad)\sqrt{dx^2+c}}$$

input
```
Int[(e + f*x^2)^2/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

output
```
-((d*e^2*x*Sqrt[a + b*x^2])/(a*(b*c - a*d)*Sqrt[c + d*x^2])) + (2*d*e*f*x*
Sqrt[a + b*x^2])/(b*(b*c - a*d)*Sqrt[c + d*x^2]) + ((b*c - 2*a*d)*f^2*x*Sq
rt[a + b*x^2])/(b^2*(b*c - a*d)*Sqrt[c + d*x^2]) + (b*e^2*x*Sqrt[c + d*x^2
])/(a*(b*c - a*d)*Sqrt[a + b*x^2]) - (2*e*f*x*Sqrt[c + d*x^2])/((b*c - a*d
)*Sqrt[a + b*x^2]) + (a*f^2*x*Sqrt[c + d*x^2])/(b*(b*c - a*d)*Sqrt[a + b*x
^2]) + (Sqrt[c]*Sqrt[d]*e^2*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/S
qrt[c]], 1 - (b*c)/(a*d)])/(a*(b*c - a*d)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x
^2))]*Sqrt[c + d*x^2]) - (2*Sqrt[c]*Sqrt[d]*e*f*Sqrt[a + b*x^2]*EllipticE[
ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*(b*c - a*d)*Sqrt[(c*(a +
 b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) - (Sqrt[c]*(b*c - 2*a*d)*f^2*Sq
rt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b^
2*Sqrt[d]*(b*c - a*d)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2
]) - (Sqrt[c]*Sqrt[d]*e^2*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqr
t[c]], 1 - (b*c)/(a*d)])/(a*(b*c - a*d)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2
))]*Sqrt[c + d*x^2]) + (2*c^(3/2)*e*f*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sq
rt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*(b*c - a*d)*Sqrt[(c*(a + b
*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) - (c^(3/2)*f^2*Sqrt[a + b*x^2]*El
lipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*(b*c - a
*d)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Defintions of rubi rules used

rule 433
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)
^q*(e + f*x^2)^r, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, f, p,
 q, r}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 6.22 (sec) , antiderivative size = 448, normalized size of antiderivative = 1.61

method	result
elliptic	$\frac{\sqrt{(b\,x^2+a)(x^2d+c)}\left(-\frac{(bd\,x^2+bc)x(a^2f^2-2abfe+b^2e^2)}{b^2a(ad-bc)\sqrt{\left(x^2+\frac{a}{b}\right)(bd\,x^2+bc)}}+\frac{\left(-\frac{f(af-2be)}{b^2}+\frac{a^2f^2-2abfe+b^2e^2}{b^2a}+\frac{c(a^2f^2-2abfe+b^2e^2)}{ba(ad-bc)}\right)\sqrt{1+\frac{b\,x^2}{a}}\sqrt{1+d}}{\sqrt{-\frac{b}{a}}\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}\right.}{}$
default	$\left(-\sqrt{-\frac{b}{a}}\,a^2d^2f^2x^3+2\sqrt{-\frac{b}{a}}\,abd^2ef\,x^3-\sqrt{-\frac{b}{a}}\,b^2d^2e^2x^3-\sqrt{\frac{b\,x^2+a}{a}}\sqrt{\frac{x^2d+c}{c}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)a^2cd\,f^2+\sqrt{\frac{b\,x^2+a}{a}}\sqrt{\frac{x^2d+}{c}}\right.$

input
```
int((f*x^2+e)^2/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-(b*d*x^2+b*c
)/b^2/a/(a*d-b*c)*x*(a^2*f^2-2*a*b*e*f+b^2*e^2)/((x^2+a/b)*(b*d*x^2+b*c))^
(1/2)+(-f*(a*f-2*b*e)/b^2+(a^2*f^2-2*a*b*e*f+b^2*e^2)/b^2/a+1/b*c/a/(a*d-b
*c)*(a^2*f^2-2*a*b*e*f+b^2*e^2))/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c
)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(
a*d+b*c)/c/b)^(1/2))-(f^2/b+(a^2*f^2-2*a*b*e*f+b^2*e^2)/b/(a*d-b*c)*d/a)*c
/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2
+a*c)^(1/2)/d*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-Elliptic
E(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 500, normalized size of antiderivative = 1.80

$$\int \frac{(e+fx^2)^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \frac{((b^3c^2de^2-2\,ab^2c^2def-(ab^2c^3-2\,a^2bc^2d)f^2)x^3+(ab^2c^2de^2-2\,a^2bc^2def}{}$$

input
```
integrate((f*x^2+e)^2/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="fricas
")
```

output
```
(((b^3*c^2*d*e^2 - 2*a*b^2*c^2*d*e*f - (a*b^2*c^3 - 2*a^2*b*c^2*d)*f^2)*x^
3 + (a*b^2*c^2*d*e^2 - 2*a^2*b*c^2*d*e*f - (a^2*b*c^3 - 2*a^3*c^2*d)*f^2)*
x)*sqrt(b*d)*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - (((b
^3*c^2*d + a*b^2*d^3)*e^2 - 2*(a*b^2*c^2*d + a*b^2*c*d^2)*e*f - (a*b^2*c^3
 - 2*a^2*b*c^2*d - a^2*b*c*d^2)*f^2)*x^3 + ((a*b^2*c^2*d + a^2*b*d^3)*e^2
- 2*(a^2*b*c^2*d + a^2*b*c*d^2)*e*f - (a^2*b*c^3 - 2*a^3*c^2*d - a^3*c*d^2
)*f^2)*x)*sqrt(b*d)*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)/x), a*d/(b*c))
 - (a*b^2*c*d^2*e^2 - 2*a^2*b*c*d^2*e*f - (a*b^2*c^2*d - a^2*b*c*d^2)*f^2*
x^2 - (a^2*b*c^2*d - 2*a^3*c*d^2)*f^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/((
a*b^4*c^2*d^2 - a^2*b^3*c*d^3)*x^3 + (a^2*b^3*c^2*d^2 - a^3*b^2*c*d^3)*x)
```

Sympy [F]

$$\int \frac{(e+fx^2)^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \int \frac{(e+fx^2)^2}{(a+bx^2)^{\frac{3}{2}}\sqrt{c+dx^2}}\,dx$$

input
```
integrate((f*x**2+e)**2/(b*x**2+a)**(3/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral((e + f*x**2)**2/((a + b*x**2)**(3/2)*sqrt(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{(e+fx^2)^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \int \frac{(fx^2+e)^2}{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}}\,dx$$

input
```
integrate((f*x^2+e)^2/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="maxima
")
```

output
```
integrate((f*x^2 + e)^2/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)), x)
```

Giac [F]

$$\int \frac{(e+fx^2)^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \int \frac{(fx^2+e)^2}{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}}\,dx$$

input
```
integrate((f*x^2+e)^2/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((f*x^2 + e)^2/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx^2)^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \int \frac{(f\,x^2+e)^2}{(b\,x^2+a)^{3/2}\sqrt{d\,x^2+c}}\,dx$$

input
```
int((e + f*x^2)^2/((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int((e + f*x^2)^2/((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{(e+fx^2)^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,efx+\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^4}{b^2d\,x^6+2abd\,x^4+b^2c\,x^4+a^2d\,x^2+2abc\,x^2+a^2c}dx\right)a^2d\,f^2}{}$$

input
```
int((f*x^2+e)^2/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x)
```

output
```
(sqrt(c + d*x**2)*sqrt(a + b*x**2)*e*f*x + int((sqrt(c + d*x**2)*sqrt(a +
b*x**2)*x**4)/(a**2*c + a**2*d*x**2 + 2*a*b*c*x**2 + 2*a*b*d*x**4 + b**2*c
*x**4 + b**2*d*x**6),x)*a**2*d*f**2 - int((sqrt(c + d*x**2)*sqrt(a + b*x**
2)*x**4)/(a**2*c + a**2*d*x**2 + 2*a*b*c*x**2 + 2*a*b*d*x**4 + b**2*c*x**4
 + b**2*d*x**6),x)*a*b*d*e*f + int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4
)/(a**2*c + a**2*d*x**2 + 2*a*b*c*x**2 + 2*a*b*d*x**4 + b**2*c*x**4 + b**2
*d*x**6),x)*a*b*d*f**2*x**2 - int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)
/(a**2*c + a**2*d*x**2 + 2*a*b*c*x**2 + 2*a*b*d*x**4 + b**2*c*x**4 + b**2*
d*x**6),x)*b**2*d*e*f*x**2 - int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a**2
*c + a**2*d*x**2 + 2*a*b*c*x**2 + 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6
),x)*a**2*c*e*f + int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a**2*c + a**2*d
*x**2 + 2*a*b*c*x**2 + 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a**2*d
*e**2 - int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a**2*c + a**2*d*x**2 + 2*
a*b*c*x**2 + 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a*b*c*e*f*x**2 +
 int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a**2*c + a**2*d*x**2 + 2*a*b*c*x
**2 + 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a*b*d*e**2*x**2)/(a*d*(
a + b*x**2))
```

3.237 $\int \frac{(e+fx^2)^2}{x^2(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 283

$$\int \frac{(e+fx^2)^2}{x^2(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = -\frac{e^2\sqrt{c+dx^2}}{acx\sqrt{a+bx^2}} - \frac{(2b^2ce^2+a^2cf^2-abe(de+2cf))\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{a^{3/2}\sqrt{b}c(bc-ad)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} + \frac{(bde^2-af(2de-cf))\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{b}c(bc-ad)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
-e^2*(d*x^2+c)^(1/2)/a/c/x/(b*x^2+a)^(1/2)-(2*b^2*c*e^2+a^2*c*f^2-a*b*e*(2
*c*f+d*e))*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(
1-a*d/b/c)^(1/2))/a^(3/2)/b^(1/2)/c/(-a*d+b*c)/(b*x^2+a)^(1/2)/(a*(d*x^2+c
)/c/(b*x^2+a))^(1/2)+(b*d*e^2-a*f*(-c*f+2*d*e))*(d*x^2+c)^(1/2)*InverseJac
obiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/a^(1/2)/b^(1/2)/c/(-a*d
+b*c)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 9.36 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.03

$$\int \frac{(e+fx^2)^2}{x^2(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \frac{\sqrt{\frac{b}{a}}(c+dx^2)\left(2b^2ce^2x^2+a^2(-de^2+cf^2x^2)+abe(-dex^2+c(e-2fx^2)\right)}{}$$

input
```
Integrate[(e + f*x^2)^2/(x^2*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

output
```
(Sqrt[b/a]*(c + d*x^2)*(2*b^2*c*e^2*x^2 + a^2*(-(d*e^2) + c*f^2*x^2) + a*b
*e*(-(d*e*x^2) + c*(e - 2*f*x^2))) + I*c*(2*b^2*c*e^2 + a^2*c*f^2 - a*b*e*
(d*e + 2*c*f))*x*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSi
nh[Sqrt[b/a]*x], (a*d)/(b*c)] - (2*I)*c*(-(b*c) + a*d)*e*(-(b*e) + a*f)*x*
Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x],
(a*d)/(b*c)])/(a^2*Sqrt[b/a]*c*(-(b*c) + a*d)*x*Sqrt[a + b*x^2]*Sqrt[c + d
*x^2])
```

Rubi [A] (warning: unable to verify)

Time = 0.91 (sec) , antiderivative size = 564, normalized size of antiderivative = 1.99, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.371$, Rules used = {448, 400, 313, 320, 441, 25, 445, 25, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx^2)^2}{x^2(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx$$

$$\Big\downarrow \text{448}$$

$$\frac{f\int \frac{fx^2+e}{(bx^2+a)^{3/2}\sqrt{dx^2+c}}dx}{e^2}+e\int \frac{fx^2+e}{x^2(bx^2+a)^{3/2}\sqrt{dx^2+c}}dx$$

$$\Big\downarrow \text{400}$$

$$\frac{f\left(\frac{(be-af)\int \frac{\sqrt{dx^2+c}}{\left(bx^2+a\right)^{3/2}}dx}{bc-ad}-\frac{(de-cf)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}\right)}{e^2}+e\int \frac{fx^2+e}{x^2\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}}dx$$

$$\Big\downarrow \text{313}$$

$$\frac{f\left(\frac{\sqrt{c+dx^2}(be-af)E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{b}\sqrt{a+bx^2}(bc-ad)\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}-\frac{(de-cf)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}\right)}{e^2}+e\int \frac{fx^2+e}{x^2\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}}dx$$

$$\Big\downarrow \text{320}$$

$$e\int \frac{fx^2+e}{x^2\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}}dx+\frac{f\left(\frac{\sqrt{c+dx^2}(be-af)E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{b}\sqrt{a+bx^2}(bc-ad)\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}-\frac{\sqrt{c}\sqrt{a+bx^2}(de-cf)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{e^2}$$

$$\Big\downarrow \text{441}$$

$$e\left(\frac{\sqrt{c+dx^2}(be-af)}{ax\sqrt{a+bx^2}(bc-ad)}-\frac{\int -\frac{d(be-af)x^2+2bce-ade-acf}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{a(bc-ad)}\right)+\frac{f\left(\frac{\sqrt{c+dx^2}(be-af)E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{b}\sqrt{a+bx^2}(bc-ad)\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}-\frac{\sqrt{c}\sqrt{a+bx^2}(de-cf)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{e^2}$$

$$\Big\downarrow \text{25}$$

$$e\left(\frac{\int \frac{d(be-af)x^2+2bce-a(de+cf)}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{a(bc-ad)}+\frac{\sqrt{c+dx^2}(be-af)}{ax\sqrt{a+bx^2}(bc-ad)}\right)+\frac{f\left(\frac{\sqrt{c+dx^2}(be-af)E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{b}\sqrt{a+bx^2}(bc-ad)\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}-\frac{\sqrt{c}\sqrt{a+bx^2}(de-cf)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{e^2}$$

$$\Big\downarrow \text{445}$$

$$e\left(\frac{-\frac{\int -\frac{d\left(b(2bce-a(de+cf))x^2+ac(be-af)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(-acf-ade+2bce)}{acx}}{a(bc-ad)}+\frac{\sqrt{c+dx^2}(be-af)}{ax\sqrt{a+bx^2}(bc-ad)}\right)+\frac{f\left(\frac{\sqrt{c+dx^2}(be-af)E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{b}\sqrt{a+bx^2}(bc-ad)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}-\frac{\sqrt{c}\sqrt{a+bx^2}(de-cf)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{e^2}$$

$$\Big\downarrow \text{ 25}$$

$$e\left(\frac{\frac{\int \frac{d\left(b(2bce-a(de+cf))x^2+ac(be-af)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(-acf-ade+2bce)}{acx}}{a(bc-ad)}+\frac{\sqrt{c+dx^2}(be-af)}{ax\sqrt{a+bx^2}(bc-ad)}\right)+\frac{f\left(\frac{\sqrt{c+dx^2}(be-af)E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{b}\sqrt{a+bx^2}(bc-ad)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}-\frac{\sqrt{c}\sqrt{a+bx^2}(de-cf)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{e^2}$$

$$\Big\downarrow \text{ 27}$$

$$e\left(\frac{\frac{d\int \frac{b(2bce-a(de+cf))x^2+ac(be-af)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(-acf-ade+2bce)}{acx}}{a(bc-ad)}+\frac{\sqrt{c+dx^2}(be-af)}{ax\sqrt{a+bx^2}(bc-ad)}\right)+\frac{f\left(\frac{\sqrt{c+dx^2}(be-af)E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{b}\sqrt{a+bx^2}(bc-ad)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}-\frac{\sqrt{c}\sqrt{a+bx^2}(de-cf)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{e^2}$$

$$\Big\downarrow \text{ 406}$$

$$e\left(\frac{\frac{d\left(ac(be-af)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+b(2bce-a(cf+de))\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(-acf-ade+2bce)}{acx}}{a(bc-ad)}+\frac{\sqrt{c+dx^2}(be-af)}{ax\sqrt{a+bx^2}(bc-ad)}\right)+\frac{f\left(\frac{\sqrt{c+dx^2}(be-af)E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{b}\sqrt{a+bx^2}(bc-ad)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}-\frac{\sqrt{c}\sqrt{a+bx^2}(de-cf)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{e^2}$$

$$\Big\downarrow \text{ 320}$$

$$e\left(\frac{\frac{d\left(b(2bce-a(cf+de))\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}(be-af)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(-acf-ade+2bce)}{acx}}{a(bc-ad)}+\right.$$

$$\frac{f\left(\frac{\sqrt{c+dx^2}(be-af)E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{b}\sqrt{a+bx^2}(bc-ad)\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}-\frac{\sqrt{c}\sqrt{a+bx^2}(de-cf)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{e^2}$$

$\downarrow$ 388

$$e\left(\frac{\frac{d\left(b(2bce-a(cf+de))\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(be-af)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(-acf-a}{acx}}{a(bc-ad)}\right.$$

$$\frac{f\left(\frac{\sqrt{c+dx^2}(be-af)E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{b}\sqrt{a+bx^2}(bc-ad)\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}-\frac{\sqrt{c}\sqrt{a+bx^2}(de-cf)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{e^2}$$

$\downarrow$ 313

$$e\left(\frac{\frac{d\left(\frac{c^{3/2}\sqrt{a+bx^2}(be-af)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+b(2bce-a(cf+de))\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)\right)}{ac}-\frac{\sqrt{a+bx^2}\sqrt{}}{}}{a(bc-ad)}\right.$$

$$\frac{f\left(\frac{\sqrt{c+dx^2}(be-af)E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{b}\sqrt{a+bx^2}(bc-ad)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}-\frac{\sqrt{c}\sqrt{a+bx^2}(de-cf)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{e^2}$$

input
```
Int[(e + f*x^2)^2/(x^2*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

output
```
(f*(((b*e - a*f)*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]], 1
- (a*d)/(b*c)])/(Sqrt[a]*Sqrt[b]*(b*c - a*d)*Sqrt[a + b*x^2]*Sqrt[(a*(c +
d*x^2))/(c*(a + b*x^2))]) - (Sqrt[c]*(d*e - c*f)*Sqrt[a + b*x^2]*EllipticF
[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*(b*c - a*d)*Sqr
t[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])))/e^2 + e*(((b*e - a*f
)*Sqrt[c + d*x^2])/(a*(b*c - a*d)*x*Sqrt[a + b*x^2]) + (-(((2*b*c*e - a*d*
e - a*c*f)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(a*c*x)) + (d*(b*(2*b*c*e - a*
(d*e + c*f))*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a +
b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]
*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (c^(3/2)*(b*e -
 a*f)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*
d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])))/(a*
c))/(a*(b*c - a*d)))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 400
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 441
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
+ b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(a*g*2*(b*c - a*d)*(p + 1))), x] + Si
mp[1/(a*2*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2
)^q*Simp[c*(b*e - a*f)*(m + 1) + e*2*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(m
 + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, q},
 x] && LtQ[p, -1]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 448
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[e   Int[(g*x)^m*(a + b*x
^2)^p*(c + d*x^2)^q*(e + f*x^2)^(r - 1), x], x] + Simp[f/e^2   Int[(g*x)^(m
 + 2)*(a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^(r - 1), x], x] /; FreeQ[{a,
b, c, d, e, f, g, m, p, q}, x] && IGtQ[r, 0]
```

Maple [A] (verified)

Time = 10.75 (sec) , antiderivative size = 480, normalized size of antiderivative = 1.70

method	result
elliptic	$\frac{\sqrt{(b x^2+a)(x^2 d+c)}\left(\frac{\left(bd x^2+bc\right)x\left(a^2 f^2-2abfe+b^2e^2\right)}{b a^2(ad-bc)\sqrt{\left(x^2+\frac{a}{b}\right)\left(bd x^2+bc\right)}}-\frac{e^2\sqrt{bd x^4+ad x^2+x^2bc+ac}}{a^2cx}+\frac{\left(\frac{f^2}{b}-\frac{a^2f^2-2abfe+b^2e^2}{b a^2}-\frac{c\left(a^2f^2-2abfe+b^2e^2\right)}{a^2(ad-bc)}\right.}{\sqrt{-\frac{b}{a}}\sqrt{bd x^4 \ldots}}\right.}{\ldots}$
risch	$-\frac{e^2\sqrt{b x^2+a}\sqrt{x^2d+c}}{a^2cx}+\frac{\left(\frac{a^2cf^2\sqrt{1+\frac{b x^2}{a}}\sqrt{1+\frac{d x^2}{c}}\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{b\sqrt{-\frac{b}{a}}\sqrt{bd x^4+ad x^2+x^2bc+ac}}-\frac{b e^2c\sqrt{1+\frac{b x^2}{a}}\sqrt{1+\frac{d x^2}{c}}\left(\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{- \ldots}\right.\right.}{\sqrt{-\frac{b}{a}}\sqrt{bd x^4+ \ldots}}\right.}{\ldots}$
default	$\frac{\left(\sqrt{-\frac{b}{a}}a^2cdf^2x^4-2\sqrt{-\frac{b}{a}}abcdefx^4-\sqrt{-\frac{b}{a}}abd^2e^2x^4+2\sqrt{-\frac{b}{a}}b^2cde^2x^4+2\sqrt{\frac{b x^2+a}{a}}\sqrt{\frac{x^2d+c}{c}}\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)a^2cdef \ldots\right.}{\ldots}$

input
```
int((f*x^2+e)^2/x^2/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOS
E)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*((b*d*x^2+b*c)
/b/a^2/(a*d-b*c)*x*(a^2*f^2-2*a*b*e*f+b^2*e^2)/((x^2+a/b)*(b*d*x^2+b*c))^(
1/2)-e^2/a^2/c*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/x+(f^2/b-1/b*(a^2*f^2-2
*a*b*e*f+b^2*e^2)/a^2-c/a^2/(a*d-b*c)*(a^2*f^2-2*a*b*e*f+b^2*e^2))/(-b/a)^
(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1
/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-(-(a^2*f^2-2*a*b*e*
f+b^2*e^2)*d/(a*d-b*c)/a^2+b*d*e^2/a^2/c)*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)
*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(-b/
a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/
c/b)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 434, normalized size of antiderivative = 1.53

$$\int \frac{(e+fx^2)^2}{x^2(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx =$$

$$-\frac{((2\,ab^3cef-a^2b^2cf^2-(2\,b^4c-ab^3d)e^2)x^3+(2\,a^2b^2cef-a^3bcf^2-(2\,ab^3c-a^2b^2d)e^2)x)\sqrt{ac}\sqrt{-\frac{b}{a}}E(\mathrm{ar}$$

input
```
integrate((f*x^2+e)^2/x^2/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="fr
icas")
```

output
```
-(((2*a*b^3*c*e*f - a^2*b^2*c*f^2 - (2*b^4*c - a*b^3*d)*e^2)*x^3 + (2*a^2*
b^2*c*e*f - a^3*b*c*f^2 - (2*a*b^3*c - a^2*b^2*d)*e^2)*x)*sqrt(a*c)*sqrt(-
b/a)*elliptic_e(arcsin(x*sqrt(-b/a)), a*d/(b*c)) + (((a^3*b + a^2*b^2)*c*f
^2 + (2*b^4*c + (a^2*b^2 - a*b^3)*d)*e^2 - 2*(a*b^3*c + a^3*b*d)*e*f)*x^3
+ ((a^4 + a^3*b)*c*f^2 + (2*a*b^3*c + (a^3*b - a^2*b^2)*d)*e^2 - 2*(a^2*b^
2*c + a^4*d)*e*f)*x)*sqrt(a*c)*sqrt(-b/a)*elliptic_f(arcsin(x*sqrt(-b/a)),
 a*d/(b*c)) + ((a^2*b^2*c - a^3*b*d)*e^2 - (2*a^2*b^2*c*e*f - a^3*b*c*f^2
- (2*a*b^3*c - a^2*b^2*d)*e^2)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/((a^3
*b^3*c^2 - a^4*b^2*c*d)*x^3 + (a^4*b^2*c^2 - a^5*b*c*d)*x)
```

Sympy [F]

$$\int \frac{(e+fx^2)^2}{x^2(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \int \frac{(e+fx^2)^2}{x^2(a+bx^2)^{\frac{3}{2}}\sqrt{c+dx^2}}\,dx$$

input
```
integrate((f*x**2+e)**2/x**2/(b*x**2+a)**(3/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral((e + f*x**2)**2/(x**2*(a + b*x**2)**(3/2)*sqrt(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{(e+fx^2)^2}{x^2(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \int \frac{(fx^2+e)^2}{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}x^2}\,dx$$

input
```
integrate((f*x^2+e)^2/x^2/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate((f*x^2 + e)^2/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*x^2), x)
```

Giac [F]

$$\int \frac{(e+fx^2)^2}{x^2(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \int \frac{(fx^2+e)^2}{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}x^2}\,dx$$

input
```
integrate((f*x^2+e)^2/x^2/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate((f*x^2 + e)^2/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx^2)^2}{x^2 (a+bx^2)^{3/2} \sqrt{c+dx^2}} \, dx = \int \frac{(f\,x^2+e)^2}{x^2\,(b\,x^2+a)^{3/2}\,\sqrt{d\,x^2+c}} \, dx$$

input
```
int((e + f*x^2)^2/(x^2*(a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int((e + f*x^2)^2/(x^2*(a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{(e+fx^2)^2}{x^2 (a+bx^2)^{3/2} \sqrt{c+dx^2}} \, dx = \frac{-\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,e^2 + \left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^2}{b^2 d\,x^6+2abd\,x^4+b^2 c\,x^4+a^2 d\,x^2+2abc\,x^2+a^2 c}dx\right) a^2 c$$

input
```
int((f*x^2+e)^2/x^2/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x)
```

output
```
( - sqrt(c + d*x**2)*sqrt(a + b*x**2)*e**2 + int((sqrt(c + d*x**2)*sqrt(a
+ b*x**2)*x**2)/(a**2*c + a**2*d*x**2 + 2*a*b*c*x**2 + 2*a*b*d*x**4 + b**2
*c*x**4 + b**2*d*x**6),x)*a**2*c*f**2*x + int((sqrt(c + d*x**2)*sqrt(a + b
*x**2)*x**2)/(a**2*c + a**2*d*x**2 + 2*a*b*c*x**2 + 2*a*b*d*x**4 + b**2*c*
x**4 + b**2*d*x**6),x)*a*b*c*f**2*x**3 - int((sqrt(c + d*x**2)*sqrt(a + b*
x**2)*x**2)/(a**2*c + a**2*d*x**2 + 2*a*b*c*x**2 + 2*a*b*d*x**4 + b**2*c*x
**4 + b**2*d*x**6),x)*a*b*d*e**2*x - int((sqrt(c + d*x**2)*sqrt(a + b*x**2
)*x**2)/(a**2*c + a**2*d*x**2 + 2*a*b*c*x**2 + 2*a*b*d*x**4 + b**2*c*x**4
+ b**2*d*x**6),x)*b**2*d*e**2*x**3 + 2*int((sqrt(c + d*x**2)*sqrt(a + b*x*
*2))/(a**2*c + a**2*d*x**2 + 2*a*b*c*x**2 + 2*a*b*d*x**4 + b**2*c*x**4 + b
**2*d*x**6),x)*a**2*c*e*f*x - 2*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a
**2*c + a**2*d*x**2 + 2*a*b*c*x**2 + 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x
**6),x)*a*b*c*e**2*x + 2*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a**2*c +
 a**2*d*x**2 + 2*a*b*c*x**2 + 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)
*a*b*c*e*f*x**3 - 2*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a**2*c + a**2
*d*x**2 + 2*a*b*c*x**2 + 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*b**2
*c*e**2*x**3)/(a*c*x*(a + b*x**2))
```

3.238 $\int \frac{(e+fx^2)^2}{x^4(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 377

$$\int \frac{(e+fx^2)^2}{x^4(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = -\frac{e^2\sqrt{c+dx^2}}{3acx^3\sqrt{a+bx^2}}+\frac{2e(2bce+ade-3acf)\sqrt{c+dx^2}}{3a^2c^2x\sqrt{a+bx^2}}$$
$$+\frac{\sqrt{b}(8b^2c^2e^2-3abce(de+4cf)-a^2(2d^2e^2-6cdef-3c^2f^2))\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{3a^{5/2}c^2(bc-ad)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$
$$-\frac{d(4b^2ce^2+3a^2cf^2-abe(de+6cf))\sqrt{c+dx^2}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{3a^{3/2}\sqrt{b}c^2(bc-ad)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output

```
-1/3*e^2*(d*x^2+c)^(1/2)/a/c/x^3/(b*x^2+a)^(1/2)+2/3*e*(-3*a*c*f+a*d*e+2*b
*c*e)*(d*x^2+c)^(1/2)/a^2/c^2/x/(b*x^2+a)^(1/2)+1/3*b^(1/2)*(8*b^2*c^2*e^2
-3*a*b*c*e*(4*c*f+d*e)-a^2*(-3*c^2*f^2-6*c*d*e*f+2*d^2*e^2))*(d*x^2+c)^(1/
2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/a^(5/2
)/c^2/(-a*d+b*c)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)-1/3*d*(4*
b^2*c*e^2+3*a^2*c*f^2-a*b*e*(6*c*f+d*e))*(d*x^2+c)^(1/2)*InverseJacobiAM(a
rctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/a^(3/2)/b^(1/2)/c^2/(-a*d+b*c)
/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.83 (sec) , antiderivative size = 415, normalized size of antiderivative = 1.10

$$\int \frac{(e+fx^2)^2}{x^4(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \frac{-\sqrt{\frac{b}{a}}(c+dx^2)\left(8b^3c^2e^2x^4+ab^2cex^2(-3dex^2+4c(e-3fx^2))+a^3de(-2\right.}{}$$

input
```
Integrate[(e + f*x^2)^2/(x^4*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

output
```
(-(Sqrt[b/a]*(c + d*x^2)*(8*b^3*c^2*e^2*x^4 + a*b^2*c*e*x^2*(-3*d*e*x^2 +
4*c*(e - 3*f*x^2)) + a^3*d*e*(-2*d*e*x^2 + c*(e + 6*f*x^2)) - a^2*b*(2*d^2
*e^2*x^4 + 2*c*d*e*x^2*(e - 3*f*x^2) + c^2*(e^2 + 6*e*f*x^2 - 3*f^2*x^4)))
) - I*b*c*(8*b^2*c^2*e^2 - 3*a*b*c*e*(d*e + 4*c*f) + a^2*(-2*d^2*e^2 + 6*c
*d*e*f + 3*c^2*f^2))*x^3*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE
[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*c*(-(b*c) + a*d)*(8*b^2*c*e^2 +
3*a^2*c*f^2 + a*b*e*(d*e - 12*c*f))*x^3*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^
2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(3*a^3*Sqrt[b/a]*c^2
*(-(b*c) + a*d)*x^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 810 vs. 2(377) = 754.

Time = 1.44 (sec) , antiderivative size = 810, normalized size of antiderivative = 2.15, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.486$, Rules used = {448, 441, 25, 445, 25, 27, 406, 320, 388, 313, 445, 25, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx^2)^2}{x^4(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx$$

$$\Big\downarrow \text{448}$$

$$\frac{f\int \frac{fx^2+e}{x^2(bx^2+a)^{3/2}\sqrt{dx^2+c}}dx}{e^2}+e\int \frac{fx^2+e}{x^4\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}}dx$$

$$\Big\downarrow \text{441}$$

$$\frac{f\left(\frac{\sqrt{c+dx^2}(be-af)}{ax\sqrt{a+bx^2}(bc-ad)}-\frac{\int -\frac{d(be-af)x^2+2bce-ade-acf}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{a(bc-ad)}\right)}{e^2}+e\left(\frac{\sqrt{c+dx^2}(be-af)}{ax^3\sqrt{a+bx^2}(bc-ad)}-\frac{\int -\frac{3d(be-af)x^2+4bce-ade-3acf}{x^4\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{a(bc-ad)}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{f\left(\frac{\int \frac{d(be-af)x^2+2bce-a(de+cf)}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{a(bc-ad)}+\frac{\sqrt{c+dx^2}(be-af)}{ax\sqrt{a+bx^2}(bc-ad)}\right)}{e^2}+e\left(\frac{\int \frac{3d(be-af)x^2+4bce-ade-3acf}{x^4\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{a(bc-ad)}+\frac{\sqrt{c+dx^2}(be-af)}{ax^3\sqrt{a+bx^2}(bc-ad)}\right)$$

$$\Big\downarrow \text{445}$$

$$e\left(\frac{-\frac{\int \frac{-d(2de-3cf)a^2-3bc(de+2cf)a+bd(4bce-ade-3acf)x^2+8b^2c^2e}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(-3acf-ade+4bce)}{3acx^3}}{a(bc-ad)}+\frac{\sqrt{c+dx^2}(be-af)}{ax^3\sqrt{a+bx^2}(bc-ad)}\right)+\frac{f\left(\frac{-\frac{\int -\frac{d\left(b(2bce-a(de+cf))x^2+ac(be-af)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(-acf-ade+2bce)}{acx}}{a(bc-ad)}+\frac{\sqrt{c+dx^2}(be-af)}{ax\sqrt{a+bx^2}(bc-ad)}\right)}{e^2}$$

$$\Big\downarrow \text{25}$$

$$e\left(\frac{-\frac{\int \frac{-d(2de-3cf)a^2-3bc(de+2cf)a+bd(4bce-ade-3acf)x^2+8b^2c^2e}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(-3acf-ade+4bce)}{3acx^3}}{a(bc-ad)}+\frac{\sqrt{c+dx^2}(be-af)}{ax^3\sqrt{a+bx^2}(bc-ad)}\right)+\frac{f\left(\frac{\frac{\int \frac{d\left(b(2bce-a(de+cf))x^2+ac(be-af)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(-acf-ade+2bce)}{acx}}{a(bc-ad)}+\frac{\sqrt{c+dx^2}(be-af)}{ax\sqrt{a+bx^2}(bc-ad)}\right)}{e^2}$$

$$\Big\downarrow \text{27}$$

$$e\left(\frac{-\frac{\int \frac{-d(2de-3cf)a^2-3bc(de+2cf)a+bd(4bce-ade-3acf)x^2+8b^2c^2e}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(-3acf-ade+4bce)}{3acx^3}}{a(bc-ad)}+\frac{\sqrt{c+dx^2}(be-af)}{ax^3\sqrt{a+bx^2}(bc-ad)}\right.$$

$$\frac{f\left(\frac{\frac{d\int \frac{b(2bce-a(de+cf))x^2+ac(be-af)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(-acf-ade+2bce)}{acx}}{a(bc-ad)}+\frac{\sqrt{c+dx^2}(be-af)}{ax\sqrt{a+bx^2}(bc-ad)}\right)}{e^2}$$

$\Big\downarrow$ 406

$$e\left(\frac{-\frac{\int \frac{-d(2de-3cf)a^2-3bc(de+2cf)a+bd(4bce-ade-3acf)x^2+8b^2c^2e}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(-3acf-ade+4bce)}{3acx^3}}{a(bc-ad)}+\frac{\sqrt{c+dx^2}(be-af)}{ax^3\sqrt{a+bx^2}(bc-ad)}\right.$$

$$\frac{f\left(\frac{\frac{d\left(ac(be-af)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+b(2bce-a(cf+de))\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(-acf-ade+2bce)}{acx}}{a(bc-ad)}+\frac{\sqrt{c+dx^2}(be-af)}{ax\sqrt{a+bx^2}(bc-ad)}\right)}{e^2}$$

$\Big\downarrow$ 320

$$e\left(\frac{-\frac{\int \frac{-d(2de-3cf)a^2-3bc(de+2cf)a+bd(4bce-ade-3acf)x^2+8b^2c^2e}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(-3acf-ade+4bce)}{3acx^3}}{a(bc-ad)}+\frac{\sqrt{c+dx^2}(be-af)}{ax^3\sqrt{a+bx^2}(bc-ad)}\right.$$

$$\frac{f\left(\frac{\frac{d\left(b(2bce-a(cf+de))\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}(be-af)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(-acf-ade+2bce)}{acx}}{a(bc-ad)}+\frac{\sqrt{c+a\ldots}}{ax\sqrt{a+\ldots}}\right.}{e^2}$$

$\Big\downarrow$ 388

$$\frac{e\left(\frac{-\frac{\int \frac{-d(2de-3cf)a^2-3bc(de+2cf)a+bd(4bce-ade-3acf)x^2+8b^2c^2e}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(-3acf-ade+4bce)}{3acx^3}}{a(bc-ad)}+\frac{\sqrt{c+dx^2}(be-af)}{ax^3\sqrt{a+bx^2}(bc-ad)}\right. \quad f\left(\frac{\frac{d\left(b(2bce-a(cf+de))\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(be-af)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(-acf-ade+2bce)}{acx}}{a(bc-ad)}\right.}{e^2}$$

$\downarrow$ 313

$$\frac{e\left(\frac{-\frac{\int \frac{-d(2de-3cf)a^2-3bc(de+2cf)a+bd(4bce-ade-3acf)x^2+8b^2c^2e}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(-3acf-ade+4bce)}{3acx^3}}{a(bc-ad)}+\frac{\sqrt{c+dx^2}(be-af)}{ax^3\sqrt{a+bx^2}(bc-ad)}\right. \quad f\left(\frac{\frac{d\left(\frac{c^{3/2}\sqrt{a+bx^2}(be-af)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+b(2bce-a(cf+de))\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)\right)}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\cdots}{\cdots}}{a(bc-ad)}\right.}{e^2}$$

$\downarrow$ 445

$$\frac{e\left(-\frac{-\frac{\int -\frac{bd\left(\left(-d(2de-3cf)a^2-3bc(de+2cf)a+8b^2c^2e\right)x^2+ac(4bce-ade-3acf)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{8b^2ce}{a}-\frac{ad(2de-3cf)}{c}-3b(2cf+de)\right)}{x}}{3ac}}{a(bc-ad)}-\frac{\sqrt{a+bx^2}\cdots}{\cdots}\right)\cdots \quad f\left(\frac{\frac{d\left(\frac{c^{3/2}\sqrt{a+bx^2}(be-af)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+b(2bce-a(cf+de))\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)\right)}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\cdots}{\cdots}}{a(bc-ad)}\cdots\right)\cdots}{e^2}$$

$\downarrow$ 25

$$\frac{e\left(-\frac{\frac{\int \frac{bd\left(\left(-d(2de-3cf)a^2-3bc(de+2cf)a+8b^2c^2e\right)x^2+ac(4bce-ade-3acf)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{8b^2ce}{a}-\frac{ad(2de-3cf)}{c}-3b(2cf+de)\right)}{x}}{3ac}}{a(bc-ad)}-\frac{\sqrt{a+bx^2}\sqrt{c\cdots}}{\cdots}\right)\cdots \quad f\left(\frac{\frac{d\left(\frac{c^{3/2}\sqrt{a+bx^2}(be-af)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+b(2bce-a(cf+de))\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)\right)}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\cdots}{\cdots}}{a(bc-ad)}\cdots\right)\cdots}{e^2}$$

$\downarrow$ 27

$$\frac{e\left(-\frac{\frac{bd\int\frac{\left(-d(2de-3cf)a^2-3bc(de+2cf)a+8b^2c^2e\right)x^2+ac(4bce-ade-3acf)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{8b^2ce}{a}-\frac{ad(2de-3cf)}{c}-3b(2cf+de)\right)}{x}}{3ac}-\frac{\sqrt{a+bx^2}\sqrt{c+}}{}\right.}{a(bc-ad)}$$

$$f\left(\frac{\frac{d\left(\frac{c^{3/2}\sqrt{a+bx^2}(be-af)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+b(2bce-a(cf+de))\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)\right)}{ac}}{a(bc-ad)}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{}\right.$$

$$\overline{\qquad e^2 \qquad}$$

$\downarrow$ 406

$$\frac{e\left(-\frac{\frac{bd\left(\left(a^2(-d)(2de-3cf)-3abc(2cf+de)+8b^2c^2e\right)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+ac(-3acf-ade+4bce)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{8b^2ce}{a}-\frac{ad(2}{}\right.}{x}}{3ac}\right.}{a(bc-ad)}$$

$$f\left(\frac{\frac{d\left(\frac{c^{3/2}\sqrt{a+bx^2}(be-af)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+b(2bce-a(cf+de))\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)\right)}{ac}}{a(bc-ad)}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{}\right.$$

$$\overline{\qquad e^2 \qquad}$$

$\downarrow$ 320

$$\frac{e\left(\frac{-\frac{\frac{bd\left(\left(a^2(-d)(2de-3cf)-3abc(2cf+de)+8b^2c^2e\right)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}(-3acf-ade+4bce)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{ac}-\frac{\sqrt{a+bx^2}\cdots}{\cdots}}{3ac}\cdots}{a(bc-ad)}\cdots\right)+f\left(\frac{\frac{d\left(\frac{c^{3/2}\sqrt{a+bx^2}(be-af)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+b(2bce-a(cf+de))\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)\right)}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\cdots}{\cdots}}{a(bc-ad)}\cdots\right)\cdots}{e^2}$$

$\downarrow$ 388

$$\frac{e\left(\frac{-\frac{\frac{bd\left(\left(a^2(-d)(2de-3cf)-3abc(2cf+de)+8b^2c^2e\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(-3acf-ade+4bce)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\cdots\right)}{ac}\cdots}{3ac}\cdots}{a(bc-ad)}\cdots\right)+f\left(\frac{\frac{d\left(\frac{c^{3/2}\sqrt{a+bx^2}(be-af)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+b(2bce-a(cf+de))\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)\right)}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\cdots}{\cdots}}{a(bc-ad)}\cdots\right)\cdots}{e^2}$$

$\downarrow$ 313

$$f\left(\frac{\sqrt{dx^2+c}(be-af)}{a(bc-ad)x\sqrt{bx^2+a}}+\frac{\frac{d\left(\frac{(be-af)\sqrt{bx^2+a}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+b(2bce-a(de+cf))\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right.}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right.\right.}{ac}}{a(bc-ad)}\right.$$

$$e^2$$

$$e\left(\frac{\sqrt{dx^2+c}(be-af)}{a(bc-ad)x^3\sqrt{bx^2+a}}+\frac{-\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}(4bce-ade-3acf)}{3acx^3}-\frac{bd\left(\frac{(4bce-ade-3acf)\sqrt{bx^2+a}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\right.}{}}{}\right.$$

input
```
Int[(e + f*x^2)^2/(x^4*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

output
```
(f*(((b*e - a*f)*Sqrt[c + d*x^2])/(a*(b*c - a*d)*x*Sqrt[a + b*x^2]) + (-((
(2*b*c*e - a*d*e - a*c*f)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(a*c*x)) + (d*(
b*(2*b*c*e - a*(d*e + c*f))*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sq
rt[c]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*
d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) +
(c^(3/2)*(b*e - a*f)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]]
, 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c
+ d*x^2])))/(a*c))/(a*(b*c - a*d))))/e^2 + e*(((b*e - a*f)*Sqrt[c + d*x^2]
)/(a*(b*c - a*d)*x^3*Sqrt[a + b*x^2]) + (-1/3*((4*b*c*e - a*d*e - 3*a*c*f)
*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(a*c*x^3) - (-((((8*b^2*c*e)/a - (a*d*(2
*d*e - 3*c*f))/c - 3*b*(d*e + 2*c*f))*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/x)
+ (b*d*((8*b^2*c^2*e - a^2*d*(2*d*e - 3*c*f) - 3*a*b*c*(d*e + 2*c*f))*((x*
Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[
ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x
^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (c^(3/2)*(4*b*c*e - a*d*e - 3*a*
c*f)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d
)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])))/(a*c
))/(3*a*c))/(a*(b*c - a*d))))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 441
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
+ b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(a*g*2*(b*c - a*d)*(p + 1))), x] + Si
mp[1/(a*2*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2
)^q*Simp[c*(b*e - a*f)*(m + 1) + e*2*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(m
 + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, q},
 x] && LtQ[p, -1]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 448
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[e   Int[(g*x)^m*(a + b*x
^2)^p*(c + d*x^2)^q*(e + f*x^2)^(r - 1), x], x] + Simp[f/e^2   Int[(g*x)^(m
 + 2)*(a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^(r - 1), x], x] /; FreeQ[{a,
b, c, d, e, f, g, m, p, q}, x] && IGtQ[r, 0]
```

Maple [A] (verified)

Time = 11.70 (sec) , antiderivative size = 547, normalized size of antiderivative = 1.45

method	result
elliptic	$\frac{\sqrt{(b\,x^2+a)(x^2 d+c)}\left(-\frac{e^2\sqrt{b d\,x^4+a d\,x^2+x^2 b c+a c}}{3a^2 c\,x^3}-\frac{e(6acf-2ade-5bce)\sqrt{b d\,x^4+a d\,x^2+x^2 b c+a c}}{3a^3 c^2 x}-\frac{\left(b d\,x^2+b c\right)x\left(a^2 f^2-2abfe+b^2 e^2\right)}{a^3(ad-bc)\sqrt{\left(x^2+\frac{a}{b}\right)\left(b d\,x^2+b c\right)}}+\frac{\left(\cdots\right.}{}\right.}{}$
risch	$-\frac{\sqrt{b\,x^2+a}\,\sqrt{x^2 d+c}\,e\left(6acf\,x^2-2ade\,x^2-5bce\,x^2+ace\right)}{3a^3 c^2 x^3}+\frac{\left(-\frac{be(6acf-2ade-5bce)c\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\left(\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+\cdots}{cb}}\right.\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{b d\,x^4+a d\,x^2+x^2 b c+a c}}\cdots\right.}{}$
default	Expression too large to display

input

```
int((f*x^2+e)^2/x^4/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOS
E)
```

output

```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-1/3*e^2/a^2/
c*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/x^3-1/3/a^3/c^2*e*(6*a*c*f-2*a*d*e-5
*b*c*e)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/x-(b*d*x^2+b*c)/a^3/(a*d-b*c)*
x*(a^2*f^2-2*a*b*e*f+b^2*e^2)/((x^2+a/b)*(b*d*x^2+b*c))^(1/2)+(-1/3*b*d*e^
2/a^2/c+(a^2*f^2-2*a*b*e*f+b^2*e^2)/a^3+b*c/a^3/(a*d-b*c)*(a^2*f^2-2*a*b*e
*f+b^2*e^2))/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d
*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))
-(1/3*b*d*e*(6*a*c*f-2*a*d*e-5*b*c*e)/a^3/c^2+(a^2*f^2-2*a*b*e*f+b^2*e^2)*
b*d/a^3/(a*d-b*c))*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d
*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/
c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 701 vs. 2(348) = 696.

Time = 0.17 (sec) , antiderivative size = 701, normalized size of antiderivative = 1.86

$$\int \frac{(e+fx^2)^2}{x^4(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx =$$

$$-\frac{((3a^2b^3c^2f^2+(8b^5c^2-3ab^4cd-2a^2b^3d^2)e^2-6(2ab^4c^2-a^2b^3cd)ef)x^5+(3a^3b^2c^2f^2+(8ab^4c^2-3a^2}{}$$

input
```
integrate((f*x^2+e)^2/x^4/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="fr
icas")
```

output
```
-1/3*(((3*a^2*b^3*c^2*f^2 + (8*b^5*c^2 - 3*a*b^4*c*d - 2*a^2*b^3*d^2)*e^2
- 6*(2*a*b^4*c^2 - a^2*b^3*c*d)*e*f)*x^5 + (3*a^3*b^2*c^2*f^2 + (8*a*b^4*c
^2 - 3*a^2*b^3*c*d - 2*a^3*b^2*d^2)*e^2 - 6*(2*a^2*b^3*c^2 - a^3*b^2*c*d)*
e*f)*x^3)*sqrt(a*c)*sqrt(-b/a)*elliptic_e(arcsin(x*sqrt(-b/a)), a*d/(b*c))
 - (((8*b^5*c^2 + (4*a^2*b^3 - 3*a*b^4)*c*d - (a^3*b^2 + 2*a^2*b^3)*d^2)*e
^2 - 6*(2*a*b^4*c^2 + (a^3*b^2 - a^2*b^3)*c*d)*e*f + 3*(a^2*b^3*c^2 + a^4*
b*c*d)*f^2)*x^5 + ((8*a*b^4*c^2 + (4*a^3*b^2 - 3*a^2*b^3)*c*d - (a^4*b + 2
*a^3*b^2)*d^2)*e^2 - 6*(2*a^2*b^3*c^2 + (a^4*b - a^3*b^2)*c*d)*e*f + 3*(a^
3*b^2*c^2 + a^5*c*d)*f^2)*x^3)*sqrt(a*c)*sqrt(-b/a)*elliptic_f(arcsin(x*sq
rt(-b/a)), a*d/(b*c)) - ((3*a^3*b^2*c^2*f^2 + (8*a*b^4*c^2 - 3*a^2*b^3*c*d
 - 2*a^3*b^2*d^2)*e^2 - 6*(2*a^2*b^3*c^2 - a^3*b^2*c*d)*e*f)*x^4 - (a^3*b^
2*c^2 - a^4*b*c*d)*e^2 + 2*((2*a^2*b^3*c^2 - a^3*b^2*c*d - a^4*b*d^2)*e^2
- 3*(a^3*b^2*c^2 - a^4*b*c*d)*e*f)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(
(a^4*b^3*c^3 - a^5*b^2*c^2*d)*x^5 + (a^5*b^2*c^3 - a^6*b*c^2*d)*x^3)
```

Sympy [F]

$$\int \frac{(e+fx^2)^2}{x^4(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \int \frac{(e+fx^2)^2}{x^4(a+bx^2)^{\frac{3}{2}}\sqrt{c+dx^2}}\,dx$$

input
```
integrate((f*x**2+e)**2/x**4/(b*x**2+a)**(3/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral((e + f*x**2)**2/(x**4*(a + b*x**2)**(3/2)*sqrt(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{(e+fx^2)^2}{x^4(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx=\int \frac{(fx^2+e)^2}{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}x^4}\,dx$$

input
```
integrate((f*x^2+e)^2/x^4/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate((f*x^2 + e)^2/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*x^4), x)
```

Giac [F]

$$\int \frac{(e+fx^2)^2}{x^4(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx=\int \frac{(fx^2+e)^2}{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}x^4}\,dx$$

input
```
integrate((f*x^2+e)^2/x^4/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate((f*x^2 + e)^2/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx^2)^2}{x^4(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx=\int \frac{(f\,x^2+e)^2}{x^4\,(b\,x^2+a)^{3/2}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int((e + f*x^2)^2/(x^4*(a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int((e + f*x^2)^2/(x^4*(a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{(e+fx^2)^2}{x^4 (a+bx^2)^{3/2} \sqrt{c+dx^2}} \, dx = \text{too large to display}$$

input
```
int((f*x^2+e)^2/x^4/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x)
```

output
```
( - sqrt(c + d*x**2)*sqrt(a + b*x**2)*e**2 + 6*int((sqrt(c + d*x**2)*sqrt(
a + b*x**2))/(a**3*c*d*x**2 + a**3*d**2*x**4 + 2*a**2*b*c**2*x**2 + 4*a**2
*b*c*d*x**4 + 2*a**2*b*d**2*x**6 + 4*a*b**2*c**2*x**4 + 5*a*b**2*c*d*x**6
+ a*b**2*d**2*x**8 + 2*b**3*c**2*x**6 + 2*b**3*c*d*x**8),x)*a**3*c*d*e*f*x
**3 - 2*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a**3*c*d*x**2 + a**3*d**2
*x**4 + 2*a**2*b*c**2*x**2 + 4*a**2*b*c*d*x**4 + 2*a**2*b*d**2*x**6 + 4*a*
b**2*c**2*x**4 + 5*a*b**2*c*d*x**6 + a*b**2*d**2*x**8 + 2*b**3*c**2*x**6 +
 2*b**3*c*d*x**8),x)*a**3*d**2*e**2*x**3 + 12*int((sqrt(c + d*x**2)*sqrt(a
 + b*x**2))/(a**3*c*d*x**2 + a**3*d**2*x**4 + 2*a**2*b*c**2*x**2 + 4*a**2*
b*c*d*x**4 + 2*a**2*b*d**2*x**6 + 4*a*b**2*c**2*x**4 + 5*a*b**2*c*d*x**6 +
 a*b**2*d**2*x**8 + 2*b**3*c**2*x**6 + 2*b**3*c*d*x**8),x)*a**2*b*c**2*e*f
*x**3 - 8*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a**3*c*d*x**2 + a**3*d*
*2*x**4 + 2*a**2*b*c**2*x**2 + 4*a**2*b*c*d*x**4 + 2*a**2*b*d**2*x**6 + 4*
a*b**2*c**2*x**4 + 5*a*b**2*c*d*x**6 + a*b**2*d**2*x**8 + 2*b**3*c**2*x**6
 + 2*b**3*c*d*x**8),x)*a**2*b*c*d*e**2*x**3 + 6*int((sqrt(c + d*x**2)*sqrt
(a + b*x**2))/(a**3*c*d*x**2 + a**3*d**2*x**4 + 2*a**2*b*c**2*x**2 + 4*a**
2*b*c*d*x**4 + 2*a**2*b*d**2*x**6 + 4*a*b**2*c**2*x**4 + 5*a*b**2*c*d*x**6
 + a*b**2*d**2*x**8 + 2*b**3*c**2*x**6 + 2*b**3*c*d*x**8),x)*a**2*b*c*d*e*
f*x**5 - 2*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a**3*c*d*x**2 + a**3*d
**2*x**4 + 2*a**2*b*c**2*x**2 + 4*a**2*b*c*d*x**4 + 2*a**2*b*d**2*x**6 ...
```

3.239 $\int \frac{(e+fx^2)^2}{x^6(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 516

$$\int \frac{(e+fx^2)^2}{x^6(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = -\frac{e^2\sqrt{c+dx^2}}{5acx^5\sqrt{a+bx^2}} + \frac{2e(3bce+2ade-5acf)\sqrt{c+dx^2}}{15a^2c^2x^3\sqrt{a+bx^2}}$$
$$-\frac{(4(2bc+ad)e(3bce+2ade-5acf)-15ac(bde^2-acf^2))\sqrt{c+dx^2}}{15a^3c^3x\sqrt{a+bx^2}}$$
$$-\frac{\sqrt{b}(48b^3c^3e^2-16ab^2c^2e(de+5cf)-3a^2bc(3d^2e^2-10cdef-10c^2f^2)-a^3d(8d^2e^2-20cdef+15c^2f^2))}{15a^{7/2}c^3(bc-ad)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$
$$+\frac{\sqrt{b}d(24b^2c^2e^2-5abce(de+8cf)-a^2(4d^2e^2-10cdef-15c^2f^2))\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\right.}{15a^{5/2}c^3(bc-ad)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
-1/5*e^2*(d*x^2+c)^(1/2)/a/c/x^5/(b*x^2+a)^(1/2)+2/15*e*(-5*a*c*f+2*a*d*e+
3*b*c*e)*(d*x^2+c)^(1/2)/a^2/c^2/x^3/(b*x^2+a)^(1/2)-1/15*(4*(a*d+2*b*c)*e
*(-5*a*c*f+2*a*d*e+3*b*c*e)-15*a*c*(-a*c*f^2+b*d*e^2))*(d*x^2+c)^(1/2)/a^3
/c^3/x/(b*x^2+a)^(1/2)-1/15*b^(1/2)*(48*b^3*c^3*e^2-16*a*b^2*c^2*e*(5*c*f+
d*e)-3*a^2*b*c*(-10*c^2*f^2-10*c*d*e*f+3*d^2*e^2)-a^3*d*(15*c^2*f^2-20*c*d
*e*f+8*d^2*e^2))*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(
1/2),(1-a*d/b/c)^(1/2))/a^(7/2)/c^3/(-a*d+b*c)/(b*x^2+a)^(1/2)/(a*(d*x^2+c
)/c/(b*x^2+a))^(1/2)+1/15*b^(1/2)*d*(24*b^2*c^2*e^2-5*a*b*c*e*(8*c*f+d*e)-
a^2*(-15*c^2*f^2-10*c*d*e*f+4*d^2*e^2))*(d*x^2+c)^(1/2)*InverseJacobiAM(ar
ctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/a^(5/2)/c^3/(-a*d+b*c)/(b*x^2+a
)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 12.18 (sec) , antiderivative size = 608, normalized size of antiderivative = 1.18

$$\int \frac{(e+fx^2)^2}{x^6(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \frac{\sqrt{\frac{b}{a}}(c+dx^2)\left(48b^4c^3e^2x^6+8ab^3c^2ex^4(3ce-2dex^2-10cfx^2)-a^2b^2cx^2(\right.}{}$$

input
```
Integrate[(e + f*x^2)^2/(x^6*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

output
```
(Sqrt[b/a]*(c + d*x^2)*(48*b^4*c^3*e^2*x^6 + 8*a*b^3*c^2*e*x^4*(3*c*e - 2*
d*e*x^2 - 10*c*f*x^2) - a^2*b^2*c*x^2*(9*d^2*e^2*x^4 + c*d*e*x^2*(11*e - 3
0*f*x^2) + c^2*(6*e^2 + 40*e*f*x^2 - 30*f^2*x^4)) - a^4*d*(8*d^2*e^2*x^4 -
 4*c*d*e*x^2*(e + 5*f*x^2) + c^2*(3*e^2 + 10*e*f*x^2 + 15*f^2*x^4)) + a^3*
b*(-8*d^3*e^2*x^6 - 5*c*d^2*e*x^4*(e - 4*f*x^2) + c^2*d*x^2*(2*e^2 + 20*e*
f*x^2 - 15*f^2*x^4) + c^3*(3*e^2 + 10*e*f*x^2 + 15*f^2*x^4))) - I*b*c*(-48
*b^3*c^3*e^2 + 16*a*b^2*c^2*e*(d*e + 5*c*f) - 3*a^2*b*c*(-3*d^2*e^2 + 10*c
*d*e*f + 10*c^2*f^2) + a^3*d*(8*d^2*e^2 - 20*c*d*e*f + 15*c^2*f^2))*x^5*Sq
rt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a
*d)/(b*c)] + (2*I)*b*c*(-(b*c) + a*d)*(24*b^2*c^2*e^2 + 4*a*b*c*e*(d*e - 1
0*c*f) + a^2*(2*d^2*e^2 - 5*c*d*e*f + 15*c^2*f^2))*x^5*Sqrt[1 + (b*x^2)/a]
*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(15*a
^4*Sqrt[b/a]*c^3*(-(b*c) + a*d)*x^5*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1072 vs. 2(516) = 1032.

Time = 2.00 (sec) , antiderivative size = 1072, normalized size of antiderivative = 2.08, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.514$, Rules used = {448, 441, 25, 445, 445, 25, 27, 406, 320, 388, 313, 445, 25, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(e+fx^2\right)^2}{x^6\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \text{448}$$

$$\frac{f\int \frac{fx^2+e}{x^4(bx^2+a)^{3/2}\sqrt{dx^2+c}}dx}{e^2}+e\int \frac{fx^2+e}{x^6\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}}dx$$

$$\Big\downarrow \text{441}$$

$$\frac{f\left(\frac{\sqrt{c+dx^2}(be-af)}{ax^3\sqrt{a+bx^2}(bc-ad)}-\frac{\int -\frac{3d(be-af)x^2+4bce-ade-3acf}{x^4\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{a(bc-ad)}\right)}{e^2}+e\left(\frac{\sqrt{c+dx^2}(be-af)}{ax^5\sqrt{a+bx^2}(bc-ad)}-\frac{\int -\frac{5d(be-af)x^2+6bce-ade-5acf}{x^6\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{a(bc-ad)}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{f\left(\frac{\int \frac{3d(be-af)x^2+4bce-ade-3acf}{x^4\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{a(bc-ad)}+\frac{\sqrt{c+dx^2}(be-af)}{ax^3\sqrt{a+bx^2}(bc-ad)}\right)}{e^2}+e\left(\frac{\int \frac{5d(be-af)x^2+6bce-ade-5acf}{x^6\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{a(bc-ad)}+\frac{\sqrt{c+dx^2}(be-af)}{ax^5\sqrt{a+bx^2}(bc-ad)}\right)$$

$$\Big\downarrow \text{445}$$

$$\frac{f\left(\frac{-\frac{\int \frac{-d(2de-3cf)a^2-3bc(de+2cf)a+bd(4bce-ade-3acf)x^2+8b^2c^2e}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(-3acf-ade+4bce)}{3acx^3}}{a(bc-ad)}+\frac{\sqrt{c+dx^2}(be-af)}{ax^3\sqrt{a+bx^2}(bc-ad)}\right)}{e^2}+$$

$$e\left(\frac{-\frac{\int \frac{-d(4de-5cf)a^2-5bc(de+4cf)a+3bd(6bce-ade-5acf)x^2+24b^2c^2e}{x^4\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{5ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(-5acf-ade+6bce)}{5acx^5}}{a(bc-ad)}+\frac{\sqrt{c+dx^2}(be-af)}{ax^5\sqrt{a+bx^2}(bc-a\cdots}\cdots\right.$$

$\downarrow$ 445

$$\frac{f\left(\frac{-\frac{-\frac{\int -\frac{bd\left(\left(-d(2de-3cf)a^2-3bc(de+2cf)a+8b^2c^2e\right)x^2+ac(4bce-ade-3acf)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{8b^2ce}{a}-\frac{ad(2de-3cf)}{c}-3b(2cf+de)\right)}{x}}{3ac}-\frac{\sqrt{a+bx^2}\sqrt{\cdots}}{\cdots}}{a(bc-ad)}\cdots\right.}{e^2}\cdots$$

$$e\left(\frac{-\frac{-\frac{\int \frac{-2d^2(4de-5cf)a^3-3bcd(3de-5cf)a^2-8b^2c^2(2de+5cf)a+bd\left(-d(4de-5cf)a^2-5bc(de+4cf)a+24b^2c^2e\right)x^2+48b^3c^3e}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{24b^2ce}{a}\cdots\right.}{\cdots}}{5ac}\cdots}{a(bc-ad)}\cdots\right.$$

$\downarrow$ 25

$$\frac{f\left(\frac{-\frac{-\frac{\int \frac{bd\left(\left(-d(2de-3cf)a^2-3bc(de+2cf)a+8b^2c^2e\right)x^2+ac(4bce-ade-3acf)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{8b^2ce}{a}-\frac{ad(2de-3cf)}{c}-3b(2cf+de)\right)}{x}}{3ac}-\frac{\sqrt{a+bx^2}\sqrt{c+d\cdots}}{\cdots}}{a(bc-ad)}\cdots\right.}{e^2}\cdots$$

$$e\left(\frac{-\frac{-\frac{\int \frac{-2d^2(4de-5cf)a^3-3bcd(3de-5cf)a^2-8b^2c^2(2de+5cf)a+bd\left(-d(4de-5cf)a^2-5bc(de+4cf)a+24b^2c^2e\right)x^2+48b^3c^3e}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{24b^2ce}{a}\cdots\right.}{\cdots}}{5ac}\cdots}{a(bc-ad)}\cdots\right.$$

$\downarrow$ 27

$$\frac{f\left(-\frac{\frac{bd\int\frac{\left(-d(2de-3cf)a^2-3bc(de+2cf)a+8b^2c^2e\right)x^2+ac(4bce-ade-3acf)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{8b^2ce}{a}-\frac{ad(2de-3cf)}{c}-3b(2cf+de)\right)}{x}}{3ac}}{a(bc-ad)}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{3}\cdots\right)}{e^2}$$

$$e\left(-\frac{-\frac{\int\frac{-2d^2(4de-5cf)a^3-3bcd(3de-5cf)a^2-8b^2c^2(2de+5cf)a+bd\left(-d(4de-5cf)a^2-5bc(de+4cf)a+24b^2c^2e\right)x^2+48b^3c^3e}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{24b^2ce}{a}\cdots\right.}{}}{5ac}}{a(bc-ad)}\cdots\right.$$

$$\Big\downarrow \ 406$$

$$\frac{f\left(-\frac{\frac{bd\left(\left(a^2(-d)(2de-3cf)-3abc(2cf+de)+8b^2c^2e\right)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+ac(-3acf-ade+4bce)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{8b^2ce}{a}-\frac{ad(2\cdots}{}\right.}{x}}{3ac}}{a(bc-ad)}\cdots\right.}{e^2}$$

$$e\left(-\frac{-\frac{\int\frac{-2d^2(4de-5cf)a^3-3bcd(3de-5cf)a^2-8b^2c^2(2de+5cf)a+bd\left(-d(4de-5cf)a^2-5bc(de+4cf)a+24b^2c^2e\right)x^2+48b^3c^3e}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{24b^2ce}{a}\cdots\right.}{}}{5ac}}{a(bc-ad)}\cdots\right.$$

$$\Big\downarrow \ 320$$

$$\frac{f\left(-\frac{\frac{bd\left(\left(a^2(-d)(2de-3cf)-3abc(2cf+de)+8b^2c^2e\right)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}(-3acf-ade+4bce)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{ac}-\frac{\sqrt{a+bx^2}\cdots}{}}{3ac}}{a(bc-ad)}\cdots\right.}{e^2}$$

$$e\left(-\frac{-\frac{\int\frac{-2d^2(4de-5cf)a^3-3bcd(3de-5cf)a^2-8b^2c^2(2de+5cf)a+bd\left(-d(4de-5cf)a^2-5bc(de+4cf)a+24b^2c^2e\right)x^2+48b^3c^3e}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{24b^2ce}{a}\cdots\right.}{}}{5ac}}{a(bc-ad)}\cdots\right.$$

$$\Big\downarrow \ 388$$

$$\frac{f\left(-\frac{\frac{bd\left(\left(a^2(-d)(2de-3cf)-3abc(2cf+de)+8b^2c^2e\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(-3acf-ade+4bce)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\cdots\right)}{ac}\cdots}{3ac}\cdots}{a(bc-ad)}\cdots\right)}{e^2}\cdots$$

$$e\left(-\frac{-\frac{-\frac{\int\frac{-2d^2(4de-5cf)a^3-3bcd(3de-5cf)a^2-8b^2c^2(2de+5cf)a+bd\left(-d(4de-5cf)a^2-5bc(de+4cf)a+24b^2c^2e\right)x^2+48b^3c^3e}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{24b^2ce}{a}\cdots\right.}{\cdots}}{5ac}\cdots}{a(bc-ad)}\cdots\right.$$

$$\Big\downarrow \; 313$$

$$\frac{f\left(\frac{\sqrt{dx^2+c}(be-af)}{a(bc-ad)x^3\sqrt{bx^2+a}}+\frac{-\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}(4bce-ade-3acf)}{3acx^3}-\frac{bd\left(\frac{(4bce-ade-3acf)\sqrt{bx^2+a}\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\left(-d(2de-3cf)a\cdots\right.\right.}{ac}\cdots}{\cdots}\cdots\right)}{e^2}\cdots$$

$$e\left(\frac{\sqrt{dx^2+c}(be-af)}{a(bc-ad)x^5\sqrt{bx^2+a}}+\frac{-\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}(6bce-ade-5acf)}{5acx^5}-\frac{-\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(\frac{24ceb^2}{a}-5(de+4cf)b-\frac{ad(4de-5cf)}{c}\right)}{3x^3}-\frac{\int\frac{-2d^2(4d\cdots}{\cdots}}{\cdots}}{\cdots}}{a(bc-a\cdots}\cdots\right.$$

$$\Big\downarrow \; 445$$

$$\frac{f\left(\frac{\sqrt{dx^2+c}(be-af)}{a(bc-ad)x^3\sqrt{bx^2+a}}+\frac{-\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}(4bce-ade-3acf)}{3acx^3}-\frac{bd\left(\frac{(4bce-ade-3acf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\left(-d(2de-3cf)a\cdots\right.\right.}{ac\cdots}}{\cdots}\right.}{e^2\cdots}$$

$$e\left(\frac{\sqrt{dx^2+c}(be-af)}{a(bc-ad)x^5\sqrt{bx^2+a}}+\frac{-\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}(6bce-ade-5acf)}{5acx^5}-\frac{-\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(\frac{24ceb^2}{a}-5(de+4cf)b-\frac{ad(4de-5cf)}{c}\right)}{3x^3}-\frac{-\frac{\sqrt{bx^2+a}\cdots}{\cdots}}{\cdots}}{\cdots}}{\cdots}\right.$$

$$\Big\downarrow \; 25$$

$$\frac{f\left(\frac{\sqrt{dx^2+c}(be-af)}{a(bc-ad)x^3\sqrt{bx^2+a}}+\frac{-\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}(4bce-ade-3acf)}{3acx^3}-\frac{bd\left(\frac{(4bce-ade-3acf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\left(-d(2de-3cf)a\cdots\right.\right.}{ac\cdots}}{\cdots}\right.}{e^2\cdots}$$

$$e\left(\frac{\sqrt{dx^2+c}(be-af)}{a(bc-ad)x^5\sqrt{bx^2+a}}+\frac{-\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}(6bce-ade-5acf)}{5acx^5}-\frac{-\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(\frac{24ceb^2}{a}-5(de+4cf)b-\frac{ad(4de-5cf)}{c}\right)}{3x^3}-\frac{\int\frac{bd\left(\left(-2\cdots\right.\right.}{\cdots}}{\cdots}}{\cdots}}{\cdots}\right.$$

$$\Big\downarrow \; 27$$

$$\frac{f\left(\frac{\sqrt{dx^2+c}(be-af)}{a(bc-ad)x^3\sqrt{bx^2+a}}+\frac{-\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}(4bce-ade-3acf)}{3acx^3}-\frac{bd\left(\frac{(4bce-ade-3acf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\left(-d(2de-3cf)a\right.\right.}{ac}}{}\right.}{e\left(\frac{\sqrt{dx^2+c}(be-af)}{a(bc-ad)x^5\sqrt{bx^2+a}}+\frac{-\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}(6bce-ade-5acf)}{5acx^5}-\frac{-\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(\frac{24ceb^2}{a}-5(de+4cf)b-\frac{ad(4de-5cf)}{c}\right)}{3x^3}-\frac{bd\int\frac{(-2d}{}}{}}{}}{}\right.e^2}$$

$\downarrow$ 406

$$\frac{f\left(\frac{\sqrt{dx^2+c}(be-af)}{a(bc-ad)x^3\sqrt{bx^2+a}}+\frac{-\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}(4bce-ade-3acf)}{3acx^3}-\frac{bd\left(\frac{(4bce-ade-3acf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\left(-d(2de-3cf)a\right.\right.}{ac}}{}\right.}{e\left(\frac{\sqrt{dx^2+c}(be-af)}{a(bc-ad)x^5\sqrt{bx^2+a}}+\frac{-\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}(6bce-ade-5acf)}{5acx^5}-\frac{-\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(\frac{24ceb^2}{a}-5(de+4cf)b-\frac{ad(4de-5cf)}{c}\right)}{3x^3}-\frac{bd\left(ac\left(-d\right.\right.}{}}{}}{}\right.e^2}$$

$\downarrow$ 320

$$f\left(\frac{\sqrt{dx^2+c}(be-af)}{a(bc-ad)x^3\sqrt{bx^2+a}}+\frac{-\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}(4bce-ade-3acf)}{3acx^3}-\frac{bd\left(\frac{(4bce-ade-3acf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\left(-d(2de-3cf)a\right.\cdots}{ac\cdots}}{\cdots}\right.$$

$$\overline{e\left(\frac{\sqrt{dx^2+c}(be-af)}{a(bc-ad)x^5\sqrt{bx^2+a}}+\frac{-\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}(6bce-ade-5acf)}{5acx^5}-\frac{-\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(\frac{24ceb^2}{a}-5(de+4cf)b-\frac{ad(4de-5cf)}{c}\right)}{3x^3}-\frac{bd\left(\frac{\left(-d(4\cdots\right.}{\cdots}\right.}{\cdots}}{\cdots}\right.} \quad e^2$$

$\downarrow$ 388

$$f\left(\frac{\sqrt{dx^2+c}(be-af)}{a(bc-ad)x^3\sqrt{bx^2+a}}+\frac{-\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}(4bce-ade-3acf)}{3acx^3}-\frac{bd\left(\frac{(4bce-ade-3acf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}+\left(-d(2de-3cf)a\right.\cdots}{ac\cdots}}{\cdots}\right.$$

$$\overline{e\left(\frac{\sqrt{dx^2+c}(be-af)}{a(bc-ad)x^5\sqrt{bx^2+a}}+\frac{-\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}(6bce-ade-5acf)}{5acx^5}-\frac{-\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(\frac{24ceb^2}{a}-5(de+4cf)b-\frac{ad(4de-5cf)}{c}\right)}{3x^3}-\frac{bd\left(\frac{\left(-d(4\cdots\right.}{\cdots}\right.}{\cdots}}{\cdots}\right.} \quad e^2$$

$\big\downarrow$ 313

$$\frac{f\left(\frac{\sqrt{dx^2+c}(be-af)}{a(bc-ad)x^3\sqrt{bx^2+a}}+\frac{-\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}(4bce-ade-3acf)}{3acx^3}-\frac{bd\left(\frac{(4bce-ade-3acf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)c^{3/2}}{\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}+\left(-d(2de-3cf)a\right.}{ac}}{}\right)}{e\left(\frac{\sqrt{dx^2+c}(be-af)}{a(bc-ad)x^5\sqrt{bx^2+a}}+\frac{-\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}(6bce-ade-5acf)}{5acx^5}-\frac{-\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(\frac{24ceb^2}{a}-5(de+4cf)b-\frac{ad(4de-5cf)}{c}\right)}{3x^3}-\frac{e^2\, bd\left(\frac{\left(-d(4\right.}{}\right.}{}}{}}{}\right)}$$

input
```
Int[(e + f*x^2)^2/(x^6*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

output
```
(f*(((b*e - a*f)*Sqrt[c + d*x^2])/(a*(b*c - a*d)*x^3*Sqrt[a + b*x^2]) + (-
1/3*((4*b*c*e - a*d*e - 3*a*c*f)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(a*c*x^3
) - (-((((8*b^2*c*e)/a - (a*d*(2*d*e - 3*c*f))/c - 3*b*(d*e + 2*c*f))*Sqrt
[a + b*x^2]*Sqrt[c + d*x^2])/x) + (b*d*((8*b^2*c^2*e - a^2*d*(2*d*e - 3*c*
f) - 3*a*b*c*(d*e + 2*c*f))*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sq
rt[c]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*
d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) +
(c^(3/2)*(4*b*c*e - a*d*e - 3*a*c*f)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqr
t[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c +
d*x^2))]*Sqrt[c + d*x^2])))/(a*c))/(3*a*c))/(a*(b*c - a*d))))/e^2 + e*(((b
*e - a*f)*Sqrt[c + d*x^2])/(a*(b*c - a*d)*x^5*Sqrt[a + b*x^2]) + (-1/5*((6
*b*c*e - a*d*e - 5*a*c*f)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(a*c*x^5) - (-1
/3*(((24*b^2*c*e)/a - (a*d*(4*d*e - 5*c*f))/c - 5*b*(d*e + 4*c*f))*Sqrt[a
+ b*x^2]*Sqrt[c + d*x^2])/x^3 - (-(((48*b^3*c^3*e - 3*a^2*b*c*d*(3*d*e - 5
*c*f) - 2*a^3*d^2*(4*d*e - 5*c*f) - 8*a*b^2*c^2*(2*d*e + 5*c*f))*Sqrt[a +
b*x^2]*Sqrt[c + d*x^2])/(a*c*x)) + (b*d*((48*b^3*c^3*e - 3*a^2*b*c*d*(3*d*
e - 5*c*f) - 2*a^3*d^2*(4*d*e - 5*c*f) - 8*a*b^2*c^2*(2*d*e + 5*c*f))*((x*
Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[
ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x
^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (c^(3/2)*(24*b^2*c^2*e - a^2*...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 441
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
+ b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(a*g*2*(b*c - a*d)*(p + 1))), x] + Si
mp[1/(a*2*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2
)^q*Simp[c*(b*e - a*f)*(m + 1) + e*2*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(m
 + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, q},
 x] && LtQ[p, -1]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 448
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[e   Int[(g*x)^m*(a + b*x
^2)^p*(c + d*x^2)^q*(e + f*x^2)^(r - 1), x], x] + Simp[f/e^2   Int[(g*x)^(m
 + 2)*(a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^(r - 1), x], x] /; FreeQ[{a,
b, c, d, e, f, g, m, p, q}, x] && IGtQ[r, 0]
```

Maple [A] (verified)

Time = 22.03 (sec) , antiderivative size = 709, normalized size of antiderivative = 1.37

method	result
elliptic	$\sqrt{(b\,x^2+a)(x^2d+c)}\left(-\frac{e^2\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{5a^2c\,x^5}-\frac{e(10acf-4ade-9bce)\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{15a^3c^2x^3}-\frac{(15a^2c^2f^2-20a^2cdef+8a^2d^2e^2-50ab\cdots}{\cdots}\right.$
risch	$-\frac{\sqrt{b\,x^2+a}\,\sqrt{x^2d+c}\,(15f^2x^4a^2c^2-20a^2cdef\,x^4+8a^2d^2e^2x^4-50ab\,c^2ef\,x^4+17abcd\,e^2x^4+33b^2c^2e^2x^4+10a^2c^2ef\,x^2-4a^2cd\,e^2x^2-\cdots}{15a^4c^3x^5}$
default	Expression too large to display

input
```
int((f*x^2+e)^2/x^6/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOS
E)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-1/5/a^2/c*e^
2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/x^5-1/15/a^3/c^2*e*(10*a*c*f-4*a*d*e
-9*b*c*e)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/x^3-1/15/a^4/c^3*(15*a^2*c^2
*f^2-20*a^2*c*d*e*f+8*a^2*d^2*e^2-50*a*b*c^2*e*f+17*a*b*c*d*e^2+33*b^2*c^2
*e^2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/x+(b*d*x^2+b*c)*b/a^4/(a*d-b*c)*
x*(a^2*f^2-2*a*b*e*f+b^2*e^2)/((x^2+a/b)*(b*d*x^2+b*c))^(1/2)+(-1/15*b*d*e
*(10*a*c*f-4*a*d*e-9*b*c*e)/a^3/c^2-b*(a^2*f^2-2*a*b*e*f+b^2*e^2)/a^4-b^2*
c/a^4/(a*d-b*c)*(a^2*f^2-2*a*b*e*f+b^2*e^2))/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2
)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)
^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-(1/15*b*d*(15*a^2*c^2*f^2-20*a^2*c*d*e*f+
8*a^2*d^2*e^2-50*a*b*c^2*e*f+17*a*b*c*d*e^2+33*b^2*c^2*e^2)/a^4/c^3-d*b^2*
(a^2*f^2-2*a*b*e*f+b^2*e^2)/(a*d-b*c)/a^4)*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2
)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(-b
/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)
/c/b)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1029 vs. $2(481) = 962$.

Time = 0.16 (sec) , antiderivative size = 1029, normalized size of antiderivative = 1.99

$$\int \frac{(e+fx^2)^2}{x^6(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \text{Too large to display}$$

input
```
integrate((f*x^2+e)^2/x^6/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="fr
icas")
```

output
```
1/15*((((48*b^5*c^3 - 16*a*b^4*c^2*d - 9*a^2*b^3*c*d^2 - 8*a^3*b^2*d^3)*e^
2 - 10*(8*a*b^4*c^3 - 3*a^2*b^3*c^2*d - 2*a^3*b^2*c*d^2)*e*f + 15*(2*a^2*b
^3*c^3 - a^3*b^2*c^2*d)*f^2)*x^7 + ((48*a*b^4*c^3 - 16*a^2*b^3*c^2*d - 9*a
^3*b^2*c*d^2 - 8*a^4*b*d^3)*e^2 - 10*(8*a^2*b^3*c^3 - 3*a^3*b^2*c^2*d - 2*
a^4*b*c*d^2)*e*f + 15*(2*a^3*b^2*c^3 - a^4*b*c^2*d)*f^2)*x^5)*sqrt(a*c)*sq
rt(-b/a)*elliptic_e(arcsin(x*sqrt(-b/a)), a*d/(b*c)) - (((48*b^5*c^3 + 8*(
3*a^2*b^3 - 2*a*b^4)*c^2*d - (5*a^3*b^2 + 9*a^2*b^3)*c*d^2 - 4*(a^4*b + 2*
a^3*b^2)*d^3)*e^2 - 10*(8*a*b^4*c^3 + (4*a^3*b^2 - 3*a^2*b^3)*c^2*d - (a^4
*b + 2*a^3*b^2)*c*d^2)*e*f + 15*(2*a^2*b^3*c^3 + (a^4*b - a^3*b^2)*c^2*d)*
f^2)*x^7 + ((48*a*b^4*c^3 + 8*(3*a^3*b^2 - 2*a^2*b^3)*c^2*d - (5*a^4*b + 9
*a^3*b^2)*c*d^2 - 4*(a^5 + 2*a^4*b)*d^3)*e^2 - 10*(8*a^2*b^3*c^3 + (4*a^4*
b - 3*a^3*b^2)*c^2*d - (a^5 + 2*a^4*b)*c*d^2)*e*f + 15*(2*a^3*b^2*c^3 + (a
^5 - a^4*b)*c^2*d)*f^2)*x^5)*sqrt(a*c)*sqrt(-b/a)*elliptic_f(arcsin(x*sqrt
(-b/a)), a*d/(b*c)) - (((48*a*b^4*c^3 - 16*a^2*b^3*c^2*d - 9*a^3*b^2*c*d^2
 - 8*a^4*b*d^3)*e^2 - 10*(8*a^2*b^3*c^3 - 3*a^3*b^2*c^2*d - 2*a^4*b*c*d^2)
*e*f + 15*(2*a^3*b^2*c^3 - a^4*b*c^2*d)*f^2)*x^6 + ((24*a^2*b^3*c^3 - 11*a
^3*b^2*c^2*d - 5*a^4*b*c*d^2 - 8*a^5*d^3)*e^2 - 20*(2*a^3*b^2*c^3 - a^4*b*
c^2*d - a^5*c*d^2)*e*f + 15*(a^4*b*c^3 - a^5*c^2*d)*f^2)*x^4 + 3*(a^4*b*c^
3 - a^5*c^2*d)*e^2 - 2*((3*a^3*b^2*c^3 - a^4*b*c^2*d - 2*a^5*c*d^2)*e^2 -
5*(a^4*b*c^3 - a^5*c^2*d)*e*f)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/((...
```

Sympy [F]

$$\int \frac{(e+fx^2)^2}{x^6(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \int \frac{(e+fx^2)^2}{x^6(a+bx^2)^{\frac{3}{2}}\sqrt{c+dx^2}}\,dx$$

input
```
integrate((f*x**2+e)**2/x**6/(b*x**2+a)**(3/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral((e + f*x**2)**2/(x**6*(a + b*x**2)**(3/2)*sqrt(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{(e+fx^2)^2}{x^6(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \int \frac{(fx^2+e)^2}{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}x^6}\,dx$$

input
```
integrate((f*x^2+e)^2/x^6/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate((f*x^2 + e)^2/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*x^6), x)
```

Giac [F]

$$\int \frac{(e+fx^2)^2}{x^6(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \int \frac{(fx^2+e)^2}{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}x^6}\,dx$$

input
```
integrate((f*x^2+e)^2/x^6/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate((f*x^2 + e)^2/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx^2)^2}{x^6(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx=\int \frac{(f\,x^2+e)^2}{x^6\,(b\,x^2+a)^{3/2}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int((e + f*x^2)^2/(x^6*(a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int((e + f*x^2)^2/(x^6*(a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{(e+fx^2)^2}{x^6(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx=\text{too large to display}$$

input
```
int((f*x^2+e)^2/x^6/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x)
```

output

```
( - 2*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*d*e**2 - 10*sqrt(c + d*x**2)*sqr
t(a + b*x**2)*a*d*f**2*x**4 - 3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b*c*e**2
 - 15*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b*c*f**2*x**4 + 25*sqrt(c + d*x**2
)*sqrt(a + b*x**2)*b*d*e*f*x**4 - 20*int((sqrt(c + d*x**2)*sqrt(a + b*x**2
)*x**2)/(2*a**3*c*d + 2*a**3*d**2*x**2 + 3*a**2*b*c**2 + 7*a**2*b*c*d*x**2
 + 4*a**2*b*d**2*x**4 + 6*a*b**2*c**2*x**2 + 8*a*b**2*c*d*x**4 + 2*a*b**2*
d**2*x**6 + 3*b**3*c**2*x**4 + 3*b**3*c*d*x**6),x)*a**3*b*d**3*f**2*x**5 -
 60*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(2*a**3*c*d + 2*a**3*d**2
*x**2 + 3*a**2*b*c**2 + 7*a**2*b*c*d*x**2 + 4*a**2*b*d**2*x**4 + 6*a*b**2*
c**2*x**2 + 8*a*b**2*c*d*x**4 + 2*a*b**2*d**2*x**6 + 3*b**3*c**2*x**4 + 3*
b**3*c*d*x**6),x)*a**2*b**2*c*d**2*f**2*x**5 + 50*int((sqrt(c + d*x**2)*sq
rt(a + b*x**2)*x**2)/(2*a**3*c*d + 2*a**3*d**2*x**2 + 3*a**2*b*c**2 + 7*a*
*2*b*c*d*x**2 + 4*a**2*b*d**2*x**4 + 6*a*b**2*c**2*x**2 + 8*a*b**2*c*d*x**
4 + 2*a*b**2*d**2*x**6 + 3*b**3*c**2*x**4 + 3*b**3*c*d*x**6),x)*a**2*b**2*
d**3*e*f*x**5 - 20*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(2*a**3*c*
d + 2*a**3*d**2*x**2 + 3*a**2*b*c**2 + 7*a**2*b*c*d*x**2 + 4*a**2*b*d**2*x
**4 + 6*a*b**2*c**2*x**2 + 8*a*b**2*c*d*x**4 + 2*a*b**2*d**2*x**6 + 3*b**3
*c**2*x**4 + 3*b**3*c*d*x**6),x)*a**2*b**2*d**3*f**2*x**7 - 45*int((sqrt(c
 + d*x**2)*sqrt(a + b*x**2)*x**2)/(2*a**3*c*d + 2*a**3*d**2*x**2 + 3*a**2*
b*c**2 + 7*a**2*b*c*d*x**2 + 4*a**2*b*d**2*x**4 + 6*a*b**2*c**2*x**2 + ...
```

3.240 $\int \frac{x^6(e+fx^2)}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 459

$$\begin{aligned}
\int \frac{x^6(e+fx^2)}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx &= \frac{(24a^2d^2f-abd(20de-13cf)-2b^2c(5de-4cf))\,x\sqrt{c+dx^2}}{15b^3d^3\sqrt{a+bx^2}} \\
&+ \frac{(5bde-4bcf-6adf)x^3\sqrt{c+dx^2}}{15b^2d^2\sqrt{a+bx^2}} + \frac{fx^5\sqrt{c+dx^2}}{5bd\sqrt{a+bx^2}} \\
&+ \frac{\sqrt{a}(48a^3d^3f+2b^3c^2(5de-4cf)+3ab^2cd(5de-3cf)-8a^2bd^2(5de+2cf))\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1\right.}{15b^{7/2}d^3(bc-ad)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} \\
&- \frac{a^{3/2}(24a^2d^2f+b^2c(5de-4cf)-5abd(4de+cf))\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{15b^{7/2}d^2(bc-ad)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}
\end{aligned}$$

output
```
1/15*(24*a^2*d^2*f-a*b*d*(-13*c*f+20*d*e)-2*b^2*c*(-4*c*f+5*d*e))*x*(d*x^2
+c)^(1/2)/b^3/d^3/(b*x^2+a)^(1/2)+1/15*(-6*a*d*f-4*b*c*f+5*b*d*e)*x^3*(d*x
^2+c)^(1/2)/b^2/d^2/(b*x^2+a)^(1/2)+1/5*f*x^5*(d*x^2+c)^(1/2)/b/d/(b*x^2+a
)^(1/2)+1/15*a^(1/2)*(48*a^3*d^3*f+2*b^3*c^2*(-4*c*f+5*d*e)+3*a*b^2*c*d*(-
3*c*f+5*d*e)-8*a^2*b*d^2*(2*c*f+5*d*e))*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*
x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/b^(7/2)/d^3/(-a*d+b*c)/(b*x
^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)-1/15*a^(3/2)*(24*a^2*d^2*f+b^2
*c*(-4*c*f+5*d*e)-5*a*b*d*(c*f+4*d*e))*(d*x^2+c)^(1/2)*InverseJacobiAM(arc
tan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/b^(7/2)/d^2/(-a*d+b*c)/(b*x^2+a)
^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.84 (sec) , antiderivative size = 396, normalized size of antiderivative = 0.86

$$\int \frac{x^6(e+fx^2)}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \frac{\sqrt{\frac{b}{a}}dx(c+dx^2)\left(24a^3d^2f+b^3cx^2(5de-4cf+3dfx^2)+a^2bd(-20de-5cf\right.}{}$$

input
```
Integrate[(x^6*(e + f*x^2))/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

output
```
(Sqrt[b/a]*d*x*(c + d*x^2)*(24*a^3*d^2*f + b^3*c*x^2*(5*d*e - 4*c*f + 3*d*
f*x^2) + a^2*b*d*(-20*d*e - 5*c*f + 6*d*f*x^2) - a*b^2*(4*c^2*f + c*d*(-5*
e + 2*f*x^2) + d^2*x^2*(5*e + 3*f*x^2))) + I*c*(48*a^3*d^3*f + 2*b^3*c^2*(
5*d*e - 4*c*f) + 3*a*b^2*c*d*(5*d*e - 3*c*f) - 8*a^2*b*d^2*(5*d*e + 2*c*f)
)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x]
, (a*d)/(b*c)] - I*c*(-(b*c) + a*d)*(24*a^2*d^2*f + 2*b^2*c*(-5*d*e + 4*c*
f) + a*b*d*(-20*d*e + 13*c*f))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Ell
ipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(15*b^3*Sqrt[b/a]*d^3*(b*c -
a*d)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 500, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {440, 444, 444, 25, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6 (e+fx^2)}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \ 440$$

$$\frac{\int \frac{x^4((5bde+bcf-6adf)x^2+5c(be-af))}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{b(bc-ad)}-\frac{x^5\sqrt{c+dx^2}(be-af)}{b\sqrt{a+bx^2}(bc-ad)}$$

$$\Big\downarrow \ 444$$

$$\frac{\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}(-6adf+bcf+5bde)}{5bd}-\frac{\int \frac{x^2\left(3ac(5bde+bcf-6adf)-\left(c(5de-4cf)b^2-5ad(4de+cf)b+24a^2d^2f\right)x^2\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{5bd}}{b(bc-ad)}-\frac{x^5\sqrt{c+dx^2}(be-af)}{b\sqrt{a+bx^2}(bc-ad)}$$

$$\Big\downarrow \ 444$$

$$\frac{\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}(-6adf+bcf+5bde)}{5bd}-\frac{-\frac{\int -\frac{\left(2c^2(5de-4cf)b^3+3acd(5de-3cf)b^2-8a^2d^2(5de+2cf)b+48a^3d^3f\right)x^2+ac\left(c(5de-4cf)b^2-5ad(4de+cf)b \cdots\right.}{\sqrt{bx^2+a}\sqrt{dx^2+c}}}{3bd} \cdots}{5bd}}{b(bc-ad)}$$

$$\frac{x^5\sqrt{c+dx^2}(be-af)}{b\sqrt{a+bx^2}(bc-ad)}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}(-6adf+bcf+5bde)}{5bd}-\frac{\frac{\int \frac{\left(2c^2(5de-4cf)b^3+3acd(5de-3cf)b^2-8a^2d^2(5de+2cf)b+48a^3d^3f\right)x^2+ac\left(c(5de-4cf)b^2-5ad(4de+cf)b+24 \cdots\right.}{\sqrt{bx^2+a}\sqrt{dx^2+c}}}{3bd} \cdots}{5bd}}{b(bc-ad)}$$

$$\frac{x^5\sqrt{c+dx^2}(be-af)}{b\sqrt{a+bx^2}(bc-ad)}$$

$$\Big\downarrow \ 406$$

$$\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}(-6adf+bcf+5bde)}{5bd}-\frac{\frac{ac\left(24a^2d^2f-5abd(cf+4de)+b^2c(5de-4cf)\right)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\left(48a^3d^3f-8a^2bd^2(2cf+5de)+3ab^2cd(5de\right.}{3bd}}{b(bc-ad)}$$

$$\frac{x^5\sqrt{c+dx^2}(be-af)}{b\sqrt{a+bx^2}(bc-ad)}$$

$\big\downarrow$ 320

$$\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}(-6adf+bcf+5bde)}{5bd}-\frac{\frac{\left(48a^3d^3f-8a^2bd^2(2cf+5de)+3ab^2cd(5de-3cf)+2b^3c^2(5de-4cf)\right)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}\left(24a^2\right.}{}}{3bd}}{b(bc-ad)}$$

$$\frac{x^5\sqrt{c+dx^2}(be-af)}{b\sqrt{a+bx^2}(bc-ad)}$$

$\big\downarrow$ 388

$$\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}(-6adf+bcf+5bde)}{5bd}-\frac{\frac{\left(48a^3d^3f-8a^2bd^2(2cf+5de)+3ab^2cd(5de-3cf)+2b^3c^2(5de-4cf)\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{}}{}}{3bd}}{b(bc-a}$$

$$\frac{x^5\sqrt{c+dx^2}(be-af)}{b\sqrt{a+bx^2}(bc-ad)}$$

$\big\downarrow$ 313

$$\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}(-6adf+bcf+5bde)}{5bd}-\frac{\frac{\frac{c^{3/2}\sqrt{a+bx^2}\left(24a^2d^2f-5abd(cf+4de)+b^2c(5de-4cf)\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+\left(48a^3d^3f-8a^2bd^2\right.}{3bd}}{b(}$$

$$\frac{x^5\sqrt{c+dx^2}(be-af)}{b\sqrt{a+bx^2}(bc-ad)}$$

input

```
Int[(x^6*(e + f*x^2))/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

output
```
-(((b*e - a*f)*x^5*Sqrt[c + d*x^2])/(b*(b*c - a*d)*Sqrt[a + b*x^2])) + (((
5*b*d*e + b*c*f - 6*a*d*f)*x^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(5*b*d) -
(-1/3*(((24*a^2*d*f)/b + (b*c*(5*d*e - 4*c*f))/d - 5*a*(4*d*e + c*f))*x*Sq
rt[a + b*x^2]*Sqrt[c + d*x^2]) + ((48*a^3*d^3*f + 2*b^3*c^2*(5*d*e - 4*c*f
) + 3*a*b^2*c*d*(5*d*e - 3*c*f) - 8*a^2*b*d^2*(5*d*e + 2*c*f))*((x*Sqrt[a
+ b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcTan[
(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a
*(c + d*x^2))]*Sqrt[c + d*x^2])) + (c^(3/2)*(24*a^2*d^2*f + b^2*c*(5*d*e -
 4*c*f) - 5*a*b*d*(4*d*e + c*f))*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]
*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^
2))]*Sqrt[c + d*x^2]))/(3*b*d))/(5*b*d))/(b*(b*c - a*d))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 440
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[g*(b*e - a*f)*(g*x)^(m - 1)*(a +
 b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] - Simp[
g^2/(2*b*(b*c - a*d)*(p + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c +
d*x^2)^q*Simp[c*(b*e - a*f)*(m - 1) + (d*(b*e - a*f)*(m + 2*q + 1) - b*2*(c
*f - d*e)*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, q}, x] &&
 LtQ[p, -1] && GtQ[m, 1]
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

Maple [A] (verified)

Time = 20.85 (sec) , antiderivative size = 576, normalized size of antiderivative = 1.25

method	result
elliptic	$\frac{\sqrt{(b x^2+a)(x^2 d+c)}\left(-\frac{\left(b d x^2+b c\right) a^2 x (a f-b e)}{b^4 (a d-b c)\sqrt{\left(x^2+\frac{a}{b}\right)\left(b d x^2+b c\right)}}+\frac{f x^3 \sqrt{b d x^4+a d x^2+x^2 b c+a c}}{5 b^2 d}+\frac{\left(-\frac{a f-b e}{b^2}-\frac{f(4 a d+4 b c)}{5 b^2 d}\right) x \sqrt{b d x^4+a d x^2+x^2 b c+}}{3 b d}\right.}{}$
risch	$-\frac{x\left(-3 b d f x^2+9 a d f+4 b c f-5 b d e\right)\sqrt{b x^2+a}\sqrt{x^2 d+c}}{15 b^3 d^2}+\frac{\left(-\frac{\left(33 f d^2 a^2+17 f d c b a-25 a b d^2 e+8 f c^2 b^2-10 d b^2 c e\right) c\sqrt{1+\frac{b x^2}{a}}\sqrt{1+\frac{d x^2}{c}}\left(\text{Ell}\right.}{\sqrt{-\frac{b}{a}}\sqrt{b d x^4+a d x^2}}\right.}{}$
default	Expression too large to display

input
```
int(x^6*(f*x^2+e)/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-(b*d*x^2+b*c
)/b^4*a^2/(a*d-b*c)*x*(a*f-b*e)/((x^2+a/b)*(b*d*x^2+b*c))^(1/2)+1/5*f/b^2/
d*x^3*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+1/3*(-1/b^2*(a*f-b*e)-1/5*f/b^2/
d*(4*a*d+4*b*c))/b/d*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+(1/b^3*c*a^2/(a
*d-b*c)*(a*f-b*e)-1/3*(-1/b^2*(a*f-b*e)-1/5*f/b^2/d*(4*a*d+4*b*c))/b/d*a*c
)/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^
2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-(a/b^3*(a*
f-b*e)+1/b^3*d*a^2*(a*f-b*e)/(a*d-b*c)-3/5*f/b^2/d*a*c-1/3*(-1/b^2*(a*f-b*
e)-1/5*f/b^2/d*(4*a*d+4*b*c))/b/d*(2*a*d+2*b*c))*c/(-b/a)^(1/2)*(1+b*x^2/a
)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*(EllipticF
(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*
d+b*c)/c/b)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 846, normalized size of antiderivative = 1.84

$$\int \frac{x^6(e+fx^2)}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^6*(f*x^2+e)/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="fric
as")
```

output
```
1/15*(((5*(2*b^4*c^3*d + 3*a*b^3*c^2*d^2 - 8*a^2*b^2*c*d^3)*e - (8*b^4*c^4
 + 9*a*b^3*c^3*d + 16*a^2*b^2*c^2*d^2 - 48*a^3*b*c*d^3)*f)*x^3 + (5*(2*a*b
^3*c^3*d + 3*a^2*b^2*c^2*d^2 - 8*a^3*b*c*d^3)*e - (8*a*b^3*c^4 + 9*a^2*b^2
*c^3*d + 16*a^3*b*c^2*d^2 - 48*a^4*c*d^3)*f)*x)*sqrt(b*d)*sqrt(-c/d)*ellip
tic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - ((5*(2*b^4*c^3*d + 3*a*b^3*c^2*d^
2 - 4*a^2*b^2*d^4 - (8*a^2*b^2 - a*b^3)*c*d^3)*e - (8*b^4*c^4 + 9*a*b^3*c^
3*d - 24*a^3*b*d^4 + 4*(4*a^2*b^2 + a*b^3)*c^2*d^2 - (48*a^3*b - 5*a^2*b^2
)*c*d^3)*f)*x^3 + (5*(2*a*b^3*c^3*d + 3*a^2*b^2*c^2*d^2 - 4*a^3*b*d^4 - (8
*a^3*b - a^2*b^2)*c*d^3)*e - (8*a*b^3*c^4 + 9*a^2*b^2*c^3*d - 24*a^4*d^4 +
 4*(4*a^3*b + a^2*b^2)*c^2*d^2 - (48*a^4 - 5*a^3*b)*c*d^3)*f)*x)*sqrt(b*d)
*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)/x), a*d/(b*c)) + (3*(b^4*c*d^3 -
a*b^3*d^4)*f*x^6 + (5*(b^4*c*d^3 - a*b^3*d^4)*e - 2*(2*b^4*c^2*d^2 + a*b^3
*c*d^3 - 3*a^2*b^2*d^4)*f)*x^4 - (10*(b^4*c^2*d^2 + a*b^3*c*d^3 - 2*a^2*b^
2*d^4)*e - (8*b^4*c^3*d + 5*a*b^3*c^2*d^2 + 11*a^2*b^2*c*d^3 - 24*a^3*b*d^
4)*f)*x^2 - 5*(2*a*b^3*c^2*d^2 + 3*a^2*b^2*c*d^3 - 8*a^3*b*d^4)*e + (8*a*b
^3*c^3*d + 9*a^2*b^2*c^2*d^2 + 16*a^3*b*c*d^3 - 48*a^4*d^4)*f)*sqrt(b*x^2
+ a)*sqrt(d*x^2 + c))/((b^6*c*d^4 - a*b^5*d^5)*x^3 + (a*b^5*c*d^4 - a^2*b^
4*d^5)*x)
```

Sympy [F]

$$\int \frac{x^6(e+fx^2)}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx=\int \frac{x^6(e+fx^2)}{(a+bx^2)^{\frac{3}{2}}\sqrt{c+dx^2}}\,dx$$

input
```
integrate(x**6*(f*x**2+e)/(b*x**2+a)**(3/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(x**6*(e + f*x**2)/((a + b*x**2)**(3/2)*sqrt(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{x^6(e+fx^2)}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx=\int \frac{(fx^2+e)x^6}{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}}\,dx$$

input
```
integrate(x^6*(f*x^2+e)/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate((f*x^2 + e)*x^6/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)), x)
```

Giac [F]

$$\int \frac{x^6(e+fx^2)}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx=\int \frac{(fx^2+e)x^6}{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}}\,dx$$

input
```
integrate(x^6*(f*x^2+e)/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="giac
")
```

output
```
integrate((f*x^2 + e)*x^6/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6(e+fx^2)}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx=\int \frac{x^6\,(f\,x^2+e)}{(b\,x^2+a)^{3/2}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int((x^6*(e + f*x^2))/((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int((x^6*(e + f*x^2))/((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^6(e+fx^2)}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \text{Too large to display}$$

input
```
int(x^6*(f*x^2+e)/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x)
```

output
```
(18*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*c*d*f*x - 12*sqrt(c + d*x**2)*sqrt
(a + b*x**2)*a*d**2*f*x**3 + 12*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b*c**2*f
*x - 15*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b*c*d*e*x - 8*sqrt(c + d*x**2)*s
qrt(a + b*x**2)*b*c*d*f*x**3 + 10*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b*d**2
*e*x**3 + 6*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b*d**2*f*x**5 + 48*int((sqrt
(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a**2*c + a**2*d*x**2 + 2*a*b*c*x**2 +
 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a**3*d**3*f + 8*int((sqrt(c
+ d*x**2)*sqrt(a + b*x**2)*x**4)/(a**2*c + a**2*d*x**2 + 2*a*b*c*x**2 + 2*
a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a**2*b*c*d**2*f - 40*int((sqrt(
c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a**2*c + a**2*d*x**2 + 2*a*b*c*x**2 +
2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a**2*b*d**3*e + 48*int((sqrt(
c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a**2*c + a**2*d*x**2 + 2*a*b*c*x**2 +
2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a**2*b*d**3*f*x**2 + 4*int((s
qrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a**2*c + a**2*d*x**2 + 2*a*b*c*x**
2 + 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a*b**2*c**2*d*f - 5*int((
sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a**2*c + a**2*d*x**2 + 2*a*b*c*x*
*2 + 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a*b**2*c*d**2*e + 8*int(
(sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a**2*c + a**2*d*x**2 + 2*a*b*c*x
**2 + 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a*b**2*c*d**2*f*x**2 -
40*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a**2*c + a**2*d*x**2 +...
```

3.241 $\int \frac{x^4(e+fx^2)}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 338

$$\int \frac{x^4(e+fx^2)}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \frac{(3bde-2bcf-4adf)x\sqrt{c+dx^2}}{3b^2d^2\sqrt{a+bx^2}}+\frac{fx^3\sqrt{c+dx^2}}{3bd\sqrt{a+bx^2}} - \frac{\sqrt{a}(8a^2d^2f+b^2c(3de-2cf)-3abd(2de+cf))\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{3b^{5/2}d^2(bc-ad)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} - \frac{a^{3/2}(3bde+bcf-4adf)\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{3b^{5/2}d(bc-ad)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
1/3*(-4*a*d*f-2*b*c*f+3*b*d*e)*x*(d*x^2+c)^(1/2)/b^2/d^2/(b*x^2+a)^(1/2)+1
/3*f*x^3*(d*x^2+c)^(1/2)/b/d/(b*x^2+a)^(1/2)-1/3*a^(1/2)*(8*a^2*d^2*f+b^2*
c*(-2*c*f+3*d*e)-3*a*b*d*(c*f+2*d*e))*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/
a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/b^(5/2)/d^2/(-a*d+b*c)/(b*x^2
+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)-1/3*a^(3/2)*(-4*a*d*f+b*c*f+3*b*
d*e)*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)
^(1/2))/b^(5/2)/d/(-a*d+b*c)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/
2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 9.49 (sec) , antiderivative size = 285, normalized size of antiderivative = 0.84

$$\int \frac{x^4(e+fx^2)}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \frac{\sqrt{\frac{b}{a}}dx(c+dx^2)\left(-4a^2df+b^2cfx^2+ab(3de+cf-dfx^2)\right)-ic(8a^2d^2f+b^2\cdots}{\cdots}$$

input
```
Integrate[(x^4*(e + f*x^2))/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

output
```
(Sqrt[b/a]*d*x*(c + d*x^2)*(-4*a^2*d*f + b^2*c*f*x^2 + a*b*(3*d*e + c*f -
d*f*x^2)) - I*c*(8*a^2*d^2*f + b^2*c*(3*d*e - 2*c*f) - 3*a*b*d*(2*d*e + c*
f))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*
x], (a*d)/(b*c)] + I*c*(-(b*c) + a*d)*(-3*b*d*e + 2*b*c*f + 4*a*d*f)*Sqrt[
1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)
/(b*c)])/(3*a^2*(b/a)^(5/2)*d^2*(b*c - a*d)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2
])
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {440, 444, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(e+fx^2)}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx$$

$$\Big\downarrow \; 440$$

$$\frac{\int \frac{x^2((3bde+bcf-4adf)x^2+3c(be-af))}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{b(bc-ad)}-\frac{x^3\sqrt{c+dx^2}(be-af)}{b\sqrt{a+bx^2}(bc-ad)}$$

$$\Big\downarrow \; 444$$

$$\frac{\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(-4adf+bcf+3bde)}{3bd}-\frac{\int \frac{ac(3bde+bcf-4adf)-\left(c(3de-2cf)b^2-3ad(2de+cf)b+8a^2d^2f\right)x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3bd}}{b(bc-ad)}-\frac{x^3\sqrt{c+dx^2}(be-af)}{b\sqrt{a+bx^2}(bc-ad)}$$

$\big\downarrow$ 406

$$\frac{\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(-4adf+bcf+3bde)}{3bd}-\frac{ac(-4adf+bcf+3bde)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx-\left(8a^2d^2f-3abd(cf+2de)+b^2c(3de-2cf)\right)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2}}}{3bd}}{b(bc-ad)}\frac{x^3\sqrt{c+dx^2}(be-af)}{b\sqrt{a+bx^2}(bc-ad)}$$

$\big\downarrow$ 320

$$\frac{\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(-4adf+bcf+3bde)}{3bd}-\frac{\frac{c^{3/2}\sqrt{a+bx^2}(-4adf+bcf+3bde)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\left(8a^2d^2f-3abd(cf+2de)+b^2c(3de-2cf\right.}{3bd}}{b(bc-ad)}\frac{x^3\sqrt{c+dx^2}(be-af)}{b\sqrt{a+bx^2}(bc-ad)}$$

$\big\downarrow$ 388

$$\frac{\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(-4adf+bcf+3bde)}{3bd}-\frac{\frac{c^{3/2}\sqrt{a+bx^2}(-4adf+bcf+3bde)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\left(8a^2d^2f-3abd(cf+2de)+b^2c(3de-2cf\right.}{3bd}}{b(bc-ad)}\frac{x^3\sqrt{c+dx^2}(be-af)}{b\sqrt{a+bx^2}(bc-ad)}$$

$\big\downarrow$ 313

$$\frac{\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(-4adf+bcf+3bde)}{3bd}-\frac{\frac{c^{3/2}\sqrt{a+bx^2}(-4adf+bcf+3bde)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\left(8a^2d^2f-3abd(cf+2de)+b^2c(3de-2cf\right.}{3bd}}{b(bc-ad)}\frac{x^3\sqrt{c+dx^2}(be-af)}{b\sqrt{a+bx^2}(bc-ad)}$$

input
```
Int[(x^4*(e + f*x^2))/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

output
```
-(((b*e - a*f)*x^3*Sqrt[c + d*x^2])/(b*(b*c - a*d)*Sqrt[a + b*x^2])) + (((
3*b*d*e + b*c*f - 4*a*d*f)*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(3*b*d) - (-
((8*a^2*d^2*f + b^2*c*(3*d*e - 2*c*f) - 3*a*b*d*(2*d*e + c*f))*((x*Sqrt[a
+ b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcTan[
(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a
*(c + d*x^2))]*Sqrt[c + d*x^2]))) + (c^(3/2)*(3*b*d*e + b*c*f - 4*a*d*f)*S
qrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(S
qrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))/(3*b*d))/(b
*(b*c - a*d))
```

Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 440
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[g*(b*e - a*f)*(g*x)^(m - 1)*(a +
 b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] - Simp[
g^2/(2*b*(b*c - a*d)*(p + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c +
d*x^2)^q*Simp[c*(b*e - a*f)*(m - 1) + (d*(b*e - a*f)*(m + 2*q + 1) - b*2*(c
*f - d*e)*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, q}, x] &&
 LtQ[p, -1] && GtQ[m, 1]
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

Maple [A] (verified)

Time = 10.19 (sec) , antiderivative size = 431, normalized size of antiderivative = 1.28

method	result
elliptic	$\frac{\sqrt{(b x^2+a)(x^2 d+c)}\left(\frac{\left(bd x^2+bc\right)ax(af-be)}{b^3(ad-bc)\sqrt{\left(x^2+\frac{a}{b}\right)\left(bd x^2+bc\right)}}+\frac{fx\sqrt{bd x^4+ad x^2+x^2bc+ac}}{3d b^2}+\frac{\left(-\frac{ca(af-be)}{b^2(ad-bc)}-\frac{fac}{3b^2d}\right)\sqrt{1+\frac{b x^2}{a}}\sqrt{1+\frac{d x^2}{c}}\ \mathrm{EllipticF}\left(\right.}{\sqrt{-\frac{b}{a}}\sqrt{bd x^4+ad x^2+x^2bc+ac}}\right.}{\sqrt{}}$
risch	$\frac{fx\sqrt{b x^2+a}\sqrt{x^2d+c}}{3b^2d}-\frac{\left(-\frac{(5adf+2bcf-3bde)c\sqrt{1+\frac{b x^2}{a}}\sqrt{1+\frac{d x^2}{c}}\left(\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)-\mathrm{EllipticE}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)\right)}{\sqrt{-\frac{b}{a}}\sqrt{bd x^4+ad x^2+x^2bc+ac}\,d}\right.}{}$
default	$\frac{\left(\sqrt{-\frac{b}{a}}\,abd^3f x^5-\sqrt{-\frac{b}{a}}\,b^2cd^2f x^5+4\sqrt{-\frac{b}{a}}\,a^2d^3f x^3-3\sqrt{-\frac{b}{a}}\,abd^3e x^3-\sqrt{-\frac{b}{a}}\,b^2c^2df x^3+4\sqrt{\frac{b x^2+a}{a}}\sqrt{\frac{x^2d+c}{c}}\ \mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}}\right.\right.}{}$

input
```
int(x^4*(f*x^2+e)/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*((b*d*x^2+b*c)
/b^3*a/(a*d-b*c)*x*(a*f-b*e)/((x^2+a/b)*(b*d*x^2+b*c))^(1/2)+1/3*f/d/b^2*x
*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+(-1/b^2*c*a/(a*d-b*c)*(a*f-b*e)-1/3*f
/b^2/d*a*c)/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*
x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-
(-1/b^2*(a*f-b*e)-1/b^2*d*a*(a*f-b*e)/(a*d-b*c)-1/3*f/d/b^2*(2*a*d+2*b*c))
*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x
^2+a*c)^(1/2)/d*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-Ellipt
icE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 569, normalized size of antiderivative = 1.68

$$\int \frac{x^4(e+fx^2)}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx =$$

$$-\frac{((3(b^3c^2d-2ab^2cd^2)e-(2b^3c^3+3ab^2c^2d-8a^2bcd^2)f)x^3+(3(ab^2c^2d-2a^2bcd^2)e-(2ab^2c^3+3a^2bc}{}$$

input
```
integrate(x^4*(f*x^2+e)/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="fric
as")
```

output
```
-1/3*(((3*(b^3*c^2*d - 2*a*b^2*c*d^2)*e - (2*b^3*c^3 + 3*a*b^2*c^2*d - 8*a
^2*b*c*d^2)*f)*x^3 + (3*(a*b^2*c^2*d - 2*a^2*b*c*d^2)*e - (2*a*b^2*c^3 + 3
*a^2*b*c^2*d - 8*a^3*c*d^2)*f)*x)*sqrt(b*d)*sqrt(-c/d)*elliptic_e(arcsin(s
qrt(-c/d)/x), a*d/(b*c)) - ((3*(b^3*c^2*d - 2*a*b^2*c*d^2 - a*b^2*d^3)*e -
 (2*b^3*c^3 + 3*a*b^2*c^2*d - 4*a^2*b*d^3 - (8*a^2*b - a*b^2)*c*d^2)*f)*x^
3 + (3*(a*b^2*c^2*d - 2*a^2*b*c*d^2 - a^2*b*d^3)*e - (2*a*b^2*c^3 + 3*a^2*
b*c^2*d - 4*a^3*d^3 - (8*a^3 - a^2*b)*c*d^2)*f)*x)*sqrt(b*d)*sqrt(-c/d)*el
liptic_f(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - ((b^3*c*d^2 - a*b^2*d^3)*f*x^4
 + (3*(b^3*c*d^2 - a*b^2*d^3)*e - 2*(b^3*c^2*d + a*b^2*c*d^2 - 2*a^2*b*d^3
)*f)*x^2 + 3*(a*b^2*c*d^2 - 2*a^2*b*d^3)*e - (2*a*b^2*c^2*d + 3*a^2*b*c*d^
2 - 8*a^3*d^3)*f)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/((b^5*c*d^3 - a*b^4*d^4
)*x^3 + (a*b^4*c*d^3 - a^2*b^3*d^4)*x)
```

Sympy [F]

$$\int \frac{x^4(e+fx^2)}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \int \frac{x^4(e+fx^2)}{(a+bx^2)^{\frac{3}{2}}\sqrt{c+dx^2}}\,dx$$

input
```
integrate(x**4*(f*x**2+e)/(b*x**2+a)**(3/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(x**4*(e + f*x**2)/((a + b*x**2)**(3/2)*sqrt(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{x^4(e+fx^2)}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \int \frac{(fx^2+e)x^4}{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}}\,dx$$

input
```
integrate(x^4*(f*x^2+e)/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate((f*x^2 + e)*x^4/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)), x)
```

Giac [F]

$$\int \frac{x^4(e+fx^2)}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \int \frac{(fx^2+e)x^4}{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}}\,dx$$

input
```
integrate(x^4*(f*x^2+e)/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="giac
")
```

output
```
integrate((f*x^2 + e)*x^4/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(e+fx^2)}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \int \frac{x^4\,(f\,x^2+e)}{(b\,x^2+a)^{3/2}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int((x^4*(e + f*x^2))/((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int((x^4*(e + f*x^2))/((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^4(e+fx^2)}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \frac{-3\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,cfx+2\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,df\,x^3-8\left(\int \frac{\sqrt{}}{b^2d\,x^6+2abd\,x}\right.}{}$$

input
```
int(x^4*(f*x^2+e)/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x)
```

output
```
( - 3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*c*f*x + 2*sqrt(c + d*x**2)*sqrt(a
+ b*x**2)*d*f*x**3 - 8*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a**2*
c + a**2*d*x**2 + 2*a*b*c*x**2 + 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6)
,x)*a**2*d**2*f - int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a**2*c + a
**2*d*x**2 + 2*a*b*c*x**2 + 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a
*b*c*d*f + 6*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a**2*c + a**2*d
*x**2 + 2*a*b*c*x**2 + 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a*b*d*
*2*e - 8*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a**2*c + a**2*d*x**
2 + 2*a*b*c*x**2 + 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a*b*d**2*f
*x**2 - int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a**2*c + a**2*d*x**2
 + 2*a*b*c*x**2 + 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*b**2*c*d*f*
x**2 + 6*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a**2*c + a**2*d*x**
2 + 2*a*b*c*x**2 + 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*b**2*d**2*
e*x**2 + 3*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a**2*c + a**2*d*x**2 +
 2*a*b*c*x**2 + 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a**2*c**2*f +
 3*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a**2*c + a**2*d*x**2 + 2*a*b*c
*x**2 + 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a*b*c**2*f*x**2)/(6*b
*d**2*(a + b*x**2))
```

3.242 $\int \frac{x^2(e+fx^2)}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 246

$$\int \frac{x^2(e+fx^2)}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \frac{fx\sqrt{c+dx^2}}{bd\sqrt{a+bx^2}} - \frac{\sqrt{a}(bde+bcf-2adf)\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{b^{3/2}d(bc-ad)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} + \frac{\sqrt{a}(be-af)\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{b^{3/2}(bc-ad)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
f*x*(d*x^2+c)^(1/2)/b/d/(b*x^2+a)^(1/2)-a^(1/2)*(-2*a*d*f+b*c*f+b*d*e)*(d*
x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/
2))/b^(3/2)/d/(-a*d+b*c)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)+a
^(1/2)*(-a*f+b*e)*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)
),(1-a*d/b/c)^(1/2))/b^(3/2)/(-a*d+b*c)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*
x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.97 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.89

$$\int \frac{x^2(e+fx^2)}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \frac{\sqrt{\frac{b}{a}}d(-be+af)x(c+dx^2)+ic(2adf-b(de+cf))\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\Big(i}{b\sqrt{\frac{b}{a}}d(bc-}$$

input
```
Integrate[(x^2*(e + f*x^2))/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

output
```
(Sqrt[b/a]*d*(-(b*e) + a*f)*x*(c + d*x^2) + I*c*(2*a*d*f - b*(d*e + c*f))*
Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x],
(a*d)/(b*c)] - I*c*(-(b*c) + a*d)*f*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c
]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(b*Sqrt[b/a]*d*(b*c - a*
d)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.17, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {440, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(e+fx^2)}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \; 440$$

$$\frac{\int \frac{(bde+bcf-2adf)x^2+c(be-af)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{b(bc-ad)}-\frac{x\sqrt{c+dx^2}(be-af)}{b\sqrt{a+bx^2}(bc-ad)}$$

$$\Big\downarrow \; 406$$

$$\frac{c(be-af)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+(-2adf+bcf+bde)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{b(bc-ad)}-\frac{x\sqrt{c+dx^2}(be-af)}{b\sqrt{a+bx^2}(bc-ad)}$$

$$\Big\downarrow \; 320$$

$$\frac{(-2adf+bcf+bde)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}(be-af)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}}{b(bc-ad)}-\frac{x\sqrt{c+dx^2}(be-af)}{b\sqrt{a+bx^2}(bc-ad)}$$

$$\Big\downarrow \; 388$$

$$\frac{(-2adf+bcf+bde)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(be-af)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}}{b(bc-ad)}-\frac{x\sqrt{c+dx^2}(be-af)}{b\sqrt{a+bx^2}(bc-ad)}$$

$$\Big\downarrow \; 313$$

$$\frac{\frac{c^{3/2}\sqrt{a+bx^2}(be-af)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+(-2adf+bcf+bde)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{b(bc-ad)}-\frac{x\sqrt{c+dx^2}(be-af)}{b\sqrt{a+bx^2}(bc-ad)}$$

input
```
Int[(x^2*(e + f*x^2))/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

output
```
-(((b*e - a*f)*x*Sqrt[c + d*x^2])/(b*(b*c - a*d)*Sqrt[a + b*x^2])) + ((b*d
*e + b*c*f - 2*a*d*f)*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*
Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(
b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (c^(3/
2)*(b*e - a*f)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 -
(b*c)/(a*d)])/(a*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*
x^2]))/(b*(b*c - a*d))
```

Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 440
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[g*(b*e - a*f)*(g*x)^(m - 1)*(a +
 b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] - Simp[
g^2/(2*b*(b*c - a*d)*(p + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c +
d*x^2)^q*Simp[c*(b*e - a*f)*(m - 1) + (d*(b*e - a*f)*(m + 2*q + 1) - b*2*(c
*f - d*e)*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, q}, x] &&
 LtQ[p, -1] && GtQ[m, 1]
```

Maple [A] (verified)

Time = 6.14 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.44

method	result
elliptic	$\frac{\sqrt{(b\,x^2+a)(x^2d+c)}\left(-\frac{\left(bd\,x^2+bc\right)x(af-be)}{b^2(ad-bc)\sqrt{\left(x^2+\frac{a}{b}\right)\left(bd\,x^2+bc\right)}}+\frac{c(af-be)\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{b(ad-bc)\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}-\frac{\left(\frac{f}{b}+\frac{d(af-be)}{b(ad-bc)}\right.}{}\right.}{\sqrt{b\,x^2+a}\,\sqrt{x^2d+c}}$
default	$\frac{\left(-\sqrt{-\frac{b}{a}}\,a\,d^2f\,x^3+\sqrt{-\frac{b}{a}}\,b\,d^2e\,x^3-\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{x^2d+c}{c}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)acdf+\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{x^2d+c}{c}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right.\right.}{}$

input
```
int(x^2*(f*x^2+e)/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-(b*d*x^2+b*c
)/b^2/(a*d-b*c)*x*(a*f-b*e)/((x^2+a/b)*(b*d*x^2+b*c))^(1/2)+1/b*c/(a*d-b*c
)*(a*f-b*e)/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*
x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-
(f/b+d/b*(a*f-b*e)/(a*d-b*c))*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)
^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(-b/a)^(1/2),(-1
+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2)))
)
```

Fricas [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.34

$$\int \frac{x^2(e+fx^2)}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx =$$

$$-\frac{((b^2cde+(b^2c^2-2\,abcd)f)x^3+(abcde+(abc^2-2\,a^2cd)f)x)\sqrt{bd}\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ad}{bc})-(((b^2cd}{}$$

input
```
integrate(x^2*(f*x^2+e)/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="fric
as")
```

output
```
-(((b^2*c*d*e + (b^2*c^2 - 2*a*b*c*d)*f)*x^3 + (a*b*c*d*e + (a*b*c^2 - 2*a
^2*c*d)*f)*x)*sqrt(b*d)*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), a*d/(b
*c)) - (((b^2*c*d + b^2*d^2)*e + (b^2*c^2 - 2*a*b*c*d - a*b*d^2)*f)*x^3 +
((a*b*c*d + a*b*d^2)*e + (a*b*c^2 - 2*a^2*c*d - a^2*d^2)*f)*x)*sqrt(b*d)*s
qrt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - (a*b*d^2*e + (b^2*
c*d - a*b*d^2)*f*x^2 + (a*b*c*d - 2*a^2*d^2)*f)*sqrt(b*x^2 + a)*sqrt(d*x^2
 + c))/((b^4*c*d^2 - a*b^3*d^3)*x^3 + (a*b^3*c*d^2 - a^2*b^2*d^3)*x)
```

Sympy [F]

$$\int \frac{x^2(e+fx^2)}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \int \frac{x^2(e+fx^2)}{(a+bx^2)^{\frac{3}{2}}\sqrt{c+dx^2}}\,dx$$

input
```
integrate(x**2*(f*x**2+e)/(b*x**2+a)**(3/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(x**2*(e + f*x**2)/((a + b*x**2)**(3/2)*sqrt(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{x^2(e+fx^2)}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \int \frac{(fx^2+e)x^2}{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}}\,dx$$

input
```
integrate(x^2*(f*x^2+e)/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate((f*x^2 + e)*x^2/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)), x)
```

Giac [F]

$$\int \frac{x^2(e+fx^2)}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \int \frac{(fx^2+e)x^2}{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}}\,dx$$

input
```
integrate(x^2*(f*x^2+e)/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="giac
")
```

output
```
integrate((f*x^2 + e)*x^2/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(e+fx^2)}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \int \frac{x^2\,(f\,x^2+e)}{(b\,x^2+a)^{3/2}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int((x^2*(e + f*x^2))/((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int((x^2*(e + f*x^2))/((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^2(e+fx^2)}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,ex+2\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^4}{b^2d\,x^6+2abd\,x^4+b^2c\,x^4+a^2d\,x^2+2abc\,x^2+a^2c}dx\right)a^2df\,-$$

input
```
int(x^2*(f*x^2+e)/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x)
```

output

```
(sqrt(c + d*x**2)*sqrt(a + b*x**2)*e*x + 2*int((sqrt(c + d*x**2)*sqrt(a +
b*x**2)*x**4)/(a**2*c + a**2*d*x**2 + 2*a*b*c*x**2 + 2*a*b*d*x**4 + b**2*c
*x**4 + b**2*d*x**6),x)*a**2*d*f - int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*
x**4)/(a**2*c + a**2*d*x**2 + 2*a*b*c*x**2 + 2*a*b*d*x**4 + b**2*c*x**4 +
b**2*d*x**6),x)*a*b*d*e + 2*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(
a**2*c + a**2*d*x**2 + 2*a*b*c*x**2 + 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*
x**6),x)*a*b*d*f*x**2 - int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a**2
*c + a**2*d*x**2 + 2*a*b*c*x**2 + 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6
),x)*b**2*d*e*x**2 - int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a**2*c + a**
2*d*x**2 + 2*a*b*c*x**2 + 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a**
2*c*e - int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a**2*c + a**2*d*x**2 + 2*
a*b*c*x**2 + 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a*b*c*e*x**2)/(2
*a*d*(a + b*x**2))
```

3.243 $\int \frac{e+fx^2}{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 209

$$\int \frac{e+fx^2}{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}\,dx = \frac{(be-af)\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{b}(bc-ad)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} - \frac{\sqrt{a}(de-cf)\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}c(bc-ad)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output

```
(-a*f+b*e)*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(
1-a*d/b/c)^(1/2))/a^(1/2)/b^(1/2)/(-a*d+b*c)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/
c/(b*x^2+a))^(1/2)-a^(1/2)*(-c*f+d*e)*(d*x^2+c)^(1/2)*InverseJacobiAM(arct
an(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/b^(1/2)/c/(-a*d+b*c)/(b*x^2+a)^(1
/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.13 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.99

$$\int \frac{e+fx^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \frac{\sqrt{\frac{b}{a}}\left(\sqrt{\frac{b}{a}}(-be+af)x(c+dx^2)+ic(-be+af)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(i\text{arcsi}\right.\right.}{b(-bc+a}$$

input
```
Integrate[(e + f*x^2)/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

output
```
(Sqrt[b/a]*(Sqrt[b/a]*(-(b*e) + a*f)*x*(c + d*x^2) + I*c*(-(b*e) + a*f)*Sq
rt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a
*d)/(b*c)] - I*(-(b*c) + a*d)*e*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*El
lipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(b*(-(b*c) + a*d)*Sqrt[a +
b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {400, 313, 320}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \text{ 400}$$

$$\frac{(be-af)\int \frac{\sqrt{dx^2+c}}{(bx^2+a)^{3/2}}dx}{bc-ad} - \frac{(de-cf)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}$$

$$\Big\downarrow \text{ 313}$$

$$\frac{\sqrt{c+dx^2}(be-af)E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{b}\sqrt{a+bx^2}(bc-ad)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} - \frac{(de-cf)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}$$

$$\Big\downarrow \text{320}$$

$$\frac{\sqrt{c+dx^2}(be-af)E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{b}\sqrt{a+bx^2}(bc-ad)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}-\frac{\sqrt{c}\sqrt{a+bx^2}(de-cf)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

input
```
Int[(e + f*x^2)/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

output
```
((b*e - a*f)*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]], 1 - (a
*d)/(b*c)])/(Sqrt[a]*Sqrt[b]*(b*c - a*d)*Sqrt[a + b*x^2]*Sqrt[(a*(c + d*x^
2))/(c*(a + b*x^2))]) - (Sqrt[c]*(d*e - c*f)*Sqrt[a + b*x^2]*EllipticF[Arc
Tan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*(b*c - a*d)*Sqrt[(c
*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
 + d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 400
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

Maple [A] (verified)

Time = 5.89 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.60

method	result
default	$\frac{\left(\sqrt{-\frac{b}{a}}\, adf\, x^3-\sqrt{-\frac{b}{a}}\, bde\, x^3+\sqrt{\frac{b\, x^2+a}{a}}\, \sqrt{\frac{x^2 d+c}{c}}\, \text{EllipticF}\left(x\sqrt{-\frac{b}{a}}, \sqrt{\frac{ad}{bc}}\right) ade-\sqrt{\frac{b\, x^2+a}{a}}\, \sqrt{\frac{x^2 d+c}{c}}\, \text{EllipticF}\left(x\sqrt{-\frac{b}{a}}, \sqrt{\frac{ad}{bc}}\right) bce-}{\sqrt{-\frac{b}{a}}\, a(ad-}$
elliptic	$\frac{\sqrt{(b\, x^2+a)(x^2 d+c)}\left(\frac{\left(bd\, x^2+bc\right)x(af-be)}{ba(ad-bc)\sqrt{\left(x^2+\frac{a}{b}\right)\left(bd\, x^2+bc\right)}}+\frac{\left(\frac{f}{b}-\frac{af-be}{ba}-\frac{c(af-be)}{a(ad-bc)}\right)\sqrt{1+\frac{b\, x^2}{a}}\, \sqrt{1+\frac{d\, x^2}{c}}\, \text{EllipticF}\left(x\sqrt{-\frac{b}{a}}, \sqrt{-1+\frac{ad+bc}{cb}}\right)}{\sqrt{-\frac{b}{a}}\, \sqrt{bd\, x^4+ad\, x^2+x^2 bc+ac}}+\right.}{\sqrt{b\, x^2+a}\, \sqrt{x^2 d+c}}$

input
```
int((f*x^2+e)/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((-b/a)^(1/2)*a*d*f*x^3-(-b/a)^(1/2)*b*d*e*x^3+((b*x^2+a)/a)^(1/2)*((d*x^2
+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*d*e-((b*x^2+a)/a)
^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b*c*e
-((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c
)^(1/2))*a*c*f+((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^
(1/2),(a*d/b/c)^(1/2))*b*c*e+(-b/a)^(1/2)*a*c*f*x-(-b/a)^(1/2)*b*c*e*x)*(d
*x^2+c)^(1/2)*(b*x^2+a)^(1/2)/(-b/a)^(1/2)/a/(a*d-b*c)/(b*d*x^4+a*d*x^2+b*
c*x^2+a*c)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.21

$$\int \frac{e+fx^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \frac{(ab^2ce-a^2bcf)\sqrt{bx^2+a}\sqrt{dx^2+c}x-(ab^2ce-a^2bcf+(b^3ce-ab^2cf)x^2)\sqrt{}}{}$$

input
```
integrate((f*x^2+e)/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
((a*b^2*c*e - a^2*b*c*f)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x - (a*b^2*c*e -
a^2*b*c*f + (b^3*c*e - a*b^2*c*f)*x^2)*sqrt(a*c)*sqrt(-b/a)*elliptic_e(arc
sin(x*sqrt(-b/a)), a*d/(b*c)) - ((a^3 + a^2*b)*c*f + ((a^2*b + a*b^2)*c*f
- (b^3*c + a^2*b*d)*e)*x^2 - (a*b^2*c + a^3*d)*e)*sqrt(a*c)*sqrt(-b/a)*ell
iptic_f(arcsin(x*sqrt(-b/a)), a*d/(b*c)))/(a^3*b^2*c^2 - a^4*b*c*d + (a^2*
b^3*c^2 - a^3*b^2*c*d)*x^2)
```

Sympy [F]

$$\int \frac{e+fx^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \int \frac{e+fx^2}{(a+bx^2)^{\frac{3}{2}}\sqrt{c+dx^2}}\,dx$$

input
```
integrate((f*x**2+e)/(b*x**2+a)**(3/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral((e + f*x**2)/((a + b*x**2)**(3/2)*sqrt(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{e+fx^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \int \frac{fx^2+e}{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}}\,dx$$

input
```
integrate((f*x^2+e)/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((f*x^2 + e)/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)), x)
```

Giac [F]

$$\int \frac{e+fx^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \int \frac{fx^2+e}{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}}\,dx$$

input
```
integrate((f*x^2+e)/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((f*x^2 + e)/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \int \frac{f\,x^2+e}{(b\,x^2+a)^{3/2}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int((e + f*x^2)/((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int((e + f*x^2)/((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{e+fx^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^2}{b^2d\,x^6+2abd\,x^4+b^2c\,x^4+a^2d\,x^2+2abc\,x^2+a^2c}dx\right)f + \left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{b^2d\,x^6+2abd\,x^4+b^2c\,x^4+a^2d\,x^2+2abc\,x^2+a^2c}dx\right)e$$

input
```
int((f*x^2+e)/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a**2*c + a**2*d*x**2 + 2*a*b
*c*x**2 + 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*f + int((sqrt(c + d
*x**2)*sqrt(a + b*x**2))/(a**2*c + a**2*d*x**2 + 2*a*b*c*x**2 + 2*a*b*d*x*
*4 + b**2*c*x**4 + b**2*d*x**6),x)*e
```

3.244 $\int \frac{e+fx^2}{x^2\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 255

$$\int \frac{e+fx^2}{x^2\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}\,dx=-\frac{e\sqrt{c+dx^2}}{acx\sqrt{a+bx^2}}-\frac{\sqrt{b}(2bce-ade-acf)\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{a^{3/2}c(bc-ad)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}+\frac{d(be-af)\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{b}c(bc-ad)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
-e*(d*x^2+c)^(1/2)/a/c/x/(b*x^2+a)^(1/2)-b^(1/2)*(-a*c*f-a*d*e+2*b*c*e)*(d
*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1
/2))/a^(3/2)/c/(-a*d+b*c)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)+
d*(-a*f+b*e)*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-
a*d/b/c)^(1/2))/a^(1/2)/b^(1/2)/c/(-a*d+b*c)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/
c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 7.15 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.01

$$\int \frac{e+fx^2}{x^2\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}\,dx = \frac{-\sqrt{\frac{b}{a}}(c+dx^2)\left(a^2de-2b^2cex^2+ab(-ce+dex^2+cfx^2)\right)-ibc(-2bce\ldots}{\ldots}$$

input
```
Integrate[(e + f*x^2)/(x^2*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

output
```
(-(Sqrt[b/a]*(c + d*x^2)*(a^2*d*e - 2*b^2*c*e*x^2 + a*b*(-(c*e) + d*e*x^2
+ c*f*x^2))) - I*b*c*(-2*b*c*e + a*d*e + a*c*f)*x*Sqrt[1 + (b*x^2)/a]*Sqrt
[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*c*(-(b*
c) + a*d)*(-2*b*e + a*f)*x*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Ellipti
cF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(a^2*Sqrt[b/a]*c*(-(b*c) + a*d)*x
*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.36, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {441, 25, 445, 25, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx^2}{x^2\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \text{441}$$

$$\frac{\sqrt{c+dx^2}(be-af)}{ax\sqrt{a+bx^2}(bc-ad)}-\frac{\int -\frac{d(be-af)x^2+2bce-ade-acf}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{a(bc-ad)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{d(be-af)x^2+2bce-a(de+cf)}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{a(bc-ad)}+\frac{\sqrt{c+dx^2}(be-af)}{ax\sqrt{a+bx^2}(bc-ad)}$$

$\Big\downarrow$ 445

$$\frac{-\frac{\int -\frac{d\left(b(2bce-a(de+cf))x^2+ac(be-af)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(-acf-ade+2bce)}{acx}}{a(bc-ad)}+\frac{\sqrt{c+dx^2}(be-af)}{ax\sqrt{a+bx^2}(bc-ad)}$$

$\Big\downarrow$ 25

$$\frac{\frac{\int \frac{d\left(b(2bce-a(de+cf))x^2+ac(be-af)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(-acf-ade+2bce)}{acx}}{a(bc-ad)}+\frac{\sqrt{c+dx^2}(be-af)}{ax\sqrt{a+bx^2}(bc-ad)}$$

$\Big\downarrow$ 27

$$\frac{\frac{d\int \frac{b(2bce-a(de+cf))x^2+ac(be-af)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(-acf-ade+2bce)}{acx}}{a(bc-ad)}+\frac{\sqrt{c+dx^2}(be-af)}{ax\sqrt{a+bx^2}(bc-ad)}$$

$\Big\downarrow$ 406

$$\frac{\frac{d\left(ac(be-af)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+b(2bce-a(cf+de))\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(-acf-ade+2bce)}{acx}}{a(bc-ad)}+\frac{\sqrt{c+dx^2}(be-af)}{ax\sqrt{a+bx^2}(bc-ad)}$$

$\Big\downarrow$ 320

$$\frac{\frac{d\left(b(2bce-a(cf+de))\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}(be-af)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(-acf-ade+2bce)}{acx}}{a(bc-ad)}+\frac{\sqrt{c+dx^2}(be-af)}{ax\sqrt{a+bx^2}(bc-ad)}$$

$\Big\downarrow$ 388

$$\frac{\frac{d\left(b(2bce-a(cf+de))\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(be-af)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(-acf-ade+}{acx}}{a(bc-ad)}$$

$$\frac{\sqrt{c+dx^2}(be-af)}{ax\sqrt{a+bx^2}(bc-ad)}$$

$$\Big\downarrow \; 313$$

$$\frac{\frac{d\left(\frac{c^{3/2}\sqrt{a+bx^2}(be-af)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+b(2bce-a(cf+de))\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)\right)}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+}}{}}{a(bc-ad)}$$

$$\frac{\sqrt{c+dx^2}(be-af)}{ax\sqrt{a+bx^2}(bc-ad)}$$

input
```
Int[(e + f*x^2)/(x^2*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

output
```
((b*e - a*f)*Sqrt[c + d*x^2])/(a*(b*c - a*d)*x*Sqrt[a + b*x^2]) + (-(((2*b
*c*e - a*d*e - a*c*f)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(a*c*x)) + (d*(b*(2
*b*c*e - a*(d*e + c*f))*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c
]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])
/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (c^(
3/2)*(b*e - a*f)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1
- (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*
x^2])))/(a*c))/(a*(b*c - a*d))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
 + d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 441
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
+ b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(a*g*2*(b*c - a*d)*(p + 1))), x] + Si
mp[1/(a*2*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2
)^q*Simp[c*(b*e - a*f)*(m + 1) + e*2*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(m
 + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, q},
 x] && LtQ[p, -1]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 10.29 (sec) , antiderivative size = 411, normalized size of antiderivative = 1.61

method	result
elliptic	$\frac{\sqrt{(b\,x^2+a)(x^2d+c)}\left(-\frac{(bd\,x^2+bc)x(af-be)}{a^2(ad-bc)\sqrt{\left(x^2+\frac{a}{b}\right)(bd\,x^2+bc)}}-\frac{e\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{a^2cx}+\frac{\left(\frac{af-be}{a^2}+\frac{bc(af-be)}{a^2(ad-bc)}\right)\sqrt{1+\frac{b\,x^2}{a}}\sqrt{1+\frac{d\,x^2}{c}}\,\mathrm{EllipticF}}{\sqrt{-\frac{b}{a}}\sqrt{bd\,x^4+ad\,x^2+x^2bc+a}}\right.}{\sqrt{b\,x^2+a}}$
risch	$-\frac{e\sqrt{b\,x^2+a}\,\sqrt{x^2d+c}}{a^2cx}+\frac{\left(ac(af-be)\left(-\frac{(bd\,x^2+bc)x}{a(ad-bc)\sqrt{\left(x^2+\frac{a}{b}\right)(bd\,x^2+bc)}}+\frac{\left(\frac{1}{a}+\frac{bc}{(ad-bc)a}\right)\sqrt{1+\frac{b\,x^2}{a}}\sqrt{1+\frac{d\,x^2}{c}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+}\right.}{\sqrt{-\frac{b}{a}}\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}\right.\right.}{}$
default	$\frac{\left(-\sqrt{-\frac{b}{a}}\,abcdf\,x^4-\sqrt{-\frac{b}{a}}\,ab\,d^2e\,x^4+2\sqrt{-\frac{b}{a}}\,b^2cde\,x^4+\sqrt{\frac{b\,x^2+a}{a}}\sqrt{\frac{x^2d+c}{c}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)a^2cdfx-\sqrt{\frac{b\,x^2+a}{a}}\sqrt{\frac{x^2d+c}{c}}\,\mathrm{E}\right.}{}$

input
```
int((f*x^2+e)/x^2/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-(b*d*x^2+b*c
)/a^2/(a*d-b*c)*x*(a*f-b*e)/((x^2+a/b)*(b*d*x^2+b*c))^(1/2)-1/a^2*e/c*(b*d
*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/x+((a*f-b*e)/a^2+b*c/a^2/(a*d-b*c)*(a*f-b*
e))/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*
x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-(b*(a*f-
b*e)*d/a^2/(a*d-b*c)+b*d*e/a^2/c)*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^
2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(-b/a)^(1/2)
,(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/
2))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.38

$$\int \frac{e+fx^2}{x^2\,(a+bx^2)^{3/2}\,\sqrt{c+dx^2}}\,dx =$$

$$-\frac{((ab^3cf-(2\,b^4c-ab^3d)e)x^3+(a^2b^2cf-(2\,ab^3c-a^2b^2d)e)x)\sqrt{ac}\sqrt{-\frac{b}{a}}E(\arcsin\left(x\sqrt{-\frac{b}{a}}\right)\mid\frac{ad}{bc})+(((2\,b}{}$$

input
```
integrate((f*x^2+e)/x^2/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="fric
as")
```

output
```
-(((a*b^3*c*f - (2*b^4*c - a*b^3*d)*e)*x^3 + (a^2*b^2*c*f - (2*a*b^3*c - a
^2*b^2*d)*e)*x)*sqrt(a*c)*sqrt(-b/a)*elliptic_e(arcsin(x*sqrt(-b/a)), a*d/
(b*c)) + (((2*b^4*c + (a^2*b^2 - a*b^3)*d)*e - (a*b^3*c + a^3*b*d)*f)*x^3
+ ((2*a*b^3*c + (a^3*b - a^2*b^2)*d)*e - (a^2*b^2*c + a^4*d)*f)*x)*sqrt(a*
c)*sqrt(-b/a)*elliptic_f(arcsin(x*sqrt(-b/a)), a*d/(b*c)) - ((a^2*b^2*c*f
- (2*a*b^3*c - a^2*b^2*d)*e)*x^2 - (a^2*b^2*c - a^3*b*d)*e)*sqrt(b*x^2 + a
)*sqrt(d*x^2 + c))/((a^3*b^3*c^2 - a^4*b^2*c*d)*x^3 + (a^4*b^2*c^2 - a^5*b
*c*d)*x)
```

Sympy [F]

$$\int \frac{e+fx^2}{x^2(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \int \frac{e+fx^2}{x^2(a+bx^2)^{\frac{3}{2}}\sqrt{c+dx^2}}\,dx$$

input
```
integrate((f*x**2+e)/x**2/(b*x**2+a)**(3/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral((e + f*x**2)/(x**2*(a + b*x**2)**(3/2)*sqrt(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{e+fx^2}{x^2(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \int \frac{fx^2+e}{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}\,x^2}\,dx$$

input
```
integrate((f*x^2+e)/x^2/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate((f*x^2 + e)/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*x^2), x)
```

Giac [F]

$$\int \frac{e+fx^2}{x^2 (a+bx^2)^{3/2} \sqrt{c+dx^2}}\,dx = \int \frac{fx^2+e}{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}x^2}\,dx$$

input
```
integrate((f*x^2+e)/x^2/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="giac
")
```

output
```
integrate((f*x^2 + e)/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx^2}{x^2 (a+bx^2)^{3/2} \sqrt{c+dx^2}}\,dx = \int \frac{f\,x^2+e}{x^2\,(b\,x^2+a)^{3/2}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int((e + f*x^2)/(x^2*(a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int((e + f*x^2)/(x^2*(a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{e+fx^2}{x^2 (a+bx^2)^{3/2} \sqrt{c+dx^2}}\,dx = \frac{-\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,e-\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^2}{b^2 d\,x^6+2abd\,x^4+b^2 c\,x^4+a^2 d\,x^2+2abc\,x^2+a^2 c}dx\right) abde}{}$$

input
```
int((f*x^2+e)/x^2/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x)
```

output
```
( - sqrt(c + d*x**2)*sqrt(a + b*x**2)*e - int((sqrt(c + d*x**2)*sqrt(a + b
*x**2)*x**2)/(a**2*c + a**2*d*x**2 + 2*a*b*c*x**2 + 2*a*b*d*x**4 + b**2*c*
x**4 + b**2*d*x**6),x)*a*b*d*e*x - int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*
x**2)/(a**2*c + a**2*d*x**2 + 2*a*b*c*x**2 + 2*a*b*d*x**4 + b**2*c*x**4 +
b**2*d*x**6),x)*b**2*d*e*x**3 + int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a
**2*c + a**2*d*x**2 + 2*a*b*c*x**2 + 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x
**6),x)*a**2*c*f*x - 2*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a**2*c + a
**2*d*x**2 + 2*a*b*c*x**2 + 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a
*b*c*e*x + int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a**2*c + a**2*d*x**2 +
 2*a*b*c*x**2 + 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a*b*c*f*x**3
- 2*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a**2*c + a**2*d*x**2 + 2*a*b*
c*x**2 + 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*b**2*c*e*x**3)/(a*c*
x*(a + b*x**2))
```

3.245 $\int \frac{e+fx^2}{x^4(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 343

$$\int \frac{e+fx^2}{x^4(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = -\frac{e\sqrt{c+dx^2}}{3acx^3\sqrt{a+bx^2}}+\frac{(4bce+2ade-3acf)\sqrt{c+dx^2}}{3a^2c^2x\sqrt{a+bx^2}}+\frac{\sqrt{b}(8b^2c^2e-a^2d(2de-3cf)-3abc(de+2cf))\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{3a^{5/2}c^2(bc-ad)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}-\frac{\sqrt{b}d(4bce-ade-3acf)\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{3a^{3/2}c^2(bc-ad)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
-1/3*e*(d*x^2+c)^(1/2)/a/c/x^3/(b*x^2+a)^(1/2)+1/3*(-3*a*c*f+2*a*d*e+4*b*c
*e)*(d*x^2+c)^(1/2)/a^2/c^2/x/(b*x^2+a)^(1/2)+1/3*b^(1/2)*(8*b^2*c^2*e-a^2
*d*(-3*c*f+2*d*e)-3*a*b*c*(2*c*f+d*e))*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x
/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/a^(5/2)/c^2/(-a*d+b*c)/(b*x^
2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)-1/3*b^(1/2)*d*(-3*a*c*f-a*d*e+4
*b*c*e)*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b
/c)^(1/2))/a^(3/2)/c^2/(-a*d+b*c)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a)
)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 9.56 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.07

$$\int \frac{e+fx^2}{x^4 (a+bx^2)^{3/2} \sqrt{c+dx^2}} \, dx = \frac{\sqrt{\frac{b}{a}}(c+dx^2) \left(-8b^3c^2ex^4+ab^2cx^2(-4ce+3dex^2+6cfx^2)+a^3d(2dex^2\right.}{}$$

input
```
Integrate[(e + f*x^2)/(x^4*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

output
```
(Sqrt[b/a]*(c + d*x^2)*(-8*b^3*c^2*e*x^4 + a*b^2*c*x^2*(-4*c*e + 3*d*e*x^2
 + 6*c*f*x^2) + a^3*d*(2*d*e*x^2 - c*(e + 3*f*x^2)) + a^2*b*(2*d^2*e*x^4 +
 c*d*x^2*(2*e - 3*f*x^2) + c^2*(e + 3*f*x^2))) - I*b*c*(8*b^2*c^2*e - 3*a*
b*c*(d*e + 2*c*f) + a^2*d*(-2*d*e + 3*c*f))*x^3*Sqrt[1 + (b*x^2)/a]*Sqrt[1
 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + I*b*c*(-(b*
c) + a*d)*(-8*b*c*e - a*d*e + 6*a*c*f)*x^3*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d
*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(3*a^3*Sqrt[b/a]*
c^2*(-(b*c) + a*d)*x^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 455, normalized size of antiderivative = 1.33, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.303$, Rules used = {441, 25, 445, 445, 25, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx^2}{x^4 (a+bx^2)^{3/2} \sqrt{c+dx^2}} \, dx$$

$$\Big\downarrow \text{ 441}$$

$$\frac{\sqrt{c+dx^2}(be-af)}{ax^3\sqrt{a+bx^2}(bc-ad)} - \frac{\int -\frac{3d(be-af)x^2+4bce-ade-3acf}{x^4\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{a(bc-ad)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{3d(be-af)x^2+4bce-ade-3acf}{x^4\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{a(bc-ad)}+\frac{\sqrt{c+dx^2}(be-af)}{ax^3\sqrt{a+bx^2}(bc-ad)}$$

$\downarrow$ 445

$$\frac{-\frac{\int \frac{-d(2de-3cf)a^2-3bc(de+2cf)a+bd(4bce-ade-3acf)x^2+8b^2c^2e}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(-3acf-ade+4bce)}{3acx^3}}{a(bc-ad)}+\frac{\sqrt{c+dx^2}(be-af)}{ax^3\sqrt{a+bx^2}(bc-ad)}$$

$\downarrow$ 445

$$\frac{-\frac{-\frac{\int -\frac{bd\left(\left(-d(2de-3cf)a^2-3bc(de+2cf)a+8b^2c^2e\right)x^2+ac(4bce-ade-3acf)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{8b^2ce}{a}-\frac{ad(2de-3cf)}{c}-3b(2cf+de)\right)}{x}}{3ac}-\sqrt{a+bx^2}\sqrt{c\cdots}}{a(bc-ad)}$$

$$\frac{\sqrt{c+dx^2}(be-af)}{ax^3\sqrt{a+bx^2}(bc-ad)}$$

$\downarrow$ 25

$$\frac{-\frac{\frac{\int \frac{bd\left(\left(-d(2de-3cf)a^2-3bc(de+2cf)a+8b^2c^2e\right)x^2+ac(4bce-ade-3acf)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{8b^2ce}{a}-\frac{ad(2de-3cf)}{c}-3b(2cf+de)\right)}{x}}{3ac}-\sqrt{a+bx^2}\sqrt{c+dx\cdots}}{a(bc-ad)}$$

$$\frac{\sqrt{c+dx^2}(be-af)}{ax^3\sqrt{a+bx^2}(bc-ad)}$$

$\downarrow$ 27

$$\frac{-\frac{\frac{bd\int \frac{\left(-d(2de-3cf)a^2-3bc(de+2cf)a+8b^2c^2e\right)x^2+ac(4bce-ade-3acf)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{8b^2ce}{a}-\frac{ad(2de-3cf)}{c}-3b(2cf+de)\right)}{x}}{3ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\cdots}{3\cdots}}{a(bc-ad)}$$

$$\frac{\sqrt{c+dx^2}(be-af)}{ax^3\sqrt{a+bx^2}(bc-ad)}$$

$\downarrow$ 406

$$\frac{-\frac{\frac{bd\left(\left(a^2(-d)(2de-3cf)-3abc(2cf+de)+8b^2c^2e\right)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+ac(-3acf-ade+4bce)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{8b^2ce}{a}-\frac{ad(2de-\cdots}{c}\cdots\right.}{x}}{3ac}}{a(bc-ad)}$$

$$\frac{\sqrt{c+dx^2}(be-af)}{ax^3\sqrt{a+bx^2}(bc-ad)}$$

$\Big\downarrow$ 320

$$-\frac{\frac{bd\left(\left(a^2(-d)(2de-3cf)-3abc(2cf+de)+8b^2c^2e\right)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}(-3acf-ade+4bce)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c-}}{}}{3ac}}{a(bc-ad)}$$

$$\frac{\sqrt{c+dx^2}(be-af)}{ax^3\sqrt{a+bx^2}(bc-ad)}$$

$\Big\downarrow$ 388

$$-\frac{\frac{bd\left(\left(a^2(-d)(2de-3cf)-3abc(2cf+de)+8b^2c^2e\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(-3acf-ade+4bce)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{ac}-}{3ac}}{a(bc-ad)}$$

$$\frac{\sqrt{c+dx^2}(be-af)}{ax^3\sqrt{a+bx^2}(bc-ad)}$$

$\Big\downarrow$ 313

$$-\frac{\frac{bd\left(\left(a^2(-d)(2de-3cf)-3abc(2cf+de)+8b^2c^2e\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(-3acf-ade+4bce)\,\text{EllipticF}\left(\arctan\left(\right.\right.}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}}{ac}}{3ac}}{a(bc-ad)}$$

$$\frac{\sqrt{c+dx^2}(be-af)}{ax^3\sqrt{a+bx^2}(bc-ad)}$$

input

```
Int[(e + f*x^2)/(x^4*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

output
```
((b*e - a*f)*Sqrt[c + d*x^2])/(a*(b*c - a*d)*x^3*Sqrt[a + b*x^2]) + (-1/3*
((4*b*c*e - a*d*e - 3*a*c*f)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(a*c*x^3) -
(-((((8*b^2*c*e)/a - (a*d*(2*d*e - 3*c*f))/c - 3*b*(d*e + 2*c*f))*Sqrt[a +
 b*x^2]*Sqrt[c + d*x^2])/x) + (b*d*((8*b^2*c^2*e - a^2*d*(2*d*e - 3*c*f) -
 3*a*b*c*(d*e + 2*c*f))*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c
]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])
/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (c^(
3/2)*(4*b*c*e - a*d*e - 3*a*c*f)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]
*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^
2))]*Sqrt[c + d*x^2])))/(a*c))/(3*a*c))/(a*(b*c - a*d))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 441
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
+ b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(a*g*2*(b*c - a*d)*(p + 1))), x] + Si
mp[1/(a*2*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2
)^q*Simp[c*(b*e - a*f)*(m + 1) + e*2*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(m
 + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, q},
 x] && LtQ[p, -1]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 10.71 (sec) , antiderivative size = 497, normalized size of antiderivative = 1.45

method	result
elliptic	$\frac{\sqrt{(b x^2+a)(x^2 d+c)}\left(-\frac{e\sqrt{bd x^4+ad x^2+x^2 bc+ac}}{3a^2 c x^3}-\frac{(3acf-2ade-5bce)\sqrt{bd x^4+ad x^2+x^2 bc+ac}}{3a^3 c^2 x}+\frac{\left(bd x^2+bc\right)bx(af-be)}{a^3(ad-bc)\sqrt{\left(x^2+\frac{a}{b}\right)\left(bd x^2+bc\right)}}+\frac{\left(-\frac{bd}{3a^2}\right.}{}\right.}{}$
risch	$-\frac{\sqrt{b x^2+a}\,\sqrt{x^2 d+c}\,(3acf x^2-2ade x^2-5bce x^2+ace)}{3a^3c^2x^3}+\frac{b\left(-\frac{(3acf-2ade-5bce)c\sqrt{1+\frac{b x^2}{a}}\,\sqrt{1+\frac{d x^2}{c}}\,\left(\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right.\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{bd x^4+ad x^2+x^2 bc+ac}}\right.}{}$
default	Expression too large to display

input
```
int((f*x^2+e)/x^4/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-1/3/a^2*e/c*
(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/x^3-1/3/a^3/c^2*(3*a*c*f-2*a*d*e-5*b*c
*e)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/x+(b*d*x^2+b*c)*b/a^3/(a*d-b*c)*x*
(a*f-b*e)/((x^2+a/b)*(b*d*x^2+b*c))^(1/2)+(-1/3*b*d*e/a^2/c-(a*f-b*e)*b/a^
3-b^2*c/a^3/(a*d-b*c)*(a*f-b*e))/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c
)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(
a*d+b*c)/c/b)^(1/2))-(1/3*b*d*(3*a*c*f-2*a*d*e-5*b*c*e)/a^3/c^2-(a*f-b*e)*
b^2*d/(a*d-b*c)/a^3)*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b
*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c
)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 558, normalized size of antiderivative = 1.63

$$\int \frac{e+fx^2}{x^4\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}\,dx =$$

$$-\frac{(((8\,b^4c^2-3\,ab^3cd-2\,a^2b^2d^2)e-3\,(2\,ab^3c^2-a^2b^2cd)f)x^5+((8\,ab^3c^2-3\,a^2b^2cd-2\,a^3bd^2)e-3\,(2\,a^2b^2}{}$$

input
```
integrate((f*x^2+e)/x^4/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="fric
as")
```

output
```
-1/3*((((8*b^4*c^2 - 3*a*b^3*c*d - 2*a^2*b^2*d^2)*e - 3*(2*a*b^3*c^2 - a^2
*b^2*c*d)*f)*x^5 + ((8*a*b^3*c^2 - 3*a^2*b^2*c*d - 2*a^3*b*d^2)*e - 3*(2*a
^2*b^2*c^2 - a^3*b*c*d)*f)*x^3)*sqrt(a*c)*sqrt(-b/a)*elliptic_e(arcsin(x*s
qrt(-b/a)), a*d/(b*c)) - (((8*b^4*c^2 + (4*a^2*b^2 - 3*a*b^3)*c*d - (a^3*b
 + 2*a^2*b^2)*d^2)*e - 3*(2*a*b^3*c^2 + (a^3*b - a^2*b^2)*c*d)*f)*x^5 + ((
8*a*b^3*c^2 + (4*a^3*b - 3*a^2*b^2)*c*d - (a^4 + 2*a^3*b)*d^2)*e - 3*(2*a^
2*b^2*c^2 + (a^4 - a^3*b)*c*d)*f)*x^3)*sqrt(a*c)*sqrt(-b/a)*elliptic_f(arc
sin(x*sqrt(-b/a)), a*d/(b*c)) - (((8*a*b^3*c^2 - 3*a^2*b^2*c*d - 2*a^3*b*d
^2)*e - 3*(2*a^2*b^2*c^2 - a^3*b*c*d)*f)*x^4 + (2*(2*a^2*b^2*c^2 - a^3*b*c
*d - a^4*d^2)*e - 3*(a^3*b*c^2 - a^4*c*d)*f)*x^2 - (a^3*b*c^2 - a^4*c*d)*e
)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/((a^4*b^2*c^3 - a^5*b*c^2*d)*x^5 + (a^5
*b*c^3 - a^6*c^2*d)*x^3)
```

Sympy [F]

$$\int \frac{e+fx^2}{x^4 (a+bx^2)^{3/2} \sqrt{c+dx^2}}\,dx = \int \frac{e+fx^2}{x^4 (a+bx^2)^{\frac{3}{2}} \sqrt{c+dx^2}}\,dx$$

input
```
integrate((f*x**2+e)/x**4/(b*x**2+a)**(3/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral((e + f*x**2)/(x**4*(a + b*x**2)**(3/2)*sqrt(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{e+fx^2}{x^4 (a+bx^2)^{3/2} \sqrt{c+dx^2}}\,dx = \int \frac{fx^2+e}{(bx^2+a)^{\frac{3}{2}} \sqrt{dx^2+cx^4}}\,dx$$

input
```
integrate((f*x^2+e)/x^4/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate((f*x^2 + e)/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*x^4), x)
```

Giac [F]

$$\int \frac{e+fx^2}{x^4 (a+bx^2)^{3/2} \sqrt{c+dx^2}}\,dx = \int \frac{fx^2+e}{(bx^2+a)^{\frac{3}{2}} \sqrt{dx^2+cx^4}}\,dx$$

input
```
integrate((f*x^2+e)/x^4/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="giac
")
```

output
```
integrate((f*x^2 + e)/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx^2}{x^4 (a+bx^2)^{3/2} \sqrt{c+dx^2}}\,dx = \int \frac{f\,x^2+e}{x^4\,(b\,x^2+a)^{3/2}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int((e + f*x^2)/(x^4*(a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int((e + f*x^2)/(x^4*(a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{e+fx^2}{x^4 (a+bx^2)^{3/2} \sqrt{c+dx^2}}\,dx = \frac{-\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,e-3\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,f\,x^2-3\left(\int \frac{\sqrt{d\,x}}{b^2 d\,x^6+2abd\,x^4+}\right.}{}$$

input
```
int((f*x^2+e)/x^4/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x)
```

output

```
( - sqrt(c + d*x**2)*sqrt(a + b*x**2)*e - 3*sqrt(c + d*x**2)*sqrt(a + b*x*
*2)*f*x**2 - 3*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a**2*c + a**2
*d*x**2 + 2*a*b*c*x**2 + 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a*b*
d*f*x**3 - 3*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a**2*c + a**2*d
*x**2 + 2*a*b*c*x**2 + 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*b**2*d
*f*x**5 - 2*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a**2*c*x**2 + a**2*d*
x**4 + 2*a*b*c*x**4 + 2*a*b*d*x**6 + b**2*c*x**6 + b**2*d*x**8),x)*a**2*d*
e*x**3 - 4*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a**2*c*x**2 + a**2*d*x
**4 + 2*a*b*c*x**4 + 2*a*b*d*x**6 + b**2*c*x**6 + b**2*d*x**8),x)*a*b*c*e*
x**3 - 2*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a**2*c*x**2 + a**2*d*x**
4 + 2*a*b*c*x**4 + 2*a*b*d*x**6 + b**2*c*x**6 + b**2*d*x**8),x)*a*b*d*e*x*
*5 - 4*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a**2*c*x**2 + a**2*d*x**4
+ 2*a*b*c*x**4 + 2*a*b*d*x**6 + b**2*c*x**6 + b**2*d*x**8),x)*b**2*c*e*x**
5 - 6*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a**2*c + a**2*d*x**2 + 2*a*
b*c*x**2 + 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a*b*c*f*x**3 - 3*i
nt((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a**2*c + a**2*d*x**2 + 2*a*b*c*x**
2 + 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a*b*d*e*x**3 - 6*int((sqr
t(c + d*x**2)*sqrt(a + b*x**2))/(a**2*c + a**2*d*x**2 + 2*a*b*c*x**2 + 2*a
*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*b**2*c*f*x**5 - 3*int((sqrt(c +
d*x**2)*sqrt(a + b*x**2))/(a**2*c + a**2*d*x**2 + 2*a*b*c*x**2 + 2*a*b*...
```

3.246 $\int \frac{e+fx^2}{x^6(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 458

$$\int \frac{e+fx^2}{x^6(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = -\frac{e\sqrt{c+dx^2}}{5acx^5\sqrt{a+bx^2}} + \frac{(6bce+4ade-5acf)\sqrt{c+dx^2}}{15a^2c^2x^3\sqrt{a+bx^2}}$$
$$+ \frac{(15abcde-2(2bc+ad)(6bce+4ade-5acf))\sqrt{c+dx^2}}{15a^3c^3x\sqrt{a+bx^2}}$$
$$- \frac{\sqrt{b}(48b^3c^3e-3a^2bcd(3de-5cf)-2a^3d^2(4de-5cf)-8ab^2c^2(2de+5cf))\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1\right.}{15a^{7/2}c^3(bc-ad)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$
$$+ \frac{\sqrt{b}d(24b^2c^2e-a^2d(4de-5cf)-5abc(de+4cf))\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{15a^{5/2}c^3(bc-ad)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
-1/5*e*(d*x^2+c)^(1/2)/a/c/x^5/(b*x^2+a)^(1/2)+1/15*(-5*a*c*f+4*a*d*e+6*b*
c*e)*(d*x^2+c)^(1/2)/a^2/c^2/x^3/(b*x^2+a)^(1/2)+1/15*(15*a*b*c*d*e-2*(a*d
+2*b*c)*(-5*a*c*f+4*a*d*e+6*b*c*e))*(d*x^2+c)^(1/2)/a^3/c^3/x/(b*x^2+a)^(1
/2)-1/15*b^(1/2)*(48*b^3*c^3*e-3*a^2*b*c*d*(-5*c*f+3*d*e)-2*a^3*d^2*(-5*c*
f+4*d*e)-8*a*b^2*c^2*(5*c*f+2*d*e))*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^
(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/a^(7/2)/c^3/(-a*d+b*c)/(b*x^2+a
)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)+1/15*b^(1/2)*d*(24*b^2*c^2*e-a^2*d
*(-5*c*f+4*d*e)-5*a*b*c*(4*c*f+d*e))*(d*x^2+c)^(1/2)*InverseJacobiAM(arcta
n(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/a^(5/2)/c^3/(-a*d+b*c)/(b*x^2+a)^(
1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.79 (sec) , antiderivative size = 511, normalized size of antiderivative = 1.12

$$\int \frac{e+fx^2}{x^6 (a+bx^2)^{3/2} \sqrt{c+dx^2}}\,dx = \frac{-\sqrt{\frac{b}{a}}(c+dx^2)\left(-48b^4c^3ex^6+8ab^3c^2x^4(-3ce+2dex^2+5cfx^2)+a^4d(8\right.}{}$$

input
```
Integrate[(e + f*x^2)/(x^6*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

output
```
(-(Sqrt[b/a]*(c + d*x^2)*(-48*b^4*c^3*e*x^6 + 8*a*b^3*c^2*x^4*(-3*c*e + 2*
d*e*x^2 + 5*c*f*x^2) + a^4*d*(8*d^2*e*x^4 - 2*c*d*x^2*(2*e + 5*f*x^2) + c^
2*(3*e + 5*f*x^2)) - a^3*b*(-8*d^3*e*x^6 - 5*c*d^2*x^4*(e - 2*f*x^2) + 2*c
^2*d*x^2*(e + 5*f*x^2) + c^3*(3*e + 5*f*x^2)) + a^2*b^2*c*x^2*(9*d^2*e*x^4
 + c*d*x^2*(11*e - 15*f*x^2) + c^2*(6*e + 20*f*x^2)))) + I*b*c*(48*b^3*c^3
*e + 2*a^3*d^2*(-4*d*e + 5*c*f) + 3*a^2*b*c*d*(-3*d*e + 5*c*f) - 8*a*b^2*c
^2*(2*d*e + 5*c*f))*x^5*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[
I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*b*c*(b*c - a*d)*(48*b^2*c^2*e + 8
*a*b*c*(d*e - 5*c*f) + a^2*d*(4*d*e - 5*c*f))*x^5*Sqrt[1 + (b*x^2)/a]*Sqrt
[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(15*a^4*Sq
rt[b/a]*c^3*(-(b*c) + a*d)*x^5*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 609, normalized size of antiderivative = 1.33, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {441, 25, 445, 445, 445, 25, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx^2}{x^6\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \ 441$$

$$\frac{\sqrt{c+dx^2}(be-af)}{ax^5\sqrt{a+bx^2}(bc-ad)}-\frac{\int -\frac{5d(be-af)x^2+6bce-ade-5acf}{x^6\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{a(bc-ad)}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{5d(be-af)x^2+6bce-ade-5acf}{x^6\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{a(bc-ad)}+\frac{\sqrt{c+dx^2}(be-af)}{ax^5\sqrt{a+bx^2}(bc-ad)}$$

$$\Big\downarrow \ 445$$

$$\frac{-\frac{\int \frac{-d(4de-5cf)a^2-5bc(de+4cf)a+3bd(6bce-ade-5acf)x^2+24b^2c^2e}{x^4\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{5ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(-5acf-ade+6bce)}{5acx^5}}{a(bc-ad)}+\frac{\sqrt{c+dx^2}(be-af)}{ax^5\sqrt{a+bx^2}(bc-ad)}$$

$$\Big\downarrow \ 445$$

$$\frac{-\frac{-\frac{\int \frac{-2d^2(4de-5cf)a^3-3bcd(3de-5cf)a^2-8b^2c^2(2de+5cf)a+bd\left(-d(4de-5cf)a^2-5bc(de+4cf)a+24b^2c^2e\right)x^2+48b^3c^3e}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{24b^2ce}{a}-\frac{ad\cdots}{\cdots}\right)}{3x^3\cdots}}{5ac}\cdots}{a(bc-ad)}\cdots\frac{\sqrt{c+dx^2}(be-af)}{ax^5\sqrt{a+bx^2}(bc-ad)}$$

$$\Big\downarrow \ 445$$

$$-\frac{-\frac{-\frac{\frac{\int -\frac{bd\left(\left(-2d^2(4de-5cf)a^3-3bcd(3de-5cf)a^2-8b^2c^2(2de+5cf)a+48b^3c^3e\right)x^2+ac\left(-d(4de-5cf)a^2-5bc(de+4cf)a+24b^2c^2e\right)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(-\cdots\right.}{\cdots}}{3ac}}{5ac}}{a\cdots}$$

$$\frac{\sqrt{c+dx^2}(be-af)}{ax^5\sqrt{a+bx^2}(bc-ad)}$$

$\big\downarrow$ 25

$$-\frac{-\frac{\frac{\frac{\int \frac{bd\left(\left(-2d^2(4de-5cf)a^3-3bcd(3de-5cf)a^2-8b^2c^2(2de+5cf)a+48b^3c^3e\right)x^2+ac\left(-d(4de-5cf)a^2-5bc(de+4cf)a+24b^2c^2e\right)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(-2a^3\cdots\right.}{\cdots}}{3ac}}{5ac}}{a(\cdots}$$

$$\frac{\sqrt{c+dx^2}(be-af)}{ax^5\sqrt{a+bx^2}(bc-ad)}$$

$\big\downarrow$ 27

$$-\frac{-\frac{\frac{\frac{bd\int \frac{\left(-2d^2(4de-5cf)a^3-3bcd(3de-5cf)a^2-8b^2c^2(2de+5cf)a+48b^3c^3e\right)x^2+ac\left(-d(4de-5cf)a^2-5bc(de+4cf)a+24b^2c^2e\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(-2a^3d\cdots\right.}{\cdots}}{3ac}}{5ac}}{a(b\cdots}$$

$$\frac{\sqrt{c+dx^2}(be-af)}{ax^5\sqrt{a+bx^2}(bc-ad)}$$

$\big\downarrow$ 406

$$-\frac{-\frac{\frac{bd\left(ac\left(a^2(-d)(4de-5cf)-5abc(4cf+de)+24b^2c^2e\right)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\left(-2a^3d^2(4de-5cf)-3a^2bcd(3de-5cf)-8ab^2c^2(5cf+2de)+48b^3c^3e\right)\int \frac{\cdots}{\sqrt{bx^2+\cdots}}\cdots\right.}{ac}\cdots}{3ac}\cdots}{\cdots}$$

$$\frac{\sqrt{c+dx^2}(be-af)}{ax^5\sqrt{a+bx^2}(bc-ad)}$$

$\big\downarrow$ 320

$$-\frac{-\frac{bd\left(\left(-2a^3d^2(4de-5cf)-3a^2bcd(3de-5cf)-8ab^2c^2(5cf+2de)+48b^3c^3e\right)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}\left(a^2(-d)(4de-5cf)-5abc(4cf+de)+24b^2c^2e\right.}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right.}{ac}}{3ac}$$

$$\frac{\sqrt{c+dx^2}(be-af)}{ax^5\sqrt{a+bx^2}(bc-ad)}$$

$\big\downarrow$ 388

$$-\frac{-\frac{bd\left(\left(-2a^3d^2(4de-5cf)-3a^2bcd(3de-5cf)-8ab^2c^2(5cf+2de)+48b^3c^3e\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}\left(a^2(-d)(4de-5cf)-5abc(4cf\right.}{\sqrt{d}\sqrt{c+dx}}\right.}{ac}}{3ac}$$

$$\frac{\sqrt{c+dx^2}(be-af)}{ax^5\sqrt{a+bx^2}(bc-ad)}$$

$\big\downarrow$ 313

$$-\frac{-\frac{bd\left(\frac{c^{3/2}\sqrt{a+bx^2}\left(a^2(-d)(4de-5cf)-5abc(4cf+de)+24b^2c^2e\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+\left(-2a^3d^2(4de-5cf)-3a^2bcd(3de-5cf)-8ab^2c^2(5cf+2d\right.\right.}{ac}}{}$$

$$\frac{\sqrt{c+dx^2}(be-af)}{ax^5\sqrt{a+bx^2}(bc-ad)}$$

input

```
Int[(e + f*x^2)/(x^6*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

output
```
((b*e - a*f)*Sqrt[c + d*x^2])/(a*(b*c - a*d)*x^5*Sqrt[a + b*x^2]) + (-1/5*
((6*b*c*e - a*d*e - 5*a*c*f)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(a*c*x^5) -
(-1/3*(((24*b^2*c*e)/a - (a*d*(4*d*e - 5*c*f))/c - 5*b*(d*e + 4*c*f))*Sqrt
[a + b*x^2]*Sqrt[c + d*x^2])/x^3 - (-(((48*b^3*c^3*e - 3*a^2*b*c*d*(3*d*e
- 5*c*f) - 2*a^3*d^2*(4*d*e - 5*c*f) - 8*a*b^2*c^2*(2*d*e + 5*c*f))*Sqrt[a
 + b*x^2]*Sqrt[c + d*x^2])/(a*c*x)) + (b*d*((48*b^3*c^3*e - 3*a^2*b*c*d*(3
*d*e - 5*c*f) - 2*a^3*d^2*(4*d*e - 5*c*f) - 8*a*b^2*c^2*(2*d*e + 5*c*f))*(
(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*Ellipti
cE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a +
b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (c^(3/2)*(24*b^2*c^2*e - a^2*
d*(4*d*e - 5*c*f) - 5*a*b*c*(d*e + 4*c*f))*Sqrt[a + b*x^2]*EllipticF[ArcTa
n[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a
*(c + d*x^2))]*Sqrt[c + d*x^2])))/(a*c))/(3*a*c))/(5*a*c))/(a*(b*c - a*d))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 441
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
+ b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(a*g*2*(b*c - a*d)*(p + 1))), x] + Si
mp[1/(a*2*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2
)^q*Simp[c*(b*e - a*f)*(m + 1) + e*2*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(m
 + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, q},
 x] && LtQ[p, -1]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 20.82 (sec) , antiderivative size = 617, normalized size of antiderivative = 1.35

method	result
elliptic	$\frac{\sqrt{(b x^2+a)(x^2 d+c)}\left(-\frac{e\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{5a^2c\,x^5}-\frac{(5acf-4ade-9bce)\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{15a^3c^2x^3}+\frac{(10a^2cfd-8a^2d^2e+25ab\,c^2f-17abcde-33}{15a^4c^3x}\right.}{}$
risch	$-\frac{\sqrt{b\,x^2+a}\,\sqrt{x^2d+c}\,(-10a^2cdf\,x^4+8a^2d^2e\,x^4-25ab\,c^2f\,x^4+17abcde\,x^4+33b^2c^2e\,x^4+5a^2c^2f\,x^2-4a^2cde\,x^2-9b\,c^2e\,x^2a+3a^2c^2e)}{15a^4c^3x^5}$
default	Expression too large to display

input
```
int((f*x^2+e)/x^6/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-1/5/a^2/c*e*
(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/x^5-1/15/a^3/c^2*(5*a*c*f-4*a*d*e-9*b*
c*e)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/x^3+1/15/a^4/c^3*(10*a^2*c*d*f-8*
a^2*d^2*e+25*a*b*c^2*f-17*a*b*c*d*e-33*b^2*c^2*e)*(b*d*x^4+a*d*x^2+b*c*x^2
+a*c)^(1/2)/x-(b*d*x^2+b*c)*b^2/a^4/(a*d-b*c)*x*(a*f-b*e)/((x^2+a/b)*(b*d*
x^2+b*c))^(1/2)+(-1/15*b*d*(5*a*c*f-4*a*d*e-9*b*c*e)/a^3/c^2+b^2*(a*f-b*e)
/a^4+b^3*c/a^4/(a*d-b*c)*(a*f-b*e))/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^
2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-
1+(a*d+b*c)/c/b)^(1/2))-(-1/15*b*d*(10*a^2*c*d*f-8*a^2*d^2*e+25*a*b*c^2*f-
17*a*b*c*d*e-33*b^2*c^2*e)/a^4/c^3+b^3*d*(a*f-b*e)/(a*d-b*c)/a^4)*c/(-b/a)
^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(
1/2)/d*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b
/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 833, normalized size of antiderivative = 1.82

$$\int \frac{e+fx^2}{x^6\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}\,dx = \text{Too large to display}$$

input
```
integrate((f*x^2+e)/x^6/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="fric
as")
```

output
```
1/15*((((48*b^5*c^3 - 16*a*b^4*c^2*d - 9*a^2*b^3*c*d^2 - 8*a^3*b^2*d^3)*e
- 5*(8*a*b^4*c^3 - 3*a^2*b^3*c^2*d - 2*a^3*b^2*c*d^2)*f)*x^7 + ((48*a*b^4*
c^3 - 16*a^2*b^3*c^2*d - 9*a^3*b^2*c*d^2 - 8*a^4*b*d^3)*e - 5*(8*a^2*b^3*c
^3 - 3*a^3*b^2*c^2*d - 2*a^4*b*c*d^2)*f)*x^5)*sqrt(a*c)*sqrt(-b/a)*ellipti
c_e(arcsin(x*sqrt(-b/a)), a*d/(b*c)) - (((48*b^5*c^3 + 8*(3*a^2*b^3 - 2*a*
b^4)*c^2*d - (5*a^3*b^2 + 9*a^2*b^3)*c*d^2 - 4*(a^4*b + 2*a^3*b^2)*d^3)*e
- 5*(8*a*b^4*c^3 + (4*a^3*b^2 - 3*a^2*b^3)*c^2*d - (a^4*b + 2*a^3*b^2)*c*d
^2)*f)*x^7 + ((48*a*b^4*c^3 + 8*(3*a^3*b^2 - 2*a^2*b^3)*c^2*d - (5*a^4*b +
 9*a^3*b^2)*c*d^2 - 4*(a^5 + 2*a^4*b)*d^3)*e - 5*(8*a^2*b^3*c^3 + (4*a^4*b
 - 3*a^3*b^2)*c^2*d - (a^5 + 2*a^4*b)*c*d^2)*f)*x^5)*sqrt(a*c)*sqrt(-b/a)*
elliptic_f(arcsin(x*sqrt(-b/a)), a*d/(b*c)) - (((48*a*b^4*c^3 - 16*a^2*b^3
*c^2*d - 9*a^3*b^2*c*d^2 - 8*a^4*b*d^3)*e - 5*(8*a^2*b^3*c^3 - 3*a^3*b^2*c
^2*d - 2*a^4*b*c*d^2)*f)*x^6 + ((24*a^2*b^3*c^3 - 11*a^3*b^2*c^2*d - 5*a^4
*b*c*d^2 - 8*a^5*d^3)*e - 10*(2*a^3*b^2*c^3 - a^4*b*c^2*d - a^5*c*d^2)*f)*
x^4 - (2*(3*a^3*b^2*c^3 - a^4*b*c^2*d - 2*a^5*c*d^2)*e - 5*(a^4*b*c^3 - a^
5*c^2*d)*f)*x^2 + 3*(a^4*b*c^3 - a^5*c^2*d)*e)*sqrt(b*x^2 + a)*sqrt(d*x^2
+ c))/((a^5*b^2*c^4 - a^6*b*c^3*d)*x^7 + (a^6*b*c^4 - a^7*c^3*d)*x^5)
```

Sympy [F]

$$\int \frac{e+fx^2}{x^6 (a+bx^2)^{3/2} \sqrt{c+dx^2}}\,dx = \int \frac{e+fx^2}{x^6 (a+bx^2)^{\frac{3}{2}} \sqrt{c+dx^2}}\,dx$$

input
```
integrate((f*x**2+e)/x**6/(b*x**2+a)**(3/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral((e + f*x**2)/(x**6*(a + b*x**2)**(3/2)*sqrt(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{e+fx^2}{x^6 (a+bx^2)^{3/2} \sqrt{c+dx^2}}\,dx = \int \frac{fx^2+e}{(bx^2+a)^{\frac{3}{2}} \sqrt{dx^2+c}\,x^6}\,dx$$

input
```
integrate((f*x^2+e)/x^6/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate((f*x^2 + e)/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*x^6), x)
```

