# Computer Algebra Independent Integration Tests

Summer 2024

1-Algebraic-functions/1.1-Binomial/1.1.2-Quadratic-binomial/36-1.1.2.8

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# Contents

# CHAPTER 1

## Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [ **270** ]. This is test number [ 36 ].

## 1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

## 1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

| System | % solved | % Failed |
|---|---|---|
| Rubi | 100.00 ( 270 ) | 0.00 ( 0 ) |
| Mathematica | 100.00 ( 270 ) | 0.00 ( 0 ) |
| Maple | 92.96 ( 251 ) | 7.04 ( 19 ) |
| Fricas | 92.96 ( 251 ) | 7.04 ( 19 ) |
| Reduce | 92.59 ( 250 ) | 7.41 ( 20 ) |
| Giac | 91.48 ( 247 ) | 8.52 ( 23 ) |
| Maxima | 90.37 ( 244 ) | 9.63 ( 26 ) |
| Sympy | 89.63 ( 242 ) | 10.37 ( 28 ) |
| Mupad | 45.56 ( 123 ) | 54.44 ( 147 ) |

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

| grade | description |
|---|---|
| A | Integral was solved and antiderivative is optimal in quality and leaf size. |
| B | Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size. |
| C | Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons<br>1. antiderivative contains a hypergeometric function and the optimal antiderivative does not.<br>2. antiderivative contains a special function and the optimal antiderivative does not.<br>3. antiderivative contains the imaginary unit and the optimal antiderivative does not. |
| F | Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised. |

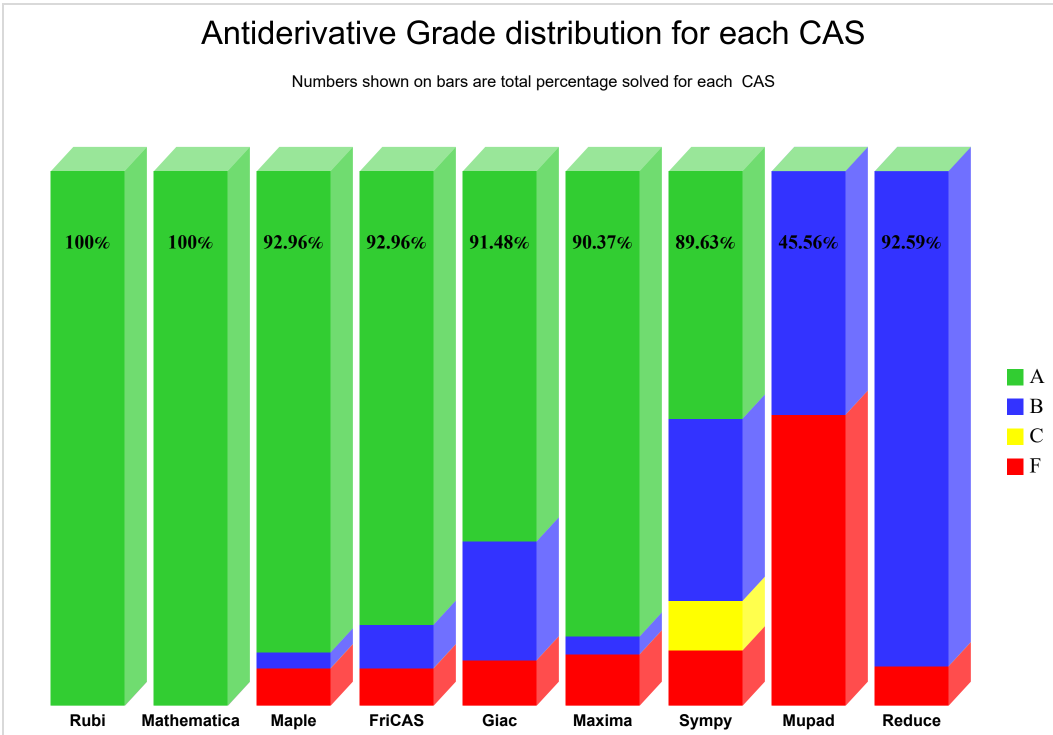
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

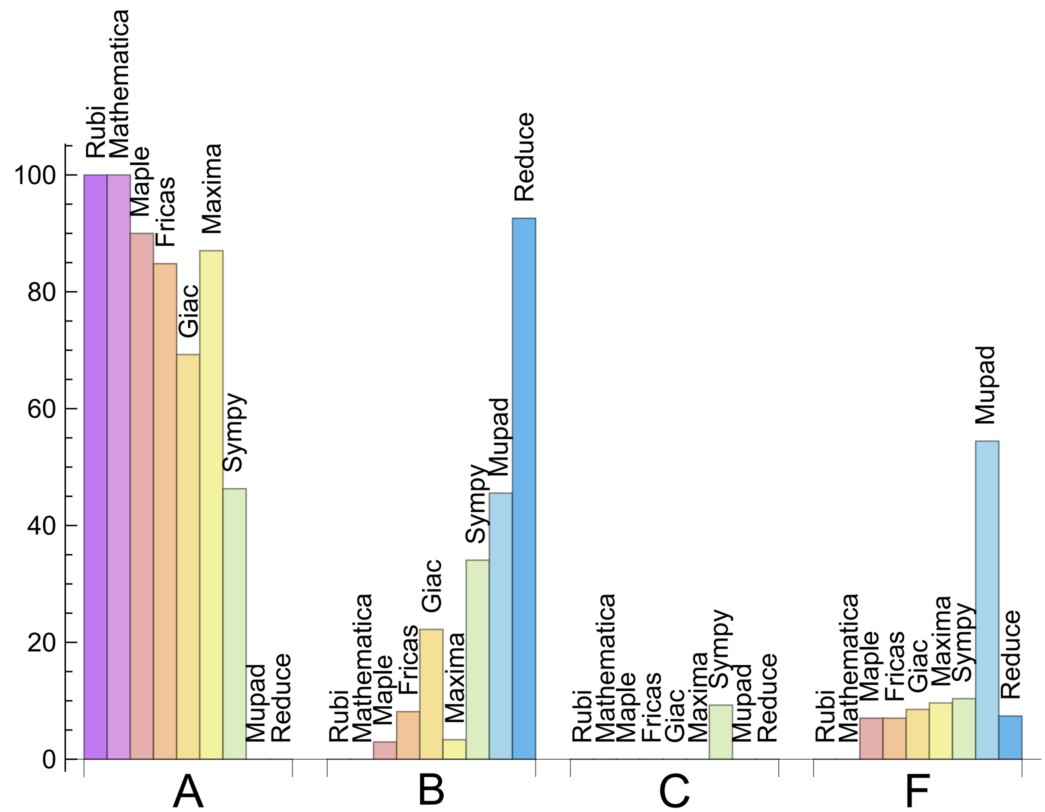
| System | % A grade | % B grade | % C grade | % F grade |
|---|---|---|---|---|
| Rubi | 100.000 | 0.000 | 0.000 | 0.000 |
| Mathematica | 100.000 | 0.000 | 0.000 | 0.000 |
| Maple | 90.000 | 2.963 | 0.000 | 7.037 |
| Maxima | 87.037 | 3.333 | 0.000 | 9.630 |
| Fricas | 84.815 | 8.148 | 0.000 | 7.037 |
| Giac | 69.259 | 22.222 | 0.000 | 8.519 |
| Sympy | 46.296 | 34.074 | 9.259 | 10.370 |
| Mupad | 0.000 | 45.556 | 0.000 | 54.444 |
| Reduce | 0.000 | 92.593 | 0.000 | 7.407 |

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.




The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

| System | Number failed | Percentage normal failure | Percentage timeout failure | Percentage exception failure |
|---|---|---|---|---|
| Rubi | 0 | 0.00 | 0.00 | 0.00 |
| Mathematica | 0 | 0.00 | 0.00 | 0.00 |
| Fricas | 19 | 100.00 | 0.00 | 0.00 |
| Maple | 19 | 100.00 | 0.00 | 0.00 |
| Reduce | 20 | 100.00 | 0.00 | 0.00 |
| Giac | 23 | 82.61 | 0.00 | 17.39 |
| Maxima | 26 | 73.08 | 0.00 | 26.92 |
| Sympy | 28 | 14.29 | 85.71 | 0.00 |
| Mupad | 147 | 0.00 | 100.00 | 0.00 |

Table 1.4: Failure statistics for each CAS

## 1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

| System | Mean time (sec) |
|---|---|
| Maxima | 0.06 |
| Fricas | 0.12 |
| Giac | 0.13 |
| Reduce | 0.23 |
| Rubi | 0.50 |
| Maple | 0.54 |
| Mathematica | 0.64 |
| Mupad | 2.34 |
| Sympy | 17.04 |

Table 1.5: Time performance for each CAS

| System | Mean size | Normalized mean | Median size | Normalized median |
|---|---|---|---|---|
| Mathematica | 135.40 | 0.83 | 128.00 | 0.84 |
| Mupad | 157.80 | 1.15 | 143.00 | 1.11 |
| Maple | 168.88 | 1.03 | 140.00 | 0.93 |
| Rubi | 178.48 | 1.05 | 162.00 | 1.02 |
| Maxima | 207.54 | 1.20 | 173.00 | 1.09 |
| Giac | 280.10 | 1.62 | 168.00 | 1.06 |
| Reduce | 305.78 | 1.76 | 258.50 | 1.66 |
| Fricas | 491.59 | 2.49 | 296.00 | 2.04 |
| Sympy | 968.90 | 5.03 | 296.50 | 1.96 |

Table 1.6: Leaf size performance for each CAS

## 1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the $y$ axis is the percentage solved which Rubi itself needed the number of rules given the $x$ axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

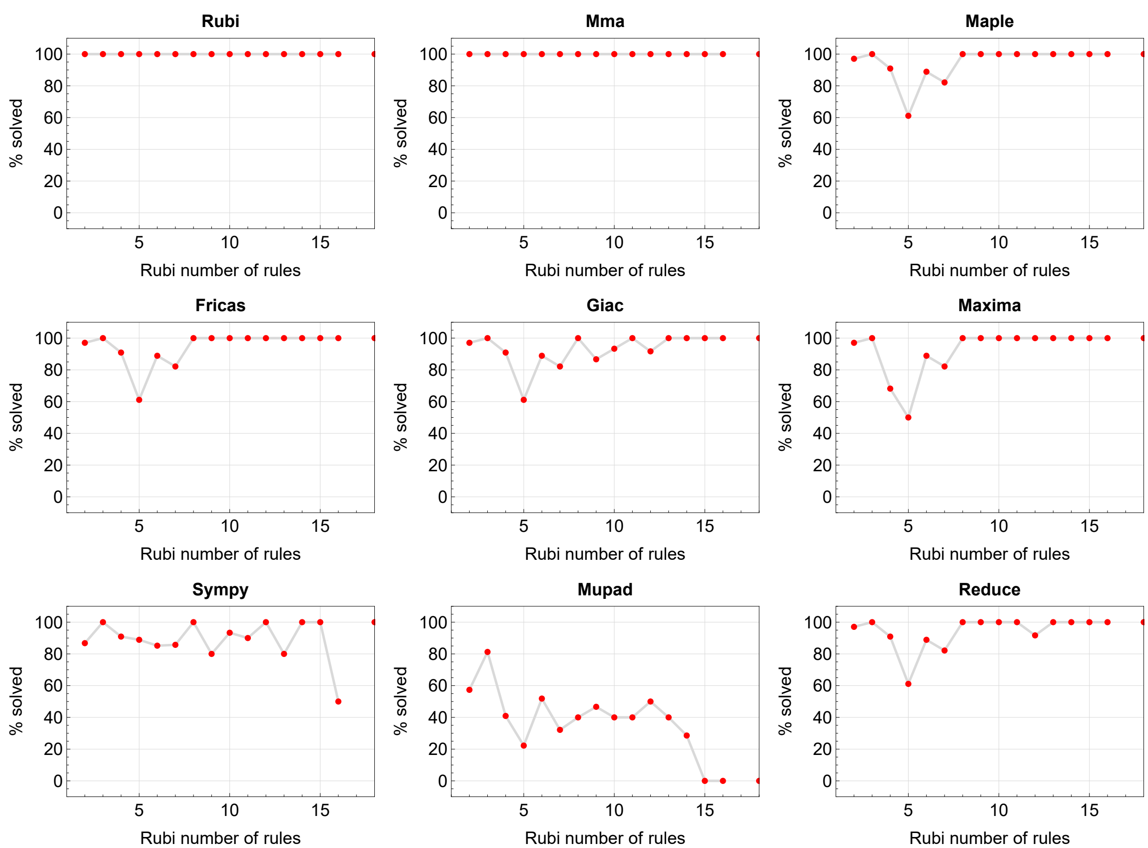



Figure 1.1: Solving statistics per number of Rubi rules used

# 1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

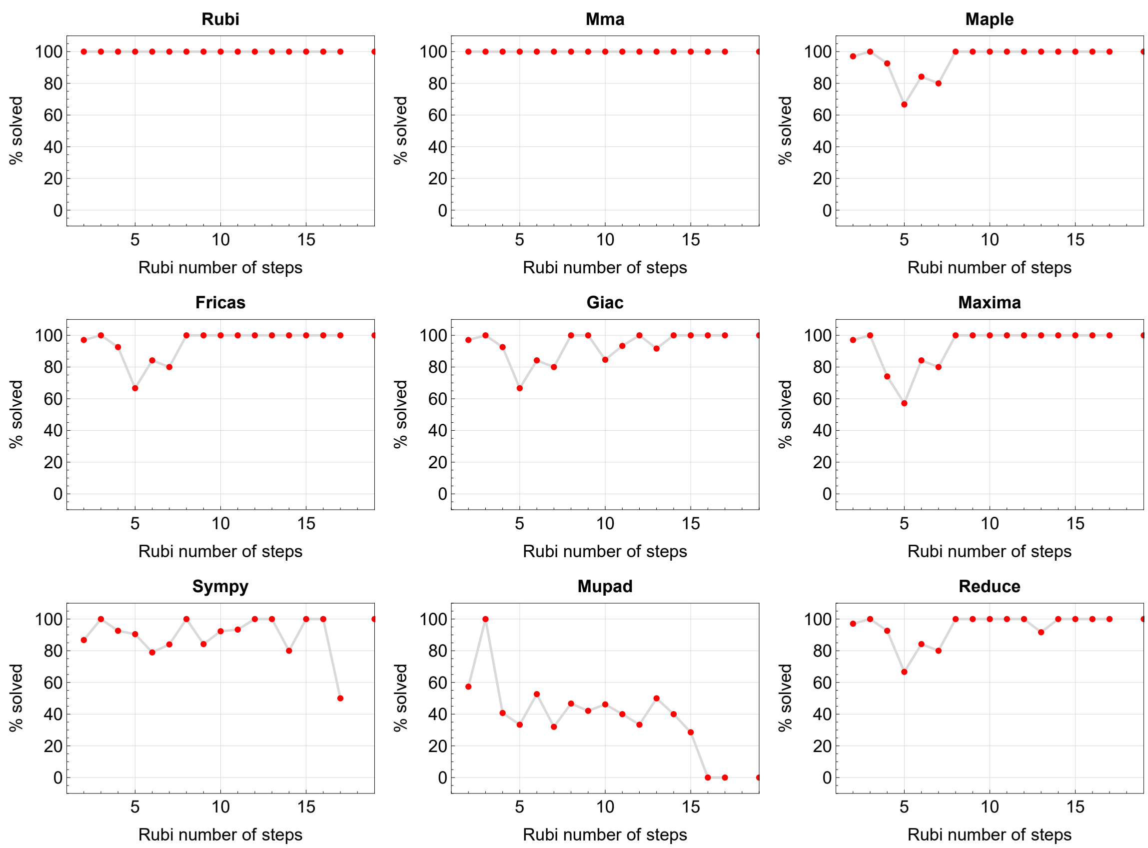



Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

## 1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

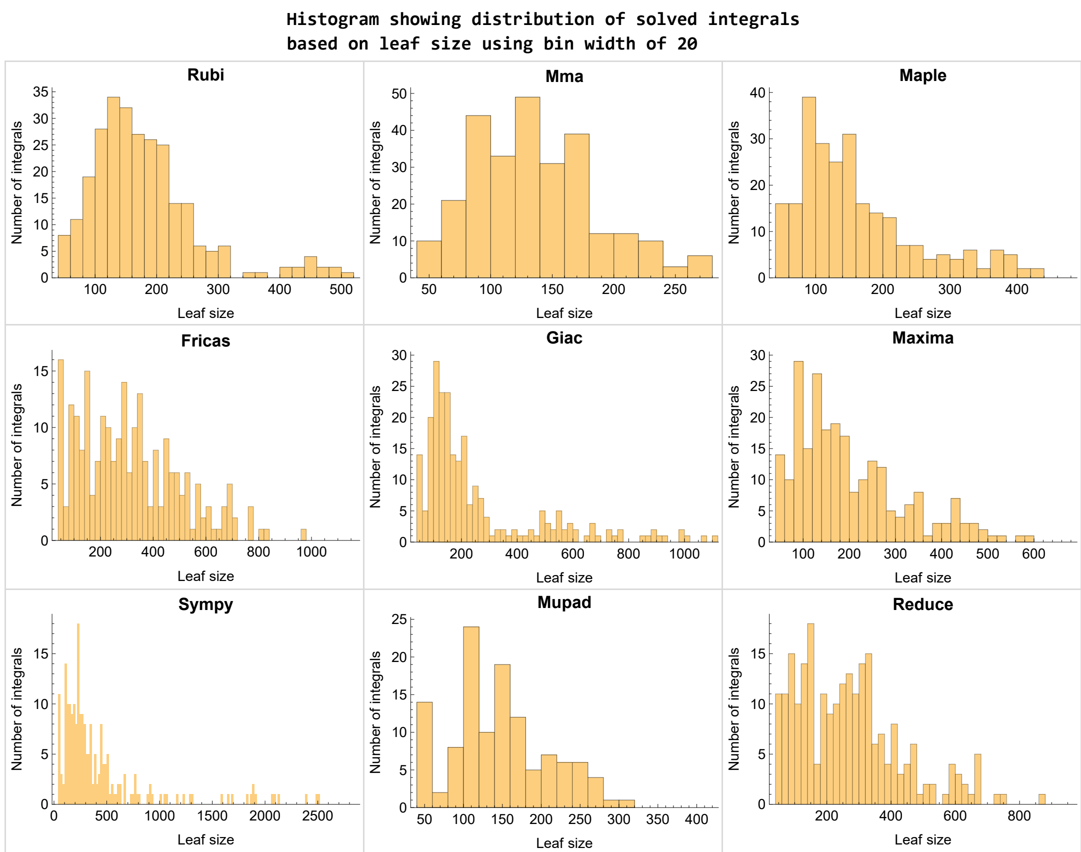



Figure 1.3: Solved integrals based on leaf size distribution

## 1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

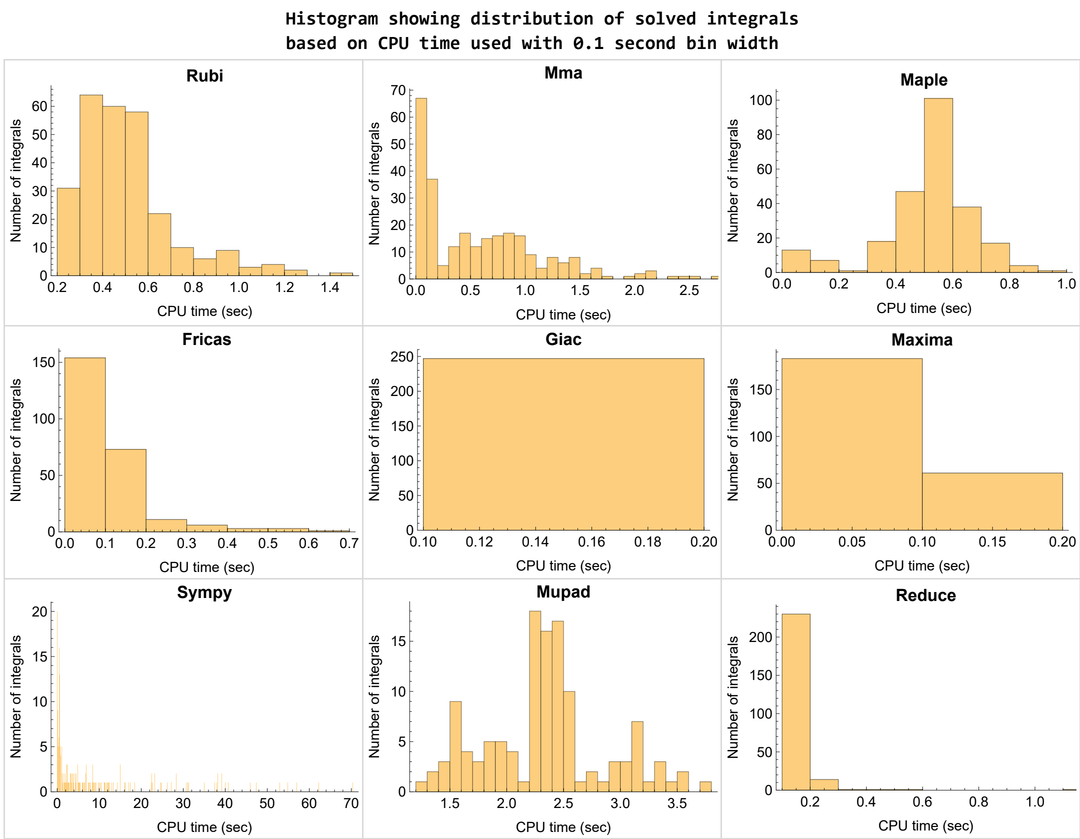



Figure 1.4: Solved integrals histogram based on CPU time used

## 1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

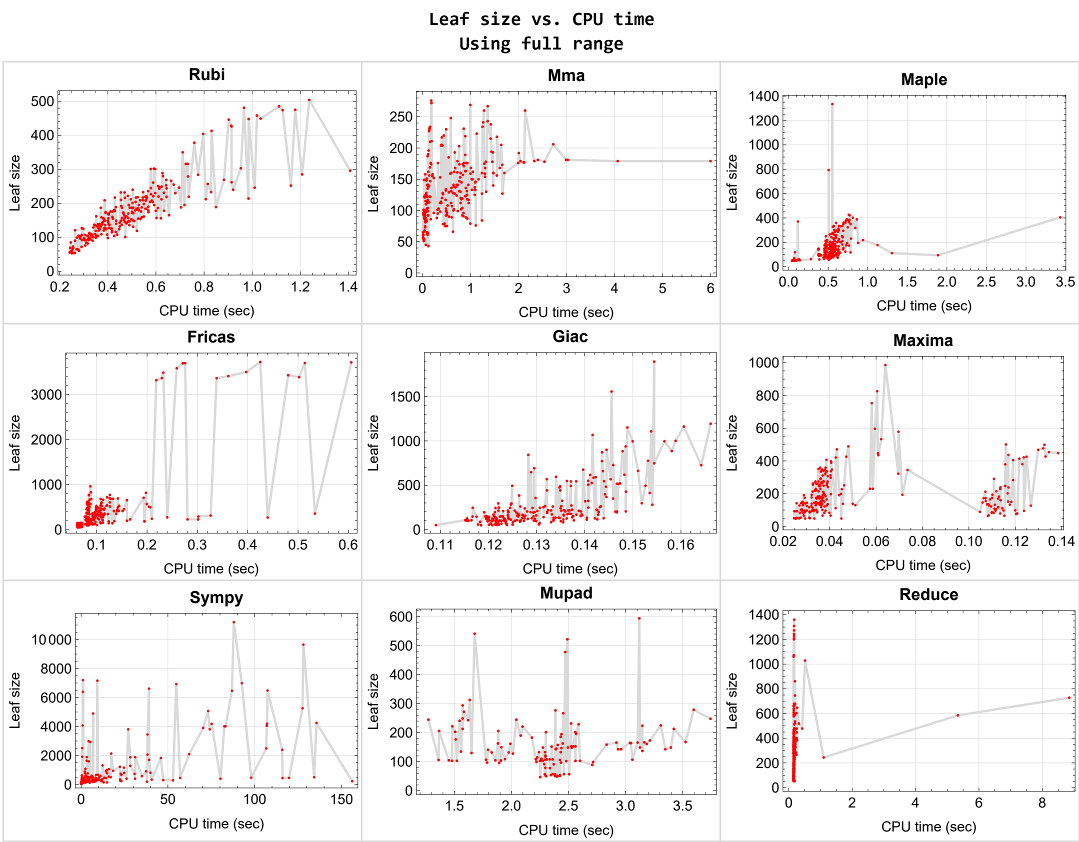



Figure 1.5: Leaf size vs. CPU time. Full range

## 1.9 list of integrals with no known antiderivative

{}

## 1.10 List of integrals solved by CAS but has no known antiderivative

**Rubi** {}

**Mathematica** {}

**Maple** {}

**Maxima** {}

**Fricas** {}

**Sympy** {}

**Giac** {}

**Reduce** {}

**Mupad** {}

## 1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

**Rubi** {212, 213, 214, 215, 228, 229, 230, 231}

**Mathematica** {}

**Maple** {}

**Maxima** Verification phase not currently implemented.

**Fricas** Verification phase not currently implemented.

**Sympy** Verification phase not currently implemented.

**Giac** Verification phase not currently implemented.

**Reduce** Verification phase not currently implemented.

**Mupad** Verification phase not currently implemented.

## 1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

## 1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

# 1.14 Important notes about some of the results

## Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

## Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

## Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

## Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

## 1.15 Current tree layout of integration tests

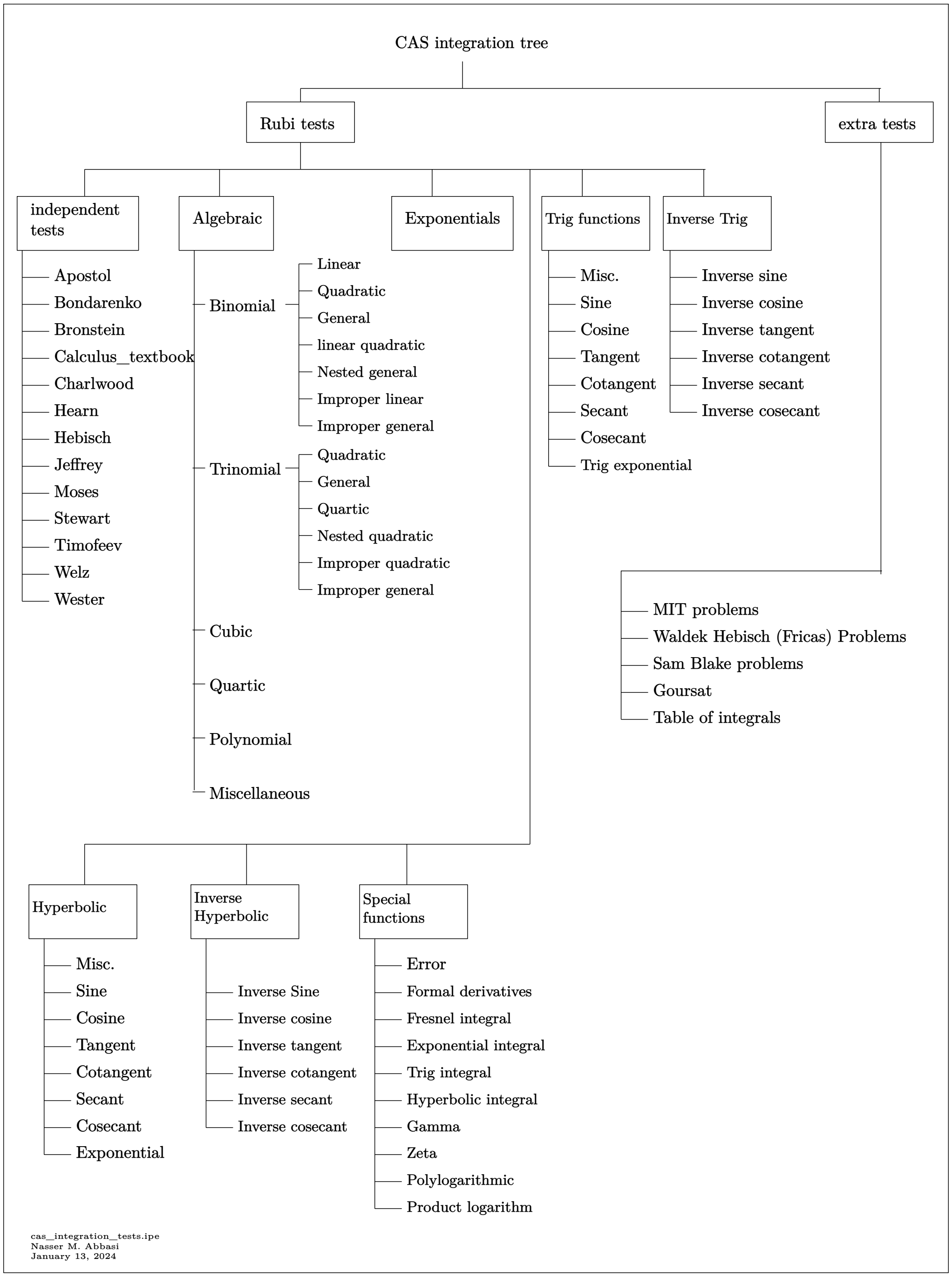
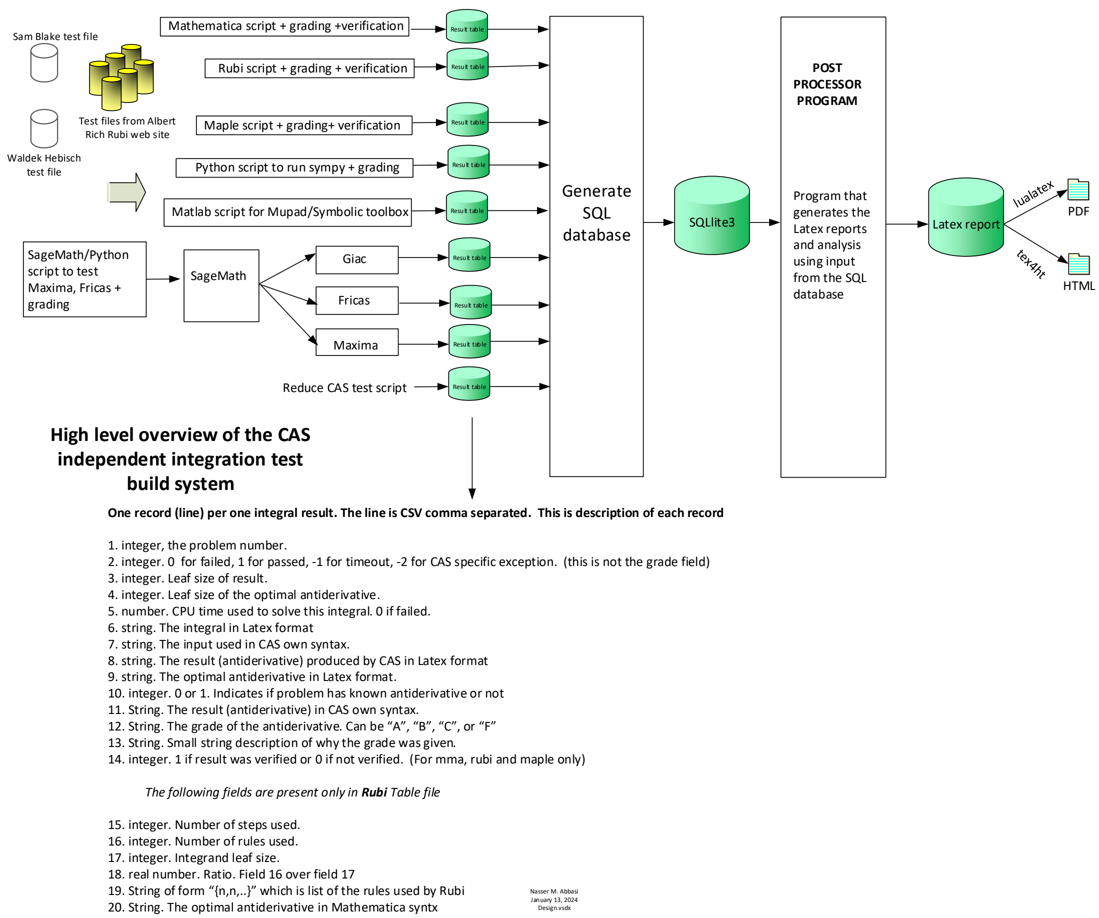



