Computer Algebra
Independent Integration Tests

Summer 2024

1-Algebraic-functions/1.1-Binomial/1.1.2-Quadratic-binomial/35-1.1.2.7

Nasser M. Abbasi

May 18, 2024 Compiled on May 18, 2024 at 5:14pm

Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**151**]. This is test number [35].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (151)	0.00 (0)
Mathematica	100.00 (151)	0.00 (0)
Sympy	100.00 (151)	0.00 (0)
Maple	70.86 (107)	29.14 (44)
Fricas	70.86 (107)	29.14 (44)
Giac	70.86 (107)	29.14 (44)
Maxima	70.86 (107)	29.14 (44)
Reduce	70.86 (107)	29.14 (44)
Mupad	60.26 (91)	39.74 (60)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

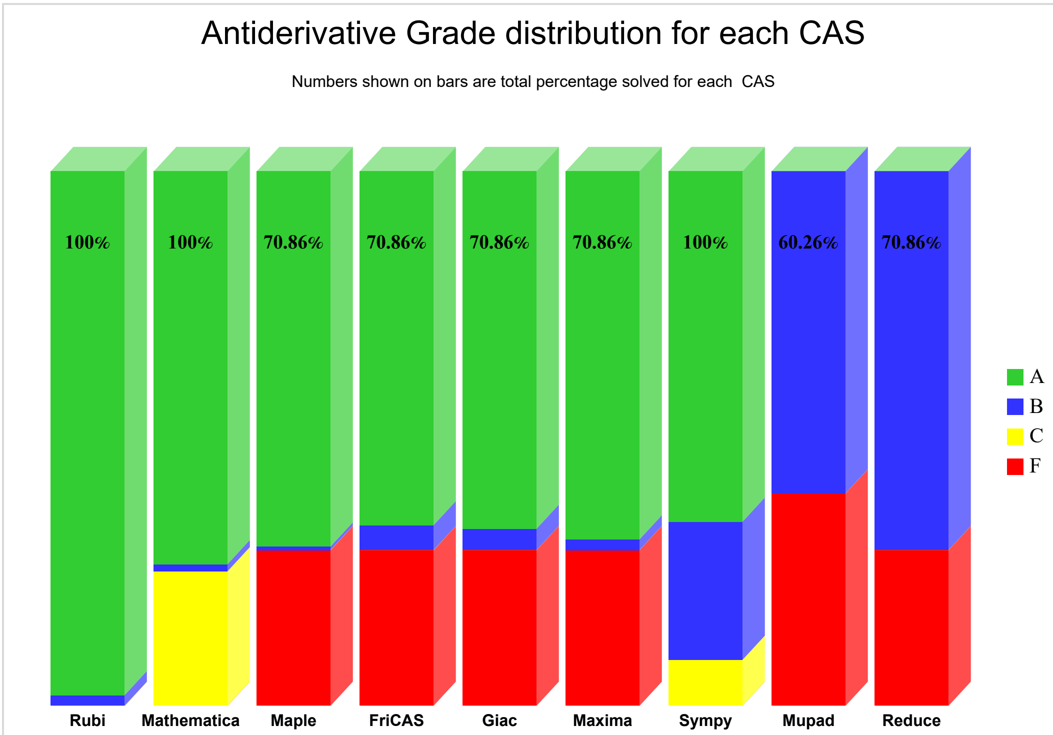
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

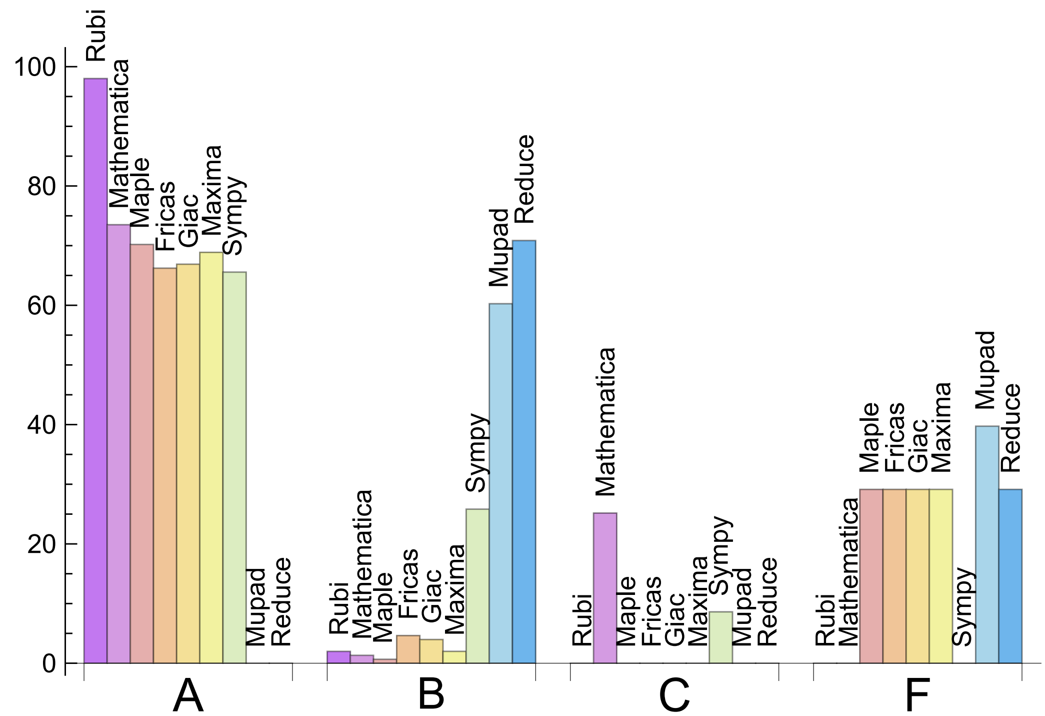
System	% A grade	% B grade	% C grade	% F grade
Rubi	98.013	1.987	0.000	0.000
Mathematica	73.510	1.325	25.166	0.000
Maple	70.199	0.662	0.000	29.139
Maxima	68.874	1.987	0.000	29.139
Giac	66.887	3.974	0.000	29.139
Fricas	66.225	4.636	0.000	29.139
Sympy	65.563	25.828	8.609	0.000
Mupad	0.000	60.265	0.000	39.735
Reduce	0.000	70.861	0.000	29.139

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Sympy	0	0.00	0.00	0.00
Fricas	44	100.00	0.00	0.00
Maple	44	100.00	0.00	0.00
Giac	44	100.00	0.00	0.00
Maxima	44	100.00	0.00	0.00
Reduce	44	100.00	0.00	0.00
Mupad	60	0.00	100.00	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.05
Fricas	0.08
Giac	0.13
Reduce	0.21
Rubi	0.29
Mupad	0.37
Maple	0.47
Mathematica	2.31
Sympy	4.83

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mupad	62.87	0.94	54.00	0.91
Maple	69.78	0.88	68.00	0.86
Giac	78.31	1.02	72.00	0.88
Mathematica	82.86	0.81	83.00	0.88
Maxima	92.33	1.09	74.00	0.99
Reduce	120.41	1.44	84.00	1.19
Fricas	147.27	1.76	110.00	1.71
Rubi	168.82	1.08	103.00	1.00
Sympy	293.19	2.70	117.00	1.17

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

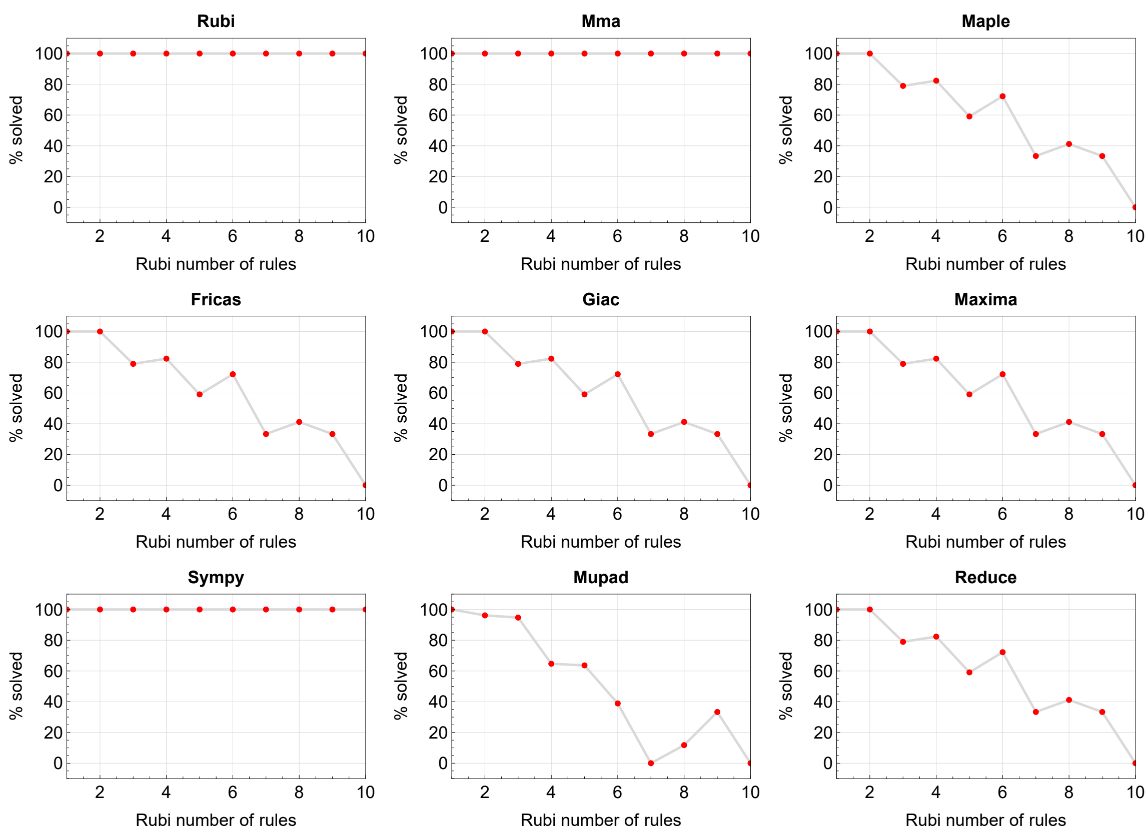


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

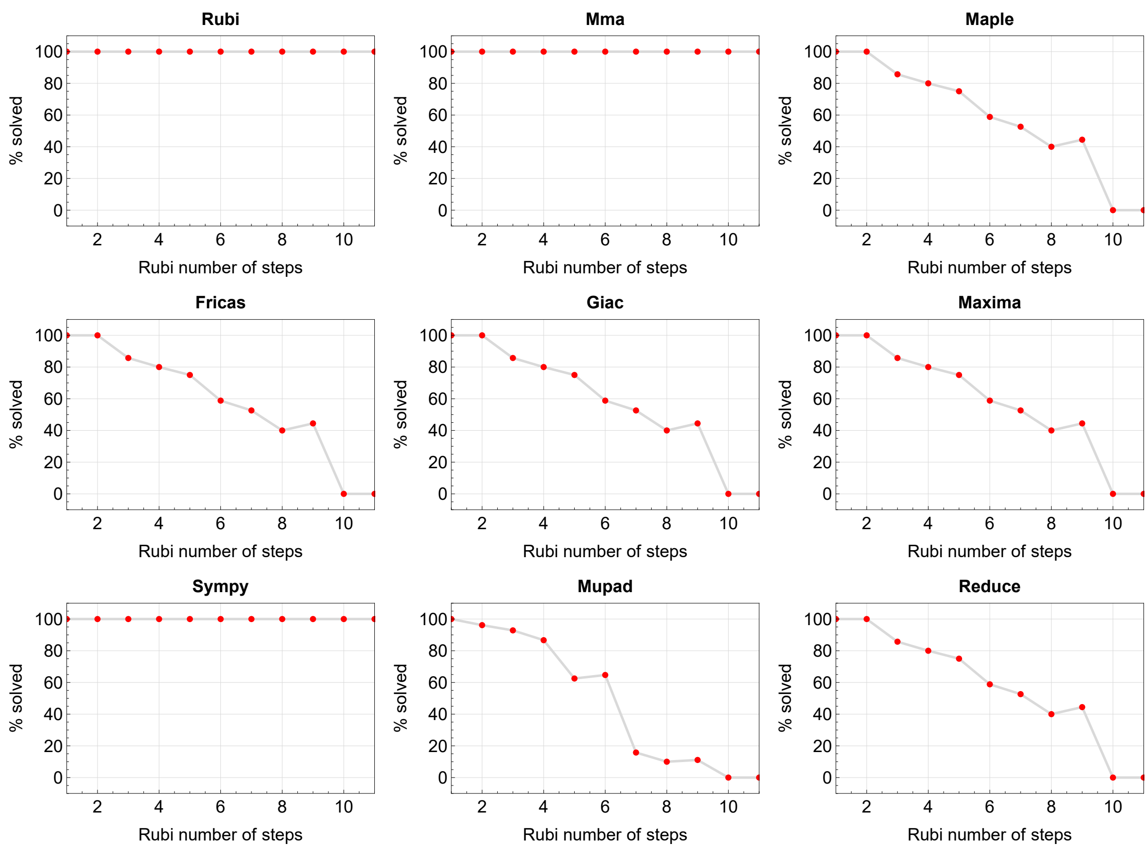


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

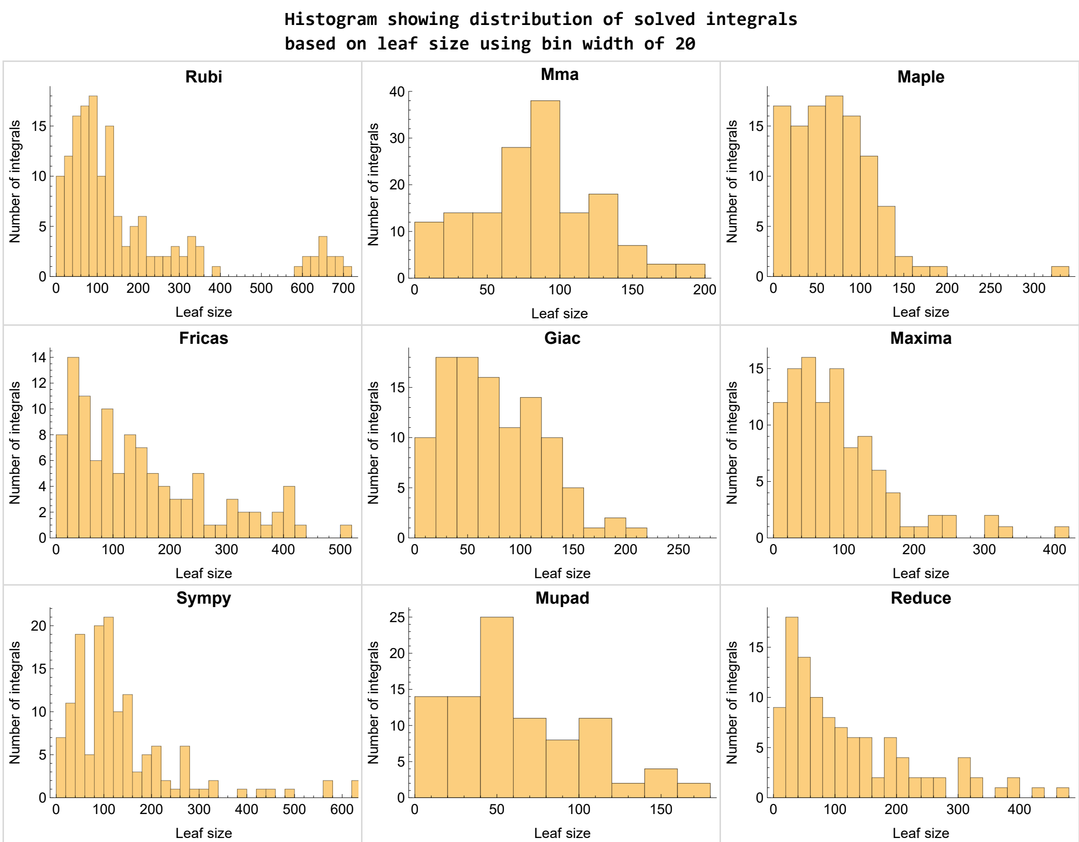


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

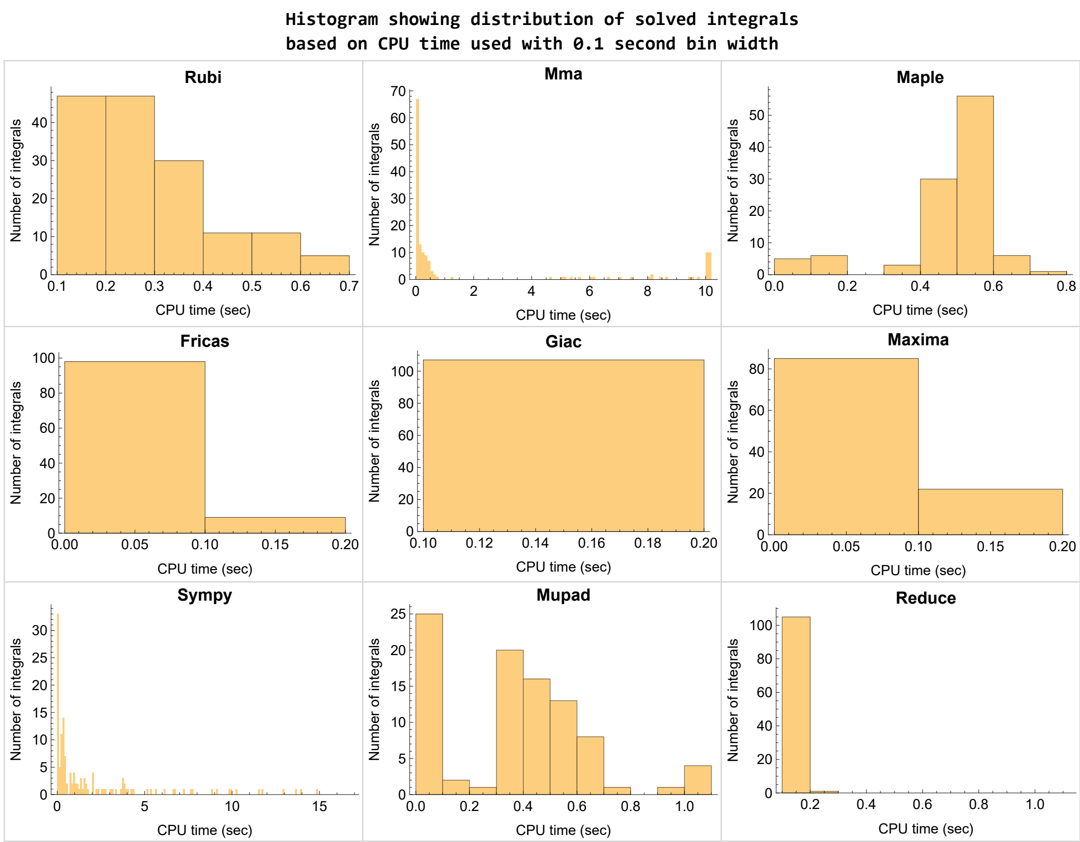


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

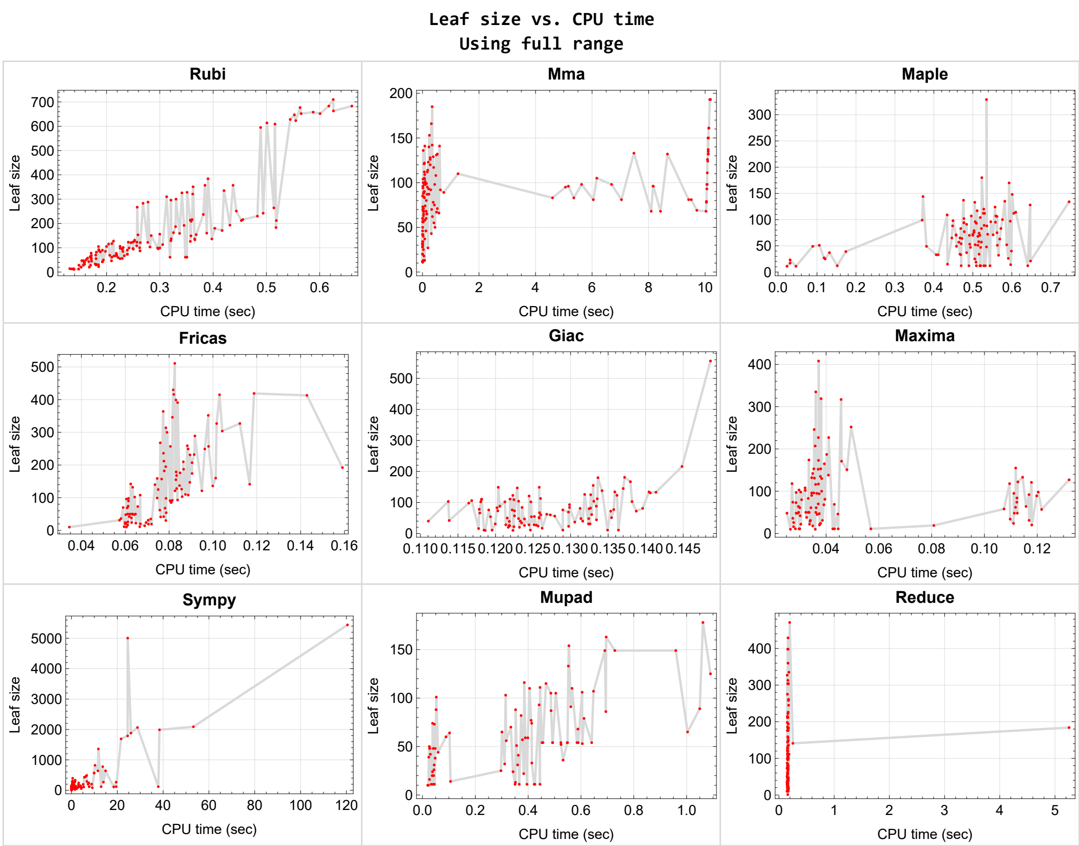


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi $\{19, 20, 21, 41, 42, 43, 44, 67, 68, 69, 70, 71, 124, 125\}$

Mathematica {}

Maple {}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

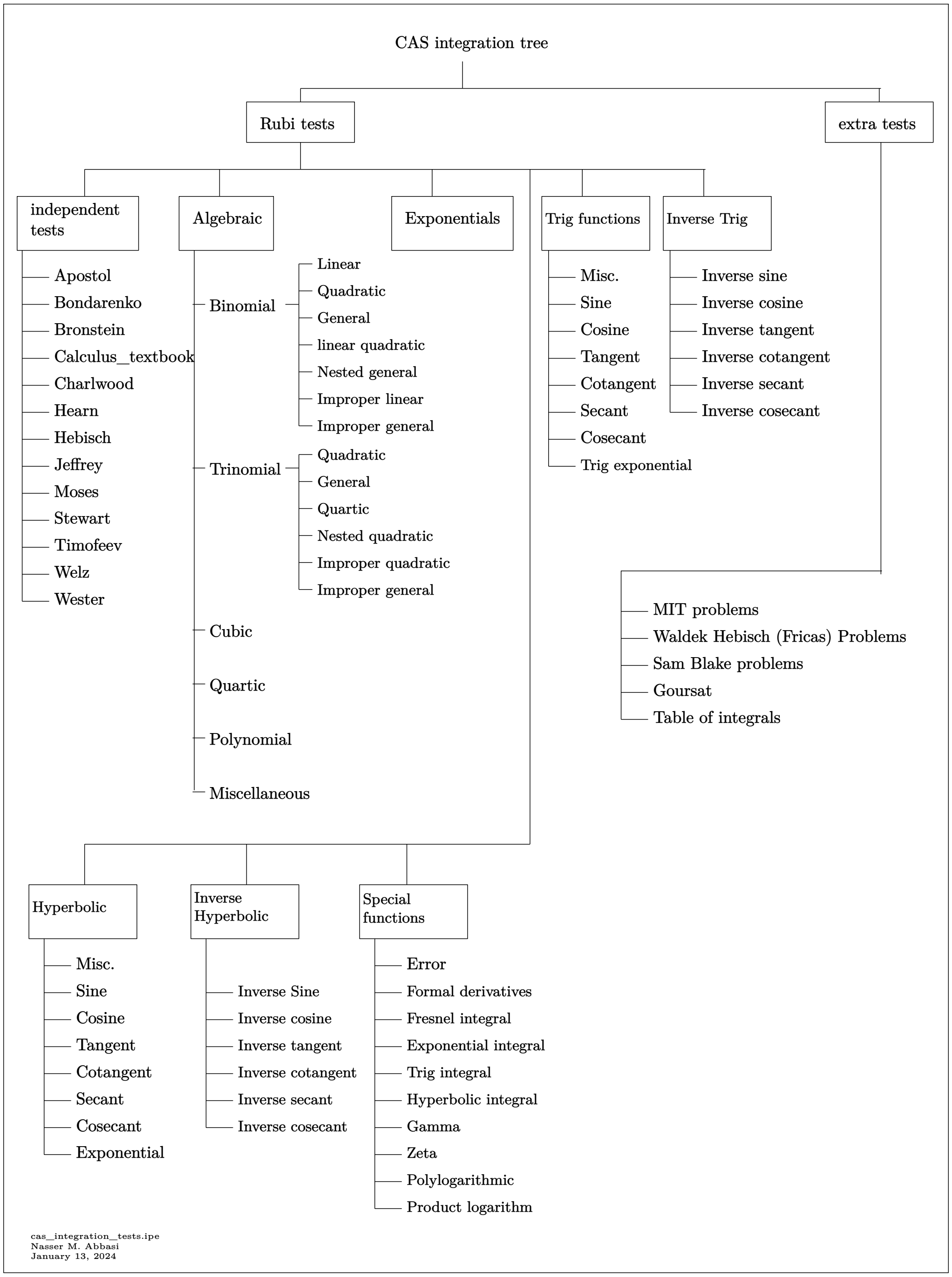
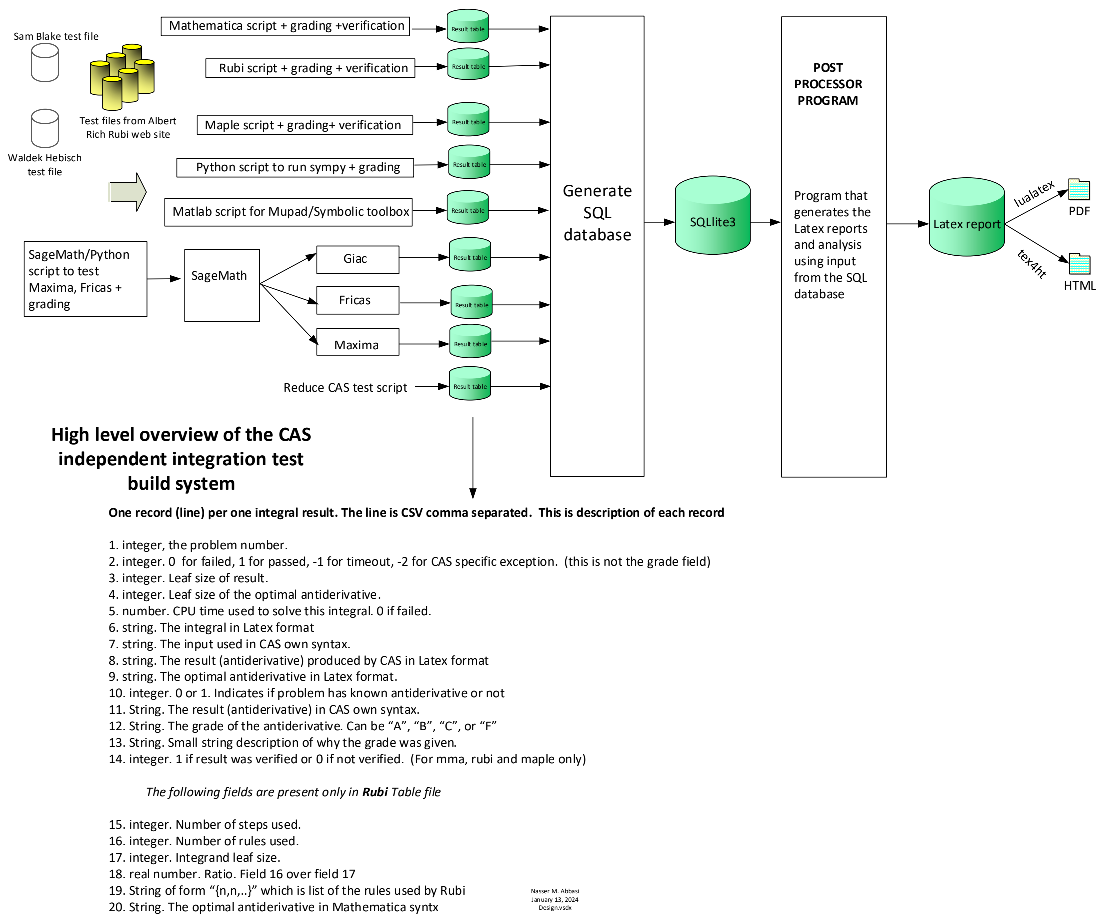


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 99, 100, 101, 102, 103, 104, 105, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151 }

B grade { 97, 98, 106 }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 151 }

B grade { 97, 98 }

C grade { 15, 16, 17, 18, 19, 20, 21, 38, 39, 40, 41, 42, 43, 44, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 122, 123, 124, 125, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140 }

B grade { 73 }

C grade { }

F normal fail { 15, 16, 17, 18, 19, 20, 21, 22, 38, 39, 40, 41, 42, 43, 44, 45, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 108, 122, 123, 124, 125, 126, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 99, 100, 101, 104, 105, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140 }

B grade { 73, 97, 98, 102, 103, 106, 107 }

C grade { }

F normal fail { 15, 16, 17, 18, 19, 20, 21, 22, 38, 39, 40, 41, 42, 43, 44, 45, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 108, 122, 123, 124, 125, 126, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 99, 100, 101, 102, 103, 104, 105, 106, 107, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 139, 140 }

B grade { 97, 98, 138 }

C grade { }

F normal fail { 15, 16, 17, 18, 19, 20, 21, 22, 38, 39, 40, 41, 42, 43, 44, 45, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 108, 122, 123, 124, 125, 126, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151 }

F(-1) timedout fail { }

F(-2) exception fail { }

Giac

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 99, 100, 101, 104, 105, 106, 107, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140 }

B grade { 73, 96, 97, 98, 102, 103 }

C grade { }

F normal fail { 15, 16, 17, 18, 19, 20, 21, 22, 38, 39, 40, 41, 42, 43, 44, 45, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 108, 122, 123, 124, 125, 126, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 33, 34, 35, 36, 37, 46, 47, 48, 49, 50, 51, 52, 53, 56, 57, 58, 59, 60, 61, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 109, 110, 111, 112, 113, 114, 120, 121, 127, 128, 129 }

C grade { }

F normal fail { }

F(-1) timedout fail { 31, 32, 38, 39, 40, 41, 42, 43, 44, 45, 54, 55, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 88, 89, 90, 108, 115, 116, 117, 118, 119, 122, 123, 124, 125, 126, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151 }

F(-2) exception fail { }

Sympy

A grade { 1, 2, 3, 4, 6, 7, 8, 9, 10, 11, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 28, 29, 30, 31, 32, 33, 34, 35, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 52, 53, 55, 56, 57, 58, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 86, 87, 89, 90, 91, 92, 100, 109, 110, 111, 114, 115, 116, 117, 118, 122, 123, 124, 125, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136 }

B grade { 5, 12, 13, 14, 27, 36, 37, 50, 51, 54, 59, 60, 73, 84, 85, 88, 93, 94, 95, 96, 97, 98, 99, 101, 102, 103, 104, 105, 106, 107, 112, 113, 119, 120, 121, 137, 138, 139, 140 }

C grade { 108, 126, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Reduce

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140 }

C grade { }

F normal fail { 15, 16, 17, 18, 19, 20, 21, 22, 38, 39, 40, 41, 42, 43, 44, 45, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 108, 122, 123, 124, 125, 126, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	55	85	74	73	73	85	73	73	73
N.S.	1	0.98	1.52	1.32	1.30	1.30	1.52	1.30	1.30	1.30
time (sec)	N/A	0.180	0.002	0.516	0.029	0.066	0.022	0.123	0.146	0.045

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	60	51	50	50	58	50	51	50
N.S.	1	1.00	1.33	1.13	1.11	1.11	1.29	1.11	1.13	1.11
time (sec)	N/A	0.172	0.002	0.563	0.032	0.065	0.023	0.126	0.148	0.026

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	32	32	27	26	26	29	26	29	26
N.S.	1	1.03	1.03	0.87	0.84	0.84	0.94	0.84	0.94	0.84
time (sec)	N/A	0.159	0.001	0.118	0.028	0.061	0.017	0.124	0.149	0.044

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	12	11	10	10	8	10	10	10
N.S.	1	1.00	0.86	0.79	0.71	0.71	0.57	0.71	0.71	0.71
time (sec)	N/A	0.131	0.000	0.023	0.036	0.035	0.017	0.119	0.147	0.020

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	32	31	98	124	31	34	32
N.S.	1	1.00	1.00	0.76	0.74	2.33	2.95	0.74	0.81	0.76
time (sec)	N/A	0.159	0.015	0.566	0.117	0.079	0.121	0.133	0.146	0.311

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	57	49	48	140	90	47	68	44
N.S.	1	1.00	1.00	0.86	0.84	2.46	1.58	0.82	1.19	0.77
time (sec)	N/A	0.164	0.019	0.458	0.111	0.074	0.143	0.124	0.161	0.059

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	82	71	70	74	212	124	60	119	64
N.S.	1	1.09	0.95	0.93	0.99	2.83	1.65	0.80	1.59	0.85
time (sec)	N/A	0.181	0.034	0.457	0.112	0.078	0.213	0.126	0.157	0.102

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	90	87	70	61	176	119	76	111	54
N.S.	1	1.03	1.00	0.80	0.70	2.02	1.37	0.87	1.28	0.62
time (sec)	N/A	0.192	0.217	0.491	0.036	0.085	0.389	0.129	0.183	0.585

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	68	68	54	45	128	90	55	73	52
N.S.	1	1.01	1.01	0.81	0.67	1.91	1.34	0.82	1.09	0.78
time (sec)	N/A	0.172	0.130	0.470	0.037	0.083	0.320	0.125	0.167	0.524

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	46	37	29	92	71	39	38	36
N.S.	1	1.00	1.07	0.86	0.67	2.14	1.65	0.91	0.88	0.84
time (sec)	N/A	0.167	0.121	0.484	0.034	0.081	0.298	0.122	0.154	0.532

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	27	26	31	35	46	23	47	24
N.S.	1	1.00	0.96	0.93	1.11	1.25	1.64	0.82	1.68	0.86
time (sec)	N/A	0.148	0.192	0.591	0.039	0.072	2.006	0.130	0.170	0.343

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	43	40	48	62	146	37	96	41
N.S.	1	1.00	0.84	0.78	0.94	1.22	2.86	0.73	1.88	0.80
time (sec)	N/A	0.162	0.313	0.599	0.029	0.076	3.759	0.131	0.161	0.363

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	78	55	52	64	85	486	52	141	59
N.S.	1	1.10	0.77	0.73	0.90	1.20	6.85	0.73	1.99	0.83
time (sec)	N/A	0.171	0.405	0.587	0.030	0.081	6.648	0.126	0.252	0.399

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	105	67	64	80	108	1360	67	184	74
N.S.	1	1.15	0.74	0.70	0.88	1.19	14.95	0.74	2.02	0.81
time (sec)	N/A	0.185	0.496	0.472	0.031	0.089	11.765	0.126	5.256	0.413

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	A	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	307	310	105	0	0	0	332	0	95	54
N.S.	1	1.01	0.34	0.00	0.00	0.00	1.08	0.00	0.31	0.18
time (sec)	N/A	0.313	6.168	0.000	0.000	0.000	1.652	0.000	0.232	0.548

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	A	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	287	288	83	0	0	0	54	0	57	54
N.S.	1	1.00	0.29	0.00	0.00	0.00	0.19	0.00	0.20	0.19
time (sec)	N/A	0.278	4.601	0.000	0.000	0.000	0.836	0.000	0.202	0.453

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	A	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	68	0	0	0	53	0	25	54
N.S.	1	1.00	0.25	0.00	0.00	0.00	0.20	0.00	0.09	0.20
time (sec)	N/A	0.258	8.093	0.000	0.000	0.000	0.758	0.000	0.176	0.492

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	A	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	283	283	68	0	0	0	54	0	67	54
N.S.	1	1.00	0.24	0.00	0.00	0.00	0.19	0.00	0.24	0.19
time (sec)	N/A	0.269	10.032	0.000	0.000	0.000	2.234	0.000	0.193	0.549

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	A	F	F	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	573	614	83	0	0	0	54	0	57	54
N.S.	1	1.07	0.14	0.00	0.00	0.00	0.09	0.00	0.10	0.09
time (sec)	N/A	0.501	5.355	0.000	0.000	0.000	0.921	0.000	0.204	0.456

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	A	F	F	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	551	595	68	0	0	0	53	0	29	54
N.S.	1	1.08	0.12	0.00	0.00	0.00	0.10	0.00	0.05	0.10
time (sec)	N/A	0.489	8.422	0.000	0.000	0.000	0.754	0.000	0.177	0.493

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	A	F	F	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	566	609	69	0	0	0	54	0	67	54
N.S.	1	1.08	0.12	0.00	0.00	0.00	0.10	0.00	0.12	0.10
time (sec)	N/A	0.516	9.707	0.000	0.000	0.000	2.009	0.000	0.178	0.524

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	A	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	98	0	0	0	61	0	213	65
N.S.	1	1.00	1.40	0.00	0.00	0.00	0.87	0.00	3.04	0.93
time (sec)	N/A	0.180	0.108	0.000	0.000	0.000	2.384	0.000	0.165	1.004

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	100	109	108	108	122	111	111	103
N.S.	1	1.00	1.15	1.25	1.24	1.24	1.40	1.28	1.28	1.18
time (sec)	N/A	0.263	0.023	0.434	0.032	0.067	0.025	0.118	0.155	0.315

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	69	75	74	74	83	76	77	74
N.S.	1	1.00	1.03	1.12	1.10	1.10	1.24	1.13	1.15	1.10
time (sec)	N/A	0.233	0.015	0.562	0.027	0.061	0.023	0.122	0.155	0.038

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	39	38	38	42	40	43	39
N.S.	1	1.00	1.00	0.85	0.83	0.83	0.91	0.87	0.93	0.85
time (sec)	N/A	0.209	0.006	0.174	0.033	0.063	0.019	0.123	0.162	0.024

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	16	16	15	16	17	16
N.S.	1	1.00	1.00	0.85	0.80	0.80	0.75	0.80	0.85	0.80
time (sec)	N/A	0.152	0.000	0.031	0.028	0.061	0.015	0.118	0.157	0.027

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	56	47	48	125	156	48	61	56
N.S.	1	1.00	1.02	0.85	0.87	2.27	2.84	0.87	1.11	1.02
time (sec)	N/A	0.225	0.028	0.520	0.112	0.075	0.233	0.123	0.159	0.317

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	68	65	62	195	116	60	120	60
N.S.	1	1.00	0.99	0.94	0.90	2.83	1.68	0.87	1.74	0.87
time (sec)	N/A	0.206	0.042	0.467	0.112	0.078	0.319	0.127	0.160	0.089

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	90	83	98	314	156	84	212	88
N.S.	1	1.00	0.92	0.85	1.00	3.20	1.59	0.86	2.16	0.90
time (sec)	N/A	0.226	0.050	0.469	0.120	0.079	0.547	0.124	0.164	0.352

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	123	112	100	133	430	196	109	304	116
N.S.	1	0.98	0.89	0.79	1.06	3.41	1.56	0.87	2.41	0.92
time (sec)	N/A	0.241	0.064	0.576	0.114	0.082	0.890	0.122	0.159	0.386

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	123	117	112	131	259	209	125	194	0
N.S.	1	0.90	0.85	0.82	0.96	1.89	1.53	0.91	1.42	0.00
time (sec)	N/A	0.249	0.395	0.527	0.036	0.088	0.364	0.133	0.168	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	101	87	76	96	187	124	84	136	0
N.S.	1	0.95	0.82	0.72	0.91	1.76	1.17	0.79	1.28	0.00
time (sec)	N/A	0.231	0.228	0.503	0.035	0.087	0.314	0.130	0.171	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	77	70	53	61	121	92	56	81	107
N.S.	1	1.04	0.95	0.72	0.82	1.64	1.24	0.76	1.09	1.45
time (sec)	N/A	0.218	0.325	0.492	0.030	0.083	0.276	0.128	0.158	0.647

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	62	70	61	179	87	61	133	68
N.S.	1	1.00	1.02	1.15	1.00	2.93	1.43	1.00	2.18	1.11
time (sec)	N/A	0.210	0.351	0.518	0.034	0.083	2.737	0.133	0.160	0.588

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	50	47	83	68	194	48	146	59
N.S.	1	1.00	0.75	0.70	1.24	1.01	2.90	0.72	2.18	0.88
time (sec)	N/A	0.217	0.441	0.494	0.033	0.078	5.102	0.131	0.160	0.389

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	94	71	72	118	103	638	80	226	93
N.S.	1	0.97	0.73	0.74	1.22	1.06	6.58	0.82	2.33	0.96
time (sec)	N/A	0.229	0.525	0.500	0.027	0.086	11.556	0.132	0.167	0.442

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	121	92	96	153	137	1880	112	303	115
N.S.	1	0.95	0.72	0.76	1.20	1.08	14.80	0.88	2.39	0.91
time (sec)	N/A	0.246	0.633	0.506	0.038	0.085	25.958	0.134	0.166	0.467

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	326	321	81	0	0	0	87	0	114	0
N.S.	1	0.98	0.25	0.00	0.00	0.00	0.27	0.00	0.35	0.00
time (sec)	N/A	0.362	6.045	0.000	0.000	0.000	1.092	0.000	0.244	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	297	300	81	0	0	0	83	0	42	0
N.S.	1	1.01	0.27	0.00	0.00	0.00	0.28	0.00	0.14	0.00
time (sec)	N/A	0.330	9.512	0.000	0.000	0.000	0.969	0.000	0.193	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	296	296	78	0	0	0	85	0	103	0
N.S.	1	1.00	0.26	0.00	0.00	0.00	0.29	0.00	0.35	0.00
time (sec)	N/A	0.321	10.048	0.000	0.000	0.000	2.682	0.000	0.219	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	A	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	631	647	81	0	0	0	87	0	114	0
N.S.	1	1.03	0.13	0.00	0.00	0.00	0.14	0.00	0.18	0.00
time (sec)	N/A	0.552	7.039	0.000	0.000	0.000	1.097	0.000	0.247	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	A	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	602	628	81	0	0	0	83	0	46	0
N.S.	1	1.04	0.13	0.00	0.00	0.00	0.14	0.00	0.08	0.00
time (sec)	N/A	0.545	9.416	0.000	0.000	0.000	0.938	0.000	0.203	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	A	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	596	623	79	0	0	0	85	0	103	0
N.S.	1	1.05	0.13	0.00	0.00	0.00	0.14	0.00	0.17	0.00
time (sec)	N/A	0.555	10.047	0.000	0.000	0.000	2.506	0.000	0.206	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	A	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	633	652	111	0	0	0	105	0	163	0
N.S.	1	1.03	0.18	0.00	0.00	0.00	0.17	0.00	0.26	0.00
time (sec)	N/A	0.601	10.076	0.000	0.000	0.000	5.359	0.000	0.231	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	109	121	114	0	0	0	90	0	778	0
N.S.	1	1.11	1.05	0.00	0.00	0.00	0.83	0.00	7.14	0.00
time (sec)	N/A	0.263	0.185	0.000	0.000	0.000	4.393	0.000	0.173	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	121	144	142	142	158	149	149	149
N.S.	1	1.00	0.91	1.08	1.07	1.07	1.19	1.12	1.12	1.12
time (sec)	N/A	0.366	0.055	0.372	0.035	0.062	0.027	0.126	0.168	0.690

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	88	99	98	98	107	102	103	105
N.S.	1	1.00	0.89	1.00	0.99	0.99	1.08	1.03	1.04	1.06
time (sec)	N/A	0.300	0.030	0.370	0.030	0.061	0.024	0.118	0.169	0.505

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	60	51	50	50	56	54	57	54
N.S.	1	1.00	1.00	0.85	0.83	0.83	0.93	0.90	0.95	0.90
time (sec)	N/A	0.221	0.006	0.106	0.036	0.063	0.018	0.125	0.158	0.590

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	23	22	22	22	22	23	22
N.S.	1	1.00	1.00	0.82	0.79	0.79	0.79	0.79	0.82	0.79
time (sec)	N/A	0.156	0.000	0.031	0.038	0.060	0.016	0.122	0.161	0.383

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	68	65	64	157	219	66	79	79
N.S.	1	1.00	0.93	0.89	0.88	2.15	3.00	0.90	1.08	1.08
time (sec)	N/A	0.239	0.035	0.446	0.116	0.077	0.400	0.118	0.168	0.612

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	99	83	88	89	257	233	88	156	110
N.S.	1	1.06	0.89	0.95	0.96	2.76	2.51	0.95	1.68	1.18
time (sec)	N/A	0.242	0.068	0.448	0.120	0.081	0.902	0.121	0.165	0.566

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	127	104	98	122	346	184	106	222	163
N.S.	1	1.09	0.90	0.84	1.05	2.98	1.59	0.91	1.91	1.41
time (sec)	N/A	0.252	0.078	0.449	0.113	0.082	3.064	0.117	0.162	0.695

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	22	20	19	18	24	15	25	35	20
N.S.	1	1.10	1.00	0.95	0.90	1.20	0.75	1.25	1.75	1.00
time (sec)	N/A	0.185	0.007	0.508	0.035	0.063	0.037	0.122	0.160	0.037

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	159	141	180	165	327	303	180	261	0
N.S.	1	0.98	0.87	1.10	1.01	2.01	1.86	1.10	1.60	0.00
time (sec)	N/A	0.337	0.603	0.523	0.039	0.102	0.454	0.134	0.174	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	137	108	148	130	231	160	121	183	0
N.S.	1	1.04	0.82	1.12	0.98	1.75	1.21	0.92	1.39	0.00
time (sec)	N/A	0.321	0.471	0.601	0.038	0.090	0.424	0.124	0.174	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	113	78	112	95	147	116	77	109	149
N.S.	1	1.13	0.78	1.12	0.95	1.47	1.16	0.77	1.09	1.49
time (sec)	N/A	0.305	0.379	0.605	0.036	0.089	0.366	0.136	0.160	0.959

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	90	75	104	94	210	131	88	159	91
N.S.	1	1.06	0.88	1.22	1.11	2.47	1.54	1.04	1.87	1.07
time (sec)	N/A	0.239	0.421	0.519	0.037	0.087	3.905	0.133	0.160	0.562

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	66	68	117	91	289	72	176	89
N.S.	1	1.00	0.78	0.80	1.38	1.07	3.40	0.85	2.07	1.05
time (sec)	N/A	0.231	0.582	0.513	0.037	0.080	6.746	0.135	0.168	1.050

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	123	89	93	152	126	777	103	258	125
N.S.	1	1.07	0.77	0.81	1.32	1.10	6.76	0.90	2.24	1.09
time (sec)	N/A	0.257	0.753	0.509	0.037	0.086	13.649	0.138	0.171	1.090

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	150	110	117	187	160	2064	138	335	178
N.S.	1	1.03	0.76	0.81	1.29	1.10	14.23	0.95	2.31	1.23
time (sec)	N/A	0.284	1.257	0.511	0.040	0.101	28.819	0.134	0.180	1.062

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	29	18	15	22	14	22	17	13	14
N.S.	1	1.26	0.78	0.65	0.96	0.61	0.96	0.74	0.57	0.61
time (sec)	N/A	0.180	0.049	0.435	0.034	0.069	0.102	0.126	0.158	0.106

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	352	357	96	0	0	0	321	0	174	0
N.S.	1	1.01	0.27	0.00	0.00	0.00	0.91	0.00	0.49	0.00
time (sec)	N/A	0.437	8.168	0.000	0.000	0.000	1.604	0.000	0.251	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	323	335	96	0	0	0	272	0	59	0
N.S.	1	1.04	0.30	0.00	0.00	0.00	0.84	0.00	0.18	0.00
time (sec)	N/A	0.420	5.163	0.000	0.000	0.000	1.433	0.000	0.198	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	323	328	93	0	0	0	134	0	139	0
N.S.	1	1.02	0.29	0.00	0.00	0.00	0.41	0.00	0.43	0.00
time (sec)	N/A	0.351	10.061	0.000	0.000	0.000	4.030	0.000	0.232	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	346	357	124	0	0	0	201	0	219	0
N.S.	1	1.03	0.36	0.00	0.00	0.00	0.58	0.00	0.63	0.00
time (sec)	N/A	0.385	10.093	0.000	0.000	0.000	8.812	0.000	0.249	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	377	384	150	0	0	0	267	0	299	0
N.S.	1	1.02	0.40	0.00	0.00	0.00	0.71	0.00	0.79	0.00
time (sec)	N/A	0.390	10.119	0.000	0.000	0.000	19.518	0.000	0.254	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	A	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	657	683	96	0	0	0	156	0	174	0
N.S.	1	1.04	0.15	0.00	0.00	0.00	0.24	0.00	0.26	0.00
time (sec)	N/A	0.660	8.157	0.000	0.000	0.000	1.358	0.000	0.288	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	A	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	628	663	95	0	0	0	272	0	63	0
N.S.	1	1.06	0.15	0.00	0.00	0.00	0.43	0.00	0.10	0.00
time (sec)	N/A	0.626	5.059	0.000	0.000	0.000	1.396	0.000	0.206	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	A	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	621	658	98	0	0	0	134	0	139	0
N.S.	1	1.06	0.16	0.00	0.00	0.00	0.22	0.00	0.22	0.00
time (sec)	N/A	0.588	10.062	0.000	0.000	0.000	3.678	0.000	0.204	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	A	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	651	683	126	0	0	0	201	0	219	0
N.S.	1	1.05	0.19	0.00	0.00	0.00	0.31	0.00	0.34	0.00
time (sec)	N/A	0.617	10.083	0.000	0.000	0.000	7.229	0.000	0.213	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	A	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	682	710	150	0	0	0	267	0	299	0
N.S.	1	1.04	0.22	0.00	0.00	0.00	0.39	0.00	0.44	0.00
time (sec)	N/A	0.625	10.102	0.000	0.000	0.000	13.954	0.000	0.220	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	140	171	142	0	0	0	427	0	1220	0
N.S.	1	1.22	1.01	0.00	0.00	0.00	3.05	0.00	8.71	0.00
time (sec)	N/A	0.417	0.342	0.000	0.000	0.000	5.616	0.000	0.170	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	B	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	136	166	329	252	399	5005	556	398	0
N.S.	1	0.94	1.15	2.28	1.75	2.77	34.76	3.86	2.76	0.00
time (sec)	N/A	0.398	0.314	0.535	0.049	0.083	24.546	0.149	0.167	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	33	25	21	20	20	20	20	21	20
N.S.	1	1.32	1.00	0.84	0.80	0.80	0.80	0.80	0.84	0.80
time (sec)	N/A	0.179	0.034	0.648	0.118	0.067	0.039	0.122	0.156	0.042

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	22	24	24	24
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.73	0.80	0.80	0.80
time (sec)	N/A	0.185	0.006	0.505	0.111	0.070	0.044	0.126	0.152	0.038

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	61	34	33	50	50	48	28	47	48
N.S.	1	1.15	0.64	0.62	0.94	0.94	0.91	0.53	0.89	0.91
time (sec)	N/A	0.351	0.011	0.469	0.033	0.063	0.063	0.125	0.166	0.041

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	61	34	33	50	50	48	28	47	48
N.S.	1	1.15	0.64	0.62	0.94	0.94	0.91	0.53	0.89	0.91
time (sec)	N/A	0.319	0.007	0.411	0.030	0.061	0.060	0.121	0.153	0.028

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	61	34	33	50	50	48	28	47	48
N.S.	1	1.15	0.64	0.62	0.94	0.94	0.91	0.53	0.89	0.91
time (sec)	N/A	0.348	0.005	0.406	0.034	0.062	0.056	0.124	0.162	0.027

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	94	95	96	96	107	98	57	88
N.S.	1	1.00	1.00	1.01	1.02	1.02	1.14	1.04	0.61	0.94
time (sec)	N/A	0.250	0.015	0.558	0.035	0.061	0.033	0.116	0.154	0.051

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	70	71	70	70	76	73	46	65
N.S.	1	1.00	1.00	1.01	1.00	1.00	1.09	1.04	0.66	0.93
time (sec)	N/A	0.219	0.010	0.457	0.037	0.059	0.027	0.120	0.168	0.301

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	49	48	48	53	50	35	48
N.S.	1	1.00	1.00	0.98	0.96	0.96	1.06	1.00	0.70	0.96
time (sec)	N/A	0.194	0.006	0.381	0.029	0.060	0.020	0.122	0.158	0.048

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	25	24	24	26	26	24	25
N.S.	1	1.00	1.00	0.89	0.86	0.86	0.93	0.93	0.86	0.89
time (sec)	N/A	0.170	0.004	0.121	0.038	0.060	0.018	0.122	0.155	0.297

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	10	8	10	12	10
N.S.	1	1.00	1.00	0.92	0.83	0.83	0.67	0.83	1.00	0.83
time (sec)	N/A	0.148	0.000	0.046	0.026	0.061	0.017	0.122	0.156	0.022

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	40	34	34	99	82	34	1	31
N.S.	1	1.00	1.03	0.87	0.87	2.54	2.10	0.87	0.03	0.79
time (sec)	N/A	0.158	0.023	0.467	0.110	0.075	0.135	0.123	0.160	0.360

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	63	57	57	182	112	57	23	51
N.S.	1	1.00	1.00	0.90	0.90	2.89	1.78	0.90	0.37	0.81
time (sec)	N/A	0.169	0.034	0.464	0.122	0.078	0.202	0.123	0.163	0.356

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	84	77	92	300	150	78	61	82
N.S.	1	1.00	0.91	0.84	1.00	3.26	1.63	0.85	0.66	0.89
time (sec)	N/A	0.186	0.050	0.478	0.117	0.079	0.280	0.118	0.158	0.373

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	117	105	94	127	416	189	103	113	110
N.S.	1	0.98	0.88	0.78	1.06	3.47	1.58	0.86	0.94	0.92
time (sec)	N/A	0.210	0.064	0.477	0.132	0.082	0.341	0.114	0.155	0.405

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	127	123	106	151	257	279	134	99	0
N.S.	1	0.85	0.83	0.71	1.01	1.72	1.87	0.90	0.66	0.00
time (sec)	N/A	0.213	0.206	0.562	0.048	0.098	0.392	0.140	0.170	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	105	99	88	116	207	175	102	80	0
N.S.	1	0.89	0.84	0.75	0.98	1.75	1.48	0.86	0.68	0.00
time (sec)	N/A	0.200	0.140	0.536	0.036	0.090	0.337	0.125	0.160	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	83	74	63	81	155	104	69	61	0
N.S.	1	0.95	0.85	0.72	0.93	1.78	1.20	0.79	0.70	0.00
time (sec)	N/A	0.188	0.093	0.569	0.040	0.083	0.264	0.130	0.159	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	65	48	47	110	82	48	40	86
N.S.	1	1.00	1.12	0.83	0.81	1.90	1.41	0.83	0.69	1.48
time (sec)	N/A	0.171	0.153	0.535	0.037	0.087	0.230	0.132	0.156	0.694

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	58	51	46	168	60	51	25	53
N.S.	1	1.00	1.07	0.94	0.85	3.11	1.11	0.94	0.46	0.98
time (sec)	N/A	0.172	0.086	0.513	0.044	0.083	2.020	0.135	0.160	0.605

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	47	37	34	68	54	144	40	39	33
N.S.	1	0.77	0.61	0.56	1.11	0.89	2.36	0.66	0.64	0.54
time (sec)	N/A	0.157	0.081	0.516	0.039	0.079	3.869	0.132	0.166	0.414

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	88	60	52	103	87	566	72	84	87
N.S.	1	0.97	0.66	0.57	1.13	0.96	6.22	0.79	0.92	0.96
time (sec)	N/A	0.186	0.128	0.506	0.030	0.081	9.890	0.139	0.158	0.487

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	115	80	71	138	121	1787	104	127	105
N.S.	1	0.95	0.66	0.59	1.14	1.00	14.77	0.86	1.05	0.87
time (sec)	N/A	0.204	0.135	0.552	0.041	0.095	24.521	0.133	0.159	0.486

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	19	19	24	27	18	13
N.S.	1	1.00	1.00	1.08	1.46	1.46	1.85	2.08	1.38	1.00
time (sec)	N/A	0.138	0.072	0.597	0.081	0.072	1.518	0.125	0.158	0.352

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	B	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	42	42	12	42	42	42	42	42	42
N.S.	1	3.82	3.82	1.09	3.82	3.82	3.82	3.82	3.82	3.82
time (sec)	N/A	0.192	0.003	0.544	0.030	0.062	0.021	0.114	0.165	0.030

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	B	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	31	31	12	31	31	31	31	31	31
N.S.	1	2.82	2.82	1.09	2.82	2.82	2.82	2.82	2.82	2.82
time (sec)	N/A	0.181	0.001	0.532	0.034	0.057	0.019	0.120	0.161	0.044

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	20	20	12	20	20	19	20	20	11
N.S.	1	1.82	1.82	1.09	1.82	1.82	1.73	1.82	1.82	1.00
time (sec)	N/A	0.168	0.002	0.152	0.038	0.060	0.019	0.123	0.169	0.033

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	11	11	7	11	11	11
N.S.	1	1.00	1.00	1.09	1.00	1.00	0.64	1.00	1.00	1.00
time (sec)	N/A	0.139	0.006	0.470	0.057	0.059	0.085	0.125	0.167	0.040

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	22	22	19	11	22	11
N.S.	1	1.00	1.00	1.09	2.00	2.00	1.73	1.00	2.00	1.00
time (sec)	N/A	0.139	0.006	0.641	0.030	0.065	0.128	0.120	0.158	0.045

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	41	58	44	39	11
N.S.	1	1.00	1.00	0.92	0.85	3.15	4.46	3.38	3.00	0.85
time (sec)	N/A	0.138	0.071	0.528	0.045	0.076	0.406	0.133	0.167	0.396

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	30	44	31	28	11
N.S.	1	1.00	1.00	0.92	0.85	2.31	3.38	2.38	2.15	0.85
time (sec)	N/A	0.138	0.057	0.490	0.030	0.079	0.330	0.132	0.160	0.366

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	19	34	11	17	11
N.S.	1	1.00	1.00	0.92	0.85	1.46	2.62	0.85	1.31	0.85
time (sec)	N/A	0.138	0.042	0.518	0.043	0.071	0.254	0.118	0.174	0.350

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	11	27	11	10	11
N.S.	1	1.00	1.00	0.92	0.85	0.85	2.08	0.85	0.77	0.85
time (sec)	N/A	0.137	0.026	0.519	0.030	0.067	0.235	0.129	0.169	0.426

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	47	13	12	11	31	146	11	68	11
N.S.	1	3.62	1.00	0.92	0.85	2.38	11.23	0.85	5.23	0.85
time (sec)	N/A	0.154	0.057	0.509	0.028	0.070	3.830	0.136	0.163	0.423

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	42	568	11	41	11
N.S.	1	1.00	1.00	0.92	0.85	3.23	43.69	0.85	3.15	0.85
time (sec)	N/A	0.134	0.067	0.520	0.043	0.078	9.906	0.135	0.164	0.445

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	90	0	0	0	53	0	301	0
N.S.	1	1.00	1.06	0.00	0.00	0.00	0.62	0.00	3.54	0.00
time (sec)	N/A	0.198	0.072	0.000	0.000	0.000	4.277	0.000	0.156	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	104	102	102	102	112	111	84	101
N.S.	1	1.00	1.01	0.99	0.99	0.99	1.09	1.08	0.82	0.98
time (sec)	N/A	0.282	0.019	0.477	0.032	0.064	0.026	0.122	0.160	0.052

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	73	70	69	69	78	76	61	70
N.S.	1	1.00	1.00	0.96	0.95	0.95	1.07	1.04	0.84	0.96
time (sec)	N/A	0.239	0.010	0.500	0.043	0.061	0.024	0.119	0.156	0.335

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	37	36	36	39	40	38	38
N.S.	1	1.00	1.00	0.88	0.86	0.86	0.93	0.95	0.90	0.90
time (sec)	N/A	0.192	0.005	0.132	0.027	0.058	0.018	0.111	0.152	0.048

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	65	57	58	159	117	62	46	57
N.S.	1	1.00	0.98	0.86	0.88	2.41	1.77	0.94	0.70	0.86
time (sec)	N/A	0.214	0.040	0.522	0.107	0.075	0.224	0.127	0.158	0.378

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	97	88	79	84	268	153	83	128	77
N.S.	1	1.17	1.06	0.95	1.01	3.23	1.84	1.00	1.54	0.93
time (sec)	N/A	0.250	0.038	0.503	0.112	0.076	0.407	0.120	0.154	0.411

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	122	109	107	121	391	196	109	214	111
N.S.	1	1.06	0.95	0.93	1.05	3.40	1.70	0.95	1.86	0.97
time (sec)	N/A	0.277	0.077	0.496	0.118	0.084	0.716	0.118	0.155	0.445

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	153	140	124	207	304	274	155	183	0
N.S.	1	0.87	0.80	0.71	1.18	1.74	1.57	0.89	1.05	0.00
time (sec)	N/A	0.259	0.229	0.557	0.036	0.104	0.425	0.133	0.161	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	131	112	96	153	232	153	113	144	0
N.S.	1	0.98	0.84	0.72	1.14	1.73	1.14	0.84	1.07	0.00
time (sec)	N/A	0.253	0.170	0.546	0.035	0.091	0.382	0.126	0.156	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	106	92	72	100	174	114	81	105	0
N.S.	1	1.08	0.94	0.73	1.02	1.78	1.16	0.83	1.07	0.00
time (sec)	N/A	0.225	0.292	0.538	0.038	0.090	0.317	0.131	0.160	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	96	87	83	97	249	134	82	176	0
N.S.	1	1.07	0.97	0.92	1.08	2.77	1.49	0.91	1.96	0.00
time (sec)	N/A	0.229	0.161	0.572	0.029	0.089	3.780	0.133	0.153	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	108	95	97	135	289	450	93	214	0
N.S.	1	1.07	0.94	0.96	1.34	2.86	4.46	0.92	2.12	0.00
time (sec)	N/A	0.257	0.173	0.596	0.038	0.092	6.185	0.130	0.150	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	99	65	58	173	93	639	81	198	133
N.S.	1	0.86	0.57	0.50	1.50	0.81	5.56	0.70	1.72	1.16
time (sec)	N/A	0.296	0.153	0.515	0.037	0.084	14.884	0.140	0.151	0.552

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	129	93	83	227	136	1989	123	275	154
N.S.	1	0.83	0.60	0.54	1.46	0.88	12.83	0.79	1.77	0.99
time (sec)	N/A	0.321	0.217	0.522	0.041	0.100	38.413	0.136	0.154	0.554

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	353	351	98	0	0	0	92	0	126	0
N.S.	1	0.99	0.28	0.00	0.00	0.00	0.26	0.00	0.36	0.00
time (sec)	N/A	0.362	5.628	0.000	0.000	0.000	1.286	0.000	0.239	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	317	326	98	0	0	0	87	0	48	0
N.S.	1	1.03	0.31	0.00	0.00	0.00	0.27	0.00	0.15	0.00
time (sec)	N/A	0.342	10.072	0.000	0.000	0.000	1.170	0.000	0.186	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	A	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	676	677	98	0	0	0	92	0	126	0
N.S.	1	1.00	0.14	0.00	0.00	0.00	0.14	0.00	0.19	0.00
time (sec)	N/A	0.563	6.691	0.000	0.000	0.000	1.311	0.000	0.256	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	A	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	638	652	98	0	0	0	87	0	48	0
N.S.	1	1.02	0.15	0.00	0.00	0.00	0.14	0.00	0.08	0.00
time (sec)	N/A	0.565	10.072	0.000	0.000	0.000	1.154	0.000	0.214	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	165	156	101	0	0	0	82	0	990	0
N.S.	1	0.95	0.61	0.00	0.00	0.00	0.50	0.00	6.00	0.00
time (sec)	N/A	0.300	0.097	0.000	0.000	0.000	9.113	0.000	0.153	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	136	133	132	132	148	149	122	149
N.S.	1	1.00	1.00	0.98	0.97	0.97	1.09	1.10	0.90	1.10
time (sec)	N/A	0.365	0.025	0.505	0.035	0.063	0.028	0.120	0.152	0.728

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	96	91	90	90	104	102	87	106
N.S.	1	1.00	1.00	0.95	0.94	0.94	1.08	1.06	0.91	1.10
time (sec)	N/A	0.297	0.013	0.484	0.035	0.061	0.024	0.121	0.149	0.604

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	56	49	48	48	53	54	52	54
N.S.	1	1.00	1.00	0.88	0.86	0.86	0.95	0.96	0.93	0.96
time (sec)	N/A	0.230	0.006	0.089	0.025	0.064	0.019	0.119	0.152	0.641

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	98	94	95	236	160	104	99	0
N.S.	1	1.00	0.98	0.94	0.95	2.36	1.60	1.04	0.99	0.00
time (sec)	N/A	0.262	0.069	0.477	0.111	0.077	0.338	0.123	0.150	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	126	122	112	118	364	201	123	211	0
N.S.	1	1.07	1.03	0.95	1.00	3.08	1.70	1.04	1.79	0.00
time (sec)	N/A	0.357	0.069	0.530	0.110	0.077	0.739	0.122	0.157	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	160	141	137	155	511	243	147	327	0
N.S.	1	1.09	0.96	0.93	1.05	3.48	1.65	1.00	2.22	0.00
time (sec)	N/A	0.388	0.083	0.476	0.112	0.083	3.155	0.123	0.149	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	209	185	170	319	413	396	216	304	0
N.S.	1	0.87	0.77	0.71	1.32	1.71	1.64	0.90	1.26	0.00
time (sec)	N/A	0.359	0.341	0.593	0.038	0.143	0.523	0.145	0.171	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	187	153	135	246	327	216	167	245	0
N.S.	1	0.98	0.80	0.71	1.29	1.71	1.13	0.87	1.28	0.00
time (sec)	N/A	0.329	0.246	0.582	0.035	0.112	0.454	0.138	0.175	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	162	129	107	174	249	155	126	186	0
N.S.	1	1.11	0.88	0.73	1.19	1.71	1.06	0.86	1.27	0.00
time (sec)	N/A	0.326	0.382	0.559	0.033	0.096	0.408	0.132	0.155	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	149	120	114	171	352	238	134	313	0
N.S.	1	1.09	0.88	0.83	1.25	2.57	1.74	0.98	2.28	0.00
time (sec)	N/A	0.358	0.273	0.610	0.046	0.098	7.611	0.136	0.162	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	154	128	128	227	415	821	133	398	0
N.S.	1	1.03	0.86	0.86	1.52	2.79	5.51	0.89	2.67	0.00
time (sec)	N/A	0.368	0.273	0.647	0.037	0.103	10.275	0.141	0.159	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	180	132	134	317	419	1690	145	429	0
N.S.	1	1.08	0.79	0.80	1.90	2.51	10.12	0.87	2.57	0.00
time (sec)	N/A	0.403	0.526	0.747	0.046	0.119	21.648	0.137	0.164	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	183	98	89	335	141	2088	131	360	0
N.S.	1	0.96	0.52	0.47	1.76	0.74	10.99	0.69	1.89	0.00
time (sec)	N/A	0.518	0.432	0.529	0.036	0.117	53.227	0.141	0.164	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	212	131	120	408	192	5440	181	471	0
N.S.	1	0.89	0.55	0.50	1.71	0.81	22.86	0.76	1.98	0.00
time (sec)	N/A	0.519	0.493	0.528	0.037	0.159	120.466	0.137	0.194	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	268	237	132	0	0	0	124	0	219	0
N.S.	1	0.88	0.49	0.00	0.00	0.00	0.46	0.00	0.82	0.00
time (sec)	N/A	0.382	8.668	0.000	0.000	0.000	2.085	0.000	0.245	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	220	216	133	0	0	0	124	0	219	0
N.S.	1	0.98	0.60	0.00	0.00	0.00	0.56	0.00	1.00	0.00
time (sec)	N/A	0.361	7.482	0.000	0.000	0.000	1.710	0.000	0.240	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	221	214	132	0	0	0	117	0	65	0
N.S.	1	0.97	0.60	0.00	0.00	0.00	0.53	0.00	0.29	0.00
time (sec)	N/A	0.357	10.106	0.000	0.000	0.000	1.554	0.000	0.194	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	176	192	132	0	0	0	117	0	65	0
N.S.	1	1.09	0.75	0.00	0.00	0.00	0.66	0.00	0.37	0.00
time (sec)	N/A	0.345	10.115	0.000	0.000	0.000	1.537	0.000	0.199	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	173	216	135	0	0	0	117	0	141	0
N.S.	1	1.25	0.78	0.00	0.00	0.00	0.68	0.00	0.82	0.00
time (sec)	N/A	0.456	10.110	0.000	0.000	0.000	3.317	0.000	0.201	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	179	193	137	0	0	0	117	0	141	0
N.S.	1	1.08	0.77	0.00	0.00	0.00	0.65	0.00	0.79	0.00
time (sec)	N/A	0.432	10.109	0.000	0.000	0.000	3.733	0.000	0.210	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	177	230	161	0	0	0	117	0	221	0
N.S.	1	1.30	0.91	0.00	0.00	0.00	0.66	0.00	1.25	0.00
time (sec)	N/A	0.483	10.136	0.000	0.000	0.000	7.729	0.000	0.201	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	198	212	161	0	0	0	117	0	221	0
N.S.	1	1.07	0.81	0.00	0.00	0.00	0.59	0.00	1.12	0.00
time (sec)	N/A	0.453	10.127	0.000	0.000	0.000	12.953	0.000	0.205	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	196	264	193	0	0	0	117	0	301	0
N.S.	1	1.35	0.98	0.00	0.00	0.00	0.60	0.00	1.54	0.00
time (sec)	N/A	0.514	10.163	0.000	0.000	0.000	19.540	0.000	0.215	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	227	242	193	0	0	0	117	0	301	0
N.S.	1	1.07	0.85	0.00	0.00	0.00	0.52	0.00	1.33	0.00
time (sec)	N/A	0.494	10.190	0.000	0.000	0.000	37.915	0.000	0.217	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	262	251	126	0	0	0	110	0	0	0
N.S.	1	0.96	0.48	0.00	0.00	0.00	0.42	0.00	0.00	0.00
time (sec)	N/A	0.444	0.274	0.000	0.000	0.000	18.438	0.000	0.178	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [76] had the largest ratio of [.470588000000000006]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	3	3	0.98	15	0.200
2	A	3	3	1.00	15	0.200
3	A	2	2	1.03	13	0.154
4	A	1	1	1.00	5	0.200
5	A	3	3	1.00	15	0.200
6	A	2	2	1.00	15	0.133
7	A	3	3	1.09	15	0.200
8	A	6	5	1.03	17	0.294
9	A	5	4	1.01	17	0.235
10	A	4	3	1.00	17	0.176
11	A	1	1	1.00	17	0.059
12	A	2	2	1.00	17	0.118
13	A	3	3	1.10	17	0.176
14	A	4	4	1.15	17	0.235
15	A	6	5	1.01	17	0.294
16	A	5	4	1.00	17	0.235
17	A	4	3	1.00	17	0.176
18	A	4	3	1.00	17	0.176
19	A	7	6	1.07	17	0.353
20	A	6	5	1.08	17	0.294
21	A	6	5	1.08	17	0.294

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	3	3	1.00	15	0.200
23	A	3	3	1.00	20	0.150
24	A	3	3	1.00	20	0.150
25	A	2	2	1.00	18	0.111
26	A	1	1	1.00	10	0.100
27	A	2	2	1.00	20	0.100
28	A	4	4	1.00	20	0.200
29	A	5	5	1.00	20	0.250
30	A	6	6	0.98	20	0.300
31	A	7	6	0.90	22	0.273
32	A	6	5	0.95	22	0.227
33	A	5	4	1.04	22	0.182
34	A	6	5	1.00	22	0.227
35	A	4	4	1.00	22	0.182
36	A	5	5	0.97	22	0.227
37	A	6	6	0.95	22	0.273
38	A	7	6	0.98	22	0.273
39	A	6	5	1.01	22	0.227
40	A	5	4	1.00	22	0.182
41	A	9	8	1.03	22	0.364
42	A	8	7	1.04	22	0.318
43	A	7	6	1.05	22	0.273
44	A	8	7	1.03	22	0.318
45	A	5	5	1.11	20	0.250
46	A	3	3	1.00	25	0.120
47	A	3	3	1.00	25	0.120
48	A	2	2	1.00	23	0.087
49	A	1	1	1.00	15	0.067
50	A	2	2	1.00	25	0.080
51	A	6	6	1.06	25	0.240
52	A	5	5	1.09	25	0.200
53	A	6	5	1.10	17	0.294

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	9	8	0.98	27	0.296
55	A	8	7	1.04	27	0.259
56	A	7	6	1.13	27	0.222
57	A	7	6	1.06	27	0.222
58	A	4	4	1.00	27	0.148
59	A	5	5	1.07	27	0.185
60	A	6	6	1.03	27	0.222
61	A	6	5	1.26	17	0.294
62	A	9	8	1.01	27	0.296
63	A	8	7	1.04	27	0.259
64	A	6	5	1.02	27	0.185
65	A	6	5	1.03	27	0.185
66	A	7	6	1.02	27	0.222
67	A	11	10	1.04	27	0.370
68	A	10	9	1.06	27	0.333
69	A	8	7	1.06	27	0.259
70	A	8	7	1.05	27	0.259
71	A	9	8	1.04	27	0.296
72	A	8	8	1.22	25	0.320
73	A	5	4	0.94	29	0.138
74	A	4	4	1.32	21	0.190
75	A	3	3	1.00	15	0.200
76	A	8	8	1.15	17	0.471
77	A	8	8	1.15	32	0.250
78	A	9	9	1.15	35	0.257
79	A	2	2	1.00	17	0.118
80	A	2	2	1.00	17	0.118
81	A	2	2	1.00	17	0.118
82	A	2	2	1.00	15	0.133
83	A	1	1	1.00	7	0.143
84	A	2	2	1.00	17	0.118
85	A	2	2	1.00	17	0.118

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	3	3	1.00	17	0.176
87	A	4	4	0.98	17	0.235
88	A	7	6	0.85	19	0.316
89	A	6	5	0.89	19	0.263
90	A	5	4	0.95	19	0.211
91	A	4	3	1.00	19	0.158
92	A	4	3	1.00	19	0.158
93	A	2	2	0.77	19	0.105
94	A	3	3	0.97	19	0.158
95	A	4	4	0.95	19	0.211
96	A	1	1	1.00	22	0.045
97	B	2	2	3.82	18	0.111
98	B	2	2	2.82	18	0.111
99	A	2	2	1.82	16	0.125
100	A	1	1	1.00	18	0.056
101	A	1	1	1.00	18	0.056
102	A	1	1	1.00	20	0.050
103	A	1	1	1.00	20	0.050
104	A	1	1	1.00	20	0.050
105	A	1	1	1.00	20	0.050
106	B	2	2	3.62	20	0.100
107	A	1	1	1.00	20	0.050
108	A	3	3	1.00	17	0.176
109	A	2	2	1.00	22	0.091
110	A	2	2	1.00	22	0.091
111	A	2	2	1.00	20	0.100
112	A	2	2	1.00	22	0.091
113	A	5	5	1.17	22	0.227
114	A	5	5	1.06	22	0.227
115	A	7	6	0.87	24	0.250
116	A	7	6	0.98	24	0.250
117	A	5	4	1.08	24	0.167

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
118	A	7	6	1.07	24	0.250
119	A	7	6	1.07	24	0.250
120	A	4	4	0.86	24	0.167
121	A	5	5	0.83	24	0.208
122	A	7	6	0.99	24	0.250
123	A	6	5	1.03	24	0.208
124	A	9	8	1.00	24	0.333
125	A	8	7	1.02	24	0.292
126	A	4	4	0.95	22	0.182
127	A	2	2	1.00	27	0.074
128	A	2	2	1.00	27	0.074
129	A	2	2	1.00	25	0.080
130	A	2	2	1.00	27	0.074
131	A	4	4	1.07	27	0.148
132	A	7	7	1.09	27	0.259
133	A	9	8	0.87	29	0.276
134	A	8	7	0.98	29	0.241
135	A	7	6	1.11	29	0.207
136	A	8	7	1.09	29	0.241
137	A	8	7	1.03	29	0.241
138	A	9	8	1.08	29	0.276
139	A	8	8	0.96	29	0.276
140	A	8	8	0.89	29	0.276
141	A	9	9	0.88	29	0.310
142	A	8	8	0.98	29	0.276
143	A	8	8	0.97	29	0.276
144	A	7	7	1.09	29	0.241
145	A	8	8	1.25	29	0.276
146	A	7	7	1.08	29	0.241
147	A	8	8	1.30	29	0.276
148	A	7	7	1.07	29	0.241
149	A	8	8	1.35	29	0.276

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
150	A	7	7	1.07	29	0.241
151	A	5	5	0.96	27	0.185

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int (A+Bx)\left(a+bx^2\right)^3\,dx$

Optimal result

Integrand size = 15, antiderivative size = 56

$$\int (A+Bx)\left(a+bx^2\right)^3\,dx=a^3Ax+a^2Abx^3+\frac{3}{5}aAb^2x^5+\frac{1}{7}Ab^3x^7+\frac{B(a+bx^2)^4}{8b}$$

output
```
a^3*A*x+a^2*A*b*x^3+3/5*a*A*b^2*x^5+1/7*A*b^3*x^7+1/8*B*(b*x^2+a)^4/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.52

$$\int (A+Bx)\left(a+bx^2\right)^3\,dx=a^3Ax+\frac{1}{2}a^3Bx^2+a^2Abx^3+\frac{3}{4}a^2bBx^4+\frac{3}{5}aAb^2x^5+\frac{1}{2}ab^2Bx^6+\frac{1}{7}Ab^3x^7+\frac{1}{8}b^3Bx^8$$

input
```
Integrate[(A + B*x)*(a + b*x^2)^3,x]
```

output
```
a^3*A*x + (a^3*B*x^2)/2 + a^2*A*b*x^3 + (3*a^2*b*B*x^4)/4 + (3*a*A*b^2*x^5
)/5 + (a*b^2*B*x^6)/2 + (A*b^3*x^7)/7 + (b^3*B*x^8)/8
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {455, 210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^2)^3 (A+Bx)\,dx$$

$$\Big\downarrow \text{ 455}$$

$$A\int (bx^2+a)^3\,dx+\frac{B(a+bx^2)^4}{8b}$$

$$\Big\downarrow \text{ 210}$$

$$A\int (b^3x^6+3ab^2x^4+3a^2bx^2+a^3)\,dx+\frac{B(a+bx^2)^4}{8b}$$

$$\Big\downarrow \text{ 2009}$$

$$A\left(a^3x+a^2bx^3+\frac{3}{5}ab^2x^5+\frac{b^3x^7}{7}\right)+\frac{B(a+bx^2)^4}{8b}$$

input
```
Int[(A + B*x)*(a + b*x^2)^3,x]
```

output
```
(B*(a + b*x^2)^4)/(8*b) + A*(a^3*x + a^2*b*x^3 + (3*a*b^2*x^5)/5 + (b^3*x^
7)/7)
```

Defintions of rubi rules used

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.32

method	result	siz
gosper	$\frac{1}{8}b^3B\,x^8+\frac{1}{7}A\,b^3x^7+\frac{1}{2}Ba\,b^2x^6+\frac{3}{5}aA\,b^2x^5+\frac{3}{4}B\,a^2b\,x^4+a^2Ab\,x^3+\frac{1}{2}B\,a^3x^2+a^3Ax$	74
default	$\frac{1}{8}b^3B\,x^8+\frac{1}{7}A\,b^3x^7+\frac{1}{2}Ba\,b^2x^6+\frac{3}{5}aA\,b^2x^5+\frac{3}{4}B\,a^2b\,x^4+a^2Ab\,x^3+\frac{1}{2}B\,a^3x^2+a^3Ax$	74
norman	$\frac{1}{8}b^3B\,x^8+\frac{1}{7}A\,b^3x^7+\frac{1}{2}Ba\,b^2x^6+\frac{3}{5}aA\,b^2x^5+\frac{3}{4}B\,a^2b\,x^4+a^2Ab\,x^3+\frac{1}{2}B\,a^3x^2+a^3Ax$	74
risch	$\frac{1}{8}b^3B\,x^8+\frac{1}{7}A\,b^3x^7+\frac{1}{2}Ba\,b^2x^6+\frac{3}{5}aA\,b^2x^5+\frac{3}{4}B\,a^2b\,x^4+a^2Ab\,x^3+\frac{1}{2}B\,a^3x^2+a^3Ax$	74
parallelrisch	$\frac{1}{8}b^3B\,x^8+\frac{1}{7}A\,b^3x^7+\frac{1}{2}Ba\,b^2x^6+\frac{3}{5}aA\,b^2x^5+\frac{3}{4}B\,a^2b\,x^4+a^2Ab\,x^3+\frac{1}{2}B\,a^3x^2+a^3Ax$	74
orering	$\frac{x\left(35b^3B\,x^7+40A\,b^3x^6+140Ba\,b^2x^5+168aA\,b^2x^4+210B\,a^2b\,x^3+280a^2Ab\,x^2+140B\,a^3x+280a^3A\right)}{280}$	76

input
```
int((B*x+A)*(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/8*b^3*B*x^8+1/7*A*b^3*x^7+1/2*B*a*b^2*x^6+3/5*a*A*b^2*x^5+3/4*B*a^2*b*x^
4+a^2*A*b*x^3+1/2*B*a^3*x^2+a^3*A*x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.30

$$\int (A+Bx)\left(a+bx^2\right)^3\,dx=\frac{1}{8}\,Bb^3x^8+\frac{1}{7}\,Ab^3x^7+\frac{1}{2}\,Bab^2x^6+\frac{3}{5}\,Aab^2x^5+\frac{3}{4}\,Ba^2bx^4+Aa^2bx^3+\frac{1}{2}\,Ba^3x^2+Aa^3x$$

input
```
integrate((B*x+A)*(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
1/8*B*b^3*x^8 + 1/7*A*b^3*x^7 + 1/2*B*a*b^2*x^6 + 3/5*A*a*b^2*x^5 + 3/4*B*
a^2*b*x^4 + A*a^2*b*x^3 + 1/2*B*a^3*x^2 + A*a^3*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.52

$$\int (A+Bx)\left(a+bx^2\right)^3\,dx = Aa^3x + Aa^2bx^3 + \frac{3Aab^2x^5}{5} + \frac{Ab^3x^7}{7} + \frac{Ba^3x^2}{2} + \frac{3Ba^2bx^4}{4} + \frac{Bab^2x^6}{2} + \frac{Bb^3x^8}{8}$$

input
```
integrate((B*x+A)*(b*x**2+a)**3,x)
```

output
```
A*a**3*x + A*a**2*b*x**3 + 3*A*a*b**2*x**5/5 + A*b**3*x**7/7 + B*a**3*x**2
/2 + 3*B*a**2*b*x**4/4 + B*a*b**2*x**6/2 + B*b**3*x**8/8
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.30

$$\int (A+Bx)\left(a+bx^2\right)^3\,dx = \frac{1}{8}Bb^3x^8 + \frac{1}{7}Ab^3x^7 + \frac{1}{2}Bab^2x^6 + \frac{3}{5}Aab^2x^5 + \frac{3}{4}Ba^2bx^4 + Aa^2bx^3 + \frac{1}{2}Ba^3x^2 + Aa^3x$$

input
```
integrate((B*x+A)*(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/8*B*b^3*x^8 + 1/7*A*b^3*x^7 + 1/2*B*a*b^2*x^6 + 3/5*A*a*b^2*x^5 + 3/4*B*
a^2*b*x^4 + A*a^2*b*x^3 + 1/2*B*a^3*x^2 + A*a^3*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.30

$$\int (A+Bx)\left(a+bx^2\right)^3\,dx=\frac{1}{8}Bb^3x^8+\frac{1}{7}Ab^3x^7+\frac{1}{2}Bab^2x^6+\frac{3}{5}Aab^2x^5+\frac{3}{4}Ba^2bx^4+Aa^2bx^3+\frac{1}{2}Ba^3x^2+Aa^3x$$

input
```
integrate((B*x+A)*(b*x^2+a)^3,x, algorithm="giac")
```

output
```
1/8*B*b^3*x^8 + 1/7*A*b^3*x^7 + 1/2*B*a*b^2*x^6 + 3/5*A*a*b^2*x^5 + 3/4*B*
a^2*b*x^4 + A*a^2*b*x^3 + 1/2*B*a^3*x^2 + A*a^3*x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.30

$$\int (A+Bx)\left(a+bx^2\right)^3\,dx=\frac{B\,a^3\,x^2}{2}+A\,a^3\,x+\frac{3\,B\,a^2\,b\,x^4}{4}+A\,a^2\,b\,x^3+\frac{B\,a\,b^2\,x^6}{2}+\frac{3\,A\,a\,b^2\,x^5}{5}+\frac{B\,b^3\,x^8}{8}+\frac{A\,b^3\,x^7}{7}$$

input
```
int((a + b*x^2)^3*(A + B*x),x)
```

output
```
(B*a^3*x^2)/2 + (A*b^3*x^7)/7 + (B*b^3*x^8)/8 + A*a^3*x + A*a^2*b*x^3 + (3
*A*a*b^2*x^5)/5 + (3*B*a^2*b*x^4)/4 + (B*a*b^2*x^6)/2
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.30

$$\int (A+Bx)\left(a+bx^2\right)^3\,dx = \frac{x(35b^4x^7+40a\,b^3x^6+140a\,b^3x^5+168a^2b^2x^4+210a^2b^2x^3+280a^3b\,x^2+140a^3bx+280a^4)}{280}$$

input
```
int((B*x+A)*(b*x^2+a)^3,x)
```

output
```
(x*(280*a**4 + 280*a**3*b*x**2 + 140*a**3*b*x + 168*a**2*b**2*x**4 + 210*a
**2*b**2*x**3 + 40*a*b**3*x**6 + 140*a*b**3*x**5 + 35*b**4*x**7))/280
```

3.2 $\int (A+Bx)\left(a+bx^2\right)^2\,dx$

Optimal result

Integrand size = 15, antiderivative size = 45

$$\int (A+Bx)\left(a+bx^2\right)^2\,dx=a^2Ax+\frac{2}{3}aAbx^3+\frac{1}{5}Ab^2x^5+\frac{B\left(a+bx^2\right)^3}{6b}$$

output
```
a^2*A*x+2/3*a*A*b*x^3+1/5*A*b^2*x^5+1/6*B*(b*x^2+a)^3/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.33

$$\int (A+Bx)\left(a+bx^2\right)^2\,dx=a^2Ax+\frac{1}{2}a^2Bx^2+\frac{2}{3}aAbx^3+\frac{1}{2}abBx^4+\frac{1}{5}Ab^2x^5+\frac{1}{6}b^2Bx^6$$

input
```
Integrate[(A + B*x)*(a + b*x^2)^2,x]
```

output
```
a^2*A*x + (a^2*B*x^2)/2 + (2*a*A*b*x^3)/3 + (a*b*B*x^4)/2 + (A*b^2*x^5)/5
+ (b^2*B*x^6)/6
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {455, 210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^2)^2 (A+Bx)\,dx$$

$$\Big\downarrow \ 455$$

$$A\int (bx^2+a)^2\,dx+\frac{B(a+bx^2)^3}{6b}$$

$$\Big\downarrow \ 210$$

$$A\int (b^2x^4+2abx^2+a^2)\,dx+\frac{B(a+bx^2)^3}{6b}$$

$$\Big\downarrow \ 2009$$

$$A\left(a^2x+\frac{2}{3}abx^3+\frac{b^2x^5}{5}\right)+\frac{B(a+bx^2)^3}{6b}$$

input
```
Int[(A + B*x)*(a + b*x^2)^2,x]
```

output
```
(B*(a + b*x^2)^3)/(6*b) + A*(a^2*x + (2*a*b*x^3)/3 + (b^2*x^5)/5)
```

Defintions of rubi rules used

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] && !LeQ[p, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.13

method	result	size
gosper	$\frac{1}{6}b^2 B\,x^6+\frac{1}{5}A\,b^2x^5+\frac{1}{2}Bab\,x^4+\frac{2}{3}aAb\,x^3+\frac{1}{2}B\,a^2x^2+a^2Ax$	51
default	$\frac{1}{6}b^2 B\,x^6+\frac{1}{5}A\,b^2x^5+\frac{1}{2}Bab\,x^4+\frac{2}{3}aAb\,x^3+\frac{1}{2}B\,a^2x^2+a^2Ax$	51
norman	$\frac{1}{6}b^2 B\,x^6+\frac{1}{5}A\,b^2x^5+\frac{1}{2}Bab\,x^4+\frac{2}{3}aAb\,x^3+\frac{1}{2}B\,a^2x^2+a^2Ax$	51
risch	$\frac{1}{6}b^2 B\,x^6+\frac{1}{5}A\,b^2x^5+\frac{1}{2}Bab\,x^4+\frac{2}{3}aAb\,x^3+\frac{1}{2}B\,a^2x^2+a^2Ax$	51
parallelrisch	$\frac{1}{6}b^2 B\,x^6+\frac{1}{5}A\,b^2x^5+\frac{1}{2}Bab\,x^4+\frac{2}{3}aAb\,x^3+\frac{1}{2}B\,a^2x^2+a^2Ax$	51
orering	$\frac{x(5b^2B\,x^5+6A\,b^2x^4+15Bab\,x^3+20aAb\,x^2+15B\,a^2x+30a^2A)}{30}$	52

input
```
int((B*x+A)*(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/6*b^2*B*x^6+1/5*A*b^2*x^5+1/2*B*a*b*x^4+2/3*a*A*b*x^3+1/2*B*a^2*x^2+a^2*
A*x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.11

$$\int (A+Bx)\left(a+bx^2\right)^2\,dx=\frac{1}{6}\,Bb^2x^6+\frac{1}{5}\,Ab^2x^5+\frac{1}{2}\,Babx^4+\frac{2}{3}\,Aabx^3+\frac{1}{2}\,Ba^2x^2+Aa^2x$$

input
```
integrate((B*x+A)*(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
1/6*B*b^2*x^6 + 1/5*A*b^2*x^5 + 1/2*B*a*b*x^4 + 2/3*A*a*b*x^3 + 1/2*B*a^2*
x^2 + A*a^2*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.29

$$\int (A+Bx)\left(a+bx^2\right)^2\,dx = Aa^2x + \frac{2Aabx^3}{3} + \frac{Ab^2x^5}{5} + \frac{Ba^2x^2}{2} + \frac{Babx^4}{2} + \frac{Bb^2x^6}{6}$$

input
```
integrate((B*x+A)*(b*x**2+a)**2,x)
```

output
```
A*a**2*x + 2*A*a*b*x**3/3 + A*b**2*x**5/5 + B*a**2*x**2/2 + B*a*b*x**4/2 +
 B*b**2*x**6/6
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.11

$$\int (A+Bx)\left(a+bx^2\right)^2\,dx = \frac{1}{6}\,Bb^2x^6 + \frac{1}{5}\,Ab^2x^5 + \frac{1}{2}\,Babx^4 + \frac{2}{3}\,Aabx^3 + \frac{1}{2}\,Ba^2x^2 + Aa^2x$$

input
```
integrate((B*x+A)*(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/6*B*b^2*x^6 + 1/5*A*b^2*x^5 + 1/2*B*a*b*x^4 + 2/3*A*a*b*x^3 + 1/2*B*a^2*
x^2 + A*a^2*x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.11

$$\int (A+Bx)\left(a+bx^2\right)^2\,dx = \frac{1}{6}\,Bb^2x^6 + \frac{1}{5}\,Ab^2x^5 + \frac{1}{2}\,Babx^4 + \frac{2}{3}\,Aabx^3 + \frac{1}{2}\,Ba^2x^2 + Aa^2x$$

input
```
integrate((B*x+A)*(b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/6*B*b^2*x^6 + 1/5*A*b^2*x^5 + 1/2*B*a*b*x^4 + 2/3*A*a*b*x^3 + 1/2*B*a^2*
x^2 + A*a^2*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.11

$$\int (A+Bx)\left(a+bx^2\right)^2\,dx=\frac{B\,a^2\,x^2}{2}+A\,a^2\,x+\frac{B\,a\,b\,x^4}{2}+\frac{2\,A\,a\,b\,x^3}{3}+\frac{B\,b^2\,x^6}{6}+\frac{A\,b^2\,x^5}{5}$$

input
```
int((a + b*x^2)^2*(A + B*x),x)
```

output
```
(B*a^2*x^2)/2 + (A*b^2*x^5)/5 + (B*b^2*x^6)/6 + A*a^2*x + (2*A*a*b*x^3)/3
+ (B*a*b*x^4)/2
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.13

$$\int (A+Bx)\left(a+bx^2\right)^2\,dx=\frac{x(5b^3x^5+6a\,b^2x^4+15a\,b^2x^3+20a^2b\,x^2+15a^2bx+30a^3)}{30}$$

input
```
int((B*x+A)*(b*x^2+a)^2,x)
```

output
```
(x*(30*a**3 + 20*a**2*b*x**2 + 15*a**2*b*x + 6*a*b**2*x**4 + 15*a*b**2*x**
3 + 5*b**3*x**5))/30
```

3.3 $\int (A+Bx)\left(a+bx^2\right)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 31

$$\int (A+Bx)\left(a+bx^2\right)\,dx=aAx+\frac{1}{3}Abx^3+\frac{B(a+bx^2)^2}{4b}$$

output
```
a*A*x+1/3*A*b*x^3+1/4*B*(b*x^2+a)^2/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int (A+Bx)\left(a+bx^2\right)\,dx=aAx+\frac{1}{2}aBx^2+\frac{1}{3}Abx^3+\frac{1}{4}bBx^4$$

input
```
Integrate[(A + B*x)*(a + b*x^2),x]
```

output
```
a*A*x + (a*B*x^2)/2 + (A*b*x^3)/3 + (b*B*x^4)/4
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {455, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^2)(A+Bx)\,dx$$

$$\Big\downarrow \text{455}$$

$$A\int (bx^2+a)\,dx+\frac{B(a+bx^2)^2}{4b}$$

$$\Big\downarrow \text{2009}$$

$$A\left(ax+\frac{bx^3}{3}\right)+\frac{B(a+bx^2)^2}{4b}$$

input
```
Int[(A + B*x)*(a + b*x^2),x]
```

output
```
(B*(a + b*x^2)^2)/(4*b) + A*(a*x + (b*x^3)/3)
```

Defintions of rubi rules used

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

method	result	size
gosper	$\frac{1}{4}bB\,x^4+\frac{1}{3}Ab\,x^3+\frac{1}{2}Ba\,x^2+aAx$	27
default	$\frac{1}{4}bB\,x^4+\frac{1}{3}Ab\,x^3+\frac{1}{2}Ba\,x^2+aAx$	27
norman	$\frac{1}{4}bB\,x^4+\frac{1}{3}Ab\,x^3+\frac{1}{2}Ba\,x^2+aAx$	27
risch	$\frac{1}{4}bB\,x^4+\frac{1}{3}Ab\,x^3+\frac{1}{2}Ba\,x^2+aAx$	27
parallelrisch	$\frac{1}{4}bB\,x^4+\frac{1}{3}Ab\,x^3+\frac{1}{2}Ba\,x^2+aAx$	27
orering	$\frac{x(3bB\,x^3+4Ab\,x^2+6Bax+12Aa)}{12}$	28

input
```
int((B*x+A)*(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/4*b*B*x^4+1/3*A*b*x^3+1/2*B*a*x^2+a*A*x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int (A+Bx)\left(a+bx^2\right)\,dx=\frac{1}{4}\,Bbx^4+\frac{1}{3}\,Abx^3+\frac{1}{2}\,Bax^2+Aax$$

input
```
integrate((B*x+A)*(b*x^2+a),x, algorithm="fricas")
```

output
```
1/4*B*b*x^4 + 1/3*A*b*x^3 + 1/2*B*a*x^2 + A*a*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int (A+Bx)\left(a+bx^2\right)\,dx=Aax+\frac{Abx^3}{3}+\frac{Bax^2}{2}+\frac{Bbx^4}{4}$$

input
```
integrate((B*x+A)*(b*x**2+a),x)
```

output
```
A*a*x + A*b*x**3/3 + B*a*x**2/2 + B*b*x**4/4
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int (A+Bx)\left(a+bx^2\right)\,dx=\frac{1}{4}\,Bbx^4+\frac{1}{3}\,Abx^3+\frac{1}{2}\,Bax^2+Aax$$

input
```
integrate((B*x+A)*(b*x^2+a),x, algorithm="maxima")
```

output
```
1/4*B*b*x^4 + 1/3*A*b*x^3 + 1/2*B*a*x^2 + A*a*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int (A+Bx)\left(a+bx^2\right)\,dx=\frac{1}{4}\,Bbx^4+\frac{1}{3}\,Abx^3+\frac{1}{2}\,Bax^2+Aax$$

input
```
integrate((B*x+A)*(b*x^2+a),x, algorithm="giac")
```

output
```
1/4*B*b*x^4 + 1/3*A*b*x^3 + 1/2*B*a*x^2 + A*a*x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int (A+Bx)\left(a+bx^2\right)\,dx=\frac{B\,b\,x^4}{4}+\frac{A\,b\,x^3}{3}+\frac{B\,a\,x^2}{2}+A\,a\,x$$

input
```
int((a + b*x^2)*(A + B*x),x)
```

output
```
A*a*x + (B*a*x^2)/2 + (A*b*x^3)/3 + (B*b*x^4)/4
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int (A+Bx)\left(a+bx^2\right)\,dx=\frac{x(3b^2x^3+4ab\,x^2+6abx+12a^2)}{12}$$

input
```
int((B*x+A)*(b*x^2+a),x)
```

output
```
(x*(12*a**2 + 4*a*b*x**2 + 6*a*b*x + 3*b**2*x**3))/12
```

3.4 $\int (A + Bx) \, dx$

Optimal result

Integrand size = 5, antiderivative size = 14

$$\int (A + Bx) \, dx = \frac{(A + Bx)^2}{2B}$$

output
```
1/2*(B*x+A)^2/B
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int (A + Bx) \, dx = Ax + \frac{Bx^2}{2}$$

input
```
Integrate[A + B*x,x]
```

output
```
A*x + (B*x^2)/2
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (A + Bx)\,dx$$

$$\Big\downarrow \; 17$$

$$\frac{(A + Bx)^2}{2B}$$

input
```
Int[A + B*x,x]
```

output
```
(A + B*x)^2/(2*B)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

method	result	size
gosper	$\frac{1}{2}x^2B + xA$	11
default	$\frac{1}{2}x^2B + xA$	11
norman	$\frac{1}{2}x^2B + xA$	11
risch	$\frac{1}{2}x^2B + xA$	11
parallelrisch	$\frac{1}{2}x^2B + xA$	11
parts	$\frac{1}{2}x^2B + xA$	11
orering	$\frac{x(Bx+2A)}{2}$	11

input
```
int(B*x+A,x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2*B+x*A
```

Fricas [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int (A + Bx)\,dx = \frac{1}{2}x^2B + xA$$

input
```
integrate(B*x+A,x, algorithm="fricas")
```

output
```
1/2*x^2*B + x*A
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.57

$$\int (A+Bx)\,dx = Ax + \frac{Bx^2}{2}$$

input
```
integrate(B*x+A,x)
```

output
```
A*x + B*x**2/2
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int (A+Bx)\,dx = \frac{1}{2}\,Bx^2 + Ax$$

input
```
integrate(B*x+A,x, algorithm="maxima")
```

output
```
1/2*B*x^2 + A*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int (A+Bx)\,dx = \frac{1}{2}\,Bx^2 + Ax$$

input
```
integrate(B*x+A,x, algorithm="giac")
```

output
```
1/2*B*x^2 + A*x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int (A + Bx)\, dx = \frac{B\, x^2}{2} + A\, x$$

input
```
int(A + B*x,x)
```

output
```
A*x + (B*x^2)/2
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int (A + Bx)\, dx = \frac{x(bx + 2a)}{2}$$

input
```
int(B*x+A,x)
```

output
```
(x*(2*a + b*x))/2
```

3.5 $\int \frac{A+Bx}{a+bx^2} dx$

Optimal result

Integrand size = 15, antiderivative size = 42

$$\int \frac{A+Bx}{a+bx^2} dx = \frac{A \arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}} + \frac{B \log(a+bx^2)}{2b}$$

output
```
A*arctan(b^(1/2)*x/a^(1/2))/a^(1/2)/b^(1/2)+1/2*B*ln(b*x^2+a)/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx}{a+bx^2} dx = \frac{A \arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}} + \frac{B \log(a+bx^2)}{2b}$$

input
```
Integrate[(A + B*x)/(a + b*x^2),x]
```

output
```
(A*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*Sqrt[b]) + (B*Log[a + b*x^2])/(2*
b)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {452, 218, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{a+bx^2}\,dx$$

$$\Big\downarrow \text{452}$$

$$A\int \frac{1}{bx^2+a}dx+B\int \frac{x}{bx^2+a}dx$$

$$\Big\downarrow \text{218}$$

$$B\int \frac{x}{bx^2+a}dx+\frac{A\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

$$\Big\downarrow \text{240}$$

$$\frac{A\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}+\frac{B\log\left(a+bx^2\right)}{2b}$$

input
```
Int[(A + B*x)/(a + b*x^2),x]
```

output
```
(A*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*Sqrt[b]) + (B*Log[a + b*x^2])/(2*
b)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 452
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[c   Int[1/
(a + b*x^2), x], x] + Simp[d   Int[x/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c^2 + a*d^2, 0]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.76

method	result	size
default	$\frac{B\ln\left(b\,x^{2}+a\right)}{2b}+\frac{A\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}$	32
risch	$\frac{\ln\left(-\sqrt{-ab}\,x+a\right)A\sqrt{-ab}}{2ab}+\frac{\ln\left(-\sqrt{-ab}\,x+a\right)B}{2b}-\frac{\ln\left(\sqrt{-ab}\,x+a\right)A\sqrt{-ab}}{2ab}+\frac{\ln\left(\sqrt{-ab}\,x+a\right)B}{2b}$	90

input
```
int((B*x+A)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*B*ln(b*x^2+a)/b+A/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 98, normalized size of antiderivative = 2.33

$$\int \frac{A+Bx}{a+bx^2}\,dx = \left[\frac{Ba\log\left(bx^2+a\right)-\sqrt{-ab}A\log\left(\frac{bx^2-2\sqrt{-ab}x-a}{bx^2+a}\right)}{2\,ab}, \frac{Ba\log\left(bx^2+a\right)+2\sqrt{ab}A\arctan\left(\frac{\sqrt{ab}x}{a}\right)}{2\,ab}\right]$$

input
```
integrate((B*x+A)/(b*x^2+a),x, algorithm="fricas")
```

output
```
[1/2*(B*a*log(b*x^2 + a) - sqrt(-a*b)*A*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(
b*x^2 + a)))/(a*b), 1/2*(B*a*log(b*x^2 + a) + 2*sqrt(a*b)*A*arctan(sqrt(a*
b)*x/a))/(a*b)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(37) = 74$.

Time = 0.12 (sec) , antiderivative size = 124, normalized size of antiderivative = 2.95

$$\int \frac{A+Bx}{a+bx^2}\,dx = \left(-\frac{A\sqrt{-ab^3}}{2ab^2}+\frac{B}{2b}\right)\log\left(x+\frac{-Ba+2ab\left(-\frac{A\sqrt{-ab^3}}{2ab^2}+\frac{B}{2b}\right)}{Ab}\right) + \left(\frac{A\sqrt{-ab^3}}{2ab^2}+\frac{B}{2b}\right)\log\left(x+\frac{-Ba+2ab\left(\frac{A\sqrt{-ab^3}}{2ab^2}+\frac{B}{2b}\right)}{Ab}\right)$$

input
```
integrate((B*x+A)/(b*x**2+a),x)
```

output
```
(-A*sqrt(-a*b**3)/(2*a*b**2) + B/(2*b))*log(x + (-B*a + 2*a*b*(-A*sqrt(-a*
b**3)/(2*a*b**2) + B/(2*b)))/(A*b)) + (A*sqrt(-a*b**3)/(2*a*b**2) + B/(2*b
))*log(x + (-B*a + 2*a*b*(A*sqrt(-a*b**3)/(2*a*b**2) + B/(2*b)))/(A*b))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.74

$$\int \frac{A+Bx}{a+bx^2}\,dx = \frac{A\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}} + \frac{B\log\left(bx^2+a\right)}{2\,b}$$

input
```
integrate((B*x+A)/(b*x^2+a),x, algorithm="maxima")
```

output
```
A*arctan(b*x/sqrt(a*b))/sqrt(a*b) + 1/2*B*log(b*x^2 + a)/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.74

$$\int \frac{A+Bx}{a+bx^2}\,dx = \frac{A\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}} + \frac{B\log\left(bx^2+a\right)}{2\,b}$$

input
```
integrate((B*x+A)/(b*x^2+a),x, algorithm="giac")
```

output
```
A*arctan(b*x/sqrt(a*b))/sqrt(a*b) + 1/2*B*log(b*x^2 + a)/b
```

Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.76

$$\int \frac{A+Bx}{a+bx^2}\,dx = \frac{B\,\ln\left(b\,x^2+a\right)}{2\,b} + \frac{A\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

input
```
int((A + B*x)/(a + b*x^2),x)
```

output
```
(B*log(a + b*x^2))/(2*b) + (A*atan((b^(1/2)*x)/a^(1/2)))/(a^(1/2)*b^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.81

$$\int \frac{A+Bx}{a+bx^2}\,dx = \frac{2\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)+\log(b\,x^2+a)\,b}{2b}$$

input
```
int((B*x+A)/(b*x^2+a),x)
```

output
```
(2*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a))) + log(a + b*x**2)*b)/(2*b
)
```

3.6 $\int \frac{A+Bx}{(a+bx^2)^2} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 57

$$\int \frac{A+Bx}{(a+bx^2)^2} \, dx = \frac{-aB+Abx}{2ab(a+bx^2)} + \frac{A\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}$$

output
```
1/2*(A*b*x-B*a)/a/b/(b*x^2+a)+1/2*A*arctan(b^(1/2)*x/a^(1/2))/a^(3/2)/b^(1
/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx}{(a+bx^2)^2} \, dx = \frac{-aB+Abx}{2ab(a+bx^2)} + \frac{A\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}$$

input
```
Integrate[(A + B*x)/(a + b*x^2)^2,x]
```

output
```
(-(a*B) + A*b*x)/(2*a*b*(a + b*x^2)) + (A*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*
a^(3/2)*Sqrt[b])
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {454, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{(a+bx^2)^2} dx$$

$$\Big\downarrow \text{454}$$

$$\frac{A\int \frac{1}{bx^2+a}dx}{2a} - \frac{aB-Abx}{2ab(a+bx^2)}$$

$$\Big\downarrow \text{218}$$

$$\frac{A\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}} - \frac{aB-Abx}{2ab(a+bx^2)}$$

input
```
Int[(A + B*x)/(a + b*x^2)^2,x]
```

output
```
-1/2*(a*B - A*b*x)/(a*b*(a + b*x^2)) + (A*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*
a^(3/2)*Sqrt[b])
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 454
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*d
 - b*c*x)/(2*a*b*(p + 1)))*(a + b*x^2)^(p + 1), x] + Simp[c*((2*p + 3)/(2*a
*(p + 1)))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d}, x] && L
tQ[p, -1] && NeQ[p, -3/2]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{2Abx-2Ba}{4ab(b\,x^2+a)}+\frac{A\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2a\sqrt{ab}}$	49
risch	$\frac{\frac{Ax}{2a}-\frac{B}{2b}}{b\,x^2+a}-\frac{A\ln\left(bx+\sqrt{-ab}\right)}{4\sqrt{-ab}\,a}+\frac{A\ln\left(-bx+\sqrt{-ab}\right)}{4\sqrt{-ab}\,a}$	73

input
```
int((B*x+A)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*(2*A*b*x-2*B*a)/a/b/(b*x^2+a)+1/2*A/a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/
2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 140, normalized size of antiderivative = 2.46

$$\int \frac{A+Bx}{(a+bx^2)^2}\,dx$$
$$=\left[\frac{2\,Aabx-2\,Ba^2-(Abx^2+Aa)\sqrt{-ab}\log\left(\frac{bx^2-2\,\sqrt{-ab}x-a}{bx^2+a}\right)}{4\,(a^2b^2x^2+a^3b)},\frac{Aabx-Ba^2+(Abx^2+Aa)\sqrt{ab}\arctan\left(\sqrt{\cdots}\right.}{2\,(a^2b^2x^2+a^3b)}\right.$$

input
```
integrate((B*x+A)/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[1/4*(2*A*a*b*x - 2*B*a^2 - (A*b*x^2 + A*a)*sqrt(-a*b)*log((b*x^2 - 2*sqrt
(-a*b)*x - a)/(b*x^2 + a)))/(a^2*b^2*x^2 + a^3*b), 1/2*(A*a*b*x - B*a^2 +
(A*b*x^2 + A*a)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a^2*b^2*x^2 + a^3*b)]
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.58

$$\int \frac{A+Bx}{(a+bx^2)^2}\,dx = A\left(-\frac{\sqrt{-\frac{1}{a^3b}}\log\left(-a^2\sqrt{-\frac{1}{a^3b}}+x\right)}{4}+\frac{\sqrt{-\frac{1}{a^3b}}\log\left(a^2\sqrt{-\frac{1}{a^3b}}+x\right)}{4}\right) + \frac{Abx-Ba}{2a^2b+2ab^2x^2}$$

input
```
integrate((B*x+A)/(b*x**2+a)**2,x)
```

output
```
A*(-sqrt(-1/(a**3*b))*log(-a**2*sqrt(-1/(a**3*b)) + x)/4 + sqrt(-1/(a**3*b
))*log(a**2*sqrt(-1/(a**3*b)) + x)/4) + (A*b*x - B*a)/(2*a**2*b + 2*a*b**2
*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.84

$$\int \frac{A+Bx}{(a+bx^2)^2}\,dx = \frac{A\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}a}+\frac{Abx-Ba}{2\,(ab^2x^2+a^2b)}$$

input
```
integrate((B*x+A)/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*A*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) + 1/2*(A*b*x - B*a)/(a*b^2*x^2 +
 a^2*b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.82

$$\int \frac{A+Bx}{(a+bx^2)^2}\,dx = \frac{A\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}a} + \frac{Abx-Ba}{2\,(bx^2+a)ab}$$

input
```
integrate((B*x+A)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*A*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) + 1/2*(A*b*x - B*a)/((b*x^2 + a)
*a*b)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.77

$$\int \frac{A+Bx}{(a+bx^2)^2}\,dx = \frac{A\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,a^{3/2}\,\sqrt{b}} - \frac{\frac{B}{2\,b}-\frac{A\,x}{2\,a}}{b\,x^2+a}$$

input
```
int((A + B*x)/(a + b*x^2)^2,x)
```

output
```
(A*atan((b^(1/2)*x)/a^(1/2)))/(2*a^(3/2)*b^(1/2)) - (B/(2*b) - (A*x)/(2*a)
)/(a + b*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.19

$$\int \frac{A+Bx}{(a+bx^2)^2}\,dx = \frac{\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a+\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)b\,x^2+abx+b^2x^2}{2ab\,(b\,x^2+a)}$$

input
```
int((B*x+A)/(b*x^2+a)^2,x)
```

output
```
(sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a + sqrt(b)*sqrt(a)*atan((b
*x)/(sqrt(b)*sqrt(a)))*b*x**2 + a*b*x + b**2*x**2)/(2*a*b*(a + b*x**2))
```

3.7 $\int \frac{A+Bx}{(a+bx^2)^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 75

$$\int \frac{A+Bx}{(a+bx^2)^3}\,dx = \frac{-aB+Abx}{4ab\,(a+bx^2)^2} + \frac{3Ax}{8a^2\,(a+bx^2)} + \frac{3A\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}}$$

output
```
1/4*(A*b*x-B*a)/a/b/(b*x^2+a)^2+3/8*A*x/a^2/(b*x^2+a)+3/8*A*arctan(b^(1/2)
*x/a^(1/2))/a^(5/2)/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.95

$$\int \frac{A+Bx}{(a+bx^2)^3}\,dx = \frac{\frac{\sqrt{a}(-2a^2B+5aAbx+3Ab^2x^3)}{(a+bx^2)^2} + 3A\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b}$$

input
```
Integrate[(A + B*x)/(a + b*x^2)^3,x]
```

output
```
((Sqrt[a]*(-2*a^2*B + 5*a*A*b*x + 3*A*b^2*x^3))/(a + b*x^2)^2 + 3*A*Sqrt[b
]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(5/2)*b)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {454, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{(a+bx^2)^3}\,dx$$

$$\Big\downarrow \text{454}$$

$$\frac{3A\int \frac{1}{(bx^2+a)^2}dx}{4a}-\frac{aB-Abx}{4ab(a+bx^2)^2}$$

$$\Big\downarrow \text{215}$$

$$\frac{3A\left(\frac{\int \frac{1}{bx^2+a}dx}{2a}+\frac{x}{2a(a+bx^2)}\right)}{4a}-\frac{aB-Abx}{4ab(a+bx^2)^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{3A\left(\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{x}{2a(a+bx^2)}\right)}{4a}-\frac{aB-Abx}{4ab(a+bx^2)^2}$$

input
```
Int[(A + B*x)/(a + b*x^2)^3,x]
```

output
```
-1/4*(a*B - A*b*x)/(a*b*(a + b*x^2)^2) + (3*A*(x/(2*a*(a + b*x^2)) + ArcTa
n[(Sqrt[b]*x)/Sqrt[a]]/(2*a^(3/2)*Sqrt[b])))/(4*a)
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 454
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*d
 - b*c*x)/(2*a*b*(p + 1)))*(a + b*x^2)^(p + 1), x] + Simp[c*((2*p + 3)/(2*a
*(p + 1)))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d}, x] && L
tQ[p, -1] && NeQ[p, -3/2]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{2Abx-2Ba}{8ab(b\,x^{2}+a)^{2}}+\frac{3A\left(\frac{x}{2a\left(b\,x^{2}+a\right)}+\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2a\sqrt{ab}}\right)}{4a}$	70
risch	$\frac{\frac{3Ab\,x^{3}}{8a^{2}}+\frac{5Ax}{8a}-\frac{B}{4b}}{(b\,x^{2}+a)^{2}}-\frac{3A\ln\left(bx+\sqrt{-ab}\right)}{16\sqrt{-ab}\,a^{2}}+\frac{3A\ln\left(-bx+\sqrt{-ab}\right)}{16\sqrt{-ab}\,a^{2}}$	83

input
```
int((B*x+A)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/8*(2*A*b*x-2*B*a)/a/b/(b*x^2+a)^2+3/4*A/a*(1/2*x/a/(b*x^2+a)+1/2/a/(a*b)
^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 212, normalized size of antiderivative = 2.83

$$\int \frac{A+Bx}{(a+bx^2)^3}\,dx$$
$$= \left[\frac{6\,Aab^2x^3+10\,Aa^2bx-4\,Ba^3-3\,(Ab^2x^4+2\,Aabx^2+Aa^2)\sqrt{-ab}\log\left(\frac{bx^2-2\,\sqrt{-ab}x-a}{bx^2+a}\right)}{16\,(a^3b^3x^4+2\,a^4b^2x^2+a^5b)}, \frac{3\,Aab^2x^3+5\,A\right.$$

input
```
integrate((B*x+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
[1/16*(6*A*a*b^2*x^3 + 10*A*a^2*b*x - 4*B*a^3 - 3*(A*b^2*x^4 + 2*A*a*b*x^2
 + A*a^2)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^3*b
^3*x^4 + 2*a^4*b^2*x^2 + a^5*b), 1/8*(3*A*a*b^2*x^3 + 5*A*a^2*b*x - 2*B*a^
3 + 3*(A*b^2*x^4 + 2*A*a*b*x^2 + A*a^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(
a^3*b^3*x^4 + 2*a^4*b^2*x^2 + a^5*b)]
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.65

$$\int \frac{A+Bx}{(a+bx^2)^3}\,dx = A\left(-\frac{3\sqrt{-\frac{1}{a^5b}}\log\left(-a^3\sqrt{-\frac{1}{a^5b}}+x\right)}{16} + \frac{3\sqrt{-\frac{1}{a^5b}}\log\left(a^3\sqrt{-\frac{1}{a^5b}}+x\right)}{16}\right) + \frac{5Aabx+3Ab^2x^3-2Ba^2}{8a^4b+16a^3b^2x^2+8a^2b^3x^4}$$

input
```
integrate((B*x+A)/(b*x**2+a)**3,x)
```

output
```
A*(-3*sqrt(-1/(a**5*b))*log(-a**3*sqrt(-1/(a**5*b)) + x)/16 + 3*sqrt(-1/(a
**5*b))*log(a**3*sqrt(-1/(a**5*b)) + x)/16) + (5*A*a*b*x + 3*A*b**2*x**3 -
 2*B*a**2)/(8*a**4*b + 16*a**3*b**2*x**2 + 8*a**2*b**3*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.99

$$\int \frac{A+Bx}{(a+bx^2)^3}\,dx = \frac{3\,Ab^2x^3+5\,Aabx-2\,Ba^2}{8\,(a^2b^3x^4+2\,a^3b^2x^2+a^4b)} + \frac{3\,A\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^2}$$

input
```
integrate((B*x+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/8*(3*A*b^2*x^3 + 5*A*a*b*x - 2*B*a^2)/(a^2*b^3*x^4 + 2*a^3*b^2*x^2 + a^4
*b) + 3/8*A*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.80

$$\int \frac{A+Bx}{(a+bx^2)^3}\,dx = \frac{3\,A\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^2} + \frac{3\,Ab^2x^3+5\,Aabx-2\,Ba^2}{8\,(bx^2+a)^2a^2b}$$

input
```
integrate((B*x+A)/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
3/8*A*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) + 1/8*(3*A*b^2*x^3 + 5*A*a*b*x
 - 2*B*a^2)/((b*x^2 + a)^2*a^2*b)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.85

$$\int \frac{A+Bx}{(a+bx^2)^3}\,dx = \frac{\frac{5\,A\,x}{8\,a}-\frac{B}{4\,b}+\frac{3\,A\,b\,x^3}{8\,a^2}}{a^2+2\,a\,b\,x^2+b^2\,x^4} + \frac{3\,A\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,a^{5/2}\,\sqrt{b}}$$

input
```
int((A + B*x)/(a + b*x^2)^3,x)
```

output
```
((5*A*x)/(8*a) - B/(4*b) + (3*A*b*x^3)/(8*a^2))/(a^2 + b^2*x^4 + 2*a*b*x^2
) + (3*A*atan((b^(1/2)*x)/a^(1/2)))/(8*a^(5/2)*b^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.59

$$\int \frac{A+Bx}{(a+bx^2)^3}\,dx$$
$$= \frac{3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^2 + 6\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)ab\,x^2 + 3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)b^2x^4 + 5a^2bx - 2a^2b + 3a\,b}{8a^2b\,(b^2x^4 + 2ab\,x^2 + a^2)}$$

input
```
int((B*x+A)/(b*x^2+a)^3,x)
```

output
```
(3*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2 + 6*sqrt(b)*sqrt(a)*
atan((b*x)/(sqrt(b)*sqrt(a)))*a*b*x**2 + 3*sqrt(b)*sqrt(a)*atan((b*x)/(sqr
t(b)*sqrt(a)))*b**2*x**4 + 5*a**2*b*x - 2*a**2*b + 3*a*b**2*x**3)/(8*a**2*
b*(a**2 + 2*a*b*x**2 + b**2*x**4))
```

3.8 $\int (A+Bx) \left(a+bx^2\right)^{3/2} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 87

$$\int (A+Bx) \left(a+bx^2\right)^{3/2} \, dx = \frac{3}{8}aAx\sqrt{a+bx^2} + \frac{1}{4}Ax\left(a+bx^2\right)^{3/2} + \frac{B\left(a+bx^2\right)^{5/2}}{5b} + \frac{3a^2A\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}}$$

output
```
3/8*a*A*x*(b*x^2+a)^(1/2)+1/4*A*x*(b*x^2+a)^(3/2)+1/5*B*(b*x^2+a)^(5/2)/b+
3/8*a^2*A*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00

$$\int (A+Bx) \left(a+bx^2\right)^{3/2} \, dx = \frac{\sqrt{a+bx^2}\left(8a^2B+2b^2x^3(5A+4Bx)+abx(25A+16Bx)\right)-15a^2A\sqrt{b}\log\left(-\sqrt{b}x+\sqrt{a+
$$

input
```
Integrate[(A + B*x)*(a + b*x^2)^(3/2),x]
```

output
```
(Sqrt[a + b*x^2]*(8*a^2*B + 2*b^2*x^3*(5*A + 4*B*x) + a*b*x*(25*A + 16*B*x
)) - 15*a^2*A*Sqrt[b]*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(40*b)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {455, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^{3/2} (A+Bx) \, dx$$

$$\Big\downarrow \text{455}$$

$$A\int \left(bx^2+a\right)^{3/2}dx+\frac{B\left(a+bx^2\right)^{5/2}}{5b}$$

$$\Big\downarrow \text{211}$$

$$A\left(\frac{3}{4}a\int \sqrt{bx^2+a}dx+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{B\left(a+bx^2\right)^{5/2}}{5b}$$

$$\Big\downarrow \text{211}$$

$$A\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{B\left(a+bx^2\right)^{5/2}}{5b}$$

$$\Big\downarrow \text{224}$$

$$A\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{B\left(a+bx^2\right)^{5/2}}{5b}$$

$$\Big\downarrow \text{219}$$

$$A\left(\frac{3}{4}a\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{B\left(a+bx^2\right)^{5/2}}{5b}$$

input
```
Int[(A + B*x)*(a + b*x^2)^(3/2),x]
```

output
```
(B*(a + b*x^2)^(5/2))/(5*b) + A*((x*(a + b*x^2)^(3/2))/4 + (3*a*((x*Sqrt[a
 + b*x^2])/2 + (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b])))/4)
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.80

method	result	size
default	$A\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)}{4}\right)+\frac{B\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{5b}$	70
risch	$\frac{\left(8b^{2}B\,x^{4}+10A\,b^{2}x^{3}+16Bab\,x^{2}+25aAbx+8a^{2}B\right)\sqrt{b\,x^{2}+a}}{40b}+\frac{3a^{2}A\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{8\sqrt{b}}$	80

input
```
int((B*x+A)*(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
A*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(
1/2)*x+(b*x^2+a)^(1/2))))+1/5*B*(b*x^2+a)^(5/2)/b
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 176, normalized size of antiderivative = 2.02

$$\int (A+Bx)\left(a+bx^2\right)^{3/2}\,dx=\left[\frac{15\,Aa^2\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,(8\,Bb^2x^4+10\,Ab^2x^3+16\,Babx^2+25\,A}{80\,b}\right.$$
$$\left.-\frac{15\,Aa^2\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(8\,Bb^2x^4+10\,Ab^2x^3+16\,Babx^2+25\,Aabx+8\,Ba^2)\sqrt{bx^2+a}}{40\,b}\right]$$

input
```
integrate((B*x+A)*(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/80*(15*A*a^2*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) +
2*(8*B*b^2*x^4 + 10*A*b^2*x^3 + 16*B*a*b*x^2 + 25*A*a*b*x + 8*B*a^2)*sqrt(
b*x^2 + a))/b, -1/40*(15*A*a^2*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a))
 - (8*B*b^2*x^4 + 10*A*b^2*x^3 + 16*B*a*b*x^2 + 25*A*a*b*x + 8*B*a^2)*sqrt
(b*x^2 + a))/b]
```

Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.37

$$\int (A+Bx)\left(a+bx^2\right)^{3/2}\,dx=\begin{cases}\frac{3Aa^2\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for}\ a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right)}{8}+\sqrt{a+bx^2}\cdot\left(\frac{5Aax}{8}+\frac{Abx^3}{4}+\frac{Ba^2}{5b}+\frac{2Bax^2}{5}+\frac{Bbx^4}{5}\right) \\ a^{\frac{3}{2}}\left(Ax+\frac{Bx^2}{2}\right)\end{cases}$$

input
```
integrate((B*x+A)*(b*x**2+a)**(3/2),x)
```

output
```
Piecewise((3*A*a**2*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqr
t(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True))/8 + sqrt(a + b*x**2)*(5*A*
a*x/8 + A*b*x**3/4 + B*a**2/(5*b) + 2*B*a*x**2/5 + B*b*x**4/5), Ne(b, 0)),
 (a**(3/2)*(A*x + B*x**2/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.70

$$\int (A+Bx) \left(a+bx^2\right)^{3/2} \, dx = \frac{1}{4} \left(bx^2+a\right)^{\frac{3}{2}} Ax + \frac{3}{8} \sqrt{bx^2+a} Aax + \frac{3 \, Aa^2 \operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8 \sqrt{b}} + \frac{\left(bx^2+a\right)^{\frac{5}{2}} B}{5 \, b}$$

input
```
integrate((B*x+A)*(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
1/4*(b*x^2 + a)^(3/2)*A*x + 3/8*sqrt(b*x^2 + a)*A*a*x + 3/8*A*a^2*arcsinh(
b*x/sqrt(a*b))/sqrt(b) + 1/5*(b*x^2 + a)^(5/2)*B/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.87

$$\int (A+Bx) \left(a+bx^2\right)^{3/2} \, dx = -\frac{3 \, Aa^2 \log\left(\left|-\sqrt{b}x + \sqrt{bx^2+a}\right|\right)}{8 \sqrt{b}} + \frac{1}{40} \sqrt{bx^2+a} \left(\frac{8 \, Ba^2}{b} + (25 \, Aa + 2 \, (8 \, Ba + (4 \, Bbx + 5 \, Ab)x)x)x\right)$$

input
```
integrate((B*x+A)*(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
-3/8*A*a^2*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b) + 1/40*sqrt(b*x^
2 + a)*(8*B*a^2/b + (25*A*a + 2*(8*B*a + (4*B*b*x + 5*A*b)*x)*x)*x)
```

Mupad [B] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.62

$$\int (A+Bx)\left(a+bx^2\right)^{3/2}\,dx=\frac{B\,(b\,x^2+a)^{5/2}}{5\,b}+\frac{A\,x\,(b\,x^2+a)^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{\left(\frac{b\,x^2}{a}+1\right)^{3/2}}$$

input
```
int((a + b*x^2)^(3/2)*(A + B*x),x)
```

output
```
(B*(a + b*x^2)^(5/2))/(5*b) + (A*x*(a + b*x^2)^(3/2)*hypergeom([-3/2, 1/2]
, 3/2, -(b*x^2)/a))/((b*x^2)/a + 1)^(3/2)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.28

$$\int (A+Bx)\,(a+bx^2)^{3/2}\,dx=\frac{25\sqrt{b\,x^2+a}\,a^2bx+8\sqrt{b\,x^2+a}\,a^2b+10\sqrt{b\,x^2+a}\,a\,b^2x^3+16\sqrt{b\,x^2+a}\,a\,b^2x^2+8\sqrt{b\,x^2+a}}{40b}$$

input
```
int((B*x+A)*(b*x^2+a)^(3/2),x)
```

output
```
(25*sqrt(a + b*x**2)*a**2*b*x + 8*sqrt(a + b*x**2)*a**2*b + 10*sqrt(a + b*
x**2)*a*b**2*x**3 + 16*sqrt(a + b*x**2)*a*b**2*x**2 + 8*sqrt(a + b*x**2)*b
**3*x**4 + 15*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**3)/(4
0*b)
```

3.9 $\int (A+Bx)\sqrt{a+bx^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 67

$$\int (A+Bx)\sqrt{a+bx^2}\,dx=\frac{1}{2}Ax\sqrt{a+bx^2}+\frac{B(a+bx^2)^{3/2}}{3b}+\frac{aA\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}$$

output
```
1/2*A*x*(b*x^2+a)^(1/2)+1/3*B*(b*x^2+a)^(3/2)/b+1/2*a*A*arctanh(b^(1/2)*x/
(b*x^2+a)^(1/2))/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.01

$$\int (A+Bx)\sqrt{a+bx^2}\,dx=\frac{\sqrt{a+bx^2}(2aB+3Abx+2bBx^2)}{6b}-\frac{aA\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{2\sqrt{b}}$$

input
```
Integrate[(A + B*x)*Sqrt[a + b*x^2],x]
```

output
```
(Sqrt[a + b*x^2]*(2*a*B + 3*A*b*x + 2*b*B*x^2))/(6*b) - (a*A*Log[-(Sqrt[b]
*x) + Sqrt[a + b*x^2]])/(2*Sqrt[b])
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {455, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx^2}(A+Bx)\,dx$$

$$\Big\downarrow \text{455}$$

$$A\int \sqrt{bx^2+a}dx+\frac{B\left(a+bx^2\right)^{3/2}}{3b}$$

$$\Big\downarrow \text{211}$$

$$A\left(\frac{1}{2}a\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{B\left(a+bx^2\right)^{3/2}}{3b}$$

$$\Big\downarrow \text{224}$$

$$A\left(\frac{1}{2}a\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{B\left(a+bx^2\right)^{3/2}}{3b}$$

$$\Big\downarrow \text{219}$$

$$A\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{B\left(a+bx^2\right)^{3/2}}{3b}$$

input
```
Int[(A + B*x)*Sqrt[a + b*x^2],x]
```

output
```
(B*(a + b*x^2)^(3/2))/(3*b) + A*((x*Sqrt[a + b*x^2])/2 + (a*ArcTanh[(Sqrt[
b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b]))
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.81

method	result	size
default	$A\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)+\frac{B\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{3b}$	54
risch	$\frac{\left(2bB\,x^{2}+3Abx+2Ba\right)\sqrt{b\,x^{2}+a}}{6b}+\frac{Aa\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}$	56

input
```
int((B*x+A)*(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
A*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))+1/3*
B*(b*x^2+a)^(3/2)/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.91

$$\int (A+Bx)\sqrt{a+bx^2}\,dx = \left[\frac{3\,Aa\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,(2\,Bbx^2+3\,Abx+2\,Ba)\sqrt{bx^2+a}}{12\,b},\right.$$
$$\left.-\frac{3\,Aa\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(2\,Bbx^2+3\,Abx+2\,Ba)\sqrt{bx^2+a}}{6\,b}\right]$$

input
```
integrate((B*x+A)*(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/12*(3*A*a*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(
2*B*b*x^2 + 3*A*b*x + 2*B*a)*sqrt(b*x^2 + a))/b, -1/6*(3*A*a*sqrt(-b)*arct
an(sqrt(-b)*x/sqrt(b*x^2 + a)) - (2*B*b*x^2 + 3*A*b*x + 2*B*a)*sqrt(b*x^2
+ a))/b]
```

Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.34

$$\int (A+Bx)\sqrt{a+bx^2}\,dx = \begin{cases} \frac{Aa\left(\begin{cases} \frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0 \\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise} \end{cases}\right)}{2}+\sqrt{a+bx^2}\left(\frac{Ax}{2}+\frac{Ba}{3b}+\frac{Bx^2}{3}\right) & \text{for } b\neq 0 \\ \sqrt{a}\left(Ax+\frac{Bx^2}{2}\right) & \text{otherwise} \end{cases}$$

input
```
integrate((B*x+A)*(b*x**2+a)**(1/2),x)
```

output
```
Piecewise((A*a*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b),
 Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True))/2 + sqrt(a + b*x**2)*(A*x/2 + B
*a/(3*b) + B*x**2/3), Ne(b, 0)), (sqrt(a)*(A*x + B*x**2/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.67

$$\int (A+Bx)\sqrt{a+bx^2}\,dx = \frac{1}{2}\sqrt{bx^2+a}Ax + \frac{Aa\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{b}} + \frac{(bx^2+a)^{\frac{3}{2}}B}{3\,b}$$

input
```
integrate((B*x+A)*(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(b*x^2 + a)*A*x + 1/2*A*a*arcsinh(b*x/sqrt(a*b))/sqrt(b) + 1/3*(b*
x^2 + a)^(3/2)*B/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.82

$$\int (A+Bx)\sqrt{a+bx^2}\,dx = -\frac{Aa\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2\sqrt{b}} + \frac{1}{6}\sqrt{bx^2+a}\left((2\,Bx+3\,A)x+\frac{2\,Ba}{b}\right)$$

input
```
integrate((B*x+A)*(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
-1/2*A*a*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b) + 1/6*sqrt(b*x^2 +
 a)*((2*B*x + 3*A)*x + 2*B*a/b)
```

Mupad [B] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.78

$$\int (A+Bx)\sqrt{a+bx^2}\,dx = \frac{B\,(b\,x^2+a)^{3/2}}{3\,b}+\frac{A\,x\,\sqrt{b\,x^2+a}}{2}+\frac{A\,a\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\,\sqrt{b}}$$

input
```
int((a + b*x^2)^(1/2)*(A + B*x),x)
```

output
```
(B*(a + b*x^2)^(3/2))/(3*b) + (A*x*(a + b*x^2)^(1/2))/2 + (A*a*log(b^(1/2)
*x + (a + b*x^2)^(1/2)))/(2*b^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.09

$$\int (A+Bx)\sqrt{a+bx^2}\,dx = \frac{3\sqrt{b\,x^2+a}\,abx+2\sqrt{b\,x^2+a}\,ab+2\sqrt{b\,x^2+a}\,b^2x^2+3\sqrt{b}\log\left(\frac{\sqrt{b\,x^2+a}+\sqrt{b}\,x}{\sqrt{a}}\right)a^2}{6b}$$

input
```
int((B*x+A)*(b*x^2+a)^(1/2),x)
```

output
```
(3*sqrt(a + b*x**2)*a*b*x + 2*sqrt(a + b*x**2)*a*b + 2*sqrt(a + b*x**2)*b*
*2*x**2 + 3*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2)/(6*b
)
```

3.10 $\int \frac{A+Bx}{\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 43

$$\int \frac{A+Bx}{\sqrt{a+bx^2}}\,dx=\frac{B\sqrt{a+bx^2}}{b}+\frac{A\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

output
```
B*(b*x^2+a)^(1/2)/b+A*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.07

$$\int \frac{A+Bx}{\sqrt{a+bx^2}}\,dx=\frac{B\sqrt{a+bx^2}}{b}-\frac{A\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{\sqrt{b}}$$

input
```
Integrate[(A + B*x)/Sqrt[a + b*x^2],x]
```

output
```
(B*Sqrt[a + b*x^2])/b - (A*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/Sqrt[b]
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {455, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{\sqrt{a+bx^2}}dx$$

$$\Big\downarrow \text{455}$$

$$A\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{B\sqrt{a+bx^2}}{b}$$

$$\Big\downarrow \text{224}$$

$$A\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{B\sqrt{a+bx^2}}{b}$$

$$\Big\downarrow \text{219}$$

$$\frac{A\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}+\frac{B\sqrt{a+bx^2}}{b}$$

input
```
Int[(A + B*x)/Sqrt[a + b*x^2],x]
```

output
```
(B*Sqrt[a + b*x^2])/b + (A*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/Sqrt[b]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{A\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{\sqrt{b}}+\frac{B\sqrt{b\,x^{2}+a}}{b}$	37
risch	$\frac{A\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{\sqrt{b}}+\frac{B\sqrt{b\,x^{2}+a}}{b}$	37

input
```
int((B*x+A)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
A*ln(b^(1/2)*x+(b*x^2+a)^(1/2))/b^(1/2)+B*(b*x^2+a)^(1/2)/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.14

$$\int \frac{A+Bx}{\sqrt{a+bx^2}}\,dx = \left[\frac{A\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,\sqrt{bx^2+a}B}{2\,b},\; -\frac{A\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-\sqrt{bx^2+a}B}{b}\right]$$

input
```
integrate((B*x+A)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(A*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*sqrt(b
*x^2 + a)*B)/b, -(A*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - sqrt(b*x
^2 + a)*B)/b]
```

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.65

$$\int \frac{A+Bx}{\sqrt{a+bx^2}}\,dx = \begin{cases} A\left(\begin{cases} \frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a \neq 0 \\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise} \end{cases}\right) + \frac{B\sqrt{a+bx^2}}{b} & \text{for } b \neq 0 \\ \frac{Ax+\frac{Bx^2}{2}}{\sqrt{a}} & \text{otherwise} \end{cases}$$

input
```
integrate((B*x+A)/(b*x**2+a)**(1/2),x)
```

output
```
Piecewise((A*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), N
e(a, 0)), (x*log(x)/sqrt(b*x**2), True)) + B*sqrt(a + b*x**2)/b, Ne(b, 0))
, ((A*x + B*x**2/2)/sqrt(a), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.67

$$\int \frac{A+Bx}{\sqrt{a+bx^2}}\,dx = \frac{A\,\text{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}} + \frac{\sqrt{bx^2+a}B}{b}$$

input
```
integrate((B*x+A)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
A*arcsinh(b*x/sqrt(a*b))/sqrt(b) + sqrt(b*x^2 + a)*B/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

$$\int \frac{A+Bx}{\sqrt{a+bx^2}}\,dx = -\frac{A\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{\sqrt{b}} + \frac{\sqrt{bx^2+a}B}{b}$$

input
```
integrate((B*x+A)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
-A*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b) + sqrt(b*x^2 + a)*B/b
```

Mupad [B] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

$$\int \frac{A+Bx}{\sqrt{a+bx^2}}\,dx = \frac{B\sqrt{b\,x^2+a}}{b} + \frac{A\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}$$

input
```
int((A + B*x)/(a + b*x^2)^(1/2),x)
```

output
```
(B*(a + b*x^2)^(1/2))/b + (A*log(b^(1/2)*x + (a + b*x^2)^(1/2)))/b^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.88

$$\int \frac{A+Bx}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{b\,x^2+a}\,b+\sqrt{b}\log\left(\frac{\sqrt{b\,x^2+a}+\sqrt{b}\,x}{\sqrt{a}}\right)a}{b}$$

input
```
int((B*x+A)/(b*x^2+a)^(1/2),x)
```

output
```
(sqrt(a + b*x**2)*b + sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*
a)/b
```

3.11 $\int \frac{A+Bx}{(a+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 28

$$\int \frac{A+Bx}{(a+bx^2)^{3/2}}\,dx = -\frac{aB-Abx}{ab\sqrt{a+bx^2}}$$

output
```
-(-A*b*x+B*a)/a/b/(b*x^2+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{A+Bx}{(a+bx^2)^{3/2}}\,dx = \frac{-aB+Abx}{ab\sqrt{a+bx^2}}$$

input
```
Integrate[(A + B*x)/(a + b*x^2)^(3/2),x]
```

output
```
(-(a*B) + A*b*x)/(a*b*Sqrt[a + b*x^2])
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {453}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{(a+bx^2)^{3/2}} \, dx$$

$$\Big\downarrow \; 453$$

$$-\frac{aB-Abx}{ab\sqrt{a+bx^2}}$$

input
```
Int[(A + B*x)/(a + b*x^2)^(3/2),x]
```

output
```
-((a*B - A*b*x)/(a*b*Sqrt[a + b*x^2]))
```

Defintions of rubi rules used

rule 453
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[-(a*
d - b*c*x)/(a*b*Sqrt[a + b*x^2]), x] /; FreeQ[{a, b, c, d}, x]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

method	result	size
gosper	$\frac{Abx-Ba}{\sqrt{b\,x^2+a}\,ab}$	26
trager	$\frac{Abx-Ba}{\sqrt{b\,x^2+a}\,ab}$	26
orering	$\frac{Abx-Ba}{\sqrt{b\,x^2+a}\,ab}$	26
default	$\frac{Ax}{a\sqrt{b\,x^2+a}}-\frac{B}{b\sqrt{b\,x^2+a}}$	32

input
```
int((B*x+A)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(A*b*x-B*a)/(b*x^2+a)^(1/2)/a/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.25

$$\int \frac{A+Bx}{(a+bx^2)^{3/2}}\,dx = \frac{(Abx-Ba)\sqrt{bx^2+a}}{ab^2x^2+a^2b}$$

input
```
integrate((B*x+A)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
(A*b*x - B*a)*sqrt(b*x^2 + a)/(a*b^2*x^2 + a^2*b)
```

Sympy [A] (verification not implemented)

Time = 2.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.64

$$\int \frac{A+Bx}{(a+bx^2)^{3/2}}\,dx = \frac{Ax}{a^{\frac{3}{2}}\sqrt{1+\frac{bx^2}{a}}} + B\left(\begin{cases} -\frac{1}{b\sqrt{a+bx^2}} & \text{for } b\neq 0 \\ \frac{x^2}{2a^{\frac{3}{2}}} & \text{otherwise} \end{cases}\right)$$

input
```
integrate((B*x+A)/(b*x**2+a)**(3/2),x)
```

output
```
A*x/(a**(3/2)*sqrt(1 + b*x**2/a)) + B*Piecewise((-1/(b*sqrt(a + b*x**2)),
Ne(b, 0)), (x**2/(2*a**(3/2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.11

$$\int \frac{A+Bx}{(a+bx^2)^{3/2}}\,dx = \frac{Ax}{\sqrt{bx^2+a}a} - \frac{B}{\sqrt{bx^2+a}b}$$

input
```
integrate((B*x+A)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
A*x/(sqrt(b*x^2 + a)*a) - B/(sqrt(b*x^2 + a)*b)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.82

$$\int \frac{A+Bx}{(a+bx^2)^{3/2}}\,dx = \frac{\frac{Ax}{a} - \frac{B}{b}}{\sqrt{bx^2+a}}$$

input
```
integrate((B*x+A)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
(A*x/a - B/b)/sqrt(b*x^2 + a)
```

Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{A+Bx}{(a+bx^2)^{3/2}}\,dx = -\frac{\frac{B}{b} - \frac{A\,x}{a}}{\sqrt{b\,x^2+a}}$$

input
```
int((A + B*x)/(a + b*x^2)^(3/2),x)
```

output
```
-(B/b - (A*x)/a)/(a + b*x^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.68

$$\int \frac{A+Bx}{(a+bx^2)^{3/2}}\,dx = \frac{\sqrt{b\,x^2+a}\,bx-\sqrt{b\,x^2+a}\,b+\sqrt{b}\,a+\sqrt{b}\,b\,x^2}{b\,(b\,x^2+a)}$$

input
```
int((B*x+A)/(b*x^2+a)^(3/2),x)
```

output
```
(sqrt(a + b*x**2)*b*x - sqrt(a + b*x**2)*b + sqrt(b)*a + sqrt(b)*b*x**2)/(
b*(a + b*x**2))
```

3.12 $\int \frac{A+Bx}{(a+bx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 51

$$\int \frac{A+Bx}{(a+bx^2)^{5/2}}\,dx = \frac{-aB+Abx}{3ab\,(a+bx^2)^{3/2}} + \frac{2Ax}{3a^2\sqrt{a+bx^2}}$$

output
```
1/3*(A*b*x-B*a)/a/b/(b*x^2+a)^(3/2)+2/3*A*x/a^2/(b*x^2+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.84

$$\int \frac{A+Bx}{(a+bx^2)^{5/2}}\,dx = \frac{-a^2B+3aAbx+2Ab^2x^3}{3a^2b\,(a+bx^2)^{3/2}}$$

input
```
Integrate[(A + B*x)/(a + b*x^2)^(5/2),x]
```

output
```
(-(a^2*B) + 3*a*A*b*x + 2*A*b^2*x^3)/(3*a^2*b*(a + b*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {454, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{(a+bx^2)^{5/2}}dx$$

$$\Big\downarrow \text{ 454}$$

$$\frac{2A\int \frac{1}{(bx^2+a)^{3/2}}dx}{3a}-\frac{aB-Abx}{3ab(a+bx^2)^{3/2}}$$

$$\Big\downarrow \text{ 208}$$

$$\frac{2Ax}{3a^2\sqrt{a+bx^2}}-\frac{aB-Abx}{3ab(a+bx^2)^{3/2}}$$

input
```
Int[(A + B*x)/(a + b*x^2)^(5/2),x]
```

output
```
-1/3*(a*B - A*b*x)/(a*b*(a + b*x^2)^(3/2)) + (2*A*x)/(3*a^2*Sqrt[a + b*x^2
])
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 454
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*d
 - b*c*x)/(2*a*b*(p + 1)))*(a + b*x^2)^(p + 1), x] + Simp[c*((2*p + 3)/(2*a
*(p + 1)))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d}, x] && L
tQ[p, -1] && NeQ[p, -3/2]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.78

method	result	size
gosper	$\frac{2A\,b^{2}x^{3}+3aAbx-a^{2}B}{3\left(b\,x^{2}+a\right)^{\frac{3}{2}}a^{2}b}$	40
trager	$\frac{2A\,b^{2}x^{3}+3aAbx-a^{2}B}{3\left(b\,x^{2}+a\right)^{\frac{3}{2}}a^{2}b}$	40
orering	$\frac{2A\,b^{2}x^{3}+3aAbx-a^{2}B}{3\left(b\,x^{2}+a\right)^{\frac{3}{2}}a^{2}b}$	40
default	$A\left(\frac{x}{3a\left(b\,x^{2}+a\right)^{\frac{3}{2}}}+\frac{2x}{3a^{2}\sqrt{b\,x^{2}+a}}\right)-\frac{B}{3b\left(b\,x^{2}+a\right)^{\frac{3}{2}}}$	50

input
```
int((B*x+A)/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(2*A*b^2*x^3+3*A*a*b*x-B*a^2)/(b*x^2+a)^(3/2)/a^2/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.22

$$\int \frac{A+Bx}{\left(a+bx^2\right)^{5/2}}\,dx = \frac{\left(2\,Ab^2x^3+3\,Aabx-Ba^2\right)\sqrt{bx^2+a}}{3\left(a^2b^3x^4+2\,a^3b^2x^2+a^4b\right)}$$

input
```
integrate((B*x+A)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
1/3*(2*A*b^2*x^3 + 3*A*a*b*x - B*a^2)*sqrt(b*x^2 + a)/(a^2*b^3*x^4 + 2*a^3
*b^2*x^2 + a^4*b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(44) = 88$.

Time = 3.76 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.86

$$\int \frac{A+Bx}{(a+bx^2)^{5/2}}\,dx = A\left(\frac{3ax}{3a^{\frac{7}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{5}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}} + \frac{2bx^3}{3a^{\frac{7}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{5}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}}\right) + B\left(\begin{cases} -\frac{1}{3ab\sqrt{a+bx^2}+3b^2x^2\sqrt{a+bx^2}} & \text{for } b\neq 0 \\ \frac{x^2}{2a^{\frac{5}{2}}} & \text{otherwise} \end{cases}\right)$$

input
```
integrate((B*x+A)/(b*x**2+a)**(5/2),x)
```

output
```
A*(3*a*x/(3*a**(7/2)*sqrt(1 + b*x**2/a) + 3*a**(5/2)*b*x**2*sqrt(1 + b*x**
2/a)) + 2*b*x**3/(3*a**(7/2)*sqrt(1 + b*x**2/a) + 3*a**(5/2)*b*x**2*sqrt(1
 + b*x**2/a))) + B*Piecewise((-1/(3*a*b*sqrt(a + b*x**2) + 3*b**2*x**2*sqr
t(a + b*x**2)), Ne(b, 0)), (x**2/(2*a**(5/2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.94

$$\int \frac{A+Bx}{(a+bx^2)^{5/2}}\,dx = \frac{2\,Ax}{3\,\sqrt{bx^2+a}a^2} + \frac{Ax}{3\,(bx^2+a)^{\frac{3}{2}}a} - \frac{B}{3\,(bx^2+a)^{\frac{3}{2}}b}$$

input
```
integrate((B*x+A)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
2/3*A*x/(sqrt(b*x^2 + a)*a^2) + 1/3*A*x/((b*x^2 + a)^(3/2)*a) - 1/3*B/((b*
x^2 + a)^(3/2)*b)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.73

$$\int \frac{A+Bx}{(a+bx^2)^{5/2}}\,dx = \frac{\left(\frac{2\,Abx^2}{a^2}+\frac{3\,A}{a}\right)x-\frac{B}{b}}{3\,(bx^2+a)^{\frac{3}{2}}}$$

input
```
integrate((B*x+A)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
1/3*((2*A*b*x^2/a^2 + 3*A/a)*x - B/b)/(b*x^2 + a)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.80

$$\int \frac{A+Bx}{(a+bx^2)^{5/2}}\,dx = \frac{2\,A\,b\,x\,(b\,x^2+a)-B\,a^2+A\,a\,b\,x}{3\,a^2\,b\,(b\,x^2+a)^{3/2}}$$

input
```
int((A + B*x)/(a + b*x^2)^(5/2),x)
```

output
```
(2*A*b*x*(a + b*x^2) - B*a^2 + A*a*b*x)/(3*a^2*b*(a + b*x^2)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.88

$$\int \frac{A+Bx}{(a+bx^2)^{5/2}}\,dx = \frac{3\sqrt{b\,x^2+a}\,abx-\sqrt{b\,x^2+a}\,ab+2\sqrt{b\,x^2+a}\,b^2x^3-2\sqrt{b}\,a^2-4\sqrt{b}\,ab\,x^2-2\sqrt{b}\,b^2x^4}{3ab\,(b^2x^4+2ab\,x^2+a^2)}$$

input
```
int((B*x+A)/(b*x^2+a)^(5/2),x)
```

output
```
(3*sqrt(a + b*x**2)*a*b*x - sqrt(a + b*x**2)*a*b + 2*sqrt(a + b*x**2)*b**2
*x**3 - 2*sqrt(b)*a**2 - 4*sqrt(b)*a*b*x**2 - 2*sqrt(b)*b**2*x**4)/(3*a*b*
(a**2 + 2*a*b*x**2 + b**2*x**4))
```

3.13 $\int \frac{A+Bx}{(a+bx^2)^{7/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 71

$$\int \frac{A+Bx}{(a+bx^2)^{7/2}}\,dx = \frac{-aB+Abx}{5ab\,(a+bx^2)^{5/2}} + \frac{4Ax}{15a^2\,(a+bx^2)^{3/2}} + \frac{8Ax}{15a^3\sqrt{a+bx^2}}$$

output
```
1/5*(A*b*x-B*a)/a/b/(b*x^2+a)^(5/2)+4/15*A*x/a^2/(b*x^2+a)^(3/2)+8/15*A*x/
a^3/(b*x^2+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.77

$$\int \frac{A+Bx}{(a+bx^2)^{7/2}}\,dx = \frac{-3a^3B+15a^2Abx+20aAb^2x^3+8Ab^3x^5}{15a^3b\,(a+bx^2)^{5/2}}$$

input
```
Integrate[(A + B*x)/(a + b*x^2)^(7/2),x]
```

output
```
(-3*a^3*B + 15*a^2*A*b*x + 20*a*A*b^2*x^3 + 8*A*b^3*x^5)/(15*a^3*b*(a + b*
x^2)^(5/2))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.10, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {454, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{(a+bx^2)^{7/2}}\,dx$$

$$\Big\downarrow \text{454}$$

$$\frac{4A\int \frac{1}{(bx^2+a)^{5/2}}dx}{5a}-\frac{aB-Abx}{5ab\,(a+bx^2)^{5/2}}$$

$$\Big\downarrow \text{209}$$

$$\frac{4A\left(\frac{2\int \frac{1}{\left(bx^2+a\right)^{3/2}}dx}{3a}+\frac{x}{3a(a+bx^2)^{3/2}}\right)}{5a}-\frac{aB-Abx}{5ab\,(a+bx^2)^{5/2}}$$

$$\Big\downarrow \text{208}$$

$$\frac{4A\left(\frac{2x}{3a^2\sqrt{a+bx^2}}+\frac{x}{3a(a+bx^2)^{3/2}}\right)}{5a}-\frac{aB-Abx}{5ab\,(a+bx^2)^{5/2}}$$

input
```
Int[(A + B*x)/(a + b*x^2)^(7/2),x]
```

output
```
-1/5*(a*B - A*b*x)/(a*b*(a + b*x^2)^(5/2)) + (4*A*(x/(3*a*(a + b*x^2)^(3/2
)) + (2*x)/(3*a^2*Sqrt[a + b*x^2])))/(5*a)
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 454
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*d
 - b*c*x)/(2*a*b*(p + 1)))*(a + b*x^2)^(p + 1), x] + Simp[c*((2*p + 3)/(2*a
*(p + 1)))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d}, x] && L
tQ[p, -1] && NeQ[p, -3/2]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.73

method	result	size
gosper	$\frac{8A\,b^{3}x^{5}+20aA\,b^{2}x^{3}+15a^{2}Abx-3a^{3}B}{15\left(b\,x^{2}+a\right)^{\frac{5}{2}}a^{3}b}$	52
trager	$\frac{8A\,b^{3}x^{5}+20aA\,b^{2}x^{3}+15a^{2}Abx-3a^{3}B}{15\left(b\,x^{2}+a\right)^{\frac{5}{2}}a^{3}b}$	52
orering	$\frac{8A\,b^{3}x^{5}+20aA\,b^{2}x^{3}+15a^{2}Abx-3a^{3}B}{15\left(b\,x^{2}+a\right)^{\frac{5}{2}}a^{3}b}$	52
default	$A\left(\frac{x}{5a\left(b\,x^{2}+a\right)^{\frac{5}{2}}}+\frac{\frac{4x}{15a\left(b\,x^{2}+a\right)^{\frac{3}{2}}}+\frac{8x}{15a^{2}\sqrt{b\,x^{2}+a}}}{a}\right)-\frac{B}{5b\left(b\,x^{2}+a\right)^{\frac{5}{2}}}$	71

input
```
int((B*x+A)/(b*x^2+a)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*(8*A*b^3*x^5+20*A*a*b^2*x^3+15*A*a^2*b*x-3*B*a^3)/(b*x^2+a)^(5/2)/a^3
/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.20

$$\int \frac{A+Bx}{(a+bx^2)^{7/2}}\,dx = \frac{(8\,Ab^3x^5+20\,Aab^2x^3+15\,Aa^2bx-3\,Ba^3)\sqrt{bx^2+a}}{15\,(a^3b^4x^6+3\,a^4b^3x^4+3\,a^5b^2x^2+a^6b)}$$

input
```
integrate((B*x+A)/(b*x^2+a)^(7/2),x, algorithm="fricas")
```

output
```
1/15*(8*A*b^3*x^5 + 20*A*a*b^2*x^3 + 15*A*a^2*b*x - 3*B*a^3)*sqrt(b*x^2 +
a)/(a^3*b^4*x^6 + 3*a^4*b^3*x^4 + 3*a^5*b^2*x^2 + a^6*b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 415 vs. 2(65) = 130.

Time = 6.65 (sec) , antiderivative size = 486, normalized size of antiderivative = 6.85

$$\int \frac{A+Bx}{(a+bx^2)^{7/2}}\,dx = A\left(\frac{15a^5x}{15a^{\frac{17}{2}}\sqrt{1+\frac{bx^2}{a}}+45a^{\frac{15}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}+45a^{\frac{13}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}}+15a^{\frac{11}{2}}b^3x^6\sqrt{1+\frac{bx^2}{a}}} + \frac{35a^4bx^3}{15a^{\frac{17}{2}}\sqrt{1+\frac{bx^2}{a}}+45a^{\frac{15}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}+45a^{\frac{13}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}}+15a^{\frac{11}{2}}b^3x^6\sqrt{1+\frac{bx^2}{a}}} + \frac{28a^3b^2x^5}{15a^{\frac{17}{2}}\sqrt{1+\frac{bx^2}{a}}+45a^{\frac{15}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}+45a^{\frac{13}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}}+15a^{\frac{11}{2}}b^3x^6\sqrt{1+\frac{bx^2}{a}}} + \frac{8a^2b^3x^7}{15a^{\frac{17}{2}}\sqrt{1+\frac{bx^2}{a}}+45a^{\frac{15}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}+45a^{\frac{13}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}}+15a^{\frac{11}{2}}b^3x^6\sqrt{1+\frac{bx^2}{a}}}\right) + B\left(\begin{cases} -\frac{1}{5a^2b\sqrt{a+bx^2}+10ab^2x^2\sqrt{a+bx^2}+5b^3x^4\sqrt{a+bx^2}} & \text{for } b \neq 0 \\ \frac{x^2}{2a^{\frac{7}{2}}} & \text{otherwise} \end{cases}\right)$$

input
```
integrate((B*x+A)/(b*x**2+a)**(7/2),x)
```

output
```
A*(15*a**5*x/(15*a**(17/2)*sqrt(1 + b*x**2/a) + 45*a**(15/2)*b*x**2*sqrt(1
 + b*x**2/a) + 45*a**(13/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 15*a**(11/2)*b*
*3*x**6*sqrt(1 + b*x**2/a)) + 35*a**4*b*x**3/(15*a**(17/2)*sqrt(1 + b*x**2
/a) + 45*a**(15/2)*b*x**2*sqrt(1 + b*x**2/a) + 45*a**(13/2)*b**2*x**4*sqrt
(1 + b*x**2/a) + 15*a**(11/2)*b**3*x**6*sqrt(1 + b*x**2/a)) + 28*a**3*b**2
*x**5/(15*a**(17/2)*sqrt(1 + b*x**2/a) + 45*a**(15/2)*b*x**2*sqrt(1 + b*x*
*2/a) + 45*a**(13/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 15*a**(11/2)*b**3*x**6
*sqrt(1 + b*x**2/a)) + 8*a**2*b**3*x**7/(15*a**(17/2)*sqrt(1 + b*x**2/a) +
 45*a**(15/2)*b*x**2*sqrt(1 + b*x**2/a) + 45*a**(13/2)*b**2*x**4*sqrt(1 +
b*x**2/a) + 15*a**(11/2)*b**3*x**6*sqrt(1 + b*x**2/a))) + B*Piecewise((-1/
(5*a**2*b*sqrt(a + b*x**2) + 10*a*b**2*x**2*sqrt(a + b*x**2) + 5*b**3*x**4
*sqrt(a + b*x**2)), Ne(b, 0)), (x**2/(2*a**(7/2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.90

$$\int \frac{A+Bx}{(a+bx^2)^{7/2}}\,dx = \frac{8\,Ax}{15\,\sqrt{bx^2+a}a^3} + \frac{4\,Ax}{15\,(bx^2+a)^{\frac{3}{2}}a^2} + \frac{Ax}{5\,(bx^2+a)^{\frac{5}{2}}a} - \frac{B}{5\,(bx^2+a)^{\frac{5}{2}}b}$$

input
```
integrate((B*x+A)/(b*x^2+a)^(7/2),x, algorithm="maxima")
```

output
```
8/15*A*x/(sqrt(b*x^2 + a)*a^3) + 4/15*A*x/((b*x^2 + a)^(3/2)*a^2) + 1/5*A*
x/((b*x^2 + a)^(5/2)*a) - 1/5*B/((b*x^2 + a)^(5/2)*b)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.73

$$\int \frac{A+Bx}{(a+bx^2)^{7/2}}\,dx = \frac{\left(4\left(\frac{2\,Ab^2x^2}{a^3}+\frac{5\,Ab}{a^2}\right)x^2+\frac{15\,A}{a}\right)x-\frac{3\,B}{b}}{15\,(bx^2+a)^{\frac{5}{2}}}$$

input
```
integrate((B*x+A)/(b*x^2+a)^(7/2),x, algorithm="giac")
```

output
```
1/15*((4*(2*A*b^2*x^2/a^3 + 5*A*b/a^2)*x^2 + 15*A/a)*x - 3*B/b)/(b*x^2 + a
)^(5/2)
```

Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.83

$$\int \frac{A+Bx}{(a+bx^2)^{7/2}}\,dx = \frac{8\,A\,b\,x\,(b\,x^2+a)^2-3\,B\,a^3+3\,A\,a^2\,b\,x+4\,A\,a\,b\,x\,(b\,x^2+a)}{15\,a^3\,b\,(b\,x^2+a)^{5/2}}$$

input
```
int((A + B*x)/(a + b*x^2)^(7/2),x)
```

output
```
(8*A*b*x*(a + b*x^2)^2 - 3*B*a^3 + 3*A*a^2*b*x + 4*A*a*b*x*(a + b*x^2))/(1
5*a^3*b*(a + b*x^2)^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.99

$$\int \frac{A+Bx}{(a+bx^2)^{7/2}}\,dx = \frac{15\sqrt{b\,x^2+a}\,a^2bx-3\sqrt{b\,x^2+a}\,a^2b+20\sqrt{b\,x^2+a}\,a\,b^2x^3+8\sqrt{b\,x^2+a}\,b^3x^5-8\sqrt{b}\,a^3}{15a^2b\,(b^3x^6+3a\,b^2x^4+3a^2b\,x^2+a^3)}$$

input
```
int((B*x+A)/(b*x^2+a)^(7/2),x)
```

output
```
(15*sqrt(a + b*x**2)*a**2*b*x - 3*sqrt(a + b*x**2)*a**2*b + 20*sqrt(a + b*
x**2)*a*b**2*x**3 + 8*sqrt(a + b*x**2)*b**3*x**5 - 8*sqrt(b)*a**3 - 24*sqr
t(b)*a**2*b*x**2 - 24*sqrt(b)*a*b**2*x**4 - 8*sqrt(b)*b**3*x**6)/(15*a**2*
b*(a**3 + 3*a**2*b*x**2 + 3*a*b**2*x**4 + b**3*x**6))
```

3.14 $\int \frac{A+Bx}{(a+bx^2)^{9/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 91

$$\int \frac{A+Bx}{(a+bx^2)^{9/2}}\,dx = \frac{-aB+Abx}{7ab\,(a+bx^2)^{7/2}} + \frac{6Ax}{35a^2\,(a+bx^2)^{5/2}} + \frac{8Ax}{35a^3\,(a+bx^2)^{3/2}} + \frac{16Ax}{35a^4\sqrt{a+bx^2}}$$

output
```
1/7*(A*b*x-B*a)/a/b/(b*x^2+a)^(7/2)+6/35*A*x/a^2/(b*x^2+a)^(5/2)+8/35*A*x/
a^3/(b*x^2+a)^(3/2)+16/35*A*x/a^4/(b*x^2+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.74

$$\int \frac{A+Bx}{(a+bx^2)^{9/2}}\,dx = \frac{-5a^4B+35a^3Abx+70a^2Ab^2x^3+56aAb^3x^5+16Ab^4x^7}{35a^4b\,(a+bx^2)^{7/2}}$$

input
```
Integrate[(A + B*x)/(a + b*x^2)^(9/2),x]
```

output
```
(-5*a^4*B + 35*a^3*A*b*x + 70*a^2*A*b^2*x^3 + 56*a*A*b^3*x^5 + 16*A*b^4*x^
7)/(35*a^4*b*(a + b*x^2)^(7/2))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.15, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {454, 209, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{(a+bx^2)^{9/2}}\,dx$$

$$\Big\downarrow \text{454}$$

$$\frac{6A\int \frac{1}{(bx^2+a)^{7/2}}dx}{7a}-\frac{aB-Abx}{7ab\,(a+bx^2)^{7/2}}$$

$$\Big\downarrow \text{209}$$

$$\frac{6A\left(\frac{4\int \frac{1}{\left(bx^2+a\right)^{5/2}}dx}{5a}+\frac{x}{5a(a+bx^2)^{5/2}}\right)}{7a}-\frac{aB-Abx}{7ab\,(a+bx^2)^{7/2}}$$

$$\Big\downarrow \text{209}$$

$$\frac{6A\left(\frac{4\left(\frac{2\int \frac{1}{\left(bx^2+a\right)^{3/2}}dx}{3a}+\frac{x}{3a\left(a+bx^2\right)^{3/2}}\right)}{5a}+\frac{x}{5a(a+bx^2)^{5/2}}\right)}{7a}-\frac{aB-Abx}{7ab\,(a+bx^2)^{7/2}}$$

$$\Big\downarrow \text{208}$$

$$\frac{6A\left(\frac{4\left(\frac{2x}{3a^2\sqrt{a+bx^2}}+\frac{x}{3a\left(a+bx^2\right)^{3/2}}\right)}{5a}+\frac{x}{5a(a+bx^2)^{5/2}}\right)}{7a}-\frac{aB-Abx}{7ab\,(a+bx^2)^{7/2}}$$

input
```
Int[(A + B*x)/(a + b*x^2)^(9/2),x]
```

output
```
-1/7*(a*B - A*b*x)/(a*b*(a + b*x^2)^(7/2)) + (6*A*(x/(5*a*(a + b*x^2)^(5/2
)) + (4*(x/(3*a*(a + b*x^2)^(3/2)) + (2*x)/(3*a^2*Sqrt[a + b*x^2])))/(5*a)
))/(7*a)
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 454
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*d
 - b*c*x)/(2*a*b*(p + 1)))*(a + b*x^2)^(p + 1), x] + Simp[c*((2*p + 3)/(2*a
*(p + 1)))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d}, x] && L
tQ[p, -1] && NeQ[p, -3/2]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.70

method	result	size
gosper	$\frac{16A\,b^4x^7+56Aa\,b^3x^5+70A\,a^2b^2x^3+35A\,a^3bx-5B\,a^4}{35(b\,x^2+a)^{\frac{7}{2}}a^4b}$	64
trager	$\frac{16A\,b^4x^7+56Aa\,b^3x^5+70A\,a^2b^2x^3+35A\,a^3bx-5B\,a^4}{35(b\,x^2+a)^{\frac{7}{2}}a^4b}$	64
orering	$\frac{16A\,b^4x^7+56Aa\,b^3x^5+70A\,a^2b^2x^3+35A\,a^3bx-5B\,a^4}{35(b\,x^2+a)^{\frac{7}{2}}a^4b}$	64
default	$A\left(\frac{x}{7a(b\,x^2+a)^{\frac{7}{2}}}+\frac{\frac{6x}{35a\left(b\,x^2+a\right)^{\frac{5}{2}}}+\frac{6\left(\frac{4x}{15a\left(b\,x^2+a\right)^{\frac{3}{2}}}+\frac{8x}{15a^2\sqrt{b\,x^2+a}}\right)}{7a}}{a}\right)-\frac{B}{7b(b\,x^2+a)^{\frac{7}{2}}}$	92

input
```
int((B*x+A)/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
1/35*(16*A*b^4*x^7+56*A*a*b^3*x^5+70*A*a^2*b^2*x^3+35*A*a^3*b*x-5*B*a^4)/(
b*x^2+a)^(7/2)/a^4/b
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.19

$$\int \frac{A+Bx}{(a+bx^2)^{9/2}}\,dx = \frac{(16\,Ab^4x^7+56\,Aab^3x^5+70\,Aa^2b^2x^3+35\,Aa^3bx-5\,Ba^4)\sqrt{bx^2+a}}{35\,(a^4b^5x^8+4\,a^5b^4x^6+6\,a^6b^3x^4+4\,a^7b^2x^2+a^8b)}$$

input
```
integrate((B*x+A)/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

output
```
1/35*(16*A*b^4*x^7 + 56*A*a*b^3*x^5 + 70*A*a^2*b^2*x^3 + 35*A*a^3*b*x - 5*
B*a^4)*sqrt(b*x^2 + a)/(a^4*b^5*x^8 + 4*a^5*b^4*x^6 + 6*a^6*b^3*x^4 + 4*a^
7*b^2*x^2 + a^8*b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1266 vs. $2(85) = 170$.