Giac [F]

$$\int \frac{e+fx^2}{x^6(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx=\int \frac{fx^2+e}{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}x^6}\,dx$$

input
```
integrate((f*x^2+e)/x^6/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="giac
")
```

output
```
integrate((f*x^2 + e)/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx^2}{x^6(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx=\int \frac{f\,x^2+e}{x^6\,(b\,x^2+a)^{3/2}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int((e + f*x^2)/(x^6*(a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int((e + f*x^2)/(x^6*(a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{e+fx^2}{x^6(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx=\text{Too large to display}$$

input
```
int((f*x^2+e)/x^6/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x)
```

output
```
( - 3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*e - 5*sqrt(c + d*x**2)*sqrt(a + b*
x**2)*f*x**2 - 12*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a**2*c*x**4 + a
**2*d*x**6 + 2*a*b*c*x**6 + 2*a*b*d*x**8 + b**2*c*x**8 + b**2*d*x**10),x)*
a**2*d*e*x**5 - 18*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a**2*c*x**4 +
a**2*d*x**6 + 2*a*b*c*x**6 + 2*a*b*d*x**8 + b**2*c*x**8 + b**2*d*x**10),x)
*a*b*c*e*x**5 - 12*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a**2*c*x**4 +
a**2*d*x**6 + 2*a*b*c*x**6 + 2*a*b*d*x**8 + b**2*c*x**8 + b**2*d*x**10),x)
*a*b*d*e*x**7 - 18*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a**2*c*x**4 +
a**2*d*x**6 + 2*a*b*c*x**6 + 2*a*b*d*x**8 + b**2*c*x**8 + b**2*d*x**10),x)
*b**2*c*e*x**7 - 10*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a**2*c*x**2 +
 a**2*d*x**4 + 2*a*b*c*x**4 + 2*a*b*d*x**6 + b**2*c*x**6 + b**2*d*x**8),x)
*a**2*d*f*x**5 - 20*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a**2*c*x**2 +
 a**2*d*x**4 + 2*a*b*c*x**4 + 2*a*b*d*x**6 + b**2*c*x**6 + b**2*d*x**8),x)
*a*b*c*f*x**5 - 15*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a**2*c*x**2 +
a**2*d*x**4 + 2*a*b*c*x**4 + 2*a*b*d*x**6 + b**2*c*x**6 + b**2*d*x**8),x)*
a*b*d*e*x**5 - 10*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a**2*c*x**2 + a
**2*d*x**4 + 2*a*b*c*x**4 + 2*a*b*d*x**6 + b**2*c*x**6 + b**2*d*x**8),x)*a
*b*d*f*x**7 - 20*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a**2*c*x**2 + a*
*2*d*x**4 + 2*a*b*c*x**4 + 2*a*b*d*x**6 + b**2*c*x**6 + b**2*d*x**8),x)*b*
*2*c*f*x**7 - 15*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a**2*c*x**2 +...
```

3.247 $\int \frac{x^{10}}{(a+bx^2)^{3/2}\sqrt{c+dx^2}(e+fx^2)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 788

$$\int \frac{x^{10}}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \frac{\left(10bce+20ade+\frac{15bde^2}{f}+13acf+\frac{8bc^2f}{d}+\frac{24a^2df}{b}\right)x\sqrt{c+dx^2}}{15b^2d^2f^2\sqrt{a+bx^2}}$$

$$-\frac{(5bde+4bcf+6adf)x^3\sqrt{c+dx^2}}{15b^2d^2f^2\sqrt{a+bx^2}}+\frac{x^5\sqrt{c+dx^2}}{5bdf\sqrt{a+bx^2}}$$

$$-\frac{\sqrt{a}(48a^4d^3f^3-8a^3bd^2f^2(de+2cf)+b^4ce(15d^2e^2+10cdef+8c^2f^2)-a^2b^2df(10d^2e^2-cdef+9c^2f^2)}{15b^{7/2}d^3(bc-ad)f^3(be-af)\sqrt{a+bx^2}\sqrt{}}$$

$$-\frac{a^{3/2}(24a^4cd^2f^4-a^3bcdf^3(28de+5cf)-a^2b^2cf^2(d^2e^2-5cdef+4c^2f^2)-b^4ce^2(15d^2e^2+5cdef+4c^2f}{15b^{7/2}cd^2(bc-ad)f^3(be-af)^2}$$

$$-\frac{a^{3/2}e^4\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}cf^3(be-af)^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
1/15*(10*b*c*e+20*a*d*e+15*b*d*e^2/f+13*a*c*f+8*b*c^2*f/d+24*a^2*d*f/b)*x*
(d*x^2+c)^(1/2)/b^2/d^2/f^2/(b*x^2+a)^(1/2)-1/15*(6*a*d*f+4*b*c*f+5*b*d*e)
*x^3*(d*x^2+c)^(1/2)/b^2/d^2/f^2/(b*x^2+a)^(1/2)+1/5*x^5*(d*x^2+c)^(1/2)/b
/d/f/(b*x^2+a)^(1/2)-1/15*a^(1/2)*(48*a^4*d^3*f^3-8*a^3*b*d^2*f^2*(2*c*f+d
*e)+b^4*c*e*(8*c^2*f^2+10*c*d*e*f+15*d^2*e^2)-a^2*b^2*d*f*(9*c^2*f^2-c*d*e
*f+10*d^2*e^2)-a*b^3*(8*c^3*f^3+c^2*d*e*f^2+15*d^3*e^3))*(d*x^2+c)^(1/2)*E
llipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/b^(7/2)/d^
3/(-a*d+b*c)/f^3/(-a*f+b*e)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2
)-1/15*a^(3/2)*(24*a^4*c*d^2*f^4-a^3*b*c*d*f^3*(5*c*f+28*d*e)-a^2*b^2*c*f^
2*(4*c^2*f^2-5*c*d*e*f+d^2*e^2)-b^4*c*e^2*(4*c^2*f^2+5*c*d*e*f+15*d^2*e^2)
+a*b^3*e*(8*c^3*f^3+5*c^2*d*e*f^2+5*c*d^2*e^2*f+15*d^3*e^3))*(d*x^2+c)^(1/
2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/b^(7/2)/c/
d^2/(-a*d+b*c)/f^3/(-a*f+b*e)^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^
(1/2)-a^(3/2)*e^4*(d*x^2+c)^(1/2)*EllipticPi(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)
^(1/2),1-a*f/b/e,(1-a*d/b/c)^(1/2))/b^(1/2)/c/f^3/(-a*f+b*e)^2/(b*x^2+a)^(
1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 7.78 (sec) , antiderivative size = 701, normalized size of antiderivative = 0.89

$$\int \frac{x^{10}}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \frac{\sqrt{\frac{b}{a}}df^2x(c+dx^2)\left(-24a^4d^2f^2+a^3bdf(4de+5cf-6dfx^2)+b^4ce\right.}{}$$

input
```
Integrate[x^10/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]*(e + f*x^2)),x]
```

output
```
(Sqrt[b/a]*d*f^2*x*(c + d*x^2)*(-24*a^4*d^2*f^2 + a^3*b*d*f*(4*d*e + 5*c*f
 - 6*d*f*x^2) + b^4*c*e*x^2*(-5*d*e - 4*c*f + 3*d*f*x^2) + a*b^3*(d^2*e*x^
2*(5*e - 3*f*x^2) + 4*c^2*f*(-e + f*x^2) + c*d*(-5*e^2 + 3*e*f*x^2 - 3*f^2
*x^4)) + a^2*b^2*(4*c^2*f^2 + 2*c*d*f^2*x^2 + d^2*(5*e^2 + e*f*x^2 + 3*f^2
*x^4))) - I*c*f*(48*a^4*d^3*f^3 - 8*a^3*b*d^2*f^2*(d*e + 2*c*f) + a^2*b^2*
d*f*(-10*d^2*e^2 + c*d*e*f - 9*c^2*f^2) + b^4*c*e*(15*d^2*e^2 + 10*c*d*e*f
 + 8*c^2*f^2) - a*b^3*(15*d^3*e^3 + c^2*d*e*f^2 + 8*c^3*f^3))*Sqrt[1 + (b*
x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]
 + I*(-(b*c) + a*d)*(24*a^3*c*d^2*f^4 + a^2*b*c*d*f^3*(-4*d*e + 13*c*f) +
a*b^2*c*f^2*(-5*d^2*e^2 - 3*c*d*e*f + 8*c^2*f^2) - b^3*e*(15*d^3*e^3 + 15*
c*d^2*e^2*f + 10*c^2*d*e*f^2 + 8*c^3*f^3))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d
*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (15*I)*b^3*d^3*(
-(b*c) + a*d)*e^4*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)
/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(15*b^3*Sqrt[b/a]*d^3*(b*c -
 a*d)*f^4*(b*e - a*f)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{10}}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx$$
$$\Big\downarrow \; 450$$
$$\int \frac{x^{10}}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}dx$$

input
```
Int[x^10/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]*(e + f*x^2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [A] (verified)

Time = 25.52 (sec) , antiderivative size = 897, normalized size of antiderivative = 1.14

method	result
risch	$-\frac{x\left(-3bdf\,x^2+9adf+4bcf+5bde\right)\sqrt{b\,x^2+a}\,\sqrt{x^2d+c}}{15b^3d^2f^2}+\frac{\left(-\frac{\left(33a^2d^2f^2+17abcd\,f^2+25ab\,d^2ef+8b^2c^2f^2+10b^2cdef+15b^2d^2e^2\right)c\sqrt{1+\frac{b\,x}{a}}}{f\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x}}\right.}{}$
elliptic	Expression too large to display
default	Expression too large to display

input
```
int(x^10/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x,method=_RETURNVERBOSE
)
```

output
```
-1/15*x*(-3*b*d*f*x^2+9*a*d*f+4*b*c*f+5*b*d*e)*(b*x^2+a)^(1/2)*(d*x^2+c)^(
1/2)/b^3/d^2/f^2+1/15/d^2/f^2/b^3*(-1/f*(33*a^2*d^2*f^2+17*a*b*c*d*f^2+25*
a*b*d^2*e*f+8*b^2*c^2*f^2+10*b^2*c*d*e*f+15*b^2*d^2*e^2)*c/(-b/a)^(1/2)*(1
+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*(E
llipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2)
,(-1+(a*d+b*c)/c/b)^(1/2)))-(15*a^3*d^2*f^3-9*a^2*b*c*d*f^3+15*a^2*b*d^2*e
*f^2-4*a*b^2*c^2*f^3-5*a*b^2*c*d*e*f^2+15*a*b^2*d^2*e^2*f+15*b^3*d^2*e^3)/
f^2/b/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*
c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+15*a^5
*f^2*d^2/(a*f-b*e)/b*(-(b*d*x^2+b*c)/a/(a*d-b*c)*x/((x^2+a/b)*(b*d*x^2+b*c
))^(1/2)+(1/a+b*c/(a*d-b*c)/a)/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^
(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*
d+b*c)/c/b)^(1/2))-b/(a*d-b*c)/a*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2
/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(EllipticF(x*(-b/a)^(1/2),(-
1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))
))-15*b^3*d^2*e^4/f^2/(a*f-b*e)/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)
^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b
/e,(-1/c*d)^(1/2)/(-b/a)^(1/2)))*((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/
2)/(d*x^2+c)^(1/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^{10}}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate(x^10/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="fri
cas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^{10}}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx=\int \frac{x^{10}}{(a+bx^2)^{\frac{3}{2}}\sqrt{c+dx^2}\,(e+fx^2)}\,dx$$

input
```
integrate(x**10/(b*x**2+a)**(3/2)/(d*x**2+c)**(1/2)/(f*x**2+e),x)
```

output
```
Integral(x**10/((a + b*x**2)**(3/2)*sqrt(c + d*x**2)*(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{x^{10}}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx=\int \frac{x^{10}}{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}(fx^2+e)}\,dx$$

input
```
integrate(x^10/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="max
ima")
```

output
```
integrate(x^10/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*(f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{x^{10}}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx=\int \frac{x^{10}}{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}(fx^2+e)}\,dx$$

input
```
integrate(x^10/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="gia
c")
```

output
```
integrate(x^10/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{10}}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx=\int \frac{x^{10}}{(b\,x^2+a)^{3/2}\,\sqrt{d\,x^2+c}\,(f\,x^2+e)}\,dx$$

input
```
int(x^10/((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)*(e + f*x^2)),x)
```

output
```
int(x^10/((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)*(e + f*x^2)), x)
```

Reduce [F]

$$\int \frac{x^{10}}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx=\int \frac{x^{10}}{(b\,x^2+a)^{\frac{3}{2}}\,\sqrt{d\,x^2+c}\,(f\,x^2+e)}dx$$

input
```
int(x^10/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x)
```

output
```
int(x^10/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x)
```

3.248 $\int \frac{x^8}{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}\left(e+fx^2\right)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 576

$$\int \frac{x^8}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = -\frac{(3bde+2bcf+4adf)x\sqrt{c+dx^2}}{3b^2d^2f^2\sqrt{a+bx^2}}+\frac{x^3\sqrt{c+dx^2}}{3bdf\sqrt{a+bx^2}}$$
$$+\frac{\sqrt{a}(8a^3d^2f^2+b^3ce(3de+2cf)-a^2bdf(2de+3cf)-ab^2(3d^2e^2+2c^2f^2))\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\right.}{3b^{5/2}d^2(bc-ad)f^2(be-af)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$
$$+\frac{a^{3/2}(4a^3cdf^3-b^3ce^2(3de+cf)-a^2bcf^2(5de+cf)+ab^2e(3d^2e^2+cdef+2c^2f^2))\sqrt{c+dx^2}\,\text{EllipticF}\left(\right.}{3b^{5/2}cd(bc-ad)f^2(be-af)^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$
$$+\frac{a^{3/2}e^3\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}cf^2(be-af)^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
-1/3*(4*a*d*f+2*b*c*f+3*b*d*e)*x*(d*x^2+c)^(1/2)/b^2/d^2/f^2/(b*x^2+a)^(1/
2)+1/3*x^3*(d*x^2+c)^(1/2)/b/d/f/(b*x^2+a)^(1/2)+1/3*a^(1/2)*(8*a^3*d^2*f^
2+b^3*c*e*(2*c*f+3*d*e)-a^2*b*d*f*(3*c*f+2*d*e)-a*b^2*(2*c^2*f^2+3*d^2*e^2
))*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/
c)^(1/2))/b^(5/2)/d^2/(-a*d+b*c)/f^2/(-a*f+b*e)/(b*x^2+a)^(1/2)/(a*(d*x^2+
c)/c/(b*x^2+a))^(1/2)+1/3*a^(3/2)*(4*a^3*c*d*f^3-b^3*c*e^2*(c*f+3*d*e)-a^2
*b*c*f^2*(c*f+5*d*e)+a*b^2*e*(2*c^2*f^2+c*d*e*f+3*d^2*e^2))*(d*x^2+c)^(1/2
)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/b^(5/2)/c/d
/(-a*d+b*c)/f^2/(-a*f+b*e)^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/
2)+a^(3/2)*e^3*(d*x^2+c)^(1/2)*EllipticPi(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1
/2),1-a*f/b/e,(1-a*d/b/c)^(1/2))/b^(1/2)/c/f^2/(-a*f+b*e)^2/(b*x^2+a)^(1/2
)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.05 (sec) , antiderivative size = 480, normalized size of antiderivative = 0.83

$$\int \frac{x^8}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \frac{icf(8a^3d^2f^2+b^3ce(3de+2cf)-a^2bdf(2de+3cf)-ab^2(3d^2e^2+}{}$$

input
```
Integrate[x^8/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]*(e + f*x^2)),x]
```

output
```
(I*c*f*(8*a^3*d^2*f^2 + b^3*c*e*(3*d*e + 2*c*f) - a^2*b*d*f*(2*d*e + 3*c*f
) - a*b^2*(3*d^2*e^2 + 2*c^2*f^2))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]
*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*(-(b*c) + a*d)*(4*a^2*
c*d*f^3 + a*b*c*f^2*(-(d*e) + 2*c*f) - b^2*e*(3*d^2*e^2 + 3*c*d*e*f + 2*c^
2*f^2))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b
/a]*x], (a*d)/(b*c)] + d*(Sqrt[b/a]*f^2*x*(c + d*x^2)*(4*a^3*d*f + b^3*c*e
*x^2 + a*b^2*(-(d*e*x^2) + c*(e - f*x^2)) - a^2*b*(c*f + d*(e - f*x^2))) -
 (3*I)*b^2*d*(-(b*c) + a*d)*e^3*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*El
lipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(3*a^2*(b/a)^
(5/2)*d^2*(b*c - a*d)*f^3*(b*e - a*f)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx$$

$$\Big\downarrow \; 450$$

$$\int \frac{x^8}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}dx$$

input
```
Int[x^8/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]*(e + f*x^2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [A] (verified)

Time = 23.23 (sec) , antiderivative size = 772, normalized size of antiderivative = 1.34

method	result
risch	$\frac{x\sqrt{b\,x^2+a}\,\sqrt{x^2d+c}}{3f\,b^2d} - \frac{\left(-\frac{(5adf+2bcf+3bde)c\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\left(\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)-\mathrm{EllipticE}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)\right)}{f\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}\,d} - \right.}{}$
elliptic	Expression too large to display
default	Expression too large to display

input
```
int(x^8/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
1/3/f/b^2/d*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)-1/3/d/f/b^2*(-1/f*(5*a*d*f+2
*b*c*f+3*b*d*e)*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^
4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b
)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2)))-(3*a^2*d*f^2-
a*b*c*f^2+3*a*b*d*e*f+3*b^2*d*e^2)/f^2/b/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1
+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/
2),(-1+(a*d+b*c)/c/b)^(1/2))+3*f/b*a^4*d/(a*f-b*e)*(-(b*d*x^2+b*c)/a/(a*d-
b*c)*x/((x^2+a/b)*(b*d*x^2+b*c))^(1/2)+(1/a+b*c/(a*d-b*c)/a)/(-b/a)^(1/2)*
(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*El
lipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-b/(a*d-b*c)/a*c/(-b/a)^(1
/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2
)*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(
1/2),(-1+(a*d+b*c)/c/b)^(1/2))))-3/f^2*b^2*d*e^3/(a*f-b*e)/(-b/a)^(1/2)*(1
+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*Elli
pticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2)))*((b*x^2+a)*(d*
x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^8}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate(x^8/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^8}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{x^8}{(a+bx^2)^{\frac{3}{2}}\sqrt{c+dx^2}\,(e+fx^2)}\,dx$$

input
```
integrate(x**8/(b*x**2+a)**(3/2)/(d*x**2+c)**(1/2)/(f*x**2+e),x)
```

output
```
Integral(x**8/((a + b*x**2)**(3/2)*sqrt(c + d*x**2)*(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{x^8}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{x^8}{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}(fx^2+e)}\,dx$$

input
```
integrate(x^8/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="maxi
ma")
```

output
```
integrate(x^8/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*(f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{x^8}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{x^8}{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}(fx^2+e)}\,dx$$

input
```
integrate(x^8/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="giac
")
```

output
```
integrate(x^8/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^8}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{x^8}{(b\,x^2+a)^{3/2}\,\sqrt{d\,x^2+c}\,(f\,x^2+e)}\,dx$$

input
```
int(x^8/((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)*(e + f*x^2)),x)
```

output
```
int(x^8/((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)*(e + f*x^2)), x)
```

Reduce [F]

$$\int \frac{x^8}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \text{too large to display}$$

input
```
int(x^8/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x)
```

output
```
( - 3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*c*f*x - 4*sqrt(c + d*x**2)*sqrt(
a + b*x**2)*a*d*e*x + 2*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*d*f*x**3 - 2*s
qrt(c + d*x**2)*sqrt(a + b*x**2)*b*c*e*x + sqrt(c + d*x**2)*sqrt(a + b*x**
2)*b*d*e*x**3 - 16*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**6)/(2*a**3*c*
e*f + 2*a**3*c*f**2*x**2 + 2*a**3*d*e*f*x**2 + 2*a**3*d*f**2*x**4 + a**2*b
*c*e**2 + 5*a**2*b*c*e*f*x**2 + 4*a**2*b*c*f**2*x**4 + a**2*b*d*e**2*x**2
+ 5*a**2*b*d*e*f*x**4 + 4*a**2*b*d*f**2*x**6 + 2*a*b**2*c*e**2*x**2 + 4*a*
b**2*c*e*f*x**4 + 2*a*b**2*c*f**2*x**6 + 2*a*b**2*d*e**2*x**4 + 4*a*b**2*d
*e*f*x**6 + 2*a*b**2*d*f**2*x**8 + b**3*c*e**2*x**4 + b**3*c*e*f*x**6 + b*
*3*d*e**2*x**6 + b**3*d*e*f*x**8),x)*a**4*d**2*f**3 - 2*int((sqrt(c + d*x*
*2)*sqrt(a + b*x**2)*x**6)/(2*a**3*c*e*f + 2*a**3*c*f**2*x**2 + 2*a**3*d*e
*f*x**2 + 2*a**3*d*f**2*x**4 + a**2*b*c*e**2 + 5*a**2*b*c*e*f*x**2 + 4*a**
2*b*c*f**2*x**4 + a**2*b*d*e**2*x**2 + 5*a**2*b*d*e*f*x**4 + 4*a**2*b*d*f*
*2*x**6 + 2*a*b**2*c*e**2*x**2 + 4*a*b**2*c*e*f*x**4 + 2*a*b**2*c*f**2*x**
6 + 2*a*b**2*d*e**2*x**4 + 4*a*b**2*d*e*f*x**6 + 2*a*b**2*d*f**2*x**8 + b*
*3*c*e**2*x**4 + b**3*c*e*f*x**6 + b**3*d*e**2*x**6 + b**3*d*e*f*x**8),x)*
a**3*b*c*d*f**3 - 20*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**6)/(2*a**3*
c*e*f + 2*a**3*c*f**2*x**2 + 2*a**3*d*e*f*x**2 + 2*a**3*d*f**2*x**4 + a**2
*b*c*e**2 + 5*a**2*b*c*e*f*x**2 + 4*a**2*b*c*f**2*x**4 + a**2*b*d*e**2*x**
2 + 5*a**2*b*d*e*f*x**4 + 4*a**2*b*d*f**2*x**6 + 2*a*b**2*c*e**2*x**2 +...
```

3.249 $\int \frac{x^6}{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}\left(e+fx^2\right)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 427

$$\begin{aligned}\int \frac{x^6}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx &= \frac{x\sqrt{c+dx^2}}{bdf\sqrt{a+bx^2}} \\ &\quad - \frac{\sqrt{a}(b^2ce+2a^2df-ab(de+cf))\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{b^{3/2}d(bc-ad)f(be-af)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} \\ &\quad + \frac{a^{3/2}(b^2ce^2-a^2cf^2-abe(de-cf))\sqrt{c+dx^2}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{b^{3/2}c(bc-ad)f(be-af)^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} \\ &\quad - \frac{a^{3/2}e^2\sqrt{c+dx^2}\,\mathrm{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}cf(be-af)^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}\end{aligned}$$

output
```
x*(d*x^2+c)^(1/2)/b/d/f/(b*x^2+a)^(1/2)-a^(1/2)*(b^2*c*e+2*a^2*d*f-a*b*(c*
f+d*e))*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a
*d/b/c)^(1/2))/b^(3/2)/d/(-a*d+b*c)/f/(-a*f+b*e)/(b*x^2+a)^(1/2)/(a*(d*x^2
+c)/c/(b*x^2+a))^(1/2)+a^(3/2)*(b^2*c*e^2-a^2*c*f^2-a*b*e*(-c*f+d*e))*(d*x
^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/b
^(3/2)/c/(-a*d+b*c)/f/(-a*f+b*e)^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a
))^(1/2)-a^(3/2)*e^2*(d*x^2+c)^(1/2)*EllipticPi(b^(1/2)*x/a^(1/2)/(1+b*x^2
/a)^(1/2),1-a*f/b/e,(1-a*d/b/c)^(1/2))/b^(1/2)/c/f/(-a*f+b*e)^2/(b*x^2+a)^
(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.26 (sec) , antiderivative size = 337, normalized size of antiderivative = 0.79

$$\int \frac{x^6}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \frac{-a^2\sqrt{\frac{b}{a}}df^2x(c+dx^2)-icf(b^2ce+2a^2df-ab(de+cf))\sqrt{1+\frac{b}{}}}{}$$

input
```
Integrate[x^6/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]*(e + f*x^2)),x]
```

output
```
(-(a^2*Sqrt[b/a]*d*f^2*x*(c + d*x^2)) - I*c*f*(b^2*c*e + 2*a^2*d*f - a*b*(
d*e + c*f))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sq
rt[b/a]*x], (a*d)/(b*c)] + I*(-(b*c) + a*d)*(a*c*f^2 - b*e*(d*e + c*f))*Sq
rt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a
*d)/(b*c)] + I*b*d*(-(b*c) + a*d)*e^2*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)
/c]*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(b*Sqrt[
b/a]*d*(b*c - a*d)*f^2*(b*e - a*f)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx$$
$$\Big\downarrow \text{ 450}$$
$$\int \frac{x^6}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx$$

input
```
Int[x^6/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]*(e + f*x^2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [A] (verified)

Time = 8.68 (sec) , antiderivative size = 806, normalized size of antiderivative = 1.89

method	result
elliptic	$\frac{\sqrt{(b x^2+a)(x^2 d+c)}\left(-\frac{\left(bd\,x^2+bc\right)a^2 x}{b^2(ad-bc)(af-be)\sqrt{\left(x^2+\frac{a}{b}\right)\left(bd\,x^2+bc\right)}}-\frac{\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\text{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)a}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}\,f\,b^2}-\frac{\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\cdots}}{\sqrt{-\frac{b}{a}}\cdots}\right.}{}$
default	$\frac{\left(-\sqrt{-\frac{b}{a}}\,a^2d^2f^2x^3-\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{x^2d+c}{c}}\,\text{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)a^2cd\,f^2+\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{x^2d+c}{c}}\,\text{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)ab\,c^2f^2+\cdots\right.}{}$

input
```
int(x^6/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-(b*d*x^2+b*c
)/b^2*a^2/(a*d-b*c)*x/(a*f-b*e)/((x^2+a/b)*(b*d*x^2+b*c))^(1/2)-1/(-b/a)^(
1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/
2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))/f/b^2*a-1/(-b/a)^(1/
2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))/f^2/b*e+1/(-b/a)^(1/2)
*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*E
llipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))*a^2/b^2/(a*f-b*e)-c/(-b/
a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)
^(1/2)/d/f/b*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+c/(-b/a)^(
1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/
2)/d/f/b*EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+c/(-b/a)^(1/2)
*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*a
^2/b/(a*d-b*c)/(a*f-b*e)*EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2)
)-e^2/(a*f-b*e)/f^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*
x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^
(1/2)/(-b/a)^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^6}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate(x^6/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^6}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{x^6}{(a+bx^2)^{\frac{3}{2}}\sqrt{c+dx^2}\,(e+fx^2)}\,dx$$

input
```
integrate(x**6/(b*x**2+a)**(3/2)/(d*x**2+c)**(1/2)/(f*x**2+e),x)
```

output
```
Integral(x**6/((a + b*x**2)**(3/2)*sqrt(c + d*x**2)*(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{x^6}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{x^6}{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}(fx^2+e)}\,dx$$

input
```
integrate(x^6/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="maxi
ma")
```

output
```
integrate(x^6/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*(f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{x^6}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{x^6}{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}(fx^2+e)}\,dx$$

input
```
integrate(x^6/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="giac
")
```

output
```
integrate(x^6/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx=\int \frac{x^6}{{\left(b\,x^2+a\right)}^{3/2}\,\sqrt{d\,x^2+c}\,\left(f\,x^2+e\right)}\,dx$$

input
```
int(x^6/((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)*(e + f*x^2)),x)
```

output
```
int(x^6/((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)*(e + f*x^2)), x)
```

Reduce [F]

$$\int \frac{x^6}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx=\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,\cdots}}{b^2df\,x^8+2abdf\,x^6+b^2cf\,x^6+b^2de\,x^6+a^2df\,x^4+2abcf\,x^4+\cdots}$$

input
```
int(x^6/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**6)/(a**2*c*e + a**2*c*f*x**2 + a
**2*d*e*x**2 + a**2*d*f*x**4 + 2*a*b*c*e*x**2 + 2*a*b*c*f*x**4 + 2*a*b*d*e
*x**4 + 2*a*b*d*f*x**6 + b**2*c*e*x**4 + b**2*c*f*x**6 + b**2*d*e*x**6 + b
**2*d*f*x**8),x)
```

3.250 $\int \frac{x^4}{(a+bx^2)^{3/2}\sqrt{c+dx^2}(e+fx^2)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 339

$$\int \frac{x^4}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \frac{a^{3/2}\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{b}(bc-ad)(be-af)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$
$$-\frac{a^{3/2}(2bce-ade-acf)\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}c(bc-ad)(be-af)^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$
$$+\frac{a^{3/2}e\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}c(be-af)^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
a^(3/2)*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a
*d/b/c)^(1/2))/b^(1/2)/(-a*d+b*c)/(-a*f+b*e)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/
c/(b*x^2+a))^(1/2)-a^(3/2)*(-a*c*f-a*d*e+2*b*c*e)*(d*x^2+c)^(1/2)*InverseJ
acobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/b^(1/2)/c/(-a*d+b*c)/
(-a*f+b*e)^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)+a^(3/2)*e*(d*
x^2+c)^(1/2)*EllipticPi(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),1-a*f/b/e,(1-a
*d/b/c)^(1/2))/b^(1/2)/c/(-a*f+b*e)^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^
2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.85 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.04

$$\int \frac{x^4}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \frac{a\sqrt{\frac{b}{a}}cfx + a\sqrt{\frac{b}{a}}dfx^3 + iacf\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(i\text{arcsinh}\left(\sqrt{\frac{b}{a}}x\right)\right.}{}$$

input
```
Integrate[x^4/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]*(e + f*x^2)),x]
```

output
```
(a*Sqrt[b/a]*c*f*x + a*Sqrt[b/a]*d*f*x^3 + I*a*c*f*Sqrt[1 + (b*x^2)/a]*Sqr
t[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + I*(-(b*c
) + a*d)*e*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqr
t[b/a]*x], (a*d)/(b*c)] + I*b*c*e*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*
EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*a*d*e*Sqr
t[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqr
t[b/a]*x], (a*d)/(b*c)])/(Sqrt[b/a]*(b*c - a*d)*f*(b*e - a*f)*Sqrt[a + b*x
^2]*Sqrt[c + d*x^2])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx$$

$$\Big\downarrow \ 450$$

$$\int \frac{x^4}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}dx$$

input
```
Int[x^4/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]*(e + f*x^2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [A] (verified)

Time = 8.43 (sec) , antiderivative size = 397, normalized size of antiderivative = 1.17

method	result
default	$\frac{\left(\sqrt{-\frac{b}{a}}\, a d f\, x^{3}-\sqrt{\frac{b x^{2}+a}{a}}\,\sqrt{\frac{x^{2} d+c}{c}}\,\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{a d}{b c}}\right) a d e+\sqrt{\frac{b x^{2}+a}{a}}\,\sqrt{\frac{x^{2} d+c}{c}}\,\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{a d}{b c}}\right) b c e-\sqrt{\frac{b x^{2}+a}{a}}\,\sqrt{x\cdots}\right.}{\cdots}$
elliptic	$\frac{\sqrt{(b x^{2}+a)(x^{2} d+c)}\left(\frac{(b d x^{2}+b c) a x}{b(a d-b c)(a f-b e)\sqrt{\left(x^{2}+\frac{a}{b}\right)(b d x^{2}+b c)}}+\frac{\sqrt{1+\frac{b x^{2}}{a}}\,\sqrt{1+\frac{d x^{2}}{c}}\,\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{a d+b c}{c b}}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{b d x^{4}+a d x^{2}+x^{2} b c+a c}\, f b}-\frac{\sqrt{1+\frac{b x^{2}}{a}}\,\sqrt{1+\frac{d x^{2}}{c}}\cdots}{\sqrt{-\frac{b}{a}}\,\sqrt{b d x^{4}\cdots}}\right.}{\sqrt{b x^{2}\cdots}}$

input
```
int(x^4/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
((-b/a)^(1/2)*a*d*f*x^3-((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(
x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*d*e+((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1
/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b*c*e-((b*x^2+a)/a)^(1/2)*((
d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*c*f+((b*x^2+
a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)
^(1/2)/(-b/a)^(1/2))*a*d*e-((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*Ellipti
cPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*b*c*e+(-b/a)^(1/2)
*a*c*f*x)*(d*x^2+c)^(1/2)*(b*x^2+a)^(1/2)/f/(-b/a)^(1/2)/(a*f-b*e)/(a*d-b*
c)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^4}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate(x^4/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^4}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{x^4}{(a+bx^2)^{\frac{3}{2}}\sqrt{c+dx^2}\,(e+fx^2)}\,dx$$

input
```
integrate(x**4/(b*x**2+a)**(3/2)/(d*x**2+c)**(1/2)/(f*x**2+e),x)
```

output
```
Integral(x**4/((a + b*x**2)**(3/2)*sqrt(c + d*x**2)*(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{x^4}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{x^4}{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}(fx^2+e)}\,dx$$

input
```
integrate(x^4/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="maxi
ma")
```

output
```
integrate(x^4/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*(f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{x^4}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{x^4}{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}(fx^2+e)}\,dx$$

input
```
integrate(x^4/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="giac
")
```

output
```
integrate(x^4/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{x^4}{(b\,x^2+a)^{3/2}\,\sqrt{d\,x^2+c}\,(f\,x^2+e)}\,dx$$

input
```
int(x^4/((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)*(e + f*x^2)),x)
```

output
```
int(x^4/((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)*(e + f*x^2)), x)
```

Reduce [F]

$$\int \frac{x^4}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,}}{b^2df\,x^8+2abdf\,x^6+b^2cf\,x^6+b^2de\,x^6+a^2df\,x^4+2abcf\,x^4+}$$

input
```
int(x^4/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a**2*c*e + a**2*c*f*x**2 + a
**2*d*e*x**2 + a**2*d*f*x**4 + 2*a*b*c*e*x**2 + 2*a*b*c*f*x**4 + 2*a*b*d*e
*x**4 + 2*a*b*d*f*x**6 + b**2*c*e*x**4 + b**2*c*f*x**6 + b**2*d*e*x**6 + b
**2*d*f*x**8),x)
```

3.251 $\int \frac{x^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}(e+fx^2)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 338

$$\int \frac{x^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = -\frac{\sqrt{a}\sqrt{b}\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{(bc-ad)(be-af)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} + \frac{\sqrt{a}(b^2ce-a^2df)\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}c(bc-ad)(be-af)^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} - \frac{a^{3/2}f\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}c(be-af)^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output

```
-a^(1/2)*b^(1/2)*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(
1/2),(1-a*d/b/c)^(1/2))/(-a*d+b*c)/(-a*f+b*e)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)
/c/(b*x^2+a))^(1/2)+a^(1/2)*(-a^2*d*f+b^2*c*e)*(d*x^2+c)^(1/2)*InverseJaco
biAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/b^(1/2)/c/(-a*d+b*c)/(-a
*f+b*e)^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)-a^(3/2)*f*(d*x^2
+c)^(1/2)*EllipticPi(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),1-a*f/b/e,(1-a*d/
b/c)^(1/2))/b^(1/2)/c/(-a*f+b*e)^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a
))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.98 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.03

$$\int \frac{x^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \frac{-b\sqrt{\frac{b}{a}}cx - b\sqrt{\frac{b}{a}}dx^3 - ibc\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(i\operatorname{arcsinh}\left(\sqrt{\frac{b}{a}}x\right)\right.}{}$$

input
```
Integrate[x^2/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]*(e + f*x^2)),x]
```

output
```
(-(b*Sqrt[b/a]*c*x) - b*Sqrt[b/a]*d*x^3 - I*b*c*Sqrt[1 + (b*x^2)/a]*Sqrt[1
 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*(-(b*c) +
 a*d)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a
]*x], (a*d)/(b*c)] - I*b*c*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Ellipti
cPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + I*a*d*Sqrt[1 + (b*
x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x]
, (a*d)/(b*c)])/(Sqrt[b/a]*(b*c - a*d)*(b*e - a*f)*Sqrt[a + b*x^2]*Sqrt[c
+ d*x^2])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx$$
$$\Big\downarrow \; 450$$
$$\int \frac{x^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}dx$$

input
```
Int[x^2/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]*(e + f*x^2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [A] (verified)

Time = 9.20 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.15

method	result
default	$\frac{\left(-\sqrt{-\frac{b}{a}}\, b d\, x^{3}+a \sqrt{\frac{b\, x^{2}+a}{a}}\, \sqrt{\frac{x^{2} d+c}{c}}\, \operatorname{EllipticF}\left(x \sqrt{-\frac{b}{a}}, \sqrt{\frac{a d}{b c}}\right) d-\sqrt{\frac{b\, x^{2}+a}{a}}\, \sqrt{\frac{x^{2} d+c}{c}}\, \operatorname{EllipticF}\left(x \sqrt{-\frac{b}{a}}, \sqrt{\frac{a d}{b c}}\right) b c+\sqrt{\frac{b\, x^{2}+a}{a}}\, \sqrt{x^{2}\cdots}\right.}{(a\cdots}$
elliptic	$\frac{\sqrt{\left(b\, x^{2}+a\right)\left(x^{2} d+c\right)}\, \left(-\frac{\left(b d\, x^{2}+b c\right) x}{(a d-b c)(a f-b e) \sqrt{\left(x^{2}+\frac{a}{b}\right)\left(b d\, x^{2}+b c\right)}}+\frac{\sqrt{1+\frac{b\, x^{2}}{a}}\, \sqrt{1+\frac{d\, x^{2}}{c}}\, \operatorname{EllipticF}\left(x \sqrt{-\frac{b}{a}}, \sqrt{-1+\frac{a d+b c}{c b}}\right)}{\sqrt{-\frac{b}{a}}\, \sqrt{b d\, x^{4}+a d\, x^{2}+x^{2} b c+a c}\,(a f-b e)}+\frac{b c \sqrt{1+\frac{b\, x^{2}}{a}}\, \sqrt{1+\cdots}}{(a f-b e)(a d-b\cdots}\right.}{\sqrt{b\, x^{2}+a}\, \sqrt{x^{2} d+c}}$

input
```
int(x^2/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
(-(-b/a)^(1/2)*b*d*x^3+a*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF
(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*d-((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)
*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b*c+((b*x^2+a)/a)^(1/2)*((d*x^2
+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b*c-((b*x^2+a)/a)^(
1/2)*((d*x^2+c)/c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/
(-b/a)^(1/2))*a*d+((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticPi(x*(-b
/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*b*c-(-b/a)^(1/2)*b*c*x)*(d*
x^2+c)^(1/2)*(b*x^2+a)^(1/2)/(a*f-b*e)/(-b/a)^(1/2)/(a*d-b*c)/(b*d*x^4+a*d
*x^2+b*c*x^2+a*c)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate(x^2/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{x^2}{(a+bx^2)^{\frac{3}{2}}\sqrt{c+dx^2}\,(e+fx^2)}\,dx$$

input
```
integrate(x**2/(b*x**2+a)**(3/2)/(d*x**2+c)**(1/2)/(f*x**2+e),x)
```

output
```
Integral(x**2/((a + b*x**2)**(3/2)*sqrt(c + d*x**2)*(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{x^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{x^2}{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}(fx^2+e)}\,dx$$

input
```
integrate(x^2/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="maxi
ma")
```

output
```
integrate(x^2/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*(f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{x^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{x^2}{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}(fx^2+e)}\,dx$$

input
```
integrate(x^2/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="giac
")
```

output
```
integrate(x^2/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{x^2}{{\left(b\,x^2+a\right)}^{3/2}\,\sqrt{d\,x^2+c}\,\left(f\,x^2+e\right)}\,dx$$

input
```
int(x^2/((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)*(e + f*x^2)),x)
```

output
```
int(x^2/((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)*(e + f*x^2)), x)
```

Reduce [F]

$$\int \frac{x^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{\sqrt{d\,x^2+c}\,\sqrt{b}}{b^2df\,x^8+2abdf\,x^6+b^2cf\,x^6+b^2de\,x^6+a^2df\,x^4+2abcf\,x^4+}$$

input
```
int(x^2/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a**2*c*e + a**2*c*f*x**2 + a
**2*d*e*x**2 + a**2*d*f*x**4 + 2*a*b*c*e*x**2 + 2*a*b*c*f*x**4 + 2*a*b*d*e
*x**4 + 2*a*b*d*f*x**6 + b**2*c*e*x**4 + b**2*c*f*x**6 + b**2*d*e*x**6 + b
**2*d*f*x**8),x)
```

3.252 $\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}(e+fx^2)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 342

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \frac{b^{3/2}\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}(bc-ad)(be-af)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} - \frac{\sqrt{a}\sqrt{b}(bde+bcf-2adf)\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{c(bc-ad)(be-af)^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} + \frac{a^{3/2}f^2\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce(be-af)^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
b^(3/2)*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a
*d/b/c)^(1/2))/a^(1/2)/(-a*d+b*c)/(-a*f+b*e)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/
c/(b*x^2+a))^(1/2)-a^(1/2)*b^(1/2)*(-2*a*d*f+b*c*f+b*d*e)*(d*x^2+c)^(1/2)*
InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/c/(-a*d+b*c)/
(-a*f+b*e)^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)+a^(3/2)*f^2*(
d*x^2+c)^(1/2)*EllipticPi(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),1-a*f/b/e,(1
-a*d/b/c)^(1/2))/b^(1/2)/c/e/(-a*f+b*e)^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(
b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.15 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.07

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \frac{\sqrt{\frac{b}{a}}\left(ab\left(\frac{b}{a}\right)^{3/2}cex+ab\left(\frac{b}{a}\right)^{3/2}dex^3+ib^2ce\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(i\right.\right.}{}$$

input
```
Integrate[1/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]*(e + f*x^2)),x]
```

output
```
(Sqrt[b/a]*(a*b*(b/a)^(3/2)*c*e*x + a*b*(b/a)^(3/2)*d*e*x^3 + I*b^2*c*e*Sq
rt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a
*d)/(b*c)] + I*b*(-(b*c) + a*d)*e*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*
EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + I*a*b*c*f*Sqrt[1 + (b*x^2
)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (
a*d)/(b*c)] - I*a^2*d*f*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi
[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(b*(b*c - a*d)*e*(b*e
 - a*f)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {421, 25, 400, 313, 320, 414}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx$$

$$\Big\downarrow \text{421}$$

$$\frac{f^2\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)}dx}{(be-af)^2}-\frac{b\int -\frac{-bfx^2+be-2af}{(bx^2+a)^{3/2}\sqrt{dx^2+c}}dx}{(be-af)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{f^2\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)}dx}{(be-af)^2}+\frac{b\int \frac{-bfx^2+be-2af}{(bx^2+a)^{3/2}\sqrt{dx^2+c}}dx}{(be-af)^2}$$

$$\Big\downarrow \; 400$$

$$\frac{f^2\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)}dx}{(be-af)^2}+\frac{b\left(\frac{b(be-af)\int \frac{\sqrt{dx^2+c}}{\left(bx^2+a\right)^{3/2}}dx}{bc-ad}-\frac{(-2adf+bcf+bde)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}\right)}{(be-af)^2}$$

$$\Big\downarrow \; 313$$

$$\frac{b\left(\frac{\sqrt{b}\sqrt{c+dx^2}(be-af)E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{a+bx^2}(bc-ad)\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}-\frac{(-2adf+bcf+bde)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}\right)}{(be-af)^2}+\frac{f^2\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)}dx}{(be-af)^2}$$

$$\Big\downarrow \; 320$$

$$\frac{f^2\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)}dx}{(be-af)^2}+\frac{b\left(\frac{\sqrt{b}\sqrt{c+dx^2}(be-af)E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{a+bx^2}(bc-ad)\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}-\frac{\sqrt{c}\sqrt{a+bx^2}(-2adf+bcf+bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{(be-af)^2}$$

$$\Big\downarrow \; 414$$

$$\frac{a^{3/2}f^2\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce\sqrt{a+bx^2}(be-af)^2\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}+\frac{b\left(\frac{\sqrt{b}\sqrt{c+dx^2}(be-af)E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{a+bx^2}(bc-ad)\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}-\frac{\sqrt{c}\sqrt{a+bx^2}(-2adf+bcf+bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{(be-af)^2}$$

input
```
Int[1/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]*(e + f*x^2)),x]
```

output
```
(b*((Sqrt[b]*(b*e - a*f)*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt
[a]], 1 - (a*d)/(b*c)])/(Sqrt[a]*(b*c - a*d)*Sqrt[a + b*x^2]*Sqrt[(a*(c +
d*x^2))/(c*(a + b*x^2))]) - (Sqrt[c]*(b*d*e + b*c*f - 2*a*d*f)*Sqrt[a + b*
x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*(
b*c - a*d)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])))/(b*e -
 a*f)^2 + (a^(3/2)*f^2*Sqrt[c + d*x^2]*EllipticPi[1 - (a*f)/(b*e), ArcTan[
(Sqrt[b]*x)/Sqrt[a]], 1 - (a*d)/(b*c)])/(Sqrt[b]*c*e*(b*e - a*f)^2*Sqrt[a
+ b*x^2]*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
 + d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 400
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 421
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b^2/(b*c - a*d)^2   Int[(c + d*x^2)^(q + 2)*((e +
 f*x^2)^r/(a + b*x^2)), x], x] - Simp[d/(b*c - a*d)^2   Int[(c + d*x^2)^q*(
e + f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r}
, x] && LtQ[q, -1]
```

Maple [A] (verified)

Time = 8.44 (sec) , antiderivative size = 412, normalized size of antiderivative = 1.20

method	result
default	$\frac{\left(\sqrt{-\frac{b}{a}}\, b^{2} d e\, x^{3}-\sqrt{\frac{b\,x^{2}+a}{a}}\, \sqrt{\frac{x^{2} d+c}{c}}\, \operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}}, \sqrt{\frac{ad}{bc}}\right) abde+\sqrt{\frac{b\,x^{2}+a}{a}}\, \sqrt{\frac{x^{2} d+c}{c}}\, \operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}}, \sqrt{\frac{ad}{bc}}\right) b^{2} c e-\sqrt{\frac{b\,x^{2}+a}{a}}\right.}{}$
elliptic	$\frac{\sqrt{(b\,x^{2}+a)(x^{2} d+c)}\left(\frac{\left(bd\,x^{2}+bc\right) b x}{a(ad-bc)(af-be)\sqrt{\left(x^{2}+\frac{a}{b}\right)\left(bd\,x^{2}+bc\right)}}-\frac{\sqrt{1+\frac{b\,x^{2}}{a}}\, \sqrt{1+\frac{d\,x^{2}}{c}}\, \operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}}, \sqrt{-1+\frac{ad+bc}{cb}}\right) b}{\sqrt{-\frac{b}{a}}\, \sqrt{bd\,x^{4}+ad\,x^{2}+x^{2} bc+ac}\; a(af-be)}-\frac{b^{2} c\sqrt{1+\frac{b\,x^{2}}{a}}\, \sqrt{1+}}{a(ad-bc)(af-}\right.}{\sqrt{b\,x^{2}+a}\, \sqrt{x^{2} d+c}}$

input
```
int(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
((-b/a)^(1/2)*b^2*d*e*x^3-((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*Elliptic
F(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b*d*e+((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c
)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^2*c*e-((b*x^2+a)/a)^(1
/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^2*c*e+
((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,
(-1/c*d)^(1/2)/(-b/a)^(1/2))*a^2*d*f-((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/
2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*a*b*c*f+
(-b/a)^(1/2)*b^2*c*e*x)*(d*x^2+c)^(1/2)*(b*x^2+a)^(1/2)/e/(-b/a)^(1/2)/a/(
a*d-b*c)/(a*f-b*e)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{1}{(a+bx^2)^{\frac{3}{2}}\sqrt{c+dx^2}\,(e+fx^2)}\,dx$$

input
```
integrate(1/(b*x**2+a)**(3/2)/(d*x**2+c)**(1/2)/(f*x**2+e),x)
```

output
```
Integral(1/((a + b*x**2)**(3/2)*sqrt(c + d*x**2)*(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{1}{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}(fx^2+e)}\,dx$$

input
```
integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="maxima
")
```

output
```
integrate(1/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*(f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{1}{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}(fx^2+e)}\,dx$$

input
```
integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{1}{(b\,x^2+a)^{3/2}\,\sqrt{d\,x^2+c}\,(f\,x^2+e)}\,dx$$

input
```
int(1/((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)*(e + f*x^2)),x)
```

output
```
int(1/((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)*(e + f*x^2)), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{\sqrt{d\,x^2+c}\,\sqrt{b}}{b^2df\,x^8+2abdf\,x^6+b^2cf\,x^6+b^2de\,x^6+a^2df\,x^4+2abcf\,x^4+}$$

input
```
int(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a**2*c*e + a**2*c*f*x**2 + a**2*d
*e*x**2 + a**2*d*f*x**4 + 2*a*b*c*e*x**2 + 2*a*b*c*f*x**4 + 2*a*b*d*e*x**4
 + 2*a*b*d*f*x**6 + b**2*c*e*x**4 + b**2*c*f*x**6 + b**2*d*e*x**6 + b**2*d
*f*x**8),x)
```

3.253 $\int \frac{1}{x^2(a+bx^2)^{3/2}\sqrt{c+dx^2}(e+fx^2)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 430

$$\int \frac{1}{x^2\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}\left(e+fx^2\right)}\,dx=-\frac{\sqrt{c+dx^2}}{acex\sqrt{a+bx^2}}-\frac{\sqrt{b}(2b^2ce+a^2df-ab(de+cf))\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{a^{3/2}c(bc-ad)e(be-af)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}+\frac{\sqrt{b}(b^2de^2-a^2df^2-abf(de-cf))\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{a}c(bc-ad)e(be-af)^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}-\frac{a^{3/2}f^3\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce^2(be-af)^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
-(d*x^2+c)^(1/2)/a/c/e/x/(b*x^2+a)^(1/2)-b^(1/2)*(2*b^2*c*e+a^2*d*f-a*b*(c
*f+d*e))*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-
a*d/b/c)^(1/2))/a^(3/2)/c/(-a*d+b*c)/e/(-a*f+b*e)/(b*x^2+a)^(1/2)/(a*(d*x^
2+c)/c/(b*x^2+a))^(1/2)+b^(1/2)*(b^2*d*e^2-a^2*d*f^2-a*b*f*(-c*f+d*e))*(d*
x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/
a^(1/2)/c/(-a*d+b*c)/e/(-a*f+b*e)^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+
a))^(1/2)-a^(3/2)*f^3*(d*x^2+c)^(1/2)*EllipticPi(b^(1/2)*x/a^(1/2)/(1+b*x^
2/a)^(1/2),1-a*f/b/e,(1-a*d/b/c)^(1/2))/b^(1/2)/c/e^2/(-a*f+b*e)^2/(b*x^2+
a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.04 (sec) , antiderivative size = 636, normalized size of antiderivative = 1.48

$$\int \frac{1}{x^2 (a+bx^2)^{3/2} \sqrt{c+dx^2} (e+fx^2)} dx = \frac{-ab^2\sqrt{\frac{b}{a}}c^2e^2 + a^3\left(\frac{b}{a}\right)^{3/2}cde^2 + a^3\left(\frac{b}{a}\right)^{3/2}c^2ef - a^3\sqrt{\frac{b}{a}}cdef - 2}{}$$

input
```
Integrate[1/(x^2*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2]*(e + f*x^2)),x]
```

output
```
(-(a*b^2*Sqrt[b/a]*c^2*e^2) + a^3*(b/a)^(3/2)*c*d*e^2 + a^3*(b/a)^(3/2)*c^
2*e*f - a^3*Sqrt[b/a]*c*d*e*f - 2*b^3*Sqrt[b/a]*c^2*e^2*x^2 + a^3*(b/a)^(3
/2)*d^2*e^2*x^2 + a*b^2*Sqrt[b/a]*c^2*e*f*x^2 - a^3*Sqrt[b/a]*d^2*e*f*x^2
 - 2*b^3*Sqrt[b/a]*c*d*e^2*x^4 + a*b^2*Sqrt[b/a]*d^2*e^2*x^4 + a*b^2*Sqrt[b
/a]*c*d*e*f*x^4 - a^3*(b/a)^(3/2)*d^2*e*f*x^4 - I*b*c*e*(2*b^2*c*e + a^2*d
*f - a*b*(d*e + c*f))*x*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[
I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + I*b*c*(-(b*c) + a*d)*e*(-2*b*e + a*
f)*x*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]
*x], (a*d)/(b*c)] - I*a^2*b*c^2*f^2*x*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)
/c]*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + I*a^3*c
*d*f^2*x*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e), I
*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(a^2*Sqrt[b/a]*c*(b*c - a*d)*e^2*(b*e
 - a*f)*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a+bx^2)^{3/2} \sqrt{c+dx^2} (e+fx^2)} dx$$
$$\Big\downarrow \text{450}$$
$$\int \frac{1}{x^2 (a+bx^2)^{3/2} \sqrt{c+dx^2} (e+fx^2)} dx$$

input
```
Int[1/(x^2*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2]*(e + f*x^2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [A] (verified)

Time = 22.86 (sec) , antiderivative size = 622, normalized size of antiderivative = 1.45

method	result
risch	$-\frac{\sqrt{b\,x^2+a}\,\sqrt{x^2 d+c}}{a^2 c e x}+\frac{\left(-\frac{bc\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\left(\text{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)-\text{EllipticE}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}+\frac{ace\,b^2\left(-\frac{}{a(ad\ldots}\right.}{}\right.}{}$
elliptic	$\frac{\sqrt{(b\,x^2+a)(x^2d+c)}\left(-\frac{\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}}{a^2cex}-\frac{\left(bd\,x^2+bc\right)b^2x}{a^2(ad-bc)(af-be)\sqrt{\left(x^2+\frac{a}{b}\right)\left(bd\,x^2+bc\right)}}+\frac{\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\text{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\ldots}\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+x^2bc+ac}\,a^2(af-\ldots}\right.}{}$
default	$\frac{\left(-\sqrt{-\frac{b}{a}}\,a^2b\,d^2ef\,x^4+\sqrt{-\frac{b}{a}}\,a\,b^2cdef\,x^4+\sqrt{-\frac{b}{a}}\,a\,b^2d^2e^2x^4-2\sqrt{-\frac{b}{a}}\,b^3cd\,e^2x^4-\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{x^2d+c}{c}}\,\text{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)a^2bc\ldots\right.}{}$

input

```
int(1/x^2/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x,method=_RETURNVERBOS
E)
```

output

```
-1/a^2/c/e*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x+1/a^2/c/e*(-b*c/(-b/a)^(1/2)*
(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(E
llipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2)
,(-1+(a*d+b*c)/c/b)^(1/2)))+a*c*e*b^2/(a*f-b*e)*(-(b*d*x^2+b*c)/a/(a*d-b*c
)*x/((x^2+a/b)*(b*d*x^2+b*c))^(1/2)+(1/a+b*c/(a*d-b*c)/a)/(-b/a)^(1/2)*(1+
b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*Ellip
ticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-b/(a*d-b*c)/a*c/(-b/a)^(1/2)
*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(
EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2
),(-1+(a*d+b*c)/c/b)^(1/2))))-a^2*c*f^2/(a*f-b*e)/e/(-b/a)^(1/2)*(1+b*x^2/
a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(
x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2)))*((b*x^2+a)*(d*x^2+c))
^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a+bx^2)^{3/2} \sqrt{c+dx^2} (e+fx^2)} \, dx = \text{Timed out}$$

input
```
integrate(1/x^2/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{x^2 (a+bx^2)^{3/2} \sqrt{c+dx^2} (e+fx^2)} \, dx = \int \frac{1}{x^2 (a+bx^2)^{\frac{3}{2}} \sqrt{c+dx^2} (e+fx^2)} \, dx$$

input
```
integrate(1/x**2/(b*x**2+a)**(3/2)/(d*x**2+c)**(1/2)/(f*x**2+e),x)
```

output
```
Integral(1/(x**2*(a + b*x**2)**(3/2)*sqrt(c + d*x**2)*(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{x^2 (a+bx^2)^{3/2} \sqrt{c+dx^2} (e+fx^2)} \, dx = \int \frac{1}{(bx^2+a)^{\frac{3}{2}} \sqrt{dx^2+c}(fx^2+e)x^2} \, dx$$

input
```
integrate(1/x^2/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="ma
xima")
```

output
```
integrate(1/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*(f*x^2 + e)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2 (a + bx^2)^{3/2} \sqrt{c + dx^2} (e + fx^2)} dx = \int \frac{1}{(bx^2 + a)^{\frac{3}{2}} \sqrt{dx^2 + c}(fx^2 + e)x^2} dx$$

input
```
integrate(1/x^2/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="gi
ac")
```

output
```
integrate(1/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*(f*x^2 + e)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a + bx^2)^{3/2} \sqrt{c + dx^2} (e + fx^2)} dx = \int \frac{1}{x^2 (b x^2 + a)^{3/2} \sqrt{d x^2 + c} (f x^2 + e)} dx$$

input
```
int(1/(x^2*(a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)*(e + f*x^2)),x)
```

output
```
int(1/(x^2*(a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)*(e + f*x^2)), x)
```

Reduce [F]

$$\int \frac{1}{x^2 (a + bx^2)^{3/2} \sqrt{c + dx^2} (e + fx^2)} dx = \text{Too large to display}$$

input
```
int(1/x^2/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x)
```

output

```
( - sqrt(c + d*x**2)*sqrt(a + b*x**2) - int((sqrt(c + d*x**2)*sqrt(a + b*x
**2)*x**4)/(a**2*c*e + a**2*c*f*x**2 + a**2*d*e*x**2 + a**2*d*f*x**4 + 2*a
*b*c*e*x**2 + 2*a*b*c*f*x**4 + 2*a*b*d*e*x**4 + 2*a*b*d*f*x**6 + b**2*c*e*
x**4 + b**2*c*f*x**6 + b**2*d*e*x**6 + b**2*d*f*x**8),x)*a*b*d*f*x - int((
sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a**2*c*e + a**2*c*f*x**2 + a**2*d
*e*x**2 + a**2*d*f*x**4 + 2*a*b*c*e*x**2 + 2*a*b*c*f*x**4 + 2*a*b*d*e*x**4
 + 2*a*b*d*f*x**6 + b**2*c*e*x**4 + b**2*c*f*x**6 + b**2*d*e*x**6 + b**2*d
*f*x**8),x)*b**2*d*f*x**3 - 2*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)
/(a**2*c*e + a**2*c*f*x**2 + a**2*d*e*x**2 + a**2*d*f*x**4 + 2*a*b*c*e*x**
2 + 2*a*b*c*f*x**4 + 2*a*b*d*e*x**4 + 2*a*b*d*f*x**6 + b**2*c*e*x**4 + b**
2*c*f*x**6 + b**2*d*e*x**6 + b**2*d*f*x**8),x)*a*b*c*f*x - int((sqrt(c + d
*x**2)*sqrt(a + b*x**2)*x**2)/(a**2*c*e + a**2*c*f*x**2 + a**2*d*e*x**2 +
a**2*d*f*x**4 + 2*a*b*c*e*x**2 + 2*a*b*c*f*x**4 + 2*a*b*d*e*x**4 + 2*a*b*d
*f*x**6 + b**2*c*e*x**4 + b**2*c*f*x**6 + b**2*d*e*x**6 + b**2*d*f*x**8),x
)*a*b*d*e*x - 2*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a**2*c*e + a
**2*c*f*x**2 + a**2*d*e*x**2 + a**2*d*f*x**4 + 2*a*b*c*e*x**2 + 2*a*b*c*f*
x**4 + 2*a*b*d*e*x**4 + 2*a*b*d*f*x**6 + b**2*c*e*x**4 + b**2*c*f*x**6 + b
**2*d*e*x**6 + b**2*d*f*x**8),x)*b**2*c*f*x**3 - int((sqrt(c + d*x**2)*sqr
t(a + b*x**2)*x**2)/(a**2*c*e + a**2*c*f*x**2 + a**2*d*e*x**2 + a**2*d*f*x
**4 + 2*a*b*c*e*x**2 + 2*a*b*c*f*x**4 + 2*a*b*d*e*x**4 + 2*a*b*d*f*x**6...
```

3.254 $\int \frac{1}{x^4(a+bx^2)^{3/2}\sqrt{c+dx^2}(e+fx^2)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 575

$$\int \frac{1}{x^4(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = -\frac{\sqrt{c+dx^2}}{3acex^3\sqrt{a+bx^2}}+\frac{(4bce+2ade+3acf)\sqrt{c+dx^2}}{3a^2c^2e^2x\sqrt{a+bx^2}}$$
$$+\frac{\sqrt{b}(8b^3c^2e^2-ab^2ce(3de+2cf)+a^3df(2de+3cf)-a^2b(2d^2e^2+3c^2f^2))\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\right.}{3a^{5/2}c^2(bc-ad)e^2(be-af)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$
$$-\frac{\sqrt{b}(4b^3cde^3-a^3df^2(de+3cf)-ab^2de^2(de+5cf)+a^2bf(2d^2e^2+cdef+3c^2f^2))\sqrt{c+dx^2}\,\text{EllipticF}\left(\right.}{3a^{3/2}c^2(bc-ad)e^2(be-af)^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$
$$+\frac{a^{3/2}f^4\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce^3(be-af)^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
-1/3*(d*x^2+c)^(1/2)/a/c/e/x^3/(b*x^2+a)^(1/2)+1/3*(3*a*c*f+2*a*d*e+4*b*c*
e)*(d*x^2+c)^(1/2)/a^2/c^2/e^2/x/(b*x^2+a)^(1/2)+1/3*b^(1/2)*(8*b^3*c^2*e^
2-a*b^2*c*e*(2*c*f+3*d*e)+a^3*d*f*(3*c*f+2*d*e)-a^2*b*(3*c^2*f^2+2*d^2*e^2
))*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/
c)^(1/2))/a^(5/2)/c^2/(-a*d+b*c)/e^2/(-a*f+b*e)/(b*x^2+a)^(1/2)/(a*(d*x^2+
c)/c/(b*x^2+a))^(1/2)-1/3*b^(1/2)*(4*b^3*c*d*e^3-a^3*d*f^2*(3*c*f+d*e)-a*b
^2*d*e^2*(5*c*f+d*e)+a^2*b*f*(3*c^2*f^2+c*d*e*f+2*d^2*e^2))*(d*x^2+c)^(1/2
)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/a^(3/2)/c^2
/(-a*d+b*c)/e^2/(-a*f+b*e)^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/
2)+a^(3/2)*f^4*(d*x^2+c)^(1/2)*EllipticPi(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1
/2),1-a*f/b/e,(1-a*d/b/c)^(1/2))/b^(1/2)/c/e^3/(-a*f+b*e)^2/(b*x^2+a)^(1/2
)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.94 (sec) , antiderivative size = 963, normalized size of antiderivative = 1.67

$$\int \frac{1}{x^4 (a+bx^2)^{3/2} \sqrt{c+dx^2} (e+fx^2)} \, dx = \frac{-a^2b^3c^3e^3 + a^3b^2c^2de^3 + a^3b^2c^3e^2f - a^4bc^2de^2f + 4ab^4c^3e^3x^2 -}{}$$

input
```
Integrate[1/(x^4*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2]*(e + f*x^2)),x]
```

output
```
(-(a^2*b^3*c^3*e^3) + a^3*b^2*c^2*d*e^3 + a^3*b^2*c^3*e^2*f - a^4*b*c^2*d*
e^2*f + 4*a*b^4*c^3*e^3*x^2 - 3*a^2*b^3*c^2*d*e^3*x^2 - a^3*b^2*c*d^2*e^3*
x^2 - a^2*b^3*c^3*e^2*f*x^2 + a^4*b*c*d^2*e^2*f*x^2 - 3*a^3*b^2*c^3*e*f^2*
x^2 + 3*a^4*b*c^2*d*e*f^2*x^2 + 8*b^5*c^3*e^3*x^4 + a*b^4*c^2*d*e^3*x^4 -
4*a^2*b^3*c*d^2*e^3*x^4 - 2*a^3*b^2*d^3*e^3*x^4 - 2*a*b^4*c^3*e^2*f*x^4 -
a^2*b^3*c^2*d*e^2*f*x^4 + a^3*b^2*c*d^2*e^2*f*x^4 + 2*a^4*b*d^3*e^2*f*x^4
- 3*a^2*b^3*c^3*e*f^2*x^4 + 3*a^4*b*c*d^2*e*f^2*x^4 + 8*b^5*c^2*d*e^3*x^6
- 3*a*b^4*c*d^2*e^3*x^6 - 2*a^2*b^3*d^3*e^3*x^6 - 2*a*b^4*c^2*d*e^2*f*x^6
+ 2*a^3*b^2*d^3*e^2*f*x^6 - 3*a^2*b^3*c^2*d*e*f^2*x^6 + 3*a^3*b^2*c*d^2*e*
f^2*x^6 + I*a*b*Sqrt[b/a]*c*e*(8*b^3*c^2*e^2 - a*b^2*c*e*(3*d*e + 2*c*f) +
 a^3*d*f*(2*d*e + 3*c*f) - a^2*b*(2*d^2*e^2 + 3*c^2*f^2))*x^3*Sqrt[1 + (b*
x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]
 - I*a*b*Sqrt[b/a]*c*(-(b*c) + a*d)*e*(-8*b^2*c*e^2 + a*b*e*(-(d*e) + 2*c*
f) + a^2*f*(d*e + 3*c*f))*x^3*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Elli
pticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (3*I)*a^5*(b/a)^(3/2)*c^3*f^3
*x^3*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e), I*Arc
Sinh[Sqrt[b/a]*x], (a*d)/(b*c)] - (3*I)*a^5*Sqrt[b/a]*c^2*d*f^3*x^3*Sqrt[1
 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b
/a]*x], (a*d)/(b*c)])/(3*a^3*b*c^2*(b*c - a*d)*e^3*(b*e - a*f)*x^3*Sqrt[a
+ b*x^2]*Sqrt[c + d*x^2])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (a+bx^2)^{3/2} \sqrt{c+dx^2} (e+fx^2)} \, dx$$
$$\Big\downarrow \; 450$$
$$\int \frac{1}{x^4 (a+bx^2)^{3/2} \sqrt{c+dx^2} (e+fx^2)} dx$$

input
```
Int[1/(x^4*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2]*(e + f*x^2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [A] (verified)

Time = 24.54 (sec) , antiderivative size = 766, normalized size of antiderivative = 1.33

method	result
risch	$-\frac{\sqrt{b\,x^{2}+a}\,\sqrt{x^{2}d+c}\,\left(-3acf\,x^{2}-2ade\,x^{2}-5bce\,x^{2}+ace\right)}{3a^{3}c^{2}e^{2}x^{3}}-\frac{\left(-\frac{b(3acf+2ade+5bce)c\sqrt{1+\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,\left(\text{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+}{cb}}\right.\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^{4}+ad\,x^{2}+x^{2}bc+ac}}\right.}{}$
elliptic	Expression too large to display
default	Expression too large to display

input
```
int(1/x^4/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x,method=_RETURNVERBOS
E)
```

output
```
-1/3*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(-3*a*c*f*x^2-2*a*d*e*x^2-5*b*c*e*x^2
+a*c*e)/a^3/c^2/e^2/x^3-1/3/a^3/c^2/e^2*(-b*(3*a*c*f+2*a*d*e+5*b*c*e)*c/(-
b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*
c)^(1/2)*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(
-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2)))+a*c*d*e*b/(-b/a)^(1/2)*(1+b*x^2/a)^
(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-
b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-3*a^3*c^2*f^3/(a*f-b*e)/e/(-b/a)^(1/2
)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*
EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))+3*a*b^3*c^2
*e^2/(a*f-b*e)*(-(b*d*x^2+b*c)/a/(a*d-b*c)*x/((x^2+a/b)*(b*d*x^2+b*c))^(1/
2)+(1/a+b*c/(a*d-b*c)/a)/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/
(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)
/c/b)^(1/2))-b/(a*d-b*c)/a*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1
/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d
+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2)))))*((
b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (a+bx^2)^{3/2} \sqrt{c+dx^2} (e+fx^2)} dx = \text{Timed out}$$

input
```
integrate(1/x^4/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{x^4 (a+bx^2)^{3/2} \sqrt{c+dx^2} (e+fx^2)} dx = \int \frac{1}{x^4 (a+bx^2)^{\frac{3}{2}} \sqrt{c+dx^2} (e+fx^2)} dx$$

input
```
integrate(1/x**4/(b*x**2+a)**(3/2)/(d*x**2+c)**(1/2)/(f*x**2+e),x)
```

output
```
Integral(1/(x**4*(a + b*x**2)**(3/2)*sqrt(c + d*x**2)*(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{x^4 (a+bx^2)^{3/2} \sqrt{c+dx^2} (e+fx^2)} \, dx = \int \frac{1}{(bx^2+a)^{\frac{3}{2}} \sqrt{dx^2+c}(fx^2+e)x^4} \, dx$$

input
```
integrate(1/x^4/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="ma
xima")
```

output
```
integrate(1/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*(f*x^2 + e)*x^4), x)
```

Giac [F]

$$\int \frac{1}{x^4 (a+bx^2)^{3/2} \sqrt{c+dx^2} (e+fx^2)} \, dx = \int \frac{1}{(bx^2+a)^{\frac{3}{2}} \sqrt{dx^2+c}(fx^2+e)x^4} \, dx$$

input
```
integrate(1/x^4/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="gi
ac")
```

output
```
integrate(1/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*(f*x^2 + e)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (a+bx^2)^{3/2} \sqrt{c+dx^2} (e+fx^2)} \, dx = \int \frac{1}{x^4 \left(b\,x^2+a\right)^{3/2} \sqrt{d\,x^2+c} \left(f\,x^2+e\right)} \, dx$$

input
```
int(1/(x^4*(a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)*(e + f*x^2)),x)
```

output
```
int(1/(x^4*(a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)*(e + f*x^2)), x)
```

Reduce [F]

$$\int \frac{1}{x^4 (a+bx^2)^{3/2} \sqrt{c+dx^2}\,(e+fx^2)}\,dx = \text{too large to display}$$

input
```
int(1/x^4/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x)
```

output
```
( - sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*c*e + 3*sqrt(c + d*x**2)*sqrt(a +
b*x**2)*a*c*f*x**2 + 2*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*d*e*x**2 + 4*sq
rt(c + d*x**2)*sqrt(a + b*x**2)*b*c*e*x**2 + 3*int((sqrt(c + d*x**2)*sqrt(
a + b*x**2)*x**4)/(a**2*c*e + a**2*c*f*x**2 + a**2*d*e*x**2 + a**2*d*f*x**
4 + 2*a*b*c*e*x**2 + 2*a*b*c*f*x**4 + 2*a*b*d*e*x**4 + 2*a*b*d*f*x**6 + b*
*2*c*e*x**4 + b**2*c*f*x**6 + b**2*d*e*x**6 + b**2*d*f*x**8),x)*a**2*b*c*d
*f**2*x**3 + 2*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a**2*c*e + a*
*2*c*f*x**2 + a**2*d*e*x**2 + a**2*d*f*x**4 + 2*a*b*c*e*x**2 + 2*a*b*c*f*x
**4 + 2*a*b*d*e*x**4 + 2*a*b*d*f*x**6 + b**2*c*e*x**4 + b**2*c*f*x**6 + b*
*2*d*e*x**6 + b**2*d*f*x**8),x)*a**2*b*d**2*e*f*x**3 + 4*int((sqrt(c + d*x
**2)*sqrt(a + b*x**2)*x**4)/(a**2*c*e + a**2*c*f*x**2 + a**2*d*e*x**2 + a*
*2*d*f*x**4 + 2*a*b*c*e*x**2 + 2*a*b*c*f*x**4 + 2*a*b*d*e*x**4 + 2*a*b*d*f
*x**6 + b**2*c*e*x**4 + b**2*c*f*x**6 + b**2*d*e*x**6 + b**2*d*f*x**8),x)*
a*b**2*c*d*e*f*x**3 + 3*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a**2
*c*e + a**2*c*f*x**2 + a**2*d*e*x**2 + a**2*d*f*x**4 + 2*a*b*c*e*x**2 + 2*
a*b*c*f*x**4 + 2*a*b*d*e*x**4 + 2*a*b*d*f*x**6 + b**2*c*e*x**4 + b**2*c*f*
x**6 + b**2*d*e*x**6 + b**2*d*f*x**8),x)*a*b**2*c*d*f**2*x**5 + 2*int((sqr
t(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a**2*c*e + a**2*c*f*x**2 + a**2*d*e*
x**2 + a**2*d*f*x**4 + 2*a*b*c*e*x**2 + 2*a*b*c*f*x**4 + 2*a*b*d*e*x**4 +
2*a*b*d*f*x**6 + b**2*c*e*x**4 + b**2*c*f*x**6 + b**2*d*e*x**6 + b**2*d...
```

3.255 $\int \frac{1}{x^6(a+bx^2)^{3/2}\sqrt{c+dx^2}(e+fx^2)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 787

$$\int \frac{1}{x^6(a+bx^2)^{3/2}\sqrt{c+dx^2}(e+fx^2)}\,dx = -\frac{\sqrt{c+dx^2}}{5acex^5\sqrt{a+bx^2}}+\frac{(6bce+4ade+5acf)\sqrt{c+dx^2}}{15a^2c^2e^2x^3\sqrt{a+bx^2}}$$
$$-\frac{\left(\frac{24b^2ce}{a}+13bde+\frac{8ad^2e}{c}+20bcf+10adf+\frac{15acf^2}{e}\right)\sqrt{c+dx^2}}{15a^2c^2e^2x\sqrt{a+bx^2}}$$
$$-\frac{\sqrt{b}(48b^4c^3e^3-8ab^3c^2e^2(2de+cf)-a^2b^2ce(9d^2e^2-cdef+10c^2f^2)+a^4df(8d^2e^2+10cdef+15c^2f^2) - \cdots}{15a^{7/2}c^3(bc-ad)e^3(be-af)\sqrt{a+bx^2}\sqrt{\frac{a\cdots}{c\cdots}}}$$
$$+\frac{\sqrt{b}(24b^4c^2de^4-ab^3cde^3(5de+28cf)-a^2b^2de^2(4d^2e^2-5cdef+c^2f^2)-a^4df^2(4d^2e^2+5cdef+15c^2f^2)\cdots}{15a^{5/2}c^3(bc-ad)e^3(be-af)^2\sqrt{\cdots}}$$
$$-\frac{a^{3/2}f^5\sqrt{c+dx^2}\,\mathrm{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce^4(be-af)^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
-1/5*(d*x^2+c)^(1/2)/a/c/e/x^5/(b*x^2+a)^(1/2)+1/15*(5*a*c*f+4*a*d*e+6*b*c
*e)*(d*x^2+c)^(1/2)/a^2/c^2/e^2/x^3/(b*x^2+a)^(1/2)-1/15*(24*b^2*c*e/a+13*
b*d*e+8*a*d^2*e/c+20*b*c*f+10*a*d*f+15*a*c*f^2/e)*(d*x^2+c)^(1/2)/a^2/c^2/
e^2/x/(b*x^2+a)^(1/2)-1/15*b^(1/2)*(48*b^4*c^3*e^3-8*a*b^3*c^2*e^2*(c*f+2*
d*e)-a^2*b^2*c*e*(10*c^2*f^2-c*d*e*f+9*d^2*e^2)+a^4*d*f*(15*c^2*f^2+10*c*d
*e*f+8*d^2*e^2)-a^3*b*(15*c^3*f^3+c*d^2*e^2*f+8*d^3*e^3))*(d*x^2+c)^(1/2)*
EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/a^(7/2)/c
^3/(-a*d+b*c)/e^3/(-a*f+b*e)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/
2)+1/15*b^(1/2)*(24*b^4*c^2*d*e^4-a*b^3*c*d*e^3*(28*c*f+5*d*e)-a^2*b^2*d*e
^2*(c^2*f^2-5*c*d*e*f+4*d^2*e^2)-a^4*d*f^2*(15*c^2*f^2+5*c*d*e*f+4*d^2*e^2
)+a^3*b*f*(15*c^3*f^3+5*c^2*d*e*f^2+5*c*d^2*e^2*f+8*d^3*e^3))*(d*x^2+c)^(1
/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/a^(5/2)/c
^3/(-a*d+b*c)/e^3/(-a*f+b*e)^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(
1/2)-a^(3/2)*f^5*(d*x^2+c)^(1/2)*EllipticPi(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^
(1/2),1-a*f/b/e,(1-a*d/b/c)^(1/2))/b^(1/2)/c/e^4/(-a*f+b*e)^2/(b*x^2+a)^(1
/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 8.92 (sec) , antiderivative size = 1515, normalized size of antiderivative = 1.93

$$\int \frac{1}{x^6 (a+bx^2)^{3/2} \sqrt{c+dx^2} (e+fx^2)} dx = \text{Too large to display}$$

input
```
Integrate[1/(x^6*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2]*(e + f*x^2)),x]
```

output
```
(-3*a^3*b^3*c^4*e^4 + 3*a^4*b^2*c^3*d*e^4 + 3*a^4*b^2*c^4*e^3*f - 3*a^5*b*
c^3*d*e^3*f + 6*a^2*b^4*c^4*e^4*x^2 - 5*a^3*b^3*c^3*d*e^4*x^2 - a^4*b^2*c^
2*d^2*e^4*x^2 - a^3*b^3*c^4*e^3*f*x^2 + a^5*b*c^2*d^2*e^3*f*x^2 - 5*a^4*b^
2*c^4*e^2*f^2*x^2 + 5*a^5*b*c^3*d*e^2*f^2*x^2 - 24*a*b^5*c^4*e^4*x^4 + 17*
a^2*b^4*c^3*d*e^4*x^4 + 3*a^3*b^3*c^2*d^2*e^4*x^4 + 4*a^4*b^2*c*d^3*e^4*x^
4 + 4*a^2*b^4*c^4*e^3*f*x^4 - 2*a^3*b^3*c^3*d*e^3*f*x^4 + 2*a^4*b^2*c^2*d^
2*e^3*f*x^4 - 4*a^5*b*c*d^3*e^3*f*x^4 + 5*a^3*b^3*c^4*e^2*f^2*x^4 - 5*a^5*
b*c^2*d^2*e^2*f^2*x^4 + 15*a^4*b^2*c^4*e*f^3*x^4 - 15*a^5*b*c^3*d*e*f^3*x^
4 - 48*b^6*c^4*e^4*x^6 - 8*a*b^5*c^3*d*e^4*x^6 + 20*a^2*b^4*c^2*d^2*e^4*x^
6 + 13*a^3*b^3*c*d^3*e^4*x^6 + 8*a^4*b^2*d^4*e^4*x^6 + 8*a*b^5*c^4*e^3*f*x
^6 + 3*a^2*b^4*c^3*d*e^3*f*x^6 - 3*a^4*b^2*c*d^3*e^3*f*x^6 - 8*a^5*b*d^4*e
^3*f*x^6 + 10*a^2*b^4*c^4*e^2*f^2*x^6 + 5*a^3*b^3*c^3*d*e^2*f^2*x^6 - 5*a^
4*b^2*c^2*d^2*e^2*f^2*x^6 - 10*a^5*b*c*d^3*e^2*f^2*x^6 + 15*a^3*b^3*c^4*e*
f^3*x^6 - 15*a^5*b*c^2*d^2*e*f^3*x^6 - 48*b^6*c^3*d*e^4*x^8 + 16*a*b^5*c^2
*d^2*e^4*x^8 + 9*a^2*b^4*c*d^3*e^4*x^8 + 8*a^3*b^3*d^4*e^4*x^8 + 8*a*b^5*c
^3*d*e^3*f*x^8 - a^2*b^4*c^2*d^2*e^3*f*x^8 + a^3*b^3*c*d^3*e^3*f*x^8 - 8*a
^4*b^2*d^4*e^3*f*x^8 + 10*a^2*b^4*c^3*d*e^2*f^2*x^8 - 10*a^4*b^2*c*d^3*e^2
*f^2*x^8 + 15*a^3*b^3*c^3*d*e*f^3*x^8 - 15*a^4*b^2*c^2*d^2*e*f^3*x^8 - I*a
*b*Sqrt[b/a]*c*e*(48*b^4*c^3*e^3 - 8*a*b^3*c^2*e^2*(2*d*e + c*f) + a^2*b^2
*c*e*(-9*d^2*e^2 + c*d*e*f - 10*c^2*f^2) + a^4*d*f*(8*d^2*e^2 + 10*c*d*...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6 (a+bx^2)^{3/2} \sqrt{c+dx^2} (e+fx^2)} dx$$

$$\Big\downarrow \text{450}$$

$$\int \frac{1}{x^6 (a+bx^2)^{3/2} \sqrt{c+dx^2} (e+fx^2)} dx$$

input
```
Int[1/(x^6*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2]*(e + f*x^2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [A] (verified)

Time = 26.81 (sec) , antiderivative size = 1113, normalized size of antiderivative = 1.41

method	result	size
risch	Expression too large to display	1113
elliptic	Expression too large to display	1793
default	Expression too large to display	3169

input
```
int(1/x^6/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x,method=_RETURNVERBOS
E)
```

output
```
-1/15*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(15*a^2*c^2*f^2*x^4+10*a^2*c*d*e*f*x
^4+8*a^2*d^2*e^2*x^4+25*a*b*c^2*e*f*x^4+17*a*b*c*d*e^2*x^4+33*b^2*c^2*e^2*
x^4-5*a^2*c^2*e*f*x^2-4*a^2*c*d*e^2*x^2-9*a*b*c^2*e^2*x^2+3*a^2*c^2*e^2)/a
^4/c^3/e^3/x^5+1/15/e^3/a^4/c^3*(-b*(15*a^2*c^2*f^2+10*a^2*c*d*e*f+8*a^2*d
^2*e^2+25*a*b*c^2*e*f+17*a*b*c*d*e^2+33*b^2*c^2*e^2)*c/(-b/a)^(1/2)*(1+b*x
^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(Ellipti
cF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(
a*d+b*c)/c/b)^(1/2)))+9*a*b^2*c^2*d*e^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+
d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2
),(-1+(a*d+b*c)/c/b)^(1/2))+4*a^2*b*c*d^2*e^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/
2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a
)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-15*a^4*c^3*f^4/(a*f-b*e)/e/(-b/a)^(1/2)*
(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*El
lipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))+15*a*b^4*c^3*
e^3/(a*f-b*e)*(-(b*d*x^2+b*c)/a/(a*d-b*c)*x/((x^2+a/b)*(b*d*x^2+b*c))^(1/2
)+(1/a+b*c/(a*d-b*c)/a)/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(
b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/
c/b)^(1/2))-b/(a*d-b*c)/a*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/
2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+
b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))))+5...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^6 (a+bx^2)^{3/2} \sqrt{c+dx^2} (e+fx^2)} dx = \text{Timed out}$$

input
```
integrate(1/x^6/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{x^6 \left(a+bx^2\right)^{3/2} \sqrt{c+dx^2} \left(e+fx^2\right)} \, dx = \int \frac{1}{x^6 \left(a+bx^2\right)^{\frac{3}{2}} \sqrt{c+dx^2} \left(e+fx^2\right)} \, dx$$

input
```
integrate(1/x**6/(b*x**2+a)**(3/2)/(d*x**2+c)**(1/2)/(f*x**2+e),x)
```

output
```
Integral(1/(x**6*(a + b*x**2)**(3/2)*sqrt(c + d*x**2)*(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{x^6 \left(a+bx^2\right)^{3/2} \sqrt{c+dx^2} \left(e+fx^2\right)} \, dx = \int \frac{1}{\left(bx^2+a\right)^{\frac{3}{2}} \sqrt{dx^2+c}\left(fx^2+e\right)x^6} \, dx$$

input
```
integrate(1/x^6/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="ma
xima")
```

output
```
integrate(1/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*(f*x^2 + e)*x^6), x)
```

Giac [F]

$$\int \frac{1}{x^6 \left(a+bx^2\right)^{3/2} \sqrt{c+dx^2} \left(e+fx^2\right)} \, dx = \int \frac{1}{\left(bx^2+a\right)^{\frac{3}{2}} \sqrt{dx^2+c}\left(fx^2+e\right)x^6} \, dx$$

input
```
integrate(1/x^6/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="gi
ac")
```

output
```
integrate(1/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*(f*x^2 + e)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^6 (a+bx^2)^{3/2} \sqrt{c+dx^2} (e+fx^2)} \, dx = \int \frac{1}{x^6 \left(b x^2+a\right)^{3/2} \sqrt{d x^2+c} \left(f x^2+e\right)} \, dx$$

input
```
int(1/(x^6*(a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)*(e + f*x^2)),x)
```

output
```
int(1/(x^6*(a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)*(e + f*x^2)), x)
```

Reduce [F]

$$\int \frac{1}{x^6 (a+bx^2)^{3/2} \sqrt{c+dx^2} (e+fx^2)} \, dx = \text{too large to display}$$

input
```
int(1/x^6/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x)
```

output
```
( - 3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*c*e + 5*sqrt(c + d*x**2)*sqrt(a
+ b*x**2)*a*c*f*x**2 + 4*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*d*e*x**2 + 12
*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*d*f*x**4 + 6*sqrt(c + d*x**2)*sqrt(a
+ b*x**2)*b*c*e*x**2 + 18*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b*c*f*x**4 + 1
5*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b*d*e*x**4 + 12*int((sqrt(c + d*x**2)*
sqrt(a + b*x**2)*x**4)/(a**2*c*e + a**2*c*f*x**2 + a**2*d*e*x**2 + a**2*d*
f*x**4 + 2*a*b*c*e*x**2 + 2*a*b*c*f*x**4 + 2*a*b*d*e*x**4 + 2*a*b*d*f*x**6
 + b**2*c*e*x**4 + b**2*c*f*x**6 + b**2*d*e*x**6 + b**2*d*f*x**8),x)*a**2*
b*d**2*f**2*x**5 + 18*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a**2*c
*e + a**2*c*f*x**2 + a**2*d*e*x**2 + a**2*d*f*x**4 + 2*a*b*c*e*x**2 + 2*a*
b*c*f*x**4 + 2*a*b*d*e*x**4 + 2*a*b*d*f*x**6 + b**2*c*e*x**4 + b**2*c*f*x*
*6 + b**2*d*e*x**6 + b**2*d*f*x**8),x)*a*b**2*c*d*f**2*x**5 + 15*int((sqrt
(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a**2*c*e + a**2*c*f*x**2 + a**2*d*e*x
**2 + a**2*d*f*x**4 + 2*a*b*c*e*x**2 + 2*a*b*c*f*x**4 + 2*a*b*d*e*x**4 + 2
*a*b*d*f*x**6 + b**2*c*e*x**4 + b**2*c*f*x**6 + b**2*d*e*x**6 + b**2*d*f*x
**8),x)*a*b**2*d**2*e*f*x**5 + 12*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x
**4)/(a**2*c*e + a**2*c*f*x**2 + a**2*d*e*x**2 + a**2*d*f*x**4 + 2*a*b*c*e
*x**2 + 2*a*b*c*f*x**4 + 2*a*b*d*e*x**4 + 2*a*b*d*f*x**6 + b**2*c*e*x**4 +
 b**2*c*f*x**6 + b**2*d*e*x**6 + b**2*d*f*x**8),x)*a*b**2*d**2*f**2*x**7 +
 18*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a**2*c*e + a**2*c*f*x...
```

3.256 $\int \frac{x^8}{(a+bx^2)^{3/2}\sqrt{c+dx^2}(e+fx^2)^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 634

$$\int \frac{x^8}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \frac{x\sqrt{c+dx^2}}{bdf^2\sqrt{a+bx^2}}$$
$$+\frac{e^3x\sqrt{c+dx^2}}{2f^2(be-af)(de-cf)\sqrt{a+bx^2}\,(e+fx^2)}$$
$$-\frac{\sqrt{a}(b^3ce^2(3de-2cf)-4a^3df^2(de-cf)-ab^2e(3d^2e^2+2cdef-4c^2f^2)+2a^2bf(2d^2e^2-cdef-c^2f^2))}{2b^{3/2}d(bc-ad)f^2(be-af)^2(de-cf)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$
$$+\frac{a^{3/2}(3b^3ce^3+2a^3cf^3+2a^2bef(3de-cf)-3ab^2e^2(de+2cf))\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{2b^{3/2}c(bc-ad)f^2(be-af)^3\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$
$$+\frac{a^{3/2}e^2(af(6de-7cf)-be(3de-4cf))\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{2\sqrt{b}cf^2(be-af)^3(de-cf)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
x*(d*x^2+c)^(1/2)/b/d/f^2/(b*x^2+a)^(1/2)+1/2*e^3*x*(d*x^2+c)^(1/2)/f^2/(-
a*f+b*e)/(-c*f+d*e)/(b*x^2+a)^(1/2)/(f*x^2+e)-1/2*a^(1/2)*(b^3*c*e^2*(-2*c
*f+3*d*e)-4*a^3*d*f^2*(-c*f+d*e)-a*b^2*e*(-4*c^2*f^2+2*c*d*e*f+3*d^2*e^2)+
2*a^2*b*f*(-c^2*f^2-c*d*e*f+2*d^2*e^2))*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*
x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/b^(3/2)/d/(-a*d+b*c)/f^2/(-
a*f+b*e)^2/(-c*f+d*e)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)+1/2*
a^(3/2)*(3*b^3*c*e^3+2*a^3*c*f^3+2*a^2*b*e*f*(-c*f+3*d*e)-3*a*b^2*e^2*(2*c
*f+d*e))*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/
b/c)^(1/2))/b^(3/2)/c/(-a*d+b*c)/f^2/(-a*f+b*e)^3/(b*x^2+a)^(1/2)/(a*(d*x^
2+c)/c/(b*x^2+a))^(1/2)+1/2*a^(3/2)*e^2*(a*f*(-7*c*f+6*d*e)-b*e*(-4*c*f+3*
d*e))*(d*x^2+c)^(1/2)*EllipticPi(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),1-a*f
/b/e,(1-a*d/b/c)^(1/2))/b^(1/2)/c/f^2/(-a*f+b*e)^3/(-c*f+d*e)/(b*x^2+a)^(1
/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.89 (sec) , antiderivative size = 1797, normalized size of antiderivative = 2.83

$$\int \frac{x^8}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \text{Too large to display}$$

input
```
Integrate[x^8/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]*(e + f*x^2)^2),x]
```

output
```
(-(a*b^2*Sqrt[b/a]*c^2*d*e^3*f^2*x) + a^3*(b/a)^(3/2)*c*d^2*e^3*f^2*x + 2*
a^3*Sqrt[b/a]*c*d^2*e^2*f^3*x - 2*a^3*Sqrt[b/a]*c^2*d*e*f^4*x - b^3*Sqrt[b
/a]*c^2*d*e^3*f^2*x^3 + a^3*(b/a)^(3/2)*d^3*e^3*f^2*x^3 + 2*a^3*Sqrt[b/a]*
d^3*e^2*f^3*x^3 - 2*a^3*Sqrt[b/a]*c^2*d*f^5*x^3 - b^3*Sqrt[b/a]*c*d^2*e^3*
f^2*x^5 + a*b^2*Sqrt[b/a]*d^3*e^3*f^2*x^5 + 2*a^3*Sqrt[b/a]*d^3*e*f^4*x^5
- 2*a^3*Sqrt[b/a]*c*d^2*f^5*x^5 - I*c*f*(b^3*c*e^2*(3*d*e - 2*c*f) + 4*a^3
*d*f^2*(-(d*e) + c*f) - 2*a^2*b*f*(-2*d^2*e^2 + c*d*e*f + c^2*f^2) + a*b^2
*e*(-3*d^2*e^2 - 2*c*d*e*f + 4*c^2*f^2))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x
^2)/c]*(e + f*x^2)*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + I*(-(b
*c) + a*d)*(-(d*e) + c*f)*(2*a^2*c*f^3 + b^2*e^2*(3*d*e + 2*c*f) - 2*a*b*e
*f*(3*d*e + 2*c*f))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(e + f*x^2)*El
lipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - (3*I)*b^3*c*d^2*e^5*Sqrt[1
+ (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/
a]*x], (a*d)/(b*c)] + (3*I)*a*b^2*d^3*e^5*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*
x^2)/c]*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (4*
I)*b^3*c^2*d*e^4*f*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f
)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (2*I)*a*b^2*c*d^2*e^4*f*Sq
rt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sq
rt[b/a]*x], (a*d)/(b*c)] - (6*I)*a^2*b*d^3*e^4*f*Sqrt[1 + (b*x^2)/a]*Sqrt[
1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx$$
$$\Big\downarrow \text{450}$$
$$\int \frac{x^8}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^2}dx$$

input
```
Int[x^8/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]*(e + f*x^2)^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1729 vs. $2(604) = 1208$.

Time = 11.15 (sec) , antiderivative size = 1730, normalized size of antiderivative = 2.73

method	result	size
elliptic	Expression too large to display	1730
default	Expression too large to display	4421

input
```
int(x^8/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x,method=_RETURNVERBOS
E)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-(b*d*x^2+b*c
)/b^2*a^3/(a*d-b*c)*x/(a*f-b*e)^2/((x^2+a/b)*(b*d*x^2+b*c))^(1/2)+1/2/f/(a
*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)*e^3*x/(a*f-b*e)*(b*d*x^4+a*d*x^2+b*c*x^2+a
*c)^(1/2)/(f*x^2+e)+c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*
d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*a^3/b/(a*d-b*c)/(a*f-b*e)^2*EllipticE(x*(
-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-7/2*e^2/(a*c*f^2-a*d*e*f-b*c*e*f+b*d
*e^2)/(a*f-b*e)/f/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^
4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1
/2)/(-b/a)^(1/2))*a*c+3*e^3/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/f^2/(a*f-b*e
)/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^
2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2)
)*a*d+2*e^3/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/f^2/(a*f-b*e)/(-b/a)^(1/2)*(
1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*Ell
ipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*b*c-3/2*e^4/(a
*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/f^3/(a*f-b*e)/(-b/a)^(1/2)*(1+b*x^2/a)^(1/
2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/
a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*b*d+1/2*c/(-b/a)^(1/2)*(1+b*
x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*b/f^2*e
^3/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/(a*f-b*e)*EllipticF(x*(-b/a)^(1/2),(-
1+(a*d+b*c)/c/b)^(1/2))-1/2*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^8}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(x^8/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^8}{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}\left(e+fx^2\right)^2}\,dx = \int \frac{x^8}{\left(a+bx^2\right)^{\frac{3}{2}}\sqrt{c+dx^2}\left(e+fx^2\right)^2}\,dx$$

input
```
integrate(x**8/(b*x**2+a)**(3/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**2,x)
```

output
```
Integral(x**8/((a + b*x**2)**(3/2)*sqrt(c + d*x**2)*(e + f*x**2)**2), x)
```

Maxima [F]

$$\int \frac{x^8}{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}\left(e+fx^2\right)^2}\,dx = \int \frac{x^8}{\left(bx^2+a\right)^{\frac{3}{2}}\sqrt{dx^2+c}\left(fx^2+e\right)^2}\,dx$$

input
```
integrate(x^8/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="ma
xima")
```

output
```
integrate(x^8/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*(f*x^2 + e)^2), x)
```

Giac [F]

$$\int \frac{x^8}{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}\left(e+fx^2\right)^2}\,dx = \int \frac{x^8}{\left(bx^2+a\right)^{\frac{3}{2}}\sqrt{dx^2+c}\left(fx^2+e\right)^2}\,dx$$

input
```
integrate(x^8/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="gi
ac")
```

output
```
integrate(x^8/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*(f*x^2 + e)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^8}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{x^8}{(b\,x^2+a)^{3/2}\,\sqrt{d\,x^2+c}\,(f\,x^2+e)^2}\,dx$$

input
```
int(x^8/((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^2),x)
```

output
```
int(x^8/((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^2), x)
```

Reduce [F]

$$\int \frac{x^8}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{x^8}{(b\,x^2+a)^{\frac{3}{2}}\,\sqrt{d\,x^2+c}\,(f\,x^2+e)^2}dx$$

input
```
int(x^8/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x)
```

output
```
int(x^8/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x)
```

3.257 $\int \frac{x^6}{(a+bx^2)^{3/2}\sqrt{c+dx^2}(e+fx^2)^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 514

$$\int \frac{x^6}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = -\frac{e^2x\sqrt{c+dx^2}}{2f(be-af)(de-cf)\sqrt{a+bx^2}\,(e+fx^2)}$$
$$+\frac{\sqrt{a}(b^2ce^2-abde^2-2a^2f(de-cf))\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{2\sqrt{b}(bc-ad)f(be-af)^2(de-cf)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$
$$-\frac{a^{3/2}(b^2ce^2+2a^2f(2de+cf)-abe(de+6cf))\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{2\sqrt{b}c(bc-ad)f(be-af)^3\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$
$$-\frac{a^{3/2}e(af(4de-5cf)-be(de-2cf))\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{2\sqrt{b}cf(be-af)^3(de-cf)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
-1/2*e^2*x*(d*x^2+c)^(1/2)/f/(-a*f+b*e)/(-c*f+d*e)/(b*x^2+a)^(1/2)/(f*x^2+
e)+1/2*a^(1/2)*(b^2*c*e^2-a*b*d*e^2-2*a^2*f*(-c*f+d*e))*(d*x^2+c)^(1/2)*El
lipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/b^(1/2)/(-a
*d+b*c)/f/(-a*f+b*e)^2/(-c*f+d*e)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a)
)^(1/2)-1/2*a^(3/2)*(b^2*c*e^2+2*a^2*f*(c*f+2*d*e)-a*b*e*(6*c*f+d*e))*(d*x
^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/b
^(1/2)/c/(-a*d+b*c)/f/(-a*f+b*e)^3/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a
))^(1/2)-1/2*a^(3/2)*e*(a*f*(-5*c*f+4*d*e)-b*e*(-2*c*f+d*e))*(d*x^2+c)^(1/
2)*EllipticPi(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),1-a*f/b/e,(1-a*d/b/c)^(1
/2))/b^(1/2)/c/f/(-a*f+b*e)^3/(-c*f+d*e)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b
*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.12 (sec) , antiderivative size = 1596, normalized size of antiderivative = 3.11

$$\int \frac{x^6}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \text{Too large to display}$$

input
```
Integrate[x^6/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]*(e + f*x^2)^2),x]
```

output
```
(a*b*Sqrt[b/a]*c^2*e^2*f^2*x - 3*a^2*Sqrt[b/a]*c*d*e^2*f^2*x + 2*a^2*Sqrt[
b/a]*c^2*e*f^3*x + a*b*(b/a)^(3/2)*c^2*e^2*f^2*x^3 - 3*a^2*Sqrt[b/a]*d^2*e
^2*f^2*x^3 + 2*a^2*Sqrt[b/a]*c^2*f^4*x^3 + a*b*(b/a)^(3/2)*c*d*e^2*f^2*x^5
 - a*b*Sqrt[b/a]*d^2*e^2*f^2*x^5 - 2*a^2*Sqrt[b/a]*d^2*e*f^3*x^5 + 2*a^2*S
qrt[b/a]*c*d*f^4*x^5 + I*c*f*(b^2*c*e^2 - a*b*d*e^2 + 2*a^2*f*(-(d*e) + c*
f))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(e + f*x^2)*EllipticE[I*ArcSin
h[Sqrt[b/a]*x], (a*d)/(b*c)] + I*(-(b*c) + a*d)*e*(-(b*e) + 4*a*f)*(-(d*e)
 + c*f)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(e + f*x^2)*EllipticF[I*Ar
cSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + I*b^2*c*d*e^4*Sqrt[1 + (b*x^2)/a]*Sqrt[
1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)
] - I*a*b*d^2*e^4*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)
/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - (2*I)*b^2*c^2*e^3*f*Sqrt[1
+ (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/
a]*x], (a*d)/(b*c)] - (2*I)*a*b*c*d*e^3*f*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*
x^2)/c]*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (4*
I)*a^2*d^2*e^3*f*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/
(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (5*I)*a*b*c^2*e^2*f^2*Sqrt[1
 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b
/a]*x], (a*d)/(b*c)] - (5*I)*a^2*c*d*e^2*f^2*Sqrt[1 + (b*x^2)/a]*Sqrt[1 +
(d*x^2)/c]*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx$$
$$\Big\downarrow \; 450$$
$$\int \frac{x^6}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^2}dx$$

input
```
Int[x^6/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]*(e + f*x^2)^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1430 vs. $2(488) = 976$.

Time = 10.99 (sec) , antiderivative size = 1431, normalized size of antiderivative = 2.78

method	result	size
elliptic	Expression too large to display	1431
default	Expression too large to display	2731

input
```
int(x^6/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x,method=_RETURNVERBOS
E)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*((b*d*x^2+b*c)
/b*a^2/(a*d-b*c)*x/(a*f-b*e)^2/((x^2+a/b)*(b*d*x^2+b*c))^(1/2)-1/2/(a*c*f^
2-a*d*e*f-b*c*e*f+b*d*e^2)*e^2*x/(a*f-b*e)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(
1/2)/(f*x^2+e)+1/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4
+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1
/2))/f^2/b-1/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d
*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))
/b*a^2/(a*f-b*e)^2+1/2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b
*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c
/b)^(1/2))*b*d/f^2*e^3/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/(a*f-b*e)-c/(-b/a
)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^
(1/2)*a^2/(a*d-b*c)/(a*f-b*e)^2*EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b
)^(1/2))-1/2*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a
*d*x^2+b*c*x^2+a*c)^(1/2)*b*e^2/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/(a*f-b*e
)/f*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+1/2*c/(-b/a)^(1/2)*
(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*b*
e^2/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/(a*f-b*e)/f*EllipticE(x*(-b/a)^(1/2)
,(-1+(a*d+b*c)/c/b)^(1/2))+5/2*e/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/(a*f-b*
e)/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x
^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^6}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(x^6/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^6}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{x^6}{(a+bx^2)^{\frac{3}{2}}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx$$

input
```
integrate(x**6/(b*x**2+a)**(3/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**2,x)
```

output
```
Integral(x**6/((a + b*x**2)**(3/2)*sqrt(c + d*x**2)*(e + f*x**2)**2), x)
```

Maxima [F]

$$\int \frac{x^6}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{x^6}{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}(fx^2+e)^2}\,dx$$

input
```
integrate(x^6/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="ma
xima")
```

output
```
integrate(x^6/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*(f*x^2 + e)^2), x)
```

Giac [F]

$$\int \frac{x^6}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{x^6}{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}(fx^2+e)^2}\,dx$$

input
```
integrate(x^6/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="gi
ac")
```

output
```
integrate(x^6/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*(f*x^2 + e)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{x^6}{(b\,x^2+a)^{3/2}\,\sqrt{d\,x^2+c}\,(f\,x^2+e)^2}\,dx$$

input
```
int(x^6/((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^2),x)
```

output
```
int(x^6/((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^2), x)
```

Reduce [F]

$$\int \frac{x^6}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{}{b^2d\,f^2x^{10}+2abd\,f^2x^8+b^2c\,f^2x^8+2b^2def\,x^8+a^2d\,f^2x^6+2a}$$

input
```
int(x^6/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**6)/(a**2*c*e**2 + 2*a**2*c*e*f*x
**2 + a**2*c*f**2*x**4 + a**2*d*e**2*x**2 + 2*a**2*d*e*f*x**4 + a**2*d*f**
2*x**6 + 2*a*b*c*e**2*x**2 + 4*a*b*c*e*f*x**4 + 2*a*b*c*f**2*x**6 + 2*a*b*
d*e**2*x**4 + 4*a*b*d*e*f*x**6 + 2*a*b*d*f**2*x**8 + b**2*c*e**2*x**4 + 2*
b**2*c*e*f*x**6 + b**2*c*f**2*x**8 + b**2*d*e**2*x**6 + 2*b**2*d*e*f*x**8
+ b**2*d*f**2*x**10),x)
```

3.258 $\int \frac{x^4}{(a+bx^2)^{3/2}\sqrt{c+dx^2}(e+fx^2)^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 462

$$\int \frac{x^4}{(a+bx^2)^{3/2}\sqrt{c+dx^2}(e+fx^2)^2}\,dx = \frac{ex\sqrt{c+dx^2}}{2(be-af)(de-cf)\sqrt{a+bx^2}(e+fx^2)} - \frac{\sqrt{a}\sqrt{b}(bce-3ade+2acf)\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{2(bc-ad)(be-af)^2(de-cf)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} - \frac{a^{3/2}(3b^2ce-abde-2a^2df)\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{2\sqrt{b}c(bc-ad)(be-af)^3\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} + \frac{a^{3/2}(bde^2+af(2de-3cf))\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{2\sqrt{b}c(be-af)^3(de-cf)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
1/2*e*x*(d*x^2+c)^(1/2)/(-a*f+b*e)/(-c*f+d*e)/(b*x^2+a)^(1/2)/(f*x^2+e)-1/
2*a^(1/2)*b^(1/2)*(2*a*c*f-3*a*d*e+b*c*e)*(d*x^2+c)^(1/2)*EllipticE(b^(1/2
)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/(-a*d+b*c)/(-a*f+b*e)^2/(
-c*f+d*e)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)-1/2*a^(3/2)*(-2*
a^2*d*f-a*b*d*e+3*b^2*c*e)*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*
x/a^(1/2)),(1-a*d/b/c)^(1/2))/b^(1/2)/c/(-a*d+b*c)/(-a*f+b*e)^3/(b*x^2+a)^
(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)+1/2*a^(3/2)*(b*d*e^2+a*f*(-3*c*f+2*d
*e))*(d*x^2+c)^(1/2)*EllipticPi(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),1-a*f/
b/e,(1-a*d/b/c)^(1/2))/b^(1/2)/c/(-a*f+b*e)^3/(-c*f+d*e)/(b*x^2+a)^(1/2)/(
a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.88 (sec) , antiderivative size = 1394, normalized size of antiderivative = 3.02

$$\int \frac{x^4}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \text{Too large to display}$$

input
```
Integrate[x^4/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]*(e + f*x^2)^2),x]
```

output
```
(2*a*b*Sqrt[b/a]*c*d*e^2*f*x - 3*a*b*Sqrt[b/a]*c^2*e*f^2*x + a^2*Sqrt[b/a]
*c*d*e*f^2*x + 2*a*b*Sqrt[b/a]*d^2*e^2*f*x^3 - a*b*(b/a)^(3/2)*c^2*e*f^2*x
^3 + a^2*Sqrt[b/a]*d^2*e*f^2*x^3 - 2*a*b*Sqrt[b/a]*c^2*f^3*x^3 - a*b*(b/a)
^(3/2)*c*d*e*f^2*x^5 + 3*a*b*Sqrt[b/a]*d^2*e*f^2*x^5 - 2*a*b*Sqrt[b/a]*c*d
*f^3*x^5 - I*b*c*f*(b*c*e - 3*a*d*e + 2*a*c*f)*Sqrt[1 + (b*x^2)/a]*Sqrt[1
+ (d*x^2)/c]*(e + f*x^2)*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] -
I*(-(b*c) + a*d)*(b*e + 2*a*f)*(-(d*e) + c*f)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 +
 (d*x^2)/c]*(e + f*x^2)*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + I
*b^2*c*d*e^3*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e
), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*a*b*d^2*e^3*Sqrt[1 + (b*x^2)/a
]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d
)/(b*c)] + (2*I)*a*b*c*d*e^2*f*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Ell
ipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - (2*I)*a^2*d^2*
e^2*f*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e), I*Ar
cSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - (3*I)*a*b*c^2*e*f^2*Sqrt[1 + (b*x^2)/a]
*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)
/(b*c)] + (3*I)*a^2*c*d*e*f^2*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Elli
pticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + I*b^2*c*d*e^2*f
*x^2*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e), I*Arc
Sinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*a*b*d^2*e^2*f*x^2*Sqrt[1 + (b*x^2)/...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx$$
$$\Big\downarrow 450$$
$$\int \frac{x^4}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^2}dx$$

input
```
Int[x^4/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]*(e + f*x^2)^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1169 vs. $2(436) = 872$.

Time = 11.16 (sec) , antiderivative size = 1170, normalized size of antiderivative = 2.53

method	result	size
elliptic	Expression too large to display	1170
default	Expression too large to display	2409

input
```
int(x^4/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x,method=_RETURNVERBOS
E)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-(b*d*x^2+b*c
)*a/(a*d-b*c)*x/(a*f-b*e)^2/((x^2+a/b)*(b*d*x^2+b*c))^(1/2)+1/2*f/(a*c*f^2
-a*d*e*f-b*c*e*f+b*d*e^2)*e*x/(a*f-b*e)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2
)/(f*x^2+e)+1/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*
d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2)
)*a/(a*f-b*e)^2-1/2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*
x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)
^(1/2))*b*d*e^2/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/(a*f-b*e)/f+c/(-b/a)^(1/
2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
*a*b/(a*f-b*e)^2/(a*d-b*c)*EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/
2))+1/2*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^
2+b*c*x^2+a*c)^(1/2)*b*e/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/(a*f-b*e)*Ellip
ticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-1/2*c/(-b/a)^(1/2)*(1+b*x^2/
a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*b*e/(a*c*f^
2-a*d*e*f-b*c*e*f+b*d*e^2)/(a*f-b*e)*EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c
)/c/b)^(1/2))-3/2/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)*f/(a*f-b*e)/(-b/a)^(1/
2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*a*c+e/(a*c
*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/(a*f-b*e)/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+
d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^4}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(x^4/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^4}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{x^4}{(a+bx^2)^{\frac{3}{2}}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx$$

input
```
integrate(x**4/(b*x**2+a)**(3/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**2,x)
```

output
```
Integral(x**4/((a + b*x**2)**(3/2)*sqrt(c + d*x**2)*(e + f*x**2)**2), x)
```

Maxima [F]

$$\int \frac{x^4}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{x^4}{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}(fx^2+e)^2}\,dx$$

input
```
integrate(x^4/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="ma
xima")
```

output
```
integrate(x^4/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*(f*x^2 + e)^2), x)
```

Giac [F]

$$\int \frac{x^4}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{x^4}{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}(fx^2+e)^2}\,dx$$

input
```
integrate(x^4/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="gi
ac")
```

output
```
integrate(x^4/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*(f*x^2 + e)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx=\int \frac{x^4}{{\left(b\,x^2+a\right)}^{3/2}\,\sqrt{d\,x^2+c}\,{\left(f\,x^2+e\right)}^2}\,dx$$

input
```
int(x^4/((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^2),x)
```

output
```
int(x^4/((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^2), x)
```

Reduce [F]

$$\int \frac{x^4}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx=\int \frac{}{b^2d\,f^2x^{10}+2abd\,f^2x^8+b^2c\,f^2x^8+2b^2def\,x^8+a^2d\,f^2x^6+2a}$$

input
```
int(x^4/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a**2*c*e**2 + 2*a**2*c*e*f*x
**2 + a**2*c*f**2*x**4 + a**2*d*e**2*x**2 + 2*a**2*d*e*f*x**4 + a**2*d*f**
2*x**6 + 2*a*b*c*e**2*x**2 + 4*a*b*c*e*f*x**4 + 2*a*b*c*f**2*x**6 + 2*a*b*
d*e**2*x**4 + 4*a*b*d*e*f*x**6 + 2*a*b*d*f**2*x**8 + b**2*c*e**2*x**4 + 2*
b**2*c*e*f*x**6 + b**2*c*f**2*x**8 + b**2*d*e**2*x**6 + 2*b**2*d*e*f*x**8
+ b**2*d*f**2*x**10),x)
```

3.259 $\int \frac{x^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}(e+fx^2)^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 466

$$\int \frac{x^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = -\frac{fx\sqrt{c+dx^2}}{2(be-af)(de-cf)\sqrt{a+bx^2}\,(e+fx^2)} - \frac{\sqrt{a}\sqrt{b}(2bde-3bcf+adf)\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{2(bc-ad)(be-af)^2(de-cf)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} + \frac{\sqrt{a}\sqrt{b}(2b^2ce+abcf-3a^2df)\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{2c(bc-ad)(be-af)^3\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} + \frac{a^{3/2}f(acf^2-be(3de-2cf))\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{2\sqrt{b}ce(be-af)^3(de-cf)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
-1/2*f*x*(d*x^2+c)^(1/2)/(-a*f+b*e)/(-c*f+d*e)/(b*x^2+a)^(1/2)/(f*x^2+e)-1
/2*a^(1/2)*b^(1/2)*(a*d*f-3*b*c*f+2*b*d*e)*(d*x^2+c)^(1/2)*EllipticE(b^(1/
2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/(-a*d+b*c)/(-a*f+b*e)^2/
(-c*f+d*e)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)+1/2*a^(1/2)*b^(
1/2)*(-3*a^2*d*f+a*b*c*f+2*b^2*c*e)*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan
(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/c/(-a*d+b*c)/(-a*f+b*e)^3/(b*x^2+a)
^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)+1/2*a^(3/2)*f*(a*c*f^2-b*e*(-2*c*f+
3*d*e))*(d*x^2+c)^(1/2)*EllipticPi(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),1-a
*f/b/e,(1-a*d/b/c)^(1/2))/b^(1/2)/c/e/(-a*f+b*e)^3/(-c*f+d*e)/(b*x^2+a)^(1
/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.38 (sec) , antiderivative size = 1388, normalized size of antiderivative = 2.98

$$\int \frac{x^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \text{Too large to display}$$

input
```
Integrate[x^2/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]*(e + f*x^2)^2),x]
```

output
```
(-2*a*b*(b/a)^(3/2)*c*d*e^3*x + 2*a*b*(b/a)^(3/2)*c^2*e^2*f*x + a*b*Sqrt[b
/a]*c^2*e*f^2*x - a^2*Sqrt[b/a]*c*d*e*f^2*x - 2*a*b*(b/a)^(3/2)*d^2*e^3*x^
3 + 3*a*b*(b/a)^(3/2)*c^2*e*f^2*x^3 - a^2*Sqrt[b/a]*d^2*e*f^2*x^3 - 2*a*b*
(b/a)^(3/2)*d^2*e^2*f*x^5 + 3*a*b*(b/a)^(3/2)*c*d*e*f^2*x^5 - a*b*Sqrt[b/a
]*d^2*e*f^2*x^5 - I*b*c*e*(2*b*d*e - 3*b*c*f + a*d*f)*Sqrt[1 + (b*x^2)/a]*
Sqrt[1 + (d*x^2)/c]*(e + f*x^2)*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b
*c)] + (3*I)*b*(-(b*c) + a*d)*e*(-(d*e) + c*f)*Sqrt[1 + (b*x^2)/a]*Sqrt[1
+ (d*x^2)/c]*(e + f*x^2)*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] -
(3*I)*b^2*c*d*e^3*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)
/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (3*I)*a*b*d^2*e^3*Sqrt[1 +
(b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]
*x], (a*d)/(b*c)] + (2*I)*b^2*c^2*e^2*f*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^
2)/c]*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - (2*I)
*a*b*c*d*e^2*f*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b
*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + I*a*b*c^2*e*f^2*Sqrt[1 + (b*x^
2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x],
(a*d)/(b*c)] - I*a^2*c*d*e*f^2*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Ell
ipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - (3*I)*b^2*c*d*
e^2*f*x^2*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e),
I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (3*I)*a*b*d^2*e^2*f*x^2*Sqrt[1 +...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx$$

$$\Big\downarrow \ 450$$

$$\int \frac{x^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^2}dx$$

input
```
Int[x^2/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]*(e + f*x^2)^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1167 vs. $2(440) = 880$.

Time = 9.78 (sec) , antiderivative size = 1168, normalized size of antiderivative = 2.51

method	result	size
elliptic	Expression too large to display	1168
default	Expression too large to display	2052

input
```
int(x^2/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x,method=_RETURNVERBOS
E)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*((b*d*x^2+b*c)
*b/(a*d-b*c)*x/(a*f-b*e)^2/((x^2+a/b)*(b*d*x^2+b*c))^(1/2)-1/2*f^2/(a*c*f^
2-a*d*e*f-b*c*e*f+b*d*e^2)*x/(a*f-b*e)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
/(f*x^2+e)-1/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d
*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))
*b/(a*f-b*e)^2+1/2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x
^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^
(1/2))*b*d*e/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/(a*f-b*e)-c/(-b/a)^(1/2)*(1
+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*b^2/
(a*d-b*c)/(a*f-b*e)^2*EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-1
/2*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c
*x^2+a*c)^(1/2)*b*f/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/(a*f-b*e)*EllipticF(
x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+1/2*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1
/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*b*f/(a*c*f^2-a*d
*e*f-b*c*e*f+b*d*e^2)/(a*f-b*e)*EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b
)^(1/2))+1/2/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/(a*f-b*e)/e*f^2/(-b/a)^(1/2
)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*
EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*a*c+1/(a*c*
f^2-a*d*e*f-b*c*e*f+b*d*e^2)/(a*f-b*e)*f/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1
+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(x^2/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{x^2}{(a+bx^2)^{\frac{3}{2}}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx$$

input
```
integrate(x**2/(b*x**2+a)**(3/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**2,x)
```

output
```
Integral(x**2/((a + b*x**2)**(3/2)*sqrt(c + d*x**2)*(e + f*x**2)**2), x)
```

Maxima [F]

$$\int \frac{x^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{x^2}{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}(fx^2+e)^2}\,dx$$

input
```
integrate(x^2/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="ma
xima")
```

output
```
integrate(x^2/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*(f*x^2 + e)^2), x)
```

Giac [F]

$$\int \frac{x^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{x^2}{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}(fx^2+e)^2}\,dx$$

input
```
integrate(x^2/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="gi
ac")
```

output
```
integrate(x^2/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*(f*x^2 + e)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{x^2}{(b\,x^2+a)^{3/2}\,\sqrt{d\,x^2+c}\,(f\,x^2+e)^2}\,dx$$

input
```
int(x^2/((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^2),x)
```

output
```
int(x^2/((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^2), x)
```

Reduce [F]

$$\int \frac{x^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{}{b^2d\,f^2x^{10}+2abd\,f^2x^8+b^2c\,f^2x^8+2b^2def\,x^8+a^2d\,f^2x^6+2a}$$

input
```
int(x^2/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a**2*c*e**2 + 2*a**2*c*e*f*x
**2 + a**2*c*f**2*x**4 + a**2*d*e**2*x**2 + 2*a**2*d*e*f*x**4 + a**2*d*f**
2*x**6 + 2*a*b*c*e**2*x**2 + 4*a*b*c*e*f*x**4 + 2*a*b*c*f**2*x**6 + 2*a*b*
d*e**2*x**4 + 4*a*b*d*e*f*x**6 + 2*a*b*d*f**2*x**8 + b**2*c*e**2*x**4 + 2*
b**2*c*e*f*x**6 + b**2*c*f**2*x**8 + b**2*d*e**2*x**6 + 2*b**2*d*e*f*x**8
+ b**2*d*f**2*x**10),x)
```

3.260 $\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}(e+fx^2)^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 517

$$\begin{aligned}
\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx &= \frac{f^2x\sqrt{c+dx^2}}{2e(be-af)(de-cf)\sqrt{a+bx^2}\,(e+fx^2)} \\
&- \frac{\sqrt{b}(abcf^2-a^2df^2-2b^2e(de-cf))\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{2\sqrt{a}(bc-ad)e(be-af)^2(de-cf)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} \\
&- \frac{\sqrt{a}\sqrt{b}(a^2df^2-abf(6de+cf)+2b^2e(de+2cf))\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{2c(bc-ad)e(be-af)^3\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} \\
&+ \frac{a^{3/2}f^2(be(5de-4cf)-af(2de-cf))\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{2\sqrt{b}ce^2(be-af)^3(de-cf)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}
\end{aligned}$$

output
```
1/2*f^2*x*(d*x^2+c)^(1/2)/e/(-a*f+b*e)/(-c*f+d*e)/(b*x^2+a)^(1/2)/(f*x^2+e
)-1/2*b^(1/2)*(a*b*c*f^2-a^2*d*f^2-2*b^2*e*(-c*f+d*e))*(d*x^2+c)^(1/2)*Ell
ipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/a^(1/2)/(-a*
d+b*c)/e/(-a*f+b*e)^2/(-c*f+d*e)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))
^(1/2)-1/2*a^(1/2)*b^(1/2)*(a^2*d*f^2-a*b*f*(c*f+6*d*e)+2*b^2*e*(2*c*f+d*e
))*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(
1/2))/c/(-a*d+b*c)/e/(-a*f+b*e)^3/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a)
)^(1/2)+1/2*a^(3/2)*f^2*(b*e*(-4*c*f+5*d*e)-a*f*(-c*f+2*d*e))*(d*x^2+c)^(1
/2)*EllipticPi(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),1-a*f/b/e,(1-a*d/b/c)^(
1/2))/b^(1/2)/c/e^2/(-a*f+b*e)^3/(-c*f+d*e)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c
/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.23 (sec) , antiderivative size = 1659, normalized size of antiderivative = 3.21

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \text{Too large to display}$$

input
```
Integrate[1/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]*(e + f*x^2)^2),x]
```

output
```
(Sqrt[b/a]*(2*b^3*Sqrt[b/a]*c*d*e^4*x - 2*b^3*Sqrt[b/a]*c^2*e^3*f*x - a^3*
(b/a)^(3/2)*c^2*e*f^3*x + a^3*Sqrt[b/a]*c*d*e*f^3*x + 2*b^3*Sqrt[b/a]*d^2*
e^4*x^3 - 2*b^3*Sqrt[b/a]*c^2*e^2*f^2*x^3 - a*b^2*Sqrt[b/a]*c^2*e*f^3*x^3
+ a^3*Sqrt[b/a]*d^2*e*f^3*x^3 + 2*b^3*Sqrt[b/a]*d^2*e^3*f*x^5 - 2*b^3*Sqrt
[b/a]*c*d*e^2*f^2*x^5 - a*b^2*Sqrt[b/a]*c*d*e*f^3*x^5 + a^3*(b/a)^(3/2)*d^
2*e*f^3*x^5 + I*b*c*e*(-(a*b*c*f^2) + a^2*d*f^2 + 2*b^2*e*(d*e - c*f))*Sqr
t[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(e + f*x^2)*EllipticE[I*ArcSinh[Sqrt[
b/a]*x], (a*d)/(b*c)] - I*b*(-(b*c) + a*d)*e*(2*b*e + a*f)*(-(d*e) + c*f)*
Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(e + f*x^2)*EllipticF[I*ArcSinh[Sq
rt[b/a]*x], (a*d)/(b*c)] + (5*I)*a*b^2*c*d*e^3*f*Sqrt[1 + (b*x^2)/a]*Sqrt[
1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)
] - (5*I)*a^2*b*d^2*e^3*f*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Elliptic
Pi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - (4*I)*a*b^2*c^2*e^2
*f^2*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e), I*Arc
Sinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (2*I)*a^2*b*c*d*e^2*f^2*Sqrt[1 + (b*x^2)
/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a
*d)/(b*c)] + (2*I)*a^3*d^2*e^2*f^2*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]
*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + I*a^2*b*c^
2*e*f^3*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e), I*
ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*a^3*c*d*e*f^3*Sqrt[1 + (b*x^2)/a...
```

Rubi [A] (verified)

Time = 1.16 (sec) , antiderivative size = 817, normalized size of antiderivative = 1.58, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.469$, Rules used = {426, 421, 25, 400, 313, 320, 414, 424, 406, 320, 388, 313, 413, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^2}dx$$

$$\Big\downarrow \; 426$$

$$\frac{b\int \frac{1}{(bx^2+a)^{3/2}\sqrt{dx^2+c}(fx^2+e)}dx}{be-af}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^2}dx}{be-af}$$

$$\Big\downarrow \; 421$$

$$\frac{b\left(\frac{f^2\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(be-af)^2}-\frac{b\int -\frac{-bfx^2+be-2af}{\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}}dx}{(be-af)^2}\right)}{be-af}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^2}dx}{be-af}$$

$\downarrow$ 25

$$\frac{b\left(\frac{f^2\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(be-af)^2}+\frac{b\int \frac{-bfx^2+be-2af}{\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}}dx}{(be-af)^2}\right)}{be-af}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^2}dx}{be-af}$$

$\downarrow$ 400

$$\frac{b\left(\frac{f^2\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(be-af)^2}+\frac{b\left(\frac{b(be-af)\int \frac{\sqrt{dx^2+c}}{\left(bx^2+a\right)^{3/2}}dx}{bc-ad}-\frac{(-2adf+bcf+bde)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}\right)}{(be-af)^2}\right)}{be-af}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^2}dx}{be-af}$$

$\downarrow$ 313

$$\frac{b\left(\frac{b\left(\frac{\sqrt{b}\sqrt{c+dx^2}(be-af)E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{a+bx^2}(bc-ad)\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}-\frac{(-2adf+bcf+bde)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}\right)}{(be-af)^2}+\frac{f^2\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(be-af)^2}\right)}{be-af}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^2}dx}{be-af}$$

$\downarrow$ 320

$$\frac{b\left(\frac{f^2\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)}dx}{(be-af)^2}+\frac{b\left(\frac{\sqrt{b}\sqrt{c+dx^2}(be-af)E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{a+bx^2}(bc-ad)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}-\frac{\sqrt{c}\sqrt{a+bx^2}(-2adf+bcf+bde)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{(be-af)^2}\right)}{be-af}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^2}dx}{be-af}$$

$\downarrow$ 414

$$\frac{b\left(\frac{a^{3/2}f^2\sqrt{c+dx^2}\operatorname{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce\sqrt{a+bx^2}(be-af)^2\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}+\frac{b\left(\frac{\sqrt{b}\sqrt{c+dx^2}(be-af)E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{a+bx^2}(bc-ad)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}-\frac{\sqrt{c}\sqrt{a+bx^2}(-2adf+bcf+bde)\operatorname{Elliptic}}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)}\right)}{(be-af)^2}\right)}{be-af}$$

$$\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^2}dx}{be-af}$$

$\downarrow$ 424

$$\frac{b\left(\frac{a^{3/2}f^2\sqrt{c+dx^2}\operatorname{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce\sqrt{a+bx^2}(be-af)^2\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}+\frac{b\left(\frac{\sqrt{b}\sqrt{c+dx^2}(be-af)E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{a+bx^2}(bc-ad)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}-\frac{\sqrt{c}\sqrt{a+bx^2}(-2adf+bcf+bde)\operatorname{Elliptic}}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)}\right)}{(be-af)^2}\right)}{be-af}$$

$$\frac{f\left(\frac{(be(3de-2cf)-af(2de-cf))\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)}dx}{2e(be-af)(de-cf)}-\frac{bd\int\frac{fx^2+e}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{2e(be-af)(de-cf)}+\frac{f^2x\sqrt{a+bx^2}\sqrt{c+dx^2}}{2e(e+fx^2)(be-af)(de-cf)}\right)}{be-af}$$

$\downarrow$ 406

$$\frac{b\left(\frac{a^{3/2}f^2\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce\sqrt{a+bx^2}(be-af)^2\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}+\frac{b\left(\frac{\sqrt{b}\sqrt{c+dx^2}(be-af)E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{a+bx^2}(bc-ad)\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}-\frac{\sqrt{c}\sqrt{a+bx^2}(-2adf+bcf+bde)\,\text{Elliptic}}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{}}\right.}{(be-af)^2}\right.}{be-af}$$

$$\frac{f\left(-\frac{bd\left(e\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+f\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{2e(be-af)(de-cf)}+\frac{(be(3de-2cf)-af(2de-cf))\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{2e(be-af)(de-cf)}+\frac{f^2x\sqrt{a+bx^2}\sqrt{c}}{2e(e+fx^2)(be-af}\right.}{be-af}$$

$$\Big\downarrow \; 320$$

$$\frac{b\left(\frac{a^{3/2}f^2\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce\sqrt{a+bx^2}(be-af)^2\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}+\frac{b\left(\frac{\sqrt{b}\sqrt{c+dx^2}(be-af)E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{a+bx^2}(bc-ad)\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}-\frac{\sqrt{c}\sqrt{a+bx^2}(-2adf+bcf+bde)\,\text{Elliptic}}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{}}\right.}{(be-af)^2}\right.}{be-af}$$

$$\frac{f\left(-\frac{bd\left(f\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{\sqrt{c}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{2e(be-af)(de-cf)}+\frac{(be(3de-2cf)-af(2de-cf))\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{2e(be-af)(de-cf)}\right.}{be-af}$$

$$\Big\downarrow \; 388$$

$$\frac{b\left(\frac{a^{3/2}f^2\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce\sqrt{a+bx^2}(be-af)^2\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}+\frac{b\left(\frac{\sqrt{b}\sqrt{c+dx^2}(be-af)E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{a+bx^2}(bc-ad)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}-\frac{\sqrt{c}\sqrt{a+bx^2}(-2adf+bcf+bde)\,\text{Elliptic}}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)}\right.}{(be-af)^2}}{be-af}$$

$$\frac{f\left(-\frac{bd\left(f\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}}dx}{b}\right)+\frac{\sqrt{c}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{2e(be-af)(de-cf)}+\frac{(be(3de-2cf)-af(2de-cf))\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(}}{2e(be-af)(de-cf)}\right.}{be-af}$$

$\downarrow$ 313

$$\frac{b\left(\frac{a^{3/2}f^2\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce\sqrt{a+bx^2}(be-af)^2\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}+\frac{b\left(\frac{\sqrt{b}\sqrt{c+dx^2}(be-af)E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{a+bx^2}(bc-ad)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}-\frac{\sqrt{c}\sqrt{a+bx^2}(-2adf+bcf+bde)\,\text{Elliptic}}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)}\right.}{(be-af)^2}}{be-af}$$

$$\frac{f\left(\frac{(be(3de-2cf)-af(2de-cf))\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)}dx}{2e(be-af)(de-cf)}-\frac{bd\left(\frac{\sqrt{c}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+f\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\right.}{b\sqrt{d}\sqrt{c+}}\right.\right.}{2e(be-af)(de-cf)}\right.}{be-af}$$

$\downarrow$ 413

$$\frac{b\left(\frac{a^{3/2}f^2\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce\sqrt{a+bx^2}(be-af)^2\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}+\frac{b\left(\frac{\sqrt{b}\sqrt{c+dx^2}(be-af)E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{a+bx^2}(bc-ad)\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}-\frac{\sqrt{c}\sqrt{a+bx^2}(-2adf+bcf+bde)\,\text{Elliptic}\ldots}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\ldots}\right.}{(be-af)^2}\ldots\right.}{be-af}$$

$$\frac{f\left(\frac{\sqrt{\frac{bx^2}{a}+1}(be(3de-2cf)-af(2de-cf))\int\frac{1}{\sqrt{\frac{bx^2}{a}+1}\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{2e\sqrt{a+bx^2}(be-af)(de-cf)}-\frac{bd\left(\frac{\sqrt{c}e\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+f\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\ldots}{\ldots}\right.\right.}{2e(be-af)(de-cf)}\ldots\right.}{be-af}$$

$\downarrow$ 413

$$\frac{b\left(\frac{a^{3/2}\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)f^2}{\sqrt{b}ce(be-af)^2\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}+\frac{b\left(\frac{\sqrt{b}(be-af)\sqrt{dx^2+c}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}(bc-ad)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}-\frac{\sqrt{c}(bde+bcf-2adf)\sqrt{bx^2+a}\,\text{EllipticF}\ldots}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\ldots}\right.}{(be-af)^2}\ldots\right.}{be-af}$$

$$\frac{f\left(\frac{x\sqrt{bx^2+a}\sqrt{dx^2+c}f^2}{2e(be-af)(de-cf)(fx^2+e)}-\frac{bd\left(f\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)+\frac{\sqrt{c}e\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{dx^2+c}}\right)}{2e(be-af)(de-cf)}+\ldots\right.}{be-af}$$

$\downarrow$ 412

$$\frac{b\left(\frac{a^{3/2}\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)f^2}{\sqrt{b}ce(be-af)^2\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}+\frac{b\left(\frac{\sqrt{b}(be-af)\sqrt{dx^2+c}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}(bc-ad)\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}-\frac{\sqrt{c}(bde+bcf-2adf)\sqrt{bx^2+a}\,\text{EllipticF}}{a\sqrt{d}(bc-ad)\sqrt{\frac{c(bx^2+c}{a(dx^2+c}}}\right.}{(be-af)^2}\right.}{be-af}$$

$$\frac{f\left(\frac{x\sqrt{bx^2+a}\sqrt{dx^2+c}f^2}{2e(be-af)(de-cf)(fx^2+e)}-\frac{bd\left(f\left(\frac{x\sqrt{bx^2+a}}{b\sqrt{dx^2+c}}-\frac{\sqrt{c}\sqrt{bx^2+a}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)+\frac{\sqrt{c}e\sqrt{bx^2+a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{dx^2+c}}\right)}{2e(be-af)(de-cf)}+\cdots\right.}{be-af}$$

input
```
Int[1/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]*(e + f*x^2)^2),x]
```

output
```
-((f*((f^2*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(2*e*(b*e - a*f)*(d*e - c*f)
*(e + f*x^2)) - (b*d*(f*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c
]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])
/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (Sqr
t[c]*e*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a
*d)])/(a*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])))/
(2*e*(b*e - a*f)*(d*e - c*f)) + (Sqrt[-a]*(b*e*(3*d*e - 2*c*f) - a*f*(2*d*
e - c*f))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[(a*f)/(b*e),
ArcSin[(Sqrt[b]*x)/Sqrt[-a]], (a*d)/(b*c)])/(2*Sqrt[b]*e^2*(b*e - a*f)*(d*
e - c*f)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])))/(b*e - a*f)) + (b*((b*((Sqrt[b
]*(b*e - a*f)*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]], 1 - (
a*d)/(b*c)])/(Sqrt[a]*(b*c - a*d)*Sqrt[a + b*x^2]*Sqrt[(a*(c + d*x^2))/(c*
(a + b*x^2))]) - (Sqrt[c]*(b*d*e + b*c*f - 2*a*d*f)*Sqrt[a + b*x^2]*Ellipt
icF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*(b*c - a*d)*
Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])))/(b*e - a*f)^2 + (
a^(3/2)*f^2*Sqrt[c + d*x^2]*EllipticPi[1 - (a*f)/(b*e), ArcTan[(Sqrt[b]*x)
/Sqrt[a]], 1 - (a*d)/(b*c)])/(Sqrt[b]*c*e*(b*e - a*f)^2*Sqrt[a + b*x^2]*Sq
rt[(a*(c + d*x^2))/(c*(a + b*x^2))])))/(b*e - a*f)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 400
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 421
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b^2/(b*c - a*d)^2   Int[(c + d*x^2)^(q + 2)*((e +
 f*x^2)^r/(a + b*x^2)), x], x] - Simp[d/(b*c - a*d)^2   Int[(c + d*x^2)^q*(
e + f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r}
, x] && LtQ[q, -1]
```

rule 424
```
Int[1/(((a_) + (b_.)*(x_)^2)^2*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*
(x_)^2]), x_Symbol] :> Simp[b^2*x*Sqrt[c + d*x^2]*(Sqrt[e + f*x^2]/(2*a*(b*
c - a*d)*(b*e - a*f)*(a + b*x^2))), x] + (Simp[(b^2*c*e + 3*a^2*d*f - 2*a*b
*(d*e + c*f))/(2*a*(b*c - a*d)*(b*e - a*f))   Int[1/((a + b*x^2)*Sqrt[c + d
*x^2]*Sqrt[e + f*x^2]), x], x] - Simp[d*(f/(2*a*(b*c - a*d)*(b*e - a*f)))
 Int[(a + b*x^2)/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x], x]) /; FreeQ[{a, b,
 c, d, e, f}, x]
```

rule 426
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[b/(b*c - a*d)   Int[(a + b*x^2)^p*(c + d*x^2)^
(q + 1)*(e + f*x^2)^r, x], x] - Simp[d/(b*c - a*d)   Int[(a + b*x^2)^(p + 1
)*(c + d*x^2)^q*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] &&
 ILtQ[p, 0] && LeQ[q, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1338 vs. $2(491) = 982$.

Time = 11.02 (sec) , antiderivative size = 1339, normalized size of antiderivative = 2.59

method	result	size
elliptic	Expression too large to display	1339
default	Expression too large to display	2813

input
```
int(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-(b*d*x^2+b*c
)*b^2/a/(a*d-b*c)*x/(a*f-b*e)^2/((x^2+a/b)*(b*d*x^2+b*c))^(1/2)+1/2*f^3/(a
*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/e*x/(a*f-b*e)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c
)^(1/2)/(f*x^2+e)+1/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*
x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)
^(1/2))/a*b^2/(a*f-b*e)^2-1/2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(
1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d
+b*c)/c/b)^(1/2))*b*d*f/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/(a*f-b*e)+c/(-b/
a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)
^(1/2)*b^3/(a*d-b*c)/a/(a*f-b*e)^2*EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/
c/b)^(1/2))+1/2*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^
4+a*d*x^2+b*c*x^2+a*c)^(1/2)*b*f^2/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/e/(a*
f-b*e)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-1/2*c/(-b/a)^(1/
2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
*b*f^2/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/e/(a*f-b*e)*EllipticE(x*(-b/a)^(1
/2),(-1+(a*d+b*c)/c/b)^(1/2))+1/2/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/e^2/(a
*f-b*e)*f^3/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*
x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-
b/a)^(1/2))*a*c-1/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/(a*f-b*e)*f^2/e/(-b/a)
^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{1}{(a+bx^2)^{\frac{3}{2}}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx$$

input
```
integrate(1/(b*x**2+a)**(3/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**2,x)
```

output
```
Integral(1/((a + b*x**2)**(3/2)*sqrt(c + d*x**2)*(e + f*x**2)**2), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{1}{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}(fx^2+e)^2}\,dx$$

input
```
integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="maxi
ma")
```

output
```
integrate(1/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*(f*x^2 + e)^2), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{1}{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}(fx^2+e)^2}\,dx$$

input
```
integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="giac
")
```

output
```
integrate(1/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*(f*x^2 + e)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{1}{(b\,x^2+a)^{3/2}\,\sqrt{d\,x^2+c}\,(f\,x^2+e)^2}\,dx$$

input
```
int(1/((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^2),x)
```

output
```
int(1/((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^2), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{1}{(b\,x^2+a)^{\frac{3}{2}}\,\sqrt{d\,x^2+c}\,(f\,x^2+e)^2}dx$$

input
```
int(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x)
```

output
```
int(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x)
```

3.261 $\int \frac{1}{x^2(a+bx^2)^{3/2}\sqrt{c+dx^2}(e+fx^2)^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 672

$$\int \frac{1}{x^2(a+bx^2)^{3/2}\sqrt{c+dx^2}(e+fx^2)^2}\,dx = -\frac{\sqrt{c+dx^2}}{acex\sqrt{a+bx^2}(e+fx^2)}$$

$$+\frac{f(af(2de-3cf)-2be(de-cf))x\sqrt{c+dx^2}}{2ace^2(be-af)(de-cf)\sqrt{a+bx^2}(e+fx^2)}$$

$$+\frac{\sqrt{b}(a^3df^2(2de-3cf)-4b^3ce^2(de-cf)-a^2bf(4d^2e^2-2cdef-3c^2f^2)+2ab^2e(d^2e^2+cdef-2c^2f^2))}{2a^{3/2}c(bc-ad)e^2(be-af)^2(de-cf)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

$$+\frac{\sqrt{b}(2b^3de^3+3a^3df^3-2ab^2ef(de-3cf)-3a^2bf^2(2de+cf))\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right.}{2\sqrt{a}c(bc-ad)e^2(be-af)^3\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

$$-\frac{a^{3/2}f^3(be(7de-6cf)-af(4de-3cf))\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{2\sqrt{b}ce^3(be-af)^3(de-cf)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
-(d*x^2+c)^(1/2)/a/c/e/x/(b*x^2+a)^(1/2)/(f*x^2+e)+1/2*f*(a*f*(-3*c*f+2*d*
e)-2*b*e*(-c*f+d*e))*x*(d*x^2+c)^(1/2)/a/c/e^2/(-a*f+b*e)/(-c*f+d*e)/(b*x^
2+a)^(1/2)/(f*x^2+e)+1/2*b^(1/2)*(a^3*d*f^2*(-3*c*f+2*d*e)-4*b^3*c*e^2*(-c
*f+d*e)-a^2*b*f*(-3*c^2*f^2-2*c*d*e*f+4*d^2*e^2)+2*a*b^2*e*(-2*c^2*f^2+c*d
*e*f+d^2*e^2))*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/
2),(1-a*d/b/c)^(1/2))/a^(3/2)/c/(-a*d+b*c)/e^2/(-a*f+b*e)^2/(-c*f+d*e)/(b*
x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)+1/2*b^(1/2)*(2*b^3*d*e^3+3*a^
3*d*f^3-2*a*b^2*e*f*(-3*c*f+d*e)-3*a^2*b*f^2*(c*f+2*d*e))*(d*x^2+c)^(1/2)*
InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/a^(1/2)/c/(-a
*d+b*c)/e^2/(-a*f+b*e)^3/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)-1
/2*a^(3/2)*f^3*(b*e*(-6*c*f+7*d*e)-a*f*(-3*c*f+4*d*e))*(d*x^2+c)^(1/2)*Ell
ipticPi(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),1-a*f/b/e,(1-a*d/b/c)^(1/2))/b
^(1/2)/c/e^3/(-a*f+b*e)^3/(-c*f+d*e)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2
+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.10 (sec) , antiderivative size = 737, normalized size of antiderivative = 1.10

$$\int \frac{1}{x^2 (a+bx^2)^{3/2} \sqrt{c+dx^2} (e+fx^2)^2} \, dx = \frac{\sqrt{\frac{b}{a}} e(c+dx^2) \left(4b^4ce^2(-de+cf)x^2(e+fx^2)+2ab^3e(de-c\right.}{}$$

input
```
Integrate[1/(x^2*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2]*(e + f*x^2)^2),x]
```

output
```
(Sqrt[b/a]*e*(c + d*x^2)*(4*b^4*c*e^2*(-(d*e) + c*f)*x^2*(e + f*x^2) + 2*a
*b^3*e*(d*e - c*f)*(e + f*x^2)*(-(c*e) + d*e*x^2 + 2*c*f*x^2) + a^4*d*f^2*
(2*d*e*(e + f*x^2) - c*f*(2*e + 3*f*x^2)) + a^3*b*f*(c^2*f^2*(2*e + 3*f*x^
2) + c*d*f*(2*e^2 - 3*f^2*x^4) - 2*d^2*e*(2*e^2 + e*f*x^2 - f^2*x^4)) + a^
2*b^2*(2*c*d*e*f*(e + f*x^2)^2 + 2*d^2*e^2*(e^2 - e*f*x^2 - 2*f^2*x^4) + c
^2*f^2*(-4*e^2 - 2*e*f*x^2 + 3*f^2*x^4))) - I*b*c*e*(4*b^3*c*e^2*(d*e - c*
f) + a^3*d*f^2*(-2*d*e + 3*c*f) + a^2*b*f*(4*d^2*e^2 - 2*c*d*e*f - 3*c^2*f
^2) - 2*a*b^2*e*(d^2*e^2 + c*d*e*f - 2*c^2*f^2))*x*Sqrt[1 + (b*x^2)/a]*Sqr
t[1 + (d*x^2)/c]*(e + f*x^2)*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)
] + I*b*c*(-(b*c) + a*d)*e*(-(d*e) + c*f)*(4*b^2*e^2 - 4*a*b*e*f + 3*a^2*f
^2)*x*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(e + f*x^2)*EllipticF[I*ArcS
inh[Sqrt[b/a]*x], (a*d)/(b*c)] + I*a^2*c*(-(b*c) + a*d)*f^2*(b*e*(7*d*e -
6*c*f) + a*f*(-4*d*e + 3*c*f))*x*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(
e + f*x^2)*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(
2*a^2*Sqrt[b/a]*c*(-(b*c) + a*d)*e^3*(b*e - a*f)^2*(-(d*e) + c*f)*x*Sqrt[a
 + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a+bx^2)^{3/2} \sqrt{c+dx^2} (e+fx^2)^2} \, dx$$

$$\Big\downarrow \; 450$$

$$\int \frac{1}{x^2 (a+bx^2)^{3/2} \sqrt{c+dx^2} (e+fx^2)^2} dx$$

input
```
Int[1/(x^2*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2]*(e + f*x^2)^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1574 vs. $2(642) = 1284$.

Time = 28.46 (sec) , antiderivative size = 1575, normalized size of antiderivative = 2.34

method	result	size
elliptic	Expression too large to display	1575
risch	Expression too large to display	1589
default	Expression too large to display	4739

input
```
int(1/x^2/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x,method=_RETURNVERB
OSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*((b*d*x^2+b*c)
*b^3/a^2/(a*d-b*c)*x/(a*f-b*e)^2/((x^2+a/b)*(b*d*x^2+b*c))^(1/2)-1/2*f^4/(
a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/e^2*x/(a*f-b*e)*(b*d*x^4+a*d*x^2+b*c*x^2+
a*c)^(1/2)/(f*x^2+e)-1/a^2/c/e^2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/x+1/2
/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2
+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))*b*d*f^2/(a*
c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/e/(a*f-b*e)-c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2
)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*b^4/(a*d-b*c)/a^2/
(a*f-b*e)^2*EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-3/2*f^4/(a*
c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/e^3/(a*f-b*e)/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2
)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a
)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*a*c+2*f^3/(a*c*f^2-a*d*e*f-b*
c*e*f+b*d*e^2)/e^2/(a*f-b*e)/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1
/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,
(-1/c*d)^(1/2)/(-b/a)^(1/2))*a*d-1/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2
/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*b/a^2/e^2*EllipticF(x*(-b/a)
^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+1/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2
/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*b/a^2/e^2*EllipticE(x*(-b/a)
^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+3*f^3/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/e
^2/(a*f-b*e)/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a+bx^2)^{3/2} \sqrt{c+dx^2} (e+fx^2)^2} \, dx = \text{Timed out}$$

input
```
integrate(1/x^2/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="
fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{x^2 (a+bx^2)^{3/2} \sqrt{c+dx^2} (e+fx^2)^2} \, dx = \int \frac{1}{x^2 (a+bx^2)^{\frac{3}{2}} \sqrt{c+dx^2} (e+fx^2)^2} \, dx$$

input
```
integrate(1/x**2/(b*x**2+a)**(3/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**2,x)
```

output
```
Integral(1/(x**2*(a + b*x**2)**(3/2)*sqrt(c + d*x**2)*(e + f*x**2)**2), x)
```

Maxima [F]

$$\int \frac{1}{x^2 (a+bx^2)^{3/2} \sqrt{c+dx^2} (e+fx^2)^2} \, dx = \int \frac{1}{(bx^2+a)^{\frac{3}{2}} \sqrt{dx^2+c} (fx^2+e)^2 x^2} \, dx$$

input
```
integrate(1/x^2/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="
maxima")
```

output
```
integrate(1/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*(f*x^2 + e)^2*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2 (a+bx^2)^{3/2} \sqrt{c+dx^2} (e+fx^2)^2} \, dx = \int \frac{1}{(bx^2+a)^{\frac{3}{2}} \sqrt{dx^2+c} (fx^2+e)^2 x^2} \, dx$$

input
```
integrate(1/x^2/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="
giac")
```

output
```
integrate(1/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*(f*x^2 + e)^2*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a+bx^2)^{3/2} \sqrt{c+dx^2} (e+fx^2)^2} \, dx = \int \frac{1}{x^2 \left(b\,x^2+a\right)^{3/2} \sqrt{d\,x^2+c} \left(f\,x^2+e\right)^2} \, dx$$

input
```
int(1/(x^2*(a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^2),x)
```

output
```
int(1/(x^2*(a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^2), x)
```

Reduce [F]

$$\int \frac{1}{x^2 (a+bx^2)^{3/2} \sqrt{c+dx^2} (e+fx^2)^2} \, dx = \text{too large to display}$$

input
```
int(1/x^2/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x)
```

output

```
( - sqrt(c + d*x**2)*sqrt(a + b*x**2) - 3*int((sqrt(c + d*x**2)*sqrt(a + b
*x**2)*x**4)/(a**2*c*e**2 + 2*a**2*c*e*f*x**2 + a**2*c*f**2*x**4 + a**2*d*
e**2*x**2 + 2*a**2*d*e*f*x**4 + a**2*d*f**2*x**6 + 2*a*b*c*e**2*x**2 + 4*a
*b*c*e*f*x**4 + 2*a*b*c*f**2*x**6 + 2*a*b*d*e**2*x**4 + 4*a*b*d*e*f*x**6 +
 2*a*b*d*f**2*x**8 + b**2*c*e**2*x**4 + 2*b**2*c*e*f*x**6 + b**2*c*f**2*x*
*8 + b**2*d*e**2*x**6 + 2*b**2*d*e*f*x**8 + b**2*d*f**2*x**10),x)*a*b*d*e*
f*x - 3*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a**2*c*e**2 + 2*a**2
*c*e*f*x**2 + a**2*c*f**2*x**4 + a**2*d*e**2*x**2 + 2*a**2*d*e*f*x**4 + a*
*2*d*f**2*x**6 + 2*a*b*c*e**2*x**2 + 4*a*b*c*e*f*x**4 + 2*a*b*c*f**2*x**6
+ 2*a*b*d*e**2*x**4 + 4*a*b*d*e*f*x**6 + 2*a*b*d*f**2*x**8 + b**2*c*e**2*x
**4 + 2*b**2*c*e*f*x**6 + b**2*c*f**2*x**8 + b**2*d*e**2*x**6 + 2*b**2*d*e
*f*x**8 + b**2*d*f**2*x**10),x)*a*b*d*f**2*x**3 - 3*int((sqrt(c + d*x**2)*
sqrt(a + b*x**2)*x**4)/(a**2*c*e**2 + 2*a**2*c*e*f*x**2 + a**2*c*f**2*x**4
 + a**2*d*e**2*x**2 + 2*a**2*d*e*f*x**4 + a**2*d*f**2*x**6 + 2*a*b*c*e**2*
x**2 + 4*a*b*c*e*f*x**4 + 2*a*b*c*f**2*x**6 + 2*a*b*d*e**2*x**4 + 4*a*b*d*
e*f*x**6 + 2*a*b*d*f**2*x**8 + b**2*c*e**2*x**4 + 2*b**2*c*e*f*x**6 + b**2
*c*f**2*x**8 + b**2*d*e**2*x**6 + 2*b**2*d*e*f*x**8 + b**2*d*f**2*x**10),x
)*b**2*d*e*f*x**3 - 3*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a**2*c
*e**2 + 2*a**2*c*e*f*x**2 + a**2*c*f**2*x**4 + a**2*d*e**2*x**2 + 2*a**2*d
*e*f*x**4 + a**2*d*f**2*x**6 + 2*a*b*c*e**2*x**2 + 4*a*b*c*e*f*x**4 + 2...
```

3.262 $\int \frac{1}{x^4(a+bx^2)^{3/2}\sqrt{c+dx^2}(e+fx^2)^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 917

$$\int \frac{1}{x^4(a+bx^2)^{3/2}\sqrt{c+dx^2}(e+fx^2)^2}\,dx =$$

$$-\frac{\sqrt{c+dx^2}}{3acex^3\sqrt{a+bx^2}(e+fx^2)}+\frac{(4bce+2ade+5acf)\sqrt{c+dx^2}}{3a^2c^2e^2x\sqrt{a+bx^2}(e+fx^2)}$$

$$+\frac{f(8b^2ce^2(de-cf)-a^2f(4d^2e^2+8cdef-15c^2f^2)+4abe(d^2e^2-c^2f^2))x\sqrt{c+dx^2}}{6a^2c^2e^3(be-af)(de-cf)\sqrt{a+bx^2}(e+fx^2)}$$

$$+\frac{\sqrt{b}(16b^4c^2e^3(de-cf)-a^4df^2(4d^2e^2+8cdef-15c^2f^2)-2ab^3ce^2(3d^2e^2+cdef-4c^2f^2)+a^3bf(8d^3e^3\cdots}{6a^{5/2}c^2(bc-ad)e^3(be-\cdots}$$

$$-\frac{\sqrt{b}(8b^4cde^4-2ab^3de^3(de+6cf)+a^4df^3(2de+15cf)+6a^2b^2ef(d^2e^2+cdef+4c^2f^2)-a^3bf^2(6d^2e^2+\cdots}{6a^{3/2}c^2(bc-ad)e^3(be-af)^3\sqrt{a+bx^2}\sqrt{\frac{a(c+\cdots}{c(a+\cdots}}}$$

$$+\frac{a^{3/2}f^4(be(9de-8cf)-af(6de-5cf))\sqrt{c+dx^2}\,\mathrm{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{2\sqrt{b}ce^4(be-af)^3(de-cf)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
-1/3*(d*x^2+c)^(1/2)/a/c/e/x^3/(b*x^2+a)^(1/2)/(f*x^2+e)+1/3*(5*a*c*f+2*a*
d*e+4*b*c*e)*(d*x^2+c)^(1/2)/a^2/c^2/e^2/x/(b*x^2+a)^(1/2)/(f*x^2+e)+1/6*f
*(8*b^2*c*e^2*(-c*f+d*e)-a^2*f*(-15*c^2*f^2+8*c*d*e*f+4*d^2*e^2)+4*a*b*e*(
-c^2*f^2+d^2*e^2))*x*(d*x^2+c)^(1/2)/a^2/c^2/e^3/(-a*f+b*e)/(-c*f+d*e)/(b*
x^2+a)^(1/2)/(f*x^2+e)+1/6*b^(1/2)*(16*b^4*c^2*e^3*(-c*f+d*e)-a^4*d*f^2*(-
15*c^2*f^2+8*c*d*e*f+4*d^2*e^2)-2*a*b^3*c*e^2*(-4*c^2*f^2+c*d*e*f+3*d^2*e^
2)+a^3*b*f*(-15*c^3*f^3-6*c^2*d*e*f^2+10*c*d^2*e^2*f+8*d^3*e^3)-2*a^2*b^2*
e*(-7*c^3*f^3+7*c^2*d*e*f^2-2*c*d^2*e^2*f+2*d^3*e^3))*(d*x^2+c)^(1/2)*Elli
pticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/a^(5/2)/c^2/(
-a*d+b*c)/e^3/(-a*f+b*e)^2/(-c*f+d*e)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^
2+a))^(1/2)-1/6*b^(1/2)*(8*b^4*c*d*e^4-2*a*b^3*d*e^3*(6*c*f+d*e)+a^4*d*f^3
*(15*c*f+2*d*e)+6*a^2*b^2*e*f*(4*c^2*f^2+c*d*e*f+d^2*e^2)-a^3*b*f^2*(15*c^
2*f^2+26*c*d*e*f+6*d^2*e^2))*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2
)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/a^(3/2)/c^2/(-a*d+b*c)/e^3/(-a*f+b*e)^3/(b
*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)+1/2*a^(3/2)*f^4*(b*e*(-8*c*f
+9*d*e)-a*f*(-5*c*f+6*d*e))*(d*x^2+c)^(1/2)*EllipticPi(b^(1/2)*x/a^(1/2)/(
1+b*x^2/a)^(1/2),1-a*f/b/e,(1-a*d/b/c)^(1/2))/b^(1/2)/c/e^4/(-a*f+b*e)^3/(
-c*f+d*e)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 7.32 (sec) , antiderivative size = 693, normalized size of antiderivative = 0.76

$$\int \frac{1}{x^4 (a+bx^2)^{3/2} \sqrt{c+dx^2} (e+fx^2)^2}\,dx = \frac{-\sqrt{\frac{b}{a}}e(c+dx^2)\left(3a^3c^2(bc-ad)f^5x^4(a+bx^2)+6b^5c^2e^3(-de\right.}{}$$

input
```
Integrate[1/(x^4*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2]*(e + f*x^2)^2),x]
```

output
```
(-(Sqrt[b/a]*e*(c + d*x^2)*(3*a^3*c^2*(b*c - a*d)*f^5*x^4*(a + b*x^2) + 6*
b^5*c^2*e^3*(-(d*e) + c*f)*x^4*(e + f*x^2) + 2*a*c*(-(b*c) + a*d)*e*(b*e -
 a*f)^2*(-(d*e) + c*f)*(a + b*x^2)*(e + f*x^2) + 2*(b*c - a*d)*(b*e - a*f)
^2*(-(d*e) + c*f)*(5*b*c*e + 2*a*d*e + 6*a*c*f)*x^2*(a + b*x^2)*(e + f*x^2
))) - I*c*x^3*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(e + f*x^2)*(b*e*(16
*b^4*c^2*e^3*(-(d*e) + c*f) + a^4*d*f^2*(4*d^2*e^2 + 8*c*d*e*f - 15*c^2*f^
2) + 2*a*b^3*c*e^2*(3*d^2*e^2 + c*d*e*f - 4*c^2*f^2) + 2*a^2*b^2*e*(2*d^3*
e^3 - 2*c*d^2*e^2*f + 7*c^2*d*e*f^2 - 7*c^3*f^3) + a^3*b*f*(-8*d^3*e^3 - 1
0*c*d^2*e^2*f + 6*c^2*d*e*f^2 + 15*c^3*f^3))*EllipticE[I*ArcSinh[Sqrt[b/a]
*x], (a*d)/(b*c)] - (b*c - a*d)*(b*e*(-(d*e) + c*f)*(16*b^3*c*e^3 + 2*a*b^
2*e^2*(d*e - 4*c*f) - 2*a^2*b*e*f*(2*d*e + 7*c*f) + a^3*f^2*(2*d*e + 15*c*
f))*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + 3*a^3*c*f^3*(b*e*(9*d
*e - 8*c*f) + a*f*(-6*d*e + 5*c*f))*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt
[b/a]*x], (a*d)/(b*c)])))/(6*a^3*Sqrt[b/a]*c^2*(b*c - a*d)*e^4*(b*e - a*f)
^2*(d*e - c*f)*x^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (a+bx^2)^{3/2} \sqrt{c+dx^2} (e+fx^2)^2} \, dx$$
$$\Big\downarrow \; 450$$
$$\int \frac{1}{x^4 (a+bx^2)^{3/2} \sqrt{c+dx^2} (e+fx^2)^2} dx$$

input
```
Int[1/(x^4*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2]*(e + f*x^2)^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [A] (verified)

Time = 29.42 (sec) , antiderivative size = 1733, normalized size of antiderivative = 1.89

method	result	size
risch	Expression too large to display	1733
elliptic	Expression too large to display	2019
default	Expression too large to display	7728

input
```
int(1/x^4/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x,method=_RETURNVERB
OSE)
```

output
```
-1/3*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(-6*a*c*f*x^2-2*a*d*e*x^2-5*b*c*e*x^2
+a*c*e)/a^3/c^2/e^3/x^3-1/3/a^3/e^3/c^2*(-b*(6*a*c*f+2*a*d*e+5*b*c*e)*c/(-
b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*
c)^(1/2)*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(
-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2)))+a*c*d*e*b/(-b/a)^(1/2)*(1+b*x^2/a)^
(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-
b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-3*a*b^4*c^2*e^3/(a*f-b*e)^2*(-(b*d*x^
2+b*c)/a/(a*d-b*c)*x/((x^2+a/b)*(b*d*x^2+b*c))^(1/2)+(1/a+b*c/(a*d-b*c)/a)
/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2
+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-b/(a*d-b*c)
/a*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c
*x^2+a*c)^(1/2)*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-Ellipt
icE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))))-3*a^3*c^2*e*f^3/(a*f-b*e)*(
1/2*f^2/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/e*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c
)^(1/2)/(f*x^2+e)-1/2*d*b/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/(-b/a)^(1/2)*(
1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*Ell
ipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+1/2*f*b/(a*c*f^2-a*d*e*f-b
*c*e*f+b*d*e^2)/e*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*
x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)
^(1/2))-1/2*f*b/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/e*c/(-b/a)^(1/2)*(1+b...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (a+bx^2)^{3/2} \sqrt{c+dx^2} (e+fx^2)^2} \, dx = \text{Timed out}$$

input
```
integrate(1/x^4/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="
fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{x^4 (a+bx^2)^{3/2} \sqrt{c+dx^2} (e+fx^2)^2} \, dx = \int \frac{1}{x^4 (a+bx^2)^{\frac{3}{2}} \sqrt{c+dx^2} (e+fx^2)^2} \, dx$$

input
```
integrate(1/x**4/(b*x**2+a)**(3/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**2,x)
```

output
```
Integral(1/(x**4*(a + b*x**2)**(3/2)*sqrt(c + d*x**2)*(e + f*x**2)**2), x)
```

Maxima [F]

$$\int \frac{1}{x^4 (a+bx^2)^{3/2} \sqrt{c+dx^2} (e+fx^2)^2} \, dx = \int \frac{1}{(bx^2+a)^{\frac{3}{2}} \sqrt{dx^2+c} (fx^2+e)^2 x^4} \, dx$$

input
```
integrate(1/x^4/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="
maxima")
```

output
```
integrate(1/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*(f*x^2 + e)^2*x^4), x)
```

Giac [F]

$$\int \frac{1}{x^4 (a+bx^2)^{3/2} \sqrt{c+dx^2} (e+fx^2)^2} \, dx = \int \frac{1}{(bx^2+a)^{\frac{3}{2}} \sqrt{dx^2+c} (fx^2+e)^2 x^4} \, dx$$

input
```
integrate(1/x^4/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="
giac")
```

output
```
integrate(1/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*(f*x^2 + e)^2*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (a+bx^2)^{3/2} \sqrt{c+dx^2} (e+fx^2)^2} \, dx = \int \frac{1}{x^4 \left(b\,x^2+a\right)^{3/2} \sqrt{d\,x^2+c} \left(f\,x^2+e\right)^2} \, dx$$

input
```
int(1/(x^4*(a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^2),x)
```

output
```
int(1/(x^4*(a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^2), x)
```

Reduce [F]

$$\int \frac{1}{x^4 (a+bx^2)^{3/2} \sqrt{c+dx^2} (e+fx^2)^2} \, dx = \int \frac{1}{x^4 \left(b\,x^2+a\right)^{\frac{3}{2}} \sqrt{d\,x^2+c}\, \left(f\,x^2+e\right)^2} dx$$

input
```
int(1/x^4/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x)
```

output
```
int(1/x^4/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x)
```

3.263 $\int \frac{x^{10}}{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}\left(e+fx^2\right)^3}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 951

$$\int \frac{x^{10}}{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}\left(e+fx^2\right)^3}\,dx = \frac{x\sqrt{c+dx^2}}{bdf^3\sqrt{a+bx^2}}$$
$$-\frac{e^4x\sqrt{c+dx^2}}{4f^3(be-af)(de-cf)\sqrt{a+bx^2}\left(e+fx^2\right)^2}$$
$$-\frac{e^3(af(14de-17cf)-3be(3de-4cf))x\sqrt{c+dx^2}}{8f^3(be-af)^2(de-cf)^2\sqrt{a+bx^2}\left(e+fx^2\right)}$$
$$-\frac{\sqrt{a}(16a^4df^3(de-cf)^2-8a^3bf^2(de-cf)^2(3de+cf)+b^4ce^3(15d^2e^2-26cdef+8c^2f^2)-3ab^3e^2(5d^3e^3}{8b^{3/2}d(bc-ad)f^3(be}$$
$$+\frac{a^{3/2}(b^4ce^4(15de-16cf)-8a^4cf^4(de-cf)-ab^3e^3(15d^2e^2+32cdef-49c^2f^2)+a^2b^2e^2f(48d^2e^2-cde}{8b^{3/2}c(bc-ad)f^3(be-af)^4(de-c}$$
$$-\frac{3a^{3/2}e^2(b^2e^2(5d^2e^2-12cdef+8c^2f^2)-2abef(8d^2e^2-19cdef+12c^2f^2)+a^2f^2(16d^2e^2-36cdef+21}{8\sqrt{b}cf^3(be-af)^4(de-cf)^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2}{c(a+bx^2}}}$$

output
```
x*(d*x^2+c)^(1/2)/b/d/f^3/(b*x^2+a)^(1/2)-1/4*e^4*x*(d*x^2+c)^(1/2)/f^3/(-
a*f+b*e)/(-c*f+d*e)/(b*x^2+a)^(1/2)/(f*x^2+e)^2-1/8*e^3*(a*f*(-17*c*f+14*d
*e)-3*b*e*(-4*c*f+3*d*e))*x*(d*x^2+c)^(1/2)/f^3/(-a*f+b*e)^2/(-c*f+d*e)^2/
(b*x^2+a)^(1/2)/(f*x^2+e)-1/8*a^(1/2)*(16*a^4*d*f^3*(-c*f+d*e)^2-8*a^3*b*f
^2*(-c*f+d*e)^2*(c*f+3*d*e)+b^4*c*e^3*(8*c^2*f^2-26*c*d*e*f+15*d^2*e^2)-3*
a*b^3*e^2*(8*c^3*f^3-19*c^2*d*e*f^2+4*c*d^2*e^2*f+5*d^3*e^3)+a^2*b^2*e*f*(
24*c^3*f^3-24*c^2*d*e*f^2-41*c*d^2*e^2*f+38*d^3*e^3))*(d*x^2+c)^(1/2)*Elli
pticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/b^(3/2)/d/(-a
*d+b*c)/f^3/(-a*f+b*e)^3/(-c*f+d*e)^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^
2+a))^(1/2)+1/8*a^(3/2)*(b^4*c*e^4*(-16*c*f+15*d*e)-8*a^4*c*f^4*(-c*f+d*e)
-a*b^3*e^3*(-49*c^2*f^2+32*c*d*e*f+15*d^2*e^2)+a^2*b^2*e^2*f*(-48*c^2*f^2-
c*d*e*f+48*d^2*e^2)-8*a^3*b*e*f^2*(c^2*f^2-7*c*d*e*f+6*d^2*e^2))*(d*x^2+c)
^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/b^(3/2
)/c/(-a*d+b*c)/f^3/(-a*f+b*e)^4/(-c*f+d*e)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/
(b*x^2+a))^(1/2)-3/8*a^(3/2)*e^2*(b^2*e^2*(8*c^2*f^2-12*c*d*e*f+5*d^2*e^2)
-2*a*b*e*f*(12*c^2*f^2-19*c*d*e*f+8*d^2*e^2)+a^2*f^2*(21*c^2*f^2-36*c*d*e*
f+16*d^2*e^2))*(d*x^2+c)^(1/2)*EllipticPi(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1
/2),1-a*f/b/e,(1-a*d/b/c)^(1/2))/b^(1/2)/c/f^3/(-a*f+b*e)^4/(-c*f+d*e)^2/(
b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 7.77 (sec) , antiderivative size = 756, normalized size of antiderivative = 0.79

$$\int \frac{x^{10}}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \frac{\sqrt{\frac{b}{a}}df^2x(c+dx^2)\left(2b(bc-ad)e^4(be-af)(de-cf)\left(a+bx^2\right)+\right.}{}$$

input
```
Integrate[x^10/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]*(e + f*x^2)^3),x]
```

output
```
(Sqrt[b/a]*d*f^2*x*(c + d*x^2)*(2*b*(b*c - a*d)*e^4*(b*e - a*f)*(d*e - c*f
)*(a + b*x^2) + b*(b*c - a*d)*e^3*(a*f*(14*d*e - 17*c*f) + b*e*(-7*d*e + 1
0*c*f))*(a + b*x^2)*(e + f*x^2) - 8*a^4*f^2*(d*e - c*f)^2*(e + f*x^2)^2) -
 I*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(e + f*x^2)^2*(-(c*f*(-16*a^4*d
*f^3*(d*e - c*f)^2 + 8*a^3*b*f^2*(d*e - c*f)^2*(3*d*e + c*f) + b^4*c*e^3*(
-15*d^2*e^2 + 26*c*d*e*f - 8*c^2*f^2) + a^2*b^2*e*f*(-38*d^3*e^3 + 41*c*d^
2*e^2*f + 24*c^2*d*e*f^2 - 24*c^3*f^3) + 3*a*b^3*e^2*(5*d^3*e^3 + 4*c*d^2*
e^2*f - 19*c^2*d*e*f^2 + 8*c^3*f^3))*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*
d)/(b*c)]) - (b*c - a*d)*((d*e - c*f)*(8*a^3*c*f^4*(-(d*e) + c*f) + b^3*e^
3*(15*d^2*e^2 - 6*c*d*e*f - 8*c^2*f^2) + 24*a^2*b*e*f^2*(2*d^2*e^2 - c*d*e
*f - c^2*f^2) + a*b^2*e^2*f*(-48*d^2*e^2 + 23*c*d*e*f + 24*c^2*f^2))*Ellip
ticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - 3*b*d*e^2*(b^2*e^2*(5*d^2*e^2
- 12*c*d*e*f + 8*c^2*f^2) - 2*a*b*e*f*(8*d^2*e^2 - 19*c*d*e*f + 12*c^2*f^2
) + a^2*f^2*(16*d^2*e^2 - 36*c*d*e*f + 21*c^2*f^2))*EllipticPi[(a*f)/(b*e)
, I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])))/(8*b*Sqrt[b/a]*d*(b*c - a*d)*f^4
*(b*e - a*f)^3*(d*e - c*f)^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^2
)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{10}}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx$$
$$\Big\downarrow \ 450$$
$$\int \frac{x^{10}}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^3}dx$$

input
```
Int[x^10/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]*(e + f*x^2)^3),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3517 vs. $2(915) = 1830$.