Figure 1.6: CAS integration tests tree

## 1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



# CHAPTER 2

## DETAILED SUMMARY TABLES OF RESULTS

## 2.1 List of integrals sorted by grade for each CAS



### Rubi

**A grade** { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270 }

**B grade** { }

**C grade** { }

**F normal fail** { }

**F(-1) timedout fail** { }

**F(-2) exception fail** { }

## Mma

**A grade** { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270 }

**B grade** { }

**C grade** { }

**F normal fail** { }

**F(-1) timedout fail** { }

**F(-2) exception fail** { }

## Maple

**A grade** { 1, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 83, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 255, 266, 267, 268, 269, 270 }

**B grade** { 2, 69, 82, 84, 85, 252, 253, 254 }

**C grade** { }

**F normal fail** { 139, 140, 141, 142, 143, 144, 145, 146, 147, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265 }

**F(-1) timedout fail** { }

**F(-2) exception fail** { }

## Fricas

**A grade** { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 55, 56, 57, 59, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 138, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 255, 266, 267, 268, 269, 270 }

**B grade** { 54, 58, 60, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 252, 253, 254 }

**C grade** { }

**F normal fail** { 139, 140, 141, 142, 143, 144, 145, 146, 147, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265 }

**F(-1) timedout fail** { }

**F(-2) exception fail** { }

## Maxima

**A grade** { 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 268, 269, 270 }

**B grade** { 1, 2, 3, 4, 240, 241, 242, 266, 267 }

**C grade** { }

**F normal fail** { 139, 140, 141, 142, 143, 144, 145, 146, 147, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265 }

**F(-1) timedout fail** { }

**F(-2) exception fail** { 171, 172, 173, 174, 175, 176, 177 }

## Giac

**A grade** { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 66, 67, 74, 75, 76, 77, 79, 80, 81, 89, 90, 91, 92, 93, 95, 100, 101, 102, 103, 105, 109, 110, 111, 112, 113, 114, 115, 117, 118, 119, 120, 121, 129, 130, 132, 134, 135, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 198, 199, 200, 201, 208, 209, 210, 211, 212, 213, 214, 216, 217, 218, 219, 220, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 240, 241, 242, 243, 244, 245, 249, 250, 251, 266, 267, 268, 269 }

**B grade** { 68, 69, 70, 71, 72, 73, 82, 83, 84, 85, 86, 87, 88, 96, 97, 98, 99, 106, 107, 108, 116, 122, 123, 124, 125, 126, 127, 128, 131, 133, 136, 137, 138, 193, 194, 195, 196, 197, 202, 203, 204, 205, 206, 207, 215, 221, 222, 223, 236, 237, 238, 239, 246, 247, 248, 252, 253, 254, 255, 270 }

**C grade** { }

**F normal fail** { 139, 140, 141, 142, 143, 144, 145, 146, 147, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265 }

**F(-1) timedout fail** { }

**F(-2) exception fail** { 65, 78, 94, 104 }

## Mupad

**A grade** { }

**B grade** { 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 39, 41, 42, 43, 44, 47, 49, 51, 52, 53, 55, 57, 59, 60, 66, 67, 68, 69, 70, 72, 79, 80, 93, 94, 95, 96, 97, 98, 99, 103, 104, 105, 106, 107, 108, 113, 115, 116, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 164, 165, 166, 167, 168, 169, 176, 177, 186, 187, 193, 194, 208, 209, 210, 211, 212, 213, 214, 215, 220, 221, 222, 227, 228, 229, 230, 236, 237, 238, 243, 244, 245, 246, 247, 248, 249, 250, 251, 270 }

**C grade** { }

**F normal fail** { }

**F(-1) timedout fail** { 1, 2, 36, 37, 38, 40, 45, 46, 48, 50, 54, 56, 58, 61, 62, 63, 64, 65, 71, 73, 74, 75, 76, 77, 78, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 100, 101, 102, 109, 110, 111, 112, 114, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 160, 161, 162, 163, 170, 171, 172, 173, 174, 175, 178, 179, 180, 181, 182, 183, 184, 185, 188, 189, 190, 191, 192, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 216, 217, 218, 219, 223, 224, 225, 226, 231, 232, 233, 234, 235, 239, 240, 241, 242, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269 }

**F(-2) exception fail** { }

## Sympy

**A grade** { 5, 7, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 56, 57, 61, 62, 63, 64, 65, 66, 67, 68, 76, 78, 79, 80, 81, 82, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 100, 101, 102, 103, 105, 109, 110, 111, 112, 113, 115, 117, 118, 119, 120, 121, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 162, 163, 166, 170, 171, 172, 173, 174, 175, 176, 179, 180, 181, 182, 183, 184, 185, 186, 189, 190, 191, 192, 193, 198, 199, 200, 201, 202, 203, 211, 212, 213, 216, 217, 218, 219, 220, 221, 227, 228, 229, 233, 234, 235 }

**B grade** { 1, 2, 3, 4, 6, 8, 9, 10, 11, 36, 37, 38, 39, 45, 46, 47, 48, 49, 53, 54, 55, 69, 70, 71, 72, 73, 74, 75, 77, 83, 84, 85, 86, 87, 99, 104, 106, 107, 108, 114, 116, 136, 137, 138, 161, 164, 165, 167, 168, 169, 177, 178, 187, 194, 195, 196, 197, 204, 205, 206, 207, 208, 209, 210, 214, 215, 222, 223, 224, 225, 226, 230, 232, 236, 237, 238, 239, 240, 241, 242, 243, 244, 249, 250, 251, 252, 253, 254, 255, 266, 267, 268 }

**C grade** { 122, 123, 124, 129, 130, 131, 132, 133, 134, 135, 139, 140, 141, 142, 143, 144, 145, 146, 256, 257, 258, 259, 260, 261, 262 }

**F normal fail** { 125, 126, 127, 128 }

**F(-1) timedout fail** { 40, 41, 42, 43, 44, 50, 51, 52, 58, 59, 60, 88, 147, 188, 231, 245, 246, 247, 248, 263, 264, 265, 269, 270 }

**F(-2) exception fail** { }

## Reduce

**A grade** { }

**B grade** { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 266, 267, 268, 269, 270 }

**C grade** { }

**F normal fail** { 139, 140, 141, 142, 143, 144, 145, 146, 147, 240, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265 }

**F(-1) timedout fail** { }

**F(-2) exception fail** { }

## 2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

| Problem 1 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | B | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 183 | 233 | 156 | 295 | 435 | 522 | 3806 | 204 | 478 | 0 |
| N.S. | 1 | 1.27 | 0.85 | 1.61 | 2.38 | 2.85 | 20.80 | 1.11 | 2.61 | 0.00 |
| time (sec) | N/A | 0.829 | 1.548 | 0.691 | 0.061 | 0.110 | 27.137 | 0.128 | 0.252 | 0.000 |

| Problem 2 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | A | B | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 172 | 178 | 127 | 334 | 447 | 467 | 3448 | 138 | 467 | 0 |
| N.S. | 1 | 1.03 | 0.74 | 1.94 | 2.60 | 2.72 | 20.05 | 0.80 | 2.72 | 0.00 |
| time (sec) | N/A | 0.599 | 1.238 | 0.571 | 0.061 | 0.108 | 38.259 | 0.137 | 0.197 | 0.000 |

| Problem 3 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 135 | 154 | 89 | 95 | 240 | 137 | 740 | 112 | 244 | 196 |
| N.S. | 1 | 1.14 | 0.66 | 0.70 | 1.78 | 1.01 | 5.48 | 0.83 | 1.81 | 1.45 |
| time (sec) | N/A | 0.535 | 0.901 | 0.580 | 0.039 | 0.091 | 22.793 | 0.140 | 1.101 | 2.567 |

| Problem 4 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 103 | 162 | 79 | 76 | 253 | 122 | 575 | 81 | 265 | 186 |
| N.S. | 1 | 1.57 | 0.77 | 0.74 | 2.46 | 1.18 | 5.58 | 0.79 | 2.57 | 1.81 |
| time (sec) | N/A | 0.477 | 0.995 | 0.562 | 0.039 | 0.088 | 35.014 | 0.136 | 0.244 | 2.430 |

| Problem 5 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 120 | 145 | 84 | 85 | 179 | 131 | 660 | 95 | 224 | 133 |
| N.S. | 1 | 1.21 | 0.70 | 0.71 | 1.49 | 1.09 | 5.50 | 0.79 | 1.87 | 1.11 |
| time (sec) | N/A | 0.380 | 1.237 | 0.543 | 0.036 | 0.088 | 23.243 | 0.130 | 0.172 | 2.294 |

| Problem 6 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 133 | 139 | 87 | 88 | 197 | 134 | 904 | 94 | 285 | 133 |
| N.S. | 1 | 1.05 | 0.65 | 0.66 | 1.48 | 1.01 | 6.80 | 0.71 | 2.14 | 1.00 |
| time (sec) | N/A | 0.365 | 0.933 | 0.574 | 0.041 | 0.092 | 37.441 | 0.135 | 0.226 | 2.241 |

| Problem 7 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 108 | 125 | 76 | 73 | 123 | 119 | 796 | 82 | 204 | 99 |
| N.S. | 1 | 1.16 | 0.70 | 0.68 | 1.14 | 1.10 | 7.37 | 0.76 | 1.89 | 0.92 |
| time (sec) | N/A | 0.298 | 1.109 | 0.620 | 0.032 | 0.096 | 24.766 | 0.133 | 0.166 | 2.219 |

| Problem 8 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 127 | 121 | 92 | 96 | 153 | 137 | 1880 | 112 | 303 | 115 |
| N.S. | 1 | 0.95 | 0.72 | 0.76 | 1.20 | 1.08 | 14.80 | 0.88 | 2.39 | 0.91 |
| time (sec) | N/A | 0.264 | 0.826 | 0.508 | 0.036 | 0.085 | 31.186 | 0.136 | 0.162 | 2.226 |

| Problem 9 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 138 | 156 | 130 | 193 | 157 | 468 | 6613 | 152 | 588 | 159 |
| N.S. | 1 | 1.13 | 0.94 | 1.40 | 1.14 | 3.39 | 47.92 | 1.10 | 4.26 | 1.15 |
| time (sec) | N/A | 0.382 | 1.326 | 0.546 | 0.037 | 0.115 | 39.147 | 0.138 | 0.185 | 2.833 |

| Problem 10 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 207 | 212 | 168 | 277 | 228 | 528 | 6922 | 239 | 746 | 225 |
| N.S. | 1 | 1.02 | 0.81 | 1.34 | 1.10 | 2.55 | 33.44 | 1.15 | 3.60 | 1.09 |
| time (sec) | N/A | 0.807 | 1.304 | 0.562 | 0.045 | 0.119 | 54.802 | 0.135 | 0.196 | 3.308 |

| Problem 11 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 230 | 246 | 173 | 327 | 265 | 691 | 11198 | 325 | 1029 | 279 |
| N.S. | 1 | 1.07 | 0.75 | 1.42 | 1.15 | 3.00 | 48.69 | 1.41 | 4.47 | 1.21 |
| time (sec) | N/A | 1.010 | 1.661 | 0.559 | 0.039 | 0.142 | 87.983 | 0.135 | 0.513 | 3.596 |

| Problem 12 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 65 | 65 | 65 | 54 | 53 | 53 | 60 | 57 | 59 | 57 |
| N.S. | 1 | 1.00 | 1.00 | 0.83 | 0.82 | 0.82 | 0.92 | 0.88 | 0.91 | 0.88 |
| time (sec) | N/A | 0.275 | 0.018 | 0.122 | 0.027 | 0.066 | 0.030 | 0.124 | 0.152 | 2.382 |

| Problem 13 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 65 | 65 | 65 | 54 | 53 | 53 | 60 | 57 | 59 | 57 |
| N.S. | 1 | 1.00 | 1.00 | 0.83 | 0.82 | 0.82 | 0.92 | 0.88 | 0.91 | 0.88 |
| time (sec) | N/A | 0.270 | 0.018 | 0.112 | 0.031 | 0.062 | 0.025 | 0.121 | 0.153 | 2.361 |

| Problem 14 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 65 | 65 | 65 | 54 | 53 | 53 | 60 | 57 | 59 | 57 |
| N.S. | 1 | 1.00 | 1.00 | 0.83 | 0.82 | 0.82 | 0.92 | 0.88 | 0.91 | 0.88 |
| time (sec) | N/A | 0.253 | 0.010 | 0.110 | 0.028 | 0.064 | 0.028 | 0.118 | 0.153 | 2.291 |

| Problem 15 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 60 | 60 | 60 | 51 | 50 | 50 | 56 | 54 | 57 | 54 |
| N.S. | 1 | 1.00 | 1.00 | 0.85 | 0.83 | 0.83 | 0.93 | 0.90 | 0.95 | 0.90 |
| time (sec) | N/A | 0.249 | 0.011 | 0.096 | 0.031 | 0.065 | 0.024 | 0.122 | 0.152 | 2.345 |

| Problem 16 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 56 | 56 | 56 | 51 | 48 | 48 | 54 | 53 | 54 | 52 |
| N.S. | 1 | 1.00 | 1.00 | 0.91 | 0.86 | 0.86 | 0.96 | 0.95 | 0.96 | 0.93 |
| time (sec) | N/A | 0.242 | 0.015 | 0.056 | 0.025 | 0.064 | 0.098 | 0.120 | 0.151 | 2.299 |

| Problem 17 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 54 | 54 | 54 | 50 | 48 | 55 | 49 | 50 | 61 | 49 |
| N.S. | 1 | 1.00 | 1.00 | 0.93 | 0.89 | 1.02 | 0.91 | 0.93 | 1.13 | 0.91 |
| time (sec) | N/A | 0.250 | 0.036 | 0.045 | 0.025 | 0.069 | 0.104 | 0.121 | 0.155 | 2.397 |

| Problem 18 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 54 | 54 | 51 | 48 | 48 | 55 | 51 | 48 | 63 | 47 |
| N.S. | 1 | 1.00 | 0.94 | 0.89 | 0.89 | 1.02 | 0.94 | 0.89 | 1.17 | 0.87 |
| time (sec) | N/A | 0.248 | 0.027 | 0.046 | 0.029 | 0.064 | 0.183 | 0.118 | 0.157 | 2.252 |

| Problem 19 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 54 | 54 | 55 | 49 | 49 | 55 | 54 | 50 | 63 | 50 |
| N.S. | 1 | 1.00 | 1.02 | 0.91 | 0.91 | 1.02 | 1.00 | 0.93 | 1.17 | 0.93 |
| time (sec) | N/A | 0.252 | 0.020 | 0.043 | 0.027 | 0.062 | 0.463 | 0.109 | 0.156 | 2.304 |

| Problem 20 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 109 | 109 | 98 | 102 | 101 | 101 | 110 | 105 | 105 | 108 |
| N.S. | 1 | 1.00 | 0.90 | 0.94 | 0.93 | 0.93 | 1.01 | 0.96 | 0.96 | 0.99 |
| time (sec) | N/A | 0.347 | 0.047 | 0.383 | 0.028 | 0.061 | 0.034 | 0.121 | 0.155 | 2.308 |

| Problem 21 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 109 | 109 | 92 | 102 | 101 | 101 | 110 | 105 | 105 | 108 |
| N.S. | 1 | 1.00 | 0.84 | 0.94 | 0.93 | 0.93 | 1.01 | 0.96 | 0.96 | 0.99 |
| time (sec) | N/A | 0.331 | 0.052 | 0.393 | 0.033 | 0.064 | 0.028 | 0.115 | 0.153 | 2.213 |

| Problem 22 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 104 | 104 | 92 | 102 | 101 | 101 | 110 | 105 | 105 | 107 |
| N.S. | 1 | 1.00 | 0.88 | 0.98 | 0.97 | 0.97 | 1.06 | 1.01 | 1.01 | 1.03 |
| time (sec) | N/A | 0.316 | 0.041 | 0.380 | 0.037 | 0.066 | 0.037 | 0.123 | 0.155 | 2.295 |

| Problem 23 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 99 | 99 | 88 | 99 | 98 | 98 | 107 | 102 | 103 | 105 |
| N.S. | 1 | 1.00 | 0.89 | 1.00 | 0.99 | 0.99 | 1.08 | 1.03 | 1.04 | 1.06 |
| time (sec) | N/A | 0.324 | 0.034 | 0.378 | 0.033 | 0.064 | 0.038 | 0.124 | 0.153 | 2.262 |

| Problem 24 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 92 | 92 | 88 | 96 | 96 | 96 | 104 | 100 | 99 | 103 |
| N.S. | 1 | 1.00 | 0.96 | 1.04 | 1.04 | 1.04 | 1.13 | 1.09 | 1.08 | 1.12 |
| time (sec) | N/A | 0.309 | 0.043 | 0.378 | 0.029 | 0.071 | 0.110 | 0.122 | 0.153 | 2.250 |

| Problem 25 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 90 | 90 | 88 | 98 | 96 | 103 | 99 | 98 | 107 | 92 |
| N.S. | 1 | 1.00 | 0.98 | 1.09 | 1.07 | 1.14 | 1.10 | 1.09 | 1.19 | 1.02 |
| time (sec) | N/A | 0.337 | 0.063 | 0.374 | 0.032 | 0.066 | 0.140 | 0.125 | 0.152 | 2.286 |

| Problem 26 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 98 | 98 | 87 | 95 | 96 | 103 | 100 | 97 | 109 | 103 |
| N.S. | 1 | 1.00 | 0.89 | 0.97 | 0.98 | 1.05 | 1.02 | 0.99 | 1.11 | 1.05 |
| time (sec) | N/A | 0.301 | 0.039 | 0.471 | 0.038 | 0.065 | 0.231 | 0.127 | 0.152 | 2.274 |

| Problem 27 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 98 | 98 | 83 | 92 | 97 | 103 | 100 | 97 | 109 | 106 |
| N.S. | 1 | 1.00 | 0.85 | 0.94 | 0.99 | 1.05 | 1.02 | 0.99 | 1.11 | 1.08 |
| time (sec) | N/A | 0.320 | 0.054 | 0.379 | 0.030 | 0.068 | 0.574 | 0.116 | 0.150 | 2.425 |

| Problem 28 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 149 | 149 | 149 | 148 | 145 | 145 | 163 | 153 | 151 | 153 |
| N.S. | 1 | 1.00 | 1.00 | 0.99 | 0.97 | 0.97 | 1.09 | 1.03 | 1.01 | 1.03 |
| time (sec) | N/A | 0.435 | 0.032 | 0.525 | 0.033 | 0.062 | 0.041 | 0.127 | 0.151 | 2.524 |

| Problem 29 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 149 | 149 | 125 | 148 | 145 | 145 | 165 | 153 | 151 | 153 |
| N.S. | 1 | 1.00 | 0.84 | 0.99 | 0.97 | 0.97 | 1.11 | 1.03 | 1.01 | 1.03 |
| time (sec) | N/A | 0.387 | 0.068 | 0.377 | 0.030 | 0.065 | 0.037 | 0.121 | 0.151 | 2.542 |

| Problem 30 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 138 | 138 | 124 | 148 | 145 | 145 | 163 | 153 | 151 | 153 |
| N.S. | 1 | 1.00 | 0.90 | 1.07 | 1.05 | 1.05 | 1.18 | 1.11 | 1.09 | 1.11 |
| time (sec) | N/A | 0.365 | 0.062 | 0.472 | 0.029 | 0.070 | 0.041 | 0.120 | 0.154 | 2.561 |

| Problem 31 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 133 | 133 | 121 | 144 | 142 | 142 | 158 | 149 | 149 | 149 |
| N.S. | 1 | 1.00 | 0.91 | 1.08 | 1.07 | 1.07 | 1.19 | 1.12 | 1.12 | 1.12 |
| time (sec) | N/A | 0.354 | 0.051 | 0.378 | 0.026 | 0.067 | 0.040 | 0.123 | 0.152 | 2.494 |

| Problem 32 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 129 | 129 | 121 | 142 | 140 | 140 | 158 | 148 | 145 | 147 |
| N.S. | 1 | 1.00 | 0.94 | 1.10 | 1.09 | 1.09 | 1.22 | 1.15 | 1.12 | 1.14 |
| time (sec) | N/A | 0.340 | 0.062 | 0.372 | 0.029 | 0.071 | 0.160 | 0.119 | 0.155 | 2.438 |

| Problem 33 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 124 | 124 | 123 | 145 | 139 | 147 | 150 | 145 | 153 | 121 |
| N.S. | 1 | 1.00 | 0.99 | 1.17 | 1.12 | 1.19 | 1.21 | 1.17 | 1.23 | 0.98 |
| time (sec) | N/A | 0.366 | 0.091 | 0.377 | 0.032 | 0.069 | 0.180 | 0.118 | 0.156 | 2.297 |

| Problem 34 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 135 | 135 | 124 | 142 | 139 | 147 | 151 | 144 | 155 | 143 |
| N.S. | 1 | 1.00 | 0.92 | 1.05 | 1.03 | 1.09 | 1.12 | 1.07 | 1.15 | 1.06 |
| time (sec) | N/A | 0.358 | 0.068 | 0.381 | 0.037 | 0.070 | 0.283 | 0.120 | 0.151 | 2.387 |

| Problem 35 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 139 | 139 | 124 | 141 | 142 | 147 | 155 | 146 | 155 | 148 |
| N.S. | 1 | 1.00 | 0.89 | 1.01 | 1.02 | 1.06 | 1.12 | 1.05 | 1.12 | 1.06 |
| time (sec) | N/A | 0.360 | 0.050 | 0.380 | 0.028 | 0.069 | 0.589 | 0.117 | 0.154 | 2.529 |

| Problem 36 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 130 | 130 | 114 | 128 | 127 | 270 | 274 | 137 | 151 | 0 |
| N.S. | 1 | 1.00 | 0.88 | 0.98 | 0.98 | 2.08 | 2.11 | 1.05 | 1.16 | 0.00 |
| time (sec) | N/A | 0.359 | 0.096 | 0.454 | 0.109 | 0.095 | 0.691 | 0.120 | 0.156 | 0.000 |

| Problem 37 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 111 | 111 | 95 | 95 | 98 | 238 | 245 | 112 | 121 | 0 |
| N.S. | 1 | 1.00 | 0.86 | 0.86 | 0.88 | 2.14 | 2.21 | 1.01 | 1.09 | 0.00 |
| time (sec) | N/A | 0.322 | 0.054 | 0.452 | 0.120 | 0.084 | 0.647 | 0.120 | 0.151 | 0.000 |

| Problem 38 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 92 | 92 | 81 | 85 | 82 | 180 | 211 | 88 | 104 | 0 |
| N.S. | 1 | 1.00 | 0.88 | 0.92 | 0.89 | 1.96 | 2.29 | 0.96 | 1.13 | 0.00 |
| time (sec) | N/A | 0.293 | 0.062 | 0.486 | 0.111 | 0.080 | 0.564 | 0.122 | 0.149 | 0.000 |

| Problem 39 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 73 | 73 | 68 | 65 | 64 | 157 | 219 | 66 | 79 | 79 |
| N.S. | 1 | 1.00 | 0.93 | 0.89 | 0.88 | 2.15 | 3.00 | 0.90 | 1.08 | 1.08 |
| time (sec) | N/A | 0.255 | 0.043 | 0.450 | 0.108 | 0.083 | 0.523 | 0.122 | 0.152 | 2.297 |

| Problem 40 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F(-1) | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 72 | 72 | 73 | 73 | 65 | 157 | 0 | 66 | 90 | 0 |
| N.S. | 1 | 1.00 | 1.01 | 1.01 | 0.90 | 2.18 | 0.00 | 0.92 | 1.25 | 0.00 |
| time (sec) | N/A | 0.291 | 0.056 | 0.441 | 0.115 | 0.107 | 0.000 | 0.123 | 0.151 | 0.000 |

| Problem 41 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F(-1) | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 76 | 76 | 75 | 67 | 67 | 165 | 0 | 68 | 90 | 78 |
| N.S. | 1 | 1.00 | 0.99 | 0.88 | 0.88 | 2.17 | 0.00 | 0.89 | 1.18 | 1.03 |
| time (sec) | N/A | 0.306 | 0.052 | 0.492 | 0.108 | 0.098 | 0.000 | 0.120 | 0.150 | 2.337 |

| Problem 42 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F(-1) | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 92 | 92 | 84 | 89 | 76 | 207 | 0 | 80 | 125 | 97 |
| N.S. | 1 | 1.00 | 0.91 | 0.97 | 0.83 | 2.25 | 0.00 | 0.87 | 1.36 | 1.05 |
| time (sec) | N/A | 0.321 | 0.079 | 0.459 | 0.120 | 0.112 | 0.000 | 0.122 | 0.151 | 2.434 |

| Problem 43 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F(-1) | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 108 | 108 | 103 | 100 | 97 | 218 | 0 | 98 | 127 | 118 |
| N.S. | 1 | 1.00 | 0.95 | 0.93 | 0.90 | 2.02 | 0.00 | 0.91 | 1.18 | 1.09 |
| time (sec) | N/A | 0.333 | 0.090 | 0.455 | 0.115 | 0.111 | 0.000 | 0.127 | 0.153 | 2.298 |

| Problem 44 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F(-1) | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 127 | 127 | 113 | 124 | 117 | 282 | 0 | 128 | 158 | 145 |
| N.S. | 1 | 1.00 | 0.89 | 0.98 | 0.92 | 2.22 | 0.00 | 1.01 | 1.24 | 1.14 |
| time (sec) | N/A | 0.361 | 0.140 | 0.458 | 0.113 | 0.113 | 0.000 | 0.122 | 0.155 | 2.393 |

| Problem 45 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 155 | 179 | 139 | 134 | 150 | 468 | 335 | 159 | 264 | 0 |
| N.S. | 1 | 1.15 | 0.90 | 0.86 | 0.97 | 3.02 | 2.16 | 1.03 | 1.70 | 0.00 |
| time (sec) | N/A | 0.568 | 0.137 | 0.543 | 0.118 | 0.082 | 2.424 | 0.127 | 0.155 | 0.000 |

| Problem 46 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 130 | 155 | 128 | 124 | 127 | 372 | 289 | 131 | 243 | 0 |
| N.S. | 1 | 1.19 | 0.98 | 0.95 | 0.98 | 2.86 | 2.22 | 1.01 | 1.87 | 0.00 |
| time (sec) | N/A | 0.526 | 0.080 | 0.471 | 0.127 | 0.080 | 2.216 | 0.121 | 0.157 | 0.000 |

| Problem 47 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 115 | 131 | 100 | 103 | 108 | 357 | 284 | 111 | 204 | 152 |
| N.S. | 1 | 1.14 | 0.87 | 0.90 | 0.94 | 3.10 | 2.47 | 0.97 | 1.77 | 1.32 |
| time (sec) | N/A | 0.505 | 0.078 | 0.461 | 0.111 | 0.084 | 2.178 | 0.125 | 0.153 | 2.352 |

| Problem 48 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 92 | 109 | 92 | 78 | 84 | 286 | 212 | 81 | 182 | 0 |
| N.S. | 1 | 1.18 | 1.00 | 0.85 | 0.91 | 3.11 | 2.30 | 0.88 | 1.98 | 0.00 |
| time (sec) | N/A | 0.373 | 0.052 | 0.450 | 0.110 | 0.076 | 1.658 | 0.122 | 0.153 | 0.000 |

| Problem 49 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 93 | 99 | 83 | 88 | 89 | 257 | 233 | 88 | 156 | 110 |
| N.S. | 1 | 1.06 | 0.89 | 0.95 | 0.96 | 2.76 | 2.51 | 0.95 | 1.68 | 1.18 |
| time (sec) | N/A | 0.271 | 0.083 | 0.467 | 0.105 | 0.080 | 1.800 | 0.118 | 0.153 | 2.262 |

| Problem 50 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F(-1) | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 95 | 104 | 85 | 99 | 87 | 296 | 0 | 93 | 192 | 0 |
| N.S. | 1 | 1.09 | 0.89 | 1.04 | 0.92 | 3.12 | 0.00 | 0.98 | 2.02 | 0.00 |
| time (sec) | N/A | 0.359 | 0.081 | 0.470 | 0.110 | 0.097 | 0.000 | 0.120 | 0.152 | 0.000 |

| Problem 51 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F(-1) | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 111 | 115 | 110 | 96 | 105 | 336 | 0 | 103 | 195 | 133 |
| N.S. | 1 | 1.04 | 0.99 | 0.86 | 0.95 | 3.03 | 0.00 | 0.93 | 1.76 | 1.20 |
| time (sec) | N/A | 0.417 | 0.071 | 0.558 | 0.112 | 0.094 | 0.000 | 0.120 | 0.151 | 2.431 |

| Problem 52 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F(-1) | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 131 | 136 | 112 | 127 | 117 | 442 | 0 | 126 | 289 | 158 |
| N.S. | 1 | 1.04 | 0.85 | 0.97 | 0.89 | 3.37 | 0.00 | 0.96 | 2.21 | 1.21 |
| time (sec) | N/A | 0.489 | 0.101 | 0.468 | 0.113 | 0.115 | 0.000 | 0.124 | 0.153 | 2.405 |

| Problem 53 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 158 | 186 | 139 | 140 | 165 | 574 | 357 | 157 | 353 | 232 |
| N.S. | 1 | 1.18 | 0.88 | 0.89 | 1.04 | 3.63 | 2.26 | 0.99 | 2.23 | 1.47 |
| time (sec) | N/A | 0.608 | 0.112 | 0.464 | 0.109 | 0.084 | 12.231 | 0.126 | 0.152 | 2.522 |

| Problem 54 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | B | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 131 | 161 | 126 | 115 | 136 | 480 | 282 | 122 | 303 | 0 |
| N.S. | 1 | 1.23 | 0.96 | 0.88 | 1.04 | 3.66 | 2.15 | 0.93 | 2.31 | 0.00 |
| time (sec) | N/A | 0.550 | 0.079 | 0.457 | 0.108 | 0.083 | 11.331 | 0.122 | 0.157 | 0.000 |

| Problem 55 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 134 | 148 | 122 | 123 | 146 | 447 | 304 | 128 | 278 | 195 |
| N.S. | 1 | 1.10 | 0.91 | 0.92 | 1.09 | 3.34 | 2.27 | 0.96 | 2.07 | 1.46 |
| time (sec) | N/A | 0.510 | 0.096 | 0.463 | 0.106 | 0.084 | 10.080 | 0.123 | 0.148 | 2.438 |

| Problem 56 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 105 | 127 | 99 | 97 | 111 | 357 | 178 | 97 | 229 | 0 |
| N.S. | 1 | 1.21 | 0.94 | 0.92 | 1.06 | 3.40 | 1.70 | 0.92 | 2.18 | 0.00 |
| time (sec) | N/A | 0.388 | 0.114 | 0.600 | 0.114 | 0.080 | 6.734 | 0.121 | 0.155 | 0.000 |

| Problem 57 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 116 | 127 | 104 | 98 | 122 | 346 | 184 | 106 | 222 | 163 |
| N.S. | 1 | 1.09 | 0.90 | 0.84 | 1.05 | 2.98 | 1.59 | 0.91 | 1.91 | 1.41 |
| time (sec) | N/A | 0.292 | 0.112 | 0.461 | 0.110 | 0.084 | 3.771 | 0.122 | 0.156 | 2.450 |