Time = 11.76 (sec) , antiderivative size = 1360, normalized size of antiderivative = 14.95

$$\int \frac{A + Bx}{(a + bx^2)^{9/2}} \, dx = \text{Too large to display}$$

input
```
integrate((B*x+A)/(b*x**2+a)**(9/2),x)
```

output
```
A*(35*a**14*x/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt
(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2
)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a
) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*
sqrt(1 + b*x**2/a)) + 175*a**13*b*x**3/(35*a**(37/2)*sqrt(1 + b*x**2/a) +
210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 +
 b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b*
*4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) +
 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a)) + 371*a**12*b**2*x**5/(35*a**
(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*
a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 +
b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**
5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a)) +
 429*a**11*b**3*x**7/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x*
*2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a
**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b
*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6
*x**12*sqrt(1 + b*x**2/a)) + 286*a**10*b**4*x**9/(35*a**(37/2)*sqrt(1 + b*
x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**
4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.88

$$\int \frac{A+Bx}{(a+bx^2)^{9/2}}\,dx = \frac{16\,Ax}{35\,\sqrt{bx^2+a}a^4} + \frac{8\,Ax}{35\,(bx^2+a)^{\frac{3}{2}}a^3} + \frac{6\,Ax}{35\,(bx^2+a)^{\frac{5}{2}}a^2} + \frac{Ax}{7\,(bx^2+a)^{\frac{7}{2}}a} - \frac{B}{7\,(bx^2+a)^{\frac{7}{2}}b}$$

input
```
integrate((B*x+A)/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

output
```
16/35*A*x/(sqrt(b*x^2 + a)*a^4) + 8/35*A*x/((b*x^2 + a)^(3/2)*a^3) + 6/35*
A*x/((b*x^2 + a)^(5/2)*a^2) + 1/7*A*x/((b*x^2 + a)^(7/2)*a) - 1/7*B/((b*x^
2 + a)^(7/2)*b)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.74

$$\int \frac{A+Bx}{(a+bx^2)^{9/2}}\,dx = \frac{\left(2\left(4\left(\frac{2\,Ab^3x^2}{a^4} + \frac{7\,Ab^2}{a^3}\right)x^2 + \frac{35\,Ab}{a^2}\right)x^2 + \frac{35\,A}{a}\right)x - \frac{5\,B}{b}}{35\,(bx^2+a)^{\frac{7}{2}}}$$

input
```
integrate((B*x+A)/(b*x^2+a)^(9/2),x, algorithm="giac")
```

output
```
1/35*((2*(4*(2*A*b^3*x^2/a^4 + 7*A*b^2/a^3)*x^2 + 35*A*b/a^2)*x^2 + 35*A/a
)*x - 5*B/b)/(b*x^2 + a)^(7/2)
```

Mupad [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.81

$$\int \frac{A+Bx}{(a+bx^2)^{9/2}}\,dx = \frac{16\,A\,x}{35\,a^4\,\sqrt{b\,x^2+a}} - \frac{\frac{B}{7\,b}-\frac{A\,x}{7\,a}}{\left(b\,x^2+a\right)^{7/2}} + \frac{8\,A\,x}{35\,a^3\,\left(b\,x^2+a\right)^{3/2}} + \frac{6\,A\,x}{35\,a^2\,\left(b\,x^2+a\right)^{5/2}}$$

input
```
int((A + B*x)/(a + b*x^2)^(9/2),x)
```

output
```
(16*A*x)/(35*a^4*(a + b*x^2)^(1/2)) - (B/(7*b) - (A*x)/(7*a))/(a + b*x^2)^
(7/2) + (8*A*x)/(35*a^3*(a + b*x^2)^(3/2)) + (6*A*x)/(35*a^2*(a + b*x^2)^(
5/2))
```

Reduce [B] (verification not implemented)

Time = 5.26 (sec) , antiderivative size = 184, normalized size of antiderivative = 2.02

$$\int \frac{A+Bx}{(a+bx^2)^{9/2}}\,dx = \frac{35\sqrt{b\,x^2+a}\,a^3bx - 5\sqrt{b\,x^2+a}\,a^3b + 70\sqrt{b\,x^2+a}\,a^2b^2x^3 + 56\sqrt{b\,x^2+a}\,a\,b^3x^5 + 16\sqrt{}}{35a^3b\left(b^4x^8 + 4a\,b^3x^6 + 6a\right.}$$

input
```
int((B*x+A)/(b*x^2+a)^(9/2),x)
```

output
```
(35*sqrt(a + b*x**2)*a**3*b*x - 5*sqrt(a + b*x**2)*a**3*b + 70*sqrt(a + b*
x**2)*a**2*b**2*x**3 + 56*sqrt(a + b*x**2)*a*b**3*x**5 + 16*sqrt(a + b*x**
2)*b**4*x**7 - 16*sqrt(b)*a**4 - 64*sqrt(b)*a**3*b*x**2 - 96*sqrt(b)*a**2*
b**2*x**4 - 64*sqrt(b)*a*b**3*x**6 - 16*sqrt(b)*b**4*x**8)/(35*a**3*b*(a**
4 + 4*a**3*b*x**2 + 6*a**2*b**2*x**4 + 4*a*b**3*x**6 + b**4*x**8))
```

3.15 $\int (A+Bx)\left(a+bx^2\right)^{4/3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 307

$$\int (A+Bx)\left(a+bx^2\right)^{4/3}\,dx=\frac{24}{55}aAx\sqrt[3]{a+bx^2}+\frac{3}{11}Ax\left(a+bx^2\right)^{4/3}+\frac{3B\left(a+bx^2\right)^{7/3}}{14b}$$

$$-\frac{16\ 3^{3/4}\sqrt{2-\sqrt{3}}a^2A\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\ \text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{}}\right.\right.}{55bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

output
```
24/55*a*A*x*(b*x^2+a)^(1/3)+3/11*A*x*(b*x^2+a)^(4/3)+3/14*B*(b*x^2+a)^(7/3
)/b-16/55*3^(3/4)*(1/2*6^(1/2)-1/2*2^(1/2))*a^2*A*(a^(1/3)-(b*x^2+a)^(1/3)
)*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/((1-3^(1/2))*a^(1/3)-
(b*x^2+a)^(1/3))^2)^(1/2)*EllipticF(((1+3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))/
((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3)),2*I-I*3^(1/2))/b/x/(-a^(1/3)*(a^(1/3
)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.17 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.34

$$\int (A+Bx)\left(a+bx^2\right)^{4/3}\,dx=\frac{3(a+bx^2)\left(55a^2B+5b^2x^3(14A+11Bx)+2abx(91A+55Bx)\right)+224a^2Abx\left(1+\frac{bx^2}{a}\right)^{2/3}\text{H}}{770b\left(a+bx^2\right)^{2/3}}$$

input
```
Integrate[(A + B*x)*(a + b*x^2)^(4/3),x]
```

output
```
(3*(a + b*x^2)*(55*a^2*B + 5*b^2*x^3*(14*A + 11*B*x) + 2*a*b*x*(91*A + 55*
B*x)) + 224*a^2*A*b*x*(1 + (b*x^2)/a)^(2/3)*Hypergeometric2F1[1/2, 2/3, 3/
2, -((b*x^2)/a)])/(770*b*(a + b*x^2)^(2/3))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {455, 211, 211, 234, 760}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^{4/3}(A+Bx)\,dx$$

$$\Big\downarrow \text{455}$$

$$A\int \left(bx^2+a\right)^{4/3}dx+\frac{3B\left(a+bx^2\right)^{7/3}}{14b}$$

$$\Big\downarrow \text{211}$$

$$A\left(\frac{8}{11}a\int \sqrt[3]{bx^2+a}dx+\frac{3}{11}x\left(a+bx^2\right)^{4/3}\right)+\frac{3B\left(a+bx^2\right)^{7/3}}{14b}$$

$$\Big\downarrow \text{211}$$

$$A\left(\frac{8}{11}a\left(\frac{2}{5}a\int\frac{1}{(bx^2+a)^{2/3}}dx+\frac{3}{5}x\sqrt[3]{a+bx^2}\right)+\frac{3}{11}x\left(a+bx^2\right)^{4/3}\right)+\frac{3B\left(a+bx^2\right)^{7/3}}{14b}$$

$$\Big\downarrow \text{234}$$

$$A\left(\frac{8}{11}a\left(\frac{3a\sqrt{bx^2}\int\frac{1}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}}{5bx}+\frac{3}{5}x\sqrt[3]{a+bx^2}\right)+\frac{3}{11}x\left(a+bx^2\right)^{4/3}\right)+\frac{3B\left(a+bx^2\right)^{7/3}}{14b}$$

$$\Big\downarrow \text{760}$$

$$A\left(\frac{8}{11}a\left(\frac{3}{5}x\sqrt[3]{a+bx^2}-\frac{2\,3^{3/4}\sqrt{2-\sqrt{3}}a\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\text{EllipticF}\left(\arcsin\right.}{5bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}\right.\right.$$

$$\frac{3B\left(a+bx^2\right)^{7/3}}{14b}$$

input
```
Int[(A + B*x)*(a + b*x^2)^(4/3),x]
```

output
```
(3*B*(a + b*x^2)^(7/3))/(14*b) + A*((3*x*(a + b*x^2)^(4/3))/11 + (8*a*((3*
x*(a + b*x^2)^(1/3))/5 - (2*3^(3/4)*Sqrt[2 - Sqrt[3]]*a*(a^(1/3) - (a + b*
x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))
/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqr
t[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1
/3))], -7 + 4*Sqrt[3]])/(5*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3
)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)])))/11)
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 234
```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Maple [F]

$$\int (Bx+A)\left(b\,x^{2}+a\right)^{\frac{4}{3}}dx$$

input
```
int((B*x+A)*(b*x^2+a)^(4/3),x)
```

output
```
int((B*x+A)*(b*x^2+a)^(4/3),x)
```

Fricas [F]

$$\int (A+Bx)\left(a+bx^2\right)^{4/3}\,dx=\int \left(bx^2+a\right)^{\frac{4}{3}}(Bx+A)\,dx$$

input
```
integrate((B*x+A)*(b*x^2+a)^(4/3),x, algorithm="fricas")
```

output
```
integral((B*b*x^3 + A*b*x^2 + B*a*x + A*a)*(b*x^2 + a)^(1/3), x)
```

Sympy [A] (verification not implemented)

Time = 1.65 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.08

$$\int (A+Bx)\left(a+bx^2\right)^{4/3}\,dx = Aa^{\frac{4}{3}}x\,{}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right) + \frac{A\sqrt[3]{a}bx^3\,{}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{3} - \frac{9Ba^{\frac{13}{3}}b\sqrt[3]{1+\frac{bx^2}{a}}}{56a^2b^2+56ab^3x^2} + \frac{9Ba^{\frac{13}{3}}b}{56a^2b^2+56ab^3x^2} - \frac{6Ba^{\frac{10}{3}}b^2x^2\sqrt[3]{1+\frac{bx^2}{a}}}{56a^2b^2+56ab^3x^2} + \frac{9Ba^{\frac{10}{3}}b^2x^2}{56a^2b^2+56ab^3x^2} + \frac{15Ba^{\frac{7}{3}}b^3x^4\sqrt[3]{1+\frac{bx^2}{a}}}{56a^2b^2+56ab^3x^2} + \frac{12Ba^{\frac{4}{3}}b^4x^6\sqrt[3]{1+\frac{bx^2}{a}}}{56a^2b^2+56ab^3x^2} + Ba\left(\begin{cases}\frac{\sqrt[3]{a}x^2}{2} & \text{for } b=0\\ \frac{3\left(a+bx^2\right)^{\frac{4}{3}}}{8b} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((B*x+A)*(b*x**2+a)**(4/3),x)
```

output
```
A*a**(4/3)*x*hyper((-1/3, 1/2), (3/2,), b*x**2*exp_polar(I*pi)/a) + A*a**(
1/3)*b*x**3*hyper((-1/3, 3/2), (5/2,), b*x**2*exp_polar(I*pi)/a)/3 - 9*B*a
**(13/3)*b*(1 + b*x**2/a)**(1/3)/(56*a**2*b**2 + 56*a*b**3*x**2) + 9*B*a**
(13/3)*b/(56*a**2*b**2 + 56*a*b**3*x**2) - 6*B*a**(10/3)*b**2*x**2*(1 + b*
x**2/a)**(1/3)/(56*a**2*b**2 + 56*a*b**3*x**2) + 9*B*a**(10/3)*b**2*x**2/(
56*a**2*b**2 + 56*a*b**3*x**2) + 15*B*a**(7/3)*b**3*x**4*(1 + b*x**2/a)**(
1/3)/(56*a**2*b**2 + 56*a*b**3*x**2) + 12*B*a**(4/3)*b**4*x**6*(1 + b*x**2
/a)**(1/3)/(56*a**2*b**2 + 56*a*b**3*x**2) + B*a*Piecewise((a**(1/3)*x**2/
2, Eq(b, 0)), (3*(a + b*x**2)**(4/3)/(8*b), True))
```

Maxima [F]

$$\int (A+Bx)\left(a+bx^2\right)^{4/3}\,dx=\int \left(bx^2+a\right)^{\frac{4}{3}}(Bx+A)\,dx$$

input
```
integrate((B*x+A)*(b*x^2+a)^(4/3),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(4/3)*(B*x + A), x)
```

Giac [F]

$$\int (A+Bx)\left(a+bx^2\right)^{4/3}\,dx=\int \left(bx^2+a\right)^{\frac{4}{3}}(Bx+A)\,dx$$

input
```
integrate((B*x+A)*(b*x^2+a)^(4/3),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(4/3)*(B*x + A), x)
```

Mupad [B] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.18

$$\int (A+Bx)\left(a+bx^2\right)^{4/3}\,dx=\frac{3\,B\,\left(b\,x^2+a\right)^{7/3}}{14\,b}+\frac{A\,x\,\left(b\,x^2+a\right)^{4/3}\,{}_2\mathrm{F}_1\left(-\frac{4}{3},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{\left(\frac{b\,x^2}{a}+1\right)^{4/3}}$$

input
```
int((a + b*x^2)^(4/3)*(A + B*x),x)
```

output
```
(3*B*(a + b*x^2)^(7/3))/(14*b) + (A*x*(a + b*x^2)^(4/3)*hypergeom([-4/3, 1
/2], 3/2, -(b*x^2)/a))/((b*x^2)/a + 1)^(4/3)
```

Reduce [F]

$$\int (A+Bx)\left(a+bx^2\right)^{4/3}\,dx = \frac{39(b\,x^2+a)^{\frac{1}{3}}\,a^2x}{55}+\frac{3(b\,x^2+a)^{\frac{1}{3}}\,a^2}{14}+\frac{3(b\,x^2+a)^{\frac{1}{3}}\,ab\,x^3}{11}+\frac{3(b\,x^2+a)^{\frac{1}{3}}\,ab\,x^2}{7}+\frac{3(b\,x^2+a)^{\frac{1}{3}}\,b^2x^4}{14}+\frac{16\left(\int \frac{1}{(b\,x^2+a)^{\frac{2}{3}}}dx\right)a^3}{55}$$

input
```
int((B*x+A)*(b*x^2+a)^(4/3),x)
```

output
```
(546*(a + b*x**2)**(1/3)*a**2*x + 165*(a + b*x**2)**(1/3)*a**2 + 210*(a +
b*x**2)**(1/3)*a*b*x**3 + 330*(a + b*x**2)**(1/3)*a*b*x**2 + 165*(a + b*x*
*2)**(1/3)*b**2*x**4 + 224*int((a + b*x**2)**(1/3)/(a + b*x**2),x)*a**3)/7
70
```

3.16 $\int (A+Bx)\sqrt[3]{a+bx^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 287

$$\int (A+Bx)\sqrt[3]{a+bx^2}\,dx=\frac{3}{5}Ax\sqrt[3]{a+bx^2}+\frac{3B(a+bx^2)^{4/3}}{8b} -\frac{2\,3^{3/4}\sqrt{2-\sqrt{3}}aA\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right),\ldots\right)}{5bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

output
```
3/5*A*x*(b*x^2+a)^(1/3)+3/8*B*(b*x^2+a)^(4/3)/b-2/5*3^(3/4)*(1/2*6^(1/2)-1
/2*2^(1/2))*a*A*(a^(1/3)-(b*x^2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3
)+(b*x^2+a)^(2/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)*Elliptic
F(((1+3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/
3)),2*I-I*3^(1/2))/b/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^
(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.60 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.29

$$\int (A+Bx)\sqrt[3]{a+bx^2}\,dx$$
$$=\frac{3(a+bx^2)\left(5aB+bx(8A+5Bx)\right)+16aAbx\left(1+\frac{bx^2}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{3}{2},-\frac{bx^2}{a}\right)}{40b\left(a+bx^2\right)^{2/3}}$$

input
```
Integrate[(A + B*x)*(a + b*x^2)^(1/3),x]
```

output
```
(3*(a + b*x^2)*(5*a*B + b*x*(8*A + 5*B*x)) + 16*a*A*b*x*(1 + (b*x^2)/a)^(2
/3)*Hypergeometric2F1[1/2, 2/3, 3/2, -((b*x^2)/a)])/(40*b*(a + b*x^2)^(2/3
))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {455, 211, 234, 760}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{a+bx^2}(A+Bx)\,dx$$
$$\Big\downarrow \text{455}$$
$$A\int \sqrt[3]{bx^2+a}dx+\frac{3B\left(a+bx^2\right)^{4/3}}{8b}$$
$$\Big\downarrow \text{211}$$
$$A\left(\frac{2}{5}a\int \frac{1}{\left(bx^2+a\right)^{2/3}}dx+\frac{3}{5}x\sqrt[3]{a+bx^2}\right)+\frac{3B\left(a+bx^2\right)^{4/3}}{8b}$$

$$\Big\downarrow \; 234$$

$$A\left(\frac{3a\sqrt{bx^2}\int \frac{1}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}}{5bx}+\frac{3}{5}x\sqrt[3]{a+bx^2}\right)+\frac{3B\left(a+bx^2\right)^{4/3}}{8b}$$

$$\Big\downarrow \; 760$$

$$A\left(\frac{3}{5}x\sqrt[3]{a+bx^2}-\frac{2\, 3^{3/4}\sqrt{2-\sqrt{3}}a\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\right.}{\left(1-\right.}\right.\right.}{5bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}\right.$$

$$\frac{3B\left(a+bx^2\right)^{4/3}}{8b}$$

input
```
Int[(A + B*x)*(a + b*x^2)^(1/3),x]
```

output
```
(3*B*(a + b*x^2)^(4/3))/(8*b) + A*((3*x*(a + b*x^2)^(1/3))/5 - (2*3^(3/4)*
Sqrt[2 - Sqrt[3]]*a*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*
(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2
)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/(
(1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(5*b*x*Sqrt[
-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*
x^2)^(1/3))^2)]))
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 234
```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Maple [F]

$$\int (Bx + A) \left(b\,x^{2}+a \right)^{\frac{1}{3}}dx$$

input
```
int((B*x+A)*(b*x^2+a)^(1/3),x)
```

output
```
int((B*x+A)*(b*x^2+a)^(1/3),x)
```

Fricas [F]

$$\int (A + Bx)\sqrt[3]{a + bx^2}\,dx = \int (bx^2 + a)^{\frac{1}{3}}(Bx + A)\,dx$$

input
```
integrate((B*x+A)*(b*x^2+a)^(1/3),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(1/3)*(B*x + A), x)
```

Sympy [A] (verification not implemented)

Time = 0.84 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.19

$$\int (A+Bx)\sqrt[3]{a+bx^2}\,dx = A\sqrt[3]{a}x\,{}_2F_1\left(\begin{matrix} -\frac{1}{3},\frac{1}{2} \\ \frac{3}{2} \end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)+B\left(\begin{cases}\frac{\sqrt[3]{a}x^2}{2} & \text{for}\ b=0\\ \frac{3(a+bx^2)^{\frac{4}{3}}}{8b} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((B*x+A)*(b*x**2+a)**(1/3),x)
```

output
```
A*a**(1/3)*x*hyper((-1/3, 1/2), (3/2,), b*x**2*exp_polar(I*pi)/a) + B*Piec
ewise((a**(1/3)*x**2/2, Eq(b, 0)), (3*(a + b*x**2)**(4/3)/(8*b), True))
```

Maxima [F]

$$\int (A+Bx)\sqrt[3]{a+bx^2}\,dx = \int (bx^2+a)^{\frac{1}{3}}(Bx+A)\,dx$$

input
```
integrate((B*x+A)*(b*x^2+a)^(1/3),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(1/3)*(B*x + A), x)
```

Giac [F]

$$\int (A+Bx)\sqrt[3]{a+bx^2}\,dx = \int (bx^2+a)^{\frac{1}{3}}(Bx+A)\,dx$$

input
```
integrate((B*x+A)*(b*x^2+a)^(1/3),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(1/3)*(B*x + A), x)
```

Mupad [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.19

$$\int (A+Bx)\sqrt[3]{a+bx^2}\,dx = \frac{3\,B\,{\left(b\,x^2+a\right)}^{4/3}}{8\,b}+\frac{A\,x\,{\left(b\,x^2+a\right)}^{1/3}\,{}_2\mathrm{F}_1\left(-\frac{1}{3},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{{\left(\frac{b\,x^2}{a}+1\right)}^{1/3}}$$

input
```
int((a + b*x^2)^(1/3)*(A + B*x),x)
```

output
```
(3*B*(a + b*x^2)^(4/3))/(8*b) + (A*x*(a + b*x^2)^(1/3)*hypergeom([-1/3, 1/
2], 3/2, -(b*x^2)/a))/((b*x^2)/a + 1)^(1/3)
```

Reduce [F]

$$\int (A+Bx)\sqrt[3]{a+bx^2}\,dx = \frac{3(b\,x^2+a)^{\frac{1}{3}}\,ax}{5}+\frac{3(b\,x^2+a)^{\frac{1}{3}}\,a}{8}+\frac{3(b\,x^2+a)^{\frac{1}{3}}\,b\,x^2}{8}+\frac{2\left(\int \frac{1}{(b\,x^2+a)^{\frac{2}{3}}}dx\right)a^2}{5}$$

input
```
int((B*x+A)*(b*x^2+a)^(1/3),x)
```

output
```
(24*(a + b*x**2)**(1/3)*a*x + 15*(a + b*x**2)**(1/3)*a + 15*(a + b*x**2)**
(1/3)*b*x**2 + 16*int((a + b*x**2)**(1/3)/(a + b*x**2),x)*a**2)/40
```

3.17 $\int \frac{A+Bx}{\left(a+bx^2\right)^{2/3}} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 267

$$\int \frac{A+Bx}{\left(a+bx^2\right)^{2/3}} \, dx = \frac{3B\sqrt[3]{a+bx^2}}{2b} - \frac{3^{3/4}\sqrt{2-\sqrt{3}}A\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right),-7+4\sqrt{3}\right)}{bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

output
```
3/2*B*(b*x^2+a)^(1/3)/b-3^(3/4)*(1/2*6^(1/2)-1/2*2^(1/2))*A*(a^(1/3)-(b*x^
2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/((1-3^(1/2)
)*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)*EllipticF(((1+3^(1/2))*a^(1/3)-(b*x^2+
a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3)),2*I-I*3^(1/2))/b/x/(-a^(1/
3)*(a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2
)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.09 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.25

$$\int \frac{A+Bx}{(a+bx^2)^{2/3}}\,dx=\frac{3B\sqrt[3]{a+bx^2}}{2b}+\frac{Ax\left(\frac{a+bx^2}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{3}{2},-\frac{bx^2}{a}\right)}{(a+bx^2)^{2/3}}$$

input
```
Integrate[(A + B*x)/(a + b*x^2)^(2/3),x]
```

output
```
(3*B*(a + b*x^2)^(1/3))/(2*b) + (A*x*((a + b*x^2)/a)^(2/3)*Hypergeometric2
F1[1/2, 2/3, 3/2, -((b*x^2)/a)])/(a + b*x^2)^(2/3)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {455, 234, 760}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{(a+bx^2)^{2/3}}\,dx$$

$$\Big\downarrow \text{455}$$

$$A\int \frac{1}{(bx^2+a)^{2/3}}dx+\frac{3B\sqrt[3]{a+bx^2}}{2b}$$

$$\Big\downarrow \text{234}$$

$$\frac{3A\sqrt{bx^2}\int \frac{1}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}}{2bx}+\frac{3B\sqrt[3]{a+bx^2}}{2b}$$

$$\Big\downarrow \text{760}$$

$$\frac{3B\sqrt[3]{a+bx^2}}{2b} - \frac{3^{3/4}\sqrt{2-\sqrt{3}}A\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right),\right.}{bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

input
```
Int[(A + B*x)/(a + b*x^2)^(2/3),x]
```

output
```
(3*B*(a + b*x^2)^(1/3))/(2*b) - (3^(3/4)*Sqrt[2 - Sqrt[3]]*A*(a^(1/3) - (a
 + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(
2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1
+ Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^
2)^(1/3))], -7 + 4*Sqrt[3]])/(b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(
1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)])
```

Defintions of rubi rules used

rule 234
```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Maple [F]

$$\int \frac{Bx+A}{(b\,x^2+a)^{\frac{2}{3}}}dx$$

input
```
int((B*x+A)/(b*x^2+a)^(2/3),x)
```

output
```
int((B*x+A)/(b*x^2+a)^(2/3),x)
```

Fricas [F]

$$\int \frac{A+Bx}{(a+bx^2)^{2/3}}\,dx = \int \frac{Bx+A}{(bx^2+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate((B*x+A)/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

output
```
integral((B*x + A)/(b*x^2 + a)^(2/3), x)
```

Sympy [A] (verification not implemented)

Time = 0.76 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.20

$$\int \frac{A+Bx}{(a+bx^2)^{2/3}}\,dx = \frac{Ax\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\\frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{a^{\frac{2}{3}}} + B\left(\begin{cases}\frac{x^2}{2a^{\frac{2}{3}}} & \text{for } b=0\\ \frac{3\sqrt[3]{a+bx^2}}{2b} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((B*x+A)/(b*x**2+a)**(2/3),x)
```

output
```
A*x*hyper((1/2, 2/3), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(2/3) + B*Piece
wise((x**2/(2*a**(2/3)), Eq(b, 0)), (3*(a + b*x**2)**(1/3)/(2*b), True))
```

Maxima [F]

$$\int \frac{A+Bx}{(a+bx^2)^{2/3}}\,dx = \int \frac{Bx+A}{(bx^2+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate((B*x+A)/(b*x^2+a)^(2/3),x, algorithm="maxima")
```

output
```
integrate((B*x + A)/(b*x^2 + a)^(2/3), x)
```

Giac [F]

$$\int \frac{A+Bx}{(a+bx^2)^{2/3}}\,dx = \int \frac{Bx+A}{(bx^2+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate((B*x+A)/(b*x^2+a)^(2/3),x, algorithm="giac")
```

output
```
integrate((B*x + A)/(b*x^2 + a)^(2/3), x)
```

Mupad [B] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.20

$$\int \frac{A+Bx}{(a+bx^2)^{2/3}}\,dx = \frac{3\,B\,(b\,x^2+a)^{1/3}}{2\,b} + \frac{A\,x\,\left(\frac{b\,x^2}{a}+1\right)^{2/3}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{2}{3};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{(b\,x^2+a)^{2/3}}$$

input
```
int((A + B*x)/(a + b*x^2)^(2/3),x)
```

output
```
(3*B*(a + b*x^2)^(1/3))/(2*b) + (A*x*((b*x^2)/a + 1)^(2/3)*hypergeom([1/2,
 2/3], 3/2, -(b*x^2)/a))/(a + b*x^2)^(2/3)
```

Reduce [F]

$$\int \frac{A+Bx}{(a+bx^2)^{2/3}}\,dx = \frac{3(b\,x^2+a)^{\frac{1}{3}}}{2} + \left(\int \frac{1}{(b\,x^2+a)^{\frac{2}{3}}}dx\right)a$$

input
```
int((B*x+A)/(b*x^2+a)^(2/3),x)
```

output
```
(3*(a + b*x**2)**(1/3) + 2*int(1/(a + b*x**2)**(2/3),x)*a)/2
```

3.18 $\int \frac{A+Bx}{\left(a+bx^2\right)^{5/3}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 283

$$\int \frac{A+Bx}{\left(a+bx^2\right)^{5/3}}\,dx=-\frac{3(aB-Abx)}{4ab\left(a+bx^2\right)^{2/3}}$$

$$-\frac{3^{3/4}\sqrt{2-\sqrt{3}}A\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+b}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+b}}\right.\right.}{4abx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

output
```
1/4*(3*A*b*x-3*B*a)/a/b/(b*x^2+a)^(2/3)-1/4*3^(3/4)*(1/2*6^(1/2)-1/2*2^(1/
2))*A*(a^(1/3)-(b*x^2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a
)^(2/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)*EllipticF(((1+3^(1
/2))*a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3)),2*I-I*
3^(1/2))/a/b/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b
*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.24

$$\int \frac{A+Bx}{(a+bx^2)^{5/3}}\,dx=\frac{-3aB+3Abx+Abx\left(1+\frac{bx^2}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{3}{2},-\frac{bx^2}{a}\right)}{4ab\,(a+bx^2)^{2/3}}$$

input
```
Integrate[(A + B*x)/(a + b*x^2)^(5/3),x]
```

output
```
(-3*a*B + 3*A*b*x + A*b*x*(1 + (b*x^2)/a)^(2/3)*Hypergeometric2F1[1/2, 2/3
, 3/2, -((b*x^2)/a)])/(4*a*b*(a + b*x^2)^(2/3))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {454, 234, 760}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{(a+bx^2)^{5/3}}\,dx$$

$$\Big\downarrow \text{454}$$

$$\frac{A\int \frac{1}{(bx^2+a)^{2/3}}dx}{4a}-\frac{3(aB-Abx)}{4ab\,(a+bx^2)^{2/3}}$$

$$\Big\downarrow \text{234}$$

$$\frac{3A\sqrt{bx^2}\int \frac{1}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}}{8abx}-\frac{3(aB-Abx)}{4ab\,(a+bx^2)^{2/3}}$$

$$\Big\downarrow \text{760}$$

$$-\frac{3^{3/4}\sqrt{2-\sqrt{3}}A\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)\right.}{4abx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

$$\frac{3(aB-Abx)}{4ab\left(a+bx^2\right)^{2/3}}$$

input
```
Int[(A + B*x)/(a + b*x^2)^(5/3),x]
```

output
```
(-3*(a*B - A*b*x))/(4*a*b*(a + b*x^2)^(2/3)) - (3^(3/4)*Sqrt[2 - Sqrt[3]]*
A*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3)
+ (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*Ellipt
icF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1
/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(4*a*b*x*Sqrt[-((a^(1/3)*(a^(1
/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)])
```

Defintions of rubi rules used

rule 234
```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 454
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*d
 - b*c*x)/(2*a*b*(p + 1)))*(a + b*x^2)^(p + 1), x] + Simp[c*((2*p + 3)/(2*a
*(p + 1)))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d}, x] && L
tQ[p, -1] && NeQ[p, -3/2]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Maple [F]

$$\int \frac{Bx+A}{\left(b\,x^{2}+a\right)^{\frac{5}{3}}}dx$$

input
```
int((B*x+A)/(b*x^2+a)^(5/3),x)
```

output
```
int((B*x+A)/(b*x^2+a)^(5/3),x)
```

Fricas [F]

$$\int \frac{A+Bx}{(a+bx^2)^{5/3}}\,dx = \int \frac{Bx+A}{(bx^2+a)^{\frac{5}{3}}}\,dx$$

input
```
integrate((B*x+A)/(b*x^2+a)^(5/3),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(1/3)*(B*x + A)/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [A] (verification not implemented)

Time = 2.23 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.19

$$\int \frac{A+Bx}{(a+bx^2)^{5/3}}\,dx = \frac{Ax\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{3}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{a^{\frac{5}{3}}} + B\left(\begin{cases}-\frac{3}{4b(a+bx^2)^{\frac{2}{3}}} & \text{for } b\neq 0\\ \frac{x^2}{2a^{\frac{5}{3}}} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((B*x+A)/(b*x**2+a)**(5/3),x)
```

output
```
A*x*hyper((1/2, 5/3), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(5/3) + B*Piece
wise((-3/(4*b*(a + b*x**2)**(2/3)), Ne(b, 0)), (x**2/(2*a**(5/3)), True))
```

Maxima [F]

$$\int \frac{A+Bx}{(a+bx^2)^{5/3}}\,dx = \int \frac{Bx+A}{(bx^2+a)^{\frac{5}{3}}}\,dx$$

input
```
integrate((B*x+A)/(b*x^2+a)^(5/3),x, algorithm="maxima")
```

output
```
integrate((B*x + A)/(b*x^2 + a)^(5/3), x)
```

Giac [F]

$$\int \frac{A+Bx}{(a+bx^2)^{5/3}}\,dx = \int \frac{Bx+A}{(bx^2+a)^{\frac{5}{3}}}\,dx$$

input
```
integrate((B*x+A)/(b*x^2+a)^(5/3),x, algorithm="giac")
```

output
```
integrate((B*x + A)/(b*x^2 + a)^(5/3), x)
```

Mupad [B] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.19

$$\int \frac{A+Bx}{\left(a+bx^2\right)^{5/3}}\,dx=\frac{A\,x\,\left(\frac{b\,x^2}{a}+1\right)^{5/3}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{5}{3};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{\left(b\,x^2+a\right)^{5/3}}-\frac{3\,B}{4\,b\,\left(b\,x^2+a\right)^{2/3}}$$

input
```
int((A + B*x)/(a + b*x^2)^(5/3),x)
```

output
```
(A*x*((b*x^2)/a + 1)^(5/3)*hypergeom([1/2, 5/3], 3/2, -(b*x^2)/a))/(a + b*
x^2)^(5/3) - (3*B)/(4*b*(a + b*x^2)^(2/3))
```

Reduce [F]

$$\int \frac{A+Bx}{\left(a+bx^2\right)^{5/3}}\,dx=\left(\int \frac{x}{\left(b\,x^2+a\right)^{\frac{2}{3}} a+\left(b\,x^2+a\right)^{\frac{2}{3}} b\,x^2}dx\right) b+\left(\int \frac{1}{\left(b\,x^2+a\right)^{\frac{2}{3}} a+\left(b\,x^2+a\right)^{\frac{2}{3}} b\,x^2}dx\right) a$$

input
```
int((B*x+A)/(b*x^2+a)^(5/3),x)
```

output
```
int(x/((a + b*x**2)**(2/3)*a + (a + b*x**2)**(2/3)*b*x**2),x)*b + int(1/((
a + b*x**2)**(2/3)*a + (a + b*x**2)**(2/3)*b*x**2),x)*a
```

3.19 $\int (A+Bx)\left(a+bx^2\right)^{2/3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 573

$$\int (A+Bx)\left(a+bx^2\right)^{2/3}\,dx = \frac{3}{7}Ax\left(a+bx^2\right)^{2/3} + \frac{3B\left(a+bx^2\right)^{5/3}}{10b} - \frac{12aAx}{7\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}$$
$$+ \frac{6\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^{4/3}A\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right)\right.}{7bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$
$$- \frac{4\sqrt{2}3^{3/4}a^{4/3}A\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx}}\right.\right.}{7bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

output
```
3/7*A*x*(b*x^2+a)^(2/3)+3/10*B*(b*x^2+a)^(5/3)/b-12*a*A*x/(7*(1-3^(1/2))*a
^(1/3)-7*(b*x^2+a)^(1/3))+6/7*3^(1/4)*(1/2*6^(1/2)+1/2*2^(1/2))*a^(4/3)*A*
(a^(1/3)-(b*x^2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3
))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)*EllipticE(((1+3^(1/2))*a
^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3)),2*I-I*3^(1/2
))/b/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^
(1/3))^2)^(1/2)-4/7*2^(1/2)*3^(3/4)*a^(4/3)*A*(a^(1/3)-(b*x^2+a)^(1/3))*((
a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/((1-3^(1/2))*a^(1/3)-(b*x
^2+a)^(1/3))^2)^(1/2)*EllipticF(((1+3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))/((1-
3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3)),2*I-I*3^(1/2))/b/x/(-a^(1/3)*(a^(1/3)-(b
*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.35 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.14

$$\int (A+Bx)\left(a+bx^2\right)^{2/3}\,dx=\frac{3(a+bx^2)\left(7aB+bx(10A+7Bx)\right)+40aAbx\sqrt[3]{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{70b\sqrt[3]{a+bx^2}}$$

input
```
Integrate[(A + B*x)*(a + b*x^2)^(2/3),x]
```

output
```
(3*(a + b*x^2)*(7*a*B + b*x*(10*A + 7*B*x)) + 40*a*A*b*x*(1 + (b*x^2)/a)^(
1/3)*Hypergeometric2F1[1/3, 1/2, 3/2, -((b*x^2)/a)])/(70*b*(a + b*x^2)^(1/
3))
```

Rubi [A] (warning: unable to verify)

Time = 0.50 (sec) , antiderivative size = 614, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {455, 211, 233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^{2/3} (A+Bx) \, dx$$

$$\Big\downarrow \text{455}$$

$$A \int \left(bx^2+a\right)^{2/3} dx + \frac{3B\left(a+bx^2\right)^{5/3}}{10b}$$

$$\Big\downarrow \text{211}$$

$$A\left(\frac{4}{7}a\int \frac{1}{\sqrt[3]{bx^2+a}}dx + \frac{3}{7}x\left(a+bx^2\right)^{2/3}\right) + \frac{3B\left(a+bx^2\right)^{5/3}}{10b}$$

$$\Big\downarrow \text{233}$$

$$A\left(\frac{6a\sqrt{bx^2}\int \frac{\sqrt[3]{bx^2+a}}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}}{7bx} + \frac{3}{7}x\left(a+bx^2\right)^{2/3}\right) + \frac{3B\left(a+bx^2\right)^{5/3}}{10b}$$

$$\Big\downarrow \text{833}$$

$$A\left(\frac{6a\sqrt{bx^2}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a} - \int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}\right)}{7bx} + \frac{3}{7}x\left(a+bx^2\right)^{2/3}\right) + \frac{3B\left(a+bx^2\right)^{5/3}}{10b}$$

$$\Big\downarrow \text{760}$$

$$A\left(\frac{6a\sqrt{bx^2}\left(-\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\ldots}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+\ldots}\right.}}}{\sqrt[4]{3}\sqrt{bx^2}\sqrt{-\frac{\sqrt[3]{a}\left(\ldots\right.}{\left(\left(1-\sqrt{3}\right)\ldots\right.}}}\right.}{7bx}\right.$$

$$\frac{3B\left(a+bx^2\right)^{5/3}}{10b}$$

$\downarrow$ 2418

$$A\left(\frac{6a\sqrt{bx^2}\left(-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b\ldots}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b\ldots}}\right.\right.}{\sqrt[4]{3}\sqrt{bx^2}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}\right.}{\ldots}\right.$$

$$\frac{3B\left(a+bx^2\right)^{5/3}}{10b}$$

input
```
Int[(A + B*x)*(a + b*x^2)^(2/3),x]
```

output
```
(3*B*(a + b*x^2)^(5/3))/(10*b) + A*((3*x*(a + b*x^2)^(2/3))/7 + (6*a*Sqrt[
b*x^2]*((-2*Sqrt[b*x^2])/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3)) + (3^
(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3
) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3)
- (a + b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x
^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/
(Sqrt[b*x^2]*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])
*a^(1/3) - (a + b*x^2)^(1/3))^2)]) - (2*Sqrt[2 - Sqrt[3]]*(1 + Sqrt[3])*a^
(1/3)*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1
/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*El
lipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*
a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[b*x^2]*Sqrt[
-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*
x^2)^(1/3))^2)])))/(7*b*x))
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int (Bx+A)\left(b\,x^{2}+a\right)^{\frac{2}{3}}dx$$

input
```
int((B*x+A)*(b*x^2+a)^(2/3),x)
```

output
```
int((B*x+A)*(b*x^2+a)^(2/3),x)
```

Fricas [F]

$$\int (A+Bx)\left(a+bx^2\right)^{2/3}\,dx=\int \left(bx^{2}+a\right)^{\frac{2}{3}}(Bx+A)\,dx$$

input
```
integrate((B*x+A)*(b*x^2+a)^(2/3),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(2/3)*(B*x + A), x)
```

Sympy [A] (verification not implemented)

Time = 0.92 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.09

$$\int (A+Bx)\left(a+bx^2\right)^{2/3}\,dx = Aa^{\frac{2}{3}}x_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)+B\left(\begin{cases}\frac{a^{\frac{2}{3}}x^2}{2} & \text{for } b=0\\ \frac{3(a+bx^2)^{\frac{5}{3}}}{10b} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((B*x+A)*(b*x**2+a)**(2/3),x)
```

output
```
A*a**(2/3)*x*hyper((-2/3, 1/2), (3/2,), b*x**2*exp_polar(I*pi)/a) + B*Piec
ewise((a**(2/3)*x**2/2, Eq(b, 0)), (3*(a + b*x**2)**(5/3)/(10*b), True))
```

Maxima [F]

$$\int (A+Bx)\left(a+bx^2\right)^{2/3}\,dx = \int \left(bx^2+a\right)^{\frac{2}{3}}(Bx+A)\,dx$$

input
```
integrate((B*x+A)*(b*x^2+a)^(2/3),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(2/3)*(B*x + A), x)
```

Giac [F]

$$\int (A+Bx)\left(a+bx^2\right)^{2/3}\,dx = \int \left(bx^2+a\right)^{\frac{2}{3}}(Bx+A)\,dx$$

input
```
integrate((B*x+A)*(b*x^2+a)^(2/3),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(2/3)*(B*x + A), x)
```

Mupad [B] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.09

$$\int (A+Bx) \left(a+bx^2\right)^{2/3} \, dx = \frac{3 B \left(b x^2+a\right)^{5/3}}{10 b} + \frac{A x \left(b x^2+a\right)^{2/3} {}_2\mathrm{F}_1\left(-\frac{2}{3}, \frac{1}{2};\ \frac{3}{2};\ -\frac{b x^2}{a}\right)}{\left(\frac{b x^2}{a}+1\right)^{2/3}}$$

input
```
int((a + b*x^2)^(2/3)*(A + B*x),x)
```

output
```
(3*B*(a + b*x^2)^(5/3))/(10*b) + (A*x*(a + b*x^2)^(2/3)*hypergeom([-2/3, 1
/2], 3/2, -(b*x^2)/a))/((b*x^2)/a + 1)^(2/3)
```

Reduce [F]

$$\int (A+Bx) \left(a+bx^2\right)^{2/3} \, dx = \frac{3(b x^2+a)^{\frac{2}{3}} a x}{7} + \frac{3(b x^2+a)^{\frac{2}{3}} a}{10} + \frac{3(b x^2+a)^{\frac{2}{3}} b x^2}{10} + \frac{4\left(\int \frac{1}{(b x^2+a)^{\frac{1}{3}}} dx\right) a^2}{7}$$

input
```
int((B*x+A)*(b*x^2+a)^(2/3),x)
```

output
```
(30*(a + b*x**2)**(2/3)*a*x + 21*(a + b*x**2)**(2/3)*a + 21*(a + b*x**2)**
(2/3)*b*x**2 + 40*int((a + b*x**2)**(2/3)/(a + b*x**2),x)*a**2)/70
```

3.20 $\int \frac{A+Bx}{\sqrt[3]{a+bx^2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 551

$$\int \frac{A+Bx}{\sqrt[3]{a+bx^2}}\,dx=\frac{3B(a+bx^2)^{2/3}}{4b}-\frac{3Ax}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}$$
$$+\frac{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}A\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx}}\right.\right.}{2bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$
$$-\frac{\sqrt{2}3^{3/4}\sqrt[3]{a}A\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+b}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+b}}\right.\right.}{bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

output
```
3/4*B*(b*x^2+a)^(2/3)/b-3*A*x/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))+3/2*3^
(1/4)*(1/2*6^(1/2)+1/2*2^(1/2))*a^(1/3)*A*(a^(1/3)-(b*x^2+a)^(1/3))*((a^(2
/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a
)^(1/3))^2)^(1/2)*EllipticE(((1+3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1
/2))*a^(1/3)-(b*x^2+a)^(1/3)),2*I-I*3^(1/2))/b/x/(-a^(1/3)*(a^(1/3)-(b*x^2
+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)-2^(1/2)*3^(3/4)*
a^(1/3)*A*(a^(1/3)-(b*x^2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x
^2+a)^(2/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)*EllipticF(((1+
3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3)),2*
I-I*3^(1/2))/b/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-
(b*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.42 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.12

$$\int \frac{A+Bx}{\sqrt[3]{a+bx^2}}\,dx = \frac{3B(a+bx^2)^{2/3}}{4b} + \frac{Ax\sqrt[3]{\frac{a+bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{\sqrt[3]{a+bx^2}}$$

input
```
Integrate[(A + B*x)/(a + b*x^2)^(1/3),x]
```

output
```
(3*B*(a + b*x^2)^(2/3))/(4*b) + (A*x*((a + b*x^2)/a)^(1/3)*Hypergeometric2
F1[1/3, 1/2, 3/2, -((b*x^2)/a)])/(a + b*x^2)^(1/3)
```

Rubi [A] (warning: unable to verify)

Time = 0.49 (sec) , antiderivative size = 595, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {455, 233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{\sqrt[3]{a+bx^2}}\,dx$$

$$\Big\downarrow \; 455$$

$$A\int \frac{1}{\sqrt[3]{bx^2+a}}dx+\frac{3B\left(a+bx^2\right)^{2/3}}{4b}$$

$$\Big\downarrow \; 233$$

$$\frac{3A\sqrt{bx^2}\int \frac{\sqrt[3]{bx^2+a}}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}}{2bx}+\frac{3B\left(a+bx^2\right)^{2/3}}{4b}$$

$$\Big\downarrow \; 833$$

$$\frac{3A\sqrt{bx^2}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}-\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}\right)}{2bx}+\frac{3B\left(a+bx^2\right)^{2/3}}{4b}$$

$$\Big\downarrow \; 760$$

$$\frac{3A\sqrt{bx^2}\left(-\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+\cdots\right.}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\cdots\right.}}}{\sqrt[4]{3}\sqrt{bx^2}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}\cdots\right.}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}\cdots\right.}}}\cdots\right.}{2bx}\cdots$$

$$\frac{3B\left(a+bx^2\right)^{2/3}}{4b}$$

$$\Big\downarrow \; 2418$$

$$\frac{3A\sqrt{bx^2}\left(-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2}\cdots}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2}\cdots}\right.\right.}{\sqrt[4]{3}\sqrt{bx^2}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}\cdots}{\cdots}$$

$$\frac{3B\left(a+bx^2\right)^{2/3}}{4b}$$

input
```
Int[(A + B*x)/(a + b*x^2)^(1/3),x]
```

output
```
(3*B*(a + b*x^2)^(2/3))/(4*b) + (3*A*Sqrt[b*x^2]*((-2*Sqrt[b*x^2])/((1 - S
qrt[3])*a^(1/3) - (a + b*x^2)^(1/3)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*
(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) +
(a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*Elliptic
E[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3
) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(Sqrt[b*x^2]*Sqrt[-((a^(1/3)*(a^
(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)
]) - (2*Sqrt[2 - Sqrt[3]]*(1 + Sqrt[3])*a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/
3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - S
qrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^
(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -
7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[b*x^2]*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2
)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)])))/(2*b*x)
```

Defintions of rubi rules used

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] && !LeQ[p, -1]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int \frac{Bx + A}{(b\,x^2 + a)^{\frac{1}{3}}}dx$$

input
```
int((B*x+A)/(b*x^2+a)^(1/3),x)
```

output
```
int((B*x+A)/(b*x^2+a)^(1/3),x)
```

Fricas [F]

$$\int \frac{A+Bx}{\sqrt[3]{a+bx^2}}\,dx = \int \frac{Bx+A}{(bx^2+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate((B*x+A)/(b*x^2+a)^(1/3),x, algorithm="fricas")
```

output
```
integral((B*x + A)/(b*x^2 + a)^(1/3), x)
```

Sympy [A] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.10

$$\int \frac{A+Bx}{\sqrt[3]{a+bx^2}}\,dx = \frac{Ax{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{\sqrt[3]{a}} + B\left(\begin{cases}\frac{x^2}{2\sqrt[3]{a}} & \text{for } b=0\\ \frac{3(a+bx^2)^{\frac{2}{3}}}{4b} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((B*x+A)/(b*x**2+a)**(1/3),x)
```

output
```
A*x*hyper((1/3, 1/2), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(1/3) + B*Piece
wise((x**2/(2*a**(1/3)), Eq(b, 0)), (3*(a + b*x**2)**(2/3)/(4*b), True))
```

Maxima [F]

$$\int \frac{A+Bx}{\sqrt[3]{a+bx^2}}\,dx = \int \frac{Bx+A}{(bx^2+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate((B*x+A)/(b*x^2+a)^(1/3),x, algorithm="maxima")
```

output
```
integrate((B*x + A)/(b*x^2 + a)^(1/3), x)
```

Giac [F]

$$\int \frac{A+Bx}{\sqrt[3]{a+bx^2}}\,dx=\int \frac{Bx+A}{(bx^2+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate((B*x+A)/(b*x^2+a)^(1/3),x, algorithm="giac")
```

output
```
integrate((B*x + A)/(b*x^2 + a)^(1/3), x)
```

Mupad [B] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.10

$$\int \frac{A+Bx}{\sqrt[3]{a+bx^2}}\,dx=\frac{3\,B\,(b\,x^2+a)^{2/3}}{4\,b}+\frac{A\,x\,\left(\frac{b\,x^2}{a}+1\right)^{1/3}\,{}_2\mathrm{F}_1\left(\frac{1}{3},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{(b\,x^2+a)^{1/3}}$$

input
```
int((A + B*x)/(a + b*x^2)^(1/3),x)
```

output
```
(3*B*(a + b*x^2)^(2/3))/(4*b) + (A*x*((b*x^2)/a + 1)^(1/3)*hypergeom([1/3,
 1/2], 3/2, -(b*x^2)/a))/(a + b*x^2)^(1/3)
```

Reduce [F]

$$\int \frac{A+Bx}{\sqrt[3]{a+bx^2}}\,dx=\left(\int \frac{x}{(b\,x^2+a)^{\frac{1}{3}}}dx\right)b+\left(\int \frac{1}{(b\,x^2+a)^{\frac{1}{3}}}dx\right)a$$

input
```
int((B*x+A)/(b*x^2+a)^(1/3),x)
```

output
```
int(x/(a + b*x**2)**(1/3),x)*b + int(1/(a + b*x**2)**(1/3),x)*a
```

3.21 $\int \frac{A+Bx}{(a+bx^2)^{4/3}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 566

$$\int \frac{A+Bx}{(a+bx^2)^{4/3}}\,dx = -\frac{3(aB-Abx)}{2ab\sqrt[3]{a+bx^2}}+\frac{3Ax}{2a\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}$$

$$-\frac{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}A\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right)|-}{4a^{2/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

$$+\frac{3^{3/4}A\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right),-7\right.}{\sqrt{2}a^{2/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

output
```
1/2*(3*A*b*x-3*B*a)/a/b/(b*x^2+a)^(1/3)+3/2*A*x/a/((1-3^(1/2))*a^(1/3)-(b*
x^2+a)^(1/3))-3/4*3^(1/4)*(1/2*6^(1/2)+1/2*2^(1/2))*A*(a^(1/3)-(b*x^2+a)^(
1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/((1-3^(1/2))*a^(1
/3)-(b*x^2+a)^(1/3))^2)^(1/2)*EllipticE(((1+3^(1/2))*a^(1/3)-(b*x^2+a)^(1/
3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3)),2*I-I*3^(1/2))/a^(2/3)/b/x/(-a^(
1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1
/2)+1/2*3^(3/4)*A*(a^(1/3)-(b*x^2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1
/3)+(b*x^2+a)^(2/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)*Ellipt
icF(((1+3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(
1/3)),2*I-I*3^(1/2))*2^(1/2)/a^(2/3)/b/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3
))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 9.71 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.12

$$\int \frac{A+Bx}{(a+bx^2)^{4/3}}\,dx = \frac{-3aB+3Abx-Abx\sqrt[3]{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{2ab\sqrt[3]{a+bx^2}}$$

input
```
Integrate[(A + B*x)/(a + b*x^2)^(4/3),x]
```

output
```
(-3*a*B + 3*A*b*x - A*b*x*(1 + (b*x^2)/a)^(1/3)*Hypergeometric2F1[1/3, 1/2
, 3/2, -((b*x^2)/a)])/(2*a*b*(a + b*x^2)^(1/3))
```

Rubi [A] (warning: unable to verify)

Time = 0.52 (sec) , antiderivative size = 609, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {454, 233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{(a+bx^2)^{4/3}}\,dx$$

$\Big\downarrow$ 454

$$-\frac{A\int \frac{1}{\sqrt[3]{bx^2+a}}dx}{2a}-\frac{3(aB-Abx)}{2ab\sqrt[3]{a+bx^2}}$$

$\Big\downarrow$ 233

$$-\frac{3A\sqrt{bx^2}\int \frac{\sqrt[3]{bx^2+a}}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}}{4abx}-\frac{3(aB-Abx)}{2ab\sqrt[3]{a+bx^2}}$$

$\Big\downarrow$ 833

$$-\frac{3A\sqrt{bx^2}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}\int\frac{1}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}-\int\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}\right)}{4abx}-\frac{3(aB-Abx)}{2ab\sqrt[3]{a+bx^2}}$$

$\Big\downarrow$ 760

$$-\frac{3A\sqrt{bx^2}\left(-\int\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(\cdots}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+\cdots}\right.}}}{\sqrt[4]{3}\sqrt{bx^2}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{\cdots}\right.}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{\cdots}\right.}}}\cdots\right)}{4abx}-\frac{3(aB-Abx)}{2ab\sqrt[3]{a+bx^2}}$$

$\Big\downarrow$ 2418

$$-\frac{3A\sqrt{bx^2}\left(-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx}}\right.\right.}{\sqrt[4]{3}\sqrt{bx^2}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}\right.}{}$$

$$\frac{3(aB-Abx)}{2ab\sqrt[3]{a+bx^2}}$$

input
```
Int[(A + B*x)/(a + b*x^2)^(4/3),x]
```

output
```
(-3*(a*B - A*b*x))/(2*a*b*(a + b*x^2)^(1/3)) - (3*A*Sqrt[b*x^2]*((-2*Sqrt[
b*x^2])/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3)) + (3^(1/4)*Sqrt[2 + Sq
rt[3]]*a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a +
b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/
3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 -
Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(Sqrt[b*x^2]*Sqrt
[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b
*x^2)^(1/3))^2)]) - (2*Sqrt[2 - Sqrt[3]]*(1 + Sqrt[3])*a^(1/3)*(a^(1/3) -
(a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)
^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticF[ArcSin[((
1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*
x^2)^(1/3))], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[b*x^2]*Sqrt[-((a^(1/3)*(a^(1/
3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)]))
)/(4*a*b*x)
```

Defintions of rubi rules used

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 454
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*d
 - b*c*x)/(2*a*b*(p + 1)))*(a + b*x^2)^(p + 1), x] + Simp[c*((2*p + 3)/(2*a
*(p + 1)))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d}, x] && L
tQ[p, -1] && NeQ[p, -3/2]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int \frac{Bx+A}{\left(b\,x^{2}+a\right)^{\frac{4}{3}}}d x$$

input
```
int((B*x+A)/(b*x^2+a)^(4/3),x)
```

output
```
int((B*x+A)/(b*x^2+a)^(4/3),x)
```

Fricas [F]

$$\int \frac{A+Bx}{(a+bx^2)^{4/3}}\,dx = \int \frac{Bx+A}{(bx^2+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate((B*x+A)/(b*x^2+a)^(4/3),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(2/3)*(B*x + A)/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [A] (verification not implemented)

Time = 2.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.10

$$\int \frac{A+Bx}{(a+bx^2)^{4/3}}\,dx = \frac{Ax\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{4}{3}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{a^{\frac{4}{3}}} + B\left(\begin{cases}-\frac{3}{2b\sqrt[3]{a+bx^2}} & \text{for}\ b\neq 0\\ \frac{x^2}{2a^{\frac{4}{3}}} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((B*x+A)/(b*x**2+a)**(4/3),x)
```

output
```
A*x*hyper((1/2, 4/3), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(4/3) + B*Piece
wise((-3/(2*b*(a + b*x**2)**(1/3)), Ne(b, 0)), (x**2/(2*a**(4/3)), True))
```

Maxima [F]

$$\int \frac{A+Bx}{(a+bx^2)^{4/3}}\,dx = \int \frac{Bx+A}{(bx^2+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate((B*x+A)/(b*x^2+a)^(4/3),x, algorithm="maxima")
```

output
```
integrate((B*x + A)/(b*x^2 + a)^(4/3), x)
```

Giac [F]

$$\int \frac{A+Bx}{(a+bx^2)^{4/3}}\,dx = \int \frac{Bx+A}{(bx^2+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate((B*x+A)/(b*x^2+a)^(4/3),x, algorithm="giac")
```

output
```
integrate((B*x + A)/(b*x^2 + a)^(4/3), x)
```

Mupad [B] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.10

$$\int \frac{A+Bx}{(a+bx^2)^{4/3}}\,dx = \frac{A\,x\,\left(\frac{b\,x^2}{a}+1\right)^{4/3}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{4}{3};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{(b\,x^2+a)^{4/3}} - \frac{3\,B}{2\,b\,(b\,x^2+a)^{1/3}}$$

input
```
int((A + B*x)/(a + b*x^2)^(4/3),x)
```

output
```
(A*x*((b*x^2)/a + 1)^(4/3)*hypergeom([1/2, 4/3], 3/2, -(b*x^2)/a))/(a + b*
x^2)^(4/3) - (3*B)/(2*b*(a + b*x^2)^(1/3))
```

Reduce [F]

$$\int \frac{A+Bx}{(a+bx^2)^{4/3}}\,dx = \left(\int \frac{x}{(b\,x^2+a)^{\frac{1}{3}}\,a+(b\,x^2+a)^{\frac{1}{3}}\,b\,x^2}dx\right)b + \left(\int \frac{1}{(b\,x^2+a)^{\frac{1}{3}}\,a+(b\,x^2+a)^{\frac{1}{3}}\,b\,x^2}dx\right)a$$

input
```
int((B*x+A)/(b*x^2+a)^(4/3),x)
```

output
```
int(x/((a + b*x**2)**(1/3)*a + (a + b*x**2)**(1/3)*b*x**2),x)*b + int(1/((
a + b*x**2)**(1/3)*a + (a + b*x**2)**(1/3)*b*x**2),x)*a
```

3.22 $\int (A+Bx)\left(a+bx^2\right)^p\,dx$

Optimal result

Integrand size = 15, antiderivative size = 70

$$\int (A+Bx)\left(a+bx^2\right)^p\,dx=\frac{B\left(a+bx^2\right)^{1+p}}{2b(1+p)}+Ax\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right)$$

output
```
1/2*B*(b*x^2+a)^(p+1)/b/(p+1)+A*x*(b*x^2+a)^p*hypergeom([1/2, -p],[3/2],-b
*x^2/a)/((1+b*x^2/a)^p)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.40

$$\int (A+Bx)\left(a+bx^2\right)^p\,dx=\frac{\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\left(bBx^2\left(1+\frac{bx^2}{a}\right)^p+aB\left(-1+\left(1+\frac{bx^2}{a}\right)^p\right)+2Ab(1+p)x\,\text{Hypergeometric2F1}\right.}{2b(1+p)}$$

input
```
Integrate[(A + B*x)*(a + b*x^2)^p,x]
```

output
```
((a + b*x^2)^p*(b*B*x^2*(1 + (b*x^2)/a)^p + a*B*(-1 + (1 + (b*x^2)/a)^p) +
 2*A*b*(1 + p)*x*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)]))/(2*b*(1 +
 p)*(1 + (b*x^2)/a)^p)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {455, 238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (A+Bx)\left(a+bx^2\right)^p dx$$

$$\Big\downarrow \text{455}$$

$$A\int \left(bx^2+a\right)^p dx+\frac{B\left(a+bx^2\right)^{p+1}}{2b(p+1)}$$

$$\Big\downarrow \text{238}$$

$$A\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\int \left(\frac{bx^2}{a}+1\right)^p dx+\frac{B\left(a+bx^2\right)^{p+1}}{2b(p+1)}$$

$$\Big\downarrow \text{237}$$

$$Ax\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right)+\frac{B\left(a+bx^2\right)^{p+1}}{2b(p+1)}$$

input
```
Int[(A + B*x)*(a + b*x^2)^p,x]
```

output
```
(B*(a + b*x^2)^(1 + p))/(2*b*(1 + p)) + (A*x*(a + b*x^2)^p*Hypergeometric2
F1[1/2, -p, 3/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p
```

Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

Maple [F]

$$\int (Bx+A)\left(b\,x^{2}+a\right)^{p}dx$$

input
```
int((B*x+A)*(b*x^2+a)^p,x)
```

output
```
int((B*x+A)*(b*x^2+a)^p,x)
```

Fricas [F]

$$\int (A+Bx)\left(a+bx^2\right)^p\,dx=\int (Bx+A)(bx^2+a)^p\,dx$$

input
```
integrate((B*x+A)*(b*x^2+a)^p,x, algorithm="fricas")
```

output
```
integral((B*x + A)*(b*x^2 + a)^p, x)
```

Sympy [A] (verification not implemented)

Time = 2.38 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.87

$$\int (A+Bx)\left(a+bx^2\right)^p\,dx = Aa^px\,{}_2F_1\left(\begin{matrix}\frac{1}{2},-p\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right) + B\left(\begin{cases}\frac{a^px^2}{2} & \text{for } b=0\\ \frac{\begin{cases}\frac{(a+bx^2)^{p+1}}{p+1} & \text{for } p\neq -1\\ \log\left(a+bx^2\right) & \text{otherwise}\end{cases}}{2b} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((B*x+A)*(b*x**2+a)**p,x)
```

output
```
A*a**p*x*hyper((1/2, -p), (3/2,), b*x**2*exp_polar(I*pi)/a) + B*Piecewise(
(a**p*x**2/2, Eq(b, 0)), (Piecewise(((a + b*x**2)**(p + 1)/(p + 1), Ne(p,
-1)), (log(a + b*x**2), True))/(2*b), True))
```

Maxima [F]

$$\int (A+Bx)\left(a+bx^2\right)^p\,dx = \int (Bx+A)\left(bx^2+a\right)^p\,dx$$

input
```
integrate((B*x+A)*(b*x^2+a)^p,x, algorithm="maxima")
```

output
```
integrate((B*x + A)*(b*x^2 + a)^p, x)
```

Giac [F]

$$\int (A+Bx)\left(a+bx^2\right)^p\,dx=\int (Bx+A)\left(bx^2+a\right)^p\,dx$$

input
```
integrate((B*x+A)*(b*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate((B*x + A)*(b*x^2 + a)^p, x)
```

Mupad [B] (verification not implemented)

Time = 1.00 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.93

$$\int (A+Bx)\left(a+bx^2\right)^p\,dx=\frac{B\,{\left(b\,x^2+a\right)}^{p+1}}{2\,b\,\left(p+1\right)}+\frac{A\,x\,{\left(b\,x^2+a\right)}^p\,{}_2\mathrm{F}_1\left(\frac{1}{2},-p;\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{{\left(\frac{b\,x^2}{a}+1\right)}^p}$$

input
```
int((a + b*x^2)^p*(A + B*x),x)
```

output
```
(B*(a + b*x^2)^(p + 1))/(2*b*(p + 1)) + (A*x*(a + b*x^2)^p*hypergeom([1/2,
 -p], 3/2, -(b*x^2)/a))/((b*x^2)/a + 1)^p
```

Reduce [F]

$$\int (A+Bx)\left(a+bx^2\right)^p\,dx$$
$$=\frac{2(b\,x^2+a)^p\,apx+2(b\,x^2+a)^p\,ap+2(b\,x^2+a)^p\,ax+(b\,x^2+a)^p\,a+2(b\,x^2+a)^p\,bp\,x^2+(b\,x^2+a)^p\,b\,x^2}{4p^2+6p+}$$

input
```
int((B*x+A)*(b*x^2+a)^p,x)
```

output
```
(2*(a + b*x**2)**p*a*p*x + 2*(a + b*x**2)**p*a*p + 2*(a + b*x**2)**p*a*x +
 (a + b*x**2)**p*a + 2*(a + b*x**2)**p*b*p*x**2 + (a + b*x**2)**p*b*x**2 +
 8*int((a + b*x**2)**p/(2*a*p + a + 2*b*p*x**2 + b*x**2),x)*a**2*p**3 + 12
*int((a + b*x**2)**p/(2*a*p + a + 2*b*p*x**2 + b*x**2),x)*a**2*p**2 + 4*in
t((a + b*x**2)**p/(2*a*p + a + 2*b*p*x**2 + b*x**2),x)*a**2*p)/(2*(2*p**2
+ 3*p + 1))
```

3.23 $\int (a+bx^2)^3 (A+Bx+Cx^2)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 87

$$\int (a+bx^2)^3 (A+Bx+Cx^2)\,dx = a^3Ax+\frac{1}{3}a^2(3Ab+aC)x^3+\frac{3}{5}ab(Ab+aC)x^5+\frac{1}{7}b^2(Ab+3aC)x^7+\frac{1}{9}b^3Cx^9+\frac{B(a+bx^2)^4}{8b}$$

output
```
a^3*A*x+1/3*a^2*(3*A*b+C*a)*x^3+3/5*a*b*(A*b+C*a)*x^5+1/7*b^2*(A*b+3*C*a)*
x^7+1/9*b^3*C*x^9+1/8*B*(b*x^2+a)^4/b
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.15

$$\begin{aligned}\int (a+bx^2)^3 (A+Bx+Cx^2)\,dx &= \frac{1}{6}a^3x(6A+x(3B+2Cx))\\&+\frac{1}{20}a^2bx^3(20A+3x(5B+4Cx))\\&+\frac{1}{70}ab^2x^5(42A+5x(7B+6Cx))\\&+\frac{1}{504}b^3x^7(72A+7x(9B+8Cx))\end{aligned}$$

input
```
Integrate[(a + b*x^2)^3*(A + B*x + C*x^2),x]
```

output
```
(a^3*x*(6*A + x*(3*B + 2*C*x)))/6 + (a^2*b*x^3*(20*A + 3*x*(5*B + 4*C*x)))
/20 + (a*b^2*x^5*(42*A + 5*x*(7*B + 6*C*x)))/70 + (b^3*x^7*(72*A + 7*x*(9*
B + 8*C*x)))/504
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {2017, 290, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^2)^3 (A+Bx+Cx^2)\,dx$$

$$\Big\downarrow \text{2017}$$

$$\int (bx^2+a)^3 (Cx^2+A)\,dx+\frac{B(a+bx^2)^4}{8b}$$

$$\Big\downarrow \text{290}$$

$$\int (b^3Cx^8+b^2(Ab+3aC)x^6+3ab(Ab+aC)x^4+a^2(3Ab+aC)x^2+a^3A)\,dx+\frac{B(a+bx^2)^4}{8b}$$

$$\Big\downarrow \text{2009}$$

$$a^3Ax+\frac{1}{3}a^2x^3(aC+3Ab)+\frac{1}{7}b^2x^7(3aC+Ab)+\frac{3}{5}abx^5(aC+Ab)+\frac{B(a+bx^2)^4}{8b}+\frac{1}{9}b^3Cx^9$$

input
```
Int[(a + b*x^2)^3*(A + B*x + C*x^2),x]
```

output
```
a^3*A*x + (a^2*(3*A*b + a*C)*x^3)/3 + (3*a*b*(A*b + a*C)*x^5)/5 + (b^2*(A*
b + 3*a*C)*x^7)/7 + (b^3*C*x^9)/9 + (B*(a + b*x^2)^4)/(8*b)
```

Defintions of rubi rules used

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2017
```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.25

method	result
norman	$\frac{b^{3} C \,x^{9}}{9}+\frac{b^{3} B \,x^{8}}{8}+\left(\frac{1}{7} b^{3} A +\frac{3}{7} a C \,b^{2}\right) x^{7}+\frac{B a \,b^{2} x^{6}}{2}+\left(\frac{3}{5} a \,b^{2} A +\frac{3}{5} a^{2} b C \right) x^{5}+\frac{3 B \,a^{2} b \,x^{4}}{4}+\left(a^{2} b A \right.$
default	$\frac{b^{3} C \,x^{9}}{9}+\frac{b^{3} B \,x^{8}}{8}+\frac{\left(b^{3} A +3 a C \,b^{2}\right) x^{7}}{7}+\frac{B a \,b^{2} x^{6}}{2}+\frac{\left(3 a \,b^{2} A +3 a^{2} b C \right) x^{5}}{5}+\frac{3 B \,a^{2} b \,x^{4}}{4}+\frac{\left(3 a^{2} b A +C \,a^{3}\right) x^{3}}{3}+\frac{B \,a^{3}}{2}$
gosper	$\frac{1}{9} b^{3} C \,x^{9}+\frac{1}{8} b^{3} B \,x^{8}+\frac{1}{7} A \,b^{3} x^{7}+\frac{3}{7} x^{7} a C \,b^{2}+\frac{1}{2} B a \,b^{2} x^{6}+\frac{3}{5} a A \,b^{2} x^{5}+\frac{3}{5} x^{5} a^{2} b C +\frac{3}{4} B \,a^{2} b \,x^{4}$
risch	$\frac{1}{9} b^{3} C \,x^{9}+\frac{1}{8} b^{3} B \,x^{8}+\frac{1}{7} A \,b^{3} x^{7}+\frac{3}{7} x^{7} a C \,b^{2}+\frac{1}{2} B a \,b^{2} x^{6}+\frac{3}{5} a A \,b^{2} x^{5}+\frac{3}{5} x^{5} a^{2} b C +\frac{3}{4} B \,a^{2} b \,x^{4}$
parallelrisch	$\frac{1}{9} b^{3} C \,x^{9}+\frac{1}{8} b^{3} B \,x^{8}+\frac{1}{7} A \,b^{3} x^{7}+\frac{3}{7} x^{7} a C \,b^{2}+\frac{1}{2} B a \,b^{2} x^{6}+\frac{3}{5} a A \,b^{2} x^{5}+\frac{3}{5} x^{5} a^{2} b C +\frac{3}{4} B \,a^{2} b \,x^{4}$
orering	$\frac{x \left(280 b^{3} C \,x^{8}+315 b^{3} B \,x^{7}+360 A \,b^{3} x^{6}+1080 C a \,b^{2} x^{6}+1260 B a \,b^{2} x^{5}+1512 a A \,b^{2} x^{4}+1512 C \,a^{2} b \,x^{4}+1890 B \,a^{2} b \,x^{3}+2520 a^{2} A b \right.}{2520}$

input
```
int((b*x^2+a)^3*(C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
1/9*b^3*C*x^9+1/8*b^3*B*x^8+(1/7*b^3*A+3/7*a*C*b^2)*x^7+1/2*B*a*b^2*x^6+(3
/5*a*b^2*A+3/5*a^2*b*C)*x^5+3/4*B*a^2*b*x^4+(a^2*b*A+1/3*C*a^3)*x^3+1/2*B*
a^3*x^2+a^3*A*x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.24

$$\int \left(a+bx^2\right)^3 \left(A+Bx+Cx^2\right) \,dx = \frac{1}{9}\,Cb^3x^9 + \frac{1}{8}\,Bb^3x^8 + \frac{1}{2}\,Bab^2x^6 + \frac{3}{4}\,Ba^2bx^4 + \frac{1}{7}\,(3\,Cab^2 + Ab^3)x^7 + \frac{1}{2}\,Ba^3x^2 + \frac{3}{5}\,(Ca^2b + Aab^2)x^5 + Aa^3x + \frac{1}{3}\,(Ca^3 + 3\,Aa^2b)x^3$$

input
```
integrate((b*x^2+a)^3*(C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
1/9*C*b^3*x^9 + 1/8*B*b^3*x^8 + 1/2*B*a*b^2*x^6 + 3/4*B*a^2*b*x^4 + 1/7*(3
*C*a*b^2 + A*b^3)*x^7 + 1/2*B*a^3*x^2 + 3/5*(C*a^2*b + A*a*b^2)*x^5 + A*a^
3*x + 1/3*(C*a^3 + 3*A*a^2*b)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.40

$$\int \left(a+bx^2\right)^3 \left(A+Bx+Cx^2\right) \,dx = Aa^3x + \frac{Ba^3x^2}{2} + \frac{3Ba^2bx^4}{4} + \frac{Bab^2x^6}{2} + \frac{Bb^3x^8}{8} + \frac{Cb^3x^9}{9} + x^7\left(\frac{Ab^3}{7} + \frac{3Cab^2}{7}\right) + x^5 \cdot \left(\frac{3Aab^2}{5} + \frac{3Ca^2b}{5}\right) + x^3\left(Aa^2b + \frac{Ca^3}{3}\right)$$

input
```
integrate((b*x**2+a)**3*(C*x**2+B*x+A),x)
```

output
```
A*a**3*x + B*a**3*x**2/2 + 3*B*a**2*b*x**4/4 + B*a*b**2*x**6/2 + B*b**3*x*
*8/8 + C*b**3*x**9/9 + x**7*(A*b**3/7 + 3*C*a*b**2/7) + x**5*(3*A*a*b**2/5
 + 3*C*a**2*b/5) + x**3*(A*a**2*b + C*a**3/3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.24

$$\int \left(a+bx^2\right)^3 \left(A+Bx+Cx^2\right)\,dx = \frac{1}{9}\,Cb^3x^9 + \frac{1}{8}\,Bb^3x^8 + \frac{1}{2}\,Bab^2x^6 + \frac{3}{4}\,Ba^2bx^4 + \frac{1}{7}\,\left(3\,Cab^2 + Ab^3\right)x^7 + \frac{1}{2}\,Ba^3x^2 + \frac{3}{5}\,\left(Ca^2b + Aab^2\right)x^5 + Aa^3x + \frac{1}{3}\,\left(Ca^3 + 3\,Aa^2b\right)x^3$$

input
```
integrate((b*x^2+a)^3*(C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
1/9*C*b^3*x^9 + 1/8*B*b^3*x^8 + 1/2*B*a*b^2*x^6 + 3/4*B*a^2*b*x^4 + 1/7*(3
*C*a*b^2 + A*b^3)*x^7 + 1/2*B*a^3*x^2 + 3/5*(C*a^2*b + A*a*b^2)*x^5 + A*a^
3*x + 1/3*(C*a^3 + 3*A*a^2*b)*x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.28

$$\int \left(a+bx^2\right)^3 \left(A+Bx+Cx^2\right)\,dx = \frac{1}{9}\,Cb^3x^9 + \frac{1}{8}\,Bb^3x^8 + \frac{3}{7}\,Cab^2x^7 + \frac{1}{7}\,Ab^3x^7 + \frac{1}{2}\,Bab^2x^6 + \frac{3}{5}\,Ca^2bx^5 + \frac{3}{5}\,Aab^2x^5 + \frac{3}{4}\,Ba^2bx^4 + \frac{1}{3}\,Ca^3x^3 + Aa^2bx^3 + \frac{1}{2}\,Ba^3x^2 + Aa^3x$$

input
```
integrate((b*x^2+a)^3*(C*x^2+B*x+A),x, algorithm="giac")
```

output
```
1/9*C*b^3*x^9 + 1/8*B*b^3*x^8 + 3/7*C*a*b^2*x^7 + 1/7*A*b^3*x^7 + 1/2*B*a*
b^2*x^6 + 3/5*C*a^2*b*x^5 + 3/5*A*a*b^2*x^5 + 3/4*B*a^2*b*x^4 + 1/3*C*a^3*
x^3 + A*a^2*b*x^3 + 1/2*B*a^3*x^2 + A*a^3*x
```

Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.18

$$\int (a+bx^2)^3 (A+Bx+Cx^2)\,dx = x^3\left(\frac{C\,a^3}{3}+A\,b\,a^2\right)+x^7\left(\frac{A\,b^3}{7}+\frac{3\,C\,a\,b^2}{7}\right)+\frac{B\,a^3\,x^2}{2}+\frac{B\,b^3\,x^8}{8}+\frac{C\,b^3\,x^9}{9}+A\,a^3\,x+\frac{3\,a\,b\,x^5\,(A\,b+C\,a)}{5}+\frac{3\,B\,a^2\,b\,x^4}{4}+\frac{B\,a\,b^2\,x^6}{2}$$

input
```
int((a + b*x^2)^3*(A + B*x + C*x^2),x)
```

output
```
x^3*((C*a^3)/3 + A*a^2*b) + x^7*((A*b^3)/7 + (3*C*a*b^2)/7) + (B*a^3*x^2)/
2 + (B*b^3*x^8)/8 + (C*b^3*x^9)/9 + A*a^3*x + (3*a*b*x^5*(A*b + C*a))/5 +
(3*B*a^2*b*x^4)/4 + (B*a*b^2*x^6)/2
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.28

$$\int (a+bx^2)^3 (A+Bx+Cx^2)\,dx = \frac{x(280b^3c\,x^8+315b^4x^7+360a\,b^3x^6+1080a\,b^2c\,x^6+1260a\,b^3x^5+1512a^2b^2x^4+1512a^2bc\,x^4+1890a^2b^2}{2520}$$

input
```
int((b*x^2+a)^3*(C*x^2+B*x+A),x)
```

output
```
(x*(2520*a**4 + 2520*a**3*b*x**2 + 1260*a**3*b*x + 840*a**3*c*x**2 + 1512*
a**2*b**2*x**4 + 1890*a**2*b**2*x**3 + 1512*a**2*b*c*x**4 + 360*a*b**3*x**
6 + 1260*a*b**3*x**5 + 1080*a*b**2*c*x**6 + 315*b**4*x**7 + 280*b**3*c*x**
8))/2520
```

3.24 $\int (a+bx^2)^2 (A+Bx+Cx^2)\, dx$

Optimal result

Integrand size = 20, antiderivative size = 67

$$\int (a+bx^2)^2 (A+Bx+Cx^2)\, dx = a^2Ax + \frac{1}{3}a(2Ab+aC)x^3 + \frac{1}{5}b(Ab+2aC)x^5 + \frac{1}{7}b^2Cx^7 + \frac{B(a+bx^2)^3}{6b}$$

output
```
a^2*A*x+1/3*a*(2*A*b+C*a)*x^3+1/5*b*(A*b+2*C*a)*x^5+1/7*b^2*C*x^7+1/6*B*(b
*x^2+a)^3/b
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.03

$$\int (a+bx^2)^2 (A+Bx+Cx^2)\, dx = \frac{1}{210}x\big(35a^2(6A+x(3B+2Cx)) + 7abx^2(20A+3x(5B+4Cx)) + b^2x^4(42A+5x(7B+6Cx))\big)$$

input
```
Integrate[(a + b*x^2)^2*(A + B*x + C*x^2),x]
```

output
```
(x*(35*a^2*(6*A + x*(3*B + 2*C*x)) + 7*a*b*x^2*(20*A + 3*x*(5*B + 4*C*x))
+ b^2*x^4*(42*A + 5*x*(7*B + 6*C*x))))/210
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2017, 290, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^2 \left(A+Bx+Cx^2\right) \, dx$$

$$\Big\downarrow \text{2017}$$

$$\int \left(bx^2+a\right)^2 \left(Cx^2+A\right)dx+\frac{B\left(a+bx^2\right)^3}{6b}$$

$$\Big\downarrow \text{290}$$

$$\int \left(b^2Cx^6+b(Ab+2aC)x^4+a(2Ab+aC)x^2+a^2A\right)dx+\frac{B\left(a+bx^2\right)^3}{6b}$$

$$\Big\downarrow \text{2009}$$

$$a^2Ax+\frac{1}{5}bx^5(2aC+Ab)+\frac{1}{3}ax^3(aC+2Ab)+\frac{B\left(a+bx^2\right)^3}{6b}+\frac{1}{7}b^2Cx^7$$

input
```
Int[(a + b*x^2)^2*(A + B*x + C*x^2),x]
```

output
```
a^2*A*x + (a*(2*A*b + a*C)*x^3)/3 + (b*(A*b + 2*a*C)*x^5)/5 + (b^2*C*x^7)/
7 + (B*(a + b*x^2)^3)/(6*b)
```

Defintions of rubi rules used

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2017
```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.12

method	result
default	$\frac{b^2 C x^7}{7}+\frac{b^2 B x^6}{6}+\frac{(b^2 A+2Cab)x^5}{5}+\frac{Bab\,x^4}{2}+\frac{(2abA+a^2C)x^3}{3}+\frac{B a^2x^2}{2}+a^2Ax$
norman	$\frac{b^2 C x^7}{7}+\frac{b^2 B x^6}{6}+\left(\frac{1}{5}b^2A+\frac{2}{5}Cab\right)x^5+\frac{Bab\,x^4}{2}+\left(\frac{2}{3}abA+\frac{1}{3}a^2C\right)x^3+\frac{B a^2x^2}{2}+a^2Ax$
gosper	$\frac{1}{7}b^2C\,x^7+\frac{1}{6}b^2B\,x^6+\frac{1}{5}A\,b^2x^5+\frac{2}{5}x^5Cab+\frac{1}{2}Bab\,x^4+\frac{2}{3}aAb\,x^3+\frac{1}{3}x^3a^2C+\frac{1}{2}B\,a^2x^2+a^2A$
risch	$\frac{1}{7}b^2C\,x^7+\frac{1}{6}b^2B\,x^6+\frac{1}{5}A\,b^2x^5+\frac{2}{5}x^5Cab+\frac{1}{2}Bab\,x^4+\frac{2}{3}aAb\,x^3+\frac{1}{3}x^3a^2C+\frac{1}{2}B\,a^2x^2+a^2A$
parallelrisch	$\frac{1}{7}b^2C\,x^7+\frac{1}{6}b^2B\,x^6+\frac{1}{5}A\,b^2x^5+\frac{2}{5}x^5Cab+\frac{1}{2}Bab\,x^4+\frac{2}{3}aAb\,x^3+\frac{1}{3}x^3a^2C+\frac{1}{2}B\,a^2x^2+a^2A$
orering	$\frac{x(30b^2C\,x^6+35b^2B\,x^5+42A\,b^2x^4+84Cab\,x^4+105Bab\,x^3+140aAb\,x^2+70C\,a^2x^2+105B\,a^2x+210a^2A)}{210}$

input
```
int((b*x^2+a)^2*(C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
1/7*b^2*C*x^7+1/6*b^2*B*x^6+1/5*(A*b^2+2*C*a*b)*x^5+1/2*B*a*b*x^4+1/3*(2*A
*a*b+C*a^2)*x^3+1/2*B*a^2*x^2+a^2*A*x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.10

$$\int (a+bx^2)^2 (A+Bx+Cx^2)\, dx = \frac{1}{7}Cb^2x^7 + \frac{1}{6}Bb^2x^6 + \frac{1}{2}Babx^4 + \frac{1}{5}(2Cab + Ab^2)x^5 + \frac{1}{2}Ba^2x^2 + Aa^2x + \frac{1}{3}(Ca^2 + 2Aab)x^3$$

input
```
integrate((b*x^2+a)^2*(C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
1/7*C*b^2*x^7 + 1/6*B*b^2*x^6 + 1/2*B*a*b*x^4 + 1/5*(2*C*a*b + A*b^2)*x^5
+ 1/2*B*a^2*x^2 + A*a^2*x + 1/3*(C*a^2 + 2*A*a*b)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.24

$$\int (a+bx^2)^2 (A+Bx+Cx^2)\, dx = Aa^2x + \frac{Ba^2x^2}{2} + \frac{Babx^4}{2} + \frac{Bb^2x^6}{6} + \frac{Cb^2x^7}{7} + x^5\left(\frac{Ab^2}{5} + \frac{2Cab}{5}\right) + x^3\cdot\left(\frac{2Aab}{3} + \frac{Ca^2}{3}\right)$$

input
```
integrate((b*x**2+a)**2*(C*x**2+B*x+A),x)
```

output
```
A*a**2*x + B*a**2*x**2/2 + B*a*b*x**4/2 + B*b**2*x**6/6 + C*b**2*x**7/7 +
x**5*(A*b**2/5 + 2*C*a*b/5) + x**3*(2*A*a*b/3 + C*a**2/3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.10

$$\int (a+bx^2)^2 (A+Bx+Cx^2)\, dx = \frac{1}{7}Cb^2x^7 + \frac{1}{6}Bb^2x^6 + \frac{1}{2}Babx^4 + \frac{1}{5}(2Cab + Ab^2)x^5 + \frac{1}{2}Ba^2x^2 + Aa^2x + \frac{1}{3}(Ca^2 + 2Aab)x^3$$

input
```
integrate((b*x^2+a)^2*(C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
1/7*C*b^2*x^7 + 1/6*B*b^2*x^6 + 1/2*B*a*b*x^4 + 1/5*(2*C*a*b + A*b^2)*x^5
+ 1/2*B*a^2*x^2 + A*a^2*x + 1/3*(C*a^2 + 2*A*a*b)*x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.13

$$\int (a+bx^2)^2 (A+Bx+Cx^2)\, dx = \frac{1}{7}Cb^2x^7 + \frac{1}{6}Bb^2x^6 + \frac{2}{5}Cabx^5 + \frac{1}{5}Ab^2x^5 + \frac{1}{2}Babx^4 + \frac{1}{3}Ca^2x^3 + \frac{2}{3}Aabx^3 + \frac{1}{2}Ba^2x^2 + Aa^2x$$

input
```
integrate((b*x^2+a)^2*(C*x^2+B*x+A),x, algorithm="giac")
```

output
```
1/7*C*b^2*x^7 + 1/6*B*b^2*x^6 + 2/5*C*a*b*x^5 + 1/5*A*b^2*x^5 + 1/2*B*a*b*
x^4 + 1/3*C*a^2*x^3 + 2/3*A*a*b*x^3 + 1/2*B*a^2*x^2 + A*a^2*x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.10

$$\int (a+bx^2)^2 (A+Bx+Cx^2)\, dx = x^3\left(\frac{C\,a^2}{3} + \frac{2\,A\,b\,a}{3}\right) + x^5\left(\frac{A\,b^2}{5} + \frac{2\,C\,a\,b}{5}\right) + \frac{B\,a^2\,x^2}{2} + \frac{B\,b^2\,x^6}{6} + \frac{C\,b^2\,x^7}{7} + A\,a^2\,x + \frac{B\,a\,b\,x^4}{2}$$

input
```
int((a + b*x^2)^2*(A + B*x + C*x^2),x)
```

output
```
x^3*((C*a^2)/3 + (2*A*a*b)/3) + x^5*((A*b^2)/5 + (2*C*a*b)/5) + (B*a^2*x^2
)/2 + (B*b^2*x^6)/6 + (C*b^2*x^7)/7 + A*a^2*x + (B*a*b*x^4)/2
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.15

$$\int (a+bx^2)^2 (A+Bx+Cx^2)\,dx = \frac{x(30b^2c\,x^6+35b^3x^5+42a\,b^2x^4+84abc\,x^4+105a\,b^2x^3+140a^2b\,x^2+70a^2c\,x^2+105a^2bx+210a^3)}{210}$$

input
```
int((b*x^2+a)^2*(C*x^2+B*x+A),x)
```

output
```
(x*(210*a**3 + 140*a**2*b*x**2 + 105*a**2*b*x + 70*a**2*c*x**2 + 42*a*b**2
*x**4 + 105*a*b**2*x**3 + 84*a*b*c*x**4 + 35*b**3*x**5 + 30*b**2*c*x**6))/
210
```

3.25 $\int (a+bx^2)(A+Bx+Cx^2)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 46

$$\int (a+bx^2)(A+Bx+Cx^2)\,dx = aAx+\frac{1}{2}aBx^2+\frac{1}{3}(Ab+aC)x^3+\frac{1}{4}bBx^4+\frac{1}{5}bCx^5$$

output
```
a*A*x+1/2*a*B*x^2+1/3*(A*b+C*a)*x^3+1/4*b*B*x^4+1/5*b*C*x^5
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int (a+bx^2)(A+Bx+Cx^2)\,dx = aAx+\frac{1}{2}aBx^2+\frac{1}{3}(Ab+aC)x^3+\frac{1}{4}bBx^4+\frac{1}{5}bCx^5$$

input
```
Integrate[(a + b*x^2)*(A + B*x + C*x^2),x]
```

output
```
a*A*x + (a*B*x^2)/2 + ((A*b + a*C)*x^3)/3 + (b*B*x^4)/4 + (b*C*x^5)/5
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a + bx^2) (A + Bx + Cx^2) \, dx$$

$$\Big\downarrow \text{ 2188}$$

$$\int (x^2(aC + Ab) + aA + aBx + bBx^3 + bCx^4) \, dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3}x^3(aC + Ab) + aAx + \frac{1}{2}aBx^2 + \frac{1}{4}bBx^4 + \frac{1}{5}bCx^5$$

input
```
Int[(a + b*x^2)*(A + B*x + C*x^2),x]
```

output
```
a*A*x + (a*B*x^2)/2 + ((A*b + a*C)*x^3)/3 + (b*B*x^4)/4 + (b*C*x^5)/5
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

method	result	size
default	$aAx+\frac{Ba\,x^{2}}{2}+\frac{(Ab+Ca)x^{3}}{3}+\frac{bB\,x^{4}}{4}+\frac{bC\,x^{5}}{5}$	39
norman	$\frac{bC\,x^{5}}{5}+\frac{bB\,x^{4}}{4}+\left(\frac{Ab}{3}+\frac{Ca}{3}\right)x^{3}+\frac{Ba\,x^{2}}{2}+aAx$	40
gosper	$\frac{1}{5}bC\,x^{5}+\frac{1}{4}bB\,x^{4}+\frac{1}{3}Ab\,x^{3}+\frac{1}{3}x^{3}Ca+\frac{1}{2}Ba\,x^{2}+aAx$	41
risch	$\frac{1}{5}bC\,x^{5}+\frac{1}{4}bB\,x^{4}+\frac{1}{3}Ab\,x^{3}+\frac{1}{3}x^{3}Ca+\frac{1}{2}Ba\,x^{2}+aAx$	41
parallelrisch	$\frac{1}{5}bC\,x^{5}+\frac{1}{4}bB\,x^{4}+\frac{1}{3}Ab\,x^{3}+\frac{1}{3}x^{3}Ca+\frac{1}{2}Ba\,x^{2}+aAx$	41
orering	$\frac{x\left(12Cb\,x^{4}+15bB\,x^{3}+20Ab\,x^{2}+20Ca\,x^{2}+30Bax+60Aa\right)}{60}$	42

input
```
int((b*x^2+a)*(C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
a*A*x+1/2*B*a*x^2+1/3*(A*b+C*a)*x^3+1/4*b*B*x^4+1/5*b*C*x^5
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

$$\int \left(a+bx^{2}\right)\left(A+Bx+Cx^{2}\right)\,dx=\frac{1}{5}\,Cbx^{5}+\frac{1}{4}\,Bbx^{4}+\frac{1}{2}\,Bax^{2}+\frac{1}{3}\,(Ca+Ab)x^{3}+Aax$$

input
```
integrate((b*x^2+a)*(C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
1/5*C*b*x^5 + 1/4*B*b*x^4 + 1/2*B*a*x^2 + 1/3*(C*a + A*b)*x^3 + A*a*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91

$$\int \left(a+bx^2\right)\left(A+Bx+Cx^2\right)\,dx = Aax+\frac{Bax^2}{2}+\frac{Bbx^4}{4}+\frac{Cbx^5}{5}+x^3\left(\frac{Ab}{3}+\frac{Ca}{3}\right)$$

input
```
integrate((b*x**2+a)*(C*x**2+B*x+A),x)
```

output
```
A*a*x + B*a*x**2/2 + B*b*x**4/4 + C*b*x**5/5 + x**3*(A*b/3 + C*a/3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

$$\int \left(a+bx^2\right)\left(A+Bx+Cx^2\right)\,dx = \frac{1}{5}Cbx^5+\frac{1}{4}Bbx^4+\frac{1}{2}Bax^2+\frac{1}{3}(Ca+Ab)x^3+Aax$$

input
```
integrate((b*x^2+a)*(C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
1/5*C*b*x^5 + 1/4*B*b*x^4 + 1/2*B*a*x^2 + 1/3*(C*a + A*b)*x^3 + A*a*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87

$$\int \left(a+bx^2\right)\left(A+Bx+Cx^2\right)\,dx = \frac{1}{5}Cbx^5+\frac{1}{4}Bbx^4+\frac{1}{3}Cax^3+\frac{1}{3}Abx^3+\frac{1}{2}Bax^2+Aax$$

input
```
integrate((b*x^2+a)*(C*x^2+B*x+A),x, algorithm="giac")
```

output
```
1/5*C*b*x^5 + 1/4*B*b*x^4 + 1/3*C*a*x^3 + 1/3*A*b*x^3 + 1/2*B*a*x^2 + A*a*
x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

$$\int (a+bx^2) (A+Bx+Cx^2) \, dx = \frac{C\,b\,x^5}{5} + \frac{B\,b\,x^4}{4} + \left(\frac{A\,b}{3} + \frac{C\,a}{3}\right) x^3 + \frac{B\,a\,x^2}{2} + A\,a\,x$$

input
```
int((a + b*x^2)*(A + B*x + C*x^2),x)
```

output
```
x^3*((A*b)/3 + (C*a)/3) + A*a*x + (B*a*x^2)/2 + (B*b*x^4)/4 + (C*b*x^5)/5
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.93

$$\int (a+bx^2) (A+Bx+Cx^2) \, dx = \frac{x(12bc\,x^4 + 15b^2x^3 + 20ab\,x^2 + 20ac\,x^2 + 30abx + 60a^2)}{60}$$

input
```
int((b*x^2+a)*(C*x^2+B*x+A),x)
```

output
```
(x*(60*a**2 + 20*a*b*x**2 + 30*a*b*x + 20*a*c*x**2 + 15*b**2*x**3 + 12*b*c
*x**4))/60
```

3.26 $\int (A+Bx+Cx^2)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 20

$$\int (A+Bx+Cx^2)\,dx=Ax+\frac{Bx^2}{2}+\frac{Cx^3}{3}$$

output
```
A*x+1/2*B*x^2+1/3*C*x^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int (A+Bx+Cx^2)\,dx=Ax+\frac{Bx^2}{2}+\frac{Cx^3}{3}$$

input
```
Integrate[A + B*x + C*x^2,x]
```

output
```
A*x + (B*x^2)/2 + (C*x^3)/3
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (A + Bx + Cx^2) \, dx$$

$$\Big\downarrow \; 2009$$

$$Ax + \frac{Bx^2}{2} + \frac{Cx^3}{3}$$

input
```
Int[A + B*x + C*x^2,x]
```

output
```
A*x + (B*x^2)/2 + (C*x^3)/3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

method	result	size
gosper	$xA+\frac{1}{2}x^2B+\frac{1}{3}C\,x^3$	17
default	$xA+\frac{1}{2}x^2B+\frac{1}{3}C\,x^3$	17
norman	$xA+\frac{1}{2}x^2B+\frac{1}{3}C\,x^3$	17
risch	$xA+\frac{1}{2}x^2B+\frac{1}{3}C\,x^3$	17
parallelrisch	$xA+\frac{1}{2}x^2B+\frac{1}{3}C\,x^3$	17
parts	$xA+\frac{1}{2}x^2B+\frac{1}{3}C\,x^3$	17
orering	$\frac{x(2C\,x^2+3Bx+6A)}{6}$	18

input
```
int(C*x^2+B*x+A,x,method=_RETURNVERBOSE)
```

output
```
x*A+1/2*x^2*B+1/3*C*x^3
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int (A+Bx+Cx^2)\,dx=\frac{1}{3}\,Cx^3+\frac{1}{2}\,Bx^2+Ax$$

input
```
integrate(C*x^2+B*x+A,x, algorithm="fricas")
```

output
```
1/3*C*x^3 + 1/2*B*x^2 + A*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

$$\int \left(A+Bx+Cx^2\right)\,dx=Ax+\frac{Bx^2}{2}+\frac{Cx^3}{3}$$

input
```
integrate(C*x**2+B*x+A,x)
```

output
```
A*x + B*x**2/2 + C*x**3/3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \left(A+Bx+Cx^2\right)\,dx=\frac{1}{3}\,Cx^3+\frac{1}{2}\,Bx^2+Ax$$

input
```
integrate(C*x^2+B*x+A,x, algorithm="maxima")
```

output
```
1/3*C*x^3 + 1/2*B*x^2 + A*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \left(A+Bx+Cx^2\right)\,dx=\frac{1}{3}\,Cx^3+\frac{1}{2}\,Bx^2+Ax$$

input
```
integrate(C*x^2+B*x+A,x, algorithm="giac")
```

output
```
1/3*C*x^3 + 1/2*B*x^2 + A*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int (A + Bx + Cx^2) \, dx = \frac{C x^3}{3} + \frac{B x^2}{2} + A x$$

input
```
int(A + B*x + C*x^2,x)
```

output
```
A*x + (B*x^2)/2 + (C*x^3)/3
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int (A + Bx + Cx^2) \, dx = \frac{x(2c x^2 + 3bx + 6a)}{6}$$

input
```
int(C*x^2+B*x+A,x)
```

output
```
(x*(6*a + 3*b*x + 2*c*x**2))/6
```

3.27 $\int \frac{A+Bx+Cx^2}{a+bx^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 55

$$\int \frac{A+Bx+Cx^2}{a+bx^2}\,dx=\frac{Cx}{b}+\frac{(Ab-aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}+\frac{B\log\left(a+bx^2\right)}{2b}$$

output
```
C*x/b+(A*b-C*a)*arctan(b^(1/2)*x/a^(1/2))/a^(1/2)/b^(3/2)+1/2*B*ln(b*x^2+a
)/b
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.02

$$\int \frac{A+Bx+Cx^2}{a+bx^2}\,dx=\frac{Cx}{b}-\frac{(-Ab+aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}+\frac{B\log\left(a+bx^2\right)}{2b}$$

input
```
Integrate[(A + B*x + C*x^2)/(a + b*x^2),x]
```

output
```
(C*x)/b - ((-(A*b) + a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(3/2)) +
 (B*Log[a + b*x^2])/(2*b)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2341, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{a+bx^2} \, dx$$

$$\Big\downarrow \text{2341}$$

$$\int \left(\frac{-aC+Ab+bBx}{b\left(a+bx^2\right)}+\frac{C}{b}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{(Ab-aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}+\frac{B\log\left(a+bx^2\right)}{2b}+\frac{Cx}{b}$$

input
```
Int[(A + B*x + C*x^2)/(a + b*x^2),x]
```

output
```
(C*x)/b + ((A*b - a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(3/2)) + (B
*Log[a + b*x^2])/(2*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2341
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.85

method	result
default	$\frac{Cx}{b}+\frac{\frac{B\ln\left(b\,x^2+a\right)}{2}+\frac{(Ab-Ca)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}}{b}$
risch	$\frac{Cx}{b}+\frac{\ln\left(abA-a^2C-\sqrt{-ab(Ab-Ca)^2}\,x\right)B}{2b}+\frac{\ln\left(abA-a^2C-\sqrt{-ab(Ab-Ca)^2}\,x\right)\sqrt{-ab(Ab-Ca)^2}}{2b^2a}+\frac{\ln\left(abA-a^2C+\sqrt{-a\right.}{2b}$

input
```
int((C*x^2+B*x+A)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
C*x/b+1/b*(1/2*B*ln(b*x^2+a)+(A*b-C*a)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 125, normalized size of antiderivative = 2.27

$$\int \frac{A+Bx+Cx^2}{a+bx^2}\,dx = \left[\frac{2\,Cabx+Bab\log\left(bx^2+a\right)+(Ca-Ab)\sqrt{-ab}\log\left(\frac{bx^2-2\,\sqrt{-ab}x-a}{bx^2+a}\right)}{2\,ab^2},\ \frac{2\,Cabx+Bab\log\left(bx^2+a\right)-2\,(C}{2\,ab^2}\right.$$

input
```
integrate((C*x^2+B*x+A)/(b*x^2+a),x, algorithm="fricas")
```

output
```
[1/2*(2*C*a*b*x + B*a*b*log(b*x^2 + a) + (C*a - A*b)*sqrt(-a*b)*log((b*x^2
 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a*b^2), 1/2*(2*C*a*b*x + B*a*b*log(b
*x^2 + a) - 2*(C*a - A*b)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a*b^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(48) = 96$.

Time = 0.23 (sec) , antiderivative size = 156, normalized size of antiderivative = 2.84

$$\int \frac{A + Bx + Cx^2}{a + bx^2}\,dx = \frac{Cx}{b} + \left(\frac{B}{2b} - \frac{\sqrt{-ab^3}(-Ab + Ca)}{2ab^3}\right)\log\left(x + \frac{Ba - 2ab\left(\frac{B}{2b} - \frac{\sqrt{-ab^3}(-Ab+Ca)}{2ab^3}\right)}{-Ab + Ca}\right) + \left(\frac{B}{2b} + \frac{\sqrt{-ab^3}(-Ab + Ca)}{2ab^3}\right)\log\left(x + \frac{Ba - 2ab\left(\frac{B}{2b} + \frac{\sqrt{-ab^3}(-Ab+Ca)}{2ab^3}\right)}{-Ab + Ca}\right)$$

input
```
integrate((C*x**2+B*x+A)/(b*x**2+a),x)
```

output
```
C*x/b + (B/(2*b) - sqrt(-a*b**3)*(-A*b + C*a)/(2*a*b**3))*log(x + (B*a - 2
*a*b*(B/(2*b) - sqrt(-a*b**3)*(-A*b + C*a)/(2*a*b**3)))/(-A*b + C*a)) + (B
/(2*b) + sqrt(-a*b**3)*(-A*b + C*a)/(2*a*b**3))*log(x + (B*a - 2*a*b*(B/(2
*b) + sqrt(-a*b**3)*(-A*b + C*a)/(2*a*b**3)))/(-A*b + C*a))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.87

$$\int \frac{A + Bx + Cx^2}{a + bx^2}\,dx = \frac{Cx}{b} + \frac{B\log\left(bx^2 + a\right)}{2\,b} - \frac{(Ca - Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b}$$

input
```
integrate((C*x^2+B*x+A)/(b*x^2+a),x, algorithm="maxima")
```

output
```
C*x/b + 1/2*B*log(b*x^2 + a)/b - (C*a - A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a
*b)*b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.87

$$\int \frac{A+Bx+Cx^2}{a+bx^2}\,dx = \frac{Cx}{b} + \frac{B\log\left(bx^2+a\right)}{2\,b} - \frac{(Ca-Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b}$$

input
```
integrate((C*x^2+B*x+A)/(b*x^2+a),x, algorithm="giac")
```

output
```
C*x/b + 1/2*B*log(b*x^2 + a)/b - (C*a - A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a
*b)*b)
```

Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.02

$$\int \frac{A+Bx+Cx^2}{a+bx^2}\,dx = \frac{B\,\ln\left(b\,x^2+a\right)}{2\,b} + \frac{C\,x}{b} + \frac{A\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}} - \frac{C\,\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{3/2}}$$

input
```
int((A + B*x + C*x^2)/(a + b*x^2),x)
```

output
```
(B*log(a + b*x^2))/(2*b) + (C*x)/b + (A*atan((b^(1/2)*x)/a^(1/2)))/(a^(1/2
)*b^(1/2)) - (C*a^(1/2)*atan((b^(1/2)*x)/a^(1/2)))/b^(3/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.11

$$\int \frac{A+Bx+Cx^2}{a+bx^2}\,dx = \frac{2\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)b - 2\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)c + \log\left(b\,x^2+a\right)b^2 + 2bcx}{2b^2}$$

input
```
int((C*x^2+B*x+A)/(b*x^2+a),x)
```

output
```
(2*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b - 2*sqrt(b)*sqrt(a)*ata
n((b*x)/(sqrt(b)*sqrt(a)))*c + log(a + b*x**2)*b**2 + 2*b*c*x)/(2*b**2)
```

3.28 $\int \frac{A+Bx+Cx^2}{(a+bx^2)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 69

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^2}\,dx=-\frac{aB-(Ab-aC)x}{2ab(a+bx^2)}+\frac{(Ab+aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}$$

output
```
-1/2*(B*a-(A*b-C*a)*x)/a/b/(b*x^2+a)+1/2*(A*b+C*a)*arctan(b^(1/2)*x/a^(1/2
))/a^(3/2)/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.99

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^2}\,dx=\frac{-aB+Abx-aCx}{2ab(a+bx^2)}+\frac{(Ab+aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}$$

input
```
Integrate[(A + B*x + C*x^2)/(a + b*x^2)^2,x]
```

output
```
(-(a*B) + A*b*x - a*C*x)/(2*a*b*(a + b*x^2)) + ((A*b + a*C)*ArcTan[(Sqrt[b
]*x)/Sqrt[a]])/(2*a^(3/2)*b^(3/2))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2345, 25, 27, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^2}dx$$

$$\Big\downarrow \text{2345}$$

$$-\frac{\int -\frac{Ab+aC}{b(bx^2+a)}dx}{2a}-\frac{aB-x(Ab-aC)}{2ab\,(a+bx^2)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{Ab+aC}{b(bx^2+a)}dx}{2a}-\frac{aB-x(Ab-aC)}{2ab\,(a+bx^2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{(aC+Ab)\int \frac{1}{bx^2+a}dx}{2ab}-\frac{aB-x(Ab-aC)}{2ab\,(a+bx^2)}$$

$$\Big\downarrow \text{218}$$

$$\frac{(aC+Ab)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}-\frac{aB-x(Ab-aC)}{2ab\,(a+bx^2)}$$

input
```
Int[(A + B*x + C*x^2)/(a + b*x^2)^2,x]
```

output
```
-1/2*(a*B - (A*b - a*C)*x)/(a*b*(a + b*x^2)) + ((A*b + a*C)*ArcTan[(Sqrt[b
]*x)/Sqrt[a]])/(2*a^(3/2)*b^(3/2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{\frac{(Ab-Ca)x}{2ab}-\frac{B}{2b}}{b\,x^{2}+a}+\frac{(Ab+Ca)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2ab\sqrt{ab}}$	65
risch	$\frac{\frac{(Ab-Ca)x}{2ab}-\frac{B}{2b}}{b\,x^{2}+a}-\frac{A\ln\left(bx+\sqrt{-ab}\right)}{4\sqrt{-ab}\,a}-\frac{\ln\left(bx+\sqrt{-ab}\right)C}{4\sqrt{-ab}\,b}+\frac{A\ln\left(-bx+\sqrt{-ab}\right)}{4\sqrt{-ab}\,a}+\frac{\ln\left(-bx+\sqrt{-ab}\right)C}{4\sqrt{-ab}\,b}$	130

input
```
int((C*x^2+B*x+A)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/2*(A*b-C*a)/a/b*x-1/2*B/b)/(b*x^2+a)+1/2*(A*b+C*a)/a/b/(a*b)^(1/2)*arct
an(b*x/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 195, normalized size of antiderivative = 2.83

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^2}\,dx$$
$$= \left[-\frac{2\,Ba^2b+(Ca^2+Aab+(Cab+Ab^2)x^2)\sqrt{-ab}\log\left(\frac{bx^2-2\,\sqrt{-ab}x-a}{bx^2+a}\right)+2\,(Ca^2b-Aab^2)x}{4\,(a^2b^3x^2+a^3b^2)},\right.$$
$$\left.-\frac{Ba^2b-(Ca^2+Aab+(Cab+Ab^2)x^2)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}x}{a}\right)+(Ca^2b-Aab^2)x}{2\,(a^2b^3x^2+a^3b^2)}\right]$$

input
```
integrate((C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[-1/4*(2*B*a^2*b + (C*a^2 + A*a*b + (C*a*b + A*b^2)*x^2)*sqrt(-a*b)*log((b
*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 2*(C*a^2*b - A*a*b^2)*x)/(a^2*b^
3*x^2 + a^3*b^2), -1/2*(B*a^2*b - (C*a^2 + A*a*b + (C*a*b + A*b^2)*x^2)*sq
rt(a*b)*arctan(sqrt(a*b)*x/a) + (C*a^2*b - A*a*b^2)*x)/(a^2*b^3*x^2 + a^3*
b^2)]
```

Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.68

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^2}\,dx = -\frac{\sqrt{-\frac{1}{a^3b^3}}(Ab+Ca)\log\left(-a^2b\sqrt{-\frac{1}{a^3b^3}}+x\right)}{4} + \frac{\sqrt{-\frac{1}{a^3b^3}}(Ab+Ca)\log\left(a^2b\sqrt{-\frac{1}{a^3b^3}}+x\right)}{4} + \frac{-Ba+x(Ab-Ca)}{2a^2b+2ab^2x^2}$$

input
```
integrate((C*x**2+B*x+A)/(b*x**2+a)**2,x)
```

output
```
-sqrt(-1/(a**3*b**3))*(A*b + C*a)*log(-a**2*b*sqrt(-1/(a**3*b**3)) + x)/4
+ sqrt(-1/(a**3*b**3))*(A*b + C*a)*log(a**2*b*sqrt(-1/(a**3*b**3)) + x)/4
+ (-B*a + x*(A*b - C*a))/(2*a**2*b + 2*a*b**2*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.90

$$\int \frac{A + Bx + Cx^2}{(a + bx^2)^2}\, dx = -\frac{Ba + (Ca - Ab)x}{2\,(ab^2x^2 + a^2b)} + \frac{(Ca + Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}ab}$$

input
```
integrate((C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
-1/2*(B*a + (C*a - A*b)*x)/(a*b^2*x^2 + a^2*b) + 1/2*(C*a + A*b)*arctan(b*
x/sqrt(a*b))/(sqrt(a*b)*a*b)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.87

$$\int \frac{A + Bx + Cx^2}{(a + bx^2)^2}\, dx = \frac{(Ca + Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}ab} - \frac{Cax - Abx + Ba}{2\,(bx^2 + a)ab}$$

input
```
integrate((C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*(C*a + A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b) - 1/2*(C*a*x - A*b*x
 + B*a)/((b*x^2 + a)*a*b)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.87

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^2}\,dx=\frac{\operatorname{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,(A\,b+C\,a)}{2\,a^{3/2}\,b^{3/2}}-\frac{\frac{B}{2\,b}-\frac{x\,(A\,b-C\,a)}{2\,a\,b}}{b\,x^2+a}$$

input
```
int((A + B*x + C*x^2)/(a + b*x^2)^2,x)
```

output
```
(atan((b^(1/2)*x)/a^(1/2))*(A*b + C*a))/(2*a^(3/2)*b^(3/2)) - (B/(2*b) - (
x*(A*b - C*a))/(2*a*b))/(a + b*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.74

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^2}\,dx$$
$$=\frac{\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)ab+\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)ac+\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)b^2x^2+\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)bc\,x^2}{2a\,b^2\,(b\,x^2+a)}$$

input
```
int((C*x^2+B*x+A)/(b*x^2+a)^2,x)
```

output
```
(sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*b + sqrt(b)*sqrt(a)*atan(
(b*x)/(sqrt(b)*sqrt(a)))*a*c + sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)
))*b**2*x**2 + sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b*c*x**2 + a*
b**2*x - a*b*c*x + b**3*x**2)/(2*a*b**2*(a + b*x**2))
```

3.29 $\int \frac{A+Bx+Cx^2}{(a+bx^2)^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 98

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^3}\,dx=-\frac{aB-(Ab-aC)x}{4ab\,(a+bx^2)^2}+\frac{(3Ab+aC)x}{8a^2b\,(a+bx^2)}+\frac{(3Ab+aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}}$$

output
```
-1/4*(B*a-(A*b-C*a)*x)/a/b/(b*x^2+a)^2+1/8*(3*A*b+C*a)*x/a^2/b/(b*x^2+a)+1
/8*(3*A*b+C*a)*arctan(b^(1/2)*x/a^(1/2))/a^(5/2)/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.92

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^3}\,dx=\frac{3Ab^2x^3-a^2(2B+Cx)+abx(5A+Cx^2)}{8a^2b\,(a+bx^2)^2}+\frac{(3Ab+aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}}$$

input
```
Integrate[(A + B*x + C*x^2)/(a + b*x^2)^3,x]
```

output
```
(3*A*b^2*x^3 - a^2*(2*B + C*x) + a*b*x*(5*A + C*x^2))/(8*a^2*b*(a + b*x^2)
^2) + ((3*A*b + a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(5/2)*b^(3/2))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2345, 25, 27, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^3}dx$$

$$\Big\downarrow \text{ 2345}$$

$$-\frac{\int -\frac{3Ab+aC}{b(bx^2+a)^2}dx}{4a}-\frac{aB-x(Ab-aC)}{4ab\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{3Ab+aC}{b(bx^2+a)^2}dx}{4a}-\frac{aB-x(Ab-aC)}{4ab\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{(aC+3Ab)\int \frac{1}{(bx^2+a)^2}dx}{4ab}-\frac{aB-x(Ab-aC)}{4ab\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{ 215}$$

$$\frac{(aC+3Ab)\left(\frac{\int \frac{1}{bx^2+a}dx}{2a}+\frac{x}{2a(a+bx^2)}\right)}{4ab}-\frac{aB-x(Ab-aC)}{4ab\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{(aC+3Ab)\left(\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{x}{2a(a+bx^2)}\right)}{4ab}-\frac{aB-x(Ab-aC)}{4ab\left(a+bx^2\right)^2}$$

input
```
Int[(A + B*x + C*x^2)/(a + b*x^2)^3,x]
```

output
```
-1/4*(a*B - (A*b - a*C)*x)/(a*b*(a + b*x^2)^2) + ((3*A*b + a*C)*(x/(2*a*(a
 + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*a^(3/2)*Sqrt[b])))/(4*a*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.85

method	result	size
default	$\frac{\frac{(3Ab+Ca)x^3}{8a^2}+\frac{(5Ab-Ca)x}{8ab}-\frac{B}{4b}}{(b\,x^2+a)^2}+\frac{(3Ab+Ca)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8a^2b\sqrt{ab}}$	83
risch	$\frac{\frac{(3Ab+Ca)x^3}{8a^2}+\frac{(5Ab-Ca)x}{8ab}-\frac{B}{4b}}{(b\,x^2+a)^2}-\frac{3A\ln\left(bx+\sqrt{-ab}\right)}{16\sqrt{-ab}\,a^2}-\frac{\ln\left(bx+\sqrt{-ab}\right)C}{16\sqrt{-ab}\,ba}+\frac{3A\ln\left(-bx+\sqrt{-ab}\right)}{16\sqrt{-ab}\,a^2}+\frac{\ln\left(-bx+\sqrt{-ab}\right)C}{16\sqrt{-ab}\,ba}$	153

input
```
int((C*x^2+B*x+A)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/8*(3*A*b+C*a)/a^2*x^3+1/8*(5*A*b-C*a)/a/b*x-1/4*B/b)/(b*x^2+a)^2+1/8*(3
*A*b+C*a)/a^2/b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 314, normalized size of antiderivative = 3.20

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^3}\,dx$$

$$= \left[-\frac{4\,Ba^3b-2\,(Ca^2b^2+3\,Aab^3)x^3+((Cab^2+3\,Ab^3)x^4+Ca^3+3\,Aa^2b+2\,(Ca^2b+3\,Aab^2)x^2)\sqrt{-ab}\,\mathrm{l}\ldots}{16\,(a^3b^4x^4+2\,a^4b^3x^2+a^5b^2)}\right.$$

$$-\frac{2\,Ba^3b-(Ca^2b^2+3\,Aab^3)x^3-((Cab^2+3\,Ab^3)x^4+Ca^3+3\,Aa^2b+2\,(Ca^2b+3\,Aab^2)x^2)\sqrt{ab}\,\mathrm{arcta}\ldots}{8\,(a^3b^4x^4+2\,a^4b^3x^2+a^5b^2)}$$

input
```
integrate((C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
[-1/16*(4*B*a^3*b - 2*(C*a^2*b^2 + 3*A*a*b^3)*x^3 + ((C*a*b^2 + 3*A*b^3)*x
^4 + C*a^3 + 3*A*a^2*b + 2*(C*a^2*b + 3*A*a*b^2)*x^2)*sqrt(-a*b)*log((b*x^
2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 2*(C*a^3*b - 5*A*a^2*b^2)*x)/(a^3*b
^4*x^4 + 2*a^4*b^3*x^2 + a^5*b^2), -1/8*(2*B*a^3*b - (C*a^2*b^2 + 3*A*a*b^
3)*x^3 - ((C*a*b^2 + 3*A*b^3)*x^4 + C*a^3 + 3*A*a^2*b + 2*(C*a^2*b + 3*A*a
*b^2)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) + (C*a^3*b - 5*A*a^2*b^2)*x)/(a
^3*b^4*x^4 + 2*a^4*b^3*x^2 + a^5*b^2)]
```

Sympy [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.59

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^3}\,dx = -\frac{\sqrt{-\frac{1}{a^5b^3}}\cdot(3Ab+Ca)\log\left(-a^3b\sqrt{-\frac{1}{a^5b^3}}+x\right)}{16} + \frac{\sqrt{-\frac{1}{a^5b^3}}\cdot(3Ab+Ca)\log\left(a^3b\sqrt{-\frac{1}{a^5b^3}}+x\right)}{16} + \frac{-2Ba^2+x^3\cdot(3Ab^2+Cab)+x(5Aab-Ca^2)}{8a^4b+16a^3b^2x^2+8a^2b^3x^4}$$

input
```
integrate((C*x**2+B*x+A)/(b*x**2+a)**3,x)
```

output
```
-sqrt(-1/(a**5*b**3))*(3*A*b + C*a)*log(-a**3*b*sqrt(-1/(a**5*b**3)) + x)/
16 + sqrt(-1/(a**5*b**3))*(3*A*b + C*a)*log(a**3*b*sqrt(-1/(a**5*b**3)) +
x)/16 + (-2*B*a**2 + x**3*(3*A*b**2 + C*a*b) + x*(5*A*a*b - C*a**2))/(8*a*
*4*b + 16*a**3*b**2*x**2 + 8*a**2*b**3*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^3}\,dx = \frac{(Cab+3\,Ab^2)x^3-2\,Ba^2-(Ca^2-5\,Aab)x}{8\,(a^2b^3x^4+2\,a^3b^2x^2+a^4b)} + \frac{(Ca+3\,Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^2b}$$

input
```
integrate((C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/8*((C*a*b + 3*A*b^2)*x^3 - 2*B*a^2 - (C*a^2 - 5*A*a*b)*x)/(a^2*b^3*x^4 +
 2*a^3*b^2*x^2 + a^4*b) + 1/8*(C*a + 3*A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*
b)*a^2*b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.86

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^3}\,dx = \frac{(Ca+3\,Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^2b} + \frac{Cabx^3+3\,Ab^2x^3-Ca^2x+5\,Aabx-2\,Ba^2}{8\,(bx^2+a)^2a^2b}$$

input
```
integrate((C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
1/8*(C*a + 3*A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b) + 1/8*(C*a*b*x^3
 + 3*A*b^2*x^3 - C*a^2*x + 5*A*a*b*x - 2*B*a^2)/((b*x^2 + a)^2*a^2*b)
```

Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.90

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^3}\,dx = \frac{\frac{x^3\,(3\,A\,b+C\,a)}{8\,a^2}-\frac{B}{4\,b}+\frac{x\,(5\,A\,b-C\,a)}{8\,a\,b}}{a^2+2\,a\,b\,x^2+b^2\,x^4} + \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,(3\,A\,b+C\,a)}{8\,a^{5/2}\,b^{3/2}}$$

input
```
int((A + B*x + C*x^2)/(a + b*x^2)^3,x)
```

output
```
((x^3*(3*A*b + C*a))/(8*a^2) - B/(4*b) + (x*(5*A*b - C*a))/(8*a*b))/(a^2 +
 b^2*x^4 + 2*a*b*x^2) + (atan((b^(1/2)*x)/a^(1/2))*(3*A*b + C*a))/(8*a^(5/
2)*b^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 212, normalized size of antiderivative = 2.16

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^3}\,dx$$
$$= \frac{3\sqrt{b}\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\sqrt{a}}\right)a^2b+\sqrt{b}\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\sqrt{a}}\right)a^2c+6\sqrt{b}\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\sqrt{a}}\right)a\,b^2x^2+2\sqrt{b}\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\sqrt{a}}\right.}{8a^2b^2\,(b^2}$$

input
```
int((C*x^2+B*x+A)/(b*x^2+a)^3,x)
```

output
```
(3*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2*b + sqrt(b)*sqrt(a)*
atan((b*x)/(sqrt(b)*sqrt(a)))*a**2*c + 6*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(
b)*sqrt(a)))*a*b**2*x**2 + 2*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))
*a*b*c*x**2 + 3*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**3*x**4 +
sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**2*c*x**4 + 5*a**2*b**2*x
- 2*a**2*b**2 - a**2*b*c*x + 3*a*b**3*x**3 + a*b**2*c*x**3)/(8*a**2*b**2*(
a**2 + 2*a*b*x**2 + b**2*x**4))
```

3.30 $\int \frac{A+Bx+Cx^2}{(a+bx^2)^4}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 126

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^4}\,dx=-\frac{aB-(Ab-aC)x}{6ab\,(a+bx^2)^3}+\frac{(5Ab+aC)x}{24a^2b\,(a+bx^2)^2}+\frac{(5Ab+aC)x}{16a^3b\,(a+bx^2)}+\frac{(5Ab+aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16a^{7/2}b^{3/2}}$$

output
```
-1/6*(B*a-(A*b-C*a)*x)/a/b/(b*x^2+a)^3+1/24*(5*A*b+C*a)*x/a^2/b/(b*x^2+a)^
2+1/16*(5*A*b+C*a)*x/a^3/b/(b*x^2+a)+1/16*(5*A*b+C*a)*arctan(b^(1/2)*x/a^(
1/2))/a^(7/2)/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.89

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^4}\,dx=\frac{15Ab^3x^5-a^3(8B+3Cx)+ab^2x^3(40A+3Cx^2)+a^2bx(33A+8Cx^2)}{48a^3b\,(a+bx^2)^3}+\frac{(5Ab+aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16a^{7/2}b^{3/2}}$$

input
```
Integrate[(A + B*x + C*x^2)/(a + b*x^2)^4,x]
```

output
```
(15*A*b^3*x^5 - a^3*(8*B + 3*C*x) + a*b^2*x^3*(40*A + 3*C*x^2) + a^2*b*x*(
33*A + 8*C*x^2))/(48*a^3*b*(a + b*x^2)^3) + ((5*A*b + a*C)*ArcTan[(Sqrt[b]
*x)/Sqrt[a]])/(16*a^(7/2)*b^(3/2))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {2345, 25, 27, 215, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{\left(a+bx^2\right)^4}dx$$

$$\Big\downarrow \text{ 2345}$$

$$-\frac{\int -\frac{5Ab+aC}{b(bx^2+a)^3}dx}{6a}-\frac{aB-x(Ab-aC)}{6ab\left(a+bx^2\right)^3}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{5Ab+aC}{b(bx^2+a)^3}dx}{6a}-\frac{aB-x(Ab-aC)}{6ab\left(a+bx^2\right)^3}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{(aC+5Ab)\int \frac{1}{(bx^2+a)^3}dx}{6ab}-\frac{aB-x(Ab-aC)}{6ab\left(a+bx^2\right)^3}$$

$$\Big\downarrow \text{ 215}$$

$$\frac{(aC+5Ab)\left(\frac{3\int \frac{1}{\left(bx^2+a\right)^2}dx}{4a}+\frac{x}{4a(a+bx^2)^2}\right)}{6ab}-\frac{aB-x(Ab-aC)}{6ab\left(a+bx^2\right)^3}$$

$$\Big\downarrow \text{ 215}$$

$$\frac{(aC+5Ab)\left(\frac{3\left(\frac{\int \frac{1}{bx^2+a}dx}{2a}+\frac{x}{2a\left(a+bx^2\right)}\right)}{4a}+\frac{x}{4a(a+bx^2)^2}\right)}{6ab}-\frac{aB-x(Ab-aC)}{6ab\left(a+bx^2\right)^3}$$

$\Big\downarrow$ 218

$$\frac{(aC+5Ab)\left(\frac{3\left(\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{x}{2a\left(a+bx^2\right)}\right)}{4a}+\frac{x}{4a(a+bx^2)^2}\right)}{6ab}-\frac{aB-x(Ab-aC)}{6ab\left(a+bx^2\right)^3}$$

input
```
Int[(A + B*x + C*x^2)/(a + b*x^2)^4,x]
```

output
```
-1/6*(a*B - (A*b - a*C)*x)/(a*b*(a + b*x^2)^3) + ((5*A*b + a*C)*(x/(4*a*(a
 + b*x^2)^2) + (3*(x/(2*a*(a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*a^
(3/2)*Sqrt[b])))/(4*a)))/(6*a*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.79

method	result
default	$\frac{\frac{(5Ab+Ca)b\,x^5}{16a^3}+\frac{(5Ab+Ca)x^3}{6a^2}+\frac{(11Ab-Ca)x}{16ab}-\frac{B}{6b}}{(b\,x^2+a)^3}+\frac{(5Ab+Ca)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16a^3b\sqrt{ab}}$
risch	$\frac{\frac{(5Ab+Ca)b\,x^5}{16a^3}+\frac{(5Ab+Ca)x^3}{6a^2}+\frac{(11Ab-Ca)x}{16ab}-\frac{B}{6b}}{(b\,x^2+a)^3}-\frac{5\ln(bx+\sqrt{-ab})A}{32\sqrt{-ab}\,a^3}-\frac{\ln(bx+\sqrt{-ab})C}{32\sqrt{-ab}\,b\,a^2}+\frac{5\ln(-bx+\sqrt{-ab})A}{32\sqrt{-ab}\,a^3}+\frac{\ln(-bx+\sqrt{-}}{32\sqrt{-ab}}$

input
```
int((C*x^2+B*x+A)/(b*x^2+a)^4,x,method=_RETURNVERBOSE)
```

output
```
(1/16*(5*A*b+C*a)/a^3*b*x^5+1/6/a^2*(5*A*b+C*a)*x^3+1/16*(11*A*b-C*a)/a/b*
x-1/6*B/b)/(b*x^2+a)^3+1/16*(5*A*b+C*a)/a^3/b/(a*b)^(1/2)*arctan(b*x/(a*b)
^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 430, normalized size of antiderivative = 3.41

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^4}\,dx$$
$$= \left[-\frac{16\,Ba^4b-6\,(Ca^2b^3+5\,Aab^4)x^5-16\,(Ca^3b^2+5\,Aa^2b^3)x^3+3\,((Cab^3+5\,Ab^4)x^6+Ca^4+5\,Aa^3b+}{96\,(a^4b^5x^6+3\,a^5b^4x^4}\right.$$
$$-\frac{8\,Ba^4b-3\,(Ca^2b^3+5\,Aab^4)x^5-8\,(Ca^3b^2+5\,Aa^2b^3)x^3-3\,((Cab^3+5\,Ab^4)x^6+Ca^4+5\,Aa^3b+3\,(}{48\,(a^4b^5x^6+3\,a^5b^4x^4+3\,a}$$

input
```
integrate((C*x^2+B*x+A)/(b*x^2+a)^4,x, algorithm="fricas")
```

output
```
[-1/96*(16*B*a^4*b - 6*(C*a^2*b^3 + 5*A*a*b^4)*x^5 - 16*(C*a^3*b^2 + 5*A*a
^2*b^3)*x^3 + 3*((C*a*b^3 + 5*A*b^4)*x^6 + C*a^4 + 5*A*a^3*b + 3*(C*a^2*b^
2 + 5*A*a*b^3)*x^4 + 3*(C*a^3*b + 5*A*a^2*b^2)*x^2)*sqrt(-a*b)*log((b*x^2
- 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 6*(C*a^4*b - 11*A*a^3*b^2)*x)/(a^4*b^
5*x^6 + 3*a^5*b^4*x^4 + 3*a^6*b^3*x^2 + a^7*b^2), -1/48*(8*B*a^4*b - 3*(C*
a^2*b^3 + 5*A*a*b^4)*x^5 - 8*(C*a^3*b^2 + 5*A*a^2*b^3)*x^3 - 3*((C*a*b^3 +
 5*A*b^4)*x^6 + C*a^4 + 5*A*a^3*b + 3*(C*a^2*b^2 + 5*A*a*b^3)*x^4 + 3*(C*a
^3*b + 5*A*a^2*b^2)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) + 3*(C*a^4*b - 11
*A*a^3*b^2)*x)/(a^4*b^5*x^6 + 3*a^5*b^4*x^4 + 3*a^6*b^3*x^2 + a^7*b^2)]
```

Sympy [A] (verification not implemented)

Time = 0.89 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.56

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^4}\,dx = -\frac{\sqrt{-\frac{1}{a^7b^3}}\cdot(5Ab+Ca)\log\left(-a^4b\sqrt{-\frac{1}{a^7b^3}}+x\right)}{32} + \frac{\sqrt{-\frac{1}{a^7b^3}}\cdot(5Ab+Ca)\log\left(a^4b\sqrt{-\frac{1}{a^7b^3}}+x\right)}{32} + \frac{-8Ba^3+x^5\cdot(15Ab^3+3Cab^2)+x^3\cdot(40Aab^2+8Ca^2b)+x(33Aa^2b-3Ca^3)}{48a^6b+144a^5b^2x^2+144a^4b^3x^4+48a^3b^4x^6}$$

input
```
integrate((C*x**2+B*x+A)/(b*x**2+a)**4,x)
```

output
```
-sqrt(-1/(a**7*b**3))*(5*A*b + C*a)*log(-a**4*b*sqrt(-1/(a**7*b**3)) + x)/
32 + sqrt(-1/(a**7*b**3))*(5*A*b + C*a)*log(a**4*b*sqrt(-1/(a**7*b**3)) +
x)/32 + (-8*B*a**3 + x**5*(15*A*b**3 + 3*C*a*b**2) + x**3*(40*A*a*b**2 + 8
*C*a**2*b) + x*(33*A*a**2*b - 3*C*a**3))/(48*a**6*b + 144*a**5*b**2*x**2 +
 144*a**4*b**3*x**4 + 48*a**3*b**4*x**6)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.06

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^4}\,dx = \frac{3\,(Cab^2+5\,Ab^3)x^5-8\,Ba^3+8\,(Ca^2b+5\,Aab^2)x^3-3\,(Ca^3-11\,Aa^2b)x}{48\,(a^3b^4x^6+3\,a^4b^3x^4+3\,a^5b^2x^2+a^6b)} + \frac{(Ca+5\,Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{ab}a^3b}$$

input
```
integrate((C*x^2+B*x+A)/(b*x^2+a)^4,x, algorithm="maxima")
```

output
```
1/48*(3*(C*a*b^2 + 5*A*b^3)*x^5 - 8*B*a^3 + 8*(C*a^2*b + 5*A*a*b^2)*x^3 -
3*(C*a^3 - 11*A*a^2*b)*x)/(a^3*b^4*x^6 + 3*a^4*b^3*x^4 + 3*a^5*b^2*x^2 + a
^6*b) + 1/16*(C*a + 5*A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3*b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.87

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^4}\,dx = \frac{(Ca+5\,Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{ab}a^3b} + \frac{3\,Cab^2x^5+15\,Ab^3x^5+8\,Ca^2bx^3+40\,Aab^2x^3-3\,Ca^3x+33\,Aa^2bx-8\,Ba^3}{48\,(bx^2+a)^3a^3b}$$

input
```
integrate((C*x^2+B*x+A)/(b*x^2+a)^4,x, algorithm="giac")
```

output
```
1/16*(C*a + 5*A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3*b) + 1/48*(3*C*a*b
^2*x^5 + 15*A*b^3*x^5 + 8*C*a^2*b*x^3 + 40*A*a*b^2*x^3 - 3*C*a^3*x + 33*A*
a^2*b*x - 8*B*a^3)/((b*x^2 + a)^3*a^3*b)
```

Mupad [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.92

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^4}\,dx = \frac{\frac{x^3\,(5\,A\,b+C\,a)}{6\,a^2}-\frac{B}{6\,b}+\frac{b\,x^5\,(5\,A\,b+C\,a)}{16\,a^3}+\frac{x\,(11\,A\,b-C\,a)}{16\,a\,b}}{a^3+3\,a^2\,b\,x^2+3\,a\,b^2\,x^4+b^3\,x^6} + \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,(5\,A\,b+C\,a)}{16\,a^{7/2}\,b^{3/2}}$$

input
```
int((A + B*x + C*x^2)/(a + b*x^2)^4,x)
```

output
```
((x^3*(5*A*b + C*a))/(6*a^2) - B/(6*b) + (b*x^5*(5*A*b + C*a))/(16*a^3) +
(x*(11*A*b - C*a))/(16*a*b))/(a^3 + b^3*x^6 + 3*a^2*b*x^2 + 3*a*b^2*x^4) +
 (atan((b^(1/2)*x)/a^(1/2))*(5*A*b + C*a))/(16*a^(7/2)*b^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 304, normalized size of antiderivative = 2.41

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^4}\,dx = \frac{15\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^3b+3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^3c+45\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^2b^2x^2+9\sqrt{b}\,\sqrt{a}\,atan\left(\cdots\right.}{}$$

input
```
int((C*x^2+B*x+A)/(b*x^2+a)^4,x)
```

output
```
(15*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**3*b + 3*sqrt(b)*sqrt(
a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**3*c + 45*sqrt(b)*sqrt(a)*atan((b*x)/(s
qrt(b)*sqrt(a)))*a**2*b**2*x**2 + 9*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sq
rt(a)))*a**2*b*c*x**2 + 45*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a
*b**3*x**4 + 9*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*b**2*c*x**4
 + 15*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**4*x**6 + 3*sqrt(b)*
sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**3*c*x**6 + 33*a**3*b**2*x - 8*a**
3*b**2 - 3*a**3*b*c*x + 40*a**2*b**3*x**3 + 8*a**2*b**2*c*x**3 + 15*a*b**4
*x**5 + 3*a*b**3*c*x**5)/(48*a**3*b**2*(a**3 + 3*a**2*b*x**2 + 3*a*b**2*x*
*4 + b**3*x**6))
```

3.31 $\int (a+bx^2)^{3/2} (A+Bx+Cx^2) \, dx$

Optimal result

Integrand size = 22, antiderivative size = 137

$$\int (a+bx^2)^{3/2} (A+Bx+Cx^2) \, dx = \frac{a(6Ab-aC)x\sqrt{a+bx^2}}{16b} + \frac{(6Ab-aC)x(a+bx^2)^{3/2}}{24b} + \frac{B(a+bx^2)^{5/2}}{5b} + \frac{Cx(a+bx^2)^{5/2}}{6b} + \frac{a^2(6Ab-aC)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{3/2}}$$

output

```
1/16*a*(6*A*b-C*a)*x*(b*x^2+a)^(1/2)/b+1/24*(6*A*b-C*a)*x*(b*x^2+a)^(3/2)/
b+1/5*B*(b*x^2+a)^(5/2)/b+1/6*C*x*(b*x^2+a)^(5/2)/b+1/16*a^2*(6*A*b-C*a)*a
rctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.85

$$\int (a+bx^2)^{3/2} (A+Bx+Cx^2) \, dx = \frac{\sqrt{b}\sqrt{a+bx^2}(3a^2(16B+5Cx)+4b^2x^3(15A+2x(6B+5Cx))+2abx(75A+x(48B+35Cx)}{240b^{3/2}}$$

input
```
Integrate[(a + b*x^2)^(3/2)*(A + B*x + C*x^2),x]
```

output
```
(Sqrt[b]*Sqrt[a + b*x^2]*(3*a^2*(16*B + 5*C*x) + 4*b^2*x^3*(15*A + 2*x*(6*
B + 5*C*x)) + 2*a*b*x*(75*A + x*(48*B + 35*C*x))) + 15*a^2*(-6*A*b + a*C)*
Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(240*b^(3/2))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.90, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {2346, 455, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^{3/2} \left(A+Bx+Cx^2\right) \, dx$$

$$\Big\downarrow \text{2346}$$

$$\frac{\int (6Ab+6Bxb-aC) \left(bx^2+a\right)^{3/2}dx}{6b}+\frac{Cx\left(a+bx^2\right)^{5/2}}{6b}$$

$$\Big\downarrow \text{455}$$

$$\frac{(6Ab-aC)\int \left(bx^2+a\right)^{3/2}dx+\frac{6}{5}B\left(a+bx^2\right)^{5/2}}{6b}+\frac{Cx\left(a+bx^2\right)^{5/2}}{6b}$$

$$\Big\downarrow \text{211}$$

$$\frac{(6Ab-aC)\left(\frac{3}{4}a\int \sqrt{bx^2+a}dx+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{6}{5}B\left(a+bx^2\right)^{5/2}}{6b}+\frac{Cx\left(a+bx^2\right)^{5/2}}{6b}$$

$$\Big\downarrow \text{211}$$

$$\frac{(6Ab-aC)\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{6}{5}B\left(a+bx^2\right)^{5/2}}{6b}+$$
$$\frac{Cx\left(a+bx^2\right)^{5/2}}{6b}$$

$$\Big\downarrow \text{224}$$

$$\frac{(6Ab-aC)\left(\frac{3}{4}a\left(\frac{1}{2}a\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{6}{5}B\left(a+bx^2\right)^{5/2}}{6b}+\frac{Cx\left(a+bx^2\right)^{5/2}}{6b}$$

$$\Big\downarrow \text{219}$$

$$\frac{(6Ab-aC)\left(\frac{3}{4}a\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{6}{5}B\left(a+bx^2\right)^{5/2}}{6b}+\frac{Cx\left(a+bx^2\right)^{5/2}}{6b}$$

input
```
Int[(a + b*x^2)^(3/2)*(A + B*x + C*x^2),x]
```

output
```
(C*x*(a + b*x^2)^(5/2))/(6*b) + ((6*B*(a + b*x^2)^(5/2))/5 + (6*A*b - a*C)
*((x*(a + b*x^2)^(3/2))/4 + (3*a*((x*Sqrt[a + b*x^2])/2 + (a*ArcTanh[(Sqrt
[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b])))/4))/(6*b)
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.82

method	result
risch	$\frac{(40b^2Cx^5+48b^2Bx^4+60Ab^2x^3+70Cabx^3+96Babx^2+150aAbx+15Ca^2x+48a^2B)\sqrt{bx^2+a}}{240b}+\frac{a^2(6Ab-Ca)\ln\left(\sqrt{b}\,x+\sqrt{bx^2+\cdots}\right.}{16b^{\frac{3}{2}}}$
default	$A\left(\frac{x(bx^2+a)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{bx^2+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{bx^2+a}\right)}{2\sqrt{b}}\right)}{4}\right)+\frac{B(bx^2+a)^{\frac{5}{2}}}{5b}+C\left(\frac{x(bx^2+a)^{\frac{5}{2}}}{6b}-\frac{a\left(\frac{x(bx^2+a)^{\frac{3}{2}}}{4}+\frac{3a\left(\cdots\right.}{\cdots}\right.}{\cdots}\right.$

input
```
int((b*x^2+a)^(3/2)*(C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
1/240/b*(40*C*b^2*x^5+48*B*b^2*x^4+60*A*b^2*x^3+70*C*a*b*x^3+96*B*a*b*x^2+
150*A*a*b*x+15*C*a^2*x+48*B*a^2)*(b*x^2+a)^(1/2)+1/16*a^2*(6*A*b-C*a)/b^(3
/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.89

$$\int \left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)\,dx=\left[-\frac{15\,(Ca^3-6\,Aa^2b)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\,(40\,Cb^3x^5+48\,Bb^3x^4+96\,B}{480\,b^2}\right.$$

input
```
integrate((b*x^2+a)^(3/2)*(C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
[-1/480*(15*(C*a^3 - 6*A*a^2*b)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*s
qrt(b)*x - a) - 2*(40*C*b^3*x^5 + 48*B*b^3*x^4 + 96*B*a*b^2*x^2 + 48*B*a^2
*b + 10*(7*C*a*b^2 + 6*A*b^3)*x^3 + 15*(C*a^2*b + 10*A*a*b^2)*x)*sqrt(b*x^
2 + a))/b^2, 1/240*(15*(C*a^3 - 6*A*a^2*b)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt
(b*x^2 + a)) + (40*C*b^3*x^5 + 48*B*b^3*x^4 + 96*B*a*b^2*x^2 + 48*B*a^2*b
+ 10*(7*C*a*b^2 + 6*A*b^3)*x^3 + 15*(C*a^2*b + 10*A*a*b^2)*x)*sqrt(b*x^2 +
 a))/b^2]
```

Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.53

$$\int \left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)\,dx=\begin{cases}\sqrt{a+bx^2}\left(\frac{Ba^2}{5b}+\frac{2Bax^2}{5}+\frac{Bbx^4}{5}+\frac{Cbx^5}{6}+\frac{x^3\left(Ab^2+\frac{7Cab}{6}\right)}{4b}+\frac{x\left(2Aab+Ca^2-\frac{3a\left(Ab^2+\frac{7Cab}{6}\right)}{4b}\right)}{2b}\right)+\left(Aa\right. \\ a^{\frac{3}{2}}\left(Ax+\frac{Bx^2}{2}+\frac{Cx^3}{3}\right)\end{cases}$$

input
```
integrate((b*x**2+a)**(3/2)*(C*x**2+B*x+A),x)
```

output
```
Piecewise((sqrt(a + b*x**2)*(B*a**2/(5*b) + 2*B*a*x**2/5 + B*b*x**4/5 + C*
b*x**5/6 + x**3*(A*b**2 + 7*C*a*b/6)/(4*b) + x*(2*A*a*b + C*a**2 - 3*a*(A*
b**2 + 7*C*a*b/6)/(4*b))/(2*b)) + (A*a**2 - a*(2*A*a*b + C*a**2 - 3*a*(A*b
**2 + 7*C*a*b/6)/(4*b))/(2*b))*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) +
 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True)), Ne(b, 0)), (a*
*(3/2)*(A*x + B*x**2/2 + C*x**3/3), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.96

$$\int \left(a+bx^2\right)^{3/2} \left(A+Bx+Cx^2\right)\,dx = \frac{1}{4}\left(bx^2+a\right)^{\frac{3}{2}}Ax + \frac{3}{8}\sqrt{bx^2+a}Aax + \frac{\left(bx^2+a\right)^{\frac{5}{2}}Cx}{6\,b} - \frac{\left(bx^2+a\right)^{\frac{3}{2}}Cax}{24\,b} - \frac{\sqrt{bx^2+a}Ca^2x}{16\,b} - \frac{Ca^3 \operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{3}{2}}} + \frac{3\,Aa^2 \operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{b}} + \frac{\left(bx^2+a\right)^{\frac{5}{2}}B}{5\,b}$$

input
```
integrate((b*x^2+a)^(3/2)*(C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
1/4*(b*x^2 + a)^(3/2)*A*x + 3/8*sqrt(b*x^2 + a)*A*a*x + 1/6*(b*x^2 + a)^(5
/2)*C*x/b - 1/24*(b*x^2 + a)^(3/2)*C*a*x/b - 1/16*sqrt(b*x^2 + a)*C*a^2*x/
b - 1/16*C*a^3*arcsinh(b*x/sqrt(a*b))/b^(3/2) + 3/8*A*a^2*arcsinh(b*x/sqrt
(a*b))/sqrt(b) + 1/5*(b*x^2 + a)^(5/2)*B/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.91

$$\int \left(a+bx^2\right)^{3/2} \left(A+Bx+Cx^2\right)\,dx = \frac{1}{240}\sqrt{bx^2+a}\left(\frac{48\,Ba^2}{b} + \left(2\left(48\,Ba + \left(4\,(5\,Cbx+6\,Bb)x + \frac{5\,(7\,Cab^4+6\,Ab^5)}{b^4}\right)x\right)x + \frac{15}{}\right.\right. + \frac{\left(Ca^3-6\,Aa^2b\right)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{16\,b^{\frac{3}{2}}}$$

input
```
integrate((b*x^2+a)^(3/2)*(C*x^2+B*x+A),x, algorithm="giac")
```

output
```
1/240*sqrt(b*x^2 + a)*(48*B*a^2/b + (2*(48*B*a + (4*(5*C*b*x + 6*B*b)*x +
5*(7*C*a*b^4 + 6*A*b^5)/b^4)*x)*x + 15*(C*a^2*b^3 + 10*A*a*b^4)/b^4)*x) +
1/16*(C*a^3 - 6*A*a^2*b)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)\,dx=\int \left(b\,x^2+a\right)^{3/2}\left(C\,x^2+B\,x+A\right)\,dx$$

input
```
int((a + b*x^2)^(3/2)*(A + B*x + C*x^2),x)
```

output
```
int((a + b*x^2)^(3/2)*(A + B*x + C*x^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.42

$$\int \left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)\,dx=\frac{150\sqrt{b\,x^2+a}\,a^2b^2x+48\sqrt{b\,x^2+a}\,a^2b^2+15\sqrt{b\,x^2+a}\,a^2bcx+60\sqrt{b\,x^2+a}\,a\,b^3x^3+96\sqrt{b\,x^2}}{}$$

input
```
int((b*x^2+a)^(3/2)*(C*x^2+B*x+A),x)
```

output
```
(150*sqrt(a + b*x**2)*a**2*b**2*x + 48*sqrt(a + b*x**2)*a**2*b**2 + 15*sqr
t(a + b*x**2)*a**2*b*c*x + 60*sqrt(a + b*x**2)*a*b**3*x**3 + 96*sqrt(a + b
*x**2)*a*b**3*x**2 + 70*sqrt(a + b*x**2)*a*b**2*c*x**3 + 48*sqrt(a + b*x**
2)*b**4*x**4 + 40*sqrt(a + b*x**2)*b**3*c*x**5 + 90*sqrt(b)*log((sqrt(a +
b*x**2) + sqrt(b)*x)/sqrt(a))*a**3*b - 15*sqrt(b)*log((sqrt(a + b*x**2) +
sqrt(b)*x)/sqrt(a))*a**3*c)/(240*b**2)
```

3.32 $\int \sqrt{a+bx^2}(A+Bx+Cx^2)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 106

$$\int \sqrt{a+bx^2}(A+Bx+Cx^2)\,dx = \frac{(4Ab-aC)x\sqrt{a+bx^2}}{8b}+\frac{B(a+bx^2)^{3/2}}{3b}+\frac{Cx(a+bx^2)^{3/2}}{4b}+\frac{a(4Ab-aC)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{3/2}}$$

output
```
1/8*(4*A*b-C*a)*x*(b*x^2+a)^(1/2)/b+1/3*B*(b*x^2+a)^(3/2)/b+1/4*C*x*(b*x^2
+a)^(3/2)/b+1/8*a*(4*A*b-C*a)*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.82

$$\int \sqrt{a+bx^2}(A+Bx+Cx^2)\,dx = \frac{\sqrt{a+bx^2}(8aB+12Abx+3aCx+8bBx^2+6bCx^3)}{24b}+\frac{a(-4Ab+aC)\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{8b^{3/2}}$$

input
```
Integrate[Sqrt[a + b*x^2]*(A + B*x + C*x^2),x]
```

output
```
(Sqrt[a + b*x^2]*(8*a*B + 12*A*b*x + 3*a*C*x + 8*b*B*x^2 + 6*b*C*x^3))/(24
*b) + (a*(-4*A*b + a*C)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(8*b^(3/2))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {2346, 455, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx^2}\left(A+Bx+Cx^2\right) dx$$

$$\Big\downarrow \text{2346}$$

$$\frac{\int (4Ab+4Bxb-aC)\sqrt{bx^2+a}dx}{4b}+\frac{Cx\left(a+bx^2\right)^{3/2}}{4b}$$

$$\Big\downarrow \text{455}$$

$$\frac{(4Ab-aC)\int \sqrt{bx^2+a}dx+\frac{4}{3}B\left(a+bx^2\right)^{3/2}}{4b}+\frac{Cx\left(a+bx^2\right)^{3/2}}{4b}$$

$$\Big\downarrow \text{211}$$

$$\frac{(4Ab-aC)\left(\frac{1}{2}a\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{4}{3}B\left(a+bx^2\right)^{3/2}}{4b}+\frac{Cx\left(a+bx^2\right)^{3/2}}{4b}$$

$$\Big\downarrow \text{224}$$

$$\frac{(4Ab-aC)\left(\frac{1}{2}a\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{4}{3}B\left(a+bx^2\right)^{3/2}}{4b}+\frac{Cx\left(a+bx^2\right)^{3/2}}{4b}$$

$$\Big\downarrow \text{219}$$

$$\frac{(4Ab-aC)\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{4}{3}B\left(a+bx^2\right)^{3/2}}{4b}+\frac{Cx\left(a+bx^2\right)^{3/2}}{4b}$$

input
```
Int[Sqrt[a + b*x^2]*(A + B*x + C*x^2),x]
```

output
```
(C*x*(a + b*x^2)^(3/2))/(4*b) + ((4*B*(a + b*x^2)^(3/2))/3 + (4*A*b - a*C)
*((x*Sqrt[a + b*x^2])/2 + (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt
[b])))/(4*b)
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.72

method	result
risch	$\frac{(6Cb\,x^3+8bB\,x^2+12Abx+3Cax+8Ba)\sqrt{b\,x^2+a}}{24b}+\frac{a(4Ab-Ca)\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{8b^{\frac{3}{2}}}$
default	$A\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)+\frac{B(b\,x^2+a)^{\frac{3}{2}}}{3b}+C\left(\frac{x(b\,x^2+a)^{\frac{3}{2}}}{4b}-\frac{a\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)}{4b}\right)$

input
```
int((b*x^2+a)^(1/2)*(C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
1/24*(6*C*b*x^3+8*B*b*x^2+12*A*b*x+3*C*a*x+8*B*a)/b*(b*x^2+a)^(1/2)+1/8*a*
(4*A*b-C*a)/b^(3/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.76

$$\int\sqrt{a+bx^2}\left(A+Bx+Cx^2\right)\,dx$$
$$=\left[-\frac{3\,(Ca^2-4\,Aab)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\,(6\,Cb^2x^3+8\,Bb^2x^2+8\,Bab+3\,(Cab+\right.}{48\,b^2}\right.$$

input
```
integrate((b*x^2+a)^(1/2)*(C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
[-1/48*(3*(C*a^2 - 4*A*a*b)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(
b)*x - a) - 2*(6*C*b^2*x^3 + 8*B*b^2*x^2 + 8*B*a*b + 3*(C*a*b + 4*A*b^2)*x
)*sqrt(b*x^2 + a))/b^2, 1/24*(3*(C*a^2 - 4*A*a*b)*sqrt(-b)*arctan(sqrt(-b)
*x/sqrt(b*x^2 + a)) + (6*C*b^2*x^3 + 8*B*b^2*x^2 + 8*B*a*b + 3*(C*a*b + 4*
A*b^2)*x)*sqrt(b*x^2 + a))/b^2]
```

Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.17

$$\int \sqrt{a+bx^2}(A+Bx+Cx^2)\,dx = \begin{cases} \sqrt{a+bx^2}\left(\frac{Ba}{3b}+\frac{Bx^2}{3}+\frac{Cx^3}{4}+\frac{x\left(Ab+\frac{Ca}{4}\right)}{2b}\right)+\left(Aa-\frac{a\left(Ab+\frac{Ca}{4}\right)}{2b}\right)\left(\begin{cases} \frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0 \\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise} \end{cases}\right) \\ \sqrt{a}\left(Ax+\frac{Bx^2}{2}+\frac{Cx^3}{3}\right) \end{cases}$$

input
```
integrate((b*x**2+a)**(1/2)*(C*x**2+B*x+A),x)
```

output
```
Piecewise((sqrt(a + b*x**2)*(B*a/(3*b) + B*x**2/3 + C*x**3/4 + x*(A*b + C*
a/4)/(2*b)) + (A*a - a*(A*b + C*a/4)/(2*b))*Piecewise((log(2*sqrt(b)*sqrt(
a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True)), N
e(b, 0)), (sqrt(a)*(A*x + B*x**2/2 + C*x**3/3), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.91

$$\int \sqrt{a+bx^2}(A+Bx+Cx^2)\,dx = \frac{1}{2}\sqrt{bx^2+a}Ax+\frac{(bx^2+a)^{\frac{3}{2}}Cx}{4\,b} - \frac{\sqrt{bx^2+a}Cax}{8\,b} - \frac{Ca^2\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{3}{2}}} + \frac{Aa\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{b}} + \frac{(bx^2+a)^{\frac{3}{2}}B}{3\,b}$$

input
```
integrate((b*x^2+a)^(1/2)*(C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
1/2*sqrt(b*x^2 + a)*A*x + 1/4*(b*x^2 + a)^(3/2)*C*x/b - 1/8*sqrt(b*x^2 + a
)*C*a*x/b - 1/8*C*a^2*arcsinh(b*x/sqrt(a*b))/b^(3/2) + 1/2*A*a*arcsinh(b*x
/sqrt(a*b))/sqrt(b) + 1/3*(b*x^2 + a)^(3/2)*B/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.79

$$\int \sqrt{a+bx^2}\left(A+Bx+Cx^2\right)\,dx = \frac{1}{24}\sqrt{bx^2+a}\left(\left(2\,(3\,Cx+4\,B)x+\frac{3\,(Cab+4\,Ab^2)}{b^2}\right)x+\frac{8\,Ba}{b}\right) + \frac{(Ca^2-4\,Aab)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{8\,b^{\frac{3}{2}}}$$

input
```
integrate((b*x^2+a)^(1/2)*(C*x^2+B*x+A),x, algorithm="giac")
```

output
```
1/24*sqrt(b*x^2 + a)*((2*(3*C*x + 4*B)*x + 3*(C*a*b + 4*A*b^2)/b^2)*x + 8*
B*a/b) + 1/8*(C*a^2 - 4*A*a*b)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3
/2)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+bx^2}\left(A+Bx+Cx^2\right)\,dx = \int \sqrt{b\,x^2+a}\,\left(C\,x^2+B\,x+A\right)\,dx$$

input
```
int((a + b*x^2)^(1/2)*(A + B*x + C*x^2),x)
```

output
```
int((a + b*x^2)^(1/2)*(A + B*x + C*x^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.28

$$\int \sqrt{a+bx^2}\left(A+Bx+Cx^2\right)\,dx$$

$$= \frac{12\sqrt{b\,x^2+a}\,a\,b^2x+8\sqrt{b\,x^2+a}\,a\,b^2+3\sqrt{b\,x^2+a}\,abcx+8\sqrt{b\,x^2+a}\,b^3x^2+6\sqrt{b\,x^2+a}\,b^2c\,x^3+12\sqrt{b}\,\mathrm{lo}}{24b^2}$$

input
```
int((b*x^2+a)^(1/2)*(C*x^2+B*x+A),x)
```

output
```
(12*sqrt(a + b*x**2)*a*b**2*x + 8*sqrt(a + b*x**2)*a*b**2 + 3*sqrt(a + b*x
**2)*a*b*c*x + 8*sqrt(a + b*x**2)*b**3*x**2 + 6*sqrt(a + b*x**2)*b**2*c*x*
*3 + 12*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2*b - 3*sqr
t(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2*c)/(24*b**2)
```

3.33 $\int \frac{A+Bx+Cx^2}{\sqrt{a+bx^2}} dx$

Optimal result

Integrand size = 22, antiderivative size = 74

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx^2}} dx = \frac{B\sqrt{a+bx^2}}{b} + \frac{Cx\sqrt{a+bx^2}}{2b} + \frac{(2Ab-aC)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}$$

output
```
B*(b*x^2+a)^(1/2)/b+1/2*C*x*(b*x^2+a)^(1/2)/b+1/2*(2*A*b-C*a)*arctanh(b^(1
/2)*x/(b*x^2+a)^(1/2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.95

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx^2}} dx = \frac{(2B+Cx)\sqrt{a+bx^2}}{2b} + \frac{(2Ab-aC)\text{arctanh}\left(\frac{\sqrt{b}x}{-\sqrt{a}+\sqrt{a+bx^2}}\right)}{b^{3/2}}$$

input
```
Integrate[(A + B*x + C*x^2)/Sqrt[a + b*x^2],x]
```

output
```
((2*B + C*x)*Sqrt[a + b*x^2])/(2*b) + ((2*A*b - a*C)*ArcTanh[(Sqrt[b]*x)/(
-Sqrt[a] + Sqrt[a + b*x^2])])/b^(3/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2346, 455, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx^2}}dx$$

$$\Big\downarrow \ 2346$$

$$\frac{\int \frac{2Ab+2Bxb-aC}{\sqrt{bx^2+a}}dx}{2b}+\frac{Cx\sqrt{a+bx^2}}{2b}$$

$$\Big\downarrow \ 455$$

$$\frac{(2Ab-aC)\int \frac{1}{\sqrt{bx^2+a}}dx+2B\sqrt{a+bx^2}}{2b}+\frac{Cx\sqrt{a+bx^2}}{2b}$$

$$\Big\downarrow \ 224$$

$$\frac{(2Ab-aC)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+2B\sqrt{a+bx^2}}{2b}+\frac{Cx\sqrt{a+bx^2}}{2b}$$

$$\Big\downarrow \ 219$$

$$\frac{\frac{(2Ab-aC)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}+2B\sqrt{a+bx^2}}{2b}+\frac{Cx\sqrt{a+bx^2}}{2b}$$

input
```
Int[(A + B*x + C*x^2)/Sqrt[a + b*x^2],x]
```

output
```
(C*x*Sqrt[a + b*x^2])/(2*b) + (2*B*Sqrt[a + b*x^2] + ((2*A*b - a*C)*ArcTan
h[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/Sqrt[b])/(2*b)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] && !LeQ[p, -1]
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.72

method	result	size
risch	$\frac{(Cx+2B)\sqrt{b\,x^2+a}}{2b}+\frac{(2Ab-Ca)\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2b^{\frac{3}{2}}}$	53
default	$\frac{A\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}+\frac{B\sqrt{b\,x^2+a}}{b}+C\left(\frac{x\sqrt{b\,x^2+a}}{2b}-\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2b^{\frac{3}{2}}}\right)$	77

input
```
int((C*x^2+B*x+A)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(C*x+2*B)/b*(b*x^2+a)^(1/2)+1/2*(2*A*b-C*a)/b^(3/2)*ln(b^(1/2)*x+(b*x^
2+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.64

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx^2}}\,dx$$
$$= \left[-\frac{(Ca-2\,Ab)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\,(Cbx+2\,Bb)\sqrt{bx^2+a}}{4\,b^2}, \frac{(Ca-2\,Ab)\sqrt{-b}\,\mathrm{ar}\right.$$

input
```
integrate((C*x^2+B*x+A)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/4*((C*a - 2*A*b)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x -
a) - 2*(C*b*x + 2*B*b)*sqrt(b*x^2 + a))/b^2, 1/2*((C*a - 2*A*b)*sqrt(-b)*a
rctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (C*b*x + 2*B*b)*sqrt(b*x^2 + a))/b^2]
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.24

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx^2}}\,dx$$
$$= \begin{cases} \left(A-\frac{Ca}{2b}\right)\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right)+\sqrt{a+bx^2}\left(\frac{B}{b}+\frac{Cx}{2b}\right) & \text{for } b\neq 0\\ \frac{Ax+\frac{Bx^2}{2}+\frac{Cx^3}{3}}{\sqrt{a}} & \text{otherwise}\end{cases}$$

input
```
integrate((C*x**2+B*x+A)/(b*x**2+a)**(1/2),x)
```

output
```
Piecewise(((A - C*a/(2*b))*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b
*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True)) + sqrt(a + b*x**2)*
(B/b + C*x/(2*b)), Ne(b, 0)), ((A*x + B*x**2/2 + C*x**3/3)/sqrt(a), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.82

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{bx^2+a}Cx}{2\,b} - \frac{Ca\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{3}{2}}} + \frac{A\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}} + \frac{\sqrt{bx^2+a}B}{b}$$

input
```
integrate((C*x^2+B*x+A)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(b*x^2 + a)*C*x/b - 1/2*C*a*arcsinh(b*x/sqrt(a*b))/b^(3/2) + A*arc
sinh(b*x/sqrt(a*b))/sqrt(b) + sqrt(b*x^2 + a)*B/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.76

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx^2}}\,dx = \frac{1}{2}\sqrt{bx^2+a}\left(\frac{Cx}{b}+\frac{2\,B}{b}\right) + \frac{(Ca-2\,Ab)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2\,b^{\frac{3}{2}}}$$

input
```
integrate((C*x^2+B*x+A)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(b*x^2 + a)*(C*x/b + 2*B/b) + 1/2*(C*a - 2*A*b)*log(abs(-sqrt(b)*x
 + sqrt(b*x^2 + a)))/b^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.45

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx^2}}\,dx = \begin{cases} \frac{2\,C\,x^3+3\,B\,x^2+6\,A\,x}{6\,\sqrt{a}} & \text{if } b=0 \\ \frac{B\,\sqrt{b\,x^2+a}}{b} + \frac{A\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{\sqrt{b}} - \frac{C\,a\,\ln\left(2\,\sqrt{b}\,x+2\,\sqrt{b\,x^2+a}\right)}{2\,b^{3/2}} + \frac{C\,x\,\sqrt{b\,x^2+a}}{2\,b} & \text{if } b\neq 0 \end{cases}$$

input
```
int((A + B*x + C*x^2)/(a + b*x^2)^(1/2),x)
```

output
```
piecewise(b == 0, (6*A*x + 3*B*x^2 + 2*C*x^3)/(6*a^(1/2)), b ~= 0, (B*(a +
 b*x^2)^(1/2))/b + (A*log(b^(1/2)*x + (a + b*x^2)^(1/2)))/b^(1/2) - (C*a*l
og(2*b^(1/2)*x + 2*(a + b*x^2)^(1/2)))/(2*b^(3/2)) + (C*x*(a + b*x^2)^(1/2
))/(2*b))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.09

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx^2}}\,dx = \frac{2\sqrt{b\,x^2+a}\,b^2+\sqrt{b\,x^2+a}\,bcx+2\sqrt{b}\log\left(\frac{\sqrt{b\,x^2+a}+\sqrt{b}\,x}{\sqrt{a}}\right)ab-\sqrt{b}\log\left(\frac{\sqrt{b\,x^2+a}+\sqrt{b}\,x}{\sqrt{a}}\right)ac}{2b^2}$$

input
```
int((C*x^2+B*x+A)/(b*x^2+a)^(1/2),x)
```

output
```
(2*sqrt(a + b*x**2)*b**2 + sqrt(a + b*x**2)*b*c*x + 2*sqrt(b)*log((sqrt(a
+ b*x**2) + sqrt(b)*x)/sqrt(a))*a*b - sqrt(b)*log((sqrt(a + b*x**2) + sqrt
(b)*x)/sqrt(a))*a*c)/(2*b**2)
```

3.34 $\int \frac{A+Bx+Cx^2}{(a+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 61

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^{3/2}}\,dx = -\frac{aB-(Ab-aC)x}{ab\sqrt{a+bx^2}} + \frac{C\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}$$

output
```
-(B*a-(A*b-C*a)*x)/a/b/(b*x^2+a)^(1/2)+C*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2)
)/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.02

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^{3/2}}\,dx = \frac{-aB+Abx-aCx}{ab\sqrt{a+bx^2}} - \frac{C\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{b^{3/2}}$$

input
```
Integrate[(A + B*x + C*x^2)/(a + b*x^2)^(3/2),x]
```

output
```
(-(a*B) + A*b*x - a*C*x)/(a*b*Sqrt[a + b*x^2]) - (C*Log[-(Sqrt[b]*x) + Sqr
t[a + b*x^2]])/b^(3/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {2345, 25, 27, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{2345}$$

$$-\frac{\int -\frac{aC}{b\sqrt{bx^2+a}}dx}{a}-\frac{aB-x(Ab-aC)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{aC}{b\sqrt{bx^2+a}}dx}{a}-\frac{aB-x(Ab-aC)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{C\int \frac{1}{\sqrt{bx^2+a}}dx}{b}-\frac{aB-x(Ab-aC)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{224}$$

$$\frac{C\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{b}-\frac{aB-x(Ab-aC)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{C\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}-\frac{aB-x(Ab-aC)}{ab\sqrt{a+bx^2}}$$

input
```
Int[(A + B*x + C*x^2)/(a + b*x^2)^(3/2),x]
```

output
```
-((a*B - (A*b - a*C)*x)/(a*b*Sqrt[a + b*x^2])) + (C*ArcTanh[(Sqrt[b]*x)/Sq
rt[a + b*x^2]])/b^(3/2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.15

method	result	size
default	$\frac{Ax}{a\sqrt{b\,x^{2}+a}}-\frac{B}{b\sqrt{b\,x^{2}+a}}+C\left(-\frac{x}{b\sqrt{b\,x^{2}+a}}+\frac{\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{b^{\frac{3}{2}}}\right)$	70

input
```
int((C*x^2+B*x+A)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
A*x/a/(b*x^2+a)^(1/2)-B/b/(b*x^2+a)^(1/2)+C*(-x/b/(b*x^2+a)^(1/2)+1/b^(3/2
)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 179, normalized size of antiderivative = 2.93

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^{3/2}}\,dx = \left[\frac{(Cabx^2+Ca^2)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\,(Bab+(Cab-Ab^2)x}{2\,(ab^3x^2+a^2b^2)}\right.$$

$$\left.-\frac{(Cabx^2+Ca^2)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)+(Bab+(Cab-Ab^2)x)\sqrt{bx^2+a}}{ab^3x^2+a^2b^2}\right]$$

input
```
integrate((C*x^2+B*x+A)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/2*((C*a*b*x^2 + C*a^2)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)
*x - a) - 2*(B*a*b + (C*a*b - A*b^2)*x)*sqrt(b*x^2 + a))/(a*b^3*x^2 + a^2*
b^2), -((C*a*b*x^2 + C*a^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) +
(B*a*b + (C*a*b - A*b^2)*x)*sqrt(b*x^2 + a))/(a*b^3*x^2 + a^2*b^2)]
```

Sympy [A] (verification not implemented)

Time = 2.74 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.43

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^{3/2}}\,dx = \frac{Ax}{a^{\frac{3}{2}}\sqrt{1+\frac{bx^2}{a}}} + B\left(\begin{cases}-\frac{1}{b\sqrt{a+bx^2}} & \text{for } b\neq 0\\ \frac{x^2}{2a^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right) + C\left(\frac{\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{\frac{3}{2}}} - \frac{x}{\sqrt{a}b\sqrt{1+\frac{bx^2}{a}}}\right)$$

input
```
integrate((C*x**2+B*x+A)/(b*x**2+a)**(3/2),x)
```

output
```
A*x/(a**(3/2)*sqrt(1 + b*x**2/a)) + B*Piecewise((-1/(b*sqrt(a + b*x**2)),
Ne(b, 0)), (x**2/(2*a**(3/2)), True)) + C*(asinh(sqrt(b)*x/sqrt(a))/b**(3/
2) - x/(sqrt(a)*b*sqrt(1 + b*x**2/a)))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^{3/2}}\,dx = \frac{Ax}{\sqrt{bx^2+a}a} - \frac{Cx}{\sqrt{bx^2+a}b} + \frac{C\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{3}{2}}} - \frac{B}{\sqrt{bx^2+a}b}$$

input
```
integrate((C*x^2+B*x+A)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
A*x/(sqrt(b*x^2 + a)*a) - C*x/(sqrt(b*x^2 + a)*b) + C*arcsinh(b*x/sqrt(a*b
))/b^(3/2) - B/(sqrt(b*x^2 + a)*b)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^{3/2}}\,dx = -\frac{\frac{B}{b} + \frac{(Cab-Ab^2)x}{ab^2}}{\sqrt{bx^2+a}} - \frac{C\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{b^{\frac{3}{2}}}$$

input
```
integrate((C*x^2+B*x+A)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
-(B/b + (C*a*b - A*b^2)*x/(a*b^2))/sqrt(b*x^2 + a) - C*log(abs(-sqrt(b)*x
+ sqrt(b*x^2 + a)))/b^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.11

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^{3/2}}\,dx = \frac{C\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{b^{3/2}} - \frac{B}{b\,\sqrt{b\,x^2+a}} + \frac{A\,x}{a\,\sqrt{b\,x^2+a}} - \frac{C\,x}{b\,\sqrt{b\,x^2+a}}$$

input
```
int((A + B*x + C*x^2)/(a + b*x^2)^(3/2),x)
```

output
```
(C*log(b^(1/2)*x + (a + b*x^2)^(1/2)))/b^(3/2) - B/(b*(a + b*x^2)^(1/2)) +
 (A*x)/(a*(a + b*x^2)^(1/2)) - (C*x)/(b*(a + b*x^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 133, normalized size of antiderivative = 2.18

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^{3/2}}\,dx = \frac{\sqrt{b\,x^2+a}\,b^2x-\sqrt{b\,x^2+a}\,b^2-\sqrt{b\,x^2+a}\,bcx+\sqrt{b}\log\left(\frac{\sqrt{b\,x^2+a}+\sqrt{b}\,x}{\sqrt{a}}\right)ac+\sqrt{b}\log\left(\right.}{b^2\,(b\,x^2+a)}$$

input
```
int((C*x^2+B*x+A)/(b*x^2+a)^(3/2),x)
```

output
```
(sqrt(a + b*x**2)*b**2*x - sqrt(a + b*x**2)*b**2 - sqrt(a + b*x**2)*b*c*x
+ sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a*c + sqrt(b)*log((s
qrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*b*c*x**2 + sqrt(b)*a*b - sqrt(b)*a*c
 + sqrt(b)*b**2*x**2 - sqrt(b)*b*c*x**2)/(b**2*(a + b*x**2))
```

3.35 $\int \frac{A+Bx+Cx^2}{\left(a+bx^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 67

$$\int \frac{A+Bx+Cx^2}{\left(a+bx^2\right)^{5/2}}\,dx=-\frac{aB-(Ab-aC)x}{3ab\left(a+bx^2\right)^{3/2}}+\frac{(2Ab+aC)x}{3a^2b\sqrt{a+bx^2}}$$

output
```
-1/3*(B*a-(A*b-C*a)*x)/a/b/(b*x^2+a)^(3/2)+1/3*(2*A*b+C*a)*x/a^2/b/(b*x^2+
a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.75

$$\int \frac{A+Bx+Cx^2}{\left(a+bx^2\right)^{5/2}}\,dx=\frac{-a^2B+3aAbx+2Ab^2x^3+abCx^3}{3a^2b\left(a+bx^2\right)^{3/2}}$$

input
```
Integrate[(A + B*x + C*x^2)/(a + b*x^2)^(5/2),x]
```

output
```
(-(a^2*B) + 3*a*A*b*x + 2*A*b^2*x^3 + a*b*C*x^3)/(3*a^2*b*(a + b*x^2)^(3/2
))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2345, 25, 27, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^{5/2}}dx$$

$$\Big\downarrow \text{ 2345}$$

$$-\frac{\int -\frac{2Ab+aC}{b(bx^2+a)^{3/2}}dx}{3a}-\frac{aB-x(Ab-aC)}{3ab(a+bx^2)^{3/2}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{2Ab+aC}{b(bx^2+a)^{3/2}}dx}{3a}-\frac{aB-x(Ab-aC)}{3ab(a+bx^2)^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{(aC+2Ab)\int \frac{1}{(bx^2+a)^{3/2}}dx}{3ab}-\frac{aB-x(Ab-aC)}{3ab(a+bx^2)^{3/2}}$$

$$\Big\downarrow \text{ 208}$$

$$\frac{x(aC+2Ab)}{3a^2b\sqrt{a+bx^2}}-\frac{aB-x(Ab-aC)}{3ab(a+bx^2)^{3/2}}$$

input
```
Int[(A + B*x + C*x^2)/(a + b*x^2)^(5/2),x]
```

output
```
-1/3*(a*B - (A*b - a*C)*x)/(a*b*(a + b*x^2)^(3/2)) + ((2*A*b + a*C)*x)/(3*
a^2*b*Sqrt[a + b*x^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.70

method	result	size
gosper	$\frac{2A\,b^2x^3+Cab\,x^3+3aAbx-a^2B}{3(b\,x^2+a)^{\frac{3}{2}}a^2b}$	47
trager	$\frac{2A\,b^2x^3+Cab\,x^3+3aAbx-a^2B}{3(b\,x^2+a)^{\frac{3}{2}}a^2b}$	47
orering	$\frac{2A\,b^2x^3+Cab\,x^3+3aAbx-a^2B}{3(b\,x^2+a)^{\frac{3}{2}}a^2b}$	47
default	$A\left(\frac{x}{3a(b\,x^2+a)^{\frac{3}{2}}}+\frac{2x}{3a^2\sqrt{b\,x^2+a}}\right)-\frac{B}{3b(b\,x^2+a)^{\frac{3}{2}}}+C\left(-\frac{x}{2b(b\,x^2+a)^{\frac{3}{2}}}+\frac{a\left(\frac{x}{3a\left(b\,x^2+a\right)^{\frac{3}{2}}}+\frac{2x}{3a^2\sqrt{b\,x^2+a}}\right)}{2b}\right)$	105

input
```
int((C*x^2+B*x+A)/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(2*A*b^2*x^3+C*a*b*x^3+3*A*a*b*x-B*a^2)/(b*x^2+a)^(3/2)/a^2/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.01

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^{5/2}}\,dx = \frac{(3\,Aabx+(Cab+2\,Ab^2)x^3-Ba^2)\sqrt{bx^2+a}}{3\,(a^2b^3x^4+2\,a^3b^2x^2+a^4b)}$$

input
```
integrate((C*x^2+B*x+A)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
1/3*(3*A*a*b*x + (C*a*b + 2*A*b^2)*x^3 - B*a^2)*sqrt(b*x^2 + a)/(a^2*b^3*x
^4 + 2*a^3*b^2*x^2 + a^4*b)
```

Sympy [A] (verification not implemented)

Time = 5.10 (sec) , antiderivative size = 194, normalized size of antiderivative = 2.90

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^{5/2}}\,dx = A\left(\frac{3ax}{3a^{\frac{7}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{5}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}} + \frac{2bx^3}{3a^{\frac{7}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{5}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}}\right) + B\left(\begin{cases}-\frac{1}{3ab\sqrt{a+bx^2}+3b^2x^2\sqrt{a+bx^2}} & \text{for } b\neq 0\\ \frac{x^2}{2a^{\frac{5}{2}}} & \text{otherwise}\end{cases}\right) + \frac{Cx^3}{3a^{\frac{5}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{3}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}}$$

input
```
integrate((C*x**2+B*x+A)/(b*x**2+a)**(5/2),x)
```

output
```
A*(3*a*x/(3*a**(7/2)*sqrt(1 + b*x**2/a) + 3*a**(5/2)*b*x**2*sqrt(1 + b*x**
2/a)) + 2*b*x**3/(3*a**(7/2)*sqrt(1 + b*x**2/a) + 3*a**(5/2)*b*x**2*sqrt(1
 + b*x**2/a))) + B*Piecewise((-1/(3*a*b*sqrt(a + b*x**2) + 3*b**2*x**2*sqr
t(a + b*x**2)), Ne(b, 0)), (x**2/(2*a**(5/2)), True)) + C*x**3/(3*a**(5/2)
*sqrt(1 + b*x**2/a) + 3*a**(3/2)*b*x**2*sqrt(1 + b*x**2/a))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.24

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^{5/2}}\,dx = \frac{2\,Ax}{3\,\sqrt{bx^2+a}a^2} + \frac{Ax}{3\,(bx^2+a)^{\frac{3}{2}}a} - \frac{Cx}{3\,(bx^2+a)^{\frac{3}{2}}b} + \frac{Cx}{3\,\sqrt{bx^2+a}ab} - \frac{B}{3\,(bx^2+a)^{\frac{3}{2}}b}$$

input
```
integrate((C*x^2+B*x+A)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
2/3*A*x/(sqrt(b*x^2 + a)*a^2) + 1/3*A*x/((b*x^2 + a)^(3/2)*a) - 1/3*C*x/((
b*x^2 + a)^(3/2)*b) + 1/3*C*x/(sqrt(b*x^2 + a)*a*b) - 1/3*B/((b*x^2 + a)^(
3/2)*b)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.72

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^{5/2}}\,dx = \frac{x\left(\frac{3\,A}{a} + \frac{(Cab+2\,Ab^2)x^2}{a^2b}\right) - \frac{B}{b}}{3\,(bx^2+a)^{\frac{3}{2}}}$$

input
```
integrate((C*x^2+B*x+A)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
1/3*(x*(3*A/a + (C*a*b + 2*A*b^2)*x^2/(a^2*b)) - B/b)/(b*x^2 + a)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.88

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^{5/2}}\,dx = \frac{2\,A\,b\,x\,(b\,x^2+a) - C\,a^2\,x - B\,a^2 + C\,a\,x\,(b\,x^2+a) + A\,a\,b\,x}{3\,a^2\,b\,(b\,x^2+a)^{3/2}}$$

input
```
int((A + B*x + C*x^2)/(a + b*x^2)^(5/2),x)
```

output
```
(2*A*b*x*(a + b*x^2) - C*a^2*x - B*a^2 + C*a*x*(a + b*x^2) + A*a*b*x)/(3*a
^2*b*(a + b*x^2)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.18

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^{5/2}}\,dx = \frac{3\sqrt{b\,x^2+a}\,a\,b^2x-\sqrt{b\,x^2+a}\,a\,b^2+2\sqrt{b\,x^2+a}\,b^3x^3+\sqrt{b\,x^2+a}\,b^2c\,x^3-2\sqrt{b}\,a^2b}{3a\,b^2\left(b^2x^4+2ab\,x^2+a\right.}$$

input
```
int((C*x^2+B*x+A)/(b*x^2+a)^(5/2),x)
```

output
```
(3*sqrt(a + b*x**2)*a*b**2*x - sqrt(a + b*x**2)*a*b**2 + 2*sqrt(a + b*x**2
)*b**3*x**3 + sqrt(a + b*x**2)*b**2*c*x**3 - 2*sqrt(b)*a**2*b + sqrt(b)*a*
*2*c - 4*sqrt(b)*a*b**2*x**2 + 2*sqrt(b)*a*b*c*x**2 - 2*sqrt(b)*b**3*x**4
+ sqrt(b)*b**2*c*x**4)/(3*a*b**2*(a**2 + 2*a*b*x**2 + b**2*x**4))
```

3.36 $\int \frac{A+Bx+Cx^2}{(a+bx^2)^{7/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 97

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^{7/2}}\,dx = -\frac{aB-(Ab-aC)x}{5ab(a+bx^2)^{5/2}} + \frac{(4Ab+aC)x}{15a^2b(a+bx^2)^{3/2}} + \frac{2(4Ab+aC)x}{15a^3b\sqrt{a+bx^2}}$$

output
```
-1/5*(B*a-(A*b-C*a)*x)/a/b/(b*x^2+a)^(5/2)+1/15*(4*A*b+C*a)*x/a^2/b/(b*x^2
+a)^(3/2)+2/15*(4*A*b+C*a)*x/a^3/b/(b*x^2+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.73

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^{7/2}}\,dx = \frac{-3a^3B+8Ab^3x^5+5a^2bx(3A+Cx^2)+2ab^2x^3(10A+Cx^2)}{15a^3b(a+bx^2)^{5/2}}$$

input
```
Integrate[(A + B*x + C*x^2)/(a + b*x^2)^(7/2),x]
```

output
```
(-3*a^3*B + 8*A*b^3*x^5 + 5*a^2*b*x*(3*A + C*x^2) + 2*a*b^2*x^3*(10*A + C*
x^2))/(15*a^3*b*(a + b*x^2)^(5/2))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {2345, 25, 27, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{\left(a+bx^2\right)^{7/2}}dx$$

$$\Big\downarrow \text{ 2345}$$

$$-\frac{\int -\frac{4Ab+aC}{b\left(bx^2+a\right)^{5/2}}dx}{5a}-\frac{aB-x(Ab-aC)}{5ab\left(a+bx^2\right)^{5/2}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{4Ab+aC}{b\left(bx^2+a\right)^{5/2}}dx}{5a}-\frac{aB-x(Ab-aC)}{5ab\left(a+bx^2\right)^{5/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{(aC+4Ab)\int \frac{1}{\left(bx^2+a\right)^{5/2}}dx}{5ab}-\frac{aB-x(Ab-aC)}{5ab\left(a+bx^2\right)^{5/2}}$$

$$\Big\downarrow \text{ 209}$$

$$\frac{(aC+4Ab)\left(\frac{2\int \frac{1}{\left(bx^2+a\right)^{3/2}}dx}{3a}+\frac{x}{3a\left(a+bx^2\right)^{3/2}}\right)}{5ab}-\frac{aB-x(Ab-aC)}{5ab\left(a+bx^2\right)^{5/2}}$$

$$\Big\downarrow \text{ 208}$$

$$\frac{\left(\frac{2x}{3a^2\sqrt{a+bx^2}}+\frac{x}{3a\left(a+bx^2\right)^{3/2}}\right)(aC+4Ab)}{5ab}-\frac{aB-x(Ab-aC)}{5ab\left(a+bx^2\right)^{5/2}}$$

input
```
Int[(A + B*x + C*x^2)/(a + b*x^2)^(7/2),x]
```

output
```
-1/5*(a*B - (A*b - a*C)*x)/(a*b*(a + b*x^2)^(5/2)) + ((4*A*b + a*C)*(x/(3*
a*(a + b*x^2)^(3/2)) + (2*x)/(3*a^2*Sqrt[a + b*x^2])))/(5*a*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.74

method	result
gosper	$\frac{8A\,b^3x^5+2Ca\,b^2x^5+20aA\,b^2x^3+5C\,a^2b\,x^3+15a^2Abx-3a^3B}{15(b\,x^2+a)^{\frac{5}{2}}a^3b}$
trager	$\frac{8A\,b^3x^5+2Ca\,b^2x^5+20aA\,b^2x^3+5C\,a^2b\,x^3+15a^2Abx-3a^3B}{15(b\,x^2+a)^{\frac{5}{2}}a^3b}$
orering	$\frac{8A\,b^3x^5+2Ca\,b^2x^5+20aA\,b^2x^3+5C\,a^2b\,x^3+15a^2Abx-3a^3B}{15(b\,x^2+a)^{\frac{5}{2}}a^3b}$
default	$A\left(\frac{x}{5a(b\,x^2+a)^{\frac{5}{2}}}+\frac{\frac{4x}{15a\left(b\,x^2+a\right)^{\frac{3}{2}}}+\frac{8x}{15a^2\sqrt{b\,x^2+a}}}{a}\right)-\frac{B}{5b(b\,x^2+a)^{\frac{5}{2}}}+C\left(-\frac{x}{4b(b\,x^2+a)^{\frac{5}{2}}}+\frac{a\left(\frac{x}{5a\left(b\,x^2+a\right)^{\frac{5}{2}}}+\frac{4x}{15a\left(b\,x^2\right.}\right.}{4b}\right.$

input
```
int((C*x^2+B*x+A)/(b*x^2+a)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*(8*A*b^3*x^5+2*C*a*b^2*x^5+20*A*a*b^2*x^3+5*C*a^2*b*x^3+15*A*a^2*b*x-
3*B*a^3)/(b*x^2+a)^(5/2)/a^3/b
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.06

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^{7/2}}\,dx = \frac{(2\,(Cab^2+4\,Ab^3)x^5+15\,Aa^2bx-3\,Ba^3+5\,(Ca^2b+4\,Aab^2)x^3)\sqrt{bx^2+a}}{15\,(a^3b^4x^6+3\,a^4b^3x^4+3\,a^5b^2x^2+a^6b)}$$

input
```
integrate((C*x^2+B*x+A)/(b*x^2+a)^(7/2),x, algorithm="fricas")
```

output
```
1/15*(2*(C*a*b^2 + 4*A*b^3)*x^5 + 15*A*a^2*b*x - 3*B*a^3 + 5*(C*a^2*b + 4*
A*a*b^2)*x^3)*sqrt(b*x^2 + a)/(a^3*b^4*x^6 + 3*a^4*b^3*x^4 + 3*a^5*b^2*x^2
 + a^6*b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 415 vs. $2(87) = 174$.

Time = 11.56 (sec) , antiderivative size = 638, normalized size of antiderivative = 6.58

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^{7/2}}\,dx = A\left(\frac{15a^5x}{15a^{\frac{17}{2}}\sqrt{1+\frac{bx^2}{a}}+45a^{\frac{15}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}+45a^{\frac{13}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}}+15a^{\frac{11}{2}}b^3x^6\sqrt{1+\frac{bx^2}{a}}}\right.$$
$$+\frac{35a^4bx^3}{15a^{\frac{17}{2}}\sqrt{1+\frac{bx^2}{a}}+45a^{\frac{15}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}+45a^{\frac{13}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}}+15a^{\frac{11}{2}}b^3x^6\sqrt{1+\frac{bx^2}{a}}}$$
$$+\frac{28a^3b^2x^5}{15a^{\frac{17}{2}}\sqrt{1+\frac{bx^2}{a}}+45a^{\frac{15}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}+45a^{\frac{13}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}}+15a^{\frac{11}{2}}b^3x^6\sqrt{1+\frac{bx^2}{a}}}$$
$$\left.+\frac{8a^2b^3x^7}{15a^{\frac{17}{2}}\sqrt{1+\frac{bx^2}{a}}+45a^{\frac{15}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}+45a^{\frac{13}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}}+15a^{\frac{11}{2}}b^3x^6\sqrt{1+\frac{bx^2}{a}}}\right)$$
$$+B\left(\begin{cases}-\frac{1}{5a^2b\sqrt{a+bx^2}+10ab^2x^2\sqrt{a+bx^2}+5b^3x^4\sqrt{a+bx^2}} & \text{for } b\neq 0\\ \frac{x^2}{2a^{\frac{7}{2}}} & \text{otherwise}\end{cases}\right)$$
$$+C\left(\frac{5ax^3}{15a^{\frac{9}{2}}\sqrt{1+\frac{bx^2}{a}}+30a^{\frac{7}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}+15a^{\frac{5}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}}}\right.$$
$$\left.+\frac{2bx^5}{15a^{\frac{9}{2}}\sqrt{1+\frac{bx^2}{a}}+30a^{\frac{7}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}+15a^{\frac{5}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}}}\right)$$

input
```
integrate((C*x**2+B*x+A)/(b*x**2+a)**(7/2),x)
```

output
```
A*(15*a**5*x/(15*a**(17/2)*sqrt(1 + b*x**2/a) + 45*a**(15/2)*b*x**2*sqrt(1
 + b*x**2/a) + 45*a**(13/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 15*a**(11/2)*b*
*3*x**6*sqrt(1 + b*x**2/a)) + 35*a**4*b*x**3/(15*a**(17/2)*sqrt(1 + b*x**2
/a) + 45*a**(15/2)*b*x**2*sqrt(1 + b*x**2/a) + 45*a**(13/2)*b**2*x**4*sqrt
(1 + b*x**2/a) + 15*a**(11/2)*b**3*x**6*sqrt(1 + b*x**2/a)) + 28*a**3*b**2
*x**5/(15*a**(17/2)*sqrt(1 + b*x**2/a) + 45*a**(15/2)*b*x**2*sqrt(1 + b*x*
*2/a) + 45*a**(13/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 15*a**(11/2)*b**3*x**6
*sqrt(1 + b*x**2/a)) + 8*a**2*b**3*x**7/(15*a**(17/2)*sqrt(1 + b*x**2/a) +
 45*a**(15/2)*b*x**2*sqrt(1 + b*x**2/a) + 45*a**(13/2)*b**2*x**4*sqrt(1 +
b*x**2/a) + 15*a**(11/2)*b**3*x**6*sqrt(1 + b*x**2/a))) + B*Piecewise((-1/
(5*a**2*b*sqrt(a + b*x**2) + 10*a*b**2*x**2*sqrt(a + b*x**2) + 5*b**3*x**4
*sqrt(a + b*x**2)), Ne(b, 0)), (x**2/(2*a**(7/2)), True)) + C*(5*a*x**3/(1
5*a**(9/2)*sqrt(1 + b*x**2/a) + 30*a**(7/2)*b*x**2*sqrt(1 + b*x**2/a) + 15
*a**(5/2)*b**2*x**4*sqrt(1 + b*x**2/a)) + 2*b*x**5/(15*a**(9/2)*sqrt(1 + b
*x**2/a) + 30*a**(7/2)*b*x**2*sqrt(1 + b*x**2/a) + 15*a**(5/2)*b**2*x**4*s
qrt(1 + b*x**2/a)))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.22

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^{7/2}}\,dx = \frac{8\,Ax}{15\,\sqrt{bx^2+a}a^3} + \frac{4\,Ax}{15\,(bx^2+a)^{\frac{3}{2}}a^2} + \frac{Ax}{5\,(bx^2+a)^{\frac{5}{2}}a} - \frac{Cx}{5\,(bx^2+a)^{\frac{5}{2}}b} + \frac{2\,Cx}{15\,\sqrt{bx^2+a}a^2b} + \frac{Cx}{15\,(bx^2+a)^{\frac{3}{2}}ab} - \frac{B}{5\,(bx^2+a)^{\frac{5}{2}}b}$$

input
```
integrate((C*x^2+B*x+A)/(b*x^2+a)^(7/2),x, algorithm="maxima")
```

output
```
8/15*A*x/(sqrt(b*x^2 + a)*a^3) + 4/15*A*x/((b*x^2 + a)^(3/2)*a^2) + 1/5*A*
x/((b*x^2 + a)^(5/2)*a) - 1/5*C*x/((b*x^2 + a)^(5/2)*b) + 2/15*C*x/(sqrt(b
*x^2 + a)*a^2*b) + 1/15*C*x/((b*x^2 + a)^(3/2)*a*b) - 1/5*B/((b*x^2 + a)^(
5/2)*b)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.82

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^{7/2}}\,dx = \frac{\left(x^2\left(\frac{2\,(Cab^3+4\,Ab^4)x^2}{a^3b^2}+\frac{5\,(Ca^2b^2+4\,Aab^3)}{a^3b^2}\right)+\frac{15\,A}{a}\right)x-\frac{3\,B}{b}}{15\,(bx^2+a)^{\frac{5}{2}}}$$

input
```
integrate((C*x^2+B*x+A)/(b*x^2+a)^(7/2),x, algorithm="giac")
```

output
```
1/15*((x^2*(2*(C*a*b^3 + 4*A*b^4)*x^2/(a^3*b^2) + 5*(C*a^2*b^2 + 4*A*a*b^3
)/(a^3*b^2)) + 15*A/a)*x - 3*B/b)/(b*x^2 + a)^(5/2)
```

Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.96

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^{7/2}}\,dx = \frac{8\,A\,b\,x\,(b\,x^2+a)^2-3\,C\,a^3\,x-3\,B\,a^3+2\,C\,a\,x\,(b\,x^2+a)^2+C\,a^2\,x\,(b\,x^2+a)+3}{15\,a^3\,b\,(b\,x^2+a)^{5/2}}$$

input
```
int((A + B*x + C*x^2)/(a + b*x^2)^(7/2),x)
```

output
```
(8*A*b*x*(a + b*x^2)^2 - 3*C*a^3*x - 3*B*a^3 + 2*C*a*x*(a + b*x^2)^2 + C*a
^2*x*(a + b*x^2) + 3*A*a^2*b*x + 4*A*a*b*x*(a + b*x^2))/(15*a^3*b*(a + b*x
^2)^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 226, normalized size of antiderivative = 2.33

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^{7/2}}\,dx = \frac{15\sqrt{b\,x^2+a}\,a^2b^2x-3\sqrt{b\,x^2+a}\,a^2b^2+20\sqrt{b\,x^2+a}\,a\,b^3x^3+5\sqrt{b\,x^2+a}\,a\,b^2c\,x^3+}{}$$

input
```
int((C*x^2+B*x+A)/(b*x^2+a)^(7/2),x)
```

output
```
(15*sqrt(a + b*x**2)*a**2*b**2*x - 3*sqrt(a + b*x**2)*a**2*b**2 + 20*sqrt(
a + b*x**2)*a*b**3*x**3 + 5*sqrt(a + b*x**2)*a*b**2*c*x**3 + 8*sqrt(a + b*
x**2)*b**4*x**5 + 2*sqrt(a + b*x**2)*b**3*c*x**5 - 8*sqrt(b)*a**3*b - 2*sq
rt(b)*a**3*c - 24*sqrt(b)*a**2*b**2*x**2 - 6*sqrt(b)*a**2*b*c*x**2 - 24*sq
rt(b)*a*b**3*x**4 - 6*sqrt(b)*a*b**2*c*x**4 - 8*sqrt(b)*b**4*x**6 - 2*sqrt
(b)*b**3*c*x**6)/(15*a**2*b**2*(a**3 + 3*a**2*b*x**2 + 3*a*b**2*x**4 + b**
3*x**6))
```

3.37 $\int \frac{A+Bx+Cx^2}{\left(a+bx^2\right)^{9/2}} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 127

$$\int \frac{A+Bx+Cx^2}{\left(a+bx^2\right)^{9/2}} \, dx = -\frac{aB-(Ab-aC)x}{7ab\left(a+bx^2\right)^{7/2}} + \frac{(6Ab+aC)x}{35a^2b\left(a+bx^2\right)^{5/2}} + \frac{4(6Ab+aC)x}{105a^3b\left(a+bx^2\right)^{3/2}} + \frac{8(6Ab+aC)x}{105a^4b\sqrt{a+bx^2}}$$

output
```
-1/7*(B*a-(A*b-C*a)*x)/a/b/(b*x^2+a)^(7/2)+1/35*(6*A*b+C*a)*x/a^2/b/(b*x^2
+a)^(5/2)+4/105*(6*A*b+C*a)*x/a^3/b/(b*x^2+a)^(3/2)+8/105*(6*A*b+C*a)*x/a^
4/b/(b*x^2+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.72

$$\int \frac{A+Bx+Cx^2}{\left(a+bx^2\right)^{9/2}} \, dx = \frac{-15a^4B+48Ab^4x^7+35a^3bx(3A+Cx^2)+8ab^3x^5(21A+Cx^2)+14a^2b^2x^3(15A}{105a^4b\left(a+bx^2\right)^{7/2}}$$

input
```
Integrate[(A + B*x + C*x^2)/(a + b*x^2)^(9/2),x]
```

output
```
(-15*a^4*B + 48*A*b^4*x^7 + 35*a^3*b*x*(3*A + C*x^2) + 8*a*b^3*x^5*(21*A +
 C*x^2) + 14*a^2*b^2*x^3*(15*A + 2*C*x^2))/(105*a^4*b*(a + b*x^2)^(7/2))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {2345, 25, 27, 209, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^{9/2}}dx$$

$$\Big\downarrow \text{2345}$$

$$-\frac{\int -\frac{6Ab+aC}{b(bx^2+a)^{7/2}}dx}{7a}-\frac{aB-x(Ab-aC)}{7ab(a+bx^2)^{7/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{6Ab+aC}{b(bx^2+a)^{7/2}}dx}{7a}-\frac{aB-x(Ab-aC)}{7ab(a+bx^2)^{7/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(aC+6Ab)\int \frac{1}{(bx^2+a)^{7/2}}dx}{7ab}-\frac{aB-x(Ab-aC)}{7ab(a+bx^2)^{7/2}}$$

$$\Big\downarrow \text{209}$$

$$\frac{(aC+6Ab)\left(\frac{4\int \frac{1}{(bx^2+a)^{5/2}}dx}{5a}+\frac{x}{5a(a+bx^2)^{5/2}}\right)}{7ab}-\frac{aB-x(Ab-aC)}{7ab(a+bx^2)^{7/2}}$$

$$\Big\downarrow \text{209}$$

$$\frac{(aC+6Ab)\left(\frac{4\left(\frac{2\int \frac{1}{\left(bx^2+a\right)^{3/2}}dx}{3a}+\frac{x}{3a\left(a+bx^2\right)^{3/2}}\right)}{5a}+\frac{x}{5a(a+bx^2)^{5/2}}\right)}{7ab}-\frac{aB-x(Ab-aC)}{7ab\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \text{208}$$

$$\frac{\left(\frac{4\left(\frac{2x}{3a^2\sqrt{a+bx^2}}+\frac{x}{3a\left(a+bx^2\right)^{3/2}}\right)}{5a}+\frac{x}{5a(a+bx^2)^{5/2}}\right)(aC+6Ab)}{7ab}-\frac{aB-x(Ab-aC)}{7ab\left(a+bx^2\right)^{7/2}}$$

input
```
Int[(A + B*x + C*x^2)/(a + b*x^2)^(9/2),x]
```

output
```
-1/7*(a*B - (A*b - a*C)*x)/(a*b*(a + b*x^2)^(7/2)) + ((6*A*b + a*C)*(x/(5*
a*(a + b*x^2)^(5/2)) + (4*(x/(3*a*(a + b*x^2)^(3/2)) + (2*x)/(3*a^2*Sqrt[a
 + b*x^2])))/(5*a)))/(7*a*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.76

method	result
gosper	$\frac{48A\,b^{4}x^{7}+8Ca\,b^{3}x^{7}+168Aa\,b^{3}x^{5}+28C\,a^{2}b^{2}x^{5}+210A\,a^{2}b^{2}x^{3}+35C\,a^{3}b\,x^{3}+105A\,a^{3}bx-15B\,a^{4}}{105(b\,x^{2}+a)^{\frac{7}{2}}a^{4}b}$
trager	$\frac{48A\,b^{4}x^{7}+8Ca\,b^{3}x^{7}+168Aa\,b^{3}x^{5}+28C\,a^{2}b^{2}x^{5}+210A\,a^{2}b^{2}x^{3}+35C\,a^{3}b\,x^{3}+105A\,a^{3}bx-15B\,a^{4}}{105(b\,x^{2}+a)^{\frac{7}{2}}a^{4}b}$
orering	$\frac{48A\,b^{4}x^{7}+8Ca\,b^{3}x^{7}+168Aa\,b^{3}x^{5}+28C\,a^{2}b^{2}x^{5}+210A\,a^{2}b^{2}x^{3}+35C\,a^{3}b\,x^{3}+105A\,a^{3}bx-15B\,a^{4}}{105(b\,x^{2}+a)^{\frac{7}{2}}a^{4}b}$
default	$A\left(\frac{x}{7a(b\,x^{2}+a)^{\frac{7}{2}}}+\frac{\frac{6x}{35a\left(b\,x^{2}+a\right)^{\frac{5}{2}}}+\frac{6\left(\frac{4x}{15a\left(b\,x^{2}+a\right)^{\frac{3}{2}}}+\frac{8x}{15a^{2}\sqrt{b\,x^{2}+a}}\right)}{7a}}{a}\right)-\frac{B}{7b(b\,x^{2}+a)^{\frac{7}{2}}}+C\left(-\frac{x}{6b(b\,x^{2}+a)^{\frac{7}{2}}}+\frac{a\left(\frac{\ }{7a(}\right.}{}\right.$

input
```
int((C*x^2+B*x+A)/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
1/105*(48*A*b^4*x^7+8*C*a*b^3*x^7+168*A*a*b^3*x^5+28*C*a^2*b^2*x^5+210*A*a
^2*b^2*x^3+35*C*a^3*b*x^3+105*A*a^3*b*x-15*B*a^4)/(b*x^2+a)^(7/2)/a^4/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.08

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^{9/2}}\,dx = \frac{(8\,(Cab^3+6\,Ab^4)x^7+105\,Aa^3bx+28\,(Ca^2b^2+6\,Aab^3)x^5-15\,Ba^4+35\,(Ca^3b+}{105\,(a^4b^5x^8+4\,a^5b^4x^6+6\,a^6b^3x^4+4\,a^7b^2x^2+a^8b)}$$

input
```
integrate((C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

output
```
1/105*(8*(C*a*b^3 + 6*A*b^4)*x^7 + 105*A*a^3*b*x + 28*(C*a^2*b^2 + 6*A*a*b
^3)*x^5 - 15*B*a^4 + 35*(C*a^3*b + 6*A*a^2*b^2)*x^3)*sqrt(b*x^2 + a)/(a^4*
b^5*x^8 + 4*a^5*b^4*x^6 + 6*a^6*b^3*x^4 + 4*a^7*b^2*x^2 + a^8*b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1266 vs. 2(117) = 234.

Time = 25.96 (sec) , antiderivative size = 1880, normalized size of antiderivative = 14.80

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^{9/2}}\,dx = \text{Too large to display}$$

input
```
integrate((C*x**2+B*x+A)/(b*x**2+a)**(9/2),x)
```

output
```
A*(35*a**14*x/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt
(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2
)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a
) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*
sqrt(1 + b*x**2/a)) + 175*a**13*b*x**3/(35*a**(37/2)*sqrt(1 + b*x**2/a) +
210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 +
 b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b*
*4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) +
 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a)) + 371*a**12*b**2*x**5/(35*a**
(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*
a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 +
b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**
5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a)) +
 429*a**11*b**3*x**7/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x*
*2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a
**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b
*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6
*x**12*sqrt(1 + b*x**2/a)) + 286*a**10*b**4*x**9/(35*a**(37/2)*sqrt(1 + b*
x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**
4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.20

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^{9/2}}\,dx = \frac{16\,Ax}{35\,\sqrt{bx^2+a}a^4} + \frac{8\,Ax}{35\,(bx^2+a)^{\frac{3}{2}}a^3} + \frac{6\,Ax}{35\,(bx^2+a)^{\frac{5}{2}}a^2} + \frac{Ax}{7\,(bx^2+a)^{\frac{7}{2}}a} - \frac{Cx}{7\,(bx^2+a)^{\frac{7}{2}}b} + \frac{8\,Cx}{105\,\sqrt{bx^2+a}a^3b} + \frac{4\,Cx}{105\,(bx^2+a)^{\frac{3}{2}}a^2b} + \frac{Cx}{35\,(bx^2+a)^{\frac{5}{2}}ab} - \frac{B}{7\,(bx^2+a)^{\frac{7}{2}}b}$$

input
```
integrate((C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

output
```
16/35*A*x/(sqrt(b*x^2 + a)*a^4) + 8/35*A*x/((b*x^2 + a)^(3/2)*a^3) + 6/35*
A*x/((b*x^2 + a)^(5/2)*a^2) + 1/7*A*x/((b*x^2 + a)^(7/2)*a) - 1/7*C*x/((b*
x^2 + a)^(7/2)*b) + 8/105*C*x/(sqrt(b*x^2 + a)*a^3*b) + 4/105*C*x/((b*x^2
+ a)^(3/2)*a^2*b) + 1/35*C*x/((b*x^2 + a)^(5/2)*a*b) - 1/7*B/((b*x^2 + a)^
(7/2)*b)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.88

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^{9/2}}\,dx = \frac{\left(\left(4\,x^2\left(\frac{2\,(Cab^5+6\,Ab^6)x^2}{a^4b^3}+\frac{7\,(Ca^2b^4+6\,Aab^5)}{a^4b^3}\right)+\frac{35\,(Ca^3b^3+6\,Aa^2b^4)}{a^4b^3}\right)x^2+\frac{105\,A}{a}\right)x-\frac{15\,B}{b}}{105\,(bx^2+a)^{\frac{7}{2}}}$$

input
```
integrate((C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="giac")
```

output
```
1/105*(((4*x^2*(2*(C*a*b^5 + 6*A*b^6)*x^2/(a^4*b^3) + 7*(C*a^2*b^4 + 6*A*a
*b^5)/(a^4*b^3)) + 35*(C*a^3*b^3 + 6*A*a^2*b^4)/(a^4*b^3))*x^2 + 105*A/a)*
x - 15*B/b)/(b*x^2 + a)^(7/2)
```

Mupad [B] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.91

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^{9/2}}\,dx = \frac{x\,(6\,A\,b+C\,a)}{35\,a^2\,b\,(b\,x^2+a)^{5/2}}-\frac{\frac{B}{7\,b}-x\,\left(\frac{A}{7\,a}-\frac{C}{7\,b}\right)}{(b\,x^2+a)^{7/2}} + \frac{x\,(24\,A\,b+4\,C\,a)}{105\,a^3\,b\,(b\,x^2+a)^{3/2}}+\frac{x\,(48\,A\,b+8\,C\,a)}{105\,a^4\,b\,\sqrt{b\,x^2+a}}$$

input
```
int((A + B*x + C*x^2)/(a + b*x^2)^(9/2),x)
```

output
```
(x*(6*A*b + C*a))/(35*a^2*b*(a + b*x^2)^(5/2)) - (B/(7*b) - x*(A/(7*a) - C
/(7*b)))/(a + b*x^2)^(7/2) + (x*(24*A*b + 4*C*a))/(105*a^3*b*(a + b*x^2)^(
3/2)) + (x*(48*A*b + 8*C*a))/(105*a^4*b*(a + b*x^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 303, normalized size of antiderivative = 2.39

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^{9/2}}\,dx = \frac{105\sqrt{b\,x^2+a}\,a^3b^2x - 15\sqrt{b\,x^2+a}\,a^3b^2 + 210\sqrt{b\,x^2+a}\,a^2b^3x^3 + 35\sqrt{b\,x^2+a}\,a^2b^2}{}$$

input
```
int((C*x^2+B*x+A)/(b*x^2+a)^(9/2),x)
```

output
```
(105*sqrt(a + b*x**2)*a**3*b**2*x - 15*sqrt(a + b*x**2)*a**3*b**2 + 210*sq
rt(a + b*x**2)*a**2*b**3*x**3 + 35*sqrt(a + b*x**2)*a**2*b**2*c*x**3 + 168
*sqrt(a + b*x**2)*a*b**4*x**5 + 28*sqrt(a + b*x**2)*a*b**3*c*x**5 + 48*sqr
t(a + b*x**2)*b**5*x**7 + 8*sqrt(a + b*x**2)*b**4*c*x**7 - 48*sqrt(b)*a**4
*b - 8*sqrt(b)*a**4*c - 192*sqrt(b)*a**3*b**2*x**2 - 32*sqrt(b)*a**3*b*c*x
**2 - 288*sqrt(b)*a**2*b**3*x**4 - 48*sqrt(b)*a**2*b**2*c*x**4 - 192*sqrt(
b)*a*b**4*x**6 - 32*sqrt(b)*a*b**3*c*x**6 - 48*sqrt(b)*b**5*x**8 - 8*sqrt(
b)*b**4*c*x**8)/(105*a**3*b**2*(a**4 + 4*a**3*b*x**2 + 6*a**2*b**2*x**4 +
4*a*b**3*x**6 + b**4*x**8))
```

3.38 $\int \sqrt[3]{a+bx^2}\left(A+Bx+Cx^2\right)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 326

$$\int \sqrt[3]{a+bx^2}\left(A+Bx+Cx^2\right)\,dx$$
$$=\frac{3(11Ab-3aC)x\sqrt[3]{a+bx^2}}{55b}+\frac{3B\left(a+bx^2\right)^{4/3}}{8b}+\frac{3Cx\left(a+bx^2\right)^{4/3}}{11b}$$
$$-\frac{2\,3^{3/4}\sqrt{2-\sqrt{3}}a(11Ab-3aC)\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\right.\right.}{55b^2x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

output
```
3/55*(11*A*b-3*C*a)*x*(b*x^2+a)^(1/3)/b+3/8*B*(b*x^2+a)^(4/3)/b+3/11*C*x*(
b*x^2+a)^(4/3)/b-2/55*3^(3/4)*(1/2*6^(1/2)-1/2*2^(1/2))*a*(11*A*b-3*C*a)*(
a^(1/3)-(b*x^2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3)
)/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)*EllipticF(((1+3^(1/2))*a^
(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3)),2*I-I*3^(1/2)
)/b^2/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)
^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.05 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.25

$$\int \sqrt[3]{a+bx^2}\left(A+Bx+Cx^2\right)\,dx = \frac{\sqrt[3]{a+bx^2}\left(3(11B+8Cx)\left(a+bx^2\right)+\frac{8(11Ab-3aC)x\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{\sqrt[3]{1+\frac{bx^2}{a}}}\right)}{88b}$$

input

```
Integrate[(a + b*x^2)^(1/3)*(A + B*x + C*x^2),x]
```

output

```
((a + b*x^2)^(1/3)*(3*(11*B + 8*C*x)*(a + b*x^2) + (8*(11*A*b - 3*a*C)*x*H
ypergeometric2F1[-1/3, 1/2, 3/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^(1/3)))/(8
8*b)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 321, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {2346, 27, 455, 211, 234, 760}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{a+bx^2}\left(A+Bx+Cx^2\right)\,dx$$

$$\Big\downarrow \text{ 2346}$$

$$\frac{3\int \frac{1}{3}(11Ab+11Bxb-3aC)\sqrt[3]{bx^2+a}dx}{11b}+\frac{3Cx\left(a+bx^2\right)^{4/3}}{11b}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int (11Ab+11Bxb-3aC)\sqrt[3]{bx^2+a}dx}{11b}+\frac{3Cx\left(a+bx^2\right)^{4/3}}{11b}$$

$$\Big\downarrow \; 455$$

$$\frac{(11Ab-3aC)\int \sqrt[3]{bx^2+a}dx+\frac{33}{8}B\left(a+bx^2\right)^{4/3}}{11b}+\frac{3Cx\left(a+bx^2\right)^{4/3}}{11b}$$

$$\Big\downarrow \; 211$$

$$\frac{(11Ab-3aC)\left(\frac{2}{5}a\int \frac{1}{(bx^2+a)^{2/3}}dx+\frac{3}{5}x\sqrt[3]{a+bx^2}\right)+\frac{33}{8}B\left(a+bx^2\right)^{4/3}}{11b}+\frac{3Cx\left(a+bx^2\right)^{4/3}}{11b}$$

$$\Big\downarrow \; 234$$

$$\frac{(11Ab-3aC)\left(\frac{3a\sqrt{bx^2}\int \frac{1}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}}{5bx}+\frac{3}{5}x\sqrt[3]{a+bx^2}\right)+\frac{33}{8}B\left(a+bx^2\right)^{4/3}}{11b}+\frac{3Cx\left(a+bx^2\right)^{4/3}}{11b}$$

$$\Big\downarrow \; 760$$

$$\frac{(11Ab-3aC)\left(\frac{3}{5}x\sqrt[3]{a+bx^2}-\frac{2\ 3^{3/4}\sqrt{2-\sqrt{3}}a\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{(1+\cdots}{(1-\cdots}\right.\right.}{5bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}\right.}{11b}$$

$$\frac{3Cx\left(a+bx^2\right)^{4/3}}{11b}$$

input
```
Int[(a + b*x^2)^(1/3)*(A + B*x + C*x^2),x]
```

output
```
(3*C*x*(a + b*x^2)^(4/3))/(11*b) + ((33*B*(a + b*x^2)^(4/3))/8 + (11*A*b -
 3*a*C)*((3*x*(a + b*x^2)^(1/3))/5 - (2*3^(3/4)*Sqrt[2 - Sqrt[3]]*a*(a^(1/
3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b
*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticF[ArcS
in[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a
 + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(5*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a +
 b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)])))/(11*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 234
```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Maple [F]

$$\int \left(b\,x^{2}+a\right)^{\frac{1}{3}} \left(C\,x^{2}+Bx+A\right)dx$$

input
```
int((b*x^2+a)^(1/3)*(C*x^2+B*x+A),x)
```

output
```
int((b*x^2+a)^(1/3)*(C*x^2+B*x+A),x)
```

Fricas [F]

$$\int \sqrt[3]{a+bx^2}\left(A+Bx+Cx^2\right)\,dx=\int \left(Cx^{2}+Bx+A\right)\left(bx^{2}+a\right)^{\frac{1}{3}}\,dx$$

input
```
integrate((b*x^2+a)^(1/3)*(C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
integral((C*x^2 + B*x + A)*(b*x^2 + a)^(1/3), x)
```

Sympy [A] (verification not implemented)

Time = 1.09 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.27

$$\int \sqrt[3]{a+bx^2}\left(A+Bx+Cx^2\right)\,dx=A\sqrt[3]{a}x\,{}_{2}F_{1}\left(\begin{matrix}-\frac{1}{3},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)+B\left(\begin{cases}\frac{\sqrt[3]{a}x^{2}}{2} & \text{for}\ b=0\\ \frac{3\left(a+bx^{2}\right)^{\frac{4}{3}}}{8b} & \text{otherwise}\end{cases}\right)+\frac{C\sqrt[3]{a}x^{3}\,{}_{2}F_{1}\left(\begin{matrix}-\frac{1}{3},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{3}$$

input
```
integrate((b*x**2+a)**(1/3)*(C*x**2+B*x+A),x)
```

output
```
A*a**(1/3)*x*hyper((-1/3, 1/2), (3/2,), b*x**2*exp_polar(I*pi)/a) + B*Piec
ewise((a**(1/3)*x**2/2, Eq(b, 0)), (3*(a + b*x**2)**(4/3)/(8*b), True)) +
C*a**(1/3)*x**3*hyper((-1/3, 3/2), (5/2,), b*x**2*exp_polar(I*pi)/a)/3
```

Maxima [F]

$$\int \sqrt[3]{a+bx^2}\left(A+Bx+Cx^2\right)\,dx = \int \left(Cx^2+Bx+A\right)\left(bx^2+a\right)^{\frac{1}{3}}\,dx$$

input
```
integrate((b*x^2+a)^(1/3)*(C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)*(b*x^2 + a)^(1/3), x)
```

Giac [F]

$$\int \sqrt[3]{a+bx^2}\left(A+Bx+Cx^2\right)\,dx = \int \left(Cx^2+Bx+A\right)\left(bx^2+a\right)^{\frac{1}{3}}\,dx$$

input
```
integrate((b*x^2+a)^(1/3)*(C*x^2+B*x+A),x, algorithm="giac")
```

output
```
integrate((C*x^2 + B*x + A)*(b*x^2 + a)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{a+bx^2}\left(A+Bx+Cx^2\right)\,dx = \int \left(b\,x^2+a\right)^{1/3}\,\left(C\,x^2+B\,x+A\right)\,dx$$

input
```
int((a + b*x^2)^(1/3)*(A + B*x + C*x^2),x)
```

output
```
int((a + b*x^2)^(1/3)*(A + B*x + C*x^2), x)
```

Reduce [F]

$$\int \sqrt[3]{a+bx^2}\left(A+Bx+Cx^2\right)\,dx$$
$$=\frac{264(b\,x^2+a)^{\frac{1}{3}}\,abx+165(b\,x^2+a)^{\frac{1}{3}}\,ab+48(b\,x^2+a)^{\frac{1}{3}}\,acx+165(b\,x^2+a)^{\frac{1}{3}}\,b^2x^2+120(b\,x^2+a)^{\frac{1}{3}}\,bc\,x^3}{440b}$$

input
```
int((b*x^2+a)^(1/3)*(C*x^2+B*x+A),x)
```

output
```
(264*(a + b*x**2)**(1/3)*a*b*x + 165*(a + b*x**2)**(1/3)*a*b + 48*(a + b*x
**2)**(1/3)*a*c*x + 165*(a + b*x**2)**(1/3)*b**2*x**2 + 120*(a + b*x**2)**
(1/3)*b*c*x**3 + 176*int((a + b*x**2)**(1/3)/(a + b*x**2),x)*a**2*b - 48*i
nt((a + b*x**2)**(1/3)/(a + b*x**2),x)*a**2*c)/(440*b)
```

3.39 $\int \frac{A+Bx+Cx^2}{\left(a+bx^2\right)^{2/3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 297

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^{2/3}}\,dx = \frac{3B\sqrt[3]{a+bx^2}}{2b}+\frac{3Cx\sqrt[3]{a+bx^2}}{5b} - \frac{3^{3/4}\sqrt{2-\sqrt{3}}(5Ab-3aC)\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)}{\left(1-\sqrt{3}\right)}\right.\right.}{5b^2x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

output

```
3/2*B*(b*x^2+a)^(1/3)/b+3/5*C*x*(b*x^2+a)^(1/3)/b-1/5*3^(3/4)*(1/2*6^(1/2)
-1/2*2^(1/2))*(5*A*b-3*C*a)*(a^(1/3)-(b*x^2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b
*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1
/2)*EllipticF(((1+3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(
b*x^2+a)^(1/3)),2*I-I*3^(1/2))/b^2/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(
(1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 9.51 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.27

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^{2/3}}\,dx = \frac{3(5B+2Cx)(a+bx^2)+2(5Ab-3aC)x\left(1+\frac{bx^2}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\right.}{10b(a+bx^2)^{2/3}}$$

input
```
Integrate[(A + B*x + C*x^2)/(a + b*x^2)^(2/3),x]
```

output
```
(3*(5*B + 2*C*x)*(a + b*x^2) + 2*(5*A*b - 3*a*C)*x*(1 + (b*x^2)/a)^(2/3)*H
ypergeometric2F1[1/2, 2/3, 3/2, -((b*x^2)/a)])/(10*b*(a + b*x^2)^(2/3))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {2346, 27, 455, 234, 760}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^{2/3}}dx$$

$$\Big\downarrow \text{ 2346}$$

$$\frac{3\int \frac{5Ab+5Bxb-3aC}{3(bx^2+a)^{2/3}}dx}{5b}+\frac{3Cx\sqrt[3]{a+bx^2}}{5b}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{5Ab+5Bxb-3aC}{(bx^2+a)^{2/3}}dx}{5b}+\frac{3Cx\sqrt[3]{a+bx^2}}{5b}$$

$$\Big\downarrow \text{ 455}$$

$$\frac{(5Ab-3aC)\int \frac{1}{(bx^2+a)^{2/3}}dx+\frac{15}{2}B\sqrt[3]{a+bx^2}}{5b}+\frac{3Cx\sqrt[3]{a+bx^2}}{5b}$$

$$\Big\downarrow \text{234}$$

$$\frac{\frac{3\sqrt{bx^2}(5Ab-3aC)\int \frac{1}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}}{2bx}+\frac{15}{2}B\sqrt[3]{a+bx^2}}{5b}+\frac{3Cx\sqrt[3]{a+bx^2}}{5b}$$

$$\Big\downarrow \text{760}$$

$$\frac{\frac{15}{2}B\sqrt[3]{a+bx^2}-\frac{3^{3/4}\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}(5Ab-3aC)\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right),-7+4\sqrt{3}\right)}{bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}}{5b}+\frac{3Cx\sqrt[3]{a+bx^2}}{5b}$$

input
```
Int[(A + B*x + C*x^2)/(a + b*x^2)^(2/3),x]
```

output
```
(3*C*x*(a + b*x^2)^(1/3))/(5*b) + ((15*B*(a + b*x^2)^(1/3))/2 - (3^(3/4)*S
qrt[2 - Sqrt[3]]*(5*A*b - 3*a*C)*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/
3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3)
 - (a + b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*
x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])
/(b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3
) - (a + b*x^2)^(1/3))^2)]))/(5*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 234
```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Maple [F]

$$\int \frac{C\,x^{2}+Bx+A}{\left(b\,x^{2}+a\right)^{\frac{2}{3}}}dx$$

input
```
int((C*x^2+B*x+A)/(b*x^2+a)^(2/3),x)
```

output
```
int((C*x^2+B*x+A)/(b*x^2+a)^(2/3),x)
```

Fricas [F]