Time = 21.50 (sec) , antiderivative size = 3518, normalized size of antiderivative = 3.70

method	result	size
elliptic	Expression too large to display	3518
default	Expression too large to display	12740

input
```
int(x^10/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x,method=_RETURNVERBO
SE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-(b*d*x^2+b*c
)/b^2*a^4/(a*d-b*c)*x/(a*f-b*e)^3/((x^2+a/b)*(b*d*x^2+b*c))^(1/2)+2/(-b/a)
^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(
1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))*b*d^2*e^5/f^3/(a*c
*f^2-a*d*e*f-b*c*e*f+b*d*e^2)^2/(a*f-b*e)*a+19/8/(-b/a)^(1/2)*(1+b*x^2/a)^
(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-
b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))*b^2*d*e^5/f^3/(a*c*f^2-a*d*e*f-b*c*e*
f+b*d*e^2)^2/(a*f-b*e)*c-5/4*c^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c
)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*e^4*b^2/f^2/(a*c*f^2-a*d*e*f-b
*c*e*f+b*d*e^2)^2/(a*f-b*e)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1
/2))+5/4*c^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d
*x^2+b*c*x^2+a*c)^(1/2)*e^4*b^2/f^2/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)^2/(a
*f-b*e)*EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-7/8*c/(-b/a)^(1
/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2
)*d*e^5*b^2/f^3/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)^2/(a*f-b*e)*EllipticE(x*
(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+27/2*e^3/(a*c*f^2-a*d*e*f-b*c*e*f+b
*d*e^2)^2/(a*f-b*e)/f/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*
d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d
)^(1/2)/(-b/a)^(1/2))*a^2*c*d+9*e^3/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)^2/(a
*f-b*e)/f/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^{10}}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(x^10/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x, algorithm="f
ricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{10}}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx=\text{Timed out}$$

input
```
integrate(x**10/(b*x**2+a)**(3/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{10}}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx=\int \frac{x^{10}}{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}(fx^2+e)^3}\,dx$$

input
```
integrate(x^10/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x, algorithm="m
axima")
```

output
```
integrate(x^10/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*(f*x^2 + e)^3), x)
```

Giac [F]

$$\int \frac{x^{10}}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx=\int \frac{x^{10}}{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}(fx^2+e)^3}\,dx$$

input
```
integrate(x^10/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x, algorithm="g
iac")
```

output
```
integrate(x^10/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*(f*x^2 + e)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{10}}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \int \frac{x^{10}}{(b\,x^2+a)^{3/2}\,\sqrt{d\,x^2+c}\,(f\,x^2+e)^3}\,dx$$

input
```
int(x^10/((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^3),x)
```

output
```
int(x^10/((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^3), x)
```

Reduce [F]

$$\int \frac{x^{10}}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \int \frac{x^{10}}{(b\,x^2+a)^{\frac{3}{2}}\,\sqrt{d\,x^2+c}\,(f\,x^2+e)^3}dx$$

input
```
int(x^10/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x)
```

output
```
int(x^10/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x)
```

3.264 $\int \frac{x^8}{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}\left(e+fx^2\right)^3}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 808

$$\int \frac{x^8}{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}\left(e+fx^2\right)^3}\,dx=\frac{e^3x\sqrt{c+dx^2}}{4f^2(be-af)(de-cf)\sqrt{a+bx^2}\left(e+fx^2\right)^2}$$

$$+\frac{e^2(af(10de-13cf)-be(5de-8cf))x\sqrt{c+dx^2}}{8f^2(be-af)^2(de-cf)^2\sqrt{a+bx^2}\left(e+fx^2\right)}$$

$$+\frac{\sqrt{a}(a^2bde^2f(10de-13cf)+3b^3ce^3(de-2cf)+8a^3f^2(de-cf)^2-ab^2e^2(3d^2e^2+4cdef-13c^2f^2))\sqrt{c \cdots}}{8\sqrt{b}(bc-ad)f^2(be-af)^3(de-cf)^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

$$-\frac{a^{3/2}(b^3ce^3(3de-4cf)+a^2bef(12d^2e^2+19cdef-32c^2f^2)-ab^2e^2(3d^2e^2+8cdef-13c^2f^2)-8a^3f^2(3 \cdots}{8\sqrt{b}c(bc-ad)f^2(be-af)^4(de-cf)\sqrt{a+bx^2} \cdots}$$

$$+\frac{a^{3/2}e(b^2e^2(3d^2e^2-8cdef+8c^2f^2)-2abef(6d^2e^2-17cdef+14c^2f^2)+a^2f^2(24d^2e^2-56cdef+35c^2f^{\cdots}}{8\sqrt{b}cf^2(be-af)^4(de-cf)^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
1/4*e^3*x*(d*x^2+c)^(1/2)/f^2/(-a*f+b*e)/(-c*f+d*e)/(b*x^2+a)^(1/2)/(f*x^2
+e)^2+1/8*e^2*(a*f*(-13*c*f+10*d*e)-b*e*(-8*c*f+5*d*e))*x*(d*x^2+c)^(1/2)/
f^2/(-a*f+b*e)^2/(-c*f+d*e)^2/(b*x^2+a)^(1/2)/(f*x^2+e)+1/8*a^(1/2)*(a^2*b
*d*e^2*f*(-13*c*f+10*d*e)+3*b^3*c*e^3*(-2*c*f+d*e)+8*a^3*f^2*(-c*f+d*e)^2-
a*b^2*e^2*(-13*c^2*f^2+4*c*d*e*f+3*d^2*e^2))*(d*x^2+c)^(1/2)*EllipticE(b^(
1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/b^(1/2)/(-a*d+b*c)/f^2
/(-a*f+b*e)^3/(-c*f+d*e)^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
-1/8*a^(3/2)*(b^3*c*e^3*(-4*c*f+3*d*e)+a^2*b*e*f*(-32*c^2*f^2+19*c*d*e*f+1
2*d^2*e^2)-a*b^2*e^2*(-13*c^2*f^2+8*c*d*e*f+3*d^2*e^2)-8*a^3*f^2*(-c^2*f^2
-2*c*d*e*f+3*d^2*e^2))*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^
(1/2)),(1-a*d/b/c)^(1/2))/b^(1/2)/c/(-a*d+b*c)/f^2/(-a*f+b*e)^4/(-c*f+d*e)
/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)+1/8*a^(3/2)*e*(b^2*e^2*(8
*c^2*f^2-8*c*d*e*f+3*d^2*e^2)-2*a*b*e*f*(14*c^2*f^2-17*c*d*e*f+6*d^2*e^2)+
a^2*f^2*(35*c^2*f^2-56*c*d*e*f+24*d^2*e^2))*(d*x^2+c)^(1/2)*EllipticPi(b^(
1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),1-a*f/b/e,(1-a*d/b/c)^(1/2))/b^(1/2)/c/f^
2/(-a*f+b*e)^4/(-c*f+d*e)^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2
)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.86 (sec) , antiderivative size = 590, normalized size of antiderivative = 0.73

$$\int \frac{x^8}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \frac{-\sqrt{\frac{b}{a}}f^2x(c+dx^2)\left(2(bc-ad)e^3(be-af)(de-cf)\left(a+bx^2\right)+\right.}{}$$

input
```
Integrate[x^8/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]*(e + f*x^2)^3),x]
```

output
```
(-(Sqrt[b/a]*f^2*x*(c + d*x^2)*(2*(b*c - a*d)*e^3*(b*e - a*f)*(d*e - c*f)*
(a + b*x^2) + (b*c - a*d)*e^2*(a*f*(10*d*e - 13*c*f) - 3*b*e*(d*e - 2*c*f)
)*(a + b*x^2)*(e + f*x^2) - 8*a^3*f*(d*e - c*f)^2*(e + f*x^2)^2)) + I*Sqrt
[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(e + f*x^2)^2*(c*f*(a^2*b*d*e^2*f*(10*
d*e - 13*c*f) + 3*b^3*c*e^3*(d*e - 2*c*f) + 8*a^3*f^2*(d*e - c*f)^2 + a*b^
2*e^2*(-3*d^2*e^2 - 4*c*d*e*f + 13*c^2*f^2))*EllipticE[I*ArcSinh[Sqrt[b/a]
*x], (a*d)/(b*c)] - (b*c - a*d)*e*((d*e - c*f)*(b^2*e^2*(3*d*e - 2*c*f) +
24*a^2*f^2*(d*e - c*f) + a*b*e*f*(-12*d*e + 11*c*f))*EllipticF[I*ArcSinh[S
qrt[b/a]*x], (a*d)/(b*c)] + (a^2*f^2*(-24*d^2*e^2 + 56*c*d*e*f - 35*c^2*f^
2) + b^2*e^2*(-3*d^2*e^2 + 8*c*d*e*f - 8*c^2*f^2) + 2*a*b*e*f*(6*d^2*e^2 -
 17*c*d*e*f + 14*c^2*f^2))*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x],
 (a*d)/(b*c)])))/(8*Sqrt[b/a]*(b*c - a*d)*f^3*(b*e - a*f)^3*(d*e - c*f)^2*
Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^2)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx$$
$$\Big\downarrow \; 450$$
$$\int \frac{x^8}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^3}dx$$

input
```
Int[x^8/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]*(e + f*x^2)^3),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3212 vs. $2(776) = 1552$.

Time = 20.70 (sec) , antiderivative size = 3213, normalized size of antiderivative = 3.98

method	result	size
elliptic	Expression too large to display	3213
default	Expression too large to display	9193

input
```
int(x^8/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x,method=_RETURNVERBOS
E)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*((b*d*x^2+b*c)
/b*a^3/(a*d-b*c)*x/(a*f-b*e)^3/((x^2+a/b)*(b*d*x^2+b*c))^(1/2)+1/4/f/(a*c*
f^2-a*d*e*f-b*c*e*f+b*d*e^2)*e^3*x/(a*f-b*e)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)
^(1/2)/(f*x^2+e)^2-1/8*e^2*(13*a*c*f^2-10*a*d*e*f-6*b*c*e*f+3*b*d*e^2)/f/(
a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)^2/(a*f-b*e)*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*
c)^(1/2)/(f*x^2+e)-c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d
*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*a^3/(a*d-b*c)/(a*f-b*e)^3*EllipticE(x*(-b/
a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-1/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x
^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(
-1+(a*d+b*c)/c/b)^(1/2))/b*a^3/(a*f-b*e)^3+1/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2
)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)
^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))/b/f^3+5/8/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*
(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(
1/2),(-1+(a*d+b*c)/c/b)^(1/2))*b^2*d^2*e^5/f^3/(a*c*f^2-a*d*e*f-b*c*e*f+b*
d*e^2)^2/(a*f-b*e)+35/8*e/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)^2*f/(a*f-b*e)/
(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+
a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*
a^2*c^2-7*e^2/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)^2/(a*f-b*e)/(-b/a)^(1/2)*(
1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*Ell
ipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*a^2*c*d+3*e...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^8}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(x^8/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^8}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**8/(b*x**2+a)**(3/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^8}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \int \frac{x^8}{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}(fx^2+e)^3}\,dx$$

input
```
integrate(x^8/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x, algorithm="ma
xima")
```

output
```
integrate(x^8/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*(f*x^2 + e)^3), x)
```

Giac [F]

$$\int \frac{x^8}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \int \frac{x^8}{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}(fx^2+e)^3}\,dx$$

input
```
integrate(x^8/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x, algorithm="gi
ac")
```

output
```
integrate(x^8/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*(f*x^2 + e)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^8}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx=\int \frac{x^8}{{\left(b\,x^2+a\right)}^{3/2}\,\sqrt{d\,x^2+c}\,{\left(f\,x^2+e\right)}^3}\,dx$$

input
```
int(x^8/((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^3),x)
```

output
```
int(x^8/((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^3), x)
```

Reduce [F]

$$\int \frac{x^8}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx=\int \frac{x^8}{\left(b\,x^2+a\right)^{\frac{3}{2}}\sqrt{d\,x^2+c}\,\left(f\,x^2+e\right)^3}dx$$

input
```
int(x^8/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x)
```

output
```
int(x^8/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x)
```

3.265 $\int \frac{x^6}{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}\left(e+fx^2\right)^3}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 728

$$\int \frac{x^6}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = -\frac{e^2x\sqrt{c+dx^2}}{4f(be-af)(de-cf)\sqrt{a+bx^2}\,(e+fx^2)^2}$$
$$+\frac{e(be(de-4cf)-3af(2de-3cf))x\sqrt{c+dx^2}}{8f(be-af)^2(de-cf)^2\sqrt{a+bx^2}\,(e+fx^2)}$$
$$+\frac{\sqrt{a}\sqrt{b}(b^2ce^2(de+2cf)-a^2f(14d^2e^2-25cdef+8c^2f^2)-abe(d^2e^2-4cdef+9c^2f^2))\sqrt{c+dx^2}\,E\left(\text{arct}\right.}{8(bc-ad)f(be-af)^3(de-cf)^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$
$$-\frac{a^{3/2}(b^3cde^3+a^2bdef(8de-7cf)+8a^3df^2(de-cf)-ab^2e(d^2e^2+16cdef-15c^2f^2))\sqrt{c+dx^2}\,\text{Elliptic}}{8\sqrt{b}c(bc-ad)f(be-af)^4(de-cf)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$
$$-\frac{a^{3/2}(2abde^2f(4de-7cf)-b^2de^3(de-4cf)+a^2f^2(8d^2e^2-20cdef+15c^2f^2))\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\right.}{8\sqrt{b}cf(be-af)^4(de-cf)^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
-1/4*e^2*x*(d*x^2+c)^(1/2)/f/(-a*f+b*e)/(-c*f+d*e)/(b*x^2+a)^(1/2)/(f*x^2+
e)^2+1/8*e*(b*e*(-4*c*f+d*e)-3*a*f*(-3*c*f+2*d*e))*x*(d*x^2+c)^(1/2)/f/(-a
*f+b*e)^2/(-c*f+d*e)^2/(b*x^2+a)^(1/2)/(f*x^2+e)+1/8*a^(1/2)*b^(1/2)*(b^2*
c*e^2*(2*c*f+d*e)-a^2*f*(8*c^2*f^2-25*c*d*e*f+14*d^2*e^2)-a*b*e*(9*c^2*f^2
-4*c*d*e*f+d^2*e^2))*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/
a)^(1/2),(1-a*d/b/c)^(1/2))/(-a*d+b*c)/f/(-a*f+b*e)^3/(-c*f+d*e)^2/(b*x^2+
a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)-1/8*a^(3/2)*(b^3*c*d*e^3+a^2*b*d*
e*f*(-7*c*f+8*d*e)+8*a^3*d*f^2*(-c*f+d*e)-a*b^2*e*(-15*c^2*f^2+16*c*d*e*f+
d^2*e^2))*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d
/b/c)^(1/2))/b^(1/2)/c/(-a*d+b*c)/f/(-a*f+b*e)^4/(-c*f+d*e)/(b*x^2+a)^(1/2
)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)-1/8*a^(3/2)*(2*a*b*d*e^2*f*(-7*c*f+4*d*e
)-b^2*d*e^3*(-4*c*f+d*e)+a^2*f^2*(15*c^2*f^2-20*c*d*e*f+8*d^2*e^2))*(d*x^2
+c)^(1/2)*EllipticPi(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),1-a*f/b/e,(1-a*d/
b/c)^(1/2))/b^(1/2)/c/f/(-a*f+b*e)^4/(-c*f+d*e)^2/(b*x^2+a)^(1/2)/(a*(d*x^
2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.49 (sec) , antiderivative size = 548, normalized size of antiderivative = 0.75

$$\int \frac{x^6}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \frac{\sqrt{\frac{b}{a}}f^2x(c+dx^2)\left(2(bc-ad)e^2(be-af)(de-cf)\left(a+bx^2\right)+(b\right.}{}$$

input
```
Integrate[x^6/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]*(e + f*x^2)^3),x]
```

output
```
(Sqrt[b/a]*f^2*x*(c + d*x^2)*(2*(b*c - a*d)*e^2*(b*e - a*f)*(d*e - c*f)*(a
 + b*x^2) + (b*c - a*d)*e*(3*a*f*(2*d*e - 3*c*f) + b*e*(d*e + 2*c*f))*(a +
 b*x^2)*(e + f*x^2) - 8*a^2*b*(d*e - c*f)^2*(e + f*x^2)^2) + I*Sqrt[1 + (b
*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(e + f*x^2)^2*(b*c*f*(b^2*c*e^2*(d*e + 2*c*f)
 + a^2*f*(-14*d^2*e^2 + 25*c*d*e*f - 8*c^2*f^2) - a*b*e*(d^2*e^2 - 4*c*d*e
*f + 9*c^2*f^2))*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - (b*c - a
*d)*((d*e - c*f)*(b^2*e^2*(d*e - 2*c*f) + 8*a^2*f^2*(-(d*e) + c*f) + a*b*e
*f*(-8*d*e + 9*c*f))*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (2*a
*b*d*e^2*f*(4*d*e - 7*c*f) + b^2*d*e^3*(-(d*e) + 4*c*f) + a^2*f^2*(8*d^2*e
^2 - 20*c*d*e*f + 15*c^2*f^2))*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]
*x], (a*d)/(b*c)])))/(8*Sqrt[b/a]*(b*c - a*d)*f^2*(b*e - a*f)^3*(d*e - c*f
)^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^2)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx$$

$$\Big\downarrow \; 450$$

$$\int \frac{x^6}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^3}dx$$

input
```
Int[x^6/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]*(e + f*x^2)^3),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2802 vs. $2(696) = 1392$.

Time = 21.26 (sec) , antiderivative size = 2803, normalized size of antiderivative = 3.85

method	result	size
elliptic	Expression too large to display	2803
default	Expression too large to display	8149

input
```
int(x^6/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x,method=_RETURNVERBOS
E)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-(b*d*x^2+b*c
)*a^2/(a*d-b*c)*x/(a*f-b*e)^3/((x^2+a/b)*(b*d*x^2+b*c))^(1/2)-1/4/(a*c*f^2
-a*d*e*f-b*c*e*f+b*d*e^2)*e^2*x/(a*f-b*e)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1
/2)/(f*x^2+e)^2+1/8*e*(9*a*c*f^2-6*a*d*e*f-2*b*c*e*f-b*d*e^2)/(a*c*f^2-a*d
*e*f-b*c*e*f+b*d*e^2)^2/(a*f-b*e)*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(f
*x^2+e)-17/8/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d
*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))
*b*d*e^2/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)^2/(a*f-b*e)*a*c+1/(-b/a)^(1/2)*
(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*El
lipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))*b*d^2*e^3/(a*c*f^2-a*d*e*
f-b*c*e*f+b*d*e^2)^2/(a*f-b*e)/f*a-15/8/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)^
2*f^2/(a*f-b*e)/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+
a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2
)/(-b/a)^(1/2))*a^2*c^2-e^2/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)^2/(a*f-b*e)/
(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+
a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/2)/(-b/a)^(1/2))*
a^2*d^2+3/8/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*
x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))*
b^2*d*e^3/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)^2/(a*f-b*e)/f*c+3/4*c/(-b/a)^(
1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^6}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(x^6/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^6}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx=\text{Timed out}$$

input
```
integrate(x**6/(b*x**2+a)**(3/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^6}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx=\int \frac{x^6}{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}(fx^2+e)^3}\,dx$$

input
```
integrate(x^6/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x, algorithm="ma
xima")
```

output
```
integrate(x^6/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*(f*x^2 + e)^3), x)
```

Giac [F]

$$\int \frac{x^6}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx=\int \frac{x^6}{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}(fx^2+e)^3}\,dx$$

input
```
integrate(x^6/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x, algorithm="gi
ac")
```

output
```
integrate(x^6/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*(f*x^2 + e)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx=\int \frac{x^6}{(b\,x^2+a)^{3/2}\,\sqrt{d\,x^2+c}\,(f\,x^2+e)^3}\,dx$$

input
```
int(x^6/((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^3),x)
```

output
```
int(x^6/((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^3), x)
```

Reduce [F]

$$\int \frac{x^6}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx=\int \frac{}{b^2d\,f^3x^{12}+2abd\,f^3x^{10}+b^2c\,f^3x^{10}+3b^2de\,f^2x^{10}+a^2d\,f^3x^8}$$

input
```
int(x^6/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**6)/(a**2*c*e**3 + 3*a**2*c*e**2*
f*x**2 + 3*a**2*c*e*f**2*x**4 + a**2*c*f**3*x**6 + a**2*d*e**3*x**2 + 3*a*
*2*d*e**2*f*x**4 + 3*a**2*d*e*f**2*x**6 + a**2*d*f**3*x**8 + 2*a*b*c*e**3*
x**2 + 6*a*b*c*e**2*f*x**4 + 6*a*b*c*e*f**2*x**6 + 2*a*b*c*f**3*x**8 + 2*a
*b*d*e**3*x**4 + 6*a*b*d*e**2*f*x**6 + 6*a*b*d*e*f**2*x**8 + 2*a*b*d*f**3*
x**10 + b**2*c*e**3*x**4 + 3*b**2*c*e**2*f*x**6 + 3*b**2*c*e*f**2*x**8 + b
**2*c*f**3*x**10 + b**2*d*e**3*x**6 + 3*b**2*d*e**2*f*x**8 + 3*b**2*d*e*f*
*2*x**10 + b**2*d*f**3*x**12),x)
```

3.266 $\int \frac{x^4}{(a+bx^2)^{3/2}\sqrt{c+dx^2}(e+fx^2)^3} dx$

Optimal result

Integrand size = 35, antiderivative size = 663

$$\int \frac{x^4}{(a+bx^2)^{3/2}\sqrt{c+dx^2}(e+fx^2)^3} dx = \frac{ex\sqrt{c+dx^2}}{4(be-af)(de-cf)\sqrt{a+bx^2}(e+fx^2)^2}$$
$$+ \frac{(3bde^2+af(2de-5cf))x\sqrt{c+dx^2}}{8(be-af)^2(de-cf)^2\sqrt{a+bx^2}(e+fx^2)}$$
$$+ \frac{\sqrt{a}\sqrt{b}(a^2df(2de-5cf)-b^2ce(5de-2cf)+ab(13d^2e^2-20cdef+13c^2f^2))\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|\right.}{8(bc-ad)(be-af)^3(de-cf)^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$
$$- \frac{a^{3/2}\sqrt{b}(b^2ce(11de-12cf)-a^2df(12de-11cf)-ab(3d^2e^2-8cdef+3c^2f^2))\sqrt{c+dx^2}\text{EllipticF}\left(\text{arct}\right.}{8c(bc-ad)(be-af)^4(de-cf)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$
$$+ \frac{3a^{3/2}(b^2d^2e^4+a^2c^2f^4+2abef(2d^2e^2-5cdef+2c^2f^2))\sqrt{c+dx^2}\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\right.}{8\sqrt{b}ce(be-af)^4(de-cf)^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
1/4*e*x*(d*x^2+c)^(1/2)/(-a*f+b*e)/(-c*f+d*e)/(b*x^2+a)^(1/2)/(f*x^2+e)^2+
1/8*(3*b*d*e^2+a*f*(-5*c*f+2*d*e))*x*(d*x^2+c)^(1/2)/(-a*f+b*e)^2/(-c*f+d*
e)^2/(b*x^2+a)^(1/2)/(f*x^2+e)+1/8*a^(1/2)*b^(1/2)*(a^2*d*f*(-5*c*f+2*d*e)
-b^2*c*e*(-2*c*f+5*d*e)+a*b*(13*c^2*f^2-20*c*d*e*f+13*d^2*e^2))*(d*x^2+c)^
(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/(-a
*d+b*c)/(-a*f+b*e)^3/(-c*f+d*e)^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a)
)^(1/2)-1/8*a^(3/2)*b^(1/2)*(b^2*c*e*(-12*c*f+11*d*e)-a^2*d*f*(-11*c*f+12*
d*e)-a*b*(3*c^2*f^2-8*c*d*e*f+3*d^2*e^2))*(d*x^2+c)^(1/2)*InverseJacobiAM(
arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/c/(-a*d+b*c)/(-a*f+b*e)^4/(-c
*f+d*e)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)+3/8*a^(3/2)*(b^2*d
^2*e^4+a^2*c^2*f^4+2*a*b*e*f*(2*c^2*f^2-5*c*d*e*f+2*d^2*e^2))*(d*x^2+c)^(1
/2)*EllipticPi(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),1-a*f/b/e,(1-a*d/b/c)^(
1/2))/b^(1/2)/c/e/(-a*f+b*e)^4/(-c*f+d*e)^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c
/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.22 (sec) , antiderivative size = 502, normalized size of antiderivative = 0.76

$$\int \frac{x^4}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \frac{-\sqrt{\frac{b}{a}}efx(c+dx^2)\left(2(bc-ad)ef(be-af)(de-cf)\left(a+bx^2\right)+\right.}{}$$

input
```
Integrate[x^4/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]*(e + f*x^2)^3),x]
```

output
```
(-(Sqrt[b/a]*e*f*x*(c + d*x^2)*(2*(b*c - a*d)*e*f*(b*e - a*f)*(d*e - c*f)*
(a + b*x^2) + (b*c - a*d)*f*(a*f*(2*d*e - 5*c*f) + b*e*(5*d*e - 2*c*f))*(a
 + b*x^2)*(e + f*x^2) - 8*a*b^2*(d*e - c*f)^2*(e + f*x^2)^2)) + I*Sqrt[1 +
 (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(e + f*x^2)^2*(b*c*e*f*(a^2*d*f*(2*d*e - 5
*c*f) + b^2*c*e*(-5*d*e + 2*c*f) + a*b*(13*d^2*e^2 - 20*c*d*e*f + 13*c^2*f
^2))*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - (b*c - a*d)*(b*e*(d*
e - c*f)*(a*f*(12*d*e - 13*c*f) + b*e*(3*d*e - 2*c*f))*EllipticF[I*ArcSinh
[Sqrt[b/a]*x], (a*d)/(b*c)] - 3*(b^2*d^2*e^4 + a^2*c^2*f^4 + 2*a*b*e*f*(2*
d^2*e^2 - 5*c*d*e*f + 2*c^2*f^2))*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b
/a]*x], (a*d)/(b*c)])))/(8*Sqrt[b/a]*(b*c - a*d)*e*f*(b*e - a*f)^3*(d*e -
c*f)^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^2)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx$$
$$\Big\downarrow \text{450}$$
$$\int \frac{x^4}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^3}dx$$

input
```
Int[x^4/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]*(e + f*x^2)^3),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2506 vs. 2(631) = 1262.

Time = 21.10 (sec) , antiderivative size = 2507, normalized size of antiderivative = 3.78

method	result	size
elliptic	Expression too large to display	2507
default	Expression too large to display	6729

input
```
int(x^4/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x,method=_RETURNVERBOS
E)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*((b*d*x^2+b*c)
*b*a/(a*d-b*c)*x/(a*f-b*e)^3/((x^2+a/b)*(b*d*x^2+b*c))^(1/2)+1/4*f/(a*c*f^
2-a*d*e*f-b*c*e*f+b*d*e^2)*e*x/(a*f-b*e)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/
2)/(f*x^2+e)^2-1/8*f*(5*a*c*f^2-2*a*d*e*f+2*b*c*e*f-5*b*d*e^2)/(a*c*f^2-a*
d*e*f-b*c*e*f+b*d*e^2)^2/(a*f-b*e)*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(
f*x^2+e)-3/8/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d
*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))
*b^2*d^2*e^3/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)^2/(a*f-b*e)/f-1/2/(-b/a)^(1
/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2
)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))*b*d^2*e^2/(a*c*f^2-a*
d*e*f-b*c*e*f+b*d*e^2)^2/(a*f-b*e)*a+5/8*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*
(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*d*b^2/(a*c*f^2-a*d*e
*f-b*c*e*f+b*d*e^2)^2/(a*f-b*e)*e^2*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)
/c/b)^(1/2))-c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a
*d*x^2+b*c*x^2+a*c)^(1/2)*a*b^2/(a*d-b*c)/(a*f-b*e)^3*EllipticE(x*(-b/a)^(
1/2),(-1+(a*d+b*c)/c/b)^(1/2))-5/8*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x
^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*d*b^2/(a*c*f^2-a*d*e*f-b*c
*e*f+b*d*e^2)^2/(a*f-b*e)*e^2*EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^
(1/2))-5/8*c^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a
*d*x^2+b*c*x^2+a*c)^(1/2)*b/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)^2/(a*f-b*...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^4}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(x^4/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^4}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**4/(b*x**2+a)**(3/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^4}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \int \frac{x^4}{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}(fx^2+e)^3}\,dx$$

input
```
integrate(x^4/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x, algorithm="ma
xima")
```

output
```
integrate(x^4/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*(f*x^2 + e)^3), x)
```

Giac [F]

$$\int \frac{x^4}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \int \frac{x^4}{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}(fx^2+e)^3}\,dx$$

input
```
integrate(x^4/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x, algorithm="gi
ac")
```

output
```
integrate(x^4/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*(f*x^2 + e)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx=\int \frac{x^4}{(b\,x^2+a)^{3/2}\,\sqrt{d\,x^2+c}\,(f\,x^2+e)^3}\,dx$$

input
```
int(x^4/((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^3),x)
```

output
```
int(x^4/((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^3), x)
```

Reduce [F]

$$\int \frac{x^4}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx=\int \frac{}{b^2d\,f^3x^{12}+2abd\,f^3x^{10}+b^2c\,f^3x^{10}+3b^2de\,f^2x^{10}+a^2d\,f^3x^8 -}$$

input
```
int(x^4/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a**2*c*e**3 + 3*a**2*c*e**2*
f*x**2 + 3*a**2*c*e*f**2*x**4 + a**2*c*f**3*x**6 + a**2*d*e**3*x**2 + 3*a*
*2*d*e**2*f*x**4 + 3*a**2*d*e*f**2*x**6 + a**2*d*f**3*x**8 + 2*a*b*c*e**3*
x**2 + 6*a*b*c*e**2*f*x**4 + 6*a*b*c*e*f**2*x**6 + 2*a*b*c*f**3*x**8 + 2*a
*b*d*e**3*x**4 + 6*a*b*d*e**2*f*x**6 + 6*a*b*d*e*f**2*x**8 + 2*a*b*d*f**3*
x**10 + b**2*c*e**3*x**4 + 3*b**2*c*e**2*f*x**6 + 3*b**2*c*e*f**2*x**8 + b
**2*c*f**3*x**10 + b**2*d*e**3*x**6 + 3*b**2*d*e**2*f*x**8 + 3*b**2*d*e*f*
*2*x**10 + b**2*d*f**3*x**12),x)
```

3.267 $\int \frac{x^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}(e+fx^2)^3}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 727

$$\int \frac{x^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = -\frac{fx\sqrt{c+dx^2}}{4(be-af)(de-cf)\sqrt{a+bx^2}\,(e+fx^2)^2}$$
$$-\frac{f(be(7de-4cf)-af(2de+cf))x\sqrt{c+dx^2}}{8e(be-af)^2(de-cf)^2\sqrt{a+bx^2}\,(e+fx^2)}$$
$$+\frac{\sqrt{a}\sqrt{b}(a^2df^2(2de+cf)-abf(9d^2e^2-4cdef+c^2f^2)-b^2e(8d^2e^2-25cdef+14c^2f^2))\sqrt{c+dx^2}E\Big(\text{arct}}{8(bc-ad)e(be-af)^3(de-cf)^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$
$$-\frac{\sqrt{a}\sqrt{b}(a^3cdf^3-ab^2cef(7de-8cf)-8b^3ce^2(de-cf)+a^2bf(15d^2e^2-16cdef-c^2f^2))\sqrt{c+dx^2}\,\text{Ellipt}}{8c(bc-ad)e(be-af)^4(de-cf)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$
$$+\frac{a^{3/2}f(2abcef^2(7de-4cf)-a^2cf^3(4de-cf)-b^2e^2(15d^2e^2-20cdef+8c^2f^2))\sqrt{c+dx^2}\,\text{EllipticPi}\Big(1-}{8\sqrt{b}ce^2(be-af)^4(de-cf)^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
-1/4*f*x*(d*x^2+c)^(1/2)/(-a*f+b*e)/(-c*f+d*e)/(b*x^2+a)^(1/2)/(f*x^2+e)^2
-1/8*f*(b*e*(-4*c*f+7*d*e)-a*f*(c*f+2*d*e))*x*(d*x^2+c)^(1/2)/e/(-a*f+b*e)
^2/(-c*f+d*e)^2/(b*x^2+a)^(1/2)/(f*x^2+e)+1/8*a^(1/2)*b^(1/2)*(a^2*d*f^2*(
c*f+2*d*e)-a*b*f*(c^2*f^2-4*c*d*e*f+9*d^2*e^2)-b^2*e*(14*c^2*f^2-25*c*d*e*
f+8*d^2*e^2))*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2
),(1-a*d/b/c)^(1/2))/(-a*d+b*c)/e/(-a*f+b*e)^3/(-c*f+d*e)^2/(b*x^2+a)^(1/2
)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)-1/8*a^(1/2)*b^(1/2)*(a^3*c*d*f^3-a*b^2*c
*e*f*(-8*c*f+7*d*e)-8*b^3*c*e^2*(-c*f+d*e)+a^2*b*f*(-c^2*f^2-16*c*d*e*f+15
*d^2*e^2))*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*
d/b/c)^(1/2))/c/(-a*d+b*c)/e/(-a*f+b*e)^4/(-c*f+d*e)/(b*x^2+a)^(1/2)/(a*(d
*x^2+c)/c/(b*x^2+a))^(1/2)+1/8*a^(3/2)*f*(2*a*b*c*e*f^2*(-4*c*f+7*d*e)-a^2
*c*f^3*(-c*f+4*d*e)-b^2*e^2*(8*c^2*f^2-20*c*d*e*f+15*d^2*e^2))*(d*x^2+c)^(
1/2)*EllipticPi(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),1-a*f/b/e,(1-a*d/b/c)^
(1/2))/b^(1/2)/c/e^2/(-a*f+b*e)^4/(-c*f+d*e)^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c
)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.11 (sec) , antiderivative size = 526, normalized size of antiderivative = 0.72

$$\int \frac{x^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \frac{\sqrt{\frac{b}{a}}ex(c+dx^2)\left(2(bc-ad)ef^2(be-af)(de-cf)\left(a+bx^2\right)-(\right.}{}$$

input
```
Integrate[x^2/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]*(e + f*x^2)^3),x]
```

output
```
(Sqrt[b/a]*e*x*(c + d*x^2)*(2*(b*c - a*d)*e*f^2*(b*e - a*f)*(d*e - c*f)*(a
 + b*x^2) - (b*c - a*d)*f^2*(a*f*(2*d*e + c*f) + b*(-9*d*e^2 + 6*c*e*f))*(
a + b*x^2)*(e + f*x^2) - 8*b^3*e*(d*e - c*f)^2*(e + f*x^2)^2) - I*Sqrt[1 +
 (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(e + f*x^2)^2*(b*c*e*(-(a^2*d*f^2*(2*d*e +
 c*f)) + a*b*f*(9*d^2*e^2 - 4*c*d*e*f + c^2*f^2) + b^2*e*(8*d^2*e^2 - 25*c
*d*e*f + 14*c^2*f^2))*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - (b*
c - a*d)*(b*e*(d*e - c*f)*(-(a*c*f^2) + b*e*(15*d*e - 14*c*f))*EllipticF[I
*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (2*a*b*c*e*f^2*(7*d*e - 4*c*f) + a^2
*c*f^3*(-4*d*e + c*f) + b^2*e^2*(-15*d^2*e^2 + 20*c*d*e*f - 8*c^2*f^2))*El
lipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])))/(8*Sqrt[b/a]
*(b*c - a*d)*e^2*(b*e - a*f)^3*(d*e - c*f)^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^
2]*(e + f*x^2)^2)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx$$
$$\Big\downarrow \text{ 450}$$
$$\int \frac{x^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^3}dx$$

input
```
Int[x^2/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]*(e + f*x^2)^3),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2668 vs. $2(695) = 1390$.

Time = 21.30 (sec) , antiderivative size = 2669, normalized size of antiderivative = 3.67

method	result	size
elliptic	Expression too large to display	2669
default	Expression too large to display	7326

input
```
int(x^2/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x,method=_RETURNVERBOS
E)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-(b*d*x^2+b*c
)*b^2/(a*d-b*c)*x/(a*f-b*e)^3/((x^2+a/b)*(b*d*x^2+b*c))^(1/2)-1/4*f^2/(a*c
*f^2-a*d*e*f-b*c*e*f+b*d*e^2)*x/(a*f-b*e)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1
/2)/(f*x^2+e)^2+1/8*f^2*(a*c*f^2+2*a*d*e*f+6*b*c*e*f-9*b*d*e^2)/(a*c*f^2-a
*d*e*f-b*c*e*f+b*d*e^2)^2/e/(a*f-b*e)*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2
)/(f*x^2+e)+1/8*c^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*
x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*f^3*b/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)^2/e
/(a*f-b*e)*a*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-1/8*c^2/(-
b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*
c)^(1/2)*f^3*b/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)^2/e/(a*f-b*e)*a*EllipticE
(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+1/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)
*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^
(1/2),(-1+(a*d+b*c)/c/b)^(1/2))*b^2/(a*f-b*e)^3+7/8/(-b/a)^(1/2)*(1+b*x^2/
a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x
*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))*b^2*d^2/(a*c*f^2-a*d*e*f-b*c*e*f+b
*d*e^2)^2/(a*f-b*e)*e^2+c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)
/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*b^3/(a*d-b*c)/(a*f-b*e)^3*EllipticE(x
*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+7/4/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e
^2)^2/(a*f-b*e)*f^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*
x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(x^2/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**2/(b*x**2+a)**(3/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \int \frac{x^2}{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}(fx^2+e)^3}\,dx$$

input
```
integrate(x^2/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x, algorithm="ma
xima")
```

output
```
integrate(x^2/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*(f*x^2 + e)^3), x)
```

Giac [F]

$$\int \frac{x^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \int \frac{x^2}{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}(fx^2+e)^3}\,dx$$

input
```
integrate(x^2/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x, algorithm="gi
ac")
```

output
```
integrate(x^2/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*(f*x^2 + e)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx=\int \frac{x^2}{(b\,x^2+a)^{3/2}\,\sqrt{d\,x^2+c}\,(f\,x^2+e)^3}\,dx$$

input
```
int(x^2/((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^3),x)
```

output
```
int(x^2/((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^3), x)
```

Reduce [F]

$$\int \frac{x^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx=\int \frac{}{b^2d\,f^3x^{12}+2abd\,f^3x^{10}+b^2c\,f^3x^{10}+3b^2de\,f^2x^{10}+a^2d\,f^3x^8}$$

input
```
int(x^2/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a**2*c*e**3 + 3*a**2*c*e**2*
f*x**2 + 3*a**2*c*e*f**2*x**4 + a**2*c*f**3*x**6 + a**2*d*e**3*x**2 + 3*a*
*2*d*e**2*f*x**4 + 3*a**2*d*e*f**2*x**6 + a**2*d*f**3*x**8 + 2*a*b*c*e**3*
x**2 + 6*a*b*c*e**2*f*x**4 + 6*a*b*c*e*f**2*x**6 + 2*a*b*c*f**3*x**8 + 2*a
*b*d*e**3*x**4 + 6*a*b*d*e**2*f*x**6 + 6*a*b*d*e*f**2*x**8 + 2*a*b*d*f**3*
x**10 + b**2*c*e**3*x**4 + 3*b**2*c*e**2*f*x**6 + 3*b**2*c*e*f**2*x**8 + b
**2*c*f**3*x**10 + b**2*d*e**3*x**6 + 3*b**2*d*e**2*f*x**8 + 3*b**2*d*e*f*
*2*x**10 + b**2*d*f**3*x**12),x)
```

3.268 $\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}(e+fx^2)^3}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 810

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \frac{f^2x\sqrt{c+dx^2}}{4e(be-af)(de-cf)\sqrt{a+bx^2}\,(e+fx^2)^2}$$
$$+\frac{f^2(be(11de-8cf)-3af(2de-cf))x\sqrt{c+dx^2}}{8e^2(be-af)^2(de-cf)^2\sqrt{a+bx^2}\,(e+fx^2)}$$
$$-\frac{\sqrt{b}(ab^2cef^2(13de-10cf)-8b^3e^2(de-cf)^2+3a^3df^3(2de-cf)-a^2bf^2(13d^2e^2-4cdef-3c^2f^2))\sqrt{c}}{8\sqrt{a}(bc-ad)e^2(be-af)^3(de-cf)^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$
$$+\frac{\sqrt{a}\sqrt{b}(a^3df^3(4de-3cf)+ab^2ef(32d^2e^2-19cdef-12c^2f^2)-a^2bf^2(13d^2e^2-8cdef-3c^2f^2)-8b^3e^2(}{8c(bc-ad)e^2(be-af)^4(de-cf)\sqrt{a+bx^2}\sqrt{}}$$
$$+\frac{a^{3/2}f^2(a^2f^2(8d^2e^2-8cdef+3c^2f^2)-2abef(14d^2e^2-17cdef+6c^2f^2)+b^2e^2(35d^2e^2-56cdef+24c^2}{8\sqrt{b}ce^3(be-af)^4(de-cf)^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
1/4*f^2*x*(d*x^2+c)^(1/2)/e/(-a*f+b*e)/(-c*f+d*e)/(b*x^2+a)^(1/2)/(f*x^2+e
)^2+1/8*f^2*(b*e*(-8*c*f+11*d*e)-3*a*f*(-c*f+2*d*e))*x*(d*x^2+c)^(1/2)/e^2
/(-a*f+b*e)^2/(-c*f+d*e)^2/(b*x^2+a)^(1/2)/(f*x^2+e)-1/8*b^(1/2)*(a*b^2*c*
e*f^2*(-10*c*f+13*d*e)-8*b^3*e^2*(-c*f+d*e)^2+3*a^3*d*f^3*(-c*f+2*d*e)-a^2
*b*f^2*(-3*c^2*f^2-4*c*d*e*f+13*d^2*e^2))*(d*x^2+c)^(1/2)*EllipticE(b^(1/2
)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/a^(1/2)/(-a*d+b*c)/e^2/(-
a*f+b*e)^3/(-c*f+d*e)^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)+1/
8*a^(1/2)*b^(1/2)*(a^3*d*f^3*(-3*c*f+4*d*e)+a*b^2*e*f*(-12*c^2*f^2-19*c*d*
e*f+32*d^2*e^2)-a^2*b*f^2*(-3*c^2*f^2-8*c*d*e*f+13*d^2*e^2)-8*b^3*e^2*(-3*
c^2*f^2+2*c*d*e*f+d^2*e^2))*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan(b^(1/2)
*x/a^(1/2)),(1-a*d/b/c)^(1/2))/c/(-a*d+b*c)/e^2/(-a*f+b*e)^4/(-c*f+d*e)/(b
*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)+1/8*a^(3/2)*f^2*(a^2*f^2*(3*
c^2*f^2-8*c*d*e*f+8*d^2*e^2)-2*a*b*e*f*(6*c^2*f^2-17*c*d*e*f+14*d^2*e^2)+b
^2*e^2*(24*c^2*f^2-56*c*d*e*f+35*d^2*e^2))*(d*x^2+c)^(1/2)*EllipticPi(b^(1
/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),1-a*f/b/e,(1-a*d/b/c)^(1/2))/b^(1/2)/c/e^3
/(-a*f+b*e)^4/(-c*f+d*e)^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.68 (sec) , antiderivative size = 600, normalized size of antiderivative = 0.74

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \frac{\sqrt{\frac{b}{a}}\left(-\sqrt{\frac{b}{a}}ex(c+dx^2)\left(2a(bc-ad)ef^3(be-af)(de-cf)\left(a+\right.\right.\right.}{}$$

input
```
Integrate[1/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]*(e + f*x^2)^3),x]
```

output

```
(Sqrt[b/a]*(-(Sqrt[b/a]*e*x*(c + d*x^2)*(2*a*(b*c - a*d)*e*f^3*(b*e - a*f)
*(d*e - c*f)*(a + b*x^2) + a*(b*c - a*d)*f^3*(b*e*(13*d*e - 10*c*f) + 3*a*
f*(-2*d*e + c*f))*(a + b*x^2)*(e + f*x^2) - 8*b^4*e^2*(d*e - c*f)^2*(e + f
*x^2)^2)) + I*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(e + f*x^2)^2*(b*c*e
*(8*b^3*e^2*(d*e - c*f)^2 + 3*a^3*d*f^3*(-2*d*e + c*f) + a*b^2*c*e*f^2*(-1
3*d*e + 10*c*f) + a^2*b*f^2*(13*d^2*e^2 - 4*c*d*e*f - 3*c^2*f^2))*Elliptic
E[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - (b*c - a*d)*(b*e*(d*e - c*f)*(a*b
*e*f*(11*d*e - 10*c*f) + 8*b^2*e^2*(d*e - c*f) + a^2*f^2*(-4*d*e + 3*c*f))
*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + a*f*(b^2*e^2*(-35*d^2*e^
2 + 56*c*d*e*f - 24*c^2*f^2) + a^2*f^2*(-8*d^2*e^2 + 8*c*d*e*f - 3*c^2*f^2
) + 2*a*b*e*f*(14*d^2*e^2 - 17*c*d*e*f + 6*c^2*f^2))*EllipticPi[(a*f)/(b*e
), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))))/(8*b*(b*c - a*d)*e^3*(b*e - a*
f)^3*(d*e - c*f)^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^2)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx$$

$$\Big\downarrow \text{426}$$

$$\frac{b\int \frac{1}{(bx^2+a)^{3/2}\sqrt{dx^2+c}(fx^2+e)^2}dx}{be-af}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^3}dx}{be-af}$$

$$\Big\downarrow \text{426}$$

$$\frac{b\left(\frac{b\int \frac{1}{\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{be-af}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^2}dx}{be-af}\right)}{be-af}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^3}dx}{be-af}$$

$$\Big\downarrow \text{421}$$

$$\frac{b\left(\frac{b\left(\frac{f^2\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(be-af)^2}-\frac{b\int -\frac{-bfx^2+be-2af}{\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}}dx}{(be-af)^2}\right)}{be-af}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^2}dx}{be-af}\right)}{be-af}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^3}dx}{be-af}$$

$\big\downarrow$ 25

$$\frac{b\left(\frac{b\left(\frac{f^2\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(be-af)^2}+\frac{b\int \frac{-bfx^2+be-2af}{\left(bx^2+a\right)^{3/2}\sqrt{dx^2+c}}dx}{(be-af)^2}\right)}{be-af}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^2}dx}{be-af}\right)}{be-af}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^3}dx}{be-af}$$

$\big\downarrow$ 400

$$\frac{b\left(\frac{b\left(\frac{f^2\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(be-af)^2}+\frac{b\left(\frac{b(be-af)\int \frac{\sqrt{dx^2+c}}{\left(bx^2+a\right)^{3/2}}dx}{bc-ad}-\frac{(-2adf+bcf+bde)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}\right)}{(be-af)^2}\right)}{be-af}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^2}dx}{be-af}\right)}{be-af}-\frac{f\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^3}dx}{be-af}$$

$\big\downarrow$ 313

$$\frac{b\left(\frac{b\left(\frac{b\left(\frac{\sqrt{b}\sqrt{c+dx^2}(be-af)E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{a+bx^2}(bc-ad)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}-\frac{(-2adf+bcf+bde)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{bc-ad}\right)}{(be-af)^2}+\frac{f^2\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)}dx}{(be-af)^2}\right)}{be-af}-\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\ldots}}{be-af}\ldots\right)}{be-af}$$

$$\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^3}dx}{be-af}$$

$\Big\downarrow$ 320

$$\frac{b\left(\frac{b\left(\frac{f^2\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}(fx^2+e)}dx}{(be-af)^2}+\frac{b\left(\frac{\sqrt{b}\sqrt{c+dx^2}(be-af)E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{a+bx^2}(bc-ad)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}-\frac{\sqrt{c}\sqrt{a+bx^2}(-2adf+bcf+bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{(be-af)^2}\right)}{be-af}-\ldots\right)}{be-af}$$

$$\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^3}dx}{be-af}$$

$\Big\downarrow$ 414

$$\frac{b\left(\frac{b\left(\frac{a^{3/2}f^2\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce\sqrt{a+bx^2}(be-af)^2\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}+\frac{b\left(\frac{\sqrt{b}\sqrt{c+dx^2}(be-af)E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{a+bx^2}(bc-ad)\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}-\frac{\sqrt{c}\sqrt{a+bx^2}(-2adf+bcf+bde)\,\text{EllipticF}\left(\ldots\right.}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(\ldots}{a(\ldots}}}\right.}{(be-af)^2}\right.}{be-af}\right.}{be-af}$$

$$\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^3}dx}{be-af}$$

$\downarrow$ 424

$$\frac{b\left(\frac{b\left(\frac{a^{3/2}f^2\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce\sqrt{a+bx^2}(be-af)^2\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}+\frac{b\left(\frac{\sqrt{b}\sqrt{c+dx^2}(be-af)E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{a+bx^2}(bc-ad)\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}-\frac{\sqrt{c}\sqrt{a+bx^2}(-2adf+bcf+bde)\,\text{EllipticF}\left(\ldots\right.}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(\ldots}{a(\ldots}}}\right.}{(be-af)^2}\right.}{be-af}\right.}{be-\ldots}$$

$$\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^3}dx}{be-af}$$

$\downarrow$ 406

$$b\left(\frac{b\left(\frac{a^{3/2}f^2\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce\sqrt{a+bx^2}(be-af)^2\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}+\frac{b\left(\frac{\sqrt{b}\sqrt{c+dx^2}(be-af)E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{a+bx^2}(bc-ad)\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}-\frac{\sqrt{c}\sqrt{a+bx^2}(-2adf+bcf+bde)\,\text{EllipticF}\left(\right.}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(\right.}{a\left(\right.}}}\right.}{(be-af)^2}\right.}{be-af}\right.$$

$$\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^3}dx}{be-af}$$

$\downarrow$ 320

$$b\left(\frac{b\left(\frac{a^{3/2}f^2\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce\sqrt{a+bx^2}(be-af)^2\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}+\frac{b\left(\frac{\sqrt{b}\sqrt{c+dx^2}(be-af)E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{a+bx^2}(bc-ad)\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}-\frac{\sqrt{c}\sqrt{a+bx^2}(-2adf+bcf+bde)\,\text{EllipticF}\left(\right.}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(\right.}{a\left(\right.}}}\right.}{(be-af)^2}\right.}{be-af}\right.$$

$$\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^3}dx}{be-af}$$

$\downarrow$ 388

$$b\left(\frac{b\left(\frac{a^{3/2}f^2\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}ce\sqrt{a+bx^2}(be-af)^2\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}+\frac{b\left(\frac{\sqrt{b}\sqrt{c+dx^2}(be-af)E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{a+bx^2}(bc-ad)\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}-\frac{\sqrt{c}\sqrt{a+bx^2}(-2adf+bcf+bde)\,\text{EllipticF}\left(\cdots\right.}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c\left(\cdots\right)}{a\left(\cdots\right)}}\cdots}\cdots\right)}{(be-af)^2}\cdots\right)}{be-af}\cdots\right.\cdots$$

$$\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^3}dx}{be-af}$$

$\Big\downarrow$ 313

$$b\left(\frac{b\left(\frac{a^{3/2}\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)f^2}{\sqrt{b}ce(be-af)^2\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}+\frac{b\left(\frac{\sqrt{b}(be-af)\sqrt{dx^2+c}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}(bc-ad)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}-\frac{\sqrt{c}(bde+bcf-2adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\text{ar}\cdots\right.}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\sqrt{\cdots}}\cdots\right)}{(be-af)^2}\cdots\right)}{be-af}\cdots\right.\cdots$$

$$\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^3}dx}{be-af}$$

$\Big\downarrow$ 413

$$b\left(\frac{b\left(\frac{b\left(\frac{a^{3/2}\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)f^2}{\sqrt{b}ce(be-af)^2\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}+\frac{b\left(\frac{\sqrt{b}(be-af)\sqrt{dx^2+c}\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}(bc-ad)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}-\frac{\sqrt{c}(bde+bcf-2adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\text{ar}\cdots\right.}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\cdots}\cdots\right.}{(be-af)^2}\cdots\right.}{be-af}\cdots\right.}{\cdots}\cdots\right.$$

$$\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^3}dx}{be-af}$$

$\Big\downarrow$ 413

$$b\left(\frac{b\left(\frac{b\left(\frac{a^{3/2}\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)f^2}{\sqrt{b}ce(be-af)^2\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}+\frac{b\left(\frac{\sqrt{b}(be-af)\sqrt{dx^2+c}\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}(bc-ad)\sqrt{bx^2+a}\sqrt{\frac{a\left(dx^2+c\right)}{c\left(bx^2+a\right)}}}-\frac{\sqrt{c}(bde+bcf-2adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\text{ar}\cdots\right.}{a\sqrt{d}(bc-ad)\sqrt{\frac{c\left(bx^2+a\right)}{a\left(dx^2+c\right)}}\cdots}\cdots\right.}{(be-af)^2}\cdots\right.}{be-af}\cdots\right.}{\cdots}\cdots\right.$$

$$\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^3}dx}{be-af}$$

$\Big\downarrow$ 412

$$b\left(\frac{b\left(\frac{a^{3/2}\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)f^2}{\sqrt{b}ce(be-af)^2\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}+\frac{b\left(\frac{\sqrt{b}(be-af)\sqrt{dx^2+c}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}(bc-ad)\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}-\frac{\sqrt{c}(bde+bcf-2adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\text{ar}\right.}{a\sqrt{d}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{}}\right)}{(be-af)^2}\right)}{be-af}\right.$$

$$\frac{f\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^3}dx}{be-af}$$

$\downarrow$ 433

$$b\left(\frac{b\left(\frac{a^{3/2}\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)f^2}{\sqrt{b}ce(be-af)^2\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}+\frac{b\left(\frac{\sqrt{b}(be-af)\sqrt{dx^2+c}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}(bc-ad)\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}-\frac{\sqrt{c}(bde+bcf-2adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\text{ar}\right.}{a\sqrt{d}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\sqrt{}}\right)}{(be-af)^2}\right)}{be-af}\right.$$

$$\frac{f\int\left(-\frac{f^{3/2}}{8(-e)^{3/2}\left(\sqrt{-e}\sqrt{f}-fx\right)^3\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{f^{3/2}}{8(-e)^{3/2}\left(fx+\sqrt{-e}\sqrt{f}\right)^3\sqrt{bx^2+a}\sqrt{dx^2+c}}-\frac{3f}{16e^2\left(\sqrt{-e}\sqrt{f}-fx\right)^2\sqrt{bx^2+a}\sqrt{dx^2+c}}-\right.}{be-af}$$

$\downarrow$ 2009

$$b\left(\frac{b\left(\frac{b\left(\frac{a^{3/2}\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{af}{be},\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)f^2}{\sqrt{b}ce(be-af)^2\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}+\frac{b\left(\frac{\sqrt{b}(be-af)\sqrt{dx^2+c}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}(bc-ad)\sqrt{bx^2+a}\sqrt{\frac{a(dx^2+c)}{c(bx^2+a)}}}-\frac{\sqrt{c}(bde+bcf-2adf)\sqrt{bx^2+a}\,\text{EllipticF}\left(\text{ar}\cdots\right.}{a\sqrt{d}(bc-ad)\sqrt{\frac{c(bx^2+a)}{a(dx^2+c)}}\cdots}\right)}{(be-af)^2}\cdots\right)}{be-af}\cdots\right)}{f\left(-\frac{\int\frac{1}{(\sqrt{-e}\sqrt{f}-fx)^3\sqrt{bx^2+a}\sqrt{dx^2+c}}dxf^{3/2}}{8(-e)^{3/2}}-\frac{\int\frac{1}{(fx+\sqrt{-e}\sqrt{f})^3\sqrt{bx^2+a}\sqrt{dx^2+c}}dxf^{3/2}}{8(-e)^{3/2}}-\frac{3\int\frac{1}{(\sqrt{-e}\sqrt{f}-fx)^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dxf}{16e^2}-\frac{3\int\frac{\cdots}{(fx+\cdots}}{\cdots}\cdots\right.}{be-af}\cdots$$

input

```
Int[1/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]*(e + f*x^2)^3),x]
```

output

```
$Aborted
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 400
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 421
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b^2/(b*c - a*d)^2   Int[(c + d*x^2)^(q + 2)*((e +
 f*x^2)^r/(a + b*x^2)), x], x] - Simp[d/(b*c - a*d)^2   Int[(c + d*x^2)^q*(
e + f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r}
, x] && LtQ[q, -1]
```

rule 424
```
Int[1/(((a_) + (b_.)*(x_)^2)^2*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*
(x_)^2]), x_Symbol] :> Simp[b^2*x*Sqrt[c + d*x^2]*(Sqrt[e + f*x^2]/(2*a*(b*
c - a*d)*(b*e - a*f)*(a + b*x^2))), x] + (Simp[(b^2*c*e + 3*a^2*d*f - 2*a*b
*(d*e + c*f))/(2*a*(b*c - a*d)*(b*e - a*f))   Int[1/((a + b*x^2)*Sqrt[c + d
*x^2]*Sqrt[e + f*x^2]), x], x] - Simp[d*(f/(2*a*(b*c - a*d)*(b*e - a*f)))
 Int[(a + b*x^2)/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x], x]) /; FreeQ[{a, b,
 c, d, e, f}, x]
```

rule 426
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[b/(b*c - a*d)   Int[(a + b*x^2)^p*(c + d*x^2)^
(q + 1)*(e + f*x^2)^r, x], x] - Simp[d/(b*c - a*d)   Int[(a + b*x^2)^(p + 1
)*(c + d*x^2)^q*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] &&
 ILtQ[p, 0] && LeQ[q, -1]
```

rule 433
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)
^q*(e + f*x^2)^r, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, f, p,
 q, r}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3116 vs. $2(778) = 1556$.

Time = 21.02 (sec) , antiderivative size = 3117, normalized size of antiderivative = 3.85

method	result	size
elliptic	Expression too large to display	3117
default	Expression too large to display	9458

input

```
int(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x,method=_RETURNVERBOSE)
```

output

```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(3/8*c^2/(-b/a
)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^
(1/2)*f^4*b/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)^2/e^2/(a*f-b*e)*a*EllipticF(
x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+(b*d*x^2+b*c)*b^3/a/(a*d-b*c)*x/(
a*f-b*e)^3/((x^2+a/b)*(b*d*x^2+b*c))^(1/2)+17/4/(a*c*f^2-a*d*e*f-b*c*e*f+b
*d*e^2)^2/(a*f-b*e)*f^3/e/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)
/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1
/c*d)^(1/2)/(-b/a)^(1/2))*a*b*c*d-1/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^
2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-
1+(a*d+b*c)/c/b)^(1/2))*b^3/a/(a*f-b*e)^3-13/8*c/(-b/a)^(1/2)*(1+b*x^2/a)^
(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*d*f^2*b^2/(a*c
*f^2-a*d*e*f-b*c*e*f+b*d*e^2)^2/(a*f-b*e)*EllipticE(x*(-b/a)^(1/2),(-1+(a*
d+b*c)/c/b)^(1/2))-5/4*c^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2
)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*f^3*b^2/(a*c*f^2-a*d*e*f-b*c*e*f+b*d
*e^2)^2/e/(a*f-b*e)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+5/4
*c^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c
*x^2+a*c)^(1/2)*f^3*b^2/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)^2/e/(a*f-b*e)*El
lipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+1/8*f^3*(3*a*c*f^2-6*a*d*
e*f-10*b*c*e*f+13*b*d*e^2)/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)^2/e^2/(a*f-b*
e)*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(f*x^2+e)+1/4*f^3/(a*c*f^2-a*d...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x**2+a)**(3/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^3}\,dx = \int \frac{1}{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}(fx^2+e)^3}\,dx$$

input
```
integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x, algorithm="maxi
ma")
```

output
```
integrate(1/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*(f*x^2 + e)^3), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}(e+fx^2)^3}\,dx = \int \frac{1}{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}(fx^2+e)^3}\,dx$$

input
```
integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x, algorithm="giac
")
```

output
```
integrate(1/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*(f*x^2 + e)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}(e+fx^2)^3}\,dx = \int \frac{1}{(b\,x^2+a)^{3/2}\sqrt{d\,x^2+c}\,(f\,x^2+e)^3}\,dx$$

input
```
int(1/((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^3),x)
```

output
```
int(1/((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^3), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}(e+fx^2)^3}\,dx = \int \frac{1}{(b\,x^2+a)^{\frac{3}{2}}\sqrt{d\,x^2+c}\,(f\,x^2+e)^3}dx$$

input
```
int(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x)
```

output
```
int(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x)
```

3.269 $\int \frac{1}{x^2\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}\left(e+fx^2\right)^3}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 1047

$$\int \frac{1}{x^2\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}\left(e+fx^2\right)^3}\,dx = -\frac{\sqrt{c+dx^2}}{acex\sqrt{a+bx^2}\left(e+fx^2\right)^2}$$

$$+\frac{f(af(4de-5cf)-4be(de-cf))x\sqrt{c+dx^2}}{4ace^2(be-af)(de-cf)\sqrt{a+bx^2}\left(e+fx^2\right)^2}$$

$$-\frac{f(8b^2e^2(de-cf)^2+a^2f^2(8d^2e^2-26cdef+15c^2f^2)-abef(16d^2e^2-47cdef+28c^2f^2))\,x\sqrt{c+dx^2}}{8ace^3(be-af)^2(de-cf)^2\sqrt{a+bx^2}\left(e+fx^2\right)}$$

$$-\frac{\sqrt{b}(16b^4ce^3(de-cf)^2-8ab^3e^2(de-cf)^2(de+3cf)+a^4df^3(8d^2e^2-26cdef+15c^2f^2)-3a^3bf^2(8d^3e^3}{8a^{3/2}c(bc-ad)e^3(be}$$

$$-\frac{\sqrt{b}(a^4df^4(16de-15cf)-8b^4de^4(de-cf)+a^2b^2ef^2(48d^2e^2+cdef-48c^2f^2)-a^3bf^3(49d^2e^2-32cde}{8\sqrt{a}c(bc-ad)e^3(be-af)^4(de-c}$$

$$-\frac{3a^{3/2}f^3(a^2f^2(8d^2e^2-12cdef+5c^2f^2)-2abef(12d^2e^2-19cdef+8c^2f^2)+b^2e^2(21d^2e^2-36cdef+16}{8\sqrt{b}ce^4(be-af)^4(de-cf)^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2}{c(a+bx^2}}}$$

output
```
-(d*x^2+c)^(1/2)/a/c/e/x/(b*x^2+a)^(1/2)/(f*x^2+e)^2+1/4*f*(a*f*(-5*c*f+4*
d*e)-4*b*e*(-c*f+d*e))*x*(d*x^2+c)^(1/2)/a/c/e^2/(-a*f+b*e)/(-c*f+d*e)/(b*
x^2+a)^(1/2)/(f*x^2+e)^2-1/8*f*(8*b^2*e^2*(-c*f+d*e)^2+a^2*f^2*(15*c^2*f^2
-26*c*d*e*f+8*d^2*e^2)-a*b*e*f*(28*c^2*f^2-47*c*d*e*f+16*d^2*e^2))*x*(d*x^
2+c)^(1/2)/a/c/e^3/(-a*f+b*e)^2/(-c*f+d*e)^2/(b*x^2+a)^(1/2)/(f*x^2+e)-1/8
*b^(1/2)*(16*b^4*c*e^3*(-c*f+d*e)^2-8*a*b^3*e^2*(-c*f+d*e)^2*(3*c*f+d*e)+a
^4*d*f^3*(15*c^2*f^2-26*c*d*e*f+8*d^2*e^2)-3*a^3*b*f^2*(5*c^3*f^3+4*c^2*d*
e*f^2-19*c*d^2*e^2*f+8*d^3*e^3)+a^2*b^2*e*f*(38*c^3*f^3-41*c^2*d*e*f^2-24*
c*d^2*e^2*f+24*d^3*e^3))*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*
x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/a^(3/2)/c/(-a*d+b*c)/e^3/(-a*f+b*e)^3/(-c*
f+d*e)^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)-1/8*b^(1/2)*(a^4*
d*f^4*(-15*c*f+16*d*e)-8*b^4*d*e^4*(-c*f+d*e)+a^2*b^2*e*f^2*(-48*c^2*f^2+c
*d*e*f+48*d^2*e^2)-a^3*b*f^3*(-15*c^2*f^2-32*c*d*e*f+49*d^2*e^2)+8*a*b^3*e
^2*f*(6*c^2*f^2-7*c*d*e*f+d^2*e^2))*(d*x^2+c)^(1/2)*InverseJacobiAM(arctan
(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/a^(1/2)/c/(-a*d+b*c)/e^3/(-a*f+b*e)
^4/(-c*f+d*e)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)-3/8*a^(3/2)*
f^3*(a^2*f^2*(5*c^2*f^2-12*c*d*e*f+8*d^2*e^2)-2*a*b*e*f*(8*c^2*f^2-19*c*d*
e*f+12*d^2*e^2)+b^2*e^2*(16*c^2*f^2-36*c*d*e*f+21*d^2*e^2))*(d*x^2+c)^(1/2
)*EllipticPi(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),1-a*f/b/e,(1-a*d/b/c)^(1/
2))/b^(1/2)/c/e^4/(-a*f+b*e)^4/(-c*f+d*e)^2/(b*x^2+a)^(1/2)/(a*(d*x^2+c...
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 8.29 (sec) , antiderivative size = 778, normalized size of antiderivative = 0.74

$$\int \frac{1}{x^2 (a+bx^2)^{3/2} \sqrt{c+dx^2} (e+fx^2)^3} \, dx = \frac{\sqrt{\frac{b}{a}} e(c+dx^2) \left(2a^2c(bc-ad)ef^4(be-af)(de-cf)x^2(a+bx\right.}{}$$

input
```
Integrate[1/(x^2*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2]*(e + f*x^2)^3),x]
```

output
```
(Sqrt[b/a]*e*(c + d*x^2)*(2*a^2*c*(b*c - a*d)*e*f^4*(b*e - a*f)*(d*e - c*f
)*x^2*(a + b*x^2) + a^2*c*(b*c - a*d)*f^4*(b*e*(17*d*e - 14*c*f) + a*f*(-1
0*d*e + 7*c*f))*x^2*(a + b*x^2)*(e + f*x^2) - 8*b^5*c*e^3*(d*e - c*f)^2*x^
2*(e + f*x^2)^2 - 8*(b*c - a*d)*(b*e - a*f)^3*(d*e - c*f)^2*(a + b*x^2)*(e
 + f*x^2)^2) + I*c*x*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(e + f*x^2)^2
*(-(b*e*(16*b^4*c*e^3*(d*e - c*f)^2 - 8*a*b^3*e^2*(d*e - c*f)^2*(d*e + 3*c
*f) + a^4*d*f^3*(8*d^2*e^2 - 26*c*d*e*f + 15*c^2*f^2) - 3*a^3*b*f^2*(8*d^3
*e^3 - 19*c*d^2*e^2*f + 4*c^2*d*e*f^2 + 5*c^3*f^3) + a^2*b^2*e*f*(24*d^3*e
^3 - 24*c*d^2*e^2*f - 41*c^2*d*e*f^2 + 38*c^3*f^3))*EllipticE[I*ArcSinh[Sq
rt[b/a]*x], (a*d)/(b*c)]) + (b*c - a*d)*(b*e*(d*e - c*f)*(a^2*b*e*f^2*(39*
d*e - 38*c*f) + 16*b^3*e^3*(d*e - c*f) - 24*a*b^2*e^2*f*(d*e - c*f) + a^3*
f^3*(-16*d*e + 15*c*f))*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - 3
*a^2*f^2*(a^2*f^2*(8*d^2*e^2 - 12*c*d*e*f + 5*c^2*f^2) - 2*a*b*e*f*(12*d^2
*e^2 - 19*c*d*e*f + 8*c^2*f^2) + b^2*e^2*(21*d^2*e^2 - 36*c*d*e*f + 16*c^2
*f^2))*EllipticPi[(a*f)/(b*e), I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])))/(8*
a^2*Sqrt[b/a]*c*(b*c - a*d)*e^4*(b*e - a*f)^3*(d*e - c*f)^2*x*Sqrt[a + b*x
^2]*Sqrt[c + d*x^2]*(e + f*x^2)^2)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a+bx^2)^{3/2} \sqrt{c+dx^2} (e+fx^2)^3} \, dx$$
$$\Big\downarrow \; 450$$
$$\int \frac{1}{x^2 (a+bx^2)^{3/2} \sqrt{c+dx^2} (e+fx^2)^3} dx$$

input
```
Int[1/(x^2*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2]*(e + f*x^2)^3),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3358 vs. 2(1011) = 2022.

Time = 39.73 (sec) , antiderivative size = 3359, normalized size of antiderivative = 3.21

method	result	size
elliptic	Expression too large to display	3359
risch	Expression too large to display	4211
default	Expression too large to display	13531

input
```
int(1/x^2/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x,method=_RETURNVERB
OSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(15/8/(-b/a)^(
1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/
2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))*f^2*d^2*b^2/(a*c*f^2
-a*d*e*f-b*c*e*f+b*d*e^2)^2/(a*f-b*e)-1/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+
d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/a^2*b/e^3*EllipticF(x*(
-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+1/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+
d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/a^2*b/e^3*EllipticE(x*(
-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-1/8*f^4*(7*a*c*f^2-10*a*d*e*f-14*b*c
*e*f+17*b*d*e^2)/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)^2/e^3/(a*f-b*e)*x*(b*d*
x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(f*x^2+e)-63/8*f^2/(a*c*f^2-a*d*e*f-b*c*e*f
+b*d*e^2)^2/(a*f-b*e)/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*
d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d
)^(1/2)/(-b/a)^(1/2))*b^2*d^2+1/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)
^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a
*d+b*c)/c/b)^(1/2))*b^4/a^2/(a*f-b*e)^3+9/2*f^5/(a*c*f^2-a*d*e*f-b*c*e*f+b
*d*e^2)^2/e^3/(a*f-b*e)/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(
b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c
*d)^(1/2)/(-b/a)^(1/2))*a^2*c*d+6*f^5/(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)^2/
e^3/(a*f-b*e)/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*
d*x^2+b*c*x^2+a*c)^(1/2)*EllipticPi(x*(-b/a)^(1/2),a*f/b/e,(-1/c*d)^(1/...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^2 \left(a+bx^2\right)^{3/2} \sqrt{c+dx^2} \left(e+fx^2\right)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/x^2/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x, algorithm="
fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a+bx^2)^{3/2} \sqrt{c+dx^2} (e+fx^2)^3} \, dx = \text{Timed out}$$

input
```
integrate(1/x**2/(b*x**2+a)**(3/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{x^2 (a+bx^2)^{3/2} \sqrt{c+dx^2} (e+fx^2)^3} \, dx = \int \frac{1}{(bx^2+a)^{\frac{3}{2}} \sqrt{dx^2+c} (fx^2+e)^3 x^2} \, dx$$

input
```
integrate(1/x^2/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x, algorithm="
maxima")
```

output
```
integrate(1/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*(f*x^2 + e)^3*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2 (a+bx^2)^{3/2} \sqrt{c+dx^2} (e+fx^2)^3} \, dx = \int \frac{1}{(bx^2+a)^{\frac{3}{2}} \sqrt{dx^2+c} (fx^2+e)^3 x^2} \, dx$$

input
```
integrate(1/x^2/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x, algorithm="
giac")
```

output
```
integrate(1/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*(f*x^2 + e)^3*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a+bx^2)^{3/2} \sqrt{c+dx^2} (e+fx^2)^3} \, dx = \int \frac{1}{x^2 \left(b\,x^2+a\right)^{3/2} \sqrt{d\,x^2+c} \left(f\,x^2+e\right)^3} \, dx$$

input
```
int(1/(x^2*(a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^3),x)
```

output
```
int(1/(x^2*(a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^3), x)
```

Reduce [F]

$$\int \frac{1}{x^2 (a+bx^2)^{3/2} \sqrt{c+dx^2} (e+fx^2)^3} \, dx = \text{too large to display}$$

input
```
int(1/x^2/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^3,x)
```

output

```
( - sqrt(c + d*x**2)*sqrt(a + b*x**2) - 5*int((sqrt(c + d*x**2)*sqrt(a + b
*x**2)*x**4)/(a**2*c*e**3 + 3*a**2*c*e**2*f*x**2 + 3*a**2*c*e*f**2*x**4 +
a**2*c*f**3*x**6 + a**2*d*e**3*x**2 + 3*a**2*d*e**2*f*x**4 + 3*a**2*d*e*f*
*2*x**6 + a**2*d*f**3*x**8 + 2*a*b*c*e**3*x**2 + 6*a*b*c*e**2*f*x**4 + 6*a
*b*c*e*f**2*x**6 + 2*a*b*c*f**3*x**8 + 2*a*b*d*e**3*x**4 + 6*a*b*d*e**2*f*
x**6 + 6*a*b*d*e*f**2*x**8 + 2*a*b*d*f**3*x**10 + b**2*c*e**3*x**4 + 3*b**
2*c*e**2*f*x**6 + 3*b**2*c*e*f**2*x**8 + b**2*c*f**3*x**10 + b**2*d*e**3*x
**6 + 3*b**2*d*e**2*f*x**8 + 3*b**2*d*e*f**2*x**10 + b**2*d*f**3*x**12),x)
*a*b*d*e**2*f*x - 10*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a**2*c*
e**3 + 3*a**2*c*e**2*f*x**2 + 3*a**2*c*e*f**2*x**4 + a**2*c*f**3*x**6 + a*
*2*d*e**3*x**2 + 3*a**2*d*e**2*f*x**4 + 3*a**2*d*e*f**2*x**6 + a**2*d*f**3
*x**8 + 2*a*b*c*e**3*x**2 + 6*a*b*c*e**2*f*x**4 + 6*a*b*c*e*f**2*x**6 + 2*
a*b*c*f**3*x**8 + 2*a*b*d*e**3*x**4 + 6*a*b*d*e**2*f*x**6 + 6*a*b*d*e*f**2
*x**8 + 2*a*b*d*f**3*x**10 + b**2*c*e**3*x**4 + 3*b**2*c*e**2*f*x**6 + 3*b
**2*c*e*f**2*x**8 + b**2*c*f**3*x**10 + b**2*d*e**3*x**6 + 3*b**2*d*e**2*f
*x**8 + 3*b**2*d*e*f**2*x**10 + b**2*d*f**3*x**12),x)*a*b*d*e*f**2*x**3 -
5*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a**2*c*e**3 + 3*a**2*c*e**
2*f*x**2 + 3*a**2*c*e*f**2*x**4 + a**2*c*f**3*x**6 + a**2*d*e**3*x**2 + 3*
a**2*d*e**2*f*x**4 + 3*a**2*d*e*f**2*x**6 + a**2*d*f**3*x**8 + 2*a*b*c*e**
3*x**2 + 6*a*b*c*e**2*f*x**4 + 6*a*b*c*e*f**2*x**6 + 2*a*b*c*f**3*x**8 ...
```

3.270 $\int \frac{x^4(e+fx^2)^2}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 546

$$\int \frac{x^4(e+fx^2)^2}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \frac{(5b^2de^2+6a^2df^2+abf(10de+cf))\,x^3\sqrt{c+dx^2}}{5b^2d(bc+ad)\sqrt{a-bx^2}}$$
$$-\frac{f^2x^5\sqrt{c+dx^2}}{5bd\sqrt{a-bx^2}}$$
$$+\frac{\left(\frac{24a^2df^2}{b}+5af(8de+cf)+b\left(15de^2+10cef-\frac{4c^2f^2}{d}\right)\right)x\sqrt{a-bx^2}\sqrt{c+dx^2}}{15b^2d(bc+ad)}$$
$$-\frac{\sqrt{a}(48a^3d^3f^2+16a^2bd^2f(5de+cf)+3ab^2d(10d^2e^2+10cdef-3c^2f^2)+b^3c(15d^2e^2-20cdef+8c^2f^2)}{15b^{7/2}d^3(bc+ad)\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}}$$
$$+\frac{\sqrt{a}c(24a^2d^2f^2+abdf(40de-13cf)+b^2(15d^2e^2-20cdef+8c^2f^2))\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticF}\left(\text{arcsi}\right.}{15b^{7/2}d^3\sqrt{a-bx^2}\sqrt{c+dx^2}}$$

output
```
1/5*(5*b^2*d*e^2+6*a^2*d*f^2+a*b*f*(c*f+10*d*e))*x^3*(d*x^2+c)^(1/2)/b^2/d
/(a*d+b*c)/(-b*x^2+a)^(1/2)-1/5*f^2*x^5*(d*x^2+c)^(1/2)/b/d/(-b*x^2+a)^(1/
2)+1/15*(24*a^2*d*f^2/b+5*a*f*(c*f+8*d*e)+b*(15*d*e^2+10*c*e*f-4*c^2*f^2/d
))*x*(-b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b^2/d/(a*d+b*c)-1/15*a^(1/2)*(48*a^3
*d^3*f^2+16*a^2*b*d^2*f*(c*f+5*d*e)+3*a*b^2*d*(-3*c^2*f^2+10*c*d*e*f+10*d^
2*e^2)+b^3*c*(8*c^2*f^2-20*c*d*e*f+15*d^2*e^2))*(1-b*x^2/a)^(1/2)*(d*x^2+c
)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2),(-a*d/b/c)^(1/2))/b^(7/2)/d^3/(a*d+b*c
)/(-b*x^2+a)^(1/2)/(1+d*x^2/c)^(1/2)+1/15*a^(1/2)*c*(24*a^2*d^2*f^2+a*b*d*
f*(-13*c*f+40*d*e)+b^2*(8*c^2*f^2-20*c*d*e*f+15*d^2*e^2))*(1-b*x^2/a)^(1/2
)*(1+d*x^2/c)^(1/2)*EllipticF(b^(1/2)*x/a^(1/2),(-a*d/b/c)^(1/2))/b^(7/2)/
d^3/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.29 (sec) , antiderivative size = 459, normalized size of antiderivative = 0.84

$$\int \frac{x^4(e+fx^2)^2}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \frac{-\sqrt{-\frac{b}{a}}dx(c+dx^2)\left(-24a^3d^2f^2+b^3cfx^2(10de-4cf+3dfx^2)+a^2bdf(-4\right.}{}$$

input
```
Integrate[(x^4*(e + f*x^2)^2)/((a - b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

output
```
(-(Sqrt[-(b/a)]*d*x*(c + d*x^2)*(-24*a^3*d^2*f^2 + b^3*c*f*x^2*(10*d*e - 4
*c*f + 3*d*f*x^2) + a^2*b*d*f*(-40*d*e - 5*c*f + 6*d*f*x^2) + a*b^2*(4*c^2
*f^2 + 2*c*d*f*(-5*e + f*x^2) + d^2*(-15*e^2 + 10*e*f*x^2 + 3*f^2*x^4))))
+ I*c*(48*a^3*d^3*f^2 + 16*a^2*b*d^2*f*(5*d*e + c*f) + 3*a*b^2*d*(10*d^2*e
^2 + 10*c*d*e*f - 3*c^2*f^2) + b^3*c*(15*d^2*e^2 - 20*c*d*e*f + 8*c^2*f^2)
)*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[-(b/a)]
*x], -((a*d)/(b*c))] - I*c*(b*c + a*d)*(24*a^2*d^2*f^2 + a*b*d*f*(40*d*e -
 13*c*f) + b^2*(15*d^2*e^2 - 20*c*d*e*f + 8*c^2*f^2))*Sqrt[1 - (b*x^2)/a]*
Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))])/
(15*b^3*Sqrt[-(b/a)]*d^3*(b*c + a*d)*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (warning: unable to verify)

Time = 1.86 (sec) , antiderivative size = 861, normalized size of antiderivative = 1.58, number of steps used = 19, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.528$, Rules used = {448, 440, 25, 444, 399, 323, 323, 321, 331, 330, 327, 444, 399, 323, 323, 321, 331, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 \left(e+f x^2\right)^2}{\left(a-b x^2\right)^{3/2} \sqrt{c+d x^2}} \, dx$$

$$\Big\downarrow \ 448$$

$$\frac{f \int \frac{x^6 (f x^2+e)}{(a-b x^2)^{3/2} \sqrt{d x^2+c}}dx}{e^2}+e \int \frac{x^4 \left(f x^2+e\right)}{\left(a-b x^2\right)^{3/2} \sqrt{d x^2+c}}dx$$

$$\Big\downarrow \ 440$$

$$\frac{f \left(\frac{\int -\frac{x^4 \left((5 b d e+b c f+6 a d f) x^2+5 c (b e+a f)\right)}{\sqrt{a-b x^2} \sqrt{d x^2+c}}dx}{b (a d+b c)}+\frac{x^5 \sqrt{c+d x^2} (a f+b e)}{b \sqrt{a-b x^2} (a d+b c)}\right)}{e^2}+e \left(\frac{\int -\frac{x^2 \left((3 b d e+b c f+4 a d f) x^2+3 c (b e+a f)\right)}{\sqrt{a-b x^2} \sqrt{d x^2+c}}dx}{b (a d+b c)}+\frac{x^3 \sqrt{c+d x^2} (a f+b e)}{b \sqrt{a-b x^2} (a d+b c)}\right)$$

$$\Big\downarrow \ 25$$

$$\frac{f \left(\frac{x^5 \sqrt{c+d x^2} (a f+b e)}{b \sqrt{a-b x^2} (a d+b c)}-\frac{\int \frac{x^4 \left((5 b d e+b c f+6 a d f) x^2+5 c (b e+a f)\right)}{\sqrt{a-b x^2} \sqrt{d x^2+c}}dx}{b (a d+b c)}\right)}{e^2}+e \left(\frac{x^3 \sqrt{c+d x^2} (a f+b e)}{b \sqrt{a-b x^2} (a d+b c)}-\frac{\int \frac{x^2 \left((3 b d e+b c f+4 a d f) x^2+3 c (b e+a f)\right)}{\sqrt{a-b x^2} \sqrt{d x^2+c}}dx}{b (a d+b c)}\right)$$

$$\Big\downarrow \ 444$$

$$\frac{f\left(\frac{x^5\sqrt{c+dx^2}(af+be)}{b\sqrt{a-bx^2}(ad+bc)}-\frac{\frac{\int\frac{x^2\left(\left(c(5de-4cf)b^2+5ad(4de+cf)b+24a^2d^2f\right)x^2+3ac(5bde+bcf+6adf)\right)}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{5bd}-\frac{x^3\sqrt{a-bx^2}\sqrt{c+dx^2}(6adf+bcf+5bde)}{5bd}}{b(ad+bc)}\right)}{e^2}+$$

$$e\left(\frac{x^3\sqrt{c+dx^2}(af+be)}{b\sqrt{a-bx^2}(ad+bc)}-\frac{\frac{\int\frac{\left(c(3de-2cf)b^2+3ad(2de+cf)b+8a^2d^2f\right)x^2+ac(3bde+bcf+4adf)}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{3bd}-\frac{x\sqrt{a-bx^2}\sqrt{c+dx^2}(4adf+bcf+3bde)}{3bd}}{b(ad+bc)}\right)$$

$$\Big\downarrow \; 399$$

$$\frac{f\left(\frac{x^5\sqrt{c+dx^2}(af+be)}{b\sqrt{a-bx^2}(ad+bc)}-\frac{\frac{\int\frac{x^2\left(\left(c(5de-4cf)b^2+5ad(4de+cf)b+24a^2d^2f\right)x^2+3ac(5bde+bcf+6adf)\right)}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{5bd}-\frac{x^3\sqrt{a-bx^2}\sqrt{c+dx^2}(6adf+bcf+5bde)}{5bd}}{b(ad+bc)}\right)}{e^2}+$$

$$e\left(\frac{x^3\sqrt{c+dx^2}(af+be)}{b\sqrt{a-bx^2}(ad+bc)}-\frac{\frac{\frac{\left(8a^2d^2f+3abd(cf+2de)+b^2c(3de-2cf)\right)\int\frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}-\frac{c(ad+bc)(4adf-2bcf+3bde)\int\frac{1}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{d}}{3bd}-x\sqrt{a-\cdots}}{b(ad+bc)}\right.$$

$$\Big\downarrow \; 323$$

$$\frac{f\left(\frac{x^5\sqrt{c+dx^2}(af+be)}{b\sqrt{a-bx^2}(ad+bc)}-\frac{\frac{\int\frac{x^2\left(\left(c(5de-4cf)b^2+5ad(4de+cf)b+24a^2d^2f\right)x^2+3ac(5bde+bcf+6adf)\right)}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{5bd}-\frac{x^3\sqrt{a-bx^2}\sqrt{c+dx^2}(6adf+bcf+5bde)}{5bd}}{b(ad+bc)}\right)}{e^2}+$$

$$e\left(\frac{x^3\sqrt{c+dx^2}(af+be)}{b\sqrt{a-bx^2}(ad+bc)}-\frac{\frac{\frac{\left(8a^2d^2f+3abd(cf+2de)+b^2c(3de-2cf)\right)\int\frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}-\frac{c\sqrt{\frac{dx^2}{c}+1}(ad+bc)(4adf-2bcf+3bde)\int\frac{1}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}dx}{d\sqrt{c+dx^2}}}{3bd}\cdots}{b(ad+bc)}\right.$$

$$\Big\downarrow \; 323$$

$$\frac{f\left(\frac{x^5\sqrt{c+dx^2}(af+be)}{b\sqrt{a-bx^2}(ad+bc)}-\frac{\frac{\int \frac{x^2\left(\left(c(5de-4cf)b^2+5ad(4de+cf)b+24a^2d^2f\right)x^2+3ac(5bde+bcf+6adf)\right)}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{5bd}-\frac{x^3\sqrt{a-bx^2}\sqrt{c+dx^2}(6adf+bcf+5bde)}{5bd}}{b(ad+bc)}\right)}{e^2}+$$

$$e\left(\frac{x^3\sqrt{c+dx^2}(af+be)}{b\sqrt{a-bx^2}(ad+bc)}-\frac{\frac{\frac{\left(8a^2d^2f+3abd(cf+2de)+b^2c(3de-2cf)\right)\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}-\frac{c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(4adf-2bcf+3bde)\int \frac{1}{\sqrt{1-\frac{bx^2}{a}}\sqrt{\ldots}}}{d\sqrt{a-bx^2}\sqrt{c+dx^2}}}{3bd}}{b(ad+bc)}\right.$$

$$\downarrow \text{321}$$

$$e\left(\frac{x^3\sqrt{c+dx^2}(af+be)}{b\sqrt{a-bx^2}(ad+bc)}-\frac{\frac{\frac{\left(8a^2d^2f+3abd(cf+2de)+b^2c(3de-2cf)\right)\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}-\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(4adf-2bcf+3bde)\,\text{EllipticF}\left(\ldots\right.}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}}{3bd}}{b(ad+bc)}\right.$$

$$\frac{f\left(\frac{x^5\sqrt{c+dx^2}(af+be)}{b\sqrt{a-bx^2}(ad+bc)}-\frac{\frac{\int \frac{x^2\left(\left(c(5de-4cf)b^2+5ad(4de+cf)b+24a^2d^2f\right)x^2+3ac(5bde+bcf+6adf)\right)}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{5bd}-\frac{x^3\sqrt{a-bx^2}\sqrt{c+dx^2}(6adf+bcf+5bde)}{5bd}}{b(ad+bc)}\right)}{e^2}$$

$$\downarrow \text{331}$$

$$e\left(\frac{x^3\sqrt{c+dx^2}(af+be)}{b\sqrt{a-bx^2}(ad+bc)}-\frac{\frac{\frac{\sqrt{1-\frac{bx^2}{a}}\left(8a^2d^2f+3abd(cf+2de)+b^2c(3de-2cf)\right)\int \frac{\sqrt{dx^2+c}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(4adf-2bcf+3bde)\,\text{El}\ldots}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}}{3bd}}{b(ad+bc)}\right.$$

$$\frac{f\left(\frac{x^5\sqrt{c+dx^2}(af+be)}{b\sqrt{a-bx^2}(ad+bc)}-\frac{\frac{\int \frac{x^2\left(\left(c(5de-4cf)b^2+5ad(4de+cf)b+24a^2d^2f\right)x^2+3ac(5bde+bcf+6adf)\right)}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{5bd}-\frac{x^3\sqrt{a-bx^2}\sqrt{c+dx^2}(6adf+bcf+5bde)}{5bd}}{b(ad+bc)}\right)}{e^2}$$

$$\downarrow \text{330}$$

$$\frac{e\left(\frac{x^3\sqrt{c+dx^2}(af+be)}{b\sqrt{a-bx^2}(ad+bc)}-\frac{\frac{\frac{\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\left(8a^2d^2f+3abd(cf+2de)+b^2c(3de-2cf)\right)\int\frac{\sqrt{\frac{dx^2}{c}+1}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}-\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(4adf-2bcf\cdots}{\sqrt{b}d\sqrt{a-bx^2}\cdots}}{3bd}}{b(ad+bc)}\right) f\left(\frac{x^5\sqrt{c+dx^2}(af+be)}{b\sqrt{a-bx^2}(ad+bc)}-\frac{\frac{\int\frac{x^2\left(\left(c(5de-4cf)b^2+5ad(4de+cf)b+24a^2d^2f\right)x^2+3ac(5bde+bcf+6adf)\right)}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{5bd}-\frac{x^3\sqrt{a-bx^2}\sqrt{c+dx^2}(6adf+bcf+5bde)}{5bd}}{b(ad+bc)}\right)}{e^2}$$

$\big\downarrow$ 327

$$\frac{f\left(\frac{x^5\sqrt{c+dx^2}(af+be)}{b\sqrt{a-bx^2}(ad+bc)}-\frac{\frac{\int\frac{x^2\left(\left(c(5de-4cf)b^2+5ad(4de+cf)b+24a^2d^2f\right)x^2+3ac(5bde+bcf+6adf)\right)}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{5bd}-\frac{x^3\sqrt{a-bx^2}\sqrt{c+dx^2}(6adf+bcf+5bde)}{5bd}}{b(ad+bc)}\right)}{e^2}+$$

$$e\left(\frac{x^3\sqrt{c+dx^2}(af+be)}{b\sqrt{a-bx^2}(ad+bc)}-\frac{\frac{\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\left(8a^2d^2f+3abd(cf+2de)+b^2c(3de-2cf)\right)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}-\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc\cdots}{\cdots}}{3bd}}{b(ad+bc)}\cdots\right.$$

$\big\downarrow$ 444

$$\frac{f\left(\frac{x^5\sqrt{c+dx^2}(af+be)}{b\sqrt{a-bx^2}(ad+bc)}-\frac{\frac{\frac{\int\frac{\left(-2c^2(5de-4cf)b^3+3acd(5de-3cf)b^2+8a^2d^2(5de+2cf)b+48a^3d^3f\right)x^2+ac\left(c(5de-4cf)b^2+5ad(4de+cf)b+24a^2d^2f\right)}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{3bd}-\frac{1}{3}x\cdots}{5bd}}{b(ad+bc)}\cdots\right.}{e^2\cdots}$$

$$e\left(\frac{x^3\sqrt{c+dx^2}(af+be)}{b\sqrt{a-bx^2}(ad+bc)}-\frac{\frac{\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\left(8a^2d^2f+3abd(cf+2de)+b^2c(3de-2cf)\right)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}-\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc\cdots}{\cdots}}{3bd}}{b(ad+bc)}\cdots\right.$$

$\big\downarrow$ 399

$$f\left(\frac{x^5\sqrt{c+dx^2}(af+be)}{b\sqrt{a-bx^2}(ad+bc)}-\frac{\frac{\frac{\frac{\left(48a^3d^3f+8a^2bd^2(2cf+5de)+3ab^2cd(5de-3cf)-2b^3c^2(5de-4cf)\right)\int\frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}-\frac{c(ad+bc)\left(24a^2d^2f+abd(20de-13cf)-2b^2c\cdots\right.}{d}}{3bd}\cdots}{5bd}\cdots}{b\cdots}\cdots\right.$$

$$e^2\cdots$$

$$e\left(\frac{x^3\sqrt{c+dx^2}(af+be)}{b\sqrt{a-bx^2}(ad+bc)}-\frac{\frac{\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\left(8a^2d^2f+3abd(cf+2de)+b^2c(3de-2cf)\right)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}-\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\cdots}{\cdots}}{3bd}\cdots}{b(ad+bc)}\cdots\right.$$

$$\Big\downarrow \text{ 323}$$

$$f\left(\frac{x^5\sqrt{c+dx^2}(af+be)}{b\sqrt{a-bx^2}(ad+bc)}-\frac{\frac{\frac{\frac{\left(48a^3d^3f+8a^2bd^2(2cf+5de)+3ab^2cd(5de-3cf)-2b^3c^2(5de-4cf)\right)\int\frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}-\frac{c\sqrt{\frac{dx^2}{c}+1}(ad+bc)\left(24a^2d^2f+abd(20de-13c\cdots\right.}{d\sqrt{c\cdots}}}{3bd}\cdots}{5bd}\cdots}{\cdots}\cdots\right.$$

$$e^2\cdots$$

$$e\left(\frac{x^3\sqrt{c+dx^2}(af+be)}{b\sqrt{a-bx^2}(ad+bc)}-\frac{\frac{\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\left(8a^2d^2f+3abd(cf+2de)+b^2c(3de-2cf)\right)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}-\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\cdots}{\cdots}}{3bd}\cdots}{b(ad+bc)}\cdots\right.$$

$$\Big\downarrow \text{ 323}$$

$$e\left(\frac{(be+af)x^3\sqrt{dx^2+c}}{b(bc+ad)\sqrt{a-bx^2}}-\frac{\frac{\frac{\sqrt{a}\left(c(3de-2cf)b^2+3ad(2de+cf)b+8a^2d^2f\right)\sqrt{1-\frac{bx^2}{a}}\sqrt{dx^2+c}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}-\frac{\sqrt{a}c(bc+ad)(3bde-2bcf+4adf)}{}}{3bd}}{b(bc+ad)}\right.$$

$$f\left(\frac{(be+af)x^5\sqrt{dx^2+c}}{b(bc+ad)\sqrt{a-bx^2}}-\frac{\frac{\frac{\left(-2c^2(5de-4cf)b^3+3acd(5de-3cf)b^2+8a^2d^2(5de+2cf)b+48a^3d^3f\right)\int\frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}-\frac{c(bc+ad)\left(-2c(5de-4cf)b^2+ad(20de-13c\right.}{d}}{3bd}}{5bd}\right.$$

$\Big\downarrow$ 321

$$e\left(\frac{(be+af)x^3\sqrt{dx^2+c}}{b(bc+ad)\sqrt{a-bx^2}}-\frac{\frac{\frac{\sqrt{a}\left(c(3de-2cf)b^2+3ad(2de+cf)b+8a^2d^2f\right)\sqrt{1-\frac{bx^2}{a}}\sqrt{dx^2+c}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}-\frac{\sqrt{a}c(bc+ad)(3bde-2bcf+4adf)}{}}{3bd}}{b(bc+ad)}\right.$$

$$f\left(\frac{(be+af)x^5\sqrt{dx^2+c}}{b(bc+ad)\sqrt{a-bx^2}}-\frac{\frac{\frac{\left(-2c^2(5de-4cf)b^3+3acd(5de-3cf)b^2+8a^2d^2(5de+2cf)b+48a^3d^3f\right)\int\frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}-\frac{\sqrt{a}c(bc+ad)\left(-2c(5de-4cf)b^2+ad(20de-\right.}{}}{3bd}}{5}\right.$$

$\Big\downarrow$ 331

$$e\left(\frac{(be+af)x^3\sqrt{dx^2+c}}{b(bc+ad)\sqrt{a-bx^2}}-\frac{\frac{\frac{\sqrt{a}\left(c(3de-2cf)b^2+3ad(2de+cf)b+8a^2d^2f\right)\sqrt{1-\frac{bx^2}{a}}\sqrt{dx^2+c}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}-\frac{\sqrt{a}c(bc+ad)(3bde-2bcf+4adf)\cdots}{\cdots}}{3bd}\cdots}{b(bc+ad)}\cdots\right.$$

$$f\left(\frac{(be+af)x^5\sqrt{dx^2+c}}{b(bc+ad)\sqrt{a-bx^2}}-\frac{\frac{\frac{\left(-2c^2(5de-4cf)b^3+3acd(5de-3cf)b^2+8a^2d^2(5de+2cf)b+48a^3d^3f\right)\sqrt{1-\frac{bx^2}{a}}\int\frac{\sqrt{dx^2+c}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{\sqrt{a}c(bc+ad)\left(-2c(5de-4cf)b^2+\cdots\right.}{\cdots}}{3bd}\cdots}{\cdots}\cdots\right.$$

$\downarrow$ 330

$$e\left(\frac{(be+af)x^3\sqrt{dx^2+c}}{b(bc+ad)\sqrt{a-bx^2}}-\frac{\frac{\frac{\sqrt{a}\left(c(3de-2cf)b^2+3ad(2de+cf)b+8a^2d^2f\right)\sqrt{1-\frac{bx^2}{a}}\sqrt{dx^2+c}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}-\frac{\sqrt{a}c(bc+ad)(3bde-2bcf+4adf)\cdots}{\cdots}}{3bd}\cdots}{b(bc+ad)}\cdots\right.$$

$$f\left(\frac{(be+af)x^5\sqrt{dx^2+c}}{b(bc+ad)\sqrt{a-bx^2}}-\frac{\frac{\frac{\left(-2c^2(5de-4cf)b^3+3acd(5de-3cf)b^2+8a^2d^2(5de+2cf)b+48a^3d^3f\right)\sqrt{1-\frac{bx^2}{a}}\sqrt{dx^2+c}\int\frac{\sqrt{\frac{dx^2}{c}+1}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}-\frac{\sqrt{a}c(bc+ad)\left(-2c(5de-\cdots\right.}{\cdots}}{3bd}\cdots}{\cdots}\cdots\right.$$

$\downarrow$ 327

$$e\left(\frac{(be+af)x^3\sqrt{dx^2+c}}{b(bc+ad)\sqrt{a-bx^2}}-\frac{\frac{\frac{\sqrt{a}\left(c(3de-2cf)b^2+3ad(2de+cf)b+8a^2d^2f\right)\sqrt{1-\frac{bx^2}{a}}\sqrt{dx^2+c}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}-\sqrt{a}c(bc+ad)(3bde-2bcf+4adf)}{3bd}}{b(bc+ad)}\right.$$

$$f\left(\frac{(be+af)x^5\sqrt{dx^2+c}}{b(bc+ad)\sqrt{a-bx^2}}-\frac{\frac{\frac{\sqrt{a}\left(-2c^2(5de-4cf)b^3+3acd(5de-3cf)b^2+8a^2d^2(5de+2cf)b+48a^3d^3f\right)\sqrt{1-\frac{bx^2}{a}}\sqrt{dx^2+c}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}-\sqrt{a}c(bc+a}{3bd}}{}\right.$$

input
```
Int[(x^4*(e + f*x^2)^2)/((a - b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

output
```
e*(((b*e + a*f)*x^3*Sqrt[c + d*x^2])/(b*(b*c + a*d)*Sqrt[a - b*x^2]) - (-1
/3*((3*b*d*e + b*c*f + 4*a*d*f)*x*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])/(b*d) +
 ((Sqrt[a]*(8*a^2*d^2*f + b^2*c*(3*d*e - 2*c*f) + 3*a*b*d*(2*d*e + c*f))*S
qrt[1 - (b*x^2)/a]*Sqrt[c + d*x^2]*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]],
-((a*d)/(b*c))])/(Sqrt[b]*d*Sqrt[a - b*x^2]*Sqrt[1 + (d*x^2)/c]) - (Sqrt[a
]*c*(b*c + a*d)*(3*b*d*e - 2*b*c*f + 4*a*d*f)*Sqrt[1 - (b*x^2)/a]*Sqrt[1 +
 (d*x^2)/c]*EllipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[
b]*d*Sqrt[a - b*x^2]*Sqrt[c + d*x^2]))/(3*b*d))/(b*(b*c + a*d))) + (f*(((b
*e + a*f)*x^5*Sqrt[c + d*x^2])/(b*(b*c + a*d)*Sqrt[a - b*x^2]) - (-1/5*((5
*b*d*e + b*c*f + 6*a*d*f)*x^3*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])/(b*d) + (-1
/3*(((24*a^2*d*f)/b + (b*c*(5*d*e - 4*c*f))/d + 5*a*(4*d*e + c*f))*x*Sqrt[
a - b*x^2]*Sqrt[c + d*x^2]) + ((Sqrt[a]*(48*a^3*d^3*f - 2*b^3*c^2*(5*d*e -
 4*c*f) + 3*a*b^2*c*d*(5*d*e - 3*c*f) + 8*a^2*b*d^2*(5*d*e + 2*c*f))*Sqrt[
1 - (b*x^2)/a]*Sqrt[c + d*x^2]*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a
*d)/(b*c))])/(Sqrt[b]*d*Sqrt[a - b*x^2]*Sqrt[1 + (d*x^2)/c]) - (Sqrt[a]*c*
(b*c + a*d)*(24*a^2*d^2*f + a*b*d*(20*d*e - 13*c*f) - 2*b^2*c*(5*d*e - 4*c
*f))*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[ArcSin[(Sqrt[b]*x)/
Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*d*Sqrt[a - b*x^2]*Sqrt[c + d*x^2]))/(3
*b*d))/(5*b*d))/(b*(b*c + a*d))))/e^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 440
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[g*(b*e - a*f)*(g*x)^(m - 1)*(a +
 b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] - Simp[
g^2/(2*b*(b*c - a*d)*(p + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c +
d*x^2)^q*Simp[c*(b*e - a*f)*(m - 1) + (d*(b*e - a*f)*(m + 2*q + 1) - b*2*(c
*f - d*e)*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, q}, x] &&
 LtQ[p, -1] && GtQ[m, 1]
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

rule 448
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[e   Int[(g*x)^m*(a + b*x
^2)^p*(c + d*x^2)^q*(e + f*x^2)^(r - 1), x], x] + Simp[f/e^2   Int[(g*x)^(m
 + 2)*(a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^(r - 1), x], x] /; FreeQ[{a,
b, c, d, e, f, g, m, p, q}, x] && IGtQ[r, 0]
```

Maple [A] (verified)

Time = 21.08 (sec) , antiderivative size = 652, normalized size of antiderivative = 1.19

method	result
elliptic	$\frac{\sqrt{(-b\,x^2+a)(x^2d+c)}\left(-\frac{\left(-bd\,x^2-bc\right)ax\left(a^2f^2+2abfe+b^2e^2\right)}{b^4(ad+bc)\sqrt{\left(x^2-\frac{a}{b}\right)\left(-bd\,x^2-bc\right)}}+\frac{f^2x^3\sqrt{-bd\,x^4+ad\,x^2-x^2bc+ac}}{5b^2d}-\frac{\left(-\frac{f(af+2be)}{b^2}-\frac{f^2(4ad-4bc)}{5b^2d}\right)x\sqrt{-bd\,x}}{3bd}\right.}{}$
risch	$\frac{fx\left(3bdf\,x^2+9adf-4bcf+10bde\right)\sqrt{-b\,x^2+a}\,\sqrt{x^2d+c}}{15b^3d^2}-\frac{\left(-\frac{\left(33a^2d^2f^2-17abcd\,f^2+50ab\,d^2ef+8b^2c^2f^2-20b^2cdef+15b^2d^2e^2\right)c\sqrt{1-\frac{b\,x}{a}}}{\sqrt{\frac{b}{a}}\,\sqrt{-bd\,x^4}}\right.}{}$
default	Expression too large to display

input

```
int(x^4*(f*x^2+e)^2/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBO
SE)
```

output

```
((-b*x^2+a)*(d*x^2+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-(-b*d*x^2-
b*c)/b^4*a/(a*d+b*c)*x*(a^2*f^2+2*a*b*e*f+b^2*e^2)/((x^2-a/b)*(-b*d*x^2-b*
c))^(1/2)+1/5*f^2/b^2/d*x^3*(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)-1/3*(-1/b
^2*f*(a*f+2*b*e)-1/5*f^2/b^2/d*(4*a*d-4*b*c))/b/d*x*(-b*d*x^4+a*d*x^2-b*c*
x^2+a*c)^(1/2)+(-1/b^3*c*a/(a*d+b*c)*(a^2*f^2+2*a*b*e*f+b^2*e^2)+1/3*(-1/b
^2*f*(a*f+2*b*e)-1/5*f^2/b^2/d*(4*a*d-4*b*c))/b/d*a*c)/(b/a)^(1/2)*(1-b*x^
2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)*Elliptic
F(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2))-(-1/b^3*(a^2*f^2+2*a*b*e*f+b^2*e
^2)-1/b^3*d*a*(a^2*f^2+2*a*b*e*f+b^2*e^2)/(a*d+b*c)-3/5*f^2/b^2/d*a*c+1/3*
(-1/b^2*f*(a*f+2*b*e)-1/5*f^2/b^2/d*(4*a*d-4*b*c))/b/d*(2*a*d-2*b*c))*c/(b
/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*
c)^(1/2)/d*(EllipticF(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2))-EllipticE(x*
(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1025 vs. $2(494) = 988$.

Time = 0.12 (sec) , antiderivative size = 1025, normalized size of antiderivative = 1.88

$$\int \frac{x^4(e+fx^2)^2}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^4*(f*x^2+e)^2/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="f
ricas")
```

output
```
1/15*(((15*(a*b^4*c*d^2 + 2*a^2*b^3*d^3)*e^2 - 10*(2*a*b^4*c^2*d - 3*a^2*b
^3*c*d^2 - 8*a^3*b^2*d^3)*e*f + (8*a*b^4*c^3 - 9*a^2*b^3*c^2*d + 16*a^3*b^
2*c*d^2 + 48*a^4*b*d^3)*f^2)*x^3 - (15*(a^2*b^3*c*d^2 + 2*a^3*b^2*d^3)*e^2
 - 10*(2*a^2*b^3*c^2*d - 3*a^3*b^2*c*d^2 - 8*a^4*b*d^3)*e*f + (8*a^2*b^3*c
^3 - 9*a^3*b^2*c^2*d + 16*a^4*b*c*d^2 + 48*a^5*d^3)*f^2)*x)*sqrt(-b*d)*sqr
t(a/b)*elliptic_e(arcsin(sqrt(a/b)/x), -b*c/(a*d)) - ((15*(2*a^2*b^3*d^3 +
 (a*b^4 + b^5)*c*d^2)*e^2 + 10*(8*a^3*b^2*d^3 - (2*a*b^4 - b^5)*c^2*d + (3
*a^2*b^3 + 4*a*b^4)*c*d^2)*e*f + (48*a^4*b*d^3 + 4*(2*a*b^4 - b^5)*c^3 - (
9*a^2*b^3 - 5*a*b^4)*c^2*d + 8*(2*a^3*b^2 + 3*a^2*b^3)*c*d^2)*f^2)*x^3 - (
15*(2*a^3*b^2*d^3 + (a^2*b^3 + a*b^4)*c*d^2)*e^2 + 10*(8*a^4*b*d^3 - (2*a^
2*b^3 - a*b^4)*c^2*d + (3*a^3*b^2 + 4*a^2*b^3)*c*d^2)*e*f + (48*a^5*d^3 +
4*(2*a^2*b^3 - a*b^4)*c^3 - (9*a^3*b^2 - 5*a^2*b^3)*c^2*d + 8*(2*a^4*b + 3
*a^3*b^2)*c*d^2)*f^2)*x)*sqrt(-b*d)*sqrt(a/b)*elliptic_f(arcsin(sqrt(a/b)/
x), -b*c/(a*d)) + (3*(b^5*c*d^2 + a*b^4*d^3)*f^2*x^6 + 2*(5*(b^5*c*d^2 + a
*b^4*d^3)*e*f - (2*b^5*c^2*d - a*b^4*c*d^2 - 3*a^2*b^3*d^3)*f^2)*x^4 - 15*
(a*b^4*c*d^2 + 2*a^2*b^3*d^3)*e^2 + 10*(2*a*b^4*c^2*d - 3*a^2*b^3*c*d^2 -
8*a^3*b^2*d^3)*e*f - (8*a*b^4*c^3 - 9*a^2*b^3*c^2*d + 16*a^3*b^2*c*d^2 + 4
8*a^4*b*d^3)*f^2 + (15*(b^5*c*d^2 + a*b^4*d^3)*e^2 - 20*(b^5*c^2*d - a*b^4
*c*d^2 - 2*a^2*b^3*d^3)*e*f + (8*b^5*c^3 - 5*a*b^4*c^2*d + 11*a^2*b^3*c*d^
2 + 24*a^3*b^2*d^3)*f^2)*x^2)*sqrt(-b*x^2 + a)*sqrt(d*x^2 + c))/((b^7*c...
```

Sympy [F]

$$\int \frac{x^4(e+fx^2)^2}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx=\int \frac{x^4(e+fx^2)^2}{(a-bx^2)^{\frac{3}{2}}\sqrt{c+dx^2}}\,dx$$

input
```
integrate(x**4*(f*x**2+e)**2/(-b*x**2+a)**(3/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(x**4*(e + f*x**2)**2/((a - b*x**2)**(3/2)*sqrt(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{x^4(e+fx^2)^2}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx=\int \frac{(fx^2+e)^2x^4}{(-bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}}\,dx$$

input
```
integrate(x^4*(f*x^2+e)^2/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="m
axima")
```

output
```
integrate((f*x^2 + e)^2*x^4/((-b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)), x)
```

Giac [F]

$$\int \frac{x^4(e+fx^2)^2}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx=\int \frac{(fx^2+e)^2x^4}{(-bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}}\,dx$$

input
```
integrate(x^4*(f*x^2+e)^2/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="g
iac")
```

output
```
integrate((f*x^2 + e)^2*x^4/((-b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(e+fx^2)^2}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \int \frac{x^4\,(f\,x^2+e)^2}{(a-b\,x^2)^{3/2}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int((x^4*(e + f*x^2)^2)/((a - b*x^2)^(3/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int((x^4*(e + f*x^2)^2)/((a - b*x^2)^(3/2)*(c + d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^4(e+fx^2)^2}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \text{too large to display}$$

input
```
int(x^4*(f*x^2+e)^2/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x)
```

output

```
(9*sqrt(c + d*x**2)*sqrt(a - b*x**2)*a*c*d*f**2*x - 6*sqrt(c + d*x**2)*sqr
t(a - b*x**2)*a*d**2*f**2*x**3 - 6*sqrt(c + d*x**2)*sqrt(a - b*x**2)*b*c**
2*f**2*x + 15*sqrt(c + d*x**2)*sqrt(a - b*x**2)*b*c*d*e*f*x + 4*sqrt(c + d
*x**2)*sqrt(a - b*x**2)*b*c*d*f**2*x**3 - 10*sqrt(c + d*x**2)*sqrt(a - b*x
**2)*b*d**2*e*f*x**3 - 3*sqrt(c + d*x**2)*sqrt(a - b*x**2)*b*d**2*f**2*x**
5 + 24*int((sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**4)/(a**2*c + a**2*d*x**2
- 2*a*b*c*x**2 - 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a**3*d**3*f*
*2 - 4*int((sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**4)/(a**2*c + a**2*d*x**2
- 2*a*b*c*x**2 - 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a**2*b*c*d**
2*f**2 + 40*int((sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**4)/(a**2*c + a**2*d*
x**2 - 2*a*b*c*x**2 - 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a**2*b*
d**3*e*f - 24*int((sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**4)/(a**2*c + a**2*
d*x**2 - 2*a*b*c*x**2 - 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a**2*
b*d**3*f**2*x**2 + 2*int((sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**4)/(a**2*c
+ a**2*d*x**2 - 2*a*b*c*x**2 - 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x
)*a*b**2*c**2*d*f**2 - 5*int((sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**4)/(a**
2*c + a**2*d*x**2 - 2*a*b*c*x**2 - 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**
6),x)*a*b**2*c*d**2*e*f + 4*int((sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**4)/(
a**2*c + a**2*d*x**2 - 2*a*b*c*x**2 - 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*
x**6),x)*a*b**2*c*d**2*f**2*x**2 + 15*int((sqrt(c + d*x**2)*sqrt(a - b*...
```

3.271 $\int \frac{x^2(e+fx^2)^2}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 380

$$\int \frac{x^2(e+fx^2)^2}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \frac{(3b^2de^2+4a^2df^2+abf(6de+cf))\,x\sqrt{c+dx^2}}{3b^2d(bc+ad)\sqrt{a-bx^2}} - \frac{f^2x^3\sqrt{c+dx^2}}{3bd\sqrt{a-bx^2}} - \frac{\sqrt{a}(8a^2d^2f^2+3abdf(4de+cf)+b^2(3d^2e^2+6cdef-2c^2f^2))\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{3b^{5/2}d^2(bc+ad)\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}} + \frac{2\sqrt{a}cf(3bde-bcf+2adf)\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{3b^{5/2}d^2\sqrt{a-bx^2}\sqrt{c+dx^2}}$$

output
```
1/3*(3*b^2*d*e^2+4*a^2*d*f^2+a*b*f*(c*f+6*d*e))*x*(d*x^2+c)^(1/2)/b^2/d/(a
*d+b*c)/(-b*x^2+a)^(1/2)-1/3*f^2*x^3*(d*x^2+c)^(1/2)/b/d/(-b*x^2+a)^(1/2)-
1/3*a^(1/2)*(8*a^2*d^2*f^2+3*a*b*d*f*(c*f+4*d*e)+b^2*(-2*c^2*f^2+6*c*d*e*f
+3*d^2*e^2))*(1-b*x^2/a)^(1/2)*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)
,(-a*d/b/c)^(1/2))/b^(5/2)/d^2/(a*d+b*c)/(-b*x^2+a)^(1/2)/(1+d*x^2/c)^(1/2
)+2/3*a^(1/2)*c*f*(2*a*d*f-b*c*f+3*b*d*e)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1
/2)*EllipticF(b^(1/2)*x/a^(1/2),(-a*d/b/c)^(1/2))/b^(5/2)/d^2/(-b*x^2+a)^(
1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.65 (sec) , antiderivative size = 323, normalized size of antiderivative = 0.85

$$\int \frac{x^2(e+fx^2)^2}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \frac{\sqrt{-\frac{b}{a}}dx(c+dx^2)\left(4a^2df^2+abf(6de+cf-dfx^2)+b^2(3de^2-cf^2x^2)\right)+ic}{}$$

input
```
Integrate[(x^2*(e + f*x^2)^2)/((a - b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

output
```
(Sqrt[-(b/a)]*d*x*(c + d*x^2)*(4*a^2*d*f^2 + a*b*f*(6*d*e + c*f - d*f*x^2)
 + b^2*(3*d*e^2 - c*f^2*x^2)) + I*c*(8*a^2*d^2*f^2 + 3*a*b*d*f*(4*d*e + c*
f) + b^2*(3*d^2*e^2 + 6*c*d*e*f - 2*c^2*f^2))*Sqrt[1 - (b*x^2)/a]*Sqrt[1 +
 (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))] - (2*I)*c
*(b*c + a*d)*f*(3*b*d*e - b*c*f + 2*a*d*f)*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d
*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))])/(3*b^2*Sqrt
[-(b/a)]*d^2*(b*c + a*d)*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (warning: unable to verify)

Time = 1.37 (sec) , antiderivative size = 636, normalized size of antiderivative = 1.67, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {448, 440, 25, 399, 323, 323, 321, 331, 330, 327, 444, 399, 323, 323, 321, 331, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(e+fx^2)^2}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \text{448}$$

$$\frac{f\int \frac{x^4(fx^2+e)}{(a-bx^2)^{3/2}\sqrt{dx^2+c}}dx}{e^2}+e\int \frac{x^2(fx^2+e)}{(a-bx^2)^{3/2}\sqrt{dx^2+c}}dx$$

$$\Big\downarrow \ 440$$

$$\frac{f\left(\frac{\int -\frac{x^2\left((3bde+bcf+4adf)x^2+3c(be+af)\right)}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{b(ad+bc)}+\frac{x^3\sqrt{c+dx^2}(af+be)}{b\sqrt{a-bx^2}(ad+bc)}\right)}{e^2}+e\left(\frac{\int -\frac{(bde+bcf+2adf)x^2+c(be+af)}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{b(ad+bc)}+\frac{x\sqrt{c+dx^2}(af+be)}{b\sqrt{a-bx^2}(ad+bc)}\right)$$

$$\Big\downarrow \ 25$$

$$\frac{f\left(\frac{x^3\sqrt{c+dx^2}(af+be)}{b\sqrt{a-bx^2}(ad+bc)}-\frac{\int \frac{x^2\left((3bde+bcf+4adf)x^2+3c(be+af)\right)}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{b(ad+bc)}\right)}{e^2}+e\left(\frac{x\sqrt{c+dx^2}(af+be)}{b\sqrt{a-bx^2}(ad+bc)}-\frac{\int \frac{(bde+bcf+2adf)x^2+c(be+af)}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{b(ad+bc)}\right)$$

$$\Big\downarrow \ 399$$

$$\frac{f\left(\frac{x^3\sqrt{c+dx^2}(af+be)}{b\sqrt{a-bx^2}(ad+bc)}-\frac{\int \frac{x^2\left((3bde+bcf+4adf)x^2+3c(be+af)\right)}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{b(ad+bc)}\right)}{e^2}+e\left(\frac{x\sqrt{c+dx^2}(af+be)}{b\sqrt{a-bx^2}(ad+bc)}-\frac{\frac{(2adf+bcf+bde)\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}-\frac{cf(ad+bc)\int \frac{1}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{d}}{b(ad+bc)}\right)$$

$$\Big\downarrow \ 323$$

$$\frac{f\left(\frac{x^3\sqrt{c+dx^2}(af+be)}{b\sqrt{a-bx^2}(ad+bc)}-\frac{\int \frac{x^2\left((3bde+bcf+4adf)x^2+3c(be+af)\right)}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{b(ad+bc)}\right)}{e^2}+e\left(\frac{x\sqrt{c+dx^2}(af+be)}{b\sqrt{a-bx^2}(ad+bc)}-\frac{\frac{(2adf+bcf+bde)\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}-\frac{cf\sqrt{\frac{dx^2}{c}+1}(ad+bc)\int \frac{1}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}dx}{d\sqrt{c+dx^2}}}{b(ad+bc)}\right)$$

$$\Big\downarrow \ 323$$

$$\frac{f\left(\frac{x^3\sqrt{c+dx^2}(af+be)}{b\sqrt{a-bx^2}(ad+bc)}-\frac{\int \frac{x^2\left((3bde+bcf+4adf)x^2+3c(be+af)\right)}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{b(ad+bc)}\right)}{e^2}+$$

$$e\left(\frac{x\sqrt{c+dx^2}(af+be)}{b\sqrt{a-bx^2}(ad+bc)}-\frac{\frac{(2adf+bcf+bde)\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}-\frac{cf\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\int \frac{1}{\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}}dx}{d\sqrt{a-bx^2}\sqrt{c+dx^2}}}{b(ad+bc)}\right)$$

$$\Big\downarrow \text{321}$$

$$e\left(\frac{x\sqrt{c+dx^2}(af+be)}{b\sqrt{a-bx^2}(ad+bc)}-\frac{\frac{(2adf+bcf+bde)\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}-\frac{\sqrt{a}cf\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}}{b(ad+bc)}\right)+$$

$$\frac{f\left(\frac{x^3\sqrt{c+dx^2}(af+be)}{b\sqrt{a-bx^2}(ad+bc)}-\frac{\int \frac{x^2\left((3bde+bcf+4adf)x^2+3c(be+af)\right)}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{b(ad+bc)}\right)}{e^2}$$

$$\Big\downarrow \text{331}$$

$$e\left(\frac{x\sqrt{c+dx^2}(af+be)}{b\sqrt{a-bx^2}(ad+bc)}-\frac{\frac{\sqrt{1-\frac{bx^2}{a}}(2adf+bcf+bde)\int \frac{\sqrt{dx^2+c}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{\sqrt{a}cf\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}}{b(ad+bc)}\right)$$

$$\frac{f\left(\frac{x^3\sqrt{c+dx^2}(af+be)}{b\sqrt{a-bx^2}(ad+bc)}-\frac{\int \frac{x^2\left((3bde+bcf+4adf)x^2+3c(be+af)\right)}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{b(ad+bc)}\right)}{e^2}$$

$$\Big\downarrow \text{330}$$

$$e\left(\frac{x\sqrt{c+dx^2}(af+be)}{b\sqrt{a-bx^2}(ad+bc)}-\frac{\frac{\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(2adf+bcf+bde)\int\frac{\sqrt{\frac{dx^2}{c}+1}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}-\frac{\sqrt{a}cf\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\right.}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}}{b(ad+bc)}\right.$$

$$\frac{f\left(\frac{x^3\sqrt{c+dx^2}(af+be)}{b\sqrt{a-bx^2}(ad+bc)}-\frac{\int\frac{x^2\left((3bde+bcf+4adf)x^2+3c(be+af)\right)}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{b(ad+bc)}\right)}{e^2}$$

$\downarrow$ 327

$$\frac{f\left(\frac{x^3\sqrt{c+dx^2}(af+be)}{b\sqrt{a-bx^2}(ad+bc)}-\frac{\int\frac{x^2\left((3bde+bcf+4adf)x^2+3c(be+af)\right)}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{b(ad+bc)}\right)}{e^2}+$$

$$e\left(\frac{x\sqrt{c+dx^2}(af+be)}{b\sqrt{a-bx^2}(ad+bc)}-\frac{\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(2adf+bcf+bde)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}-\frac{\sqrt{a}cf\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\text{EllipticF}\left(a\right.}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}}{b(ad+bc)}\right.$$

$\downarrow$ 444

$$\frac{f\left(\frac{x^3\sqrt{c+dx^2}(af+be)}{b\sqrt{a-bx^2}(ad+bc)}-\frac{\frac{\int\frac{\left(c(3de-2cf)b^2+3ad(2de+cf)b+8a^2d^2f\right)x^2+ac(3bde+bcf+4adf)}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{3bd}-\frac{x\sqrt{a-bx^2}\sqrt{c+dx^2}(4adf+bcf+3bde)}{3bd}}{b(ad+bc)}\right)}{e^2}+$$

$$e\left(\frac{x\sqrt{c+dx^2}(af+be)}{b\sqrt{a-bx^2}(ad+bc)}-\frac{\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(2adf+bcf+bde)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}-\frac{\sqrt{a}cf\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\text{EllipticF}\left(a\right.}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}}{b(ad+bc)}\right.$$

$\downarrow$ 399

$$\frac{f\left(\frac{x^3\sqrt{c+dx^2}(af+be)}{b\sqrt{a-bx^2}(ad+bc)}-\frac{\frac{\frac{\left(8a^2d^2f+3abd(cf+2de)+b^2c(3de-2cf)\right)\int\frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}-\frac{c(ad+bc)(4adf-2bcf+3bde)\int\frac{1}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{d}}{3bd}-\frac{x\sqrt{a-bx^2}\sqrt{c+dx^2}\cdots}{\cdots}}{b(ad+bc)}\right.}{e^2}$$

$$e\left(\frac{x\sqrt{c+dx^2}(af+be)}{b\sqrt{a-bx^2}(ad+bc)}-\frac{\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(2adf+bcf+bde)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}-\frac{\sqrt{a}cf\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\operatorname{EllipticF}\left(\cdots\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}}{b(ad+bc)}\right.$$

$\downarrow$ 323

$$\frac{f\left(\frac{x^3\sqrt{c+dx^2}(af+be)}{b\sqrt{a-bx^2}(ad+bc)}-\frac{\frac{\frac{\left(8a^2d^2f+3abd(cf+2de)+b^2c(3de-2cf)\right)\int\frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}-\frac{c\sqrt{\frac{dx^2}{c}+1}(ad+bc)(4adf-2bcf+3bde)\int\frac{1}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}dx}{d\sqrt{c+dx^2}}}{3bd}-\frac{x\sqrt{a-\cdots}}{\cdots}}{b(ad+bc)}\right.}{e^2}$$

$$e\left(\frac{x\sqrt{c+dx^2}(af+be)}{b\sqrt{a-bx^2}(ad+bc)}-\frac{\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(2adf+bcf+bde)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}-\frac{\sqrt{a}cf\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\operatorname{EllipticF}\left(\cdots\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}}{b(ad+bc)}\right.$$

$\downarrow$ 323

$$\frac{f\left(\frac{x^3\sqrt{c+dx^2}(af+be)}{b\sqrt{a-bx^2}(ad+bc)}-\frac{\frac{\frac{\left(8a^2d^2f+3abd(cf+2de)+b^2c(3de-2cf)\right)\int\frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}-\frac{c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(4adf-2bcf+3bde)\int\frac{1}{\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}}dx}{d\sqrt{a-bx^2}\sqrt{c+dx^2}}}{3bd}\cdots}{b(ad+bc)}\right.}{e^2}$$

$$e\left(\frac{x\sqrt{c+dx^2}(af+be)}{b\sqrt{a-bx^2}(ad+bc)}-\frac{\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(2adf+bcf+bde)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}-\frac{\sqrt{a}cf\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\operatorname{EllipticF}\left(\cdots\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}}{b(ad+bc)}\right.$$

$\downarrow$ 321

$$\frac{f\left(\frac{x^3\sqrt{c+dx^2}(af+be)}{b\sqrt{a-bx^2}(ad+bc)}-\frac{\frac{\frac{\left(8a^2d^2f+3abd(cf+2de)+b^2c(3de-2cf)\right)\int\frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}-\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(4adf-2bcf+3bde)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\cdot}}{\sqrt{\cdot}}\right.\right.}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}}{3bd}}{b(ad+bc)}\right.}{e\left(\frac{x\sqrt{c+dx^2}(af+be)}{b\sqrt{a-bx^2}(ad+bc)}-\frac{\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(2adf+bcf+bde)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}-\frac{\sqrt{a}cf\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\text{EllipticF}\left(\cdot\right.}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}}{b(ad+bc)}\right.}$$

e^2

$\downarrow$ 331

$$\frac{f\left(\frac{x^3\sqrt{c+dx^2}(af+be)}{b\sqrt{a-bx^2}(ad+bc)}-\frac{\frac{\frac{\sqrt{1-\frac{bx^2}{a}}\left(8a^2d^2f+3abd(cf+2de)+b^2c(3de-2cf)\right)\int\frac{\sqrt{dx^2+c}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(4adf-2bcf+3bde)\text{EllipticF}\left(\cdot\right.}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}}{3bd}}{b(ad+bc)}\right.}{e\left(\frac{x\sqrt{c+dx^2}(af+be)}{b\sqrt{a-bx^2}(ad+bc)}-\frac{\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(2adf+bcf+bde)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}-\frac{\sqrt{a}cf\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\text{EllipticF}\left(\cdot\right.}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}}{b(ad+bc)}\right.}$$

e^2

$\downarrow$ 330

$$\frac{f\left(\frac{x^3\sqrt{c+dx^2}(af+be)}{b\sqrt{a-bx^2}(ad+bc)}-\frac{\frac{\frac{\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\left(8a^2d^2f+3abd(cf+2de)+b^2c(3de-2cf)\right)\int\frac{\sqrt{\frac{dx^2}{c}+1}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}-\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(4adf-2bcf+3bde)\text{E}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}}{3bd}}{b(ad+bc)}\right.}{e\left(\frac{x\sqrt{c+dx^2}(af+be)}{b\sqrt{a-bx^2}(ad+bc)}-\frac{\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(2adf+bcf+bde)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}-\frac{\sqrt{a}cf\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\text{EllipticF}\left(\cdot\right.}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}}{b(ad+bc)}\right.}$$

e^2

$\downarrow$ 327

$$\frac{f\left(\frac{x^3\sqrt{c+dx^2}(af+be)}{b\sqrt{a-bx^2}(ad+bc)}-\frac{\frac{\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\left(8a^2d^2f+3abd(cf+2de)+b^2c(3de-2cf)\right)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}-\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(4adf-2}{\sqrt{b}d\sqrt{a-}}}{3bd}}{b(ad+bc)}\right.}{e^2}$$

$$e\left(\frac{x\sqrt{c+dx^2}(af+be)}{b\sqrt{a-bx^2}(ad+bc)}-\frac{\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(2adf+bcf+bde)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}-\frac{\sqrt{a}cf\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\,\mathrm{EllipticF}\left(\right.}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}}{b(ad+bc)}\right.$$

input
```
Int[(x^2*(e + f*x^2)^2)/((a - b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

output
```
e*(((b*e + a*f)*x*Sqrt[c + d*x^2])/(b*(b*c + a*d)*Sqrt[a - b*x^2]) - ((Sqr
t[a]*(b*d*e + b*c*f + 2*a*d*f)*Sqrt[1 - (b*x^2)/a]*Sqrt[c + d*x^2]*Ellipti
cE[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*d*Sqrt[a - b*x^2
]*Sqrt[1 + (d*x^2)/c]) - (Sqrt[a]*c*(b*c + a*d)*f*Sqrt[1 - (b*x^2)/a]*Sqrt
[1 + (d*x^2)/c]*EllipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(S
qrt[b]*d*Sqrt[a - b*x^2]*Sqrt[c + d*x^2]))/(b*(b*c + a*d))) + (f*(((b*e +
a*f)*x^3*Sqrt[c + d*x^2])/(b*(b*c + a*d)*Sqrt[a - b*x^2]) - (-1/3*((3*b*d*
e + b*c*f + 4*a*d*f)*x*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])/(b*d) + ((Sqrt[a]*
(8*a^2*d^2*f + b^2*c*(3*d*e - 2*c*f) + 3*a*b*d*(2*d*e + c*f))*Sqrt[1 - (b*
x^2)/a]*Sqrt[c + d*x^2]*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*
c))])/(Sqrt[b]*d*Sqrt[a - b*x^2]*Sqrt[1 + (d*x^2)/c]) - (Sqrt[a]*c*(b*c +
a*d)*(3*b*d*e - 2*b*c*f + 4*a*d*f)*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]
*EllipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*d*Sqrt[a
 - b*x^2]*Sqrt[c + d*x^2]))/(3*b*d))/(b*(b*c + a*d))))/e^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 440
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[g*(b*e - a*f)*(g*x)^(m - 1)*(a +
 b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] - Simp[
g^2/(2*b*(b*c - a*d)*(p + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c +
d*x^2)^q*Simp[c*(b*e - a*f)*(m - 1) + (d*(b*e - a*f)*(m + 2*q + 1) - b*2*(c
*f - d*e)*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, q}, x] &&
 LtQ[p, -1] && GtQ[m, 1]
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

rule 448
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[e   Int[(g*x)^m*(a + b*x
^2)^p*(c + d*x^2)^q*(e + f*x^2)^(r - 1), x], x] + Simp[f/e^2   Int[(g*x)^(m
 + 2)*(a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^(r - 1), x], x] /; FreeQ[{a,
b, c, d, e, f, g, m, p, q}, x] && IGtQ[r, 0]
```

Maple [A] (verified)

Time = 10.67 (sec) , antiderivative size = 488, normalized size of antiderivative = 1.28

method	result
elliptic	$\frac{\sqrt{(-b\,x^2+a)(x^2d+c)}\left(-\frac{\left(-bd\,x^2-bc\right)x\left(a^2f^2+2abfe+b^2e^2\right)}{b^3(ad+bc)\sqrt{\left(x^2-\frac{a}{b}\right)\left(-bd\,x^2-bc\right)}}+\frac{f^2x\sqrt{-bd\,x^4+ad\,x^2-x^2bc+ac}}{3d\,b^2}+\frac{\left(-\frac{c\left(a^2f^2+2abfe+b^2e^2\right)}{b^2(ad+bc)}-\frac{f^2ac}{3b^2d}\right)\sqrt{1-\frac{b}{}}}{\sqrt{\frac{b}{a}}\sqrt{-bd\,x^4+}}\right.}{}$
risch	$\frac{f^2x\sqrt{-b\,x^2+a}\,\sqrt{x^2d+c}}{3b^2d}-\frac{\left(\frac{\left(3a^2d\,f^2+abc\,f^2+6abdef+3b^2d\,e^2\right)\sqrt{1-\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{b}{a}},\sqrt{-1-\frac{ad-bc}{cb}}\right)}{b\sqrt{\frac{b}{a}}\,\sqrt{-bd\,x^4+ad\,x^2-x^2bc+ac}}-\frac{f(5adf-2bcf+6bde}{}\right.}{}$
default	$\frac{\left(-\sqrt{\frac{b}{a}}\,ab\,d^3f^2x^5-\sqrt{\frac{b}{a}}\,b^2c\,d^2f^2x^5+4\sqrt{\frac{b}{a}}\,a^2d^3f^2x^3+6\sqrt{\frac{b}{a}}\,ab\,d^3ef\,x^3-\sqrt{\frac{b}{a}}\,b^2c^2d\,f^2x^3+3\sqrt{\frac{b}{a}}\,b^2d^3e^2x^3+4\sqrt{\frac{-b\,x^2+a}{a}}\,\sqrt{\frac{x^2d+c}{c}}\,\mathrm{E}\right.}{}$

input
```
int(x^2*(f*x^2+e)^2/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBO
SE)
```

output
```
((-b*x^2+a)*(d*x^2+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-(-b*d*x^2-
b*c)/b^3/(a*d+b*c)*x*(a^2*f^2+2*a*b*e*f+b^2*e^2)/((x^2-a/b)*(-b*d*x^2-b*c)
)^(1/2)+1/3*f^2/d/b^2*x*(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)+(-1/b^2*c/(a*
d+b*c)*(a^2*f^2+2*a*b*e*f+b^2*e^2)-1/3*f^2/b^2/d*a*c)/(b/a)^(1/2)*(1-b*x^2
/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)*EllipticF
(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2))-(-1/b^2*f*(a*f+2*b*e)-1/b^2*d*(a^
2*f^2+2*a*b*e*f+b^2*e^2)/(a*d+b*c)-1/3*f^2/d/b^2*(2*a*d-2*b*c))*c/(b/a)^(1
/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/
2)/d*(EllipticF(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2))-EllipticE(x*(b/a)^
(1/2),(-1-(a*d-b*c)/c/b)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 697 vs. $2(334) = 668$.

Time = 0.09 (sec) , antiderivative size = 697, normalized size of antiderivative = 1.83

$$\int \frac{x^2(e+fx^2)^2}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \frac{((3\,a^2b^3d^2e^2+6\,(a^2b^3cd+2\,a^3b^2d^2)ef-(2\,a^2b^3c^2-3\,a^3b^2cd-8\,a^4bd^2)f^2)x}{}$$

input
```
integrate(x^2*(f*x^2+e)^2/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="f
ricas")
```

output
```
1/3*(((3*a^2*b^3*d^2*e^2 + 6*(a^2*b^3*c*d + 2*a^3*b^2*d^2)*e*f - (2*a^2*b^
3*c^2 - 3*a^3*b^2*c*d - 8*a^4*b*d^2)*f^2)*x^3 - (3*a^3*b^2*d^2*e^2 + 6*(a^
3*b^2*c*d + 2*a^4*b*d^2)*e*f - (2*a^3*b^2*c^2 - 3*a^4*b*c*d - 8*a^5*d^2)*f
^2)*x)*sqrt(-b*d)*sqrt(a/b)*elliptic_e(arcsin(sqrt(a/b)/x), -b*c/(a*d)) -
((3*(b^5*c*d + a^2*b^3*d^2)*e^2 + 6*(2*a^3*b^2*d^2 + (a^2*b^3 + a*b^4)*c*d
)*e*f + (8*a^4*b*d^2 - (2*a^2*b^3 - a*b^4)*c^2 + (3*a^3*b^2 + 4*a^2*b^3)*c
*d)*f^2)*x^3 - (3*(a*b^4*c*d + a^3*b^2*d^2)*e^2 + 6*(2*a^4*b*d^2 + (a^3*b^
2 + a^2*b^3)*c*d)*e*f + (8*a^5*d^2 - (2*a^3*b^2 - a^2*b^3)*c^2 + (3*a^4*b
+ 4*a^3*b^2)*c*d)*f^2)*x)*sqrt(-b*d)*sqrt(a/b)*elliptic_f(arcsin(sqrt(a/b)
/x), -b*c/(a*d)) - (3*a^2*b^3*d^2*e^2 - (a*b^4*c*d + a^2*b^3*d^2)*f^2*x^4
+ 6*(a^2*b^3*c*d + 2*a^3*b^2*d^2)*e*f - (2*a^2*b^3*c^2 - 3*a^3*b^2*c*d - 8
*a^4*b*d^2)*f^2 - 2*(3*(a*b^4*c*d + a^2*b^3*d^2)*e*f - (a*b^4*c^2 - a^2*b^
3*c*d - 2*a^3*b^2*d^2)*f^2)*x^2)*sqrt(-b*x^2 + a)*sqrt(d*x^2 + c))/((a*b^6
*c*d^2 + a^2*b^5*d^3)*x^3 - (a^2*b^5*c*d^2 + a^3*b^4*d^3)*x)
```

Sympy [F]

$$\int \frac{x^2(e+fx^2)^2}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \int \frac{x^2(e+fx^2)^2}{(a-bx^2)^{\frac{3}{2}}\sqrt{c+dx^2}}\,dx$$

input
```
integrate(x**2*(f*x**2+e)**2/(-b*x**2+a)**(3/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(x**2*(e + f*x**2)**2/((a - b*x**2)**(3/2)*sqrt(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{x^2(e+fx^2)^2}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \int \frac{(fx^2+e)^2x^2}{(-bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}}\,dx$$

input
```
integrate(x^2*(f*x^2+e)^2/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="m
axima")
```

output
```
integrate((f*x^2 + e)^2*x^2/((-b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)), x)
```

Giac [F]

$$\int \frac{x^2(e+fx^2)^2}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \int \frac{(fx^2+e)^2x^2}{(-bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}}\,dx$$

input
```
integrate(x^2*(f*x^2+e)^2/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="g
iac")
```

output
```
integrate((f*x^2 + e)^2*x^2/((-b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(e+fx^2)^2}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \int \frac{x^2\,(f\,x^2+e)^2}{(a-b\,x^2)^{3/2}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int((x^2*(e + f*x^2)^2)/((a - b*x^2)^(3/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int((x^2*(e + f*x^2)^2)/((a - b*x^2)^(3/2)*(c + d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^2(e+fx^2)^2}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \text{Too large to display}$$

input
```
int(x^2*(f*x^2+e)^2/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x)
```

output
```
(3*sqrt(c + d*x**2)*sqrt(a - b*x**2)*a*c*f**2*x - 2*sqrt(c + d*x**2)*sqrt(
a - b*x**2)*a*d*f**2*x**3 + 3*sqrt(c + d*x**2)*sqrt(a - b*x**2)*b*d*e**2*x
 + 8*int((sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**4)/(a**2*c + a**2*d*x**2 -
2*a*b*c*x**2 - 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a**3*d**2*f**2
 - int((sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**4)/(a**2*c + a**2*d*x**2 - 2*
a*b*c*x**2 - 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a**2*b*c*d*f**2
+ 12*int((sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**4)/(a**2*c + a**2*d*x**2 -
2*a*b*c*x**2 - 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a**2*b*d**2*e*
f - 8*int((sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**4)/(a**2*c + a**2*d*x**2 -
 2*a*b*c*x**2 - 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a**2*b*d**2*f
**2*x**2 + int((sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**4)/(a**2*c + a**2*d*x
**2 - 2*a*b*c*x**2 - 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a*b**2*c
*d*f**2*x**2 + 3*int((sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**4)/(a**2*c + a*
*2*d*x**2 - 2*a*b*c*x**2 - 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a*
b**2*d**2*e**2 - 12*int((sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**4)/(a**2*c +
 a**2*d*x**2 - 2*a*b*c*x**2 - 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)
*a*b**2*d**2*e*f*x**2 - 3*int((sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**4)/(a*
*2*c + a**2*d*x**2 - 2*a*b*c*x**2 - 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x*
*6),x)*b**3*d**2*e**2*x**2 - 3*int((sqrt(c + d*x**2)*sqrt(a - b*x**2))/(a*
*2*c + a**2*d*x**2 - 2*a*b*c*x**2 - 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d...
```

3.272 $\int \frac{(e+fx^2)^2}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 283

$$\int \frac{(e+fx^2)^2}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \frac{(be+af)^2x\sqrt{c+dx^2}}{ab(bc+ad)\sqrt{a-bx^2}} - \frac{(b^2de^2+2a^2df^2+abf(2de+cf))\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\,E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{a}b^{3/2}d(bc+ad)\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}} + \frac{(bde^2+acf^2)\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{a}b^{3/2}d\sqrt{a-bx^2}\sqrt{c+dx^2}}$$

output
```
(a*f+b*e)^2*x*(d*x^2+c)^(1/2)/a/b/(a*d+b*c)/(-b*x^2+a)^(1/2)-(b^2*d*e^2+2*
a^2*d*f^2+a*b*f*(c*f+2*d*e))*(1-b*x^2/a)^(1/2)*(d*x^2+c)^(1/2)*EllipticE(b
^(1/2)*x/a^(1/2),(-a*d/b/c)^(1/2))/a^(1/2)/b^(3/2)/d/(a*d+b*c)/(-b*x^2+a)^
(1/2)/(1+d*x^2/c)^(1/2)+(a*c*f^2+b*d*e^2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1
/2)*EllipticF(b^(1/2)*x/a^(1/2),(-a*d/b/c)^(1/2))/a^(1/2)/b^(3/2)/d/(-b*x^
2+a)^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 9.40 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.89

$$\int \frac{(e+fx^2)^2}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \frac{b\left(\sqrt{-\frac{b}{a}}d(be+af)^2x(c+dx^2)+ic(b^2de^2+2a^2df^2+abf(2de+cf))\sqrt{1-\right.}{}$$

input
```
Integrate[(e + f*x^2)^2/((a - b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

output
```
(b*(Sqrt[-(b/a)]*d*(b*e + a*f)^2*x*(c + d*x^2) + I*c*(b^2*d*e^2 + 2*a^2*d*
f^2 + a*b*f*(2*d*e + c*f))*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Ellipti
cE[I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))] - I*(b*c + a*d)*(b*d*e^2 + a
*c*f^2)*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[-
(b/a)]*x], -((a*d)/(b*c))]))/(a^3*(-(b/a))^(5/2)*d*(b*c + a*d)*Sqrt[a - b*
x^2]*Sqrt[c + d*x^2])
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 614 vs. 2(283) = 566.

Time = 0.83 (sec) , antiderivative size = 614, normalized size of antiderivative = 2.17, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.061$, Rules used = {433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx^2)^2}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx$$

$$\Big\downarrow \; 433$$

$$\int \left(\frac{e^2}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}+\frac{2efx^2}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}+\frac{f^2x^4}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{\sqrt{a}cf^2\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{b^{3/2}d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}f^2\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(2ad+bc)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{b^{3/2}d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}(ad+bc)}+\frac{e^2\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{b}e^2\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}(ad+bc)}-\frac{2\sqrt{a}ef\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}(ad+bc)}+\frac{be^2x\sqrt{c+dx^2}}{a\sqrt{a-bx^2}(ad+bc)}+\frac{2efx\sqrt{c+dx^2}}{\sqrt{a-bx^2}(ad+bc)}+\frac{af^2x\sqrt{c+dx^2}}{b\sqrt{a-bx^2}(ad+bc)}$$

input
```
Int[(e + f*x^2)^2/((a - b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

output
```
(b*e^2*x*Sqrt[c + d*x^2])/(a*(b*c + a*d)*Sqrt[a - b*x^2]) + (2*e*f*x*Sqrt[
c + d*x^2])/((b*c + a*d)*Sqrt[a - b*x^2]) + (a*f^2*x*Sqrt[c + d*x^2])/(b*(
b*c + a*d)*Sqrt[a - b*x^2]) - (Sqrt[b]*e^2*Sqrt[1 - (b*x^2)/a]*Sqrt[c + d*
x^2]*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[a]*(b*c
 + a*d)*Sqrt[a - b*x^2]*Sqrt[1 + (d*x^2)/c]) - (2*Sqrt[a]*e*f*Sqrt[1 - (b*
x^2)/a]*Sqrt[c + d*x^2]*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*
c))])/(Sqrt[b]*(b*c + a*d)*Sqrt[a - b*x^2]*Sqrt[1 + (d*x^2)/c]) - (Sqrt[a]
*(b*c + 2*a*d)*f^2*Sqrt[1 - (b*x^2)/a]*Sqrt[c + d*x^2]*EllipticE[ArcSin[(S
qrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(b^(3/2)*d*(b*c + a*d)*Sqrt[a - b*x^2
]*Sqrt[1 + (d*x^2)/c]) + (e^2*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Elli
pticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[a]*Sqrt[b]*Sqrt[
a - b*x^2]*Sqrt[c + d*x^2]) + (Sqrt[a]*c*f^2*Sqrt[1 - (b*x^2)/a]*Sqrt[1 +
(d*x^2)/c]*EllipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(b^(3/2
)*d*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])
```

Defintions of rubi rules used

rule 433
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)
^q*(e + f*x^2)^r, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, f, p,
 q, r}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 6.21 (sec) , antiderivative size = 462, normalized size of antiderivative = 1.63

method	result
elliptic	$\frac{\sqrt{(-b x^2+a)(x^2 d+c)} \Bigg(-\frac{(-bd x^2-bc) x (a^2 f^2+2abfe+b^2e^2)}{b^2 a(ad+bc)\sqrt{(x^2-\frac{a}{b})(-bd x^2-bc)}} + \frac{\left(-\frac{f(af+2be)}{b^2}+\frac{a^2f^2+2abfe+b^2e^2}{b^2 a}-\frac{c(a^2f^2+2abfe+b^2e^2)}{ba(ad+bc)}\right)\sqrt{1-\frac{b x^2}{a}}\sqrt{1}}{\sqrt{\frac{b}{a}}\sqrt{-bd x^4+ad x^2-x^2bc+ac}}}{}$
default	$\frac{\Big(\sqrt{\frac{b}{a}}\, a^2 d^2 f^2 x^3+2\sqrt{\frac{b}{a}}\, ab d^2 ef x^3+\sqrt{\frac{b}{a}}\, b^2 d^2 e^2 x^3+\sqrt{\frac{-b x^2+a}{a}}\sqrt{\frac{x^2d+c}{c}}\, \mathrm{EllipticF}\left(x\sqrt{\frac{b}{a}},\sqrt{-\frac{ad}{bc}}\right) a^2 cd f^2+\sqrt{\frac{-b x^2+a}{a}}\sqrt{\frac{x^2d+c}{c}}\ \mathrm{E}}{}$

input
```
int((f*x^2+e)^2/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((-b*x^2+a)*(d*x^2+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-(-b*d*x^2-
b*c)/b^2/a/(a*d+b*c)*x*(a^2*f^2+2*a*b*e*f+b^2*e^2)/((x^2-a/b)*(-b*d*x^2-b*
c))^(1/2)+(-1/b^2*f*(a*f+2*b*e)+(a^2*f^2+2*a*b*e*f+b^2*e^2)/b^2/a-1/b*c/a/
(a*d+b*c)*(a^2*f^2+2*a*b*e*f+b^2*e^2))/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*
x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)*EllipticF(x*(b/a)^(1/2),
(-1-(a*d-b*c)/c/b)^(1/2))-(-f^2/b-d/b*(a^2*f^2+2*a*b*e*f+b^2*e^2)/(a*d+b*c
)/a)*c/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b
*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2))-Ell
ipticE(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 430, normalized size of antiderivative = 1.52

$$\int \frac{(e+fx^2)^2}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \frac{((ab^3de^2+2\,a^2b^2def+(a^2b^2c+2\,a^3bd)f^2)x^3-(a^2b^2de^2+2\,a^3bdef+(a^3b\cdots}{\cdots}$$

input
```
integrate((f*x^2+e)^2/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="frica
s")
```

output
```
(((a*b^3*d*e^2 + 2*a^2*b^2*d*e*f + (a^2*b^2*c + 2*a^3*b*d)*f^2)*x^3 - (a^2
*b^2*d*e^2 + 2*a^3*b*d*e*f + (a^3*b*c + 2*a^4*d)*f^2)*x)*sqrt(-b*d)*sqrt(a
/b)*elliptic_e(arcsin(sqrt(a/b)/x), -b*c/(a*d)) - (((a*b^3 - b^4)*d*e^2 +
2*(b^4*c + a^2*b^2*d)*e*f + (2*a^3*b*d + (a^2*b^2 + a*b^3)*c)*f^2)*x^3 - (
(a^2*b^2 - a*b^3)*d*e^2 + 2*(a*b^3*c + a^3*b*d)*e*f + (2*a^4*d + (a^3*b +
a^2*b^2)*c)*f^2)*x)*sqrt(-b*d)*sqrt(a/b)*elliptic_f(arcsin(sqrt(a/b)/x), -
b*c/(a*d)) - (a*b^3*d*e^2 + 2*a^2*b^2*d*e*f - (a*b^3*c + a^2*b^2*d)*f^2*x^
2 + (a^2*b^2*c + 2*a^3*b*d)*f^2)*sqrt(-b*x^2 + a)*sqrt(d*x^2 + c))/((a*b^5
*c*d + a^2*b^4*d^2)*x^3 - (a^2*b^4*c*d + a^3*b^3*d^2)*x)
```

Sympy [F]

$$\int \frac{(e+fx^2)^2}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \int \frac{(e+fx^2)^2}{(a-bx^2)^{\frac{3}{2}}\sqrt{c+dx^2}}\,dx$$

input
```
integrate((f*x**2+e)**2/(-b*x**2+a)**(3/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral((e + f*x**2)**2/((a - b*x**2)**(3/2)*sqrt(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{(e+fx^2)^2}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \int \frac{(fx^2+e)^2}{(-bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}}\,dx$$

input
```
integrate((f*x^2+e)^2/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="maxim
a")
```

output
```
integrate((f*x^2 + e)^2/((-b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)), x)
```

Giac [F]

$$\int \frac{(e+fx^2)^2}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \int \frac{(fx^2+e)^2}{(-bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}}\,dx$$

input
```
integrate((f*x^2+e)^2/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="giac"
)
```

output
```
integrate((f*x^2 + e)^2/((-b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx^2)^2}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \int \frac{(f\,x^2+e)^2}{(a-b\,x^2)^{3/2}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int((e + f*x^2)^2/((a - b*x^2)^(3/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int((e + f*x^2)^2/((a - b*x^2)^(3/2)*(c + d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{(e+fx^2)^2}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \frac{\sqrt{d\,x^2+c}\,\sqrt{-b\,x^2+a}\,efx + \left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{-b\,x^2+a}\,x^4}{b^2d\,x^6-2abd\,x^4+b^2c\,x^4+a^2d\,x^2-2abc\,x^2+a^2c}dx\right)a^2d}{}$$

input
```
int((f*x^2+e)^2/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x)
```

output
```
(sqrt(c + d*x**2)*sqrt(a - b*x**2)*e*f*x + int((sqrt(c + d*x**2)*sqrt(a -
b*x**2)*x**4)/(a**2*c + a**2*d*x**2 - 2*a*b*c*x**2 - 2*a*b*d*x**4 + b**2*c
*x**4 + b**2*d*x**6),x)*a**2*d*f**2 + int((sqrt(c + d*x**2)*sqrt(a - b*x**
2)*x**4)/(a**2*c + a**2*d*x**2 - 2*a*b*c*x**2 - 2*a*b*d*x**4 + b**2*c*x**4
 + b**2*d*x**6),x)*a*b*d*e*f - int((sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**4
)/(a**2*c + a**2*d*x**2 - 2*a*b*c*x**2 - 2*a*b*d*x**4 + b**2*c*x**4 + b**2
*d*x**6),x)*a*b*d*f**2*x**2 - int((sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**4)
/(a**2*c + a**2*d*x**2 - 2*a*b*c*x**2 - 2*a*b*d*x**4 + b**2*c*x**4 + b**2*
d*x**6),x)*b**2*d*e*f*x**2 - int((sqrt(c + d*x**2)*sqrt(a - b*x**2))/(a**2
*c + a**2*d*x**2 - 2*a*b*c*x**2 - 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6
),x)*a**2*c*e*f + int((sqrt(c + d*x**2)*sqrt(a - b*x**2))/(a**2*c + a**2*d
*x**2 - 2*a*b*c*x**2 - 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a**2*d
*e**2 + int((sqrt(c + d*x**2)*sqrt(a - b*x**2))/(a**2*c + a**2*d*x**2 - 2*
a*b*c*x**2 - 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a*b*c*e*f*x**2 -
 int((sqrt(c + d*x**2)*sqrt(a - b*x**2))/(a**2*c + a**2*d*x**2 - 2*a*b*c*x
**2 - 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a*b*d*e**2*x**2)/(a*d*(
a - b*x**2))
```

3.273 $\int \frac{(e+fx^2)^2}{x^2(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 334

$$\int \frac{(e+fx^2)^2}{x^2(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = -\frac{e^2\sqrt{c+dx^2}}{acx\sqrt{a-bx^2}} + \frac{(2b^2ce^2+a^2cf^2+abe(de+2cf))\,x\sqrt{c+dx^2}}{a^2c(bc+ad)\sqrt{a-bx^2}} - \frac{(2b^2ce^2+a^2cf^2+abe(de+2cf))\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\,E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{a^{3/2}\sqrt{b}c(bc+ad)\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}} + \frac{2e(be+af)\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{a^{3/2}\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}$$

output

```
-e^2*(d*x^2+c)^(1/2)/a/c/x/(-b*x^2+a)^(1/2)+(2*b^2*c*e^2+a^2*c*f^2+a*b*e*(
2*c*f+d*e))*x*(d*x^2+c)^(1/2)/a^2/c/(a*d+b*c)/(-b*x^2+a)^(1/2)-(2*b^2*c*e^
2+a^2*c*f^2+a*b*e*(2*c*f+d*e))*(1-b*x^2/a)^(1/2)*(d*x^2+c)^(1/2)*EllipticE
(b^(1/2)*x/a^(1/2),(-a*d/b/c)^(1/2))/a^(3/2)/b^(1/2)/c/(a*d+b*c)/(-b*x^2+a
)^(1/2)/(1+d*x^2/c)^(1/2)+2*e*(a*f+b*e)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2
)*EllipticF(b^(1/2)*x/a^(1/2),(-a*d/b/c)^(1/2))/a^(3/2)/b^(1/2)/(-b*x^2+a)
^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 9.54 (sec) , antiderivative size = 296, normalized size of antiderivative = 0.89

$$\int \frac{(e+fx^2)^2}{x^2(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \frac{\sqrt{-\frac{b}{a}}(c+dx^2)\left(2b^2ce^2x^2+abe(-ce+dex^2+2cfx^2)+a^2(-de^2+cf^2x^2\right.}{}$$

input
```
Integrate[(e + f*x^2)^2/(x^2*(a - b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

output
```
(Sqrt[-(b/a)]*(c + d*x^2)*(2*b^2*c*e^2*x^2 + a*b*e*(-(c*e) + d*e*x^2 + 2*c
*f*x^2) + a^2*(-(d*e^2) + c*f^2*x^2)) + I*c*(2*b^2*c*e^2 + a^2*c*f^2 + a*b
*e*(d*e + 2*c*f))*x*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*Ar
cSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))] - (2*I)*c*(b*c + a*d)*e*(b*e + a*f)
*x*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[-(b/a)
]*x], -((a*d)/(b*c))])/(a^2*Sqrt[-(b/a)]*c*(b*c + a*d)*x*Sqrt[a - b*x^2]*S
qrt[c + d*x^2])
```

Rubi [A] (warning: unable to verify)

Time = 1.29 (sec) , antiderivative size = 586, normalized size of antiderivative = 1.75, number of steps used = 20, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {448, 402, 399, 323, 323, 321, 331, 330, 327, 441, 445, 25, 27, 399, 323, 323, 321, 331, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx^2)^2}{x^2(a-bx^2)^{3/2}\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \text{448}$$

$$\frac{f\int \frac{fx^2+e}{(a-bx^2)^{3/2}\sqrt{dx^2+c}}dx}{e^2}+e\int \frac{fx^2+e}{x^2(a-bx^2)^{3/2}\sqrt{dx^2+c}}dx$$

$$\Big\downarrow \; 402$$

$$\frac{f\left(\frac{\int \frac{a(de-cf)-d(be+af)x^2}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{a(ad+bc)}+\frac{x\sqrt{c+dx^2}(af+be)}{a\sqrt{a-bx^2}(ad+bc)}\right)}{e^2}+e\int \frac{fx^2+e}{x^2\left(a-bx^2\right)^{3/2}\sqrt{dx^2+c}}dx$$

$$\Big\downarrow \; 399$$

$$\frac{f\left(\frac{e(ad+bc)\int \frac{1}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx-(af+be)\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{a(ad+bc)}+\frac{x\sqrt{c+dx^2}(af+be)}{a\sqrt{a-bx^2}(ad+bc)}\right)}{e^2}+e\int \frac{fx^2+e}{x^2\left(a-bx^2\right)^{3/2}\sqrt{dx^2+c}}dx$$

$$\Big\downarrow \; 323$$

$$\frac{f\left(\frac{\frac{e\sqrt{\frac{dx^2}{c}+1}(ad+bc)\int \frac{1}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}dx}{\sqrt{c+dx^2}}-(af+be)\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{a(ad+bc)}+\frac{x\sqrt{c+dx^2}(af+be)}{a\sqrt{a-bx^2}(ad+bc)}\right)}{e^2}+e\int \frac{fx^2+e}{x^2\left(a-bx^2\right)^{3/2}\sqrt{dx^2+c}}dx$$

$$\Big\downarrow \; 323$$

$$\frac{f\left(\frac{\frac{e\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\int \frac{1}{\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}}dx}{\sqrt{a-bx^2}\sqrt{c+dx^2}}-(af+be)\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{a(ad+bc)}+\frac{x\sqrt{c+dx^2}(af+be)}{a\sqrt{a-bx^2}(ad+bc)}\right)}{e^2}+e\int \frac{fx^2+e}{x^2\left(a-bx^2\right)^{3/2}\sqrt{dx^2+c}}dx$$

$$\Big\downarrow \; 321$$

$$\frac{f\left(\frac{\frac{\sqrt{a}e\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-(af+be)\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{a(ad+bc)}+\frac{x\sqrt{c+dx^2}(af+be)}{a\sqrt{a-bx^2}(ad+bc)}\right)}{e^2}+e\int \frac{fx^2+e}{x^2\left(a-bx^2\right)^{3/2}\sqrt{dx^2+c}}dx$$

$$\Big\downarrow \; 331$$

$$\frac{f\left(\frac{\frac{\sqrt{a}e\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{1-\frac{bx^2}{a}}(af+be)\int\frac{\sqrt{dx^2+c}}{\sqrt{1-\frac{bx^2}{a}}}dx}{\sqrt{a-bx^2}}}{a(ad+bc)}+\frac{x\sqrt{c+dx^2}(af+be)}{a\sqrt{a-bx^2}(ad+bc)}\right)}{e^2}+e\int\frac{fx^2+e}{x^2\left(a-bx^2\right)^{3/2}\sqrt{dx^2+c}}dx$$

$$\Big\downarrow \; 330$$

$$\frac{f\left(\frac{\frac{\sqrt{a}e\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(af+be)\int\frac{\sqrt{\frac{dx^2}{c}+1}}{\sqrt{1-\frac{bx^2}{a}}}dx}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}}{a(ad+bc)}+\frac{x\sqrt{c+dx^2}(af+be)}{a\sqrt{a-bx^2}(ad+bc)}\right)}{e^2}+e\int\frac{fx^2+e}{x^2\left(a-bx^2\right)^{3/2}\sqrt{dx^2+c}}dx$$

$$\Big\downarrow \; 327$$

$$e\int\frac{fx^2+e}{x^2\left(a-bx^2\right)^{3/2}\sqrt{dx^2+c}}dx+\frac{f\left(\frac{\frac{\sqrt{a}e\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(af+be)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}}{a(ad+bc)}+\frac{x\sqrt{c+dx^2}(af+be)}{a\sqrt{a-bx^2}(ad+bc)}\right)}{e^2}$$

$$\Big\downarrow \; 441$$

$$e\left(\frac{\int\frac{d(be+af)x^2+2bce+ade+acf}{x^2\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{a(ad+bc)}+\frac{\sqrt{c+dx^2}(af+be)}{ax\sqrt{a-bx^2}(ad+bc)}\right)+\frac{f\left(\frac{\frac{\sqrt{a}e\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(af+be)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}}{a(ad+bc)}+\frac{x\sqrt{c+dx^2}(af+be)}{a\sqrt{a-bx^2}(ad+bc)}\right)}{e^2}$$

$$\Big\downarrow \; 445$$

$$\frac{e\left(\frac{-\frac{\int -\frac{d\left(ac(be+af)-b(2bce+ade+acf)x^2\right)}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}(acf+ade+2bce)}{acx}}{a(ad+bc)}+\frac{\sqrt{c+dx^2}(af+be)}{ax\sqrt{a-bx^2}(ad+bc)}\right)+f\left(\frac{\frac{\sqrt{a}e\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(af+be)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}}{a(ad+bc)}+\frac{x\sqrt{c+dx^2}(af+be)}{a\sqrt{a-bx^2}(ad+bc)}\right)}{e^2}$$

$\Big\downarrow$ 25

$$\frac{e\left(\frac{\frac{\int \frac{d\left(ac(be+af)-b(2bce+ade+acf)x^2\right)}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}(acf+ade+2bce)}{acx}}{a(ad+bc)}+\frac{\sqrt{c+dx^2}(af+be)}{ax\sqrt{a-bx^2}(ad+bc)}\right)+f\left(\frac{\frac{\sqrt{a}e\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(af+be)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}}{a(ad+bc)}+\frac{x\sqrt{c+dx^2}(af+be)}{a\sqrt{a-bx^2}(ad+bc)}\right)}{e^2}$$

$\Big\downarrow$ 27

$$\frac{e\left(\frac{\frac{d\int \frac{ac(be+af)-b(2bce+ade+acf)x^2}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}(acf+ade+2bce)}{acx}}{a(ad+bc)}+\frac{\sqrt{c+dx^2}(af+be)}{ax\sqrt{a-bx^2}(ad+bc)}\right)+f\left(\frac{\frac{\sqrt{a}e\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(af+be)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}}{a(ad+bc)}+\frac{x\sqrt{c+dx^2}(af+be)}{a\sqrt{a-bx^2}(ad+bc)}\right)}{e^2}$$

$\Big\downarrow$ 399

$$\frac{e\left(\frac{\frac{d\left(\frac{c(ad+bc)(af+2be)\int \frac{1}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{d}-\frac{b(acf+ade+2bce)\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}\right)}{ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}(acf+ade+2bce)}{acx}}{a(ad+bc)}+\frac{\sqrt{c+dx^2}(af+b}{ax\sqrt{a-bx^2}(ad+}\right)+f\left(\frac{\frac{\sqrt{a}e\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(af+be)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}}{a(ad+bc)}+\frac{x\sqrt{c+dx^2}(af+be)}{a\sqrt{a-bx^2}(ad+bc)}\right)}{e^2}$$

$\downarrow$ 323

$$\frac{e\left(\frac{\frac{d\left(\frac{c\sqrt{\frac{dx^2}{c}+1}(ad+bc)(af+2be)\int \frac{1}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}dx}{d\sqrt{c+dx^2}}-\frac{b(acf+ade+2bce)\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}\right)}{ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}(acf+ade+2bce)}{acx}}{a(ad+bc)}+\frac{\sqrt{c+dx^2}}{ax\sqrt{a-bx}}\right)+f\left(\frac{\frac{\sqrt{a}e\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(af+be)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}}{a(ad+bc)}+\frac{x\sqrt{c+dx^2}(af+be)}{a\sqrt{a-bx^2}(ad+bc)}\right)}{e^2}$$

$\downarrow$ 323

$$\frac{e\left(\frac{\frac{d\left(\frac{c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(af+2be)\int\frac{1}{\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}}dx}{d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{b(acf+ade+2bce)\int\frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}\right)}{ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}(acf+ade+2bce)}{acx}}{a(ad+bc)}+\frac{\sqrt{c\cdots}}{ax\sqrt{\cdots}}\right)+f\left(\frac{\frac{\sqrt{a}e\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(af+be)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}}{a(ad+bc)}+\frac{x\sqrt{c+dx^2}(af+be)}{a\sqrt{a-bx^2}(ad+bc)}\right)}{e^2}$$

$\downarrow$ 321

$$\frac{e\left(\frac{\frac{d\left(\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(af+2be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{b(acf+ade+2bce)\int\frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}\right)}{ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}(acf+ade+2bce)}{acx}\cdots}{a(ad+bc)}\cdots\right)+f\left(\frac{\frac{\sqrt{a}e\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(af+be)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}}{a(ad+bc)}+\frac{x\sqrt{c+dx^2}(af+be)}{a\sqrt{a-bx^2}(ad+bc)}\right)}{e^2}$$

$\downarrow$ 331

$$\frac{e\left(\frac{\frac{d\left(\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(af+2be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{b\sqrt{1-\frac{bx^2}{a}}(acf+ade+2bce)\int\frac{\sqrt{dx^2+c}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}\right)}{ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}(acf+a}{acx}}{a(ad+bc)}\right.}{}$$

$$\frac{f\left(\frac{\frac{\sqrt{a}e\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(af+be)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}}{a(ad+bc)}+\frac{x\sqrt{c+dx^2}(af+be)}{a\sqrt{a-bx^2}(ad+bc)}\right)}{e^2}$$

$\downarrow$ 330

$$\frac{e\left(\frac{\frac{d\left(\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(af+2be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{b\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(acf+ade+2bce)\int\frac{\sqrt{\frac{dx^2}{c}+1}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}\right)}{ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}}{ac}}{a(ad+bc)}\right.}{}$$

$$\frac{f\left(\frac{\frac{\sqrt{a}e\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(af+be)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}}{a(ad+bc)}+\frac{x\sqrt{c+dx^2}(af+be)}{a\sqrt{a-bx^2}(ad+bc)}\right)}{e^2}$$

$\downarrow$ 327

$$\frac{f\left(\frac{\frac{\sqrt{a}e\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(af+be)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}}{a(ad+bc)}+\frac{x\sqrt{c+dx^2}(af+be)}{a\sqrt{a-bx^2}(ad+bc)}\right)}{e^2}+$$

$$e\left(\frac{\frac{d\left(\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(af+2be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(acf+ade+2bce)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}\right)}{ac}-\sqrt{a-\cdots}}{a(ad+bc)}\right.$$

input
```
Int[(e + f*x^2)^2/(x^2*(a - b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

output
```
(f*(((b*e + a*f)*x*Sqrt[c + d*x^2])/(a*(b*c + a*d)*Sqrt[a - b*x^2]) + (-((
Sqrt[a]*(b*e + a*f)*Sqrt[1 - (b*x^2)/a]*Sqrt[c + d*x^2]*EllipticE[ArcSin[(
Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*Sqrt[a - b*x^2]*Sqrt[1 + (d
*x^2)/c])) + (Sqrt[a]*(b*c + a*d)*e*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c
]*EllipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*Sqrt[a
- b*x^2]*Sqrt[c + d*x^2]))/(a*(b*c + a*d))))/e^2 + e*(((b*e + a*f)*Sqrt[c
+ d*x^2])/(a*(b*c + a*d)*x*Sqrt[a - b*x^2]) + (-(((2*b*c*e + a*d*e + a*c*f
)*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])/(a*c*x)) + (d*(-((Sqrt[a]*Sqrt[b]*(2*b*
c*e + a*d*e + a*c*f)*Sqrt[1 - (b*x^2)/a]*Sqrt[c + d*x^2]*EllipticE[ArcSin[
(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(d*Sqrt[a - b*x^2]*Sqrt[1 + (d*x^2)
/c])) + (Sqrt[a]*c*(b*c + a*d)*(2*b*e + a*f)*Sqrt[1 - (b*x^2)/a]*Sqrt[1 +
(d*x^2)/c]*EllipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b
]*d*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])))/(a*c))/(a*(b*c + a*d)))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 441
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
+ b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(a*g*2*(b*c - a*d)*(p + 1))), x] + Si
mp[1/(a*2*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2
)^q*Simp[c*(b*e - a*f)*(m + 1) + e*2*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(m
 + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, q},
 x] && LtQ[p, -1]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 448
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[e   Int[(g*x)^m*(a + b*x
^2)^p*(c + d*x^2)^q*(e + f*x^2)^(r - 1), x], x] + Simp[f/e^2   Int[(g*x)^(m
 + 2)*(a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^(r - 1), x], x] /; FreeQ[{a,
b, c, d, e, f, g, m, p, q}, x] && IGtQ[r, 0]
```

Maple [A] (verified)

Time = 11.02 (sec) , antiderivative size = 495, normalized size of antiderivative = 1.48

method	result
elliptic	$\frac{\sqrt{(-b x^2+a)(x^2 d+c)}\left(-\frac{(-bd x^2-bc)x(a^2f^2+2abfe+b^2e^2)}{b a^2(ad+bc)\sqrt{(x^2-\frac{a}{b})(-bd x^2-bc)}}-\frac{e^2\sqrt{-bd x^4+ad x^2-x^2bc+ac}}{a^2cx}+\frac{\left(-\frac{f^2}{b}+\frac{a^2f^2+2abfe+b^2e^2}{b a^2}-\frac{c(a^2f^2+2abf}{a^2(ad+b}\right.}{\sqrt{\frac{b}{a}}\sqrt{}}\right.}{}$
risch	$-\frac{e^2\sqrt{-b x^2+a}\sqrt{x^2d+c}}{a^2cx}-\frac{\left(\frac{a^2cf^2\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{b}{a}},\sqrt{-1-\frac{ad-bc}{cb}}\right)}{b\sqrt{\frac{b}{a}}\sqrt{-bd x^4+ad x^2-x^2bc+ac}}-\frac{b e^2c\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\left(\mathrm{EllipticF}\left(x\sqrt{\frac{b}{a}},\sqrt{-1}\right.\right.}{\sqrt{\frac{b}{a}}\sqrt{-bd x^4+a}}\right.}{}$
default	$\frac{\left(\sqrt{\frac{b}{a}}\,a^2cd f^2x^4+2\sqrt{\frac{b}{a}}\,abcdef x^4+\sqrt{\frac{b}{a}}\,ab d^2e^2x^4+2\sqrt{\frac{b}{a}}\,b^2cd e^2x^4+2\sqrt{\frac{-b x^2+a}{a}}\sqrt{\frac{x^2d+c}{c}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{b}{a}},\sqrt{-\frac{ad}{bc}}\right)a^2cdefx+2\sqrt{\right.}{}$

input
```
int((f*x^2+e)^2/x^2/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBO
SE)
```

output
```
((-b*x^2+a)*(d*x^2+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-(-b*d*x^2-
b*c)/b/a^2/(a*d+b*c)*x*(a^2*f^2+2*a*b*e*f+b^2*e^2)/((x^2-a/b)*(-b*d*x^2-b*
c))^(1/2)-e^2/a^2/c*(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)/x+(-f^2/b+1/b*(a^
2*f^2+2*a*b*e*f+b^2*e^2)/a^2-c/a^2/(a*d+b*c)*(a^2*f^2+2*a*b*e*f+b^2*e^2))/
(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+
a*c)^(1/2)*EllipticF(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2))-(-(a^2*f^2+2*
a*b*e*f+b^2*e^2)*d/(a*d+b*c)/a^2-b*d*e^2/a^2/c)*c/(b/a)^(1/2)*(1-b*x^2/a)^
(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)/d*(EllipticF(
x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2))-EllipticE(x*(b/a)^(1/2),(-1-(a*d-b
*c)/c/b)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 427, normalized size of antiderivative = 1.28

$$\int \frac{(e+fx^2)^2}{x^2(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx =$$

$$-\frac{((2\,ab^3cef+a^2b^2cf^2+(2\,b^4c+ab^3d)e^2)x^3-(2\,a^2b^2cef+a^3bcf^2+(2\,ab^3c+a^2b^2d)e^2)x)\sqrt{ac}\sqrt{\frac{b}{a}}E(\mathrm{arcs}}{}$$

input
```
integrate((f*x^2+e)^2/x^2/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="f
ricas")
```

output
```
-(((2*a*b^3*c*e*f + a^2*b^2*c*f^2 + (2*b^4*c + a*b^3*d)*e^2)*x^3 - (2*a^2*
b^2*c*e*f + a^3*b*c*f^2 + (2*a*b^3*c + a^2*b^2*d)*e^2)*x)*sqrt(a*c)*sqrt(b
/a)*elliptic_e(arcsin(x*sqrt(b/a)), -a*d/(b*c)) + (((a^3*b - a^2*b^2)*c*f^
2 - (2*b^4*c + (a^2*b^2 + a*b^3)*d)*e^2 - 2*(a*b^3*c + a^3*b*d)*e*f)*x^3 -
 ((a^4 - a^3*b)*c*f^2 - (2*a*b^3*c + (a^3*b + a^2*b^2)*d)*e^2 - 2*(a^2*b^2
*c + a^4*d)*e*f)*x)*sqrt(a*c)*sqrt(b/a)*elliptic_f(arcsin(x*sqrt(b/a)), -a
*d/(b*c)) - ((a^2*b^2*c + a^3*b*d)*e^2 - (2*a^2*b^2*c*e*f + a^3*b*c*f^2 +
(2*a*b^3*c + a^2*b^2*d)*e^2)*x^2)*sqrt(-b*x^2 + a)*sqrt(d*x^2 + c))/((a^3*
b^3*c^2 + a^4*b^2*c*d)*x^3 - (a^4*b^2*c^2 + a^5*b*c*d)*x)
```

Sympy [F]

$$\int \frac{(e+fx^2)^2}{x^2(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \int \frac{(e+fx^2)^2}{x^2(a-bx^2)^{\frac{3}{2}}\sqrt{c+dx^2}}\,dx$$

input
```
integrate((f*x**2+e)**2/x**2/(-b*x**2+a)**(3/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral((e + f*x**2)**2/(x**2*(a - b*x**2)**(3/2)*sqrt(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{(e+fx^2)^2}{x^2(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \int \frac{(fx^2+e)^2}{(-bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}x^2}\,dx$$

input
```
integrate((f*x^2+e)^2/x^2/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="m
axima")
```

output
```
integrate((f*x^2 + e)^2/((-b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*x^2), x)
```