| Problem 58 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | F(-1) | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 130 | 144 | 117 | 130 | 133 | 488 | 0 | 128 | 321 | 0 |
| N.S. | 1 | 1.11 | 0.90 | 1.00 | 1.02 | 3.75 | 0.00 | 0.98 | 2.47 | 0.00 |
| time (sec) | N/A | 0.402 | 0.112 | 0.460 | 0.118 | 0.102 | 0.000 | 0.127 | 0.153 | 0.000 |

| Problem 59 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F(-1) | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 144 | 157 | 141 | 125 | 152 | 524 | 0 | 141 | 328 | 202 |
| N.S. | 1 | 1.09 | 0.98 | 0.87 | 1.06 | 3.64 | 0.00 | 0.98 | 2.28 | 1.40 |
| time (sec) | N/A | 0.602 | 0.097 | 0.481 | 0.117 | 0.099 | 0.000 | 0.120 | 0.153 | 2.559 |

| Problem 60 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | F(-1) | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 170 | 188 | 147 | 169 | 172 | 696 | 0 | 162 | 468 | 229 |
| N.S. | 1 | 1.11 | 0.86 | 0.99 | 1.01 | 4.09 | 0.00 | 0.95 | 2.75 | 1.35 |
| time (sec) | N/A | 0.704 | 0.162 | 0.476 | 0.107 | 0.126 | 0.000 | 0.124 | 0.154 | 2.587 |

| Problem 61 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 227 | 244 | 160 | 286 | 255 | 387 | 264 | 207 | 305 | 0 |
| N.S. | 1 | 1.07 | 0.70 | 1.26 | 1.12 | 1.70 | 1.16 | 0.91 | 1.34 | 0.00 |
| time (sec) | N/A | 0.644 | 0.793 | 0.586 | 0.033 | 0.097 | 0.797 | 0.123 | 0.189 | 0.000 |

| Problem 62 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 194 | 208 | 141 | 238 | 213 | 326 | 228 | 177 | 261 | 0 |
| N.S. | 1 | 1.07 | 0.73 | 1.23 | 1.10 | 1.68 | 1.18 | 0.91 | 1.35 | 0.00 |
| time (sec) | N/A | 0.557 | 0.646 | 0.599 | 0.033 | 0.094 | 0.694 | 0.126 | 0.168 | 0.000 |

| Problem 63 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 167 | 173 | 128 | 194 | 173 | 288 | 196 | 149 | 227 | 0 |
| N.S. | 1 | 1.04 | 0.77 | 1.16 | 1.04 | 1.72 | 1.17 | 0.89 | 1.36 | 0.00 |
| time (sec) | N/A | 0.464 | 0.638 | 0.561 | 0.035 | 0.095 | 0.663 | 0.123 | 0.164 | 0.000 |

| Problem 64 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 132 | 137 | 108 | 148 | 130 | 231 | 160 | 121 | 183 | 0 |
| N.S. | 1 | 1.04 | 0.82 | 1.12 | 0.98 | 1.75 | 1.21 | 0.92 | 1.39 | 0.00 |
| time (sec) | N/A | 0.344 | 0.523 | 0.511 | 0.051 | 0.089 | 0.477 | 0.120 | 0.156 | 0.000 |

| Problem 65 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | F(-2) | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 138 | 143 | 127 | 153 | 125 | 533 | 265 | 0 | 209 | 0 |
| N.S. | 1 | 1.04 | 0.92 | 1.11 | 0.91 | 3.86 | 1.92 | 0.00 | 1.51 | 0.00 |
| time (sec) | N/A | 0.449 | 0.671 | 0.513 | 0.030 | 0.108 | 3.420 | 0.000 | 0.158 | 0.000 |

| Problem 66 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 135 | 141 | 131 | 155 | 103 | 477 | 235 | 133 | 204 | 166 |
| N.S. | 1 | 1.04 | 0.97 | 1.15 | 0.76 | 3.53 | 1.74 | 0.99 | 1.51 | 1.23 |
| time (sec) | N/A | 0.478 | 0.767 | 0.532 | 0.028 | 0.103 | 2.656 | 0.140 | 0.163 | 3.105 |

| Problem 67 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 150 | 147 | 119 | 204 | 141 | 539 | 238 | 194 | 245 | 161 |
| N.S. | 1 | 0.98 | 0.79 | 1.36 | 0.94 | 3.59 | 1.59 | 1.29 | 1.63 | 1.07 |
| time (sec) | N/A | 0.490 | 0.829 | 0.527 | 0.029 | 0.121 | 3.399 | 0.147 | 0.164 | 3.185 |

| Problem 68 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 143 | 145 | 120 | 185 | 130 | 494 | 216 | 247 | 400 | 143 |
| N.S. | 1 | 1.01 | 0.84 | 1.29 | 0.91 | 3.45 | 1.51 | 1.73 | 2.80 | 1.00 |
| time (sec) | N/A | 0.470 | 0.796 | 0.550 | 0.034 | 0.099 | 3.606 | 0.141 | 0.187 | 3.348 |

| Problem 69 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | A | A | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 141 | 148 | 133 | 233 | 177 | 582 | 253 | 492 | 269 | 168 |
| N.S. | 1 | 1.05 | 0.94 | 1.65 | 1.26 | 4.13 | 1.79 | 3.49 | 1.91 | 1.19 |
| time (sec) | N/A | 0.487 | 1.097 | 0.566 | 0.036 | 0.109 | 4.812 | 0.137 | 0.168 | 3.524 |

| Problem 70 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 140 | 140 | 125 | 210 | 185 | 257 | 265 | 494 | 287 | 149 |
| N.S. | 1 | 1.00 | 0.89 | 1.50 | 1.32 | 1.84 | 1.89 | 3.53 | 2.05 | 1.06 |
| time (sec) | N/A | 0.435 | 1.086 | 0.585 | 0.038 | 0.094 | 4.463 | 0.125 | 0.174 | 3.396 |

| Problem 71 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | B | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 172 | 175 | 140 | 258 | 229 | 314 | 347 | 648 | 310 | 0 |
| N.S. | 1 | 1.02 | 0.81 | 1.50 | 1.33 | 1.83 | 2.02 | 3.77 | 1.80 | 0.00 |
| time (sec) | N/A | 0.501 | 1.472 | 0.602 | 0.034 | 0.103 | 9.082 | 0.129 | 0.191 | 0.000 |

| Problem 72 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 200 | 210 | 160 | 302 | 269 | 353 | 670 | 676 | 369 | 248 |
| N.S. | 1 | 1.05 | 0.80 | 1.51 | 1.34 | 1.76 | 3.35 | 3.38 | 1.84 | 1.24 |
| time (sec) | N/A | 0.559 | 1.475 | 0.635 | 0.036 | 0.116 | 9.740 | 0.140 | 0.202 | 3.743 |

| Problem 73 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | B | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 235 | 248 | 176 | 350 | 311 | 410 | 729 | 844 | 400 | 0 |
| N.S. | 1 | 1.06 | 0.75 | 1.49 | 1.32 | 1.74 | 3.10 | 3.59 | 1.70 | 0.00 |
| time (sec) | N/A | 0.614 | 1.993 | 0.783 | 0.037 | 0.134 | 30.784 | 0.128 | 0.241 | 0.000 |

| Problem 74 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 260 | 266 | 189 | 318 | 293 | 477 | 483 | 261 | 383 | 0 |
| N.S. | 1 | 1.02 | 0.73 | 1.22 | 1.13 | 1.83 | 1.86 | 1.00 | 1.47 | 0.00 |
| time (sec) | N/A | 0.663 | 0.890 | 0.843 | 0.036 | 0.102 | 0.883 | 0.143 | 0.263 | 0.000 |

| Problem 75 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 227 | 230 | 174 | 270 | 251 | 428 | 422 | 238 | 339 | 0 |
| N.S. | 1 | 1.01 | 0.77 | 1.19 | 1.11 | 1.89 | 1.86 | 1.05 | 1.49 | 0.00 |
| time (sec) | N/A | 0.597 | 0.770 | 0.543 | 0.046 | 0.100 | 0.827 | 0.128 | 0.185 | 0.000 |

| Problem 76 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 200 | 195 | 161 | 226 | 211 | 390 | 364 | 210 | 305 | 0 |
| N.S. | 1 | 0.98 | 0.80 | 1.13 | 1.06 | 1.95 | 1.82 | 1.05 | 1.52 | 0.00 |
| time (sec) | N/A | 0.487 | 0.726 | 0.635 | 0.037 | 0.103 | 0.771 | 0.130 | 0.186 | 0.000 |

| Problem 77 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 163 | 159 | 141 | 180 | 165 | 327 | 303 | 180 | 261 | 0 |
| N.S. | 1 | 0.98 | 0.87 | 1.10 | 1.01 | 2.01 | 1.86 | 1.10 | 1.60 | 0.00 |
| time (sec) | N/A | 0.373 | 0.714 | 0.510 | 0.039 | 0.095 | 0.612 | 0.137 | 0.173 | 0.000 |

| Problem 78 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | F(-2) | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 175 | 180 | 163 | 199 | 173 | 719 | 600 | 0 | 291 | 0 |
| N.S. | 1 | 1.03 | 0.93 | 1.14 | 0.99 | 4.11 | 3.43 | 0.00 | 1.66 | 0.00 |
| time (sec) | N/A | 0.529 | 0.901 | 0.517 | 0.037 | 0.120 | 6.098 | 0.000 | 0.175 | 0.000 |

| Problem 79 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 168 | 180 | 157 | 201 | 148 | 663 | 520 | 197 | 288 | 165 |
| N.S. | 1 | 1.07 | 0.93 | 1.20 | 0.88 | 3.95 | 3.10 | 1.17 | 1.71 | 0.98 |
| time (sec) | N/A | 0.528 | 0.841 | 0.524 | 0.034 | 0.113 | 3.948 | 0.143 | 0.176 | 3.037 |

| Problem 80 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 195 | 204 | 150 | 264 | 199 | 683 | 500 | 254 | 327 | 173 |
| N.S. | 1 | 1.05 | 0.77 | 1.35 | 1.02 | 3.50 | 2.56 | 1.30 | 1.68 | 0.89 |
| time (sec) | N/A | 0.575 | 1.024 | 0.519 | 0.045 | 0.132 | 5.497 | 0.143 | 0.181 | 3.212 |

| Problem 81 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 205 | 204 | 150 | 312 | 223 | 603 | 447 | 323 | 314 | 0 |
| N.S. | 1 | 1.00 | 0.73 | 1.52 | 1.09 | 2.94 | 2.18 | 1.58 | 1.53 | 0.00 |
| time (sec) | N/A | 0.587 | 1.041 | 0.630 | 0.034 | 0.126 | 6.260 | 0.141 | 0.178 | 0.000 |

| Problem 82 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | A | A | A | B | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 204 | 208 | 150 | 360 | 275 | 682 | 462 | 515 | 325 | 0 |
| N.S. | 1 | 1.02 | 0.74 | 1.76 | 1.35 | 3.34 | 2.26 | 2.52 | 1.59 | 0.00 |
| time (sec) | N/A | 0.582 | 1.167 | 0.599 | 0.034 | 0.128 | 8.356 | 0.147 | 0.189 | 0.000 |

| Problem 83 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | B | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 182 | 185 | 160 | 301 | 248 | 680 | 401 | 494 | 332 | 0 |
| N.S. | 1 | 1.02 | 0.88 | 1.65 | 1.36 | 3.74 | 2.20 | 2.71 | 1.82 | 0.00 |
| time (sec) | N/A | 0.523 | 1.237 | 0.577 | 0.037 | 0.108 | 8.433 | 0.153 | 0.190 | 0.000 |

| Problem 84 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | A | A | B | B | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 178 | 185 | 168 | 349 | 297 | 772 | 439 | 726 | 349 | 0 |
| N.S. | 1 | 1.04 | 0.94 | 1.96 | 1.67 | 4.34 | 2.47 | 4.08 | 1.96 | 0.00 |
| time (sec) | N/A | 0.525 | 1.608 | 0.691 | 0.040 | 0.126 | 14.386 | 0.164 | 0.189 | 0.000 |

| Problem 85 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | A | A | B | B | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 169 | 164 | 174 | 286 | 265 | 353 | 774 | 728 | 369 | 0 |
| N.S. | 1 | 0.97 | 1.03 | 1.69 | 1.57 | 2.09 | 4.58 | 4.31 | 2.18 | 0.00 |
| time (sec) | N/A | 0.469 | 0.839 | 0.625 | 0.036 | 0.116 | 14.976 | 0.146 | 0.207 | 0.000 |

| Problem 86 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | B | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 205 | 201 | 177 | 334 | 307 | 412 | 808 | 872 | 400 | 0 |
| N.S. | 1 | 0.98 | 0.86 | 1.63 | 1.50 | 2.01 | 3.94 | 4.25 | 1.95 | 0.00 |
| time (sec) | N/A | 0.503 | 2.103 | 0.678 | 0.039 | 0.142 | 39.960 | 0.144 | 0.253 | 0.000 |

| Problem 87 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | B | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 233 | 236 | 192 | 378 | 347 | 451 | 1697 | 900 | 447 | 0 |
| N.S. | 1 | 1.01 | 0.82 | 1.62 | 1.49 | 1.94 | 7.28 | 3.86 | 1.92 | 0.00 |
| time (sec) | N/A | 0.568 | 2.005 | 0.723 | 0.037 | 0.154 | 39.191 | 0.145 | 0.252 | 0.000 |

| Problem 88 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F(-1) | B | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 262 | 269 | 206 | 426 | 389 | 506 | 0 | 1068 | 478 | 0 |
| N.S. | 1 | 1.03 | 0.79 | 1.63 | 1.48 | 1.93 | 0.00 | 4.08 | 1.82 | 0.00 |
| time (sec) | N/A | 0.643 | 2.722 | 0.760 | 0.041 | 0.204 | 0.000 | 0.142 | 0.422 | 0.000 |

| Problem 89 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 221 | 255 | 141 | 296 | 259 | 333 | 218 | 187 | 261 | 0 |
| N.S. | 1 | 1.15 | 0.64 | 1.34 | 1.17 | 1.51 | 0.99 | 0.85 | 1.18 | 0.00 |
| time (sec) | N/A | 0.674 | 0.737 | 0.606 | 0.035 | 0.095 | 0.709 | 0.132 | 0.175 | 0.000 |

| Problem 90 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 192 | 220 | 128 | 248 | 217 | 295 | 194 | 159 | 227 | 0 |
| N.S. | 1 | 1.15 | 0.67 | 1.29 | 1.13 | 1.54 | 1.01 | 0.83 | 1.18 | 0.00 |
| time (sec) | N/A | 0.611 | 0.632 | 0.680 | 0.034 | 0.092 | 0.629 | 0.136 | 0.170 | 0.000 |

| Problem 91 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 159 | 184 | 118 | 200 | 175 | 238 | 167 | 132 | 183 | 0 |
| N.S. | 1 | 1.16 | 0.74 | 1.26 | 1.10 | 1.50 | 1.05 | 0.83 | 1.15 | 0.00 |
| time (sec) | N/A | 0.559 | 0.928 | 0.556 | 0.029 | 0.093 | 0.608 | 0.134 | 0.159 | 0.000 |

| Problem 92 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 131 | 149 | 92 | 155 | 134 | 204 | 143 | 106 | 149 | 0 |
| N.S. | 1 | 1.14 | 0.70 | 1.18 | 1.02 | 1.56 | 1.09 | 0.81 | 1.14 | 0.00 |
| time (sec) | N/A | 0.455 | 0.487 | 0.540 | 0.036 | 0.086 | 0.635 | 0.126 | 0.158 | 0.000 |

| Problem 93 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 100 | 113 | 78 | 112 | 95 | 147 | 116 | 77 | 109 | 149 |
| N.S. | 1 | 1.13 | 0.78 | 1.12 | 0.95 | 1.47 | 1.16 | 0.77 | 1.09 | 1.49 |
| time (sec) | N/A | 0.324 | 0.430 | 0.621 | 0.033 | 0.086 | 0.455 | 0.124 | 0.155 | 3.125 |

| Problem 94 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | F(-2) | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 100 | 104 | 107 | 106 | 79 | 410 | 168 | 0 | 139 | 137 |
| N.S. | 1 | 1.04 | 1.07 | 1.06 | 0.79 | 4.10 | 1.68 | 0.00 | 1.39 | 1.37 |
| time (sec) | N/A | 0.426 | 0.616 | 0.522 | 0.029 | 0.104 | 1.966 | 0.000 | 0.153 | 3.147 |

| Problem 95 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 89 | 98 | 96 | 84 | 65 | 379 | 114 | 103 | 203 | 103 |
| N.S. | 1 | 1.10 | 1.08 | 0.94 | 0.73 | 4.26 | 1.28 | 1.16 | 2.28 | 1.16 |
| time (sec) | N/A | 0.400 | 0.509 | 0.523 | 0.029 | 0.096 | 1.399 | 0.138 | 0.154 | 2.561 |

| Problem 96 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 104 | 108 | 102 | 119 | 88 | 451 | 138 | 176 | 191 | 99 |
| N.S. | 1 | 1.04 | 0.98 | 1.14 | 0.85 | 4.34 | 1.33 | 1.69 | 1.84 | 0.95 |
| time (sec) | N/A | 0.438 | 0.653 | 0.519 | 0.029 | 0.099 | 2.492 | 0.137 | 0.160 | 2.715 |

| Problem 97 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 110 | 116 | 89 | 136 | 111 | 173 | 138 | 220 | 209 | 107 |
| N.S. | 1 | 1.05 | 0.81 | 1.24 | 1.01 | 1.57 | 1.25 | 2.00 | 1.90 | 0.97 |
| time (sec) | N/A | 0.433 | 0.608 | 0.537 | 0.043 | 0.086 | 2.755 | 0.136 | 0.159 | 3.059 |

| Problem 98 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 142 | 153 | 104 | 180 | 151 | 226 | 221 | 442 | 241 | 143 |
| N.S. | 1 | 1.08 | 0.73 | 1.27 | 1.06 | 1.59 | 1.56 | 3.11 | 1.70 | 1.01 |
| time (sec) | N/A | 0.498 | 0.831 | 0.555 | 0.028 | 0.095 | 5.089 | 0.132 | 0.166 | 2.960 |

| Problem 99 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 173 | 188 | 125 | 224 | 191 | 261 | 507 | 442 | 287 | 164 |
| N.S. | 1 | 1.09 | 0.72 | 1.29 | 1.10 | 1.51 | 2.93 | 2.55 | 1.66 | 0.95 |
| time (sec) | N/A | 0.541 | 0.928 | 0.573 | 0.029 | 0.096 | 5.148 | 0.135 | 0.168 | 3.047 |

| Problem 100 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 165 | 212 | 124 | 242 | 214 | 358 | 292 | 156 | 338 | 0 |
| N.S. | 1 | 1.28 | 0.75 | 1.47 | 1.30 | 2.17 | 1.77 | 0.95 | 2.05 | 0.00 |
| time (sec) | N/A | 0.623 | 0.711 | 0.683 | 0.035 | 0.092 | 11.456 | 0.128 | 0.165 | 0.000 |

| Problem 101 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 138 | 176 | 112 | 194 | 170 | 293 | 230 | 127 | 288 | 0 |
| N.S. | 1 | 1.28 | 0.81 | 1.41 | 1.23 | 2.12 | 1.67 | 0.92 | 2.09 | 0.00 |
| time (sec) | N/A | 0.538 | 0.792 | 0.740 | 0.034 | 0.092 | 7.390 | 0.129 | 0.160 | 0.000 |

| Problem 102 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 108 | 137 | 90 | 151 | 130 | 253 | 185 | 100 | 251 | 0 |
| N.S. | 1 | 1.27 | 0.83 | 1.40 | 1.20 | 2.34 | 1.71 | 0.93 | 2.32 | 0.00 |
| time (sec) | N/A | 0.407 | 0.550 | 0.553 | 0.028 | 0.093 | 6.999 | 0.129 | 0.156 | 0.000 |

| Problem 103 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 85 | 90 | 75 | 104 | 94 | 210 | 131 | 88 | 159 | 91 |
| N.S. | 1 | 1.06 | 0.88 | 1.22 | 1.11 | 2.47 | 1.54 | 1.04 | 1.87 | 1.07 |
| time (sec) | N/A | 0.263 | 0.475 | 0.539 | 0.033 | 0.090 | 4.942 | 0.127 | 0.157 | 2.370 |

| Problem 104 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | F(-2) | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 89 | 99 | 100 | 114 | 93 | 602 | 274 | 0 | 280 | 102 |
| N.S. | 1 | 1.11 | 1.12 | 1.28 | 1.04 | 6.76 | 3.08 | 0.00 | 3.15 | 1.15 |
| time (sec) | N/A | 0.350 | 0.644 | 0.526 | 0.032 | 0.110 | 8.458 | 0.000 | 0.159 | 2.479 |

| Problem 105 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 88 | 92 | 93 | 112 | 98 | 220 | 282 | 125 | 256 | 103 |
| N.S. | 1 | 1.05 | 1.06 | 1.27 | 1.11 | 2.50 | 3.20 | 1.42 | 2.91 | 1.17 |
| time (sec) | N/A | 0.303 | 0.511 | 0.526 | 0.033 | 0.090 | 8.497 | 0.136 | 0.160 | 2.600 |

| Problem 106 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 113 | 124 | 99 | 165 | 136 | 281 | 332 | 198 | 417 | 143 |
| N.S. | 1 | 1.10 | 0.88 | 1.46 | 1.20 | 2.49 | 2.94 | 1.75 | 3.69 | 1.27 |
| time (sec) | N/A | 0.447 | 0.734 | 0.539 | 0.034 | 0.093 | 7.867 | 0.132 | 0.163 | 2.933 |

| Problem 107 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 150 | 165 | 130 | 210 | 177 | 321 | 547 | 296 | 450 | 167 |
| N.S. | 1 | 1.10 | 0.87 | 1.40 | 1.18 | 2.14 | 3.65 | 1.97 | 3.00 | 1.11 |
| time (sec) | N/A | 0.654 | 0.789 | 0.628 | 0.032 | 0.094 | 16.003 | 0.152 | 0.162 | 3.164 |

| Problem 108 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 176 | 189 | 137 | 258 | 221 | 384 | 466 | 521 | 481 | 213 |
| N.S. | 1 | 1.07 | 0.78 | 1.47 | 1.26 | 2.18 | 2.65 | 2.96 | 2.73 | 1.21 |
| time (sec) | N/A | 0.849 | 1.088 | 0.585 | 0.034 | 0.105 | 13.381 | 0.147 | 0.175 | 3.422 |

| Problem 109 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 186 | 230 | 145 | 286 | 290 | 455 | 1003 | 204 | 437 | 0 |
| N.S. | 1 | 1.24 | 0.78 | 1.54 | 1.56 | 2.45 | 5.39 | 1.10 | 2.35 | 0.00 |
| time (sec) | N/A | 0.681 | 1.096 | 0.624 | 0.036 | 0.111 | 14.922 | 0.136 | 0.178 | 0.000 |

| Problem 110 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 151 | 195 | 123 | 239 | 248 | 417 | 911 | 174 | 400 | 0 |
| N.S. | 1 | 1.29 | 0.81 | 1.58 | 1.64 | 2.76 | 6.03 | 1.15 | 2.65 | 0.00 |
| time (sec) | N/A | 0.593 | 0.891 | 0.599 | 0.036 | 0.099 | 14.291 | 0.136 | 0.165 | 0.000 |

| Problem 111 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 129 | 150 | 118 | 210 | 191 | 334 | 588 | 134 | 262 | 0 |
| N.S. | 1 | 1.16 | 0.91 | 1.63 | 1.48 | 2.59 | 4.56 | 1.04 | 2.03 | 0.00 |
| time (sec) | N/A | 0.503 | 0.805 | 0.681 | 0.044 | 0.095 | 11.110 | 0.135 | 0.160 | 0.000 |

| Problem 112 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 100 | 120 | 91 | 167 | 152 | 291 | 496 | 101 | 245 | 0 |
| N.S. | 1 | 1.20 | 0.91 | 1.67 | 1.52 | 2.91 | 4.96 | 1.01 | 2.45 | 0.00 |
| time (sec) | N/A | 0.383 | 0.848 | 0.599 | 0.038 | 0.095 | 10.414 | 0.133 | 0.156 | 0.000 |

| Problem 113 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 85 | 85 | 66 | 68 | 117 | 91 | 289 | 72 | 176 | 89 |
| N.S. | 1 | 1.00 | 0.78 | 0.80 | 1.38 | 1.07 | 3.40 | 0.85 | 2.07 | 1.05 |
| time (sec) | N/A | 0.246 | 0.635 | 0.532 | 0.031 | 0.081 | 8.956 | 0.131 | 0.149 | 2.707 |

| Problem 114 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 102 | 108 | 96 | 168 | 130 | 296 | 937 | 113 | 380 | 0 |
| N.S. | 1 | 1.06 | 0.94 | 1.65 | 1.27 | 2.90 | 9.19 | 1.11 | 3.73 | 0.00 |
| time (sec) | N/A | 0.333 | 0.854 | 0.526 | 0.034 | 0.098 | 18.481 | 0.134 | 0.153 | 0.000 |

| Problem 115 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 142 | 137 | 131 | 170 | 147 | 342 | 1057 | 168 | 429 | 166 |
| N.S. | 1 | 0.96 | 0.92 | 1.20 | 1.04 | 2.41 | 7.44 | 1.18 | 3.02 | 1.17 |
| time (sec) | N/A | 0.458 | 0.924 | 0.548 | 0.036 | 0.100 | 16.107 | 0.138 | 0.160 | 2.919 |

| Problem 116 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 156 | 178 | 133 | 242 | 201 | 440 | 1875 | 271 | 675 | 224 |
| N.S. | 1 | 1.14 | 0.85 | 1.55 | 1.29 | 2.82 | 12.02 | 1.74 | 4.33 | 1.44 |
| time (sec) | N/A | 0.634 | 1.030 | 0.541 | 0.037 | 0.109 | 28.330 | 0.137 | 0.191 | 3.146 |

| Problem 117 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 107 | 107 | 62 | 59 | 83 | 66 | 133 | 100 | 64 | 0 |
| N.S. | 1 | 1.00 | 0.58 | 0.55 | 0.78 | 0.62 | 1.24 | 0.93 | 0.60 | 0.00 |
| time (sec) | N/A | 0.306 | 0.102 | 0.389 | 0.040 | 0.067 | 0.299 | 0.123 | 0.151 | 0.000 |

| Problem 118 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 107 | 107 | 65 | 56 | 83 | 58 | 133 | 95 | 61 | 0 |
| N.S. | 1 | 1.00 | 0.61 | 0.52 | 0.78 | 0.54 | 1.24 | 0.89 | 0.57 | 0.00 |
| time (sec) | N/A | 0.291 | 0.090 | 0.510 | 0.027 | 0.066 | 0.268 | 0.121 | 0.155 | 0.000 |

| Problem 119 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 105 | 105 | 65 | 61 | 83 | 58 | 131 | 97 | 63 | 0 |
| N.S. | 1 | 1.00 | 0.62 | 0.58 | 0.79 | 0.55 | 1.25 | 0.92 | 0.60 | 0.00 |
| time (sec) | N/A | 0.278 | 0.102 | 0.283 | 0.038 | 0.069 | 0.320 | 0.122 | 0.156 | 0.000 |

| Problem 120 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 103 | 103 | 64 | 61 | 86 | 61 | 129 | 119 | 65 | 0 |
| N.S. | 1 | 1.00 | 0.62 | 0.59 | 0.83 | 0.59 | 1.25 | 1.16 | 0.63 | 0.00 |
| time (sec) | N/A | 0.285 | 0.110 | 0.128 | 0.035 | 0.071 | 0.408 | 0.128 | 0.156 | 0.000 |

| Problem 121 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 103 | 103 | 61 | 61 | 86 | 61 | 128 | 116 | 68 | 0 |
| N.S. | 1 | 1.00 | 0.59 | 0.59 | 0.83 | 0.59 | 1.24 | 1.13 | 0.66 | 0.00 |
| time (sec) | N/A | 0.286 | 0.105 | 0.127 | 0.031 | 0.070 | 0.331 | 0.123 | 0.150 | 0.000 |

| Problem 122 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | C | B | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 366 | 475 | 248 | 405 | 477 | 3486 | 1285 | 595 | 639 | 0 |
| N.S. | 1 | 1.30 | 0.68 | 1.11 | 1.30 | 9.52 | 3.51 | 1.63 | 1.75 | 0.00 |
| time (sec) | N/A | 1.179 | 0.591 | 3.432 | 0.132 | 0.232 | 28.345 | 0.134 | 0.159 | 0.000 |

| Problem 123 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | C | B | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 340 | 449 | 230 | 384 | 422 | 3365 | 1222 | 544 | 610 | 0 |
| N.S. | 1 | 1.32 | 0.68 | 1.13 | 1.24 | 9.90 | 3.59 | 1.60 | 1.79 | 0.00 |
| time (sec) | N/A | 1.035 | 0.492 | 0.770 | 0.124 | 0.229 | 22.327 | 0.138 | 0.155 | 0.000 |

| Problem 124 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | C | B | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 318 | 426 | 217 | 365 | 427 | 3318 | 1164 | 524 | 590 | 0 |
| N.S. | 1 | 1.34 | 0.68 | 1.15 | 1.34 | 10.43 | 3.66 | 1.65 | 1.86 | 0.00 |
| time (sec) | N/A | 0.914 | 0.401 | 0.764 | 0.133 | 0.218 | 24.677 | 0.141 | 0.156 | 0.000 |

| Problem 125 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | F | B | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 294 | 404 | 204 | 323 | 380 | 3363 | 0 | 487 | 577 | 0 |
| N.S. | 1 | 1.37 | 0.69 | 1.10 | 1.29 | 11.44 | 0.00 | 1.66 | 1.96 | 0.00 |
| time (sec) | N/A | 0.797 | 0.415 | 0.748 | 0.123 | 0.338 | 0.000 | 0.135 | 0.157 | 0.000 |

| Problem 126 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | F | B | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 293 | 428 | 205 | 335 | 405 | 3429 | 0 | 483 | 606 | 0 |
| N.S. | 1 | 1.46 | 0.70 | 1.14 | 1.38 | 11.70 | 0.00 | 1.65 | 2.07 | 0.00 |
| time (sec) | N/A | 0.913 | 0.396 | 0.637 | 0.119 | 0.480 | 0.000 | 0.144 | 0.160 | 0.000 |

| Problem 127 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | F | B | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 295 | 458 | 204 | 332 | 379 | 3409 | 0 | 475 | 630 | 0 |
| N.S. | 1 | 1.55 | 0.69 | 1.13 | 1.28 | 11.56 | 0.00 | 1.61 | 2.14 | 0.00 |
| time (sec) | N/A | 1.020 | 0.386 | 0.658 | 0.116 | 0.361 | 0.000 | 0.135 | 0.160 | 0.000 |

| Problem 128 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | F | B | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 321 | 485 | 220 | 362 | 426 | 3386 | 0 | 506 | 679 | 0 |
| N.S. | 1 | 1.51 | 0.69 | 1.13 | 1.33 | 10.55 | 0.00 | 1.58 | 2.12 | 0.00 |
| time (sec) | N/A | 1.111 | 0.444 | 0.718 | 0.125 | 0.502 | 0.000 | 0.141 | 0.165 | 0.000 |