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^{2/3}}\,dx=\int \frac{Cx^2+Bx+A}{(bx^2+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

output
```
integral((C*x^2 + B*x + A)/(b*x^2 + a)^(2/3), x)
```

Sympy [A] (verification not implemented)

Time = 0.97 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.28

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^{2/3}}\,dx = \frac{Ax\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{a^{\frac{2}{3}}} + B\left(\begin{cases}\frac{x^2}{2a^{\frac{2}{3}}} & \text{for } b=0\\ \frac{3\sqrt[3]{a+bx^2}}{2b} & \text{otherwise}\end{cases}\right) + \frac{Cx^3\,{}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}}$$

input
```
integrate((C*x**2+B*x+A)/(b*x**2+a)**(2/3),x)
```

output
```
A*x*hyper((1/2, 2/3), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(2/3) + B*Piece
wise((x**2/(2*a**(2/3)), Eq(b, 0)), (3*(a + b*x**2)**(1/3)/(2*b), True)) +
 C*x**3*hyper((2/3, 3/2), (5/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(2/3))
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^{2/3}}\,dx = \int \frac{Cx^2+Bx+A}{(bx^2+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(b*x^2+a)^(2/3),x, algorithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)/(b*x^2 + a)^(2/3), x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^{2/3}}\,dx = \int \frac{Cx^2+Bx+A}{(bx^2+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(b*x^2+a)^(2/3),x, algorithm="giac")
```

output
```
integrate((C*x^2 + B*x + A)/(b*x^2 + a)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^{2/3}}\,dx = \int \frac{C\,x^2+B\,x+A}{(b\,x^2+a)^{2/3}}\,dx$$

input
```
int((A + B*x + C*x^2)/(a + b*x^2)^(2/3),x)
```

output
```
int((A + B*x + C*x^2)/(a + b*x^2)^(2/3), x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^{2/3}}\,dx = \frac{3(b\,x^2+a)^{\frac{1}{3}}}{2} + \left(\int \frac{x^2}{(b\,x^2+a)^{\frac{2}{3}}}dx\right)c + \left(\int \frac{1}{(b\,x^2+a)^{\frac{2}{3}}}dx\right)a$$

input
```
int((C*x^2+B*x+A)/(b*x^2+a)^(2/3),x)
```

output
```
(3*(a + b*x**2)**(1/3) + 2*int(x**2/(a + b*x**2)**(2/3),x)*c + 2*int(1/(a
+ b*x**2)**(2/3),x)*a)/2
```

3.40 $\int \frac{A+Bx+Cx^2}{(a+bx^2)^{5/3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 296

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^{5/3}}\,dx = -\frac{3(aB-(Ab-aC)x)}{4ab\,(a+bx^2)^{2/3}}$$
$$-\frac{3^{3/4}\sqrt{2-\sqrt{3}}(Ab+3aC)\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{\ldots}}{\left(1-\sqrt{3}\right)\sqrt[3]{\ldots}}\right.\right.}{4ab^2x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

output
```
1/4*(-3*B*a+3*(A*b-C*a)*x)/a/b/(b*x^2+a)^(2/3)-1/4*3^(3/4)*(1/2*6^(1/2)-1/
2*2^(1/2))*(A*b+3*C*a)*(a^(1/3)-(b*x^2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+
a)^(1/3)+(b*x^2+a)^(2/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)*E
llipticF(((1+3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2
+a)^(1/3)),2*I-I*3^(1/2))/a/b^2/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/((1-
3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.26

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^{5/3}}\,dx = \frac{3Abx-3a(B+Cx)+(Ab+3aC)x\left(1+\frac{bx^2}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{3}{2},-\frac{b}{}\right.}{4ab\left(a+bx^2\right)^{2/3}}$$

input
```
Integrate[(A + B*x + C*x^2)/(a + b*x^2)^(5/3),x]
```

output
```
(3*A*b*x - 3*a*(B + C*x) + (A*b + 3*a*C)*x*(1 + (b*x^2)/a)^(2/3)*Hypergeom
etric2F1[1/2, 2/3, 3/2, -((b*x^2)/a)])/(4*a*b*(a + b*x^2)^(2/3))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2345, 27, 234, 760}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^{5/3}}dx$$

$$\Big\downarrow \; 2345$$

$$-\frac{3\int -\frac{Ab+3aC}{3b(bx^2+a)^{2/3}}dx}{4a}-\frac{3(aB-x(Ab-aC))}{4ab\left(a+bx^2\right)^{2/3}}$$

$$\Big\downarrow \; 27$$

$$\frac{(3aC+Ab)\int \frac{1}{(bx^2+a)^{2/3}}dx}{4ab}-\frac{3(aB-x(Ab-aC))}{4ab\left(a+bx^2\right)^{2/3}}$$

$$\Big\downarrow \; 234$$

$$\frac{3\sqrt{bx^2}(3aC+Ab)\int \frac{1}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}}{8ab^2x}-\frac{3(aB-x(Ab-aC))}{4ab\left(a+bx^2\right)^{2/3}}$$

$\big\downarrow$ 760

$$-\frac{3^{3/4}\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}(3aC+Ab)\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right),-7+4\sqrt{3}\right)}{4ab^2x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}-\frac{3(aB-x(Ab-aC))}{4ab\left(a+bx^2\right)^{2/3}}$$

input
```
Int[(A + B*x + C*x^2)/(a + b*x^2)^(5/3),x]
```

output
```
(-3*(a*B - (A*b - a*C)*x))/(4*a*b*(a + b*x^2)^(2/3)) - (3^(3/4)*Sqrt[2 - S
qrt[3]]*(A*b + 3*a*C)*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3
)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x
^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))
/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(4*a*b^2*x
*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (
a + b*x^2)^(1/3))^2)])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 234
```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Maple [F]

$$\int \frac{C\,x^{2}+B x +A}{\left(b\,x^{2}+a \right)^{\frac{5}{3}}}d x$$

input
```
int((C*x^2+B*x+A)/(b*x^2+a)^(5/3),x)
```

output
```
int((C*x^2+B*x+A)/(b*x^2+a)^(5/3),x)
```

Fricas [F]

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^{5/3}}\,dx = \int \frac{Cx^{2} + Bx + A}{(bx^{2} + a)^{\frac{5}{3}}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(b*x^2+a)^(5/3),x, algorithm="fricas")
```

output
```
integral((C*x^2 + B*x + A)*(b*x^2 + a)^(1/3)/(b^2*x^4 + 2*a*b*x^2 + a^2),
x)
```

Sympy [A] (verification not implemented)

Time = 2.68 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.29

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^{5/3}}\,dx = \frac{Ax\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{3}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{a^{\frac{5}{3}}} + B\left(\begin{cases}-\frac{3}{4b(a+bx^2)^{\frac{2}{3}}} & \text{for } b\neq 0\\ \frac{x^2}{2a^{\frac{5}{3}}} & \text{otherwise}\end{cases}\right) + \frac{Cx^3\,{}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{5}{3}\\ \frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{3a^{\frac{5}{3}}}$$

input
```
integrate((C*x**2+B*x+A)/(b*x**2+a)**(5/3),x)
```

output
```
A*x*hyper((1/2, 5/3), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(5/3) + B*Piece
wise((-3/(4*b*(a + b*x**2)**(2/3)), Ne(b, 0)), (x**2/(2*a**(5/3)), True))
+ C*x**3*hyper((3/2, 5/3), (5/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(5/3))
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^{5/3}}\,dx = \int \frac{Cx^2+Bx+A}{(bx^2+a)^{\frac{5}{3}}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(b*x^2+a)^(5/3),x, algorithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)/(b*x^2 + a)^(5/3), x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^{5/3}}\,dx = \int \frac{Cx^2+Bx+A}{(bx^2+a)^{\frac{5}{3}}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(b*x^2+a)^(5/3),x, algorithm="giac")
```

output
```
integrate((C*x^2 + B*x + A)/(b*x^2 + a)^(5/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^{5/3}}\,dx = \int \frac{C\,x^2+B\,x+A}{(b\,x^2+a)^{5/3}}\,dx$$

input
```
int((A + B*x + C*x^2)/(a + b*x^2)^(5/3),x)
```

output
```
int((A + B*x + C*x^2)/(a + b*x^2)^(5/3), x)
```

Reduce [F]

$$\begin{aligned}\int \frac{A+Bx+Cx^2}{(a+bx^2)^{5/3}}\,dx &= \left(\int \frac{x^2}{(b\,x^2+a)^{\frac{2}{3}}\,a+(b\,x^2+a)^{\frac{2}{3}}\,b\,x^2}dx\right)c \\ &+ \left(\int \frac{x}{(b\,x^2+a)^{\frac{2}{3}}\,a+(b\,x^2+a)^{\frac{2}{3}}\,b\,x^2}dx\right)b \\ &+ \left(\int \frac{1}{(b\,x^2+a)^{\frac{2}{3}}\,a+(b\,x^2+a)^{\frac{2}{3}}\,b\,x^2}dx\right)a\end{aligned}$$

input
```
int((C*x^2+B*x+A)/(b*x^2+a)^(5/3),x)
```

output
```
int(x**2/((a + b*x**2)**(2/3)*a + (a + b*x**2)**(2/3)*b*x**2),x)*c + int(x
/((a + b*x**2)**(2/3)*a + (a + b*x**2)**(2/3)*b*x**2),x)*b + int(1/((a + b
*x**2)**(2/3)*a + (a + b*x**2)**(2/3)*b*x**2),x)*a
```

3.41 $\int (a+bx^2)^{2/3} (A+Bx+Cx^2)\, dx$

Optimal result

Integrand size = 22, antiderivative size = 631

$$\int (a+bx^2)^{2/3} (A+Bx+Cx^2)\, dx = \frac{3(13Ab-3aC)x(a+bx^2)^{2/3}}{91b} + \frac{3B(a+bx^2)^{5/3}}{10b} + \frac{3Cx(a+bx^2)^{5/3}}{13b} - \frac{12a(13Ab-3aC)x}{91b\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}$$
$$+ \frac{6\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^{4/3}(13Ab-3aC)\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}\cdots}{\left(1-\sqrt{3}\right)\sqrt[3]{a}\cdots}\right.\right.}{91b^2x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$
$$- \frac{4\sqrt{2}3^{3/4}a^{4/3}(13Ab-3aC)\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\cdots}{\left(1-\sqrt{3}\right)\cdots}\right.\right.}{91b^2x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

output
```
3/91*(13*A*b-3*C*a)*x*(b*x^2+a)^(2/3)/b+3/10*B*(b*x^2+a)^(5/3)/b+3/13*C*x*
(b*x^2+a)^(5/3)/b-12/91*a*(13*A*b-3*C*a)*x/b/((1-3^(1/2))*a^(1/3)-(b*x^2+a
)^(1/3))+6/91*3^(1/4)*(1/2*6^(1/2)+1/2*2^(1/2))*a^(4/3)*(13*A*b-3*C*a)*(a^
(1/3)-(b*x^2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/
((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)*EllipticE(((1+3^(1/2))*a^(1
/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3)),2*I-I*3^(1/2))/
b^2/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(
1/3))^2)^(1/2)-4/91*2^(1/2)*3^(3/4)*a^(4/3)*(13*A*b-3*C*a)*(a^(1/3)-(b*x^2
+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/((1-3^(1/2))
*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)*EllipticF(((1+3^(1/2))*a^(1/3)-(b*x^2+a
)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3)),2*I-I*3^(1/2))/b^2/x/(-a^(1
/3)*(a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/
2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.04 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.13

$$\int \left(a+bx^2\right)^{2/3} \left(A+Bx+Cx^2\right) \, dx=\frac{\left(a+bx^2\right)^{2/3} \left(3(13B+10Cx)\left(a+bx^2\right)+\frac{10(13Ab-3aC)x \text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{\left(1+\frac{bx^2}{a}\right)^{2/3}}\right)}{130b}$$

input
```
Integrate[(a + b*x^2)^(2/3)*(A + B*x + C*x^2),x]
```

output
```
((a + b*x^2)^(2/3)*(3*(13*B + 10*C*x)*(a + b*x^2) + (10*(13*A*b - 3*a*C)*x
*Hypergeometric2F1[-2/3, 1/2, 3/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^(2/3)))/
(130*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.55 (sec) , antiderivative size = 647, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2346, 27, 455, 211, 233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^{2/3} \left(A+Bx+Cx^2\right) \, dx$$

$$\Big\downarrow \ 2346$$

$$\frac{3\int \frac{1}{3}(13Ab+13Bxb-3aC)\left(bx^2+a\right)^{2/3}dx}{13b}+\frac{3Cx\left(a+bx^2\right)^{5/3}}{13b}$$

$$\Big\downarrow \ 27$$

$$\frac{\int (13Ab+13Bxb-3aC)\left(bx^2+a\right)^{2/3}dx}{13b}+\frac{3Cx\left(a+bx^2\right)^{5/3}}{13b}$$

$$\Big\downarrow \ 455$$

$$\frac{(13Ab-3aC)\int \left(bx^2+a\right)^{2/3}dx+\frac{39}{10}B\left(a+bx^2\right)^{5/3}}{13b}+\frac{3Cx\left(a+bx^2\right)^{5/3}}{13b}$$

$$\Big\downarrow \ 211$$

$$\frac{(13Ab-3aC)\left(\frac{4}{7}a\int \frac{1}{\sqrt[3]{bx^2+a}}dx+\frac{3}{7}x\left(a+bx^2\right)^{2/3}\right)+\frac{39}{10}B\left(a+bx^2\right)^{5/3}}{13b}+\frac{3Cx\left(a+bx^2\right)^{5/3}}{13b}$$

$$\Big\downarrow \ 233$$

$$\frac{(13Ab-3aC)\left(\frac{6a\sqrt{bx^2}\int \frac{\sqrt[3]{bx^2+a}}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}}{7bx}+\frac{3}{7}x\left(a+bx^2\right)^{2/3}\right)+\frac{39}{10}B\left(a+bx^2\right)^{5/3}}{13b}+\frac{3Cx\left(a+bx^2\right)^{5/3}}{13b}$$

$$\Big\downarrow \ 833$$

$$\frac{(13Ab-3aC)\left(\frac{6a\sqrt{bx^2}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}-\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}\right)}{7bx}+\frac{3}{7}x\left(a+bx^2\right)^{2/3}\right)+}{13b}$$

$$\frac{3Cx\left(a+bx^2\right)^{5/3}}{13b}$$

$\downarrow$ 760

$$\frac{(13Ab-3aC)\left(\frac{6a\sqrt{bx^2}\left(-\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{}\right.}}}{\sqrt[4]{3}\sqrt{bx^2}\sqrt{-\frac{\sqrt[3]{}}{\left(\left(1-\right.\right.}}}\right.}{7bx}\right.}{13b}$$

$$\frac{3Cx\left(a+bx^2\right)^{5/3}}{13b}$$

$\downarrow$ 2418

$$(13Ab-3aC)\frac{6a\sqrt{bx^2}\left(-\frac{2\sqrt{2-\sqrt{3}}(1+\sqrt{3})\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{(1+\sqrt{3})\sqrt[3]{a}}{(1-\sqrt{3})\sqrt[3]{a}}\right.\right.}{\sqrt[4]{3}\sqrt{bx^2}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}\right.}{}$$

$$\frac{3Cx(a+bx^2)^{5/3}}{13b}$$

input
```
Int[(a + b*x^2)^(2/3)*(A + B*x + C*x^2),x]
```

output
```
(3*C*x*(a + b*x^2)^(5/3))/(13*b) + ((39*B*(a + b*x^2)^(5/3))/10 + (13*A*b
- 3*a*C)*((3*x*(a + b*x^2)^(2/3))/7 + (6*a*Sqrt[b*x^2]*((-2*Sqrt[b*x^2])/(
(1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*a^
(1/3)*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1
/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*El
lipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*
a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(Sqrt[b*x^2]*Sqrt[-((a^(1/
3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/
3))^2)]) - (2*Sqrt[2 - Sqrt[3]]*(1 + Sqrt[3])*a^(1/3)*(a^(1/3) - (a + b*x^
2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/(
(1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[
3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3
))], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[b*x^2]*Sqrt[-((a^(1/3)*(a^(1/3) - (a +
 b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)])))/(7*b*x)
))/(13*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int \left(b\,x^{2}+a\right)^{\frac{2}{3}} \left(C\,x^{2}+Bx+A\right)dx$$

input
```
int((b*x^2+a)^(2/3)*(C*x^2+B*x+A),x)
```

output
```
int((b*x^2+a)^(2/3)*(C*x^2+B*x+A),x)
```

Fricas [F]

$$\int \left(a+bx^2\right)^{2/3} \left(A+Bx+Cx^2\right)\, dx=\int \left(Cx^{2}+Bx+A\right)\left(bx^{2}+a\right)^{\frac{2}{3}}\, dx$$

input
```
integrate((b*x^2+a)^(2/3)*(C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
integral((C*x^2 + B*x + A)*(b*x^2 + a)^(2/3), x)
```

Sympy [A] (verification not implemented)

Time = 1.10 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.14

$$\int \left(a+bx^2\right)^{2/3}\left(A+Bx+Cx^2\right)\,dx = Aa^{\frac{2}{3}}x\,{}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right) + B\left(\begin{cases}\frac{a^{\frac{2}{3}}x^2}{2} & \text{for } b=0\\ \frac{3\left(a+bx^2\right)^{\frac{5}{3}}}{10b} & \text{otherwise}\end{cases}\right) + \frac{Ca^{\frac{2}{3}}x^3\,{}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{3}$$

input
```
integrate((b*x**2+a)**(2/3)*(C*x**2+B*x+A),x)
```

output
```
A*a**(2/3)*x*hyper((-2/3, 1/2), (3/2,), b*x**2*exp_polar(I*pi)/a) + B*Piec
ewise((a**(2/3)*x**2/2, Eq(b, 0)), (3*(a + b*x**2)**(5/3)/(10*b), True)) +
 C*a**(2/3)*x**3*hyper((-2/3, 3/2), (5/2,), b*x**2*exp_polar(I*pi)/a)/3
```

Maxima [F]

$$\int \left(a+bx^2\right)^{2/3}\left(A+Bx+Cx^2\right)\,dx = \int \left(Cx^2+Bx+A\right)\left(bx^2+a\right)^{\frac{2}{3}}\,dx$$

input
```
integrate((b*x^2+a)^(2/3)*(C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)*(b*x^2 + a)^(2/3), x)
```

Giac [F]

$$\int \left(a+bx^2\right)^{2/3} \left(A+Bx+Cx^2\right) \, dx = \int \left(Cx^2+Bx+A\right)\left(bx^2+a\right)^{\frac{2}{3}} \, dx$$

input
```
integrate((b*x^2+a)^(2/3)*(C*x^2+B*x+A),x, algorithm="giac")
```

output
```
integrate((C*x^2 + B*x + A)*(b*x^2 + a)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^2\right)^{2/3} \left(A+Bx+Cx^2\right) \, dx = \int \left(b\,x^2+a\right)^{2/3}\,\left(C\,x^2+B\,x+A\right) \, dx$$

input
```
int((a + b*x^2)^(2/3)*(A + B*x + C*x^2),x)
```

output
```
int((a + b*x^2)^(2/3)*(A + B*x + C*x^2), x)
```

Reduce [F]

$$\int \left(a+bx^2\right)^{2/3} \left(A+Bx+Cx^2\right) \, dx = \frac{390(b\,x^2+a)^{\frac{2}{3}}\,abx+273(b\,x^2+a)^{\frac{2}{3}}\,ab+120(b\,x^2+a)^{\frac{2}{3}}\,acx+273(b\,x^2+a)^{\frac{2}{3}}\,b^2x^2+210(b\,x^2}{910b}$$

input
```
int((b*x^2+a)^(2/3)*(C*x^2+B*x+A),x)
```

output
```
(390*(a + b*x**2)**(2/3)*a*b*x + 273*(a + b*x**2)**(2/3)*a*b + 120*(a + b*
x**2)**(2/3)*a*c*x + 273*(a + b*x**2)**(2/3)*b**2*x**2 + 210*(a + b*x**2)*
*(2/3)*b*c*x**3 + 520*int((a + b*x**2)**(2/3)/(a + b*x**2),x)*a**2*b - 120
*int((a + b*x**2)**(2/3)/(a + b*x**2),x)*a**2*c)/(910*b)
```

3.42 $\int \frac{A+Bx+Cx^2}{\sqrt[3]{a+bx^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 602

$$\int \frac{A+Bx+Cx^2}{\sqrt[3]{a+bx^2}}\,dx$$

$$= \frac{3B(a+bx^2)^{2/3}}{4b} + \frac{3Cx(a+bx^2)^{2/3}}{7b} - \frac{3(7Ab-3aC)x}{7b\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}$$

$$+ \frac{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}(7Ab-3aC)\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{}}{\left(1-\sqrt{3}\right)\sqrt[3]{}}\right.\right.}{14b^2x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

$$- \frac{\sqrt{2}3^{3/4}\sqrt[3]{a}(7Ab-3aC)\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{}}{\left(1-\sqrt{3}\right)\sqrt[3]{}}\right.\right.}{7b^2x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

output
```
3/4*B*(b*x^2+a)^(2/3)/b+3/7*C*x*(b*x^2+a)^(2/3)/b-3/7*(7*A*b-3*C*a)*x/b/((
1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))+3/14*3^(1/4)*(1/2*6^(1/2)+1/2*2^(1/2))
*a^(1/3)*(7*A*b-3*C*a)*(a^(1/3)-(b*x^2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+
a)^(1/3)+(b*x^2+a)^(2/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)*E
llipticE(((1+3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2
+a)^(1/3)),2*I-I*3^(1/2))/b^2/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/((1-3^
(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)-1/7*2^(1/2)*3^(3/4)*a^(1/3)*(7*A*
b-3*C*a)*(a^(1/3)-(b*x^2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^
2+a)^(2/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)*EllipticF(((1+3
^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3)),2*I
-I*3^(1/2))/b^2/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)
-(b*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 9.42 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.13

$$\int \frac{A+Bx+Cx^2}{\sqrt[3]{a+bx^2}}\,dx$$
$$=\frac{3(7B+4Cx)\left(a+bx^2\right)+4(7Ab-3aC)x\sqrt[3]{1+\frac{bx^2}{a}}\ \text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{28b\sqrt[3]{a+bx^2}}$$

input
```
Integrate[(A + B*x + C*x^2)/(a + b*x^2)^(1/3),x]
```

output
```
(3*(7*B + 4*C*x)*(a + b*x^2) + 4*(7*A*b - 3*a*C)*x*(1 + (b*x^2)/a)^(1/3)*H
ypergeometric2F1[1/3, 1/2, 3/2, -((b*x^2)/a)])/(28*b*(a + b*x^2)^(1/3))
```

Rubi [A] (warning: unable to verify)

Time = 0.54 (sec) , antiderivative size = 628, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {2346, 27, 455, 233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{\sqrt[3]{a+bx^2}}dx$$

$$\Big\downarrow \text{ 2346}$$

$$\frac{3\int \frac{7Ab+7Bxb-3aC}{3\sqrt[3]{bx^2+a}}dx}{7b}+\frac{3Cx(a+bx^2)^{2/3}}{7b}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{7Ab+7Bxb-3aC}{\sqrt[3]{bx^2+a}}dx}{7b}+\frac{3Cx(a+bx^2)^{2/3}}{7b}$$

$$\Big\downarrow \text{ 455}$$

$$\frac{(7Ab-3aC)\int \frac{1}{\sqrt[3]{bx^2+a}}dx+\frac{21}{4}B(a+bx^2)^{2/3}}{7b}+\frac{3Cx(a+bx^2)^{2/3}}{7b}$$

$$\Big\downarrow \text{ 233}$$

$$\frac{\frac{3\sqrt{bx^2}(7Ab-3aC)\int \frac{\sqrt[3]{bx^2+a}}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}}{2bx}+\frac{21}{4}B(a+bx^2)^{2/3}}{7b}+\frac{3Cx(a+bx^2)^{2/3}}{7b}$$

$$\Big\downarrow \text{ 833}$$

$$\frac{\frac{3\sqrt{bx^2}(7Ab-3aC)\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}-\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}\right)}{2bx}+\frac{21}{4}B(a+bx^2)^{2/3}}{7b}+\frac{3Cx(a+bx^2)^{2/3}}{7b}$$

$$\Big\downarrow \text{ 760}$$

$$\frac{3\sqrt{bx^2}(7Ab-3aC)\left(-\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+b}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx}\right.}}}{\sqrt[4]{3}\sqrt{bx^2}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\right.}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\right.}}}}{2bx}}{7b}$$

$$\frac{3Cx\left(a+bx^2\right)^{2/3}}{7b}$$

$\downarrow$ 2418

$$\frac{3\sqrt{bx^2}(7Ab-3aC)\left(-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2}}\right.\right.}{\sqrt[4]{3}\sqrt{bx^2}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}\right.}{}$$

$$\frac{3Cx\left(a+bx^2\right)^{2/3}}{7b}$$

input
```
Int[(A + B*x + C*x^2)/(a + b*x^2)^(1/3),x]
```

output
```
(3*C*x*(a + b*x^2)^(2/3))/(7*b) + ((21*B*(a + b*x^2)^(2/3))/4 + (3*(7*A*b
- 3*a*C)*Sqrt[b*x^2]*((-2*Sqrt[b*x^2])/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2
)^(1/3)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)
)*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqr
t[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1
/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7
+ 4*Sqrt[3]])/(Sqrt[b*x^2]*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/
((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]) - (2*Sqrt[2 - Sqrt[3]]*(1
 + Sqrt[3])*a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*
(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2
)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/(
(1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqr
t[b*x^2]*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(
1/3) - (a + b*x^2)^(1/3))^2)])))/(2*b*x))/(7*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int \frac{C\,x^{2}+Bx+A}{\left(b\,x^{2}+a\right)^{\frac{1}{3}}}dx$$

input
```
int((C*x^2+B*x+A)/(b*x^2+a)^(1/3),x)
```

output
```
int((C*x^2+B*x+A)/(b*x^2+a)^(1/3),x)
```

Fricas [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt[3]{a+bx^2}}\,dx=\int \frac{Cx^2+Bx+A}{(bx^2+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(b*x^2+a)^(1/3),x, algorithm="fricas")
```

output
```
integral((C*x^2 + B*x + A)/(b*x^2 + a)^(1/3), x)
```

Sympy [A] (verification not implemented)

Time = 0.94 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.14

$$\int \frac{A+Bx+Cx^2}{\sqrt[3]{a+bx^2}}\,dx=\frac{Ax\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{\sqrt[3]{a}}+B\left(\begin{cases}\frac{x^2}{2\sqrt[3]{a}} & \text{for } b=0\\ \frac{3(a+bx^2)^{\frac{2}{3}}}{4b} & \text{otherwise}\end{cases}\right)+\frac{Cx^3\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{3}{2}\\\frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{3\sqrt[3]{a}}$$

input
```
integrate((C*x**2+B*x+A)/(b*x**2+a)**(1/3),x)
```

output
```
A*x*hyper((1/3, 1/2), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(1/3) + B*Piece
wise((x**2/(2*a**(1/3)), Eq(b, 0)), (3*(a + b*x**2)**(2/3)/(4*b), True)) +
 C*x**3*hyper((1/3, 3/2), (5/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(1/3))
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt[3]{a+bx^2}}\,dx = \int \frac{Cx^2+Bx+A}{(bx^2+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(b*x^2+a)^(1/3),x, algorithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)/(b*x^2 + a)^(1/3), x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt[3]{a+bx^2}}\,dx = \int \frac{Cx^2+Bx+A}{(bx^2+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(b*x^2+a)^(1/3),x, algorithm="giac")
```

output
```
integrate((C*x^2 + B*x + A)/(b*x^2 + a)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{\sqrt[3]{a+bx^2}}\,dx = \int \frac{C\,x^2+B\,x+A}{\left(b\,x^2+a\right)^{1/3}}\,dx$$

input
```
int((A + B*x + C*x^2)/(a + b*x^2)^(1/3),x)
```

output
```
int((A + B*x + C*x^2)/(a + b*x^2)^(1/3), x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt[3]{a+bx^2}}\,dx = \left(\int \frac{x^2}{(b\,x^2+a)^{\frac{1}{3}}}dx\right)c + \left(\int \frac{x}{(b\,x^2+a)^{\frac{1}{3}}}dx\right)b + \left(\int \frac{1}{(b\,x^2+a)^{\frac{1}{3}}}dx\right)a$$

input
```
int((C*x^2+B*x+A)/(b*x^2+a)^(1/3),x)
```

output
```
int(x**2/(a + b*x**2)**(1/3),x)*c + int(x/(a + b*x**2)**(1/3),x)*b + int(1
/(a + b*x**2)**(1/3),x)*a
```

3.43 $\int \frac{A+Bx+Cx^2}{(a+bx^2)^{4/3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 596

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^{4/3}}\,dx = -\frac{3(aB-(Ab-aC)x)}{2ab\sqrt[3]{a+bx^2}} + \frac{3(Ab-3aC)x}{2ab\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}$$

$$-\frac{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}(Ab-3aC)\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a}\cdots}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a}\cdots}\right.\right.}{4a^{2/3}b^2x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

$$+\frac{3^{3/4}(Ab-3aC)\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+b}\cdots}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+b}\cdots}\right.\right.}{\sqrt{2}a^{2/3}b^2x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

output
```
1/2*(-3*B*a+3*(A*b-C*a)*x)/a/b/(b*x^2+a)^(1/3)+3/2*(A*b-3*C*a)*x/a/b/((1-3
^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))-3/4*3^(1/4)*(1/2*6^(1/2)+1/2*2^(1/2))*(A*
b-3*C*a)*(a^(1/3)-(b*x^2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^
2+a)^(2/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)*EllipticE(((1+3
^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3)),2*I
-I*3^(1/2))/a^(2/3)/b^2/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))
*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)+1/2*3^(3/4)*(A*b-3*C*a)*(a^(1/3)-(b*x^2
+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/((1-3^(1/2))
*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)*EllipticF(((1+3^(1/2))*a^(1/3)-(b*x^2+a
)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3)),2*I-I*3^(1/2))*2^(1/2)/a^(2
/3)/b^2/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+
a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.13

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^{4/3}}\,dx = \frac{3Abx-3a(B+Cx)+(-Ab+3aC)x\sqrt[3]{1+\frac{bx^2}{a}}\ \text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{2ab\sqrt[3]{a+bx^2}}$$

input
```
Integrate[(A + B*x + C*x^2)/(a + b*x^2)^(4/3),x]
```

output
```
(3*A*b*x - 3*a*(B + C*x) + (-(A*b) + 3*a*C)*x*(1 + (b*x^2)/a)^(1/3)*Hyperg
eometric2F1[1/3, 1/2, 3/2, -((b*x^2)/a)])/(2*a*b*(a + b*x^2)^(1/3))
```

Rubi [A] (warning: unable to verify)

Time = 0.56 (sec) , antiderivative size = 623, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {2345, 27, 233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{\left(a+bx^2\right)^{4/3}}dx$$

$$\Big\downarrow \text{ 2345}$$

$$-\frac{3\int \frac{A-\frac{3aC}{b}}{3\sqrt[3]{bx^2+a}}dx}{2a}-\frac{3(aB-x(Ab-aC))}{2ab\sqrt[3]{a+bx^2}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\left(A-\frac{3aC}{b}\right)\int \frac{1}{\sqrt[3]{bx^2+a}}dx}{2a}-\frac{3(aB-x(Ab-aC))}{2ab\sqrt[3]{a+bx^2}}$$

$$\Big\downarrow \text{ 233}$$

$$-\frac{3\sqrt{bx^2}\left(A-\frac{3aC}{b}\right)\int \frac{\sqrt[3]{bx^2+a}}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}}{4abx}-\frac{3(aB-x(Ab-aC))}{2ab\sqrt[3]{a+bx^2}}$$

$$\Big\downarrow \text{ 833}$$

$$-\frac{3\sqrt{bx^2}\left(A-\frac{3aC}{b}\right)\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}-\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}\right)}{4abx}-\frac{3(aB-x(Ab-aC))}{2ab\sqrt[3]{a+bx^2}}$$

$$\Big\downarrow \text{ 760}$$

$$-\frac{3\sqrt{bx^2}\left(A-\frac{3aC}{b}\right)\left(-\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a \cdots}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{\cdots}\right.}}}{\sqrt[4]{3}\sqrt{bx^2}\sqrt{\cdots}}\right.}{4abx}-\frac{3(aB-x(Ab-aC))}{2ab\sqrt[3]{a+bx^2}}$$

$$\Big\downarrow \text{ 2418}$$

$$-\frac{3\sqrt{bx^2}\left(A-\frac{3aC}{b}\right)\Bigg(-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,\text{EllipticF}\Bigg(\arcsin\Bigg(\frac{\left(1+\sqrt{3}\right)\cdots}{\left(1-\sqrt{3}\right)\cdots}}{\sqrt[4]{3}\sqrt{bx^2}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}\cdots}{\cdots}$$

$$\frac{3(aB-x(Ab-aC))}{2ab\sqrt[3]{a+bx^2}}$$

input
```
Int[(A + B*x + C*x^2)/(a + b*x^2)^(4/3),x]
```

output
```
(-3*(a*B - (A*b - a*C)*x))/(2*a*b*(a + b*x^2)^(1/3)) - (3*(A - (3*a*C)/b)*
Sqrt[b*x^2]*((-2*Sqrt[b*x^2])/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))
+ (3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a
^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(
1/3) - (a + b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a
+ b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[
3]])/(Sqrt[b*x^2]*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqr
t[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)]) - (2*Sqrt[2 - Sqrt[3]]*(1 + Sqrt[3
])*a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^
2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^
2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt
[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[b*x^2]*
Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a
 + b*x^2)^(1/3))^2)])))/(4*a*b*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int \frac{C\,x^2 + Bx + A}{(b\,x^2 + a)^{\frac{4}{3}}} dx$$

input
```
int((C*x^2+B*x+A)/(b*x^2+a)^(4/3),x)
```

output
```
int((C*x^2+B*x+A)/(b*x^2+a)^(4/3),x)
```

Fricas [F]

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^{4/3}}\,dx=\int \frac{Cx^2+Bx+A}{(bx^2+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(b*x^2+a)^(4/3),x, algorithm="fricas")
```

output
```
integral((C*x^2 + B*x + A)*(b*x^2 + a)^(2/3)/(b^2*x^4 + 2*a*b*x^2 + a^2),
x)
```

Sympy [A] (verification not implemented)

Time = 2.51 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.14

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^{4/3}}\,dx=\frac{Ax\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{4}{3}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{a^{\frac{4}{3}}}+B\left(\begin{cases}-\frac{3}{2b\sqrt[3]{a+bx^2}} & \text{for}\ b\neq 0\\ \frac{x^2}{2a^{\frac{4}{3}}} & \text{otherwise}\end{cases}\right)+\frac{Cx^3\,{}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{3a^{\frac{4}{3}}}$$

input
```
integrate((C*x**2+B*x+A)/(b*x**2+a)**(4/3),x)
```

output
```
A*x*hyper((1/2, 4/3), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(4/3) + B*Piece
wise((-3/(2*b*(a + b*x**2)**(1/3)), Ne(b, 0)), (x**2/(2*a**(4/3)), True))
+ C*x**3*hyper((4/3, 3/2), (5/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(4/3))
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^{4/3}}\,dx = \int \frac{Cx^2+Bx+A}{(bx^2+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(b*x^2+a)^(4/3),x, algorithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)/(b*x^2 + a)^(4/3), x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^{4/3}}\,dx = \int \frac{Cx^2+Bx+A}{(bx^2+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(b*x^2+a)^(4/3),x, algorithm="giac")
```

output
```
integrate((C*x^2 + B*x + A)/(b*x^2 + a)^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^{4/3}}\,dx = \int \frac{C\,x^2+B\,x+A}{(b\,x^2+a)^{4/3}}\,dx$$

input
```
int((A + B*x + C*x^2)/(a + b*x^2)^(4/3),x)
```

output
```
int((A + B*x + C*x^2)/(a + b*x^2)^(4/3), x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^{4/3}}\,dx = \left(\int \frac{x^2}{(b\,x^2+a)^{\frac{1}{3}}\,a+(b\,x^2+a)^{\frac{1}{3}}\,b\,x^2}dx\right)c + \left(\int \frac{x}{(b\,x^2+a)^{\frac{1}{3}}\,a+(b\,x^2+a)^{\frac{1}{3}}\,b\,x^2}dx\right)b + \left(\int \frac{1}{(b\,x^2+a)^{\frac{1}{3}}\,a+(b\,x^2+a)^{\frac{1}{3}}\,b\,x^2}dx\right)a$$

input
```
int((C*x^2+B*x+A)/(b*x^2+a)^(4/3),x)
```

output
```
int(x**2/((a + b*x**2)**(1/3)*a + (a + b*x**2)**(1/3)*b*x**2),x)*c + int(x
/((a + b*x**2)**(1/3)*a + (a + b*x**2)**(1/3)*b*x**2),x)*b + int(1/((a + b
*x**2)**(1/3)*a + (a + b*x**2)**(1/3)*b*x**2),x)*a
```

3.44 $\int \frac{A+Bx+Cx^2}{(a+bx^2)^{7/3}} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 633

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^{7/3}} \, dx = -\frac{3(aB-(Ab-aC)x)}{8ab(a+bx^2)^{4/3}} + \frac{3(5Ab+3aC)x}{16a^2b\sqrt[3]{a+bx^2}} + \frac{3(5Ab+3aC)x}{16a^2b\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}$$
$$-\frac{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}(5Ab+3aC)\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a}}\right.\right.}{32a^{5/3}b^2x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$
$$+\frac{3^{3/4}(5Ab+3aC)\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+}}\right.\right.}{8\sqrt{2}a^{5/3}b^2x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

output
```
1/8*(-3*B*a+3*(A*b-C*a)*x)/a/b/(b*x^2+a)^(4/3)+3/16*(5*A*b+3*C*a)*x/a^2/b/
(b*x^2+a)^(1/3)+3/16*(5*A*b+3*C*a)*x/a^2/b/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^
(1/3))-3/32*3^(1/4)*(1/2*6^(1/2)+1/2*2^(1/2))*(5*A*b+3*C*a)*(a^(1/3)-(b*x^
2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/((1-3^(1/2)
)*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)*EllipticE(((1+3^(1/2))*a^(1/3)-(b*x^2+
a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3)),2*I-I*3^(1/2))/a^(5/3)/b^2
/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3
))^2)^(1/2)+1/16*3^(3/4)*(5*A*b+3*C*a)*(a^(1/3)-(b*x^2+a)^(1/3))*((a^(2/3)
+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(
1/3))^2)^(1/2)*EllipticF(((1+3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2)
)*a^(1/3)-(b*x^2+a)^(1/3)),2*I-I*3^(1/2))*2^(1/2)/a^(5/3)/b^2/x/(-a^(1/3)*
(a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.08 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.18

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^{7/3}}\,dx=\frac{3(5Ab^2x^3+a^2(-2B+Cx)+abx(7A+3Cx^2))-(5Ab+3aC)x(a+bx^2)\sqrt[3]{1+\cdots}}{16a^2b\,(a+bx^2)^{4/3}}$$

input
```
Integrate[(A + B*x + C*x^2)/(a + b*x^2)^(7/3),x]
```

output
```
(3*(5*A*b^2*x^3 + a^2*(-2*B + C*x) + a*b*x*(7*A + 3*C*x^2)) - (5*A*b + 3*a
*C)*x*(a + b*x^2)*(1 + (b*x^2)/a)^(1/3)*Hypergeometric2F1[1/3, 1/2, 3/2, -
((b*x^2)/a)])/(16*a^2*b*(a + b*x^2)^(4/3))
```

Rubi [A] (warning: unable to verify)

Time = 0.60 (sec) , antiderivative size = 652, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {2345, 27, 215, 233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^{7/3}}dx$$

$$\Big\downarrow \text{2345}$$

$$-\frac{3\int -\frac{5A+\frac{3aC}{b}}{3(bx^2+a)^{4/3}}dx}{8a}-\frac{3(aB-x(Ab-aC))}{8ab(a+bx^2)^{4/3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\left(\frac{3aC}{b}+5A\right)\int \frac{1}{(bx^2+a)^{4/3}}dx}{8a}-\frac{3(aB-x(Ab-aC))}{8ab(a+bx^2)^{4/3}}$$

$$\Big\downarrow \text{215}$$

$$\frac{\left(\frac{3aC}{b}+5A\right)\left(\frac{3x}{2a\sqrt[3]{a+bx^2}}-\frac{\int \frac{1}{\sqrt[3]{bx^2+a}}dx}{2a}\right)}{8a}-\frac{3(aB-x(Ab-aC))}{8ab(a+bx^2)^{4/3}}$$

$$\Big\downarrow \text{233}$$

$$\frac{\left(\frac{3aC}{b}+5A\right)\left(\frac{3x}{2a\sqrt[3]{a+bx^2}}-\frac{3\sqrt{bx^2}\int \frac{\sqrt[3]{bx^2+a}}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}}{4abx}\right)}{8a}-\frac{3(aB-x(Ab-aC))}{8ab(a+bx^2)^{4/3}}$$

$$\Big\downarrow \text{833}$$

$$\frac{\left(\frac{3aC}{b}+5A\right)\left(\frac{3x}{2a\sqrt[3]{a+bx^2}}-\frac{3\sqrt{bx^2}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}\int\frac{1}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}-\int\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}\right)}{4abx}\right)}{8a}-\frac{3(aB-x(Ab-aC))}{8ab\left(a+bx^2\right)^{4/3}}$$

$\big\downarrow$ 760

$$\frac{\left(\frac{3aC}{b}+5A\right)\left(\frac{3x}{2a\sqrt[3]{a+bx^2}}-\frac{3\sqrt{bx^2}\left(-\int\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}\cdots}{\left(\cdots\right)}}\cdots}{\sqrt[4]{\cdots}}\right)}{4abx}\right)}{8a}-\frac{3(aB-x(Ab-aC))}{8ab\left(a+bx^2\right)^{4/3}}$$

$\big\downarrow$ 2418

$$\frac{\left(\frac{3aC}{b}+5A\right)\left(\frac{3x}{2a\sqrt[3]{a+bx^2}}-\frac{3\sqrt{bx^2}\left(-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\text{EllipticF}\left(\text{arc}\right.}{\sqrt[4]{3}\sqrt{bx^2}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}\right.}{}\right.}{}$$

$$\frac{3(aB-x(Ab-aC))}{8ab\left(a+bx^2\right)^{4/3}}$$

input
```
Int[(A + B*x + C*x^2)/(a + b*x^2)^(7/3),x]
```

output
```
(-3*(a*B - (A*b - a*C)*x))/(8*a*b*(a + b*x^2)^(4/3)) + ((5*A + (3*a*C)/b)*
((3*x)/(2*a*(a + b*x^2)^(1/3)) - (3*Sqrt[b*x^2]*((-2*Sqrt[b*x^2])/((1 - Sq
rt[3])*a^(1/3) - (a + b*x^2)^(1/3)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(
a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (
a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticE
[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3)
 - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(Sqrt[b*x^2]*Sqrt[-((a^(1/3)*(a^(
1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)]
) - (2*Sqrt[2 - Sqrt[3]]*(1 + Sqrt[3])*a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3
))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sq
rt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(
1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7
 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[b*x^2]*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)
^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)])))/(4*a*b*x)))/(8
*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int \frac{C\,x^{2}+Bx+A}{\left(b\,x^{2}+a\right)^{\frac{7}{3}}}dx$$

input
```
int((C*x^2+B*x+A)/(b*x^2+a)^(7/3),x)
```

output
```
int((C*x^2+B*x+A)/(b*x^2+a)^(7/3),x)
```

Fricas [F]

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^{7/3}}\,dx=\int \frac{Cx^{2}+Bx+A}{\left(bx^{2}+a\right)^{\frac{7}{3}}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(b*x^2+a)^(7/3),x, algorithm="fricas")
```

output
```
integral((C*x^2 + B*x + A)*(b*x^2 + a)^(2/3)/(b^3*x^6 + 3*a*b^2*x^4 + 3*a^
2*b*x^2 + a^3), x)
```

Sympy [A] (verification not implemented)

Time = 5.36 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.17

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^{7/3}}\,dx = \frac{Ax\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{3}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{a^{\frac{7}{3}}} + B\left(\begin{cases}-\frac{3}{8ab\sqrt[3]{a+bx^2}+8b^2x^2\sqrt[3]{a+bx^2}} & \text{for}\ b\neq 0\\ \frac{x^2}{2a^{\frac{7}{3}}} & \text{otherwise}\end{cases}\right) + \frac{Cx^3\,{}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{7}{3}\\ \frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{3a^{\frac{7}{3}}}$$

input
```
integrate((C*x**2+B*x+A)/(b*x**2+a)**(7/3),x)
```

output
```
A*x*hyper((1/2, 7/3), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(7/3) + B*Piece
wise((-3/(8*a*b*(a + b*x**2)**(1/3) + 8*b**2*x**2*(a + b*x**2)**(1/3)), Ne
(b, 0)), (x**2/(2*a**(7/3)), True)) + C*x**3*hyper((3/2, 7/3), (5/2,), b*x
**2*exp_polar(I*pi)/a)/(3*a**(7/3))
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^{7/3}}\,dx = \int \frac{Cx^2+Bx+A}{(bx^2+a)^{\frac{7}{3}}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(b*x^2+a)^(7/3),x, algorithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)/(b*x^2 + a)^(7/3), x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^{7/3}}\,dx=\int \frac{Cx^2+Bx+A}{(bx^2+a)^{\frac{7}{3}}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(b*x^2+a)^(7/3),x, algorithm="giac")
```

output
```
integrate((C*x^2 + B*x + A)/(b*x^2 + a)^(7/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^{7/3}}\,dx=\int \frac{C\,x^2+B\,x+A}{(b\,x^2+a)^{7/3}}\,dx$$

input
```
int((A + B*x + C*x^2)/(a + b*x^2)^(7/3),x)
```

output
```
int((A + B*x + C*x^2)/(a + b*x^2)^(7/3), x)
```

Reduce [F]

$$\begin{aligned}\int \frac{A+Bx+Cx^2}{(a+bx^2)^{7/3}}\,dx=&\left(\int \frac{x^2}{(b\,x^2+a)^{\frac{1}{3}}\,a^2+2\,(b\,x^2+a)^{\frac{1}{3}}\,ab\,x^2+(b\,x^2+a)^{\frac{1}{3}}\,b^2x^4}dx\right)c\\&+\left(\int \frac{x}{(b\,x^2+a)^{\frac{1}{3}}\,a^2+2\,(b\,x^2+a)^{\frac{1}{3}}\,ab\,x^2+(b\,x^2+a)^{\frac{1}{3}}\,b^2x^4}dx\right)b\\&+\left(\int \frac{1}{(b\,x^2+a)^{\frac{1}{3}}\,a^2+2\,(b\,x^2+a)^{\frac{1}{3}}\,ab\,x^2+(b\,x^2+a)^{\frac{1}{3}}\,b^2x^4}dx\right)a\end{aligned}$$

input
```
int((C*x^2+B*x+A)/(b*x^2+a)^(7/3),x)
```

output
```
int(x**2/((a + b*x**2)**(1/3)*a**2 + 2*(a + b*x**2)**(1/3)*a*b*x**2 + (a +
 b*x**2)**(1/3)*b**2*x**4),x)*c + int(x/((a + b*x**2)**(1/3)*a**2 + 2*(a +
 b*x**2)**(1/3)*a*b*x**2 + (a + b*x**2)**(1/3)*b**2*x**4),x)*b + int(1/((a
 + b*x**2)**(1/3)*a**2 + 2*(a + b*x**2)**(1/3)*a*b*x**2 + (a + b*x**2)**(1
/3)*b**2*x**4),x)*a
```

3.45 $\int (a+bx^2)^p (A+Bx+Cx^2)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 109

$$\int (a+bx^2)^p (A+Bx+Cx^2)\,dx = \frac{B(a+bx^2)^{1+p}}{2b(1+p)} + \frac{Cx(a+bx^2)^{1+p}}{b(3+2p)} + \left(A-\frac{aC}{3b+2bp}\right)x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right)$$

output
```
1/2*B*(b*x^2+a)^(p+1)/b/(p+1)+C*x*(b*x^2+a)^(p+1)/b/(3+2*p)+(A-a*C/(2*b*p+
3*b))*x*(b*x^2+a)^p*hypergeom([1/2, -p],[3/2],-b*x^2/a)/((1+b*x^2/a)^p)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.05

$$\int (a+bx^2)^p (A+Bx+Cx^2)\,dx = \frac{(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\left(3B(a+bx^2)\left(1+\frac{bx^2}{a}\right)^p + 6Ab(1+p)x\,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right)+\right.}{6b(1+p)}$$

input
```
Integrate[(a + b*x^2)^p*(A + B*x + C*x^2),x]
```

output
```
((a + b*x^2)^p*(3*B*(a + b*x^2)*(1 + (b*x^2)/a)^p + 6*A*b*(1 + p)*x*Hyperg
eometric2F1[1/2, -p, 3/2, -((b*x^2)/a)] + 2*b*C*(1 + p)*x^3*Hypergeometric
2F1[3/2, -p, 5/2, -((b*x^2)/a)]))/(6*b*(1 + p)*(1 + (b*x^2)/a)^p)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2346, 25, 455, 238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^2)^p\left(A+Bx+Cx^2\right)dx$$

$$\Big\downarrow \text{2346}$$

$$\frac{\int -\left((aC-Ab(2p+3)-bB(2p+3)x)\left(bx^2+a\right)^p\right)dx}{b(2p+3)}+\frac{Cx\left(a+bx^2\right)^{p+1}}{b(2p+3)}$$

$$\Big\downarrow \text{25}$$

$$\frac{Cx\left(a+bx^2\right)^{p+1}}{b(2p+3)}-\frac{\int (aC-Ab(2p+3)-bB(2p+3)x)\left(bx^2+a\right)^p dx}{b(2p+3)}$$

$$\Big\downarrow \text{455}$$

$$\frac{Cx\left(a+bx^2\right)^{p+1}}{b(2p+3)}-\frac{(aC-Ab(2p+3))\int \left(bx^2+a\right)^p dx-\frac{B(2p+3)\left(a+bx^2\right)^{p+1}}{2(p+1)}}{b(2p+3)}$$

$$\Big\downarrow \text{238}$$

$$\frac{Cx\left(a+bx^2\right)^{p+1}}{b(2p+3)}-\frac{\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}(aC-Ab(2p+3))\int \left(\frac{bx^2}{a}+1\right)^p dx-\frac{B(2p+3)\left(a+bx^2\right)^{p+1}}{2(p+1)}}{b(2p+3)}$$

$$\Big\downarrow \text{237}$$

$$\frac{Cx(a+bx^2)^{p+1}}{b(2p+3)} - \frac{x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(aC-Ab(2p+3))\,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right) - \frac{B(2p+3)(a+bx^2)^{p+1}}{2(p+1)}}{b(2p+3)}$$

input
```
Int[(a + b*x^2)^p*(A + B*x + C*x^2),x]
```

output
```
(C*x*(a + b*x^2)^(1 + p))/(b*(3 + 2*p)) - (-1/2*(B*(3 + 2*p)*(a + b*x^2)^(
1 + p))/(1 + p) + ((a*C - A*b*(3 + 2*p))*x*(a + b*x^2)^p*Hypergeometric2F1
[1/2, -p, 3/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p)/(b*(3 + 2*p))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Maple [F]

$$\int \left(b\,x^{2}+a\right)^{p} \left(C\,x^{2}+B x +A \right)d x$$

input
```
int((b*x^2+a)^p*(C*x^2+B*x+A),x)
```

output
```
int((b*x^2+a)^p*(C*x^2+B*x+A),x)
```

Fricas [F]

$$\int (a+bx^2)^p (A+Bx+Cx^2)\,dx = \int (Cx^2+Bx+A)(bx^2+a)^p\,dx$$

input
```
integrate((b*x^2+a)^p*(C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
integral((C*x^2 + B*x + A)*(b*x^2 + a)^p, x)
```

Sympy [A] (verification not implemented)

Time = 4.39 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.83

$$\int (a+bx^2)^p (A+Bx+Cx^2)\,dx = Aa^p x {}_2F_1\left(\begin{matrix}\frac{1}{2}, -p \\ \frac{3}{2}\end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right) + B\left(\begin{cases}\frac{a^p x^2}{2} & \text{for}\ b=0 \\ \frac{\begin{cases}\frac{(a+bx^2)^{p+1}}{p+1} & \text{for}\ p\neq -1 \\ \log(a+bx^2) & \text{otherwise}\end{cases}}{2b} & \text{otherwise}\end{cases}\right) + \frac{Ca^p x^3 {}_2F_1\left(\begin{matrix}\frac{3}{2}, -p \\ \frac{5}{2}\end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)}{3}$$

input
```
integrate((b*x**2+a)**p*(C*x**2+B*x+A),x)
```

output
```
A*a**p*x*hyper((1/2, -p), (3/2,), b*x**2*exp_polar(I*pi)/a) + B*Piecewise(
(a**p*x**2/2, Eq(b, 0)), (Piecewise(((a + b*x**2)**(p + 1)/(p + 1), Ne(p,
-1)), (log(a + b*x**2), True))/(2*b), True)) + C*a**p*x**3*hyper((3/2, -p)
, (5/2,), b*x**2*exp_polar(I*pi)/a)/3
```

Maxima [F]

$$\int (a+bx^2)^p (A+Bx+Cx^2)\, dx = \int (Cx^2+Bx+A)(bx^2+a)^p\, dx$$

input
```
integrate((b*x^2+a)^p*(C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)*(b*x^2 + a)^p, x)
```

Giac [F]

$$\int (a+bx^2)^p (A+Bx+Cx^2)\, dx = \int (Cx^2+Bx+A)(bx^2+a)^p\, dx$$

input
```
integrate((b*x^2+a)^p*(C*x^2+B*x+A),x, algorithm="giac")
```

output
```
integrate((C*x^2 + B*x + A)*(b*x^2 + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx^2)^p (A+Bx+Cx^2) \, dx = \int (b\,x^2+a)^p \left(C\,x^2+B\,x+A\right) \, dx$$

input
```
int((a + b*x^2)^p*(A + B*x + C*x^2),x)
```

output
```
int((a + b*x^2)^p*(A + B*x + C*x^2), x)
```

Reduce [F]

$$\int (a+bx^2)^p (A+Bx+Cx^2) \, dx$$
$$= \frac{4(b\,x^2+a)^p\,ab\,p^2x+4(b\,x^2+a)^p\,ab\,p^2+10(b\,x^2+a)^p\,abpx+8(b\,x^2+a)^p\,abp+6(b\,x^2+a)^p\,abx+3(b\,x}{}$$

input
```
int((b*x^2+a)^p*(C*x^2+B*x+A),x)
```

output

```
(4*(a + b*x**2)**p*a*b*p**2*x + 4*(a + b*x**2)**p*a*b*p**2 + 10*(a + b*x**
2)**p*a*b*p*x + 8*(a + b*x**2)**p*a*b*p + 6*(a + b*x**2)**p*a*b*x + 3*(a +
 b*x**2)**p*a*b + 4*(a + b*x**2)**p*a*c*p**2*x + 4*(a + b*x**2)**p*a*c*p*x
 + 4*(a + b*x**2)**p*b**2*p**2*x**2 + 8*(a + b*x**2)**p*b**2*p*x**2 + 3*(a
 + b*x**2)**p*b**2*x**2 + 4*(a + b*x**2)**p*b*c*p**2*x**3 + 6*(a + b*x**2)
**p*b*c*p*x**3 + 2*(a + b*x**2)**p*b*c*x**3 + 32*int((a + b*x**2)**p/(4*a*
p**2 + 8*a*p + 3*a + 4*b*p**2*x**2 + 8*b*p*x**2 + 3*b*x**2),x)*a**2*b*p**5
 + 144*int((a + b*x**2)**p/(4*a*p**2 + 8*a*p + 3*a + 4*b*p**2*x**2 + 8*b*p
*x**2 + 3*b*x**2),x)*a**2*b*p**4 + 232*int((a + b*x**2)**p/(4*a*p**2 + 8*a
*p + 3*a + 4*b*p**2*x**2 + 8*b*p*x**2 + 3*b*x**2),x)*a**2*b*p**3 + 156*int
((a + b*x**2)**p/(4*a*p**2 + 8*a*p + 3*a + 4*b*p**2*x**2 + 8*b*p*x**2 + 3*
b*x**2),x)*a**2*b*p**2 + 36*int((a + b*x**2)**p/(4*a*p**2 + 8*a*p + 3*a +
4*b*p**2*x**2 + 8*b*p*x**2 + 3*b*x**2),x)*a**2*b*p - 16*int((a + b*x**2)**
p/(4*a*p**2 + 8*a*p + 3*a + 4*b*p**2*x**2 + 8*b*p*x**2 + 3*b*x**2),x)*a**2
*c*p**4 - 48*int((a + b*x**2)**p/(4*a*p**2 + 8*a*p + 3*a + 4*b*p**2*x**2 +
 8*b*p*x**2 + 3*b*x**2),x)*a**2*c*p**3 - 44*int((a + b*x**2)**p/(4*a*p**2
+ 8*a*p + 3*a + 4*b*p**2*x**2 + 8*b*p*x**2 + 3*b*x**2),x)*a**2*c*p**2 - 12
*int((a + b*x**2)**p/(4*a*p**2 + 8*a*p + 3*a + 4*b*p**2*x**2 + 8*b*p*x**2
+ 3*b*x**2),x)*a**2*c*p)/(2*b*(4*p**3 + 12*p**2 + 11*p + 3))
```

3.46 $\int (a+bx^2)^3 (A+Bx+Cx^2+Dx^3) \, dx$

Optimal result

Integrand size = 25, antiderivative size = 133

$$\int (a+bx^2)^3 (A+Bx+Cx^2+Dx^3) \, dx = a^3Ax + \frac{1}{3}a^2(3Ab+aC)x^3 + \frac{1}{4}a^3Dx^4 + \frac{3}{5}ab(Ab+aC)x^5 + \frac{1}{2}a^2bDx^6 + \frac{1}{7}b^2(Ab+3aC)x^7 + \frac{3}{8}ab^2Dx^8 + \frac{1}{9}b^3Cx^9 + \frac{1}{10}b^3Dx^{10} + \frac{B(a+bx^2)^4}{8b}$$

output
```
a^3*A*x+1/3*a^2*(3*A*b+C*a)*x^3+1/4*a^3*D*x^4+3/5*a*b*(A*b+C*a)*x^5+1/2*a^
2*b*D*x^6+1/7*b^2*(A*b+3*C*a)*x^7+3/8*a*b^2*D*x^8+1/9*b^3*C*x^9+1/10*b^3*D
*x^10+1/8*B*(b*x^2+a)^4/b
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.91

$$\int \left(a+bx^2\right)^3 \left(A+Bx+Cx^2+Dx^3\right) dx$$
$$= \frac{210a^3x(12A+x(6B+x(4C+3Dx)))+126a^2bx^3(20A+x(15B+2x(6C+5Dx)))+9ab^2x^5(168A+}{2520}$$

input
```
Integrate[(a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(210*a^3*x*(12*A + x*(6*B + x*(4*C + 3*D*x))) + 126*a^2*b*x^3*(20*A + x*(1
5*B + 2*x*(6*C + 5*D*x))) + 9*a*b^2*x^5*(168*A + 5*x*(28*B + 3*x*(8*C + 7*
D*x))) + b^3*x^7*(360*A + 7*x*(45*B + 4*x*(10*C + 9*D*x))))/2520
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2017, 2341, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^3 \left(A+Bx+Cx^2+Dx^3\right) dx$$
$$\Big\downarrow \text{ 2017}$$
$$\int \left(bx^2+a\right)^3 \left(Dx^3+Cx^2+A\right) dx+\frac{B\left(a+bx^2\right)^4}{8b}$$
$$\Big\downarrow \text{ 2341}$$
$$\int \left(b^3Dx^9+b^3Cx^8+3ab^2Dx^7+b^2(Ab+3aC)x^6+3a^2bDx^5+3ab(Ab+aC)x^4+a^3Dx^3+a^2(3Ab+aC)x^2+\right.$$
$$\frac{B\left(a+bx^2\right)^4}{8b}$$
$$\Big\downarrow \text{ 2009}$$

$$a^3Ax + \frac{1}{4}a^3Dx^4 + \frac{1}{3}a^2x^3(aC + 3Ab) + \frac{1}{2}a^2bDx^6 + \frac{1}{7}b^2x^7(3aC + Ab) + \frac{3}{5}abx^5(aC + Ab) + \frac{3}{8}ab^2Dx^8 + \frac{B(a + bx^2)^4}{8b} + \frac{1}{9}b^3Cx^9 + \frac{1}{10}b^3Dx^{10}$$

input
```
Int[(a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
a^3*A*x + (a^2*(3*A*b + a*C)*x^3)/3 + (a^3*D*x^4)/4 + (3*a*b*(A*b + a*C)*x
^5)/5 + (a^2*b*D*x^6)/2 + (b^2*(A*b + 3*a*C)*x^7)/7 + (3*a*b^2*D*x^8)/8 +
(b^3*C*x^9)/9 + (b^3*D*x^10)/10 + (B*(a + b*x^2)^4)/(8*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2017
```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

rule 2341
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.08

method	result
norman	$\frac{b^3 D x^{10}}{10}+\frac{b^3 C x^9}{9}+\left(\frac{1}{8} B b^3+\frac{3}{8} a b^2 D\right) x^8+\left(\frac{1}{7} b^3 A+\frac{3}{7} a C b^2\right) x^7+\left(\frac{1}{2} a b^2 B+\frac{1}{2} a^2 b D\right) x^6+\left(\frac{3}{5}\right.$
default	$\frac{b^3 D x^{10}}{10}+\frac{b^3 C x^9}{9}+\frac{(B b^3+3 a b^2 D) x^8}{8}+\frac{(b^3 A+3 a C b^2) x^7}{7}+\frac{(3 a b^2 B+3 a^2 b D) x^6}{6}+\frac{(3 a b^2 A+3 a^2 b C) x^5}{5}+\frac{(3 a^2 b}{}$
gosper	$\frac{1}{10} b^3 D x^{10}+\frac{1}{9} b^3 C x^9+\frac{1}{8} b^3 B x^8+\frac{3}{8} a b^2 D x^8+\frac{1}{7} A b^3 x^7+\frac{3}{7} x^7 a C b^2+\frac{1}{2} B a b^2 x^6+\frac{1}{2} a^2 b D x^6$
parallelrisch	$\frac{1}{10} b^3 D x^{10}+\frac{1}{9} b^3 C x^9+\frac{1}{8} b^3 B x^8+\frac{3}{8} a b^2 D x^8+\frac{1}{7} A b^3 x^7+\frac{3}{7} x^7 a C b^2+\frac{1}{2} B a b^2 x^6+\frac{1}{2} a^2 b D x^6$
orering	$\frac{x(252 b^3 D x^9+280 b^3 C x^8+315 b^3 B x^7+945 D a b^2 x^7+360 A b^3 x^6+1080 C a b^2 x^6+1260 B a b^2 x^5+1260 D a^2 b x^5+1512 a A b^2 x^4+}{2520}$

input
```
int((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
1/10*b^3*D*x^10+1/9*b^3*C*x^9+(1/8*B*b^3+3/8*a*b^2*D)*x^8+(1/7*b^3*A+3/7*a
*C*b^2)*x^7+(1/2*a*b^2*B+1/2*a^2*b*D)*x^6+(3/5*a*b^2*A+3/5*a^2*b*C)*x^5+(3
/4*a^2*b*B+1/4*a^3*D)*x^4+(a^2*b*A+1/3*C*a^3)*x^3+1/2*B*a^3*x^2+a^3*A*x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.07

$$\int (a+bx^2)^3 (A+Bx+Cx^2+Dx^3)\,dx = \frac{1}{10}\,Db^3x^{10}+\frac{1}{9}\,Cb^3x^9+\frac{1}{8}\,(3\,Dab^2+Bb^3)x^8+\frac{1}{7}\,(3\,Cab^2+Ab^3)x^7+\frac{1}{2}\,(Da^2b+Bab^2)x^6+\frac{1}{2}\,Ba^3x^2+\frac{3}{5}\,(Ca^2b+Aab^2)x^5+Aa^3x+\frac{1}{4}\,(Da^3+3\,Ba^2b)x^4+\frac{1}{3}\,(Ca^3+3\,Aa^2b)x^3$$

input
```
integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
1/10*D*b^3*x^10 + 1/9*C*b^3*x^9 + 1/8*(3*D*a*b^2 + B*b^3)*x^8 + 1/7*(3*C*a
*b^2 + A*b^3)*x^7 + 1/2*(D*a^2*b + B*a*b^2)*x^6 + 1/2*B*a^3*x^2 + 3/5*(C*a
^2*b + A*a*b^2)*x^5 + A*a^3*x + 1/4*(D*a^3 + 3*B*a^2*b)*x^4 + 1/3*(C*a^3 +
 3*A*a^2*b)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.19

$$\int (a+bx^2)^3 (A+Bx+Cx^2+Dx^3)\ dx = Aa^3x+\frac{Ba^3x^2}{2}+\frac{Cb^3x^9}{9}+\frac{Db^3x^{10}}{10}+x^8\left(\frac{Bb^3}{8}+\frac{3Dab^2}{8}\right)+x^7\left(\frac{Ab^3}{7}+\frac{3Cab^2}{7}\right)+x^6\left(\frac{Bab^2}{2}+\frac{Da^2b}{2}\right)+x^5\cdot\left(\frac{3Aab^2}{5}+\frac{3Ca^2b}{5}\right)+x^4\cdot\left(\frac{3Ba^2b}{4}+\frac{Da^3}{4}\right)+x^3\left(Aa^2b+\frac{Ca^3}{3}\right)$$

input
```
integrate((b*x**2+a)**3*(D*x**3+C*x**2+B*x+A),x)
```

output
```
A*a**3*x + B*a**3*x**2/2 + C*b**3*x**9/9 + D*b**3*x**10/10 + x**8*(B*b**3/
8 + 3*D*a*b**2/8) + x**7*(A*b**3/7 + 3*C*a*b**2/7) + x**6*(B*a*b**2/2 + D*
a**2*b/2) + x**5*(3*A*a*b**2/5 + 3*C*a**2*b/5) + x**4*(3*B*a**2*b/4 + D*a*
*3/4) + x**3*(A*a**2*b + C*a**3/3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.07

$$\int (a+bx^2)^3 (A+Bx+Cx^2+Dx^3)\ dx = \frac{1}{10}\,Db^3x^{10}+\frac{1}{9}\,Cb^3x^9+\frac{1}{8}\,(3\,Dab^2+Bb^3)x^8+\frac{1}{7}\,(3\,Cab^2+Ab^3)x^7+\frac{1}{2}\,(Da^2b+Bab^2)x^6+\frac{1}{2}\,Ba^3x^2+\frac{3}{5}\,(Ca^2b+Aab^2)x^5+Aa^3x+\frac{1}{4}\,(Da^3+3\,Ba^2b)x^4+\frac{1}{3}\,(Ca^3+3\,Aa^2b)x^3$$

input
```
integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
1/10*D*b^3*x^10 + 1/9*C*b^3*x^9 + 1/8*(3*D*a*b^2 + B*b^3)*x^8 + 1/7*(3*C*a
*b^2 + A*b^3)*x^7 + 1/2*(D*a^2*b + B*a*b^2)*x^6 + 1/2*B*a^3*x^2 + 3/5*(C*a
^2*b + A*a*b^2)*x^5 + A*a^3*x + 1/4*(D*a^3 + 3*B*a^2*b)*x^4 + 1/3*(C*a^3 +
 3*A*a^2*b)*x^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.12

$$\int (a+bx^2)^3 (A+Bx+Cx^2+Dx^3)\ dx = \frac{1}{10}Db^3x^{10} + \frac{1}{9}Cb^3x^9 + \frac{3}{8}Dab^2x^8 + \frac{1}{8}Bb^3x^8 + \frac{3}{7}Cab^2x^7 + \frac{1}{7}Ab^3x^7 + \frac{1}{2}Da^2bx^6 + \frac{1}{2}Bab^2x^6 + \frac{3}{5}Ca^2bx^5 + \frac{3}{5}Aab^2x^5 + \frac{1}{4}Da^3x^4 + \frac{3}{4}Ba^2bx^4 + \frac{1}{3}Ca^3x^3 + Aa^2bx^3 + \frac{1}{2}Ba^3x^2 + Aa^3x$$

input
```
integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

output
```
1/10*D*b^3*x^10 + 1/9*C*b^3*x^9 + 3/8*D*a*b^2*x^8 + 1/8*B*b^3*x^8 + 3/7*C*
a*b^2*x^7 + 1/7*A*b^3*x^7 + 1/2*D*a^2*b*x^6 + 1/2*B*a*b^2*x^6 + 3/5*C*a^2*
b*x^5 + 3/5*A*a*b^2*x^5 + 1/4*D*a^3*x^4 + 3/4*B*a^2*b*x^4 + 1/3*C*a^3*x^3
+ A*a^2*b*x^3 + 1/2*B*a^3*x^2 + A*a^3*x
```

Mupad [B] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.12

$$\int (a+bx^2)^3 (A+Bx+Cx^2+Dx^3)\,dx = \frac{B\,a^3\,x^2}{2}+\frac{A\,b^3\,x^7}{7}+\frac{C\,a^3\,x^3}{3}+\frac{B\,b^3\,x^8}{8}+\frac{C\,b^3\,x^9}{9}+\frac{a^3\,x^4\,D}{4}+\frac{b^3\,x^{10}\,D}{10}+A\,a^3\,x+\frac{a^2\,b\,x^6\,D}{2}+\frac{3\,a\,b^2\,x^8\,D}{8}+A\,a^2\,b\,x^3+\frac{3\,A\,a\,b^2\,x^5}{5}+\frac{3\,B\,a^2\,b\,x^4}{4}+\frac{B\,a\,b^2\,x^6}{2}+\frac{3\,C\,a^2\,b\,x^5}{5}+\frac{3\,C\,a\,b^2\,x^7}{7}$$

input
```
int((a + b*x^2)^3*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
(B*a^3*x^2)/2 + (A*b^3*x^7)/7 + (C*a^3*x^3)/3 + (B*b^3*x^8)/8 + (C*b^3*x^9
)/9 + (a^3*x^4*D)/4 + (b^3*x^10*D)/10 + A*a^3*x + (a^2*b*x^6*D)/2 + (3*a*b
^2*x^8*D)/8 + A*a^2*b*x^3 + (3*A*a*b^2*x^5)/5 + (3*B*a^2*b*x^4)/4 + (B*a*b
^2*x^6)/2 + (3*C*a^2*b*x^5)/5 + (3*C*a*b^2*x^7)/7
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.12

$$\int (a+bx^2)^3 (A+Bx+Cx^2+Dx^3)\,dx = \frac{x(252b^3d\,x^9+280b^3c\,x^8+945a\,b^2d\,x^7+315b^4x^7+360a\,b^3x^6+1080a\,b^2c\,x^6+1260a^2bd\,x^5+1260a\,b^3x^5}{25}$$

input
```
int((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x)
```

output
```
(x*(2520*a**4 + 2520*a**3*b*x**2 + 1260*a**3*b*x + 840*a**3*c*x**2 + 630*a
**3*d*x**3 + 1512*a**2*b**2*x**4 + 1890*a**2*b**2*x**3 + 1512*a**2*b*c*x**
4 + 1260*a**2*b*d*x**5 + 360*a*b**3*x**6 + 1260*a*b**3*x**5 + 1080*a*b**2*
c*x**6 + 945*a*b**2*d*x**7 + 315*b**4*x**7 + 280*b**3*c*x**8 + 252*b**3*d*
x**9))/2520
```

3.47 $\int (a+bx^2)^2 (A+Bx+Cx^2+Dx^3)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 99

$$\int (a+bx^2)^2 (A+Bx+Cx^2+Dx^3)\,dx = a^2Ax+\frac{1}{3}a(2Ab+aC)x^3+\frac{1}{4}a^2Dx^4 + \frac{1}{5}b(Ab+2aC)x^5+\frac{1}{3}abDx^6 + \frac{1}{7}b^2Cx^7+\frac{1}{8}b^2Dx^8+\frac{B(a+bx^2)^3}{6b}$$

output
```
a^2*A*x+1/3*a*(2*A*b+C*a)*x^3+1/4*a^2*D*x^4+1/5*b*(A*b+2*C*a)*x^5+1/3*a*b*
D*x^6+1/7*b^2*C*x^7+1/8*b^2*D*x^8+1/6*B*(b*x^2+a)^3/b
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.89

$$\int (a+bx^2)^2 (A+Bx+Cx^2+Dx^3)\,dx = \frac{1}{840}(70a^2x(12A+x(6B+x(4C+3Dx))) + 28abx^3(20A+x(15B+2x(6C+5Dx))) + b^2x^5(168A+5x(28B+3x(8C+7Dx))))$$

input
```
Integrate[(a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(70*a^2*x*(12*A + x*(6*B + x*(4*C + 3*D*x))) + 28*a*b*x^3*(20*A + x*(15*B
+ 2*x*(6*C + 5*D*x))) + b^2*x^5*(168*A + 5*x*(28*B + 3*x*(8*C + 7*D*x))))/
840
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2017, 2341, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^2 \left(A+Bx+Cx^2+Dx^3\right) \, dx$$

$$\Big\downarrow \ 2017$$

$$\int \left(bx^2+a\right)^2 \left(Dx^3+Cx^2+A\right)dx+\frac{B\left(a+bx^2\right)^3}{6b}$$

$$\Big\downarrow \ 2341$$

$$\int \left(b^2Dx^7+b^2Cx^6+2abDx^5+b(Ab+2aC)x^4+a^2Dx^3+a(2Ab+aC)x^2+a^2A\right)dx+\frac{B\left(a+bx^2\right)^3}{6b}$$

$$\Big\downarrow \ 2009$$

$$a^2Ax+\frac{1}{4}a^2Dx^4+\frac{1}{5}bx^5(2aC+Ab)+\frac{1}{3}ax^3(aC+2Ab)+\frac{B\left(a+bx^2\right)^3}{6b}+\frac{1}{3}abDx^6+\frac{1}{7}b^2Cx^7+\frac{1}{8}b^2Dx^8$$

input
```
Int[(a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
a^2*A*x + (a*(2*A*b + a*C)*x^3)/3 + (a^2*D*x^4)/4 + (b*(A*b + 2*a*C)*x^5)/
5 + (a*b*D*x^6)/3 + (b^2*C*x^7)/7 + (b^2*D*x^8)/8 + (B*(a + b*x^2)^3)/(6*b
)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2017
```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

rule 2341
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00

method	result
default	$\frac{b^2Dx^8}{8}+\frac{b^2C\,x^7}{7}+\frac{(B\,b^2+2abD)x^6}{6}+\frac{(b^2A+2Cab)x^5}{5}+\frac{(2abB+Da^2)x^4}{4}+\frac{(2abA+a^2C)x^3}{3}+\frac{B\,a^2x^2}{2}+a^2Ax$
norman	$\frac{b^2Dx^8}{8}+\frac{b^2C\,x^7}{7}+\left(\frac{1}{6}B\,b^2+\frac{1}{3}abD\right)x^6+\left(\frac{1}{5}b^2A+\frac{2}{5}Cab\right)x^5+\left(\frac{1}{2}abB+\frac{1}{4}Da^2\right)x^4+\left(\frac{2}{3}abA+\right.$
gosper	$\frac{1}{8}b^2Dx^8+\frac{1}{7}b^2C\,x^7+\frac{1}{6}b^2B\,x^6+\frac{1}{3}abDx^6+\frac{1}{5}A\,b^2x^5+\frac{2}{5}x^5Cab+\frac{1}{2}Bab\,x^4+\frac{1}{4}a^2Dx^4+\frac{2}{3}aA$
parallelrisch	$\frac{1}{8}b^2Dx^8+\frac{1}{7}b^2C\,x^7+\frac{1}{6}b^2B\,x^6+\frac{1}{3}abDx^6+\frac{1}{5}A\,b^2x^5+\frac{2}{5}x^5Cab+\frac{1}{2}Bab\,x^4+\frac{1}{4}a^2Dx^4+\frac{2}{3}aA$
orering	$\frac{x(105b^2Dx^7+120b^2C\,x^6+140b^2B\,x^5+280Dab\,x^5+168A\,b^2x^4+336Cab\,x^4+420Bab\,x^3+210Da^2x^3+560aAb\,x^2+280C\,a^2x^2}{840}$

input
```
int((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
1/8*b^2*D*x^8+1/7*b^2*C*x^7+1/6*(B*b^2+2*D*a*b)*x^6+1/5*(A*b^2+2*C*a*b)*x^
5+1/4*(2*B*a*b+D*a^2)*x^4+1/3*(2*A*a*b+C*a^2)*x^3+1/2*B*a^2*x^2+a^2*A*x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.99

$$\int \left(a+bx^2\right)^2 \left(A+Bx+Cx^2+Dx^3\right)\,dx = \frac{1}{8}\,Db^2x^8+\frac{1}{7}\,Cb^2x^7+\frac{1}{6}\,\left(2\,Dab+Bb^2\right)x^6+\frac{1}{5}\,\left(2\,Cab+Ab^2\right)x^5+\frac{1}{2}\,Ba^2x^2+\frac{1}{4}\,\left(Da^2+2\,Bab\right)x^4+Aa^2x+\frac{1}{3}\,\left(Ca^2+2\,Aab\right)x^3$$

input
```
integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
1/8*D*b^2*x^8 + 1/7*C*b^2*x^7 + 1/6*(2*D*a*b + B*b^2)*x^6 + 1/5*(2*C*a*b +
 A*b^2)*x^5 + 1/2*B*a^2*x^2 + 1/4*(D*a^2 + 2*B*a*b)*x^4 + A*a^2*x + 1/3*(C
*a^2 + 2*A*a*b)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.08

$$\int \left(a+bx^2\right)^2 \left(A+Bx+Cx^2+Dx^3\right)\,dx = Aa^2x+\frac{Ba^2x^2}{2}+\frac{Cb^2x^7}{7}+\frac{Db^2x^8}{8}+x^6\left(\frac{Bb^2}{6}+\frac{Dab}{3}\right)+x^5\left(\frac{Ab^2}{5}+\frac{2Cab}{5}\right)+x^4\left(\frac{Bab}{2}+\frac{Da^2}{4}\right)+x^3\cdot\left(\frac{2Aab}{3}+\frac{Ca^2}{3}\right)$$

input
```
integrate((b*x**2+a)**2*(D*x**3+C*x**2+B*x+A),x)
```

output
```
A*a**2*x + B*a**2*x**2/2 + C*b**2*x**7/7 + D*b**2*x**8/8 + x**6*(B*b**2/6
+ D*a*b/3) + x**5*(A*b**2/5 + 2*C*a*b/5) + x**4*(B*a*b/2 + D*a**2/4) + x**
3*(2*A*a*b/3 + C*a**2/3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.99

$$\int \left(a+bx^2\right)^2 \left(A+Bx+Cx^2+Dx^3\right)\,dx = \frac{1}{8}\,Db^2x^8+\frac{1}{7}\,Cb^2x^7+\frac{1}{6}\,\left(2\,Dab+Bb^2\right)x^6+\frac{1}{5}\,\left(2\,Cab+Ab^2\right)x^5+\frac{1}{2}\,Ba^2x^2+\frac{1}{4}\,\left(Da^2+2\,Bab\right)x^4+Aa^2x+\frac{1}{3}\,\left(Ca^2+2\,Aab\right)x^3$$

input
```
integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
1/8*D*b^2*x^8 + 1/7*C*b^2*x^7 + 1/6*(2*D*a*b + B*b^2)*x^6 + 1/5*(2*C*a*b +
 A*b^2)*x^5 + 1/2*B*a^2*x^2 + 1/4*(D*a^2 + 2*B*a*b)*x^4 + A*a^2*x + 1/3*(C
*a^2 + 2*A*a*b)*x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.03

$$\int \left(a+bx^2\right)^2 \left(A+Bx+Cx^2+Dx^3\right)\,dx = \frac{1}{8}\,Db^2x^8+\frac{1}{7}\,Cb^2x^7+\frac{1}{3}\,Dabx^6+\frac{1}{6}\,Bb^2x^6+\frac{2}{5}\,Cabx^5+\frac{1}{5}\,Ab^2x^5+\frac{1}{4}\,Da^2x^4+\frac{1}{2}\,Babx^4+\frac{1}{3}\,Ca^2x^3+\frac{2}{3}\,Aabx^3+\frac{1}{2}\,Ba^2x^2+Aa^2x$$

input
```
integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

output
```
1/8*D*b^2*x^8 + 1/7*C*b^2*x^7 + 1/3*D*a*b*x^6 + 1/6*B*b^2*x^6 + 2/5*C*a*b*
x^5 + 1/5*A*b^2*x^5 + 1/4*D*a^2*x^4 + 1/2*B*a*b*x^4 + 1/3*C*a^2*x^3 + 2/3*
A*a*b*x^3 + 1/2*B*a^2*x^2 + A*a^2*x
```

Mupad [B] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.06

$$\int (a+bx^2)^2 (A+Bx+Cx^2+Dx^3)\ dx = \frac{A\,x\,(15\,a^2+10\,a\,b\,x^2+3\,b^2\,x^4)}{15}+\frac{a^2\,x^4\,D}{4}+\frac{b^2\,x^8\,D}{8}+\frac{B\,x^2\,(3\,a^2+3\,a\,b\,x^2+b^2\,x^4)}{6}+\frac{C\,x^3\,(35\,a^2+42\,a\,b\,x^2+15\,b^2\,x^4)}{105}+\frac{a\,b\,x^6\,D}{3}$$

input
```
int((a + b*x^2)^2*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
(A*x*(15*a^2 + 3*b^2*x^4 + 10*a*b*x^2))/15 + (a^2*x^4*D)/4 + (b^2*x^8*D)/8
 + (B*x^2*(3*a^2 + b^2*x^4 + 3*a*b*x^2))/6 + (C*x^3*(35*a^2 + 15*b^2*x^4 +
 42*a*b*x^2))/105 + (a*b*x^6*D)/3
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.04

$$\int (a+bx^2)^2 (A+Bx+Cx^2+Dx^3)\ dx = \frac{x(105b^2d\,x^7+120b^2c\,x^6+280abd\,x^5+140b^3x^5+168a\,b^2x^4+336abc\,x^4+210a^2d\,x^3+420a\,b^2x^3+560a}{840}$$

input
```
int((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x)
```

output
```
(x*(840*a**3 + 560*a**2*b*x**2 + 420*a**2*b*x + 280*a**2*c*x**2 + 210*a**2
*d*x**3 + 168*a*b**2*x**4 + 420*a*b**2*x**3 + 336*a*b*c*x**4 + 280*a*b*d*x
**5 + 140*b**3*x**5 + 120*b**2*c*x**6 + 105*b**2*d*x**7))/840
```

3.48 $\int (a+bx^2)(A+Bx+Cx^2+Dx^3)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 60

$$\int (a+bx^2)(A+Bx+Cx^2+Dx^3)\,dx = aAx+\frac{1}{2}aBx^2+\frac{1}{3}(Ab+aC)x^3+\frac{1}{4}(bB+aD)x^4+\frac{1}{5}bCx^5+\frac{1}{6}bDx^6$$

output
```
a*A*x+1/2*a*B*x^2+1/3*(A*b+C*a)*x^3+1/4*(B*b+D*a)*x^4+1/5*b*C*x^5+1/6*b*D*
x^6
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$\int (a+bx^2)(A+Bx+Cx^2+Dx^3)\,dx = aAx+\frac{1}{2}aBx^2+\frac{1}{3}(Ab+aC)x^3+\frac{1}{4}(bB+aD)x^4+\frac{1}{5}bCx^5+\frac{1}{6}bDx^6$$

input
```
Integrate[(a + b*x^2)*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
a*A*x + (a*B*x^2)/2 + ((A*b + a*C)*x^3)/3 + ((b*B + a*D)*x^4)/4 + (b*C*x^5
)/5 + (b*D*x^6)/6
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2341, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)\left(A+Bx+Cx^2+Dx^3\right)\,dx$$

$$\Big\downarrow \text{2341}$$

$$\int \left(x^2(aC+Ab)+aA+x^3(aD+bB)+aBx+bCx^4+bDx^5\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}x^3(aC+Ab)+aAx+\frac{1}{4}x^4(aD+bB)+\frac{1}{2}aBx^2+\frac{1}{5}bCx^5+\frac{1}{6}bDx^6$$

input
```
Int[(a + b*x^2)*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
a*A*x + (a*B*x^2)/2 + ((A*b + a*C)*x^3)/3 + ((b*B + a*D)*x^4)/4 + (b*C*x^5
)/5 + (b*D*x^6)/6
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2341
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.85

method	result	size
default	$aAx+\frac{Ba\,x^{2}}{2}+\frac{(Ab+Ca)x^{3}}{3}+\frac{(Bb+Da)x^{4}}{4}+\frac{bC\,x^{5}}{5}+\frac{bD x^{6}}{6}$	51
norman	$\frac{bD x^{6}}{6}+\frac{bC\,x^{5}}{5}+\left(\frac{Bb}{4}+\frac{Da}{4}\right)x^{4}+\left(\frac{Ab}{3}+\frac{Ca}{3}\right)x^{3}+\frac{Ba\,x^{2}}{2}+aAx$	53
gosper	$\frac{1}{6}bD x^{6}+\frac{1}{5}bC\,x^{5}+\frac{1}{4}bB\,x^{4}+\frac{1}{4}x^{4}Da+\frac{1}{3}Ab\,x^{3}+\frac{1}{3}x^{3}Ca+\frac{1}{2}Ba\,x^{2}+aAx$	55
parallelrisch	$\frac{1}{6}bD x^{6}+\frac{1}{5}bC\,x^{5}+\frac{1}{4}bB\,x^{4}+\frac{1}{4}x^{4}Da+\frac{1}{3}Ab\,x^{3}+\frac{1}{3}x^{3}Ca+\frac{1}{2}Ba\,x^{2}+aAx$	55
orering	$\frac{x\left(10Db\,x^{5}+12Cb\,x^{4}+15bB\,x^{3}+15Da\,x^{3}+20Ab\,x^{2}+20Ca\,x^{2}+30Bax+60Aa\right)}{60}$	56

input
```
int((b*x^2+a)*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
a*A*x+1/2*B*a*x^2+1/3*(A*b+C*a)*x^3+1/4*(B*b+D*a)*x^4+1/5*b*C*x^5+1/6*b*D*
x^6
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.83

$$\int \left(a+bx^2\right)\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{1}{6}\,Dbx^6+\frac{1}{5}\,Cbx^5+\frac{1}{4}\,(Da+Bb)x^4+\frac{1}{2}\,Bax^2+\frac{1}{3}\,(Ca+Ab)x^3+Aax$$

input
```
integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
1/6*D*b*x^6 + 1/5*C*b*x^5 + 1/4*(D*a + B*b)*x^4 + 1/2*B*a*x^2 + 1/3*(C*a +
 A*b)*x^3 + A*a*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.93

$$\int (a+bx^2)\left(A+Bx+Cx^2+Dx^3\right)dx = Aax+\frac{Bax^2}{2}+\frac{Cbx^5}{5}+\frac{Dbx^6}{6}+x^4\left(\frac{Bb}{4}+\frac{Da}{4}\right)+x^3\left(\frac{Ab}{3}+\frac{Ca}{3}\right)$$

input
```
integrate((b*x**2+a)*(D*x**3+C*x**2+B*x+A),x)
```

output
```
A*a*x + B*a*x**2/2 + C*b*x**5/5 + D*b*x**6/6 + x**4*(B*b/4 + D*a/4) + x**3
*(A*b/3 + C*a/3)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.83

$$\int (a+bx^2)\left(A+Bx+Cx^2+Dx^3\right)dx = \frac{1}{6}Dbx^6+\frac{1}{5}Cbx^5+\frac{1}{4}(Da+Bb)x^4+\frac{1}{2}Bax^2+\frac{1}{3}(Ca+Ab)x^3+Aax$$

input
```
integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
1/6*D*b*x^6 + 1/5*C*b*x^5 + 1/4*(D*a + B*b)*x^4 + 1/2*B*a*x^2 + 1/3*(C*a +
 A*b)*x^3 + A*a*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.90

$$\int (a+bx^2)(A+Bx+Cx^2+Dx^3)\,dx = \frac{1}{6}Dbx^6+\frac{1}{5}Cbx^5+\frac{1}{4}Dax^4+\frac{1}{4}Bbx^4+\frac{1}{3}Cax^3+\frac{1}{3}Abx^3+\frac{1}{2}Bax^2+Aax$$

input
```
integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

output
```
1/6*D*b*x^6 + 1/5*C*b*x^5 + 1/4*D*a*x^4 + 1/4*B*b*x^4 + 1/3*C*a*x^3 + 1/3*
A*b*x^3 + 1/2*B*a*x^2 + A*a*x
```

Mupad [B] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.90

$$\int (a+bx^2)(A+Bx+Cx^2+Dx^3)\,dx = \frac{a\,x^4\,D}{4}+\frac{b\,x^6\,D}{6}+A\,a\,x+\frac{B\,a\,x^2}{2}+\frac{A\,b\,x^3}{3}+\frac{C\,a\,x^3}{3}+\frac{B\,b\,x^4}{4}+\frac{C\,b\,x^5}{5}$$

input
```
int((a + b*x^2)*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
(a*x^4*D)/4 + (b*x^6*D)/6 + A*a*x + (B*a*x^2)/2 + (A*b*x^3)/3 + (C*a*x^3)/
3 + (B*b*x^4)/4 + (C*b*x^5)/5
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.95

$$\int (a + bx^2) (A + Bx + Cx^2 + Dx^3) \, dx = \frac{x(10bd\,x^5 + 12bc\,x^4 + 15ad\,x^3 + 15b^2x^3 + 20ab\,x^2 + 20ac\,x^2 + 30abx + 60a^2)}{60}$$

input
```
int((b*x^2+a)*(D*x^3+C*x^2+B*x+A),x)
```

output
```
(x*(60*a**2 + 20*a*b*x**2 + 30*a*b*x + 20*a*c*x**2 + 15*a*d*x**3 + 15*b**2
*x**3 + 12*b*c*x**4 + 10*b*d*x**5))/60
```

3.49 $\int (A+Bx+Cx^2+Dx^3)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 28

$$\int (A+Bx+Cx^2+Dx^3)\,dx = Ax+\frac{Bx^2}{2}+\frac{Cx^3}{3}+\frac{Dx^4}{4}$$

output
```
A*x+1/2*B*x^2+1/3*C*x^3+1/4*D*x^4
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int (A+Bx+Cx^2+Dx^3)\,dx = Ax+\frac{Bx^2}{2}+\frac{Cx^3}{3}+\frac{Dx^4}{4}$$

input
```
Integrate[A + B*x + C*x^2 + D*x^3,x]
```

output
```
A*x + (B*x^2)/2 + (C*x^3)/3 + (D*x^4)/4
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(A+Bx+Cx^2+Dx^3\right) \, dx$$

$$\Big\downarrow \text{2009}$$

$$Ax+\frac{Bx^2}{2}+\frac{Cx^3}{3}+\frac{Dx^4}{4}$$

input
```
Int[A + B*x + C*x^2 + D*x^3,x]
```

output
```
A*x + (B*x^2)/2 + (C*x^3)/3 + (D*x^4)/4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.82

method	result	size
gosper	$xA+\frac{1}{2}x^2B+\frac{1}{3}C\,x^3+\frac{1}{4}Dx^4$	23
default	$xA+\frac{1}{2}x^2B+\frac{1}{3}C\,x^3+\frac{1}{4}Dx^4$	23
norman	$xA+\frac{1}{2}x^2B+\frac{1}{3}C\,x^3+\frac{1}{4}Dx^4$	23
parallelrisch	$xA+\frac{1}{2}x^2B+\frac{1}{3}C\,x^3+\frac{1}{4}Dx^4$	23
parts	$xA+\frac{1}{2}x^2B+\frac{1}{3}C\,x^3+\frac{1}{4}Dx^4$	23
orering	$\frac{x(3Dx^3+4C\,x^2+6Bx+12A)}{12}$	24

input
```
int(D*x^3+C*x^2+B*x+A,x,method=_RETURNVERBOSE)
```

output
```
x*A+1/2*x^2*B+1/3*C*x^3+1/4*D*x^4
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int \left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{1}{4}\,Dx^4+\frac{1}{3}\,Cx^3+\frac{1}{2}\,Bx^2+Ax$$

input
```
integrate(D*x^3+C*x^2+B*x+A,x, algorithm="fricas")
```

output
```
1/4*D*x^4 + 1/3*C*x^3 + 1/2*B*x^2 + A*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int \left(A+Bx+Cx^2+Dx^3\right) dx = Ax+\frac{Bx^2}{2}+\frac{Cx^3}{3}+\frac{Dx^4}{4}$$

input
```
integrate(D*x**3+C*x**2+B*x+A,x)
```

output
```
A*x + B*x**2/2 + C*x**3/3 + D*x**4/4
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int \left(A+Bx+Cx^2+Dx^3\right) dx = \frac{1}{4}Dx^4+\frac{1}{3}Cx^3+\frac{1}{2}Bx^2+Ax$$

input
```
integrate(D*x^3+C*x^2+B*x+A,x, algorithm="maxima")
```

output
```
1/4*D*x^4 + 1/3*C*x^3 + 1/2*B*x^2 + A*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int \left(A+Bx+Cx^2+Dx^3\right) dx = \frac{1}{4}Dx^4+\frac{1}{3}Cx^3+\frac{1}{2}Bx^2+Ax$$

input
```
integrate(D*x^3+C*x^2+B*x+A,x, algorithm="giac")
```

output
```
1/4*D*x^4 + 1/3*C*x^3 + 1/2*B*x^2 + A*x
```

Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int \left(A + Bx + Cx^2 + Dx^3\right) \, dx = A\,x + \frac{x^4\,D}{4} + \frac{B\,x^2}{2} + \frac{C\,x^3}{3}$$

input
```
int(A + B*x + C*x^2 + x^3*D,x)
```

output
```
A*x + (x^4*D)/4 + (B*x^2)/2 + (C*x^3)/3
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.82

$$\int \left(A + Bx + Cx^2 + Dx^3\right) \, dx = \frac{x(3d\,x^3 + 4c\,x^2 + 6bx + 12a)}{12}$$

input
```
int(D*x^3+C*x^2+B*x+A,x)
```

output
```
(x*(12*a + 6*b*x + 4*c*x**2 + 3*d*x**3))/12
```

3.50 $\int \frac{A+Bx+Cx^2+Dx^3}{a+bx^2} dx$

Optimal result

Integrand size = 25, antiderivative size = 73

$$\int \frac{A+Bx+Cx^2+Dx^3}{a+bx^2} dx = \frac{Cx}{b} + \frac{Dx^2}{2b} + \frac{(Ab-aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}} + \frac{(bB-aD)\log(a+bx^2)}{2b^2}$$

output
```
C*x/b+1/2*D*x^2/b+(A*b-C*a)*arctan(b^(1/2)*x/a^(1/2))/a^(1/2)/b^(3/2)+1/2*
(B*b-D*a)*ln(b*x^2+a)/b^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.93

$$\int \frac{A+Bx+Cx^2+Dx^3}{a+bx^2} dx = \frac{bx(2C+Dx) + \frac{2\sqrt{b}(Ab-aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}} + (bB-aD)\log(a+bx^2)}{2b^2}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/(a + b*x^2),x]
```

output
```
(b*x*(2*C + D*x) + (2*Sqrt[b]*(A*b - a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqr
t[a] + (b*B - a*D)*Log[a + b*x^2])/(2*b^2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2341, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{a+bx^2}\,dx$$

$$\Big\downarrow \text{2341}$$

$$\int \left(\frac{x(bB-aD)-aC+Ab}{b\,(a+bx^2)}+\frac{C}{b}+\frac{Dx}{b}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{(Ab-aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}+\frac{(bB-aD)\log\left(a+bx^2\right)}{2b^2}+\frac{Cx}{b}+\frac{Dx^2}{2b}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/(a + b*x^2),x]
```

output
```
(C*x)/b + (D*x^2)/(2*b) + ((A*b - a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[
a]*b^(3/2)) + ((b*B - a*D)*Log[a + b*x^2])/(2*b^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2341
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.89

method	result	size
default	$\frac{\frac{1}{2}Dx^2+Cx}{b}+\frac{\frac{(Bb-Da)\ln\left(b\,x^2+a\right)}{2b}+\frac{(Ab-Ca)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}}{b}$	65

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/2*D*x^2+C*x)+1/b*(1/2*(B*b-D*a)/b*ln(b*x^2+a)+(A*b-C*a)/(a*b)^(1/2)
*arctan(b*x/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 157, normalized size of antiderivative = 2.15

$$\int \frac{A+Bx+Cx^2+Dx^3}{a+bx^2}\,dx$$
$$=\left[\frac{Dabx^2+2\,Cabx+(Ca-Ab)\sqrt{-ab}\log\left(\frac{bx^2-2\,\sqrt{-ab}x-a}{bx^2+a}\right)-(Da^2-Bab)\log\left(bx^2+a\right)}{2\,ab^2},\ \frac{Dabx^2+2\,Cab}{}\right.$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="fricas")
```

output
```
[1/2*(D*a*b*x^2 + 2*C*a*b*x + (C*a - A*b)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-
a*b)*x - a)/(b*x^2 + a)) - (D*a^2 - B*a*b)*log(b*x^2 + a))/(a*b^2), 1/2*(D
*a*b*x^2 + 2*C*a*b*x - 2*(C*a - A*b)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) - (D*
a^2 - B*a*b)*log(b*x^2 + a))/(a*b^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 219 vs. $2(65) = 130$.

Time = 0.40 (sec) , antiderivative size = 219, normalized size of antiderivative = 3.00

$$\int \frac{A + Bx + Cx^2 + Dx^3}{a + bx^2}\,dx$$
$$= \frac{Cx}{b} + \frac{Dx^2}{2b} + \left(-\frac{-Bb + Da}{2b^2} - \frac{\sqrt{-ab^5}(-Ab + Ca)}{2ab^4}\right)\log\left(x + \frac{Bab - Da^2 - 2ab^2\left(-\frac{-Bb+Da}{2b^2} - \frac{\sqrt{-ab^5}(-Ab+Ca)}{2ab^4}\right)}{-Ab^2 + Cab}\right)$$
$$+ \left(-\frac{-Bb + Da}{2b^2} + \frac{\sqrt{-ab^5}(-Ab + Ca)}{2ab^4}\right)\log\left(x + \frac{Bab - Da^2 - 2ab^2\left(-\frac{-Bb+Da}{2b^2} + \frac{\sqrt{-ab^5}(-Ab+Ca)}{2ab^4}\right)}{-Ab^2 + Cab}\right)$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(b*x**2+a),x)
```

output
```
C*x/b + D*x**2/(2*b) + (-(-B*b + D*a)/(2*b**2) - sqrt(-a*b**5)*(-A*b + C*a
)/(2*a*b**4))*log(x + (B*a*b - D*a**2 - 2*a*b**2*(-(-B*b + D*a)/(2*b**2) -
 sqrt(-a*b**5)*(-A*b + C*a)/(2*a*b**4)))/(-A*b**2 + C*a*b)) + (-(-B*b + D*
a)/(2*b**2) + sqrt(-a*b**5)*(-A*b + C*a)/(2*a*b**4))*log(x + (B*a*b - D*a*
*2 - 2*a*b**2*(-(-B*b + D*a)/(2*b**2) + sqrt(-a*b**5)*(-A*b + C*a)/(2*a*b*
*4)))/(-A*b**2 + C*a*b))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.88

$$\int \frac{A+Bx+Cx^2+Dx^3}{a+bx^2}\,dx = -\frac{(Ca-Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b} + \frac{Dx^2+2\,Cx}{2\,b} - \frac{(Da-Bb)\log\left(bx^2+a\right)}{2\,b^2}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="maxima")
```

output
```
-(C*a - A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b) + 1/2*(D*x^2 + 2*C*x)/b -
 1/2*(D*a - B*b)*log(b*x^2 + a)/b^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.90

$$\int \frac{A+Bx+Cx^2+Dx^3}{a+bx^2}\,dx = -\frac{(Ca-Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b} - \frac{(Da-Bb)\log\left(bx^2+a\right)}{2\,b^2} + \frac{Dbx^2+2\,Cbx}{2\,b^2}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="giac")
```

output
```
-(C*a - A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b) - 1/2*(D*a - B*b)*log(b*x
^2 + a)/b^2 + 1/2*(D*b*x^2 + 2*C*b*x)/b^2
```

Mupad [B] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.08

$$\int \frac{A+Bx+Cx^2+Dx^3}{a+bx^2}\,dx = \frac{B\,\ln\left(b\,x^2+a\right)}{2\,b}-\frac{\left(a\,\ln\left(b\,x^2+a\right)-b\,x^2\right)\,D}{2\,b^2}+\frac{C\,x}{b}+\frac{A\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}-\frac{C\,\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{3/2}}$$

input
```
int((A + B*x + C*x^2 + x^3*D)/(a + b*x^2),x)
```

output
```
(B*log(a + b*x^2))/(2*b) - ((a*log(a + b*x^2) - b*x^2)*D)/(2*b^2) + (C*x)/
b + (A*atan((b^(1/2)*x)/a^(1/2)))/(a^(1/2)*b^(1/2)) - (C*a^(1/2)*atan((b^(
1/2)*x)/a^(1/2)))/b^(3/2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.08

$$\int \frac{A+Bx+Cx^2+Dx^3}{a+bx^2}\,dx = \frac{2\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)b-2\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)c-\log(b\,x^2+a)\,ad+\log(b\,x^2+a)\,b^2+2bcx+bd\,x^2}{2b^2}$$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x^2+a),x)
```

output
```
(2*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b - 2*sqrt(b)*sqrt(a)*ata
n((b*x)/(sqrt(b)*sqrt(a)))*c - log(a + b*x**2)*a*d + log(a + b*x**2)*b**2
+ 2*b*c*x + b*d*x**2)/(2*b**2)
```

3.51 $\int \frac{A+Bx+Cx^2+Dx^3}{\left(a+bx^2\right)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 93

$$\int \frac{A+Bx+Cx^2+Dx^3}{\left(a+bx^2\right)^2}\,dx=-\frac{a(bB-aD)-b(Ab-aC)x}{2ab^2\left(a+bx^2\right)}+\frac{(Ab+aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}+\frac{D\log\left(a+bx^2\right)}{2b^2}$$

output
```
-1/2*(a*(B*b-D*a)-b*(A*b-C*a)*x)/a/b^2/(b*x^2+a)+1/2*(A*b+C*a)*arctan(b^(1
/2)*x/a^(1/2))/a^(3/2)/b^(3/2)+1/2*D*ln(b*x^2+a)/b^2
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.89

$$\int \frac{A+Bx+Cx^2+Dx^3}{\left(a+bx^2\right)^2}\,dx=\frac{\frac{a^2D+Ab^2x-ab(B+Cx)}{a(a+bx^2)}+\frac{\sqrt{b}(Ab+aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}}+D\log\left(a+bx^2\right)}{2b^2}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/(a + b*x^2)^2,x]
```

output
```
((a^2*D + A*b^2*x - a*b*(B + C*x))/(a*(a + b*x^2)) + (Sqrt[b]*(A*b + a*C)*
ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(3/2) + D*Log[a + b*x^2])/(2*b^2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2345, 25, 27, 452, 218, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^2}dx$$

$$\Big\downarrow \text{2345}$$

$$-\frac{\int -\frac{Ab+aC+2aDx}{b(bx^2+a)}dx}{2a}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{2ab\left(a+bx^2\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{Ab+aC+2aDx}{b(bx^2+a)}dx}{2a}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{2ab\left(a+bx^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{Ab+aC+2aDx}{bx^2+a}dx}{2ab}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{2ab\left(a+bx^2\right)}$$

$$\Big\downarrow \text{452}$$

$$\frac{(aC+Ab)\int \frac{1}{bx^2+a}dx+2aD\int \frac{x}{bx^2+a}dx}{2ab}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{2ab\left(a+bx^2\right)}$$

$$\Big\downarrow \text{218}$$

$$\frac{2aD\int \frac{x}{bx^2+a}dx+\frac{(aC+Ab)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}}{2ab}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{2ab\left(a+bx^2\right)}$$

$$\Big\downarrow \text{240}$$

$$\frac{\frac{(aC+Ab)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}+\frac{aD\log(a+bx^2)}{b}}{2ab}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{2ab\,(a+bx^2)}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/(a + b*x^2)^2,x]
```

output
```
-1/2*(a*(B - (a*D)/b) - (A*b - a*C)*x)/(a*b*(a + b*x^2)) + (((A*b + a*C)*A
rcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*Sqrt[b]) + (a*D*Log[a + b*x^2])/b)/(2
*a*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 452
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[c   Int[1/
(a + b*x^2), x], x] + Simp[d   Int[x/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c^2 + a*d^2, 0]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{\frac{(Ab-Ca)x}{2ab}-\frac{Bb-Da}{2b^2}}{b\,x^2+a}+\frac{\frac{Da\ln\left(b\,x^2+a\right)}{b}+\frac{(Ab+Ca)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}}{2ab}$	88

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/2*(A*b-C*a)/a/b*x-1/2*(B*b-D*a)/b^2)/(b*x^2+a)+1/2/a/b*(D*a/b*ln(b*x^2+
a)+(A*b+C*a)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 257, normalized size of antiderivative = 2.76

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^2}\,dx$$
$$= \left[\frac{2\,Da^3-2\,Ba^2b-(Ca^2+Aab+(Cab+Ab^2)x^2)\sqrt{-ab}\log\left(\frac{bx^2-2\,\sqrt{-ab}x-a}{bx^2+a}\right)-2\,(Ca^2b-Aab^2)x+2\,(I}{4\,(a^2b^3x^2+a^3b^2)}\right.$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[1/4*(2*D*a^3 - 2*B*a^2*b - (C*a^2 + A*a*b + (C*a*b + A*b^2)*x^2)*sqrt(-a*
b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) - 2*(C*a^2*b - A*a*b^2)*x
 + 2*(D*a^2*b*x^2 + D*a^3)*log(b*x^2 + a))/(a^2*b^3*x^2 + a^3*b^2), 1/2*(D
*a^3 - B*a^2*b + (C*a^2 + A*a*b + (C*a*b + A*b^2)*x^2)*sqrt(a*b)*arctan(sq
rt(a*b)*x/a) - (C*a^2*b - A*a*b^2)*x + (D*a^2*b*x^2 + D*a^3)*log(b*x^2 + a
))/(a^2*b^3*x^2 + a^3*b^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 233 vs. $2(82) = 164$.

Time = 0.90 (sec) , antiderivative size = 233, normalized size of antiderivative = 2.51

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(a + bx^2)^2} \, dx$$
$$= \left(\frac{D}{2b^2} - \frac{\sqrt{-a^3b^5}(Ab + Ca)}{4a^3b^4}\right) \log\left(x + \frac{-2Da^2 + 4a^2b^2\left(\frac{D}{2b^2} - \frac{\sqrt{-a^3b^5}(Ab+Ca)}{4a^3b^4}\right)}{Ab^2 + Cab}\right)$$
$$+ \left(\frac{D}{2b^2} + \frac{\sqrt{-a^3b^5}(Ab + Ca)}{4a^3b^4}\right) \log\left(x + \frac{-2Da^2 + 4a^2b^2\left(\frac{D}{2b^2} + \frac{\sqrt{-a^3b^5}(Ab+Ca)}{4a^3b^4}\right)}{Ab^2 + Cab}\right)$$
$$+ \frac{-Bab + Da^2 + x(Ab^2 - Cab)}{2a^2b^2 + 2ab^3x^2}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(b*x**2+a)**2,x)
```

output
```
(D/(2*b**2) - sqrt(-a**3*b**5)*(A*b + C*a)/(4*a**3*b**4))*log(x + (-2*D*a*
*2 + 4*a**2*b**2*(D/(2*b**2) - sqrt(-a**3*b**5)*(A*b + C*a)/(4*a**3*b**4))
)/(A*b**2 + C*a*b)) + (D/(2*b**2) + sqrt(-a**3*b**5)*(A*b + C*a)/(4*a**3*b
**4))*log(x + (-2*D*a**2 + 4*a**2*b**2*(D/(2*b**2) + sqrt(-a**3*b**5)*(A*b
 + C*a)/(4*a**3*b**4)))/(A*b**2 + C*a*b)) + (-B*a*b + D*a**2 + x*(A*b**2 -
 C*a*b))/(2*a**2*b**2 + 2*a*b**3*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.96

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^2}\,dx = \frac{Da^2-Bab-(Cab-Ab^2)x}{2\,(ab^3x^2+a^2b^2)} + \frac{D\log(bx^2+a)}{2\,b^2} + \frac{(Ca+Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}ab}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*(D*a^2 - B*a*b - (C*a*b - A*b^2)*x)/(a*b^3*x^2 + a^2*b^2) + 1/2*D*log(
b*x^2 + a)/b^2 + 1/2*(C*a + A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.95

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^2}\,dx = \frac{D\log(bx^2+a)}{2\,b^2} + \frac{(Ca+Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}ab} - \frac{(Ca-Ab)x-\frac{Da^2-Bab}{b}}{2\,(bx^2+a)ab}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*D*log(b*x^2 + a)/b^2 + 1/2*(C*a + A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b
)*a*b) - 1/2*((C*a - A*b)*x - (D*a^2 - B*a*b)/b)/((b*x^2 + a)*a*b)
```

Mupad [B] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.18

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^2}\,dx = \frac{\left(\ln\left(b\,x^2+a\right)+\frac{a}{b\,x^2+a}\right)\,D}{2\,b^2}-\frac{B}{2\,b\,\left(b\,x^2+a\right)}+\frac{A\,x}{2\,a\,\left(b\,x^2+a\right)}-\frac{C\,x}{2\,b\,\left(b\,x^2+a\right)}+\frac{A\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,a^{3/2}\,\sqrt{b}}+\frac{C\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,\sqrt{a}\,b^{3/2}}$$

input
```
int((A + B*x + C*x^2 + x^3*D)/(a + b*x^2)^2,x)
```

output
```
((log(a + b*x^2) + a/(a + b*x^2))*D)/(2*b^2) - B/(2*b*(a + b*x^2)) + (A*x)
/(2*a*(a + b*x^2)) - (C*x)/(2*b*(a + b*x^2)) + (A*atan((b^(1/2)*x)/a^(1/2)
))/(2*a^(3/2)*b^(1/2)) + (C*atan((b^(1/2)*x)/a^(1/2)))/(2*a^(1/2)*b^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.68

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^2}\,dx = \frac{\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)ab+\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)ac+\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)b^2x^2+\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)bc\,x^2}{2a\,b^2\left(b\,x^2+a\right)}$$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x)
```

output
```
(sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*b + sqrt(b)*sqrt(a)*atan(
(b*x)/(sqrt(b)*sqrt(a)))*a*c + sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)
))*b**2*x**2 + sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b*c*x**2 + lo
g(a + b*x**2)*a**2*d + log(a + b*x**2)*a*b*d*x**2 + a*b**2*x - a*b*c*x - a
*b*d*x**2 + b**3*x**2)/(2*a*b**2*(a + b*x**2))
```

3.52 $\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^3} dx$

Optimal result

Integrand size = 25, antiderivative size = 116

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^3} dx = -\frac{a(bB-aD)-b(Ab-aC)x}{4ab^2(a+bx^2)^2} - \frac{4a^2D-b(3Ab+aC)x}{8a^2b^2(a+bx^2)} + \frac{(3Ab+aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}}$$

output
```
-1/4*(a*(B*b-D*a)-b*(A*b-C*a)*x)/a/b^2/(b*x^2+a)^2-1/8*(4*a^2*D-b*(3*A*b+C
*a)*x)/a^2/b^2/(b*x^2+a)+1/8*(3*A*b+C*a)*arctan(b^(1/2)*x/a^(1/2))/a^(5/2)
/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.90

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^3} dx = \frac{\frac{\sqrt{a}(-2a^3D+3Ab^3x^3+ab^2x(5A+Cx^2)-a^2b(2B+x(C+4Dx)))}{(a+bx^2)^2}+\sqrt{b}(3Ab+aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^2}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/(a + b*x^2)^3,x]
```

output
```
((Sqrt[a]*(-2*a^3*D + 3*A*b^3*x^3 + a*b^2*x*(5*A + C*x^2) - a^2*b*(2*B + x
*(C + 4*D*x))))/(a + b*x^2)^2 + Sqrt[b]*(3*A*b + a*C)*ArcTan[(Sqrt[b]*x)/S
qrt[a]])/(8*a^(5/2)*b^2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2345, 25, 27, 454, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^3}dx$$

$$\Big\downarrow \text{ 2345}$$

$$-\frac{\int -\frac{b\left(3A+\frac{aC}{b}\right)+4aDx}{b(bx^2+a)^2}dx}{4a}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{4ab\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{3Ab+aC+4aDx}{b(bx^2+a)^2}dx}{4a}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{4ab\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{3Ab+aC+4aDx}{(bx^2+a)^2}dx}{4ab}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{4ab\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{ 454}$$

$$\frac{\frac{(aC+3Ab)\int \frac{1}{bx^2+a}dx}{2a}-\frac{4a^2D-bx(aC+3Ab)}{2ab(a+bx^2)}}{4ab}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{4ab\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{\frac{(aC+3Ab)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}-\frac{4a^2D-bx(aC+3Ab)}{2ab(a+bx^2)}}{4ab}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{4ab\left(a+bx^2\right)^2}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/(a + b*x^2)^3,x]
```

output
```
-1/4*(a*(B - (a*D)/b) - (A*b - a*C)*x)/(a*b*(a + b*x^2)^2) + (-1/2*(4*a^2*
D - b*(3*A*b + a*C)*x)/(a*b*(a + b*x^2)) + ((3*A*b + a*C)*ArcTan[(Sqrt[b]*
x)/Sqrt[a]])/(2*a^(3/2)*Sqrt[b]))/(4*a*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 454
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*d
 - b*c*x)/(2*a*b*(p + 1)))*(a + b*x^2)^(p + 1), x] + Simp[c*((2*p + 3)/(2*a
*(p + 1)))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d}, x] && L
tQ[p, -1] && NeQ[p, -3/2]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.84

method	result	size
default	$\frac{\frac{(3Ab+Ca)x^3}{8a^2}-\frac{Dx^2}{2b}+\frac{(5Ab-Ca)x}{8ab}-\frac{Bb+Da}{4b^2}}{(b\,x^2+a)^2}+\frac{(3Ab+Ca)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8a^2b\sqrt{ab}}$	98

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/8*(3*A*b+C*a)/a^2*x^3-1/2*D*x^2/b+1/8*(5*A*b-C*a)/a/b*x-1/4*(B*b+D*a)/b
^2)/(b*x^2+a)^2+1/8*(3*A*b+C*a)/a^2/b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 346, normalized size of antiderivative = 2.98

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^3}\,dx$$
$$= \left[-\frac{8\,Da^3bx^2+4\,Da^4+4\,Ba^3b-2\,(Ca^2b^2+3\,Aab^3)x^3+((Cab^2+3\,Ab^3)x^4+Ca^3+3\,Aa^2b+2\,(Ca^2b}{16\,(a^3b^4x^4+2\,a^4b^3x^2+a^5b^2)}\right.$$
$$-\frac{4\,Da^3bx^2+2\,Da^4+2\,Ba^3b-(Ca^2b^2+3\,Aab^3)x^3-((Cab^2+3\,Ab^3)x^4+Ca^3+3\,Aa^2b+2\,(Ca^2b+}{8\,(a^3b^4x^4+2\,a^4b^3x^2+a^5b^2)}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
[-1/16*(8*D*a^3*b*x^2 + 4*D*a^4 + 4*B*a^3*b - 2*(C*a^2*b^2 + 3*A*a*b^3)*x^
3 + ((C*a*b^2 + 3*A*b^3)*x^4 + C*a^3 + 3*A*a^2*b + 2*(C*a^2*b + 3*A*a*b^2)
*x^2)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 2*(C*a^3*
b - 5*A*a^2*b^2)*x)/(a^3*b^4*x^4 + 2*a^4*b^3*x^2 + a^5*b^2), -1/8*(4*D*a^3
*b*x^2 + 2*D*a^4 + 2*B*a^3*b - (C*a^2*b^2 + 3*A*a*b^3)*x^3 - ((C*a*b^2 + 3
*A*b^3)*x^4 + C*a^3 + 3*A*a^2*b + 2*(C*a^2*b + 3*A*a*b^2)*x^2)*sqrt(a*b)*a
rctan(sqrt(a*b)*x/a) + (C*a^3*b - 5*A*a^2*b^2)*x)/(a^3*b^4*x^4 + 2*a^4*b^3
*x^2 + a^5*b^2)]
```

Sympy [A] (verification not implemented)

Time = 3.06 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.59

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^3}\,dx = -\frac{\sqrt{-\frac{1}{a^5b^3}}\cdot(3Ab+Ca)\log\left(-a^3b\sqrt{-\frac{1}{a^5b^3}}+x\right)}{16} + \frac{\sqrt{-\frac{1}{a^5b^3}}\cdot(3Ab+Ca)\log\left(a^3b\sqrt{-\frac{1}{a^5b^3}}+x\right)}{16} + \frac{-2Ba^2b-2Da^3-4Da^2bx^2+x^3\cdot(3Ab^3+Cab^2)+x(5Aab^2-Ca^2b)}{8a^4b^2+16a^3b^3x^2+8a^2b^4x^4}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(b*x**2+a)**3,x)
```

output
```
-sqrt(-1/(a**5*b**3))*(3*A*b + C*a)*log(-a**3*b*sqrt(-1/(a**5*b**3)) + x)/
16 + sqrt(-1/(a**5*b**3))*(3*A*b + C*a)*log(a**3*b*sqrt(-1/(a**5*b**3)) +
x)/16 + (-2*B*a**2*b - 2*D*a**3 - 4*D*a**2*b*x**2 + x**3*(3*A*b**3 + C*a*b
**2) + x*(5*A*a*b**2 - C*a**2*b))/(8*a**4*b**2 + 16*a**3*b**3*x**2 + 8*a**
2*b**4*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.05

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^3}\,dx = -\frac{4\,Da^2bx^2+2\,Da^3+2\,Ba^2b-(Cab^2+3\,Ab^3)x^3+(Ca^2b-5\,Aab^2)x}{8\,(a^2b^4x^4+2\,a^3b^3x^2+a^4b^2)} + \frac{(Ca+3\,Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^2b}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
-1/8*(4*D*a^2*b*x^2 + 2*D*a^3 + 2*B*a^2*b - (C*a*b^2 + 3*A*b^3)*x^3 + (C*a
^2*b - 5*A*a*b^2)*x)/(a^2*b^4*x^4 + 2*a^3*b^3*x^2 + a^4*b^2) + 1/8*(C*a +
3*A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.91

$$\int \frac{A+Bx+Cx^2+Dx^3}{\left(a+bx^2\right)^3}\,dx = \frac{(Ca+3\,Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^2b} + \frac{Cab^2x^3+3\,Ab^3x^3-4\,Da^2bx^2-Ca^2bx+5\,Aab^2x-2\,Da^3-2\,Ba^2b}{8\,\left(bx^2+a\right)^2a^2b^2}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
1/8*(C*a + 3*A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b) + 1/8*(C*a*b^2*x
^3 + 3*A*b^3*x^3 - 4*D*a^2*b*x^2 - C*a^2*b*x + 5*A*a*b^2*x - 2*D*a^3 - 2*B
*a^2*b)/((b*x^2 + a)^2*a^2*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.41

$$\int \frac{A+Bx+Cx^2+Dx^3}{\left(a+bx^2\right)^3}\,dx = \frac{\frac{C\,x^3}{8\,a}-\frac{C\,x}{8\,b}}{a^2+2\,a\,b\,x^2+b^2\,x^4} + \frac{\frac{5\,A\,x}{8\,a}+\frac{3\,A\,b\,x^3}{8\,a^2}}{a^2+2\,a\,b\,x^2+b^2\,x^4} - \frac{B}{4\,b\,\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)} - \frac{\left(2\,b\,x^2+a\right)\,D}{4\,b^2\,\left(b\,x^2+a\right)^2} + \frac{3\,A\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,a^{5/2}\,\sqrt{b}} + \frac{C\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,a^{3/2}\,b^{3/2}}$$

input
```
int((A + B*x + C*x^2 + x^3*D)/(a + b*x^2)^3,x)
```

output
```
((C*x^3)/(8*a) - (C*x)/(8*b))/(a^2 + b^2*x^4 + 2*a*b*x^2) + ((5*A*x)/(8*a)
 + (3*A*b*x^3)/(8*a^2))/(a^2 + b^2*x^4 + 2*a*b*x^2) - B/(4*b*(a^2 + b^2*x^
4 + 2*a*b*x^2)) - ((a + 2*b*x^2)*D)/(4*b^2*(a + b*x^2)^2) + (3*A*atan((b^(
1/2)*x)/a^(1/2)))/(8*a^(5/2)*b^(1/2)) + (C*atan((b^(1/2)*x)/a^(1/2)))/(8*a
^(3/2)*b^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.91

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^3}\,dx$$
$$= \frac{3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^2b+\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^2c+6\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a\,b^2x^2+2\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right.}{8a^2}$$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x)
```

output
```
(3*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2*b + sqrt(b)*sqrt(a)*
atan((b*x)/(sqrt(b)*sqrt(a)))*a**2*c + 6*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(
b)*sqrt(a)))*a*b**2*x**2 + 2*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))
*a*b*c*x**2 + 3*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**3*x**4 +
sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**2*c*x**4 + 5*a**2*b**2*x
- 2*a**2*b**2 - a**2*b*c*x + 3*a*b**3*x**3 + a*b**2*c*x**3 + 2*a*b**2*d*x*
*4)/(8*a**2*b**2*(a**2 + 2*a*b*x**2 + b**2*x**4))
```

3.53 $\int \frac{-x+4x^3}{(5+x^2)^2} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 20

$$\int \frac{-x+4x^3}{(5+x^2)^2} \, dx = \frac{21}{2(5+x^2)} + 2\log(5+x^2)$$

output
```
21/(2*x^2+10)+2*ln(x^2+5)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{-x+4x^3}{(5+x^2)^2} \, dx = \frac{21}{2(5+x^2)} + 2\log(5+x^2)$$

input
```
Integrate[(-x + 4*x^3)/(5 + x^2)^2,x]
```

output
```
21/(2*(5 + x^2)) + 2*Log[5 + x^2]
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {2027, 353, 25, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4x^3-x}{(x^2+5)^2}dx$$

$$\Big\downarrow \text{2027}$$

$$\int \frac{x(4x^2-1)}{(x^2+5)^2}dx$$

$$\Big\downarrow \text{353}$$

$$\frac{1}{2}\int -\frac{1-4x^2}{(x^2+5)^2}dx^2$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{2}\int \frac{1-4x^2}{(x^2+5)^2}dx^2$$

$$\Big\downarrow \text{49}$$

$$-\frac{1}{2}\int \left(\frac{21}{(x^2+5)^2}-\frac{4}{x^2+5}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{21}{x^2+5}+4\log\left(x^2+5\right)\right)$$

input
```
Int[(-x + 4*x^3)/(5 + x^2)^2,x]
```

output
```
(21/(5 + x^2) + 4*Log[5 + x^2])/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

method	result	size
default	$2\ln\left(x^2+5\right)+\frac{21}{2(x^2+5)}$	19
norman	$2\ln\left(x^2+5\right)+\frac{21}{2(x^2+5)}$	19
risch	$2\ln\left(x^2+5\right)+\frac{21}{2(x^2+5)}$	19
meijerg	$-\frac{21x^2}{50\left(1+\frac{x^2}{5}\right)}+2\ln\left(1+\frac{x^2}{5}\right)$	26
parallelrisch	$\frac{4\ln(x^2+5)x^2+21+20\ln(x^2+5)}{2x^2+10}$	31

input
```
int((4*x^3-x)/(x^2+5)^2,x,method=_RETURNVERBOSE)
```

output
```
2*ln(x^2+5)+21/2/(x^2+5)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int \frac{-x+4x^3}{(5+x^2)^2}\,dx = \frac{4\left(x^2+5\right)\log\left(x^2+5\right)+21}{2\left(x^2+5\right)}$$

input
```
integrate((4*x^3-x)/(x^2+5)^2,x, algorithm="fricas")
```

output
```
1/2*(4*(x^2 + 5)*log(x^2 + 5) + 21)/(x^2 + 5)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

$$\int \frac{-x+4x^3}{(5+x^2)^2}\,dx = 2\log\left(x^2+5\right)+\frac{21}{2x^2+10}$$

input
```
integrate((4*x**3-x)/(x**2+5)**2,x)
```

output
```
2*log(x**2 + 5) + 21/(2*x**2 + 10)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{-x+4x^3}{(5+x^2)^2}\,dx = \frac{21}{2\left(x^2+5\right)}+2\,\log\left(x^2+5\right)$$

input
```
integrate((4*x^3-x)/(x^2+5)^2,x, algorithm="maxima")
```

output
```
21/2/(x^2 + 5) + 2*log(x^2 + 5)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.25

$$\int \frac{-x+4x^3}{(5+x^2)^2}\,dx = -\frac{4\,x^2-1}{2\,(x^2+5)} + 2\,\log\left(x^2+5\right)$$

input
```
integrate((4*x^3-x)/(x^2+5)^2,x, algorithm="giac")
```

output
```
-1/2*(4*x^2 - 1)/(x^2 + 5) + 2*log(x^2 + 5)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{-x+4x^3}{(5+x^2)^2}\,dx = 2\,\ln\left(x^2+5\right) + \frac{21}{2\,(x^2+5)}$$

input
```
int(-(x - 4*x^3)/(x^2 + 5)^2,x)
```

output
```
2*log(x^2 + 5) + 21/(2*(x^2 + 5))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.75

$$\int \frac{-x+4x^3}{(5+x^2)^2}\,dx = \frac{20\log(x^2+5)\,x^2+100\log(x^2+5)-21x^2}{10x^2+50}$$

input
```
int((4*x^3-x)/(x^2+5)^2,x)
```

output
```
(20*log(x**2 + 5)*x**2 + 100*log(x**2 + 5) - 21*x**2)/(10*(x**2 + 5))
```

3.54 $\int (a+bx^2)^{3/2} (A+Bx+Cx^2+Dx^3) \, dx$

Optimal result

Integrand size = 27, antiderivative size = 163

$$\int (a+bx^2)^{3/2} (A+Bx+Cx^2+Dx^3) \, dx = \frac{a(6Ab-aC)x\sqrt{a+bx^2}}{16b} + \frac{(6Ab-aC)x(a+bx^2)^{3/2}}{24b} + \frac{(bB-aD)(a+bx^2)^{5/2}}{5b^2} + \frac{Cx(a+bx^2)^{5/2}}{6b} + \frac{D(a+bx^2)^{7/2}}{7b^2} + \frac{a^2(6Ab-aC)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{3/2}}$$

output
```
1/16*a*(6*A*b-C*a)*x*(b*x^2+a)^(1/2)/b+1/24*(6*A*b-C*a)*x*(b*x^2+a)^(3/2)/
b+1/5*(B*b-D*a)*(b*x^2+a)^(5/2)/b^2+1/6*C*x*(b*x^2+a)^(5/2)/b+1/7*D*(b*x^2
+a)^(7/2)/b^2+1/16*a^2*(6*A*b-C*a)*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(3
/2)
```

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.87

$$\int \left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{\sqrt{a+bx^2}(-96a^3D+4b^3x^3(105A+84Bx+70Cx^2+60Dx^3)+3a^2b(112B+x(35C+16Dx}{}$$

input
```
Integrate[(a + b*x^2)^(3/2)*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(Sqrt[a + b*x^2]*(-96*a^3*D + 4*b^3*x^3*(105*A + 84*B*x + 70*C*x^2 + 60*D*
x^3) + 3*a^2*b*(112*B + x*(35*C + 16*D*x)) + 2*a*b^2*x*(525*A + x*(336*B +
 245*C*x + 192*D*x^2))) + 105*a^2*Sqrt[b]*(-6*A*b + a*C)*Log[-(Sqrt[b]*x)
+ Sqrt[a + b*x^2]])/(1680*b^2)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.98, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2346, 2346, 27, 455, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$

$$\Big\downarrow \; 2346$$

$$\frac{\int \left(bx^2+a\right)^{3/2}\left(7bCx^2+(7bB-2aD)x+7Ab\right)dx}{7b}+\frac{Dx^2\left(a+bx^2\right)^{5/2}}{7b}$$

$$\Big\downarrow \; 2346$$

$$\frac{\frac{\int b(7(6Ab-aC)+6(7bB-2aD)x)\left(bx^2+a\right)^{3/2}dx}{6b}+\frac{7}{6}Cx\left(a+bx^2\right)^{5/2}}{7b}+\frac{Dx^2\left(a+bx^2\right)^{5/2}}{7b}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{1}{6}\int (7(6Ab-aC)+6(7bB-2aD)x)\left(bx^2+a\right)^{3/2}dx+\frac{7}{6}Cx\left(a+bx^2\right)^{5/2}}{7b}+\frac{Dx^2\left(a+bx^2\right)^{5/2}}{7b}$$

$\downarrow$ 455

$$\frac{\frac{1}{6}\left(7(6Ab-aC)\int \left(bx^2+a\right)^{3/2}dx+\frac{6\left(a+bx^2\right)^{5/2}(7bB-2aD)}{5b}\right)+\frac{7}{6}Cx\left(a+bx^2\right)^{5/2}}{7b}+\frac{Dx^2\left(a+bx^2\right)^{5/2}}{7b}$$

$\downarrow$ 211

$$\frac{\frac{1}{6}\left(7(6Ab-aC)\left(\frac{3}{4}a\int \sqrt{bx^2+a}dx+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{6\left(a+bx^2\right)^{5/2}(7bB-2aD)}{5b}\right)+\frac{7}{6}Cx\left(a+bx^2\right)^{5/2}}{7b}+\frac{Dx^2\left(a+bx^2\right)^{5/2}}{7b}$$

$\downarrow$ 211

$$\frac{\frac{1}{6}\left(7(6Ab-aC)\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{6\left(a+bx^2\right)^{5/2}(7bB-2aD)}{5b}\right)+\frac{7}{6}Cx\left(a+bx^2\right)^{5/2}}{7b}+\frac{Dx^2\left(a+bx^2\right)^{5/2}}{7b}$$

$\downarrow$ 224

$$\frac{\frac{1}{6}\left(7(6Ab-aC)\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{6\left(a+bx^2\right)^{5/2}(7bB-2aD)}{5b}\right)+\frac{7}{6}Cx\left(a+bx^2\right)^{5/2}}{7b}+\frac{Dx^2\left(a+bx^2\right)^{5/2}}{7b}$$

$\downarrow$ 219

$$\frac{\frac{1}{6}\left(7(6Ab-aC)\left(\frac{3}{4}a\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{6\left(a+bx^2\right)^{5/2}(7bB-2aD)}{5b}\right)+\frac{7}{6}Cx\left(a+bx^2\right)^{5/2}}{7b}+\frac{Dx^2\left(a+bx^2\right)^{5/2}}{7b}$$

input
```
Int[(a + b*x^2)^(3/2)*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(D*x^2*(a + b*x^2)^(5/2))/(7*b) + ((7*C*x*(a + b*x^2)^(5/2))/6 + ((6*(7*b*
B - 2*a*D)*(a + b*x^2)^(5/2))/(5*b) + 7*(6*A*b - a*C)*((x*(a + b*x^2)^(3/2
))/4 + (3*a*((x*Sqrt[a + b*x^2])/2 + (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2
]])/(2*Sqrt[b])))/4))/6)/(7*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.10

method	result
default	$A\left(\frac{x(b\,x^2+a)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)}{4}\right)+\frac{B(b\,x^2+a)^{\frac{5}{2}}}{5b}+C\left(\frac{x(b\,x^2+a)^{\frac{5}{2}}}{6b}-\frac{a\left(\frac{x\left(b\,x^2+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\right.}{}\right.}{}\right.$

input
```
int((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
A*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(
1/2)*x+(b*x^2+a)^(1/2))))+1/5*B*(b*x^2+a)^(5/2)/b+C*(1/6*x*(b*x^2+a)^(5/2)
/b-1/6*a/b*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/
2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))))+D*(1/7*x^2*(b*x^2+a)^(5/2)/b-2/35*a/b^
2*(b*x^2+a)^(5/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 327, normalized size of antiderivative = 2.01

$$\int \left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx=\left[-\frac{105\,(Ca^3-6\,Aa^2b)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\,(240\,Db^3x^6+280\,Cb^3x^5+\right.$$

input
```
integrate((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
[-1/3360*(105*(C*a^3 - 6*A*a^2*b)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)
*sqrt(b)*x - a) - 2*(240*D*b^3*x^6 + 280*C*b^3*x^5 + 48*(8*D*a*b^2 + 7*B*b
^3)*x^4 - 96*D*a^3 + 336*B*a^2*b + 70*(7*C*a*b^2 + 6*A*b^3)*x^3 + 48*(D*a^
2*b + 14*B*a*b^2)*x^2 + 105*(C*a^2*b + 10*A*a*b^2)*x)*sqrt(b*x^2 + a))/b^2
, 1/1680*(105*(C*a^3 - 6*A*a^2*b)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 +
a)) + (240*D*b^3*x^6 + 280*C*b^3*x^5 + 48*(8*D*a*b^2 + 7*B*b^3)*x^4 - 96*D
*a^3 + 336*B*a^2*b + 70*(7*C*a*b^2 + 6*A*b^3)*x^3 + 48*(D*a^2*b + 14*B*a*b
^2)*x^2 + 105*(C*a^2*b + 10*A*a*b^2)*x)*sqrt(b*x^2 + a))/b^2]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 303 vs. $2(143) = 286$.

Time = 0.45 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.86

$$\int \left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx = \begin{cases} \sqrt{a+bx^2}\left(\frac{Cbx^5}{6}+\frac{Dbx^6}{7}+\frac{x^4\left(Bb^2+\frac{8Dab}{7}\right)}{5b}+\frac{x^3\left(Ab^2+\frac{7Cab}{6}\right)}{4b}+\frac{x^2\cdot\left(2Bab+Da^2-\frac{4a\left(Bb^2+\frac{8Dab}{7}\right)}{5b}\right)}{3b}+\frac{x\left(2A\right.}{} \right. \\ a^{\frac{3}{2}}\left(Ax+\frac{Bx^2}{2}+\frac{Cx^3}{3}+\frac{Dx^4}{4}\right) \end{cases}$$

input
```
integrate((b*x**2+a)**(3/2)*(D*x**3+C*x**2+B*x+A),x)
```

output
```
Piecewise((sqrt(a + b*x**2)*(C*b*x**5/6 + D*b*x**6/7 + x**4*(B*b**2 + 8*D*
a*b/7)/(5*b) + x**3*(A*b**2 + 7*C*a*b/6)/(4*b) + x**2*(2*B*a*b + D*a**2 -
4*a*(B*b**2 + 8*D*a*b/7)/(5*b))/(3*b) + x*(2*A*a*b + C*a**2 - 3*a*(A*b**2
+ 7*C*a*b/6)/(4*b))/(2*b) + (B*a**2 - 2*a*(2*B*a*b + D*a**2 - 4*a*(B*b**2
+ 8*D*a*b/7)/(5*b))/(3*b))/b) + (A*a**2 - a*(2*A*a*b + C*a**2 - 3*a*(A*b**
2 + 7*C*a*b/6)/(4*b))/(2*b))*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2
*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True)), Ne(b, 0)), (a**(
3/2)*(A*x + B*x**2/2 + C*x**3/3 + D*x**4/4), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.01

$$\int (a+bx^2)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{(bx^2+a)^{\frac{5}{2}}Dx^2}{7\,b}+\frac{1}{4}\,(bx^2+a)^{\frac{3}{2}}Ax+\frac{3}{8}\,\sqrt{bx^2+a}Aax+\frac{(bx^2+a)^{\frac{5}{2}}Cx}{6\,b}-\frac{(bx^2+a)^{\frac{3}{2}}Cax}{24\,b}-\frac{\sqrt{bx^2+a}Ca^2x}{16\,b}-\frac{Ca^3\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{3}{2}}}+\frac{3\,Aa^2\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{b}}-\frac{2\,(bx^2+a)^{\frac{5}{2}}Da}{35\,b^2}+\frac{(bx^2+a)^{\frac{5}{2}}B}{5\,b}$$

input
```
integrate((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
1/7*(b*x^2 + a)^(5/2)*D*x^2/b + 1/4*(b*x^2 + a)^(3/2)*A*x + 3/8*sqrt(b*x^2
 + a)*A*a*x + 1/6*(b*x^2 + a)^(5/2)*C*x/b - 1/24*(b*x^2 + a)^(3/2)*C*a*x/b
 - 1/16*sqrt(b*x^2 + a)*C*a^2*x/b - 1/16*C*a^3*arcsinh(b*x/sqrt(a*b))/b^(3
/2) + 3/8*A*a^2*arcsinh(b*x/sqrt(a*b))/sqrt(b) - 2/35*(b*x^2 + a)^(5/2)*D*
a/b^2 + 1/5*(b*x^2 + a)^(5/2)*B/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.10

$$\int (a+bx^2)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{1}{1680}\,\sqrt{bx^2+a}\left(\left(2\left(\left(4\left(5\,(6\,Dbx+7\,Cb)x+\frac{6\,(8\,Dab^5+7\,Bb^6)}{b^5}\right)x+\frac{35\,(7\,Cab^5+6\,Ab^6)}{b^5}\right.\right.\right.\right.+\frac{(Ca^3-6\,Aa^2b)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{16\,b^{\frac{3}{2}}}$$

input
```
integrate((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

output
```
1/1680*sqrt(b*x^2 + a)*((2*((4*(5*(6*D*b*x + 7*C*b)*x + 6*(8*D*a*b^5 + 7*B
*b^6)/b^5)*x + 35*(7*C*a*b^5 + 6*A*b^6)/b^5)*x + 24*(D*a^2*b^4 + 14*B*a*b^
5)/b^5)*x + 105*(C*a^2*b^4 + 10*A*a*b^5)/b^5)*x - 48*(2*D*a^3*b^3 - 7*B*a^
2*b^4)/b^5) + 1/16*(C*a^3 - 6*A*a^2*b)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a
)))/b^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx=\int \left(b\,x^2+a\right)^{3/2}\left(A+B\,x+C\,x^2+x^3\,D\right)\,dx$$

input
```
int((a + b*x^2)^(3/2)*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
int((a + b*x^2)^(3/2)*(A + B*x + C*x^2 + x^3*D), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.60

$$\int \left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{-96\sqrt{b\,x^2+a}\,a^3d+1050\sqrt{b\,x^2+a}\,a^2b^2x+336\sqrt{b\,x^2+a}\,a^2b^2+105\sqrt{b\,x^2+a}\,a^2bcx+48\sqrt{b}}{}$$

input
```
int((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A),x)
```

output
```
( - 96*sqrt(a + b*x**2)*a**3*d + 1050*sqrt(a + b*x**2)*a**2*b**2*x + 336*s
qrt(a + b*x**2)*a**2*b**2 + 105*sqrt(a + b*x**2)*a**2*b*c*x + 48*sqrt(a +
b*x**2)*a**2*b*d*x**2 + 420*sqrt(a + b*x**2)*a*b**3*x**3 + 672*sqrt(a + b*
x**2)*a*b**3*x**2 + 490*sqrt(a + b*x**2)*a*b**2*c*x**3 + 384*sqrt(a + b*x*
*2)*a*b**2*d*x**4 + 336*sqrt(a + b*x**2)*b**4*x**4 + 280*sqrt(a + b*x**2)*
b**3*c*x**5 + 240*sqrt(a + b*x**2)*b**3*d*x**6 + 630*sqrt(b)*log((sqrt(a +
 b*x**2) + sqrt(b)*x)/sqrt(a))*a**3*b - 105*sqrt(b)*log((sqrt(a + b*x**2)
+ sqrt(b)*x)/sqrt(a))*a**3*c)/(1680*b**2)
```

3.55 $\int \sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx$

Optimal result

Integrand size = 27, antiderivative size = 132

$$\int \sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx = \frac{(4Ab-aC)x\sqrt{a+bx^2}}{8b} + \frac{(bB-aD)\left(a+bx^2\right)^{3/2}}{3b^2} + \frac{Cx(a+bx^2)^{3/2}}{4b} + \frac{D(a+bx^2)^{5/2}}{5b^2} + \frac{a(4Ab-aC)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{3/2}}$$

output
```
1/8*(4*A*b-C*a)*x*(b*x^2+a)^(1/2)/b+1/3*(B*b-D*a)*(b*x^2+a)^(3/2)/b^2+1/4*
C*x*(b*x^2+a)^(3/2)/b+1/5*D*(b*x^2+a)^(5/2)/b^2+1/8*a*(4*A*b-C*a)*arctanh(
b^(1/2)*x/(b*x^2+a)^(1/2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.82

$$\int \sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$=\frac{\sqrt{a+bx^2}(-16a^2D+ab(40B+x(15C+8Dx))+2b^2x(30A+x(20B+3x(5C+4Dx))))+15a\sqrt{b}(-4}{120b^2}$$

input
```
Integrate[Sqrt[a + b*x^2]*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(Sqrt[a + b*x^2]*(-16*a^2*D + a*b*(40*B + x*(15*C + 8*D*x)) + 2*b^2*x*(30*
A + x*(20*B + 3*x*(5*C + 4*D*x)))) + 15*a*Sqrt[b]*(-4*A*b + a*C)*Log[-(Sqr
t[b]*x) + Sqrt[a + b*x^2]])/(120*b^2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2346, 2346, 27, 455, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$

$$\Big\downarrow \text{2346}$$

$$\frac{\int \sqrt{bx^2+a}\left(5bCx^2+(5bB-2aD)x+5Ab\right)dx}{5b}+\frac{Dx^2\left(a+bx^2\right)^{3/2}}{5b}$$

$$\Big\downarrow \text{2346}$$

$$\frac{\frac{\int b(5(4Ab-aC)+4(5bB-2aD)x)\sqrt{bx^2+a}dx}{4b}+\frac{5}{4}Cx\left(a+bx^2\right)^{3/2}}{5b}+\frac{Dx^2\left(a+bx^2\right)^{3/2}}{5b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{4}\int (5(4Ab-aC)+4(5bB-2aD)x)\sqrt{bx^2+a}dx+\frac{5}{4}Cx\left(a+bx^2\right)^{3/2}}{5b}+\frac{Dx^2\left(a+bx^2\right)^{3/2}}{5b}$$

$$\Big\downarrow \; 455$$

$$\frac{\frac{1}{4}\left(5(4Ab-aC)\int \sqrt{bx^2+a}dx+\frac{4(a+bx^2)^{3/2}(5bB-2aD)}{3b}\right)+\frac{5}{4}Cx\left(a+bx^2\right)^{3/2}}{5b}+\frac{Dx^2\left(a+bx^2\right)^{3/2}}{5b}$$

$$\Big\downarrow \; 211$$

$$\frac{\frac{1}{4}\left(5(4Ab-aC)\left(\frac{1}{2}a\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{4(a+bx^2)^{3/2}(5bB-2aD)}{3b}\right)+\frac{5}{4}Cx\left(a+bx^2\right)^{3/2}}{5b}+\frac{Dx^2\left(a+bx^2\right)^{3/2}}{5b}$$

$$\Big\downarrow \; 224$$

$$\frac{\frac{1}{4}\left(5(4Ab-aC)\left(\frac{1}{2}a\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{4(a+bx^2)^{3/2}(5bB-2aD)}{3b}\right)+\frac{5}{4}Cx\left(a+bx^2\right)^{3/2}}{5b}+\frac{Dx^2\left(a+bx^2\right)^{3/2}}{5b}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{1}{4}\left(5(4Ab-aC)\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{4(a+bx^2)^{3/2}(5bB-2aD)}{3b}\right)+\frac{5}{4}Cx\left(a+bx^2\right)^{3/2}}{5b}+\frac{Dx^2\left(a+bx^2\right)^{3/2}}{5b}$$

input
```
Int[Sqrt[a + b*x^2]*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(D*x^2*(a + b*x^2)^(3/2))/(5*b) + ((5*C*x*(a + b*x^2)^(3/2))/4 + ((4*(5*b*
B - 2*a*D)*(a + b*x^2)^(3/2))/(3*b) + 5*(4*A*b - a*C)*((x*Sqrt[a + b*x^2])
/2 + (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b])))/4)/(5*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.12

method	result
default	$A\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)+\frac{B\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{3b}+C\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{4b}-\frac{a\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)}{4b}\right)$

input
```
int((b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
A*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))+1/3*
B*(b*x^2+a)^(3/2)/b+C*(1/4*x*(b*x^2+a)^(3/2)/b-1/4*a/b*(1/2*x*(b*x^2+a)^(1
/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))))+D*(1/5*x^2*(b*x^2+a)^(3/
2)/b-2/15*a/b^2*(b*x^2+a)^(3/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.75

$$\int \sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$

$$= \left[-\frac{15\,(Ca^2-4\,Aab)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\,(24\,Db^2x^4+30\,Cb^2x^3-16\,Da^2+40\,B}{240\,b^2}\right.$$

input
```
integrate((b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
[-1/240*(15*(C*a^2 - 4*A*a*b)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqr
t(b)*x - a) - 2*(24*D*b^2*x^4 + 30*C*b^2*x^3 - 16*D*a^2 + 40*B*a*b + 8*(D*
a*b + 5*B*b^2)*x^2 + 15*(C*a*b + 4*A*b^2)*x)*sqrt(b*x^2 + a))/b^2, 1/120*(
15*(C*a^2 - 4*A*a*b)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (24*D*b
^2*x^4 + 30*C*b^2*x^3 - 16*D*a^2 + 40*B*a*b + 8*(D*a*b + 5*B*b^2)*x^2 + 15
*(C*a*b + 4*A*b^2)*x)*sqrt(b*x^2 + a))/b^2]
```

Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.21

$$\int \sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$

$$= \begin{cases} \sqrt{a+bx^2}\left(\frac{Cx^3}{4}+\frac{Dx^4}{5}+\frac{x^2\left(Bb+\frac{Da}{5}\right)}{3b}+\frac{x\left(Ab+\frac{Ca}{4}\right)}{2b}+\frac{Ba-\frac{2a\left(Bb+\frac{Da}{5}\right)}{3b}}{b}\right)+\left(Aa-\frac{a\left(Ab+\frac{Ca}{4}\right)}{2b}\right)\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+}\right.}{\sqrt{b}} \\ \frac{x\log(x)}{\sqrt{bx^2}}\end{cases}\right. \\ \sqrt{a}\left(Ax+\frac{Bx^2}{2}+\frac{Cx^3}{3}+\frac{Dx^4}{4}\right) \end{cases}$$

input
```
integrate((b*x**2+a)**(1/2)*(D*x**3+C*x**2+B*x+A),x)
```

output
```
Piecewise((sqrt(a + b*x**2)*(C*x**3/4 + D*x**4/5 + x**2*(B*b + D*a/5)/(3*b
) + x*(A*b + C*a/4)/(2*b) + (B*a - 2*a*(B*b + D*a/5)/(3*b))/b) + (A*a - a*
(A*b + C*a/4)/(2*b))*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sq
rt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True)), Ne(b, 0)), (sqrt(a)*(A*x
 + B*x**2/2 + C*x**3/3 + D*x**4/4), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.98

$$\int \sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx = \frac{(bx^2+a)^{\frac{3}{2}}Dx^2}{5\,b}+\frac{1}{2}\sqrt{bx^2+a}Ax + \frac{(bx^2+a)^{\frac{3}{2}}Cx}{4\,b}-\frac{\sqrt{bx^2+a}Cax}{8\,b} - \frac{Ca^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{3}{2}}}+\frac{Aa\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{b}} - \frac{2\,(bx^2+a)^{\frac{3}{2}}Da}{15\,b^2}+\frac{(bx^2+a)^{\frac{3}{2}}B}{3\,b}$$

input
```
integrate((b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
1/5*(b*x^2 + a)^(3/2)*D*x^2/b + 1/2*sqrt(b*x^2 + a)*A*x + 1/4*(b*x^2 + a)^
(3/2)*C*x/b - 1/8*sqrt(b*x^2 + a)*C*a*x/b - 1/8*C*a^2*arcsinh(b*x/sqrt(a*b
))/b^(3/2) + 1/2*A*a*arcsinh(b*x/sqrt(a*b))/sqrt(b) - 2/15*(b*x^2 + a)^(3/
2)*D*a/b^2 + 1/3*(b*x^2 + a)^(3/2)*B/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.92

$$\int \sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$=\frac{1}{120}\sqrt{bx^2+a}\left(\left(2\left(3\left(4\,Dx+5\,C\right)x+\frac{4\left(Dab^2+5\,Bb^3\right)}{b^3}\right)x+\frac{15\left(Cab^2+4\,Ab^3\right)}{b^3}\right)x-\frac{8\left(2\,Da^2b-5\,B\right.}{b^3}\right.$$
$$+\frac{\left(Ca^2-4\,Aab\right)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{8\,b^{\frac{3}{2}}}$$

input
```
integrate((b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

output
```
1/120*sqrt(b*x^2 + a)*((2*(3*(4*D*x + 5*C)*x + 4*(D*a*b^2 + 5*B*b^3)/b^3)*
x + 15*(C*a*b^2 + 4*A*b^3)/b^3)*x - 8*(2*D*a^2*b - 5*B*a*b^2)/b^3) + 1/8*(
C*a^2 - 4*A*a*b)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx=\int \sqrt{b\,x^2+a}\,\left(A+B\,x+C\,x^2+x^3\,D\right)\,dx$$

input
```
int((a + b*x^2)^(1/2)*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
int((a + b*x^2)^(1/2)*(A + B*x + C*x^2 + x^3*D), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.39

$$\int \sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$= \frac{-16\sqrt{b\,x^2+a}\,a^2d+60\sqrt{b\,x^2+a}\,a\,b^2x+40\sqrt{b\,x^2+a}\,a\,b^2+15\sqrt{b\,x^2+a}\,abcx+8\sqrt{b\,x^2+a}\,abd\,x^2+40}{}$$

input
```
int((b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A),x)
```

output
```
( - 16*sqrt(a + b*x**2)*a**2*d + 60*sqrt(a + b*x**2)*a*b**2*x + 40*sqrt(a
+ b*x**2)*a*b**2 + 15*sqrt(a + b*x**2)*a*b*c*x + 8*sqrt(a + b*x**2)*a*b*d*
x**2 + 40*sqrt(a + b*x**2)*b**3*x**2 + 30*sqrt(a + b*x**2)*b**2*c*x**3 + 2
4*sqrt(a + b*x**2)*b**2*d*x**4 + 60*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b
)*x)/sqrt(a))*a**2*b - 15*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(
a))*a**2*c)/(120*b**2)
```

3.56 $\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{a+bx^2}} dx$

Optimal result

Integrand size = 27, antiderivative size = 100

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{a+bx^2}}\,dx=\frac{(bB-aD)\sqrt{a+bx^2}}{b^2}+\frac{Cx\sqrt{a+bx^2}}{2b}+\frac{D(a+bx^2)^{3/2}}{3b^2}+\frac{(2Ab-aC)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}$$

output
```
(B*b-D*a)*(b*x^2+a)^(1/2)/b^2+1/2*C*x*(b*x^2+a)^(1/2)/b+1/3*D*(b*x^2+a)^(3
/2)/b^2+1/2*(2*A*b-C*a)*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.78

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{a+bx^2}}\,dx=\frac{\sqrt{a+bx^2}(6bB-4aD+3bCx+2bDx^2)}{6b^2}+\frac{(-2Ab+aC)\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{2b^{3/2}}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/Sqrt[a + b*x^2],x]
```

output
```
(Sqrt[a + b*x^2]*(6*b*B - 4*a*D + 3*b*C*x + 2*b*D*x^2))/(6*b^2) + ((-2*A*b
 + a*C)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(2*b^(3/2))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.13, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2346, 2346, 27, 455, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{a+bx^2}}dx$$

$$\Big\downarrow \text{2346}$$

$$\frac{\int \frac{3bCx^2+(3bB-2aD)x+3Ab}{\sqrt{bx^2+a}}dx}{3b}+\frac{Dx^2\sqrt{a+bx^2}}{3b}$$

$$\Big\downarrow \text{2346}$$

$$\frac{\frac{\int \frac{b(3(2Ab-aC)+2(3bB-2aD)x)}{\sqrt{bx^2+a}}dx}{2b}+\frac{3}{2}Cx\sqrt{a+bx^2}}{3b}+\frac{Dx^2\sqrt{a+bx^2}}{3b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{2}\int \frac{3(2Ab-aC)+2(3bB-2aD)x}{\sqrt{bx^2+a}}dx+\frac{3}{2}Cx\sqrt{a+bx^2}}{3b}+\frac{Dx^2\sqrt{a+bx^2}}{3b}$$

$$\Big\downarrow \text{455}$$

$$\frac{\frac{1}{2}\left(3(2Ab-aC)\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{2\sqrt{a+bx^2}(3bB-2aD)}{b}\right)+\frac{3}{2}Cx\sqrt{a+bx^2}}{3b}+\frac{Dx^2\sqrt{a+bx^2}}{3b}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{1}{2}\left(3(2Ab-aC)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{2\sqrt{a+bx^2}(3bB-2aD)}{b}\right)+\frac{3}{2}Cx\sqrt{a+bx^2}}{3b}+\frac{Dx^2\sqrt{a+bx^2}}{3b}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{1}{2}\left(\frac{3(2Ab-aC)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}+\frac{2\sqrt{a+bx^2}(3bB-2aD)}{b}\right)+\frac{3}{2}Cx\sqrt{a+bx^2}}{3b}+\frac{Dx^2\sqrt{a+bx^2}}{3b}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/Sqrt[a + b*x^2],x]
```

output
```
(D*x^2*Sqrt[a + b*x^2])/(3*b) + ((3*C*x*Sqrt[a + b*x^2])/2 + ((2*(3*b*B -
2*a*D)*Sqrt[a + b*x^2])/b + (3*(2*A*b - a*C)*ArcTanh[(Sqrt[b]*x)/Sqrt[a +
b*x^2]])/Sqrt[b])/2)/(3*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.12

method	result
default	$\frac{A\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{\sqrt{b}}+\frac{B\sqrt{b\,x^{2}+a}}{b}+C\left(\frac{x\sqrt{b\,x^{2}+a}}{2b}-\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{2b^{\frac{3}{2}}}\right)+D\left(\frac{x^{2}\sqrt{b\,x^{2}+a}}{3b}-\frac{2a\sqrt{b\,x^{2}+a}}{3b^{2}}\right)$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
A*ln(b^(1/2)*x+(b*x^2+a)^(1/2))/b^(1/2)+B*(b*x^2+a)^(1/2)/b+C*(1/2*x*(b*x^
2+a)^(1/2)/b-1/2*a/b^(3/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))+D*(1/3*x^2/b*(b*
x^2+a)^(1/2)-2/3*a/b^2*(b*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.47

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{a+bx^2}}\,dx = \left[-\frac{3\,(Ca-2\,Ab)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\,(2\,Dbx^2+3\,Cbx-4\,Da+6\,Bb)\sqrt{bx^2+a}}{12\,b^2},\right.$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/12*(3*(C*a - 2*A*b)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x
 - a) - 2*(2*D*b*x^2 + 3*C*b*x - 4*D*a + 6*B*b)*sqrt(b*x^2 + a))/b^2, 1/6*
(3*(C*a - 2*A*b)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (2*D*b*x^2
+ 3*C*b*x - 4*D*a + 6*B*b)*sqrt(b*x^2 + a))/b^2]
```

Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.16

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{a+bx^2}}\,dx = \begin{cases} \left(A-\frac{Ca}{2b}\right)\left(\begin{cases} \frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for}\ a\neq 0 \\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise} \end{cases}\right)+\sqrt{a+bx^2}\left(\frac{Cx}{2b}+\frac{Dx^2}{3b}+\frac{B-\frac{2Da}{3b}}{b}\right) & \text{for}\ b\neq 0 \\ \frac{Ax+\frac{Bx^2}{2}+\frac{Cx^3}{3}+\frac{Dx^4}{4}}{\sqrt{a}} & \text{otherwise} \end{cases}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(b*x**2+a)**(1/2),x)
```

output
```
Piecewise(((A - C*a/(2*b))*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b
*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True)) + sqrt(a + b*x**2)*
(C*x/(2*b) + D*x**2/(3*b) + (B - 2*D*a/(3*b))/b), Ne(b, 0)), ((A*x + B*x**
2/2 + C*x**3/3 + D*x**4/4)/sqrt(a), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.95

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{bx^2+a}Dx^2}{3\,b}+\frac{\sqrt{bx^2+a}Cx}{2\,b}-\frac{Ca\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{3}{2}}}+\frac{A\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}}-\frac{2\,\sqrt{bx^2+a}Da}{3\,b^2}+\frac{\sqrt{bx^2+a}B}{b}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/3*sqrt(b*x^2 + a)*D*x^2/b + 1/2*sqrt(b*x^2 + a)*C*x/b - 1/2*C*a*arcsinh(
b*x/sqrt(a*b))/b^(3/2) + A*arcsinh(b*x/sqrt(a*b))/sqrt(b) - 2/3*sqrt(b*x^2
 + a)*D*a/b^2 + sqrt(b*x^2 + a)*B/b
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.77

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{a+bx^2}}\,dx = \frac{1}{6}\sqrt{bx^2+a}\left(\left(\frac{2\,Dx}{b}+\frac{3\,C}{b}\right)x-\frac{2\,(2\,Dab-3\,Bb^2)}{b^3}\right) + \frac{(Ca-2\,Ab)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2\,b^{\frac{3}{2}}}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/6*sqrt(b*x^2 + a)*((2*D*x/b + 3*C/b)*x - 2*(2*D*a*b - 3*B*b^2)/b^3) + 1/
2*(C*a - 2*A*b)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.96 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.49

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{a+bx^2}}\,dx = \begin{cases} \frac{B\,x^2}{2\,\sqrt{a}}+\frac{C\,x^3}{3\,\sqrt{a}}+\frac{x^4\,D}{4\,\sqrt{a}}+\frac{A\,x}{\sqrt{a}} & \text{if } b = \\ \frac{{\left(b\,x^2+a\right)}^{3/2}\,D-3\,a\,\sqrt{b\,x^2+a}\,D}{3\,b^2}+\frac{B\,\sqrt{b\,x^2+a}}{b}+\frac{A\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}-\frac{C\,a\,\ln\left(2\,\sqrt{b}\,x+2\,\sqrt{b\,x^2+a}\right)}{2\,b^{3/2}}+\frac{C\,x\,\sqrt{b\,x^2+a}}{2\,b} & \text{if } b \neq \end{cases}$$

input
```
int((A + B*x + C*x^2 + x^3*D)/(a + b*x^2)^(1/2),x)
```

output
```
piecewise(b == 0, (B*x^2)/(2*a^(1/2)) + (C*x^3)/(3*a^(1/2)) + (x^4*D)/(4*a
^(1/2)) + (A*x)/a^(1/2), b ~= 0, ((a + b*x^2)^(3/2)*D - 3*a*(a + b*x^2)^(1
/2)*D)/(3*b^2) + (B*(a + b*x^2)^(1/2))/b + (A*log(b^(1/2)*x + (a + b*x^2)^
(1/2)))/b^(1/2) - (C*a*log(2*b^(1/2)*x + 2*(a + b*x^2)^(1/2)))/(2*b^(3/2))
 + (C*x*(a + b*x^2)^(1/2))/(2*b))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.09

$$\int \frac{A + Bx + Cx^2 + Dx^3}{\sqrt{a + bx^2}}\, dx$$
$$= \frac{-4\sqrt{b\,x^2+a}\,ad + 6\sqrt{b\,x^2+a}\,b^2 + 3\sqrt{b\,x^2+a}\,bcx + 2\sqrt{b\,x^2+a}\,bd\,x^2 + 6\sqrt{b}\log\left(\frac{\sqrt{b\,x^2+a}+\sqrt{b}\,x}{\sqrt{a}}\right)ab - 3\sqrt{b}}{6b^2}$$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(1/2),x)
```

output
```
( - 4*sqrt(a + b*x**2)*a*d + 6*sqrt(a + b*x**2)*b**2 + 3*sqrt(a + b*x**2)*
b*c*x + 2*sqrt(a + b*x**2)*b*d*x**2 + 6*sqrt(b)*log((sqrt(a + b*x**2) + sq
rt(b)*x)/sqrt(a))*a*b - 3*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(
a))*a*c)/(6*b**2)
```

3.57 $\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 85

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{3/2}}\,dx=-\frac{a(bB-aD)-b(Ab-aC)x}{ab^2\sqrt{a+bx^2}}+\frac{D\sqrt{a+bx^2}}{b^2}+\frac{C\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}$$

output
```
-(a*(B*b-D*a)-b*(A*b-C*a)*x)/a/b^2/(b*x^2+a)^(1/2)+D*(b*x^2+a)^(1/2)/b^2+C
*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.88

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{3/2}}\,dx=\frac{2a^2D+Ab^2x-ab(B+x(C-Dx))}{ab^2\sqrt{a+bx^2}}-\frac{C\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{b^{3/2}}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/(a + b*x^2)^(3/2),x]
```

output
```
(2*a^2*D + A*b^2*x - a*b*(B + x*(C - D*x)))/(a*b^2*Sqrt[a + b*x^2]) - (C*L
og[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/b^(3/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2345, 25, 27, 455, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{3/2}}dx$$

$$\Big\downarrow \text{2345}$$

$$-\frac{\int -\frac{a(C+Dx)}{b\sqrt{bx^2+a}}dx}{a}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{a(C+Dx)}{b\sqrt{bx^2+a}}dx}{a}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{C+Dx}{\sqrt{bx^2+a}}dx}{b}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{455}$$

$$\frac{C\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{D\sqrt{a+bx^2}}{b}}{b}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{224}$$

$$\frac{C\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{D\sqrt{a+bx^2}}{b}}{b}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{C\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}+\frac{D\sqrt{a+bx^2}}{b}}{b}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{ab\sqrt{a+bx^2}}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/(a + b*x^2)^(3/2),x]
```

output
```
-((a*(B - (a*D)/b) - (A*b - a*C)*x)/(a*b*Sqrt[a + b*x^2])) + ((D*Sqrt[a +
b*x^2])/b + (C*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/Sqrt[b])/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.22

method	result	size
default	$\frac{Ax}{a\sqrt{b\,x^2+a}}-\frac{B}{b\sqrt{b\,x^2+a}}+C\left(-\frac{x}{b\sqrt{b\,x^2+a}}+\frac{\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{b^{\frac{3}{2}}}\right)+D\left(\frac{x^2}{b\sqrt{b\,x^2+a}}+\frac{2a}{b^2\sqrt{b\,x^2+a}}\right)$	104

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
A*x/a/(b*x^2+a)^(1/2)-B/b/(b*x^2+a)^(1/2)+C*(-x/b/(b*x^2+a)^(1/2)+1/b^(3/2
)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))+D*(x^2/b/(b*x^2+a)^(1/2)+2*a/b^2/(b*x^2+a
)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 210, normalized size of antiderivative = 2.47

$$\int \frac{A+Bx+Cx^2+Dx^3}{\left(a+bx^2\right)^{3/2}}\,dx = \left[\frac{(Cabx^2+Ca^2)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,(Dabx^2+2\,D}{2\,(ab^3x^2+a^2b^2)}\right.$$

$$\left.-\frac{(Cabx^2+Ca^2)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(Dabx^2+2\,Da^2-Bab-(Cab-Ab^2)x)\sqrt{bx^2+a}}{ab^3x^2+a^2b^2}\right]$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/2*((C*a*b*x^2 + C*a^2)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)
*x - a) + 2*(D*a*b*x^2 + 2*D*a^2 - B*a*b - (C*a*b - A*b^2)*x)*sqrt(b*x^2 +
 a))/(a*b^3*x^2 + a^2*b^2), -((C*a*b*x^2 + C*a^2)*sqrt(-b)*arctan(sqrt(-b)
*x/sqrt(b*x^2 + a)) - (D*a*b*x^2 + 2*D*a^2 - B*a*b - (C*a*b - A*b^2)*x)*sq
rt(b*x^2 + a))/(a*b^3*x^2 + a^2*b^2)]
```

Sympy [A] (verification not implemented)

Time = 3.90 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.54

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{3/2}}\,dx = \frac{Ax}{a^{\frac{3}{2}}\sqrt{1+\frac{bx^2}{a}}} + B\left(\begin{cases} -\frac{1}{b\sqrt{a+bx^2}} & \text{for}\ b\neq 0 \\ \frac{x^2}{2a^{\frac{3}{2}}} & \text{otherwise} \end{cases}\right) + C\left(\frac{\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{\frac{3}{2}}} - \frac{x}{\sqrt{a}b\sqrt{1+\frac{bx^2}{a}}}\right) + D\left(\begin{cases} \frac{2a}{b^2\sqrt{a+bx^2}} + \frac{x^2}{b\sqrt{a+bx^2}} & \text{for}\ b\neq 0 \\ \frac{x^4}{4a^{\frac{3}{2}}} & \text{otherwise} \end{cases}\right)$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(b*x**2+a)**(3/2),x)
```

output
```
A*x/(a**(3/2)*sqrt(1 + b*x**2/a)) + B*Piecewise((-1/(b*sqrt(a + b*x**2)),
Ne(b, 0)), (x**2/(2*a**(3/2)), True)) + C*(asinh(sqrt(b)*x/sqrt(a))/b**(3/
2) - x/(sqrt(a)*b*sqrt(1 + b*x**2/a))) + D*Piecewise((2*a/(b**2*sqrt(a + b
*x**2)) + x**2/(b*sqrt(a + b*x**2)), Ne(b, 0)), (x**4/(4*a**(3/2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.11

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{3/2}}\,dx = \frac{Dx^2}{\sqrt{bx^2+a}b} + \frac{Ax}{\sqrt{bx^2+a}a} - \frac{Cx}{\sqrt{bx^2+a}b} + \frac{C\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{3}{2}}} + \frac{2\,Da}{\sqrt{bx^2+a}b^2} - \frac{B}{\sqrt{bx^2+a}b}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
D*x^2/(sqrt(b*x^2 + a)*b) + A*x/(sqrt(b*x^2 + a)*a) - C*x/(sqrt(b*x^2 + a)
*b) + C*arcsinh(b*x/sqrt(a*b))/b^(3/2) + 2*D*a/(sqrt(b*x^2 + a)*b^2) - B/(
sqrt(b*x^2 + a)*b)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.04

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(a + bx^2)^{3/2}}\,dx = \frac{\left(\frac{Dx}{b} - \frac{Cab^2 - Ab^3}{ab^3}\right)x + \frac{2\,Da^2b - Bab^2}{ab^3}}{\sqrt{bx^2 + a}} - \frac{C\log\left(\left|-\sqrt{b}x + \sqrt{bx^2 + a}\right|\right)}{b^{\frac{3}{2}}}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
((D*x/b - (C*a*b^2 - A*b^3)/(a*b^3))*x + (2*D*a^2*b - B*a*b^2)/(a*b^3))/sq
rt(b*x^2 + a) - C*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.07

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(a + bx^2)^{3/2}}\,dx = \frac{C\,\ln\left(\sqrt{b}\,x + \sqrt{b\,x^2 + a}\right)}{b^{3/2}} - \frac{B}{b\,\sqrt{b\,x^2 + a}} + \frac{(b\,x^2 + 2\,a)\,D}{b^2\,\sqrt{b\,x^2 + a}} + \frac{A\,x}{a\,\sqrt{b\,x^2 + a}} - \frac{C\,x}{b\,\sqrt{b\,x^2 + a}}$$

input
```
int((A + B*x + C*x^2 + x^3*D)/(a + b*x^2)^(3/2),x)
```

output
```
(C*log(b^(1/2)*x + (a + b*x^2)^(1/2)))/b^(3/2) - B/(b*(a + b*x^2)^(1/2)) +
 ((2*a + b*x^2)*D)/(b^2*(a + b*x^2)^(1/2)) + (A*x)/(a*(a + b*x^2)^(1/2)) -
 (C*x)/(b*(a + b*x^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.87

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{3/2}}\,dx = \frac{2\sqrt{b\,x^2+a}\,ad+\sqrt{b\,x^2+a}\,b^2x-\sqrt{b\,x^2+a}\,b^2-\sqrt{b\,x^2+a}\,bcx+\sqrt{b\,x^2+a}$$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(3/2),x)
```

output
```
(2*sqrt(a + b*x**2)*a*d + sqrt(a + b*x**2)*b**2*x - sqrt(a + b*x**2)*b**2
- sqrt(a + b*x**2)*b*c*x + sqrt(a + b*x**2)*b*d*x**2 + sqrt(b)*log((sqrt(a
 + b*x**2) + sqrt(b)*x)/sqrt(a))*a*c + sqrt(b)*log((sqrt(a + b*x**2) + sqr
t(b)*x)/sqrt(a))*b*c*x**2 + sqrt(b)*a*b - sqrt(b)*a*c + sqrt(b)*b**2*x**2
- sqrt(b)*b*c*x**2)/(b**2*(a + b*x**2))
```

3.58 $\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 85

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{5/2}}\,dx = -\frac{a(bB-aD)-b(Ab-aC)x}{3ab^2(a+bx^2)^{3/2}} - \frac{3a^2D-b(2Ab+aC)x}{3a^2b^2\sqrt{a+bx^2}}$$

output
```
-1/3*(a*(B*b-D*a)-b*(A*b-C*a)*x)/a/b^2/(b*x^2+a)^(3/2)-1/3*(3*a^2*D-b*(2*A
*b+C*a)*x)/a^2/b^2/(b*x^2+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.78

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{5/2}}\,dx = \frac{-2a^3D+2Ab^3x^3+ab^2x(3A+Cx^2)-a^2b(B+3Dx^2)}{3a^2b^2(a+bx^2)^{3/2}}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/(a + b*x^2)^(5/2),x]
```

output
```
(-2*a^3*D + 2*A*b^3*x^3 + a*b^2*x*(3*A + C*x^2) - a^2*b*(B + 3*D*x^2))/(3*
a^2*b^2*(a + b*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2345, 25, 27, 453}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{5/2}}dx$$

$$\Big\downarrow \text{2345}$$

$$-\frac{\int -\frac{b\left(2A+\frac{aC}{b}\right)+3aDx}{b(bx^2+a)^{3/2}}dx}{3a}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{3ab\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{2Ab+aC+3aDx}{b(bx^2+a)^{3/2}}dx}{3a}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{3ab\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{2Ab+aC+3aDx}{(bx^2+a)^{3/2}}dx}{3ab}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{3ab\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{453}$$

$$-\frac{3a^2D-bx(aC+2Ab)}{3a^2b^2\sqrt{a+bx^2}}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{3ab\left(a+bx^2\right)^{3/2}}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/(a + b*x^2)^(5/2),x]
```

output
```
-1/3*(a*(B - (a*D)/b) - (A*b - a*C)*x)/(a*b*(a + b*x^2)^(3/2)) - (3*a^2*D
- b*(2*A*b + a*C)*x)/(3*a^2*b^2*Sqrt[a + b*x^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 453
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[-(a*
d - b*c*x)/(a*b*Sqrt[a + b*x^2]), x] /; FreeQ[{a, b, c, d}, x]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.80

method	result
gosper	$\frac{2A\,b^{3}x^{3}+C a\,b^{2}x^{3}-3D x^{2}a^{2}b+3A x a\,b^{2}-a^{2}b B-2a^{3}D}{3\left(b\,x^{2}+a\right)^{\frac{3}{2}}a^{2}b^{2}}$
trager	$\frac{2A\,b^{3}x^{3}+C a\,b^{2}x^{3}-3D x^{2}a^{2}b+3A x a\,b^{2}-a^{2}b B-2a^{3}D}{3\left(b\,x^{2}+a\right)^{\frac{3}{2}}a^{2}b^{2}}$
orering	$\frac{2A\,b^{3}x^{3}+C a\,b^{2}x^{3}-3D x^{2}a^{2}b+3A x a\,b^{2}-a^{2}b B-2a^{3}D}{3\left(b\,x^{2}+a\right)^{\frac{3}{2}}a^{2}b^{2}}$
default	$A\left(\frac{x}{3a\left(b\,x^{2}+a\right)^{\frac{3}{2}}}+\frac{2x}{3a^{2}\sqrt{b\,x^{2}+a}}\right)-\frac{B}{3b\left(b\,x^{2}+a\right)^{\frac{3}{2}}}+C\left(-\frac{x}{2b\left(b\,x^{2}+a\right)^{\frac{3}{2}}}+\frac{a\left(\frac{x}{3a\left(b\,x^{2}+a\right)^{\frac{3}{2}}}+\frac{2x}{3a^{2}\sqrt{b\,x^{2}+a}}\right)}{2b}\right)+D\Big($

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(2*A*b^3*x^3+C*a*b^2*x^3-3*D*a^2*b*x^2+3*A*a*b^2*x-B*a^2*b-2*D*a^3)/(b
*x^2+a)^(3/2)/a^2/b^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.07

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{5/2}}\,dx = -\frac{(3\,Da^2bx^2-3\,Aab^2x+2\,Da^3+Ba^2b-(Cab^2+2\,Ab^3)x^3)\sqrt{bx^2+a}}{3\,(a^2b^4x^4+2\,a^3b^3x^2+a^4b^2)}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
-1/3*(3*D*a^2*b*x^2 - 3*A*a*b^2*x + 2*D*a^3 + B*a^2*b - (C*a*b^2 + 2*A*b^3
)*x^3)*sqrt(b*x^2 + a)/(a^2*b^4*x^4 + 2*a^3*b^3*x^2 + a^4*b^2)
```

Sympy [A] (verification not implemented)

Time = 6.75 (sec) , antiderivative size = 289, normalized size of antiderivative = 3.40

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{5/2}}\,dx = A\left(\frac{3ax}{3a^{\frac{7}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{5}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}} + \frac{2bx^3}{3a^{\frac{7}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{5}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}}\right) + B\left(\begin{cases}-\frac{1}{3ab\sqrt{a+bx^2}+3b^2x^2\sqrt{a+bx^2}} & \text{for } b\neq 0\\ \frac{x^2}{2a^{\frac{5}{2}}} & \text{otherwise}\end{cases}\right) + \frac{Cx^3}{3a^{\frac{5}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{3}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}} + D\left(\begin{cases}-\frac{2a}{3ab^2\sqrt{a+bx^2}+3b^3x^2\sqrt{a+bx^2}}-\frac{3bx^2}{3ab^2\sqrt{a+bx^2}+3b^3x^2\sqrt{a+bx^2}} & \text{for } b\neq 0\\ \frac{x^4}{4a^{\frac{5}{2}}} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(b*x**2+a)**(5/2),x)
```

output
```
A*(3*a*x/(3*a**(7/2)*sqrt(1 + b*x**2/a) + 3*a**(5/2)*b*x**2*sqrt(1 + b*x**
2/a)) + 2*b*x**3/(3*a**(7/2)*sqrt(1 + b*x**2/a) + 3*a**(5/2)*b*x**2*sqrt(1
 + b*x**2/a))) + B*Piecewise((-1/(3*a*b*sqrt(a + b*x**2) + 3*b**2*x**2*sqr
t(a + b*x**2)), Ne(b, 0)), (x**2/(2*a**(5/2)), True)) + C*x**3/(3*a**(5/2)
*sqrt(1 + b*x**2/a) + 3*a**(3/2)*b*x**2*sqrt(1 + b*x**2/a)) + D*Piecewise(
(-2*a/(3*a*b**2*sqrt(a + b*x**2) + 3*b**3*x**2*sqrt(a + b*x**2)) - 3*b*x**
2/(3*a*b**2*sqrt(a + b*x**2) + 3*b**3*x**2*sqrt(a + b*x**2)), Ne(b, 0)), (
x**4/(4*a**(5/2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.38

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{5/2}}\,dx = -\frac{Dx^2}{(bx^2+a)^{\frac{3}{2}}b}+\frac{2\,Ax}{3\sqrt{bx^2+a}a^2}+\frac{Ax}{3\,(bx^2+a)^{\frac{3}{2}}a} - \frac{Cx}{3\,(bx^2+a)^{\frac{3}{2}}b}+\frac{Cx}{3\sqrt{bx^2+a}ab}-\frac{2\,Da}{3\,(bx^2+a)^{\frac{3}{2}}b^2}-\frac{B}{3\,(bx^2+a)^{\frac{3}{2}}b}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
-D*x^2/((b*x^2 + a)^(3/2)*b) + 2/3*A*x/(sqrt(b*x^2 + a)*a^2) + 1/3*A*x/((b
*x^2 + a)^(3/2)*a) - 1/3*C*x/((b*x^2 + a)^(3/2)*b) + 1/3*C*x/(sqrt(b*x^2 +
 a)*a*b) - 2/3*D*a/((b*x^2 + a)^(3/2)*b^2) - 1/3*B/((b*x^2 + a)^(3/2)*b)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.85

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{5/2}}\,dx = -\frac{\left(x\left(\frac{3\,D}{b}-\frac{(Cab^2+2\,Ab^3)x}{a^2b^2}\right)-\frac{3\,A}{a}\right)x+\frac{2\,Da^3+Ba^2b}{a^2b^2}}{3\,(bx^2+a)^{\frac{3}{2}}}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
-1/3*((x*(3*D/b - (C*a*b^2 + 2*A*b^3)*x/(a^2*b^2)) - 3*A/a)*x + (2*D*a^3 +
 B*a^2*b)/(a^2*b^2))/(b*x^2 + a)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 1.05 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.05

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{5/2}}\,dx = \frac{2\,A\,x\,(b\,x^2+a)+A\,a\,x}{3\,a^2\,(b\,x^2+a)^{3/2}} - \frac{B}{3\,b\,(b\,x^2+a)^{3/2}} - \frac{(3\,b\,x^2+2\,a)\ D}{3\,b^2\,(b\,x^2+a)^{3/2}} + \frac{C\,x^3}{3\,a\,(b\,x^2+a)^{3/2}}$$

input
```
int((A + B*x + C*x^2 + x^3*D)/(a + b*x^2)^(5/2),x)
```

output
```
(2*A*x*(a + b*x^2) + A*a*x)/(3*a^2*(a + b*x^2)^(3/2)) - B/(3*b*(a + b*x^2)
^(3/2)) - ((2*a + 3*b*x^2)*D)/(3*b^2*(a + b*x^2)^(3/2)) + (C*x^3)/(3*a*(a
+ b*x^2)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 176, normalized size of antiderivative = 2.07

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{5/2}}\,dx = \frac{-2\sqrt{b\,x^2+a}\,a^2d+3\sqrt{b\,x^2+a}\,a\,b^2x-\sqrt{b\,x^2+a}\,a\,b^2-3\sqrt{b\,x^2+a}\,abd\,x^2 -$$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(5/2),x)
```

output
```
( - 2*sqrt(a + b*x**2)*a**2*d + 3*sqrt(a + b*x**2)*a*b**2*x - sqrt(a + b*x
**2)*a*b**2 - 3*sqrt(a + b*x**2)*a*b*d*x**2 + 2*sqrt(a + b*x**2)*b**3*x**3
 + sqrt(a + b*x**2)*b**2*c*x**3 - 2*sqrt(b)*a**2*b + sqrt(b)*a**2*c - 4*sq
rt(b)*a*b**2*x**2 + 2*sqrt(b)*a*b*c*x**2 - 2*sqrt(b)*b**3*x**4 + sqrt(b)*b
**2*c*x**4)/(3*a*b**2*(a**2 + 2*a*b*x**2 + b**2*x**4))
```

3.59 $\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{7/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 115

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{7/2}}\,dx = -\frac{a(bB-aD)-b(Ab-aC)x}{5ab^2(a+bx^2)^{5/2}} - \frac{5a^2D-b(4Ab+aC)x}{15a^2b^2(a+bx^2)^{3/2}} + \frac{2(4Ab+aC)x}{15a^3b\sqrt{a+bx^2}}$$

output
```
-1/5*(a*(B*b-D*a)-b*(A*b-C*a)*x)/a/b^2/(b*x^2+a)^(5/2)-1/15*(5*a^2*D-b*(4*
A*b+C*a)*x)/a^2/b^2/(b*x^2+a)^(3/2)+2/15*(4*A*b+C*a)*x/a^3/b/(b*x^2+a)^(1/
2)
```

Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.77

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{7/2}}\,dx = \frac{-2a^4D+8Ab^4x^5+5a^2b^2x(3A+Cx^2)+2ab^3x^3(10A+Cx^2)-a^3b(3B+}{15a^3b^2(a+bx^2)^{5/2}}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/(a + b*x^2)^(7/2),x]
```

output
```
(-2*a^4*D + 8*A*b^4*x^5 + 5*a^2*b^2*x*(3*A + C*x^2) + 2*a*b^3*x^3*(10*A +
C*x^2) - a^3*b*(3*B + 5*D*x^2))/(15*a^3*b^2*(a + b*x^2)^(5/2))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {2345, 25, 27, 454, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{7/2}}dx$$

$$\Big\downarrow \text{2345}$$

$$-\frac{\int -\frac{b\left(4A+\frac{aC}{b}\right)+5aDx}{b(bx^2+a)^{5/2}}dx}{5a}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{5ab\left(a+bx^2\right)^{5/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{4Ab+aC+5aDx}{b(bx^2+a)^{5/2}}dx}{5a}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{5ab\left(a+bx^2\right)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{4Ab+aC+5aDx}{(bx^2+a)^{5/2}}dx}{5ab}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{5ab\left(a+bx^2\right)^{5/2}}$$

$$\Big\downarrow \text{454}$$

$$\frac{\frac{2(aC+4Ab)\int \frac{1}{\left(bx^2+a\right)^{3/2}}dx}{3a}-\frac{5a^2D-bx(aC+4Ab)}{3ab(a+bx^2)^{3/2}}}{5ab}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{5ab\left(a+bx^2\right)^{5/2}}$$

$$\Big\downarrow \text{208}$$

$$\frac{\frac{2x(aC+4Ab)}{3a^2\sqrt{a+bx^2}}-\frac{5a^2D-bx(aC+4Ab)}{3ab(a+bx^2)^{3/2}}}{5ab}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{5ab\left(a+bx^2\right)^{5/2}}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/(a + b*x^2)^(7/2),x]
```

output
```
-1/5*(a*(B - (a*D)/b) - (A*b - a*C)*x)/(a*b*(a + b*x^2)^(5/2)) + (-1/3*(5*
a^2*D - b*(4*A*b + a*C)*x)/(a*b*(a + b*x^2)^(3/2)) + (2*(4*A*b + a*C)*x)/(
3*a^2*Sqrt[a + b*x^2]))/(5*a*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 454
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*d
 - b*c*x)/(2*a*b*(p + 1)))*(a + b*x^2)^(p + 1), x] + Simp[c*((2*p + 3)/(2*a
*(p + 1)))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d}, x] && L
tQ[p, -1] && NeQ[p, -3/2]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.81

method	result
gosper	$\frac{8A\,b^4x^5+2Ca\,b^3x^5+20Aa\,b^3x^3+5C\,a^2b^2x^3-5Dx^2a^3b+15Ax\,a^2b^2-3B\,a^3b-2Da^4}{15(b\,x^2+a)^{\frac{5}{2}}a^3b^2}$
trager	$\frac{8A\,b^4x^5+2Ca\,b^3x^5+20Aa\,b^3x^3+5C\,a^2b^2x^3-5Dx^2a^3b+15Ax\,a^2b^2-3B\,a^3b-2Da^4}{15(b\,x^2+a)^{\frac{5}{2}}a^3b^2}$
orering	$\frac{8A\,b^4x^5+2Ca\,b^3x^5+20Aa\,b^3x^3+5C\,a^2b^2x^3-5Dx^2a^3b+15Ax\,a^2b^2-3B\,a^3b-2Da^4}{15(b\,x^2+a)^{\frac{5}{2}}a^3b^2}$
default	$A\left(\frac{x}{5a(b\,x^2+a)^{\frac{5}{2}}}+\frac{\frac{4x}{15a\left(b\,x^2+a\right)^{\frac{3}{2}}}+\frac{8x}{15a^2\sqrt{b\,x^2+a}}}{a}\right)-\frac{B}{5b(b\,x^2+a)^{\frac{5}{2}}}+C\left(-\frac{x}{4b(b\,x^2+a)^{\frac{5}{2}}}+\frac{a\left(\frac{x}{5a\left(b\,x^2+a\right)^{\frac{5}{2}}}+\frac{4x}{15a\left(b\,x^2\right.}\right.}{4b}\right.$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*(8*A*b^4*x^5+2*C*a*b^3*x^5+20*A*a*b^3*x^3+5*C*a^2*b^2*x^3-5*D*a^3*b*x
^2+15*A*a^2*b^2*x-3*B*a^3*b-2*D*a^4)/(b*x^2+a)^(5/2)/a^3/b^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.10

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{7/2}}\,dx =$$
$$-\frac{\left(5\,Da^3bx^2-15\,Aa^2b^2x-2\,(Cab^3+4\,Ab^4)x^5+2\,Da^4+3\,Ba^3b-5\,(Ca^2b^2+4\,Aab^3)x^3\right)\sqrt{bx^2+a}}{15\,(a^3b^5x^6+3\,a^4b^4x^4+3\,a^5b^3x^2+a^6b^2)}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(7/2),x, algorithm="fricas")
```

output
```
-1/15*(5*D*a^3*b*x^2 - 15*A*a^2*b^2*x - 2*(C*a*b^3 + 4*A*b^4)*x^5 + 2*D*a^
4 + 3*B*a^3*b - 5*(C*a^2*b^2 + 4*A*a*b^3)*x^3)*sqrt(b*x^2 + a)/(a^3*b^5*x^
6 + 3*a^4*b^4*x^4 + 3*a^5*b^3*x^2 + a^6*b^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 415 vs. $2(104) = 208$.