Giac [F]

$$\int \frac{(e+fx^2)^2}{x^2(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx=\int \frac{(fx^2+e)^2}{(-bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}x^2}\,dx$$

input
```
integrate((f*x^2+e)^2/x^2/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="g
iac")
```

output
```
integrate((f*x^2 + e)^2/((-b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx^2)^2}{x^2(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx=\int \frac{(f\,x^2+e)^2}{x^2\,(a-b\,x^2)^{3/2}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int((e + f*x^2)^2/(x^2*(a - b*x^2)^(3/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int((e + f*x^2)^2/(x^2*(a - b*x^2)^(3/2)*(c + d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{(e+fx^2)^2}{x^2(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx=\frac{-\sqrt{d\,x^2+c}\,\sqrt{-b\,x^2+a}\,e^2+\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{-b\,x^2+a}\,x^2}{b^2d\,x^6-2abd\,x^4+b^2c\,x^4+a^2d\,x^2-2abc\,x^2+a^2c}dx\right)a}{}$$

input
```
int((f*x^2+e)^2/x^2/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x)
```

output
```
( - sqrt(c + d*x**2)*sqrt(a - b*x**2)*e**2 + int((sqrt(c + d*x**2)*sqrt(a
- b*x**2)*x**2)/(a**2*c + a**2*d*x**2 - 2*a*b*c*x**2 - 2*a*b*d*x**4 + b**2
*c*x**4 + b**2*d*x**6),x)*a**2*c*f**2*x - int((sqrt(c + d*x**2)*sqrt(a - b
*x**2)*x**2)/(a**2*c + a**2*d*x**2 - 2*a*b*c*x**2 - 2*a*b*d*x**4 + b**2*c*
x**4 + b**2*d*x**6),x)*a*b*c*f**2*x**3 + int((sqrt(c + d*x**2)*sqrt(a - b*
x**2)*x**2)/(a**2*c + a**2*d*x**2 - 2*a*b*c*x**2 - 2*a*b*d*x**4 + b**2*c*x
**4 + b**2*d*x**6),x)*a*b*d*e**2*x - int((sqrt(c + d*x**2)*sqrt(a - b*x**2
)*x**2)/(a**2*c + a**2*d*x**2 - 2*a*b*c*x**2 - 2*a*b*d*x**4 + b**2*c*x**4
+ b**2*d*x**6),x)*b**2*d*e**2*x**3 + 2*int((sqrt(c + d*x**2)*sqrt(a - b*x*
*2))/(a**2*c + a**2*d*x**2 - 2*a*b*c*x**2 - 2*a*b*d*x**4 + b**2*c*x**4 + b
**2*d*x**6),x)*a**2*c*e*f*x + 2*int((sqrt(c + d*x**2)*sqrt(a - b*x**2))/(a
**2*c + a**2*d*x**2 - 2*a*b*c*x**2 - 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x
**6),x)*a*b*c*e**2*x - 2*int((sqrt(c + d*x**2)*sqrt(a - b*x**2))/(a**2*c +
 a**2*d*x**2 - 2*a*b*c*x**2 - 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)
*a*b*c*e*f*x**3 - 2*int((sqrt(c + d*x**2)*sqrt(a - b*x**2))/(a**2*c + a**2
*d*x**2 - 2*a*b*c*x**2 - 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*b**2
*c*e**2*x**3)/(a*c*x*(a - b*x**2))
```

3.274 $\int \frac{(e+fx^2)^2}{x^4(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 472

$$\int \frac{(e+fx^2)^2}{x^4(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = -\frac{e^2\sqrt{c+dx^2}}{3acx^3\sqrt{a-bx^2}} - \frac{2e(2bce-ade+3acf)\sqrt{c+dx^2}}{3a^2c^2x\sqrt{a-bx^2}}$$
$$+ \frac{b(8b^2c^2e^2+3abce(de+4cf)-a^2(2d^2e^2-6cdef-3c^2f^2))\,x\sqrt{c+dx^2}}{3a^3c^2(bc+ad)\sqrt{a-bx^2}}$$
$$- \frac{\sqrt{b}(8b^2c^2e^2+3abce(de+4cf)-a^2(2d^2e^2-6cdef-3c^2f^2))\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\,E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|-\frac{ad}{bc}\right)}{3a^{5/2}c^2(bc+ad)\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}}$$
$$+ \frac{(8b^2ce^2+3a^2cf^2-abe(de-12cf))\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{3a^{5/2}\sqrt{b}c\sqrt{a-bx^2}\sqrt{c+dx^2}}$$

output
```
-1/3*e^2*(d*x^2+c)^(1/2)/a/c/x^3/(-b*x^2+a)^(1/2)-2/3*e*(3*a*c*f-a*d*e+2*b
*c*e)*(d*x^2+c)^(1/2)/a^2/c^2/x/(-b*x^2+a)^(1/2)+1/3*b*(8*b^2*c^2*e^2+3*a*
b*c*e*(4*c*f+d*e)-a^2*(-3*c^2*f^2-6*c*d*e*f+2*d^2*e^2))*x*(d*x^2+c)^(1/2)/
a^3/c^2/(a*d+b*c)/(-b*x^2+a)^(1/2)-1/3*b^(1/2)*(8*b^2*c^2*e^2+3*a*b*c*e*(4
*c*f+d*e)-a^2*(-3*c^2*f^2-6*c*d*e*f+2*d^2*e^2))*(1-b*x^2/a)^(1/2)*(d*x^2+c
)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2),(-a*d/b/c)^(1/2))/a^(5/2)/c^2/(a*d+b*c
)/(-b*x^2+a)^(1/2)/(1+d*x^2/c)^(1/2)+1/3*(8*b^2*c*e^2+3*a^2*c*f^2-a*b*e*(-
12*c*f+d*e))*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(b^(1/2)*x/a^(1/
2),(-a*d/b/c)^(1/2))/a^(5/2)/b^(1/2)/c/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.70 (sec) , antiderivative size = 422, normalized size of antiderivative = 0.89

$$\int \frac{(e+fx^2)^2}{x^4 (a-bx^2)^{3/2} \sqrt{c+dx^2}} \, dx = \frac{-\sqrt{-\frac{b}{a}}(c+dx^2)\left(-8b^3c^2e^2x^4+ab^2cex^2(-3dex^2+4c(e-3fx^2))+a^3de\cdots\right.}{\cdots}$$

input
```
Integrate[(e + f*x^2)^2/(x^4*(a - b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

output
```
(-(Sqrt[-(b/a)]*(c + d*x^2)*(-8*b^3*c^2*e^2*x^4 + a*b^2*c*e*x^2*(-3*d*e*x^
2 + 4*c*(e - 3*f*x^2)) + a^3*d*e*(-2*d*e*x^2 + c*(e + 6*f*x^2)) + a^2*b*(2
*d^2*e^2*x^4 + 2*c*d*e*x^2*(e - 3*f*x^2) + c^2*(e^2 + 6*e*f*x^2 - 3*f^2*x^
4)))) + I*b*c*(8*b^2*c^2*e^2 + 3*a*b*c*e*(d*e + 4*c*f) + a^2*(-2*d^2*e^2 +
 6*c*d*e*f + 3*c^2*f^2))*x^3*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Ellip
ticE[I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))] - I*c*(b*c + a*d)*(8*b^2*c
*e^2 + 3*a^2*c*f^2 + a*b*e*(-(d*e) + 12*c*f))*x^3*Sqrt[1 - (b*x^2)/a]*Sqrt
[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))])/(3*a
^3*Sqrt[-(b/a)]*c^2*(b*c + a*d)*x^3*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (warning: unable to verify)

Time = 1.78 (sec) , antiderivative size = 777, normalized size of antiderivative = 1.65, number of steps used = 22, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.611, Rules used = {448, 441, 445, 25, 27, 399, 323, 323, 321, 331, 330, 327, 445, 25, 27, 399, 323, 323, 321, 331, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(e+fx^2\right)^2}{x^4 \left(a-bx^2\right)^{3/2} \sqrt{c+dx^2}}dx$$

$$\Big\downarrow \; 448$$

$$\frac{f\int \frac{fx^2+e}{x^2\left(a-bx^2\right)^{3/2}\sqrt{dx^2+c}}dx}{e^2}+e\int \frac{fx^2+e}{x^4\left(a-bx^2\right)^{3/2}\sqrt{dx^2+c}}dx$$

$$\Big\downarrow \; 441$$

$$\frac{f\left(\frac{\int \frac{d(be+af)x^2+2bce+ade+acf}{x^2\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{a(ad+bc)}+\frac{\sqrt{c+dx^2}(af+be)}{ax\sqrt{a-bx^2}(ad+bc)}\right)}{e^2}+e\left(\frac{\int \frac{3d(be+af)x^2+4bce+ade+3acf}{x^4\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{a(ad+bc)}+\frac{\sqrt{c+dx^2}(af+be)}{ax^3\sqrt{a-bx^2}(ad+bc)}\right)$$

$$\Big\downarrow \; 445$$

$$e\left(\frac{-\frac{\int -\frac{-d(2de-3cf)a^2+3bc(de+2cf)a+bd(4bce+ade+3acf)x^2+8b^2c^2e}{x^2\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{3ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}(3acf+ade+4bce)}{3acx^3}}{a(ad+bc)}+\frac{\sqrt{c+dx^2}(af+be)}{ax^3\sqrt{a-bx^2}(ad+bc)}\right.$$
$$\frac{f\left(\frac{-\frac{\int -\frac{d\left(ac(be+af)-b(2bce+ade+acf)x^2\right)}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}(acf+ade+2bce)}{acx}}{a(ad+bc)}+\frac{\sqrt{c+dx^2}(af+be)}{ax\sqrt{a-bx^2}(ad+bc)}\right)}{e^2}$$

$$\Big\downarrow \; 25$$

$$\frac{e\left(\frac{\frac{\int \frac{-d(2de-3cf)a^2+3bc(de+2cf)a+bd(4bce+ade+3acf)x^2+8b^2c^2e}{x^2\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{3ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}(3acf+ade+4bce)}{3acx^3}}{a(ad+bc)}+\frac{\sqrt{c+dx^2}(af+be)}{ax^3\sqrt{a-bx^2}(ad+bc)}\right)-f\left(\frac{\frac{\int \frac{d\left(ac(be+af)-b(2bce+ade+acf)x^2\right)}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}(acf+ade+2bce)}{acx}}{a(ad+bc)}+\frac{\sqrt{c+dx^2}(af+be)}{ax\sqrt{a-bx^2}(ad+bc)}\right)}{e^2}$$

$\downarrow$ 27

$$\frac{e\left(\frac{\frac{\int \frac{-d(2de-3cf)a^2+3bc(de+2cf)a+bd(4bce+ade+3acf)x^2+8b^2c^2e}{x^2\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{3ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}(3acf+ade+4bce)}{3acx^3}}{a(ad+bc)}+\frac{\sqrt{c+dx^2}(af+be)}{ax^3\sqrt{a-bx^2}(ad+bc)}\right)-f\left(\frac{\frac{d\int \frac{ac(be+af)-b(2bce+ade+acf)x^2}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}(acf+ade+2bce)}{acx}}{a(ad+bc)}+\frac{\sqrt{c+dx^2}(af+be)}{ax\sqrt{a-bx^2}(ad+bc)}\right)}{e^2}$$

$\downarrow$ 399

$$\frac{e\left(\frac{\frac{\int \frac{-d(2de-3cf)a^2+3bc(de+2cf)a+bd(4bce+ade+3acf)x^2+8b^2c^2e}{x^2\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{3ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}(3acf+ade+4bce)}{3acx^3}}{a(ad+bc)}+\frac{\sqrt{c+dx^2}(af+be)}{ax^3\sqrt{a-bx^2}(ad+bc)}\right)-f\left(\frac{\frac{d\left(\frac{c(ad+bc)(af+2be)\int \frac{1}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{d}-\frac{b(acf+ade+2bce)\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}\right)}{ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}(acf+ade+2bce)}{acx}}{a(ad+bc)}+\frac{\sqrt{c+dx^2}(af+be)}{ax\sqrt{a-bx^2}(ad+bc)}\right)}{e^2}$$

$\downarrow$ 323

$$\frac{e\left(\frac{\frac{\int \frac{-d(2de-3cf)a^2+3bc(de+2cf)a+bd(4bce+ade+3acf)x^2+8b^2c^2e}{x^2\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{3ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}(3acf+ade+4bce)}{3acx^3}}{a(ad+bc)}+\frac{\sqrt{c+dx^2}(af+be)}{ax^3\sqrt{a-bx^2}(ad+bc)}\right)- \qquad f\left(\frac{\frac{d\left(\frac{c\sqrt{\frac{dx^2}{c}+1}(ad+bc)(af+2be)\int \frac{1}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}dx}{d\sqrt{c+dx^2}}-\frac{b(acf+ade+2bce)\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}\right)}{ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}(acf+ade+2bce)}{acx}}{a(ad+bc)}+\frac{\sqrt{c+dx^2}(af+be)}{ax\sqrt{a-bx^2}(ad+bc)}\right.}{e^2}$$

$\downarrow$ 323

$$\frac{e\left(\frac{\frac{\int \frac{-d(2de-3cf)a^2+3bc(de+2cf)a+bd(4bce+ade+3acf)x^2+8b^2c^2e}{x^2\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{3ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}(3acf+ade+4bce)}{3acx^3}}{a(ad+bc)}+\frac{\sqrt{c+dx^2}(af+be)}{ax^3\sqrt{a-bx^2}(ad+bc)}\right)- \qquad f\left(\frac{\frac{d\left(\frac{c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(af+2be)\int \frac{1}{\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}}dx}{d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{b(acf+ade+2bce)\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}\right)}{ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}(acf+ade+2bce)}{acx}}{a(ad+bc)}+\frac{\sqrt{c+dx^2}(\ldots}{ax\sqrt{a-bx^2}\ldots}\right.}{e^2}$$

$\downarrow$ 321

$$\frac{e\left(\frac{\frac{\int \frac{-d(2de-3cf)a^2+3bc(de+2cf)a+bd(4bce+ade+3acf)x^2+8b^2c^2e}{x^2\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{3ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}(3acf+ade+4bce)}{3acx^3}}{a(ad+bc)}+\frac{\sqrt{c+dx^2}(af+be)}{ax^3\sqrt{a-bx^2}(ad+bc)}\right)- \qquad f\left(\frac{\frac{d\left(\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(af+2be)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{b(acf+ade+2bce)\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}\right)}{ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}(acf+ade+2bce)}{acx}}{a(ad+bc)}+\ldots\right.}{e^2}$$

$\downarrow$ 331

$$\frac{e\left(\frac{\frac{\int \frac{-d(2de-3cf)a^2+3bc(de+2cf)a+bd(4bce+ade+3acf)x^2+8b^2c^2e}{x^2\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{3ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}(3acf+ade+4bce)}{3acx^3}}{a(ad+bc)}+\frac{\sqrt{c+dx^2}(af+be)}{ax^3\sqrt{a-bx^2}(ad+bc)}\right)-\cdots}{}$$
$$\frac{f\left(\frac{\frac{d\left(\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(af+2be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{b\sqrt{1-\frac{bx^2}{a}}(acf+ade+2bce)\int\frac{\sqrt{dx^2+c}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}\right)}{ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}(acf+ade+2\cdots}{acx}}{a(ad+bc)}\cdots\right.}{e^2}$$

$\downarrow$ 330

$$\frac{e\left(\frac{\frac{\int \frac{-d(2de-3cf)a^2+3bc(de+2cf)a+bd(4bce+ade+3acf)x^2+8b^2c^2e}{x^2\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{3ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}(3acf+ade+4bce)}{3acx^3}}{a(ad+bc)}+\frac{\sqrt{c+dx^2}(af+be)}{ax^3\sqrt{a-bx^2}(ad+bc)}\right)-\cdots}{}$$
$$\frac{f\left(\frac{\frac{d\left(\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(af+2be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{b\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(acf+ade+2bce)\int\frac{\sqrt{\frac{dx^2}{c}+1}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}\right)}{ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}(acf\cdots}{acx}}{a(ad+bc)}\cdots\right.}{e^2}$$

$\downarrow$ 327

$$\frac{e\left(\frac{\frac{\int \frac{-d(2de-3cf)a^2+3bc(de+2cf)a+bd(4bce+ade+3acf)x^2+8b^2c^2e}{x^2\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{3ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}(3acf+ade+4bce)}{3acx^3}}{a(ad+bc)}+\frac{\sqrt{c+dx^2}(af+be)}{ax^3\sqrt{a-bx^2}(ad+bc)}\right)-\cdots}{}$$
$$\frac{f\left(\frac{\frac{d\left(\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(af+2be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(acf+ade+2bce)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}\right)}{ac}-\frac{\sqrt{a-b\cdots}}{}}{a(ad+bc)}\cdots\right.}{e^2}$$

$$\Big\downarrow \; 445$$

$$\frac{e\left(\frac{-\frac{\int -\frac{bd\left(ac(4bce+ade+3acf)-\left(-d(2de-3cf)a^2+3bc(de+2cf)a+8b^2c^2e\right)x^2\right)}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}\left(\frac{8b^2ce}{a}-\frac{2ad^2e}{c}+3adf+6bcf+3bde\right)}{x}}{3ac}}{a(ad+bc)}-\frac{\sqrt{a-bx^2}\sqrt{c\ldots}}{\ldots}\right)}{e^2}$$

$$f\left(\frac{\frac{d\left(\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(af+2be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(acf+ade+2bce)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}\right)}{ac}}{a(ad+bc)}-\frac{\sqrt{a-bx\ldots}}{\ldots}\right)$$

$$\Big\downarrow \; 25$$

$$\frac{e\left(\frac{\frac{\int \frac{bd\left(ac(4bce+ade+3acf)-\left(-d(2de-3cf)a^2+3bc(de+2cf)a+8b^2c^2e\right)x^2\right)}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}\left(\frac{8b^2ce}{a}-\frac{2ad^2e}{c}+3adf+6bcf+3bde\right)}{x}}{3ac}}{a(ad+bc)}-\frac{\sqrt{a-bx^2}\sqrt{c+d\ldots}}{3\ldots}\right)}{e^2}$$

$$f\left(\frac{\frac{d\left(\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(af+2be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(acf+ade+2bce)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}\right)}{ac}}{a(ad+bc)}-\frac{\sqrt{a-bx\ldots}}{\ldots}\right)$$

$$\Big\downarrow \; 27$$

$$\frac{e\left(\frac{\frac{bd\int \frac{ac(4bce+ade+3acf)-\left(-d(2de-3cf)a^2+3bc(de+2cf)a+8b^2c^2e\right)x^2}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}\left(\frac{8b^2ce}{a}-\frac{2ad^2e}{c}+3adf+6bcf+3bde\right)}{x}}{3ac}}{a(ad+bc)}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}}{3a\ldots}\right)}{e^2}$$

$$f\left(\frac{\frac{d\left(\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(af+2be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(acf+ade+2bce)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}\right)}{ac}}{a(ad+bc)}-\frac{\sqrt{a-bx\ldots}}{\ldots}\right)$$

$\downarrow$ 399

$$\frac{e\left(\frac{\frac{bd\left(\frac{c(ad+bc)(6acf-ade+8bce)\int \frac{1}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{d}-\frac{\left(a^2(-d)(2de-3cf)+3abc(2cf+de)+8b^2c^2e\right)\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}\right)}{ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}\left(\frac{8b^2ce}{a}-\frac{2ad^2e}{c}+\cdots\right.}{x}}{3ac}}{a(ad+bc)}\right. \quad f\left(\frac{\frac{d\left(\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(af+2be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(acf+ade+2bce)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}\right)}{ac}-\frac{\sqrt{a-bx\cdots}}{}}{a(ad+bc)}\right.}{e^2}$$

$\downarrow$ 323

$$\frac{e\left(\frac{\frac{bd\left(\frac{c\sqrt{\frac{dx^2}{c}+1}(ad+bc)(6acf-ade+8bce)\int \frac{1}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}dx}{d\sqrt{c+dx^2}}-\frac{\left(a^2(-d)(2de-3cf)+3abc(2cf+de)+8b^2c^2e\right)\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}\right)}{ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}\left(\frac{8b^2ce}{a}\cdots\right.}{}}{3ac}}{a(ad+bc)}\right. \quad f\left(\frac{\frac{d\left(\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(af+2be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(acf+ade+2bce)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}\right)}{ac}-\frac{\sqrt{a-bx\cdots}}{}}{a(ad+bc)}\right.}{e^2}$$

$\downarrow$ 323

$$\frac{e\left(\frac{\frac{\frac{bd\left(\frac{c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(6acf-ade+8bce)\int\frac{1}{\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}}dx}{d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\left(a^2(-d)(2de-3cf)+3abc(2cf+de)+8b^2c^2e\right)\int\frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}\right)}{ac}-\frac{\sqrt{a-bx^2}\sqrt{c+d\cdots}}{\cdots}}{3ac}\cdots}{a(ad+bc)}\cdots\right)\cdots f\left(\frac{\frac{d\left(\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(af+2be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(acf+ade+2bce)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}\right)}{ac}-\frac{\sqrt{a-b\cdots}}{\cdots}}{a(ad+bc)}\cdots\right)\cdots}{e^2}$$

$\downarrow$ 321

$$\frac{e\left(\frac{\frac{\frac{bd\left(\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(6acf-ade+8bce)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\left(a^2(-d)(2de-3cf)+3abc(2cf+de)+8b^2c^2e\right)\int\frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}\right)}{ac}-\frac{\sqrt{a-\cdots}}{\cdots}}{3ac}\cdots}{a(ad+bc)}\cdots\right)\cdots f\left(\frac{\frac{d\left(\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(af+2be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(acf+ade+2bce)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}\right)}{ac}-\frac{\sqrt{a-b\cdots}}{\cdots}}{a(ad+bc)}\cdots\right)\cdots}{e^2}$$

$\downarrow$ 331

$$\frac{e\left(\frac{\frac{bd\left(\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(6acf-ade+8bce)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{1-\frac{bx^2}{a}}\left(a^2(-d)(2de-3cf)+3abc(2cf+de)+8b^2c^2e\right)\int\frac{\sqrt{dx^2+c}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}\right)}{ac}}{3ac}}{a(ad+bc)}\right)\quad f\left(\frac{\frac{d\left(\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(af+2be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(acf+ade+2bce)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}\right)}{ac}}{a(ad+bc)}-\frac{\sqrt{a-bx^2}\cdots}{\cdots}\right)}{e^2}$$

$\big\downarrow$ 330

$$\frac{e\left(\frac{\frac{bd\left(\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(6acf-ade+8bce)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\left(a^2(-d)(2de-3cf)+3abc(2cf+de)+8b^2c^2e\right)\int\frac{\sqrt{\cdots}}{\sqrt{1\cdots}}}{d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}\right)}{ac}}{3ac}}{a(ad+bc)}\right)\quad f\left(\frac{\frac{d\left(\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(af+2be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(acf+ade+2bce)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}\right)}{ac}}{a(ad+bc)}-\frac{\sqrt{a-bx\cdots}}{\cdots}\right)}{e^2}$$

$\big\downarrow$ 327

$$\frac{e\left(\frac{\frac{bd\left(\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(6acf-ade+8bce)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\left(a^2(-d)(2de-3cf)+3abc(2cf+de)+8b^2c^2e\right)E\left(\cdots\right.}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}\right.}{ac}}{3ac}}{a(ad+bc)}\right.\cdots + f\left(\frac{\frac{d\left(\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(af+2be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(acf+ade+2bce)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}\right)}{ac}}{a(ad+bc)}-\frac{\sqrt{a-bx}\cdots}{\cdots}\right.\cdots}{e^2}$$

input
```
Int[(e + f*x^2)^2/(x^4*(a - b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

output
```
(f*(((b*e + a*f)*Sqrt[c + d*x^2])/(a*(b*c + a*d)*x*Sqrt[a - b*x^2]) + (-((
(2*b*c*e + a*d*e + a*c*f)*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])/(a*c*x)) + (d*(
-((Sqrt[a]*Sqrt[b]*(2*b*c*e + a*d*e + a*c*f)*Sqrt[1 - (b*x^2)/a]*Sqrt[c +
d*x^2]*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(d*Sqrt[a -
 b*x^2]*Sqrt[1 + (d*x^2)/c])) + (Sqrt[a]*c*(b*c + a*d)*(2*b*e + a*f)*Sqrt[
1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]],
-((a*d)/(b*c))])/(Sqrt[b]*d*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])))/(a*c))/(a*(
b*c + a*d))))/e^2 + e*(((b*e + a*f)*Sqrt[c + d*x^2])/(a*(b*c + a*d)*x^3*Sq
rt[a - b*x^2]) + (-1/3*((4*b*c*e + a*d*e + 3*a*c*f)*Sqrt[a - b*x^2]*Sqrt[c
 + d*x^2])/(a*c*x^3) + (-((((8*b^2*c*e)/a + 3*b*d*e - (2*a*d^2*e)/c + 6*b*
c*f + 3*a*d*f)*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])/x) + (b*d*(-((Sqrt[a]*(8*b
^2*c^2*e - a^2*d*(2*d*e - 3*c*f) + 3*a*b*c*(d*e + 2*c*f))*Sqrt[1 - (b*x^2)
/a]*Sqrt[c + d*x^2]*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))]
)/(Sqrt[b]*d*Sqrt[a - b*x^2]*Sqrt[1 + (d*x^2)/c])) + (Sqrt[a]*c*(b*c + a*d
)*(8*b*c*e - a*d*e + 6*a*c*f)*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Elli
pticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*d*Sqrt[a - b*
x^2]*Sqrt[c + d*x^2])))/(a*c))/(3*a*c))/(a*(b*c + a*d)))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 441
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
+ b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(a*g*2*(b*c - a*d)*(p + 1))), x] + Si
mp[1/(a*2*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2
)^q*Simp[c*(b*e - a*f)*(m + 1) + e*2*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(m
 + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, q},
 x] && LtQ[p, -1]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 448
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[e   Int[(g*x)^m*(a + b*x
^2)^p*(c + d*x^2)^q*(e + f*x^2)^(r - 1), x], x] + Simp[f/e^2   Int[(g*x)^(m
 + 2)*(a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^(r - 1), x], x] /; FreeQ[{a,
b, c, d, e, f, g, m, p, q}, x] && IGtQ[r, 0]
```

Maple [A] (verified)

Time = 11.92 (sec) , antiderivative size = 564, normalized size of antiderivative = 1.19

method	result
elliptic	$\frac{\sqrt{(-b\,x^2+a)(x^2d+c)}\left(-\frac{e^2\sqrt{-bd\,x^4+ad\,x^2-x^2bc+ac}}{3a^2c\,x^3}-\frac{e(6acf-2ade+5bce)\sqrt{-bd\,x^4+ad\,x^2-x^2bc+ac}}{3a^3c^2x}-\frac{\left(-bd\,x^2-bc\right)x\left(a^2f^2+2abfe+b^2e^2\right.}{a^3(ad+bc)\sqrt{\left(x^2-\frac{a}{b}\right)\left(-bd\,x^2-bc\right.}}\right.}{}$
risch	$-\frac{\sqrt{-b\,x^2+a}\,\sqrt{x^2d+c}\,e(6acf\,x^2-2ade\,x^2+5bce\,x^2+ace)}{3a^3c^2x^3}-\frac{\left(-\frac{be(6acf-2ade+5bce)c\sqrt{1-\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\left(\mathrm{EllipticF}\left(x\sqrt{\frac{b}{a}},\sqrt{-1-\frac{ad-}{c}}\right.\right.}{\sqrt{\frac{b}{a}}\,\sqrt{-bd\,x^4+ad\,x^2-x^2bc+ac}}\right.}{}$
default	Expression too large to display

input
```
int((f*x^2+e)^2/x^4/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBO
SE)
```

output
```
((-b*x^2+a)*(d*x^2+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-1/3*e^2/a^
2/c*(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)/x^3-1/3/a^3/c^2*e*(6*a*c*f-2*a*d*
e+5*b*c*e)*(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)/x-(-b*d*x^2-b*c)/a^3/(a*d+
b*c)*x*(a^2*f^2+2*a*b*e*f+b^2*e^2)/((x^2-a/b)*(-b*d*x^2-b*c))^(1/2)+(1/3*b
*d*e^2/a^2/c+(a^2*f^2+2*a*b*e*f+b^2*e^2)/a^3-b*c/a^3/(a*d+b*c)*(a^2*f^2+2*
a*b*e*f+b^2*e^2))/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^
4+a*d*x^2-b*c*x^2+a*c)^(1/2)*EllipticF(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1
/2))-(-1/3*b*d*e*(6*a*c*f-2*a*d*e+5*b*c*e)/a^3/c^2-(a^2*f^2+2*a*b*e*f+b^2*
e^2)*b*d/a^3/(a*d+b*c))*c/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/
(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(b/a)^(1/2),(-1-(a*d-b
*c)/c/b)^(1/2))-EllipticE(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 690, normalized size of antiderivative = 1.46

$$\int \frac{(e+fx^2)^2}{x^4\,(a-bx^2)^{3/2}\,\sqrt{c+dx^2}}\,dx =$$

$$-\frac{((3\,a^2b^3c^2f^2+(8\,b^5c^2+3\,ab^4cd-2\,a^2b^3d^2)e^2+6\,(2\,ab^4c^2+a^2b^3cd)ef)x^5-(3\,a^3b^2c^2f^2+(8\,ab^4c^2+3\,a^2}{}$$

input
```
integrate((f*x^2+e)^2/x^4/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="f
ricas")
```

output
```
-1/3*(((3*a^2*b^3*c^2*f^2 + (8*b^5*c^2 + 3*a*b^4*c*d - 2*a^2*b^3*d^2)*e^2
+ 6*(2*a*b^4*c^2 + a^2*b^3*c*d)*e*f)*x^5 - (3*a^3*b^2*c^2*f^2 + (8*a*b^4*c
^2 + 3*a^2*b^3*c*d - 2*a^3*b^2*d^2)*e^2 + 6*(2*a^2*b^3*c^2 + a^3*b^2*c*d)*
e*f)*x^3)*sqrt(a*c)*sqrt(b/a)*elliptic_e(arcsin(x*sqrt(b/a)), -a*d/(b*c))
- (((8*b^5*c^2 + (4*a^2*b^3 + 3*a*b^4)*c*d + (a^3*b^2 - 2*a^2*b^3)*d^2)*e^
2 + 6*(2*a*b^4*c^2 + (a^3*b^2 + a^2*b^3)*c*d)*e*f + 3*(a^2*b^3*c^2 + a^4*b
*c*d)*f^2)*x^5 - ((8*a*b^4*c^2 + (4*a^3*b^2 + 3*a^2*b^3)*c*d + (a^4*b - 2*
a^3*b^2)*d^2)*e^2 + 6*(2*a^2*b^3*c^2 + (a^4*b + a^3*b^2)*c*d)*e*f + 3*(a^3
*b^2*c^2 + a^5*c*d)*f^2)*x^3)*sqrt(a*c)*sqrt(b/a)*elliptic_f(arcsin(x*sqrt
(b/a)), -a*d/(b*c)) + ((3*a^3*b^2*c^2*f^2 + (8*a*b^4*c^2 + 3*a^2*b^3*c*d -
 2*a^3*b^2*d^2)*e^2 + 6*(2*a^2*b^3*c^2 + a^3*b^2*c*d)*e*f)*x^4 - (a^3*b^2*
c^2 + a^4*b*c*d)*e^2 - 2*((2*a^2*b^3*c^2 + a^3*b^2*c*d - a^4*b*d^2)*e^2 +
3*(a^3*b^2*c^2 + a^4*b*c*d)*e*f)*x^2)*sqrt(-b*x^2 + a)*sqrt(d*x^2 + c))/((
a^4*b^3*c^3 + a^5*b^2*c^2*d)*x^5 - (a^5*b^2*c^3 + a^6*b*c^2*d)*x^3)
```

Sympy [F]

$$\int \frac{(e+fx^2)^2}{x^4(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \int \frac{(e+fx^2)^2}{x^4(a-bx^2)^{\frac{3}{2}}\sqrt{c+dx^2}}\,dx$$

input
```
integrate((f*x**2+e)**2/x**4/(-b*x**2+a)**(3/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral((e + f*x**2)**2/(x**4*(a - b*x**2)**(3/2)*sqrt(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{(e+fx^2)^2}{x^4(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \int \frac{(fx^2+e)^2}{(-bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}x^4}\,dx$$

input
```
integrate((f*x^2+e)^2/x^4/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="m
axima")
```

output
```
integrate((f*x^2 + e)^2/((-b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*x^4), x)
```

Giac [F]

$$\int \frac{(e+fx^2)^2}{x^4(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \int \frac{(fx^2+e)^2}{(-bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}x^4}\,dx$$

input
```
integrate((f*x^2+e)^2/x^4/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="g
iac")
```

output
```
integrate((f*x^2 + e)^2/((-b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx^2)^2}{x^4(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \int \frac{(f\,x^2+e)^2}{x^4\,(a-b\,x^2)^{3/2}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int((e + f*x^2)^2/(x^4*(a - b*x^2)^(3/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int((e + f*x^2)^2/(x^4*(a - b*x^2)^(3/2)*(c + d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{(e+fx^2)^2}{x^4(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \text{too large to display}$$

input

```
int((f*x^2+e)^2/x^4/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x)
```

output

```
( - sqrt(c + d*x**2)*sqrt(a - b*x**2)*e**2 + 6*int((sqrt(c + d*x**2)*sqrt(
a - b*x**2))/(a**3*c*d*x**2 + a**3*d**2*x**4 - 2*a**2*b*c**2*x**2 - 4*a**2
*b*c*d*x**4 - 2*a**2*b*d**2*x**6 + 4*a*b**2*c**2*x**4 + 5*a*b**2*c*d*x**6
+ a*b**2*d**2*x**8 - 2*b**3*c**2*x**6 - 2*b**3*c*d*x**8),x)*a**3*c*d*e*f*x
**3 - 2*int((sqrt(c + d*x**2)*sqrt(a - b*x**2))/(a**3*c*d*x**2 + a**3*d**2
*x**4 - 2*a**2*b*c**2*x**2 - 4*a**2*b*c*d*x**4 - 2*a**2*b*d**2*x**6 + 4*a*
b**2*c**2*x**4 + 5*a*b**2*c*d*x**6 + a*b**2*d**2*x**8 - 2*b**3*c**2*x**6 -
 2*b**3*c*d*x**8),x)*a**3*d**2*e**2*x**3 - 12*int((sqrt(c + d*x**2)*sqrt(a
 - b*x**2))/(a**3*c*d*x**2 + a**3*d**2*x**4 - 2*a**2*b*c**2*x**2 - 4*a**2*
b*c*d*x**4 - 2*a**2*b*d**2*x**6 + 4*a*b**2*c**2*x**4 + 5*a*b**2*c*d*x**6 +
 a*b**2*d**2*x**8 - 2*b**3*c**2*x**6 - 2*b**3*c*d*x**8),x)*a**2*b*c**2*e*f
*x**3 + 8*int((sqrt(c + d*x**2)*sqrt(a - b*x**2))/(a**3*c*d*x**2 + a**3*d*
*2*x**4 - 2*a**2*b*c**2*x**2 - 4*a**2*b*c*d*x**4 - 2*a**2*b*d**2*x**6 + 4*
a*b**2*c**2*x**4 + 5*a*b**2*c*d*x**6 + a*b**2*d**2*x**8 - 2*b**3*c**2*x**6
 - 2*b**3*c*d*x**8),x)*a**2*b*c*d*e**2*x**3 - 6*int((sqrt(c + d*x**2)*sqrt
(a - b*x**2))/(a**3*c*d*x**2 + a**3*d**2*x**4 - 2*a**2*b*c**2*x**2 - 4*a**
2*b*c*d*x**4 - 2*a**2*b*d**2*x**6 + 4*a*b**2*c**2*x**4 + 5*a*b**2*c*d*x**6
 + a*b**2*d**2*x**8 - 2*b**3*c**2*x**6 - 2*b**3*c*d*x**8),x)*a**2*b*c*d*e*
f*x**5 + 2*int((sqrt(c + d*x**2)*sqrt(a - b*x**2))/(a**3*c*d*x**2 + a**3*d
**2*x**4 - 2*a**2*b*c**2*x**2 - 4*a**2*b*c*d*x**4 - 2*a**2*b*d**2*x**6 ...
```

3.275 $\int \frac{x^6(e+fx^2)}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 483

$$\int \frac{x^6(e+fx^2)}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \frac{(be+af)x^5\sqrt{c+dx^2}}{b(bc+ad)\sqrt{a-bx^2}}$$
$$+\frac{(24a^2d^2f+b^2c(5de-4cf)+5abd(4de+cf))\,x\sqrt{a-bx^2}\sqrt{c+dx^2}}{15b^3d^2(bc+ad)}$$
$$+\frac{(5bde+bcf+6adf)x^3\sqrt{a-bx^2}\sqrt{c+dx^2}}{5b^2d(bc+ad)}$$
$$-\frac{\sqrt{a}(48a^3d^3f-2b^3c^2(5de-4cf)+3ab^2cd(5de-3cf)+8a^2bd^2(5de+2cf))\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\text{arcsi}\right.}{15b^{7/2}d^3(bc+ad)\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}}$$
$$+\frac{\sqrt{a}c(24a^2d^2f+abd(20de-13cf)-2b^2c(5de-4cf))\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{15b^{7/2}d^3\sqrt{a-bx^2}\sqrt{c+dx^2}}$$

output
```
(a*f+b*e)*x^5*(d*x^2+c)^(1/2)/b/(a*d+b*c)/(-b*x^2+a)^(1/2)+1/15*(24*a^2*d^
2*f+b^2*c*(-4*c*f+5*d*e)+5*a*b*d*(c*f+4*d*e))*x*(-b*x^2+a)^(1/2)*(d*x^2+c)
^(1/2)/b^3/d^2/(a*d+b*c)+1/5*(6*a*d*f+b*c*f+5*b*d*e)*x^3*(-b*x^2+a)^(1/2)*
(d*x^2+c)^(1/2)/b^2/d/(a*d+b*c)-1/15*a^(1/2)*(48*a^3*d^3*f-2*b^3*c^2*(-4*c
*f+5*d*e)+3*a*b^2*c*d*(-3*c*f+5*d*e)+8*a^2*b*d^2*(2*c*f+5*d*e))*(1-b*x^2/a
)^(1/2)*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2),(-a*d/b/c)^(1/2))/b^(7
/2)/d^3/(a*d+b*c)/(-b*x^2+a)^(1/2)/(1+d*x^2/c)^(1/2)+1/15*a^(1/2)*c*(24*a^
2*d^2*f+a*b*d*(-13*c*f+20*d*e)-2*b^2*c*(-4*c*f+5*d*e))*(1-b*x^2/a)^(1/2)*(
1+d*x^2/c)^(1/2)*EllipticF(b^(1/2)*x/a^(1/2),(-a*d/b/c)^(1/2))/b^(7/2)/d^3
/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.09 (sec) , antiderivative size = 403, normalized size of antiderivative = 0.83

$$\int \frac{x^6(e+fx^2)}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \frac{-\sqrt{-\frac{b}{a}}dx(c+dx^2)\left(-24a^3d^2f+b^3cx^2(5de-4cf+3dfx^2)+a^2bd(-20de\right.}{}$$

input
```
Integrate[(x^6*(e + f*x^2))/((a - b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

output
```
(-(Sqrt[-(b/a)]*d*x*(c + d*x^2)*(-24*a^3*d^2*f + b^3*c*x^2*(5*d*e - 4*c*f
+ 3*d*f*x^2) + a^2*b*d*(-20*d*e - 5*c*f + 6*d*f*x^2) + a*b^2*(4*c^2*f + c*
d*(-5*e + 2*f*x^2) + d^2*x^2*(5*e + 3*f*x^2)))) + I*c*(48*a^3*d^3*f + 3*a*
b^2*c*d*(5*d*e - 3*c*f) + 8*a^2*b*d^2*(5*d*e + 2*c*f) + 2*b^3*c^2*(-5*d*e
+ 4*c*f))*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt
[-(b/a)]*x], -((a*d)/(b*c))] - I*c*(b*c + a*d)*(24*a^2*d^2*f + a*b*d*(20*d
*e - 13*c*f) + 2*b^2*c*(-5*d*e + 4*c*f))*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x
^2)/c]*EllipticF[I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))])/(15*b^3*Sqrt[
-(b/a)]*d^3*(b*c + a*d)*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 489, normalized size of antiderivative = 1.01, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.324, Rules used = {440, 25, 444, 444, 399, 323, 323, 321, 331, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6 (e+fx^2)}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \text{440}$$

$$\frac{\int -\frac{x^4((5bde+bcf+6adf)x^2+5c(be+af))}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{b(ad+bc)}+\frac{x^5\sqrt{c+dx^2}(af+be)}{b\sqrt{a-bx^2}(ad+bc)}$$

$$\Big\downarrow \text{25}$$

$$\frac{x^5\sqrt{c+dx^2}(af+be)}{b\sqrt{a-bx^2}(ad+bc)}-\frac{\int \frac{x^4((5bde+bcf+6adf)x^2+5c(be+af))}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{b(ad+bc)}$$

$$\Big\downarrow \text{444}$$

$$\frac{x^5\sqrt{c+dx^2}(af+be)}{b\sqrt{a-bx^2}(ad+bc)}-\frac{\frac{\int \frac{x^2\left(\left(c(5de-4cf)b^2+5ad(4de+cf)b+24a^2d^2f\right)x^2+3ac(5bde+bcf+6adf)\right)}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{5bd}-\frac{x^3\sqrt{a-bx^2}\sqrt{c+dx^2}(6adf+bcf+5bde)}{5bd}}{b(ad+bc)}$$

$$\Big\downarrow \text{444}$$

$$\frac{x^5\sqrt{c+dx^2}(af+be)}{b\sqrt{a-bx^2}(ad+bc)}-\frac{\frac{\frac{\int \frac{\left(-2c^2(5de-4cf)b^3+3acd(5de-3cf)b^2+8a^2d^2(5de+2cf)b+48a^3d^3f\right)x^2+ac\left(c(5de-4cf)b^2+5ad(4de+cf)b+24a^2d^2f\right)}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{3bd}-\frac{1}{3}x\sqrt{a-bx^2}\sqrt{c+dx^2}\left(\frac{24a^2df}{b}+\right.}{5bd}}{b(ad+bc)}$$

$$\Big\downarrow \text{399}$$

$$\frac{\frac{x^5\sqrt{c+dx^2}(af+be)}{b\sqrt{a-bx^2}(ad+bc)}-\frac{\frac{\frac{\left(48a^3d^3f+8a^2bd^2(2cf+5de)+3ab^2cd(5de-3cf)-2b^3c^2(5de-4cf)\right)\int\frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}-\frac{c(ad+bc)\left(24a^2d^2f+abd(20de-13cf)-2b^2c(5de-4cf)\right)\int\frac{1}{\sqrt{a-bx^2}\sqrt{dx^2+c}}}{d}}{3bd}}{5bd}}{b(ad+bc)}$$

$\downarrow$ 323

$$\frac{\frac{x^5\sqrt{c+dx^2}(af+be)}{b\sqrt{a-bx^2}(ad+bc)}-\frac{\frac{\frac{\left(48a^3d^3f+8a^2bd^2(2cf+5de)+3ab^2cd(5de-3cf)-2b^3c^2(5de-4cf)\right)\int\frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}-\frac{c\sqrt{\frac{dx^2}{c}+1}(ad+bc)\left(24a^2d^2f+abd(20de-13cf)-2b^2c(5de-4cf)\right)\int\frac{}{\sqrt{a-bx^2}}}{d\sqrt{c+dx^2}}}{3bd}}{5bd}}{b(ad+bc)}$$

$\downarrow$ 323

$$\frac{\frac{x^5\sqrt{c+dx^2}(af+be)}{b\sqrt{a-bx^2}(ad+bc)}-\frac{\frac{\frac{\left(48a^3d^3f+8a^2bd^2(2cf+5de)+3ab^2cd(5de-3cf)-2b^3c^2(5de-4cf)\right)\int\frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}-\frac{c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\left(24a^2d^2f+abd(20de-13cf)-2b^2c(5de-4cf)\right)\int}{d\sqrt{a-bx^2}\sqrt{c+dx^2}}}{3bd}}{5bd}}{b(ad+bc)}$$

$\downarrow$ 321

$$\frac{\frac{x^5\sqrt{c+dx^2}(af+be)}{b\sqrt{a-bx^2}(ad+bc)}-\frac{\frac{\frac{\left(48a^3d^3f+8a^2bd^2(2cf+5de)+3ab^2cd(5de-3cf)-2b^3c^2(5de-4cf)\right)\int\frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}-\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\left(24a^2d^2f+abd(20de-13cf)-2b^2c(5de-4cf\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}}{3bd}}{5bd}}{b(ad+bc}$$

$\downarrow$ 331

$$\frac{\frac{x^5\sqrt{c+dx^2}(af+be)}{b\sqrt{a-bx^2}(ad+bc)}-\frac{\frac{\frac{\sqrt{1-\frac{bx^2}{a}}\left(48a^3d^3f+8a^2bd^2(2cf+5de)+3ab^2cd(5de-3cf)-2b^3c^2(5de-4cf)\right)\int\frac{\sqrt{dx^2+c}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\left(24a^2d^2f+abd(20de-13cf)-2b^2c\right.}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}}{3bd}}{5bd}}{b(ad+}$$

$\downarrow$ 330

$$\frac{x^5\sqrt{c+dx^2}(af+be)}{b\sqrt{a-bx^2}(ad+bc)} - \frac{\frac{\frac{\frac{\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\left(48a^3d^3f+8a^2bd^2(2cf+5de)+3ab^2cd(5de-3cf)-2b^3c^2(5de-4cf)\right)\int\frac{\sqrt{\frac{dx^2}{c}+1}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}} - \frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\left(24a^2d^2f+abd(20de-13c\ldots\right.}{\sqrt{b}d\sqrt{a-bx^2}\ldots}}{3bd}\ldots}{5bd}\ldots}{b(a\ldots}$$

$$\Big\downarrow \text{327}$$

$$\frac{x^5\sqrt{c+dx^2}(af+be)}{b\sqrt{a-bx^2}(ad+bc)} - \frac{\frac{\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\left(48a^3d^3f+8a^2bd^2(2cf+5de)+3ab^2cd(5de-3cf)-2b^3c^2(5de-4cf)\right)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}} - \frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\left(24a^2d^2f+a\ldots\right.}{\ldots}}{3bd}\ldots}{5bd}\ldots$$

input
```
Int[(x^6*(e + f*x^2))/((a - b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

output
```
((b*e + a*f)*x^5*Sqrt[c + d*x^2])/(b*(b*c + a*d)*Sqrt[a - b*x^2]) - (-1/5*
((5*b*d*e + b*c*f + 6*a*d*f)*x^3*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])/(b*d) +
(-1/3*(((24*a^2*d*f)/b + (b*c*(5*d*e - 4*c*f))/d + 5*a*(4*d*e + c*f))*x*Sq
rt[a - b*x^2]*Sqrt[c + d*x^2]) + ((Sqrt[a]*(48*a^3*d^3*f - 2*b^3*c^2*(5*d*
e - 4*c*f) + 3*a*b^2*c*d*(5*d*e - 3*c*f) + 8*a^2*b*d^2*(5*d*e + 2*c*f))*Sq
rt[1 - (b*x^2)/a]*Sqrt[c + d*x^2]*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -
((a*d)/(b*c))])/(Sqrt[b]*d*Sqrt[a - b*x^2]*Sqrt[1 + (d*x^2)/c]) - (Sqrt[a]
*c*(b*c + a*d)*(24*a^2*d^2*f + a*b*d*(20*d*e - 13*c*f) - 2*b^2*c*(5*d*e -
4*c*f))*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[ArcSin[(Sqrt[b]*
x)/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*d*Sqrt[a - b*x^2]*Sqrt[c + d*x^2]))
/(3*b*d))/(5*b*d))/(b*(b*c + a*d))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 440
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[g*(b*e - a*f)*(g*x)^(m - 1)*(a +
 b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] - Simp[
g^2/(2*b*(b*c - a*d)*(p + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c +
d*x^2)^q*Simp[c*(b*e - a*f)*(m - 1) + (d*(b*e - a*f)*(m + 2*q + 1) - b*2*(c
*f - d*e)*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, q}, x] &&
 LtQ[p, -1] && GtQ[m, 1]
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

Maple [A] (verified)

Time = 20.69 (sec) , antiderivative size = 587, normalized size of antiderivative = 1.22

method	result
elliptic	$\frac{\sqrt{(-b\,x^2+a)(x^2d+c)}\left(-\frac{\left(-bd\,x^2-bc\right)a^2x(af+be)}{b^4(ad+bc)\sqrt{\left(x^2-\frac{a}{b}\right)\left(-bd\,x^2-bc\right)}}+\frac{f\,x^3\sqrt{-bd\,x^4+ad\,x^2-x^2bc+ac}}{5b^2d}-\frac{\left(-\frac{af+be}{b^2}-\frac{f(4ad-4bc)}{5b^2d}\right)x\sqrt{-bd\,x^4+ad\,x^2-}}{3bd}\right.}{}$
risch	$\frac{x\left(3bdf\,x^2+9adf-4bcf+5bde\right)\sqrt{-b\,x^2+a}\,\sqrt{x^2d+c}}{15b^3d^2}-\frac{\left(-\frac{\left(33f\,d^2a^2-17fdcba+25ab\,d^2e+8f\,c^2b^2-10d\,b^2ce\right)c\sqrt{1-\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\left(\text{Ellipt}\right.}{\sqrt{\frac{b}{a}}\,\sqrt{-bd\,x^4+ad\,x^2-x^2b}}\right.}{}$
default	Expression too large to display

input
```
int(x^6*(f*x^2+e)/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE
)
```

output
```
((-b*x^2+a)*(d*x^2+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-(-b*d*x^2-
b*c)/b^4*a^2/(a*d+b*c)*x*(a*f+b*e)/((x^2-a/b)*(-b*d*x^2-b*c))^(1/2)+1/5*f/
b^2/d*x^3*(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)-1/3*(-1/b^2*(a*f+b*e)-1/5*f
/b^2/d*(4*a*d-4*b*c))/b/d*x*(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)+(-1/b^3*c
*a^2/(a*d+b*c)*(a*f+b*e)+1/3*(-1/b^2*(a*f+b*e)-1/5*f/b^2/d*(4*a*d-4*b*c))/
b/d*a*c)/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2
-b*c*x^2+a*c)^(1/2)*EllipticF(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2))-(-a/
b^3*(a*f+b*e)-1/b^3*d*a^2*(a*f+b*e)/(a*d+b*c)-3/5*f/b^2/d*a*c+1/3*(-1/b^2*
(a*f+b*e)-1/5*f/b^2/d*(4*a*d-4*b*c))/b/d*(2*a*d-2*b*c))*c/(b/a)^(1/2)*(1-b
*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)/d*(El
lipticF(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2))-EllipticE(x*(b/a)^(1/2),(-
1-(a*d-b*c)/c/b)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 837, normalized size of antiderivative = 1.73

$$\int \frac{x^6(e+fx^2)}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^6*(f*x^2+e)/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="fri
cas")
```

output
```
-1/15*(((5*(2*a*b^4*c^2*d - 3*a^2*b^3*c*d^2 - 8*a^3*b^2*d^3)*e - (8*a*b^4*
c^3 - 9*a^2*b^3*c^2*d + 16*a^3*b^2*c*d^2 + 48*a^4*b*d^3)*f)*x^3 - (5*(2*a^
2*b^3*c^2*d - 3*a^3*b^2*c*d^2 - 8*a^4*b*d^3)*e - (8*a^2*b^3*c^3 - 9*a^3*b^
2*c^2*d + 16*a^4*b*c*d^2 + 48*a^5*d^3)*f)*x)*sqrt(-b*d)*sqrt(a/b)*elliptic
_e(arcsin(sqrt(a/b)/x), -b*c/(a*d)) + ((5*(8*a^3*b^2*d^3 - (2*a*b^4 - b^5)
*c^2*d + (3*a^2*b^3 + 4*a*b^4)*c*d^2)*e + (48*a^4*b*d^3 + 4*(2*a*b^4 - b^5
)*c^3 - (9*a^2*b^3 - 5*a*b^4)*c^2*d + 8*(2*a^3*b^2 + 3*a^2*b^3)*c*d^2)*f)*
x^3 - (5*(8*a^4*b*d^3 - (2*a^2*b^3 - a*b^4)*c^2*d + (3*a^3*b^2 + 4*a^2*b^3
)*c*d^2)*e + (48*a^5*d^3 + 4*(2*a^2*b^3 - a*b^4)*c^3 - (9*a^3*b^2 - 5*a^2*
b^3)*c^2*d + 8*(2*a^4*b + 3*a^3*b^2)*c*d^2)*f)*x)*sqrt(-b*d)*sqrt(a/b)*ell
iptic_f(arcsin(sqrt(a/b)/x), -b*c/(a*d)) - (3*(b^5*c*d^2 + a*b^4*d^3)*f*x^
6 + (5*(b^5*c*d^2 + a*b^4*d^3)*e - 2*(2*b^5*c^2*d - a*b^4*c*d^2 - 3*a^2*b^
3*d^3)*f)*x^4 - (10*(b^5*c^2*d - a*b^4*c*d^2 - 2*a^2*b^3*d^3)*e - (8*b^5*c
^3 - 5*a*b^4*c^2*d + 11*a^2*b^3*c*d^2 + 24*a^3*b^2*d^3)*f)*x^2 + 5*(2*a*b^
4*c^2*d - 3*a^2*b^3*c*d^2 - 8*a^3*b^2*d^3)*e - (8*a*b^4*c^3 - 9*a^2*b^3*c^
2*d + 16*a^3*b^2*c*d^2 + 48*a^4*b*d^3)*f)*sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)
)/((b^7*c*d^3 + a*b^6*d^4)*x^3 - (a*b^6*c*d^3 + a^2*b^5*d^4)*x)
```

Sympy [F]

$$\int \frac{x^6(e+fx^2)}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \int \frac{x^6(e+fx^2)}{(a-bx^2)^{\frac{3}{2}}\sqrt{c+dx^2}}\,dx$$

input
```
integrate(x**6*(f*x**2+e)/(-b*x**2+a)**(3/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(x**6*(e + f*x**2)/((a - b*x**2)**(3/2)*sqrt(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{x^6(e+fx^2)}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \int \frac{(fx^2+e)x^6}{(-bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}}\,dx$$

input
```
integrate(x^6*(f*x^2+e)/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="max
ima")
```

output
```
integrate((f*x^2 + e)*x^6/((-b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)), x)
```

Giac [F]

$$\int \frac{x^6(e+fx^2)}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx=\int \frac{(fx^2+e)x^6}{(-bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}}\,dx$$

input
```
integrate(x^6*(f*x^2+e)/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="gia
c")
```

output
```
integrate((f*x^2 + e)*x^6/((-b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6(e+fx^2)}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx=\int \frac{x^6\,(f\,x^2+e)}{{(a-b\,x^2)}^{3/2}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int((x^6*(e + f*x^2))/((a - b*x^2)^(3/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int((x^6*(e + f*x^2))/((a - b*x^2)^(3/2)*(c + d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^6(e+fx^2)}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx=\text{Too large to display}$$

input
```
int(x^6*(f*x^2+e)/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x)
```

output

```
(18*sqrt(c + d*x**2)*sqrt(a - b*x**2)*a*c*d*f*x - 12*sqrt(c + d*x**2)*sqrt
(a - b*x**2)*a*d**2*f*x**3 - 12*sqrt(c + d*x**2)*sqrt(a - b*x**2)*b*c**2*f
*x + 15*sqrt(c + d*x**2)*sqrt(a - b*x**2)*b*c*d*e*x + 8*sqrt(c + d*x**2)*s
qrt(a - b*x**2)*b*c*d*f*x**3 - 10*sqrt(c + d*x**2)*sqrt(a - b*x**2)*b*d**2
*e*x**3 - 6*sqrt(c + d*x**2)*sqrt(a - b*x**2)*b*d**2*f*x**5 + 48*int((sqrt
(c + d*x**2)*sqrt(a - b*x**2)*x**4)/(a**2*c + a**2*d*x**2 - 2*a*b*c*x**2 -
 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a**3*d**3*f - 8*int((sqrt(c
+ d*x**2)*sqrt(a - b*x**2)*x**4)/(a**2*c + a**2*d*x**2 - 2*a*b*c*x**2 - 2*
a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a**2*b*c*d**2*f + 40*int((sqrt(
c + d*x**2)*sqrt(a - b*x**2)*x**4)/(a**2*c + a**2*d*x**2 - 2*a*b*c*x**2 -
2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a**2*b*d**3*e - 48*int((sqrt(
c + d*x**2)*sqrt(a - b*x**2)*x**4)/(a**2*c + a**2*d*x**2 - 2*a*b*c*x**2 -
2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a**2*b*d**3*f*x**2 + 4*int((s
qrt(c + d*x**2)*sqrt(a - b*x**2)*x**4)/(a**2*c + a**2*d*x**2 - 2*a*b*c*x**
2 - 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a*b**2*c**2*d*f - 5*int((
sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**4)/(a**2*c + a**2*d*x**2 - 2*a*b*c*x*
*2 - 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a*b**2*c*d**2*e + 8*int(
(sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**4)/(a**2*c + a**2*d*x**2 - 2*a*b*c*x
**2 - 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a*b**2*c*d**2*f*x**2 -
40*int((sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**4)/(a**2*c + a**2*d*x**2 -...
```

3.276 $\int \frac{x^4(e+fx^2)}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 354

$$\int \frac{x^4(e+fx^2)}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \frac{(be+af)x^3\sqrt{c+dx^2}}{b(bc+ad)\sqrt{a-bx^2}} + \frac{(3bde+bcf+4adf)x\sqrt{a-bx^2}\sqrt{c+dx^2}}{3b^2d(bc+ad)} - \frac{\sqrt{a}(8a^2d^2f+b^2c(3de-2cf)+3abd(2de+cf))\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\,E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{3b^{5/2}d^2(bc+ad)\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}} + \frac{\sqrt{a}c(3bde-2bcf+4adf)\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{3b^{5/2}d^2\sqrt{a-bx^2}\sqrt{c+dx^2}}$$

output
```
(a*f+b*e)*x^3*(d*x^2+c)^(1/2)/b/(a*d+b*c)/(-b*x^2+a)^(1/2)+1/3*(4*a*d*f+b*
c*f+3*b*d*e)*x*(-b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b^2/d/(a*d+b*c)-1/3*a^(1/2
)*(8*a^2*d^2*f+b^2*c*(-2*c*f+3*d*e)+3*a*b*d*(c*f+2*d*e))*(1-b*x^2/a)^(1/2)
*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2),(-a*d/b/c)^(1/2))/b^(5/2)/d^2
/(a*d+b*c)/(-b*x^2+a)^(1/2)/(1+d*x^2/c)^(1/2)+1/3*a^(1/2)*c*(4*a*d*f-2*b*c
*f+3*b*d*e)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(b^(1/2)*x/a^(1/2
),(-a*d/b/c)^(1/2))/b^(5/2)/d^2/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 9.55 (sec) , antiderivative size = 293, normalized size of antiderivative = 0.83

$$\int \frac{x^4(e+fx^2)}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \frac{\sqrt{-\frac{b}{a}}dx(c+dx^2)\left(4a^2df-b^2cfx^2+ab(3de+cf-dfx^2)\right)+ic(8a^2d^2f+b}{}$$

input
```
Integrate[(x^4*(e + f*x^2))/((a - b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

output
```
(Sqrt[-(b/a)]*d*x*(c + d*x^2)*(4*a^2*d*f - b^2*c*f*x^2 + a*b*(3*d*e + c*f
- d*f*x^2)) + I*c*(8*a^2*d^2*f + b^2*c*(3*d*e - 2*c*f) + 3*a*b*d*(2*d*e +
c*f))*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[-(b
/a)]*x], -((a*d)/(b*c))] - I*c*(b*c + a*d)*(3*b*d*e - 2*b*c*f + 4*a*d*f)*S
qrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[-(b/a)]*x]
, -((a*d)/(b*c))])/(3*b^2*Sqrt[-(b/a)]*d^2*(b*c + a*d)*Sqrt[a - b*x^2]*Sqr
t[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {440, 25, 444, 399, 323, 323, 321, 331, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(e+fx^2)}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx$$

$$\Big\downarrow \ 440$$

$$\frac{\int -\frac{x^2((3bde+bcf+4adf)x^2+3c(be+af))}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{b(ad+bc)}+\frac{x^3\sqrt{c+dx^2}(af+be)}{b\sqrt{a-bx^2}(ad+bc)}$$

$$\Big\downarrow \ 25$$

$$\frac{x^3\sqrt{c+dx^2}(af+be)}{b\sqrt{a-bx^2}(ad+bc)}-\frac{\int \frac{x^2\left((3bde+bcf+4adf)x^2+3c(be+af)\right)}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{b(ad+bc)}$$

$\big\downarrow$ 444

$$\frac{x^3\sqrt{c+dx^2}(af+be)}{b\sqrt{a-bx^2}(ad+bc)}-\frac{\frac{\int \frac{\left(c(3de-2cf)b^2+3ad(2de+cf)b+8a^2d^2f\right)x^2+ac(3bde+bcf+4adf)}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{3bd}-\frac{x\sqrt{a-bx^2}\sqrt{c+dx^2}(4adf+bcf+3bde)}{3bd}}{b(ad+bc)}$$

$\big\downarrow$ 399

$$\frac{x^3\sqrt{c+dx^2}(af+be)}{b\sqrt{a-bx^2}(ad+bc)}-\frac{\frac{\frac{\left(8a^2d^2f+3abd(cf+2de)+b^2c(3de-2cf)\right)\int\frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}-\frac{c(ad+bc)(4adf-2bcf+3bde)\int\frac{1}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{d}}{3bd}-\frac{x\sqrt{a-bx^2}\sqrt{c+dx^2}(4adf+bcf+3bde)}{3bd}}{b(ad+bc)}$$

$\big\downarrow$ 323

$$\frac{x^3\sqrt{c+dx^2}(af+be)}{b\sqrt{a-bx^2}(ad+bc)}-\frac{\frac{\frac{\left(8a^2d^2f+3abd(cf+2de)+b^2c(3de-2cf)\right)\int\frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}-\frac{c\sqrt{\frac{dx^2}{c}+1}(ad+bc)(4adf-2bcf+3bde)\int\frac{1}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}dx}{d\sqrt{c+dx^2}}}{3bd}-\frac{x\sqrt{a-bx^2}\sqrt{c+dx^2}(4adf+bcf+3}{3bd}}{b(ad+bc)}$$

$\big\downarrow$ 323

$$\frac{x^3\sqrt{c+dx^2}(af+be)}{b\sqrt{a-bx^2}(ad+bc)}-\frac{\frac{\frac{\left(8a^2d^2f+3abd(cf+2de)+b^2c(3de-2cf)\right)\int\frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}-\frac{c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(4adf-2bcf+3bde)\int\frac{1}{\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}}dx}{d\sqrt{a-bx^2}\sqrt{c+dx^2}}}{3bd}-\frac{x\sqrt{a-bx^2}\sqrt{c+dx^2}(4ad}{3bd}}{b(ad+bc)}$$

$\big\downarrow$ 321

$$\frac{x^3\sqrt{c+dx^2}(af+be)}{b\sqrt{a-bx^2}(ad+bc)}-\frac{\frac{\frac{\left(8a^2d^2f+3abd(cf+2de)+b^2c(3de-2cf)\right)\int\frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}-\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(4adf-2bcf+3bde)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}}{3bd}-x\sqrt{a-bx^2}\sqrt{c-}{b(ad+bc)}$$

$\big\downarrow$ 331

$$\frac{x^3\sqrt{c+dx^2}(af+be)}{b\sqrt{a-bx^2}(ad+bc)} - \frac{\frac{\frac{\sqrt{1-\frac{bx^2}{a}}\left(8a^2d^2f+3abd(cf+2de)+b^2c(3de-2cf)\right)\int \frac{\sqrt{dx^2+c}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}} - \frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(4adf-2bcf+3bde)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}}{3bd} - x\sqrt{a}}{b(ad+bc)}$$

$\downarrow$ 330

$$\frac{x^3\sqrt{c+dx^2}(af+be)}{b\sqrt{a-bx^2}(ad+bc)} - \frac{\frac{\frac{\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\left(8a^2d^2f+3abd(cf+2de)+b^2c(3de-2cf)\right)\int \frac{\sqrt{\frac{dx^2}{c}+1}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}} - \frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(4adf-2bcf+3bde)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}}{3bd}}{b(ad+bc)}$$

$\downarrow$ 327

$$\frac{x^3\sqrt{c+dx^2}(af+be)}{b\sqrt{a-bx^2}(ad+bc)} - \frac{\frac{\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\left(8a^2d^2f+3abd(cf+2de)+b^2c(3de-2cf)\right)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}} - \frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(4adf-2bcf+3bde)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}}{\sqrt{}}\right.\right.}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}}{3bd}}{b(ad+bc)}$$

input

```
Int[(x^4*(e + f*x^2))/((a - b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

output

```
((b*e + a*f)*x^3*Sqrt[c + d*x^2])/(b*(b*c + a*d)*Sqrt[a - b*x^2]) - (-1/3*
((3*b*d*e + b*c*f + 4*a*d*f)*x*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])/(b*d) + ((
Sqrt[a]*(8*a^2*d^2*f + b^2*c*(3*d*e - 2*c*f) + 3*a*b*d*(2*d*e + c*f))*Sqrt
[1 - (b*x^2)/a]*Sqrt[c + d*x^2]*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((
a*d)/(b*c))])/(Sqrt[b]*d*Sqrt[a - b*x^2]*Sqrt[1 + (d*x^2)/c]) - (Sqrt[a]*c
*(b*c + a*d)*(3*b*d*e - 2*b*c*f + 4*a*d*f)*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d
*x^2)/c]*EllipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*
d*Sqrt[a - b*x^2]*Sqrt[c + d*x^2]))/(3*b*d))/(b*(b*c + a*d))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 440
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[g*(b*e - a*f)*(g*x)^(m - 1)*(a +
 b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] - Simp[
g^2/(2*b*(b*c - a*d)*(p + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c +
d*x^2)^q*Simp[c*(b*e - a*f)*(m - 1) + (d*(b*e - a*f)*(m + 2*q + 1) - b*2*(c
*f - d*e)*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, q}, x] &&
 LtQ[p, -1] && GtQ[m, 1]
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

Maple [A] (verified)

Time = 10.68 (sec) , antiderivative size = 441, normalized size of antiderivative = 1.25

method	result
elliptic	$\frac{\sqrt{(-b x^2+a)(x^2 d+c)}\left(-\frac{\left(-bd x^2-bc\right)ax(af+be)}{b^3(ad+bc)\sqrt{\left(x^2-\frac{a}{b}\right)\left(-bd x^2-bc\right)}}+\frac{fx\sqrt{-bd x^4+ad x^2-x^2bc+ac}}{3d b^2}+\frac{\left(-\frac{ca(af+be)}{b^2(ad+bc)}-\frac{fac}{3b^2d}\right)\sqrt{1-\frac{b x^2}{a}}\sqrt{1+\frac{d x^2}{c}}\,\mathrm{El}\cdots}{\sqrt{\frac{b}{a}}\sqrt{-bd x^4+ad x^2-x\cdots}}\cdots\right.}{\cdots}$
risch	$\frac{fx\sqrt{-b x^2+a}\sqrt{x^2d+c}}{3b^2d}-\frac{\left(-\frac{(5adf-2bcf+3bde)c\sqrt{1-\frac{b x^2}{a}}\sqrt{1+\frac{d x^2}{c}}\left(\mathrm{EllipticF}\left(x\sqrt{\frac{b}{a}},\sqrt{-1-\frac{ad-bc}{cb}}\right)-\mathrm{EllipticE}\left(x\sqrt{\frac{b}{a}},\sqrt{-1-\frac{ad-bc}{cb}}\right)\right)}{\sqrt{\frac{b}{a}}\sqrt{-bd x^4+ad x^2-x^2bc+ac}\,d}\cdots\right.}{\cdots}$
default	$\frac{\left(-\sqrt{\frac{b}{a}}\,ab d^3 f x^5-\sqrt{\frac{b}{a}}\,b^2c d^2 f x^5+4\sqrt{\frac{b}{a}}\,a^2 d^3 f x^3+3\sqrt{\frac{b}{a}}\,ab d^3 e x^3-\sqrt{\frac{b}{a}}\,b^2c^2df x^3+4\sqrt{\frac{-b x^2+a}{a}}\sqrt{\frac{x^2d+c}{c}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{b}{a}},\sqrt{-\frac{ac}{bc}\cdots}\right.\right.}{\cdots}$

input
```
int(x^4*(f*x^2+e)/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE
)
```

output
```
((-b*x^2+a)*(d*x^2+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-(-b*d*x^2-
b*c)/b^3*a/(a*d+b*c)*x*(a*f+b*e)/((x^2-a/b)*(-b*d*x^2-b*c))^(1/2)+1/3*f/d/
b^2*x*(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)+(-1/b^2*c*a/(a*d+b*c)*(a*f+b*e)
-1/3*f/b^2/d*a*c)/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^
4+a*d*x^2-b*c*x^2+a*c)^(1/2)*EllipticF(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1
/2))-(-1/b^2*(a*f+b*e)-1/b^2*d*a*(a*f+b*e)/(a*d+b*c)-1/3*f/d/b^2*(2*a*d-2*
b*c))*c/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-
b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2))-El
lipticE(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 551, normalized size of antiderivative = 1.56

$$\int \frac{x^4(e+fx^2)}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \frac{((3\,(ab^3cd+2\,a^2b^2d^2)e-(2\,ab^3c^2-3\,a^2b^2cd-8\,a^3bd^2)f)x^3-(3\,(a^2b^2cd+}{}$$

input
```
integrate(x^4*(f*x^2+e)/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="fri
cas")
```

output
```
1/3*(((3*(a*b^3*c*d + 2*a^2*b^2*d^2)*e - (2*a*b^3*c^2 - 3*a^2*b^2*c*d - 8*
a^3*b*d^2)*f)*x^3 - (3*(a^2*b^2*c*d + 2*a^3*b*d^2)*e - (2*a^2*b^2*c^2 - 3*
a^3*b*c*d - 8*a^4*d^2)*f)*x)*sqrt(-b*d)*sqrt(a/b)*elliptic_e(arcsin(sqrt(a
/b)/x), -b*c/(a*d)) - ((3*(2*a^2*b^2*d^2 + (a*b^3 + b^4)*c*d)*e + (8*a^3*b
*d^2 - (2*a*b^3 - b^4)*c^2 + (3*a^2*b^2 + 4*a*b^3)*c*d)*f)*x^3 - (3*(2*a^3
*b*d^2 + (a^2*b^2 + a*b^3)*c*d)*e + (8*a^4*d^2 - (2*a^2*b^2 - a*b^3)*c^2 +
 (3*a^3*b + 4*a^2*b^2)*c*d)*f)*x)*sqrt(-b*d)*sqrt(a/b)*elliptic_f(arcsin(s
qrt(a/b)/x), -b*c/(a*d)) + ((b^4*c*d + a*b^3*d^2)*f*x^4 + (3*(b^4*c*d + a*
b^3*d^2)*e - 2*(b^4*c^2 - a*b^3*c*d - 2*a^2*b^2*d^2)*f)*x^2 - 3*(a*b^3*c*d
 + 2*a^2*b^2*d^2)*e + (2*a*b^3*c^2 - 3*a^2*b^2*c*d - 8*a^3*b*d^2)*f)*sqrt(
-b*x^2 + a)*sqrt(d*x^2 + c))/((b^6*c*d^2 + a*b^5*d^3)*x^3 - (a*b^5*c*d^2 +
 a^2*b^4*d^3)*x)
```

Sympy [F]

$$\int \frac{x^4(e+fx^2)}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx=\int \frac{x^4(e+fx^2)}{(a-bx^2)^{\frac{3}{2}}\sqrt{c+dx^2}}\,dx$$

input
```
integrate(x**4*(f*x**2+e)/(-b*x**2+a)**(3/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(x**4*(e + f*x**2)/((a - b*x**2)**(3/2)*sqrt(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{x^4(e+fx^2)}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx=\int \frac{(fx^2+e)x^4}{(-bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}}\,dx$$

input
```
integrate(x^4*(f*x^2+e)/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="max
ima")
```

output
```
integrate((f*x^2 + e)*x^4/((-b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)), x)
```

Giac [F]

$$\int \frac{x^4(e+fx^2)}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx=\int \frac{(fx^2+e)x^4}{(-bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}}\,dx$$

input
```
integrate(x^4*(f*x^2+e)/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="gia
c")
```

output
```
integrate((f*x^2 + e)*x^4/((-b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(e+fx^2)}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx=\int \frac{x^4\,(f\,x^2+e)}{{(a-b\,x^2)}^{3/2}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int((x^4*(e + f*x^2))/((a - b*x^2)^(3/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int((x^4*(e + f*x^2))/((a - b*x^2)^(3/2)*(c + d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^4(e+fx^2)}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx=\frac{3\sqrt{d\,x^2+c}\,\sqrt{-b\,x^2+a}\,cfx-2\sqrt{d\,x^2+c}\,\sqrt{-b\,x^2+a}\,df\,x^3+8\Big(\int \frac{}{b^2d\,x^6-2ab}$$

input
```
int(x^4*(f*x^2+e)/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x)
```

output
```
(3*sqrt(c + d*x**2)*sqrt(a - b*x**2)*c*f*x - 2*sqrt(c + d*x**2)*sqrt(a - b
*x**2)*d*f*x**3 + 8*int((sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**4)/(a**2*c +
 a**2*d*x**2 - 2*a*b*c*x**2 - 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)
*a**2*d**2*f - int((sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**4)/(a**2*c + a**2
*d*x**2 - 2*a*b*c*x**2 - 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a*b*
c*d*f + 6*int((sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**4)/(a**2*c + a**2*d*x*
*2 - 2*a*b*c*x**2 - 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a*b*d**2*
e - 8*int((sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**4)/(a**2*c + a**2*d*x**2 -
 2*a*b*c*x**2 - 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a*b*d**2*f*x*
*2 + int((sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**4)/(a**2*c + a**2*d*x**2 -
2*a*b*c*x**2 - 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*b**2*c*d*f*x**
2 - 6*int((sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**4)/(a**2*c + a**2*d*x**2 -
 2*a*b*c*x**2 - 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*b**2*d**2*e*x
**2 - 3*int((sqrt(c + d*x**2)*sqrt(a - b*x**2))/(a**2*c + a**2*d*x**2 - 2*
a*b*c*x**2 - 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a**2*c**2*f + 3*
int((sqrt(c + d*x**2)*sqrt(a - b*x**2))/(a**2*c + a**2*d*x**2 - 2*a*b*c*x*
*2 - 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a*b*c**2*f*x**2)/(6*b*d*
*2*(a - b*x**2))
```

3.277 $\int \frac{x^2(e+fx^2)}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 251

$$\int \frac{x^2(e+fx^2)}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \frac{(be+af)x\sqrt{c+dx^2}}{b(bc+ad)\sqrt{a-bx^2}} - \frac{\sqrt{a}(bde+bcf+2adf)\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|-\frac{ad}{bc}\right)}{b^{3/2}d(bc+ad)\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}} + \frac{\sqrt{a}cf\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{b^{3/2}d\sqrt{a-bx^2}\sqrt{c+dx^2}}$$

output
```
(a*f+b*e)*x*(d*x^2+c)^(1/2)/b/(a*d+b*c)/(-b*x^2+a)^(1/2)-a^(1/2)*(2*a*d*f+
b*c*f+b*d*e)*(1-b*x^2/a)^(1/2)*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)
,(-a*d/b/c)^(1/2))/b^(3/2)/d/(a*d+b*c)/(-b*x^2+a)^(1/2)/(1+d*x^2/c)^(1/2)+
a^(1/2)*c*f*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(b^(1/2)*x/a^(1/2
),(-a*d/b/c)^(1/2))/b^(3/2)/d/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.98 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.89

$$\int \frac{x^2(e+fx^2)}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \frac{\sqrt{-\frac{b}{a}}d(be+af)x(c+dx^2)+ic(bde+bcf+2adf)\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\Big(ia}{b\sqrt{-\frac{b}{a}}d(}$$

input
```
Integrate[(x^2*(e + f*x^2))/((a - b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

output
```
(Sqrt[-(b/a)]*d*(b*e + a*f)*x*(c + d*x^2) + I*c*(b*d*e + b*c*f + 2*a*d*f)*
Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[-(b/a)]*x
], -((a*d)/(b*c))] - I*c*(b*c + a*d)*f*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2
)/c]*EllipticF[I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))])/(b*Sqrt[-(b/a)]
*d*(b*c + a*d)*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.265$, Rules used = {440, 25, 399, 323, 323, 321, 331, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(e+fx^2)}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx$$

$$\Big\downarrow \text{440}$$

$$\frac{\int -\frac{(bde+bcf+2adf)x^2+c(be+af)}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{b(ad+bc)}+\frac{x\sqrt{c+dx^2}(af+be)}{b\sqrt{a-bx^2}(ad+bc)}$$

$$\Big\downarrow \text{25}$$

$$\frac{x\sqrt{c+dx^2}(af+be)}{b\sqrt{a-bx^2}(ad+bc)}-\frac{\int \frac{(bde+bcf+2adf)x^2+c(be+af)}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{b(ad+bc)}$$

$$\Big\downarrow \; 399$$

$$\frac{x\sqrt{c+dx^2}(af+be)}{b\sqrt{a-bx^2}(ad+bc)}-\frac{\frac{(2adf+bcf+bde)\int\frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}-\frac{cf(ad+bc)\int\frac{1}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{d}}{b(ad+bc)}$$

$$\Big\downarrow \; 323$$

$$\frac{x\sqrt{c+dx^2}(af+be)}{b\sqrt{a-bx^2}(ad+bc)}-\frac{\frac{(2adf+bcf+bde)\int\frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}-\frac{cf\sqrt{\frac{dx^2}{c}+1}(ad+bc)\int\frac{1}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}dx}{d\sqrt{c+dx^2}}}{b(ad+bc)}$$

$$\Big\downarrow \; 323$$

$$\frac{x\sqrt{c+dx^2}(af+be)}{b\sqrt{a-bx^2}(ad+bc)}-\frac{\frac{(2adf+bcf+bde)\int\frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}-\frac{cf\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\int\frac{1}{\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}}dx}{d\sqrt{a-bx^2}\sqrt{c+dx^2}}}{b(ad+bc)}$$

$$\Big\downarrow \; 321$$

$$\frac{x\sqrt{c+dx^2}(af+be)}{b\sqrt{a-bx^2}(ad+bc)}-\frac{\frac{(2adf+bcf+bde)\int\frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}-\frac{\sqrt{a}cf\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}}{b(ad+bc)}$$

$$\Big\downarrow \; 331$$

$$\frac{x\sqrt{c+dx^2}(af+be)}{b\sqrt{a-bx^2}(ad+bc)}-\frac{\frac{\sqrt{1-\frac{bx^2}{a}}(2adf+bcf+bde)\int\frac{\sqrt{dx^2+c}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{\sqrt{a}cf\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}}{b(ad+bc)}$$

$$\Big\downarrow \; 330$$

$$\frac{x\sqrt{c+dx^2}(af+be)}{b\sqrt{a-bx^2}(ad+bc)}-\frac{\frac{\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(2adf+bcf+bde)\int\frac{\sqrt{\frac{dx^2}{c}+1}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}-\frac{\sqrt{a}cf\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}}{b(ad+bc)}$$

$$\Big\downarrow \; 327$$

$$\frac{\frac{x\sqrt{c+dx^2}(af+be)}{b\sqrt{a-bx^2}(ad+bc)}-\frac{\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(2adf+bcf+bde)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}-\frac{\sqrt{a}cf\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}}{b(ad+bc)}}{}$$

input
```
Int[(x^2*(e + f*x^2))/((a - b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

output
```
((b*e + a*f)*x*Sqrt[c + d*x^2])/(b*(b*c + a*d)*Sqrt[a - b*x^2]) - ((Sqrt[a
]*(b*d*e + b*c*f + 2*a*d*f)*Sqrt[1 - (b*x^2)/a]*Sqrt[c + d*x^2]*EllipticE[
ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*d*Sqrt[a - b*x^2]*S
qrt[1 + (d*x^2)/c]) - (Sqrt[a]*c*(b*c + a*d)*f*Sqrt[1 - (b*x^2)/a]*Sqrt[1
+ (d*x^2)/c]*EllipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt
[b]*d*Sqrt[a - b*x^2]*Sqrt[c + d*x^2]))/(b*(b*c + a*d))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 440
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[g*(b*e - a*f)*(g*x)^(m - 1)*(a +
 b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] - Simp[
g^2/(2*b*(b*c - a*d)*(p + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c +
d*x^2)^q*Simp[c*(b*e - a*f)*(m - 1) + (d*(b*e - a*f)*(m + 2*q + 1) - b*2*(c
*f - d*e)*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, q}, x] &&
 LtQ[p, -1] && GtQ[m, 1]
```

Maple [A] (verified)

Time = 6.23 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.45

method	result
elliptic	$\frac{\sqrt{(-b x^2+a)(x^2 d+c)}\left(-\frac{\left(-bd x^2-bc\right)x(af+be)}{b^2(ad+bc)\sqrt{\left(x^2-\frac{a}{b}\right)\left(-bd x^2-bc\right)}}-\frac{c(af+be)\sqrt{1-\frac{b x^2}{a}}\sqrt{1+\frac{d x^2}{c}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{b}{a}},\sqrt{-1-\frac{ad-bc}{cb}}\right)}{b(ad+bc)\sqrt{\frac{b}{a}}\sqrt{-bd x^4+ad x^2-x^2bc+ac}}-\frac{\left(-\frac{f}{b}-\frac{d(af+}{b(ad+}\cdots\right.}{}\cdots\right.}{\sqrt{-b x^2+a}\,\sqrt{x^2 d+c}}$
default	$\frac{\left(\sqrt{\frac{b}{a}}\,a d^2 f x^3+\sqrt{\frac{b}{a}}\,b d^2 e x^3+\sqrt{\frac{-b x^2+a}{a}}\sqrt{\frac{x^2 d+c}{c}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{b}{a}},\sqrt{-\frac{ad}{bc}}\right)acdf+\sqrt{\frac{-b x^2+a}{a}}\sqrt{\frac{x^2 d+c}{c}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{b}{a}},\sqrt{-\frac{ad}{bc}}\right)\cdots\right.}{\cdots}$

input
```
int(x^2*(f*x^2+e)/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE
)
```

output
```
((-b*x^2+a)*(d*x^2+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-(-b*d*x^2-
b*c)/b^2/(a*d+b*c)*x*(a*f+b*e)/((x^2-a/b)*(-b*d*x^2-b*c))^(1/2)-1/b*c/(a*d
+b*c)*(a*f+b*e)/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+
a*d*x^2-b*c*x^2+a*c)^(1/2)*EllipticF(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2
))-(-f/b-d/b*(a*f+b*e)/(a*d+b*c))*c/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2
/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(b/a)^(1/2),
(-1-(a*d-b*c)/c/b)^(1/2))-EllipticE(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2)
)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.37

$$\int \frac{x^2(e+fx^2)}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \frac{((a^2b^2de+(a^2b^2c+2\,a^3bd)f)x^3-(a^3bde+(a^3bc+2\,a^4d)f)x)\sqrt{-bd}\sqrt{\frac{a}{b}}E(}{}$$

input
```
integrate(x^2*(f*x^2+e)/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="fri
cas")
```

output
```
(((a^2*b^2*d*e + (a^2*b^2*c + 2*a^3*b*d)*f)*x^3 - (a^3*b*d*e + (a^3*b*c +
2*a^4*d)*f)*x)*sqrt(-b*d)*sqrt(a/b)*elliptic_e(arcsin(sqrt(a/b)/x), -b*c/(
a*d)) - (((b^4*c + a^2*b^2*d)*e + (2*a^3*b*d + (a^2*b^2 + a*b^3)*c)*f)*x^3
 - ((a*b^3*c + a^3*b*d)*e + (2*a^4*d + (a^3*b + a^2*b^2)*c)*f)*x)*sqrt(-b*
d)*sqrt(a/b)*elliptic_f(arcsin(sqrt(a/b)/x), -b*c/(a*d)) - (a^2*b^2*d*e -
(a*b^3*c + a^2*b^2*d)*f*x^2 + (a^2*b^2*c + 2*a^3*b*d)*f)*sqrt(-b*x^2 + a)*
sqrt(d*x^2 + c))/((a*b^5*c*d + a^2*b^4*d^2)*x^3 - (a^2*b^4*c*d + a^3*b^3*d
^2)*x)
```

Sympy [F]

$$\int \frac{x^2(e+fx^2)}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \int \frac{x^2(e+fx^2)}{(a-bx^2)^{\frac{3}{2}}\sqrt{c+dx^2}}\,dx$$

input
```
integrate(x**2*(f*x**2+e)/(-b*x**2+a)**(3/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(x**2*(e + f*x**2)/((a - b*x**2)**(3/2)*sqrt(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{x^2(e+fx^2)}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \int \frac{(fx^2+e)x^2}{(-bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}}\,dx$$

input
```
integrate(x^2*(f*x^2+e)/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="max
ima")
```

output
```
integrate((f*x^2 + e)*x^2/((-b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)), x)
```

Giac [F]

$$\int \frac{x^2(e+fx^2)}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \int \frac{(fx^2+e)x^2}{(-bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}}\,dx$$

input
```
integrate(x^2*(f*x^2+e)/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="gia
c")
```

output
```
integrate((f*x^2 + e)*x^2/((-b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(e+fx^2)}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx=\int \frac{x^2\,\left(f\,x^2+e\right)}{{\left(a-b\,x^2\right)}^{3/2}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int((x^2*(e + f*x^2))/((a - b*x^2)^(3/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int((x^2*(e + f*x^2))/((a - b*x^2)^(3/2)*(c + d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^2(e+fx^2)}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx=\frac{\sqrt{d\,x^2+c}\,\sqrt{-b\,x^2+a}\,ex+2\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{-b\,x^2+a}\,x^4}{b^2d\,x^6-2abd\,x^4+b^2c\,x^4+a^2d\,x^2-2abc\,x^2+a^2c}dx\right)a^2df}{}$$

input
```
int(x^2*(f*x^2+e)/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x)
```

output
```
(sqrt(c + d*x**2)*sqrt(a - b*x**2)*e*x + 2*int((sqrt(c + d*x**2)*sqrt(a -
b*x**2)*x**4)/(a**2*c + a**2*d*x**2 - 2*a*b*c*x**2 - 2*a*b*d*x**4 + b**2*c
*x**4 + b**2*d*x**6),x)*a**2*d*f + int((sqrt(c + d*x**2)*sqrt(a - b*x**2)*
x**4)/(a**2*c + a**2*d*x**2 - 2*a*b*c*x**2 - 2*a*b*d*x**4 + b**2*c*x**4 +
b**2*d*x**6),x)*a*b*d*e - 2*int((sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**4)/(
a**2*c + a**2*d*x**2 - 2*a*b*c*x**2 - 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*
x**6),x)*a*b*d*f*x**2 - int((sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**4)/(a**2
*c + a**2*d*x**2 - 2*a*b*c*x**2 - 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6
),x)*b**2*d*e*x**2 - int((sqrt(c + d*x**2)*sqrt(a - b*x**2))/(a**2*c + a**
2*d*x**2 - 2*a*b*c*x**2 - 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a**
2*c*e + int((sqrt(c + d*x**2)*sqrt(a - b*x**2))/(a**2*c + a**2*d*x**2 - 2*
a*b*c*x**2 - 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a*b*c*e*x**2)/(2
*a*d*(a - b*x**2))
```

3.278 $\int \frac{e+fx^2}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 237

$$\begin{aligned}\int \frac{e+fx^2}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx =& \frac{(be+af)x\sqrt{c+dx^2}}{a(bc+ad)\sqrt{a-bx^2}} \\ &- \frac{(be+af)\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{b}(bc+ad)\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}} \\ &+ \frac{e\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}\end{aligned}$$

output

```
(a*f+b*e)*x*(d*x^2+c)^(1/2)/a/(a*d+b*c)/(-b*x^2+a)^(1/2)-(a*f+b*e)*(1-b*x^
2/a)^(1/2)*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2),(-a*d/b/c)^(1/2))/a
^(1/2)/b^(1/2)/(a*d+b*c)/(-b*x^2+a)^(1/2)/(1+d*x^2/c)^(1/2)+e*(1-b*x^2/a)^
(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(b^(1/2)*x/a^(1/2),(-a*d/b/c)^(1/2))/a^(1
/2)/b^(1/2)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.05 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.89

$$\int \frac{e+fx^2}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \frac{\sqrt{-\frac{b}{a}}(be+af)x(c+dx^2)+ic(be+af)\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(i\text{arcsinh}\left(\sqrt{-\right.\right.}{a\sqrt{-\frac{b}{a}}(bc+a}$$

input
```
Integrate[(e + f*x^2)/((a - b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

output
```
(Sqrt[-(b/a)]*(b*e + a*f)*x*(c + d*x^2) + I*c*(b*e + a*f)*Sqrt[1 - (b*x^2)
/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c)
)] - I*(b*c + a*d)*e*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*A
rcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))])/(a*Sqrt[-(b/a)]*(b*c + a*d)*Sqrt[
a - b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$, Rules used = {402, 399, 323, 323, 321, 331, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx^2}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx$$

$$\Big\downarrow \ 402$$

$$\frac{\int \frac{a(de-cf)-d(be+af)x^2}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{a(ad+bc)}+\frac{x\sqrt{c+dx^2}(af+be)}{a\sqrt{a-bx^2}(ad+bc)}$$

$$\Big\downarrow \ 399$$

$$\frac{e(ad+bc)\int \frac{1}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx-(af+be)\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{a(ad+bc)}+\frac{x\sqrt{c+dx^2}(af+be)}{a\sqrt{a-bx^2}(ad+bc)}$$

$$\Big\downarrow \text{323}$$

$$\frac{\frac{e\sqrt{\frac{dx^2}{c}+1}(ad+bc)\int \frac{1}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}dx}{\sqrt{c+dx^2}}-(af+be)\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{a(ad+bc)}+\frac{x\sqrt{c+dx^2}(af+be)}{a\sqrt{a-bx^2}(ad+bc)}$$

$$\Big\downarrow \text{323}$$

$$\frac{\frac{e\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\int \frac{1}{\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}}dx}{\sqrt{a-bx^2}\sqrt{c+dx^2}}-(af+be)\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{a(ad+bc)}+\frac{x\sqrt{c+dx^2}(af+be)}{a\sqrt{a-bx^2}(ad+bc)}$$

$$\Big\downarrow \text{321}$$

$$\frac{\frac{\sqrt{a}e\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-(af+be)\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{a(ad+bc)}+\frac{x\sqrt{c+dx^2}(af+be)}{a\sqrt{a-bx^2}(ad+bc)}$$

$$\Big\downarrow \text{331}$$

$$\frac{\frac{\sqrt{a}e\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{1-\frac{bx^2}{a}}(af+be)\int \frac{\sqrt{dx^2+c}}{\sqrt{1-\frac{bx^2}{a}}}dx}{\sqrt{a-bx^2}}}{a(ad+bc)}+\frac{x\sqrt{c+dx^2}(af+be)}{a\sqrt{a-bx^2}(ad+bc)}$$

$$\Big\downarrow \text{330}$$

$$\frac{\frac{\sqrt{a}e\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(af+be)\int \frac{\sqrt{\frac{dx^2}{c}+1}}{\sqrt{1-\frac{bx^2}{a}}}dx}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}}{a(ad+bc)}+\frac{x\sqrt{c+dx^2}(af+be)}{a\sqrt{a-bx^2}(ad+bc)}$$

$$\Big\downarrow \text{327}$$

$$\frac{\frac{\sqrt{a}e\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(af+be)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}}{a(ad+bc)}+\frac{x\sqrt{c+dx^2}(af+be)}{a\sqrt{a-bx^2}(ad+bc)}$$

input
```
Int[(e + f*x^2)/((a - b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

output
```
((b*e + a*f)*x*Sqrt[c + d*x^2])/(a*(b*c + a*d)*Sqrt[a - b*x^2]) + (-((Sqrt
[a]*(b*e + a*f)*Sqrt[1 - (b*x^2)/a]*Sqrt[c + d*x^2]*EllipticE[ArcSin[(Sqrt
[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*Sqrt[a - b*x^2]*Sqrt[1 + (d*x^2
)/c])) + (Sqrt[a]*(b*c + a*d)*e*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*El
lipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*Sqrt[a - b*
x^2]*Sqrt[c + d*x^2]))/(a*(b*c + a*d))
```

Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 5.92 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.41

method	result
default	$\frac{(\sqrt{\frac{b}{a}}\,adf\,x^3+\sqrt{\frac{b}{a}}\,bde\,x^3+\sqrt{\frac{-b\,x^2+a}{a}}\,\sqrt{\frac{x^2d+c}{c}}\,\operatorname{EllipticF}(x\sqrt{\frac{b}{a}},\sqrt{-\frac{ad}{bc}})ade+\sqrt{\frac{-b\,x^2+a}{a}}\,\sqrt{\frac{x^2d+c}{c}}\,\operatorname{EllipticF}(x\sqrt{\frac{b}{a}},\sqrt{-\frac{ad}{bc}})bce-}{\sqrt{\frac{b}{a}}\,a(ad+b}$
elliptic	$\frac{\sqrt{(-b\,x^2+a)(x^2d+c)}\,(-\frac{(-bd\,x^2-bc)x(af+be)}{ba(ad+bc)\sqrt{(x^2-\frac{a}{b})(-bd\,x^2-bc)}}+\frac{(-\frac{f}{b}+\frac{af+be}{ba}-\frac{c(af+be)}{a(ad+bc)})\sqrt{1-\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\operatorname{EllipticF}(x\sqrt{\frac{b}{a}},\sqrt{-1-\frac{ad-bc}{cb}}}{\sqrt{\frac{b}{a}}\,\sqrt{-bd\,x^4+ad\,x^2-x^2bc+ac}}}{\sqrt{-b\,x^2+a}\,\sqrt{x^2d+c}}$

input
```
int((f*x^2+e)/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((b/a)^(1/2)*a*d*f*x^3+(b/a)^(1/2)*b*d*e*x^3+((-b*x^2+a)/a)^(1/2)*((d*x^2+
c)/c)^(1/2)*EllipticF(x*(b/a)^(1/2),(-a*d/b/c)^(1/2))*a*d*e+((-b*x^2+a)/a)
^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(b/a)^(1/2),(-a*d/b/c)^(1/2))*b*c*e
-((-b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(b/a)^(1/2),(-a*d/b/
c)^(1/2))*a*c*f-((-b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(b/a)
^(1/2),(-a*d/b/c)^(1/2))*b*c*e+(b/a)^(1/2)*a*c*f*x+(b/a)^(1/2)*b*c*e*x)*(d
*x^2+c)^(1/2)*(-b*x^2+a)^(1/2)/(b/a)^(1/2)/a/(a*d+b*c)/(-b*d*x^4+a*d*x^2-b
*c*x^2+a*c)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.06

$$\int \frac{e+fx^2}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \frac{(ab^2ce+a^2bcf)\sqrt{-bx^2+a}\sqrt{dx^2+c}x-(ab^2ce+a^2bcf-(b^3ce+ab^2cf)x^2}{}$$

input
```
integrate((f*x^2+e)/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="fricas"
)
```

output
```
((a*b^2*c*e + a^2*b*c*f)*sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)*x - (a*b^2*c*e +
 a^2*b*c*f - (b^3*c*e + a*b^2*c*f)*x^2)*sqrt(a*c)*sqrt(b/a)*elliptic_e(arc
sin(x*sqrt(b/a)), -a*d/(b*c)) - ((a^3 - a^2*b)*c*f - ((a^2*b - a*b^2)*c*f
- (b^3*c + a^2*b*d)*e)*x^2 - (a*b^2*c + a^3*d)*e)*sqrt(a*c)*sqrt(b/a)*elli
ptic_f(arcsin(x*sqrt(b/a)), -a*d/(b*c)))/(a^3*b^2*c^2 + a^4*b*c*d - (a^2*b
^3*c^2 + a^3*b^2*c*d)*x^2)
```

Sympy [F]

$$\int \frac{e+fx^2}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \int \frac{e+fx^2}{(a-bx^2)^{\frac{3}{2}}\sqrt{c+dx^2}}\,dx$$

input
```
integrate((f*x**2+e)/(-b*x**2+a)**(3/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral((e + f*x**2)/((a - b*x**2)**(3/2)*sqrt(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{e+fx^2}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx=\int \frac{fx^2+e}{(-bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}}\,dx$$

input
```
integrate((f*x^2+e)/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="maxima"
)
```

output
```
integrate((f*x^2 + e)/((-b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)), x)
```

Giac [F]

$$\int \frac{e+fx^2}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx=\int \frac{fx^2+e}{(-bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}}\,dx$$

input
```
integrate((f*x^2+e)/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((f*x^2 + e)/((-b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx^2}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx=\int \frac{f\,x^2+e}{(a-b\,x^2)^{3/2}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int((e + f*x^2)/((a - b*x^2)^(3/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int((e + f*x^2)/((a - b*x^2)^(3/2)*(c + d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{e+fx^2}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{-b\,x^2+a}\,x^2}{b^2d\,x^6-2abd\,x^4+b^2c\,x^4+a^2d\,x^2-2abc\,x^2+a^2c}dx\right)f + \left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{-b\,x^2+a}}{b^2d\,x^6-2abd\,x^4+b^2c\,x^4+a^2d\,x^2-2abc\,x^2+a^2c}dx\right)e$$

input
```
int((f*x^2+e)/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**2)/(a**2*c + a**2*d*x**2 - 2*a*b
*c*x**2 - 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*f + int((sqrt(c + d
*x**2)*sqrt(a - b*x**2))/(a**2*c + a**2*d*x**2 - 2*a*b*c*x**2 - 2*a*b*d*x*
*4 + b**2*c*x**4 + b**2*d*x**6),x)*e
```

3.279 $\int \frac{e+fx^2}{x^2\left(a-bx^2\right)^{3/2}\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 300

$$\int \frac{e+fx^2}{x^2\left(a-bx^2\right)^{3/2}\sqrt{c+dx^2}}\,dx = -\frac{e\sqrt{c+dx^2}}{acx\sqrt{a-bx^2}}+\frac{b(2bce+ade+acf)x\sqrt{c+dx^2}}{a^2c(bc+ad)\sqrt{a-bx^2}} - \frac{\sqrt{b}(2bce+ade+acf)\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{a^{3/2}c(bc+ad)\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}} + \frac{(2be+af)\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{a^{3/2}\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}$$

output

```
-e*(d*x^2+c)^(1/2)/a/c/x/(-b*x^2+a)^(1/2)+b*(a*c*f+a*d*e+2*b*c*e)*x*(d*x^2
+c)^(1/2)/a^2/c/(a*d+b*c)/(-b*x^2+a)^(1/2)-b^(1/2)*(a*c*f+a*d*e+2*b*c*e)*(
1-b*x^2/a)^(1/2)*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2),(-a*d/b/c)^(1
/2))/a^(3/2)/c/(a*d+b*c)/(-b*x^2+a)^(1/2)/(1+d*x^2/c)^(1/2)+(a*f+2*b*e)*(1
-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(b^(1/2)*x/a^(1/2),(-a*d/b/c)^(
1/2))/a^(3/2)/b^(1/2)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 7.47 (sec) , antiderivative size = 266, normalized size of antiderivative = 0.89

$$\int \frac{e+fx^2}{x^2(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \frac{-\sqrt{-\frac{b}{a}}(c+dx^2)\left(a^2de-2b^2cex^2+ab(-dex^2+c(e-fx^2))\right)+ibc(2bce}{}$$

input
```
Integrate[(e + f*x^2)/(x^2*(a - b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

output
```
(-(Sqrt[-(b/a)]*(c + d*x^2)*(a^2*d*e - 2*b^2*c*e*x^2 + a*b*(-(d*e*x^2) + c
*(e - f*x^2)))) + I*b*c*(2*b*c*e + a*d*e + a*c*f)*x*Sqrt[1 - (b*x^2)/a]*Sq
rt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))] - I
*c*(b*c + a*d)*(2*b*e + a*f)*x*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Ell
ipticF[I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))])/(a^2*Sqrt[-(b/a)]*c*(b*
c + a*d)*x*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.10, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.324$, Rules used = {441, 445, 25, 27, 399, 323, 323, 321, 331, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx^2}{x^2(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx$$

$$\Big\downarrow \; 441$$

$$\frac{\int \frac{d(be+af)x^2+2bce+ade+acf}{x^2\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{a(ad+bc)}+\frac{\sqrt{c+dx^2}(af+be)}{ax\sqrt{a-bx^2}(ad+bc)}$$

$$\Big\downarrow \; 445$$

$$\frac{-\frac{\int -\frac{d\left(ac(be+af)-b(2bce+ade+acf)x^2\right)}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}(acf+ade+2bce)}{acx}}{a(ad+bc)}+\frac{\sqrt{c+dx^2}(af+be)}{ax\sqrt{a-bx^2}(ad+bc)}$$

$\Big\downarrow$ 25

$$\frac{\frac{\int \frac{d\left(ac(be+af)-b(2bce+ade+acf)x^2\right)}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}(acf+ade+2bce)}{acx}}{a(ad+bc)}+\frac{\sqrt{c+dx^2}(af+be)}{ax\sqrt{a-bx^2}(ad+bc)}$$

$\Big\downarrow$ 27

$$\frac{\frac{d\int \frac{ac(be+af)-b(2bce+ade+acf)x^2}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}(acf+ade+2bce)}{acx}}{a(ad+bc)}+\frac{\sqrt{c+dx^2}(af+be)}{ax\sqrt{a-bx^2}(ad+bc)}$$

$\Big\downarrow$ 399

$$\frac{\frac{d\left(\frac{c(ad+bc)(af+2be)\int \frac{1}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{d}-\frac{b(acf+ade+2bce)\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}\right)}{ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}(acf+ade+2bce)}{acx}}{a(ad+bc)}+\frac{\sqrt{c+dx^2}(af+be)}{ax\sqrt{a-bx^2}(ad+bc)}$$

$\Big\downarrow$ 323

$$\frac{\frac{d\left(\frac{c\sqrt{\frac{dx^2}{c}+1}(ad+bc)(af+2be)\int \frac{1}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}dx}{d\sqrt{c+dx^2}}-\frac{b(acf+ade+2bce)\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}\right)}{ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}(acf+ade+2bce)}{acx}}{a(ad+bc)}+\frac{\sqrt{c+dx^2}(af+be)}{ax\sqrt{a-bx^2}(ad+bc)}$$

$\Big\downarrow$ 323

$$\frac{\frac{d\left(\frac{c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(af+2be)\int \frac{1}{\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}}dx}{d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{b(acf+ade+2bce)\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}\right)}{ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}(acf+ade+2bce)}{acx}}{a(ad+bc)}+\frac{\sqrt{c+dx^2}(af+be)}{ax\sqrt{a-bx^2}(ad+bc)}$$

$\Big\downarrow$ 321

$$\frac{\frac{d\left(\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(af+2be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{b(acf+ade+2bce)\int\frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}\right)}{ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}(acf+ade+2bce)}{acx}}{a(ad+bc)}+$$

$$\frac{\sqrt{c+dx^2}(af+be)}{ax\sqrt{a-bx^2}(ad+bc)}$$

$\Big\downarrow$ 331

$$\frac{\frac{d\left(\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(af+2be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{b\sqrt{1-\frac{bx^2}{a}}(acf+ade+2bce)\int\frac{\sqrt{dx^2+c}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}\right)}{ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}(acf+ade+}{acx}}{a(ad+bc)}$$

$$\frac{\sqrt{c+dx^2}(af+be)}{ax\sqrt{a-bx^2}(ad+bc)}$$

$\Big\downarrow$ 330

$$\frac{\frac{d\left(\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(af+2be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{b\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(acf+ade+2bce)\int\frac{\sqrt{\frac{dx^2}{c}+1}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}\right)}{ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}(acf}{acx}}{a(ad+bc)}$$

$$\frac{\sqrt{c+dx^2}(af+be)}{ax\sqrt{a-bx^2}(ad+bc)}$$

$\Big\downarrow$ 327

$$\frac{\frac{d\left(\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(af+2be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(acf+ade+2bce)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}\right)}{ac}-\sqrt{a-bx^2}}{a(ad+bc)}$$

$$\frac{\sqrt{c+dx^2}(af+be)}{ax\sqrt{a-bx^2}(ad+bc)}$$

input

```
Int[(e + f*x^2)/(x^2*(a - b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

output
```
((b*e + a*f)*Sqrt[c + d*x^2])/(a*(b*c + a*d)*x*Sqrt[a - b*x^2]) + (-(((2*b
*c*e + a*d*e + a*c*f)*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])/(a*c*x)) + (d*(-((S
qrt[a]*Sqrt[b]*(2*b*c*e + a*d*e + a*c*f)*Sqrt[1 - (b*x^2)/a]*Sqrt[c + d*x^
2]*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(d*Sqrt[a - b*x
^2]*Sqrt[1 + (d*x^2)/c])) + (Sqrt[a]*c*(b*c + a*d)*(2*b*e + a*f)*Sqrt[1 -
(b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a
*d)/(b*c))])/(Sqrt[b]*d*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])))/(a*c))/(a*(b*c
+ a*d))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 441
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
+ b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(a*g*2*(b*c - a*d)*(p + 1))), x] + Si
mp[1/(a*2*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2
)^q*Simp[c*(b*e - a*f)*(m + 1) + e*2*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(m
 + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, q},
 x] && LtQ[p, -1]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 10.42 (sec) , antiderivative size = 423, normalized size of antiderivative = 1.41

method	result
elliptic	$\frac{\sqrt{(-b\,x^2+a)(x^2d+c)}\left(-\frac{\left(-bd\,x^2-bc\right)x(af+be)}{a^2(ad+bc)\sqrt{\left(x^2-\frac{a}{b}\right)\left(-bd\,x^2-bc\right)}}-\frac{e\sqrt{-bd\,x^4+ad\,x^2-x^2bc+ac}}{a^2cx}+\frac{\left(\frac{af+be}{a^2}-\frac{bc(af+be)}{a^2(ad+bc)}\right)\sqrt{1-\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\ \mathrm{Elli}}{\sqrt{\frac{b}{a}}\,\sqrt{-bd\,x^4+ad\,x^2-x^2}}\right.}{\sqrt{-b\,x^2+}}$
risch	$-\frac{\sqrt{-b\,x^2+a}\,\sqrt{x^2d+c}\,e}{a^2cx}-\frac{\left(ac(af+be)\left(\frac{\left(-bd\,x^2-bc\right)x}{a(ad+bc)\sqrt{\left(x^2-\frac{a}{b}\right)\left(-bd\,x^2-bc\right)}}+\frac{\left(-\frac{1}{a}+\frac{bc}{a(ad+bc)}\right)\sqrt{1-\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\ \mathrm{EllipticF}\left(x\sqrt{\frac{b}{a}},\sqrt{-\right.\right.}{\sqrt{\frac{b}{a}}\,\sqrt{-bd\,x^4+ad\,x^2-x^2bc+ac}}\right.\right.}{}$
default	$\frac{\left(\sqrt{\frac{b}{a}}\,abcdf\,x^4+\sqrt{\frac{b}{a}}\,ab\,d^2e\,x^4+2\sqrt{\frac{b}{a}}\,b^2cde\,x^4+\sqrt{\frac{-b\,x^2+a}{a}}\,\sqrt{\frac{x^2d+c}{c}}\ \mathrm{EllipticF}\left(x\sqrt{\frac{b}{a}},\sqrt{-\frac{ad}{bc}}\right)a^2cdfx+\sqrt{\frac{-b\,x^2+a}{a}}\,\sqrt{\frac{x^2d+c}{c}}\ \mathrm{Ellipt}\right.}{}$

input
```
int((f*x^2+e)/x^2/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE
)
```

output
```
((-b*x^2+a)*(d*x^2+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-(-b*d*x^2-
b*c)/a^2/(a*d+b*c)*x*(a*f+b*e)/((x^2-a/b)*(-b*d*x^2-b*c))^(1/2)-1/a^2*e/c*
(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)/x+((a*f+b*e)/a^2-b*c/a^2/(a*d+b*c)*(a
*f+b*e))/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2
-b*c*x^2+a*c)^(1/2)*EllipticF(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2))-(-b*
d*(a*f+b*e)/a^2/(a*d+b*c)-b*d*e/a^2/c)*c/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+
d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(b/a)^(
1/2),(-1-(a*d-b*c)/c/b)^(1/2))-EllipticE(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^
(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.13

$$\int \frac{e+fx^2}{x^2\left(a-bx^2\right)^{3/2}\sqrt{c+dx^2}}\,dx =$$
$$-\frac{\left(\left(ab^3cf+\left(2\,b^4c+ab^3d\right)e\right)x^3-\left(a^2b^2cf+\left(2\,ab^3c+a^2b^2d\right)e\right)x\right)\sqrt{ac}\sqrt{\frac{b}{a}}E\left(\arcsin\left(x\sqrt{\frac{b}{a}}\right)\mid-\frac{ad}{bc}\right)-\left(\left(\left(2\,b^4c\right.\right.\right.}{}$$

input
```
integrate((f*x^2+e)/x^2/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="fri
cas")
```

output
```
-(((a*b^3*c*f + (2*b^4*c + a*b^3*d)*e)*x^3 - (a^2*b^2*c*f + (2*a*b^3*c + a
^2*b^2*d)*e)*x)*sqrt(a*c)*sqrt(b/a)*elliptic_e(arcsin(x*sqrt(b/a)), -a*d/(
b*c)) - (((2*b^4*c + (a^2*b^2 + a*b^3)*d)*e + (a*b^3*c + a^3*b*d)*f)*x^3 -
 ((2*a*b^3*c + (a^3*b + a^2*b^2)*d)*e + (a^2*b^2*c + a^4*d)*f)*x)*sqrt(a*c
)*sqrt(b/a)*elliptic_f(arcsin(x*sqrt(b/a)), -a*d/(b*c)) + ((a^2*b^2*c*f +
(2*a*b^3*c + a^2*b^2*d)*e)*x^2 - (a^2*b^2*c + a^3*b*d)*e)*sqrt(-b*x^2 + a)
*sqrt(d*x^2 + c))/((a^3*b^3*c^2 + a^4*b^2*c*d)*x^3 - (a^4*b^2*c^2 + a^5*b*
c*d)*x)
```

Sympy [F]

$$\int \frac{e+fx^2}{x^2\left(a-bx^2\right)^{3/2}\sqrt{c+dx^2}}\,dx = \int \frac{e+fx^2}{x^2\left(a-bx^2\right)^{\frac{3}{2}}\sqrt{c+dx^2}}\,dx$$

input
```
integrate((f*x**2+e)/x**2/(-b*x**2+a)**(3/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral((e + f*x**2)/(x**2*(a - b*x**2)**(3/2)*sqrt(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{e+fx^2}{x^2\left(a-bx^2\right)^{3/2}\sqrt{c+dx^2}}\,dx = \int \frac{fx^2+e}{\left(-bx^2+a\right)^{\frac{3}{2}}\sqrt{dx^2+c}\,x^2}\,dx$$

input
```
integrate((f*x^2+e)/x^2/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="max
ima")
```

output
```
integrate((f*x^2 + e)/((-b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*x^2), x)
```

Giac [F]

$$\int \frac{e+fx^2}{x^2\left(a-bx^2\right)^{3/2}\sqrt{c+dx^2}}\,dx=\int \frac{fx^2+e}{\left(-bx^2+a\right)^{\frac{3}{2}}\sqrt{dx^2+c}\,x^2}\,dx$$

input
```
integrate((f*x^2+e)/x^2/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="gia
c")
```

output
```
integrate((f*x^2 + e)/((-b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx^2}{x^2\left(a-bx^2\right)^{3/2}\sqrt{c+dx^2}}\,dx=\int \frac{f\,x^2+e}{x^2\left(a-b\,x^2\right)^{3/2}\sqrt{d\,x^2+c}}\,dx$$

input
```
int((e + f*x^2)/(x^2*(a - b*x^2)^(3/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int((e + f*x^2)/(x^2*(a - b*x^2)^(3/2)*(c + d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{e+fx^2}{x^2\left(a-bx^2\right)^{3/2}\sqrt{c+dx^2}}\,dx=\frac{-\sqrt{d\,x^2+c}\,\sqrt{-b\,x^2+a}\,e+\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{-b\,x^2+a}\,x^2}{b^2d\,x^6-2abd\,x^4+b^2c\,x^4+a^2d\,x^2-2abc\,x^2+a^2c}dx\right)ab}{}$$

input
```
int((f*x^2+e)/x^2/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x)
```

output

```
( - sqrt(c + d*x**2)*sqrt(a - b*x**2)*e + int((sqrt(c + d*x**2)*sqrt(a - b
*x**2)*x**2)/(a**2*c + a**2*d*x**2 - 2*a*b*c*x**2 - 2*a*b*d*x**4 + b**2*c*
x**4 + b**2*d*x**6),x)*a*b*d*e*x - int((sqrt(c + d*x**2)*sqrt(a - b*x**2)*
x**2)/(a**2*c + a**2*d*x**2 - 2*a*b*c*x**2 - 2*a*b*d*x**4 + b**2*c*x**4 +
b**2*d*x**6),x)*b**2*d*e*x**3 + int((sqrt(c + d*x**2)*sqrt(a - b*x**2))/(a
**2*c + a**2*d*x**2 - 2*a*b*c*x**2 - 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x
**6),x)*a**2*c*f*x + 2*int((sqrt(c + d*x**2)*sqrt(a - b*x**2))/(a**2*c + a
**2*d*x**2 - 2*a*b*c*x**2 - 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a
*b*c*e*x - int((sqrt(c + d*x**2)*sqrt(a - b*x**2))/(a**2*c + a**2*d*x**2 -
 2*a*b*c*x**2 - 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a*b*c*f*x**3
- 2*int((sqrt(c + d*x**2)*sqrt(a - b*x**2))/(a**2*c + a**2*d*x**2 - 2*a*b*
c*x**2 - 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*b**2*c*e*x**3)/(a*c*
x*(a - b*x**2))
```

3.280 $\int \frac{e+fx^2}{x^4(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 421

$$\int \frac{e+fx^2}{x^4(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx=-\frac{e\sqrt{c+dx^2}}{3acx^3\sqrt{a-bx^2}}-\frac{(4bce-2ade+3acf)\sqrt{c+dx^2}}{3a^2c^2x\sqrt{a-bx^2}}+\frac{b(8b^2c^2e-a^2d(2de-3cf)+3abc(de+2cf))\,x\sqrt{c+dx^2}}{3a^3c^2(bc+ad)\sqrt{a-bx^2}}-\frac{\sqrt{b}(8b^2c^2e-a^2d(2de-3cf)+3abc(de+2cf))\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\,E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{3a^{5/2}c^2(bc+ad)\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}}+\frac{\sqrt{b}(8bce-ade+6acf)\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{3a^{5/2}c\sqrt{a-bx^2}\sqrt{c+dx^2}}$$

output

```
-1/3*e*(d*x^2+c)^(1/2)/a/c/x^3/(-b*x^2+a)^(1/2)-1/3*(3*a*c*f-2*a*d*e+4*b*c
*e)*(d*x^2+c)^(1/2)/a^2/c^2/x/(-b*x^2+a)^(1/2)+1/3*b*(8*b^2*c^2*e-a^2*d*(-
3*c*f+2*d*e)+3*a*b*c*(2*c*f+d*e))*x*(d*x^2+c)^(1/2)/a^3/c^2/(a*d+b*c)/(-b*
x^2+a)^(1/2)-1/3*b^(1/2)*(8*b^2*c^2*e-a^2*d*(-3*c*f+2*d*e)+3*a*b*c*(2*c*f+
d*e))*(1-b*x^2/a)^(1/2)*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2),(-a*d/
b/c)^(1/2))/a^(5/2)/c^2/(a*d+b*c)/(-b*x^2+a)^(1/2)/(1+d*x^2/c)^(1/2)+1/3*b
^(1/2)*(6*a*c*f-a*d*e+8*b*c*e)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)*Ellipti
cF(b^(1/2)*x/a^(1/2),(-a*d/b/c)^(1/2))/a^(5/2)/c/(-b*x^2+a)^(1/2)/(d*x^2+c
)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.27 (sec) , antiderivative size = 374, normalized size of antiderivative = 0.89

$$\int \frac{e+fx^2}{x^4 (a-bx^2)^{3/2} \sqrt{c+dx^2}}\,dx = \frac{-\sqrt{-\frac{b}{a}}(c+dx^2)\left(-8b^3c^2ex^4+ab^2cx^2(4ce-3dex^2-6cfx^2)+a^3d(-2d\right.}{}$$

input
```
Integrate[(e + f*x^2)/(x^4*(a - b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

output
```
(-(Sqrt[-(b/a)]*(c + d*x^2)*(-8*b^3*c^2*e*x^4 + a*b^2*c*x^2*(4*c*e - 3*d*e
*x^2 - 6*c*f*x^2) + a^3*d*(-2*d*e*x^2 + c*(e + 3*f*x^2)) + a^2*b*(2*d^2*e*
x^4 + c*d*x^2*(2*e - 3*f*x^2) + c^2*(e + 3*f*x^2)))) + I*b*c*(8*b^2*c^2*e
+ 3*a*b*c*(d*e + 2*c*f) + a^2*d*(-2*d*e + 3*c*f))*x^3*Sqrt[1 - (b*x^2)/a]*
Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))] -
 I*b*c*(b*c + a*d)*(8*b*c*e - a*d*e + 6*a*c*f)*x^3*Sqrt[1 - (b*x^2)/a]*Sqr
t[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))])/(3*
a^3*Sqrt[-(b/a)]*c^2*(b*c + a*d)*x^3*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 440, normalized size of antiderivative = 1.05, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.382$, Rules used = {441, 445, 25, 445, 25, 27, 399, 323, 323, 321, 331, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx^2}{x^4 (a-bx^2)^{3/2} \sqrt{c+dx^2}}dx$$

$$\Big\downarrow \text{441}$$

$$\frac{\int \frac{3d(be+af)x^2+4bce+ade+3acf}{x^4\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{a(ad+bc)}+\frac{\sqrt{c+dx^2}(af+be)}{ax^3\sqrt{a-bx^2}(ad+bc)}$$

$$\Big\downarrow \text{445}$$

$$\frac{-\frac{\int -\frac{-d(2de-3cf)a^2+3bc(de+2cf)a+bd(4bce+ade+3acf)x^2+8b^2c^2e}{x^2\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{3ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}(3acf+ade+4bce)}{3acx^3}}{a(ad+bc)}+\frac{\sqrt{c+dx^2}(af+be)}{ax^3\sqrt{a-bx^2}(ad+bc)}$$

$\Big\downarrow$ 25

$$\frac{\frac{\int \frac{-d(2de-3cf)a^2+3bc(de+2cf)a+bd(4bce+ade+3acf)x^2+8b^2c^2e}{x^2\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{3ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}(3acf+ade+4bce)}{3acx^3}}{a(ad+bc)}+\frac{\sqrt{c+dx^2}(af+be)}{ax^3\sqrt{a-bx^2}(ad+bc)}$$

$\Big\downarrow$ 445

$$\frac{\frac{-\frac{\int -\frac{bd\left(ac(4bce+ade+3acf)-\left(-d(2de-3cf)a^2+3bc(de+2cf)a+8b^2c^2e\right)x^2\right)}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}\left(\frac{8b^2ce}{a}-\frac{2ad^2e}{c}+3adf+6bcf+3bde\right)}{x}}{3ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx}}{3}}{a(ad+bc)}$$
$$\frac{\sqrt{c+dx^2}(af+be)}{ax^3\sqrt{a-bx^2}(ad+bc)}$$

$\Big\downarrow$ 25

$$\frac{\frac{\frac{\int \frac{bd\left(ac(4bce+ade+3acf)-\left(-d(2de-3cf)a^2+3bc(de+2cf)a+8b^2c^2e\right)x^2\right)}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}\left(\frac{8b^2ce}{a}-\frac{2ad^2e}{c}+3adf+6bcf+3bde\right)}{x}}{3ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}(3}{3aca}}{a(ad+bc)}$$
$$\frac{\sqrt{c+dx^2}(af+be)}{ax^3\sqrt{a-bx^2}(ad+bc)}$$

$\Big\downarrow$ 27

$$\frac{\frac{\frac{bd\int \frac{ac(4bce+ade+3acf)-\left(-d(2de-3cf)a^2+3bc(de+2cf)a+8b^2c^2e\right)x^2}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}\left(\frac{8b^2ce}{a}-\frac{2ad^2e}{c}+3adf+6bcf+3bde\right)}{x}}{3ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}(3ac}{3acx^3}}{a(ad+bc)}$$
$$\frac{\sqrt{c+dx^2}(af+be)}{ax^3\sqrt{a-bx^2}(ad+bc)}$$

$\Big\downarrow$ 399

$$\frac{\frac{\frac{bd\left(\frac{c(ad+bc)(6acf-ade+8bce)\int \frac{1}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{d}-\frac{\left(a^2(-d)(2de-3cf)+3abc(2cf+de)+8b^2c^2e\right)\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}\right)}{ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}\left(\frac{8b^2ce}{a}-\frac{2ad^2e}{c}+3adf \cdots\right.}{x}}{3ac}}{a(ad+bc)}$$

$$\frac{\sqrt{c+dx^2}(af+be)}{ax^3\sqrt{a-bx^2}(ad+bc)}$$

$$\Big\downarrow \text{323}$$

$$\frac{\frac{\frac{bd\left(\frac{c\sqrt{\frac{dx^2}{c}+1}(ad+bc)(6acf-ade+8bce)\int \frac{1}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}dx}{d\sqrt{c+dx^2}}-\frac{\left(a^2(-d)(2de-3cf)+3abc(2cf+de)+8b^2c^2e\right)\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}\right)}{ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}\left(\frac{8b^2ce}{a}-\frac{2a \cdots}{}\right.}{x}}{3ac}}{a(ad+bc)}$$

$$\frac{\sqrt{c+dx^2}(af+be)}{ax^3\sqrt{a-bx^2}(ad+bc)}$$

$$\Big\downarrow \text{323}$$

$$\frac{\frac{\frac{bd\left(\frac{c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(6acf-ade+8bce)\int \frac{1}{\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}}dx}{d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\left(a^2(-d)(2de-3cf)+3abc(2cf+de)+8b^2c^2e\right)\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}\right)}{ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}\left(\cdots\right.}{\cdots}}{3ac}}{a(ad+bc)}$$

$$\frac{\sqrt{c+dx^2}(af+be)}{ax^3\sqrt{a-bx^2}(ad+bc)}$$

$$\Big\downarrow \text{321}$$

$$\frac{\frac{\frac{bd\left(\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(6acf-ade+8bce)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\left(a^2(-d)(2de-3cf)+3abc(2cf+de)+8b^2c^2e\right)\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}\right)}{ac}-\frac{\sqrt{a-bx^2}\cdots}{\cdots}}{3ac}}{a(ad+bc)}$$

$$\frac{\sqrt{c+dx^2}(af+be)}{ax^3\sqrt{a-bx^2}(ad+bc)}$$

$$\Big\downarrow \text{331}$$

$$\frac{\frac{\frac{bd\left(\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(6acf-ade+8bce)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{1-\frac{bx^2}{a}}\left(a^2(-d)(2de-3cf)+3abc(2cf+de)+8b^2c^2e\right)\int\frac{\sqrt{dx^2+c}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}\right)}{ac}-}{3ac}}{a(ad+bc)}$$

$$\frac{\sqrt{c+dx^2}(af+be)}{ax^3\sqrt{a-bx^2}(ad+bc)}$$

$\Big\downarrow$ 330

$$\frac{\frac{\frac{bd\left(\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(6acf-ade+8bce)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\left(a^2(-d)(2de-3cf)+3abc(2cf+de)+8b^2c^2e\right)\int\frac{\sqrt{\frac{dx^2}{c}+\cdots}}{\sqrt{1-\frac{bx\cdots}{a}}}}{d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}}{ac}}{3ac}}{a(ad+bc)}$$

$$\frac{\sqrt{c+dx^2}(af+be)}{ax^3\sqrt{a-bx^2}(ad+bc)}$$

$\Big\downarrow$ 327

$$\frac{\frac{\frac{bd\left(\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(6acf-ade+8bce)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\left(a^2(-d)(2de-3cf)+3abc(2cf+de)+8b^2c^2e\right)E\left(\operatorname{arcs}\cdots\right.}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}}{ac}}{3ac}}{a(ad+bc)}$$

$$\frac{\sqrt{c+dx^2}(af+be)}{ax^3\sqrt{a-bx^2}(ad+bc)}$$

input
```
Int[(e + f*x^2)/(x^4*(a - b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

output
```
((b*e + a*f)*Sqrt[c + d*x^2])/(a*(b*c + a*d)*x^3*Sqrt[a - b*x^2]) + (-1/3*
((4*b*c*e + a*d*e + 3*a*c*f)*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])/(a*c*x^3) +
(-((((8*b^2*c*e)/a + 3*b*d*e - (2*a*d^2*e)/c + 6*b*c*f + 3*a*d*f)*Sqrt[a -
 b*x^2]*Sqrt[c + d*x^2])/x) + (b*d*(-((Sqrt[a]*(8*b^2*c^2*e - a^2*d*(2*d*e
 - 3*c*f) + 3*a*b*c*(d*e + 2*c*f))*Sqrt[1 - (b*x^2)/a]*Sqrt[c + d*x^2]*Ell
ipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*d*Sqrt[a - b
*x^2]*Sqrt[1 + (d*x^2)/c])) + (Sqrt[a]*c*(b*c + a*d)*(8*b*c*e - a*d*e + 6*
a*c*f)*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[ArcSin[(Sqrt[b]*x
)/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*d*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])))
/(a*c))/(3*a*c))/(a*(b*c + a*d))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 441
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
+ b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(a*g*2*(b*c - a*d)*(p + 1))), x] + Si
mp[1/(a*2*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2
)^q*Simp[c*(b*e - a*f)*(m + 1) + e*2*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(m
 + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, q},
 x] && LtQ[p, -1]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 11.99 (sec) , antiderivative size = 508, normalized size of antiderivative = 1.21

method	result
elliptic	$\frac{\sqrt{(-b x^2+a)(x^2 d+c)}\, \left(-\frac{e\sqrt{-bd\,x^4+ad\,x^2-x^2bc+ac}}{3a^2c\,x^3}-\frac{(3acf-2ade+5bce)\sqrt{-bd\,x^4+ad\,x^2-x^2bc+ac}}{3a^3c^2x}-\frac{\left(-bd\,x^2-bc\right)bx(af+be)}{a^3(ad+bc)\sqrt{\left(x^2-\frac{a}{b}\right)\left(-bd\,x^2-bc\right)}}+\cdots\right.}{\cdots}$
risch	$-\frac{\sqrt{-b\,x^2+a}\,\sqrt{x^2d+c}\,(3acf\,x^2-2ade\,x^2+5bce\,x^2+ace)}{3a^3c^2x^3}-\frac{b\left(-\frac{(3acf-2ade+5bce)c\sqrt{1-\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\left(\mathrm{EllipticF}\left(x\sqrt{\frac{b}{a}},\sqrt{-1-\frac{ad-bc}{cb}}\right)\cdots\right.}{\sqrt{\frac{b}{a}}\,\sqrt{-bd\,x^4+ad\,x^2-x^2bc+ac}}\cdots\right.}{\cdots}$
default	Expression too large to display

input

```
int((f*x^2+e)/x^4/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE
)
```

output

```
((-b*x^2+a)*(d*x^2+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-1/3/a^2*e/
c*(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)/x^3-1/3/a^3/c^2*(3*a*c*f-2*a*d*e+5*
b*c*e)*(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)/x-(-b*d*x^2-b*c)*b/a^3/(a*d+b*
c)*x*(a*f+b*e)/((x^2-a/b)*(-b*d*x^2-b*c))^(1/2)+(1/3*b*d*e/a^2/c+b*(a*f+b*
e)/a^3-b^2*c/a^3/(a*d+b*c)*(a*f+b*e))/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x
^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)*EllipticF(x*(b/a)^(1/2),(
-1-(a*d-b*c)/c/b)^(1/2))-(-1/3*b*d*(3*a*c*f-2*a*d*e+5*b*c*e)/a^3/c^2-d*b^2
*(a*f+b*e)/(a*d+b*c)/a^3)*c/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2
)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(b/a)^(1/2),(-1-(a*d
-b*c)/c/b)^(1/2))-EllipticE(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 548, normalized size of antiderivative = 1.30

$$\int \frac{e+fx^2}{x^4\left(a-bx^2\right)^{3/2}\sqrt{c+dx^2}}\,dx =$$

$$-\frac{(((8\,b^4c^2+3\,ab^3cd-2\,a^2b^2d^2)e+3\,(2\,ab^3c^2+a^2b^2cd)f)x^5-((8\,ab^3c^2+3\,a^2b^2cd-2\,a^3bd^2)e+3\,(2\,a^2b^2\cdots}{\cdots}$$

input
```
integrate((f*x^2+e)/x^4/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="fri
cas")
```

output
```
-1/3*((((8*b^4*c^2 + 3*a*b^3*c*d - 2*a^2*b^2*d^2)*e + 3*(2*a*b^3*c^2 + a^2
*b^2*c*d)*f)*x^5 - ((8*a*b^3*c^2 + 3*a^2*b^2*c*d - 2*a^3*b*d^2)*e + 3*(2*a
^2*b^2*c^2 + a^3*b*c*d)*f)*x^3)*sqrt(a*c)*sqrt(b/a)*elliptic_e(arcsin(x*sq
rt(b/a)), -a*d/(b*c)) - (((8*b^4*c^2 + (4*a^2*b^2 + 3*a*b^3)*c*d + (a^3*b
 - 2*a^2*b^2)*d^2)*e + 3*(2*a*b^3*c^2 + (a^3*b + a^2*b^2)*c*d)*f)*x^5 - ((8
*a*b^3*c^2 + (4*a^3*b + 3*a^2*b^2)*c*d + (a^4 - 2*a^3*b)*d^2)*e + 3*(2*a^2
*b^2*c^2 + (a^4 + a^3*b)*c*d)*f)*x^3)*sqrt(a*c)*sqrt(b/a)*elliptic_f(arcsi
n(x*sqrt(b/a)), -a*d/(b*c)) + (((8*a*b^3*c^2 + 3*a^2*b^2*c*d - 2*a^3*b*d^2
)*e + 3*(2*a^2*b^2*c^2 + a^3*b*c*d)*f)*x^4 - (2*(2*a^2*b^2*c^2 + a^3*b*c*d
 - a^4*d^2)*e + 3*(a^3*b*c^2 + a^4*c*d)*f)*x^2 - (a^3*b*c^2 + a^4*c*d)*e)*
sqrt(-b*x^2 + a)*sqrt(d*x^2 + c))/((a^4*b^2*c^3 + a^5*b*c^2*d)*x^5 - (a^5*
b*c^3 + a^6*c^2*d)*x^3)
```

Sympy [F]

$$\int \frac{e+fx^2}{x^4 (a-bx^2)^{3/2} \sqrt{c+dx^2}}\,dx = \int \frac{e+fx^2}{x^4 (a-bx^2)^{\frac{3}{2}} \sqrt{c+dx^2}}\,dx$$

input
```
integrate((f*x**2+e)/x**4/(-b*x**2+a)**(3/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral((e + f*x**2)/(x**4*(a - b*x**2)**(3/2)*sqrt(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{e+fx^2}{x^4 (a-bx^2)^{3/2} \sqrt{c+dx^2}}\,dx = \int \frac{fx^2+e}{(-bx^2+a)^{\frac{3}{2}} \sqrt{dx^2+cx^4}}\,dx$$

input
```
integrate((f*x^2+e)/x^4/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="max
ima")
```

output
```
integrate((f*x^2 + e)/((-b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*x^4), x)
```

Giac [F]

$$\int \frac{e+fx^2}{x^4\left(a-bx^2\right)^{3/2}\sqrt{c+dx^2}}\,dx=\int \frac{fx^2+e}{\left(-bx^2+a\right)^{\frac{3}{2}}\sqrt{dx^2+c}\,x^4}\,dx$$

input
```
integrate((f*x^2+e)/x^4/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="gia
c")
```

output
```
integrate((f*x^2 + e)/((-b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx^2}{x^4\left(a-bx^2\right)^{3/2}\sqrt{c+dx^2}}\,dx=\int \frac{f\,x^2+e}{x^4\,{\left(a-b\,x^2\right)}^{3/2}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int((e + f*x^2)/(x^4*(a - b*x^2)^(3/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int((e + f*x^2)/(x^4*(a - b*x^2)^(3/2)*(c + d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{e+fx^2}{x^4\left(a-bx^2\right)^{3/2}\sqrt{c+dx^2}}\,dx=\frac{-\sqrt{d\,x^2+c}\,\sqrt{-b\,x^2+a}\,e-3\sqrt{d\,x^2+c}\,\sqrt{-b\,x^2+a}\,f\,x^2+3\left(\int \frac{}{b^2d\,x^6-2ab\ldots}\right.}{}$$

input
```
int((f*x^2+e)/x^4/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x)
```

output

```
( - sqrt(c + d*x**2)*sqrt(a - b*x**2)*e - 3*sqrt(c + d*x**2)*sqrt(a - b*x*
*2)*f*x**2 + 3*int((sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**2)/(a**2*c + a**2
*d*x**2 - 2*a*b*c*x**2 - 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a*b*
d*f*x**3 - 3*int((sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**2)/(a**2*c + a**2*d
*x**2 - 2*a*b*c*x**2 - 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*b**2*d
*f*x**5 - 2*int((sqrt(c + d*x**2)*sqrt(a - b*x**2))/(a**2*c*x**2 + a**2*d*
x**4 - 2*a*b*c*x**4 - 2*a*b*d*x**6 + b**2*c*x**6 + b**2*d*x**8),x)*a**2*d*
e*x**3 + 4*int((sqrt(c + d*x**2)*sqrt(a - b*x**2))/(a**2*c*x**2 + a**2*d*x
**4 - 2*a*b*c*x**4 - 2*a*b*d*x**6 + b**2*c*x**6 + b**2*d*x**8),x)*a*b*c*e*
x**3 + 2*int((sqrt(c + d*x**2)*sqrt(a - b*x**2))/(a**2*c*x**2 + a**2*d*x**
4 - 2*a*b*c*x**4 - 2*a*b*d*x**6 + b**2*c*x**6 + b**2*d*x**8),x)*a*b*d*e*x*
*5 - 4*int((sqrt(c + d*x**2)*sqrt(a - b*x**2))/(a**2*c*x**2 + a**2*d*x**4
- 2*a*b*c*x**4 - 2*a*b*d*x**6 + b**2*c*x**6 + b**2*d*x**8),x)*b**2*c*e*x**
5 + 6*int((sqrt(c + d*x**2)*sqrt(a - b*x**2))/(a**2*c + a**2*d*x**2 - 2*a*
b*c*x**2 - 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a*b*c*f*x**3 + 3*i
nt((sqrt(c + d*x**2)*sqrt(a - b*x**2))/(a**2*c + a**2*d*x**2 - 2*a*b*c*x**
2 - 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a*b*d*e*x**3 - 6*int((sqr
t(c + d*x**2)*sqrt(a - b*x**2))/(a**2*c + a**2*d*x**2 - 2*a*b*c*x**2 - 2*a
*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*b**2*c*f*x**5 - 3*int((sqrt(c +
d*x**2)*sqrt(a - b*x**2))/(a**2*c + a**2*d*x**2 - 2*a*b*c*x**2 - 2*a*b*...
```

3.281 $\int \frac{x^6}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 319

$$\begin{aligned}\int \frac{x^6}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx &= \frac{ax^3\sqrt{c+dx^2}}{b(bc+ad)\sqrt{a-bx^2}} \\ &+ \frac{(bc+4ad)x\sqrt{a-bx^2}\sqrt{c+dx^2}}{3b^2d(bc+ad)} \\ &+ \frac{\sqrt{a}(2b^2c^2-3abcd-8a^2d^2)\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{3b^{5/2}d^2(bc+ad)\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}} \\ &- \frac{2\sqrt{a}c(bc-2ad)\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{3b^{5/2}d^2\sqrt{a-bx^2}\sqrt{c+dx^2}}\end{aligned}$$

output

```
a*x^3*(d*x^2+c)^(1/2)/b/(a*d+b*c)/(-b*x^2+a)^(1/2)+1/3*(4*a*d+b*c)*x*(-b*x
^2+a)^(1/2)*(d*x^2+c)^(1/2)/b^2/d/(a*d+b*c)+1/3*a^(1/2)*(-8*a^2*d^2-3*a*b*
c*d+2*b^2*c^2)*(1-b*x^2/a)^(1/2)*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/
2),(-a*d/b/c)^(1/2))/b^(5/2)/d^2/(a*d+b*c)/(-b*x^2+a)^(1/2)/(1+d*x^2/c)^(1
/2)-2/3*a^(1/2)*c*(-2*a*d+b*c)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)*Ellipti
cF(b^(1/2)*x/a^(1/2),(-a*d/b/c)^(1/2))/b^(5/2)/d^2/(-b*x^2+a)^(1/2)/(d*x^2
+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.28 (sec) , antiderivative size = 269, normalized size of antiderivative = 0.84

$$\int \frac{x^6}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \frac{\sqrt{-\frac{b}{a}}dx(c+dx^2)\left(4a^2d-b^2cx^2+ab(c-dx^2)\right)+ic(-2b^2c^2+3abcd+8a^2c}{}$$

input
```
Integrate[x^6/((a - b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

output
```
(Sqrt[-(b/a)]*d*x*(c + d*x^2)*(4*a^2*d - b^2*c*x^2 + a*b*(c - d*x^2)) + I*
c*(-2*b^2*c^2 + 3*a*b*c*d + 8*a^2*d^2)*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2
)/c]*EllipticE[I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))] - (2*I)*c*(-(b^2
*c^2) + a*b*c*d + 2*a^2*d^2)*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Ellip
ticF[I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))])/(3*b^2*Sqrt[-(b/a)]*d^2*(
b*c + a*d)*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {372, 444, 399, 323, 323, 321, 331, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \text{372}$$

$$\frac{ax^3\sqrt{c+dx^2}}{b\sqrt{a-bx^2}(ad+bc)}-\frac{\int \frac{x^2\left((bc+4ad)x^2+3ac\right)}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{b(ad+bc)}$$

$$\Big\downarrow \text{444}$$

$$\frac{ax^3\sqrt{c+dx^2}}{b\sqrt{a-bx^2}(ad+bc)}-\frac{\frac{\int \frac{ac(bc+4ad)-\left(2b^2c^2-3abdc-8a^2d^2\right)x^2}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{3bd}-\frac{x\sqrt{a-bx^2}\sqrt{c+dx^2}(4ad+bc)}{3bd}}{b(ad+bc)}$$

$\downarrow$ 399

$$\frac{ax^3\sqrt{c+dx^2}}{b\sqrt{a-bx^2}(ad+bc)}-\frac{\frac{\frac{2c(bc-2ad)(ad+bc)\int \frac{1}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{d}-\frac{\left(-8a^2d^2-3abcd+2b^2c^2\right)\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}}{3bd}-\frac{x\sqrt{a-bx^2}\sqrt{c+dx^2}(4ad+bc)}{3bd}}{b(ad+bc)}$$

$\downarrow$ 323

$$\frac{ax^3\sqrt{c+dx^2}}{b\sqrt{a-bx^2}(ad+bc)}-\frac{\frac{\frac{2c\sqrt{\frac{dx^2}{c}+1}(bc-2ad)(ad+bc)\int \frac{1}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}dx}{d\sqrt{c+dx^2}}-\frac{\left(-8a^2d^2-3abcd+2b^2c^2\right)\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}}{3bd}-\frac{x\sqrt{a-bx^2}\sqrt{c+dx^2}(4ad+bc)}{3bd}}{b(ad+bc)}$$

$\downarrow$ 323

$$\frac{ax^3\sqrt{c+dx^2}}{b\sqrt{a-bx^2}(ad+bc)}-\frac{\frac{\frac{2c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(bc-2ad)(ad+bc)\int \frac{1}{\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}}dx}{d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\left(-8a^2d^2-3abcd+2b^2c^2\right)\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}}{3bd}-\frac{x\sqrt{a-bx^2}\sqrt{c+dx^2}(4ad+bc)}{3bd}}{b(ad+bc)}$$

$\downarrow$ 321

$$\frac{ax^3\sqrt{c+dx^2}}{b\sqrt{a-bx^2}(ad+bc)}-\frac{\frac{\frac{2\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(bc-2ad)(ad+bc)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\left(-8a^2d^2-3abcd+2b^2c^2\right)\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}}{3bd}-\frac{x\sqrt{a-bx^2}\sqrt{c+dx^2}(4ad+bc)}{3bd}}{b(ad+bc)}$$

$\downarrow$ 331

$$\frac{ax^3\sqrt{c+dx^2}}{b\sqrt{a-bx^2}(ad+bc)}-\frac{\frac{\frac{2\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(bc-2ad)(ad+bc)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{1-\frac{bx^2}{a}}\left(-8a^2d^2-3abcd+2b^2c^2\right)\int \frac{\sqrt{dx^2+c}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}}{3bd}-\frac{x\sqrt{a-bx^2}\sqrt{c+dx^2}(4ad+bc}{3bd}}{b(ad+bc)}$$

$$\Big\downarrow \ 330$$

$$\frac{ax^3\sqrt{c+dx^2}}{b\sqrt{a-bx^2}(ad+bc)} - \frac{\frac{\frac{2\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(bc-2ad)(ad+bc)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}} - \frac{\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\left(-8a^2d^2-3abcd+2b^2c^2\right)\int\frac{\sqrt{\frac{dx^2}{c}+1}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}}{3bd} - \frac{x\sqrt{a-bx^2}\sqrt{c+dx^2}}{3bd}}{b(ad+bc)}$$

$$\Big\downarrow \ 327$$

$$\frac{ax^3\sqrt{c+dx^2}}{b\sqrt{a-bx^2}(ad+bc)} - \frac{\frac{\frac{2\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(bc-2ad)(ad+bc)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}} - \frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\left(-8a^2d^2-3abcd+2b^2c^2\right)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}}{3bd} - \frac{x\sqrt{a-b}}{}}{b(ad+bc)}$$

input
```
Int[x^6/((a - b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

output
```
(a*x^3*Sqrt[c + d*x^2])/(b*(b*c + a*d)*Sqrt[a - b*x^2]) - (-1/3*((b*c + 4*
a*d)*x*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])/(b*d) + (-((Sqrt[a]*(2*b^2*c^2 - 3
*a*b*c*d - 8*a^2*d^2)*Sqrt[1 - (b*x^2)/a]*Sqrt[c + d*x^2]*EllipticE[ArcSin
[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*d*Sqrt[a - b*x^2]*Sqrt[1
+ (d*x^2)/c])) + (2*Sqrt[a]*c*(b*c - 2*a*d)*(b*c + a*d)*Sqrt[1 - (b*x^2)/a
]*Sqrt[1 + (d*x^2)/c]*EllipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c)
)])/(Sqrt[b]*d*Sqrt[a - b*x^2]*Sqrt[c + d*x^2]))/(3*b*d))/(b*(b*c + a*d))
```

Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

Maple [A] (verified)

Time = 12.18 (sec) , antiderivative size = 417, normalized size of antiderivative = 1.31

method	result
elliptic	$\frac{\sqrt{(-b x^2+a)(x^2 d+c)}\left(-\frac{(-bd x^2-bc)a^2 x}{b^3(ad+bc)\sqrt{\left(x^2-\frac{a}{b}\right)(-bd x^2-bc)}}+\frac{x\sqrt{-bd x^4+ad x^2-x^2bc+ac}}{3d b^2}+\frac{\left(-\frac{c a^2}{b^2(ad+bc)}-\frac{ac}{3b^2 d}\right)\sqrt{1-\frac{b x^2}{a}}\sqrt{1+\frac{d x^2}{c}}\,\mathrm{Elli}}{\sqrt{\frac{b}{a}}\sqrt{-bd x^4+ad x^2-x^2 b}}\right.}{\sqrt{-b}}$
default	$\frac{\left(-\sqrt{\frac{b}{a}}ab d^3 x^5-\sqrt{\frac{b}{a}}b^2 c d^2 x^5+4\sqrt{\frac{b}{a}}a^2 d^3 x^3-\sqrt{\frac{b}{a}}b^2 c^2 d x^3+4\sqrt{\frac{-b x^2+a}{a}}\sqrt{\frac{x^2 d+c}{c}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{b}{a}},\sqrt{-\frac{ad}{bc}}\right)a^2 c d^2+2\sqrt{\frac{-b x^2+a}{a}}\right.}{}$
risch	$\frac{\sqrt{-b x^2+a}\sqrt{x^2 d+c}\,x}{3b^2 d}-\frac{\left(-\frac{b(5ad-2bc)c\sqrt{1-\frac{b x^2}{a}}\sqrt{1+\frac{d x^2}{c}}\left(\mathrm{EllipticF}\left(x\sqrt{\frac{b}{a}},\sqrt{-1-\frac{ad-bc}{cb}}\right)-\mathrm{EllipticE}\left(x\sqrt{\frac{b}{a}},\sqrt{-1-\frac{ad-bc}{cb}}\right)\right)}{\sqrt{\frac{b}{a}}\sqrt{-bd x^4+ad x^2-x^2bc+ac}\,d}+\frac{abc\sqrt{1-}}{b}\right.}{}$

input
```
int(x^6/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((-b*x^2+a)*(d*x^2+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-(-b*d*x^2-
b*c)/b^3*a^2/(a*d+b*c)*x/((x^2-a/b)*(-b*d*x^2-b*c))^(1/2)+1/3/d/b^2*x*(-b*
d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)+(-1/b^2*c*a^2/(a*d+b*c)-1/3/b^2/d*a*c)/(b
/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*
c)^(1/2)*EllipticF(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2))-(-a/b^2-1/b^2*d
*a^2/(a*d+b*c)-1/3/d/b^2*(2*a*d-2*b*c))*c/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1
+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(b/a)^
(1/2),(-1-(a*d-b*c)/c/b)^(1/2))-EllipticE(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)
^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 390, normalized size of antiderivative = 1.22

$$\int \frac{x^6}{\left(a-bx^2\right)^{3/2}\sqrt{c+dx^2}}\,dx =$$

$$-\frac{((2\,ab^3c^2-3\,a^2b^2cd-8\,a^3bd^2)x^3-(2\,a^2b^2c^2-3\,a^3bcd-8\,a^4d^2)x)\sqrt{-bd}\sqrt{\frac{a}{b}}E(\arcsin\left(\frac{\sqrt{\frac{a}{b}}}{x}\right)|-\frac{bc}{ad})+((}{}$$

input
```
integrate(x^6/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*(((2*a*b^3*c^2 - 3*a^2*b^2*c*d - 8*a^3*b*d^2)*x^3 - (2*a^2*b^2*c^2 -
3*a^3*b*c*d - 8*a^4*d^2)*x)*sqrt(-b*d)*sqrt(a/b)*elliptic_e(arcsin(sqrt(a/
b)/x), -b*c/(a*d)) + ((8*a^3*b*d^2 - (2*a*b^3 - b^4)*c^2 + (3*a^2*b^2 + 4*
a*b^3)*c*d)*x^3 - (8*a^4*d^2 - (2*a^2*b^2 - a*b^3)*c^2 + (3*a^3*b + 4*a^2*
b^2)*c*d)*x)*sqrt(-b*d)*sqrt(a/b)*elliptic_f(arcsin(sqrt(a/b)/x), -b*c/(a*
d)) - (2*a*b^3*c^2 - 3*a^2*b^2*c*d - 8*a^3*b*d^2 + (b^4*c*d + a*b^3*d^2)*x
^4 - 2*(b^4*c^2 - a*b^3*c*d - 2*a^2*b^2*d^2)*x^2)*sqrt(-b*x^2 + a)*sqrt(d*
x^2 + c))/((b^6*c*d^2 + a*b^5*d^3)*x^3 - (a*b^5*c*d^2 + a^2*b^4*d^3)*x)
```

Sympy [F]

$$\int \frac{x^6}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \int \frac{x^6}{(a-bx^2)^{\frac{3}{2}}\sqrt{c+dx^2}}\,dx$$

input
```
integrate(x**6/(-b*x**2+a)**(3/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(x**6/((a - b*x**2)**(3/2)*sqrt(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{x^6}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \int \frac{x^6}{(-bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}}\,dx$$

input
```
integrate(x^6/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^6/((-b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)), x)
```

Giac [F]

$$\int \frac{x^6}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \int \frac{x^6}{(-bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}}\,dx$$

input
```
integrate(x^6/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^6/((-b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \int \frac{x^6}{(a-b\,x^2)^{3/2}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int(x^6/((a - b*x^2)^(3/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int(x^6/((a - b*x^2)^(3/2)*(c + d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^6}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \frac{3\sqrt{d\,x^2+c}\,\sqrt{-b\,x^2+a}\,cx-2\sqrt{d\,x^2+c}\,\sqrt{-b\,x^2+a}\,d\,x^3+8\left(\int \frac{\sqrt{d}}{b^2d\,x^6-2abd\,x^4}\right.}{}$$

input
```
int(x^6/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x)
```

output
```
(3*sqrt(c + d*x**2)*sqrt(a - b*x**2)*c*x - 2*sqrt(c + d*x**2)*sqrt(a - b*x
**2)*d*x**3 + 8*int((sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**4)/(a**2*c + a**
2*d*x**2 - 2*a*b*c*x**2 - 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a**
2*d**2 - int((sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**4)/(a**2*c + a**2*d*x**
2 - 2*a*b*c*x**2 - 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a*b*c*d -
8*int((sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**4)/(a**2*c + a**2*d*x**2 - 2*a
*b*c*x**2 - 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a*b*d**2*x**2 + i
nt((sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**4)/(a**2*c + a**2*d*x**2 - 2*a*b*
c*x**2 - 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*b**2*c*d*x**2 - 3*in
t((sqrt(c + d*x**2)*sqrt(a - b*x**2))/(a**2*c + a**2*d*x**2 - 2*a*b*c*x**2
 - 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a**2*c**2 + 3*int((sqrt(c
+ d*x**2)*sqrt(a - b*x**2))/(a**2*c + a**2*d*x**2 - 2*a*b*c*x**2 - 2*a*b*d
*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a*b*c**2*x**2)/(6*b*d**2*(a - b*x**2
))
```

3.282 $\int \frac{x^4}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 238

$$\int \frac{x^4}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \frac{ax\sqrt{c+dx^2}}{b(bc+ad)\sqrt{a-bx^2}} - \frac{\sqrt{a}(bc+2ad)\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\,E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{b^{3/2}d(bc+ad)\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}} + \frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{b^{3/2}d\sqrt{a-bx^2}\sqrt{c+dx^2}}$$

output
```
a*x*(d*x^2+c)^(1/2)/b/(a*d+b*c)/(-b*x^2+a)^(1/2)-a^(1/2)*(2*a*d+b*c)*(1-b*
x^2/a)^(1/2)*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2),(-a*d/b/c)^(1/2))
/b^(3/2)/d/(a*d+b*c)/(-b*x^2+a)^(1/2)/(1+d*x^2/c)^(1/2)+a^(1/2)*c*(1-b*x^2
/a)^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(b^(1/2)*x/a^(1/2),(-a*d/b/c)^(1/2))/
b^(3/2)/d/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.50 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.88

$$\int \frac{x^4}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \frac{a\sqrt{-\frac{b}{a}}dx(c+dx^2)+ic(bc+2ad)\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(i\text{arcsinh}\left(\sqrt{-\frac{b}{a}}x\right)\right.}{b\sqrt{-\frac{b}{a}}d(bc+ad)}$$

input
```
Integrate[x^4/((a - b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

output
```
(a*Sqrt[-(b/a)]*d*x*(c + d*x^2) + I*c*(b*c + 2*a*d)*Sqrt[1 - (b*x^2)/a]*Sq
rt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))] - I
*c*(b*c + a*d)*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh
[Sqrt[-(b/a)]*x], -((a*d)/(b*c))])/(b*Sqrt[-(b/a)]*d*(b*c + a*d)*Sqrt[a -
b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {372, 399, 323, 323, 321, 331, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \; 372$$

$$\frac{ax\sqrt{c+dx^2}}{b\sqrt{a-bx^2}(ad+bc)}-\frac{\int \frac{(bc+2ad)x^2+ac}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{b(ad+bc)}$$

$$\Big\downarrow \; 399$$

$$\frac{ax\sqrt{c+dx^2}}{b\sqrt{a-bx^2}(ad+bc)}-\frac{\frac{(2ad+bc)\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}-\frac{c(ad+bc)\int \frac{1}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{d}}{b(ad+bc)}$$

$$\Big\downarrow 323$$

$$\frac{ax\sqrt{c+dx^2}}{b\sqrt{a-bx^2}(ad+bc)}-\frac{\frac{(2ad+bc)\int\frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}-\frac{c\sqrt{\frac{dx^2}{c}+1}(ad+bc)\int\frac{1}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}dx}{d\sqrt{c+dx^2}}}{b(ad+bc)}$$

$$\Big\downarrow 323$$

$$\frac{ax\sqrt{c+dx^2}}{b\sqrt{a-bx^2}(ad+bc)}-\frac{\frac{(2ad+bc)\int\frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}-\frac{c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\int\frac{1}{\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}}dx}{d\sqrt{a-bx^2}\sqrt{c+dx^2}}}{b(ad+bc)}$$

$$\Big\downarrow 321$$

$$\frac{ax\sqrt{c+dx^2}}{b\sqrt{a-bx^2}(ad+bc)}-\frac{\frac{(2ad+bc)\int\frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}-\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}}{b(ad+bc)}$$

$$\Big\downarrow 331$$

$$\frac{ax\sqrt{c+dx^2}}{b\sqrt{a-bx^2}(ad+bc)}-\frac{\frac{\sqrt{1-\frac{bx^2}{a}}(2ad+bc)\int\frac{\sqrt{dx^2+c}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}}{b(ad+bc)}$$

$$\Big\downarrow 330$$

$$\frac{ax\sqrt{c+dx^2}}{b\sqrt{a-bx^2}(ad+bc)}-\frac{\frac{\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(2ad+bc)\int\frac{\sqrt{\frac{dx^2}{c}+1}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}-\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}}{b(ad+bc)}$$

$$\Big\downarrow 327$$

$$\frac{ax\sqrt{c+dx^2}}{b\sqrt{a-bx^2}(ad+bc)}-\frac{\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(2ad+bc)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}-\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}}{b(ad+bc)}$$

input
```
Int[x^4/((a - b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

output
```
(a*x*Sqrt[c + d*x^2])/(b*(b*c + a*d)*Sqrt[a - b*x^2]) - ((Sqrt[a]*(b*c + 2
*a*d)*Sqrt[1 - (b*x^2)/a]*Sqrt[c + d*x^2]*EllipticE[ArcSin[(Sqrt[b]*x)/Sqr
t[a]], -((a*d)/(b*c))])/(Sqrt[b]*d*Sqrt[a - b*x^2]*Sqrt[1 + (d*x^2)/c]) -
(Sqrt[a]*c*(b*c + a*d)*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[A
rcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*d*Sqrt[a - b*x^2]*Sq
rt[c + d*x^2]))/(b*(b*c + a*d))
```

Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

Maple [A] (verified)

Time = 6.23 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.32

method	result
default	$\frac{\left(\sqrt{\frac{b}{a}}\, a\, d^{2} x^{3}+\sqrt{\frac{-b\, x^{2}+a}{a}}\, \sqrt{\frac{x^{2} d+c}{c}}\, \operatorname{EllipticF}\left(x \sqrt{\frac{b}{a}}, \sqrt{-\frac{a d}{b c}}\right) a c d+\sqrt{\frac{-b\, x^{2}+a}{a}}\, \sqrt{\frac{x^{2} d+c}{c}}\, \operatorname{EllipticF}\left(x \sqrt{\frac{b}{a}}, \sqrt{-\frac{a d}{b c}}\right) b\, c^{2}-2 \sqrt{\frac{-b\, x^{2}+a}{a}}\right.}{b d \sqrt{\frac{b}{a}}\,(a d+b c)(-b d\, x^{4}}$
elliptic	$\frac{\sqrt{(-b\, x^{2}+a)(x^{2} d+c)}\left(-\frac{\left(-b d\, x^{2}-b c\right) a x}{b^{2}(a d+b c) \sqrt{\left(x^{2}-\frac{a}{b}\right)\left(-b d\, x^{2}-b c\right)}}-\frac{c a \sqrt{1-\frac{b\, x^{2}}{a}}\, \sqrt{1+\frac{d\, x^{2}}{c}}\, \operatorname{EllipticF}\left(x \sqrt{\frac{b}{a}}, \sqrt{-1-\frac{a d-b c}{c b}}\right)}{b(a d+b c) \sqrt{\frac{b}{a}}\, \sqrt{-b d\, x^{4}+a d\, x^{2}-x^{2} b c+a c}}-\frac{\left(-\frac{1}{b}-\frac{a d}{b(a d+b c)}\right) c}{}\right.}{\sqrt{-b\, x^{2}+a}\, \sqrt{x^{2} d+c}}$

input
```
int(x^4/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((b/a)^(1/2)*a*d^2*x^3+((-b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(
x*(b/a)^(1/2),(-a*d/b/c)^(1/2))*a*c*d+((-b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(
1/2)*EllipticF(x*(b/a)^(1/2),(-a*d/b/c)^(1/2))*b*c^2-2*((-b*x^2+a)/a)^(1/2
)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(b/a)^(1/2),(-a*d/b/c)^(1/2))*a*c*d-((-b
*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(b/a)^(1/2),(-a*d/b/c)^(1
/2))*b*c^2+(b/a)^(1/2)*a*c*d*x)*(d*x^2+c)^(1/2)*(-b*x^2+a)^(1/2)/b/d/(b/a)
^(1/2)/(a*d+b*c)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.07

$$\int \frac{x^4}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \frac{((ab^2c+2a^2bd)x^3-(a^2bc+2a^3d)x)\sqrt{-bd}\sqrt{\frac{a}{b}}E(\arcsin\left(\frac{\sqrt{\frac{a}{b}}}{x}\right)|-\frac{bc}{ad})-((}{}$$

input
```
integrate(x^4/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
(((a*b^2*c + 2*a^2*b*d)*x^3 - (a^2*b*c + 2*a^3*d)*x)*sqrt(-b*d)*sqrt(a/b)*
elliptic_e(arcsin(sqrt(a/b)/x), -b*c/(a*d)) - ((2*a^2*b*d + (a*b^2 + b^3)*
c)*x^3 - (2*a^3*d + (a^2*b + a*b^2)*c)*x)*sqrt(-b*d)*sqrt(a/b)*elliptic_f(
arcsin(sqrt(a/b)/x), -b*c/(a*d)) - (a*b^2*c + 2*a^2*b*d - (b^3*c + a*b^2*d
)*x^2)*sqrt(-b*x^2 + a)*sqrt(d*x^2 + c))/((b^5*c*d + a*b^4*d^2)*x^3 - (a*b
^4*c*d + a^2*b^3*d^2)*x)
```

Sympy [F]

$$\int \frac{x^4}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \int \frac{x^4}{(a-bx^2)^{\frac{3}{2}}\sqrt{c+dx^2}}\,dx$$

input
```
integrate(x**4/(-b*x**2+a)**(3/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(x**4/((a - b*x**2)**(3/2)*sqrt(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{x^4}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \int \frac{x^4}{(-bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}}\,dx$$

input
```
integrate(x^4/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^4/((-b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)), x)
```

Giac [F]

$$\int \frac{x^4}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \int \frac{x^4}{(-bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}}\,dx$$

input
```
integrate(x^4/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^4/((-b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \int \frac{x^4}{(a-b\,x^2)^{3/2}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int(x^4/((a - b*x^2)^(3/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int(x^4/((a - b*x^2)^(3/2)*(c + d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^4}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{d\,x^2+c}\,\sqrt{-b\,x^2+a}\,x^4}{b^2d\,x^6-2abd\,x^4+b^2c\,x^4+a^2d\,x^2-2abc\,x^2+a^2c}dx$$

input
```
int(x^4/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**4)/(a**2*c + a**2*d*x**2 - 2*a*b
*c*x**2 - 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)
```

3.283 $\int \frac{x^2}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 132

$$\int \frac{x^2}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx=\frac{x\sqrt{c+dx^2}}{(bc+ad)\sqrt{a-bx^2}}-\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}(bc+ad)\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}}$$

output
```
x*(d*x^2+c)^(1/2)/(a*d+b*c)/(-b*x^2+a)^(1/2)-a^(1/2)*(1-b*x^2/a)^(1/2)*(d*
x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2),(-a*d/b/c)^(1/2))/b^(1/2)/(a*d+b*
c)/(-b*x^2+a)^(1/2)/(1+d*x^2/c)^(1/2)
```

Mathematica [A] (verified)

Time = 1.65 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.92

$$\int \frac{x^2}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx=\frac{\sqrt{c+dx^2}\left(\sqrt{\frac{b}{a}}x\sqrt{1+\frac{dx^2}{c}}-\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\sqrt{\frac{b}{a}}x\right)|-\frac{ad}{bc}\right)\right)}{\sqrt{\frac{b}{a}}(bc+ad)\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}}$$

input
```
Integrate[x^2/((a - b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

output
```
(Sqrt[c + d*x^2]*(Sqrt[b/a]*x*Sqrt[1 + (d*x^2)/c] - Sqrt[1 - (b*x^2)/a]*El
lipticE[ArcSin[Sqrt[b/a]*x], -((a*d)/(b*c))]))/(Sqrt[b/a]*(b*c + a*d)*Sqrt
[a - b*x^2]*Sqrt[1 + (d*x^2)/c])
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {373, 331, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \text{373}$$

$$\frac{x\sqrt{c+dx^2}}{\sqrt{a-bx^2}(ad+bc)}-\frac{\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{ad+bc}$$

$$\Big\downarrow \text{331}$$

$$\frac{x\sqrt{c+dx^2}}{\sqrt{a-bx^2}(ad+bc)}-\frac{\sqrt{1-\frac{bx^2}{a}}\int \frac{\sqrt{dx^2+c}}{\sqrt{1-\frac{bx^2}{a}}}dx}{\sqrt{a-bx^2}(ad+bc)}$$

$$\Big\downarrow \text{330}$$

$$\frac{x\sqrt{c+dx^2}}{\sqrt{a-bx^2}(ad+bc)}-\frac{\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\int \frac{\sqrt{\frac{dx^2}{c}+1}}{\sqrt{1-\frac{bx^2}{a}}}dx}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}(ad+bc)}$$

$$\Big\downarrow \text{327}$$

$$\frac{x\sqrt{c+dx^2}}{\sqrt{a-bx^2}(ad+bc)}-\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}(ad+bc)}$$

input
```
Int[x^2/((a - b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

output
```
(x*Sqrt[c + d*x^2])/((b*c + a*d)*Sqrt[a - b*x^2]) - (Sqrt[a]*Sqrt[1 - (b*x
^2)/a]*Sqrt[c + d*x^2]*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c
))])/(Sqrt[b]*(b*c + a*d)*Sqrt[a - b*x^2]*Sqrt[1 + (d*x^2)/c])
```

Defintions of rubi rules used

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

Maple [A] (verified)

Time = 5.86 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.05

method	result
default	$\frac{\left(\sqrt{\frac{b}{a}}\, d\, x^{3}-\sqrt{\frac{-b\, x^{2}+a}{a}}\, \sqrt{\frac{x^{2} d+c}{c}}\, \operatorname{EllipticE}\left(x \sqrt{\frac{b}{a}}, \sqrt{-\frac{a d}{b c}}\right) c+\sqrt{\frac{b}{a}}\, c x\right) \sqrt{x^{2} d+c}\, \sqrt{-b\, x^{2}+a}}{\sqrt{\frac{b}{a}}\, (a d+b c)(-b d\, x^{4}+a d\, x^{2}-x^{2} b c+a c)}$
elliptic	$\frac{\sqrt{(-b\, x^{2}+a)(x^{2} d+c)}\, \left(-\frac{\left(-b d\, x^{2}-b c\right) x}{b(a d+b c) \sqrt{\left(x^{2}-\frac{a}{b}\right)\left(-b d\, x^{2}-b c\right)}}-\frac{c \sqrt{1-\frac{b\, x^{2}}{a}}\, \sqrt{1+\frac{d\, x^{2}}{c}}\, \operatorname{EllipticF}\left(x \sqrt{\frac{b}{a}}, \sqrt{-1-\frac{a d-b c}{c b}}\right)}{(a d+b c) \sqrt{\frac{b}{a}}\, \sqrt{-b d\, x^{4}+a d\, x^{2}-x^{2} b c+a c}}+\frac{c \sqrt{1-\frac{b\, x^{2}}{a}}\, \sqrt{1+\frac{d\, x^{2}}{c}}\, (\cdots}{\cdots}\right.}{\sqrt{-b\, x^{2}+a}\, \sqrt{x^{2} d+c}}$

input
```
int(x^2/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((b/a)^(1/2)*d*x^3-((-b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(b
/a)^(1/2),(-a*d/b/c)^(1/2))*c+(b/a)^(1/2)*c*x)*(d*x^2+c)^(1/2)*(-b*x^2+a)^
(1/2)/(b/a)^(1/2)/(a*d+b*c)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.24

$$\int \frac{x^2}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \frac{\sqrt{-bx^2+a}\sqrt{dx^2+c}abx+(b^2x^2-ab)\sqrt{ac}\sqrt{\frac{b}{a}}E(\arcsin\left(x\sqrt{\frac{b}{a}}\right)\mid -\frac{ad}{bc})+(\cdots}{a^2b^2c+a^3bd-(ab^3c+a^2b\cdots}$$

input
```
integrate(x^2/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
(sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)*a*b*x + (b^2*x^2 - a*b)*sqrt(a*c)*sqrt(b
/a)*elliptic_e(arcsin(x*sqrt(b/a)), -a*d/(b*c)) + ((a*b - b^2)*x^2 - a^2 +
 a*b)*sqrt(a*c)*sqrt(b/a)*elliptic_f(arcsin(x*sqrt(b/a)), -a*d/(b*c)))/(a^
2*b^2*c + a^3*b*d - (a*b^3*c + a^2*b^2*d)*x^2)
```

Sympy [F]

$$\int \frac{x^2}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \int \frac{x^2}{(a-bx^2)^{\frac{3}{2}}\sqrt{c+dx^2}}\,dx$$

input
```
integrate(x**2/(-b*x**2+a)**(3/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(x**2/((a - b*x**2)**(3/2)*sqrt(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{x^2}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \int \frac{x^2}{(-bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}}\,dx$$

input
```
integrate(x^2/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/((-b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)), x)
```

Giac [F]

$$\int \frac{x^2}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \int \frac{x^2}{(-bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}}\,dx$$

input
```
integrate(x^2/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2/((-b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \int \frac{x^2}{(a-b\,x^2)^{3/2}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int(x^2/((a - b*x^2)^(3/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int(x^2/((a - b*x^2)^(3/2)*(c + d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^2}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{d\,x^2+c}\,\sqrt{-b\,x^2+a}\,x^2}{b^2d\,x^6-2abd\,x^4+b^2c\,x^4+a^2d\,x^2-2abc\,x^2+a^2c}dx$$

input
```
int(x^2/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**2)/(a**2*c + a**2*d*x**2 - 2*a*b
*c*x**2 - 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)
```

3.284 $\int \frac{1}{\left(a-bx^2\right)^{3/2}\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 224

$$\int \frac{1}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \frac{bx\sqrt{c+dx^2}}{a(bc+ad)\sqrt{a-bx^2}} - \frac{\sqrt{b}c\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{a^{3/2}(bc+ad)\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}} + \frac{\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{a^{3/2}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}}$$

output
```
b*x*(d*x^2+c)^(1/2)/a/(a*d+b*c)/(-b*x^2+a)^(1/2)-b^(1/2)*c*(-b*x^2+a)^(1/2
)*(1+d*x^2/c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2),(-a*d/b/c)^(1/2))/a^(3/2)/
(a*d+b*c)/(1-b*x^2/a)^(1/2)/(d*x^2+c)^(1/2)+(-b*x^2+a)^(1/2)*(1+d*x^2/c)^(
1/2)*EllipticF(b^(1/2)*x/a^(1/2),(-a*d/b/c)^(1/2))/a^(3/2)/b^(1/2)/(1-b*x^
2/a)^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [A] (verified)

Time = 1.62 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.50

$$\int \frac{1}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \frac{bx(c+dx^2)+\frac{ad\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(\arcsin\left(\sqrt{-\frac{d}{c}}x\right)|-\frac{bc}{ad}\right)}{\sqrt{-\frac{d}{c}}}}{a(bc+ad)\sqrt{a-bx^2}\sqrt{c+dx^2}}$$

input
```
Integrate[1/((a - b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

output
```
(b*x*(c + d*x^2) + (a*d*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[
ArcSin[Sqrt[-(d/c)]*x], -((b*c)/(a*d))])/Sqrt[-(d/c)])/(a*(b*c + a*d)*Sqrt
[a - b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {316, 27, 326, 323, 323, 321, 331, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \text{316}$$

$$\frac{\int \frac{d\sqrt{a-bx^2}}{\sqrt{dx^2+c}}dx}{a(ad+bc)}+\frac{bx\sqrt{c+dx^2}}{a\sqrt{a-bx^2}(ad+bc)}$$

$$\Big\downarrow \text{27}$$

$$\frac{d\int \frac{\sqrt{a-bx^2}}{\sqrt{dx^2+c}}dx}{a(ad+bc)}+\frac{bx\sqrt{c+dx^2}}{a\sqrt{a-bx^2}(ad+bc)}$$

$$\Big\downarrow \text{326}$$

$$\frac{d\left(\frac{(ad+bc)\int \frac{1}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{d}-\frac{b\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}\right)}{a(ad+bc)}+\frac{bx\sqrt{c+dx^2}}{a\sqrt{a-bx^2}(ad+bc)}$$

$\Big\downarrow$ 323

$$\frac{d\left(\frac{\sqrt{\frac{dx^2}{c}+1}(ad+bc)\int \frac{1}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}dx}{d\sqrt{c+dx^2}}-\frac{b\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}\right)}{a(ad+bc)}+\frac{bx\sqrt{c+dx^2}}{a\sqrt{a-bx^2}(ad+bc)}$$

$\Big\downarrow$ 323

$$\frac{d\left(\frac{\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\int \frac{1}{\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}}dx}{d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{b\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}\right)}{a(ad+bc)}+\frac{bx\sqrt{c+dx^2}}{a\sqrt{a-bx^2}(ad+bc)}$$

$\Big\downarrow$ 321

$$\frac{d\left(\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{b\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}\right)}{a(ad+bc)}+\frac{bx\sqrt{c+dx^2}}{a\sqrt{a-bx^2}(ad+bc)}$$

$\Big\downarrow$ 331

$$\frac{d\left(\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{b\sqrt{1-\frac{bx^2}{a}}\int \frac{\sqrt{dx^2+c}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}\right)}{a(ad+bc)}+\frac{bx\sqrt{c+dx^2}}{a\sqrt{a-bx^2}(ad+bc)}$$

$\Big\downarrow$ 330

$$\frac{d\left(\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{b\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\int \frac{\sqrt{\frac{dx^2}{c}+1}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}\right)}{a(ad+bc)}+\frac{bx\sqrt{c+dx^2}}{a\sqrt{a-bx^2}(ad+bc)}$$

$\Big\downarrow$ 327

$$\frac{d\left(\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}\right)}{a(ad+bc)}+\frac{bx\sqrt{c+dx^2}}{a\sqrt{a-bx^2}(ad+bc)}$$

input
```
Int[1/((a - b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

output
```
(b*x*Sqrt[c + d*x^2])/(a*(b*c + a*d)*Sqrt[a - b*x^2]) + (d*(-((Sqrt[a]*Sqr
t[b]*Sqrt[1 - (b*x^2)/a]*Sqrt[c + d*x^2]*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt
[a]], -((a*d)/(b*c))])/(d*Sqrt[a - b*x^2]*Sqrt[1 + (d*x^2)/c])) + (Sqrt[a]
*(b*c + a*d)*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[ArcSin[(Sqr
t[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*d*Sqrt[a - b*x^2]*Sqrt[c + d*x
^2])))/(a*(b*c + a*d))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 326
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
b/d   Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Simp[(b*c - a*d)/d   In
t[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&
PosQ[d/c] && NegQ[b/a]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Maple [A] (verified)

Time = 5.82 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.11

method	result
default	$\frac{\left(\sqrt{\frac{b}{a}}\, bd\, x^{3}+a\sqrt{\frac{-b\, x^{2}+a}{a}}\, \sqrt{\frac{x^{2} d+c}{c}}\, \operatorname{EllipticF}\left(x\sqrt{\frac{b}{a}}, \sqrt{-\frac{ad}{bc}}\right) d+\sqrt{\frac{-b\, x^{2}+a}{a}}\, \sqrt{\frac{x^{2} d+c}{c}}\, \operatorname{EllipticF}\left(x\sqrt{\frac{b}{a}}, \sqrt{-\frac{ad}{bc}}\right) bc-\sqrt{\frac{-b\, x^{2}+a}{a}}\, \sqrt{x^{2}\cdots}\right.}{\sqrt{\frac{b}{a}}\, a(ad+bc)(-bd\, x^{4}+ad\, x^{2}-x^{2} bc+ac)}$
elliptic	$\frac{\sqrt{(-b\, x^{2}+a)(x^{2} d+c)}\, \left(-\frac{\left(-bd\, x^{2}-bc\right) x}{a(ad+bc)\sqrt{\left(x^{2}-\frac{a}{b}\right)\left(-bd\, x^{2}-bc\right)}}+\frac{\left(\frac{1}{a}-\frac{bc}{a(ad+bc)}\right)\sqrt{1-\frac{b\, x^{2}}{a}}\, \sqrt{1+\frac{d\, x^{2}}{c}}\, \operatorname{EllipticF}\left(x\sqrt{\frac{b}{a}}, \sqrt{-1-\frac{ad-bc}{cb}}\right)}{\sqrt{\frac{b}{a}}\, \sqrt{-bd\, x^{4}+ad\, x^{2}-x^{2} bc+ac}}+\frac{bc\sqrt{1-\cdots}}{\cdots}\right.}{\sqrt{-b\, x^{2}+a}\, \sqrt{x^{2} d+c}}$

input
```
int(1/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((b/a)^(1/2)*b*d*x^3+a*((-b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(
x*(b/a)^(1/2),(-a*d/b/c)^(1/2))*d+((-b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)
*EllipticF(x*(b/a)^(1/2),(-a*d/b/c)^(1/2))*b*c-((-b*x^2+a)/a)^(1/2)*((d*x^
2+c)/c)^(1/2)*EllipticE(x*(b/a)^(1/2),(-a*d/b/c)^(1/2))*b*c+(b/a)^(1/2)*b*
c*x)*(d*x^2+c)^(1/2)*(-b*x^2+a)^(1/2)/(b/a)^(1/2)/a/(a*d+b*c)/(-b*d*x^4+a*
d*x^2-b*c*x^2+a*c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.83

$$\int \frac{1}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \frac{\sqrt{-bx^2+a}\sqrt{dx^2+c}ab^2cx+(b^3cx^2-ab^2c)\sqrt{ac}\sqrt{\frac{b}{a}}E\left(\arcsin\left(x\sqrt{\frac{b}{a}}\right)|-\frac{ad}{bc}\right.}{a^3b^2c^2+a^4bcd-(a^2b^3c}$$

input
```
integrate(1/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
(sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)*a*b^2*c*x + (b^3*c*x^2 - a*b^2*c)*sqrt(a
*c)*sqrt(b/a)*elliptic_e(arcsin(x*sqrt(b/a)), -a*d/(b*c)) + (a*b^2*c + a^3
*d - (b^3*c + a^2*b*d)*x^2)*sqrt(a*c)*sqrt(b/a)*elliptic_f(arcsin(x*sqrt(b
/a)), -a*d/(b*c)))/(a^3*b^2*c^2 + a^4*b*c*d - (a^2*b^3*c^2 + a^3*b^2*c*d)*
x^2)
```