| Problem 129 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | C | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 405 | 504 | 260 | 402 | 501 | 3697 | 4189 | 617 | 1244 | 0 |
| N.S. | 1 | 1.24 | 0.64 | 0.99 | 1.24 | 9.13 | 10.34 | 1.52 | 3.07 | 0.00 |
| time (sec) | N/A | 1.237 | 1.281 | 0.812 | 0.116 | 0.276 | 107.111 | 0.141 | 0.165 | 0.000 |

| Problem 130 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | C | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 374 | 474 | 241 | 381 | 448 | 3694 | 4012 | 568 | 1227 | 0 |
| N.S. | 1 | 1.27 | 0.64 | 1.02 | 1.20 | 9.88 | 10.73 | 1.52 | 3.28 | 0.00 |
| time (sec) | N/A | 1.127 | 1.263 | 0.763 | 0.138 | 0.271 | 83.209 | 0.145 | 0.166 | 0.000 |

| Problem 131 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | C | B | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 342 | 448 | 234 | 380 | 468 | 3584 | 3900 | 550 | 1208 | 0 |
| N.S. | 1 | 1.31 | 0.68 | 1.11 | 1.37 | 10.48 | 11.40 | 1.61 | 3.53 | 0.00 |
| time (sec) | N/A | 0.985 | 1.250 | 0.733 | 0.130 | 0.259 | 70.348 | 0.141 | 0.163 | 0.000 |

| Problem 132 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | C | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 340 | 378 | 223 | 362 | 414 | 3500 | 4184 | 517 | 1201 | 0 |
| N.S. | 1 | 1.11 | 0.66 | 1.06 | 1.22 | 10.29 | 12.31 | 1.52 | 3.53 | 0.00 |
| time (sec) | N/A | 0.759 | 1.128 | 0.695 | 0.122 | 0.397 | 75.227 | 0.147 | 0.165 | 0.000 |

| Problem 133 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | C | B | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 350 | 413 | 238 | 389 | 454 | 3700 | 4014 | 543 | 1273 | 0 |
| N.S. | 1 | 1.18 | 0.68 | 1.11 | 1.30 | 10.57 | 11.47 | 1.55 | 3.64 | 0.00 |
| time (sec) | N/A | 0.831 | 1.423 | 0.736 | 0.134 | 0.514 | 82.537 | 0.137 | 0.164 | 0.000 |

| Problem 134 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | C | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 384 | 446 | 243 | 388 | 437 | 3722 | 4048 | 546 | 1309 | 0 |
| N.S. | 1 | 1.16 | 0.63 | 1.01 | 1.14 | 9.69 | 10.54 | 1.42 | 3.41 | 0.00 |
| time (sec) | N/A | 0.903 | 1.357 | 0.862 | 0.117 | 0.425 | 106.932 | 0.144 | 0.164 | 0.000 |

| Problem 135 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | C | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 411 | 481 | 267 | 421 | 499 | 3715 | 4245 | 585 | 1360 | 0 |
| N.S. | 1 | 1.17 | 0.65 | 1.02 | 1.21 | 9.04 | 10.33 | 1.42 | 3.31 | 0.00 |
| time (sec) | N/A | 0.966 | 1.356 | 0.773 | 0.133 | 0.606 | 135.629 | 0.142 | 0.170 | 0.000 |

| Problem 136 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | B | B | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 181 | 181 | 127 | 180 | 230 | 829 | 6390 | 1558 | 1061 | 0 |
| N.S. | 1 | 1.00 | 0.70 | 0.99 | 1.27 | 4.58 | 35.30 | 8.61 | 5.86 | 0.00 |
| time (sec) | N/A | 0.427 | 1.663 | 0.538 | 0.057 | 0.088 | 1.035 | 0.146 | 0.151 | 0.000 |

| Problem 137 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | B | B | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 119 | 119 | 79 | 118 | 138 | 339 | 2504 | 692 | 453 | 0 |
| N.S. | 1 | 1.00 | 0.66 | 0.99 | 1.16 | 2.85 | 21.04 | 5.82 | 3.81 | 0.00 |
| time (sec) | N/A | 0.350 | 0.457 | 0.079 | 0.050 | 0.086 | 0.607 | 0.130 | 0.156 | 0.000 |

| Problem 138 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | B | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 69 | 69 | 43 | 68 | 66 | 117 | 604 | 214 | 129 | 0 |
| N.S. | 1 | 1.00 | 0.62 | 0.99 | 0.96 | 1.70 | 8.75 | 3.10 | 1.87 | 0.00 |
| time (sec) | N/A | 0.243 | 0.124 | 0.071 | 0.037 | 0.077 | 0.481 | 0.122 | 0.154 | 0.000 |

| Problem 139 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | C | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 151 | 151 | 112 | 0 | 0 | 0 | 379 | 0 | 401 | 0 |
| N.S. | 1 | 1.00 | 0.74 | 0.00 | 0.00 | 0.00 | 2.51 | 0.00 | 2.66 | 0.00 |
| time (sec) | N/A | 0.385 | 0.963 | 0.000 | 0.000 | 0.000 | 3.968 | 0.000 | 0.157 | 0.000 |

| Problem 140 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | C | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 188 | 174 | 160 | 0 | 0 | 0 | 1828 | 0 | 0 | 0 |
| N.S. | 1 | 0.93 | 0.85 | 0.00 | 0.00 | 0.00 | 9.72 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.391 | 1.474 | 0.000 | 0.000 | 0.000 | 45.927 | 0.000 | 0.179 | 0.000 |

| Problem 141 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | C | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 194 | 176 | 160 | 0 | 0 | 0 | 5258 | 0 | 0 | 0 |
| N.S. | 1 | 0.91 | 0.82 | 0.00 | 0.00 | 0.00 | 27.10 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.411 | 1.705 | 0.000 | 0.000 | 0.000 | 127.581 | 0.000 | 0.198 | 0.000 |

| Problem 142 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | C | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 239 | 253 | 177 | 0 | 0 | 0 | 468 | 0 | 162 | 0 |
| N.S. | 1 | 1.06 | 0.74 | 0.00 | 0.00 | 0.00 | 1.96 | 0.00 | 0.68 | 0.00 |
| time (sec) | N/A | 0.642 | 2.118 | 0.000 | 0.000 | 0.000 | 10.288 | 0.000 | 1.517 | 0.000 |

| Problem 143 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | C | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 229 | 251 | 176 | 0 | 0 | 0 | 230 | 0 | 76 | 0 |
| N.S. | 1 | 1.10 | 0.77 | 0.00 | 0.00 | 0.00 | 1.00 | 0.00 | 0.33 | 0.00 |
| time (sec) | N/A | 0.609 | 0.934 | 0.000 | 0.000 | 0.000 | 3.765 | 0.000 | 0.419 | 0.000 |

| Problem 144 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | C | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 229 | 243 | 176 | 0 | 0 | 0 | 223 | 0 | 84 | 0 |
| N.S. | 1 | 1.06 | 0.77 | 0.00 | 0.00 | 0.00 | 0.97 | 0.00 | 0.37 | 0.00 |
| time (sec) | N/A | 0.614 | 0.913 | 0.000 | 0.000 | 0.000 | 3.162 | 0.000 | 0.226 | 0.000 |

| Problem 145 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | C | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 228 | 217 | 179 | 0 | 0 | 0 | 223 | 0 | 148 | 0 |
| N.S. | 1 | 0.95 | 0.79 | 0.00 | 0.00 | 0.00 | 0.98 | 0.00 | 0.65 | 0.00 |
| time (sec) | N/A | 0.429 | 2.326 | 0.000 | 0.000 | 0.000 | 11.762 | 0.000 | 0.223 | 0.000 |

| Problem 146 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | C | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 252 | 232 | 179 | 0 | 0 | 0 | 223 | 0 | 224 | 0 |
| N.S. | 1 | 0.92 | 0.71 | 0.00 | 0.00 | 0.00 | 0.88 | 0.00 | 0.89 | 0.00 |
| time (sec) | N/A | 0.469 | 4.064 | 0.000 | 0.000 | 0.000 | 156.205 | 0.000 | 0.352 | 0.000 |

| Problem 147 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 252 | 232 | 179 | 0 | 0 | 0 | 0 | 0 | 300 | 0 |
| N.S. | 1 | 0.92 | 0.71 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 1.19 | 0.00 |
| time (sec) | N/A | 0.454 | 5.994 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.728 | 0.000 |

| Problem 148 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 61 | 61 | 61 | 52 | 51 | 51 | 56 | 57 | 54 | 57 |
| N.S. | 1 | 1.00 | 1.00 | 0.85 | 0.84 | 0.84 | 0.92 | 0.93 | 0.89 | 0.93 |
| time (sec) | N/A | 0.280 | 0.019 | 0.143 | 0.026 | 0.062 | 0.026 | 0.122 | 0.156 | 2.506 |

| Problem 149 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 61 | 61 | 61 | 52 | 51 | 51 | 56 | 57 | 54 | 57 |
| N.S. | 1 | 1.00 | 1.00 | 0.85 | 0.84 | 0.84 | 0.92 | 0.93 | 0.89 | 0.93 |
| time (sec) | N/A | 0.280 | 0.014 | 0.095 | 0.026 | 0.067 | 0.022 | 0.130 | 0.154 | 2.459 |

| Problem 150 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 56 | 56 | 56 | 49 | 48 | 48 | 53 | 54 | 52 | 54 |
| N.S. | 1 | 1.00 | 1.00 | 0.88 | 0.86 | 0.86 | 0.95 | 0.96 | 0.93 | 0.96 |
| time (sec) | N/A | 0.241 | 0.012 | 0.094 | 0.045 | 0.062 | 0.022 | 0.125 | 0.178 | 2.425 |

| Problem 151 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 54 | 54 | 54 | 52 | 48 | 53 | 48 | 51 | 54 | 51 |
| N.S. | 1 | 1.00 | 1.00 | 0.96 | 0.89 | 0.98 | 0.89 | 0.94 | 1.00 | 0.94 |
| time (sec) | N/A | 0.256 | 0.024 | 0.053 | 0.033 | 0.067 | 0.069 | 0.122 | 0.163 | 2.424 |

| Problem 152 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 54 | 54 | 55 | 50 | 50 | 53 | 53 | 53 | 54 | 51 |
| N.S. | 1 | 1.00 | 1.02 | 0.93 | 0.93 | 0.98 | 0.98 | 0.98 | 1.00 | 0.94 |
| time (sec) | N/A | 0.263 | 0.025 | 0.067 | 0.025 | 0.062 | 0.125 | 0.124 | 0.156 | 2.381 |

| Problem 153 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 54 | 54 | 53 | 48 | 51 | 53 | 56 | 54 | 54 | 51 |
| N.S. | 1 | 1.00 | 0.98 | 0.89 | 0.94 | 0.98 | 1.04 | 1.00 | 1.00 | 0.94 |
| time (sec) | N/A | 0.251 | 0.030 | 0.069 | 0.026 | 0.062 | 0.421 | 0.122 | 0.184 | 2.406 |

| Problem 154 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 101 | 101 | 101 | 94 | 93 | 93 | 107 | 105 | 89 | 108 |
| N.S. | 1 | 1.00 | 1.00 | 0.93 | 0.92 | 0.92 | 1.06 | 1.04 | 0.88 | 1.07 |
| time (sec) | N/A | 0.340 | 0.025 | 0.413 | 0.026 | 0.063 | 0.034 | 0.116 | 0.156 | 2.361 |

| Problem 155 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 101 | 101 | 101 | 94 | 93 | 93 | 107 | 105 | 89 | 108 |
| N.S. | 1 | 1.00 | 1.00 | 0.93 | 0.92 | 0.92 | 1.06 | 1.04 | 0.88 | 1.07 |
| time (sec) | N/A | 0.337 | 0.022 | 0.388 | 0.025 | 0.067 | 0.029 | 0.120 | 0.157 | 2.335 |

| Problem 156 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 96 | 96 | 96 | 91 | 90 | 90 | 104 | 102 | 87 | 106 |
| N.S. | 1 | 1.00 | 1.00 | 0.95 | 0.94 | 0.94 | 1.08 | 1.06 | 0.91 | 1.10 |
| time (sec) | N/A | 0.286 | 0.019 | 0.398 | 0.031 | 0.064 | 0.032 | 0.122 | 0.160 | 1.362 |

| Problem 157 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 94 | 94 | 94 | 96 | 90 | 95 | 99 | 100 | 89 | 105 |
| N.S. | 1 | 1.00 | 1.00 | 1.02 | 0.96 | 1.01 | 1.05 | 1.06 | 0.95 | 1.12 |
| time (sec) | N/A | 0.302 | 0.031 | 0.401 | 0.035 | 0.064 | 0.103 | 0.115 | 0.155 | 1.451 |

| Problem 158 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 94 | 94 | 93 | 95 | 92 | 95 | 100 | 99 | 89 | 103 |
| N.S. | 1 | 1.00 | 0.99 | 1.01 | 0.98 | 1.01 | 1.06 | 1.05 | 0.95 | 1.10 |
| time (sec) | N/A | 0.318 | 0.032 | 0.394 | 0.028 | 0.068 | 0.186 | 0.123 | 0.157 | 1.522 |

| Problem 159 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 94 | 94 | 93 | 90 | 93 | 95 | 102 | 100 | 89 | 103 |
| N.S. | 1 | 1.00 | 0.99 | 0.96 | 0.99 | 1.01 | 1.09 | 1.06 | 0.95 | 1.10 |
| time (sec) | N/A | 0.328 | 0.021 | 0.384 | 0.026 | 0.066 | 0.652 | 0.123 | 0.158 | 1.479 |

| Problem 160 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 210 | 210 | 210 | 233 | 212 | 452 | 384 | 243 | 175 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 1.11 | 1.01 | 2.15 | 1.83 | 1.16 | 0.83 | 0.00 |
| time (sec) | N/A | 0.454 | 0.159 | 0.569 | 0.107 | 0.080 | 0.540 | 0.129 | 0.157 | 0.000 |

| Problem 161 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 172 | 172 | 162 | 185 | 173 | 368 | 337 | 196 | 151 | 0 |
| N.S. | 1 | 1.00 | 0.94 | 1.08 | 1.01 | 2.14 | 1.96 | 1.14 | 0.88 | 0.00 |
| time (sec) | N/A | 0.390 | 0.122 | 0.465 | 0.106 | 0.080 | 0.489 | 0.121 | 0.154 | 0.000 |

| Problem 162 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 136 | 136 | 135 | 137 | 132 | 286 | 185 | 147 | 127 | 0 |
| N.S. | 1 | 1.00 | 0.99 | 1.01 | 0.97 | 2.10 | 1.36 | 1.08 | 0.93 | 0.00 |
| time (sec) | N/A | 0.346 | 0.105 | 0.463 | 0.111 | 0.080 | 0.457 | 0.123 | 0.153 | 0.000 |

| Problem 163 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 100 | 100 | 98 | 94 | 95 | 236 | 160 | 104 | 99 | 0 |
| N.S. | 1 | 1.00 | 0.98 | 0.94 | 0.95 | 2.36 | 1.60 | 1.04 | 0.99 | 0.00 |
| time (sec) | N/A | 0.283 | 0.081 | 0.468 | 0.124 | 0.080 | 0.414 | 0.116 | 0.149 | 0.000 |

| Problem 164 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 84 | 84 | 83 | 78 | 79 | 211 | 150 | 83 | 81 | 97 |
| N.S. | 1 | 1.00 | 0.99 | 0.93 | 0.94 | 2.51 | 1.79 | 0.99 | 0.96 | 1.15 |
| time (sec) | N/A | 0.310 | 0.084 | 0.480 | 0.121 | 0.083 | 0.547 | 0.125 | 0.151 | 1.786 |

| Problem 165 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 82 | 82 | 83 | 79 | 79 | 216 | 151 | 80 | 72 | 107 |
| N.S. | 1 | 1.00 | 1.01 | 0.96 | 0.96 | 2.63 | 1.84 | 0.98 | 0.88 | 1.30 |
| time (sec) | N/A | 0.316 | 0.082 | 0.480 | 0.114 | 0.078 | 1.036 | 0.124 | 0.151 | 1.776 |

| Problem 166 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 105 | 105 | 103 | 94 | 96 | 246 | 167 | 102 | 72 | 135 |
| N.S. | 1 | 1.00 | 0.98 | 0.90 | 0.91 | 2.34 | 1.59 | 0.97 | 0.69 | 1.29 |
| time (sec) | N/A | 0.352 | 0.093 | 1.888 | 0.111 | 0.079 | 2.520 | 0.127 | 0.152 | 1.558 |

| Problem 167 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 138 | 138 | 139 | 128 | 135 | 292 | 301 | 149 | 86 | 177 |
| N.S. | 1 | 1.00 | 1.01 | 0.93 | 0.98 | 2.12 | 2.18 | 1.08 | 0.62 | 1.28 |
| time (sec) | N/A | 0.376 | 0.129 | 0.576 | 0.108 | 0.078 | 7.714 | 0.129 | 0.154 | 1.512 |

| Problem 168 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 176 | 176 | 174 | 165 | 174 | 374 | 354 | 196 | 111 | 213 |
| N.S. | 1 | 1.00 | 0.99 | 0.94 | 0.99 | 2.12 | 2.01 | 1.11 | 0.63 | 1.21 |
| time (sec) | N/A | 0.419 | 0.181 | 0.468 | 0.110 | 0.081 | 12.171 | 0.130 | 0.155 | 1.572 |

| Problem 169 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 212 | 212 | 211 | 199 | 215 | 458 | 398 | 245 | 137 | 249 |
| N.S. | 1 | 1.00 | 1.00 | 0.94 | 1.01 | 2.16 | 1.88 | 1.16 | 0.65 | 1.17 |
| time (sec) | N/A | 0.495 | 0.191 | 0.477 | 0.112 | 0.084 | 26.169 | 0.124 | 0.155 | 1.572 |

| Problem 170 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 229 | 249 | 227 | 230 | 229 | 572 | 444 | 246 | 318 | 0 |
| N.S. | 1 | 1.09 | 0.99 | 1.00 | 1.00 | 2.50 | 1.94 | 1.07 | 1.39 | 0.00 |
| time (sec) | N/A | 0.623 | 0.136 | 0.789 | 0.113 | 0.081 | 1.075 | 0.117 | 0.156 | 0.000 |

| Problem 171 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-2) | A | A | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 188 | 209 | 187 | 180 | 0 | 478 | 257 | 196 | 347 | 0 |
| N.S. | 1 | 1.11 | 0.99 | 0.96 | 0.00 | 2.54 | 1.37 | 1.04 | 1.85 | 0.00 |
| time (sec) | N/A | 0.534 | 0.107 | 0.586 | 0.000 | 0.081 | 1.141 | 0.129 | 0.149 | 0.000 |

| Problem 172 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-2) | A | A | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 149 | 171 | 148 | 137 | 0 | 424 | 221 | 149 | 314 | 0 |
| N.S. | 1 | 1.15 | 0.99 | 0.92 | 0.00 | 2.85 | 1.48 | 1.00 | 2.11 | 0.00 |
| time (sec) | N/A | 0.506 | 0.082 | 0.574 | 0.000 | 0.085 | 1.014 | 0.124 | 0.160 | 0.000 |

| Problem 173 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-2) | A | A | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 124 | 127 | 122 | 112 | 0 | 364 | 201 | 123 | 283 | 0 |
| N.S. | 1 | 1.02 | 0.98 | 0.90 | 0.00 | 2.94 | 1.62 | 0.99 | 2.28 | 0.00 |
| time (sec) | N/A | 0.390 | 0.078 | 0.469 | 0.000 | 0.080 | 0.777 | 0.121 | 0.154 | 0.000 |

| Problem 174 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-2) | A | A | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 115 | 119 | 115 | 107 | 0 | 353 | 197 | 119 | 279 | 0 |
| N.S. | 1 | 1.03 | 1.00 | 0.93 | 0.00 | 3.07 | 1.71 | 1.03 | 2.43 | 0.00 |
| time (sec) | N/A | 0.417 | 0.062 | 0.474 | 0.000 | 0.082 | 2.225 | 0.121 | 0.152 | 0.000 |

| Problem 175 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-2) | A | A | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 125 | 126 | 125 | 116 | 0 | 378 | 212 | 120 | 305 | 0 |
| N.S. | 1 | 1.01 | 1.00 | 0.93 | 0.00 | 3.02 | 1.70 | 0.96 | 2.44 | 0.00 |
| time (sec) | N/A | 0.412 | 0.067 | 0.481 | 0.000 | 0.084 | 4.808 | 0.124 | 0.146 | 0.000 |

| Problem 176 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-2) | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 151 | 156 | 151 | 139 | 0 | 432 | 226 | 147 | 335 | 203 |
| N.S. | 1 | 1.03 | 1.00 | 0.92 | 0.00 | 2.86 | 1.50 | 0.97 | 2.22 | 1.34 |
| time (sec) | N/A | 0.620 | 0.091 | 0.583 | 0.000 | 0.083 | 11.951 | 0.126 | 0.147 | 1.508 |

| Problem 177 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-2) | A | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 189 | 195 | 190 | 173 | 0 | 488 | 394 | 197 | 367 | 241 |
| N.S. | 1 | 1.03 | 1.01 | 0.92 | 0.00 | 2.58 | 2.08 | 1.04 | 1.94 | 1.28 |
| time (sec) | N/A | 0.719 | 0.117 | 0.488 | 0.000 | 0.085 | 31.079 | 0.124 | 0.152 | 1.546 |

| Problem 178 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 234 | 240 | 230 | 212 | 238 | 582 | 449 | 247 | 305 | 0 |
| N.S. | 1 | 1.03 | 0.98 | 0.91 | 1.02 | 2.49 | 1.92 | 1.06 | 1.30 | 0.00 |
| time (sec) | N/A | 0.921 | 0.143 | 0.483 | 0.117 | 0.083 | 57.082 | 0.122 | 0.154 | 0.000 |

| Problem 179 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 277 | 303 | 272 | 267 | 283 | 762 | 503 | 294 | 447 | 0 |
| N.S. | 1 | 1.09 | 0.98 | 0.96 | 1.02 | 2.75 | 1.82 | 1.06 | 1.61 | 0.00 |
| time (sec) | N/A | 0.953 | 0.188 | 0.566 | 0.119 | 0.085 | 6.716 | 0.125 | 0.154 | 0.000 |

| Problem 180 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 235 | 262 | 232 | 219 | 239 | 668 | 316 | 244 | 410 | 0 |
| N.S. | 1 | 1.11 | 0.99 | 0.93 | 1.02 | 2.84 | 1.34 | 1.04 | 1.74 | 0.00 |
| time (sec) | N/A | 0.914 | 0.149 | 0.487 | 0.114 | 0.086 | 6.967 | 0.121 | 0.152 | 0.000 |

| Problem 181 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 195 | 219 | 176 | 176 | 194 | 614 | 280 | 195 | 370 | 0 |
| N.S. | 1 | 1.12 | 0.90 | 0.90 | 0.99 | 3.15 | 1.44 | 1.00 | 1.90 | 0.00 |
| time (sec) | N/A | 0.736 | 0.171 | 0.460 | 0.110 | 0.081 | 6.458 | 0.119 | 0.155 | 0.000 |

| Problem 182 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 167 | 174 | 156 | 148 | 170 | 556 | 260 | 169 | 347 | 0 |
| N.S. | 1 | 1.04 | 0.93 | 0.89 | 1.02 | 3.33 | 1.56 | 1.01 | 2.08 | 0.00 |
| time (sec) | N/A | 0.569 | 0.141 | 0.452 | 0.113 | 0.081 | 5.332 | 0.121 | 0.155 | 0.000 |

| Problem 183 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 147 | 160 | 141 | 137 | 155 | 511 | 243 | 147 | 327 | 0 |
| N.S. | 1 | 1.09 | 0.96 | 0.93 | 1.05 | 3.48 | 1.65 | 1.00 | 2.22 | 0.00 |
| time (sec) | N/A | 0.419 | 0.121 | 0.463 | 0.107 | 0.086 | 3.401 | 0.122 | 0.156 | 0.000 |

| Problem 184 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 156 | 170 | 152 | 142 | 160 | 512 | 250 | 149 | 332 | 0 |
| N.S. | 1 | 1.09 | 0.97 | 0.91 | 1.03 | 3.28 | 1.60 | 0.96 | 2.13 | 0.00 |
| time (sec) | N/A | 0.465 | 0.238 | 0.460 | 0.114 | 0.084 | 8.702 | 0.123 | 0.154 | 0.000 |

| Problem 185 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 176 | 190 | 169 | 153 | 182 | 569 | 270 | 166 | 352 | 0 |
| N.S. | 1 | 1.08 | 0.96 | 0.87 | 1.03 | 3.23 | 1.53 | 0.94 | 2.00 | 0.00 |
| time (sec) | N/A | 0.568 | 0.155 | 0.483 | 0.113 | 0.088 | 22.480 | 0.126 | 0.157 | 0.000 |

| Problem 186 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 200 | 214 | 196 | 177 | 202 | 628 | 284 | 194 | 371 | 272 |
| N.S. | 1 | 1.07 | 0.98 | 0.88 | 1.01 | 3.14 | 1.42 | 0.97 | 1.86 | 1.36 |
| time (sec) | N/A | 0.985 | 0.127 | 0.454 | 0.117 | 0.084 | 52.947 | 0.124 | 0.155 | 1.584 |

| Problem 187 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 238 | 252 | 234 | 211 | 247 | 678 | 450 | 245 | 394 | 313 |
| N.S. | 1 | 1.06 | 0.98 | 0.89 | 1.04 | 2.85 | 1.89 | 1.03 | 1.66 | 1.32 |
| time (sec) | N/A | 1.161 | 0.157 | 0.467 | 0.116 | 0.089 | 119.809 | 0.131 | 0.152 | 1.634 |

| Problem 188 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F(-1) | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 282 | 296 | 276 | 250 | 291 | 772 | 0 | 295 | 434 | 0 |
| N.S. | 1 | 1.05 | 0.98 | 0.89 | 1.03 | 2.74 | 0.00 | 1.05 | 1.54 | 0.00 |
| time (sec) | N/A | 1.408 | 0.183 | 0.470 | 0.111 | 0.084 | 0.000 | 0.122 | 0.150 | 0.000 |

| Problem 189 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 245 | 216 | 194 | 169 | 331 | 412 | 279 | 215 | 304 | 0 |
| N.S. | 1 | 0.88 | 0.79 | 0.69 | 1.35 | 1.68 | 1.14 | 0.88 | 1.24 | 0.00 |
| time (sec) | N/A | 0.471 | 1.436 | 0.592 | 0.038 | 0.151 | 0.602 | 0.137 | 0.172 | 0.000 |

| Problem 190 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 191 | 187 | 153 | 135 | 246 | 327 | 216 | 167 | 245 | 0 |
| N.S. | 1 | 0.98 | 0.80 | 0.71 | 1.29 | 1.71 | 1.13 | 0.87 | 1.28 | 0.00 |
| time (sec) | N/A | 0.371 | 0.368 | 0.632 | 0.035 | 0.108 | 0.453 | 0.143 | 0.161 | 0.000 |

| Problem 191 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 170 | 167 | 138 | 118 | 182 | 268 | 340 | 153 | 230 | 0 |
| N.S. | 1 | 0.98 | 0.81 | 0.69 | 1.07 | 1.58 | 2.00 | 0.90 | 1.35 | 0.00 |
| time (sec) | N/A | 0.404 | 0.702 | 0.587 | 0.035 | 0.096 | 1.553 | 0.139 | 0.163 | 0.000 |

| Problem 192 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 143 | 137 | 117 | 110 | 128 | 257 | 269 | 201 | 198 | 0 |
| N.S. | 1 | 0.96 | 0.82 | 0.77 | 0.90 | 1.80 | 1.88 | 1.41 | 1.38 | 0.00 |
| time (sec) | N/A | 0.369 | 0.482 | 0.558 | 0.039 | 0.093 | 1.728 | 0.146 | 0.164 | 0.000 |

| Problem 193 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 136 | 143 | 112 | 110 | 114 | 260 | 250 | 398 | 181 | 150 |
| N.S. | 1 | 1.05 | 0.82 | 0.81 | 0.84 | 1.91 | 1.84 | 2.93 | 1.33 | 1.10 |
| time (sec) | N/A | 0.377 | 0.596 | 0.536 | 0.033 | 0.094 | 1.919 | 0.144 | 0.164 | 1.912 |

| Problem 194 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 135 | 156 | 132 | 124 | 143 | 271 | 551 | 662 | 183 | 190 |
| N.S. | 1 | 1.16 | 0.98 | 0.92 | 1.06 | 2.01 | 4.08 | 4.90 | 1.36 | 1.41 |
| time (sec) | N/A | 0.403 | 0.486 | 0.549 | 0.035 | 0.109 | 2.338 | 0.151 | 0.168 | 2.080 |

| Problem 195 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | B | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 141 | 157 | 99 | 91 | 193 | 138 | 1078 | 772 | 213 | 0 |
| N.S. | 1 | 1.11 | 0.70 | 0.65 | 1.37 | 0.98 | 7.65 | 5.48 | 1.51 | 0.00 |
| time (sec) | N/A | 0.471 | 0.490 | 0.543 | 0.032 | 0.131 | 2.482 | 0.144 | 0.166 | 0.000 |

| Problem 196 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | B | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 191 | 187 | 137 | 123 | 275 | 182 | 1890 | 884 | 274 | 0 |
| N.S. | 1 | 0.98 | 0.72 | 0.64 | 1.44 | 0.95 | 9.90 | 4.63 | 1.43 | 0.00 |
| time (sec) | N/A | 0.491 | 0.686 | 0.559 | 0.035 | 0.195 | 3.248 | 0.148 | 0.173 | 0.000 |

| Problem 197 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | B | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 243 | 217 | 171 | 154 | 359 | 224 | 2990 | 996 | 333 | 0 |
| N.S. | 1 | 0.89 | 0.70 | 0.63 | 1.48 | 0.92 | 12.30 | 4.10 | 1.37 | 0.00 |
| time (sec) | N/A | 0.516 | 0.862 | 0.571 | 0.036 | 0.301 | 4.320 | 0.150 | 0.179 | 0.000 |

| Problem 198 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 297 | 240 | 260 | 201 | 407 | 496 | 505 | 264 | 363 | 0 |
| N.S. | 1 | 0.81 | 0.88 | 0.68 | 1.37 | 1.67 | 1.70 | 0.89 | 1.22 | 0.00 |
| time (sec) | N/A | 0.503 | 2.136 | 0.611 | 0.039 | 0.206 | 0.703 | 0.135 | 0.193 | 0.000 |

| Problem 199 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 241 | 209 | 185 | 170 | 319 | 413 | 396 | 216 | 304 | 0 |
| N.S. | 1 | 0.87 | 0.77 | 0.71 | 1.32 | 1.71 | 1.64 | 0.90 | 1.26 | 0.00 |
| time (sec) | N/A | 0.386 | 0.546 | 0.567 | 0.038 | 0.141 | 0.540 | 0.137 | 0.174 | 0.000 |

| Problem 200 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 219 | 189 | 175 | 150 | 252 | 356 | 777 | 204 | 294 | 0 |
| N.S. | 1 | 0.86 | 0.80 | 0.68 | 1.15 | 1.63 | 3.55 | 0.93 | 1.34 | 0.00 |
| time (sec) | N/A | 0.433 | 1.287 | 0.562 | 0.039 | 0.109 | 2.261 | 0.149 | 0.176 | 0.000 |

| Problem 201 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 220 | 187 | 153 | 136 | 230 | 313 | 624 | 279 | 271 | 0 |
| N.S. | 1 | 0.85 | 0.70 | 0.62 | 1.05 | 1.42 | 2.84 | 1.27 | 1.23 | 0.00 |
| time (sec) | N/A | 0.459 | 0.784 | 0.589 | 0.036 | 0.106 | 2.827 | 0.154 | 0.178 | 0.000 |