Time = 13.65 (sec) , antiderivative size = 777, normalized size of antiderivative = 6.76

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(a + bx^2)^{7/2}}\, dx = \text{Too large to display}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(b*x**2+a)**(7/2),x)
```

output
```
A*(15*a**5*x/(15*a**(17/2)*sqrt(1 + b*x**2/a) + 45*a**(15/2)*b*x**2*sqrt(1
 + b*x**2/a) + 45*a**(13/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 15*a**(11/2)*b*
*3*x**6*sqrt(1 + b*x**2/a)) + 35*a**4*b*x**3/(15*a**(17/2)*sqrt(1 + b*x**2
/a) + 45*a**(15/2)*b*x**2*sqrt(1 + b*x**2/a) + 45*a**(13/2)*b**2*x**4*sqrt
(1 + b*x**2/a) + 15*a**(11/2)*b**3*x**6*sqrt(1 + b*x**2/a)) + 28*a**3*b**2
*x**5/(15*a**(17/2)*sqrt(1 + b*x**2/a) + 45*a**(15/2)*b*x**2*sqrt(1 + b*x*
*2/a) + 45*a**(13/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 15*a**(11/2)*b**3*x**6
*sqrt(1 + b*x**2/a)) + 8*a**2*b**3*x**7/(15*a**(17/2)*sqrt(1 + b*x**2/a) +
 45*a**(15/2)*b*x**2*sqrt(1 + b*x**2/a) + 45*a**(13/2)*b**2*x**4*sqrt(1 +
b*x**2/a) + 15*a**(11/2)*b**3*x**6*sqrt(1 + b*x**2/a))) + B*Piecewise((-1/
(5*a**2*b*sqrt(a + b*x**2) + 10*a*b**2*x**2*sqrt(a + b*x**2) + 5*b**3*x**4
*sqrt(a + b*x**2)), Ne(b, 0)), (x**2/(2*a**(7/2)), True)) + C*(5*a*x**3/(1
5*a**(9/2)*sqrt(1 + b*x**2/a) + 30*a**(7/2)*b*x**2*sqrt(1 + b*x**2/a) + 15
*a**(5/2)*b**2*x**4*sqrt(1 + b*x**2/a)) + 2*b*x**5/(15*a**(9/2)*sqrt(1 + b
*x**2/a) + 30*a**(7/2)*b*x**2*sqrt(1 + b*x**2/a) + 15*a**(5/2)*b**2*x**4*s
qrt(1 + b*x**2/a))) + D*Piecewise((-2*a/(15*a**2*b**2*sqrt(a + b*x**2) + 3
0*a*b**3*x**2*sqrt(a + b*x**2) + 15*b**4*x**4*sqrt(a + b*x**2)) - 5*b*x**2
/(15*a**2*b**2*sqrt(a + b*x**2) + 30*a*b**3*x**2*sqrt(a + b*x**2) + 15*b**
4*x**4*sqrt(a + b*x**2)), Ne(b, 0)), (x**4/(4*a**(7/2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.32

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{7/2}}\,dx = -\frac{Dx^2}{3\,(bx^2+a)^{\frac{5}{2}}b}+\frac{8\,Ax}{15\,\sqrt{bx^2+a}a^3} + \frac{4\,Ax}{15\,(bx^2+a)^{\frac{3}{2}}a^2}+\frac{Ax}{5\,(bx^2+a)^{\frac{5}{2}}a}-\frac{Cx}{5\,(bx^2+a)^{\frac{5}{2}}b}+\frac{2\,Cx}{15\,\sqrt{bx^2+a}a^2b} + \frac{Cx}{15\,(bx^2+a)^{\frac{3}{2}}ab}-\frac{2\,Da}{15\,(bx^2+a)^{\frac{5}{2}}b^2}-\frac{B}{5\,(bx^2+a)^{\frac{5}{2}}b}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(7/2),x, algorithm="maxima")
```

output
```
-1/3*D*x^2/((b*x^2 + a)^(5/2)*b) + 8/15*A*x/(sqrt(b*x^2 + a)*a^3) + 4/15*A
*x/((b*x^2 + a)^(3/2)*a^2) + 1/5*A*x/((b*x^2 + a)^(5/2)*a) - 1/5*C*x/((b*x
^2 + a)^(5/2)*b) + 2/15*C*x/(sqrt(b*x^2 + a)*a^2*b) + 1/15*C*x/((b*x^2 + a
)^(3/2)*a*b) - 2/15*D*a/((b*x^2 + a)^(5/2)*b^2) - 1/5*B/((b*x^2 + a)^(5/2)
*b)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.90

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{7/2}}\,dx = \frac{\left(\left(x\left(\frac{2\,(Cab^3+4\,Ab^4)x^2}{a^3b^2}+\frac{5\,(Ca^2b^2+4\,Aab^3)}{a^3b^2}\right)-\frac{5\,D}{b}\right)x+\frac{15\,A}{a}\right)x-\frac{2\,Da^4+3\,Ba^3b}{a^3b^2}}{15\,(bx^2+a)^{\frac{5}{2}}}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(7/2),x, algorithm="giac")
```

output
```
1/15*(((x*(2*(C*a*b^3 + 4*A*b^4)*x^2/(a^3*b^2) + 5*(C*a^2*b^2 + 4*A*a*b^3)
/(a^3*b^2)) - 5*D/b)*x + 15*A/a)*x - (2*D*a^4 + 3*B*a^3*b)/(a^3*b^2))/(b*x
^2 + a)^(5/2)
```

Mupad [B] (verification not implemented)

Time = 1.09 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.09

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{7/2}}\,dx = \frac{8\,A\,x\,\left(b\,x^2+a\right)^2+3\,A\,a^2\,x+4\,A\,a\,x\,\left(b\,x^2+a\right)}{15\,a^3\,\left(b\,x^2+a\right)^{5/2}} - \frac{B}{5\,b\,\left(b\,x^2+a\right)^{5/2}} - \frac{\left(5\,b\,x^2+2\,a\right)\,D}{15\,b^2\,\left(b\,x^2+a\right)^{5/2}} + \frac{C\,x^3}{3\,a\,\left(b\,x^2+a\right)^{5/2}} + \frac{2\,C\,b\,x^5}{15\,a^2\,\left(b\,x^2+a\right)^{5/2}}$$

input
```
int((A + B*x + C*x^2 + x^3*D)/(a + b*x^2)^(7/2),x)
```

output
```
(8*A*x*(a + b*x^2)^2 + 3*A*a^2*x + 4*A*a*x*(a + b*x^2))/(15*a^3*(a + b*x^2
)^(5/2)) - B/(5*b*(a + b*x^2)^(5/2)) - ((2*a + 5*b*x^2)*D)/(15*b^2*(a + b*
x^2)^(5/2)) + (C*x^3)/(3*a*(a + b*x^2)^(5/2)) + (2*C*b*x^5)/(15*a^2*(a + b
*x^2)^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 258, normalized size of antiderivative = 2.24

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{7/2}}\,dx = \frac{-2\sqrt{b\,x^2+a}\,a^3d+15\sqrt{b\,x^2+a}\,a^2b^2x-3\sqrt{b\,x^2+a}\,a^2b^2-5\sqrt{b\,x^2+a}\,a^2ba}{}$$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(7/2),x)
```

output
```
( - 2*sqrt(a + b*x**2)*a**3*d + 15*sqrt(a + b*x**2)*a**2*b**2*x - 3*sqrt(a
 + b*x**2)*a**2*b**2 - 5*sqrt(a + b*x**2)*a**2*b*d*x**2 + 20*sqrt(a + b*x*
*2)*a*b**3*x**3 + 5*sqrt(a + b*x**2)*a*b**2*c*x**3 + 8*sqrt(a + b*x**2)*b*
*4*x**5 + 2*sqrt(a + b*x**2)*b**3*c*x**5 - 8*sqrt(b)*a**3*b - 2*sqrt(b)*a*
*3*c - 24*sqrt(b)*a**2*b**2*x**2 - 6*sqrt(b)*a**2*b*c*x**2 - 24*sqrt(b)*a*
b**3*x**4 - 6*sqrt(b)*a*b**2*c*x**4 - 8*sqrt(b)*b**4*x**6 - 2*sqrt(b)*b**3
*c*x**6)/(15*a**2*b**2*(a**3 + 3*a**2*b*x**2 + 3*a*b**2*x**4 + b**3*x**6))
```

3.60 $\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{9/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 145

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{9/2}}\,dx = -\frac{a(bB-aD)-b(Ab-aC)x}{7ab^2(a+bx^2)^{7/2}} - \frac{7a^2D-b(6Ab+aC)x}{35a^2b^2(a+bx^2)^{5/2}} + \frac{4(6Ab+aC)x}{105a^3b(a+bx^2)^{3/2}} + \frac{8(6Ab+aC)x}{105a^4b\sqrt{a+bx^2}}$$

output
```
-1/7*(a*(B*b-D*a)-b*(A*b-C*a)*x)/a/b^2/(b*x^2+a)^(7/2)-1/35*(7*a^2*D-b*(6*
A*b+C*a)*x)/a^2/b^2/(b*x^2+a)^(5/2)+4/105*(6*A*b+C*a)*x/a^3/b/(b*x^2+a)^(3
/2)+8/105*(6*A*b+C*a)*x/a^4/b/(b*x^2+a)^(1/2)
```

Mathematica [A] (verified)

Time = 1.26 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.76

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{9/2}}\,dx = \frac{-6a^5D+48Ab^5x^7+35a^3b^2x(3A+Cx^2)+8ab^4x^5(21A+Cx^2)+14a^2b^3x}{105a^4b^2(a+bx^2)^{7/2}}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/(a + b*x^2)^(9/2),x]
```

output
```
(-6*a^5*D + 48*A*b^5*x^7 + 35*a^3*b^2*x*(3*A + C*x^2) + 8*a*b^4*x^5*(21*A
+ C*x^2) + 14*a^2*b^3*x^3*(15*A + 2*C*x^2) - 3*a^4*b*(5*B + 7*D*x^2))/(105
*a^4*b^2*(a + b*x^2)^(7/2))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2345, 25, 27, 454, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{9/2}}\,dx$$

$$\Big\downarrow \text{2345}$$

$$-\frac{\int -\frac{b\left(6A+\frac{aC}{b}\right)+7aDx}{b(bx^2+a)^{7/2}}dx}{7a}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{7ab\,(a+bx^2)^{7/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{6Ab+aC+7aDx}{b(bx^2+a)^{7/2}}dx}{7a}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{7ab\,(a+bx^2)^{7/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{6Ab+aC+7aDx}{(bx^2+a)^{7/2}}dx}{7ab}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{7ab\,(a+bx^2)^{7/2}}$$

$$\Big\downarrow \text{454}$$

$$\frac{\frac{4(aC+6Ab)\int \frac{1}{\left(bx^2+a\right)^{5/2}}dx}{5a}-\frac{7a^2D-bx(aC+6Ab)}{5ab(a+bx^2)^{5/2}}}{7ab}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{7ab\,(a+bx^2)^{7/2}}$$

$$\Big\downarrow \text{209}$$

$$\frac{\frac{4(aC+6Ab)\left(\frac{2\int \frac{1}{\left(bx^2+a\right)^{3/2}}dx}{3a}+\frac{x}{3a\left(a+bx^2\right)^{3/2}}\right)}{5a}-\frac{7a^2D-bx(aC+6Ab)}{5ab(a+bx^2)^{5/2}}}{7ab}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{7ab\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \text{208}$$

$$\frac{\frac{4\left(\frac{2x}{3a^2\sqrt{a+bx^2}}+\frac{x}{3a\left(a+bx^2\right)^{3/2}}\right)(aC+6Ab)}{5a}-\frac{7a^2D-bx(aC+6Ab)}{5ab(a+bx^2)^{5/2}}}{7ab}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{7ab\left(a+bx^2\right)^{7/2}}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/(a + b*x^2)^(9/2),x]
```

output
```
-1/7*(a*(B - (a*D)/b) - (A*b - a*C)*x)/(a*b*(a + b*x^2)^(7/2)) + (-1/5*(7*
a^2*D - b*(6*A*b + a*C)*x)/(a*b*(a + b*x^2)^(5/2)) + (4*(6*A*b + a*C)*(x/(
3*a*(a + b*x^2)^(3/2)) + (2*x)/(3*a^2*Sqrt[a + b*x^2])))/(5*a))/(7*a*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 454
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*d
 - b*c*x)/(2*a*b*(p + 1)))*(a + b*x^2)^(p + 1), x] + Simp[c*((2*p + 3)/(2*a
*(p + 1)))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d}, x] && L
tQ[p, -1] && NeQ[p, -3/2]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.81

method	result
gosper	$\frac{48A\,b^5x^7+8Ca\,b^4x^7+168Aa\,b^4x^5+28C\,a^2b^3x^5+210a^2A\,b^3x^3+35C\,a^3b^2x^3-21Dx^2a^4b+105a^3b^2Ax-15a^4bB-6Da^5}{105(b\,x^2+a)^{\frac{7}{2}}a^4b^2}$
trager	$\frac{48A\,b^5x^7+8Ca\,b^4x^7+168Aa\,b^4x^5+28C\,a^2b^3x^5+210a^2A\,b^3x^3+35C\,a^3b^2x^3-21Dx^2a^4b+105a^3b^2Ax-15a^4bB-6Da^5}{105(b\,x^2+a)^{\frac{7}{2}}a^4b^2}$
orering	$\frac{48A\,b^5x^7+8Ca\,b^4x^7+168Aa\,b^4x^5+28C\,a^2b^3x^5+210a^2A\,b^3x^3+35C\,a^3b^2x^3-21Dx^2a^4b+105a^3b^2Ax-15a^4bB-6Da^5}{105(b\,x^2+a)^{\frac{7}{2}}a^4b^2}$
default	$A\left(\frac{x}{7a(b\,x^2+a)^{\frac{7}{2}}}+\frac{\frac{6x}{35a\left(b\,x^2+a\right)^{\frac{5}{2}}}+\frac{6\left(\frac{4x}{15a\left(b\,x^2+a\right)^{\frac{3}{2}}}+\frac{8x}{15a^2\sqrt{b\,x^2+a}}\right)}{7a}}{a}\right)-\frac{B}{7b(b\,x^2+a)^{\frac{7}{2}}}+C\left(-\frac{x}{6b(b\,x^2+a)^{\frac{7}{2}}}+\frac{a\left(\frac{\cdots}{7a(\cdots}\right.}{\cdots}\right.$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
1/105*(48*A*b^5*x^7+8*C*a*b^4*x^7+168*A*a*b^4*x^5+28*C*a^2*b^3*x^5+210*A*a
^2*b^3*x^3+35*C*a^3*b^2*x^3-21*D*a^4*b*x^2+105*A*a^3*b^2*x-15*B*a^4*b-6*D*
a^5)/(b*x^2+a)^(7/2)/a^4/b^2
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.10

$$\int \frac{A+Bx+Cx^2+Dx^3}{\left(a+bx^2\right)^{9/2}}\,dx =$$
$$-\frac{(21\,Da^4bx^2 - 8\,(Cab^4 + 6\,Ab^5)x^7 - 105\,Aa^3b^2x + 6\,Da^5 + 15\,Ba^4b - 28\,(Ca^2b^3 + 6\,Aab^4)x^5 - 35\,(Ca^3b}{105\,(a^4b^6x^8 + 4\,a^5b^5x^6 + 6\,a^6b^4x^4 + 4\,a^7b^3x^2 + a^8b^2)}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

output
```
-1/105*(21*D*a^4*b*x^2 - 8*(C*a*b^4 + 6*A*b^5)*x^7 - 105*A*a^3*b^2*x + 6*D
*a^5 + 15*B*a^4*b - 28*(C*a^2*b^3 + 6*A*a*b^4)*x^5 - 35*(C*a^3*b^2 + 6*A*a
^2*b^3)*x^3)*sqrt(b*x^2 + a)/(a^4*b^6*x^8 + 4*a^5*b^5*x^6 + 6*a^6*b^4*x^4
+ 4*a^7*b^3*x^2 + a^8*b^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1266 vs. 2(134) = 268.

Time = 28.82 (sec) , antiderivative size = 2064, normalized size of antiderivative = 14.23

$$\int \frac{A+Bx+Cx^2+Dx^3}{\left(a+bx^2\right)^{9/2}}\,dx = \text{Too large to display}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(b*x**2+a)**(9/2),x)
```

output
```
A*(35*a**14*x/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt
(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2
)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a
) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*
sqrt(1 + b*x**2/a)) + 175*a**13*b*x**3/(35*a**(37/2)*sqrt(1 + b*x**2/a) +
210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 +
 b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b*
*4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) +
 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a)) + 371*a**12*b**2*x**5/(35*a**
(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*
a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 +
b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**
5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a)) +
 429*a**11*b**3*x**7/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x*
*2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a
**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b
*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6
*x**12*sqrt(1 + b*x**2/a)) + 286*a**10*b**4*x**9/(35*a**(37/2)*sqrt(1 + b*
x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**
4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.29

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{9/2}}\,dx = -\frac{Dx^2}{5\,(bx^2+a)^{\frac{7}{2}}b}+\frac{16\,Ax}{35\,\sqrt{bx^2+a}a^4}+\frac{8\,Ax}{35\,(bx^2+a)^{\frac{3}{2}}a^3}+\frac{6\,Ax}{35\,(bx^2+a)^{\frac{5}{2}}a^2}+\frac{Ax}{7\,(bx^2+a)^{\frac{7}{2}}a}-\frac{Cx}{7\,(bx^2+a)^{\frac{7}{2}}b}+\frac{8\,Cx}{105\,\sqrt{bx^2+a}a^3b}+\frac{4\,Cx}{105\,(bx^2+a)^{\frac{3}{2}}a^2b}+\frac{Cx}{35\,(bx^2+a)^{\frac{5}{2}}ab}-\frac{2\,Da}{35\,(bx^2+a)^{\frac{7}{2}}b^2}-\frac{B}{7\,(bx^2+a)^{\frac{7}{2}}b}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

output
```
-1/5*D*x^2/((b*x^2 + a)^(7/2)*b) + 16/35*A*x/(sqrt(b*x^2 + a)*a^4) + 8/35*
A*x/((b*x^2 + a)^(3/2)*a^3) + 6/35*A*x/((b*x^2 + a)^(5/2)*a^2) + 1/7*A*x/(
(b*x^2 + a)^(7/2)*a) - 1/7*C*x/((b*x^2 + a)^(7/2)*b) + 8/105*C*x/(sqrt(b*x
^2 + a)*a^3*b) + 4/105*C*x/((b*x^2 + a)^(3/2)*a^2*b) + 1/35*C*x/((b*x^2 +
a)^(5/2)*a*b) - 2/35*D*a/((b*x^2 + a)^(7/2)*b^2) - 1/7*B/((b*x^2 + a)^(7/2
)*b)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.95

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{9/2}}\,dx = \frac{\left(\left(\left(4\,x^2\left(\frac{2\,(Cab^5+6\,Ab^6)x^2}{a^4b^3}+\frac{7\,(Ca^2b^4+6\,Aab^5)}{a^4b^3}\right)+\frac{35\,(Ca^3b^3+6\,Aa^2b^4)}{a^4b^3}\right)x-\frac{21\,D}{b}\right)x+ \cdots}{105\,(bx^2+a)^{\frac{7}{2}}}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="giac")
```

output
```
1/105*((((4*x^2*(2*(C*a*b^5 + 6*A*b^6)*x^2/(a^4*b^3) + 7*(C*a^2*b^4 + 6*A*
a*b^5)/(a^4*b^3)) + 35*(C*a^3*b^3 + 6*A*a^2*b^4)/(a^4*b^3))*x - 21*D/b)*x
+ 105*A/a)*x - 3*(2*D*a^5*b + 5*B*a^4*b^2)/(a^4*b^3))/(b*x^2 + a)^(7/2)
```

Mupad [B] (verification not implemented)

Time = 1.06 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.23

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{9/2}}\,dx = \frac{16\,A\,x}{35\,a^4\,\sqrt{b\,x^2+a}} - \frac{(7\,b\,x^2+2\,a)\,D}{35\,b^2\,(b\,x^2+a)^{7/2}} - \frac{B}{7\,b\,(b\,x^2+a)^{7/2}} + \frac{8\,A\,x}{35\,a^3\,(b\,x^2+a)^{3/2}} + \frac{6\,A\,x}{35\,a^2\,(b\,x^2+a)^{5/2}} + \frac{A\,x}{7\,a\,(b\,x^2+a)^{7/2}} - \frac{C\,x}{7\,b\,(b\,x^2+a)^{7/2}} + \frac{8\,C\,x}{105\,a^3\,b\,\sqrt{b\,x^2+a}} + \frac{4\,C\,x}{105\,a^2\,b\,(b\,x^2+a)^{3/2}} + \frac{C\,x}{35\,a\,b\,(b\,x^2+a)^{5/2}}$$

input
```
int((A + B*x + C*x^2 + x^3*D)/(a + b*x^2)^(9/2),x)
```

output
```
(16*A*x)/(35*a^4*(a + b*x^2)^(1/2)) - ((2*a + 7*b*x^2)*D)/(35*b^2*(a + b*x
^2)^(7/2)) - B/(7*b*(a + b*x^2)^(7/2)) + (8*A*x)/(35*a^3*(a + b*x^2)^(3/2)
) + (6*A*x)/(35*a^2*(a + b*x^2)^(5/2)) + (A*x)/(7*a*(a + b*x^2)^(7/2)) - (
C*x)/(7*b*(a + b*x^2)^(7/2)) + (8*C*x)/(105*a^3*b*(a + b*x^2)^(1/2)) + (4*
C*x)/(105*a^2*b*(a + b*x^2)^(3/2)) + (C*x)/(35*a*b*(a + b*x^2)^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 335, normalized size of antiderivative = 2.31

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(a + bx^2)^{9/2}} \, dx = \frac{-6\sqrt{b\,x^2+a}\,a^4d + 105\sqrt{b\,x^2+a}\,a^3b^2x - 15\sqrt{b\,x^2+a}\,a^3b^2 - 21\sqrt{b\,x^2+a}\,a}{}$$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(9/2),x)
```

output
```
( - 6*sqrt(a + b*x**2)*a**4*d + 105*sqrt(a + b*x**2)*a**3*b**2*x - 15*sqrt
(a + b*x**2)*a**3*b**2 - 21*sqrt(a + b*x**2)*a**3*b*d*x**2 + 210*sqrt(a +
b*x**2)*a**2*b**3*x**3 + 35*sqrt(a + b*x**2)*a**2*b**2*c*x**3 + 168*sqrt(a
 + b*x**2)*a*b**4*x**5 + 28*sqrt(a + b*x**2)*a*b**3*c*x**5 + 48*sqrt(a + b
*x**2)*b**5*x**7 + 8*sqrt(a + b*x**2)*b**4*c*x**7 - 48*sqrt(b)*a**4*b - 8*
sqrt(b)*a**4*c - 192*sqrt(b)*a**3*b**2*x**2 - 32*sqrt(b)*a**3*b*c*x**2 - 2
88*sqrt(b)*a**2*b**3*x**4 - 48*sqrt(b)*a**2*b**2*c*x**4 - 192*sqrt(b)*a*b*
*4*x**6 - 32*sqrt(b)*a*b**3*c*x**6 - 48*sqrt(b)*b**5*x**8 - 8*sqrt(b)*b**4
*c*x**8)/(105*a**3*b**2*(a**4 + 4*a**3*b*x**2 + 6*a**2*b**2*x**4 + 4*a*b**
3*x**6 + b**4*x**8))
```

3.61 $\int \frac{-x+x^3}{\sqrt{-2+x^2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 23

$$\int \frac{-x+x^3}{\sqrt{-2+x^2}}\,dx = \sqrt{-2+x^2}+\frac{1}{3}\left(-2+x^2\right)^{3/2}$$

output
```
(x^2-2)^(1/2)+1/3*(x^2-2)^(3/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \frac{-x+x^3}{\sqrt{-2+x^2}}\,dx = \frac{1}{3}\sqrt{-2+x^2}\left(1+x^2\right)$$

input
```
Integrate[(-x + x^3)/Sqrt[-2 + x^2],x]
```

output
```
(Sqrt[-2 + x^2]*(1 + x^2))/3
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.26, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {2027, 353, 25, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3-x}{\sqrt{x^2-2}}dx$$

$$\Big\downarrow \text{2027}$$

$$\int \frac{x\left(x^2-1\right)}{\sqrt{x^2-2}}dx$$

$$\Big\downarrow \text{353}$$

$$\frac{1}{2}\int -\frac{1-x^2}{\sqrt{x^2-2}}dx^2$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{2}\int \frac{1-x^2}{\sqrt{x^2-2}}dx^2$$

$$\Big\downarrow \text{53}$$

$$-\frac{1}{2}\int \left(-\sqrt{x^2-2}-\frac{1}{\sqrt{x^2-2}}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{2}{3}\left(x^2-2\right)^{3/2}+2\sqrt{x^2-2}\right)$$

input
```
Int[(-x + x^3)/Sqrt[-2 + x^2],x]
```

output
```
(2*Sqrt[-2 + x^2] + (2*(-2 + x^2)^(3/2))/3)/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.65

method	result	size
gosper	$\frac{(x^2+1)\sqrt{x^2-2}}{3}$	15
risch	$\frac{(x^2+1)\sqrt{x^2-2}}{3}$	15
pseudoelliptic	$\frac{(x^2+1)\sqrt{x^2-2}}{3}$	15
trager	$\left(\frac{x^2}{3}+\frac{1}{3}\right)\sqrt{x^2-2}$	16
default	$\frac{x^2\sqrt{x^2-2}}{3}+\frac{\sqrt{x^2-2}}{3}$	23
orering	$\frac{\sqrt{x^2-2}\,(x^2+1)\,(x^3-x)}{3(-1+x)x(1+x)}$	35
meijerg	$\frac{\sqrt{2}\,\sqrt{-\operatorname{signum}\left(-1+\frac{x^2}{2}\right)}\left(\frac{4\sqrt{\pi}}{3}-\frac{\sqrt{\pi}\,(2x^2+8)\sqrt{1-\frac{x^2}{2}}}{6}\right)}{\sqrt{\pi}\,\sqrt{\operatorname{signum}\left(-1+\frac{x^2}{2}\right)}}+\frac{\sqrt{2}\,\sqrt{-\operatorname{signum}\left(-1+\frac{x^2}{2}\right)}\left(-2\sqrt{\pi}+2\sqrt{\pi}\,\sqrt{1-\frac{x^2}{2}}\right)}{2\sqrt{\pi}\,\sqrt{\operatorname{signum}\left(-1+\frac{x^2}{2}\right)}}$	108

input
```
int((x^3-x)/(x^2-2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(x^2+1)*(x^2-2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.61

$$\int \frac{-x+x^3}{\sqrt{-2+x^2}}\,dx=\frac{1}{3}\left(x^2+1\right)\sqrt{x^2-2}$$

input
```
integrate((x^3-x)/(x^2-2)^(1/2),x, algorithm="fricas")
```

output
```
1/3*(x^2 + 1)*sqrt(x^2 - 2)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{-x+x^3}{\sqrt{-2+x^2}}\,dx = \frac{x^2\sqrt{x^2-2}}{3} + \frac{\sqrt{x^2-2}}{3}$$

input
```
integrate((x**3-x)/(x**2-2)**(1/2),x)
```

output
```
x**2*sqrt(x**2 - 2)/3 + sqrt(x**2 - 2)/3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{-x+x^3}{\sqrt{-2+x^2}}\,dx = \frac{1}{3}\sqrt{x^2-2}x^2 + \frac{1}{3}\sqrt{x^2-2}$$

input
```
integrate((x^3-x)/(x^2-2)^(1/2),x, algorithm="maxima")
```

output
```
1/3*sqrt(x^2 - 2)*x^2 + 1/3*sqrt(x^2 - 2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{-x+x^3}{\sqrt{-2+x^2}}\,dx = \frac{1}{3}\left(x^2-2\right)^{\frac{3}{2}} + \sqrt{x^2-2}$$

input
```
integrate((x^3-x)/(x^2-2)^(1/2),x, algorithm="giac")
```

output
```
1/3*(x^2 - 2)^(3/2) + sqrt(x^2 - 2)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.61

$$\int \frac{-x + x^3}{\sqrt{-2 + x^2}} \, dx = \frac{(x^2 + 1)\sqrt{x^2 - 2}}{3}$$

input
```
int(-(x - x^3)/(x^2 - 2)^(1/2),x)
```

output
```
((x^2 + 1)*(x^2 - 2)^(1/2))/3
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.57

$$\int \frac{-x + x^3}{\sqrt{-2 + x^2}} \, dx = \frac{\sqrt{x^2 - 2}\,(x^2 + 1)}{3}$$

input
```
int((x^3-x)/(x^2-2)^(1/2),x)
```

output
```
(sqrt(x**2 - 2)*(x**2 + 1))/3
```

3.62 $\int \sqrt[3]{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx$

Optimal result

Integrand size = 27, antiderivative size = 352

$$\int \sqrt[3]{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{3(11Ab-3aC)x\sqrt[3]{a+bx^2}}{55b}+\frac{3(bB-aD)\left(a+bx^2\right)^{4/3}}{8b^2}+\frac{3Cx\left(a+bx^2\right)^{4/3}}{11b}+\frac{3D\left(a+bx^2\right)^{7/3}}{14b^2}-\frac{2\,3^{3/4}\sqrt{2-\sqrt{3}}a(11Ab-3aC)\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\right.\right.}{55b^2x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

output
```
3/55*(11*A*b-3*C*a)*x*(b*x^2+a)^(1/3)/b+3/8*(B*b-D*a)*(b*x^2+a)^(4/3)/b^2+
3/11*C*x*(b*x^2+a)^(4/3)/b+3/14*D*(b*x^2+a)^(7/3)/b^2-2/55*3^(3/4)*(1/2*6^
(1/2)-1/2*2^(1/2))*a*(11*A*b-3*C*a)*(a^(1/3)-(b*x^2+a)^(1/3))*((a^(2/3)+a^
(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3
))^2)^(1/2)*EllipticF(((1+3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a
^(1/3)-(b*x^2+a)^(1/3)),2*I-I*3^(1/2))/b^2/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^
(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.17 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.27

$$\int \sqrt[3]{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$=\frac{\sqrt[3]{a+bx^2}\left(-3(a+bx^2)\left(33aD-b(77B+56Cx+44Dx^2)\right)+\frac{56b(11Ab-3aC)x\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{\sqrt[3]{1+\frac{bx^2}{a}}}\right)}{616b^2}$$

input
```
Integrate[(a + b*x^2)^(1/3)*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
((a + b*x^2)^(1/3)*(-3*(a + b*x^2)*(33*a*D - b*(77*B + 56*C*x + 44*D*x^2))
 + (56*b*(11*A*b - 3*a*C)*x*Hypergeometric2F1[-1/3, 1/2, 3/2, -((b*x^2)/a)
])/(1 + (b*x^2)/a)^(1/3)))/(616*b^2)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2346, 27, 2346, 27, 455, 211, 234, 760}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$\Big\downarrow \ 2346$$
$$\frac{3\int \frac{2}{3}\sqrt[3]{bx^2+a}\left(7bCx^2+(7bB-3aD)x+7Ab\right)dx}{14b}+\frac{3Dx^2\left(a+bx^2\right)^{4/3}}{14b}$$
$$\Big\downarrow \ 27$$
$$\frac{\int \sqrt[3]{bx^2+a}\left(7bCx^2+(7bB-3aD)x+7Ab\right)dx}{7b}+\frac{3Dx^2\left(a+bx^2\right)^{4/3}}{14b}$$

$\Big\downarrow$ 2346

$$\frac{\frac{3\int \frac{1}{3}b(7(11Ab-3aC)+11(7bB-3aD)x)\sqrt[3]{bx^2+a}dx}{11b}+\frac{21}{11}Cx\left(a+bx^2\right)^{4/3}}{7b}+\frac{3Dx^2\left(a+bx^2\right)^{4/3}}{14b}$$

$\Big\downarrow$ 27

$$\frac{\frac{1}{11}\int(7(11Ab-3aC)+11(7bB-3aD)x)\sqrt[3]{bx^2+a}dx+\frac{21}{11}Cx\left(a+bx^2\right)^{4/3}}{7b}+\frac{3Dx^2\left(a+bx^2\right)^{4/3}}{14b}$$

$\Big\downarrow$ 455

$$\frac{\frac{1}{11}\left(7(11Ab-3aC)\int\sqrt[3]{bx^2+a}dx+\frac{33\left(a+bx^2\right)^{4/3}(7bB-3aD)}{8b}\right)+\frac{21}{11}Cx\left(a+bx^2\right)^{4/3}}{7b}+\frac{3Dx^2\left(a+bx^2\right)^{4/3}}{14b}$$

$\Big\downarrow$ 211

$$\frac{\frac{1}{11}\left(7(11Ab-3aC)\left(\frac{2}{5}a\int\frac{1}{\left(bx^2+a\right)^{2/3}}dx+\frac{3}{5}x\sqrt[3]{a+bx^2}\right)+\frac{33\left(a+bx^2\right)^{4/3}(7bB-3aD)}{8b}\right)+\frac{21}{11}Cx\left(a+bx^2\right)^{4/3}}{7b}+\frac{3Dx^2\left(a+bx^2\right)^{4/3}}{14b}$$

$\Big\downarrow$ 234

$$\frac{\frac{1}{11}\left(7(11Ab-3aC)\left(\frac{3a\sqrt{bx^2}\int\frac{1}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}}{5bx}+\frac{3}{5}x\sqrt[3]{a+bx^2}\right)+\frac{33\left(a+bx^2\right)^{4/3}(7bB-3aD)}{8b}\right)+\frac{21}{11}Cx\left(a+bx^2\right)^{4/3}}{7b}+\frac{3Dx^2\left(a+bx^2\right)^{4/3}}{14b}$$

$\Big\downarrow$ 760

$$\frac{\frac{1}{11}\left(7(11Ab-3aC)\left(\frac{3}{5}x\sqrt[3]{a+bx^2}-\frac{2\,3^{3/4}\sqrt{2-\sqrt{3}}a\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\text{EllipticF}\left(\text{arcsi}\right.}{5bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}\right.\right.}{7b}$$

$$\frac{3Dx^2\left(a+bx^2\right)^{4/3}}{14b}$$

input
```
Int[(a + b*x^2)^(1/3)*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(3*D*x^2*(a + b*x^2)^(4/3))/(14*b) + ((21*C*x*(a + b*x^2)^(4/3))/11 + ((33
*(7*b*B - 3*a*D)*(a + b*x^2)^(4/3))/(8*b) + 7*(11*A*b - 3*a*C)*((3*x*(a +
b*x^2)^(1/3))/5 - (2*3^(3/4)*Sqrt[2 - Sqrt[3]]*a*(a^(1/3) - (a + b*x^2)^(1
/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 -
Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a
^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))],
-7 + 4*Sqrt[3]])/(5*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1
 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)])))/11)/(7*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 234
```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Maple [F]

$$\int \left(b\,x^{2}+a\right)^{\frac{1}{3}} \left(D x^{3}+C\,x^{2}+B x +A\right)d x$$

input
```
int((b*x^2+a)^(1/3)*(D*x^3+C*x^2+B*x+A),x)
```

output
```
int((b*x^2+a)^(1/3)*(D*x^3+C*x^2+B*x+A),x)
```

Fricas [F]

$$\int \sqrt[3]{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx=\int \left(Dx^{3}+Cx^{2}+Bx+A\right)\left(bx^{2}+a\right)^{\frac{1}{3}}\,dx$$

input
```
integrate((b*x^2+a)^(1/3)*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
integral((D*x^3 + C*x^2 + B*x + A)*(b*x^2 + a)^(1/3), x)
```

Sympy [A] (verification not implemented)

Time = 1.60 (sec) , antiderivative size = 321, normalized size of antiderivative = 0.91

$$\int \sqrt[3]{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$= A\sqrt[3]{a}x\,{}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)+B\left(\begin{cases}\frac{\sqrt[3]{a}x^2}{2} & \text{for } b=0\\ \frac{3(a+bx^2)^{\frac{4}{3}}}{8b} & \text{otherwise}\end{cases}\right)$$
$$+\frac{C\sqrt[3]{a}x^3\,{}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{3}-\frac{9Da^{\frac{13}{3}}\sqrt[3]{1+\frac{bx^2}{a}}}{56a^2b^2+56ab^3x^2}$$
$$+\frac{9Da^{\frac{13}{3}}}{56a^2b^2+56ab^3x^2}-\frac{6Da^{\frac{10}{3}}bx^2\sqrt[3]{1+\frac{bx^2}{a}}}{56a^2b^2+56ab^3x^2}+\frac{9Da^{\frac{10}{3}}bx^2}{56a^2b^2+56ab^3x^2}$$
$$+\frac{15Da^{\frac{7}{3}}b^2x^4\sqrt[3]{1+\frac{bx^2}{a}}}{56a^2b^2+56ab^3x^2}+\frac{12Da^{\frac{4}{3}}b^3x^6\sqrt[3]{1+\frac{bx^2}{a}}}{56a^2b^2+56ab^3x^2}$$

input
```
integrate((b*x**2+a)**(1/3)*(D*x**3+C*x**2+B*x+A),x)
```

output
```
A*a**(1/3)*x*hyper((-1/3, 1/2), (3/2,), b*x**2*exp_polar(I*pi)/a) + B*Piec
ewise((a**(1/3)*x**2/2, Eq(b, 0)), (3*(a + b*x**2)**(4/3)/(8*b), True)) +
C*a**(1/3)*x**3*hyper((-1/3, 3/2), (5/2,), b*x**2*exp_polar(I*pi)/a)/3 - 9
*D*a**(13/3)*(1 + b*x**2/a)**(1/3)/(56*a**2*b**2 + 56*a*b**3*x**2) + 9*D*a
**(13/3)/(56*a**2*b**2 + 56*a*b**3*x**2) - 6*D*a**(10/3)*b*x**2*(1 + b*x**
2/a)**(1/3)/(56*a**2*b**2 + 56*a*b**3*x**2) + 9*D*a**(10/3)*b*x**2/(56*a**
2*b**2 + 56*a*b**3*x**2) + 15*D*a**(7/3)*b**2*x**4*(1 + b*x**2/a)**(1/3)/(
56*a**2*b**2 + 56*a*b**3*x**2) + 12*D*a**(4/3)*b**3*x**6*(1 + b*x**2/a)**(
1/3)/(56*a**2*b**2 + 56*a*b**3*x**2)
```

Maxima [F]

$$\int \sqrt[3]{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx=\int \left(Dx^3+Cx^2+Bx+A\right)\left(bx^2+a\right)^{\frac{1}{3}}\,dx$$

input
```
integrate((b*x^2+a)^(1/3)*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)*(b*x^2 + a)^(1/3), x)
```

Giac [F]

$$\int \sqrt[3]{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx=\int \left(Dx^3+Cx^2+Bx+A\right)\left(bx^2+a\right)^{\frac{1}{3}}\,dx$$

input
```
integrate((b*x^2+a)^(1/3)*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)*(b*x^2 + a)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx=\int {\left(b\,x^2+a\right)}^{1/3}\,\left(A+B\,x+C\,x^2+x^3\,D\right)\,dx$$

input
```
int((a + b*x^2)^(1/3)*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
int((a + b*x^2)^(1/3)*(A + B*x + C*x^2 + x^3*D), x)
```

Reduce [F]

$$\int \sqrt[3]{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$=\frac{-495(b\,x^2+a)^{\frac{1}{3}}\,a^2d+1848(b\,x^2+a)^{\frac{1}{3}}\,a\,b^2x+1155(b\,x^2+a)^{\frac{1}{3}}\,a\,b^2+336(b\,x^2+a)^{\frac{1}{3}}\,abcx+165(b\,x^2+a}{}$$

input
```
int((b*x^2+a)^(1/3)*(D*x^3+C*x^2+B*x+A),x)
```

output
```
( - 495*(a + b*x**2)**(1/3)*a**2*d + 1848*(a + b*x**2)**(1/3)*a*b**2*x + 1
155*(a + b*x**2)**(1/3)*a*b**2 + 336*(a + b*x**2)**(1/3)*a*b*c*x + 165*(a
+ b*x**2)**(1/3)*a*b*d*x**2 + 1155*(a + b*x**2)**(1/3)*b**3*x**2 + 840*(a
+ b*x**2)**(1/3)*b**2*c*x**3 + 660*(a + b*x**2)**(1/3)*b**2*d*x**4 + 1232*
int((a + b*x**2)**(1/3)/(a + b*x**2),x)*a**2*b**2 - 336*int((a + b*x**2)**
(1/3)/(a + b*x**2),x)*a**2*b*c)/(3080*b**2)
```

3.63 $\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{2/3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 323

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{2/3}}\,dx=\frac{3(bB-aD)\sqrt[3]{a+bx^2}}{2b^2}+\frac{3Cx\sqrt[3]{a+bx^2}}{5b}+\frac{3D(a+bx^2)^{4/3}}{8b^2}-\frac{3^{3/4}\sqrt{2-\sqrt{3}}(5Ab-3aC)\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\ldots}{\left(1-\sqrt{3}\right)\ldots}\right.\right.}{5b^2x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

output
```
3/2*(B*b-D*a)*(b*x^2+a)^(1/3)/b^2+3/5*C*x*(b*x^2+a)^(1/3)/b+3/8*D*(b*x^2+a
)^(4/3)/b^2-1/5*3^(3/4)*(1/2*6^(1/2)-1/2*2^(1/2))*(5*A*b-3*C*a)*(a^(1/3)-(
b*x^2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/((1-3^(
1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)*EllipticF(((1+3^(1/2))*a^(1/3)-(b*
x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3)),2*I-I*3^(1/2))/b^2/x/(
-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2
)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.16 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.30

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{2/3}}\,dx = \frac{-3(a+bx^2)\,(15aD-b(20B+8Cx+5Dx^2))+8b(5Ab-3aC)x\left(1+\frac{bx^2}{a}\right.}{40b^2\,(a+bx^2)^{2/3}}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/(a + b*x^2)^(2/3),x]
```

output
```
(-3*(a + b*x^2)*(15*a*D - b*(20*B + 8*C*x + 5*D*x^2)) + 8*b*(5*A*b - 3*a*C
)*x*(1 + (b*x^2)/a)^(2/3)*Hypergeometric2F1[1/2, 2/3, 3/2, -((b*x^2)/a)])/
(40*b^2*(a + b*x^2)^(2/3))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2346, 27, 2346, 27, 455, 234, 760}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{2/3}}dx$$

$$\Big\downarrow \text{ 2346}$$

$$\frac{3\int \frac{2(4bCx^2+(4bB-3aD)x+4Ab)}{3(bx^2+a)^{2/3}}dx}{8b}+\frac{3Dx^2\sqrt[3]{a+bx^2}}{8b}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{4bCx^2+(4bB-3aD)x+4Ab}{(bx^2+a)^{2/3}}dx}{4b}+\frac{3Dx^2\sqrt[3]{a+bx^2}}{8b}$$

$$\Big\downarrow \text{ 2346}$$

$$\frac{\frac{3\int \frac{b(4(5Ab-3aC)+5(4bB-3aD)x)}{3\left(bx^2+a\right)^{2/3}}dx}{5b}+\frac{12}{5}Cx\sqrt[3]{a+bx^2}}{4b}+\frac{3Dx^2\sqrt[3]{a+bx^2}}{8b}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{1}{5}\int \frac{4(5Ab-3aC)+5(4bB-3aD)x}{\left(bx^2+a\right)^{2/3}}dx+\frac{12}{5}Cx\sqrt[3]{a+bx^2}}{4b}+\frac{3Dx^2\sqrt[3]{a+bx^2}}{8b}$$

$$\Big\downarrow \; 455$$

$$\frac{\frac{1}{5}\left(4(5Ab-3aC)\int \frac{1}{\left(bx^2+a\right)^{2/3}}dx+\frac{15\sqrt[3]{a+bx^2}(4bB-3aD)}{2b}\right)+\frac{12}{5}Cx\sqrt[3]{a+bx^2}}{4b}+\frac{3Dx^2\sqrt[3]{a+bx^2}}{8b}$$

$$\Big\downarrow \; 234$$

$$\frac{\frac{1}{5}\left(\frac{6\sqrt{bx^2}(5Ab-3aC)\int \frac{1}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}}{bx}+\frac{15\sqrt[3]{a+bx^2}(4bB-3aD)}{2b}\right)+\frac{12}{5}Cx\sqrt[3]{a+bx^2}}{4b}+\frac{3Dx^2\sqrt[3]{a+bx^2}}{8b}$$

$$\Big\downarrow \; 760$$

$$\frac{\frac{1}{5}\left(\frac{15\sqrt[3]{a+bx^2}(4bB-3aD)}{2b}-\frac{4\,3^{3/4}\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}(5Ab-3aC)\,\text{EllipticF}\left(\arcsin\right.}{bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}\right.}{4b}$$

$$\frac{3Dx^2\sqrt[3]{a+bx^2}}{8b}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/(a + b*x^2)^(2/3),x]
```

output
```
(3*D*x^2*(a + b*x^2)^(1/3))/(8*b) + ((12*C*x*(a + b*x^2)^(1/3))/5 + ((15*(
4*b*B - 3*a*D)*(a + b*x^2)^(1/3))/(2*b) - (4*3^(3/4)*Sqrt[2 - Sqrt[3]]*(5*
A*b - 3*a*C)*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*
x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3)
)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sq
rt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(b*x*Sqrt[-((a^(1/3
)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3
))^2)]))/5)/(4*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 234
```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Maple [F]

$$\int \frac{D x^{3}+C \,x^{2}+B x +A}{\left(b \,x^{2}+a \right)^{\frac{2}{3}}}d x$$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(2/3),x)
```

output
```
int((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(2/3),x)
```

Fricas [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{2/3}}\,dx=\int \frac{Dx^{3}+Cx^{2}+Bx+A}{(bx^{2}+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

output
```
integral((D*x^3 + C*x^2 + B*x + A)/(b*x^2 + a)^(2/3), x)
```

Sympy [A] (verification not implemented)

Time = 1.43 (sec) , antiderivative size = 272, normalized size of antiderivative = 0.84

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{2/3}}\,dx=\frac{Ax\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{a^{\frac{2}{3}}}+B\left(\begin{cases}\frac{x^2}{2a^{\frac{2}{3}}} & \text{for}\ b=0\\ \frac{3\sqrt[3]{a+bx^2}}{2b} & \text{otherwise}\end{cases}\right)+\frac{Cx^3\,{}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}}-\frac{9Da^{\frac{10}{3}}\sqrt[3]{1+\frac{bx^2}{a}}}{8a^2b^2+8ab^3x^2}+\frac{9Da^{\frac{10}{3}}}{8a^2b^2+8ab^3x^2}-\frac{6Da^{\frac{7}{3}}bx^2\sqrt[3]{1+\frac{bx^2}{a}}}{8a^2b^2+8ab^3x^2}+\frac{9Da^{\frac{7}{3}}bx^2}{8a^2b^2+8ab^3x^2}+\frac{3Da^{\frac{4}{3}}b^2x^4\sqrt[3]{1+\frac{bx^2}{a}}}{8a^2b^2+8ab^3x^2}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(b*x**2+a)**(2/3),x)
```

output
```
A*x*hyper((1/2, 2/3), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(2/3) + B*Piece
wise((x**2/(2*a**(2/3)), Eq(b, 0)), (3*(a + b*x**2)**(1/3)/(2*b), True)) +
 C*x**3*hyper((2/3, 3/2), (5/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(2/3)) -
 9*D*a**(10/3)*(1 + b*x**2/a)**(1/3)/(8*a**2*b**2 + 8*a*b**3*x**2) + 9*D*a
**(10/3)/(8*a**2*b**2 + 8*a*b**3*x**2) - 6*D*a**(7/3)*b*x**2*(1 + b*x**2/a
)**(1/3)/(8*a**2*b**2 + 8*a*b**3*x**2) + 9*D*a**(7/3)*b*x**2/(8*a**2*b**2
+ 8*a*b**3*x**2) + 3*D*a**(4/3)*b**2*x**4*(1 + b*x**2/a)**(1/3)/(8*a**2*b*
*2 + 8*a*b**3*x**2)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{2/3}}\,dx=\int \frac{Dx^3+Cx^2+Bx+A}{(bx^2+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(2/3),x, algorithm="maxima")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)/(b*x^2 + a)^(2/3), x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{2/3}}\,dx=\int \frac{Dx^3+Cx^2+Bx+A}{(bx^2+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(2/3),x, algorithm="giac")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)/(b*x^2 + a)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{\left(a+bx^2\right)^{2/3}}\,dx=\int \frac{A+B\,x+C\,x^2+x^3\,D}{\left(b\,x^2+a\right)^{2/3}}\,dx$$

input
```
int((A + B*x + C*x^2 + x^3*D)/(a + b*x^2)^(2/3),x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/(a + b*x^2)^(2/3), x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{\left(a+bx^2\right)^{2/3}}\,dx=\frac{3(b\,x^2+a)^{\frac{1}{3}}}{2}+\left(\int \frac{x^3}{(b\,x^2+a)^{\frac{2}{3}}}dx\right)d+\left(\int \frac{x^2}{(b\,x^2+a)^{\frac{2}{3}}}dx\right)c+\left(\int \frac{1}{(b\,x^2+a)^{\frac{2}{3}}}dx\right)a$$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(2/3),x)
```

output
```
(3*(a + b*x**2)**(1/3) + 2*int(x**3/(a + b*x**2)**(2/3),x)*d + 2*int(x**2/
(a + b*x**2)**(2/3),x)*c + 2*int(1/(a + b*x**2)**(2/3),x)*a)/2
```

3.64 $\int \frac{A+Bx+Cx^2+Dx^3}{\left(a+bx^2\right)^{5/3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 323

$$\int \frac{A+Bx+Cx^2+Dx^3}{\left(a+bx^2\right)^{5/3}}\,dx=-\frac{3(a(bB-aD)-b(Ab-aC)x)}{4ab^2\left(a+bx^2\right)^{2/3}}+\frac{3D\sqrt[3]{a+bx^2}}{2b^2}$$
$$-\frac{3^{3/4}\sqrt{2-\sqrt{3}}(Ab+3aC)\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{\ldots}}{\left(1-\sqrt{3}\right)\sqrt[3]{\ldots}}\right.\right.}{4ab^2x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

output
```
1/4*(-3*a*(B*b-D*a)+3*b*(A*b-C*a)*x)/a/b^2/(b*x^2+a)^(2/3)+3/2*D*(b*x^2+a)
^(1/3)/b^2-1/4*3^(3/4)*(1/2*6^(1/2)-1/2*2^(1/2))*(A*b+3*C*a)*(a^(1/3)-(b*x
^2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/((1-3^(1/2
))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)*EllipticF(((1+3^(1/2))*a^(1/3)-(b*x^2
+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3)),2*I-I*3^(1/2))/a/b^2/x/(-
a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)
^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.29

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{5/3}}\,dx = \frac{9a^2D+3Ab^2x-3ab(B+x(C-2Dx))+b(Ab+3aC)x\left(1+\frac{bx^2}{a}\right)^{2/3}\text{Hyp}}{4ab^2\,(a+bx^2)^{2/3}}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/(a + b*x^2)^(5/3),x]
```

output
```
(9*a^2*D + 3*A*b^2*x - 3*a*b*(B + x*(C - 2*D*x)) + b*(A*b + 3*a*C)*x*(1 +
(b*x^2)/a)^(2/3)*Hypergeometric2F1[1/2, 2/3, 3/2, -((b*x^2)/a)])/(4*a*b^2*
(a + b*x^2)^(2/3))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {2345, 27, 455, 234, 760}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{5/3}}dx$$

$$\Big\downarrow \text{2345}$$

$$-\frac{3\int -\frac{b\left(A+\frac{3aC}{b}\right)+4aDx}{3b(bx^2+a)^{2/3}}dx}{4a}-\frac{3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab\,(a+bx^2)^{2/3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{Ab+3aC+4aDx}{(bx^2+a)^{2/3}}dx}{4ab}-\frac{3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab\,(a+bx^2)^{2/3}}$$

$$\Big\downarrow \text{455}$$

$$\frac{(3aC+Ab)\int \frac{1}{(bx^2+a)^{2/3}}dx+\frac{6aD\sqrt[3]{a+bx^2}}{b}}{4ab}-\frac{3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab\left(a+bx^2\right)^{2/3}}$$

$$\Big\downarrow \; 234$$

$$\frac{\frac{3\sqrt{bx^2}(3aC+Ab)\int \frac{1}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}}{2bx}+\frac{6aD\sqrt[3]{a+bx^2}}{b}}{4ab}-\frac{3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab\left(a+bx^2\right)^{2/3}}$$

$$\Big\downarrow \; 760$$

$$\frac{\frac{6aD\sqrt[3]{a+bx^2}}{b}-\frac{3^{3/4}\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}(3aC+Ab)\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right),-7+4\sqrt{3}\right)}{bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}}{4ab}-\frac{3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab\left(a+bx^2\right)^{2/3}}$$

input

```
Int[(A + B*x + C*x^2 + D*x^3)/(a + b*x^2)^(5/3),x]
```

output

```
(-3*(a*(B - (a*D)/b) - (A*b - a*C)*x))/(4*a*b*(a + b*x^2)^(2/3)) + ((6*a*D
*(a + b*x^2)^(1/3))/b - (3^(3/4)*Sqrt[2 - Sqrt[3]]*(A*b + 3*a*C)*(a^(1/3)
- (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^
2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticF[ArcSin[
((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a +
b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^
2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)]))/(4*a*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 234
```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Maple [F]

$$\int \frac{Dx^3 + C\,x^2 + Bx + A}{\left(b\,x^2 + a\right)^{\frac{5}{3}}}dx$$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(5/3),x)
```

output
```
int((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(5/3),x)
```

Fricas [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{5/3}}\,dx=\int \frac{Dx^3+Cx^2+Bx+A}{(bx^2+a)^{\frac{5}{3}}}\,dx$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(5/3),x, algorithm="fricas")
```

output
```
integral((D*x^3 + C*x^2 + B*x + A)*(b*x^2 + a)^(1/3)/(b^2*x^4 + 2*a*b*x^2
+ a^2), x)
```

Sympy [A] (verification not implemented)

Time = 4.03 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.41

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{5/3}}\,dx=\frac{Ax\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{5}{3}\\ \frac{3}{2}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{a^{\frac{5}{3}}}+B\left(\begin{cases}-\frac{3}{4b(a+bx^2)^{\frac{2}{3}}} & \text{for } b\neq 0\\ \frac{x^2}{2a^{\frac{5}{3}}} & \text{otherwise}\end{cases}\right)+\frac{Cx^3\,{}_2F_1\left(\left.\begin{matrix}\frac{3}{2},\frac{5}{3}\\ \frac{5}{2}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{3a^{\frac{5}{3}}}+D\left(\begin{cases}\frac{9a}{4b^2(a+bx^2)^{\frac{2}{3}}}+\frac{3x^2}{2b(a+bx^2)^{\frac{2}{3}}} & \text{for } b\neq 0\\ \frac{x^4}{4a^{\frac{5}{3}}} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(b*x**2+a)**(5/3),x)
```

output
```
A*x*hyper((1/2, 5/3), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(5/3) + B*Piece
wise((-3/(4*b*(a + b*x**2)**(2/3)), Ne(b, 0)), (x**2/(2*a**(5/3)), True))
+ C*x**3*hyper((3/2, 5/3), (5/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(5/3))
+ D*Piecewise((9*a/(4*b**2*(a + b*x**2)**(2/3)) + 3*x**2/(2*b*(a + b*x**2)
**(2/3)), Ne(b, 0)), (x**4/(4*a**(5/3)), True))
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{5/3}}\,dx=\int \frac{Dx^3+Cx^2+Bx+A}{(bx^2+a)^{\frac{5}{3}}}\,dx$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(5/3),x, algorithm="maxima")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)/(b*x^2 + a)^(5/3), x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{5/3}}\,dx=\int \frac{Dx^3+Cx^2+Bx+A}{(bx^2+a)^{\frac{5}{3}}}\,dx$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(5/3),x, algorithm="giac")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)/(b*x^2 + a)^(5/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{5/3}}\,dx = \int \frac{A+B\,x+C\,x^2+x^3\,D}{(b\,x^2+a)^{5/3}}\,dx$$

input
```
int((A + B*x + C*x^2 + x^3*D)/(a + b*x^2)^(5/3),x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/(a + b*x^2)^(5/3), x)
```

Reduce [F]

$$\begin{aligned}\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{5/3}}\,dx &= \left(\int \frac{x^3}{(b\,x^2+a)^{\frac{2}{3}}\,a+(b\,x^2+a)^{\frac{2}{3}}\,b\,x^2}dx\right)d \\ &+ \left(\int \frac{x^2}{(b\,x^2+a)^{\frac{2}{3}}\,a+(b\,x^2+a)^{\frac{2}{3}}\,b\,x^2}dx\right)c \\ &+ \left(\int \frac{x}{(b\,x^2+a)^{\frac{2}{3}}\,a+(b\,x^2+a)^{\frac{2}{3}}\,b\,x^2}dx\right)b \\ &+ \left(\int \frac{1}{(b\,x^2+a)^{\frac{2}{3}}\,a+(b\,x^2+a)^{\frac{2}{3}}\,b\,x^2}dx\right)a\end{aligned}$$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(5/3),x)
```

output
```
int(x**3/((a + b*x**2)**(2/3)*a + (a + b*x**2)**(2/3)*b*x**2),x)*d + int(x
**2/((a + b*x**2)**(2/3)*a + (a + b*x**2)**(2/3)*b*x**2),x)*c + int(x/((a
+ b*x**2)**(2/3)*a + (a + b*x**2)**(2/3)*b*x**2),x)*b + int(1/((a + b*x**2
)**(2/3)*a + (a + b*x**2)**(2/3)*b*x**2),x)*a
```

3.65 $\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{8/3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 346

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{8/3}}\,dx = -\frac{3(a(bB-aD)-b(Ab-aC)x)}{10ab^2(a+bx^2)^{5/3}} - \frac{3(10a^2D-b(7Ab+3aC)x)}{40a^2b^2(a+bx^2)^{2/3}}$$
$$-\frac{3^{3/4}\sqrt{2-\sqrt{3}}(7Ab+3aC)\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)}{\left(1-\sqrt{3}\right)}\right)\right)}{40a^2b^2x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

output
```
1/10*(-3*a*(B*b-D*a)+3*b*(A*b-C*a)*x)/a/b^2/(b*x^2+a)^(5/3)-3/40*(10*a^2*D
-b*(7*A*b+3*C*a)*x)/a^2/b^2/(b*x^2+a)^(2/3)-1/40*3^(3/4)*(1/2*6^(1/2)-1/2*
2^(1/2))*(7*A*b+3*C*a)*(a^(1/3)-(b*x^2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+
a)^(1/3)+(b*x^2+a)^(2/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)*E
llipticF(((1+3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2
+a)^(1/3)),2*I-I*3^(1/2))/a^2/b^2/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/((
1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.09 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.36

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{8/3}}\,dx = \frac{-18a^3D+21Ab^3x^3+3ab^2x(11A+3Cx^2)-3a^2b(4B+x(C+10Dx))+ \cdots}{40a^2b^2\left(a \cdots\right.}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/(a + b*x^2)^(8/3),x]
```

output
```
(-18*a^3*D + 21*A*b^3*x^3 + 3*a*b^2*x*(11*A + 3*C*x^2) - 3*a^2*b*(4*B + x*
(C + 10*D*x)) + b*(7*A*b + 3*a*C)*x*(a + b*x^2)*(1 + (b*x^2)/a)^(2/3)*Hype
rgeometric2F1[1/2, 2/3, 3/2, -((b*x^2)/a)])/(40*a^2*b^2*(a + b*x^2)^(5/3))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {2345, 27, 454, 234, 760}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{8/3}}dx$$

$$\Big\downarrow \text{2345}$$

$$-\frac{3\int -\frac{b\left(7A+\frac{3aC}{b}\right)+10aDx}{3b(bx^2+a)^{5/3}}dx}{10a}-\frac{3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{10ab\left(a+bx^2\right)^{5/3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{7Ab+3aC+10aDx}{(bx^2+a)^{5/3}}dx}{10ab}-\frac{3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{10ab\left(a+bx^2\right)^{5/3}}$$

$$\Big\downarrow \text{454}$$

$$\frac{\frac{(3aC+7Ab)\int \frac{1}{\left(bx^2+a\right)^{2/3}}dx}{4a}-\frac{3(10a^2D-bx(3aC+7Ab))}{4ab(a+bx^2)^{2/3}}}{10ab}-\frac{3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{10ab\left(a+bx^2\right)^{5/3}}$$

$\Big\downarrow$ 234

$$\frac{\frac{3\sqrt{bx^2}(3aC+7Ab)\int \frac{1}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}}{8abx}-\frac{3(10a^2D-bx(3aC+7Ab))}{4ab(a+bx^2)^{2/3}}}{10ab}-\frac{3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{10ab\left(a+bx^2\right)^{5/3}}$$

$\Big\downarrow$ 760

$$-\frac{\frac{3^{3/4}\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}(3aC+7Ab)\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right),-7+\ldots\right)}{4abx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}}{10ab}$$

$$\frac{3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{10ab\left(a+bx^2\right)^{5/3}}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/(a + b*x^2)^(8/3),x]
```

output
```
(-3*(a*(B - (a*D)/b) - (A*b - a*C)*x))/(10*a*b*(a + b*x^2)^(5/3)) + ((-3*(
10*a^2*D - b*(7*A*b + 3*a*C)*x))/(4*a*b*(a + b*x^2)^(2/3)) - (3^(3/4)*Sqrt
[2 - Sqrt[3]]*(7*A*b + 3*a*C)*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3)
+ a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) -
(a + b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2
)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(4
*a*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/
3) - (a + b*x^2)^(1/3))^2)]))/(10*a*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 234
```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 454
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*d
 - b*c*x)/(2*a*b*(p + 1)))*(a + b*x^2)^(p + 1), x] + Simp[c*((2*p + 3)/(2*a
*(p + 1)))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d}, x] && L
tQ[p, -1] && NeQ[p, -3/2]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Maple [F]

$$\int \frac{Dx^{3}+C\,x^{2}+Bx+A}{\left(b\,x^{2}+a\right)^{\frac{8}{3}}}dx$$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(8/3),x)
```

output
```
int((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(8/3),x)
```

Fricas [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{8/3}}\,dx=\int \frac{Dx^{3}+Cx^{2}+Bx+A}{\left(bx^{2}+a\right)^{\frac{8}{3}}}\,dx$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(8/3),x, algorithm="fricas")
```

output
```
integral((D*x^3 + C*x^2 + B*x + A)*(b*x^2 + a)^(1/3)/(b^3*x^6 + 3*a*b^2*x^
4 + 3*a^2*b*x^2 + a^3), x)
```

Sympy [A] (verification not implemented)

Time = 8.81 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.58

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{8/3}}\,dx=\frac{Ax\,{}_{2}F_{1}\left(\begin{matrix}\frac{1}{2},\frac{8}{3}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{a^{\frac{8}{3}}}+B\left(\begin{cases}-\frac{3}{10ab(a+bx^{2})^{\frac{2}{3}}+10b^{2}x^{2}(a+bx^{2})^{\frac{2}{3}}} & \text{for}\;b\neq 0\\ \frac{x^{2}}{2a^{\frac{8}{3}}} & \text{otherwise}\end{cases}\right)+\frac{Cx^{3}\,{}_{2}F_{1}\left(\begin{matrix}\frac{3}{2},\frac{8}{3}\\ \frac{5}{2}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{3a^{\frac{8}{3}}}+D\left(\begin{cases}-\frac{9a}{20ab^{2}(a+bx^{2})^{\frac{2}{3}}+20b^{3}x^{2}(a+bx^{2})^{\frac{2}{3}}}-\frac{15bx^{2}}{20ab^{2}(a+bx^{2})^{\frac{2}{3}}+20b^{3}x^{2}(a+bx^{2})^{\frac{2}{3}}} & \text{for}\;b\neq 0\\ \frac{x^{4}}{4a^{\frac{8}{3}}} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(b*x**2+a)**(8/3),x)
```

output
```
A*x*hyper((1/2, 8/3), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(8/3) + B*Piece
wise((-3/(10*a*b*(a + b*x**2)**(2/3) + 10*b**2*x**2*(a + b*x**2)**(2/3)),
Ne(b, 0)), (x**2/(2*a**(8/3)), True)) + C*x**3*hyper((3/2, 8/3), (5/2,), b
*x**2*exp_polar(I*pi)/a)/(3*a**(8/3)) + D*Piecewise((-9*a/(20*a*b**2*(a +
b*x**2)**(2/3) + 20*b**3*x**2*(a + b*x**2)**(2/3)) - 15*b*x**2/(20*a*b**2*
(a + b*x**2)**(2/3) + 20*b**3*x**2*(a + b*x**2)**(2/3)), Ne(b, 0)), (x**4/
(4*a**(8/3)), True))
```

Maxima [F]

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(a + bx^2)^{8/3}}\,dx = \int \frac{Dx^3 + Cx^2 + Bx + A}{(bx^2 + a)^{\frac{8}{3}}}\,dx$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(8/3),x, algorithm="maxima")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)/(b*x^2 + a)^(8/3), x)
```

Giac [F]

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(a + bx^2)^{8/3}}\,dx = \int \frac{Dx^3 + Cx^2 + Bx + A}{(bx^2 + a)^{\frac{8}{3}}}\,dx$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(8/3),x, algorithm="giac")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)/(b*x^2 + a)^(8/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{8/3}}\,dx = \int \frac{A+B\,x+C\,x^2+x^3\,D}{(b\,x^2+a)^{8/3}}\,dx$$

input
```
int((A + B*x + C*x^2 + x^3*D)/(a + b*x^2)^(8/3),x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/(a + b*x^2)^(8/3), x)
```

Reduce [F]

$$\begin{aligned}\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{8/3}}\,dx &= \left(\int \frac{x^3}{(b\,x^2+a)^{\frac{2}{3}}\,a^2+2\,(b\,x^2+a)^{\frac{2}{3}}\,ab\,x^2+(b\,x^2+a)^{\frac{2}{3}}\,b^2x^4}dx\right)d \\ &+ \left(\int \frac{x^2}{(b\,x^2+a)^{\frac{2}{3}}\,a^2+2\,(b\,x^2+a)^{\frac{2}{3}}\,ab\,x^2+(b\,x^2+a)^{\frac{2}{3}}\,b^2x^4}dx\right)c \\ &+ \left(\int \frac{x}{(b\,x^2+a)^{\frac{2}{3}}\,a^2+2\,(b\,x^2+a)^{\frac{2}{3}}\,ab\,x^2+(b\,x^2+a)^{\frac{2}{3}}\,b^2x^4}dx\right)b \\ &+ \left(\int \frac{1}{(b\,x^2+a)^{\frac{2}{3}}\,a^2+2\,(b\,x^2+a)^{\frac{2}{3}}\,ab\,x^2+(b\,x^2+a)^{\frac{2}{3}}\,b^2x^4}dx\right)a\end{aligned}$$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(8/3),x)
```

output
```
int(x**3/((a + b*x**2)**(2/3)*a**2 + 2*(a + b*x**2)**(2/3)*a*b*x**2 + (a +
 b*x**2)**(2/3)*b**2*x**4),x)*d + int(x**2/((a + b*x**2)**(2/3)*a**2 + 2*(
a + b*x**2)**(2/3)*a*b*x**2 + (a + b*x**2)**(2/3)*b**2*x**4),x)*c + int(x/
((a + b*x**2)**(2/3)*a**2 + 2*(a + b*x**2)**(2/3)*a*b*x**2 + (a + b*x**2)*
*(2/3)*b**2*x**4),x)*b + int(1/((a + b*x**2)**(2/3)*a**2 + 2*(a + b*x**2)*
*(2/3)*a*b*x**2 + (a + b*x**2)**(2/3)*b**2*x**4),x)*a
```

3.66 $\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{11/3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 377

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{11/3}}\,dx=-\frac{3(a(bB-aD)-b(Ab-aC)x)}{16ab^2(a+bx^2)^{8/3}} -\frac{3(16a^2D-b(13Ab+3aC)x)}{160a^2b^2(a+bx^2)^{5/3}}+\frac{21(13Ab+3aC)x}{640a^3b(a+bx^2)^{2/3}} -\frac{7\,3^{3/4}\sqrt{2-\sqrt{3}}(13Ab+3aC)\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{(1+\cdots}{(1-\cdots}\right.\right.}{640a^3b^2x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

output
```
1/16*(-3*a*(B*b-D*a)+3*b*(A*b-C*a)*x)/a/b^2/(b*x^2+a)^(8/3)-3/160*(16*a^2*
D-b*(13*A*b+3*C*a)*x)/a^2/b^2/(b*x^2+a)^(5/3)+21/640*(13*A*b+3*C*a)*x/a^3/
b/(b*x^2+a)^(2/3)-7/640*3^(3/4)*(1/2*6^(1/2)-1/2*2^(1/2))*(13*A*b+3*C*a)*(
a^(1/3)-(b*x^2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3)
)/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)*EllipticF(((1+3^(1/2))*a^
(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3)),2*I-I*3^(1/2)
)/a^3/b^2/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^
2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.12 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.40

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{11/3}}\,dx = \frac{-72a^4D+273Ab^4x^5+9ab^3x^3(78A+7Cx^2)+9a^2b^2x(61A+18Cx^2)-3a$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/(a + b*x^2)^(11/3),x]
```

output
```
(-72*a^4*D + 273*A*b^4*x^5 + 9*a*b^3*x^3*(78*A + 7*C*x^2) + 9*a^2*b^2*x*(6
1*A + 18*C*x^2) - 3*a^3*b*(40*B + x*(7*C + 64*D*x)) + 7*b*(13*A*b + 3*a*C)
*x*(a + b*x^2)^2*(1 + (b*x^2)/a)^(2/3)*Hypergeometric2F1[1/2, 2/3, 3/2, -(
(b*x^2)/a)])/(640*a^3*b^2*(a + b*x^2)^(8/3))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2345, 27, 454, 215, 234, 760}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{11/3}}dx$$

$$\Big\downarrow \text{2345}$$

$$-\frac{3\int -\frac{b\left(13A+\frac{3aC}{b}\right)+16aDx}{3b(bx^2+a)^{8/3}}dx}{16a}-\frac{3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{16ab\,(a+bx^2)^{8/3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{13Ab+3aC+16aDx}{(bx^2+a)^{8/3}}dx}{16ab}-\frac{3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{16ab\,(a+bx^2)^{8/3}}$$

$$\Big\downarrow \; 454$$

$$\frac{\frac{7(3aC+13Ab)\int \frac{1}{\left(bx^2+a\right)^{5/3}}dx}{10a}-\frac{3\left(16a^2D-bx(3aC+13Ab)\right)}{10ab(a+bx^2)^{5/3}}}{16ab}-\frac{3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{16ab\left(a+bx^2\right)^{8/3}}$$

$$\Big\downarrow \; 215$$

$$\frac{\frac{7(3aC+13Ab)\left(\frac{\int \frac{1}{\left(bx^2+a\right)^{2/3}}dx}{4a}+\frac{3x}{4a\left(a+bx^2\right)^{2/3}}\right)}{10a}-\frac{3\left(16a^2D-bx(3aC+13Ab)\right)}{10ab(a+bx^2)^{5/3}}}{16ab}-\frac{3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{16ab\left(a+bx^2\right)^{8/3}}$$

$$\Big\downarrow \; 234$$

$$\frac{\frac{7(3aC+13Ab)\left(\frac{3\sqrt{bx^2}\int \frac{1}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}}{8abx}+\frac{3x}{4a\left(a+bx^2\right)^{2/3}}\right)}{10a}-\frac{3\left(16a^2D-bx(3aC+13Ab)\right)}{10ab(a+bx^2)^{5/3}}}{16ab}-\frac{3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{16ab\left(a+bx^2\right)^{8/3}}$$

$$\Big\downarrow \; 760$$

$$\frac{\frac{7(3aC+13Ab)\left(\frac{3x}{4a\left(a+bx^2\right)^{2/3}}-\frac{3^{3/4}\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2}}\right.\right.}{4abx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}\right.}{10a}}{16ab}$$

$$\frac{3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{16ab\left(a+bx^2\right)^{8/3}}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/(a + b*x^2)^(11/3),x]
```

output
```
(-3*(a*(B - (a*D)/b) - (A*b - a*C)*x))/(16*a*b*(a + b*x^2)^(8/3)) + ((-3*(
16*a^2*D - b*(13*A*b + 3*a*C)*x))/(10*a*b*(a + b*x^2)^(5/3)) + (7*(13*A*b
+ 3*a*C)*((3*x)/(4*a*(a + b*x^2)^(2/3)) - (3^(3/4)*Sqrt[2 - Sqrt[3]]*(a^(1
/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a +
b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticF[Arc
Sin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (
a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(4*a*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (
a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)])))/(10*
a))/(16*a*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 234
```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 454
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*d
 - b*c*x)/(2*a*b*(p + 1)))*(a + b*x^2)^(p + 1), x] + Simp[c*((2*p + 3)/(2*a
*(p + 1)))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d}, x] && L
tQ[p, -1] && NeQ[p, -3/2]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Maple [F]

$$\int \frac{D x^{3}+C \,x^{2}+B x +A}{\left(b \,x^{2}+a \right)^{\frac{11}{3}}}d x$$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(11/3),x)
```

output
```
int((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(11/3),x)
```

Fricas [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{11/3}}\,dx=\int \frac{D x^{3} + C x^{2} + B x + A}{{\left(b x^{2} + a\right)}^{\frac{11}{3}}}\,{d x}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(11/3),x, algorithm="fricas")
```

output
```
integral((D*x^3 + C*x^2 + B*x + A)*(b*x^2 + a)^(1/3)/(b^4*x^8 + 4*a*b^3*x^
6 + 6*a^2*b^2*x^4 + 4*a^3*b*x^2 + a^4), x)
```

Sympy [A] (verification not implemented)

Time = 19.52 (sec) , antiderivative size = 267, normalized size of antiderivative = 0.71

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{11/3}}\,dx = \frac{Ax\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{11}{3}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{a^{\frac{11}{3}}} + B\left(\begin{cases}-\frac{3}{16a^2b(a+bx^2)^{\frac{2}{3}}+32ab^2x^2(a+bx^2)^{\frac{2}{3}}+16b^3x^4(a+bx^2)^{\frac{2}{3}}} & \text{for}\ b\neq 0\\ \frac{x^2}{2a^{\frac{11}{3}}} & \text{otherwise}\end{cases}\right) + \frac{Cx^3\,{}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{11}{3}\\ \frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{3a^{\frac{11}{3}}} + D\left(\begin{cases}-\frac{9a}{80a^2b^2(a+bx^2)^{\frac{2}{3}}+160ab^3x^2(a+bx^2)^{\frac{2}{3}}+80b^4x^4(a+bx^2)^{\frac{2}{3}}} - \frac{24bx^2}{80a^2b^2(a+bx^2)^{\frac{2}{3}}+160ab^3x^2(a+bx^2)^{\frac{2}{3}}+80b^4x^4(a+bx^2)^{\frac{2}{3}}} & \text{for}\ b\neq \\ \frac{x^4}{4a^{\frac{11}{3}}} & \text{otherw}\end{cases}\right.$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(b*x**2+a)**(11/3),x)
```

output
```
A*x*hyper((1/2, 11/3), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(11/3) + B*Pie
cewise((-3/(16*a**2*b*(a + b*x**2)**(2/3) + 32*a*b**2*x**2*(a + b*x**2)**(
2/3) + 16*b**3*x**4*(a + b*x**2)**(2/3)), Ne(b, 0)), (x**2/(2*a**(11/3)),
True)) + C*x**3*hyper((3/2, 11/3), (5/2,), b*x**2*exp_polar(I*pi)/a)/(3*a*
*(11/3)) + D*Piecewise((-9*a/(80*a**2*b**2*(a + b*x**2)**(2/3) + 160*a*b**
3*x**2*(a + b*x**2)**(2/3) + 80*b**4*x**4*(a + b*x**2)**(2/3)) - 24*b*x**2
/(80*a**2*b**2*(a + b*x**2)**(2/3) + 160*a*b**3*x**2*(a + b*x**2)**(2/3) +
 80*b**4*x**4*(a + b*x**2)**(2/3)), Ne(b, 0)), (x**4/(4*a**(11/3)), True))
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{11/3}}\,dx=\int \frac{Dx^3+Cx^2+Bx+A}{(bx^2+a)^{\frac{11}{3}}}\,dx$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(11/3),x, algorithm="maxima")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)/(b*x^2 + a)^(11/3), x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{11/3}}\,dx=\int \frac{Dx^3+Cx^2+Bx+A}{(bx^2+a)^{\frac{11}{3}}}\,dx$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(11/3),x, algorithm="giac")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)/(b*x^2 + a)^(11/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{11/3}}\,dx=\int \frac{A+B\,x+C\,x^2+x^3\,D}{(b\,x^2+a)^{11/3}}\,dx$$

input
```
int((A + B*x + C*x^2 + x^3*D)/(a + b*x^2)^(11/3),x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/(a + b*x^2)^(11/3), x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{11/3}}\,dx = \left(\int \frac{x^3}{(b\,x^2+a)^{\frac{2}{3}}\,a^3+3\,(b\,x^2+a)^{\frac{2}{3}}\,a^2b\,x^2+3\,(b\,x^2+a)^{\frac{2}{3}}\,a\,b^2x^4+(b\,x^2+a)}\right.$$

$$+\left(\int \frac{x^2}{(b\,x^2+a)^{\frac{2}{3}}\,a^3+3\,(b\,x^2+a)^{\frac{2}{3}}\,a^2b\,x^2+3\,(b\,x^2+a)^{\frac{2}{3}}\,a\,b^2x^4+(b\,x^2+a)^{\frac{2}{3}}\,b^3x^6}dx\right)c$$

$$+\left(\int \frac{x}{(b\,x^2+a)^{\frac{2}{3}}\,a^3+3\,(b\,x^2+a)^{\frac{2}{3}}\,a^2b\,x^2+3\,(b\,x^2+a)^{\frac{2}{3}}\,a\,b^2x^4+(b\,x^2+a)^{\frac{2}{3}}\,b^3x^6}dx\right)b$$

$$+\left(\int \frac{1}{(b\,x^2+a)^{\frac{2}{3}}\,a^3+3\,(b\,x^2+a)^{\frac{2}{3}}\,a^2b\,x^2+3\,(b\,x^2+a)^{\frac{2}{3}}\,a\,b^2x^4+(b\,x^2+a)^{\frac{2}{3}}\,b^3x^6}dx\right)a$$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(11/3),x)
```

output
```
int(x**3/((a + b*x**2)**(2/3)*a**3 + 3*(a + b*x**2)**(2/3)*a**2*b*x**2 + 3
*(a + b*x**2)**(2/3)*a*b**2*x**4 + (a + b*x**2)**(2/3)*b**3*x**6),x)*d + i
nt(x**2/((a + b*x**2)**(2/3)*a**3 + 3*(a + b*x**2)**(2/3)*a**2*b*x**2 + 3*
(a + b*x**2)**(2/3)*a*b**2*x**4 + (a + b*x**2)**(2/3)*b**3*x**6),x)*c + in
t(x/((a + b*x**2)**(2/3)*a**3 + 3*(a + b*x**2)**(2/3)*a**2*b*x**2 + 3*(a +
 b*x**2)**(2/3)*a*b**2*x**4 + (a + b*x**2)**(2/3)*b**3*x**6),x)*b + int(1/
((a + b*x**2)**(2/3)*a**3 + 3*(a + b*x**2)**(2/3)*a**2*b*x**2 + 3*(a + b*x
**2)**(2/3)*a*b**2*x**4 + (a + b*x**2)**(2/3)*b**3*x**6),x)*a
```

3.67 $\int (a+bx^2)^{2/3} (A+Bx+Cx^2+Dx^3)\,dx$

Optimal result

Integrand size = 27, antiderivative size = 657

$$\int (a+bx^2)^{2/3}(A+Bx+Cx^2+Dx^3)\,dx=\frac{3(13Ab-3aC)x(a+bx^2)^{2/3}}{91b}$$
$$+\frac{3(bB-aD)(a+bx^2)^{5/3}}{10b^2}+\frac{3Cx(a+bx^2)^{5/3}}{13b}$$
$$+\frac{3D(a+bx^2)^{8/3}}{16b^2}-\frac{12a(13Ab-3aC)x}{91b\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}$$
$$+\frac{6\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^{4/3}(13Ab-3aC)\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}\cdots}{\left(1-\sqrt{3}\right)\sqrt[3]{a}\cdots}\right.\right.}{91b^2x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$
$$-\frac{4\sqrt{2}3^{3/4}a^{4/3}(13Ab-3aC)\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\cdots}{\left(1-\sqrt{3}\right)\cdots}\right.\right.}{91b^2x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

output
```
3/91*(13*A*b-3*C*a)*x*(b*x^2+a)^(2/3)/b+3/10*(B*b-D*a)*(b*x^2+a)^(5/3)/b^2
+3/13*C*x*(b*x^2+a)^(5/3)/b+3/16*D*(b*x^2+a)^(8/3)/b^2-12/91*a*(13*A*b-3*C
*a)*x/b/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))+6/91*3^(1/4)*(1/2*6^(1/2)+1/
2*2^(1/2))*a^(4/3)*(13*A*b-3*C*a)*(a^(1/3)-(b*x^2+a)^(1/3))*((a^(2/3)+a^(1
/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))
^2)^(1/2)*EllipticE(((1+3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(
1/3)-(b*x^2+a)^(1/3)),2*I-I*3^(1/2))/b^2/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1
/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)-4/91*2^(1/2)*3^(3/4)*a
^(4/3)*(13*A*b-3*C*a)*(a^(1/3)-(b*x^2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a
)^(1/3)+(b*x^2+a)^(2/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)*El
lipticF(((1+3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+
a)^(1/3)),2*I-I*3^(1/2))/b^2/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(
1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.16 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.15

$$\int (a+bx^2)^{2/3} (A+Bx+Cx^2+Dx^3)\,dx = \frac{(a+bx^2)^{2/3}\left(-3(a+bx^2)(39aD-b(104B+80Cx+65Dx^2))+\frac{80b(13Ab-3aC)x\,\text{Hypergeometric2}}{\left(1+\frac{bx^2}{a}\right)^{2/3}}\right.}{1040b^2}$$

input
```
Integrate[(a + b*x^2)^(2/3)*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
((a + b*x^2)^(2/3)*(-3*(a + b*x^2)*(39*a*D - b*(104*B + 80*C*x + 65*D*x^2)
) + (80*b*(13*A*b - 3*a*C)*x*Hypergeometric2F1[-2/3, 1/2, 3/2, -((b*x^2)/a
)]))/(1 + (b*x^2)/a)^(2/3)))/(1040*b^2)
```

Rubi [A] (warning: unable to verify)

Time = 0.66 (sec) , antiderivative size = 683, normalized size of antiderivative = 1.04, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.370, Rules used = {2346, 27, 2346, 27, 455, 211, 233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^{2/3} \left(A+Bx+Cx^2+Dx^3\right) \, dx$$

$$\Big\downarrow \text{ 2346}$$

$$\frac{3\int \frac{2}{3}\left(bx^2+a\right)^{2/3}\left(8bCx^2+(8bB-3aD)x+8Ab\right)dx}{16b}+\frac{3Dx^2\left(a+bx^2\right)^{5/3}}{16b}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \left(bx^2+a\right)^{2/3}\left(8bCx^2+(8bB-3aD)x+8Ab\right)dx}{8b}+\frac{3Dx^2\left(a+bx^2\right)^{5/3}}{16b}$$

$$\Big\downarrow \text{ 2346}$$

$$\frac{\frac{3\int \frac{1}{3}b(8(13Ab-3aC)+13(8bB-3aD)x)\left(bx^2+a\right)^{2/3}dx}{13b}+\frac{24}{13}Cx\left(a+bx^2\right)^{5/3}}{8b}+\frac{3Dx^2\left(a+bx^2\right)^{5/3}}{16b}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{1}{13}\int (8(13Ab-3aC)+13(8bB-3aD)x)\left(bx^2+a\right)^{2/3}dx+\frac{24}{13}Cx\left(a+bx^2\right)^{5/3}}{8b}+\frac{3Dx^2\left(a+bx^2\right)^{5/3}}{16b}$$

$$\Big\downarrow \text{ 455}$$

$$\frac{\frac{1}{13}\left(8(13Ab-3aC)\int \left(bx^2+a\right)^{2/3}dx+\frac{39\left(a+bx^2\right)^{5/3}(8bB-3aD)}{10b}\right)+\frac{24}{13}Cx\left(a+bx^2\right)^{5/3}}{8b}+\frac{3Dx^2\left(a+bx^2\right)^{5/3}}{16b}$$

$$\Big\downarrow \text{ 211}$$

$$\frac{\frac{1}{13}\left(8(13Ab-3aC)\left(\frac{4}{7}a\int\frac{1}{\sqrt[3]{bx^2+a}}dx+\frac{3}{7}x\left(a+bx^2\right)^{2/3}\right)+\frac{39\left(a+bx^2\right)^{5/3}(8bB-3aD)}{10b}\right)+\frac{24}{13}Cx\left(a+bx^2\right)^{5/3}}{8b}+\frac{3Dx^2\left(a+bx^2\right)^{5/3}}{16b}$$

$\Big\downarrow$ 233

$$\frac{\frac{1}{13}\left(8(13Ab-3aC)\left(\frac{6a\sqrt{bx^2}\int\frac{\sqrt[3]{bx^2+a}}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}}{7bx}+\frac{3}{7}x\left(a+bx^2\right)^{2/3}\right)+\frac{39\left(a+bx^2\right)^{5/3}(8bB-3aD)}{10b}\right)+\frac{24}{13}Cx\left(a+\right.}{8b}$$
$$\frac{3Dx^2\left(a+bx^2\right)^{5/3}}{16b}$$

$\Big\downarrow$ 833

$$\frac{\frac{1}{13}\left(8(13Ab-3aC)\left(\frac{6a\sqrt{bx^2}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}\int\frac{1}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}-\int\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}\right)}{7bx}+\frac{3}{7}x\left(a+bx^2\right)^{2/}\right.\right.}{8b}$$
$$\frac{3Dx^2\left(a+bx^2\right)^{5/3}}{16b}$$

$\Big\downarrow$ 760

$$\frac{1}{13}\left(8(13Ab-3aC)\left(\frac{6a\sqrt{bx^2}\left(-\int\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{\cdots}}{\left(\left(1-\sqrt{3}\right)\cdots\right.}}}{\sqrt[4]{3}\sqrt{bx^2}\cdots}\right.}{7bx}\right.\right.$$

$$\frac{3Dx^2\left(a+bx^2\right)^{5/3}}{16b}$$

$\downarrow$ 2418

$$\frac{1}{13}\left(8(13Ab-3aC)\left(\frac{6a\sqrt{bx^2}\left(-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\cdots\right.}{\left(1-\cdots\right.}\right.\right.}{\sqrt[4]{3}\sqrt{bx^2}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}\right.}{\cdots}\right.\right.$$

$$\frac{3Dx^2\left(a+bx^2\right)^{5/3}}{16b}$$

input
```
Int[(a + b*x^2)^(2/3)*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(3*D*x^2*(a + b*x^2)^(5/3))/(16*b) + ((24*C*x*(a + b*x^2)^(5/3))/13 + ((39
*(8*b*B - 3*a*D)*(a + b*x^2)^(5/3))/(10*b) + 8*(13*A*b - 3*a*C)*((3*x*(a +
 b*x^2)^(2/3))/7 + (6*a*Sqrt[b*x^2]*((-2*Sqrt[b*x^2])/((1 - Sqrt[3])*a^(1/
3) - (a + b*x^2)^(1/3)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) - (a
 + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(
2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1
+ Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^
2)^(1/3))], -7 + 4*Sqrt[3]])/(Sqrt[b*x^2]*Sqrt[-((a^(1/3)*(a^(1/3) - (a +
b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)]) - (2*Sqrt[
2 - Sqrt[3]]*(1 + Sqrt[3])*a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(
2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/
3) - (a + b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a +
b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]
])/(3^(1/4)*Sqrt[b*x^2]*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1
 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)])))/(7*b*x)))/13)/(8*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int \left(b\,x^{2}+a\right)^{\frac{2}{3}} \left(D x^{3}+C\,x^{2}+B x +A\right)dx$$

input
```
int((b*x^2+a)^(2/3)*(D*x^3+C*x^2+B*x+A),x)
```

output
```
int((b*x^2+a)^(2/3)*(D*x^3+C*x^2+B*x+A),x)
```

Fricas [F]

$$\int \left(a+bx^2\right)^{2/3}\left(A+Bx+Cx^2+Dx^3\right)\,dx=\int \left(Dx^3+Cx^2+Bx+A\right)\left(bx^2+a\right)^{\frac{2}{3}}\,dx$$

input
```
integrate((b*x^2+a)^(2/3)*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
integral((D*x^3 + C*x^2 + B*x + A)*(b*x^2 + a)^(2/3), x)
```

Sympy [A] (verification not implemented)

Time = 1.36 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.24

$$\int \left(a+bx^2\right)^{2/3}\left(A+Bx+Cx^2+Dx^3\right)\,dx=Aa^{\frac{2}{3}}x\,{}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)+B\left(\begin{cases}\frac{a^{\frac{2}{3}}x^2}{2} & \text{for}\ b=0\\ \frac{3\left(a+bx^2\right)^{\frac{5}{3}}}{10b} & \text{otherwise}\end{cases}\right)+\frac{Ca^{\frac{2}{3}}x^3\,{}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{3}+D\left(\begin{cases}-\frac{9a^2\left(a+bx^2\right)^{\frac{2}{3}}}{80b^2}+\frac{3ax^2\left(a+bx^2\right)^{\frac{2}{3}}}{40b}+\frac{3x^4\left(a+bx^2\right)^{\frac{2}{3}}}{16} & \text{for}\ b\neq 0\\ \frac{a^{\frac{2}{3}}x^4}{4} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((b*x**2+a)**(2/3)*(D*x**3+C*x**2+B*x+A),x)
```

output
```
A*a**(2/3)*x*hyper((-2/3, 1/2), (3/2,), b*x**2*exp_polar(I*pi)/a) + B*Piec
ewise((a**(2/3)*x**2/2, Eq(b, 0)), (3*(a + b*x**2)**(5/3)/(10*b), True)) +
 C*a**(2/3)*x**3*hyper((-2/3, 3/2), (5/2,), b*x**2*exp_polar(I*pi)/a)/3 +
D*Piecewise((-9*a**2*(a + b*x**2)**(2/3)/(80*b**2) + 3*a*x**2*(a + b*x**2)
**(2/3)/(40*b) + 3*x**4*(a + b*x**2)**(2/3)/16, Ne(b, 0)), (a**(2/3)*x**4/
4, True))
```

Maxima [F]

$$\int \left(a+bx^2\right)^{2/3} \left(A+Bx+Cx^2+Dx^3\right) \, dx = \int \left(Dx^3+Cx^2+Bx+A\right)\left(bx^2+a\right)^{\frac{2}{3}} \, dx$$

input
```
integrate((b*x^2+a)^(2/3)*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)*(b*x^2 + a)^(2/3), x)
```

Giac [F]

$$\int \left(a+bx^2\right)^{2/3} \left(A+Bx+Cx^2+Dx^3\right) \, dx = \int \left(Dx^3+Cx^2+Bx+A\right)\left(bx^2+a\right)^{\frac{2}{3}} \, dx$$

input
```
integrate((b*x^2+a)^(2/3)*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)*(b*x^2 + a)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^2\right)^{2/3} \left(A+Bx+Cx^2+Dx^3\right) \, dx = \int \left(b\,x^2+a\right)^{2/3}\,\left(A+B\,x+C\,x^2+x^3\,D\right) \,d x$$

input
```
int((a + b*x^2)^(2/3)*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
int((a + b*x^2)^(2/3)*(A + B*x + C*x^2 + x^3*D), x)
```

Reduce [F]

$$\int (a+bx^2)^{2/3} (A+Bx+Cx^2+Dx^3)\, dx = \frac{-819(b\,x^2+a)^{\frac{2}{3}}\,a^2d+3120(b\,x^2+a)^{\frac{2}{3}}\,a\,b^2x+2184(b\,x^2+a)^{\frac{2}{3}}\,a\,b^2+960(b\,x^2+a)^{\frac{2}{3}}\,abcx+5}{}$$

input
```
int((b*x^2+a)^(2/3)*(D*x^3+C*x^2+B*x+A),x)
```

output
```
( - 819*(a + b*x**2)**(2/3)*a**2*d + 3120*(a + b*x**2)**(2/3)*a*b**2*x + 2
184*(a + b*x**2)**(2/3)*a*b**2 + 960*(a + b*x**2)**(2/3)*a*b*c*x + 546*(a
+ b*x**2)**(2/3)*a*b*d*x**2 + 2184*(a + b*x**2)**(2/3)*b**3*x**2 + 1680*(a
 + b*x**2)**(2/3)*b**2*c*x**3 + 1365*(a + b*x**2)**(2/3)*b**2*d*x**4 + 416
0*int((a + b*x**2)**(2/3)/(a + b*x**2),x)*a**2*b**2 - 960*int((a + b*x**2)
**(2/3)/(a + b*x**2),x)*a**2*b*c)/(7280*b**2)
```

3.68 $\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt[3]{a+bx^2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 628

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt[3]{a+bx^2}}\,dx = \frac{3(bB-aD)\left(a+bx^2\right)^{2/3}}{4b^2}$$

$$+\frac{3Cx(a+bx^2)^{2/3}}{7b}+\frac{3D(a+bx^2)^{5/3}}{10b^2}-\frac{3(7Ab-3aC)x}{7b\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}$$

$$+\frac{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}(7Ab-3aC)\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{\cdots}}{\left(1-\sqrt{3}\right)\sqrt[3]{\cdots}}\right.\right.}{14b^2x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

$$-\frac{\sqrt{2}3^{3/4}\sqrt[3]{a}(7Ab-3aC)\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{\cdots}}{\left(1-\sqrt{3}\right)\sqrt[3]{\cdots}}\right.\right.}{7b^2x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

output
```
3/4*(B*b-D*a)*(b*x^2+a)^(2/3)/b^2+3/7*C*x*(b*x^2+a)^(2/3)/b+3/10*D*(b*x^2+
a)^(5/3)/b^2-3/7*(7*A*b-3*C*a)*x/b/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))+3
/14*3^(1/4)*(1/2*6^(1/2)+1/2*2^(1/2))*a^(1/3)*(7*A*b-3*C*a)*(a^(1/3)-(b*x^
2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/((1-3^(1/2)
)*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)*EllipticE(((1+3^(1/2))*a^(1/3)-(b*x^2+
a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3)),2*I-I*3^(1/2))/b^2/x/(-a^(
1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1
/2)-1/7*2^(1/2)*3^(3/4)*a^(1/3)*(7*A*b-3*C*a)*(a^(1/3)-(b*x^2+a)^(1/3))*((
a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/((1-3^(1/2))*a^(1/3)-(b*x
^2+a)^(1/3))^2)^(1/2)*EllipticF(((1+3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))/((1-
3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3)),2*I-I*3^(1/2))/b^2/x/(-a^(1/3)*(a^(1/3)-
(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.06 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.15

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt[3]{a+bx^2}}\,dx$$
$$= \frac{-3(a+bx^2)\left(-35bB+21aD-2bx(10C+7Dx)\right)+20b(7Ab-3aC)x\sqrt[3]{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\right.}{140b^2\sqrt[3]{a+bx^2}}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/(a + b*x^2)^(1/3),x]
```

output
```
(-3*(a + b*x^2)*(-35*b*B + 21*a*D - 2*b*x*(10*C + 7*D*x)) + 20*b*(7*A*b -
3*a*C)*x*(1 + (b*x^2)/a)^(1/3)*Hypergeometric2F1[1/3, 1/2, 3/2, -((b*x^2)/
a)])/(140*b^2*(a + b*x^2)^(1/3))
```

Rubi [A] (warning: unable to verify)

Time = 0.63 (sec) , antiderivative size = 663, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2346, 27, 2346, 27, 455, 233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt[3]{a+bx^2}}dx$$

$$\Big\downarrow \text{2346}$$

$$\frac{3\int \frac{2(5bCx^2+(5bB-3aD)x+5Ab)}{3\sqrt[3]{bx^2+a}}dx}{10b}+\frac{3Dx^2(a+bx^2)^{2/3}}{10b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{5bCx^2+(5bB-3aD)x+5Ab}{\sqrt[3]{bx^2+a}}dx}{5b}+\frac{3Dx^2(a+bx^2)^{2/3}}{10b}$$

$$\Big\downarrow \text{2346}$$

$$\frac{\frac{3\int \frac{b(5(7Ab-3aC)+7(5bB-3aD)x)}{3\sqrt[3]{bx^2+a}}dx}{7b}+\frac{15}{7}Cx(a+bx^2)^{2/3}}{5b}+\frac{3Dx^2(a+bx^2)^{2/3}}{10b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{7}\int \frac{5(7Ab-3aC)+7(5bB-3aD)x}{\sqrt[3]{bx^2+a}}dx+\frac{15}{7}Cx(a+bx^2)^{2/3}}{5b}+\frac{3Dx^2(a+bx^2)^{2/3}}{10b}$$

$$\Big\downarrow \text{455}$$

$$\frac{\frac{1}{7}\left(5(7Ab-3aC)\int \frac{1}{\sqrt[3]{bx^2+a}}dx+\frac{21(a+bx^2)^{2/3}(5bB-3aD)}{4b}\right)+\frac{15}{7}Cx(a+bx^2)^{2/3}}{5b}+\frac{3Dx^2(a+bx^2)^{2/3}}{10b}$$

$$\Big\downarrow \text{233}$$

$$\frac{\frac{1}{7}\left(\frac{15\sqrt{bx^2}(7Ab-3aC)\int \frac{\sqrt[3]{bx^2+a}}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}}{2bx}+\frac{21\left(a+bx^2\right)^{2/3}(5bB-3aD)}{4b}\right)+\frac{15}{7}Cx\left(a+bx^2\right)^{2/3}}{5b}+\frac{3Dx^2\left(a+bx^2\right)^{2/3}}{10b}$$

$$\Big\downarrow \; 833$$

$$\frac{\frac{1}{7}\left(\frac{15\sqrt{bx^2}(7Ab-3aC)\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}-\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}\right)}{2bx}+\frac{21\left(a+bx^2\right)^{2/3}(5bB-3aD)}{4b}\right)}{5b}$$

$$\frac{3Dx^2\left(a+bx^2\right)^{2/3}}{10b}$$

$$\Big\downarrow \; 760$$

$$\frac{\frac{1}{7}\left(\frac{15\sqrt{bx^2}(7Ab-3aC)\left(-\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}\ldots}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a\ldots}\right.}}}{\sqrt[4]{3}\sqrt{bx^2}\sqrt{-\frac{\sqrt[3]{a}\left(\ldots\right.}{\left(\left(1-\sqrt{3}\right)\ldots\right.}}}\ldots\right.}{2bx}\ldots\right.}{5b}$$

$$\frac{3Dx^2\left(a+bx^2\right)^{2/3}}{10b}$$

$$\Big\downarrow \; 2418$$

$$\frac{1}{7}\left(\frac{15\sqrt{bx^2}(7Ab-3aC)\left(-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{\cdots}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{\cdots}}\right.\right.}{\sqrt[4]{3}\sqrt{bx^2}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}\right.}{\cdots}\right.$$

$$\frac{3Dx^2\left(a+bx^2\right)^{2/3}}{10b}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/(a + b*x^2)^(1/3),x]
```

output
```
(3*D*x^2*(a + b*x^2)^(2/3))/(10*b) + ((15*C*x*(a + b*x^2)^(2/3))/7 + ((21*
(5*b*B - 3*a*D)*(a + b*x^2)^(2/3))/(4*b) + (15*(7*A*b - 3*a*C)*Sqrt[b*x^2]
*((-2*Sqrt[b*x^2])/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3)) + (3^(1/4)*
Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^
(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a +
 b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1
/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(Sqrt[
b*x^2]*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/
3) - (a + b*x^2)^(1/3))^2)]) - (2*Sqrt[2 - Sqrt[3]]*(1 + Sqrt[3])*a^(1/3)*
(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) +
(a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*Elliptic
F[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3
) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[b*x^2]*Sqrt[-((a^(
1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(
1/3))^2)])))/(2*b*x))/7)/(5*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int \frac{Dx^3 + C\,x^2 + Bx + A}{(b\,x^2 + a)^{\frac{1}{3}}}dx$$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(1/3),x)
```

output
```
int((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(1/3),x)
```

Fricas [F]

$$\int \frac{A + Bx + Cx^2 + Dx^3}{\sqrt[3]{a + bx^2}}\,dx = \int \frac{Dx^3 + Cx^2 + Bx + A}{(bx^2 + a)^{\frac{1}{3}}}\,dx$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(1/3),x, algorithm="fricas")
```

output
```
integral((D*x^3 + C*x^2 + B*x + A)/(b*x^2 + a)^(1/3), x)
```

Sympy [A] (verification not implemented)

Time = 1.40 (sec) , antiderivative size = 272, normalized size of antiderivative = 0.43

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt[3]{a+bx^2}}\,dx = \frac{Ax\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{\sqrt[3]{a}} + B\left(\begin{cases}\frac{x^2}{2\sqrt[3]{a}} & \text{for } b=0\\ \frac{3(a+bx^2)^{\frac{2}{3}}}{4b} & \text{otherwise}\end{cases}\right) + \frac{Cx^3\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{3\sqrt[3]{a}} - \frac{9Da^{\frac{11}{3}}\left(1+\frac{bx^2}{a}\right)^{\frac{2}{3}}}{20a^2b^2+20ab^3x^2} + \frac{9Da^{\frac{11}{3}}}{20a^2b^2+20ab^3x^2} - \frac{3Da^{\frac{8}{3}}bx^2\left(1+\frac{bx^2}{a}\right)^{\frac{2}{3}}}{20a^2b^2+20ab^3x^2} + \frac{9Da^{\frac{8}{3}}bx^2}{20a^2b^2+20ab^3x^2} + \frac{6Da^{\frac{5}{3}}b^2x^4\left(1+\frac{bx^2}{a}\right)^{\frac{2}{3}}}{20a^2b^2+20ab^3x^2}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(b*x**2+a)**(1/3),x)
```

output
```
A*x*hyper((1/3, 1/2), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(1/3) + B*Piece
wise((x**2/(2*a**(1/3)), Eq(b, 0)), (3*(a + b*x**2)**(2/3)/(4*b), True)) +
 C*x**3*hyper((1/3, 3/2), (5/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(1/3)) -
 9*D*a**(11/3)*(1 + b*x**2/a)**(2/3)/(20*a**2*b**2 + 20*a*b**3*x**2) + 9*D
*a**(11/3)/(20*a**2*b**2 + 20*a*b**3*x**2) - 3*D*a**(8/3)*b*x**2*(1 + b*x*
*2/a)**(2/3)/(20*a**2*b**2 + 20*a*b**3*x**2) + 9*D*a**(8/3)*b*x**2/(20*a**
2*b**2 + 20*a*b**3*x**2) + 6*D*a**(5/3)*b**2*x**4*(1 + b*x**2/a)**(2/3)/(2
0*a**2*b**2 + 20*a*b**3*x**2)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt[3]{a+bx^2}}\,dx=\int \frac{Dx^3+Cx^2+Bx+A}{(bx^2+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(1/3),x, algorithm="maxima")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)/(b*x^2 + a)^(1/3), x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt[3]{a+bx^2}}\,dx=\int \frac{Dx^3+Cx^2+Bx+A}{(bx^2+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(1/3),x, algorithm="giac")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)/(b*x^2 + a)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt[3]{a+bx^2}}\,dx=\int \frac{A+B\,x+C\,x^2+x^3\,D}{{\left(b\,x^2+a\right)}^{1/3}}\,dx$$

input
```
int((A + B*x + C*x^2 + x^3*D)/(a + b*x^2)^(1/3),x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/(a + b*x^2)^(1/3), x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt[3]{a+bx^2}}\,dx = \left(\int \frac{x^3}{(b\,x^2+a)^{\frac{1}{3}}}dx\right)d + \left(\int \frac{x^2}{(b\,x^2+a)^{\frac{1}{3}}}dx\right)c + \left(\int \frac{x}{(b\,x^2+a)^{\frac{1}{3}}}dx\right)b + \left(\int \frac{1}{(b\,x^2+a)^{\frac{1}{3}}}dx\right)a$$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(1/3),x)
```

output
```
int(x**3/(a + b*x**2)**(1/3),x)*d + int(x**2/(a + b*x**2)**(1/3),x)*c + in
t(x/(a + b*x**2)**(1/3),x)*b + int(1/(a + b*x**2)**(1/3),x)*a
```

3.69 $\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{4/3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 621

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{4/3}}\,dx = -\frac{3(a(bB-aD)-b(Ab-aC)x)}{2ab^2\sqrt[3]{a+bx^2}}$$

$$+\frac{3D(a+bx^2)^{2/3}}{4b^2}+\frac{3\left(\frac{A}{a}-\frac{3C}{b}\right)x}{2\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}$$

$$-\frac{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}(Ab-3aC)\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a}}\right.\right.}{4a^{2/3}b^2x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

$$+\frac{3^{3/4}(Ab-3aC)\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+b}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+b}}\right.\right.}{\sqrt{2}a^{2/3}b^2x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

output
```
1/2*(-3*a*(B*b-D*a)+3*b*(A*b-C*a)*x)/a/b^2/(b*x^2+a)^(1/3)+3/4*D*(b*x^2+a)
^(2/3)/b^2+3*(A/a-3*C/b)*x/(2*(1-3^(1/2))*a^(1/3)-2*(b*x^2+a)^(1/3))-3/4*3
^(1/4)*(1/2*6^(1/2)+1/2*2^(1/2))*(A*b-3*C*a)*(a^(1/3)-(b*x^2+a)^(1/3))*((a
^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/((1-3^(1/2))*a^(1/3)-(b*x^
2+a)^(1/3))^2)^(1/2)*EllipticE(((1+3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))/((1-3
^(1/2))*a^(1/3)-(b*x^2+a)^(1/3)),2*I-I*3^(1/2))/a^(2/3)/b^2/x/(-a^(1/3)*(a
^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)+1/2
*3^(3/4)*(A*b-3*C*a)*(a^(1/3)-(b*x^2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)
^(1/3)+(b*x^2+a)^(2/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)*Ell
ipticF(((1+3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a
)^(1/3)),2*I-I*3^(1/2))*2^(1/2)/a^(2/3)/b^2/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)
^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.16

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{4/3}}\,dx = \frac{9a^2D+6Ab^2x+3ab(-2B-2Cx+Dx^2)+2b(-Ab+3aC)x\sqrt[3]{1+\frac{bx^2}{a}}\,\mathrm{H}}{4ab^2\sqrt[3]{a+bx^2}}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/(a + b*x^2)^(4/3),x]
```

output
```
(9*a^2*D + 6*A*b^2*x + 3*a*b*(-2*B - 2*C*x + D*x^2) + 2*b*(-(A*b) + 3*a*C)
*x*(1 + (b*x^2)/a)^(1/3)*Hypergeometric2F1[1/3, 1/2, 3/2, -((b*x^2)/a)])/(
4*a*b^2*(a + b*x^2)^(1/3))
```

Rubi [A] (warning: unable to verify)

Time = 0.59 (sec) , antiderivative size = 658, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2345, 27, 455, 233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{4/3}}dx$$

$$\Big\downarrow \text{2345}$$

$$-\frac{3\int \frac{b\left(A-\frac{3aC}{b}\right)-2aDx}{3b\sqrt[3]{bx^2+a}}dx}{2a}-\frac{3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{2ab\sqrt[3]{a+bx^2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{Ab-3aC-2aDx}{\sqrt[3]{bx^2+a}}dx}{2ab}-\frac{3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{2ab\sqrt[3]{a+bx^2}}$$

$$\Big\downarrow \text{455}$$

$$-\frac{(Ab-3aC)\int \frac{1}{\sqrt[3]{bx^2+a}}dx-\frac{3aD(a+bx^2)^{2/3}}{2b}}{2ab}-\frac{3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{2ab\sqrt[3]{a+bx^2}}$$

$$\Big\downarrow \text{233}$$

$$-\frac{\frac{3\sqrt{bx^2}(Ab-3aC)\int \frac{\sqrt[3]{bx^2+a}}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}}{2bx}-\frac{3aD(a+bx^2)^{2/3}}{2b}}{2ab}-\frac{3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{2ab\sqrt[3]{a+bx^2}}$$

$$\Big\downarrow \text{833}$$

$$-\frac{\frac{3\sqrt{bx^2}(Ab-3aC)\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}-\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}\right)}{2bx}-\frac{3aD(a+bx^2)^{2/3}}{2b}}{2ab}-\frac{3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{2ab\sqrt[3]{a+bx^2}}$$

$\downarrow$ 760

$$-\frac{3\sqrt{bx^2}(Ab-3aC)\left(-\int\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+b}\right.}}}{\sqrt[4]{3}\sqrt{bx^2}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\right.}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}\right.}}}\right)}{\frac{2bx}{2ab}}$$

$$\frac{3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{2ab\sqrt[3]{a+bx^2}}$$

$\downarrow$ 2418

$$-\frac{3\sqrt{bx^2}(Ab-3aC)\left(-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2}}\right.\right.}{\sqrt[4]{3}\sqrt{bx^2}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}\right)}{}$$

$$\frac{3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{2ab\sqrt[3]{a+bx^2}}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/(a + b*x^2)^(4/3),x]
```

output
```
(-3*(a*(B - (a*D)/b) - (A*b - a*C)*x))/(2*a*b*(a + b*x^2)^(1/3)) - ((-3*a*
D*(a + b*x^2)^(2/3))/(2*b) + (3*(A*b - 3*a*C)*Sqrt[b*x^2]*((-2*Sqrt[b*x^2]
)/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]
*a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)
^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]
*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3
])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(Sqrt[b*x^2]*Sqrt[-((a^
(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^
(1/3))^2)]) - (2*Sqrt[2 - Sqrt[3]]*(1 + Sqrt[3])*a^(1/3)*(a^(1/3) - (a + b
*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3)
)/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sq
rt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(
1/3))], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[b*x^2]*Sqrt[-((a^(1/3)*(a^(1/3) - (
a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)]))))/(2*b
*x))/(2*a*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int \frac{Dx^3 + C\,x^2 + Bx + A}{\left(b\,x^2 + a\right)^{\frac{4}{3}}}dx$$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(4/3),x)
```

output
```
int((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(4/3),x)
```

Fricas [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{4/3}}\,dx=\int \frac{Dx^3+Cx^2+Bx+A}{(bx^2+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(4/3),x, algorithm="fricas")
```

output
```
integral((D*x^3 + C*x^2 + B*x + A)*(b*x^2 + a)^(2/3)/(b^2*x^4 + 2*a*b*x^2
+ a^2), x)
```

Sympy [A] (verification not implemented)

Time = 3.68 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.22

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{4/3}}\,dx=\frac{Ax\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{4}{3}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{a^{\frac{4}{3}}}+B\left(\begin{cases}-\frac{3}{2b\sqrt[3]{a+bx^2}} & \text{for } b\neq 0\\ \frac{x^2}{2a^{\frac{4}{3}}} & \text{otherwise}\end{cases}\right)+\frac{Cx^3\,{}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{3a^{\frac{4}{3}}}+D\left(\begin{cases}\frac{9a}{4b^2\sqrt[3]{a+bx^2}}+\frac{3x^2}{4b\sqrt[3]{a+bx^2}} & \text{for } b\neq 0\\ \frac{x^4}{4a^{\frac{4}{3}}} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(b*x**2+a)**(4/3),x)
```

output
```
A*x*hyper((1/2, 4/3), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(4/3) + B*Piece
wise((-3/(2*b*(a + b*x**2)**(1/3)), Ne(b, 0)), (x**2/(2*a**(4/3)), True))
+ C*x**3*hyper((4/3, 3/2), (5/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(4/3))
+ D*Piecewise((9*a/(4*b**2*(a + b*x**2)**(1/3)) + 3*x**2/(4*b*(a + b*x**2)
**(1/3)), Ne(b, 0)), (x**4/(4*a**(4/3)), True))
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{4/3}}\,dx=\int \frac{Dx^3+Cx^2+Bx+A}{(bx^2+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(4/3),x, algorithm="maxima")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)/(b*x^2 + a)^(4/3), x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{4/3}}\,dx=\int \frac{Dx^3+Cx^2+Bx+A}{(bx^2+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(4/3),x, algorithm="giac")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)/(b*x^2 + a)^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{4/3}}\,dx=\int \frac{A+B\,x+C\,x^2+x^3\,D}{(b\,x^2+a)^{4/3}}\,dx$$

input
```
int((A + B*x + C*x^2 + x^3*D)/(a + b*x^2)^(4/3),x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/(a + b*x^2)^(4/3), x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{4/3}}\,dx = \left(\int \frac{x^3}{(b\,x^2+a)^{\frac{1}{3}}\,a+(b\,x^2+a)^{\frac{1}{3}}\,b\,x^2}dx\right)d + \left(\int \frac{x^2}{(b\,x^2+a)^{\frac{1}{3}}\,a+(b\,x^2+a)^{\frac{1}{3}}\,b\,x^2}dx\right)c + \left(\int \frac{x}{(b\,x^2+a)^{\frac{1}{3}}\,a+(b\,x^2+a)^{\frac{1}{3}}\,b\,x^2}dx\right)b + \left(\int \frac{1}{(b\,x^2+a)^{\frac{1}{3}}\,a+(b\,x^2+a)^{\frac{1}{3}}\,b\,x^2}dx\right)a$$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(4/3),x)
```

output
```
int(x**3/((a + b*x**2)**(1/3)*a + (a + b*x**2)**(1/3)*b*x**2),x)*d + int(x
**2/((a + b*x**2)**(1/3)*a + (a + b*x**2)**(1/3)*b*x**2),x)*c + int(x/((a
+ b*x**2)**(1/3)*a + (a + b*x**2)**(1/3)*b*x**2),x)*b + int(1/((a + b*x**2
)**(1/3)*a + (a + b*x**2)**(1/3)*b*x**2),x)*a
```

3.70 $\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{7/3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 651

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{7/3}}\,dx = -\frac{3(a(bB-aD)-b(Ab-aC)x)}{8ab^2(a+bx^2)^{4/3}} - \frac{3(8a^2D-b(5Ab+3aC)x)}{16a^2b^2\sqrt[3]{a+bx^2}} + \frac{3(5Ab+3aC)x}{16a^2b\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}$$

$$-\frac{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}(5Ab+3aC)\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a}\cdots}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a}\cdots}\right.\right.}{32a^{5/3}b^2x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

$$+\frac{3^{3/4}(5Ab+3aC)\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+}\cdots}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+}\cdots}\right.\right.}{8\sqrt{2}a^{5/3}b^2x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

output
```
1/8*(-3*a*(B*b-D*a)+3*b*(A*b-C*a)*x)/a/b^2/(b*x^2+a)^(4/3)-3/16*(8*a^2*D-b
*(5*A*b+3*C*a)*x)/a^2/b^2/(b*x^2+a)^(1/3)+3/16*(5*A*b+3*C*a)*x/a^2/b/((1-3
^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))-3/32*3^(1/4)*(1/2*6^(1/2)+1/2*2^(1/2))*(5
*A*b+3*C*a)*(a^(1/3)-(b*x^2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b
*x^2+a)^(2/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)*EllipticE(((
1+3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3)),
2*I-I*3^(1/2))/a^(5/3)/b^2/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/
2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)+1/16*3^(3/4)*(5*A*b+3*C*a)*(a^(1/3)-
(b*x^2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/((1-3^
(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)*EllipticF(((1+3^(1/2))*a^(1/3)-(b
*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3)),2*I-I*3^(1/2))*2^(1/2
)/a^(5/3)/b^2/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(
b*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.08 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.19

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{7/3}}\,dx = \frac{3(-6a^3D+5Ab^3x^3+ab^2x(7A+3Cx^2)+a^2b(-2B+x(C-8Dx)))-b(5}{16a^2b^2\,(a+b}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/(a + b*x^2)^(7/3),x]
```

output
```
(3*(-6*a^3*D + 5*A*b^3*x^3 + a*b^2*x*(7*A + 3*C*x^2) + a^2*b*(-2*B + x*(C
- 8*D*x))) - b*(5*A*b + 3*a*C)*x*(a + b*x^2)*(1 + (b*x^2)/a)^(1/3)*Hyperge
ometric2F1[1/3, 1/2, 3/2, -((b*x^2)/a)])/(16*a^2*b^2*(a + b*x^2)^(4/3))
```

Rubi [A] (warning: unable to verify)

Time = 0.62 (sec) , antiderivative size = 683, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2345, 27, 454, 233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{7/3}}dx$$

$$\Big\downarrow \text{2345}$$

$$-\frac{3\int -\frac{b\left(5A+\frac{3aC}{b}\right)+8aDx}{3b(bx^2+a)^{4/3}}dx}{8a}-\frac{3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{8ab\left(a+bx^2\right)^{4/3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{5Ab+3aC+8aDx}{(bx^2+a)^{4/3}}dx}{8ab}-\frac{3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{8ab\left(a+bx^2\right)^{4/3}}$$

$$\Big\downarrow \text{454}$$

$$\frac{-\frac{(3aC+5Ab)\int \frac{1}{\sqrt[3]{bx^2+a}}dx}{2a}-\frac{3(8a^2D-bx(3aC+5Ab))}{2ab\sqrt[3]{a+bx^2}}}{8ab}-\frac{3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{8ab\left(a+bx^2\right)^{4/3}}$$

$$\Big\downarrow \text{233}$$

$$\frac{-\frac{3\sqrt{bx^2}(3aC+5Ab)\int \frac{\sqrt[3]{bx^2+a}}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}}{4abx}-\frac{3(8a^2D-bx(3aC+5Ab))}{2ab\sqrt[3]{a+bx^2}}}{8ab}-\frac{3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{8ab\left(a+bx^2\right)^{4/3}}$$

$$\Big\downarrow \text{833}$$

$$-\frac{-\frac{3\sqrt{bx^2}(3aC+5Ab)\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}\int\frac{1}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}-\int\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}\right)}{4abx}-\frac{3(8a^2D-bx(3aC+5Ab))}{2ab\sqrt[3]{a+bx^2}}}{8ab}-\frac{3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{8ab\left(a+bx^2\right)^{4/3}}$$

$\Big\downarrow$ 760

$$-\frac{-\frac{3\sqrt{bx^2}(3aC+5Ab)\left(-\int\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a\cdots}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+b\cdots}\right.}}}{\sqrt[4]{3}\sqrt{bx^2}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}\cdots\right.}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}\cdots\right.}}}\right.}{4abx}\cdots}{8ab}$$

$$\frac{3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{8ab\left(a+bx^2\right)^{4/3}}$$

$\Big\downarrow$ 2418

$$-\frac{3\sqrt{bx^2}(3aC+5Ab)\left(-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx\cdots}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx\cdots}}\right.\right.}{\sqrt[4]{3}\sqrt{bx^2}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}\right.}{\cdots}$$

$$\frac{3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{8ab\left(a+bx^2\right)^{4/3}}$$

input

```
Int[(A + B*x + C*x^2 + D*x^3)/(a + b*x^2)^(7/3),x]
```

output
```
(-3*(a*(B - (a*D)/b) - (A*b - a*C)*x))/(8*a*b*(a + b*x^2)^(4/3)) + ((-3*(8
*a^2*D - b*(5*A*b + 3*a*C)*x))/(2*a*b*(a + b*x^2)^(1/3)) - (3*(5*A*b + 3*a
*C)*Sqrt[b*x^2]*((-2*Sqrt[b*x^2])/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/
3)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3))*Sqr
t[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])
*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) -
 (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*S
qrt[3]])/(Sqrt[b*x^2]*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 -
 Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)]) - (2*Sqrt[2 - Sqrt[3]]*(1 + Sq
rt[3])*a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a +
b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/
3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 -
Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[b*x
^2]*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3)
- (a + b*x^2)^(1/3))^2)])))/(4*a*b*x))/(8*a*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 454
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*d
 - b*c*x)/(2*a*b*(p + 1)))*(a + b*x^2)^(p + 1), x] + Simp[c*((2*p + 3)/(2*a
*(p + 1)))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d}, x] && L
tQ[p, -1] && NeQ[p, -3/2]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int \frac{D x^{3}+C\,x^{2}+B x +A}{\left(b\,x^{2}+a \right)^{\frac{7}{3}}}d x$$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(7/3),x)
```

output
```
int((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(7/3),x)
```

Fricas [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{7/3}}\,dx = \int \frac{Dx^3+Cx^2+Bx+A}{(bx^2+a)^{\frac{7}{3}}}\,dx$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(7/3),x, algorithm="fricas")
```

output
```
integral((D*x^3 + C*x^2 + B*x + A)*(b*x^2 + a)^(2/3)/(b^3*x^6 + 3*a*b^2*x^
4 + 3*a^2*b*x^2 + a^3), x)
```

Sympy [A] (verification not implemented)

Time = 7.23 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.31

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{7/3}}\,dx = \frac{Ax\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{3}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{a^{\frac{7}{3}}} + B\left(\begin{cases}-\frac{3}{8ab\sqrt[3]{a+bx^2}+8b^2x^2\sqrt[3]{a+bx^2}} & \text{for}\ b\neq 0\\ \frac{x^2}{2a^{\frac{7}{3}}} & \text{otherwise}\end{cases}\right) + \frac{Cx^3\,{}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{7}{3}\\ \frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{3a^{\frac{7}{3}}} + D\left(\begin{cases}-\frac{9a}{8ab^2\sqrt[3]{a+bx^2}+8b^3x^2\sqrt[3]{a+bx^2}} - \frac{12bx^2}{8ab^2\sqrt[3]{a+bx^2}+8b^3x^2\sqrt[3]{a+bx^2}} & \text{for}\ b\neq 0\\ \frac{x^4}{4a^{\frac{7}{3}}} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(b*x**2+a)**(7/3),x)
```

output
```
A*x*hyper((1/2, 7/3), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(7/3) + B*Piece
wise((-3/(8*a*b*(a + b*x**2)**(1/3) + 8*b**2*x**2*(a + b*x**2)**(1/3)), Ne
(b, 0)), (x**2/(2*a**(7/3)), True)) + C*x**3*hyper((3/2, 7/3), (5/2,), b*x
**2*exp_polar(I*pi)/a)/(3*a**(7/3)) + D*Piecewise((-9*a/(8*a*b**2*(a + b*x
**2)**(1/3) + 8*b**3*x**2*(a + b*x**2)**(1/3)) - 12*b*x**2/(8*a*b**2*(a +
b*x**2)**(1/3) + 8*b**3*x**2*(a + b*x**2)**(1/3)), Ne(b, 0)), (x**4/(4*a**
(7/3)), True))
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{7/3}}\,dx = \int \frac{Dx^3+Cx^2+Bx+A}{(bx^2+a)^{\frac{7}{3}}}\,dx$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(7/3),x, algorithm="maxima")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)/(b*x^2 + a)^(7/3), x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{7/3}}\,dx = \int \frac{Dx^3+Cx^2+Bx+A}{(bx^2+a)^{\frac{7}{3}}}\,dx$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(7/3),x, algorithm="giac")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)/(b*x^2 + a)^(7/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{7/3}}\,dx=\int \frac{A+B\,x+C\,x^2+x^3\,D}{(b\,x^2+a)^{7/3}}\,dx$$

input
```
int((A + B*x + C*x^2 + x^3*D)/(a + b*x^2)^(7/3),x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/(a + b*x^2)^(7/3), x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{7/3}}\,dx=\left(\int \frac{x^3}{(b\,x^2+a)^{\frac{1}{3}}\,a^2+2\,(b\,x^2+a)^{\frac{1}{3}}\,ab\,x^2+(b\,x^2+a)^{\frac{1}{3}}\,b^2x^4}dx\right)d$$
$$+\left(\int \frac{x^2}{(b\,x^2+a)^{\frac{1}{3}}\,a^2+2\,(b\,x^2+a)^{\frac{1}{3}}\,ab\,x^2+(b\,x^2+a)^{\frac{1}{3}}\,b^2x^4}dx\right)c$$
$$+\left(\int \frac{x}{(b\,x^2+a)^{\frac{1}{3}}\,a^2+2\,(b\,x^2+a)^{\frac{1}{3}}\,ab\,x^2+(b\,x^2+a)^{\frac{1}{3}}\,b^2x^4}dx\right)b$$
$$+\left(\int \frac{1}{(b\,x^2+a)^{\frac{1}{3}}\,a^2+2\,(b\,x^2+a)^{\frac{1}{3}}\,ab\,x^2+(b\,x^2+a)^{\frac{1}{3}}\,b^2x^4}dx\right)a$$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(7/3),x)
```

output
```
int(x**3/((a + b*x**2)**(1/3)*a**2 + 2*(a + b*x**2)**(1/3)*a*b*x**2 + (a +
 b*x**2)**(1/3)*b**2*x**4),x)*d + int(x**2/((a + b*x**2)**(1/3)*a**2 + 2*(
a + b*x**2)**(1/3)*a*b*x**2 + (a + b*x**2)**(1/3)*b**2*x**4),x)*c + int(x/
((a + b*x**2)**(1/3)*a**2 + 2*(a + b*x**2)**(1/3)*a*b*x**2 + (a + b*x**2)*
*(1/3)*b**2*x**4),x)*b + int(1/((a + b*x**2)**(1/3)*a**2 + 2*(a + b*x**2)*
*(1/3)*a*b*x**2 + (a + b*x**2)**(1/3)*b**2*x**4),x)*a
```

3.71 $\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{10/3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 682

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{10/3}}\,dx = -\frac{3(a(bB-aD)-b(Ab-aC)x)}{14ab^2(a+bx^2)^{7/3}} - \frac{3(14a^2D-b(11Ab+3aC)x)}{112a^2b^2(a+bx^2)^{4/3}} + \frac{15(11Ab+3aC)x}{224a^3b\sqrt[3]{a+bx^2}}$$
$$+ \frac{15(11Ab+3aC)x}{224a^3b\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}$$
$$- \frac{15\sqrt[4]{3}\sqrt{2+\sqrt{3}}(11Ab+3aC)\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\cdots}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\cdots}\right.\right.}{448a^{8/3}b^2x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$
$$+ \frac{5\,3^{3/4}(11Ab+3aC)\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{\cdots}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{\cdots}}\right.\right.}{112\sqrt{2}a^{8/3}b^2x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

output
```
1/14*(-3*a*(B*b-D*a)+3*b*(A*b-C*a)*x)/a/b^2/(b*x^2+a)^(7/3)-3/112*(14*a^2*
D-b*(11*A*b+3*C*a)*x)/a^2/b^2/(b*x^2+a)^(4/3)+15/224*(11*A*b+3*C*a)*x/a^3/
b/(b*x^2+a)^(1/3)+15/224*(11*A*b+3*C*a)*x/a^3/b/((1-3^(1/2))*a^(1/3)-(b*x^
2+a)^(1/3))-15/448*3^(1/4)*(1/2*6^(1/2)+1/2*2^(1/2))*(11*A*b+3*C*a)*(a^(1/
3)-(b*x^2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/((1
-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)*EllipticE(((1+3^(1/2))*a^(1/3)
-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3)),2*I-I*3^(1/2))/a^(
8/3)/b^2/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2
+a)^(1/3))^2)^(1/2)+5/224*3^(3/4)*(11*A*b+3*C*a)*(a^(1/3)-(b*x^2+a)^(1/3))
*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/((1-3^(1/2))*a^(1/3)-(
b*x^2+a)^(1/3))^2)^(1/2)*EllipticF(((1+3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))/(
(1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3)),2*I-I*3^(1/2))*2^(1/2)/a^(8/3)/b^2/x/
(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^
2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.10 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.22

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{10/3}}\,dx = \frac{-36a^4D+165Ab^4x^5+9ab^3x^3(44A+5Cx^2)+9a^2b^2x(31A+12Cx^2)-3a}{}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/(a + b*x^2)^(10/3),x]
```

output
```
(-36*a^4*D + 165*A*b^4*x^5 + 9*a*b^3*x^3*(44*A + 5*C*x^2) + 9*a^2*b^2*x*(3
1*A + 12*C*x^2) - 3*a^3*b*(16*B + x*(-5*C + 28*D*x)) - 5*b*(11*A*b + 3*a*C
)*x*(a + b*x^2)^2*(1 + (b*x^2)/a)^(1/3)*Hypergeometric2F1[1/3, 1/2, 3/2, -
((b*x^2)/a)])/(224*a^3*b^2*(a + b*x^2)^(7/3))
```

Rubi [A] (warning: unable to verify)

Time = 0.63 (sec) , antiderivative size = 710, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {2345, 27, 454, 215, 233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{10/3}}\,dx$$

$$\Big\downarrow \text{ 2345}$$

$$-\frac{3\int -\frac{b\left(11A+\frac{3aC}{b}\right)+14aDx}{3b(bx^2+a)^{7/3}}dx}{14a}-\frac{3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{14ab\,(a+bx^2)^{7/3}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{11Ab+3aC+14aDx}{(bx^2+a)^{7/3}}dx}{14ab}-\frac{3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{14ab\,(a+bx^2)^{7/3}}$$

$$\Big\downarrow \text{ 454}$$

$$\frac{\frac{5(3aC+11Ab)\int \frac{1}{(bx^2+a)^{4/3}}dx}{8a}-\frac{3(14a^2D-bx(3aC+11Ab))}{8ab(a+bx^2)^{4/3}}}{14ab}-\frac{3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{14ab\,(a+bx^2)^{7/3}}$$

$$\Big\downarrow \text{ 215}$$

$$\frac{\frac{5(3aC+11Ab)\left(\frac{3x}{2a\sqrt[3]{a+bx^2}}-\frac{\int \frac{1}{\sqrt[3]{bx^2+a}}dx}{2a}\right)}{8a}-\frac{3(14a^2D-bx(3aC+11Ab))}{8ab(a+bx^2)^{4/3}}}{14ab}-\frac{3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{14ab\,(a+bx^2)^{7/3}}$$

$$\Big\downarrow \text{ 233}$$

$$\frac{\frac{5(3aC+11Ab)\left(\frac{3x}{2a\sqrt[3]{a+bx^2}}-\frac{3\sqrt{bx^2}\int\frac{\sqrt[3]{bx^2+a}}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}}{4abx}\right)}{8a}-\frac{3(14a^2D-bx(3aC+11Ab))}{8ab(a+bx^2)^{4/3}}}{14ab}-$$

$$\frac{3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{14ab\left(a+bx^2\right)^{7/3}}$$

$\downarrow$ 833

$$\frac{\frac{5(3aC+11Ab)\left(\frac{3x}{2a\sqrt[3]{a+bx^2}}-\frac{3\sqrt{bx^2}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}\int\frac{1}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}-\int\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}\right)}{4abx}\right)}{8a}-\frac{3(14a^2D-bx(\ldots}{8ab(a+b\ldots}}{14ab}$$

$$\frac{3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{14ab\left(a+bx^2\right)^{7/3}}$$

$\downarrow$ 760

$$\frac{\frac{5(3aC+11Ab)\left(\frac{3x}{2a\sqrt[3]{a+bx^2}}-\frac{3\sqrt{bx^2}\left(-\int\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{\ldots}}{\left(\left(1-\ldots\right.\right.}}\ldots}{\sqrt[4]{3}\sqrt{b\ldots}}\right)}{4abx}\right)}{8a}\ldots}{14ab}$$

$$\frac{3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{14ab\left(a+bx^2\right)^{7/3}}$$

$\downarrow$ 2418

$$\frac{5(3aC+11Ab)\left(\frac{3x}{2a\sqrt[3]{a+bx^2}}-\frac{3\sqrt{bx^2}\left(-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\text{EllipticF}\left(\arcsin\left(\cdots\right.\right.}{\sqrt[4]{3}\sqrt{bx^2}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}\cdots\right.}{\cdots}\right.}{\cdots}$$

$$\frac{3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{14ab\left(a+bx^2\right)^{7/3}}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/(a + b*x^2)^(10/3),x]
```

output
```
(-3*(a*(B - (a*D)/b) - (A*b - a*C)*x))/(14*a*b*(a + b*x^2)^(7/3)) + ((-3*(
14*a^2*D - b*(11*A*b + 3*a*C)*x))/(8*a*b*(a + b*x^2)^(4/3)) + (5*(11*A*b +
 3*a*C)*((3*x)/(2*a*(a + b*x^2)^(1/3)) - (3*Sqrt[b*x^2]*((-2*Sqrt[b*x^2])/
((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*a
^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(
1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*E
llipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])
*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(Sqrt[b*x^2]*Sqrt[-((a^(1
/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1
/3))^2]]) - (2*Sqrt[2 - Sqrt[3]]*(1 + Sqrt[3])*a^(1/3)*(a^(1/3) - (a + b*x
^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/
((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt
[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/
3))], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[b*x^2]*Sqrt[-((a^(1/3)*(a^(1/3) - (a
+ b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)])))/(4*a*b
*x)))/(8*a))/(14*a*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 454
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*d
 - b*c*x)/(2*a*b*(p + 1)))*(a + b*x^2)^(p + 1), x] + Simp[c*((2*p + 3)/(2*a
*(p + 1)))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d}, x] && L
tQ[p, -1] && NeQ[p, -3/2]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int \frac{D x^{3}+C\,x^{2}+B x +A}{\left(b\,x^{2}+a \right)^{\frac{10}{3}}}d x$$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(10/3),x)
```

output
```
int((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(10/3),x)
```

Fricas [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{10/3}}\,dx=\int \frac{Dx^{3}+Cx^{2}+Bx+A}{(bx^{2}+a)^{\frac{10}{3}}}\,dx$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(10/3),x, algorithm="fricas")
```

output
```
integral((D*x^3 + C*x^2 + B*x + A)*(b*x^2 + a)^(2/3)/(b^4*x^8 + 4*a*b^3*x^
6 + 6*a^2*b^2*x^4 + 4*a^3*b*x^2 + a^4), x)
```

Sympy [A] (verification not implemented)

Time = 13.95 (sec) , antiderivative size = 267, normalized size of antiderivative = 0.39

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{10/3}}\,dx = \frac{Ax\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{10}{3}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{a^{\frac{10}{3}}} + B\left(\begin{cases}-\frac{3}{14a^2b\sqrt[3]{a+bx^2}+28ab^2x^2\sqrt[3]{a+bx^2}+14b^3x^4\sqrt[3]{a+bx^2}} & \text{for } b\neq 0\\ \frac{x^2}{2a^{\frac{10}{3}}} & \text{otherwise}\end{cases}\right) + \frac{Cx^3\,{}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{10}{3}\\ \frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{3a^{\frac{10}{3}}} + D\left(\begin{cases}-\frac{9a}{56a^2b^2\sqrt[3]{a+bx^2}+112ab^3x^2\sqrt[3]{a+bx^2}+56b^4x^4\sqrt[3]{a+bx^2}} - \frac{21bx^2}{56a^2b^2\sqrt[3]{a+bx^2}+112ab^3x^2\sqrt[3]{a+bx^2}+56b^4x^4\sqrt[3]{a+ \\ \frac{x^4}{4a^{\frac{10}{3}}}\end{cases}\right.$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(b*x**2+a)**(10/3),x)
```

output
```
A*x*hyper((1/2, 10/3), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(10/3) + B*Pie
cewise((-3/(14*a**2*b*(a + b*x**2)**(1/3) + 28*a*b**2*x**2*(a + b*x**2)**(
1/3) + 14*b**3*x**4*(a + b*x**2)**(1/3)), Ne(b, 0)), (x**2/(2*a**(10/3)),
True)) + C*x**3*hyper((3/2, 10/3), (5/2,), b*x**2*exp_polar(I*pi)/(3*a*
*(10/3)) + D*Piecewise((-9*a/(56*a**2*b**2*(a + b*x**2)**(1/3) + 112*a*b**
3*x**2*(a + b*x**2)**(1/3) + 56*b**4*x**4*(a + b*x**2)**(1/3)) - 21*b*x**2
/(56*a**2*b**2*(a + b*x**2)**(1/3) + 112*a*b**3*x**2*(a + b*x**2)**(1/3) +
 56*b**4*x**4*(a + b*x**2)**(1/3)), Ne(b, 0)), (x**4/(4*a**(10/3)), True))
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{10/3}}\,dx=\int \frac{Dx^3+Cx^2+Bx+A}{(bx^2+a)^{\frac{10}{3}}}\,dx$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(10/3),x, algorithm="maxima")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)/(b*x^2 + a)^(10/3), x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{10/3}}\,dx=\int \frac{Dx^3+Cx^2+Bx+A}{(bx^2+a)^{\frac{10}{3}}}\,dx$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(10/3),x, algorithm="giac")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)/(b*x^2 + a)^(10/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{10/3}}\,dx=\int \frac{A+B\,x+C\,x^2+x^3\,D}{(b\,x^2+a)^{10/3}}\,dx$$

input
```
int((A + B*x + C*x^2 + x^3*D)/(a + b*x^2)^(10/3),x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/(a + b*x^2)^(10/3), x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{10/3}}\,dx = \left(\int \frac{x^3}{(b\,x^2+a)^{\frac{1}{3}}\,a^3+3\,(b\,x^2+a)^{\frac{1}{3}}\,a^2b\,x^2+3\,(b\,x^2+a)^{\frac{1}{3}}\,a\,b^2x^4+(b\,x^2+a)}\right.$$

$$+\left(\int \frac{x^2}{(b\,x^2+a)^{\frac{1}{3}}\,a^3+3\,(b\,x^2+a)^{\frac{1}{3}}\,a^2b\,x^2+3\,(b\,x^2+a)^{\frac{1}{3}}\,a\,b^2x^4+(b\,x^2+a)^{\frac{1}{3}}\,b^3x^6}dx\right)c$$

$$+\left(\int \frac{x}{(b\,x^2+a)^{\frac{1}{3}}\,a^3+3\,(b\,x^2+a)^{\frac{1}{3}}\,a^2b\,x^2+3\,(b\,x^2+a)^{\frac{1}{3}}\,a\,b^2x^4+(b\,x^2+a)^{\frac{1}{3}}\,b^3x^6}dx\right)b$$

$$+\left(\int \frac{1}{(b\,x^2+a)^{\frac{1}{3}}\,a^3+3\,(b\,x^2+a)^{\frac{1}{3}}\,a^2b\,x^2+3\,(b\,x^2+a)^{\frac{1}{3}}\,a\,b^2x^4+(b\,x^2+a)^{\frac{1}{3}}\,b^3x^6}dx\right)a$$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(10/3),x)
```

output
```
int(x**3/((a + b*x**2)**(1/3)*a**3 + 3*(a + b*x**2)**(1/3)*a**2*b*x**2 + 3
*(a + b*x**2)**(1/3)*a*b**2*x**4 + (a + b*x**2)**(1/3)*b**3*x**6),x)*d + i
nt(x**2/((a + b*x**2)**(1/3)*a**3 + 3*(a + b*x**2)**(1/3)*a**2*b*x**2 + 3*
(a + b*x**2)**(1/3)*a*b**2*x**4 + (a + b*x**2)**(1/3)*b**3*x**6),x)*c + in
t(x/((a + b*x**2)**(1/3)*a**3 + 3*(a + b*x**2)**(1/3)*a**2*b*x**2 + 3*(a +
 b*x**2)**(1/3)*a*b**2*x**4 + (a + b*x**2)**(1/3)*b**3*x**6),x)*b + int(1/
((a + b*x**2)**(1/3)*a**3 + 3*(a + b*x**2)**(1/3)*a**2*b*x**2 + 3*(a + b*x
**2)**(1/3)*a*b**2*x**4 + (a + b*x**2)**(1/3)*b**3*x**6),x)*a
```

3.72 $\int (a+bx^2)^p (A+Bx+Cx^2+Dx^3)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 140

$$\int (a+bx^2)^p (A+Bx+Cx^2+Dx^3)\,dx = \frac{(bB-aD)(a+bx^2)^{1+p}}{2b^2(1+p)}+\frac{Cx(a+bx^2)^{1+p}}{b(3+2p)}+\frac{D(a+bx^2)^{2+p}}{2b^2(2+p)} + \left(A-\frac{aC}{3b+2bp}\right)x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right)$$

output
```
1/2*(B*b-D*a)*(b*x^2+a)^(p+1)/b^2/(p+1)+C*x*(b*x^2+a)^(p+1)/b/(3+2*p)+1/2*
D*(b*x^2+a)^(2+p)/b^2/(2+p)+(A-a*C/(2*b*p+3*b))*x*(b*x^2+a)^p*hypergeom([1
/2, -p],[3/2],-b*x^2/a)/((1+b*x^2/a)^p)
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.01

$$\int (a+bx^2)^p (A+Bx+Cx^2+Dx^3)\,dx$$
$$=\frac{1}{6}(a+bx^2)^p\left(6Ax\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right)\right.$$
$$\left.+\frac{\frac{3(a+bx^2)(-aD+bB(2+p)+bD(1+p)x^2)}{b^2}+2C(2+3p+p^2)x^3\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{2},-p,\frac{5}{2},-\frac{bx^2}{a}\right)}{(1+p)(2+p)}\right)$$

input
```
Integrate[(a + b*x^2)^p*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
((a + b*x^2)^p*((6*A*x*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(1 +
 (b*x^2)/a)^p + ((3*(a + b*x^2)*(-(a*D) + b*B*(2 + p) + b*D*(1 + p)*x^2))/
b^2 + (2*C*(2 + 3*p + p^2)*x^3*Hypergeometric2F1[3/2, -p, 5/2, -((b*x^2)/a
)])/(1 + (b*x^2)/a)^p)/((1 + p)*(2 + p))))/6
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.22, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {2346, 27, 2346, 25, 27, 455, 238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^2)^p (A+Bx+Cx^2+Dx^3)\,dx$$

$$\Big\downarrow \text{2346}$$

$$\frac{\int 2(bx^2+a)^p (bC(p+2)x^2-(aD-bB(p+2))x+Ab(p+2))\,dx}{2b(p+2)}+\frac{Dx^2(a+bx^2)^{p+1}}{2b(p+2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \left(bx^2+a\right)^p \left(bC(p+2)x^2-(aD-bB(p+2))x+Ab(p+2)\right)dx}{b(p+2)}+\frac{Dx^2\left(a+bx^2\right)^{p+1}}{2b(p+2)}$$

$\Big\downarrow$ 2346

$$\frac{\frac{\int -b((p+2)(aC-Ab(2p+3))+(2p+3)(aD-bB(p+2))x)\left(bx^2+a\right)^p dx}{b(2p+3)}+\frac{C(p+2)x\left(a+bx^2\right)^{p+1}}{2p+3}}{b(p+2)}+\frac{Dx^2\left(a+bx^2\right)^{p+1}}{2b(p+2)}$$

$\Big\downarrow$ 25

$$\frac{\frac{C(p+2)x\left(a+bx^2\right)^{p+1}}{2p+3}-\frac{\int b((p+2)(aC-Ab(2p+3))+(2p+3)(aD-bB(p+2))x)\left(bx^2+a\right)^p dx}{b(2p+3)}}{b(p+2)}+\frac{Dx^2\left(a+bx^2\right)^{p+1}}{2b(p+2)}$$

$\Big\downarrow$ 27

$$\frac{\frac{C(p+2)x\left(a+bx^2\right)^{p+1}}{2p+3}-\frac{\int ((p+2)(aC-Ab(2p+3))+(2p+3)(aD-bB(p+2))x)\left(bx^2+a\right)^p dx}{2p+3}}{b(p+2)}+\frac{Dx^2\left(a+bx^2\right)^{p+1}}{2b(p+2)}$$

$\Big\downarrow$ 455

$$\frac{\frac{C(p+2)x\left(a+bx^2\right)^{p+1}}{2p+3}-\frac{(p+2)(aC-Ab(2p+3))\int \left(bx^2+a\right)^p dx+\frac{(2p+3)\left(a+bx^2\right)^{p+1}(aD-bB(p+2))}{2b(p+1)}}{2p+3}}{b(p+2)}+\frac{Dx^2\left(a+bx^2\right)^{p+1}}{2b(p+2)}$$

$\Big\downarrow$ 238

$$\frac{\frac{C(p+2)x\left(a+bx^2\right)^{p+1}}{2p+3}-\frac{(p+2)\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}(aC-Ab(2p+3))\int \left(\frac{bx^2}{a}+1\right)^p dx+\frac{(2p+3)\left(a+bx^2\right)^{p+1}(aD-bB(p+2))}{2b(p+1)}}{2p+3}}{b(p+2)}+\frac{Dx^2\left(a+bx^2\right)^{p+1}}{2b(p+2)}$$

$\Big\downarrow$ 237

$$\frac{\frac{C(p+2)x\left(a+bx^2\right)^{p+1}}{2p+3}-\frac{(p+2)x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}(aC-Ab(2p+3))\,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right)+\frac{(2p+3)\left(a+bx^2\right)^{p+1}(aD-bB(p+2))}{2b(p+1)}}{2p+3}}{b(p+2)}+\frac{Dx^2\left(a+bx^2\right)^{p+1}}{2b(p+2)}$$

input
```
Int[(a + b*x^2)^p*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(D*x^2*(a + b*x^2)^(1 + p))/(2*b*(2 + p)) + ((C*(2 + p)*x*(a + b*x^2)^(1 +
 p))/(3 + 2*p) - (((3 + 2*p)*(a*D - b*B*(2 + p))*(a + b*x^2)^(1 + p))/(2*b
*(1 + p)) + ((2 + p)*(a*C - A*b*(3 + 2*p))*x*(a + b*x^2)^p*Hypergeometric2
F1[1/2, -p, 3/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p)/(3 + 2*p))/(b*(2 + p))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Maple [F]

$$\int \left(b\,x^{2}+a\right)^{p} \left(Dx^{3}+C\,x^{2}+Bx+A\right)dx$$

input
```
int((b*x^2+a)^p*(D*x^3+C*x^2+B*x+A),x)
```

output
```
int((b*x^2+a)^p*(D*x^3+C*x^2+B*x+A),x)
```

Fricas [F]

$$\int \left(a+bx^2\right)^p \left(A+Bx+Cx^2+Dx^3\right)\,dx=\int \left(Dx^{3}+Cx^{2}+Bx+A\right)\left(bx^{2}+a\right)^{p}dx$$

input
```
integrate((b*x^2+a)^p*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
integral((D*x^3 + C*x^2 + B*x + A)*(b*x^2 + a)^p, x)
```

Sympy [A] (verification not implemented)

Time = 5.62 (sec) , antiderivative size = 427, normalized size of antiderivative = 3.05

$$\int (a+bx^2)^p (A+Bx+Cx^2+Dx^3)\,dx = Aa^p x\,{}_2F_1\left(\begin{matrix}\frac{1}{2},-p\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right) + B\left(\begin{cases}\frac{a^px^2}{2} & \text{for } b=0\\ \frac{\begin{cases}\frac{(a+bx^2)^{p+1}}{p+1} & \text{for } p\neq -1\\ \log(a+bx^2) & \text{otherwise}\end{cases}}{2b} & \text{otherwise}\end{cases}\right) + \frac{Ca^px^3\,{}_2F_1\left(\begin{matrix}\frac{3}{2},-p\\ \frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{3} + D\left(\begin{cases}\frac{a^px^4}{4} & \text{for } b=0\\ \frac{a\log\left(x-\sqrt{-\frac{a}{b}}\right)}{2ab^2+2b^3x^2}+\frac{a\log\left(x+\sqrt{-\frac{a}{b}}\right)}{2ab^2+2b^3x^2}+\frac{a}{2ab^2+2b^3x^2}+\frac{bx^2\log\left(x-\sqrt{-\frac{a}{b}}\right)}{2ab^2+2b^3x^2}+\frac{bx^2\log\left(x+\sqrt{-\frac{a}{b}}\right)}{2ab^2+2b^3x^2} & \text{for } p=-2\\ -\frac{a\log\left(x-\sqrt{-\frac{a}{b}}\right)}{2b^2}-\frac{a\log\left(x+\sqrt{-\frac{a}{b}}\right)}{2b^2}+\frac{x^2}{2b} & \text{for } p=-1\\ -\frac{a^2(a+bx^2)^p}{2b^2p^2+6b^2p+4b^2}+\frac{abpx^2(a+bx^2)^p}{2b^2p^2+6b^2p+4b^2}+\frac{b^2px^4(a+bx^2)^p}{2b^2p^2+6b^2p+4b^2}+\frac{b^2x^4(a+bx^2)^p}{2b^2p^2+6b^2p+4b^2} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((b*x**2+a)**p*(D*x**3+C*x**2+B*x+A),x)
```

output
```
A*a**p*x*hyper((1/2, -p), (3/2,), b*x**2*exp_polar(I*pi)/a) + B*Piecewise(
(a**p*x**2/2, Eq(b, 0)), (Piecewise(((a + b*x**2)**(p + 1)/(p + 1), Ne(p,
-1)), (log(a + b*x**2), True))/(2*b), True)) + C*a**p*x**3*hyper((3/2, -p)
, (5/2,), b*x**2*exp_polar(I*pi)/a)/3 + D*Piecewise((a**p*x**4/4, Eq(b, 0)
), (a*log(x - sqrt(-a/b))/(2*a*b**2 + 2*b**3*x**2) + a*log(x + sqrt(-a/b))
/(2*a*b**2 + 2*b**3*x**2) + a/(2*a*b**2 + 2*b**3*x**2) + b*x**2*log(x - sq
rt(-a/b))/(2*a*b**2 + 2*b**3*x**2) + b*x**2*log(x + sqrt(-a/b))/(2*a*b**2
+ 2*b**3*x**2), Eq(p, -2)), (-a*log(x - sqrt(-a/b))/(2*b**2) - a*log(x + s
qrt(-a/b))/(2*b**2) + x**2/(2*b), Eq(p, -1)), (-a**2*(a + b*x**2)**p/(2*b*
*2*p**2 + 6*b**2*p + 4*b**2) + a*b*p*x**2*(a + b*x**2)**p/(2*b**2*p**2 + 6
*b**2*p + 4*b**2) + b**2*p*x**4*(a + b*x**2)**p/(2*b**2*p**2 + 6*b**2*p +
4*b**2) + b**2*x**4*(a + b*x**2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2), Tru
e))
```

Maxima [F]

$$\int (a+bx^2)^p (A+Bx+Cx^2+Dx^3) \, dx = \int (Dx^3+Cx^2+Bx+A)(bx^2+a)^p \, dx$$

input
```
integrate((b*x^2+a)^p*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)*(b*x^2 + a)^p, x)
```

Giac [F]

$$\int (a+bx^2)^p (A+Bx+Cx^2+Dx^3) \, dx = \int (Dx^3+Cx^2+Bx+A)(bx^2+a)^p \, dx$$

input
```
integrate((b*x^2+a)^p*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)*(b*x^2 + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx^2)^p (A+Bx+Cx^2+Dx^3) \, dx = \int (b\,x^2+a)^p \left(A+B\,x+C\,x^2+x^3\,D\right) \, dx$$

input
```
int((a + b*x^2)^p*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
int((a + b*x^2)^p*(A + B*x + C*x^2 + x^3*D), x)
```

Reduce [F]

$$\int (a+bx^2)^p (A+Bx+Cx^2+Dx^3)\,dx = \text{Too large to display}$$

input
```
int((b*x^2+a)^p*(D*x^3+C*x^2+B*x+A),x)
```

output
```
( - 4*(a + b*x**2)**p*a**2*d*p**2 - 8*(a + b*x**2)**p*a**2*d*p - 3*(a + b*
x**2)**p*a**2*d + 4*(a + b*x**2)**p*a*b**2*p**3*x + 4*(a + b*x**2)**p*a*b*
*2*p**3 + 18*(a + b*x**2)**p*a*b**2*p**2*x + 16*(a + b*x**2)**p*a*b**2*p**
2 + 26*(a + b*x**2)**p*a*b**2*p*x + 19*(a + b*x**2)**p*a*b**2*p + 12*(a +
b*x**2)**p*a*b**2*x + 6*(a + b*x**2)**p*a*b**2 + 4*(a + b*x**2)**p*a*b*c*p
**3*x + 12*(a + b*x**2)**p*a*b*c*p**2*x + 8*(a + b*x**2)**p*a*b*c*p*x + 4*
(a + b*x**2)**p*a*b*d*p**3*x**2 + 8*(a + b*x**2)**p*a*b*d*p**2*x**2 + 3*(a
 + b*x**2)**p*a*b*d*p*x**2 + 4*(a + b*x**2)**p*b**3*p**3*x**2 + 16*(a + b*
x**2)**p*b**3*p**2*x**2 + 19*(a + b*x**2)**p*b**3*p*x**2 + 6*(a + b*x**2)*
*p*b**3*x**2 + 4*(a + b*x**2)**p*b**2*c*p**3*x**3 + 14*(a + b*x**2)**p*b**
2*c*p**2*x**3 + 14*(a + b*x**2)**p*b**2*c*p*x**3 + 4*(a + b*x**2)**p*b**2*
c*x**3 + 4*(a + b*x**2)**p*b**2*d*p**3*x**4 + 12*(a + b*x**2)**p*b**2*d*p*
*2*x**4 + 11*(a + b*x**2)**p*b**2*d*p*x**4 + 3*(a + b*x**2)**p*b**2*d*x**4
 + 32*int((a + b*x**2)**p/(4*a*p**2 + 8*a*p + 3*a + 4*b*p**2*x**2 + 8*b*p*
x**2 + 3*b*x**2),x)*a**2*b**2*p**6 + 208*int((a + b*x**2)**p/(4*a*p**2 + 8
*a*p + 3*a + 4*b*p**2*x**2 + 8*b*p*x**2 + 3*b*x**2),x)*a**2*b**2*p**5 + 52
0*int((a + b*x**2)**p/(4*a*p**2 + 8*a*p + 3*a + 4*b*p**2*x**2 + 8*b*p*x**2
 + 3*b*x**2),x)*a**2*b**2*p**4 + 620*int((a + b*x**2)**p/(4*a*p**2 + 8*a*p
 + 3*a + 4*b*p**2*x**2 + 8*b*p*x**2 + 3*b*x**2),x)*a**2*b**2*p**3 + 348*in
t((a + b*x**2)**p/(4*a*p**2 + 8*a*p + 3*a + 4*b*p**2*x**2 + 8*b*p*x**2 ...
```

3.73 $\int (a+bx^2)^p (Ax+Bx^3+Cx^5+Dx^7) \, dx$

Optimal result

Integrand size = 29, antiderivative size = 144

$$\begin{aligned}\int (a+bx^2)^p (Ax+Bx^3+Cx^5+Dx^7) \, dx &= \frac{(Ab^3-a(b^2B-abC+a^2D))(a+bx^2)^{1+p}}{2b^4(1+p)} + \frac{(b^2B-2abC+3a^2D)(a+bx^2)^{2+p}}{2b^4(2+p)} \\ &+ \frac{(bC-3aD)(a+bx^2)^{3+p}}{2b^4(3+p)} + \frac{D(a+bx^2)^{4+p}}{2b^4(4+p)}\end{aligned}$$

output
```
1/2*(A*b^3-a*(B*b^2-C*a*b+D*a^2))*(b*x^2+a)^(p+1)/b^4/(p+1)+1/2*(B*b^2-2*C
*a*b+3*D*a^2)*(b*x^2+a)^(2+p)/b^4/(2+p)+1/2*(C*b-3*D*a)*(b*x^2+a)^(3+p)/b^
4/(3+p)+1/2*D*(b*x^2+a)^(4+p)/b^4/(4+p)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.15

$$\int (a+bx^2)^p (Ax+Bx^3+Cx^5+Dx^7) \, dx = \frac{(a+bx^2)^{1+p} (-6a^3D+Ab^3(24+26p+9p^2+p^3)+2a^2b(C(4+p)+3D(1+p)x^2)-ab^2(B(12+7p+}{2b^4(1+p)}$$

input
```
Integrate[(a + b*x^2)^p*(A*x + B*x^3 + C*x^5 + D*x^7),x]
```

output
```
((a + b*x^2)^(1 + p)*(-6*a^3*D + A*b^3*(24 + 26*p + 9*p^2 + p^3) + 2*a^2*b
*(C*(4 + p) + 3*D*(1 + p)*x^2) - a*b^2*(B*(12 + 7*p + p^2) + (1 + p)*x^2*(
2*C*(4 + p) + 3*D*(2 + p)*x^2)) + b^3*(1 + p)*x^2*(B*(12 + 7*p + p^2) + (2
 + p)*x^2*(C*(4 + p) + D*(3 + p)*x^2))))/(2*b^4*(1 + p)*(2 + p)*(3 + p)*(4
 + p))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2029, 2331, 2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^2)^p \left(Ax+Bx^3+Cx^5+Dx^7\right) dx$$

$$\Big\downarrow \text{2029}$$

$$\int x(a+bx^2)^p \left(A+Bx^2+Cx^4+Dx^6\right) dx$$

$$\Big\downarrow \text{2331}$$

$$\frac{1}{2}\int (bx^2+a)^p \left(Dx^6+Cx^4+Bx^2+A\right) dx^2$$

$$\Big\downarrow \text{2389}$$

$$\frac{1}{2}\int \left(\frac{\left(Ab^3-a\left(Da^2-bCa+b^2B\right)\right)(bx^2+a)^p}{b^3}+\frac{\left(3Da^2-2bCa+b^2B\right)(bx^2+a)^{p+1}}{b^3}+\frac{(bC-3aD)\,(bx^2+a}{b^3}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{(a+bx^2)^{p+1}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{b^4(p+1)}+\frac{(a+bx^2)^{p+2}\left(3a^2D-2abC+b^2B\right)}{b^4(p+2)}+\frac{(bC-3aD)\,(a+bx^2)}{b^4(p+3)}\right.$$

input
```
Int[(a + b*x^2)^p*(A*x + B*x^3 + C*x^5 + D*x^7),x]
```

output
```
(((A*b^3 - a*(b^2*B - a*b*C + a^2*D))*(a + b*x^2)^(1 + p))/(b^4*(1 + p)) +
 ((b^2*B - 2*a*b*C + 3*a^2*D)*(a + b*x^2)^(2 + p))/(b^4*(2 + p)) + ((b*C -
 3*a*D)*(a + b*x^2)^(3 + p))/(b^4*(3 + p)) + (D*(a + b*x^2)^(4 + p))/(b^4*
(4 + p)))/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2029
```
Int[(Fx_.)*((d_.)*(x_)^(q_.) + (a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.) + (c_.)*
(x_)^(t_.))^(p_.), x_Symbol] :> Int[x^(p*r)*(a + b*x^(s - r) + c*x^(t - r)
+ d*x^(q - r))^p*Fx, x] /; FreeQ[{a, b, c, d, r, s, t, q}, x] && IntegerQ[p
] && PosQ[s - r] && PosQ[t - r] && PosQ[q - r] &&  !(EqQ[p, 1] && EqQ[u, 1]
)
```

rule 2331
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/2   S
ubst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
 FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 328 vs. $2(136) = 272$.

Time = 0.54 (sec) , antiderivative size = 329, normalized size of antiderivative = 2.28

method	result
gosper	$\frac{(b x^2+a)^{p+1} (D b^3 p^3 x^6+6 D b^3 p^2 x^6+C b^3 p^3 x^4+11 D b^3 p x^6+7 C b^3 p^2 x^4-3 D a b^2 p^2 x^4+6 D x^6 b^3+B b^3 p^3 x^2+14 C b^3 p x^4-9 D a}{}$
orering	$\frac{(D b^3 p^3 x^6+6 D b^3 p^2 x^6+C b^3 p^3 x^4+11 D b^3 p x^6+7 C b^3 p^2 x^4-3 D a b^2 p^2 x^4+6 D x^6 b^3+B b^3 p^3 x^2+14 C b^3 p x^4-9 D a b^2 p x^4+8 B b}{}$
norman	$\frac{D x^8 e^{p \ln(b x^2+a)}}{2p+8}+\frac{a (A b^3 p^3+9 A b^3 p^2-B a b^2 p^2+26 A b^3 p-7 B a b^2 p+2 C a^2 b p+24 b^3 A-12 a b^2 B+8 a^2 b C-6 a^3 D) e^{p \ln(b x^2}}{2 b^4 (p^4+10 p^3+35 p^2+50 p+24)}$
parallelrisch	$\frac{24 A (b x^2+a)^p a^2 b^3-12 B (b x^2+a)^p a^3 b^2+7 C x^6 (b x^2+a)^p a b^4 p^2+3 D x^6 (b x^2+a)^p a^2 b^3 p^2+B x^4 (b x^2+a)^p a b^4 p^3+14 C x^6 (b}{}$

input
```
int((b*x^2+a)^p*(D*x^7+C*x^5+B*x^3+A*x),x,method=_RETURNVERBOSE)
```

output
```
1/2/b^4*(b*x^2+a)^(p+1)/(p^4+10*p^3+35*p^2+50*p+24)*(D*b^3*p^3*x^6+6*D*b^3
*p^2*x^6+C*b^3*p^3*x^4+11*D*b^3*p*x^6+7*C*b^3*p^2*x^4-3*D*a*b^2*p^2*x^4+6*
D*b^3*x^6+B*b^3*p^3*x^2+14*C*b^3*p*x^4-9*D*a*b^2*p*x^4+8*B*b^3*p^2*x^2-2*C
*a*b^2*p^2*x^2+8*C*b^3*x^4-6*D*a*b^2*x^4+A*b^3*p^3+19*B*b^3*p*x^2-10*C*a*b
^2*p*x^2+6*D*a^2*b*p*x^2+9*A*b^3*p^2-B*a*b^2*p^2+12*B*b^3*x^2-8*C*a*b^2*x^
2+6*D*a^2*b*x^2+26*A*b^3*p-7*B*a*b^2*p+2*C*a^2*b*p+24*A*b^3-12*B*a*b^2+8*C
*a^2*b-6*D*a^3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 399 vs. $2(137) = 274$.

Time = 0.08 (sec) , antiderivative size = 399, normalized size of antiderivative = 2.77

$$\int (a+bx^2)^p (Ax+Bx^3+Cx^5+Dx^7)\ dx = \frac{((Db^4p^3+6\,Db^4p^2+11\,Db^4p+6\,Db^4)x^8+Aab^3p^3+(8\,Cb^4+(Dab^3+Cb^4)p^3+(3\,Dab^3+7\,Cb^4)p^2+}{}$$

input
```
integrate((b*x^2+a)^p*(D*x^7+C*x^5+B*x^3+A*x),x, algorithm="fricas")
```

output
```
1/2*((D*b^4*p^3 + 6*D*b^4*p^2 + 11*D*b^4*p + 6*D*b^4)*x^8 + A*a*b^3*p^3 +
(8*C*b^4 + (D*a*b^3 + C*b^4)*p^3 + (3*D*a*b^3 + 7*C*b^4)*p^2 + 2*(D*a*b^3
+ 7*C*b^4)*p)*x^6 - 6*D*a^4 + 8*C*a^3*b - 12*B*a^2*b^2 + 24*A*a*b^3 + (12*
B*b^4 + (C*a*b^3 + B*b^4)*p^3 - (3*D*a^2*b^2 - 5*C*a*b^3 - 8*B*b^4)*p^2 -
(3*D*a^2*b^2 - 4*C*a*b^3 - 19*B*b^4)*p)*x^4 - (B*a^2*b^2 - 9*A*a*b^3)*p^2
+ (24*A*b^4 + (B*a*b^3 + A*b^4)*p^3 - (2*C*a^2*b^2 - 7*B*a*b^3 - 9*A*b^4)*
p^2 + 2*(3*D*a^3*b - 4*C*a^2*b^2 + 6*B*a*b^3 + 13*A*b^4)*p)*x^2 + (2*C*a^3
*b - 7*B*a^2*b^2 + 26*A*a*b^3)*p)*(b*x^2 + a)^p/(b^4*p^4 + 10*b^4*p^3 + 35
*b^4*p^2 + 50*b^4*p + 24*b^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5005 vs. $2(128) = 256$.

Time = 24.55 (sec) , antiderivative size = 5005, normalized size of antiderivative = 34.76

$$\int (a+bx^2)^p (Ax+Bx^3+Cx^5+Dx^7)\,dx = \text{Too large to display}$$

input
```
integrate((b*x**2+a)**p*(D*x**7+C*x**5+B*x**3+A*x),x)
```

output
```
Piecewise((a**p*(A*x**2/2 + B*x**4/4 + C*x**6/6 + D*x**8/8), Eq(b, 0)), (-
2*A*b**3/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6
) - B*a*b**2/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*
x**6) - 3*B*b**3*x**2/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 +
 12*b**7*x**6) - 2*C*a**2*b/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*
x**4 + 12*b**7*x**6) - 6*C*a*b**2*x**2/(12*a**3*b**4 + 36*a**2*b**5*x**2 +
 36*a*b**6*x**4 + 12*b**7*x**6) - 6*C*b**3*x**4/(12*a**3*b**4 + 36*a**2*b*
*5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 6*D*a**3*log(x - sqrt(-a/b))/(1
2*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 6*D*a**
3*log(x + sqrt(-a/b))/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 +
 12*b**7*x**6) + 11*D*a**3/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x
**4 + 12*b**7*x**6) + 18*D*a**2*b*x**2*log(x - sqrt(-a/b))/(12*a**3*b**4 +
 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 18*D*a**2*b*x**2*log
(x + sqrt(-a/b))/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b
**7*x**6) + 27*D*a**2*b*x**2/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6
*x**4 + 12*b**7*x**6) + 18*D*a*b**2*x**4*log(x - sqrt(-a/b))/(12*a**3*b**4
 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 18*D*a*b**2*x**4*l
og(x + sqrt(-a/b))/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12
*b**7*x**6) + 18*D*a*b**2*x**4/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b*
*6*x**4 + 12*b**7*x**6) + 6*D*b**3*x**6*log(x - sqrt(-a/b))/(12*a**3*b*...
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.75

$$\int \left(a+bx^2\right)^p \left(Ax+Bx^3+Cx^5+Dx^7\right)\,dx = \frac{(b^2(p+1)x^4+abpx^2-a^2)(bx^2+a)^pB}{2\,(p^2+3\,p+2)b^2} + \frac{(bx^2+a)^{p+1}A}{2\,b(p+1)} + \frac{((p^2+3\,p+2)b^3x^6+(p^2+p)ab^2x^4-2\,a^2bpx^2+2\,a^3)(bx^2+a)^pC}{2\,(p^3+6\,p^2+11\,p+6)b^3} + \frac{((p^3+6\,p^2+11\,p+6)b^4x^8+(p^3+3\,p^2+2\,p)ab^3x^6-3\,(p^2+p)a^2b^2x^4+6\,a^3bpx^2-6\,a^4)(bx^2+a)^pD}{2\,(p^4+10\,p^3+35\,p^2+50\,p+24)b^4}$$

input
```
integrate((b*x^2+a)^p*(D*x^7+C*x^5+B*x^3+A*x),x, algorithm="maxima")
```

output
```
1/2*(b^2*(p + 1)*x^4 + a*b*p*x^2 - a^2)*(b*x^2 + a)^p*B/((p^2 + 3*p + 2)*b
^2) + 1/2*(b*x^2 + a)^(p + 1)*A/(b*(p + 1)) + 1/2*((p^2 + 3*p + 2)*b^3*x^6
 + (p^2 + p)*a*b^2*x^4 - 2*a^2*b*p*x^2 + 2*a^3)*(b*x^2 + a)^p*C/((p^3 + 6*
p^2 + 11*p + 6)*b^3) + 1/2*((p^3 + 6*p^2 + 11*p + 6)*b^4*x^8 + (p^3 + 3*p^
2 + 2*p)*a*b^3*x^6 - 3*(p^2 + p)*a^2*b^2*x^4 + 6*a^3*b*p*x^2 - 6*a^4)*(b*x
^2 + a)^p*D/((p^4 + 10*p^3 + 35*p^2 + 50*p + 24)*b^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 556 vs. $2(137) = 274$.

Time = 0.15 (sec) , antiderivative size = 556, normalized size of antiderivative = 3.86

$$\int (a+bx^2)^p (Ax+Bx^3+Cx^5+Dx^7)\,dx$$

$$= \frac{(bx^2+a)^4(bx^2+a)^pDp^2 - 3\,(bx^2+a)^3(bx^2+a)^pDap^2 + 3\,(bx^2+a)^2(bx^2+a)^pDa^2p^2 + (bx^2+a)^3(bx^2}{}$$

$$-\frac{\frac{(bx^2+a)^{p+1}Da^3}{p+1} - \frac{(bx^2+a)^{p+1}Ca^2b}{p+1} + \frac{(bx^2+a)^{p+1}Bab^2}{p+1} - \frac{(bx^2+a)^{p+1}Ab^3}{p+1}}{2\,b^4}$$

input
```
integrate((b*x^2+a)^p*(D*x^7+C*x^5+B*x^3+A*x),x, algorithm="giac")
```

output
```
1/2*((b*x^2 + a)^4*(b*x^2 + a)^p*D*p^2 - 3*(b*x^2 + a)^3*(b*x^2 + a)^p*D*a
*p^2 + 3*(b*x^2 + a)^2*(b*x^2 + a)^p*D*a^2*p^2 + (b*x^2 + a)^3*(b*x^2 + a)
^p*C*b*p^2 - 2*(b*x^2 + a)^2*(b*x^2 + a)^p*C*a*b*p^2 + (b*x^2 + a)^2*(b*x^
2 + a)^p*B*b^2*p^2 + 5*(b*x^2 + a)^4*(b*x^2 + a)^p*D*p - 18*(b*x^2 + a)^3*
(b*x^2 + a)^p*D*a*p + 21*(b*x^2 + a)^2*(b*x^2 + a)^p*D*a^2*p + 6*(b*x^2 +
a)^3*(b*x^2 + a)^p*C*b*p - 14*(b*x^2 + a)^2*(b*x^2 + a)^p*C*a*b*p + 7*(b*x
^2 + a)^2*(b*x^2 + a)^p*B*b^2*p + 6*(b*x^2 + a)^4*(b*x^2 + a)^p*D - 24*(b*
x^2 + a)^3*(b*x^2 + a)^p*D*a + 36*(b*x^2 + a)^2*(b*x^2 + a)^p*D*a^2 + 8*(b
*x^2 + a)^3*(b*x^2 + a)^p*C*b - 24*(b*x^2 + a)^2*(b*x^2 + a)^p*C*a*b + 12*
(b*x^2 + a)^2*(b*x^2 + a)^p*B*b^2)/(b^4*p^3 + 9*b^4*p^2 + 26*b^4*p + 24*b^
4) - 1/2*((b*x^2 + a)^(p + 1)*D*a^3/(p + 1) - (b*x^2 + a)^(p + 1)*C*a^2*b/
(p + 1) + (b*x^2 + a)^(p + 1)*B*a*b^2/(p + 1) - (b*x^2 + a)^(p + 1)*A*b^3/
(p + 1))/b^4
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^2\right)^p \left(Ax+Bx^3+Cx^5+Dx^7\right)\,dx=\int \left(b\,x^2+a\right)^p\,\left(A\,x+B\,x^3+C\,x^5+x^7\,D\right)\,dx$$

input
```
int((a + b*x^2)^p*(A*x + B*x^3 + C*x^5 + x^7*D),x)
```

output
```
int((a + b*x^2)^p*(A*x + B*x^3 + C*x^5 + x^7*D), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 398, normalized size of antiderivative = 2.76

$$\int \left(a+bx^2\right)^p \left(Ax+Bx^3+Cx^5+Dx^7\right)\,dx$$
$$=\frac{(b\,x^2+a)^p\,(b^4d\,p^3x^8+6b^4d\,p^2x^8+a\,b^3d\,p^3x^6+b^4c\,p^3x^6+11b^4dp\,x^8+3a\,b^3d\,p^2x^6+7b^4c\,p^2x^6+6b^4d\,x^8}{}$$

input
```
int((b*x^2+a)^p*(D*x^7+C*x^5+B*x^3+A*x),x)
```

output
```
((a + b*x**2)**p*( - 6*a**4*d + 2*a**3*b*c*p + 8*a**3*b*c + 6*a**3*b*d*p*x
**2 + a**2*b**3*p**3 + 8*a**2*b**3*p**2 + 19*a**2*b**3*p + 12*a**2*b**3 -
2*a**2*b**2*c*p**2*x**2 - 8*a**2*b**2*c*p*x**2 - 3*a**2*b**2*d*p**2*x**4 -
 3*a**2*b**2*d*p*x**4 + 2*a*b**4*p**3*x**2 + 16*a*b**4*p**2*x**2 + 38*a*b*
*4*p*x**2 + 24*a*b**4*x**2 + a*b**3*c*p**3*x**4 + 5*a*b**3*c*p**2*x**4 + 4
*a*b**3*c*p*x**4 + a*b**3*d*p**3*x**6 + 3*a*b**3*d*p**2*x**6 + 2*a*b**3*d*
p*x**6 + b**5*p**3*x**4 + 8*b**5*p**2*x**4 + 19*b**5*p*x**4 + 12*b**5*x**4
 + b**4*c*p**3*x**6 + 7*b**4*c*p**2*x**6 + 14*b**4*c*p*x**6 + 8*b**4*c*x**
6 + b**4*d*p**3*x**8 + 6*b**4*d*p**2*x**8 + 11*b**4*d*p*x**8 + 6*b**4*d*x*
*8))/(2*b**4*(p**4 + 10*p**3 + 35*p**2 + 50*p + 24))
```

3.74 $\int \frac{-x^2+2x^4}{1+2x^2} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 25

$$\int \frac{-x^2+2x^4}{1+2x^2} \, dx = -x+\frac{x^3}{3}+\frac{\arctan\left(\sqrt{2}x\right)}{\sqrt{2}}$$

output
```
-x+1/3*x^3+1/2*arctan(x*2^(1/2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{-x^2+2x^4}{1+2x^2} \, dx = -x+\frac{x^3}{3}+\frac{\arctan\left(\sqrt{2}x\right)}{\sqrt{2}}$$

input
```
Integrate[(-x^2 + 2*x^4)/(1 + 2*x^2),x]
```

output
```
-x + x^3/3 + ArcTan[Sqrt[2]*x]/Sqrt[2]
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.32, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2027, 363, 262, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x^4-x^2}{2x^2+1}dx$$

$$\Big\downarrow \ 2027$$

$$\int \frac{x^2(2x^2-1)}{2x^2+1}dx$$

$$\Big\downarrow \ 363$$

$$\frac{x^3}{3}-2\int \frac{x^2}{2x^2+1}dx$$

$$\Big\downarrow \ 262$$

$$\frac{x^3}{3}-2\left(\frac{x}{2}-\frac{1}{2}\int \frac{1}{2x^2+1}dx\right)$$

$$\Big\downarrow \ 216$$

$$\frac{x^3}{3}-2\left(\frac{x}{2}-\frac{\arctan\left(\sqrt{2}x\right)}{2\sqrt{2}}\right)$$

input
```
Int[(-x^2 + 2*x^4)/(1 + 2*x^2),x]
```

output
```
x^3/3 - 2*(x/2 - ArcTan[Sqrt[2]*x]/(2*Sqrt[2]))
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] && !(EqQ[p, 1] && EqQ[u, 1])
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

method	result	size
default	$-x+\frac{x^{3}}{3}+\frac{\arctan\left(\sqrt{2}\, x\right)\sqrt{2}}{2}$	21
risch	$-x+\frac{x^{3}}{3}+\frac{\arctan\left(\sqrt{2}\, x\right)\sqrt{2}}{2}$	21
meijerg	$\frac{\sqrt{2}\left(-\frac{2x\sqrt{2}\left(-10x^{2}+15\right)}{15}+2\arctan\left(\sqrt{2}\, x\right)\right)}{8}-\frac{\sqrt{2}\left(2\sqrt{2}\, x-2\arctan\left(\sqrt{2}\, x\right)\right)}{8}$	49

input
```
int((2*x^4-x^2)/(2*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-x+1/3*x^3+1/2*arctan(2^(1/2)*x)*2^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \frac{-x^2 + 2x^4}{1 + 2x^2} \, dx = \frac{1}{3} \, x^3 + \frac{1}{2} \, \sqrt{2} \arctan\left(\sqrt{2}x\right) - x$$

input
```
integrate((2*x^4-x^2)/(2*x^2+1),x, algorithm="fricas")
```

output
```
1/3*x^3 + 1/2*sqrt(2)*arctan(sqrt(2)*x) - x
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \frac{-x^2 + 2x^4}{1 + 2x^2} \, dx = \frac{x^3}{3} - x + \frac{\sqrt{2} \operatorname{atan}\left(\sqrt{2}x\right)}{2}$$

input
```
integrate((2*x**4-x**2)/(2*x**2+1),x)
```

output
```
x**3/3 - x + sqrt(2)*atan(sqrt(2)*x)/2
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \frac{-x^2 + 2x^4}{1 + 2x^2} \, dx = \frac{1}{3} \, x^3 + \frac{1}{2} \, \sqrt{2} \arctan\left(\sqrt{2}x\right) - x$$

input
```
integrate((2*x^4-x^2)/(2*x^2+1),x, algorithm="maxima")
```

output
```
1/3*x^3 + 1/2*sqrt(2)*arctan(sqrt(2)*x) - x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \frac{-x^2+2x^4}{1+2x^2}\,dx = \frac{1}{3}\,x^3 + \frac{1}{2}\,\sqrt{2}\arctan\left(\sqrt{2}x\right) - x$$

input
```
integrate((2*x^4-x^2)/(2*x^2+1),x, algorithm="giac")
```

output
```
1/3*x^3 + 1/2*sqrt(2)*arctan(sqrt(2)*x) - x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \frac{-x^2+2x^4}{1+2x^2}\,dx = \frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\right)}{2} - x + \frac{x^3}{3}$$

input
```
int(-(x^2 - 2*x^4)/(2*x^2 + 1),x)
```

output
```
(2^(1/2)*atan(2^(1/2)*x))/2 - x + x^3/3
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{-x^2+2x^4}{1+2x^2}\,dx = \frac{\sqrt{2}\,atan\left(\frac{2x}{\sqrt{2}}\right)}{2} + \frac{x^3}{3} - x$$

input
```
int((2*x^4-x^2)/(2*x^2+1),x)
```

output
```
(3*sqrt(2)*atan((2*x)/sqrt(2)) + 2*x**3 - 6*x)/6
```

3.75 $\int \frac{x^3+x^4}{1+x^2} dx$

Optimal result

Integrand size = 15, antiderivative size = 30

$$\int \frac{x^3+x^4}{1+x^2} dx = -x+\frac{x^2}{2}+\frac{x^3}{3}+\arctan(x)-\frac{1}{2}\log\left(1+x^2\right)$$

output
```
-x+1/2*x^2+1/3*x^3+arctan(x)-1/2*ln(x^2+1)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{x^3+x^4}{1+x^2} dx = -x+\frac{x^2}{2}+\frac{x^3}{3}+\arctan(x)-\frac{1}{2}\log\left(1+x^2\right)$$

input
```
Integrate[(x^3 + x^4)/(1 + x^2),x]
```

output
```
-x + x^2/2 + x^3/3 + ArcTan[x] - Log[1 + x^2]/2
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2027, 523, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4+x^3}{x^2+1} \, dx$$

$$\Big\downarrow \text{ 2027}$$

$$\int \frac{x^3(x+1)}{x^2+1}dx$$

$$\Big\downarrow \text{ 523}$$

$$\int \left(x^2+\frac{1-x}{x^2+1}+x-1\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\arctan(x)+\frac{x^3}{3}+\frac{x^2}{2}-\frac{1}{2}\log\left(x^2+1\right)-x$$

input
```
Int[(x^3 + x^4)/(1 + x^2),x]
```

output
```
-x + x^2/2 + x^3/3 + ArcTan[x] - Log[1 + x^2]/2
```

Defintions of rubi rules used

rule 523
```
Int[((x_)^(m_.)*((c_) + (d_.)*(x_)))/((a_) + (b_.)*(x_)^2), x_Symbol] :> In
t[ExpandIntegrand[x^m*((c + d*x)/(a + b*x^2)), x], x] /; FreeQ[{a, b, c, d}
, x] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
default	$-x+\frac{x^{2}}{2}+\frac{x^{3}}{3}+\arctan\left(x\right)-\frac{\ln\left(x^{2}+1\right)}{2}$	25
risch	$-x+\frac{x^{2}}{2}+\frac{x^{3}}{3}+\arctan\left(x\right)-\frac{\ln\left(x^{2}+1\right)}{2}$	25
meijerg	$-\frac{x\left(-5x^{2}+15\right)}{15}+\arctan\left(x\right)+\frac{x^{2}}{2}-\frac{\ln\left(x^{2}+1\right)}{2}$	27
parallelrisch	$\frac{x^{3}}{3}+\frac{x^{2}}{2}-x-\frac{\ln\left(x-i\right)}{2}-\frac{i\ln\left(x-i\right)}{2}-\frac{\ln\left(x+i\right)}{2}+\frac{i\ln\left(x+i\right)}{2}$	45

input
```
int((x^4+x^3)/(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-x+1/2*x^2+1/3*x^3+arctan(x)-1/2*ln(x^2+1)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{x^3+x^4}{1+x^2}\,dx=\frac{1}{3}\,x^{3}+\frac{1}{2}\,x^{2}-x+\arctan\left(x\right)-\frac{1}{2}\,\log\left(x^{2}+1\right)$$

input
```
integrate((x^4+x^3)/(x^2+1),x, algorithm="fricas")
```

output
```
1/3*x^3 + 1/2*x^2 - x + arctan(x) - 1/2*log(x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int \frac{x^3+x^4}{1+x^2}\,dx = \frac{x^3}{3}+\frac{x^2}{2}-x-\frac{\log\left(x^2+1\right)}{2}+\operatorname{atan}(x)$$

input
```
integrate((x**4+x**3)/(x**2+1),x)
```

output
```
x**3/3 + x**2/2 - x - log(x**2 + 1)/2 + atan(x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{x^3+x^4}{1+x^2}\,dx = \frac{1}{3}x^3+\frac{1}{2}x^2-x+\arctan(x)-\frac{1}{2}\log\left(x^2+1\right)$$

input
```
integrate((x^4+x^3)/(x^2+1),x, algorithm="maxima")
```

output
```
1/3*x^3 + 1/2*x^2 - x + arctan(x) - 1/2*log(x^2 + 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{x^3+x^4}{1+x^2}\,dx = \frac{1}{3}x^3+\frac{1}{2}x^2-x+\arctan(x)-\frac{1}{2}\log\left(x^2+1\right)$$

input
```
integrate((x^4+x^3)/(x^2+1),x, algorithm="giac")
```

output
```
1/3*x^3 + 1/2*x^2 - x + arctan(x) - 1/2*log(x^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{x^3+x^4}{1+x^2}\,dx = \operatorname{atan}(x)-\frac{\ln(x^2+1)}{2}-x+\frac{x^2}{2}+\frac{x^3}{3}$$

input
```
int((x^3 + x^4)/(x^2 + 1),x)
```

output
```
atan(x) - log(x^2 + 1)/2 - x + x^2/2 + x^3/3
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{x^3+x^4}{1+x^2}\,dx = atan(x)-\frac{\log(x^2+1)}{2}+\frac{x^3}{3}+\frac{x^2}{2}-x$$

input
```
int((x^4+x^3)/(x^2+1),x)
```

output
```
(6*atan(x) - 3*log(x**2 + 1) + 2*x**3 + 3*x**2 - 6*x)/6
```

3.76 $\int \frac{(-1+x)x(1+x)^5}{(1+x^2)^5} dx$

Optimal result

Integrand size = 17, antiderivative size = 53

$$\int \frac{(-1+x)x(1+x)^5}{(1+x^2)^5} dx = -\frac{1}{(1+x^2)^4} + \frac{2(3+2x)}{3(1+x^2)^3} - \frac{3+8x}{6(1+x^2)^2} - \frac{1}{2(1+x^2)}$$

output
```
-1/(x^2+1)^4+2/3*(3+2*x)/(x^2+1)^3-1/6*(3+8*x)/(x^2+1)^2-1/(2*x^2+2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.64

$$\int \frac{(-1+x)x(1+x)^5}{(1+x^2)^5} dx = -\frac{x^2(3+8x+12x^2+8x^3+3x^4)}{6(1+x^2)^4}$$

input
```
Integrate[((-1 + x)*x*(1 + x)^5)/(1 + x^2)^5,x]
```

output
```
-1/6*(x^2*(3 + 8*x + 12*x^2 + 8*x^3 + 3*x^4))/(1 + x^2)^4
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.15, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {2335, 27, 2342, 2335, 27, 2345, 27, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x-1)x(x+1)^5}{(x^2+1)^5}dx$$

$$\Big\downarrow \ 2335$$

$$\frac{x^2}{(x^2+1)^4}-\frac{1}{8}\int \frac{8(-x^5-4x^4-4x^3+4x^2+3x)}{(x^2+1)^4}dx$$

$$\Big\downarrow \ 27$$

$$\frac{x^2}{(x^2+1)^4}-\int \frac{-x^5-4x^4-4x^3+4x^2+3x}{(x^2+1)^4}dx$$

$$\Big\downarrow \ 2342$$

$$\frac{x^2}{(x^2+1)^4}-\int \frac{x(-x^4-4x^3-4x^2+4x+3)}{(x^2+1)^4}dx$$

$$\Big\downarrow \ 2335$$

$$\frac{1}{6}\int -\frac{2(-3x^3-12x^2+3x+4)}{(x^2+1)^3}dx+\frac{x^2}{(x^2+1)^4}+\frac{(4-3x)x}{3(x^2+1)^3}$$

$$\Big\downarrow \ 27$$

$$-\frac{1}{3}\int \frac{-3x^3-12x^2+3x+4}{(x^2+1)^3}dx+\frac{x^2}{(x^2+1)^4}+\frac{(4-3x)x}{3(x^2+1)^3}$$

$$\Big\downarrow \ 2345$$

$$\frac{1}{3}\left(\frac{1}{4}\int \frac{12x}{(x^2+1)^2}dx+\frac{3-8x}{2(x^2+1)^2}\right)+\frac{x^2}{(x^2+1)^4}+\frac{(4-3x)x}{3(x^2+1)^3}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{3}\left(3\int \frac{x}{(x^2+1)^2}dx+\frac{3-8x}{2(x^2+1)^2}\right)+\frac{x^2}{(x^2+1)^4}+\frac{(4-3x)x}{3(x^2+1)^3}$$

$$\Big\downarrow \ 241$$

$$\frac{x^2}{(x^2+1)^4}+\frac{(4-3x)x}{3(x^2+1)^3}+\frac{1}{3}\left(\frac{3-8x}{2(x^2+1)^2}-\frac{3}{2(x^2+1)}\right)$$

input
```
Int[((-1 + x)*x*(1 + x)^5)/(1 + x^2)^5,x]
```

output
```
x^2/(1 + x^2)^4 + ((4 - 3*x)*x)/(3*(1 + x^2)^3) + ((3 - 8*x)/(2*(1 + x^2)^
2) - 3/(2*(1 + x^2)))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 2335
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Simp[c/(2*a*b*(p + 1))   Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSu
m[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
 b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

rule 2342
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[x*PolynomialQuotient
[Pq, x, x]*(a + b*x^2)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && EqQ[
Coeff[Pq, x, 0], 0] &&  !MatchQ[Pq, x^(m_.)*(u_.) /; IntegerQ[m]]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.62

method	result
gosper	$-\frac{x^{2} \left(3 x^{4}+8 x^{3}+12 x^{2}+8 x +3\right)}{6 \left(x^{2}+1\right)^{4}}$
orering	$-\frac{x^{2} \left(3 x^{4}+8 x^{3}+12 x^{2}+8 x +3\right)}{6 \left(x^{2}+1\right)^{4}}$
default	$\frac{-\frac{1}{2} x^{6}-\frac{4}{3} x^{5}-2 x^{4}-\frac{4}{3} x^{3}-\frac{1}{2} x^{2}}{\left(x^{2}+1\right)^{4}}$
norman	$\frac{-\frac{1}{2} x^{6}-\frac{4}{3} x^{5}-2 x^{4}-\frac{4}{3} x^{3}-\frac{1}{2} x^{2}}{\left(x^{2}+1\right)^{4}}$
risch	$\frac{-\frac{1}{2} x^{6}-\frac{4}{3} x^{5}-2 x^{4}-\frac{4}{3} x^{3}-\frac{1}{2} x^{2}}{\left(x^{2}+1\right)^{4}}$
parallelrisch	$\frac{-3 x^{6}-8 x^{5}-12 x^{4}-8 x^{3}-3 x^{2}}{6 \left(x^{2}+1\right)^{4}}$
meijerg	$-\frac{x^{2} \left(x^{6}+4 x^{4}+6 x^{2}+4\right)}{8 \left(x^{2}+1\right)^{4}}+\frac{x \left(-15 x^{6}-55 x^{4}-73 x^{2}+15\right)}{96 \left(x^{2}+1\right)^{4}}-\frac{5 x^{4} \left(x^{4}+4 x^{2}+6\right)}{24 \left(x^{2}+1\right)^{4}}+\frac{5 x^{6} \left(x^{2}+4\right)}{24 \left(x^{2}+1\right)^{4}}-\frac{x \left(-105 x^{6}+511 x^{4}+385 x^{2}\right.}{672 \left(x^{2}+1\right)^{4}}$

input
```
int((-1+x)*x*(1+x)^5/(x^2+1)^5,x,method=_RETURNVERBOSE)
```

output
```
-1/6*x^2*(3*x^4+8*x^3+12*x^2+8*x+3)/(x^2+1)^4
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.94

$$\int \frac{(-1+x)x(1+x)^5}{(1+x^2)^5}\,dx = -\frac{3\,x^6+8\,x^5+12\,x^4+8\,x^3+3\,x^2}{6\,(x^8+4\,x^6+6\,x^4+4\,x^2+1)}$$

input
```
integrate((-1+x)*x*(1+x)^5/(x^2+1)^5,x, algorithm="fricas")
```

output
```
-1/6*(3*x^6 + 8*x^5 + 12*x^4 + 8*x^3 + 3*x^2)/(x^8 + 4*x^6 + 6*x^4 + 4*x^2
 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.91

$$\int \frac{(-1+x)x(1+x)^5}{(1+x^2)^5}\,dx = \frac{-3x^6-8x^5-12x^4-8x^3-3x^2}{6x^8+24x^6+36x^4+24x^2+6}$$

input
```
integrate((-1+x)*x*(1+x)**5/(x**2+1)**5,x)
```

output
```
(-3*x**6 - 8*x**5 - 12*x**4 - 8*x**3 - 3*x**2)/(6*x**8 + 24*x**6 + 36*x**4
 + 24*x**2 + 6)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.94

$$\int \frac{(-1+x)x(1+x)^5}{(1+x^2)^5}\,dx = -\frac{3\,x^6+8\,x^5+12\,x^4+8\,x^3+3\,x^2}{6\,(x^8+4\,x^6+6\,x^4+4\,x^2+1)}$$

input
```
integrate((-1+x)*x*(1+x)^5/(x^2+1)^5,x, algorithm="maxima")
```

output
```
-1/6*(3*x^6 + 8*x^5 + 12*x^4 + 8*x^3 + 3*x^2)/(x^8 + 4*x^6 + 6*x^4 + 4*x^2
 + 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.53

$$\int \frac{(-1+x)x(1+x)^5}{(1+x^2)^5}\,dx = -\frac{3\left(x+\frac{1}{x}\right)^2+8\,x+\frac{8}{x}+6}{6\left(x+\frac{1}{x}\right)^4}$$

input
```
integrate((-1+x)*x*(1+x)^5/(x^2+1)^5,x, algorithm="giac")
```

output
```
-1/6*(3*(x + 1/x)^2 + 8*x + 8/x + 6)/(x + 1/x)^4
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.91

$$\int \frac{(-1+x)x(1+x)^5}{(1+x^2)^5}\,dx = \frac{\frac{4\,x}{3}+2}{(x^2+1)^3} - \frac{\frac{4\,x}{3}+\frac{1}{2}}{(x^2+1)^2} - \frac{1}{2\,(x^2+1)} - \frac{1}{(x^2+1)^4}$$

input
```
int((x*(x - 1)*(x + 1)^5)/(x^2 + 1)^5,x)
```

output
```
((4*x)/3 + 2)/(x^2 + 1)^3 - ((4*x)/3 + 1/2)/(x^2 + 1)^2 - 1/(2*(x^2 + 1))
- 1/(x^2 + 1)^4
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.89

$$\int \frac{(-1+x)x(1+x)^5}{(1+x^2)^5}\,dx = \frac{3x^8-32x^5-30x^4-32x^3+3}{24x^8+96x^6+144x^4+96x^2+24}$$

input
```
int((-1+x)*x*(1+x)^5/(x^2+1)^5,x)
```

output
```
(3*x**8 - 32*x**5 - 30*x**4 - 32*x**3 + 3)/(24*(x**8 + 4*x**6 + 6*x**4 + 4
*x**2 + 1))
```

3.77 $\int \frac{x(-1-4x-5x^2+5x^4+4x^5+x^6)}{(1+x^2)^5} \, dx$

Optimal result

Integrand size = 32, antiderivative size = 53

$$\int \frac{x(-1-4x-5x^2+5x^4+4x^5+x^6)}{(1+x^2)^5} \, dx = -\frac{1}{(1+x^2)^4}+\frac{2(3+2x)}{3(1+x^2)^3} - \frac{3+8x}{6(1+x^2)^2}-\frac{1}{2(1+x^2)}$$

output
```
-1/(x^2+1)^4+2/3*(3+2*x)/(x^2+1)^3-1/6*(3+8*x)/(x^2+1)^2-1/(2*x^2+2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.64

$$\int \frac{x(-1-4x-5x^2+5x^4+4x^5+x^6)}{(1+x^2)^5} \, dx = -\frac{x^2(3+8x+12x^2+8x^3+3x^4)}{6(1+x^2)^4}$$

input
```
Integrate[(x*(-1 - 4*x - 5*x^2 + 5*x^4 + 4*x^5 + x^6))/(1 + x^2)^5,x]
```

output
```
-1/6*(x^2*(3 + 8*x + 12*x^2 + 8*x^3 + 3*x^4))/(1 + x^2)^4
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.15, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2335, 27, 2342, 2335, 27, 2345, 27, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x \left(x^6+4 x^5+5 x^4-5 x^2-4 x-1\right)}{\left(x^2+1\right)^5} \, dx$$

$$\Big\downarrow \text{2335}$$

$$\frac{x^2}{\left(x^2+1\right)^4}-\frac{1}{8} \int \frac{8 \left(-x^5-4 x^4-4 x^3+4 x^2+3 x\right)}{\left(x^2+1\right)^4}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{x^2}{\left(x^2+1\right)^4}-\int \frac{-x^5-4 x^4-4 x^3+4 x^2+3 x}{\left(x^2+1\right)^4}dx$$

$$\Big\downarrow \text{2342}$$

$$\frac{x^2}{\left(x^2+1\right)^4}-\int \frac{x \left(-x^4-4 x^3-4 x^2+4 x+3\right)}{\left(x^2+1\right)^4}dx$$

$$\Big\downarrow \text{2335}$$

$$\frac{1}{6} \int -\frac{2 \left(-3 x^3-12 x^2+3 x+4\right)}{\left(x^2+1\right)^3}dx+\frac{x^2}{\left(x^2+1\right)^4}+\frac{(4-3 x) x}{3 \left(x^2+1\right)^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{3} \int \frac{-3 x^3-12 x^2+3 x+4}{\left(x^2+1\right)^3}dx+\frac{x^2}{\left(x^2+1\right)^4}+\frac{(4-3 x) x}{3 \left(x^2+1\right)^3}$$

$$\Big\downarrow \text{2345}$$

$$\frac{1}{3} \left(\frac{1}{4} \int \frac{12 x}{\left(x^2+1\right)^2}dx+\frac{3-8 x}{2 \left(x^2+1\right)^2}\right)+\frac{x^2}{\left(x^2+1\right)^4}+\frac{(4-3 x) x}{3 \left(x^2+1\right)^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3} \left(3 \int \frac{x}{\left(x^2+1\right)^2}dx+\frac{3-8 x}{2 \left(x^2+1\right)^2}\right)+\frac{x^2}{\left(x^2+1\right)^4}+\frac{(4-3 x) x}{3 \left(x^2+1\right)^3}$$

$$\Big\downarrow \text{241}$$

$$\frac{x^2}{(x^2+1)^4}+\frac{(4-3x)x}{3\,(x^2+1)^3}+\frac{1}{3}\left(\frac{3-8x}{2\,(x^2+1)^2}-\frac{3}{2\,(x^2+1)}\right)$$

input
```
Int[(x*(-1 - 4*x - 5*x^2 + 5*x^4 + 4*x^5 + x^6))/(1 + x^2)^5,x]
```

output
```
x^2/(1 + x^2)^4 + ((4 - 3*x)*x)/(3*(1 + x^2)^3) + ((3 - 8*x)/(2*(1 + x^2)^
2) - 3/(2*(1 + x^2)))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 2335
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Simp[c/(2*a*b*(p + 1))   Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSu
m[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
 b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

rule 2342
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[x*PolynomialQuotient
[Pq, x, x]*(a + b*x^2)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && EqQ[
Coeff[Pq, x, 0], 0] &&  !MatchQ[Pq, x^(m_.)*(u_.) /; IntegerQ[m]]
```

rule 2345

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.62

method	result
gosper	$-\frac{x^{2}\left(3x^{4}+8x^{3}+12x^{2}+8x+3\right)}{6\left(x^{2}+1\right)^{4}}$
default	$\frac{-\frac{1}{2}x^{6}-\frac{4}{3}x^{5}-2x^{4}-\frac{4}{3}x^{3}-\frac{1}{2}x^{2}}{\left(x^{2}+1\right)^{4}}$
norman	$\frac{-\frac{1}{2}x^{6}-\frac{4}{3}x^{5}-2x^{4}-\frac{4}{3}x^{3}-\frac{1}{2}x^{2}}{\left(x^{2}+1\right)^{4}}$
risch	$\frac{-\frac{1}{2}x^{6}-\frac{4}{3}x^{5}-2x^{4}-\frac{4}{3}x^{3}-\frac{1}{2}x^{2}}{\left(x^{2}+1\right)^{4}}$
parallelrisch	$\frac{-3x^{6}-8x^{5}-12x^{4}-8x^{3}-3x^{2}}{6\left(x^{2}+1\right)^{4}}$
orering	$-\frac{x^{2}\left(3x^{4}+8x^{3}+12x^{2}+8x+3\right)\left(x^{6}+4x^{5}+5x^{4}-5x^{2}-4x-1\right)}{6\left(x^{2}+1\right)^{4}\left(-1+x\right)\left(1+x\right)^{5}}$
meijerg	$-\frac{x^{2}\left(x^{6}+4x^{4}+6x^{2}+4\right)}{8\left(x^{2}+1\right)^{4}}+\frac{x\left(-15x^{6}-55x^{4}-73x^{2}+15\right)}{96\left(x^{2}+1\right)^{4}}-\frac{5x^{4}\left(x^{4}+4x^{2}+6\right)}{24\left(x^{2}+1\right)^{4}}+\frac{5x^{6}\left(x^{2}+4\right)}{24\left(x^{2}+1\right)^{4}}-\frac{x\left(-105x^{6}+511x^{4}+385x^{2}\right.}{672\left(x^{2}+1\right)^{4}}$

input

```
int(x*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)/(x^2+1)^5,x,method=_RETURNVERBOSE)
```

output

```
-1/6*x^2*(3*x^4+8*x^3+12*x^2+8*x+3)/(x^2+1)^4
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.94

$$\int \frac{x(-1-4x-5x^2+5x^4+4x^5+x^6)}{(1+x^2)^5}\,dx = -\frac{3\,x^6+8\,x^5+12\,x^4+8\,x^3+3\,x^2}{6\,(x^8+4\,x^6+6\,x^4+4\,x^2+1)}$$

input
```
integrate(x*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)/(x^2+1)^5,x, algorithm="fricas")
```

output
```
-1/6*(3*x^6 + 8*x^5 + 12*x^4 + 8*x^3 + 3*x^2)/(x^8 + 4*x^6 + 6*x^4 + 4*x^2
 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.91

$$\int \frac{x(-1-4x-5x^2+5x^4+4x^5+x^6)}{(1+x^2)^5}\,dx = \frac{-3x^6-8x^5-12x^4-8x^3-3x^2}{6x^8+24x^6+36x^4+24x^2+6}$$

input
```
integrate(x*(x**6+4*x**5+5*x**4-5*x**2-4*x-1)/(x**2+1)**5,x)
```

output
```
(-3*x**6 - 8*x**5 - 12*x**4 - 8*x**3 - 3*x**2)/(6*x**8 + 24*x**6 + 36*x**4
 + 24*x**2 + 6)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.94

$$\int \frac{x(-1-4x-5x^2+5x^4+4x^5+x^6)}{(1+x^2)^5}\,dx = -\frac{3\,x^6+8\,x^5+12\,x^4+8\,x^3+3\,x^2}{6\,(x^8+4\,x^6+6\,x^4+4\,x^2+1)}$$

input
```
integrate(x*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)/(x^2+1)^5,x, algorithm="maxima")
```

output
```
-1/6*(3*x^6 + 8*x^5 + 12*x^4 + 8*x^3 + 3*x^2)/(x^8 + 4*x^6 + 6*x^4 + 4*x^2
 + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.53

$$\int \frac{x(-1-4x-5x^2+5x^4+4x^5+x^6)}{(1+x^2)^5}\,dx = -\frac{3\left(x+\frac{1}{x}\right)^2+8\,x+\frac{8}{x}+6}{6\left(x+\frac{1}{x}\right)^4}$$

input
```
integrate(x*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)/(x^2+1)^5,x, algorithm="giac")
```

output
```
-1/6*(3*(x + 1/x)^2 + 8*x + 8/x + 6)/(x + 1/x)^4
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.91

$$\int \frac{x(-1-4x-5x^2+5x^4+4x^5+x^6)}{(1+x^2)^5}\,dx = \frac{\frac{4\,x}{3}+2}{(x^2+1)^3} - \frac{\frac{4\,x}{3}+\frac{1}{2}}{(x^2+1)^2} - \frac{1}{2\,(x^2+1)} - \frac{1}{(x^2+1)^4}$$

input
```
int(-(x*(4*x + 5*x^2 - 5*x^4 - 4*x^5 - x^6 + 1))/(x^2 + 1)^5,x)
```

output
```
((4*x)/3 + 2)/(x^2 + 1)^3 - ((4*x)/3 + 1/2)/(x^2 + 1)^2 - 1/(2*(x^2 + 1))
- 1/(x^2 + 1)^4
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.89

$$\int \frac{x(-1-4x-5x^2+5x^4+4x^5+x^6)}{(1+x^2)^5}\,dx = \frac{3x^8-32x^5-30x^4-32x^3+3}{24x^8+96x^6+144x^4+96x^2+24}$$

input
```
int(x*(x^6+4*x^5+5*x^4-5*x^2-4*x-1)/(x^2+1)^5,x)
```

output
```
(3*x**8 - 32*x**5 - 30*x**4 - 32*x**3 + 3)/(24*(x**8 + 4*x**6 + 6*x**4 + 4
*x**2 + 1))
```