Sympy [F]

$$\int \frac{1}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \int \frac{1}{(a-bx^2)^{\frac{3}{2}}\sqrt{c+dx^2}}\,dx$$

input
```
integrate(1/(-b*x**2+a)**(3/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(1/((a - b*x**2)**(3/2)*sqrt(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \int \frac{1}{(-bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}}\,dx$$

input
```
integrate(1/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((-b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)), x)
```

Giac [F]

$$\int \frac{1}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \int \frac{1}{(-bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}}\,dx$$

input
```
integrate(1/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((-b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \int \frac{1}{(a-b\,x^2)^{3/2}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int(1/((a - b*x^2)^(3/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int(1/((a - b*x^2)^(3/2)*(c + d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{d\,x^2+c}\,\sqrt{-b\,x^2+a}}{b^2d\,x^6-2abd\,x^4+b^2c\,x^4+a^2d\,x^2-2abc\,x^2+a^2c}dx$$

input
```
int(1/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a - b*x**2))/(a**2*c + a**2*d*x**2 - 2*a*b*c*x*
*2 - 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)
```

3.285 $\int \frac{1}{x^2(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 280

$$\int \frac{1}{x^2(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = -\frac{\sqrt{c+dx^2}}{acx\sqrt{a-bx^2}}+\frac{b(2bc+ad)x\sqrt{c+dx^2}}{a^2c(bc+ad)\sqrt{a-bx^2}} - \frac{\sqrt{b}(2bc+ad)\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\,E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{a^{3/2}c(bc+ad)\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}} + \frac{2\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{a^{3/2}\sqrt{a-bx^2}\sqrt{c+dx^2}}$$

output
```
-(d*x^2+c)^(1/2)/a/c/x/(-b*x^2+a)^(1/2)+b*(a*d+2*b*c)*x*(d*x^2+c)^(1/2)/a^
2/c/(a*d+b*c)/(-b*x^2+a)^(1/2)-b^(1/2)*(a*d+2*b*c)*(1-b*x^2/a)^(1/2)*(d*x^
2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2),(-a*d/b/c)^(1/2))/a^(3/2)/c/(a*d+b*
c)/(-b*x^2+a)^(1/2)/(1+d*x^2/c)^(1/2)+2*b^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2
/c)^(1/2)*EllipticF(b^(1/2)*x/a^(1/2),(-a*d/b/c)^(1/2))/a^(3/2)/(-b*x^2+a)
^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.68 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.86

$$\int \frac{1}{x^2 (a-bx^2)^{3/2} \sqrt{c+dx^2}} \, dx = \frac{-\sqrt{-\frac{b}{a}}(c+dx^2)\left(a^2d-2b^2cx^2+ab(c-dx^2)\right)+ibc(2bc+ad)x\sqrt{1-\frac{bx}{a}}}{}$$

input
```
Integrate[1/(x^2*(a - b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

output
```
(-(Sqrt[-(b/a)]*(c + d*x^2)*(a^2*d - 2*b^2*c*x^2 + a*b*(c - d*x^2))) + I*b
*c*(2*b*c + a*d)*x*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*Arc
Sinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))] - (2*I)*b*c*(b*c + a*d)*x*Sqrt[1 - (
b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)
/(b*c))])/(a^2*Sqrt[-(b/a)]*c*(b*c + a*d)*x*Sqrt[a - b*x^2]*Sqrt[c + d*x^2
])
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.09, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.407$, Rules used = {374, 445, 25, 27, 399, 323, 323, 321, 331, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a-bx^2)^{3/2} \sqrt{c+dx^2}} \, dx$$

$$\Big\downarrow \text{374}$$

$$\frac{\int \frac{bdx^2+2bc+ad}{x^2\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{a(ad+bc)}+\frac{b\sqrt{c+dx^2}}{ax\sqrt{a-bx^2}(ad+bc)}$$

$$\Big\downarrow \text{445}$$

$$\frac{-\frac{\int -\frac{bd\left(ac-(2bc+ad)x^2\right)}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}(ad+2bc)}{acx}}{a(ad+bc)}+\frac{b\sqrt{c+dx^2}}{ax\sqrt{a-bx^2}(ad+bc)}$$

$$\Big\downarrow \textbf{25}$$

$$\frac{\frac{\int \frac{bd\left(ac-(2bc+ad)x^2\right)}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}(ad+2bc)}{acx}}{a(ad+bc)}+\frac{b\sqrt{c+dx^2}}{ax\sqrt{a-bx^2}(ad+bc)}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{\frac{bd\int \frac{ac-(2bc+ad)x^2}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}(ad+2bc)}{acx}}{a(ad+bc)}+\frac{b\sqrt{c+dx^2}}{ax\sqrt{a-bx^2}(ad+bc)}$$

$$\Big\downarrow \textbf{399}$$

$$\frac{\frac{bd\left(\frac{2c(ad+bc)\int \frac{1}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{d}-\frac{(ad+2bc)\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}\right)}{ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}(ad+2bc)}{acx}}{a(ad+bc)}+\frac{b\sqrt{c+dx^2}}{ax\sqrt{a-bx^2}(ad+bc)}$$

$$\Big\downarrow \textbf{323}$$

$$\frac{\frac{bd\left(\frac{2c\sqrt{\frac{dx^2}{c}+1}(ad+bc)\int \frac{1}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}dx}{d\sqrt{c+dx^2}}-\frac{(ad+2bc)\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}\right)}{ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}(ad+2bc)}{acx}}{a(ad+bc)}+\frac{b\sqrt{c+dx^2}}{ax\sqrt{a-bx^2}(ad+bc)}$$

$$\Big\downarrow \textbf{323}$$

$$\frac{\frac{bd\left(\frac{2c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\int \frac{1}{\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}}dx}{d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{(ad+2bc)\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}\right)}{ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}(ad+2bc)}{acx}}{a(ad+bc)}+\frac{b\sqrt{c+dx^2}}{ax\sqrt{a-bx^2}(ad+bc)}$$

$$\Big\downarrow \textbf{321}$$

$$\frac{\frac{bd\left(\frac{2\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{(ad+2bc)\int\frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}\right)}{ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}(ad+2bc)}{acx}}{a(ad+bc)}+\frac{b\sqrt{c+dx^2}}{ax\sqrt{a-bx^2}(ad+bc)}$$

$$\Big\downarrow \text{331}$$

$$\frac{\frac{bd\left(\frac{2\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{1-\frac{bx^2}{a}}(ad+2bc)\int\frac{\sqrt{dx^2+c}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}\right)}{ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}(ad+2bc)}{acx}}{a(ad+bc)}+\frac{b\sqrt{c+dx^2}}{ax\sqrt{a-bx^2}(ad+bc)}$$

$$\Big\downarrow \text{330}$$

$$\frac{\frac{bd\left(\frac{2\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(ad+2bc)\int\frac{\sqrt{\frac{dx^2}{c}+1}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}\right)}{ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}(ad+2bc)}{acx}}{a(ad+bc)}+\frac{b\sqrt{c+dx^2}}{ax\sqrt{a-bx^2}(ad+bc)}$$

$$\Big\downarrow \text{327}$$

$$\frac{\frac{bd\left(\frac{2\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(ad+2bc)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}\right)}{ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}(ad+2bc)}{acx}}{a(ad+bc)}+\frac{b\sqrt{c+dx^2}}{ax\sqrt{a-bx^2}(ad+bc)}$$

input

```
Int[1/(x^2*(a - b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

output
```
(b*Sqrt[c + d*x^2])/(a*(b*c + a*d)*x*Sqrt[a - b*x^2]) + (-(((2*b*c + a*d)*
Sqrt[a - b*x^2]*Sqrt[c + d*x^2])/(a*c*x)) + (b*d*(-((Sqrt[a]*(2*b*c + a*d)
*Sqrt[1 - (b*x^2)/a]*Sqrt[c + d*x^2]*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]]
, -((a*d)/(b*c))])/(Sqrt[b]*d*Sqrt[a - b*x^2]*Sqrt[1 + (d*x^2)/c])) + (2*S
qrt[a]*c*(b*c + a*d)*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[Arc
Sin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*d*Sqrt[a - b*x^2]*Sqrt
[c + d*x^2])))/(a*c))/(a*(b*c + a*d))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 10.35 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.42

method	result
default	$\frac{\left(\sqrt{\frac{b}{a}}\, ab\, d^2 x^4+2\sqrt{\frac{b}{a}}\, b^2 cd\, x^4+2\sqrt{\frac{-b\,x^2+a}{a}}\,\sqrt{\frac{x^2 d+c}{c}}\ \text{EllipticF}\left(x\sqrt{\frac{b}{a}},\sqrt{-\frac{ad}{bc}}\right) abcdx+2\sqrt{\frac{-b\,x^2+a}{a}}\,\sqrt{\frac{x^2d+c}{c}}\ \text{EllipticF}\left(x\sqrt{\frac{b}{a}},\sqrt{-\frac{a}{b}}\right.\right.}{}$
elliptic	$\frac{\sqrt{(-b\,x^2+a)(x^2d+c)}\left(-\frac{\left(-bd\,x^2-bc\right)bx}{a^2(ad+bc)\sqrt{\left(x^2-\frac{a}{b}\right)\left(-bd\,x^2-bc\right)}}-\frac{\sqrt{-bd\,x^4+ad\,x^2-x^2bc+ac}}{a^2cx}+\frac{\left(\frac{b}{a^2}-\frac{b^2c}{a^2(ad+bc)}\right)\sqrt{1-\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\ \text{EllipticF}}{\sqrt{\frac{b}{a}}\,\sqrt{-bd\,x^4+ad\,x^2-x^2bc+}}\right.}{\sqrt{-b\,x^2+a}}$
risch	$-\frac{\sqrt{-b\,x^2+a}\,\sqrt{x^2d+c}}{a^2cx}-\frac{b\left(-\frac{c\sqrt{1-\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\left(\text{EllipticF}\left(x\sqrt{\frac{b}{a}},\sqrt{-1-\frac{ad-bc}{cb}}\right)-\text{EllipticE}\left(x\sqrt{\frac{b}{a}},\sqrt{-1-\frac{ad-bc}{cb}}\right)\right)}{\sqrt{\frac{b}{a}}\,\sqrt{-bd\,x^4+ad\,x^2-x^2bc+ac}}+ac\left(\frac{(}{a(ad+bc)\sqrt{}}\right.\right.}{}$

input
```
int(1/x^2/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((b/a)^(1/2)*a*b*d^2*x^4+2*(b/a)^(1/2)*b^2*c*d*x^4+2*((-b*x^2+a)/a)^(1/2)*
((d*x^2+c)/c)^(1/2)*EllipticF(x*(b/a)^(1/2),(-a*d/b/c)^(1/2))*a*b*c*d*x+2*
((-b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(b/a)^(1/2),(-a*d/b/c
)^(1/2))*b^2*c^2*x-((-b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(b
/a)^(1/2),(-a*d/b/c)^(1/2))*a*b*c*d*x-2*((-b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)
^(1/2)*EllipticE(x*(b/a)^(1/2),(-a*d/b/c)^(1/2))*b^2*c^2*x-(b/a)^(1/2)*a^2
*d^2*x^2+2*(b/a)^(1/2)*b^2*c^2*x^2-(b/a)^(1/2)*a^2*c*d-(b/a)^(1/2)*a*b*c^2
)*(d*x^2+c)^(1/2)*(-b*x^2+a)^(1/2)/(a*d+b*c)/(b/a)^(1/2)/x/c/a^2/(-b*d*x^4
+a*d*x^2-b*c*x^2+a*c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^2\left(a-bx^2\right)^{3/2}\sqrt{c+dx^2}}\,dx =$$

$$-\frac{((2\,b^3c+ab^2d)x^3-(2\,ab^2c+a^2bd)x)\sqrt{ac}\sqrt{\frac{b}{a}}E(\arcsin\left(x\sqrt{\frac{b}{a}}\right)\mid-\frac{ad}{bc})-((2\,b^3c+(a^2b+ab^2)d)x^3-(2\,a}{(a^3b^2c^2+a^4bcd)}$$

input
```
integrate(1/x^2/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
-(((2*b^3*c + a*b^2*d)*x^3 - (2*a*b^2*c + a^2*b*d)*x)*sqrt(a*c)*sqrt(b/a)*
elliptic_e(arcsin(x*sqrt(b/a)), -a*d/(b*c)) - ((2*b^3*c + (a^2*b + a*b^2)*
d)*x^3 - (2*a*b^2*c + (a^3 + a^2*b)*d)*x)*sqrt(a*c)*sqrt(b/a)*elliptic_f(a
rcsin(x*sqrt(b/a)), -a*d/(b*c)) - (a^2*b*c + a^3*d - (2*a*b^2*c + a^2*b*d)
*x^2)*sqrt(-b*x^2 + a)*sqrt(d*x^2 + c))/((a^3*b^2*c^2 + a^4*b*c*d)*x^3 - (
a^4*b*c^2 + a^5*c*d)*x)
```

Sympy [F]

$$\int \frac{1}{x^2 (a-bx^2)^{3/2} \sqrt{c+dx^2}}\,dx = \int \frac{1}{x^2 (a-bx^2)^{\frac{3}{2}} \sqrt{c+dx^2}}\,dx$$

input
```
integrate(1/x**2/(-b*x**2+a)**(3/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(1/(x**2*(a - b*x**2)**(3/2)*sqrt(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{x^2 (a-bx^2)^{3/2} \sqrt{c+dx^2}}\,dx = \int \frac{1}{(-bx^2+a)^{\frac{3}{2}} \sqrt{dx^2+c}\,x^2}\,dx$$

input
```
integrate(1/x^2/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((-b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2 \left(a-bx^2\right)^{3/2} \sqrt{c+dx^2}}\,dx = \int \frac{1}{\left(-bx^2+a\right)^{\frac{3}{2}} \sqrt{dx^2+c}\,x^2}\,dx$$

input
```
integrate(1/x^2/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((-b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 \left(a-bx^2\right)^{3/2} \sqrt{c+dx^2}}\,dx = \int \frac{1}{x^2 \left(a-b\,x^2\right)^{3/2} \sqrt{d\,x^2+c}}\,dx$$

input
```
int(1/(x^2*(a - b*x^2)^(3/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int(1/(x^2*(a - b*x^2)^(3/2)*(c + d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^2 \left(a-bx^2\right)^{3/2} \sqrt{c+dx^2}}\,dx = \frac{-\sqrt{d\,x^2+c}\,\sqrt{-b\,x^2+a}+\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{-b\,x^2+a}\,x^2}{b^2d\,x^6-2abd\,x^4+b^2c\,x^4+a^2d\,x^2-2abc\,x^2+a^2c}dx\right) abd

output
```
( - sqrt(c + d*x**2)*sqrt(a - b*x**2) + int((sqrt(c + d*x**2)*sqrt(a - b*x
**2)*x**2)/(a**2*c + a**2*d*x**2 - 2*a*b*c*x**2 - 2*a*b*d*x**4 + b**2*c*x*
*4 + b**2*d*x**6),x)*a*b*d*x - int((sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**2
)/(a**2*c + a**2*d*x**2 - 2*a*b*c*x**2 - 2*a*b*d*x**4 + b**2*c*x**4 + b**2
*d*x**6),x)*b**2*d*x**3 + 2*int((sqrt(c + d*x**2)*sqrt(a - b*x**2))/(a**2*
c + a**2*d*x**2 - 2*a*b*c*x**2 - 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6)
,x)*a*b*c*x - 2*int((sqrt(c + d*x**2)*sqrt(a - b*x**2))/(a**2*c + a**2*d*x
**2 - 2*a*b*c*x**2 - 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*b**2*c*x
**3)/(a*c*x*(a - b*x**2))
```

3.286 $\int \frac{1}{x^4(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 376

$$\begin{aligned}\int \frac{1}{x^4(a-bx^2)^{3/2}\sqrt{c+dx^2}}\,dx &= -\frac{\sqrt{c+dx^2}}{3acx^3\sqrt{a-bx^2}}\\ &\quad-\frac{2(2bc-ad)\sqrt{c+dx^2}}{3a^2c^2x\sqrt{a-bx^2}}+\frac{b(8b^2c^2+3abcd-2a^2d^2)\,x\sqrt{c+dx^2}}{3a^3c^2(bc+ad)\sqrt{a-bx^2}}\\ &\quad-\frac{\sqrt{b}(8b^2c^2+3abcd-2a^2d^2)\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\,E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{3a^{5/2}c^2(bc+ad)\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}}\\ &\quad+\frac{\sqrt{b}(8bc-ad)\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{3a^{5/2}c\sqrt{a-bx^2}\sqrt{c+dx^2}}\end{aligned}$$

output

```
-1/3*(d*x^2+c)^(1/2)/a/c/x^3/(-b*x^2+a)^(1/2)-2/3*(-a*d+2*b*c)*(d*x^2+c)^(
1/2)/a^2/c^2/x/(-b*x^2+a)^(1/2)+1/3*b*(-2*a^2*d^2+3*a*b*c*d+8*b^2*c^2)*x*(
d*x^2+c)^(1/2)/a^3/c^2/(a*d+b*c)/(-b*x^2+a)^(1/2)-1/3*b^(1/2)*(-2*a^2*d^2+
3*a*b*c*d+8*b^2*c^2)*(1-b*x^2/a)^(1/2)*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x
/a^(1/2),(-a*d/b/c)^(1/2))/a^(5/2)/c^2/(a*d+b*c)/(-b*x^2+a)^(1/2)/(1+d*x^2
/c)^(1/2)+1/3*b^(1/2)*(-a*d+8*b*c)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)*Ell
ipticF(b^(1/2)*x/a^(1/2),(-a*d/b/c)^(1/2))/a^(5/2)/c/(-b*x^2+a)^(1/2)/(d*x
^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.77 (sec) , antiderivative size = 320, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^4 (a-bx^2)^{3/2} \sqrt{c+dx^2}}\,dx = \frac{-\sqrt{-\frac{b}{a}}(c+dx^2)\left(-8b^3c^2x^4+ab^2cx^2(4c-3dx^2)+a^3d(c-2dx^2)+a^2b(\right.}{}$$

input
```
Integrate[1/(x^4*(a - b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

output
```
(-(Sqrt[-(b/a)]*(c + d*x^2)*(-8*b^3*c^2*x^4 + a*b^2*c*x^2*(4*c - 3*d*x^2)
+ a^3*d*(c - 2*d*x^2) + a^2*b*(c^2 + 2*c*d*x^2 + 2*d^2*x^4))) - I*b*c*(-8*
b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^3*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/
c]*EllipticE[I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))] + I*b*c*(-8*b^2*c^
2 - 7*a*b*c*d + a^2*d^2)*x^3*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Ellip
ticF[I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))])/(3*a^3*Sqrt[-(b/a)]*c^2*(
b*c + a*d)*x^3*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 392, normalized size of antiderivative = 1.04, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.481$, Rules used = {374, 445, 25, 445, 25, 27, 399, 323, 323, 321, 331, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (a-bx^2)^{3/2} \sqrt{c+dx^2}}\,dx$$

$$\Big\downarrow \text{ 374}$$

$$\frac{\int \frac{3bdx^2+4bc+ad}{x^4\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{a(ad+bc)}+\frac{b\sqrt{c+dx^2}}{ax^3\sqrt{a-bx^2}(ad+bc)}$$

$$\Big\downarrow \text{ 445}$$

$$\frac{-\frac{\int -\frac{8b^2c^2+3abdc-2a^2d^2+bd(4bc+ad)x^2}{x^2\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{3ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}(ad+4bc)}{3acx^3}}{a(ad+bc)}+\frac{b\sqrt{c+dx^2}}{ax^3\sqrt{a-bx^2}(ad+bc)}$$

$\big\downarrow$ 25

$$\frac{\frac{\int \frac{8b^2c^2+3abdc-2a^2d^2+bd(4bc+ad)x^2}{x^2\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{3ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}(ad+4bc)}{3acx^3}}{a(ad+bc)}+\frac{b\sqrt{c+dx^2}}{ax^3\sqrt{a-bx^2}(ad+bc)}$$

$\big\downarrow$ 445

$$\frac{\frac{-\frac{\int -\frac{bd\left(ac(4bc+ad)-\left(8b^2c^2+3abdc-2a^2d^2\right)x^2\right)}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}\left(\frac{8b^2c}{a}-\frac{2ad^2}{c}+3bd\right)}{x}}{3ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}(ad+4bc)}{3acx^3}}{a(ad+bc)}+\frac{b\sqrt{c+dx^2}}{ax^3\sqrt{a-bx^2}(ad+bc)}$$

$\big\downarrow$ 25

$$\frac{\frac{\frac{\int \frac{bd\left(ac(4bc+ad)-\left(8b^2c^2+3abdc-2a^2d^2\right)x^2\right)}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}\left(\frac{8b^2c}{a}-\frac{2ad^2}{c}+3bd\right)}{x}}{3ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}(ad+4bc)}{3acx^3}}{a(ad+bc)}+\frac{b\sqrt{c+dx^2}}{ax^3\sqrt{a-bx^2}(ad+bc)}$$

$\big\downarrow$ 27

$$\frac{\frac{\frac{bd\int \frac{ac(4bc+ad)-\left(8b^2c^2+3abdc-2a^2d^2\right)x^2}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}\left(\frac{8b^2c}{a}-\frac{2ad^2}{c}+3bd\right)}{x}}{3ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}(ad+4bc)}{3acx^3}}{a(ad+bc)}+\frac{b\sqrt{c+dx^2}}{ax^3\sqrt{a-bx^2}(ad+bc)}$$

$\big\downarrow$ 399

$$\frac{\frac{\frac{bd\left(\frac{c(8bc-ad)(ad+bc)\int \frac{1}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{d}-\frac{\left(-2a^2d^2+3abcd+8b^2c^2\right)\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}\right)}{ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}\left(\frac{8b^2c}{a}-\frac{2ad^2}{c}+3bd\right)}{x}}{3ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}(ad+}{3acx^3}}{a(ad+bc)}$$

$$\frac{b\sqrt{c+dx^2}}{ax^3\sqrt{a-bx^2}(ad+bc)}$$

$\big\downarrow$ 323

$$\frac{\frac{\frac{bd\left(\frac{c\sqrt{\frac{dx^2}{c}+1}(8bc-ad)(ad+bc)\int \frac{1}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}dx}{d\sqrt{c+dx^2}}-\frac{\left(-2a^2d^2+3abcd+8b^2c^2\right)\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}\right)}{ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}\left(\frac{8b^2c}{a}-\frac{2ad^2}{c}+3bd\right)}{x}}{3ac}-\frac{\sqrt{a-bx^2}\sqrt{c+\cdots}}{3a\cdots}}{a(ad+bc)}$$

$$\frac{b\sqrt{c+dx^2}}{ax^3\sqrt{a-bx^2}(ad+bc)}$$

$\downarrow$ 323

$$\frac{\frac{\frac{bd\left(\frac{c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(8bc-ad)(ad+bc)\int \frac{1}{\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}}dx}{d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\left(-2a^2d^2+3abcd+8b^2c^2\right)\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}\right)}{ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}\left(\frac{8b^2c}{a}-\frac{2ad^2}{c}+3bd\right)}{x}}{3ac}-\frac{\sqrt{a\cdots}}{\cdots}}{a(ad+bc)}$$

$$\frac{b\sqrt{c+dx^2}}{ax^3\sqrt{a-bx^2}(ad+bc)}$$

$\downarrow$ 321

$$\frac{\frac{\frac{bd\left(\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(8bc-ad)(ad+bc)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\left(-2a^2d^2+3abcd+8b^2c^2\right)\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}\right)}{ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}\left(\frac{8b^2c}{a}-\frac{2ad^2}{c}+3bd\cdots\right.}{x}}{3ac}\cdots}{a(ad+bc)}$$

$$\frac{b\sqrt{c+dx^2}}{ax^3\sqrt{a-bx^2}(ad+bc)}$$

$\downarrow$ 331

$$\frac{\frac{\frac{bd\left(\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(8bc-ad)(ad+bc)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{1-\frac{bx^2}{a}}\left(-2a^2d^2+3abcd+8b^2c^2\right)\int \frac{\sqrt{dx^2+c}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}\right)}{ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}\left(\frac{8b^2c}{a}-\cdots\right.}{x}}{3ac}\cdots}{a(ad+bc)}$$

$$\frac{b\sqrt{c+dx^2}}{ax^3\sqrt{a-bx^2}(ad+bc)}$$

$\downarrow$ 330

$$\frac{\frac{\frac{bd\left(\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(8bc-ad)(ad+bc)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\left(-2a^2d^2+3abcd+8b^2c^2\right)\int\frac{\sqrt{\frac{dx^2}{c}+1}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}\right)}{ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx^2}(}{}}{3ac}}{a(ad+bc)}$$

$$\frac{b\sqrt{c+dx^2}}{ax^3\sqrt{a-bx^2}(ad+bc)}$$

$$\Big\downarrow \; 327$$

$$\frac{\frac{\frac{bd\left(\frac{\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(8bc-ad)(ad+bc)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\left(-2a^2d^2+3abcd+8b^2c^2\right)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}\right)}{ac}-\frac{\sqrt{a-}}{}}{3ac}}{a(ad+bc)}$$

$$\frac{b\sqrt{c+dx^2}}{ax^3\sqrt{a-bx^2}(ad+bc)}$$

input
```
Int[1/(x^4*(a - b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

output
```
(b*Sqrt[c + d*x^2])/(a*(b*c + a*d)*x^3*Sqrt[a - b*x^2]) + (-1/3*((4*b*c +
a*d)*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])/(a*c*x^3) + (-((((8*b^2*c)/a + 3*b*d
 - (2*a*d^2)/c)*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])/x) + (b*d*(-((Sqrt[a]*(8*
b^2*c^2 + 3*a*b*c*d - 2*a^2*d^2)*Sqrt[1 - (b*x^2)/a]*Sqrt[c + d*x^2]*Ellip
ticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*d*Sqrt[a - b*x
^2]*Sqrt[1 + (d*x^2)/c])) + (Sqrt[a]*c*(8*b*c - a*d)*(b*c + a*d)*Sqrt[1 -
(b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a
*d)/(b*c))])/(Sqrt[b]*d*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])))/(a*c))/(3*a*c))
/(a*(b*c + a*d))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 10.60 (sec) , antiderivative size = 468, normalized size of antiderivative = 1.24

method	result
elliptic	$\frac{\sqrt{(-b\,x^2+a)(x^2d+c)}\left(-\frac{\sqrt{-bd\,x^4+ad\,x^2-x^2bc+ac}}{3a^2c\,x^3}+\frac{(2ad-5bc)\sqrt{-bd\,x^4+ad\,x^2-x^2bc+ac}}{3a^3c^2x}-\frac{\left(-bd\,x^2-bc\right)b^2x}{a^3(ad+bc)\sqrt{\left(x^2-\frac{a}{b}\right)\left(-bd\,x^2-bc\right)}}+\frac{\left(\frac{bd}{3a^2c}+\right.}{}\right.}{}$
default	$-\frac{\left(2\sqrt{\frac{b}{a}}\,a^2b\,d^3x^6-3\sqrt{\frac{b}{a}}\,a\,b^2c\,d^2x^6-8\sqrt{\frac{b}{a}}\,b^3c^2d\,x^6+\sqrt{\frac{-b\,x^2+a}{a}}\,\sqrt{\frac{x^2d+c}{c}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{b}{a}},\sqrt{-\frac{ad}{bc}}\right)a^2bc\,d^2x^3-7\sqrt{\frac{-b\,x^2+a}{a}}\,\sqrt{\frac{x^2}{}}\right.}{}$
risch	$-\frac{\sqrt{-b\,x^2+a}\,\sqrt{x^2d+c}\,\left(-2ad\,x^2+5x^2bc+ac\right)}{3a^3c^2x^3}+\frac{b\left(\frac{acd\sqrt{1-\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{b}{a}},\sqrt{-1-\frac{ad-bc}{cb}}\right)}{\sqrt{\frac{b}{a}}\,\sqrt{-bd\,x^4+ad\,x^2-x^2bc+ac}}-\frac{2adc\sqrt{1-\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\left(\right.}{}\right.}{}$

input
```
int(1/x^4/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((-b*x^2+a)*(d*x^2+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-1/3/a^2/c*
(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)/x^3+1/3/a^3/c^2*(2*a*d-5*b*c)*(-b*d*x
^4+a*d*x^2-b*c*x^2+a*c)^(1/2)/x-(-b*d*x^2-b*c)*b^2/a^3/(a*d+b*c)*x/((x^2-a
/b)*(-b*d*x^2-b*c))^(1/2)+(1/3/a^2/c*b*d+b^2/a^3-b^3*c/a^3/(a*d+b*c))/(b/a
)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)
^(1/2)*EllipticF(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2))-(1/3*d*(2*a*d-5*b
*c)*b/a^3/c^2-b^3*d/(a*d+b*c)/a^3)*c/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^
2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(b/a)^(1/2)
,(-1-(a*d-b*c)/c/b)^(1/2))-EllipticE(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2
))))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 387, normalized size of antiderivative = 1.03

$$\int \frac{1}{x^4 (a-bx^2)^{3/2} \sqrt{c+dx^2}}\,dx =$$

$$-\frac{((8\,b^4c^2+3\,ab^3cd-2\,a^2b^2d^2)x^5-(8\,ab^3c^2+3\,a^2b^2cd-2\,a^3bd^2)x^3)\sqrt{ac}\sqrt{\frac{b}{a}}E(\arcsin\left(x\sqrt{\frac{b}{a}}\right)\,|-\frac{ad}{bc})-((}{}$$

input
```
integrate(1/x^4/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*(((8*b^4*c^2 + 3*a*b^3*c*d - 2*a^2*b^2*d^2)*x^5 - (8*a*b^3*c^2 + 3*a^
2*b^2*c*d - 2*a^3*b*d^2)*x^3)*sqrt(a*c)*sqrt(b/a)*elliptic_e(arcsin(x*sqrt
(b/a)), -a*d/(b*c)) - ((8*b^4*c^2 + (4*a^2*b^2 + 3*a*b^3)*c*d + (a^3*b - 2
*a^2*b^2)*d^2)*x^5 - (8*a*b^3*c^2 + (4*a^3*b + 3*a^2*b^2)*c*d + (a^4 - 2*a
^3*b)*d^2)*x^3)*sqrt(a*c)*sqrt(b/a)*elliptic_f(arcsin(x*sqrt(b/a)), -a*d/(
b*c)) - (a^3*b*c^2 + a^4*c*d - (8*a*b^3*c^2 + 3*a^2*b^2*c*d - 2*a^3*b*d^2)
*x^4 + 2*(2*a^2*b^2*c^2 + a^3*b*c*d - a^4*d^2)*x^2)*sqrt(-b*x^2 + a)*sqrt(
d*x^2 + c))/((a^4*b^2*c^3 + a^5*b*c^2*d)*x^5 - (a^5*b*c^3 + a^6*c^2*d)*x^3
)
```

Sympy [F]

$$\int \frac{1}{x^4 (a-bx^2)^{3/2} \sqrt{c+dx^2}}\,dx = \int \frac{1}{x^4 (a-bx^2)^{\frac{3}{2}} \sqrt{c+dx^2}}\,dx$$

input
```
integrate(1/x**4/(-b*x**2+a)**(3/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(1/(x**4*(a - b*x**2)**(3/2)*sqrt(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{x^4 (a-bx^2)^{3/2} \sqrt{c+dx^2}}\,dx = \int \frac{1}{(-bx^2+a)^{\frac{3}{2}} \sqrt{dx^2+c}x^4}\,dx$$

input
```
integrate(1/x^4/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((-b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*x^4), x)
```

Giac [F]

$$\int \frac{1}{x^4 (a-bx^2)^{3/2} \sqrt{c+dx^2}}\,dx = \int \frac{1}{(-bx^2+a)^{\frac{3}{2}} \sqrt{dx^2+c}x^4}\,dx$$

input
```
integrate(1/x^4/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((-b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (a-bx^2)^{3/2} \sqrt{c+dx^2}}\,dx = \int \frac{1}{x^4 \left(a-b\,x^2\right)^{3/2} \sqrt{d\,x^2+c}}\,dx$$

input
```
int(1/(x^4*(a - b*x^2)^(3/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int(1/(x^4*(a - b*x^2)^(3/2)*(c + d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^4 (a-bx^2)^{3/2} \sqrt{c+dx^2}}\,dx = \frac{-\sqrt{d\,x^2+c}\,\sqrt{-b\,x^2+a}\,ac+2\sqrt{d\,x^2+c}\,\sqrt{-b\,x^2+a}\,ad\,x^2-4\sqrt{d\,x^2+c}}{}$$

input
```
int(1/x^4/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x)
```

output
```
( - sqrt(c + d*x**2)*sqrt(a - b*x**2)*a*c + 2*sqrt(c + d*x**2)*sqrt(a - b*
x**2)*a*d*x**2 - 4*sqrt(c + d*x**2)*sqrt(a - b*x**2)*b*c*x**2 - 2*int((sqr
t(c + d*x**2)*sqrt(a - b*x**2)*x**2)/(a**2*c + a**2*d*x**2 - 2*a*b*c*x**2
- 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a**2*b*d**2*x**3 + 4*int((s
qrt(c + d*x**2)*sqrt(a - b*x**2)*x**2)/(a**2*c + a**2*d*x**2 - 2*a*b*c*x**
2 - 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a*b**2*c*d*x**3 + 2*int((
sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**2)/(a**2*c + a**2*d*x**2 - 2*a*b*c*x*
*2 - 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a*b**2*d**2*x**5 - 4*int
((sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**2)/(a**2*c + a**2*d*x**2 - 2*a*b*c*
x**2 - 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*b**3*c*d*x**5 - int((s
qrt(c + d*x**2)*sqrt(a - b*x**2))/(a**2*c + a**2*d*x**2 - 2*a*b*c*x**2 - 2
*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a**2*b*c*d*x**3 + 8*int((sqrt(
c + d*x**2)*sqrt(a - b*x**2))/(a**2*c + a**2*d*x**2 - 2*a*b*c*x**2 - 2*a*b
*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a*b**2*c**2*x**3 + int((sqrt(c + d
*x**2)*sqrt(a - b*x**2))/(a**2*c + a**2*d*x**2 - 2*a*b*c*x**2 - 2*a*b*d*x*
*4 + b**2*c*x**4 + b**2*d*x**6),x)*a*b**2*c*d*x**5 - 8*int((sqrt(c + d*x**
2)*sqrt(a - b*x**2))/(a**2*c + a**2*d*x**2 - 2*a*b*c*x**2 - 2*a*b*d*x**4 +
 b**2*c*x**4 + b**2*d*x**6),x)*b**3*c**2*x**5)/(3*a**2*c**2*x**3*(a - b*x*
*2))
```

3.287 $\int \frac{x^8}{(a-bx^2)^{3/2}\sqrt{c+dx^2}(e+fx^2)}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 540

$$\int \frac{x^8}{(a-bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \frac{a^3x\sqrt{c+dx^2}}{b^2(bc+ad)(be+af)\sqrt{a-bx^2}}$$
$$+\frac{x\sqrt{a-bx^2}\sqrt{c+dx^2}}{3b^2df}$$
$$-\frac{\sqrt{a}(8a^3d^2f^2-b^3ce(3de+2cf)+a^2bdf(2de+3cf)-ab^2(3d^2e^2+2c^2f^2))\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\arcsin\right.}{3b^{5/2}d^2(bc+ad)f^2(be+af)\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}}$$
$$+\frac{\sqrt{a}(4a^2cdf^3+abcf^2(de-2cf)-b^2e(3d^2e^2+3cdef+2c^2f^2))\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right.\right.}{3b^{5/2}d^2f^3(be+af)\sqrt{a-bx^2}\sqrt{c+dx^2}}$$
$$+\frac{\sqrt{a}e^3\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}f^3(be+af)\sqrt{a-bx^2}\sqrt{c+dx^2}}$$

output
```
a^3*x*(d*x^2+c)^(1/2)/b^2/(a*d+b*c)/(a*f+b*e)/(-b*x^2+a)^(1/2)+1/3*x*(-b*x
^2+a)^(1/2)*(d*x^2+c)^(1/2)/b^2/d/f-1/3*a^(1/2)*(8*a^3*d^2*f^2-b^3*c*e*(2*
c*f+3*d*e)+a^2*b*d*f*(3*c*f+2*d*e)-a*b^2*(2*c^2*f^2+3*d^2*e^2))*(1-b*x^2/a
)^(1/2)*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2),(-a*d/b/c)^(1/2))/b^(5
/2)/d^2/(a*d+b*c)/f^2/(a*f+b*e)/(-b*x^2+a)^(1/2)/(1+d*x^2/c)^(1/2)+1/3*a^(
1/2)*(4*a^2*c*d*f^3+a*b*c*f^2*(-2*c*f+d*e)-b^2*e*(2*c^2*f^2+3*c*d*e*f+3*d^
2*e^2))*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(b^(1/2)*x/a^(1/2),(-
a*d/b/c)^(1/2))/b^(5/2)/d^2/f^3/(a*f+b*e)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
+a^(1/2)*e^3*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)*EllipticPi(b^(1/2)*x/a^(1
/2),-a*f/b/e,(-a*d/b/c)^(1/2))/b^(1/2)/f^3/(a*f+b*e)/(-b*x^2+a)^(1/2)/(d*x
^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.62 (sec) , antiderivative size = 488, normalized size of antiderivative = 0.90

$$\int \frac{x^8}{(a-bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \frac{\sqrt{-\frac{b}{a}}df^2x(c+dx^2)\left(4a^3df-b^3cex^2-ab^2(-ce+dex^2+cfx^2)+\right.}{}$$

input
```
Integrate[x^8/((a - b*x^2)^(3/2)*Sqrt[c + d*x^2]*(e + f*x^2)),x]
```

output
```
(Sqrt[-(b/a)]*d*f^2*x*(c + d*x^2)*(4*a^3*d*f - b^3*c*e*x^2 - a*b^2*(-(c*e)
 + d*e*x^2 + c*f*x^2) + a^2*b*(c*f + d*(e - f*x^2))) + I*c*f*(8*a^3*d^2*f^
2 - b^3*c*e*(3*d*e + 2*c*f) + a^2*b*d*f*(2*d*e + 3*c*f) - a*b^2*(3*d^2*e^2
 + 2*c^2*f^2))*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh
[Sqrt[-(b/a)]*x], -((a*d)/(b*c))] - I*(b*c + a*d)*(4*a^2*c*d*f^3 + a*b*c*f
^2*(d*e - 2*c*f) - b^2*e*(3*d^2*e^2 + 3*c*d*e*f + 2*c^2*f^2))*Sqrt[1 - (b*
x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(
b*c))] - (3*I)*b^2*d^2*(b*c + a*d)*e^3*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2
)/c]*EllipticPi[-((a*f)/(b*e)), I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))]
)/(3*b^2*Sqrt[-(b/a)]*d^2*(b*c + a*d)*f^3*(b*e + a*f)*Sqrt[a - b*x^2]*Sqrt
[c + d*x^2])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{(a-bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx$$
$$\Big\downarrow \; 450$$
$$\int \frac{x^8}{(a-bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx$$

input
```
Int[x^8/((a - b*x^2)^(3/2)*Sqrt[c + d*x^2]*(e + f*x^2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [A] (verified)

Time = 22.50 (sec) , antiderivative size = 789, normalized size of antiderivative = 1.46

method	result
risch	$\frac{x\sqrt{-b\,x^2+a}\,\sqrt{x^2 d+c}}{3b^2 df} - \frac{\left(-\frac{(5adf-2bcf-3bde)c\sqrt{1-\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\left(\mathrm{EllipticF}\left(x\sqrt{\frac{b}{a}},\sqrt{-1-\frac{ad-bc}{cb}}\right)-\mathrm{EllipticE}\left(x\sqrt{\frac{b}{a}},\sqrt{-1-\frac{ad-bc}{cb}}\right)\right)}{f\sqrt{\frac{b}{a}}\,\sqrt{-bd\,x^4+ad\,x^2-x^2bc+ac}\,d}+\right.}{}$
elliptic	Expression too large to display
default	Expression too large to display

input
```
int(x^8/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x,method=_RETURNVERBOSE
)
```

output
```
1/3*x*(-b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b^2/d/f-1/3/f/b^2/d*(-1/f*(5*a*d*f-
2*b*c*f-3*b*d*e)*c/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x
^4+a*d*x^2-b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b
)^(1/2))-EllipticE(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2)))+(3*a^2*d*f^2+a
*b*c*f^2-3*a*b*d*e*f+3*b^2*d*e^2)/f^2/b/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d
*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)*EllipticF(x*(b/a)^(1/2)
,(-1-(a*d-b*c)/c/b)^(1/2))+3*f/b*a^4*d/(a*f+b*e)*((-b*d*x^2-b*c)/a/(a*d+b*
c)*x/((x^2-a/b)*(-b*d*x^2-b*c))^(1/2)+(-1/a+b*c/a/(a*d+b*c))/(b/a)^(1/2)*(
1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)*El
lipticF(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2))-b/a/(a*d+b*c)*c/(b/a)^(1/2
)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)
*(EllipticF(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2))-EllipticE(x*(b/a)^(1/2
),(-1-(a*d-b*c)/c/b)^(1/2))))-3/f^2*b^2*d*e^3/(a*f+b*e)/(b/a)^(1/2)*(1-b*x
^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)*Ellipti
cPi(x*(b/a)^(1/2),-a*f/b/e,(-1/c*d)^(1/2)/(b/a)^(1/2)))*((-b*x^2+a)*(d*x^2
+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^8}{(a-bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate(x^8/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="fri
cas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^8}{(a-bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{x^8}{(a-bx^2)^{\frac{3}{2}}\sqrt{c+dx^2}\,(e+fx^2)}\,dx$$

input
```
integrate(x**8/(-b*x**2+a)**(3/2)/(d*x**2+c)**(1/2)/(f*x**2+e),x)
```

output
```
Integral(x**8/((a - b*x**2)**(3/2)*sqrt(c + d*x**2)*(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{x^8}{(a-bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{x^8}{(-bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}(fx^2+e)}\,dx$$

input
```
integrate(x^8/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="max
ima")
```

output
```
integrate(x^8/((-b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*(f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{x^8}{(a-bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{x^8}{(-bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}(fx^2+e)}\,dx$$

input
```
integrate(x^8/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="gia
c")
```

output
```
integrate(x^8/((-b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^8}{(a-bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{x^8}{(a-b\,x^2)^{3/2}\,\sqrt{d\,x^2+c}\,(f\,x^2+e)}\,dx$$

input
```
int(x^8/((a - b*x^2)^(3/2)*(c + d*x^2)^(1/2)*(e + f*x^2)),x)
```

output
```
int(x^8/((a - b*x^2)^(3/2)*(c + d*x^2)^(1/2)*(e + f*x^2)), x)
```

Reduce [F]

$$\int \frac{x^8}{(a-bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \text{too large to display}$$

input
```
int(x^8/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x)
```

output
```
(3*sqrt(c + d*x**2)*sqrt(a - b*x**2)*a*c*f*x + 4*sqrt(c + d*x**2)*sqrt(a -
 b*x**2)*a*d*e*x - 2*sqrt(c + d*x**2)*sqrt(a - b*x**2)*a*d*f*x**3 - 2*sqrt
(c + d*x**2)*sqrt(a - b*x**2)*b*c*e*x + sqrt(c + d*x**2)*sqrt(a - b*x**2)*
b*d*e*x**3 + 16*int((sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**6)/(2*a**3*c*e*f
 + 2*a**3*c*f**2*x**2 + 2*a**3*d*e*f*x**2 + 2*a**3*d*f**2*x**4 - a**2*b*c*
e**2 - 5*a**2*b*c*e*f*x**2 - 4*a**2*b*c*f**2*x**4 - a**2*b*d*e**2*x**2 - 5
*a**2*b*d*e*f*x**4 - 4*a**2*b*d*f**2*x**6 + 2*a*b**2*c*e**2*x**2 + 4*a*b**
2*c*e*f*x**4 + 2*a*b**2*c*f**2*x**6 + 2*a*b**2*d*e**2*x**4 + 4*a*b**2*d*e*
f*x**6 + 2*a*b**2*d*f**2*x**8 - b**3*c*e**2*x**4 - b**3*c*e*f*x**6 - b**3*
d*e**2*x**6 - b**3*d*e*f*x**8),x)*a**4*d**2*f**3 - 2*int((sqrt(c + d*x**2)
*sqrt(a - b*x**2)*x**6)/(2*a**3*c*e*f + 2*a**3*c*f**2*x**2 + 2*a**3*d*e*f*
x**2 + 2*a**3*d*f**2*x**4 - a**2*b*c*e**2 - 5*a**2*b*c*e*f*x**2 - 4*a**2*b
*c*f**2*x**4 - a**2*b*d*e**2*x**2 - 5*a**2*b*d*e*f*x**4 - 4*a**2*b*d*f**2*
x**6 + 2*a*b**2*c*e**2*x**2 + 4*a*b**2*c*e*f*x**4 + 2*a*b**2*c*f**2*x**6 +
 2*a*b**2*d*e**2*x**4 + 4*a*b**2*d*e*f*x**6 + 2*a*b**2*d*f**2*x**8 - b**3*
c*e**2*x**4 - b**3*c*e*f*x**6 - b**3*d*e**2*x**6 - b**3*d*e*f*x**8),x)*a**
3*b*c*d*f**3 - 20*int((sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**6)/(2*a**3*c*e
*f + 2*a**3*c*f**2*x**2 + 2*a**3*d*e*f*x**2 + 2*a**3*d*f**2*x**4 - a**2*b*
c*e**2 - 5*a**2*b*c*e*f*x**2 - 4*a**2*b*c*f**2*x**4 - a**2*b*d*e**2*x**2 -
 5*a**2*b*d*e*f*x**4 - 4*a**2*b*d*f**2*x**6 + 2*a*b**2*c*e**2*x**2 + 4*...
```

3.288 $\int \frac{x^6}{(a-bx^2)^{3/2}\sqrt{c+dx^2}(e+fx^2)}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 418

$$\int \frac{x^6}{(a-bx^2)^{3/2}\sqrt{c+dx^2}(e+fx^2)}\,dx = \frac{a^2x\sqrt{c+dx^2}}{b(bc+ad)(be+af)\sqrt{a-bx^2}} - \frac{\sqrt{a}(b^2ce+2a^2df+ab(de+cf))\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{b^{3/2}d(bc+ad)f(be+af)\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}} + \frac{\sqrt{a}(acf^2+be(de+cf))\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{b^{3/2}df^2(be+af)\sqrt{a-bx^2}\sqrt{c+dx^2}} - \frac{\sqrt{a}e^2\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}f^2(be+af)\sqrt{a-bx^2}\sqrt{c+dx^2}}$$

output
```
a^2*x*(d*x^2+c)^(1/2)/b/(a*d+b*c)/(a*f+b*e)/(-b*x^2+a)^(1/2)-a^(1/2)*(b^2*
c*e+2*a^2*d*f+a*b*(c*f+d*e))*(1-b*x^2/a)^(1/2)*(d*x^2+c)^(1/2)*EllipticE(b
^(1/2)*x/a^(1/2),(-a*d/b/c)^(1/2))/b^(3/2)/d/(a*d+b*c)/f/(a*f+b*e)/(-b*x^2
+a)^(1/2)/(1+d*x^2/c)^(1/2)+a^(1/2)*(a*c*f^2+b*e*(c*f+d*e))*(1-b*x^2/a)^(1
/2)*(1+d*x^2/c)^(1/2)*EllipticF(b^(1/2)*x/a^(1/2),(-a*d/b/c)^(1/2))/b^(3/2
)/d/f^2/(a*f+b*e)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)-a^(1/2)*e^2*(1-b*x^2/a)
^(1/2)*(1+d*x^2/c)^(1/2)*EllipticPi(b^(1/2)*x/a^(1/2),-a*f/b/e,(-a*d/b/c)^
(1/2))/b^(1/2)/f^2/(a*f+b*e)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.26 (sec) , antiderivative size = 344, normalized size of antiderivative = 0.82

$$\int \frac{x^6}{(a-bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \frac{icf(b^2ce+2a^2df+ab(de+cf))\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\Big(i\mathrm{arcsinh}\Big(}{}$$

input
```
Integrate[x^6/((a - b*x^2)^(3/2)*Sqrt[c + d*x^2]*(e + f*x^2)),x]
```

output
```
(I*c*f*(b^2*c*e + 2*a^2*d*f + a*b*(d*e + c*f))*Sqrt[1 - (b*x^2)/a]*Sqrt[1
+ (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))] - I*(b*c
 + a*d)*(a*c*f^2 + b*e*(d*e + c*f))*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c
]*EllipticF[I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))] + d*(a^2*Sqrt[-(b/a
)]*f^2*x*(c + d*x^2) + I*b*(b*c + a*d)*e^2*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d
*x^2)/c]*EllipticPi[-((a*f)/(b*e)), I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*
c))]))/(b*Sqrt[-(b/a)]*d*(b*c + a*d)*f^2*(b*e + a*f)*Sqrt[a - b*x^2]*Sqrt[
c + d*x^2])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(a-bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx$$

$$\Big\downarrow \; 450$$

$$\int \frac{x^6}{(a-bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}dx$$

input
```
Int[x^6/((a - b*x^2)^(3/2)*Sqrt[c + d*x^2]*(e + f*x^2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 825 vs. $2(369) = 738$.

Time = 8.66 (sec) , antiderivative size = 826, normalized size of antiderivative = 1.98

method	result
elliptic	$\frac{\sqrt{(-b x^2+a)(x^2 d+c)}\left(-\frac{(-bd x^2-bc)a^2 x}{b^2(ad+bc)(af+be)\sqrt{\left(x^2-\frac{a}{b}\right)(-bd x^2-bc)}}-\frac{\sqrt{1-\frac{b x^2}{a}}\sqrt{1+\frac{d x^2}{c}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{b}{a}},\sqrt{-1-\frac{ad-bc}{cb}}\right)a}{\sqrt{\frac{b}{a}}\sqrt{-bd x^4+ad x^2-x^2bc+ac}\,f\,b^2}+\frac{\sqrt{1-\frac{b x^2}{a}}\sqrt{1\cdots}}{\sqrt{\frac{b}{a}}\cdots}\cdots\right.}{\cdots}$
default	$\frac{\left(\sqrt{\frac{b}{a}}\,a^2d^2f^2x^3+\sqrt{\frac{-b x^2+a}{a}}\sqrt{\frac{x^2d+c}{c}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{b}{a}},\sqrt{-\frac{ad}{bc}}\right)a^2cd\,f^2+\sqrt{\frac{-b x^2+a}{a}}\sqrt{\frac{x^2d+c}{c}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{b}{a}},\sqrt{-\frac{ad}{bc}}\right)ab\,c^2f^2+\cdots\right.}{\cdots}$

input
```
int(x^6/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x,method=_RETURNVERBOSE
)
```

output
```
((-b*x^2+a)*(d*x^2+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-(-b*d*x^2-
b*c)/b^2*a^2/(a*d+b*c)*x/(a*f+b*e)/((x^2-a/b)*(-b*d*x^2-b*c))^(1/2)-1/(b/a
)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)
^(1/2)*EllipticF(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2))/f/b^2*a+1/(b/a)^(
1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1
/2)*EllipticF(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2))/f^2/b*e+1/(b/a)^(1/2
)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)
*EllipticF(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2))*a^2/b^2/(a*f+b*e)+c/(b/
a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c
)^(1/2)/d/f/b*EllipticF(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2))-c/(b/a)^(1
/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/
2)/d/f/b*EllipticE(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2))-c/(b/a)^(1/2)*(
1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)*a^
2/b/(a*d+b*c)/(a*f+b*e)*EllipticE(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2))-
e^2/(a*f+b*e)/f^2/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^
4+a*d*x^2-b*c*x^2+a*c)^(1/2)*EllipticPi(x*(b/a)^(1/2),-a*f/b/e,(-1/c*d)^(1
/2)/(b/a)^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^6}{(a-bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate(x^6/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="fri
cas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^6}{(a-bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{x^6}{(a-bx^2)^{\frac{3}{2}}\sqrt{c+dx^2}\,(e+fx^2)}\,dx$$

input
```
integrate(x**6/(-b*x**2+a)**(3/2)/(d*x**2+c)**(1/2)/(f*x**2+e),x)
```

output
```
Integral(x**6/((a - b*x**2)**(3/2)*sqrt(c + d*x**2)*(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{x^6}{(a-bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{x^6}{(-bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}(fx^2+e)}\,dx$$

input
```
integrate(x^6/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="max
ima")
```

output
```
integrate(x^6/((-b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*(f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{x^6}{(a-bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{x^6}{(-bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}(fx^2+e)}\,dx$$

input
```
integrate(x^6/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="gia
c")
```

output
```
integrate(x^6/((-b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{(a-bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{x^6}{(a-b\,x^2)^{3/2}\,\sqrt{d\,x^2+c}\,(f\,x^2+e)}\,dx$$

input
```
int(x^6/((a - b*x^2)^(3/2)*(c + d*x^2)^(1/2)*(e + f*x^2)),x)
```

output
```
int(x^6/((a - b*x^2)^(3/2)*(c + d*x^2)^(1/2)*(e + f*x^2)), x)
```

Reduce [F]

$$\int \frac{x^6}{(a-bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{\sqrt{d\,x^2+c}\,\sqrt{-b}}{b^2df\,x^8-2abdf\,x^6+b^2cf\,x^6+b^2de\,x^6+a^2df\,x^4-2abcf\,x^4-}$$

input
```
int(x^6/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**6)/(a**2*c*e + a**2*c*f*x**2 + a
**2*d*e*x**2 + a**2*d*f*x**4 - 2*a*b*c*e*x**2 - 2*a*b*c*f*x**4 - 2*a*b*d*e
*x**4 - 2*a*b*d*f*x**6 + b**2*c*e*x**4 + b**2*c*f*x**6 + b**2*d*e*x**6 + b
**2*d*f*x**8),x)
```

3.289 $\int \frac{x^4}{(a-bx^2)^{3/2}\sqrt{c+dx^2}(e+fx^2)}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 362

$$\begin{aligned}\int \frac{x^4}{(a-bx^2)^{3/2}\sqrt{c+dx^2}(e+fx^2)}\,dx &= \frac{ax\sqrt{c+dx^2}}{(bc+ad)(be+af)\sqrt{a-bx^2}} \\ &- \frac{a^{3/2}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}(bc+ad)(be+af)\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}} \\ &- \frac{\sqrt{a}e\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}f(be+af)\sqrt{a-bx^2}\sqrt{c+dx^2}} \\ &+ \frac{\sqrt{a}e\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}f(be+af)\sqrt{a-bx^2}\sqrt{c+dx^2}}\end{aligned}$$

output

```
a*x*(d*x^2+c)^(1/2)/(a*d+b*c)/(a*f+b*e)/(-b*x^2+a)^(1/2)-a^(3/2)*(1-b*x^2/
a)^(1/2)*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2),(-a*d/b/c)^(1/2))/b^(
1/2)/(a*d+b*c)/(a*f+b*e)/(-b*x^2+a)^(1/2)/(1+d*x^2/c)^(1/2)-a^(1/2)*e*(1-b
*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(b^(1/2)*x/a^(1/2),(-a*d/b/c)^(1/
2))/b^(1/2)/f/(a*f+b*e)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)+a^(1/2)*e*(1-b*x^
2/a)^(1/2)*(1+d*x^2/c)^(1/2)*EllipticPi(b^(1/2)*x/a^(1/2),-a*f/b/e,(-a*d/b
/c)^(1/2))/b^(1/2)/f/(a*f+b*e)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.81 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.02

$$\int \frac{x^4}{(a-bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \frac{a\sqrt{-\frac{b}{a}}cfx + a\sqrt{-\frac{b}{a}}dfx^3 + iacf\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\Big(i\text{arcsinh}\Big(}{}$$

input
```
Integrate[x^4/((a - b*x^2)^(3/2)*Sqrt[c + d*x^2]*(e + f*x^2)),x]
```

output
```
(a*Sqrt[-(b/a)]*c*f*x + a*Sqrt[-(b/a)]*d*f*x^3 + I*a*c*f*Sqrt[1 - (b*x^2)/
a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))
] + I*(b*c + a*d)*e*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*Ar
cSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))] - I*b*c*e*Sqrt[1 - (b*x^2)/a]*Sqrt[
1 + (d*x^2)/c]*EllipticPi[-((a*f)/(b*e)), I*ArcSinh[Sqrt[-(b/a)]*x], -((a*
d)/(b*c))] - I*a*d*e*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[-(
(a*f)/(b*e)), I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))])/(Sqrt[-(b/a)]*(b
*c + a*d)*f*(b*e + a*f)*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(a-bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx$$

$$\Big\downarrow \; 450$$

$$\int \frac{x^4}{(a-bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}dx$$

input
```
Int[x^4/((a - b*x^2)^(3/2)*Sqrt[c + d*x^2]*(e + f*x^2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [A] (verified)

Time = 8.41 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.10

method	result
default	$\frac{\left(\sqrt{\frac{b}{a}}\, adf\, x^{3}-\sqrt{\frac{-b\, x^{2}+a}{a}}\, \sqrt{\frac{x^{2} d+c}{c}}\, \operatorname{EllipticF}\left(x \sqrt{\frac{b}{a}}, \sqrt{-\frac{ad}{bc}}\right) ade-\sqrt{\frac{-b\, x^{2}+a}{a}}\, \sqrt{\frac{x^{2} d+c}{c}}\, \operatorname{EllipticF}\left(x \sqrt{\frac{b}{a}}, \sqrt{-\frac{ad}{bc}}\right) bce-\sqrt{\frac{-b\, x^{2}+a}{a}}\right.}{}$
elliptic	$\frac{\sqrt{(-b\, x^{2}+a)(x^{2} d+c)}\left(-\frac{\left(-bd\, x^{2}-bc\right) ax}{b(ad+bc)(af+be)\sqrt{\left(x^{2}-\frac{a}{b}\right)\left(-bd\, x^{2}-bc\right)}}-\frac{\sqrt{1-\frac{b\, x^{2}}{a}}\, \sqrt{1+\frac{d\, x^{2}}{c}}\, \operatorname{EllipticF}\left(x \sqrt{\frac{b}{a}}, \sqrt{-1-\frac{ad-bc}{cb}}\right)}{\sqrt{\frac{b}{a}}\, \sqrt{-bd\, x^{4}+ad\, x^{2}-x^{2} bc+ac}\, fb}+\frac{\sqrt{1-\frac{b\, x^{2}}{a}}\, \sqrt{1+}}{\sqrt{\frac{b}{a}}\, \sqrt{-b}}\right.}{\sqrt{-b}}$

input
```
int(x^4/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x,method=_RETURNVERBOSE
)
```

output
```
((b/a)^(1/2)*a*d*f*x^3-((-b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(
x*(b/a)^(1/2),(-a*d/b/c)^(1/2))*a*d*e-((-b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(
1/2)*EllipticF(x*(b/a)^(1/2),(-a*d/b/c)^(1/2))*b*c*e-((-b*x^2+a)/a)^(1/2)*
((d*x^2+c)/c)^(1/2)*EllipticE(x*(b/a)^(1/2),(-a*d/b/c)^(1/2))*a*c*f+((-b*x
^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticPi(x*(b/a)^(1/2),-a*f/b/e,(-1/c
*d)^(1/2)/(b/a)^(1/2))*a*d*e+((-b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*Elli
pticPi(x*(b/a)^(1/2),-a*f/b/e,(-1/c*d)^(1/2)/(b/a)^(1/2))*b*c*e+(b/a)^(1/2
)*a*c*f*x)*(d*x^2+c)^(1/2)*(-b*x^2+a)^(1/2)/f/(b/a)^(1/2)/(a*d+b*c)/(a*f+b
*e)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^4}{(a-bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate(x^4/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="fri
cas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^4}{(a-bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{x^4}{(a-bx^2)^{\frac{3}{2}}\sqrt{c+dx^2}\,(e+fx^2)}\,dx$$

input
```
integrate(x**4/(-b*x**2+a)**(3/2)/(d*x**2+c)**(1/2)/(f*x**2+e),x)
```

output
```
Integral(x**4/((a - b*x**2)**(3/2)*sqrt(c + d*x**2)*(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{x^4}{(a-bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{x^4}{(-bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}(fx^2+e)}\,dx$$

input
```
integrate(x^4/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="max
ima")
```

output
```
integrate(x^4/((-b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*(f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{x^4}{(a-bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{x^4}{(-bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}(fx^2+e)}\,dx$$

input
```
integrate(x^4/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="gia
c")
```

output
```
integrate(x^4/((-b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(a-bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{x^4}{(a-b\,x^2)^{3/2}\,\sqrt{d\,x^2+c}\,(f\,x^2+e)}\,dx$$

input
```
int(x^4/((a - b*x^2)^(3/2)*(c + d*x^2)^(1/2)*(e + f*x^2)),x)
```

output
```
int(x^4/((a - b*x^2)^(3/2)*(c + d*x^2)^(1/2)*(e + f*x^2)), x)
```

Reduce [F]

$$\int \frac{x^4}{(a-bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{\sqrt{d\,x^2+c}\,\sqrt{-b}}{b^2df\,x^8-2abdf\,x^6+b^2cf\,x^6+b^2de\,x^6+a^2df\,x^4-2abcf\,x^4-}$$

input
```
int(x^4/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**4)/(a**2*c*e + a**2*c*f*x**2 + a
**2*d*e*x**2 + a**2*d*f*x**4 - 2*a*b*c*e*x**2 - 2*a*b*c*f*x**4 - 2*a*b*d*e
*x**4 - 2*a*b*d*f*x**6 + b**2*c*e*x**4 + b**2*c*f*x**6 + b**2*d*e*x**6 + b
**2*d*f*x**8),x)
```

3.290 $\int \frac{x^2}{(a-bx^2)^{3/2}\sqrt{c+dx^2}(e+fx^2)}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 355

$$\begin{aligned}\int \frac{x^2}{(a-bx^2)^{3/2}\sqrt{c+dx^2}(e+fx^2)}\,dx &= \frac{bx\sqrt{c+dx^2}}{(bc+ad)(be+af)\sqrt{a-bx^2}} \\ &- \frac{\sqrt{b}c\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{a}(bc+ad)(be+af)\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}} \\ &+ \frac{\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{b}(be+af)\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}} \\ &- \frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}(be+af)\sqrt{a-bx^2}\sqrt{c+dx^2}}\end{aligned}$$

output
```
b*x*(d*x^2+c)^(1/2)/(a*d+b*c)/(a*f+b*e)/(-b*x^2+a)^(1/2)-b^(1/2)*c*(-b*x^2
+a)^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2),(-a*d/b/c)^(1/2))/
a^(1/2)/(a*d+b*c)/(a*f+b*e)/(1-b*x^2/a)^(1/2)/(d*x^2+c)^(1/2)+(-b*x^2+a)^(
1/2)*(1+d*x^2/c)^(1/2)*EllipticF(b^(1/2)*x/a^(1/2),(-a*d/b/c)^(1/2))/a^(1/
2)/b^(1/2)/(a*f+b*e)/(1-b*x^2/a)^(1/2)/(d*x^2+c)^(1/2)-a^(1/2)*(1-b*x^2/a)
^(1/2)*(1+d*x^2/c)^(1/2)*EllipticPi(b^(1/2)*x/a^(1/2),-a*f/b/e,(-a*d/b/c)^
(1/2))/b^(1/2)/(a*f+b*e)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.98 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.01

$$\int \frac{x^2}{(a-bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \frac{b\sqrt{-\frac{b}{a}}cx+b\sqrt{-\frac{b}{a}}dx^3+ibc\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(i\operatorname{arcsinh}\left(\sqrt{-\frac{b}{a}}\right.\right.}{}$$

input
```
Integrate[x^2/((a - b*x^2)^(3/2)*Sqrt[c + d*x^2]*(e + f*x^2)),x]
```

output
```
(b*Sqrt[-(b/a)]*c*x + b*Sqrt[-(b/a)]*d*x^3 + I*b*c*Sqrt[1 - (b*x^2)/a]*Sqr
t[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))] - I*
(b*c + a*d)*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sq
rt[-(b/a)]*x], -((a*d)/(b*c))] + I*b*c*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2
)/c]*EllipticPi[-((a*f)/(b*e)), I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))]
 + I*a*d*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[-((a*f)/(b*e))
, I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))])/(Sqrt[-(b/a)]*(b*c + a*d)*(b
*e + a*f)*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a-bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx$$
$$\Big\downarrow \ 450$$
$$\int \frac{x^2}{(a-bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}dx$$

input
```
Int[x^2/((a - b*x^2)^(3/2)*Sqrt[c + d*x^2]*(e + f*x^2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [A] (verified)

Time = 8.48 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.09

method	result
default	$\frac{\left(\sqrt{\frac{b}{a}}\, b d\, x^{3}+a \sqrt{\frac{-b\, x^{2}+a}{a}}\, \sqrt{\frac{x^{2} d+c}{c}}\, \operatorname{EllipticF}\left(x \sqrt{\frac{b}{a}}, \sqrt{-\frac{a d}{b c}}\right) d+\sqrt{\frac{-b\, x^{2}+a}{a}}\, \sqrt{\frac{x^{2} d+c}{c}}\, \operatorname{EllipticF}\left(x \sqrt{\frac{b}{a}}, \sqrt{-\frac{a d}{b c}}\right) b c-\sqrt{\frac{-b\, x^{2}+a}{a}}\, \sqrt{x \cdots}\right.}{\cdots}$
elliptic	$\frac{\sqrt{\left(-b\, x^{2}+a\right)\left(x^{2} d+c\right)}\, \left(-\frac{\left(-b d\, x^{2}-b c\right) x}{\left(a d+b c\right)\left(a f+b e\right)\sqrt{\left(x^{2}-\frac{a}{b}\right)\left(-b d\, x^{2}-b c\right)}}+\frac{\sqrt{1-\frac{b\, x^{2}}{a}}\, \sqrt{1+\frac{d\, x^{2}}{c}}\, \operatorname{EllipticF}\left(x \sqrt{\frac{b}{a}}, \sqrt{-1-\frac{a d-b c}{c b}}\right)}{\sqrt{\frac{b}{a}}\, \sqrt{-b d\, x^{4}+a d\, x^{2}-x^{2} b c+a c}\, \left(a f+b e\right)}-\frac{b c \sqrt{1-\frac{b\, x^{2}}{a}}\, \sqrt{1+\cdots}}{\left(a d+b c\right)\left(a f+\cdots\right.}\cdots\right.}{\sqrt{-b\, x^{2}+a}\, \sqrt{x^{2} d+c}}$

input
```
int(x^2/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x,method=_RETURNVERBOSE
)
```

output
```
((b/a)^(1/2)*b*d*x^3+a*((-b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(
x*(b/a)^(1/2),(-a*d/b/c)^(1/2))*d+((-b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)
*EllipticF(x*(b/a)^(1/2),(-a*d/b/c)^(1/2))*b*c-((-b*x^2+a)/a)^(1/2)*((d*x^
2+c)/c)^(1/2)*EllipticE(x*(b/a)^(1/2),(-a*d/b/c)^(1/2))*b*c-((-b*x^2+a)/a)
^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticPi(x*(b/a)^(1/2),-a*f/b/e,(-1/c*d)^(1/2
)/(b/a)^(1/2))*a*d-((-b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticPi(x*(
b/a)^(1/2),-a*f/b/e,(-1/c*d)^(1/2)/(b/a)^(1/2))*b*c+(b/a)^(1/2)*b*c*x)*(d*
x^2+c)^(1/2)*(-b*x^2+a)^(1/2)/(b/a)^(1/2)/(a*f+b*e)/(a*d+b*c)/(-b*d*x^4+a*
d*x^2-b*c*x^2+a*c)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^2}{(a-bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate(x^2/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="fri
cas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^2}{(a-bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{x^2}{(a-bx^2)^{\frac{3}{2}}\sqrt{c+dx^2}\,(e+fx^2)}\,dx$$

input
```
integrate(x**2/(-b*x**2+a)**(3/2)/(d*x**2+c)**(1/2)/(f*x**2+e),x)
```

output
```
Integral(x**2/((a - b*x**2)**(3/2)*sqrt(c + d*x**2)*(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{x^2}{(a-bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{x^2}{(-bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}(fx^2+e)}\,dx$$

input
```
integrate(x^2/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="max
ima")
```

output
```
integrate(x^2/((-b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*(f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{x^2}{(a-bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx=\int \frac{x^2}{(-bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}(fx^2+e)}\,dx$$

input
```
integrate(x^2/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="gia
c")
```

output
```
integrate(x^2/((-b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a-bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx=\int \frac{x^2}{(a-b\,x^2)^{3/2}\,\sqrt{d\,x^2+c}\,(f\,x^2+e)}\,dx$$

input
```
int(x^2/((a - b*x^2)^(3/2)*(c + d*x^2)^(1/2)*(e + f*x^2)),x)
```

output
```
int(x^2/((a - b*x^2)^(3/2)*(c + d*x^2)^(1/2)*(e + f*x^2)), x)
```

Reduce [F]

$$\int \frac{x^2}{(a-bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx=\int \frac{\sqrt{d\,x^2+c}\,\sqrt{-b}}{b^2df\,x^8-2abdf\,x^6+b^2cf\,x^6+b^2de\,x^6+a^2df\,x^4-2abcf\,x^4-}$$

input
```
int(x^2/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**2)/(a**2*c*e + a**2*c*f*x**2 + a
**2*d*e*x**2 + a**2*d*f*x**4 - 2*a*b*c*e*x**2 - 2*a*b*c*f*x**4 - 2*a*b*d*e
*x**4 - 2*a*b*d*f*x**6 + b**2*c*e*x**4 + b**2*c*f*x**6 + b**2*d*e*x**6 + b
**2*d*f*x**8),x)
```

3.291 $\int \frac{1}{(a-bx^2)^{3/2}\sqrt{c+dx^2}(e+fx^2)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 363

$$\int \frac{1}{(a-bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \frac{b^2x\sqrt{c+dx^2}}{a(bc+ad)(be+af)\sqrt{a-bx^2}} - \frac{b^{3/2}c\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{a^{3/2}(bc+ad)(be+af)\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}} + \frac{\sqrt{b}\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{a^{3/2}(be+af)\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}} + \frac{\sqrt{a}f\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}e(be+af)\sqrt{a-bx^2}\sqrt{c+dx^2}}$$

output
```
b^2*x*(d*x^2+c)^(1/2)/a/(a*d+b*c)/(a*f+b*e)/(-b*x^2+a)^(1/2)-b^(3/2)*c*(-b
*x^2+a)^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2),(-a*d/b/c)^(1/
2))/a^(3/2)/(a*d+b*c)/(a*f+b*e)/(1-b*x^2/a)^(1/2)/(d*x^2+c)^(1/2)+b^(1/2)*
(-b*x^2+a)^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(b^(1/2)*x/a^(1/2),(-a*d/b/c)^
(1/2))/a^(3/2)/(a*f+b*e)/(1-b*x^2/a)^(1/2)/(d*x^2+c)^(1/2)+a^(1/2)*f*(1-b*
x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)*EllipticPi(b^(1/2)*x/a^(1/2),-a*f/b/e,(-a*d
/b/c)^(1/2))/b^(1/2)/e/(a*f+b*e)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.10 (sec) , antiderivative size = 382, normalized size of antiderivative = 1.05

$$\int \frac{1}{(a-bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \frac{b^2\sqrt{-\frac{b}{a}}cex+b^2\sqrt{-\frac{b}{a}}dex^3+ib^2ce\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\Big(i\text{arcsinh}}{}$$

input
```
Integrate[1/((a - b*x^2)^(3/2)*Sqrt[c + d*x^2]*(e + f*x^2)),x]
```

output
```
(b^2*Sqrt[-(b/a)]*c*e*x + b^2*Sqrt[-(b/a)]*d*e*x^3 + I*b^2*c*e*Sqrt[1 - (b
*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/
(b*c))] - I*b*(b*c + a*d)*e*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Ellipt
icF[I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))] - I*a*b*c*f*Sqrt[1 - (b*x^2
)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[-((a*f)/(b*e)), I*ArcSinh[Sqrt[-(b/a)]
*x], -((a*d)/(b*c))] - I*a^2*d*f*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*E
llipticPi[-((a*f)/(b*e)), I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))])/(a*S
qrt[-(b/a)]*(b*c + a*d)*e*(b*e + a*f)*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 484, normalized size of antiderivative = 1.33, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.485$, Rules used = {421, 402, 399, 323, 323, 321, 331, 330, 327, 415, 323, 323, 321, 413, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx$$

$$\Big\downarrow \; 421$$

$$\frac{f^2\int \frac{\sqrt{a-bx^2}}{\sqrt{dx^2+c}(fx^2+e)}dx}{(af+be)^2}+\frac{b\int \frac{-bfx^2+be+2af}{(a-bx^2)^{3/2}\sqrt{dx^2+c}}dx}{(af+be)^2}$$

$$\Big\downarrow \; 402$$

$$\frac{f^2\int \frac{\sqrt{a-bx^2}}{\sqrt{dx^2+c}(fx^2+e)}dx}{(af+be)^2}+\frac{b\left(\frac{\int \frac{a(bde+bcf+2adf)-bd(be+af)x^2}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{a(ad+bc)}+\frac{bx\sqrt{c+dx^2}(af+be)}{a\sqrt{a-bx^2}(ad+bc)}\right)}{(af+be)^2}$$

$$\Big\downarrow \; 399$$

$$\frac{f^2\int \frac{\sqrt{a-bx^2}}{\sqrt{dx^2+c}(fx^2+e)}dx}{(af+be)^2}+\frac{b\left(\frac{(ad+bc)(2af+be)\int \frac{1}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx-b(af+be)\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{a(ad+bc)}+\frac{bx\sqrt{c+dx^2}(af+be)}{a\sqrt{a-bx^2}(ad+bc)}\right)}{(af+be)^2}$$

$$\Big\downarrow \; 323$$

$$\frac{f^2\int \frac{\sqrt{a-bx^2}}{\sqrt{dx^2+c}(fx^2+e)}dx}{(af+be)^2}+\frac{b\left(\frac{\frac{\sqrt{\frac{dx^2}{c}+1}(ad+bc)(2af+be)\int \frac{1}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}dx}{\sqrt{c+dx^2}}-b(af+be)\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{a(ad+bc)}+\frac{bx\sqrt{c+dx^2}(af+be)}{a\sqrt{a-bx^2}(ad+bc)}\right)}{(af+be)^2}$$

$$\Big\downarrow \; 323$$

$$\frac{f^2\int \frac{\sqrt{a-bx^2}}{\sqrt{dx^2+c}(fx^2+e)}dx}{(af+be)^2}+\frac{b\left(\frac{\frac{\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(2af+be)\int \frac{1}{\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}}dx}{\sqrt{a-bx^2}\sqrt{c+dx^2}}-b(af+be)\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{a(ad+bc)}+\frac{bx\sqrt{c+dx^2}(af+be)}{a\sqrt{a-bx^2}(ad+bc)}\right)}{(af+be)^2}$$

$$\Big\downarrow \; 321$$

$$\frac{b\left(\frac{\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(2af+be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-b(af+be)\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{a(ad+bc)}+\frac{bx\sqrt{c+dx^2}(af+be)}{a\sqrt{a-bx^2}(ad+bc)}\right)}{(af+be)^2}+\frac{f^2\int \frac{\sqrt{a-bx^2}}{\sqrt{dx^2+c}(fx^2+e)}dx}{(af+be)^2}$$

$\downarrow$ 331

$$\frac{b\left(\frac{\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(2af+be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{b\sqrt{1-\frac{bx^2}{a}}(af+be)\int\frac{\sqrt{dx^2+c}}{\sqrt{1-\frac{bx^2}{a}}}dx}{\sqrt{a-bx^2}}}{a(ad+bc)}+\frac{bx\sqrt{c+dx^2}(af+be)}{a\sqrt{a-bx^2}(ad+bc)}\right)}{(af+be)^2}+\frac{f^2\int\frac{\sqrt{a-bx^2}}{\sqrt{dx^2+c}(fx^2+e)}dx}{(af+be)^2}$$

$\downarrow$ 330

$$\frac{b\left(\frac{\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(2af+be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{b\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(af+be)\int\frac{\sqrt{\frac{dx^2}{c}+1}}{\sqrt{1-\frac{bx^2}{a}}}dx}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}}{a(ad+bc)}+\frac{bx\sqrt{c+dx^2}(af+be)}{a\sqrt{a-bx^2}(ad+bc)}\right)}{(af+be)^2}+\frac{f^2\int\frac{\sqrt{a-bx^2}}{\sqrt{dx^2+c}(fx^2+e)}dx}{(af+be)^2}$$

$\downarrow$ 327

$$\frac{f^2\int\frac{\sqrt{a-bx^2}}{\sqrt{dx^2+c}(fx^2+e)}dx}{(af+be)^2}+\frac{b\Bigg(\frac{\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(2af+be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(af+be)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}}{a(ad+bc)}+\frac{bx\sqrt{c+dx^2}(af+be}{a\sqrt{a-bx^2}(ad+bc}}{(af+be)^2}$$

$\downarrow$ 415

$$\frac{f^2\left(\frac{(af+be)\int \frac{1}{\sqrt{a-bx^2}\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{f}-\frac{b\int \frac{1}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{f}\right)}{(af+be)^2}+\frac{b\left(\frac{\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(2af+be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(af+be)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}}{a(ad+bc)}+\frac{bx\sqrt{c+dx^2}(af+be}{a\sqrt{a-bx^2}(ad+bc}\right.}{(af+be)^2}$$

$$\Big\downarrow \text{323}$$

$$\frac{f^2\left(\frac{(af+be)\int \frac{1}{\sqrt{a-bx^2}\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{f}-\frac{b\sqrt{\frac{dx^2}{c}+1}\int \frac{1}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}dx}{f\sqrt{c+dx^2}}\right)}{(af+be)^2}+\frac{b\left(\frac{\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(2af+be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(af+be)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}}{a(ad+bc)}+\frac{bx\sqrt{c+dx^2}(af+be}{a\sqrt{a-bx^2}(ad+bc}\right.}{(af+be)^2}$$

$$\Big\downarrow \text{323}$$

$$\frac{f^2\left(\frac{(af+be)\int \frac{1}{\sqrt{a-bx^2}\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{f}-\frac{b\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\int \frac{1}{\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}}dx}{f\sqrt{a-bx^2}\sqrt{c+dx^2}}\right)}{(af+be)^2}+\frac{b\left(\frac{\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(2af+be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(af+be)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}}{a(ad+bc)}+\frac{bx\sqrt{c+dx^2}(af+be}{a\sqrt{a-bx^2}(ad+bc}\right.}{(af+be)^2}$$

$$\Big\downarrow \text{321}$$

$$\frac{f^2\left(\frac{(af+be)\int \frac{1}{\sqrt{a-bx^2}\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{f}-\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{f\sqrt{a-bx^2}\sqrt{c+dx^2}}\right)}{(af+be)^2}+\frac{b\left(\frac{\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(2af+be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(af+be)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}}{a(ad+bc)}+\frac{bx\sqrt{c+dx^2}(af+be}{a\sqrt{a-bx^2}(ad+bc}\right.}{(af+be)^2}$$

$\Big\downarrow$ 413

$$\frac{f^2\left(\frac{\sqrt{1-\frac{bx^2}{a}}(af+be)\int \frac{1}{\sqrt{1-\frac{bx^2}{a}}\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{f\sqrt{a-bx^2}}-\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{f\sqrt{a-bx^2}\sqrt{c+dx^2}}\right)}{(af+be)^2}+\frac{b\left(\frac{\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(2af+be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(af+be)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}}{a(ad+bc)}+\frac{bx\sqrt{c+dx^2}(af+be}{a\sqrt{a-bx^2}(ad+bc}\right.}{(af+be)^2}$$

$\Big\downarrow$ 413

$$\frac{f^2\left(\frac{\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(af+be)\int \frac{1}{\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\left(fx^2+e\right)}dx}{f\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{f\sqrt{a-bx^2}\sqrt{c+dx^2}}\right)}{(af+be)^2}+\frac{b\left(\frac{\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(2af+be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(af+be)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}}{a(ad+bc)}+\frac{bx\sqrt{c+dx^2}(af+be}{a\sqrt{a-bx^2}(ad+bc}\right.}{(af+be)^2}$$

$\Big\downarrow$ 412

$$\frac{f^2\left(\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(af+be)\,\text{EllipticPi}\left(-\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}ef\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{f\sqrt{a-bx^2}\sqrt{c+dx^2}}\right)}{(af+be)^2}+$$

$$\frac{b\left(\frac{\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(2af+be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(af+be)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}}{a(ad+bc)}+\frac{bx\sqrt{c+dx^2}(af+be}{a\sqrt{a-bx^2}(ad+bc}\right.}{(af+be)^2}$$

input
```
Int[1/((a - b*x^2)^(3/2)*Sqrt[c + d*x^2]*(e + f*x^2)),x]
```

output
```
(b*((b*(b*e + a*f)*x*Sqrt[c + d*x^2])/(a*(b*c + a*d)*Sqrt[a - b*x^2]) + (-
((Sqrt[a]*Sqrt[b]*(b*e + a*f)*Sqrt[1 - (b*x^2)/a]*Sqrt[c + d*x^2]*Elliptic
E[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[a - b*x^2]*Sqrt[1 +
(d*x^2)/c])) + (Sqrt[a]*(b*c + a*d)*(b*e + 2*a*f)*Sqrt[1 - (b*x^2)/a]*Sqrt
[1 + (d*x^2)/c]*EllipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(S
qrt[b]*Sqrt[a - b*x^2]*Sqrt[c + d*x^2]))/(a*(b*c + a*d))))/(b*e + a*f)^2 +
 (f^2*(-((Sqrt[a]*Sqrt[b]*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Elliptic
F[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(f*Sqrt[a - b*x^2]*Sqrt[c
+ d*x^2])) + (Sqrt[a]*(b*e + a*f)*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*
EllipticPi[-((a*f)/(b*e)), ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(
Sqrt[b]*e*f*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])))/(b*e + a*f)^2
```

Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 415
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[d/b   Int[1/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x],
x] + Simp[(b*c - a*d)/b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NegQ[d/c]
```

rule 421
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b^2/(b*c - a*d)^2   Int[(c + d*x^2)^(q + 2)*((e +
 f*x^2)^r/(a + b*x^2)), x], x] - Simp[d/(b*c - a*d)^2   Int[(c + d*x^2)^q*(
e + f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r}
, x] && LtQ[q, -1]
```

Maple [A] (verified)

Time = 8.90 (sec) , antiderivative size = 411, normalized size of antiderivative = 1.13

method	result
default	$\frac{\left(\sqrt{\frac{b}{a}}\, b^{2} d e\, x^{3}+\sqrt{\frac{-b\, x^{2}+a}{a}}\, \sqrt{\frac{x^{2} d+c}{c}}\, \operatorname{EllipticF}\left(x \sqrt{\frac{b}{a}}, \sqrt{-\frac{a d}{b c}}\right) a b d e+\sqrt{\frac{-b\, x^{2}+a}{a}}\, \sqrt{\frac{x^{2} d+c}{c}}\, \operatorname{EllipticF}\left(x \sqrt{\frac{b}{a}}, \sqrt{-\frac{a d}{b c}}\right) b^{2} c e-\sqrt{\frac{-b\, x^{2}+}{a}}\right.}{}$
elliptic	$\frac{\sqrt{\left(-b\, x^{2}+a\right)\left(x^{2} d+c\right)}\, \left(-\frac{\left(-b d\, x^{2}-b c\right) b x}{a\left(a d+b c\right)\left(a f+b e\right) \sqrt{\left(x^{2}-\frac{a}{b}\right)\left(-b d\, x^{2}-b c\right)}}+\frac{\sqrt{1-\frac{b\, x^{2}}{a}}\, \sqrt{1+\frac{d\, x^{2}}{c}}\, \operatorname{EllipticF}\left(x \sqrt{\frac{b}{a}}, \sqrt{-1-\frac{a d-b c}{c b}}\right) b}{\sqrt{\frac{b}{a}}\, \sqrt{-b d\, x^{4}+a d\, x^{2}-x^{2} b c+a c}\, a\left(a f+b e\right)}-\frac{b^{2} c \sqrt{1-\frac{b\, x^{2}}{a}}}{a\left(a d+b c\right)\left(a\right.}\right.}{\sqrt{-b\, x^{2}+a}\, \sqrt{x^{2} d+c}}$

input
```
int(1/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x,method=_RETURNVERBOSE)
```

output
```
((b/a)^(1/2)*b^2*d*e*x^3+((-b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*Elliptic
F(x*(b/a)^(1/2),(-a*d/b/c)^(1/2))*a*b*d*e+((-b*x^2+a)/a)^(1/2)*((d*x^2+c)/
c)^(1/2)*EllipticF(x*(b/a)^(1/2),(-a*d/b/c)^(1/2))*b^2*c*e-((-b*x^2+a)/a)^
(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(b/a)^(1/2),(-a*d/b/c)^(1/2))*b^2*c*
e+((-b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticPi(x*(b/a)^(1/2),-a*f/b
/e,(-1/c*d)^(1/2)/(b/a)^(1/2))*a^2*d*f+((-b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^
(1/2)*EllipticPi(x*(b/a)^(1/2),-a*f/b/e,(-1/c*d)^(1/2)/(b/a)^(1/2))*a*b*c*
f+(b/a)^(1/2)*b^2*c*e*x)*(d*x^2+c)^(1/2)*(-b*x^2+a)^(1/2)/e/(b/a)^(1/2)/a/
(a*f+b*e)/(a*d+b*c)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a-bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate(1/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="frica
s")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(a-bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{1}{(a-bx^2)^{\frac{3}{2}}\sqrt{c+dx^2}\,(e+fx^2)}\,dx$$

input
```
integrate(1/(-b*x**2+a)**(3/2)/(d*x**2+c)**(1/2)/(f*x**2+e),x)
```

output
```
Integral(1/((a - b*x**2)**(3/2)*sqrt(c + d*x**2)*(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{(a-bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{1}{(-bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}(fx^2+e)}\,dx$$

input
```
integrate(1/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="maxim
a")
```

output
```
integrate(1/((-b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*(f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{1}{(a-bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{1}{(-bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}(fx^2+e)}\,dx$$

input
```
integrate(1/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="giac"
)
```

output
```
integrate(1/((-b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a-bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{1}{(a-b\,x^2)^{3/2}\,\sqrt{d\,x^2+c}\,(f\,x^2+e)}\,dx$$

input
```
int(1/((a - b*x^2)^(3/2)*(c + d*x^2)^(1/2)*(e + f*x^2)),x)
```

output
```
int(1/((a - b*x^2)^(3/2)*(c + d*x^2)^(1/2)*(e + f*x^2)), x)
```

Reduce [F]

$$\int \frac{1}{(a-bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{\sqrt{d\,x^2+c}\,\sqrt{-}}{b^2df\,x^8-2abdf\,x^6+b^2cf\,x^6+b^2de\,x^6+a^2df\,x^4-2abcf\,x^4-}$$

input

```
int(1/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x)
```

output

```
int((sqrt(c + d*x**2)*sqrt(a - b*x**2))/(a**2*c*e + a**2*c*f*x**2 + a**2*d
*e*x**2 + a**2*d*f*x**4 - 2*a*b*c*e*x**2 - 2*a*b*c*f*x**4 - 2*a*b*d*e*x**4
 - 2*a*b*d*f*x**6 + b**2*c*e*x**4 + b**2*c*f*x**6 + b**2*d*e*x**6 + b**2*d
*f*x**8),x)
```

3.292 $\int \frac{1}{x^2(a-bx^2)^{3/2}\sqrt{c+dx^2}(e+fx^2)}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 471

$$\begin{aligned}\int \frac{1}{x^2\left(a-bx^2\right)^{3/2}\sqrt{c+dx^2}\left(e+fx^2\right)}\,dx &= -\frac{\sqrt{c+dx^2}}{acex\sqrt{a-bx^2}} \\ &+ \frac{b(2b^2ce+a^2df+ab(de+cf))\,x\sqrt{c+dx^2}}{a^2c(bc+ad)e(be+af)\sqrt{a-bx^2}} \\ &- \frac{\sqrt{b}(2b^2ce+a^2df+ab(de+cf))\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\,E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{a^{3/2}c(bc+ad)e(be+af)\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}} \\ &+ \frac{\sqrt{b}(2be+af)\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{a^{3/2}e(be+af)\sqrt{a-bx^2}\sqrt{c+dx^2}} \\ &- \frac{\sqrt{a}f^2\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}e^2(be+af)\sqrt{a-bx^2}\sqrt{c+dx^2}}\end{aligned}$$

output
```
-(d*x^2+c)^(1/2)/a/c/e/x/(-b*x^2+a)^(1/2)+b*(2*b^2*c*e+a^2*d*f+a*b*(c*f+d*
e))*x*(d*x^2+c)^(1/2)/a^2/c/(a*d+b*c)/e/(a*f+b*e)/(-b*x^2+a)^(1/2)-b^(1/2)
*(2*b^2*c*e+a^2*d*f+a*b*(c*f+d*e))*(1-b*x^2/a)^(1/2)*(d*x^2+c)^(1/2)*Ellip
ticE(b^(1/2)*x/a^(1/2),(-a*d/b/c)^(1/2))/a^(3/2)/c/(a*d+b*c)/e/(a*f+b*e)/(
-b*x^2+a)^(1/2)/(1+d*x^2/c)^(1/2)+b^(1/2)*(a*f+2*b*e)*(1-b*x^2/a)^(1/2)*(1
+d*x^2/c)^(1/2)*EllipticF(b^(1/2)*x/a^(1/2),(-a*d/b/c)^(1/2))/a^(3/2)/e/(a
*f+b*e)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)-a^(1/2)*f^2*(1-b*x^2/a)^(1/2)*(1+
d*x^2/c)^(1/2)*EllipticPi(b^(1/2)*x/a^(1/2),-a*f/b/e,(-a*d/b/c)^(1/2))/b^(
1/2)/e^2/(a*f+b*e)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.51 (sec) , antiderivative size = 399, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^2 (a-bx^2)^{3/2} \sqrt{c+dx^2} (e+fx^2)} \, dx = \frac{-\sqrt{-\frac{b}{a}} e(c+dx^2) \left(a^3df - 2b^3cex^2 - ab^2(-ce+dex^2+cfx^2) \right.}{}$$

input
```
Integrate[1/(x^2*(a - b*x^2)^(3/2)*Sqrt[c + d*x^2]*(e + f*x^2)),x]
```

output
```
(-(Sqrt[-(b/a)]*e*(c + d*x^2)*(a^3*d*f - 2*b^3*c*e*x^2 - a*b^2*(-(c*e) + d
*e*x^2 + c*f*x^2) + a^2*b*(c*f + d*(e - f*x^2)))) + I*b*c*e*(2*b^2*c*e + a
^2*d*f + a*b*(d*e + c*f))*x*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Ellipt
icE[I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))] - I*b*c*(b*c + a*d)*e*(2*b*
e + a*f)*x*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqr
t[-(b/a)]*x], -((a*d)/(b*c))] + I*a^2*c*(b*c + a*d)*f^2*x*Sqrt[1 - (b*x^2)
/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[-((a*f)/(b*e)), I*ArcSinh[Sqrt[-(b/a)]*
x], -((a*d)/(b*c))])/(a^2*Sqrt[-(b/a)]*c*(b*c + a*d)*e^2*(b*e + a*f)*x*Sqr
t[a - b*x^2]*Sqrt[c + d*x^2])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a-bx^2)^{3/2} \sqrt{c+dx^2} (e+fx^2)} \, dx$$
$$\Big\downarrow \text{450}$$
$$\int \frac{1}{x^2 (a-bx^2)^{3/2} \sqrt{c+dx^2} (e+fx^2)} dx$$

input
```
Int[1/(x^2*(a - b*x^2)^(3/2)*Sqrt[c + d*x^2]*(e + f*x^2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [A] (verified)

Time = 21.65 (sec) , antiderivative size = 638, normalized size of antiderivative = 1.35

method	result
risch	$-\frac{\sqrt{-b x^{2}+a}\,\sqrt{x^{2} d+c}}{a^{2} c e x}-\frac{\left(-\frac{b c \sqrt{1-\frac{b x^{2}}{a}}\,\sqrt{1+\frac{d x^{2}}{c}}\,\left(\operatorname{EllipticF}\left(x \sqrt{\frac{b}{a}}, \sqrt{-1-\frac{a d-b c}{c b}}\right)-\operatorname{EllipticE}\left(x \sqrt{\frac{b}{a}}, \sqrt{-1-\frac{a d-b c}{c b}}\right)\right)}{\sqrt{\frac{b}{a}}\,\sqrt{-b d x^{4}+a d x^{2}-x^{2} b c+a c}}+\frac{a^{2} c f^{2} \sqrt{1-\frac{b x^{2}}{a}} \cdots}{(a f+b e \cdots} \cdots$
elliptic	$\frac{\sqrt{\left(-b x^{2}+a\right)\left(x^{2} d+c\right)}\,\left(-\frac{\left(-b d x^{2}-b c\right) b^{2} x}{a^{2}(a d+b c)(a f+b e) \sqrt{\left(x^{2}-\frac{a}{b}\right)\left(-b d x^{2}-b c\right)}}-\frac{\sqrt{-b d x^{4}+a d x^{2}-x^{2} b c+a c}}{a^{2} c e x}+\frac{\sqrt{1-\frac{b x^{2}}{a}}\,\sqrt{1+\frac{d x^{2}}{c}}\,\operatorname{EllipticF}\left(x \sqrt{\frac{b}{a}}, \sqrt{- \cdots}\right.}{\sqrt{\frac{b}{a}}\,\sqrt{-b d x^{4}+a d x^{2}-x^{2} b c+a c}\,a^{2} \cdots} \cdots}{\cdots}$
default	$\frac{\left(\sqrt{\frac{b}{a}}\,a^{2} b d^{2} e f x^{4}+\sqrt{\frac{b}{a}}\,a b^{2} c d e f x^{4}+\sqrt{\frac{b}{a}}\,a b^{2} d^{2} e^{2} x^{4}+2 \sqrt{\frac{b}{a}}\,b^{3} c d e^{2} x^{4}+\sqrt{\frac{-b x^{2}+a}{a}}\,\sqrt{\frac{x^{2} d+c}{c}}\,\operatorname{EllipticF}\left(x \sqrt{\frac{b}{a}}, \sqrt{-\frac{a d}{b c}}\right) a^{2} b c d e f x+ \cdots\right.}{\cdots}$

input
```
int(1/x^2/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x,method=_RETURNVERBO
SE)
```

output
```
-(-b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a^2/c/e/x-1/a^2/c/e*(-b*c/(b/a)^(1/2)*(1
-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)*(El
lipticF(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2))-EllipticE(x*(b/a)^(1/2),(-
1-(a*d-b*c)/c/b)^(1/2)))+a^2*c*f^2/(a*f+b*e)/e/(b/a)^(1/2)*(1-b*x^2/a)^(1/
2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)*EllipticPi(x*(b/
a)^(1/2),-a*f/b/e,(-1/c*d)^(1/2)/(b/a)^(1/2))+a*c*e*b^2/(a*f+b*e)*((-b*d*x
^2-b*c)/a/(a*d+b*c)*x/((x^2-a/b)*(-b*d*x^2-b*c))^(1/2)+(-1/a+b*c/a/(a*d+b*
c))/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*
x^2+a*c)^(1/2)*EllipticF(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2))-b/a/(a*d+
b*c)*c/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b
*c*x^2+a*c)^(1/2)*(EllipticF(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2))-Ellip
ticE(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2)))))*((-b*x^2+a)*(d*x^2+c))^(1/
2)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a-bx^2)^{3/2} \sqrt{c+dx^2}\,(e+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate(1/x^2/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="f
ricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{x^2 (a-bx^2)^{3/2} \sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{1}{x^2 (a-bx^2)^{\frac{3}{2}} \sqrt{c+dx^2}\,(e+fx^2)}\,dx$$

input
```
integrate(1/x**2/(-b*x**2+a)**(3/2)/(d*x**2+c)**(1/2)/(f*x**2+e),x)
```

output
```
Integral(1/(x**2*(a - b*x**2)**(3/2)*sqrt(c + d*x**2)*(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{x^2 (a-bx^2)^{3/2} \sqrt{c+dx^2}\,(e+fx^2)}\,dx = \int \frac{1}{(-bx^2+a)^{\frac{3}{2}} \sqrt{dx^2+c}(fx^2+e)x^2}\,dx$$

input
```
integrate(1/x^2/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="m
axima")
```

output
```
integrate(1/((-b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*(f*x^2 + e)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2 (a-bx^2)^{3/2} \sqrt{c+dx^2} (e+fx^2)}\,dx = \int \frac{1}{(-bx^2+a)^{\frac{3}{2}} \sqrt{dx^2+c}(fx^2+e)x^2}\,dx$$

input
```
integrate(1/x^2/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="g
iac")
```

output
```
integrate(1/((-b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*(f*x^2 + e)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a-bx^2)^{3/2} \sqrt{c+dx^2} (e+fx^2)}\,dx = \int \frac{1}{x^2 (a-b\,x^2)^{3/2} \sqrt{d\,x^2+c}\,(f\,x^2+e)}\,dx$$

input
```
int(1/(x^2*(a - b*x^2)^(3/2)*(c + d*x^2)^(1/2)*(e + f*x^2)),x)
```

output
```
int(1/(x^2*(a - b*x^2)^(3/2)*(c + d*x^2)^(1/2)*(e + f*x^2)), x)
```

Reduce [F]

$$\int \frac{1}{x^2 (a-bx^2)^{3/2} \sqrt{c+dx^2} (e+fx^2)}\,dx = \text{Too large to display}$$

input
```
int(1/x^2/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x)
```

output

```
( - sqrt(c + d*x**2)*sqrt(a - b*x**2) + int((sqrt(c + d*x**2)*sqrt(a - b*x
**2)*x**4)/(a**2*c*e + a**2*c*f*x**2 + a**2*d*e*x**2 + a**2*d*f*x**4 - 2*a
*b*c*e*x**2 - 2*a*b*c*f*x**4 - 2*a*b*d*e*x**4 - 2*a*b*d*f*x**6 + b**2*c*e*
x**4 + b**2*c*f*x**6 + b**2*d*e*x**6 + b**2*d*f*x**8),x)*a*b*d*f*x - int((
sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**4)/(a**2*c*e + a**2*c*f*x**2 + a**2*d
*e*x**2 + a**2*d*f*x**4 - 2*a*b*c*e*x**2 - 2*a*b*c*f*x**4 - 2*a*b*d*e*x**4
 - 2*a*b*d*f*x**6 + b**2*c*e*x**4 + b**2*c*f*x**6 + b**2*d*e*x**6 + b**2*d
*f*x**8),x)*b**2*d*f*x**3 + 2*int((sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**2)
/(a**2*c*e + a**2*c*f*x**2 + a**2*d*e*x**2 + a**2*d*f*x**4 - 2*a*b*c*e*x**
2 - 2*a*b*c*f*x**4 - 2*a*b*d*e*x**4 - 2*a*b*d*f*x**6 + b**2*c*e*x**4 + b**
2*c*f*x**6 + b**2*d*e*x**6 + b**2*d*f*x**8),x)*a*b*c*f*x + int((sqrt(c + d
*x**2)*sqrt(a - b*x**2)*x**2)/(a**2*c*e + a**2*c*f*x**2 + a**2*d*e*x**2 +
a**2*d*f*x**4 - 2*a*b*c*e*x**2 - 2*a*b*c*f*x**4 - 2*a*b*d*e*x**4 - 2*a*b*d
*f*x**6 + b**2*c*e*x**4 + b**2*c*f*x**6 + b**2*d*e*x**6 + b**2*d*f*x**8),x
)*a*b*d*e*x - 2*int((sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**2)/(a**2*c*e + a
**2*c*f*x**2 + a**2*d*e*x**2 + a**2*d*f*x**4 - 2*a*b*c*e*x**2 - 2*a*b*c*f*
x**4 - 2*a*b*d*e*x**4 - 2*a*b*d*f*x**6 + b**2*c*e*x**4 + b**2*c*f*x**6 + b
**2*d*e*x**6 + b**2*d*f*x**8),x)*b**2*c*f*x**3 - int((sqrt(c + d*x**2)*sqr
t(a - b*x**2)*x**2)/(a**2*c*e + a**2*c*f*x**2 + a**2*d*e*x**2 + a**2*d*f*x
**4 - 2*a*b*c*e*x**2 - 2*a*b*c*f*x**4 - 2*a*b*d*e*x**4 - 2*a*b*d*f*x**6...
```

3.293 $\int \frac{1}{x^4(a-bx^2)^{3/2}\sqrt{c+dx^2}(e+fx^2)}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 653

$$\int \frac{1}{x^4\left(a-bx^2\right)^{3/2}\sqrt{c+dx^2}\left(e+fx^2\right)}\,dx =$$
$$-\frac{\sqrt{c+dx^2}}{3acex^3\sqrt{a-bx^2}}-\frac{(4bce-2ade-3acf)\sqrt{c+dx^2}}{3a^2c^2e^2x\sqrt{a-bx^2}}$$
$$+\frac{b(8b^3c^2e^2+ab^2ce(3de+2cf)-a^3df(2de+3cf)-a^2b(2d^2e^2+3c^2f^2))\,x\sqrt{c+dx^2}}{3a^3c^2(bc+ad)e^2(be+af)\sqrt{a-bx^2}}$$
$$-\frac{\sqrt{b}(8b^3c^2e^2+ab^2ce(3de+2cf)-a^3df(2de+3cf)-a^2b(2d^2e^2+3c^2f^2))\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\,E\Big(\arcsin\Big(}{3a^{5/2}c^2(bc+ad)e^2(be+af)\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}}$$
$$+\frac{\sqrt{b}(8b^2ce^2-abe(de-2cf)-a^2f(de+3cf))\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{3a^{5/2}ce^2(be+af)\sqrt{a-bx^2}\sqrt{c+dx^2}}$$
$$+\frac{\sqrt{a}f^3\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}e^3(be+af)\sqrt{a-bx^2}\sqrt{c+dx^2}}$$

output
```
-1/3*(d*x^2+c)^(1/2)/a/c/e/x^3/(-b*x^2+a)^(1/2)-1/3*(-3*a*c*f-2*a*d*e+4*b*
c*e)*(d*x^2+c)^(1/2)/a^2/c^2/e^2/x/(-b*x^2+a)^(1/2)+1/3*b*(8*b^3*c^2*e^2+a
*b^2*c*e*(2*c*f+3*d*e)-a^3*d*f*(3*c*f+2*d*e)-a^2*b*(3*c^2*f^2+2*d^2*e^2))*
x*(d*x^2+c)^(1/2)/a^3/c^2/(a*d+b*c)/e^2/(a*f+b*e)/(-b*x^2+a)^(1/2)-1/3*b^(
1/2)*(8*b^3*c^2*e^2+a*b^2*c*e*(2*c*f+3*d*e)-a^3*d*f*(3*c*f+2*d*e)-a^2*b*(3
*c^2*f^2+2*d^2*e^2))*(1-b*x^2/a)^(1/2)*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x
/a^(1/2),(-a*d/b/c)^(1/2))/a^(5/2)/c^2/(a*d+b*c)/e^2/(a*f+b*e)/(-b*x^2+a)^
(1/2)/(1+d*x^2/c)^(1/2)+1/3*b^(1/2)*(8*b^2*c*e^2-a*b*e*(-2*c*f+d*e)-a^2*f*
(3*c*f+d*e))*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(b^(1/2)*x/a^(1/
2),(-a*d/b/c)^(1/2))/a^(5/2)/c/e^2/(a*f+b*e)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1
/2)+a^(1/2)*f^3*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)*EllipticPi(b^(1/2)*x/a
^(1/2),-a*f/b/e,(-a*d/b/c)^(1/2))/b^(1/2)/e^3/(a*f+b*e)/(-b*x^2+a)^(1/2)/(
d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 7.32 (sec) , antiderivative size = 974, normalized size of antiderivative = 1.49

$$\int \frac{1}{x^4 (a-bx^2)^{3/2} \sqrt{c+dx^2} (e+fx^2)} \, dx =$$

$$-\frac{a^2b^3c^3e^3 + a^3b^2c^2de^3 + a^3b^2c^3e^2f + a^4bc^2de^2f + 4ab^4c^3e^3x^2 + 3a^2b^3c^2de^3x^2 - a^3b^2cd^2e^3x^2 + a^2b^3c^3e^2fx^2}{}$$

input
```
Integrate[1/(x^4*(a - b*x^2)^(3/2)*Sqrt[c + d*x^2]*(e + f*x^2)),x]
```

output
```
-1/3*(a^2*b^3*c^3*e^3 + a^3*b^2*c^2*d*e^3 + a^3*b^2*c^3*e^2*f + a^4*b*c^2*
d*e^2*f + 4*a*b^4*c^3*e^3*x^2 + 3*a^2*b^3*c^2*d*e^3*x^2 - a^3*b^2*c*d^2*e^
3*x^2 + a^2*b^3*c^3*e^2*f*x^2 - a^4*b*c*d^2*e^2*f*x^2 - 3*a^3*b^2*c^3*e*f^
2*x^2 - 3*a^4*b*c^2*d*e*f^2*x^2 - 8*b^5*c^3*e^3*x^4 + a*b^4*c^2*d*e^3*x^4
+ 4*a^2*b^3*c*d^2*e^3*x^4 - 2*a^3*b^2*d^3*e^3*x^4 - 2*a*b^4*c^3*e^2*f*x^4
+ a^2*b^3*c^2*d*e^2*f*x^4 + a^3*b^2*c*d^2*e^2*f*x^4 - 2*a^4*b*d^3*e^2*f*x^
4 + 3*a^2*b^3*c^3*e*f^2*x^4 - 3*a^4*b*c*d^2*e*f^2*x^4 - 8*b^5*c^2*d*e^3*x^
6 - 3*a*b^4*c*d^2*e^3*x^6 + 2*a^2*b^3*d^3*e^3*x^6 - 2*a*b^4*c^2*d*e^2*f*x^
6 + 2*a^3*b^2*d^3*e^2*f*x^6 + 3*a^2*b^3*c^2*d*e*f^2*x^6 + 3*a^3*b^2*c*d^2*
e*f^2*x^6 - I*a*b*Sqrt[-(b/a)]*c*e*(-8*b^3*c^2*e^2 - a*b^2*c*e*(3*d*e + 2*
c*f) + a^3*d*f*(2*d*e + 3*c*f) + a^2*b*(2*d^2*e^2 + 3*c^2*f^2))*x^3*Sqrt[1
 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[-(b/a)]*x], -((
a*d)/(b*c))] + I*a*b*Sqrt[-(b/a)]*c*(b*c + a*d)*e*(-8*b^2*c*e^2 + a*b*e*(d
*e - 2*c*f) + a^2*f*(d*e + 3*c*f))*x^3*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2
)/c]*EllipticF[I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))] + (3*I)*a^5*(-(b
/a))^(3/2)*c^3*f^3*x^3*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[
-((a*f)/(b*e)), I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))] - (3*I)*a^5*Sqr
t[-(b/a)]*c^2*d*f^3*x^3*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi
[-((a*f)/(b*e)), I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))])/(a^3*b*c^2*(b
*c + a*d)*e^3*(b*e + a*f)*x^3*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (a-bx^2)^{3/2} \sqrt{c+dx^2} (e+fx^2)} \, dx$$

$$\Big\downarrow \; 450$$

$$\int \frac{1}{x^4 (a-bx^2)^{3/2} \sqrt{c+dx^2} (e+fx^2)} dx$$

input
```
Int[1/(x^4*(a - b*x^2)^(3/2)*Sqrt[c + d*x^2]*(e + f*x^2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [A] (verified)

Time = 23.75 (sec) , antiderivative size = 785, normalized size of antiderivative = 1.20

method	result
risch	$-\frac{\sqrt{-b x^2+a}\,\sqrt{x^2 d+c}\,(-3acf\,x^2-2ade\,x^2+5bce\,x^2+ace)}{3a^3c^2e^2x^3}+\frac{\left(-\frac{b(3acf+2ade-5bce)c\sqrt{1-\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\left(\mathrm{EllipticF}\left(x\sqrt{\frac{b}{a}},\sqrt{-1-\frac{ad-}{cb}}\right.\right.}{\sqrt{\frac{b}{a}}\,\sqrt{-bd\,x^4+ad\,x^2-x^2bc+ac}}\right.}{}$
elliptic	Expression too large to display
default	Expression too large to display

input
```
int(1/x^4/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x,method=_RETURNVERBO
SE)
```

output
```
-1/3*(-b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(-3*a*c*f*x^2-2*a*d*e*x^2+5*b*c*e*x^
2+a*c*e)/a^3/c^2/e^2/x^3+1/3/a^3/c^2/e^2*(-b*(3*a*c*f+2*a*d*e-5*b*c*e)*c/(
b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a
*c)^(1/2)*(EllipticF(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2))-EllipticE(x*(
b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2)))+a*b*c*d*e/(b/a)^(1/2)*(1-b*x^2/a)^(1
/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)*EllipticF(x*(b/
a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2))+3*a^3*c^2*f^3/(a*f+b*e)/e/(b/a)^(1/2)*(
1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)*El
lipticPi(x*(b/a)^(1/2),-a*f/b/e,(-1/c*d)^(1/2)/(b/a)^(1/2))-3*a*b^3*c^2*e^
2/(a*f+b*e)*((-b*d*x^2-b*c)/a/(a*d+b*c)*x/((x^2-a/b)*(-b*d*x^2-b*c))^(1/2)
+(-1/a+b*c/a/(a*d+b*c))/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-
b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)*EllipticF(x*(b/a)^(1/2),(-1-(a*d-b*c)/c
/b)^(1/2))-b/a/(a*d+b*c)*c/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)
/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)*(EllipticF(x*(b/a)^(1/2),(-1-(a*d-b*
c)/c/b)^(1/2))-EllipticE(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2)))))*((-b*x
^2+a)*(d*x^2+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (a-bx^2)^{3/2} \sqrt{c+dx^2} (e+fx^2)} dx = \text{Timed out}$$

input
```
integrate(1/x^4/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="f
ricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{x^4 (a-bx^2)^{3/2} \sqrt{c+dx^2} (e+fx^2)} dx = \int \frac{1}{x^4 (a-bx^2)^{\frac{3}{2}} \sqrt{c+dx^2} (e+fx^2)} dx$$

input
```
integrate(1/x**4/(-b*x**2+a)**(3/2)/(d*x**2+c)**(1/2)/(f*x**2+e),x)
```

output
```
Integral(1/(x**4*(a - b*x**2)**(3/2)*sqrt(c + d*x**2)*(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{x^4 (a-bx^2)^{3/2} \sqrt{c+dx^2} (e+fx^2)} \, dx = \int \frac{1}{(-bx^2+a)^{\frac{3}{2}} \sqrt{dx^2+c}(fx^2+e)x^4} \, dx$$

input
```
integrate(1/x^4/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="m
axima")
```

output
```
integrate(1/((-b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*(f*x^2 + e)*x^4), x)
```

Giac [F]

$$\int \frac{1}{x^4 (a-bx^2)^{3/2} \sqrt{c+dx^2} (e+fx^2)} \, dx = \int \frac{1}{(-bx^2+a)^{\frac{3}{2}} \sqrt{dx^2+c}(fx^2+e)x^4} \, dx$$

input
```
integrate(1/x^4/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="g
iac")
```

output
```
integrate(1/((-b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*(f*x^2 + e)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (a-bx^2)^{3/2} \sqrt{c+dx^2} (e+fx^2)} \, dx = \int \frac{1}{x^4 (a-b\,x^2)^{3/2} \sqrt{d\,x^2+c}\,(f\,x^2+e)} \, dx$$

input
```
int(1/(x^4*(a - b*x^2)^(3/2)*(c + d*x^2)^(1/2)*(e + f*x^2)),x)
```

output
```
int(1/(x^4*(a - b*x^2)^(3/2)*(c + d*x^2)^(1/2)*(e + f*x^2)), x)
```

Reduce [F]

$$\int \frac{1}{x^4 (a-bx^2)^{3/2} \sqrt{c+dx^2}\,(e+fx^2)}\,dx = \text{too large to display}$$

input
```
int(1/x^4/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x)
```

output
```
( - sqrt(c + d*x**2)*sqrt(a - b*x**2)*a*c*e + 3*sqrt(c + d*x**2)*sqrt(a -
b*x**2)*a*c*f*x**2 + 2*sqrt(c + d*x**2)*sqrt(a - b*x**2)*a*d*e*x**2 - 4*sq
rt(c + d*x**2)*sqrt(a - b*x**2)*b*c*e*x**2 - 3*int((sqrt(c + d*x**2)*sqrt(
a - b*x**2)*x**4)/(a**2*c*e + a**2*c*f*x**2 + a**2*d*e*x**2 + a**2*d*f*x**
4 - 2*a*b*c*e*x**2 - 2*a*b*c*f*x**4 - 2*a*b*d*e*x**4 - 2*a*b*d*f*x**6 + b*
*2*c*e*x**4 + b**2*c*f*x**6 + b**2*d*e*x**6 + b**2*d*f*x**8),x)*a**2*b*c*d
*f**2*x**3 - 2*int((sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**4)/(a**2*c*e + a*
*2*c*f*x**2 + a**2*d*e*x**2 + a**2*d*f*x**4 - 2*a*b*c*e*x**2 - 2*a*b*c*f*x
**4 - 2*a*b*d*e*x**4 - 2*a*b*d*f*x**6 + b**2*c*e*x**4 + b**2*c*f*x**6 + b*
*2*d*e*x**6 + b**2*d*f*x**8),x)*a**2*b*d**2*e*f*x**3 + 4*int((sqrt(c + d*x
**2)*sqrt(a - b*x**2)*x**4)/(a**2*c*e + a**2*c*f*x**2 + a**2*d*e*x**2 + a*
*2*d*f*x**4 - 2*a*b*c*e*x**2 - 2*a*b*c*f*x**4 - 2*a*b*d*e*x**4 - 2*a*b*d*f
*x**6 + b**2*c*e*x**4 + b**2*c*f*x**6 + b**2*d*e*x**6 + b**2*d*f*x**8),x)*
a*b**2*c*d*e*f*x**3 + 3*int((sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**4)/(a**2
*c*e + a**2*c*f*x**2 + a**2*d*e*x**2 + a**2*d*f*x**4 - 2*a*b*c*e*x**2 - 2*
a*b*c*f*x**4 - 2*a*b*d*e*x**4 - 2*a*b*d*f*x**6 + b**2*c*e*x**4 + b**2*c*f*
x**6 + b**2*d*e*x**6 + b**2*d*f*x**8),x)*a*b**2*c*d*f**2*x**5 + 2*int((sqr
t(c + d*x**2)*sqrt(a - b*x**2)*x**4)/(a**2*c*e + a**2*c*f*x**2 + a**2*d*e*
x**2 + a**2*d*f*x**4 - 2*a*b*c*e*x**2 - 2*a*b*c*f*x**4 - 2*a*b*d*e*x**4 -
2*a*b*d*f*x**6 + b**2*c*e*x**4 + b**2*c*f*x**6 + b**2*d*e*x**6 + b**2*d...
```

3.294 $\int \frac{1}{x^6(a-bx^2)^{3/2}\sqrt{c+dx^2}(e+fx^2)}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 905

$$\int \frac{1}{x^6\left(a-bx^2\right)^{3/2}\sqrt{c+dx^2}\left(e+fx^2\right)}\,dx =$$

$$-\frac{\sqrt{c+dx^2}}{5acex^5\sqrt{a-bx^2}}-\frac{(6bce-4ade-5acf)\sqrt{c+dx^2}}{15a^2c^2e^2x^3\sqrt{a-bx^2}}$$

$$-\frac{\left(\frac{24b^2ce}{a}-13bde+\frac{8ad^2e}{c}-20bcf+10adf+\frac{15acf^2}{e}\right)\sqrt{c+dx^2}}{15a^2c^2e^2x\sqrt{a-bx^2}}$$

$$+\frac{b(48b^4c^3e^3+8ab^3c^2e^2(2de+cf)-a^2b^2ce(9d^2e^2-cdef+10c^2f^2)+a^4df(8d^2e^2+10cdef+15c^2f^2)+a}{15a^4c^3(bc+ad)e^3(be+af)\sqrt{a-bx^2}}$$

$$-\frac{\sqrt{b}(48b^4c^3e^3+8ab^3c^2e^2(2de+cf)-a^2b^2ce(9d^2e^2-cdef+10c^2f^2)+a^4df(8d^2e^2+10cdef+15c^2f^2)+}{15a^{7/2}c^3(bc+ad)e^3(be+af)\sqrt{a-bx^2}}$$

$$+\frac{\sqrt{b}(48b^3c^2e^3-8ab^2ce^2(de-cf)+a^2be(4d^2e^2-3cdef-10c^2f^2)+a^3f(4d^2e^2+5cdef+15c^2f^2))\sqrt{1-}}{15a^{7/2}c^2e^3(be+af)\sqrt{a-bx^2}\sqrt{c+dx^2}}$$

$$-\frac{\sqrt{a}f^4\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}e^4(be+af)\sqrt{a-bx^2}\sqrt{c+dx^2}}$$

output
```
-1/5*(d*x^2+c)^(1/2)/a/c/e/x^5/(-b*x^2+a)^(1/2)-1/15*(-5*a*c*f-4*a*d*e+6*b
*c*e)*(d*x^2+c)^(1/2)/a^2/c^2/e^2/x^3/(-b*x^2+a)^(1/2)-1/15*(24*b^2*c*e/a-
13*b*d*e+8*a*d^2*e/c-20*b*c*f+10*a*d*f+15*a*c*f^2/e)*(d*x^2+c)^(1/2)/a^2/c
^2/e^2/x/(-b*x^2+a)^(1/2)+1/15*b*(48*b^4*c^3*e^3+8*a*b^3*c^2*e^2*(c*f+2*d*
e)-a^2*b^2*c*e*(10*c^2*f^2-c*d*e*f+9*d^2*e^2)+a^4*d*f*(15*c^2*f^2+10*c*d*e
*f+8*d^2*e^2)+a^3*b*(15*c^3*f^3+c*d^2*e^2*f+8*d^3*e^3))*x*(d*x^2+c)^(1/2)/
a^4/c^3/(a*d+b*c)/e^3/(a*f+b*e)/(-b*x^2+a)^(1/2)-1/15*b^(1/2)*(48*b^4*c^3*
e^3+8*a*b^3*c^2*e^2*(c*f+2*d*e)-a^2*b^2*c*e*(10*c^2*f^2-c*d*e*f+9*d^2*e^2)
+a^4*d*f*(15*c^2*f^2+10*c*d*e*f+8*d^2*e^2)+a^3*b*(15*c^3*f^3+c*d^2*e^2*f+8
*d^3*e^3))*(1-b*x^2/a)^(1/2)*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2),(
-a*d/b/c)^(1/2))/a^(7/2)/c^3/(a*d+b*c)/e^3/(a*f+b*e)/(-b*x^2+a)^(1/2)/(1+d
*x^2/c)^(1/2)+1/15*b^(1/2)*(48*b^3*c^2*e^3-8*a*b^2*c*e^2*(-c*f+d*e)+a^2*b*
e*(-10*c^2*f^2-3*c*d*e*f+4*d^2*e^2)+a^3*f*(15*c^2*f^2+5*c*d*e*f+4*d^2*e^2)
)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(b^(1/2)*x/a^(1/2),(-a*d/b/
c)^(1/2))/a^(7/2)/c^2/e^3/(a*f+b*e)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)-a^(1/
2)*f^4*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)*EllipticPi(b^(1/2)*x/a^(1/2),-a
*f/b/e,(-a*d/b/c)^(1/2))/b^(1/2)/e^4/(a*f+b*e)/(-b*x^2+a)^(1/2)/(d*x^2+c)^
(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 8.92 (sec) , antiderivative size = 1531, normalized size of antiderivative = 1.69

$$\int \frac{1}{x^6 (a-bx^2)^{3/2} \sqrt{c+dx^2} (e+fx^2)} dx = \text{Too large to display}$$

input
```
Integrate[1/(x^6*(a - b*x^2)^(3/2)*Sqrt[c + d*x^2]*(e + f*x^2)),x]
```

output
```
-1/15*(3*a^3*b^3*c^4*e^4 + 3*a^4*b^2*c^3*d*e^4 + 3*a^4*b^2*c^4*e^3*f + 3*a
^5*b*c^3*d*e^3*f + 6*a^2*b^4*c^4*e^4*x^2 + 5*a^3*b^3*c^3*d*e^4*x^2 - a^4*b
^2*c^2*d^2*e^4*x^2 + a^3*b^3*c^4*e^3*f*x^2 - a^5*b*c^2*d^2*e^3*f*x^2 - 5*a
^4*b^2*c^4*e^2*f^2*x^2 - 5*a^5*b*c^3*d*e^2*f^2*x^2 + 24*a*b^5*c^4*e^4*x^4
+ 17*a^2*b^4*c^3*d*e^4*x^4 - 3*a^3*b^3*c^2*d^2*e^4*x^4 + 4*a^4*b^2*c*d^3*e
^4*x^4 + 4*a^2*b^4*c^4*e^3*f*x^4 + 2*a^3*b^3*c^3*d*e^3*f*x^4 + 2*a^4*b^2*c
^2*d^2*e^3*f*x^4 + 4*a^5*b*c*d^3*e^3*f*x^4 - 5*a^3*b^3*c^4*e^2*f^2*x^4 + 5
*a^5*b*c^2*d^2*e^2*f^2*x^4 + 15*a^4*b^2*c^4*e*f^3*x^4 + 15*a^5*b*c^3*d*e*f
^3*x^4 - 48*b^6*c^4*e^4*x^6 + 8*a*b^5*c^3*d*e^4*x^6 + 20*a^2*b^4*c^2*d^2*e
^4*x^6 - 13*a^3*b^3*c*d^3*e^4*x^6 + 8*a^4*b^2*d^4*e^4*x^6 - 8*a*b^5*c^4*e^
3*f*x^6 + 3*a^2*b^4*c^3*d*e^3*f*x^6 - 3*a^4*b^2*c*d^3*e^3*f*x^6 + 8*a^5*b*
d^4*e^3*f*x^6 + 10*a^2*b^4*c^4*e^2*f^2*x^6 - 5*a^3*b^3*c^3*d*e^2*f^2*x^6 -
 5*a^4*b^2*c^2*d^2*e^2*f^2*x^6 + 10*a^5*b*c*d^3*e^2*f^2*x^6 - 15*a^3*b^3*c
^4*e*f^3*x^6 + 15*a^5*b*c^2*d^2*e*f^3*x^6 - 48*b^6*c^3*d*e^4*x^8 - 16*a*b^
5*c^2*d^2*e^4*x^8 + 9*a^2*b^4*c*d^3*e^4*x^8 - 8*a^3*b^3*d^4*e^4*x^8 - 8*a*
b^5*c^3*d*e^3*f*x^8 - a^2*b^4*c^2*d^2*e^3*f*x^8 - a^3*b^3*c*d^3*e^3*f*x^8
- 8*a^4*b^2*d^4*e^3*f*x^8 + 10*a^2*b^4*c^3*d*e^2*f^2*x^8 - 10*a^4*b^2*c*d^
3*e^2*f^2*x^8 - 15*a^3*b^3*c^3*d*e*f^3*x^8 - 15*a^4*b^2*c^2*d^2*e*f^3*x^8
+ I*a*b*Sqrt[-(b/a)]*c*e*(48*b^4*c^3*e^3 + 8*a*b^3*c^2*e^2*(2*d*e + c*f) +
 a^2*b^2*c*e*(-9*d^2*e^2 + c*d*e*f - 10*c^2*f^2) + a^4*d*f*(8*d^2*e^2 +...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6 (a-bx^2)^{3/2} \sqrt{c+dx^2} (e+fx^2)} \, dx$$

$$\Big\downarrow \ 450$$

$$\int \frac{1}{x^6 (a-bx^2)^{3/2} \sqrt{c+dx^2} (e+fx^2)}dx$$

input
```
Int[1/(x^6*(a - b*x^2)^(3/2)*Sqrt[c + d*x^2]*(e + f*x^2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [A] (verified)

Time = 26.17 (sec) , antiderivative size = 1138, normalized size of antiderivative = 1.26

method	result	size
risch	Expression too large to display	1138
elliptic	Expression too large to display	1844
default	Expression too large to display	3145

input
```
int(1/x^6/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x,method=_RETURNVERBO
SE)
```

output
```
-1/15*(-b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(15*a^2*c^2*f^2*x^4+10*a^2*c*d*e*f*
x^4+8*a^2*d^2*e^2*x^4-25*a*b*c^2*e*f*x^4-17*a*b*c*d*e^2*x^4+33*b^2*c^2*e^2
*x^4-5*a^2*c^2*e*f*x^2-4*a^2*c*d*e^2*x^2+9*a*b*c^2*e^2*x^2+3*a^2*c^2*e^2)/
a^4/c^3/e^3/x^5-1/15/a^4/c^3/e^3*(-b*(15*a^2*c^2*f^2+10*a^2*c*d*e*f+8*a^2*
d^2*e^2-25*a*b*c^2*e*f-17*a*b*c*d*e^2+33*b^2*c^2*e^2)*c/(b/a)^(1/2)*(1-b*x
^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)*(Ellipt
icF(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2))-EllipticE(x*(b/a)^(1/2),(-1-(a
*d-b*c)/c/b)^(1/2)))-9*a*b^2*c^2*d*e^2/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*
x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)*EllipticF(x*(b/a)^(1/2),
(-1-(a*d-b*c)/c/b)^(1/2))+4*a^2*b*c*d^2*e^2/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*
(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)*EllipticF(x*(b/a)^(
1/2),(-1-(a*d-b*c)/c/b)^(1/2))+15*a^4*c^3*f^4/(a*f+b*e)/e/(b/a)^(1/2)*(1-b
*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)*Ellip
ticPi(x*(b/a)^(1/2),-a*f/b/e,(-1/c*d)^(1/2)/(b/a)^(1/2))+15*a*b^4*c^3*e^3/
(a*f+b*e)*((-b*d*x^2-b*c)/a/(a*d+b*c)*x/((x^2-a/b)*(-b*d*x^2-b*c))^(1/2)+(
-1/a+b*c/a/(a*d+b*c))/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*
d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)*EllipticF(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b
)^(1/2))-b/a/(a*d+b*c)*c/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(
-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)*(EllipticF(x*(b/a)^(1/2),(-1-(a*d-b*c)
/c/b)^(1/2))-EllipticE(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2))))+5*a^2*...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^6 (a-bx^2)^{3/2} \sqrt{c+dx^2} (e+fx^2)} dx = \text{Timed out}$$

input
```
integrate(1/x^6/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="f
ricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{x^6 (a-bx^2)^{3/2} \sqrt{c+dx^2} (e+fx^2)} dx = \int \frac{1}{x^6 (a-bx^2)^{\frac{3}{2}} \sqrt{c+dx^2} (e+fx^2)} dx$$

input
```
integrate(1/x**6/(-b*x**2+a)**(3/2)/(d*x**2+c)**(1/2)/(f*x**2+e),x)
```

output
```
Integral(1/(x**6*(a - b*x**2)**(3/2)*sqrt(c + d*x**2)*(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{x^6 (a-bx^2)^{3/2} \sqrt{c+dx^2} (e+fx^2)} dx = \int \frac{1}{(-bx^2+a)^{\frac{3}{2}} \sqrt{dx^2+c}(fx^2+e)x^6} dx$$

input
```
integrate(1/x^6/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="m
axima")
```

output
```
integrate(1/((-b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*(f*x^2 + e)*x^6), x)
```

Giac [F]

$$\int \frac{1}{x^6 (a-bx^2)^{3/2} \sqrt{c+dx^2} (e+fx^2)} dx = \int \frac{1}{(-bx^2+a)^{\frac{3}{2}} \sqrt{dx^2+c}(fx^2+e)x^6} dx$$

input
```
integrate(1/x^6/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="g
iac")
```

output
```
integrate(1/((-b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*(f*x^2 + e)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^6 (a-bx^2)^{3/2} \sqrt{c+dx^2} (e+fx^2)} \, dx = \int \frac{1}{x^6 \left(a-b\,x^2\right)^{3/2} \sqrt{d\,x^2+c} \left(f\,x^2+e\right)} \, dx$$

input
```
int(1/(x^6*(a - b*x^2)^(3/2)*(c + d*x^2)^(1/2)*(e + f*x^2)),x)
```

output
```
int(1/(x^6*(a - b*x^2)^(3/2)*(c + d*x^2)^(1/2)*(e + f*x^2)), x)
```

Reduce [F]

$$\int \frac{1}{x^6 (a-bx^2)^{3/2} \sqrt{c+dx^2} (e+fx^2)} \, dx = \text{too large to display}$$

input
```
int(1/x^6/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e),x)
```

output

```
( - 3*sqrt(c + d*x**2)*sqrt(a - b*x**2)*a*c*e + 5*sqrt(c + d*x**2)*sqrt(a
- b*x**2)*a*c*f*x**2 + 4*sqrt(c + d*x**2)*sqrt(a - b*x**2)*a*d*e*x**2 + 12
*sqrt(c + d*x**2)*sqrt(a - b*x**2)*a*d*f*x**4 - 6*sqrt(c + d*x**2)*sqrt(a
- b*x**2)*b*c*e*x**2 - 18*sqrt(c + d*x**2)*sqrt(a - b*x**2)*b*c*f*x**4 - 1
5*sqrt(c + d*x**2)*sqrt(a - b*x**2)*b*d*e*x**4 - 12*int((sqrt(c + d*x**2)*
sqrt(a - b*x**2)*x**4)/(a**2*c*e + a**2*c*f*x**2 + a**2*d*e*x**2 + a**2*d*
f*x**4 - 2*a*b*c*e*x**2 - 2*a*b*c*f*x**4 - 2*a*b*d*e*x**4 - 2*a*b*d*f*x**6
 + b**2*c*e*x**4 + b**2*c*f*x**6 + b**2*d*e*x**6 + b**2*d*f*x**8),x)*a**2*
b*d**2*f**2*x**5 + 18*int((sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**4)/(a**2*c
*e + a**2*c*f*x**2 + a**2*d*e*x**2 + a**2*d*f*x**4 - 2*a*b*c*e*x**2 - 2*a*
b*c*f*x**4 - 2*a*b*d*e*x**4 - 2*a*b*d*f*x**6 + b**2*c*e*x**4 + b**2*c*f*x*
*6 + b**2*d*e*x**6 + b**2*d*f*x**8),x)*a*b**2*c*d*f**2*x**5 + 15*int((sqrt
(c + d*x**2)*sqrt(a - b*x**2)*x**4)/(a**2*c*e + a**2*c*f*x**2 + a**2*d*e*x
**2 + a**2*d*f*x**4 - 2*a*b*c*e*x**2 - 2*a*b*c*f*x**4 - 2*a*b*d*e*x**4 - 2
*a*b*d*f*x**6 + b**2*c*e*x**4 + b**2*c*f*x**6 + b**2*d*e*x**6 + b**2*d*f*x
**8),x)*a*b**2*d**2*e*f*x**5 + 12*int((sqrt(c + d*x**2)*sqrt(a - b*x**2)*x
**4)/(a**2*c*e + a**2*c*f*x**2 + a**2*d*e*x**2 + a**2*d*f*x**4 - 2*a*b*c*e
*x**2 - 2*a*b*c*f*x**4 - 2*a*b*d*e*x**4 - 2*a*b*d*f*x**6 + b**2*c*e*x**4 +
 b**2*c*f*x**6 + b**2*d*e*x**6 + b**2*d*f*x**8),x)*a*b**2*d**2*f**2*x**7 -
 18*int((sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**4)/(a**2*c*e + a**2*c*f*x...
```

3.295 $\int \frac{x^8}{(a-bx^2)^{3/2}\sqrt{c+dx^2}(e+fx^2)^2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 771

$$\int \frac{x^8}{(a-bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = -\frac{x\sqrt{c+dx^2}}{bdf^2\sqrt{a-bx^2}}$$
$$+\frac{(b^3ce^2(3de-2cf)+4a^3df^2(de-cf)+ab^2e(3d^2e^2+2cdef-4c^2f^2)+2a^2bf(2d^2e^2-cdef-c^2f^2))\,x\sqrt{}}{2bd(bc+ad)f^2(be+af)^2(de-cf)\sqrt{a-bx^2}}$$
$$-\frac{e^3x\sqrt{c+dx^2}}{2f^2(be+af)(de-cf)\sqrt{a-bx^2}\,(e+fx^2)}$$
$$-\frac{\sqrt{a}(b^3ce^2(3de-2cf)+4a^3df^2(de-cf)+ab^2e(3d^2e^2+2cdef-4c^2f^2)+2a^2bf(2d^2e^2-cdef-c^2f^2))}{2b^{3/2}d(bc+ad)f^2(be+af)^2(de-cf)\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}}$$
$$+\frac{\sqrt{a}(2a^2cf^3+b^2e^2(3de+2cf)+2abef(3de+2cf))\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{2b^{3/2}df^3(be+af)^2\sqrt{a-bx^2}\sqrt{c+dx^2}}$$
$$-\frac{\sqrt{a}e^2(af(6de-7cf)+be(3de-4cf))\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{2\sqrt{b}f^3(be+af)^2(de-cf)\sqrt{a-bx^2}\sqrt{c+dx^2}}$$

output
```
-x*(d*x^2+c)^(1/2)/b/d/f^2/(-b*x^2+a)^(1/2)+1/2*(b^3*c*e^2*(-2*c*f+3*d*e)+
4*a^3*d*f^2*(-c*f+d*e)+a*b^2*e*(-4*c^2*f^2+2*c*d*e*f+3*d^2*e^2)+2*a^2*b*f*
(-c^2*f^2-c*d*e*f+2*d^2*e^2))*x*(d*x^2+c)^(1/2)/b/d/(a*d+b*c)/f^2/(a*f+b*e
)^2/(-c*f+d*e)/(-b*x^2+a)^(1/2)-1/2*e^3*x*(d*x^2+c)^(1/2)/f^2/(a*f+b*e)/(-
c*f+d*e)/(-b*x^2+a)^(1/2)/(f*x^2+e)-1/2*a^(1/2)*(b^3*c*e^2*(-2*c*f+3*d*e)+
4*a^3*d*f^2*(-c*f+d*e)+a*b^2*e*(-4*c^2*f^2+2*c*d*e*f+3*d^2*e^2)+2*a^2*b*f*
(-c^2*f^2-c*d*e*f+2*d^2*e^2))*(1-b*x^2/a)^(1/2)*(d*x^2+c)^(1/2)*EllipticE(
b^(1/2)*x/a^(1/2),(-a*d/b/c)^(1/2))/b^(3/2)/d/(a*d+b*c)/f^2/(a*f+b*e)^2/(-
c*f+d*e)/(-b*x^2+a)^(1/2)/(1+d*x^2/c)^(1/2)+1/2*a^(1/2)*(2*a^2*c*f^3+b^2*e
^2*(2*c*f+3*d*e)+2*a*b*e*f*(2*c*f+3*d*e))*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1
/2)*EllipticF(b^(1/2)*x/a^(1/2),(-a*d/b/c)^(1/2))/b^(3/2)/d/f^3/(a*f+b*e)^
2/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)-1/2*a^(1/2)*e^2*(a*f*(-7*c*f+6*d*e)+b*e
*(-4*c*f+3*d*e))*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)*EllipticPi(b^(1/2)*x/
a^(1/2),-a*f/b/e,(-a*d/b/c)^(1/2))/b^(1/2)/f^3/(a*f+b*e)^2/(-c*f+d*e)/(-b*
x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.23 (sec) , antiderivative size = 567, normalized size of antiderivative = 0.74

$$\int \frac{x^8}{(a-bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \frac{\sqrt{-\frac{b}{a}}df^2x(c+dx^2)\left(a^2bde^3-b^3ce^3x^2+ab^2e^3(c-dx^2)+2a^3f(\cdot\right.}{}$$

input
```
Integrate[x^8/((a - b*x^2)^(3/2)*Sqrt[c + d*x^2]*(e + f*x^2)^2),x]
```

output
```
(Sqrt[-(b/a)]*d*f^2*x*(c + d*x^2)*(a^2*b*d*e^3 - b^3*c*e^3*x^2 + a*b^2*e^3
*(c - d*x^2) + 2*a^3*f*(-(d*e) + c*f)*(e + f*x^2)) + I*c*f*(4*a^3*d*f^2*(-
(d*e) + c*f) + b^3*c*e^2*(-3*d*e + 2*c*f) + 2*a^2*b*f*(-2*d^2*e^2 + c*d*e*
f + c^2*f^2) + a*b^2*e*(-3*d^2*e^2 - 2*c*d*e*f + 4*c^2*f^2))*Sqrt[1 - (b*x
^2)/a]*Sqrt[1 + (d*x^2)/c]*(e + f*x^2)*EllipticE[I*ArcSinh[Sqrt[-(b/a)]*x]
, -((a*d)/(b*c))] - I*(b*c + a*d)*(-(d*e) + c*f)*(2*a^2*c*f^3 + b^2*e^2*(3
*d*e + 2*c*f) + 2*a*b*e*f*(3*d*e + 2*c*f))*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d
*x^2)/c]*(e + f*x^2)*EllipticF[I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))]
+ I*b*d*(b*c + a*d)*e^2*(b*e*(-3*d*e + 4*c*f) + a*f*(-6*d*e + 7*c*f))*Sqrt
[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(e + f*x^2)*EllipticPi[-((a*f)/(b*e)),
 I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))])/(2*b*Sqrt[-(b/a)]*d*(b*c + a*
d)*f^3*(b*e + a*f)^2*(-(d*e) + c*f)*Sqrt[a - b*x^2]*Sqrt[c + d*x^2]*(e + f
*x^2))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{(a-bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx$$

$$\Big\downarrow \text{450}$$

$$\int \frac{x^8}{(a-bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^2}dx$$

input
```
Int[x^8/((a - b*x^2)^(3/2)*Sqrt[c + d*x^2]*(e + f*x^2)^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1757 vs. $2(704) = 1408$.

Time = 11.16 (sec) , antiderivative size = 1758, normalized size of antiderivative = 2.28

method	result	size
elliptic	Expression too large to display	1758
default	Expression too large to display	4457

input
```
int(x^8/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x,method=_RETURNVERBO
SE)
```

output
```
((-b*x^2+a)*(d*x^2+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-(-b*d*x^2-
b*c)/b^2*a^3/(a*d+b*c)*x/(a*f+b*e)^2/((x^2-a/b)*(-b*d*x^2-b*c))^(1/2)+1/2/
f/(a*c*f^2-a*d*e*f+b*c*e*f-b*d*e^2)*e^3*x/(a*f+b*e)*(-b*d*x^4+a*d*x^2-b*c*
x^2+a*c)^(1/2)/(f*x^2+e)-1/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)
/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)*EllipticF(x*(b/a)^(1/2),(-1-(a*d-b*c
)/c/b)^(1/2))/f^2/b^2*a+2/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/
(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)*EllipticF(x*(b/a)^(1/2),(-1-(a*d-b*c)
/c/b)^(1/2))/f^3/b*e+1/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b
*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)*EllipticF(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/
b)^(1/2))/b^2*a^3/(a*f+b*e)^2-1/2*c/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2
/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)*b/f^2*e^3/(a*c*f^2-a*d*e*f+
b*c*e*f-b*d*e^2)/(a*f+b*e)*EllipticF(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2
))+1/2*c/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2
-b*c*x^2+a*c)^(1/2)*b/f^2*e^3/(a*c*f^2-a*d*e*f+b*c*e*f-b*d*e^2)/(a*f+b*e)*
EllipticE(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2))+1/2/(b/a)^(1/2)*(1-b*x^2
/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)*EllipticF
(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2))*d*b*e^4/f^3/(a*c*f^2-a*d*e*f+b*c*
e*f-b*d*e^2)/(a*f+b*e)-c/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(
-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)*a^3/b/(a*d+b*c)/(a*f+b*e)^2*EllipticE(
x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2))-7/2*e^2/(a*c*f^2-a*d*e*f+b*c*e*...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^8}{(a-bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(x^8/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="f
ricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^8}{(a-bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**8/(-b*x**2+a)**(3/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^8}{(a-bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{x^8}{(-bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}(fx^2+e)^2}\,dx$$

input
```
integrate(x^8/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="m
axima")
```

output
```
integrate(x^8/((-b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*(f*x^2 + e)^2), x)
```

Giac [F]

$$\int \frac{x^8}{(a-bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{x^8}{(-bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}(fx^2+e)^2}\,dx$$

input
```
integrate(x^8/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="g
iac")
```

output
```
integrate(x^8/((-b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*(f*x^2 + e)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^8}{(a-bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{x^8}{(a-b\,x^2)^{3/2}\,\sqrt{d\,x^2+c}\,(f\,x^2+e)^2}\,dx$$

input
```
int(x^8/((a - b*x^2)^(3/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^2),x)
```

output
```
int(x^8/((a - b*x^2)^(3/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^2), x)
```

Reduce [F]

$$\int \frac{x^8}{(a-bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{x^8}{(-b\,x^2+a)^{\frac{3}{2}}\,\sqrt{d\,x^2+c}\,\left(f\,x^2+e\right)^2}dx$$

input
```
int(x^8/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x)
```

output
```
int(x^8/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x)
```

3.296 $\int \frac{x^6}{\left(a-bx^2\right)^{3/2}\sqrt{c+dx^2}\left(e+fx^2\right)^2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 561

$$\int \frac{x^6}{(a-bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = -\frac{(b^2ce^2+abde^2-2a^2f(de-cf))\,x\sqrt{c+dx^2}}{2(bc+ad)f(be+af)^2(de-cf)\sqrt{a-bx^2}}$$
$$+\frac{e^2x\sqrt{c+dx^2}}{2f(be+af)(de-cf)\sqrt{a-bx^2}\,(e+fx^2)}$$
$$+\frac{\sqrt{a}(b^2ce^2+abde^2-2a^2f(de-cf))\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\,E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{2\sqrt{b}(bc+ad)f(be+af)^2(de-cf)\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}}$$
$$-\frac{\sqrt{a}e(be+4af)\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{2\sqrt{b}f^2(be+af)^2\sqrt{a-bx^2}\sqrt{c+dx^2}}$$
$$+\frac{\sqrt{a}e(af(4de-5cf)+be(de-2cf))\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{2\sqrt{b}f^2(be+af)^2(de-cf)\sqrt{a-bx^2}\sqrt{c+dx^2}}$$

output
```
-1/2*(b^2*c*e^2+a*b*d*e^2-2*a^2*f*(-c*f+d*e))*x*(d*x^2+c)^(1/2)/(a*d+b*c)/
f/(a*f+b*e)^2/(-c*f+d*e)/(-b*x^2+a)^(1/2)+1/2*e^2*x*(d*x^2+c)^(1/2)/f/(a*f
+b*e)/(-c*f+d*e)/(-b*x^2+a)^(1/2)/(f*x^2+e)+1/2*a^(1/2)*(b^2*c*e^2+a*b*d*e
^2-2*a^2*f*(-c*f+d*e))*(1-b*x^2/a)^(1/2)*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)
*x/a^(1/2),(-a*d/b/c)^(1/2))/b^(1/2)/(a*d+b*c)/f/(a*f+b*e)^2/(-c*f+d*e)/(-
b*x^2+a)^(1/2)/(1+d*x^2/c)^(1/2)-1/2*a^(1/2)*e*(4*a*f+b*e)*(1-b*x^2/a)^(1/
2)*(1+d*x^2/c)^(1/2)*EllipticF(b^(1/2)*x/a^(1/2),(-a*d/b/c)^(1/2))/b^(1/2)
/f^2/(a*f+b*e)^2/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)+1/2*a^(1/2)*e*(a*f*(-5*c
*f+4*d*e)+b*e*(-2*c*f+d*e))*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)*EllipticPi
(b^(1/2)*x/a^(1/2),-a*f/b/e,(-a*d/b/c)^(1/2))/b^(1/2)/f^2/(a*f+b*e)^2/(-c*
f+d*e)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 7.41 (sec) , antiderivative size = 1664, normalized size of antiderivative = 2.97

$$\int \frac{x^6}{(a-bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \text{Too large to display}$$

input
```
Integrate[x^6/((a - b*x^2)^(3/2)*Sqrt[c + d*x^2]*(e + f*x^2)^2),x]
```

output
```
(a^2*(-(b/a))^(3/2)*c^2*e^2*f^2*x + (3*a*b*c*d*e^2*f^2*x)/Sqrt[-(b/a)] + 2
*a^2*Sqrt[-(b/a)]*c^2*e*f^3*x + b^2*Sqrt[-(b/a)]*c^2*e^2*f^2*x^3 + (3*a*b*
d^2*e^2*f^2*x^3)/Sqrt[-(b/a)] + 2*a^2*Sqrt[-(b/a)]*c^2*f^4*x^3 + b^2*Sqrt[
-(b/a)]*c*d*e^2*f^2*x^5 + a*b*Sqrt[-(b/a)]*d^2*e^2*f^2*x^5 + (2*a*b*d^2*e*
f^3*x^5)/Sqrt[-(b/a)] + 2*a^2*Sqrt[-(b/a)]*c*d*f^4*x^5 + I*c*f*(b^2*c*e^2
+ a*b*d*e^2 + 2*a^2*f*(-(d*e) + c*f))*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)
/c]*(e + f*x^2)*EllipticE[I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))] + I*(
b*c + a*d)*e*(b*e + 4*a*f)*(-(d*e) + c*f)*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*
x^2)/c]*(e + f*x^2)*EllipticF[I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))] +
 I*b^2*c*d*e^4*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[-((a*f)/
(b*e)), I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))] + I*a*b*d^2*e^4*Sqrt[1
- (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[-((a*f)/(b*e)), I*ArcSinh[Sqrt
[-(b/a)]*x], -((a*d)/(b*c))] - (2*I)*b^2*c^2*e^3*f*Sqrt[1 - (b*x^2)/a]*Sqr
t[1 + (d*x^2)/c]*EllipticPi[-((a*f)/(b*e)), I*ArcSinh[Sqrt[-(b/a)]*x], -((
a*d)/(b*c))] + (2*I)*a*b*c*d*e^3*f*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]
*EllipticPi[-((a*f)/(b*e)), I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))] + (
4*I)*a^2*d^2*e^3*f*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[-((a
*f)/(b*e)), I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))] - (5*I)*a*b*c^2*e^2
*f^2*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[-((a*f)/(b*e)), I*
ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))] - (5*I)*a^2*c*d*e^2*f^2*Sqrt[1...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(a-bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx$$

$$\Big\downarrow \; 450$$

$$\int \frac{x^6}{(a-bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^2}dx$$

input
```
Int[x^6/((a - b*x^2)^(3/2)*Sqrt[c + d*x^2]*(e + f*x^2)^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1448 vs. $2(498) = 996$.