| Problem 202 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | B | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 195 | 170 | 136 | 129 | 178 | 318 | 483 | 412 | 241 | 0 |
| N.S. | 1 | 0.87 | 0.70 | 0.66 | 0.91 | 1.63 | 2.48 | 2.11 | 1.24 | 0.00 |
| time (sec) | N/A | 0.404 | 0.902 | 0.582 | 0.036 | 0.104 | 3.195 | 0.154 | 0.184 | 0.000 |

| Problem 203 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | B | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 168 | 169 | 145 | 142 | 170 | 318 | 765 | 747 | 228 | 0 |
| N.S. | 1 | 1.01 | 0.86 | 0.85 | 1.01 | 1.89 | 4.55 | 4.45 | 1.36 | 0.00 |
| time (sec) | N/A | 0.401 | 0.812 | 0.627 | 0.036 | 0.114 | 3.849 | 0.154 | 0.185 | 0.000 |

| Problem 204 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | B | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 158 | 177 | 163 | 155 | 183 | 352 | 1593 | 886 | 244 | 0 |
| N.S. | 1 | 1.12 | 1.03 | 0.98 | 1.16 | 2.23 | 10.08 | 5.61 | 1.54 | 0.00 |
| time (sec) | N/A | 0.429 | 0.887 | 0.570 | 0.035 | 0.139 | 4.350 | 0.158 | 0.182 | 0.000 |

| Problem 205 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | B | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 141 | 157 | 102 | 92 | 193 | 181 | 2932 | 996 | 274 | 0 |
| N.S. | 1 | 1.11 | 0.72 | 0.65 | 1.37 | 1.28 | 20.79 | 7.06 | 1.94 | 0.00 |
| time (sec) | N/A | 0.459 | 0.803 | 0.617 | 0.071 | 0.195 | 5.266 | 0.157 | 0.183 | 0.000 |

| Problem 206 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | B | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 191 | 187 | 141 | 123 | 275 | 225 | 4896 | 1108 | 333 | 0 |
| N.S. | 1 | 0.98 | 0.74 | 0.64 | 1.44 | 1.18 | 25.63 | 5.80 | 1.74 | 0.00 |
| time (sec) | N/A | 0.499 | 0.953 | 0.716 | 0.037 | 0.280 | 6.915 | 0.154 | 0.187 | 0.000 |

| Problem 207 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | B | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 243 | 217 | 168 | 153 | 359 | 266 | 7166 | 1194 | 392 | 0 |
| N.S. | 1 | 0.89 | 0.69 | 0.63 | 1.48 | 1.09 | 29.49 | 4.91 | 1.61 | 0.00 |
| time (sec) | N/A | 0.545 | 1.007 | 0.694 | 0.038 | 0.440 | 9.364 | 0.166 | 0.202 | 0.000 |

| Problem 208 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 216 | 221 | 158 | 142 | 347 | 177 | 442 | 260 | 159 | 245 |
| N.S. | 1 | 1.02 | 0.73 | 0.66 | 1.61 | 0.82 | 2.05 | 1.20 | 0.74 | 1.13 |
| time (sec) | N/A | 0.560 | 0.209 | 0.687 | 0.041 | 0.087 | 0.430 | 0.133 | 0.157 | 2.043 |

| Problem 209 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 168 | 172 | 122 | 111 | 263 | 135 | 340 | 192 | 124 | 206 |
| N.S. | 1 | 1.02 | 0.73 | 0.66 | 1.57 | 0.80 | 2.02 | 1.14 | 0.74 | 1.23 |
| time (sec) | N/A | 0.464 | 0.147 | 1.306 | 0.037 | 0.084 | 0.343 | 0.126 | 0.163 | 1.885 |

| Problem 210 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 121 | 126 | 92 | 79 | 180 | 94 | 238 | 128 | 89 | 132 |
| N.S. | 1 | 1.04 | 0.76 | 0.65 | 1.49 | 0.78 | 1.97 | 1.06 | 0.74 | 1.09 |
| time (sec) | N/A | 0.384 | 0.118 | 0.539 | 0.036 | 0.084 | 0.271 | 0.127 | 0.167 | 1.827 |

| Problem 211 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 103 | 108 | 86 | 82 | 122 | 210 | 134 | 125 | 154 | 96 |
| N.S. | 1 | 1.05 | 0.83 | 0.80 | 1.18 | 2.04 | 1.30 | 1.21 | 1.50 | 0.93 |
| time (sec) | N/A | 0.397 | 0.173 | 0.506 | 0.038 | 0.094 | 4.896 | 0.116 | 0.167 | 1.893 |

| Problem 212 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 100 | 101 | 88 | 88 | 104 | 214 | 138 | 114 | 139 | 104 |
| N.S. | 1 | 1.01 | 0.88 | 0.88 | 1.04 | 2.14 | 1.38 | 1.14 | 1.39 | 1.04 |
| time (sec) | N/A | 0.472 | 0.298 | 0.516 | 0.028 | 0.091 | 14.929 | 0.115 | 0.169 | 1.908 |

| Problem 213 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 114 | 145 | 102 | 97 | 128 | 226 | 194 | 141 | 187 | 162 |
| N.S. | 1 | 1.27 | 0.89 | 0.85 | 1.12 | 1.98 | 1.70 | 1.24 | 1.64 | 1.42 |
| time (sec) | N/A | 0.482 | 0.331 | 0.516 | 0.036 | 0.104 | 37.967 | 0.121 | 0.173 | 1.991 |

| Problem 214 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | A | B | B |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 147 | 182 | 126 | 116 | 193 | 265 | 303 | 214 | 267 | 202 |
| N.S. | 1 | 1.24 | 0.86 | 0.79 | 1.31 | 1.80 | 2.06 | 1.46 | 1.82 | 1.37 |
| time (sec) | N/A | 0.518 | 0.427 | 0.532 | 0.037 | 0.096 | 47.331 | 0.124 | 0.183 | 2.302 |

| Problem 215 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | B | B | B |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 197 | 218 | 160 | 148 | 275 | 344 | 444 | 356 | 327 | 277 |
| N.S. | 1 | 1.11 | 0.81 | 0.75 | 1.40 | 1.75 | 2.25 | 1.81 | 1.66 | 1.41 |
| time (sec) | N/A | 0.524 | 0.599 | 0.530 | 0.039 | 0.122 | 116.022 | 0.130 | 0.213 | 2.385 |

| Problem 216 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 248 | 222 | 218 | 168 | 339 | 413 | 243 | 219 | 304 | 0 |
| N.S. | 1 | 0.90 | 0.88 | 0.68 | 1.37 | 1.67 | 0.98 | 0.88 | 1.23 | 0.00 |
| time (sec) | N/A | 0.483 | 1.550 | 0.589 | 0.037 | 0.144 | 0.671 | 0.132 | 0.176 | 0.000 |

| Problem 217 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 196 | 192 | 162 | 136 | 255 | 330 | 199 | 171 | 245 | 0 |
| N.S. | 1 | 0.98 | 0.83 | 0.69 | 1.30 | 1.68 | 1.02 | 0.87 | 1.25 | 0.00 |
| time (sec) | N/A | 0.456 | 1.051 | 0.575 | 0.036 | 0.112 | 0.561 | 0.133 | 0.167 | 0.000 |

| Problem 218 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 146 | 162 | 129 | 107 | 174 | 249 | 155 | 126 | 186 | 0 |
| N.S. | 1 | 1.11 | 0.88 | 0.73 | 1.19 | 1.71 | 1.06 | 0.86 | 1.27 | 0.00 |
| time (sec) | N/A | 0.352 | 0.548 | 0.542 | 0.034 | 0.102 | 0.470 | 0.142 | 0.160 | 0.000 |

| Problem 219 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 117 | 132 | 101 | 96 | 118 | 218 | 246 | 114 | 178 | 0 |
| N.S. | 1 | 1.13 | 0.86 | 0.82 | 1.01 | 1.86 | 2.10 | 0.97 | 1.52 | 0.00 |
| time (sec) | N/A | 0.371 | 0.313 | 0.550 | 0.036 | 0.088 | 0.865 | 0.143 | 0.162 | 0.000 |

| Problem 220 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 110 | 120 | 95 | 94 | 102 | 207 | 196 | 170 | 137 | 149 |
| N.S. | 1 | 1.09 | 0.86 | 0.85 | 0.93 | 1.88 | 1.78 | 1.55 | 1.25 | 1.35 |
| time (sec) | N/A | 0.363 | 0.318 | 0.587 | 0.034 | 0.089 | 1.164 | 0.139 | 0.161 | 2.275 |

| Problem 221 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 118 | 136 | 98 | 97 | 128 | 219 | 427 | 319 | 125 | 105 |
| N.S. | 1 | 1.15 | 0.83 | 0.82 | 1.08 | 1.86 | 3.62 | 2.70 | 1.06 | 0.89 |
| time (sec) | N/A | 0.388 | 0.369 | 0.539 | 0.027 | 0.091 | 1.504 | 0.140 | 0.164 | 1.860 |

| Problem 222 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 141 | 157 | 103 | 92 | 193 | 99 | 891 | 548 | 153 | 163 |
| N.S. | 1 | 1.11 | 0.73 | 0.65 | 1.37 | 0.70 | 6.32 | 3.89 | 1.09 | 1.16 |
| time (sec) | N/A | 0.485 | 0.335 | 0.520 | 0.038 | 0.101 | 2.097 | 0.136 | 0.166 | 1.881 |

| Problem 223 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | B | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 191 | 187 | 134 | 123 | 275 | 141 | 1642 | 660 | 213 | 0 |
| N.S. | 1 | 0.98 | 0.70 | 0.64 | 1.44 | 0.74 | 8.60 | 3.46 | 1.12 | 0.00 |
| time (sec) | N/A | 0.506 | 0.489 | 0.557 | 0.037 | 0.124 | 3.226 | 0.144 | 0.165 | 0.000 |

| Problem 224 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 215 | 219 | 155 | 144 | 347 | 188 | 442 | 282 | 168 | 0 |
| N.S. | 1 | 1.02 | 0.72 | 0.67 | 1.61 | 0.87 | 2.06 | 1.31 | 0.78 | 0.00 |
| time (sec) | N/A | 0.557 | 0.205 | 0.503 | 0.038 | 0.088 | 0.497 | 0.125 | 0.160 | 0.000 |

| Problem 225 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 164 | 170 | 122 | 113 | 262 | 146 | 337 | 211 | 133 | 0 |
| N.S. | 1 | 1.04 | 0.74 | 0.69 | 1.60 | 0.89 | 2.05 | 1.29 | 0.81 | 0.00 |
| time (sec) | N/A | 0.485 | 0.163 | 0.514 | 0.033 | 0.087 | 0.441 | 0.135 | 0.157 | 0.000 |

| Problem 226 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 119 | 124 | 92 | 82 | 180 | 106 | 235 | 140 | 98 | 0 |
| N.S. | 1 | 1.04 | 0.77 | 0.69 | 1.51 | 0.89 | 1.97 | 1.18 | 0.82 | 0.00 |
| time (sec) | N/A | 0.393 | 0.120 | 0.500 | 0.036 | 0.084 | 0.312 | 0.131 | 0.156 | 0.000 |

| Problem 227 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 108 | 132 | 96 | 98 | 136 | 284 | 141 | 120 | 224 | 109 |
| N.S. | 1 | 1.22 | 0.89 | 0.91 | 1.26 | 2.63 | 1.31 | 1.11 | 2.07 | 1.01 |
| time (sec) | N/A | 0.450 | 0.233 | 0.631 | 0.039 | 0.093 | 12.977 | 0.130 | 0.162 | 1.943 |

| Problem 228 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 119 | 119 | 112 | 114 | 134 | 320 | 333 | 166 | 235 | 128 |
| N.S. | 1 | 1.00 | 0.94 | 0.96 | 1.13 | 2.69 | 2.80 | 1.39 | 1.97 | 1.08 |
| time (sec) | N/A | 0.521 | 0.364 | 0.649 | 0.036 | 0.097 | 40.668 | 0.127 | 0.171 | 2.012 |

| Problem 229 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 141 | 159 | 128 | 118 | 177 | 367 | 394 | 160 | 371 | 183 |
| N.S. | 1 | 1.13 | 0.91 | 0.84 | 1.26 | 2.60 | 2.79 | 1.13 | 2.63 | 1.30 |
| time (sec) | N/A | 0.541 | 0.420 | 0.533 | 0.037 | 0.109 | 80.285 | 0.130 | 0.169 | 2.179 |

| Problem 230 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | A | B | B |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 188 | 219 | 159 | 152 | 259 | 440 | 500 | 253 | 536 | 267 |
| N.S. | 1 | 1.16 | 0.85 | 0.81 | 1.38 | 2.34 | 2.66 | 1.35 | 2.85 | 1.42 |
| time (sec) | N/A | 0.644 | 0.578 | 0.532 | 0.035 | 0.141 | 134.279 | 0.131 | 0.195 | 2.454 |

| Problem 231 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F(-1) | A | B | F(-1) |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 238 | 279 | 193 | 182 | 345 | 528 | 0 | 384 | 603 | 0 |
| N.S. | 1 | 1.17 | 0.81 | 0.76 | 1.45 | 2.22 | 0.00 | 1.61 | 2.53 | 0.00 |
| time (sec) | N/A | 0.733 | 0.671 | 0.680 | 0.074 | 0.145 | 0.000 | 0.126 | 0.240 | 0.000 |

| Problem 232 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 238 | 227 | 193 | 175 | 337 | 512 | 466 | 216 | 475 | 0 |
| N.S. | 1 | 0.95 | 0.81 | 0.74 | 1.42 | 2.15 | 1.96 | 0.91 | 2.00 | 0.00 |
| time (sec) | N/A | 0.579 | 1.467 | 0.727 | 0.039 | 0.126 | 98.008 | 0.133 | 0.187 | 0.000 |

| Problem 233 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 188 | 198 | 159 | 142 | 251 | 419 | 348 | 167 | 412 | 0 |
| N.S. | 1 | 1.05 | 0.85 | 0.76 | 1.34 | 2.23 | 1.85 | 0.89 | 2.19 | 0.00 |
| time (sec) | N/A | 0.550 | 0.999 | 0.583 | 0.036 | 0.106 | 22.479 | 0.136 | 0.164 | 0.000 |

| Problem 234 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 137 | 149 | 120 | 114 | 171 | 352 | 238 | 134 | 313 | 0 |
| N.S. | 1 | 1.09 | 0.88 | 0.83 | 1.25 | 2.57 | 1.74 | 0.98 | 2.28 | 0.00 |
| time (sec) | N/A | 0.399 | 0.407 | 0.539 | 0.036 | 0.101 | 8.512 | 0.134 | 0.163 | 0.000 |

| Problem 235 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 123 | 128 | 114 | 115 | 132 | 305 | 184 | 127 | 285 | 0 |
| N.S. | 1 | 1.04 | 0.93 | 0.93 | 1.07 | 2.48 | 1.50 | 1.03 | 2.32 | 0.00 |
| time (sec) | N/A | 0.405 | 0.385 | 0.652 | 0.036 | 0.092 | 7.551 | 0.144 | 0.165 | 0.000 |

| Problem 236 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 123 | 138 | 112 | 117 | 137 | 293 | 345 | 224 | 234 | 130 |
| N.S. | 1 | 1.12 | 0.91 | 0.95 | 1.11 | 2.38 | 2.80 | 1.82 | 1.90 | 1.06 |
| time (sec) | N/A | 0.403 | 0.390 | 0.782 | 0.036 | 0.097 | 6.580 | 0.142 | 0.162 | 1.650 |

| Problem 237 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 134 | 151 | 103 | 92 | 184 | 112 | 663 | 427 | 209 | 141 |
| N.S. | 1 | 1.13 | 0.77 | 0.69 | 1.37 | 0.84 | 4.95 | 3.19 | 1.56 | 1.05 |
| time (sec) | N/A | 0.467 | 0.489 | 0.527 | 0.036 | 0.092 | 8.608 | 0.149 | 0.162 | 1.795 |

| Problem 238 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 183 | 179 | 136 | 125 | 267 | 151 | 1316 | 776 | 256 | 221 |
| N.S. | 1 | 0.98 | 0.74 | 0.68 | 1.46 | 0.83 | 7.19 | 4.24 | 1.40 | 1.21 |
| time (sec) | N/A | 0.514 | 0.603 | 0.543 | 0.034 | 0.112 | 12.356 | 0.153 | 0.169 | 2.097 |

| Problem 239 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | B | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 235 | 209 | 168 | 156 | 351 | 195 | 2134 | 1001 | 320 | 0 |
| N.S. | 1 | 0.89 | 0.71 | 0.66 | 1.49 | 0.83 | 9.08 | 4.26 | 1.36 | 0.00 |
| time (sec) | N/A | 0.546 | 0.679 | 0.577 | 0.035 | 0.160 | 17.365 | 0.159 | 0.176 | 0.000 |

| Problem 240 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | B | A | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 289 | 245 | 215 | 201 | 986 | 816 | 9649 | 265 | 32 | 0 |
| N.S. | 1 | 0.85 | 0.74 | 0.70 | 3.41 | 2.82 | 33.39 | 0.92 | 0.11 | 0.00 |
| time (sec) | N/A | 0.579 | 1.416 | 0.658 | 0.064 | 0.198 | 128.106 | 0.132 | 200.015 | 0.000 |

| Problem 241 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | B | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 249 | 231 | 178 | 173 | 753 | 653 | 6467 | 203 | 728 | 0 |
| N.S. | 1 | 0.93 | 0.71 | 0.69 | 3.02 | 2.62 | 25.97 | 0.82 | 2.92 | 0.00 |
| time (sec) | N/A | 0.633 | 1.453 | 0.760 | 0.058 | 0.160 | 86.914 | 0.147 | 8.849 | 0.000 |

| Problem 242 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | B | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 219 | 221 | 147 | 146 | 533 | 491 | 3803 | 160 | 519 | 0 |
| N.S. | 1 | 1.01 | 0.67 | 0.67 | 2.43 | 2.24 | 17.37 | 0.73 | 2.37 | 0.00 |
| time (sec) | N/A | 0.589 | 0.765 | 0.578 | 0.062 | 0.142 | 74.112 | 0.139 | 0.319 | 0.000 |

| Problem 243 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 191 | 184 | 99 | 89 | 335 | 141 | 2088 | 131 | 351 | 222 |
| N.S. | 1 | 0.96 | 0.52 | 0.47 | 1.75 | 0.74 | 10.93 | 0.69 | 1.84 | 1.16 |
| time (sec) | N/A | 0.556 | 0.499 | 0.541 | 0.036 | 0.108 | 62.264 | 0.135 | 0.207 | 1.481 |

| Problem 244 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 217 | 186 | 131 | 123 | 313 | 182 | 2392 | 211 | 414 | 243 |
| N.S. | 1 | 0.86 | 0.60 | 0.57 | 1.44 | 0.84 | 11.02 | 0.97 | 1.91 | 1.12 |
| time (sec) | N/A | 0.503 | 0.596 | 0.572 | 0.040 | 0.124 | 115.872 | 0.139 | 0.166 | 1.620 |

| Problem 245 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F(-1) | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 246 | 216 | 162 | 154 | 337 | 225 | 0 | 349 | 474 | 294 |
| N.S. | 1 | 0.88 | 0.66 | 0.63 | 1.37 | 0.91 | 0.00 | 1.42 | 1.93 | 1.20 |
| time (sec) | N/A | 0.562 | 0.657 | 0.584 | 0.041 | 0.167 | 0.000 | 0.146 | 0.179 | 1.572 |

| Problem 246 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F(-1) | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 277 | 233 | 203 | 185 | 398 | 270 | 0 | 592 | 521 | 541 |
| N.S. | 1 | 0.84 | 0.73 | 0.67 | 1.44 | 0.97 | 0.00 | 2.14 | 1.88 | 1.95 |
| time (sec) | N/A | 0.547 | 0.761 | 0.585 | 0.041 | 0.240 | 0.000 | 0.142 | 0.194 | 1.678 |

| Problem 247 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F(-1) | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 330 | 261 | 231 | 218 | 489 | 311 | 0 | 938 | 585 | 522 |
| N.S. | 1 | 0.79 | 0.70 | 0.66 | 1.48 | 0.94 | 0.00 | 2.84 | 1.77 | 1.58 |
| time (sec) | N/A | 0.588 | 0.858 | 0.941 | 0.048 | 0.326 | 0.000 | 0.148 | 5.335 | 2.490 |

| Problem 248 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F(-1) | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 388 | 289 | 269 | 249 | 579 | 354 | 0 | 1162 | 647 | 594 |
| N.S. | 1 | 0.74 | 0.69 | 0.64 | 1.49 | 0.91 | 0.00 | 2.99 | 1.67 | 1.53 |
| time (sec) | N/A | 0.627 | 0.996 | 0.674 | 0.070 | 0.534 | 0.000 | 0.161 | 0.263 | 3.120 |

| Problem 249 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 216 | 221 | 158 | 142 | 347 | 177 | 442 | 260 | 159 | 245 |
| N.S. | 1 | 1.02 | 0.73 | 0.66 | 1.61 | 0.82 | 2.05 | 1.20 | 0.74 | 1.13 |
| time (sec) | N/A | 0.589 | 0.057 | 0.606 | 0.037 | 0.083 | 0.445 | 0.127 | 0.156 | 1.272 |

| Problem 250 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 168 | 172 | 122 | 111 | 263 | 135 | 340 | 192 | 124 | 206 |
| N.S. | 1 | 1.02 | 0.73 | 0.66 | 1.57 | 0.80 | 2.02 | 1.14 | 0.74 | 1.23 |
| time (sec) | N/A | 0.501 | 0.035 | 0.604 | 0.034 | 0.081 | 0.368 | 0.129 | 0.153 | 1.366 |

| Problem 251 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 121 | 126 | 92 | 79 | 180 | 94 | 238 | 128 | 89 | 132 |
| N.S. | 1 | 1.04 | 0.76 | 0.65 | 1.49 | 0.78 | 1.97 | 1.06 | 0.74 | 1.09 |
| time (sec) | N/A | 0.416 | 0.030 | 0.617 | 0.036 | 0.083 | 0.310 | 0.120 | 0.157 | 1.976 |

| Problem 252 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | A | B | B | B | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 204 | 204 | 159 | 1335 | 322 | 962 | 7201 | 1895 | 1072 | 0 |
| N.S. | 1 | 1.00 | 0.78 | 6.54 | 1.58 | 4.72 | 35.30 | 9.29 | 5.25 | 0.00 |
| time (sec) | N/A | 0.515 | 1.299 | 0.553 | 0.070 | 0.087 | 1.094 | 0.154 | 0.157 | 0.000 |

| Problem 253 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | A | B | B | B | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 155 | 155 | 117 | 793 | 230 | 575 | 4068 | 1152 | 661 | 0 |
| N.S. | 1 | 1.00 | 0.75 | 5.12 | 1.48 | 3.71 | 26.25 | 7.43 | 4.26 | 0.00 |
| time (sec) | N/A | 0.421 | 0.957 | 0.506 | 0.059 | 0.086 | 0.866 | 0.149 | 0.151 | 0.000 |

| Problem 254 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | A | B | B | B | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 106 | 106 | 75 | 371 | 138 | 272 | 1911 | 569 | 338 | 0 |
| N.S. | 1 | 1.00 | 0.71 | 3.50 | 1.30 | 2.57 | 18.03 | 5.37 | 3.19 | 0.00 |
| time (sec) | N/A | 0.330 | 0.508 | 0.117 | 0.046 | 0.084 | 0.609 | 0.132 | 0.154 | 0.000 |

| Problem 255 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | B | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 69 | 69 | 45 | 68 | 66 | 117 | 624 | 214 | 137 | 0 |
| N.S. | 1 | 1.00 | 0.65 | 0.99 | 0.96 | 1.70 | 9.04 | 3.10 | 1.99 | 0.00 |
| time (sec) | N/A | 0.256 | 0.069 | 0.070 | 0.040 | 0.077 | 0.450 | 0.125 | 0.152 | 0.000 |

| Problem 256 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | C | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 155 | 155 | 123 | 0 | 0 | 0 | 405 | 0 | 417 | 0 |
| N.S. | 1 | 1.00 | 0.79 | 0.00 | 0.00 | 0.00 | 2.61 | 0.00 | 2.69 | 0.00 |
| time (sec) | N/A | 0.382 | 0.900 | 0.000 | 0.000 | 0.000 | 4.555 | 0.000 | 0.155 | 0.000 |

| Problem 257 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | C | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 186 | 195 | 146 | 0 | 0 | 0 | 2062 | 0 | 0 | 0 |
| N.S. | 1 | 1.05 | 0.78 | 0.00 | 0.00 | 0.00 | 11.09 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.529 | 1.104 | 0.000 | 0.000 | 0.000 | 38.211 | 0.000 | 0.164 | 0.000 |

| Problem 258 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | C | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 242 | 219 | 154 | 0 | 0 | 0 | 6484 | 0 | 0 | 0 |
| N.S. | 1 | 0.90 | 0.64 | 0.00 | 0.00 | 0.00 | 26.79 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.556 | 1.300 | 0.000 | 0.000 | 0.000 | 107.428 | 0.000 | 0.172 | 0.000 |

| Problem 259 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | C | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 295 | 302 | 179 | 0 | 0 | 0 | 495 | 0 | 145 | 0 |
| N.S. | 1 | 1.02 | 0.61 | 0.00 | 0.00 | 0.00 | 1.68 | 0.00 | 0.49 | 0.00 |
| time (sec) | N/A | 0.591 | 2.041 | 0.000 | 0.000 | 0.000 | 17.089 | 0.000 | 7.140 | 0.000 |

| Problem 260 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | C | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 294 | 301 | 178 | 0 | 0 | 0 | 243 | 0 | 78 | 0 |
| N.S. | 1 | 1.02 | 0.61 | 0.00 | 0.00 | 0.00 | 0.83 | 0.00 | 0.27 | 0.00 |
| time (sec) | N/A | 0.597 | 1.319 | 0.000 | 0.000 | 0.000 | 5.937 | 0.000 | 1.046 | 0.000 |

| Problem 261 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | C | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 294 | 301 | 178 | 0 | 0 | 0 | 236 | 0 | 86 | 0 |
| N.S. | 1 | 1.02 | 0.61 | 0.00 | 0.00 | 0.00 | 0.80 | 0.00 | 0.29 | 0.00 |
| time (sec) | N/A | 0.578 | 2.539 | 0.000 | 0.000 | 0.000 | 4.575 | 0.000 | 0.372 | 0.000 |

| Problem 262 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | C | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 290 | 280 | 181 | 0 | 0 | 0 | 236 | 0 | 150 | 0 |
| N.S. | 1 | 0.97 | 0.62 | 0.00 | 0.00 | 0.00 | 0.81 | 0.00 | 0.52 | 0.00 |
| time (sec) | N/A | 0.629 | 2.402 | 0.000 | 0.000 | 0.000 | 23.206 | 0.000 | 0.325 | 0.000 |

| Problem 263 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 301 | 316 | 181 | 0 | 0 | 0 | 0 | 0 | 226 | 0 |
| N.S. | 1 | 1.05 | 0.60 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.75 | 0.00 |
| time (sec) | N/A | 0.724 | 3.025 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.458 | 0.000 |

| Problem 264 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 278 | 316 | 181 | 0 | 0 | 0 | 0 | 0 | 302 | 0 |
| N.S. | 1 | 1.14 | 0.65 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 1.09 | 0.00 |
| time (sec) | N/A | 0.732 | 2.991 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.709 | 0.000 |

| Problem 265 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 346 | 350 | 176 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.01 | 0.51 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.711 | 0.322 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.224 | 0.000 |

| Problem 266 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | B | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 285 | 285 | 205 | 195 | 826 | 705 | 6987 | 224 | 861 | 0 |
| N.S. | 1 | 1.00 | 0.72 | 0.68 | 2.90 | 2.47 | 24.52 | 0.79 | 3.02 | 0.00 |
| time (sec) | N/A | 1.208 | 1.639 | 0.879 | 0.060 | 0.193 | 92.603 | 0.137 | 0.193 | 0.000 |

| Problem 267 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | B | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 251 | 269 | 176 | 177 | 597 | 567 | 5071 | 204 | 660 | 0 |
| N.S. | 1 | 1.07 | 0.70 | 0.71 | 2.38 | 2.26 | 20.20 | 0.81 | 2.63 | 0.00 |
| time (sec) | N/A | 0.883 | 0.862 | 1.123 | 0.060 | 0.199 | 73.193 | 0.139 | 0.169 | 0.000 |

| Problem 268 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 253 | 246 | 138 | 131 | 421 | 187 | 2490 | 220 | 512 | 0 |
| N.S. | 1 | 0.97 | 0.55 | 0.52 | 1.66 | 0.74 | 9.84 | 0.87 | 2.02 | 0.00 |
| time (sec) | N/A | 0.696 | 0.652 | 0.625 | 0.043 | 0.135 | 106.666 | 0.143 | 0.170 | 0.000 |

| Problem 269 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F(-1) | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 286 | 257 | 179 | 166 | 426 | 240 | 0 | 367 | 599 | 0 |
| N.S. | 1 | 0.90 | 0.63 | 0.58 | 1.49 | 0.84 | 0.00 | 1.28 | 2.09 | 0.00 |
| time (sec) | N/A | 0.815 | 0.727 | 0.631 | 0.047 | 0.209 | 0.000 | 0.145 | 0.180 | 0.000 |

| Problem 270 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F(-1) | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 324 | 284 | 219 | 203 | 471 | 289 | 0 | 618 | 678 | 478 |
| N.S. | 1 | 0.88 | 0.68 | 0.63 | 1.45 | 0.89 | 0.00 | 1.91 | 2.09 | 1.48 |
| time (sec) | N/A | 0.775 | 0.890 | 0.648 | 0.043 | 0.302 | 0.000 | 0.148 | 0.200 | 2.472 |