3.78 $\int \frac{-x-4x^2-5x^3+5x^5+4x^6+x^7}{(1+x^2)^5}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 53

$$\int \frac{-x-4x^2-5x^3+5x^5+4x^6+x^7}{(1+x^2)^5}\,dx = -\frac{1}{(1+x^2)^4}+\frac{2(3+2x)}{3(1+x^2)^3}-\frac{3+8x}{6(1+x^2)^2}-\frac{1}{2(1+x^2)}$$

output
```
-1/(x^2+1)^4+2/3*(3+2*x)/(x^2+1)^3-1/6*(3+8*x)/(x^2+1)^2-1/(2*x^2+2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.64

$$\int \frac{-x-4x^2-5x^3+5x^5+4x^6+x^7}{(1+x^2)^5}\,dx = -\frac{x^2(3+8x+12x^2+8x^3+3x^4)}{6(1+x^2)^4}$$

input
```
Integrate[(-x - 4*x^2 - 5*x^3 + 5*x^5 + 4*x^6 + x^7)/(1 + x^2)^5,x]
```

output
```
-1/6*(x^2*(3 + 8*x + 12*x^2 + 8*x^3 + 3*x^4))/(1 + x^2)^4
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.15, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {2342, 2335, 27, 2342, 2335, 27, 2345, 27, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7+4x^6+5x^5-5x^3-4x^2-x}{(x^2+1)^5}dx$$

$$\Big\downarrow \text{ 2342}$$

$$\int \frac{x(x^6+4x^5+5x^4-5x^2-4x-1)}{(x^2+1)^5}dx$$

$$\Big\downarrow \text{ 2335}$$

$$\frac{x^2}{(x^2+1)^4}-\frac{1}{8}\int \frac{8(-x^5-4x^4-4x^3+4x^2+3x)}{(x^2+1)^4}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{x^2}{(x^2+1)^4}-\int \frac{-x^5-4x^4-4x^3+4x^2+3x}{(x^2+1)^4}dx$$

$$\Big\downarrow \text{ 2342}$$

$$\frac{x^2}{(x^2+1)^4}-\int \frac{x(-x^4-4x^3-4x^2+4x+3)}{(x^2+1)^4}dx$$

$$\Big\downarrow \text{ 2335}$$

$$\frac{1}{6}\int -\frac{2(-3x^3-12x^2+3x+4)}{(x^2+1)^3}dx+\frac{x^2}{(x^2+1)^4}+\frac{(4-3x)x}{3(x^2+1)^3}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{1}{3}\int \frac{-3x^3-12x^2+3x+4}{(x^2+1)^3}dx+\frac{x^2}{(x^2+1)^4}+\frac{(4-3x)x}{3(x^2+1)^3}$$

$$\Big\downarrow \text{ 2345}$$

$$\frac{1}{3}\left(\frac{1}{4}\int \frac{12x}{(x^2+1)^2}dx+\frac{3-8x}{2(x^2+1)^2}\right)+\frac{x^2}{(x^2+1)^4}+\frac{(4-3x)x}{3(x^2+1)^3}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{3}\left(3\int \frac{x}{\left(x^2+1\right)^2}dx+\frac{3-8x}{2\left(x^2+1\right)^2}\right)+\frac{x^2}{\left(x^2+1\right)^4}+\frac{(4-3x)x}{3\left(x^2+1\right)^3}$$

$$\Big\downarrow \ 241$$

$$\frac{x^2}{\left(x^2+1\right)^4}+\frac{(4-3x)x}{3\left(x^2+1\right)^3}+\frac{1}{3}\left(\frac{3-8x}{2\left(x^2+1\right)^2}-\frac{3}{2\left(x^2+1\right)}\right)$$

input
```
Int[(-x - 4*x^2 - 5*x^3 + 5*x^5 + 4*x^6 + x^7)/(1 + x^2)^5,x]
```

output
```
x^2/(1 + x^2)^4 + ((4 - 3*x)*x)/(3*(1 + x^2)^3) + ((3 - 8*x)/(2*(1 + x^2)^
2) - 3/(2*(1 + x^2)))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 2335
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Simp[c/(2*a*b*(p + 1))   Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSu
m[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
 b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

rule 2342
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[x*PolynomialQuotient
[Pq, x, x]*(a + b*x^2)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && EqQ[
Coeff[Pq, x, 0], 0] &&  !MatchQ[Pq, x^(m_.)*(u_.) /; IntegerQ[m]]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.62

method	result
gosper	$-\frac{x^{2}\left(3x^{4}+8x^{3}+12x^{2}+8x+3\right)}{6\left(x^{2}+1\right)^{4}}$
default	$\frac{-\frac{1}{2}x^{6}-\frac{4}{3}x^{5}-2x^{4}-\frac{4}{3}x^{3}-\frac{1}{2}x^{2}}{\left(x^{2}+1\right)^{4}}$
norman	$\frac{-\frac{1}{2}x^{6}-\frac{4}{3}x^{5}-2x^{4}-\frac{4}{3}x^{3}-\frac{1}{2}x^{2}}{\left(x^{2}+1\right)^{4}}$
risch	$\frac{-\frac{1}{2}x^{6}-\frac{4}{3}x^{5}-2x^{4}-\frac{4}{3}x^{3}-\frac{1}{2}x^{2}}{\left(x^{2}+1\right)^{4}}$
parallelrisch	$\frac{-3x^{6}-8x^{5}-12x^{4}-8x^{3}-3x^{2}}{6\left(x^{2}+1\right)^{4}}$
orering	$-\frac{x\left(3x^{4}+8x^{3}+12x^{2}+8x+3\right)\left(x^{7}+4x^{6}+5x^{5}-5x^{3}-4x^{2}-x\right)}{6\left(x^{2}+1\right)^{4}\left(-1+x\right)\left(1+x\right)^{5}}$
meijerg	$-\frac{x^{2}\left(x^{6}+4x^{4}+6x^{2}+4\right)}{8\left(x^{2}+1\right)^{4}}+\frac{x\left(-15x^{6}-55x^{4}-73x^{2}+15\right)}{96\left(x^{2}+1\right)^{4}}-\frac{5x^{4}\left(x^{4}+4x^{2}+6\right)}{24\left(x^{2}+1\right)^{4}}+\frac{5x^{6}\left(x^{2}+4\right)}{24\left(x^{2}+1\right)^{4}}-\frac{x\left(-105x^{6}+511x^{4}+385x^{2}\right.}{672\left(x^{2}+1\right)^{4}}$

input
```
int((x^7+4*x^6+5*x^5-5*x^3-4*x^2-x)/(x^2+1)^5,x,method=_RETURNVERBOSE)
```

output
```
-1/6*x^2*(3*x^4+8*x^3+12*x^2+8*x+3)/(x^2+1)^4
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.94

$$\int \frac{-x-4x^2-5x^3+5x^5+4x^6+x^7}{(1+x^2)^5}\,dx = -\frac{3\,x^6+8\,x^5+12\,x^4+8\,x^3+3\,x^2}{6\,(x^8+4\,x^6+6\,x^4+4\,x^2+1)}$$

input
```
integrate((x^7+4*x^6+5*x^5-5*x^3-4*x^2-x)/(x^2+1)^5,x, algorithm="fricas")
```

output
```
-1/6*(3*x^6 + 8*x^5 + 12*x^4 + 8*x^3 + 3*x^2)/(x^8 + 4*x^6 + 6*x^4 + 4*x^2
 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.91

$$\int \frac{-x-4x^2-5x^3+5x^5+4x^6+x^7}{(1+x^2)^5}\,dx = \frac{-3x^6-8x^5-12x^4-8x^3-3x^2}{6x^8+24x^6+36x^4+24x^2+6}$$

input
```
integrate((x**7+4*x**6+5*x**5-5*x**3-4*x**2-x)/(x**2+1)**5,x)
```

output
```
(-3*x**6 - 8*x**5 - 12*x**4 - 8*x**3 - 3*x**2)/(6*x**8 + 24*x**6 + 36*x**4
 + 24*x**2 + 6)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.94

$$\int \frac{-x-4x^2-5x^3+5x^5+4x^6+x^7}{(1+x^2)^5}\,dx = -\frac{3\,x^6+8\,x^5+12\,x^4+8\,x^3+3\,x^2}{6\,(x^8+4\,x^6+6\,x^4+4\,x^2+1)}$$

input
```
integrate((x^7+4*x^6+5*x^5-5*x^3-4*x^2-x)/(x^2+1)^5,x, algorithm="maxima")
```

output
```
-1/6*(3*x^6 + 8*x^5 + 12*x^4 + 8*x^3 + 3*x^2)/(x^8 + 4*x^6 + 6*x^4 + 4*x^2
 + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.53

$$\int \frac{-x-4x^2-5x^3+5x^5+4x^6+x^7}{(1+x^2)^5}\,dx=-\frac{3\left(x+\frac{1}{x}\right)^2+8\,x+\frac{8}{x}+6}{6\left(x+\frac{1}{x}\right)^4}$$

input
```
integrate((x^7+4*x^6+5*x^5-5*x^3-4*x^2-x)/(x^2+1)^5,x, algorithm="giac")
```

output
```
-1/6*(3*(x + 1/x)^2 + 8*x + 8/x + 6)/(x + 1/x)^4
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.91

$$\int \frac{-x-4x^2-5x^3+5x^5+4x^6+x^7}{(1+x^2)^5}\,dx=\frac{\frac{4\,x}{3}+2}{\left(x^2+1\right)^3}-\frac{\frac{4\,x}{3}+\frac{1}{2}}{\left(x^2+1\right)^2}-\frac{1}{2\,\left(x^2+1\right)}-\frac{1}{\left(x^2+1\right)^4}$$

input
```
int(-(x + 4*x^2 + 5*x^3 - 5*x^5 - 4*x^6 - x^7)/(x^2 + 1)^5,x)
```

output
```
((4*x)/3 + 2)/(x^2 + 1)^3 - ((4*x)/3 + 1/2)/(x^2 + 1)^2 - 1/(2*(x^2 + 1))
- 1/(x^2 + 1)^4
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.89

$$\int \frac{-x-4x^2-5x^3+5x^5+4x^6+x^7}{(1+x^2)^5}\,dx=\frac{3x^8-32x^5-30x^4-32x^3+3}{24x^8+96x^6+144x^4+96x^2+24}$$

input
```
int((x^7+4*x^6+5*x^5-5*x^3-4*x^2-x)/(x^2+1)^5,x)
```

output
```
(3*x**8 - 32*x**5 - 30*x**4 - 32*x**3 + 3)/(24*(x**8 + 4*x**6 + 6*x**4 + 4
*x**2 + 1))
```

3.79 $\int (a+bx^2)^4 (A+Bx^2) \, dx$

Optimal result

Integrand size = 17, antiderivative size = 94

$$\int (a+bx^2)^4 (A+Bx^2) \, dx = a^4Ax + \frac{1}{3}a^3(4Ab+aB)x^3 + \frac{2}{5}a^2b(3Ab+2aB)x^5 + \frac{2}{7}ab^2(2Ab+3aB)x^7 + \frac{1}{9}b^3(Ab+4aB)x^9 + \frac{1}{11}b^4Bx^{11}$$

output
```
a^4*A*x+1/3*a^3*(4*A*b+B*a)*x^3+2/5*a^2*b*(3*A*b+2*B*a)*x^5+2/7*a*b^2*(2*A
*b+3*B*a)*x^7+1/9*b^3*(A*b+4*B*a)*x^9+1/11*b^4*B*x^11
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00

$$\int (a+bx^2)^4 (A+Bx^2) \, dx = a^4Ax + \frac{1}{3}a^3(4Ab+aB)x^3 + \frac{2}{5}a^2b(3Ab+2aB)x^5 + \frac{2}{7}ab^2(2Ab+3aB)x^7 + \frac{1}{9}b^3(Ab+4aB)x^9 + \frac{1}{11}b^4Bx^{11}$$

input
```
Integrate[(a + b*x^2)^4*(A + B*x^2),x]
```

output
```
a^4*A*x + (a^3*(4*A*b + a*B)*x^3)/3 + (2*a^2*b*(3*A*b + 2*a*B)*x^5)/5 + (2
*a*b^2*(2*A*b + 3*a*B)*x^7)/7 + (b^3*(A*b + 4*a*B)*x^9)/9 + (b^4*B*x^11)/1
1
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {290, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^4 \left(A+Bx^2\right) \, dx$$

$$\Big\downarrow \text{ 290}$$

$$\int \left(a^4A+a^3x^2(aB+4Ab)+2a^2bx^4(2aB+3Ab)+b^3x^8(4aB+Ab)+2ab^2x^6(3aB+2Ab)+b^4Bx^{10}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$a^4Ax+\frac{1}{3}a^3x^3(aB+4Ab)+\frac{2}{5}a^2bx^5(2aB+3Ab)+\frac{1}{9}b^3x^9(4aB+Ab)+\frac{2}{7}ab^2x^7(3aB+2Ab)+\frac{1}{11}b^4Bx^{11}$$

input
```
Int[(a + b*x^2)^4*(A + B*x^2),x]
```

output

```
a^4*A*x + (a^3*(4*A*b + a*B)*x^3)/3 + (2*a^2*b*(3*A*b + 2*a*B)*x^5)/5 + (2
*a*b^2*(2*A*b + 3*a*B)*x^7)/7 + (b^3*(A*b + 4*a*B)*x^9)/9 + (b^4*B*x^11)/1
1
```

Defintions of rubi rules used

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.01

method	result
norman	$\frac{b^4 B x^{11}}{11}+\left(\frac{1}{9}A b^4+\frac{4}{9}B a b^3\right)x^9+\left(\frac{4}{7}a b^3 A+\frac{6}{7}a^2b^2B\right)x^7+\left(\frac{6}{5}A a^2b^2+\frac{4}{5}B a^3b\right)x^5+\left(\frac{4}{3}A a^3b\right.$ …
default	$\frac{b^4 B x^{11}}{11}+\frac{(A b^4+4B a b^3)x^9}{9}+\frac{(4a b^3 A+6a^2b^2B)x^7}{7}+\frac{(6A a^2b^2+4B a^3b)x^5}{5}+\frac{(4A a^3b+B a^4)x^3}{3}+a^4Ax$
gosper	$\frac{1}{11}b^4B x^{11}+\frac{1}{9}x^9A b^4+\frac{4}{9}x^9B a b^3+\frac{4}{7}x^7a b^3A+\frac{6}{7}x^7a^2b^2B+\frac{6}{5}x^5A a^2b^2+\frac{4}{5}x^5B a^3b+\frac{4}{3}x^3A$ …
risch	$\frac{1}{11}b^4B x^{11}+\frac{1}{9}x^9A b^4+\frac{4}{9}x^9B a b^3+\frac{4}{7}x^7a b^3A+\frac{6}{7}x^7a^2b^2B+\frac{6}{5}x^5A a^2b^2+\frac{4}{5}x^5B a^3b+\frac{4}{3}x^3A$ …
parallelrisch	$\frac{1}{11}b^4B x^{11}+\frac{1}{9}x^9A b^4+\frac{4}{9}x^9B a b^3+\frac{4}{7}x^7a b^3A+\frac{6}{7}x^7a^2b^2B+\frac{6}{5}x^5A a^2b^2+\frac{4}{5}x^5B a^3b+\frac{4}{3}x^3A$ …
orering	$\frac{x(315B b^4x^{10}+385A x^8b^4+1540B x^8a b^3+1980A x^6a b^3+2970B x^6a^2b^2+4158A x^4a^2b^2+2772B x^4a^3b+4620A x^2a^3b+1155 \ldots}{3465}$

input
```
int((b*x^2+a)^4*(B*x^2+A),x,method=_RETURNVERBOSE)
```

output
```
1/11*b^4*B*x^11+(1/9*A*b^4+4/9*B*a*b^3)*x^9+(4/7*a*b^3*A+6/7*a^2*b^2*B)*x^
7+(6/5*A*a^2*b^2+4/5*B*a^3*b)*x^5+(4/3*A*a^3*b+1/3*B*a^4)*x^3+a^4*A*x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.02

$$\int (a+bx^2)^4 (A+Bx^2)\,dx = \frac{1}{11}Bb^4x^{11}+\frac{1}{9}(4Bab^3+Ab^4)x^9+\frac{2}{7}(3Ba^2b^2+2Aab^3)x^7 + Aa^4x+\frac{2}{5}(2Ba^3b+3Aa^2b^2)x^5+\frac{1}{3}(Ba^4+4Aa^3b)x^3$$

input
```
integrate((b*x^2+a)^4*(B*x^2+A),x, algorithm="fricas")
```

output
```
1/11*B*b^4*x^11 + 1/9*(4*B*a*b^3 + A*b^4)*x^9 + 2/7*(3*B*a^2*b^2 + 2*A*a*b
^3)*x^7 + A*a^4*x + 2/5*(2*B*a^3*b + 3*A*a^2*b^2)*x^5 + 1/3*(B*a^4 + 4*A*a
^3*b)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.14

$$\int \left(a+bx^2\right)^4 \left(A+Bx^2\right)\,dx = Aa^4x + \frac{Bb^4x^{11}}{11} + x^9\left(\frac{Ab^4}{9} + \frac{4Bab^3}{9}\right) + x^7\cdot\left(\frac{4Aab^3}{7} + \frac{6Ba^2b^2}{7}\right) + x^5 \cdot\left(\frac{6Aa^2b^2}{5} + \frac{4Ba^3b}{5}\right) + x^3\cdot\left(\frac{4Aa^3b}{3} + \frac{Ba^4}{3}\right)$$

input
```
integrate((b*x**2+a)**4*(B*x**2+A),x)
```

output
```
A*a**4*x + B*b**4*x**11/11 + x**9*(A*b**4/9 + 4*B*a*b**3/9) + x**7*(4*A*a*
b**3/7 + 6*B*a**2*b**2/7) + x**5*(6*A*a**2*b**2/5 + 4*B*a**3*b/5) + x**3*(
4*A*a**3*b/3 + B*a**4/3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.02

$$\int \left(a+bx^2\right)^4 \left(A+Bx^2\right)\,dx = \frac{1}{11}\,Bb^4x^{11} + \frac{1}{9}\,\left(4\,Bab^3 + Ab^4\right)x^9 + \frac{2}{7}\,\left(3\,Ba^2b^2 + 2\,Aab^3\right)x^7 + Aa^4x + \frac{2}{5}\,\left(2\,Ba^3b + 3\,Aa^2b^2\right)x^5 + \frac{1}{3}\,\left(Ba^4 + 4\,Aa^3b\right)x^3$$

input
```
integrate((b*x^2+a)^4*(B*x^2+A),x, algorithm="maxima")
```

output
```
1/11*B*b^4*x^11 + 1/9*(4*B*a*b^3 + A*b^4)*x^9 + 2/7*(3*B*a^2*b^2 + 2*A*a*b
^3)*x^7 + A*a^4*x + 2/5*(2*B*a^3*b + 3*A*a^2*b^2)*x^5 + 1/3*(B*a^4 + 4*A*a
^3*b)*x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.04

$$\int (a+bx^2)^4 (A+Bx^2) \, dx = \frac{1}{11} Bb^4x^{11} + \frac{4}{9} Bab^3x^9 + \frac{1}{9} Ab^4x^9 + \frac{6}{7} Ba^2b^2x^7 + \frac{4}{7} Aab^3x^7 + \frac{4}{5} Ba^3bx^5 + \frac{6}{5} Aa^2b^2x^5 + \frac{1}{3} Ba^4x^3 + \frac{4}{3} Aa^3bx^3 + Aa^4x$$

input
```
integrate((b*x^2+a)^4*(B*x^2+A),x, algorithm="giac")
```

output
```
1/11*B*b^4*x^11 + 4/9*B*a*b^3*x^9 + 1/9*A*b^4*x^9 + 6/7*B*a^2*b^2*x^7 + 4/
7*A*a*b^3*x^7 + 4/5*B*a^3*b*x^5 + 6/5*A*a^2*b^2*x^5 + 1/3*B*a^4*x^3 + 4/3*
A*a^3*b*x^3 + A*a^4*x
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.94

$$\int (a+bx^2)^4 (A+Bx^2) \, dx = x^3 \left(\frac{B\,a^4}{3} + \frac{4\,A\,b\,a^3}{3}\right) + x^9 \left(\frac{A\,b^4}{9} + \frac{4\,B\,a\,b^3}{9}\right) + \frac{B\,b^4\,x^{11}}{11} + A\,a^4\,x + \frac{2\,a^2\,b\,x^5\,(3\,A\,b+2\,B\,a)}{5} + \frac{2\,a\,b^2\,x^7\,(2\,A\,b+3\,B\,a)}{7}$$

input
```
int((A + B*x^2)*(a + b*x^2)^4,x)
```

output
```
x^3*((B*a^4)/3 + (4*A*a^3*b)/3) + x^9*((A*b^4)/9 + (4*B*a*b^3)/9) + (B*b^4
*x^11)/11 + A*a^4*x + (2*a^2*b*x^5*(3*A*b + 2*B*a))/5 + (2*a*b^2*x^7*(2*A*
b + 3*B*a))/7
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.61

$$\int (a+bx^2)^4 (A+Bx^2)\, dx = \frac{x(63b^5x^{10}+385a\,b^4x^8+990a^2b^3x^6+1386a^3b^2x^4+1155a^4b\,x^2+693a^5)}{693}$$

input
```
int((b*x^2+a)^4*(B*x^2+A),x)
```

output
```
(x*(693*a**5 + 1155*a**4*b*x**2 + 1386*a**3*b**2*x**4 + 990*a**2*b**3*x**6
 + 385*a*b**4*x**8 + 63*b**5*x**10))/693
```

3.80 $\int (a+bx^2)^3 (A+Bx^2)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 70

$$\int (a+bx^2)^3 (A+Bx^2)\,dx = a^3Ax + \frac{1}{3}a^2(3Ab+aB)x^3 + \frac{3}{5}ab(Ab+aB)x^5 + \frac{1}{7}b^2(Ab+3aB)x^7 + \frac{1}{9}b^3Bx^9$$

output
```
a^3*A*x+1/3*a^2*(3*A*b+B*a)*x^3+3/5*a*b*(A*b+B*a)*x^5+1/7*b^2*(A*b+3*B*a)*
x^7+1/9*b^3*B*x^9
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00

$$\int (a+bx^2)^3 (A+Bx^2)\,dx = a^3Ax + \frac{1}{3}a^2(3Ab+aB)x^3 + \frac{3}{5}ab(Ab+aB)x^5 + \frac{1}{7}b^2(Ab+3aB)x^7 + \frac{1}{9}b^3Bx^9$$

input
```
Integrate[(a + b*x^2)^3*(A + B*x^2),x]
```

output
```
a^3*A*x + (a^2*(3*A*b + a*B)*x^3)/3 + (3*a*b*(A*b + a*B)*x^5)/5 + (b^2*(A*
b + 3*a*B)*x^7)/7 + (b^3*B*x^9)/9
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {290, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^3 \left(A+Bx^2\right) \, dx$$

$$\Big\downarrow \text{ 290}$$

$$\int \left(a^3A+a^2x^2(aB+3Ab)+b^2x^6(3aB+Ab)+3abx^4(aB+Ab)+b^3Bx^8\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$a^3Ax+\frac{1}{3}a^2x^3(aB+3Ab)+\frac{1}{7}b^2x^7(3aB+Ab)+\frac{3}{5}abx^5(aB+Ab)+\frac{1}{9}b^3Bx^9$$

input
```
Int[(a + b*x^2)^3*(A + B*x^2),x]
```

output
```
a^3*A*x + (a^2*(3*A*b + a*B)*x^3)/3 + (3*a*b*(A*b + a*B)*x^5)/5 + (b^2*(A*
b + 3*a*B)*x^7)/7 + (b^3*B*x^9)/9
```

Defintions of rubi rules used

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.01

method	result	size
norman	$\frac{b^3 B x^9}{9}+\left(\frac{1}{7}b^3 A+\frac{3}{7}a b^2 B\right)x^7+\left(\frac{3}{5}a b^2 A+\frac{3}{5}a^2 b B\right)x^5+\left(a^2 b A+\frac{1}{3}a^3 B\right)x^3+a^3 A x$	71
default	$\frac{b^3 B x^9}{9}+\frac{(b^3 A+3a b^2 B)x^7}{7}+\frac{(3a b^2 A+3a^2 b B)x^5}{5}+\frac{(3a^2 b A+a^3 B)x^3}{3}+a^3 A x$	73
gosper	$\frac{1}{9}b^3 B x^9+\frac{1}{7}A b^3 x^7+\frac{3}{7}x^7 a b^2 B+\frac{3}{5}a A b^2 x^5+\frac{3}{5}x^5 a^2 b B+a^2 A b x^3+\frac{1}{3}x^3 a^3 B+a^3 A x$	74
risch	$\frac{1}{9}b^3 B x^9+\frac{1}{7}A b^3 x^7+\frac{3}{7}x^7 a b^2 B+\frac{3}{5}a A b^2 x^5+\frac{3}{5}x^5 a^2 b B+a^2 A b x^3+\frac{1}{3}x^3 a^3 B+a^3 A x$	74
parallelrisch	$\frac{1}{9}b^3 B x^9+\frac{1}{7}A b^3 x^7+\frac{3}{7}x^7 a b^2 B+\frac{3}{5}a A b^2 x^5+\frac{3}{5}x^5 a^2 b B+a^2 A b x^3+\frac{1}{3}x^3 a^3 B+a^3 A x$	74
orering	$\frac{x(35b^3 B x^8+45A b^3 x^6+135B a b^2 x^6+189a A b^2 x^4+189B a^2 b x^4+315a^2 A b x^2+105B a^3 x^2+315a^3 A)}{315}$	78

input
```
int((b*x^2+a)^3*(B*x^2+A),x,method=_RETURNVERBOSE)
```

output
```
1/9*b^3*B*x^9+(1/7*b^3*A+3/7*a*b^2*B)*x^7+(3/5*a*b^2*A+3/5*a^2*b*B)*x^5+(a
^2*b*A+1/3*a^3*B)*x^3+a^3*A*x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00

$$\int (a+bx^2)^3 (A+Bx^2)\,dx = \frac{1}{9}Bb^3x^9+\frac{1}{7}(3Bab^2+Ab^3)x^7+\frac{3}{5}(Ba^2b+Aab^2)x^5+Aa^3x+\frac{1}{3}(Ba^3+3Aa^2b)x^3$$

input
```
integrate((b*x^2+a)^3*(B*x^2+A),x, algorithm="fricas")
```

output
```
1/9*B*b^3*x^9 + 1/7*(3*B*a*b^2 + A*b^3)*x^7 + 3/5*(B*a^2*b + A*a*b^2)*x^5
+ A*a^3*x + 1/3*(B*a^3 + 3*A*a^2*b)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.09

$$\int (a+bx^2)^3 (A+Bx^2)\,dx = Aa^3x + \frac{Bb^3x^9}{9} + x^7\left(\frac{Ab^3}{7} + \frac{3Bab^2}{7}\right) + x^5 \cdot \left(\frac{3Aab^2}{5} + \frac{3Ba^2b}{5}\right) + x^3\left(Aa^2b + \frac{Ba^3}{3}\right)$$

input
```
integrate((b*x**2+a)**3*(B*x**2+A),x)
```

output
```
A*a**3*x + B*b**3*x**9/9 + x**7*(A*b**3/7 + 3*B*a*b**2/7) + x**5*(3*A*a*b*
*2/5 + 3*B*a**2*b/5) + x**3*(A*a**2*b + B*a**3/3)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00

$$\int (a+bx^2)^3 (A+Bx^2)\,dx = \frac{1}{9}Bb^3x^9 + \frac{1}{7}(3Bab^2 + Ab^3)x^7 + \frac{3}{5}(Ba^2b + Aab^2)x^5 + Aa^3x + \frac{1}{3}(Ba^3 + 3Aa^2b)x^3$$

input
```
integrate((b*x^2+a)^3*(B*x^2+A),x, algorithm="maxima")
```

output
```
1/9*B*b^3*x^9 + 1/7*(3*B*a*b^2 + A*b^3)*x^7 + 3/5*(B*a^2*b + A*a*b^2)*x^5
+ A*a^3*x + 1/3*(B*a^3 + 3*A*a^2*b)*x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.04

$$\int \left(a+bx^2\right)^3 \left(A+Bx^2\right)\,dx = \frac{1}{9}Bb^3x^9 + \frac{3}{7}Bab^2x^7 + \frac{1}{7}Ab^3x^7 + \frac{3}{5}Ba^2bx^5 + \frac{3}{5}Aab^2x^5 + \frac{1}{3}Ba^3x^3 + Aa^2bx^3 + Aa^3x$$

input
```
integrate((b*x^2+a)^3*(B*x^2+A),x, algorithm="giac")
```

output
```
1/9*B*b^3*x^9 + 3/7*B*a*b^2*x^7 + 1/7*A*b^3*x^7 + 3/5*B*a^2*b*x^5 + 3/5*A*
a*b^2*x^5 + 1/3*B*a^3*x^3 + A*a^2*b*x^3 + A*a^3*x
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.93

$$\int \left(a+bx^2\right)^3 \left(A+Bx^2\right)\,dx = x^3\left(\frac{B\,a^3}{3}+A\,b\,a^2\right)+x^7\left(\frac{A\,b^3}{7}+\frac{3\,B\,a\,b^2}{7}\right)+\frac{B\,b^3\,x^9}{9}+A\,a^3\,x+\frac{3\,a\,b\,x^5\,(A\,b+B\,a)}{5}$$

input
```
int((A + B*x^2)*(a + b*x^2)^3,x)
```

output
```
x^3*((B*a^3)/3 + A*a^2*b) + x^7*((A*b^3)/7 + (3*B*a*b^2)/7) + (B*b^3*x^9)/
9 + A*a^3*x + (3*a*b*x^5*(A*b + B*a))/5
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.66

$$\int (a+bx^2)^3 (A+Bx^2)\,dx = \frac{x(35b^4x^8+180a\,b^3x^6+378a^2b^2x^4+420a^3b\,x^2+315a^4)}{315}$$

input
```
int((b*x^2+a)^3*(B*x^2+A),x)
```

output
```
(x*(315*a**4 + 420*a**3*b*x**2 + 378*a**2*b**2*x**4 + 180*a*b**3*x**6 + 35
*b**4*x**8))/315
```

3.81 $\int (a+bx^2)^2 (A+Bx^2) \, dx$

Optimal result

Integrand size = 17, antiderivative size = 50

$$\int (a+bx^2)^2 (A+Bx^2) \, dx = a^2Ax + \frac{1}{3}a(2Ab+aB)x^3 + \frac{1}{5}b(Ab+2aB)x^5 + \frac{1}{7}b^2Bx^7$$

output
```
a^2*A*x+1/3*a*(2*A*b+B*a)*x^3+1/5*b*(A*b+2*B*a)*x^5+1/7*b^2*B*x^7
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int (a+bx^2)^2 (A+Bx^2) \, dx = a^2Ax + \frac{1}{3}a(2Ab+aB)x^3 + \frac{1}{5}b(Ab+2aB)x^5 + \frac{1}{7}b^2Bx^7$$

input
```
Integrate[(a + b*x^2)^2*(A + B*x^2),x]
```

output
```
a^2*A*x + (a*(2*A*b + a*B)*x^3)/3 + (b*(A*b + 2*a*B)*x^5)/5 + (b^2*B*x^7)/
7
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {290, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^2 \left(A+Bx^2\right) \, dx$$

$$\Big\downarrow \text{ 290}$$

$$\int \left(a^2A+bx^4(2aB+Ab)+ax^2(aB+2Ab)+b^2Bx^6\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$a^2Ax+\frac{1}{5}bx^5(2aB+Ab)+\frac{1}{3}ax^3(aB+2Ab)+\frac{1}{7}b^2Bx^7$$

input
```
Int[(a + b*x^2)^2*(A + B*x^2),x]
```

output
```
a^2*A*x + (a*(2*A*b + a*B)*x^3)/3 + (b*(A*b + 2*a*B)*x^5)/5 + (b^2*B*x^7)/
7
```

Defintions of rubi rules used

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

method	result	size
default	$\frac{b^2 B\,x^7}{7}+\frac{(b^2 A+2abB)x^5}{5}+\frac{(2abA+a^2 B)x^3}{3}+a^2 Ax$	49
norman	$\frac{b^2 B\,x^7}{7}+\left(\frac{1}{5}b^2 A+\frac{2}{5}abB\right)x^5+\left(\frac{2}{3}abA+\frac{1}{3}a^2 B\right)x^3+a^2 Ax$	49
gosper	$\frac{1}{7}b^2 B\,x^7+\frac{1}{5}A\,b^2 x^5+\frac{2}{5}x^5 abB+\frac{2}{3}aAb\,x^3+\frac{1}{3}x^3 a^2 B+a^2 Ax$	51
risch	$\frac{1}{7}b^2 B\,x^7+\frac{1}{5}A\,b^2 x^5+\frac{2}{5}x^5 abB+\frac{2}{3}aAb\,x^3+\frac{1}{3}x^3 a^2 B+a^2 Ax$	51
parallelrisch	$\frac{1}{7}b^2 B\,x^7+\frac{1}{5}A\,b^2 x^5+\frac{2}{5}x^5 abB+\frac{2}{3}aAb\,x^3+\frac{1}{3}x^3 a^2 B+a^2 Ax$	51
orering	$\frac{x(15b^2 B\,x^6+21A\,b^2 x^4+42Bab\,x^4+70aAb\,x^2+35B\,a^2 x^2+105a^2 A)}{105}$	54

input
```
int((b*x^2+a)^2*(B*x^2+A),x,method=_RETURNVERBOSE)
```

output
```
1/7*b^2*B*x^7+1/5*(A*b^2+2*B*a*b)*x^5+1/3*(2*A*a*b+B*a^2)*x^3+a^2*A*x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int (a+bx^2)^2 (A+Bx^2)\,dx=\frac{1}{7}\,Bb^2x^7+\frac{1}{5}\,(2\,Bab+Ab^2)x^5+Aa^2x+\frac{1}{3}\,(Ba^2+2\,Aab)x^3$$

input
```
integrate((b*x^2+a)^2*(B*x^2+A),x, algorithm="fricas")
```

output
```
1/7*B*b^2*x^7 + 1/5*(2*B*a*b + A*b^2)*x^5 + A*a^2*x + 1/3*(B*a^2 + 2*A*a*b
)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06

$$\int (a+bx^2)^2 (A+Bx^2) \, dx = Aa^2x + \frac{Bb^2x^7}{7} + x^5\left(\frac{Ab^2}{5} + \frac{2Bab}{5}\right) + x^3 \cdot \left(\frac{2Aab}{3} + \frac{Ba^2}{3}\right)$$

input
```
integrate((b*x**2+a)**2*(B*x**2+A),x)
```

output
```
A*a**2*x + B*b**2*x**7/7 + x**5*(A*b**2/5 + 2*B*a*b/5) + x**3*(2*A*a*b/3 +
 B*a**2/3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int (a+bx^2)^2 (A+Bx^2) \, dx = \frac{1}{7} Bb^2x^7 + \frac{1}{5} (2 Bab + Ab^2)x^5 + Aa^2x + \frac{1}{3} (Ba^2 + 2 Aab)x^3$$

input
```
integrate((b*x^2+a)^2*(B*x^2+A),x, algorithm="maxima")
```

output
```
1/7*B*b^2*x^7 + 1/5*(2*B*a*b + A*b^2)*x^5 + A*a^2*x + 1/3*(B*a^2 + 2*A*a*b
)*x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int (a+bx^2)^2 (A+Bx^2) \, dx = \frac{1}{7} Bb^2x^7 + \frac{2}{5} Babx^5 + \frac{1}{5} Ab^2x^5 + \frac{1}{3} Ba^2x^3 + \frac{2}{3} Aabx^3 + Aa^2x$$

input
```
integrate((b*x^2+a)^2*(B*x^2+A),x, algorithm="giac")
```

output
```
1/7*B*b^2*x^7 + 2/5*B*a*b*x^5 + 1/5*A*b^2*x^5 + 1/3*B*a^2*x^3 + 2/3*A*a*b*
x^3 + A*a^2*x
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int (a+bx^2)^2 (A+Bx^2) \, dx = x^3 \left(\frac{B a^2}{3}+\frac{2 A b a}{3}\right)+x^5 \left(\frac{A b^2}{5}+\frac{2 B a b}{5}\right)+\frac{B b^2 x^7}{7}+A a^2 x$$

input
```
int((A + B*x^2)*(a + b*x^2)^2,x)
```

output
```
x^3*((B*a^2)/3 + (2*A*a*b)/3) + x^5*((A*b^2)/5 + (2*B*a*b)/5) + (B*b^2*x^7
)/7 + A*a^2*x
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.70

$$\int (a+bx^2)^2 (A+Bx^2) \, dx = \frac{x(5b^3x^6+21a\,b^2x^4+35a^2b\,x^2+35a^3)}{35}$$

input
```
int((b*x^2+a)^2*(B*x^2+A),x)
```

output
```
(x*(35*a**3 + 35*a**2*b*x**2 + 21*a*b**2*x**4 + 5*b**3*x**6))/35
```

3.82 $\int (a+bx^2) (A+Bx^2) \, dx$

Optimal result

Integrand size = 15, antiderivative size = 28

$$\int (a+bx^2) (A+Bx^2) \, dx = aAx + \frac{1}{3}(Ab+aB)x^3 + \frac{1}{5}bBx^5$$

output
```
a*A*x+1/3*(A*b+B*a)*x^3+1/5*b*B*x^5
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int (a+bx^2) (A+Bx^2) \, dx = aAx + \frac{1}{3}(Ab+aB)x^3 + \frac{1}{5}bBx^5$$

input
```
Integrate[(a + b*x^2)*(A + B*x^2),x]
```

output
```
a*A*x + ((A*b + a*B)*x^3)/3 + (b*B*x^5)/5
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {290, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^2)\left(A+Bx^2\right) dx$$

$$\Big\downarrow \text{ 290}$$

$$\int \left(x^2(aB+Ab)+aA+bBx^4\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3}x^3(aB+Ab)+aAx+\frac{1}{5}bBx^5$$

input
```
Int[(a + b*x^2)*(A + B*x^2),x]
```

output
```
a*A*x + ((A*b + a*B)*x^3)/3 + (b*B*x^5)/5
```

Defintions of rubi rules used

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

method	result	size
default	$aAx+\frac{(Ab+Ba)x^3}{3}+\frac{bB\,x^5}{5}$	25
norman	$\frac{bB\,x^5}{5}+\left(\frac{Ab}{3}+\frac{Ba}{3}\right)x^3+aAx$	26
gosper	$\frac{1}{5}bB\,x^5+\frac{1}{3}Ab\,x^3+\frac{1}{3}x^3Ba+aAx$	27
risch	$\frac{1}{5}bB\,x^5+\frac{1}{3}Ab\,x^3+\frac{1}{3}x^3Ba+aAx$	27
parallelrisch	$\frac{1}{5}bB\,x^5+\frac{1}{3}Ab\,x^3+\frac{1}{3}x^3Ba+aAx$	27
orering	$\frac{x(3bB\,x^4+5Ab\,x^2+5Ba\,x^2+15Aa)}{15}$	30

input
```
int((b*x^2+a)*(B*x^2+A),x,method=_RETURNVERBOSE)
```

output
```
a*A*x+1/3*(A*b+B*a)*x^3+1/5*b*B*x^5
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \left(a+bx^2\right)\left(A+Bx^2\right)\,dx=\frac{1}{5}\,Bbx^5+\frac{1}{3}\,(Ba+Ab)x^3+Aax$$

input
```
integrate((b*x^2+a)*(B*x^2+A),x, algorithm="fricas")
```

output
```
1/5*B*b*x^5 + 1/3*(B*a + A*b)*x^3 + A*a*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int (a+bx^2)\left(A+Bx^2\right)\,dx = Aax + \frac{Bbx^5}{5} + x^3\left(\frac{Ab}{3} + \frac{Ba}{3}\right)$$

input
```
integrate((b*x**2+a)*(B*x**2+A),x)
```

output
```
A*a*x + B*b*x**5/5 + x**3*(A*b/3 + B*a/3)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int (a+bx^2)\left(A+Bx^2\right)\,dx = \frac{1}{5}\,Bbx^5 + \frac{1}{3}\,(Ba+Ab)x^3 + Aax$$

input
```
integrate((b*x^2+a)*(B*x^2+A),x, algorithm="maxima")
```

output
```
1/5*B*b*x^5 + 1/3*(B*a + A*b)*x^3 + A*a*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int (a+bx^2)\left(A+Bx^2\right)\,dx = \frac{1}{5}\,Bbx^5 + \frac{1}{3}\,Bax^3 + \frac{1}{3}\,Abx^3 + Aax$$

input
```
integrate((b*x^2+a)*(B*x^2+A),x, algorithm="giac")
```

output
```
1/5*B*b*x^5 + 1/3*B*a*x^3 + 1/3*A*b*x^3 + A*a*x
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int (a+bx^2)(A+Bx^2)\,dx = \frac{B\,b\,x^5}{5} + \left(\frac{A\,b}{3} + \frac{B\,a}{3}\right)x^3 + A\,a\,x$$

input
```
int((A + B*x^2)*(a + b*x^2),x)
```

output
```
x^3*((A*b)/3 + (B*a)/3) + A*a*x + (B*b*x^5)/5
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int (a+bx^2)(A+Bx^2)\,dx = \frac{x(3b^2x^4 + 10ab\,x^2 + 15a^2)}{15}$$

input
```
int((b*x^2+a)*(B*x^2+A),x)
```

output
```
(x*(15*a**2 + 10*a*b*x**2 + 3*b**2*x**4))/15
```

3.83 $\int (A+Bx^2)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 12

$$\int (A+Bx^2)\,dx = Ax+\frac{Bx^3}{3}$$

output
```
A*x+1/3*B*x^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int (A+Bx^2)\,dx = Ax+\frac{Bx^3}{3}$$

input
```
Integrate[A + B*x^2,x]
```

output
```
A*x + (B*x^3)/3
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = $\{2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(A+Bx^2\right)\,dx$$

$$\Big\downarrow \; 2009$$

$$Ax+\frac{Bx^3}{3}$$

input
```
Int[A + B*x^2,x]
```

output
```
A*x + (B*x^3)/3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
gosper	$xA + \frac{1}{3}B\,x^3$	11
default	$xA + \frac{1}{3}B\,x^3$	11
norman	$xA + \frac{1}{3}B\,x^3$	11
risch	$xA + \frac{1}{3}B\,x^3$	11
parallelrisch	$xA + \frac{1}{3}B\,x^3$	11
parts	$xA + \frac{1}{3}B\,x^3$	11
orering	$\frac{x(x^2B+3A)}{3}$	13

input
```
int(B*x^2+A,x,method=_RETURNVERBOSE)
```

output
```
x*A+1/3*B*x^3
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \left(A + Bx^2\right)\,dx = \frac{1}{3}\,Bx^3 + Ax$$

input
```
integrate(B*x^2+A,x, algorithm="fricas")
```

output
```
1/3*B*x^3 + A*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int (A + Bx^2) \, dx = Ax + \frac{Bx^3}{3}$$

input
```
integrate(B*x**2+A,x)
```

output
```
A*x + B*x**3/3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int (A + Bx^2) \, dx = \frac{1}{3} Bx^3 + Ax$$

input
```
integrate(B*x^2+A,x, algorithm="maxima")
```

output
```
1/3*B*x^3 + A*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int (A + Bx^2) \, dx = \frac{1}{3} Bx^3 + Ax$$

input
```
integrate(B*x^2+A,x, algorithm="giac")
```

output
```
1/3*B*x^3 + A*x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int (A + Bx^2) \, dx = \frac{B\,x^3}{3} + A\,x$$

input
```
int(A + B*x^2,x)
```

output
```
A*x + (B*x^3)/3
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int (A + Bx^2) \, dx = \frac{x(b\,x^2 + 3a)}{3}$$

input
```
int(B*x^2+A,x)
```

output
```
(x*(3*a + b*x**2))/3
```

3.84 $\int \frac{A+Bx^2}{a+bx^2} dx$

Optimal result

Integrand size = 17, antiderivative size = 39

$$\int \frac{A+Bx^2}{a+bx^2}\,dx=\frac{Bx}{b}+\frac{(Ab-aB)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}$$

output
```
B*x/b+(A*b-B*a)*arctan(b^(1/2)*x/a^(1/2))/a^(1/2)/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.03

$$\int \frac{A+Bx^2}{a+bx^2}\,dx=\frac{Bx}{b}-\frac{(-Ab+aB)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}$$

input
```
Integrate[(A + B*x^2)/(a + b*x^2),x]
```

output
```
(B*x)/b - ((-(A*b) + a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(3/2))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {299, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{a+bx^2} \, dx$$

$$\Big\downarrow \text{ 299}$$

$$\frac{(Ab-aB)\int \frac{1}{bx^2+a}dx}{b}+\frac{Bx}{b}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{(Ab-aB)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}+\frac{Bx}{b}$$

input
```
Int[(A + B*x^2)/(a + b*x^2),x]
```

output
```
(B*x)/b + ((A*b - a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(3/2))
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.87

method	result	size
default	$\frac{Bx}{b}+\frac{(Ab-Ba)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{b\sqrt{ab}}$	34
risch	$\frac{Bx}{b}-\frac{\ln\left(bx+\sqrt{-ab}\right)A}{2\sqrt{-ab}}+\frac{\ln\left(bx+\sqrt{-ab}\right)Ba}{2b\sqrt{-ab}}+\frac{\ln\left(-bx+\sqrt{-ab}\right)A}{2\sqrt{-ab}}-\frac{\ln\left(-bx+\sqrt{-ab}\right)Ba}{2b\sqrt{-ab}}$	98

input
```
int((B*x^2+A)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
B*x/b+(A*b-B*a)/b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.54

$$\int\frac{A+Bx^2}{a+bx^2}\,dx=\left[\frac{2\,Babx+(Ba-Ab)\sqrt{-ab}\log\left(\frac{bx^2-2\sqrt{-ab}x-a}{bx^2+a}\right)}{2\,ab^2},\frac{Babx-(Ba-Ab)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}x}{a}\right)}{ab^2}\right]$$

input
```
integrate((B*x^2+A)/(b*x^2+a),x, algorithm="fricas")
```

output
```
[1/2*(2*B*a*b*x + (B*a - A*b)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/
(b*x^2 + a)))/(a*b^2), (B*a*b*x - (B*a - A*b)*sqrt(a*b)*arctan(sqrt(a*b)*x
/a))/(a*b^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(34) = 68$.

Time = 0.13 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.10

$$\int \frac{A+Bx^2}{a+bx^2}\,dx = \frac{Bx}{b} + \frac{\sqrt{-\frac{1}{ab^3}}(-Ab+Ba)\log\left(-ab\sqrt{-\frac{1}{ab^3}}+x\right)}{2} - \frac{\sqrt{-\frac{1}{ab^3}}(-Ab+Ba)\log\left(ab\sqrt{-\frac{1}{ab^3}}+x\right)}{2}$$

input
```
integrate((B*x**2+A)/(b*x**2+a),x)
```

output
```
B*x/b + sqrt(-1/(a*b**3))*(-A*b + B*a)*log(-a*b*sqrt(-1/(a*b**3)) + x)/2 -
 sqrt(-1/(a*b**3))*(-A*b + B*a)*log(a*b*sqrt(-1/(a*b**3)) + x)/2
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.87

$$\int \frac{A+Bx^2}{a+bx^2}\,dx = \frac{Bx}{b} - \frac{(Ba-Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b}$$

input
```
integrate((B*x^2+A)/(b*x^2+a),x, algorithm="maxima")
```

output
```
B*x/b - (B*a - A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.87

$$\int \frac{A+Bx^2}{a+bx^2}\,dx=\frac{Bx}{b}-\frac{(Ba-Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b}$$

input
```
integrate((B*x^2+A)/(b*x^2+a),x, algorithm="giac")
```

output
```
B*x/b - (B*a - A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b)
```

Mupad [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

$$\int \frac{A+Bx^2}{a+bx^2}\,dx=\frac{B\,x}{b}+\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,(A\,b-B\,a)}{\sqrt{a}\,b^{3/2}}$$

input
```
int((A + B*x^2)/(a + b*x^2),x)
```

output
```
(B*x)/b + (atan((b^(1/2)*x)/a^(1/2))*(A*b - B*a))/(a^(1/2)*b^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 1, normalized size of antiderivative = 0.03

$$\int \frac{A+Bx^2}{a+bx^2}\,dx=x$$

input
```
int((B*x^2+A)/(b*x^2+a),x)
```

output
```
x
```

3.85 $\int \frac{A+Bx^2}{(a+bx^2)^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 63

$$\int \frac{A+Bx^2}{(a+bx^2)^2}\,dx = \frac{(Ab-aB)x}{2ab\,(a+bx^2)} + \frac{(Ab+aB)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}$$

output
```
1/2*(A*b-B*a)*x/a/b/(b*x^2+a)+1/2*(A*b+B*a)*arctan(b^(1/2)*x/a^(1/2))/a^(3
/2)/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx^2}{(a+bx^2)^2}\,dx = -\frac{(-Ab+aB)x}{2ab\,(a+bx^2)} + \frac{(Ab+aB)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}$$

input
```
Integrate[(A + B*x^2)/(a + b*x^2)^2,x]
```

output
```
-1/2*((-(A*b) + a*B)*x)/(a*b*(a + b*x^2)) + ((A*b + a*B)*ArcTan[(Sqrt[b]*x
)/Sqrt[a]])/(2*a^(3/2)*b^(3/2))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {298, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{(a+bx^2)^2} \, dx$$

$$\Big\downarrow \ 298$$

$$\frac{(aB+Ab)\int \frac{1}{bx^2+a}dx}{2ab} + \frac{x(Ab-aB)}{2ab\,(a+bx^2)}$$

$$\Big\downarrow \ 218$$

$$\frac{(aB+Ab)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}} + \frac{x(Ab-aB)}{2ab\,(a+bx^2)}$$

input
```
Int[(A + B*x^2)/(a + b*x^2)^2,x]
```

output
```
((A*b - a*B)*x)/(2*a*b*(a + b*x^2)) + ((A*b + a*B)*ArcTan[(Sqrt[b]*x)/Sqrt
[a]])/(2*a^(3/2)*b^(3/2))
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.90

method	result	size
default	$\frac{(Ab-Ba)x}{2ab(b\,x^2+a)}+\frac{(Ab+Ba)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2ab\sqrt{ab}}$	57
risch	$\frac{(Ab-Ba)x}{2ab(b\,x^2+a)}-\frac{A\ln\left(bx+\sqrt{-ab}\right)}{4\sqrt{-ab}\,a}-\frac{\ln\left(bx+\sqrt{-ab}\right)B}{4\sqrt{-ab}\,b}+\frac{A\ln\left(-bx+\sqrt{-ab}\right)}{4\sqrt{-ab}\,a}+\frac{\ln\left(-bx+\sqrt{-ab}\right)B}{4\sqrt{-ab}\,b}$	122

input
```
int((B*x^2+A)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*(A*b-B*a)*x/a/b/(b*x^2+a)+1/2*(A*b+B*a)/a/b/(a*b)^(1/2)*arctan(b*x/(a*
b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 182, normalized size of antiderivative = 2.89

$$\int \frac{A+Bx^2}{(a+bx^2)^2}\,dx = \left[-\frac{(Ba^2+Aab+(Bab+Ab^2)x^2)\sqrt{-ab}\log\left(\frac{bx^2-2\sqrt{-ab}x-a}{bx^2+a}\right)+2\,(Ba^2b-Aab^2)x}{4\,(a^2b^3x^2+a^3b^2)}, \frac{(Ba^2+Aab+(Bab-\right.$$

input
```
integrate((B*x^2+A)/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[-1/4*((B*a^2 + A*a*b + (B*a*b + A*b^2)*x^2)*sqrt(-a*b)*log((b*x^2 - 2*sqr
t(-a*b)*x - a)/(b*x^2 + a)) + 2*(B*a^2*b - A*a*b^2)*x)/(a^2*b^3*x^2 + a^3*
b^2), 1/2*((B*a^2 + A*a*b + (B*a*b + A*b^2)*x^2)*sqrt(a*b)*arctan(sqrt(a*b
)*x/a) - (B*a^2*b - A*a*b^2)*x)/(a^2*b^3*x^2 + a^3*b^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(54) = 108$.

Time = 0.20 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.78

$$\int \frac{A+Bx^2}{(a+bx^2)^2}\,dx = \frac{x(Ab-Ba)}{2a^2b+2ab^2x^2} - \frac{\sqrt{-\frac{1}{a^3b^3}}(Ab+Ba)\log\left(-a^2b\sqrt{-\frac{1}{a^3b^3}}+x\right)}{4} + \frac{\sqrt{-\frac{1}{a^3b^3}}(Ab+Ba)\log\left(a^2b\sqrt{-\frac{1}{a^3b^3}}+x\right)}{4}$$

input
```
integrate((B*x**2+A)/(b*x**2+a)**2,x)
```

output
```
x*(A*b - B*a)/(2*a**2*b + 2*a*b**2*x**2) - sqrt(-1/(a**3*b**3))*(A*b + B*a
)*log(-a**2*b*sqrt(-1/(a**3*b**3)) + x)/4 + sqrt(-1/(a**3*b**3))*(A*b + B*
a)*log(a**2*b*sqrt(-1/(a**3*b**3)) + x)/4
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.90

$$\int \frac{A+Bx^2}{(a+bx^2)^2}\,dx = -\frac{(Ba-Ab)x}{2(ab^2x^2+a^2b)} + \frac{(Ba+Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}ab}$$

input
```
integrate((B*x^2+A)/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
-1/2*(B*a - A*b)*x/(a*b^2*x^2 + a^2*b) + 1/2*(B*a + A*b)*arctan(b*x/sqrt(a
*b))/(sqrt(a*b)*a*b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.90

$$\int \frac{A+Bx^2}{(a+bx^2)^2}\,dx = \frac{(Ba+Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}ab} - \frac{Bax-Abx}{2\,(bx^2+a)ab}$$

input
```
integrate((B*x^2+A)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*(B*a + A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b) - 1/2*(B*a*x - A*b*x
)/((b*x^2 + a)*a*b)
```

Mupad [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int \frac{A+Bx^2}{(a+bx^2)^2}\,dx = \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,(A\,b+B\,a)}{2\,a^{3/2}\,b^{3/2}} + \frac{x\,(A\,b-B\,a)}{2\,a\,b\,\left(b\,x^2+a\right)}$$

input
```
int((A + B*x^2)/(a + b*x^2)^2,x)
```

output
```
(atan((b^(1/2)*x)/a^(1/2))*(A*b + B*a))/(2*a^(3/2)*b^(3/2)) + (x*(A*b - B*
a))/(2*a*b*(a + b*x^2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.37

$$\int \frac{A+Bx^2}{(a+bx^2)^2}\,dx = \frac{\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)}{ab}$$

input
```
int((B*x^2+A)/(b*x^2+a)^2,x)
```

output
```
(sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a))))/(a*b)
```

3.86 $\int \frac{A+Bx^2}{(a+bx^2)^3} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 92

$$\int \frac{A+Bx^2}{(a+bx^2)^3} \, dx = \frac{(Ab-aB)x}{4ab(a+bx^2)^2} + \frac{(3Ab+aB)x}{8a^2b(a+bx^2)} + \frac{(3Ab+aB)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}}$$

output
```
1/4*(A*b-B*a)*x/a/b/(b*x^2+a)^2+1/8*(3*A*b+B*a)*x/a^2/b/(b*x^2+a)+1/8*(3*A
*b+B*a)*arctan(b^(1/2)*x/a^(1/2))/a^(5/2)/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.91

$$\int \frac{A+Bx^2}{(a+bx^2)^3} \, dx = \frac{x(-a^2B+3Ab^2x^2+ab(5A+Bx^2))}{8a^2b(a+bx^2)^2} + \frac{(3Ab+aB)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}}$$

input
```
Integrate[(A + B*x^2)/(a + b*x^2)^3,x]
```

output
```
(x*(-(a^2*B) + 3*A*b^2*x^2 + a*b*(5*A + B*x^2)))/(8*a^2*b*(a + b*x^2)^2) +
 ((3*A*b + a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(5/2)*b^(3/2))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {298, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{(a+bx^2)^3}dx$$

$$\Big\downarrow \text{298}$$

$$\frac{(aB+3Ab)\int \frac{1}{(bx^2+a)^2}dx}{4ab}+\frac{x(Ab-aB)}{4ab(a+bx^2)^2}$$

$$\Big\downarrow \text{215}$$

$$\frac{(aB+3Ab)\left(\frac{\int \frac{1}{bx^2+a}dx}{2a}+\frac{x}{2a(a+bx^2)}\right)}{4ab}+\frac{x(Ab-aB)}{4ab(a+bx^2)^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{(aB+3Ab)\left(\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{x}{2a(a+bx^2)}\right)}{4ab}+\frac{x(Ab-aB)}{4ab(a+bx^2)^2}$$

input
```
Int[(A + B*x^2)/(a + b*x^2)^3,x]
```

output
```
((A*b - a*B)*x)/(4*a*b*(a + b*x^2)^2) + ((3*A*b + a*B)*(x/(2*a*(a + b*x^2)
) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*a^(3/2)*Sqrt[b])))/(4*a*b)
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.84

method	result	size
default	$\frac{\frac{(3Ab+Ba)x^3}{8a^2}+\frac{(5Ab-Ba)x}{8ab}}{(b\,x^2+a)^2}+\frac{(3Ab+Ba)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8a^2b\sqrt{ab}}$	77
risch	$\frac{\frac{(3Ab+Ba)x^3}{8a^2}+\frac{(5Ab-Ba)x}{8ab}}{(b\,x^2+a)^2}-\frac{3A\ln\left(bx+\sqrt{-ab}\right)}{16\sqrt{-ab}\,a^2}-\frac{\ln\left(bx+\sqrt{-ab}\right)B}{16\sqrt{-ab}\,ba}+\frac{3A\ln\left(-bx+\sqrt{-ab}\right)}{16\sqrt{-ab}\,a^2}+\frac{\ln\left(-bx+\sqrt{-ab}\right)B}{16\sqrt{-ab}\,ba}$	147

input
```
int((B*x^2+A)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/8*(3*A*b+B*a)/a^2*x^3+1/8*(5*A*b-B*a)/a/b*x)/(b*x^2+a)^2+1/8*(3*A*b+B*a
)/a^2/b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 300, normalized size of antiderivative = 3.26

$$\int \frac{A+Bx^2}{(a+bx^2)^3}\,dx = \left[\frac{2\,(Ba^2b^2+3\,Aab^3)x^3-((Bab^2+3\,Ab^3)x^4+Ba^3+3\,Aa^2b+2\,(Ba^2b+3\,Aab^2)x^2)\sqrt{-ab}\log\left(\frac{bx^2-2\sqrt{-}}{bx^2+}\right.}{16\,(a^3b^4x^4+2\,a^4b^3x^2+a^5b^2)}\right.$$

input
```
integrate((B*x^2+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
[1/16*(2*(B*a^2*b^2 + 3*A*a*b^3)*x^3 - ((B*a*b^2 + 3*A*b^3)*x^4 + B*a^3 +
3*A*a^2*b + 2*(B*a^2*b + 3*A*a*b^2)*x^2)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a
*b)*x - a)/(b*x^2 + a)) - 2*(B*a^3*b - 5*A*a^2*b^2)*x)/(a^3*b^4*x^4 + 2*a^
4*b^3*x^2 + a^5*b^2), 1/8*((B*a^2*b^2 + 3*A*a*b^3)*x^3 + ((B*a*b^2 + 3*A*b
^3)*x^4 + B*a^3 + 3*A*a^2*b + 2*(B*a^2*b + 3*A*a*b^2)*x^2)*sqrt(a*b)*arcta
n(sqrt(a*b)*x/a) - (B*a^3*b - 5*A*a^2*b^2)*x)/(a^3*b^4*x^4 + 2*a^4*b^3*x^2
 + a^5*b^2)]
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.63

$$\int \frac{A+Bx^2}{(a+bx^2)^3}\,dx = -\frac{\sqrt{-\frac{1}{a^5b^3}}\cdot(3Ab+Ba)\log\left(-a^3b\sqrt{-\frac{1}{a^5b^3}}+x\right)}{16} + \frac{\sqrt{-\frac{1}{a^5b^3}}\cdot(3Ab+Ba)\log\left(a^3b\sqrt{-\frac{1}{a^5b^3}}+x\right)}{16} + \frac{x^3\cdot(3Ab^2+Bab)+x(5Aab-Ba^2)}{8a^4b+16a^3b^2x^2+8a^2b^3x^4}$$

input
```
integrate((B*x**2+A)/(b*x**2+a)**3,x)
```

output
```
-sqrt(-1/(a**5*b**3))*(3*A*b + B*a)*log(-a**3*b*sqrt(-1/(a**5*b**3)) + x)/
16 + sqrt(-1/(a**5*b**3))*(3*A*b + B*a)*log(a**3*b*sqrt(-1/(a**5*b**3)) +
x)/16 + (x**3*(3*A*b**2 + B*a*b) + x*(5*A*a*b - B*a**2))/(8*a**4*b + 16*a*
*3*b**2*x**2 + 8*a**2*b**3*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx^2}{(a+bx^2)^3}\,dx = \frac{(Bab+3\,Ab^2)x^3-(Ba^2-5\,Aab)x}{8\,(a^2b^3x^4+2\,a^3b^2x^2+a^4b)} + \frac{(Ba+3\,Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^2b}$$

input
```
integrate((B*x^2+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/8*((B*a*b + 3*A*b^2)*x^3 - (B*a^2 - 5*A*a*b)*x)/(a^2*b^3*x^4 + 2*a^3*b^2
*x^2 + a^4*b) + 1/8*(B*a + 3*A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.85

$$\int \frac{A+Bx^2}{(a+bx^2)^3}\,dx = \frac{(Ba+3\,Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^2b} + \frac{Babx^3+3\,Ab^2x^3-Ba^2x+5\,Aabx}{8\,(bx^2+a)^2a^2b}$$

input
```
integrate((B*x^2+A)/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
1/8*(B*a + 3*A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b) + 1/8*(B*a*b*x^3
 + 3*A*b^2*x^3 - B*a^2*x + 5*A*a*b*x)/((b*x^2 + a)^2*a^2*b)
```

Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.89

$$\int \frac{A+Bx^2}{\left(a+bx^2\right)^3}\,dx=\frac{\frac{x^3\,(3\,A\,b+B\,a)}{8\,a^2}+\frac{x\,(5\,A\,b-B\,a)}{8\,a\,b}}{a^2+2\,a\,b\,x^2+b^2\,x^4}+\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,(3\,A\,b+B\,a)}{8\,a^{5/2}\,b^{3/2}}$$

input
```
int((A + B*x^2)/(a + b*x^2)^3,x)
```

output
```
((x^3*(3*A*b + B*a))/(8*a^2) + (x*(5*A*b - B*a))/(8*a*b))/(a^2 + b^2*x^4 +
 2*a*b*x^2) + (atan((b^(1/2)*x)/a^(1/2))*(3*A*b + B*a))/(8*a^(5/2)*b^(3/2)
)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.66

$$\int \frac{A+Bx^2}{\left(a+bx^2\right)^3}\,dx=\frac{\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a+\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)b\,x^2+abx}{2a^2b\,(b\,x^2+a)}$$

input
```
int((B*x^2+A)/(b*x^2+a)^3,x)
```

output
```
(sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a + sqrt(b)*sqrt(a)*atan((b
*x)/(sqrt(b)*sqrt(a)))*b*x**2 + a*b*x)/(2*a**2*b*(a + b*x**2))
```

3.87 $\int \frac{A+Bx^2}{\left(a+bx^2\right)^4} dx$

Optimal result

Integrand size = 17, antiderivative size = 120

$$\int \frac{A+Bx^2}{\left(a+bx^2\right)^4} dx = \frac{(Ab-aB)x}{6ab\left(a+bx^2\right)^3}+\frac{(5Ab+aB)x}{24a^2b\left(a+bx^2\right)^2}+\frac{(5Ab+aB)x}{16a^3b\left(a+bx^2\right)}+\frac{(5Ab+aB)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16a^{7/2}b^{3/2}}$$

output
```
1/6*(A*b-B*a)*x/a/b/(b*x^2+a)^3+1/24*(5*A*b+B*a)*x/a^2/b/(b*x^2+a)^2+1/16*
(5*A*b+B*a)*x/a^3/b/(b*x^2+a)+1/16*(5*A*b+B*a)*arctan(b^(1/2)*x/a^(1/2))/a
^(7/2)/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.88

$$\int \frac{A+Bx^2}{\left(a+bx^2\right)^4} dx = \frac{x(-3a^3B+15Ab^3x^4+ab^2x^2(40A+3Bx^2)+a^2b(33A+8Bx^2))}{48a^3b\left(a+bx^2\right)^3}+\frac{(5Ab+aB)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16a^{7/2}b^{3/2}}$$

input
```
Integrate[(A + B*x^2)/(a + b*x^2)^4,x]
```

output
```
(x*(-3*a^3*B + 15*A*b^3*x^4 + a*b^2*x^2*(40*A + 3*B*x^2) + a^2*b*(33*A + 8
*B*x^2)))/(48*a^3*b*(a + b*x^2)^3) + ((5*A*b + a*B)*ArcTan[(Sqrt[b]*x)/Sqr
t[a]])/(16*a^(7/2)*b^(3/2))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {298, 215, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{(a+bx^2)^4}dx$$

$$\Big\downarrow \text{298}$$

$$\frac{(aB+5Ab)\int \frac{1}{(bx^2+a)^3}dx}{6ab}+\frac{x(Ab-aB)}{6ab(a+bx^2)^3}$$

$$\Big\downarrow \text{215}$$

$$\frac{(aB+5Ab)\left(\frac{3\int \frac{1}{(bx^2+a)^2}dx}{4a}+\frac{x}{4a(a+bx^2)^2}\right)}{6ab}+\frac{x(Ab-aB)}{6ab(a+bx^2)^3}$$

$$\Big\downarrow \text{215}$$

$$\frac{(aB+5Ab)\left(\frac{3\left(\frac{\int \frac{1}{bx^2+a}dx}{2a}+\frac{x}{2a(a+bx^2)}\right)}{4a}+\frac{x}{4a(a+bx^2)^2}\right)}{6ab}+\frac{x(Ab-aB)}{6ab(a+bx^2)^3}$$

$$\Big\downarrow \text{218}$$

$$\frac{(aB+5Ab)\left(\frac{3\left(\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{x}{2a\left(a+bx^2\right)}\right)}{4a}+\frac{x}{4a(a+bx^2)^2}\right)}{6ab}+\frac{x(Ab-aB)}{6ab\left(a+bx^2\right)^3}$$

input
```
Int[(A + B*x^2)/(a + b*x^2)^4,x]
```

output
```
((A*b - a*B)*x)/(6*a*b*(a + b*x^2)^3) + ((5*A*b + a*B)*(x/(4*a*(a + b*x^2)
^2) + (3*(x/(2*a*(a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*a^(3/2)*Sqr
t[b])))/(4*a)))/(6*a*b)
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.78

method	result
default	$\frac{\frac{(5Ab+Ba)b\,x^5}{16a^3}+\frac{(5Ab+Ba)x^3}{6a^2}+\frac{(11Ab-Ba)x}{16ab}}{(b\,x^2+a)^3}+\frac{(5Ab+Ba)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16a^3b\sqrt{ab}}$
risch	$\frac{\frac{(5Ab+Ba)b\,x^5}{16a^3}+\frac{(5Ab+Ba)x^3}{6a^2}+\frac{(11Ab-Ba)x}{16ab}}{(b\,x^2+a)^3}-\frac{5\ln(bx+\sqrt{-ab})A}{32\sqrt{-ab}\,a^3}-\frac{\ln(bx+\sqrt{-ab})B}{32\sqrt{-ab}\,b\,a^2}+\frac{5\ln(-bx+\sqrt{-ab})A}{32\sqrt{-ab}\,a^3}+\frac{\ln(-bx+\sqrt{-ab})}{32\sqrt{-ab}\,b\,a^2}$

input
```
int((B*x^2+A)/(b*x^2+a)^4,x,method=_RETURNVERBOSE)
```

output
```
(1/16*(5*A*b+B*a)/a^3*b*x^5+1/6/a^2*(5*A*b+B*a)*x^3+1/16*(11*A*b-B*a)/a/b*
x)/(b*x^2+a)^3+1/16*(5*A*b+B*a)/a^3/b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 416, normalized size of antiderivative = 3.47

$$\int \frac{A+Bx^2}{(a+bx^2)^4}\,dx$$
$$= \left[\frac{6\,(Ba^2b^3+5\,Aab^4)x^5+16\,(Ba^3b^2+5\,Aa^2b^3)x^3-3\,((Bab^3+5\,Ab^4)x^6+Ba^4+5\,Aa^3b+3\,(Ba^2b^2+5}{96\,(a^4b^5x^6+3\,a^5b^4x^4+3\,a^6b}\right.$$

input
```
integrate((B*x^2+A)/(b*x^2+a)^4,x, algorithm="fricas")
```

output
```
[1/96*(6*(B*a^2*b^3 + 5*A*a*b^4)*x^5 + 16*(B*a^3*b^2 + 5*A*a^2*b^3)*x^3 -
3*((B*a*b^3 + 5*A*b^4)*x^6 + B*a^4 + 5*A*a^3*b + 3*(B*a^2*b^2 + 5*A*a*b^3)
*x^4 + 3*(B*a^3*b + 5*A*a^2*b^2)*x^2)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)
*x - a)/(b*x^2 + a)) - 6*(B*a^4*b - 11*A*a^3*b^2)*x)/(a^4*b^5*x^6 + 3*a^5*
b^4*x^4 + 3*a^6*b^3*x^2 + a^7*b^2), 1/48*(3*(B*a^2*b^3 + 5*A*a*b^4)*x^5 +
8*(B*a^3*b^2 + 5*A*a^2*b^3)*x^3 + 3*((B*a*b^3 + 5*A*b^4)*x^6 + B*a^4 + 5*A
*a^3*b + 3*(B*a^2*b^2 + 5*A*a*b^3)*x^4 + 3*(B*a^3*b + 5*A*a^2*b^2)*x^2)*sq
rt(a*b)*arctan(sqrt(a*b)*x/a) - 3*(B*a^4*b - 11*A*a^3*b^2)*x)/(a^4*b^5*x^6
 + 3*a^5*b^4*x^4 + 3*a^6*b^3*x^2 + a^7*b^2)]
```

Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.58

$$\int \frac{A+Bx^2}{(a+bx^2)^4}\,dx = -\frac{\sqrt{-\frac{1}{a^7b^3}}\cdot(5Ab+Ba)\log\left(-a^4b\sqrt{-\frac{1}{a^7b^3}}+x\right)}{32} + \frac{\sqrt{-\frac{1}{a^7b^3}}\cdot(5Ab+Ba)\log\left(a^4b\sqrt{-\frac{1}{a^7b^3}}+x\right)}{32} + \frac{x^5\cdot(15Ab^3+3Bab^2)+x^3\cdot(40Aab^2+8Ba^2b)+x(33Aa^2b-3Ba^3)}{48a^6b+144a^5b^2x^2+144a^4b^3x^4+48a^3b^4x^6}$$

input
```
integrate((B*x**2+A)/(b*x**2+a)**4,x)
```

output
```
-sqrt(-1/(a**7*b**3))*(5*A*b + B*a)*log(-a**4*b*sqrt(-1/(a**7*b**3)) + x)/
32 + sqrt(-1/(a**7*b**3))*(5*A*b + B*a)*log(a**4*b*sqrt(-1/(a**7*b**3)) +
x)/32 + (x**5*(15*A*b**3 + 3*B*a*b**2) + x**3*(40*A*a*b**2 + 8*B*a**2*b) +
 x*(33*A*a**2*b - 3*B*a**3))/(48*a**6*b + 144*a**5*b**2*x**2 + 144*a**4*b*
*3*x**4 + 48*a**3*b**4*x**6)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.06

$$\int \frac{A+Bx^2}{(a+bx^2)^4}\,dx = \frac{3\,(Bab^2+5\,Ab^3)x^5+8\,(Ba^2b+5\,Aab^2)x^3-3\,(Ba^3-11\,Aa^2b)x}{48\,(a^3b^4x^6+3\,a^4b^3x^4+3\,a^5b^2x^2+a^6b)} + \frac{(Ba+5\,Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{ab}a^3b}$$

input
```
integrate((B*x^2+A)/(b*x^2+a)^4,x, algorithm="maxima")
```

output
```
1/48*(3*(B*a*b^2 + 5*A*b^3)*x^5 + 8*(B*a^2*b + 5*A*a*b^2)*x^3 - 3*(B*a^3 -
 11*A*a^2*b)*x)/(a^3*b^4*x^6 + 3*a^4*b^3*x^4 + 3*a^5*b^2*x^2 + a^6*b) + 1/
16*(B*a + 5*A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3*b)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.86

$$\int \frac{A+Bx^2}{(a+bx^2)^4}\,dx$$
$$= \frac{(Ba+5\,Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{ab}a^3b} + \frac{3\,Bab^2x^5+15\,Ab^3x^5+8\,Ba^2bx^3+40\,Aab^2x^3-3\,Ba^3x+33\,Aa^2bx}{48\,(bx^2+a)^3a^3b}$$

input
```
integrate((B*x^2+A)/(b*x^2+a)^4,x, algorithm="giac")
```

output
```
1/16*(B*a + 5*A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3*b) + 1/48*(3*B*a*b
^2*x^5 + 15*A*b^3*x^5 + 8*B*a^2*b*x^3 + 40*A*a*b^2*x^3 - 3*B*a^3*x + 33*A*
a^2*b*x)/((b*x^2 + a)^3*a^3*b)
```

Mupad [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.92

$$\int \frac{A+Bx^2}{(a+bx^2)^4}\,dx = \frac{\frac{x^3\,(5\,A\,b+B\,a)}{6\,a^2}+\frac{b\,x^5\,(5\,A\,b+B\,a)}{16\,a^3}+\frac{x\,(11\,A\,b-B\,a)}{16\,a\,b}}{a^3+3\,a^2\,b\,x^2+3\,a\,b^2\,x^4+b^3\,x^6} + \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,(5\,A\,b+B\,a)}{16\,a^{7/2}\,b^{3/2}}$$

input
```
int((A + B*x^2)/(a + b*x^2)^4,x)
```

output
```
((x^3*(5*A*b + B*a))/(6*a^2) + (b*x^5*(5*A*b + B*a))/(16*a^3) + (x*(11*A*b
 - B*a))/(16*a*b))/(a^3 + b^3*x^6 + 3*a^2*b*x^2 + 3*a*b^2*x^4) + (atan((b^
(1/2)*x)/a^(1/2))*(5*A*b + B*a))/(16*a^(7/2)*b^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.94

$$\int \frac{A+Bx^2}{(a+bx^2)^4}\,dx$$
$$=\frac{3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^2+6\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)ab\,x^2+3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)b^2x^4+5a^2bx+3a\,b^2x^3}{8a^3b\,(b^2x^4+2ab\,x^2+a^2)}$$

input
```
int((B*x^2+A)/(b*x^2+a)^4,x)
```

output
```
(3*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2 + 6*sqrt(b)*sqrt(a)*
atan((b*x)/(sqrt(b)*sqrt(a)))*a*b*x**2 + 3*sqrt(b)*sqrt(a)*atan((b*x)/(sqr
t(b)*sqrt(a)))*b**2*x**4 + 5*a**2*b*x + 3*a*b**2*x**3)/(8*a**3*b*(a**2 + 2
*a*b*x**2 + b**2*x**4))
```

3.88 $\int (a+bx^2)^{5/2} (A+Bx^2) \, dx$

Optimal result

Integrand size = 19, antiderivative size = 149

$$\int (a+bx^2)^{5/2} (A+Bx^2) \, dx = \frac{5a^2(8Ab-aB)x\sqrt{a+bx^2}}{128b} + \frac{5a(8Ab-aB)x(a+bx^2)^{3/2}}{192b} + \frac{(8Ab-aB)x(a+bx^2)^{5/2}}{48b} + \frac{Bx(a+bx^2)^{7/2}}{8b} + \frac{5a^3(8Ab-aB)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{3/2}}$$

output
```
5/128*a^2*(8*A*b-B*a)*x*(b*x^2+a)^(1/2)/b+5/192*a*(8*A*b-B*a)*x*(b*x^2+a)^
(3/2)/b+1/48*(8*A*b-B*a)*x*(b*x^2+a)^(5/2)/b+1/8*B*x*(b*x^2+a)^(7/2)/b+5/1
28*a^3*(8*A*b-B*a)*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.83

$$\int (a+bx^2)^{5/2} (A+Bx^2) \, dx = \frac{x\sqrt{a+bx^2}(264a^2Ab+15a^3B+208aAb^2x^2+118a^2bBx^2+64Ab^3x^4+136ab^2Bx^4+48b^3Bx}{384b} + \frac{5a^3(-8Ab+aB)\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{128b^{3/2}}$$

input
```
Integrate[(a + b*x^2)^(5/2)*(A + B*x^2),x]
```

output
```
(x*Sqrt[a + b*x^2]*(264*a^2*A*b + 15*a^3*B + 208*a*A*b^2*x^2 + 118*a^2*b*B
*x^2 + 64*A*b^3*x^4 + 136*a*b^2*B*x^4 + 48*b^3*B*x^6))/(384*b) + (5*a^3*(-
8*A*b + a*B)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(128*b^(3/2))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.85, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {299, 211, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^{5/2} \left(A+Bx^2\right) \, dx$$

$$\Big\downarrow \text{299}$$

$$\frac{(8Ab-aB)\int \left(bx^2+a\right)^{5/2}dx}{8b}+\frac{Bx\left(a+bx^2\right)^{7/2}}{8b}$$

$$\Big\downarrow \text{211}$$

$$\frac{(8Ab-aB)\left(\frac{5}{6}a\int \left(bx^2+a\right)^{3/2}dx+\frac{1}{6}x\left(a+bx^2\right)^{5/2}\right)}{8b}+\frac{Bx\left(a+bx^2\right)^{7/2}}{8b}$$

$$\Big\downarrow \text{211}$$

$$\frac{(8Ab-aB)\left(\frac{5}{6}a\left(\frac{3}{4}a\int \sqrt{bx^2+a}dx+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{1}{6}x\left(a+bx^2\right)^{5/2}\right)}{8b}+\frac{Bx\left(a+bx^2\right)^{7/2}}{8b}$$

$$\Big\downarrow \text{211}$$

$$\frac{(8Ab-aB)\left(\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{1}{6}x\left(a+bx^2\right)^{5/2}\right)}{8b}+\frac{Bx\left(a+bx^2\right)^{7/2}}{8b}$$

$$\Big\downarrow \text{224}$$

$$\frac{(8Ab-aB)\left(\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{1}{2}a\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{1}{6}x\left(a+bx^2\right)^{5/2}\right)}{8b}+\frac{Bx\left(a+bx^2\right)^{7/2}}{8b}$$

$$\Big\downarrow \text{219}$$

$$\frac{(8Ab-aB)\left(\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{1}{6}x\left(a+bx^2\right)^{5/2}\right)}{8b}+\frac{Bx\left(a+bx^2\right)^{7/2}}{8b}$$

input
```
Int[(a + b*x^2)^(5/2)*(A + B*x^2),x]
```

output
```
(B*x*(a + b*x^2)^(7/2))/(8*b) + ((8*A*b - a*B)*((x*(a + b*x^2)^(5/2))/6 +
(5*a*((x*(a + b*x^2)^(3/2))/4 + (3*a*((x*Sqrt[a + b*x^2])/2 + (a*ArcTanh[(
Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b])))/4))/6))/(8*b)
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.71

method	result
pseudoelliptic	$\frac{\frac{5\left(Ab-\frac{Ba}{8}\right)a^3\operatorname{arctanh}\left(\frac{\sqrt{b\,x^2+a}}{x\sqrt{b}}\right)}{16}+\frac{13\left(\frac{33a^2\left(\frac{59x^2B}{132}+A\right)b^{\frac{3}{2}}}{26}+x^2a\left(\frac{17x^2B}{26}+A\right)b^{\frac{5}{2}}+\frac{4x^4\left(\frac{3x^2B}{4}+A\right)b^{\frac{7}{2}}}{13}+\frac{15B\,a^3\sqrt{b}}{208}\right)\sqrt{b\,x^2+a}\,x}{24}}{b^{\frac{3}{2}}}$
risch	$\frac{x\left(48b^3B\,x^6+64A\,x^4b^3+136B\,x^4a\,b^2+208aA\,b^2x^2+118B\,a^2b\,x^2+264a^2bA+15a^3B\right)\sqrt{b\,x^2+a}}{384b}+\frac{5a^3(8Ab-Ba)\ln\left(\sqrt{b}\,x-\right.}{128b^{\frac{3}{2}}}$
default	$A\left(\frac{x\left(b\,x^2+a\right)^{\frac{5}{2}}}{6}+\frac{5a\left(\frac{x\left(b\,x^2+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)}{4}\right)}{6}\right)+B\left(\frac{x\left(b\,x^2+a\right)^{\frac{7}{2}}}{8b}-\frac{a\left(\frac{x\left(b\,x^2+\right.}{6}\right.}{}\right.$

input
```
int((b*x^2+a)^(5/2)*(B*x^2+A),x,method=_RETURNVERBOSE)
```

output
```
13/24*(15/26*(A*b-1/8*B*a)*a^3*arctanh((b*x^2+a)^(1/2)/x/b^(1/2))+(33/26*a
^2*(59/132*x^2*B+A)*b^(3/2)+x^2*a*(17/26*x^2*B+A)*b^(5/2)+4/13*x^4*(3/4*x^
2*B+A)*b^(7/2)+15/208*B*a^3*b^(1/2))*(b*x^2+a)^(1/2)*x)/b^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.72

$$\int (a+bx^2)^{5/2}(A+Bx^2)\,dx = \left[-\frac{15\,(Ba^4-8\,Aa^3b)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\,(48\,Bb^4x^7+8\,(17\,Bab^3+8\,\ldots}{768\,b^2}\right.$$

input
```
integrate((b*x^2+a)^(5/2)*(B*x^2+A),x, algorithm="fricas")
```

output
```
[-1/768*(15*(B*a^4 - 8*A*a^3*b)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*s
qrt(b)*x - a) - 2*(48*B*b^4*x^7 + 8*(17*B*a*b^3 + 8*A*b^4)*x^5 + 2*(59*B*a
^2*b^2 + 104*A*a*b^3)*x^3 + 3*(5*B*a^3*b + 88*A*a^2*b^2)*x)*sqrt(b*x^2 + a
))/b^2, 1/384*(15*(B*a^4 - 8*A*a^3*b)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^
2 + a)) + (48*B*b^4*x^7 + 8*(17*B*a*b^3 + 8*A*b^4)*x^5 + 2*(59*B*a^2*b^2 +
 104*A*a*b^3)*x^3 + 3*(5*B*a^3*b + 88*A*a^2*b^2)*x)*sqrt(b*x^2 + a))/b^2]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 279 vs. 2(134) = 268.

Time = 0.39 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.87

$$\int (a+bx^2)^{5/2}(A+Bx^2)\,dx = \begin{cases} \sqrt{a+bx^2}\left(\frac{Bb^2x^7}{8}+\frac{x^5\left(Ab^3+\frac{17Bab^2}{8}\right)}{6b}+\frac{x^3\cdot\left(3Aab^2+3Ba^2b-\frac{5a\left(Ab^3+\frac{17Bab^2}{8}\right)}{6b}\right)}{4b}+\frac{x\left(3Aa^2b+Ba^3-\frac{3a\left(3Aab^2-\ldots\right.}{\ldots}\right.}{\ldots}\right. \\ a^{\frac{5}{2}}\left(Ax+\frac{Bx^3}{3}\right) \end{cases}$$

input
```
integrate((b*x**2+a)**(5/2)*(B*x**2+A),x)
```

output
```
Piecewise((sqrt(a + b*x**2)*(B*b**2*x**7/8 + x**5*(A*b**3 + 17*B*a*b**2/8)
/(6*b) + x**3*(3*A*a*b**2 + 3*B*a**2*b - 5*a*(A*b**3 + 17*B*a*b**2/8)/(6*b
))/(4*b) + x*(3*A*a**2*b + B*a**3 - 3*a*(3*A*a*b**2 + 3*B*a**2*b - 5*a*(A*
b**3 + 17*B*a*b**2/8)/(6*b))/(4*b))/(2*b)) + (A*a**3 - a*(3*A*a**2*b + B*a
**3 - 3*a*(3*A*a*b**2 + 3*B*a**2*b - 5*a*(A*b**3 + 17*B*a*b**2/8)/(6*b))/(
4*b))/(2*b))*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), N
e(a, 0)), (x*log(x)/sqrt(b*x**2), True)), Ne(b, 0)), (a**(5/2)*(A*x + B*x*
*3/3), True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.01

$$\int (a+bx^2)^{5/2}(A+Bx^2)\,dx = \frac{1}{6}\left(bx^2+a\right)^{\frac{5}{2}}Ax + \frac{5}{24}\left(bx^2+a\right)^{\frac{3}{2}}Aax + \frac{5}{16}\sqrt{bx^2+a}Aa^2x + \frac{(bx^2+a)^{\frac{7}{2}}Bx}{8\,b} - \frac{(bx^2+a)^{\frac{5}{2}}Bax}{48\,b} - \frac{5\,(bx^2+a)^{\frac{3}{2}}Ba^2x}{192\,b} - \frac{5\,\sqrt{bx^2+a}Ba^3x}{128\,b} - \frac{5\,Ba^4\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\,b^{\frac{3}{2}}} + \frac{5\,Aa^3\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{b}}$$

input
```
integrate((b*x^2+a)^(5/2)*(B*x^2+A),x, algorithm="maxima")
```

output
```
1/6*(b*x^2 + a)^(5/2)*A*x + 5/24*(b*x^2 + a)^(3/2)*A*a*x + 5/16*sqrt(b*x^2
 + a)*A*a^2*x + 1/8*(b*x^2 + a)^(7/2)*B*x/b - 1/48*(b*x^2 + a)^(5/2)*B*a*x
/b - 5/192*(b*x^2 + a)^(3/2)*B*a^2*x/b - 5/128*sqrt(b*x^2 + a)*B*a^3*x/b -
 5/128*B*a^4*arcsinh(b*x/sqrt(a*b))/b^(3/2) + 5/16*A*a^3*arcsinh(b*x/sqrt(
a*b))/sqrt(b)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.90

$$\int (a+bx^2)^{5/2} (A+Bx^2)\, dx = \frac{1}{384}\left(2\left(4\left(6\,Bb^2x^2+\frac{17\,Bab^7+8\,Ab^8}{b^6}\right)x^2+\frac{59\,Ba^2b^6+104\,Aab^7}{b^6}\right)x^2+\frac{3\left(5\,Ba^3b^5+88\,A\right.}{b^6}\right. + \frac{5\left(Ba^4-8\,Aa^3b\right)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{128\,b^{\frac{3}{2}}}$$

input
```
integrate((b*x^2+a)^(5/2)*(B*x^2+A),x, algorithm="giac")
```

output
```
1/384*(2*(4*(6*B*b^2*x^2 + (17*B*a*b^7 + 8*A*b^8)/b^6)*x^2 + (59*B*a^2*b^6
 + 104*A*a*b^7)/b^6)*x^2 + 3*(5*B*a^3*b^5 + 88*A*a^2*b^6)/b^6)*sqrt(b*x^2
+ a)*x + 5/128*(B*a^4 - 8*A*a^3*b)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/
b^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx^2)^{5/2} (A+Bx^2)\, dx = \int (B\,x^2+A)\,(b\,x^2+a)^{5/2}\, dx$$

input
```
int((A + B*x^2)*(a + b*x^2)^(5/2),x)
```

output
```
int((A + B*x^2)*(a + b*x^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.66

$$\int \left(a+bx^2\right)^{5/2}\left(A+Bx^2\right)\,dx=\frac{279\sqrt{b\,x^2+a}\,a^3bx+326\sqrt{b\,x^2+a}\,a^2b^2x^3+200\sqrt{b\,x^2+a}\,a\,b^3x^5+48\sqrt{b\,x^2+a}\,b^4x^7+105\sqrt{b}}{384b}$$

input
```
int((b*x^2+a)^(5/2)*(B*x^2+A),x)
```

output
```
(279*sqrt(a + b*x**2)*a**3*b*x + 326*sqrt(a + b*x**2)*a**2*b**2*x**3 + 200
*sqrt(a + b*x**2)*a*b**3*x**5 + 48*sqrt(a + b*x**2)*b**4*x**7 + 105*sqrt(b
)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**4)/(384*b)
```

3.89 $\int (a+bx^2)^{3/2}(A+Bx^2)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 118

$$\int (a+bx^2)^{3/2}(A+Bx^2)\,dx = \frac{a(6Ab-aB)x\sqrt{a+bx^2}}{16b} + \frac{(6Ab-aB)x(a+bx^2)^{3/2}}{24b} + \frac{Bx(a+bx^2)^{5/2}}{6b} + \frac{a^2(6Ab-aB)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{3/2}}$$

output
```
1/16*a*(6*A*b-B*a)*x*(b*x^2+a)^(1/2)/b+1/24*(6*A*b-B*a)*x*(b*x^2+a)^(3/2)/
b+1/6*B*x*(b*x^2+a)^(5/2)/b+1/16*a^2*(6*A*b-B*a)*arctanh(b^(1/2)*x/(b*x^2+
a)^(1/2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.84

$$\int (a+bx^2)^{3/2}(A+Bx^2)\,dx = \frac{x\sqrt{a+bx^2}(30aAb+3a^2B+12Ab^2x^2+14abBx^2+8b^2Bx^4)}{48b} + \frac{a^2(-6Ab+aB)\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{16b^{3/2}}$$

input
```
Integrate[(a + b*x^2)^(3/2)*(A + B*x^2),x]
```

output
```
(x*Sqrt[a + b*x^2]*(30*a*A*b + 3*a^2*B + 12*A*b^2*x^2 + 14*a*b*B*x^2 + 8*b
^2*B*x^4))/(48*b) + (a^2*(-6*A*b + a*B)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]
])/(16*b^(3/2))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.89, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {299, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^{3/2} \left(A+Bx^2\right) \, dx$$

$$\Big\downarrow \text{299}$$

$$\frac{(6Ab-aB)\int \left(bx^2+a\right)^{3/2}dx}{6b}+\frac{Bx\left(a+bx^2\right)^{5/2}}{6b}$$

$$\Big\downarrow \text{211}$$

$$\frac{(6Ab-aB)\left(\frac{3}{4}a\int \sqrt{bx^2+a}dx+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)}{6b}+\frac{Bx\left(a+bx^2\right)^{5/2}}{6b}$$

$$\Big\downarrow \text{211}$$

$$\frac{(6Ab-aB)\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)}{6b}+\frac{Bx\left(a+bx^2\right)^{5/2}}{6b}$$

$$\Big\downarrow \text{224}$$

$$\frac{(6Ab-aB)\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)}{6b}+\frac{Bx\left(a+bx^2\right)^{5/2}}{6b}$$

$$\Big\downarrow \text{219}$$

$$\frac{(6Ab-aB)\left(\frac{3}{4}a\left(\frac{a\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x(a+bx^2)^{3/2}\right)}{6b}+\frac{Bx(a+bx^2)^{5/2}}{6b}$$

input
```
Int[(a + b*x^2)^(3/2)*(A + B*x^2),x]
```

output
```
(B*x*(a + b*x^2)^(5/2))/(6*b) + ((6*A*b - a*B)*((x*(a + b*x^2)^(3/2))/4 +
(3*a*((x*Sqrt[a + b*x^2])/2 + (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*
Sqrt[b])))/4))/(6*b)
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.75

method	result
risch	$\frac{x(8b^2B\,x^4+12A\,b^2x^2+14Bab\,x^2+30abA+3a^2B)\sqrt{b\,x^2+a}}{48b}+\frac{a^2(6Ab-Ba)\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{16b^{\frac{3}{2}}}$
pseudoelliptic	$\frac{\left(\frac{3}{2}a^2bA-\frac{1}{4}a^3B\right)\operatorname{arctanh}\left(\frac{\sqrt{b\,x^2+a}}{x\sqrt{b}}\right)+\left(\frac{5\left(\frac{7x^2B}{15}+A\right)a\,b^{\frac{3}{2}}}{2}+x^2\left(\frac{2x^2B}{3}+A\right)b^{\frac{5}{2}}+\frac{B\,a^2\sqrt{b}}{4}\right)\sqrt{b\,x^2+a}\,x}{4b^{\frac{3}{2}}}$
default	$A\left(\frac{x(b\,x^2+a)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)}{4}\right)+B\left(\frac{x(b\,x^2+a)^{\frac{5}{2}}}{6b}-\frac{a\left(\frac{x(b\,x^2+a)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^2+a}}{2}\right.}{\ldots}\right.}{6b}\right.$

input
```
int((b*x^2+a)^(3/2)*(B*x^2+A),x,method=_RETURNVERBOSE)
```

output
```
1/48/b*x*(8*B*b^2*x^4+12*A*b^2*x^2+14*B*a*b*x^2+30*A*a*b+3*B*a^2)*(b*x^2+a
)^(1/2)+1/16*a^2*(6*A*b-B*a)/b^(3/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.75

$$\int (a+bx^2)^{3/2}(A+Bx^2)\,dx = \left[-\frac{3\,(Ba^3-6\,Aa^2b)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\,(8\,Bb^3x^5+2\,(7\,Bab^2+6\,Ab^3}{96\,b^2}\right.$$

input
```
integrate((b*x^2+a)^(3/2)*(B*x^2+A),x, algorithm="fricas")
```

output
```
[-1/96*(3*(B*a^3 - 6*A*a^2*b)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqr
t(b)*x - a) - 2*(8*B*b^3*x^5 + 2*(7*B*a*b^2 + 6*A*b^3)*x^3 + 3*(B*a^2*b +
10*A*a*b^2)*x)*sqrt(b*x^2 + a))/b^2, 1/48*(3*(B*a^3 - 6*A*a^2*b)*sqrt(-b)*
arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (8*B*b^3*x^5 + 2*(7*B*a*b^2 + 6*A*b^3
)*x^3 + 3*(B*a^2*b + 10*A*a*b^2)*x)*sqrt(b*x^2 + a))/b^2]
```

Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.48

$$\int \left(a+bx^2\right)^{3/2}\left(A+Bx^2\right)\,dx=\begin{cases}\sqrt{a+bx^2}\left(\frac{Bbx^5}{6}+\frac{x^3\left(Ab^2+\frac{7Bab}{6}\right)}{4b}+\frac{x\left(2Aab+Ba^2-\frac{3a\left(Ab^2+\frac{7Bab}{6}\right)}{4b}\right)}{2b}\right)+\left(Aa^2-\frac{a\left(2Aab+Ba^2-\frac{3a\left(Ab^2+\right.}{4b}\right.}{2b}\right. \\ a^{\frac{3}{2}}\left(Ax+\frac{Bx^3}{3}\right)\end{cases}$$

input
```
integrate((b*x**2+a)**(3/2)*(B*x**2+A),x)
```

output
```
Piecewise((sqrt(a + b*x**2)*(B*b*x**5/6 + x**3*(A*b**2 + 7*B*a*b/6)/(4*b)
+ x*(2*A*a*b + B*a**2 - 3*a*(A*b**2 + 7*B*a*b/6)/(4*b))/(2*b)) + (A*a**2 -
 a*(2*A*a*b + B*a**2 - 3*a*(A*b**2 + 7*B*a*b/6)/(4*b))/(2*b))*Piecewise((l
og(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(
b*x**2), True)), Ne(b, 0)), (a**(3/2)*(A*x + B*x**3/3), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.98

$$\int \left(a+bx^2\right)^{3/2}\left(A+Bx^2\right)\,dx=\frac{1}{4}\left(bx^2+a\right)^{\frac{3}{2}}Ax+\frac{3}{8}\sqrt{bx^2+a}Aax+\frac{\left(bx^2+a\right)^{\frac{5}{2}}Bx}{6\,b}-\frac{\left(bx^2+a\right)^{\frac{3}{2}}Bax}{24\,b}-\frac{\sqrt{bx^2+a}Ba^2x}{16\,b}-\frac{Ba^3\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{3}{2}}}+\frac{3\,Aa^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{b}}$$

input
```
integrate((b*x^2+a)^(3/2)*(B*x^2+A),x, algorithm="maxima")
```

output
```
1/4*(b*x^2 + a)^(3/2)*A*x + 3/8*sqrt(b*x^2 + a)*A*a*x + 1/6*(b*x^2 + a)^(5
/2)*B*x/b - 1/24*(b*x^2 + a)^(3/2)*B*a*x/b - 1/16*sqrt(b*x^2 + a)*B*a^2*x/
b - 1/16*B*a^3*arcsinh(b*x/sqrt(a*b))/b^(3/2) + 3/8*A*a^2*arcsinh(b*x/sqrt
(a*b))/sqrt(b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.86

$$\int (a+bx^2)^{3/2}(A+Bx^2)\,dx = \frac{1}{48}\left(2\left(4\,Bbx^2+\frac{7\,Bab^4+6\,Ab^5}{b^4}\right)x^2+\frac{3\,(Ba^2b^3+10\,Aab^4)}{b^4}\right)\sqrt{bx^2+a}\,x + \frac{(Ba^3-6\,Aa^2b)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{16\,b^{\frac{3}{2}}}$$

input
```
integrate((b*x^2+a)^(3/2)*(B*x^2+A),x, algorithm="giac")
```

output
```
1/48*(2*(4*B*b*x^2 + (7*B*a*b^4 + 6*A*b^5)/b^4)*x^2 + 3*(B*a^2*b^3 + 10*A*
a*b^4)/b^4)*sqrt(b*x^2 + a)*x + 1/16*(B*a^3 - 6*A*a^2*b)*log(abs(-sqrt(b)*
x + sqrt(b*x^2 + a)))/b^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx^2)^{3/2}(A+Bx^2)\,dx = \int (B\,x^2+A)\,(b\,x^2+a)^{3/2}\,dx$$

input
```
int((A + B*x^2)*(a + b*x^2)^(3/2),x)
```

output
```
int((A + B*x^2)*(a + b*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.68

$$\int \left(a+bx^2\right)^{3/2} \left(A+Bx^2\right) \, dx = \frac{33\sqrt{b\,x^2+a}\,a^2bx+26\sqrt{b\,x^2+a}\,a\,b^2x^3+8\sqrt{b\,x^2+a}\,b^3x^5+15\sqrt{b}\log\left(\frac{\sqrt{b\,x^2+a}+\sqrt{b}\,x}{\sqrt{a}}\right)a^3}{48b}$$

input
```
int((b*x^2+a)^(3/2)*(B*x^2+A),x)
```

output
```
(33*sqrt(a + b*x**2)*a**2*b*x + 26*sqrt(a + b*x**2)*a*b**2*x**3 + 8*sqrt(a
 + b*x**2)*b**3*x**5 + 15*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(
a))*a**3)/(48*b)
```

3.90 $\int \sqrt{a+bx^2}(A+Bx^2)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 87

$$\int \sqrt{a+bx^2}\left(A+Bx^2\right)\,dx = \frac{(4Ab-aB)x\sqrt{a+bx^2}}{8b} + \frac{Bx(a+bx^2)^{3/2}}{4b} + \frac{a(4Ab-aB)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{3/2}}$$

output
```
1/8*(4*A*b-B*a)*x*(b*x^2+a)^(1/2)/b+1/4*B*x*(b*x^2+a)^(3/2)/b+1/8*a*(4*A*b
-B*a)*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.85

$$\int \sqrt{a+bx^2}\left(A+Bx^2\right)\,dx = \frac{x\sqrt{a+bx^2}(4Ab+aB+2bBx^2)}{8b} + \frac{a(-4Ab+aB)\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{8b^{3/2}}$$

input
```
Integrate[Sqrt[a + b*x^2]*(A + B*x^2),x]
```

output
```
(x*Sqrt[a + b*x^2]*(4*A*b + a*B + 2*b*B*x^2))/(8*b) + (a*(-4*A*b + a*B)*Lo
g[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(8*b^(3/2))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {299, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx^2}\left(A+Bx^2\right)\,dx$$

$$\Big\downarrow \text{ 299}$$

$$\frac{(4Ab-aB)\int \sqrt{bx^2+a}dx}{4b}+\frac{Bx\left(a+bx^2\right)^{3/2}}{4b}$$

$$\Big\downarrow \text{ 211}$$

$$\frac{(4Ab-aB)\left(\frac{1}{2}a\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)}{4b}+\frac{Bx\left(a+bx^2\right)^{3/2}}{4b}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{(4Ab-aB)\left(\frac{1}{2}a\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)}{4b}+\frac{Bx\left(a+bx^2\right)^{3/2}}{4b}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{(4Ab-aB)\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)}{4b}+\frac{Bx\left(a+bx^2\right)^{3/2}}{4b}$$

input
```
Int[Sqrt[a + b*x^2]*(A + B*x^2),x]
```

output
```
(B*x*(a + b*x^2)^(3/2))/(4*b) + ((4*A*b - a*B)*((x*Sqrt[a + b*x^2])/2 + (a
*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b])))/(4*b)
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.72

method	result	size
risch	$\frac{x\left(2bB\,x^{2}+4Ab+Ba\right)\sqrt{b\,x^{2}+a}}{8b}+\frac{a\left(4Ab-Ba\right)\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{8b^{\frac{3}{2}}}$	63
pseudoelliptic	$\frac{a\left(Ab-\frac{Ba}{4}\right)\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{x\sqrt{b}}\right)+\sqrt{b\,x^{2}+a}\,\left(\left(\frac{x^{2}B}{2}+A\right)b^{\frac{3}{2}}+\frac{Ba\sqrt{b}}{4}\right)x}{2b^{\frac{3}{2}}}$	65
default	$A\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)+B\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{4b}-\frac{a\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)}{4b}\right)$	98

input
```
int((b*x^2+a)^(1/2)*(B*x^2+A),x,method=_RETURNVERBOSE)
```

output
```
1/8*x*(2*B*b*x^2+4*A*b+B*a)*(b*x^2+a)^(1/2)/b+1/8*a*(4*A*b-B*a)/b^(3/2)*ln
(b^(1/2)*x+(b*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.78

$$\int \sqrt{a+bx^2}\left(A+Bx^2\right)\,dx$$
$$=\left[-\frac{(Ba^2-4\,Aab)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\,(2\,Bb^2x^3+(Bab+4\,Ab^2)x)\sqrt{bx^2+a}}{16\,b^2}, \frac{(B\cdots}{}\right.$$

input
```
integrate((b*x^2+a)^(1/2)*(B*x^2+A),x, algorithm="fricas")
```

output
```
[-1/16*((B*a^2 - 4*A*a*b)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)
*x - a) - 2*(2*B*b^2*x^3 + (B*a*b + 4*A*b^2)*x)*sqrt(b*x^2 + a))/b^2, 1/8*
((B*a^2 - 4*A*a*b)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (2*B*b^2*
x^3 + (B*a*b + 4*A*b^2)*x)*sqrt(b*x^2 + a))/b^2]
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.20

$$\int \sqrt{a+bx^2}\left(A+Bx^2\right)\,dx$$
$$=\begin{cases}\sqrt{a+bx^2}\left(\frac{Bx^3}{4}+\frac{x\left(Ab+\frac{Ba}{4}\right)}{2b}\right)+\left(Aa-\frac{a\left(Ab+\frac{Ba}{4}\right)}{2b}\right)\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right) & \text{for } b\neq 0\\ \sqrt{a}\left(Ax+\frac{Bx^3}{3}\right) & \text{otherwise}\end{cases}$$

input
```
integrate((b*x**2+a)**(1/2)*(B*x**2+A),x)
```

output
```
Piecewise((sqrt(a + b*x**2)*(B*x**3/4 + x*(A*b + B*a/4)/(2*b)) + (A*a - a*
(A*b + B*a/4)/(2*b))*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sq
rt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True)), Ne(b, 0)), (sqrt(a)*(A*x
 + B*x**3/3), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.93

$$\int \sqrt{a+bx^2}\left(A+Bx^2\right)\,dx = \frac{1}{2}\sqrt{bx^2+a}Ax + \frac{\left(bx^2+a\right)^{\frac{3}{2}}Bx}{4\,b} - \frac{\sqrt{bx^2+a}Bax}{8\,b} - \frac{Ba^2\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{3}{2}}} + \frac{Aa\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{b}}$$

input
```
integrate((b*x^2+a)^(1/2)*(B*x^2+A),x, algorithm="maxima")
```

output
```
1/2*sqrt(b*x^2 + a)*A*x + 1/4*(b*x^2 + a)^(3/2)*B*x/b - 1/8*sqrt(b*x^2 + a
)*B*a*x/b - 1/8*B*a^2*arcsinh(b*x/sqrt(a*b))/b^(3/2) + 1/2*A*a*arcsinh(b*x
/sqrt(a*b))/sqrt(b)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.79

$$\int \sqrt{a+bx^2}\left(A+Bx^2\right)\,dx = \frac{1}{8}\left(2\,Bx^2 + \frac{Bab+4\,Ab^2}{b^2}\right)\sqrt{bx^2+a}x + \frac{\left(Ba^2-4\,Aab\right)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{8\,b^{\frac{3}{2}}}$$

input
```
integrate((b*x^2+a)^(1/2)*(B*x^2+A),x, algorithm="giac")
```

output
```
1/8*(2*B*x^2 + (B*a*b + 4*A*b^2)/b^2)*sqrt(b*x^2 + a)*x + 1/8*(B*a^2 - 4*A
*a*b)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+bx^2}\,(A+Bx^2)\,dx=\int (B\,x^2+A)\,\sqrt{b\,x^2+a}\,dx$$

input
```
int((A + B*x^2)*(a + b*x^2)^(1/2),x)
```

output
```
int((A + B*x^2)*(a + b*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.70

$$\int \sqrt{a+bx^2}\,(A+Bx^2)\,dx = \frac{5\sqrt{b\,x^2+a}\,abx+2\sqrt{b\,x^2+a}\,b^2x^3+3\sqrt{b}\log\left(\frac{\sqrt{b\,x^2+a}+\sqrt{b}\,x}{\sqrt{a}}\right)a^2}{8b}$$

input
```
int((b*x^2+a)^(1/2)*(B*x^2+A),x)
```

output
```
(5*sqrt(a + b*x**2)*a*b*x + 2*sqrt(a + b*x**2)*b**2*x**3 + 3*sqrt(b)*log((
sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2)/(8*b)
```

3.91 $\int \frac{A+Bx^2}{\sqrt{a+bx^2}} dx$

Optimal result

Integrand size = 19, antiderivative size = 58

$$\int \frac{A+Bx^2}{\sqrt{a+bx^2}} dx = \frac{Bx\sqrt{a+bx^2}}{2b} + \frac{(2Ab-aB)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}$$

output
```
1/2*B*x*(b*x^2+a)^(1/2)/b+1/2*(2*A*b-B*a)*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2
))/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.12

$$\int \frac{A+Bx^2}{\sqrt{a+bx^2}} dx = \frac{Bx\sqrt{a+bx^2}}{2b} + \frac{(2Ab-aB)\text{arctanh}\left(\frac{\sqrt{b}x}{-\sqrt{a}+\sqrt{a+bx^2}}\right)}{b^{3/2}}$$

input
```
Integrate[(A + B*x^2)/Sqrt[a + b*x^2],x]
```

output
```
(B*x*Sqrt[a + b*x^2])/(2*b) + ((2*A*b - a*B)*ArcTanh[(Sqrt[b]*x)/(-Sqrt[a]
 + Sqrt[a + b*x^2])])/b^(3/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {299, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{\sqrt{a+bx^2}}\,dx$$

$$\Big\downarrow \text{ 299}$$

$$\frac{(2Ab-aB)\int \frac{1}{\sqrt{bx^2+a}}dx}{2b}+\frac{Bx\sqrt{a+bx^2}}{2b}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{(2Ab-aB)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2b}+\frac{Bx\sqrt{a+bx^2}}{2b}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{(2Ab-aB)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}+\frac{Bx\sqrt{a+bx^2}}{2b}$$

input
```
Int[(A + B*x^2)/Sqrt[a + b*x^2],x]
```

output
```
(B*x*Sqrt[a + b*x^2])/(2*b) + ((2*A*b - a*B)*ArcTanh[(Sqrt[b]*x)/Sqrt[a +
b*x^2]])/(2*b^(3/2))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.83

method	result	size
risch	$\frac{Bx\sqrt{b\,x^2+a}}{2b}+\frac{(2Ab-Ba)\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2b^{\frac{3}{2}}}$	48
pseudoelliptic	$\frac{Bx\sqrt{b\,x^2+a}}{2b}+\frac{(2Ab-Ba)\,\mathrm{arctanh}\left(\frac{\sqrt{b\,x^2+a}}{x\sqrt{b}}\right)}{2b^{\frac{3}{2}}}$	49
default	$\frac{A\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}+B\left(\frac{x\sqrt{b\,x^2+a}}{2b}-\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2b^{\frac{3}{2}}}\right)$	63

input
```
int((B*x^2+A)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*B*x*(b*x^2+a)^(1/2)/b+1/2*(2*A*b-B*a)/b^(3/2)*ln(b^(1/2)*x+(b*x^2+a)^(
1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.90

$$\int \frac{A+Bx^2}{\sqrt{a+bx^2}}\,dx$$
$$=\left[\frac{2\,\sqrt{bx^2+a}Bbx-(Ba-2\,Ab)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)}{4\,b^2},\frac{\sqrt{bx^2+a}Bbx+(Ba-2\,Ab)\sqrt{}}{2\,b^2}\right.$$

input
```
integrate((B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(2*sqrt(b*x^2 + a)*B*b*x - (B*a - 2*A*b)*sqrt(b)*log(-2*b*x^2 - 2*sqr
t(b*x^2 + a)*sqrt(b)*x - a))/b^2, 1/2*(sqrt(b*x^2 + a)*B*b*x + (B*a - 2*A*
b)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)))/b^2]
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.41

$$\int \frac{A+Bx^2}{\sqrt{a+bx^2}}\,dx = \begin{cases} \frac{Bx\sqrt{a+bx^2}}{2b} + \left(A - \frac{Ba}{2b}\right)\left(\begin{cases} \frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for}\ a \neq 0 \\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise} \end{cases}\right) & \text{for}\ b \neq 0 \\ \frac{Ax+\frac{Bx^3}{3}}{\sqrt{a}} & \text{otherwise} \end{cases}$$

input
```
integrate((B*x**2+A)/(b*x**2+a)**(1/2),x)
```

output
```
Piecewise((B*x*sqrt(a + b*x**2)/(2*b) + (A - B*a/(2*b))*Piecewise((log(2*s
qrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2
), True)), Ne(b, 0)), ((A*x + B*x**3/3)/sqrt(a), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.81

$$\int \frac{A+Bx^2}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{bx^2+a}Bx}{2\,b} - \frac{Ba\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{3}{2}}} + \frac{A\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}}$$

input
```
integrate((B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(b*x^2 + a)*B*x/b - 1/2*B*a*arcsinh(b*x/sqrt(a*b))/b^(3/2) + A*arc
sinh(b*x/sqrt(a*b))/sqrt(b)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.83

$$\int \frac{A+Bx^2}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{bx^2+a}Bx}{2\,b} + \frac{(Ba-2\,Ab)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2\,b^{\frac{3}{2}}}$$

input
```
integrate((B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(b*x^2 + a)*B*x/b + 1/2*(B*a - 2*A*b)*log(abs(-sqrt(b)*x + sqrt(b*
x^2 + a)))/b^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.48

$$\int \frac{A+Bx^2}{\sqrt{a+bx^2}}\,dx = \begin{cases} \frac{B\,x^3+3\,A\,x}{3\,\sqrt{a}} & \text{if } b=0 \\ \frac{A\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{\sqrt{b}} - \frac{B\,a\,\ln\left(2\,\sqrt{b}\,x+2\,\sqrt{b\,x^2+a}\right)}{2\,b^{3/2}} + \frac{B\,x\,\sqrt{b\,x^2+a}}{2\,b} & \text{if } b\neq 0 \end{cases}$$

input
```
int((A + B*x^2)/(a + b*x^2)^(1/2),x)
```

output
```
piecewise(b == 0, (3*A*x + B*x^3)/(3*a^(1/2)), b ~= 0, (A*log(b^(1/2)*x +
(a + b*x^2)^(1/2)))/b^(1/2) - (B*a*log(2*b^(1/2)*x + 2*(a + b*x^2)^(1/2)))
/(2*b^(3/2)) + (B*x*(a + b*x^2)^(1/2))/(2*b))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.69

$$\int \frac{A + Bx^2}{\sqrt{a + bx^2}}\,dx = \frac{\sqrt{b\,x^2 + a}\,bx + \sqrt{b}\log\left(\frac{\sqrt{b\,x^2+a}+\sqrt{b}\,x}{\sqrt{a}}\right)a}{2b}$$

input
```
int((B*x^2+A)/(b*x^2+a)^(1/2),x)
```

output
```
(sqrt(a + b*x**2)*b*x + sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a)
)*a)/(2*b)
```

3.92 $\int \frac{A+Bx^2}{(a+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 54

$$\int \frac{A+Bx^2}{(a+bx^2)^{3/2}}\,dx=\frac{(Ab-aB)x}{ab\sqrt{a+bx^2}}+\frac{B\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}$$

output
```
(A*b-B*a)*x/a/b/(b*x^2+a)^(1/2)+B*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(3/
2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.07

$$\int \frac{A+Bx^2}{(a+bx^2)^{3/2}}\,dx=\frac{Abx-aBx}{ab\sqrt{a+bx^2}}-\frac{B\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{b^{3/2}}$$

input
```
Integrate[(A + B*x^2)/(a + b*x^2)^(3/2),x]
```

output
```
(A*b*x - a*B*x)/(a*b*Sqrt[a + b*x^2]) - (B*Log[-(Sqrt[b]*x) + Sqrt[a + b*x
^2]])/b^(3/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {298, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{(a+bx^2)^{3/2}}dx$$

$$\Big\downarrow \text{298}$$

$$\frac{B\int \frac{1}{\sqrt{bx^2+a}}dx}{b}+\frac{x(Ab-aB)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{224}$$

$$\frac{B\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{b}+\frac{x(Ab-aB)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{x(Ab-aB)}{ab\sqrt{a+bx^2}}+\frac{B\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}$$

input
```
Int[(A + B*x^2)/(a + b*x^2)^(3/2),x]
```

output
```
((A*b - a*B)*x)/(a*b*Sqrt[a + b*x^2]) + (B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*
x^2]])/b^(3/2)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.94

method	result	size
pseudoelliptic	$\frac{\frac{(Ab-Ba)x}{b\sqrt{b\,x^2+a}}+\frac{Ba\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^2+a}}{x\sqrt{b}}\right)}{b^{\frac{3}{2}}}}{a}$	51
default	$\frac{Ax}{a\sqrt{b\,x^2+a}}+B\left(-\frac{x}{b\sqrt{b\,x^2+a}}+\frac{\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{b^{\frac{3}{2}}}\right)$	55

input
```
int((B*x^2+A)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((A*b-B*a)/b*x/(b*x^2+a)^(1/2)+B*a/b^(3/2)*arctanh((b*x^2+a)^(1/2)/x/b^(1/
2)))/a
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 168, normalized size of antiderivative = 3.11

$$\int \frac{A+Bx^2}{(a+bx^2)^{3/2}}\,dx = \left[-\frac{2\,(Bab-Ab^2)\sqrt{bx^2+a}x-(Babx^2+Ba^2)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)}{2\,(ab^3x^2+a^2b^2)}\right.$$

$$\left.-\frac{(Bab-Ab^2)\sqrt{bx^2+a}x+(Babx^2+Ba^2)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)}{ab^3x^2+a^2b^2}\right]$$

input
```
integrate((B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[-1/2*(2*(B*a*b - A*b^2)*sqrt(b*x^2 + a)*x - (B*a*b*x^2 + B*a^2)*sqrt(b)*l
og(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a))/(a*b^3*x^2 + a^2*b^2), -((
B*a*b - A*b^2)*sqrt(b*x^2 + a)*x + (B*a*b*x^2 + B*a^2)*sqrt(-b)*arctan(sqr
t(-b)*x/sqrt(b*x^2 + a)))/(a*b^3*x^2 + a^2*b^2)]
```

Sympy [A] (verification not implemented)

Time = 2.02 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.11

$$\int \frac{A+Bx^2}{(a+bx^2)^{3/2}}\,dx = \frac{Ax}{a^{\frac{3}{2}}\sqrt{1+\frac{bx^2}{a}}}+B\left(\frac{\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{\frac{3}{2}}}-\frac{x}{\sqrt{a}b\sqrt{1+\frac{bx^2}{a}}}\right)$$

input
```
integrate((B*x**2+A)/(b*x**2+a)**(3/2),x)
```

output
```
A*x/(a**(3/2)*sqrt(1 + b*x**2/a)) + B*(asinh(sqrt(b)*x/sqrt(a))/b**(3/2) -
 x/(sqrt(a)*b*sqrt(1 + b*x**2/a)))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int \frac{A+Bx^2}{(a+bx^2)^{3/2}}\,dx = \frac{Ax}{\sqrt{bx^2+a}a} - \frac{Bx}{\sqrt{bx^2+a}b} + \frac{B\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{3}{2}}}$$

input
```
integrate((B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
A*x/(sqrt(b*x^2 + a)*a) - B*x/(sqrt(b*x^2 + a)*b) + B*arcsinh(b*x/sqrt(a*b
))/b^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.94

$$\int \frac{A+Bx^2}{(a+bx^2)^{3/2}}\,dx = -\frac{B\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{b^{\frac{3}{2}}} - \frac{(Ba-Ab)x}{\sqrt{bx^2+a}ab}$$

input
```
integrate((B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
-B*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2) - (B*a - A*b)*x/(sqrt(b*
x^2 + a)*a*b)
```

Mupad [B] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.98

$$\int \frac{A+Bx^2}{(a+bx^2)^{3/2}}\,dx = \frac{B\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{b^{3/2}} + \frac{A\,x}{a\,\sqrt{b\,x^2+a}} - \frac{B\,x}{b\,\sqrt{b\,x^2+a}}$$

input
```
int((A + B*x^2)/(a + b*x^2)^(3/2),x)
```

output
```
(B*log(b^(1/2)*x + (a + b*x^2)^(1/2)))/b^(3/2) + (A*x)/(a*(a + b*x^2)^(1/2
)) - (B*x)/(b*(a + b*x^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.46

$$\int \frac{A+Bx^2}{(a+bx^2)^{3/2}}\,dx = \frac{\sqrt{b}\log\left(\frac{\sqrt{b\,x^2+a}+\sqrt{b}\,x}{\sqrt{a}}\right)}{b}$$

input
```
int((B*x^2+A)/(b*x^2+a)^(3/2),x)
```

output
```
(sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a)))/b
```

3.93 $\int \frac{A+Bx^2}{(a+bx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 61

$$\int \frac{A+Bx^2}{(a+bx^2)^{5/2}}\,dx=\frac{(Ab-aB)x}{3ab\left(a+bx^2\right)^{3/2}}+\frac{(2Ab+aB)x}{3a^2b\sqrt{a+bx^2}}$$

output
```
1/3*(A*b-B*a)*x/a/b/(b*x^2+a)^(3/2)+1/3*(2*A*b+B*a)*x/a^2/b/(b*x^2+a)^(1/2
)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.61

$$\int \frac{A+Bx^2}{(a+bx^2)^{5/2}}\,dx=\frac{x(3aA+2Abx^2+aBx^2)}{3a^2\left(a+bx^2\right)^{3/2}}$$

input
```
Integrate[(A + B*x^2)/(a + b*x^2)^(5/2),x]
```

output
```
(x*(3*a*A + 2*A*b*x^2 + a*B*x^2))/(3*a^2*(a + b*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.77, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {292, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{(a+bx^2)^{5/2}}dx$$

$$\Big\downarrow \text{292}$$

$$\frac{2A\int \frac{1}{(bx^2+a)^{3/2}}dx}{3a}+\frac{x(A+Bx^2)}{3a(a+bx^2)^{3/2}}$$

$$\Big\downarrow \text{208}$$

$$\frac{2Ax}{3a^2\sqrt{a+bx^2}}+\frac{x(A+Bx^2)}{3a(a+bx^2)^{3/2}}$$

input
```
Int[(A + B*x^2)/(a + b*x^2)^(5/2),x]
```

output
```
(2*A*x)/(3*a^2*Sqrt[a + b*x^2]) + (x*(A + B*x^2))/(3*a*(a + b*x^2)^(3/2))
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 292
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> Si
mp[(-x)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*(p + 1))), x] - Simp[c*(q/(
a*(p + 1)))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1), x], x] /; FreeQ[
{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && EqQ[2*(p + q + 1) + 1, 0] && Gt
Q[q, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.56

method	result	size
gosper	$\frac{x(2Ab\,x^2+Ba\,x^2+3Aa)}{3(b\,x^2+a)^{\frac{3}{2}}a^2}$	34
trager	$\frac{x(2Ab\,x^2+Ba\,x^2+3Aa)}{3(b\,x^2+a)^{\frac{3}{2}}a^2}$	34
pseudoelliptic	$\frac{x(2Ab\,x^2+Ba\,x^2+3Aa)}{3(b\,x^2+a)^{\frac{3}{2}}a^2}$	34
orering	$\frac{x(2Ab\,x^2+Ba\,x^2+3Aa)}{3(b\,x^2+a)^{\frac{3}{2}}a^2}$	34
default	$A\left(\frac{x}{3a(b\,x^2+a)^{\frac{3}{2}}}+\frac{2x}{3a^2\sqrt{b\,x^2+a}}\right)+B\left(-\frac{x}{2b(b\,x^2+a)^{\frac{3}{2}}}+\frac{a\left(\frac{x}{3a\left(b\,x^2+a\right)^{\frac{3}{2}}}+\frac{2x}{3a^2\sqrt{b\,x^2+a}}\right)}{2b}\right)$	90

input
```
int((B*x^2+A)/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*x*(2*A*b*x^2+B*a*x^2+3*A*a)/(b*x^2+a)^(3/2)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.89

$$\int \frac{A+Bx^2}{(a+bx^2)^{5/2}}\,dx=\frac{((Ba+2\,Ab)x^3+3\,Aax)\sqrt{bx^2+a}}{3\,(a^2b^2x^4+2\,a^3bx^2+a^4)}$$

input
```
integrate((B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
1/3*((B*a + 2*A*b)*x^3 + 3*A*a*x)*sqrt(b*x^2 + a)/(a^2*b^2*x^4 + 2*a^3*b*x
^2 + a^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 144 vs. $2(53) = 106$.

Time = 3.87 (sec) , antiderivative size = 144, normalized size of antiderivative = 2.36

$$\int \frac{A+Bx^2}{(a+bx^2)^{5/2}}\,dx = A\left(\frac{3ax}{3a^{\frac{7}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{5}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}} + \frac{2bx^3}{3a^{\frac{7}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{5}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}}\right) + \frac{Bx^3}{3a^{\frac{5}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{3}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}}$$

input
```
integrate((B*x**2+A)/(b*x**2+a)**(5/2),x)
```

output
```
A*(3*a*x/(3*a**(7/2)*sqrt(1 + b*x**2/a) + 3*a**(5/2)*b*x**2*sqrt(1 + b*x**
2/a)) + 2*b*x**3/(3*a**(7/2)*sqrt(1 + b*x**2/a) + 3*a**(5/2)*b*x**2*sqrt(1
 + b*x**2/a))) + B*x**3/(3*a**(5/2)*sqrt(1 + b*x**2/a) + 3*a**(3/2)*b*x**2
*sqrt(1 + b*x**2/a))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.11

$$\int \frac{A+Bx^2}{(a+bx^2)^{5/2}}\,dx = \frac{2\,Ax}{3\sqrt{bx^2+a}a^2} + \frac{Ax}{3\,(bx^2+a)^{\frac{3}{2}}a} - \frac{Bx}{3\,(bx^2+a)^{\frac{3}{2}}b} + \frac{Bx}{3\sqrt{bx^2+a}ab}$$

input
```
integrate((B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
2/3*A*x/(sqrt(b*x^2 + a)*a^2) + 1/3*A*x/((b*x^2 + a)^(3/2)*a) - 1/3*B*x/((
b*x^2 + a)^(3/2)*b) + 1/3*B*x/(sqrt(b*x^2 + a)*a*b)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.66

$$\int \frac{A+Bx^2}{(a+bx^2)^{5/2}}\,dx = \frac{x\left(\frac{3A}{a}+\frac{(Bab+2Ab^2)x^2}{a^2b}\right)}{3(bx^2+a)^{\frac{3}{2}}}$$

input
```
integrate((B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
1/3*x*(3*A/a + (B*a*b + 2*A*b^2)*x^2/(a^2*b))/(b*x^2 + a)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.54

$$\int \frac{A+Bx^2}{(a+bx^2)^{5/2}}\,dx = \frac{3\,A\,a\,x+2\,A\,b\,x^3+B\,a\,x^3}{3\,a^2\,(b\,x^2+a)^{3/2}}$$

input
```
int((A + B*x^2)/(a + b*x^2)^(5/2),x)
```

output
```
(3*A*a*x + 2*A*b*x^3 + B*a*x^3)/(3*a^2*(a + b*x^2)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.64

$$\int \frac{A+Bx^2}{(a+bx^2)^{5/2}}\,dx = \frac{\sqrt{b\,x^2+a}\,bx+\sqrt{b}\,a+\sqrt{b}\,b\,x^2}{ab\,(b\,x^2+a)}$$

input
```
int((B*x^2+A)/(b*x^2+a)^(5/2),x)
```

output
```
(sqrt(a + b*x**2)*b*x + sqrt(b)*a + sqrt(b)*b*x**2)/(a*b*(a + b*x**2))
```

3.94 $\int \frac{A+Bx^2}{(a+bx^2)^{7/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 91

$$\int \frac{A+Bx^2}{(a+bx^2)^{7/2}}\,dx=\frac{(Ab-aB)x}{5ab\,(a+bx^2)^{5/2}}+\frac{(4Ab+aB)x}{15a^2b\,(a+bx^2)^{3/2}}+\frac{2(4Ab+aB)x}{15a^3b\sqrt{a+bx^2}}$$

output
```
1/5*(A*b-B*a)*x/a/b/(b*x^2+a)^(5/2)+1/15*(4*A*b+B*a)*x/a^2/b/(b*x^2+a)^(3/
2)+2/15*(4*A*b+B*a)*x/a^3/b/(b*x^2+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.66

$$\int \frac{A+Bx^2}{(a+bx^2)^{7/2}}\,dx=\frac{15a^2Ax+20aAbx^3+5a^2Bx^3+8Ab^2x^5+2abBx^5}{15a^3\,(a+bx^2)^{5/2}}$$

input
```
Integrate[(A + B*x^2)/(a + b*x^2)^(7/2),x]
```

output
```
(15*a^2*A*x + 20*a*A*b*x^3 + 5*a^2*B*x^3 + 8*A*b^2*x^5 + 2*a*b*B*x^5)/(15*
a^3*(a + b*x^2)^(5/2))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {298, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{(a+bx^2)^{7/2}}dx$$

$$\Big\downarrow \text{298}$$

$$\frac{(aB+4Ab)\int \frac{1}{(bx^2+a)^{5/2}}dx}{5ab}+\frac{x(Ab-aB)}{5ab\left(a+bx^2\right)^{5/2}}$$

$$\Big\downarrow \text{209}$$

$$\frac{(aB+4Ab)\left(\frac{2\int \frac{1}{(bx^2+a)^{3/2}}dx}{3a}+\frac{x}{3a(a+bx^2)^{3/2}}\right)}{5ab}+\frac{x(Ab-aB)}{5ab\left(a+bx^2\right)^{5/2}}$$

$$\Big\downarrow \text{208}$$

$$\frac{\left(\frac{2x}{3a^2\sqrt{a+bx^2}}+\frac{x}{3a(a+bx^2)^{3/2}}\right)(aB+4Ab)}{5ab}+\frac{x(Ab-aB)}{5ab\left(a+bx^2\right)^{5/2}}$$

input
```
Int[(A + B*x^2)/(a + b*x^2)^(7/2),x]
```

output
```
((A*b - a*B)*x)/(5*a*b*(a + b*x^2)^(5/2)) + ((4*A*b + a*B)*(x/(3*a*(a + b*
x^2)^(3/2)) + (2*x)/(3*a^2*Sqrt[a + b*x^2])))/(5*a*b)
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.57

method	result
pseudoelliptic	$\frac{\left(\left(\frac{x^{2} B}{3}+A\right) a^{2}+\frac{4 b \left(\frac{x^{2} B}{10}+A\right) x^{2} a}{3}+\frac{8 A \,b^{2} x^{4}}{15}\right) x}{\left(b \,x^{2}+a \right)^{\frac{5}{2}} a^{3}}$
gosper	$\frac{x \left(8 A \,b^{2} x^{4}+2 B a b \,x^{4}+20 a A b \,x^{2}+5 B \,a^{2} x^{2}+15 a^{2} A \right)}{15 \left(b \,x^{2}+a \right)^{\frac{5}{2}} a^{3}}$
trager	$\frac{x \left(8 A \,b^{2} x^{4}+2 B a b \,x^{4}+20 a A b \,x^{2}+5 B \,a^{2} x^{2}+15 a^{2} A \right)}{15 \left(b \,x^{2}+a \right)^{\frac{5}{2}} a^{3}}$
orering	$\frac{x \left(8 A \,b^{2} x^{4}+2 B a b \,x^{4}+20 a A b \,x^{2}+5 B \,a^{2} x^{2}+15 a^{2} A \right)}{15 \left(b \,x^{2}+a \right)^{\frac{5}{2}} a^{3}}$
default	$A \left(\frac{x}{5 a \left(b \,x^{2}+a \right)^{\frac{5}{2}}}+\frac{\frac{4 x}{15 a \left(b \,x^{2}+a \right)^{\frac{3}{2}}}+\frac{8 x}{15 a^{2} \sqrt{b \,x^{2}+a}}}{a}\right)+B \left(-\frac{x}{4 b \left(b \,x^{2}+a \right)^{\frac{5}{2}}}+\frac{a \left(\frac{x}{5 a \left(b \,x^{2}+a \right)^{\frac{5}{2}}}+\frac{\frac{4 x}{15 a \left(b \,x^{2}+a \right)^{\frac{3}{2}}}+}{a}\right.}{4 b}\right.$

input
```
int((B*x^2+A)/(b*x^2+a)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
((1/3*x^2*B+A)*a^2+4/3*b*(1/10*x^2*B+A)*x^2*a+8/15*A*b^2*x^4)/(b*x^2+a)^(5
/2)*x/a^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.96

$$\int \frac{A+Bx^2}{(a+bx^2)^{7/2}}\,dx = \frac{(2\,(Bab+4\,Ab^2)x^5+15\,Aa^2x+5\,(Ba^2+4\,Aab)x^3)\sqrt{bx^2+a}}{15\,(a^3b^3x^6+3\,a^4b^2x^4+3\,a^5bx^2+a^6)}$$

input
```
integrate((B*x^2+A)/(b*x^2+a)^(7/2),x, algorithm="fricas")
```

output
```
1/15*(2*(B*a*b + 4*A*b^2)*x^5 + 15*A*a^2*x + 5*(B*a^2 + 4*A*a*b)*x^3)*sqrt
(b*x^2 + a)/(a^3*b^3*x^6 + 3*a^4*b^2*x^4 + 3*a^5*b*x^2 + a^6)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 566 vs. 2(83) = 166.

Time = 9.89 (sec) , antiderivative size = 566, normalized size of antiderivative = 6.22

$$\int \frac{A+Bx^2}{(a+bx^2)^{7/2}}\,dx = A\left(\frac{15a^5x}{15a^{\frac{17}{2}}\sqrt{1+\frac{bx^2}{a}}+45a^{\frac{15}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}+45a^{\frac{13}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}}+15a^{\frac{11}{2}}b^3x^6\sqrt{1+\frac{bx^2}{a}}}\right.$$
$$+\frac{35a^4bx^3}{15a^{\frac{17}{2}}\sqrt{1+\frac{bx^2}{a}}+45a^{\frac{15}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}+45a^{\frac{13}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}}+15a^{\frac{11}{2}}b^3x^6\sqrt{1+\frac{bx^2}{a}}}$$
$$+\frac{28a^3b^2x^5}{15a^{\frac{17}{2}}\sqrt{1+\frac{bx^2}{a}}+45a^{\frac{15}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}+45a^{\frac{13}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}}+15a^{\frac{11}{2}}b^3x^6\sqrt{1+\frac{bx^2}{a}}}$$
$$\left.+\frac{8a^2b^3x^7}{15a^{\frac{17}{2}}\sqrt{1+\frac{bx^2}{a}}+45a^{\frac{15}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}+45a^{\frac{13}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}}+15a^{\frac{11}{2}}b^3x^6\sqrt{1+\frac{bx^2}{a}}}\right)$$
$$+B\left(\frac{5ax^3}{15a^{\frac{9}{2}}\sqrt{1+\frac{bx^2}{a}}+30a^{\frac{7}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}+15a^{\frac{5}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}}}\right.$$
$$\left.+\frac{2bx^5}{15a^{\frac{9}{2}}\sqrt{1+\frac{bx^2}{a}}+30a^{\frac{7}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}+15a^{\frac{5}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}}}\right)$$

input
```
integrate((B*x**2+A)/(b*x**2+a)**(7/2),x)
```

output
```
A*(15*a**5*x/(15*a**(17/2)*sqrt(1 + b*x**2/a) + 45*a**(15/2)*b*x**2*sqrt(1
 + b*x**2/a) + 45*a**(13/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 15*a**(11/2)*b*
*3*x**6*sqrt(1 + b*x**2/a)) + 35*a**4*b*x**3/(15*a**(17/2)*sqrt(1 + b*x**2
/a) + 45*a**(15/2)*b*x**2*sqrt(1 + b*x**2/a) + 45*a**(13/2)*b**2*x**4*sqrt
(1 + b*x**2/a) + 15*a**(11/2)*b**3*x**6*sqrt(1 + b*x**2/a)) + 28*a**3*b**2
*x**5/(15*a**(17/2)*sqrt(1 + b*x**2/a) + 45*a**(15/2)*b*x**2*sqrt(1 + b*x*
*2/a) + 45*a**(13/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 15*a**(11/2)*b**3*x**6
*sqrt(1 + b*x**2/a)) + 8*a**2*b**3*x**7/(15*a**(17/2)*sqrt(1 + b*x**2/a) +
 45*a**(15/2)*b*x**2*sqrt(1 + b*x**2/a) + 45*a**(13/2)*b**2*x**4*sqrt(1 +
b*x**2/a) + 15*a**(11/2)*b**3*x**6*sqrt(1 + b*x**2/a))) + B*(5*a*x**3/(15*
a**(9/2)*sqrt(1 + b*x**2/a) + 30*a**(7/2)*b*x**2*sqrt(1 + b*x**2/a) + 15*a
**(5/2)*b**2*x**4*sqrt(1 + b*x**2/a)) + 2*b*x**5/(15*a**(9/2)*sqrt(1 + b*x
**2/a) + 30*a**(7/2)*b*x**2*sqrt(1 + b*x**2/a) + 15*a**(5/2)*b**2*x**4*sqr
t(1 + b*x**2/a)))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.13

$$\int \frac{A+Bx^2}{(a+bx^2)^{7/2}}\,dx = \frac{8\,Ax}{15\,\sqrt{bx^2+a}a^3} + \frac{4\,Ax}{15\,(bx^2+a)^{\frac{3}{2}}a^2} + \frac{Ax}{5\,(bx^2+a)^{\frac{5}{2}}a} - \frac{Bx}{5\,(bx^2+a)^{\frac{5}{2}}b} + \frac{2\,Bx}{15\,\sqrt{bx^2+a}a^2b} + \frac{Bx}{15\,(bx^2+a)^{\frac{3}{2}}ab}$$

input
```
integrate((B*x^2+A)/(b*x^2+a)^(7/2),x, algorithm="maxima")
```

output
```
8/15*A*x/(sqrt(b*x^2 + a)*a^3) + 4/15*A*x/((b*x^2 + a)^(3/2)*a^2) + 1/5*A*
x/((b*x^2 + a)^(5/2)*a) - 1/5*B*x/((b*x^2 + a)^(5/2)*b) + 2/15*B*x/(sqrt(b
*x^2 + a)*a^2*b) + 1/15*B*x/((b*x^2 + a)^(3/2)*a*b)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.79

$$\int \frac{A+Bx^2}{(a+bx^2)^{7/2}}\,dx = \frac{\left(x^2\left(\frac{2\,(Bab^3+4\,Ab^4)x^2}{a^3b^2} + \frac{5\,(Ba^2b^2+4\,Aab^3)}{a^3b^2}\right) + \frac{15\,A}{a}\right)x}{15\,(bx^2+a)^{\frac{5}{2}}}$$

input
```
integrate((B*x^2+A)/(b*x^2+a)^(7/2),x, algorithm="giac")
```

output
```
1/15*(x^2*(2*(B*a*b^3 + 4*A*b^4)*x^2/(a^3*b^2) + 5*(B*a^2*b^2 + 4*A*a*b^3)
/(a^3*b^2)) + 15*A/a)*x/(b*x^2 + a)^(5/2)
```

Mupad [B] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.96

$$\int \frac{A+Bx^2}{(a+bx^2)^{7/2}}\,dx = \frac{8\,A\,b\,x\,(b\,x^2+a)^2-3\,B\,a^3\,x+2\,B\,a\,x\,(b\,x^2+a)^2+B\,a^2\,x\,(b\,x^2+a)+3\,A\,a^2\,b\,x+4}{15\,a^3\,b\,(b\,x^2+a)^{5/2}}$$

input
```
int((A + B*x^2)/(a + b*x^2)^(7/2),x)
```

output
```
(8*A*b*x*(a + b*x^2)^2 - 3*B*a^3*x + 2*B*a*x*(a + b*x^2)^2 + B*a^2*x*(a +
b*x^2) + 3*A*a^2*b*x + 4*A*a*b*x*(a + b*x^2))/(15*a^3*b*(a + b*x^2)^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.92

$$\int \frac{A+Bx^2}{(a+bx^2)^{7/2}}\,dx = \frac{3\sqrt{b\,x^2+a}\,abx+2\sqrt{b\,x^2+a}\,b^2x^3-2\sqrt{b}\,a^2-4\sqrt{b}\,ab\,x^2-2\sqrt{b}\,b^2x^4}{3a^2b\,(b^2x^4+2ab\,x^2+a^2)}$$

input
```
int((B*x^2+A)/(b*x^2+a)^(7/2),x)
```

output
```
(3*sqrt(a + b*x**2)*a*b*x + 2*sqrt(a + b*x**2)*b**2*x**3 - 2*sqrt(b)*a**2
- 4*sqrt(b)*a*b*x**2 - 2*sqrt(b)*b**2*x**4)/(3*a**2*b*(a**2 + 2*a*b*x**2 +
 b**2*x**4))
```

3.95 $\int \frac{A+Bx^2}{(a+bx^2)^{9/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 121

$$\int \frac{A+Bx^2}{(a+bx^2)^{9/2}}\,dx = \frac{(Ab-aB)x}{7ab\,(a+bx^2)^{7/2}} + \frac{(6Ab+aB)x}{35a^2b\,(a+bx^2)^{5/2}} + \frac{4(6Ab+aB)x}{105a^3b\,(a+bx^2)^{3/2}} + \frac{8(6Ab+aB)x}{105a^4b\sqrt{a+bx^2}}$$

output
```
1/7*(A*b-B*a)*x/a/b/(b*x^2+a)^(7/2)+1/35*(6*A*b+B*a)*x/a^2/b/(b*x^2+a)^(5/
2)+4/105*(6*A*b+B*a)*x/a^3/b/(b*x^2+a)^(3/2)+8/105*(6*A*b+B*a)*x/a^4/b/(b*
x^2+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.66

$$\int \frac{A+Bx^2}{(a+bx^2)^{9/2}}\,dx = \frac{48Ab^3x^7 + 8ab^2x^5(21A+Bx^2) + 14a^2bx^3(15A+2Bx^2) + 35a^3(3Ax+Bx^3)}{105a^4\,(a+bx^2)^{7/2}}$$

input
```
Integrate[(A + B*x^2)/(a + b*x^2)^(9/2),x]
```

output
```
(48*A*b^3*x^7 + 8*a*b^2*x^5*(21*A + B*x^2) + 14*a^2*b*x^3*(15*A + 2*B*x^2)
 + 35*a^3*(3*A*x + B*x^3))/(105*a^4*(a + b*x^2)^(7/2))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {298, 209, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{(a+bx^2)^{9/2}}\,dx$$

$$\Big\downarrow \text{298}$$

$$\frac{(aB+6Ab)\int \frac{1}{(bx^2+a)^{7/2}}dx}{7ab}+\frac{x(Ab-aB)}{7ab\,(a+bx^2)^{7/2}}$$

$$\Big\downarrow \text{209}$$

$$\frac{(aB+6Ab)\left(\frac{4\int \frac{1}{(bx^2+a)^{5/2}}dx}{5a}+\frac{x}{5a(a+bx^2)^{5/2}}\right)}{7ab}+\frac{x(Ab-aB)}{7ab\,(a+bx^2)^{7/2}}$$

$$\Big\downarrow \text{209}$$

$$\frac{(aB+6Ab)\left(\frac{4\left(\frac{2\int \frac{1}{(bx^2+a)^{3/2}}dx}{3a}+\frac{x}{3a(a+bx^2)^{3/2}}\right)}{5a}+\frac{x}{5a(a+bx^2)^{5/2}}\right)}{7ab}+\frac{x(Ab-aB)}{7ab\,(a+bx^2)^{7/2}}$$

$$\Big\downarrow \text{208}$$

$$\frac{\left(\frac{4\left(\frac{2x}{3a^2\sqrt{a+bx^2}}+\frac{x}{3a\left(a+bx^2\right)^{3/2}}\right)}{5a}+\frac{x}{5a(a+bx^2)^{5/2}}\right)(aB+6Ab)}{7ab}+\frac{x(Ab-aB)}{7ab\left(a+bx^2\right)^{7/2}}$$

input
```
Int[(A + B*x^2)/(a + b*x^2)^(9/2),x]
```

output
```
((A*b - a*B)*x)/(7*a*b*(a + b*x^2)^(7/2)) + ((6*A*b + a*B)*(x/(5*a*(a + b*
x^2)^(5/2)) + (4*(x/(3*a*(a + b*x^2)^(3/2)) + (2*x)/(3*a^2*Sqrt[a + b*x^2]
)))/(5*a)))/(7*a*b)
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.59

method	result
pseudoelliptic	$\frac{\left(\left(\frac{x^2 B}{3}+A\right)a^3+2\left(\frac{2x^2 B}{15}+A\right)b\,x^2 a^2+\frac{8b^2\left(\frac{x^2 B}{21}+A\right)x^4 a}{5}+\frac{16A\,b^3 x^6}{35}\right)x}{\left(b\,x^2+a\right)^{\frac{7}{2}} a^4}$
gosper	$\frac{x\left(48A\,b^3x^6+8Ba\,b^2x^6+168aA\,b^2x^4+28B\,a^2b\,x^4+210a^2Ab\,x^2+35B\,a^3x^2+105a^3A\right)}{105\left(b\,x^2+a\right)^{\frac{7}{2}} a^4}$
trager	$\frac{x\left(48A\,b^3x^6+8Ba\,b^2x^6+168aA\,b^2x^4+28B\,a^2b\,x^4+210a^2Ab\,x^2+35B\,a^3x^2+105a^3A\right)}{105\left(b\,x^2+a\right)^{\frac{7}{2}} a^4}$
orering	$\frac{x\left(48A\,b^3x^6+8Ba\,b^2x^6+168aA\,b^2x^4+28B\,a^2b\,x^4+210a^2Ab\,x^2+35B\,a^3x^2+105a^3A\right)}{105\left(b\,x^2+a\right)^{\frac{7}{2}} a^4}$
default	$A\left(\frac{x}{7a\left(b\,x^2+a\right)^{\frac{7}{2}}}+\frac{\frac{6x}{35a\left(b\,x^2+a\right)^{\frac{5}{2}}}+\frac{6\left(\frac{4x}{15a\left(b\,x^2+a\right)^{\frac{3}{2}}}+\frac{8x}{15a^2\sqrt{b\,x^2+a}}\right)}{7a}}{a}\right)+B\left(-\frac{x}{6b\left(b\,x^2+a\right)^{\frac{7}{2}}}+\frac{a\left(\frac{x}{7a\left(b\,x^2+a\right)^{\frac{7}{2}}}\right.}{}\right.$

input
```
int((B*x^2+A)/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
((1/3*x^2*B+A)*a^3+2*(2/15*x^2*B+A)*b*x^2*a^2+8/5*b^2*(1/21*x^2*B+A)*x^4*a
+16/35*A*b^3*x^6)/(b*x^2+a)^(7/2)*x/a^4
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx^2}{(a+bx^2)^{9/2}}\,dx = \frac{(8\,(Bab^2+6\,Ab^3)x^7+28\,(Ba^2b+6\,Aab^2)x^5+105\,Aa^3x+35\,(Ba^3+6\,Aa^2b)x^3)\sqrt{bx}}{105\,(a^4b^4x^8+4\,a^5b^3x^6+6\,a^6b^2x^4+4\,a^7bx^2+a^8)}$$

input
```
integrate((B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

output
```
1/105*(8*(B*a*b^2 + 6*A*b^3)*x^7 + 28*(B*a^2*b + 6*A*a*b^2)*x^5 + 105*A*a^
3*x + 35*(B*a^3 + 6*A*a^2*b)*x^3)*sqrt(b*x^2 + a)/(a^4*b^4*x^8 + 4*a^5*b^3
*x^6 + 6*a^6*b^2*x^4 + 4*a^7*b*x^2 + a^8)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1787 vs. 2(114) = 228.

Time = 24.52 (sec) , antiderivative size = 1787, normalized size of antiderivative = 14.77

$$\int \frac{A+Bx^2}{(a+bx^2)^{9/2}}\,dx = \text{Too large to display}$$

input
```
integrate((B*x**2+A)/(b*x**2+a)**(9/2),x)
```

output
```
A*(35*a**14*x/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt
(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2
)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a
) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*
sqrt(1 + b*x**2/a)) + 175*a**13*b*x**3/(35*a**(37/2)*sqrt(1 + b*x**2/a) +
210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 +
 b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b*
*4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) +
 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a)) + 371*a**12*b**2*x**5/(35*a**
(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*
a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 +
b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**
5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a)) +
 429*a**11*b**3*x**7/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x*
*2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a
**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b
*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6
*x**12*sqrt(1 + b*x**2/a)) + 286*a**10*b**4*x**9/(35*a**(37/2)*sqrt(1 + b*
x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**
4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.14

$$\int \frac{A+Bx^2}{(a+bx^2)^{9/2}}\,dx = \frac{16\,Ax}{35\,\sqrt{bx^2+a}a^4} + \frac{8\,Ax}{35\,(bx^2+a)^{\frac{3}{2}}a^3} + \frac{6\,Ax}{35\,(bx^2+a)^{\frac{5}{2}}a^2} + \frac{Ax}{7\,(bx^2+a)^{\frac{7}{2}}a} - \frac{Bx}{7\,(bx^2+a)^{\frac{7}{2}}b} + \frac{8\,Bx}{105\,\sqrt{bx^2+a}a^3b} + \frac{4\,Bx}{105\,(bx^2+a)^{\frac{3}{2}}a^2b} + \frac{Bx}{35\,(bx^2+a)^{\frac{5}{2}}ab}$$

input
```
integrate((B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

output
```
16/35*A*x/(sqrt(b*x^2 + a)*a^4) + 8/35*A*x/((b*x^2 + a)^(3/2)*a^3) + 6/35*
A*x/((b*x^2 + a)^(5/2)*a^2) + 1/7*A*x/((b*x^2 + a)^(7/2)*a) - 1/7*B*x/((b*
x^2 + a)^(7/2)*b) + 8/105*B*x/(sqrt(b*x^2 + a)*a^3*b) + 4/105*B*x/((b*x^2
+ a)^(3/2)*a^2*b) + 1/35*B*x/((b*x^2 + a)^(5/2)*a*b)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.86

$$\int \frac{A+Bx^2}{(a+bx^2)^{9/2}}\,dx = \frac{\left(\left(4\,x^2\left(\frac{2\,(Bab^5+6\,Ab^6)x^2}{a^4b^3}+\frac{7\,(Ba^2b^4+6\,Aab^5)}{a^4b^3}\right)+\frac{35\,(Ba^3b^3+6\,Aa^2b^4)}{a^4b^3}\right)x^2+\frac{105\,A}{a}\right)x}{105\,(bx^2+a)^{\frac{7}{2}}}$$

input
```
integrate((B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="giac")
```

output
```
1/105*((4*x^2*(2*(B*a*b^5 + 6*A*b^6)*x^2/(a^4*b^3) + 7*(B*a^2*b^4 + 6*A*a*
b^5)/(a^4*b^3)) + 35*(B*a^3*b^3 + 6*A*a^2*b^4)/(a^4*b^3))*x^2 + 105*A/a)*x
/(b*x^2 + a)^(7/2)
```

Mupad [B] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.87

$$\int \frac{A+Bx^2}{(a+bx^2)^{9/2}}\,dx = \frac{x\left(\frac{A}{7a}-\frac{B}{7b}\right)}{(b\,x^2+a)^{7/2}}+\frac{x\,(6\,A\,b+B\,a)}{35\,a^2\,b\,(b\,x^2+a)^{5/2}} + \frac{x\,(24\,A\,b+4\,B\,a)}{105\,a^3\,b\,(b\,x^2+a)^{3/2}}+\frac{x\,(48\,A\,b+8\,B\,a)}{105\,a^4\,b\,\sqrt{b\,x^2+a}}$$

input
```
int((A + B*x^2)/(a + b*x^2)^(9/2),x)
```

output
```
(x*(A/(7*a) - B/(7*b)))/(a + b*x^2)^(7/2) + (x*(6*A*b + B*a))/(35*a^2*b*(a
 + b*x^2)^(5/2)) + (x*(24*A*b + 4*B*a))/(105*a^3*b*(a + b*x^2)^(3/2)) + (x
*(48*A*b + 8*B*a))/(105*a^4*b*(a + b*x^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.05

$$\int \frac{A+Bx^2}{(a+bx^2)^{9/2}}\,dx = \frac{15\sqrt{b\,x^2+a}\,a^2bx+20\sqrt{b\,x^2+a}\,a\,b^2x^3+8\sqrt{b\,x^2+a}\,b^3x^5-8\sqrt{b}\,a^3-24\sqrt{b}\,a^2b\,x^2-2}{15a^3b\left(b^3x^6+3a\,b^2x^4+3a^2b\,x^2+a^3\right)}$$

input
```
int((B*x^2+A)/(b*x^2+a)^(9/2),x)
```

output
```
(15*sqrt(a + b*x**2)*a**2*b*x + 20*sqrt(a + b*x**2)*a*b**2*x**3 + 8*sqrt(a
 + b*x**2)*b**3*x**5 - 8*sqrt(b)*a**3 - 24*sqrt(b)*a**2*b*x**2 - 24*sqrt(b
)*a*b**2*x**4 - 8*sqrt(b)*b**3*x**6)/(15*a**3*b*(a**3 + 3*a**2*b*x**2 + 3*
a*b**2*x**4 + b**3*x**6))
```

3.96 $\int (a+bx^2)^p (a+b(3+2p)x^2) \, dx$

Optimal result

Integrand size = 22, antiderivative size = 13

$$\int (a+bx^2)^p (a+b(3+2p)x^2) \, dx = x(a+bx^2)^{1+p}$$

output
```
x*(b*x^2+a)^(p+1)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int (a+bx^2)^p (a+b(3+2p)x^2) \, dx = x(a+bx^2)^{1+p}$$

input
```
Integrate[(a + b*x^2)^p*(a + b*(3 + 2*p)*x^2),x]
```

output
```
x*(a + b*x^2)^(1 + p)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {297}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^2)^p \left(a+b(2p+3)x^2\right) dx$$

$$\Big\downarrow \; 297$$

$$x\left(a+bx^2\right)^{p+1}$$

input
```
Int[(a + b*x^2)^p*(a + b*(3 + 2*p)*x^2),x]
```

output
```
x*(a + b*x^2)^(1 + p)
```

Defintions of rubi rules used

rule 297
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[c*
x*((a + b*x^2)^(p + 1)/a), x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d
, 0] && EqQ[a*d - b*c*(2*p + 3), 0]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

method	result	size
gosper	$x(b\,x^{2}+a)^{p+1}$	14
risch	$(b\,x^{2}+a)^{p}\,x(b\,x^{2}+a)$	19
norman	$ax\,{\mathrm e}^{p\ln(b\,x^{2}+a)}+b\,x^{3}{\mathrm e}^{p\ln(b\,x^{2}+a)}$	32
parallelrisch	$\frac{ab(b\,x^{2}+a)^{p}x^{3}+a^{2}(b\,x^{2}+a)^{p}x}{a}$	35
orering	$\frac{(b\,x^{2}+a)x(b\,x^{2}+a)^{p}(a+b(3+2p)x^{2})}{2bp\,x^{2}+3b\,x^{2}+a}$	48

input
```
int((b*x^2+a)^p*(a+b*(3+2*p)*x^2),x,method=_RETURNVERBOSE)
```

output
```
x*(b*x^2+a)^(p+1)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.46

$$\int (a+bx^2)^p\,(a+b(3+2p)x^2)\,dx=(bx^3+ax)(bx^2+a)^p$$

input
```
integrate((b*x^2+a)^p*(a+b*(3+2*p)*x^2),x, algorithm="fricas")
```

output
```
(b*x^3 + a*x)*(b*x^2 + a)^p
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 24 vs. $2(10) = 20$.

Time = 1.52 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.85

$$\int (a+bx^2)^p\,(a+b(3+2p)x^2)\,dx = ax(a+bx^2)^p + bx^3(a+bx^2)^p$$

input
```
integrate((b*x**2+a)**p*(a+b*(3+2*p)*x**2),x)
```

output
```
a*x*(a + b*x**2)**p + b*x**3*(a + b*x**2)**p
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.46

$$\int (a+bx^2)^p\,(a+b(3+2p)x^2)\,dx = (bx^3+ax)\,(bx^2+a)^p$$

input
```
integrate((b*x^2+a)^p*(a+b*(3+2*p)*x^2),x, algorithm="maxima")
```

output
```
(b*x^3 + a*x)*(b*x^2 + a)^p
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(13) = 26$.

Time = 0.13 (sec) , antiderivative size = 27, normalized size of antiderivative = 2.08

$$\int (a+bx^2)^p\,(a+b(3+2p)x^2)\,dx = (bx^2+a)^p bx^3 + (bx^2+a)^p ax$$

input
```
integrate((b*x^2+a)^p*(a+b*(3+2*p)*x^2),x, algorithm="giac")
```

output
```
(b*x^2 + a)^p*b*x^3 + (b*x^2 + a)^p*a*x
```

Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int (a+bx^2)^p (a+b(3+2p)x^2) \, dx = x (b\,x^2+a)^{p+1}$$

input
```
int((a + b*x^2)^p*(a + b*x^2*(2*p + 3)),x)
```

output
```
x*(a + b*x^2)^(p + 1)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.38

$$\int (a+bx^2)^p (a+b(3+2p)x^2) \, dx = (b\,x^2+a)^p\,x(b\,x^2+a)$$

input
```
int((b*x^2+a)^p*(a+b*(3+2*p)*x^2),x)
```

output
```
(a + b*x**2)**p*x*(a + b*x**2)
```

3.97 $\int (a+bx^2)^3 (a+9bx^2)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 11

$$\int (a+bx^2)^3 (a+9bx^2)\,dx = x(a+bx^2)^4$$

output
```
x*(b*x^2+a)^4
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 42 vs. 2(11) = 22.

Time = 0.00 (sec) , antiderivative size = 42, normalized size of antiderivative = 3.82

$$\int (a+bx^2)^3 (a+9bx^2)\,dx = a^4x + 4a^3bx^3 + 6a^2b^2x^5 + 4ab^3x^7 + b^4x^9$$

input
```
Integrate[(a + b*x^2)^3*(a + 9*b*x^2),x]
```

output
```
a^4*x + 4*a^3*b*x^3 + 6*a^2*b^2*x^5 + 4*a*b^3*x^7 + b^4*x^9
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 42 vs. 2(11) = 22.

Time = 0.19 (sec) , antiderivative size = 42, normalized size of antiderivative = 3.82, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {290, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^3 \left(a+9bx^2\right) \, dx$$

$$\Big\downarrow \text{290}$$

$$\int \left(a^4+12a^3bx^2+30a^2b^2x^4+28ab^3x^6+9b^4x^8\right) dx$$

$$\Big\downarrow \text{2009}$$

$$a^4x+4a^3bx^3+6a^2b^2x^5+4ab^3x^7+b^4x^9$$

input
```
Int[(a + b*x^2)^3*(a + 9*b*x^2),x]
```

output
```
a^4*x + 4*a^3*b*x^3 + 6*a^2*b^2*x^5 + 4*a*b^3*x^7 + b^4*x^9
```

Defintions of rubi rules used

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
orering	$x(b\,x^2+a)^4$	12
gosper	$b^4x^9+4a\,b^3x^7+6a^2b^2x^5+4a^3b\,x^3+a^4x$	43
default	$b^4x^9+4a\,b^3x^7+6a^2b^2x^5+4a^3b\,x^3+a^4x$	43
norman	$b^4x^9+4a\,b^3x^7+6a^2b^2x^5+4a^3b\,x^3+a^4x$	43
risch	$b^4x^9+4a\,b^3x^7+6a^2b^2x^5+4a^3b\,x^3+a^4x$	43
parallelrisch	$b^4x^9+4a\,b^3x^7+6a^2b^2x^5+4a^3b\,x^3+a^4x$	43

input
```
int((b*x^2+a)^3*(9*b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
x*(b*x^2+a)^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. 2(11) = 22.

Time = 0.06 (sec) , antiderivative size = 42, normalized size of antiderivative = 3.82

$$\int \left(a+bx^2\right)^3\left(a+9bx^2\right)\,dx=b^4x^9+4\,ab^3x^7+6\,a^2b^2x^5+4\,a^3bx^3+a^4x$$

input
```
integrate((b*x^2+a)^3*(9*b*x^2+a),x, algorithm="fricas")
```

output
```
b^4*x^9 + 4*a*b^3*x^7 + 6*a^2*b^2*x^5 + 4*a^3*b*x^3 + a^4*x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(8) = 16$.

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 3.82

$$\int \left(a+bx^2\right)^3 \left(a+9bx^2\right)\,dx = a^4x+4a^3bx^3+6a^2b^2x^5+4ab^3x^7+b^4x^9$$

input
```
integrate((b*x**2+a)**3*(9*b*x**2+a),x)
```

output
```
a**4*x + 4*a**3*b*x**3 + 6*a**2*b**2*x**5 + 4*a*b**3*x**7 + b**4*x**9
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(11) = 22$.

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 3.82

$$\int \left(a+bx^2\right)^3 \left(a+9bx^2\right)\,dx = b^4x^9+4\,ab^3x^7+6\,a^2b^2x^5+4\,a^3bx^3+a^4x$$

input
```
integrate((b*x^2+a)^3*(9*b*x^2+a),x, algorithm="maxima")
```

output
```
b^4*x^9 + 4*a*b^3*x^7 + 6*a^2*b^2*x^5 + 4*a^3*b*x^3 + a^4*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(11) = 22$.

Time = 0.11 (sec) , antiderivative size = 42, normalized size of antiderivative = 3.82

$$\int \left(a+bx^2\right)^3 \left(a+9bx^2\right)\,dx = b^4x^9+4\,ab^3x^7+6\,a^2b^2x^5+4\,a^3bx^3+a^4x$$

input
```
integrate((b*x^2+a)^3*(9*b*x^2+a),x, algorithm="giac")
```

output
```
b^4*x^9 + 4*a*b^3*x^7 + 6*a^2*b^2*x^5 + 4*a^3*b*x^3 + a^4*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 3.82

$$\int (a+bx^2)^3 (a+9bx^2) \, dx = a^4 \, x + 4 \, a^3 \, b \, x^3 + 6 \, a^2 \, b^2 \, x^5 + 4 \, a \, b^3 \, x^7 + b^4 \, x^9$$

input
```
int((a + b*x^2)^3*(a + 9*b*x^2),x)
```

output
```
a^4*x + b^4*x^9 + 4*a^3*b*x^3 + 4*a*b^3*x^7 + 6*a^2*b^2*x^5
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 42, normalized size of antiderivative = 3.82

$$\int (a+bx^2)^3 (a+9bx^2) \, dx = x(b^4x^8 + 4a \, b^3x^6 + 6a^2b^2x^4 + 4a^3b \, x^2 + a^4)$$

input
```
int((b*x^2+a)^3*(9*b*x^2+a),x)
```

output
```
x*(a**4 + 4*a**3*b*x**2 + 6*a**2*b**2*x**4 + 4*a*b**3*x**6 + b**4*x**8)
```

3.98 $\int (a+bx^2)^2 (a+7bx^2) \, dx$

Optimal result

Integrand size = 18, antiderivative size = 11

$$\int (a+bx^2)^2 (a+7bx^2) \, dx = x(a+bx^2)^3$$

output
```
x*(b*x^2+a)^3
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 31 vs. 2(11) = 22.

Time = 0.00 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.82

$$\int (a+bx^2)^2 (a+7bx^2) \, dx = a^3x + 3a^2bx^3 + 3ab^2x^5 + b^3x^7$$

input
```
Integrate[(a + b*x^2)^2*(a + 7*b*x^2),x]
```

output
```
a^3*x + 3*a^2*b*x^3 + 3*a*b^2*x^5 + b^3*x^7
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 31 vs. 2(11) = 22.

Time = 0.18 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.82, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {290, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^2 \left(a+7bx^2\right) \, dx$$

$$\Big\downarrow \text{290}$$

$$\int \left(a^3+9a^2bx^2+15ab^2x^4+7b^3x^6\right)dx$$

$$\Big\downarrow \text{2009}$$

$$a^3x+3a^2bx^3+3ab^2x^5+b^3x^7$$

input
```
Int[(a + b*x^2)^2*(a + 7*b*x^2),x]
```

output
```
a^3*x + 3*a^2*b*x^3 + 3*a*b^2*x^5 + b^3*x^7
```

Defintions of rubi rules used

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
orering	$x(b\,x^2+a)^3$	12
gosper	$b^3x^7+3a\,b^2x^5+3a^2b\,x^3+a^3x$	32
default	$b^3x^7+3a\,b^2x^5+3a^2b\,x^3+a^3x$	32
norman	$b^3x^7+3a\,b^2x^5+3a^2b\,x^3+a^3x$	32
risch	$b^3x^7+3a\,b^2x^5+3a^2b\,x^3+a^3x$	32
parallelrisch	$b^3x^7+3a\,b^2x^5+3a^2b\,x^3+a^3x$	32

input
```
int((b*x^2+a)^2*(7*b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
x*(b*x^2+a)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(11) = 22.

Time = 0.06 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.82

$$\int (a+bx^2)^2\,(a+7bx^2)\;dx = b^3x^7+3\,ab^2x^5+3\,a^2bx^3+a^3x$$

input
```
integrate((b*x^2+a)^2*(7*b*x^2+a),x, algorithm="fricas")
```

output
```
b^3*x^7 + 3*a*b^2*x^5 + 3*a^2*b*x^3 + a^3*x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(8) = 16$.

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.82

$$\int (a+bx^2)^2 (a+7bx^2)\, dx = a^3x + 3a^2bx^3 + 3ab^2x^5 + b^3x^7$$

input
```
integrate((b*x**2+a)**2*(7*b*x**2+a),x)
```

output
```
a**3*x + 3*a**2*b*x**3 + 3*a*b**2*x**5 + b**3*x**7
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(11) = 22$.

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.82

$$\int (a+bx^2)^2 (a+7bx^2)\, dx = b^3x^7 + 3\,ab^2x^5 + 3\,a^2bx^3 + a^3x$$

input
```
integrate((b*x^2+a)^2*(7*b*x^2+a),x, algorithm="maxima")
```

output
```
b^3*x^7 + 3*a*b^2*x^5 + 3*a^2*b*x^3 + a^3*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(11) = 22$.

Time = 0.12 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.82

$$\int (a+bx^2)^2 (a+7bx^2)\, dx = b^3x^7 + 3\,ab^2x^5 + 3\,a^2bx^3 + a^3x$$

input
```
integrate((b*x^2+a)^2*(7*b*x^2+a),x, algorithm="giac")
```

output
```
b^3*x^7 + 3*a*b^2*x^5 + 3*a^2*b*x^3 + a^3*x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.82

$$\int (a+bx^2)^2 (a+7bx^2)\ dx = a^3\,x + 3\,a^2\,b\,x^3 + 3\,a\,b^2\,x^5 + b^3\,x^7$$

input
```
int((a + b*x^2)^2*(a + 7*b*x^2),x)
```

output
```
a^3*x + b^3*x^7 + 3*a^2*b*x^3 + 3*a*b^2*x^5
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.82

$$\int (a+bx^2)^2 (a+7bx^2)\ dx = x\left(b^3x^6 + 3a\,b^2x^4 + 3a^2b\,x^2 + a^3\right)$$

input
```
int((b*x^2+a)^2*(7*b*x^2+a),x)
```

output
```
x*(a**3 + 3*a**2*b*x**2 + 3*a*b**2*x**4 + b**3*x**6)
```

3.99 $\int (a+bx^2)(a+5bx^2)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 11

$$\int (a+bx^2)(a+5bx^2)\,dx = x(a+bx^2)^2$$

output
```
x*(b*x^2+a)^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.82

$$\int (a+bx^2)(a+5bx^2)\,dx = a^2x+2abx^3+b^2x^5$$

input
```
Integrate[(a + b*x^2)*(a + 5*b*x^2),x]
```

output
```
a^2*x + 2*a*b*x^3 + b^2*x^5
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.82, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {290, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^2)(a+5bx^2)\,dx$$

$$\Big\downarrow \text{290}$$

$$\int (a^2+6abx^2+5b^2x^4)\,dx$$

$$\Big\downarrow \text{2009}$$

$$a^2x+2abx^3+b^2x^5$$

input
```
Int[(a + b*x^2)*(a + 5*b*x^2),x]
```

output
```
a^2*x + 2*a*b*x^3 + b^2*x^5
```

Defintions of rubi rules used

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
orering	$x(b\,x^2+a)^2$	12
gosper	$b^2x^5+2ab\,x^3+a^2x$	21
default	$b^2x^5+2ab\,x^3+a^2x$	21
norman	$b^2x^5+2ab\,x^3+a^2x$	21
risch	$b^2x^5+2ab\,x^3+a^2x$	21
parallelrisch	$b^2x^5+2ab\,x^3+a^2x$	21

input
```
int((b*x^2+a)*(5*b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
x*(b*x^2+a)^2
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.82

$$\int (a+bx^2)\,(a+5bx^2)\;dx=b^2x^5+2\,abx^3+a^2x$$

input
```
integrate((b*x^2+a)*(5*b*x^2+a),x, algorithm="fricas")
```

output
```
b^2*x^5 + 2*a*b*x^3 + a^2*x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(8) = 16.

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.73

$$\int (a+bx^2) (a+5bx^2) \, dx = a^2x+2abx^3+b^2x^5$$

input
```
integrate((b*x**2+a)*(5*b*x**2+a),x)
```

output
```
a**2*x + 2*a*b*x**3 + b**2*x**5
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.82

$$\int (a+bx^2) (a+5bx^2) \, dx = b^2x^5+2\,abx^3+a^2x$$

input
```
integrate((b*x^2+a)*(5*b*x^2+a),x, algorithm="maxima")
```

output
```
b^2*x^5 + 2*a*b*x^3 + a^2*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.82

$$\int (a+bx^2) (a+5bx^2) \, dx = b^2x^5+2\,abx^3+a^2x$$

input
```
integrate((b*x^2+a)*(5*b*x^2+a),x, algorithm="giac")
```

output
```
b^2*x^5 + 2*a*b*x^3 + a^2*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \left(a+bx^2\right)\left(a+5bx^2\right)\,dx=x\,{\left(b\,x^2+a\right)}^2$$

input
```
int((a + b*x^2)*(a + 5*b*x^2),x)
```

output
```
x*(a + b*x^2)^2
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.82

$$\int \left(a+bx^2\right)\left(a+5bx^2\right)\,dx=x\left(b^2x^4+2ab\,x^2+a^2\right)$$

input
```
int((b*x^2+a)*(5*b*x^2+a),x)
```

output
```
x*(a**2 + 2*a*b*x**2 + b**2*x**4)
```

3.100 $\int \frac{a-bx^2}{(a+bx^2)^2} dx$

Optimal result

Integrand size = 18, antiderivative size = 11

$$\int \frac{a-bx^2}{(a+bx^2)^2} dx = \frac{x}{a+bx^2}$$

output
```
x/(b*x^2+a)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{a-bx^2}{(a+bx^2)^2} dx = \frac{x}{a+bx^2}$$

input
```
Integrate[(a - b*x^2)/(a + b*x^2)^2,x]
```

output
```
x/(a + b*x^2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {297}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a-bx^2}{(a+bx^2)^2} \, dx$$
$$\Big\downarrow \ 297$$
$$\frac{x}{a+bx^2}$$

input
```
Int[(a - b*x^2)/(a + b*x^2)^2,x]
```

output
```
x/(a + b*x^2)
```

Defintions of rubi rules used

rule 297
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[c*
x*((a + b*x^2)^(p + 1)/a), x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d
, 0] && EqQ[a*d - b*c*(2*p + 3), 0]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
gosper	$\frac{x}{b\,x^{2}+a}$	12
default	$\frac{x}{b\,x^{2}+a}$	12
norman	$\frac{x}{b\,x^{2}+a}$	12
risch	$\frac{x}{b\,x^{2}+a}$	12
parallelrisch	$\frac{x}{b\,x^{2}+a}$	12
orering	$\frac{x}{b\,x^{2}+a}$	12

input
```
int((-b*x^2+a)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
x/(b*x^2+a)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{a-bx^2}{(a+bx^2)^2}\,dx=\frac{x}{bx^2+a}$$

input
```
integrate((-b*x^2+a)/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
x/(b*x^2 + a)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \frac{a-bx^2}{(a+bx^2)^2} \, dx = \frac{x}{a+bx^2}$$

input
```
integrate((-b*x**2+a)/(b*x**2+a)**2,x)
```

output
```
x/(a + b*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{a-bx^2}{(a+bx^2)^2} \, dx = \frac{x}{bx^2+a}$$

input
```
integrate((-b*x^2+a)/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
x/(b*x^2 + a)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{a-bx^2}{(a+bx^2)^2} \, dx = \frac{x}{bx^2+a}$$

input
```
integrate((-b*x^2+a)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
x/(b*x^2 + a)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{a-bx^2}{(a+bx^2)^2}\,dx = \frac{x}{b\,x^2+a}$$

input
```
int((a - b*x^2)/(a + b*x^2)^2,x)
```

output
```
x/(a + b*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{a-bx^2}{(a+bx^2)^2}\,dx = \frac{x}{b\,x^2+a}$$

input
```
int((-b*x^2+a)/(b*x^2+a)^2,x)
```

output
```
x/(a + b*x**2)
```

3.101 $\int \frac{a-3bx^2}{(a+bx^2)^3} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 11

$$\int \frac{a-3bx^2}{(a+bx^2)^3} \, dx = \frac{x}{(a+bx^2)^2}$$

output
```
x/(b*x^2+a)^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{a-3bx^2}{(a+bx^2)^3} \, dx = \frac{x}{(a+bx^2)^2}$$

input
```
Integrate[(a - 3*b*x^2)/(a + b*x^2)^3,x]
```

output
```
x/(a + b*x^2)^2
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {297}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a-3bx^2}{(a+bx^2)^3}\,dx$$
$$\Big\downarrow \; 297$$
$$\frac{x}{(a+bx^2)^2}$$

input
```
Int[(a - 3*b*x^2)/(a + b*x^2)^3,x]
```

output
```
x/(a + b*x^2)^2
```

Defintions of rubi rules used

rule 297
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[c*
x*((a + b*x^2)^(p + 1)/a), x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d
, 0] && EqQ[a*d - b*c*(2*p + 3), 0]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
gosper	$\frac{x}{(b\,x^2+a)^2}$	12
default	$\frac{x}{(b\,x^2+a)^2}$	12
norman	$\frac{x}{(b\,x^2+a)^2}$	12
risch	$\frac{x}{(b\,x^2+a)^2}$	12
parallelrisch	$\frac{x}{(b\,x^2+a)^2}$	12
orering	$\frac{x}{(b\,x^2+a)^2}$	12

input
```
int((-3*b*x^2+a)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
x/(b*x^2+a)^2
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 2.00

$$\int \frac{a-3bx^2}{(a+bx^2)^3}\,dx = \frac{x}{b^2x^4+2\,abx^2+a^2}$$

input
```
integrate((-3*b*x^2+a)/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
x/(b^2*x^4 + 2*a*b*x^2 + a^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(8) = 16.

Time = 0.13 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.73

$$\int \frac{a-3bx^2}{(a+bx^2)^3}\,dx = \frac{x}{a^2+2abx^2+b^2x^4}$$

input
```
integrate((-3*b*x**2+a)/(b*x**2+a)**3,x)
```

output
```
x/(a**2 + 2*a*b*x**2 + b**2*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 2.00

$$\int \frac{a-3bx^2}{(a+bx^2)^3}\,dx = \frac{x}{b^2x^4+2\,abx^2+a^2}$$

input
```
integrate((-3*b*x^2+a)/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
x/(b^2*x^4 + 2*a*b*x^2 + a^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{a-3bx^2}{(a+bx^2)^3}\,dx = \frac{x}{(bx^2+a)^2}$$

input
```
integrate((-3*b*x^2+a)/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
x/(b*x^2 + a)^2
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{a-3bx^2}{(a+bx^2)^3}\,dx = \frac{x}{(b\,x^2+a)^2}$$

input
```
int((a - 3*b*x^2)/(a + b*x^2)^3,x)
```

output
```
x/(a + b*x^2)^2
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 2.00

$$\int \frac{a-3bx^2}{(a+bx^2)^3}\,dx = \frac{x}{b^2x^4+2ab\,x^2+a^2}$$

input
```
int((-3*b*x^2+a)/(b*x^2+a)^3,x)
```

output
```
x/(a**2 + 2*a*b*x**2 + b**2*x**4)
```

3.102 $\int (a+bx^2)^{5/2} (a+8bx^2) \, dx$

Optimal result

Integrand size = 20, antiderivative size = 13

$$\int (a+bx^2)^{5/2} (a+8bx^2) \, dx = x(a+bx^2)^{7/2}$$

output
```
x*(b*x^2+a)^(7/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int (a+bx^2)^{5/2} (a+8bx^2) \, dx = x(a+bx^2)^{7/2}$$

input
```
Integrate[(a + b*x^2)^(5/2)*(a + 8*b*x^2),x]
```

output
```
x*(a + b*x^2)^(7/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {297}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^2)^{5/2} (a+8bx^2)\, dx$$

$$\Big\downarrow \; 297$$

$$x(a+bx^2)^{7/2}$$

input
```
Int[(a + b*x^2)^(5/2)*(a + 8*b*x^2),x]
```

output
```
x*(a + b*x^2)^(7/2)
```

Defintions of rubi rules used

rule 297
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[c*
x*((a + b*x^2)^(p + 1)/a), x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d
, 0] && EqQ[a*d - b*c*(2*p + 3), 0]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result
gosper	$x(b\,x^2+a)^{\frac{7}{2}}$
pseudoelliptic	$x(b\,x^2+a)^{\frac{7}{2}}$
orering	$x(b\,x^2+a)^{\frac{7}{2}}$
trager	$x(b^3x^6+3a\,b^2x^4+3a^2b\,x^2+a^3)\sqrt{b\,x^2+a}$
risch	$x(b^3x^6+3a\,b^2x^4+3a^2b\,x^2+a^3)\sqrt{b\,x^2+a}$
default	$a\left(\frac{x(b\,x^2+a)^{\frac{5}{2}}}{6}+\frac{5a\left(\frac{x(b\,x^2+a)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)}{4}\right)}{6}\right)+8b\left(\frac{x(b\,x^2+a)^{\frac{7}{2}}}{8b}-\frac{a\left(\frac{x(b\,x^2+}{6}\right.}{}\right.$

input
```
int((b*x^2+a)^(5/2)*(8*b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
x*(b*x^2+a)^(7/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. 2(11) = 22.

Time = 0.08 (sec) , antiderivative size = 41, normalized size of antiderivative = 3.15

$$\int (a+bx^2)^{5/2}\,(a+8bx^2)\;dx=(b^3x^7+3\,ab^2x^5+3\,a^2bx^3+a^3x)\sqrt{bx^2+a}$$

input
```
integrate((b*x^2+a)^(5/2)*(8*b*x^2+a),x, algorithm="fricas")
```

output
```
(b^3*x^7 + 3*a*b^2*x^5 + 3*a^2*b*x^3 + a^3*x)*sqrt(b*x^2 + a)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(10) = 20$.

Time = 0.41 (sec) , antiderivative size = 58, normalized size of antiderivative = 4.46

$$\int \left(a+bx^2\right)^{5/2}\left(a+8bx^2\right)\,dx = \begin{cases} \sqrt{a+bx^2}\,(a^3x+3a^2bx^3+3ab^2x^5+b^3x^7) & \text{for } b\neq 0 \\ a^{\frac{5}{2}}\left(ax+\frac{8bx^3}{3}\right) & \text{otherwise} \end{cases}$$

input
```
integrate((b*x**2+a)**(5/2)*(8*b*x**2+a),x)
```

output
```
Piecewise((sqrt(a + b*x**2)*(a**3*x + 3*a**2*b*x**3 + 3*a*b**2*x**5 + b**3
*x**7), Ne(b, 0)), (a**(5/2)*(a*x + 8*b*x**3/3), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \left(a+bx^2\right)^{5/2}\left(a+8bx^2\right)\,dx = \left(bx^2+a\right)^{\frac{7}{2}}x$$

input
```
integrate((b*x^2+a)^(5/2)*(8*b*x^2+a),x, algorithm="maxima")
```

output
```
(b*x^2 + a)^(7/2)*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(11) = 22$.

Time = 0.13 (sec) , antiderivative size = 44, normalized size of antiderivative = 3.38

$$\int \left(a+bx^2\right)^{5/2} \left(a+8bx^2\right) \, dx = \left(a^3+\left(3\,a^2b+\left(b^3x^2+3\,ab^2\right)x^2\right)x^2\right)\sqrt{bx^2+a}x$$

input
```
integrate((b*x^2+a)^(5/2)*(8*b*x^2+a),x, algorithm="giac")
```

output
```
(a^3 + (3*a^2*b + (b^3*x^2 + 3*a*b^2)*x^2)*x^2)*sqrt(b*x^2 + a)*x
```

Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \left(a+bx^2\right)^{5/2} \left(a+8bx^2\right) \, dx = x\,\left(b\,x^2+a\right)^{7/2}$$

input
```
int((a + b*x^2)^(5/2)*(a + 8*b*x^2),x)
```

output
```
x*(a + b*x^2)^(7/2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 39, normalized size of antiderivative = 3.00

$$\int \left(a+bx^2\right)^{5/2} \left(a+8bx^2\right) \, dx = \sqrt{b\,x^2+a}\,x\left(b^3x^6+3a\,b^2x^4+3a^2b\,x^2+a^3\right)$$

input
```
int((b*x^2+a)^(5/2)*(8*b*x^2+a),x)
```

output
```
sqrt(a + b*x**2)*x*(a**3 + 3*a**2*b*x**2 + 3*a*b**2*x**4 + b**3*x**6)
```

3.103 $\int (a+bx^2)^{3/2} (a+6bx^2)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 13

$$\int (a+bx^2)^{3/2} (a+6bx^2)\,dx = x(a+bx^2)^{5/2}$$

output
```
x*(b*x^2+a)^(5/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int (a+bx^2)^{3/2} (a+6bx^2)\,dx = x(a+bx^2)^{5/2}$$

input
```
Integrate[(a + b*x^2)^(3/2)*(a + 6*b*x^2),x]
```

output
```
x*(a + b*x^2)^(5/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {297}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^2)^{3/2} (a+6bx^2) \, dx$$

$$\Big\downarrow \; 297$$

$$x(a+bx^2)^{5/2}$$

input
```
Int[(a + b*x^2)^(3/2)*(a + 6*b*x^2),x]
```

output
```
x*(a + b*x^2)^(5/2)
```

Defintions of rubi rules used

rule 297
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[c*
x*((a + b*x^2)^(p + 1)/a), x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d
, 0] && EqQ[a*d - b*c*(2*p + 3), 0]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result
gosper	$x(b\,x^{2}+a)^{\frac{5}{2}}$
pseudoelliptic	$x(b\,x^{2}+a)^{\frac{5}{2}}$
orering	$x(b\,x^{2}+a)^{\frac{5}{2}}$
trager	$x(b^{2}x^{4}+2ab\,x^{2}+a^{2})\sqrt{b\,x^{2}+a}$
risch	$x(b^{2}x^{4}+2ab\,x^{2}+a^{2})\sqrt{b\,x^{2}+a}$
default	$a\left(\frac{x(b\,x^{2}+a)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)}{4}\right)+6b\left(\frac{x(b\,x^{2}+a)^{\frac{5}{2}}}{6b}-\frac{a\left(\frac{x(b\,x^{2}+a)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^{2}+a}}{2}\right.}{}\right.}{6b}\right.$

input
```
int((b*x^2+a)^(3/2)*(6*b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
x*(b*x^2+a)^(5/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 30 vs. $2(11) = 22$.

Time = 0.08 (sec) , antiderivative size = 30, normalized size of antiderivative = 2.31

$$\int \left(a+bx^2\right)^{3/2}\left(a+6bx^2\right)\,dx=\left(b^2x^5+2\,abx^3+a^2x\right)\sqrt{bx^2+a}$$

input
```
integrate((b*x^2+a)^(3/2)*(6*b*x^2+a),x, algorithm="fricas")
```

output
```
(b^2*x^5 + 2*a*b*x^3 + a^2*x)*sqrt(b*x^2 + a)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(10) = 20$.

Time = 0.33 (sec) , antiderivative size = 44, normalized size of antiderivative = 3.38

$$\int \left(a+bx^2\right)^{3/2} \left(a+6bx^2\right) \, dx = \begin{cases} \sqrt{a+bx^2}(a^2x+2abx^3+b^2x^5) & \text{for } b \neq 0 \\ a^{\frac{3}{2}}(ax+2bx^3) & \text{otherwise} \end{cases}$$

input
```
integrate((b*x**2+a)**(3/2)*(6*b*x**2+a),x)
```

output
```
Piecewise((sqrt(a + b*x**2)*(a**2*x + 2*a*b*x**3 + b**2*x**5), Ne(b, 0)),
(a**(3/2)*(a*x + 2*b*x**3), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \left(a+bx^2\right)^{3/2} \left(a+6bx^2\right) \, dx = \left(bx^2+a\right)^{\frac{5}{2}}x$$

input
```
integrate((b*x^2+a)^(3/2)*(6*b*x^2+a),x, algorithm="maxima")
```

output
```
(b*x^2 + a)^(5/2)*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(11) = 22$.

Time = 0.13 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.38

$$\int \left(a+bx^2\right)^{3/2} \left(a+6bx^2\right) \, dx = \left(\left(b^2x^2+2\,ab\right)x^2+a^2\right)\sqrt{bx^2+a}x$$

input
```
integrate((b*x^2+a)^(3/2)*(6*b*x^2+a),x, algorithm="giac")
```

output
```
((b^2*x^2 + 2*a*b)*x^2 + a^2)*sqrt(b*x^2 + a)*x
```

Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \left(a+bx^2\right)^{3/2} \left(a+6bx^2\right) \, dx = x \left(b\,x^2+a\right)^{5/2}$$

input
```
int((a + b*x^2)^(3/2)*(a + 6*b*x^2),x)
```

output
```
x*(a + b*x^2)^(5/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 28, normalized size of antiderivative = 2.15

$$\int \left(a+bx^2\right)^{3/2} \left(a+6bx^2\right) \, dx = \sqrt{b\,x^2+a}\,x\left(b^2x^4+2ab\,x^2+a^2\right)$$

input
```
int((b*x^2+a)^(3/2)*(6*b*x^2+a),x)
```

output
```
sqrt(a + b*x**2)*x*(a**2 + 2*a*b*x**2 + b**2*x**4)
```

3.104 $\int \sqrt{a+bx^2}(a+4bx^2)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 13

$$\int \sqrt{a+bx^2}\left(a+4bx^2\right)\,dx = x\left(a+bx^2\right)^{3/2}$$

output
```
x*(b*x^2+a)^(3/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \sqrt{a+bx^2}\left(a+4bx^2\right)\,dx = x\left(a+bx^2\right)^{3/2}$$

input
```
Integrate[Sqrt[a + b*x^2]*(a + 4*b*x^2),x]
```

output
```
x*(a + b*x^2)^(3/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {297}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx^2}\left(a+4bx^2\right)\,dx$$

$$\Big\downarrow \text{297}$$

$$x\left(a+bx^2\right)^{3/2}$$

input
```
Int[Sqrt[a + b*x^2]*(a + 4*b*x^2),x]
```

output
```
x*(a + b*x^2)^(3/2)
```

Defintions of rubi rules used

rule 297
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[c*
x*((a + b*x^2)^(p + 1)/a), x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d
, 0] && EqQ[a*d - b*c*(2*p + 3), 0]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
gosper	$x(b\,x^2+a)^{\frac{3}{2}}$	12
trager	$x(b\,x^2+a)^{\frac{3}{2}}$	12
risch	$x(b\,x^2+a)^{\frac{3}{2}}$	12
pseudoelliptic	$x(b\,x^2+a)^{\frac{3}{2}}$	12
orering	$x(b\,x^2+a)^{\frac{3}{2}}$	12
default	$a\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)+4b\left(\frac{x(b\,x^2+a)^{\frac{3}{2}}}{4b}-\frac{a\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)}{4b}\right)$	99

input
```
int((b*x^2+a)^(1/2)*(4*b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
x*(b*x^2+a)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.46

$$\int \sqrt{a+bx^2}\left(a+4bx^2\right)\,dx=\left(bx^3+ax\right)\sqrt{bx^2+a}$$

input
```
integrate((b*x^2+a)^(1/2)*(4*b*x^2+a),x, algorithm="fricas")
```

output
```
(b*x^3 + a*x)*sqrt(b*x^2 + a)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(10) = 20$.

Time = 0.25 (sec) , antiderivative size = 34, normalized size of antiderivative = 2.62

$$\int \sqrt{a+bx^2}\left(a+4bx^2\right)\,dx = \begin{cases} \sqrt{a+bx^2}\left(ax+bx^3\right) & \text{for } b \neq 0 \\ \sqrt{a}\left(ax+\frac{4bx^3}{3}\right) & \text{otherwise} \end{cases}$$

input
```
integrate((b*x**2+a)**(1/2)*(4*b*x**2+a),x)
```

output
```
Piecewise((sqrt(a + b*x**2)*(a*x + b*x**3), Ne(b, 0)), (sqrt(a)*(a*x + 4*b
*x**3/3), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \sqrt{a+bx^2}\left(a+4bx^2\right)\,dx = \left(bx^2+a\right)^{\frac{3}{2}}x$$

input
```
integrate((b*x^2+a)^(1/2)*(4*b*x^2+a),x, algorithm="maxima")
```

output
```
(b*x^2 + a)^(3/2)*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \sqrt{a+bx^2}\left(a+4bx^2\right)\,dx = \left(bx^2+a\right)^{\frac{3}{2}}x$$

input
```
integrate((b*x^2+a)^(1/2)*(4*b*x^2+a),x, algorithm="giac")
```

output
```
(b*x^2 + a)^(3/2)*x
```

Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \sqrt{a+bx^2}\left(a+4bx^2\right)\,dx=x\,{\left(b\,x^2+a\right)}^{3/2}$$

input
```
int((a + b*x^2)^(1/2)*(a + 4*b*x^2),x)
```

output
```
x*(a + b*x^2)^(3/2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

$$\int \sqrt{a+bx^2}\left(a+4bx^2\right)\,dx=\sqrt{b\,x^2+a}\,x\left(b\,x^2+a\right)$$

input
```
int((b*x^2+a)^(1/2)*(4*b*x^2+a),x)
```

output
```
sqrt(a + b*x**2)*x*(a + b*x**2)
```

3.105 $\int \frac{a+2bx^2}{\sqrt{a+bx^2}} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 13

$$\int \frac{a+2bx^2}{\sqrt{a+bx^2}} \, dx = x\sqrt{a+bx^2}$$

output
```
x*(b*x^2+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{a+2bx^2}{\sqrt{a+bx^2}} \, dx = x\sqrt{a+bx^2}$$

input
```
Integrate[(a + 2*b*x^2)/Sqrt[a + b*x^2],x]
```

output
```
x*Sqrt[a + b*x^2]
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {297}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+2bx^2}{\sqrt{a+bx^2}}\,dx$$

$$\Big\downarrow \; 297$$

$$x\sqrt{a+bx^2}$$

input
```
Int[(a + 2*b*x^2)/Sqrt[a + b*x^2],x]
```

output
```
x*Sqrt[a + b*x^2]
```

Defintions of rubi rules used

rule 297
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[c*
x*((a + b*x^2)^(p + 1)/a), x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d
, 0] && EqQ[a*d - b*c*(2*p + 3), 0]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
gosper	$x\sqrt{b\,x^{2}+a}$	12
trager	$x\sqrt{b\,x^{2}+a}$	12
risch	$x\sqrt{b\,x^{2}+a}$	12
pseudoelliptic	$x\sqrt{b\,x^{2}+a}$	12
orering	$x\sqrt{b\,x^{2}+a}$	12
default	$\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{\sqrt{b}}+2b\left(\frac{x\sqrt{b\,x^{2}+a}}{2b}-\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{2b^{\frac{3}{2}}}\right)$	64

input
```
int((2*b*x^2+a)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
x*(b*x^2+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{a+2bx^2}{\sqrt{a+bx^2}}\,dx=\sqrt{bx^2+a}x$$

input
```
integrate((2*b*x^2+a)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
sqrt(b*x^2 + a)*x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(10) = 20$.