Time = 10.83 (sec) , antiderivative size = 1449, normalized size of antiderivative = 2.58

method	result	size
elliptic	Expression too large to display	1449
default	Expression too large to display	2741

input
```
int(x^6/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x,method=_RETURNVERBO
SE)
```

output
```
((-b*x^2+a)*(d*x^2+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-(-b*d*x^2-
b*c)/b*a^2/(a*d+b*c)*x/(a*f+b*e)^2/((x^2-a/b)*(-b*d*x^2-b*c))^(1/2)-1/2/(a
*c*f^2-a*d*e*f+b*c*e*f-b*d*e^2)*e^2*x/(a*f+b*e)*(-b*d*x^4+a*d*x^2-b*c*x^2+
a*c)^(1/2)/(f*x^2+e)-1/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b
*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)*EllipticF(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/
b)^(1/2))/f^2/b+1/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^
4+a*d*x^2-b*c*x^2+a*c)^(1/2)*EllipticF(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1
/2))/b*a^2/(a*f+b*e)^2-1/2/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)
/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)*EllipticF(x*(b/a)^(1/2),(-1-(a*d-b*c
)/c/b)^(1/2))*b*d/f^2*e^3/(a*c*f^2-a*d*e*f+b*c*e*f-b*d*e^2)/(a*f+b*e)-c/(b
/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*
c)^(1/2)*a^2/(a*d+b*c)/(a*f+b*e)^2*EllipticE(x*(b/a)^(1/2),(-1-(a*d-b*c)/c
/b)^(1/2))+1/2*c/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4
+a*d*x^2-b*c*x^2+a*c)^(1/2)*b*e^2/(a*c*f^2-a*d*e*f+b*c*e*f-b*d*e^2)/(a*f+b
*e)/f*EllipticF(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2))-1/2*c/(b/a)^(1/2)*
(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)*b
*e^2/(a*c*f^2-a*d*e*f+b*c*e*f-b*d*e^2)/(a*f+b*e)/f*EllipticE(x*(b/a)^(1/2)
,(-1-(a*d-b*c)/c/b)^(1/2))+5/2*e/(a*c*f^2-a*d*e*f+b*c*e*f-b*d*e^2)/(a*f+b*
e)/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x
^2+a*c)^(1/2)*EllipticPi(x*(b/a)^(1/2),-a*f/b/e,(-1/c*d)^(1/2)/(b/a)^(1...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^6}{(a-bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(x^6/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="f
ricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^6}{(a-bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**6/(-b*x**2+a)**(3/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^6}{(a-bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{x^6}{(-bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}(fx^2+e)^2}\,dx$$

input
```
integrate(x^6/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="m
axima")
```

output
```
integrate(x^6/((-b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*(f*x^2 + e)^2), x)
```

Giac [F]

$$\int \frac{x^6}{(a-bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{x^6}{(-bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}(fx^2+e)^2}\,dx$$

input
```
integrate(x^6/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="g
iac")
```

output
```
integrate(x^6/((-b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*(f*x^2 + e)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{(a-bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx=\int \frac{x^6}{(a-b\,x^2)^{3/2}\,\sqrt{d\,x^2+c}\,(f\,x^2+e)^2}\,dx$$

input
```
int(x^6/((a - b*x^2)^(3/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^2),x)
```

output
```
int(x^6/((a - b*x^2)^(3/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^2), x)
```

Reduce [F]

$$\int \frac{x^6}{(a-bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx=\int \frac{}{b^2d\,f^2x^{10}-2abd\,f^2x^8+b^2c\,f^2x^8+2b^2def\,x^8+a^2d\,f^2x^6-2a}$$

input
```
int(x^6/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**6)/(a**2*c*e**2 + 2*a**2*c*e*f*x
**2 + a**2*c*f**2*x**4 + a**2*d*e**2*x**2 + 2*a**2*d*e*f*x**4 + a**2*d*f**
2*x**6 - 2*a*b*c*e**2*x**2 - 4*a*b*c*e*f*x**4 - 2*a*b*c*f**2*x**6 - 2*a*b*
d*e**2*x**4 - 4*a*b*d*e*f*x**6 - 2*a*b*d*f**2*x**8 + b**2*c*e**2*x**4 + 2*
b**2*c*e*f*x**6 + b**2*c*f**2*x**8 + b**2*d*e**2*x**6 + 2*b**2*d*e*f*x**8
+ b**2*d*f**2*x**10),x)
```

3.297 $\int \frac{x^4}{(a-bx^2)^{3/2}\sqrt{c+dx^2}(e+fx^2)^2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 515

$$\begin{aligned}
\int \frac{x^4}{(a-bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx &= \frac{b(bce+3ade-2acf)x\sqrt{c+dx^2}}{2(bc+ad)(be+af)^2(de-cf)\sqrt{a-bx^2}} \\
&- \frac{ex\sqrt{c+dx^2}}{2(be+af)(de-cf)\sqrt{a-bx^2}\,(e+fx^2)} \\
&- \frac{\sqrt{a}\sqrt{b}(bce+3ade-2acf)\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\,E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|-\frac{ad}{bc}\right)}{2(bc+ad)(be+af)^2(de-cf)\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}} \\
&- \frac{\sqrt{a}(be-2af)\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{2\sqrt{b}f(be+af)^2\sqrt{a-bx^2}\sqrt{c+dx^2}} \\
&+ \frac{\sqrt{a}(bde^2-af(2de-3cf))\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{2\sqrt{b}f(be+af)^2(de-cf)\sqrt{a-bx^2}\sqrt{c+dx^2}}
\end{aligned}$$

output
```
1/2*b*(-2*a*c*f+3*a*d*e+b*c*e)*x*(d*x^2+c)^(1/2)/(a*d+b*c)/(a*f+b*e)^2/(-c
*f+d*e)/(-b*x^2+a)^(1/2)-1/2*e*x*(d*x^2+c)^(1/2)/(a*f+b*e)/(-c*f+d*e)/(-b*
x^2+a)^(1/2)/(f*x^2+e)-1/2*a^(1/2)*b^(1/2)*(-2*a*c*f+3*a*d*e+b*c*e)*(1-b*x
^2/a)^(1/2)*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2),(-a*d/b/c)^(1/2))/
(a*d+b*c)/(a*f+b*e)^2/(-c*f+d*e)/(-b*x^2+a)^(1/2)/(1+d*x^2/c)^(1/2)-1/2*a^
(1/2)*(-2*a*f+b*e)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(b^(1/2)*x
/a^(1/2),(-a*d/b/c)^(1/2))/b^(1/2)/f/(a*f+b*e)^2/(-b*x^2+a)^(1/2)/(d*x^2+c
)^(1/2)+1/2*a^(1/2)*(b*d*e^2-a*f*(-3*c*f+2*d*e))*(1-b*x^2/a)^(1/2)*(1+d*x^
2/c)^(1/2)*EllipticPi(b^(1/2)*x/a^(1/2),-a*f/b/e,(-a*d/b/c)^(1/2))/b^(1/2)
/f/(a*f+b*e)^2/(-c*f+d*e)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.96 (sec) , antiderivative size = 1457, normalized size of antiderivative = 2.83

$$\int \frac{x^4}{(a-bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \text{Too large to display}$$

input
```
Integrate[x^4/((a - b*x^2)^(3/2)*Sqrt[c + d*x^2]*(e + f*x^2)^2),x]
```

output
```
(-2*a*b*Sqrt[-(b/a)]*c*d*e^2*f*x + 3*a*b*Sqrt[-(b/a)]*c^2*e*f^2*x + a^2*Sq
rt[-(b/a)]*c*d*e*f^2*x - 2*a*b*Sqrt[-(b/a)]*d^2*e^2*f*x^3 + a*b*(-(b/a))^(
3/2)*c^2*e*f^2*x^3 + a^2*Sqrt[-(b/a)]*d^2*e*f^2*x^3 + 2*a*b*Sqrt[-(b/a)]*c
^2*f^3*x^3 + a*b*(-(b/a))^(3/2)*c*d*e*f^2*x^5 - 3*a*b*Sqrt[-(b/a)]*d^2*e*f
^2*x^5 + 2*a*b*Sqrt[-(b/a)]*c*d*f^3*x^5 + I*b*c*f*(-(b*c*e) - 3*a*d*e + 2*
a*c*f)*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(e + f*x^2)*EllipticE[I*Arc
Sinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))] - I*(b*c + a*d)*(-(b*e) + 2*a*f)*(-(
d*e) + c*f)*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(e + f*x^2)*EllipticF[
I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))] + I*b^2*c*d*e^3*Sqrt[1 - (b*x^2
)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[-((a*f)/(b*e)), I*ArcSinh[Sqrt[-(b/a)]
*x], -((a*d)/(b*c))] + I*a*b*d^2*e^3*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/
c]*EllipticPi[-((a*f)/(b*e)), I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))] -
 (2*I)*a*b*c*d*e^2*f*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[-(
(a*f)/(b*e)), I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))] - (2*I)*a^2*d^2*e
^2*f*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[-((a*f)/(b*e)), I*
ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))] + (3*I)*a*b*c^2*e*f^2*Sqrt[1 - (b
*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[-((a*f)/(b*e)), I*ArcSinh[Sqrt[-(b
/a)]*x], -((a*d)/(b*c))] + (3*I)*a^2*c*d*e*f^2*Sqrt[1 - (b*x^2)/a]*Sqrt[1
+ (d*x^2)/c]*EllipticPi[-((a*f)/(b*e)), I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)
/(b*c))] + I*b^2*c*d*e^2*f*x^2*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(a-bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx$$
$$\Big\downarrow \; 450$$
$$\int \frac{x^4}{(a-bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^2}dx$$

input
```
Int[x^4/((a - b*x^2)^(3/2)*Sqrt[c + d*x^2]*(e + f*x^2)^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1185 vs. $2(452) = 904$.

Time = 10.89 (sec) , antiderivative size = 1186, normalized size of antiderivative = 2.30

method	result	size
elliptic	Expression too large to display	1186
default	Expression too large to display	2419

input
```
int(x^4/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x,method=_RETURNVERBO
SE)
```

output
```
((-b*x^2+a)*(d*x^2+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-(-b*d*x^2-
b*c)*a/(a*d+b*c)*x/(a*f+b*e)^2/((x^2-a/b)*(-b*d*x^2-b*c))^(1/2)+1/2*f/(a*c
*f^2-a*d*e*f+b*c*e*f-b*d*e^2)*e*x/(a*f+b*e)*(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)
^(1/2)/(f*x^2+e)+1/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x
^4+a*d*x^2-b*c*x^2+a*c)^(1/2)*EllipticF(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(
1/2))*a/(a*f+b*e)^2+1/2/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-
b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)*EllipticF(x*(b/a)^(1/2),(-1-(a*d-b*c)/c
/b)^(1/2))*b*d*e^2/(a*c*f^2-a*d*e*f+b*c*e*f-b*d*e^2)/(a*f+b*e)/f-c/(b/a)^(
1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1
/2)*a*b/(a*d+b*c)/(a*f+b*e)^2*EllipticE(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(
1/2))-1/2*c/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*
x^2-b*c*x^2+a*c)^(1/2)*b*e/(a*c*f^2-a*d*e*f+b*c*e*f-b*d*e^2)/(a*f+b*e)*Ell
ipticF(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2))+1/2*c/(b/a)^(1/2)*(1-b*x^2/
a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)*b*e/(a*c*f
^2-a*d*e*f+b*c*e*f-b*d*e^2)/(a*f+b*e)*EllipticE(x*(b/a)^(1/2),(-1-(a*d-b*c
)/c/b)^(1/2))-3/2/(a*c*f^2-a*d*e*f+b*c*e*f-b*d*e^2)*f/(a*f+b*e)/(b/a)^(1/2
)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)
*EllipticPi(x*(b/a)^(1/2),-a*f/b/e,(-1/c*d)^(1/2)/(b/a)^(1/2))*a*c+e/(a*c*
f^2-a*d*e*f+b*c*e*f-b*d*e^2)/(a*f+b*e)/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*
x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)*EllipticPi(x*(b/a)^(1...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^4}{(a-bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(x^4/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="f
ricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^4}{(a-bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx=\text{Timed out}$$

input
```
integrate(x**4/(-b*x**2+a)**(3/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^4}{(a-bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx=\int \frac{x^4}{(-bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}(fx^2+e)^2}\,dx$$

input
```
integrate(x^4/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="m
axima")
```

output
```
integrate(x^4/((-b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*(f*x^2 + e)^2), x)
```

Giac [F]

$$\int \frac{x^4}{(a-bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx=\int \frac{x^4}{(-bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}(fx^2+e)^2}\,dx$$

input
```
integrate(x^4/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="g
iac")
```

output
```
integrate(x^4/((-b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*(f*x^2 + e)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(a-bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{x^4}{(a-b\,x^2)^{3/2}\,\sqrt{d\,x^2+c}\,(f\,x^2+e)^2}\,dx$$

input
```
int(x^4/((a - b*x^2)^(3/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^2),x)
```

output
```
int(x^4/((a - b*x^2)^(3/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^2), x)
```

Reduce [F]

$$\int \frac{x^4}{(a-bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{}{b^2d\,f^2x^{10}-2abd\,f^2x^8+b^2c\,f^2x^8+2b^2def\,x^8+a^2d\,f^2x^6-2a}$$

input
```
int(x^4/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**4)/(a**2*c*e**2 + 2*a**2*c*e*f*x
**2 + a**2*c*f**2*x**4 + a**2*d*e**2*x**2 + 2*a**2*d*e*f*x**4 + a**2*d*f**
2*x**6 - 2*a*b*c*e**2*x**2 - 4*a*b*c*e*f*x**4 - 2*a*b*c*f**2*x**6 - 2*a*b*
d*e**2*x**4 - 4*a*b*d*e*f*x**6 - 2*a*b*d*f**2*x**8 + b**2*c*e**2*x**4 + 2*
b**2*c*e*f*x**6 + b**2*c*f**2*x**8 + b**2*d*e**2*x**6 + 2*b**2*d*e*f*x**8
+ b**2*d*f**2*x**10),x)
```

3.298 $\int \frac{x^2}{(a-bx^2)^{3/2}\sqrt{c+dx^2}(e+fx^2)^2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 505

$$\begin{aligned}
&\int \frac{x^2}{(a-bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \frac{b(2bde-3bcf-adf)x\sqrt{c+dx^2}}{2(bc+ad)(be+af)^2(de-cf)\sqrt{a-bx^2}}\\
&+\frac{fx\sqrt{c+dx^2}}{2(be+af)(de-cf)\sqrt{a-bx^2}\,(e+fx^2)}\\
&-\frac{\sqrt{a}\sqrt{b}(2bde-3bcf-adf)\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\,E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{2(bc+ad)(be+af)^2(de-cf)\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}}\\
&+\frac{3\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{2(be+af)^2\sqrt{a-bx^2}\sqrt{c+dx^2}}\\
&-\frac{\sqrt{a}(acf^2+be(3de-2cf))\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{2\sqrt{b}e(be+af)^2(de-cf)\sqrt{a-bx^2}\sqrt{c+dx^2}}
\end{aligned}$$

output
```
1/2*b*(-a*d*f-3*b*c*f+2*b*d*e)*x*(d*x^2+c)^(1/2)/(a*d+b*c)/(a*f+b*e)^2/(-c
*f+d*e)/(-b*x^2+a)^(1/2)+1/2*f*x*(d*x^2+c)^(1/2)/(a*f+b*e)/(-c*f+d*e)/(-b*
x^2+a)^(1/2)/(f*x^2+e)-1/2*a^(1/2)*b^(1/2)*(-a*d*f-3*b*c*f+2*b*d*e)*(1-b*x
^2/a)^(1/2)*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2),(-a*d/b/c)^(1/2))/
(a*d+b*c)/(a*f+b*e)^2/(-c*f+d*e)/(-b*x^2+a)^(1/2)/(1+d*x^2/c)^(1/2)+3/2*a^
(1/2)*b^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(b^(1/2)*x/a^(1
/2),(-a*d/b/c)^(1/2))/(a*f+b*e)^2/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)-1/2*a^(
1/2)*(a*c*f^2+b*e*(-2*c*f+3*d*e))*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)*Elli
pticPi(b^(1/2)*x/a^(1/2),-a*f/b/e,(-a*d/b/c)^(1/2))/b^(1/2)/e/(a*f+b*e)^2/
(-c*f+d*e)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 8.90 (sec) , antiderivative size = 1450, normalized size of antiderivative = 2.87

$$\int \frac{x^2}{(a-bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \text{Too large to display}$$

input
```
Integrate[x^2/((a - b*x^2)^(3/2)*Sqrt[c + d*x^2]*(e + f*x^2)^2),x]
```

output
```
(2*a*b*(-(b/a))^(3/2)*c*d*e^3*x + 2*b^2*Sqrt[-(b/a)]*c^2*e^2*f*x + a^2*(-(
b/a))^(3/2)*c^2*e*f^2*x + (a*b*c*d*e*f^2*x)/Sqrt[-(b/a)] + 2*a*b*(-(b/a))^
(3/2)*d^2*e^3*x^3 + 3*b^2*Sqrt[-(b/a)]*c^2*e*f^2*x^3 + (a*b*d^2*e*f^2*x^3)
/Sqrt[-(b/a)] + 2*a*b*(-(b/a))^(3/2)*d^2*e^2*f*x^5 + 3*b^2*Sqrt[-(b/a)]*c*
d*e*f^2*x^5 + a*b*Sqrt[-(b/a)]*d^2*e*f^2*x^5 + I*b*c*e*(-2*b*d*e + 3*b*c*f
 + a*d*f)*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(e + f*x^2)*EllipticE[I*
ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))] - (3*I)*b*(b*c + a*d)*e*(-(d*e) +
 c*f)*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(e + f*x^2)*EllipticF[I*ArcS
inh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))] - (3*I)*b^2*c*d*e^3*Sqrt[1 - (b*x^2)/
a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[-((a*f)/(b*e)), I*ArcSinh[Sqrt[-(b/a)]*x
], -((a*d)/(b*c))] - (3*I)*a*b*d^2*e^3*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2
)/c]*EllipticPi[-((a*f)/(b*e)), I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))]
 + (2*I)*b^2*c^2*e^2*f*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[
-((a*f)/(b*e)), I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))] + (2*I)*a*b*c*d
*e^2*f*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[-((a*f)/(b*e)),
I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))] - I*a*b*c^2*e*f^2*Sqrt[1 - (b*x
^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[-((a*f)/(b*e)), I*ArcSinh[Sqrt[-(b/a
)]*x], -((a*d)/(b*c))] - I*a^2*c*d*e*f^2*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x
^2)/c]*EllipticPi[-((a*f)/(b*e)), I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c)
)] - (3*I)*b^2*c*d*e^2*f*x^2*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*El...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a-bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx$$

$$\Big\downarrow \text{ 450}$$

$$\int \frac{x^2}{(a-bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^2}dx$$

input
```
Int[x^2/((a - b*x^2)^(3/2)*Sqrt[c + d*x^2]*(e + f*x^2)^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1183 vs. $2(442) = 884$.

Time = 10.79 (sec) , antiderivative size = 1184, normalized size of antiderivative = 2.34

method	result	size
elliptic	Expression too large to display	1184
default	Expression too large to display	2056

input
```
int(x^2/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x,method=_RETURNVERBO
SE)
```

output

```
((-b*x^2+a)*(d*x^2+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-(-b*d*x^2-
b*c)*b/(a*d+b*c)*x/(a*f+b*e)^2/((x^2-a/b)*(-b*d*x^2-b*c))^(1/2)-1/2*f^2/(a
*c*f^2-a*d*e*f+b*c*e*f-b*d*e^2)*x/(a*f+b*e)*(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)
^(1/2)/(f*x^2+e)+1/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x
^4+a*d*x^2-b*c*x^2+a*c)^(1/2)*EllipticF(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(
1/2))*b/(a*f+b*e)^2-1/2/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-
b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)*EllipticF(x*(b/a)^(1/2),(-1-(a*d-b*c)/c
/b)^(1/2))*b*d*e/(a*c*f^2-a*d*e*f+b*c*e*f-b*d*e^2)/(a*f+b*e)-c/(b/a)^(1/2)
*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)*
b^2/(a*d+b*c)/(a*f+b*e)^2*EllipticE(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2)
)+1/2*c/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-
b*c*x^2+a*c)^(1/2)*b*f/(a*c*f^2-a*d*e*f+b*c*e*f-b*d*e^2)/(a*f+b*e)*Ellipti
cF(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2))-1/2*c/(b/a)^(1/2)*(1-b*x^2/a)^(
1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)*b*f/(a*c*f^2-a
*d*e*f+b*c*e*f-b*d*e^2)/(a*f+b*e)*EllipticE(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/
b)^(1/2))+1/2/(a*c*f^2-a*d*e*f+b*c*e*f-b*d*e^2)/(a*f+b*e)/e*f^2/(b/a)^(1/2
)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)
*EllipticPi(x*(b/a)^(1/2),-a*f/b/e,(-1/c*d)^(1/2)/(b/a)^(1/2))*a*c-1/(a*c*
f^2-a*d*e*f+b*c*e*f-b*d*e^2)/(a*f+b*e)*f/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+
d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)*EllipticPi(x*(b/a)^...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^2}{(a-bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \text{Timed out}$$

input

```
integrate(x^2/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="f
ricas")
```

output

```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2}{(a-bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**2/(-b*x**2+a)**(3/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^2}{(a-bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{x^2}{(-bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}(fx^2+e)^2}\,dx$$

input
```
integrate(x^2/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="m
axima")
```

output
```
integrate(x^2/((-b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*(f*x^2 + e)^2), x)
```

Giac [F]

$$\int \frac{x^2}{(a-bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{x^2}{(-bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}(fx^2+e)^2}\,dx$$

input
```
integrate(x^2/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="g
iac")
```

output
```
integrate(x^2/((-b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*(f*x^2 + e)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a-bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{x^2}{(a-b\,x^2)^{3/2}\,\sqrt{d\,x^2+c}\,(f\,x^2+e)^2}\,dx$$

input
```
int(x^2/((a - b*x^2)^(3/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^2),x)
```

output
```
int(x^2/((a - b*x^2)^(3/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^2), x)
```

Reduce [F]

$$\int \frac{x^2}{(a-bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{}{b^2d\,f^2x^{10}-2abd\,f^2x^8+b^2c\,f^2x^8+2b^2def\,x^8+a^2d\,f^2x^6-2a}$$

input
```
int(x^2/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**2)/(a**2*c*e**2 + 2*a**2*c*e*f*x
**2 + a**2*c*f**2*x**4 + a**2*d*e**2*x**2 + 2*a**2*d*e*f*x**4 + a**2*d*f**
2*x**6 - 2*a*b*c*e**2*x**2 - 4*a*b*c*e*f*x**4 - 2*a*b*c*f**2*x**6 - 2*a*b*
d*e**2*x**4 - 4*a*b*d*e*f*x**6 - 2*a*b*d*f**2*x**8 + b**2*c*e**2*x**4 + 2*
b**2*c*e*f*x**6 + b**2*c*f**2*x**8 + b**2*d*e**2*x**6 + 2*b**2*d*e*f*x**8
+ b**2*d*f**2*x**10),x)
```

3.299 $\int \frac{1}{\left(a-bx^2\right)^{3/2}\sqrt{c+dx^2}\left(e+fx^2\right)^2}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 566

$$\int \frac{1}{\left(a-bx^2\right)^{3/2}\sqrt{c+dx^2}\left(e+fx^2\right)^2}\,dx = \frac{b\left(abcf^2+a^2df^2+2b^2e(de-cf)\right)x\sqrt{c+dx^2}}{2a(bc+ad)e(be+af)^2(de-cf)\sqrt{a-bx^2}}$$
$$-\frac{f^2x\sqrt{c+dx^2}}{2e(be+af)(de-cf)\sqrt{a-bx^2}\,(e+fx^2)}$$
$$-\frac{\sqrt{b}\left(abcf^2+a^2df^2+2b^2e(de-cf)\right)\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\,E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|-\frac{ad}{bc}\right)}{2\sqrt{a}(bc+ad)e(be+af)^2(de-cf)\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}}$$
$$+\frac{\sqrt{b}(2be-af)\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{2\sqrt{a}e(be+af)^2\sqrt{a-bx^2}\sqrt{c+dx^2}}$$
$$+\frac{\sqrt{a}f(be(5de-4cf)+af(2de-cf))\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{2\sqrt{b}e^2(be+af)^2(de-cf)\sqrt{a-bx^2}\sqrt{c+dx^2}}$$

output
```
1/2*b*(a*b*c*f^2+a^2*d*f^2+2*b^2*e*(-c*f+d*e))*x*(d*x^2+c)^(1/2)/a/(a*d+b*
c)/e/(a*f+b*e)^2/(-c*f+d*e)/(-b*x^2+a)^(1/2)-1/2*f^2*x*(d*x^2+c)^(1/2)/e/(
a*f+b*e)/(-c*f+d*e)/(-b*x^2+a)^(1/2)/(f*x^2+e)-1/2*b^(1/2)*(a*b*c*f^2+a^2*
d*f^2+2*b^2*e*(-c*f+d*e))*(1-b*x^2/a)^(1/2)*(d*x^2+c)^(1/2)*EllipticE(b^(1
/2)*x/a^(1/2),(-a*d/b/c)^(1/2))/a^(1/2)/(a*d+b*c)/e/(a*f+b*e)^2/(-c*f+d*e)
/(-b*x^2+a)^(1/2)/(1+d*x^2/c)^(1/2)+1/2*b^(1/2)*(-a*f+2*b*e)*(1-b*x^2/a)^(
1/2)*(1+d*x^2/c)^(1/2)*EllipticF(b^(1/2)*x/a^(1/2),(-a*d/b/c)^(1/2))/a^(1/
2)/e/(a*f+b*e)^2/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)+1/2*a^(1/2)*f*(b*e*(-4*c
*f+5*d*e)+a*f*(-c*f+2*d*e))*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)*EllipticPi
(b^(1/2)*x/a^(1/2),-a*f/b/e,(-a*d/b/c)^(1/2))/b^(1/2)/e^2/(a*f+b*e)^2/(-c*
f+d*e)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 7.13 (sec) , antiderivative size = 1728, normalized size of antiderivative = 3.05

$$\int \frac{1}{(a-bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \text{Too large to display}$$

input
```
Integrate[1/((a - b*x^2)^(3/2)*Sqrt[c + d*x^2]*(e + f*x^2)^2),x]
```

output
```
(-2*b^3*Sqrt[-(b/a)]*c*d*e^4*x + 2*b^3*Sqrt[-(b/a)]*c^2*e^3*f*x + a^2*b*Sq
rt[-(b/a)]*c^2*e*f^3*x + a^3*Sqrt[-(b/a)]*c*d*e*f^3*x - 2*b^3*Sqrt[-(b/a)]
*d^2*e^4*x^3 + 2*b^3*Sqrt[-(b/a)]*c^2*e^2*f^2*x^3 + (b^3*c^2*e*f^3*x^3)/Sq
rt[-(b/a)] + a^3*Sqrt[-(b/a)]*d^2*e*f^3*x^3 - 2*b^3*Sqrt[-(b/a)]*d^2*e^3*f
*x^5 + 2*b^3*Sqrt[-(b/a)]*c*d*e^2*f^2*x^5 + (b^3*c*d*e*f^3*x^5)/Sqrt[-(b/a
)] + (a*b^2*d^2*e*f^3*x^5)/Sqrt[-(b/a)] - I*b*c*e*(a*b*c*f^2 + a^2*d*f^2 +
 2*b^2*e*(d*e - c*f))*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(e + f*x^2)*
EllipticE[I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))] + I*b*(b*c + a*d)*e*(
-2*b*e + a*f)*(-(d*e) + c*f)*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(e +
f*x^2)*EllipticF[I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))] + (5*I)*a*b^2*
c*d*e^3*f*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[-((a*f)/(b*e)
), I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))] + (5*I)*a^2*b*d^2*e^3*f*Sqrt
[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[-((a*f)/(b*e)), I*ArcSinh[S
qrt[-(b/a)]*x], -((a*d)/(b*c))] - (4*I)*a*b^2*c^2*e^2*f^2*Sqrt[1 - (b*x^2)
/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[-((a*f)/(b*e)), I*ArcSinh[Sqrt[-(b/a)]*
x], -((a*d)/(b*c))] - (2*I)*a^2*b*c*d*e^2*f^2*Sqrt[1 - (b*x^2)/a]*Sqrt[1 +
 (d*x^2)/c]*EllipticPi[-((a*f)/(b*e)), I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/
(b*c))] + (2*I)*a^3*d^2*e^2*f^2*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*El
lipticPi[-((a*f)/(b*e)), I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))] - I*a^
2*b*c^2*e*f^3*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[-((a*f...
```

Rubi [A] (verified)

Time = 1.91 (sec) , antiderivative size = 935, normalized size of antiderivative = 1.65, number of steps used = 28, number of rules used = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.848$, Rules used = {426, 421, 402, 399, 323, 323, 321, 331, 330, 327, 415, 323, 323, 321, 413, 413, 412, 424, 399, 323, 323, 321, 331, 330, 327, 413, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^2}dx$$

$$\Big\downarrow \text{426}$$

$$\frac{f\int \frac{1}{\sqrt{a-bx^2}\sqrt{dx^2+c}(fx^2+e)^2}dx}{af+be}+\frac{b\int \frac{1}{(a-bx^2)^{3/2}\sqrt{dx^2+c}(fx^2+e)}dx}{af+be}$$

$$\Big\downarrow \text{421}$$

$$\frac{b\left(\frac{f^2\int \frac{\sqrt{a-bx^2}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(af+be)^2}+\frac{b\int \frac{-bfx^2+be+2af}{\left(a-bx^2\right)^{3/2}\sqrt{dx^2+c}}dx}{(af+be)^2}\right)}{af+be}+\frac{f\int \frac{1}{\sqrt{a-bx^2}\sqrt{dx^2+c}(fx^2+e)^2}dx}{af+be}$$

$\Big\downarrow$ 402

$$\frac{b\left(\frac{f^2\int \frac{\sqrt{a-bx^2}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(af+be)^2}+\frac{b\left(\frac{\int \frac{a(bde+bcf+2adf)-bd(be+af)x^2}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{a(ad+bc)}+\frac{bx\sqrt{c+dx^2}(af+be)}{a\sqrt{a-bx^2}(ad+bc)}\right)}{(af+be)^2}\right)}{af+be}+\frac{f\int \frac{1}{\sqrt{a-bx^2}\sqrt{dx^2+c}(fx^2+e)^2}dx}{af+be}$$

$\Big\downarrow$ 399

$$\frac{b\left(\frac{f^2\int \frac{\sqrt{a-bx^2}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(af+be)^2}+\frac{b\left(\frac{(ad+bc)(2af+be)\int \frac{1}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx-b(af+be)\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{a(ad+bc)}+\frac{bx\sqrt{c+dx^2}(af+be)}{a\sqrt{a-bx^2}(ad+bc)}\right)}{(af+be)^2}\right)}{af+be}+\frac{f\int \frac{1}{\sqrt{a-bx^2}\sqrt{dx^2+c}(fx^2+e)^2}dx}{af+be}$$

$\Big\downarrow$ 323

$$\frac{b\left(\frac{f^2\int \frac{\sqrt{a-bx^2}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(af+be)^2}+\frac{b\left(\frac{\frac{\sqrt{\frac{dx^2}{c}+1}(ad+bc)(2af+be)\int \frac{1}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}dx}{\sqrt{c+dx^2}}-b(af+be)\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{a(ad+bc)}+\frac{bx\sqrt{c+dx^2}(af+be)}{a\sqrt{a-bx^2}(ad+bc)}\right)}{(af+be)^2}\right)}{af+be}+\frac{f\int \frac{1}{\sqrt{a-bx^2}\sqrt{dx^2+c}(fx^2+e)^2}dx}{af+be}$$

$\Big\downarrow$ 323

$$\frac{b\left(\frac{f^2\int \frac{\sqrt{a-bx^2}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(af+be)^2}+\frac{b\left(\frac{\frac{\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(2af+be)\int \frac{1}{\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}}dx}{\sqrt{a-bx^2}\sqrt{c+dx^2}}-b(af+be)\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{a(ad+bc)}+\frac{bx\sqrt{c+dx^2}(af+be)}{a\sqrt{a-bx^2}(ad+bc)}\right)}{(af+be)^2}\right)}{af+be}+\frac{f\int \frac{1}{\sqrt{a-bx^2}\sqrt{dx^2+c}(fx^2+e)^2}dx}{af+be}$$

$$\Big\downarrow \; 321$$

$$\frac{b\left(\frac{b\left(\frac{\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(2af+be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-b(af+be)\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{a(ad+bc)}+\frac{bx\sqrt{c+dx^2}(af+be)}{a\sqrt{a-bx^2}(ad+bc)}\right)}{(af+be)^2}+\frac{f^2\int \frac{\sqrt{a-bx^2}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(af+be)^2}\right)}{af+be}+\frac{f\int \frac{1}{\sqrt{a-bx^2}\sqrt{dx^2+c}(fx^2+e)^2}dx}{af+be}$$

$$\Big\downarrow \; 331$$

$$\frac{b\left(\frac{b\left(\frac{\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(2af+be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{b\sqrt{1-\frac{bx^2}{a}}(af+be)\int \frac{\sqrt{dx^2+c}}{\sqrt{1-\frac{bx^2}{a}}}dx}{\sqrt{a-bx^2}}}{a(ad+bc)}+\frac{bx\sqrt{c+dx^2}(af+be)}{a\sqrt{a-bx^2}(ad+bc)}\right)}{(af+be)^2}+\frac{f^2\int \frac{\sqrt{a-bx^2}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(af+be)^2}\right)}{af+be}+\frac{f\int \frac{1}{\sqrt{a-bx^2}\sqrt{dx^2+c}(fx^2+e)^2}dx}{af+be}$$

$$\Big\downarrow \; 330$$

$$b\left(\frac{b\left(\frac{\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(2af+be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{b\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(af+be)\int\frac{\sqrt{\frac{dx^2}{c}+1}}{\sqrt{1-\frac{bx^2}{a}}}dx}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}}{a(ad+bc)}+\frac{bx\sqrt{c+dx^2}(af+be)}{a\sqrt{a-bx^2}(ad+bc)}\right)}{(af+be)^2}+\frac{f^2\int \cdots}{\cdots}\right)$$

$$\overline{\qquad af+be \qquad}$$

$$\frac{f\int\frac{1}{\sqrt{a-bx^2}\sqrt{dx^2+c}(fx^2+e)^2}dx}{af+be}$$

$\downarrow$ 327

$$b\left(\frac{f^2\int\frac{\sqrt{a-bx^2}}{\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{(af+be)^2}+\frac{b\left(\frac{\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(2af+be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(af+be)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\cdots\right.}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}}{a(ad+bc)}\cdots\right)}{(af+be)^2}\cdots\right)$$

$$\overline{\qquad af+be \qquad}$$

$$\frac{f\int\frac{1}{\sqrt{a-bx^2}\sqrt{dx^2+c}(fx^2+e)^2}dx}{af+be}$$

$\downarrow$ 415

$$\frac{b\left(\frac{f^2\left(\frac{(af+be)\int \frac{1}{\sqrt{a-bx^2}\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{f}-\frac{b\int \frac{1}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{f}\right)}{(af+be)^2}+\frac{b\left(\frac{\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(2af+be)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}\cdots}{a(ad+bc)\cdots}\cdots\right)}{(a\cdots}\cdots\right)}{af+be\cdots}+\frac{f\int \frac{1}{\sqrt{a-bx^2}\sqrt{dx^2+c}(fx^2+e)^2}dx}{af+be}$$

↓ 323

$$\frac{b\left(\frac{f^2\left(\frac{(af+be)\int \frac{1}{\sqrt{a-bx^2}\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{f}-\frac{b\sqrt{\frac{dx^2}{c}+1}\int \frac{1}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}dx}{f\sqrt{c+dx^2}}\right)}{(af+be)^2}+\frac{b\left(\frac{\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(2af+be)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\cdots\right.\right.}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}\cdots}{a\cdots}\cdots\right)}{\cdots}\cdots\right)}{af+be\cdots}+\frac{f\int \frac{1}{\sqrt{a-bx^2}\sqrt{dx^2+c}(fx^2+e)^2}dx}{af+be}$$

↓ 323

$$\frac{b\left(\frac{f^2\left(\frac{(af+be)\int \frac{1}{\sqrt{a-bx^2}\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{f}-\frac{b\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\int \frac{1}{\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}}dx}{f\sqrt{a-bx^2}\sqrt{c+dx^2}}\right)}{(af+be)^2}+\frac{b\left(\frac{\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(2af+be)\text{EllipticF}\left(\text{ar}\cdots\right.}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}\cdots}{\cdots}\cdots\right)}{\cdots}\cdots\right)}{af+be\cdots}+\frac{f\int \frac{1}{\sqrt{a-bx^2}\sqrt{dx^2+c}(fx^2+e)^2}dx}{af+be}$$

$$\Big\downarrow 321$$

$$\frac{b\left(\frac{f^2\left(\frac{(af+be)\int \frac{1}{\sqrt{a-bx^2}\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{f}-\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{f\sqrt{a-bx^2}\sqrt{c+dx^2}}\right)}{(af+be)^2}+\frac{b\left(\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)(2af+be)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{}}\right.}{}\right.}{af+be}$$

$$\frac{f\int \frac{1}{\sqrt{a-bx^2}\sqrt{dx^2+c}(fx^2+e)^2}dx}{af+be}$$

$$\Big\downarrow 413$$

$$\frac{b\left(\frac{f^2\left(\frac{\sqrt{1-\frac{bx^2}{a}}(af+be)\int \frac{1}{\sqrt{1-\frac{bx^2}{a}}\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{f\sqrt{a-bx^2}}-\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{f\sqrt{a-bx^2}\sqrt{c+dx^2}}\right)}{(af+be)^2}+\frac{b\left(\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc}{\sqrt{b}\sqrt{}}\right.}{}\right.}{af+be}$$

$$\frac{f\int \frac{1}{\sqrt{a-bx^2}\sqrt{dx^2+c}(fx^2+e)^2}dx}{af+be}$$

$$\Big\downarrow 413$$

$$b\left(\frac{f^2\left(\frac{\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(af+be)\int \frac{1}{\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(fx^2+e)}dx}{f\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{f\sqrt{a-bx^2}\sqrt{c+dx^2}}\right)}{(af+be)^2}+\frac{b\left(\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}}}{}\right.}{}\right.$$
$$\overline{\qquad\qquad af+be}$$

$$\frac{f\int \frac{1}{\sqrt{a-bx^2}\sqrt{dx^2+c}(fx^2+e)^2}dx}{af+be}$$

$$\Big\downarrow \text{412}$$

$$\frac{f\int \frac{1}{\sqrt{a-bx^2}\sqrt{dx^2+c}(fx^2+e)^2}dx}{af+be}+$$

$$b\left(\frac{f^2\left(\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(af+be)\text{EllipticPi}\left(-\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}ef\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{f\sqrt{a-bx^2}\sqrt{c+dx^2}}\right)}{(af+be)^2}+\frac{b\left(\frac{\sqrt{a}\sqrt{1-\frac{bx}{a}}}{}\right.}{}\right.$$
$$\overline{\qquad\qquad af+be}$$

$$\Big\downarrow \text{424}$$

$$\frac{f\left(\frac{(af(2de-cf)+be(3de-2cf))\int \frac{1}{\sqrt{a-bx^2}\sqrt{dx^2+c}(fx^2+e)}dx}{2e(af+be)(de-cf)}-\frac{bd\int \frac{fx^2+e}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{2e(af+be)(de-cf)}-\frac{f^2x\sqrt{a-bx^2}\sqrt{c+dx^2}}{2e(e+fx^2)(af+be)(de-cf)}\right)}{af+be}+$$

$$b\left(\frac{f^2\left(\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(af+be)\text{EllipticPi}\left(-\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}ef\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{f\sqrt{a-bx^2}\sqrt{c+dx^2}}\right)}{(af+be)^2}+\frac{b\left(\frac{\sqrt{a}\sqrt{1-\frac{bx}{a}}}{}\right.}{}\right.$$
$$\overline{\qquad\qquad af+be}$$

$$\Big\downarrow \text{399}$$

$$\frac{f\left(-\frac{bd\left(\frac{(de-cf)\int \frac{1}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx}{d}+\frac{f\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}\right)}{2e(af+be)(de-cf)}+\frac{(af(2de-cf)+be(3de-2cf))\int \frac{1}{\sqrt{a-bx^2}\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{2e(af+be)(de-cf)}-\frac{f^2x\sqrt{a-bx^2}\sqrt{c\cdots}}{2e(e+fx^2)(af+be)\cdots}\right)}{af+be}$$

$$\frac{b\left(\frac{f^2\left(\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(af+be)\,\text{EllipticPi}\left(-\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}ef\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{f\sqrt{a-bx^2}\sqrt{c+dx^2}}\right)}{(af+be)^2}+\frac{b\left(\frac{\sqrt{a}\sqrt{1-\frac{bx\cdots}{a}}\cdots}{\cdots}\right)}{\cdots}\right)}{af+be}$$

$$\Big\downarrow \text{323}$$

$$\frac{f\left(-\frac{bd\left(\frac{\sqrt{\frac{dx^2}{c}+1}(de-cf)\int \frac{1}{\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}dx}{d\sqrt{c+dx^2}}+\frac{f\int \frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}\right)}{2e(af+be)(de-cf)}+\frac{(af(2de-cf)+be(3de-2cf))\int \frac{1}{\sqrt{a-bx^2}\sqrt{dx^2+c}\left(fx^2+e\right)}dx}{2e(af+be)(de-cf)}-\frac{f^2x\sqrt{c\cdots}}{2e(e+fx^2\cdots}\right)}{af+be}$$

$$\frac{b\left(\frac{f^2\left(\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(af+be)\,\text{EllipticPi}\left(-\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}ef\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{f\sqrt{a-bx^2}\sqrt{c+dx^2}}\right)}{(af+be)^2}+\frac{b\left(\frac{\sqrt{a}\sqrt{1-\frac{bx\cdots}{a}}\cdots}{\cdots}\right)}{\cdots}\right)}{af+be}$$

$$\Big\downarrow \text{323}$$

$$b\left(\frac{\left(\frac{\sqrt{a}(be+af)\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(-\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}ef\sqrt{a-bx^2}\sqrt{dx^2+c}}-\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{f\sqrt{a-bx^2}\sqrt{dx^2+c}}\right)f^2}{(be+af)^2}+\frac{b\left(\frac{b(be+af)\sqrt{}}{a(bc+ad)\sqrt{}}\right.}{}\right.$$

$$\overline{be+af}$$

$$\frac{f\left(-\frac{x\sqrt{a-bx^2}\sqrt{dx^2+c}f^2}{2e(be+af)(de-cf)(fx^2+e)}-\frac{bd\left(\frac{f\int\frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}+\frac{(de-cf)\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\int\frac{1}{\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}}dx}{d\sqrt{a-bx^2}\sqrt{dx^2+c}}\right)}{2e(be+af)(de-cf)}+\frac{(be(3de-2cf)+af(2de-cf))\int}{2e(be+af)(}\right.}{be+af}$$

$\downarrow$ 321

$$b\left(\frac{\left(\frac{\sqrt{a}(be+af)\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(-\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}ef\sqrt{a-bx^2}\sqrt{dx^2+c}}-\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{f\sqrt{a-bx^2}\sqrt{dx^2+c}}\right)f^2}{(be+af)^2}+\frac{b\left(\frac{b(be+af)\sqrt{}}{a(bc+ad)\sqrt{}}\right.}{}\right.$$

$$\overline{be+af}$$

$$\frac{f\left(-\frac{x\sqrt{a-bx^2}\sqrt{dx^2+c}f^2}{2e(be+af)(de-cf)(fx^2+e)}-\frac{bd\left(\frac{\sqrt{a}(de-cf)\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{dx^2+c}}+\frac{f\int\frac{\sqrt{dx^2+c}}{\sqrt{a-bx^2}}dx}{d}\right)}{2e(be+af)(de-cf)}+\frac{(be(3de-2cf)+af(2d}{2e}\right.}{be+af}$$

$\downarrow$ 331

$$\frac{b\left(\frac{\left(\frac{\sqrt{a}(be+af)\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(-\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}ef\sqrt{a-bx^2}\sqrt{dx^2+c}}-\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{f\sqrt{a-bx^2}\sqrt{dx^2+c}}\right)f^2}{(be+af)^2}+\frac{b\left(\frac{b(be+af)\sqrt{\cdots}}{a(bc+ad)\sqrt{\cdots}}\cdots\right.}{\cdots}\cdots\right.}{be+af}$$

$$\frac{f\left(-\frac{x\sqrt{a-bx^2}\sqrt{dx^2+c}f^2}{2e(be+af)(de-cf)(fx^2+e)}-\frac{bd\left(\frac{\sqrt{a}(de-cf)\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{dx^2+c}}+\frac{f\sqrt{1-\frac{bx^2}{a}}\int\frac{\sqrt{dx^2+c}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}\right)}{2e(be+af)(de-cf)}+\frac{(be(3de-2cf\cdots}{\cdots}\cdots\right.}{be+af}$$

$\downarrow$ 330

$$\frac{b\left(\frac{\left(\frac{\sqrt{a}(be+af)\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(-\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}ef\sqrt{a-bx^2}\sqrt{dx^2+c}}-\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{f\sqrt{a-bx^2}\sqrt{dx^2+c}}\right)f^2}{(be+af)^2}+\frac{b\left(\frac{b(be+af)\sqrt{\cdots}}{a(bc+ad)\sqrt{\cdots}}\cdots\right.}{\cdots}\cdots\right.}{be+af}$$

$$\frac{f\left(-\frac{x\sqrt{a-bx^2}\sqrt{dx^2+c}f^2}{2e(be+af)(de-cf)(fx^2+e)}-\frac{bd\left(\frac{\sqrt{a}(de-cf)\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{dx^2+c}}+\frac{f\sqrt{1-\frac{bx^2}{a}}\sqrt{dx^2+c}\int\frac{\sqrt{\frac{dx^2}{c}+1}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}\right)}{2e(be+af)(de-cf)}+\frac{(be(3\cdots}{\cdots}\cdots\right.}{be+af}$$

$\downarrow$ 327

$$\frac{b\left(\frac{\left(\frac{\sqrt{a}(be+af)\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(-\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}ef\sqrt{a-bx^2}\sqrt{dx^2+c}}-\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{f\sqrt{a-bx^2}\sqrt{dx^2+c}}\right)f^2}{(be+af)^2}+\frac{b\left(\frac{b(be+af)\sqrt{}}{a(bc+ad)\sqrt{}}\right.}{}\right.}{be+af}$$

$$\frac{f\left(-\frac{x\sqrt{a-bx^2}\sqrt{dx^2+c}f^2}{2e(be+af)(de-cf)(fx^2+e)}-\frac{bd\left(\frac{\sqrt{a}f\sqrt{1-\frac{bx^2}{a}}\sqrt{dx^2+c}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}+\frac{\sqrt{a}(de-cf)\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{dx^2+c}}\right.}{2e(be+af)(de-cf)}\right.}{be+af}$$

$\downarrow$ 413

$$\frac{b\left(\frac{\left(\frac{\sqrt{a}(be+af)\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(-\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}ef\sqrt{a-bx^2}\sqrt{dx^2+c}}-\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{f\sqrt{a-bx^2}\sqrt{dx^2+c}}\right)f^2}{(be+af)^2}+\frac{b\left(\frac{b(be+af)\sqrt{}}{a(bc+ad)\sqrt{}}\right.}{}\right.}{be+af}$$

$$\frac{f\left(-\frac{x\sqrt{a-bx^2}\sqrt{dx^2+c}f^2}{2e(be+af)(de-cf)(fx^2+e)}-\frac{bd\left(\frac{\sqrt{a}f\sqrt{1-\frac{bx^2}{a}}\sqrt{dx^2+c}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}+\frac{\sqrt{a}(de-cf)\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{dx^2+c}}\right.}{2e(be+af)(de-cf)}\right.}{be+af}$$

$\downarrow$ 413

$$\frac{b\left(\frac{\left(\frac{\sqrt{a}(be+af)\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(-\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}ef\sqrt{a-bx^2}\sqrt{dx^2+c}}-\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{f\sqrt{a-bx^2}\sqrt{dx^2+c}}\right)f^2}{(be+af)^2}+\frac{b\left(\frac{b(be+af)\sqrt{}}{a(bc+ad)\sqrt{}}\right.}{}\right.}{be+af}$$

$$\frac{f\left(-\frac{x\sqrt{a-bx^2}\sqrt{dx^2+c}f^2}{2e(be+af)(de-cf)(fx^2+e)}-\frac{bd\left(\frac{\sqrt{a}f\sqrt{1-\frac{bx^2}{a}}\sqrt{dx^2+c}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}+\frac{\sqrt{a}(de-cf)\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{dx^2+c}}\right.}{2e(be+af)(de-cf)}\right.}{be+af}$$

$\Big\downarrow$ 412

$$\frac{f\left(-\frac{x\sqrt{a-bx^2}\sqrt{dx^2+c}f^2}{2e(be+af)(de-cf)(fx^2+e)}-\frac{bd\left(\frac{\sqrt{a}f\sqrt{1-\frac{bx^2}{a}}\sqrt{dx^2+c}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}+\frac{\sqrt{a}(de-cf)\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{dx^2+c}}\right.}{2e(be+af)(de-cf)}\right.}{be+af}$$

$$\frac{b\left(\frac{\left(\frac{\sqrt{a}(be+af)\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticPi}\left(-\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}ef\sqrt{a-bx^2}\sqrt{dx^2+c}}-\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{f\sqrt{a-bx^2}\sqrt{dx^2+c}}\right)f^2}{(be+af)^2}+\frac{b\left(\frac{b(be+af)\sqrt{}}{a(bc+ad)\sqrt{}}\right.}{}\right.}{be+af}$$

input
```
Int[1/((a - b*x^2)^(3/2)*Sqrt[c + d*x^2]*(e + f*x^2)^2),x]
```

output
```
(f*(-1/2*(f^2*x*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])/(e*(b*e + a*f)*(d*e - c*f
)*(e + f*x^2)) - (b*d*((Sqrt[a]*f*Sqrt[1 - (b*x^2)/a]*Sqrt[c + d*x^2]*Elli
pticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*d*Sqrt[a - b*
x^2]*Sqrt[1 + (d*x^2)/c]) + (Sqrt[a]*(d*e - c*f)*Sqrt[1 - (b*x^2)/a]*Sqrt[
1 + (d*x^2)/c]*EllipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(Sq
rt[b]*d*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])))/(2*e*(b*e + a*f)*(d*e - c*f)) +
 (Sqrt[a]*(b*e*(3*d*e - 2*c*f) + a*f*(2*d*e - c*f))*Sqrt[1 - (b*x^2)/a]*Sq
rt[1 + (d*x^2)/c]*EllipticPi[-((a*f)/(b*e)), ArcSin[(Sqrt[b]*x)/Sqrt[a]],
-((a*d)/(b*c))])/(2*Sqrt[b]*e^2*(b*e + a*f)*(d*e - c*f)*Sqrt[a - b*x^2]*Sq
rt[c + d*x^2])))/(b*e + a*f) + (b*((b*((b*(b*e + a*f)*x*Sqrt[c + d*x^2])/(
a*(b*c + a*d)*Sqrt[a - b*x^2]) + (-((Sqrt[a]*Sqrt[b]*(b*e + a*f)*Sqrt[1 -
(b*x^2)/a]*Sqrt[c + d*x^2]*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/
(b*c))])/(Sqrt[a - b*x^2]*Sqrt[1 + (d*x^2)/c])) + (Sqrt[a]*(b*c + a*d)*(b*
e + 2*a*f)*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[ArcSin[(Sqrt[
b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])
)/(a*(b*c + a*d))))/(b*e + a*f)^2 + (f^2*(-((Sqrt[a]*Sqrt[b]*Sqrt[1 - (b*x
^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/
(b*c))])/(f*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])) + (Sqrt[a]*(b*e + a*f)*Sqrt[
1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticPi[-((a*f)/(b*e)), ArcSin[(Sqrt
[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*e*f*Sqrt[a - b*x^2]*Sqrt[c +...
```

Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 415
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[d/b   Int[1/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x],
x] + Simp[(b*c - a*d)/b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NegQ[d/c]
```

rule 421
```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Simp[b^2/(b*c - a*d)^2   Int[(c + d*x^2)^(q + 2)*((e +
 f*x^2)^r/(a + b*x^2)), x], x] - Simp[d/(b*c - a*d)^2   Int[(c + d*x^2)^q*(
e + f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r}
, x] && LtQ[q, -1]
```

rule 424
```
Int[1/(((a_) + (b_.)*(x_)^2)^2*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*
(x_)^2]), x_Symbol] :> Simp[b^2*x*Sqrt[c + d*x^2]*(Sqrt[e + f*x^2]/(2*a*(b*
c - a*d)*(b*e - a*f)*(a + b*x^2))), x] + (Simp[(b^2*c*e + 3*a^2*d*f - 2*a*b
*(d*e + c*f))/(2*a*(b*c - a*d)*(b*e - a*f))   Int[1/((a + b*x^2)*Sqrt[c + d
*x^2]*Sqrt[e + f*x^2]), x], x] - Simp[d*(f/(2*a*(b*c - a*d)*(b*e - a*f)))
 Int[(a + b*x^2)/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x], x]) /; FreeQ[{a, b,
 c, d, e, f}, x]
```

rule 426
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> Simp[b/(b*c - a*d)   Int[(a + b*x^2)^p*(c + d*x^2)^
(q + 1)*(e + f*x^2)^r, x], x] - Simp[d/(b*c - a*d)   Int[(a + b*x^2)^(p + 1
)*(c + d*x^2)^q*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] &&
 ILtQ[p, 0] && LeQ[q, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1354 vs. $2(503) = 1006$.

Time = 10.86 (sec) , antiderivative size = 1355, normalized size of antiderivative = 2.39

method	result	size
elliptic	Expression too large to display	1355
default	Expression too large to display	2825

input
```
int(1/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x,method=_RETURNVERBOSE
)
```

output
```
((-b*x^2+a)*(d*x^2+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-(-b*d*x^2-
b*c)*b^2/a/(a*d+b*c)*x/(a*f+b*e)^2/((x^2-a/b)*(-b*d*x^2-b*c))^(1/2)+1/2*f^
3/(a*c*f^2-a*d*e*f+b*c*e*f-b*d*e^2)/e*x/(a*f+b*e)*(-b*d*x^4+a*d*x^2-b*c*x^
2+a*c)^(1/2)/(f*x^2+e)+1/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(
-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)*EllipticF(x*(b/a)^(1/2),(-1-(a*d-b*c)/
c/b)^(1/2))/a*b^2/(a*f+b*e)^2+1/2/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c
)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)*EllipticF(x*(b/a)^(1/2),(-1-(
a*d-b*c)/c/b)^(1/2))*b*d*f/(a*c*f^2-a*d*e*f+b*c*e*f-b*d*e^2)/(a*f+b*e)-c/(
b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a
*c)^(1/2)*b^3/a/(a*d+b*c)/(a*f+b*e)^2*EllipticE(x*(b/a)^(1/2),(-1-(a*d-b*c
)/c/b)^(1/2))-1/2*c/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*
x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)*b*f^2/(a*c*f^2-a*d*e*f+b*c*e*f-b*d*e^2)/e/(
a*f+b*e)*EllipticF(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2))+1/2*c/(b/a)^(1/
2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2
)*b*f^2/(a*c*f^2-a*d*e*f+b*c*e*f-b*d*e^2)/e/(a*f+b*e)*EllipticE(x*(b/a)^(1
/2),(-1-(a*d-b*c)/c/b)^(1/2))+1/2/(a*c*f^2-a*d*e*f+b*c*e*f-b*d*e^2)/e^2/(a
*f+b*e)*f^3/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*
x^2-b*c*x^2+a*c)^(1/2)*EllipticPi(x*(b/a)^(1/2),-a*f/b/e,(-1/c*d)^(1/2)/(b
/a)^(1/2))*a*c-1/(a*c*f^2-a*d*e*f+b*c*e*f-b*d*e^2)/(a*f+b*e)*f^2/e/(b/a)^(
1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a-bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="fri
cas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a-bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx=\text{Timed out}$$

input
```
integrate(1/(-b*x**2+a)**(3/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{(a-bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx=\int \frac{1}{(-bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}(fx^2+e)^2}\,dx$$

input
```
integrate(1/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="max
ima")
```

output
```
integrate(1/((-b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*(f*x^2 + e)^2), x)
```

Giac [F]

$$\int \frac{1}{(a-bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx=\int \frac{1}{(-bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}(fx^2+e)^2}\,dx$$

input
```
integrate(1/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm="gia
c")
```

output
```
integrate(1/((-b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*(f*x^2 + e)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a-bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{1}{(a-b\,x^2)^{3/2}\,\sqrt{d\,x^2+c}\,(f\,x^2+e)^2}\,dx$$

input
```
int(1/((a - b*x^2)^(3/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^2),x)
```

output
```
int(1/((a - b*x^2)^(3/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^2), x)
```

Reduce [F]

$$\int \frac{1}{(a-bx^2)^{3/2}\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{1}{(-b\,x^2+a)^{\frac{3}{2}}\,\sqrt{d\,x^2+c}\,(f\,x^2+e)^2}dx$$

input
```
int(1/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x)
```

output
```
int(1/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x)
```

3.300 $\int \frac{1}{x^2(a-bx^2)^{3/2}\sqrt{c+dx^2}(e+fx^2)^2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 787

$$\int \frac{1}{x^2\left(a-bx^2\right)^{3/2}\sqrt{c+dx^2}\left(e+fx^2\right)^2}\,dx = \frac{b(a^3df^2(2de-3cf)+4b^3ce^2(de-cf)+a^2bf(4d^2e^2-2cdef}{2a^2c(bc+ad)e^2(be+af)^2(d}$$

$$-\frac{\sqrt{c+dx^2}}{acex\sqrt{a-bx^2}\left(e+fx^2\right)}-\frac{f(af(2de-3cf)+2be(de-cf))x\sqrt{c+dx^2}}{2ace^2(be+af)(de-cf)\sqrt{a-bx^2}\left(e+fx^2\right)}$$

$$-\frac{\sqrt{b}(a^3df^2(2de-3cf)+4b^3ce^2(de-cf)+a^2bf(4d^2e^2-2cdef-3c^2f^2)+2ab^2e(d^2e^2+cdef-2c^2f^2))}{2a^{3/2}c(bc+ad)e^2(be+af)^2(de-cf)\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}}$$

$$+\frac{\sqrt{b}(4b^2e^2+4abef+3a^2f^2)\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{2a^{3/2}e^2(be+af)^2\sqrt{a-bx^2}\sqrt{c+dx^2}}$$

$$-\frac{\sqrt{a}f^2(be(7de-6cf)+af(4de-3cf))\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{2\sqrt{b}e^3(be+af)^2(de-cf)\sqrt{a-bx^2}\sqrt{c+dx^2}}$$

output
```
1/2*b*(a^3*d*f^2*(-3*c*f+2*d*e)+4*b^3*c*e^2*(-c*f+d*e)+a^2*b*f*(-3*c^2*f^2
-2*c*d*e*f+4*d^2*e^2)+2*a*b^2*e*(-2*c^2*f^2+c*d*e*f+d^2*e^2))*x*(d*x^2+c)^
(1/2)/a^2/c/(a*d+b*c)/e^2/(a*f+b*e)^2/(-c*f+d*e)/(-b*x^2+a)^(1/2)-(d*x^2+c
)^(1/2)/a/c/e/x/(-b*x^2+a)^(1/2)/(f*x^2+e)-1/2*f*(a*f*(-3*c*f+2*d*e)+2*b*e
*(-c*f+d*e))*x*(d*x^2+c)^(1/2)/a/c/e^2/(a*f+b*e)/(-c*f+d*e)/(-b*x^2+a)^(1/
2)/(f*x^2+e)-1/2*b^(1/2)*(a^3*d*f^2*(-3*c*f+2*d*e)+4*b^3*c*e^2*(-c*f+d*e)+
a^2*b*f*(-3*c^2*f^2-2*c*d*e*f+4*d^2*e^2)+2*a*b^2*e*(-2*c^2*f^2+c*d*e*f+d^2
*e^2))*(1-b*x^2/a)^(1/2)*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/2),(-a*d
/b/c)^(1/2))/a^(3/2)/c/(a*d+b*c)/e^2/(a*f+b*e)^2/(-c*f+d*e)/(-b*x^2+a)^(1/
2)/(1+d*x^2/c)^(1/2)+1/2*b^(1/2)*(3*a^2*f^2+4*a*b*e*f+4*b^2*e^2)*(1-b*x^2/
a)^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(b^(1/2)*x/a^(1/2),(-a*d/b/c)^(1/2))/a
^(3/2)/e^2/(a*f+b*e)^2/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)-1/2*a^(1/2)*f^2*(b
*e*(-6*c*f+7*d*e)+a*f*(-3*c*f+4*d*e))*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)*
EllipticPi(b^(1/2)*x/a^(1/2),-a*f/b/e,(-a*d/b/c)^(1/2))/b^(1/2)/e^3/(a*f+b
*e)^2/(-c*f+d*e)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 9.36 (sec) , antiderivative size = 747, normalized size of antiderivative = 0.95

$$\int \frac{1}{x^2 (a-bx^2)^{3/2} \sqrt{c+dx^2} (e+fx^2)^2} \, dx = \frac{-\sqrt{-\frac{b}{a}} e(c+dx^2) \left(4b^4ce^2(de-cf)x^2(e+fx^2)+2ab^3e(de-\right.}{}$$

input
```
Integrate[1/(x^2*(a - b*x^2)^(3/2)*Sqrt[c + d*x^2]*(e + f*x^2)^2),x]
```

output
```
(-(Sqrt[-(b/a)]*e*(c + d*x^2)*(4*b^4*c*e^2*(d*e - c*f)*x^2*(e + f*x^2) + 2
*a*b^3*e*(d*e - c*f)*(e + f*x^2)*(-(c*e) + d*e*x^2 + 2*c*f*x^2) + a^4*d*f^
2*(-2*d*e*(e + f*x^2) + c*f*(2*e + 3*f*x^2)) + a^3*b*f*(c^2*f^2*(2*e + 3*f
*x^2) + c*d*f*(2*e^2 - 3*f^2*x^4) - 2*d^2*e*(2*e^2 + e*f*x^2 - f^2*x^4)) +
 a^2*b^2*(-2*c*d*e*f*(e + f*x^2)^2 + c^2*f^2*(4*e^2 + 2*e*f*x^2 - 3*f^2*x^
4) + 2*d^2*e^2*(-e^2 + e*f*x^2 + 2*f^2*x^4)))) + I*b*c*e*(4*b^3*c*e^2*(-(d
*e) + c*f) + a^3*d*f^2*(-2*d*e + 3*c*f) - 2*a*b^2*e*(d^2*e^2 + c*d*e*f - 2
*c^2*f^2) + a^2*b*f*(-4*d^2*e^2 + 2*c*d*e*f + 3*c^2*f^2))*x*Sqrt[1 - (b*x^
2)/a]*Sqrt[1 + (d*x^2)/c]*(e + f*x^2)*EllipticE[I*ArcSinh[Sqrt[-(b/a)]*x],
 -((a*d)/(b*c))] - I*b*c*(b*c + a*d)*e*(-(d*e) + c*f)*(4*b^2*e^2 + 4*a*b*e
*f + 3*a^2*f^2)*x*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(e + f*x^2)*Elli
pticF[I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))] + I*a^2*c*(b*c + a*d)*f^2
*(a*f*(-4*d*e + 3*c*f) + b*e*(-7*d*e + 6*c*f))*x*Sqrt[1 - (b*x^2)/a]*Sqrt[
1 + (d*x^2)/c]*(e + f*x^2)*EllipticPi[-((a*f)/(b*e)), I*ArcSinh[Sqrt[-(b/a
)]*x], -((a*d)/(b*c))])/(2*a^2*Sqrt[-(b/a)]*c*(b*c + a*d)*e^3*(b*e + a*f)^
2*(-(d*e) + c*f)*x*Sqrt[a - b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a-bx^2)^{3/2} \sqrt{c+dx^2} (e+fx^2)^2} \, dx$$

$$\Big\downarrow \text{ 450}$$

$$\int \frac{1}{x^2 (a-bx^2)^{3/2} \sqrt{c+dx^2} (e+fx^2)^2}dx$$

input
```
Int[1/(x^2*(a - b*x^2)^(3/2)*Sqrt[c + d*x^2]*(e + f*x^2)^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1597 vs. $2(720) = 1440$.

Time = 28.24 (sec) , antiderivative size = 1598, normalized size of antiderivative = 2.03

method	result	size
elliptic	Expression too large to display	1598
risch	Expression too large to display	1616
default	Expression too large to display	4733

input
```
int(1/x^2/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x,method=_RETURNVER
BOSE)
```

output
```
((-b*x^2+a)*(d*x^2+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-(-b*d*x^2-
b*c)*b^3/a^2/(a*d+b*c)*x/(a*f+b*e)^2/((x^2-a/b)*(-b*d*x^2-b*c))^(1/2)-1/2*
f^4/(a*c*f^2-a*d*e*f+b*c*e*f-b*d*e^2)/e^2*x/(a*f+b*e)*(-b*d*x^4+a*d*x^2-b*
c*x^2+a*c)^(1/2)/(f*x^2+e)-1/a^2/c/e^2*(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2
)/x+1/2*c/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^
2-b*c*x^2+a*c)^(1/2)*b*f^3/(a*c*f^2-a*d*e*f+b*c*e*f-b*d*e^2)/e^2/(a*f+b*e)
*EllipticF(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2))-1/2*c/(b/a)^(1/2)*(1-b*
x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)*b*f^3/
(a*c*f^2-a*d*e*f+b*c*e*f-b*d*e^2)/e^2/(a*f+b*e)*EllipticE(x*(b/a)^(1/2),(-
1-(a*d-b*c)/c/b)^(1/2))+1/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/
(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)*b/a^2/e^2*EllipticF(x*(b/a)^(1/2),(-1
-(a*d-b*c)/c/b)^(1/2))-1/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(
-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)*b/a^2/e^2*EllipticE(x*(b/a)^(1/2),(-1-
(a*d-b*c)/c/b)^(1/2))+1/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-
b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)*EllipticF(x*(b/a)^(1/2),(-1-(a*d-b*c)/c
/b)^(1/2))/a^2*b^3/(a*f+b*e)^2+2*f^3/(a*c*f^2-a*d*e*f+b*c*e*f-b*d*e^2)/e^2
/(a*f+b*e)/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x
^2-b*c*x^2+a*c)^(1/2)*EllipticPi(x*(b/a)^(1/2),-a*f/b/e,(-1/c*d)^(1/2)/(b/
a)^(1/2))*a*d-3*f^3/(a*c*f^2-a*d*e*f+b*c*e*f-b*d*e^2)/e^2/(a*f+b*e)/(b/a)^
(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a-bx^2)^{3/2} \sqrt{c+dx^2} (e+fx^2)^2} \, dx = \text{Timed out}$$

input
```
integrate(1/x^2/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm=
"fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^2 \left(a-b x^2\right)^{3/2} \sqrt{c+d x^2} \left(e+f x^2\right)^2}\, dx = \text{Timed out}$$

input
```
integrate(1/x**2/(-b*x**2+a)**(3/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{x^2 \left(a-b x^2\right)^{3/2} \sqrt{c+d x^2} \left(e+f x^2\right)^2}\, dx = \int \frac{1}{\left(-b x^2+a\right)^{\frac{3}{2}} \sqrt{d x^2+c} \left(f x^2+e\right)^2 x^2}\, dx$$

input
```
integrate(1/x^2/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm=
"maxima")
```

output
```
integrate(1/((-b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*(f*x^2 + e)^2*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2 \left(a-b x^2\right)^{3/2} \sqrt{c+d x^2} \left(e+f x^2\right)^2}\, dx = \int \frac{1}{\left(-b x^2+a\right)^{\frac{3}{2}} \sqrt{d x^2+c} \left(f x^2+e\right)^2 x^2}\, dx$$

input
```
integrate(1/x^2/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm=
"giac")
```

output
```
integrate(1/((-b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*(f*x^2 + e)^2*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a-bx^2)^{3/2} \sqrt{c+dx^2} (e+fx^2)^2} \, dx = \int \frac{1}{x^2 \left(a-b\,x^2\right)^{3/2} \sqrt{d\,x^2+c} \left(f\,x^2+e\right)^2} \, dx$$

input
```
int(1/(x^2*(a - b*x^2)^(3/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^2),x)
```

output
```
int(1/(x^2*(a - b*x^2)^(3/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^2), x)
```

Reduce [F]

$$\int \frac{1}{x^2 (a-bx^2)^{3/2} \sqrt{c+dx^2} (e+fx^2)^2} \, dx = \text{too large to display}$$

input
```
int(1/x^2/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x)
```

output

```
( - sqrt(c + d*x**2)*sqrt(a - b*x**2) + 3*int((sqrt(c + d*x**2)*sqrt(a - b
*x**2)*x**4)/(a**2*c*e**2 + 2*a**2*c*e*f*x**2 + a**2*c*f**2*x**4 + a**2*d*
e**2*x**2 + 2*a**2*d*e*f*x**4 + a**2*d*f**2*x**6 - 2*a*b*c*e**2*x**2 - 4*a
*b*c*e*f*x**4 - 2*a*b*c*f**2*x**6 - 2*a*b*d*e**2*x**4 - 4*a*b*d*e*f*x**6 -
 2*a*b*d*f**2*x**8 + b**2*c*e**2*x**4 + 2*b**2*c*e*f*x**6 + b**2*c*f**2*x*
*8 + b**2*d*e**2*x**6 + 2*b**2*d*e*f*x**8 + b**2*d*f**2*x**10),x)*a*b*d*e*
f*x + 3*int((sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**4)/(a**2*c*e**2 + 2*a**2
*c*e*f*x**2 + a**2*c*f**2*x**4 + a**2*d*e**2*x**2 + 2*a**2*d*e*f*x**4 + a*
*2*d*f**2*x**6 - 2*a*b*c*e**2*x**2 - 4*a*b*c*e*f*x**4 - 2*a*b*c*f**2*x**6
- 2*a*b*d*e**2*x**4 - 4*a*b*d*e*f*x**6 - 2*a*b*d*f**2*x**8 + b**2*c*e**2*x
**4 + 2*b**2*c*e*f*x**6 + b**2*c*f**2*x**8 + b**2*d*e**2*x**6 + 2*b**2*d*e
*f*x**8 + b**2*d*f**2*x**10),x)*a*b*d*f**2*x**3 - 3*int((sqrt(c + d*x**2)*
sqrt(a - b*x**2)*x**4)/(a**2*c*e**2 + 2*a**2*c*e*f*x**2 + a**2*c*f**2*x**4
 + a**2*d*e**2*x**2 + 2*a**2*d*e*f*x**4 + a**2*d*f**2*x**6 - 2*a*b*c*e**2*
x**2 - 4*a*b*c*e*f*x**4 - 2*a*b*c*f**2*x**6 - 2*a*b*d*e**2*x**4 - 4*a*b*d*
e*f*x**6 - 2*a*b*d*f**2*x**8 + b**2*c*e**2*x**4 + 2*b**2*c*e*f*x**6 + b**2
*c*f**2*x**8 + b**2*d*e**2*x**6 + 2*b**2*d*e*f*x**8 + b**2*d*f**2*x**10),x
)*b**2*d*e*f*x**3 - 3*int((sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**4)/(a**2*c
*e**2 + 2*a**2*c*e*f*x**2 + a**2*c*f**2*x**4 + a**2*d*e**2*x**2 + 2*a**2*d
*e*f*x**4 + a**2*d*f**2*x**6 - 2*a*b*c*e**2*x**2 - 4*a*b*c*e*f*x**4 - 2...
```

3.301 $\int \frac{1}{x^4(a-bx^2)^{3/2}\sqrt{c+dx^2}(e+fx^2)^2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 1102

$$\int \frac{1}{x^4(a-bx^2)^{3/2}\sqrt{c+dx^2}(e+fx^2)^2}\,dx = \frac{b(16b^4c^2e^3(de-cf)-a^4df^2(4d^2e^2+8cdef-15c^2f^2)+2ab^3 \cdots}{\cdots}$$

$$-\frac{\sqrt{c+dx^2}}{3acex^3\sqrt{a-bx^2}(e+fx^2)}-\frac{(4bce-2ade-5acf)\sqrt{c+dx^2}}{3a^2c^2e^2x\sqrt{a-bx^2}(e+fx^2)}$$

$$-\frac{f(8b^2ce^2(de-cf)-a^2f(4d^2e^2+8cdef-15c^2f^2)-ab(4d^2e^3-4c^2ef^2))x\sqrt{c+dx^2}}{6a^2c^2e^3(be+af)(de-cf)\sqrt{a-bx^2}(e+fx^2)}$$

$$-\frac{\sqrt{b}(16b^4c^2e^3(de-cf)-a^4df^2(4d^2e^2+8cdef-15c^2f^2)+2ab^3ce^2(3d^2e^2+cdef-4c^2f^2)-a^3bf(8d^3e^3 \cdots}{6a^{5/2}c^2(bc+ad)e^3(\cdots}$$

$$+\frac{\sqrt{b}(16b^3ce^3-2ab^2e^2(de-4cf)-2a^2bef(2de+7cf)-a^3f^2(2de+15cf))\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\mathrm{EllipticF} \cdots}{6a^{5/2}ce^3(be+af)^2\sqrt{a-bx^2}\sqrt{c+dx^2}}$$

$$+\frac{\sqrt{a}f^3(be(9de-8cf)+af(6de-5cf))\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\mathrm{EllipticPi}\left(-\frac{af}{be},\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{2\sqrt{b}e^4(be+af)^2(de-cf)\sqrt{a-bx^2}\sqrt{c+dx^2}}$$

output
```
1/6*b*(16*b^4*c^2*e^3*(-c*f+d*e)-a^4*d*f^2*(-15*c^2*f^2+8*c*d*e*f+4*d^2*e^
2)+2*a*b^3*c*e^2*(-4*c^2*f^2+c*d*e*f+3*d^2*e^2)-a^3*b*f*(-15*c^3*f^3-6*c^2
*d*e*f^2+10*c*d^2*e^2*f+8*d^3*e^3)-2*a^2*b^2*e*(-7*c^3*f^3+7*c^2*d*e*f^2-2
*c*d^2*e^2*f+2*d^3*e^3))*x*(d*x^2+c)^(1/2)/a^3/c^2/(a*d+b*c)/e^3/(a*f+b*e)
^2/(-c*f+d*e)/(-b*x^2+a)^(1/2)-1/3*(d*x^2+c)^(1/2)/a/c/e/x^3/(-b*x^2+a)^(1
/2)/(f*x^2+e)-1/3*(-5*a*c*f-2*a*d*e+4*b*c*e)*(d*x^2+c)^(1/2)/a^2/c^2/e^2/x
/(-b*x^2+a)^(1/2)/(f*x^2+e)-1/6*f*(8*b^2*c*e^2*(-c*f+d*e)-a^2*f*(-15*c^2*f
^2+8*c*d*e*f+4*d^2*e^2)-a*b*(-4*c^2*e*f^2+4*d^2*e^3))*x*(d*x^2+c)^(1/2)/a^
2/c^2/e^3/(a*f+b*e)/(-c*f+d*e)/(-b*x^2+a)^(1/2)/(f*x^2+e)-1/6*b^(1/2)*(16*
b^4*c^2*e^3*(-c*f+d*e)-a^4*d*f^2*(-15*c^2*f^2+8*c*d*e*f+4*d^2*e^2)+2*a*b^3
*c*e^2*(-4*c^2*f^2+c*d*e*f+3*d^2*e^2)-a^3*b*f*(-15*c^3*f^3-6*c^2*d*e*f^2+1
0*c*d^2*e^2*f+8*d^3*e^3)-2*a^2*b^2*e*(-7*c^3*f^3+7*c^2*d*e*f^2-2*c*d^2*e^2
*f+2*d^3*e^3))*(1-b*x^2/a)^(1/2)*(d*x^2+c)^(1/2)*EllipticE(b^(1/2)*x/a^(1/
2),(-a*d/b/c)^(1/2))/a^(5/2)/c^2/(a*d+b*c)/e^3/(a*f+b*e)^2/(-c*f+d*e)/(-b*
x^2+a)^(1/2)/(1+d*x^2/c)^(1/2)+1/6*b^(1/2)*(16*b^3*c*e^3-2*a*b^2*e^2*(-4*c
*f+d*e)-2*a^2*b*e*f*(7*c*f+2*d*e)-a^3*f^2*(15*c*f+2*d*e))*(1-b*x^2/a)^(1/2
)*(1+d*x^2/c)^(1/2)*EllipticF(b^(1/2)*x/a^(1/2),(-a*d/b/c)^(1/2))/a^(5/2)/
c/e^3/(a*f+b*e)^2/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)+1/2*a^(1/2)*f^3*(b*e*(-
8*c*f+9*d*e)+a*f*(-5*c*f+6*d*e))*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)*Ellip
ticPi(b^(1/2)*x/a^(1/2),-a*f/b/e,(-a*d/b/c)^(1/2))/b^(1/2)/e^4/(a*f+b*e...
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 7.53 (sec) , antiderivative size = 700, normalized size of antiderivative = 0.64

$$\int \frac{1}{x^4 (a-bx^2)^{3/2} \sqrt{c+dx^2} (e+fx^2)^2} dx = \frac{-\sqrt{-\frac{b}{a}}e(c+dx^2) (3a^3c^2(bc+ad)f^5x^4(a-bx^2)+6b^5c^2e^3(-c$$

input
```
Integrate[1/(x^4*(a - b*x^2)^(3/2)*Sqrt[c + d*x^2]*(e + f*x^2)^2),x]
```

output
```
(-(Sqrt[-(b/a)]*e*(c + d*x^2)*(3*a^3*c^2*(b*c + a*d)*f^5*x^4*(a - b*x^2) +
 6*b^5*c^2*e^3*(-(d*e) + c*f)*x^4*(e + f*x^2) + 2*a*c*(b*c + a*d)*e*(b*e +
 a*f)^2*(d*e - c*f)*(a - b*x^2)*(e + f*x^2) + 2*(b*c + a*d)*(b*e + a*f)^2*
(d*e - c*f)*(5*b*c*e - 2*a*(d*e + 3*c*f))*x^2*(a - b*x^2)*(e + f*x^2))) +
I*c*x^3*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(e + f*x^2)*(b*e*(-16*b^4*
c^2*e^3*(-(d*e) + c*f) + 2*a*b^3*c*e^2*(3*d^2*e^2 + c*d*e*f - 4*c^2*f^2) +
 a^4*d*f^2*(-4*d^2*e^2 - 8*c*d*e*f + 15*c^2*f^2) - 2*a^2*b^2*e*(2*d^3*e^3
- 2*c*d^2*e^2*f + 7*c^2*d*e*f^2 - 7*c^3*f^3) + a^3*b*f*(-8*d^3*e^3 - 10*c*
d^2*e^2*f + 6*c^2*d*e*f^2 + 15*c^3*f^3))*EllipticE[I*ArcSinh[Sqrt[-(b/a)]*
x], -((a*d)/(b*c))] + (b*c + a*d)*(b*e*(-(d*e) + c*f)*(16*b^3*c*e^3 - 2*a*
b^2*e^2*(d*e - 4*c*f) - 2*a^2*b*e*f*(2*d*e + 7*c*f) - a^3*f^2*(2*d*e + 15*
c*f))*EllipticF[I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))] + 3*a^3*c*f^3*(
a*f*(-6*d*e + 5*c*f) + b*e*(-9*d*e + 8*c*f))*EllipticPi[-((a*f)/(b*e)), I*
ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))])))/(6*a^3*Sqrt[-(b/a)]*c^2*(b*c +
 a*d)*e^4*(b*e + a*f)^2*(d*e - c*f)*x^3*Sqrt[a - b*x^2]*Sqrt[c + d*x^2]*(e
 + f*x^2))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (a-bx^2)^{3/2} \sqrt{c+dx^2} (e+fx^2)^2} \, dx$$
$$\Big\downarrow \; 450$$
$$\int \frac{1}{x^4 (a-bx^2)^{3/2} \sqrt{c+dx^2} (e+fx^2)^2} dx$$

input
```
Int[1/(x^4*(a - b*x^2)^(3/2)*Sqrt[c + d*x^2]*(e + f*x^2)^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [A] (verified)

Time = 28.33 (sec) , antiderivative size = 1765, normalized size of antiderivative = 1.60

method	result	size
risch	Expression too large to display	1765
elliptic	Expression too large to display	2057
default	Expression too large to display	7708

input
```
int(1/x^4/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x,method=_RETURNVER
BOSE)
```

output
```
-1/3*(-b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(-6*a*c*f*x^2-2*a*d*e*x^2+5*b*c*e*x^
2+a*c*e)/a^3/c^2/e^3/x^3+1/3/a^3/e^3/c^2*(-b*(6*a*c*f+2*a*d*e-5*b*c*e)*c/(
b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a
*c)^(1/2)*(EllipticF(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2))-EllipticE(x*(
b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2)))+a*b*c*d*e/(b/a)^(1/2)*(1-b*x^2/a)^(1
/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)*EllipticF(x*(b/
a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2))-3*a*b^4*c^2*e^3/(a*f+b*e)^2*((-b*d*x^2-
b*c)/a/(a*d+b*c)*x/((x^2-a/b)*(-b*d*x^2-b*c))^(1/2)+(-1/a+b*c/a/(a*d+b*c))
/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2
+a*c)^(1/2)*EllipticF(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2))-b/a/(a*d+b*c
)*c/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*
x^2+a*c)^(1/2)*(EllipticF(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2))-Elliptic
E(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2))))+3*a^3*c^2*e*f^3/(a*f+b*e)*(1/2
*f^2/(a*c*f^2-a*d*e*f+b*c*e*f-b*d*e^2)/e*x*(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^
(1/2)/(f*x^2+e)+1/2*d*b/(a*c*f^2-a*d*e*f+b*c*e*f-b*d*e^2)/(b/a)^(1/2)*(1-b
*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)*Ellip
ticF(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2))-1/2*b*f/(a*c*f^2-a*d*e*f+b*c*
e*f-b*d*e^2)/e*c/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4
+a*d*x^2-b*c*x^2+a*c)^(1/2)*EllipticF(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/
2))+1/2*b*f/(a*c*f^2-a*d*e*f+b*c*e*f-b*d*e^2)/e*c/(b/a)^(1/2)*(1-b*x^2/...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (a-bx^2)^{3/2} \sqrt{c+dx^2} (e+fx^2)^2} \, dx = \text{Timed out}$$

input
```
integrate(1/x^4/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm=
"fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^4 \left(a-b x^2\right)^{3/2} \sqrt{c+d x^2} \left(e+f x^2\right)^2} \, dx=\text{Timed out}$$

input
```
integrate(1/x**4/(-b*x**2+a)**(3/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{x^4 \left(a-b x^2\right)^{3/2} \sqrt{c+d x^2} \left(e+f x^2\right)^2} \, dx=\int \frac{1}{{\left(-b x^{2}+a\right)}^{\frac{3}{2}} \sqrt{d x^{2}+c} {\left(f x^{2}+e\right)}^{2} x^{4}} \, dx$$

input
```
integrate(1/x^4/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm=
"maxima")
```

output
```
integrate(1/((-b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*(f*x^2 + e)^2*x^4), x)
```

Giac [F]

$$\int \frac{1}{x^4 \left(a-b x^2\right)^{3/2} \sqrt{c+d x^2} \left(e+f x^2\right)^2} \, dx=\int \frac{1}{{\left(-b x^{2}+a\right)}^{\frac{3}{2}} \sqrt{d x^{2}+c} {\left(f x^{2}+e\right)}^{2} x^{4}} \, dx$$

input
```
integrate(1/x^4/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x, algorithm=
"giac")
```

output
```
integrate(1/((-b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*(f*x^2 + e)^2*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (a-bx^2)^{3/2} \sqrt{c+dx^2} (e+fx^2)^2}\,dx = \int \frac{1}{x^4 \left(a-b\,x^2\right)^{3/2} \sqrt{d\,x^2+c}\,\left(f\,x^2+e\right)^2}\,dx$$

input
```
int(1/(x^4*(a - b*x^2)^(3/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^2),x)
```

output
```
int(1/(x^4*(a - b*x^2)^(3/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^2), x)
```

Reduce [F]

$$\int \frac{1}{x^4 (a-bx^2)^{3/2} \sqrt{c+dx^2} (e+fx^2)^2}\,dx = \int \frac{1}{x^4 \left(-b\,x^2+a\right)^{\frac{3}{2}} \sqrt{d\,x^2+c}\,\left(f\,x^2+e\right)^2}dx$$

input
```
int(1/x^4/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x)
```

output
```
int(1/x^4/(-b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^2,x)
```

3.302 $\int x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 1152

$$\int x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}\,dx = \text{Too large to display}$$

output
```
1/384*(15*a^3*d^3*f^3-7*a^2*b*d^2*f^2*(c*f+d*e)-a*b^2*d*f*(7*c^2*f^2-6*c*d
*e*f+7*d^2*e^2)+b^3*(15*c^3*f^3-7*c^2*d*e*f^2-7*c*d^2*e^2*f+15*d^3*e^3))*x
*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/b^2/d^3/f^3/(b*x^2+a)^(1/2)-1/192*(5*a^2*
d*f/b-2*a*(c*f+d*e)-b*(2*c*e-5*d*e^2/f-5*c^2*f/d))*x*(b*x^2+a)^(1/2)*(d*x^
2+c)^(1/2)*(f*x^2+e)^(1/2)/b/d/f+1/48*(a/b+c/d+e/f)*x^3*(b*x^2+a)^(1/2)*(d
*x^2+c)^(1/2)*(f*x^2+e)^(1/2)+1/8*x^5*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x
^2+e)^(1/2)-1/384*(-a*d+b*c)^(1/2)*e*(15*a^3*d^3*f^3-7*a^2*b*d^2*f^2*(c*f+
d*e)-a*b^2*d*f*(7*c^2*f^2-6*c*d*e*f+7*d^2*e^2)+b^3*(15*c^3*f^3-7*c^2*d*e*f
^2-7*c*d^2*e^2*f+15*d^3*e^3))*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1
/2)*EllipticE((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2),(c*(-a*f+b*e)/(-a
*d+b*c)/e)^(1/2))/b^3/c^(1/2)/d^3/f^3/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(f*x
^2+e)^(1/2)-1/384*a*(-a*d+b*c)^(1/2)*(15*a^3*d^3*f^3-3*a^2*b*d^2*f^2*(-c*f
+9*d*e)+a*b^2*d*f*(-3*c^2*f^2+2*c*d*e*f+d^2*e^2)-b^3*(15*c^3*f^3-17*c^2*d*
e*f^2-3*c*d^2*e^2*f+5*d^3*e^3))*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^
(1/2)*EllipticF((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2),(c*(-a*f+b*e)/(
-a*d+b*c)/e)^(1/2))/b^4/c^(1/2)/d^3/f^2/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(f
*x^2+e)^(1/2)-1/128*a*(5*a^4*d^4*f^4-2*a^2*b^2*d^2*f^2*(-c*f+d*e)^2-4*a^3*
b*d^3*f^3*(c*f+d*e)-4*a*b^3*d*f*(-c*f+d*e)^2*(c*f+d*e)+b^4*(-c*f+d*e)^2*(5
*c^2*f^2+6*c*d*e*f+5*d^2*e^2))*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(
1/2)*EllipticPi((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2),b*c/(-a*d+b*...
```

Mathematica [F]

$$\int x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}\,dx=\int x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}\,dx$$

input
```
Integrate[x^4*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*Sqrt[e + f*x^2],x]
```

output
```
Integrate[x^4*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*Sqrt[e + f*x^2], x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}\,dx$$

$$\Big\downarrow \text{450}$$

$$\int x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}dx$$

input
```
Int[x^4*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*Sqrt[e + f*x^2],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [F]

$$\int x^4\sqrt{b\,x^2+a}\,\sqrt{x^2d+c}\,\sqrt{f\,x^2+e}dx$$

input
```
int(x^4*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2),x)
```

output
```
int(x^4*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}\,dx=\text{Timed out}$$

input
```
integrate(x^4*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2),x, algorithm
="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}\,dx=\int x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}\,dx$$

input
```
integrate(x**4*(b*x**2+a)**(1/2)*(d*x**2+c)**(1/2)*(f*x**2+e)**(1/2),x)
```

output
```
Integral(x**4*sqrt(a + b*x**2)*sqrt(c + d*x**2)*sqrt(e + f*x**2), x)
```

Maxima [F]

$$\int x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}\,dx=\int \sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{fx^2+e}x^4\,dx$$

input
```
integrate(x^4*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2),x, algorithm
="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)*x^4, x)
```

Giac [F]

$$\int x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}\,dx=\int \sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{fx^2+e}x^4\,dx$$

input
```
integrate(x^4*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2),x, algorithm
="giac")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)*x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}\,dx=\int x^4\,\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}\,dx$$

input
```
int(x^4*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(1/2),x)
```

output
```
int(x^4*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(1/2), x)
```

Reduce [F]

$$\int x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}\,dx=\int x^4\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}dx$$

input
```
int(x^4*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2),x)
```

output
```
int(x^4*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2),x)
```

3.303 $\int x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 819

$$\int x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}\,dx$$

$$= -\frac{\left(3a^2df-2ab(de+cf)-b^2\left(2ce-\frac{3de^2}{f}-\frac{3c^2f}{d}\right)\right)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{48bdf\sqrt{a+bx^2}}$$

$$+\frac{1}{24}\left(\frac{a}{b}+\frac{c}{d}+\frac{e}{f}\right)x\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}+\frac{1}{6}x^3\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}$$

$$+\frac{\sqrt{bc-ad}\,e(3a^2d^2f^2-2abdf(de+cf)+b^2(3d^2e^2-2cdef+3c^2f^2))\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}E\left(\arcsin\left(\cdots\right.\right.}{48b^2\sqrt{c}d^2f^2\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$

$$-\frac{a\sqrt{bc-ad}(6abd^2ef-3a^2d^2f^2+b^2(d^2e^2-4cdef+3c^2f^2))\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,\text{EllipticF}\left(\arcsin\left(\cdots\right.\right.}{48b^3\sqrt{c}d^2f\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$

$$-\frac{a(bde-bcf-adf)(bde-bcf+adf)(adf-b(de+cf))\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,\text{EllipticPi}\left(\frac{bc}{bc-ad},\arcsin\left(\cdots\right.\right.}{16b^3\sqrt{c}d^2\sqrt{bc-ad}f^2\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$

output
```
-1/48*(3*a^2*d*f-2*a*b*(c*f+d*e)-b^2*(2*c*e-3*d*e^2/f-3*c^2*f/d))*x*(d*x^2
+c)^(1/2)*(f*x^2+e)^(1/2)/b/d/f/(b*x^2+a)^(1/2)+1/24*(a/b+c/d+e/f)*x*(b*x^
2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)+1/6*x^3*(b*x^2+a)^(1/2)*(d*x^2+
c)^(1/2)*(f*x^2+e)^(1/2)+1/48*(-a*d+b*c)^(1/2)*e*(3*a^2*d^2*f^2-2*a*b*d*f*
(c*f+d*e)+b^2*(3*c^2*f^2-2*c*d*e*f+3*d^2*e^2))*(d*x^2+c)^(1/2)*(a*(f*x^2+e
)/e/(b*x^2+a))^(1/2)*EllipticE((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2),
(c*(-a*f+b*e)/(-a*d+b*c)/e)^(1/2))/b^2/c^(1/2)/d^2/f^2/(a*(d*x^2+c)/c/(b*x
^2+a))^(1/2)/(f*x^2+e)^(1/2)-1/48*a*(-a*d+b*c)^(1/2)*(6*a*b*d^2*e*f-3*a^2*
d^2*f^2+b^2*(3*c^2*f^2-4*c*d*e*f+d^2*e^2))*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/
(b*x^2+a))^(1/2)*EllipticF((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2),(c*(
-a*f+b*e)/(-a*d+b*c)/e)^(1/2))/b^3/c^(1/2)/d^2/f/(a*(d*x^2+c)/c/(b*x^2+a))
^(1/2)/(f*x^2+e)^(1/2)-1/16*a*(-a*d*f-b*c*f+b*d*e)*(a*d*f-b*c*f+b*d*e)*(a*
d*f-b*(c*f+d*e))*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)*EllipticP
i((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2),b*c/(-a*d+b*c),(c*(-a*f+b*e)/
(-a*d+b*c)/e)^(1/2))/b^3/c^(1/2)/d^2/(-a*d+b*c)^(1/2)/f^2/(a*(d*x^2+c)/c/(
b*x^2+a))^(1/2)/(f*x^2+e)^(1/2)
```

Mathematica [F]

$$\int x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}\,dx=\int x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}\,dx$$

input
```
Integrate[x^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*Sqrt[e + f*x^2],x]
```

output
```
Integrate[x^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*Sqrt[e + f*x^2], x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}\,dx$$

$$\Big\downarrow \; 450$$

$$\int x^2 \sqrt{a+bx^2} \sqrt{c+dx^2} \sqrt{e+fx^2} dx$$

input
```
Int[x^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*Sqrt[e + f*x^2],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [F]

$$\int x^2 \sqrt{b\,x^2+a}\,\sqrt{x^2 d+c}\,\sqrt{f\,x^2+e}dx$$

input
```
int(x^2*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2),x)
```

output
```
int(x^2*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int x^2 \sqrt{a+bx^2} \sqrt{c+dx^2} \sqrt{e+fx^2}\,dx = \text{Timed out}$$

input
```
integrate(x^2*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2),x, algorithm
="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}\,dx=\int x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}\,dx$$

input
```
integrate(x**2*(b*x**2+a)**(1/2)*(d*x**2+c)**(1/2)*(f*x**2+e)**(1/2),x)
```

output
```
Integral(x**2*sqrt(a + b*x**2)*sqrt(c + d*x**2)*sqrt(e + f*x**2), x)
```

Maxima [F]

$$\int x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}\,dx=\int \sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{fx^2+e}x^2\,dx$$

input
```
integrate(x^2*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2),x, algorithm
="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)*x^2, x)
```

Giac [F]

$$\int x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}\,dx=\int \sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{fx^2+e}x^2\,dx$$

input
```
integrate(x^2*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2),x, algorithm
="giac")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}\,dx=\int x^2\,\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}\,dx$$

input
```
int(x^2*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(1/2),x)
```

output
```
int(x^2*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(1/2), x)
```

Reduce [F]

$$\int x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}\,dx=\int x^2\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}dx$$

input
```
int(x^2*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2),x)
```

output
```
int(x^2*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2),x)
```

3.304 $\int \sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 646

$$\begin{aligned}
&\int \sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}\,dx \\
&= \frac{(bde+bcf+adf)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{8df\sqrt{a+bx^2}} + \frac{1}{4}x\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2} \\
&\quad - \frac{\sqrt{bc-ad}e(bde+bcf+adf)\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}E\left(\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)|\frac{c(be-af)}{(bc-ad)e}\right)}{8b\sqrt{c}df\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}} \\
&\quad + \frac{a\sqrt{bc-ad}(3bde+bcf-adf)\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right),\frac{c(be-af)}{(bc-ad)e}\right)}{8b^2\sqrt{c}d\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}} \\
&\quad - \frac{a\left(a^2d^2f^2+b^2(de-cf)^2-2abdf(de+cf)\right)\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\operatorname{EllipticPi}\left(\frac{bc}{bc-ad},\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right),\right.}{8b^2\sqrt{c}d\sqrt{bc-ad}f\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}
\end{aligned}$$

output
```
1/8*(a*d*f+b*c*f+b*d*e)*x*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/d/f/(b*x^2+a)^(1
/2)+1/4*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)-1/8*(-a*d+b*c)^(
1/2)*e*(a*d*f+b*c*f+b*d*e)*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)
*EllipticE((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2),(c*(-a*f+b*e)/(-a*d+
b*c)/e)^(1/2))/b/c^(1/2)/d/f/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(f*x^2+e)^(1/
2)+1/8*a*(-a*d+b*c)^(1/2)*(-a*d*f+b*c*f+3*b*d*e)*(d*x^2+c)^(1/2)*(a*(f*x^2
+e)/e/(b*x^2+a))^(1/2)*EllipticF((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2
),(c*(-a*f+b*e)/(-a*d+b*c)/e)^(1/2))/b^2/c^(1/2)/d/(a*(d*x^2+c)/c/(b*x^2+a
))^(1/2)/(f*x^2+e)^(1/2)-1/8*a*(a^2*d^2*f^2+b^2*(-c*f+d*e)^2-2*a*b*d*f*(c*
f+d*e))*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)*EllipticPi((-a*d+b
*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2),b*c/(-a*d+b*c),(c*(-a*f+b*e)/(-a*d+b*c
)/e)^(1/2))/b^2/c^(1/2)/d/(-a*d+b*c)^(1/2)/f/(a*(d*x^2+c)/c/(b*x^2+a))^(1/
2)/(f*x^2+e)^(1/2)
```

Mathematica [F]

$$\int \sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}\,dx = \int \sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}\,dx$$

input
```
Integrate[Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*Sqrt[e + f*x^2],x]
```

output
```
Integrate[Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*Sqrt[e + f*x^2], x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}\,dx$$

$$\Big\downarrow \; 434$$

$$\int \sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}dx$$

input
```
Int[Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*Sqrt[e + f*x^2],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \sqrt{b\,x^{2}+a}\,\sqrt{x^{2}d+c}\,\sqrt{f\,x^{2}+e}dx$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2),x)
```

output
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}\,dx=\text{Timed out}$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2),x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F]

$$\int \sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}\,dx = \int \sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(1/2)*(f*x**2+e)**(1/2),x)
```

output
```
Integral(sqrt(a + b*x**2)*sqrt(c + d*x**2)*sqrt(e + f*x**2), x)
```

Maxima [F]

$$\int \sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}\,dx = \int \sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{fx^2+e}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e), x)
```

Giac [F]

$$\int \sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}\,dx = \int \sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{fx^2+e}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}\,dx = \int \sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}\,dx$$

input
```
int((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(1/2),x)
```

output
```
int((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(1/2), x)
```

Reduce [F]

$$\begin{aligned}
&\int \sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}\,dx \\
&= \frac{\sqrt{f\,x^2+e}\,\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x}{4} \\
&+ \frac{\left(\int \frac{\sqrt{f\,x^2+e}\,\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^4}{bdf\,x^6+adf\,x^4+bcf\,x^4+bde\,x^4+acf\,x^2+ade\,x^2+bce\,x^2+ace}dx\right)adf}{4} \\
&+ \frac{\left(\int \frac{\sqrt{f\,x^2+e}\,\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^4}{bdf\,x^6+adf\,x^4+bcf\,x^4+bde\,x^4+acf\,x^2+ade\,x^2+bce\,x^2+ace}dx\right)bcf}{4} \\
&+ \frac{\left(\int \frac{\sqrt{f\,x^2+e}\,\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^4}{bdf\,x^6+adf\,x^4+bcf\,x^4+bde\,x^4+acf\,x^2+ade\,x^2+bce\,x^2+ace}dx\right)bde}{4} \\
&+ \frac{\left(\int \frac{\sqrt{f\,x^2+e}\,\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^2}{bdf\,x^6+adf\,x^4+bcf\,x^4+bde\,x^4+acf\,x^2+ade\,x^2+bce\,x^2+ace}dx\right)acf}{2} \\
&+ \frac{\left(\int \frac{\sqrt{f\,x^2+e}\,\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^2}{bdf\,x^6+adf\,x^4+bcf\,x^4+bde\,x^4+acf\,x^2+ade\,x^2+bce\,x^2+ace}dx\right)ade}{2} \\
&+ \frac{\left(\int \frac{\sqrt{f\,x^2+e}\,\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^2}{bdf\,x^6+adf\,x^4+bcf\,x^4+bde\,x^4+acf\,x^2+ade\,x^2+bce\,x^2+ace}dx\right)bce}{2} \\
&+ \frac{3\left(\int \frac{\sqrt{f\,x^2+e}\,\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{bdf\,x^6+adf\,x^4+bcf\,x^4+bde\,x^4+acf\,x^2+ade\,x^2+bce\,x^2+ace}dx\right)ace}{4}
\end{aligned}$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2),x)
```

output
```
(sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*x + int((sqrt(e + f*x*
*2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*e*x*
*2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*a*
d*f + int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c*e
 + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*
x**4 + b*d*f*x**6),x)*b*c*f + int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(
a + b*x**2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x*
*2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*b*d*e + 2*int((sqrt(e + f*x*
*2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c*e + a*c*f*x**2 + a*d*e*x*
*2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*a*
c*f + 2*int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c
*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*
e*x**4 + b*d*f*x**6),x)*a*d*e + 2*int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*s
qrt(a + b*x**2)*x**2)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*
e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*b*c*e + 3*int((sqrt(e +
f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c*e + a*c*f*x**2 + a*d*e*x**
2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*a*c
*e)/4
```

3.305 $\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{x^2} dx$

Optimal result

Integrand size = 37, antiderivative size = 565

$$\begin{aligned}
\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{x^2}\,dx &= -\frac{a\sqrt{c+dx^2}\sqrt{e+fx^2}}{x\sqrt{a+bx^2}}+\frac{bx\sqrt{c+dx^2}\sqrt{e+fx^2}}{2\sqrt{a+bx^2}} \\
&-\frac{3\sqrt{bc-ad}e\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}E\left(\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)|\frac{c(be-af)}{(bc-ad)e}\right)}{2\sqrt{c}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}} \\
&+\frac{\sqrt{bc-ad}(2be+af)\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right),\frac{c(be-af)}{(bc-ad)e}\right)}{2b\sqrt{c}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}} \\
&+\frac{a(bde+bcf+adf)\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\operatorname{EllipticPi}\left(\frac{bc}{bc-ad},\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right),\frac{c(be-af)}{(bc-ad)e}\right)}{2b\sqrt{c}\sqrt{bc-ad}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}
\end{aligned}$$

output
```
-a*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/x/(b*x^2+a)^(1/2)+1/2*b*x*(d*x^2+c)^(1/
2)*(f*x^2+e)^(1/2)/(b*x^2+a)^(1/2)-3/2*(-a*d+b*c)^(1/2)*e*(d*x^2+c)^(1/2)*
(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)*EllipticE((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^
2+a)^(1/2),(c*(-a*f+b*e)/(-a*d+b*c)/e)^(1/2))/c^(1/2)/(a*(d*x^2+c)/c/(b*x^
2+a))^(1/2)/(f*x^2+e)^(1/2)+1/2*(-a*d+b*c)^(1/2)*(a*f+2*b*e)*(d*x^2+c)^(1/
2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)*EllipticF((-a*d+b*c)^(1/2)*x/c^(1/2)/(b
*x^2+a)^(1/2),(c*(-a*f+b*e)/(-a*d+b*c)/e)^(1/2))/b/c^(1/2)/(a*(d*x^2+c)/c/
(b*x^2+a))^(1/2)/(f*x^2+e)^(1/2)+1/2*a*(a*d*f+b*c*f+b*d*e)*(d*x^2+c)^(1/2)
*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)*EllipticPi((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*
x^2+a)^(1/2),b*c/(-a*d+b*c),(c*(-a*f+b*e)/(-a*d+b*c)/e)^(1/2))/b/c^(1/2)/(
-a*d+b*c)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(f*x^2+e)^(1/2)
```

Mathematica [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{x^2}\,dx = \int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{x^2}\,dx$$

input
```
Integrate[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/x^2,x]
```

output
```
Integrate[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/x^2, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{x^2}\,dx$$
$$\Big\downarrow \; 450$$
$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{x^2}dx$$

input
```
Int[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/x^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [F]

$$\int \frac{\sqrt{b\,x^{2}+a}\,\sqrt{x^{2} d+c}\,\sqrt{f\,x^{2}+e}}{x^{2}}dx$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/x^2,x)
```

output
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/x^2,x)
```

Fricas [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{x^2}\,dx=\int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{fx^2+e}}{x^2}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/x^2,x, algorithm
="fricas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/x^2, x)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{x^2}\,dx = \int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{x^2}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(1/2)*(f*x**2+e)**(1/2)/x**2,x)
```

output
```
Integral(sqrt(a + b*x**2)*sqrt(c + d*x**2)*sqrt(e + f*x**2)/x**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{x^2}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{fx^2+e}}{x^2}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/x^2,x, algorithm
="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/x^2, x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{x^2}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{fx^2+e}}{x^2}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/x^2,x, algorithm
="giac")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{x^2}\,dx=\int \frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}}{x^2}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(1/2))/x^2,x)
```

output
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(1/2))/x^2, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{x^2}\,dx=\int \frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}}{x^2}dx$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/x^2,x)
```

output
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/x^2,x)
```

3.306 $\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{x^4}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 583

$$\begin{aligned}
&\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{x^4}\,dx \\
&= -\frac{a\sqrt{c+dx^2}\sqrt{e+fx^2}}{3x^3\sqrt{a+bx^2}} - \frac{(2bce+ade+acf)\sqrt{c+dx^2}\sqrt{e+fx^2}}{3cex\sqrt{a+bx^2}} \\
&\quad - \frac{\sqrt{bc-ad}(bce+ade+acf)\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}E\left(\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)|\frac{c(be-af)}{(bc-ad)e}\right)}{3ac^{3/2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}} \\
&\quad + \frac{\sqrt{bc-ad}(de+2cf)\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right),\frac{c(be-af)}{(bc-ad)e}\right)}{3c^{3/2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}} \\
&\quad + \frac{adf\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\operatorname{EllipticPi}\left(\frac{bc}{bc-ad},\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right),\frac{c(be-af)}{(bc-ad)e}\right)}{\sqrt{c}\sqrt{bc-ad}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}
\end{aligned}$$

output
```
-1/3*a*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/x^3/(b*x^2+a)^(1/2)-1/3*(a*c*f+a*d*
e+2*b*c*e)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/c/e/x/(b*x^2+a)^(1/2)-1/3*(-a*d
+b*c)^(1/2)*(a*c*f+a*d*e+b*c*e)*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^
(1/2)*EllipticE((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2),(c*(-a*f+b*e)/(
-a*d+b*c)/e)^(1/2))/a/c^(3/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(f*x^2+e)^(1
/2)+1/3*(-a*d+b*c)^(1/2)*(2*c*f+d*e)*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2
+a))^(1/2)*EllipticF((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2),(c*(-a*f+b
*e)/(-a*d+b*c)/e)^(1/2))/c^(3/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(f*x^2+e)
^(1/2)+a*d*f*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)*EllipticPi((-
a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2),b*c/(-a*d+b*c),(c*(-a*f+b*e)/(-a*
d+b*c)/e)^(1/2))/c^(1/2)/(-a*d+b*c)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/
(f*x^2+e)^(1/2)
```

Mathematica [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{x^4}\,dx = \int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{x^4}\,dx$$

input
```
Integrate[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/x^4,x]
```

output
```
Integrate[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/x^4, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{x^4}\,dx$$
$$\Big\downarrow \text{450}$$
$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{x^4}dx$$

input
```
Int[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/x^4,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [F]

$$\int \frac{\sqrt{b\,x^{2}+a}\,\sqrt{x^{2} d+c}\,\sqrt{f\,x^{2}+e}}{x^{4}}dx$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/x^4,x)
```

output
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/x^4,x)
```

Fricas [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{x^4}\,dx=\int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{fx^2+e}}{x^4}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/x^4,x, algorithm
="fricas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/x^4, x)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{x^4}\,dx = \int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{x^4}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(1/2)*(f*x**2+e)**(1/2)/x**4,x)
```

output
```
Integral(sqrt(a + b*x**2)*sqrt(c + d*x**2)*sqrt(e + f*x**2)/x**4, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{x^4}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{fx^2+e}}{x^4}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/x^4,x, algorithm
="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/x^4, x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{x^4}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{fx^2+e}}{x^4}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/x^4,x, algorithm
="giac")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{x^4}\,dx = \int \frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}}{x^4}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(1/2))/x^4,x)
```

output
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(1/2))/x^4, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{x^4}\,dx = \int \frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}}{x^4}dx$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/x^4,x)
```

output
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/x^4,x)
```

3.307 $\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{x^6}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 574

$$
\begin{aligned}
&\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{x^6}\,dx \\
&= -\frac{a\sqrt{c+dx^2}\sqrt{e+fx^2}}{5x^5\sqrt{a+bx^2}} - \frac{(4bce+ade+acf)\sqrt{c+dx^2}\sqrt{e+fx^2}}{15cex^3\sqrt{a+bx^2}} \\
&\quad + \frac{\left(b^2ce-3ab(de+cf)+2a^2\left(\frac{d^2e}{c}-df+\frac{cf^2}{e}\right)\right)\sqrt{c+dx^2}\sqrt{e+fx^2}}{15acex\sqrt{a+bx^2}} \\
&\quad + \frac{2\sqrt{bc-ad}(b^2c^2e^2-abce(de+cf)+a^2(d^2e^2-cdef+c^2f^2))\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}E\left(\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right.\right.}{15a^2c^{5/2}e\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}} \\
&\quad + \frac{\sqrt{bc-ad}(de-cf)(bce-2ade+acf)\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right),\frac{c(be-af)}{(bc-ad)e}\right)}{15ac^{5/2}e\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}
\end{aligned}
$$

output
```
-1/5*a*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/x^5/(b*x^2+a)^(1/2)-1/15*(a*c*f+a*d
*e+4*b*c*e)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/c/e/x^3/(b*x^2+a)^(1/2)+1/15*(
b^2*c*e-3*a*b*(c*f+d*e)+2*a^2*(d^2*e/c-d*f+c*f^2/e))*(d*x^2+c)^(1/2)*(f*x^
2+e)^(1/2)/a/c/e/x/(b*x^2+a)^(1/2)+2/15*(-a*d+b*c)^(1/2)*(b^2*c^2*e^2-a*b*
c*e*(c*f+d*e)+a^2*(c^2*f^2-c*d*e*f+d^2*e^2))*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/
e/(b*x^2+a))^(1/2)*EllipticE((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2),(c
*(-a*f+b*e)/(-a*d+b*c)/e)^(1/2))/a^2/c^(5/2)/e/(a*(d*x^2+c)/c/(b*x^2+a))^(
1/2)/(f*x^2+e)^(1/2)+1/15*(-a*d+b*c)^(1/2)*(-c*f+d*e)*(a*c*f-2*a*d*e+b*c*e
)*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)*EllipticF((-a*d+b*c)^(1/
2)*x/c^(1/2)/(b*x^2+a)^(1/2),(c*(-a*f+b*e)/(-a*d+b*c)/e)^(1/2))/a/c^(5/2)/
e/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(f*x^2+e)^(1/2)
```

Mathematica [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{x^6}\,dx=\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{x^6}\,dx$$

input
```
Integrate[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/x^6,x]
```

output
```
Integrate[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/x^6, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{x^6}\,dx$$
$$\Big\downarrow \ 450$$
$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{x^6}dx$$

input
```
Int[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/x^6,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [F]

$$\int \frac{\sqrt{b\,x^{2}+a}\,\sqrt{x^{2} d+c}\,\sqrt{f\,x^{2}+e}}{x^{6}}dx$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/x^6,x)
```

output
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/x^6,x)
```

Fricas [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{x^6}\,dx=\int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{fx^2+e}}{x^6}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/x^6,x, algorithm
="fricas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/x^6, x)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{x^6}\,dx = \int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{x^6}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(1/2)*(f*x**2+e)**(1/2)/x**6,x)
```

output
```
Integral(sqrt(a + b*x**2)*sqrt(c + d*x**2)*sqrt(e + f*x**2)/x**6, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{x^6}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{fx^2+e}}{x^6}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/x^6,x, algorithm
="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/x^6, x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{x^6}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{fx^2+e}}{x^6}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/x^6,x, algorithm
="giac")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{x^6}\,dx=\int \frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}}{x^6}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(1/2))/x^6,x)
```

output
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(1/2))/x^6, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{x^6}\,dx=\int \frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}}{x^6}dx$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/x^6,x)
```

output
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/x^6,x)
```

3.308 $\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{x^8}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 821

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{x^8}\,dx$$
$$= -\frac{a\sqrt{c+dx^2}\sqrt{e+fx^2}}{7x^7\sqrt{a+bx^2}} - \frac{(6bce+ade+acf)\sqrt{c+dx^2}\sqrt{e+fx^2}}{35cex^5\sqrt{a+bx^2}}$$
$$+ \frac{\left(b^2ce-5ab(de+cf)+2a^2\left(\frac{2d^2e}{c}-df+\frac{2cf^2}{e}\right)\right)\sqrt{c+dx^2}\sqrt{e+fx^2}}{105acex^3\sqrt{a+bx^2}}$$
$$- \frac{(4b^3c^3e^3-3ab^2c^2e^2(de+cf)-a^2bce(9d^2e^2-8cdef+9c^2f^2)+a^3(8d^3e^3-5cd^2e^2f-5c^2def^2+8c^3 \ldots}{105a^2c^3e^3x\sqrt{a+bx^2}}$$
$$- \frac{\sqrt{bc-ad}(8b^3c^3e^3-5ab^2c^2e^2(de+cf)-a^2bce(5d^2e^2-6cdef+5c^2f^2)+a^3(8d^3e^3-5cd^2e^2f-5c^2de \ldots}{105a^3c^{7/2}e^2\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$
$$- \frac{\sqrt{bc-ad}(de-cf)\left(4b^2c^2e^2+abce(de-2cf)-a^2(8d^2e^2-cdef-4c^2f^2)\right)\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,\text{Ellipt} \ldots}{105a^2c^{7/2}e^2\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$

output
```
-1/7*a*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/x^7/(b*x^2+a)^(1/2)-1/35*(a*c*f+a*d
*e+6*b*c*e)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/c/e/x^5/(b*x^2+a)^(1/2)+1/105*
(b^2*c*e-5*a*b*(c*f+d*e)+2*a^2*(2*d^2*e/c-d*f+2*c*f^2/e))*(d*x^2+c)^(1/2)*
(f*x^2+e)^(1/2)/a/c/e/x^3/(b*x^2+a)^(1/2)-1/105*(4*b^3*c^3*e^3-3*a*b^2*c^2
*e^2*(c*f+d*e)-a^2*b*c*e*(9*c^2*f^2-8*c*d*e*f+9*d^2*e^2)+a^3*(8*c^3*f^3-5*
c^2*d*e*f^2-5*c*d^2*e^2*f+8*d^3*e^3))*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/a^2/
c^3/e^3/x/(b*x^2+a)^(1/2)-1/105*(-a*d+b*c)^(1/2)*(8*b^3*c^3*e^3-5*a*b^2*c^
2*e^2*(c*f+d*e)-a^2*b*c*e*(5*c^2*f^2-6*c*d*e*f+5*d^2*e^2)+a^3*(8*c^3*f^3-5
*c^2*d*e*f^2-5*c*d^2*e^2*f+8*d^3*e^3))*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x
^2+a))^(1/2)*EllipticE((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2),(c*(-a*f
+b*e)/(-a*d+b*c)/e)^(1/2))/a^3/c^(7/2)/e^2/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
/(f*x^2+e)^(1/2)-1/105*(-a*d+b*c)^(1/2)*(-c*f+d*e)*(4*b^2*c^2*e^2+a*b*c*e*
(-2*c*f+d*e)-a^2*(-4*c^2*f^2-c*d*e*f+8*d^2*e^2))*(d*x^2+c)^(1/2)*(a*(f*x^2
+e)/e/(b*x^2+a))^(1/2)*EllipticF((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2
),(c*(-a*f+b*e)/(-a*d+b*c)/e)^(1/2))/a^2/c^(7/2)/e^2/(a*(d*x^2+c)/c/(b*x^2
+a))^(1/2)/(f*x^2+e)^(1/2)
```

Mathematica [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{x^8}\,dx=\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{x^8}\,dx$$

input
```
Integrate[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/x^8,x]
```

output
```
Integrate[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/x^8, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{x^8}\,dx$$

$$\Big\downarrow \; 450$$

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{x^8}\,dx$$

input
```
Int[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/x^8,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [F]

$$\int \frac{\sqrt{b\,x^{2}+a}\,\sqrt{x^{2} d+c}\,\sqrt{f\,x^{2}+e}}{x^{8}}d x$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/x^8,x)
```

output
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/x^8,x)
```

Fricas [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{x^8}\,dx=\int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{fx^2+e}}{x^8}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/x^8,x, algorithm
="fricas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/x^8, x)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{x^8}\,dx = \int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{x^8}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(1/2)*(f*x**2+e)**(1/2)/x**8,x)
```

output
```
Integral(sqrt(a + b*x**2)*sqrt(c + d*x**2)*sqrt(e + f*x**2)/x**8, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{x^8}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{fx^2+e}}{x^8}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/x^8,x, algorithm
="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/x^8, x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{x^8}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{fx^2+e}}{x^8}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/x^8,x, algorithm
="giac")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/x^8, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{x^8}\,dx = \int \frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}}{x^8}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(1/2))/x^8,x)
```

output
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(1/2))/x^8, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{x^8}\,dx = \int \frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}}{x^8}dx$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/x^8,x)
```

output
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/x^8,x)
```

3.309 $\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{x^{10}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 1126

$$\begin{aligned}
&\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{x^{10}}\,dx \\
&= -\frac{a\sqrt{c+dx^2}\sqrt{e+fx^2}}{9x^9\sqrt{a+bx^2}} - \frac{(8bce+ade+acf)\sqrt{c+dx^2}\sqrt{e+fx^2}}{63cex^7\sqrt{a+bx^2}} \\
&+ \frac{\left(b^2ce-7ab(de+cf)+2a^2\left(\frac{3d^2e}{c}-df+\frac{3cf^2}{e}\right)\right)\sqrt{c+dx^2}\sqrt{e+fx^2}}{315acex^5\sqrt{a+bx^2}} \\
&- \frac{(2b^3c^3e^3-ab^2c^2e^2(de+cf)-a^2bce(9d^2e^2-4cdef+9c^2f^2)+a^3(8d^3e^3-3cd^2e^2f-3c^2def^2+8c^3f^3 \cdots}{315a^2c^3e^3x^3\sqrt{a+bx^2}} \\
&+ \frac{(8b^4c^4e^4-5ab^3c^3e^3(de+cf)-a^2b^2c^2e^2(3d^2e^2-4cdef+3c^2f^2)-a^3bce(16d^3e^3-9cd^2e^2f-9c^2def^2 \cdots}{315a^3c^4e^4x\sqrt{a+ \cdots}} \\
&+ \frac{2\sqrt{bc-ad}(8b^4c^4e^4-4ab^3c^3e^3(de+cf)-3a^2b^2c^2e^2(d^2e^2-cdef+c^2f^2)-a^3bce(4d^3e^3-3cd^2e^2f- \cdots}{315 \cdots} \\
&+ \frac{\sqrt{bc-ad}(de-cf)\left(8b^3c^3e^3-3a^2bc^3ef^2+3ab^2c^2e^2(de-cf)-a^3(16d^3e^3-3c^2def^2-8c^3f^3)\right)\sqrt{c+ \cdots}}{315a^3c^{9/2}e^3\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}
\end{aligned}$$

output
```
-1/9*a*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/x^9/(b*x^2+a)^(1/2)-1/63*(a*c*f+a*d
*e+8*b*c*e)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/c/e/x^7/(b*x^2+a)^(1/2)+1/315*
(b^2*c*e-7*a*b*(c*f+d*e)+2*a^2*(3*d^2*e/c-d*f+3*c*f^2/e))*(d*x^2+c)^(1/2)*
(f*x^2+e)^(1/2)/a/c/e/x^5/(b*x^2+a)^(1/2)-1/315*(2*b^3*c^3*e^3-a*b^2*c^2*e
^2*(c*f+d*e)-a^2*b*c*e*(9*c^2*f^2-4*c*d*e*f+9*d^2*e^2)+a^3*(8*c^3*f^3-3*c^
2*d*e*f^2-3*c*d^2*e^2*f+8*d^3*e^3))*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/a^2/c^
3/e^3/x^3/(b*x^2+a)^(1/2)+1/315*(8*b^4*c^4*e^4-5*a*b^3*c^3*e^3*(c*f+d*e)-a
^2*b^2*c^2*e^2*(3*c^2*f^2-4*c*d*e*f+3*d^2*e^2)-a^3*b*c*e*(16*c^3*f^3-9*c^2
*d*e*f^2-9*c*d^2*e^2*f+16*d^3*e^3)+2*a^4*(8*c^4*f^4-4*c^3*d*e*f^3-3*c^2*d^
2*e^2*f^2-4*c*d^3*e^3*f+8*d^4*e^4))*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/a^3/c^
4/e^4/x/(b*x^2+a)^(1/2)+2/315*(-a*d+b*c)^(1/2)*(8*b^4*c^4*e^4-4*a*b^3*c^3*
e^3*(c*f+d*e)-3*a^2*b^2*c^2*e^2*(c^2*f^2-c*d*e*f+d^2*e^2)-a^3*b*c*e*(4*c^3
*f^3-3*c^2*d*e*f^2-3*c*d^2*e^2*f+4*d^3*e^3)+a^4*(8*c^4*f^4-4*c^3*d*e*f^3-3
*c^2*d^2*e^2*f^2-4*c*d^3*e^3*f+8*d^4*e^4))*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/
(b*x^2+a))^(1/2)*EllipticE((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2),(c*(
-a*f+b*e)/(-a*d+b*c)/e)^(1/2))/a^4/c^(9/2)/e^3/(a*(d*x^2+c)/c/(b*x^2+a))^(
1/2)/(f*x^2+e)^(1/2)+1/315*(-a*d+b*c)^(1/2)*(-c*f+d*e)*(8*b^3*c^3*e^3-3*a^
2*b*c^3*e*f^2+3*a*b^2*c^2*e^2*(-c*f+d*e)-a^3*(-8*c^3*f^3-3*c^2*d*e*f^2+16*
d^3*e^3))*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)*EllipticF((-a*d+
b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2),(c*(-a*f+b*e)/(-a*d+b*c)/e)^(1/2))...
```

Mathematica [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{x^{10}}\,dx = \int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{x^{10}}\,dx$$

input
```
Integrate[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/x^10,x]
```

output
```
Integrate[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/x^10, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{x^{10}}\,dx$$

$$\Big\downarrow \; 450$$

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{x^{10}}dx$$

input
```
Int[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/x^10,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [F]

$$\int \frac{\sqrt{b\,x^{2}+a}\,\sqrt{x^{2} d+c}\,\sqrt{f\,x^{2}+e}}{x^{10}}dx$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/x^10,x)
```

output
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/x^10,x)
```

Fricas [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{x^{10}}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{fx^2+e}}{x^{10}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/x^10,x, algorith
m="fricas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/x^10, x)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{x^{10}}\,dx = \int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{x^{10}}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(1/2)*(f*x**2+e)**(1/2)/x**10,x)
```

output
```
Integral(sqrt(a + b*x**2)*sqrt(c + d*x**2)*sqrt(e + f*x**2)/x**10, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{x^{10}}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{fx^2+e}}{x^{10}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/x^10,x, algorith
m="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/x^10, x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{x^{10}}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{fx^2+e}}{x^{10}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/x^10,x, algorith
m="giac")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/x^10, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{x^{10}}\,dx = \int \frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}}{x^{10}}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(1/2))/x^10,x)
```

output
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(1/2))/x^10, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{x^{10}}\,dx = \int \frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}}{x^{10}}dx$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/x^10,x)
```

output
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/x^10,x)
```

3.310 $\int x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 1544

$$\int x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}\,dx=\text{Too large to display}$$

output
```
-1/3840*(105*a^4*d^4*f^4-10*a^3*b*d^3*f^3*(4*c*f+19*d*e)+2*a^2*b^2*d^2*f^2
*(-17*c^2*f^2+47*c*d*e*f+18*d^2*e^2)+2*a*b^3*d*f*(-20*c^3*f^3+47*c^2*d*e*f
^2-18*c*d^2*e^2*f+15*d^3*e^3)-b^4*(-105*c^4*f^4+190*c^3*d*e*f^3-36*c^2*d^2
*e^2*f^2-30*c*d^3*e^3*f+45*d^4*e^4))*x*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/b^3
/d^4/f^3/(b*x^2+a)^(1/2)+1/1920*(35*a^3*d^3*f^3-a^2*b*d^2*f^2*(11*c*f+61*d
*e)+a*b^2*d*f*(-11*c^2*f^2+26*c*d*e*f+9*d^2*e^2)-b^3*(-35*c^3*f^3+61*c^2*d
*e*f^2-9*c*d^2*e^2*f+15*d^3*e^3))*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2
+e)^(1/2)/b^3/d^3/f^2-1/480*(7*a^2*d*f^2/b-2*a*f*(c*f+6*d*e)-b*(3*d*e^2+12
*c*e*f-7*c^2*f^2/d))*x^3*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/b
/d/f+1/80*(a*d*f+b*c*f+11*b*d*e)*x^5*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^
2+e)^(1/2)/b/d+1/10*f*x^7*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)+
1/3840*(-a*d+b*c)^(1/2)*e*(105*a^4*d^4*f^4-10*a^3*b*d^3*f^3*(4*c*f+19*d*e)
+2*a^2*b^2*d^2*f^2*(-17*c^2*f^2+47*c*d*e*f+18*d^2*e^2)+2*a*b^3*d*f*(-20*c^
3*f^3+47*c^2*d*e*f^2-18*c*d^2*e^2*f+15*d^3*e^3)-b^4*(-105*c^4*f^4+190*c^3*
d*e*f^3-36*c^2*d^2*e^2*f^2-30*c*d^3*e^3*f+45*d^4*e^4))*(d*x^2+c)^(1/2)*(a*
(f*x^2+e)/e/(b*x^2+a))^(1/2)*EllipticE((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a
)^(1/2),(c*(-a*f+b*e)/(-a*d+b*c)/e)^(1/2))/b^4/c^(1/2)/d^4/f^3/(a*(d*x^2+c
)/c/(b*x^2+a))^(1/2)/(f*x^2+e)^(1/2)+1/3840*a*(-a*d+b*c)^(1/2)*(105*a^4*d^
4*f^4+40*a^2*b^2*d^3*e*f^2*(-c*f+7*d*e)-30*a^3*b*d^3*f^3*(-c*f+11*d*e)-2*a
*b^3*d*f*(15*c^3*f^3-31*c^2*d*e*f^2+13*c*d^2*e^2*f+3*d^3*e^3)+b^4*(-105...
```

Mathematica [F]

$$\int x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}\,dx=\int x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}\,dx$$

input
```
Integrate[x^4*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^(3/2),x]
```

output
```
Integrate[x^4*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^(3/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}\,dx$$

$$\Big\downarrow \text{ 450}$$

$$\int x^4 \sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}dx$$

input
```
Int[x^4*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^(3/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [F]

$$\int x^4 \sqrt{b\,x^2+a}\,\sqrt{x^2 d+c}\,\left(f\,x^2+e\right)^{\frac{3}{2}}dx$$

input
```
int(x^4*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(3/2),x)
```

output
```
int(x^4*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(3/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}\,dx=\text{Timed out}$$

input
```
integrate(x^4*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(3/2),x, algorithm
="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}\,dx=\int x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x**4*(b*x**2+a)**(1/2)*(d*x**2+c)**(1/2)*(f*x**2+e)**(3/2),x)
```

output
```
Integral(x**4*sqrt(a + b*x**2)*sqrt(c + d*x**2)*(e + f*x**2)**(3/2), x)
```

Maxima [F]

$$\int x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}\,dx=\int \sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^{\frac{3}{2}}x^4\,dx$$

input
```
integrate(x^4*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(3/2),x, algorithm
="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)*x^4, x)
```

Giac [F]

$$\int x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}\,dx=\int \sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^{\frac{3}{2}}x^4\,dx$$

input
```
integrate(x^4*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(3/2),x, algorithm
="giac")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)*x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}\,dx=\int x^4\,\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,{\left(f\,x^2+e\right)}^{3/2}\,dx$$

input
```
int(x^4*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(3/2),x)
```

output
```
int(x^4*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(3/2), x)
```

Reduce [F]

$$\int x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}\,dx=\int x^4\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,\left(f\,x^2+e\right)^{\frac{3}{2}}dx$$

input
```
int(x^4*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(3/2),x)
```

output
```
int(x^4*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(3/2),x)
```

3.311 $\int x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 1159

$$\int x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}\,dx=\text{Too large to display}$$

output
```
1/384*(15*a^3*d^3*f^3-a^2*b*d^2*f^2*(7*c*f+31*d*e)+a*b^2*d*f*(-7*c^2*f^2+2
2*c*d*e*f+9*d^2*e^2)-b^3*(-15*c^3*f^3+31*c^2*d*e*f^2-9*c*d^2*e^2*f+9*d^3*e
^3))*x*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/b^2/d^3/f^2/(b*x^2+a)^(1/2)-1/192*(
5*a^2*d*f^2/b-2*a*f*(c*f+5*d*e)-b*(3*d*e^2+10*c*e*f-5*c^2*f^2/d))*x*(b*x^2
+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/b/d/f+1/48*(a*d*f+b*c*f+9*b*d*e)
*x^3*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/b/d+1/8*f*x^5*(b*x^2+
a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)-1/384*(-a*d+b*c)^(1/2)*e*(15*a^3*
d^3*f^3-a^2*b*d^2*f^2*(7*c*f+31*d*e)+a*b^2*d*f*(-7*c^2*f^2+22*c*d*e*f+9*d^
2*e^2)-b^3*(-15*c^3*f^3+31*c^2*d*e*f^2-9*c*d^2*e^2*f+9*d^3*e^3))*(d*x^2+c)
^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)*EllipticE((-a*d+b*c)^(1/2)*x/c^(1/2
)/(b*x^2+a)^(1/2),(c*(-a*f+b*e)/(-a*d+b*c)/e)^(1/2))/b^3/c^(1/2)/d^3/f^2/(
a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(f*x^2+e)^(1/2)-1/384*a*(-a*d+b*c)^(1/2)*(1
5*a^3*d^3*f^3-3*a^2*b*d^2*f^2*(-c*f+17*d*e)+a*b^2*d*f*(-3*c^2*f^2+2*c*d*e*
f+49*d^2*e^2)+b^3*(-15*c^3*f^3+41*c^2*d*e*f^2-29*c*d^2*e^2*f+3*d^3*e^3))*(
d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)*EllipticF((-a*d+b*c)^(1/2)*
x/c^(1/2)/(b*x^2+a)^(1/2),(c*(-a*f+b*e)/(-a*d+b*c)/e)^(1/2))/b^4/c^(1/2)/d
^3/f/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(f*x^2+e)^(1/2)-1/128*a*(5*a^4*d^4*f^
4+4*a*b^3*d*f*(-c*f+d*e)^3-4*a^3*b*d^3*f^3*(c*f+3*d*e)-b^4*(-c*f+d*e)^3*(5
*c*f+3*d*e)+2*a^2*b^2*d^2*f^2*(-c^2*f^2+6*c*d*e*f+3*d^2*e^2))*(d*x^2+c)^(1
/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)*EllipticPi((-a*d+b*c)^(1/2)*x/c^(1/...
```

Mathematica [F]

$$\int x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}\,dx=\int x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}\,dx$$

input
```
Integrate[x^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^(3/2),x]
```

output
```
Integrate[x^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^(3/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \sqrt{a+bx^2} \sqrt{c+dx^2} \left(e+fx^2\right)^{3/2} \, dx$$

$$\Big\downarrow \; 450$$

$$\int x^2 \sqrt{a+bx^2} \sqrt{c+dx^2} \left(e+fx^2\right)^{3/2} dx$$

input
```
Int[x^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^(3/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [F]

$$\int x^2 \sqrt{b\,x^2+a}\,\sqrt{x^2 d+c}\,\left(f\,x^2+e\right)^{\frac{3}{2}}dx$$

input
```
int(x^2*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(3/2),x)
```

output
```
int(x^2*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(3/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}\,dx=\text{Timed out}$$

input
```
integrate(x^2*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(3/2),x, algorithm
="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}\,dx=\int x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x**2*(b*x**2+a)**(1/2)*(d*x**2+c)**(1/2)*(f*x**2+e)**(3/2),x)
```

output
```
Integral(x**2*sqrt(a + b*x**2)*sqrt(c + d*x**2)*(e + f*x**2)**(3/2), x)
```

Maxima [F]

$$\int x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}\,dx=\int\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^{\frac{3}{2}}x^2\,dx$$

input
```
integrate(x^2*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(3/2),x, algorithm
="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)*x^2, x)
```

Giac [F]

$$\int x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}\,dx=\int \sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^{\frac{3}{2}}x^2\,dx$$

input
```
integrate(x^2*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(3/2),x, algorithm
="giac")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}\,dx=\int x^2\,\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,{\left(f\,x^2+e\right)}^{3/2}\,dx$$

input
```
int(x^2*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(3/2),x)
```

output
```
int(x^2*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(3/2), x)
```

Reduce [F]

$$\int x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}\,dx=\int x^2\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,\left(f\,x^2+e\right)^{\frac{3}{2}}dx$$

input
```
int(x^2*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(3/2),x)
```

output
```
int(x^2*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(3/2),x)
```

3.312 $\int \sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 850

$$\int \sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)^{3/2}\,dx = \frac{\left(8ae+\frac{8bce}{d}+\frac{3be^2}{f}-\frac{3a^2f}{b}-\frac{3bc^2f}{d^2}+\frac{2acf}{d}\right)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{48\sqrt{a+bx^2}} + \frac{(7bde+bcf+adf)x\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{24bd} + \frac{1}{6}fx^3\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2} + \frac{\sqrt{bc-ad}e(3a^2d^2f^2-2abdf(4de+cf)-b^2(3d^2e^2+8cdef-3c^2f^2))\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}E\left(\arcsin\left(\frac{\sqrt{bc\cdots}}{\sqrt{c}\sqrt{\cdots}}\right.\right.}{48b^2\sqrt{c}d^2f\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}} - \frac{a\sqrt{bc-ad}(12abd^2ef-3a^2d^2f^2-b^2(17d^2e^2+10cdef-3c^2f^2))\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\operatorname{EllipticF}\left(\arcsin\left(\cdots\right.\right.}{48b^3\sqrt{c}d^2\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}} + \frac{a(a^3d^3f^3-b^3(de-cf)^3-a^2bd^2f^2(3de+cf)+ab^2df(3d^2e^2+6cdef-c^2f^2))\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\text{Ellipt}\cdots}{16b^3\sqrt{c}d^2\sqrt{bc-ad}f\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$

output
```
1/48*(8*a*e+8*b*c*e/d+3*b*e^2/f-3*a^2*f/b-3*b*c^2*f/d^2+2*a*c*f/d)*x*(d*x^
2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^(1/2)+1/24*(a*d*f+b*c*f+7*b*d*e)*x*(b
*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/b/d+1/6*f*x^3*(b*x^2+a)^(1/2
)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)+1/48*(-a*d+b*c)^(1/2)*e*(3*a^2*d^2*f^2-2
*a*b*d*f*(c*f+4*d*e)-b^2*(-3*c^2*f^2+8*c*d*e*f+3*d^2*e^2))*(d*x^2+c)^(1/2)
*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)*EllipticE((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x
^2+a)^(1/2),(c*(-a*f+b*e)/(-a*d+b*c)/e)^(1/2))/b^2/c^(1/2)/d^2/f/(a*(d*x^2
+c)/c/(b*x^2+a))^(1/2)/(f*x^2+e)^(1/2)-1/48*a*(-a*d+b*c)^(1/2)*(12*a*b*d^2
*e*f-3*a^2*d^2*f^2-b^2*(-3*c^2*f^2+10*c*d*e*f+17*d^2*e^2))*(d*x^2+c)^(1/2)
*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)*EllipticF((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x
^2+a)^(1/2),(c*(-a*f+b*e)/(-a*d+b*c)/e)^(1/2))/b^3/c^(1/2)/d^2/(a*(d*x^2+c
)/c/(b*x^2+a))^(1/2)/(f*x^2+e)^(1/2)+1/16*a*(a^3*d^3*f^3-b^3*(-c*f+d*e)^3-
a^2*b*d^2*f^2*(c*f+3*d*e)+a*b^2*d*f*(-c^2*f^2+6*c*d*e*f+3*d^2*e^2))*(d*x^2
+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)*EllipticPi((-a*d+b*c)^(1/2)*x/c^
(1/2)/(b*x^2+a)^(1/2),b*c/(-a*d+b*c),(c*(-a*f+b*e)/(-a*d+b*c)/e)^(1/2))/b^
3/c^(1/2)/d^2/(-a*d+b*c)^(1/2)/f/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(f*x^2+e)
^(1/2)
```

Mathematica [F]

$$\int \sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}\,dx=\int \sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}\,dx$$

input
```
Integrate[Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^(3/2),x]
```

output
```
Integrate[Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^(3/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}\,dx$$

$$\Big\downarrow \text{434}$$

$$\int \sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}\,dx$$

input
```
Int[Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^(3/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \sqrt{b\,x^{2}+a}\,\sqrt{x^{2}d+c}\,\left(f\,x^{2}+e\right)^{\frac{3}{2}}d x$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(3/2),x)
```

output
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(3/2),x)
```

Fricas [F]

$$\int \sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}\,dx=\int \sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(3/2),x, algorithm="fr
icas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2), x)
```

Sympy [F]

$$\int \sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}\,dx=\int \sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(1/2)*(f*x**2+e)**(3/2),x)
```

output
```
Integral(sqrt(a + b*x**2)*sqrt(c + d*x**2)*(e + f*x**2)**(3/2), x)
```

Maxima [F]

$$\int \sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}\,dx=\int \sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(3/2),x, algorithm="ma
xima")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2), x)
```

Giac [F]

$$\int \sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}\,dx=\int \sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(3/2),x, algorithm="gi
ac")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}\,dx=\int \sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,{\left(f\,x^2+e\right)}^{3/2}\,dx$$

input
```
int((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(3/2),x)
```

output
```
int((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(3/2), x)
```

Reduce [F]

$$\int \sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}\,dx=\int \sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,\left(f\,x^2+e\right)^{\frac{3}{2}}dx$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(3/2),x)
```

output
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(3/2),x)
```

3.313 $\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}}{x^2}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 709

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)^{3/2}}{x^2}\,dx = \frac{(13bde+bcf+adf)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{8d\sqrt{a+bx^2}}$$
$$-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{x}+\frac{1}{4}fx\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}$$
$$-\frac{\sqrt{bc-ad}e(13bde+bcf+adf)\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}E\left(\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)|\frac{c(be-af)}{(bc-ad)e}\right)}{8b\sqrt{c}d\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$
$$+\frac{\sqrt{bc-ad}(8b^2de^2-a^2df^2+abf(7de+cf))\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right),\frac{c(be-af)}{(bc-ad)e}\right)}{8b^2\sqrt{c}d\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$
$$-\frac{a(a^2d^2f^2-2abdf(3de+cf)-b^2(3d^2e^2+6cdef-c^2f^2))\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,\text{EllipticPi}\left(\frac{bc}{bc-ad},\arcsin\left(\right.\right.}{8b^2\sqrt{c}d\sqrt{bc-ad}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$

output
```
1/8*(a*d*f+b*c*f+13*b*d*e)*x*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/d/(b*x^2+a)^(
1/2)-e*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/x+1/4*f*x*(b*x^2+a)
^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)-1/8*(-a*d+b*c)^(1/2)*e*(a*d*f+b*c*f
+13*b*d*e)*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)*EllipticE((-a*d
+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2),(c*(-a*f+b*e)/(-a*d+b*c)/e)^(1/2))/b
/c^(1/2)/d/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(f*x^2+e)^(1/2)+1/8*(-a*d+b*c)^
(1/2)*(8*b^2*d*e^2-a^2*d*f^2+a*b*f*(c*f+7*d*e))*(d*x^2+c)^(1/2)*(a*(f*x^2+
e)/e/(b*x^2+a))^(1/2)*EllipticF((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2)
,(c*(-a*f+b*e)/(-a*d+b*c)/e)^(1/2))/b^2/c^(1/2)/d/(a*(d*x^2+c)/c/(b*x^2+a)
)^(1/2)/(f*x^2+e)^(1/2)-1/8*a*(a^2*d^2*f^2-2*a*b*d*f*(c*f+3*d*e)-b^2*(-c^2
*f^2+6*c*d*e*f+3*d^2*e^2))*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)
*EllipticPi((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2),b*c/(-a*d+b*c),(c*(
-a*f+b*e)/(-a*d+b*c)/e)^(1/2))/b^2/c^(1/2)/d/(-a*d+b*c)^(1/2)/(a*(d*x^2+c)
/c/(b*x^2+a))^(1/2)/(f*x^2+e)^(1/2)
```

Mathematica [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)^{3/2}}{x^2}\,dx = \int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)^{3/2}}{x^2}\,dx$$

input
```
Integrate[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^(3/2))/x^2,x]
```

output
```
Integrate[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^(3/2))/x^2, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}}{x^2}\,dx$$

$$\Big\downarrow \text{450}$$

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}}{x^2}dx$$

input
```
Int[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^(3/2))/x^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [F]

$$\int \frac{\sqrt{b\,x^{2}+a}\,\sqrt{x^{2} d+c}\,\left(f\,x^{2}+e\right)^{\frac{3}{2}}}{x^{2}}d x$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(3/2)/x^2,x)
```

output
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(3/2)/x^2,x)
```

Fricas [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)^{3/2}}{x^2}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(3/2)/x^2,x, algorithm
="fricas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)/x^2, x)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)^{3/2}}{x^2}\,dx = \int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(1/2)*(f*x**2+e)**(3/2)/x**2,x)
```

output
```
Integral(sqrt(a + b*x**2)*sqrt(c + d*x**2)*(e + f*x**2)**(3/2)/x**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)^{3/2}}{x^2}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(3/2)/x^2,x, algorithm
="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)/x^2, x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)^{3/2}}{x^2}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(3/2)/x^2,x, algorithm
="giac")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)^{3/2}}{x^2}\,dx = \int \frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,(f\,x^2+e)^{3/2}}{x^2}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(3/2))/x^2,x)
```

output
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(3/2))/x^2, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)^{3/2}}{x^2}\,dx = \int \frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,(f\,x^2+e)^{\frac{3}{2}}}{x^2}dx$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(3/2)/x^2,x)
```

output
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(3/2)/x^2,x)
```

3.314 $\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)^{3/2}}{x^4}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 651

$$\begin{aligned}\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)^{3/2}}{x^4}\,dx &= -\frac{e(2bce+2ade+11acf)\sqrt{a+bx^2}\sqrt{c+dx^2}}{6acx\sqrt{e+fx^2}} \\ &- \frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{3x^3} + \frac{f\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{2x} \\ &- \frac{\sqrt{-be+af}(2bce+2ade+11acf)\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}E\left(\arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right)|\frac{a(de-cf)}{c(be-af)}\right)}{6a^{3/2}\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}} \\ &+ \frac{\sqrt{-be+af}(2bce+7ade+6acf)\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right),\frac{a(de-cf)}{c(be-af)}\right)}{6a^{3/2}\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}} \\ &+ \frac{e(3bde+bcf+adf)\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\text{EllipticPi}\left(-\frac{af}{be-af},\arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right),\frac{a(de-cf)}{c(be-af)}\right)}{2\sqrt{a}\sqrt{-be+af}\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}\end{aligned}$$

output
```
-1/6*e*(11*a*c*f+2*a*d*e+2*b*c*e)*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a/c/x/(f
*x^2+e)^(1/2)-1/3*e*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/x^3+1/
2*f*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/x-1/6*(a*f-b*e)^(1/2)*
(11*a*c*f+2*a*d*e+2*b*c*e)*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)
*EllipticE((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f
+b*e))^(1/2))/a^(3/2)/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)+1/6*
(a*f-b*e)^(1/2)*(6*a*c*f+7*a*d*e+2*b*c*e)*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(
f*x^2+e))^(1/2)*EllipticF((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c
*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(3/2)/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2
+e))^(1/2)+1/2*e*(a*d*f+b*c*f+3*b*d*e)*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x
^2+e))^(1/2)*EllipticPi((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),-a*f/(-a
*f+b*e),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(1/2)/(a*f-b*e)^(1/2)/(d*x^2+
c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)
```

Mathematica [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)^{3/2}}{x^4}\,dx = \int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)^{3/2}}{x^4}\,dx$$

input
```
Integrate[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^(3/2))/x^4,x]
```

output
```
Integrate[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^(3/2))/x^4, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)^{3/2}}{x^4}\,dx$$

$$\Big\downarrow \text{450}$$

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)^{3/2}}{x^4}dx$$

input
```
Int[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^(3/2))/x^4,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [F]

$$\int \frac{\sqrt{b\,x^{2}+a}\,\sqrt{x^{2} d+c}\,\left(f\,x^{2}+e\right)^{\frac{3}{2}}}{x^{4}}d x$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(3/2)/x^4,x)
```

output
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(3/2)/x^4,x)
```

Fricas [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)^{3/2}}{x^4}\,dx=\int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(3/2)/x^4,x, algorithm
="fricas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)/x^4, x)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)^{3/2}}{x^4}\,dx = \int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(1/2)*(f*x**2+e)**(3/2)/x**4,x)
```

output
```
Integral(sqrt(a + b*x**2)*sqrt(c + d*x**2)*(e + f*x**2)**(3/2)/x**4, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)^{3/2}}{x^4}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(3/2)/x^4,x, algorithm
="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)/x^4, x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)^{3/2}}{x^4}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(3/2)/x^4,x, algorithm
="giac")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)^{3/2}}{x^4}\,dx = \int \frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,(f\,x^2+e)^{3/2}}{x^4}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(3/2))/x^4,x)
```

output
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(3/2))/x^4, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)^{3/2}}{x^4}\,dx = \int \frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,(f\,x^2+e)^{\frac{3}{2}}}{x^4}dx$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(3/2)/x^4,x)
```

output
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(3/2)/x^4,x)
```

3.315 $\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)^{3/2}}{x^6}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 760

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)^{3/2}}{x^6}\,dx = \frac{(2b^2c^2e^2-abce(2de+7cf)+a^2(2d^2e^2-7cdef-3c^2f^2))\sqrt{a+b}}{15a^2c^2x\sqrt{e+fx^2}}$$
$$-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{5x^5}-\frac{(bce+ade+6acf)\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{15acx^3}$$
$$+\frac{\sqrt{-be+af}(2b^2c^2e^2-abce(2de+7cf)+a^2(2d^2e^2-7cdef-3c^2f^2))\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}E\left(\arcsin\left(\frac{\sqrt{-b}}{\sqrt{a}\sqrt{}}\right.\right.}{15a^{5/2}ce\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$
$$-\frac{\sqrt{-be+af}(2b^2c^2e+a^2d(de-9cf)-3abc(de+2cf))\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{-be+afx}}{\sqrt{a}\sqrt{e+fx^2}}\right)\right.}{15a^{5/2}c\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$
$$+\frac{bdef\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\operatorname{EllipticPi}\left(-\frac{af}{be-af},\arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right),\frac{a(de-cf)}{c(be-af)}\right)}{\sqrt{a}\sqrt{-be+af}\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$

output
```
1/15*(2*b^2*c^2*e^2-a*b*c*e*(7*c*f+2*d*e)+a^2*(-3*c^2*f^2-7*c*d*e*f+2*d^2*
e^2))*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a^2/c^2/x/(f*x^2+e)^(1/2)-1/5*e*(b*x
^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/x^5-1/15*(6*a*c*f+a*d*e+b*c*e)
*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/a/c/x^3+1/15*(a*f-b*e)^(1
/2)*(2*b^2*c^2*e^2-a*b*c*e*(7*c*f+2*d*e)+a^2*(-3*c^2*f^2-7*c*d*e*f+2*d^2*e
^2))*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticE((a*f-b*e)^(
1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(5/2)/
c/e/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)-1/15*(a*f-b*e)^(1/2)*(
2*b^2*c^2*e+a^2*d*(-9*c*f+d*e)-3*a*b*c*(2*c*f+d*e))*(b*x^2+a)^(1/2)*(e*(d*
x^2+c)/c/(f*x^2+e))^(1/2)*EllipticF((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1
/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(5/2)/c/(d*x^2+c)^(1/2)/(e*(b*x^2
+a)/a/(f*x^2+e))^(1/2)+b*d*e*f*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(
1/2)*EllipticPi((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),-a*f/(-a*f+b*e),
(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(1/2)/(a*f-b*e)^(1/2)/(d*x^2+c)^(1/2)
/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)
```

Mathematica [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)^{3/2}}{x^6}\,dx = \int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)^{3/2}}{x^6}\,dx$$

input
```
Integrate[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^(3/2))/x^6,x]
```

output
```
Integrate[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^(3/2))/x^6, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)^{3/2}}{x^6}\,dx$$

$$\Big\downarrow \text{ 450}$$

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}}{x^6}dx$$

input
```
Int[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^(3/2))/x^6,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [F]

$$\int \frac{\sqrt{b\,x^{2}+a}\,\sqrt{x^{2}d+c}\,\left(f\,x^{2}+e\right)^{\frac{3}{2}}}{x^{6}}dx$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(3/2)/x^6,x)
```

output
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(3/2)/x^6,x)
```

Fricas [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)^{3/2}}{x^6}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^{\frac{3}{2}}}{x^6}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(3/2)/x^6,x, algorithm
="fricas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)/x^6, x)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)^{3/2}}{x^6}\,dx = \int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)^{\frac{3}{2}}}{x^6}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(1/2)*(f*x**2+e)**(3/2)/x**6,x)
```

output
```
Integral(sqrt(a + b*x**2)*sqrt(c + d*x**2)*(e + f*x**2)**(3/2)/x**6, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)^{3/2}}{x^6}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^{\frac{3}{2}}}{x^6}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(3/2)/x^6,x, algorithm
="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)/x^6, x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)^{3/2}}{x^6}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^{\frac{3}{2}}}{x^6}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(3/2)/x^6,x, algorithm
="giac")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)^{3/2}}{x^6}\,dx = \int \frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,(f\,x^2+e)^{3/2}}{x^6}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(3/2))/x^6,x)
```

output
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(3/2))/x^6, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)^{3/2}}{x^6}\,dx = \int \frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,(f\,x^2+e)^{\frac{3}{2}}}{x^6}dx$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(3/2)/x^6,x)
```

output
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(3/2)/x^6,x)
```

3.316 $\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}}{x^8}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 803

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}}{x^8}\,dx =$$
$$-\frac{(bce+ade-2acf)\left(8b^2c^2e^2-abce(13de+3cf)+a^2(8d^2e^2-3cdef+3c^2f^2)\right)\sqrt{a+bx^2}\sqrt{c+dx^2}}{105a^3c^3ex\sqrt{e+fx^2}}$$
$$-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{7x^7}-\frac{f\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{2x^5}$$
$$-\frac{(2bce+2ade-19acf)\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{70acx^5}$$
$$+\frac{(4b^2c^2e^2-abce(2de+9cf)+a^2(4d^2e^2-9cdef-3c^2f^2))\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{105a^2c^2ex^3}$$
$$-\frac{\sqrt{-be+af}(bce+ade-2acf)\left(8b^2c^2e^2-abce(13de+3cf)+a^2(8d^2e^2-3cdef+3c^2f^2)\right)\sqrt{a+bx^2}\sqrt{\frac{e(\ldots}{c(\ldots}}}{105a^{7/2}c^2e^2\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$
$$+\frac{(bc-ad)\sqrt{-be+af}\left(8b^2c^2e^2-abce(de+15cf)-a^2(4d^2e^2-9cdef-3c^2f^2)\right)\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,\text{Ellipt}\ldots}{105a^{7/2}c^2e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$

output
```
-1/105*(-2*a*c*f+a*d*e+b*c*e)*(8*b^2*c^2*e^2-a*b*c*e*(3*c*f+13*d*e)+a^2*(3
*c^2*f^2-3*c*d*e*f+8*d^2*e^2))*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a^3/c^3/e/x
/(f*x^2+e)^(1/2)-1/7*e*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/x^7
-1/2*f*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/x^5-1/70*(-19*a*c*f
+2*a*d*e+2*b*c*e)*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/a/c/x^5+
1/105*(4*b^2*c^2*e^2-a*b*c*e*(9*c*f+2*d*e)+a^2*(-3*c^2*f^2-9*c*d*e*f+4*d^2
*e^2))*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/a^2/c^2/e/x^3-1/105
*(a*f-b*e)^(1/2)*(-2*a*c*f+a*d*e+b*c*e)*(8*b^2*c^2*e^2-a*b*c*e*(3*c*f+13*d
*e)+a^2*(3*c^2*f^2-3*c*d*e*f+8*d^2*e^2))*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f
*x^2+e))^(1/2)*EllipticE((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*
f+d*e)/c/(-a*f+b*e))^(1/2))/a^(7/2)/c^2/e^2/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a
/(f*x^2+e))^(1/2)+1/105*(-a*d+b*c)*(a*f-b*e)^(1/2)*(8*b^2*c^2*e^2-a*b*c*e*
(15*c*f+d*e)-a^2*(-3*c^2*f^2-9*c*d*e*f+4*d^2*e^2))*(b*x^2+a)^(1/2)*(e*(d*x
^2+c)/c/(f*x^2+e))^(1/2)*EllipticF((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/
2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(7/2)/c^2/e/(d*x^2+c)^(1/2)/(e*(b*
x^2+a)/a/(f*x^2+e))^(1/2)
```

Mathematica [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)^{3/2}}{x^8}\,dx=\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)^{3/2}}{x^8}\,dx$$

input
```
Integrate[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^(3/2))/x^8,x]
```

output
```
Integrate[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^(3/2))/x^8, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}}{x^8}\,dx$$

$$\Big\downarrow \ 450$$

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}}{x^8}dx$$

input
```
Int[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^(3/2))/x^8,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [F]

$$\int \frac{\sqrt{b\,x^{2}+a}\,\sqrt{x^{2} d+c}\,\left(f\,x^{2}+e\right)^{\frac{3}{2}}}{x^{8}}d x$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(3/2)/x^8,x)
```

output
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(3/2)/x^8,x)
```

Fricas [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)^{3/2}}{x^8}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^{\frac{3}{2}}}{x^8}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(3/2)/x^8,x, algorithm
="fricas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)/x^8, x)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)^{3/2}}{x^8}\,dx = \int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{\frac{3}{2}}}{x^8}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(1/2)*(f*x**2+e)**(3/2)/x**8,x)
```

output
```
Integral(sqrt(a + b*x**2)*sqrt(c + d*x**2)*(e + f*x**2)**(3/2)/x**8, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)^{3/2}}{x^8}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}\,(fx^2+e)^{\frac{3}{2}}}{x^8}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(3/2)/x^8,x, algorithm
="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)/x^8, x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)^{3/2}}{x^8}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}\,(fx^2+e)^{\frac{3}{2}}}{x^8}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(3/2)/x^8,x, algorithm
="giac")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)/x^8, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)^{3/2}}{x^8}\,dx = \int \frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,(f\,x^2+e)^{3/2}}{x^8}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(3/2))/x^8,x)
```

output
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(3/2))/x^8, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)^{3/2}}{x^8}\,dx = \int \frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,(f\,x^2+e)^{\frac{3}{2}}}{x^8}dx$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(3/2)/x^8,x)
```

output
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(3/2)/x^8,x)
```

3.317 $\int \frac{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 884

$$
\begin{aligned}
&\int \frac{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx \\
&= \frac{\left(2ac-\frac{3bc^2}{d}-\frac{3a^2d}{b}+\frac{15bde^2}{f^2}-\frac{4bce}{f}-\frac{4ade}{f}\right)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{48df\sqrt{a+bx^2}} \\
&\quad-\frac{(5bde+3bcf-adf)x\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{24bdf^2} \\
&\quad+\frac{x\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}}{6df} \\
&\quad+\frac{\sqrt{bc-ad}e\left(3a^2d^2f^2+2abdf(2de-cf)-b^2(15d^2e^2-4cdef-3c^2f^2)\right)\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}E\left(\arcsin\left(\cdots\right.\right.}{48b^2\sqrt{c}d^2f^3\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}} \\
&\quad+\frac{a\sqrt{bc-ad}\left(3a^2d^2f^2+b^2(5d^2e^2-2cdef-3c^2f^2)\right)\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right),\cdots\right.}{48b^3\sqrt{c}d^2f^2\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}} \\
&\quad+\frac{a\left(a^3d^3f^3+a^2bd^2f^2(de-cf)+ab^2df(3d^2e^2-2cdef-c^2f^2)-b^3(5d^3e^3-3cd^2e^2f-c^2def^2-c^3f^3)\right)\cdots}{16b^3\sqrt{c}d^2\sqrt{bc-ad}f^3\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+\cdots}}
\end{aligned}
$$

output
```
1/48*(2*a*c-3*b*c^2/d-3*a^2*d/b+15*b*d*e^2/f^2-4*b*c*e/f-4*a*d*e/f)*x*(d*x
^2+c)^(1/2)*(f*x^2+e)^(1/2)/d/f/(b*x^2+a)^(1/2)-1/24*(-a*d*f+3*b*c*f+5*b*d
*e)*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/b/d/f^2+1/6*x*(b*x^2
+a)^(1/2)*(d*x^2+c)^(3/2)*(f*x^2+e)^(1/2)/d/f+1/48*(-a*d+b*c)^(1/2)*e*(3*a
^2*d^2*f^2+2*a*b*d*f*(-c*f+2*d*e)-b^2*(-3*c^2*f^2-4*c*d*e*f+15*d^2*e^2))*(
d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)*EllipticE((-a*d+b*c)^(1/2)*
x/c^(1/2)/(b*x^2+a)^(1/2),(c*(-a*f+b*e)/(-a*d+b*c)/e)^(1/2))/b^2/c^(1/2)/d
^2/f^3/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(f*x^2+e)^(1/2)+1/48*a*(-a*d+b*c)^(
1/2)*(3*a^2*d^2*f^2+b^2*(-3*c^2*f^2-2*c*d*e*f+5*d^2*e^2))*(d*x^2+c)^(1/2)*
(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)*EllipticF((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^
2+a)^(1/2),(c*(-a*f+b*e)/(-a*d+b*c)/e)^(1/2))/b^3/c^(1/2)/d^2/f^2/(a*(d*x^
2+c)/c/(b*x^2+a))^(1/2)/(f*x^2+e)^(1/2)+1/16*a*(a^3*d^3*f^3+a^2*b*d^2*f^2*
(-c*f+d*e)+a*b^2*d*f*(-c^2*f^2-2*c*d*e*f+3*d^2*e^2)-b^3*(-c^3*f^3-c^2*d*e*
f^2-3*c*d^2*e^2*f+5*d^3*e^3))*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1
/2)*EllipticPi((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2),b*c/(-a*d+b*c),(
c*(-a*f+b*e)/(-a*d+b*c)/e)^(1/2))/b^3/c^(1/2)/d^2/(-a*d+b*c)^(1/2)/f^3/(a*
(d*x^2+c)/c/(b*x^2+a))^(1/2)/(f*x^2+e)^(1/2)
```

Mathematica [F]

$$\int \frac{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx=\int \frac{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx$$

input
```
Integrate[(x^4*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/Sqrt[e + f*x^2],x]
```

output
```
Integrate[(x^4*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/Sqrt[e + f*x^2], x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx$$

$$\downarrow 450$$

$$\int \frac{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}dx$$

input
```
Int[(x^4*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/Sqrt[e + f*x^2],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [F]

$$\int \frac{x^4\sqrt{b\,x^2+a}\,\sqrt{x^2d+c}}{\sqrt{f\,x^2+e}}dx$$

input
```
int(x^4*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

output
```
int(x^4*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx = \text{Timed out}$$

input
```
integrate(x^4*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm
="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx = \int \frac{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx$$

input
```
integrate(x**4*(b*x**2+a)**(1/2)*(d*x**2+c)**(1/2)/(f*x**2+e)**(1/2),x)
```

output
```
Integral(x**4*sqrt(a + b*x**2)*sqrt(c + d*x**2)/sqrt(e + f*x**2), x)
```

Maxima [F]

$$\int \frac{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}x^4}{\sqrt{fx^2+e}}\,dx$$

input
```
integrate(x^4*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm
="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x^4/sqrt(f*x^2 + e), x)
```

Giac [F]

$$\int \frac{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}x^4}{\sqrt{fx^2+e}}\,dx$$

input
```
integrate(x^4*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm
="giac")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x^4/sqrt(f*x^2 + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx=\int \frac{x^4\,\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{\sqrt{f\,x^2+e}}\,dx$$

input
```
int((x^4*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2))/(e + f*x^2)^(1/2),x)
```

output
```
int((x^4*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2))/(e + f*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx=\text{too large to display}$$

input
```
int(x^4*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

output

```
(sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*d*f*x + sqrt(e + f*x
**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b*c*f*x - 5*sqrt(e + f*x**2)*sqrt(c
 + d*x**2)*sqrt(a + b*x**2)*b*d*e*x + 4*sqrt(e + f*x**2)*sqrt(c + d*x**2)*
sqrt(a + b*x**2)*b*d*f*x**3 - 3*int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqr
t(a + b*x**2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*
x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*a**2*d**2*f**2 + 2*int((sq
rt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c*e + a*c*f*x**2
 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*
x**6),x)*a*b*c*d*f**2 - 4*int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a +
b*x**2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 +
 b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*a*b*d**2*e*f - 3*int((sqrt(e + f
*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*e
*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)
*b**2*c**2*f**2 - 4*int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2
)*x**4)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f
*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*b**2*c*d*e*f + 15*int((sqrt(e + f*x**2
)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*e*x**2
 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*b**2
*d**2*e**2 - 2*int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**
2)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x...
```

3.318 $\int \frac{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 672

$$\begin{aligned}
&\int \frac{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx \\
&= -\frac{(3bde-bcf-adf)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{8df^2\sqrt{a+bx^2}} + \frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{4f} \\
&\quad + \frac{\sqrt{bc-ad}e(3bde-bcf-adf)\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}E\left(\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)|\frac{c(be-af)}{(bc-ad)e}\right)}{8b\sqrt{c}df^2\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}} \\
&\quad - \frac{a\sqrt{bc-ad}(bde-bcf+adf)\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right),\frac{c(be-af)}{(bc-ad)e}\right)}{8b^2\sqrt{c}df\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}} \\
&\quad - \frac{a(a^2d^2f^2+2abdf(de-cf)-b^2(3d^2e^2-2cdef-c^2f^2))\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,\text{EllipticPi}\left(\frac{bc}{bc-ad},\arcsin\right.}{8b^2\sqrt{c}d\sqrt{bc-ad}f^2\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}
\end{aligned}$$

output
```
-1/8*(-a*d*f-b*c*f+3*b*d*e)*x*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/d/f^2/(b*x^2
+a)^(1/2)+1/4*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/f+1/8*(-a*
d+b*c)^(1/2)*e*(-a*d*f-b*c*f+3*b*d*e)*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^
2+a))^(1/2)*EllipticE((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2),(c*(-a*f+
b*e)/(-a*d+b*c)/e)^(1/2))/b/c^(1/2)/d/f^2/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/
(f*x^2+e)^(1/2)-1/8*a*(-a*d+b*c)^(1/2)*(a*d*f-b*c*f+b*d*e)*(d*x^2+c)^(1/2)
*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)*EllipticF((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x
^2+a)^(1/2),(c*(-a*f+b*e)/(-a*d+b*c)/e)^(1/2))/b^2/c^(1/2)/d/f/(a*(d*x^2+c
)/c/(b*x^2+a))^(1/2)/(f*x^2+e)^(1/2)-1/8*a*(a^2*d^2*f^2+2*a*b*d*f*(-c*f+d*
e)-b^2*(-c^2*f^2-2*c*d*e*f+3*d^2*e^2))*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x
^2+a))^(1/2)*EllipticPi((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2),b*c/(-a
*d+b*c),(c*(-a*f+b*e)/(-a*d+b*c)/e)^(1/2))/b^2/c^(1/2)/d/(-a*d+b*c)^(1/2)/
f^2/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(f*x^2+e)^(1/2)
```

Mathematica [F]

$$\int \frac{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx = \int \frac{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx$$

input
```
Integrate[(x^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/Sqrt[e + f*x^2],x]
```

output
```
Integrate[(x^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/Sqrt[e + f*x^2], x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx$$

$$\Big\downarrow \; 450$$

$$\int \frac{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}dx$$

input
```
Int[(x^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/Sqrt[e + f*x^2],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [F]

$$\int \frac{x^2\sqrt{b\,x^2+a}\,\sqrt{x^2d+c}}{\sqrt{f\,x^2+e}}dx$$

input
```
int(x^2*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

output
```
int(x^2*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx=\text{Timed out}$$

input
```
integrate(x^2*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm
="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx = \int \frac{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx$$

input
```
integrate(x**2*(b*x**2+a)**(1/2)*(d*x**2+c)**(1/2)/(f*x**2+e)**(1/2),x)
```

output
```
Integral(x**2*sqrt(a + b*x**2)*sqrt(c + d*x**2)/sqrt(e + f*x**2), x)
```

Maxima [F]

$$\int \frac{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}x^2}{\sqrt{fx^2+e}}\,dx$$

input
```
integrate(x^2*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm
="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x^2/sqrt(f*x^2 + e), x)
```

Giac [F]

$$\int \frac{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}x^2}{\sqrt{fx^2+e}}\,dx$$

input
```
integrate(x^2*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm
="giac")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x^2/sqrt(f*x^2 + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx = \int \frac{x^2\,\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{\sqrt{f\,x^2+e}}\,dx$$

input
```
int((x^2*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2))/(e + f*x^2)^(1/2),x)
```

output
```
int((x^2*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2))/(e + f*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx$$
$$= \frac{\sqrt{f\,x^2+e}\,\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x+\left(\int \frac{\sqrt{f\,x^2+e}\,\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^4}{bdf\,x^6+adf\,x^4+bcf\,x^4+bde\,x^4+acf\,x^2+ade\,x^2+bce\,x^2+ace}dx\right)adf+\left(\int \frac{}{bdf\,x^6+a}\right.}{}$$

input
```
int(x^2*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

output
```
(sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*x + int((sqrt(e + f*x*
*2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*e*x*
*2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*a*
d*f + int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c*e
 + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*
x**4 + b*d*f*x**6),x)*b*c*f - 3*int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqr
t(a + b*x**2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*
x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*b*d*e + 2*int((sqrt(e + f*
x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c*e + a*c*f*x**2 + a*d*e*
x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*
a*c*f - 2*int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a
*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*
d*e*x**4 + b*d*f*x**6),x)*a*d*e - 2*int((sqrt(e + f*x**2)*sqrt(c + d*x**2)
*sqrt(a + b*x**2)*x**2)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*
c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*b*c*e - int((sqrt(e +
f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c*e + a*c*f*x**2 + a*d*e*x**
2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*a*c
*e)/(4*f)
```

3.319 $\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 530

$$\begin{aligned}
&\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx \\
&= \frac{bx\sqrt{c+dx^2}\sqrt{e+fx^2}}{2f\sqrt{a+bx^2}} - \frac{\sqrt{bc-ad}e\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}E\left(\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)|\frac{c(be-af)}{(bc-ad)e}\right)}{2\sqrt{c}f\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}} \\
&+ \frac{a\sqrt{bc-ad}\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right),\frac{c(be-af)}{(bc-ad)e}\right)}{2b\sqrt{c}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}} \\
&- \frac{a(bde-bcf-adf)\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,\text{EllipticPi}\left(\frac{bc}{bc-ad},\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right),\frac{c(be-af)}{(bc-ad)e}\right)}{2b\sqrt{c}\sqrt{bc-ad}f\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}
\end{aligned}$$

output
```
1/2*b*x*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/f/(b*x^2+a)^(1/2)-1/2*(-a*d+b*c)^(
1/2)*e*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)*EllipticE((-a*d+b*c
)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2),(c*(-a*f+b*e)/(-a*d+b*c)/e)^(1/2))/c^(1/
2)/f/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(f*x^2+e)^(1/2)+1/2*a*(-a*d+b*c)^(1/2
)*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)*EllipticF((-a*d+b*c)^(1/
2)*x/c^(1/2)/(b*x^2+a)^(1/2),(c*(-a*f+b*e)/(-a*d+b*c)/e)^(1/2))/b/c^(1/2)/
(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(f*x^2+e)^(1/2)-1/2*a*(-a*d*f-b*c*f+b*d*e)
*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)*EllipticPi((-a*d+b*c)^(1/
2)*x/c^(1/2)/(b*x^2+a)^(1/2),b*c/(-a*d+b*c),(c*(-a*f+b*e)/(-a*d+b*c)/e)^(1
/2))/b/c^(1/2)/(-a*d+b*c)^(1/2)/f/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(f*x^2+e
)^(1/2)
```

Mathematica [A] (verified)

Time = 3.31 (sec) , antiderivative size = 503, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx = \frac{\frac{x\sqrt{a+bx^2}(c+dx^2)}{\sqrt{e+fx^2}} - \frac{\sqrt{c}\sqrt{-de+cf}\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,E\left(\arcsin\left(\frac{\sqrt{-de+cf}\,x}{\sqrt{c}\sqrt{e+fx^2}}\right)|\frac{bce-acf}{ade-acf}\right)}{f\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}} + \frac{(be-2af)(de-cf)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}\,\text{Ellip}}{\sqrt{e}f^2\sqrt{be-af}\sqrt{}}}{2\sqrt{c+d}}$$

input
```
Integrate[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/Sqrt[e + f*x^2],x]
```

output
```
((x*Sqrt[a + b*x^2]*(c + d*x^2))/Sqrt[e + f*x^2] - (Sqrt[c]*Sqrt[-(d*e) +
c*f]*Sqrt[a + b*x^2]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*EllipticE[ArcSi
n[(Sqrt[-(d*e) + c*f]*x)/(Sqrt[c]*Sqrt[e + f*x^2])], (b*c*e - a*c*f)/(a*d*
e - a*c*f)])/(f*Sqrt[(e*(a + b*x^2))/(a*(e + f*x^2))]) + ((b*e - 2*a*f)*(d
*e - c*f)*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]*Sqrt[e + f*x^2]*EllipticF[
ArcSin[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])], (b*c*e - a*d*e)/(b*
c*e - a*c*f)])/(Sqrt[e]*f^2*Sqrt[b*e - a*f]*Sqrt[(a*(e + f*x^2))/(e*(a + b
*x^2))]) + (e*(-(b*d*e) + b*c*f + a*d*f)*Sqrt[a + b*x^2]*Sqrt[(e*(c + d*x^
2))/(c*(e + f*x^2))]*EllipticPi[(a*f)/(-(b*e) + a*f), ArcSin[(Sqrt[-(b*e)
+ a*f]*x)/(Sqrt[a]*Sqrt[e + f*x^2])], (a*d*e - a*c*f)/(b*c*e - a*c*f)])/(S
qrt[a]*f^2*Sqrt[-(b*e) + a*f]*Sqrt[(e*(a + b*x^2))/(a*(e + f*x^2))]))/(2*S
qrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 545, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.206$, Rules used = {430, 427, 321, 428, 412, 429, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}dx$$

$$\Big\downarrow \text{430}$$

$$-\frac{c(de-cf)\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}\sqrt{fx^2+e}}dx}{2f}+\frac{bc(de-cf)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{fx^2+e}}dx}{2df}-\frac{(-adf-bcf+bde)\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\sqrt{fx^2+e}}dx}{2df}+\frac{dx\sqrt{a+bx^2}\sqrt{e+fx^2}}{2f\sqrt{c+dx^2}}$$

$$\Big\downarrow \text{427}$$

$$-\frac{c(de-cf)\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}\sqrt{fx^2+e}}dx}{2f}-\frac{(-adf-bcf+bde)\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\sqrt{fx^2+e}}dx}{2df}+\frac{b\sqrt{c+dx^2}(de-cf)\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\int \frac{1}{\sqrt{1-\frac{(bc-ad)x^2}{c\left(bx^2+a\right)}}\sqrt{1-\frac{(be-af)x^2}{e\left(bx^2+a\right)}}}d\frac{x}{\sqrt{bx^2+a}}}{2df\sqrt{e+fx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}+\frac{dx\sqrt{a+bx^2}\sqrt{e+fx^2}}{2f\sqrt{c+dx^2}}$$

$$\Big\downarrow \text{321}$$

$$-\frac{c(de-cf)\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}\sqrt{fx^2+e}}dx}{2f}-\frac{(-adf-bcf+bde)\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\sqrt{fx^2+e}}dx}{2df}+\frac{b\sqrt{e}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right),\frac{(bc-ad)e}{c(be-af)}\right)}{2df\sqrt{e+fx^2}\sqrt{be-af}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}+\frac{dx\sqrt{a+bx^2}\sqrt{e+fx^2}}{2f\sqrt{c+dx^2}}$$

$$\Big\downarrow \text{428}$$

$$-\frac{c(de-cf)\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}\sqrt{fx^2+e}}dx}{2f}-\frac{c\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}(-adf-bcf+bde)\int \frac{1}{\left(1-\frac{dx^2}{dx^2+c}\right)\sqrt{\frac{(bc-ad)x^2}{a(dx^2+c)}+1}\sqrt{1-\frac{(de-cf)x^2}{e(dx^2+c)}}}d\frac{x}{\sqrt{dx^2+c}}}{2adf\sqrt{e+fx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{b\sqrt{e}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right),\frac{(bc-ad)e}{c(be-af)}\right)}{2df\sqrt{e+fx^2}\sqrt{be-af}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}+\frac{dx\sqrt{a+bx^2}\sqrt{e+fx^2}}{2f\sqrt{c+dx^2}}$$

$\Big\downarrow$ 412

$$-\frac{c(de-cf)\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}\sqrt{fx^2+e}}dx}{2f}+\frac{b\sqrt{e}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right),\frac{(bc-ad)e}{c(be-af)}\right)}{2df\sqrt{e+fx^2}\sqrt{be-af}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}-\frac{c\sqrt{e}\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}(-adf-bcf+bde)\text{EllipticPi}\left(\frac{de}{de-cf},\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{dx^2+c}}\right),-\frac{(bc-ad)e}{a(de-cf)}\right)}{2adf\sqrt{e+fx^2}\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{dx\sqrt{a+bx^2}\sqrt{e+fx^2}}{2f\sqrt{c+dx^2}}$$

$\Big\downarrow$ 429

$$-\frac{\sqrt{a+bx^2}(de-cf)\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\int \frac{\sqrt{\frac{(bc-ad)x^2}{a(dx^2+c)}+1}}{\sqrt{1-\frac{(de-cf)x^2}{e(dx^2+c)}}}d\frac{x}{\sqrt{dx^2+c}}}{2f\sqrt{e+fx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{b\sqrt{e}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right),\frac{(bc-ad)e}{c(be-af)}\right)}{2df\sqrt{e+fx^2}\sqrt{be-af}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}-\frac{c\sqrt{e}\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}(-adf-bcf+bde)\text{EllipticPi}\left(\frac{de}{de-cf},\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{dx^2+c}}\right),-\frac{(bc-ad)e}{a(de-cf)}\right)}{2adf\sqrt{e+fx^2}\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{dx\sqrt{a+bx^2}\sqrt{e+fx^2}}{2f\sqrt{c+dx^2}}$$

$\Big\downarrow$ 327

$$\frac{b\sqrt{e}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right),\frac{(bc-ad)e}{c(be-af)}\right)}{2df\sqrt{e+fx^2}\sqrt{be-af}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}-\frac{\sqrt{e}\sqrt{a+bx^2}\sqrt{de-cf}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}E\left(\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{dx^2+c}}\right)|-\frac{(bc-ad)e}{a(de-cf)}\right)}{2f\sqrt{e+fx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{c\sqrt{e}\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}(-adf-bcf+bde)\,\text{EllipticPi}\left(\frac{de}{de-cf},\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{dx^2+c}}\right),-\frac{(bc-ad)e}{a(de-cf)}\right)}{2adf\sqrt{e+fx^2}\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{dx\sqrt{a+bx^2}\sqrt{e+fx^2}}{2f\sqrt{c+dx^2}}$$

input
```
Int[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/Sqrt[e + f*x^2],x]
```

output
```
(d*x*Sqrt[a + b*x^2]*Sqrt[e + f*x^2])/(2*f*Sqrt[c + d*x^2]) - (Sqrt[e]*Sqr
t[d*e - c*f]*Sqrt[a + b*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))]*Ellipti
cE[ArcSin[(Sqrt[d*e - c*f]*x)/(Sqrt[e]*Sqrt[c + d*x^2])], -(((b*c - a*d)*e
)/(a*(d*e - c*f)))])/(2*f*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[e + f
*x^2]) + (b*Sqrt[e]*(d*e - c*f)*Sqrt[c + d*x^2]*Sqrt[(a*(e + f*x^2))/(e*(a
 + b*x^2))]*EllipticF[ArcSin[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])
], ((b*c - a*d)*e)/(c*(b*e - a*f))])/(2*d*f*Sqrt[b*e - a*f]*Sqrt[(a*(c + d
*x^2))/(c*(a + b*x^2))]*Sqrt[e + f*x^2]) - (c*Sqrt[e]*(b*d*e - b*c*f - a*d
*f)*Sqrt[a + b*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))]*EllipticPi[(d*e)
/(d*e - c*f), ArcSin[(Sqrt[d*e - c*f]*x)/(Sqrt[e]*Sqrt[c + d*x^2])], -(((b
*c - a*d)*e)/(a*(d*e - c*f)))])/(2*a*d*f*Sqrt[d*e - c*f]*Sqrt[(c*(a + b*x^
2))/(a*(c + d*x^2))]*Sqrt[e + f*x^2])
```

Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 427
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.
)*(x_)^2]), x_Symbol] :> Simp[Sqrt[c + d*x^2]*(Sqrt[a*((e + f*x^2)/(e*(a +
b*x^2)))]/(c*Sqrt[e + f*x^2]*Sqrt[a*((c + d*x^2)/(c*(a + b*x^2)))]))   Subs
t[Int[1/(Sqrt[1 - (b*c - a*d)*(x^2/c)]*Sqrt[1 - (b*e - a*f)*(x^2/e)]), x],
x, x/Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 428
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/(Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*
(x_)^2]), x_Symbol] :> Simp[a*Sqrt[c + d*x^2]*(Sqrt[a*((e + f*x^2)/(e*(a +
b*x^2)))]/(c*Sqrt[e + f*x^2]*Sqrt[a*((c + d*x^2)/(c*(a + b*x^2)))]))   Subs
t[Int[1/((1 - b*x^2)*Sqrt[1 - (b*c - a*d)*(x^2/c)]*Sqrt[1 - (b*e - a*f)*(x^
2/e)]), x], x, x/Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 429
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)^(3/2)*Sqrt[(e_) + (f_.
)*(x_)^2]), x_Symbol] :> Simp[Sqrt[c + d*x^2]*(Sqrt[a*((e + f*x^2)/(e*(a +
b*x^2)))]/(a*Sqrt[e + f*x^2]*Sqrt[a*((c + d*x^2)/(c*(a + b*x^2)))]))   Subs
t[Int[Sqrt[1 - (b*c - a*d)*(x^2/c)]/Sqrt[1 - (b*e - a*f)*(x^2/e)], x], x, x
/Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 430
```
Int[(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2])/Sqrt[(e_) + (f_.)
*(x_)^2], x_Symbol] :> Simp[d*x*Sqrt[a + b*x^2]*(Sqrt[e + f*x^2]/(2*f*Sqrt[
c + d*x^2])), x] + (-Simp[c*((d*e - c*f)/(2*f))   Int[Sqrt[a + b*x^2]/((c +
 d*x^2)^(3/2)*Sqrt[e + f*x^2]), x], x] - Simp[(b*d*e - b*c*f - a*d*f)/(2*d*
f)   Int[Sqrt[c + d*x^2]/(Sqrt[a + b*x^2]*Sqrt[e + f*x^2]), x], x] + Simp[b
*c*((d*e - c*f)/(2*d*f))   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[(d*e - c*f)/c]
```

Maple [F]

$$\int \frac{\sqrt{b\,x^{2}+a}\,\sqrt{x^{2} d +c}}{\sqrt{f\,x^{2}+e}}dx$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

output
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

Fricas [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx=\int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{\sqrt{fx^2+e}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="fr
icas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/sqrt(f*x^2 + e), x)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx = \int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(1/2)/(f*x**2+e)**(1/2),x)
```

output
```
Integral(sqrt(a + b*x**2)*sqrt(c + d*x**2)/sqrt(e + f*x**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{\sqrt{fx^2+e}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/sqrt(f*x^2 + e), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{\sqrt{fx^2+e}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/sqrt(f*x^2 + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx = \int \frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{\sqrt{f\,x^2+e}}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2))/(e + f*x^2)^(1/2),x)
```

output
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2))/(e + f*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx = \int \frac{\sqrt{f\,x^2+e}\,\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{f\,x^2+e}dx$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

output
```
int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2))/(e + f*x**2),x)
```

3.320 $\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^2\sqrt{e+fx^2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 503

$$\begin{aligned}
&\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^2\sqrt{e+fx^2}}\,dx \\
&= -\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x\sqrt{e+fx^2}} \\
&\quad - \frac{c\sqrt{-be+af}\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}E\left(\arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right)|\frac{a(de-cf)}{c(be-af)}\right)}{\sqrt{a}e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}} \\
&\quad + \frac{d\sqrt{-be+af}\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right),\frac{a(de-cf)}{c(be-af)}\right)}{\sqrt{a}f\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}} \\
&\quad + \frac{bde\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\operatorname{EllipticPi}\left(-\frac{af}{be-af},\arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right),\frac{a(de-cf)}{c(be-af)}\right)}{\sqrt{a}f\sqrt{-be+af}\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}
\end{aligned}$$

output
```
-(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x/(f*x^2+e)^(1/2)-c*(a*f-b*e)^(1/2)*(b*x^
2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticE((a*f-b*e)^(1/2)*x/a^(
1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(1/2)/e/(d*x^2+c
)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)+d*(a*f-b*e)^(1/2)*(b*x^2+a)^(1/2)*
(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticF((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2
+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(1/2)/f/(d*x^2+c)^(1/2)/(e*
(b*x^2+a)/a/(f*x^2+e))^(1/2)+b*d*e*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e
))^(1/2)*EllipticPi((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),-a*f/(-a*f+b
*e),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(1/2)/f/(a*f-b*e)^(1/2)/(d*x^2+c)
^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)
```

Mathematica [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^2\sqrt{e+fx^2}}\,dx=\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^2\sqrt{e+fx^2}}\,dx$$

input
```
Integrate[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(x^2*Sqrt[e + f*x^2]),x]
```

output
```
Integrate[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(x^2*Sqrt[e + f*x^2]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^2\sqrt{e+fx^2}}\,dx$$
$$\Big\downarrow \ 450$$
$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^2\sqrt{e+fx^2}}dx$$

input
```
Int[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(x^2*Sqrt[e + f*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [F]

$$\int \frac{\sqrt{b\,x^{2}+a}\,\sqrt{x^{2} d+c}}{x^{2} \sqrt{f\,x^{2}+e}}d x$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^2/(f*x^2+e)^(1/2),x)
```

output
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^2/(f*x^2+e)^(1/2),x)
```

Fricas [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^2\sqrt{e+fx^2}}\,dx=\int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{\sqrt{fx^2+e}x^2}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^2/(f*x^2+e)^(1/2),x, algorithm
="fricas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(f*x^4 + e*x^2),
x)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^2\sqrt{e+fx^2}}\,dx = \int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^2\sqrt{e+fx^2}}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(1/2)/x**2/(f*x**2+e)**(1/2),x)
```

output
```
Integral(sqrt(a + b*x**2)*sqrt(c + d*x**2)/(x**2*sqrt(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^2\sqrt{e+fx^2}}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{\sqrt{fx^2+e}x^2}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^2/(f*x^2+e)^(1/2),x, algorithm
="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/(sqrt(f*x^2 + e)*x^2), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^2\sqrt{e+fx^2}}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{\sqrt{fx^2+e}x^2}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^2/(f*x^2+e)^(1/2),x, algorithm
="giac")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/(sqrt(f*x^2 + e)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^2\sqrt{e+fx^2}}\,dx=\int \frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{x^2\,\sqrt{f\,x^2+e}}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2))/(x^2*(e + f*x^2)^(1/2)),x)
```

output
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2))/(x^2*(e + f*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^2\sqrt{e+fx^2}}\,dx=\int \frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{x^2\sqrt{f\,x^2+e}}dx$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^2/(f*x^2+e)^(1/2),x)
```

output
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^2/(f*x^2+e)^(1/2),x)
```

3.321 $\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^4\sqrt{e+fx^2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 425

$$
\begin{aligned}
\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^4\sqrt{e+fx^2}}\,dx
&= -\frac{\left(\frac{b}{a}+\frac{d}{c}-\frac{2f}{e}\right)\sqrt{a+bx^2}\sqrt{c+dx^2}}{3x\sqrt{e+fx^2}} - \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{3ex^3} \\
&\quad - \frac{\sqrt{-be+af}(bce+ade-2acf)\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}E\left(\arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right)|\frac{a(de-cf)}{c(be-af)}\right)}{3a^{3/2}e^2\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}} \\
&\quad + \frac{(bc-ad)\sqrt{-be+af}\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right),\frac{a(de-cf)}{c(be-af)}\right)}{3a^{3/2}e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}
\end{aligned}
$$

output

```
-1/3*(b/a+d/c-2*f/e)*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x/(f*x^2+e)^(1/2)-1/3
*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/e/x^3-1/3*(a*f-b*e)^(1/2)
*(-2*a*c*f+a*d*e+b*c*e)*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*El
lipticE((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*
e))^(1/2))/a^(3/2)/e^2/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)+1/3
*(-a*d+b*c)*(a*f-b*e)^(1/2)*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2
)*EllipticF((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*
f+b*e))^(1/2))/a^(3/2)/e/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)
```

Mathematica [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^4\sqrt{e+fx^2}}\,dx = \int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^4\sqrt{e+fx^2}}\,dx$$

input
```
Integrate[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(x^4*Sqrt[e + f*x^2]),x]
```

output
```
Integrate[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(x^4*Sqrt[e + f*x^2]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^4\sqrt{e+fx^2}}\,dx$$

$$\Big\downarrow \ 450$$

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^4\sqrt{e+fx^2}}dx$$

input
```
Int[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(x^4*Sqrt[e + f*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [F]

$$\int \frac{\sqrt{b\,x^{2}+a}\,\sqrt{x^{2} d +c}}{x^{4} \sqrt{f\,x^{2}+e}}d x$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^4/(f*x^2+e)^(1/2),x)
```

output
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^4/(f*x^2+e)^(1/2),x)
```

Fricas [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^4\sqrt{e+fx^2}}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{\sqrt{fx^2+e}x^4}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^4/(f*x^2+e)^(1/2),x, algorithm
="fricas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(f*x^6 + e*x^4),
x)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^4\sqrt{e+fx^2}}\,dx = \int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^4\sqrt{e+fx^2}}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(1/2)/x**4/(f*x**2+e)**(1/2),x)
```

output
```
Integral(sqrt(a + b*x**2)*sqrt(c + d*x**2)/(x**4*sqrt(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^4\sqrt{e+fx^2}}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{\sqrt{fx^2+e}x^4}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^4/(f*x^2+e)^(1/2),x, algorithm
="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/(sqrt(f*x^2 + e)*x^4), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^4\sqrt{e+fx^2}}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{\sqrt{fx^2+e}x^4}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^4/(f*x^2+e)^(1/2),x, algorithm
="giac")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/(sqrt(f*x^2 + e)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^4\sqrt{e+fx^2}}\,dx=\int \frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{x^4\,\sqrt{f\,x^2+e}}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2))/(x^4*(e + f*x^2)^(1/2)),x)
```

output
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2))/(x^4*(e + f*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^4\sqrt{e+fx^2}}\,dx=\int \frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{x^4\sqrt{f\,x^2+e}}dx$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^4/(f*x^2+e)^(1/2),x)
```

output
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^4/(f*x^2+e)^(1/2),x)
```

3.322 $\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^6\sqrt{e+fx^2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 593

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^6\sqrt{e+fx^2}}\,dx$$
$$= \frac{(2b^2c^2e^2-abce(2de-3cf)+a^2(2d^2e^2+3cdef-8c^2f^2))\sqrt{a+bx^2}\sqrt{c+dx^2}}{15a^2c^2e^2x\sqrt{e+fx^2}}$$
$$-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{5ex^5}-\frac{\left(\frac{b}{a}+\frac{d}{c}-\frac{4f}{e}\right)\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{15ex^3}$$
$$+\frac{\sqrt{-be+af}(2b^2c^2e^2-abce(2de-3cf)+a^2(2d^2e^2+3cdef-8c^2f^2))\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}E\left(\arcsin\left(\frac{\cdots}{\cdots}\right.\right.}{15a^{5/2}ce^3\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$
$$-\frac{(bc-ad)\sqrt{-be+af}(2bce-ade+4acf)\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-be+afx}}{\sqrt{a}\sqrt{e+fx^2}}\right),\frac{a(de-cf)}{c(be-af)}\right)}{15a^{5/2}ce^2\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$

output
```
1/15*(2*b^2*c^2*e^2-a*b*c*e*(-3*c*f+2*d*e)+a^2*(-8*c^2*f^2+3*c*d*e*f+2*d^2
*e^2))*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a^2/c^2/e^2/x/(f*x^2+e)^(1/2)-1/5*(
b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/e/x^5-1/15*(b/a+d/c-4*f/e)*
(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/e/x^3+1/15*(a*f-b*e)^(1/2)
*(2*b^2*c^2*e^2-a*b*c*e*(-3*c*f+2*d*e)+a^2*(-8*c^2*f^2+3*c*d*e*f+2*d^2*e^2
))*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticE((a*f-b*e)^(1/
2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(5/2)/c/
e^3/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)-1/15*(-a*d+b*c)*(a*f-b
*e)^(1/2)*(4*a*c*f-a*d*e+2*b*c*e)*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e)
)^(1/2)*EllipticF((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/
c/(-a*f+b*e))^(1/2))/a^(5/2)/c/e^2/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e
))^(1/2)
```

Mathematica [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^6\sqrt{e+fx^2}}\,dx = \int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^6\sqrt{e+fx^2}}\,dx$$

input
```
Integrate[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(x^6*Sqrt[e + f*x^2]),x]
```

output
```
Integrate[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(x^6*Sqrt[e + f*x^2]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^6\sqrt{e+fx^2}}\,dx$$
$$\Big\downarrow \ 450$$
$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^6\sqrt{e+fx^2}}dx$$

input
```
Int[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(x^6*Sqrt[e + f*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [F]

$$\int \frac{\sqrt{b\,x^{2}+a}\,\sqrt{x^{2} d+c}}{x^{6} \sqrt{f\,x^{2}+e}}dx$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^6/(f*x^2+e)^(1/2),x)
```

output
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^6/(f*x^2+e)^(1/2),x)
```

Fricas [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^6\sqrt{e+fx^2}}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{\sqrt{fx^2+e}x^6}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^6/(f*x^2+e)^(1/2),x, algorithm
="fricas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(f*x^8 + e*x^6),
x)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^6\sqrt{e+fx^2}}\,dx=\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^6\sqrt{e+fx^2}}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(1/2)/x**6/(f*x**2+e)**(1/2),x)
```

output
```
Integral(sqrt(a + b*x**2)*sqrt(c + d*x**2)/(x**6*sqrt(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^6\sqrt{e+fx^2}}\,dx=\int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{\sqrt{fx^2+e}x^6}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^6/(f*x^2+e)^(1/2),x, algorithm
="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/(sqrt(f*x^2 + e)*x^6), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^6\sqrt{e+fx^2}}\,dx=\int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{\sqrt{fx^2+e}x^6}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^6/(f*x^2+e)^(1/2),x, algorithm
="giac")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/(sqrt(f*x^2 + e)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^6\sqrt{e+fx^2}}\,dx=\int \frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{x^6\,\sqrt{f\,x^2+e}}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2))/(x^6*(e + f*x^2)^(1/2)),x)
```

output
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2))/(x^6*(e + f*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^6\sqrt{e+fx^2}}\,dx=\int \frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{x^6\sqrt{f\,x^2+e}}dx$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^6/(f*x^2+e)^(1/2),x)
```

output
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^6/(f*x^2+e)^(1/2),x)
```

3.323 $\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^8\sqrt{e+fx^2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 828

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^8\sqrt{e+fx^2}}\,dx = -\frac{(8b^3c^3e^3 - ab^2c^2e^2(5de-9cf) - a^2bce(5d^2e^2+8cdef-16c^2f^2) + a^3(8d^3e^3+9cd^2e^2f+16c^2def^2 -}{105a^3c^3e^3x\sqrt{e+fx^2}}$$
$$-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{7ex^7} - \frac{\left(\frac{b}{a}+\frac{d}{c}-\frac{6f}{e}\right)\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{35ex^5}$$
$$+\frac{\left(\frac{4b^2ce}{a}-2bde+\frac{4ad^2e}{c}+5bcf+5adf-\frac{24acf^2}{e}\right)\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{105ace^2x^3}$$
$$-\frac{\sqrt{-be+af}\,(8b^3c^3e^3-ab^2c^2e^2(5de-9cf)-a^2bce(5d^2e^2+8cdef-16c^2f^2)+a^3(8d^3e^3+9cd^2e^2f+}{105a^{7/2}c^2e^4\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$
$$+\frac{(bc-ad)\sqrt{-be+af}\,(8b^2c^2e^2-abce(de-13cf)-a^2(4d^2e^2+5cdef-24c^2f^2))\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,\mathrm{E}}{105a^{7/2}c^2e^3\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$

output
```
-1/105*(8*b^3*c^3*e^3-a*b^2*c^2*e^2*(-9*c*f+5*d*e)-a^2*b*c*e*(-16*c^2*f^2+
8*c*d*e*f+5*d^2*e^2)+a^3*(-48*c^3*f^3+16*c^2*d*e*f^2+9*c*d^2*e^2*f+8*d^3*e
^3))*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a^3/c^3/e^3/x/(f*x^2+e)^(1/2)-1/7*(b*
x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/e/x^7-1/35*(b/a+d/c-6*f/e)*(b
*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/e/x^5+1/105*(4*b^2*c*e/a-2*b
*d*e+4*a*d^2*e/c+5*b*c*f+5*a*d*f-24*a*c*f^2/e)*(b*x^2+a)^(1/2)*(d*x^2+c)^(
1/2)*(f*x^2+e)^(1/2)/a/c/e^2/x^3-1/105*(a*f-b*e)^(1/2)*(8*b^3*c^3*e^3-a*b^
2*c^2*e^2*(-9*c*f+5*d*e)-a^2*b*c*e*(-16*c^2*f^2+8*c*d*e*f+5*d^2*e^2)+a^3*(
-48*c^3*f^3+16*c^2*d*e*f^2+9*c*d^2*e^2*f+8*d^3*e^3))*(b*x^2+a)^(1/2)*(e*(d
*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticE((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(
1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(7/2)/c^2/e^4/(d*x^2+c)^(1/2)/(e
*(b*x^2+a)/a/(f*x^2+e))^(1/2)+1/105*(-a*d+b*c)*(a*f-b*e)^(1/2)*(8*b^2*c^2*
e^2-a*b*c*e*(-13*c*f+d*e)-a^2*(-24*c^2*f^2+5*c*d*e*f+4*d^2*e^2))*(b*x^2+a)
^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticF((a*f-b*e)^(1/2)*x/a^(1/2)
/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(7/2)/c^2/e^3/(d*x^2
+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)
```

Mathematica [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^8\sqrt{e+fx^2}}\,dx=\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^8\sqrt{e+fx^2}}\,dx$$

input
```
Integrate[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(x^8*Sqrt[e + f*x^2]),x]
```

output
```
Integrate[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(x^8*Sqrt[e + f*x^2]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^8\sqrt{e+fx^2}}\,dx$$

$$\Big\downarrow \; 450$$

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^8\sqrt{e+fx^2}}dx$$

input
```
Int[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(x^8*Sqrt[e + f*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [F]

$$\int \frac{\sqrt{b\,x^2+a}\,\sqrt{x^2d+c}}{x^8\sqrt{f\,x^2+e}}dx$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^8/(f*x^2+e)^(1/2),x)
```

output
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^8/(f*x^2+e)^(1/2),x)
```

Fricas [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^8\sqrt{e+fx^2}}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{\sqrt{fx^2+e}x^8}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^8/(f*x^2+e)^(1/2),x, algorithm
="fricas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(f*x^10 + e*x^8),
 x)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^8\sqrt{e+fx^2}}\,dx=\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^8\sqrt{e+fx^2}}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(1/2)/x**8/(f*x**2+e)**(1/2),x)
```

output
```
Integral(sqrt(a + b*x**2)*sqrt(c + d*x**2)/(x**8*sqrt(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^8\sqrt{e+fx^2}}\,dx=\int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{\sqrt{fx^2+e}x^8}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^8/(f*x^2+e)^(1/2),x, algorithm
="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/(sqrt(f*x^2 + e)*x^8), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^8\sqrt{e+fx^2}}\,dx=\int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{\sqrt{fx^2+e}x^8}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^8/(f*x^2+e)^(1/2),x, algorithm
="giac")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/(sqrt(f*x^2 + e)*x^8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^8\sqrt{e+fx^2}}\,dx = \int \frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{x^8\,\sqrt{f\,x^2+e}}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2))/(x^8*(e + f*x^2)^(1/2)),x)
```

output
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2))/(x^8*(e + f*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^8\sqrt{e+fx^2}}\,dx = \int \frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{x^8\sqrt{f\,x^2+e}}dx$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^8/(f*x^2+e)^(1/2),x)
```

output
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^8/(f*x^2+e)^(1/2),x)
```