## 2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [135] had the largest ratio of [.5625000000000000]

Table 2.1: Rubi specific breakdown of results for each integral

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 1 | A | 11 | 10 | 1.27 | 25 | 0.400 |
| 2 | A | 11 | 10 | 1.03 | 25 | 0.400 |
| 3 | A | 7 | 7 | 1.14 | 25 | 0.280 |
| 4 | A | 6 | 6 | 1.57 | 25 | 0.240 |
| 5 | A | 7 | 7 | 1.21 | 25 | 0.280 |
| 6 | A | 7 | 7 | 1.05 | 25 | 0.280 |
| 7 | A | 5 | 5 | 1.16 | 23 | 0.217 |
| 8 | A | 6 | 6 | 0.95 | 22 | 0.273 |
| 9 | A | 12 | 11 | 1.13 | 25 | 0.440 |
| 10 | A | 13 | 12 | 1.02 | 25 | 0.480 |
| 11 | A | 15 | 14 | 1.07 | 25 | 0.560 |
| 12 | A | 2 | 2 | 1.00 | 26 | 0.077 |
| 13 | A | 2 | 2 | 1.00 | 26 | 0.077 |
| 14 | A | 2 | 2 | 1.00 | 24 | 0.083 |
| 15 | A | 2 | 2 | 1.00 | 23 | 0.087 |
| 16 | A | 2 | 2 | 1.00 | 26 | 0.077 |
| 17 | A | 2 | 2 | 1.00 | 26 | 0.077 |
| 18 | A | 2 | 2 | 1.00 | 26 | 0.077 |
| 19 | A | 2 | 2 | 1.00 | 26 | 0.077 |
| 20 | A | 2 | 2 | 1.00 | 28 | 0.071 |
| 21 | A | 2 | 2 | 1.00 | 28 | 0.071 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 22 | A | 3 | 3 | 1.00 | 26 | 0.115 |
| 23 | A | 3 | 3 | 1.00 | 25 | 0.120 |
| 24 | A | 3 | 3 | 1.00 | 28 | 0.107 |
| 25 | A | 3 | 3 | 1.00 | 28 | 0.107 |
| 26 | A | 2 | 2 | 1.00 | 28 | 0.071 |
| 27 | A | 2 | 2 | 1.00 | 28 | 0.071 |
| 28 | A | 2 | 2 | 1.00 | 28 | 0.071 |
| 29 | A | 2 | 2 | 1.00 | 28 | 0.071 |
| 30 | A | 3 | 3 | 1.00 | 26 | 0.115 |
| 31 | A | 3 | 3 | 1.00 | 25 | 0.120 |
| 32 | A | 3 | 3 | 1.00 | 28 | 0.107 |
| 33 | A | 3 | 3 | 1.00 | 28 | 0.107 |
| 34 | A | 2 | 2 | 1.00 | 28 | 0.071 |
| 35 | A | 2 | 2 | 1.00 | 28 | 0.071 |
| 36 | A | 2 | 2 | 1.00 | 28 | 0.071 |
| 37 | A | 2 | 2 | 1.00 | 28 | 0.071 |
| 38 | A | 2 | 2 | 1.00 | 26 | 0.077 |
| 39 | A | 2 | 2 | 1.00 | 25 | 0.080 |
| 40 | A | 2 | 2 | 1.00 | 28 | 0.071 |
| 41 | A | 2 | 2 | 1.00 | 28 | 0.071 |
| 42 | A | 2 | 2 | 1.00 | 28 | 0.071 |
| 43 | A | 2 | 2 | 1.00 | 28 | 0.071 |
| 44 | A | 2 | 2 | 1.00 | 28 | 0.071 |
| 45 | A | 4 | 4 | 1.15 | 28 | 0.143 |
| 46 | A | 4 | 4 | 1.19 | 28 | 0.143 |
| 47 | A | 4 | 4 | 1.14 | 28 | 0.143 |
| 48 | A | 4 | 4 | 1.18 | 26 | 0.154 |
| 49 | A | 6 | 6 | 1.06 | 25 | 0.240 |
| 50 | A | 5 | 5 | 1.09 | 28 | 0.179 |
| 51 | A | 4 | 4 | 1.04 | 28 | 0.143 |
| 52 | A | 4 | 4 | 1.04 | 28 | 0.143 |
| 53 | A | 6 | 6 | 1.18 | 28 | 0.214 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 54 | A | 7 | 7 | 1.23 | 28 | 0.250 |
| 55 | A | 8 | 8 | 1.10 | 28 | 0.286 |
| 56 | A | 6 | 6 | 1.21 | 26 | 0.231 |
| 57 | A | 5 | 5 | 1.09 | 25 | 0.200 |
| 58 | A | 7 | 7 | 1.11 | 28 | 0.250 |
| 59 | A | 6 | 6 | 1.09 | 28 | 0.214 |
| 60 | A | 6 | 6 | 1.11 | 28 | 0.214 |
| 61 | A | 14 | 13 | 1.07 | 30 | 0.433 |
| 62 | A | 12 | 11 | 1.07 | 30 | 0.367 |
| 63 | A | 10 | 9 | 1.04 | 28 | 0.321 |
| 64 | A | 8 | 7 | 1.04 | 27 | 0.259 |
| 65 | A | 11 | 10 | 1.04 | 30 | 0.333 |
| 66 | A | 12 | 11 | 1.04 | 30 | 0.367 |
| 67 | A | 14 | 13 | 0.98 | 30 | 0.433 |
| 68 | A | 13 | 12 | 1.01 | 30 | 0.400 |
| 69 | A | 13 | 12 | 1.05 | 30 | 0.400 |
| 70 | A | 10 | 9 | 1.00 | 30 | 0.300 |
| 71 | A | 12 | 11 | 1.02 | 30 | 0.367 |
| 72 | A | 14 | 13 | 1.05 | 30 | 0.433 |
| 73 | A | 16 | 15 | 1.06 | 30 | 0.500 |
| 74 | A | 16 | 15 | 1.02 | 30 | 0.500 |
| 75 | A | 12 | 11 | 1.01 | 30 | 0.367 |
| 76 | A | 10 | 9 | 0.98 | 28 | 0.321 |
| 77 | A | 9 | 8 | 0.98 | 27 | 0.296 |
| 78 | A | 13 | 12 | 1.03 | 30 | 0.400 |
| 79 | A | 13 | 12 | 1.07 | 30 | 0.400 |
| 80 | A | 15 | 14 | 1.05 | 30 | 0.467 |
| 81 | A | 15 | 14 | 1.00 | 30 | 0.467 |
| 82 | A | 16 | 15 | 1.02 | 30 | 0.500 |
| 83 | A | 15 | 14 | 1.02 | 30 | 0.467 |
| 84 | A | 15 | 14 | 1.04 | 30 | 0.467 |
| 85 | A | 11 | 10 | 0.97 | 30 | 0.333 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 86 | A | 13 | 12 | 0.98 | 30 | 0.400 |
| 87 | A | 15 | 14 | 1.01 | 30 | 0.467 |
| 88 | A | 17 | 16 | 1.03 | 30 | 0.533 |
| 89 | A | 14 | 13 | 1.15 | 30 | 0.433 |
| 90 | A | 13 | 12 | 1.15 | 30 | 0.400 |
| 91 | A | 11 | 10 | 1.16 | 30 | 0.333 |
| 92 | A | 8 | 7 | 1.14 | 28 | 0.250 |
| 93 | A | 7 | 6 | 1.13 | 27 | 0.222 |
| 94 | A | 10 | 9 | 1.04 | 30 | 0.300 |
| 95 | A | 11 | 10 | 1.10 | 30 | 0.333 |
| 96 | A | 11 | 10 | 1.04 | 30 | 0.333 |
| 97 | A | 9 | 8 | 1.05 | 30 | 0.267 |
| 98 | A | 11 | 10 | 1.08 | 30 | 0.333 |
| 99 | A | 13 | 12 | 1.09 | 30 | 0.400 |
| 100 | A | 11 | 10 | 1.28 | 30 | 0.333 |
| 101 | A | 10 | 9 | 1.28 | 30 | 0.300 |
| 102 | A | 7 | 6 | 1.27 | 28 | 0.214 |
| 103 | A | 7 | 6 | 1.06 | 27 | 0.222 |
| 104 | A | 10 | 9 | 1.11 | 30 | 0.300 |
| 105 | A | 7 | 6 | 1.05 | 30 | 0.200 |
| 106 | A | 9 | 8 | 1.10 | 30 | 0.267 |
| 107 | A | 11 | 10 | 1.10 | 30 | 0.333 |
| 108 | A | 13 | 12 | 1.07 | 30 | 0.400 |
| 109 | A | 12 | 11 | 1.24 | 30 | 0.367 |
| 110 | A | 9 | 8 | 1.29 | 30 | 0.267 |
| 111 | A | 9 | 8 | 1.16 | 30 | 0.267 |
| 112 | A | 7 | 6 | 1.20 | 28 | 0.214 |
| 113 | A | 4 | 4 | 1.00 | 27 | 0.148 |
| 114 | A | 9 | 8 | 1.06 | 30 | 0.267 |
| 115 | A | 9 | 8 | 0.96 | 30 | 0.267 |
| 116 | A | 11 | 10 | 1.14 | 30 | 0.333 |
| 117 | A | 2 | 2 | 1.00 | 30 | 0.067 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | number of rules / integrand leaf size |
|---|---|---|---|---|---|---|
| 118 | A | 2 | 2 | 1.00 | 30 | 0.067 |
| 119 | A | 2 | 2 | 1.00 | 30 | 0.067 |
| 120 | A | 2 | 2 | 1.00 | 30 | 0.067 |
| 121 | A | 2 | 2 | 1.00 | 30 | 0.067 |
| 122 | A | 2 | 2 | 1.30 | 32 | 0.062 |
| 123 | A | 2 | 2 | 1.32 | 32 | 0.062 |
| 124 | A | 2 | 2 | 1.34 | 32 | 0.062 |
| 125 | A | 2 | 2 | 1.37 | 32 | 0.062 |
| 126 | A | 2 | 2 | 1.46 | 32 | 0.062 |
| 127 | A | 2 | 2 | 1.55 | 32 | 0.062 |
| 128 | A | 2 | 2 | 1.51 | 32 | 0.062 |
| 129 | A | 4 | 4 | 1.24 | 32 | 0.125 |
| 130 | A | 4 | 4 | 1.27 | 32 | 0.125 |
| 131 | A | 4 | 4 | 1.31 | 32 | 0.125 |
| 132 | A | 13 | 12 | 1.11 | 32 | 0.375 |
| 133 | A | 15 | 14 | 1.18 | 32 | 0.438 |
| 134 | A | 17 | 16 | 1.16 | 32 | 0.500 |
| 135 | A | 19 | 18 | 1.17 | 32 | 0.562 |
| 136 | A | 2 | 2 | 1.00 | 30 | 0.067 |
| 137 | A | 2 | 2 | 1.00 | 28 | 0.071 |
| 138 | A | 2 | 2 | 1.00 | 21 | 0.095 |
| 139 | A | 2 | 2 | 1.00 | 30 | 0.067 |
| 140 | A | 5 | 5 | 0.93 | 30 | 0.167 |
| 141 | A | 5 | 5 | 0.91 | 30 | 0.167 |
| 142 | A | 7 | 7 | 1.06 | 32 | 0.219 |
| 143 | A | 7 | 7 | 1.10 | 32 | 0.219 |
| 144 | A | 7 | 7 | 1.06 | 32 | 0.219 |
| 145 | A | 5 | 5 | 0.95 | 32 | 0.156 |
| 146 | A | 6 | 6 | 0.92 | 32 | 0.188 |
| 147 | A | 6 | 6 | 0.92 | 32 | 0.188 |
| 148 | A | 2 | 2 | 1.00 | 28 | 0.071 |
| 149 | A | 2 | 2 | 1.00 | 28 | 0.071 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 150 | A | 2 | 2 | 1.00 | 25 | 0.080 |
| 151 | A | 2 | 2 | 1.00 | 28 | 0.071 |
| 152 | A | 2 | 2 | 1.00 | 28 | 0.071 |
| 153 | A | 2 | 2 | 1.00 | 28 | 0.071 |
| 154 | A | 2 | 2 | 1.00 | 30 | 0.067 |
| 155 | A | 2 | 2 | 1.00 | 30 | 0.067 |
| 156 | A | 2 | 2 | 1.00 | 27 | 0.074 |
| 157 | A | 2 | 2 | 1.00 | 30 | 0.067 |
| 158 | A | 2 | 2 | 1.00 | 30 | 0.067 |
| 159 | A | 2 | 2 | 1.00 | 30 | 0.067 |
| 160 | A | 2 | 2 | 1.00 | 30 | 0.067 |
| 161 | A | 2 | 2 | 1.00 | 30 | 0.067 |
| 162 | A | 2 | 2 | 1.00 | 30 | 0.067 |
| 163 | A | 2 | 2 | 1.00 | 27 | 0.074 |
| 164 | A | 2 | 2 | 1.00 | 30 | 0.067 |
| 165 | A | 2 | 2 | 1.00 | 30 | 0.067 |
| 166 | A | 2 | 2 | 1.00 | 30 | 0.067 |
| 167 | A | 2 | 2 | 1.00 | 30 | 0.067 |
| 168 | A | 2 | 2 | 1.00 | 30 | 0.067 |
| 169 | A | 2 | 2 | 1.00 | 30 | 0.067 |
| 170 | A | 4 | 4 | 1.09 | 30 | 0.133 |
| 171 | A | 5 | 5 | 1.11 | 30 | 0.167 |
| 172 | A | 5 | 5 | 1.15 | 30 | 0.167 |
| 173 | A | 4 | 4 | 1.02 | 27 | 0.148 |
| 174 | A | 4 | 4 | 1.03 | 30 | 0.133 |
| 175 | A | 4 | 4 | 1.01 | 30 | 0.133 |
| 176 | A | 4 | 4 | 1.03 | 30 | 0.133 |
| 177 | A | 4 | 4 | 1.03 | 30 | 0.133 |
| 178 | A | 4 | 4 | 1.03 | 30 | 0.133 |
| 179 | A | 5 | 5 | 1.09 | 30 | 0.167 |
| 180 | A | 6 | 6 | 1.11 | 30 | 0.200 |
| 181 | A | 5 | 5 | 1.12 | 30 | 0.167 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 182 | A | 7 | 7 | 1.04 | 30 | 0.233 |
| 183 | A | 7 | 7 | 1.09 | 27 | 0.259 |
| 184 | A | 6 | 6 | 1.09 | 30 | 0.200 |
| 185 | A | 5 | 5 | 1.08 | 30 | 0.167 |
| 186 | A | 6 | 6 | 1.07 | 30 | 0.200 |
| 187 | A | 6 | 6 | 1.06 | 30 | 0.200 |
| 188 | A | 6 | 6 | 1.05 | 30 | 0.200 |
| 189 | A | 9 | 8 | 0.88 | 32 | 0.250 |
| 190 | A | 8 | 7 | 0.98 | 29 | 0.241 |
| 191 | A | 10 | 9 | 0.98 | 32 | 0.281 |
| 192 | A | 11 | 10 | 0.96 | 32 | 0.312 |
| 193 | A | 9 | 8 | 1.05 | 32 | 0.250 |
| 194 | A | 8 | 7 | 1.16 | 32 | 0.219 |
| 195 | A | 6 | 6 | 1.11 | 32 | 0.188 |
| 196 | A | 7 | 7 | 0.98 | 32 | 0.219 |
| 197 | A | 7 | 7 | 0.89 | 32 | 0.219 |
| 198 | A | 10 | 9 | 0.81 | 32 | 0.281 |
| 199 | A | 9 | 8 | 0.87 | 29 | 0.276 |
| 200 | A | 10 | 9 | 0.86 | 32 | 0.281 |
| 201 | A | 11 | 10 | 0.85 | 32 | 0.312 |
| 202 | A | 12 | 11 | 0.87 | 32 | 0.344 |
| 203 | A | 10 | 9 | 1.01 | 32 | 0.281 |
| 204 | A | 9 | 8 | 1.12 | 32 | 0.250 |
| 205 | A | 5 | 5 | 1.11 | 32 | 0.156 |
| 206 | A | 6 | 6 | 0.98 | 32 | 0.188 |
| 207 | A | 8 | 8 | 0.89 | 32 | 0.250 |
| 208 | A | 4 | 3 | 1.02 | 32 | 0.094 |
| 209 | A | 4 | 3 | 1.02 | 32 | 0.094 |
| 210 | A | 4 | 3 | 1.04 | 30 | 0.100 |
| 211 | A | 4 | 3 | 1.05 | 32 | 0.094 |
| 212 | A | 8 | 7 | 1.01 | 32 | 0.219 |
| 213 | A | 9 | 8 | 1.27 | 32 | 0.250 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 214 | A | 10 | 9 | 1.24 | 32 | 0.281 |
| 215 | A | 10 | 9 | 1.11 | 32 | 0.281 |
| 216 | A | 8 | 7 | 0.90 | 32 | 0.219 |
| 217 | A | 7 | 6 | 0.98 | 32 | 0.188 |
| 218 | A | 7 | 6 | 1.11 | 29 | 0.207 |
| 219 | A | 9 | 8 | 1.13 | 32 | 0.250 |
| 220 | A | 8 | 7 | 1.09 | 32 | 0.219 |
| 221 | A | 7 | 6 | 1.15 | 32 | 0.188 |
| 222 | A | 5 | 5 | 1.11 | 32 | 0.156 |
| 223 | A | 7 | 7 | 0.98 | 32 | 0.219 |
| 224 | A | 4 | 3 | 1.02 | 32 | 0.094 |
| 225 | A | 4 | 3 | 1.04 | 32 | 0.094 |
| 226 | A | 4 | 3 | 1.04 | 30 | 0.100 |
| 227 | A | 4 | 3 | 1.22 | 32 | 0.094 |
| 228 | A | 8 | 7 | 1.00 | 32 | 0.219 |
| 229 | A | 9 | 8 | 1.13 | 32 | 0.250 |
| 230 | A | 12 | 11 | 1.16 | 32 | 0.344 |
| 231 | A | 14 | 13 | 1.17 | 32 | 0.406 |
| 232 | A | 9 | 8 | 0.95 | 32 | 0.250 |
| 233 | A | 8 | 7 | 1.05 | 32 | 0.219 |
| 234 | A | 8 | 7 | 1.09 | 29 | 0.241 |
| 235 | A | 9 | 8 | 1.04 | 32 | 0.250 |
| 236 | A | 8 | 7 | 1.12 | 32 | 0.219 |
| 237 | A | 5 | 5 | 1.13 | 32 | 0.156 |
| 238 | A | 6 | 6 | 0.98 | 32 | 0.188 |
| 239 | A | 7 | 7 | 0.89 | 32 | 0.219 |
| 240 | A | 13 | 12 | 0.85 | 32 | 0.375 |
| 241 | A | 13 | 12 | 0.93 | 32 | 0.375 |
| 242 | A | 12 | 11 | 1.01 | 32 | 0.344 |
| 243 | A | 8 | 8 | 0.96 | 29 | 0.276 |
| 244 | A | 6 | 6 | 0.86 | 32 | 0.188 |
| 245 | A | 7 | 7 | 0.88 | 32 | 0.219 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 246 | A | 9 | 9 | 0.84 | 32 | 0.281 |
| 247 | A | 10 | 10 | 0.79 | 32 | 0.312 |
| 248 | A | 11 | 11 | 0.74 | 32 | 0.344 |
| 249 | A | 5 | 4 | 1.02 | 33 | 0.121 |
| 250 | A | 5 | 4 | 1.02 | 33 | 0.121 |
| 251 | A | 5 | 4 | 1.04 | 31 | 0.129 |
| 252 | A | 2 | 2 | 1.00 | 32 | 0.062 |
| 253 | A | 2 | 2 | 1.00 | 32 | 0.062 |
| 254 | A | 2 | 2 | 1.00 | 30 | 0.067 |
| 255 | A | 2 | 2 | 1.00 | 23 | 0.087 |
| 256 | A | 2 | 2 | 1.00 | 32 | 0.062 |
| 257 | A | 4 | 4 | 1.05 | 32 | 0.125 |
| 258 | A | 4 | 4 | 0.90 | 32 | 0.125 |
| 259 | A | 5 | 5 | 1.02 | 34 | 0.147 |
| 260 | A | 5 | 5 | 1.02 | 34 | 0.147 |
| 261 | A | 5 | 5 | 1.02 | 34 | 0.147 |
| 262 | A | 6 | 6 | 0.97 | 34 | 0.176 |
| 263 | A | 7 | 7 | 1.05 | 34 | 0.206 |
| 264 | A | 7 | 7 | 1.14 | 34 | 0.206 |
| 265 | A | 5 | 5 | 1.01 | 32 | 0.156 |
| 266 | A | 16 | 15 | 1.00 | 37 | 0.405 |
| 267 | A | 11 | 10 | 1.07 | 34 | 0.294 |
| 268 | A | 8 | 8 | 0.97 | 37 | 0.216 |
| 269 | A | 9 | 9 | 0.90 | 37 | 0.243 |
| 270 | A | 9 | 9 | 0.88 | 37 | 0.243 |

# CHAPTER 3

## LISTING OF INTEGRALS

# 3.1 $\int \frac{x^7(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 183

$$\int \frac{x^7(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = \frac{a^3(Ab-aC+bBx)}{7b^5\,(a+bx^2)^{7/2}} - \frac{a^2(7(3Ab-4aC)+22bBx)}{35b^5\,(a+bx^2)^{5/2}} + \frac{a(105(Ab-2aC)+122bBx)}{105b^5\,(a+bx^2)^{3/2}} - \frac{105(Ab-4aC)+176bBx}{105b^5\sqrt{a+bx^2}} + \frac{C\sqrt{a+bx^2}}{b^5} + \frac{B\,\mathrm{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{9/2}}$$

output
```
1/7*a^3*(B*b*x+A*b-C*a)/b^5/(b*x^2+a)^(7/2)-1/35*a^2*(22*B*b*x+21*A*b-28*C
*a)/b^5/(b*x^2+a)^(5/2)+1/105*a*(122*B*b*x+105*A*b-210*C*a)/b^5/(b*x^2+a)^
(3/2)-1/105*(176*B*b*x+105*A*b-420*C*a)/b^5/(b*x^2+a)^(1/2)+C*(b*x^2+a)^(1
/2)/b^5+B*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(9/2)
```

## Mathematica [A] (verified)

Time = 1.55 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.85

$$\int \frac{x^7(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = \frac{384a^4C-3a^3b(16A+7x(5B-64Cx))+14a^2b^2x^2(-12A+5x(-5B+24Cx}{}$$

input
```
Integrate[(x^7*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

output
```
(384*a^4*C - 3*a^3*b*(16*A + 7*x*(5*B - 64*C*x)) + 14*a^2*b^2*x^2*(-12*A +
 5*x*(-5*B + 24*C*x)) + 14*a*b^3*x^4*(-15*A + x*(-29*B + 60*C*x)) + b^4*x^
6*(-105*A + x*(-176*B + 105*C*x)) - 105*Sqrt[b]*B*(a + b*x^2)^(7/2)*Log[-(
Sqrt[b]*x) + Sqrt[a + b*x^2]])/(105*b^5*(a + b*x^2)^(7/2))
```

## Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.27, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2335, 25, 530, 25, 2345, 2345, 27, 455, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}dx$$

$$\Big\downarrow \text{2335}$$

$$-\frac{\int -\frac{x^6(7aB-(Ab-8aC)x)}{(bx^2+a)^{7/2}}dx}{7ab}-\frac{x^7(aB-x(Ab-aC))}{7ab(a+bx^2)^{7/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{x^6(7aB-(Ab-8aC)x)}{(bx^2+a)^{7/2}}dx}{7ab}-\frac{x^7(aB-x(Ab-aC))}{7ab(a+bx^2)^{7/2}}$$

$$\Big\downarrow \text{530}$$

$$-\frac{\frac{\int -\frac{-5a\left(A-\frac{8aC}{b}\right)x^5+\frac{35a^2Bx^4}{b}+\frac{5a^2(Ab-8aC)x^3}{b^2}-\frac{35a^3Bx^2}{b^2}-\frac{5a^3(Ab-8aC)x}{b^3}+\frac{7a^4B}{b^3}}{\left(bx^2+a\right)^{5/2}}dx}{5a}-\frac{a^3(-8aC+Ab+7bBx)}{5b^4(a+bx^2)^{5/2}}}{7ab}-\frac{x^7(aB-x(Ab-aC))}{7ab\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{-5a\left(A-\frac{8aC}{b}\right)x^5+\frac{35a^2Bx^4}{b}+\frac{5a^2(Ab-8aC)x^3}{b^2}-\frac{35a^3Bx^2}{b^2}-\frac{5a^3(Ab-8aC)x}{b^3}+\frac{7a^4B}{b^3}}{\left(bx^2+a\right)^{5/2}}dx}{5a}-\frac{a^3(-8aC+Ab+7bBx)}{5b^4(a+bx^2)^{5/2}}}{7ab}-\frac{x^7(aB-x(Ab-aC))}{7ab\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \; 2345$$

$$\frac{\frac{\frac{a^3(15(Ab-8aC)+77bBx)}{3b^4\left(a+bx^2\right)^{3/2}}-\frac{\int \frac{\frac{56Ba^4}{b^3}-\frac{105Bx^2a^3}{b^2}-\frac{30(Ab-8aC)xa^3}{b^3}+\frac{15(Ab-8aC)x^3a^2}{b^2}}{\left(bx^2+a\right)^{3/2}}dx}{3a}}{5a}-\frac{a^3(-8aC+Ab+7bBx)}{5b^4(a+bx^2)^{5/2}}}{7ab}-\frac{x^7(aB-x(Ab-aC))}{7ab\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \; 2345$$

$$\frac{\frac{\frac{a^3(15(Ab-8aC)+77bBx)}{3b^4\left(a+bx^2\right)^{3/2}}-\frac{\frac{a^3(45(Ab-8aC)+161bBx)}{b^4\sqrt{a+bx^2}}-\frac{\int \frac{15a^3(7aB-(Ab-8aC)x)}{b^3\sqrt{bx^2+a}}dx}{a}}{3a}}{5a}-\frac{a^3(-8aC+Ab+7bBx)}{5b^4(a+bx^2)^{5/2}}}{7ab}-\frac{x^7(aB-x(Ab-aC))}{7ab\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{a^3(15(Ab-8aC)+77bBx)}{3b^4\left(a+bx^2\right)^{3/2}}-\frac{\frac{a^3(45(Ab-8aC)+161bBx)}{b^4\sqrt{a+bx^2}}-\frac{15a^2\int \frac{7aB-(Ab-8aC)x}{\sqrt{bx^2+a}}dx}{b^3}}{3a}}{5a}-\frac{a^3(-8aC+Ab+7bBx)}{5b^4(a+bx^2)^{5/2}}}{7ab}-\frac{x^7(aB-x(Ab-aC))}{7ab\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \; 455$$

$$\frac{\frac{\frac{a^3(15(Ab-8aC)+77bBx)}{3b^4\left(a+bx^2\right)^{3/2}}-\frac{\frac{a^3(45(Ab-8aC)+161bBx)}{b^4\sqrt{a+bx^2}}-\frac{15a^2\left(7aB\int\frac{1}{\sqrt{bx^2+a}}dx-\frac{\sqrt{a+bx^2}(Ab-8aC)}{b}\right)}{b^3}}{3a}}{5a}-\frac{a^3(-8aC+Ab+7bBx)}{5b^4(a+bx^2)^{5/2}}}{7ab}-\frac{x^7(aB-x(Ab-aC))}{7ab\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \; 224$$

$$\frac{\frac{\frac{a^3(15(Ab-8aC)+77bBx)}{3b^4\left(a+bx^2\right)^{3/2}}-\frac{\frac{a^3(45(Ab-8aC)+161bBx)}{b^4\sqrt{a+bx^2}}-\frac{15a^2\left(7aB\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}-\frac{\sqrt{a+bx^2}(Ab-8aC)}{b}\right)}{b^3}}{3a}}{5a}-\frac{a^3(-8aC+Ab+7bBx)}{5b^4(a+bx^2)^{5/2}}}{7ab}-\frac{x^7(aB-x(Ab-aC))}{7ab\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{\frac{a^3(15(Ab-8aC)+77bBx)}{3b^4\left(a+bx^2\right)^{3/2}}-\frac{\frac{a^3(45(Ab-8aC)+161bBx)}{b^4\sqrt{a+bx^2}}-\frac{15a^2\left(\frac{7aB\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}-\frac{\sqrt{a+bx^2}(Ab-8aC)}{b}\right)}{b^3}}{3a}}{5a}-\frac{a^3(-8aC+Ab+7bBx)}{5b^4(a+bx^2)^{5/2}}}{7ab}-\frac{x^7(aB-x(Ab-aC))}{7ab\left(a+bx^2\right)^{7/2}}$$

input
```
Int[(x^7*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

output
```
-1/7*(x^7*(a*B - (A*b - a*C)*x))/(a*b*(a + b*x^2)^(7/2)) + (-1/5*(a^3*(A*b
 - 8*a*C + 7*b*B*x))/(b^4*(a + b*x^2)^(5/2)) + ((a^3*(15*(A*b - 8*a*C) + 7
7*b*B*x))/(3*b^4*(a + b*x^2)^(3/2)) - ((a^3*(45*(A*b - 8*a*C) + 161*b*B*x)
)/(b^4*Sqrt[a + b*x^2]) - (15*a^2*(-(((A*b - 8*a*C)*Sqrt[a + b*x^2])/b) +
(7*a*B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/Sqrt[b]))/b^3)/(3*a))/(5*a))/
(7*a*b)
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 530
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symb
ol] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Co
eff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Po
lynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x
)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(a
 + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Qx + e*(2*p + 3), x], x], x]] /;
FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && IGtQ[m, 0] && LtQ[p, -1] && EqQ[n,
1] && IntegerQ[2*p]
```

rule 2335
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Simp[c/(2*a*b*(p + 1))   Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSu
m[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
 b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

## Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.61

| method | result |
|---|---|
| default | $A\left(-\frac{x^{6}}{b\left(b\,x^{2}+a\right)^{\frac{7}{2}}}+\frac{6a\left(-\frac{x^{4}}{3b\left(b\,x^{2}+a\right)^{\frac{7}{2}}}+\frac{4a\left(-\frac{x^{2}}{5b\left(b\,x^{2}+a\right)^{\frac{7}{2}}}-\frac{2a}{35b^{2}\left(b\,x^{2}+a\right)^{\frac{7}{2}}}\right)}{3b}\right)}{b}\right)+B\left(-\frac{x^{7}}{7b\left(b\,x^{2}+a\right)^{\frac{7}{2}}}+\frac{-\frac{x^{5}}{5b\left(b\,x^{2}+a\right.}}{}\right.$ |
| risch | Expression too large to display |

input
```
int(x^7*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
A*(-x^6/b/(b*x^2+a)^(7/2)+6*a/b*(-1/3*x^4/b/(b*x^2+a)^(7/2)+4/3*a/b*(-1/5*
x^2/b/(b*x^2+a)^(7/2)-2/35*a/b^2/(b*x^2+a)^(7/2))))+B*(-1/7*x^7/b/(b*x^2+a
)^(7/2)+1/b*(-1/5*x^5/b/(b*x^2+a)^(5/2)+1/b*(-1/3*x^3/b/(b*x^2+a)^(3/2)+1/
b*(-x/b/(b*x^2+a)^(1/2)+1/b^(3/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))))))+C*(x^8
/b/(b*x^2+a)^(7/2)-8*a/b*(-x^6/b/(b*x^2+a)^(7/2)+6*a/b*(-1/3*x^4/b/(b*x^2+
a)^(7/2)+4/3*a/b*(-1/5*x^2/b/(b*x^2+a)^(7/2)-2/35*a/b^2/(b*x^2+a)^(7/2))))
)
```

## Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 522, normalized size of antiderivative = 2.85

$$\int \frac{x^7(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = \left[\frac{105\,(Bb^4x^8+4\,Bab^3x^6+6\,Ba^2b^2x^4+4\,Ba^3bx^2+Ba^4)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{\right.}}{}\right.$$

$$-\frac{105\,(Bb^4x^8+4\,Bab^3x^6+6\,Ba^2b^2x^4+4\,Ba^3bx^2+Ba^4)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(105\,Cb^4x^8-176\,Bb^4x^7}{}$$

input
```
integrate(x^7*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

output
```
[1/210*(105*(B*b^4*x^8 + 4*B*a*b^3*x^6 + 6*B*a^2*b^2*x^4 + 4*B*a^3*b*x^2 +
 B*a^4)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(105*C
*b^4*x^8 - 176*B*b^4*x^7 - 406*B*a*b^3*x^5 - 350*B*a^2*b^2*x^3 + 105*(8*C*
a*b^3 - A*b^4)*x^6 - 105*B*a^3*b*x + 384*C*a^4 - 48*A*a^3*b + 210*(8*C*a^2
*b^2 - A*a*b^3)*x^4 + 168*(8*C*a^3*b - A*a^2*b^2)*x^2)*sqrt(b*x^2 + a))/(b
^9*x^8 + 4*a*b^8*x^6 + 6*a^2*b^7*x^4 + 4*a^3*b^6*x^2 + a^4*b^5), -1/105*(1
05*(B*b^4*x^8 + 4*B*a*b^3*x^6 + 6*B*a^2*b^2*x^4 + 4*B*a^3*b*x^2 + B*a^4)*s
qrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (105*C*b^4*x^8 - 176*B*b^4*x^
7 - 406*B*a*b^3*x^5 - 350*B*a^2*b^2*x^3 + 105*(8*C*a*b^3 - A*b^4)*x^6 - 10
5*B*a^3*b*x + 384*C*a^4 - 48*A*a^3*b + 210*(8*C*a^2*b^2 - A*a*b^3)*x^4 + 1
68*(8*C*a^3*b - A*a^2*b^2)*x^2)*sqrt(b*x^2 + a))/(b^9*x^8 + 4*a*b^8*x^6 +
6*a^2*b^7*x^4 + 4*a^3*b^6*x^2 + a^4*b^5)]
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 366 vs. $2(170) = 340$.