Time = 0.23 (sec) , antiderivative size = 27, normalized size of antiderivative = 2.08

$$\int \frac{a + 2bx^2}{\sqrt{a + bx^2}}\,dx = \begin{cases} x\sqrt{a + bx^2} & \text{for } b \neq 0 \\ \frac{ax + \frac{2bx^3}{3}}{\sqrt{a}} & \text{otherwise} \end{cases}$$

input
```
integrate((2*b*x**2+a)/(b*x**2+a)**(1/2),x)
```

output
```
Piecewise((x*sqrt(a + b*x**2), Ne(b, 0)), ((a*x + 2*b*x**3/3)/sqrt(a), Tru
e))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{a + 2bx^2}{\sqrt{a + bx^2}}\,dx = \sqrt{bx^2 + a}x$$

input
```
integrate((2*b*x^2+a)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
sqrt(b*x^2 + a)*x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{a + 2bx^2}{\sqrt{a + bx^2}}\,dx = \sqrt{bx^2 + a}x$$

input
```
integrate((2*b*x^2+a)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
sqrt(b*x^2 + a)*x
```

Mupad [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{a+2bx^2}{\sqrt{a+bx^2}}\,dx = x\,\sqrt{b\,x^2+a}$$

input
```
int((a + 2*b*x^2)/(a + b*x^2)^(1/2),x)
```

output
```
x*(a + b*x^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{a+2bx^2}{\sqrt{a+bx^2}}\,dx = \sqrt{b\,x^2+a}\,x$$

input
```
int((2*b*x^2+a)/(b*x^2+a)^(1/2),x)
```

output
```
sqrt(a + b*x**2)*x
```

3.106 $\int \frac{a-2bx^2}{\left(a+bx^2\right)^{5/2}} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 13

$$\int \frac{a-2bx^2}{(a+bx^2)^{5/2}} \, dx = \frac{x}{(a+bx^2)^{3/2}}$$

output
```
x/(b*x^2+a)^(3/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{a-2bx^2}{(a+bx^2)^{5/2}} \, dx = \frac{x}{(a+bx^2)^{3/2}}$$

input
```
Integrate[(a - 2*b*x^2)/(a + b*x^2)^(5/2),x]
```

output
```
x/(a + b*x^2)^(3/2)
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 47 vs. $2(13) = 26$.

Time = 0.15 (sec) , antiderivative size = 47, normalized size of antiderivative = 3.62, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = $\{292, 208\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a-2bx^2}{(a+bx^2)^{5/2}}dx$$

$$\Big\downarrow \text{292}$$

$$\frac{2}{3}\int \frac{1}{(bx^2+a)^{3/2}}dx+\frac{x(a-2bx^2)}{3a(a+bx^2)^{3/2}}$$

$$\Big\downarrow \text{208}$$

$$\frac{2x}{3a\sqrt{a+bx^2}}+\frac{x(a-2bx^2)}{3a(a+bx^2)^{3/2}}$$

input
```
Int[(a - 2*b*x^2)/(a + b*x^2)^(5/2),x]
```

output
```
(x*(a - 2*b*x^2))/(3*a*(a + b*x^2)^(3/2)) + (2*x)/(3*a*Sqrt[a + b*x^2])
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 292
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> Si
mp[(-x)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*(p + 1))), x] - Simp[c*(q/(
a*(p + 1)))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1), x], x] /; FreeQ[
{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && EqQ[2*(p + q + 1) + 1, 0] && Gt
Q[q, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
gosper	$\frac{x}{(b\,x^2+a)^{\frac{3}{2}}}$	12
trager	$\frac{x}{(b\,x^2+a)^{\frac{3}{2}}}$	12
pseudoelliptic	$\frac{x}{(b\,x^2+a)^{\frac{3}{2}}}$	12
orering	$\frac{x}{(b\,x^2+a)^{\frac{3}{2}}}$	12
default	$a\left(\frac{x}{3a(b\,x^2+a)^{\frac{3}{2}}}+\frac{2x}{3a^2\sqrt{b\,x^2+a}}\right)-2b\left(-\frac{x}{2b(b\,x^2+a)^{\frac{3}{2}}}+\frac{a\left(\frac{x}{3a\left(b\,x^2+a\right)^{\frac{3}{2}}}+\frac{2x}{3a^2\sqrt{b\,x^2+a}}\right)}{2b}\right)$	91

input
```
int((-2*b*x^2+a)/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
x/(b*x^2+a)^(3/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(11) = 22.

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.38

$$\int \frac{a-2bx^2}{(a+bx^2)^{5/2}}\,dx = \frac{\sqrt{bx^2+a}\,x}{b^2x^4+2\,abx^2+a^2}$$

input
```
integrate((-2*b*x^2+a)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
sqrt(b*x^2 + a)*x/(b^2*x^4 + 2*a*b*x^2 + a^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 146 vs. $2(10) = 20$.

Time = 3.83 (sec) , antiderivative size = 146, normalized size of antiderivative = 11.23

$$\int \frac{a-2bx^2}{(a+bx^2)^{5/2}}\,dx = a\left(\frac{3ax}{3a^{\frac{7}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{5}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}} + \frac{2bx^3}{3a^{\frac{7}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{5}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}}\right) - \frac{2bx^3}{3a^{\frac{5}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{3}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}}$$

input
```
integrate((-2*b*x**2+a)/(b*x**2+a)**(5/2),x)
```

output
```
a*(3*a*x/(3*a**(7/2)*sqrt(1 + b*x**2/a) + 3*a**(5/2)*b*x**2*sqrt(1 + b*x**
2/a)) + 2*b*x**3/(3*a**(7/2)*sqrt(1 + b*x**2/a) + 3*a**(5/2)*b*x**2*sqrt(1
 + b*x**2/a))) - 2*b*x**3/(3*a**(5/2)*sqrt(1 + b*x**2/a) + 3*a**(3/2)*b*x*
*2*sqrt(1 + b*x**2/a))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{a-2bx^2}{(a+bx^2)^{5/2}}\,dx = \frac{x}{(bx^2+a)^{\frac{3}{2}}}$$

input
```
integrate((-2*b*x^2+a)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
x/(b*x^2 + a)^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{a-2bx^2}{(a+bx^2)^{5/2}}\,dx = \frac{x}{(bx^2+a)^{\frac{3}{2}}}$$

input
```
integrate((-2*b*x^2+a)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
x/(b*x^2 + a)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{a-2bx^2}{(a+bx^2)^{5/2}}\,dx = \frac{x}{(b\,x^2+a)^{3/2}}$$

input
```
int((a - 2*b*x^2)/(a + b*x^2)^(5/2),x)
```

output
```
x/(a + b*x^2)^(3/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 68, normalized size of antiderivative = 5.23

$$\int \frac{a-2bx^2}{(a+bx^2)^{5/2}}\,dx = \frac{3\sqrt{b\,x^2+a}\,abx-4\sqrt{b}\,a^2-8\sqrt{b}\,ab\,x^2-4\sqrt{b}\,b^2x^4}{3ab\,(b^2x^4+2ab\,x^2+a^2)}$$

input
```
int((-2*b*x^2+a)/(b*x^2+a)^(5/2),x)
```

output
```
(3*sqrt(a + b*x**2)*a*b*x - 4*sqrt(b)*a**2 - 8*sqrt(b)*a*b*x**2 - 4*sqrt(b
)*b**2*x**4)/(3*a*b*(a**2 + 2*a*b*x**2 + b**2*x**4))
```

3.107 $\int \frac{a-4bx^2}{(a+bx^2)^{7/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 13

$$\int \frac{a-4bx^2}{(a+bx^2)^{7/2}}\,dx = \frac{x}{(a+bx^2)^{5/2}}$$

output
```
x/(b*x^2+a)^(5/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{a-4bx^2}{(a+bx^2)^{7/2}}\,dx = \frac{x}{(a+bx^2)^{5/2}}$$

input
```
Integrate[(a - 4*b*x^2)/(a + b*x^2)^(7/2),x]
```

output
```
x/(a + b*x^2)^(5/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {297}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a-4bx^2}{(a+bx^2)^{7/2}}\,dx$$

$$\Big\downarrow \; 297$$

$$\frac{x}{(a+bx^2)^{5/2}}$$

input
```
Int[(a - 4*b*x^2)/(a + b*x^2)^(7/2),x]
```

output
```
x/(a + b*x^2)^(5/2)
```

Defintions of rubi rules used

rule 297
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[c*
x*((a + b*x^2)^(p + 1)/a), x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d
, 0] && EqQ[a*d - b*c*(2*p + 3), 0]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result
gosper	$\frac{x}{(b\,x^2+a)^{\frac{5}{2}}}$
trager	$\frac{x}{(b\,x^2+a)^{\frac{5}{2}}}$
pseudoelliptic	$\frac{x}{(b\,x^2+a)^{\frac{5}{2}}}$
orering	$\frac{x}{(b\,x^2+a)^{\frac{5}{2}}}$
default	$a\left(\frac{x}{5a(b\,x^2+a)^{\frac{5}{2}}}+\frac{\frac{4x}{15a\left(b\,x^2+a\right)^{\frac{3}{2}}}+\frac{8x}{15a^2\sqrt{b\,x^2+a}}}{a}\right)-4b\left(-\frac{x}{4b(b\,x^2+a)^{\frac{5}{2}}}+\frac{a\left(\frac{x}{5a\left(b\,x^2+a\right)^{\frac{5}{2}}}+\frac{\frac{4x}{15a\left(b\,x^2+a\right)^{\frac{3}{2}}}+\frac{}{15}}{a}\right.}{4b}\right.$

input
```
int((-4*b*x^2+a)/(b*x^2+a)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
x/(b*x^2+a)^(5/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. 2(11) = 22.

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 3.23

$$\int \frac{a-4bx^2}{(a+bx^2)^{7/2}}\,dx=\frac{\sqrt{bx^2+a}\,x}{b^3x^6+3\,ab^2x^4+3\,a^2bx^2+a^3}$$

input
```
integrate((-4*b*x^2+a)/(b*x^2+a)^(7/2),x, algorithm="fricas")
```

output
```
sqrt(b*x^2 + a)*x/(b^3*x^6 + 3*a*b^2*x^4 + 3*a^2*b*x^2 + a^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 568 vs. $2(10) = 20$.

Time = 9.91 (sec) , antiderivative size = 568, normalized size of antiderivative = 43.69

$$\int \frac{a-4bx^2}{(a+bx^2)^{7/2}}\,dx = a\left(\frac{15a^5x}{15a^{\frac{17}{2}}\sqrt{1+\frac{bx^2}{a}}+45a^{\frac{15}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}+45a^{\frac{13}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}}+15a^{\frac{11}{2}}b^3x^6\sqrt{1+\frac{bx^2}{a}}}\right.$$
$$+\frac{35a^4bx^3}{15a^{\frac{17}{2}}\sqrt{1+\frac{bx^2}{a}}+45a^{\frac{15}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}+45a^{\frac{13}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}}+15a^{\frac{11}{2}}b^3x^6\sqrt{1+\frac{bx^2}{a}}}$$
$$+\frac{28a^3b^2x^5}{15a^{\frac{17}{2}}\sqrt{1+\frac{bx^2}{a}}+45a^{\frac{15}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}+45a^{\frac{13}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}}+15a^{\frac{11}{2}}b^3x^6\sqrt{1+\frac{bx^2}{a}}}$$
$$\left.+\frac{8a^2b^3x^7}{15a^{\frac{17}{2}}\sqrt{1+\frac{bx^2}{a}}+45a^{\frac{15}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}+45a^{\frac{13}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}}+15a^{\frac{11}{2}}b^3x^6\sqrt{1+\frac{bx^2}{a}}}\right)$$
$$-4b\left(\frac{5ax^3}{15a^{\frac{9}{2}}\sqrt{1+\frac{bx^2}{a}}+30a^{\frac{7}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}+15a^{\frac{5}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}}}\right.$$
$$\left.+\frac{2bx^5}{15a^{\frac{9}{2}}\sqrt{1+\frac{bx^2}{a}}+30a^{\frac{7}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}+15a^{\frac{5}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}}}\right)$$

input
```
integrate((-4*b*x**2+a)/(b*x**2+a)**(7/2),x)
```

output
```
a*(15*a**5*x/(15*a**(17/2)*sqrt(1 + b*x**2/a) + 45*a**(15/2)*b*x**2*sqrt(1
 + b*x**2/a) + 45*a**(13/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 15*a**(11/2)*b*
*3*x**6*sqrt(1 + b*x**2/a)) + 35*a**4*b*x**3/(15*a**(17/2)*sqrt(1 + b*x**2
/a) + 45*a**(15/2)*b*x**2*sqrt(1 + b*x**2/a) + 45*a**(13/2)*b**2*x**4*sqrt
(1 + b*x**2/a) + 15*a**(11/2)*b**3*x**6*sqrt(1 + b*x**2/a)) + 28*a**3*b**2
*x**5/(15*a**(17/2)*sqrt(1 + b*x**2/a) + 45*a**(15/2)*b*x**2*sqrt(1 + b*x*
*2/a) + 45*a**(13/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 15*a**(11/2)*b**3*x**6
*sqrt(1 + b*x**2/a)) + 8*a**2*b**3*x**7/(15*a**(17/2)*sqrt(1 + b*x**2/a) +
 45*a**(15/2)*b*x**2*sqrt(1 + b*x**2/a) + 45*a**(13/2)*b**2*x**4*sqrt(1 +
b*x**2/a) + 15*a**(11/2)*b**3*x**6*sqrt(1 + b*x**2/a))) - 4*b*(5*a*x**3/(1
5*a**(9/2)*sqrt(1 + b*x**2/a) + 30*a**(7/2)*b*x**2*sqrt(1 + b*x**2/a) + 15
*a**(5/2)*b**2*x**4*sqrt(1 + b*x**2/a)) + 2*b*x**5/(15*a**(9/2)*sqrt(1 + b
*x**2/a) + 30*a**(7/2)*b*x**2*sqrt(1 + b*x**2/a) + 15*a**(5/2)*b**2*x**4*s
qrt(1 + b*x**2/a)))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{a-4bx^2}{(a+bx^2)^{7/2}}\,dx = \frac{x}{(bx^2+a)^{\frac{5}{2}}}$$

input
```
integrate((-4*b*x^2+a)/(b*x^2+a)^(7/2),x, algorithm="maxima")
```

output
```
x/(b*x^2 + a)^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{a-4bx^2}{(a+bx^2)^{7/2}}\,dx = \frac{x}{(bx^2+a)^{\frac{5}{2}}}$$

input
```
integrate((-4*b*x^2+a)/(b*x^2+a)^(7/2),x, algorithm="giac")
```

output
```
x/(b*x^2 + a)^(5/2)
```

Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{a-4bx^2}{(a+bx^2)^{7/2}}\,dx=\frac{x}{(b\,x^2+a)^{5/2}}$$

input
```
int((a - 4*b*x^2)/(a + b*x^2)^(7/2),x)
```

output
```
x/(a + b*x^2)^(5/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 41, normalized size of antiderivative = 3.15

$$\int \frac{a-4bx^2}{(a+bx^2)^{7/2}}\,dx=\frac{\sqrt{b\,x^2+a}\,x}{b^3x^6+3a\,b^2x^4+3a^2b\,x^2+a^3}$$

input
```
int((-4*b*x^2+a)/(b*x^2+a)^(7/2),x)
```

output
```
(sqrt(a + b*x**2)*x)/(a**3 + 3*a**2*b*x**2 + 3*a*b**2*x**4 + b**3*x**6)
```

3.108 $\int (a+bx^2)^p (A+Bx^2)\, dx$

Optimal result

Integrand size = 17, antiderivative size = 85

$$\int (a+bx^2)^p (A+Bx^2)\, dx = \frac{Bx(a+bx^2)^{1+p}}{b(3+2p)} + \left(A - \frac{aB}{3b+2bp}\right) x(a+bx^2)^p \left(1 + \frac{bx^2}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{2}, -p, \frac{3}{2}, -\frac{bx^2}{a}\right)$$

output
```
B*x*(b*x^2+a)^(p+1)/b/(3+2*p)+(A-a*B/(2*b*p+3*b))*x*(b*x^2+a)^p*hypergeom(
[1/2, -p],[3/2],-b*x^2/a)/((1+b*x^2/a)^p)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.06

$$\int (a+bx^2)^p (A+Bx^2)\, dx = \frac{x(a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} \left(B(a+bx^2)\left(1+\frac{bx^2}{a}\right)^p + (-aB+Ab(3+2p))\,\text{Hypergeometric2F1}\left(\frac{1}{2}, -p, \frac{3}{2},\right.\right.}{b(3+2p)}$$

input
```
Integrate[(a + b*x^2)^p*(A + B*x^2),x]
```

output
```
(x*(a + b*x^2)^p*(B*(a + b*x^2)*(1 + (b*x^2)/a)^p + (-(a*B) + A*b*(3 + 2*p
))*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)]))/(b*(3 + 2*p)*(1 + (b*x^
2)/a)^p)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {299, 238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (A+Bx^2)(a+bx^2)^p\,dx$$

$$\Big\downarrow \text{ 299}$$

$$\left(A-\frac{aB}{2bp+3b}\right)\int (bx^2+a)^p\,dx+\frac{Bx(a+bx^2)^{p+1}}{b(2p+3)}$$

$$\Big\downarrow \text{ 238}$$

$$(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(A-\frac{aB}{2bp+3b}\right)\int \left(\frac{bx^2}{a}+1\right)^p dx+\frac{Bx(a+bx^2)^{p+1}}{b(2p+3)}$$

$$\Big\downarrow \text{ 237}$$

$$x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(A-\frac{aB}{2bp+3b}\right)\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right)+\frac{Bx(a+bx^2)^{p+1}}{b(2p+3)}$$

input
```
Int[(a + b*x^2)^p*(A + B*x^2),x]
```

output
```
(B*x*(a + b*x^2)^(1 + p))/(b*(3 + 2*p)) + ((A - (a*B)/(3*b + 2*b*p))*x*(a
+ b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^
p
```

Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

Maple [F]

$$\int \left(b\,x^{2}+a\right)^{p} \left(x^{2} B+A\right)dx$$

input
```
int((b*x^2+a)^p*(B*x^2+A),x)
```

output
```
int((b*x^2+a)^p*(B*x^2+A),x)
```

Fricas [F]

$$\int \left(a+bx^2\right)^p \left(A+Bx^2\right)\,dx=\int \left(Bx^2+A\right)\left(bx^2+a\right)^p\,dx$$

input
```
integrate((b*x^2+a)^p*(B*x^2+A),x, algorithm="fricas")
```

output
```
integral((B*x^2 + A)*(b*x^2 + a)^p, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.62

$$\int (a+bx^2)^p (A+Bx^2)\, dx = Aa^p x\, {}_2F_1\left(\begin{matrix} \frac{1}{2}, -p \\ \frac{3}{2} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right) + \frac{Ba^p x^3\, {}_2F_1\left(\begin{matrix} \frac{3}{2}, -p \\ \frac{5}{2} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)}{3}$$

input
```
integrate((b*x**2+a)**p*(B*x**2+A),x)
```

output
```
A*a**p*x*hyper((1/2, -p), (3/2,), b*x**2*exp_polar(I*pi)/a) + B*a**p*x**3*
hyper((3/2, -p), (5/2,), b*x**2*exp_polar(I*pi)/a)/3
```

Maxima [F]

$$\int (a+bx^2)^p (A+Bx^2)\, dx = \int (Bx^2+A)(bx^2+a)^p\, dx$$

input
```
integrate((b*x^2+a)^p*(B*x^2+A),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*(b*x^2 + a)^p, x)
```

Giac [F]

$$\int (a+bx^2)^p (A+Bx^2)\, dx = \int (Bx^2+A)(bx^2+a)^p\, dx$$

input
```
integrate((b*x^2+a)^p*(B*x^2+A),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*(b*x^2 + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^2\right)^p \left(A+Bx^2\right)\,dx=\int \left(B\,x^2+A\right)\,\left(b\,x^2+a\right)^p\,dx$$

input
```
int((A + B*x^2)*(a + b*x^2)^p,x)
```

output
```
int((A + B*x^2)*(a + b*x^2)^p, x)
```

Reduce [F]

$$\int \left(a+bx^2\right)^p \left(A+Bx^2\right)\,dx = \frac{4(b\,x^2+a)^p\,apx+3(b\,x^2+a)^p\,ax+2(b\,x^2+a)^p\,bp\,x^3+(b\,x^2+a)^p\,b\,x^3+16\left(\int \frac{(b\,x^2+a)^p}{4b\,p^2x^2+8bp\,x^2+4a\,p^2+3b\,x^2+8}\right.}{}$$

input
```
int((b*x^2+a)^p*(B*x^2+A),x)
```

output
```
(4*(a + b*x**2)**p*a*p*x + 3*(a + b*x**2)**p*a*x + 2*(a + b*x**2)**p*b*p*x
**3 + (a + b*x**2)**p*b*x**3 + 16*int((a + b*x**2)**p/(4*a*p**2 + 8*a*p +
3*a + 4*b*p**2*x**2 + 8*b*p*x**2 + 3*b*x**2),x)*a**2*p**4 + 48*int((a + b*
x**2)**p/(4*a*p**2 + 8*a*p + 3*a + 4*b*p**2*x**2 + 8*b*p*x**2 + 3*b*x**2),
x)*a**2*p**3 + 44*int((a + b*x**2)**p/(4*a*p**2 + 8*a*p + 3*a + 4*b*p**2*x
**2 + 8*b*p*x**2 + 3*b*x**2),x)*a**2*p**2 + 12*int((a + b*x**2)**p/(4*a*p*
*2 + 8*a*p + 3*a + 4*b*p**2*x**2 + 8*b*p*x**2 + 3*b*x**2),x)*a**2*p)/(4*p*
*2 + 8*p + 3)
```

3.109 $\int (a+bx^2)^3 (A+Bx^2+Cx^4)\, dx$

Optimal result

Integrand size = 22, antiderivative size = 103

$$\int (a+bx^2)^3 (A+Bx^2+Cx^4)\, dx = a^3Ax + \frac{1}{3}a^2(3Ab+aB)x^3 + \frac{1}{5}a\left(3Ab^2+a(3bB+aC)\right)x^5 + \frac{1}{7}b\left(Ab^2+3a(bB+aC)\right)x^7 + \frac{1}{9}b^2(bB+3aC)x^9 + \frac{1}{11}b^3Cx^{11}$$

output
```
a^3*A*x+1/3*a^2*(3*A*b+B*a)*x^3+1/5*a*(3*A*b^2+a*(3*B*b+C*a))*x^5+1/7*b*(A
*b^2+3*a*(B*b+C*a))*x^7+1/9*b^2*(B*b+3*C*a)*x^9+1/11*b^3*C*x^11
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.01

$$\int \left(a+bx^2\right)^3 \left(A+Bx^2+Cx^4\right) \, dx = a^3Ax+\frac{1}{3}a^2(3Ab+aB)x^3 +\frac{1}{5}a\left(3Ab^2+3abB+a^2C\right)x^5 +\frac{1}{7}b\left(Ab^2+3abB+3a^2C\right)x^7 +\frac{1}{9}b^2(bB+3aC)x^9+\frac{1}{11}b^3Cx^{11}$$

input
```
Integrate[(a + b*x^2)^3*(A + B*x^2 + C*x^4),x]
```

output
```
a^3*A*x + (a^2*(3*A*b + a*B)*x^3)/3 + (a*(3*A*b^2 + 3*a*b*B + a^2*C)*x^5)/
5 + (b*(A*b^2 + 3*a*b*B + 3*a^2*C)*x^7)/7 + (b^2*(b*B + 3*a*C)*x^9)/9 + (b
^3*C*x^11)/11
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1467, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^3 \left(A+Bx^2+Cx^4\right) \, dx$$

$$\Big\downarrow \text{ 1467}$$

$$\int \left(a^3A+a^2x^2(aB+3Ab)+bx^6\left(3a(aC+bB)+Ab^2\right)+ax^4\left(a(aC+3bB)+3Ab^2\right)+b^2x^8(3aC+bB)+b^3Cx$$

$$\Big\downarrow \text{ 2009}$$

$$a^3Ax+\frac{1}{3}a^2x^3(aB+3Ab)+\frac{1}{7}bx^7\left(3a(aC+bB)+Ab^2\right)+\frac{1}{5}ax^5\left(a(aC+3bB)+3Ab^2\right)+\frac{1}{9}b^2x^9(3aC+bB)+\frac{1}{11}b^3Cx^{11}$$

input
```
Int[(a + b*x^2)^3*(A + B*x^2 + C*x^4),x]
```

output
```
a^3*A*x + (a^2*(3*A*b + a*B)*x^3)/3 + (a*(3*A*b^2 + a*(3*b*B + a*C))*x^5)/
5 + (b*(A*b^2 + 3*a*(b*B + a*C))*x^7)/7 + (b^2*(b*B + 3*a*C)*x^9)/9 + (b^3
*C*x^11)/11
```

Defintions of rubi rules used

rule 1467
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.99

method	result
norman	$\frac{b^3Cx^{11}}{11}+\left(\frac{1}{9}Bb^3+\frac{1}{3}aCb^2\right)x^9+\left(\frac{1}{7}b^3A+\frac{3}{7}ab^2B+\frac{3}{7}a^2bC\right)x^7+(\frac{3}{5}ab^2A+\frac{3}{5}a^2bB+\frac{1}{5}Ca^3$
default	$\frac{b^3Cx^{11}}{11}+\frac{(Bb^3+3aCb^2)x^9}{9}+\frac{(b^3A+3ab^2B+3a^2bC)x^7}{7}+\frac{(3ab^2A+3a^2bB+Ca^3)x^5}{5}+\frac{(3a^2bA+a^3B)x^3}{3}+a^3A$
gosper	$\frac{1}{11}b^3Cx^{11}+\frac{1}{9}b^3Bx^9+\frac{1}{3}x^9aCb^2+\frac{1}{7}Ab^3x^7+\frac{3}{7}x^7ab^2B+\frac{3}{7}x^7a^2bC+\frac{3}{5}aAb^2x^5+\frac{3}{5}x^5a^2bB$
risch	$\frac{1}{11}b^3Cx^{11}+\frac{1}{9}b^3Bx^9+\frac{1}{3}x^9aCb^2+\frac{1}{7}Ab^3x^7+\frac{3}{7}x^7ab^2B+\frac{3}{7}x^7a^2bC+\frac{3}{5}aAb^2x^5+\frac{3}{5}x^5a^2bB$
parallelrisch	$\frac{1}{11}b^3Cx^{11}+\frac{1}{9}b^3Bx^9+\frac{1}{3}x^9aCb^2+\frac{1}{7}Ab^3x^7+\frac{3}{7}x^7ab^2B+\frac{3}{7}x^7a^2bC+\frac{3}{5}aAb^2x^5+\frac{3}{5}x^5a^2bB$
orering	$\frac{x(315b^3Cx^{10}+385b^3Bx^8+1155Cab^2x^8+495Ab^3x^6+1485Bab^2x^6+1485Ca^2bx^6+2079aAb^2x^4+2079Ba^2bx^4+693Ca^3x}{3465}$

input
```
int((b*x^2+a)^3*(C*x^4+B*x^2+A),x,method=_RETURNVERBOSE)
```

output
```
1/11*b^3*C*x^11+(1/9*B*b^3+1/3*a*C*b^2)*x^9+(1/7*b^3*A+3/7*a*b^2*B+3/7*a^2
*b*C)*x^7+(3/5*a*b^2*A+3/5*a^2*b*B+1/5*C*a^3)*x^5+(a^2*b*A+1/3*a^3*B)*x^3+
a^3*A*x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.99

$$\int (a+bx^2)^3 (A+Bx^2+Cx^4)\,dx = \frac{1}{11}\,Cb^3x^{11} + \frac{1}{9}\left(3\,Cab^2 + Bb^3\right)x^9 + \frac{1}{7}\left(3\,Ca^2b + 3\,Bab^2 + Ab^3\right)x^7 + \frac{1}{5}\left(Ca^3 + 3\,Ba^2b + 3\,Aab^2\right)x^5 + Aa^3x + \frac{1}{3}\left(Ba^3 + 3\,Aa^2b\right)x^3$$

input
```
integrate((b*x^2+a)^3*(C*x^4+B*x^2+A),x, algorithm="fricas")
```

output
```
1/11*C*b^3*x^11 + 1/9*(3*C*a*b^2 + B*b^3)*x^9 + 1/7*(3*C*a^2*b + 3*B*a*b^2
 + A*b^3)*x^7 + 1/5*(C*a^3 + 3*B*a^2*b + 3*A*a*b^2)*x^5 + A*a^3*x + 1/3*(B
*a^3 + 3*A*a^2*b)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.09

$$\int (a+bx^2)^3 (A+Bx^2+Cx^4)\,dx = Aa^3x + \frac{Cb^3x^{11}}{11} + x^9\left(\frac{Bb^3}{9} + \frac{Cab^2}{3}\right) + x^7\left(\frac{Ab^3}{7} + \frac{3Bab^2}{7} + \frac{3Ca^2b}{7}\right) + x^5 \cdot \left(\frac{3Aab^2}{5} + \frac{3Ba^2b}{5} + \frac{Ca^3}{5}\right) + x^3\left(Aa^2b + \frac{Ba^3}{3}\right)$$

input
```
integrate((b*x**2+a)**3*(C*x**4+B*x**2+A),x)
```

output

```
A*a**3*x + C*b**3*x**11/11 + x**9*(B*b**3/9 + C*a*b**2/3) + x**7*(A*b**3/7
 + 3*B*a*b**2/7 + 3*C*a**2*b/7) + x**5*(3*A*a*b**2/5 + 3*B*a**2*b/5 + C*a*
*3/5) + x**3*(A*a**2*b + B*a**3/3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.99

$$\int (a+bx^2)^3 (A+Bx^2+Cx^4)\, dx = \frac{1}{11}\,Cb^3x^{11} + \frac{1}{9}\,(3\,Cab^2+Bb^3)x^9 + \frac{1}{7}\,(3\,Ca^2b+3\,Bab^2+Ab^3)x^7 + \frac{1}{5}\,(Ca^3+3\,Ba^2b+3\,Aab^2)x^5 + Aa^3x + \frac{1}{3}\,(Ba^3+3\,Aa^2b)x^3$$

input

```
integrate((b*x^2+a)^3*(C*x^4+B*x^2+A),x, algorithm="maxima")
```

output

```
1/11*C*b^3*x^11 + 1/9*(3*C*a*b^2 + B*b^3)*x^9 + 1/7*(3*C*a^2*b + 3*B*a*b^2
 + A*b^3)*x^7 + 1/5*(C*a^3 + 3*B*a^2*b + 3*A*a*b^2)*x^5 + A*a^3*x + 1/3*(B
*a^3 + 3*A*a^2*b)*x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.08

$$\int (a+bx^2)^3 (A+Bx^2+Cx^4)\, dx = \frac{1}{11}\,Cb^3x^{11} + \frac{1}{3}\,Cab^2x^9 + \frac{1}{9}\,Bb^3x^9 + \frac{3}{7}\,Ca^2bx^7 + \frac{3}{7}\,Bab^2x^7 + \frac{1}{7}\,Ab^3x^7 + \frac{1}{5}\,Ca^3x^5 + \frac{3}{5}\,Ba^2bx^5 + \frac{3}{5}\,Aab^2x^5 + \frac{1}{3}\,Ba^3x^3 + Aa^2bx^3 + Aa^3x$$

input

```
integrate((b*x^2+a)^3*(C*x^4+B*x^2+A),x, algorithm="giac")
```

output
```
1/11*C*b^3*x^11 + 1/3*C*a*b^2*x^9 + 1/9*B*b^3*x^9 + 3/7*C*a^2*b*x^7 + 3/7*
B*a*b^2*x^7 + 1/7*A*b^3*x^7 + 1/5*C*a^3*x^5 + 3/5*B*a^2*b*x^5 + 3/5*A*a*b^
2*x^5 + 1/3*B*a^3*x^3 + A*a^2*b*x^3 + A*a^3*x
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.98

$$\int (a+bx^2)^3 (A+Bx^2+Cx^4)\,dx = x^5\left(\frac{C\,a^3}{5}+\frac{3\,B\,a^2\,b}{5}+\frac{3\,A\,a\,b^2}{5}\right) + x^7\left(\frac{3\,C\,a^2\,b}{7}+\frac{3\,B\,a\,b^2}{7}+\frac{A\,b^3}{7}\right) + x^3\left(\frac{B\,a^3}{3}+A\,b\,a^2\right) + x^9\left(\frac{B\,b^3}{9}+\frac{C\,a\,b^2}{3}\right)+\frac{C\,b^3\,x^{11}}{11}+A\,a^3\,x$$

input
```
int((a + b*x^2)^3*(A + B*x^2 + C*x^4),x)
```

output
```
x^5*((C*a^3)/5 + (3*A*a*b^2)/5 + (3*B*a^2*b)/5) + x^7*((A*b^3)/7 + (3*B*a*
b^2)/7 + (3*C*a^2*b)/7) + x^3*((B*a^3)/3 + A*a^2*b) + x^9*((B*b^3)/9 + (C*
a*b^2)/3) + (C*b^3*x^11)/11 + A*a^3*x
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.82

$$\int (a+bx^2)^3 (A+Bx^2+Cx^4)\,dx = \frac{x(315b^3c\,x^{10}+1155a\,b^2c\,x^8+385b^4x^8+1485a^2bc\,x^6+1980a\,b^3x^6+693a^3c\,x^4+4158a^2b^2x^4+4620a^3b}{3465}$$

input
```
int((b*x^2+a)^3*(C*x^4+B*x^2+A),x)
```

output
```
(x*(3465*a**4 + 4620*a**3*b*x**2 + 693*a**3*c*x**4 + 4158*a**2*b**2*x**4 +
 1485*a**2*b*c*x**6 + 1980*a*b**3*x**6 + 1155*a*b**2*c*x**8 + 385*b**4*x**
8 + 315*b**3*c*x**10))/3465
```

3.110 $\int (a+bx^2)^2 (A+Bx^2+Cx^4)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 73

$$\int (a+bx^2)^2 (A+Bx^2+Cx^4)\,dx = a^2Ax+\frac{1}{3}a(2Ab+aB)x^3+\frac{1}{5}\left(Ab^2+a(2bB+aC)\right)x^5 + \frac{1}{7}b(bB+2aC)x^7+\frac{1}{9}b^2Cx^9$$

output
```
a^2*A*x+1/3*a*(2*A*b+B*a)*x^3+1/5*(A*b^2+a*(2*B*b+C*a))*x^5+1/7*b*(B*b+2*C
*a)*x^7+1/9*b^2*C*x^9
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00

$$\int (a+bx^2)^2 (A+Bx^2+Cx^4)\,dx = a^2Ax+\frac{1}{3}a(2Ab+aB)x^3+\frac{1}{5}\left(Ab^2+2abB+a^2C\right)x^5 + \frac{1}{7}b(bB+2aC)x^7+\frac{1}{9}b^2Cx^9$$

input
```
Integrate[(a + b*x^2)^2*(A + B*x^2 + C*x^4),x]
```

output
```
a^2*A*x + (a*(2*A*b + a*B)*x^3)/3 + ((A*b^2 + 2*a*b*B + a^2*C)*x^5)/5 + (b
*(b*B + 2*a*C)*x^7)/7 + (b^2*C*x^9)/9
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1467, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^2 \left(A+Bx^2+Cx^4\right) \, dx$$

$$\Big\downarrow \text{1467}$$

$$\int \left(a^2A+x^4\left(a(aC+2bB)+Ab^2\right)+ax^2(aB+2Ab)+bx^6(2aC+bB)+b^2Cx^8\right)dx$$

$$\Big\downarrow \text{2009}$$

$$a^2Ax+\frac{1}{5}x^5\left(a(aC+2bB)+Ab^2\right)+\frac{1}{3}ax^3(aB+2Ab)+\frac{1}{7}bx^7(2aC+bB)+\frac{1}{9}b^2Cx^9$$

input
```
Int[(a + b*x^2)^2*(A + B*x^2 + C*x^4),x]
```

output
```
a^2*A*x + (a*(2*A*b + a*B)*x^3)/3 + ((A*b^2 + a*(2*b*B + a*C))*x^5)/5 + (b
*(b*B + 2*a*C)*x^7)/7 + (b^2*C*x^9)/9
```

Defintions of rubi rules used

rule 1467
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.96

method	result
default	$\frac{b^2 C x^9}{9}+\frac{(B b^2+2Cab)x^7}{7}+\frac{(b^2 A+2abB+a^2 C)x^5}{5}+\frac{(2abA+a^2 B)x^3}{3}+a^2 A x$
norman	$\frac{b^2 C x^9}{9}+\left(\frac{1}{7}B b^2+\frac{2}{7}Cab\right)x^7+\left(\frac{1}{5}b^2 A+\frac{2}{5}abB+\frac{1}{5}a^2 C\right)x^5+\left(\frac{2}{3}abA+\frac{1}{3}a^2 B\right)x^3+a^2 A x$
gosper	$\frac{1}{9}b^2 C x^9+\frac{1}{7}b^2 B x^7+\frac{2}{7}x^7 Cab+\frac{1}{5}A b^2 x^5+\frac{2}{5}x^5 abB+\frac{1}{5}x^5 a^2 C+\frac{2}{3}aAb x^3+\frac{1}{3}x^3 a^2 B+a^2 A$
risch	$\frac{1}{9}b^2 C x^9+\frac{1}{7}b^2 B x^7+\frac{2}{7}x^7 Cab+\frac{1}{5}A b^2 x^5+\frac{2}{5}x^5 abB+\frac{1}{5}x^5 a^2 C+\frac{2}{3}aAb x^3+\frac{1}{3}x^3 a^2 B+a^2 A$
parallelrisch	$\frac{1}{9}b^2 C x^9+\frac{1}{7}b^2 B x^7+\frac{2}{7}x^7 Cab+\frac{1}{5}A b^2 x^5+\frac{2}{5}x^5 abB+\frac{1}{5}x^5 a^2 C+\frac{2}{3}aAb x^3+\frac{1}{3}x^3 a^2 B+a^2 A$
orering	$\frac{x(35b^2 C x^8+45b^2 B x^6+90Cab x^6+63A b^2 x^4+126Bab x^4+63C a^2 x^4+210aAb x^2+105B a^2 x^2+315a^2 A)}{315}$

input
```
int((b*x^2+a)^2*(C*x^4+B*x^2+A),x,method=_RETURNVERBOSE)
```

output
```
1/9*b^2*C*x^9+1/7*(B*b^2+2*C*a*b)*x^7+1/5*(A*b^2+2*B*a*b+C*a^2)*x^5+1/3*(2
*A*a*b+B*a^2)*x^3+a^2*A*x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.95

$$\int (a+bx^2)^2 (A+Bx^2+Cx^4)\,dx=\frac{1}{9}Cb^2x^9+\frac{1}{7}(2Cab+Bb^2)x^7+\frac{1}{5}(Ca^2+2Bab+Ab^2)x^5+Aa^2x+\frac{1}{3}(Ba^2+2Aab)x^3$$

input
```
integrate((b*x^2+a)^2*(C*x^4+B*x^2+A),x, algorithm="fricas")
```

output
```
1/9*C*b^2*x^9 + 1/7*(2*C*a*b + B*b^2)*x^7 + 1/5*(C*a^2 + 2*B*a*b + A*b^2)*
x^5 + A*a^2*x + 1/3*(B*a^2 + 2*A*a*b)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.07

$$\int \left(a+bx^2\right)^2 \left(A+Bx^2+Cx^4\right)\,dx = Aa^2x+\frac{Cb^2x^9}{9}+x^7\left(\frac{Bb^2}{7}+\frac{2Cab}{7}\right) +x^5\left(\frac{Ab^2}{5}+\frac{2Bab}{5}+\frac{Ca^2}{5}\right)+x^3\cdot\left(\frac{2Aab}{3}+\frac{Ba^2}{3}\right)$$

input
```
integrate((b*x**2+a)**2*(C*x**4+B*x**2+A),x)
```

output
```
A*a**2*x + C*b**2*x**9/9 + x**7*(B*b**2/7 + 2*C*a*b/7) + x**5*(A*b**2/5 +
2*B*a*b/5 + C*a**2/5) + x**3*(2*A*a*b/3 + B*a**2/3)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.95

$$\int \left(a+bx^2\right)^2 \left(A+Bx^2+Cx^4\right)\,dx = \frac{1}{9}\,Cb^2x^9+\frac{1}{7}\left(2\,Cab+Bb^2\right)x^7 +\frac{1}{5}\left(Ca^2+2\,Bab+Ab^2\right)x^5 +Aa^2x+\frac{1}{3}\left(Ba^2+2\,Aab\right)x^3$$

input
```
integrate((b*x^2+a)^2*(C*x^4+B*x^2+A),x, algorithm="maxima")
```

output
```
1/9*C*b^2*x^9 + 1/7*(2*C*a*b + B*b^2)*x^7 + 1/5*(C*a^2 + 2*B*a*b + A*b^2)*
x^5 + A*a^2*x + 1/3*(B*a^2 + 2*A*a*b)*x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.04

$$\int \left(a+bx^2\right)^2 \left(A+Bx^2+Cx^4\right) dx = \frac{1}{9}Cb^2x^9+\frac{2}{7}Cabx^7+\frac{1}{7}Bb^2x^7+\frac{1}{5}Ca^2x^5+\frac{2}{5}Babx^5 + \frac{1}{5}Ab^2x^5+\frac{1}{3}Ba^2x^3+\frac{2}{3}Aabx^3+Aa^2x$$

input
```
integrate((b*x^2+a)^2*(C*x^4+B*x^2+A),x, algorithm="giac")
```

output
```
1/9*C*b^2*x^9 + 2/7*C*a*b*x^7 + 1/7*B*b^2*x^7 + 1/5*C*a^2*x^5 + 2/5*B*a*b*
x^5 + 1/5*A*b^2*x^5 + 1/3*B*a^2*x^3 + 2/3*A*a*b*x^3 + A*a^2*x
```

Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.96

$$\int \left(a+bx^2\right)^2 \left(A+Bx^2+Cx^4\right) dx = x^3\left(\frac{B\,a^2}{3}+\frac{2\,A\,b\,a}{3}\right)+x^7\left(\frac{B\,b^2}{7}+\frac{2\,C\,a\,b}{7}\right) + x^5\left(\frac{C\,a^2}{5}+\frac{2\,B\,a\,b}{5}+\frac{A\,b^2}{5}\right)+\frac{C\,b^2\,x^9}{9}+A\,a^2\,x$$

input
```
int((a + b*x^2)^2*(A + B*x^2 + C*x^4),x)
```

output
```
x^3*((B*a^2)/3 + (2*A*a*b)/3) + x^7*((B*b^2)/7 + (2*C*a*b)/7) + x^5*((A*b^
2)/5 + (C*a^2)/5 + (2*B*a*b)/5) + (C*b^2*x^9)/9 + A*a^2*x
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.84

$$\int \left(a+bx^2\right)^2 \left(A+Bx^2+Cx^4\right)\,dx = \frac{x(35b^2c\,x^8+90abc\,x^6+45b^3x^6+63a^2c\,x^4+189a\,b^2x^4+315a^2b\,x^2+315a^3)}{315}$$

input
```
int((b*x^2+a)^2*(C*x^4+B*x^2+A),x)
```

output
```
(x*(315*a**3 + 315*a**2*b*x**2 + 63*a**2*c*x**4 + 189*a*b**2*x**4 + 90*a*b
*c*x**6 + 45*b**3*x**6 + 35*b**2*c*x**8))/315
```

3.111 $\int (a+bx^2)(A+Bx^2+Cx^4)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 42

$$\int (a+bx^2)(A+Bx^2+Cx^4)\,dx = aAx+\frac{1}{3}(Ab+aB)x^3+\frac{1}{5}(bB+aC)x^5+\frac{1}{7}bCx^7$$

output
```
a*A*x+1/3*(A*b+B*a)*x^3+1/5*(B*b+C*a)*x^5+1/7*b*C*x^7
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int (a+bx^2)(A+Bx^2+Cx^4)\,dx = aAx+\frac{1}{3}(Ab+aB)x^3+\frac{1}{5}(bB+aC)x^5+\frac{1}{7}bCx^7$$

input
```
Integrate[(a + b*x^2)*(A + B*x^2 + C*x^4),x]
```

output
```
a*A*x + ((A*b + a*B)*x^3)/3 + ((b*B + a*C)*x^5)/5 + (b*C*x^7)/7
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1467, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)\left(A+Bx^2+Cx^4\right)\,dx$$

$$\Big\downarrow \text{ 1467}$$

$$\int \left(x^2(aB+Ab)+aA+x^4(aC+bB)+bCx^6\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3}x^3(aB+Ab)+aAx+\frac{1}{5}x^5(aC+bB)+\frac{1}{7}bCx^7$$

input
```
Int[(a + b*x^2)*(A + B*x^2 + C*x^4),x]
```

output
```
a*A*x + ((A*b + a*B)*x^3)/3 + ((b*B + a*C)*x^5)/5 + (b*C*x^7)/7
```

Defintions of rubi rules used

rule 1467
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

method	result	size
default	$aAx+\frac{(Ab+Ba)x^3}{3}+\frac{(Bb+Ca)x^5}{5}+\frac{bC\,x^7}{7}$	37
norman	$\frac{bC\,x^7}{7}+\left(\frac{Bb}{5}+\frac{Ca}{5}\right)x^5+\left(\frac{Ab}{3}+\frac{Ba}{3}\right)x^3+aAx$	39
gosper	$\frac{1}{7}bC\,x^7+\frac{1}{5}bB\,x^5+\frac{1}{5}x^5Ca+\frac{1}{3}Ab\,x^3+\frac{1}{3}x^3Ba+aAx$	41
risch	$\frac{1}{7}bC\,x^7+\frac{1}{5}bB\,x^5+\frac{1}{5}x^5Ca+\frac{1}{3}Ab\,x^3+\frac{1}{3}x^3Ba+aAx$	41
parallelrisch	$\frac{1}{7}bC\,x^7+\frac{1}{5}bB\,x^5+\frac{1}{5}x^5Ca+\frac{1}{3}Ab\,x^3+\frac{1}{3}x^3Ba+aAx$	41
orering	$\frac{x(15Cb\,x^6+21bB\,x^4+21Ca\,x^4+35Ab\,x^2+35Ba\,x^2+105Aa)}{105}$	44

input
```
int((b*x^2+a)*(C*x^4+B*x^2+A),x,method=_RETURNVERBOSE)
```

output
```
a*A*x+1/3*(A*b+B*a)*x^3+1/5*(B*b+C*a)*x^5+1/7*b*C*x^7
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

$$\int \left(a+bx^2\right)\left(A+Bx^2+Cx^4\right)\,dx=\frac{1}{7}Cbx^7+\frac{1}{5}(Ca+Bb)x^5+\frac{1}{3}(Ba+Ab)x^3+Aax$$

input
```
integrate((b*x^2+a)*(C*x^4+B*x^2+A),x, algorithm="fricas")
```

output
```
1/7*C*b*x^7 + 1/5*(C*a + B*b)*x^5 + 1/3*(B*a + A*b)*x^3 + A*a*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.93

$$\int (a+bx^2)\left(A+Bx^2+Cx^4\right) dx = Aax + \frac{Cbx^7}{7} + x^5\left(\frac{Bb}{5} + \frac{Ca}{5}\right) + x^3\left(\frac{Ab}{3} + \frac{Ba}{3}\right)$$

input
```
integrate((b*x**2+a)*(C*x**4+B*x**2+A),x)
```

output
```
A*a*x + C*b*x**7/7 + x**5*(B*b/5 + C*a/5) + x**3*(A*b/3 + B*a/3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

$$\int (a+bx^2)\left(A+Bx^2+Cx^4\right) dx = \frac{1}{7}Cbx^7 + \frac{1}{5}(Ca+Bb)x^5 + \frac{1}{3}(Ba+Ab)x^3 + Aax$$

input
```
integrate((b*x^2+a)*(C*x^4+B*x^2+A),x, algorithm="maxima")
```

output
```
1/7*C*b*x^7 + 1/5*(C*a + B*b)*x^5 + 1/3*(B*a + A*b)*x^3 + A*a*x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.95

$$\int (a+bx^2)\left(A+Bx^2+Cx^4\right) dx = \frac{1}{7}Cbx^7 + \frac{1}{5}Cax^5 + \frac{1}{5}Bbx^5 + \frac{1}{3}Bax^3 + \frac{1}{3}Abx^3 + Aax$$

input
```
integrate((b*x^2+a)*(C*x^4+B*x^2+A),x, algorithm="giac")
```

output
```
1/7*C*b*x^7 + 1/5*C*a*x^5 + 1/5*B*b*x^5 + 1/3*B*a*x^3 + 1/3*A*b*x^3 + A*a*
x
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.90

$$\int (a+bx^2)\left(A+Bx^2+Cx^4\right)\,dx=\frac{C\,b\,x^7}{7}+\left(\frac{B\,b}{5}+\frac{C\,a}{5}\right)\,x^5+\left(\frac{A\,b}{3}+\frac{B\,a}{3}\right)\,x^3+A\,a\,x$$

input
```
int((a + b*x^2)*(A + B*x^2 + C*x^4),x)
```

output
```
x^3*((A*b)/3 + (B*a)/3) + x^5*((B*b)/5 + (C*a)/5) + A*a*x + (C*b*x^7)/7
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.90

$$\int (a+bx^2)\left(A+Bx^2+Cx^4\right)\,dx=\frac{x(15bc\,x^6+21ac\,x^4+21b^2x^4+70ab\,x^2+105a^2)}{105}$$

input
```
int((b*x^2+a)*(C*x^4+B*x^2+A),x)
```

output
```
(x*(105*a**2 + 70*a*b*x**2 + 21*a*c*x**4 + 21*b**2*x**4 + 15*b*c*x**6))/10
5
```

3.112 $\int \frac{A+Bx^2+Cx^4}{a+bx^2} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 66

$$\int \frac{A+Bx^2+Cx^4}{a+bx^2} \, dx = \frac{(bB-aC)x}{b^2}+\frac{Cx^3}{3b}+\frac{(Ab^2-a(bB-aC))\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{5/2}}$$

output
```
(B*b-C*a)*x/b^2+1/3*C*x^3/b+(A*b^2-a*(B*b-C*a))*arctan(b^(1/2)*x/a^(1/2))/
a^(1/2)/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.98

$$\int \frac{A+Bx^2+Cx^4}{a+bx^2} \, dx = \frac{(bB-aC)x}{b^2}+\frac{Cx^3}{3b}+\frac{(Ab^2-abB+a^2C)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{5/2}}$$

input
```
Integrate[(A + B*x^2 + C*x^4)/(a + b*x^2),x]
```

output
```
((b*B - a*C)*x)/b^2 + (C*x^3)/(3*b) + ((A*b^2 - a*b*B + a^2*C)*ArcTan[(Sqr
t[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(5/2))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1467, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{a+bx^2}\,dx$$

$$\Big\downarrow \text{1467}$$

$$\int \left(\frac{a^2C-abB+Ab^2}{b^2\left(a+bx^2\right)}+\frac{bB-aC}{b^2}+\frac{Cx^2}{b}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(Ab^2-a(bB-aC)\right)}{\sqrt{a}b^{5/2}}+\frac{x(bB-aC)}{b^2}+\frac{Cx^3}{3b}$$

input
```
Int[(A + B*x^2 + C*x^4)/(a + b*x^2),x]
```

output
```
((b*B - a*C)*x)/b^2 + (C*x^3)/(3*b) + ((A*b^2 - a*(b*B - a*C))*ArcTan[(Sqr
t[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(5/2))
```

Defintions of rubi rules used

rule 1467
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.86

method	result
default	$\frac{\frac{1}{3}Cb\,x^{3}+bBx-Cax}{b^{2}}+\frac{\left(b^{2}A-abB+a^{2}C\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{b^{2}\sqrt{ab}}$
risch	$\frac{C\,x^{3}}{3b}+\frac{Bx}{b}-\frac{Cax}{b^{2}}-\frac{\ln\left(bx+\sqrt{-ab}\right)A}{2\sqrt{-ab}}+\frac{\ln\left(bx+\sqrt{-ab}\right)aB}{2b\sqrt{-ab}}-\frac{\ln\left(bx+\sqrt{-ab}\right)a^{2}C}{2b^{2}\sqrt{-ab}}+\frac{\ln\left(-bx+\sqrt{-ab}\right)A}{2\sqrt{-ab}}-\frac{\ln\left(-bx+\sqrt{-ab}\right)}{2b\sqrt{-ab}}$

input
```
int((C*x^4+B*x^2+A)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/b^2*(1/3*C*b*x^3+b*B*x-C*a*x)+(A*b^2-B*a*b+C*a^2)/b^2/(a*b)^(1/2)*arctan
(b*x/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 159, normalized size of antiderivative = 2.41

$$\int \frac{A+Bx^2+Cx^4}{a+bx^2}\,dx = \left[\frac{2\,Cab^2x^3-3\,(Ca^2-Bab+Ab^2)\sqrt{-ab}\log\left(\frac{bx^2-2\sqrt{-ab}x-a}{bx^2+a}\right)-6\,(Ca^2b-Bab^2)x}{6\,ab^3},\frac{Cab^2x^3+3\,(Ca^2-B}{}\right.$$

input
```
integrate((C*x^4+B*x^2+A)/(b*x^2+a),x, algorithm="fricas")
```

output
```
[1/6*(2*C*a*b^2*x^3 - 3*(C*a^2 - B*a*b + A*b^2)*sqrt(-a*b)*log((b*x^2 - 2*
sqrt(-a*b)*x - a)/(b*x^2 + a)) - 6*(C*a^2*b - B*a*b^2)*x)/(a*b^3), 1/3*(C*
a*b^2*x^3 + 3*(C*a^2 - B*a*b + A*b^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) - 3*
(C*a^2*b - B*a*b^2)*x)/(a*b^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(56) = 112$.

Time = 0.22 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.77

$$\int \frac{A + Bx^2 + Cx^4}{a + bx^2}\,dx = \frac{Cx^3}{3b} + x\left(\frac{B}{b} - \frac{Ca}{b^2}\right) - \frac{\sqrt{-\frac{1}{ab^5}}\,(Ab^2 - Bab + Ca^2)\log\left(-ab^2\sqrt{-\frac{1}{ab^5}} + x\right)}{2} + \frac{\sqrt{-\frac{1}{ab^5}}\,(Ab^2 - Bab + Ca^2)\log\left(ab^2\sqrt{-\frac{1}{ab^5}} + x\right)}{2}$$

input
```
integrate((C*x**4+B*x**2+A)/(b*x**2+a),x)
```

output
```
C*x**3/(3*b) + x*(B/b - C*a/b**2) - sqrt(-1/(a*b**5))*(A*b**2 - B*a*b + C*
a**2)*log(-a*b**2*sqrt(-1/(a*b**5)) + x)/2 + sqrt(-1/(a*b**5))*(A*b**2 - B
*a*b + C*a**2)*log(a*b**2*sqrt(-1/(a*b**5)) + x)/2
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.88

$$\int \frac{A + Bx^2 + Cx^4}{a + bx^2}\,dx = \frac{(Ca^2 - Bab + Ab^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^2} + \frac{Cbx^3 - 3\,(Ca - Bb)x}{3\,b^2}$$

input
```
integrate((C*x^4+B*x^2+A)/(b*x^2+a),x, algorithm="maxima")
```

output
```
(C*a^2 - B*a*b + A*b^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/3*(C*b*x
^3 - 3*(C*a - B*b)*x)/b^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.94

$$\int \frac{A+Bx^2+Cx^4}{a+bx^2}\,dx = \frac{(Ca^2-Bab+Ab^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^2} + \frac{Cb^2x^3-3\,Cabx+3\,Bb^2x}{3\,b^3}$$

input
```
integrate((C*x^4+B*x^2+A)/(b*x^2+a),x, algorithm="giac")
```

output
```
(C*a^2 - B*a*b + A*b^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/3*(C*b^2
*x^3 - 3*C*a*b*x + 3*B*b^2*x)/b^3
```

Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.86

$$\int \frac{A+Bx^2+Cx^4}{a+bx^2}\,dx = x\left(\frac{B}{b}-\frac{C\,a}{b^2}\right)+\frac{C\,x^3}{3\,b}+\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,\left(C\,a^2-B\,a\,b+A\,b^2\right)}{\sqrt{a}\,b^{5/2}}$$

input
```
int((A + B*x^2 + C*x^4)/(a + b*x^2),x)
```

output
```
x*(B/b - (C*a)/b^2) + (C*x^3)/(3*b) + (atan((b^(1/2)*x)/a^(1/2))*(A*b^2 +
C*a^2 - B*a*b))/(a^(1/2)*b^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.70

$$\int \frac{A+Bx^2+Cx^4}{a+bx^2}\,dx = \frac{3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)ac-3abcx+3b^3x+b^2c\,x^3}{3b^3}$$

input
```
int((C*x^4+B*x^2+A)/(b*x^2+a),x)
```

output
```
(3*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*c - 3*a*b*c*x + 3*b**3*
x + b**2*c*x**3)/(3*b**3)
```

3.113 $\int \frac{A+Bx^2+Cx^4}{\left(a+bx^2\right)^2} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 83

$$\int \frac{A+Bx^2+Cx^4}{\left(a+bx^2\right)^2} \, dx=\frac{Cx}{b^2}+\frac{\left(\frac{A}{a}-\frac{bB-aC}{b^2}\right)x}{2\left(a+bx^2\right)}+\frac{\left(Ab^2+a(bB-3aC)\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{5/2}}$$

output
```
C*x/b^2+(A/a-(B*b-C*a)/b^2)*x/(2*b*x^2+2*a)+1/2*(A*b^2+a*(B*b-3*C*a))*arct
an(b^(1/2)*x/a^(1/2))/a^(3/2)/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.06

$$\int \frac{A+Bx^2+Cx^4}{\left(a+bx^2\right)^2} \, dx=\frac{Cx}{b^2}+\frac{\left(Ab^2-abB+a^2C\right)x}{2ab^2\left(a+bx^2\right)}-\frac{\left(-Ab^2-abB+3a^2C\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{5/2}}$$

input
```
Integrate[(A + B*x^2 + C*x^4)/(a + b*x^2)^2,x]
```

output
```
(C*x)/b^2 + ((A*b^2 - a*b*B + a^2*C)*x)/(2*a*b^2*(a + b*x^2)) - ((-(A*b^2)
 - a*b*B + 3*a^2*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*b^(5/2))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.17, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1471, 25, 27, 299, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{\left(a+bx^2\right)^2}dx$$

$$\Big\downarrow \text{ 1471}$$

$$\frac{x\left(Ab^2-a(bB-aC)\right)}{2ab^2\left(a+bx^2\right)}-\frac{\int -\frac{2aCx^2+b\left(A+\frac{a(bB-aC)}{b^2}\right)}{b(bx^2+a)}dx}{2a}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{2aCx^2+Ab+a\left(B-\frac{aC}{b}\right)}{b(bx^2+a)}dx}{2a}+\frac{x\left(Ab^2-a(bB-aC)\right)}{2ab^2\left(a+bx^2\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{2aCx^2+Ab+a\left(B-\frac{aC}{b}\right)}{bx^2+a}dx}{2ab}+\frac{x\left(Ab^2-a(bB-aC)\right)}{2ab^2\left(a+bx^2\right)}$$

$$\Big\downarrow \text{ 299}$$

$$\frac{\frac{\left(a(bB-3aC)+Ab^2\right)\int \frac{1}{bx^2+a}dx}{b}+\frac{2aCx}{b}}{2ab}+\frac{x\left(Ab^2-a(bB-aC)\right)}{2ab^2\left(a+bx^2\right)}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a(bB-3aC)+Ab^2\right)}{\sqrt{a}b^{3/2}}+\frac{2aCx}{b}}{2ab}+\frac{x\left(Ab^2-a(bB-aC)\right)}{2ab^2\left(a+bx^2\right)}$$

input
```
Int[(A + B*x^2 + C*x^4)/(a + b*x^2)^2,x]
```

output
```
((A*b^2 - a*(b*B - a*C))*x)/(2*a*b^2*(a + b*x^2)) + ((2*a*C*x)/b + ((A*b^2
 + a*(b*B - 3*a*C))*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(3/2)))/(2*a*b
)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.95

method	result
default	$\frac{Cx}{b^2}+\frac{\frac{\left(b^2A-abB+a^2C\right)x}{2a\left(b\,x^2+a\right)}+\frac{\left(b^2A+abB-3a^2C\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2a\sqrt{ab}}}{b^2}$
risch	$\frac{Cx}{b^2}+\frac{(b^2A-abB+a^2C)x}{2a\,b^2(b\,x^2+a)}-\frac{A\ln(bx+\sqrt{-ab})}{4\sqrt{-ab}\,a}-\frac{\ln(bx+\sqrt{-ab})B}{4b\sqrt{-ab}}+\frac{3a\ln(bx+\sqrt{-ab})C}{4b^2\sqrt{-ab}}+\frac{A\ln(-bx+\sqrt{-ab})}{4\sqrt{-ab}\,a}+\frac{\ln(-bx+}{4b\sqrt{}}$

input
```
int((C*x^4+B*x^2+A)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
C*x/b^2+1/b^2*(1/2*(A*b^2-B*a*b+C*a^2)/a*x/(b*x^2+a)+1/2*(A*b^2+B*a*b-3*C*
a^2)/a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 268, normalized size of antiderivative = 3.23

$$\int \frac{A+Bx^2+Cx^4}{(a+bx^2)^2}\,dx$$
$$=\left[\frac{4\,Ca^2b^2x^3+(3\,Ca^3-Ba^2b-Aab^2+(3\,Ca^2b-Bab^2-Ab^3)x^2)\sqrt{-ab}\log\left(\frac{bx^2-2\,\sqrt{-ab}x-a}{bx^2+a}\right)+2\,(3\,Ca^3}{4\,(a^2b^4x^2+a^3b^3)}\right.$$

input
```
integrate((C*x^4+B*x^2+A)/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[1/4*(4*C*a^2*b^2*x^3 + (3*C*a^3 - B*a^2*b - A*a*b^2 + (3*C*a^2*b - B*a*b^
2 - A*b^3)*x^2)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) +
 2*(3*C*a^3*b - B*a^2*b^2 + A*a*b^3)*x)/(a^2*b^4*x^2 + a^3*b^3), 1/2*(2*C*
a^2*b^2*x^3 - (3*C*a^3 - B*a^2*b - A*a*b^2 + (3*C*a^2*b - B*a*b^2 - A*b^3)
*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) + (3*C*a^3*b - B*a^2*b^2 + A*a*b^3)*
x)/(a^2*b^4*x^2 + a^3*b^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(73) = 146$.

Time = 0.41 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.84

$$\int \frac{A + Bx^2 + Cx^4}{(a + bx^2)^2}\, dx = \frac{Cx}{b^2} + \frac{x(Ab^2 - Bab + Ca^2)}{2a^2b^2 + 2ab^3x^2} + \frac{\sqrt{-\frac{1}{a^3b^5}}(-Ab^2 - Bab + 3Ca^2)\log\left(-a^2b^2\sqrt{-\frac{1}{a^3b^5}} + x\right)}{4} - \frac{\sqrt{-\frac{1}{a^3b^5}}(-Ab^2 - Bab + 3Ca^2)\log\left(a^2b^2\sqrt{-\frac{1}{a^3b^5}} + x\right)}{4}$$

input
```
integrate((C*x**4+B*x**2+A)/(b*x**2+a)**2,x)
```

output
```
C*x/b**2 + x*(A*b**2 - B*a*b + C*a**2)/(2*a**2*b**2 + 2*a*b**3*x**2) + sqr
t(-1/(a**3*b**5))*(-A*b**2 - B*a*b + 3*C*a**2)*log(-a**2*b**2*sqrt(-1/(a**
3*b**5)) + x)/4 - sqrt(-1/(a**3*b**5))*(-A*b**2 - B*a*b + 3*C*a**2)*log(a*
*2*b**2*sqrt(-1/(a**3*b**5)) + x)/4
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.01

$$\int \frac{A + Bx^2 + Cx^4}{(a + bx^2)^2}\, dx = \frac{(Ca^2 - Bab + Ab^2)x}{2\,(ab^3x^2 + a^2b^2)} + \frac{Cx}{b^2} - \frac{(3\,Ca^2 - Bab - Ab^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}ab^2}$$

input
```
integrate((C*x^4+B*x^2+A)/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*(C*a^2 - B*a*b + A*b^2)*x/(a*b^3*x^2 + a^2*b^2) + C*x/b^2 - 1/2*(3*C*a
^2 - B*a*b - A*b^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx^2+Cx^4}{(a+bx^2)^2}\,dx = \frac{Cx}{b^2} - \frac{(3\,Ca^2-Bab-Ab^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}ab^2} + \frac{Ca^2x-Babx+Ab^2x}{2\,(bx^2+a)ab^2}$$

input
```
integrate((C*x^4+B*x^2+A)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
C*x/b^2 - 1/2*(3*C*a^2 - B*a*b - A*b^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a
*b^2) + 1/2*(C*a^2*x - B*a*b*x + A*b^2*x)/((b*x^2 + a)*a*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.93

$$\int \frac{A+Bx^2+Cx^4}{(a+bx^2)^2}\,dx = \frac{C\,x}{b^2} + \frac{\operatorname{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,\left(-3\,C\,a^2+B\,a\,b+A\,b^2\right)}{2\,a^{3/2}\,b^{5/2}} + \frac{x\,\left(C\,a^2-B\,a\,b+A\,b^2\right)}{2\,a\,\left(b^3\,x^2+a\,b^2\right)}$$

input
```
int((A + B*x^2 + C*x^4)/(a + b*x^2)^2,x)
```

output
```
(C*x)/b^2 + (atan((b^(1/2)*x)/a^(1/2))*(A*b^2 - 3*C*a^2 + B*a*b))/(2*a^(3/
2)*b^(5/2)) + (x*(A*b^2 + C*a^2 - B*a*b))/(2*a*(a*b^2 + b^3*x^2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.54

$$\int \frac{A+Bx^2+Cx^4}{(a+bx^2)^2}\,dx$$
$$= \frac{-3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^2c + 2\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a\,b^2 - 3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)abc\,x^2 + 2\sqrt{b}\,\sqrt{a}\,atan\left(\cdots\right.}{2a\,b^3\,(b\,x^2+a)}$$

input
```
int((C*x^4+B*x^2+A)/(b*x^2+a)^2,x)
```

output
```
( - 3*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2*c + 2*sqrt(b)*sqr
t(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*b**2 - 3*sqrt(b)*sqrt(a)*atan((b*x)/(
sqrt(b)*sqrt(a)))*a*b*c*x**2 + 2*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(
a)))*b**3*x**2 + 3*a**2*b*c*x + 2*a*b**2*c*x**3)/(2*a*b**3*(a + b*x**2))
```

3.114 $\int \frac{A+Bx^2+Cx^4}{(a+bx^2)^3} dx$

Optimal result

Integrand size = 22, antiderivative size = 115

$$\int \frac{A+Bx^2+Cx^4}{(a+bx^2)^3} dx = \frac{\left(\frac{A}{a}-\frac{bB-aC}{b^2}\right)x}{4(a+bx^2)^2}+\frac{(3Ab^2+a(bB-5aC))x}{8a^2b^2(a+bx^2)} + \frac{(3Ab^2+a(bB+3aC))\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{5/2}}$$

output
```
1/4*(A/a-(B*b-C*a)/b^2)*x/(b*x^2+a)^2+1/8*(3*A*b^2+a*(B*b-5*C*a))*x/a^2/b^
2/(b*x^2+a)+1/8*(3*A*b^2+a*(B*b+3*C*a))*arctan(b^(1/2)*x/a^(1/2))/a^(5/2)/
b^(5/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.95

$$\int \frac{A+Bx^2+Cx^4}{(a+bx^2)^3} dx = \frac{x(-3a^3C+3Ab^3x^2+ab^2(5A+Bx^2)-a^2b(B+5Cx^2))}{8a^2b^2(a+bx^2)^2} + \frac{(3Ab^2+a(bB+3aC))\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{5/2}}$$

input
```
Integrate[(A + B*x^2 + C*x^4)/(a + b*x^2)^3,x]
```

output
```
(x*(-3*a^3*C + 3*A*b^3*x^2 + a*b^2*(5*A + B*x^2) - a^2*b*(B + 5*C*x^2)))/(
8*a^2*b^2*(a + b*x^2)^2) + ((3*A*b^2 + a*(b*B + 3*a*C))*ArcTan[(Sqrt[b]*x)
/Sqrt[a]])/(8*a^(5/2)*b^(5/2))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {1471, 25, 27, 298, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{(a+bx^2)^3}dx$$

$$\Big\downarrow \text{ 1471}$$

$$\frac{x\left(Ab^2-a(bB-aC)\right)}{4ab^2\left(a+bx^2\right)^2}-\frac{\int -\frac{4aCx^2+b\left(3A+\frac{a(bB-aC)}{b^2}\right)}{b(bx^2+a)^2}dx}{4a}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{4aCx^2+3Ab+a\left(B-\frac{aC}{b}\right)}{b(bx^2+a)^2}dx}{4a}+\frac{x\left(Ab^2-a(bB-aC)\right)}{4ab^2\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{4aCx^2+3Ab+a\left(B-\frac{aC}{b}\right)}{(bx^2+a)^2}dx}{4ab}+\frac{x\left(Ab^2-a(bB-aC)\right)}{4ab^2\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{ 298}$$

$$\frac{\frac{1}{2}\left(\frac{3Ab}{a}+\frac{3aC}{b}+B\right)\int \frac{1}{bx^2+a}dx+\frac{x\left(\frac{3Ab}{a}-\frac{5aC}{b}+B\right)}{2(a+bx^2)}}{4ab}+\frac{x\left(Ab^2-a(bB-aC)\right)}{4ab^2\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(\frac{3Ab}{a}+\frac{3aC}{b}+B\right)}{2\sqrt{a}\sqrt{b}}+\frac{x\left(\frac{3Ab}{a}-\frac{5aC}{b}+B\right)}{2(a+bx^2)}}{4ab}+\frac{x(Ab^2-a(bB-aC))}{4ab^2\left(a+bx^2\right)^2}$$

input
```
Int[(A + B*x^2 + C*x^4)/(a + b*x^2)^3,x]
```

output
```
((A*b^2 - a*(b*B - a*C))*x)/(4*a*b^2*(a + b*x^2)^2) + ((((3*A*b)/a + B - (
5*a*C)/b)*x)/(2*(a + b*x^2)) + (((3*A*b)/a + B + (3*a*C)/b)*ArcTan[(Sqrt[b
]*x)/Sqrt[a]])/(2*Sqrt[a]*Sqrt[b]))/(4*a*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.93

method	result
default	$\frac{\frac{\left(3b^2A+abB-5a^2C\right)x^3}{8a^2b}+\frac{\left(5b^2A-abB-3a^2C\right)x}{8a\,b^2}}{\left(b\,x^2+a\right)^2}+\frac{\left(3b^2A+abB+3a^2C\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8a^2b^2\sqrt{ab}}$
risch	$\frac{\frac{\left(3b^2A+abB-5a^2C\right)x^3}{8a^2b}+\frac{\left(5b^2A-abB-3a^2C\right)x}{8a\,b^2}}{\left(b\,x^2+a\right)^2}-\frac{3A\ln\left(bx+\sqrt{-ab}\right)}{16\sqrt{-ab}\,a^2}-\frac{\ln\left(bx+\sqrt{-ab}\right)B}{16\sqrt{-ab}\,ba}-\frac{3\ln\left(bx+\sqrt{-ab}\right)C}{16\sqrt{-ab}\,b^2}+\frac{3A\ln\left(-bx+\sqrt{-\right.}{16\sqrt{-ab}\,a^2}$

input
```
int((C*x^4+B*x^2+A)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/8*(3*A*b^2+B*a*b-5*C*a^2)/a^2/b*x^3+1/8*(5*A*b^2-B*a*b-3*C*a^2)/a/b^2*x
)/(b*x^2+a)^2+1/8*(3*A*b^2+B*a*b+3*C*a^2)/a^2/b^2/(a*b)^(1/2)*arctan(b*x/(
a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 391, normalized size of antiderivative = 3.40

$$\int \frac{A+Bx^2+Cx^4}{\left(a+bx^2\right)^3}\,dx$$

$$= \left[-\frac{2\,(5\,Ca^3b^2-Ba^2b^3-3\,Aab^4)x^3+(3\,Ca^4+Ba^3b+3\,Aa^2b^2+(3\,Ca^2b^2+Bab^3+3\,Ab^4)x^4+2\,(3\,Ca}{16\,(a^3b^5x^4+2\,a^4b^4x^2-}\right.$$

$$-\frac{(5\,Ca^3b^2-Ba^2b^3-3\,Aab^4)x^3-(3\,Ca^4+Ba^3b+3\,Aa^2b^2+(3\,Ca^2b^2+Bab^3+3\,Ab^4)x^4+2\,(3\,Ca^3b}{8\,(a^3b^5x^4+2\,a^4b^4x^2+a^5b^3)}$$

input
```
integrate((C*x^4+B*x^2+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
[-1/16*(2*(5*C*a^3*b^2 - B*a^2*b^3 - 3*A*a*b^4)*x^3 + (3*C*a^4 + B*a^3*b +
 3*A*a^2*b^2 + (3*C*a^2*b^2 + B*a*b^3 + 3*A*b^4)*x^4 + 2*(3*C*a^3*b + B*a^
2*b^2 + 3*A*a*b^3)*x^2)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2
 + a)) + 2*(3*C*a^4*b + B*a^3*b^2 - 5*A*a^2*b^3)*x)/(a^3*b^5*x^4 + 2*a^4*b
^4*x^2 + a^5*b^3), -1/8*((5*C*a^3*b^2 - B*a^2*b^3 - 3*A*a*b^4)*x^3 - (3*C*
a^4 + B*a^3*b + 3*A*a^2*b^2 + (3*C*a^2*b^2 + B*a*b^3 + 3*A*b^4)*x^4 + 2*(3
*C*a^3*b + B*a^2*b^2 + 3*A*a*b^3)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) + (
3*C*a^4*b + B*a^3*b^2 - 5*A*a^2*b^3)*x)/(a^3*b^5*x^4 + 2*a^4*b^4*x^2 + a^5
*b^3)]
```

Sympy [A] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.70

$$\int \frac{A+Bx^2+Cx^4}{(a+bx^2)^3}\,dx = -\frac{\sqrt{-\frac{1}{a^5b^5}}\cdot(3Ab^2+Bab+3Ca^2)\log\left(-a^3b^2\sqrt{-\frac{1}{a^5b^5}}+x\right)}{16} + \frac{\sqrt{-\frac{1}{a^5b^5}}\cdot(3Ab^2+Bab+3Ca^2)\log\left(a^3b^2\sqrt{-\frac{1}{a^5b^5}}+x\right)}{16} + \frac{x^3\cdot(3Ab^3+Bab^2-5Ca^2b)+x(5Aab^2-Ba^2b-3Ca^3)}{8a^4b^2+16a^3b^3x^2+8a^2b^4x^4}$$

input
```
integrate((C*x**4+B*x**2+A)/(b*x**2+a)**3,x)
```

output
```
-sqrt(-1/(a**5*b**5))*(3*A*b**2 + B*a*b + 3*C*a**2)*log(-a**3*b**2*sqrt(-1
/(a**5*b**5)) + x)/16 + sqrt(-1/(a**5*b**5))*(3*A*b**2 + B*a*b + 3*C*a**2)
*log(a**3*b**2*sqrt(-1/(a**5*b**5)) + x)/16 + (x**3*(3*A*b**3 + B*a*b**2 -
 5*C*a**2*b) + x*(5*A*a*b**2 - B*a**2*b - 3*C*a**3))/(8*a**4*b**2 + 16*a**
3*b**3*x**2 + 8*a**2*b**4*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.05

$$\int \frac{A+Bx^2+Cx^4}{(a+bx^2)^3}\,dx = -\frac{(5\,Ca^2b-Bab^2-3\,Ab^3)x^3+(3\,Ca^3+Ba^2b-5\,Aab^2)x}{8\,(a^2b^4x^4+2\,a^3b^3x^2+a^4b^2)} + \frac{(3\,Ca^2+Bab+3\,Ab^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^2b^2}$$

input
```
integrate((C*x^4+B*x^2+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
-1/8*((5*C*a^2*b - B*a*b^2 - 3*A*b^3)*x^3 + (3*C*a^3 + B*a^2*b - 5*A*a*b^2
)*x)/(a^2*b^4*x^4 + 2*a^3*b^3*x^2 + a^4*b^2) + 1/8*(3*C*a^2 + B*a*b + 3*A*
b^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.95

$$\int \frac{A+Bx^2+Cx^4}{(a+bx^2)^3}\,dx = \frac{(3\,Ca^2+Bab+3\,Ab^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^2b^2} - \frac{5\,Ca^2bx^3-Bab^2x^3-3\,Ab^3x^3+3\,Ca^3x+Ba^2bx-5\,Aab^2x}{8\,(bx^2+a)^2a^2b^2}$$

input
```
integrate((C*x^4+B*x^2+A)/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
1/8*(3*C*a^2 + B*a*b + 3*A*b^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b^2)
- 1/8*(5*C*a^2*b*x^3 - B*a*b^2*x^3 - 3*A*b^3*x^3 + 3*C*a^3*x + B*a^2*b*x -
 5*A*a*b^2*x)/((b*x^2 + a)^2*a^2*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.97

$$\int \frac{A+Bx^2+Cx^4}{(a+bx^2)^3}\,dx = \frac{\frac{x^3\,(-5\,C\,a^2+B\,a\,b+3\,A\,b^2)}{8\,a^2\,b}-\frac{x\,(3\,C\,a^2+B\,a\,b-5\,A\,b^2)}{8\,a\,b^2}}{a^2+2\,a\,b\,x^2+b^2\,x^4} + \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,(3\,C\,a^2+B\,a\,b+3\,A\,b^2)}{8\,a^{5/2}\,b^{5/2}}$$

input
```
int((A + B*x^2 + C*x^4)/(a + b*x^2)^3,x)
```

output
```
((x^3*(3*A*b^2 - 5*C*a^2 + B*a*b))/(8*a^2*b) - (x*(3*C*a^2 - 5*A*b^2 + B*a
*b))/(8*a*b^2))/(a^2 + b^2*x^4 + 2*a*b*x^2) + (atan((b^(1/2)*x)/a^(1/2))*(
3*A*b^2 + 3*C*a^2 + B*a*b))/(8*a^(5/2)*b^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.86

$$\int \frac{A+Bx^2+Cx^4}{(a+bx^2)^3}\,dx = \frac{3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^3c+4\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^2b^2+6\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^2bc\,x^2+8\sqrt{b}\,\sqrt{a}\,atan\left(\ldots\right.}{8a^2b^3\,(b^2\ldots}$$

input
```
int((C*x^4+B*x^2+A)/(b*x^2+a)^3,x)
```

output
```
(3*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**3*c + 4*sqrt(b)*sqrt(a
)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2*b**2 + 6*sqrt(b)*sqrt(a)*atan((b*x)/(
sqrt(b)*sqrt(a)))*a**2*b*c*x**2 + 8*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sq
rt(a)))*a*b**3*x**2 + 3*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*b*
*2*c*x**4 + 4*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**4*x**4 - 3*
a**3*b*c*x + 4*a**2*b**3*x - 5*a**2*b**2*c*x**3 + 4*a*b**4*x**3)/(8*a**2*b
**3*(a**2 + 2*a*b*x**2 + b**2*x**4))
```

3.115 $\int (a+bx^2)^{3/2} (A+Bx^2+Cx^4)\, dx$

Optimal result

Integrand size = 24, antiderivative size = 175

$$\int (a+bx^2)^{3/2} (A+Bx^2+Cx^4)\, dx = \frac{a(48Ab^2-a(8bB-3aC))\,x\sqrt{a+bx^2}}{128b^2} + \frac{1}{192}\left(48A-\frac{a(8bB-3aC)}{b^2}\right)x(a+bx^2)^{3/2} + \frac{(8bB-3aC)x(a+bx^2)^{5/2}}{48b^2} + \frac{Cx^3(a+bx^2)^{5/2}}{8b} + \frac{a^2(48Ab^2-a(8bB-3aC))\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{5/2}}$$

output
```
1/128*a*(48*A*b^2-a*(8*B*b-3*C*a))*x*(b*x^2+a)^(1/2)/b^2+1/192*(48*A-a*(8*
B*b-3*C*a)/b^2)*x*(b*x^2+a)^(3/2)+1/48*(8*B*b-3*C*a)*x*(b*x^2+a)^(5/2)/b^2
+1/8*C*x^3*(b*x^2+a)^(5/2)/b+1/128*a^2*(48*A*b^2-a*(8*B*b-3*C*a))*arctanh(
b^(1/2)*x/(b*x^2+a)^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.80

$$\int \left(a+bx^2\right)^{3/2} \left(A+Bx^2+Cx^4\right) \, dx = \frac{\sqrt{b}x\sqrt{a+bx^2}(-9a^3C+6a^2b(4B+Cx^2)+16b^3x^2(6A+4Bx^2+3Cx^4)+8ab^2(30A+14Bx^2}{384b^{5/2}}$$

input
```
Integrate[(a + b*x^2)^(3/2)*(A + B*x^2 + C*x^4),x]
```

output
```
(Sqrt[b]*x*Sqrt[a + b*x^2]*(-9*a^3*C + 6*a^2*b*(4*B + C*x^2) + 16*b^3*x^2*
(6*A + 4*B*x^2 + 3*C*x^4) + 8*a*b^2*(30*A + 14*B*x^2 + 9*C*x^4)) - 3*a^2*(
48*A*b^2 + a*(-8*b*B + 3*a*C))*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(384*b
^(5/2))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.87, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1473, 299, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^{3/2} \left(A+Bx^2+Cx^4\right) \, dx$$

$$\Big\downarrow \ 1473$$

$$\frac{\int \left(bx^2+a\right)^{3/2} \left((8bB-3aC)x^2+8Ab\right) dx}{8b}+\frac{Cx^3\left(a+bx^2\right)^{5/2}}{8b}$$

$$\Big\downarrow \ 299$$

$$\frac{\frac{\left(48Ab^2-a(8bB-3aC)\right)\int \left(bx^2+a\right)^{3/2}dx}{6b}+\frac{x\left(a+bx^2\right)^{5/2}(8bB-3aC)}{6b}}{8b}+\frac{Cx^3\left(a+bx^2\right)^{5/2}}{8b}$$

$$\Big\downarrow \ 211$$

$$\frac{\frac{\left(48Ab^2-a(8bB-3aC)\right)\left(\frac{3}{4}a\int\sqrt{bx^2+a}dx+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)}{6b}+\frac{x\left(a+bx^2\right)^{5/2}(8bB-3aC)}{6b}}{8b}+\frac{Cx^3\left(a+bx^2\right)^{5/2}}{8b}$$

$\Big\downarrow$ 211

$$\frac{\frac{\left(48Ab^2-a(8bB-3aC)\right)\left(\frac{3}{4}a\left(\frac{1}{2}a\int\frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)}{6b}+\frac{x\left(a+bx^2\right)^{5/2}(8bB-3aC)}{6b}}{8b}+\frac{Cx^3\left(a+bx^2\right)^{5/2}}{8b}$$

$\Big\downarrow$ 224

$$\frac{\frac{\left(48Ab^2-a(8bB-3aC)\right)\left(\frac{3}{4}a\left(\frac{1}{2}a\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)}{6b}+\frac{x\left(a+bx^2\right)^{5/2}(8bB-3aC)}{6b}}{8b}+\frac{Cx^3\left(a+bx^2\right)^{5/2}}{8b}$$

$\Big\downarrow$ 219

$$\frac{\frac{\left(\frac{3}{4}a\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)\left(48Ab^2-a(8bB-3aC)\right)}{6b}+\frac{x\left(a+bx^2\right)^{5/2}(8bB-3aC)}{6b}}{8b}+\frac{Cx^3\left(a+bx^2\right)^{5/2}}{8b}$$

input

```
Int[(a + b*x^2)^(3/2)*(A + B*x^2 + C*x^4),x]
```

output

```
(C*x^3*(a + b*x^2)^(5/2))/(8*b) + (((8*b*B - 3*a*C)*x*(a + b*x^2)^(5/2))/(
6*b) + ((48*A*b^2 - a*(8*b*B - 3*a*C))*((x*(a + b*x^2)^(3/2))/4 + (3*a*((x
*Sqrt[a + b*x^2])/2 + (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b])
))/4))/(6*b))/(8*b)
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1473
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[c^p*x^(4*p - 1)*((d + e*x^2)^(q + 1)/(e*(4*p + 2*q + 1)))
, x] + Simp[1/(e*(4*p + 2*q + 1))   Int[(d + e*x^2)^q*ExpandToSum[e*(4*p +
2*q + 1)*(a + b*x^2 + c*x^4)^p - d*c^p*(4*p - 1)*x^(4*p - 2) - e*c^p*(4*p +
 2*q + 1)*x^(4*p), x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] &&  !LtQ[q, -1]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.71

method	result
pseudoelliptic	$\frac{\frac{3\left(b^{2} A-\frac{1}{6} a b B+\frac{1}{16} a^{2} C\right) a^{2} \operatorname{arctanh}\left(\frac{\sqrt{b x^{2}+a}}{x \sqrt{b}}\right)}{8}+\frac{5 \sqrt{b x^{2}+a}\left(a\left(\frac{3}{10} C x^{4}+\frac{7}{15} x^{2} B+A\right) b^{\frac{5}{2}}+\frac{2\left(\frac{1}{2} C x^{4}+\frac{2}{3} x^{2} B+A\right) x^{2} b^{\frac{7}{2}}}{5}+\frac{\left(\left(\frac{C x^{2}}{4}+B \cdots\right.\right.}{\cdots}\right.}{8}}{b^{\frac{5}{2}}}$
risch	$\frac{x\left(48 b^{3} C x^{6}+64 B b^{3} x^{4}+72 C a b^{2} x^{4}+96 A x^{2} b^{3}+112 B x^{2} a b^{2}+6 C a^{2} b x^{2}+240 a b^{2} A+24 a^{2} b B-9 C a^{3}\right) \sqrt{b x^{2}+a}}{384 b^{2}}+\frac{a^{2}(48 b \cdots}{\cdots}$
default	$A\left(\frac{x\left(b x^{2}+a\right)^{\frac{3}{2}}}{4}+\frac{3 a\left(\frac{x \sqrt{b x^{2}+a}}{2}+\frac{a \ln \left(\sqrt{b}\, x+\sqrt{b x^{2}+a}\right)}{2 \sqrt{b}}\right)}{4}\right)+C\left(\frac{x^{3}\left(b x^{2}+a\right)^{\frac{5}{2}}}{8 b}-\frac{3 a\left(\frac{x\left(b x^{2}+a\right)^{\frac{5}{2}}}{6 b}-\frac{a\left(\frac{x\left(b x^{2}+a \cdots\right.}{4} \cdots\right)}{\cdots}\right)}{\cdots} \cdots\right.$

input
```
int((b*x^2+a)^(3/2)*(C*x^4+B*x^2+A),x,method=_RETURNVERBOSE)
```

output
```
5/8/b^(5/2)*(3/5*(b^2*A-1/6*a*b*B+1/16*a^2*C)*a^2*arctanh((b*x^2+a)^(1/2)/
x/b^(1/2))+(b*x^2+a)^(1/2)*(a*(3/10*C*x^4+7/15*x^2*B+A)*b^(5/2)+2/5*(1/2*C
*x^4+2/3*x^2*B+A)*x^2*b^(7/2)+1/10*((1/4*C*x^2+B)*b^(3/2)-3/8*C*a*b^(1/2))
*a^2)*x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.74

$$\int \left(a+bx^2\right)^{3/2}\left(A+Bx^2+Cx^4\right)\,dx = \left[\frac{3\,(3\,Ca^4-8\,Ba^3b+48\,Aa^2b^2)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,(48\,Cb^4x^7+8\,(9\,C\right.$$

$$-\frac{3\,(3\,Ca^4-8\,Ba^3b+48\,Aa^2b^2)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(48\,Cb^4x^7+8\,(9\,Cab^3+8\,Bb^4)x^5+2\,(3\,Ca^2b^2+}{384\,b^3}$$

input
```
integrate((b*x^2+a)^(3/2)*(C*x^4+B*x^2+A),x, algorithm="fricas")
```

output
```
[1/768*(3*(3*C*a^4 - 8*B*a^3*b + 48*A*a^2*b^2)*sqrt(b)*log(-2*b*x^2 - 2*sq
rt(b*x^2 + a)*sqrt(b)*x - a) + 2*(48*C*b^4*x^7 + 8*(9*C*a*b^3 + 8*B*b^4)*x
^5 + 2*(3*C*a^2*b^2 + 56*B*a*b^3 + 48*A*b^4)*x^3 - 3*(3*C*a^3*b - 8*B*a^2*
b^2 - 80*A*a*b^3)*x)*sqrt(b*x^2 + a))/b^3, -1/384*(3*(3*C*a^4 - 8*B*a^3*b
+ 48*A*a^2*b^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (48*C*b^4*x^
7 + 8*(9*C*a*b^3 + 8*B*b^4)*x^5 + 2*(3*C*a^2*b^2 + 56*B*a*b^3 + 48*A*b^4)*
x^3 - 3*(3*C*a^3*b - 8*B*a^2*b^2 - 80*A*a*b^3)*x)*sqrt(b*x^2 + a))/b^3]
```

Sympy [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.57

$$\int \left(a+bx^2\right)^{3/2}\left(A+Bx^2+Cx^4\right)\,dx = \begin{cases} \sqrt{a+bx^2}\left(\frac{Cbx^7}{8}+\frac{x^5\left(Bb^2+\frac{9Cab}{8}\right)}{6b}+\frac{x^3\left(Ab^2+2Bab+Ca^2-\frac{5a\left(Bb^2+\frac{9Cab}{8}\right)}{6b}\right)}{4b}+\frac{x\left(2Aab+Ba^2-\frac{3a\left(Ab^2+2Bab+C\right.}{}\right.}{2b}\right. \\ a^{\frac{3}{2}}\left(Ax+\frac{Bx^3}{3}+\frac{Cx^5}{5}\right) \end{cases}$$

input
```
integrate((b*x**2+a)**(3/2)*(C*x**4+B*x**2+A),x)
```

output
```
Piecewise((sqrt(a + b*x**2)*(C*b*x**7/8 + x**5*(B*b**2 + 9*C*a*b/8)/(6*b)
+ x**3*(A*b**2 + 2*B*a*b + C*a**2 - 5*a*(B*b**2 + 9*C*a*b/8)/(6*b))/(4*b)
+ x*(2*A*a*b + B*a**2 - 3*a*(A*b**2 + 2*B*a*b + C*a**2 - 5*a*(B*b**2 + 9*C
*a*b/8)/(6*b))/(4*b))/(2*b)) + (A*a**2 - a*(2*A*a*b + B*a**2 - 3*a*(A*b**2
 + 2*B*a*b + C*a**2 - 5*a*(B*b**2 + 9*C*a*b/8)/(6*b))/(4*b))/(2*b))*Piecew
ise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)
/sqrt(b*x**2), True)), Ne(b, 0)), (a**(3/2)*(A*x + B*x**3/3 + C*x**5/5), T
rue))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.18

$$\int \left(a+bx^2\right)^{3/2}\left(A+Bx^2+Cx^4\right)\,dx=\frac{(bx^2+a)^{\frac{5}{2}}Cx^3}{8\,b}+\frac{1}{4}\,(bx^2+a)^{\frac{3}{2}}Ax+\frac{3}{8}\,\sqrt{bx^2+a}Aax-\frac{(bx^2+a)^{\frac{5}{2}}Cax}{16\,b^2}+\frac{(bx^2+a)^{\frac{3}{2}}Ca^2x}{64\,b^2}+\frac{3\,\sqrt{bx^2+a}Ca^3x}{128\,b^2}+\frac{(bx^2+a)^{\frac{5}{2}}Bx}{6\,b}-\frac{(bx^2+a)^{\frac{3}{2}}Bax}{24\,b}-\frac{\sqrt{bx^2+a}Ba^2x}{16\,b}+\frac{3\,Ca^4\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\,b^{\frac{5}{2}}}-\frac{Ba^3\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{3}{2}}}+\frac{3\,Aa^2\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{b}}$$

input
```
integrate((b*x^2+a)^(3/2)*(C*x^4+B*x^2+A),x, algorithm="maxima")
```

output
```
1/8*(b*x^2 + a)^(5/2)*C*x^3/b + 1/4*(b*x^2 + a)^(3/2)*A*x + 3/8*sqrt(b*x^2
 + a)*A*a*x - 1/16*(b*x^2 + a)^(5/2)*C*a*x/b^2 + 1/64*(b*x^2 + a)^(3/2)*C*
a^2*x/b^2 + 3/128*sqrt(b*x^2 + a)*C*a^3*x/b^2 + 1/6*(b*x^2 + a)^(5/2)*B*x/
b - 1/24*(b*x^2 + a)^(3/2)*B*a*x/b - 1/16*sqrt(b*x^2 + a)*B*a^2*x/b + 3/12
8*C*a^4*arcsinh(b*x/sqrt(a*b))/b^(5/2) - 1/16*B*a^3*arcsinh(b*x/sqrt(a*b))
/b^(3/2) + 3/8*A*a^2*arcsinh(b*x/sqrt(a*b))/sqrt(b)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.89

$$\int \left(a+bx^2\right)^{3/2} \left(A+Bx^2+Cx^4\right) dx = \frac{1}{384}\left(2\left(4\left(6\,Cbx^2+\frac{9\,Cab^6+8\,Bb^7}{b^6}\right)x^2+\frac{3\,Ca^2b^5+56\,Bab^6+48\,Ab^7}{b^6}\right)x^2-\frac{3\,(3\,Ca^3b^4-\right.$$

$$-\frac{(3\,Ca^4-8\,Ba^3b+48\,Aa^2b^2)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{128\,b^{\frac{5}{2}}}$$

input
```
integrate((b*x^2+a)^(3/2)*(C*x^4+B*x^2+A),x, algorithm="giac")
```

output
```
1/384*(2*(4*(6*C*b*x^2 + (9*C*a*b^6 + 8*B*b^7)/b^6)*x^2 + (3*C*a^2*b^5 + 5
6*B*a*b^6 + 48*A*b^7)/b^6)*x^2 - 3*(3*C*a^3*b^4 - 8*B*a^2*b^5 - 80*A*a*b^6
)/b^6)*sqrt(b*x^2 + a)*x - 1/128*(3*C*a^4 - 8*B*a^3*b + 48*A*a^2*b^2)*log(
abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^2\right)^{3/2} \left(A+Bx^2+Cx^4\right) dx = \int \left(b\,x^2+a\right)^{3/2}\,\left(C\,x^4+B\,x^2+A\right) dx$$

input
```
int((a + b*x^2)^(3/2)*(A + B*x^2 + C*x^4),x)
```

output
```
int((a + b*x^2)^(3/2)*(A + B*x^2 + C*x^4), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.05

$$\int (a+bx^2)^{3/2} (A+Bx^2+Cx^4)\,dx = \frac{-9\sqrt{b\,x^2+a}\,a^3bcx+264\sqrt{b\,x^2+a}\,a^2b^3x+6\sqrt{b\,x^2+a}\,a^2b^2c\,x^3+208\sqrt{b\,x^2+a}\,a\,b^4x^3+72\sqrt{}}{}$$

input
```
int((b*x^2+a)^(3/2)*(C*x^4+B*x^2+A),x)
```

output
```
( - 9*sqrt(a + b*x**2)*a**3*b*c*x + 264*sqrt(a + b*x**2)*a**2*b**3*x + 6*s
qrt(a + b*x**2)*a**2*b**2*c*x**3 + 208*sqrt(a + b*x**2)*a*b**4*x**3 + 72*s
qrt(a + b*x**2)*a*b**3*c*x**5 + 64*sqrt(a + b*x**2)*b**5*x**5 + 48*sqrt(a
+ b*x**2)*b**4*c*x**7 + 9*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(
a))*a**4*c + 120*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**3*
b**2)/(384*b**3)
```

3.116 $\int \sqrt{a+bx^2}\left(A+Bx^2+Cx^4\right)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 134

$$\int \sqrt{a+bx^2}\left(A+Bx^2+Cx^4\right)\,dx=\frac{1}{16}\left(8A-\frac{a(2bB-aC)}{b^2}\right)x\sqrt{a+bx^2}+\frac{(2bB-aC)x\left(a+bx^2\right)^{3/2}}{8b^2}+\frac{Cx^3\left(a+bx^2\right)^{3/2}}{6b}+\frac{a\left(8Ab^2-a(2bB-aC)\right)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{5/2}}$$

output
```
1/16*(8*A-a*(2*B*b-C*a)/b^2)*x*(b*x^2+a)^(1/2)+1/8*(2*B*b-C*a)*x*(b*x^2+a)
^(3/2)/b^2+1/6*C*x^3*(b*x^2+a)^(3/2)/b+1/16*a*(8*A*b^2-a*(2*B*b-C*a))*arct
anh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.84

$$\int \sqrt{a+bx^2}\left(A+Bx^2+Cx^4\right)\,dx=\frac{x\sqrt{a+bx^2}\left(24Ab^2+6abB-3a^2C+12b^2Bx^2+2abCx^2+8b^2Cx^4\right)}{48b^2}-\frac{a\left(8Ab^2-2abB+a^2C\right)\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{16b^{5/2}}$$

input
```
Integrate[Sqrt[a + b*x^2]*(A + B*x^2 + C*x^4),x]
```

output
```
(x*Sqrt[a + b*x^2]*(24*A*b^2 + 6*a*b*B - 3*a^2*C + 12*b^2*B*x^2 + 2*a*b*C*
x^2 + 8*b^2*C*x^4))/(48*b^2) - (a*(8*A*b^2 - 2*a*b*B + a^2*C)*Log[-(Sqrt[b
]*x) + Sqrt[a + b*x^2]])/(16*b^(5/2))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1473, 27, 299, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx^2}\left(A+Bx^2+Cx^4\right)\,dx$$

$$\Big\downarrow \text{1473}$$

$$\frac{\int 3\sqrt{bx^2+a}\left((2bB-aC)x^2+2Ab\right)dx}{6b}+\frac{Cx^3\left(a+bx^2\right)^{3/2}}{6b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \sqrt{bx^2+a}\left((2bB-aC)x^2+2Ab\right)dx}{2b}+\frac{Cx^3\left(a+bx^2\right)^{3/2}}{6b}$$

$$\Big\downarrow \text{299}$$

$$\frac{\frac{\left(8Ab^2-a(2bB-aC)\right)\int \sqrt{bx^2+a}dx}{4b}+\frac{x\left(a+bx^2\right)^{3/2}(2bB-aC)}{4b}}{2b}+\frac{Cx^3\left(a+bx^2\right)^{3/2}}{6b}$$

$$\Big\downarrow \text{211}$$

$$\frac{\frac{\left(8Ab^2-a(2bB-aC)\right)\left(\frac{1}{2}a\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)}{4b}+\frac{x\left(a+bx^2\right)^{3/2}(2bB-aC)}{4b}}{2b}+\frac{Cx^3\left(a+bx^2\right)^{3/2}}{6b}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{\left(8Ab^2-a(2bB-aC)\right)\left(\frac{1}{2}a\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)}{4b}+\frac{x\left(a+bx^2\right)^{3/2}(2bB-aC)}{4b}}{2b}+\frac{Cx^3\left(a+bx^2\right)^{3/2}}{6b}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\frac{\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)\left(8Ab^2-a(2bB-aC)\right)}{4b}+\frac{x\left(a+bx^2\right)^{3/2}(2bB-aC)}{4b}}{2b}+\frac{Cx^3\left(a+bx^2\right)^{3/2}}{6b}$$

input
```
Int[Sqrt[a + b*x^2]*(A + B*x^2 + C*x^4),x]
```

output
```
(C*x^3*(a + b*x^2)^(3/2))/(6*b) + (((2*b*B - a*C)*x*(a + b*x^2)^(3/2))/(4*
b) + ((8*A*b^2 - a*(2*b*B - a*C))*((x*Sqrt[a + b*x^2])/2 + (a*ArcTanh[(Sqr
t[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b])))/(4*b))/(2*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1473
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[c^p*x^(4*p - 1)*((d + e*x^2)^(q + 1)/(e*(4*p + 2*q + 1)))
, x] + Simp[1/(e*(4*p + 2*q + 1))   Int[(d + e*x^2)^q*ExpandToSum[e*(4*p +
2*q + 1)*(a + b*x^2 + c*x^4)^p - d*c^p*(4*p - 1)*x^(4*p - 2) - e*c^p*(4*p +
 2*q + 1)*x^(4*p), x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] &&  !LtQ[q, -1]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.72

method	result
pseudoelliptic	$\frac{a\left(b^{2} A-\frac{1}{4} a b B+\frac{1}{8} a^{2} C\right) \operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{x \sqrt{b}}\right)+\sqrt{b\,x^{2}+a}\,x\left(\left(\frac{1}{3} C\,x^{4}+\frac{1}{2} x^{2} B+A\right) b^{\frac{5}{2}}+\frac{a\left(\left(\frac{C\,x^{2}}{3}+B\right) b^{\frac{3}{2}}-\frac{C a \sqrt{b}}{2}\right)}{4}\right)}{2 b^{\frac{5}{2}}}$
risch	$\frac{x\left(8 C\,b^{2} x^{4}+12 b^{2} B\,x^{2}+2 x^{2} a C b+24 b^{2} A+6 a b B-3 a^{2} C\right) \sqrt{b\,x^{2}+a}}{48 b^{2}}+\frac{a\left(8 b^{2} A-2 a b B+a^{2} C\right) \ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{16 b^{\frac{5}{2}}}$
default	$A\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)+C\left(\frac{x^{3}\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{6b}-\frac{a\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{4b}-\frac{a\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+\cdots}\right.}{2\sqrt{b}}\right.}{4b}\right.}{2b}\right.$

input
```
int((b*x^2+a)^(1/2)*(C*x^4+B*x^2+A),x,method=_RETURNVERBOSE)
```

output
```
1/2*(a*(b^2*A-1/4*a*b*B+1/8*a^2*C)*arctanh((b*x^2+a)^(1/2)/x/b^(1/2))+(b*x
^2+a)^(1/2)*x*((1/3*C*x^4+1/2*x^2*B+A)*b^(5/2)+1/4*a*((1/3*C*x^2+B)*b^(3/2
)-1/2*C*a*b^(1/2))))/b^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.73

$$\int \sqrt{a+bx^2}\left(A+Bx^2+Cx^4\right)\,dx$$
$$=\left[\frac{3\,(Ca^3-2\,Ba^2b+8\,Aab^2)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,(8\,Cb^3x^5+2\,(Cab^2+6\,Bb^3)x^3-}{96\,b^3}\right.$$
$$-\frac{3\,(Ca^3-2\,Ba^2b+8\,Aab^2)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(8\,Cb^3x^5+2\,(Cab^2+6\,Bb^3)x^3-3\,(Ca^2b-2\,Bab}{48\,b^3}$$

input
```
integrate((b*x^2+a)^(1/2)*(C*x^4+B*x^2+A),x, algorithm="fricas")
```

output
```
[1/96*(3*(C*a^3 - 2*B*a^2*b + 8*A*a*b^2)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x
^2 + a)*sqrt(b)*x - a) + 2*(8*C*b^3*x^5 + 2*(C*a*b^2 + 6*B*b^3)*x^3 - 3*(C
*a^2*b - 2*B*a*b^2 - 8*A*b^3)*x)*sqrt(b*x^2 + a))/b^3, -1/48*(3*(C*a^3 - 2
*B*a^2*b + 8*A*a*b^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (8*C*b
^3*x^5 + 2*(C*a*b^2 + 6*B*b^3)*x^3 - 3*(C*a^2*b - 2*B*a*b^2 - 8*A*b^3)*x)*
sqrt(b*x^2 + a))/b^3]
```

Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.14

$$\int \sqrt{a+bx^2}\left(A+Bx^2+Cx^4\right)\,dx$$
$$=\begin{cases}\sqrt{a+bx^2}\left(\frac{Cx^5}{6}+\frac{x^3\left(Bb+\frac{Ca}{6}\right)}{4b}+\frac{x\left(Ab+Ba-\frac{3a\left(Bb+\frac{Ca}{6}\right)}{4b}\right)}{2b}\right)+\left(Aa-\frac{a\left(Ab+Ba-\frac{3a\left(Bb+\frac{Ca}{6}\right)}{4b}\right)}{2b}\right)\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+}\right.}{\sqrt{b}}\\ \frac{x\log(x)}{\sqrt{bx^2}}\end{cases}\right.\\ \sqrt{a}\left(Ax+\frac{Bx^3}{3}+\frac{Cx^5}{5}\right)\end{cases}$$

input
```
integrate((b*x**2+a)**(1/2)*(C*x**4+B*x**2+A),x)
```

output
```
Piecewise((sqrt(a + b*x**2)*(C*x**5/6 + x**3*(B*b + C*a/6)/(4*b) + x*(A*b
+ B*a - 3*a*(B*b + C*a/6)/(4*b))/(2*b)) + (A*a - a*(A*b + B*a - 3*a*(B*b +
 C*a/6)/(4*b))/(2*b))*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/s
qrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True)), Ne(b, 0)), (sqrt(a)*(A*
x + B*x**3/3 + C*x**5/5), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.14

$$\int \sqrt{a+bx^2}\left(A+Bx^2+Cx^4\right)\,dx = \frac{(bx^2+a)^{\frac{3}{2}}Cx^3}{6\,b} + \frac{1}{2}\sqrt{bx^2+a}Ax - \frac{(bx^2+a)^{\frac{3}{2}}Cax}{8\,b^2} + \frac{\sqrt{bx^2+a}Ca^2x}{16\,b^2} + \frac{(bx^2+a)^{\frac{3}{2}}Bx}{4\,b} - \frac{\sqrt{bx^2+a}Bax}{8\,b} + \frac{Ca^3\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{5}{2}}} - \frac{Ba^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{3}{2}}} + \frac{Aa\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{b}}$$

input
```
integrate((b*x^2+a)^(1/2)*(C*x^4+B*x^2+A),x, algorithm="maxima")
```

output
```
1/6*(b*x^2 + a)^(3/2)*C*x^3/b + 1/2*sqrt(b*x^2 + a)*A*x - 1/8*(b*x^2 + a)^
(3/2)*C*a*x/b^2 + 1/16*sqrt(b*x^2 + a)*C*a^2*x/b^2 + 1/4*(b*x^2 + a)^(3/2)
*B*x/b - 1/8*sqrt(b*x^2 + a)*B*a*x/b + 1/16*C*a^3*arcsinh(b*x/sqrt(a*b))/b
^(5/2) - 1/8*B*a^2*arcsinh(b*x/sqrt(a*b))/b^(3/2) + 1/2*A*a*arcsinh(b*x/sq
rt(a*b))/sqrt(b)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.84

$$\int \sqrt{a+bx^2}\left(A+Bx^2+Cx^4\right)\,dx$$
$$=\frac{1}{48}\left(2\left(4\,Cx^2+\frac{Cab^3+6\,Bb^4}{b^4}\right)x^2-\frac{3\,(Ca^2b^2-2\,Bab^3-8\,Ab^4)}{b^4}\right)\sqrt{bx^2+a}x$$
$$-\frac{(Ca^3-2\,Ba^2b+8\,Aab^2)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{16\,b^{\frac{5}{2}}}$$

input
```
integrate((b*x^2+a)^(1/2)*(C*x^4+B*x^2+A),x, algorithm="giac")
```

output
```
1/48*(2*(4*C*x^2 + (C*a*b^3 + 6*B*b^4)/b^4)*x^2 - 3*(C*a^2*b^2 - 2*B*a*b^3
 - 8*A*b^4)/b^4)*sqrt(b*x^2 + a)*x - 1/16*(C*a^3 - 2*B*a^2*b + 8*A*a*b^2)*
log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+bx^2}\left(A+Bx^2+Cx^4\right)\,dx=\int \sqrt{b\,x^2+a}\,\left(C\,x^4+B\,x^2+A\right)\,dx$$

input
```
int((a + b*x^2)^(1/2)*(A + B*x^2 + C*x^4),x)
```

output
```
int((a + b*x^2)^(1/2)*(A + B*x^2 + C*x^4), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.07

$$\int \sqrt{a+bx^2}\left(A+Bx^2+Cx^4\right)\,dx$$
$$= \frac{-3\sqrt{b\,x^2+a}\,a^2bcx+30\sqrt{b\,x^2+a}\,a\,b^3x+2\sqrt{b\,x^2+a}\,a\,b^2c\,x^3+12\sqrt{b\,x^2+a}\,b^4x^3+8\sqrt{b\,x^2+a}\,b^3c\,x^5+}{48b^3}$$

input
```
int((b*x^2+a)^(1/2)*(C*x^4+B*x^2+A),x)
```

output
```
( - 3*sqrt(a + b*x**2)*a**2*b*c*x + 30*sqrt(a + b*x**2)*a*b**3*x + 2*sqrt(
a + b*x**2)*a*b**2*c*x**3 + 12*sqrt(a + b*x**2)*b**4*x**3 + 8*sqrt(a + b*x
**2)*b**3*c*x**5 + 3*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a
**3*c + 18*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2*b**2)/
(48*b**3)
```

3.117 $\int \frac{A+Bx^2+Cx^4}{\sqrt{a+bx^2}} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 98

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{a+bx^2}} \, dx = \frac{(4bB-3aC)x\sqrt{a+bx^2}}{8b^2} + \frac{Cx^3\sqrt{a+bx^2}}{4b} + \frac{(8Ab^2-a(4bB-3aC))\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{5/2}}$$

output
```
1/8*(4*B*b-3*C*a)*x*(b*x^2+a)^(1/2)/b^2+1/4*C*x^3*(b*x^2+a)^(1/2)/b+1/8*(8
*A*b^2-a*(4*B*b-3*C*a))*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.94

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{a+bx^2}} \, dx = \frac{x\sqrt{a+bx^2}(4bB-3aC+2bCx^2)}{8b^2} + \frac{(8Ab^2-4abB+3a^2C)\,\text{arctanh}\left(\frac{\sqrt{b}x}{-\sqrt{a}+\sqrt{a+bx^2}}\right)}{4b^{5/2}}$$

input
```
Integrate[(A + B*x^2 + C*x^4)/Sqrt[a + b*x^2],x]
```

output
```
(x*Sqrt[a + b*x^2]*(4*b*B - 3*a*C + 2*b*C*x^2))/(8*b^2) + ((8*A*b^2 - 4*a*
b*B + 3*a^2*C)*ArcTanh[(Sqrt[b]*x)/(-Sqrt[a] + Sqrt[a + b*x^2])])/(4*b^(5/
2))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1473, 299, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{a+bx^2}}dx$$

$$\Big\downarrow \text{1473}$$

$$\frac{\int \frac{(4bB-3aC)x^2+4Ab}{\sqrt{bx^2+a}}dx}{4b}+\frac{Cx^3\sqrt{a+bx^2}}{4b}$$

$$\Big\downarrow \text{299}$$

$$\frac{\frac{(8Ab^2-a(4bB-3aC))\int \frac{1}{\sqrt{bx^2+a}}dx}{2b}+\frac{x\sqrt{a+bx^2}(4bB-3aC)}{2b}}{4b}+\frac{Cx^3\sqrt{a+bx^2}}{4b}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{(8Ab^2-a(4bB-3aC))\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2b}+\frac{x\sqrt{a+bx^2}(4bB-3aC)}{2b}}{4b}+\frac{Cx^3\sqrt{a+bx^2}}{4b}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(8Ab^2-a(4bB-3aC))}{2b^{3/2}}+\frac{x\sqrt{a+bx^2}(4bB-3aC)}{2b}}{4b}+\frac{Cx^3\sqrt{a+bx^2}}{4b}$$

input
```
Int[(A + B*x^2 + C*x^4)/Sqrt[a + b*x^2],x]
```

output
```
(C*x^3*Sqrt[a + b*x^2])/(4*b) + (((4*b*B - 3*a*C)*x*Sqrt[a + b*x^2])/(2*b)
 + ((8*A*b^2 - a*(4*b*B - 3*a*C))*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2
*b^(3/2)))/(4*b)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1473
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[c^p*x^(4*p - 1)*((d + e*x^2)^(q + 1)/(e*(4*p + 2*q + 1)))
, x] + Simp[1/(e*(4*p + 2*q + 1))   Int[(d + e*x^2)^q*ExpandToSum[e*(4*p +
2*q + 1)*(a + b*x^2 + c*x^4)^p - d*c^p*(4*p - 1)*x^(4*p - 2) - e*c^p*(4*p +
 2*q + 1)*x^(4*p), x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] &&  !LtQ[q, -1]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.73

method	result
risch	$\frac{x(2Cb\,x^2+4Bb-3Ca)\sqrt{b\,x^2+a}}{8b^2}+\frac{(8b^2A-4abB+3a^2C)\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{8b^{\frac{5}{2}}}$
pseudoelliptic	$\frac{(b^2A-\frac{1}{2}abB+\frac{3}{8}a^2C)\,\mathrm{arctanh}\left(\frac{\sqrt{b\,x^2+a}}{x\sqrt{b}}\right)+\frac{\left(\left(\frac{C\,x^2}{2}+B\right)b^{\frac{3}{2}}-\frac{3Ca\sqrt{b}}{4}\right)\sqrt{b\,x^2+a}\,x}{2}}{b^{\frac{5}{2}}}$
default	$\frac{A\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}+C\left(\frac{x^3\sqrt{b\,x^2+a}}{4b}-\frac{3a\left(\frac{x\sqrt{b\,x^2+a}}{2b}-\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2b^{\frac{3}{2}}}\right)}{4b}\right)+B\left(\frac{x\sqrt{b\,x^2+a}}{2b}-\frac{a\ln(\sqrt{}}{}\right.$

input
```
int((C*x^4+B*x^2+A)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*x*(2*C*b*x^2+4*B*b-3*C*a)/b^2*(b*x^2+a)^(1/2)+1/8*(8*A*b^2-4*B*a*b+3*C
*a^2)/b^(5/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.78

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{a+bx^2}}\,dx$$
$$=\left[\frac{(3\,Ca^2-4\,Bab+8\,Ab^2)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,(2\,Cb^2x^3-(3\,Cab-4\,Bb^2)x)\sqrt{bx}}{16\,b^3}\right.$$
$$\left.-\frac{(3\,Ca^2-4\,Bab+8\,Ab^2)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(2\,Cb^2x^3-(3\,Cab-4\,Bb^2)x)\sqrt{bx^2+a}}{8\,b^3}\right]$$

input
```
integrate((C*x^4+B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/16*((3*C*a^2 - 4*B*a*b + 8*A*b^2)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 +
 a)*sqrt(b)*x - a) + 2*(2*C*b^2*x^3 - (3*C*a*b - 4*B*b^2)*x)*sqrt(b*x^2 +
a))/b^3, -1/8*((3*C*a^2 - 4*B*a*b + 8*A*b^2)*sqrt(-b)*arctan(sqrt(-b)*x/sq
rt(b*x^2 + a)) - (2*C*b^2*x^3 - (3*C*a*b - 4*B*b^2)*x)*sqrt(b*x^2 + a))/b^
3]
```

Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.16

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{a+bx^2}}\,dx = \begin{cases} \left(A-\frac{a\left(B-\frac{3Ca}{4b}\right)}{2b}\right)\left(\begin{cases} \frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for}\ a\neq 0 \\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise} \end{cases}\right)+\sqrt{a+bx^2}\left(\frac{Cx^3}{4b}+\frac{x\left(B-\frac{3Ca}{4b}\right)}{2b}\right) & \text{for}\ b\neq 0 \\ \frac{Ax+\frac{Bx^3}{3}+\frac{Cx^5}{5}}{\sqrt{a}} & \text{otherwise} \end{cases}$$

input
```
integrate((C*x**4+B*x**2+A)/(b*x**2+a)**(1/2),x)
```

output
```
Piecewise(((A - a*(B - 3*C*a/(4*b))/(2*b))*Piecewise((log(2*sqrt(b)*sqrt(a
 + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True)) + s
qrt(a + b*x**2)*(C*x**3/(4*b) + x*(B - 3*C*a/(4*b))/(2*b)), Ne(b, 0)), ((A
*x + B*x**3/3 + C*x**5/5)/sqrt(a), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.02

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{bx^2+a}Cx^3}{4\,b}-\frac{3\,\sqrt{bx^2+a}Cax}{8\,b^2}+\frac{\sqrt{bx^2+a}Bx}{2\,b}+\frac{3\,Ca^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{5}{2}}}-\frac{Ba\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{3}{2}}}+\frac{A\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}}$$

input
```
integrate((C*x^4+B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/4*sqrt(b*x^2 + a)*C*x^3/b - 3/8*sqrt(b*x^2 + a)*C*a*x/b^2 + 1/2*sqrt(b*x
^2 + a)*B*x/b + 3/8*C*a^2*arcsinh(b*x/sqrt(a*b))/b^(5/2) - 1/2*B*a*arcsinh
(b*x/sqrt(a*b))/b^(3/2) + A*arcsinh(b*x/sqrt(a*b))/sqrt(b)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.83

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{a+bx^2}}\,dx = \frac{1}{8}\sqrt{bx^2+a}\left(\frac{2\,Cx^2}{b}-\frac{3\,Cab-4\,Bb^2}{b^3}\right)x - \frac{(3\,Ca^2-4\,Bab+8\,Ab^2)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{8\,b^{\frac{5}{2}}}$$

input
```
integrate((C*x^4+B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/8*sqrt(b*x^2 + a)*(2*C*x^2/b - (3*C*a*b - 4*B*b^2)/b^3)*x - 1/8*(3*C*a^2
 - 4*B*a*b + 8*A*b^2)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{a+bx^2}}\,dx = \int \frac{C\,x^4+B\,x^2+A}{\sqrt{b\,x^2+a}}\,dx$$

input
```
int((A + B*x^2 + C*x^4)/(a + b*x^2)^(1/2),x)
```

output
```
int((A + B*x^2 + C*x^4)/(a + b*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.07

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{a+bx^2}}\,dx$$
$$= \frac{-3\sqrt{b\,x^2+a}\,abcx+4\sqrt{b\,x^2+a}\,b^3x+2\sqrt{b\,x^2+a}\,b^2c\,x^3+3\sqrt{b}\log\left(\frac{\sqrt{b\,x^2+a}+\sqrt{b}\,x}{\sqrt{a}}\right)a^2c+4\sqrt{b}\log\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right.}{8b^3}$$

input
```
int((C*x^4+B*x^2+A)/(b*x^2+a)^(1/2),x)
```

output
```
( - 3*sqrt(a + b*x**2)*a*b*c*x + 4*sqrt(a + b*x**2)*b**3*x + 2*sqrt(a + b*
x**2)*b**2*c*x**3 + 3*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*
a**2*c + 4*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a*b**2)/(8*
b**3)
```

3.118 $\int \frac{A+Bx^2+Cx^4}{(a+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 90

$$\int \frac{A+Bx^2+Cx^4}{(a+bx^2)^{3/2}}\,dx=\frac{\left(\frac{A}{a}-\frac{bB-aC}{b^2}\right)x}{\sqrt{a+bx^2}}+\frac{Cx\sqrt{a+bx^2}}{2b^2}+\frac{(2bB-3aC)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{5/2}}$$

output
```
(A/a-(B*b-C*a)/b^2)*x/(b*x^2+a)^(1/2)+1/2*C*x*(b*x^2+a)^(1/2)/b^2+1/2*(2*B
*b-3*C*a)*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.97

$$\int \frac{A+Bx^2+Cx^4}{(a+bx^2)^{3/2}}\,dx=\frac{x(2Ab^2-2abB+3a^2C+abCx^2)}{2ab^2\sqrt{a+bx^2}}+\frac{(-2bB+3aC)\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{2b^{5/2}}$$

input
```
Integrate[(A + B*x^2 + C*x^4)/(a + b*x^2)^(3/2),x]
```

output
```
(x*(2*A*b^2 - 2*a*b*B + 3*a^2*C + a*b*C*x^2))/(2*a*b^2*Sqrt[a + b*x^2]) +
((-2*b*B + 3*a*C)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(2*b^(5/2))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1471, 25, 27, 299, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{(a+bx^2)^{3/2}}dx$$

$$\Big\downarrow \text{1471}$$

$$\frac{x(Ab^2-a(bB-aC))}{ab^2\sqrt{a+bx^2}}-\frac{\int -\frac{a(bCx^2+bB-aC)}{b^2\sqrt{bx^2+a}}dx}{a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{a(bCx^2+bB-aC)}{b^2\sqrt{bx^2+a}}dx}{a}+\frac{x(Ab^2-a(bB-aC))}{ab^2\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{bCx^2+bB-aC}{\sqrt{bx^2+a}}dx}{b^2}+\frac{x(Ab^2-a(bB-aC))}{ab^2\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{299}$$

$$\frac{\frac{1}{2}(2bB-3aC)\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}Cx\sqrt{a+bx^2}}{b^2}+\frac{x(Ab^2-a(bB-aC))}{ab^2\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{1}{2}(2bB-3aC)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}Cx\sqrt{a+bx^2}}{b^2}+\frac{x(Ab^2-a(bB-aC))}{ab^2\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{x(Ab^2 - a(bB - aC))}{ab^2\sqrt{a+bx^2}} + \frac{\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bB-3aC)}{2\sqrt{b}} + \frac{1}{2}Cx\sqrt{a+bx^2}}{b^2}$$

input
```
Int[(A + B*x^2 + C*x^4)/(a + b*x^2)^(3/2),x]
```

output
```
((A*b^2 - a*(b*B - a*C))*x)/(a*b^2*Sqrt[a + b*x^2]) + ((C*x*Sqrt[a + b*x^2
])/2 + ((2*b*B - 3*a*C)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b]))
/b^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.92

method	result
pseudoelliptic	$\frac{a\sqrt{b\,x^2+a}\,\left(Bb-\frac{3Ca}{2}\right)\operatorname{arctanh}\left(\frac{\sqrt{b\,x^2+a}}{x\sqrt{b}}\right)+\left(\frac{3a^2C}{2}-\left(-\frac{C\,x^2}{2}+B\right)ba+b^2A\right)x\sqrt{b}}{\sqrt{b\,x^2+a}\,b^{\frac{5}{2}}\,a}$
risch	$\frac{Cx\sqrt{b\,x^2+a}}{2b^2}+\frac{b(2Bb-3Ca)\left(-\frac{x}{b\sqrt{b\,x^2+a}}+\frac{\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{b^{\frac{3}{2}}}\right)-\frac{aCx}{\sqrt{b\,x^2+a}}+\frac{2b^2Ax}{a\sqrt{b\,x^2+a}}}{2b^2}$
default	$\frac{Ax}{a\sqrt{b\,x^2+a}}+C\left(\frac{x^3}{2b\sqrt{b\,x^2+a}}-\frac{3a\left(-\frac{x}{b\sqrt{b\,x^2+a}}+\frac{\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{b^{\frac{3}{2}}}\right)}{2b}\right)+B\left(-\frac{x}{b\sqrt{b\,x^2+a}}+\frac{\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+\right.}}{b^{\frac{3}{2}}}\right.$

input
```
int((C*x^4+B*x^2+A)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/(b*x^2+a)^(1/2)*(a*(b*x^2+a)^(1/2)*(B*b-3/2*C*a)*arctanh((b*x^2+a)^(1/2)
/x/b^(1/2))+(3/2*a^2*C-(-1/2*C*x^2+B)*b*a+b^2*A)*x*b^(1/2))/b^(5/2)/a
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 249, normalized size of antiderivative = 2.77

$$\int \frac{A+Bx^2+Cx^4}{(a+bx^2)^{3/2}}\,dx=\left[-\frac{(3\,Ca^3-2\,Ba^2b+(3\,Ca^2b-2\,Bab^2)x^2)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right.}{4\,(ab^4x^2+a^2b^3)}\right.$$

input
```
integrate((C*x^4+B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[-1/4*((3*C*a^3 - 2*B*a^2*b + (3*C*a^2*b - 2*B*a*b^2)*x^2)*sqrt(b)*log(-2*
b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(C*a*b^2*x^3 + (3*C*a^2*b - 2
*B*a*b^2 + 2*A*b^3)*x)*sqrt(b*x^2 + a))/(a*b^4*x^2 + a^2*b^3), 1/2*((3*C*a
^3 - 2*B*a^2*b + (3*C*a^2*b - 2*B*a*b^2)*x^2)*sqrt(-b)*arctan(sqrt(-b)*x/s
qrt(b*x^2 + a)) + (C*a*b^2*x^3 + (3*C*a^2*b - 2*B*a*b^2 + 2*A*b^3)*x)*sqrt
(b*x^2 + a))/(a*b^4*x^2 + a^2*b^3)]
```

Sympy [A] (verification not implemented)

Time = 3.78 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.49

$$\int \frac{A+Bx^2+Cx^4}{(a+bx^2)^{3/2}}\,dx = \frac{Ax}{a^{\frac{3}{2}}\sqrt{1+\frac{bx^2}{a}}} + B\left(\frac{\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{\frac{3}{2}}} - \frac{x}{\sqrt{a}b\sqrt{1+\frac{bx^2}{a}}}\right) + C\left(\frac{3\sqrt{a}x}{2b^2\sqrt{1+\frac{bx^2}{a}}} - \frac{3a\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{\frac{5}{2}}} + \frac{x^3}{2\sqrt{a}b\sqrt{1+\frac{bx^2}{a}}}\right)$$

input
```
integrate((C*x**4+B*x**2+A)/(b*x**2+a)**(3/2),x)
```

output
```
A*x/(a**(3/2)*sqrt(1 + b*x**2/a)) + B*(asinh(sqrt(b)*x/sqrt(a))/b**(3/2) -
 x/(sqrt(a)*b*sqrt(1 + b*x**2/a))) + C*(3*sqrt(a)*x/(2*b**2*sqrt(1 + b*x**
2/a)) - 3*a*asinh(sqrt(b)*x/sqrt(a))/(2*b**(5/2)) + x**3/(2*sqrt(a)*b*sqrt
(1 + b*x**2/a)))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.08

$$\int \frac{A+Bx^2+Cx^4}{(a+bx^2)^{3/2}}\,dx = \frac{Cx^3}{2\sqrt{bx^2+a}b} + \frac{Ax}{\sqrt{bx^2+a}a} + \frac{3\,Cax}{2\sqrt{bx^2+a}b^2} - \frac{Bx}{\sqrt{bx^2+a}b} - \frac{3\,Ca\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{5}{2}}} + \frac{B\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{3}{2}}}$$

input
```
integrate((C*x^4+B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
1/2*C*x^3/(sqrt(b*x^2 + a)*b) + A*x/(sqrt(b*x^2 + a)*a) + 3/2*C*a*x/(sqrt(
b*x^2 + a)*b^2) - B*x/(sqrt(b*x^2 + a)*b) - 3/2*C*a*arcsinh(b*x/sqrt(a*b))
/b^(5/2) + B*arcsinh(b*x/sqrt(a*b))/b^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.91

$$\int \frac{A+Bx^2+Cx^4}{(a+bx^2)^{3/2}}\,dx = \frac{\left(\frac{Cx^2}{b}+\frac{3\,Ca^2b-2\,Bab^2+2\,Ab^3}{ab^3}\right)x}{2\,\sqrt{bx^2+a}} + \frac{(3\,Ca-2\,Bb)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2\,b^{\frac{5}{2}}}$$

input
```
integrate((C*x^4+B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
1/2*(C*x^2/b + (3*C*a^2*b - 2*B*a*b^2 + 2*A*b^3)/(a*b^3))*x/sqrt(b*x^2 + a
) + 1/2*(3*C*a - 2*B*b)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{(a+bx^2)^{3/2}}\,dx = \int \frac{C\,x^4+B\,x^2+A}{(b\,x^2+a)^{3/2}}\,dx$$

input
```
int((A + B*x^2 + C*x^4)/(a + b*x^2)^(3/2),x)
```

output
```
int((A + B*x^2 + C*x^4)/(a + b*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.96

$$\int \frac{A + Bx^2 + Cx^4}{(a + bx^2)^{3/2}}\, dx = \frac{12\sqrt{b\,x^2+a}\,abcx + 4\sqrt{b\,x^2+a}\,b^2c\,x^3 - 12\sqrt{b}\log\left(\frac{\sqrt{b\,x^2+a}+\sqrt{b}\,x}{\sqrt{a}}\right)a^2c + 8\sqrt{b}\log\left(\frac{\sqrt{}}{}\right.}{}$$

input
```
int((C*x^4+B*x^2+A)/(b*x^2+a)^(3/2),x)
```

output
```
(12*sqrt(a + b*x**2)*a*b*c*x + 4*sqrt(a + b*x**2)*b**2*c*x**3 - 12*sqrt(b)
*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2*c + 8*sqrt(b)*log((sqrt(
a + b*x**2) + sqrt(b)*x)/sqrt(a))*a*b**2 - 12*sqrt(b)*log((sqrt(a + b*x**2
) + sqrt(b)*x)/sqrt(a))*a*b*c*x**2 + 8*sqrt(b)*log((sqrt(a + b*x**2) + sqr
t(b)*x)/sqrt(a))*b**3*x**2 + 9*sqrt(b)*a**2*c + 9*sqrt(b)*a*b*c*x**2)/(8*b
**3*(a + b*x**2))
```

3.119 $\int \frac{A+Bx^2+Cx^4}{(a+bx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 101

$$\int \frac{A+Bx^2+Cx^4}{(a+bx^2)^{5/2}}\,dx=\frac{\left(\frac{A}{a}-\frac{bB-aC}{b^2}\right)x}{3\left(a+bx^2\right)^{3/2}}+\frac{\left(2Ab^2+a(bB-4aC)\right)x}{3a^2b^2\sqrt{a+bx^2}}+\frac{C\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{5/2}}$$

output
```
1/3*(A/a-(B*b-C*a)/b^2)*x/(b*x^2+a)^(3/2)+1/3*(2*A*b^2+a*(B*b-4*C*a))*x/a^
2/b^2/(b*x^2+a)^(1/2)+C*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.94

$$\int \frac{A+Bx^2+Cx^4}{(a+bx^2)^{5/2}}\,dx=\frac{3aAb^2x-3a^3Cx+2Ab^3x^3+ab^2Bx^3-4a^2bCx^3}{3a^2b^2\left(a+bx^2\right)^{3/2}}-\frac{C\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{b^{5/2}}$$

input
```
Integrate[(A + B*x^2 + C*x^4)/(a + b*x^2)^(5/2),x]
```

output
```
(3*a*A*b^2*x - 3*a^3*C*x + 2*A*b^3*x^3 + a*b^2*B*x^3 - 4*a^2*b*C*x^3)/(3*a
^2*b^2*(a + b*x^2)^(3/2)) - (C*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/b^(5/2
)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1471, 25, 27, 298, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{\left(a+bx^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 1471}$$

$$\frac{x\left(Ab^2-a(bB-aC)\right)}{3ab^2\left(a+bx^2\right)^{3/2}}-\frac{\int -\frac{3aCx^2+b\left(2A+\frac{a(bB-aC)}{b^2}\right)}{b\left(bx^2+a\right)^{3/2}}dx}{3a}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{3aCx^2+2Ab+a\left(B-\frac{aC}{b}\right)}{b\left(bx^2+a\right)^{3/2}}dx}{3a}+\frac{x\left(Ab^2-a(bB-aC)\right)}{3ab^2\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{3aCx^2+2Ab+a\left(B-\frac{aC}{b}\right)}{\left(bx^2+a\right)^{3/2}}dx}{3ab}+\frac{x\left(Ab^2-a(bB-aC)\right)}{3ab^2\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{ 298}$$

$$\frac{\frac{3aC\int \frac{1}{\sqrt{bx^2+a}}dx}{b}+\frac{x\left(\frac{2Ab}{a}-\frac{4aC}{b}+B\right)}{\sqrt{a+bx^2}}}{3ab}+\frac{x\left(Ab^2-a(bB-aC)\right)}{3ab^2\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{\frac{3aC\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{b}+\frac{x\left(\frac{2Ab}{a}-\frac{4aC}{b}+B\right)}{\sqrt{a+bx^2}}}{3ab}+\frac{x(Ab^2-a(bB-aC))}{3ab^2(a+bx^2)^{3/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{x\left(\frac{2Ab}{a}-\frac{4aC}{b}+B\right)}{\sqrt{a+bx^2}}+\frac{3aC\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}}{3ab}+\frac{x(Ab^2-a(bB-aC))}{3ab^2(a+bx^2)^{3/2}}$$

input
```
Int[(A + B*x^2 + C*x^4)/(a + b*x^2)^(5/2),x]
```

output
```
((A*b^2 - a*(b*B - a*C))*x)/(3*a*b^2*(a + b*x^2)^(3/2)) + ((((2*A*b)/a + B
 - (4*a*C)/b)*x)/Sqrt[a + b*x^2] + (3*a*C*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x
^2]])/b^(3/2))/(3*a*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.96

method	result
pseudoelliptic	$\frac{a^{2} C \operatorname{arctanh}\left(\frac{\sqrt{b x^{2}+a}}{x \sqrt{b}}\right) b^{2} \left(b x^{2}+a\right)^{\frac{3}{2}}+\frac{2 \left(A x^{2} b^{3}+\frac{3 a \left(\frac{x^{2} B}{3}+A\right) b^{2}}{2}-2 C a^{2} b x^{2}-\frac{3 C a^{3}}{2}\right) b^{\frac{5}{2}} x}{3}}{\left(b x^{2}+a\right)^{\frac{3}{2}} b^{\frac{9}{2}} a^{2}}$
default	$A \left(\frac{x}{3 a \left(b x^{2}+a\right)^{\frac{3}{2}}}+\frac{2 x}{3 a^{2} \sqrt{b x^{2}+a}}\right)+C \left(-\frac{x^{3}}{3 b \left(b x^{2}+a\right)^{\frac{3}{2}}}+\frac{-\frac{x}{b \sqrt{b x^{2}+a}}+\frac{\ln\left(\sqrt{b}\, x+\sqrt{b x^{2}+a}\right)}{b^{\frac{3}{2}}}}{b}\right)+B \left(-\frac{}{2 b (b x}\right.$

input
```
int((C*x^4+B*x^2+A)/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/(b*x^2+a)^(3/2)*(a^2*C*arctanh((b*x^2+a)^(1/2)/x/b^(1/2))*b^2*(b*x^2+a)^
(3/2)+2/3*(A*x^2*b^3+3/2*a*(1/3*x^2*B+A)*b^2-2*C*a^2*b*x^2-3/2*C*a^3)*b^(5
/2)*x)/b^(9/2)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 289, normalized size of antiderivative = 2.86

$$\int \frac{A+Bx^2+Cx^4}{(a+bx^2)^{5/2}}\,dx = \left[\frac{3\,(Ca^2b^2x^4+2\,Ca^3bx^2+Ca^4)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\,((4\,C}{6\,(a^2b^5x^4+2\,a^3b^4x^2+a^4b}\right.$$

$$-\frac{3\,(Ca^2b^2x^4+2\,Ca^3bx^2+Ca^4)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)+((4\,Ca^2b^2-Bab^3-2\,Ab^4)x^3+3\,(Ca^3b-Aab^3)}{3\,(a^2b^5x^4+2\,a^3b^4x^2+a^4b^3)}$$

input
```
integrate((C*x^4+B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
[1/6*(3*(C*a^2*b^2*x^4 + 2*C*a^3*b*x^2 + C*a^4)*sqrt(b)*log(-2*b*x^2 - 2*s
qrt(b*x^2 + a)*sqrt(b)*x - a) - 2*((4*C*a^2*b^2 - B*a*b^3 - 2*A*b^4)*x^3 +
 3*(C*a^3*b - A*a*b^3)*x)*sqrt(b*x^2 + a))/(a^2*b^5*x^4 + 2*a^3*b^4*x^2 +
a^4*b^3), -1/3*(3*(C*a^2*b^2*x^4 + 2*C*a^3*b*x^2 + C*a^4)*sqrt(-b)*arctan(
sqrt(-b)*x/sqrt(b*x^2 + a)) + ((4*C*a^2*b^2 - B*a*b^3 - 2*A*b^4)*x^3 + 3*(
C*a^3*b - A*a*b^3)*x)*sqrt(b*x^2 + a))/(a^2*b^5*x^4 + 2*a^3*b^4*x^2 + a^4*
b^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 450 vs. $2(92) = 184$.

Time = 6.19 (sec) , antiderivative size = 450, normalized size of antiderivative = 4.46

$$\int \frac{A+Bx^2+Cx^4}{(a+bx^2)^{5/2}}\,dx = A\left(\frac{3ax}{3a^{\frac{7}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{5}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}} + \frac{2bx^3}{3a^{\frac{7}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{5}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}}\right) + \frac{Bx^3}{3a^{\frac{5}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{3}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}} + C\left(\frac{3a^{\frac{39}{2}}b^{11}\sqrt{1+\frac{bx^2}{a}}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3a^{\frac{39}{2}}b^{\frac{27}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{37}{2}}b^{\frac{29}{2}}x^2\sqrt{1+\frac{bx^2}{a}}} + \frac{3a^{\frac{37}{2}}b^{12}x^2\sqrt{1+\frac{bx^2}{a}}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3a^{\frac{39}{2}}b^{\frac{27}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{37}{2}}b^{\frac{29}{2}}x^2\sqrt{1+\frac{bx^2}{a}}} - \frac{3a^{19}b^{\frac{23}{2}}x}{3a^{\frac{39}{2}}b^{\frac{27}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{37}{2}}b^{\frac{29}{2}}x^2\sqrt{1+\frac{bx^2}{a}}} - \frac{4a^{18}b^{\frac{25}{2}}x^3}{3a^{\frac{39}{2}}b^{\frac{27}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{37}{2}}b^{\frac{29}{2}}x^2\sqrt{1+\frac{bx^2}{a}}}\right)$$

input
```
integrate((C*x**4+B*x**2+A)/(b*x**2+a)**(5/2),x)
```

output
```
A*(3*a*x/(3*a**(7/2)*sqrt(1 + b*x**2/a) + 3*a**(5/2)*b*x**2*sqrt(1 + b*x**
2/a)) + 2*b*x**3/(3*a**(7/2)*sqrt(1 + b*x**2/a) + 3*a**(5/2)*b*x**2*sqrt(1
 + b*x**2/a))) + B*x**3/(3*a**(5/2)*sqrt(1 + b*x**2/a) + 3*a**(3/2)*b*x**2
*sqrt(1 + b*x**2/a)) + C*(3*a**(39/2)*b**11*sqrt(1 + b*x**2/a)*asinh(sqrt(
b)*x/sqrt(a))/(3*a**(39/2)*b**(27/2)*sqrt(1 + b*x**2/a) + 3*a**(37/2)*b**(
29/2)*x**2*sqrt(1 + b*x**2/a)) + 3*a**(37/2)*b**12*x**2*sqrt(1 + b*x**2/a)
*asinh(sqrt(b)*x/sqrt(a))/(3*a**(39/2)*b**(27/2)*sqrt(1 + b*x**2/a) + 3*a*
*(37/2)*b**(29/2)*x**2*sqrt(1 + b*x**2/a)) - 3*a**19*b**(23/2)*x/(3*a**(39
/2)*b**(27/2)*sqrt(1 + b*x**2/a) + 3*a**(37/2)*b**(29/2)*x**2*sqrt(1 + b*x
**2/a)) - 4*a**18*b**(25/2)*x**3/(3*a**(39/2)*b**(27/2)*sqrt(1 + b*x**2/a)
 + 3*a**(37/2)*b**(29/2)*x**2*sqrt(1 + b*x**2/a)))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.34

$$\int \frac{A+Bx^2+Cx^4}{(a+bx^2)^{5/2}}\,dx = -\frac{1}{3}\,Cx\left(\frac{3\,x^2}{(bx^2+a)^{\frac{3}{2}}b}+\frac{2\,a}{(bx^2+a)^{\frac{3}{2}}b^2}\right)+\frac{2\,Ax}{3\,\sqrt{bx^2+a}a^2}+\frac{Ax}{3\,(bx^2+a)^{\frac{3}{2}}a}-\frac{Cx}{3\,\sqrt{bx^2+a}b^2}-\frac{Bx}{3\,(bx^2+a)^{\frac{3}{2}}b}+\frac{Bx}{3\,\sqrt{bx^2+a}ab}+\frac{C\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{5}{2}}}$$

input

```
integrate((C*x^4+B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output

```
-1/3*C*x*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2)) + 2/3
*A*x/(sqrt(b*x^2 + a)*a^2) + 1/3*A*x/((b*x^2 + a)^(3/2)*a) - 1/3*C*x/(sqrt
(b*x^2 + a)*b^2) - 1/3*B*x/((b*x^2 + a)^(3/2)*b) + 1/3*B*x/(sqrt(b*x^2 + a
)*a*b) + C*arcsinh(b*x/sqrt(a*b))/b^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.92

$$\int \frac{A+Bx^2+Cx^4}{(a+bx^2)^{5/2}}\,dx = -\frac{x\left(\frac{(4\,Ca^2b^2-Bab^3-2\,Ab^4)x^2}{a^2b^3}+\frac{3\,(Ca^3b-Aab^3)}{a^2b^3}\right)}{3\,(bx^2+a)^{\frac{3}{2}}}-\frac{C\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{b^{\frac{5}{2}}}$$

input

```
integrate((C*x^4+B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output

```
-1/3*x*((4*C*a^2*b^2 - B*a*b^3 - 2*A*b^4)*x^2/(a^2*b^3) + 3*(C*a^3*b - A*a
*b^3)/(a^2*b^3))/(b*x^2 + a)^(3/2) - C*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a
)))/b^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{(a+bx^2)^{5/2}}\,dx = \int \frac{C\,x^4+B\,x^2+A}{(b\,x^2+a)^{5/2}}\,dx$$

input
```
int((A + B*x^2 + C*x^4)/(a + b*x^2)^(5/2),x)
```

output
```
int((A + B*x^2 + C*x^4)/(a + b*x^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 214, normalized size of antiderivative = 2.12

$$\int \frac{A+Bx^2+Cx^4}{(a+bx^2)^{5/2}}\,dx = \frac{-3\sqrt{b\,x^2+a}\,a^2bcx+3\sqrt{b\,x^2+a}\,a\,b^3x-4\sqrt{b\,x^2+a}\,a\,b^2c\,x^3+3\sqrt{b\,x^2+a}\,b^4x^3+}{}$$

input
```
int((C*x^4+B*x^2+A)/(b*x^2+a)^(5/2),x)
```

output
```
( - 3*sqrt(a + b*x**2)*a**2*b*c*x + 3*sqrt(a + b*x**2)*a*b**3*x - 4*sqrt(a
 + b*x**2)*a*b**2*c*x**3 + 3*sqrt(a + b*x**2)*b**4*x**3 + 3*sqrt(b)*log((s
qrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**3*c + 6*sqrt(b)*log((sqrt(a + b*x
**2) + sqrt(b)*x)/sqrt(a))*a**2*b*c*x**2 + 3*sqrt(b)*log((sqrt(a + b*x**2)
 + sqrt(b)*x)/sqrt(a))*a*b**2*c*x**4 - sqrt(b)*a**2*b**2 - 2*sqrt(b)*a*b**
3*x**2 - sqrt(b)*b**4*x**4)/(3*a*b**3*(a**2 + 2*a*b*x**2 + b**2*x**4))
```

3.120 $\int \frac{A+Bx^2+Cx^4}{\left(a+bx^2\right)^{7/2}} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 115

$$\int \frac{A+Bx^2+Cx^4}{\left(a+bx^2\right)^{7/2}} \, dx = \frac{\left(\frac{A}{a}-\frac{bB-aC}{b^2}\right)x}{5\left(a+bx^2\right)^{5/2}} + \frac{\left(4Ab^2+a(bB-6aC)\right)x}{15a^2b^2\left(a+bx^2\right)^{3/2}} + \frac{\left(8Ab^2+a(2bB+3aC)\right)x}{15a^3b^2\sqrt{a+bx^2}}$$

output
```
1/5*(A/a-(B*b-C*a)/b^2)*x/(b*x^2+a)^(5/2)+1/15*(4*A*b^2+a*(B*b-6*C*a))*x/a
^2/b^2/(b*x^2+a)^(3/2)+1/15*(8*A*b^2+a*(2*B*b+3*C*a))*x/a^3/b^2/(b*x^2+a)^
(1/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.57

$$\int \frac{A+Bx^2+Cx^4}{\left(a+bx^2\right)^{7/2}} \, dx = \frac{8Ab^2x^5+2abx^3\left(10A+Bx^2\right)+a^2\left(15Ax+5Bx^3+3Cx^5\right)}{15a^3\left(a+bx^2\right)^{5/2}}$$

input
```
Integrate[(A + B*x^2 + C*x^4)/(a + b*x^2)^(7/2),x]
```

output
```
(8*A*b^2*x^5 + 2*a*b*x^3*(10*A + B*x^2) + a^2*(15*A*x + 5*B*x^3 + 3*C*x^5)
)/(15*a^3*(a + b*x^2)^(5/2))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.86, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1469, 2075, 362, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{(a+bx^2)^{7/2}}dx$$

$$\Big\downarrow \text{1469}$$

$$\frac{\int \frac{x^2(4Ab+a(Cx^2+B))}{(bx^2+a)^{7/2}}dx}{a}+\frac{Ax}{a(a+bx^2)^{5/2}}$$

$$\Big\downarrow \text{2075}$$

$$\frac{\int \frac{x^2(aCx^2+4Ab+aB)}{(bx^2+a)^{7/2}}dx}{a}+\frac{Ax}{a(a+bx^2)^{5/2}}$$

$$\Big\downarrow \text{362}$$

$$\frac{\frac{1}{5}\left(\frac{8Ab}{a}+\frac{3aC}{b}+2B\right)\int \frac{x^2}{(bx^2+a)^{5/2}}dx+\frac{x^3\left(a\left(B-\frac{aC}{b}\right)+4Ab\right)}{5a(a+bx^2)^{5/2}}}{a}+\frac{Ax}{a(a+bx^2)^{5/2}}$$

$$\Big\downarrow \text{242}$$

$$\frac{\frac{x^3\left(\frac{8Ab}{a}+\frac{3aC}{b}+2B\right)}{15a(a+bx^2)^{3/2}}+\frac{x^3\left(a\left(B-\frac{aC}{b}\right)+4Ab\right)}{5a(a+bx^2)^{5/2}}}{a}+\frac{Ax}{a(a+bx^2)^{5/2}}$$

input
```
Int[(A + B*x^2 + C*x^4)/(a + b*x^2)^(7/2),x]
```

output
```
(A*x)/(a*(a + b*x^2)^(5/2)) + (((4*A*b + a*(B - (a*C)/b))*x^3)/(5*a*(a + b
*x^2)^(5/2)) + (((8*A*b)/a + 2*B + (3*a*C)/b)*x^3)/(15*a*(a + b*x^2)^(3/2)
))/a
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

rule 1469
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[a^p*x*((d + e*x^2)^(q + 1)/d), x] + Simp[1/d   Int[x^2*(d
 + e*x^2)^q*(d*PolynomialQuotient[(a + b*x^2 + c*x^4)^p - a^p, x^2, x] - e*
a^p*(2*q + 3)), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && ILtQ[q + 1/2, 0] && LtQ[4
*p + 2*q + 1, 0]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.50

method	result
pseudoelliptic	$\frac{x\left(\left(\frac{1}{5}C\,x^{4}+\frac{1}{3}x^{2}B+A\right)a^{2}+\frac{4b\left(\frac{x^{2}B}{10}+A\right)x^{2}a}{3}+\frac{8A\,b^{2}x^{4}}{15}\right)}{\left(b\,x^{2}+a\right)^{\frac{5}{2}}a^{3}}$
gosper	$\frac{x\left(8A\,b^{2}x^{4}+2Bab\,x^{4}+3C\,a^{2}x^{4}+20aAb\,x^{2}+5B\,a^{2}x^{2}+15a^{2}A\right)}{15\left(b\,x^{2}+a\right)^{\frac{5}{2}}a^{3}}$
trager	$\frac{x\left(8A\,b^{2}x^{4}+2Bab\,x^{4}+3C\,a^{2}x^{4}+20aAb\,x^{2}+5B\,a^{2}x^{2}+15a^{2}A\right)}{15\left(b\,x^{2}+a\right)^{\frac{5}{2}}a^{3}}$
orering	$\frac{x\left(8A\,b^{2}x^{4}+2Bab\,x^{4}+3C\,a^{2}x^{4}+20aAb\,x^{2}+5B\,a^{2}x^{2}+15a^{2}A\right)}{15\left(b\,x^{2}+a\right)^{\frac{5}{2}}a^{3}}$
default	$A\left(\frac{x}{5a\left(b\,x^{2}+a\right)^{\frac{5}{2}}}+\frac{\frac{4x}{15a\left(b\,x^{2}+a\right)^{\frac{3}{2}}}+\frac{8x}{15a^{2}\sqrt{b\,x^{2}+a}}}{a}\right)+C\left(-\frac{x^{3}}{2b\left(b\,x^{2}+a\right)^{\frac{5}{2}}}+\frac{3a\left(-\frac{x}{4b\left(b\,x^{2}+a\right)^{\frac{5}{2}}}+\frac{a\left(\frac{x}{5a\left(b\,x^{2}+a\right)^{\frac{5}{2}}} \cdots\right.}{\cdots}\right.}{2 \cdots}\right.$

input
```
int((C*x^4+B*x^2+A)/(b*x^2+a)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/(b*x^2+a)^(5/2)*x*((1/5*C*x^4+1/3*x^2*B+A)*a^2+4/3*b*(1/10*x^2*B+A)*x^2*
a+8/15*A*b^2*x^4)/a^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.81

$$\int \frac{A+Bx^2+Cx^4}{(a+bx^2)^{7/2}}\,dx = \frac{((3\,Ca^2+2\,Bab+8\,Ab^2)x^5+15\,Aa^2x+5\,(Ba^2+4\,Aab)x^3)\sqrt{bx^2+a}}{15\,(a^3b^3x^6+3\,a^4b^2x^4+3\,a^5bx^2+a^6)}$$

input
```
integrate((C*x^4+B*x^2+A)/(b*x^2+a)^(7/2),x, algorithm="fricas")
```

output
```
1/15*((3*C*a^2 + 2*B*a*b + 8*A*b^2)*x^5 + 15*A*a^2*x + 5*(B*a^2 + 4*A*a*b)
*x^3)*sqrt(b*x^2 + a)/(a^3*b^3*x^6 + 3*a^4*b^2*x^4 + 3*a^5*b*x^2 + a^6)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 639 vs. 2(107) = 214.

Time = 14.88 (sec) , antiderivative size = 639, normalized size of antiderivative = 5.56

$$\int \frac{A+Bx^2+Cx^4}{(a+bx^2)^{7/2}}\,dx = A\left(\frac{15a^5x}{15a^{\frac{17}{2}}\sqrt{1+\frac{bx^2}{a}}+45a^{\frac{15}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}+45a^{\frac{13}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}}+15a^{\frac{11}{2}}b^3x^6\sqrt{1+\ldots}}\right.$$
$$+\frac{35a^4bx^3}{15a^{\frac{17}{2}}\sqrt{1+\frac{bx^2}{a}}+45a^{\frac{15}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}+45a^{\frac{13}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}}+15a^{\frac{11}{2}}b^3x^6\sqrt{1+\frac{bx^2}{a}}}$$
$$+\frac{28a^3b^2x^5}{15a^{\frac{17}{2}}\sqrt{1+\frac{bx^2}{a}}+45a^{\frac{15}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}+45a^{\frac{13}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}}+15a^{\frac{11}{2}}b^3x^6\sqrt{1+\frac{bx^2}{a}}}$$
$$\left.+\frac{8a^2b^3x^7}{15a^{\frac{17}{2}}\sqrt{1+\frac{bx^2}{a}}+45a^{\frac{15}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}+45a^{\frac{13}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}}+15a^{\frac{11}{2}}b^3x^6\sqrt{1+\frac{bx^2}{a}}}\right)$$
$$+B\left(\frac{5ax^3}{15a^{\frac{9}{2}}\sqrt{1+\frac{bx^2}{a}}+30a^{\frac{7}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}+15a^{\frac{5}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}}}\right.$$
$$\left.+\frac{2bx^5}{15a^{\frac{9}{2}}\sqrt{1+\frac{bx^2}{a}}+30a^{\frac{7}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}+15a^{\frac{5}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}}}\right)$$
$$+\frac{Cx^5}{5a^{\frac{7}{2}}\sqrt{1+\frac{bx^2}{a}}+10a^{\frac{5}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}+5a^{\frac{3}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}}}$$

input
```
integrate((C*x**4+B*x**2+A)/(b*x**2+a)**(7/2),x)
```

output
```
A*(15*a**5*x/(15*a**(17/2)*sqrt(1 + b*x**2/a) + 45*a**(15/2)*b*x**2*sqrt(1
 + b*x**2/a) + 45*a**(13/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 15*a**(11/2)*b*
*3*x**6*sqrt(1 + b*x**2/a)) + 35*a**4*b*x**3/(15*a**(17/2)*sqrt(1 + b*x**2
/a) + 45*a**(15/2)*b*x**2*sqrt(1 + b*x**2/a) + 45*a**(13/2)*b**2*x**4*sqrt
(1 + b*x**2/a) + 15*a**(11/2)*b**3*x**6*sqrt(1 + b*x**2/a)) + 28*a**3*b**2
*x**5/(15*a**(17/2)*sqrt(1 + b*x**2/a) + 45*a**(15/2)*b*x**2*sqrt(1 + b*x*
*2/a) + 45*a**(13/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 15*a**(11/2)*b**3*x**6
*sqrt(1 + b*x**2/a)) + 8*a**2*b**3*x**7/(15*a**(17/2)*sqrt(1 + b*x**2/a) +
 45*a**(15/2)*b*x**2*sqrt(1 + b*x**2/a) + 45*a**(13/2)*b**2*x**4*sqrt(1 +
b*x**2/a) + 15*a**(11/2)*b**3*x**6*sqrt(1 + b*x**2/a))) + B*(5*a*x**3/(15*
a**(9/2)*sqrt(1 + b*x**2/a) + 30*a**(7/2)*b*x**2*sqrt(1 + b*x**2/a) + 15*a
**(5/2)*b**2*x**4*sqrt(1 + b*x**2/a)) + 2*b*x**5/(15*a**(9/2)*sqrt(1 + b*x
**2/a) + 30*a**(7/2)*b*x**2*sqrt(1 + b*x**2/a) + 15*a**(5/2)*b**2*x**4*sqr
t(1 + b*x**2/a))) + C*x**5/(5*a**(7/2)*sqrt(1 + b*x**2/a) + 10*a**(5/2)*b*
x**2*sqrt(1 + b*x**2/a) + 5*a**(3/2)*b**2*x**4*sqrt(1 + b*x**2/a))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.50

$$\int \frac{A+Bx^2+Cx^4}{(a+bx^2)^{7/2}}\,dx = -\frac{Cx^3}{2\,(bx^2+a)^{\frac{5}{2}}b}+\frac{8\,Ax}{15\,\sqrt{bx^2+a}a^3} + \frac{4\,Ax}{15\,(bx^2+a)^{\frac{3}{2}}a^2}+\frac{Ax}{5\,(bx^2+a)^{\frac{5}{2}}a}+\frac{Cx}{10\,(bx^2+a)^{\frac{3}{2}}b^2}+\frac{Cx}{5\,\sqrt{bx^2+a}ab^2} - \frac{3\,Cax}{10\,(bx^2+a)^{\frac{5}{2}}b^2}-\frac{Bx}{5\,(bx^2+a)^{\frac{5}{2}}b}+\frac{2\,Bx}{15\,\sqrt{bx^2+a}a^2b}+\frac{Bx}{15\,(bx^2+a)^{\frac{3}{2}}ab}$$

input
```
integrate((C*x^4+B*x^2+A)/(b*x^2+a)^(7/2),x, algorithm="maxima")
```

output
```
-1/2*C*x^3/((b*x^2 + a)^(5/2)*b) + 8/15*A*x/(sqrt(b*x^2 + a)*a^3) + 4/15*A
*x/((b*x^2 + a)^(3/2)*a^2) + 1/5*A*x/((b*x^2 + a)^(5/2)*a) + 1/10*C*x/((b*
x^2 + a)^(3/2)*b^2) + 1/5*C*x/(sqrt(b*x^2 + a)*a*b^2) - 3/10*C*a*x/((b*x^2
 + a)^(5/2)*b^2) - 1/5*B*x/((b*x^2 + a)^(5/2)*b) + 2/15*B*x/(sqrt(b*x^2 +
a)*a^2*b) + 1/15*B*x/((b*x^2 + a)^(3/2)*a*b)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.70

$$\int \frac{A+Bx^2+Cx^4}{(a+bx^2)^{7/2}}\,dx = \frac{\left(x^2\left(\frac{(3\,Ca^2b^2+2\,Bab^3+8\,Ab^4)x^2}{a^3b^2}+\frac{5\,(Ba^2b^2+4\,Aab^3)}{a^3b^2}\right)+\frac{15\,A}{a}\right)x}{15\,(bx^2+a)^{\frac{5}{2}}}$$

input
```
integrate((C*x^4+B*x^2+A)/(b*x^2+a)^(7/2),x, algorithm="giac")
```

output
```
1/15*(x^2*((3*C*a^2*b^2 + 2*B*a*b^3 + 8*A*b^4)*x^2/(a^3*b^2) + 5*(B*a^2*b^
2 + 4*A*a*b^3)/(a^3*b^2)) + 15*A/a)*x/(b*x^2 + a)^(5/2)
```

Mupad [B] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.16

$$\int \frac{A+Bx^2+Cx^4}{(a+bx^2)^{7/2}}\,dx = \frac{3\,C\,a^4\,x-6\,C\,a^3\,x\,(b\,x^2+a)-3\,B\,a^3\,b\,x+8\,A\,b^2\,x\,(b\,x^2+a)^2+3\,C\,a^2\,x\,(b\,x^2+}{15\,a^3\,b^2\,(b}$$

input
```
int((A + B*x^2 + C*x^4)/(a + b*x^2)^(7/2),x)
```

output
```
(3*C*a^4*x - 6*C*a^3*x*(a + b*x^2) - 3*B*a^3*b*x + 8*A*b^2*x*(a + b*x^2)^2
 + 3*C*a^2*x*(a + b*x^2)^2 + 3*A*a^2*b^2*x + 4*A*a*b^2*x*(a + b*x^2) + 2*B
*a*b*x*(a + b*x^2)^2 + B*a^2*b*x*(a + b*x^2))/(15*a^3*b^2*(a + b*x^2)^(5/2
))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.72

$$\int \frac{A+Bx^2+Cx^4}{(a+bx^2)^{7/2}}\,dx = \frac{15\sqrt{b\,x^2+a}\,a^2b^3x+25\sqrt{b\,x^2+a}\,a\,b^4x^3+3\sqrt{b\,x^2+a}\,a\,b^3c\,x^5+10\sqrt{b\,x^2+a}\,b^5x^5}{15a^2}$$

input
```
int((C*x^4+B*x^2+A)/(b*x^2+a)^(7/2),x)
```

output
```
(15*sqrt(a + b*x**2)*a**2*b**3*x + 25*sqrt(a + b*x**2)*a*b**4*x**3 + 3*sqr
t(a + b*x**2)*a*b**3*c*x**5 + 10*sqrt(a + b*x**2)*b**5*x**5 + 3*sqrt(b)*a*
*4*c - 10*sqrt(b)*a**3*b**2 + 9*sqrt(b)*a**3*b*c*x**2 - 30*sqrt(b)*a**2*b*
*3*x**2 + 9*sqrt(b)*a**2*b**2*c*x**4 - 30*sqrt(b)*a*b**4*x**4 + 3*sqrt(b)*
a*b**3*c*x**6 - 10*sqrt(b)*b**5*x**6)/(15*a**2*b**3*(a**3 + 3*a**2*b*x**2
+ 3*a*b**2*x**4 + b**3*x**6))
```

3.121 $\int \frac{A+Bx^2+Cx^4}{\left(a+bx^2\right)^{9/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 155

$$\int \frac{A+Bx^2+Cx^4}{\left(a+bx^2\right)^{9/2}}\,dx=\frac{\left(\frac{A}{a}-\frac{bB-aC}{b^2}\right)x}{7\left(a+bx^2\right)^{7/2}}+\frac{\left(6Ab^2+a(bB-8aC)\right)x}{35a^2b^2\left(a+bx^2\right)^{5/2}}+\frac{\left(24Ab^2+a(4bB+3aC)\right)x}{105a^3b^2\left(a+bx^2\right)^{3/2}}+\frac{2\left(24Ab^2+a(4bB+3aC)\right)x}{105a^4b^2\sqrt{a+bx^2}}$$

output
```
1/7*(A/a-(B*b-C*a)/b^2)*x/(b*x^2+a)^(7/2)+1/35*(6*A*b^2+a*(B*b-8*C*a))*x/a
^2/b^2/(b*x^2+a)^(5/2)+1/105*(24*A*b^2+a*(4*B*b+3*C*a))*x/a^3/b^2/(b*x^2+a
)^(3/2)+2/105*(24*A*b^2+a*(4*B*b+3*C*a))*x/a^4/b^2/(b*x^2+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.60

$$\int \frac{A+Bx^2+Cx^4}{\left(a+bx^2\right)^{9/2}}\,dx=\frac{48Ab^3x^7+8ab^2x^5(21A+Bx^2)+2a^2bx^3(105A+14Bx^2+3Cx^4)+7a^3(15Ax+}{105a^4\left(a+bx^2\right)^{7/2}}$$

input
```
Integrate[(A + B*x^2 + C*x^4)/(a + b*x^2)^(9/2),x]
```

output
```
(48*A*b^3*x^7 + 8*a*b^2*x^5*(21*A + B*x^2) + 2*a^2*b*x^3*(105*A + 14*B*x^2
 + 3*C*x^4) + 7*a^3*(15*A*x + 5*B*x^3 + 3*C*x^5))/(105*a^4*(a + b*x^2)^(7/
2))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.83, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1469, 2075, 362, 245, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{(a+bx^2)^{9/2}}\,dx$$

$$\Big\downarrow \text{1469}$$

$$\frac{\int \frac{x^2(6Ab+a(Cx^2+B))}{(bx^2+a)^{9/2}}dx}{a}+\frac{Ax}{a(a+bx^2)^{7/2}}$$

$$\Big\downarrow \text{2075}$$

$$\frac{\int \frac{x^2(aCx^2+6Ab+aB)}{(bx^2+a)^{9/2}}dx}{a}+\frac{Ax}{a(a+bx^2)^{7/2}}$$

$$\Big\downarrow \text{362}$$

$$\frac{\frac{1}{7}\left(\frac{4(aB+6Ab)}{a}+\frac{3aC}{b}\right)\int \frac{x^2}{(bx^2+a)^{7/2}}dx+\frac{x^3\left(a\left(B-\frac{aC}{b}\right)+6Ab\right)}{7a(a+bx^2)^{7/2}}}{a}+\frac{Ax}{a(a+bx^2)^{7/2}}$$

$$\Big\downarrow \text{245}$$

$$\frac{\frac{1}{7}\left(\frac{4(aB+6Ab)}{a}+\frac{3aC}{b}\right)\left(\frac{2b\int \frac{x^4}{(bx^2+a)^{7/2}}dx}{3a}+\frac{x^3}{3a(a+bx^2)^{5/2}}\right)+\frac{x^3\left(a\left(B-\frac{aC}{b}\right)+6Ab\right)}{7a(a+bx^2)^{7/2}}}{a}+\frac{Ax}{a(a+bx^2)^{7/2}}$$

$$\Big\downarrow \text{242}$$

$$\frac{\frac{1}{7}\left(\frac{2bx^5}{15a^2(a+bx^2)^{5/2}}+\frac{x^3}{3a(a+bx^2)^{5/2}}\right)\left(\frac{4(aB+6Ab)}{a}+\frac{3aC}{b}\right)+\frac{x^3\left(a\left(B-\frac{aC}{b}\right)+6Ab\right)}{7a(a+bx^2)^{7/2}}}{a}+\frac{Ax}{a\left(a+bx^2\right)^{7/2}}$$

input
```
Int[(A + B*x^2 + C*x^4)/(a + b*x^2)^(9/2),x]
```

output
```
(A*x)/(a*(a + b*x^2)^(7/2)) + (((6*A*b + a*(B - (a*C)/b))*x^3)/(7*a*(a + b
*x^2)^(7/2)) + (((4*(6*A*b + a*B))/a + (3*a*C)/b)*(x^3/(3*a*(a + b*x^2)^(5
/2)) + (2*b*x^5)/(15*a^2*(a + b*x^2)^(5/2))))/7)/a
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

rule 1469
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[a^p*x*((d + e*x^2)^(q + 1)/d), x] + Simp[1/d   Int[x^2*(d
 + e*x^2)^q*(d*PolynomialQuotient[(a + b*x^2 + c*x^4)^p - a^p, x^2, x] - e*
a^p*(2*q + 3)), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && ILtQ[q + 1/2, 0] && LtQ[4
*p + 2*q + 1, 0]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.54

method	result
pseudoelliptic	$\frac{\left(\left(\frac{1}{5}C\,x^4+\frac{1}{3}x^2B+A\right)a^3+2\left(\frac{1}{35}C\,x^4+\frac{2}{15}x^2B+A\right)x^2b\,a^2+\frac{8b^2\left(\frac{x^2B}{21}+A\right)x^4a}{5}+\frac{16A\,b^3x^6}{35}\right)x}{\left(b\,x^2+a\right)^{\frac{7}{2}}a^4}$
gosper	$\frac{x\left(48A\,b^3x^6+8Ba\,b^2x^6+6C\,a^2b\,x^6+168aA\,b^2x^4+28B\,a^2b\,x^4+21C\,a^3x^4+210a^2Ab\,x^2+35B\,a^3x^2+105a^3A\right)}{105\left(b\,x^2+a\right)^{\frac{7}{2}}a^4}$
trager	$\frac{x\left(48A\,b^3x^6+8Ba\,b^2x^6+6C\,a^2b\,x^6+168aA\,b^2x^4+28B\,a^2b\,x^4+21C\,a^3x^4+210a^2Ab\,x^2+35B\,a^3x^2+105a^3A\right)}{105\left(b\,x^2+a\right)^{\frac{7}{2}}a^4}$
orering	$\frac{x\left(48A\,b^3x^6+8Ba\,b^2x^6+6C\,a^2b\,x^6+168aA\,b^2x^4+28B\,a^2b\,x^4+21C\,a^3x^4+210a^2Ab\,x^2+35B\,a^3x^2+105a^3A\right)}{105\left(b\,x^2+a\right)^{\frac{7}{2}}a^4}$
default	$A\left(\frac{x}{7a\left(b\,x^2+a\right)^{\frac{7}{2}}}+\frac{\frac{6x}{35a\left(b\,x^2+a\right)^{\frac{5}{2}}}+\frac{6\left(\frac{4x}{15a\left(b\,x^2+a\right)^{\frac{3}{2}}}+\frac{8x}{15a^2\sqrt{b\,x^2+a}}\right)}{7a}}{a}\right)+C\left(-\frac{x^3}{4b\left(b\,x^2+a\right)^{\frac{7}{2}}}+\frac{3a\left(-\frac{x}{6b\left(b\,x^2+\right.}\right.}{}\right.$

input
```
int((C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
((1/5*C*x^4+1/3*x^2*B+A)*a^3+2*(1/35*C*x^4+2/15*x^2*B+A)*x^2*b*a^2+8/5*b^2
*(1/21*x^2*B+A)*x^4*a+16/35*A*b^3*x^6)/(b*x^2+a)^(7/2)*x/a^4
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.88

$$\int \frac{A+Bx^2+Cx^4}{(a+bx^2)^{9/2}}\,dx = \frac{(2\,(3\,Ca^2b+4\,Bab^2+24\,Ab^3)x^7+7\,(3\,Ca^3+4\,Ba^2b+24\,Aab^2)x^5+105\,Aa^3x+}{105\,(a^4b^4x^8+4\,a^5b^3x^6+6\,a^6b^2x^4+4\,a^7bx^2+a^8)}$$

input
```
integrate((C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

output
```
1/105*(2*(3*C*a^2*b + 4*B*a*b^2 + 24*A*b^3)*x^7 + 7*(3*C*a^3 + 4*B*a^2*b +
 24*A*a*b^2)*x^5 + 105*A*a^3*x + 35*(B*a^3 + 6*A*a^2*b)*x^3)*sqrt(b*x^2 +
a)/(a^4*b^4*x^8 + 4*a^5*b^3*x^6 + 6*a^6*b^2*x^4 + 4*a^7*b*x^2 + a^8)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1989 vs. $2(150) = 300$.

Time = 38.41 (sec) , antiderivative size = 1989, normalized size of antiderivative = 12.83

$$\int \frac{A+Bx^2+Cx^4}{(a+bx^2)^{9/2}}\,dx = \text{Too large to display}$$

input
```
integrate((C*x**4+B*x**2+A)/(b*x**2+a)**(9/2),x)
```

output
```
A*(35*a**14*x/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt
(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2
)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a
) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*
sqrt(1 + b*x**2/a)) + 175*a**13*b*x**3/(35*a**(37/2)*sqrt(1 + b*x**2/a) +
210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 +
 b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b*
*4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) +
 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a)) + 371*a**12*b**2*x**5/(35*a**
(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*
a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 +
b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**
5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a)) +
 429*a**11*b**3*x**7/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x*
*2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a
**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b
*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6
*x**12*sqrt(1 + b*x**2/a)) + 286*a**10*b**4*x**9/(35*a**(37/2)*sqrt(1 + b*
x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**
4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.46

$$\int \frac{A + Bx^2 + Cx^4}{(a + bx^2)^{9/2}}\, dx = -\frac{Cx^3}{4\,(bx^2 + a)^{\frac{7}{2}}b} + \frac{16\,Ax}{35\,\sqrt{bx^2 + a}a^4} + \frac{8\,Ax}{35\,(bx^2 + a)^{\frac{3}{2}}a^3} + \frac{6\,Ax}{35\,(bx^2 + a)^{\frac{5}{2}}a^2} + \frac{Ax}{7\,(bx^2 + a)^{\frac{7}{2}}a} + \frac{3\,Cx}{140\,(bx^2 + a)^{\frac{5}{2}}b^2} + \frac{2\,Cx}{35\,\sqrt{bx^2 + a}a^2b^2} + \frac{Cx}{35\,(bx^2 + a)^{\frac{3}{2}}ab^2} - \frac{3\,Cax}{28\,(bx^2 + a)^{\frac{7}{2}}b^2} - \frac{Bx}{7\,(bx^2 + a)^{\frac{7}{2}}b} + \frac{8\,Bx}{105\,\sqrt{bx^2 + a}a^3b} + \frac{4\,Bx}{105\,(bx^2 + a)^{\frac{3}{2}}a^2b} + \frac{Bx}{35\,(bx^2 + a)^{\frac{5}{2}}ab}$$

input
```
integrate((C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

output
```
-1/4*C*x^3/((b*x^2 + a)^(7/2)*b) + 16/35*A*x/(sqrt(b*x^2 + a)*a^4) + 8/35*
A*x/((b*x^2 + a)^(3/2)*a^3) + 6/35*A*x/((b*x^2 + a)^(5/2)*a^2) + 1/7*A*x/(
(b*x^2 + a)^(7/2)*a) + 3/140*C*x/((b*x^2 + a)^(5/2)*b^2) + 2/35*C*x/(sqrt(
b*x^2 + a)*a^2*b^2) + 1/35*C*x/((b*x^2 + a)^(3/2)*a*b^2) - 3/28*C*a*x/((b*
x^2 + a)^(7/2)*b^2) - 1/7*B*x/((b*x^2 + a)^(7/2)*b) + 8/105*B*x/(sqrt(b*x^
2 + a)*a^3*b) + 4/105*B*x/((b*x^2 + a)^(3/2)*a^2*b) + 1/35*B*x/((b*x^2 + a
)^(5/2)*a*b)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.79

$$\int \frac{A+Bx^2+Cx^4}{(a+bx^2)^{9/2}}\,dx = \frac{\left(\left(x^2\left(\frac{2\,(3\,Ca^2b^4+4\,Bab^5+24\,Ab^6)x^2}{a^4b^3}+\frac{7\,(3\,Ca^3b^3+4\,Ba^2b^4+24\,Aab^5)}{a^4b^3}\right)+\frac{35\,(Ba^3b^3+6\,Aa^2b^4)}{a^4b^3}\right)x^2 \cdots}{105\,(bx^2+a)^{\frac{7}{2}}}$$

input
```
integrate((C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="giac")
```

output
```
1/105*((x^2*(2*(3*C*a^2*b^4 + 4*B*a*b^5 + 24*A*b^6)*x^2/(a^4*b^3) + 7*(3*C
*a^3*b^3 + 4*B*a^2*b^4 + 24*A*a*b^5)/(a^4*b^3)) + 35*(B*a^3*b^3 + 6*A*a^2*
b^4)/(a^4*b^3))*x^2 + 105*A/a)*x/(b*x^2 + a)^(7/2)
```

Mupad [B] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.99

$$\int \frac{A+Bx^2+Cx^4}{(a+bx^2)^{9/2}}\,dx = \frac{x\left(\frac{A}{7\,a}-\frac{a\left(\frac{B}{7\,a}-\frac{C}{7\,b}\right)}{b}\right)}{{\left(b\,x^2+a\right)}^{7/2}}-\frac{x\left(\frac{C}{5\,b^2}-\frac{-C\,a^2+B\,a\,b+6\,A\,b^2}{35\,a^2\,b^2}\right)}{{\left(b\,x^2+a\right)}^{5/2}} + \frac{x\,\left(3\,C\,a^2+4\,B\,a\,b+24\,A\,b^2\right)}{105\,a^3\,b^2\,{\left(b\,x^2+a\right)}^{3/2}}+\frac{x\,\left(6\,C\,a^2+8\,B\,a\,b+48\,A\,b^2\right)}{105\,a^4\,b^2\,\sqrt{b\,x^2+a}}$$

input
```
int((A + B*x^2 + C*x^4)/(a + b*x^2)^(9/2),x)
```

output
```
(x*(A/(7*a) - (a*(B/(7*a) - C/(7*b)))/b))/(a + b*x^2)^(7/2) - (x*(C/(5*b^2
) - (6*A*b^2 - C*a^2 + B*a*b)/(35*a^2*b^2)))/(a + b*x^2)^(5/2) + (x*(24*A*
b^2 + 3*C*a^2 + 4*B*a*b))/(105*a^3*b^2*(a + b*x^2)^(3/2)) + (x*(48*A*b^2 +
 6*C*a^2 + 8*B*a*b))/(105*a^4*b^2*(a + b*x^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.77

$$\int \frac{A + Bx^2 + Cx^4}{(a + bx^2)^{9/2}}\,dx = \frac{105\sqrt{b\,x^2+a}\,a^3b^3x + 245\sqrt{b\,x^2+a}\,a^2b^4x^3 + 21\sqrt{b\,x^2+a}\,a^2b^3c\,x^5 + 196\sqrt{b\,x^2+}}{}$$

input
```
int((C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x)
```

output
```
(105*sqrt(a + b*x**2)*a**3*b**3*x + 245*sqrt(a + b*x**2)*a**2*b**4*x**3 +
21*sqrt(a + b*x**2)*a**2*b**3*c*x**5 + 196*sqrt(a + b*x**2)*a*b**5*x**5 +
6*sqrt(a + b*x**2)*a*b**4*c*x**7 + 56*sqrt(a + b*x**2)*b**6*x**7 - 6*sqrt(
b)*a**5*c - 56*sqrt(b)*a**4*b**2 - 24*sqrt(b)*a**4*b*c*x**2 - 224*sqrt(b)*
a**3*b**3*x**2 - 36*sqrt(b)*a**3*b**2*c*x**4 - 336*sqrt(b)*a**2*b**4*x**4
- 24*sqrt(b)*a**2*b**3*c*x**6 - 224*sqrt(b)*a*b**5*x**6 - 6*sqrt(b)*a*b**4
*c*x**8 - 56*sqrt(b)*b**6*x**8)/(105*a**3*b**3*(a**4 + 4*a**3*b*x**2 + 6*a
**2*b**2*x**4 + 4*a*b**3*x**6 + b**4*x**8))
```

3.122 $\int \sqrt[3]{a+bx^2}\left(A+Bx^2+Cx^4\right)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 353

$$\int \sqrt[3]{a+bx^2}\left(A+Bx^2+Cx^4\right)\,dx = \frac{3}{935}\left(187A-\frac{3a(17bB-9aC)}{b^2}\right)x\sqrt[3]{a+bx^2} + \frac{3(17bB-9aC)x(a+bx^2)^{4/3}}{187b^2} + \frac{3Cx^3(a+bx^2)^{4/3}}{17b} - \frac{2\,3^{3/4}\sqrt{2-\sqrt{3}}a(187Ab^2-51abB+27a^2C)\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\ \text{Ellipti}}{935b^3x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

output

```
3/935*(187*A-3*a*(17*B*b-9*C*a)/b^2)*x*(b*x^2+a)^(1/3)+3/187*(17*B*b-9*C*a
)*x*(b*x^2+a)^(4/3)/b^2+3/17*C*x^3*(b*x^2+a)^(4/3)/b-2/935*3^(3/4)*(1/2*6^
(1/2)-1/2*2^(1/2))*a*(187*A*b^2-51*B*a*b+27*C*a^2)*(a^(1/3)-(b*x^2+a)^(1/3
))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/((1-3^(1/2))*a^(1/3)
-(b*x^2+a)^(1/3))^2)^(1/2)*EllipticF(((1+3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))
/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3)),2*I-I*3^(1/2))/b^3/x/(-a^(1/3)*(a^(
1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.63 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.28

$$\int \sqrt[3]{a+bx^2}\left(A+Bx^2+Cx^4\right) dx$$
$$= \frac{x\sqrt[3]{a+bx^2}\left(-3(a+bx^2)\left(-17bB+9aC-11bCx^2\right)+\frac{(187Ab^2+3a(-17bB+9aC))\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{\sqrt[3]{1+\frac{bx^2}{a}}}\right)}{187b^2}$$

input
```
Integrate[(a + b*x^2)^(1/3)*(A + B*x^2 + C*x^4),x]
```

output
```
(x*(a + b*x^2)^(1/3)*(-3*(a + b*x^2)*(-17*b*B + 9*a*C - 11*b*C*x^2) + ((18
7*A*b^2 + 3*a*(-17*b*B + 9*a*C))*Hypergeometric2F1[-1/3, 1/2, 3/2, -((b*x^
2)/a)])/(1 + (b*x^2)/a)^(1/3)))/(187*b^2)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 351, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1473, 27, 299, 211, 234, 760}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{a+bx^2}\left(A+Bx^2+Cx^4\right) dx$$
$$\Big\downarrow \; 1473$$
$$\frac{3\int \frac{1}{3}\sqrt[3]{bx^2+a}\left((17bB-9aC)x^2+17Ab\right)dx}{17b}+\frac{3Cx^3\left(a+bx^2\right)^{4/3}}{17b}$$
$$\Big\downarrow \; 27$$
$$\frac{\int \sqrt[3]{bx^2+a}\left((17bB-9aC)x^2+17Ab\right)dx}{17b}+\frac{3Cx^3\left(a+bx^2\right)^{4/3}}{17b}$$

$$\Big\downarrow \; 299$$

$$\frac{\frac{(187Ab^2-3a(17bB-9aC))\int \sqrt[3]{bx^2+a}dx}{11b}+\frac{3x(a+bx^2)^{4/3}(17bB-9aC)}{11b}}{17b}+\frac{3Cx^3(a+bx^2)^{4/3}}{17b}$$

$$\Big\downarrow \; 211$$

$$\frac{\frac{(187Ab^2-3a(17bB-9aC))\left(\frac{2}{5}a\int \frac{1}{(bx^2+a)^{2/3}}dx+\frac{3}{5}x\sqrt[3]{a+bx^2}\right)}{11b}+\frac{3x(a+bx^2)^{4/3}(17bB-9aC)}{11b}}{17b}+\frac{3Cx^3(a+bx^2)^{4/3}}{17b}$$

$$\Big\downarrow \; 234$$

$$\frac{\frac{(187Ab^2-3a(17bB-9aC))\left(\frac{3a\sqrt{bx^2}\int \frac{1}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}}{5bx}+\frac{3}{5}x\sqrt[3]{a+bx^2}\right)}{11b}+\frac{3x(a+bx^2)^{4/3}(17bB-9aC)}{11b}}{17b}+\frac{3Cx^3(a+bx^2)^{4/3}}{17b}$$

$$\Big\downarrow \; 760$$

$$\frac{\frac{(187Ab^2-3a(17bB-9aC))\left(\frac{3}{5}x\sqrt[3]{a+bx^2}-\frac{2\,3^{3/4}\sqrt{2-\sqrt{3}}a\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{(1+}{(1-}\right.\right.}{5bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}\right.}{11b}}{17b}$$

$$\frac{3Cx^3(a+bx^2)^{4/3}}{17b}$$

input
```
Int[(a + b*x^2)^(1/3)*(A + B*x^2 + C*x^4),x]
```

output
```
(3*C*x^3*(a + b*x^2)^(4/3))/(17*b) + ((3*(17*b*B - 9*a*C)*x*(a + b*x^2)^(4
/3))/(11*b) + ((187*A*b^2 - 3*a*(17*b*B - 9*a*C))*((3*x*(a + b*x^2)^(1/3))
/5 - (2*3^(3/4)*Sqrt[2 - Sqrt[3]]*a*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^
(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1
/3) - (a + b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a +
 b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3
]])/(5*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a
^(1/3) - (a + b*x^2)^(1/3))^2)])))/(11*b))/(17*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 234
```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 1473
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[c^p*x^(4*p - 1)*((d + e*x^2)^(q + 1)/(e*(4*p + 2*q + 1)))
, x] + Simp[1/(e*(4*p + 2*q + 1))   Int[(d + e*x^2)^q*ExpandToSum[e*(4*p +
2*q + 1)*(a + b*x^2 + c*x^4)^p - d*c^p*(4*p - 1)*x^(4*p - 2) - e*c^p*(4*p +
 2*q + 1)*x^(4*p), x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] &&  !LtQ[q, -1]
```

Maple [F]

$$\int \left(b\,x^{2}+a \right)^{\frac{1}{3}} \left(C\,x^{4}+x^{2} B +A \right)d x$$

input
```
int((b*x^2+a)^(1/3)*(C*x^4+B*x^2+A),x)
```

output
```
int((b*x^2+a)^(1/3)*(C*x^4+B*x^2+A),x)
```

Fricas [F]

$$\int \sqrt[3]{a+bx^2}\left(A+Bx^2+Cx^4\right)\,dx=\int \left(Cx^{4}+Bx^{2}+A\right)\left(bx^{2}+a\right)^{\frac{1}{3}}\,dx$$

input
```
integrate((b*x^2+a)^(1/3)*(C*x^4+B*x^2+A),x, algorithm="fricas")
```

output
```
integral((C*x^4 + B*x^2 + A)*(b*x^2 + a)^(1/3), x)
```

Sympy [A] (verification not implemented)

Time = 1.29 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.26

$$\int \sqrt[3]{a+bx^2}\left(A+Bx^2+Cx^4\right)\,dx = A\sqrt[3]{a}x{}_2F_1\left(\begin{matrix} -\frac{1}{3},\frac{1}{2} \\ \frac{3}{2} \end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right) + \frac{B\sqrt[3]{a}x^3{}_2F_1\left(\begin{matrix} -\frac{1}{3},\frac{3}{2} \\ \frac{5}{2} \end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{3} + \frac{C\sqrt[3]{a}x^5{}_2F_1\left(\begin{matrix} -\frac{1}{3},\frac{5}{2} \\ \frac{7}{2} \end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{5}$$

input
```
integrate((b*x**2+a)**(1/3)*(C*x**4+B*x**2+A),x)
```

output
```
A*a**(1/3)*x*hyper((-1/3, 1/2), (3/2,), b*x**2*exp_polar(I*pi)/a) + B*a**(
1/3)*x**3*hyper((-1/3, 3/2), (5/2,), b*x**2*exp_polar(I*pi)/a)/3 + C*a**(1
/3)*x**5*hyper((-1/3, 5/2), (7/2,), b*x**2*exp_polar(I*pi)/a)/5
```

Maxima [F]

$$\int \sqrt[3]{a+bx^2}\left(A+Bx^2+Cx^4\right)\,dx = \int \left(Cx^4+Bx^2+A\right)\left(bx^2+a\right)^{\frac{1}{3}}\,dx$$

input
```
integrate((b*x^2+a)^(1/3)*(C*x^4+B*x^2+A),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)*(b*x^2 + a)^(1/3), x)
```

Giac [F]

$$\int \sqrt[3]{a+bx^2}\left(A+Bx^2+Cx^4\right)\,dx=\int \left(Cx^4+Bx^2+A\right)\left(bx^2+a\right)^{\frac{1}{3}}\,dx$$

input
```
integrate((b*x^2+a)^(1/3)*(C*x^4+B*x^2+A),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)*(b*x^2 + a)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{a+bx^2}\left(A+Bx^2+Cx^4\right)\,dx=\int \left(b\,x^2+a\right)^{1/3}\,\left(C\,x^4+B\,x^2+A\right)\,dx$$

input
```
int((a + b*x^2)^(1/3)*(A + B*x^2 + C*x^4),x)
```

output
```
int((a + b*x^2)^(1/3)*(A + B*x^2 + C*x^4), x)
```

Reduce [F]

$$\int \sqrt[3]{a+bx^2}\left(A+Bx^2+Cx^4\right)\,dx$$
$$=\frac{-54(b\,x^2+a)^{\frac{1}{3}}\,a^2cx+663(b\,x^2+a)^{\frac{1}{3}}\,a\,b^2x+30(b\,x^2+a)^{\frac{1}{3}}\,abc\,x^3+255(b\,x^2+a)^{\frac{1}{3}}\,b^3x^3+165(b\,x^2+a)}{935b^2}$$

input
```
int((b*x^2+a)^(1/3)*(C*x^4+B*x^2+A),x)
```

output
```
( - 54*(a + b*x**2)**(1/3)*a**2*c*x + 663*(a + b*x**2)**(1/3)*a*b**2*x + 3
0*(a + b*x**2)**(1/3)*a*b*c*x**3 + 255*(a + b*x**2)**(1/3)*b**3*x**3 + 165
*(a + b*x**2)**(1/3)*b**2*c*x**5 + 54*int((a + b*x**2)**(1/3)/(a + b*x**2)
,x)*a**3*c + 272*int((a + b*x**2)**(1/3)/(a + b*x**2),x)*a**2*b**2)/(935*b
**2)
```

3.123 $\int \frac{A+Bx^2+Cx^4}{(a+bx^2)^{2/3}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 317

$$\int \frac{A+Bx^2+Cx^4}{(a+bx^2)^{2/3}}\,dx = \frac{3(11bB-9aC)x\sqrt[3]{a+bx^2}}{55b^2}+\frac{3Cx^3\sqrt[3]{a+bx^2}}{11b} - \frac{3^{3/4}\sqrt{2-\sqrt{3}}(55Ab^2-33abB+27a^2C)\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,\text{EllipticF}\left(\text{ar}\right.}{55b^3x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

output
```
3/55*(11*B*b-9*C*a)*x*(b*x^2+a)^(1/3)/b^2+3/11*C*x^3*(b*x^2+a)^(1/3)/b-1/5
5*3^(3/4)*(1/2*6^(1/2)-1/2*2^(1/2))*(55*A*b^2-33*B*a*b+27*C*a^2)*(a^(1/3)-
(b*x^2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/((1-3^
(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)*EllipticF(((1+3^(1/2))*a^(1/3)-(b
*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3)),2*I-I*3^(1/2))/b^3/x/
(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^
2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.31

$$\int \frac{A+Bx^2+Cx^4}{(a+bx^2)^{2/3}}\,dx = \frac{x\left(-3(a+bx^2)\left(-11bB+9aC-5bCx^2\right)+\left(55Ab^2+3a(-11bB+9aC)\right)\left(1+\frac{bx^2}{a}\right.\right.}{55b^2\left(a+bx^2\right)^{2/3}}$$

input
```
Integrate[(A + B*x^2 + C*x^4)/(a + b*x^2)^(2/3),x]
```

output
```
(x*(-3*(a + b*x^2)*(-11*b*B + 9*a*C - 5*b*C*x^2) + (55*A*b^2 + 3*a*(-11*b*
B + 9*a*C))*(1 + (b*x^2)/a)^(2/3)*Hypergeometric2F1[1/2, 2/3, 3/2, -((b*x^
2)/a)]))/(55*b^2*(a + b*x^2)^(2/3))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1473, 27, 299, 234, 760}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{(a+bx^2)^{2/3}}dx$$

$$\Big\downarrow \ 1473$$

$$\frac{3\int \frac{(11bB-9aC)x^2+11Ab}{3(bx^2+a)^{2/3}}dx}{11b}+\frac{3Cx^3\sqrt[3]{a+bx^2}}{11b}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{(11bB-9aC)x^2+11Ab}{(bx^2+a)^{2/3}}dx}{11b}+\frac{3Cx^3\sqrt[3]{a+bx^2}}{11b}$$

$$\Big\downarrow \ 299$$

$$\frac{\frac{\left(55Ab^2-3a(11bB-9aC)\right)\int \frac{1}{\left(bx^2+a\right)^{2/3}}dx}{5b}+\frac{3x\sqrt[3]{a+bx^2}(11bB-9aC)}{5b}}{11b}+\frac{3Cx^3\sqrt[3]{a+bx^2}}{11b}$$

$$\Big\downarrow \; 234$$

$$\frac{\frac{3\sqrt{bx^2}\left(55Ab^2-3a(11bB-9aC)\right)\int \frac{1}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}}{10b^2x}+\frac{3x\sqrt[3]{a+bx^2}(11bB-9aC)}{5b}}{11b}+\frac{3Cx^3\sqrt[3]{a+bx^2}}{11b}$$

$$\Big\downarrow \; 760$$

$$\frac{\frac{3x\sqrt[3]{a+bx^2}(11bB-9aC)}{5b}-\frac{3^{3/4}\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\left(55Ab^2-3a(11bB-9aC)\right)\text{EllipticF}}{5b^2x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}}{11b}$$

$$\frac{3Cx^3\sqrt[3]{a+bx^2}}{11b}$$

input
```
Int[(A + B*x^2 + C*x^4)/(a + b*x^2)^(2/3),x]
```

output
```
(3*C*x^3*(a + b*x^2)^(1/3))/(11*b) + ((3*(11*b*B - 9*a*C)*x*(a + b*x^2)^(1
/3))/(5*b) - (3^(3/4)*Sqrt[2 - Sqrt[3]]*(55*A*b^2 - 3*a*(11*b*B - 9*a*C))*
(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) +
(a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*Elliptic
F[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3
) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(5*b^2*x*Sqrt[-((a^(1/3)*(a^(1/3
) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)]))/
(11*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 234
```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 1473
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[c^p*x^(4*p - 1)*((d + e*x^2)^(q + 1)/(e*(4*p + 2*q + 1)))
, x] + Simp[1/(e*(4*p + 2*q + 1))   Int[(d + e*x^2)^q*ExpandToSum[e*(4*p +
2*q + 1)*(a + b*x^2 + c*x^4)^p - d*c^p*(4*p - 1)*x^(4*p - 2) - e*c^p*(4*p +
 2*q + 1)*x^(4*p), x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] &&  !LtQ[q, -1]
```