3.324 $\int \frac{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 718

$$\int \frac{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx = \frac{5ex\sqrt{a+bx^2}\sqrt{c+dx^2}}{4f^2\sqrt{e+fx^2}}$$

$$+ \frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{4f\sqrt{e+fx^2}} - \frac{(15bde-bcf-adf)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{8df^3\sqrt{a+bx^2}}$$

$$+ \frac{\sqrt{bc-ad}e(15bde-bcf-adf)\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}E\left(\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)|\frac{c(be-af)}{(bc-ad)e}\right)}{8b\sqrt{c}df^3\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$

$$- \frac{a\sqrt{bc-ad}(5bde-bcf+adf)\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right),\frac{c(be-af)}{(bc-ad)e}\right)}{8b^2\sqrt{c}df^2\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$

$$- \frac{a(a^2d^2f^2+2abdf(3de-cf)-b^2(15d^2e^2-6cdef-c^2f^2))\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,\text{EllipticPi}\left(\frac{bc}{bc-ad},\arcsin\left(\right.\right.}{8b^2\sqrt{c}d\sqrt{bc-ad}f^3\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$

output
```
5/4*e*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/f^2/(f*x^2+e)^(1/2)+1/4*x^3*(b*x^2
+a)^(1/2)*(d*x^2+c)^(1/2)/f/(f*x^2+e)^(1/2)-1/8*(-a*d*f-b*c*f+15*b*d*e)*x*
(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/d/f^3/(b*x^2+a)^(1/2)+1/8*(-a*d+b*c)^(1/2)
*e*(-a*d*f-b*c*f+15*b*d*e)*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)
*EllipticE((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2),(c*(-a*f+b*e)/(-a*d+
b*c)/e)^(1/2))/b/c^(1/2)/d/f^3/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(f*x^2+e)^(
1/2)-1/8*a*(-a*d+b*c)^(1/2)*(a*d*f-b*c*f+5*b*d*e)*(d*x^2+c)^(1/2)*(a*(f*x^
2+e)/e/(b*x^2+a))^(1/2)*EllipticF((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/
2),(c*(-a*f+b*e)/(-a*d+b*c)/e)^(1/2))/b^2/c^(1/2)/d/f^2/(a*(d*x^2+c)/c/(b*
x^2+a))^(1/2)/(f*x^2+e)^(1/2)-1/8*a*(a^2*d^2*f^2+2*a*b*d*f*(-c*f+3*d*e)-b^
2*(-c^2*f^2-6*c*d*e*f+15*d^2*e^2))*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a
))^(1/2)*EllipticPi((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2),b*c/(-a*d+b
*c),(c*(-a*f+b*e)/(-a*d+b*c)/e)^(1/2))/b^2/c^(1/2)/d/(-a*d+b*c)^(1/2)/f^3/
(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(f*x^2+e)^(1/2)
```

Mathematica [F]

$$\int \frac{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx = \int \frac{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx$$

input
```
Integrate[(x^4*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(e + f*x^2)^(3/2),x]
```

output
```
Integrate[(x^4*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(e + f*x^2)^(3/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx$$

$$\Big\downarrow \; 450$$

$$\int \frac{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}dx$$

input
```
Int[(x^4*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(e + f*x^2)^(3/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [F]

$$\int \frac{x^{4} \sqrt{b\,x^{2}+a}\,\sqrt{x^{2} d +c}}{\left(f\,x^{2}+e \right)^{\frac{3}{2}}}d x$$

input
```
int(x^4*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x)
```

output
```
int(x^4*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x)
```

Fricas [F]

$$\int \frac{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}x^4}{(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm
="fricas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)*x^4/(f^2*x^4 + 2*
e*f*x^2 + e^2), x)
```

Sympy [F]

$$\int \frac{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx = \int \frac{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**4*(b*x**2+a)**(1/2)*(d*x**2+c)**(1/2)/(f*x**2+e)**(3/2),x)
```

output
```
Integral(x**4*sqrt(a + b*x**2)*sqrt(c + d*x**2)/(e + f*x**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}x^4}{(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm
="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x^4/(f*x^2 + e)^(3/2), x)
```

Giac [F]

$$\int \frac{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}x^4}{(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm
="giac")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x^4/(f*x^2 + e)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx = \int \frac{x^4\,\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{(f\,x^2+e)^{3/2}}\,dx$$

input
```
int((x^4*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2))/(e + f*x^2)^(3/2),x)
```

output
```
int((x^4*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2))/(e + f*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx = \int \frac{x^4\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{(f\,x^2+e)^{\frac{3}{2}}}dx$$

input
```
int(x^4*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x)
```

output
```
int(x^4*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x)
```

3.325 $\int \frac{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 573

$$\int \frac{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx = -\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}}{f\sqrt{e+fx^2}}+\frac{3bx\sqrt{c+dx^2}\sqrt{e+fx^2}}{2f^2\sqrt{a+bx^2}}$$
$$-\frac{3\sqrt{bc-ad}e\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}E\left(\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)|\frac{c(be-af)}{(bc-ad)e}\right)}{2\sqrt{c}f^2\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$
$$+\frac{a\sqrt{bc-ad}\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right),\frac{c(be-af)}{(bc-ad)e}\right)}{2b\sqrt{c}f\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$
$$-\frac{a(3bde-bcf-adf)\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\operatorname{EllipticPi}\left(\frac{bc}{bc-ad},\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right),\frac{c(be-af)}{(bc-ad)e}\right)}{2b\sqrt{c}\sqrt{bc-ad}f^2\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$

output
```
-x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/f/(f*x^2+e)^(1/2)+3/2*b*x*(d*x^2+c)^(1/
2)*(f*x^2+e)^(1/2)/f^2/(b*x^2+a)^(1/2)-3/2*(-a*d+b*c)^(1/2)*e*(d*x^2+c)^(1
/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)*EllipticE((-a*d+b*c)^(1/2)*x/c^(1/2)/(
b*x^2+a)^(1/2),(c*(-a*f+b*e)/(-a*d+b*c)/e)^(1/2))/c^(1/2)/f^2/(a*(d*x^2+c)
/c/(b*x^2+a))^(1/2)/(f*x^2+e)^(1/2)+1/2*a*(-a*d+b*c)^(1/2)*(d*x^2+c)^(1/2)
*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)*EllipticF((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x
^2+a)^(1/2),(c*(-a*f+b*e)/(-a*d+b*c)/e)^(1/2))/b/c^(1/2)/f/(a*(d*x^2+c)/c/
(b*x^2+a))^(1/2)/(f*x^2+e)^(1/2)-1/2*a*(-a*d*f-b*c*f+3*b*d*e)*(d*x^2+c)^(1
/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)*EllipticPi((-a*d+b*c)^(1/2)*x/c^(1/2)/
(b*x^2+a)^(1/2),b*c/(-a*d+b*c),(c*(-a*f+b*e)/(-a*d+b*c)/e)^(1/2))/b/c^(1/2
)/(-a*d+b*c)^(1/2)/f^2/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(f*x^2+e)^(1/2)
```

Mathematica [F]

$$\int \frac{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx=\int \frac{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx$$

input
```
Integrate[(x^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(e + f*x^2)^(3/2),x]
```

output
```
Integrate[(x^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(e + f*x^2)^(3/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx$$
$$\Big\downarrow \; 450$$
$$\int \frac{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}dx$$

input
```
Int[(x^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(e + f*x^2)^(3/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [F]

$$\int \frac{x^{2} \sqrt{b\,x^{2}+a}\,\sqrt{x^{2} d +c}}{\left(f\,x^{2}+e \right)^{\frac{3}{2}}}d x$$

input
```
int(x^2*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x)
```

output
```
int(x^2*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x)
```

Fricas [F]

$$\int \frac{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}x^2}{(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm
="fricas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)*x^2/(f^2*x^4 + 2*
e*f*x^2 + e^2), x)
```

Sympy [F]

$$\int \frac{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx = \int \frac{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2*(b*x**2+a)**(1/2)*(d*x**2+c)**(1/2)/(f*x**2+e)**(3/2),x)
```

output
```
Integral(x**2*sqrt(a + b*x**2)*sqrt(c + d*x**2)/(e + f*x**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}x^2}{(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm
="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x^2/(f*x^2 + e)^(3/2), x)
```

Giac [F]

$$\int \frac{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}x^2}{(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm
="giac")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x^2/(f*x^2 + e)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx = \int \frac{x^2\,\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{\left(f\,x^2+e\right)^{3/2}}\,dx$$

input
```
int((x^2*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2))/(e + f*x^2)^(3/2),x)
```

output
```
int((x^2*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2))/(e + f*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx = \int \frac{x^2\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{\left(f\,x^2+e\right)^{\frac{3}{2}}}dx$$

input
```
int(x^2*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x)
```

output
```
int(x^2*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x)
```

3.326 $\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{\left(e+fx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 481

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{\left(e+fx^2\right)^{3/2}}\,dx=\frac{\sqrt{a}\sqrt{-be+af}\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}E\left(\arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right)|\frac{a(de-cf)}{c(be-af)}\right)}{ef\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{\sqrt{a}b(de-cf)\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right),\frac{a(de-cf)}{c(be-af)}\right)}{cf^2\sqrt{-be+af}\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{\sqrt{a}bde\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}\operatorname{EllipticPi}\left(-\frac{af}{be-af},\arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right),\frac{a(de-cf)}{c(be-af)}\right)}{cf^2\sqrt{-be+af}\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$

output
```
a^(1/2)*(a*f-b*e)^(1/2)*(d*x^2+c)^(1/2)*(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)*El
lipticE((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*
e))^(1/2))/e/f/(b*x^2+a)^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)-a^(1/2)*b*(
-c*f+d*e)*(d*x^2+c)^(1/2)*(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)*EllipticF((a*f-b
*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/c/f
^2/(a*f-b*e)^(1/2)/(b*x^2+a)^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)+a^(1/2)
*b*d*e*(d*x^2+c)^(1/2)*(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)*EllipticPi((a*f-b*e
)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),-a*f/(-a*f+b*e),(a*(-c*f+d*e)/c/(-a*f+b*
e))^(1/2))/c/f^2/(a*f-b*e)^(1/2)/(b*x^2+a)^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))
^(1/2)
```

Mathematica [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{\left(e+fx^2\right)^{3/2}}\,dx=\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{\left(e+fx^2\right)^{3/2}}\,dx$$

input
```
Integrate[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(e + f*x^2)^(3/2),x]
```

output
```
Integrate[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(e + f*x^2)^(3/2), x]
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 748, normalized size of antiderivative = 1.56, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {432, 428, 412, 429, 324, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{\left(e+fx^2\right)^{3/2}}\,dx$$

$$\Big\downarrow \text{432}$$

$$\frac{b\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\sqrt{fx^2+e}}dx}{f}-\frac{(be-af)\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)^{3/2}}dx}{f}$$

$$\Big\downarrow \text{428}$$

$$\frac{bc\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\int \frac{1}{\left(1-\frac{dx^2}{dx^2+c}\right)\sqrt{\frac{(bc-ad)x^2}{a\left(dx^2+c\right)}+1}\sqrt{1-\frac{(de-cf)x^2}{e\left(dx^2+c\right)}}}d\frac{x}{\sqrt{dx^2+c}}}{af\sqrt{e+fx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{(be-af)\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\left(fx^2+e\right)^{3/2}}dx}{f}$$

$$\Big\downarrow \text{412}$$

$$\frac{bc\sqrt{e}\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\operatorname{EllipticPi}\left(\frac{de}{de-cf},\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{dx^2+c}}\right),-\frac{(bc-ad)e}{a(de-cf)}\right)}{af\sqrt{e+fx^2}\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{(be-af)\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}(fx^2+e)^{3/2}}dx}{f}$$

$\big\downarrow$ 429

$$\frac{bc\sqrt{e}\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\operatorname{EllipticPi}\left(\frac{de}{de-cf},\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{dx^2+c}}\right),-\frac{(bc-ad)e}{a(de-cf)}\right)}{af\sqrt{e+fx^2}\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{c+dx^2}(be-af)\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}\int\frac{\sqrt{\frac{(de-cf)x^2}{c(fx^2+e)}+1}}{\sqrt{\frac{(be-af)x^2}{a(fx^2+e)}+1}}d\frac{x}{\sqrt{fx^2+e}}}{ef\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$

$\big\downarrow$ 324

$$\frac{bc\sqrt{e}\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\operatorname{EllipticPi}\left(\frac{de}{de-cf},\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{dx^2+c}}\right),-\frac{(bc-ad)e}{a(de-cf)}\right)}{af\sqrt{e+fx^2}\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{c+dx^2}(be-af)\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}\left(\int\frac{1}{\sqrt{\frac{(be-af)x^2}{a(fx^2+e)}+1}\sqrt{\frac{(de-cf)x^2}{c(fx^2+e)}+1}}d\frac{x}{\sqrt{fx^2+e}}+\frac{(de-cf)\int\frac{x^2}{(fx^2+e)\sqrt{\frac{(be-af)x^2}{a(fx^2+e)}+1}\sqrt{\frac{(de-cf)x^2}{c(fx^2+e)}+1}}d\frac{x}{\sqrt{\cdots}}}{c}\right.}{ef\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$

$\big\downarrow$ 320

$$\frac{bc\sqrt{e}\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\operatorname{EllipticPi}\left(\frac{de}{de-cf},\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{dx^2+c}}\right),-\frac{(bc-ad)e}{a(de-cf)}\right)}{af\sqrt{e+fx^2}\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{c+dx^2}(be-af)\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}\left(\frac{(de-cf)\int\frac{x^2}{(fx^2+e)\sqrt{\frac{(be-af)x^2}{a(fx^2+e)}+1}\sqrt{\frac{(de-cf)x^2}{c(fx^2+e)}+1}}d\frac{x}{\sqrt{fx^2+e}}}{c}+\frac{\sqrt{c}\sqrt{\frac{x^2(be-af)}{a(e+fx^2)}+1}\operatorname{EllipticF}\left(\arctan\left(\cdots\right.\right.}{\sqrt{de-cf}\sqrt{\frac{x^2(de-cf)}{c(e+fx^2)}+1}\sqrt{\cdots}}\right.}{ef\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$

$\big\downarrow$ 388

$$\frac{bc\sqrt{e}\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\,\text{EllipticPi}\left(\frac{de}{de-cf},\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{dx^2+c}}\right),-\frac{(bc-ad)e}{a(de-cf)}\right)}{af\sqrt{e+fx^2}\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{c+dx^2}(be-af)\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}\left(\frac{(de-cf)\left(\frac{ax\sqrt{\frac{x^2(be-af)}{a(e+fx^2)}+1}}{\sqrt{e+fx^2}(be-af)\sqrt{\frac{x^2(de-cf)}{c(e+fx^2)}+1}}-\frac{a\int\frac{\sqrt{\frac{(be-af)x^2}{a(fx^2+e)}+1}}{\left(\frac{(de-cf)x^2}{c(fx^2+e)}+1\right)^{3/2}}d\frac{x}{\sqrt{fx^2+e}}}{be-af}\right)}{c}+\frac{\sqrt{c}\sqrt{\frac{x^2(be-af)}{a(e+fx^2)}+1}\,\text{E}}{\sqrt{de-cf}\,\sqrt{}}\right.}{ef\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$

$\downarrow$ 313

$$\frac{bc\sqrt{e}\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\,\text{EllipticPi}\left(\frac{de}{de-cf},\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{dx^2+c}}\right),-\frac{(bc-ad)e}{a(de-cf)}\right)}{af\sqrt{e+fx^2}\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{c+dx^2}(be-af)\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}\left(\frac{\sqrt{c}\sqrt{\frac{x^2(be-af)}{a(e+fx^2)}+1}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{de-cf}x}{\sqrt{c}\sqrt{fx^2+e}}\right),-\frac{(bc-ad)e}{a(de-cf)}\right)}{\sqrt{de-cf}\sqrt{\frac{x^2(de-cf)}{c(e+fx^2)}+1}\sqrt{\frac{c\left(\frac{x^2(be-af)}{e+fx^2}+a\right)}{a\left(\frac{x^2(de-cf)}{e+fx^2}+c\right)}}}+\frac{(de-cf)\left(\frac{ax\sqrt{\frac{x^2(be-af)}{a(e+fx^2)}+1}}{\sqrt{e+fx^2}(be-af)\sqrt{\frac{x^2(de-}{c(e+f}}}\right.}{}\right.}{ef\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$

input
```
Int[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(e + f*x^2)^(3/2),x]
```

output
```
-(((b*e - a*f)*Sqrt[c + d*x^2]*Sqrt[(e*(a + b*x^2))/(a*(e + f*x^2))]*(((d*
e - c*f)*((a*x*Sqrt[1 + ((b*e - a*f)*x^2)/(a*(e + f*x^2))])/((b*e - a*f)*S
qrt[e + f*x^2]*Sqrt[1 + ((d*e - c*f)*x^2)/(c*(e + f*x^2))]) - (a*Sqrt[c]*S
qrt[1 + ((b*e - a*f)*x^2)/(a*(e + f*x^2))]*EllipticE[ArcTan[(Sqrt[d*e - c*
f]*x)/(Sqrt[c]*Sqrt[e + f*x^2])], -(((b*c - a*d)*e)/(a*(d*e - c*f)))])/((b
*e - a*f)*Sqrt[d*e - c*f]*Sqrt[(c*(a + ((b*e - a*f)*x^2)/(e + f*x^2)))/(a*
(c + ((d*e - c*f)*x^2)/(e + f*x^2)))]*Sqrt[1 + ((d*e - c*f)*x^2)/(c*(e + f
*x^2))])))/c + (Sqrt[c]*Sqrt[1 + ((b*e - a*f)*x^2)/(a*(e + f*x^2))]*Ellipt
icF[ArcTan[(Sqrt[d*e - c*f]*x)/(Sqrt[c]*Sqrt[e + f*x^2])], -(((b*c - a*d)*
e)/(a*(d*e - c*f)))])/(Sqrt[d*e - c*f]*Sqrt[(c*(a + ((b*e - a*f)*x^2)/(e +
 f*x^2)))/(a*(c + ((d*e - c*f)*x^2)/(e + f*x^2)))]*Sqrt[1 + ((d*e - c*f)*x
^2)/(c*(e + f*x^2))])))/(e*f*Sqrt[a + b*x^2]*Sqrt[(e*(c + d*x^2))/(c*(e +
f*x^2))])) + (b*c*Sqrt[e]*Sqrt[a + b*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x
^2))]*EllipticPi[(d*e)/(d*e - c*f), ArcSin[(Sqrt[d*e - c*f]*x)/(Sqrt[e]*Sq
rt[c + d*x^2])], -(((b*c - a*d)*e)/(a*(d*e - c*f)))])/(a*f*Sqrt[d*e - c*f]
*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[e + f*x^2])
```

Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
 + d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 324
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Simp[b   Int[x^2/(Sqr
t[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c
] && PosQ[b/a]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 428
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/(Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*
(x_)^2]), x_Symbol] :> Simp[a*Sqrt[c + d*x^2]*(Sqrt[a*((e + f*x^2)/(e*(a +
b*x^2)))]/(c*Sqrt[e + f*x^2]*Sqrt[a*((c + d*x^2)/(c*(a + b*x^2)))]))   Subs
t[Int[1/((1 - b*x^2)*Sqrt[1 - (b*c - a*d)*(x^2/c)]*Sqrt[1 - (b*e - a*f)*(x^
2/e)]), x], x, x/Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 429
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)^(3/2)*Sqrt[(e_) + (f_.
)*(x_)^2]), x_Symbol] :> Simp[Sqrt[c + d*x^2]*(Sqrt[a*((e + f*x^2)/(e*(a +
b*x^2)))]/(a*Sqrt[e + f*x^2]*Sqrt[a*((c + d*x^2)/(c*(a + b*x^2)))]))   Subs
t[Int[Sqrt[1 - (b*c - a*d)*(x^2/c)]/Sqrt[1 - (b*e - a*f)*(x^2/e)], x], x, x
/Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 432
```
Int[(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2])/((e_) + (f_.)*(x_
)^2)^(3/2), x_Symbol] :> Simp[b/f   Int[Sqrt[c + d*x^2]/(Sqrt[a + b*x^2]*Sq
rt[e + f*x^2]), x], x] - Simp[(b*e - a*f)/f   Int[Sqrt[c + d*x^2]/(Sqrt[a +
 b*x^2]*(e + f*x^2)^(3/2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Maple [F]

$$\int \frac{\sqrt{b\,x^{2}+a}\,\sqrt{x^{2} d+c}}{\left(f\,x^{2}+e\right)^{\frac{3}{2}}}d x$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x)
```

output
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x)
```

Fricas [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm="fr
icas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(f^2*x^4 + 2*e*f*
x^2 + e^2), x)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx = \int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(1/2)/(f*x**2+e)**(3/2),x)
```

output
```
Integral(sqrt(a + b*x**2)*sqrt(c + d*x**2)/(e + f*x**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm="ma
xima")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/(f*x^2 + e)^(3/2), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm="gi
ac")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/(f*x^2 + e)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx = \int \frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{{\left(f\,x^2+e\right)}^{3/2}}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2))/(e + f*x^2)^(3/2),x)
```

output
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2))/(e + f*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx = \int \frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{(f\,x^2+e)^{\frac{3}{2}}}dx$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x)
```

output
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x)
```

3.327 $\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^2(e+fx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 349

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^2(e+fx^2)^{3/2}}\,dx = -\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{ex\sqrt{e+fx^2}} - \frac{2c\sqrt{-be+af}\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}E\left(\arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right)|\frac{a(de-cf)}{c(be-af)}\right)}{\sqrt{a}e^2\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}} - \frac{(bc-ad)\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right),\frac{a(de-cf)}{c(be-af)}\right)}{\sqrt{a}e\sqrt{-be+af}\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$

output
```
-(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/e/x/(f*x^2+e)^(1/2)-2*c*(a*f-b*e)^(1/2)*(
b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticE((a*f-b*e)^(1/2)*x
/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(1/2)/e^2/(d
*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)-(-a*d+b*c)*(b*x^2+a)^(1/2)*(
e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticF((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+
e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(1/2)/e/(a*f-b*e)^(1/2)/(d*x
^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)
```

Mathematica [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^2(e+fx^2)^{3/2}}\,dx = \int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^2(e+fx^2)^{3/2}}\,dx$$

input
```
Integrate[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(x^2*(e + f*x^2)^(3/2)),x]
```

output
```
Integrate[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(x^2*(e + f*x^2)^(3/2)), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^2(e+fx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 450}$$

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^2(e+fx^2)^{3/2}}dx$$

input
```
Int[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(x^2*(e + f*x^2)^(3/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [F]

$$\int \frac{\sqrt{b\,x^{2}+a}\,\sqrt{x^{2} d+c}}{x^{2} \left(f\,x^{2}+e\right)^{\frac{3}{2}}}d x$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^2/(f*x^2+e)^(3/2),x)
```

output
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^2/(f*x^2+e)^(3/2),x)
```

Fricas [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^2\left(e+fx^2\right)^{3/2}}\,dx=\int \frac{\sqrt{bx^{2}+a}\sqrt{dx^{2}+c}}{(fx^{2}+e)^{\frac{3}{2}}x^{2}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^2/(f*x^2+e)^(3/2),x, algorithm
="fricas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(f^2*x^6 + 2*e*f*
x^4 + e^2*x^2), x)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^2\,(e+fx^2)^{3/2}}\,dx = \int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^2\,(e+fx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(1/2)/x**2/(f*x**2+e)**(3/2),x)
```

output
```
Integral(sqrt(a + b*x**2)*sqrt(c + d*x**2)/(x**2*(e + f*x**2)**(3/2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^2\,(e+fx^2)^{3/2}}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{(fx^2+e)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^2/(f*x^2+e)^(3/2),x, algorithm
="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/((f*x^2 + e)^(3/2)*x^2), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^2\,(e+fx^2)^{3/2}}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{(fx^2+e)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^2/(f*x^2+e)^(3/2),x, algorithm
="giac")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/((f*x^2 + e)^(3/2)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^2(e+fx^2)^{3/2}}\,dx = \int \frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{x^2\,(f\,x^2+e)^{3/2}}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2))/(x^2*(e + f*x^2)^(3/2)),x)
```

output
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2))/(x^2*(e + f*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^2(e+fx^2)^{3/2}}\,dx = \int \frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{x^2\,(f\,x^2+e)^{\frac{3}{2}}}dx$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^2/(f*x^2+e)^(3/2),x)
```

output
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^2/(f*x^2+e)^(3/2),x)
```

3.328 $\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^4(e+fx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 436

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^4(e+fx^2)^{3/2}}\,dx = -\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{3ex^3\sqrt{e+fx^2}} - \frac{\left(\frac{b}{a}+\frac{d}{c}-\frac{4f}{e}\right)\sqrt{a+bx^2}\sqrt{c+dx^2}}{3ex\sqrt{e+fx^2}}$$
$$-\frac{\sqrt{-be+af}(bce+ade-8acf)\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}E\left(\arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right)|\frac{a(de-cf)}{c(be-af)}\right)}{3a^{3/2}e^3\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$
$$-\frac{(bc-ad)(be-4af)\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-be+afx}}{\sqrt{a}\sqrt{e+fx^2}}\right),\frac{a(de-cf)}{c(be-af)}\right)}{3a^{3/2}e^2\sqrt{-be+af}\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$

output
```
-1/3*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/e/x^3/(f*x^2+e)^(1/2)-1/3*(b/a+d/c-4*
f/e)*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/e/x/(f*x^2+e)^(1/2)-1/3*(a*f-b*e)^(1/
2)*(-8*a*c*f+a*d*e+b*c*e)*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*
EllipticE((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+
b*e))^(1/2))/a^(3/2)/e^3/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)-1
/3*(-a*d+b*c)*(-4*a*f+b*e)*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)
*EllipticF((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f
+b*e))^(1/2))/a^(3/2)/e^2/(a*f-b*e)^(1/2)/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(
f*x^2+e))^(1/2)
```

Mathematica [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^4\left(e+fx^2\right)^{3/2}}\,dx=\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^4\left(e+fx^2\right)^{3/2}}\,dx$$

input
```
Integrate[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(x^4*(e + f*x^2)^(3/2)),x]
```

output
```
Integrate[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(x^4*(e + f*x^2)^(3/2)), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^4\left(e+fx^2\right)^{3/2}}\,dx$$
$$\Big\downarrow \ 450$$
$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^4\left(e+fx^2\right)^{3/2}}dx$$

input
```
Int[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(x^4*(e + f*x^2)^(3/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [F]

$$\int \frac{\sqrt{b\,x^{2}+a}\,\sqrt{x^{2} d+c}}{x^{4} \left(f\,x^{2}+e\right)^{\frac{3}{2}}}dx$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^4/(f*x^2+e)^(3/2),x)
```

output
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^4/(f*x^2+e)^(3/2),x)
```

Fricas [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^4\left(e+fx^2\right)^{3/2}}\,dx=\int \frac{\sqrt{bx^{2}+a}\sqrt{dx^{2}+c}}{(fx^{2}+e)^{\frac{3}{2}}x^{4}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^4/(f*x^2+e)^(3/2),x, algorithm
="fricas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(f^2*x^8 + 2*e*f*
x^6 + e^2*x^4), x)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^4\left(e+fx^2\right)^{3/2}}\,dx=\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^4\left(e+fx^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(1/2)/x**4/(f*x**2+e)**(3/2),x)
```

output
```
Integral(sqrt(a + b*x**2)*sqrt(c + d*x**2)/(x**4*(e + f*x**2)**(3/2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^4\left(e+fx^2\right)^{3/2}}\,dx=\int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{\left(fx^2+e\right)^{\frac{3}{2}}x^4}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^4/(f*x^2+e)^(3/2),x, algorithm
="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/((f*x^2 + e)^(3/2)*x^4), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^4\left(e+fx^2\right)^{3/2}}\,dx=\int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{\left(fx^2+e\right)^{\frac{3}{2}}x^4}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^4/(f*x^2+e)^(3/2),x, algorithm
="giac")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/((f*x^2 + e)^(3/2)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^4\left(e+fx^2\right)^{3/2}}\,dx=\int \frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{x^4\,{\left(f\,x^2+e\right)}^{3/2}}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2))/(x^4*(e + f*x^2)^(3/2)),x)
```

output
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2))/(x^4*(e + f*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^4\left(e+fx^2\right)^{3/2}}\,dx=\int \frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{x^4\,\left(f\,x^2+e\right)^{\frac{3}{2}}}dx$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^4/(f*x^2+e)^(3/2),x)
```

output
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^4/(f*x^2+e)^(3/2),x)
```

3.329 $\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^6(e+fx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 602

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^6(e+fx^2)^{3/2}}\,dx = -\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{5ex^5\sqrt{e+fx^2}} - \frac{\left(\frac{b}{a}+\frac{d}{c}-\frac{6f}{e}\right)\sqrt{a+bx^2}\sqrt{c+dx^2}}{15ex^3\sqrt{e+fx^2}}$$
$$+\frac{\left(\frac{2b^2ce}{a}-2bde+\frac{2ad^2e}{c}+7bcf+7adf-\frac{24acf^2}{e}\right)\sqrt{a+bx^2}\sqrt{c+dx^2}}{15ace^2x\sqrt{e+fx^2}}$$
$$+\frac{2\sqrt{-be+af}(b^2c^2e^2-abce(de-4cf)+a^2(d^2e^2+4cdef-24c^2f^2))\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}E\left(\arcsin\left(\frac{\sqrt{-be+afx}}{\sqrt{a}\sqrt{e+fx^2}}\right)\dots\right.}{15a^{5/2}ce^4\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$
$$+\frac{(bc-ad)(2b^2ce^2+a^2f(de-24cf)-abe(de-7cf))\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-be+afx}}{\sqrt{a}\sqrt{e+fx^2}}\right),\dots\right.}{15a^{5/2}ce^3\sqrt{-be+af}\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$

output
```
-1/5*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/e/x^5/(f*x^2+e)^(1/2)-1/15*(b/a+d/c-6
*f/e)*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/e/x^3/(f*x^2+e)^(1/2)+1/15*(2*b^2*c*
e/a-2*b*d*e+2*a*d^2*e/c+7*b*c*f+7*a*d*f-24*a*c*f^2/e)*(b*x^2+a)^(1/2)*(d*x
^2+c)^(1/2)/a/c/e^2/x/(f*x^2+e)^(1/2)+2/15*(a*f-b*e)^(1/2)*(b^2*c^2*e^2-a*
b*c*e*(-4*c*f+d*e)+a^2*(-24*c^2*f^2+4*c*d*e*f+d^2*e^2))*(b*x^2+a)^(1/2)*(e
*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticE((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e
)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(5/2)/c/e^4/(d*x^2+c)^(1/2)/(
e*(b*x^2+a)/a/(f*x^2+e))^(1/2)+1/15*(-a*d+b*c)*(2*b^2*c*e^2+a^2*f*(-24*c*f
+d*e)-a*b*e*(-7*c*f+d*e))*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*
EllipticF((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+
b*e))^(1/2))/a^(5/2)/c/e^3/(a*f-b*e)^(1/2)/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/
(f*x^2+e))^(1/2)
```

Mathematica [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^6\left(e+fx^2\right)^{3/2}}\,dx=\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^6\left(e+fx^2\right)^{3/2}}\,dx$$

input
```
Integrate[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(x^6*(e + f*x^2)^(3/2)),x]
```

output
```
Integrate[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(x^6*(e + f*x^2)^(3/2)), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^6\left(e+fx^2\right)^{3/2}}\,dx$$

$$\Big\downarrow \text{450}$$

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^6\left(e+fx^2\right)^{3/2}}dx$$

input
```
Int[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(x^6*(e + f*x^2)^(3/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [F]

$$\int \frac{\sqrt{b\,x^{2}+a}\,\sqrt{x^{2} d+c}}{x^{6} \left(f\,x^{2}+e\right)^{\frac{3}{2}}}d x$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^6/(f*x^2+e)^(3/2),x)
```

output
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^6/(f*x^2+e)^(3/2),x)
```

Fricas [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^6 \left(e+fx^2\right)^{3/2}}\,dx = \int \frac{\sqrt{bx^{2}+a}\sqrt{dx^{2}+c}}{{\left(fx^{2}+e\right)}^{\frac{3}{2}}x^{6}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^6/(f*x^2+e)^(3/2),x, algorithm
="fricas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(f^2*x^10 + 2*e*f
*x^8 + e^2*x^6), x)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^6\left(e+fx^2\right)^{3/2}}\,dx=\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^6\left(e+fx^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(1/2)/x**6/(f*x**2+e)**(3/2),x)
```

output
```
Integral(sqrt(a + b*x**2)*sqrt(c + d*x**2)/(x**6*(e + f*x**2)**(3/2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^6\left(e+fx^2\right)^{3/2}}\,dx=\int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{\left(fx^2+e\right)^{\frac{3}{2}}x^6}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^6/(f*x^2+e)^(3/2),x, algorithm
="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/((f*x^2 + e)^(3/2)*x^6), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^6\left(e+fx^2\right)^{3/2}}\,dx=\int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{\left(fx^2+e\right)^{\frac{3}{2}}x^6}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^6/(f*x^2+e)^(3/2),x, algorithm
="giac")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/((f*x^2 + e)^(3/2)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^6\left(e+fx^2\right)^{3/2}}\,dx=\int \frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{x^6\,\left(f\,x^2+e\right)^{3/2}}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2))/(x^6*(e + f*x^2)^(3/2)),x)
```

output
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2))/(x^6*(e + f*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^6\left(e+fx^2\right)^{3/2}}\,dx=\int \frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{x^6\,\left(f\,x^2+e\right)^{\frac{3}{2}}}dx$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^6/(f*x^2+e)^(3/2),x)
```

output
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^6/(f*x^2+e)^(3/2),x)
```

3.330 $\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^8(e+fx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 855

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^8(e+fx^2)^{3/2}}\,dx = -\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{7ex^7\sqrt{e+fx^2}} - \frac{\left(\frac{b}{a}+\frac{d}{c}-\frac{8f}{e}\right)\sqrt{a+bx^2}\sqrt{c+dx^2}}{35ex^5\sqrt{e+fx^2}}$$

$$+\frac{\left(\frac{4b^2ce}{a}-2bde+\frac{4ad^2e}{c}+9bcf+9adf-\frac{48acf^2}{e}\right)\sqrt{a+bx^2}\sqrt{c+dx^2}}{105ace^2x^3\sqrt{e+fx^2}}$$

$$-\frac{\left(\frac{8b^3c^2e^2}{a}-b^2ce(5de-19cf)-5ab(d^2e^2+4cdef-12c^2f^2)+a^2\left(\frac{8d^3e^2}{c}+19d^2ef+60cdf^2-\frac{192c^2f^3}{e}\right)\right)\sqrt{\cdots}}{105a^2c^2e^3x\sqrt{e+fx^2}}$$

$$-\frac{\sqrt{-be+af}(8b^3c^3e^3-ab^2c^2e^2(5de-23cf)-a^2bce(5d^2e^2+22cdef-72c^2f^2)+a^3(8d^3e^3+23cd^2e^2f+\cdots}{105a^{7/2}c^2e^5\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}\cdots}$$

$$-\frac{(bc-ad)\left(8b^3c^2e^3-ab^2ce^2(de-19cf)-a^2be(4d^2e^2+11cdef-60c^2f^2)+4a^3f(d^2e^2+3cdef-48c^2f^{\cdots}\right.}{105a^{7/2}c^2e^4\sqrt{-be+af}\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}\cdots}$$

output
```
-1/7*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/e/x^7/(f*x^2+e)^(1/2)-1/35*(b/a+d/c-8
*f/e)*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/e/x^5/(f*x^2+e)^(1/2)+1/105*(4*b^2*c
*e/a-2*b*d*e+4*a*d^2*e/c+9*b*c*f+9*a*d*f-48*a*c*f^2/e)*(b*x^2+a)^(1/2)*(d*
x^2+c)^(1/2)/a/c/e^2/x^3/(f*x^2+e)^(1/2)-1/105*(8*b^3*c^2*e^2/a-b^2*c*e*(-
19*c*f+5*d*e)-5*a*b*(-12*c^2*f^2+4*c*d*e*f+d^2*e^2)+a^2*(8*d^3*e^2/c+19*d^
2*e*f+60*c*d*f^2-192*c^2*f^3/e))*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a^2/c^2/e
^3/x/(f*x^2+e)^(1/2)-1/105*(a*f-b*e)^(1/2)*(8*b^3*c^3*e^3-a*b^2*c^2*e^2*(-
23*c*f+5*d*e)-a^2*b*c*e*(-72*c^2*f^2+22*c*d*e*f+5*d^2*e^2)+a^3*(-384*c^3*f
^3+72*c^2*d*e*f^2+23*c*d^2*e^2*f+8*d^3*e^3))*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/
c/(f*x^2+e))^(1/2)*EllipticE((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*
(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(7/2)/c^2/e^5/(d*x^2+c)^(1/2)/(e*(b*x^2+
a)/a/(f*x^2+e))^(1/2)-1/105*(-a*d+b*c)*(8*b^3*c^2*e^3-a*b^2*c*e^2*(-19*c*f
+d*e)-a^2*b*e*(-60*c^2*f^2+11*c*d*e*f+4*d^2*e^2)+4*a^3*f*(-48*c^2*f^2+3*c*
d*e*f+d^2*e^2))*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticF(
(a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2
))/a^(7/2)/c^2/e^4/(a*f-b*e)^(1/2)/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e
))^(1/2)
```

Mathematica [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^8\left(e+fx^2\right)^{3/2}}\,dx=\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^8\left(e+fx^2\right)^{3/2}}\,dx$$

input
```
Integrate[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(x^8*(e + f*x^2)^(3/2)),x]
```

output
```
Integrate[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(x^8*(e + f*x^2)^(3/2)), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^8\left(e+fx^2\right)^{3/2}}\,dx$$

$$\downarrow 450$$

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^8\,(e+fx^2)^{3/2}}dx$$

input
```
Int[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(x^8*(e + f*x^2)^(3/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [F]

$$\int \frac{\sqrt{b\,x^2+a}\,\sqrt{x^2d+c}}{x^8\,(f\,x^2+e)^{\frac{3}{2}}}dx$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^8/(f*x^2+e)^(3/2),x)
```

output
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^8/(f*x^2+e)^(3/2),x)
```

Fricas [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^8\,(e+fx^2)^{3/2}}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{(fx^2+e)^{\frac{3}{2}}x^8}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^8/(f*x^2+e)^(3/2),x, algorithm
="fricas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(f^2*x^12 + 2*e*f
*x^10 + e^2*x^8), x)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^8\left(e+fx^2\right)^{3/2}}\,dx=\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^8\left(e+fx^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(1/2)/x**8/(f*x**2+e)**(3/2),x)
```

output
```
Integral(sqrt(a + b*x**2)*sqrt(c + d*x**2)/(x**8*(e + f*x**2)**(3/2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^8\left(e+fx^2\right)^{3/2}}\,dx=\int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{\left(fx^2+e\right)^{\frac{3}{2}}x^8}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^8/(f*x^2+e)^(3/2),x, algorithm
="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/((f*x^2 + e)^(3/2)*x^8), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^8\left(e+fx^2\right)^{3/2}}\,dx=\int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{\left(fx^2+e\right)^{\frac{3}{2}}x^8}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^8/(f*x^2+e)^(3/2),x, algorithm
="giac")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/((f*x^2 + e)^(3/2)*x^8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^8\left(e+fx^2\right)^{3/2}}\,dx=\int \frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{x^8\,\left(f\,x^2+e\right)^{3/2}}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2))/(x^8*(e + f*x^2)^(3/2)),x)
```

output
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2))/(x^8*(e + f*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^8\left(e+fx^2\right)^{3/2}}\,dx=\int \frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{x^8\,\left(f\,x^2+e\right)^{\frac{3}{2}}}dx$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^8/(f*x^2+e)^(3/2),x)
```

output
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^8/(f*x^2+e)^(3/2),x)
```

3.331 $\int \frac{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 858

$$\int \frac{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{5/2}}\,dx = -\frac{e^2x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3f^3(e+fx^2)^{3/2}}$$
$$-\frac{(9bde-bcf-adf)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{8bdf^3\sqrt{e+fx^2}}+\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{4f^2\sqrt{e+fx^2}}$$
$$-\frac{c(3a^2df^2(de-cf)+b^2e(105d^2e^2-100cdef+3c^2f^2)-abf(100d^2e^2-95cdef+3c^2f^2))\sqrt{a+bx^2}\sqrt{\frac{e(c}{c(e}}}{24\sqrt{a}bdf^4\sqrt{-be+af}(de-cf)\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$
$$+\frac{e(3a^2df^2(de-cf)+abf(30d^2e^2-101cdef+63c^2f^2)-b^2e(105d^2e^2-240cdef+127c^2f^2))\sqrt{a+bx^2}\sqrt{}}{24\sqrt{a}bf^5\sqrt{-be+af}(de-cf)\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$
$$-\frac{e(a^2d^2f^2+2abdf(5de-cf)-b^2(35d^2e^2-10cdef-c^2f^2))\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,\text{EllipticPi}\left(-\frac{af}{be-af},\text{arcs}\right.}{8\sqrt{a}bdf^5\sqrt{-be+af}\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$

output
```
-1/3*e^2*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/f^3/(f*x^2+e)^(3/2)-1/8*(-a*d*f
-b*c*f+9*b*d*e)*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b/d/f^3/(f*x^2+e)^(1/2)+
1/4*x^3*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/f^2/(f*x^2+e)^(1/2)-1/24*c*(3*a^2*
d*f^2*(-c*f+d*e)+b^2*e*(3*c^2*f^2-100*c*d*e*f+105*d^2*e^2)-a*b*f*(3*c^2*f^
2-95*c*d*e*f+100*d^2*e^2))*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)
*EllipticE((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f
+b*e))^(1/2))/a^(1/2)/b/d/f^4/(a*f-b*e)^(1/2)/(-c*f+d*e)/(d*x^2+c)^(1/2)/(
e*(b*x^2+a)/a/(f*x^2+e))^(1/2)+1/24*e*(3*a^2*d*f^2*(-c*f+d*e)+a*b*f*(63*c^
2*f^2-101*c*d*e*f+30*d^2*e^2)-b^2*e*(127*c^2*f^2-240*c*d*e*f+105*d^2*e^2))
*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticF((a*f-b*e)^(1/2)
*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(1/2)/b/f^
5/(a*f-b*e)^(1/2)/(-c*f+d*e)/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/
2)-1/8*e*(a^2*d^2*f^2+2*a*b*d*f*(-c*f+5*d*e)-b^2*(-c^2*f^2-10*c*d*e*f+35*d
^2*e^2))*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticPi((a*f-b
*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),-a*f/(-a*f+b*e),(a*(-c*f+d*e)/c/(-a*f+
b*e))^(1/2))/a^(1/2)/b/d/f^5/(a*f-b*e)^(1/2)/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/
a/(f*x^2+e))^(1/2)
```

Mathematica [F]

$$\int \frac{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{5/2}}\,dx = \int \frac{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{5/2}}\,dx$$

input
```
Integrate[(x^6*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(e + f*x^2)^(5/2),x]
```

output
```
Integrate[(x^6*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(e + f*x^2)^(5/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{5/2}}\,dx$$

$$\downarrow 450$$

$$\int \frac{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{5/2}}dx$$

input
```
Int[(x^6*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(e + f*x^2)^(5/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [F]

$$\int \frac{x^6\sqrt{b\,x^2+a}\,\sqrt{x^2d+c}}{(f\,x^2+e)^{\frac{5}{2}}}dx$$

input
```
int(x^6*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x)
```

output
```
int(x^6*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x^6*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x, algorithm
="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{5/2}}\,dx = \int \frac{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**6*(b*x**2+a)**(1/2)*(d*x**2+c)**(1/2)/(f*x**2+e)**(5/2),x)
```

output
```
Integral(x**6*sqrt(a + b*x**2)*sqrt(c + d*x**2)/(e + f*x**2)**(5/2), x)
```

Maxima [F]

$$\int \frac{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{5/2}}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}x^6}{(fx^2+e)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^6*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x, algorithm
="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x^6/(f*x^2 + e)^(5/2), x)
```

Giac [F]

$$\int \frac{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{5/2}}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}x^6}{(fx^2+e)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^6*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x, algorithm
="giac")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x^6/(f*x^2 + e)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{5/2}}\,dx=\int \frac{x^6\,\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{(f\,x^2+e)^{5/2}}\,dx$$

input
```
int((x^6*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2))/(e + f*x^2)^(5/2),x)
```

output
```
int((x^6*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2))/(e + f*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{5/2}}\,dx=\int \frac{x^6\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{(f\,x^2+e)^{\frac{5}{2}}}dx$$

input
```
int(x^6*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x)
```

output
```
int(x^6*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x)
```

3.332 $\int \frac{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 669

$$\int \frac{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{5/2}}\,dx = \frac{ex\sqrt{a+bx^2}\sqrt{c+dx^2}}{3f^2(e+fx^2)^{3/2}} + \frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}}{2f^2\sqrt{e+fx^2}}$$

$$+ \frac{c(be(15de-13cf)-af(13de-11cf))\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}E\left(\arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right)|\frac{a(de-cf)}{c(be-af)}\right)}{6\sqrt{a}f^3\sqrt{-be+af}(de-cf)\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$

$$- \frac{(af(3d^2e^2-11cdef+6c^2f^2)-be(15d^2e^2-33cdef+16c^2f^2))\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{}}{\sqrt{}}\right.\right.}{6\sqrt{a}f^4\sqrt{-be+af}(de-cf)\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$

$$- \frac{e(5bde-bcf-adf)\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\operatorname{EllipticPi}\left(-\frac{af}{be-af},\arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right),\frac{a(de-cf)}{c(be-af)}\right)}{2\sqrt{a}f^4\sqrt{-be+af}\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$

output
```
1/3*e*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/f^2/(f*x^2+e)^(3/2)+1/2*x*(b*x^2+a
)^(1/2)*(d*x^2+c)^(1/2)/f^2/(f*x^2+e)^(1/2)+1/6*c*(b*e*(-13*c*f+15*d*e)-a*
f*(-11*c*f+13*d*e))*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*Ellipt
icE((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^
(1/2))/a^(1/2)/f^3/(a*f-b*e)^(1/2)/(-c*f+d*e)/(d*x^2+c)^(1/2)/(e*(b*x^2+a)
/a/(f*x^2+e))^(1/2)-1/6*(a*f*(6*c^2*f^2-11*c*d*e*f+3*d^2*e^2)-b*e*(16*c^2*
f^2-33*c*d*e*f+15*d^2*e^2))*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2
)*EllipticF((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*
f+b*e))^(1/2))/a^(1/2)/f^4/(a*f-b*e)^(1/2)/(-c*f+d*e)/(d*x^2+c)^(1/2)/(e*(
b*x^2+a)/a/(f*x^2+e))^(1/2)-1/2*e*(-a*d*f-b*c*f+5*b*d*e)*(b*x^2+a)^(1/2)*(
e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticPi((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2
+e)^(1/2),-a*f/(-a*f+b*e),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(1/2)/f^4/(
a*f-b*e)^(1/2)/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)
```

Mathematica [F]

$$\int \frac{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{5/2}}\,dx=\int \frac{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{5/2}}\,dx$$

input
```
Integrate[(x^4*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(e + f*x^2)^(5/2),x]
```

output
```
Integrate[(x^4*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(e + f*x^2)^(5/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{5/2}}\,dx$$
$$\Big\downarrow \ 450$$
$$\int \frac{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{5/2}}dx$$

input
```
Int[(x^4*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(e + f*x^2)^(5/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [F]

$$\int \frac{x^4\sqrt{b\,x^2+a}\,\sqrt{x^2 d+c}}{(f\,x^2+e)^{\frac{5}{2}}}dx$$

input
```
int(x^4*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x)
```

output
```
int(x^4*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate(x^4*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x, algorithm
="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{5/2}}\,dx=\int \frac{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**4*(b*x**2+a)**(1/2)*(d*x**2+c)**(1/2)/(f*x**2+e)**(5/2),x)
```

output
```
Integral(x**4*sqrt(a + b*x**2)*sqrt(c + d*x**2)/(e + f*x**2)**(5/2), x)
```

Maxima [F]

$$\int \frac{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{5/2}}\,dx=\int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}x^4}{(fx^2+e)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^4*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x, algorithm
="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x^4/(f*x^2 + e)^(5/2), x)
```

Giac [F]

$$\int \frac{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{5/2}}\,dx=\int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}x^4}{(fx^2+e)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^4*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x, algorithm
="giac")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x^4/(f*x^2 + e)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{5/2}}\,dx = \int \frac{x^4\,\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{{\left(f\,x^2+e\right)}^{5/2}}\,dx$$

input
```
int((x^4*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2))/(e + f*x^2)^(5/2),x)
```

output
```
int((x^4*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2))/(e + f*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{5/2}}\,dx = \int \frac{x^4\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{(f\,x^2+e)^{\frac{5}{2}}}dx$$

input
```
int(x^4*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x)
```

output
```
int(x^4*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x)
```

3.333 $\int \frac{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 592

$$\begin{aligned}\int \frac{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{5/2}}\,dx &= -\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3f\,(e+fx^2)^{3/2}} \\ &- \frac{c(be(3de-2cf)-af(2de-cf))\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}E\left(\arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right)|\frac{a(de-cf)}{c(be-af)}\right)}{3\sqrt{a}ef^2\sqrt{-be+af}(de-cf)\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}} \\ &- \frac{(acdf^2+b(3d^2e^2-6cdef+2c^2f^2))\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right),\frac{a(de-cf)}{c(be-af)}\right)}{3\sqrt{a}f^3\sqrt{-be+af}(de-cf)\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}} \\ &+ \frac{bde\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,\text{EllipticPi}\left(-\frac{af}{be-af},\arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right),\frac{a(de-cf)}{c(be-af)}\right)}{\sqrt{a}f^3\sqrt{-be+af}\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}\end{aligned}$$

output
```
-1/3*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/f/(f*x^2+e)^(3/2)-1/3*c*(b*e*(-2*c*
f+3*d*e)-a*f*(-c*f+2*d*e))*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)
*EllipticE((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f
+b*e))^(1/2))/a^(1/2)/e/f^2/(a*f-b*e)^(1/2)/(-c*f+d*e)/(d*x^2+c)^(1/2)/(e*
(b*x^2+a)/a/(f*x^2+e))^(1/2)-1/3*(a*c*d*f^2+b*(2*c^2*f^2-6*c*d*e*f+3*d^2*e
^2))*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticF((a*f-b*e)^(
1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(1/2)/
f^3/(a*f-b*e)^(1/2)/(-c*f+d*e)/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(
1/2)+b*d*e*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticPi((a*f
-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),-a*f/(-a*f+b*e),(a*(-c*f+d*e)/c/(-a*
f+b*e))^(1/2))/a^(1/2)/f^3/(a*f-b*e)^(1/2)/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/
(f*x^2+e))^(1/2)
```

Mathematica [F]

$$\int \frac{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{5/2}}\,dx=\int \frac{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{5/2}}\,dx$$

input
```
Integrate[(x^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(e + f*x^2)^(5/2),x]
```

output
```
Integrate[(x^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(e + f*x^2)^(5/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{5/2}}\,dx$$
$$\Big\downarrow \; 450$$
$$\int \frac{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{5/2}}dx$$

input
```
Int[(x^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(e + f*x^2)^(5/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [F]

$$\int \frac{x^{2} \sqrt{b\,x^{2}+a}\,\sqrt{x^{2} d +c}}{\left(f\,x^{2}+e \right)^{\frac{5}{2}}}d x$$

input
```
int(x^2*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x)
```

output
```
int(x^2*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x)
```

Fricas [F]

$$\int \frac{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{5/2}}\,dx=\int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}x^2}{(fx^2+e)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^2*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x, algorithm
="fricas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)*x^2/(f^3*x^6 + 3*
e*f^2*x^4 + 3*e^2*f*x^2 + e^3), x)
```

Sympy [F]

$$\int \frac{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{5/2}}\,dx = \int \frac{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**2*(b*x**2+a)**(1/2)*(d*x**2+c)**(1/2)/(f*x**2+e)**(5/2),x)
```

output
```
Integral(x**2*sqrt(a + b*x**2)*sqrt(c + d*x**2)/(e + f*x**2)**(5/2), x)
```

Maxima [F]

$$\int \frac{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{5/2}}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}x^2}{(fx^2+e)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^2*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x, algorithm
="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x^2/(f*x^2 + e)^(5/2), x)
```

Giac [F]

$$\int \frac{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{5/2}}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}x^2}{(fx^2+e)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^2*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x, algorithm
="giac")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x^2/(f*x^2 + e)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{5/2}}\,dx = \int \frac{x^2\,\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{{\left(f\,x^2+e\right)}^{5/2}}\,dx$$

input
```
int((x^2*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2))/(e + f*x^2)^(5/2),x)
```

output
```
int((x^2*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2))/(e + f*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{5/2}}\,dx = \int \frac{x^2\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{(f\,x^2+e)^{\frac{5}{2}}}dx$$

input
```
int(x^2*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x)
```

output
```
int(x^2*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x)
```

3.334 $\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{\left(e+fx^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 388

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{\left(e+fx^2\right)^{5/2}}\,dx = \frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3e\left(e+fx^2\right)^{3/2}} + \frac{c(bce+ade-2acf)\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}E\left(\arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right)|\frac{a(de-cf)}{c(be-af)}\right)}{3\sqrt{a}e^2\sqrt{-be+af}(de-cf)\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}} - \frac{c(bc-ad)\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right),\frac{a(de-cf)}{c(be-af)}\right)}{3\sqrt{a}e\sqrt{-be+af}(de-cf)\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$

output

```
1/3*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/e/(f*x^2+e)^(3/2)+1/3*c*(-2*a*c*f+a*
d*e+b*c*e)*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticE((a*f-
b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^
(1/2)/e^2/(a*f-b*e)^(1/2)/(-c*f+d*e)/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2
+e))^(1/2)-1/3*c*(-a*d+b*c)*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2
)*EllipticF((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*
f+b*e))^(1/2))/a^(1/2)/e/(a*f-b*e)^(1/2)/(-c*f+d*e)/(d*x^2+c)^(1/2)/(e*(b*
x^2+a)/a/(f*x^2+e))^(1/2)
```

Mathematica [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{5/2}}\,dx=\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{5/2}}\,dx$$

input
```
Integrate[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(e + f*x^2)^(5/2),x]
```

output
```
Integrate[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(e + f*x^2)^(5/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{434}$$

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{5/2}}dx$$

input
```
Int[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(e + f*x^2)^(5/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{\sqrt{b\,x^{2}+a}\,\sqrt{x^{2} d +c}}{\left(f\,x^{2}+e \right)^{\frac{5}{2}}}d x$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x)
```

output
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x)
```

Fricas [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{5/2}}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{(fx^2+e)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x, algorithm="fr
icas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(f^3*x^6 + 3*e*f^
2*x^4 + 3*e^2*f*x^2 + e^3), x)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{5/2}}\,dx=\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(1/2)/(f*x**2+e)**(5/2),x)
```

output
```
Integral(sqrt(a + b*x**2)*sqrt(c + d*x**2)/(e + f*x**2)**(5/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{5/2}}\,dx=\int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{(fx^2+e)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x, algorithm="ma
xima")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/(f*x^2 + e)^(5/2), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{5/2}}\,dx=\int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{(fx^2+e)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x, algorithm="gi
ac")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/(f*x^2 + e)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{5/2}}\,dx = \int \frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{(f\,x^2+e)^{5/2}}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2))/(e + f*x^2)^(5/2),x)
```

output
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2))/(e + f*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{5/2}}\,dx = \int \frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{(f\,x^2+e)^{\frac{5}{2}}}dx$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x)
```

output
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x)
```

3.335 $\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^2(e+fx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 450

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^2(e+fx^2)^{5/2}}\,dx = -\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{ex(e+fx^2)^{3/2}} - \frac{4fx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3e^2(e+fx^2)^{3/2}} - \frac{c(af(7de-8cf)-be(6de-7cf))\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}E\left(\arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right)|\frac{a(de-cf)}{c(be-af)}\right)}{3\sqrt{a}e^3\sqrt{-be+af}(de-cf)\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}} - \frac{(bc-ad)(3de-4cf)\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right),\frac{a(de-cf)}{c(be-af)}\right)}{3\sqrt{a}e^2\sqrt{-be+af}(de-cf)\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$

output
```
-(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/e/x/(f*x^2+e)^(3/2)-4/3*f*x*(b*x^2+a)^(1/
2)*(d*x^2+c)^(1/2)/e^2/(f*x^2+e)^(3/2)-1/3*c*(a*f*(-8*c*f+7*d*e)-b*e*(-7*c
*f+6*d*e))*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticE((a*f-
b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^
(1/2)/e^3/(a*f-b*e)^(1/2)/(-c*f+d*e)/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2
+e))^(1/2)-1/3*(-a*d+b*c)*(-4*c*f+3*d*e)*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f
*x^2+e))^(1/2)*EllipticF((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*
f+d*e)/c/(-a*f+b*e))^(1/2))/a^(1/2)/e^2/(a*f-b*e)^(1/2)/(-c*f+d*e)/(d*x^2+
c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)
```

Mathematica [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^2\left(e+fx^2\right)^{5/2}}\,dx=\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^2\left(e+fx^2\right)^{5/2}}\,dx$$

input
```
Integrate[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(x^2*(e + f*x^2)^(5/2)),x]
```

output
```
Integrate[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(x^2*(e + f*x^2)^(5/2)), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^2\left(e+fx^2\right)^{5/2}}\,dx$$
$$\Big\downarrow \; 450$$
$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^2\left(e+fx^2\right)^{5/2}}dx$$

input
```
Int[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(x^2*(e + f*x^2)^(5/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [F]

$$\int \frac{\sqrt{b\,x^{2}+a}\,\sqrt{x^{2} d+c}}{x^{2} \left(f\,x^{2}+e\right)^{\frac{5}{2}}}d x$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^2/(f*x^2+e)^(5/2),x)
```

output
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^2/(f*x^2+e)^(5/2),x)
```

Fricas [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^2\left(e+fx^2\right)^{5/2}}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{\left(fx^2+e\right)^{\frac{5}{2}}x^2}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^2/(f*x^2+e)^(5/2),x, algorithm
="fricas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(f^3*x^8 + 3*e*f^
2*x^6 + 3*e^2*f*x^4 + e^3*x^2), x)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^2(e+fx^2)^{5/2}}\,dx = \int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^2(e+fx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(1/2)/x**2/(f*x**2+e)**(5/2),x)
```

output
```
Integral(sqrt(a + b*x**2)*sqrt(c + d*x**2)/(x**2*(e + f*x**2)**(5/2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^2(e+fx^2)^{5/2}}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{(fx^2+e)^{\frac{5}{2}}x^2}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^2/(f*x^2+e)^(5/2),x, algorithm
="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/((f*x^2 + e)^(5/2)*x^2), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^2(e+fx^2)^{5/2}}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{(fx^2+e)^{\frac{5}{2}}x^2}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^2/(f*x^2+e)^(5/2),x, algorithm
="giac")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/((f*x^2 + e)^(5/2)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^2\left(e+fx^2\right)^{5/2}}\,dx=\int \frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{x^2\,\left(f\,x^2+e\right)^{5/2}}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2))/(x^2*(e + f*x^2)^(5/2)),x)
```

output
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2))/(x^2*(e + f*x^2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^2\left(e+fx^2\right)^{5/2}}\,dx=\int \frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{x^2\,\left(f\,x^2+e\right)^{\frac{5}{2}}}dx$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^2/(f*x^2+e)^(5/2),x)
```

output
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^2/(f*x^2+e)^(5/2),x)
```

3.336 $\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^4\left(e+fx^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 592

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^4\left(e+fx^2\right)^{5/2}}\,dx = -\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{3ex^3\left(e+fx^2\right)^{3/2}}$$
$$-\frac{\left(\frac{b}{a}+\frac{d}{c}-\frac{6f}{e}\right)\sqrt{a+bx^2}\sqrt{c+dx^2}}{3ex\left(e+fx^2\right)^{3/2}}-\frac{f(bce+ade-8acf)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3ace^3\left(e+fx^2\right)^{3/2}}$$
$$+\frac{\left(b^2ce^2(de-cf)+abe(d^2e^2-16cdef+16c^2f^2)-a^2f(d^2e^2-16cdef+16c^2f^2)\right)\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}E\Big(}{3a^{3/2}e^4\sqrt{-be+af}(de-cf)\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$
$$+\frac{(bc-ad)(af(7de-8cf)-be(de-cf))\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right),\frac{a(de-cf)}{c(be-af)}\right)}{3a^{3/2}e^3\sqrt{-be+af}(de-cf)\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$

output
```
-1/3*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/e/x^3/(f*x^2+e)^(3/2)-1/3*(b/a+d/c-6*
f/e)*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/e/x/(f*x^2+e)^(3/2)-1/3*f*(-8*a*c*f+a
*d*e+b*c*e)*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a/c/e^3/(f*x^2+e)^(3/2)+1/3*
(b^2*c*e^2*(-c*f+d*e)+a*b*e*(16*c^2*f^2-16*c*d*e*f+d^2*e^2)-a^2*f*(16*c^2*
f^2-16*c*d*e*f+d^2*e^2))*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*E
llipticE((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b
*e))^(1/2))/a^(3/2)/e^4/(a*f-b*e)^(1/2)/(-c*f+d*e)/(d*x^2+c)^(1/2)/(e*(b*x
^2+a)/a/(f*x^2+e))^(1/2)+1/3*(-a*d+b*c)*(a*f*(-8*c*f+7*d*e)-b*e*(-c*f+d*e)
)*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticF((a*f-b*e)^(1/2
)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(3/2)/e^3
/(a*f-b*e)^(1/2)/(-c*f+d*e)/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2
)
```

Mathematica [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^4\left(e+fx^2\right)^{5/2}}\,dx=\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^4\left(e+fx^2\right)^{5/2}}\,dx$$

input
```
Integrate[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(x^4*(e + f*x^2)^(5/2)),x]
```

output
```
Integrate[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(x^4*(e + f*x^2)^(5/2)), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^4\left(e+fx^2\right)^{5/2}}\,dx$$

$$\Big\downarrow \; 450$$

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^4\left(e+fx^2\right)^{5/2}}dx$$

input
```
Int[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(x^4*(e + f*x^2)^(5/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [F]

$$\int \frac{\sqrt{b\,x^{2}+a}\,\sqrt{x^{2} d +c}}{x^{4} \left(f\,x^{2}+e \right)^{\frac{5}{2}}}d x$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^4/(f*x^2+e)^(5/2),x)
```

output
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^4/(f*x^2+e)^(5/2),x)
```

Fricas [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^4\left(e+fx^2\right)^{5/2}}\,dx=\int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{(fx^2+e)^{\frac{5}{2}}x^4}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^4/(f*x^2+e)^(5/2),x, algorithm
="fricas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(f^3*x^10 + 3*e*f
^2*x^8 + 3*e^2*f*x^6 + e^3*x^4), x)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^4\left(e+fx^2\right)^{5/2}}\,dx = \int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^4\left(e+fx^2\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(1/2)/x**4/(f*x**2+e)**(5/2),x)
```

output
```
Integral(sqrt(a + b*x**2)*sqrt(c + d*x**2)/(x**4*(e + f*x**2)**(5/2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^4\left(e+fx^2\right)^{5/2}}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{\left(fx^2+e\right)^{\frac{5}{2}}x^4}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^4/(f*x^2+e)^(5/2),x, algorithm
="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/((f*x^2 + e)^(5/2)*x^4), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^4\left(e+fx^2\right)^{5/2}}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{\left(fx^2+e\right)^{\frac{5}{2}}x^4}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^4/(f*x^2+e)^(5/2),x, algorithm
="giac")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/((f*x^2 + e)^(5/2)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^4\left(e+fx^2\right)^{5/2}}\,dx=\int \frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{x^4\,\left(f\,x^2+e\right)^{5/2}}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2))/(x^4*(e + f*x^2)^(5/2)),x)
```

output
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2))/(x^4*(e + f*x^2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^4\left(e+fx^2\right)^{5/2}}\,dx=\int \frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{x^4\,\left(f\,x^2+e\right)^{\frac{5}{2}}}dx$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^4/(f*x^2+e)^(5/2),x)
```

output
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^4/(f*x^2+e)^(5/2),x)
```

3.337 $\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^6\left(e+fx^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 845

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^6\left(e+fx^2\right)^{5/2}}\,dx = -\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{5ex^5\left(e+fx^2\right)^{3/2}} - \frac{\left(\frac{b}{a}+\frac{d}{c}-\frac{8f}{e}\right)\sqrt{a+bx^2}\sqrt{c+dx^2}}{15ex^3\left(e+fx^2\right)^{3/2}}$$

$$+\frac{\left(\frac{2b^2ce}{a}-2bde+\frac{2ad^2e}{c}+11bcf+11adf-\frac{48acf^2}{e}\right)\sqrt{a+bx^2}\sqrt{c+dx^2}}{15ace^2x\left(e+fx^2\right)^{3/2}}$$

$$+\frac{2f\left(b^2c^2e^2-abce(de-6cf)+a^2\left(d^2e^2+6cdef-32c^2f^2\right)\right)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{15a^2c^2e^4\left(e+fx^2\right)^{3/2}}$$

$$-\frac{\left(2b^3c^2e^3(de-cf)-ab^2ce^2\left(2d^2e^2-13cdef+11c^2f^2\right)-a^3f\left(2d^3e^3+11cd^2e^2f-136c^2def^2+128c^3f^3\right)\right.}{15a^{5/2}ce^5\sqrt{-be+af}(de-}$$

$$+\frac{(bc-ad)\left(2b^2ce^2(de-cf)-abe\left(d^2e^2-13cdef+12c^2f^2\right)+a^2f\left(d^2e^2-60cdef+64c^2f^2\right)\right)\sqrt{a+bx^2}\sqrt{\vphantom{x}}}{15a^{5/2}ce^4\sqrt{-be+af}(de-cf)\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$

output
```
-1/5*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/e/x^5/(f*x^2+e)^(3/2)-1/15*(b/a+d/c-8
*f/e)*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/e/x^3/(f*x^2+e)^(3/2)+1/15*(2*b^2*c*
e/a-2*b*d*e+2*a*d^2*e/c+11*b*c*f+11*a*d*f-48*a*c*f^2/e)*(b*x^2+a)^(1/2)*(d
*x^2+c)^(1/2)/a/c/e^2/x/(f*x^2+e)^(3/2)+2/15*f*(b^2*c^2*e^2-a*b*c*e*(-6*c*
f+d*e)+a^2*(-32*c^2*f^2+6*c*d*e*f+d^2*e^2))*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1
/2)/a^2/c^2/e^4/(f*x^2+e)^(3/2)-1/15*(2*b^3*c^2*e^3*(-c*f+d*e)-a*b^2*c*e^2
*(11*c^2*f^2-13*c*d*e*f+2*d^2*e^2)-a^3*f*(128*c^3*f^3-136*c^2*d*e*f^2+11*c
*d^2*e^2*f+2*d^3*e^3)+a^2*b*e*(136*c^3*f^3-146*c^2*d*e*f^2+13*c*d^2*e^2*f+
2*d^3*e^3))*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticE((a*f
-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a
^(5/2)/c/e^5/(a*f-b*e)^(1/2)/(-c*f+d*e)/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*
x^2+e))^(1/2)+1/15*(-a*d+b*c)*(2*b^2*c*e^2*(-c*f+d*e)-a*b*e*(12*c^2*f^2-13
*c*d*e*f+d^2*e^2)+a^2*f*(64*c^2*f^2-60*c*d*e*f+d^2*e^2))*(b*x^2+a)^(1/2)*(
e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticF((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+
e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(5/2)/c/e^4/(a*f-b*e)^(1/2)/
(-c*f+d*e)/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)
```

Mathematica [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^6\left(e+fx^2\right)^{5/2}}\,dx=\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^6\left(e+fx^2\right)^{5/2}}\,dx$$

input
```
Integrate[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(x^6*(e + f*x^2)^(5/2)),x]
```

output
```
Integrate[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(x^6*(e + f*x^2)^(5/2)), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^6\left(e+fx^2\right)^{5/2}}\,dx$$

$$\Big\downarrow \; 450$$

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^6\left(e+fx^2\right)^{5/2}}dx$$

input
```
Int[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(x^6*(e + f*x^2)^(5/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [F]

$$\int \frac{\sqrt{b\,x^2+a}\,\sqrt{x^2d+c}}{x^6\left(f\,x^2+e\right)^{\frac{5}{2}}}dx$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^6/(f*x^2+e)^(5/2),x)
```

output
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^6/(f*x^2+e)^(5/2),x)
```

Fricas [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^6\left(e+fx^2\right)^{5/2}}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{\left(fx^2+e\right)^{\frac{5}{2}}x^6}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^6/(f*x^2+e)^(5/2),x, algorithm
="fricas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(f^3*x^12 + 3*e*f
^2*x^10 + 3*e^2*f*x^8 + e^3*x^6), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^6\left(e+fx^2\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(1/2)/x**6/(f*x**2+e)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^6\left(e+fx^2\right)^{5/2}}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{\left(fx^2+e\right)^{\frac{5}{2}}x^6}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^6/(f*x^2+e)^(5/2),x, algorithm
="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/((f*x^2 + e)^(5/2)*x^6), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^6\left(e+fx^2\right)^{5/2}}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{\left(fx^2+e\right)^{\frac{5}{2}}x^6}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^6/(f*x^2+e)^(5/2),x, algorithm
="giac")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/((f*x^2 + e)^(5/2)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^6\left(e+fx^2\right)^{5/2}}\,dx = \int \frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{x^6\,\left(f\,x^2+e\right)^{5/2}}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2))/(x^6*(e + f*x^2)^(5/2)),x)
```

output
```
int(((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2))/(x^6*(e + f*x^2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^6\left(e+fx^2\right)^{5/2}}\,dx = \int \frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{x^6\,\left(f\,x^2+e\right)^{\frac{5}{2}}}dx$$

input
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^6/(f*x^2+e)^(5/2),x)
```

output
```
int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^6/(f*x^2+e)^(5/2),x)
```

3.338 $\int \frac{x^4\sqrt{e+fx^2}}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 697

$$\int \frac{x^4\sqrt{e+fx^2}}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$
$$= \frac{(bde-3bcf-3adf)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{8bd^2f\sqrt{a+bx^2}} + \frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{4bd}$$
$$- \frac{\sqrt{bc-ad}\,e(bde-3bcf-3adf)\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,E\left(\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)|\frac{c(be-af)}{(bc-ad)e}\right)}{8b^2\sqrt{c}d^2f\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$
$$- \frac{a(3a^2d^2f-abd(5de-2cf)-3b^2c(de-cf))\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right),\frac{c(be-a}{(bc-ad}\right.}{8b^3\sqrt{c}d^2\sqrt{bc-ad}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$
$$+ \frac{a(3a^2d^2f^2-2abdf(de-cf)-b^2(d^2e^2+2cdef-3c^2f^2))\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,\text{EllipticPi}\left(\frac{bc}{bc-ad},\arcsin\right.}{8b^3\sqrt{c}d^2\sqrt{bc-ad}f\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$

output
```
1/8*(-3*a*d*f-3*b*c*f+b*d*e)*x*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/b/d^2/f/(b*
x^2+a)^(1/2)+1/4*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/b/d-1/8
*(-a*d+b*c)^(1/2)*e*(-3*a*d*f-3*b*c*f+b*d*e)*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/
e/(b*x^2+a))^(1/2)*EllipticE((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2),(c
*(-a*f+b*e)/(-a*d+b*c)/e)^(1/2))/b^2/c^(1/2)/d^2/f/(a*(d*x^2+c)/c/(b*x^2+a
))^(1/2)/(f*x^2+e)^(1/2)-1/8*a*(3*a^2*d^2*f-a*b*d*(-2*c*f+5*d*e)-3*b^2*c*(
-c*f+d*e))*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)*EllipticF((-a*d
+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2),(c*(-a*f+b*e)/(-a*d+b*c)/e)^(1/2))/b
^3/c^(1/2)/d^2/(-a*d+b*c)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(f*x^2+e)^
(1/2)+1/8*a*(3*a^2*d^2*f^2-2*a*b*d*f*(-c*f+d*e)-b^2*(-3*c^2*f^2+2*c*d*e*f+
d^2*e^2))*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)*EllipticPi((-a*d
+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2),b*c/(-a*d+b*c),(c*(-a*f+b*e)/(-a*d+b
*c)/e)^(1/2))/b^3/c^(1/2)/d^2/(-a*d+b*c)^(1/2)/f/(a*(d*x^2+c)/c/(b*x^2+a))
^(1/2)/(f*x^2+e)^(1/2)
```

Mathematica [F]

$$\int \frac{x^4\sqrt{e+fx^2}}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx=\int \frac{x^4\sqrt{e+fx^2}}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

input
```
Integrate[(x^4*Sqrt[e + f*x^2])/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
Integrate[(x^4*Sqrt[e + f*x^2])/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4\sqrt{e+fx^2}}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

$$\Big\downarrow \text{ 450}$$

$$\int \frac{x^4\sqrt{e+fx^2}}{\sqrt{a+bx^2}\sqrt{c+dx^2}}dx$$

input
```
Int[(x^4*Sqrt[e + f*x^2])/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [F]

$$\int \frac{x^{4} \sqrt{f\,x^{2}+e}}{\sqrt{b\,x^{2}+a}\,\sqrt{x^{2} d+c}}dx$$

input
```
int(x^4*(f*x^2+e)^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

output
```
int(x^4*(f*x^2+e)^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^4\sqrt{e+fx^2}}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx=\text{Timed out}$$

input
```
integrate(x^4*(f*x^2+e)^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm
="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^4\sqrt{e+fx^2}}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx=\int \frac{x^4\sqrt{e+fx^2}}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

input
```
integrate(x**4*(f*x**2+e)**(1/2)/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(x**4*sqrt(e + f*x**2)/(sqrt(a + b*x**2)*sqrt(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{x^4\sqrt{e+fx^2}}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx=\int \frac{\sqrt{fx^2+e}x^4}{\sqrt{bx^2+a}\sqrt{dx^2+c}}\,dx$$

input
```
integrate(x^4*(f*x^2+e)^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm
="maxima")
```

output
```
integrate(sqrt(f*x^2 + e)*x^4/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

Giac [F]

$$\int \frac{x^4\sqrt{e+fx^2}}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx=\int \frac{\sqrt{fx^2+e}x^4}{\sqrt{bx^2+a}\sqrt{dx^2+c}}\,dx$$

input
```
integrate(x^4*(f*x^2+e)^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm
="giac")
```

output
```
integrate(sqrt(f*x^2 + e)*x^4/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4\sqrt{e+fx^2}}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{x^4\,\sqrt{f\,x^2+e}}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int((x^4*(e + f*x^2)^(1/2))/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int((x^4*(e + f*x^2)^(1/2))/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^4\sqrt{e+fx^2}}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$
$$= \frac{\sqrt{f\,x^2+e}\,\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x-3\left(\int \frac{\sqrt{f\,x^2+e}\,\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^4}{bdf\,x^6+adf\,x^4+bcf\,x^4+bde\,x^4+acf\,x^2+ade\,x^2+bce\,x^2+ace}dx\right)adf-3\left(\int \frac{}{bdf\,x^6}\right.}{}$$

input
```
int(x^4*(f*x^2+e)^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

output

```
(sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*x - 3*int((sqrt(e + f*
x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*e*
x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*
a*d*f - 3*int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a
*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*
d*e*x**4 + b*d*f*x**6),x)*b*c*f + int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*s
qrt(a + b*x**2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*
e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*b*d*e - 2*int((sqrt(e +
f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c*e + a*c*f*x**2 + a*d*
e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x
)*a*c*f - 2*int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/
(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 +
b*d*e*x**4 + b*d*f*x**6),x)*a*d*e - 2*int((sqrt(e + f*x**2)*sqrt(c + d*x**
2)*sqrt(a + b*x**2)*x**2)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 +
b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*b*c*e - int((sqrt(e
+ f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c*e + a*c*f*x**2 + a*d*e*x
**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*a
*c*e)/(4*b*d)
```

3.339 $\int \frac{x^2\sqrt{e+fx^2}}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 543

$$\int \frac{x^2\sqrt{e+fx^2}}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \frac{fx\sqrt{a+bx^2}\sqrt{c+dx^2}}{2bd\sqrt{e+fx^2}} - \frac{c\sqrt{-be+af}\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}E\left(\arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right)|\frac{a(de-cf)}{c(be-af)}\right)}{2\sqrt{a}bd\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}} + \frac{e\sqrt{-be+af}\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right),\frac{a(de-cf)}{c(be-af)}\right)}{2\sqrt{a}bf\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}} + \frac{e(bde-bcf-adf)\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,\text{EllipticPi}\left(-\frac{af}{be-af},\arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right),\frac{a(de-cf)}{c(be-af)}\right)}{2\sqrt{a}bdf\sqrt{-be+af}\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$

output
```
1/2*f*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b/d/(f*x^2+e)^(1/2)-1/2*c*(a*f-b*e
)^(1/2)*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticE((a*f-b*e
)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(1/
2)/b/d/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)+1/2*e*(a*f-b*e)^(1/
2)*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticF((a*f-b*e)^(1/
2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(1/2)/b/
f/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)+1/2*e*(-a*d*f-b*c*f+b*d*
e)*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticPi((a*f-b*e)^(1
/2)*x/a^(1/2)/(f*x^2+e)^(1/2),-a*f/(-a*f+b*e),(a*(-c*f+d*e)/c/(-a*f+b*e))^
(1/2))/a^(1/2)/b/d/f/(a*f-b*e)^(1/2)/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2
+e))^(1/2)
```

Mathematica [F]

$$\int \frac{x^2\sqrt{e+fx^2}}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{x^2\sqrt{e+fx^2}}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

input
```
Integrate[(x^2*Sqrt[e + f*x^2])/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
Integrate[(x^2*Sqrt[e + f*x^2])/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\sqrt{e+fx^2}}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$
$$\Big\downarrow \; 450$$
$$\int \frac{x^2\sqrt{e+fx^2}}{\sqrt{a+bx^2}\sqrt{c+dx^2}}dx$$

input
```
Int[(x^2*Sqrt[e + f*x^2])/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [F]

$$\int \frac{x^2\sqrt{f\,x^2+e}}{\sqrt{b\,x^2+a}\,\sqrt{x^2d+c}}dx$$

input
```
int(x^2*(f*x^2+e)^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

output
```
int(x^2*(f*x^2+e)^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{x^2\sqrt{e+fx^2}}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{fx^2+e}x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}\,dx$$

input
```
integrate(x^2*(f*x^2+e)^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm
="fricas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)*x^2/(b*d*x^4 + (b
*c + a*d)*x^2 + a*c), x)
```

Sympy [F]

$$\int \frac{x^2\sqrt{e+fx^2}}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{x^2\sqrt{e+fx^2}}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

input
```
integrate(x**2*(f*x**2+e)**(1/2)/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(x**2*sqrt(e + f*x**2)/(sqrt(a + b*x**2)*sqrt(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{x^2\sqrt{e+fx^2}}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{fx^2+e}x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}\,dx$$

input
```
integrate(x^2*(f*x^2+e)^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm
="maxima")
```

output
```
integrate(sqrt(f*x^2 + e)*x^2/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

Giac [F]

$$\int \frac{x^2\sqrt{e+fx^2}}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{fx^2+e}x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}\,dx$$

input
```
integrate(x^2*(f*x^2+e)^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm
="giac")
```

output
```
integrate(sqrt(f*x^2 + e)*x^2/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2\sqrt{e+fx^2}}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx=\int \frac{x^2\,\sqrt{f\,x^2+e}}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int((x^2*(e + f*x^2)^(1/2))/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int((x^2*(e + f*x^2)^(1/2))/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^2\sqrt{e+fx^2}}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx=\int \frac{\sqrt{f\,x^2+e}\,\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^2}{bd\,x^4+ad\,x^2+bc\,x^2+ac}dx$$

input
```
int(x^2*(f*x^2+e)^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

output
```
int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x
**2 + b*c*x**2 + b*d*x**4),x)
```

3.340 $\int \frac{\sqrt{e+fx^2}}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 160

$$\int \frac{\sqrt{e+fx^2}}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \frac{e\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,\text{EllipticPi}\left(-\frac{af}{be-af},\arcsin\left(\frac{\sqrt{-be+af}\,x}{\sqrt{a}\sqrt{e+fx^2}}\right),\frac{a(de-cf)}{c(be-af)}\right)}{\sqrt{a}\sqrt{-be+af}\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$

output
```
e*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticPi((a*f-b*e)^(1/
2)*x/a^(1/2)/(f*x^2+e)^(1/2),-a*f/(-a*f+b*e),(a*(-c*f+d*e)/c/(-a*f+b*e))^(
1/2))/a^(1/2)/(a*f-b*e)^(1/2)/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1
/2)
```

Mathematica [A] (verified)

Time = 2.65 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.99

$$\int \frac{\sqrt{e+fx^2}}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \frac{e\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,\text{EllipticPi}\left(\frac{af}{-be+af},\arcsin\left(\frac{\sqrt{-be+af}\,x}{\sqrt{a}\sqrt{e+fx^2}}\right),\frac{a(-de+cf)}{c(-be+af)}\right)}{\sqrt{a}\sqrt{-be+af}\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$

input
```
Integrate[Sqrt[e + f*x^2]/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
(e*Sqrt[a + b*x^2]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*EllipticPi[(a*f)/
(-(b*e) + a*f), ArcSin[(Sqrt[-(b*e) + a*f]*x)/(Sqrt[a]*Sqrt[e + f*x^2])],
(a*(-(d*e) + c*f))/(c*(-(b*e) + a*f))])/(Sqrt[a]*Sqrt[-(b*e) + a*f]*Sqrt[c
 + d*x^2]*Sqrt[(e*(a + b*x^2))/(a*(e + f*x^2))])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {428, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{e+fx^2}}{\sqrt{a+bx^2}\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \text{ 428}$$

$$\frac{e\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\int \frac{1}{\left(1-\frac{fx^2}{fx^2+e}\right)\sqrt{\frac{(be-af)x^2}{a\left(fx^2+e\right)}+1}\sqrt{\frac{(de-cf)x^2}{c\left(fx^2+e\right)}+1}}d\frac{x}{\sqrt{fx^2+e}}}{a\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$

$$\Big\downarrow \text{ 412}$$

$$\frac{e\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\text{EllipticPi}\left(-\frac{af}{be-af},\arcsin\left(\frac{\sqrt{af-be}x}{\sqrt{a}\sqrt{fx^2+e}}\right),\frac{a(de-cf)}{c(be-af)}\right)}{\sqrt{a}\sqrt{c+dx^2}\sqrt{af-be}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$

input
```
Int[Sqrt[e + f*x^2]/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
(e*Sqrt[a + b*x^2]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*EllipticPi[-((a*f
)/(b*e - a*f)), ArcSin[(Sqrt[-(b*e) + a*f]*x)/(Sqrt[a]*Sqrt[e + f*x^2])],
(a*(d*e - c*f))/(c*(b*e - a*f))])/(Sqrt[a]*Sqrt[-(b*e) + a*f]*Sqrt[c + d*x
^2]*Sqrt[(e*(a + b*x^2))/(a*(e + f*x^2))])
```

Defintions of rubi rules used

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 428
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/(Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*
(x_)^2]), x_Symbol] :> Simp[a*Sqrt[c + d*x^2]*(Sqrt[a*((e + f*x^2)/(e*(a +
b*x^2)))]/(c*Sqrt[e + f*x^2]*Sqrt[a*((c + d*x^2)/(c*(a + b*x^2)))]))   Subs
t[Int[1/((1 - b*x^2)*Sqrt[1 - (b*c - a*d)*(x^2/c)]*Sqrt[1 - (b*e - a*f)*(x^
2/e)]), x], x, x/Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Maple [F]

$$\int \frac{\sqrt{f\,x^{2}+e}}{\sqrt{b\,x^{2}+a}\,\sqrt{x^{2} d+c}}dx$$

input
```
int((f*x^2+e)^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

output
```
int((f*x^2+e)^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{e+fx^2}}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx=\text{Timed out}$$

input
```
integrate((f*x^2+e)^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{e+fx^2}}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{e+fx^2}}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

input
```
integrate((f*x**2+e)**(1/2)/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(sqrt(e + f*x**2)/(sqrt(a + b*x**2)*sqrt(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{e+fx^2}}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{fx^2+e}}{\sqrt{bx^2+a}\sqrt{dx^2+c}}\,dx$$

input
```
integrate((f*x^2+e)^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate(sqrt(f*x^2 + e)/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

Giac [F]

$$\int \frac{\sqrt{e+fx^2}}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{fx^2+e}}{\sqrt{bx^2+a}\sqrt{dx^2+c}}\,dx$$

input
```
integrate((f*x^2+e)^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate(sqrt(f*x^2 + e)/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{e+fx^2}}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx=\int \frac{\sqrt{f\,x^2+e}}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int((e + f*x^2)^(1/2)/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int((e + f*x^2)^(1/2)/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{e+fx^2}}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx=\int \frac{\sqrt{f\,x^2+e}\,\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{bd\,x^4+ad\,x^2+bc\,x^2+ac}dx$$

input
```
int((f*x^2+e)^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

output
```
int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c + a*d*x**2 +
 b*c*x**2 + b*d*x**4),x)
```

3.341 $\int \frac{\sqrt{e+fx^2}}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 296

$$\int \frac{\sqrt{e+fx^2}}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = -\frac{\sqrt{a-bx^2}\sqrt{e+fx^2}}{ax\sqrt{c+dx^2}}-\frac{\sqrt{bc+ad}\sqrt{e+fx^2}E\left(\arctan\left(\frac{\sqrt{bc+ad}x}{\sqrt{c}\sqrt{a-bx^2}}\right)|\frac{a(de-cf)}{(bc+ad)e}\right)}{a\sqrt{c}\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}+\frac{\sqrt{c}(be+af)\sqrt{e+fx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{bc+ad}x}{\sqrt{c}\sqrt{a-bx^2}}\right),\frac{a(de-cf)}{(bc+ad)e}\right)}{a\sqrt{bc+ad}e\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$

output
```
-(-b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/a/x/(d*x^2+c)^(1/2)-(a*d+b*c)^(1/2)*(f*x
^2+e)^(1/2)*EllipticE((a*d+b*c)^(1/2)*x/c^(1/2)/(-b*x^2+a)^(1/2)/(1+(a*d+b
*c)*x^2/c/(-b*x^2+a))^(1/2),(a*(-c*f+d*e)/(a*d+b*c)/e)^(1/2))/a/c^(1/2)/(d
*x^2+c)^(1/2)/(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)+c^(1/2)*(a*f+b*e)*(f*x^2+e)^
(1/2)*InverseJacobiAM(arctan((a*d+b*c)^(1/2)*x/c^(1/2)/(-b*x^2+a)^(1/2)),(
a*(-c*f+d*e)/(a*d+b*c)/e)^(1/2))/a/(a*d+b*c)^(1/2)/e/(d*x^2+c)^(1/2)/(c*(f
*x^2+e)/e/(d*x^2+c))^(1/2)
```

Mathematica [F]

$$\int \frac{\sqrt{e+fx^2}}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{e+fx^2}}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

input
```
Integrate[Sqrt[e + f*x^2]/(x^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
Integrate[Sqrt[e + f*x^2]/(x^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{e+fx^2}}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$
$$\Big\downarrow \; 450$$
$$\int \frac{\sqrt{e+fx^2}}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}dx$$

input
```
Int[Sqrt[e + f*x^2]/(x^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [F]

$$\int \frac{\sqrt{f\,x^2+e}}{x^2\sqrt{b\,x^2+a}\,\sqrt{x^2d+c}}dx$$

input
```
int((f*x^2+e)^(1/2)/x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

output
```
int((f*x^2+e)^(1/2)/x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{\sqrt{e+fx^2}}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{fx^2+e}}{\sqrt{bx^2+a}\sqrt{dx^2+cx^2}}\,dx$$

input
```
integrate((f*x^2+e)^(1/2)/x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm
="fricas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(b*d*x^6 + (b*c +
 a*d)*x^4 + a*c*x^2), x)
```

Sympy [F]

$$\int \frac{\sqrt{e+fx^2}}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{e+fx^2}}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

input
```
integrate((f*x**2+e)**(1/2)/x**2/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(sqrt(e + f*x**2)/(x**2*sqrt(a + b*x**2)*sqrt(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{e+fx^2}}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{fx^2+e}}{\sqrt{bx^2+a}\sqrt{dx^2+c}x^2}\,dx$$

input
```
integrate((f*x^2+e)^(1/2)/x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm
="maxima")
```

output
```
integrate(sqrt(f*x^2 + e)/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x^2), x)
```

Giac [F]

$$\int \frac{\sqrt{e+fx^2}}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{fx^2+e}}{\sqrt{bx^2+a}\sqrt{dx^2+c}x^2}\,dx$$

input
```
integrate((f*x^2+e)^(1/2)/x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm
="giac")
```

output
```
integrate(sqrt(f*x^2 + e)/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{e+fx^2}}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{f\,x^2+e}}{x^2\,\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int((e + f*x^2)^(1/2)/(x^2*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int((e + f*x^2)^(1/2)/(x^2*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{e+fx^2}}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{f\,x^2+e}\,\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{bd\,x^6+ad\,x^4+bc\,x^4+ac\,x^2}dx$$

input
```
int((f*x^2+e)^(1/2)/x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

output
```
int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c*x**2 + a*d*x
**4 + b*c*x**4 + b*d*x**6),x)
```

3.342 $\int \frac{\sqrt{e+fx^2}}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 438

$$\begin{aligned}\int \frac{\sqrt{e+fx^2}}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx &= \frac{(2bce+2ade-acf)\sqrt{a+bx^2}\sqrt{c+dx^2}}{3a^2c^2x\sqrt{e+fx^2}} - \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{3acx^3} \\ &+ \frac{\sqrt{-be+af}(2bce+2ade-acf)\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}E\left(\arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right)|\frac{a(de-cf)}{c(be-af)}\right)}{3a^{5/2}ce\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}} \\ &- \frac{(2bc+ad)\sqrt{-be+af}\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right),\frac{a(de-cf)}{c(be-af)}\right)}{3a^{5/2}c\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}\end{aligned}$$

output
```
1/3*(-a*c*f+2*a*d*e+2*b*c*e)*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a^2/c^2/x/(f*
x^2+e)^(1/2)-1/3*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/a/c/x^3+1
/3*(a*f-b*e)^(1/2)*(-a*c*f+2*a*d*e+2*b*c*e)*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c
/(f*x^2+e))^(1/2)*EllipticE((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(
-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(5/2)/c/e/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/
(f*x^2+e))^(1/2)-1/3*(a*d+2*b*c)*(a*f-b*e)^(1/2)*(b*x^2+a)^(1/2)*(e*(d*x^2
+c)/c/(f*x^2+e))^(1/2)*EllipticF((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2)
,(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(5/2)/c/(d*x^2+c)^(1/2)/(e*(b*x^2+a)
/a/(f*x^2+e))^(1/2)
```

Mathematica [F]

$$\int \frac{\sqrt{e+fx^2}}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx=\int \frac{\sqrt{e+fx^2}}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

input
```
Integrate[Sqrt[e + f*x^2]/(x^4*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
Integrate[Sqrt[e + f*x^2]/(x^4*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{e+fx^2}}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

$$\Big\downarrow \text{450}$$

$$\int \frac{\sqrt{e+fx^2}}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}dx$$

input
```
Int[Sqrt[e + f*x^2]/(x^4*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [F]

$$\int \frac{\sqrt{f\,x^2+e}}{x^4\sqrt{b\,x^2+a}\,\sqrt{x^2d+c}}dx$$

input
```
int((f*x^2+e)^(1/2)/x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

output
```
int((f*x^2+e)^(1/2)/x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{\sqrt{e+fx^2}}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx=\int \frac{\sqrt{fx^2+e}}{\sqrt{bx^2+a}\sqrt{dx^2+c}x^4}\,dx$$

input
```
integrate((f*x^2+e)^(1/2)/x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm
="fricas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(b*d*x^8 + (b*c +
 a*d)*x^6 + a*c*x^4), x)
```

Sympy [F]

$$\int \frac{\sqrt{e+fx^2}}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx=\int \frac{\sqrt{e+fx^2}}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

input
```
integrate((f*x**2+e)**(1/2)/x**4/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(sqrt(e + f*x**2)/(x**4*sqrt(a + b*x**2)*sqrt(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{e+fx^2}}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{fx^2+e}}{\sqrt{bx^2+a}\sqrt{dx^2+c}x^4}\,dx$$

input
```
integrate((f*x^2+e)^(1/2)/x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm
="maxima")
```

output
```
integrate(sqrt(f*x^2 + e)/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x^4), x)
```

Giac [F]

$$\int \frac{\sqrt{e+fx^2}}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{fx^2+e}}{\sqrt{bx^2+a}\sqrt{dx^2+c}x^4}\,dx$$

input
```
integrate((f*x^2+e)^(1/2)/x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm
="giac")
```

output
```
integrate(sqrt(f*x^2 + e)/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{e+fx^2}}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{f\,x^2+e}}{x^4\,\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int((e + f*x^2)^(1/2)/(x^4*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int((e + f*x^2)^(1/2)/(x^4*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{e+fx^2}}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{f\,x^2+e}}{x^4\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}dx$$

input
```
int((f*x^2+e)^(1/2)/x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

output
```
int((f*x^2+e)^(1/2)/x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

3.343 $\int \frac{\sqrt{e+fx^2}}{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 608

$$\int \frac{\sqrt{e+fx^2}}{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

$$= -\frac{\left(\frac{8b^2ce^2}{a}+be(7de-3cf)+a\left(\frac{8d^2e^2}{c}-3def-2cf^2\right)\right)\sqrt{a+bx^2}\sqrt{c+dx^2}}{15a^2c^2ex\sqrt{e+fx^2}}$$

$$-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{5acx^5}+\frac{(4bce+4ade-acf)\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{15a^2c^2ex^3}$$

$$-\frac{\sqrt{-be+af}\,(8b^2c^2e^2+abce(7de-3cf)+a^2(8d^2e^2-3cdef-2c^2f^2))\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,E\Big(\arcsin\Big(\cdots}{15a^{7/2}c^2e^2\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$

$$+\frac{\sqrt{-be+af}\,(8b^2c^2e+a^2d(4de-cf)+abc(3de+cf))\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,\text{EllipticF}\Big(\arcsin\Big(\frac{\sqrt{-be+af}x\cdots}{\sqrt{a}\sqrt{e+fx^2}\cdots}}{15a^{7/2}c^2e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$

output
```
-1/15*(8*b^2*c*e^2/a+b*e*(-3*c*f+7*d*e)+a*(8*d^2*e^2/c-3*d*e*f-2*c*f^2))*(
b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a^2/c^2/e/x/(f*x^2+e)^(1/2)-1/5*(b*x^2+a)^(
1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/a/c/x^5+1/15*(-a*c*f+4*a*d*e+4*b*c*e)
*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/a^2/c^2/e/x^3-1/15*(a*f-b
*e)^(1/2)*(8*b^2*c^2*e^2+a*b*c*e*(-3*c*f+7*d*e)+a^2*(-2*c^2*f^2-3*c*d*e*f+
8*d^2*e^2))*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticE((a*f
-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a
^(7/2)/c^2/e^2/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)+1/15*(a*f-b
*e)^(1/2)*(8*b^2*c^2*e+a^2*d*(-c*f+4*d*e)+a*b*c*(c*f+3*d*e))*(b*x^2+a)^(1/
2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticF((a*f-b*e)^(1/2)*x/a^(1/2)/(f*
x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(7/2)/c^2/e/(d*x^2+c)^(1
/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)
```

Mathematica [F]

$$\int \frac{\sqrt{e+fx^2}}{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx=\int \frac{\sqrt{e+fx^2}}{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

input
```
Integrate[Sqrt[e + f*x^2]/(x^6*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
Integrate[Sqrt[e + f*x^2]/(x^6*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{e+fx^2}}{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$
$$\Big\downarrow \text{450}$$
$$\int \frac{\sqrt{e+fx^2}}{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}}dx$$

input
```
Int[Sqrt[e + f*x^2]/(x^6*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [F]

$$\int \frac{\sqrt{f\,x^{2}+e}}{x^{6}\sqrt{b\,x^{2}+a}\,\sqrt{x^{2} d+c}}dx$$

input
```
int((f*x^2+e)^(1/2)/x^6/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

output
```
int((f*x^2+e)^(1/2)/x^6/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{\sqrt{e+fx^2}}{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{fx^2+e}}{\sqrt{bx^2+a}\sqrt{dx^2+c}x^6}\,dx$$

input
```
integrate((f*x^2+e)^(1/2)/x^6/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm
="fricas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(b*d*x^10 + (b*c
+ a*d)*x^8 + a*c*x^6), x)
```

Sympy [F]

$$\int \frac{\sqrt{e+fx^2}}{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{e+fx^2}}{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

input
```
integrate((f*x**2+e)**(1/2)/x**6/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(sqrt(e + f*x**2)/(x**6*sqrt(a + b*x**2)*sqrt(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{e+fx^2}}{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{fx^2+e}}{\sqrt{bx^2+a}\sqrt{dx^2+c}x^6}\,dx$$

input
```
integrate((f*x^2+e)^(1/2)/x^6/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm
="maxima")
```

output
```
integrate(sqrt(f*x^2 + e)/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x^6), x)
```

Giac [F]

$$\int \frac{\sqrt{e+fx^2}}{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{fx^2+e}}{\sqrt{bx^2+a}\sqrt{dx^2+c}x^6}\,dx$$

input
```
integrate((f*x^2+e)^(1/2)/x^6/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm
="giac")
```

output
```
integrate(sqrt(f*x^2 + e)/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{e+fx^2}}{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx=\int \frac{\sqrt{f\,x^2+e}}{x^6\,\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int((e + f*x^2)^(1/2)/(x^6*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int((e + f*x^2)^(1/2)/(x^6*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{e+fx^2}}{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx=\int \frac{\sqrt{f\,x^2+e}}{x^6\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}dx$$

input
```
int((f*x^2+e)^(1/2)/x^6/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

output
```
int((f*x^2+e)^(1/2)/x^6/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

3.344 $\int \frac{x^4(e+fx^2)^{3/2}}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 908

$$\int \frac{x^4(e+fx^2)^{3/2}}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \frac{(15a^2d^2f^2-2abdf(11de-7cf)+b^2(3d^2e^2-22cdef+15c^2f^2))\,x\sqrt{c+dx^2}\sqrt{e+fx^2}}{48b^2d^3f\sqrt{a+bx^2}}$$
$$+\frac{(7bde-5bcf-5adf)x\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{24b^2d^2}+\frac{fx^3\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{6bd}$$
$$-\frac{\sqrt{bc-ad}\,e(15a^2d^2f^2-2abdf(11de-7cf)+b^2(3d^2e^2-22cdef+15c^2f^2))\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,E\Big(\arcsin\cdots}{48b^3\sqrt{c}d^3f\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$
$$+\frac{a(15a^3d^3f^2-3a^2bd^2f(14de-3cf)+ab^2d(31d^2e^2-22cdef+9c^2f^2)+b^3c(17d^2e^2-32cdef+15c^2f^2))\cdots}{48b^4\sqrt{c}d^3\sqrt{bc-ad}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$
$$-\frac{a(5a^3d^3f^3+3ab^2df(de-cf)^2-3a^2bd^2f^2(3de-cf)+b^3(de-cf)^2(de+5cf))\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,\mathrm{Elli}\cdots}{16b^4\sqrt{c}d^3\sqrt{bc-ad}f\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$

output
```
1/48*(15*a^2*d^2*f^2-2*a*b*d*f*(-7*c*f+11*d*e)+b^2*(15*c^2*f^2-22*c*d*e*f+
3*d^2*e^2))*x*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/b^2/d^3/f/(b*x^2+a)^(1/2)+1/
24*(-5*a*d*f-5*b*c*f+7*b*d*e)*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^
(1/2)/b^2/d^2+1/6*f*x^3*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/b/
d-1/48*(-a*d+b*c)^(1/2)*e*(15*a^2*d^2*f^2-2*a*b*d*f*(-7*c*f+11*d*e)+b^2*(1
5*c^2*f^2-22*c*d*e*f+3*d^2*e^2))*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))
^(1/2)*EllipticE((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2),(c*(-a*f+b*e)/
(-a*d+b*c)/e)^(1/2))/b^3/c^(1/2)/d^3/f/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(f*
x^2+e)^(1/2)+1/48*a*(15*a^3*d^3*f^2-3*a^2*b*d^2*f*(-3*c*f+14*d*e)+a*b^2*d*
(9*c^2*f^2-22*c*d*e*f+31*d^2*e^2)+b^3*c*(15*c^2*f^2-32*c*d*e*f+17*d^2*e^2)
)*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)*EllipticF((-a*d+b*c)^(1/
2)*x/c^(1/2)/(b*x^2+a)^(1/2),(c*(-a*f+b*e)/(-a*d+b*c)/e)^(1/2))/b^4/c^(1/2
)/d^3/(-a*d+b*c)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(f*x^2+e)^(1/2)-1/1
6*a*(5*a^3*d^3*f^3+3*a*b^2*d*f*(-c*f+d*e)^2-3*a^2*b*d^2*f^2*(-c*f+3*d*e)+b
^3*(-c*f+d*e)^2*(5*c*f+d*e))*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/
2)*EllipticPi((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2),b*c/(-a*d+b*c),(c
*(-a*f+b*e)/(-a*d+b*c)/e)^(1/2))/b^4/c^(1/2)/d^3/(-a*d+b*c)^(1/2)/f/(a*(d*
x^2+c)/c/(b*x^2+a))^(1/2)/(f*x^2+e)^(1/2)
```

Mathematica [F]

$$\int \frac{x^4(e+fx^2)^{3/2}}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{x^4(e+fx^2)^{3/2}}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

input
```
Integrate[(x^4*(e + f*x^2)^(3/2))/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
Integrate[(x^4*(e + f*x^2)^(3/2))/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 (e+fx^2)^{3/2}}{\sqrt{a+bx^2}\sqrt{c+dx^2}} dx$$
$$\Big\downarrow \; 450$$
$$\int \frac{x^4 (e+fx^2)^{3/2}}{\sqrt{a+bx^2}\sqrt{c+dx^2}} dx$$

input
```
Int[(x^4*(e + f*x^2)^(3/2))/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [F]

$$\int \frac{x^4 (f\,x^2+e)^{\frac{3}{2}}}{\sqrt{b\,x^2+a}\,\sqrt{x^2 d+c}} dx$$

input
```
int(x^4*(f*x^2+e)^(3/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

output
```
int(x^4*(f*x^2+e)^(3/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{x^4(e+fx^2)^{3/2}}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{(fx^2+e)^{\frac{3}{2}}x^4}{\sqrt{bx^2+a}\sqrt{dx^2+c}}\,dx$$

input
```
integrate(x^4*(f*x^2+e)^(3/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm
="fricas")
```

output
```
integral((f*x^6 + e*x^4)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(
b*d*x^4 + (b*c + a*d)*x^2 + a*c), x)
```

Sympy [F]

$$\int \frac{x^4(e+fx^2)^{3/2}}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{x^4(e+fx^2)^{\frac{3}{2}}}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

input
```
integrate(x**4*(f*x**2+e)**(3/2)/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(x**4*(e + f*x**2)**(3/2)/(sqrt(a + b*x**2)*sqrt(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{x^4(e+fx^2)^{3/2}}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{(fx^2+e)^{\frac{3}{2}}x^4}{\sqrt{bx^2+a}\sqrt{dx^2+c}}\,dx$$

input
```
integrate(x^4*(f*x^2+e)^(3/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm
="maxima")
```

output
```
integrate((f*x^2 + e)^(3/2)*x^4/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

Giac [F]

$$\int \frac{x^4(e+fx^2)^{3/2}}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{(fx^2+e)^{\frac{3}{2}}x^4}{\sqrt{bx^2+a}\sqrt{dx^2+c}}\,dx$$

input
```
integrate(x^4*(f*x^2+e)^(3/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm
="giac")
```

output
```
integrate((f*x^2 + e)^(3/2)*x^4/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(e+fx^2)^{3/2}}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{x^4\,(f\,x^2+e)^{3/2}}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int((x^4*(e + f*x^2)^(3/2))/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int((x^4*(e + f*x^2)^(3/2))/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^4(e+fx^2)^{3/2}}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \text{too large to display}$$

input
```
int(x^4*(f*x^2+e)^(3/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

output
```
( - 5*sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*d*f*x - 5*sqrt(
e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b*c*f*x + 7*sqrt(e + f*x**2)
*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b*d*e*x + 4*sqrt(e + f*x**2)*sqrt(c + d
*x**2)*sqrt(a + b*x**2)*b*d*f*x**3 + 15*int((sqrt(e + f*x**2)*sqrt(c + d*x
**2)*sqrt(a + b*x**2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4
+ b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*a**2*d**2*f**2 + 1
4*int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c*e + a
*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4
 + b*d*f*x**6),x)*a*b*c*d*f**2 - 22*int((sqrt(e + f*x**2)*sqrt(c + d*x**2)
*sqrt(a + b*x**2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*
c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*a*b*d**2*e*f + 15*int(
(sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c*e + a*c*f*x
**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d
*f*x**6),x)*b**2*c**2*f**2 - 22*int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqr
t(a + b*x**2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*
x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*b**2*c*d*e*f + 3*int((sqrt
(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c*e + a*c*f*x**2 +
 a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x*
*6),x)*b**2*d**2*e**2 + 10*int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a +
 b*x**2)*x**2)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x*...
```

3.345 $\int \frac{x^2(e+fx^2)^{3/2}}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 696

$$\int \frac{x^2(e+fx^2)^{3/2}}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \frac{(5bde-3bcf-3adf)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{8bd^2\sqrt{a+bx^2}} + \frac{fx\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{4bd} - \frac{\sqrt{bc-ad}e(5bde-3bcf-3adf)\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}E\left(\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)|\frac{c(be-af)}{(bc-ad)e}\right)}{8b^2\sqrt{c}d^2\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}} - \frac{a(3a^2d^2f^2-abdf(9de-2cf)+b^2(8d^2e^2-7cdef+3c^2f^2))\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{bc-}}{\sqrt{c}\sqrt{a}}\right.\right.}{8b^3\sqrt{c}d^2\sqrt{bc-ad}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}} + \frac{a(3a^2d^2f^2+3b^2(de-cf)^2-2abdf(3de-cf))\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\operatorname{EllipticPi}\left(\frac{bc}{bc-ad},\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right),\right.}{8b^3\sqrt{c}d^2\sqrt{bc-ad}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$

output
```
1/8*(-3*a*d*f-3*b*c*f+5*b*d*e)*x*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/b/d^2/(b*
x^2+a)^(1/2)+1/4*f*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/b/d-1
/8*(-a*d+b*c)^(1/2)*e*(-3*a*d*f-3*b*c*f+5*b*d*e)*(d*x^2+c)^(1/2)*(a*(f*x^2
+e)/e/(b*x^2+a))^(1/2)*EllipticE((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2
),(c*(-a*f+b*e)/(-a*d+b*c)/e)^(1/2))/b^2/c^(1/2)/d^2/(a*(d*x^2+c)/c/(b*x^2
+a))^(1/2)/(f*x^2+e)^(1/2)-1/8*a*(3*a^2*d^2*f^2-a*b*d*f*(-2*c*f+9*d*e)+b^2
*(3*c^2*f^2-7*c*d*e*f+8*d^2*e^2))*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a)
)^(1/2)*EllipticF((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2),(c*(-a*f+b*e)
/(-a*d+b*c)/e)^(1/2))/b^3/c^(1/2)/d^2/(-a*d+b*c)^(1/2)/(a*(d*x^2+c)/c/(b*x
^2+a))^(1/2)/(f*x^2+e)^(1/2)+1/8*a*(3*a^2*d^2*f^2+3*b^2*(-c*f+d*e)^2-2*a*b
*d*f*(-c*f+3*d*e))*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)*Ellipti
cPi((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2),b*c/(-a*d+b*c),(c*(-a*f+b*e
)/(-a*d+b*c)/e)^(1/2))/b^3/c^(1/2)/d^2/(-a*d+b*c)^(1/2)/(a*(d*x^2+c)/c/(b*
x^2+a))^(1/2)/(f*x^2+e)^(1/2)
```

Mathematica [F]

$$\int \frac{x^2(e+fx^2)^{3/2}}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{x^2(e+fx^2)^{3/2}}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

input
```
Integrate[(x^2*(e + f*x^2)^(3/2))/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
Integrate[(x^2*(e + f*x^2)^(3/2))/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\left(e+fx^2\right)^{3/2}}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

$$\Big\downarrow \text{ 450}$$

$$\int \frac{x^2\left(e+fx^2\right)^{3/2}}{\sqrt{a+bx^2}\sqrt{c+dx^2}}dx$$

input
```
Int[(x^2*(e + f*x^2)^(3/2))/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [F]

$$\int \frac{x^{2}\left(f\,x^{2}+e\right)^{\frac{3}{2}}}{\sqrt{b\,x^{2}+a}\,\sqrt{x^{2}d+c}}dx$$

input
```
int(x^2*(f*x^2+e)^(3/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

output
```
int(x^2*(f*x^2+e)^(3/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{x^2(e+fx^2)^{3/2}}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx=\int \frac{(fx^2+e)^{\frac{3}{2}}x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}\,dx$$

input
```
integrate(x^2*(f*x^2+e)^(3/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm
="fricas")
```

output
```
integral((f*x^4 + e*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(
b*d*x^4 + (b*c + a*d)*x^2 + a*c), x)
```

Sympy [F]

$$\int \frac{x^2(e+fx^2)^{3/2}}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{x^2(e+fx^2)^{\frac{3}{2}}}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

input
```
integrate(x**2*(f*x**2+e)**(3/2)/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(x**2*(e + f*x**2)**(3/2)/(sqrt(a + b*x**2)*sqrt(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{x^2(e+fx^2)^{3/2}}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{(fx^2+e)^{\frac{3}{2}}x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}\,dx$$

input
```
integrate(x^2*(f*x^2+e)^(3/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm
="maxima")
```

output
```
integrate((f*x^2 + e)^(3/2)*x^2/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

Giac [F]

$$\int \frac{x^2(e+fx^2)^{3/2}}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{(fx^2+e)^{\frac{3}{2}}x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}\,dx$$

input
```
integrate(x^2*(f*x^2+e)^(3/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm
="giac")
```

output
```
integrate((f*x^2 + e)^(3/2)*x^2/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(e+fx^2)^{3/2}}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx=\int \frac{x^2\,{\left(f\,x^2+e\right)}^{3/2}}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int((x^2*(e + f*x^2)^(3/2))/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int((x^2*(e + f*x^2)^(3/2))/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^2(e+fx^2)^{3/2}}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx=\frac{\sqrt{f\,x^2+e}\,\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,fx-3\left(\int \frac{\sqrt{f\,x^2+e}\,\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^4}{bdf\,x^6+adf\,x^4+bcf\,x^4+bde\,x^4+acf\,x^2+ade\,x^2+b\cdots}\right.}{}$$

input
```
int(x^2*(f*x^2+e)^(3/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

output
```
(sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*f*x - 3*int((sqrt(e +
f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*
e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x
)*a*d*f**2 - 3*int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**
4)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4
 + b*d*e*x**4 + b*d*f*x**6),x)*b*c*f**2 + 5*int((sqrt(e + f*x**2)*sqrt(c +
 d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x
**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*b*d*e*f - 2*in
t((sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c*e + a*c*f
*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b
*d*f*x**6),x)*a*c*f**2 - 2*int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a +
 b*x**2)*x**2)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2
+ b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*a*d*e*f - 2*int((sqrt(e + f*x**
2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c*e + a*c*f*x**2 + a*d*e*x**
2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*b*c
*e*f + 4*int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*
c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d
*e*x**4 + b*d*f*x**6),x)*b*d*e**2 - int((sqrt(e + f*x**2)*sqrt(c + d*x**2)
*sqrt(a + b*x**2))/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x
**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*a*c*e*f)/(4*b*d)
```

3.346 $\int \frac{(e+fx^2)^{3/2}}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 541

$$\begin{aligned}\int \frac{(e+fx^2)^{3/2}}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx &= \frac{f^2x\sqrt{a+bx^2}\sqrt{c+dx^2}}{2bd\sqrt{e+fx^2}} \\
&- \frac{cf\sqrt{-be+af}\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}E\left(\arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right)|\frac{a(de-cf)}{c(be-af)}\right)}{2\sqrt{a}bd\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}} \\
&+ \frac{e\sqrt{-be+af}\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right),\frac{a(de-cf)}{c(be-af)}\right)}{2\sqrt{a}b\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}} \\
&+ \frac{e(3bde-bcf-adf)\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\operatorname{EllipticPi}\left(-\frac{af}{be-af},\arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right),\frac{a(de-cf)}{c(be-af)}\right)}{2\sqrt{a}bd\sqrt{-be+af}\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}\end{aligned}$$

output
```
1/2*f^2*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b/d/(f*x^2+e)^(1/2)-1/2*c*f*(a*f
-b*e)^(1/2)*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticE((a*f
-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a
^(1/2)/b/d/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)+1/2*e*(a*f-b*e)
^(1/2)*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticF((a*f-b*e)
^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(1/2
)/b/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)+1/2*e*(-a*d*f-b*c*f+3*
b*d*e)*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticPi((a*f-b*e
)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),-a*f/(-a*f+b*e),(a*(-c*f+d*e)/c/(-a*f+b*
e))^(1/2))/a^(1/2)/b/d/(a*f-b*e)^(1/2)/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x
^2+e))^(1/2)
```

Mathematica [F]

$$\int \frac{(e+fx^2)^{3/2}}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{(e+fx^2)^{3/2}}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

input
```
Integrate[(e + f*x^2)^(3/2)/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
Integrate[(e + f*x^2)^(3/2)/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx^2)^{3/2}}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

$$\Big\downarrow \text{434}$$

$$\int \frac{(e+fx^2)^{3/2}}{\sqrt{a+bx^2}\sqrt{c+dx^2}}dx$$

input
```
Int[(e + f*x^2)^(3/2)/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{(f\,x^{2}+e)^{\frac{3}{2}}}{\sqrt{b\,x^{2}+a}\,\sqrt{x^{2}d+c}}dx$$

input
```
int((f*x^2+e)^(3/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

output
```
int((f*x^2+e)^(3/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(e+fx^2)^{3/2}}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \text{Timed out}$$

input
```
integrate((f*x^2+e)^(3/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(e + fx^2)^{3/2}}{\sqrt{a + bx^2}\sqrt{c + dx^2}}\,dx = \int \frac{(e + fx^2)^{\frac{3}{2}}}{\sqrt{a + bx^2}\sqrt{c + dx^2}}\,dx$$

input
```
integrate((f*x**2+e)**(3/2)/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral((e + f*x**2)**(3/2)/(sqrt(a + b*x**2)*sqrt(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{(e + fx^2)^{3/2}}{\sqrt{a + bx^2}\sqrt{c + dx^2}}\,dx = \int \frac{(fx^2 + e)^{\frac{3}{2}}}{\sqrt{bx^2 + a}\sqrt{dx^2 + c}}\,dx$$

input
```
integrate((f*x^2+e)^(3/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate((f*x^2 + e)^(3/2)/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

Giac [F]

$$\int \frac{(e + fx^2)^{3/2}}{\sqrt{a + bx^2}\sqrt{c + dx^2}}\,dx = \int \frac{(fx^2 + e)^{\frac{3}{2}}}{\sqrt{bx^2 + a}\sqrt{dx^2 + c}}\,dx$$

input
```
integrate((f*x^2+e)^(3/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate((f*x^2 + e)^(3/2)/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx^2)^{3/2}}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx=\int \frac{(f\,x^2+e)^{3/2}}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int((e + f*x^2)^(3/2)/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int((e + f*x^2)^(3/2)/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{(e+fx^2)^{3/2}}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx=\left(\int \frac{\sqrt{f\,x^2+e}\,\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^2}{bd\,x^4+ad\,x^2+bc\,x^2+ac}dx\right)f+\left(\int \frac{\sqrt{f\,x^2+e}\,\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{bd\,x^4+ad\,x^2+bc\,x^2+ac}dx\right)e$$

input
```
int((f*x^2+e)^(3/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

output
```
int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x
**2 + b*c*x**2 + b*d*x**4),x)*f + int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*s
qrt(a + b*x**2))/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*e
```

3.347 $\int \frac{(e+fx^2)^{3/2}}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 527

$$
\begin{aligned}
\int \frac{(e+fx^2)^{3/2}}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = &-\frac{e\sqrt{c+dx^2}\sqrt{e+fx^2}}{cx\sqrt{a+bx^2}} \\
&-\frac{\sqrt{bc-ad}e^2\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}E\left(\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)|\frac{c(be-af)}{(bc-ad)e}\right)}{ac^{3/2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}} \\
&-\frac{(acf^2+be(de-2cf))\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right),\frac{c(be-af)}{(bc-ad)e}\right)}{bc^{3/2}\sqrt{bc-ad}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}} \\
&+\frac{af^2\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,\text{EllipticPi}\left(\frac{bc}{bc-ad},\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right),\frac{c(be-af)}{(bc-ad)e}\right)}{b\sqrt{c}\sqrt{bc-ad}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}
\end{aligned}
$$

output
```
-e*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/c/x/(b*x^2+a)^(1/2)-(-a*d+b*c)^(1/2)*e^
2*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)*EllipticE((-a*d+b*c)^(1/
2)*x/c^(1/2)/(b*x^2+a)^(1/2),(c*(-a*f+b*e)/(-a*d+b*c)/e)^(1/2))/a/c^(3/2)/
(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(f*x^2+e)^(1/2)-(a*c*f^2+b*e*(-2*c*f+d*e))
*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)*EllipticF((-a*d+b*c)^(1/2
)*x/c^(1/2)/(b*x^2+a)^(1/2),(c*(-a*f+b*e)/(-a*d+b*c)/e)^(1/2))/b/c^(3/2)/(
-a*d+b*c)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(f*x^2+e)^(1/2)+a*f^2*(d*x
^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)*EllipticPi((-a*d+b*c)^(1/2)*x/
c^(1/2)/(b*x^2+a)^(1/2),b*c/(-a*d+b*c),(c*(-a*f+b*e)/(-a*d+b*c)/e)^(1/2))/
b/c^(1/2)/(-a*d+b*c)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(f*x^2+e)^(1/2)
```

Mathematica [F]

$$\int \frac{(e+fx^2)^{3/2}}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{(e+fx^2)^{3/2}}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

input
```
Integrate[(e + f*x^2)^(3/2)/(x^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
Integrate[(e + f*x^2)^(3/2)/(x^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx^2)^{3/2}}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$
$$\Big\downarrow \text{450}$$
$$\int \frac{(e+fx^2)^{3/2}}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}dx$$

input
```
Int[(e + f*x^2)^(3/2)/(x^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [F]

$$\int \frac{\left(f\,x^{2}+e\right)^{\frac{3}{2}}}{x^{2}\sqrt{b\,x^{2}+a}\,\sqrt{x^{2}d+c}}dx$$

input
```
int((f*x^2+e)^(3/2)/x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

output
```
int((f*x^2+e)^(3/2)/x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{(e+fx^2)^{3/2}}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{\left(fx^2+e\right)^{\frac{3}{2}}}{\sqrt{bx^2+a}\sqrt{dx^2+c}x^2}\,dx$$

input
```
integrate((f*x^2+e)^(3/2)/x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm
="fricas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)/(b*d*x^6 + (b*c
 + a*d)*x^4 + a*c*x^2), x)
```

Sympy [F]

$$\int \frac{(e + fx^2)^{3/2}}{x^2\sqrt{a + bx^2}\sqrt{c + dx^2}}\,dx = \int \frac{(e + fx^2)^{\frac{3}{2}}}{x^2\sqrt{a + bx^2}\sqrt{c + dx^2}}\,dx$$

input
```
integrate((f*x**2+e)**(3/2)/x**2/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral((e + f*x**2)**(3/2)/(x**2*sqrt(a + b*x**2)*sqrt(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{(e + fx^2)^{3/2}}{x^2\sqrt{a + bx^2}\sqrt{c + dx^2}}\,dx = \int \frac{(fx^2 + e)^{\frac{3}{2}}}{\sqrt{bx^2 + a}\sqrt{dx^2 + c}x^2}\,dx$$

input
```
integrate((f*x^2+e)^(3/2)/x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm
="maxima")
```

output
```
integrate((f*x^2 + e)^(3/2)/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x^2), x)
```

Giac [F]

$$\int \frac{(e + fx^2)^{3/2}}{x^2\sqrt{a + bx^2}\sqrt{c + dx^2}}\,dx = \int \frac{(fx^2 + e)^{\frac{3}{2}}}{\sqrt{bx^2 + a}\sqrt{dx^2 + c}x^2}\,dx$$

input
```
integrate((f*x^2+e)^(3/2)/x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm
="giac")
```

output
```
integrate((f*x^2 + e)^(3/2)/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx^2)^{3/2}}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{(f\,x^2+e)^{3/2}}{x^2\,\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int((e + f*x^2)^(3/2)/(x^2*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int((e + f*x^2)^(3/2)/(x^2*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{(e+fx^2)^{3/2}}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{(f\,x^2+e)^{\frac{3}{2}}}{x^2\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}dx$$

input
```
int((f*x^2+e)^(3/2)/x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

output
```
int((f*x^2+e)^(3/2)/x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

3.348 $\int \frac{(e+fx^2)^{3/2}}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 440

$$\int \frac{(e+fx^2)^{3/2}}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \frac{2e(bce+ade-2acf)\sqrt{a+bx^2}\sqrt{c+dx^2}}{3a^2c^2x\sqrt{e+fx^2}} - \frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{3acx^3} + \frac{2\sqrt{-be+af}(bce+ade-2acf)\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}E\left(\arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right)|\frac{a(de-cf)}{c(be-af)}\right)}{3a^{5/2}c\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}} - \frac{\sqrt{-be+af}(2bce+ade-3acf)\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right),\frac{a(de-cf)}{c(be-af)}\right)}{3a^{5/2}c\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$

output

```
2/3*e*(-2*a*c*f+a*d*e+b*c*e)*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a^2/c^2/x/(f*
x^2+e)^(1/2)-1/3*e*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/a/c/x^3
+2/3*(a*f-b*e)^(1/2)*(-2*a*c*f+a*d*e+b*c*e)*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c
/(f*x^2+e))^(1/2)*EllipticE((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(
-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(5/2)/c/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f
*x^2+e))^(1/2)-1/3*(a*f-b*e)^(1/2)*(-3*a*c*f+a*d*e+2*b*c*e)*(b*x^2+a)^(1/2
)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticF((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x
^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(5/2)/c/(d*x^2+c)^(1/2)/(
e*(b*x^2+a)/a/(f*x^2+e))^(1/2)
```

Mathematica [F]

$$\int \frac{(e+fx^2)^{3/2}}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{(e+fx^2)^{3/2}}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

input
```
Integrate[(e + f*x^2)^(3/2)/(x^4*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
Integrate[(e + f*x^2)^(3/2)/(x^4*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx^2)^{3/2}}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

$$\Big\downarrow \text{450}$$

$$\int \frac{(e+fx^2)^{3/2}}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}dx$$

input
```
Int[(e + f*x^2)^(3/2)/(x^4*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [F]

$$\int \frac{\left(f\,x^{2}+e\right)^{\frac{3}{2}}}{x^{4}\sqrt{b\,x^{2}+a}\,\sqrt{x^{2}d+c}}dx$$

input
```
int((f*x^2+e)^(3/2)/x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

output
```
int((f*x^2+e)^(3/2)/x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{(e+fx^2)^{3/2}}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx=\int \frac{\left(fx^2+e\right)^{\frac{3}{2}}}{\sqrt{bx^2+a}\sqrt{dx^2+c}x^4}\,dx$$

input
```
integrate((f*x^2+e)^(3/2)/x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm
="fricas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)/(b*d*x^8 + (b*c
 + a*d)*x^6 + a*c*x^4), x)
```

Sympy [F]

$$\int \frac{(e+fx^2)^{3/2}}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{(e+fx^2)^{\frac{3}{2}}}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

input
```
integrate((f*x**2+e)**(3/2)/x**4/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral((e + f*x**2)**(3/2)/(x**4*sqrt(a + b*x**2)*sqrt(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{(e+fx^2)^{3/2}}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{(fx^2+e)^{\frac{3}{2}}}{\sqrt{bx^2+a}\sqrt{dx^2+c}x^4}\,dx$$

input
```
integrate((f*x^2+e)^(3/2)/x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm
="maxima")
```

output
```
integrate((f*x^2 + e)^(3/2)/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x^4), x)
```

Giac [F]

$$\int \frac{(e+fx^2)^{3/2}}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{(fx^2+e)^{\frac{3}{2}}}{\sqrt{bx^2+a}\sqrt{dx^2+c}x^4}\,dx$$

input
```
integrate((f*x^2+e)^(3/2)/x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm
="giac")
```

output
```
integrate((f*x^2 + e)^(3/2)/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx^2)^{3/2}}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx=\int \frac{(f\,x^2+e)^{3/2}}{x^4\,\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int((e + f*x^2)^(3/2)/(x^4*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int((e + f*x^2)^(3/2)/(x^4*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{(e+fx^2)^{3/2}}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx=\int \frac{(f\,x^2+e)^{\frac{3}{2}}}{x^4\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}dx$$

input
```
int((f*x^2+e)^(3/2)/x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

output
```
int((f*x^2+e)^(3/2)/x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

3.349 $\int \frac{(e+fx^2)^{3/2}}{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 605

$$\int \frac{(e+fx^2)^{3/2}}{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx =$$
$$-\frac{(8b^2c^2e^2+abce(7de-13cf)+a^2(8d^2e^2-13cdef+3c^2f^2))\sqrt{a+bx^2}\sqrt{c+dx^2}}{15a^3c^3x\sqrt{e+fx^2}}$$
$$-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{5acx^5}+\frac{2(2bce+2ade-3acf)\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{15a^2c^2x^3}$$
$$-\frac{\sqrt{-be+af}(8b^2c^2e^2+abce(7de-13cf)+a^2(8d^2e^2-13cdef+3c^2f^2))\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}E\left(\arcsin\left(\frac{\sqrt{}}{\sqrt{}}\right.\right.}{15a^{7/2}c^2e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$
$$+\frac{\sqrt{-be+af}(8b^2c^2e+3abc(de-3cf)+2a^2d(2de-3cf))\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{-be+af}}{\sqrt{a}\sqrt{e+fx}}\right.\right.}{15a^{7/2}c^2\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$

output
```
-1/15*(8*b^2*c^2*e^2+a*b*c*e*(-13*c*f+7*d*e)+a^2*(3*c^2*f^2-13*c*d*e*f+8*d
^2*e^2))*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a^3/c^3/x/(f*x^2+e)^(1/2)-1/5*e*(
b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/a/c/x^5+2/15*(-3*a*c*f+2*a*
d*e+2*b*c*e)*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/a^2/c^2/x^3-1
/15*(a*f-b*e)^(1/2)*(8*b^2*c^2*e^2+a*b*c*e*(-13*c*f+7*d*e)+a^2*(3*c^2*f^2-
13*c*d*e*f+8*d^2*e^2))*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*Ell
ipticE((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e
))^(1/2))/a^(7/2)/c^2/e/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)+1/
15*(a*f-b*e)^(1/2)*(8*b^2*c^2*e+3*a*b*c*(-3*c*f+d*e)+2*a^2*d*(-3*c*f+2*d*e
))*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticF((a*f-b*e)^(1/
2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(7/2)/c^
2/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)
```

Mathematica [F]

$$\int \frac{(e+fx^2)^{3/2}}{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{(e+fx^2)^{3/2}}{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

input
```
Integrate[(e + f*x^2)^(3/2)/(x^6*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
Integrate[(e + f*x^2)^(3/2)/(x^6*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx^2)^{3/2}}{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

$$\Big\downarrow \; 450$$

$$\int \frac{(e+fx^2)^{3/2}}{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}}dx$$

input
```
Int[(e + f*x^2)^(3/2)/(x^6*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [F]

$$\int \frac{(f\,x^2+e)^{\frac{3}{2}}}{x^6\sqrt{b\,x^2+a}\,\sqrt{x^2d+c}}dx$$

input
```
int((f*x^2+e)^(3/2)/x^6/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

output
```
int((f*x^2+e)^(3/2)/x^6/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{(e+fx^2)^{3/2}}{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx=\int \frac{(fx^2+e)^{\frac{3}{2}}}{\sqrt{bx^2+a}\sqrt{dx^2+c}x^6}\,dx$$

input
```
integrate((f*x^2+e)^(3/2)/x^6/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm
="fricas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)/(b*d*x^10 + (b*
c + a*d)*x^8 + a*c*x^6), x)
```

Sympy [F]

$$\int \frac{(e+fx^2)^{3/2}}{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{(e+fx^2)^{\frac{3}{2}}}{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

input
```
integrate((f*x**2+e)**(3/2)/x**6/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral((e + f*x**2)**(3/2)/(x**6*sqrt(a + b*x**2)*sqrt(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{(e+fx^2)^{3/2}}{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{(fx^2+e)^{\frac{3}{2}}}{\sqrt{bx^2+a}\sqrt{dx^2+c}x^6}\,dx$$

input
```
integrate((f*x^2+e)^(3/2)/x^6/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm
="maxima")
```

output
```
integrate((f*x^2 + e)^(3/2)/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x^6), x)
```

Giac [F]

$$\int \frac{(e+fx^2)^{3/2}}{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{(fx^2+e)^{\frac{3}{2}}}{\sqrt{bx^2+a}\sqrt{dx^2+c}x^6}\,dx$$

input
```
integrate((f*x^2+e)^(3/2)/x^6/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm
="giac")
```

output
```
integrate((f*x^2 + e)^(3/2)/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx^2)^{3/2}}{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{(f\,x^2+e)^{3/2}}{x^6\,\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int((e + f*x^2)^(3/2)/(x^6*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int((e + f*x^2)^(3/2)/(x^6*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{(e+fx^2)^{3/2}}{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{(f\,x^2+e)^{\frac{3}{2}}}{x^6\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}dx$$

input
```
int((f*x^2+e)^(3/2)/x^6/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

output
```
int((f*x^2+e)^(3/2)/x^6/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

3.350 $\int \frac{\left(e+fx^2\right)^{3/2}}{x^8\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 861

$$\int \frac{\left(e+fx^2\right)^{3/2}}{x^8\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \frac{2(24b^3c^3e^3+4ab^2c^2e^2(5de-9cf)+a^2bce(20d^2e^2-31cdef+6c^2f^2)+3a^3(\cdots}{105a^4c^4ex\sqrt{e+fx^2}\cdots}$$
$$-\frac{e\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{7acx^7}+\frac{2(3bce+3ade-4acf)\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{35a^2c^2x^5}$$
$$-\frac{\left(\frac{24b^2ce^2}{a}+be(23de-33cf)+a\left(\frac{24d^2e^2}{c}-33def+3cf^2\right)\right)\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{105a^2c^2ex^3}$$
$$+\frac{2\sqrt{-be+af}(24b^3c^3e^3+4ab^2c^2e^2(5de-9cf)+a^2bce(20d^2e^2-31cdef+6c^2f^2)+3a^3(8d^3e^3-12cd^2e^2f\cdots}{105a^{9/2}c^3e^2\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}\cdots}$$
$$-\frac{\sqrt{-be+af}(48b^3c^3e^2+16ab^2c^2e(de-3cf)+a^2bc(17d^2e^2-24cdef-3c^2f^2)+3a^3d(8d^2e^2-11cdef+\cdots}{105a^{9/2}c^3e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}\cdots}$$

output
```
2/105*(24*b^3*c^3*e^3+4*a*b^2*c^2*e^2*(-9*c*f+5*d*e)+a^2*b*c*e*(6*c^2*f^2-
31*c*d*e*f+20*d^2*e^2)+3*a^3*(c^3*f^3+2*c^2*d*e*f^2-12*c*d^2*e^2*f+8*d^3*e
^3))*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a^4/c^4/e/x/(f*x^2+e)^(1/2)-1/7*e*(b*
x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/a/c/x^7+2/35*(-4*a*c*f+3*a*d*
e+3*b*c*e)*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/a^2/c^2/x^5-1/1
05*(24*b^2*c*e^2/a+b*e*(-33*c*f+23*d*e)+a*(24*d^2*e^2/c-33*d*e*f+3*c*f^2))
*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/a^2/c^2/e/x^3+2/105*(a*f-
b*e)^(1/2)*(24*b^3*c^3*e^3+4*a*b^2*c^2*e^2*(-9*c*f+5*d*e)+a^2*b*c*e*(6*c^2
*f^2-31*c*d*e*f+20*d^2*e^2)+3*a^3*(c^3*f^3+2*c^2*d*e*f^2-12*c*d^2*e^2*f+8*
d^3*e^3))*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticE((a*f-b
*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(
9/2)/c^3/e^2/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)-1/105*(a*f-b*
e)^(1/2)*(48*b^3*c^3*e^2+16*a*b^2*c^2*e*(-3*c*f+d*e)+a^2*b*c*(-3*c^2*f^2-2
4*c*d*e*f+17*d^2*e^2)+3*a^3*d*(c^2*f^2-11*c*d*e*f+8*d^2*e^2))*(b*x^2+a)^(1
/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticF((a*f-b*e)^(1/2)*x/a^(1/2)/(f
*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(9/2)/c^3/e/(d*x^2+c)^(
1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)
```

Mathematica [F]

$$\int \frac{(e+fx^2)^{3/2}}{x^8\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{(e+fx^2)^{3/2}}{x^8\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

input
```
Integrate[(e + f*x^2)^(3/2)/(x^8*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
Integrate[(e + f*x^2)^(3/2)/(x^8*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx^2)^{3/2}}{x^8\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

$$\downarrow 450$$

$$\int \frac{(e+fx^2)^{3/2}}{x^8\sqrt{a+bx^2}\sqrt{c+dx^2}}dx$$

input
```
Int[(e + f*x^2)^(3/2)/(x^8*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [F]

$$\int \frac{(f\,x^2+e)^{\frac{3}{2}}}{x^8\sqrt{b\,x^2+a}\,\sqrt{x^2d+c}}dx$$

input
```
int((f*x^2+e)^(3/2)/x^8/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

output
```
int((f*x^2+e)^(3/2)/x^8/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{(e+fx^2)^{3/2}}{x^8\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{(fx^2+e)^{\frac{3}{2}}}{\sqrt{bx^2+a}\sqrt{dx^2+c}x^8}\,dx$$

input
```
integrate((f*x^2+e)^(3/2)/x^8/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm
="fricas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)/(b*d*x^12 + (b*
c + a*d)*x^10 + a*c*x^8), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx^2)^{3/2}}{x^8\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \text{Timed out}$$

input
```
integrate((f*x**2+e)**(3/2)/x**8/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(e+fx^2)^{3/2}}{x^8\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{(fx^2+e)^{\frac{3}{2}}}{\sqrt{bx^2+a}\sqrt{dx^2+c}x^8}\,dx$$

input
```
integrate((f*x^2+e)^(3/2)/x^8/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm
="maxima")
```

output
```
integrate((f*x^2 + e)^(3/2)/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x^8), x)
```

Giac [F]

$$\int \frac{(e+fx^2)^{3/2}}{x^8\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{(fx^2+e)^{\frac{3}{2}}}{\sqrt{bx^2+a}\sqrt{dx^2+c}x^8}\,dx$$

input
```
integrate((f*x^2+e)^(3/2)/x^8/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm
="giac")
```

output
```
integrate((f*x^2 + e)^(3/2)/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x^8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx^2)^{3/2}}{x^8\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{\left(f\,x^2+e\right)^{3/2}}{x^8\,\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int((e + f*x^2)^(3/2)/(x^8*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int((e + f*x^2)^(3/2)/(x^8*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{(e+fx^2)^{3/2}}{x^8\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{\left(f\,x^2+e\right)^{\frac{3}{2}}}{x^8\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}dx$$

input
```
int((f*x^2+e)^(3/2)/x^8/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

output
```
int((f*x^2+e)^(3/2)/x^8/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

3.351 $\int \frac{\sqrt{e+fx^2}}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 194

$$\begin{aligned}\int \frac{\sqrt{e+fx^2}}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx &= -\frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{cx\sqrt{a+bx^2}} \\ &\quad - \frac{\sqrt{e}\sqrt{be-af}\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,E\left(\arcsin\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{a+bx^2}}\right)\middle|\frac{(bc-ad)e}{c(be-af)}\right)}{ac\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}\end{aligned}$$

output
```
-(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/c/x/(b*x^2+a)^(1/2)-e^(1/2)*(-a*f+b*e)^(1
/2)*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)*EllipticE((-a*f+b*e)^(
1/2)*x/e^(1/2)/(b*x^2+a)^(1/2),((-a*d+b*c)*e/c/(-a*f+b*e))^(1/2))/a/c/(a*(
d*x^2+c)/c/(b*x^2+a))^(1/2)/(f*x^2+e)^(1/2)
```

Mathematica [F]

$$\int \frac{\sqrt{e+fx^2}}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx=\int \frac{\sqrt{e+fx^2}}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

input
```
Integrate[Sqrt[e + f*x^2]/(x^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
Integrate[Sqrt[e + f*x^2]/(x^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{e+fx^2}}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$
$$\Big\downarrow \; 450$$
$$\int \frac{\sqrt{e+fx^2}}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}dx$$

input
```
Int[Sqrt[e + f*x^2]/(x^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [F]

$$\int \frac{\sqrt{f\,x^{2}+e}}{x^{2}\sqrt{b\,x^{2}+a}\,\sqrt{x^{2}d+c}}dx$$

input
```
int((f*x^2+e)^(1/2)/x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

output
```
int((f*x^2+e)^(1/2)/x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{\sqrt{e+fx^2}}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{fx^2+e}}{\sqrt{bx^2+a}\sqrt{dx^2+c}x^2}\,dx$$

input
```
integrate((f*x^2+e)^(1/2)/x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm
="fricas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(b*d*x^6 + (b*c +
 a*d)*x^4 + a*c*x^2), x)
```

Sympy [F]

$$\int \frac{\sqrt{e+fx^2}}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{e+fx^2}}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

input
```
integrate((f*x**2+e)**(1/2)/x**2/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(sqrt(e + f*x**2)/(x**2*sqrt(a + b*x**2)*sqrt(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{e+fx^2}}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx=\int \frac{\sqrt{fx^2+e}}{\sqrt{bx^2+a}\sqrt{dx^2+c}x^2}\,dx$$

input
```
integrate((f*x^2+e)^(1/2)/x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm
="maxima")
```

output
```
integrate(sqrt(f*x^2 + e)/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x^2), x)
```

Giac [F]

$$\int \frac{\sqrt{e+fx^2}}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx=\int \frac{\sqrt{fx^2+e}}{\sqrt{bx^2+a}\sqrt{dx^2+c}x^2}\,dx$$

input
```
integrate((f*x^2+e)^(1/2)/x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm
="giac")
```

output
```
integrate(sqrt(f*x^2 + e)/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{e+fx^2}}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx=\int \frac{\sqrt{f\,x^2+e}}{x^2\,\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int((e + f*x^2)^(1/2)/(x^2*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int((e + f*x^2)^(1/2)/(x^2*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{e+fx^2}}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{f\,x^2+e}\,\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{bd\,x^6+ad\,x^4+bc\,x^4+ac\,x^2}dx$$

input
```
int((f*x^2+e)^(1/2)/x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

output
```
int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c*x**2 + a*d*x
**4 + b*c*x**4 + b*d*x**6),x)
```

3.352 $\int \frac{\sqrt{e+fx^2}}{x^2\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 296

$$\begin{aligned}\int \frac{\sqrt{e+fx^2}}{x^2\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx &= -\frac{\sqrt{a-bx^2}\sqrt{e+fx^2}}{ax\sqrt{c+dx^2}} - \frac{\sqrt{bc+ad}\sqrt{e+fx^2}\,E\left(\arctan\left(\frac{\sqrt{bc+ad}x}{\sqrt{c}\sqrt{a-bx^2}}\right)\middle|\frac{a(de-cf)}{(bc+ad)e}\right)}{a\sqrt{c}\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}} \\ &\quad + \frac{\sqrt{c}(be+af)\sqrt{e+fx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{bc+ad}x}{\sqrt{c}\sqrt{a-bx^2}}\right),\frac{a(de-cf)}{(bc+ad)e}\right)}{a\sqrt{bc+ad}\,e\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}\end{aligned}$$

output

```
-(-b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/a/x/(d*x^2+c)^(1/2)-(a*d+b*c)^(1/2)*(f*x
^2+e)^(1/2)*EllipticE((a*d+b*c)^(1/2)*x/c^(1/2)/(-b*x^2+a)^(1/2)/(1+(a*d+b
*c)*x^2/c/(-b*x^2+a))^(1/2),(a*(-c*f+d*e)/(a*d+b*c)/e)^(1/2))/a/c^(1/2)/(d
*x^2+c)^(1/2)/(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)+c^(1/2)*(a*f+b*e)*(f*x^2+e)^
(1/2)*InverseJacobiAM(arctan((a*d+b*c)^(1/2)*x/c^(1/2)/(-b*x^2+a)^(1/2)),(
a*(-c*f+d*e)/(a*d+b*c)/e)^(1/2))/a/(a*d+b*c)^(1/2)/e/(d*x^2+c)^(1/2)/(c*(f
*x^2+e)/e/(d*x^2+c))^(1/2)
```

Mathematica [F]

$$\int \frac{\sqrt{e+fx^2}}{x^2\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{e+fx^2}}{x^2\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx$$

input
```
Integrate[Sqrt[e + f*x^2]/(x^2*Sqrt[a - b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
Integrate[Sqrt[e + f*x^2]/(x^2*Sqrt[a - b*x^2]*Sqrt[c + d*x^2]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{e+fx^2}}{x^2\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx$$
$$\Big\downarrow \; 450$$
$$\int \frac{\sqrt{e+fx^2}}{x^2\sqrt{a-bx^2}\sqrt{c+dx^2}}dx$$

input
```
Int[Sqrt[e + f*x^2]/(x^2*Sqrt[a - b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [F]

$$\int \frac{\sqrt{f\,x^{2}+e}}{x^{2} \sqrt{-b\,x^{2}+a}\,\sqrt{x^{2} d+c}}dx$$

input
```
int((f*x^2+e)^(1/2)/x^2/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

output
```
int((f*x^2+e)^(1/2)/x^2/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{\sqrt{e+fx^2}}{x^2\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{fx^2+e}}{\sqrt{-bx^2+a}\sqrt{dx^2+c}x^2}\,dx$$

input
```
integrate((f*x^2+e)^(1/2)/x^2/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorith
m="fricas")
```

output
```
integral(-sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(b*d*x^6 + (b*c
 - a*d)*x^4 - a*c*x^2), x)
```

Sympy [F]

$$\int \frac{\sqrt{e+fx^2}}{x^2\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{e+fx^2}}{x^2\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx$$

input
```
integrate((f*x**2+e)**(1/2)/x**2/(-b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(sqrt(e + f*x**2)/(x**2*sqrt(a - b*x**2)*sqrt(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{e+fx^2}}{x^2\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx=\int \frac{\sqrt{fx^2+e}}{\sqrt{-bx^2+a}\sqrt{dx^2+c}x^2}\,dx$$

input
```
integrate((f*x^2+e)^(1/2)/x^2/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorith
m="maxima")
```

output
```
integrate(sqrt(f*x^2 + e)/(sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)*x^2), x)
```

Giac [F]

$$\int \frac{\sqrt{e+fx^2}}{x^2\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx=\int \frac{\sqrt{fx^2+e}}{\sqrt{-bx^2+a}\sqrt{dx^2+c}x^2}\,dx$$

input
```
integrate((f*x^2+e)^(1/2)/x^2/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorith
m="giac")
```

output
```
integrate(sqrt(f*x^2 + e)/(sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{e+fx^2}}{x^2\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx=\int \frac{\sqrt{f\,x^2+e}}{x^2\,\sqrt{a-b\,x^2}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int((e + f*x^2)^(1/2)/(x^2*(a - b*x^2)^(1/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int((e + f*x^2)^(1/2)/(x^2*(a - b*x^2)^(1/2)*(c + d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{e+fx^2}}{x^2\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{f\,x^2+e}\,\sqrt{d\,x^2+c}\,\sqrt{-b\,x^2+a}}{-bd\,x^6+ad\,x^4-bc\,x^4+ac\,x^2}dx$$

input
```
int((f*x^2+e)^(1/2)/x^2/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

output
```
int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a - b*x**2))/(a*c*x**2 + a*d*x
**4 - b*c*x**4 - b*d*x**6),x)
```

3.353 $\int \frac{\sqrt{e+fx^2}}{x^2\sqrt{a+bx^2}\sqrt{c-dx^2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 196

$$\int \frac{\sqrt{e+fx^2}}{x^2\sqrt{a+bx^2}\sqrt{c-dx^2}}\,dx = -\frac{\sqrt{c-dx^2}\sqrt{e+fx^2}}{cx\sqrt{a+bx^2}} - \frac{\sqrt{e}\sqrt{be-af}\sqrt{c-dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,E\left(\arcsin\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{a+bx^2}}\right)|\frac{(bc+ad)e}{c(be-af)}\right)}{ac\sqrt{\frac{a(c-dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$

output
```
-(-d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/c/x/(b*x^2+a)^(1/2)-e^(1/2)*(-a*f+b*e)^(
1/2)*(-d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)*EllipticE((-a*f+b*e)
^(1/2)*x/e^(1/2)/(b*x^2+a)^(1/2),((a*d+b*c)*e/c/(-a*f+b*e))^(1/2))/a/c/(a*
(-d*x^2+c)/c/(b*x^2+a))^(1/2)/(f*x^2+e)^(1/2)
```

Mathematica [F]

$$\int \frac{\sqrt{e+fx^2}}{x^2\sqrt{a+bx^2}\sqrt{c-dx^2}}\,dx = \int \frac{\sqrt{e+fx^2}}{x^2\sqrt{a+bx^2}\sqrt{c-dx^2}}\,dx$$

input
```
Integrate[Sqrt[e + f*x^2]/(x^2*Sqrt[a + b*x^2]*Sqrt[c - d*x^2]),x]
```

output
```
Integrate[Sqrt[e + f*x^2]/(x^2*Sqrt[a + b*x^2]*Sqrt[c - d*x^2]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{e+fx^2}}{x^2\sqrt{a+bx^2}\sqrt{c-dx^2}}\,dx$$

$$\Big\downarrow \text{450}$$

$$\int \frac{\sqrt{e+fx^2}}{x^2\sqrt{a+bx^2}\sqrt{c-dx^2}}dx$$

input
```
Int[Sqrt[e + f*x^2]/(x^2*Sqrt[a + b*x^2]*Sqrt[c - d*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [F]

$$\int \frac{\sqrt{f\,x^2+e}}{x^2\sqrt{b\,x^2+a}\,\sqrt{-x^2d+c}}dx$$

input
```
int((f*x^2+e)^(1/2)/x^2/(b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x)
```

output
```
int((f*x^2+e)^(1/2)/x^2/(b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{\sqrt{e+fx^2}}{x^2\sqrt{a+bx^2}\sqrt{c-dx^2}}\,dx = \int \frac{\sqrt{fx^2+e}}{\sqrt{bx^2+a}\sqrt{-dx^2+c}x^2}\,dx$$

input
```
integrate((f*x^2+e)^(1/2)/x^2/(b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x, algorith
m="fricas")
```

output
```
integral(-sqrt(b*x^2 + a)*sqrt(-d*x^2 + c)*sqrt(f*x^2 + e)/(b*d*x^6 - (b*c
 - a*d)*x^4 - a*c*x^2), x)
```

Sympy [F]

$$\int \frac{\sqrt{e+fx^2}}{x^2\sqrt{a+bx^2}\sqrt{c-dx^2}}\,dx = \int \frac{\sqrt{e+fx^2}}{x^2\sqrt{a+bx^2}\sqrt{c-dx^2}}\,dx$$

input
```
integrate((f*x**2+e)**(1/2)/x**2/(b*x**2+a)**(1/2)/(-d*x**2+c)**(1/2),x)
```

output
```
Integral(sqrt(e + f*x**2)/(x**2*sqrt(a + b*x**2)*sqrt(c - d*x**2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{e+fx^2}}{x^2\sqrt{a+bx^2}\sqrt{c-dx^2}}\,dx = \int \frac{\sqrt{fx^2+e}}{\sqrt{bx^2+a}\sqrt{-dx^2+cx^2}}\,dx$$

input
```
integrate((f*x^2+e)^(1/2)/x^2/(b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x, algorith
m="maxima")
```

output
```
integrate(sqrt(f*x^2 + e)/(sqrt(b*x^2 + a)*sqrt(-d*x^2 + c)*x^2), x)
```

Giac [F]

$$\int \frac{\sqrt{e+fx^2}}{x^2\sqrt{a+bx^2}\sqrt{c-dx^2}}\,dx = \int \frac{\sqrt{fx^2+e}}{\sqrt{bx^2+a}\sqrt{-dx^2+cx^2}}\,dx$$

input
```
integrate((f*x^2+e)^(1/2)/x^2/(b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x, algorith
m="giac")
```

output
```
integrate(sqrt(f*x^2 + e)/(sqrt(b*x^2 + a)*sqrt(-d*x^2 + c)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{e+fx^2}}{x^2\sqrt{a+bx^2}\sqrt{c-dx^2}}\,dx = \int \frac{\sqrt{f\,x^2+e}}{x^2\,\sqrt{b\,x^2+a}\,\sqrt{c-d\,x^2}}\,dx$$

input
```
int((e + f*x^2)^(1/2)/(x^2*(a + b*x^2)^(1/2)*(c - d*x^2)^(1/2)),x)
```

output
```
int((e + f*x^2)^(1/2)/(x^2*(a + b*x^2)^(1/2)*(c - d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{e+fx^2}}{x^2\sqrt{a+bx^2}\sqrt{c-dx^2}}\,dx = \int \frac{\sqrt{f\,x^2+e}\,\sqrt{-d\,x^2+c}\,\sqrt{b\,x^2+a}}{-bd\,x^6-ad\,x^4+bc\,x^4+ac\,x^2}dx$$

input
```
int((f*x^2+e)^(1/2)/x^2/(b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x)
```

output
```
int((sqrt(e + f*x**2)*sqrt(c - d*x**2)*sqrt(a + b*x**2))/(a*c*x**2 - a*d*x
**4 + b*c*x**4 - b*d*x**6),x)
```

3.354 $\int \frac{\sqrt{e+fx^2}}{x^2\sqrt{a-bx^2}\sqrt{c-dx^2}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 354

$$\int \frac{\sqrt{e+fx^2}}{x^2\sqrt{a-bx^2}\sqrt{c-dx^2}}\,dx = -\frac{\sqrt{c-dx^2}\sqrt{e+fx^2}}{cx\sqrt{a-bx^2}} - \frac{\sqrt{-bc+ad}e\sqrt{c-dx^2}\sqrt{\frac{a(e+fx^2)}{e(a-bx^2)}}E\left(\arcsin\left(\frac{\sqrt{-bc+ad}x}{\sqrt{c}\sqrt{a-bx^2}}\right)|\frac{c(be+af)}{(bc-ad)e}\right)}{ac^{3/2}\sqrt{\frac{a(c-dx^2)}{c(a-bx^2)}}\sqrt{e+fx^2}} + \frac{(de+cf)\sqrt{c-dx^2}\sqrt{\frac{a(e+fx^2)}{e(a-bx^2)}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{-bc+ad}x}{\sqrt{c}\sqrt{a-bx^2}}\right),\frac{c(be+af)}{(bc-ad)e}\right)}{c^{3/2}\sqrt{-bc+ad}\sqrt{\frac{a(c-dx^2)}{c(a-bx^2)}}\sqrt{e+fx^2}}$$

output

```
-(-d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/c/x/(-b*x^2+a)^(1/2)-(a*d-b*c)^(1/2)*e*(
-d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(-b*x^2+a))^(1/2)*EllipticE((a*d-b*c)^(1/2)
*x/c^(1/2)/(-b*x^2+a)^(1/2),(c*(a*f+b*e)/(-a*d+b*c)/e)^(1/2))/a/c^(3/2)/(a
*(-d*x^2+c)/c/(-b*x^2+a))^(1/2)/(f*x^2+e)^(1/2)+(c*f+d*e)*(-d*x^2+c)^(1/2)
*(a*(f*x^2+e)/e/(-b*x^2+a))^(1/2)*EllipticF((a*d-b*c)^(1/2)*x/c^(1/2)/(-b*
x^2+a)^(1/2),(c*(a*f+b*e)/(-a*d+b*c)/e)^(1/2))/c^(3/2)/(a*d-b*c)^(1/2)/(a*
(-d*x^2+c)/c/(-b*x^2+a))^(1/2)/(f*x^2+e)^(1/2)
```

Mathematica [F]

$$\int \frac{\sqrt{e+fx^2}}{x^2\sqrt{a-bx^2}\sqrt{c-dx^2}}\,dx=\int \frac{\sqrt{e+fx^2}}{x^2\sqrt{a-bx^2}\sqrt{c-dx^2}}\,dx$$

input
```
Integrate[Sqrt[e + f*x^2]/(x^2*Sqrt[a - b*x^2]*Sqrt[c - d*x^2]),x]
```

output
```
Integrate[Sqrt[e + f*x^2]/(x^2*Sqrt[a - b*x^2]*Sqrt[c - d*x^2]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{e+fx^2}}{x^2\sqrt{a-bx^2}\sqrt{c-dx^2}}\,dx$$
$$\Big\downarrow \; 450$$
$$\int \frac{\sqrt{e+fx^2}}{x^2\sqrt{a-bx^2}\sqrt{c-dx^2}}dx$$

input
```
Int[Sqrt[e + f*x^2]/(x^2*Sqrt[a - b*x^2]*Sqrt[c - d*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [F]

$$\int \frac{\sqrt{f\,x^{2}+e}}{x^{2}\sqrt{-b\,x^{2}+a}\,\sqrt{-x^{2}d+c}}dx$$

input
```
int((f*x^2+e)^(1/2)/x^2/(-b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x)
```

output
```
int((f*x^2+e)^(1/2)/x^2/(-b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{\sqrt{e+fx^2}}{x^2\sqrt{a-bx^2}\sqrt{c-dx^2}}\,dx = \int \frac{\sqrt{fx^2+e}}{\sqrt{-bx^2+a}\sqrt{-dx^2+c}x^2}\,dx$$

input
```
integrate((f*x^2+e)^(1/2)/x^2/(-b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x, algorit
hm="fricas")
```

output
```
integral(sqrt(-b*x^2 + a)*sqrt(-d*x^2 + c)*sqrt(f*x^2 + e)/(b*d*x^6 - (b*c
 + a*d)*x^4 + a*c*x^2), x)
```

Sympy [F]

$$\int \frac{\sqrt{e+fx^2}}{x^2\sqrt{a-bx^2}\sqrt{c-dx^2}}\,dx = \int \frac{\sqrt{e+fx^2}}{x^2\sqrt{a-bx^2}\sqrt{c-dx^2}}\,dx$$

input
```
integrate((f*x**2+e)**(1/2)/x**2/(-b*x**2+a)**(1/2)/(-d*x**2+c)**(1/2),x)
```

output
```
Integral(sqrt(e + f*x**2)/(x**2*sqrt(a - b*x**2)*sqrt(c - d*x**2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{e+fx^2}}{x^2\sqrt{a-bx^2}\sqrt{c-dx^2}}\,dx = \int \frac{\sqrt{fx^2+e}}{\sqrt{-bx^2+a}\sqrt{-dx^2+cx^2}}\,dx$$

input
```
integrate((f*x^2+e)^(1/2)/x^2/(-b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x, algorit
hm="maxima")
```

output
```
integrate(sqrt(f*x^2 + e)/(sqrt(-b*x^2 + a)*sqrt(-d*x^2 + c)*x^2), x)
```

Giac [F]

$$\int \frac{\sqrt{e+fx^2}}{x^2\sqrt{a-bx^2}\sqrt{c-dx^2}}\,dx = \int \frac{\sqrt{fx^2+e}}{\sqrt{-bx^2+a}\sqrt{-dx^2+cx^2}}\,dx$$

input
```
integrate((f*x^2+e)^(1/2)/x^2/(-b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x, algorit
hm="giac")
```

output
```
integrate(sqrt(f*x^2 + e)/(sqrt(-b*x^2 + a)*sqrt(-d*x^2 + c)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{e+fx^2}}{x^2\sqrt{a-bx^2}\sqrt{c-dx^2}}\,dx = \int \frac{\sqrt{f\,x^2+e}}{x^2\,\sqrt{a-b\,x^2}\,\sqrt{c-d\,x^2}}\,dx$$

input
```
int((e + f*x^2)^(1/2)/(x^2*(a - b*x^2)^(1/2)*(c - d*x^2)^(1/2)),x)
```

output
```
int((e + f*x^2)^(1/2)/(x^2*(a - b*x^2)^(1/2)*(c - d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{e+fx^2}}{x^2\sqrt{a-bx^2}\sqrt{c-dx^2}}\,dx = \int \frac{\sqrt{f\,x^2+e}\,\sqrt{-d\,x^2+c}\,\sqrt{-b\,x^2+a}}{bd\,x^6-ad\,x^4-bc\,x^4+ac\,x^2}dx$$

input
```
int((f*x^2+e)^(1/2)/x^2/(-b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x)
```

output
```
int((sqrt(e + f*x**2)*sqrt(c - d*x**2)*sqrt(a - b*x**2))/(a*c*x**2 - a*d*x
**4 - b*c*x**4 + b*d*x**6),x)
```

3.355 $\int \frac{\sqrt{e-fx^2}}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 194

$$\int \frac{\sqrt{e-fx^2}}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = -\frac{\sqrt{c+dx^2}\sqrt{e-fx^2}}{cx\sqrt{a+bx^2}} - \frac{\sqrt{e}\sqrt{be+af}\sqrt{c+dx^2}\sqrt{\frac{a(e-fx^2)}{e(a+bx^2)}}E\left(\arcsin\left(\frac{\sqrt{be+af}x}{\sqrt{e}\sqrt{a+bx^2}}\right)|\frac{(bc-ad)e}{c(be+af)}\right)}{ac\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e-fx^2}}$$

output
```
-(d*x^2+c)^(1/2)*(-f*x^2+e)^(1/2)/c/x/(b*x^2+a)^(1/2)-e^(1/2)*(a*f+b*e)^(1
/2)*(d*x^2+c)^(1/2)*(a*(-f*x^2+e)/e/(b*x^2+a))^(1/2)*EllipticE((a*f+b*e)^(
1/2)*x/e^(1/2)/(b*x^2+a)^(1/2),((-a*d+b*c)*e/c/(a*f+b*e))^(1/2))/a/c/(a*(d
*x^2+c)/c/(b*x^2+a))^(1/2)/(-f*x^2+e)^(1/2)
```

Mathematica [F]

$$\int \frac{\sqrt{e-fx^2}}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx=\int \frac{\sqrt{e-fx^2}}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

input
```
Integrate[Sqrt[e - f*x^2]/(x^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
Integrate[Sqrt[e - f*x^2]/(x^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{e-fx^2}}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$
$$\Big\downarrow \text{450}$$
$$\int \frac{\sqrt{e-fx^2}}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}dx$$

input
```
Int[Sqrt[e - f*x^2]/(x^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [F]

$$\int \frac{\sqrt{-f\,x^2+e}}{x^2\sqrt{b\,x^2+a}\,\sqrt{x^2d+c}}dx$$

input
```
int((-f*x^2+e)^(1/2)/x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

output
```
int((-f*x^2+e)^(1/2)/x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{\sqrt{e-fx^2}}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{-fx^2+e}}{\sqrt{bx^2+a}\sqrt{dx^2+c}x^2}\,dx$$

input
```
integrate((-f*x^2+e)^(1/2)/x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorith
m="fricas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-f*x^2 + e)/(b*d*x^6 + (b*c
+ a*d)*x^4 + a*c*x^2), x)
```

Sympy [F]

$$\int \frac{\sqrt{e-fx^2}}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{e-fx^2}}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

input
```
integrate((-f*x**2+e)**(1/2)/x**2/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(sqrt(e - f*x**2)/(x**2*sqrt(a + b*x**2)*sqrt(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{e-fx^2}}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{-fx^2+e}}{\sqrt{bx^2+a}\sqrt{dx^2+cx^2}}\,dx$$

input
```
integrate((-f*x^2+e)^(1/2)/x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorith
m="maxima")
```

output
```
integrate(sqrt(-f*x^2 + e)/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x^2), x)
```

Giac [F]

$$\int \frac{\sqrt{e-fx^2}}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{-fx^2+e}}{\sqrt{bx^2+a}\sqrt{dx^2+cx^2}}\,dx$$

input
```
integrate((-f*x^2+e)^(1/2)/x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorith
m="giac")
```

output
```
integrate(sqrt(-f*x^2 + e)/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{e-fx^2}}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{e-f\,x^2}}{x^2\,\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int((e - f*x^2)^(1/2)/(x^2*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int((e - f*x^2)^(1/2)/(x^2*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{e-fx^2}}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx=\int \frac{\sqrt{-f\,x^2+e}\,\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{bd\,x^6+ad\,x^4+bc\,x^4+ac\,x^2}dx$$

input
```
int((-f*x^2+e)^(1/2)/x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

output
```
int((sqrt(e - f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c*x**2 + a*d*x
**4 + b*c*x**4 + b*d*x**6),x)
```

3.356 $\int \frac{\sqrt{e-fx^2}}{x^2\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 200

$$\begin{aligned}\int \frac{\sqrt{e-fx^2}}{x^2\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx &= -\frac{\sqrt{c+dx^2}\sqrt{e-fx^2}}{cx\sqrt{a-bx^2}} \\ &\quad -\frac{\sqrt{e}\sqrt{-be+af}\sqrt{c+dx^2}\sqrt{\frac{a(e-fx^2)}{e(a-bx^2)}}\,E\left(\arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{e}\sqrt{a-bx^2}}\right)|\frac{(bc+ad)e}{c(be-af)}\right)}{ac\sqrt{\frac{a(c+dx^2)}{c(a-bx^2)}}\sqrt{e-fx^2}}\end{aligned}$$

output

```
-(d*x^2+c)^(1/2)*(-f*x^2+e)^(1/2)/c/x/(-b*x^2+a)^(1/2)-e^(1/2)*(a*f-b*e)^(
1/2)*(d*x^2+c)^(1/2)*(a*(-f*x^2+e)/e/(-b*x^2+a))^(1/2)*EllipticE((a*f-b*e)
^(1/2)*x/e^(1/2)/(-b*x^2+a)^(1/2),((a*d+b*c)*e/c/(-a*f+b*e))^(1/2))/a/c/(a
*(d*x^2+c)/c/(-b*x^2+a))^(1/2)/(-f*x^2+e)^(1/2)
```

Mathematica [F]

$$\int \frac{\sqrt{e-fx^2}}{x^2\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx=\int \frac{\sqrt{e-fx^2}}{x^2\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx$$

input
```
Integrate[Sqrt[e - f*x^2]/(x^2*Sqrt[a - b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
Integrate[Sqrt[e - f*x^2]/(x^2*Sqrt[a - b*x^2]*Sqrt[c + d*x^2]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{e-fx^2}}{x^2\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx$$
$$\Big\downarrow \ 450$$
$$\int \frac{\sqrt{e-fx^2}}{x^2\sqrt{a-bx^2}\sqrt{c+dx^2}}dx$$

input
```
Int[Sqrt[e - f*x^2]/(x^2*Sqrt[a - b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [F]

$$\int \frac{\sqrt{-f\,x^{2}+e}}{x^{2}\sqrt{-b\,x^{2}+a}\,\sqrt{x^{2}d+c}}dx$$

input
```
int((-f*x^2+e)^(1/2)/x^2/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

output
```
int((-f*x^2+e)^(1/2)/x^2/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{\sqrt{e-fx^2}}{x^2\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{-fx^2+e}}{\sqrt{-bx^2+a}\sqrt{dx^2+c}x^2}\,dx$$

input
```
integrate((-f*x^2+e)^(1/2)/x^2/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorit
hm="fricas")
```

output
```
integral(-sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-f*x^2 + e)/(b*d*x^6 + (b*
c - a*d)*x^4 - a*c*x^2), x)
```

Sympy [F]

$$\int \frac{\sqrt{e-fx^2}}{x^2\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{e-fx^2}}{x^2\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx$$

input
```
integrate((-f*x**2+e)**(1/2)/x**2/(-b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(sqrt(e - f*x**2)/(x**2*sqrt(a - b*x**2)*sqrt(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{e-fx^2}}{x^2\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{-fx^2+e}}{\sqrt{-bx^2+a}\sqrt{dx^2+c}x^2}\,dx$$

input
```
integrate((-f*x^2+e)^(1/2)/x^2/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorit
hm="maxima")
```

output
```
integrate(sqrt(-f*x^2 + e)/(sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)*x^2), x)
```

Giac [F]

$$\int \frac{\sqrt{e-fx^2}}{x^2\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{-fx^2+e}}{\sqrt{-bx^2+a}\sqrt{dx^2+c}x^2}\,dx$$

input
```
integrate((-f*x^2+e)^(1/2)/x^2/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorit
hm="giac")
```

output
```
integrate(sqrt(-f*x^2 + e)/(sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{e-fx^2}}{x^2\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{e-f\,x^2}}{x^2\,\sqrt{a-b\,x^2}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int((e - f*x^2)^(1/2)/(x^2*(a - b*x^2)^(1/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int((e - f*x^2)^(1/2)/(x^2*(a - b*x^2)^(1/2)*(c + d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{e-fx^2}}{x^2\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{-f\,x^2+e}\,\sqrt{d\,x^2+c}\,\sqrt{-b\,x^2+a}}{-bd\,x^6+ad\,x^4-bc\,x^4+ac\,x^2}dx$$

input
```
int((-f*x^2+e)^(1/2)/x^2/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

output
```
int((sqrt(e - f*x**2)*sqrt(c + d*x**2)*sqrt(a - b*x**2))/(a*c*x**2 + a*d*x
**4 - b*c*x**4 - b*d*x**6),x)
```

3.357 $\int \frac{\sqrt{e-fx^2}}{x^2\sqrt{a+bx^2}\sqrt{c-dx^2}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 196

$$\int \frac{\sqrt{e-fx^2}}{x^2\sqrt{a+bx^2}\sqrt{c-dx^2}}\,dx = -\frac{\sqrt{c-dx^2}\sqrt{e-fx^2}}{cx\sqrt{a+bx^2}} - \frac{\sqrt{e}\sqrt{be+af}\sqrt{c-dx^2}\sqrt{\frac{a(e-fx^2)}{e(a+bx^2)}}E\left(\arcsin\left(\frac{\sqrt{be+af}x}{\sqrt{e}\sqrt{a+bx^2}}\right)|\frac{(bc+ad)e}{c(be+af)}\right)}{ac\sqrt{\frac{a(c-dx^2)}{c(a+bx^2)}}\sqrt{e-fx^2}}$$

output
```
-(-d*x^2+c)^(1/2)*(-f*x^2+e)^(1/2)/c/x/(b*x^2+a)^(1/2)-e^(1/2)*(a*f+b*e)^(
1/2)*(-d*x^2+c)^(1/2)*(a*(-f*x^2+e)/e/(b*x^2+a))^(1/2)*EllipticE((a*f+b*e)
^(1/2)*x/e^(1/2)/(b*x^2+a)^(1/2),((a*d+b*c)*e/c/(a*f+b*e))^(1/2))/a/c/(a*(
-d*x^2+c)/c/(b*x^2+a))^(1/2)/(-f*x^2+e)^(1/2)
```

Mathematica [F]

$$\int \frac{\sqrt{e-fx^2}}{x^2\sqrt{a+bx^2}\sqrt{c-dx^2}}\,dx=\int \frac{\sqrt{e-fx^2}}{x^2\sqrt{a+bx^2}\sqrt{c-dx^2}}\,dx$$

input
```
Integrate[Sqrt[e - f*x^2]/(x^2*Sqrt[a + b*x^2]*Sqrt[c - d*x^2]),x]
```

output
```
Integrate[Sqrt[e - f*x^2]/(x^2*Sqrt[a + b*x^2]*Sqrt[c - d*x^2]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{e-fx^2}}{x^2\sqrt{a+bx^2}\sqrt{c-dx^2}}\,dx$$
$$\Big\downarrow \ 450$$
$$\int \frac{\sqrt{e-fx^2}}{x^2\sqrt{a+bx^2}\sqrt{c-dx^2}}dx$$

input
```
Int[Sqrt[e - f*x^2]/(x^2*Sqrt[a + b*x^2]*Sqrt[c - d*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [F]

$$\int \frac{\sqrt{-f\,x^{2}+e}}{x^{2}\sqrt{b\,x^{2}+a}\,\sqrt{-x^{2}d+c}}dx$$

input
```
int((-f*x^2+e)^(1/2)/x^2/(b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x)
```

output
```
int((-f*x^2+e)^(1/2)/x^2/(b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{\sqrt{e-fx^2}}{x^2\sqrt{a+bx^2}\sqrt{c-dx^2}}\,dx = \int \frac{\sqrt{-fx^2+e}}{\sqrt{bx^2+a}\sqrt{-dx^2+c}x^2}\,dx$$

input
```
integrate((-f*x^2+e)^(1/2)/x^2/(b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x, algorit
hm="fricas")
```

output
```
integral(-sqrt(b*x^2 + a)*sqrt(-d*x^2 + c)*sqrt(-f*x^2 + e)/(b*d*x^6 - (b*
c - a*d)*x^4 - a*c*x^2), x)
```

Sympy [F]

$$\int \frac{\sqrt{e-fx^2}}{x^2\sqrt{a+bx^2}\sqrt{c-dx^2}}\,dx = \int \frac{\sqrt{e-fx^2}}{x^2\sqrt{a+bx^2}\sqrt{c-dx^2}}\,dx$$

input
```
integrate((-f*x**2+e)**(1/2)/x**2/(b*x**2+a)**(1/2)/(-d*x**2+c)**(1/2),x)
```

output
```
Integral(sqrt(e - f*x**2)/(x**2*sqrt(a + b*x**2)*sqrt(c - d*x**2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{e-fx^2}}{x^2\sqrt{a+bx^2}\sqrt{c-dx^2}}\,dx=\int \frac{\sqrt{-fx^2+e}}{\sqrt{bx^2+a}\sqrt{-dx^2+cx^2}}\,dx$$

input
```
integrate((-f*x^2+e)^(1/2)/x^2/(b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x, algorit
hm="maxima")
```

output
```
integrate(sqrt(-f*x^2 + e)/(sqrt(b*x^2 + a)*sqrt(-d*x^2 + c)*x^2), x)
```

Giac [F]

$$\int \frac{\sqrt{e-fx^2}}{x^2\sqrt{a+bx^2}\sqrt{c-dx^2}}\,dx=\int \frac{\sqrt{-fx^2+e}}{\sqrt{bx^2+a}\sqrt{-dx^2+cx^2}}\,dx$$

input
```
integrate((-f*x^2+e)^(1/2)/x^2/(b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x, algorit
hm="giac")
```

output
```
integrate(sqrt(-f*x^2 + e)/(sqrt(b*x^2 + a)*sqrt(-d*x^2 + c)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{e-fx^2}}{x^2\sqrt{a+bx^2}\sqrt{c-dx^2}}\,dx=\int \frac{\sqrt{e-f\,x^2}}{x^2\,\sqrt{b\,x^2+a}\,\sqrt{c-d\,x^2}}\,dx$$

input
```
int((e - f*x^2)^(1/2)/(x^2*(a + b*x^2)^(1/2)*(c - d*x^2)^(1/2)),x)
```

output
```
int((e - f*x^2)^(1/2)/(x^2*(a + b*x^2)^(1/2)*(c - d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{e-fx^2}}{x^2\sqrt{a+bx^2}\sqrt{c-dx^2}}\,dx = \int \frac{\sqrt{-f\,x^2+e}\,\sqrt{-d\,x^2+c}\,\sqrt{b\,x^2+a}}{-bd\,x^6-ad\,x^4+bc\,x^4+ac\,x^2}dx$$

input
```
int((-f*x^2+e)^(1/2)/x^2/(b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x)
```

output
```
int((sqrt(e - f*x**2)*sqrt(c - d*x**2)*sqrt(a + b*x**2))/(a*c*x**2 - a*d*x
**4 + b*c*x**4 - b*d*x**6),x)
```

3.358 $\int \frac{\sqrt{e-fx^2}}{x^2\sqrt{a-bx^2}\sqrt{c-dx^2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 204

$$\begin{aligned}\int \frac{\sqrt{e-fx^2}}{x^2\sqrt{a-bx^2}\sqrt{c-dx^2}}\,dx &= -\frac{\sqrt{c-dx^2}\sqrt{e-fx^2}}{cx\sqrt{a-bx^2}} \\ &\quad -\frac{\sqrt{e}\sqrt{-be+af}\sqrt{c-dx^2}\sqrt{\frac{a(e-fx^2)}{e(a-bx^2)}}\,E\left(\arcsin\left(\frac{\sqrt{-be+af}\,x}{\sqrt{e}\sqrt{a-bx^2}}\right)|\frac{(bc-ad)e}{c(be-af)}\right)}{ac\sqrt{\frac{a(c-dx^2)}{c(a-bx^2)}}\sqrt{e-fx^2}}\end{aligned}$$

output
```
-(-d*x^2+c)^(1/2)*(-f*x^2+e)^(1/2)/c/x/(-b*x^2+a)^(1/2)-e^(1/2)*(a*f-b*e)^
(1/2)*(-d*x^2+c)^(1/2)*(a*(-f*x^2+e)/e/(-b*x^2+a))^(1/2)*EllipticE((a*f-b*
e)^(1/2)*x/e^(1/2)/(-b*x^2+a)^(1/2),((-a*d+b*c)*e/c/(-a*f+b*e))^(1/2))/a/c
/(a*(-d*x^2+c)/c/(-b*x^2+a))^(1/2)/(-f*x^2+e)^(1/2)
```

Mathematica [F]

$$\int \frac{\sqrt{e-fx^2}}{x^2\sqrt{a-bx^2}\sqrt{c-dx^2}}\,dx=\int \frac{\sqrt{e-fx^2}}{x^2\sqrt{a-bx^2}\sqrt{c-dx^2}}\,dx$$

input
```
Integrate[Sqrt[e - f*x^2]/(x^2*Sqrt[a - b*x^2]*Sqrt[c - d*x^2]),x]
```

output
```
Integrate[Sqrt[e - f*x^2]/(x^2*Sqrt[a - b*x^2]*Sqrt[c - d*x^2]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{e-fx^2}}{x^2\sqrt{a-bx^2}\sqrt{c-dx^2}}\,dx$$

$$\Big\downarrow \text{ 450}$$

$$\int \frac{\sqrt{e-fx^2}}{x^2\sqrt{a-bx^2}\sqrt{c-dx^2}}dx$$

input
```
Int[Sqrt[e - f*x^2]/(x^2*Sqrt[a - b*x^2]*Sqrt[c - d*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [F]

$$\int \frac{\sqrt{-f\,x^2+e}}{x^2\sqrt{-b\,x^2+a}\,\sqrt{-x^2d+c}}dx$$

input
```
int((-f*x^2+e)^(1/2)/x^2/(-b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x)
```

output
```
int((-f*x^2+e)^(1/2)/x^2/(-b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{\sqrt{e-fx^2}}{x^2\sqrt{a-bx^2}\sqrt{c-dx^2}}\,dx = \int \frac{\sqrt{-fx^2+e}}{\sqrt{-bx^2+a}\sqrt{-dx^2+c}x^2}\,dx$$

input
```
integrate((-f*x^2+e)^(1/2)/x^2/(-b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x, algori
thm="fricas")
```

output
```
integral(sqrt(-b*x^2 + a)*sqrt(-d*x^2 + c)*sqrt(-f*x^2 + e)/(b*d*x^6 - (b*
c + a*d)*x^4 + a*c*x^2), x)
```