Time = 27.14 (sec) , antiderivative size = 3806, normalized size of antiderivative = 20.80

$$\int \frac{x^7(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = \text{Too large to display}$$

input

```
integrate(x**7*(C*x**2+B*x+A)/(b*x**2+a)**(9/2),x)
```

output

```
A*Piecewise((-16*a**3/(35*a**3*b**4*sqrt(a + b*x**2) + 105*a**2*b**5*x**2*
sqrt(a + b*x**2) + 105*a*b**6*x**4*sqrt(a + b*x**2) + 35*b**7*x**6*sqrt(a
+ b*x**2)) - 56*a**2*b*x**2/(35*a**3*b**4*sqrt(a + b*x**2) + 105*a**2*b**5
*x**2*sqrt(a + b*x**2) + 105*a*b**6*x**4*sqrt(a + b*x**2) + 35*b**7*x**6*s
qrt(a + b*x**2)) - 70*a*b**2*x**4/(35*a**3*b**4*sqrt(a + b*x**2) + 105*a**
2*b**5*x**2*sqrt(a + b*x**2) + 105*a*b**6*x**4*sqrt(a + b*x**2) + 35*b**7*
x**6*sqrt(a + b*x**2)) - 35*b**3*x**6/(35*a**3*b**4*sqrt(a + b*x**2) + 105
*a**2*b**5*x**2*sqrt(a + b*x**2) + 105*a*b**6*x**4*sqrt(a + b*x**2) + 35*b
**7*x**6*sqrt(a + b*x**2)), Ne(b, 0)), (x**8/(8*a**(9/2)), True)) + B*(105
*a**(205/2)*b**45*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(105*a**(205
/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*sqrt(1 +
 b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2100*a**
(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(107/2)*x*
*8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x**2/a)
 + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) + 630*a**(203/2)*b*
*46*x**2*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(105*a**(205/2)*b**(9
9/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*sqrt(1 + b*x**2/a
) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2100*a**(199/2)*b
**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(107/2)*x**8*sqrt(1
 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x**2/a) + 105...
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 435 vs. $2(152) = 304$.

Time = 0.06 (sec) , antiderivative size = 435, normalized size of antiderivative = 2.38

$$\int \frac{x^7(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = \frac{Cx^8}{(bx^2+a)^{\frac{7}{2}}b} - \frac{1}{35}\left(\frac{35\,x^6}{(bx^2+a)^{\frac{7}{2}}b} + \frac{70\,ax^4}{(bx^2+a)^{\frac{7}{2}}b^2} + \frac{56\,a^2x^2}{(bx^2+a)^{\frac{7}{2}}b^3} + \frac{16\,a^3}{(bx^2+a)^{\frac{7}{2}}b^4}\right)Bx + \frac{8\,Cax^6}{(bx^2+a)^{\frac{7}{2}}b^2} - \frac{Ax^6}{(bx^2+a)^{\frac{7}{2}}b} - \frac{Bx\left(\frac{15\,x^4}{(bx^2+a)^{\frac{5}{2}}b} + \frac{20\,ax^2}{(bx^2+a)^{\frac{5}{2}}b^2} + \frac{8\,a^2}{(bx^2+a)^{\frac{5}{2}}b^3}\right)}{15\,b} - \frac{Bx\left(\frac{3\,x^2}{(bx^2+a)^{\frac{3}{2}}b} + \frac{2\,a}{(bx^2+a)^{\frac{3}{2}}b^2}\right)}{3\,b^2} + \frac{16\,Ca^2x^4}{(bx^2+a)^{\frac{7}{2}}b^3} - \frac{2\,Aax^4}{(bx^2+a)^{\frac{7}{2}}b^2} - \frac{Bax^3}{(bx^2+a)^{\frac{5}{2}}b^3} + \frac{64\,Ca^3x^2}{5\,(bx^2+a)^{\frac{7}{2}}b^4} - \frac{8\,Aa^2x^2}{5\,(bx^2+a)^{\frac{7}{2}}b^3} + \frac{139\,Bx}{105\,\sqrt{bx^2+a}b^4} + \frac{17\,Bax}{105\,(bx^2+a)^{\frac{3}{2}}b^4} - \frac{29\,Ba^2x}{35\,(bx^2+a)^{\frac{5}{2}}b^4} + \frac{B\,\text{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{9}{2}}} + \frac{128\,Ca^4}{35\,(bx^2+a)^{\frac{7}{2}}b^5} - \frac{16\,Aa^3}{35\,(bx^2+a)^{\frac{7}{2}}b^4}$$

input
```
integrate(x^7*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

output
```
C*x^8/((b*x^2 + a)^(7/2)*b) - 1/35*(35*x^6/((b*x^2 + a)^(7/2)*b) + 70*a*x^
4/((b*x^2 + a)^(7/2)*b^2) + 56*a^2*x^2/((b*x^2 + a)^(7/2)*b^3) + 16*a^3/((
b*x^2 + a)^(7/2)*b^4))*B*x + 8*C*a*x^6/((b*x^2 + a)^(7/2)*b^2) - A*x^6/((b
*x^2 + a)^(7/2)*b) - 1/15*B*x*(15*x^4/((b*x^2 + a)^(5/2)*b) + 20*a*x^2/((b
*x^2 + a)^(5/2)*b^2) + 8*a^2/((b*x^2 + a)^(5/2)*b^3))/b - 1/3*B*x*(3*x^2/(
(b*x^2 + a)^(3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2))/b^2 + 16*C*a^2*x^4/((b
*x^2 + a)^(7/2)*b^3) - 2*A*a*x^4/((b*x^2 + a)^(7/2)*b^2) - B*a*x^3/((b*x^2
 + a)^(5/2)*b^3) + 64/5*C*a^3*x^2/((b*x^2 + a)^(7/2)*b^4) - 8/5*A*a^2*x^2/
((b*x^2 + a)^(7/2)*b^3) + 139/105*B*x/(sqrt(b*x^2 + a)*b^4) + 17/105*B*a*x
/((b*x^2 + a)^(3/2)*b^4) - 29/35*B*a^2*x/((b*x^2 + a)^(5/2)*b^4) + B*arcsi
nh(b*x/sqrt(a*b))/b^(9/2) + 128/35*C*a^4/((b*x^2 + a)^(7/2)*b^5) - 16/35*A
*a^3/((b*x^2 + a)^(7/2)*b^4)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.11

$$\int \frac{x^7(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = \frac{\left(\left(\left(\left(\left(\left(\left(\frac{105\,Cx}{b}-\frac{176\,B}{b}\right)x+\frac{105\,(8\,Ca^4b^7-Aa^3b^8)}{a^3b^9}\right)x-\frac{406\,Ba}{b^2}\right)x+\frac{210\,(8\,Ca^5b^6-Aa^4b^7)}{a^3b^9}\right.\right.\right.\right.}{105\,(bx^2+a}$$
$$-\frac{B\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{b^{\frac{9}{2}}}$$

input
```
integrate(x^7*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="giac")
```

output
```
1/105*((((((((105*C*x/b - 176*B/b)*x + 105*(8*C*a^4*b^7 - A*a^3*b^8)/(a^3*
b^9))*x - 406*B*a/b^2)*x + 210*(8*C*a^5*b^6 - A*a^4*b^7)/(a^3*b^9))*x - 35
0*B*a^2/b^3)*x + 168*(8*C*a^6*b^5 - A*a^5*b^6)/(a^3*b^9))*x - 105*B*a^3/b^
4)*x + 48*(8*C*a^7*b^4 - A*a^6*b^5)/(a^3*b^9))/(b*x^2 + a)^(7/2) - B*log(a
bs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(9/2)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^7(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = \int \frac{x^7\,(C\,x^2+B\,x+A)}{(b\,x^2+a)^{9/2}}\,dx$$

input
```
int((x^7*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x)
```

output
```
int((x^7*(A + B*x + C*x^2))/(a + b*x^2)^(9/2), x)
```

## Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 478, normalized size of antiderivative = 2.61

$$\int \frac{x^7(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = \frac{-48\sqrt{b\,x^2+a}\,a^4b+384\sqrt{b\,x^2+a}\,a^4c-168\sqrt{b\,x^2+a}\,a^3b^2x^2-105\sqrt{b\,x^2+a}\,a}{}$$

input
```
int(x^7*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x)
```

output
```
( - 48*sqrt(a + b*x**2)*a**4*b + 384*sqrt(a + b*x**2)*a**4*c - 168*sqrt(a
+ b*x**2)*a**3*b**2*x**2 - 105*sqrt(a + b*x**2)*a**3*b**2*x + 1344*sqrt(a
+ b*x**2)*a**3*b*c*x**2 - 210*sqrt(a + b*x**2)*a**2*b**3*x**4 - 350*sqrt(a
 + b*x**2)*a**2*b**3*x**3 + 1680*sqrt(a + b*x**2)*a**2*b**2*c*x**4 - 105*s
qrt(a + b*x**2)*a*b**4*x**6 - 406*sqrt(a + b*x**2)*a*b**4*x**5 + 840*sqrt(
a + b*x**2)*a*b**3*c*x**6 - 176*sqrt(a + b*x**2)*b**5*x**7 + 105*sqrt(a +
b*x**2)*b**4*c*x**8 + 105*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(
a))*a**4*b + 420*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**3*
b**2*x**2 + 630*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2*b
**3*x**4 + 420*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a*b**4*
x**6 + 105*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*b**5*x**8 +
 56*sqrt(b)*a**4*b + 224*sqrt(b)*a**3*b**2*x**2 + 336*sqrt(b)*a**2*b**3*x*
*4 + 224*sqrt(b)*a*b**4*x**6 + 56*sqrt(b)*b**5*x**8)/(105*b**5*(a**4 + 4*a
**3*b*x**2 + 6*a**2*b**2*x**4 + 4*a*b**3*x**6 + b**4*x**8))
```

# 3.2 $\int \frac{x^6(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 172

$$\int \frac{x^6(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = \frac{a^2(aB-(Ab-aC)x)}{7b^4(a+bx^2)^{7/2}} - \frac{a(21aB-(15Ab-22aC)x)}{35b^4(a+bx^2)^{5/2}} + \frac{105aB-(45Ab-122aC)x}{105b^4(a+bx^2)^{3/2}} - \frac{105aB-(15Ab-176aC)x}{105ab^4\sqrt{a+bx^2}} + \frac{C\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{9/2}}$$

output
```
1/7*a^2*(B*a-(A*b-C*a)*x)/b^4/(b*x^2+a)^(7/2)-1/35*a*(21*B*a-(15*A*b-22*C*
a)*x)/b^4/(b*x^2+a)^(5/2)+1/105*(105*B*a-(45*A*b-122*C*a)*x)/b^4/(b*x^2+a)
^(3/2)-1/105*(105*B*a-(15*A*b-176*C*a)*x)/a/b^4/(b*x^2+a)^(1/2)+C*arctanh(
b^(1/2)*x/(b*x^2+a)^(1/2))/b^(9/2)
```

## Mathematica [A] (verified)

Time = 1.24 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.74

$$\int \frac{x^6(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = \frac{15Ab^4x^7-14a^3bx^2(12B+25Cx)-14a^2b^2x^4(15B+29Cx)-3a^4(16B+35C}{105ab^4(a+bx^2)^{7/2}} - \frac{C\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{b^{9/2}}$$

input
```
Integrate[(x^6*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

output
```
(15*A*b^4*x^7 - 14*a^3*b*x^2*(12*B + 25*C*x) - 14*a^2*b^2*x^4*(15*B + 29*C
*x) - 3*a^4*(16*B + 35*C*x) - a*b^3*x^6*(105*B + 176*C*x))/(105*a*b^4*(a +
 b*x^2)^(7/2)) - (C*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/b^(9/2)
```

## Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.03, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2335, 25, 27, 530, 25, 2345, 2345, 27, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}dx$$

$$\Big\downarrow \text{2335}$$

$$-\frac{\int -\frac{ax^5(6B+7Cx)}{(bx^2+a)^{7/2}}dx}{7ab}-\frac{x^6(aB-x(Ab-aC))}{7ab(a+bx^2)^{7/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{ax^5(6B+7Cx)}{(bx^2+a)^{7/2}}dx}{7ab}-\frac{x^6(aB-x(Ab-aC))}{7ab(a+bx^2)^{7/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{x^5(6B+7Cx)}{(bx^2+a)^{7/2}}dx}{7b}-\frac{x^6(aB-x(Ab-aC))}{7ab(a+bx^2)^{7/2}}$$

$$\Big\downarrow \text{530}$$

$$\frac{-\frac{\int -\frac{\frac{35aCx^4}{b}+\frac{30aBx^3}{b}-\frac{35a^2Cx^2}{b^2}-\frac{30a^2Bx}{b^2}+\frac{7a^3C}{b^3}}{(bx^2+a)^{5/2}}dx}{5a}-\frac{a^2(6B+7Cx)}{5b^3(a+bx^2)^{5/2}}}{7b}-\frac{x^6(aB-x(Ab-aC))}{7ab(a+bx^2)^{7/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{\frac{35aCx^4}{b}+\frac{30aBx^3}{b}-\frac{35a^2Cx^2}{b^2}-\frac{30a^2Bx}{b^2}+\frac{7a^3C}{b^3}}{(bx^2+a)^{5/2}}dx}{5a}-\frac{a^2(6B+7Cx)}{5b^3(a+bx^2)^{5/2}}}{7b}-\frac{x^6(aB-x(Ab-aC))}{7ab\,(a+bx^2)^{7/2}}$$

$$\Big\downarrow \ 2345$$

$$\frac{\frac{\frac{a^2(60B+77Cx)}{3b^3\left(a+bx^2\right)^{3/2}}-\frac{\int \frac{\frac{56Ca^3}{b^3}-\frac{105Cx^2a^2}{b^2}-\frac{90Bxa^2}{b^2}}{(bx^2+a)^{3/2}}dx}{3a}}{5a}-\frac{a^2(6B+7Cx)}{5b^3(a+bx^2)^{5/2}}}{7b}-\frac{x^6(aB-x(Ab-aC))}{7ab\,(a+bx^2)^{7/2}}$$

$$\Big\downarrow \ 2345$$

$$\frac{\frac{\frac{a^2(60B+77Cx)}{3b^3\left(a+bx^2\right)^{3/2}}-\frac{\frac{a^2(90B+161Cx)}{b^3\sqrt{a+bx^2}}-\frac{\int \frac{105a^3C}{b^3\sqrt{bx^2+a}}dx}{a}}{3a}}{5a}-\frac{a^2(6B+7Cx)}{5b^3(a+bx^2)^{5/2}}}{7b}-\frac{x^6(aB-x(Ab-aC))}{7ab\,(a+bx^2)^{7/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\frac{a^2(60B+77Cx)}{3b^3\left(a+bx^2\right)^{3/2}}-\frac{\frac{a^2(90B+161Cx)}{b^3\sqrt{a+bx^2}}-\frac{105a^2C\int \frac{1}{\sqrt{bx^2+a}}dx}{b^3}}{3a}}{5a}-\frac{a^2(6B+7Cx)}{5b^3(a+bx^2)^{5/2}}}{7b}-\frac{x^6(aB-x(Ab-aC))}{7ab\,(a+bx^2)^{7/2}}$$

$$\Big\downarrow \ 224$$

$$\frac{\frac{\frac{a^2(60B+77Cx)}{3b^3\left(a+bx^2\right)^{3/2}}-\frac{\frac{a^2(90B+161Cx)}{b^3\sqrt{a+bx^2}}-\frac{105a^2C\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{b^3}}{3a}}{5a}-\frac{a^2(6B+7Cx)}{5b^3(a+bx^2)^{5/2}}}{7b}-\frac{x^6(aB-x(Ab-aC))}{7ab\,(a+bx^2)^{7/2}}$$

$$\Big\downarrow \ 219$$

$$\frac{\frac{\frac{a^2(60B+77Cx)}{3b^3\left(a+bx^2\right)^{3/2}}-\frac{\frac{a^2(90B+161Cx)}{b^3\sqrt{a+bx^2}}-\frac{105a^2C\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{7/2}}}{3a}}{5a}-\frac{a^2(6B+7Cx)}{5b^3(a+bx^2)^{5/2}}}{7b}-\frac{x^6(aB-x(Ab-aC))}{7ab\,(a+bx^2)^{7/2}}$$

input
```
Int[(x^6*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

output
```
-1/7*(x^6*(a*B - (A*b - a*C)*x))/(a*b*(a + b*x^2)^(7/2)) + (-1/5*(a^2*(6*B
 + 7*C*x))/(b^3*(a + b*x^2)^(5/2)) + ((a^2*(60*B + 77*C*x))/(3*b^3*(a + b*
x^2)^(3/2)) - ((a^2*(90*B + 161*C*x))/(b^3*Sqrt[a + b*x^2]) - (105*a^2*C*A
rcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/b^(7/2))/(3*a))/(5*a))/(7*b)
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 530
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symb
ol] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Co
eff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Po
lynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x
)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(a
 + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Qx + e*(2*p + 3), x], x], x]] /;
FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && IGtQ[m, 0] && LtQ[p, -1] && EqQ[n,
1] && IntegerQ[2*p]
```

rule 2335
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Simp[c/(2*a*b*(p + 1))   Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSu
m[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
 b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 333 vs. $2(150) = 300$.

Time = 0.57 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.94

| method | result |
|---|---|
| default | $A\left(-\frac{x^{5}}{2b\left(b\,x^{2}+a\right)^{\frac{7}{2}}}+\frac{5a\left(-\frac{x^{3}}{4b\left(b\,x^{2}+a\right)^{\frac{7}{2}}}+\frac{3a\left(-\frac{x}{6b\left(b\,x^{2}+a\right)^{\frac{7}{2}}}+\frac{a\left(\frac{x}{7a\left(b\,x^{2}+a\right)^{\frac{7}{2}}}+\frac{\frac{6x}{35a\left(b\,x^{2}+a\right)^{\frac{5}{2}}}+\frac{6\left(\frac{4x}{15a\left(b\,x^{2}+a\right)^{\frac{3}{2}}}+\frac{8x}{15a^{2}\sqrt{b\,x^{2}\cdots}}\cdots\right.}{7a}}{a}\cdots\right.}{6b}\cdots\right.}{4b}\cdots\right.}{2b}\cdots\right.$ |

input
```
int(x^6*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
A*(-1/2*x^5/b/(b*x^2+a)^(7/2)+5/2*a/b*(-1/4*x^3/b/(b*x^2+a)^(7/2)+3/4*a/b*
(-1/6*x/b/(b*x^2+a)^(7/2)+1/6*a/b*(1/7*x/a/(b*x^2+a)^(7/2)+6/7/a*(1/5*x/a/
(b*x^2+a)^(5/2)+4/5/a*(1/3*x/a/(b*x^2+a)^(3/2)+2/3*x/a^2/(b*x^2+a)^(1/2))
))))+B*(-x^6/b/(b*x^2+a)^(7/2)+6*a/b*(-1/3*x^4/b/(b*x^2+a)^(7/2)+4/3*a/b*(
-1/5*x^2/b/(b*x^2+a)^(7/2)-2/35*a/b^2/(b*x^2+a)^(7/2))))+C*(-1/7*x^7/b/(b*
x^2+a)^(7/2)+1/b*(-1/5*x^5/b/(b*x^2+a)^(5/2)+1/b*(-1/3*x^3/b/(b*x^2+a)^(3/
2)+1/b*(-x/b/(b*x^2+a)^(1/2)+1/b^(3/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))))))
```

## Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 467, normalized size of antiderivative = 2.72

$$\int \frac{x^6(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = \left[\frac{105\,(Cab^4x^8+4\,Ca^2b^3x^6+6\,Ca^3b^2x^4+4\,Ca^4bx^2+Ca^5)\sqrt{b}\log\left(-2\,bx^2-2\right.}{}\right.$$

$$-\frac{105\,(Cab^4x^8+4\,Ca^2b^3x^6+6\,Ca^3b^2x^4+4\,Ca^4bx^2+Ca^5)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)+(105\,Bab^4x^6+406\,Ca}{105\,(ab^9x^8+4\,a^2b^8x^6}$$

input
```
integrate(x^6*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

output
```
[1/210*(105*(C*a*b^4*x^8 + 4*C*a^2*b^3*x^6 + 6*C*a^3*b^2*x^4 + 4*C*a^4*b*x
^2 + C*a^5)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(1
05*B*a*b^4*x^6 + 406*C*a^2*b^3*x^5 + 210*B*a^2*b^3*x^4 + 350*C*a^3*b^2*x^3
 + 168*B*a^3*b^2*x^2 + (176*C*a*b^4 - 15*A*b^5)*x^7 + 105*C*a^4*b*x + 48*B
*a^4*b)*sqrt(b*x^2 + a))/(a*b^9*x^8 + 4*a^2*b^8*x^6 + 6*a^3*b^7*x^4 + 4*a^
4*b^6*x^2 + a^5*b^5), -1/105*(105*(C*a*b^4*x^8 + 4*C*a^2*b^3*x^6 + 6*C*a^3
*b^2*x^4 + 4*C*a^4*b*x^2 + C*a^5)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 +
a)) + (105*B*a*b^4*x^6 + 406*C*a^2*b^3*x^5 + 210*B*a^2*b^3*x^4 + 350*C*a^3
*b^2*x^3 + 168*B*a^3*b^2*x^2 + (176*C*a*b^4 - 15*A*b^5)*x^7 + 105*C*a^4*b*
x + 48*B*a^4*b)*sqrt(b*x^2 + a))/(a*b^9*x^8 + 4*a^2*b^8*x^6 + 6*a^3*b^7*x^
4 + 4*a^4*b^6*x^2 + a^5*b^5)]
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 366 vs. 2(151) = 302.

Time = 38.26 (sec) , antiderivative size = 3448, normalized size of antiderivative = 20.05

$$\int \frac{x^6(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x**6*(C*x**2+B*x+A)/(b*x**2+a)**(9/2),x)
```

output
```
A*x**7/(7*a**(9/2)*sqrt(1 + b*x**2/a) + 21*a**(7/2)*b*x**2*sqrt(1 + b*x**2
/a) + 21*a**(5/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 7*a**(3/2)*b**3*x**6*sqrt
(1 + b*x**2/a)) + B*Piecewise((-16*a**3/(35*a**3*b**4*sqrt(a + b*x**2) + 1
05*a**2*b**5*x**2*sqrt(a + b*x**2) + 105*a*b**6*x**4*sqrt(a + b*x**2) + 35
*b**7*x**6*sqrt(a + b*x**2)) - 56*a**2*b*x**2/(35*a**3*b**4*sqrt(a + b*x**
2) + 105*a**2*b**5*x**2*sqrt(a + b*x**2) + 105*a*b**6*x**4*sqrt(a + b*x**2
) + 35*b**7*x**6*sqrt(a + b*x**2)) - 70*a*b**2*x**4/(35*a**3*b**4*sqrt(a +
 b*x**2) + 105*a**2*b**5*x**2*sqrt(a + b*x**2) + 105*a*b**6*x**4*sqrt(a +
b*x**2) + 35*b**7*x**6*sqrt(a + b*x**2)) - 35*b**3*x**6/(35*a**3*b**4*sqrt
(a + b*x**2) + 105*a**2*b**5*x**2*sqrt(a + b*x**2) + 105*a*b**6*x**4*sqrt(
a + b*x**2) + 35*b**7*x**6*sqrt(a + b*x**2)), Ne(b, 0)), (x**8/(8*a**(9/2)
), True)) + C*(105*a**(205/2)*b**45*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqr
t(a))/(105*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(10
1/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*
x**2/a) + 2100*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(19
7/2)*b**(107/2)*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*
sqrt(1 + b*x**2/a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) +
 630*a**(203/2)*b**46*x**2*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(10
5*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2
*sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a...
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 447 vs. $2(146) = 292$.

Time = 0.06 (sec) , antiderivative size = 447, normalized size of antiderivative = 2.60

$$\int \frac{x^6(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = -\frac{1}{35}\left(\frac{35\,x^6}{(bx^2+a)^{\frac{7}{2}}b}+\frac{70\,ax^4}{(bx^2+a)^{\frac{7}{2}}b^2}+\frac{56\,a^2x^2}{(bx^2+a)^{\frac{7}{2}}b^3}+\frac{16\,a^3}{(bx^2+a)^{\frac{7}{2}}b^4}\right)Cx - \frac{Bx^6}{(bx^2+a)^{\frac{7}{2}}b} - \frac{Cx\left(\frac{15\,x^4}{(bx^2+a)^{\frac{5}{2}}b}+\frac{20\,ax^2}{(bx^2+a)^{\frac{5}{2}}b^2}+\frac{8\,a^2}{(bx^2+a)^{\frac{5}{2}}b^3}\right)}{15\,b} - \frac{Ax^5}{2\,(bx^2+a)^{\frac{7}{2}}b} - \frac{Cx\left(\frac{3\,x^2}{(bx^2+a)^{\frac{3}{2}}b}+\frac{2\,a}{(bx^2+a)^{\frac{3}{2}}b^2}\right)}{3\,b^2} - \frac{2\,Bax^4}{(bx^2+a)^{\frac{7}{2}}b^2} - \frac{Cax^3}{(bx^2+a)^{\frac{5}{2}}b^3} - \frac{5\,Aax^3}{8\,(bx^2+a)^{\frac{7}{2}}b^2} - \frac{8\,Ba^2x^2}{5\,(bx^2+a)^{\frac{7}{2}}b^3} + \frac{139\,Cx}{105\,\sqrt{bx^2+a}b^4} + \frac{17\,Cax}{105\,(bx^2+a)^{\frac{3}{2}}b^4} - \frac{29\,Ca^2x}{35\,(bx^2+a)^{\frac{5}{2}}b^4} + \frac{Ax}{14\,(bx^2+a)^{\frac{3}{2}}b^3} + \frac{Ax}{7\,\sqrt{bx^2+a}ab^3} + \frac{3\,Aax}{56\,(bx^2+a)^{\frac{5}{2}}b^3} - \frac{15\,Aa^2x}{56\,(bx^2+a)^{\frac{7}{2}}b^3} + \frac{C\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{9}{2}}} - \frac{16\,Ba^3}{35\,(bx^2+a)^{\frac{7}{2}}b^4}$$

input
```
integrate(x^6*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

output
```
-1/35*(35*x^6/((b*x^2 + a)^(7/2)*b) + 70*a*x^4/((b*x^2 + a)^(7/2)*b^2) + 5
6*a^2*x^2/((b*x^2 + a)^(7/2)*b^3) + 16*a^3/((b*x^2 + a)^(7/2)*b^4))*C*x -
B*x^6/((b*x^2 + a)^(7/2)*b) - 1/15*C*x*(15*x^4/((b*x^2 + a)^(5/2)*b) + 20*
a*x^2/((b*x^2 + a)^(5/2)*b^2) + 8*a^2/((b*x^2 + a)^(5/2)*b^3))/b - 1/2*A*x
^5/((b*x^2 + a)^(7/2)*b) - 1/3*C*x*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2*a/((b*
x^2 + a)^(3/2)*b^2))/b^2 - 2*B*a*x^4/((b*x^2 + a)^(7/2)*b^2) - C*a*x^3/((b
*x^2 + a)^(5/2)*b^3) - 5/8*A*a*x^3/((b*x^2 + a)^(7/2)*b^2) - 8/5*B*a^2*x^2
/((b*x^2 + a)^(7/2)*b^3) + 139/105*C*x/(sqrt(b*x^2 + a)*b^4) + 17/105*C*a*
x/((b*x^2 + a)^(3/2)*b^4) - 29/35*C*a^2*x/((b*x^2 + a)^(5/2)*b^4) + 1/14*A
*x/((b*x^2 + a)^(3/2)*b^3) + 1/7*A*x/(sqrt(b*x^2 + a)*a*b^3) + 3/56*A*a*x/
((b*x^2 + a)^(5/2)*b^3) - 15/56*A*a^2*x/((b*x^2 + a)^(7/2)*b^3) + C*arcsin
h(b*x/sqrt(a*b))/b^(9/2) - 16/35*B*a^3/((b*x^2 + a)^(7/2)*b^4)
```

## Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.80

$$\int \frac{x^6(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = -\frac{\left(\left(\left(\left(\left(\left(x\left(\frac{105\,B}{b}+\frac{(176\,Ca^3b^7-15\,Aa^2b^8)x}{a^3b^8}\right)+\frac{406\,Ca}{b^2}\right)x+\frac{210\,Ba}{b^2}\right)x+\frac{350\,Ca^2}{b^3}\right)x+\frac{168\,Ba^2}{b^3}\right)x+\frac{105\,Ca^3}{b^4}\right)x+\frac{48\,Ba}{}}{105\,(bx^2+a)^{\frac{7}{2}}} - \frac{C\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{b^{\frac{9}{2}}}$$

input
```
integrate(x^6*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="giac")
```

output
```
-1/105*((((((x*(105*B/b + (176*C*a^3*b^7 - 15*A*a^2*b^8)*x/(a^3*b^8)) + 40
6*C*a/b^2)*x + 210*B*a/b^2)*x + 350*C*a^2/b^3)*x + 168*B*a^2/b^3)*x + 105*
C*a^3/b^4)*x + 48*B*a^3/b^4)/(b*x^2 + a)^(7/2) - C*log(abs(-sqrt(b)*x + sq
rt(b*x^2 + a)))/b^(9/2)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^6(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = \int \frac{x^6\,(C\,x^2+B\,x+A)}{(b\,x^2+a)^{9/2}}\,dx$$

input
```
int((x^6*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x)
```

output
```
int((x^6*(A + B*x + C*x^2))/(a + b*x^2)^(9/2), x)
```

## Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 467, normalized size of antiderivative = 2.72

$$\int \frac{x^6(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = \frac{-48\sqrt{b\,x^2+a}\,a^3b^2-105\sqrt{b\,x^2+a}\,a^3bcx-168\sqrt{b\,x^2+a}\,a^2b^3x^2-350\sqrt{b\,x^2+}}{}$$

input
```
int(x^6*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x)
```

output
```
( - 48*sqrt(a + b*x**2)*a**3*b**2 - 105*sqrt(a + b*x**2)*a**3*b*c*x - 168*
sqrt(a + b*x**2)*a**2*b**3*x**2 - 350*sqrt(a + b*x**2)*a**2*b**2*c*x**3 -
210*sqrt(a + b*x**2)*a*b**4*x**4 - 406*sqrt(a + b*x**2)*a*b**3*c*x**5 + 15
*sqrt(a + b*x**2)*b**5*x**7 - 105*sqrt(a + b*x**2)*b**5*x**6 - 176*sqrt(a
+ b*x**2)*b**4*c*x**7 + 105*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqr
t(a))*a**4*c + 420*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**
3*b*c*x**2 + 630*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2*
b**2*c*x**4 + 420*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a*b*
*3*c*x**6 + 105*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*b**4*c
*x**8 + 15*sqrt(b)*a**4*b + 56*sqrt(b)*a**4*c + 60*sqrt(b)*a**3*b**2*x**2
+ 224*sqrt(b)*a**3*b*c*x**2 + 90*sqrt(b)*a**2*b**3*x**4 + 336*sqrt(b)*a**2
*b**2*c*x**4 + 60*sqrt(b)*a*b**4*x**6 + 224*sqrt(b)*a*b**3*c*x**6 + 15*sqr
t(b)*b**5*x**8 + 56*sqrt(b)*b**4*c*x**8)/(105*b**5*(a**4 + 4*a**3*b*x**2 +
 6*a**2*b**2*x**4 + 4*a*b**3*x**6 + b**4*x**8))
```