Maple [F]

$$\int \frac{C\,x^4+x^2B+A}{\left(b\,x^2+a\right)^{\frac{2}{3}}}dx$$

input
```
int((C*x^4+B*x^2+A)/(b*x^2+a)^(2/3),x)
```

output
```
int((C*x^4+B*x^2+A)/(b*x^2+a)^(2/3),x)
```

Fricas [F]

$$\int \frac{A+Bx^2+Cx^4}{(a+bx^2)^{2/3}}\,dx = \int \frac{Cx^4+Bx^2+A}{(bx^2+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

output
```
integral((C*x^4 + B*x^2 + A)/(b*x^2 + a)^(2/3), x)
```

Sympy [A] (verification not implemented)

Time = 1.17 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.27

$$\int \frac{A+Bx^2+Cx^4}{(a+bx^2)^{2/3}}\,dx = \frac{Ax\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{a^{\frac{2}{3}}} + \frac{Bx^3\,{}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}} + \frac{Cx^5\,{}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{5}{2}\\ \frac{7}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{5a^{\frac{2}{3}}}$$

input
```
integrate((C*x**4+B*x**2+A)/(b*x**2+a)**(2/3),x)
```

output
```
A*x*hyper((1/2, 2/3), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(2/3) + B*x**3*
hyper((2/3, 3/2), (5/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(2/3)) + C*x**5*
hyper((2/3, 5/2), (7/2,), b*x**2*exp_polar(I*pi)/a)/(5*a**(2/3))
```

Maxima [F]

$$\int \frac{A+Bx^2+Cx^4}{(a+bx^2)^{2/3}}\,dx = \int \frac{Cx^4+Bx^2+A}{(bx^2+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/(b*x^2+a)^(2/3),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)/(b*x^2 + a)^(2/3), x)
```

Giac [F]

$$\int \frac{A+Bx^2+Cx^4}{(a+bx^2)^{2/3}}\,dx = \int \frac{Cx^4+Bx^2+A}{(bx^2+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/(b*x^2+a)^(2/3),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)/(b*x^2 + a)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{(a+bx^2)^{2/3}}\,dx = \int \frac{C\,x^4+B\,x^2+A}{(b\,x^2+a)^{2/3}}\,dx$$

input
```
int((A + B*x^2 + C*x^4)/(a + b*x^2)^(2/3),x)
```

output
```
int((A + B*x^2 + C*x^4)/(a + b*x^2)^(2/3), x)
```

Reduce [F]

$$\int \frac{A+Bx^2+Cx^4}{(a+bx^2)^{2/3}}\,dx = \left(\int \frac{x^4}{(b\,x^2+a)^{\frac{2}{3}}}dx\right)c + \left(\int \frac{x^2}{(b\,x^2+a)^{\frac{2}{3}}}dx\right)b + \left(\int \frac{1}{(b\,x^2+a)^{\frac{2}{3}}}dx\right)a$$

input
```
int((C*x^4+B*x^2+A)/(b*x^2+a)^(2/3),x)
```

output
```
int(x**4/(a + b*x**2)**(2/3),x)*c + int(x**2/(a + b*x**2)**(2/3),x)*b + in
t(1/(a + b*x**2)**(2/3),x)*a
```

3.124 $\int (a+bx^2)^{2/3} (A+Bx^2+Cx^4)\, dx$

Optimal result

Integrand size = 24, antiderivative size = 676

$$\int (a+bx^2)^{2/3} (A+Bx^2+Cx^4)\, dx = \frac{3\left(247A-\frac{3a(19bB-9aC)}{b^2}\right)x(a+bx^2)^{2/3}}{1729}$$

$$+\frac{3(19bB-9aC)x(a+bx^2)^{5/3}}{247b^2}+\frac{3Cx^3(a+bx^2)^{5/3}}{19b}$$

$$-\frac{12a(247Ab^2-57abB+27a^2C)\,x}{1729b^2\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}+\frac{6\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^{4/3}(247Ab^2-57abB+27a^2C)\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{1729b^3x\sqrt{-}}$$

$$-\frac{4\sqrt{2}3^{3/4}a^{4/3}(247Ab^2-57abB+27a^2C)\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\ \text{EllipticF}\left(\text{ar}\right.}{1729b^3x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

output
```
3/1729*(247*A-3*a*(19*B*b-9*C*a)/b^2)*x*(b*x^2+a)^(2/3)+3/247*(19*B*b-9*C*
a)*x*(b*x^2+a)^(5/3)/b^2+3/19*C*x^3*(b*x^2+a)^(5/3)/b-12/1729*a*(247*A*b^2
-57*B*a*b+27*C*a^2)*x/b^2/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))+6/1729*3^(
1/4)*(1/2*6^(1/2)+1/2*2^(1/2))*a^(4/3)*(247*A*b^2-57*B*a*b+27*C*a^2)*(a^(1
/3)-(b*x^2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/((
1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)*EllipticE(((1+3^(1/2))*a^(1/3
)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3)),2*I-I*3^(1/2))/b^
3/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/
3))^2)^(1/2)-4/1729*2^(1/2)*3^(3/4)*a^(4/3)*(247*A*b^2-57*B*a*b+27*C*a^2)*
(a^(1/3)-(b*x^2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3
))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)*EllipticF(((1+3^(1/2))*a
^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3)),2*I-I*3^(1/2
))/b^3/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a
)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.69 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.14

$$\int \left(a+bx^2\right)^{2/3}\left(A+Bx^2+Cx^4\right)\,dx=\frac{x\left(a+bx^2\right)^{2/3}\left(-3\left(a+bx^2\right)\left(-19bB+9aC-13bCx^2\right)+\frac{\left(247Ab^2+3a(-19bB+9aC)\right)\text{Hypergeometric2F}}{\left(1+\frac{bx^2}{a}\right)^{2/3}}\right.}{247b^2}$$

input
```
Integrate[(a + b*x^2)^(2/3)*(A + B*x^2 + C*x^4),x]
```

output
```
(x*(a + b*x^2)^(2/3)*(-3*(a + b*x^2)*(-19*b*B + 9*a*C - 13*b*C*x^2) + ((24
7*A*b^2 + 3*a*(-19*b*B + 9*a*C))*Hypergeometric2F1[-2/3, 1/2, 3/2, -((b*x^
2)/a)])/(1 + (b*x^2)/a)^(2/3)))/(247*b^2)
```

Rubi [A] (warning: unable to verify)

Time = 0.56 (sec) , antiderivative size = 677, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1473, 27, 299, 211, 233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^{2/3} \left(A+Bx^2+Cx^4\right) \, dx$$

$$\Big\downarrow \text{1473}$$

$$\frac{3\int \frac{1}{3}\left(bx^2+a\right)^{2/3}\left((19bB-9aC)x^2+19Ab\right)dx}{19b}+\frac{3Cx^3\left(a+bx^2\right)^{5/3}}{19b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \left(bx^2+a\right)^{2/3}\left((19bB-9aC)x^2+19Ab\right)dx}{19b}+\frac{3Cx^3\left(a+bx^2\right)^{5/3}}{19b}$$

$$\Big\downarrow \text{299}$$

$$\frac{\frac{\left(247Ab^2-3a(19bB-9aC)\right)\int \left(bx^2+a\right)^{2/3}dx}{13b}+\frac{3x\left(a+bx^2\right)^{5/3}(19bB-9aC)}{13b}}{19b}+\frac{3Cx^3\left(a+bx^2\right)^{5/3}}{19b}$$

$$\Big\downarrow \text{211}$$

$$\frac{\frac{\left(247Ab^2-3a(19bB-9aC)\right)\left(\frac{4}{7}a\int \frac{1}{\sqrt[3]{bx^2+a}}dx+\frac{3}{7}x\left(a+bx^2\right)^{2/3}\right)}{13b}+\frac{3x\left(a+bx^2\right)^{5/3}(19bB-9aC)}{13b}}{19b}+\frac{3Cx^3\left(a+bx^2\right)^{5/3}}{19b}$$

$$\Big\downarrow \text{233}$$

$$\frac{\frac{\left(247Ab^2-3a(19bB-9aC)\right)\left(\frac{6a\sqrt{bx^2}\int \frac{\sqrt[3]{bx^2+a}}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}}{7bx}+\frac{3}{7}x\left(a+bx^2\right)^{2/3}\right)}{13b}+\frac{3x\left(a+bx^2\right)^{5/3}(19bB-9aC)}{13b}}{19b}+\frac{3Cx^3\left(a+bx^2\right)^{5/3}}{19b}$$

$$\Big\downarrow \text{833}$$

$$\frac{\frac{\left(247Ab^2-3a(19bB-9aC)\right)\left(\frac{6a\sqrt{bx^2}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}\int\frac{1}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}-\int\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}\right)}{7bx}+\frac{3}{7}x\left(a+bx^2\right)^{2/3}\right)}{13b}+\frac{3\ldots}{}}{19b}$$

$$\frac{3Cx^3\left(a+bx^2\right)^{5/3}}{19b}$$

$\downarrow$ 760

$$\frac{\frac{\left(247Ab^2-3a(19bB-9aC)\right)\left(\frac{6a\sqrt{bx^2}\left(-\int\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{\ldots}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{\ldots}\right.}}}{\sqrt[4]{3}\sqrt{bx^2}\sqrt{\ldots}}\ldots\right)}{7bx}\ldots\right.}{13b}\ldots}{19b}$$

$$\frac{3Cx^3\left(a+bx^2\right)^{5/3}}{19b}$$

$\downarrow$ 2418

$$\frac{(247Ab^2-3a(19bB-9aC))\Bigg(\frac{6a\sqrt{bx^2}\Bigg(-\frac{2\sqrt{2-\sqrt{3}}(1+\sqrt{3})\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,\text{EllipticF}\Big(\arcsin\Big(\frac{(1+\cdots}{(1-\cdots}}{\sqrt[4]{3}\sqrt{bx^2}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}\cdots}{\cdots}\cdots}{\cdots}$$

$$\frac{3Cx^3(a+bx^2)^{5/3}}{19b}$$

input
```
Int[(a + b*x^2)^(2/3)*(A + B*x^2 + C*x^4),x]
```

output
```
(3*C*x^3*(a + b*x^2)^(5/3))/(19*b) + ((3*(19*b*B - 9*a*C)*x*(a + b*x^2)^(5
/3))/(13*b) + ((247*A*b^2 - 3*a*(19*b*B - 9*a*C))*((3*x*(a + b*x^2)^(2/3))
/7 + (6*a*Sqrt[b*x^2]*((-2*Sqrt[b*x^2])/((1 - Sqrt[3])*a^(1/3) - (a + b*x^
2)^(1/3)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3
))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sq
rt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(
1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7
 + 4*Sqrt[3]])/(Sqrt[b*x^2]*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))
/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)]) - (2*Sqrt[2 - Sqrt[3]]*(
1 + Sqrt[3])*a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)
*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^
2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/
((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(3^(1/4)*Sq
rt[b*x^2]*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^
(1/3) - (a + b*x^2)^(1/3))^2)])))/(7*b*x)))/(13*b))/(19*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 1473
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[c^p*x^(4*p - 1)*((d + e*x^2)^(q + 1)/(e*(4*p + 2*q + 1)))
, x] + Simp[1/(e*(4*p + 2*q + 1))   Int[(d + e*x^2)^q*ExpandToSum[e*(4*p +
2*q + 1)*(a + b*x^2 + c*x^4)^p - d*c^p*(4*p - 1)*x^(4*p - 2) - e*c^p*(4*p +
 2*q + 1)*x^(4*p), x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] &&  !LtQ[q, -1]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int \left(b\,x^{2}+a\right)^{\frac{2}{3}} \left(C\,x^{4}+x^{2} B+A\right)dx$$

input
```
int((b*x^2+a)^(2/3)*(C*x^4+B*x^2+A),x)
```

output
```
int((b*x^2+a)^(2/3)*(C*x^4+B*x^2+A),x)
```

Fricas [F]

$$\int \left(a+bx^2\right)^{2/3} \left(A+Bx^2+Cx^4\right)\,dx=\int \left(Cx^{4}+Bx^{2}+A\right)\left(bx^{2}+a\right)^{\frac{2}{3}}\,dx$$

input
```
integrate((b*x^2+a)^(2/3)*(C*x^4+B*x^2+A),x, algorithm="fricas")
```

output
```
integral((C*x^4 + B*x^2 + A)*(b*x^2 + a)^(2/3), x)
```

Sympy [A] (verification not implemented)

Time = 1.31 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.14

$$\int \left(a+bx^2\right)^{2/3}\left(A+Bx^2+Cx^4\right)\,dx = Aa^{\frac{2}{3}}x\,{}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right) + \frac{Ba^{\frac{2}{3}}x^3\,{}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{3} + \frac{Ca^{\frac{2}{3}}x^5\,{}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{5}{2}\\ \frac{7}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{5}$$

input
```
integrate((b*x**2+a)**(2/3)*(C*x**4+B*x**2+A),x)
```

output
```
A*a**(2/3)*x*hyper((-2/3, 1/2), (3/2,), b*x**2*exp_polar(I*pi)/a) + B*a**(
2/3)*x**3*hyper((-2/3, 3/2), (5/2,), b*x**2*exp_polar(I*pi)/a)/3 + C*a**(2
/3)*x**5*hyper((-2/3, 5/2), (7/2,), b*x**2*exp_polar(I*pi)/a)/5
```

Maxima [F]

$$\int \left(a+bx^2\right)^{2/3}\left(A+Bx^2+Cx^4\right)\,dx = \int \left(Cx^4+Bx^2+A\right)\left(bx^2+a\right)^{\frac{2}{3}}\,dx$$

input
```
integrate((b*x^2+a)^(2/3)*(C*x^4+B*x^2+A),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)*(b*x^2 + a)^(2/3), x)
```

Giac [F]

$$\int (a+bx^2)^{2/3} (A+Bx^2+Cx^4)\,dx = \int (Cx^4+Bx^2+A)(bx^2+a)^{\frac{2}{3}}\,dx$$

input
```
integrate((b*x^2+a)^(2/3)*(C*x^4+B*x^2+A),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)*(b*x^2 + a)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx^2)^{2/3} (A+Bx^2+Cx^4)\,dx = \int (b\,x^2+a)^{2/3}\,(C\,x^4+B\,x^2+A)\,dx$$

input
```
int((a + b*x^2)^(2/3)*(A + B*x^2 + C*x^4),x)
```

output
```
int((a + b*x^2)^(2/3)*(A + B*x^2 + C*x^4), x)
```

Reduce [F]

$$\int (a+bx^2)^{2/3} (A+Bx^2+Cx^4)\,dx = \frac{-108(b\,x^2+a)^{\frac{2}{3}}\,a^2cx+969(b\,x^2+a)^{\frac{2}{3}}\,a\,b^2x+84(b\,x^2+a)^{\frac{2}{3}}\,abc\,x^3+399(b\,x^2+a)^{\frac{2}{3}}\,b^3x^3+2}{1729b^2}$$

input
```
int((b*x^2+a)^(2/3)*(C*x^4+B*x^2+A),x)
```

output
```
( - 108*(a + b*x**2)**(2/3)*a**2*c*x + 969*(a + b*x**2)**(2/3)*a*b**2*x +
84*(a + b*x**2)**(2/3)*a*b*c*x**3 + 399*(a + b*x**2)**(2/3)*b**3*x**3 + 27
3*(a + b*x**2)**(2/3)*b**2*c*x**5 + 108*int((a + b*x**2)**(2/3)/(a + b*x**
2),x)*a**3*c + 760*int((a + b*x**2)**(2/3)/(a + b*x**2),x)*a**2*b**2)/(172
9*b**2)
```

3.125 $\int \frac{A+Bx^2+Cx^4}{\sqrt[3]{a+bx^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 638

$$\int \frac{A+Bx^2+Cx^4}{\sqrt[3]{a+bx^2}}\,dx$$

$$= \frac{3(13bB-9aC)x(a+bx^2)^{2/3}}{91b^2} + \frac{3Cx^3(a+bx^2)^{2/3}}{13b} - \frac{3\left(91A-\frac{3a(13bB-9aC)}{b^2}\right)x}{91\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}$$

$$+ \frac{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}(91Ab^2-39abB+27a^2C)\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,E\left(\mathrm{arc}\right.}{182b^3x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

$$- \frac{\sqrt{2}3^{3/4}\sqrt[3]{a}(91Ab^2-39abB+27a^2C)\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,\mathrm{EllipticF}\left(\mathrm{ar}\right.}{91b^3x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

output
```
3/91*(13*B*b-9*C*a)*x*(b*x^2+a)^(2/3)/b^2+3/13*C*x^3*(b*x^2+a)^(2/3)/b-3*(
91*A-3*a*(13*B*b-9*C*a)/b^2)*x/(91*(1-3^(1/2))*a^(1/3)-91*(b*x^2+a)^(1/3))
+3/182*3^(1/4)*(1/2*6^(1/2)+1/2*2^(1/2))*a^(1/3)*(91*A*b^2-39*B*a*b+27*C*a
^2)*(a^(1/3)-(b*x^2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^
(2/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)*EllipticE(((1+3^(1/2
))*a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3)),2*I-I*3^
(1/2))/b^3/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x
^2+a)^(1/3))^2)^(1/2)-1/91*2^(1/2)*3^(3/4)*a^(1/3)*(91*A*b^2-39*B*a*b+27*C
*a^2)*(a^(1/3)-(b*x^2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a
)^(2/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)*EllipticF(((1+3^(1
/2))*a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3)),2*I-I*
3^(1/2))/b^3/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b
*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.15

$$\int \frac{A+Bx^2+Cx^4}{\sqrt[3]{a+bx^2}}\,dx = \frac{x\left(-3(a+bx^2)\left(-13bB+9aC-7bCx^2\right)+\left(91Ab^2+3a(-13bB+9aC)\right)\sqrt[3]{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F}}{91b^2\sqrt[3]{a+bx^2}}$$

input
```
Integrate[(A + B*x^2 + C*x^4)/(a + b*x^2)^(1/3),x]
```

output
```
(x*(-3*(a + b*x^2)*(-13*b*B + 9*a*C - 7*b*C*x^2) + (91*A*b^2 + 3*a*(-13*b*
B + 9*a*C))*(1 + (b*x^2)/a)^(1/3)*Hypergeometric2F1[1/3, 1/2, 3/2, -((b*x^
2)/a)]))/(91*b^2*(a + b*x^2)^(1/3))
```

Rubi [A] (warning: unable to verify)

Time = 0.57 (sec) , antiderivative size = 652, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1473, 27, 299, 233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{\sqrt[3]{a+bx^2}}dx$$

$$\Big\downarrow \text{1473}$$

$$\frac{3\int \frac{(13bB-9aC)x^2+13Ab}{3\sqrt[3]{bx^2+a}}dx}{13b}+\frac{3Cx^3\left(a+bx^2\right)^{2/3}}{13b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(13bB-9aC)x^2+13Ab}{\sqrt[3]{bx^2+a}}dx}{13b}+\frac{3Cx^3\left(a+bx^2\right)^{2/3}}{13b}$$

$$\Big\downarrow \text{299}$$

$$\frac{\frac{(91Ab^2-3a(13bB-9aC))\int \frac{1}{\sqrt[3]{bx^2+a}}dx}{7b}+\frac{3x\left(a+bx^2\right)^{2/3}(13bB-9aC)}{7b}}{13b}+\frac{3Cx^3\left(a+bx^2\right)^{2/3}}{13b}$$

$$\Big\downarrow \text{233}$$

$$\frac{\frac{3\sqrt{bx^2}(91Ab^2-3a(13bB-9aC))\int \frac{\sqrt[3]{bx^2+a}}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}}{14b^2x}+\frac{3x\left(a+bx^2\right)^{2/3}(13bB-9aC)}{7b}}{13b}+\frac{3Cx^3\left(a+bx^2\right)^{2/3}}{13b}$$

$$\Big\downarrow \text{833}$$

$$\frac{\frac{3\sqrt{bx^2}(91Ab^2-3a(13bB-9aC))\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}-\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}\right)}{14b^2x}+\frac{3x\left(a+bx^2\right)^{2/3}(13bB-}{7b}}{13b}$$

$$\frac{3Cx^3\left(a+bx^2\right)^{2/3}}{13b}$$

$\Big\downarrow$ 760

$$\frac{3\sqrt{bx^2}\left(91Ab^2-3a(13bB-9aC)\right)\left(-\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a\cdots}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}\cdots\right.}}}{\sqrt[4]{3}\sqrt{bx^2}\sqrt{-\cdots}}\cdots\right.}{14b^2x}\cdots$$

$$\cdots 13b$$

$$\frac{3Cx^3\left(a+bx^2\right)^{2/3}}{13b}$$

$\Big\downarrow$ 2418

$$\frac{3\sqrt{bx^2}\left(91Ab^2-3a(13bB-9aC)\right)\left(-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\cdots}{\left(1-\sqrt{3}\right)\cdots}\right.\right.}{\sqrt[4]{3}\sqrt{bx^2}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}\cdots\right.}{\cdots}\cdots$$

$$\frac{3Cx^3\left(a+bx^2\right)^{2/3}}{13b}$$

input
```
Int[(A + B*x^2 + C*x^4)/(a + b*x^2)^(1/3),x]
```

output
```
(3*C*x^3*(a + b*x^2)^(2/3))/(13*b) + ((3*(13*b*B - 9*a*C)*x*(a + b*x^2)^(2
/3))/(7*b) + (3*(91*A*b^2 - 3*a*(13*b*B - 9*a*C))*Sqrt[b*x^2]*((-2*Sqrt[b*
x^2])/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3)) + (3^(1/4)*Sqrt[2 + Sqrt
[3]]*a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*
x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3)
)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sq
rt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(Sqrt[b*x^2]*Sqrt[-
((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x
^2)^(1/3))^2)]) - (2*Sqrt[2 - Sqrt[3]]*(1 + Sqrt[3])*a^(1/3)*(a^(1/3) - (a
 + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(
2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1
+ Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^
2)^(1/3))], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[b*x^2]*Sqrt[-((a^(1/3)*(a^(1/3)
 - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)])))/
(14*b^2*x))/(13*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 1473
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[c^p*x^(4*p - 1)*((d + e*x^2)^(q + 1)/(e*(4*p + 2*q + 1)))
, x] + Simp[1/(e*(4*p + 2*q + 1))   Int[(d + e*x^2)^q*ExpandToSum[e*(4*p +
2*q + 1)*(a + b*x^2 + c*x^4)^p - d*c^p*(4*p - 1)*x^(4*p - 2) - e*c^p*(4*p +
 2*q + 1)*x^(4*p), x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] &&  !LtQ[q, -1]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int \frac{C\,x^{4}+x^{2}B+A}{\left(b\,x^{2}+a\right)^{\frac{1}{3}}}dx$$

input
```
int((C*x^4+B*x^2+A)/(b*x^2+a)^(1/3),x)
```

output
```
int((C*x^4+B*x^2+A)/(b*x^2+a)^(1/3),x)
```

Fricas [F]

$$\int \frac{A+Bx^2+Cx^4}{\sqrt[3]{a+bx^2}}\,dx=\int \frac{Cx^4+Bx^2+A}{(bx^2+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/(b*x^2+a)^(1/3),x, algorithm="fricas")
```

output
```
integral((C*x^4 + B*x^2 + A)/(b*x^2 + a)^(1/3), x)
```

Sympy [A] (verification not implemented)

Time = 1.15 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.14

$$\int \frac{A+Bx^2+Cx^4}{\sqrt[3]{a+bx^2}}\,dx=\frac{Ax\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{\sqrt[3]{a}}+\frac{Bx^3\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{3\sqrt[3]{a}}+\frac{Cx^5\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{5}{2}\\ \frac{7}{2}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{5\sqrt[3]{a}}$$

input
```
integrate((C*x**4+B*x**2+A)/(b*x**2+a)**(1/3),x)
```

output
```
A*x*hyper((1/3, 1/2), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(1/3) + B*x**3*
hyper((1/3, 3/2), (5/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(1/3)) + C*x**5*
hyper((1/3, 5/2), (7/2,), b*x**2*exp_polar(I*pi)/a)/(5*a**(1/3))
```

Maxima [F]

$$\int \frac{A+Bx^2+Cx^4}{\sqrt[3]{a+bx^2}}\,dx=\int \frac{Cx^4+Bx^2+A}{(bx^2+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/(b*x^2+a)^(1/3),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)/(b*x^2 + a)^(1/3), x)
```

Giac [F]

$$\int \frac{A+Bx^2+Cx^4}{\sqrt[3]{a+bx^2}}\,dx=\int \frac{Cx^4+Bx^2+A}{(bx^2+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/(b*x^2+a)^(1/3),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)/(b*x^2 + a)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{\sqrt[3]{a+bx^2}}\,dx=\int \frac{C\,x^4+B\,x^2+A}{{\left(b\,x^2+a\right)}^{1/3}}\,dx$$

input
```
int((A + B*x^2 + C*x^4)/(a + b*x^2)^(1/3),x)
```

output
```
int((A + B*x^2 + C*x^4)/(a + b*x^2)^(1/3), x)
```

Reduce [F]

$$\int \frac{A + Bx^2 + Cx^4}{\sqrt[3]{a + bx^2}} \, dx = \left(\int \frac{x^4}{(b\,x^2 + a)^{\frac{1}{3}}} dx\right) c + \left(\int \frac{x^2}{(b\,x^2 + a)^{\frac{1}{3}}} dx\right) b + \left(\int \frac{1}{(b\,x^2 + a)^{\frac{1}{3}}} dx\right) a$$

input

```
int((C*x^4+B*x^2+A)/(b*x^2+a)^(1/3),x)
```

output

```
int(x**4/(a + b*x**2)**(1/3),x)*c + int(x**2/(a + b*x**2)**(1/3),x)*b + in
t(1/(a + b*x**2)**(1/3),x)*a
```

3.126 $\int (a+bx^2)^p (A+Bx^2+Cx^4)\, dx$

Optimal result

Integrand size = 22, antiderivative size = 165

$$\int (a+bx^2)^p (A+Bx^2+Cx^4)\, dx$$
$$= -\frac{(3aC-bB(5+2p))x(a+bx^2)^{1+p}}{b^2(3+2p)(5+2p)} + \frac{Cx^3(a+bx^2)^{1+p}}{b(5+2p)}$$
$$+ \frac{(Ab^2(15+16p+4p^2)+a(3aC-bB(5+2p)))\,x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2}\ldots\right.}{b^2(3+2p)(5+2p)}$$

output

```
-(3*C*a-b*B*(5+2*p))*x*(b*x^2+a)^(p+1)/b^2/(3+2*p)/(5+2*p)+C*x^3*(b*x^2+a)
^(p+1)/b/(5+2*p)+(A*b^2*(4*p^2+16*p+15)+a*(3*C*a-b*B*(5+2*p)))*x*(b*x^2+a)
^p*hypergeom([1/2, -p],[3/2],-b*x^2/a)/b^2/(3+2*p)/(5+2*p)/((1+b*x^2/a)^p)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.61

$$\int (a+bx^2)^p (A+Bx^2+Cx^4)\,dx = \frac{1}{15}x(a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p}\left(15A\,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right)+5Bx^2\,\text{Hypergeometric2F1}\left(\frac{3}{2},-p,\frac{5}{2},-\frac{bx^2}{a}\right)+3Cx^4\,\text{Hypergeometric2F1}\left(\frac{5}{2},-p,\frac{7}{2},-\frac{bx^2}{a}\right)\right)$$

input
```
Integrate[(a + b*x^2)^p*(A + B*x^2 + C*x^4),x]
```

output
```
(x*(a + b*x^2)^p*(15*A*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)] + 5*B
*x^2*Hypergeometric2F1[3/2, -p, 5/2, -((b*x^2)/a)] + 3*C*x^4*Hypergeometri
c2F1[5/2, -p, 7/2, -((b*x^2)/a)]))/(15*(1 + (b*x^2)/a)^p)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1473, 299, 238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^2)^p (A+Bx^2+Cx^4)\,dx$$

$$\Big\downarrow \text{1473}$$

$$\frac{\int (bx^2+a)^p (Ab(2p+5)-(3aC-bB(2p+5))x^2)\,dx}{b(2p+5)}+\frac{Cx^3(a+bx^2)^{p+1}}{b(2p+5)}$$

$$\Big\downarrow \text{299}$$

$$\frac{\left(\frac{a(3aC-bB(2p+5))}{b(2p+3)}+Ab(2p+5)\right)\int \left(bx^2+a\right)^p dx-\frac{x\left(a+bx^2\right)^{p+1}(3aC-bB(2p+5))}{b(2p+3)}}{b(2p+5)}+\frac{Cx^3\left(a+bx^2\right)^{p+1}}{b(2p+5)}$$

$\Big\downarrow$ 238

$$\frac{\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(\frac{a(3aC-bB(2p+5))}{b(2p+3)}+Ab(2p+5)\right)\int \left(\frac{bx^2}{a}+1\right)^p dx-\frac{x\left(a+bx^2\right)^{p+1}(3aC-bB(2p+5))}{b(2p+3)}}{b(2p+5)}+\frac{Cx^3\left(a+bx^2\right)^{p+1}}{b(2p+5)}$$

$\Big\downarrow$ 237

$$\frac{x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right)\left(\frac{a(3aC-bB(2p+5))}{b(2p+3)}+Ab(2p+5)\right)-\frac{x\left(a+bx^2\right)^{p+1}(3a}{b(2p+}}{b(2p+5)}$$
$$\frac{Cx^3\left(a+bx^2\right)^{p+1}}{b(2p+5)}$$

input
```
Int[(a + b*x^2)^p*(A + B*x^2 + C*x^4),x]
```

output
```
(C*x^3*(a + b*x^2)^(1 + p))/(b*(5 + 2*p)) + (-(((3*a*C - b*B*(5 + 2*p))*x*
(a + b*x^2)^(1 + p))/(b*(3 + 2*p))) + ((A*b*(5 + 2*p) + (a*(3*a*C - b*B*(5
 + 2*p)))/(b*(3 + 2*p)))*x*(a + b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -
((b*x^2)/a)])/(1 + (b*x^2)/a)^p)/(b*(5 + 2*p))
```

Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1473
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[c^p*x^(4*p - 1)*((d + e*x^2)^(q + 1)/(e*(4*p + 2*q + 1)))
, x] + Simp[1/(e*(4*p + 2*q + 1))   Int[(d + e*x^2)^q*ExpandToSum[e*(4*p +
2*q + 1)*(a + b*x^2 + c*x^4)^p - d*c^p*(4*p - 1)*x^(4*p - 2) - e*c^p*(4*p +
 2*q + 1)*x^(4*p), x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] &&  !LtQ[q, -1]
```

Maple [F]

$$\int \left(b\,x^{2}+a\right)^{p} \left(C\,x^{4}+x^{2} B+A\right)dx$$

input
```
int((b*x^2+a)^p*(C*x^4+B*x^2+A),x)
```

output
```
int((b*x^2+a)^p*(C*x^4+B*x^2+A),x)
```

Fricas [F]

$$\int \left(a+bx^2\right)^p \left(A+Bx^2+Cx^4\right)\,dx=\int \left(Cx^4+Bx^2+A\right)\left(bx^2+a\right)^p\,dx$$

input
```
integrate((b*x^2+a)^p*(C*x^4+B*x^2+A),x, algorithm="fricas")
```

output
```
integral((C*x^4 + B*x^2 + A)*(b*x^2 + a)^p, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 9.11 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.50

$$\int (a+bx^2)^p (A+Bx^2+Cx^4)\ dx = Aa^p x {}_2F_1\left(\begin{matrix} \frac{1}{2}, -p \\ \frac{3}{2} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right) + \frac{Ba^p x^3 {}_2F_1\left(\begin{matrix} \frac{3}{2}, -p \\ \frac{5}{2} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)}{3} + \frac{Ca^p x^5 {}_2F_1\left(\begin{matrix} \frac{5}{2}, -p \\ \frac{7}{2} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)}{5}$$

input
```
integrate((b*x**2+a)**p*(C*x**4+B*x**2+A),x)
```

output
```
A*a**p*x*hyper((1/2, -p), (3/2,), b*x**2*exp_polar(I*pi)/a) + B*a**p*x**3*
hyper((3/2, -p), (5/2,), b*x**2*exp_polar(I*pi)/a)/3 + C*a**p*x**5*hyper((
5/2, -p), (7/2,), b*x**2*exp_polar(I*pi)/a)/5
```

Maxima [F]

$$\int (a+bx^2)^p (A+Bx^2+Cx^4)\ dx = \int (Cx^4+Bx^2+A)(bx^2+a)^p\,dx$$

input
```
integrate((b*x^2+a)^p*(C*x^4+B*x^2+A),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)*(b*x^2 + a)^p, x)
```

Giac [F]

$$\int (a+bx^2)^p (A+Bx^2+Cx^4) \, dx = \int (Cx^4+Bx^2+A)(bx^2+a)^p \, dx$$

input
```
integrate((b*x^2+a)^p*(C*x^4+B*x^2+A),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)*(b*x^2 + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx^2)^p (A+Bx^2+Cx^4) \, dx = \int (b\,x^2+a)^p \left(C\,x^4+B\,x^2+A\right) \, dx$$

input
```
int((a + b*x^2)^p*(A + B*x^2 + C*x^4),x)
```

output
```
int((a + b*x^2)^p*(A + B*x^2 + C*x^4), x)
```

Reduce [F]

$$\int (a+bx^2)^p (A+Bx^2+Cx^4) \, dx = \text{Too large to display}$$

input
```
int((b*x^2+a)^p*(C*x^4+B*x^2+A),x)
```

output
```
( - 6*(a + b*x**2)**p*a**2*c*p*x + 8*(a + b*x**2)**p*a*b**2*p**2*x + 26*(a
 + b*x**2)**p*a*b**2*p*x + 15*(a + b*x**2)**p*a*b**2*x + 4*(a + b*x**2)**p
*a*b*c*p**2*x**3 + 2*(a + b*x**2)**p*a*b*c*p*x**3 + 4*(a + b*x**2)**p*b**3
*p**2*x**3 + 12*(a + b*x**2)**p*b**3*p*x**3 + 5*(a + b*x**2)**p*b**3*x**3
+ 4*(a + b*x**2)**p*b**2*c*p**2*x**5 + 8*(a + b*x**2)**p*b**2*c*p*x**5 + 3
*(a + b*x**2)**p*b**2*c*x**5 + 48*int((a + b*x**2)**p/(8*a*p**3 + 36*a*p**
2 + 46*a*p + 15*a + 8*b*p**3*x**2 + 36*b*p**2*x**2 + 46*b*p*x**2 + 15*b*x*
*2),x)*a**3*c*p**4 + 216*int((a + b*x**2)**p/(8*a*p**3 + 36*a*p**2 + 46*a*
p + 15*a + 8*b*p**3*x**2 + 36*b*p**2*x**2 + 46*b*p*x**2 + 15*b*x**2),x)*a*
*3*c*p**3 + 276*int((a + b*x**2)**p/(8*a*p**3 + 36*a*p**2 + 46*a*p + 15*a
+ 8*b*p**3*x**2 + 36*b*p**2*x**2 + 46*b*p*x**2 + 15*b*x**2),x)*a**3*c*p**2
 + 90*int((a + b*x**2)**p/(8*a*p**3 + 36*a*p**2 + 46*a*p + 15*a + 8*b*p**3
*x**2 + 36*b*p**2*x**2 + 46*b*p*x**2 + 15*b*x**2),x)*a**3*c*p + 64*int((a
+ b*x**2)**p/(8*a*p**3 + 36*a*p**2 + 46*a*p + 15*a + 8*b*p**3*x**2 + 36*b*
p**2*x**2 + 46*b*p*x**2 + 15*b*x**2),x)*a**2*b**2*p**6 + 512*int((a + b*x*
*2)**p/(8*a*p**3 + 36*a*p**2 + 46*a*p + 15*a + 8*b*p**3*x**2 + 36*b*p**2*x
**2 + 46*b*p*x**2 + 15*b*x**2),x)*a**2*b**2*p**5 + 1536*int((a + b*x**2)**
p/(8*a*p**3 + 36*a*p**2 + 46*a*p + 15*a + 8*b*p**3*x**2 + 36*b*p**2*x**2 +
 46*b*p*x**2 + 15*b*x**2),x)*a**2*b**2*p**4 + 2128*int((a + b*x**2)**p/(8*
a*p**3 + 36*a*p**2 + 46*a*p + 15*a + 8*b*p**3*x**2 + 36*b*p**2*x**2 + 4...
```

3.127 $\int \left(a+bx^2\right)^3 \left(A+Bx^2+Cx^4+Dx^6\right) \, dx$

Optimal result

Integrand size = 27, antiderivative size = 136

$$\begin{aligned}\int \left(a+bx^2\right)^3 \left(A+Bx^2+Cx^4+Dx^6\right) \, dx = {} & a^3Ax+\frac{1}{3}a^2(3Ab+aB)x^3 \\ & +\frac{1}{5}a\left(3Ab^2+a(3bB+aC)\right)x^5 \\ & +\frac{1}{7}\left(Ab^3+a\left(3b^2B+3abC+a^2D\right)\right)x^7 \\ & +\frac{1}{9}b\left(b^2B+3abC+3a^2D\right)x^9 \\ & +\frac{1}{11}b^2(bC+3aD)x^{11}+\frac{1}{13}b^3Dx^{13}\end{aligned}$$

output
```
a^3*A*x+1/3*a^2*(3*A*b+B*a)*x^3+1/5*a*(3*A*b^2+a*(3*B*b+C*a))*x^5+1/7*(A*b
^3+a*(3*B*b^2+3*C*a*b+D*a^2))*x^7+1/9*b*(B*b^2+3*C*a*b+3*D*a^2)*x^9+1/11*b
^2*(C*b+3*D*a)*x^11+1/13*b^3*D*x^13
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00

$$\int (a+bx^2)^3 (A+Bx^2+Cx^4+Dx^6)\,dx = a^3Ax+\frac{1}{3}a^2(3Ab+aB)x^3 + \frac{1}{5}a(3Ab^2+3abB+a^2C)x^5 + \frac{1}{7}(Ab^3+3ab^2B+3a^2bC+a^3D)x^7 + \frac{1}{9}b(b^2B+3abC+3a^2D)x^9 + \frac{1}{11}b^2(bC+3aD)x^{11}+\frac{1}{13}b^3Dx^{13}$$

input
```
Integrate[(a + b*x^2)^3*(A + B*x^2 + C*x^4 + D*x^6),x]
```

output
```
a^3*A*x + (a^2*(3*A*b + a*B)*x^3)/3 + (a*(3*A*b^2 + 3*a*b*B + a^2*C)*x^5)/
5 + ((A*b^3 + 3*a*b^2*B + 3*a^2*b*C + a^3*D)*x^7)/7 + (b*(b^2*B + 3*a*b*C
+ 3*a^2*D)*x^9)/9 + (b^2*(b*C + 3*a*D)*x^11)/11 + (b^3*D*x^13)/13
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2341, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^2)^3 (A+Bx^2+Cx^4+Dx^6)\,dx$$

$$\Big\downarrow \text{2341}$$

$$\int (a^3A+x^6(a(a^2D+3abC+3b^2B)+Ab^3)+a^2x^2(aB+3Ab)+bx^8(3a^2D+3abC+b^2B)+ax^4(a(aC+3b$$

$$\Big\downarrow \text{2009}$$

$$a^3Ax+\frac{1}{7}x^7\left(a\left(a^2D+3abC+3b^2B\right)+Ab^3\right)+\frac{1}{3}a^2x^3(aB+3Ab)+\frac{1}{9}bx^9\left(3a^2D+3abC+b^2B\right)+\frac{1}{5}ax^5\left(a(aC+3bB)+3Ab^2\right)+\frac{1}{11}b^2x^{11}(3aD+bC)+\frac{1}{13}b^3Dx^{13}$$

input
```
Int[(a + b*x^2)^3*(A + B*x^2 + C*x^4 + D*x^6),x]
```

output
```
a^3*A*x + (a^2*(3*A*b + a*B)*x^3)/3 + (a*(3*A*b^2 + a*(3*b*B + a*C))*x^5)/
5 + ((A*b^3 + a*(3*b^2*B + 3*a*b*C + a^2*D))*x^7)/7 + (b*(b^2*B + 3*a*b*C
+ 3*a^2*D)*x^9)/9 + (b^2*(b*C + 3*a*D)*x^11)/11 + (b^3*D*x^13)/13
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2341
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.98

method	result
default	$\frac{b^3Dx^{13}}{13}+\frac{(b^3C+3a\,b^2D)x^{11}}{11}+\frac{(B\,b^3+3aC\,b^2+3a^2bD)x^9}{9}+\frac{(b^3A+3a\,b^2B+3a^2bC+a^3D)x^7}{7}+\frac{(3a\,b^2A+3a^2bB+C\,a}{5}$
norman	$\frac{b^3Dx^{13}}{13}+\left(\frac{1}{11}b^3C+\frac{3}{11}a\,b^2D\right)x^{11}+\left(\frac{1}{9}B\,b^3+\frac{1}{3}aC\,b^2+\frac{1}{3}a^2bD\right)x^9+\left(\frac{1}{7}b^3A+\frac{3}{7}a\,b^2B+\frac{3}{7}a^2b\right.$
gosper	$\frac{1}{13}b^3Dx^{13}+\frac{1}{11}b^3C\,x^{11}+\frac{3}{11}x^{11}a\,b^2D+\frac{1}{9}b^3B\,x^9+\frac{1}{3}x^9aC\,b^2+\frac{1}{3}x^9a^2bD+\frac{1}{7}A\,b^3x^7+\frac{3}{7}x^7a\,b$
parallelrisch	$\frac{1}{13}b^3Dx^{13}+\frac{1}{11}b^3C\,x^{11}+\frac{3}{11}x^{11}a\,b^2D+\frac{1}{9}b^3B\,x^9+\frac{1}{3}x^9aC\,b^2+\frac{1}{3}x^9a^2bD+\frac{1}{7}A\,b^3x^7+\frac{3}{7}x^7a\,b$
orering	$\frac{x\left(3465b^3Dx^{12}+4095b^3C\,x^{10}+12285Da\,b^2x^{10}+5005b^3B\,x^8+15015Ca\,b^2x^8+15015Da^2b\,x^8+6435A\,b^3x^6+19305Ba\,b^2x^6+1\right.}{45045}$

input
```
int((b*x^2+a)^3*(D*x^6+C*x^4+B*x^2+A),x,method=_RETURNVERBOSE)
```

output
```
1/13*b^3*D*x^13+1/11*(C*b^3+3*D*a*b^2)*x^11+1/9*(B*b^3+3*C*a*b^2+3*D*a^2*b
)*x^9+1/7*(A*b^3+3*B*a*b^2+3*C*a^2*b+D*a^3)*x^7+1/5*(3*A*a*b^2+3*B*a^2*b+C
*a^3)*x^5+1/3*(3*A*a^2*b+B*a^3)*x^3+a^3*A*x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.97

$$\int \left(a+bx^2\right)^3 \left(A+Bx^2+Cx^4+Dx^6\right) \, dx = \frac{1}{13}\, Db^3x^{13} + \frac{1}{11}\left(3\, Dab^2 + Cb^3\right)x^{11} + \frac{1}{9}\left(3\, Da^2b + 3\, Cab^2 + Bb^3\right)x^9 + \frac{1}{7}\left(Da^3 + 3\, Ca^2b + 3\, Bab^2 + Ab^3\right)x^7 + \frac{1}{5}\left(Ca^3 + 3\, Ba^2b + 3\, Aab^2\right)x^5 + Aa^3x + \frac{1}{3}\left(Ba^3 + 3\, Aa^2b\right)x^3$$

input
```
integrate((b*x^2+a)^3*(D*x^6+C*x^4+B*x^2+A),x, algorithm="fricas")
```

output
```
1/13*D*b^3*x^13 + 1/11*(3*D*a*b^2 + C*b^3)*x^11 + 1/9*(3*D*a^2*b + 3*C*a*b
^2 + B*b^3)*x^9 + 1/7*(D*a^3 + 3*C*a^2*b + 3*B*a*b^2 + A*b^3)*x^7 + 1/5*(C
*a^3 + 3*B*a^2*b + 3*A*a*b^2)*x^5 + A*a^3*x + 1/3*(B*a^3 + 3*A*a^2*b)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.09

$$\int \left(a+bx^2\right)^3 \left(A+Bx^2+Cx^4+Dx^6\right) \, dx = Aa^3x + \frac{Db^3x^{13}}{13} + x^{11}\left(\frac{Cb^3}{11} + \frac{3Dab^2}{11}\right) + x^9\left(\frac{Bb^3}{9} + \frac{Cab^2}{3} + \frac{Da^2b}{3}\right) + x^7\left(\frac{Ab^3}{7} + \frac{3Bab^2}{7} + \frac{3Ca^2b}{7} + \frac{Da^3}{7}\right) + x^5 \cdot \left(\frac{3Aab^2}{5} + \frac{3Ba^2b}{5} + \frac{Ca^3}{5}\right) + x^3\left(Aa^2b + \frac{Ba^3}{3}\right)$$

input
```
integrate((b*x**2+a)**3*(D*x**6+C*x**4+B*x**2+A),x)
```

output
```
A*a**3*x + D*b**3*x**13/13 + x**11*(C*b**3/11 + 3*D*a*b**2/11) + x**9*(B*b
**3/9 + C*a*b**2/3 + D*a**2*b/3) + x**7*(A*b**3/7 + 3*B*a*b**2/7 + 3*C*a**
2*b/7 + D*a**3/7) + x**5*(3*A*a*b**2/5 + 3*B*a**2*b/5 + C*a**3/5) + x**3*(
A*a**2*b + B*a**3/3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.97

$$\int \left(a+bx^2\right)^3 \left(A+Bx^2+Cx^4+Dx^6\right)\,dx = \frac{1}{13}\,Db^3x^{13} + \frac{1}{11}\left(3\,Dab^2+Cb^3\right)x^{11} + \frac{1}{9}\left(3\,Da^2b+3\,Cab^2+Bb^3\right)x^9 + \frac{1}{7}\left(Da^3+3\,Ca^2b+3\,Bab^2+Ab^3\right)x^7 + \frac{1}{5}\left(Ca^3+3\,Ba^2b+3\,Aab^2\right)x^5 + Aa^3x + \frac{1}{3}\left(Ba^3+3\,Aa^2b\right)x^3$$

input
```
integrate((b*x^2+a)^3*(D*x^6+C*x^4+B*x^2+A),x, algorithm="maxima")
```

output
```
1/13*D*b^3*x^13 + 1/11*(3*D*a*b^2 + C*b^3)*x^11 + 1/9*(3*D*a^2*b + 3*C*a*b
^2 + B*b^3)*x^9 + 1/7*(D*a^3 + 3*C*a^2*b + 3*B*a*b^2 + A*b^3)*x^7 + 1/5*(C
*a^3 + 3*B*a^2*b + 3*A*a*b^2)*x^5 + A*a^3*x + 1/3*(B*a^3 + 3*A*a^2*b)*x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.10

$$\int (a+bx^2)^3 (A+Bx^2+Cx^4+Dx^6)\, dx = \frac{1}{13} Db^3x^{13} + \frac{3}{11} Dab^2x^{11} + \frac{1}{11} Cb^3x^{11} + \frac{1}{3} Da^2bx^9 + \frac{1}{3} Cab^2x^9 + \frac{1}{9} Bb^3x^9 + \frac{1}{7} Da^3x^7 + \frac{3}{7} Ca^2bx^7 + \frac{3}{7} Bab^2x^7 + \frac{1}{7} Ab^3x^7 + \frac{1}{5} Ca^3x^5 + \frac{3}{5} Ba^2bx^5 + \frac{3}{5} Aab^2x^5 + \frac{1}{3} Ba^3x^3 + Aa^2bx^3 + Aa^3x$$

input
```
integrate((b*x^2+a)^3*(D*x^6+C*x^4+B*x^2+A),x, algorithm="giac")
```

output
```
1/13*D*b^3*x^13 + 3/11*D*a*b^2*x^11 + 1/11*C*b^3*x^11 + 1/3*D*a^2*b*x^9 +
1/3*C*a*b^2*x^9 + 1/9*B*b^3*x^9 + 1/7*D*a^3*x^7 + 3/7*C*a^2*b*x^7 + 3/7*B*
a*b^2*x^7 + 1/7*A*b^3*x^7 + 1/5*C*a^3*x^5 + 3/5*B*a^2*b*x^5 + 3/5*A*a*b^2*
x^5 + 1/3*B*a^3*x^3 + A*a^2*b*x^3 + A*a^3*x
```

Mupad [B] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.10

$$\int (a+bx^2)^3 (A+Bx^2+Cx^4+Dx^6)\, dx = \frac{B\,a^3\,x^3}{3} + \frac{A\,b^3\,x^7}{7} + \frac{C\,a^3\,x^5}{5} + \frac{B\,b^3\,x^9}{9} + \frac{C\,b^3\,x^{11}}{11} + \frac{a^3\,x^7\,D}{7} + \frac{b^3\,x^{13}\,D}{13} + A\,a^3\,x + \frac{a^2\,b\,x^9\,D}{3} + \frac{3\,a\,b^2\,x^{11}\,D}{11} + A\,a^2\,b\,x^3 + \frac{3\,A\,a\,b^2\,x^5}{5} + \frac{3\,B\,a^2\,b\,x^5}{5} + \frac{3\,B\,a\,b^2\,x^7}{7} + \frac{3\,C\,a^2\,b\,x^7}{7} + \frac{C\,a\,b^2\,x^9}{3}$$

input
```
int((a + b*x^2)^3*(A + B*x^2 + C*x^4 + x^6*D),x)
```

output
```
(B*a^3*x^3)/3 + (A*b^3*x^7)/7 + (C*a^3*x^5)/5 + (B*b^3*x^9)/9 + (C*b^3*x^1
1)/11 + (a^3*x^7*D)/7 + (b^3*x^13*D)/13 + A*a^3*x + (a^2*b*x^9*D)/3 + (3*a
*b^2*x^11*D)/11 + A*a^2*b*x^3 + (3*A*a*b^2*x^5)/5 + (3*B*a^2*b*x^5)/5 + (3
*B*a*b^2*x^7)/7 + (3*C*a^2*b*x^7)/7 + (C*a*b^2*x^9)/3
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.90

$$\int \left(a+bx^2\right)^3 \left(A+Bx^2+Cx^4+Dx^6\right)\,dx$$
$$= \frac{x(3465b^3d\,x^{12}+12285a\,b^2d\,x^{10}+4095b^3c\,x^{10}+15015a^2bd\,x^8+15015a\,b^2c\,x^8+5005b^4x^8+6435a^3d\,x^6+}{45045}$$

input
```
int((b*x^2+a)^3*(D*x^6+C*x^4+B*x^2+A),x)
```

output
```
(x*(45045*a**4 + 60060*a**3*b*x**2 + 9009*a**3*c*x**4 + 6435*a**3*d*x**6 +
 54054*a**2*b**2*x**4 + 19305*a**2*b*c*x**6 + 15015*a**2*b*d*x**8 + 25740*
a*b**3*x**6 + 15015*a*b**2*c*x**8 + 12285*a*b**2*d*x**10 + 5005*b**4*x**8
+ 4095*b**3*c*x**10 + 3465*b**3*d*x**12))/45045
```

3.128 $\int (a+bx^2)^2 (A+Bx^2+Cx^4+Dx^6)\,dx$

Optimal result

Integrand size = 27, antiderivative size = 96

$$\begin{aligned}\int (a+bx^2)^2 (A+Bx^2+Cx^4+Dx^6)\,dx = {} & a^2Ax+\frac{1}{3}a(2Ab+aB)x^3 \\ & +\frac{1}{5}\left(Ab^2+a(2bB+aC)\right)x^5 \\ & +\frac{1}{7}\left(b^2B+2abC+a^2D\right)x^7 \\ & +\frac{1}{9}b(bC+2aD)x^9+\frac{1}{11}b^2Dx^{11}\end{aligned}$$

output
```
a^2*A*x+1/3*a*(2*A*b+B*a)*x^3+1/5*(A*b^2+a*(2*B*b+C*a))*x^5+1/7*(B*b^2+2*C
*a*b+D*a^2)*x^7+1/9*b*(C*b+2*D*a)*x^9+1/11*b^2*D*x^11
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00

$$\int (a+bx^2)^2 (A+Bx^2+Cx^4+Dx^6)\ dx = a^2Ax+\frac{1}{3}a(2Ab+aB)x^3 + \frac{1}{5}(Ab^2+2abB+a^2C)x^5 + \frac{1}{7}(b^2B+2abC+a^2D)x^7 + \frac{1}{9}b(bC+2aD)x^9+\frac{1}{11}b^2Dx^{11}$$

input
```
Integrate[(a + b*x^2)^2*(A + B*x^2 + C*x^4 + D*x^6),x]
```

output
```
a^2*A*x + (a*(2*A*b + a*B)*x^3)/3 + ((A*b^2 + 2*a*b*B + a^2*C)*x^5)/5 + ((
b^2*B + 2*a*b*C + a^2*D)*x^7)/7 + (b*(b*C + 2*a*D)*x^9)/9 + (b^2*D*x^11)/1
1
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2341, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^2)^2 (A+Bx^2+Cx^4+Dx^6)\ dx$$

$$\Big\downarrow \text{2341}$$

$$\int (a^2A+x^6(a^2D+2abC+b^2B)+x^4(a(aC+2bB)+Ab^2)+ax^2(aB+2Ab)+bx^8(2aD+bC)+b^2Dx^{10})\ dx$$

$$\Big\downarrow \text{2009}$$

$$a^2Ax + \frac{1}{7}x^7\left(a^2D + 2abC + b^2B\right) + \frac{1}{5}x^5\left(a(aC + 2bB) + Ab^2\right) + \frac{1}{3}ax^3(aB + 2Ab) + \frac{1}{9}bx^9(2aD + bC) + \frac{1}{11}b^2Dx^{11}$$

input
```
Int[(a + b*x^2)^2*(A + B*x^2 + C*x^4 + D*x^6),x]
```

output
```
a^2*A*x + (a*(2*A*b + a*B)*x^3)/3 + ((A*b^2 + a*(2*b*B + a*C))*x^5)/5 + ((
b^2*B + 2*a*b*C + a^2*D)*x^7)/7 + (b*(b*C + 2*a*D)*x^9)/9 + (b^2*D*x^11)/1
1
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2341
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.95

method	result
default	$\frac{b^2Dx^{11}}{11} + \frac{(b^2C+2abD)x^9}{9} + \frac{(Bb^2+2Cab+Da^2)x^7}{7} + \frac{(b^2A+2abB+a^2C)x^5}{5} + \frac{(2abA+a^2B)x^3}{3} + a^2Ax$
norman	$\frac{b^2Dx^{11}}{11} + \left(\frac{1}{9}b^2C + \frac{2}{9}abD\right)x^9 + \left(\frac{1}{7}Bb^2 + \frac{2}{7}Cab + \frac{1}{7}Da^2\right)x^7 + \left(\frac{1}{5}b^2A + \frac{2}{5}abB + \frac{1}{5}a^2C\right)x^5 +$
gosper	$\frac{1}{11}b^2Dx^{11} + \frac{1}{9}b^2Cx^9 + \frac{2}{9}x^9abD + \frac{1}{7}b^2Bx^7 + \frac{2}{7}x^7Cab + \frac{1}{7}x^7Da^2 + \frac{1}{5}Ab^2x^5 + \frac{2}{5}x^5abB + \frac{1}{5}x$
parallelrisch	$\frac{1}{11}b^2Dx^{11} + \frac{1}{9}b^2Cx^9 + \frac{2}{9}x^9abD + \frac{1}{7}b^2Bx^7 + \frac{2}{7}x^7Cab + \frac{1}{7}x^7Da^2 + \frac{1}{5}Ab^2x^5 + \frac{2}{5}x^5abB + \frac{1}{5}x$
orering	$\frac{x(315b^2Dx^{10}+385Cb^2x^8+770Dabx^8+495b^2Bx^6+990Cabx^6+495Da^2x^6+693Ab^2x^4+1386Babx^4+693Ca^2x^4+2310aAb}{3465}$

input
```
int((b*x^2+a)^2*(D*x^6+C*x^4+B*x^2+A),x,method=_RETURNVERBOSE)
```

output
```
1/11*b^2*D*x^11+1/9*(C*b^2+2*D*a*b)*x^9+1/7*(B*b^2+2*C*a*b+D*a^2)*x^7+1/5*
(A*b^2+2*B*a*b+C*a^2)*x^5+1/3*(2*A*a*b+B*a^2)*x^3+a^2*A*x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.94

$$\int \left(a+bx^2\right)^2 \left(A+Bx^2+Cx^4+Dx^6\right)\,dx = \frac{1}{11}\,Db^2x^{11}+\frac{1}{9}\,\left(2\,Dab+Cb^2\right)x^9 + \frac{1}{7}\,\left(Da^2+2\,Cab+Bb^2\right)x^7 + \frac{1}{5}\,\left(Ca^2+2\,Bab+Ab^2\right)x^5 + Aa^2x+\frac{1}{3}\,\left(Ba^2+2\,Aab\right)x^3$$

input
```
integrate((b*x^2+a)^2*(D*x^6+C*x^4+B*x^2+A),x, algorithm="fricas")
```

output
```
1/11*D*b^2*x^11 + 1/9*(2*D*a*b + C*b^2)*x^9 + 1/7*(D*a^2 + 2*C*a*b + B*b^2
)*x^7 + 1/5*(C*a^2 + 2*B*a*b + A*b^2)*x^5 + A*a^2*x + 1/3*(B*a^2 + 2*A*a*b
)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.08

$$\int \left(a+bx^2\right)^2 \left(A+Bx^2+Cx^4+Dx^6\right)\,dx = Aa^2x+\frac{Db^2x^{11}}{11}+x^9\left(\frac{Cb^2}{9}+\frac{2Dab}{9}\right) + x^7\left(\frac{Bb^2}{7}+\frac{2Cab}{7}+\frac{Da^2}{7}\right) + x^5\left(\frac{Ab^2}{5}+\frac{2Bab}{5}+\frac{Ca^2}{5}\right) + x^3\cdot\left(\frac{2Aab}{3}+\frac{Ba^2}{3}\right)$$

input
```
integrate((b*x**2+a)**2*(D*x**6+C*x**4+B*x**2+A),x)
```

output
```
A*a**2*x + D*b**2*x**11/11 + x**9*(C*b**2/9 + 2*D*a*b/9) + x**7*(B*b**2/7
+ 2*C*a*b/7 + D*a**2/7) + x**5*(A*b**2/5 + 2*B*a*b/5 + C*a**2/5) + x**3*(2
*A*a*b/3 + B*a**2/3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.94

$$\int (a+bx^2)^2 (A+Bx^2+Cx^4+Dx^6)\,dx = \frac{1}{11}Db^2x^{11} + \frac{1}{9}(2Dab+Cb^2)x^9 + \frac{1}{7}(Da^2+2Cab+Bb^2)x^7 + \frac{1}{5}(Ca^2+2Bab+Ab^2)x^5 + Aa^2x + \frac{1}{3}(Ba^2+2Aab)x^3$$

input
```
integrate((b*x^2+a)^2*(D*x^6+C*x^4+B*x^2+A),x, algorithm="maxima")
```

output
```
1/11*D*b^2*x^11 + 1/9*(2*D*a*b + C*b^2)*x^9 + 1/7*(D*a^2 + 2*C*a*b + B*b^2
)*x^7 + 1/5*(C*a^2 + 2*B*a*b + A*b^2)*x^5 + A*a^2*x + 1/3*(B*a^2 + 2*A*a*b
)*x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.06

$$\int (a+bx^2)^2 (A+Bx^2+Cx^4+Dx^6)\,dx = \frac{1}{11}Db^2x^{11} + \frac{2}{9}Dabx^9 + \frac{1}{9}Cb^2x^9 + \frac{1}{7}Da^2x^7 + \frac{2}{7}Cabx^7 + \frac{1}{7}Bb^2x^7 + \frac{1}{5}Ca^2x^5 + \frac{2}{5}Babx^5 + \frac{1}{5}Ab^2x^5 + \frac{1}{3}Ba^2x^3 + \frac{2}{3}Aabx^3 + Aa^2x$$

input
```
integrate((b*x^2+a)^2*(D*x^6+C*x^4+B*x^2+A),x, algorithm="giac")
```

output
```
1/11*D*b^2*x^11 + 2/9*D*a*b*x^9 + 1/9*C*b^2*x^9 + 1/7*D*a^2*x^7 + 2/7*C*a*
b*x^7 + 1/7*B*b^2*x^7 + 1/5*C*a^2*x^5 + 2/5*B*a*b*x^5 + 1/5*A*b^2*x^5 + 1/
3*B*a^2*x^3 + 2/3*A*a*b*x^3 + A*a^2*x
```

Mupad [B] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.10

$$\int (a+bx^2)^2 (A+Bx^2+Cx^4+Dx^6) \, dx = \frac{A\,x\,(15\,a^2+10\,a\,b\,x^2+3\,b^2\,x^4)}{15}+\frac{a^2\,x^7\,D}{7}+\frac{b^2\,x^{11}\,D}{11} + \frac{B\,x^3\,(35\,a^2+42\,a\,b\,x^2+15\,b^2\,x^4)}{105}+\frac{C\,x^5\,(63\,a^2+90\,a\,b\,x^2+35\,b^2\,x^4)}{315}+\frac{2\,a\,b\,x^9\,D}{9}$$

input
```
int((a + b*x^2)^2*(A + B*x^2 + C*x^4 + x^6*D),x)
```

output
```
(A*x*(15*a^2 + 3*b^2*x^4 + 10*a*b*x^2))/15 + (a^2*x^7*D)/7 + (b^2*x^11*D)/
11 + (B*x^3*(35*a^2 + 15*b^2*x^4 + 42*a*b*x^2))/105 + (C*x^5*(63*a^2 + 35*
b^2*x^4 + 90*a*b*x^2))/315 + (2*a*b*x^9*D)/9
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.91

$$\int (a+bx^2)^2 (A+Bx^2+Cx^4+Dx^6) \, dx = \frac{x(315b^2d\,x^{10}+770abd\,x^8+385b^2c\,x^8+495a^2d\,x^6+990abc\,x^6+495b^3x^6+693a^2c\,x^4+2079a\,b^2x^4+34}{3465}$$

input
```
int((b*x^2+a)^2*(D*x^6+C*x^4+B*x^2+A),x)
```

output
```
(x*(3465*a**3 + 3465*a**2*b*x**2 + 693*a**2*c*x**4 + 495*a**2*d*x**6 + 207
9*a*b**2*x**4 + 990*a*b*c*x**6 + 770*a*b*d*x**8 + 495*b**3*x**6 + 385*b**2
*c*x**8 + 315*b**2*d*x**10))/3465
```

3.129 $\int (a+bx^2)(A+Bx^2+Cx^4+Dx^6)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 56

$$\int (a+bx^2)(A+Bx^2+Cx^4+Dx^6)\,dx = aAx+\frac{1}{3}(Ab+aB)x^3+\frac{1}{5}(bB+aC)x^5+\frac{1}{7}(bC+aD)x^7+\frac{1}{9}bDx^9$$

output
```
a*A*x+1/3*(A*b+B*a)*x^3+1/5*(B*b+C*a)*x^5+1/7*(C*b+D*a)*x^7+1/9*b*D*x^9
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

$$\int (a+bx^2)(A+Bx^2+Cx^4+Dx^6)\,dx = aAx+\frac{1}{3}(Ab+aB)x^3+\frac{1}{5}(bB+aC)x^5+\frac{1}{7}(bC+aD)x^7+\frac{1}{9}bDx^9$$

input
```
Integrate[(a + b*x^2)*(A + B*x^2 + C*x^4 + D*x^6),x]
```

output
```
a*A*x + ((A*b + a*B)*x^3)/3 + ((b*B + a*C)*x^5)/5 + ((b*C + a*D)*x^7)/7 +
(b*D*x^9)/9
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2341, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right) \left(A+Bx^2+Cx^4+Dx^6\right) \, dx$$

$$\Big\downarrow \text{ 2341}$$

$$\int \left(x^2(aB+Ab)+aA+x^4(aC+bB)+x^6(aD+bC)+bDx^8\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3}x^3(aB+Ab)+aAx+\frac{1}{5}x^5(aC+bB)+\frac{1}{7}x^7(aD+bC)+\frac{1}{9}bDx^9$$

input
```
Int[(a + b*x^2)*(A + B*x^2 + C*x^4 + D*x^6),x]
```

output
```
a*A*x + ((A*b + a*B)*x^3)/3 + ((b*B + a*C)*x^5)/5 + ((b*C + a*D)*x^7)/7 +
(b*D*x^9)/9
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2341
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.88

method	result	size
default	$aAx+\frac{(Ab+Ba)x^{3}}{3}+\frac{(Bb+Ca)x^{5}}{5}+\frac{(Cb+Da)x^{7}}{7}+\frac{bDx^{9}}{9}$	49
norman	$\frac{bDx^{9}}{9}+\left(\frac{Cb}{7}+\frac{Da}{7}\right)x^{7}+\left(\frac{Bb}{5}+\frac{Ca}{5}\right)x^{5}+\left(\frac{Ab}{3}+\frac{Ba}{3}\right)x^{3}+aAx$	52
gosper	$\frac{1}{9}bDx^{9}+\frac{1}{7}bC\,x^{7}+\frac{1}{7}x^{7}Da+\frac{1}{5}bB\,x^{5}+\frac{1}{5}x^{5}Ca+\frac{1}{3}Ab\,x^{3}+\frac{1}{3}x^{3}Ba+aAx$	55
parallelrisch	$\frac{1}{9}bDx^{9}+\frac{1}{7}bC\,x^{7}+\frac{1}{7}x^{7}Da+\frac{1}{5}bB\,x^{5}+\frac{1}{5}x^{5}Ca+\frac{1}{3}Ab\,x^{3}+\frac{1}{3}x^{3}Ba+aAx$	55
orering	$\frac{x(35Db\,x^{8}+45Cb\,x^{6}+45Da\,x^{6}+63bB\,x^{4}+63Ca\,x^{4}+105Ab\,x^{2}+105Ba\,x^{2}+315Aa)}{315}$	58

input
```
int((b*x^2+a)*(D*x^6+C*x^4+B*x^2+A),x,method=_RETURNVERBOSE)
```

output
```
a*A*x+1/3*(A*b+B*a)*x^3+1/5*(B*b+C*a)*x^5+1/7*(C*b+D*a)*x^7+1/9*b*D*x^9
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

$$\int (a+bx^2)\left(A+Bx^2+Cx^4+Dx^6\right)\,dx=\frac{1}{9}\,Dbx^9+\frac{1}{7}\,(Da+Cb)x^7+\frac{1}{5}\,(Ca+Bb)x^5+\frac{1}{3}\,(Ba+Ab)x^3+Aax$$

input
```
integrate((b*x^2+a)*(D*x^6+C*x^4+B*x^2+A),x, algorithm="fricas")
```

output
```
1/9*D*b*x^9 + 1/7*(D*a + C*b)*x^7 + 1/5*(C*a + B*b)*x^5 + 1/3*(B*a + A*b)*
x^3 + A*a*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.95

$$\int (a+bx^2)(A+Bx^2+Cx^4+Dx^6)\,dx = Aax + \frac{Dbx^9}{9} + x^7\left(\frac{Cb}{7}+\frac{Da}{7}\right) + x^5\left(\frac{Bb}{5}+\frac{Ca}{5}\right) + x^3\left(\frac{Ab}{3}+\frac{Ba}{3}\right)$$

input
```
integrate((b*x**2+a)*(D*x**6+C*x**4+B*x**2+A),x)
```

output
```
A*a*x + D*b*x**9/9 + x**7*(C*b/7 + D*a/7) + x**5*(B*b/5 + C*a/5) + x**3*(A
*b/3 + B*a/3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

$$\int (a+bx^2)(A+Bx^2+Cx^4+Dx^6)\,dx = \frac{1}{9}Dbx^9 + \frac{1}{7}(Da+Cb)x^7 + \frac{1}{5}(Ca+Bb)x^5 + \frac{1}{3}(Ba+Ab)x^3 + Aax$$

input
```
integrate((b*x^2+a)*(D*x^6+C*x^4+B*x^2+A),x, algorithm="maxima")
```

output
```
1/9*D*b*x^9 + 1/7*(D*a + C*b)*x^7 + 1/5*(C*a + B*b)*x^5 + 1/3*(B*a + A*b)*
x^3 + A*a*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.96

$$\int (a+bx^2) (A+Bx^2+Cx^4+Dx^6) \, dx = \frac{1}{9} Dbx^9 + \frac{1}{7} Dax^7 + \frac{1}{7} Cbx^7 + \frac{1}{5} Cax^5 + \frac{1}{5} Bbx^5 + \frac{1}{3} Bax^3 + \frac{1}{3} Abx^3 + Aax$$

input
```
integrate((b*x^2+a)*(D*x^6+C*x^4+B*x^2+A),x, algorithm="giac")
```

output
```
1/9*D*b*x^9 + 1/7*D*a*x^7 + 1/7*C*b*x^7 + 1/5*C*a*x^5 + 1/5*B*b*x^5 + 1/3*
B*a*x^3 + 1/3*A*b*x^3 + A*a*x
```

Mupad [B] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.96

$$\int (a+bx^2) (A+Bx^2+Cx^4+Dx^6) \, dx = \frac{a\,x^7\,D}{7} + \frac{b\,x^9\,D}{9} + A\,a\,x + \frac{A\,b\,x^3}{3} + \frac{B\,a\,x^3}{3} + \frac{B\,b\,x^5}{5} + \frac{C\,a\,x^5}{5} + \frac{C\,b\,x^7}{7}$$

input
```
int((a + b*x^2)*(A + B*x^2 + C*x^4 + x^6*D),x)
```

output
```
(a*x^7*D)/7 + (b*x^9*D)/9 + A*a*x + (A*b*x^3)/3 + (B*a*x^3)/3 + (B*b*x^5)/
5 + (C*a*x^5)/5 + (C*b*x^7)/7
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.93

$$\int (a + bx^2)\left(A + Bx^2 + Cx^4 + Dx^6\right) dx = \frac{x(35bd\,x^8 + 45ad\,x^6 + 45bc\,x^6 + 63ac\,x^4 + 63b^2x^4 + 210ab\,x^2 + 315a^2)}{315}$$

input
```
int((b*x^2+a)*(D*x^6+C*x^4+B*x^2+A),x)
```

output
```
(x*(315*a**2 + 210*a*b*x**2 + 63*a*c*x**4 + 45*a*d*x**6 + 63*b**2*x**4 + 4
5*b*c*x**6 + 35*b*d*x**8))/315
```

3.130 $\int \frac{A+Bx^2+Cx^4+Dx^6}{a+bx^2} dx$

Optimal result

Integrand size = 27, antiderivative size = 100

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{a+bx^2}\,dx=\frac{(b^2B-abC+a^2D)\,x}{b^3}+\frac{(bC-aD)x^3}{3b^2}+\frac{Dx^5}{5b}+\frac{(Ab^3-a(b^2B-abC+a^2D))\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{7/2}}$$

output
```
(B*b^2-C*a*b+D*a^2)*x/b^3+1/3*(C*b-D*a)*x^3/b^2+1/5*D*x^5/b+(A*b^3-a*(B*b^
2-C*a*b+D*a^2))*arctan(b^(1/2)*x/a^(1/2))/a^(1/2)/b^(7/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.98

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{a+bx^2}\,dx=\frac{x(15a^2D-5ab(3C+Dx^2)+b^2(15B+5Cx^2+3Dx^4))}{15b^3}+\frac{(Ab^3-a(b^2B-abC+a^2D))\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{7/2}}$$

input
```
Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(a + b*x^2),x]
```

output
```
(x*(15*a^2*D - 5*a*b*(3*C + D*x^2) + b^2*(15*B + 5*C*x^2 + 3*D*x^4)))/(15*
b^3) + ((A*b^3 - a*(b^2*B - a*b*C + a^2*D))*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(
Sqrt[a]*b^(7/2))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2341, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{a+bx^2}\,dx$$

$$\Big\downarrow \text{2341}$$

$$\int \left(\frac{a^2D-abC+b^2B}{b^3}+\frac{a^3(-D)+a^2bC-ab^2B+Ab^3}{b^3\left(a+bx^2\right)}+\frac{x^2(bC-aD)}{b^2}+\frac{Dx^4}{b}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{\sqrt{a}b^{7/2}}+\frac{x\left(a^2D-abC+b^2B\right)}{b^3}+\frac{x^3(bC-aD)}{3b^2}+\frac{Dx^5}{5b}$$

input
```
Int[(A + B*x^2 + C*x^4 + D*x^6)/(a + b*x^2),x]
```

output
```
((b^2*B - a*b*C + a^2*D)*x)/b^3 + ((b*C - a*D)*x^3)/(3*b^2) + (D*x^5)/(5*b
) + ((A*b^3 - a*(b^2*B - a*b*C + a^2*D))*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqr
t[a]*b^(7/2))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2341
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{\frac{1}{5}Dx^5b^2+\frac{1}{3}Cb^2x^3-\frac{1}{3}Dab\,x^3+b^2Bx-Cabx+Da^2x}{b^3}+\frac{(b^3A-a\,b^2B+a^2bC-a^3D)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{b^3\sqrt{ab}}$	94

input
```
int((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/b^3*(1/5*D*x^5*b^2+1/3*C*b^2*x^3-1/3*D*a*b*x^3+b^2*B*x-C*a*b*x+D*a^2*x)+
(A*b^3-B*a*b^2+C*a^2*b-D*a^3)/b^3/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 236, normalized size of antiderivative = 2.36

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{a+bx^2}\,dx$$
$$=\left[\frac{6\,Dab^3x^5-10\,(Da^2b^2-Cab^3)x^3+15\,(Da^3-Ca^2b+Bab^2-Ab^3)\sqrt{-ab}\log\left(\frac{bx^2-2\,\sqrt{-ab}x-a}{bx^2+a}\right)+30\,(D}{30\,ab^4}\right.$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a),x, algorithm="fricas")
```

output
```
[1/30*(6*D*a*b^3*x^5 - 10*(D*a^2*b^2 - C*a*b^3)*x^3 + 15*(D*a^3 - C*a^2*b
+ B*a*b^2 - A*b^3)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)
) + 30*(D*a^3*b - C*a^2*b^2 + B*a*b^3)*x)/(a*b^4), 1/15*(3*D*a*b^3*x^5 - 5
*(D*a^2*b^2 - C*a*b^3)*x^3 - 15*(D*a^3 - C*a^2*b + B*a*b^2 - A*b^3)*sqrt(a
*b)*arctan(sqrt(a*b)*x/a) + 15*(D*a^3*b - C*a^2*b^2 + B*a*b^3)*x)/(a*b^4)]
```

Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.60

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{a+bx^2}\,dx = \frac{Dx^5}{5b}+x^3\left(\frac{C}{3b}-\frac{Da}{3b^2}\right)+x\left(\frac{B}{b}-\frac{Ca}{b^2}+\frac{Da^2}{b^3}\right) + \frac{\sqrt{-\frac{1}{ab^7}}(-Ab^3+Bab^2-Ca^2b+Da^3)\log\left(-ab^3\sqrt{-\frac{1}{ab^7}}+x\right)}{2} - \frac{\sqrt{-\frac{1}{ab^7}}(-Ab^3+Bab^2-Ca^2b+Da^3)\log\left(ab^3\sqrt{-\frac{1}{ab^7}}+x\right)}{2}$$

input
```
integrate((D*x**6+C*x**4+B*x**2+A)/(b*x**2+a),x)
```

output
```
D*x**5/(5*b) + x**3*(C/(3*b) - D*a/(3*b**2)) + x*(B/b - C*a/b**2 + D*a**2/
b**3) + sqrt(-1/(a*b**7))*(-A*b**3 + B*a*b**2 - C*a**2*b + D*a**3)*log(-a*
b**3*sqrt(-1/(a*b**7)) + x)/2 - sqrt(-1/(a*b**7))*(-A*b**3 + B*a*b**2 - C*
a**2*b + D*a**3)*log(a*b**3*sqrt(-1/(a*b**7)) + x)/2
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.95

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{a+bx^2}\,dx = -\frac{(Da^3-Ca^2b+Bab^2-Ab^3)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^3} + \frac{3\,Db^2x^5-5\,(Dab-Cb^2)x^3+15\,(Da^2-Cab+Bb^2)x}{15\,b^3}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a),x, algorithm="maxima")
```

output
```
-(D*a^3 - C*a^2*b + B*a*b^2 - A*b^3)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3)
 + 1/15*(3*D*b^2*x^5 - 5*(D*a*b - C*b^2)*x^3 + 15*(D*a^2 - C*a*b + B*b^2)*
x)/b^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.04

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{a+bx^2}\,dx = -\frac{(Da^3-Ca^2b+Bab^2-Ab^3)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^3} + \frac{3\,Db^4x^5-5\,Dab^3x^3+5\,Cb^4x^3+15\,Da^2b^2x-15\,Cab^3x+15\,Bb^4x}{15\,b^5}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a),x, algorithm="giac")
```

output
```
-(D*a^3 - C*a^2*b + B*a*b^2 - A*b^3)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3)
 + 1/15*(3*D*b^4*x^5 - 5*D*a*b^3*x^3 + 5*C*b^4*x^3 + 15*D*a^2*b^2*x - 15*C
*a*b^3*x + 15*B*b^4*x)/b^5
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{a+bx^2}\,dx = \int \frac{A+B\,x^2+C\,x^4+x^6\,D}{b\,x^2+a}\,dx$$

input
```
int((A + B*x^2 + C*x^4 + x^6*D)/(a + b*x^2),x)
```

output
```
int((A + B*x^2 + C*x^4 + x^6*D)/(a + b*x^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.99

$$\int \frac{A + Bx^2 + Cx^4 + Dx^6}{a + bx^2}\, dx$$
$$= \frac{-15\sqrt{b}\,\sqrt{a}\, atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right) a^2 d + 15\sqrt{b}\,\sqrt{a}\, atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right) abc + 15a^2 bdx - 15a\, b^2 cx - 5a\, b^2 d\, x^3 + 15b^4 x + 5}{15b^4}$$

input
```
int((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a),x)
```

output
```
( - 15*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2*d + 15*sqrt(b)*s
qrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*b*c + 15*a**2*b*d*x - 15*a*b**2*c*x
 - 5*a*b**2*d*x**3 + 15*b**4*x + 5*b**3*c*x**3 + 3*b**3*d*x**5)/(15*b**4)
```

3.131 $\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 118

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^2}\,dx = \frac{(bC-2aD)x}{b^3}+\frac{Dx^3}{3b^2}+\frac{\left(\frac{A}{a}-\frac{b^2B-abC+a^2D}{b^3}\right)x}{2(a+bx^2)} + \frac{(Ab^3+a(b^2B-3abC+5a^2D))\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{7/2}}$$

output
```
(C*b-2*D*a)*x/b^3+1/3*D*x^3/b^2+(A/a-(B*b^2-C*a*b+D*a^2)/b^3)*x/(2*b*x^2+2
*a)+1/2*(A*b^3+a*(B*b^2-3*C*a*b+5*D*a^2))*arctan(b^(1/2)*x/a^(1/2))/a^(3/2
)/b^(7/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.03

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^2}\,dx = \frac{(bC-2aD)x}{b^3}+\frac{Dx^3}{3b^2}-\frac{(-Ab^3+ab^2B-a^2bC+a^3D)\,x}{2ab^3(a+bx^2)} + \frac{(Ab^3+ab^2B-3a^2bC+5a^3D)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{7/2}}$$

input
```
Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(a + b*x^2)^2,x]
```

output
```
((b*C - 2*a*D)*x)/b^3 + (D*x^3)/(3*b^2) - ((-(A*b^3) + a*b^2*B - a^2*b*C +
 a^3*D)*x)/(2*a*b^3*(a + b*x^2)) + ((A*b^3 + a*b^2*B - 3*a^2*b*C + 5*a^3*D
)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*b^(7/2))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {2345, 25, 1467, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^2}\,dx$$

$$\Big\downarrow \text{2345}$$

$$\frac{x\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{2a(a+bx^2)}-\frac{\int -\frac{\frac{2aDx^4}{b}+\frac{2a(bC-aD)x^2}{b^2}+\frac{Da^3-bCa^2+b^2Ba+Ab^3}{b^3}}{bx^2+a}dx}{2a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\frac{2aDx^4}{b}+\frac{2a(bC-aD)x^2}{b^2}+A+\frac{a\left(Da^2-bCa+b^2B\right)}{b^3}}{bx^2+a}dx}{2a}+\frac{x\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{2a(a+bx^2)}$$

$$\Big\downarrow \text{1467}$$

$$\frac{\int \left(\frac{2aDx^2}{b^2}+\frac{2a(bC-2aD)}{b^3}+\frac{5Da^3-3bCa^2+b^2Ba+Ab^3}{b^3(bx^2+a)}\right)dx}{2a}+\frac{x\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{2a(a+bx^2)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a(5a^2D-3abC+b^2B)+Ab^3\right)}{\sqrt{a}b^{7/2}}+\frac{2ax(bC-2aD)}{b^3}+\frac{2aDx^3}{3b^2}}{2a}+\frac{x\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{2a(a+bx^2)}$$

input
```
Int[(A + B*x^2 + C*x^4 + D*x^6)/(a + b*x^2)^2,x]
```

output
```
((A - (a*(b^2*B - a*b*C + a^2*D))/b^3)*x)/(2*a*(a + b*x^2)) + ((2*a*(b*C -
 2*a*D)*x)/b^3 + (2*a*D*x^3)/(3*b^2) + ((A*b^3 + a*(b^2*B - 3*a*b*C + 5*a^
2*D))*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(7/2)))/(2*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1467
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{\frac{1}{3}Dx^3b+Cbx-2Dax}{b^3}+\frac{\frac{\left(b^3A-a\,b^2B+a^2bC-a^3D\right)x}{2a\left(b\,x^2+a\right)}+\frac{\left(b^3A+a\,b^2B-3a^2bC+5a^3D\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2a\sqrt{ab}}}{b^3}$	112

input
```
int((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b^3*(1/3*D*x^3*b+C*b*x-2*D*a*x)+1/b^3*(1/2*(A*b^3-B*a*b^2+C*a^2*b-D*a^3)
/a*x/(b*x^2+a)+1/2*(A*b^3+B*a*b^2-3*C*a^2*b+5*D*a^3)/a/(a*b)^(1/2)*arctan(
b*x/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 364, normalized size of antiderivative = 3.08

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^2}\,dx$$
$$= \left[\frac{4\,Da^2b^3x^5 - 4\,(5\,Da^3b^2 - 3\,Ca^2b^3)x^3 - 3\,(5\,Da^4 - 3\,Ca^3b + Ba^2b^2 + Aab^3 + (5\,Da^3b - 3\,Ca^2b^2 + Ba}{12\,(a^2b^5x^2 + a^3b^4)}\right.$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[1/12*(4*D*a^2*b^3*x^5 - 4*(5*D*a^3*b^2 - 3*C*a^2*b^3)*x^3 - 3*(5*D*a^4 -
3*C*a^3*b + B*a^2*b^2 + A*a*b^3 + (5*D*a^3*b - 3*C*a^2*b^2 + B*a*b^3 + A*b
^4)*x^2)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) - 6*(5*D
*a^4*b - 3*C*a^3*b^2 + B*a^2*b^3 - A*a*b^4)*x)/(a^2*b^5*x^2 + a^3*b^4), 1/
6*(2*D*a^2*b^3*x^5 - 2*(5*D*a^3*b^2 - 3*C*a^2*b^3)*x^3 + 3*(5*D*a^4 - 3*C*
a^3*b + B*a^2*b^2 + A*a*b^3 + (5*D*a^3*b - 3*C*a^2*b^2 + B*a*b^3 + A*b^4)*
x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) - 3*(5*D*a^4*b - 3*C*a^3*b^2 + B*a^2*
b^3 - A*a*b^4)*x)/(a^2*b^5*x^2 + a^3*b^4)]
```

Sympy [A] (verification not implemented)

Time = 0.74 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.70

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^2}\,dx = \frac{Dx^3}{3b^2}+x\left(\frac{C}{b^2}-\frac{2Da}{b^3}\right)+\frac{x(Ab^3-Bab^2+Ca^2b-Da^3)}{2a^2b^3+2ab^4x^2} - \frac{\sqrt{-\frac{1}{a^3b^7}}(Ab^3+Bab^2-3Ca^2b+5Da^3)\log\left(-a^2b^3\sqrt{-\frac{1}{a^3b^7}}+x\right)}{4} + \frac{\sqrt{-\frac{1}{a^3b^7}}(Ab^3+Bab^2-3Ca^2b+5Da^3)\log\left(a^2b^3\sqrt{-\frac{1}{a^3b^7}}+x\right)}{4}$$

input
```
integrate((D*x**6+C*x**4+B*x**2+A)/(b*x**2+a)**2,x)
```

output
```
D*x**3/(3*b**2) + x*(C/b**2 - 2*D*a/b**3) + x*(A*b**3 - B*a*b**2 + C*a**2*
b - D*a**3)/(2*a**2*b**3 + 2*a*b**4*x**2) - sqrt(-1/(a**3*b**7))*(A*b**3 +
 B*a*b**2 - 3*C*a**2*b + 5*D*a**3)*log(-a**2*b**3*sqrt(-1/(a**3*b**7)) + x
)/4 + sqrt(-1/(a**3*b**7))*(A*b**3 + B*a*b**2 - 3*C*a**2*b + 5*D*a**3)*log
(a**2*b**3*sqrt(-1/(a**3*b**7)) + x)/4
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^2}\,dx = -\frac{(Da^3-Ca^2b+Bab^2-Ab^3)x}{2\,(ab^4x^2+a^2b^3)} + \frac{Dbx^3-3\,(2\,Da-Cb)x}{3\,b^3} + \frac{(5\,Da^3-3\,Ca^2b+Bab^2+Ab^3)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}ab^3}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
-1/2*(D*a^3 - C*a^2*b + B*a*b^2 - A*b^3)*x/(a*b^4*x^2 + a^2*b^3) + 1/3*(D*
b*x^3 - 3*(2*D*a - C*b)*x)/b^3 + 1/2*(5*D*a^3 - 3*C*a^2*b + B*a*b^2 + A*b^
3)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.04

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^2}\,dx=\frac{(5\,Da^3-3\,Ca^2b+Bab^2+Ab^3)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}ab^3}-\frac{Da^3x-Ca^2bx+Bab^2x-Ab^3x}{2\,(bx^2+a)ab^3}+\frac{Db^4x^3-6\,Dab^3x+3\,Cb^4x}{3\,b^6}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*(5*D*a^3 - 3*C*a^2*b + B*a*b^2 + A*b^3)*arctan(b*x/sqrt(a*b))/(sqrt(a*
b)*a*b^3) - 1/2*(D*a^3*x - C*a^2*b*x + B*a*b^2*x - A*b^3*x)/((b*x^2 + a)*a
*b^3) + 1/3*(D*b^4*x^3 - 6*D*a*b^3*x + 3*C*b^4*x)/b^6
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^2}\,dx=\int \frac{A+B\,x^2+C\,x^4+x^6\,D}{(b\,x^2+a)^2}\,dx$$

input
```
int((A + B*x^2 + C*x^4 + x^6*D)/(a + b*x^2)^2,x)
```

output
```
int((A + B*x^2 + C*x^4 + x^6*D)/(a + b*x^2)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.79

$$\int \frac{A + Bx^2 + Cx^4 + Dx^6}{(a + bx^2)^2}\,dx$$

$$= \frac{15\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^3d - 9\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^2bc + 15\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^2bd\,x^2 + 6\sqrt{b}\,\sqrt{a}\,atan(}{}$$

input
```
int((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^2,x)
```

output
```
(15*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**3*d - 9*sqrt(b)*sqrt(
a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2*b*c + 15*sqrt(b)*sqrt(a)*atan((b*x)/
(sqrt(b)*sqrt(a)))*a**2*b*d*x**2 + 6*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*s
qrt(a)))*a*b**3 - 9*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*b**2*c
*x**2 + 6*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**4*x**2 - 15*a**
3*b*d*x + 9*a**2*b**2*c*x - 10*a**2*b**2*d*x**3 + 6*a*b**3*c*x**3 + 2*a*b*
*3*d*x**5)/(6*a*b**4*(a + b*x**2))
```

3.132 $\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 147

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^3}\,dx = \frac{Dx}{b^3} + \frac{\left(\frac{A}{a} - \frac{b^2B-abC+a^2D}{b^3}\right)x}{4(a+bx^2)^2} + \frac{(3Ab^3+a(b^2B-5abC+9a^2D))\,x}{8a^2b^3(a+bx^2)} + \frac{(3Ab^3+a(b^2B+3abC-15a^2D))\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{7/2}}$$

output
```
D*x/b^3+1/4*(A/a-(B*b^2-C*a*b+D*a^2)/b^3)*x/(b*x^2+a)^2+1/8*(3*A*b^3+a*(B*
b^2-5*C*a*b+9*D*a^2))*x/a^2/b^3/(b*x^2+a)+1/8*(3*A*b^3+a*(B*b^2+3*C*a*b-15
*D*a^2))*arctan(b^(1/2)*x/a^(1/2))/a^(5/2)/b^(7/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.96

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^3}\,dx$$
$$=\frac{x(15a^4D+3Ab^4x^2+ab^3(5A+Bx^2)+a^3b(-3C+25Dx^2)-a^2b^2(B+5Cx^2-8Dx^4))}{8a^2b^3\left(a+bx^2\right)^2}$$
$$+\frac{(3Ab^3+a(b^2B+3abC-15a^2D))\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{7/2}}$$

input
```
Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(a + b*x^2)^3,x]
```

output
```
(x*(15*a^4*D + 3*A*b^4*x^2 + a*b^3*(5*A + B*x^2) + a^3*b*(-3*C + 25*D*x^2)
 - a^2*b^2*(B + 5*C*x^2 - 8*D*x^4)))/(8*a^2*b^3*(a + b*x^2)^2) + ((3*A*b^3
 + a*(b^2*B + 3*a*b*C - 15*a^2*D))*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(5/2)
*b^(7/2))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2345, 25, 1471, 25, 27, 299, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^3}\,dx$$

$$\Big\downarrow \text{ 2345}$$

$$\frac{x\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{4a\left(a+bx^2\right)^2}-\frac{\int -\frac{\frac{4aDx^4}{b}+\frac{4a(bC-aD)x^2}{b^2}+\frac{Da^3-bCa^2+b^2Ba+3Ab^3}{b^3}}{(bx^2+a)^2}dx}{4a}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{\frac{4aDx^4}{b}+\frac{4a(bC-aD)x^2}{b^2}+\frac{Da^3-bCa^2+b^2Ba+3Ab^3}{b^3}}{(bx^2+a)^2}dx}{4a}+\frac{x\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{4a\left(a+bx^2\right)^2}$$

$$\Big\downarrow \; 1471$$

$$\frac{\frac{x\left(\frac{9a^2D-5abC+b^2B}{b^3}+\frac{3A}{a}\right)}{2(a+bx^2)}-\frac{\int -\frac{-7Da^3+8bDx^2a^2+3bCa^2+b^2Ba+3Ab^3}{b^3\left(bx^2+a\right)}dx}{2a}}{4a}+\frac{x\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{4a\left(a+bx^2\right)^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{3Ab^3+8a^2Dx^2b+a\left(-7Da^2+3bCa+b^2B\right)}{b^3\left(bx^2+a\right)}dx}{2a}+\frac{x\left(\frac{9a^2D-5abC+b^2B}{b^3}+\frac{3A}{a}\right)}{2(a+bx^2)}}{4a}+\frac{x\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{4a\left(a+bx^2\right)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{3Ab^3+8a^2Dx^2b+a\left(-7Da^2+3bCa+b^2B\right)}{bx^2+a}dx}{2ab^3}+\frac{x\left(\frac{9a^2D-5abC+b^2B}{b^3}+\frac{3A}{a}\right)}{2(a+bx^2)}}{4a}+\frac{x\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{4a\left(a+bx^2\right)^2}$$

$$\Big\downarrow \; 299$$

$$\frac{\frac{\left(a\left(-15a^2D+3abC+b^2B\right)+3Ab^3\right)\int \frac{1}{bx^2+a}dx+8a^2Dx}{2ab^3}+\frac{x\left(\frac{9a^2D-5abC+b^2B}{b^3}+\frac{3A}{a}\right)}{2(a+bx^2)}}{4a}+\frac{x\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{4a\left(a+bx^2\right)^2}$$

$$\Big\downarrow \; 218$$

$$\frac{\frac{\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a\left(-15a^2D+3abC+b^2B\right)+3Ab^3\right)}{\sqrt{a}\sqrt{b}}+8a^2Dx}{2ab^3}+\frac{x\left(\frac{9a^2D-5abC+b^2B}{b^3}+\frac{3A}{a}\right)}{2(a+bx^2)}}{4a}+\frac{x\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{4a\left(a+bx^2\right)^2}$$

input
```
Int[(A + B*x^2 + C*x^4 + D*x^6)/(a + b*x^2)^3,x]
```

output
```
((A - (a*(b^2*B - a*b*C + a^2*D))/b^3)*x)/(4*a*(a + b*x^2)^2) + ((((3*A)/a
 + (b^2*B - 5*a*b*C + 9*a^2*D)/b^3)*x)/(2*(a + b*x^2)) + (8*a^2*D*x + ((3*
A*b^3 + a*(b^2*B + 3*a*b*C - 15*a^2*D))*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt
[a]*Sqrt[b]))/(2*a*b^3))/(4*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{Dx}{b^3}+\frac{\frac{\frac{b\left(3b^3A+a\,b^2B-5a^2bC+9a^3D\right)x^3}{8a^2}+\frac{\left(5b^3A-a\,b^2B-3a^2bC+7a^3D\right)x}{8a}}{\left(b\,x^2+a\right)^2}+\frac{\left(3b^3A+a\,b^2B+3a^2bC-15a^3D\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8a^2\sqrt{ab}}}{b^3}$	137

input
```
int((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
D*x/b^3+1/b^3*((1/8*b*(3*A*b^3+B*a*b^2-5*C*a^2*b+9*D*a^3)/a^2*x^3+1/8*(5*A
*b^3-B*a*b^2-3*C*a^2*b+7*D*a^3)/a*x)/(b*x^2+a)^2+1/8*(3*A*b^3+B*a*b^2+3*C*
a^2*b-15*D*a^3)/a^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 511, normalized size of antiderivative = 3.48

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^3}\,dx$$
$$=\left[\frac{16\,Da^3b^3x^5+2\,(25\,Da^4b^2-5\,Ca^3b^3+Ba^2b^4+3\,Aab^5)x^3+(15\,Da^5-3\,Ca^4b-Ba^3b^2-3\,Aa^2b^3+(1}{}\right.$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
[1/16*(16*D*a^3*b^3*x^5 + 2*(25*D*a^4*b^2 - 5*C*a^3*b^3 + B*a^2*b^4 + 3*A*
a*b^5)*x^3 + (15*D*a^5 - 3*C*a^4*b - B*a^3*b^2 - 3*A*a^2*b^3 + (15*D*a^3*b
^2 - 3*C*a^2*b^3 - B*a*b^4 - 3*A*b^5)*x^4 + 2*(15*D*a^4*b - 3*C*a^3*b^2 -
B*a^2*b^3 - 3*A*a*b^4)*x^2)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b
*x^2 + a)) + 2*(15*D*a^5*b - 3*C*a^4*b^2 - B*a^3*b^3 + 5*A*a^2*b^4)*x)/(a^
3*b^6*x^4 + 2*a^4*b^5*x^2 + a^5*b^4), 1/8*(8*D*a^3*b^3*x^5 + (25*D*a^4*b^2
 - 5*C*a^3*b^3 + B*a^2*b^4 + 3*A*a*b^5)*x^3 - (15*D*a^5 - 3*C*a^4*b - B*a^
3*b^2 - 3*A*a^2*b^3 + (15*D*a^3*b^2 - 3*C*a^2*b^3 - B*a*b^4 - 3*A*b^5)*x^4
 + 2*(15*D*a^4*b - 3*C*a^3*b^2 - B*a^2*b^3 - 3*A*a*b^4)*x^2)*sqrt(a*b)*arc
tan(sqrt(a*b)*x/a) + (15*D*a^5*b - 3*C*a^4*b^2 - B*a^3*b^3 + 5*A*a^2*b^4)*
x)/(a^3*b^6*x^4 + 2*a^4*b^5*x^2 + a^5*b^4)]
```

Sympy [A] (verification not implemented)

Time = 3.16 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.65

$$\int \frac{A + Bx^2 + Cx^4 + Dx^6}{(a + bx^2)^3}\,dx$$
$$= \frac{Dx}{b^3} + \frac{\sqrt{-\frac{1}{a^5b^7}}(-3Ab^3 - Bab^2 - 3Ca^2b + 15Da^3)\log\left(-a^3b^3\sqrt{-\frac{1}{a^5b^7}} + x\right)}{16}$$
$$- \frac{\sqrt{-\frac{1}{a^5b^7}}(-3Ab^3 - Bab^2 - 3Ca^2b + 15Da^3)\log\left(a^3b^3\sqrt{-\frac{1}{a^5b^7}} + x\right)}{16}$$
$$+ \frac{x^3 \cdot (3Ab^4 + Bab^3 - 5Ca^2b^2 + 9Da^3b) + x(5Aab^3 - Ba^2b^2 - 3Ca^3b + 7Da^4)}{8a^4b^3 + 16a^3b^4x^2 + 8a^2b^5x^4}$$

input
```
integrate((D*x**6+C*x**4+B*x**2+A)/(b*x**2+a)**3,x)
```

output
```
D*x/b**3 + sqrt(-1/(a**5*b**7))*(-3*A*b**3 - B*a*b**2 - 3*C*a**2*b + 15*D*
a**3)*log(-a**3*b**3*sqrt(-1/(a**5*b**7)) + x)/16 - sqrt(-1/(a**5*b**7))*(
-3*A*b**3 - B*a*b**2 - 3*C*a**2*b + 15*D*a**3)*log(a**3*b**3*sqrt(-1/(a**5
*b**7)) + x)/16 + (x**3*(3*A*b**4 + B*a*b**3 - 5*C*a**2*b**2 + 9*D*a**3*b)
 + x*(5*A*a*b**3 - B*a**2*b**2 - 3*C*a**3*b + 7*D*a**4))/(8*a**4*b**3 + 16
*a**3*b**4*x**2 + 8*a**2*b**5*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.05

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^3}\,dx = \frac{(9\,Da^3b-5\,Ca^2b^2+Bab^3+3\,Ab^4)x^3+(7\,Da^4-3\,Ca^3b-Ba^2b^2+5\,Aab^3)x}{8\,(a^2b^5x^4+2\,a^3b^4x^2+a^4b^3)} + \frac{Dx}{b^3} - \frac{(15\,Da^3-3\,Ca^2b-Bab^2-3\,Ab^3)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^2b^3}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/8*((9*D*a^3*b - 5*C*a^2*b^2 + B*a*b^3 + 3*A*b^4)*x^3 + (7*D*a^4 - 3*C*a^
3*b - B*a^2*b^2 + 5*A*a*b^3)*x)/(a^2*b^5*x^4 + 2*a^3*b^4*x^2 + a^4*b^3) +
D*x/b^3 - 1/8*(15*D*a^3 - 3*C*a^2*b - B*a*b^2 - 3*A*b^3)*arctan(b*x/sqrt(a
*b))/(sqrt(a*b)*a^2*b^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^3}\,dx = \frac{Dx}{b^3} - \frac{(15\,Da^3-3\,Ca^2b-Bab^2-3\,Ab^3)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^2b^3} + \frac{9\,Da^3bx^3-5\,Ca^2b^2x^3+Bab^3x^3+3\,Ab^4x^3+7\,Da^4x-3\,Ca^3bx-Ba^2b^2x+5\,Aab^3x}{8\,(bx^2+a)^2a^2b^3}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
D*x/b^3 - 1/8*(15*D*a^3 - 3*C*a^2*b - B*a*b^2 - 3*A*b^3)*arctan(b*x/sqrt(a
*b))/(sqrt(a*b)*a^2*b^3) + 1/8*(9*D*a^3*b*x^3 - 5*C*a^2*b^2*x^3 + B*a*b^3*
x^3 + 3*A*b^4*x^3 + 7*D*a^4*x - 3*C*a^3*b*x - B*a^2*b^2*x + 5*A*a*b^3*x)/(
(b*x^2 + a)^2*a^2*b^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^3}\,dx=\int \frac{A+B\,x^2+C\,x^4+x^6\,D}{(b\,x^2+a)^3}\,dx$$

input
```
int((A + B*x^2 + C*x^4 + x^6*D)/(a + b*x^2)^3,x)
```

output
```
int((A + B*x^2 + C*x^4 + x^6*D)/(a + b*x^2)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 327, normalized size of antiderivative = 2.22

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^3}\,dx$$
$$=\frac{-15\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^4d+3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^3bc-30\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^3bd\,x^2+4\sqrt{b}\,\sqrt{a}\,ata}{}$$

input
```
int((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^3,x)
```

output
```
( - 15*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**4*d + 3*sqrt(b)*sq
rt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**3*b*c - 30*sqrt(b)*sqrt(a)*atan((b*
x)/(sqrt(b)*sqrt(a)))*a**3*b*d*x**2 + 4*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b
)*sqrt(a)))*a**2*b**3 + 6*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*
*2*b**2*c*x**2 - 15*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2*b**
2*d*x**4 + 8*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*b**4*x**2 + 3
*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*b**3*c*x**4 + 4*sqrt(b)*s
qrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**5*x**4 + 15*a**4*b*d*x - 3*a**3*b*
*2*c*x + 25*a**3*b**2*d*x**3 + 4*a**2*b**4*x - 5*a**2*b**3*c*x**3 + 8*a**2
*b**3*d*x**5 + 4*a*b**5*x**3)/(8*a**2*b**4*(a**2 + 2*a*b*x**2 + b**2*x**4)
)
```

3.133 $\int (a+bx^2)^{3/2} (A+Bx^2+Cx^4+Dx^6) \, dx$

Optimal result

Integrand size = 29, antiderivative size = 241

$$\int (a+bx^2)^{3/2} (A+Bx^2+Cx^4+Dx^6) \, dx = \frac{a(96Ab^3-a(16b^2B-6abC+3a^2D))\,x\sqrt{a+bx^2}}{256b^3} + \frac{1}{384}\left(96A-\frac{a(16b^2B-6abC+3a^2D)}{b^3}\right)x\left(a+bx^2\right)^{3/2} + \frac{(16b^2B-6abC+3a^2D)\,x(a+bx^2)^{5/2}}{96b^3} + \frac{(2bC-aD)x^3(a+bx^2)^{5/2}}{16b^2} + \frac{Dx^5(a+bx^2)^{5/2}}{10b} + \frac{a^2(96Ab^3-a(16b^2B-6abC+3a^2D))\,\mathrm{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{256b^{7/2}}$$

output
```
1/256*a*(96*A*b^3-a*(16*B*b^2-6*C*a*b+3*D*a^2))*x*(b*x^2+a)^(1/2)/b^3+1/38
4*(96*A-a*(16*B*b^2-6*C*a*b+3*D*a^2)/b^3)*x*(b*x^2+a)^(3/2)+1/96*(16*B*b^2
-6*C*a*b+3*D*a^2)*x*(b*x^2+a)^(5/2)/b^3+1/16*(2*C*b-D*a)*x^3*(b*x^2+a)^(5/
2)/b^2+1/10*D*x^5*(b*x^2+a)^(5/2)/b+1/256*a^2*(96*A*b^3-a*(16*B*b^2-6*C*a*
b+3*D*a^2))*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(7/2)
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.77

$$\int \left(a+bx^2\right)^{3/2} \left(A+Bx^2+Cx^4+Dx^6\right) \, dx = \frac{\sqrt{b}x\sqrt{a+bx^2}(45a^4D-30a^3b(3C+Dx^2)+12a^2b^2(20B+5Cx^2+2Dx^4)+32b^4x^2(30A+2\cdots}{\cdots}$$

input
```
Integrate[(a + b*x^2)^(3/2)*(A + B*x^2 + C*x^4 + D*x^6),x]
```

output
```
(Sqrt[b]*x*Sqrt[a + b*x^2]*(45*a^4*D - 30*a^3*b*(3*C + D*x^2) + 12*a^2*b^2
*(20*B + 5*C*x^2 + 2*D*x^4) + 32*b^4*x^2*(30*A + 20*B*x^2 + 15*C*x^4 + 12*
D*x^6) + 16*a*b^3*(150*A + 70*B*x^2 + 45*C*x^4 + 33*D*x^6)) + 15*a^2*(-96*
A*b^3 + a*(16*b^2*B - 6*a*b*C + 3*a^2*D))*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^
2]])/(3840*b^(7/2))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.87, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2346, 27, 1473, 299, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^{3/2} \left(A+Bx^2+Cx^4+Dx^6\right) \, dx$$

$$\Big\downarrow \text{2346}$$

$$\frac{\int 5\left(bx^2+a\right)^{3/2} \left((2bC-aD)x^4+2bBx^2+2Ab\right)dx}{10b}+\frac{Dx^5\left(a+bx^2\right)^{5/2}}{10b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \left(bx^2+a\right)^{3/2} \left((2bC-aD)x^4+2bBx^2+2Ab\right)dx}{2b}+\frac{Dx^5\left(a+bx^2\right)^{5/2}}{10b}$$

$$\Big\downarrow \text{1473}$$

$$\frac{\frac{\int (bx^2+a)^{3/2}(16Ab^2+(3Da^2-6bCa+16b^2B)x^2)dx}{8b}+\frac{x^3(a+bx^2)^{5/2}(2bC-aD)}{8b}}{2b}+\frac{Dx^5(a+bx^2)^{5/2}}{10b}$$

$\downarrow$ 299

$$\frac{\frac{\frac{\left(96Ab^3-a\left(3a^2D-6abC+16b^2B\right)\right)\int (bx^2+a)^{3/2}dx}{6b}+\frac{x(a+bx^2)^{5/2}(3a^2D-6abC+16b^2B)}{6b}}{8b}+\frac{x^3(a+bx^2)^{5/2}(2bC-aD)}{8b}}{2b}+\frac{Dx^5(a+bx^2)^{5/2}}{10b}$$

$\downarrow$ 211

$$\frac{\frac{\frac{\left(96Ab^3-a\left(3a^2D-6abC+16b^2B\right)\right)\left(\frac{3}{4}a\int \sqrt{bx^2+a}dx+\frac{1}{4}x(a+bx^2)^{3/2}\right)}{6b}+\frac{x(a+bx^2)^{5/2}(3a^2D-6abC+16b^2B)}{6b}}{8b}+\frac{x^3(a+bx^2)^{5/2}(2bC-aD)}{8b}}{2b}+\frac{Dx^5(a+bx^2)^{5/2}}{10b}$$

$\downarrow$ 211

$$\frac{\frac{\frac{\left(96Ab^3-a\left(3a^2D-6abC+16b^2B\right)\right)\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x(a+bx^2)^{3/2}\right)}{6b}+\frac{x(a+bx^2)^{5/2}(3a^2D-6abC+16b^2B)}{6b}}{8b}+\frac{x^3(a+bx^2)^{5/2}(2bC}{8b}}{2b}$$

$$\frac{Dx^5(a+bx^2)^{5/2}}{10b}$$

$\downarrow$ 224

$$\frac{\frac{\frac{\left(96Ab^3-a\left(3a^2D-6abC+16b^2B\right)\right)\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x(a+bx^2)^{3/2}\right)}{6b}+\frac{x(a+bx^2)^{5/2}(3a^2D-6abC+16b^2B)}{6b}}{8b}+x^3(a+bx^2}{2b}$$

$$\frac{Dx^5(a+bx^2)^{5/2}}{10b}$$

$\downarrow$ 219

$$\frac{\frac{\frac{\left(\frac{3}{4}a\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x(a+bx^2)^{3/2}\right)\left(96Ab^3-a\left(3a^2D-6abC+16b^2B\right)\right)}{6b}+\frac{x(a+bx^2)^{5/2}(3a^2D-6abC+16b^2B)}{6b}}{8b}+x^3(a+bx^2)}{2b}$$

$$\frac{Dx^5(a+bx^2)^{5/2}}{10b}$$

input
```
Int[(a + b*x^2)^(3/2)*(A + B*x^2 + C*x^4 + D*x^6),x]
```

output
```
(D*x^5*(a + b*x^2)^(5/2))/(10*b) + (((2*b*C - a*D)*x^3*(a + b*x^2)^(5/2))/
(8*b) + (((16*b^2*B - 6*a*b*C + 3*a^2*D)*x*(a + b*x^2)^(5/2))/(6*b) + ((96
*A*b^3 - a*(16*b^2*B - 6*a*b*C + 3*a^2*D))*((x*(a + b*x^2)^(3/2))/4 + (3*a
*((x*Sqrt[a + b*x^2])/2 + (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt
[b])))/4))/(6*b))/(8*b))/(2*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1473
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[c^p*x^(4*p - 1)*((d + e*x^2)^(q + 1)/(e*(4*p + 2*q + 1)))
, x] + Simp[1/(e*(4*p + 2*q + 1))   Int[(d + e*x^2)^q*ExpandToSum[e*(4*p +
2*q + 1)*(a + b*x^2 + c*x^4)^p - d*c^p*(4*p - 1)*x^(4*p - 2) - e*c^p*(4*p +
 2*q + 1)*x^(4*p), x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] &&  !LtQ[q, -1]
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.71

method	result
pseudoelliptic	$\frac{\frac{3\left(b^3A-\frac{1}{6}a\,b^2B+\frac{1}{16}a^2bC-\frac{1}{32}a^3D\right)a^2\,\mathrm{arctanh}\left(\frac{\sqrt{b\,x^2+a}}{x\sqrt{b}}\right)}{8}+\frac{5\sqrt{b\,x^2+a}\,x\left(a\left(\frac{11}{50}Dx^6+\frac{3}{10}C\,x^4+\frac{7}{15}x^2B+A\right)b^{\frac{7}{2}}+\frac{2\left(\frac{2}{5}Dx^6+\frac{1}{2}C\,x^4+\frac{2}{3}\right.}{5}\right.}{}}{b^{\frac{7}{2}}}$
default	$A\left(\frac{x(b\,x^2+a)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)}{4}\right)+C\left(\frac{x^3(b\,x^2+a)^{\frac{5}{2}}}{8b}-\frac{3a\left(\frac{x(b\,x^2+a)^{\frac{5}{2}}}{6b}-\frac{a\left(\frac{x(b\,x^2+a}{4}\right.}{}\right.}{}\right.$

input
```
int((b*x^2+a)^(3/2)*(D*x^6+C*x^4+B*x^2+A),x,method=_RETURNVERBOSE)
```

output
```
5/8/b^(7/2)*(3/5*(b^3*A-1/6*a*b^2*B+1/16*a^2*b*C-1/32*a^3*D)*a^2*arctanh((
b*x^2+a)^(1/2)/x/b^(1/2))+(b*x^2+a)^(1/2)*x*(a*(11/50*D*x^6+3/10*C*x^4+7/1
5*x^2*B+A)*b^(7/2)+2/5*(2/5*D*x^6+1/2*C*x^4+2/3*x^2*B+A)*x^2*b^(9/2)+3/160
*(4/3*(2/5*D*x^4+C*x^2+4*B)*b^(5/2)+(2*(-1/3*D*x^2-C)*b^(3/2)+D*a*b^(1/2))
*a)*a^2))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 413, normalized size of antiderivative = 1.71

$$\int \left(a+bx^2\right)^{3/2} \left(A+Bx^2+Cx^4+Dx^6\right) dx = \left[-\frac{15\,(3\,Da^5-6\,Ca^4b+16\,Ba^3b^2-96\,Aa^2b^3)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\,(38\right.$$

input
```
integrate((b*x^2+a)^(3/2)*(D*x^6+C*x^4+B*x^2+A),x, algorithm="fricas")
```

output
```
[-1/7680*(15*(3*D*a^5 - 6*C*a^4*b + 16*B*a^3*b^2 - 96*A*a^2*b^3)*sqrt(b)*l
og(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(384*D*b^5*x^9 + 48*(11
*D*a*b^4 + 10*C*b^5)*x^7 + 8*(3*D*a^2*b^3 + 90*C*a*b^4 + 80*B*b^5)*x^5 - 1
0*(3*D*a^3*b^2 - 6*C*a^2*b^3 - 112*B*a*b^4 - 96*A*b^5)*x^3 + 15*(3*D*a^4*b
 - 6*C*a^3*b^2 + 16*B*a^2*b^3 + 160*A*a*b^4)*x)*sqrt(b*x^2 + a))/b^4, 1/38
40*(15*(3*D*a^5 - 6*C*a^4*b + 16*B*a^3*b^2 - 96*A*a^2*b^3)*sqrt(-b)*arctan
(sqrt(-b)*x/sqrt(b*x^2 + a)) + (384*D*b^5*x^9 + 48*(11*D*a*b^4 + 10*C*b^5)
*x^7 + 8*(3*D*a^2*b^3 + 90*C*a*b^4 + 80*B*b^5)*x^5 - 10*(3*D*a^3*b^2 - 6*C
*a^2*b^3 - 112*B*a*b^4 - 96*A*b^5)*x^3 + 15*(3*D*a^4*b - 6*C*a^3*b^2 + 16*
B*a^2*b^3 + 160*A*a*b^4)*x)*sqrt(b*x^2 + a))/b^4]
```

Sympy [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.64

$$\int \left(a+bx^2\right)^{3/2}\left(A+Bx^2+Cx^4+Dx^6\right)\,dx=\begin{cases}\sqrt{a+bx^2}\left(\frac{Dbx^9}{10}+\frac{x^7\left(Cb^2+\frac{11Dab}{10}\right)}{8b}+\frac{x^5\left(Bb^2+2Cab+Da^2-\frac{7a\left(Cb^2+\frac{11Dab}{10}\right)}{8b}\right)}{6b}+\frac{x^3\left(Ab^2+2Bab+Ca^2-\frac{5a\left(Bb\cdots\right.}{\cdots}\right.}{\cdots}\right. \\ a^{\frac{3}{2}}\left(Ax+\frac{Bx^3}{3}+\frac{Cx^5}{5}+\frac{Dx^7}{7}\right)\end{cases}$$

input
```
integrate((b*x**2+a)**(3/2)*(D*x**6+C*x**4+B*x**2+A),x)
```

output
```
Piecewise((sqrt(a + b*x**2)*(D*b*x**9/10 + x**7*(C*b**2 + 11*D*a*b/10)/(8*
b) + x**5*(B*b**2 + 2*C*a*b + D*a**2 - 7*a*(C*b**2 + 11*D*a*b/10)/(8*b))/(
6*b) + x**3*(A*b**2 + 2*B*a*b + C*a**2 - 5*a*(B*b**2 + 2*C*a*b + D*a**2 -
7*a*(C*b**2 + 11*D*a*b/10)/(8*b))/(6*b))/(4*b) + x*(2*A*a*b + B*a**2 - 3*a
*(A*b**2 + 2*B*a*b + C*a**2 - 5*a*(B*b**2 + 2*C*a*b + D*a**2 - 7*a*(C*b**2
 + 11*D*a*b/10)/(8*b))/(6*b))/(4*b))/(2*b)) + (A*a**2 - a*(2*A*a*b + B*a**
2 - 3*a*(A*b**2 + 2*B*a*b + C*a**2 - 5*a*(B*b**2 + 2*C*a*b + D*a**2 - 7*a*
(C*b**2 + 11*D*a*b/10)/(8*b))/(6*b))/(4*b))/(2*b))*Piecewise((log(2*sqrt(b
)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), Tr
ue)), Ne(b, 0)), (a**(3/2)*(A*x + B*x**3/3 + C*x**5/5 + D*x**7/7), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.32

$$\int \left(a+bx^2\right)^{3/2}\left(A+Bx^2+Cx^4+Dx^6\right)\,dx = \frac{(bx^2+a)^{\frac{5}{2}}Dx^5}{10\,b} - \frac{(bx^2+a)^{\frac{5}{2}}Dax^3}{16\,b^2} + \frac{(bx^2+a)^{\frac{5}{2}}Cx^3}{8\,b} + \frac{1}{4}\,(bx^2+a)^{\frac{3}{2}}Ax + \frac{3}{8}\,\sqrt{bx^2+a}Aax + \frac{(bx^2+a)^{\frac{5}{2}}Da^2x}{32\,b^3} - \frac{(bx^2+a)^{\frac{3}{2}}Da^3x}{128\,b^3} - \frac{3\,\sqrt{bx^2+a}Da^4x}{256\,b^3} - \frac{(bx^2+a)^{\frac{5}{2}}Cax}{16\,b^2} + \frac{(bx^2+a)^{\frac{3}{2}}Ca^2x}{64\,b^2} + \frac{3\,\sqrt{bx^2+a}Ca^3x}{128\,b^2} + \frac{(bx^2+a)^{\frac{5}{2}}Bx}{6\,b} - \frac{(bx^2+a)^{\frac{3}{2}}Bax}{24\,b} - \frac{\sqrt{bx^2+a}Ba^2x}{16\,b} - \frac{3\,Da^5\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{256\,b^{\frac{7}{2}}} + \frac{3\,Ca^4\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\,b^{\frac{5}{2}}} - \frac{Ba^3\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{3}{2}}} + \frac{3\,Aa^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{b}}$$

input

```
integrate((b*x^2+a)^(3/2)*(D*x^6+C*x^4+B*x^2+A),x, algorithm="maxima")
```

output

```
1/10*(b*x^2 + a)^(5/2)*D*x^5/b - 1/16*(b*x^2 + a)^(5/2)*D*a*x^3/b^2 + 1/8*
(b*x^2 + a)^(5/2)*C*x^3/b + 1/4*(b*x^2 + a)^(3/2)*A*x + 3/8*sqrt(b*x^2 + a
)*A*a*x + 1/32*(b*x^2 + a)^(5/2)*D*a^2*x/b^3 - 1/128*(b*x^2 + a)^(3/2)*D*a
^3*x/b^3 - 3/256*sqrt(b*x^2 + a)*D*a^4*x/b^3 - 1/16*(b*x^2 + a)^(5/2)*C*a*
x/b^2 + 1/64*(b*x^2 + a)^(3/2)*C*a^2*x/b^2 + 3/128*sqrt(b*x^2 + a)*C*a^3*x
/b^2 + 1/6*(b*x^2 + a)^(5/2)*B*x/b - 1/24*(b*x^2 + a)^(3/2)*B*a*x/b - 1/16
*sqrt(b*x^2 + a)*B*a^2*x/b - 3/256*D*a^5*arcsinh(b*x/sqrt(a*b))/b^(7/2) +
3/128*C*a^4*arcsinh(b*x/sqrt(a*b))/b^(5/2) - 1/16*B*a^3*arcsinh(b*x/sqrt(a
*b))/b^(3/2) + 3/8*A*a^2*arcsinh(b*x/sqrt(a*b))/sqrt(b)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.90

$$\int \left(a+bx^2\right)^{3/2}\left(A+Bx^2+Cx^4+Dx^6\right)\,dx=\frac{1}{3840}\left(2\left(4\left(6\left(8\,Dbx^2+\frac{11\,Dab^8+10\,Cb^9}{b^8}\right)x^2+\frac{3\,Da^2b^7+90\,Cab^8+80\,Bb^9}{b^8}\right)x^2-\frac{5(3D\ldots}{}\right.\right.$$
$$+\frac{\left(3\,Da^5-6\,Ca^4b+16\,Ba^3b^2-96\,Aa^2b^3\right)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{256\,b^{\frac{7}{2}}}$$

input
```
integrate((b*x^2+a)^(3/2)*(D*x^6+C*x^4+B*x^2+A),x, algorithm="giac")
```

output
```
1/3840*(2*(4*(6*(8*D*b*x^2 + (11*D*a*b^8 + 10*C*b^9)/b^8)*x^2 + (3*D*a^2*b
^7 + 90*C*a*b^8 + 80*B*b^9)/b^8)*x^2 - 5*(3*D*a^3*b^6 - 6*C*a^2*b^7 - 112*
B*a*b^8 - 96*A*b^9)/b^8)*x^2 + 15*(3*D*a^4*b^5 - 6*C*a^3*b^6 + 16*B*a^2*b^
7 + 160*A*a*b^8)/b^8)*sqrt(b*x^2 + a)*x + 1/256*(3*D*a^5 - 6*C*a^4*b + 16*
B*a^3*b^2 - 96*A*a^2*b^3)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(7/2)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^2\right)^{3/2}\left(A+Bx^2+Cx^4+Dx^6\right)\,dx=\int \left(b\,x^2+a\right)^{3/2}\left(A+B\,x^2+C\,x^4+x^6\,D\right)\,dx$$

input
```
int((a + b*x^2)^(3/2)*(A + B*x^2 + C*x^4 + x^6*D),x)
```

output
```
int((a + b*x^2)^(3/2)*(A + B*x^2 + C*x^4 + x^6*D), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.26

$$\int \left(a+bx^2\right)^{3/2} \left(A+Bx^2+Cx^4+Dx^6\right) dx = \frac{45\sqrt{b\,x^2+a}\,a^4bdx-90\sqrt{b\,x^2+a}\,a^3b^2cx-30\sqrt{b\,x^2+a}\,a^3b^2d\,x^3+2640\sqrt{b\,x^2+a}\,a^2b^4x+60}{}$$

input
```
int((b*x^2+a)^(3/2)*(D*x^6+C*x^4+B*x^2+A),x)
```

output
```
(45*sqrt(a + b*x**2)*a**4*b*d*x - 90*sqrt(a + b*x**2)*a**3*b**2*c*x - 30*s
qrt(a + b*x**2)*a**3*b**2*d*x**3 + 2640*sqrt(a + b*x**2)*a**2*b**4*x + 60*
sqrt(a + b*x**2)*a**2*b**3*c*x**3 + 24*sqrt(a + b*x**2)*a**2*b**3*d*x**5 +
 2080*sqrt(a + b*x**2)*a*b**5*x**3 + 720*sqrt(a + b*x**2)*a*b**4*c*x**5 +
528*sqrt(a + b*x**2)*a*b**4*d*x**7 + 640*sqrt(a + b*x**2)*b**6*x**5 + 480*
sqrt(a + b*x**2)*b**5*c*x**7 + 384*sqrt(a + b*x**2)*b**5*d*x**9 - 45*sqrt(
b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**5*d + 90*sqrt(b)*log((sq
rt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**4*b*c + 1200*sqrt(b)*log((sqrt(a +
 b*x**2) + sqrt(b)*x)/sqrt(a))*a**3*b**3)/(3840*b**4)
```

3.134 $\int \sqrt{a+bx^2}(A+Bx^2+Cx^4+Dx^6)\,dx$

Optimal result

Integrand size = 29, antiderivative size = 191

$$\begin{aligned}\int \sqrt{a+bx^2}\left(A+Bx^2+Cx^4+Dx^6\right)\,dx &= \frac{1}{128}\left(64A-\frac{a(16b^2B-8abC+5a^2D)}{b^3}\right)x\sqrt{a+bx^2} \\ &\quad+\frac{(16b^2B-8abC+5a^2D)\,x(a+bx^2)^{3/2}}{64b^3}+\frac{(8bC-5aD)x^3(a+bx^2)^{3/2}}{48b^2} \\ &\quad+\frac{Dx^5(a+bx^2)^{3/2}}{8b}+\frac{a(64Ab^3-a(16b^2B-8abC+5a^2D))\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{7/2}}\end{aligned}$$

output
```
1/128*(64*A-a*(16*B*b^2-8*C*a*b+5*D*a^2)/b^3)*x*(b*x^2+a)^(1/2)+1/64*(16*B
*b^2-8*C*a*b+5*D*a^2)*x*(b*x^2+a)^(3/2)/b^3+1/48*(8*C*b-5*D*a)*x^3*(b*x^2+
a)^(3/2)/b^2+1/8*D*x^5*(b*x^2+a)^(3/2)/b+1/128*a*(64*A*b^3-a*(16*B*b^2-8*C
*a*b+5*D*a^2))*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(7/2)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.80

$$\int \sqrt{a+bx^2}\left(A+Bx^2+Cx^4+Dx^6\right)\,dx$$
$$=\frac{\sqrt{b}x\sqrt{a+bx^2}(192Ab^3+15a^3D-2a^2b(12C+5Dx^2)+8ab^2(6B+2Cx^2+Dx^4)+16b^3x^2(6B+4Cx^2}{384b^{7/2}}$$

input
```
Integrate[Sqrt[a + b*x^2]*(A + B*x^2 + C*x^4 + D*x^6),x]
```

output
```
(Sqrt[b]*x*Sqrt[a + b*x^2]*(192*A*b^3 + 15*a^3*D - 2*a^2*b*(12*C + 5*D*x^2
) + 8*a*b^2*(6*B + 2*C*x^2 + D*x^4) + 16*b^3*x^2*(6*B + 4*C*x^2 + 3*D*x^4)
) + 3*a*(-64*A*b^3 + a*(16*b^2*B - 8*a*b*C + 5*a^2*D))*Log[-(Sqrt[b]*x) +
Sqrt[a + b*x^2]])/(384*b^(7/2))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2346, 1473, 27, 299, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx^2}\left(A+Bx^2+Cx^4+Dx^6\right)\,dx$$

$$\Big\downarrow \text{2346}$$

$$\frac{\int \sqrt{bx^2+a}\left((8bC-5aD)x^4+8bBx^2+8Ab\right)dx}{8b}+\frac{Dx^5\left(a+bx^2\right)^{3/2}}{8b}$$

$$\Big\downarrow \text{1473}$$

$$\frac{\frac{\int 3\sqrt{bx^2+a}\left(16Ab^2+\left(5Da^2-8bCa+16b^2B\right)x^2\right)dx}{6b}+\frac{x^3\left(a+bx^2\right)^{3/2}(8bC-5aD)}{6b}}{8b}+\frac{Dx^5\left(a+bx^2\right)^{3/2}}{8b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \sqrt{bx^2+a}(16Ab^2+(5Da^2-8bCa+16b^2B)x^2)dx}{2b}+\frac{x^3(a+bx^2)^{3/2}(8bC-5aD)}{6b}}{8b}+\frac{Dx^5(a+bx^2)^{3/2}}{8b}$$

$\downarrow$ 299

$$\frac{\frac{\frac{\left(64Ab^3-a\left(5a^2D-8abC+16b^2B\right)\right)\int \sqrt{bx^2+a}dx}{4b}+\frac{x\left(a+bx^2\right)^{3/2}\left(5a^2D-8abC+16b^2B\right)}{4b}}{2b}+\frac{x^3(a+bx^2)^{3/2}(8bC-5aD)}{6b}}{8b}+\frac{Dx^5(a+bx^2)^{3/2}}{8b}$$

$\downarrow$ 211

$$\frac{\frac{\frac{\left(64Ab^3-a\left(5a^2D-8abC+16b^2B\right)\right)\left(\frac{1}{2}a\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)}{4b}+\frac{x\left(a+bx^2\right)^{3/2}\left(5a^2D-8abC+16b^2B\right)}{4b}}{2b}+\frac{x^3(a+bx^2)^{3/2}(8bC-5aD)}{6b}}{8b}+\frac{Dx^5(a+bx^2)^{3/2}}{8b}$$

$\downarrow$ 224

$$\frac{\frac{\frac{\left(64Ab^3-a\left(5a^2D-8abC+16b^2B\right)\right)\left(\frac{1}{2}a\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)}{4b}+\frac{x\left(a+bx^2\right)^{3/2}\left(5a^2D-8abC+16b^2B\right)}{4b}}{2b}+\frac{x^3(a+bx^2)^{3/2}(8bC-5aD)}{6b}}{8b}+\frac{Dx^5(a+bx^2)^{3/2}}{8b}$$

$\downarrow$ 219

$$\frac{\frac{\frac{\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)\left(64Ab^3-a\left(5a^2D-8abC+16b^2B\right)\right)}{4b}+\frac{x\left(a+bx^2\right)^{3/2}\left(5a^2D-8abC+16b^2B\right)}{4b}}{2b}+\frac{x^3(a+bx^2)^{3/2}(8bC-5aD)}{6b}}{8b}+\frac{Dx^5(a+bx^2)^{3/2}}{8b}$$

input
```
Int[Sqrt[a + b*x^2]*(A + B*x^2 + C*x^4 + D*x^6),x]
```

output
```
(D*x^5*(a + b*x^2)^(3/2))/(8*b) + (((8*b*C - 5*a*D)*x^3*(a + b*x^2)^(3/2))
/(6*b) + (((16*b^2*B - 8*a*b*C + 5*a^2*D)*x*(a + b*x^2)^(3/2))/(4*b) + ((6
4*A*b^3 - a*(16*b^2*B - 8*a*b*C + 5*a^2*D))*((x*Sqrt[a + b*x^2])/2 + (a*Ar
cTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b])))/(4*b))/(2*b))/(8*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1473
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[c^p*x^(4*p - 1)*((d + e*x^2)^(q + 1)/(e*(4*p + 2*q + 1)))
, x] + Simp[1/(e*(4*p + 2*q + 1))   Int[(d + e*x^2)^q*ExpandToSum[e*(4*p +
2*q + 1)*(a + b*x^2 + c*x^4)^p - d*c^p*(4*p - 1)*x^(4*p - 2) - e*c^p*(4*p +
 2*q + 1)*x^(4*p), x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] &&  !LtQ[q, -1]
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.71

method	result
pseudoelliptic	$\frac{a\left(b^3A-\frac{1}{4}ab^2B+\frac{1}{8}a^2bC-\frac{5}{64}a^3D\right)\operatorname{arctanh}\left(\frac{\sqrt{bx^2+a}}{x\sqrt{b}}\right)+\sqrt{bx^2+a}\,x\left(\left(\frac{1}{4}Dx^6+\frac{1}{3}Cx^4+\frac{1}{2}x^2B+A\right)b^{\frac{7}{2}}+\frac{5\left(\left(\frac{8}{15}Dx^4+\frac{16}{15}Cx^2\cdots\right.\right.}{\cdots}\right.}{2b^{\frac{7}{2}}}$
default	$A\left(\frac{x\sqrt{bx^2+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{bx^2+a}\right)}{2\sqrt{b}}\right)+C\left(\frac{x^3\left(bx^2+a\right)^{\frac{3}{2}}}{6b}-\frac{a\left(\frac{x\left(bx^2+a\right)^{\frac{3}{2}}}{4b}-\frac{a\left(\frac{x\sqrt{bx^2+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{bx^2+\cdots}\right)}{2\sqrt{b}}\right)}{4b}\cdots\right)}{2b}\cdots\right.$

input
```
int((b*x^2+a)^(1/2)*(D*x^6+C*x^4+B*x^2+A),x,method=_RETURNVERBOSE)
```

output
```
1/2*(a*(b^3*A-1/4*a*b^2*B+1/8*a^2*b*C-5/64*a^3*D)*arctanh((b*x^2+a)^(1/2)/
x/b^(1/2))+(b*x^2+a)^(1/2)*x*((1/4*D*x^6+1/3*C*x^4+1/2*x^2*B+A)*b^(7/2)+5/
64*((8/15*D*x^4+16/15*C*x^2+16/5*B)*b^(5/2)+a*((-2/3*D*x^2-8/5*C)*b^(3/2)+
D*a*b^(1/2)))*a))/b^(7/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.71

$$\int \sqrt{a+bx^2}\left(A+Bx^2+Cx^4+Dx^6\right)\,dx$$

$$=\left[-\frac{3\left(5\,Da^4-8\,Ca^3b+16\,Ba^2b^2-64\,Aab^3\right)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\left(48\,Db^4x^7+8\left(\right.\right.}{}\right.$$

input
```
integrate((b*x^2+a)^(1/2)*(D*x^6+C*x^4+B*x^2+A),x, algorithm="fricas")
```

output
```
[-1/768*(3*(5*D*a^4 - 8*C*a^3*b + 16*B*a^2*b^2 - 64*A*a*b^3)*sqrt(b)*log(-
2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(48*D*b^4*x^7 + 8*(D*a*b^3
+ 8*C*b^4)*x^5 - 2*(5*D*a^2*b^2 - 8*C*a*b^3 - 48*B*b^4)*x^3 + 3*(5*D*a^3*b
 - 8*C*a^2*b^2 + 16*B*a*b^3 + 64*A*b^4)*x)*sqrt(b*x^2 + a))/b^4, 1/384*(3*
(5*D*a^4 - 8*C*a^3*b + 16*B*a^2*b^2 - 64*A*a*b^3)*sqrt(-b)*arctan(sqrt(-b)
*x/sqrt(b*x^2 + a)) + (48*D*b^4*x^7 + 8*(D*a*b^3 + 8*C*b^4)*x^5 - 2*(5*D*a
^2*b^2 - 8*C*a*b^3 - 48*B*b^4)*x^3 + 3*(5*D*a^3*b - 8*C*a^2*b^2 + 16*B*a*b
^3 + 64*A*b^4)*x)*sqrt(b*x^2 + a))/b^4]
```

Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.13

$$\int \sqrt{a+bx^2}\left(A+Bx^2+Cx^4+Dx^6\right)\,dx$$

$$=\begin{cases}\sqrt{a+bx^2}\left(\frac{Dx^7}{8}+\frac{x^5\left(Cb+\frac{Da}{8}\right)}{6b}+\frac{x^3\left(Bb+Ca-\frac{5a\left(Cb+\frac{Da}{8}\right)}{6b}\right)}{4b}+\frac{x\left(Ab+Ba-\frac{3a\left(Bb+Ca-\frac{5a\left(Cb+\frac{Da}{8}\right)}{6b}\right)}{4b}\right)}{2b}\right)+\left(Aa-\frac{a\left(\right.}{}\right. \\ \sqrt{a}\left(Ax+\frac{Bx^3}{3}+\frac{Cx^5}{5}+\frac{Dx^7}{7}\right)\end{cases}$$

input
```
integrate((b*x**2+a)**(1/2)*(D*x**6+C*x**4+B*x**2+A),x)
```

output
```
Piecewise((sqrt(a + b*x**2)*(D*x**7/8 + x**5*(C*b + D*a/8)/(6*b) + x**3*(B
*b + C*a - 5*a*(C*b + D*a/8)/(6*b))/(4*b) + x*(A*b + B*a - 3*a*(B*b + C*a
- 5*a*(C*b + D*a/8)/(6*b))/(4*b))/(2*b)) + (A*a - a*(A*b + B*a - 3*a*(B*b
+ C*a - 5*a*(C*b + D*a/8)/(6*b))/(4*b))/(2*b))*Piecewise((log(2*sqrt(b)*sq
rt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True))
, Ne(b, 0)), (sqrt(a)*(A*x + B*x**3/3 + C*x**5/5 + D*x**7/7), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.29

$$\int \sqrt{a+bx^2}\left(A+Bx^2+Cx^4+Dx^6\right)\,dx = \frac{(bx^2+a)^{\frac{3}{2}}Dx^5}{8\,b} - \frac{5\,(bx^2+a)^{\frac{3}{2}}Dax^3}{48\,b^2} + \frac{(bx^2+a)^{\frac{3}{2}}Cx^3}{6\,b} + \frac{1}{2}\sqrt{bx^2+a}Ax + \frac{5\,(bx^2+a)^{\frac{3}{2}}Da^2x}{64\,b^3} - \frac{5\sqrt{bx^2+a}Da^3x}{128\,b^3} - \frac{(bx^2+a)^{\frac{3}{2}}Cax}{8\,b^2} + \frac{\sqrt{bx^2+a}Ca^2x}{16\,b^2} + \frac{(bx^2+a)^{\frac{3}{2}}Bx}{4\,b} - \frac{\sqrt{bx^2+a}Bax}{8\,b} - \frac{5\,Da^4\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\,b^{\frac{7}{2}}} + \frac{Ca^3\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{5}{2}}} - \frac{Ba^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{3}{2}}} + \frac{Aa\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{b}}$$

input
```
integrate((b*x^2+a)^(1/2)*(D*x^6+C*x^4+B*x^2+A),x, algorithm="maxima")
```

output
```
1/8*(b*x^2 + a)^(3/2)*D*x^5/b - 5/48*(b*x^2 + a)^(3/2)*D*a*x^3/b^2 + 1/6*(
b*x^2 + a)^(3/2)*C*x^3/b + 1/2*sqrt(b*x^2 + a)*A*x + 5/64*(b*x^2 + a)^(3/2
)*D*a^2*x/b^3 - 5/128*sqrt(b*x^2 + a)*D*a^3*x/b^3 - 1/8*(b*x^2 + a)^(3/2)*
C*a*x/b^2 + 1/16*sqrt(b*x^2 + a)*C*a^2*x/b^2 + 1/4*(b*x^2 + a)^(3/2)*B*x/b
 - 1/8*sqrt(b*x^2 + a)*B*a*x/b - 5/128*D*a^4*arcsinh(b*x/sqrt(a*b))/b^(7/2
) + 1/16*C*a^3*arcsinh(b*x/sqrt(a*b))/b^(5/2) - 1/8*B*a^2*arcsinh(b*x/sqrt
(a*b))/b^(3/2) + 1/2*A*a*arcsinh(b*x/sqrt(a*b))/sqrt(b)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.87

$$\int \sqrt{a+bx^2}\left(A+Bx^2+Cx^4+Dx^6\right)\,dx$$
$$=\frac{1}{384}\left(2\left(4\left(6\,Dx^2+\frac{Dab^5+8\,Cb^6}{b^6}\right)x^2-\frac{5\,Da^2b^4-8\,Cab^5-48\,Bb^6}{b^6}\right)x^2+\frac{3\,(5\,Da^3b^3-8\,Ca^2b^4+16}{b^6}\right.$$
$$+\frac{(5\,Da^4-8\,Ca^3b+16\,Ba^2b^2-64\,Aab^3)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{128\,b^{\frac{7}{2}}}$$

input
```
integrate((b*x^2+a)^(1/2)*(D*x^6+C*x^4+B*x^2+A),x, algorithm="giac")
```

output
```
1/384*(2*(4*(6*D*x^2 + (D*a*b^5 + 8*C*b^6)/b^6)*x^2 - (5*D*a^2*b^4 - 8*C*a
*b^5 - 48*B*b^6)/b^6)*x^2 + 3*(5*D*a^3*b^3 - 8*C*a^2*b^4 + 16*B*a*b^5 + 64
*A*b^6)/b^6)*sqrt(b*x^2 + a)*x + 1/128*(5*D*a^4 - 8*C*a^3*b + 16*B*a^2*b^2
 - 64*A*a*b^3)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(7/2)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+bx^2}\left(A+Bx^2+Cx^4+Dx^6\right)\,dx=\int \sqrt{b\,x^2+a}\,\left(A+B\,x^2+C\,x^4+x^6\,D\right)\,dx$$

input
```
int((a + b*x^2)^(1/2)*(A + B*x^2 + C*x^4 + x^6*D),x)
```

output
```
int((a + b*x^2)^(1/2)*(A + B*x^2 + C*x^4 + x^6*D), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.28

$$\int \sqrt{a+bx^2}\left(A+Bx^2+Cx^4+Dx^6\right)\,dx$$

$$= \frac{15\sqrt{b\,x^2+a}\,a^3bdx-24\sqrt{b\,x^2+a}\,a^2b^2cx-10\sqrt{b\,x^2+a}\,a^2b^2d\,x^3+240\sqrt{b\,x^2+a}\,a\,b^4x+16\sqrt{b\,x^2+a}\,a\,b}{}$$

input
```
int((b*x^2+a)^(1/2)*(D*x^6+C*x^4+B*x^2+A),x)
```

output
```
(15*sqrt(a + b*x**2)*a**3*b*d*x - 24*sqrt(a + b*x**2)*a**2*b**2*c*x - 10*s
qrt(a + b*x**2)*a**2*b**2*d*x**3 + 240*sqrt(a + b*x**2)*a*b**4*x + 16*sqrt
(a + b*x**2)*a*b**3*c*x**3 + 8*sqrt(a + b*x**2)*a*b**3*d*x**5 + 96*sqrt(a
+ b*x**2)*b**5*x**3 + 64*sqrt(a + b*x**2)*b**4*c*x**5 + 48*sqrt(a + b*x**2
)*b**4*d*x**7 - 15*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**
4*d + 24*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**3*b*c + 14
4*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2*b**3)/(384*b**4
)
```

3.135 $\int \frac{A+Bx^2+Cx^4+Dx^6}{\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 146

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{\sqrt{a+bx^2}}\,dx = \frac{(8b^2B-6abC+5a^2D)\,x\sqrt{a+bx^2}}{16b^3} + \frac{(6bC-5aD)x^3\sqrt{a+bx^2}}{24b^2} + \frac{Dx^5\sqrt{a+bx^2}}{6b} + \frac{(16Ab^3-a(8b^2B-6abC+5a^2D))\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{7/2}}$$

output
```
1/16*(8*B*b^2-6*C*a*b+5*D*a^2)*x*(b*x^2+a)^(1/2)/b^3+1/24*(6*C*b-5*D*a)*x^
3*(b*x^2+a)^(1/2)/b^2+1/6*D*x^5*(b*x^2+a)^(1/2)/b+1/16*(16*A*b^3-a*(8*B*b^
2-6*C*a*b+5*D*a^2))*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(7/2)
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.88

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{\sqrt{a+bx^2}}\,dx = \frac{x\sqrt{a+bx^2}(24b^2B-18abC+15a^2D+12b^2Cx^2-10abDx^2+8b^2Dx^4)}{48b^3} + \frac{(16Ab^3-8ab^2B+6a^2bC-5a^3D)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{-\sqrt{a}+\sqrt{a+bx^2}}\right)}{8b^{7/2}}$$

input
```
Integrate[(A + B*x^2 + C*x^4 + D*x^6)/Sqrt[a + b*x^2],x]
```

output
```
(x*Sqrt[a + b*x^2]*(24*b^2*B - 18*a*b*C + 15*a^2*D + 12*b^2*C*x^2 - 10*a*b
*D*x^2 + 8*b^2*D*x^4))/(48*b^3) + ((16*A*b^3 - 8*a*b^2*B + 6*a^2*b*C - 5*a
^3*D)*ArcTanh[(Sqrt[b]*x)/(-Sqrt[a] + Sqrt[a + b*x^2])])/(8*b^(7/2))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2346, 1473, 27, 299, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{\sqrt{a+bx^2}}\,dx$$

$$\Big\downarrow \text{2346}$$

$$\frac{\int \frac{(6bC-5aD)x^4+6bBx^2+6Ab}{\sqrt{bx^2+a}}dx}{6b}+\frac{Dx^5\sqrt{a+bx^2}}{6b}$$

$$\Big\downarrow \text{1473}$$

$$\frac{\frac{\int \frac{3\left(8Ab^2+\left(5Da^2-6bCa+8b^2B\right)x^2\right)}{\sqrt{bx^2+a}}dx}{4b}+\frac{x^3\sqrt{a+bx^2}(6bC-5aD)}{4b}}{6b}+\frac{Dx^5\sqrt{a+bx^2}}{6b}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{3\int \frac{8Ab^2+\left(5Da^2-6bCa+8b^2B\right)x^2}{\sqrt{bx^2+a}}dx}{4b}+\frac{x^3\sqrt{a+bx^2}(6bC-5aD)}{4b}}{6b}+\frac{Dx^5\sqrt{a+bx^2}}{6b}$$

$$\Big\downarrow \ 299$$

$$\frac{\frac{3\left(\frac{\left(16Ab^3-a\left(5a^2D-6abC+8b^2B\right)\right)\int \frac{1}{\sqrt{bx^2+a}}dx}{2b}+\frac{x\sqrt{a+bx^2}\left(5a^2D-6abC+8b^2B\right)}{2b}\right)}{4b}+\frac{x^3\sqrt{a+bx^2}(6bC-5aD)}{4b}}{6b}+\frac{Dx^5\sqrt{a+bx^2}}{6b}$$

$$\Big\downarrow \ 224$$

$$\frac{\frac{3\left(\frac{\left(16Ab^3-a\left(5a^2D-6abC+8b^2B\right)\right)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2b}+\frac{x\sqrt{a+bx^2}\left(5a^2D-6abC+8b^2B\right)}{2b}\right)}{4b}+\frac{x^3\sqrt{a+bx^2}(6bC-5aD)}{4b}}{6b}+\frac{Dx^5\sqrt{a+bx^2}}{6b}$$

$$\Big\downarrow \ 219$$

$$\frac{\frac{3\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(16Ab^3-a\left(5a^2D-6abC+8b^2B\right)\right)}{2b^{3/2}}+\frac{x\sqrt{a+bx^2}\left(5a^2D-6abC+8b^2B\right)}{2b}\right)}{4b}+\frac{x^3\sqrt{a+bx^2}(6bC-5aD)}{4b}}{6b}+\frac{Dx^5\sqrt{a+bx^2}}{6b}$$

input
```
Int[(A + B*x^2 + C*x^4 + D*x^6)/Sqrt[a + b*x^2],x]
```

output
```
(D*x^5*Sqrt[a + b*x^2])/(6*b) + (((6*b*C - 5*a*D)*x^3*Sqrt[a + b*x^2])/(4*
b) + (3*(((8*b^2*B - 6*a*b*C + 5*a^2*D)*x*Sqrt[a + b*x^2])/(2*b) + ((16*A*
b^3 - a*(8*b^2*B - 6*a*b*C + 5*a^2*D))*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]
])/(2*b^(3/2))))/(4*b))/(6*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1473
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[c^p*x^(4*p - 1)*((d + e*x^2)^(q + 1)/(e*(4*p + 2*q + 1)))
, x] + Simp[1/(e*(4*p + 2*q + 1))   Int[(d + e*x^2)^q*ExpandToSum[e*(4*p +
2*q + 1)*(a + b*x^2 + c*x^4)^p - d*c^p*(4*p - 1)*x^(4*p - 2) - e*c^p*(4*p +
 2*q + 1)*x^(4*p), x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] &&  !LtQ[q, -1]
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.73

method	result
pseudoelliptic	$\frac{\left(b^3A-\frac{1}{2}a\,b^2B+\frac{3}{8}a^2bC-\frac{5}{16}a^3D\right)\operatorname{arctanh}\left(\frac{\sqrt{b\,x^2+a}}{x\sqrt{b}}\right)+\frac{5\sqrt{b\,x^2+a}\left(\frac{4\left(\frac{2}{3}Dx^4+C\,x^2+2B\right)b^{\frac{5}{2}}}{5}+\left(2\left(-\frac{Dx^2}{3}-\frac{3C}{5}\right)b^{\frac{3}{2}}+Da\sqrt{b}\right)a\right)}{16}}{b^{\frac{7}{2}}}$
default	$\frac{A\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}+C\left(\frac{x^3\sqrt{b\,x^2+a}}{4b}-\frac{3a\left(\frac{x\sqrt{b\,x^2+a}}{2b}-\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2b^{\frac{3}{2}}}\right)}{4b}\right)+D\left(\frac{x^5\sqrt{b\,x^2+a}}{6b}-\frac{5a\left(\right.}{}\right.$

input
```
int((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/b^(7/2)*((b^3*A-1/2*a*b^2*B+3/8*a^2*b*C-5/16*a^3*D)*arctanh((b*x^2+a)^(1
/2)/x/b^(1/2))+5/16*(b*x^2+a)^(1/2)*(4/5*(2/3*D*x^4+C*x^2+2*B)*b^(5/2)+(2*
(-1/3*D*x^2-3/5*C)*b^(3/2)+D*a*b^(1/2))*a)*x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.71

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{\sqrt{a+bx^2}}\,dx$$
$$=\left[-\frac{3\left(5\,Da^3-6\,Ca^2b+8\,Bab^2-16\,Ab^3\right)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\left(8\,Db^3x^5-2\left(5\,Da\right.\right.}{96\,b^4}\right.$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/96*(3*(5*D*a^3 - 6*C*a^2*b + 8*B*a*b^2 - 16*A*b^3)*sqrt(b)*log(-2*b*x^
2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(8*D*b^3*x^5 - 2*(5*D*a*b^2 - 6*C
*b^3)*x^3 + 3*(5*D*a^2*b - 6*C*a*b^2 + 8*B*b^3)*x)*sqrt(b*x^2 + a))/b^4, 1
/48*(3*(5*D*a^3 - 6*C*a^2*b + 8*B*a*b^2 - 16*A*b^3)*sqrt(-b)*arctan(sqrt(-
b)*x/sqrt(b*x^2 + a)) + (8*D*b^3*x^5 - 2*(5*D*a*b^2 - 6*C*b^3)*x^3 + 3*(5*
D*a^2*b - 6*C*a*b^2 + 8*B*b^3)*x)*sqrt(b*x^2 + a))/b^4]
```

Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.06

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{\sqrt{a+bx^2}}\,dx = \begin{cases} \left(A-\frac{a\left(B-\frac{3a\left(C-\frac{5Da}{6b}\right)}{4b}\right)}{2b}\right)\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right)+\sqrt{a+bx^2}\left(\frac{Dx^5}{6b}+\frac{x^3\left(C-\frac{5Da}{6b}\right)}{4b}+\frac{x\left(B-\right.}{} \right. \\ \frac{Ax+\frac{Bx^3}{3}+\frac{Cx^5}{5}+\frac{Dx^7}{7}}{\sqrt{a}} \end{cases}$$

input
```
integrate((D*x**6+C*x**4+B*x**2+A)/(b*x**2+a)**(1/2),x)
```

output
```
Piecewise(((A - a*(B - 3*a*(C - 5*D*a/(6*b))/(4*b))/(2*b))*Piecewise((log(
2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x
**2), True)) + sqrt(a + b*x**2)*(D*x**5/(6*b) + x**3*(C - 5*D*a/(6*b))/(4*
b) + x*(B - 3*a*(C - 5*D*a/(6*b))/(4*b))/(2*b)), Ne(b, 0)), ((A*x + B*x**3
/3 + C*x**5/5 + D*x**7/7)/sqrt(a), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.19

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{bx^2+a}Dx^5}{6\,b} - \frac{5\,\sqrt{bx^2+a}Dax^3}{24\,b^2} + \frac{\sqrt{bx^2+a}Cx^3}{4\,b} + \frac{5\,\sqrt{bx^2+a}Da^2x}{16\,b^3} - \frac{3\,\sqrt{bx^2+a}Cax}{8\,b^2} + \frac{\sqrt{bx^2+a}Bx}{2\,b} - \frac{5\,Da^3\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{7}{2}}} + \frac{3\,Ca^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{5}{2}}} - \frac{Ba\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{3}{2}}} + \frac{A\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/6*sqrt(b*x^2 + a)*D*x^5/b - 5/24*sqrt(b*x^2 + a)*D*a*x^3/b^2 + 1/4*sqrt(
b*x^2 + a)*C*x^3/b + 5/16*sqrt(b*x^2 + a)*D*a^2*x/b^3 - 3/8*sqrt(b*x^2 + a
)*C*a*x/b^2 + 1/2*sqrt(b*x^2 + a)*B*x/b - 5/16*D*a^3*arcsinh(b*x/sqrt(a*b)
)/b^(7/2) + 3/8*C*a^2*arcsinh(b*x/sqrt(a*b))/b^(5/2) - 1/2*B*a*arcsinh(b*x
/sqrt(a*b))/b^(3/2) + A*arcsinh(b*x/sqrt(a*b))/sqrt(b)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.86

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{\sqrt{a+bx^2}}\,dx = \frac{1}{48}\left(2\left(\frac{4\,Dx^2}{b} - \frac{5\,Dab^3-6\,Cb^4}{b^5}\right)x^2 + \frac{3\,(5\,Da^2b^2-6\,Cab^3+8\,Bb^4)}{b^5}\right)\sqrt{bx^2+a}\,x + \frac{(5\,Da^3-6\,Ca^2b+8\,Bab^2-16\,Ab^3)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{16\,b^{\frac{7}{2}}}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/48*(2*(4*D*x^2/b - (5*D*a*b^3 - 6*C*b^4)/b^5)*x^2 + 3*(5*D*a^2*b^2 - 6*C
*a*b^3 + 8*B*b^4)/b^5)*sqrt(b*x^2 + a)*x + 1/16*(5*D*a^3 - 6*C*a^2*b + 8*B
*a*b^2 - 16*A*b^3)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(7/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{\sqrt{a+bx^2}}\,dx=\int \frac{A+B\,x^2+C\,x^4+x^6\,D}{\sqrt{b\,x^2+a}}\,dx$$

input
```
int((A + B*x^2 + C*x^4 + x^6*D)/(a + b*x^2)^(1/2),x)
```

output
```
int((A + B*x^2 + C*x^4 + x^6*D)/(a + b*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.27

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{\sqrt{a+bx^2}}\,dx$$
$$=\frac{15\sqrt{b\,x^2+a}\,a^2bdx-18\sqrt{b\,x^2+a}\,a\,b^2cx-10\sqrt{b\,x^2+a}\,a\,b^2d\,x^3+24\sqrt{b\,x^2+a}\,b^4x+12\sqrt{b\,x^2+a}\,b^3c\,x^3}{}$$

input
```
int((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(1/2),x)
```

output
```
(15*sqrt(a + b*x**2)*a**2*b*d*x - 18*sqrt(a + b*x**2)*a*b**2*c*x - 10*sqrt
(a + b*x**2)*a*b**2*d*x**3 + 24*sqrt(a + b*x**2)*b**4*x + 12*sqrt(a + b*x*
*2)*b**3*c*x**3 + 8*sqrt(a + b*x**2)*b**3*d*x**5 - 15*sqrt(b)*log((sqrt(a
+ b*x**2) + sqrt(b)*x)/sqrt(a))*a**3*d + 18*sqrt(b)*log((sqrt(a + b*x**2)
+ sqrt(b)*x)/sqrt(a))*a**2*b*c + 24*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b
)*x)/sqrt(a))*a*b**3)/(48*b**4)
```

3.136 $\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 137

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{3/2}}\,dx=\frac{\left(\frac{A}{a}-\frac{b^2B-abC+a^2D}{b^3}\right)x}{\sqrt{a+bx^2}}+\frac{(4bC-7aD)x\sqrt{a+bx^2}}{8b^3}+\frac{Dx^3\sqrt{a+bx^2}}{4b^2}+\frac{(8b^2B-12abC+15a^2D)\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{7/2}}$$

output
```
(A/a-(B*b^2-C*a*b+D*a^2)/b^3)*x/(b*x^2+a)^(1/2)+1/8*(4*C*b-7*D*a)*x*(b*x^2
+a)^(1/2)/b^3+1/4*D*x^3*(b*x^2+a)^(1/2)/b^2+1/8*(8*B*b^2-12*C*a*b+15*D*a^2
)*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(7/2)
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.88

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{3/2}}\,dx=\frac{\frac{\sqrt{b}x\left(8Ab^3+a\left(-15a^2D+ab\left(12C-5Dx^2\right)+b^2\left(-8B+4Cx^2+2Dx^4\right)\right)\right)}{a\sqrt{a+bx^2}}+\left(-8b^2B+12abC-\right.}{8b^{7/2}}$$

input
```
Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(a + b*x^2)^(3/2),x]
```

output
```
((Sqrt[b]*x*(8*A*b^3 + a*(-15*a^2*D + a*b*(12*C - 5*D*x^2) + b^2*(-8*B + 4
*C*x^2 + 2*D*x^4))))/(a*Sqrt[a + b*x^2]) + (-8*b^2*B + 12*a*b*C - 15*a^2*D
)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(8*b^(7/2))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2345, 25, 1473, 27, 299, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{3/2}}dx$$

$$\Big\downarrow \text{2345}$$

$$\frac{x\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{a\sqrt{a+bx^2}}-\frac{\int -\frac{\frac{aDx^4}{b}+\frac{a(bC-aD)x^2}{b^2}+\frac{a\left(Da^2-bCa+b^2B\right)}{b^3}}{\sqrt{bx^2+a}}dx}{a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\frac{aDx^4}{b}+\frac{a(bC-aD)x^2}{b^2}+\frac{a\left(Da^2-bCa+b^2B\right)}{b^3}}{\sqrt{bx^2+a}}dx}{a}+\frac{x\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{a\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{1473}$$

$$\frac{\frac{\int \frac{a\left(b^2\left(4C-\frac{7aD}{b}\right)x^2+4\left(Da^2-bCa+b^2B\right)\right)}{b^2\sqrt{bx^2+a}}dx}{4b}+\frac{aDx^3\sqrt{a+bx^2}}{4b^2}}{a}+\frac{x\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{a\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{a\int \frac{b(4bC-7aD)x^2+4\left(Da^2-bCa+b^2B\right)}{\sqrt{bx^2+a}}dx}{4b^3}+\frac{aDx^3\sqrt{a+bx^2}}{4b^2}}{a}+\frac{x\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{a\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{299}$$

$$\frac{\frac{a\left(\frac{1}{2}(15a^2D-12abC+8b^2B)\int\frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}(4bC-7aD)\right)}{4b^3}+\frac{aDx^3\sqrt{a+bx^2}}{4b^2}}{a}+\frac{x\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{a\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{a\left(\frac{1}{2}(15a^2D-12abC+8b^2B)\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}(4bC-7aD)\right)}{4b^3}+\frac{aDx^3\sqrt{a+bx^2}}{4b^2}}{a}+\frac{x\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{a\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{x\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{a\sqrt{a+bx^2}}+\frac{\frac{a\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(15a^2D-12abC+8b^2B)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}(4bC-7aD)\right)}{4b^3}+\frac{aDx^3\sqrt{a+bx^2}}{4b^2}}{a}$$

input
```
Int[(A + B*x^2 + C*x^4 + D*x^6)/(a + b*x^2)^(3/2),x]
```

output
```
((A - (a*(b^2*B - a*b*C + a^2*D))/b^3)*x)/(a*Sqrt[a + b*x^2]) + ((a*D*x^3*
Sqrt[a + b*x^2])/(4*b^2) + (a*(((4*b*C - 7*a*D)*x*Sqrt[a + b*x^2])/2 + ((8
*b^2*B - 12*a*b*C + 15*a^2*D)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqr
t[b])))/(4*b^3))/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1473
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[c^p*x^(4*p - 1)*((d + e*x^2)^(q + 1)/(e*(4*p + 2*q + 1)))
, x] + Simp[1/(e*(4*p + 2*q + 1))   Int[(d + e*x^2)^q*ExpandToSum[e*(4*p +
2*q + 1)*(a + b*x^2 + c*x^4)^p - d*c^p*(4*p - 1)*x^(4*p - 2) - e*c^p*(4*p +
 2*q + 1)*x^(4*p), x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] &&  !LtQ[q, -1]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.83

method	result
pseudoelliptic	$\frac{\sqrt{b x^2+a}\, a \left(B b^2-\frac{3}{2} C a b+\frac{15}{8} D a^2\right) \operatorname{arctanh}\left(\frac{\sqrt{b x^2+a}}{x \sqrt{b}}\right)+\left(-\frac{15 a^3 D}{8}+\frac{3\left(-\frac{5 D x^2}{12}+C\right) b a^2}{2}-\left(-\frac{1}{4} D x^4-\frac{1}{2} C x^2+B\right) b^2 a+b^3 A\right. \cdots}{\sqrt{b x^2+a}\, a b^{\frac{7}{2}}}$
default	$\frac{A x}{a \sqrt{b x^2+a}}+C\left(\frac{x^3}{2 b \sqrt{b x^2+a}}-\frac{3 a\left(-\frac{x}{b \sqrt{b x^2+a}}+\frac{\ln\left(\sqrt{b}\, x+\sqrt{b x^2+a}\right)}{b^{\frac{3}{2}}}\right)}{2 b}\right)+D\left(\frac{x^5}{4 b \sqrt{b x^2+a}}-\frac{5 a\left(\frac{x^3}{2 b \sqrt{b x^2+a}}-\cdots\right.}{\cdots}\right.$

input
```
int((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/(b*x^2+a)^(1/2)*((b*x^2+a)^(1/2)*a*(B*b^2-3/2*C*a*b+15/8*D*a^2)*arctanh(
(b*x^2+a)^(1/2)/x/b^(1/2))+(-15/8*a^3*D+3/2*(-5/12*D*x^2+C)*b*a^2-(-1/4*D*
x^4-1/2*C*x^2+B)*b^2*a+b^3*A)*b^(1/2)*x)/a/b^(7/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 352, normalized size of antiderivative = 2.57

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{\left(a+bx^2\right)^{3/2}}\, dx = \left[\frac{\left(15\, Da^4-12\, Ca^3b+8\, Ba^2b^2+\left(15\, Da^3b-12\, Ca^2b^2+8\, Bab^3\right)x^2\right)\sqrt{b}\log \cdots}{\cdots}\right.$$

$$-\frac{\left(15\, Da^4-12\, Ca^3b+8\, Ba^2b^2+\left(15\, Da^3b-12\, Ca^2b^2+8\, Bab^3\right)x^2\right)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-\left(2\, Dab^3x^5-\cdots\right.}{8\left(ab^5x^2+a^2b^4\right)}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/16*((15*D*a^4 - 12*C*a^3*b + 8*B*a^2*b^2 + (15*D*a^3*b - 12*C*a^2*b^2 +
 8*B*a*b^3)*x^2)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) +
 2*(2*D*a*b^3*x^5 - (5*D*a^2*b^2 - 4*C*a*b^3)*x^3 - (15*D*a^3*b - 12*C*a^2
*b^2 + 8*B*a*b^3 - 8*A*b^4)*x)*sqrt(b*x^2 + a))/(a*b^5*x^2 + a^2*b^4), -1/
8*((15*D*a^4 - 12*C*a^3*b + 8*B*a^2*b^2 + (15*D*a^3*b - 12*C*a^2*b^2 + 8*B
*a*b^3)*x^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (2*D*a*b^3*x^5
- (5*D*a^2*b^2 - 4*C*a*b^3)*x^3 - (15*D*a^3*b - 12*C*a^2*b^2 + 8*B*a*b^3 -
 8*A*b^4)*x)*sqrt(b*x^2 + a))/(a*b^5*x^2 + a^2*b^4)]
```

Sympy [A] (verification not implemented)

Time = 7.61 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.74

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{3/2}}\,dx = \frac{Ax}{a^{\frac{3}{2}}\sqrt{1+\frac{bx^2}{a}}}+B\left(\frac{\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{\frac{3}{2}}}-\frac{x}{\sqrt{a}b\sqrt{1+\frac{bx^2}{a}}}\right) + C\left(\frac{3\sqrt{a}x}{2b^2\sqrt{1+\frac{bx^2}{a}}}-\frac{3a\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{\frac{5}{2}}}+\frac{x^3}{2\sqrt{a}b\sqrt{1+\frac{bx^2}{a}}}\right) + D\left(-\frac{15a^{\frac{3}{2}}x}{8b^3\sqrt{1+\frac{bx^2}{a}}}-\frac{5\sqrt{a}x^3}{8b^2\sqrt{1+\frac{bx^2}{a}}}+\frac{15a^2\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{\frac{7}{2}}}+\frac{x^5}{4\sqrt{a}b\sqrt{1+\frac{bx^2}{a}}}\right)$$

input
```
integrate((D*x**6+C*x**4+B*x**2+A)/(b*x**2+a)**(3/2),x)
```

output
```
A*x/(a**(3/2)*sqrt(1 + b*x**2/a)) + B*(asinh(sqrt(b)*x/sqrt(a))/b**(3/2) -
 x/(sqrt(a)*b*sqrt(1 + b*x**2/a))) + C*(3*sqrt(a)*x/(2*b**2*sqrt(1 + b*x**
2/a)) - 3*a*asinh(sqrt(b)*x/sqrt(a))/(2*b**(5/2)) + x**3/(2*sqrt(a)*b*sqrt
(1 + b*x**2/a))) + D*(-15*a**(3/2)*x/(8*b**3*sqrt(1 + b*x**2/a)) - 5*sqrt(
a)*x**3/(8*b**2*sqrt(1 + b*x**2/a)) + 15*a**2*asinh(sqrt(b)*x/sqrt(a))/(8*
b**(7/2)) + x**5/(4*sqrt(a)*b*sqrt(1 + b*x**2/a)))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.25

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{3/2}}\,dx = \frac{Dx^5}{4\sqrt{bx^2+a}b} - \frac{5\,Dax^3}{8\sqrt{bx^2+a}b^2} + \frac{Cx^3}{2\sqrt{bx^2+a}b} + \frac{Ax}{\sqrt{bx^2+a}a} - \frac{15\,Da^2x}{8\sqrt{bx^2+a}b^3} + \frac{3\,Cax}{2\sqrt{bx^2+a}b^2} - \frac{Bx}{\sqrt{bx^2+a}b} + \frac{15\,Da^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{7}{2}}} - \frac{3\,Ca\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{5}{2}}} + \frac{B\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{3}{2}}}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
1/4*D*x^5/(sqrt(b*x^2 + a)*b) - 5/8*D*a*x^3/(sqrt(b*x^2 + a)*b^2) + 1/2*C*
x^3/(sqrt(b*x^2 + a)*b) + A*x/(sqrt(b*x^2 + a)*a) - 15/8*D*a^2*x/(sqrt(b*x
^2 + a)*b^3) + 3/2*C*a*x/(sqrt(b*x^2 + a)*b^2) - B*x/(sqrt(b*x^2 + a)*b) +
 15/8*D*a^2*arcsinh(b*x/sqrt(a*b))/b^(7/2) - 3/2*C*a*arcsinh(b*x/sqrt(a*b)
)/b^(5/2) + B*arcsinh(b*x/sqrt(a*b))/b^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.98

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{3/2}}\,dx = \frac{\left(\left(\frac{2\,Dx^2}{b} - \frac{5\,Da^2b^3-4\,Cab^4}{ab^5}\right)x^2 - \frac{15\,Da^3b^2-12\,Ca^2b^3+8\,Bab^4-8\,Ab^5}{ab^5}\right)x}{8\sqrt{bx^2+a}} - \frac{(15\,Da^2-12\,Cab+8\,Bb^2)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{8\,b^{\frac{7}{2}}}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
1/8*((2*D*x^2/b - (5*D*a^2*b^3 - 4*C*a*b^4)/(a*b^5))*x^2 - (15*D*a^3*b^2 -
 12*C*a^2*b^3 + 8*B*a*b^4 - 8*A*b^5)/(a*b^5))*x/sqrt(b*x^2 + a) - 1/8*(15*
D*a^2 - 12*C*a*b + 8*B*b^2)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(7/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{3/2}}\,dx=\int \frac{A+B\,x^2+C\,x^4+x^6\,D}{(b\,x^2+a)^{3/2}}\,dx$$

input
```
int((A + B*x^2 + C*x^4 + x^6*D)/(a + b*x^2)^(3/2),x)
```

output
```
int((A + B*x^2 + C*x^4 + x^6*D)/(a + b*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 313, normalized size of antiderivative = 2.28

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{3/2}}\,dx=\frac{-15\sqrt{b\,x^2+a}\,a^2bdx+12\sqrt{b\,x^2+a}\,a\,b^2cx-5\sqrt{b\,x^2+a}\,a\,b^2d\,x^3+4\sqrt{b\,x^2}}{}$$

input
```
int((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(3/2),x)
```

output
```
( - 15*sqrt(a + b*x**2)*a**2*b*d*x + 12*sqrt(a + b*x**2)*a*b**2*c*x - 5*sq
rt(a + b*x**2)*a*b**2*d*x**3 + 4*sqrt(a + b*x**2)*b**3*c*x**3 + 2*sqrt(a +
 b*x**2)*b**3*d*x**5 + 15*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(
a))*a**3*d - 12*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2*b
*c + 15*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2*b*d*x**2
+ 8*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a*b**3 - 12*sqrt(b
)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a*b**2*c*x**2 + 8*sqrt(b)*lo
g((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*b**4*x**2 - 10*sqrt(b)*a**3*d +
9*sqrt(b)*a**2*b*c - 10*sqrt(b)*a**2*b*d*x**2 + 9*sqrt(b)*a*b**2*c*x**2)/(
8*b**4*(a + b*x**2))
```

3.137 $\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 149

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{5/2}}\,dx = \frac{\left(\frac{A}{a}-\frac{b^2B-abC+a^2D}{b^3}\right)x}{3(a+bx^2)^{3/2}} + \frac{(2Ab^3+a(b^2B-4abC+7a^2D))\,x}{3a^2b^3\sqrt{a+bx^2}} + \frac{Dx\sqrt{a+bx^2}}{2b^3} + \frac{(2bC-5aD)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{7/2}}$$

output
```
1/3*(A/a-(B*b^2-C*a*b+D*a^2)/b^3)*x/(b*x^2+a)^(3/2)+1/3*(2*A*b^3+a*(B*b^2-
4*C*a*b+7*D*a^2))*x/a^2/b^3/(b*x^2+a)^(1/2)+1/2*D*x*(b*x^2+a)^(1/2)/b^3+1/
2*(2*C*b-5*D*a)*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(7/2)
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.86

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{5/2}}\,dx = \frac{x(15a^4D+4Ab^4x^2+2ab^3(3A+Bx^2)+a^2b^2x^2(-8C+3Dx^2)+a^3b(-6C}{6a^2b^3\,(a+bx^2)^{3/2}}$$
$$+\frac{(-2bC+5aD)\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{2b^{7/2}}$$

input
```
Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(a + b*x^2)^(5/2),x]
```

output
```
(x*(15*a^4*D + 4*A*b^4*x^2 + 2*a*b^3*(3*A + B*x^2) + a^2*b^2*x^2*(-8*C + 3
*D*x^2) + a^3*b*(-6*C + 20*D*x^2)))/(6*a^2*b^3*(a + b*x^2)^(3/2)) + ((-2*b
*C + 5*a*D)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(2*b^(7/2))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2345, 25, 1471, 27, 299, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{5/2}}\,dx$$

$$\Big\downarrow \; 2345$$

$$\frac{x\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{3a\,(a+bx^2)^{3/2}}-\frac{\int -\frac{\frac{3aDx^4}{b}+\frac{3a(bC-aD)x^2}{b^2}+\frac{Da^3-bCa^2+b^2Ba+2Ab^3}{b^3}}{(bx^2+a)^{3/2}}dx}{3a}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{\frac{3aDx^4}{b}+\frac{3a(bC-aD)x^2}{b^2}+\frac{Da^3-bCa^2+b^2Ba+2Ab^3}{b^3}}{(bx^2+a)^{3/2}}dx}{3a}+\frac{x\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{3a\,(a+bx^2)^{3/2}}$$

$$\Big\downarrow \text{1471}$$

$$\frac{\frac{x\left(\frac{7a^2D-4abC+b^2B}{b^3}+\frac{2A}{a}\right)}{\sqrt{a+bx^2}}-\frac{\int -\frac{3a^2\left(bDx^2+bC-2aD\right)}{b^3\sqrt{bx^2+a}}dx}{a}}{3a}+\frac{x\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{3a\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{3a\int \frac{bDx^2+bC-2aD}{\sqrt{bx^2+a}}dx}{b^3}+\frac{x\left(\frac{7a^2D-4abC+b^2B}{b^3}+\frac{2A}{a}\right)}{\sqrt{a+bx^2}}}{3a}+\frac{x\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{3a\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{299}$$

$$\frac{\frac{3a\left(\frac{1}{2}(2bC-5aD)\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}Dx\sqrt{a+bx^2}\right)}{b^3}+\frac{x\left(\frac{7a^2D-4abC+b^2B}{b^3}+\frac{2A}{a}\right)}{\sqrt{a+bx^2}}}{3a}+\frac{x\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{3a\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{3a\left(\frac{1}{2}(2bC-5aD)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}Dx\sqrt{a+bx^2}\right)}{b^3}+\frac{x\left(\frac{7a^2D-4abC+b^2B}{b^3}+\frac{2A}{a}\right)}{\sqrt{a+bx^2}}}{3a}+\frac{x\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{3a\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{x\left(\frac{7a^2D-4abC+b^2B}{b^3}+\frac{2A}{a}\right)}{\sqrt{a+bx^2}}+\frac{3a\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bC-5aD)}{2\sqrt{b}}+\frac{1}{2}Dx\sqrt{a+bx^2}\right)}{b^3}}{3a}+\frac{x\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{3a\left(a+bx^2\right)^{3/2}}$$

input
```
Int[(A + B*x^2 + C*x^4 + D*x^6)/(a + b*x^2)^(5/2),x]
```

output
```
((A - (a*(b^2*B - a*b*C + a^2*D))/b^3)*x)/(3*a*(a + b*x^2)^(3/2)) + ((((2*
A)/a + (b^2*B - 4*a*b*C + 7*a^2*D)/b^3)*x)/Sqrt[a + b*x^2] + (3*a*((D*x*Sq
rt[a + b*x^2])/2 + ((2*b*C - 5*a*D)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/
(2*Sqrt[b])))/b^3)/(3*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.86

method	result
pseudoelliptic	$\frac{(b\,x^2+a)^{\frac{3}{2}} a^2 \left(Cb-\frac{5Da}{2}\right) \operatorname{arctanh}\left(\frac{\sqrt{b\,x^2+a}}{x\sqrt{b}}\right)+\frac{2\left(\frac{15Da^4\sqrt{b}}{4}+b^{\frac{3}{2}}\left(\left(5Dx^2-\frac{3C}{2}\right)a^3-2\left(-\frac{3Dx^2}{8}+C\right)x^2 b\,a^2+\frac{3a\left(\frac{x^2B}{3}+A\right)b^2}{2}+\cdots\right.\right.}{3}}{b^{\frac{7}{2}}(b\,x^2+a)^{\frac{3}{2}} a^2}$
default	$A\left(\frac{x}{3a(b\,x^2+a)^{\frac{3}{2}}}+\frac{2x}{3a^2\sqrt{b\,x^2+a}}\right)+C\left(-\frac{x^3}{3b(b\,x^2+a)^{\frac{3}{2}}}+\frac{-\frac{x}{b\sqrt{b\,x^2+a}}+\frac{\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{b^{\frac{3}{2}}}}{b}\right)+D\left(\frac{x^5}{2b(b\,x^2+\cdots}\right.$

input
```
int((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/(b*x^2+a)^(3/2)*(3/2*(b*x^2+a)^(3/2)*a^2*(C*b-5/2*D*a)*arctanh((b*x^2+
a)^(1/2)/x/b^(1/2))+(15/4*D*a^4*b^(1/2)+b^(3/2)*((5*D*x^2-3/2*C)*a^3-2*(-3
/8*D*x^2+C)*x^2*b*a^2+3/2*a*(1/3*x^2*B+A)*b^2+A*x^2*b^3))*x)/b^(7/2)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 415, normalized size of antiderivative = 2.79

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{5/2}}\,dx = \left[-\frac{3\,(5\,Da^5-2\,Ca^4b+(5\,Da^3b^2-2\,Ca^2b^3)x^4+2\,(5\,Da^4b-2\,Ca^3b^2)x^2)\cdots}{\cdots}\right.$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
[-1/12*(3*(5*D*a^5 - 2*C*a^4*b + (5*D*a^3*b^2 - 2*C*a^2*b^3)*x^4 + 2*(5*D*
a^4*b - 2*C*a^3*b^2)*x^2)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)
*x - a) - 2*(3*D*a^2*b^3*x^5 + 2*(10*D*a^3*b^2 - 4*C*a^2*b^3 + B*a*b^4 + 2
*A*b^5)*x^3 + 3*(5*D*a^4*b - 2*C*a^3*b^2 + 2*A*a*b^4)*x)*sqrt(b*x^2 + a))/
(a^2*b^6*x^4 + 2*a^3*b^5*x^2 + a^4*b^4), 1/6*(3*(5*D*a^5 - 2*C*a^4*b + (5*
D*a^3*b^2 - 2*C*a^2*b^3)*x^4 + 2*(5*D*a^4*b - 2*C*a^3*b^2)*x^2)*sqrt(-b)*a
rctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (3*D*a^2*b^3*x^5 + 2*(10*D*a^3*b^2 - 4
*C*a^2*b^3 + B*a*b^4 + 2*A*b^5)*x^3 + 3*(5*D*a^4*b - 2*C*a^3*b^2 + 2*A*a*b
^4)*x)*sqrt(b*x^2 + a))/(a^2*b^6*x^4 + 2*a^3*b^5*x^2 + a^4*b^4)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 821 vs. $2(138) = 276$.