Sympy [F]

$$\int \frac{\sqrt{e-fx^2}}{x^2\sqrt{a-bx^2}\sqrt{c-dx^2}}\,dx = \int \frac{\sqrt{e-fx^2}}{x^2\sqrt{a-bx^2}\sqrt{c-dx^2}}\,dx$$

input
```
integrate((-f*x**2+e)**(1/2)/x**2/(-b*x**2+a)**(1/2)/(-d*x**2+c)**(1/2),x)
```

output
```
Integral(sqrt(e - f*x**2)/(x**2*sqrt(a - b*x**2)*sqrt(c - d*x**2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{e-fx^2}}{x^2\sqrt{a-bx^2}\sqrt{c-dx^2}}\,dx=\int \frac{\sqrt{-fx^2+e}}{\sqrt{-bx^2+a}\sqrt{-dx^2+cx^2}}\,dx$$

input
```
integrate((-f*x^2+e)^(1/2)/x^2/(-b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x, algori
thm="maxima")
```

output
```
integrate(sqrt(-f*x^2 + e)/(sqrt(-b*x^2 + a)*sqrt(-d*x^2 + c)*x^2), x)
```

Giac [F]

$$\int \frac{\sqrt{e-fx^2}}{x^2\sqrt{a-bx^2}\sqrt{c-dx^2}}\,dx=\int \frac{\sqrt{-fx^2+e}}{\sqrt{-bx^2+a}\sqrt{-dx^2+cx^2}}\,dx$$

input
```
integrate((-f*x^2+e)^(1/2)/x^2/(-b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x, algori
thm="giac")
```

output
```
integrate(sqrt(-f*x^2 + e)/(sqrt(-b*x^2 + a)*sqrt(-d*x^2 + c)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{e-fx^2}}{x^2\sqrt{a-bx^2}\sqrt{c-dx^2}}\,dx=\int \frac{\sqrt{e-f\,x^2}}{x^2\,\sqrt{a-b\,x^2}\,\sqrt{c-d\,x^2}}\,dx$$

input
```
int((e - f*x^2)^(1/2)/(x^2*(a - b*x^2)^(1/2)*(c - d*x^2)^(1/2)),x)
```

output
```
int((e - f*x^2)^(1/2)/(x^2*(a - b*x^2)^(1/2)*(c - d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{e-fx^2}}{x^2\sqrt{a-bx^2}\sqrt{c-dx^2}}\,dx=\int \frac{\sqrt{-f\,x^2+e}\,\sqrt{-d\,x^2+c}\,\sqrt{-b\,x^2+a}}{bd\,x^6-ad\,x^4-bc\,x^4+ac\,x^2}dx$$

input
```
int((-f*x^2+e)^(1/2)/x^2/(-b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x)
```

output
```
int((sqrt(e - f*x**2)*sqrt(c - d*x**2)*sqrt(a - b*x**2))/(a*c*x**2 - a*d*x
**4 - b*c*x**4 + b*d*x**6),x)
```

3.359 $\int \frac{x^6}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 696

$$\begin{aligned}
&\int \frac{x^6}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx \\
&= -\frac{3(bde+bcf+adf)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{8bd^2f^2\sqrt{a+bx^2}}+\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{4bdf} \\
&\quad+\frac{3\sqrt{bc-ad}e(bde+bcf+adf)\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}E\left(\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)|\frac{c(be-af)}{(bc-ad)e}\right)}{8b^2\sqrt{c}d^2f^2\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}} \\
&\quad-\frac{a(3a^2d^2f-abd(de-2cf)+b^2c(de+3cf))\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right),\frac{c(be-af)}{(bc-ad)e}\right)}{8b^3\sqrt{c}d^2\sqrt{bc-ad}f\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}} \\
&\quad+\frac{a(3a^2d^2f^2+2abdf(de+cf)+b^2(3d^2e^2+2cdef+3c^2f^2))\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\operatorname{EllipticPi}\left(\frac{bc}{bc-ad},\operatorname{arcsi}\right.}{8b^3\sqrt{c}d^2\sqrt{bc-ad}f^2\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}
\end{aligned}$$

output
```
-3/8*(a*d*f+b*c*f+b*d*e)*x*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/b/d^2/f^2/(b*x^
2+a)^(1/2)+1/4*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/b/d/f+3/8
*(-a*d+b*c)^(1/2)*e*(a*d*f+b*c*f+b*d*e)*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*
x^2+a))^(1/2)*EllipticE((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2),(c*(-a*
f+b*e)/(-a*d+b*c)/e)^(1/2))/b^2/c^(1/2)/d^2/f^2/(a*(d*x^2+c)/c/(b*x^2+a))^
(1/2)/(f*x^2+e)^(1/2)-1/8*a*(3*a^2*d^2*f-a*b*d*(-2*c*f+d*e)+b^2*c*(3*c*f+d
*e))*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)*EllipticF((-a*d+b*c)^
(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2),(c*(-a*f+b*e)/(-a*d+b*c)/e)^(1/2))/b^3/c^(
1/2)/d^2/(-a*d+b*c)^(1/2)/f/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(f*x^2+e)^(1/2
)+1/8*a*(3*a^2*d^2*f^2+2*a*b*d*f*(c*f+d*e)+b^2*(3*c^2*f^2+2*c*d*e*f+3*d^2*
e^2))*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)*EllipticPi((-a*d+b*c
)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2),b*c/(-a*d+b*c),(c*(-a*f+b*e)/(-a*d+b*c)/
e)^(1/2))/b^3/c^(1/2)/d^2/(-a*d+b*c)^(1/2)/f^2/(a*(d*x^2+c)/c/(b*x^2+a))^(
1/2)/(f*x^2+e)^(1/2)
```

Mathematica [F]

$$\int \frac{x^6}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{x^6}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$$

input
```
Integrate[x^6/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
```

output
```
Integrate[x^6/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$$

$$\Big\downarrow \text{ 450}$$

$$\int \frac{x^6}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}dx$$

input
```
Int[x^6/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [F]

$$\int \frac{x^{6}}{\sqrt{b\,x^{2}+a}\,\sqrt{x^{2} d+c}\,\sqrt{f\,x^{2}+e}}dx$$

input
```
int(x^6/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

output
```
int(x^6/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^6}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx=\text{Timed out}$$

input
```
integrate(x^6/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm
="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^6}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{x^6}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$$

input
```
integrate(x**6/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**(1/2),x)
```

output
```
Integral(x**6/(sqrt(a + b*x**2)*sqrt(c + d*x**2)*sqrt(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{x^6}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{x^6}{\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{fx^2+e}}\,dx$$

input
```
integrate(x^6/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm
="maxima")
```

output
```
integrate(x^6/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{x^6}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{x^6}{\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{fx^2+e}}\,dx$$

input
```
integrate(x^6/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm
="giac")
```

output
```
integrate(x^6/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx=\int \frac{x^6}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}}\,dx$$

input
```
int(x^6/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(1/2)),x)
```

output
```
int(x^6/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^6}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \frac{\sqrt{f\,x^2+e}\,\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x-3\left(\int \frac{\sqrt{f\,x^2+e}\,\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^4}{bdf\,x^6+adf\,x^4+bcf\,x^4+bde\,x^4+acf\,x^2+ade\,x^2+bce\,x^2+ace}dx\right)adf-3\left(\int \frac{}{bdf\,x^6}\right.}{}$$

input
```
int(x^6/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

output

```
(sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*x - 3*int((sqrt(e + f*
x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*e*
x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*
a*d*f - 3*int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a
*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*
d*e*x**4 + b*d*f*x**6),x)*b*c*f - 3*int((sqrt(e + f*x**2)*sqrt(c + d*x**2)
*sqrt(a + b*x**2)*x**4)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*
c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*b*d*e - 2*int((sqrt(e
+ f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c*e + a*c*f*x**2 + a*
d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6)
,x)*a*c*f - 2*int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2
)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4
+ b*d*e*x**4 + b*d*f*x**6),x)*a*d*e - 2*int((sqrt(e + f*x**2)*sqrt(c + d*x
**2)*sqrt(a + b*x**2)*x**2)/(a*c*e + a*c*f*x**2 + a*d*e*x**2 + a*d*f*x**4
+ b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)*b*c*e - int((sqrt(
e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c*e + a*c*f*x**2 + a*d*e
*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*f*x**6),x)
*a*c*e)/(4*b*d*f)
```

3.360 $\int \frac{x^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 550

$$\begin{aligned}\int \frac{x^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx &= \frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}}{2bd\sqrt{e+fx^2}}\\
&- \frac{c\sqrt{-be+af}\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}E\left(\arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right)|\frac{a(de-cf)}{c(be-af)}\right)}{2\sqrt{a}bdf\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}\\
&+ \frac{e(be+af)\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right),\frac{a(de-cf)}{c(be-af)}\right)}{2\sqrt{a}bf^2\sqrt{-be+af}\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}\\
&- \frac{e(bde+bcf+adf)\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,\text{EllipticPi}\left(-\frac{af}{be-af},\arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right),\frac{a(de-cf)}{c(be-af)}\right)}{2\sqrt{a}bdf^2\sqrt{-be+af}\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}\end{aligned}$$

output
```
1/2*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b/d/(f*x^2+e)^(1/2)-1/2*c*(a*f-b*e)^
(1/2)*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticE((a*f-b*e)^
(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(1/2)
/b/d/f/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)+1/2*e*(a*f+b*e)*(b*
x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticF((a*f-b*e)^(1/2)*x/a
^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(1/2)/b/f^2/(a
*f-b*e)^(1/2)/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)-1/2*e*(a*d*f
+b*c*f+b*d*e)*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticPi((
a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),-a*f/(-a*f+b*e),(a*(-c*f+d*e)/c/(
-a*f+b*e))^(1/2))/a^(1/2)/b/d/f^2/(a*f-b*e)^(1/2)/(d*x^2+c)^(1/2)/(e*(b*x^
2+a)/a/(f*x^2+e))^(1/2)
```

Mathematica [F]

$$\int \frac{x^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{x^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$$

input
```
Integrate[x^4/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
```

output
```
Integrate[x^4/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$$
$$\Big\downarrow \ 450$$
$$\int \frac{x^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}dx$$

input
```
Int[x^4/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [F]

$$\int \frac{x^4}{\sqrt{b\,x^2+a}\,\sqrt{x^2 d+c}\,\sqrt{f\,x^2+e}}dx$$

input
```
int(x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

output
```
int(x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx=\text{Timed out}$$

input
```
integrate(x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm
="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{x^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$$

input
```
integrate(x**4/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**(1/2),x)
```

output
```
Integral(x**4/(sqrt(a + b*x**2)*sqrt(c + d*x**2)*sqrt(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{x^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{x^4}{\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{fx^2+e}}\,dx$$

input
```
integrate(x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm
="maxima")
```

output
```
integrate(x^4/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{x^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{x^4}{\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{fx^2+e}}\,dx$$

input
```
integrate(x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm
="giac")
```

output
```
integrate(x^4/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{x^4}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}}\,dx$$

input
```
int(x^4/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(1/2)),x)
```

output
```
int(x^4/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{\sqrt{f\,x^2+e}\,\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^4}{bdf\,x^6+adf\,x^4+bcf\,x^4+bde\,x^4+acf\,x^2+ade\,x^2+bce\,x^2+ace}dx$$

input
```
int(x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

output
```
int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c*e + a*c
*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 +
 b*d*f*x**6),x)
```

3.361 $\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 313

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = -\frac{a\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{a+bx^2}}\right),\frac{c(be-af)}{(bc-ad)e}\right)}{b\sqrt{c}\sqrt{bc-ad}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}} + \frac{a\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,\text{EllipticPi}\left(\frac{bc}{bc-ad},\arcsin\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{a+bx^2}}\right),\frac{c(be-af)}{(bc-ad)e}\right)}{b\sqrt{c}\sqrt{bc-ad}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$

output

```
-a*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)*EllipticF((-a*d+b*c)^(1
/2)*x/c^(1/2)/(b*x^2+a)^(1/2),(c*(-a*f+b*e)/(-a*d+b*c)/e)^(1/2))/b/c^(1/2)
/(-a*d+b*c)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(f*x^2+e)^(1/2)+a*(d*x^2
+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)*EllipticPi((-a*d+b*c)^(1/2)*x/c^
(1/2)/(b*x^2+a)^(1/2),b*c/(-a*d+b*c),(c*(-a*f+b*e)/(-a*d+b*c)/e)^(1/2))/b/
c^(1/2)/(-a*d+b*c)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(f*x^2+e)^(1/2)
```

Mathematica [F]

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$$

input
```
Integrate[x^2/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
```

output
```
Integrate[x^2/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$$
$$\Big\downarrow \text{450}$$
$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}dx$$

input
```
Int[x^2/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [F]

$$\int \frac{x^2}{\sqrt{b\,x^2+a}\,\sqrt{x^2d+c}\,\sqrt{f\,x^2+e}}dx$$

input
```
int(x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

output
```
int(x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \text{Timed out}$$

input
```
integrate(x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm
="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$$

input
```
integrate(x**2/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**(1/2),x)
```

output
```
Integral(x**2/(sqrt(a + b*x**2)*sqrt(c + d*x**2)*sqrt(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{fx^2+e}}\,dx$$

input
```
integrate(x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm
="maxima")
```

output
```
integrate(x^2/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{fx^2+e}}\,dx$$

input
```
integrate(x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm
="giac")
```

output
```
integrate(x^2/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{x^2}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}}\,dx$$

input
```
int(x^2/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(1/2)),x)
```

output
```
int(x^2/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$$
$$= \int \frac{\sqrt{f\,x^2+e}\,\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^2}{bdf\,x^6+adf\,x^4+bcf\,x^4+bde\,x^4+acf\,x^2+ade\,x^2+bce\,x^2+ace}dx$$

input
```
int(x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

output
```
int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c*e + a*c
*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 +
 b*d*f*x**6),x)
```

3.362 $\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 145

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \frac{\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right),\frac{c(be-af)}{(bc-ad)e}\right)}{\sqrt{c}\sqrt{bc-ad}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$

output
```
(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)*EllipticF((-a*d+b*c)^(1/2)
*x/c^(1/2)/(b*x^2+a)^(1/2),(c*(-a*f+b*e)/(-a*d+b*c)/e)^(1/2))/c^(1/2)/(-a*
d+b*c)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(f*x^2+e)^(1/2)
```

Mathematica [A] (verified)

Time = 2.98 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.02

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \frac{\sqrt{e}\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{a+bx^2}}\right),\frac{(bc-ad)e}{c(be-af)}\right)}{c\sqrt{be-af}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$

input
```
Integrate[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
```

output
```
(Sqrt[e]*Sqrt[c + d*x^2]*Sqrt[(a*(e + f*x^2))/(e*(a + b*x^2))]*EllipticF[A
rcSin[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])], ((b*c - a*d)*e)/(c*(
b*e - a*f))])/(c*Sqrt[b*e - a*f]*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]*Sqr
t[e + f*x^2])
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {427, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}dx$$

$$\Big\downarrow \text{427}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\int \frac{1}{\sqrt{1-\frac{(bc-ad)x^2}{c\left(bx^2+a\right)}}\sqrt{1-\frac{(be-af)x^2}{e\left(bx^2+a\right)}}}d\frac{x}{\sqrt{bx^2+a}}}{c\sqrt{e+fx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

$$\Big\downarrow \text{321}$$

$$\frac{\sqrt{e}\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right),\frac{(bc-ad)e}{c(be-af)}\right)}{c\sqrt{e+fx^2}\sqrt{be-af}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

input
```
Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
```

output

```
(Sqrt[e]*Sqrt[c + d*x^2]*Sqrt[(a*(e + f*x^2))/(e*(a + b*x^2))]*EllipticF[A
rcSin[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])], ((b*c - a*d)*e)/(c*(
b*e - a*f))])/(c*Sqrt[b*e - a*f]*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]*Sqr
t[e + f*x^2])
```

Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 427
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.
)*(x_)^2]), x_Symbol] :> Simp[Sqrt[c + d*x^2]*(Sqrt[a*((e + f*x^2)/(e*(a +
b*x^2)))]/(c*Sqrt[e + f*x^2]*Sqrt[a*((c + d*x^2)/(c*(a + b*x^2)))]))   Subs
t[Int[1/(Sqrt[1 - (b*c - a*d)*(x^2/c)]*Sqrt[1 - (b*e - a*f)*(x^2/e)]), x],
x, x/Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Maple [F]

$$\int \frac{1}{\sqrt{b\,x^{2}+a}\,\sqrt{x^{2}d+c}\,\sqrt{f\,x^{2}+e}}dx$$

input
```
int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

output
```
int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx=\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{fx^2+e}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="
fricas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(b*d*f*x^6 + (b*d
*e + (b*c + a*d)*f)*x^4 + a*c*e + (a*c*f + (b*c + a*d)*e)*x^2), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$$

input
```
integrate(1/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**(1/2),x)
```

output
```
Integral(1/(sqrt(a + b*x**2)*sqrt(c + d*x**2)*sqrt(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{fx^2+e}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="
maxima")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{fx^2+e}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="
giac")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{1}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}}\,dx$$

input
```
int(1/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(1/2)),x)
```

output
```
int(1/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{\sqrt{f\,x^2+e}\,\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{bdf\,x^6+adf\,x^4+bcf\,x^4+bde\,x^4+acf\,x^2+ade\,x^2+bce\,x^2+ace}dx$$

input
```
int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

output
```
int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c*e + a*c*f*x*
*2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 + b*d*
f*x**6),x)
```

3.363 $\int \frac{1}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 341

$$\int \frac{1}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = -\frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{cex\sqrt{a+bx^2}} - \frac{\sqrt{bc-ad}\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}E\left(\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)|\frac{c(be-af)}{(bc-ad)e}\right)}{ac^{3/2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}} - \frac{d\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right),\frac{c(be-af)}{(bc-ad)e}\right)}{c^{3/2}\sqrt{bc-ad}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$

output
```
-(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/c/e/x/(b*x^2+a)^(1/2)-(-a*d+b*c)^(1/2)*(d
*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)*EllipticE((-a*d+b*c)^(1/2)*x
/c^(1/2)/(b*x^2+a)^(1/2),(c*(-a*f+b*e)/(-a*d+b*c)/e)^(1/2))/a/c^(3/2)/(a*(
d*x^2+c)/c/(b*x^2+a))^(1/2)/(f*x^2+e)^(1/2)-d*(d*x^2+c)^(1/2)*(a*(f*x^2+e)
/e/(b*x^2+a))^(1/2)*EllipticF((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2),(
c*(-a*f+b*e)/(-a*d+b*c)/e)^(1/2))/c^(3/2)/(-a*d+b*c)^(1/2)/(a*(d*x^2+c)/c/
(b*x^2+a))^(1/2)/(f*x^2+e)^(1/2)
```

Mathematica [F]

$$\int \frac{1}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{1}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$$

input
```
Integrate[1/(x^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
```

output
```
Integrate[1/(x^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$$
$$\Big\downarrow \; 450$$
$$\int \frac{1}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}dx$$

input
```
Int[1/(x^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [F]

$$\int \frac{1}{x^2\sqrt{b\,x^2+a}\,\sqrt{x^2d+c}\,\sqrt{f\,x^2+e}}dx$$

input
```
int(1/x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

output
```
int(1/x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

Fricas [F]

$$\int \frac{1}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{fx^2+e}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorit
hm="fricas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(b*d*f*x^8 + (b*d
*e + (b*c + a*d)*f)*x^6 + a*c*e*x^2 + (a*c*f + (b*c + a*d)*e)*x^4), x)
```

Sympy [F]

$$\int \frac{1}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{1}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$$

input
```
integrate(1/x**2/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**(1/2),x)
```

output
```
Integral(1/(x**2*sqrt(a + b*x**2)*sqrt(c + d*x**2)*sqrt(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx=\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{fx^2+e}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorit
hm="maxima")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx=\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{fx^2+e}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorit
hm="giac")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx=\int \frac{1}{x^2\,\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}}\,dx$$

input
```
int(1/(x^2*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(1/2)),x)
```

output
```
int(1/(x^2*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$$
$$= \int \frac{\sqrt{f\,x^2+e}\,\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{bdf\,x^8+adf\,x^6+bcf\,x^6+bde\,x^6+acf\,x^4+ade\,x^4+bce\,x^4+ace\,x^2}dx$$

input
```
int(1/x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

output
```
int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c*e*x**2 + a*c
*f*x**4 + a*d*e*x**4 + a*d*f*x**6 + b*c*e*x**4 + b*c*f*x**6 + b*d*e*x**6 +
 b*d*f*x**8),x)
```

3.364 $\int \frac{1}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 458

$$\begin{aligned}
&\int \frac{1}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx \\
&= \frac{2(bce+ade+acf)\sqrt{a+bx^2}\sqrt{c+dx^2}}{3a^2c^2ex\sqrt{e+fx^2}} - \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{3acex^3} \\
&\quad + \frac{2\sqrt{-be+af}(bce+ade+acf)\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}E\left(\arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right)|\frac{a(de-cf)}{c(be-af)}\right)}{3a^{5/2}ce^2\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}} \\
&\quad + \frac{(2b^2ce-a^2df+ab(de+cf))\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right),\frac{a(de-cf)}{c(be-af)}\right)}{3a^{5/2}ce\sqrt{-be+af}\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}
\end{aligned}$$

output
```
2/3*(a*c*f+a*d*e+b*c*e)*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a^2/c^2/e/x/(f*x^2
+e)^(1/2)-1/3*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/a/c/e/x^3+2/
3*(a*f-b*e)^(1/2)*(a*c*f+a*d*e+b*c*e)*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^
2+e))^(1/2)*EllipticE((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d
*e)/c/(-a*f+b*e))^(1/2))/a^(5/2)/c/e^2/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x
^2+e))^(1/2)+1/3*(2*b^2*c*e-a^2*d*f+a*b*(c*f+d*e))*(b*x^2+a)^(1/2)*(e*(d*x
^2+c)/c/(f*x^2+e))^(1/2)*EllipticF((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/
2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(5/2)/c/e/(a*f-b*e)^(1/2)/(d*x^2+c
)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)
```

Mathematica [F]

$$\int \frac{1}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx=\int \frac{1}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$$

input
```
Integrate[1/(x^4*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
```

output
```
Integrate[1/(x^4*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$$
$$\Big\downarrow \; 450$$
$$\int \frac{1}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}dx$$

input
```
Int[1/(x^4*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [F]

$$\int \frac{1}{x^4\sqrt{b\,x^2+a}\,\sqrt{x^2 d+c}\,\sqrt{f\,x^2+e}}dx$$

input
```
int(1/x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

output
```
int(1/x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

Fricas [F]

$$\int \frac{1}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{fx^2+e}x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorit
hm="fricas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(b*d*f*x^10 + (b*
d*e + (b*c + a*d)*f)*x^8 + a*c*e*x^4 + (a*c*f + (b*c + a*d)*e)*x^6), x)
```

Sympy [F]

$$\int \frac{1}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{1}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$$

input
```
integrate(1/x**4/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**(1/2),x)
```

output
```
Integral(1/(x**4*sqrt(a + b*x**2)*sqrt(c + d*x**2)*sqrt(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{fx^2+e}x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorit
hm="maxima")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)*x^4), x)
```

Giac [F]

$$\int \frac{1}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{fx^2+e}x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorit
hm="giac")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{1}{x^4\,\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}}\,dx$$

input
```
int(1/(x^4*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(1/2)),x)
```

output
```
int(1/(x^4*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$$
$$= \int \frac{\sqrt{f\,x^2+e}\,\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{bdf\,x^{10}+adf\,x^8+bcf\,x^8+bde\,x^8+acf\,x^6+ade\,x^6+bce\,x^6+ace\,x^4}dx$$

input
```
int(1/x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

output
```
int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c*e*x**4 + a*c
*f*x**6 + a*d*e*x**6 + a*d*f*x**8 + b*c*e*x**6 + b*c*f*x**8 + b*d*e*x**8 +
 b*d*f*x**10),x)
```

3.365 $\int \frac{1}{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 638

$$\int \frac{1}{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = -\frac{\left(\frac{8b^2ce^2}{a}+7be(de+cf)+a\left(\frac{8d^2e^2}{c}+7def+8cf^2\right)\right)\sqrt{a+bx^2}\sqrt{c+dx^2}}{15a^2c^2e^2x\sqrt{e+fx^2}}$$
$$-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{5acex^5}+\frac{4(bce+ade+acf)\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}{15a^2c^2e^2x^3}$$
$$-\frac{\sqrt{-be+af}\,(8b^2c^2e^2+7abce(de+cf)+a^2(8d^2e^2+7cdef+8c^2f^2))\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}E\left(\arcsin\left(\frac{\sqrt{\cdot}}{\sqrt{\cdot}}\right.\right.}{15a^{7/2}c^2e^3\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$
$$-\frac{(8b^3c^2e^2+3ab^2ce(de+cf)-4a^3df(de+cf)+a^2b(4d^2e^2+cdef+4c^2f^2))\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\text{Ellipt}}{15a^{7/2}c^2e^2\sqrt{-be+af}\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$

output
```
-1/15*(8*b^2*c*e^2/a+7*b*e*(c*f+d*e)+a*(8*d^2*e^2/c+7*d*e*f+8*c*f^2))*(b*x
^2+a)^(1/2)*(d*x^2+c)^(1/2)/a^2/c^2/e^2/x/(f*x^2+e)^(1/2)-1/5*(b*x^2+a)^(1
/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/a/c/e/x^5+4/15*(a*c*f+a*d*e+b*c*e)*(b*
x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/a^2/c^2/e^2/x^3-1/15*(a*f-b*e
)^(1/2)*(8*b^2*c^2*e^2+7*a*b*c*e*(c*f+d*e)+a^2*(8*c^2*f^2+7*c*d*e*f+8*d^2*
e^2))*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticE((a*f-b*e)^
(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(7/2)
/c^2/e^3/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)-1/15*(8*b^3*c^2*e
^2+3*a*b^2*c*e*(c*f+d*e)-4*a^3*d*f*(c*f+d*e)+a^2*b*(4*c^2*f^2+c*d*e*f+4*d^
2*e^2))*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticF((a*f-b*e
)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(7/
2)/c^2/e^2/(a*f-b*e)^(1/2)/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)
```

Mathematica [F]

$$\int \frac{1}{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{1}{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$$

input
```
Integrate[1/(x^6*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
```

output
```
Integrate[1/(x^6*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$$
$$\Big\downarrow \text{450}$$
$$\int \frac{1}{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}dx$$

input
```
Int[1/(x^6*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [F]

$$\int \frac{1}{x^{6} \sqrt{b\,x^{2}+a}\,\sqrt{x^{2} d+c}\,\sqrt{f\,x^{2}+e}}dx$$

input
```
int(1/x^6/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

output
```
int(1/x^6/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

Fricas [F]

$$\int \frac{1}{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{fx^2+e}x^6}\,dx$$

input
```
integrate(1/x^6/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorit
hm="fricas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(b*d*f*x^12 + (b*
d*e + (b*c + a*d)*f)*x^10 + a*c*e*x^6 + (a*c*f + (b*c + a*d)*e)*x^8), x)
```

Sympy [F]

$$\int \frac{1}{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{1}{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$$

input
```
integrate(1/x**6/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**(1/2),x)
```

output
```
Integral(1/(x**6*sqrt(a + b*x**2)*sqrt(c + d*x**2)*sqrt(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{fx^2+e}x^6}\,dx$$

input
```
integrate(1/x^6/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorit
hm="maxima")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)*x^6), x)
```

Giac [F]

$$\int \frac{1}{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{fx^2+e}x^6}\,dx$$

input
```
integrate(1/x^6/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorit
hm="giac")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{1}{x^6\,\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}}\,dx$$

input
```
int(1/(x^6*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(1/2)),x)
```

output
```
int(1/(x^6*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{1}{x^6\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}}dx$$

input
```
int(1/x^6/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

output
```
int(1/x^6/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

3.366 $\int \frac{x^8}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 995

$$\int \frac{x^8}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx =$$

$$-\frac{e(af(de-cf)-be(5de-cf))x\sqrt{a+bx^2}\sqrt{c+dx^2}}{4bdf^2(be-af)(de-cf)\sqrt{e+fx^2}}+\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{4bdf\sqrt{e+fx^2}}$$

$$+\frac{(3a^2df^2(de-cf)-b^2e(15d^2e^2-4cdef-3c^2f^2)+abf(4d^2e^2-cdef-3c^2f^2))\,x\sqrt{c+dx^2}\sqrt{e+fx^2}}{8bd^2f^3(be-af)(de-cf)\sqrt{a+bx^2}}$$

$$-\frac{\sqrt{bc-ad}e(3a^2df^2(de-cf)-b^2e(15d^2e^2-4cdef-3c^2f^2)+abf(4d^2e^2-cdef-3c^2f^2))\,\sqrt{c+dx^2}\sqrt{\frac{a}{e}}}{8b^2\sqrt{c}d^2f^3(be-af)(de-cf)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$

$$+\frac{a(2a^2bcdf^2+3a^3d^2f^2-b^3ce(5de+3cf)+ab^2(5d^2e^2+3cdef+3c^2f^2))\,\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,\mathrm{EllipticF}\left(a\right.}{8b^3\sqrt{c}d^2\sqrt{bc-ad}f^2(be-af)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$

$$+\frac{a(3a^2d^2f^2+2abdf(3de+cf)+3b^2(5d^2e^2+2cdef+c^2f^2))\,\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,\mathrm{EllipticPi}\left(\frac{bc}{bc-ad},\arcsin\right.}{8b^3\sqrt{c}d^2\sqrt{bc-ad}f^3\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$

output
```
-1/4*e*(a*f*(-c*f+d*e)-b*e*(-c*f+5*d*e))*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)
/b/d/f^2/(-a*f+b*e)/(-c*f+d*e)/(f*x^2+e)^(1/2)+1/4*x^3*(b*x^2+a)^(1/2)*(d*
x^2+c)^(1/2)/b/d/f/(f*x^2+e)^(1/2)+1/8*(3*a^2*d*f^2*(-c*f+d*e)-b^2*e*(-3*c
^2*f^2-4*c*d*e*f+15*d^2*e^2)+a*b*f*(-3*c^2*f^2-c*d*e*f+4*d^2*e^2))*x*(d*x^
2+c)^(1/2)*(f*x^2+e)^(1/2)/b/d^2/f^3/(-a*f+b*e)/(-c*f+d*e)/(b*x^2+a)^(1/2)
-1/8*(-a*d+b*c)^(1/2)*e*(3*a^2*d*f^2*(-c*f+d*e)-b^2*e*(-3*c^2*f^2-4*c*d*e*
f+15*d^2*e^2)+a*b*f*(-3*c^2*f^2-c*d*e*f+4*d^2*e^2))*(d*x^2+c)^(1/2)*(a*(f*
x^2+e)/e/(b*x^2+a))^(1/2)*EllipticE((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(
1/2),(c*(-a*f+b*e)/(-a*d+b*c)/e)^(1/2))/b^2/c^(1/2)/d^2/f^3/(-a*f+b*e)/(-c
*f+d*e)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(f*x^2+e)^(1/2)+1/8*a*(2*a^2*b*c*d
*f^2+3*a^3*d^2*f^2-b^3*c*e*(3*c*f+5*d*e)+a*b^2*(3*c^2*f^2+3*c*d*e*f+5*d^2*
e^2))*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)*EllipticF((-a*d+b*c)
^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2),(c*(-a*f+b*e)/(-a*d+b*c)/e)^(1/2))/b^3/c^
(1/2)/d^2/(-a*d+b*c)^(1/2)/f^2/(-a*f+b*e)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/
(f*x^2+e)^(1/2)+1/8*a*(3*a^2*d^2*f^2+2*a*b*d*f*(c*f+3*d*e)+3*b^2*(c^2*f^2+
2*c*d*e*f+5*d^2*e^2))*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)*Elli
pticPi((-a*d+b*c)^(1/2)*x/c^(1/2)/(b*x^2+a)^(1/2),b*c/(-a*d+b*c),(c*(-a*f+
b*e)/(-a*d+b*c)/e)^(1/2))/b^3/c^(1/2)/d^2/(-a*d+b*c)^(1/2)/f^3/(a*(d*x^2+c
)/c/(b*x^2+a))^(1/2)/(f*x^2+e)^(1/2)
```

Mathematica [F]

$$\int \frac{x^8}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx=\int \frac{x^8}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx$$

input
```
Integrate[x^8/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^(3/2)),x]
```

output
```
Integrate[x^8/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^(3/2)), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx$$

$$\Big\downarrow \; 450$$

$$\int \frac{x^8}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}dx$$

input
```
Int[x^8/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^(3/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [F]

$$\int \frac{x^8}{\sqrt{b\,x^2+a}\,\sqrt{x^2d+c}\,\left(f\,x^2+e\right)^{\frac{3}{2}}}dx$$

input
```
int(x^8/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x)
```

output
```
int(x^8/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x)
```

Fricas [F]

$$\int \frac{x^8}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{x^8}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^8/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm
="fricas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)*x^8/(b*d*f^2*x^8
+ (2*b*d*e*f + (b*c + a*d)*f^2)*x^6 + (b*d*e^2 + a*c*f^2 + 2*(b*c + a*d)*e
*f)*x^4 + a*c*e^2 + (2*a*c*e*f + (b*c + a*d)*e^2)*x^2), x)
```

Sympy [F]

$$\int \frac{x^8}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{x^8}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**8/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**(3/2),x)
```

output
```
Integral(x**8/(sqrt(a + b*x**2)*sqrt(c + d*x**2)*(e + f*x**2)**(3/2)), x)
```

Maxima [F]

$$\int \frac{x^8}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{x^8}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^8/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm
="maxima")
```

output
```
integrate(x^8/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)), x)
```

Giac [F]

$$\int \frac{x^8}{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}}\,dx = \int \frac{x^8}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^8/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm
="giac")
```

output
```
integrate(x^8/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^8}{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}}\,dx = \int \frac{x^8}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,{\left(f\,x^2+e\right)}^{3/2}}\,dx$$

input
```
int(x^8/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(3/2)),x)
```

output
```
int(x^8/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{x^8}{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}}\,dx = \int \frac{x^8}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,\left(f\,x^2+e\right)^{\frac{3}{2}}}dx$$

input
```
int(x^8/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x)
```

output
```
int(x^8/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x)
```

3.367 $\int \frac{x^6}{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 616

$$\int \frac{x^6}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = \frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}}{2bdf\sqrt{e+fx^2}}$$
$$-\frac{c(af(de-cf)-be(3de-cf))\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}E\left(\arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right)|\frac{a(de-cf)}{c(be-af)}\right)}{2\sqrt{a}bdf^2\sqrt{-be+af}(de-cf)\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$
$$+\frac{e(be(3de-5cf)+af(de-cf))\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right),\frac{a(de-cf)}{c(be-af)}\right)}{2\sqrt{a}bf^3\sqrt{-be+af}(de-cf)\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$
$$-\frac{e(3bde+bcf+adf)\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,\text{EllipticPi}\left(-\frac{af}{be-af},\arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right),\frac{a(de-cf)}{c(be-af)}\right)}{2\sqrt{a}bdf^3\sqrt{-be+af}\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$

output
```
1/2*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b/d/f/(f*x^2+e)^(1/2)-1/2*c*(a*f*(-c
*f+d*e)-b*e*(-c*f+3*d*e))*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*
EllipticE((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+
b*e))^(1/2))/a^(1/2)/b/d/f^2/(a*f-b*e)^(1/2)/(-c*f+d*e)/(d*x^2+c)^(1/2)/(e
*(b*x^2+a)/a/(f*x^2+e))^(1/2)+1/2*e*(b*e*(-5*c*f+3*d*e)+a*f*(-c*f+d*e))*(b
*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticF((a*f-b*e)^(1/2)*x/
a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(1/2)/b/f^3/(
a*f-b*e)^(1/2)/(-c*f+d*e)/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)-
1/2*e*(a*d*f+b*c*f+3*b*d*e)*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2
)*EllipticPi((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),-a*f/(-a*f+b*e),(a*
(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(1/2)/b/d/f^3/(a*f-b*e)^(1/2)/(d*x^2+c)^
(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)
```

Mathematica [F]

$$\int \frac{x^6}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{x^6}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx$$

input
```
Integrate[x^6/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^(3/2)),x]
```

output
```
Integrate[x^6/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^(3/2)), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx$$

$$\Big\downarrow \; 450$$

$$\int \frac{x^6}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}dx$$

input
```
Int[x^6/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^(3/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [F]

$$\int \frac{x^{6}}{\sqrt{b\,x^{2}+a}\,\sqrt{x^{2} d+c}\,\left(f\,x^{2}+e\right)^{\frac{3}{2}}}dx$$

input
```
int(x^6/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x)
```

output
```
int(x^6/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^6}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate(x^6/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm
="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^6}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{x^6}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**6/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**(3/2),x)
```

output
```
Integral(x**6/(sqrt(a + b*x**2)*sqrt(c + d*x**2)*(e + f*x**2)**(3/2)), x)
```

Maxima [F]

$$\int \frac{x^6}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{x^6}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^6/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm
="maxima")
```

output
```
integrate(x^6/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)), x)
```

Giac [F]

$$\int \frac{x^6}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{x^6}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^6/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm
="giac")
```

output
```
integrate(x^6/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx=\int \frac{x^6}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,{\left(f\,x^2+e\right)}^{3/2}}\,dx$$

input
```
int(x^6/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(3/2)),x)
```

output
```
int(x^6/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{x^6}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx=\int \frac{\sqrt{f\,x^2+e}\,\sqrt{d\,x^2+}}{bd\,f^2x^8+ad\,f^2x^6+bc\,f^2x^6+2bdef\,x^6+ac\,f^2x^4+2adef\,x^4-}$$

input
```
int(x^6/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x)
```

output
```
int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**6)/(a*c*e**2 +
2*a*c*e*f*x**2 + a*c*f**2*x**4 + a*d*e**2*x**2 + 2*a*d*e*f*x**4 + a*d*f**2
*x**6 + b*c*e**2*x**2 + 2*b*c*e*f*x**4 + b*c*f**2*x**6 + b*d*e**2*x**4 + 2
*b*d*e*f*x**6 + b*d*f**2*x**8),x)
```

3.368 $\int \frac{x^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 493

$$\int \frac{x^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}}\,dx = -\frac{ce\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}E\left(\arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right)|\frac{a(de-cf)}{c(be-af)}\right)}{\sqrt{a}f\sqrt{-be+af}(de-cf)\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}} - \frac{e(de-2cf)\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right),\frac{a(de-cf)}{c(be-af)}\right)}{\sqrt{a}f^2\sqrt{-be+af}(de-cf)\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}} + \frac{e\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,\text{EllipticPi}\left(-\frac{af}{be-af},\arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right),\frac{a(de-cf)}{c(be-af)}\right)}{\sqrt{a}f^2\sqrt{-be+af}\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$

output
```
-c*e*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticE((a*f-b*e)^(
1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(1/2)/
f/(a*f-b*e)^(1/2)/(-c*f+d*e)/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/
2)-e*(-2*c*f+d*e)*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*Elliptic
F((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1
/2))/a^(1/2)/f^2/(a*f-b*e)^(1/2)/(-c*f+d*e)/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a
/(f*x^2+e))^(1/2)+e*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*Ellipt
icPi((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),-a*f/(-a*f+b*e),(a*(-c*f+d*
e)/c/(-a*f+b*e))^(1/2))/a^(1/2)/f^2/(a*f-b*e)^(1/2)/(d*x^2+c)^(1/2)/(e*(b*
x^2+a)/a/(f*x^2+e))^(1/2)
```

Mathematica [F]

$$\int \frac{x^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{x^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx$$

input
```
Integrate[x^4/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^(3/2)),x]
```

output
```
Integrate[x^4/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^(3/2)), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx$$
$$\Big\downarrow \text{450}$$
$$\int \frac{x^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}dx$$

input
```
Int[x^4/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^(3/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [F]

$$\int \frac{x^{4}}{\sqrt{b\,x^{2}+a}\,\sqrt{x^{2} d+c}\,\left(f\,x^{2}+e\right)^{\frac{3}{2}}}dx$$

input
```
int(x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x)
```

output
```
int(x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate(x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm
="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{x^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**4/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**(3/2),x)
```

output
```
Integral(x**4/(sqrt(a + b*x**2)*sqrt(c + d*x**2)*(e + f*x**2)**(3/2)), x)
```

Maxima [F]

$$\int \frac{x^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{x^4}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm
="maxima")
```

output
```
integrate(x^4/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)), x)
```

Giac [F]

$$\int \frac{x^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{x^4}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm
="giac")
```

output
```
integrate(x^4/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx=\int \frac{x^4}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,{\left(f\,x^2+e\right)}^{3/2}}\,dx$$

input
```
int(x^4/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(3/2)),x)
```

output
```
int(x^4/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{x^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx=\int \frac{\sqrt{f\,x^2+e}\,\sqrt{d\,x^2+ \cdots}}{bd\,f^2x^8+ad\,f^2x^6+bc\,f^2x^6+2bdef\,x^6+ac\,f^2x^4+2adef\,x^4 \cdots}$$

input
```
int(x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x)
```

output
```
int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c*e**2 +
2*a*c*e*f*x**2 + a*c*f**2*x**4 + a*d*e**2*x**2 + 2*a*d*e*f*x**4 + a*d*f**2
*x**6 + b*c*e**2*x**2 + 2*b*c*e*f*x**4 + b*c*f**2*x**6 + b*d*e**2*x**4 + 2
*b*d*e*f*x**6 + b*d*f**2*x**8),x)
```

3.369 $\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 314

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = \frac{c\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}E\left(\arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right)|\frac{a(de-cf)}{c(be-af)}\right)}{\sqrt{a}\sqrt{-be+af}(de-cf)\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}} - \frac{c\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right),\frac{a(de-cf)}{c(be-af)}\right)}{\sqrt{a}\sqrt{-be+af}(de-cf)\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$

output

```
c*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticE((a*f-b*e)^(1/2
)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(1/2)/(a*
f-b*e)^(1/2)/(-c*f+d*e)/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)-c*
(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticF((a*f-b*e)^(1/2)*
x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(1/2)/(a*f-
b*e)^(1/2)/(-c*f+d*e)/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)
```

Mathematica [F]

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx$$

input
```
Integrate[x^2/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^(3/2)),x]
```

output
```
Integrate[x^2/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^(3/2)), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx$$

$$\Big\downarrow \; 450$$

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}dx$$

input
```
Int[x^2/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^(3/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [F]

$$\int \frac{x^2}{\sqrt{b\,x^2+a}\,\sqrt{x^2d+c}\,\left(f\,x^2+e\right)^{\frac{3}{2}}}dx$$

input
```
int(x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x)
```

output
```
int(x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x)
```

Fricas [F]

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx=\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm
="fricas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)*x^2/(b*d*f^2*x^8
+ (2*b*d*e*f + (b*c + a*d)*f^2)*x^6 + (b*d*e^2 + a*c*f^2 + 2*(b*c + a*d)*e
*f)*x^4 + a*c*e^2 + (2*a*c*e*f + (b*c + a*d)*e^2)*x^2), x)
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**(3/2),x)
```

output
```
Integral(x**2/(sqrt(a + b*x**2)*sqrt(c + d*x**2)*(e + f*x**2)**(3/2)), x)
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm
="maxima")
```

output
```
integrate(x^2/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm
="giac")
```

output
```
integrate(x^2/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx=\int \frac{x^2}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,{\left(f\,x^2+e\right)}^{3/2}}\,dx$$

input
```
int(x^2/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(3/2)),x)
```

output
```
int(x^2/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx=\int \frac{\sqrt{f\,x^2+e}\,\sqrt{d\,x^2+}}{bd\,f^2x^8+ad\,f^2x^6+bc\,f^2x^6+2bdef\,x^6+ac\,f^2x^4+2adef\,x^4}$$

input
```
int(x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x)
```

output
```
int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c*e**2 +
2*a*c*e*f*x**2 + a*c*f**2*x**4 + a*d*e**2*x**2 + 2*a*d*e*f*x**4 + a*d*f**2
*x**6 + b*c*e**2*x**2 + 2*b*c*e*f*x**4 + b*c*f**2*x**6 + b*d*e**2*x**4 + 2
*b*d*e*f*x**6 + b*d*f**2*x**8),x)
```

3.370 $\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 320

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = -\frac{\sqrt{a}f\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}E\left(\arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right)|\frac{a(de-cf)}{c(be-af)}\right)}{e\sqrt{-be+af}(de-cf)\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}} + \frac{\sqrt{a}d\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-be+afx}}{\sqrt{a}\sqrt{e+fx^2}}\right),\frac{a(de-cf)}{c(be-af)}\right)}{c\sqrt{-be+af}(de-cf)\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$

output

```
-a^(1/2)*f*(d*x^2+c)^(1/2)*(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)*EllipticE((a*f-
b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/e/
(a*f-b*e)^(1/2)/(-c*f+d*e)/(b*x^2+a)^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)
+a^(1/2)*d*(d*x^2+c)^(1/2)*(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)*EllipticF((a*f-
b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/c/
(a*f-b*e)^(1/2)/(-c*f+d*e)/(b*x^2+a)^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)
```

Mathematica [F]

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx$$

input
```
Integrate[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^(3/2)),x]
```

output
```
Integrate[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^(3/2)), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx$$

$$\Big\downarrow \; 434$$

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}dx$$

input
```
Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^(3/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{1}{\sqrt{b\,x^{2}+a}\,\sqrt{x^{2}d+c}\,\left(f\,x^{2}+e\right)^{\frac{3}{2}}}d x$$

input
```
int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x)
```

output
```
int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}}\,dx=\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm="
fricas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(b*d*f^2*x^8 + (2
*b*d*e*f + (b*c + a*d)*f^2)*x^6 + (b*d*e^2 + a*c*f^2 + 2*(b*c + a*d)*e*f)*
x^4 + a*c*e^2 + (2*a*c*e*f + (b*c + a*d)*e^2)*x^2), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}}\,dx=\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**(3/2),x)
```

output
```
Integral(1/(sqrt(a + b*x**2)*sqrt(c + d*x**2)*(e + f*x**2)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm="
maxima")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm="
giac")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{1}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,{\left(f\,x^2+e\right)}^{3/2}}\,dx$$

input
```
int(1/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(3/2)),x)
```

output
```
int(1/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{\sqrt{f\,x^2+e}\,\sqrt{d\,x^2}}{bd\,f^2x^8+ad\,f^2x^6+bc\,f^2x^6+2bdef\,x^6+ac\,f^2x^4+2adef\,x^4}$$

input
```
int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x)
```

output
```
int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c*e**2 + 2*a*c
*e*f*x**2 + a*c*f**2*x**4 + a*d*e**2*x**2 + 2*a*d*e*f*x**4 + a*d*f**2*x**6
 + b*c*e**2*x**2 + 2*b*c*e*f*x**4 + b*c*f**2*x**6 + b*d*e**2*x**4 + 2*b*d*
e*f*x**6 + b*d*f**2*x**8),x)
```

3.371 $\int \frac{1}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 404

$$\int \frac{1}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}}\,dx=-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{acex\sqrt{e+fx^2}}-\frac{(af(de-2cf)-be(de-cf))\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}E\left(\arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right)|\frac{a(de-cf)}{c(be-af)}\right)}{a^{3/2}e^2\sqrt{-be+af}(de-cf)\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}-\frac{(bde-bcf+adf)\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right),\frac{a(de-cf)}{c(be-af)}\right)}{a^{3/2}e\sqrt{-be+af}(de-cf)\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$

output
```
-(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a/c/e/x/(f*x^2+e)^(1/2)-(a*f*(-2*c*f+d*e)
-b*e*(-c*f+d*e))*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticE
((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/
2))/a^(3/2)/e^2/(a*f-b*e)^(1/2)/(-c*f+d*e)/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/
(f*x^2+e))^(1/2)-(a*d*f-b*c*f+b*d*e)*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2
+e))^(1/2)*EllipticF((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*
e)/c/(-a*f+b*e))^(1/2))/a^(3/2)/e/(a*f-b*e)^(1/2)/(-c*f+d*e)/(d*x^2+c)^(1/
2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)
```

Mathematica [F]

$$\int \frac{1}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{1}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx$$

input
```
Integrate[1/(x^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^(3/2)),x]
```

output
```
Integrate[1/(x^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^(3/2)), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx$$
$$\Big\downarrow \text{ 450}$$
$$\int \frac{1}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}dx$$

input
```
Int[1/(x^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^(3/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [F]

$$\int \frac{1}{x^{2}\sqrt{b\,x^{2}+a}\,\sqrt{x^{2}d+c}\,\left(f\,x^{2}+e\right)^{\frac{3}{2}}}dx$$

input
```
int(1/x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x)
```

output
```
int(1/x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x)
```

Fricas [F]

$$\int \frac{1}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx=\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorit
hm="fricas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(b*d*f^2*x^10 + (
2*b*d*e*f + (b*c + a*d)*f^2)*x^8 + (b*d*e^2 + a*c*f^2 + 2*(b*c + a*d)*e*f)
*x^6 + a*c*e^2*x^2 + (2*a*c*e*f + (b*c + a*d)*e^2)*x^4), x)
```

Sympy [F]

$$\int \frac{1}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx=\int \frac{1}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x**2/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**(3/2),x)
```

output
```
Integral(1/(x**2*sqrt(a + b*x**2)*sqrt(c + d*x**2)*(e + f*x**2)**(3/2)), x
)
```

Maxima [F]

$$\int \frac{1}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorit
hm="maxima")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorit
hm="giac")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{1}{x^2\,\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,{\left(f\,x^2+e\right)}^{3/2}}\,dx$$

input
```
int(1/(x^2*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(3/2)),x)
```

output
```
int(1/(x^2*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int(1/x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x)
```

output
```
( - sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2) - int((sqrt(e + f*x
**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c*e**2 + 2*a*c*e*f*x**2 +
a*c*f**2*x**4 + a*d*e**2*x**2 + 2*a*d*e*f*x**4 + a*d*f**2*x**6 + b*c*e**2*
x**2 + 2*b*c*e*f*x**4 + b*c*f**2*x**6 + b*d*e**2*x**4 + 2*b*d*e*f*x**6 + b
*d*f**2*x**8),x)*a*d*e*f*x - int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a
 + b*x**2)*x**2)/(a*c*e**2 + 2*a*c*e*f*x**2 + a*c*f**2*x**4 + a*d*e**2*x**
2 + 2*a*d*e*f*x**4 + a*d*f**2*x**6 + b*c*e**2*x**2 + 2*b*c*e*f*x**4 + b*c*
f**2*x**6 + b*d*e**2*x**4 + 2*b*d*e*f*x**6 + b*d*f**2*x**8),x)*a*d*f**2*x*
*3 - int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c*e*
*2 + 2*a*c*e*f*x**2 + a*c*f**2*x**4 + a*d*e**2*x**2 + 2*a*d*e*f*x**4 + a*d
*f**2*x**6 + b*c*e**2*x**2 + 2*b*c*e*f*x**4 + b*c*f**2*x**6 + b*d*e**2*x**
4 + 2*b*d*e*f*x**6 + b*d*f**2*x**8),x)*b*c*e*f*x - int((sqrt(e + f*x**2)*s
qrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c*e**2 + 2*a*c*e*f*x**2 + a*c*f*
*2*x**4 + a*d*e**2*x**2 + 2*a*d*e*f*x**4 + a*d*f**2*x**6 + b*c*e**2*x**2 +
 2*b*c*e*f*x**4 + b*c*f**2*x**6 + b*d*e**2*x**4 + 2*b*d*e*f*x**6 + b*d*f**
2*x**8),x)*b*c*f**2*x**3 + int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a +
 b*x**2)*x**2)/(a*c*e**2 + 2*a*c*e*f*x**2 + a*c*f**2*x**4 + a*d*e**2*x**2
+ 2*a*d*e*f*x**4 + a*d*f**2*x**6 + b*c*e**2*x**2 + 2*b*c*e*f*x**4 + b*c*f*
*2*x**6 + b*d*e**2*x**4 + 2*b*d*e*f*x**6 + b*d*f**2*x**8),x)*b*d*e**2*x +
int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c*e**2...
```

3.372 $\int \frac{1}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 570

$$\int \frac{1}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx =$$

$$-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{3acex^3\sqrt{e+fx^2}}+\frac{2(bce+ade+2acf)\sqrt{a+bx^2}\sqrt{c+dx^2}}{3a^2c^2e^2x\sqrt{e+fx^2}}$$

$$-\frac{(2b^2ce^2(de-cf)-a^2f(2d^2e^2+3cdef-8c^2f^2)+abe(2d^2e^2+cdef-3c^2f^2))\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}E\Big(}{3a^{5/2}ce^3\sqrt{-be+af}(de-cf)\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$

$$-\frac{(a^2df(de-4cf)-2b^2ce(de-cf)-ab(d^2e^2+3cdef-4c^2f^2))\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,\mathrm{EllipticF}\Big(\arcsin\Big(}{3a^{5/2}ce^2\sqrt{-be+af}(de-cf)\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$

output
```
-1/3*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a/c/e/x^3/(f*x^2+e)^(1/2)+2/3*(2*a*c*
f+a*d*e+b*c*e)*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a^2/c^2/e^2/x/(f*x^2+e)^(1/
2)-1/3*(2*b^2*c*e^2*(-c*f+d*e)-a^2*f*(-8*c^2*f^2+3*c*d*e*f+2*d^2*e^2)+a*b*
e*(-3*c^2*f^2+c*d*e*f+2*d^2*e^2))*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e)
)^(1/2)*EllipticE((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/
c/(-a*f+b*e))^(1/2))/a^(5/2)/c/e^3/(a*f-b*e)^(1/2)/(-c*f+d*e)/(d*x^2+c)^(1
/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)-1/3*(a^2*d*f*(-4*c*f+d*e)-2*b^2*c*e*(-
c*f+d*e)-a*b*(-4*c^2*f^2+3*c*d*e*f+d^2*e^2))*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/
c/(f*x^2+e))^(1/2)*EllipticF((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*
(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(5/2)/c/e^2/(a*f-b*e)^(1/2)/(-c*f+d*e)/(
d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)
```

Mathematica [F]

$$\int \frac{1}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx=\int \frac{1}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx$$

input
```
Integrate[1/(x^4*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^(3/2)),x]
```

output
```
Integrate[1/(x^4*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^(3/2)), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx$$
$$\Big\downarrow \text{ 450}$$
$$\int \frac{1}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}dx$$

input
```
Int[1/(x^4*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^(3/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [F]

$$\int \frac{1}{x^{4}\sqrt{b\,x^{2}+a}\,\sqrt{x^{2}d+c}\,\left(f\,x^{2}+e\right)^{\frac{3}{2}}}dx$$

input
```
int(1/x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x)
```

output
```
int(1/x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x)
```

Fricas [F]

$$\int \frac{1}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx=\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^{\frac{3}{2}}x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorit
hm="fricas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(b*d*f^2*x^12 + (
2*b*d*e*f + (b*c + a*d)*f^2)*x^10 + (b*d*e^2 + a*c*f^2 + 2*(b*c + a*d)*e*f
)*x^8 + a*c*e^2*x^4 + (2*a*c*e*f + (b*c + a*d)*e^2)*x^6), x)
```

Sympy [F]

$$\int \frac{1}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{1}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x**4/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**(3/2),x)
```

output
```
Integral(1/(x**4*sqrt(a + b*x**2)*sqrt(c + d*x**2)*(e + f*x**2)**(3/2)), x
)
```

Maxima [F]

$$\int \frac{1}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^{\frac{3}{2}}x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorit
hm="maxima")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)*x^4), x)
```

Giac [F]

$$\int \frac{1}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^{\frac{3}{2}}x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorit
hm="giac")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx=\int \frac{1}{x^4\,\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,{\left(f\,x^2+e\right)}^{3/2}}\,dx$$

input
```
int(1/(x^4*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(3/2)),x)
```

output
```
int(1/(x^4*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx=\int \frac{1}{x^4\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,\left(f\,x^2+e\right)^{\frac{3}{2}}}dx$$

input
```
int(1/x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x)
```

output
```
int(1/x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x)
```

3.373 $\int \frac{x^{10}}{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 1042

$$\int \frac{x^{10}}{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^{5/2}}\,dx = -\frac{e^2(be(7de-3cf)-3af(de-cf))x\sqrt{a+bx^2}\sqrt{c+dx^2}}{12bdf^3(be-af)(de-cf)\left(e+fx^2\right)^{3/2}} + \frac{x^5\sqrt{a+bx^2}\sqrt{c+dx^2}}{4bdf\left(e+fx^2\right)^{3/2}} - \frac{(7bde+3bcf+3adf)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{8b^2d^2f^3\sqrt{e+fx^2}} + \frac{c(9a^3df^3(de-cf)^2+3a^2bf^2(de-cf)^2(5de+3cf)+b^3e^2(105d^3e^3-145cd^2e^2f+15c^2def^2+9c^3f^3)-}{24\sqrt{a}b^2d^2f^4(-be+af)^{3/2}(de} - \frac{e(9a^3df^3(de-cf)^2+3a^2bf^2(de-cf)^2(7de+cf)+ab^2ef(75d^3e^3-236cd^2e^2f+183c^2def^2-6c^3f^3)-}{24\sqrt{a}b^2df^5(-be+af)^{3/}} + \frac{e(3a^2d^2f^2+2abdf(5de+cf)+b^2(35d^2e^2+10cdef+3c^2f^2))\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,\text{EllipticPi}\left(-\frac{af}{be-af}, \text{ar}\right.}{8\sqrt{a}b^2d^2f^5\sqrt{-be+af}\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$

output
```
-1/12*e^2*(b*e*(-3*c*f+7*d*e)-3*a*f*(-c*f+d*e))*x*(b*x^2+a)^(1/2)*(d*x^2+c
)^(1/2)/b/d/f^3/(-a*f+b*e)/(-c*f+d*e)/(f*x^2+e)^(3/2)+1/4*x^5*(b*x^2+a)^(1
/2)*(d*x^2+c)^(1/2)/b/d/f/(f*x^2+e)^(3/2)-1/8*(3*a*d*f+3*b*c*f+7*b*d*e)*x*
(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b^2/d^2/f^3/(f*x^2+e)^(1/2)+1/24*c*(9*a^3*
d*f^3*(-c*f+d*e)^2+3*a^2*b*f^2*(-c*f+d*e)^2*(3*c*f+5*d*e)+b^3*e^2*(9*c^3*f
^3+15*c^2*d*e*f^2-145*c*d^2*e^2*f+105*d^3*e^3)-a*b^2*e*f*(18*c^3*f^3+21*c^
2*d*e*f^2-200*c*d^2*e^2*f+145*d^3*e^3))*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*
x^2+e))^(1/2)*EllipticE((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f
+d*e)/c/(-a*f+b*e))^(1/2))/a^(1/2)/b^2/d^2/f^4/(a*f-b*e)^(3/2)/(-c*f+d*e)^
2/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)-1/24*e*(9*a^3*d*f^3*(-c*
f+d*e)^2+3*a^2*b*f^2*(-c*f+d*e)^2*(c*f+7*d*e)+a*b^2*e*f*(-6*c^3*f^3+183*c^
2*d*e*f^2-236*c*d^2*e^2*f+75*d^3*e^3)-b^3*e^2*(-3*c^3*f^3+199*c^2*d*e*f^2-
285*c*d^2*e^2*f+105*d^3*e^3))*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1
/2)*EllipticF((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-
a*f+b*e))^(1/2))/a^(1/2)/b^2/d/f^5/(a*f-b*e)^(3/2)/(-c*f+d*e)^2/(d*x^2+c)^
(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)+1/8*e*(3*a^2*d^2*f^2+2*a*b*d*f*(c*f+
5*d*e)+b^2*(3*c^2*f^2+10*c*d*e*f+35*d^2*e^2))*(b*x^2+a)^(1/2)*(e*(d*x^2+c)
/c/(f*x^2+e))^(1/2)*EllipticPi((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),-
a*f/(-a*f+b*e),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(1/2)/b^2/d^2/f^5/(a*f
-b*e)^(1/2)/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)
```

Mathematica [F]

$$\int \frac{x^{10}}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx = \int \frac{x^{10}}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx$$

input
```
Integrate[x^10/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^(5/2)),x]
```

output
```
Integrate[x^10/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^(5/2)), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{10}}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx$$
$$\Big\downarrow \text{450}$$
$$\int \frac{x^{10}}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}dx$$

input
```
Int[x^10/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^(5/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [F]

$$\int \frac{x^{10}}{\sqrt{b\,x^{2}+a}\,\sqrt{x^{2}d+c}\,\left(f\,x^{2}+e\right)^{\frac{5}{2}}}dx$$

input
```
int(x^10/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x)
```

output
```
int(x^10/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x)
```

Fricas [F]

$$\int \frac{x^{10}}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx=\int \frac{x^{10}}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^10/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x, algorith
m="fricas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)*x^10/(b*d*f^3*x^1
0 + (3*b*d*e*f^2 + (b*c + a*d)*f^3)*x^8 + (3*b*d*e^2*f + a*c*f^3 + 3*(b*c
+ a*d)*e*f^2)*x^6 + a*c*e^3 + (b*d*e^3 + 3*a*c*e*f^2 + 3*(b*c + a*d)*e^2*f
)*x^4 + (3*a*c*e^2*f + (b*c + a*d)*e^3)*x^2), x)
```

Sympy [F]

$$\int \frac{x^{10}}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx=\int \frac{x^{10}}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**10/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**(5/2),x)
```

output
```
Integral(x**10/(sqrt(a + b*x**2)*sqrt(c + d*x**2)*(e + f*x**2)**(5/2)), x)
```

Maxima [F]

$$\int \frac{x^{10}}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx=\int \frac{x^{10}}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^10/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x, algorith
m="maxima")
```

output
```
integrate(x^10/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^(5/2)), x)
```

Giac [F]

$$\int \frac{x^{10}}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx = \int \frac{x^{10}}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^10/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x, algorith
m="giac")
```

output
```
integrate(x^10/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{10}}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx = \int \frac{x^{10}}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,(f\,x^2+e)^{5/2}}\,dx$$

input
```
int(x^10/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(5/2)),x)
```

output
```
int(x^10/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{x^{10}}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx = \int \frac{x^{10}}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,(f\,x^2+e)^{\frac{5}{2}}}dx$$

input
```
int(x^10/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x)
```

output
```
int(x^10/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x)
```

3.374 $\int \frac{x^8}{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 819

$$\int \frac{x^8}{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^{5/2}}\,dx = \frac{e(be(5de-3cf)-3af(de-cf))x\sqrt{a+bx^2}\sqrt{c+dx^2}}{6bdf^2(be-af)(de-cf)\left(e+fx^2\right)^{3/2}}$$

$$+\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{2bdf\left(e+fx^2\right)^{3/2}}$$

$$-\frac{c(3a^2f^2(de-cf)^2+b^2e^2(15d^2e^2-22cdef+3c^2f^2)-2abef(11d^2e^2-16cdef+3c^2f^2))\sqrt{a+bx^2}\sqrt{\frac{e(c\cdots}{c(e\cdots}}\cdots}{6\sqrt{a}bdf^3(-be+af)^{3/2}(de-cf)^2\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$

$$+\frac{e(3a^2f^2(de-cf)^2+2abef(6d^2e^2-19cdef+15c^2f^2)-b^2e^2(15d^2e^2-42cdef+31c^2f^2))\sqrt{a+bx^2}\sqrt{\frac{e(\cdots}{c(\cdots}}\cdots}{6\sqrt{a}bf^4(-be+af)^{3/2}(de-cf)^2\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$

$$-\frac{e(5bde+bcf+adf)\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,\text{EllipticPi}\left(-\frac{af}{be-af},\arcsin\left(\frac{\sqrt{-be+afx}}{\sqrt{a}\sqrt{e+fx^2}}\right),\frac{a(de-cf)}{c(be-af)}\right)}{2\sqrt{a}bdf^4\sqrt{-be+af}\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$

output
```
1/6*e*(b*e*(-3*c*f+5*d*e)-3*a*f*(-c*f+d*e))*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1
/2)/b/d/f^2/(-a*f+b*e)/(-c*f+d*e)/(f*x^2+e)^(3/2)+1/2*x^3*(b*x^2+a)^(1/2)*
(d*x^2+c)^(1/2)/b/d/f/(f*x^2+e)^(3/2)-1/6*c*(3*a^2*f^2*(-c*f+d*e)^2+b^2*e^
2*(3*c^2*f^2-22*c*d*e*f+15*d^2*e^2)-2*a*b*e*f*(3*c^2*f^2-16*c*d*e*f+11*d^2
*e^2))*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticE((a*f-b*e)
^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(1/2
)/b/d/f^3/(a*f-b*e)^(3/2)/(-c*f+d*e)^2/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x
^2+e))^(1/2)+1/6*e*(3*a^2*f^2*(-c*f+d*e)^2+2*a*b*e*f*(15*c^2*f^2-19*c*d*e*
f+6*d^2*e^2)-b^2*e^2*(31*c^2*f^2-42*c*d*e*f+15*d^2*e^2))*(b*x^2+a)^(1/2)*(
e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticF((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+
e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(1/2)/b/f^4/(a*f-b*e)^(3/2)/
(-c*f+d*e)^2/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)-1/2*e*(a*d*f+
b*c*f+5*b*d*e)*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticPi(
(a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),-a*f/(-a*f+b*e),(a*(-c*f+d*e)/c/
(-a*f+b*e))^(1/2))/a^(1/2)/b/d/f^4/(a*f-b*e)^(1/2)/(d*x^2+c)^(1/2)/(e*(b*x
^2+a)/a/(f*x^2+e))^(1/2)
```

Mathematica [F]

$$\int \frac{x^8}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx = \int \frac{x^8}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx$$

input
```
Integrate[x^8/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^(5/2)),x]
```

output
```
Integrate[x^8/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^(5/2)), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{450}$$

$$\int \frac{x^8}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}dx$$

input
```
Int[x^8/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^(5/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [F]

$$\int \frac{x^8}{\sqrt{b\,x^2+a}\,\sqrt{x^2d+c}\,\left(f\,x^2+e\right)^{\frac{5}{2}}}dx$$

input
```
int(x^8/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x)
```

output
```
int(x^8/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^8}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate(x^8/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x, algorithm
="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^8}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx = \int \frac{x^8}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**8/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**(5/2),x)
```

output
```
Integral(x**8/(sqrt(a + b*x**2)*sqrt(c + d*x**2)*(e + f*x**2)**(5/2)), x)
```

Maxima [F]

$$\int \frac{x^8}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx = \int \frac{x^8}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^8/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x, algorithm
="maxima")
```

output
```
integrate(x^8/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^(5/2)), x)
```

Giac [F]

$$\int \frac{x^8}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx = \int \frac{x^8}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^8/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x, algorithm
="giac")
```

output
```
integrate(x^8/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^8}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx = \int \frac{x^8}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,(f\,x^2+e)^{5/2}}\,dx$$

input
```
int(x^8/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(5/2)),x)
```

output
```
int(x^8/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{x^8}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx = \int \frac{x^8}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,(f\,x^2+e)^{\frac{5}{2}}}dx$$

input
```
int(x^8/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x)
```

output
```
int(x^8/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x)
```

3.375 $\int \frac{x^6}{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 633

$$\int \frac{x^6}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx = -\frac{e^2x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3f(be-af)(de-cf)\,(e+fx^2)^{3/2}}$$
$$-\frac{ce(af(5de-7cf)-be(3de-5cf))\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}E\left(\arcsin\left(\frac{\sqrt{-be+afx}}{\sqrt{a}\sqrt{e+fx^2}}\right)|\frac{a(de-cf)}{c(be-af)}\right)}{3\sqrt{a}f^2(-be+af)^{3/2}(de-cf)^2\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$
$$+\frac{e(be(3d^2e^2-9cdef+8c^2f^2)-af(3d^2e^2-10cdef+9c^2f^2))\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-be}}{\sqrt{a}\sqrt{}}\right.\right.}{3\sqrt{a}f^3(-be+af)^{3/2}(de-cf)^2\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$
$$+\frac{e\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,\text{EllipticPi}\left(-\frac{af}{be-af},\arcsin\left(\frac{\sqrt{-be+afx}}{\sqrt{a}\sqrt{e+fx^2}}\right),\frac{a(de-cf)}{c(be-af)}\right)}{\sqrt{a}f^3\sqrt{-be+af}\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$

output
```
-1/3*e^2*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/f/(-a*f+b*e)/(-c*f+d*e)/(f*x^2+
e)^(3/2)-1/3*c*e*(a*f*(-7*c*f+5*d*e)-b*e*(-5*c*f+3*d*e))*(b*x^2+a)^(1/2)*(
e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticE((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+
e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(1/2)/f^2/(a*f-b*e)^(3/2)/(-
c*f+d*e)^2/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)+1/3*e*(b*e*(8*c
^2*f^2-9*c*d*e*f+3*d^2*e^2)-a*f*(9*c^2*f^2-10*c*d*e*f+3*d^2*e^2))*(b*x^2+a
)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticF((a*f-b*e)^(1/2)*x/a^(1/2
)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(1/2)/f^3/(a*f-b*e)
^(3/2)/(-c*f+d*e)^2/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)+e*(b*x
^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticPi((a*f-b*e)^(1/2)*x/a
^(1/2)/(f*x^2+e)^(1/2),-a*f/(-a*f+b*e),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/
a^(1/2)/f^3/(a*f-b*e)^(1/2)/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2
)
```

Mathematica [F]

$$\int \frac{x^6}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx = \int \frac{x^6}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx$$

input
```
Integrate[x^6/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^(5/2)),x]
```

output
```
Integrate[x^6/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^(5/2)), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{ 450}$$

$$\int \frac{x^6}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}dx$$

input
```
Int[x^6/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^(5/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [F]

$$\int \frac{x^{6}}{\sqrt{b\,x^{2}+a}\,\sqrt{x^{2}d+c}\,\left(f\,x^{2}+e\right)^{\frac{5}{2}}}d x$$

input
```
int(x^6/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x)
```

output
```
int(x^6/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x)
```

Fricas [F]

$$\int \frac{x^6}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx = \int \frac{x^{6}}{\sqrt{bx^{2}+a}\sqrt{dx^{2}+c}(fx^{2}+e)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^6/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x, algorithm
="fricas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)*x^6/(b*d*f^3*x^10
 + (3*b*d*e*f^2 + (b*c + a*d)*f^3)*x^8 + (3*b*d*e^2*f + a*c*f^3 + 3*(b*c +
 a*d)*e*f^2)*x^6 + a*c*e^3 + (b*d*e^3 + 3*a*c*e*f^2 + 3*(b*c + a*d)*e^2*f)
*x^4 + (3*a*c*e^2*f + (b*c + a*d)*e^3)*x^2), x)
```

Sympy [F]

$$\int \frac{x^6}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx = \int \frac{x^6}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**6/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**(5/2),x)
```

output
```
Integral(x**6/(sqrt(a + b*x**2)*sqrt(c + d*x**2)*(e + f*x**2)**(5/2)), x)
```

Maxima [F]

$$\int \frac{x^6}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx = \int \frac{x^6}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^6/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x, algorithm
="maxima")
```

output
```
integrate(x^6/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^(5/2)), x)
```

Giac [F]

$$\int \frac{x^6}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx = \int \frac{x^6}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^6/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x, algorithm
="giac")
```

output
```
integrate(x^6/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx = \int \frac{x^6}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,(f\,x^2+e)^{5/2}}\,dx$$

input
```
int(x^6/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(5/2)),x)
```

output
```
int(x^6/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{x^6}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx = \int \frac{x^6}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,(f\,x^2+e)^{\frac{5}{2}}}dx$$

input
```
int(x^6/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x)
```

output
```
int(x^6/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x)
```

3.376 $\int \frac{x^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 407

$$\int \frac{x^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx = \frac{ex\sqrt{a+bx^2}\sqrt{c+dx^2}}{3(be-af)(de-cf)\,(e+fx^2)^{3/2}} + \frac{2c(bce+ade-2acf)\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}E\left(\arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right)\middle|\frac{a(de-cf)}{c(be-af)}\right)}{3\sqrt{a}(-be+af)^{3/2}(de-cf)^2\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}} - \frac{c(2bce+ade-3acf)\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right),\frac{a(de-cf)}{c(be-af)}\right)}{3\sqrt{a}(-be+af)^{3/2}(de-cf)^2\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$

output
```
1/3*e*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(-a*f+b*e)/(-c*f+d*e)/(f*x^2+e)^(3
/2)+2/3*c*(-2*a*c*f+a*d*e+b*c*e)*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))
^(1/2)*EllipticE((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c
/(-a*f+b*e))^(1/2))/a^(1/2)/(a*f-b*e)^(3/2)/(-c*f+d*e)^2/(d*x^2+c)^(1/2)/(
e*(b*x^2+a)/a/(f*x^2+e))^(1/2)-1/3*c*(-3*a*c*f+a*d*e+2*b*c*e)*(b*x^2+a)^(1
/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticF((a*f-b*e)^(1/2)*x/a^(1/2)/(f
*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(1/2)/(a*f-b*e)^(3/2)/(
-c*f+d*e)^2/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)
```

Mathematica [F]

$$\int \frac{x^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx=\int \frac{x^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx$$

input
```
Integrate[x^4/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^(5/2)),x]
```

output
```
Integrate[x^4/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^(5/2)), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx$$
$$\Big\downarrow \ 450$$
$$\int \frac{x^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}dx$$

input
```
Int[x^4/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^(5/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [F]

$$\int \frac{x^4}{\sqrt{b\,x^2+a}\,\sqrt{x^2 d+c}\,\left(f\,x^2+e\right)^{\frac{5}{2}}}dx$$

input
```
int(x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x)
```

output
```
int(x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x)
```

Fricas [F]

$$\int \frac{x^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx = \int \frac{x^4}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x, algorithm
="fricas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)*x^4/(b*d*f^3*x^10
 + (3*b*d*e*f^2 + (b*c + a*d)*f^3)*x^8 + (3*b*d*e^2*f + a*c*f^3 + 3*(b*c +
 a*d)*e*f^2)*x^6 + a*c*e^3 + (b*d*e^3 + 3*a*c*e*f^2 + 3*(b*c + a*d)*e^2*f)
*x^4 + (3*a*c*e^2*f + (b*c + a*d)*e^3)*x^2), x)
```

Sympy [F]

$$\int \frac{x^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx = \int \frac{x^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**4/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**(5/2),x)
```

output
```
Integral(x**4/(sqrt(a + b*x**2)*sqrt(c + d*x**2)*(e + f*x**2)**(5/2)), x)
```

Maxima [F]

$$\int \frac{x^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx = \int \frac{x^4}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x, algorithm
="maxima")
```

output
```
integrate(x^4/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^(5/2)), x)
```

Giac [F]

$$\int \frac{x^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx = \int \frac{x^4}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x, algorithm
="giac")
```

output
```
integrate(x^4/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx = \int \frac{x^4}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,(f\,x^2+e)^{5/2}}\,dx$$

input
```
int(x^4/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(5/2)),x)
```

output
```
int(x^4/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{x^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx = \int \frac{x^4}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,(f\,x^2+e)^{\frac{5}{2}}}dx$$

input
```
int(x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x)
```

output
```
int(x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x)
```

3.377 $\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 421

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx = -\frac{fx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3(be-af)(de-cf)\,(e+fx^2)^{3/2}} - \frac{c(be(3de-cf)-af(de+cf))\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}E\left(\arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right)\middle|\frac{a(de-cf)}{c(be-af)}\right)}{3\sqrt{a}e(-be+af)^{3/2}(de-cf)^2\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}} + \frac{c(3bde-bcf-2adf)\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right),\frac{a(de-cf)}{c(be-af)}\right)}{3\sqrt{a}(-be+af)^{3/2}(de-cf)^2\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$

output
```
-1/3*f*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(-a*f+b*e)/(-c*f+d*e)/(f*x^2+e)^(
3/2)-1/3*c*(b*e*(-c*f+3*d*e)-a*f*(c*f+d*e))*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c
/(f*x^2+e))^(1/2)*EllipticE((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(
-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(1/2)/e/(a*f-b*e)^(3/2)/(-c*f+d*e)^2/(d*x
^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)+1/3*c*(-2*a*d*f-b*c*f+3*b*d*e)
*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticF((a*f-b*e)^(1/2)
*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(1/2)/(a*f
-b*e)^(3/2)/(-c*f+d*e)^2/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)
```

Mathematica [F]

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^{5/2}}\,dx = \int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^{5/2}}\,dx$$

input
```
Integrate[x^2/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^(5/2)),x]
```

output
```
Integrate[x^2/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^(5/2)), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^{5/2}}\,dx$$

$$\Big\downarrow \; 450$$

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^{5/2}}dx$$

input
```
Int[x^2/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^(5/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [F]

$$\int \frac{x^2}{\sqrt{b\,x^2+a}\,\sqrt{x^2d+c}\,\left(f\,x^2+e\right)^{\frac{5}{2}}}dx$$

input
```
int(x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x)
```

output
```
int(x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x)
```

Fricas [F]

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx = \int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x, algorithm
="fricas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)*x^2/(b*d*f^3*x^10
 + (3*b*d*e*f^2 + (b*c + a*d)*f^3)*x^8 + (3*b*d*e^2*f + a*c*f^3 + 3*(b*c +
 a*d)*e*f^2)*x^6 + a*c*e^3 + (b*d*e^3 + 3*a*c*e*f^2 + 3*(b*c + a*d)*e^2*f)
*x^4 + (3*a*c*e^2*f + (b*c + a*d)*e^3)*x^2), x)
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx = \int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**2/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**(5/2),x)
```

output
```
Integral(x**2/(sqrt(a + b*x**2)*sqrt(c + d*x**2)*(e + f*x**2)**(5/2)), x)
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx = \int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x, algorithm
="maxima")
```

output
```
integrate(x^2/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^(5/2)), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx = \int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x, algorithm
="giac")
```

output
```
integrate(x^2/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx = \int \frac{x^2}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,(f\,x^2+e)^{5/2}}\,dx$$

input
```
int(x^2/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(5/2)),x)
```

output
```
int(x^2/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx = \int \frac{x^2}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,(f\,x^2+e)^{\frac{5}{2}}}dx$$

input
```
int(x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x)
```

output
```
int(x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x)
```

3.378 $\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 449

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^{5/2}}\,dx = \frac{f^2x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3e(be-af)(de-cf)\left(e+fx^2\right)^{3/2}} + \frac{2cf(be(3de-2cf)-af(2de-cf))\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}E\left(\arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right)|\frac{a(de-cf)}{c(be-af)}\right)}{3\sqrt{a}e^2(-be+af)^{3/2}(de-cf)^2\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}} + \frac{\left(adf(3de-cf)-b(3d^2e^2-c^2f^2)\right)\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right),\frac{a(de-cf)}{c(be-af)}\right)}{3\sqrt{a}e(-be+af)^{3/2}(de-cf)^2\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$

output

```
1/3*f^2*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/e/(-a*f+b*e)/(-c*f+d*e)/(f*x^2+e
)^(3/2)+2/3*c*f*(b*e*(-2*c*f+3*d*e)-a*f*(-c*f+2*d*e))*(b*x^2+a)^(1/2)*(e*(
d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticE((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^
(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(1/2)/e^2/(a*f-b*e)^(3/2)/(-c*f
+d*e)^2/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)+1/3*(a*d*f*(-c*f+3
*d*e)-b*(-c^2*f^2+3*d^2*e^2))*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1
/2)*EllipticF((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-
a*f+b*e))^(1/2))/a^(1/2)/e/(a*f-b*e)^(3/2)/(-c*f+d*e)^2/(d*x^2+c)^(1/2)/(e
*(b*x^2+a)/a/(f*x^2+e))^(1/2)
```

Mathematica [F]

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx = \int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx$$

input
```
Integrate[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^(5/2)),x]
```

output
```
Integrate[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^(5/2)), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx$$

$$\Big\downarrow \ 434$$

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}dx$$

input
```
Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^(5/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 434
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2)^(r_.), x_Symbol] :> Unintegrable[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*
x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, p, q, r}, x]
```

Maple [F]

$$\int \frac{1}{\sqrt{b\,x^{2}+a}\,\sqrt{x^{2}d+c}\,\left(f\,x^{2}+e\right)^{\frac{5}{2}}}dx$$

input
```
int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x)
```

output
```
int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx = \int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x, algorithm="
fricas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(b*d*f^3*x^10 + (
3*b*d*e*f^2 + (b*c + a*d)*f^3)*x^8 + (3*b*d*e^2*f + a*c*f^3 + 3*(b*c + a*d
)*e*f^2)*x^6 + a*c*e^3 + (b*d*e^3 + 3*a*c*e*f^2 + 3*(b*c + a*d)*e^2*f)*x^4
 + (3*a*c*e^2*f + (b*c + a*d)*e^3)*x^2), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx = \int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**(5/2),x)
```

output
```
Integral(1/(sqrt(a + b*x**2)*sqrt(c + d*x**2)*(e + f*x**2)**(5/2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx = \int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x, algorithm="
maxima")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^(5/2)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx = \int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x, algorithm="
giac")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx = \int \frac{1}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,{\left(f\,x^2+e\right)}^{5/2}}\,dx$$

input
```
int(1/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(5/2)),x)
```

output
```
int(1/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx = \int \frac{1}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,(f\,x^2+e)^{\frac{5}{2}}}dx$$

input
```
int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x)
```

output
```
int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x)
```

3.379 $\int \frac{1}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}(e+fx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 575

$$\int \frac{1}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^{5/2}}\,dx = -\frac{f^3x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3e^2(be-af)(de-cf)\left(e+fx^2\right)^{3/2}}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{ace^2x\sqrt{e+fx^2}}$$
$$-\frac{(3b^2e^2(de-cf)^2+a^2f^2(3d^2e^2-13cdef+8c^2f^2)-abef(6d^2e^2-21cdef+13c^2f^2))\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}{3a^{3/2}e^3(-be+af)^{3/2}(de-cf)^2\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$
$$-\frac{(2a^2df^2(3de-2cf)-3b^2e(de-cf)^2-abf(3d^2e^2+3cdef-4c^2f^2))\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,\text{EllipticF}\left(\text{arcs}\right.}{3a^{3/2}e^2(-be+af)^{3/2}(de-cf)^2\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$

output
```
-1/3*f^3*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/e^2/(-a*f+b*e)/(-c*f+d*e)/(f*x^
2+e)^(3/2)-(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a/c/e^2/x/(f*x^2+e)^(1/2)-1/3*(
3*b^2*e^2*(-c*f+d*e)^2+a^2*f^2*(8*c^2*f^2-13*c*d*e*f+3*d^2*e^2)-a*b*e*f*(1
3*c^2*f^2-21*c*d*e*f+6*d^2*e^2))*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))
^(1/2)*EllipticE((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c
/(-a*f+b*e))^(1/2))/a^(3/2)/e^3/(a*f-b*e)^(3/2)/(-c*f+d*e)^2/(d*x^2+c)^(1/
2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)-1/3*(2*a^2*d*f^2*(-2*c*f+3*d*e)-3*b^2*e
*(-c*f+d*e)^2-a*b*f*(-4*c^2*f^2+3*c*d*e*f+3*d^2*e^2))*(b*x^2+a)^(1/2)*(e*(
d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticF((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^
(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(3/2)/e^2/(a*f-b*e)^(3/2)/(-c*f
+d*e)^2/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)
```

Mathematica [F]

$$\int \frac{1}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^{5/2}}\,dx=\int \frac{1}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^{5/2}}\,dx$$

input
```
Integrate[1/(x^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^(5/2)),x]
```

output
```
Integrate[1/(x^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^(5/2)), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^{5/2}}\,dx$$
$$\Big\downarrow \; 450$$
$$\int \frac{1}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^{5/2}}dx$$

input
```
Int[1/(x^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^(5/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [F]

$$\int \frac{1}{x^{2} \sqrt{b\,x^{2}+a}\,\sqrt{x^{2} d+c}\,\left(f\,x^{2}+e\right)^{\frac{5}{2}}}d x$$

input
```
int(1/x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x)
```

output
```
int(1/x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x)
```

Fricas [F]

$$\int \frac{1}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^{5/2}}\,dx=\int \frac{1}{\sqrt{bx^{2}+a}\sqrt{dx^{2}+c}(fx^{2}+e)^{\frac{5}{2}}x^{2}}\,dx$$

input
```
integrate(1/x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x, algorit
hm="fricas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(b*d*f^3*x^12 + (
3*b*d*e*f^2 + (b*c + a*d)*f^3)*x^10 + (3*b*d*e^2*f + a*c*f^3 + 3*(b*c + a*
d)*e*f^2)*x^8 + a*c*e^3*x^2 + (b*d*e^3 + 3*a*c*e*f^2 + 3*(b*c + a*d)*e^2*f
)*x^6 + (3*a*c*e^2*f + (b*c + a*d)*e^3)*x^4), x)
```

Sympy [F]

$$\int \frac{1}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx = \int \frac{1}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/x**2/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**(5/2),x)
```

output
```
Integral(1/(x**2*sqrt(a + b*x**2)*sqrt(c + d*x**2)*(e + f*x**2)**(5/2)), x
)
```

Maxima [F]

$$\int \frac{1}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx = \int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^{\frac{5}{2}}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x, algorit
hm="maxima")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^(5/2)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx = \int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^{\frac{5}{2}}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x, algorit
hm="giac")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^(5/2)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx = \int \frac{1}{x^2\,\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,(f\,x^2+e)^{5/2}}\,dx$$

input
```
int(1/(x^2*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(5/2)),x)
```

output
```
int(1/(x^2*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx = \int \frac{1}{x^2\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,\left(f\,x^2+e\right)^{\frac{5}{2}}}dx$$

input
```
int(1/x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x)
```

output
```
int(1/x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x)
```

3.380 $\int \frac{1}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 757

$$\int \frac{1}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^{5/2}}\,dx=\frac{f^4x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3e^3(be-af)(de-cf)\left(e+fx^2\right)^{3/2}}$$
$$-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{3ace^2x^3\sqrt{e+fx^2}}+\frac{(2bce+2ade+7acf)\sqrt{a+bx^2}\sqrt{c+dx^2}}{3a^2c^2e^3x\sqrt{e+fx^2}}$$
$$+\frac{2(b^3ce^3(de-cf)^2+ab^2e^2(de-cf)^2(de+2cf)+a^3f^2(d^3e^3+2cd^2e^2f-12c^2def^2+8c^3f^3)-a^2bef(2d^3\ldots}{3a^{5/2}ce^4(-be+af)^{3/2}(de-cf)^2\sqrt{\ldots}}$$
$$-\frac{(2b^3ce^2(de-cf)^2+ab^2e(de-cf)^2(de+5cf)+a^3df^2(d^2e^2-11cdef+8c^2f^2)-2a^2bf(d^3e^3-3cd^2e^2f\ldots}{3a^{5/2}ce^3(-be+af)^{3/2}(de-cf)^2\sqrt{c+\ldots}}$$

output
```
1/3*f^4*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/e^3/(-a*f+b*e)/(-c*f+d*e)/(f*x^2
+e)^(3/2)-1/3*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a/c/e^2/x^3/(f*x^2+e)^(1/2)+
1/3*(7*a*c*f+2*a*d*e+2*b*c*e)*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a^2/c^2/e^3/
x/(f*x^2+e)^(1/2)+2/3*(b^3*c*e^3*(-c*f+d*e)^2+a*b^2*e^2*(-c*f+d*e)^2*(2*c*
f+d*e)+a^3*f^2*(8*c^3*f^3-12*c^2*d*e*f^2+2*c*d^2*e^2*f+d^3*e^3)-a^2*b*e*f*
(12*c^3*f^3-18*c^2*d*e*f^2+3*c*d^2*e^2*f+2*d^3*e^3))*(b*x^2+a)^(1/2)*(e*(d
*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticE((a*f-b*e)^(1/2)*x/a^(1/2)/(f*x^2+e)^(
1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(5/2)/c/e^4/(a*f-b*e)^(3/2)/(-c*
f+d*e)^2/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)-1/3*(2*b^3*c*e^2*
(-c*f+d*e)^2+a*b^2*e*(-c*f+d*e)^2*(5*c*f+d*e)+a^3*d*f^2*(8*c^2*f^2-11*c*d*
e*f+d^2*e^2)-2*a^2*b*f*(4*c^3*f^3-3*c^2*d*e*f^2-3*c*d^2*e^2*f+d^3*e^3))*(b
*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticF((a*f-b*e)^(1/2)*x/
a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(5/2)/c/e^3/(
a*f-b*e)^(3/2)/(-c*f+d*e)^2/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2
)
```

Mathematica [F]

$$\int \frac{1}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx = \int \frac{1}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx$$

input
```
Integrate[1/(x^4*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^(5/2)),x]
```

output
```
Integrate[1/(x^4*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^(5/2)), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx$$

$$\Big\downarrow \; 450$$

$$\int \frac{1}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx$$

input
```
Int[1/(x^4*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^(5/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [F]

$$\int \frac{1}{x^{4}\sqrt{b\,x^{2}+a}\,\sqrt{x^{2}d+c}\,\left(f\,x^{2}+e\right)^{\frac{5}{2}}}\,dx$$

input
```
int(1/x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x)
```

output
```
int(1/x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x)
```

Fricas [F]

$$\int \frac{1}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx = \int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^{\frac{5}{2}}x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x, algorit
hm="fricas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(b*d*f^3*x^14 + (
3*b*d*e*f^2 + (b*c + a*d)*f^3)*x^12 + (3*b*d*e^2*f + a*c*f^3 + 3*(b*c + a*
d)*e*f^2)*x^10 + a*c*e^3*x^4 + (b*d*e^3 + 3*a*c*e*f^2 + 3*(b*c + a*d)*e^2*
f)*x^8 + (3*a*c*e^2*f + (b*c + a*d)*e^3)*x^6), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/x**4/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx = \int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^{\frac{5}{2}}x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x, algorit
hm="maxima")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^(5/2)*x^4), x)
```

Giac [F]

$$\int \frac{1}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx = \int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^{\frac{5}{2}}x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x, algorit
hm="giac")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^(5/2)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx = \int \frac{1}{x^4\,\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,{(f\,x^2+e)}^{5/2}}\,dx$$

input
```
int(1/(x^4*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(5/2)),x)
```

output
```
int(1/(x^4*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{5/2}}\,dx = \int \frac{1}{x^4\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,\left(f\,x^2+e\right)^{\frac{5}{2}}}dx$$

input
```
int(1/x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x)
```

output
```
int(1/x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(5/2),x)
```

3.381 $\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 313

$$\begin{aligned}\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx &= -\frac{a\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right),\frac{c(be-af)}{(bc-ad)e}\right)}{b\sqrt{c}\sqrt{bc-ad}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}} \\ &\quad+\frac{a\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,\text{EllipticPi}\left(\frac{bc}{bc-ad},\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right),\frac{c(be-af)}{(bc-ad)e}\right)}{b\sqrt{c}\sqrt{bc-ad}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}\end{aligned}$$

output
```
-a*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)*EllipticF((-a*d+b*c)^(1
/2)*x/c^(1/2)/(b*x^2+a)^(1/2),(c*(-a*f+b*e)/(-a*d+b*c)/e)^(1/2))/b/c^(1/2)
/(-a*d+b*c)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(f*x^2+e)^(1/2)+a*(d*x^2
+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)*EllipticPi((-a*d+b*c)^(1/2)*x/c^
(1/2)/(b*x^2+a)^(1/2),b*c/(-a*d+b*c),(c*(-a*f+b*e)/(-a*d+b*c)/e)^(1/2))/b/
c^(1/2)/(-a*d+b*c)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(f*x^2+e)^(1/2)
```

Mathematica [F]

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$$

input
```
Integrate[x^2/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
```

output
```
Integrate[x^2/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$$
$$\Big\downarrow \text{450}$$
$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}dx$$

input
```
Int[x^2/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [F]

$$\int \frac{x^2}{\sqrt{b\,x^2+a}\,\sqrt{x^2 d+c}\,\sqrt{f\,x^2+e}}dx$$

input
```
int(x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

output
```
int(x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \text{Timed out}$$

input
```
integrate(x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm
="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$$

input
```
integrate(x**2/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**(1/2),x)
```

output
```
Integral(x**2/(sqrt(a + b*x**2)*sqrt(c + d*x**2)*sqrt(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{fx^2+e}}\,dx$$

input
```
integrate(x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm
="maxima")
```

output
```
integrate(x^2/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{fx^2+e}}\,dx$$

input
```
integrate(x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm
="giac")
```

output
```
integrate(x^2/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{x^2}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}}\,dx$$

input
```
int(x^2/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(1/2)),x)
```

output
```
int(x^2/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$$
$$= \int \frac{\sqrt{f\,x^2+e}\,\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^2}{bdf\,x^6+adf\,x^4+bcf\,x^4+bde\,x^4+acf\,x^2+ade\,x^2+bce\,x^2+ace}dx$$

input
```
int(x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

output
```
int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c*e + a*c
*f*x**2 + a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 +
 b*d*f*x**6),x)
```

3.382 $\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 412

$$
\begin{aligned}
\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx
&= \frac{\sqrt{a}\sqrt{a-bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{bc+ad}x}{\sqrt{a}\sqrt{c+dx^2}}\right),\frac{a(de-cf)}{(bc+ad)e}\right)}{b\sqrt{bc+ad}\sqrt{\frac{c(a-bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}} \\
&\quad - \frac{\sqrt{c}\sqrt{bc+ad}\sqrt{e+fx^2}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{bc+ad}x}{\sqrt{c}\sqrt{a-bx^2}}\right),\frac{a(de-cf)}{(bc+ad)e}\right)}{bde\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}} \\
&\quad + \frac{c^{3/2}\sqrt{e+fx^2}\,\mathrm{EllipticPi}\left(\frac{ad}{bc+ad},\arctan\left(\frac{\sqrt{bc+ad}x}{\sqrt{c}\sqrt{a-bx^2}}\right),\frac{a(de-cf)}{(bc+ad)e}\right)}{d\sqrt{bc+ad}e\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}
\end{aligned}
$$

output
```
a^(1/2)*(-b*x^2+a)^(1/2)*(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)*EllipticF((a*d+b*
c)^(1/2)*x/a^(1/2)/(d*x^2+c)^(1/2),(a*(-c*f+d*e)/(a*d+b*c)/e)^(1/2))/b/(a*
d+b*c)^(1/2)/(c*(-b*x^2+a)/a/(d*x^2+c))^(1/2)/(f*x^2+e)^(1/2)-c^(1/2)*(a*d
+b*c)^(1/2)*(f*x^2+e)^(1/2)*InverseJacobiAM(arctan((a*d+b*c)^(1/2)*x/c^(1/
2)/(-b*x^2+a)^(1/2)),(a*(-c*f+d*e)/(a*d+b*c)/e)^(1/2))/b/d/e/(d*x^2+c)^(1/
2)/(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)+c^(3/2)*(f*x^2+e)^(1/2)*EllipticPi((a*d
+b*c)^(1/2)*x/c^(1/2)/(-b*x^2+a)^(1/2)/(1+(a*d+b*c)*x^2/c/(-b*x^2+a))^(1/2
),a*d/(a*d+b*c),(a*(-c*f+d*e)/(a*d+b*c)/e)^(1/2))/d/(a*d+b*c)^(1/2)/e/(d*x
^2+c)^(1/2)/(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)
```

Mathematica [F]

$$\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$$

input
```
Integrate[x^2/(Sqrt[a - b*x^2]*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
```

output
```
Integrate[x^2/(Sqrt[a - b*x^2]*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$$
$$\Big\downarrow \ 450$$
$$\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}dx$$

input
```
Int[x^2/(Sqrt[a - b*x^2]*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [F]

$$\int \frac{x^2}{\sqrt{-b\,x^2+a}\,\sqrt{x^2d+c}\,\sqrt{f\,x^2+e}}dx$$

input
```
int(x^2/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

output
```
int(x^2/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \text{Timed out}$$

input
```
integrate(x^2/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorith
m="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$$

input
```
integrate(x**2/(-b*x**2+a)**(1/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**(1/2),x)
```

output
```
Integral(x**2/(sqrt(a - b*x**2)*sqrt(c + d*x**2)*sqrt(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{x^2}{\sqrt{-bx^2+a}\sqrt{dx^2+c}\sqrt{fx^2+e}}\,dx$$

input
```
integrate(x^2/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorith
m="maxima")
```

output
```
integrate(x^2/(sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{x^2}{\sqrt{-bx^2+a}\sqrt{dx^2+c}\sqrt{fx^2+e}}\,dx$$

input
```
integrate(x^2/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorith
m="giac")
```

output
```
integrate(x^2/(sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{x^2}{\sqrt{a-b\,x^2}\,\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}}\,dx$$

input
```
int(x^2/((a - b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(1/2)),x)
```

output
```
int(x^2/((a - b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{\sqrt{f\,x^2+e}\,\sqrt{d\,x^2+c}\,\sqrt{-b\,x^2+a}\,x^2}{-bdf\,x^6+adf\,x^4-bcf\,x^4-bde\,x^4+acf\,x^2+ade\,x^2-bce\,x^2+ace}dx$$

input
```
int(x^2/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

output
```
int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**2)/(a*c*e + a*c
*f*x**2 + a*d*e*x**2 + a*d*f*x**4 - b*c*e*x**2 - b*c*f*x**4 - b*d*e*x**4 -
 b*d*f*x**6),x)
```

3.383 $\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c-dx^2}\sqrt{e+fx^2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 310

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c-dx^2}\sqrt{e+fx^2}}\,dx = -\frac{a\sqrt{c-dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{bc+ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right),\frac{c(be-af)}{(bc+ad)e}\right)}{b\sqrt{c}\sqrt{bc+ad}\sqrt{\frac{a(c-dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}} + \frac{a\sqrt{c-dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\operatorname{EllipticPi}\left(\frac{bc}{bc+ad},\arcsin\left(\frac{\sqrt{bc+ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right),\frac{c(be-af)}{(bc+ad)e}\right)}{b\sqrt{c}\sqrt{bc+ad}\sqrt{\frac{a(c-dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$

output
```
-a*(-d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)*EllipticF((a*d+b*c)^(1
/2)*x/c^(1/2)/(b*x^2+a)^(1/2),(c*(-a*f+b*e)/(a*d+b*c)/e)^(1/2))/b/c^(1/2)/
(a*d+b*c)^(1/2)/(a*(-d*x^2+c)/c/(b*x^2+a))^(1/2)/(f*x^2+e)^(1/2)+a*(-d*x^2
+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)*EllipticPi((a*d+b*c)^(1/2)*x/c^(
1/2)/(b*x^2+a)^(1/2),b*c/(a*d+b*c),(c*(-a*f+b*e)/(a*d+b*c)/e)^(1/2))/b/c^(
1/2)/(a*d+b*c)^(1/2)/(a*(-d*x^2+c)/c/(b*x^2+a))^(1/2)/(f*x^2+e)^(1/2)
```

Mathematica [F]

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c-dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c-dx^2}\sqrt{e+fx^2}}\,dx$$

input
```
Integrate[x^2/(Sqrt[a + b*x^2]*Sqrt[c - d*x^2]*Sqrt[e + f*x^2]),x]
```

output
```
Integrate[x^2/(Sqrt[a + b*x^2]*Sqrt[c - d*x^2]*Sqrt[e + f*x^2]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c-dx^2}\sqrt{e+fx^2}}\,dx$$

$$\Big\downarrow \; 450$$

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c-dx^2}\sqrt{e+fx^2}}dx$$

input
```
Int[x^2/(Sqrt[a + b*x^2]*Sqrt[c - d*x^2]*Sqrt[e + f*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [F]

$$\int \frac{x^2}{\sqrt{b\,x^2+a}\,\sqrt{-x^2d+c}\,\sqrt{f\,x^2+e}}dx$$

input
```
int(x^2/(b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

output
```
int(x^2/(b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c-dx^2}\sqrt{e+fx^2}}\,dx = \text{Timed out}$$

input
```
integrate(x^2/(b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorith
m="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c-dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c-dx^2}\sqrt{e+fx^2}}\,dx$$

input
```
integrate(x**2/(b*x**2+a)**(1/2)/(-d*x**2+c)**(1/2)/(f*x**2+e)**(1/2),x)
```

output
```
Integral(x**2/(sqrt(a + b*x**2)*sqrt(c - d*x**2)*sqrt(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c-dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{x^2}{\sqrt{bx^2+a}\sqrt{-dx^2+c}\sqrt{fx^2+e}}\,dx$$

input
```
integrate(x^2/(b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorith
m="maxima")
```

output
```
integrate(x^2/(sqrt(b*x^2 + a)*sqrt(-d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c-dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{x^2}{\sqrt{bx^2+a}\sqrt{-dx^2+c}\sqrt{fx^2+e}}\,dx$$

input
```
integrate(x^2/(b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorith
m="giac")
```

output
```
integrate(x^2/(sqrt(b*x^2 + a)*sqrt(-d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c-dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{x^2}{\sqrt{b\,x^2+a}\,\sqrt{c-d\,x^2}\,\sqrt{f\,x^2+e}}\,dx$$

input
```
int(x^2/((a + b*x^2)^(1/2)*(c - d*x^2)^(1/2)*(e + f*x^2)^(1/2)),x)
```

output
```
int(x^2/((a + b*x^2)^(1/2)*(c - d*x^2)^(1/2)*(e + f*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c-dx^2}\sqrt{e+fx^2}}\,dx$$
$$= \int \frac{\sqrt{f\,x^2+e}\,\sqrt{-d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^2}{-bdf\,x^6-adf\,x^4+bcf\,x^4-bde\,x^4+acf\,x^2-ade\,x^2+bce\,x^2+ace}dx$$

input
```
int(x^2/(b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

output
```
int((sqrt(e + f*x**2)*sqrt(c - d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c*e + a*c
*f*x**2 - a*d*e*x**2 - a*d*f*x**4 + b*c*e*x**2 + b*c*f*x**4 - b*d*e*x**4 -
 b*d*f*x**6),x)
```

3.384 $\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c-dx^2}\sqrt{e+fx^2}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 321

$$\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c-dx^2}\sqrt{e+fx^2}}\,dx = \frac{a\sqrt{c-dx^2}\sqrt{\frac{a(e+fx^2)}{e(a-bx^2)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-bc+ad}x}{\sqrt{c}\sqrt{a-bx^2}}\right),\frac{c(be+af)}{(bc-ad)e}\right)}{b\sqrt{c}\sqrt{-bc+ad}\sqrt{\frac{a(c-dx^2)}{c(a-bx^2)}}\sqrt{e+fx^2}} - \frac{a\sqrt{c-dx^2}\sqrt{\frac{a(e+fx^2)}{e(a-bx^2)}}\,\text{EllipticPi}\left(\frac{bc}{bc-ad},\arcsin\left(\frac{\sqrt{-bc+ad}x}{\sqrt{c}\sqrt{a-bx^2}}\right),\frac{c(be+af)}{(bc-ad)e}\right)}{b\sqrt{c}\sqrt{-bc+ad}\sqrt{\frac{a(c-dx^2)}{c(a-bx^2)}}\sqrt{e+fx^2}}$$

output
```
a*(-d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(-b*x^2+a))^(1/2)*EllipticF((a*d-b*c)^(1
/2)*x/c^(1/2)/(-b*x^2+a)^(1/2),(c*(a*f+b*e)/(-a*d+b*c)/e)^(1/2))/b/c^(1/2)
/(a*d-b*c)^(1/2)/(a*(-d*x^2+c)/c/(-b*x^2+a))^(1/2)/(f*x^2+e)^(1/2)-a*(-d*x
^2+c)^(1/2)*(a*(f*x^2+e)/e/(-b*x^2+a))^(1/2)*EllipticPi((a*d-b*c)^(1/2)*x/
c^(1/2)/(-b*x^2+a)^(1/2),b*c/(-a*d+b*c),(c*(a*f+b*e)/(-a*d+b*c)/e)^(1/2))/
b/c^(1/2)/(a*d-b*c)^(1/2)/(a*(-d*x^2+c)/c/(-b*x^2+a))^(1/2)/(f*x^2+e)^(1/2
)
```

Mathematica [F]

$$\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c-dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c-dx^2}\sqrt{e+fx^2}}\,dx$$

input
```
Integrate[x^2/(Sqrt[a - b*x^2]*Sqrt[c - d*x^2]*Sqrt[e + f*x^2]),x]
```

output
```
Integrate[x^2/(Sqrt[a - b*x^2]*Sqrt[c - d*x^2]*Sqrt[e + f*x^2]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c-dx^2}\sqrt{e+fx^2}}\,dx$$

$$\Big\downarrow \; 450$$

$$\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c-dx^2}\sqrt{e+fx^2}}dx$$

input
```
Int[x^2/(Sqrt[a - b*x^2]*Sqrt[c - d*x^2]*Sqrt[e + f*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [F]

$$\int \frac{x^2}{\sqrt{-b\,x^2+a}\,\sqrt{-x^2d+c}\,\sqrt{f\,x^2+e}}dx$$

input
```
int(x^2/(-b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

output
```
int(x^2/(-b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c-dx^2}\sqrt{e+fx^2}}\,dx = \text{Timed out}$$

input
```
integrate(x^2/(-b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorit
hm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c-dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c-dx^2}\sqrt{e+fx^2}}\,dx$$

input
```
integrate(x**2/(-b*x**2+a)**(1/2)/(-d*x**2+c)**(1/2)/(f*x**2+e)**(1/2),x)
```

output
```
Integral(x**2/(sqrt(a - b*x**2)*sqrt(c - d*x**2)*sqrt(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c-dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{x^2}{\sqrt{-bx^2+a}\sqrt{-dx^2+c}\sqrt{fx^2+e}}\,dx$$

input
```
integrate(x^2/(-b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorit
hm="maxima")
```

output
```
integrate(x^2/(sqrt(-b*x^2 + a)*sqrt(-d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c-dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{x^2}{\sqrt{-bx^2+a}\sqrt{-dx^2+c}\sqrt{fx^2+e}}\,dx$$

input
```
integrate(x^2/(-b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorit
hm="giac")
```

output
```
integrate(x^2/(sqrt(-b*x^2 + a)*sqrt(-d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c-dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{x^2}{\sqrt{a-b\,x^2}\,\sqrt{c-d\,x^2}\,\sqrt{f\,x^2+e}}\,dx$$

input
```
int(x^2/((a - b*x^2)^(1/2)*(c - d*x^2)^(1/2)*(e + f*x^2)^(1/2)),x)
```

output
```
int(x^2/((a - b*x^2)^(1/2)*(c - d*x^2)^(1/2)*(e + f*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c-dx^2}\sqrt{e+fx^2}}\,dx$$
$$= \int \frac{\sqrt{f\,x^2+e}\,\sqrt{-d\,x^2+c}\,\sqrt{-b\,x^2+a}\,x^2}{bdf\,x^6-adf\,x^4-bcf\,x^4+bde\,x^4+acf\,x^2-ade\,x^2-bce\,x^2+ace}dx$$

input
```
int(x^2/(-b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

output
```
int((sqrt(e + f*x**2)*sqrt(c - d*x**2)*sqrt(a - b*x**2)*x**2)/(a*c*e + a*c
*f*x**2 - a*d*e*x**2 - a*d*f*x**4 - b*c*e*x**2 - b*c*f*x**4 + b*d*e*x**4 +
 b*d*f*x**6),x)
```

3.385 $\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e-fx^2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 316

$$\begin{aligned}\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e-fx^2}}\,dx &= -\frac{a\sqrt{e}\sqrt{c+dx^2}\sqrt{\frac{a(e-fx^2)}{e(a+bx^2)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{be+af}x}{\sqrt{e}\sqrt{a+bx^2}}\right),\frac{(bc-ad)e}{c(be+af)}\right)}{bc\sqrt{be+af}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e-fx^2}} \\ &\quad+\frac{a\sqrt{e}\sqrt{c+dx^2}\sqrt{\frac{a(e-fx^2)}{e(a+bx^2)}}\,\text{EllipticPi}\left(\frac{be}{be+af},\arcsin\left(\frac{\sqrt{be+af}x}{\sqrt{e}\sqrt{a+bx^2}}\right),\frac{(bc-ad)e}{c(be+af)}\right)}{bc\sqrt{be+af}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e-fx^2}}\end{aligned}$$

output
```
-a*e^(1/2)*(d*x^2+c)^(1/2)*(a*(-f*x^2+e)/e/(b*x^2+a))^(1/2)*EllipticF((a*f
+b*e)^(1/2)*x/e^(1/2)/(b*x^2+a)^(1/2),((-a*d+b*c)*e/c/(a*f+b*e))^(1/2))/b/
c/(a*f+b*e)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(-f*x^2+e)^(1/2)+a*e^(1/
2)*(d*x^2+c)^(1/2)*(a*(-f*x^2+e)/e/(b*x^2+a))^(1/2)*EllipticPi((a*f+b*e)^(
1/2)*x/e^(1/2)/(b*x^2+a)^(1/2),b*e/(a*f+b*e),((-a*d+b*c)*e/c/(a*f+b*e))^(1
/2))/b/c/(a*f+b*e)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(-f*x^2+e)^(1/2)
```

Mathematica [F]

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e-fx^2}}\,dx = \int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e-fx^2}}\,dx$$

input
```
Integrate[x^2/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*Sqrt[e - f*x^2]),x]
```

output
```
Integrate[x^2/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*Sqrt[e - f*x^2]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e-fx^2}}\,dx$$
$$\Big\downarrow \text{ 450}$$
$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e-fx^2}}dx$$

input
```
Int[x^2/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*Sqrt[e - f*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [F]

$$\int \frac{x^2}{\sqrt{b\,x^2+a}\,\sqrt{x^2 d+c}\,\sqrt{-f\,x^2+e}}dx$$

input
```
int(x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(-f*x^2+e)^(1/2),x)
```

output
```
int(x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(-f*x^2+e)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e-fx^2}}\,dx = \text{Timed out}$$

input
```
integrate(x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(-f*x^2+e)^(1/2),x, algorith
m="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e-fx^2}}\,dx = \int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e-fx^2}}\,dx$$

input
```
integrate(x**2/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2)/(-f*x**2+e)**(1/2),x)
```

output
```
Integral(x**2/(sqrt(a + b*x**2)*sqrt(c + d*x**2)*sqrt(e - f*x**2)), x)
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e-fx^2}}\,dx=\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{-fx^2+e}}\,dx$$

input
```
integrate(x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(-f*x^2+e)^(1/2),x, algorith
m="maxima")
```

output
```
integrate(x^2/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e-fx^2}}\,dx=\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{-fx^2+e}}\,dx$$

input
```
integrate(x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(-f*x^2+e)^(1/2),x, algorith
m="giac")
```

output
```
integrate(x^2/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e-fx^2}}\,dx=\int \frac{x^2}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,\sqrt{e-f\,x^2}}\,dx$$

input
```
int(x^2/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e - f*x^2)^(1/2)),x)
```

output
```
int(x^2/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e - f*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e-fx^2}}\,dx$$
$$= \int \frac{\sqrt{-f\,x^2+e}\,\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^2}{-bdf\,x^6-adf\,x^4-bcf\,x^4+bde\,x^4-acf\,x^2+ade\,x^2+bce\,x^2+ace}dx$$

input
```
int(x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(-f*x^2+e)^(1/2),x)
```

output
```
int((sqrt(e - f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c*e - a*c
*f*x**2 + a*d*e*x**2 - a*d*f*x**4 + b*c*e*x**2 - b*c*f*x**4 + b*d*e*x**4 -
 b*d*f*x**6),x)
```

3.386 $\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}\sqrt{e-fx^2}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 324

$$\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}\sqrt{e-fx^2}}\,dx = \frac{a\sqrt{\frac{a(c+dx^2)}{c(a-bx^2)}}\sqrt{e-fx^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{e}\sqrt{a-bx^2}}\right),\frac{(bc+ad)e}{c(be-af)}\right)}{b\sqrt{e}\sqrt{-be+af}\sqrt{c+dx^2}\sqrt{\frac{a(e-fx^2)}{e(a-bx^2)}}} - \frac{a\sqrt{e}\sqrt{c+dx^2}\sqrt{\frac{a(e-fx^2)}{e(a-bx^2)}}\,\text{EllipticPi}\left(\frac{be}{be-af},\arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{e}\sqrt{a-bx^2}}\right),\frac{(bc+ad)e}{c(be-af)}\right)}{bc\sqrt{-be+af}\sqrt{\frac{a(c+dx^2)}{c(a-bx^2)}}\sqrt{e-fx^2}}$$

output
```
a*(a*(d*x^2+c)/c/(-b*x^2+a))^(1/2)*(-f*x^2+e)^(1/2)*EllipticF((a*f-b*e)^(1
/2)*x/e^(1/2)/(-b*x^2+a)^(1/2),((a*d+b*c)*e/c/(-a*f+b*e))^(1/2))/b/e^(1/2)
/(a*f-b*e)^(1/2)/(d*x^2+c)^(1/2)/(a*(-f*x^2+e)/e/(-b*x^2+a))^(1/2)-a*e^(1/
2)*(d*x^2+c)^(1/2)*(a*(-f*x^2+e)/e/(-b*x^2+a))^(1/2)*EllipticPi((a*f-b*e)^
(1/2)*x/e^(1/2)/(-b*x^2+a)^(1/2),b*e/(-a*f+b*e),((a*d+b*c)*e/c/(-a*f+b*e))
^(1/2))/b/c/(a*f-b*e)^(1/2)/(a*(d*x^2+c)/c/(-b*x^2+a))^(1/2)/(-f*x^2+e)^(1
/2)
```

Mathematica [F]

$$\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}\sqrt{e-fx^2}}\,dx=\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}\sqrt{e-fx^2}}\,dx$$

input
```
Integrate[x^2/(Sqrt[a - b*x^2]*Sqrt[c + d*x^2]*Sqrt[e - f*x^2]),x]
```

output
```
Integrate[x^2/(Sqrt[a - b*x^2]*Sqrt[c + d*x^2]*Sqrt[e - f*x^2]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}\sqrt{e-fx^2}}\,dx$$
$$\Big\downarrow \text{450}$$
$$\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}\sqrt{e-fx^2}}dx$$

input
```
Int[x^2/(Sqrt[a - b*x^2]*Sqrt[c + d*x^2]*Sqrt[e - f*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [F]

$$\int \frac{x^2}{\sqrt{-b\,x^2+a}\,\sqrt{x^2d+c}\,\sqrt{-f\,x^2+e}}dx$$

input
```
int(x^2/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(-f*x^2+e)^(1/2),x)
```

output
```
int(x^2/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(-f*x^2+e)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}\sqrt{e-fx^2}}\,dx = \text{Timed out}$$

input
```
integrate(x^2/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(-f*x^2+e)^(1/2),x, algorit
hm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}\sqrt{e-fx^2}}\,dx = \int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}\sqrt{e-fx^2}}\,dx$$

input
```
integrate(x**2/(-b*x**2+a)**(1/2)/(d*x**2+c)**(1/2)/(-f*x**2+e)**(1/2),x)
```

output
```
Integral(x**2/(sqrt(a - b*x**2)*sqrt(c + d*x**2)*sqrt(e - f*x**2)), x)
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}\sqrt{e-fx^2}}\,dx=\int \frac{x^2}{\sqrt{-bx^2+a}\sqrt{dx^2+c}\sqrt{-fx^2+e}}\,dx$$

input
```
integrate(x^2/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(-f*x^2+e)^(1/2),x, algorit
hm="maxima")
```

output
```
integrate(x^2/(sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}\sqrt{e-fx^2}}\,dx=\int \frac{x^2}{\sqrt{-bx^2+a}\sqrt{dx^2+c}\sqrt{-fx^2+e}}\,dx$$

input
```
integrate(x^2/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(-f*x^2+e)^(1/2),x, algorit
hm="giac")
```

output
```
integrate(x^2/(sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}\sqrt{e-fx^2}}\,dx=\int \frac{x^2}{\sqrt{a-b\,x^2}\,\sqrt{d\,x^2+c}\,\sqrt{e-f\,x^2}}\,dx$$

input
```
int(x^2/((a - b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e - f*x^2)^(1/2)),x)
```

output
```
int(x^2/((a - b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e - f*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}\sqrt{e-fx^2}}\,dx$$
$$= \int \frac{\sqrt{-f\,x^2+e}\,\sqrt{d\,x^2+c}\,\sqrt{-b\,x^2+a}\,x^2}{bdf\,x^6-adf\,x^4+bcf\,x^4-bde\,x^4-acf\,x^2+ade\,x^2-bce\,x^2+ace}dx$$

input
```
int(x^2/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(-f*x^2+e)^(1/2),x)
```

output
```
int((sqrt(e - f*x**2)*sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**2)/(a*c*e - a*c
*f*x**2 + a*d*e*x**2 - a*d*f*x**4 - b*c*e*x**2 + b*c*f*x**4 - b*d*e*x**4 +
 b*d*f*x**6),x)
```

3.387 $\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c-dx^2}\sqrt{e-fx^2}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 312

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c-dx^2}\sqrt{e-fx^2}}\,dx = -\frac{a\sqrt{c-dx^2}\sqrt{\frac{a(e-fx^2)}{e(a+bx^2)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{bc+ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right),\frac{c(be+af)}{(bc+ad)e}\right)}{b\sqrt{c}\sqrt{bc+ad}\sqrt{\frac{a(c-dx^2)}{c(a+bx^2)}}\sqrt{e-fx^2}} + \frac{a\sqrt{c-dx^2}\sqrt{\frac{a(e-fx^2)}{e(a+bx^2)}}\,\text{EllipticPi}\left(\frac{bc}{bc+ad},\arcsin\left(\frac{\sqrt{bc+ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right),\frac{c(be+af)}{(bc+ad)e}\right)}{b\sqrt{c}\sqrt{bc+ad}\sqrt{\frac{a(c-dx^2)}{c(a+bx^2)}}\sqrt{e-fx^2}}$$

output
```
-a*(-d*x^2+c)^(1/2)*(a*(-f*x^2+e)/e/(b*x^2+a))^(1/2)*EllipticF((a*d+b*c)^(
1/2)*x/c^(1/2)/(b*x^2+a)^(1/2),(c*(a*f+b*e)/(a*d+b*c)/e)^(1/2))/b/c^(1/2)/
(a*d+b*c)^(1/2)/(a*(-d*x^2+c)/c/(b*x^2+a))^(1/2)/(-f*x^2+e)^(1/2)+a*(-d*x^
2+c)^(1/2)*(a*(-f*x^2+e)/e/(b*x^2+a))^(1/2)*EllipticPi((a*d+b*c)^(1/2)*x/c
^(1/2)/(b*x^2+a)^(1/2),b*c/(a*d+b*c),(c*(a*f+b*e)/(a*d+b*c)/e)^(1/2))/b/c^
(1/2)/(a*d+b*c)^(1/2)/(a*(-d*x^2+c)/c/(b*x^2+a))^(1/2)/(-f*x^2+e)^(1/2)
```

Mathematica [F]

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c-dx^2}\sqrt{e-fx^2}}\,dx = \int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c-dx^2}\sqrt{e-fx^2}}\,dx$$

input
```
Integrate[x^2/(Sqrt[a + b*x^2]*Sqrt[c - d*x^2]*Sqrt[e - f*x^2]),x]
```

output
```
Integrate[x^2/(Sqrt[a + b*x^2]*Sqrt[c - d*x^2]*Sqrt[e - f*x^2]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c-dx^2}\sqrt{e-fx^2}}\,dx$$
$$\Big\downarrow \; 450$$
$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c-dx^2}\sqrt{e-fx^2}}dx$$

input
```
Int[x^2/(Sqrt[a + b*x^2]*Sqrt[c - d*x^2]*Sqrt[e - f*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [F]

$$\int \frac{x^2}{\sqrt{b\,x^2+a}\,\sqrt{-x^2d+c}\,\sqrt{-f\,x^2+e}}dx$$

input
```
int(x^2/(b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2)/(-f*x^2+e)^(1/2),x)
```

output
```
int(x^2/(b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2)/(-f*x^2+e)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c-dx^2}\sqrt{e-fx^2}}\,dx = \text{Timed out}$$

input
```
integrate(x^2/(b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2)/(-f*x^2+e)^(1/2),x, algorit
hm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c-dx^2}\sqrt{e-fx^2}}\,dx = \int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c-dx^2}\sqrt{e-fx^2}}\,dx$$

input
```
integrate(x**2/(b*x**2+a)**(1/2)/(-d*x**2+c)**(1/2)/(-f*x**2+e)**(1/2),x)
```

output
```
Integral(x**2/(sqrt(a + b*x**2)*sqrt(c - d*x**2)*sqrt(e - f*x**2)), x)
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c-dx^2}\sqrt{e-fx^2}}\,dx = \int \frac{x^2}{\sqrt{bx^2+a}\sqrt{-dx^2+c}\sqrt{-fx^2+e}}\,dx$$

input
```
integrate(x^2/(b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2)/(-f*x^2+e)^(1/2),x, algorit
hm="maxima")
```

output
```
integrate(x^2/(sqrt(b*x^2 + a)*sqrt(-d*x^2 + c)*sqrt(-f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c-dx^2}\sqrt{e-fx^2}}\,dx = \int \frac{x^2}{\sqrt{bx^2+a}\sqrt{-dx^2+c}\sqrt{-fx^2+e}}\,dx$$

input
```
integrate(x^2/(b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2)/(-f*x^2+e)^(1/2),x, algorit
hm="giac")
```

output
```
integrate(x^2/(sqrt(b*x^2 + a)*sqrt(-d*x^2 + c)*sqrt(-f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c-dx^2}\sqrt{e-fx^2}}\,dx = \int \frac{x^2}{\sqrt{b\,x^2+a}\,\sqrt{c-d\,x^2}\,\sqrt{e-f\,x^2}}\,dx$$

input
```
int(x^2/((a + b*x^2)^(1/2)*(c - d*x^2)^(1/2)*(e - f*x^2)^(1/2)),x)
```

output
```
int(x^2/((a + b*x^2)^(1/2)*(c - d*x^2)^(1/2)*(e - f*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c-dx^2}\sqrt{e-fx^2}}\,dx$$
$$= \int \frac{\sqrt{-f\,x^2+e}\,\sqrt{-d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^2}{bdf\,x^6+adf\,x^4-bcf\,x^4-bde\,x^4-acf\,x^2-ade\,x^2+bce\,x^2+ace}dx$$

input
```
int(x^2/(b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2)/(-f*x^2+e)^(1/2),x)
```

output
```
int((sqrt(e - f*x**2)*sqrt(c - d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c*e - a*c
*f*x**2 - a*d*e*x**2 + a*d*f*x**4 + b*c*e*x**2 - b*c*f*x**4 - b*d*e*x**4 +
 b*d*f*x**6),x)
```

3.388 $\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c-dx^2}\sqrt{e-fx^2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 327

$$\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c-dx^2}\sqrt{e-fx^2}}\,dx = \frac{a\sqrt{c-dx^2}\sqrt{\frac{a(e-fx^2)}{e(a-bx^2)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-bc+ad}x}{\sqrt{c}\sqrt{a-bx^2}}\right),\frac{c(be-af)}{(bc-ad)e}\right)}{b\sqrt{c}\sqrt{-bc+ad}\sqrt{\frac{a(c-dx^2)}{c(a-bx^2)}}\sqrt{e-fx^2}} - \frac{a\sqrt{c-dx^2}\sqrt{\frac{a(e-fx^2)}{e(a-bx^2)}}\,\text{EllipticPi}\left(\frac{bc}{bc-ad},\arcsin\left(\frac{\sqrt{-bc+ad}x}{\sqrt{c}\sqrt{a-bx^2}}\right),\frac{c(be-af)}{(bc-ad)e}\right)}{b\sqrt{c}\sqrt{-bc+ad}\sqrt{\frac{a(c-dx^2)}{c(a-bx^2)}}\sqrt{e-fx^2}}$$

output
```
a*(-d*x^2+c)^(1/2)*(a*(-f*x^2+e)/e/(-b*x^2+a))^(1/2)*EllipticF((a*d-b*c)^(
1/2)*x/c^(1/2)/(-b*x^2+a)^(1/2),(c*(-a*f+b*e)/(-a*d+b*c)/e)^(1/2))/b/c^(1/
2)/(a*d-b*c)^(1/2)/(a*(-d*x^2+c)/c/(-b*x^2+a))^(1/2)/(-f*x^2+e)^(1/2)-a*(-
d*x^2+c)^(1/2)*(a*(-f*x^2+e)/e/(-b*x^2+a))^(1/2)*EllipticPi((a*d-b*c)^(1/2
)*x/c^(1/2)/(-b*x^2+a)^(1/2),b*c/(-a*d+b*c),(c*(-a*f+b*e)/(-a*d+b*c)/e)^(1
/2))/b/c^(1/2)/(a*d-b*c)^(1/2)/(a*(-d*x^2+c)/c/(-b*x^2+a))^(1/2)/(-f*x^2+e
)^(1/2)
```

Mathematica [F]

$$\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c-dx^2}\sqrt{e-fx^2}}\,dx = \int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c-dx^2}\sqrt{e-fx^2}}\,dx$$

input
```
Integrate[x^2/(Sqrt[a - b*x^2]*Sqrt[c - d*x^2]*Sqrt[e - f*x^2]),x]
```

output
```
Integrate[x^2/(Sqrt[a - b*x^2]*Sqrt[c - d*x^2]*Sqrt[e - f*x^2]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c-dx^2}\sqrt{e-fx^2}}\,dx$$

$$\Big\downarrow \; 450$$

$$\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c-dx^2}\sqrt{e-fx^2}}dx$$

input
```
Int[x^2/(Sqrt[a - b*x^2]*Sqrt[c - d*x^2]*Sqrt[e - f*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [F]

$$\int \frac{x^2}{\sqrt{-b\,x^2+a}\,\sqrt{-x^2 d+c}\,\sqrt{-f\,x^2+e}}dx$$

input
```
int(x^2/(-b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2)/(-f*x^2+e)^(1/2),x)
```

output
```
int(x^2/(-b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2)/(-f*x^2+e)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c-dx^2}\sqrt{e-fx^2}}\,dx=\text{Timed out}$$

input
```
integrate(x^2/(-b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2)/(-f*x^2+e)^(1/2),x, algori
thm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c-dx^2}\sqrt{e-fx^2}}\,dx=\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c-dx^2}\sqrt{e-fx^2}}\,dx$$

input
```
integrate(x**2/(-b*x**2+a)**(1/2)/(-d*x**2+c)**(1/2)/(-f*x**2+e)**(1/2),x)
```

output
```
Integral(x**2/(sqrt(a - b*x**2)*sqrt(c - d*x**2)*sqrt(e - f*x**2)), x)
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c-dx^2}\sqrt{e-fx^2}}\,dx = \int \frac{x^2}{\sqrt{-bx^2+a}\sqrt{-dx^2+c}\sqrt{-fx^2+e}}\,dx$$

input
```
integrate(x^2/(-b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2)/(-f*x^2+e)^(1/2),x, algori
thm="maxima")
```

output
```
integrate(x^2/(sqrt(-b*x^2 + a)*sqrt(-d*x^2 + c)*sqrt(-f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c-dx^2}\sqrt{e-fx^2}}\,dx = \int \frac{x^2}{\sqrt{-bx^2+a}\sqrt{-dx^2+c}\sqrt{-fx^2+e}}\,dx$$

input
```
integrate(x^2/(-b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2)/(-f*x^2+e)^(1/2),x, algori
thm="giac")
```

output
```
integrate(x^2/(sqrt(-b*x^2 + a)*sqrt(-d*x^2 + c)*sqrt(-f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c-dx^2}\sqrt{e-fx^2}}\,dx = \int \frac{x^2}{\sqrt{a-b\,x^2}\,\sqrt{c-d\,x^2}\,\sqrt{e-f\,x^2}}\,dx$$

input
```
int(x^2/((a - b*x^2)^(1/2)*(c - d*x^2)^(1/2)*(e - f*x^2)^(1/2)),x)
```

output
```
int(x^2/((a - b*x^2)^(1/2)*(c - d*x^2)^(1/2)*(e - f*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c-dx^2}\sqrt{e-fx^2}}\,dx$$
$$= \int \frac{\sqrt{-f\,x^2+e}\,\sqrt{-d\,x^2+c}\,\sqrt{-b\,x^2+a}\,x^2}{-bdf\,x^6+adf\,x^4+bcf\,x^4+bde\,x^4-acf\,x^2-ade\,x^2-bce\,x^2+ace}dx$$

input
```
int(x^2/(-b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2)/(-f*x^2+e)^(1/2),x)
```

output
```
int((sqrt(e - f*x**2)*sqrt(c - d*x**2)*sqrt(a - b*x**2)*x**2)/(a*c*e - a*c
*f*x**2 - a*d*e*x**2 + a*d*f*x**4 - b*c*e*x**2 + b*c*f*x**4 + b*d*e*x**4 -
 b*d*f*x**6),x)
```

3.389 $\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 314

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}}\,dx = \frac{c\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}E\left(\arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right)|\frac{a(de-cf)}{c(be-af)}\right)}{\sqrt{a}\sqrt{-be+af}(de-cf)\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}} - \frac{c\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right),\frac{a(de-cf)}{c(be-af)}\right)}{\sqrt{a}\sqrt{-be+af}(de-cf)\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$

output

```
c*(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticE((a*f-b*e)^(1/2
)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(1/2)/(a*
f-b*e)^(1/2)/(-c*f+d*e)/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)-c*
(b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticF((a*f-b*e)^(1/2)*
x/a^(1/2)/(f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(1/2)/(a*f-
b*e)^(1/2)/(-c*f+d*e)/(d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)
```

Mathematica [F]

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx$$

input
```
Integrate[x^2/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^(3/2)),x]
```

output
```
Integrate[x^2/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^(3/2)), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx$$

$$\Big\downarrow \; 450$$

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}dx$$

input
```
Int[x^2/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^(3/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [F]

$$\int \frac{x^2}{\sqrt{b\,x^2+a}\,\sqrt{x^2 d+c}\,\left(f\,x^2+e\right)^{\frac{3}{2}}}dx$$

input
```
int(x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x)
```

output
```
int(x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x)
```

Fricas [F]

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx=\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm
="fricas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)*x^2/(b*d*f^2*x^8
+ (2*b*d*e*f + (b*c + a*d)*f^2)*x^6 + (b*d*e^2 + a*c*f^2 + 2*(b*c + a*d)*e
*f)*x^4 + a*c*e^2 + (2*a*c*e*f + (b*c + a*d)*e^2)*x^2), x)
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**(3/2),x)
```

output
```
Integral(x**2/(sqrt(a + b*x**2)*sqrt(c + d*x**2)*(e + f*x**2)**(3/2)), x)
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm
="maxima")
```

output
```
integrate(x^2/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm
="giac")
```

output
```
integrate(x^2/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{x^2}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,{\left(f\,x^2+e\right)}^{3/2}}\,dx$$

input
```
int(x^2/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(3/2)),x)
```

output
```
int(x^2/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{\sqrt{f\,x^2+e}\,\sqrt{d\,x^2+}}{bd\,f^2x^8+ad\,f^2x^6+bc\,f^2x^6+2bdef\,x^6+ac\,f^2x^4+2adef\,x^4}$$

input
```
int(x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x)
```

output
```
int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c*e**2 +
2*a*c*e*f*x**2 + a*c*f**2*x**4 + a*d*e**2*x**2 + 2*a*d*e*f*x**4 + a*d*f**2
*x**6 + b*c*e**2*x**2 + 2*b*c*e*f*x**4 + b*c*f**2*x**6 + b*d*e**2*x**4 + 2
*b*d*e*f*x**6 + b*d*f**2*x**8),x)
```

3.390 $\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}(e+fx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 314

$$\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = \frac{c\sqrt{a-bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}E\left(\arcsin\left(\frac{\sqrt{be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right)|-\frac{a(de-cf)}{c(be+af)}\right)}{\sqrt{a}\sqrt{be+af}(de-cf)\sqrt{c+dx^2}\sqrt{\frac{e(a-bx^2)}{a(e+fx^2)}}} - \frac{c\sqrt{a-bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right),-\frac{a(de-cf)}{c(be+af)}\right)}{\sqrt{a}\sqrt{be+af}(de-cf)\sqrt{c+dx^2}\sqrt{\frac{e(a-bx^2)}{a(e+fx^2)}}}$$

output

```
c*(-b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticE((a*f+b*e)^(1/
2)*x/a^(1/2)/(f*x^2+e)^(1/2),(-a*(-c*f+d*e)/c/(a*f+b*e))^(1/2))/a^(1/2)/(a
*f+b*e)^(1/2)/(-c*f+d*e)/(d*x^2+c)^(1/2)/(e*(-b*x^2+a)/a/(f*x^2+e))^(1/2)-
c*(-b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticF((a*f+b*e)^(1/
2)*x/a^(1/2)/(f*x^2+e)^(1/2),(-a*(-c*f+d*e)/c/(a*f+b*e))^(1/2))/a^(1/2)/(a
*f+b*e)^(1/2)/(-c*f+d*e)/(d*x^2+c)^(1/2)/(e*(-b*x^2+a)/a/(f*x^2+e))^(1/2)
```

Mathematica [F]

$$\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx$$

input
```
Integrate[x^2/(Sqrt[a - b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^(3/2)),x]
```

output
```
Integrate[x^2/(Sqrt[a - b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^(3/2)), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx$$
$$\Big\downarrow \; 450$$
$$\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}dx$$

input
```
Int[x^2/(Sqrt[a - b*x^2]*Sqrt[c + d*x^2]*(e + f*x^2)^(3/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [F]

$$\int \frac{x^{2}}{\sqrt{-b\,x^{2}+a}\,\sqrt{x^{2}d+c}\,\left(f\,x^{2}+e\right)^{\frac{3}{2}}}dx$$

input
```
int(x^2/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x)
```

output
```
int(x^2/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x)
```

Fricas [F]

$$\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx=\int \frac{x^{2}}{\sqrt{-bx^{2}+a}\sqrt{dx^{2}+c}(fx^{2}+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorith
m="fricas")
```

output
```
integral(-sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)*x^2/(b*d*f^2*x^
8 + (2*b*d*e*f + (b*c - a*d)*f^2)*x^6 + (b*d*e^2 - a*c*f^2 + 2*(b*c - a*d)
*e*f)*x^4 - a*c*e^2 - (2*a*c*e*f - (b*c - a*d)*e^2)*x^2), x)
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx=\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2/(-b*x**2+a)**(1/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**(3/2),x)
```

output
```
Integral(x**2/(sqrt(a - b*x**2)*sqrt(c + d*x**2)*(e + f*x**2)**(3/2)), x)
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx=\int \frac{x^2}{\sqrt{-bx^2+a}\sqrt{dx^2+c}(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorith
m="maxima")
```

output
```
integrate(x^2/(sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx=\int \frac{x^2}{\sqrt{-bx^2+a}\sqrt{dx^2+c}(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorith
m="giac")
```

output
```
integrate(x^2/(sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{x^2}{\sqrt{a-b\,x^2}\,\sqrt{d\,x^2+c}\,(f\,x^2+e)^{3/2}}\,dx$$

input
```
int(x^2/((a - b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(3/2)),x)
```

output
```
int(x^2/((a - b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{\sqrt{f\,x^2+e}\,\sqrt{d\,x^2+}}{-bd\,f^2x^8+ad\,f^2x^6-bc\,f^2x^6-2bdef\,x^6+ac\,f^2x^4+2adef\,x}$$

input
```
int(x^2/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x)
```

output
```
int((sqrt(e + f*x**2)*sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**2)/(a*c*e**2 +
2*a*c*e*f*x**2 + a*c*f**2*x**4 + a*d*e**2*x**2 + 2*a*d*e*f*x**4 + a*d*f**2
*x**6 - b*c*e**2*x**2 - 2*b*c*e*f*x**4 - b*c*f**2*x**6 - b*d*e**2*x**4 - 2
*b*d*e*f*x**6 - b*d*f**2*x**8),x)
```

3.391 $\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c-dx^2}(e+fx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 312

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c-dx^2}\,(e+fx^2)^{3/2}}\,dx = \frac{\sqrt{c}\sqrt{a+bx^2}\sqrt{\frac{e(c-dx^2)}{c(e+fx^2)}}E\left(\arcsin\left(\frac{\sqrt{de+cf}x}{\sqrt{c}\sqrt{e+fx^2}}\right)|-\frac{c(be-af)}{a(de+cf)}\right)}{(be-af)\sqrt{de+cf}\sqrt{c-dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}} - \frac{\sqrt{c}\sqrt{a+bx^2}\sqrt{\frac{e(c-dx^2)}{c(e+fx^2)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{de+cf}x}{\sqrt{c}\sqrt{e+fx^2}}\right),-\frac{c(be-af)}{a(de+cf)}\right)}{(be-af)\sqrt{de+cf}\sqrt{c-dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}}$$

output

```
c^(1/2)*(b*x^2+a)^(1/2)*(e*(-d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticE((c*f+d*
e)^(1/2)*x/c^(1/2)/(f*x^2+e)^(1/2),(-c*(-a*f+b*e)/a/(c*f+d*e))^(1/2))/(-a*
f+b*e)/(c*f+d*e)^(1/2)/(-d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)-c^
(1/2)*(b*x^2+a)^(1/2)*(e*(-d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticF((c*f+d*e)
^(1/2)*x/c^(1/2)/(f*x^2+e)^(1/2),(-c*(-a*f+b*e)/a/(c*f+d*e))^(1/2))/(-a*f+
b*e)/(c*f+d*e)^(1/2)/(-d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(f*x^2+e))^(1/2)
```

Mathematica [F]

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c-dx^2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c-dx^2}\,(e+fx^2)^{3/2}}\,dx$$

input
```
Integrate[x^2/(Sqrt[a + b*x^2]*Sqrt[c - d*x^2]*(e + f*x^2)^(3/2)),x]
```

output
```
Integrate[x^2/(Sqrt[a + b*x^2]*Sqrt[c - d*x^2]*(e + f*x^2)^(3/2)), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c-dx^2}\,(e+fx^2)^{3/2}}\,dx$$

$$\Big\downarrow \; 450$$

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c-dx^2}\,(e+fx^2)^{3/2}}dx$$

input
```
Int[x^2/(Sqrt[a + b*x^2]*Sqrt[c - d*x^2]*(e + f*x^2)^(3/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [F]

$$\int \frac{x^2}{\sqrt{b\,x^2+a}\,\sqrt{-x^2 d+c}\,\left(f\,x^2+e\right)^{\frac{3}{2}}}dx$$

input
```
int(x^2/(b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x)
```

output
```
int(x^2/(b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x)
```

Fricas [F]

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c-dx^2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{x^2}{\sqrt{bx^2+a}\sqrt{-dx^2+c}(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorith
m="fricas")
```

output
```
integral(-sqrt(b*x^2 + a)*sqrt(-d*x^2 + c)*sqrt(f*x^2 + e)*x^2/(b*d*f^2*x^
8 + (2*b*d*e*f - (b*c - a*d)*f^2)*x^6 + (b*d*e^2 - a*c*f^2 - 2*(b*c - a*d)
*e*f)*x^4 - a*c*e^2 - (2*a*c*e*f + (b*c - a*d)*e^2)*x^2), x)
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c-dx^2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c-dx^2}\,(e+fx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2/(b*x**2+a)**(1/2)/(-d*x**2+c)**(1/2)/(f*x**2+e)**(3/2),x)
```

output
```
Integral(x**2/(sqrt(a + b*x**2)*sqrt(c - d*x**2)*(e + f*x**2)**(3/2)), x)
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c-dx^2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{x^2}{\sqrt{bx^2+a}\sqrt{-dx^2+c}(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorith
m="maxima")
```

output
```
integrate(x^2/(sqrt(b*x^2 + a)*sqrt(-d*x^2 + c)*(f*x^2 + e)^(3/2)), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c-dx^2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{x^2}{\sqrt{bx^2+a}\sqrt{-dx^2+c}(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorith
m="giac")
```

output
```
integrate(x^2/(sqrt(b*x^2 + a)*sqrt(-d*x^2 + c)*(f*x^2 + e)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c-dx^2}\left(e+fx^2\right)^{3/2}}\,dx = \int \frac{x^2}{\sqrt{b\,x^2+a}\,\sqrt{c-d\,x^2}\,{\left(f\,x^2+e\right)}^{3/2}}\,dx$$

input
```
int(x^2/((a + b*x^2)^(1/2)*(c - d*x^2)^(1/2)*(e + f*x^2)^(3/2)),x)
```

output
```
int(x^2/((a + b*x^2)^(1/2)*(c - d*x^2)^(1/2)*(e + f*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c-dx^2}\left(e+fx^2\right)^{3/2}}\,dx = \int \frac{\sqrt{f\,x^2+e}\,\sqrt{-d\,x^2}}{-bd\,f^2x^8-ad\,f^2x^6+bc\,f^2x^6-2bdef\,x^6+ac\,f^2x^4-2adef\,x}$$

input
```
int(x^2/(b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x)
```

output
```
int((sqrt(e + f*x**2)*sqrt(c - d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c*e**2 +
2*a*c*e*f*x**2 + a*c*f**2*x**4 - a*d*e**2*x**2 - 2*a*d*e*f*x**4 - a*d*f**2
*x**6 + b*c*e**2*x**2 + 2*b*c*e*f*x**4 + b*c*f**2*x**6 - b*d*e**2*x**4 - 2
*b*d*e*f*x**6 - b*d*f**2*x**8),x)
```

3.392 $\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c-dx^2}(e+fx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 312

$$\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c-dx^2}\,(e+fx^2)^{3/2}}\,dx = -\frac{c\sqrt{a-bx^2}\sqrt{\frac{e(c-dx^2)}{c(e+fx^2)}}E\left(\arcsin\left(\frac{\sqrt{be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right)|\frac{a(de+cf)}{c(be+af)}\right)}{\sqrt{a}\sqrt{be+af}(de+cf)\sqrt{c-dx^2}\sqrt{\frac{e(a-bx^2)}{a(e+fx^2)}}} + \frac{c\sqrt{a-bx^2}\sqrt{\frac{e(c-dx^2)}{c(e+fx^2)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right),\frac{a(de+cf)}{c(be+af)}\right)}{\sqrt{a}\sqrt{be+af}(de+cf)\sqrt{c-dx^2}\sqrt{\frac{e(a-bx^2)}{a(e+fx^2)}}}$$

output
```
-c*(-b*x^2+a)^(1/2)*(e*(-d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticE((a*f+b*e)^(
1/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(c*f+d*e)/c/(a*f+b*e))^(1/2))/a^(1/2)/(a
*f+b*e)^(1/2)/(c*f+d*e)/(-d*x^2+c)^(1/2)/(e*(-b*x^2+a)/a/(f*x^2+e))^(1/2)+
c*(-b*x^2+a)^(1/2)*(e*(-d*x^2+c)/c/(f*x^2+e))^(1/2)*EllipticF((a*f+b*e)^(1
/2)*x/a^(1/2)/(f*x^2+e)^(1/2),(a*(c*f+d*e)/c/(a*f+b*e))^(1/2))/a^(1/2)/(a*
f+b*e)^(1/2)/(c*f+d*e)/(-d*x^2+c)^(1/2)/(e*(-b*x^2+a)/a/(f*x^2+e))^(1/2)
```

Mathematica [F]

$$\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c-dx^2}\left(e+fx^2\right)^{3/2}}\,dx=\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c-dx^2}\left(e+fx^2\right)^{3/2}}\,dx$$

input
```
Integrate[x^2/(Sqrt[a - b*x^2]*Sqrt[c - d*x^2]*(e + f*x^2)^(3/2)),x]
```

output
```
Integrate[x^2/(Sqrt[a - b*x^2]*Sqrt[c - d*x^2]*(e + f*x^2)^(3/2)), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c-dx^2}\left(e+fx^2\right)^{3/2}}\,dx$$
$$\Big\downarrow \; 450$$
$$\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c-dx^2}\left(e+fx^2\right)^{3/2}}dx$$

input
```
Int[x^2/(Sqrt[a - b*x^2]*Sqrt[c - d*x^2]*(e + f*x^2)^(3/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [F]

$$\int \frac{x^2}{\sqrt{-b\,x^2+a}\,\sqrt{-x^2d+c}\,\left(f\,x^2+e\right)^{\frac{3}{2}}}dx$$

input
```
int(x^2/(-b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x)
```

output
```
int(x^2/(-b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x)
```

Fricas [F]

$$\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c-dx^2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{x^2}{\sqrt{-bx^2+a}\sqrt{-dx^2+c}(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(-b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorit
hm="fricas")
```

output
```
integral(sqrt(-b*x^2 + a)*sqrt(-d*x^2 + c)*sqrt(f*x^2 + e)*x^2/(b*d*f^2*x^
8 + (2*b*d*e*f - (b*c + a*d)*f^2)*x^6 + (b*d*e^2 + a*c*f^2 - 2*(b*c + a*d)
*e*f)*x^4 + a*c*e^2 + (2*a*c*e*f - (b*c + a*d)*e^2)*x^2), x)
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c-dx^2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c-dx^2}\,(e+fx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2/(-b*x**2+a)**(1/2)/(-d*x**2+c)**(1/2)/(f*x**2+e)**(3/2),x)
```

output
```
Integral(x**2/(sqrt(a - b*x**2)*sqrt(c - d*x**2)*(e + f*x**2)**(3/2)), x)
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c-dx^2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{x^2}{\sqrt{-bx^2+a}\sqrt{-dx^2+c}(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(-b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorit
hm="maxima")
```

output
```
integrate(x^2/(sqrt(-b*x^2 + a)*sqrt(-d*x^2 + c)*(f*x^2 + e)^(3/2)), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c-dx^2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{x^2}{\sqrt{-bx^2+a}\sqrt{-dx^2+c}(fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(-b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorit
hm="giac")
```

output
```
integrate(x^2/(sqrt(-b*x^2 + a)*sqrt(-d*x^2 + c)*(f*x^2 + e)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c-dx^2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{x^2}{\sqrt{a-b\,x^2}\,\sqrt{c-d\,x^2}\,{(f\,x^2+e)}^{3/2}}\,dx$$

input
```
int(x^2/((a - b*x^2)^(1/2)*(c - d*x^2)^(1/2)*(e + f*x^2)^(3/2)),x)
```

output
```
int(x^2/((a - b*x^2)^(1/2)*(c - d*x^2)^(1/2)*(e + f*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c-dx^2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{\sqrt{f\,x^2+e}\,\sqrt{-d\,x^2+}}{bd\,f^2x^8-ad\,f^2x^6-bc\,f^2x^6+2bdef\,x^6+ac\,f^2x^4-2adef\,x^4}$$

input
```
int(x^2/(-b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x)
```

output
```
int((sqrt(e + f*x**2)*sqrt(c - d*x**2)*sqrt(a - b*x**2)*x**2)/(a*c*e**2 +
2*a*c*e*f*x**2 + a*c*f**2*x**4 - a*d*e**2*x**2 - 2*a*d*e*f*x**4 - a*d*f**2
*x**6 - b*c*e**2*x**2 - 2*b*c*e*f*x**4 - b*c*f**2*x**6 + b*d*e**2*x**4 + 2
*b*d*e*f*x**6 + b*d*f**2*x**8),x)
```

3.393 $\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}(e-fx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 174

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e-fx^2)^{3/2}}\,dx = \frac{x\sqrt{c+dx^2}}{(de+cf)\sqrt{a+bx^2}\sqrt{e-fx^2}} - \frac{\sqrt{a}\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{be+af}x}{\sqrt{a}\sqrt{e-fx^2}}\right)|\frac{(bc-ad)e}{c(be+af)}\right)}{\sqrt{be+af}(de+cf)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
x*(d*x^2+c)^(1/2)/(c*f+d*e)/(b*x^2+a)^(1/2)/(-f*x^2+e)^(1/2)-a^(1/2)*(d*x^
2+c)^(1/2)*EllipticE((a*f+b*e)^(1/2)*x/a^(1/2)/(-f*x^2+e)^(1/2)/(1+(a*f+b*
e)*x^2/a/(-f*x^2+e))^(1/2),((-a*d+b*c)*e/c/(a*f+b*e))^(1/2))/(a*f+b*e)^(1/
2)/(c*f+d*e)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [F]

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e-fx^2)^{3/2}}\,dx = \int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e-fx^2)^{3/2}}\,dx$$

input
```
Integrate[x^2/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e - f*x^2)^(3/2)),x]
```

output
```
Integrate[x^2/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e - f*x^2)^(3/2)), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e-fx^2)^{3/2}}\,dx$$
$$\Big\downarrow \; 450$$
$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e-fx^2)^{3/2}}dx$$

input
```
Int[x^2/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(e - f*x^2)^(3/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [F]

$$\int \frac{x^2}{\sqrt{b\,x^2+a}\,\sqrt{x^2d+c}\,\left(-f\,x^2+e\right)^{\frac{3}{2}}}dx$$

input
```
int(x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(-f*x^2+e)^(3/2),x)
```

output
```
int(x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(-f*x^2+e)^(3/2),x)
```

Fricas [F]

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e-fx^2)^{3/2}}\,dx = \int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}(-fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(-f*x^2+e)^(3/2),x, algorith
m="fricas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-f*x^2 + e)*x^2/(b*d*f^2*x^8
 - (2*b*d*e*f - (b*c + a*d)*f^2)*x^6 + (b*d*e^2 + a*c*f^2 - 2*(b*c + a*d)*
e*f)*x^4 + a*c*e^2 - (2*a*c*e*f - (b*c + a*d)*e^2)*x^2), x)
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e-fx^2)^{3/2}}\,dx = \int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e-fx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2)/(-f*x**2+e)**(3/2),x)
```

output
```
Integral(x**2/(sqrt(a + b*x**2)*sqrt(c + d*x**2)*(e - f*x**2)**(3/2)), x)
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e-fx^2)^{3/2}}\,dx = \int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}(-fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(-f*x^2+e)^(3/2),x, algorith
m="maxima")
```

output
```
integrate(x^2/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(-f*x^2 + e)^(3/2)), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e-fx^2)^{3/2}}\,dx = \int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}(-fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(-f*x^2+e)^(3/2),x, algorith
m="giac")
```

output
```
integrate(x^2/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*(-f*x^2 + e)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e-fx^2)^{3/2}}\,dx = \int \frac{x^2}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,(e-f\,x^2)^{3/2}}\,dx$$

input
```
int(x^2/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e - f*x^2)^(3/2)),x)
```

output
```
int(x^2/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e - f*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(e-fx^2)^{3/2}}\,dx = \int \frac{\sqrt{-f\,x^2+e}\,\sqrt{d\,x^2}}{bd\,f^2x^8+ad\,f^2x^6+bc\,f^2x^6-2bdef\,x^6+ac\,f^2x^4-2adef\,x^4}$$

input
```
int(x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(-f*x^2+e)^(3/2),x)
```

output
```
int((sqrt(e - f*x**2)*sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c*e**2 -
2*a*c*e*f*x**2 + a*c*f**2*x**4 + a*d*e**2*x**2 - 2*a*d*e*f*x**4 + a*d*f**2
*x**6 + b*c*e**2*x**2 - 2*b*c*e*f*x**4 + b*c*f**2*x**6 + b*d*e**2*x**4 - 2
*b*d*e*f*x**6 + b*d*f**2*x**8),x)
```

3.394 $\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}\left(e-fx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 322

$$\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}\left(e-fx^2\right)^{3/2}}\,dx=\frac{c\sqrt{a-bx^2}\sqrt{\frac{e(c+dx^2)}{c(e-fx^2)}}E\left(\arcsin\left(\frac{\sqrt{be-af}x}{\sqrt{a}\sqrt{e-fx^2}}\right)|-\frac{a(de+cf)}{c(be-af)}\right)}{\sqrt{a}\sqrt{be-af}(de+cf)\sqrt{c+dx^2}\sqrt{\frac{e(a-bx^2)}{a(e-fx^2)}}}-\frac{c\sqrt{a-bx^2}\sqrt{\frac{e(c+dx^2)}{c(e-fx^2)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{be-af}x}{\sqrt{a}\sqrt{e-fx^2}}\right),-\frac{a(de+cf)}{c(be-af)}\right)}{\sqrt{a}\sqrt{be-af}(de+cf)\sqrt{c+dx^2}\sqrt{\frac{e(a-bx^2)}{a(e-fx^2)}}}$$

output
```
c*(-b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(-f*x^2+e))^(1/2)*EllipticE((-a*f+b*e)^(
1/2)*x/a^(1/2)/(-f*x^2+e)^(1/2),(-a*(c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(1/2)
/(-a*f+b*e)^(1/2)/(c*f+d*e)/(d*x^2+c)^(1/2)/(e*(-b*x^2+a)/a/(-f*x^2+e))^(1
/2)-c*(-b*x^2+a)^(1/2)*(e*(d*x^2+c)/c/(-f*x^2+e))^(1/2)*EllipticF((-a*f+b*
e)^(1/2)*x/a^(1/2)/(-f*x^2+e)^(1/2),(-a*(c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(
1/2)/(-a*f+b*e)^(1/2)/(c*f+d*e)/(d*x^2+c)^(1/2)/(e*(-b*x^2+a)/a/(-f*x^2+e)
)^(1/2)
```

Mathematica [F]

$$\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e-fx^2)^{3/2}}\,dx = \int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e-fx^2)^{3/2}}\,dx$$

input
```
Integrate[x^2/(Sqrt[a - b*x^2]*Sqrt[c + d*x^2]*(e - f*x^2)^(3/2)),x]
```

output
```
Integrate[x^2/(Sqrt[a - b*x^2]*Sqrt[c + d*x^2]*(e - f*x^2)^(3/2)), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e-fx^2)^{3/2}}\,dx$$
$$\Big\downarrow \ 450$$
$$\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e-fx^2)^{3/2}}dx$$

input
```
Int[x^2/(Sqrt[a - b*x^2]*Sqrt[c + d*x^2]*(e - f*x^2)^(3/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [F]

$$\int \frac{x^2}{\sqrt{-b\,x^2+a}\,\sqrt{x^2d+c}\,\left(-f\,x^2+e\right)^{\frac{3}{2}}}dx$$

input
```
int(x^2/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(-f*x^2+e)^(3/2),x)
```

output
```
int(x^2/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(-f*x^2+e)^(3/2),x)
```

Fricas [F]

$$\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e-fx^2)^{3/2}}\,dx = \int \frac{x^2}{\sqrt{-bx^2+a}\sqrt{dx^2+c}(-fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(-f*x^2+e)^(3/2),x, algorit
hm="fricas")
```

output
```
integral(-sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-f*x^2 + e)*x^2/(b*d*f^2*x
^8 - (2*b*d*e*f - (b*c - a*d)*f^2)*x^6 + (b*d*e^2 - a*c*f^2 - 2*(b*c - a*d
)*e*f)*x^4 - a*c*e^2 + (2*a*c*e*f + (b*c - a*d)*e^2)*x^2), x)
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e-fx^2)^{3/2}}\,dx = \int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e-fx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2/(-b*x**2+a)**(1/2)/(d*x**2+c)**(1/2)/(-f*x**2+e)**(3/2),x)
```

output
```
Integral(x**2/(sqrt(a - b*x**2)*sqrt(c + d*x**2)*(e - f*x**2)**(3/2)), x)
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e-fx^2)^{3/2}}\,dx = \int \frac{x^2}{\sqrt{-bx^2+a}\sqrt{dx^2+c}(-fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(-f*x^2+e)^(3/2),x, algorit
hm="maxima")
```

output
```
integrate(x^2/(sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)*(-f*x^2 + e)^(3/2)), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e-fx^2)^{3/2}}\,dx = \int \frac{x^2}{\sqrt{-bx^2+a}\sqrt{dx^2+c}(-fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(-f*x^2+e)^(3/2),x, algorit
hm="giac")
```

output
```
integrate(x^2/(sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)*(-f*x^2 + e)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e-fx^2)^{3/2}}\,dx = \int \frac{x^2}{\sqrt{a-b\,x^2}\,\sqrt{d\,x^2+c}\,(e-f\,x^2)^{3/2}}\,dx$$

input
```
int(x^2/((a - b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e - f*x^2)^(3/2)),x)
```

output
```
int(x^2/((a - b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e - f*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}\,(e-fx^2)^{3/2}}\,dx = \int \frac{\sqrt{-f\,x^2+e}\,\sqrt{d\,x^2}}{-bd\,f^2x^8+ad\,f^2x^6-bc\,f^2x^6+2bdef\,x^6+ac\,f^2x^4-2adef\,x}$$

input
```
int(x^2/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(-f*x^2+e)^(3/2),x)
```

output
```
int((sqrt(e - f*x**2)*sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**2)/(a*c*e**2 -
2*a*c*e*f*x**2 + a*c*f**2*x**4 + a*d*e**2*x**2 - 2*a*d*e*f*x**4 + a*d*f**2
*x**6 - b*c*e**2*x**2 + 2*b*c*e*f*x**4 - b*c*f**2*x**6 - b*d*e**2*x**4 + 2
*b*d*e*f*x**6 - b*d*f**2*x**8),x)
```

3.395 $\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c-dx^2}(e-fx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 320

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c-dx^2}\,(e-fx^2)^{3/2}}\,dx = \frac{\sqrt{c}\sqrt{a+bx^2}\sqrt{\frac{e(c-dx^2)}{c(e-fx^2)}}E\left(\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{c}\sqrt{e-fx^2}}\right)|-\frac{c(be+af)}{a(de-cf)}\right)}{(be+af)\sqrt{de-cf}\sqrt{c-dx^2}\sqrt{\frac{e(a+bx^2)}{a(e-fx^2)}}} - \frac{\sqrt{c}\sqrt{a+bx^2}\sqrt{\frac{e(c-dx^2)}{c(e-fx^2)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{c}\sqrt{e-fx^2}}\right),-\frac{c(be+af)}{a(de-cf)}\right)}{(be+af)\sqrt{de-cf}\sqrt{c-dx^2}\sqrt{\frac{e(a+bx^2)}{a(e-fx^2)}}}$$

output
```
c^(1/2)*(b*x^2+a)^(1/2)*(e*(-d*x^2+c)/c/(-f*x^2+e))^(1/2)*EllipticE((-c*f+
d*e)^(1/2)*x/c^(1/2)/(-f*x^2+e)^(1/2),(-c*(a*f+b*e)/a/(-c*f+d*e))^(1/2))/(
a*f+b*e)/(-c*f+d*e)^(1/2)/(-d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(-f*x^2+e))^(1/2
)-c^(1/2)*(b*x^2+a)^(1/2)*(e*(-d*x^2+c)/c/(-f*x^2+e))^(1/2)*EllipticF((-c*
f+d*e)^(1/2)*x/c^(1/2)/(-f*x^2+e)^(1/2),(-c*(a*f+b*e)/a/(-c*f+d*e))^(1/2))
/(a*f+b*e)/(-c*f+d*e)^(1/2)/(-d*x^2+c)^(1/2)/(e*(b*x^2+a)/a/(-f*x^2+e))^(1
/2)
```

Mathematica [F]

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c-dx^2}\,(e-fx^2)^{3/2}}\,dx = \int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c-dx^2}\,(e-fx^2)^{3/2}}\,dx$$

input
```
Integrate[x^2/(Sqrt[a + b*x^2]*Sqrt[c - d*x^2]*(e - f*x^2)^(3/2)),x]
```

output
```
Integrate[x^2/(Sqrt[a + b*x^2]*Sqrt[c - d*x^2]*(e - f*x^2)^(3/2)), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c-dx^2}\,(e-fx^2)^{3/2}}\,dx$$

$$\Big\downarrow \; 450$$

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c-dx^2}\,(e-fx^2)^{3/2}}dx$$

input
```
Int[x^2/(Sqrt[a + b*x^2]*Sqrt[c - d*x^2]*(e - f*x^2)^(3/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [F]

$$\int \frac{x^{2}}{\sqrt{b\,x^{2}+a}\,\sqrt{-x^{2} d+c}\,\left(-f\,x^{2}+e\right)^{\frac{3}{2}}}dx$$

input
```
int(x^2/(b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2)/(-f*x^2+e)^(3/2),x)
```

output
```
int(x^2/(b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2)/(-f*x^2+e)^(3/2),x)
```

Fricas [F]

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c-dx^2}\,(e-fx^2)^{3/2}}\,dx = \int \frac{x^{2}}{\sqrt{bx^{2}+a}\sqrt{-dx^{2}+c}(-fx^{2}+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2)/(-f*x^2+e)^(3/2),x, algorit
hm="fricas")
```

output
```
integral(-sqrt(b*x^2 + a)*sqrt(-d*x^2 + c)*sqrt(-f*x^2 + e)*x^2/(b*d*f^2*x
^8 - (2*b*d*e*f + (b*c - a*d)*f^2)*x^6 + (b*d*e^2 - a*c*f^2 + 2*(b*c - a*d
)*e*f)*x^4 - a*c*e^2 + (2*a*c*e*f - (b*c - a*d)*e^2)*x^2), x)
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c-dx^2}\,(e-fx^2)^{3/2}}\,dx = \int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c-dx^2}\,(e-fx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2/(b*x**2+a)**(1/2)/(-d*x**2+c)**(1/2)/(-f*x**2+e)**(3/2),x)
```

output
```
Integral(x**2/(sqrt(a + b*x**2)*sqrt(c - d*x**2)*(e - f*x**2)**(3/2)), x)
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c-dx^2}\,(e-fx^2)^{3/2}}\,dx = \int \frac{x^2}{\sqrt{bx^2+a}\sqrt{-dx^2+c}(-fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2)/(-f*x^2+e)^(3/2),x, algorit
hm="maxima")
```

output
```
integrate(x^2/(sqrt(b*x^2 + a)*sqrt(-d*x^2 + c)*(-f*x^2 + e)^(3/2)), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c-dx^2}\,(e-fx^2)^{3/2}}\,dx = \int \frac{x^2}{\sqrt{bx^2+a}\sqrt{-dx^2+c}(-fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2)/(-f*x^2+e)^(3/2),x, algorit
hm="giac")
```

output
```
integrate(x^2/(sqrt(b*x^2 + a)*sqrt(-d*x^2 + c)*(-f*x^2 + e)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c-dx^2}\,(e-fx^2)^{3/2}}\,dx = \int \frac{x^2}{\sqrt{b\,x^2+a}\,\sqrt{c-d\,x^2}\,(e-f\,x^2)^{3/2}}\,dx$$

input
```
int(x^2/((a + b*x^2)^(1/2)*(c - d*x^2)^(1/2)*(e - f*x^2)^(3/2)),x)
```

output
```
int(x^2/((a + b*x^2)^(1/2)*(c - d*x^2)^(1/2)*(e - f*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c-dx^2}\,(e-fx^2)^{3/2}}\,dx = \int \frac{\sqrt{-f\,x^2+e}\,\sqrt{-d\,x}}{-bd\,f^2x^8-ad\,f^2x^6+bc\,f^2x^6+2bdef\,x^6+ac\,f^2x^4+2adef\,x}$$

input
```
int(x^2/(b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2)/(-f*x^2+e)^(3/2),x)
```

output
```
int((sqrt(e - f*x**2)*sqrt(c - d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c*e**2 -
2*a*c*e*f*x**2 + a*c*f**2*x**4 - a*d*e**2*x**2 + 2*a*d*e*f*x**4 - a*d*f**2
*x**6 + b*c*e**2*x**2 - 2*b*c*e*f*x**4 + b*c*f**2*x**6 - b*d*e**2*x**4 + 2
*b*d*e*f*x**6 - b*d*f**2*x**8),x)
```

3.396 $\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c-dx^2}\left(e-fx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 328

$$\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c-dx^2}\,(e-fx^2)^{3/2}}\,dx = -\frac{c\sqrt{a-bx^2}\sqrt{\frac{e(c-dx^2)}{c(e-fx^2)}}\,E\left(\arcsin\left(\frac{\sqrt{be-af}\,x}{\sqrt{a}\sqrt{e-fx^2}}\right)|\frac{a(de-cf)}{c(be-af)}\right)}{\sqrt{a}\sqrt{be-af}(de-cf)\sqrt{c-dx^2}\sqrt{\frac{e(a-bx^2)}{a(e-fx^2)}}} + \frac{c\sqrt{a-bx^2}\sqrt{\frac{e(c-dx^2)}{c(e-fx^2)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{be-af}\,x}{\sqrt{a}\sqrt{e-fx^2}}\right),\frac{a(de-cf)}{c(be-af)}\right)}{\sqrt{a}\sqrt{be-af}(de-cf)\sqrt{c-dx^2}\sqrt{\frac{e(a-bx^2)}{a(e-fx^2)}}}$$

output

```
-c*(-b*x^2+a)^(1/2)*(e*(-d*x^2+c)/c/(-f*x^2+e))^(1/2)*EllipticE((-a*f+b*e)
^(1/2)*x/a^(1/2)/(-f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2))/a^(1/
2)/(-a*f+b*e)^(1/2)/(-c*f+d*e)/(-d*x^2+c)^(1/2)/(e*(-b*x^2+a)/a/(-f*x^2+e)
)^(1/2)+c*(-b*x^2+a)^(1/2)*(e*(-d*x^2+c)/c/(-f*x^2+e))^(1/2)*EllipticF((-a
*f+b*e)^(1/2)*x/a^(1/2)/(-f*x^2+e)^(1/2),(a*(-c*f+d*e)/c/(-a*f+b*e))^(1/2)
)/a^(1/2)/(-a*f+b*e)^(1/2)/(-c*f+d*e)/(-d*x^2+c)^(1/2)/(e*(-b*x^2+a)/a/(-f
*x^2+e))^(1/2)
```

Mathematica [F]

$$\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c-dx^2}\,(e-fx^2)^{3/2}}\,dx = \int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c-dx^2}\,(e-fx^2)^{3/2}}\,dx$$

input
```
Integrate[x^2/(Sqrt[a - b*x^2]*Sqrt[c - d*x^2]*(e - f*x^2)^(3/2)),x]
```

output
```
Integrate[x^2/(Sqrt[a - b*x^2]*Sqrt[c - d*x^2]*(e - f*x^2)^(3/2)), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c-dx^2}\,(e-fx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 450}$$

$$\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c-dx^2}\,(e-fx^2)^{3/2}}dx$$

input
```
Int[x^2/(Sqrt[a - b*x^2]*Sqrt[c - d*x^2]*(e - f*x^2)^(3/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 450
```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^
(q_.)*((e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Unintegrable[(g*x)^m*(a +
b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x] /; FreeQ[{a, b, c, d, e, f, g, m,
p, q, r}, x]
```

Maple [F]

$$\int \frac{x^{2}}{\sqrt{-b\,x^{2}+a}\,\sqrt{-x^{2}d+c}\,\left(-f\,x^{2}+e\right)^{\frac{3}{2}}}dx$$

input
```
int(x^2/(-b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2)/(-f*x^2+e)^(3/2),x)
```

output
```
int(x^2/(-b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2)/(-f*x^2+e)^(3/2),x)
```

Fricas [F]

$$\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c-dx^2}\,(e-fx^2)^{3/2}}\,dx=\int \frac{x^{2}}{\sqrt{-bx^{2}+a}\sqrt{-dx^{2}+c}(-fx^{2}+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(-b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2)/(-f*x^2+e)^(3/2),x, algori
thm="fricas")
```

output
```
integral(sqrt(-b*x^2 + a)*sqrt(-d*x^2 + c)*sqrt(-f*x^2 + e)*x^2/(b*d*f^2*x
^8 - (2*b*d*e*f + (b*c + a*d)*f^2)*x^6 + (b*d*e^2 + a*c*f^2 + 2*(b*c + a*d
)*e*f)*x^4 + a*c*e^2 - (2*a*c*e*f + (b*c + a*d)*e^2)*x^2), x)
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c-dx^2}\,(e-fx^2)^{3/2}}\,dx = \int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c-dx^2}\,(e-fx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2/(-b*x**2+a)**(1/2)/(-d*x**2+c)**(1/2)/(-f*x**2+e)**(3/2),x)
```

output
```
Integral(x**2/(sqrt(a - b*x**2)*sqrt(c - d*x**2)*(e - f*x**2)**(3/2)), x)
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c-dx^2}\,(e-fx^2)^{3/2}}\,dx = \int \frac{x^2}{\sqrt{-bx^2+a}\sqrt{-dx^2+c}(-fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(-b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2)/(-f*x^2+e)^(3/2),x, algori
thm="maxima")
```

output
```
integrate(x^2/(sqrt(-b*x^2 + a)*sqrt(-d*x^2 + c)*(-f*x^2 + e)^(3/2)), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c-dx^2}\,(e-fx^2)^{3/2}}\,dx = \int \frac{x^2}{\sqrt{-bx^2+a}\sqrt{-dx^2+c}(-fx^2+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(-b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2)/(-f*x^2+e)^(3/2),x, algori
thm="giac")
```

output
```
integrate(x^2/(sqrt(-b*x^2 + a)*sqrt(-d*x^2 + c)*(-f*x^2 + e)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c-dx^2}\,(e-fx^2)^{3/2}}\,dx=\int \frac{x^2}{\sqrt{a-b\,x^2}\,\sqrt{c-d\,x^2}\,{(e-f\,x^2)}^{3/2}}\,dx$$

input
```
int(x^2/((a - b*x^2)^(1/2)*(c - d*x^2)^(1/2)*(e - f*x^2)^(3/2)),x)
```

output
```
int(x^2/((a - b*x^2)^(1/2)*(c - d*x^2)^(1/2)*(e - f*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{x^2}{\sqrt{a-bx^2}\sqrt{c-dx^2}\,(e-fx^2)^{3/2}}\,dx=\int \frac{\sqrt{-f\,x^2+e}\,\sqrt{-d\,x^2}}{bd\,f^2x^8-ad\,f^2x^6-bc\,f^2x^6-2bdef\,x^6+ac\,f^2x^4+2adef\,x^4}$$

input
```
int(x^2/(-b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2)/(-f*x^2+e)^(3/2),x)
```

output
```
int((sqrt(e - f*x**2)*sqrt(c - d*x**2)*sqrt(a - b*x**2)*x**2)/(a*c*e**2 -
2*a*c*e*f*x**2 + a*c*f**2*x**4 - a*d*e**2*x**2 + 2*a*d*e*f*x**4 - a*d*f**2
*x**6 - b*c*e**2*x**2 + 2*b*c*e*f*x**4 - b*c*f**2*x**6 + b*d*e**2*x**4 - 2
*b*d*e*f*x**6 + b*d*f**2*x**8),x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                  convert(leaf_count_result,string)," vs. $2 (",
                                  convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#           Port of original Maple grading function by
#           Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#           added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False

   except AttributeError as error:
      return False

def expnType(expn):
   debug=False
   if debug:
      print("expn=",expn,"type(expn)=",type(expn))

   if is_atom(expn):
     return 1
   elif isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if isinstance(expn.args[0],Rational):  #type(op(1,expn),'rational')
         return 1
      else:
         return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
   elif isinstance(expn,Pow):    #type(expn,'`^`')
      if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
         return expnType(expn.args[0])    #ExpnType(op(1,expn))
      elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
         if isinstance(expn.args[0],Rational):  #type(op(1,expn),'rational')
            return 1
         else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
      else:
         return max(3,expnType(expn.args[0]),expnType(expn.args[1]))  #max(3,ExpnType(op(1,expn)),Expr
   elif isinstance(expn,Add) or isinstance(expn,Mul):  #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#           Albert Rich to use with Sagemath. This is used to
#           grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#           'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#           issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
            'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
            'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
            'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
            'elliptic_pi','exp_integral_e','log_integral',
            'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
            'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
```

```
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file