Time = 10.28 (sec) , antiderivative size = 821, normalized size of antiderivative = 5.51

$$\int \frac{A + Bx^2 + Cx^4 + Dx^6}{(a + bx^2)^{5/2}} \, dx = \text{Too large to display}$$

input
```
integrate((D*x**6+C*x**4+B*x**2+A)/(b*x**2+a)**(5/2),x)
```

output
```
A*(3*a*x/(3*a**(7/2)*sqrt(1 + b*x**2/a) + 3*a**(5/2)*b*x**2*sqrt(1 + b*x**
2/a)) + 2*b*x**3/(3*a**(7/2)*sqrt(1 + b*x**2/a) + 3*a**(5/2)*b*x**2*sqrt(1
 + b*x**2/a))) + B*x**3/(3*a**(5/2)*sqrt(1 + b*x**2/a) + 3*a**(3/2)*b*x**2
*sqrt(1 + b*x**2/a)) + C*(3*a**(39/2)*b**11*sqrt(1 + b*x**2/a)*asinh(sqrt(
b)*x/sqrt(a))/(3*a**(39/2)*b**(27/2)*sqrt(1 + b*x**2/a) + 3*a**(37/2)*b**(
29/2)*x**2*sqrt(1 + b*x**2/a)) + 3*a**(37/2)*b**12*x**2*sqrt(1 + b*x**2/a)
*asinh(sqrt(b)*x/sqrt(a))/(3*a**(39/2)*b**(27/2)*sqrt(1 + b*x**2/a) + 3*a*
*(37/2)*b**(29/2)*x**2*sqrt(1 + b*x**2/a)) - 3*a**19*b**(23/2)*x/(3*a**(39
/2)*b**(27/2)*sqrt(1 + b*x**2/a) + 3*a**(37/2)*b**(29/2)*x**2*sqrt(1 + b*x
**2/a)) - 4*a**18*b**(25/2)*x**3/(3*a**(39/2)*b**(27/2)*sqrt(1 + b*x**2/a)
 + 3*a**(37/2)*b**(29/2)*x**2*sqrt(1 + b*x**2/a))) + D*(-15*a**(81/2)*b**2
2*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(6*a**(79/2)*b**(51/2)*sqrt(
1 + b*x**2/a) + 6*a**(77/2)*b**(53/2)*x**2*sqrt(1 + b*x**2/a)) - 15*a**(79
/2)*b**23*x**2*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(6*a**(79/2)*b*
*(51/2)*sqrt(1 + b*x**2/a) + 6*a**(77/2)*b**(53/2)*x**2*sqrt(1 + b*x**2/a)
) + 15*a**40*b**(45/2)*x/(6*a**(79/2)*b**(51/2)*sqrt(1 + b*x**2/a) + 6*a**
(77/2)*b**(53/2)*x**2*sqrt(1 + b*x**2/a)) + 20*a**39*b**(47/2)*x**3/(6*a**
(79/2)*b**(51/2)*sqrt(1 + b*x**2/a) + 6*a**(77/2)*b**(53/2)*x**2*sqrt(1 +
b*x**2/a)) + 3*a**38*b**(49/2)*x**5/(6*a**(79/2)*b**(51/2)*sqrt(1 + b*x**2
/a) + 6*a**(77/2)*b**(53/2)*x**2*sqrt(1 + b*x**2/a)))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.52

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{5/2}}\,dx = \frac{Dx^5}{2\,(bx^2+a)^{\frac{3}{2}}b} - \frac{1}{3}\,Cx\left(\frac{3\,x^2}{(bx^2+a)^{\frac{3}{2}}b} + \frac{2\,a}{(bx^2+a)^{\frac{3}{2}}b^2}\right) + \frac{5\,Dax\left(\frac{3\,x^2}{(bx^2+a)^{\frac{3}{2}}b} + \frac{2\,a}{(bx^2+a)^{\frac{3}{2}}b^2}\right)}{6\,b} + \frac{2\,Ax}{3\,\sqrt{bx^2+a}a^2} + \frac{Ax}{3\,(bx^2+a)^{\frac{3}{2}}a} + \frac{5\,Dax}{6\,\sqrt{bx^2+a}b^3} - \frac{Cx}{3\,\sqrt{bx^2+a}b^2} - \frac{Bx}{3\,(bx^2+a)^{\frac{3}{2}}b} + \frac{Bx}{3\,\sqrt{bx^2+a}ab} - \frac{5\,Da\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{7}{2}}} + \frac{C\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{5}{2}}}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
1/2*D*x^5/((b*x^2 + a)^(3/2)*b) - 1/3*C*x*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2
*a/((b*x^2 + a)^(3/2)*b^2)) + 5/6*D*a*x*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2*a
/((b*x^2 + a)^(3/2)*b^2))/b + 2/3*A*x/(sqrt(b*x^2 + a)*a^2) + 1/3*A*x/((b*
x^2 + a)^(3/2)*a) + 5/6*D*a*x/(sqrt(b*x^2 + a)*b^3) - 1/3*C*x/(sqrt(b*x^2
+ a)*b^2) - 1/3*B*x/((b*x^2 + a)^(3/2)*b) + 1/3*B*x/(sqrt(b*x^2 + a)*a*b)
- 5/2*D*a*arcsinh(b*x/sqrt(a*b))/b^(7/2) + C*arcsinh(b*x/sqrt(a*b))/b^(5/2
)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.89

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{5/2}}\,dx = \frac{\left(\left(\frac{3\,Dx^2}{b}+\frac{2\,(10\,Da^3b^3-4\,Ca^2b^4+Bab^5+2\,Ab^6)}{a^2b^5}\right)x^2+\frac{3\,(5\,Da^4b^2-2\,Ca^3b^3+2\,Aab^5)}{a^2b^5}\right)x}{6\,(bx^2+a)^{\frac{3}{2}}} + \frac{(5\,Da-2\,Cb)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2\,b^{\frac{7}{2}}}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
1/6*((3*D*x^2/b + 2*(10*D*a^3*b^3 - 4*C*a^2*b^4 + B*a*b^5 + 2*A*b^6)/(a^2*
b^5))*x^2 + 3*(5*D*a^4*b^2 - 2*C*a^3*b^3 + 2*A*a*b^5)/(a^2*b^5))*x/(b*x^2
+ a)^(3/2) + 1/2*(5*D*a - 2*C*b)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^
(7/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{5/2}}\,dx = \int \frac{A+B\,x^2+C\,x^4+x^6\,D}{(b\,x^2+a)^{5/2}}\,dx$$

input
```
int((A + B*x^2 + C*x^4 + x^6*D)/(a + b*x^2)^(5/2),x)
```

output
```
int((A + B*x^2 + C*x^4 + x^6*D)/(a + b*x^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 398, normalized size of antiderivative = 2.67

$$\int \frac{A + Bx^2 + Cx^4 + Dx^6}{(a + bx^2)^{5/2}}\, dx = \frac{30\sqrt{b\,x^2 + a}\,a^3 bdx - 12\sqrt{b\,x^2 + a}\,a^2 b^2 cx + 40\sqrt{b\,x^2 + a}\,a^2 b^2 d\,x^3 + 12\sqrt{b\,x}$$

input
```
int((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(5/2),x)
```

output
```
(30*sqrt(a + b*x**2)*a**3*b*d*x - 12*sqrt(a + b*x**2)*a**2*b**2*c*x + 40*s
qrt(a + b*x**2)*a**2*b**2*d*x**3 + 12*sqrt(a + b*x**2)*a*b**4*x - 16*sqrt(
a + b*x**2)*a*b**3*c*x**3 + 6*sqrt(a + b*x**2)*a*b**3*d*x**5 + 12*sqrt(a +
 b*x**2)*b**5*x**3 - 30*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a)
)*a**4*d + 12*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**3*b*c
 - 60*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**3*b*d*x**2 +
24*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2*b**2*c*x**2 -
30*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2*b**2*d*x**4 +
12*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a*b**3*c*x**4 - 5*s
qrt(b)*a**4*d - 10*sqrt(b)*a**3*b*d*x**2 - 4*sqrt(b)*a**2*b**3 - 5*sqrt(b)
*a**2*b**2*d*x**4 - 8*sqrt(b)*a*b**4*x**2 - 4*sqrt(b)*b**5*x**4)/(12*a*b**
4*(a**2 + 2*a*b*x**2 + b**2*x**4))
```

3.138 $\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{7/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 167

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{7/2}}\,dx = \frac{\left(\frac{A}{a}-\frac{b^2B-abC+a^2D}{b^3}\right)x}{5(a+bx^2)^{5/2}} + \frac{(4Ab^3+a(b^2B-6abC+11a^2D))\,x}{15a^2b^3(a+bx^2)^{3/2}} + \frac{(8Ab^3+a(2b^2B+3abC-23a^2D))\,x}{15a^3b^3\sqrt{a+bx^2}} + \frac{D\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{7/2}}$$

output
```
1/5*(A/a-(B*b^2-C*a*b+D*a^2)/b^3)*x/(b*x^2+a)^(5/2)+1/15*(4*A*b^3+a*(B*b^2
-6*C*a*b+11*D*a^2))*x/a^2/b^3/(b*x^2+a)^(3/2)+1/15*(8*A*b^3+a*(2*B*b^2+3*C
*a*b-23*D*a^2))*x/a^3/b^3/(b*x^2+a)^(1/2)+D*arctanh(b^(1/2)*x/(b*x^2+a)^(1
/2))/b^(7/2)
```

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.79

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{7/2}}\,dx = \frac{-15a^5Dx-35a^4bDx^3+8Ab^5x^5-23a^3b^2Dx^5+2ab^4x^3(10A+Bx^2)+a^2}{15a^3b^3\left(a+bx^2\right)^{5/2}}$$
$$-\frac{D\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{b^{7/2}}$$

input
```
Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(a + b*x^2)^(7/2),x]
```

output
```
(-15*a^5*D*x - 35*a^4*b*D*x^3 + 8*A*b^5*x^5 - 23*a^3*b^2*D*x^5 + 2*a*b^4*x
^3*(10*A + B*x^2) + a^2*b^3*x*(15*A + 5*B*x^2 + 3*C*x^4))/(15*a^3*b^3*(a +
 b*x^2)^(5/2)) - (D*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/b^(7/2)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2345, 25, 1471, 25, 27, 298, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{7/2}}\,dx$$

$$\Big\downarrow \text{2345}$$

$$\frac{x\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{5a\left(a+bx^2\right)^{5/2}}-\frac{\int -\frac{\frac{5aDx^4}{b}+\frac{5a(bC-aD)x^2}{b^2}+\frac{Da^3-bCa^2+b^2Ba+4Ab^3}{b^3}}{(bx^2+a)^{5/2}}dx}{5a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\frac{5aDx^4}{b}+\frac{5a(bC-aD)x^2}{b^2}+\frac{Da^3-bCa^2+b^2Ba+4Ab^3}{b^3}}{(bx^2+a)^{5/2}}dx}{5a}+\frac{x\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{5a\left(a+bx^2\right)^{5/2}}$$

$$\Big\downarrow \; 1471$$

$$\frac{\frac{x\left(\frac{11a^2D-6abC+b^2B}{b^3}+\frac{4A}{a}\right)}{3(a+bx^2)^{3/2}}-\frac{\int -\frac{-8Da^3+15bDx^2a^2+3bCa^2+2b^2Ba+8Ab^3}{b^3\left(bx^2+a\right)^{3/2}}dx}{3a}}{5a}+\frac{x\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{5a\left(a+bx^2\right)^{5/2}}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{8Ab^3+15a^2Dx^2b+a\left(-8Da^2+3bCa+2b^2B\right)}{b^3\left(bx^2+a\right)^{3/2}}dx}{3a}+\frac{x\left(\frac{11a^2D-6abC+b^2B}{b^3}+\frac{4A}{a}\right)}{3(a+bx^2)^{3/2}}}{5a}+\frac{x\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{5a\left(a+bx^2\right)^{5/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{8Ab^3+15a^2Dx^2b+a\left(-8Da^2+3bCa+2b^2B\right)}{\left(bx^2+a\right)^{3/2}}dx}{3ab^3}+\frac{x\left(\frac{11a^2D-6abC+b^2B}{b^3}+\frac{4A}{a}\right)}{3(a+bx^2)^{3/2}}}{5a}+\frac{x\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{5a\left(a+bx^2\right)^{5/2}}$$

$$\Big\downarrow \; 298$$

$$\frac{\frac{15a^2D\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{x\left(a\left(-23a^2D+3abC+2b^2B\right)+8Ab^3\right)}{a\sqrt{a+bx^2}}}{3ab^3}+\frac{x\left(\frac{11a^2D-6abC+b^2B}{b^3}+\frac{4A}{a}\right)}{3(a+bx^2)^{3/2}}}{5a}+\frac{x\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{5a\left(a+bx^2\right)^{5/2}}$$

$$\Big\downarrow \; 224$$

$$\frac{\frac{15a^2D\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{x\left(a\left(-23a^2D+3abC+2b^2B\right)+8Ab^3\right)}{a\sqrt{a+bx^2}}}{3ab^3}+\frac{x\left(\frac{11a^2D-6abC+b^2B}{b^3}+\frac{4A}{a}\right)}{3(a+bx^2)^{3/2}}}{5a}+\frac{x\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{5a\left(a+bx^2\right)^{5/2}}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{\frac{x\left(a\left(-23a^2D+3abC+2b^2B\right)+8Ab^3\right)}{a\sqrt{a+bx^2}}+\frac{15a^2D\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}}{3ab^3}+\frac{x\left(\frac{11a^2D-6abC+b^2B}{b^3}+\frac{4A}{a}\right)}{3(a+bx^2)^{3/2}}}{5a}+\frac{x\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{5a\left(a+bx^2\right)^{5/2}}$$

input
```
Int[(A + B*x^2 + C*x^4 + D*x^6)/(a + b*x^2)^(7/2),x]
```

output
```
((A - (a*(b^2*B - a*b*C + a^2*D))/b^3)*x)/(5*a*(a + b*x^2)^(5/2)) + ((((4*
A)/a + (b^2*B - 6*a*b*C + 11*a^2*D)/b^3)*x)/(3*(a + b*x^2)^(3/2)) + (((8*A
*b^3 + a*(2*b^2*B + 3*a*b*C - 23*a^2*D))*x)/(a*Sqrt[a + b*x^2]) + (15*a^2*
D*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/Sqrt[b])/(3*a*b^3))/(5*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.80

method	result
pseudoelliptic	$\frac{a^{3} D \operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{x \sqrt{b}}\right) b^{3} \left(b\,x^{2}+a\right)^{\frac{5}{2}}+\frac{8 b^{\frac{7}{2}} \left(A\,b^{5} x^{4}+\frac{5 \left(\frac{x^{2} B}{10}+A\right) x^{2} a\,b^{4}}{2}+\frac{15 \left(\frac{1}{5} C\,x^{4}+\frac{1}{3} x^{2} B+A\right) a^{2} b^{3}}{8}-\frac{23 D a^{3} b^{2} x^{4}}{8}-\frac{35 D a^{4} b}{8}\right.}{15}}{\left(b\,x^{2}+a\right)^{\frac{5}{2}} b^{\frac{13}{2}} a^{3}}$
default	$A \left(\frac{x}{5 a \left(b\,x^{2}+a\right)^{\frac{5}{2}}}+\frac{\frac{4 x}{15 a \left(b\,x^{2}+a\right)^{\frac{3}{2}}}+\frac{8 x}{15 a^{2} \sqrt{b\,x^{2}+a}}}{a}\right)+C \left(-\frac{x^{3}}{2 b \left(b\,x^{2}+a\right)^{\frac{5}{2}}}+\frac{3 a \left(-\frac{x}{4 b \left(b\,x^{2}+a\right)^{\frac{5}{2}}}+\frac{a \left(\frac{x}{5 a \left(b\,x^{2}+a\right)}\right.}{}\right.}{2}\right.$

input
```
int((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/(b*x^2+a)^(5/2)*(a^3*D*arctanh((b*x^2+a)^(1/2)/x/b^(1/2))*b^3*(b*x^2+a)^
(5/2)+8/15*b^(7/2)*(A*b^5*x^4+5/2*(1/10*x^2*B+A)*x^2*a*b^4+15/8*(1/5*C*x^4
+1/3*x^2*B+A)*a^2*b^3-23/8*D*a^3*b^2*x^4-35/8*D*a^4*b*x^2-15/8*D*a^5)*x)/b
^(13/2)/a^3
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 419, normalized size of antiderivative = 2.51

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{7/2}}\,dx = \left[\frac{15\,(Da^3b^3x^6+3\,Da^4b^2x^4+3\,Da^5bx^2+Da^6)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+\cdots}\right.}{\cdots} \right.$$

$$-\frac{15\,(Da^3b^3x^6+3\,Da^4b^2x^4+3\,Da^5bx^2+Da^6)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)+((23\,Da^3b^3-3\,Ca^2b^4-2\,Bab^5-\cdots}{15\,(a^3b^7x^6+3\,a^4b^6x^4+3\,a^5b^5x^2+\cdots}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(7/2),x, algorithm="fricas")
```

output
```
[1/30*(15*(D*a^3*b^3*x^6 + 3*D*a^4*b^2*x^4 + 3*D*a^5*b*x^2 + D*a^6)*sqrt(b
)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*((23*D*a^3*b^3 - 3*C
*a^2*b^4 - 2*B*a*b^5 - 8*A*b^6)*x^5 + 5*(7*D*a^4*b^2 - B*a^2*b^4 - 4*A*a*b
^5)*x^3 + 15*(D*a^5*b - A*a^2*b^4)*x)*sqrt(b*x^2 + a))/(a^3*b^7*x^6 + 3*a^
4*b^6*x^4 + 3*a^5*b^5*x^2 + a^6*b^4), -1/15*(15*(D*a^3*b^3*x^6 + 3*D*a^4*b
^2*x^4 + 3*D*a^5*b*x^2 + D*a^6)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)
) + ((23*D*a^3*b^3 - 3*C*a^2*b^4 - 2*B*a*b^5 - 8*A*b^6)*x^5 + 5*(7*D*a^4*b
^2 - B*a^2*b^4 - 4*A*a*b^5)*x^3 + 15*(D*a^5*b - A*a^2*b^4)*x)*sqrt(b*x^2 +
 a))/(a^3*b^7*x^6 + 3*a^4*b^6*x^4 + 3*a^5*b^5*x^2 + a^6*b^4)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1690 vs. 2(162) = 324.

Time = 21.65 (sec) , antiderivative size = 1690, normalized size of antiderivative = 10.12

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate((D*x**6+C*x**4+B*x**2+A)/(b*x**2+a)**(7/2),x)
```

output
```
A*(15*a**5*x/(15*a**(17/2)*sqrt(1 + b*x**2/a) + 45*a**(15/2)*b*x**2*sqrt(1
 + b*x**2/a) + 45*a**(13/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 15*a**(11/2)*b*
*3*x**6*sqrt(1 + b*x**2/a)) + 35*a**4*b*x**3/(15*a**(17/2)*sqrt(1 + b*x**2
/a) + 45*a**(15/2)*b*x**2*sqrt(1 + b*x**2/a) + 45*a**(13/2)*b**2*x**4*sqrt
(1 + b*x**2/a) + 15*a**(11/2)*b**3*x**6*sqrt(1 + b*x**2/a)) + 28*a**3*b**2
*x**5/(15*a**(17/2)*sqrt(1 + b*x**2/a) + 45*a**(15/2)*b*x**2*sqrt(1 + b*x*
*2/a) + 45*a**(13/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 15*a**(11/2)*b**3*x**6
*sqrt(1 + b*x**2/a)) + 8*a**2*b**3*x**7/(15*a**(17/2)*sqrt(1 + b*x**2/a) +
 45*a**(15/2)*b*x**2*sqrt(1 + b*x**2/a) + 45*a**(13/2)*b**2*x**4*sqrt(1 +
b*x**2/a) + 15*a**(11/2)*b**3*x**6*sqrt(1 + b*x**2/a))) + B*(5*a*x**3/(15*
a**(9/2)*sqrt(1 + b*x**2/a) + 30*a**(7/2)*b*x**2*sqrt(1 + b*x**2/a) + 15*a
**(5/2)*b**2*x**4*sqrt(1 + b*x**2/a)) + 2*b*x**5/(15*a**(9/2)*sqrt(1 + b*x
**2/a) + 30*a**(7/2)*b*x**2*sqrt(1 + b*x**2/a) + 15*a**(5/2)*b**2*x**4*sqr
t(1 + b*x**2/a))) + C*x**5/(5*a**(7/2)*sqrt(1 + b*x**2/a) + 10*a**(5/2)*b*
x**2*sqrt(1 + b*x**2/a) + 5*a**(3/2)*b**2*x**4*sqrt(1 + b*x**2/a)) + D*(15
*a**(99/2)*b**25*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(15*a**(99/2)
*b**(57/2)*sqrt(1 + b*x**2/a) + 45*a**(97/2)*b**(59/2)*x**2*sqrt(1 + b*x**
2/a) + 45*a**(95/2)*b**(61/2)*x**4*sqrt(1 + b*x**2/a) + 15*a**(93/2)*b**(6
3/2)*x**6*sqrt(1 + b*x**2/a)) + 45*a**(97/2)*b**26*x**2*sqrt(1 + b*x**2/a)
*asinh(sqrt(b)*x/sqrt(a))/(15*a**(99/2)*b**(57/2)*sqrt(1 + b*x**2/a) + ...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 317 vs. $2(150) = 300$.

Time = 0.05 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.90

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{7/2}}\,dx =$$
$$-\frac{1}{15}\,Dx\left(\frac{15\,x^4}{(bx^2+a)^{\frac{5}{2}}b}+\frac{20\,ax^2}{(bx^2+a)^{\frac{5}{2}}b^2}+\frac{8\,a^2}{(bx^2+a)^{\frac{5}{2}}b^3}\right)$$
$$-\frac{Dx\left(\frac{3\,x^2}{(bx^2+a)^{\frac{3}{2}}b}+\frac{2\,a}{(bx^2+a)^{\frac{3}{2}}b^2}\right)}{3\,b}-\frac{Cx^3}{2\,(bx^2+a)^{\frac{5}{2}}b}+\frac{8\,Ax}{15\,\sqrt{bx^2+a}a^3}$$
$$+\frac{4\,Ax}{15\,(bx^2+a)^{\frac{3}{2}}a^2}+\frac{Ax}{5\,(bx^2+a)^{\frac{5}{2}}a}+\frac{7\,Dx}{15\,\sqrt{bx^2+a}b^3}-\frac{4\,Dax}{15\,(bx^2+a)^{\frac{3}{2}}b^3}$$
$$+\frac{Cx}{10\,(bx^2+a)^{\frac{3}{2}}b^2}+\frac{Cx}{5\,\sqrt{bx^2+a}ab^2}-\frac{3\,Cax}{10\,(bx^2+a)^{\frac{5}{2}}b^2}-\frac{Bx}{5\,(bx^2+a)^{\frac{5}{2}}b}$$
$$+\frac{2\,Bx}{15\,\sqrt{bx^2+a}a^2b}+\frac{Bx}{15\,(bx^2+a)^{\frac{3}{2}}ab}+\frac{D\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{7}{2}}}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(7/2),x, algorithm="maxima")
```

output
```
-1/15*D*x*(15*x^4/((b*x^2 + a)^(5/2)*b) + 20*a*x^2/((b*x^2 + a)^(5/2)*b^2)
 + 8*a^2/((b*x^2 + a)^(5/2)*b^3)) - 1/3*D*x*(3*x^2/((b*x^2 + a)^(3/2)*b) +
 2*a/((b*x^2 + a)^(3/2)*b^2))/b - 1/2*C*x^3/((b*x^2 + a)^(5/2)*b) + 8/15*A
*x/(sqrt(b*x^2 + a)*a^3) + 4/15*A*x/((b*x^2 + a)^(3/2)*a^2) + 1/5*A*x/((b*
x^2 + a)^(5/2)*a) + 7/15*D*x/(sqrt(b*x^2 + a)*b^3) - 4/15*D*a*x/((b*x^2 +
a)^(3/2)*b^3) + 1/10*C*x/((b*x^2 + a)^(3/2)*b^2) + 1/5*C*x/(sqrt(b*x^2 + a
)*a*b^2) - 3/10*C*a*x/((b*x^2 + a)^(5/2)*b^2) - 1/5*B*x/((b*x^2 + a)^(5/2)
*b) + 2/15*B*x/(sqrt(b*x^2 + a)*a^2*b) + 1/15*B*x/((b*x^2 + a)^(3/2)*a*b)
+ D*arcsinh(b*x/sqrt(a*b))/b^(7/2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.87

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{7/2}}\,dx = -\frac{\left(x^2\left(\frac{(23\,Da^3b^4-3\,Ca^2b^5-2\,Bab^6-8\,Ab^7)x^2}{a^3b^5}+\frac{5\,(7\,Da^4b^3-Ba^2b^5-4\,Aab^6)}{a^3b^5}\right)+\frac{15\,(Da^5b^2-Aa^2b^5)}{a^3b^5}\right)x}{15\,(bx^2+a)^{\frac{5}{2}}} - \frac{D\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{b^{\frac{7}{2}}}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(7/2),x, algorithm="giac")
```

output
```
-1/15*(x^2*((23*D*a^3*b^4 - 3*C*a^2*b^5 - 2*B*a*b^6 - 8*A*b^7)*x^2/(a^3*b^
5) + 5*(7*D*a^4*b^3 - B*a^2*b^5 - 4*A*a*b^6)/(a^3*b^5)) + 15*(D*a^5*b^2 -
A*a^2*b^5)/(a^3*b^5))*x/(b*x^2 + a)^(5/2) - D*log(abs(-sqrt(b)*x + sqrt(b*
x^2 + a)))/b^(7/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{7/2}}\,dx = \int \frac{A+B\,x^2+C\,x^4+x^6\,D}{(b\,x^2+a)^{7/2}}\,dx$$

input
```
int((A + B*x^2 + C*x^4 + x^6*D)/(a + b*x^2)^(7/2),x)
```

output
```
int((A + B*x^2 + C*x^4 + x^6*D)/(a + b*x^2)^(7/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 429, normalized size of antiderivative = 2.57

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{7/2}}\,dx = \frac{-15\sqrt{b\,x^2+a}\,a^4bdx-35\sqrt{b\,x^2+a}\,a^3b^2d\,x^3+15\sqrt{b\,x^2+a}\,a^2b^4x-23\sqrt{b}}{}$$

input
```
int((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(7/2),x)
```

output
```
( - 15*sqrt(a + b*x**2)*a**4*b*d*x - 35*sqrt(a + b*x**2)*a**3*b**2*d*x**3
+ 15*sqrt(a + b*x**2)*a**2*b**4*x - 23*sqrt(a + b*x**2)*a**2*b**3*d*x**5 +
 25*sqrt(a + b*x**2)*a*b**5*x**3 + 3*sqrt(a + b*x**2)*a*b**4*c*x**5 + 10*s
qrt(a + b*x**2)*b**6*x**5 + 15*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/
sqrt(a))*a**5*d + 45*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a
**4*b*d*x**2 + 45*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**3
*b**2*d*x**4 + 15*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2
*b**3*d*x**6 + 5*sqrt(b)*a**5*d + 3*sqrt(b)*a**4*b*c + 15*sqrt(b)*a**4*b*d
*x**2 - 10*sqrt(b)*a**3*b**3 + 9*sqrt(b)*a**3*b**2*c*x**2 + 15*sqrt(b)*a**
3*b**2*d*x**4 - 30*sqrt(b)*a**2*b**4*x**2 + 9*sqrt(b)*a**2*b**3*c*x**4 + 5
*sqrt(b)*a**2*b**3*d*x**6 - 30*sqrt(b)*a*b**5*x**4 + 3*sqrt(b)*a*b**4*c*x*
*6 - 10*sqrt(b)*b**6*x**6)/(15*a**2*b**4*(a**3 + 3*a**2*b*x**2 + 3*a*b**2*
x**4 + b**3*x**6))
```

3.139 $\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{9/2}} dx$

Optimal result

Integrand size = 29, antiderivative size = 190

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{9/2}} dx = \frac{\left(\frac{A}{a}-\frac{b^2B-abC+a^2D}{b^3}\right)x}{7(a+bx^2)^{7/2}} + \frac{(6Ab^3+a(b^2B-8abC+15a^2D))x}{35a^2b^3(a+bx^2)^{5/2}} + \frac{(24Ab^3+a(4b^2B+3abC-45a^2D))x}{105a^3b^3(a+bx^2)^{3/2}} + \frac{(48Ab^3+a(8b^2B+6abC+15a^2D))x}{105a^4b^3\sqrt{a+bx^2}}$$

output
```
1/7*(A/a-(B*b^2-C*a*b+D*a^2)/b^3)*x/(b*x^2+a)^(7/2)+1/35*(6*A*b^3+a*(B*b^2
-8*C*a*b+15*D*a^2))*x/a^2/b^3/(b*x^2+a)^(5/2)+1/105*(24*A*b^3+a*(4*B*b^2+3
*C*a*b-45*D*a^2))*x/a^3/b^3/(b*x^2+a)^(3/2)+1/105*(48*A*b^3+a*(8*B*b^2+6*C
*a*b+15*D*a^2))*x/a^4/b^3/(b*x^2+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.52

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{9/2}}\,dx = \frac{48Ab^3x^7+8ab^2x^5(21A+Bx^2)+2a^2bx^3(105A+14Bx^2+3Cx^4)+a^3(10}{105a^4\,(a+bx^2)^{7/2}}$$

input
```
Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(a + b*x^2)^(9/2),x]
```

output
```
(48*A*b^3*x^7 + 8*a*b^2*x^5*(21*A + B*x^2) + 2*a^2*b*x^3*(105*A + 14*B*x^2
 + 3*C*x^4) + a^3*(105*A*x + 35*B*x^3 + 21*C*x^5 + 15*D*x^7))/(105*a^4*(a
+ b*x^2)^(7/2))
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.96, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {2344, 2089, 1586, 9, 25, 27, 362, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{9/2}}dx$$

$$\Big\downarrow \text{2344}$$

$$\frac{\int \frac{x^2(6Ab+a(Dx^4+Cx^2+B))}{(bx^2+a)^{9/2}}dx}{a}+\frac{Ax}{a\,(a+bx^2)^{7/2}}$$

$$\Big\downarrow \text{2089}$$

$$\frac{\int \frac{x^2(aDx^4+aCx^2+6Ab+aB)}{(bx^2+a)^{9/2}}dx}{a}+\frac{Ax}{a\,(a+bx^2)^{7/2}}$$

$$\Big\downarrow \text{1586}$$

$$\frac{\frac{x^3\left(\frac{a\left(a^2D-abC+b^2B\right)}{b^2}+6Ab\right)}{7a(a+bx^2)^{7/2}}-\frac{\int -\frac{x\left(\frac{7a^2Dx^3}{b}+\left(24Ab+\frac{a\left(-3Da^2+3bCa+4b^2B\right)}{b^2}\right)x\right)}{\left(bx^2+a\right)^{7/2}}dx}{7a}}{a}+\frac{Ax}{a\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \; 9$$

$$\frac{\frac{x^3\left(\frac{a\left(a^2D-abC+b^2B\right)}{b^2}+6Ab\right)}{7a(a+bx^2)^{7/2}}-\frac{\int -\frac{x^2\left(7a^2Dx^2+b\left(24Ab+\frac{a\left(-3Da^2+3bCa+4b^2B\right)}{b^2}\right)\right)}{b\left(bx^2+a\right)^{7/2}}dx}{7a}}{a}+\frac{Ax}{a\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{x^2\left(24Ab^2+7a^2Dx^2+a\left(-\frac{3Da^2}{b}+3Ca+4bB\right)\right)}{b\left(bx^2+a\right)^{7/2}}dx}{7a}+\frac{x^3\left(\frac{a\left(a^2D-abC+b^2B\right)}{b^2}+6Ab\right)}{7a(a+bx^2)^{7/2}}}{a}+\frac{Ax}{a\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{x^2\left(24Ab^2+7a^2Dx^2+a\left(-\frac{3Da^2}{b}+3Ca+4bB\right)\right)}{\left(bx^2+a\right)^{7/2}}dx}{7ab}+\frac{x^3\left(\frac{a\left(a^2D-abC+b^2B\right)}{b^2}+6Ab\right)}{7a(a+bx^2)^{7/2}}}{a}+\frac{Ax}{a\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \; 362$$

$$\frac{\frac{\frac{\left(a\left(15a^2D+6abC+8b^2B\right)+48Ab^3\right)\int \frac{x^2}{\left(bx^2+a\right)^{5/2}}dx}{5ab}+\frac{x^3\left(a\left(-10a^2D+3abC+4b^2B\right)+24Ab^3\right)}{5ab\left(a+bx^2\right)^{5/2}}}{7ab}+\frac{x^3\left(\frac{a\left(a^2D-abC+b^2B\right)}{b^2}+6Ab\right)}{7a(a+bx^2)^{7/2}}}{a}+\frac{Ax}{a\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \; 242$$

$$\frac{\frac{x^3\left(\frac{a\left(a^2D-abC+b^2B\right)}{b^2}+6Ab\right)}{7a(a+bx^2)^{7/2}}+\frac{\frac{x^3\left(a\left(15a^2D+6abC+8b^2B\right)+48Ab^3\right)}{15a^2b\left(a+bx^2\right)^{3/2}}+\frac{x^3\left(a\left(-10a^2D+3abC+4b^2B\right)+24Ab^3\right)}{5ab\left(a+bx^2\right)^{5/2}}}{7ab}}{a}+\frac{Ax}{a\left(a+bx^2\right)^{7/2}}$$

input
```
Int[(A + B*x^2 + C*x^4 + D*x^6)/(a + b*x^2)^(9/2),x]
```

output
```
(A*x)/(a*(a + b*x^2)^(7/2)) + (((6*A*b + (a*(b^2*B - a*b*C + a^2*D))/b^2)*
x^3)/(7*a*(a + b*x^2)^(7/2)) + (((24*A*b^3 + a*(4*b^2*B + 3*a*b*C - 10*a^2
*D))*x^3)/(5*a*b*(a + b*x^2)^(5/2)) + ((48*A*b^3 + a*(8*b^2*B + 6*a*b*C +
15*a^2*D))*x^3)/(15*a^2*b*(a + b*x^2)^(3/2)))/(7*a*b))/a
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

rule 1586
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, d + e*x^2, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^
p, d + e*x^2, x], x, 0]}, Simp[(-R)*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(2*d
*f*(q + 1))), x] + Simp[f/(2*d*(q + 1))   Int[(f*x)^(m - 1)*(d + e*x^2)^(q
+ 1)*ExpandToSum[2*d*(q + 1)*x*Qx + R*(m + 2*q + 3)*x, x], x], x]] /; FreeQ
[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[q, -1]
&& GtQ[m, 0]
```

rule 2089
```
Int[(u_)^(p_.)*((f_.)*(x_))^(m_.)*(z_)^(q_.), x_Symbol] :> Int[(f*x)^m*Expa
ndToSum[z, x]^q*ExpandToSum[u, x]^p, x] /; FreeQ[{f, m, p, q}, x] && Binomi
alQ[z, x] && TrinomialQ[u, x] &&  !(BinomialMatchQ[z, x] && TrinomialMatchQ
[u, x])
```

rule 2344
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coeff[Pq, x, 0
], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[A*x*((a + b*
x^2)^(p + 1)/a), x] + Simp[1/a   Int[x^2*(a + b*x^2)^p*(a*Q - A*b*(2*p + 3)
), x], x]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && ILtQ[p + 1/2, 0] && LtQ
[Expon[Pq, x] + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.47

method	result
pseudoelliptic	$\frac{x\left(\left(\frac{1}{7}Dx^{6}+\frac{1}{5}C\,x^{4}+\frac{1}{3}x^{2}B+A\right)a^{3}+2\left(\frac{1}{35}C\,x^{4}+\frac{2}{15}x^{2}B+A\right)x^{2}b\,a^{2}+\frac{8b^{2}\left(\frac{x^{2}B}{21}+A\right)x^{4}a}{5}+\frac{16A\,b^{3}x^{6}}{35}\right)}{\left(b\,x^{2}+a\right)^{\frac{7}{2}}a^{4}}$
gosper	$\frac{x\left(48A\,b^{3}x^{6}+8Ba\,b^{2}x^{6}+6C\,a^{2}b\,x^{6}+15Da^{3}x^{6}+168aA\,b^{2}x^{4}+28B\,a^{2}b\,x^{4}+21C\,a^{3}x^{4}+210a^{2}Ab\,x^{2}+35B\,a^{3}x^{2}+105a^{3}A\right)}{105\left(b\,x^{2}+a\right)^{\frac{7}{2}}a^{4}}$
trager	$\frac{x\left(48A\,b^{3}x^{6}+8Ba\,b^{2}x^{6}+6C\,a^{2}b\,x^{6}+15Da^{3}x^{6}+168aA\,b^{2}x^{4}+28B\,a^{2}b\,x^{4}+21C\,a^{3}x^{4}+210a^{2}Ab\,x^{2}+35B\,a^{3}x^{2}+105a^{3}A\right)}{105\left(b\,x^{2}+a\right)^{\frac{7}{2}}a^{4}}$
orering	$\frac{x\left(48A\,b^{3}x^{6}+8Ba\,b^{2}x^{6}+6C\,a^{2}b\,x^{6}+15Da^{3}x^{6}+168aA\,b^{2}x^{4}+28B\,a^{2}b\,x^{4}+21C\,a^{3}x^{4}+210a^{2}Ab\,x^{2}+35B\,a^{3}x^{2}+105a^{3}A\right)}{105\left(b\,x^{2}+a\right)^{\frac{7}{2}}a^{4}}$
default	$A\left(\frac{x}{7a\left(b\,x^{2}+a\right)^{\frac{7}{2}}}+\frac{\frac{6x}{35a\left(b\,x^{2}+a\right)^{\frac{5}{2}}}+\frac{6\left(\frac{4x}{15a\left(b\,x^{2}+a\right)^{\frac{3}{2}}}+\frac{8x}{15a^{2}\sqrt{b\,x^{2}+a}}\right)}{7a}}{a}\right)+C\left(-\frac{x^{3}}{4b\left(b\,x^{2}+a\right)^{\frac{7}{2}}}+\frac{3a\left(-\frac{x}{6b\left(b\,x^{2}+\right.}\right.}{}\right.$

input
```
int((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
1/(b*x^2+a)^(7/2)*x*((1/7*D*x^6+1/5*C*x^4+1/3*x^2*B+A)*a^3+2*(1/35*C*x^4+2
/15*x^2*B+A)*x^2*b*a^2+8/5*b^2*(1/21*x^2*B+A)*x^4*a+16/35*A*b^3*x^6)/a^4
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.74

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{9/2}}\,dx = \frac{((15\,Da^3+6\,Ca^2b+8\,Bab^2+48\,Ab^3)x^7+7\,(3\,Ca^3+4\,Ba^2b+24\,Aab^2)x}{105\,(a^4b^4x^8+4\,a^5b^3x^6+6\,a^6b^2x^4+4}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

output
```
1/105*((15*D*a^3 + 6*C*a^2*b + 8*B*a*b^2 + 48*A*b^3)*x^7 + 7*(3*C*a^3 + 4*
B*a^2*b + 24*A*a*b^2)*x^5 + 105*A*a^3*x + 35*(B*a^3 + 6*A*a^2*b)*x^3)*sqrt
(b*x^2 + a)/(a^4*b^4*x^8 + 4*a^5*b^3*x^6 + 6*a^6*b^2*x^4 + 4*a^7*b*x^2 + a
^8)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2088 vs. 2(187) = 374.

Time = 53.23 (sec) , antiderivative size = 2088, normalized size of antiderivative = 10.99

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{9/2}}\,dx = \text{Too large to display}$$

input
```
integrate((D*x**6+C*x**4+B*x**2+A)/(b*x**2+a)**(9/2),x)
```

output

```
A*(35*a**14*x/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt
(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2
)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a
) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*
sqrt(1 + b*x**2/a)) + 175*a**13*b*x**3/(35*a**(37/2)*sqrt(1 + b*x**2/a) +
210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 +
 b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b*
*4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) +
 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a)) + 371*a**12*b**2*x**5/(35*a**
(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*
a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 +
b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**
5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a)) +
 429*a**11*b**3*x**7/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x*
*2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a
**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b
*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6
*x**12*sqrt(1 + b*x**2/a)) + 286*a**10*b**4*x**9/(35*a**(37/2)*sqrt(1 + b*
x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**
4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.76

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{9/2}}\,dx = -\frac{Dx^5}{2\,(bx^2+a)^{\frac{7}{2}}b} - \frac{5\,Dax^3}{8\,(bx^2+a)^{\frac{7}{2}}b^2} - \frac{Cx^3}{4\,(bx^2+a)^{\frac{7}{2}}b} + \frac{16\,Ax}{35\,\sqrt{bx^2+a}a^4} + \frac{8\,Ax}{35\,(bx^2+a)^{\frac{3}{2}}a^3} + \frac{6\,Ax}{35\,(bx^2+a)^{\frac{5}{2}}a^2} + \frac{Ax}{7\,(bx^2+a)^{\frac{7}{2}}a} + \frac{Dx}{14\,(bx^2+a)^{\frac{3}{2}}b^3} + \frac{Dx}{7\,\sqrt{bx^2+a}ab^3} + \frac{3\,Dax}{56\,(bx^2+a)^{\frac{5}{2}}b^3} - \frac{15\,Da^2x}{56\,(bx^2+a)^{\frac{7}{2}}b^3} + \frac{3\,Cx}{140\,(bx^2+a)^{\frac{5}{2}}b^2} + \frac{2\,Cx}{35\,\sqrt{bx^2+a}a^2b^2} + \frac{Cx}{35\,(bx^2+a)^{\frac{3}{2}}ab^2} - \frac{3\,Cax}{28\,(bx^2+a)^{\frac{7}{2}}b^2} - \frac{Bx}{7\,(bx^2+a)^{\frac{7}{2}}b} + \frac{8\,Bx}{105\,\sqrt{bx^2+a}a^3b} + \frac{4\,Bx}{105\,(bx^2+a)^{\frac{3}{2}}a^2b} + \frac{Bx}{35\,(bx^2+a)^{\frac{5}{2}}ab}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

output
```
-1/2*D*x^5/((b*x^2 + a)^(7/2)*b) - 5/8*D*a*x^3/((b*x^2 + a)^(7/2)*b^2) - 1
/4*C*x^3/((b*x^2 + a)^(7/2)*b) + 16/35*A*x/(sqrt(b*x^2 + a)*a^4) + 8/35*A*
x/((b*x^2 + a)^(3/2)*a^3) + 6/35*A*x/((b*x^2 + a)^(5/2)*a^2) + 1/7*A*x/((b
*x^2 + a)^(7/2)*a) + 1/14*D*x/((b*x^2 + a)^(3/2)*b^3) + 1/7*D*x/(sqrt(b*x^
2 + a)*a*b^3) + 3/56*D*a*x/((b*x^2 + a)^(5/2)*b^3) - 15/56*D*a^2*x/((b*x^2
 + a)^(7/2)*b^3) + 3/140*C*x/((b*x^2 + a)^(5/2)*b^2) + 2/35*C*x/(sqrt(b*x^
2 + a)*a^2*b^2) + 1/35*C*x/((b*x^2 + a)^(3/2)*a*b^2) - 3/28*C*a*x/((b*x^2
+ a)^(7/2)*b^2) - 1/7*B*x/((b*x^2 + a)^(7/2)*b) + 8/105*B*x/(sqrt(b*x^2 +
a)*a^3*b) + 4/105*B*x/((b*x^2 + a)^(3/2)*a^2*b) + 1/35*B*x/((b*x^2 + a)^(5
/2)*a*b)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.69

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{9/2}}\,dx = \frac{\left(\left(x^2\left(\frac{(15\,Da^3b^3+6\,Ca^2b^4+8\,Bab^5+48\,Ab^6)x^2}{a^4b^3}+\frac{7\,(3\,Ca^3b^3+4\,Ba^2b^4+24\,Aab^5)}{a^4b^3}\right)+\frac{35\,(Ba^3b \cdots}{\cdots}\right.\right.}{105\,(bx^2+a)^{\frac{7}{2}}}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="giac")
```

output
```
1/105*((x^2*((15*D*a^3*b^3 + 6*C*a^2*b^4 + 8*B*a*b^5 + 48*A*b^6)*x^2/(a^4*
b^3) + 7*(3*C*a^3*b^3 + 4*B*a^2*b^4 + 24*A*a*b^5)/(a^4*b^3)) + 35*(B*a^3*b
^3 + 6*A*a^2*b^4)/(a^4*b^3))*x^2 + 105*A/a)*x/(b*x^2 + a)^(7/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{9/2}}\,dx = \int \frac{A+B\,x^2+C\,x^4+x^6\,D}{(b\,x^2+a)^{9/2}}\,dx$$

input
```
int((A + B*x^2 + C*x^4 + x^6*D)/(a + b*x^2)^(9/2),x)
```

output
```
int((A + B*x^2 + C*x^4 + x^6*D)/(a + b*x^2)^(9/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.89

$$\int \frac{A + Bx^2 + Cx^4 + Dx^6}{(a + bx^2)^{9/2}}\, dx = \frac{105\sqrt{b\,x^2 + a}\, a^3 b^4 x + 245\sqrt{b\,x^2 + a}\, a^2 b^5 x^3 + 21\sqrt{b\,x^2 + a}\, a^2 b^4 c\, x^5 + 15\sqrt{b}}{}$$

input
```
int((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x)
```

output
```
(105*sqrt(a + b*x**2)*a**3*b**4*x + 245*sqrt(a + b*x**2)*a**2*b**5*x**3 +
21*sqrt(a + b*x**2)*a**2*b**4*c*x**5 + 15*sqrt(a + b*x**2)*a**2*b**4*d*x**
7 + 196*sqrt(a + b*x**2)*a*b**6*x**5 + 6*sqrt(a + b*x**2)*a*b**5*c*x**7 +
56*sqrt(a + b*x**2)*b**7*x**7 + 15*sqrt(b)*a**6*d - 6*sqrt(b)*a**5*b*c + 6
0*sqrt(b)*a**5*b*d*x**2 - 56*sqrt(b)*a**4*b**3 - 24*sqrt(b)*a**4*b**2*c*x*
*2 + 90*sqrt(b)*a**4*b**2*d*x**4 - 224*sqrt(b)*a**3*b**4*x**2 - 36*sqrt(b)
*a**3*b**3*c*x**4 + 60*sqrt(b)*a**3*b**3*d*x**6 - 336*sqrt(b)*a**2*b**5*x*
*4 - 24*sqrt(b)*a**2*b**4*c*x**6 + 15*sqrt(b)*a**2*b**4*d*x**8 - 224*sqrt(
b)*a*b**6*x**6 - 6*sqrt(b)*a*b**5*c*x**8 - 56*sqrt(b)*b**7*x**8)/(105*a**3
*b**4*(a**4 + 4*a**3*b*x**2 + 6*a**2*b**2*x**4 + 4*a*b**3*x**6 + b**4*x**8
))
```

3.140 $\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{11/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 238

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{11/2}}\,dx = \frac{\left(\frac{A}{a}-\frac{b^2B-abC+a^2D}{b^3}\right)x}{9(a+bx^2)^{9/2}} + \frac{(8Ab^3+a(b^2B-10abC+19a^2D))\,x}{63a^2b^3(a+bx^2)^{7/2}} + \frac{(16Ab^3+a(2b^2B+abC-25a^2D))\,x}{105a^3b^3(a+bx^2)^{5/2}} + \frac{(64Ab^3+a(8b^2B+4abC+5a^2D))\,x}{315a^4b^3(a+bx^2)^{3/2}} + \frac{2(64Ab^3+a(8b^2B+4abC+5a^2D))\,x}{315a^5b^3\sqrt{a+bx^2}}$$

output
```
1/9*(A/a-(B*b^2-C*a*b+D*a^2)/b^3)*x/(b*x^2+a)^(9/2)+1/63*(8*A*b^3+a*(B*b^2
-10*C*a*b+19*D*a^2))*x/a^2/b^3/(b*x^2+a)^(7/2)+1/105*(16*A*b^3+a*(2*B*b^2+
C*a*b-25*D*a^2))*x/a^3/b^3/(b*x^2+a)^(5/2)+1/315*(64*A*b^3+a*(8*B*b^2+4*C*
a*b+5*D*a^2))*x/a^4/b^3/(b*x^2+a)^(3/2)+2/315*(64*A*b^3+a*(8*B*b^2+4*C*a*b
+5*D*a^2))*x/a^5/b^3/(b*x^2+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.55

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{11/2}}\,dx = \frac{128Ab^4x^9+16ab^3x^7(36A+Bx^2)+8a^2b^2x^5(126A+9Bx^2+Cx^4)+2a^3b}{315a^5}$$

input
```
Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(a + b*x^2)^(11/2),x]
```

output
```
(128*A*b^4*x^9 + 16*a*b^3*x^7*(36*A + B*x^2) + 8*a^2*b^2*x^5*(126*A + 9*B*
x^2 + C*x^4) + 2*a^3*b*x^3*(420*A + 63*B*x^2 + 18*C*x^4 + 5*D*x^6) + 3*a^4
*(105*A*x + 35*B*x^3 + 21*C*x^5 + 15*D*x^7))/(315*a^5*(a + b*x^2)^(9/2))
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.89, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {2344, 2089, 1586, 9, 27, 362, 245, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{11/2}}dx$$

$$\Big\downarrow \text{2344}$$

$$\frac{\int \frac{x^2(8Ab+a(Dx^4+Cx^2+B))}{(bx^2+a)^{11/2}}dx}{a}+\frac{Ax}{a(a+bx^2)^{9/2}}$$

$$\Big\downarrow \text{2089}$$

$$\frac{\int \frac{x^2(aDx^4+aCx^2+8Ab+aB)}{(bx^2+a)^{11/2}}dx}{a}+\frac{Ax}{a(a+bx^2)^{9/2}}$$

$$\Big\downarrow \text{1586}$$

$$\frac{\frac{x^3\left(\frac{a\left(a^2D-abC+b^2B\right)}{b^2}+8Ab\right)}{9a(a+bx^2)^{9/2}}-\frac{\int -\frac{3x\left(\frac{3a^2Dx^3}{b}+\left(16Ab+\frac{a\left(-Da^2+bCa+2b^2B\right)}{b^2}\right)x\right)}{\left(bx^2+a\right)^{9/2}}dx}{9a}}{a}+\frac{Ax}{a\left(a+bx^2\right)^{9/2}}$$

$$\downarrow \ 9$$

$$\frac{\frac{x^3\left(\frac{a\left(a^2D-abC+b^2B\right)}{b^2}+8Ab\right)}{9a(a+bx^2)^{9/2}}-\frac{\int -\frac{3x^2\left(3a^2Dx^2+b\left(16Ab+\frac{a\left(-Da^2+bCa+2b^2B\right)}{b^2}\right)\right)}{b\left(bx^2+a\right)^{9/2}}dx}{9a}}{a}+\frac{Ax}{a\left(a+bx^2\right)^{9/2}}$$

$$\downarrow \ 27$$

$$\frac{\frac{\int \frac{x^2\left(16Ab^2+3a^2Dx^2+a\left(-\frac{Da^2}{b}+Ca+2bB\right)\right)}{\left(bx^2+a\right)^{9/2}}dx}{3ab}+\frac{x^3\left(\frac{a\left(a^2D-abC+b^2B\right)}{b^2}+8Ab\right)}{9a(a+bx^2)^{9/2}}}{a}+\frac{Ax}{a\left(a+bx^2\right)^{9/2}}$$

$$\downarrow \ 362$$

$$\frac{\frac{\frac{\left(a\left(5a^2D+4abC+8b^2B\right)+64Ab^3\right)\int \frac{x^2}{\left(bx^2+a\right)^{7/2}}dx}{7ab}+\frac{x^3\left(a\left(-4a^2D+abC+2b^2B\right)+16Ab^3\right)}{7ab\left(a+bx^2\right)^{7/2}}}{3ab}+\frac{x^3\left(\frac{a\left(a^2D-abC+b^2B\right)}{b^2}+8Ab\right)}{9a(a+bx^2)^{9/2}}}{a}+\frac{Ax}{a\left(a+bx^2\right)^{9/2}}$$

$$\downarrow \ 245$$

$$\frac{\frac{\frac{\left(a\left(5a^2D+4abC+8b^2B\right)+64Ab^3\right)\left(\frac{2b\int \frac{x^4}{\left(bx^2+a\right)^{7/2}}dx}{3a}+\frac{x^3}{3a\left(a+bx^2\right)^{5/2}}\right)}{7ab}+\frac{x^3\left(a\left(-4a^2D+abC+2b^2B\right)+16Ab^3\right)}{7ab\left(a+bx^2\right)^{7/2}}}{3ab}+\frac{x^3\left(\frac{a\left(a^2D-abC+b^2B\right)}{b^2}+8Ab\right)}{9a(a+bx^2)^{9/2}}}{a}+\frac{Ax}{a\left(a+bx^2\right)^{9/2}}$$

$$\downarrow \ 242$$

$$\frac{\frac{x^3\left(\frac{a\left(a^2D-abC+b^2B\right)}{b^2}+8Ab\right)}{9a(a+bx^2)^{9/2}}+\frac{\frac{x^3\left(a\left(-4a^2D+abC+2b^2B\right)+16Ab^3\right)}{7ab\left(a+bx^2\right)^{7/2}}+\frac{\left(\frac{2bx^5}{15a^2\left(a+bx^2\right)^{5/2}}+\frac{x^3}{3a\left(a+bx^2\right)^{5/2}}\right)\left(a\left(5a^2D+4abC+8b^2B\right)+64Ab^3\right)}{7ab}}{3ab}}{a}+\frac{Ax}{a\left(a+bx^2\right)^{9/2}}$$

input
```
Int[(A + B*x^2 + C*x^4 + D*x^6)/(a + b*x^2)^(11/2),x]
```

output
```
(A*x)/(a*(a + b*x^2)^(9/2)) + (((8*A*b + (a*(b^2*B - a*b*C + a^2*D))/b^2)*
x^3)/(9*a*(a + b*x^2)^(9/2)) + (((16*A*b^3 + a*(2*b^2*B + a*b*C - 4*a^2*D)
)*x^3)/(7*a*b*(a + b*x^2)^(7/2)) + ((64*A*b^3 + a*(8*b^2*B + 4*a*b*C + 5*a
^2*D))*(x^3/(3*a*(a + b*x^2)^(5/2)) + (2*b*x^5)/(15*a^2*(a + b*x^2)^(5/2))
))/(7*a*b))/(3*a*b))/a
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

rule 1586
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, d + e*x^2, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^
p, d + e*x^2, x], x, 0]}, Simp[(-R)*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(2*d
*f*(q + 1))), x] + Simp[f/(2*d*(q + 1))   Int[(f*x)^(m - 1)*(d + e*x^2)^(q
+ 1)*ExpandToSum[2*d*(q + 1)*x*Qx + R*(m + 2*q + 3)*x, x], x], x]] /; FreeQ
[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[q, -1]
&& GtQ[m, 0]
```

rule 2089
```
Int[(u_)^(p_.)*((f_.)*(x_))^(m_.)*(z_)^(q_.), x_Symbol] :> Int[(f*x)^m*Expa
ndToSum[z, x]^q*ExpandToSum[u, x]^p, x] /; FreeQ[{f, m, p, q}, x] && Binomi
alQ[z, x] && TrinomialQ[u, x] &&  !(BinomialMatchQ[z, x] && TrinomialMatchQ
[u, x])
```

rule 2344
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coeff[Pq, x, 0
], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[A*x*((a + b*
x^2)^(p + 1)/a), x] + Simp[1/a   Int[x^2*(a + b*x^2)^p*(a*Q - A*b*(2*p + 3)
), x], x]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && ILtQ[p + 1/2, 0] && LtQ
[Expon[Pq, x] + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.50

method	result
pseudoelliptic	$\frac{x\left(\left(\frac{1}{7}Dx^{6}+\frac{1}{5}C\,x^{4}+\frac{1}{3}x^{2}B+A\right)a^{4}+\frac{8\left(\frac{1}{84}Dx^{6}+\frac{3}{70}C\,x^{4}+\frac{3}{20}x^{2}B+A\right)x^{2}b\,a^{3}}{3}+\frac{16\left(\frac{1}{126}C\,x^{4}+\frac{1}{14}x^{2}B+A\right)x^{4}b^{2}a^{2}}{5}+\frac{64\left(\frac{x^{2}B}{36}+A\right)x^{6}\cdots}{35}\cdots\right.}{(b\,x^{2}+a)^{\frac{9}{2}}a^{5}}$
gosper	$\frac{x\left(128A\,x^{8}b^{4}+16B\,x^{8}a\,b^{3}+8C\,a^{2}b^{2}x^{8}+10Da^{3}b\,x^{8}+576A\,x^{6}a\,b^{3}+72B\,x^{6}a^{2}b^{2}+36C\,a^{3}b\,x^{6}+45Da^{4}x^{6}+1008A\,x^{4}a^{2}b^{2}+12\cdots\right.}{315(b\,x^{2}+a)^{\frac{9}{2}}a^{5}}$
trager	$\frac{x\left(128A\,x^{8}b^{4}+16B\,x^{8}a\,b^{3}+8C\,a^{2}b^{2}x^{8}+10Da^{3}b\,x^{8}+576A\,x^{6}a\,b^{3}+72B\,x^{6}a^{2}b^{2}+36C\,a^{3}b\,x^{6}+45Da^{4}x^{6}+1008A\,x^{4}a^{2}b^{2}+12\cdots\right.}{315(b\,x^{2}+a)^{\frac{9}{2}}a^{5}}$
orering	$\frac{x\left(128A\,x^{8}b^{4}+16B\,x^{8}a\,b^{3}+8C\,a^{2}b^{2}x^{8}+10Da^{3}b\,x^{8}+576A\,x^{6}a\,b^{3}+72B\,x^{6}a^{2}b^{2}+36C\,a^{3}b\,x^{6}+45Da^{4}x^{6}+1008A\,x^{4}a^{2}b^{2}+12\cdots\right.}{315(b\,x^{2}+a)^{\frac{9}{2}}a^{5}}$
	$\cdots 6\left(\frac{4x}{15a\left(b\,x^{2}+a\right)^{\frac{3}{2}}}+\frac{8x}{15a^{2}\sqrt{b\,x^{2}+a}}\right)\cdots$

input
```
int((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(11/2),x,method=_RETURNVERBOSE)
```

output
```
1/(b*x^2+a)^(9/2)*x*((1/7*D*x^6+1/5*C*x^4+1/3*x^2*B+A)*a^4+8/3*(1/84*D*x^6
+3/70*C*x^4+3/20*x^2*B+A)*x^2*b*a^3+16/5*(1/126*C*x^4+1/14*x^2*B+A)*x^4*b^
2*a^2+64/35*(1/36*x^2*B+A)*x^6*b^3*a+128/315*A*x^8*b^4)/a^5
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.81

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{11/2}}\,dx = \frac{(2\,(5\,Da^3b+4\,Ca^2b^2+8\,Bab^3+64\,Ab^4)x^9+9\,(5\,Da^4+4\,Ca^3b+8\,Ba^2b^2}{315\,(a^5b^5x^{10}+5\,a^6b^4}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(11/2),x, algorithm="fricas")
```

output
```
1/315*(2*(5*D*a^3*b + 4*C*a^2*b^2 + 8*B*a*b^3 + 64*A*b^4)*x^9 + 9*(5*D*a^4
 + 4*C*a^3*b + 8*B*a^2*b^2 + 64*A*a*b^3)*x^7 + 315*A*a^4*x + 63*(C*a^4 + 2
*B*a^3*b + 16*A*a^2*b^2)*x^5 + 105*(B*a^4 + 8*A*a^3*b)*x^3)*sqrt(b*x^2 + a
)/(a^5*b^5*x^10 + 5*a^6*b^4*x^8 + 10*a^7*b^3*x^6 + 10*a^8*b^2*x^4 + 5*a^9*
b*x^2 + a^10)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5440 vs. 2(238) = 476.

Time = 120.47 (sec) , antiderivative size = 5440, normalized size of antiderivative = 22.86

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{11/2}}\,dx = \text{Too large to display}$$

input
```
integrate((D*x**6+C*x**4+B*x**2+A)/(b*x**2+a)**(11/2),x)
```

output

```
A*(315*a**30*x/(315*a**(71/2)*sqrt(1 + b*x**2/a) + 3150*a**(69/2)*b*x**2*s
qrt(1 + b*x**2/a) + 14175*a**(67/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 37800*a
**(65/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 66150*a**(63/2)*b**4*x**8*sqrt(1 +
 b*x**2/a) + 79380*a**(61/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 66150*a**(59/
2)*b**6*x**12*sqrt(1 + b*x**2/a) + 37800*a**(57/2)*b**7*x**14*sqrt(1 + b*x
**2/a) + 14175*a**(55/2)*b**8*x**16*sqrt(1 + b*x**2/a) + 3150*a**(53/2)*b*
*9*x**18*sqrt(1 + b*x**2/a) + 315*a**(51/2)*b**10*x**20*sqrt(1 + b*x**2/a)
) + 2730*a**29*b*x**3/(315*a**(71/2)*sqrt(1 + b*x**2/a) + 3150*a**(69/2)*b
*x**2*sqrt(1 + b*x**2/a) + 14175*a**(67/2)*b**2*x**4*sqrt(1 + b*x**2/a) +
37800*a**(65/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 66150*a**(63/2)*b**4*x**8*s
qrt(1 + b*x**2/a) + 79380*a**(61/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 66150*
a**(59/2)*b**6*x**12*sqrt(1 + b*x**2/a) + 37800*a**(57/2)*b**7*x**14*sqrt(
1 + b*x**2/a) + 14175*a**(55/2)*b**8*x**16*sqrt(1 + b*x**2/a) + 3150*a**(5
3/2)*b**9*x**18*sqrt(1 + b*x**2/a) + 315*a**(51/2)*b**10*x**20*sqrt(1 + b*
x**2/a)) + 10773*a**28*b**2*x**5/(315*a**(71/2)*sqrt(1 + b*x**2/a) + 3150*
a**(69/2)*b*x**2*sqrt(1 + b*x**2/a) + 14175*a**(67/2)*b**2*x**4*sqrt(1 + b
*x**2/a) + 37800*a**(65/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 66150*a**(63/2)*
b**4*x**8*sqrt(1 + b*x**2/a) + 79380*a**(61/2)*b**5*x**10*sqrt(1 + b*x**2/
a) + 66150*a**(59/2)*b**6*x**12*sqrt(1 + b*x**2/a) + 37800*a**(57/2)*b**7*
x**14*sqrt(1 + b*x**2/a) + 14175*a**(55/2)*b**8*x**16*sqrt(1 + b*x**2/a...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 408, normalized size of antiderivative = 1.71

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{11/2}}\,dx = -\frac{Dx^5}{4\,(bx^2+a)^{\frac{9}{2}}b} - \frac{5\,Dax^3}{24\,(bx^2+a)^{\frac{9}{2}}b^2} - \frac{Cx^3}{6\,(bx^2+a)^{\frac{9}{2}}b} + \frac{128\,Ax}{315\,\sqrt{bx^2+a}a^5} + \frac{64\,Ax}{315\,(bx^2+a)^{\frac{3}{2}}a^4} + \frac{16\,Ax}{105\,(bx^2+a)^{\frac{5}{2}}a^3} + \frac{8\,Ax}{63\,(bx^2+a)^{\frac{7}{2}}a^2} + \frac{Ax}{9\,(bx^2+a)^{\frac{9}{2}}a} + \frac{Dx}{84\,(bx^2+a)^{\frac{5}{2}}b^3} + \frac{2\,Dx}{63\,\sqrt{bx^2+a}a^2b^3} + \frac{Dx}{63\,(bx^2+a)^{\frac{3}{2}}ab^3} + \frac{5\,Dax}{504\,(bx^2+a)^{\frac{7}{2}}b^3} - \frac{5\,Da^2x}{72\,(bx^2+a)^{\frac{9}{2}}b^3} + \frac{Cx}{126\,(bx^2+a)^{\frac{7}{2}}b^2} + \frac{8\,Cx}{315\,\sqrt{bx^2+a}a^3b^2} + \frac{4\,Cx}{315\,(bx^2+a)^{\frac{3}{2}}a^2b^2} + \frac{Cx}{105\,(bx^2+a)^{\frac{5}{2}}ab^2} - \frac{Cax}{18\,(bx^2+a)^{\frac{9}{2}}b^2} - \frac{Bx}{9\,(bx^2+a)^{\frac{9}{2}}b} + \frac{16\,Bx}{315\,\sqrt{bx^2+a}a^4b} + \frac{8\,Bx}{315\,(bx^2+a)^{\frac{3}{2}}a^3b} + \frac{2\,Bx}{105\,(bx^2+a)^{\frac{5}{2}}a^2b} + \frac{Bx}{63\,(bx^2+a)^{\frac{7}{2}}ab}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(11/2),x, algorithm="maxima")
```

output
```
-1/4*D*x^5/((b*x^2 + a)^(9/2)*b) - 5/24*D*a*x^3/((b*x^2 + a)^(9/2)*b^2) -
1/6*C*x^3/((b*x^2 + a)^(9/2)*b) + 128/315*A*x/(sqrt(b*x^2 + a)*a^5) + 64/3
15*A*x/((b*x^2 + a)^(3/2)*a^4) + 16/105*A*x/((b*x^2 + a)^(5/2)*a^3) + 8/63
*A*x/((b*x^2 + a)^(7/2)*a^2) + 1/9*A*x/((b*x^2 + a)^(9/2)*a) + 1/84*D*x/((
b*x^2 + a)^(5/2)*b^3) + 2/63*D*x/(sqrt(b*x^2 + a)*a^2*b^3) + 1/63*D*x/((b*
x^2 + a)^(3/2)*a*b^3) + 5/504*D*a*x/((b*x^2 + a)^(7/2)*b^3) - 5/72*D*a^2*x
/((b*x^2 + a)^(9/2)*b^3) + 1/126*C*x/((b*x^2 + a)^(7/2)*b^2) + 8/315*C*x/(
sqrt(b*x^2 + a)*a^3*b^2) + 4/315*C*x/((b*x^2 + a)^(3/2)*a^2*b^2) + 1/105*C
*x/((b*x^2 + a)^(5/2)*a*b^2) - 1/18*C*a*x/((b*x^2 + a)^(9/2)*b^2) - 1/9*B*
x/((b*x^2 + a)^(9/2)*b) + 16/315*B*x/(sqrt(b*x^2 + a)*a^4*b) + 8/315*B*x/(
(b*x^2 + a)^(3/2)*a^3*b) + 2/105*B*x/((b*x^2 + a)^(5/2)*a^2*b) + 1/63*B*x/
((b*x^2 + a)^(7/2)*a*b)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.76

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{11/2}}\,dx = \frac{\left(\left(\left(x^2\left(\frac{2\,(5\,Da^3b^5+4\,Ca^2b^6+8\,Bab^7+64\,Ab^8)x^2}{a^5b^4}+\frac{9\,(5\,Da^4b^4+4\,Ca^3b^5+8\,Ba^2b^6+64\,Aab^7)}{a^5b^4}\right)\right.\right.\right.}{315\,(bx^2+a}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(11/2),x, algorithm="giac")
```

output
```
1/315*(((x^2*(2*(5*D*a^3*b^5 + 4*C*a^2*b^6 + 8*B*a*b^7 + 64*A*b^8)*x^2/(a^
5*b^4) + 9*(5*D*a^4*b^4 + 4*C*a^3*b^5 + 8*B*a^2*b^6 + 64*A*a*b^7)/(a^5*b^4
)) + 63*(C*a^4*b^4 + 2*B*a^3*b^5 + 16*A*a^2*b^6)/(a^5*b^4))*x^2 + 105*(B*a
^4*b^4 + 8*A*a^3*b^5)/(a^5*b^4))*x^2 + 315*A/a)*x/(b*x^2 + a)^(9/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{11/2}}\,dx = \int \frac{A+B\,x^2+C\,x^4+x^6\,D}{(b\,x^2+a)^{11/2}}\,dx$$

input
```
int((A + B*x^2 + C*x^4 + x^6*D)/(a + b*x^2)^(11/2),x)
```

output
```
int((A + B*x^2 + C*x^4 + x^6*D)/(a + b*x^2)^(11/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 471, normalized size of antiderivative = 1.98

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{11/2}}\,dx = \frac{144\sqrt{b\,x^2+a}\,b^8x^9-10\sqrt{b}\,a^7d-144\sqrt{b}\,a^5b^3-144\sqrt{b}\,b^8x^{10}+315\sqrt{b\,x^2+}}{}$$

input
```
int((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(11/2),x)
```

output
```
(315*sqrt(a + b*x**2)*a**4*b**4*x + 945*sqrt(a + b*x**2)*a**3*b**5*x**3 +
63*sqrt(a + b*x**2)*a**3*b**4*c*x**5 + 45*sqrt(a + b*x**2)*a**3*b**4*d*x**
7 + 1134*sqrt(a + b*x**2)*a**2*b**6*x**5 + 36*sqrt(a + b*x**2)*a**2*b**5*c
*x**7 + 10*sqrt(a + b*x**2)*a**2*b**5*d*x**9 + 648*sqrt(a + b*x**2)*a*b**7
*x**7 + 8*sqrt(a + b*x**2)*a*b**6*c*x**9 + 144*sqrt(a + b*x**2)*b**8*x**9
- 10*sqrt(b)*a**7*d - 8*sqrt(b)*a**6*b*c - 50*sqrt(b)*a**6*b*d*x**2 - 144*
sqrt(b)*a**5*b**3 - 40*sqrt(b)*a**5*b**2*c*x**2 - 100*sqrt(b)*a**5*b**2*d*
x**4 - 720*sqrt(b)*a**4*b**4*x**2 - 80*sqrt(b)*a**4*b**3*c*x**4 - 100*sqrt
(b)*a**4*b**3*d*x**6 - 1440*sqrt(b)*a**3*b**5*x**4 - 80*sqrt(b)*a**3*b**4*
c*x**6 - 50*sqrt(b)*a**3*b**4*d*x**8 - 1440*sqrt(b)*a**2*b**6*x**6 - 40*sq
rt(b)*a**2*b**5*c*x**8 - 10*sqrt(b)*a**2*b**5*d*x**10 - 720*sqrt(b)*a*b**7
*x**8 - 8*sqrt(b)*a*b**6*c*x**10 - 144*sqrt(b)*b**8*x**10)/(315*a**4*b**4*
(a**5 + 5*a**4*b*x**2 + 10*a**3*b**2*x**4 + 10*a**2*b**3*x**6 + 5*a*b**4*x
**8 + b**5*x**10))
```

3.141 $\int (a+bx^2)^{3/4}(A+Bx^2+Cx^4+Dx^6)\,dx$

Optimal result

Integrand size = 29, antiderivative size = 268

$$\int (a+bx^2)^{3/4}(A+Bx^2+Cx^4+Dx^6)\,dx = \frac{2a(1989Ab^3-2a(221b^2B-102abC+60a^2D))x}{3315b^3\sqrt[4]{a+bx^2}} + \frac{2\left(1989A-\frac{2a(221b^2B-102abC+60a^2D)}{b^3}\right)x(a+bx^2)^{3/4}}{9945} + \frac{2(221b^2B-102abC+60a^2D)x(a+bx^2)^{7/4}}{1989b^3} + \frac{2(17bC-10aD)x^3(a+bx^2)^{7/4}}{221b^2} + \frac{2Dx^5(a+bx^2)^{7/4}}{17b} - \frac{2a^{3/2}(1989Ab^3-2a(221b^2B-102abC+60a^2D))\sqrt[4]{1+\frac{bx^2}{a}}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{3315b^{7/2}\sqrt[4]{a+bx^2}}$$

output
```
2/3315*a*(1989*A*b^3-2*a*(221*B*b^2-102*C*a*b+60*D*a^2))*x/b^3/(b*x^2+a)^(
1/4)+2/9945*(1989*A-2*a*(221*B*b^2-102*C*a*b+60*D*a^2)/b^3)*x*(b*x^2+a)^(3
/4)+2/1989*(221*B*b^2-102*C*a*b+60*D*a^2)*x*(b*x^2+a)^(7/4)/b^3+2/221*(17*
C*b-10*D*a)*x^3*(b*x^2+a)^(7/4)/b^2+2/17*D*x^5*(b*x^2+a)^(7/4)/b-2/3315*a^
(3/2)*(1989*A*b^3-2*a*(221*B*b^2-102*C*a*b+60*D*a^2))*(1+b*x^2/a)^(1/4)*El
lipticE(sin(1/2*arctan(b^(1/2)*x/a^(1/2))),2^(1/2))/b^(7/2)/(b*x^2+a)^(1/4
)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.67 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.49

$$\int (a+bx^2)^{3/4} (A+Bx^2+Cx^4+Dx^6)\ dx = \frac{x(a+bx^2)^{3/4}\left(2(a+bx^2)(60a^2D-6ab(17C+15Dx^2)+b^2(221B+153Cx^2+117Dx^4))+\right.}{1989b^3}$$

input
```
Integrate[(a + b*x^2)^(3/4)*(A + B*x^2 + C*x^4 + D*x^6),x]
```

output
```
(x*(a + b*x^2)^(3/4)*(2*(a + b*x^2)*(60*a^2*D - 6*a*b*(17*C + 15*D*x^2) +
b^2*(221*B + 153*C*x^2 + 117*D*x^4)) + ((1989*A*b^3 - 2*a*(221*b^2*B - 102
*a*b*C + 60*a^2*D))*Hypergeometric2F1[-3/4, 1/2, 3/2, -((b*x^2)/a)])/(1 +
(b*x^2)/a)^(3/4)))/(1989*b^3)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.88, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.310$, Rules used = {2346, 27, 1473, 27, 299, 211, 227, 225, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^{3/4} \left(A+Bx^2+Cx^4+Dx^6\right) \, dx$$

$$\Big\downarrow \; 2346$$

$$\frac{2 \int \frac{1}{2} \left(bx^2+a\right)^{3/4} \left((17bC-10aD)x^4+17bBx^2+17Ab\right) dx}{17b}+\frac{2Dx^5 \left(a+bx^2\right)^{7/4}}{17b}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \left(bx^2+a\right)^{3/4} \left((17bC-10aD)x^4+17bBx^2+17Ab\right) dx}{17b}+\frac{2Dx^5 \left(a+bx^2\right)^{7/4}}{17b}$$

$$\Big\downarrow \; 1473$$

$$\frac{\frac{2 \int \frac{1}{2} \left(bx^2+a\right)^{3/4} \left(221Ab^2+\left(60Da^2-102bCa+221b^2B\right)x^2\right) dx}{13b}+\frac{2x^3 \left(a+bx^2\right)^{7/4} (17bC-10aD)}{13b}}{17b}+\frac{2Dx^5 \left(a+bx^2\right)^{7/4}}{17b}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \left(bx^2+a\right)^{3/4} \left(221Ab^2+\left(60Da^2-102bCa+221b^2B\right)x^2\right) dx}{13b}+\frac{2x^3 \left(a+bx^2\right)^{7/4} (17bC-10aD)}{13b}}{17b}+\frac{2Dx^5 \left(a+bx^2\right)^{7/4}}{17b}$$

$$\Big\downarrow \; 299$$

$$\frac{\frac{\frac{\left(1989Ab^3-2a\left(60a^2D-102abC+221b^2B\right)\right) \int \left(bx^2+a\right)^{3/4} dx}{9b}+\frac{2x\left(a+bx^2\right)^{7/4} \left(60a^2D-102abC+221b^2B\right)}{9b}}{13b}+\frac{2x^3 \left(a+bx^2\right)^{7/4} (17bC-10aD)}{13b}}{17b}+\frac{2Dx^5 \left(a+bx^2\right)^{7/4}}{17b}$$

$$\Big\downarrow \; 211$$

$$\frac{\frac{\frac{\left(1989Ab^3-2a\left(60a^2D-102abC+221b^2B\right)\right) \left(\frac{3}{5} a \int \frac{1}{\sqrt[4]{bx^2+a}} dx+\frac{2}{5} x \left(a+bx^2\right)^{3/4}\right)}{9b}+\frac{2x\left(a+bx^2\right)^{7/4} \left(60a^2D-102abC+221b^2B\right)}{9b}}{13b}+\frac{2x^3 \left(a+bx^2\right)^{7/4} (17b}{13b}}{17b}$$

$$\frac{2Dx^5 \left(a+bx^2\right)^{7/4}}{17b}$$

$$\Big\downarrow \; 227$$

$$\frac{\frac{\frac{\left(1989Ab^3-2a\left(60a^2D-102abC+221b^2B\right)\right)\left(\frac{3a\sqrt[4]{\frac{bx^2}{a}+1}\int \frac{1}{\sqrt[4]{\frac{bx^2}{a}+1}}dx}{5\sqrt[4]{a+bx^2}}+\frac{2}{5}x\left(a+bx^2\right)^{3/4}\right)}{9b}+\frac{2x\left(a+bx^2\right)^{7/4}\left(60a^2D-102abC+221b^2B\right)}{9b}}{13b}+\frac{2x^3(\cdots}{\cdots}}{17b}$$

$$\frac{2Dx^5\left(a+bx^2\right)^{7/4}}{17b}$$

$\downarrow$ 225

$$\frac{\frac{\frac{\left(1989Ab^3-2a\left(60a^2D-102abC+221b^2B\right)\right)\left(\frac{3a\sqrt[4]{\frac{bx^2}{a}+1}\left(\frac{2x}{\sqrt[4]{\frac{bx^2}{a}+1}}-\int \frac{1}{\left(\frac{bx^2}{a}+1\right)^{5/4}}dx\right)}{5\sqrt[4]{a+bx^2}}+\frac{2}{5}x\left(a+bx^2\right)^{3/4}\right)}{9b}+\frac{2x\left(a+bx^2\right)^{7/4}\left(60a^2D-102abC+\cdots\right.}{9b}}{13b}}{17b}$$

$$\frac{2Dx^5\left(a+bx^2\right)^{7/4}}{17b}$$

$\downarrow$ 212

$$\frac{\frac{\frac{\left(\frac{3a\sqrt[4]{\frac{bx^2}{a}+1}\left(\frac{2x}{\sqrt[4]{\frac{bx^2}{a}+1}}-\frac{2\sqrt{a}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}}\right)}{5\sqrt[4]{a+bx^2}}+\frac{2}{5}x\left(a+bx^2\right)^{3/4}\right)\left(1989Ab^3-2a\left(60a^2D-102abC+221b^2B\right)\right)}{9b}+\frac{2x\left(a+bx^2\right)^{7/4}\left(60a^2D-\cdots\right.}{9b}}{13b}}{17b}$$

$$\frac{2Dx^5\left(a+bx^2\right)^{7/4}}{17b}$$

input
```
Int[(a + b*x^2)^(3/4)*(A + B*x^2 + C*x^4 + D*x^6),x]
```

output
```
(2*D*x^5*(a + b*x^2)^(7/4))/(17*b) + ((2*(17*b*C - 10*a*D)*x^3*(a + b*x^2)
^(7/4))/(13*b) + ((2*(221*b^2*B - 102*a*b*C + 60*a^2*D)*x*(a + b*x^2)^(7/4
))/(9*b) + ((1989*A*b^3 - 2*a*(221*b^2*B - 102*a*b*C + 60*a^2*D))*((2*x*(a
 + b*x^2)^(3/4))/5 + (3*a*(1 + (b*x^2)/a)^(1/4)*((2*x)/(1 + (b*x^2)/a)^(1/
4) - (2*Sqrt[a]*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/Sqrt[b]))/(5*
(a + b*x^2)^(1/4))))/(9*b))/(13*b))/(17*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 225
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Simp[a   Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[
a, 0] && PosQ[b/a]
```

rule 227
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4)   Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1473
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[c^p*x^(4*p - 1)*((d + e*x^2)^(q + 1)/(e*(4*p + 2*q + 1)))
, x] + Simp[1/(e*(4*p + 2*q + 1))   Int[(d + e*x^2)^q*ExpandToSum[e*(4*p +
2*q + 1)*(a + b*x^2 + c*x^4)^p - d*c^p*(4*p - 1)*x^(4*p - 2) - e*c^p*(4*p +
 2*q + 1)*x^(4*p), x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] &&  !LtQ[q, -1]
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Maple [F]

$$\int \left(b\,x^{2}+a\right)^{\frac{3}{4}} \left(D x^{6}+C\,x^{4}+x^{2} B+A\right)dx$$

input
```
int((b*x^2+a)^(3/4)*(D*x^6+C*x^4+B*x^2+A),x)
```

output
```
int((b*x^2+a)^(3/4)*(D*x^6+C*x^4+B*x^2+A),x)
```

Fricas [F]

$$\int \left(a+bx^2\right)^{3/4}\left(A+Bx^2+Cx^4+Dx^6\right)\,dx=\int \left(Dx^{6}+Cx^{4}+Bx^{2}+A\right)\left(bx^{2}+a\right)^{\frac{3}{4}}\,dx$$

input
```
integrate((b*x^2+a)^(3/4)*(D*x^6+C*x^4+B*x^2+A),x, algorithm="fricas")
```

output
```
integral((D*x^6 + C*x^4 + B*x^2 + A)*(b*x^2 + a)^(3/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.08 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.46

$$\int (a+bx^2)^{3/4}(A+Bx^2+Cx^4+Dx^6)\,dx = Aa^{\frac{3}{4}}x\,{}_2F_1\left(\begin{matrix}-\frac{3}{4},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right) + \frac{Ba^{\frac{3}{4}}x^3\,{}_2F_1\left(\begin{matrix}-\frac{3}{4},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{3} + \frac{Ca^{\frac{3}{4}}x^5\,{}_2F_1\left(\begin{matrix}-\frac{3}{4},\frac{5}{2}\\ \frac{7}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{5} + \frac{Da^{\frac{3}{4}}x^7\,{}_2F_1\left(\begin{matrix}-\frac{3}{4},\frac{7}{2}\\ \frac{9}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{7}$$

input
```
integrate((b*x**2+a)**(3/4)*(D*x**6+C*x**4+B*x**2+A),x)
```

output
```
A*a**(3/4)*x*hyper((-3/4, 1/2), (3/2,), b*x**2*exp_polar(I*pi)/a) + B*a**(
3/4)*x**3*hyper((-3/4, 3/2), (5/2,), b*x**2*exp_polar(I*pi)/a)/3 + C*a**(3
/4)*x**5*hyper((-3/4, 5/2), (7/2,), b*x**2*exp_polar(I*pi)/a)/5 + D*a**(3/
4)*x**7*hyper((-3/4, 7/2), (9/2,), b*x**2*exp_polar(I*pi)/a)/7
```

Maxima [F]

$$\int (a+bx^2)^{3/4}(A+Bx^2+Cx^4+Dx^6)\,dx = \int (Dx^6+Cx^4+Bx^2+A)(bx^2+a)^{\frac{3}{4}}\,dx$$

input
```
integrate((b*x^2+a)^(3/4)*(D*x^6+C*x^4+B*x^2+A),x, algorithm="maxima")
```

output
```
integrate((D*x^6 + C*x^4 + B*x^2 + A)*(b*x^2 + a)^(3/4), x)
```

Giac [F]

$$\int (a+bx^2)^{3/4} (A+Bx^2+Cx^4+Dx^6) \, dx = \int (Dx^6+Cx^4+Bx^2+A)(bx^2+a)^{\frac{3}{4}} \, dx$$

input
```
integrate((b*x^2+a)^(3/4)*(D*x^6+C*x^4+B*x^2+A),x, algorithm="giac")
```

output
```
integrate((D*x^6 + C*x^4 + B*x^2 + A)*(b*x^2 + a)^(3/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx^2)^{3/4} (A+Bx^2+Cx^4+Dx^6) \, dx = \int (b\,x^2+a)^{3/4} (A+B\,x^2+C\,x^4+x^6\,D) \, dx$$

input
```
int((a + b*x^2)^(3/4)*(A + B*x^2 + C*x^4 + x^6*D),x)
```

output
```
int((a + b*x^2)^(3/4)*(A + B*x^2 + C*x^4 + x^6*D), x)
```

Reduce [F]

$$\int (a+bx^2)^{3/4} (A+Bx^2+Cx^4+Dx^6) \, dx = \frac{360(b\,x^2+a)^{\frac{3}{4}}\,a^3dx - 612(b\,x^2+a)^{\frac{3}{4}}\,a^2bcx - 300(b\,x^2+a)^{\frac{3}{4}}\,a^2bd\,x^3 + 5304(b\,x^2+a)^{\frac{3}{4}}\,a\,b^3x}{}$$

input
```
int((b*x^2+a)^(3/4)*(D*x^6+C*x^4+B*x^2+A),x)
```

output
```
(360*(a + b*x**2)**(3/4)*a**3*d*x - 612*(a + b*x**2)**(3/4)*a**2*b*c*x - 3
00*(a + b*x**2)**(3/4)*a**2*b*d*x**3 + 5304*(a + b*x**2)**(3/4)*a*b**3*x +
 510*(a + b*x**2)**(3/4)*a*b**2*c*x**3 + 270*(a + b*x**2)**(3/4)*a*b**2*d*
x**5 + 2210*(a + b*x**2)**(3/4)*b**4*x**3 + 1530*(a + b*x**2)**(3/4)*b**3*
c*x**5 + 1170*(a + b*x**2)**(3/4)*b**3*d*x**7 - 360*int((a + b*x**2)**(3/4
)/(a + b*x**2),x)*a**4*d + 612*int((a + b*x**2)**(3/4)/(a + b*x**2),x)*a**
3*b*c + 4641*int((a + b*x**2)**(3/4)/(a + b*x**2),x)*a**2*b**3)/(9945*b**3
)
```

3.142 $\int \sqrt[4]{a+bx^2}\left(A+Bx^2+Cx^4+Dx^6\right)\,dx$

Optimal result

Integrand size = 29, antiderivative size = 220

$$\begin{aligned}\int \sqrt[4]{a+bx^2}\left(A+Bx^2+Cx^4+Dx^6\right)\,dx &= \frac{2}{231}\left(77A-\frac{2a(11b^2B-6abC+4a^2D)}{b^3}\right)x\sqrt[4]{a+bx^2} \\ &\quad+\frac{2(11b^2B-6abC+4a^2D)\,x(a+bx^2)^{5/4}}{77b^3} \\ &\quad+\frac{2(3bC-2aD)x^3(a+bx^2)^{5/4}}{33b^2}+\frac{2Dx^5(a+bx^2)^{5/4}}{15b} \\ &\quad+\frac{2a^{3/2}(77Ab^3-2a(11b^2B-6abC+4a^2D))\left(1+\frac{bx^2}{a}\right)^{3/4}\operatorname{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{231b^{7/2}\left(a+bx^2\right)^{3/4}}\end{aligned}$$

output
```
2/231*(77*A-2*a*(11*B*b^2-6*C*a*b+4*D*a^2)/b^3)*x*(b*x^2+a)^(1/4)+2/77*(11
*B*b^2-6*C*a*b+4*D*a^2)*x*(b*x^2+a)^(5/4)/b^3+2/33*(3*C*b-2*D*a)*x^3*(b*x^
2+a)^(5/4)/b^2+2/15*D*x^5*(b*x^2+a)^(5/4)/b+2/231*a^(3/2)*(77*A*b^3-2*a*(1
1*B*b^2-6*C*a*b+4*D*a^2))*(1+b*x^2/a)^(3/4)*InverseJacobiAM(1/2*arctan(b^(
1/2)*x/a^(1/2)),2^(1/2))/b^(7/2)/(b*x^2+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.48 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.60

$$\int \sqrt[4]{a+bx^2}\left(A+Bx^2+Cx^4+Dx^6\right)\,dx$$
$$=\frac{x\sqrt[4]{a+bx^2}\left(2(a+bx^2)\left(60a^2D-10ab(9C+7Dx^2)+b^2(165B+105Cx^2+77Dx^4)\right)+\frac{15(77Ab^3-2a(11b^2B\ldots}{\ldots}\right.}{1155b^3}$$

input
```
Integrate[(a + b*x^2)^(1/4)*(A + B*x^2 + C*x^4 + D*x^6),x]
```

output
```
(x*(a + b*x^2)^(1/4)*(2*(a + b*x^2)*(60*a^2*D - 10*a*b*(9*C + 7*D*x^2) + b
^2*(165*B + 105*C*x^2 + 77*D*x^4)) + (15*(77*A*b^3 - 2*a*(11*b^2*B - 6*a*b
*C + 4*a^2*D))*Hypergeometric2F1[-1/4, 1/2, 3/2, -((b*x^2)/a)])/(1 + (b*x^
2)/a)^(1/4)))/(1155*b^3)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2346, 27, 1473, 27, 299, 211, 231, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[4]{a+bx^2}\left(A+Bx^2+Cx^4+Dx^6\right)\,dx$$

$$\Big\downarrow \text{2346}$$

$$\frac{2\int \frac{5}{2}\sqrt[4]{bx^2+a}\left((3bC-2aD)x^4+3bBx^2+3Ab\right)dx}{15b}+\frac{2Dx^5\left(a+bx^2\right)^{5/4}}{15b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \sqrt[4]{bx^2+a}\left((3bC-2aD)x^4+3bBx^2+3Ab\right)dx}{3b}+\frac{2Dx^5\left(a+bx^2\right)^{5/4}}{15b}$$

$$\Big\downarrow \; 1473$$

$$\frac{\frac{2\int \frac{3}{2}\sqrt[4]{bx^2+a}\left(11Ab^2+\left(4Da^2-6bCa+11b^2B\right)x^2\right)dx}{11b}+\frac{2x^3\left(a+bx^2\right)^{5/4}(3bC-2aD)}{11b}}{3b}+\frac{2Dx^5\left(a+bx^2\right)^{5/4}}{15b}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{3\int \sqrt[4]{bx^2+a}\left(11Ab^2+\left(4Da^2-6bCa+11b^2B\right)x^2\right)dx}{11b}+\frac{2x^3\left(a+bx^2\right)^{5/4}(3bC-2aD)}{11b}}{3b}+\frac{2Dx^5\left(a+bx^2\right)^{5/4}}{15b}$$

$$\Big\downarrow \; 299$$

$$\frac{\frac{3\left(\frac{\left(77Ab^3-2a\left(4a^2D-6abC+11b^2B\right)\right)\int \sqrt[4]{bx^2+a}dx}{7b}+\frac{2x\left(a+bx^2\right)^{5/4}\left(4a^2D-6abC+11b^2B\right)}{7b}\right)}{11b}+\frac{2x^3\left(a+bx^2\right)^{5/4}(3bC-2aD)}{11b}}{3b}+\frac{2Dx^5\left(a+bx^2\right)^{5/4}}{15b}$$

$$\Big\downarrow \; 211$$

$$\frac{\frac{3\left(\frac{\left(77Ab^3-2a\left(4a^2D-6abC+11b^2B\right)\right)\left(\frac{1}{3}a\int \frac{1}{\left(bx^2+a\right)^{3/4}}dx+\frac{2}{3}x\sqrt[4]{a+bx^2}\right)}{7b}+\frac{2x\left(a+bx^2\right)^{5/4}\left(4a^2D-6abC+11b^2B\right)}{7b}\right)}{11b}+\frac{2x^3\left(a+bx^2\right)^{5/4}(3bC-2aD)}{11b}}{3b}+\frac{2Dx^5\left(a+bx^2\right)^{5/4}}{15b}$$

$$\Big\downarrow \; 231$$

$$\frac{\frac{3\left(\frac{\left(77Ab^3-2a\left(4a^2D-6abC+11b^2B\right)\right)\left(\frac{a\left(\frac{bx^2}{a}+1\right)^{3/4}\int \frac{1}{\left(\frac{bx^2}{a}+1\right)^{3/4}}dx}{3\left(a+bx^2\right)^{3/4}}+\frac{2}{3}x\sqrt[4]{a+bx^2}\right)}{7b}+\frac{2x\left(a+bx^2\right)^{5/4}\left(4a^2D-6abC+11b^2B\right)}{7b}\right)}{11b}+\frac{2x^3\left(a+bx^2\right)^{5/4}(3bC-2aD)}{11b}}{3b}+\frac{2Dx^5\left(a+bx^2\right)^{5/4}}{15b}$$

$$\Big\downarrow \text{229}$$

$$\frac{3\left(\frac{2x\left(a+bx^2\right)^{5/4}\left(4a^2D-6abC+11b^2B\right)}{7b}+\frac{\left(\frac{2a^{3/2}\left(\frac{bx^2}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{3\sqrt{b}\left(a+bx^2\right)^{3/4}}+\frac{2}{3}x\sqrt[4]{a+bx^2}\right)\left(77Ab^3-2a\left(4a^2D-6abC+11b^2B\right)\right)}{7b}\right)}{11b}$$

$$3b$$

$$\frac{2Dx^5\left(a+bx^2\right)^{5/4}}{15b}$$

input
```
Int[(a + b*x^2)^(1/4)*(A + B*x^2 + C*x^4 + D*x^6),x]
```

output
```
(2*D*x^5*(a + b*x^2)^(5/4))/(15*b) + ((2*(3*b*C - 2*a*D)*x^3*(a + b*x^2)^(
5/4))/(11*b) + (3*((2*(11*b^2*B - 6*a*b*C + 4*a^2*D)*x*(a + b*x^2)^(5/4))/
(7*b) + ((77*A*b^3 - 2*a*(11*b^2*B - 6*a*b*C + 4*a^2*D))*((2*x*(a + b*x^2)
^(1/4))/3 + (2*a^(3/2)*(1 + (b*x^2)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x)/
Sqrt[a]]/2, 2])/(3*Sqrt[b]*(a + b*x^2)^(3/4))))/(7*b)))/(11*b))/(3*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1473
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[c^p*x^(4*p - 1)*((d + e*x^2)^(q + 1)/(e*(4*p + 2*q + 1)))
, x] + Simp[1/(e*(4*p + 2*q + 1))   Int[(d + e*x^2)^q*ExpandToSum[e*(4*p +
2*q + 1)*(a + b*x^2 + c*x^4)^p - d*c^p*(4*p - 1)*x^(4*p - 2) - e*c^p*(4*p +
 2*q + 1)*x^(4*p), x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] &&  !LtQ[q, -1]
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Maple [F]

$$\int \left(b\,x^{2}+a \right)^{\frac{1}{4}} \left(D x^{6}+C\,x^{4}+x^{2} B +A \right)d x$$

input
```
int((b*x^2+a)^(1/4)*(D*x^6+C*x^4+B*x^2+A),x)
```

output
```
int((b*x^2+a)^(1/4)*(D*x^6+C*x^4+B*x^2+A),x)
```

Fricas [F]

$$\int \sqrt[4]{a+bx^2}\left(A+Bx^2+Cx^4+Dx^6\right)\,dx=\int \left(Dx^6+Cx^4+Bx^2+A\right)\left(bx^2+a\right)^{\frac{1}{4}}\,dx$$

input
```
integrate((b*x^2+a)^(1/4)*(D*x^6+C*x^4+B*x^2+A),x, algorithm="fricas")
```

output
```
integral((D*x^6 + C*x^4 + B*x^2 + A)*(b*x^2 + a)^(1/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.71 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.56

$$\int \sqrt[4]{a+bx^2}\left(A+Bx^2+Cx^4+Dx^6\right)\,dx=A\sqrt[4]{a}x\,{}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{1}{2}\\\frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)+\frac{B\sqrt[4]{a}x^3\,{}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{3}{2}\\\frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{3}+\frac{C\sqrt[4]{a}x^5\,{}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{5}{2}\\\frac{7}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{5}+\frac{D\sqrt[4]{a}x^7\,{}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{7}{2}\\\frac{9}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{7}$$

input
```
integrate((b*x**2+a)**(1/4)*(D*x**6+C*x**4+B*x**2+A),x)
```

output
```
A*a**(1/4)*x*hyper((-1/4, 1/2), (3/2,), b*x**2*exp_polar(I*pi)/a) + B*a**(
1/4)*x**3*hyper((-1/4, 3/2), (5/2,), b*x**2*exp_polar(I*pi)/a)/3 + C*a**(1
/4)*x**5*hyper((-1/4, 5/2), (7/2,), b*x**2*exp_polar(I*pi)/a)/5 + D*a**(1/
4)*x**7*hyper((-1/4, 7/2), (9/2,), b*x**2*exp_polar(I*pi)/a)/7
```

Maxima [F]

$$\int \sqrt[4]{a+bx^2}\left(A+Bx^2+Cx^4+Dx^6\right)\,dx=\int \left(Dx^6+Cx^4+Bx^2+A\right)\left(bx^2+a\right)^{\frac{1}{4}}\,dx$$

input
```
integrate((b*x^2+a)^(1/4)*(D*x^6+C*x^4+B*x^2+A),x, algorithm="maxima")
```

output
```
integrate((D*x^6 + C*x^4 + B*x^2 + A)*(b*x^2 + a)^(1/4), x)
```

Giac [F]

$$\int \sqrt[4]{a+bx^2}\left(A+Bx^2+Cx^4+Dx^6\right)\,dx=\int \left(Dx^6+Cx^4+Bx^2+A\right)\left(bx^2+a\right)^{\frac{1}{4}}\,dx$$

input
```
integrate((b*x^2+a)^(1/4)*(D*x^6+C*x^4+B*x^2+A),x, algorithm="giac")
```

output
```
integrate((D*x^6 + C*x^4 + B*x^2 + A)*(b*x^2 + a)^(1/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt[4]{a+bx^2}\left(A+Bx^2+Cx^4+Dx^6\right)\,dx=\int {\left(b\,x^2+a\right)}^{1/4}\,\left(A+B\,x^2+C\,x^4+x^6\,D\right)\,dx$$

input
```
int((a + b*x^2)^(1/4)*(A + B*x^2 + C*x^4 + x^6*D),x)
```

output
```
int((a + b*x^2)^(1/4)*(A + B*x^2 + C*x^4 + x^6*D), x)
```

Reduce [F]

$$\int \sqrt[4]{a+bx^2}\left(A+Bx^2+Cx^4+Dx^6\right)\,dx$$

$$= \frac{40(b\,x^2+a)^{\frac{1}{4}}\,a^3dx-60(b\,x^2+a)^{\frac{1}{4}}\,a^2bcx-20(b\,x^2+a)^{\frac{1}{4}}\,a^2bd\,x^3+880(b\,x^2+a)^{\frac{1}{4}}\,a\,b^3x+30(b\,x^2+a)^{\frac{1}{4}}}{}$$

input
```
int((b*x^2+a)^(1/4)*(D*x^6+C*x^4+B*x^2+A),x)
```

output
```
(40*(a + b*x**2)**(1/4)*a**3*d*x - 60*(a + b*x**2)**(1/4)*a**2*b*c*x - 20*
(a + b*x**2)**(1/4)*a**2*b*d*x**3 + 880*(a + b*x**2)**(1/4)*a*b**3*x + 30*
(a + b*x**2)**(1/4)*a*b**2*c*x**3 + 14*(a + b*x**2)**(1/4)*a*b**2*d*x**5 +
 330*(a + b*x**2)**(1/4)*b**4*x**3 + 210*(a + b*x**2)**(1/4)*b**3*c*x**5 +
 154*(a + b*x**2)**(1/4)*b**3*d*x**7 - 40*int((a + b*x**2)**(1/4)/(a + b*x
**2),x)*a**4*d + 60*int((a + b*x**2)**(1/4)/(a + b*x**2),x)*a**3*b*c + 275
*int((a + b*x**2)**(1/4)/(a + b*x**2),x)*a**2*b**3)/(1155*b**3)
```

3.143 $\int \frac{A+Bx^2+Cx^4+Dx^6}{\sqrt[4]{a+bx^2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 221

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{\sqrt[4]{a+bx^2}}\,dx = \frac{2(195Ab^3-78ab^2B+52a^2bC-40a^3D)\,x}{195b^3\sqrt[4]{a+bx^2}} + \frac{2(39b^2B-26abC+20a^2D)\,x(a+bx^2)^{3/4}}{195b^3} + \frac{2(13bC-10aD)x^3(a+bx^2)^{3/4}}{117b^2} + \frac{2Dx^5(a+bx^2)^{3/4}}{13b} - \frac{2\sqrt{a}(195Ab^3-78ab^2B+52a^2bC-40a^3D)\sqrt[4]{1+\frac{bx^2}{a}}\,E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{195b^{7/2}\sqrt[4]{a+bx^2}}$$

output

```
2/195*(195*A*b^3-78*B*a*b^2+52*C*a^2*b-40*D*a^3)*x/b^3/(b*x^2+a)^(1/4)+2/1
95*(39*B*b^2-26*C*a*b+20*D*a^2)*x*(b*x^2+a)^(3/4)/b^3+2/117*(13*C*b-10*D*a
)*x^3*(b*x^2+a)^(3/4)/b^2+2/13*D*x^5*(b*x^2+a)^(3/4)/b-2/195*a^(1/2)*(195*
A*b^3-78*B*a*b^2+52*C*a^2*b-40*D*a^3)*(1+b*x^2/a)^(1/4)*EllipticE(sin(1/2*
arctan(b^(1/2)*x/a^(1/2))),2^(1/2))/b^(7/2)/(b*x^2+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.11 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.60

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{\sqrt[4]{a+bx^2}}\,dx$$

$$=\frac{x\left(2(a+bx^2)\left(60a^2D-2ab(39C+25Dx^2)+b^2(117B+65Cx^2+45Dx^4)\right)+3(195Ab^3-78ab^2B+52\right.}{585b^3\sqrt[4]{a+bx^2}}$$

input
```
Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(a + b*x^2)^(1/4),x]
```

output
```
(x*(2*(a + b*x^2)*(60*a^2*D - 2*a*b*(39*C + 25*D*x^2) + b^2*(117*B + 65*C*
x^2 + 45*D*x^4)) + 3*(195*A*b^3 - 78*a*b^2*B + 52*a^2*b*C - 40*a^3*D)*(1 +
 (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2, -((b*x^2)/a)]))/(585*b^
3*(a + b*x^2)^(1/4))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.97, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2346, 27, 1473, 27, 299, 227, 225, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{\sqrt[4]{a+bx^2}}\,dx$$

$$\Big\downarrow \text{2346}$$

$$\frac{2\int \frac{(13bC-10aD)x^4+13bBx^2+13Ab}{2\sqrt[4]{bx^2+a}}dx}{13b}+\frac{2Dx^5\left(a+bx^2\right)^{3/4}}{13b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(13bC-10aD)x^4+13bBx^2+13Ab}{\sqrt[4]{bx^2+a}}dx}{13b}+\frac{2Dx^5\left(a+bx^2\right)^{3/4}}{13b}$$

$$\Big\downarrow \; 1473$$

$$\frac{\frac{2\int \frac{3\left(39Ab^2+\left(20Da^2-26bCa+39b^2B\right)x^2\right)}{2\sqrt[4]{bx^2+a}}dx}{9b}+\frac{2x^3\left(a+bx^2\right)^{3/4}(13bC-10aD)}{9b}}{13b}+\frac{2Dx^5\left(a+bx^2\right)^{3/4}}{13b}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{39Ab^2+\left(20Da^2-26bCa+39b^2B\right)x^2}{\sqrt[4]{bx^2+a}}dx}{3b}+\frac{2x^3\left(a+bx^2\right)^{3/4}(13bC-10aD)}{9b}}{13b}+\frac{2Dx^5\left(a+bx^2\right)^{3/4}}{13b}$$

$$\Big\downarrow \; 299$$

$$\frac{\frac{\frac{\left(-40a^3D+52a^2bC-78ab^2B+195Ab^3\right)\int \frac{1}{\sqrt[4]{bx^2+a}}dx}{5b}+\frac{2x\left(a+bx^2\right)^{3/4}\left(20a^2D-26abC+39b^2B\right)}{5b}}{3b}+\frac{2x^3\left(a+bx^2\right)^{3/4}(13bC-10aD)}{9b}}{13b}+\frac{2Dx^5\left(a+bx^2\right)^{3/4}}{13b}$$

$$\Big\downarrow \; 227$$

$$\frac{\frac{\frac{\sqrt[4]{\frac{bx^2}{a}+1}\left(-40a^3D+52a^2bC-78ab^2B+195Ab^3\right)\int \frac{1}{\sqrt[4]{\frac{bx^2}{a}+1}}dx}{5b\sqrt[4]{a+bx^2}}+\frac{2x\left(a+bx^2\right)^{3/4}\left(20a^2D-26abC+39b^2B\right)}{5b}}{3b}+\frac{2x^3\left(a+bx^2\right)^{3/4}(13bC-10aD)}{9b}}{13b}+\frac{2Dx^5\left(a+bx^2\right)^{3/4}}{13b}$$

$$\Big\downarrow \; 225$$

$$\frac{\frac{\frac{\sqrt[4]{\frac{bx^2}{a}+1}\left(-40a^3D+52a^2bC-78ab^2B+195Ab^3\right)\left(\frac{2x}{\sqrt[4]{\frac{bx^2}{a}+1}}-\int \frac{1}{\left(\frac{bx^2}{a}+1\right)^{5/4}}dx\right)}{5b\sqrt[4]{a+bx^2}}+\frac{2x\left(a+bx^2\right)^{3/4}\left(20a^2D-26abC+39b^2B\right)}{5b}}{3b}+\frac{2x^3\left(a+bx^2\right)^{3/4}(13bC-10aD)}{9b}}{13b}+\frac{2Dx^5\left(a+bx^2\right)^{3/4}}{13b}$$

$$\Big\downarrow \text{212}$$

$$\frac{\frac{\frac{2x\left(a+bx^2\right)^{3/4}\left(20a^2D-26abC+39b^2B\right)}{5b}+\frac{\sqrt[4]{\frac{bx^2}{a}+1}\left(\frac{2x}{\sqrt[4]{\frac{bx^2}{a}+1}}-\frac{2\sqrt{a}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}}\right)\left(-40a^3D+52a^2bC-78ab^2B+195Ab^3\right)}{5b\sqrt[4]{a+bx^2}}}{3b}+\frac{2x^3(a}{}}{13b}$$

$$\frac{2Dx^5\left(a+bx^2\right)^{3/4}}{13b}$$

input
```
Int[(A + B*x^2 + C*x^4 + D*x^6)/(a + b*x^2)^(1/4),x]
```

output
```
(2*D*x^5*(a + b*x^2)^(3/4))/(13*b) + ((2*(13*b*C - 10*a*D)*x^3*(a + b*x^2)
^(3/4))/(9*b) + ((2*(39*b^2*B - 26*a*b*C + 20*a^2*D)*x*(a + b*x^2)^(3/4))/
(5*b) + ((195*A*b^3 - 78*a*b^2*B + 52*a^2*b*C - 40*a^3*D)*(1 + (b*x^2)/a)^
(1/4)*((2*x)/(1 + (b*x^2)/a)^(1/4) - (2*Sqrt[a]*EllipticE[ArcTan[(Sqrt[b]*
x)/Sqrt[a]]/2, 2])/Sqrt[b]))/(5*b*(a + b*x^2)^(1/4)))/(3*b))/(13*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 225
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Simp[a   Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[
a, 0] && PosQ[b/a]
```

rule 227
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4)   Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1473
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[c^p*x^(4*p - 1)*((d + e*x^2)^(q + 1)/(e*(4*p + 2*q + 1)))
, x] + Simp[1/(e*(4*p + 2*q + 1))   Int[(d + e*x^2)^q*ExpandToSum[e*(4*p +
2*q + 1)*(a + b*x^2 + c*x^4)^p - d*c^p*(4*p - 1)*x^(4*p - 2) - e*c^p*(4*p +
 2*q + 1)*x^(4*p), x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] &&  !LtQ[q, -1]
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Maple [F]

$$\int \frac{Dx^6 + C\,x^4 + x^2 B + A}{\left(b\,x^2 + a\right)^{\frac{1}{4}}}dx$$

input
```
int((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(1/4),x)
```

output
```
int((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(1/4),x)
```

Fricas [F]

$$\int \frac{A + Bx^2 + Cx^4 + Dx^6}{\sqrt[4]{a + bx^2}}\,dx = \int \frac{Dx^6 + Cx^4 + Bx^2 + A}{(bx^2 + a)^{\frac{1}{4}}}\,dx$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(1/4),x, algorithm="fricas")
```

output
```
integral((D*x^6 + C*x^4 + B*x^2 + A)/(b*x^2 + a)^(1/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.55 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.53

$$\int \frac{A + Bx^2 + Cx^4 + Dx^6}{\sqrt[4]{a + bx^2}}\,dx = \frac{Ax\,{}_2F_1\left(\begin{matrix}\frac{1}{4}, \frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{\sqrt[4]{a}} + \frac{Bx^3\,{}_2F_1\left(\begin{matrix}\frac{1}{4}, \frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{3\sqrt[4]{a}} + \frac{Cx^5\,{}_2F_1\left(\begin{matrix}\frac{1}{4}, \frac{5}{2}\\ \frac{7}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{5\sqrt[4]{a}} + \frac{Dx^7\,{}_2F_1\left(\begin{matrix}\frac{1}{4}, \frac{7}{2}\\ \frac{9}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{7\sqrt[4]{a}}$$

input
```
integrate((D*x**6+C*x**4+B*x**2+A)/(b*x**2+a)**(1/4),x)
```

output
```
A*x*hyper((1/4, 1/2), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(1/4) + B*x**3*
hyper((1/4, 3/2), (5/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(1/4)) + C*x**5*
hyper((1/4, 5/2), (7/2,), b*x**2*exp_polar(I*pi)/a)/(5*a**(1/4)) + D*x**7*
hyper((1/4, 7/2), (9/2,), b*x**2*exp_polar(I*pi)/a)/(7*a**(1/4))
```

Maxima [F]

$$\int \frac{A + Bx^2 + Cx^4 + Dx^6}{\sqrt[4]{a + bx^2}}\,dx = \int \frac{Dx^6 + Cx^4 + Bx^2 + A}{(bx^2 + a)^{\frac{1}{4}}}\,dx$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate((D*x^6 + C*x^4 + B*x^2 + A)/(b*x^2 + a)^(1/4), x)
```

Giac [F]

$$\int \frac{A + Bx^2 + Cx^4 + Dx^6}{\sqrt[4]{a + bx^2}}\,dx = \int \frac{Dx^6 + Cx^4 + Bx^2 + A}{(bx^2 + a)^{\frac{1}{4}}}\,dx$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(1/4),x, algorithm="giac")
```

output
```
integrate((D*x^6 + C*x^4 + B*x^2 + A)/(b*x^2 + a)^(1/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + Bx^2 + Cx^4 + Dx^6}{\sqrt[4]{a + bx^2}}\,dx = \int \frac{A + B\,x^2 + C\,x^4 + x^6\,D}{(b\,x^2 + a)^{1/4}}\,dx$$

input
```
int((A + B*x^2 + C*x^4 + x^6*D)/(a + b*x^2)^(1/4),x)
```

output
```
int((A + B*x^2 + C*x^4 + x^6*D)/(a + b*x^2)^(1/4), x)
```

Reduce [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{\sqrt[4]{a+bx^2}}\,dx = \left(\int \frac{x^6}{(b\,x^2+a)^{\frac{1}{4}}}dx\right)d+\left(\int \frac{x^4}{(b\,x^2+a)^{\frac{1}{4}}}dx\right)c + \left(\int \frac{x^2}{(b\,x^2+a)^{\frac{1}{4}}}dx\right)b+\left(\int \frac{1}{(b\,x^2+a)^{\frac{1}{4}}}dx\right)a$$

input

```
int((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(1/4),x)
```

output

```
int(x**6/(a + b*x**2)**(1/4),x)*d + int(x**4/(a + b*x**2)**(1/4),x)*c + in
t(x**2/(a + b*x**2)**(1/4),x)*b + int(1/(a + b*x**2)**(1/4),x)*a
```

3.144 $\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{3/4}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 176

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{3/4}}\,dx = \frac{2(77b^2B-66abC+60a^2D)\,x\sqrt[4]{a+bx^2}}{231b^3} + \frac{2(11bC-10aD)x^3\sqrt[4]{a+bx^2}}{77b^2} + \frac{2Dx^5\sqrt[4]{a+bx^2}}{11b} + \frac{2\sqrt{a}(231Ab^3-2a(77b^2B-66abC+60a^2D))\left(1+\frac{bx^2}{a}\right)^{3/4}\operatorname{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{231b^{7/2}\,(a+bx^2)^{3/4}}$$

output

```
2/231*(77*B*b^2-66*C*a*b+60*D*a^2)*x*(b*x^2+a)^(1/4)/b^3+2/77*(11*C*b-10*D
*a)*x^3*(b*x^2+a)^(1/4)/b^2+2/11*D*x^5*(b*x^2+a)^(1/4)/b+2/231*a^(1/2)*(23
1*A*b^3-2*a*(77*B*b^2-66*C*a*b+60*D*a^2))*(1+b*x^2/a)^(3/4)*InverseJacobiA
M(1/2*arctan(b^(1/2)*x/a^(1/2)),2^(1/2))/b^(7/2)/(b*x^2+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.12 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.75

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{3/4}}\,dx = \frac{x\Big(2(a+bx^2)\left(60a^2D-6ab(11C+5Dx^2)+b^2(77B+33Cx^2+21Dx^4)\right)}{}$$

input
```
Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(a + b*x^2)^(3/4),x]
```

output
```
(x*(2*(a + b*x^2)*(60*a^2*D - 6*a*b*(11*C + 5*D*x^2) + b^2*(77*B + 33*C*x^
2 + 21*D*x^4)) + (231*A*b^3 - 2*a*(77*b^2*B - 66*a*b*C + 60*a^2*D))*(1 + (
b*x^2)/a)^(3/4)*Hypergeometric2F1[1/2, 3/4, 3/2, -((b*x^2)/a)]))/(231*b^3*
(a + b*x^2)^(3/4))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2346, 27, 1473, 27, 299, 231, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{3/4}}\,dx$$

$$\Big\downarrow \text{2346}$$

$$\frac{2\int \frac{(11bC-10aD)x^4+11bBx^2+11Ab}{2(bx^2+a)^{3/4}}dx}{11b}+\frac{2Dx^5\sqrt[4]{a+bx^2}}{11b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(11bC-10aD)x^4+11bBx^2+11Ab}{(bx^2+a)^{3/4}}dx}{11b}+\frac{2Dx^5\sqrt[4]{a+bx^2}}{11b}$$

$$\Big\downarrow \text{1473}$$

$$\frac{\frac{2\int \frac{77Ab^2+\left(60Da^2-66bCa+77b^2B\right)x^2}{2\left(bx^2+a\right)^{3/4}}dx}{7b}+\frac{2x^3\sqrt[4]{a+bx^2}(11bC-10aD)}{7b}}{11b}+\frac{2Dx^5\sqrt[4]{a+bx^2}}{11b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{77Ab^2+\left(60Da^2-66bCa+77b^2B\right)x^2}{\left(bx^2+a\right)^{3/4}}dx}{7b}+\frac{2x^3\sqrt[4]{a+bx^2}(11bC-10aD)}{7b}}{11b}+\frac{2Dx^5\sqrt[4]{a+bx^2}}{11b}$$

$$\Big\downarrow \text{299}$$

$$\frac{\frac{\frac{\left(231Ab^3-2a\left(60a^2D-66abC+77b^2B\right)\right)\int \frac{1}{\left(bx^2+a\right)^{3/4}}dx}{3b}+\frac{2x\sqrt[4]{a+bx^2}\left(60a^2D-66abC+77b^2B\right)}{3b}}{7b}+\frac{2x^3\sqrt[4]{a+bx^2}(11bC-10aD)}{7b}}{11b}+\frac{2Dx^5\sqrt[4]{a+bx^2}}{11b}$$

$$\Big\downarrow \text{231}$$

$$\frac{\frac{\frac{\left(\frac{bx^2}{a}+1\right)^{3/4}\left(231Ab^3-2a\left(60a^2D-66abC+77b^2B\right)\right)\int \frac{1}{\left(\frac{bx^2}{a}+1\right)^{3/4}}dx}{3b\left(a+bx^2\right)^{3/4}}+\frac{2x\sqrt[4]{a+bx^2}\left(60a^2D-66abC+77b^2B\right)}{3b}}{7b}+\frac{2x^3\sqrt[4]{a+bx^2}(11bC-10aD)}{7b}}{11b}+\frac{2Dx^5\sqrt[4]{a+bx^2}}{11b}$$

$$\Big\downarrow \text{229}$$

$$\frac{\frac{\frac{2\sqrt{a}\left(\frac{bx^2}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)\left(231Ab^3-2a\left(60a^2D-66abC+77b^2B\right)\right)}{3b^{3/2}\left(a+bx^2\right)^{3/4}}+\frac{2x\sqrt[4]{a+bx^2}\left(60a^2D-66abC+77b^2B\right)}{3b}}{7b}+\frac{2x^3\sqrt[4]{a+bx^2}(1}{7b}}{11b}$$
$$\frac{2Dx^5\sqrt[4]{a+bx^2}}{11b}$$

input
```
Int[(A + B*x^2 + C*x^4 + D*x^6)/(a + b*x^2)^(3/4),x]
```

output
```
(2*D*x^5*(a + b*x^2)^(1/4))/(11*b) + ((2*(11*b*C - 10*a*D)*x^3*(a + b*x^2)
^(1/4))/(7*b) + ((2*(77*b^2*B - 66*a*b*C + 60*a^2*D)*x*(a + b*x^2)^(1/4))/
(3*b) + (2*Sqrt[a]*(231*A*b^3 - 2*a*(77*b^2*B - 66*a*b*C + 60*a^2*D))*(1 +
 (b*x^2)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3*b^(3/2)*
(a + b*x^2)^(3/4)))/(7*b))/(11*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1473
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[c^p*x^(4*p - 1)*((d + e*x^2)^(q + 1)/(e*(4*p + 2*q + 1)))
, x] + Simp[1/(e*(4*p + 2*q + 1))   Int[(d + e*x^2)^q*ExpandToSum[e*(4*p +
2*q + 1)*(a + b*x^2 + c*x^4)^p - d*c^p*(4*p - 1)*x^(4*p - 2) - e*c^p*(4*p +
 2*q + 1)*x^(4*p), x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] &&  !LtQ[q, -1]
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Maple [F]

$$\int \frac{Dx^{6}+C\,x^{4}+x^{2}B+A}{\left(b\,x^{2}+a\right)^{\frac{3}{4}}}dx$$

input
```
int((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(3/4),x)
```

output
```
int((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(3/4),x)
```

Fricas [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{3/4}}\,dx=\int \frac{Dx^{6}+Cx^{4}+Bx^{2}+A}{(bx^{2}+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(3/4),x, algorithm="fricas")
```

output
```
integral((D*x^6 + C*x^4 + B*x^2 + A)/(b*x^2 + a)^(3/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.54 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.66

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{3/4}}\,dx = \frac{Ax\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{a^{\frac{3}{4}}} + \frac{Bx^3\,{}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{3a^{\frac{3}{4}}} + \frac{Cx^5\,{}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{5}{2}\\ \frac{7}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{5a^{\frac{3}{4}}} + \frac{Dx^7\,{}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{7}{2}\\ \frac{9}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{7a^{\frac{3}{4}}}$$

input
```
integrate((D*x**6+C*x**4+B*x**2+A)/(b*x**2+a)**(3/4),x)
```

output
```
A*x*hyper((1/2, 3/4), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(3/4) + B*x**3*
hyper((3/4, 3/2), (5/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(3/4)) + C*x**5*
hyper((3/4, 5/2), (7/2,), b*x**2*exp_polar(I*pi)/a)/(5*a**(3/4)) + D*x**7*
hyper((3/4, 7/2), (9/2,), b*x**2*exp_polar(I*pi)/a)/(7*a**(3/4))
```

Maxima [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{3/4}}\,dx = \int \frac{Dx^6+Cx^4+Bx^2+A}{(bx^2+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate((D*x^6 + C*x^4 + B*x^2 + A)/(b*x^2 + a)^(3/4), x)
```

Giac [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{3/4}}\,dx=\int \frac{Dx^6+Cx^4+Bx^2+A}{(bx^2+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(3/4),x, algorithm="giac")
```

output
```
integrate((D*x^6 + C*x^4 + B*x^2 + A)/(b*x^2 + a)^(3/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{3/4}}\,dx=\int \frac{A+B\,x^2+C\,x^4+x^6\,D}{(b\,x^2+a)^{3/4}}\,dx$$

input
```
int((A + B*x^2 + C*x^4 + x^6*D)/(a + b*x^2)^(3/4),x)
```

output
```
int((A + B*x^2 + C*x^4 + x^6*D)/(a + b*x^2)^(3/4), x)
```

Reduce [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{3/4}}\,dx=\left(\int \frac{x^6}{(b\,x^2+a)^{\frac{3}{4}}}dx\right)d+\left(\int \frac{x^4}{(b\,x^2+a)^{\frac{3}{4}}}dx\right)c+\left(\int \frac{x^2}{(b\,x^2+a)^{\frac{3}{4}}}dx\right)b+\left(\int \frac{1}{(b\,x^2+a)^{\frac{3}{4}}}dx\right)a$$

input
```
int((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(3/4),x)
```

output
```
int(x**6/(a + b*x**2)**(3/4),x)*d + int(x**4/(a + b*x**2)**(3/4),x)*c + in
t(x**2/(a + b*x**2)**(3/4),x)*b + int(1/(a + b*x**2)**(3/4),x)*a
```

3.145 $\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{5/4}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 173

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{5/4}}\,dx = \frac{2(15b^2B-21abC+25a^2D)\,x}{15b^3\sqrt[4]{a+bx^2}} + \frac{2(3bC-5aD)x(a+bx^2)^{3/4}}{15b^3} + \frac{2Dx^3(a+bx^2)^{3/4}}{9b^2} + \frac{2(15Ab^3-30ab^2B+36a^2bC-40a^3D)\sqrt[4]{1+\frac{bx^2}{a}}\,E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{15\sqrt{a}b^{7/2}\sqrt[4]{a+bx^2}}$$

output

```
2/15*(15*B*b^2-21*C*a*b+25*D*a^2)*x/b^3/(b*x^2+a)^(1/4)+2/15*(3*C*b-5*D*a)
*x*(b*x^2+a)^(3/4)/b^3+2/9*D*x^3*(b*x^2+a)^(3/4)/b^2+2/15*(15*A*b^3-30*B*a
*b^2+36*C*a^2*b-40*D*a^3)*(1+b*x^2/a)^(1/4)*EllipticE(sin(1/2*arctan(b^(1/
2)*x/a^(1/2))),2^(1/2))/a^(1/2)/b^(7/2)/(b*x^2+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.11 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.78

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{5/4}}\,dx = \frac{x\left(90Ab^3+2a(-60a^2D+2ab(27C-5Dx^2)+b^2(-45B+9Cx^2+5Dx^4\right.}{}$$

input
```
Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(a + b*x^2)^(5/4),x]
```

output
```
(x*(90*A*b^3 + 2*a*(-60*a^2*D + 2*a*b*(27*C - 5*D*x^2) + b^2*(-45*B + 9*C*
x^2 + 5*D*x^4)) + 3*(-15*A*b^3 + 30*a*b^2*B - 36*a^2*b*C + 40*a^3*D)*(1 +
(b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2, -((b*x^2)/a)]))/(45*a*b^
3*(a + b*x^2)^(1/4))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.25, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {2345, 27, 1473, 27, 299, 227, 225, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{5/4}}\,dx$$

$$\Big\downarrow \text{2345}$$

$$\frac{2x\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{a\sqrt[4]{a+bx^2}}-\frac{2\int \frac{-\frac{aDx^4}{b}-\frac{a(bC-aD)x^2}{b^2}+A-\frac{2a\left(Da^2-bCa+b^2B\right)}{b^3}}{2\sqrt[4]{bx^2+a}}dx}{a}$$

$$\Big\downarrow \text{27}$$

$$\frac{2x\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{a\sqrt[4]{a+bx^2}}-\frac{\int\frac{-\frac{aDx^4}{b}-\frac{a(bC-aD)x^2}{b^2}+A-\frac{2a\left(Da^2-bCa+b^2B\right)}{b^3}}{\sqrt[4]{bx^2+a}}dx}{a}$$

$\big\downarrow$ 1473

$$\frac{2x\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{a\sqrt[4]{a+bx^2}}-\frac{\frac{2\int\frac{3\left(3\left(Ab-\frac{2a\left(Da^2-bCa+b^2B\right)}{b^2}\right)-a\left(3C-\frac{5aD}{b}\right)x^2\right)}{2\sqrt[4]{bx^2+a}}dx}{9b}-\frac{2aDx^3\left(a+bx^2\right)^{3/4}}{9b^2}}{a}$$

$\big\downarrow$ 27

$$\frac{2x\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{a\sqrt[4]{a+bx^2}}-\frac{\frac{\int\frac{3\left(Ab-\frac{2a\left(Da^2-bCa+b^2B\right)}{b^2}\right)-a\left(3C-\frac{5aD}{b}\right)x^2}{\sqrt[4]{bx^2+a}}dx}{3b}-\frac{2aDx^3\left(a+bx^2\right)^{3/4}}{9b^2}}{a}$$

$\big\downarrow$ 299

$$\frac{2x\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{a\sqrt[4]{a+bx^2}}-\frac{\frac{\frac{\left(-40a^3D+36a^2bC-30ab^2B+15Ab^3\right)\int\frac{1}{\sqrt[4]{bx^2+a}}dx}{5b^2}-\frac{2ax\left(a+bx^2\right)^{3/4}(3bC-5aD)}{5b^2}}{3b}-\frac{2aDx^3\left(a+bx^2\right)^{3/4}}{9b^2}}{a}$$

$\big\downarrow$ 227

$$\frac{2x\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{a\sqrt[4]{a+bx^2}}-\frac{\frac{\frac{\sqrt[4]{\frac{bx^2}{a}+1}\left(-40a^3D+36a^2bC-30ab^2B+15Ab^3\right)\int\frac{1}{\sqrt[4]{\frac{bx^2}{a}+1}}dx}{5b^2\sqrt[4]{a+bx^2}}-\frac{2ax\left(a+bx^2\right)^{3/4}(3bC-5aD)}{5b^2}}{3b}-\frac{2aDx^3\left(a+bx^2\right)^{3/4}}{9b^2}}{a}$$

$\big\downarrow$ 225

$$\frac{\frac{2x\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{a\sqrt[4]{a+bx^2}}-\frac{\frac{\frac{\sqrt[4]{\frac{bx^2}{a}+1}(-40a^3D+36a^2bC-30ab^2B+15Ab^3)\left(\frac{2x}{\sqrt[4]{\frac{bx^2}{a}+1}}-\int\frac{1}{\left(\frac{bx^2}{a}+1\right)^{5/4}}dx\right)}{5b^2\sqrt[4]{a+bx^2}}-\frac{2ax(a+bx^2)^{3/4}(3bC-5aD)}{5b^2}}{3b}-\frac{2aDx^3(a+bx^2)^{3/4}}{9b^2}}{a}$$

$$\Big\downarrow \text{212}$$

$$\frac{\frac{2x\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{a\sqrt[4]{a+bx^2}}-\frac{\frac{\frac{\sqrt[4]{\frac{bx^2}{a}+1}\left(\frac{2x}{\sqrt[4]{\frac{bx^2}{a}+1}}-\frac{2\sqrt{a}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}}\right)(-40a^3D+36a^2bC-30ab^2B+15Ab^3)}{5b^2\sqrt[4]{a+bx^2}}-\frac{2ax(a+bx^2)^{3/4}(3bC-5aD)}{5b^2}}{3b}-\frac{2aDx^3(a+bx^2)^{3/4}}{9b^2}}{a}$$

input
```
Int[(A + B*x^2 + C*x^4 + D*x^6)/(a + b*x^2)^(5/4),x]
```

output
```
(2*(A - (a*(b^2*B - a*b*C + a^2*D))/b^3)*x)/(a*(a + b*x^2)^(1/4)) - ((-2*a
*D*x^3*(a + b*x^2)^(3/4))/(9*b^2) + ((-2*a*(3*b*C - 5*a*D)*x*(a + b*x^2)^(
3/4))/(5*b^2) + ((15*A*b^3 - 30*a*b^2*B + 36*a^2*b*C - 40*a^3*D)*(1 + (b*x
^2)/a)^(1/4)*((2*x)/(1 + (b*x^2)/a)^(1/4) - (2*Sqrt[a]*EllipticE[ArcTan[(S
qrt[b]*x)/Sqrt[a]]/2, 2])/Sqrt[b]))/(5*b^2*(a + b*x^2)^(1/4)))/(3*b))/a
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 225
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Simp[a   Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[
a, 0] && PosQ[b/a]
```

rule 227
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4)   Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1473
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[c^p*x^(4*p - 1)*((d + e*x^2)^(q + 1)/(e*(4*p + 2*q + 1)))
, x] + Simp[1/(e*(4*p + 2*q + 1))   Int[(d + e*x^2)^q*ExpandToSum[e*(4*p +
2*q + 1)*(a + b*x^2 + c*x^4)^p - d*c^p*(4*p - 1)*x^(4*p - 2) - e*c^p*(4*p +
 2*q + 1)*x^(4*p), x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] &&  !LtQ[q, -1]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Maple [F]

$$\int \frac{Dx^{6}+C\,x^{4}+x^{2}B+A}{\left(b\,x^{2}+a\right)^{\frac{5}{4}}}dx$$

input
```
int((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(5/4),x)
```

output
```
int((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(5/4),x)
```

Fricas [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{\left(a+bx^2\right)^{5/4}}\,dx=\int \frac{Dx^6+Cx^4+Bx^2+A}{\left(bx^2+a\right)^{\frac{5}{4}}}\,dx$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

output
```
integral((D*x^6 + C*x^4 + B*x^2 + A)*(b*x^2 + a)^(3/4)/(b^2*x^4 + 2*a*b*x^
2 + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.32 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.68

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{\left(a+bx^2\right)^{5/4}}\,dx=\frac{Ax\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{a^{\frac{5}{4}}}+\frac{Bx^3\,{}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{3a^{\frac{5}{4}}}+\frac{Cx^5\,{}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{5}{2}\\ \frac{7}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{5a^{\frac{5}{4}}}+\frac{Dx^7\,{}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{7}{2}\\ \frac{9}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{7a^{\frac{5}{4}}}$$

input
```
integrate((D*x**6+C*x**4+B*x**2+A)/(b*x**2+a)**(5/4),x)
```

output
```
A*x*hyper((1/2, 5/4), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(5/4) + B*x**3*
hyper((5/4, 3/2), (5/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(5/4)) + C*x**5*
hyper((5/4, 5/2), (7/2,), b*x**2*exp_polar(I*pi)/a)/(5*a**(5/4)) + D*x**7*
hyper((5/4, 7/2), (9/2,), b*x**2*exp_polar(I*pi)/a)/(7*a**(5/4))
```

Maxima [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{5/4}}\,dx=\int \frac{Dx^6+Cx^4+Bx^2+A}{(bx^2+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(5/4),x, algorithm="maxima")
```

output
```
integrate((D*x^6 + C*x^4 + B*x^2 + A)/(b*x^2 + a)^(5/4), x)
```

Giac [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{5/4}}\,dx=\int \frac{Dx^6+Cx^4+Bx^2+A}{(bx^2+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(5/4),x, algorithm="giac")
```

output
```
integrate((D*x^6 + C*x^4 + B*x^2 + A)/(b*x^2 + a)^(5/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{5/4}}\,dx=\int \frac{A+B\,x^2+C\,x^4+x^6\,D}{(b\,x^2+a)^{5/4}}\,dx$$

input
```
int((A + B*x^2 + C*x^4 + x^6*D)/(a + b*x^2)^(5/4),x)
```

output
```
int((A + B*x^2 + C*x^4 + x^6*D)/(a + b*x^2)^(5/4), x)
```

Reduce [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{5/4}}\,dx = \left(\int \frac{x^6}{(b\,x^2+a)^{\frac{1}{4}}\,a+(b\,x^2+a)^{\frac{1}{4}}\,b\,x^2}dx\right)d + \left(\int \frac{x^4}{(b\,x^2+a)^{\frac{1}{4}}\,a+(b\,x^2+a)^{\frac{1}{4}}\,b\,x^2}dx\right)c + \left(\int \frac{x^2}{(b\,x^2+a)^{\frac{1}{4}}\,a+(b\,x^2+a)^{\frac{1}{4}}\,b\,x^2}dx\right)b + \left(\int \frac{1}{(b\,x^2+a)^{\frac{1}{4}}\,a+(b\,x^2+a)^{\frac{1}{4}}\,b\,x^2}dx\right)a$$

input
```
int((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(5/4),x)
```

output
```
int(x**6/((a + b*x**2)**(1/4)*a + (a + b*x**2)**(1/4)*b*x**2),x)*d + int(x
**4/((a + b*x**2)**(1/4)*a + (a + b*x**2)**(1/4)*b*x**2),x)*c + int(x**2/(
(a + b*x**2)**(1/4)*a + (a + b*x**2)**(1/4)*b*x**2),x)*b + int(1/((a + b*x
**2)**(1/4)*a + (a + b*x**2)**(1/4)*b*x**2),x)*a
```

3.146 $\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{7/4}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 179

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{7/4}}\,dx = \frac{2\left(\frac{A}{a}-\frac{b^2B-abC+a^2D}{b^3}\right)x}{3\,(a+bx^2)^{3/4}} + \frac{2(7bC-13aD)x\sqrt[4]{a+bx^2}}{21b^3} + \frac{2Dx^3\sqrt[4]{a+bx^2}}{7b^2} + \frac{2(7Ab^3+14ab^2B-28a^2bC+40a^3D)\left(1+\frac{bx^2}{a}\right)^{3/4}\operatorname{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{21\sqrt{a}b^{7/2}\,(a+bx^2)^{3/4}}$$

output
```
2/3*(A/a-(B*b^2-C*a*b+D*a^2)/b^3)*x/(b*x^2+a)^(3/4)+2/21*(7*C*b-13*D*a)*x*
(b*x^2+a)^(1/4)/b^3+2/7*D*x^3*(b*x^2+a)^(1/4)/b^2+2/21*(7*A*b^3+14*B*a*b^2
-28*C*a^2*b+40*D*a^3)*(1+b*x^2/a)^(3/4)*InverseJacobiAM(1/2*arctan(b^(1/2)
*x/a^(1/2)),2^(1/2))/a^(1/2)/b^(7/2)/(b*x^2+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.11 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.77

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{7/4}}\,dx = \frac{x\left(2(7Ab^3+a(-20a^2D+2ab(7C-5Dx^2)+b^2(-7B+7Cx^2+3Dx^4)))\right.}{}$$

input
```
Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(a + b*x^2)^(7/4),x]
```

output
```
(x*(2*(7*A*b^3 + a*(-20*a^2*D + 2*a*b*(7*C - 5*D*x^2) + b^2*(-7*B + 7*C*x^
2 + 3*D*x^4))) + (7*A*b^3 + 2*a*(7*b^2*B - 14*a*b*C + 20*a^2*D))*(1 + (b*x
^2)/a)^(3/4)*Hypergeometric2F1[1/2, 3/4, 3/2, -((b*x^2)/a)]))/(21*a*b^3*(a
 + b*x^2)^(3/4))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2345, 27, 1473, 27, 299, 231, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{7/4}}dx$$

$$\Big\downarrow \text{ 2345}$$

$$\frac{2x\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{3a(a+bx^2)^{3/4}}-\frac{2\int -\frac{\frac{3aDx^4}{b}+\frac{3a(bC-aD)x^2}{b^2}+A+\frac{2a\left(Da^2-bCa+b^2B\right)}{b^3}}{2(bx^2+a)^{3/4}}dx}{3a}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{\frac{3aDx^4}{b}+\frac{3a(bC-aD)x^2}{b^2}+A+\frac{2a\left(Da^2-bCa+b^2B\right)}{b^3}}{(bx^2+a)^{3/4}}dx}{3a}+\frac{2x\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{3a(a+bx^2)^{3/4}}$$

$$\Big\downarrow \ 1473$$

$$\frac{\frac{2\int \frac{3a\left(7C-\frac{13aD}{b}\right)x^2+7\left(Ab+\frac{2a\left(Da^2-bCa+b^2B\right)}{b^2}\right)}{2\left(bx^2+a\right)^{3/4}}dx}{7b}+\frac{6aDx^3\sqrt[4]{a+bx^2}}{7b^2}}{3a}+\frac{2x\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{3a\left(a+bx^2\right)^{3/4}}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\int \frac{3a\left(7C-\frac{13aD}{b}\right)x^2+7\left(Ab+\frac{2a\left(Da^2-bCa+b^2B\right)}{b^2}\right)}{\left(bx^2+a\right)^{3/4}}dx}{7b}+\frac{6aDx^3\sqrt[4]{a+bx^2}}{7b^2}}{3a}+\frac{2x\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{3a\left(a+bx^2\right)^{3/4}}$$

$$\Big\downarrow \ 299$$

$$\frac{\frac{\frac{\left(40a^3D-28a^2bC+14ab^2B+7Ab^3\right)\int \frac{1}{\left(bx^2+a\right)^{3/4}}dx}{b^2}+\frac{2ax\sqrt[4]{a+bx^2}(7bC-13aD)}{b^2}}{7b}+\frac{6aDx^3\sqrt[4]{a+bx^2}}{7b^2}}{3a}+\frac{2x\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{3a\left(a+bx^2\right)^{3/4}}$$

$$\Big\downarrow \ 231$$

$$\frac{\frac{\frac{\left(\frac{bx^2}{a}+1\right)^{3/4}\left(40a^3D-28a^2bC+14ab^2B+7Ab^3\right)\int \frac{1}{\left(\frac{bx^2}{a}+1\right)^{3/4}}dx}{b^2\left(a+bx^2\right)^{3/4}}+\frac{2ax\sqrt[4]{a+bx^2}(7bC-13aD)}{b^2}}{7b}+\frac{6aDx^3\sqrt[4]{a+bx^2}}{7b^2}}{3a}+\frac{2x\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{3a\left(a+bx^2\right)^{3/4}}$$

$$\Big\downarrow \ 229$$

$$\frac{2x\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{3a\left(a+bx^2\right)^{3/4}}+\frac{\frac{\frac{2\sqrt{a}\left(\frac{bx^2}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)\left(40a^3D-28a^2bC+14ab^2B+7Ab^3\right)}{b^{5/2}\left(a+bx^2\right)^{3/4}}+\frac{2ax\sqrt[4]{a+bx^2}(7bC-13aD)}{b^2}}{7b}+\frac{6aDx^3\sqrt[4]{a+bx^2}}{7b^2}}{3a}$$

input

```
Int[(A + B*x^2 + C*x^4 + D*x^6)/(a + b*x^2)^(7/4),x]
```

output
```
(2*(A - (a*(b^2*B - a*b*C + a^2*D))/b^3)*x)/(3*a*(a + b*x^2)^(3/4)) + ((6*
a*D*x^3*(a + b*x^2)^(1/4))/(7*b^2) + ((2*a*(7*b*C - 13*a*D)*x*(a + b*x^2)^
(1/4))/b^2 + (2*Sqrt[a]*(7*A*b^3 + 14*a*b^2*B - 28*a^2*b*C + 40*a^3*D)*(1
+ (b*x^2)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(b^(5/2)*(
a + b*x^2)^(3/4)))/(7*b))/(3*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1473
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[c^p*x^(4*p - 1)*((d + e*x^2)^(q + 1)/(e*(4*p + 2*q + 1)))
, x] + Simp[1/(e*(4*p + 2*q + 1))   Int[(d + e*x^2)^q*ExpandToSum[e*(4*p +
2*q + 1)*(a + b*x^2 + c*x^4)^p - d*c^p*(4*p - 1)*x^(4*p - 2) - e*c^p*(4*p +
 2*q + 1)*x^(4*p), x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] &&  !LtQ[q, -1]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Maple [F]

$$\int \frac{Dx^{6}+C\,x^{4}+x^{2}B+A}{\left(b\,x^{2}+a\right)^{\frac{7}{4}}}dx$$

input
```
int((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(7/4),x)
```

output
```
int((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(7/4),x)
```

Fricas [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{\left(a+bx^2\right)^{7/4}}\,dx=\int \frac{Dx^{6}+Cx^{4}+Bx^{2}+A}{\left(bx^{2}+a\right)^{\frac{7}{4}}}\,dx$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(7/4),x, algorithm="fricas")
```

output
```
integral((D*x^6 + C*x^4 + B*x^2 + A)*(b*x^2 + a)^(1/4)/(b^2*x^4 + 2*a*b*x^
2 + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.73 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.65

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{7/4}}\,dx = \frac{Ax\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{4}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{a^{\frac{7}{4}}} + \frac{Bx^3\,{}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{7}{4}\\ \frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{3a^{\frac{7}{4}}} + \frac{Cx^5\,{}_2F_1\left(\begin{matrix}\frac{7}{4},\frac{5}{2}\\ \frac{7}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{5a^{\frac{7}{4}}} + \frac{Dx^7\,{}_2F_1\left(\begin{matrix}\frac{7}{4},\frac{7}{2}\\ \frac{9}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{7a^{\frac{7}{4}}}$$

input
```
integrate((D*x**6+C*x**4+B*x**2+A)/(b*x**2+a)**(7/4),x)
```

output
```
A*x*hyper((1/2, 7/4), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(7/4) + B*x**3*
hyper((3/2, 7/4), (5/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(7/4)) + C*x**5*
hyper((7/4, 5/2), (7/2,), b*x**2*exp_polar(I*pi)/a)/(5*a**(7/4)) + D*x**7*
hyper((7/4, 7/2), (9/2,), b*x**2*exp_polar(I*pi)/a)/(7*a**(7/4))
```

Maxima [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{7/4}}\,dx = \int \frac{Dx^6+Cx^4+Bx^2+A}{(bx^2+a)^{\frac{7}{4}}}\,dx$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(7/4),x, algorithm="maxima")
```

output
```
integrate((D*x^6 + C*x^4 + B*x^2 + A)/(b*x^2 + a)^(7/4), x)
```

Giac [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{7/4}}\,dx = \int \frac{Dx^6+Cx^4+Bx^2+A}{(bx^2+a)^{\frac{7}{4}}}\,dx$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(7/4),x, algorithm="giac")
```

output
```
integrate((D*x^6 + C*x^4 + B*x^2 + A)/(b*x^2 + a)^(7/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{7/4}}\,dx = \int \frac{A+B\,x^2+C\,x^4+x^6\,D}{(b\,x^2+a)^{7/4}}\,dx$$

input
```
int((A + B*x^2 + C*x^4 + x^6*D)/(a + b*x^2)^(7/4),x)
```

output
```
int((A + B*x^2 + C*x^4 + x^6*D)/(a + b*x^2)^(7/4), x)
```

Reduce [F]

$$\begin{aligned}\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{7/4}}\,dx &= \left(\int \frac{x^6}{(b\,x^2+a)^{\frac{3}{4}}\,a+(b\,x^2+a)^{\frac{3}{4}}\,b\,x^2}dx\right)d \\ &+ \left(\int \frac{x^4}{(b\,x^2+a)^{\frac{3}{4}}\,a+(b\,x^2+a)^{\frac{3}{4}}\,b\,x^2}dx\right)c \\ &+ \left(\int \frac{x^2}{(b\,x^2+a)^{\frac{3}{4}}\,a+(b\,x^2+a)^{\frac{3}{4}}\,b\,x^2}dx\right)b \\ &+ \left(\int \frac{1}{(b\,x^2+a)^{\frac{3}{4}}\,a+(b\,x^2+a)^{\frac{3}{4}}\,b\,x^2}dx\right)a\end{aligned}$$

input
```
int((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(7/4),x)
```

output
```
int(x**6/((a + b*x**2)**(3/4)*a + (a + b*x**2)**(3/4)*b*x**2),x)*d + int(x
**4/((a + b*x**2)**(3/4)*a + (a + b*x**2)**(3/4)*b*x**2),x)*c + int(x**2/(
(a + b*x**2)**(3/4)*a + (a + b*x**2)**(3/4)*b*x**2),x)*b + int(1/((a + b*x
**2)**(3/4)*a + (a + b*x**2)**(3/4)*b*x**2),x)*a
```

3.147 $\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{9/4}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 177

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{9/4}}\,dx = \frac{2\left(\frac{A}{a}-\frac{b^2B-abC+a^2D}{b^3}\right)x}{5(a+bx^2)^{5/4}} + \frac{2(5bC-12aD)x}{5b^3\sqrt[4]{a+bx^2}} + \frac{2Dx(a+bx^2)^{3/4}}{5b^3} + \frac{2(3Ab^3+2a(b^2B-6abC+12a^2D))\sqrt[4]{1+\frac{bx^2}{a}}\,E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5a^{3/2}b^{7/2}\sqrt[4]{a+bx^2}}$$

output
```
2/5*(A/a-(B*b^2-C*a*b+D*a^2)/b^3)*x/(b*x^2+a)^(5/4)+2/5*(5*C*b-12*D*a)*x/b
^3/(b*x^2+a)^(1/4)+2/5*D*x*(b*x^2+a)^(3/4)/b^3+2/5*(3*A*b^3+2*a*(B*b^2-6*C
*a*b+12*D*a^2))*(1+b*x^2/a)^(1/4)*EllipticE(sin(1/2*arctan(b^(1/2)*x/a^(1/
2))),2^(1/2))/a^(3/2)/b^(7/2)/(b*x^2+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.14 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.91

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{9/4}}\,dx = \frac{2x(12a^4D+3Ab^4x^2+2ab^3(2A+Bx^2)+a^3b(-6C+14Dx^2)+a^2b^2(B-$$

input
```
Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(a + b*x^2)^(9/4),x]
```

output
```
(2*x*(12*a^4*D + 3*A*b^4*x^2 + 2*a*b^3*(2*A + B*x^2) + a^3*b*(-6*C + 14*D*
x^2) + a^2*b^2*(B - 7*C*x^2 + D*x^4)) - (3*A*b^3 + 2*a*(b^2*B - 6*a*b*C +
12*a^2*D))*x*(a + b*x^2)*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4, 1/2,
 3/2, -((b*x^2)/a)])/(5*a^2*b^3*(a + b*x^2)^(5/4))
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.30, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {2345, 27, 1471, 27, 299, 227, 225, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{9/4}}dx$$

$$\Big\downarrow \text{2345}$$

$$\frac{2x\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{5a(a+bx^2)^{5/4}}-\frac{2\int -\frac{\frac{5aDx^4}{b}+\frac{5a(bC-aD)x^2}{b^2}+3A+\frac{2a\left(Da^2-bCa+b^2B\right)}{b^3}}{2(bx^2+a)^{5/4}}dx}{5a}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\frac{5aDx^4}{b}+\frac{5a(bC-aD)x^2}{b^2}+3A+\frac{2a\left(Da^2-bCa+b^2B\right)}{b^3}}{(bx^2+a)^{5/4}}dx}{5a}+\frac{2x\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{5a(a+bx^2)^{5/4}}$$

$$\Big\downarrow \ 1471$$

$$\frac{\frac{2x\left(\frac{12a^2D-7abC+2b^2B}{b^3}+\frac{3A}{a}\right)}{\sqrt[4]{a+bx^2}}-\frac{2\int \frac{b^2\left(3A+\frac{2a\left(11Da^2-6bCa+b^2B\right)}{b^3}\right)-5a^2Dx^2}{2b^2\sqrt[4]{bx^2+a}}dx}{a}}{5a}+\frac{2x\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{5a\left(a+bx^2\right)^{5/4}}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{2x\left(\frac{12a^2D-7abC+2b^2B}{b^3}+\frac{3A}{a}\right)}{\sqrt[4]{a+bx^2}}-\frac{\int \frac{3Ab^2-5a^2Dx^2+2a\left(\frac{11Da^2}{b}-6Ca+bB\right)}{\sqrt[4]{bx^2+a}}dx}{ab^2}}{5a}+\frac{2x\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{5a\left(a+bx^2\right)^{5/4}}$$

$$\Big\downarrow \ 299$$

$$\frac{\frac{2x\left(\frac{12a^2D-7abC+2b^2B}{b^3}+\frac{3A}{a}\right)}{\sqrt[4]{a+bx^2}}-\frac{\frac{\left(2a\left(12a^2D-6abC+b^2B\right)+3Ab^3\right)\int \frac{1}{\sqrt[4]{bx^2+a}}dx}{b}-\frac{2a^2Dx\left(a+bx^2\right)^{3/4}}{b}}{ab^2}}{5a}+\frac{2x\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{5a\left(a+bx^2\right)^{5/4}}$$

$$\Big\downarrow \ 227$$

$$\frac{\frac{2x\left(\frac{12a^2D-7abC+2b^2B}{b^3}+\frac{3A}{a}\right)}{\sqrt[4]{a+bx^2}}-\frac{\frac{\sqrt[4]{\frac{bx^2}{a}+1}\left(2a\left(12a^2D-6abC+b^2B\right)+3Ab^3\right)\int \frac{1}{\sqrt[4]{\frac{bx^2}{a}+1}}dx}{b\sqrt[4]{a+bx^2}}-\frac{2a^2Dx\left(a+bx^2\right)^{3/4}}{b}}{ab^2}}{5a}+\frac{2x\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{5a\left(a+bx^2\right)^{5/4}}$$

$$\Big\downarrow \ 225$$

$$\frac{\frac{2x\left(\frac{12a^2D-7abC+2b^2B}{b^3}+\frac{3A}{a}\right)}{\sqrt[4]{a+bx^2}}-\frac{\frac{\sqrt[4]{\frac{bx^2}{a}+1}\left(2a\left(12a^2D-6abC+b^2B\right)+3Ab^3\right)\left(\frac{2x}{\sqrt[4]{\frac{bx^2}{a}+1}}-\int\frac{1}{\left(\frac{bx^2}{a}+1\right)^{5/4}}dx\right)}{b\sqrt[4]{a+bx^2}}-\frac{2a^2Dx\left(a+bx^2\right)^{3/4}}{b}}{ab^2}}{5a}+\frac{2x\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{5a\left(a+bx^2\right)^{5/4}}$$

$\downarrow$ 212

$$\frac{\frac{2x\left(\frac{12a^2D-7abC+2b^2B}{b^3}+\frac{3A}{a}\right)}{\sqrt[4]{a+bx^2}}-\frac{\frac{\sqrt[4]{\frac{bx^2}{a}+1}\left(\frac{2x}{\sqrt[4]{\frac{bx^2}{a}+1}}-\frac{2\sqrt{a}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}}\right)\left(2a\left(12a^2D-6abC+b^2B\right)+3Ab^3\right)}{b\sqrt[4]{a+bx^2}}-\frac{2a^2Dx\left(a+bx^2\right)^{3/4}}{b}}{ab^2}}{5a}+\frac{2x\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{5a\left(a+bx^2\right)^{5/4}}$$

input
```
Int[(A + B*x^2 + C*x^4 + D*x^6)/(a + b*x^2)^(9/4),x]
```

output
```
(2*(A - (a*(b^2*B - a*b*C + a^2*D))/b^3)*x)/(5*a*(a + b*x^2)^(5/4)) + ((2*
((3*A)/a + (2*b^2*B - 7*a*b*C + 12*a^2*D)/b^3)*x)/(a + b*x^2)^(1/4) - ((-2
*a^2*D*x*(a + b*x^2)^(3/4))/b + ((3*A*b^3 + 2*a*(b^2*B - 6*a*b*C + 12*a^2*
D))*(1 + (b*x^2)/a)^(1/4)*((2*x)/(1 + (b*x^2)/a)^(1/4) - (2*Sqrt[a]*Ellipt
icE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/Sqrt[b]))/(b*(a + b*x^2)^(1/4)))/(a
*b^2))/(5*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 225
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Simp[a   Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[
a, 0] && PosQ[b/a]
```

rule 227
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4)   Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Maple [F]

$$\int \frac{Dx^{6}+C\,x^{4}+x^{2}B+A}{\left(b\,x^{2}+a\right)^{\frac{9}{4}}}dx$$

input
```
int((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/4),x)
```

output
```
int((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/4),x)
```

Fricas [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{\left(a+bx^2\right)^{9/4}}\,dx=\int \frac{Dx^{6}+Cx^{4}+Bx^{2}+A}{\left(bx^{2}+a\right)^{\frac{9}{4}}}\,dx$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/4),x, algorithm="fricas")
```

output
```
integral((D*x^6 + C*x^4 + B*x^2 + A)*(b*x^2 + a)^(3/4)/(b^3*x^6 + 3*a*b^2*
x^4 + 3*a^2*b*x^2 + a^3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 7.73 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.66

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{9/4}}\,dx = \frac{Ax\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{9}{4}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{a^{\frac{9}{4}}} + \frac{Bx^3\,{}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{9}{4}\\ \frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{3a^{\frac{9}{4}}} + \frac{Cx^5\,{}_2F_1\left(\begin{matrix}\frac{9}{4},\frac{5}{2}\\ \frac{7}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{5a^{\frac{9}{4}}} + \frac{Dx^7\,{}_2F_1\left(\begin{matrix}\frac{9}{4},\frac{7}{2}\\ \frac{9}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{7a^{\frac{9}{4}}}$$

input

```
integrate((D*x**6+C*x**4+B*x**2+A)/(b*x**2+a)**(9/4),x)
```

output

```
A*x*hyper((1/2, 9/4), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(9/4) + B*x**3*
hyper((3/2, 9/4), (5/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(9/4)) + C*x**5*
hyper((9/4, 5/2), (7/2,), b*x**2*exp_polar(I*pi)/a)/(5*a**(9/4)) + D*x**7*
hyper((9/4, 7/2), (9/2,), b*x**2*exp_polar(I*pi)/a)/(7*a**(9/4))
```

Maxima [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{9/4}}\,dx = \int \frac{Dx^6+Cx^4+Bx^2+A}{(bx^2+a)^{\frac{9}{4}}}\,dx$$

input

```
integrate((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/4),x, algorithm="maxima")
```

output

```
integrate((D*x^6 + C*x^4 + B*x^2 + A)/(b*x^2 + a)^(9/4), x)
```

Giac [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{9/4}}\,dx = \int \frac{Dx^6+Cx^4+Bx^2+A}{(bx^2+a)^{\frac{9}{4}}}\,dx$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/4),x, algorithm="giac")
```

output
```
integrate((D*x^6 + C*x^4 + B*x^2 + A)/(b*x^2 + a)^(9/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{9/4}}\,dx = \int \frac{A+B\,x^2+C\,x^4+x^6\,D}{(b\,x^2+a)^{9/4}}\,dx$$

input
```
int((A + B*x^2 + C*x^4 + x^6*D)/(a + b*x^2)^(9/4),x)
```

output
```
int((A + B*x^2 + C*x^4 + x^6*D)/(a + b*x^2)^(9/4), x)
```

Reduce [F]

$$\begin{aligned}\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{9/4}}\,dx = &\left(\int \frac{x^6}{(b\,x^2+a)^{\frac{1}{4}}\,a^2+2\,(b\,x^2+a)^{\frac{1}{4}}\,ab\,x^2+(b\,x^2+a)^{\frac{1}{4}}\,b^2x^4}dx\right) d\\ &+\left(\int \frac{x^4}{(b\,x^2+a)^{\frac{1}{4}}\,a^2+2\,(b\,x^2+a)^{\frac{1}{4}}\,ab\,x^2+(b\,x^2+a)^{\frac{1}{4}}\,b^2x^4}dx\right) c\\ &+\left(\int \frac{x^2}{(b\,x^2+a)^{\frac{1}{4}}\,a^2+2\,(b\,x^2+a)^{\frac{1}{4}}\,ab\,x^2+(b\,x^2+a)^{\frac{1}{4}}\,b^2x^4}dx\right) b\\ &+\left(\int \frac{1}{(b\,x^2+a)^{\frac{1}{4}}\,a^2+2\,(b\,x^2+a)^{\frac{1}{4}}\,ab\,x^2+(b\,x^2+a)^{\frac{1}{4}}\,b^2x^4}dx\right) a\end{aligned}$$

input
```
int((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/4),x)
```

output
```
int(x**6/((a + b*x**2)**(1/4)*a**2 + 2*(a + b*x**2)**(1/4)*a*b*x**2 + (a +
 b*x**2)**(1/4)*b**2*x**4),x)*d + int(x**4/((a + b*x**2)**(1/4)*a**2 + 2*(
a + b*x**2)**(1/4)*a*b*x**2 + (a + b*x**2)**(1/4)*b**2*x**4),x)*c + int(x*
*2/((a + b*x**2)**(1/4)*a**2 + 2*(a + b*x**2)**(1/4)*a*b*x**2 + (a + b*x**
2)**(1/4)*b**2*x**4),x)*b + int(1/((a + b*x**2)**(1/4)*a**2 + 2*(a + b*x**
2)**(1/4)*a*b*x**2 + (a + b*x**2)**(1/4)*b**2*x**4),x)*a
```

3.148 $\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{11/4}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 198

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{11/4}}\,dx = \frac{2\left(\frac{A}{a}-\frac{b^2B-abC+a^2D}{b^3}\right)x}{7\,(a+bx^2)^{7/4}} + \frac{2(5Ab^3+a(2b^2B-9abC+16a^2D))\,x}{21a^2b^3\,(a+bx^2)^{3/4}} + \frac{2Dx\sqrt[4]{a+bx^2}}{3b^3} + \frac{2(5Ab^3+2a(b^2B+6abC-20a^2D))\left(1+\frac{bx^2}{a}\right)^{3/4}\operatorname{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{21a^{3/2}b^{7/2}\,(a+bx^2)^{3/4}}$$

output
```
2/7*(A/a-(B*b^2-C*a*b+D*a^2)/b^3)*x/(b*x^2+a)^(7/4)+2/21*(5*A*b^3+a*(2*B*b
^2-9*C*a*b+16*D*a^2))*x/a^2/b^3/(b*x^2+a)^(3/4)+2/3*D*x*(b*x^2+a)^(1/4)/b^
3+2/21*(5*A*b^3+2*a*(B*b^2+6*C*a*b-20*D*a^2))*(1+b*x^2/a)^(3/4)*InverseJac
obiAM(1/2*arctan(b^(1/2)*x/a^(1/2)),2^(1/2))/a^(3/2)/b^(7/2)/(b*x^2+a)^(3/
4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.13 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.81

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{11/4}}\,dx = \frac{2x(20a^4D+5Ab^4x^2+2ab^3(4A+Bx^2)-6a^3b(C-5Dx^2)-a^2b^2(B+9C}{}$$

input
```
Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(a + b*x^2)^(11/4),x]
```

output
```
(2*x*(20*a^4*D + 5*A*b^4*x^2 + 2*a*b^3*(4*A + B*x^2) - 6*a^3*b*(C - 5*D*x^
2) - a^2*b^2*(B + 9*C*x^2 - 7*D*x^4)) + (5*A*b^3 + 2*a*(b^2*B + 6*a*b*C -
20*a^2*D))*x*(a + b*x^2)*(1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[1/2, 3/4,
 3/2, -((b*x^2)/a)])/(21*a^2*b^3*(a + b*x^2)^(7/4))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2345, 27, 1471, 27, 299, 231, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{11/4}}\,dx$$

$$\Big\downarrow \text{ 2345}$$

$$\frac{2x\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{7a\,(a+bx^2)^{7/4}}-\frac{2\int -\frac{\frac{7aDx^4}{b}+\frac{7a(bC-aD)x^2}{b^2}+5A+\frac{2a\left(Da^2-bCa+b^2B\right)}{b^3}}{2(bx^2+a)^{7/4}}dx}{7a}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{\frac{7aDx^4}{b}+\frac{7a(bC-aD)x^2}{b^2}+5A+\frac{2a\left(Da^2-bCa+b^2B\right)}{b^3}}{(bx^2+a)^{7/4}}dx}{7a}+\frac{2x\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{7a\,(a+bx^2)^{7/4}}$$

$$\Big\downarrow \ 1471$$

$$\frac{\frac{2x\left(\frac{16a^2D-9abC+2b^2B}{b^3}+\frac{5A}{a}\right)}{3(a+bx^2)^{3/4}}-\frac{2\int -\frac{\left(5A+\frac{2a\left(-13Da^2+6bCa+b^2B\right)}{b^3}\right)b^2+21a^2Dx^2}{2b^2\left(bx^2+a\right)^{3/4}}dx}{3a}}{7a}+\frac{2x\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{7a\left(a+bx^2\right)^{7/4}}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\int \frac{5Ab^2+21a^2Dx^2+2a\left(-\frac{13Da^2}{b}+6Ca+bB\right)}{\left(bx^2+a\right)^{3/4}}dx}{3ab^2}+\frac{2x\left(\frac{16a^2D-9abC+2b^2B}{b^3}+\frac{5A}{a}\right)}{3(a+bx^2)^{3/4}}}{7a}+\frac{2x\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{7a\left(a+bx^2\right)^{7/4}}$$

$$\Big\downarrow \ 299$$

$$\frac{\frac{\frac{\left(2a\left(-20a^2D+6abC+b^2B\right)+5Ab^3\right)\int \frac{1}{\left(bx^2+a\right)^{3/4}}dx}{b}+\frac{14a^2Dx\sqrt[4]{a+bx^2}}{b}}{3ab^2}+\frac{2x\left(\frac{16a^2D-9abC+2b^2B}{b^3}+\frac{5A}{a}\right)}{3(a+bx^2)^{3/4}}}{7a}+\frac{2x\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{7a\left(a+bx^2\right)^{7/4}}$$

$$\Big\downarrow \ 231$$

$$\frac{\frac{\frac{\left(\frac{bx^2}{a}+1\right)^{3/4}\left(2a\left(-20a^2D+6abC+b^2B\right)+5Ab^3\right)\int \frac{1}{\left(\frac{bx^2}{a}+1\right)^{3/4}}dx}{b\left(a+bx^2\right)^{3/4}}+\frac{14a^2Dx\sqrt[4]{a+bx^2}}{b}}{3ab^2}+\frac{2x\left(\frac{16a^2D-9abC+2b^2B}{b^3}+\frac{5A}{a}\right)}{3(a+bx^2)^{3/4}}}{7a}+\frac{2x\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{7a\left(a+bx^2\right)^{7/4}}$$

$$\Big\downarrow \ 229$$

$$\frac{\frac{\frac{2\sqrt{a}\left(\frac{bx^2}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)\left(2a\left(-20a^2D+6abC+b^2B\right)+5Ab^3\right)}{b^{3/2}\left(a+bx^2\right)^{3/4}}+\frac{14a^2Dx\sqrt[4]{a+bx^2}}{b}}{3ab^2}+\frac{2x\left(\frac{16a^2D-9abC+2b^2B}{b^3}+\frac{5A}{a}\right)}{3(a+bx^2)^{3/4}}}{7a}+\frac{2x\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{7a\left(a+bx^2\right)^{7/4}}$$

input
```
Int[(A + B*x^2 + C*x^4 + D*x^6)/(a + b*x^2)^(11/4),x]
```

output
```
(2*(A - (a*(b^2*B - a*b*C + a^2*D))/b^3)*x)/(7*a*(a + b*x^2)^(7/4)) + ((2*
((5*A)/a + (2*b^2*B - 9*a*b*C + 16*a^2*D)/b^3)*x)/(3*(a + b*x^2)^(3/4)) +
((14*a^2*D*x*(a + b*x^2)^(1/4))/b + (2*Sqrt[a]*(5*A*b^3 + 2*a*(b^2*B + 6*a
*b*C - 20*a^2*D))*(1 + (b*x^2)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[
a]]/2, 2])/(b^(3/2)*(a + b*x^2)^(3/4)))/(3*a*b^2))/(7*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Maple [F]

$$\int \frac{Dx^{6}+C\,x^{4}+x^{2}B+A}{\left(b\,x^{2}+a\right)^{\frac{11}{4}}}dx$$

input
```
int((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(11/4),x)
```

output
```
int((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(11/4),x)
```

Fricas [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{11/4}}\,dx=\int \frac{Dx^{6}+Cx^{4}+Bx^{2}+A}{{\left(bx^{2}+a\right)}^{\frac{11}{4}}}\,dx$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(11/4),x, algorithm="fricas")
```

output
```
integral((D*x^6 + C*x^4 + B*x^2 + A)*(b*x^2 + a)^(1/4)/(b^3*x^6 + 3*a*b^2*
x^4 + 3*a^2*b*x^2 + a^3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 12.95 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.59

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{11/4}}\,dx = \frac{Ax\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{11}{4}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{a^{\frac{11}{4}}} + \frac{Bx^3\,{}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{11}{4}\\ \frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{3a^{\frac{11}{4}}} + \frac{Cx^5\,{}_2F_1\left(\begin{matrix}\frac{5}{2},\frac{11}{4}\\ \frac{7}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{5a^{\frac{11}{4}}} + \frac{Dx^7\,{}_2F_1\left(\begin{matrix}\frac{11}{4},\frac{7}{2}\\ \frac{9}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{7a^{\frac{11}{4}}}$$

input
```
integrate((D*x**6+C*x**4+B*x**2+A)/(b*x**2+a)**(11/4),x)
```

output
```
A*x*hyper((1/2, 11/4), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(11/4) + B*x**
3*hyper((3/2, 11/4), (5/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(11/4)) + C*x
**5*hyper((5/2, 11/4), (7/2,), b*x**2*exp_polar(I*pi)/a)/(5*a**(11/4)) + D
*x**7*hyper((11/4, 7/2), (9/2,), b*x**2*exp_polar(I*pi)/a)/(7*a**(11/4))
```

Maxima [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{11/4}}\,dx = \int \frac{Dx^6+Cx^4+Bx^2+A}{(bx^2+a)^{\frac{11}{4}}}\,dx$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(11/4),x, algorithm="maxima")
```

output
```
integrate((D*x^6 + C*x^4 + B*x^2 + A)/(b*x^2 + a)^(11/4), x)
```

Giac [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{11/4}}\,dx=\int \frac{Dx^6+Cx^4+Bx^2+A}{(bx^2+a)^{\frac{11}{4}}}\,dx$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(11/4),x, algorithm="giac")
```

output
```
integrate((D*x^6 + C*x^4 + B*x^2 + A)/(b*x^2 + a)^(11/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{11/4}}\,dx=\int \frac{A+B\,x^2+C\,x^4+x^6\,D}{(b\,x^2+a)^{11/4}}\,dx$$

input
```
int((A + B*x^2 + C*x^4 + x^6*D)/(a + b*x^2)^(11/4),x)
```

output
```
int((A + B*x^2 + C*x^4 + x^6*D)/(a + b*x^2)^(11/4), x)
```

Reduce [F]

$$\begin{aligned}\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{11/4}}\,dx&=\left(\int \frac{x^6}{(b\,x^2+a)^{\frac{3}{4}}\,a^2+2\,(b\,x^2+a)^{\frac{3}{4}}\,ab\,x^2+(b\,x^2+a)^{\frac{3}{4}}\,b^2x^4}dx\right)d\\&+\left(\int \frac{x^4}{(b\,x^2+a)^{\frac{3}{4}}\,a^2+2\,(b\,x^2+a)^{\frac{3}{4}}\,ab\,x^2+(b\,x^2+a)^{\frac{3}{4}}\,b^2x^4}dx\right)c\\&+\left(\int \frac{x^2}{(b\,x^2+a)^{\frac{3}{4}}\,a^2+2\,(b\,x^2+a)^{\frac{3}{4}}\,ab\,x^2+(b\,x^2+a)^{\frac{3}{4}}\,b^2x^4}dx\right)b\\&+\left(\int \frac{1}{(b\,x^2+a)^{\frac{3}{4}}\,a^2+2\,(b\,x^2+a)^{\frac{3}{4}}\,ab\,x^2+(b\,x^2+a)^{\frac{3}{4}}\,b^2x^4}dx\right)a\end{aligned}$$

input
```
int((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(11/4),x)
```

output
```
int(x**6/((a + b*x**2)**(3/4)*a**2 + 2*(a + b*x**2)**(3/4)*a*b*x**2 + (a +
 b*x**2)**(3/4)*b**2*x**4),x)*d + int(x**4/((a + b*x**2)**(3/4)*a**2 + 2*(
a + b*x**2)**(3/4)*a*b*x**2 + (a + b*x**2)**(3/4)*b**2*x**4),x)*c + int(x*
*2/((a + b*x**2)**(3/4)*a**2 + 2*(a + b*x**2)**(3/4)*a*b*x**2 + (a + b*x**
2)**(3/4)*b**2*x**4),x)*b + int(1/((a + b*x**2)**(3/4)*a**2 + 2*(a + b*x**
2)**(3/4)*a*b*x**2 + (a + b*x**2)**(3/4)*b**2*x**4),x)*a
```

3.149 $\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{13/4}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 196

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{13/4}}\,dx = \frac{2\left(\frac{A}{a}-\frac{b^2B-abC+a^2D}{b^3}\right)x}{9(a+bx^2)^{9/4}} + \frac{2(7Ab^3+a(2b^2B-11abC+20a^2D))x}{45a^2b^3(a+bx^2)^{5/4}} + \frac{2Dx}{b^3\sqrt[4]{a+bx^2}} + \frac{2(7Ab^3+2a(b^2B+2abC-20a^2D))\sqrt[4]{1+\frac{bx^2}{a}}\,E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{15a^{5/2}b^{7/2}\sqrt[4]{a+bx^2}}$$

output

```
2/9*(A/a-(B*b^2-C*a*b+D*a^2)/b^3)*x/(b*x^2+a)^(9/4)+2/45*(7*A*b^3+a*(2*B*b
^2-11*C*a*b+20*D*a^2))*x/a^2/b^3/(b*x^2+a)^(5/4)+2*D*x/b^3/(b*x^2+a)^(1/4)
+2/15*(7*A*b^3+2*a*(B*b^2+2*C*a*b-20*D*a^2))*(1+b*x^2/a)^(1/4)*EllipticE(s
in(1/2*arctan(b^(1/2)*x/a^(1/2))),2^(1/2))/a^(5/2)/b^(7/2)/(b*x^2+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.16 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.98

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{13/4}}\,dx = \frac{2x(-60a^5D+21Ab^5x^4+ab^4x^2(49A+6Bx^2)+2a^4b(3C-65Dx^2)+a^2b}{}$$

input
```
Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(a + b*x^2)^(13/4),x]
```

output
```
(2*x*(-60*a^5*D + 21*A*b^5*x^4 + a*b^4*x^2*(49*A + 6*B*x^2) + 2*a^4*b*(3*C
 - 65*D*x^2) + a^2*b^3*(33*A + 14*B*x^2 + 12*C*x^4) + a^3*b^2*(3*B + 13*C*
x^2 - 75*D*x^4)) - 3*(7*A*b^3 + 2*a*(b^2*B + 2*a*b*C - 20*a^2*D))*x*(a + b
*x^2)^2*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2, -((b*x^2)/a
)])/(45*a^3*b^3*(a + b*x^2)^(9/4))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.35, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {2345, 27, 1471, 27, 298, 227, 225, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{13/4}}dx$$

$$\Big\downarrow \ 2345$$

$$\frac{2x\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{9a\,(a+bx^2)^{9/4}}-\frac{2\int -\frac{\frac{9aDx^4}{b}+\frac{9a(bC-aD)x^2}{b^2}+7A+\frac{2a\left(Da^2-bCa+b^2B\right)}{b^3}}{2(bx^2+a)^{9/4}}dx}{9a}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{\frac{9aDx^4}{b}+\frac{9a(bC-aD)x^2}{b^2}+7A+\frac{2a\left(Da^2-bCa+b^2B\right)}{b^3}}{(bx^2+a)^{9/4}}dx}{9a}+\frac{2x\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{9a\left(a+bx^2\right)^{9/4}}$$

$\Big\downarrow$ 1471

$$\frac{\frac{2x\left(\frac{20a^2D-11abC+2b^2B}{b^3}+\frac{7A}{a}\right)}{5(a+bx^2)^{5/4}}-\frac{2\int -\frac{3\left(\left(7A+\frac{2a\left(-5Da^2+2bCa+b^2B\right)}{b^3}\right)b^2+15a^2Dx^2\right)}{2b^2\left(bx^2+a\right)^{5/4}}dx}{5a}}{9a}+\frac{2x\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{9a\left(a+bx^2\right)^{9/4}}$$

$\Big\downarrow$ 27

$$\frac{\frac{3\int \frac{7Ab^2+15a^2Dx^2+2a\left(-\frac{5Da^2}{b}+2Ca+bB\right)}{\left(bx^2+a\right)^{5/4}}dx}{5ab^2}+\frac{2x\left(\frac{20a^2D-11abC+2b^2B}{b^3}+\frac{7A}{a}\right)}{5(a+bx^2)^{5/4}}}{9a}+\frac{2x\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{9a\left(a+bx^2\right)^{9/4}}$$

$\Big\downarrow$ 298

$$\frac{\frac{3\left(\frac{2x\left(a\left(-25a^2D+4abC+2b^2B\right)+7Ab^3\right)}{ab\sqrt[4]{a+bx^2}}-\frac{\left(-40a^3D+4a^2bC+2ab^2B+7Ab^3\right)\int\frac{1}{\sqrt[4]{bx^2+a}}dx}{ab}\right)}{5ab^2}+\frac{2x\left(\frac{20a^2D-11abC+2b^2B}{b^3}+\frac{7A}{a}\right)}{5(a+bx^2)^{5/4}}}{9a}+\frac{2x\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{9a\left(a+bx^2\right)^{9/4}}$$

$\Big\downarrow$ 227

$$\frac{\frac{3\left(\frac{2x\left(a\left(-25a^2D+4abC+2b^2B\right)+7Ab^3\right)}{ab\sqrt[4]{a+bx^2}}-\frac{\sqrt[4]{\frac{bx^2}{a}+1}\left(-40a^3D+4a^2bC+2ab^2B+7Ab^3\right)\int\frac{1}{\sqrt[4]{\frac{bx^2}{a}+1}}dx}{ab\sqrt[4]{a+bx^2}}\right)}{5ab^2}+\frac{2x\left(\frac{20a^2D-11abC+2b^2B}{b^3}+\frac{7A}{a}\right)}{5(a+bx^2)^{5/4}}}{9a}+\frac{2x\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{9a\left(a+bx^2\right)^{9/4}}$$

$\Big\downarrow$ 225

$$\frac{\frac{3\left(\frac{2x\left(a\left(-25a^2D+4abC+2b^2B\right)+7Ab^3\right)}{ab\sqrt[4]{a+bx^2}}-\frac{\sqrt[4]{\frac{bx^2}{a}+1}\left(-40a^3D+4a^2bC+2ab^2B+7Ab^3\right)\left(\frac{2x}{\sqrt[4]{\frac{bx^2}{a}+1}}-\int\frac{1}{\left(\frac{bx^2}{a}+1\right)^{5/4}}dx\right)}{ab\sqrt[4]{a+bx^2}}\right)}{5ab^2}+\frac{2x\left(\frac{20a^2D-11ab}{b^3}\right.}{5(a+bx}}{9a}$$

$$\frac{2x\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{9a\left(a+bx^2\right)^{9/4}}$$

$\Big\downarrow$ 212

$$\frac{2x\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{9a\left(a+bx^2\right)^{9/4}}+$$

$$\frac{\frac{2x\left(\frac{20a^2D-11abC+2b^2B}{b^3}+\frac{7A}{a}\right)}{5(a+bx^2)^{5/4}}+\frac{3\left(\frac{2x\left(a\left(-25a^2D+4abC+2b^2B\right)+7Ab^3\right)}{ab\sqrt[4]{a+bx^2}}-\frac{\sqrt[4]{\frac{bx^2}{a}+1}\left(\frac{2x}{\sqrt[4]{\frac{bx^2}{a}+1}}-\frac{2\sqrt{a}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}}\right)\left(-40a^3D+\right.}{ab\sqrt[4]{a+bx^2}}\right.}{5ab^2}}{9a}$$

input
```
Int[(A + B*x^2 + C*x^4 + D*x^6)/(a + b*x^2)^(13/4),x]
```

output
```
(2*(A - (a*(b^2*B - a*b*C + a^2*D))/b^3)*x)/(9*a*(a + b*x^2)^(9/4)) + ((2*
((7*A)/a + (2*b^2*B - 11*a*b*C + 20*a^2*D)/b^3)*x)/(5*(a + b*x^2)^(5/4)) +
 (3*((2*(7*A*b^3 + a*(2*b^2*B + 4*a*b*C - 25*a^2*D))*x)/(a*b*(a + b*x^2)^(
1/4)) - ((7*A*b^3 + 2*a*b^2*B + 4*a^2*b*C - 40*a^3*D)*(1 + (b*x^2)/a)^(1/4
)*((2*x)/(1 + (b*x^2)/a)^(1/4) - (2*Sqrt[a]*EllipticE[ArcTan[(Sqrt[b]*x)/S
qrt[a]]/2, 2])/Sqrt[b]))/(a*b*(a + b*x^2)^(1/4))))/(5*a*b^2))/(9*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 225
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Simp[a   Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[
a, 0] && PosQ[b/a]
```

rule 227
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4)   Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Maple [F]

$$\int \frac{Dx^{6}+C\,x^{4}+x^{2}B+A}{\left(b\,x^{2}+a\right)^{\frac{13}{4}}}dx$$

input
```
int((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(13/4),x)
```

output
```
int((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(13/4),x)
```

Fricas [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{13/4}}\,dx=\int \frac{Dx^{6}+Cx^{4}+Bx^{2}+A}{(bx^{2}+a)^{\frac{13}{4}}}\,dx$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(13/4),x, algorithm="fricas")
```

output
```
integral((D*x^6 + C*x^4 + B*x^2 + A)*(b*x^2 + a)^(3/4)/(b^4*x^8 + 4*a*b^3*
x^6 + 6*a^2*b^2*x^4 + 4*a^3*b*x^2 + a^4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 19.54 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.60

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{13/4}}\,dx = \frac{Ax\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{13}{4}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{a^{\frac{13}{4}}} + \frac{Bx^3\,{}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{13}{4}\\ \frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{3a^{\frac{13}{4}}} + \frac{Cx^5\,{}_2F_1\left(\begin{matrix}\frac{5}{2},\frac{13}{4}\\ \frac{7}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{5a^{\frac{13}{4}}} + \frac{Dx^7\,{}_2F_1\left(\begin{matrix}\frac{13}{4},\frac{7}{2}\\ \frac{9}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{7a^{\frac{13}{4}}}$$

input
```
integrate((D*x**6+C*x**4+B*x**2+A)/(b*x**2+a)**(13/4),x)
```

output
```
A*x*hyper((1/2, 13/4), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(13/4) + B*x**
3*hyper((3/2, 13/4), (5/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(13/4)) + C*x
**5*hyper((5/2, 13/4), (7/2,), b*x**2*exp_polar(I*pi)/a)/(5*a**(13/4)) + D
*x**7*hyper((13/4, 7/2), (9/2,), b*x**2*exp_polar(I*pi)/a)/(7*a**(13/4))
```

Maxima [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{13/4}}\,dx = \int \frac{Dx^6+Cx^4+Bx^2+A}{(bx^2+a)^{\frac{13}{4}}}\,dx$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(13/4),x, algorithm="maxima")
```

output
```
integrate((D*x^6 + C*x^4 + B*x^2 + A)/(b*x^2 + a)^(13/4), x)
```

Giac [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{13/4}}\,dx=\int \frac{Dx^6+Cx^4+Bx^2+A}{(bx^2+a)^{\frac{13}{4}}}\,dx$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(13/4),x, algorithm="giac")
```

output
```
integrate((D*x^6 + C*x^4 + B*x^2 + A)/(b*x^2 + a)^(13/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{13/4}}\,dx=\int \frac{A+B\,x^2+C\,x^4+x^6\,D}{(b\,x^2+a)^{13/4}}\,dx$$

input
```
int((A + B*x^2 + C*x^4 + x^6*D)/(a + b*x^2)^(13/4),x)
```

output
```
int((A + B*x^2 + C*x^4 + x^6*D)/(a + b*x^2)^(13/4), x)
```

Reduce [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{13/4}}\,dx=\left(\int \frac{x^6}{(b\,x^2+a)^{\frac{1}{4}}\,a^3+3\,(b\,x^2+a)^{\frac{1}{4}}\,a^2b\,x^2+3\,(b\,x^2+a)^{\frac{1}{4}}\,a\,b^2x^4+(b\,x^2+a\right.$$

$$+\left(\int \frac{x^4}{(b\,x^2+a)^{\frac{1}{4}}\,a^3+3\,(b\,x^2+a)^{\frac{1}{4}}\,a^2b\,x^2+3\,(b\,x^2+a)^{\frac{1}{4}}\,a\,b^2x^4+(b\,x^2+a)^{\frac{1}{4}}\,b^3x^6}dx\right)c$$

$$+\left(\int \frac{x^2}{(b\,x^2+a)^{\frac{1}{4}}\,a^3+3\,(b\,x^2+a)^{\frac{1}{4}}\,a^2b\,x^2+3\,(b\,x^2+a)^{\frac{1}{4}}\,a\,b^2x^4+(b\,x^2+a)^{\frac{1}{4}}\,b^3x^6}dx\right)b$$

$$+\left(\int \frac{1}{(b\,x^2+a)^{\frac{1}{4}}\,a^3+3\,(b\,x^2+a)^{\frac{1}{4}}\,a^2b\,x^2+3\,(b\,x^2+a)^{\frac{1}{4}}\,a\,b^2x^4+(b\,x^2+a)^{\frac{1}{4}}\,b^3x^6}dx\right)a$$

input
```
int((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(13/4),x)
```

output
```
int(x**6/((a + b*x**2)**(1/4)*a**3 + 3*(a + b*x**2)**(1/4)*a**2*b*x**2 + 3
*(a + b*x**2)**(1/4)*a*b**2*x**4 + (a + b*x**2)**(1/4)*b**3*x**6),x)*d + i
nt(x**4/((a + b*x**2)**(1/4)*a**3 + 3*(a + b*x**2)**(1/4)*a**2*b*x**2 + 3*
(a + b*x**2)**(1/4)*a*b**2*x**4 + (a + b*x**2)**(1/4)*b**3*x**6),x)*c + in
t(x**2/((a + b*x**2)**(1/4)*a**3 + 3*(a + b*x**2)**(1/4)*a**2*b*x**2 + 3*(
a + b*x**2)**(1/4)*a*b**2*x**4 + (a + b*x**2)**(1/4)*b**3*x**6),x)*b + int
(1/((a + b*x**2)**(1/4)*a**3 + 3*(a + b*x**2)**(1/4)*a**2*b*x**2 + 3*(a +
b*x**2)**(1/4)*a*b**2*x**4 + (a + b*x**2)**(1/4)*b**3*x**6),x)*a
```

3.150 $\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{15/4}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 227

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{15/4}}\,dx = \frac{2\left(\frac{A}{a}-\frac{b^2B-abC+a^2D}{b^3}\right)x}{11\,(a+bx^2)^{11/4}} + \frac{2(9Ab^3+a(2b^2B-13abC+24a^2D))\,x}{77a^2b^3\,(a+bx^2)^{7/4}} + \frac{2(45Ab^3+a(10b^2B+12abC-111a^2D))\,x}{231a^3b^3\,(a+bx^2)^{3/4}} + \frac{2(45Ab^3+10ab^2B+12a^2bC+120a^3D)\left(1+\frac{bx^2}{a}\right)^{3/4}\operatorname{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{231a^{5/2}b^{7/2}\,(a+bx^2)^{3/4}}$$

output

```
2/11*(A/a-(B*b^2-C*a*b+D*a^2)/b^3)*x/(b*x^2+a)^(11/4)+2/77*(9*A*b^3+a*(2*B
*b^2-13*C*a*b+24*D*a^2))*x/a^2/b^3/(b*x^2+a)^(7/4)+2/231*(45*A*b^3+a*(10*B
*b^2+12*C*a*b-111*D*a^2))*x/a^3/b^3/(b*x^2+a)^(3/4)+2/231*(45*A*b^3+10*B*a
*b^2+12*C*a^2*b+120*D*a^3)*(1+b*x^2/a)^(3/4)*InverseJacobiAM(1/2*arctan(b^
(1/2)*x/a^(1/2)),2^(1/2))/a^(5/2)/b^(7/2)/(b*x^2+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.19 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.85

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{15/4}}\,dx = \frac{-2x(60a^5D-45Ab^5x^4-ab^4x^2(117A+10Bx^2)+6a^4b(C+25Dx^2)-a^2}{}$$

input
```
Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(a + b*x^2)^(15/4),x]
```

output
```
(-2*x*(60*a^5*D - 45*A*b^5*x^4 - a*b^4*x^2*(117*A + 10*B*x^2) + 6*a^4*b*(C
 + 25*D*x^2) - a^2*b^3*(93*A + 26*B*x^2 + 12*C*x^4) + a^3*b^2*(5*B + 15*C*
x^2 + 111*D*x^4)) + (45*A*b^3 + 2*a*(5*b^2*B + 6*a*b*C + 60*a^2*D))*x*(a +
 b*x^2)^2*(1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[1/2, 3/4, 3/2, -((b*x^2)
/a)])/(231*a^3*b^3*(a + b*x^2)^(11/4))
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2345, 27, 1471, 27, 298, 231, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{15/4}}dx$$

$$\Big\downarrow \text{ 2345}$$

$$\frac{2x\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{11a(a+bx^2)^{11/4}}-\frac{2\int -\frac{\frac{11aDx^4}{b}+\frac{11a(bC-aD)x^2}{b^2}+9A+\frac{2a\left(Da^2-bCa+b^2B\right)}{b^3}}{2(bx^2+a)^{11/4}}dx}{11a}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{\frac{11aDx^4}{b}+\frac{11a(bC-aD)x^2}{b^2}+9A+\frac{2a\left(Da^2-bCa+b^2B\right)}{b^3}}{(bx^2+a)^{11/4}}dx}{11a}+\frac{2x\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{11a\left(a+bx^2\right)^{11/4}}$$

$\Big\downarrow$ 1471

$$\frac{\frac{2x\left(\frac{24a^2D-13abC+2b^2B}{b^3}+\frac{9A}{a}\right)}{7(a+bx^2)^{7/4}}-\frac{2\int -\frac{\left(45A+\frac{2a\left(-17Da^2+6bCa+5b^2B\right)}{b^3}\right)b^2+77a^2Dx^2}{2b^2\left(bx^2+a\right)^{7/4}}dx}{7a}}{11a}+\frac{2x\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{11a\left(a+bx^2\right)^{11/4}}$$

$\Big\downarrow$ 27

$$\frac{\frac{\int \frac{45Ab^2+77a^2Dx^2+2a\left(-\frac{17Da^2}{b}+6Ca+5bB\right)}{\left(bx^2+a\right)^{7/4}}dx}{7ab^2}+\frac{2x\left(\frac{24a^2D-13abC+2b^2B}{b^3}+\frac{9A}{a}\right)}{7(a+bx^2)^{7/4}}}{11a}+\frac{2x\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{11a\left(a+bx^2\right)^{11/4}}$$

$\Big\downarrow$ 298

$$\frac{\frac{\frac{\left(120a^3D+12a^2bC+10ab^2B+45Ab^3\right)\int \frac{1}{\left(bx^2+a\right)^{3/4}}dx}{3ab}+\frac{2x\left(a\left(-111a^2D+12abC+10b^2B\right)+45Ab^3\right)}{3ab\left(a+bx^2\right)^{3/4}}}{7ab^2}+\frac{2x\left(\frac{24a^2D-13abC+2b^2B}{b^3}+\frac{9A}{a}\right)}{7(a+bx^2)^{7/4}}}{11a}+\frac{2x\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{11a\left(a+bx^2\right)^{11/4}}$$

$\Big\downarrow$ 231

$$\frac{\frac{\frac{\left(\frac{bx^2}{a}+1\right)^{3/4}\left(120a^3D+12a^2bC+10ab^2B+45Ab^3\right)\int \frac{1}{\left(\frac{bx^2}{a}+1\right)^{3/4}}dx}{3ab\left(a+bx^2\right)^{3/4}}+\frac{2x\left(a\left(-111a^2D+12abC+10b^2B\right)+45Ab^3\right)}{3ab\left(a+bx^2\right)^{3/4}}}{7ab^2}+\frac{2x\left(\frac{24a^2D-13abC+2b^2B}{b^3}+\frac{9A}{a}\right)}{7(a+bx^2)^{7/4}}}{11a}+\frac{2x\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{11a\left(a+bx^2\right)^{11/4}}$$

$\Big\downarrow$ 229

$$\frac{2x\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{11a\left(a+bx^2\right)^{11/4}}+\frac{\frac{2x\left(\frac{24a^2D-13abC+2b^2B}{b^3}+\frac{9A}{a}\right)}{7(a+bx^2)^{7/4}}+\frac{\frac{2x\left(a\left(-111a^2D+12abC+10b^2B\right)+45Ab^3\right)}{3ab\left(a+bx^2\right)^{3/4}}+\frac{2\left(\frac{bx^2}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)\left(120a^3D+12a^2bC+10ab^2B+\right.}{3\sqrt{a}b^{3/2}\left(a+bx^2\right)^{3/4}}}{7ab^2}}{11a}$$

input
```
Int[(A + B*x^2 + C*x^4 + D*x^6)/(a + b*x^2)^(15/4),x]
```

output
```
(2*(A - (a*(b^2*B - a*b*C + a^2*D))/b^3)*x)/(11*a*(a + b*x^2)^(11/4)) + ((
2*((9*A)/a + (2*b^2*B - 13*a*b*C + 24*a^2*D)/b^3)*x)/(7*(a + b*x^2)^(7/4))
 + ((2*(45*A*b^3 + a*(10*b^2*B + 12*a*b*C - 111*a^2*D))*x)/(3*a*b*(a + b*x
^2)^(3/4)) + (2*(45*A*b^3 + 10*a*b^2*B + 12*a^2*b*C + 120*a^3*D)*(1 + (b*x
^2)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3*Sqrt[a]*b^(3/
2)*(a + b*x^2)^(3/4)))/(7*a*b^2))/(11*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Maple [F]

$$\int \frac{Dx^{6}+C\,x^{4}+x^{2}B+A}{\left(b\,x^{2}+a\right)^{\frac{15}{4}}}dx$$

input
```
int((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(15/4),x)
```

output
```
int((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(15/4),x)
```

Fricas [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{15/4}}\,dx=\int \frac{Dx^{6}+Cx^{4}+Bx^{2}+A}{\left(bx^{2}+a\right)^{\frac{15}{4}}}\,dx$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(15/4),x, algorithm="fricas")
```

output
```
integral((D*x^6 + C*x^4 + B*x^2 + A)*(b*x^2 + a)^(1/4)/(b^4*x^8 + 4*a*b^3*
x^6 + 6*a^2*b^2*x^4 + 4*a^3*b*x^2 + a^4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 37.91 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.52

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{15/4}}\,dx = \frac{Ax\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{15}{4}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{a^{\frac{15}{4}}} + \frac{Bx^3\,{}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{15}{4}\\ \frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{3a^{\frac{15}{4}}} + \frac{Cx^5\,{}_2F_1\left(\begin{matrix}\frac{5}{2},\frac{15}{4}\\ \frac{7}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{5a^{\frac{15}{4}}} + \frac{Dx^7\,{}_2F_1\left(\begin{matrix}\frac{7}{2},\frac{15}{4}\\ \frac{9}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{7a^{\frac{15}{4}}}$$

input
```
integrate((D*x**6+C*x**4+B*x**2+A)/(b*x**2+a)**(15/4),x)
```

output
```
A*x*hyper((1/2, 15/4), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(15/4) + B*x**
3*hyper((3/2, 15/4), (5/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(15/4)) + C*x
**5*hyper((5/2, 15/4), (7/2,), b*x**2*exp_polar(I*pi)/a)/(5*a**(15/4)) + D
*x**7*hyper((7/2, 15/4), (9/2,), b*x**2*exp_polar(I*pi)/a)/(7*a**(15/4))
```

Maxima [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{15/4}}\,dx = \int \frac{Dx^6+Cx^4+Bx^2+A}{(bx^2+a)^{\frac{15}{4}}}\,dx$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(15/4),x, algorithm="maxima")
```

output
```
integrate((D*x^6 + C*x^4 + B*x^2 + A)/(b*x^2 + a)^(15/4), x)
```

Giac [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{15/4}}\,dx=\int \frac{Dx^6+Cx^4+Bx^2+A}{(bx^2+a)^{\frac{15}{4}}}\,dx$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(15/4),x, algorithm="giac")
```

output
```
integrate((D*x^6 + C*x^4 + B*x^2 + A)/(b*x^2 + a)^(15/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{15/4}}\,dx=\int \frac{A+B\,x^2+C\,x^4+x^6\,D}{(b\,x^2+a)^{15/4}}\,dx$$

input
```
int((A + B*x^2 + C*x^4 + x^6*D)/(a + b*x^2)^(15/4),x)
```

output
```
int((A + B*x^2 + C*x^4 + x^6*D)/(a + b*x^2)^(15/4), x)
```

Reduce [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{15/4}}\,dx=\left(\int \frac{x^6}{(b\,x^2+a)^{\frac{3}{4}}\,a^3+3\,(b\,x^2+a)^{\frac{3}{4}}\,a^2b\,x^2+3\,(b\,x^2+a)^{\frac{3}{4}}\,a\,b^2x^4+(b\,x^2+a}\right.$$
$$+\left(\int \frac{x^4}{(b\,x^2+a)^{\frac{3}{4}}\,a^3+3\,(b\,x^2+a)^{\frac{3}{4}}\,a^2b\,x^2+3\,(b\,x^2+a)^{\frac{3}{4}}\,a\,b^2x^4+(b\,x^2+a)^{\frac{3}{4}}\,b^3x^6}dx\right)c$$
$$+\left(\int \frac{x^2}{(b\,x^2+a)^{\frac{3}{4}}\,a^3+3\,(b\,x^2+a)^{\frac{3}{4}}\,a^2b\,x^2+3\,(b\,x^2+a)^{\frac{3}{4}}\,a\,b^2x^4+(b\,x^2+a)^{\frac{3}{4}}\,b^3x^6}dx\right)b$$
$$+\left(\int \frac{1}{(b\,x^2+a)^{\frac{3}{4}}\,a^3+3\,(b\,x^2+a)^{\frac{3}{4}}\,a^2b\,x^2+3\,(b\,x^2+a)^{\frac{3}{4}}\,a\,b^2x^4+(b\,x^2+a)^{\frac{3}{4}}\,b^3x^6}dx\right)a$$

input
```
int((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(15/4),x)
```

output
```
int(x**6/((a + b*x**2)**(3/4)*a**3 + 3*(a + b*x**2)**(3/4)*a**2*b*x**2 + 3
*(a + b*x**2)**(3/4)*a*b**2*x**4 + (a + b*x**2)**(3/4)*b**3*x**6),x)*d + i
nt(x**4/((a + b*x**2)**(3/4)*a**3 + 3*(a + b*x**2)**(3/4)*a**2*b*x**2 + 3*
(a + b*x**2)**(3/4)*a*b**2*x**4 + (a + b*x**2)**(3/4)*b**3*x**6),x)*c + in
t(x**2/((a + b*x**2)**(3/4)*a**3 + 3*(a + b*x**2)**(3/4)*a**2*b*x**2 + 3*(
a + b*x**2)**(3/4)*a*b**2*x**4 + (a + b*x**2)**(3/4)*b**3*x**6),x)*b + int
(1/((a + b*x**2)**(3/4)*a**3 + 3*(a + b*x**2)**(3/4)*a**2*b*x**2 + 3*(a +
b*x**2)**(3/4)*a*b**2*x**4 + (a + b*x**2)**(3/4)*b**3*x**6),x)*a
```

3.151 $\int (a+bx^2)^p (A+Bx^2+Cx^4+Dx^6)\,dx$

Optimal result

Integrand size = 27, antiderivative size = 262

$$\int (a+bx^2)^p (A+Bx^2+Cx^4+Dx^6)\,dx = \frac{(15a^2D-3abC(7+2p)+b^2B(35+24p+4p^2))\,x(a+bx^2)^{1+p}}{b^3(3+2p)(5+2p)(7+2p)} - \frac{(5aD-bC(7+2p))x^3(a+bx^2)^{1+p}}{b^2(5+2p)(7+2p)} + \frac{Dx^5(a+bx^2)^{1+p}}{b(7+2p)} + \frac{\left(Ab^3(35+24p+4p^2)-\frac{a(15a^2D-3abC(7+2p)+b^2B(35+24p+4p^2))}{3+2p}\right)x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric}}{b^3(5+2p)(7+2p)}$$

output
```
(15*a^2*D-3*a*b*C*(7+2*p)+b^2*B*(4*p^2+24*p+35))*x*(b*x^2+a)^(p+1)/b^3/(3+
2*p)/(5+2*p)/(7+2*p)-(5*D*a-b*C*(7+2*p))*x^3*(b*x^2+a)^(p+1)/b^2/(5+2*p)/(
7+2*p)+D*x^5*(b*x^2+a)^(p+1)/b/(7+2*p)+(A*b^3*(4*p^2+24*p+35)-a*(15*a^2*D-
3*a*b*C*(7+2*p)+b^2*B*(4*p^2+24*p+35))/(3+2*p))*x*(b*x^2+a)^p*hypergeom([1
/2, -p],[3/2],-b*x^2/a)/b^3/(5+2*p)/(7+2*p)/((1+b*x^2/a)^p)
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.48

$$\int (a+bx^2)^p (A+Bx^2+Cx^4+Dx^6)\ dx = \frac{1}{105}x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\left(105A\ \text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right) + 35Bx^2\ \text{Hypergeometric2F1}\left(\frac{3}{2},-p,\frac{5}{2},-\frac{bx^2}{a}\right) + 21Cx^4\ \text{Hypergeometric2F1}\left(\frac{5}{2},-p,\frac{7}{2},-\frac{bx^2}{a}\right) + 15Dx^6\ \text{Hypergeometric2F1}\left(\frac{7}{2},-p,\frac{9}{2},-\frac{bx^2}{a}\right)\right)$$

input
```
Integrate[(a + b*x^2)^p*(A + B*x^2 + C*x^4 + D*x^6),x]
```

output
```
(x*(a + b*x^2)^p*(105*A*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)] + 35
*B*x^2*Hypergeometric2F1[3/2, -p, 5/2, -((b*x^2)/a)] + 21*C*x^4*Hypergeome
tric2F1[5/2, -p, 7/2, -((b*x^2)/a)] + 15*D*x^6*Hypergeometric2F1[7/2, -p,
9/2, -((b*x^2)/a)]))/(105*(1 + (b*x^2)/a)^p)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2346, 1473, 299, 238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^2)^p (A+Bx^2+Cx^4+Dx^6)\ dx$$

$$\Big\downarrow \ 2346$$

$$\frac{\int (bx^2+a)^p \left(-((5aD-bC(2p+7))x^4)+bB(2p+7)x^2+Ab(2p+7)\right)dx}{b(2p+7)}+\frac{Dx^5(a+bx^2)^{p+1}}{b(2p+7)}$$

$\Big\downarrow$ 1473

$$\frac{\frac{\int (bx^2+a)^p \left(A(4p^2+24p+35)b^2+(15Da^2-3bC(2p+7)a+b^2B(4p^2+24p+35))x^2\right)dx}{b(2p+5)}-\frac{x^3(a+bx^2)^{p+1}(5aD-bC(2p+7))}{b(2p+5)}}{b(2p+7)}+\frac{Dx^5(a+bx^2)^{p+1}}{b(2p+7)}$$

$\Big\downarrow$ 299

$$\frac{\frac{\frac{\left(Ab^3(4p^2+24p+35)-\frac{a(15a^2D-3abC(2p+7)+b^2B(4p^2+24p+35))}{2p+3}\right)\int (bx^2+a)^p dx}{b}+\frac{x(a+bx^2)^{p+1}(15a^2D-3abC(2p+7)+b^2B(4p^2+24p+35))}{b(2p+3)}}{b(2p+5)}-\frac{x^3(a+}{}}{b(2p+7)}$$

$$\frac{Dx^5(a+bx^2)^{p+1}}{b(2p+7)}$$

$\Big\downarrow$ 238

$$\frac{\frac{\frac{(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(Ab^3(4p^2+24p+35)-\frac{a(15a^2D-3abC(2p+7)+b^2B(4p^2+24p+35))}{2p+3}\right)\int\left(\frac{bx^2}{a}+1\right)^p dx}{b}+\frac{x(a+bx^2)^{p+1}(15a^2D-3abC(2p+7)+b^2B(4p}{b(2p+3)}}{b(2p+5)}}{b(2p+7)}$$

$$\frac{Dx^5(a+bx^2)^{p+1}}{b(2p+7)}$$

$\Big\downarrow$ 237

$$\frac{\frac{\frac{x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right)\left(Ab^3(4p^2+24p+35)-\frac{a(15a^2D-3abC(2p+7)+b^2B(4p^2+24p+35))}{2p+3}\right)}{b}+\frac{x(a+bx^2)^{p+1}(15a^2D}{}}{b(2p+5)}}{b(2p+7)}$$

$$\frac{Dx^5(a+bx^2)^{p+1}}{b(2p+7)}$$

input
```
Int[(a + b*x^2)^p*(A + B*x^2 + C*x^4 + D*x^6),x]
```

output
```
(D*x^5*(a + b*x^2)^(1 + p))/(b*(7 + 2*p)) + (-(((5*a*D - b*C*(7 + 2*p))*x^
3*(a + b*x^2)^(1 + p))/(b*(5 + 2*p))) + (((15*a^2*D - 3*a*b*C*(7 + 2*p) +
b^2*B*(35 + 24*p + 4*p^2))*x*(a + b*x^2)^(1 + p))/(b*(3 + 2*p)) + ((A*b^3*
(35 + 24*p + 4*p^2) - (a*(15*a^2*D - 3*a*b*C*(7 + 2*p) + b^2*B*(35 + 24*p
+ 4*p^2)))/(3 + 2*p))*x*(a + b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b
*x^2)/a)])/(b*(1 + (b*x^2)/a)^p))/(b*(5 + 2*p)))/(b*(7 + 2*p))
```

Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] && !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] && !IntegerQ[2*p] && !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1473
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[c^p*x^(4*p - 1)*((d + e*x^2)^(q + 1)/(e*(4*p + 2*q + 1)))
, x] + Simp[1/(e*(4*p + 2*q + 1))   Int[(d + e*x^2)^q*ExpandToSum[e*(4*p +
2*q + 1)*(a + b*x^2 + c*x^4)^p - d*c^p*(4*p - 1)*x^(4*p - 2) - e*c^p*(4*p +
 2*q + 1)*x^(4*p), x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && !LtQ[q, -1]
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && !LeQ[p, -1]
```

Maple [F]

$$\int \left(b\,x^{2}+a\right)^{p} \left(D x^{6}+C\,x^{4}+x^{2} B +A \right)dx$$

input
```
int((b*x^2+a)^p*(D*x^6+C*x^4+B*x^2+A),x)
```

output
```
int((b*x^2+a)^p*(D*x^6+C*x^4+B*x^2+A),x)
```

Fricas [F]

$$\int \left(a+bx^2\right)^p \left(A+Bx^2+Cx^4+Dx^6\right)\,dx = \int \left(Dx^{6}+Cx^{4}+Bx^{2}+A\right)\left(bx^{2}+a\right)^{p}\,dx$$

input
```
integrate((b*x^2+a)^p*(D*x^6+C*x^4+B*x^2+A),x, algorithm="fricas")
```

output
```
integral((D*x^6 + C*x^4 + B*x^2 + A)*(b*x^2 + a)^p, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 18.44 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.42

$$\int (a+bx^2)^p (A+Bx^2+Cx^4+Dx^6)\, dx = Aa^p x\,{}_2F_1\left(\begin{matrix}\frac{1}{2},-p\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right) + \frac{Ba^p x^3\,{}_2F_1\left(\begin{matrix}\frac{3}{2},-p\\ \frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{3} + \frac{Ca^p x^5\,{}_2F_1\left(\begin{matrix}\frac{5}{2},-p\\ \frac{7}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{5} + \frac{Da^p x^7\,{}_2F_1\left(\begin{matrix}\frac{7}{2},-p\\ \frac{9}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{7}$$

input
```
integrate((b*x**2+a)**p*(D*x**6+C*x**4+B*x**2+A),x)
```

output
```
A*a**p*x*hyper((1/2, -p), (3/2,), b*x**2*exp_polar(I*pi)/a) + B*a**p*x**3*
hyper((3/2, -p), (5/2,), b*x**2*exp_polar(I*pi)/a)/3 + C*a**p*x**5*hyper((
5/2, -p), (7/2,), b*x**2*exp_polar(I*pi)/a)/5 + D*a**p*x**7*hyper((7/2, -p
), (9/2,), b*x**2*exp_polar(I*pi)/a)/7
```

Maxima [F]

$$\int (a+bx^2)^p (A+Bx^2+Cx^4+Dx^6)\, dx = \int (Dx^6+Cx^4+Bx^2+A)(bx^2+a)^p\, dx$$

input
```
integrate((b*x^2+a)^p*(D*x^6+C*x^4+B*x^2+A),x, algorithm="maxima")
```

output
```
integrate((D*x^6 + C*x^4 + B*x^2 + A)*(b*x^2 + a)^p, x)
```

Giac [F]

$$\int (a+bx^2)^p (A+Bx^2+Cx^4+Dx^6)\, dx = \int (Dx^6+Cx^4+Bx^2+A)(bx^2+a)^p\, dx$$

input
```
integrate((b*x^2+a)^p*(D*x^6+C*x^4+B*x^2+A),x, algorithm="giac")
```

output
```
integrate((D*x^6 + C*x^4 + B*x^2 + A)*(b*x^2 + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx^2)^p (A+Bx^2+Cx^4+Dx^6)\, dx = \int (b\,x^2+a)^p \left(A+B\,x^2+C\,x^4+x^6\,D\right) dx$$

input
```
int((a + b*x^2)^p*(A + B*x^2 + C*x^4 + x^6*D),x)
```

output
```
int((a + b*x^2)^p*(A + B*x^2 + C*x^4 + x^6*D), x)
```

Reduce [F]

$$\int (a+bx^2)^p (A+Bx^2+Cx^4+Dx^6)\, dx = \text{too large to display}$$

input
```
int((b*x^2+a)^p*(D*x^6+C*x^4+B*x^2+A),x)
```

output
```
(30*(a + b*x**2)**p*a**3*d*p*x - 12*(a + b*x**2)**p*a**2*b*c*p**2*x - 42*(
a + b*x**2)**p*a**2*b*c*p*x - 20*(a + b*x**2)**p*a**2*b*d*p**2*x**3 - 10*(
a + b*x**2)**p*a**2*b*d*p*x**3 + 16*(a + b*x**2)**p*a*b**3*p**3*x + 108*(a
 + b*x**2)**p*a*b**3*p**2*x + 212*(a + b*x**2)**p*a*b**3*p*x + 105*(a + b*
x**2)**p*a*b**3*x + 8*(a + b*x**2)**p*a*b**2*c*p**3*x**3 + 32*(a + b*x**2)
**p*a*b**2*c*p**2*x**3 + 14*(a + b*x**2)**p*a*b**2*c*p*x**3 + 8*(a + b*x**
2)**p*a*b**2*d*p**3*x**5 + 16*(a + b*x**2)**p*a*b**2*d*p**2*x**5 + 6*(a +
b*x**2)**p*a*b**2*d*p*x**5 + 8*(a + b*x**2)**p*b**4*p**3*x**3 + 52*(a + b*
x**2)**p*b**4*p**2*x**3 + 94*(a + b*x**2)**p*b**4*p*x**3 + 35*(a + b*x**2)
**p*b**4*x**3 + 8*(a + b*x**2)**p*b**3*c*p**3*x**5 + 44*(a + b*x**2)**p*b*
*3*c*p**2*x**5 + 62*(a + b*x**2)**p*b**3*c*p*x**5 + 21*(a + b*x**2)**p*b**
3*c*x**5 + 8*(a + b*x**2)**p*b**3*d*p**3*x**7 + 36*(a + b*x**2)**p*b**3*d*
p**2*x**7 + 46*(a + b*x**2)**p*b**3*d*p*x**7 + 15*(a + b*x**2)**p*b**3*d*x
**7 - 480*int((a + b*x**2)**p/(16*a*p**4 + 128*a*p**3 + 344*a*p**2 + 352*a
*p + 105*a + 16*b*p**4*x**2 + 128*b*p**3*x**2 + 344*b*p**2*x**2 + 352*b*p*
x**2 + 105*b*x**2),x)*a**4*d*p**5 - 3840*int((a + b*x**2)**p/(16*a*p**4 +
128*a*p**3 + 344*a*p**2 + 352*a*p + 105*a + 16*b*p**4*x**2 + 128*b*p**3*x*
*2 + 344*b*p**2*x**2 + 352*b*p*x**2 + 105*b*x**2),x)*a**4*d*p**4 - 10320*i
nt((a + b*x**2)**p/(16*a*p**4 + 128*a*p**3 + 344*a*p**2 + 352*a*p + 105*a
+ 16*b*p**4*x**2 + 128*b*p**3*x**2 + 344*b*p**2*x**2 + 352*b*p*x**2 + 1...
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                  print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False

   except AttributeError as error:
      return False

def expnType(expn):
   debug=False
   if debug:
      print("expn=",expn,"type(expn)=",type(expn))

   if is_atom(expn):
     return 1
   elif isinstance(expn,list):
      return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
         return 1
      else:
         return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
   elif isinstance(expn,Pow):   #type(expn,'`^`')
      if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
         return expnType(expn.args[0])   #ExpnType(op(1,expn))
      elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
         else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
      else:
         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expn
   elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
else:
    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =""
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType(


#print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#          Albert Rich to use with Sagemath. This is used to
#          grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#          'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#          issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
            'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
            'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
            'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
            'elliptic_pi','exp_integral_e','log_integral',
            'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
            'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
```

```
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file