# 3.3 $\int \frac{x^5(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 135

$$\int \frac{x^5(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = -\frac{a^2(Ab-aC+bBx)}{7b^4(a+bx^2)^{7/2}} + \frac{a(7(2Ab-3aC)+15bBx)}{35b^4(a+bx^2)^{5/2}} - \frac{7(Ab-3aC)+9bBx}{21b^4(a+bx^2)^{3/2}} - \frac{7aC-bBx}{7ab^4\sqrt{a+bx^2}}$$

output
```
-1/7*a^2*(B*b*x+A*b-C*a)/b^4/(b*x^2+a)^(7/2)+1/35*a*(15*B*b*x+14*A*b-21*C*
a)/b^4/(b*x^2+a)^(5/2)-1/21*(9*B*b*x+7*A*b-21*C*a)/b^4/(b*x^2+a)^(3/2)-1/7
*(-B*b*x+7*C*a)/a/b^4/(b*x^2+a)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.90 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.66

$$\int \frac{x^5(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = \frac{-48a^4C+15b^4Bx^7-35ab^3x^4(A+3Cx^2)-14a^2b^2x^2(2A+15Cx^2)-8a^3b(A}{105ab^4(a+bx^2)^{7/2}}$$

input
```
Integrate[(x^5*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

output
```
(-48*a^4*C + 15*b^4*B*x^7 - 35*a*b^3*x^4*(A + 3*C*x^2) - 14*a^2*b^2*x^2*(2
*A + 15*C*x^2) - 8*a^3*b*(A + 21*C*x^2))/(105*a*b^4*(a + b*x^2)^(7/2))
```

## Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.14, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {2335, 25, 530, 27, 2345, 27, 453}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}dx$$

$$\Big\downarrow \text{ 2335}$$

$$-\frac{\int -\frac{x^4(5aB+(Ab+6aC)x)}{(bx^2+a)^{7/2}}dx}{7ab}-\frac{x^5(aB-x(Ab-aC))}{7ab(a+bx^2)^{7/2}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{x^4(5aB+(Ab+6aC)x)}{(bx^2+a)^{7/2}}dx}{7ab}-\frac{x^5(aB-x(Ab-aC))}{7ab(a+bx^2)^{7/2}}$$

$$\Big\downarrow \text{ 530}$$

$$\frac{-\frac{\int \frac{5\left(\frac{Ba^3}{b^2}-\frac{5Bx^2a^2}{b}+\frac{(Ab+6aC)xa^2}{b^2}-\left(A+\frac{6aC}{b}\right)x^3a\right)}{(bx^2+a)^{5/2}}dx}{5a}-\frac{a^2(6aC+Ab-5bBx)}{5b^3(a+bx^2)^{5/2}}}{7ab}-\frac{x^5(aB-x(Ab-aC))}{7ab(a+bx^2)^{7/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{-\frac{\int \frac{\frac{Ba^3}{b^2}-\frac{5Bx^2a^2}{b}+\frac{(Ab+6aC)xa^2}{b^2}-\left(A+\frac{6aC}{b}\right)x^3a}{(bx^2+a)^{5/2}}dx}{a}-\frac{a^2(6aC+Ab-5bBx)}{5b^3(a+bx^2)^{5/2}}}{7ab}-\frac{x^5(aB-x(Ab-aC))}{7ab(a+bx^2)^{7/2}}$$

$$\Big\downarrow \text{ 2345}$$

$$-\frac{-\frac{-\frac{\int \frac{3a^2(aB+(Ab+6aC)x)}{b^2\left(bx^2+a\right)^{3/2}}dx}{3a}-\frac{2a^2(6aC+Ab-3bBx)}{3b^3\left(a+bx^2\right)^{3/2}}}{a}-\frac{a^2(6aC+Ab-5bBx)}{5b^3(a+bx^2)^{5/2}}}{7ab}-\frac{x^5(aB-x(Ab-aC))}{7ab\left(a+bx^2\right)^{7/2}}$$

$\Big\downarrow$ 27

$$-\frac{-\frac{-\frac{a\int \frac{aB+(Ab+6aC)x}{\left(bx^2+a\right)^{3/2}}dx}{b^2}-\frac{2a^2(6aC+Ab-3bBx)}{3b^3\left(a+bx^2\right)^{3/2}}}{a}-\frac{a^2(6aC+Ab-5bBx)}{5b^3(a+bx^2)^{5/2}}}{7ab}-\frac{x^5(aB-x(Ab-aC))}{7ab\left(a+bx^2\right)^{7/2}}$$

$\Big\downarrow$ 453

$$\frac{-\frac{a^2(6aC+Ab-5bBx)}{5b^3(a+bx^2)^{5/2}}-\frac{\frac{a(6aC+Ab-bBx)}{b^3\sqrt{a+bx^2}}-\frac{2a^2(6aC+Ab-3bBx)}{3b^3\left(a+bx^2\right)^{3/2}}}{a}}{7ab}-\frac{x^5(aB-x(Ab-aC))}{7ab\left(a+bx^2\right)^{7/2}}$$

input
```
Int[(x^5*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

output
```
-1/7*(x^5*(a*B - (A*b - a*C)*x))/(a*b*(a + b*x^2)^(7/2)) + (-1/5*(a^2*(A*b
 + 6*a*C - 5*b*B*x))/(b^3*(a + b*x^2)^(5/2)) - ((-2*a^2*(A*b + 6*a*C - 3*b
*B*x))/(3*b^3*(a + b*x^2)^(3/2)) + (a*(A*b + 6*a*C - b*B*x))/(b^3*Sqrt[a +
 b*x^2]))/a)/(7*a*b)
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 453
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[-(a*
d - b*c*x)/(a*b*Sqrt[a + b*x^2]), x] /; FreeQ[{a, b, c, d}, x]
```

rule 530
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symb
ol] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Co
eff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Po
lynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x
)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(a
 + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Qx + e*(2*p + 3), x], x], x]] /;
FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && IGtQ[m, 0] && LtQ[p, -1] && EqQ[n,
1] && IntegerQ[2*p]
```

rule 2335
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Simp[c/(2*a*b*(p + 1))   Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSu
m[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
 b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

## Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.70

| method | result |
|---|---|
| gosper | $-\frac{-15B\,x^{7}b^{4}+105Ca\,b^{3}x^{6}+35Aa\,b^{3}x^{4}+210C\,a^{2}b^{2}x^{4}+28A\,a^{2}b^{2}x^{2}+168C\,a^{3}b\,x^{2}+8A\,a^{3}b+48C\,a^{4}}{105(b\,x^{2}+a)^{\frac{7}{2}}\,a\,b^{4}}$ |
| trager | $-\frac{-15B\,x^{7}b^{4}+105Ca\,b^{3}x^{6}+35Aa\,b^{3}x^{4}+210C\,a^{2}b^{2}x^{4}+28A\,a^{2}b^{2}x^{2}+168C\,a^{3}b\,x^{2}+8A\,a^{3}b+48C\,a^{4}}{105(b\,x^{2}+a)^{\frac{7}{2}}\,a\,b^{4}}$ |
| orering | $-\frac{-15B\,x^{7}b^{4}+105Ca\,b^{3}x^{6}+35Aa\,b^{3}x^{4}+210C\,a^{2}b^{2}x^{4}+28A\,a^{2}b^{2}x^{2}+168C\,a^{3}b\,x^{2}+8A\,a^{3}b+48C\,a^{4}}{105(b\,x^{2}+a)^{\frac{7}{2}}\,a\,b^{4}}$ |
| default | $A\left(-\frac{x^{4}}{3b(b\,x^{2}+a)^{\frac{7}{2}}}+\frac{4a\left(-\frac{x^{2}}{5b\left(b\,x^{2}+a\right)^{\frac{7}{2}}}-\frac{2a}{35b^{2}\left(b\,x^{2}+a\right)^{\frac{7}{2}}}\right)}{3b}\right)+B\left(-\frac{x^{5}}{2b(b\,x^{2}+a)^{\frac{7}{2}}}+\frac{5a\left(-\frac{x^{3}}{4b\left(b\,x^{2}+a\right)^{\frac{7}{2}}}+\frac{3a\left(-\frac{\cdots}{6b\left(b\,x^{2}\cdots\right.}\right.}{\cdots}\right.}{\cdots}\right.$ |

input
```
int(x^5*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-1/105*(-15*B*b^4*x^7+105*C*a*b^3*x^6+35*A*a*b^3*x^4+210*C*a^2*b^2*x^4+28*
A*a^2*b^2*x^2+168*C*a^3*b*x^2+8*A*a^3*b+48*C*a^4)/(b*x^2+a)^(7/2)/a/b^4
```

## Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.01

$$\int \frac{x^5(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = \frac{(15\,Bb^4x^7 - 105\,Cab^3x^6 - 48\,Ca^4 - 8\,Aa^3b - 35\,(6\,Ca^2b^2 + Aab^3)x^4 - 28\,(6\,C}{105\,(ab^8x^8 + 4\,a^2b^7x^6 + 6\,a^3b^6x^4 + 4\,a^4b^5x^2 + a^5b^4)}$$

input
```
integrate(x^5*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

output
```
1/105*(15*B*b^4*x^7 - 105*C*a*b^3*x^6 - 48*C*a^4 - 8*A*a^3*b - 35*(6*C*a^2
*b^2 + A*a*b^3)*x^4 - 28*(6*C*a^3*b + A*a^2*b^2)*x^2)*sqrt(b*x^2 + a)/(a*b
^8*x^8 + 4*a^2*b^7*x^6 + 6*a^3*b^6*x^4 + 4*a^4*b^5*x^2 + a^5*b^4)
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 274 vs. 2(119) = 238.

Time = 22.79 (sec) , antiderivative size = 740, normalized size of antiderivative = 5.48

$$\int \frac{x^5(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = A\left(\left\{\begin{array}{l} -\frac{8a^2}{105a^3b^3\sqrt{a+bx^2}+315a^2b^4x^2\sqrt{a+bx^2}+315ab^5x^4\sqrt{a+bx^2}+105b^6x^6\sqrt{a+bx^2}} - \frac{}{105a^3b^3\sqrt{a+bx^2}} \\ \frac{x^6}{6a^{\frac{9}{2}}} \end{array}\right.\right.$$

$$+ \frac{Bx^7}{7a^{\frac{9}{2}}\sqrt{1+\frac{bx^2}{a}} + 21a^{\frac{7}{2}}bx^2\sqrt{1+\frac{bx^2}{a}} + 21a^{\frac{5}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}} + 7a^{\frac{3}{2}}b^3x^6\sqrt{1+\frac{bx^2}{a}}}$$

$$+C\left(\left\{\begin{array}{l} -\frac{16a^3}{35a^3b^4\sqrt{a+bx^2}+105a^2b^5x^2\sqrt{a+bx^2}+105ab^6x^4\sqrt{a+bx^2}+35b^7x^6\sqrt{a+bx^2}} - \frac{56a^2bx^2}{35a^3b^4\sqrt{a+bx^2}+105a^2b^5x^2\sqrt{a+bx^2}+105ab^6x^4\sqrt{a+bx^2}} \\ \frac{x^8}{8a^{\frac{9}{2}}} \end{array}\right.\right.$$

input
```
integrate(x**5*(C*x**2+B*x+A)/(b*x**2+a)**(9/2),x)
```

output
```
A*Piecewise((-8*a**2/(105*a**3*b**3*sqrt(a + b*x**2) + 315*a**2*b**4*x**2*
sqrt(a + b*x**2) + 315*a*b**5*x**4*sqrt(a + b*x**2) + 105*b**6*x**6*sqrt(a
 + b*x**2)) - 28*a*b*x**2/(105*a**3*b**3*sqrt(a + b*x**2) + 315*a**2*b**4*
x**2*sqrt(a + b*x**2) + 315*a*b**5*x**4*sqrt(a + b*x**2) + 105*b**6*x**6*s
qrt(a + b*x**2)) - 35*b**2*x**4/(105*a**3*b**3*sqrt(a + b*x**2) + 315*a**2
*b**4*x**2*sqrt(a + b*x**2) + 315*a*b**5*x**4*sqrt(a + b*x**2) + 105*b**6*
x**6*sqrt(a + b*x**2)), Ne(b, 0)), (x**6/(6*a**(9/2)), True)) + B*x**7/(7*
a**(9/2)*sqrt(1 + b*x**2/a) + 21*a**(7/2)*b*x**2*sqrt(1 + b*x**2/a) + 21*a
**(5/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 7*a**(3/2)*b**3*x**6*sqrt(1 + b*x**
2/a)) + C*Piecewise((-16*a**3/(35*a**3*b**4*sqrt(a + b*x**2) + 105*a**2*b*
*5*x**2*sqrt(a + b*x**2) + 105*a*b**6*x**4*sqrt(a + b*x**2) + 35*b**7*x**6
*sqrt(a + b*x**2)) - 56*a**2*b*x**2/(35*a**3*b**4*sqrt(a + b*x**2) + 105*a
**2*b**5*x**2*sqrt(a + b*x**2) + 105*a*b**6*x**4*sqrt(a + b*x**2) + 35*b**
7*x**6*sqrt(a + b*x**2)) - 70*a*b**2*x**4/(35*a**3*b**4*sqrt(a + b*x**2) +
 105*a**2*b**5*x**2*sqrt(a + b*x**2) + 105*a*b**6*x**4*sqrt(a + b*x**2) +
35*b**7*x**6*sqrt(a + b*x**2)) - 35*b**3*x**6/(35*a**3*b**4*sqrt(a + b*x**
2) + 105*a**2*b**5*x**2*sqrt(a + b*x**2) + 105*a*b**6*x**4*sqrt(a + b*x**2
) + 35*b**7*x**6*sqrt(a + b*x**2)), Ne(b, 0)), (x**8/(8*a**(9/2)), True))
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 240 vs. $2(113) = 226$.

Time = 0.04 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.78

$$\int \frac{x^5(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = -\frac{Cx^6}{(bx^2+a)^{\frac{7}{2}}b} - \frac{Bx^5}{2\,(bx^2+a)^{\frac{7}{2}}b} - \frac{2\,Cax^4}{(bx^2+a)^{\frac{7}{2}}b^2} - \frac{Ax^4}{3\,(bx^2+a)^{\frac{7}{2}}b} - \frac{5\,Bax^3}{8\,(bx^2+a)^{\frac{7}{2}}b^2} - \frac{8\,Ca^2x^2}{5\,(bx^2+a)^{\frac{7}{2}}b^3} - \frac{4\,Aax^2}{15\,(bx^2+a)^{\frac{7}{2}}b^2} + \frac{Bx}{14\,(bx^2+a)^{\frac{3}{2}}b^3} + \frac{Bx}{7\,\sqrt{bx^2+a}ab^3} + \frac{3\,Bax}{56\,(bx^2+a)^{\frac{5}{2}}b^3} - \frac{15\,Ba^2x}{56\,(bx^2+a)^{\frac{7}{2}}b^3} - \frac{16\,Ca^3}{35\,(bx^2+a)^{\frac{7}{2}}b^4} - \frac{8\,Aa^2}{105\,(bx^2+a)^{\frac{7}{2}}b^3}$$

input
```
integrate(x^5*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

output
```
-C*x^6/((b*x^2 + a)^(7/2)*b) - 1/2*B*x^5/((b*x^2 + a)^(7/2)*b) - 2*C*a*x^4
/((b*x^2 + a)^(7/2)*b^2) - 1/3*A*x^4/((b*x^2 + a)^(7/2)*b) - 5/8*B*a*x^3/(
(b*x^2 + a)^(7/2)*b^2) - 8/5*C*a^2*x^2/((b*x^2 + a)^(7/2)*b^3) - 4/15*A*a*
x^2/((b*x^2 + a)^(7/2)*b^2) + 1/14*B*x/((b*x^2 + a)^(3/2)*b^3) + 1/7*B*x/(
sqrt(b*x^2 + a)*a*b^3) + 3/56*B*a*x/((b*x^2 + a)^(5/2)*b^3) - 15/56*B*a^2*
x/((b*x^2 + a)^(7/2)*b^3) - 16/35*C*a^3/((b*x^2 + a)^(7/2)*b^4) - 8/105*A*
a^2/((b*x^2 + a)^(7/2)*b^3)
```

## Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.83

$$\int \frac{x^5(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = \frac{\left(5\left(3\left(\frac{Bx}{a}-\frac{7C}{b}\right)x^2-\frac{7\left(6Ca^4b^2+Aa^3b^3\right)}{a^3b^4}\right)x^2-\frac{28\left(6Ca^5b+Aa^4b^2\right)}{a^3b^4}\right)x^2-\frac{8\left(6Ca^6+Aa^5b\right)}{a^3b^4}}{105\left(bx^2+a\right)^{\frac{7}{2}}}$$

input
```
integrate(x^5*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="giac")
```

output
```
1/105*((5*(3*(B*x/a - 7*C/b)*x^2 - 7*(6*C*a^4*b^2 + A*a^3*b^3)/(a^3*b^4))*
x^2 - 28*(6*C*a^5*b + A*a^4*b^2)/(a^3*b^4))*x^2 - 8*(6*C*a^6 + A*a^5*b)/(a
^3*b^4))/(b*x^2 + a)^(7/2)
```

## Mupad [B] (verification not implemented)

Time = 2.57 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.45

$$\int \frac{x^5(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = \frac{\frac{a\left(\frac{C}{3\,b^3}-\frac{7\,A\,b-14\,C\,a}{21\,a\,b^3}\right)}{b}-\frac{3\,B\,x}{7\,b^3}}{{\left(b\,x^2+a\right)}^{3/2}}-\frac{\frac{a^2\,\left(\frac{A}{7\,b}-\frac{C\,a}{7\,b^2}\right)}{b^2}+\frac{B\,a^2\,x}{7\,b^3}}{{\left(b\,x^2+a\right)}^{7/2}}-\frac{\frac{C}{b^4}-\frac{B\,x}{7\,a\,b^3}}{\sqrt{b\,x^2+a}}-\frac{\frac{a\,\left(\frac{7\,C\,a^2-7\,A\,a\,b}{35\,a\,b^3}+\frac{a\,\left(\frac{C}{5\,b^2}-\frac{7\,A\,b^2-7\,C\,a\,b}{35\,a\,b^3}\right)}{b}\right)}{b}-\frac{3\,B\,a\,x}{7\,b^3}}{{\left(b\,x^2+a\right)}^{5/2}}$$

input
```
int((x^5*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x)
```

output
```
((a*(C/(3*b^3) - (7*A*b - 14*C*a)/(21*a*b^3)))/b - (3*B*x)/(7*b^3))/(a + b
*x^2)^(3/2) - ((a^2*(A/(7*b) - (C*a)/(7*b^2)))/b^2 + (B*a^2*x)/(7*b^3))/(a
 + b*x^2)^(7/2) - (C/b^4 - (B*x)/(7*a*b^3))/(a + b*x^2)^(1/2) - ((a*((7*C*
a^2 - 7*A*a*b)/(35*a*b^3) + (a*(C/(5*b^2) - (7*A*b^2 - 7*C*a*b)/(35*a*b^3)
))/b))/b - (3*B*a*x)/(7*b^3))/(a + b*x^2)^(5/2)
```

## Reduce [B] (verification not implemented)

Time = 1.10 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.81

$$\int \frac{x^5(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = \frac{-8\sqrt{b\,x^2+a}\,a^4b-48\sqrt{b\,x^2+a}\,a^4c-28\sqrt{b\,x^2+a}\,a^3b^2x^2-168\sqrt{b\,x^2+a}\,a^3bc}{}$$

input
```
int(x^5*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x)
```

output
```
( - 8*sqrt(a + b*x**2)*a**4*b - 48*sqrt(a + b*x**2)*a**4*c - 28*sqrt(a + b
*x**2)*a**3*b**2*x**2 - 168*sqrt(a + b*x**2)*a**3*b*c*x**2 - 35*sqrt(a + b
*x**2)*a**2*b**3*x**4 - 210*sqrt(a + b*x**2)*a**2*b**2*c*x**4 - 105*sqrt(a
 + b*x**2)*a*b**3*c*x**6 + 15*sqrt(a + b*x**2)*b**5*x**7 + 15*sqrt(b)*a**4
*b + 60*sqrt(b)*a**3*b**2*x**2 + 90*sqrt(b)*a**2*b**3*x**4 + 60*sqrt(b)*a*
b**4*x**6 + 15*sqrt(b)*b**5*x**8)/(105*a*b**4*(a**4 + 4*a**3*b*x**2 + 6*a*
*2*b**2*x**4 + 4*a*b**3*x**6 + b**4*x**8))
```

# 3.4 $\int \frac{x^4(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 103

$$\int \frac{x^4(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = -\frac{x^4(aB-(Ab-aC)x)}{7ab\,(a+bx^2)^{7/2}} - \frac{4B}{21b^3\,(a+bx^2)^{3/2}} + \frac{4a^3B+b^2(2Ab+5aC)x^5}{35a^2b^3\,(a+bx^2)^{5/2}}$$

output
```
-1/7*x^4*(B*a-(A*b-C*a)*x)/a/b/(b*x^2+a)^(7/2)-4/21*B/b^3/(b*x^2+a)^(3/2)+
1/35*(4*a^3*B+b^2*(2*A*b+5*C*a)*x^5)/a^2/b^3/(b*x^2+a)^(5/2)
```

## Mathematica [A] (verified)

Time = 0.99 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.77

$$\int \frac{x^4(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = \frac{-8a^4B-28a^3bBx^2-35a^2b^2Bx^4+21aAb^3x^5+6Ab^4x^7+15ab^3Cx^7}{105a^2b^3\,(a+bx^2)^{7/2}}$$

input
```
Integrate[(x^4*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

output
```
(-8*a^4*B - 28*a^3*b*B*x^2 - 35*a^2*b^2*B*x^4 + 21*a*A*b^3*x^5 + 6*A*b^4*x
^7 + 15*a*b^3*C*x^7)/(105*a^2*b^3*(a + b*x^2)^(7/2))
```

## Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.57, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2335, 25, 530, 2345, 27, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 \left(A+Bx+Cx^2\right)}{\left(a+bx^2\right)^{9/2}} \, dx$$

$$\Big\downarrow \ 2335$$

$$-\frac{\int -\frac{x^3(4aB+(2Ab+5aC)x)}{\left(bx^2+a\right)^{7/2}}dx}{7ab}-\frac{x^4(aB-x(Ab-aC))}{7ab\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{x^3(4aB+(2Ab+5aC)x)}{\left(bx^2+a\right)^{7/2}}dx}{7ab}-\frac{x^4(aB-x(Ab-aC))}{7ab\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \ 530$$

$$\frac{\frac{a\left(bx\left(\frac{5aC}{b}+2A\right)+4aB\right)}{5b^2\left(a+bx^2\right)^{5/2}}-\frac{\int \frac{\frac{(2Ab+5aC)a^2}{b^2}-\frac{20Bxa^2}{b}-5\left(2A+\frac{5aC}{b}\right)x^2a}{\left(bx^2+a\right)^{5/2}}dx}{5a}}{7ab}-\frac{x^4(aB-x(Ab-aC))}{7ab\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \ 2345$$

$$\frac{\frac{a\left(bx\left(\frac{5aC}{b}+2A\right)+4aB\right)}{5b^2\left(a+bx^2\right)^{5/2}}-\frac{\frac{2a\left(3bx\left(\frac{5aC}{b}+2A\right)+10aB\right)}{3b^2\left(a+bx^2\right)^{3/2}}-\frac{\int \frac{3a^2(2Ab+5aC)}{b^2\left(bx^2+a\right)^{3/2}}dx}{3a}}{5a}}{7ab}-\frac{x^4(aB-x(Ab-aC))}{7ab\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{a\left(bx\left(\frac{5aC}{b}+2A\right)+4aB\right)}{5b^2(a+bx^2)^{5/2}}-\frac{\frac{2a\left(3bx\left(\frac{5aC}{b}+2A\right)+10aB\right)}{3b^2\left(a+bx^2\right)^{3/2}}-\frac{a(5aC+2Ab)\int\frac{1}{\left(bx^2+a\right)^{3/2}}dx}{b^2}}{5a}}{7ab}-\frac{x^4(aB-x(Ab-aC))}{7ab\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \text{208}$$

$$\frac{\frac{a\left(bx\left(\frac{5aC}{b}+2A\right)+4aB\right)}{5b^2(a+bx^2)^{5/2}}-\frac{\frac{2a\left(3bx\left(\frac{5aC}{b}+2A\right)+10aB\right)}{3b^2\left(a+bx^2\right)^{3/2}}-\frac{x(5aC+2Ab)}{b^2\sqrt{a+bx^2}}}{5a}}{7ab}-\frac{x^4(aB-x(Ab-aC))}{7ab\left(a+bx^2\right)^{7/2}}$$

input
```
Int[(x^4*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

output
```
-1/7*(x^4*(a*B - (A*b - a*C)*x))/(a*b*(a + b*x^2)^(7/2)) + ((a*(4*a*B + b*
(2*A + (5*a*C)/b)*x))/(5*b^2*(a + b*x^2)^(5/2)) - ((2*a*(10*a*B + 3*b*(2*A
 + (5*a*C)/b)*x))/(3*b^2*(a + b*x^2)^(3/2)) - ((2*A*b + 5*a*C)*x)/(b^2*Sqr
t[a + b*x^2]))/(5*a))/(7*a*b)
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 530
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symb
ol] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Co
eff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Po
lynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x
)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(a
 + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Qx + e*(2*p + 3), x], x], x]] /;
FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && IGtQ[m, 0] && LtQ[p, -1] && EqQ[n,
1] && IntegerQ[2*p]
```

rule 2335
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Simp[c/(2*a*b*(p + 1))   Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSu
m[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
 b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

## Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.74

| method | result |
|---|---|
| gosper | $\frac{6A\,b^4x^7+15Ca\,x^7b^3+21Aa\,b^3x^5-35B\,x^4a^2b^2-28B\,a^3b\,x^2-8B\,a^4}{105(b\,x^2+a)^{\frac{7}{2}}a^2b^3}$ |
| trager | $\frac{6A\,b^4x^7+15Ca\,x^7b^3+21Aa\,b^3x^5-35B\,x^4a^2b^2-28B\,a^3b\,x^2-8B\,a^4}{105(b\,x^2+a)^{\frac{7}{2}}a^2b^3}$ |
| orering | $\frac{6A\,b^4x^7+15Ca\,x^7b^3+21Aa\,b^3x^5-35B\,x^4a^2b^2-28B\,a^3b\,x^2-8B\,a^4}{105(b\,x^2+a)^{\frac{7}{2}}a^2b^3}$ |
| default | $A\left(-\frac{x^3}{4b(b\,x^2+a)^{\frac{7}{2}}}+\frac{3a\left(-\frac{x}{6b\left(b\,x^2+a\right)^{\frac{7}{2}}}+\frac{a\left(\frac{x}{7a\left(b\,x^2+a\right)^{\frac{7}{2}}}+\frac{\frac{6x}{35a\left(b\,x^2+a\right)^{\frac{5}{2}}}+\frac{6\left(\frac{4x}{15a\left(b\,x^2+a\right)^{\frac{3}{2}}}+\frac{8x}{15a^2\sqrt{b\,x^2+a}}\right)}{7a}}{a}\right)}{6b}\right)}{4b}\right)+B\Big($ |

input
```
int(x^4*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
1/105*(6*A*b^4*x^7+15*C*a*b^3*x^7+21*A*a*b^3*x^5-35*B*a^2*b^2*x^4-28*B*a^3
*b*x^2-8*B*a^4)/(b*x^2+a)^(7/2)/a^2/b^3
```

## Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.18

$$\int \frac{x^4(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = \frac{(21\,Aab^3x^5 - 35\,Ba^2b^2x^4 + 3\,(5\,Cab^3 + 2\,Ab^4)x^7 - 28\,Ba^3bx^2 - 8\,Ba^4)\sqrt{bx^2 -}}{105\,(a^2b^7x^8 + 4\,a^3b^6x^6 + 6\,a^4b^5x^4 + 4\,a^5b^4x^2 + a^6b^3)}$$

input
```
integrate(x^4*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

output
```
1/105*(21*A*a*b^3*x^5 - 35*B*a^2*b^2*x^4 + 3*(5*C*a*b^3 + 2*A*b^4)*x^7 - 2
8*B*a^3*b*x^2 - 8*B*a^4)*sqrt(b*x^2 + a)/(a^2*b^7*x^8 + 4*a^3*b^6*x^6 + 6*
a^4*b^5*x^4 + 4*a^5*b^4*x^2 + a^6*b^3)
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(92) = 184$.

Time = 35.01 (sec) , antiderivative size = 575, normalized size of antiderivative = 5.58

$$\int \frac{x^4(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = A\left(\frac{7ax^5}{35a^{\frac{11}{2}}\sqrt{1+\frac{bx^2}{a}} + 105a^{\frac{9}{2}}bx^2\sqrt{1+\frac{bx^2}{a}} + 105a^{\frac{7}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}} + 35a^{\frac{5}{2}}b^3x^6\sqrt{}}\right.$$

$$\left. + \frac{2bx^7}{35a^{\frac{11}{2}}\sqrt{1+\frac{bx^2}{a}} + 105a^{\frac{9}{2}}bx^2\sqrt{1+\frac{bx^2}{a}} + 105a^{\frac{7}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}} + 35a^{\frac{5}{2}}b^3x^6\sqrt{1+\frac{bx^2}{a}}}\right)$$

$$+ B\left(\left(\begin{cases} -\frac{8a^2}{105a^3b^3\sqrt{a+bx^2}+315a^2b^4x^2\sqrt{a+bx^2}+315ab^5x^4\sqrt{a+bx^2}+105b^6x^6\sqrt{a+bx^2}} - \frac{28abx^2}{105a^3b^3\sqrt{a+bx^2}+315a^2b^4x^2\sqrt{a+bx^2}+315ab^5x^4\sqrt{a}} \\ \frac{x^6}{6a^{\frac{9}{2}}} \end{cases}\right.\right.$$

$$+ \frac{Cx^7}{7a^{\frac{9}{2}}\sqrt{1+\frac{bx^2}{a}} + 21a^{\frac{7}{2}}bx^2\sqrt{1+\frac{bx^2}{a}} + 21a^{\frac{5}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}} + 7a^{\frac{3}{2}}b^3x^6\sqrt{1+\frac{bx^2}{a}}}$$

input
```
integrate(x**4*(C*x**2+B*x+A)/(b*x**2+a)**(9/2),x)
```

output
```
A*(7*a*x**5/(35*a**(11/2)*sqrt(1 + b*x**2/a) + 105*a**(9/2)*b*x**2*sqrt(1
+ b*x**2/a) + 105*a**(7/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 35*a**(5/2)*b**3
*x**6*sqrt(1 + b*x**2/a)) + 2*b*x**7/(35*a**(11/2)*sqrt(1 + b*x**2/a) + 10
5*a**(9/2)*b*x**2*sqrt(1 + b*x**2/a) + 105*a**(7/2)*b**2*x**4*sqrt(1 + b*x
**2/a) + 35*a**(5/2)*b**3*x**6*sqrt(1 + b*x**2/a))) + B*Piecewise((-8*a**2
/(105*a**3*b**3*sqrt(a + b*x**2) + 315*a**2*b**4*x**2*sqrt(a + b*x**2) + 3
15*a*b**5*x**4*sqrt(a + b*x**2) + 105*b**6*x**6*sqrt(a + b*x**2)) - 28*a*b
*x**2/(105*a**3*b**3*sqrt(a + b*x**2) + 315*a**2*b**4*x**2*sqrt(a + b*x**2
) + 315*a*b**5*x**4*sqrt(a + b*x**2) + 105*b**6*x**6*sqrt(a + b*x**2)) - 3
5*b**2*x**4/(105*a**3*b**3*sqrt(a + b*x**2) + 315*a**2*b**4*x**2*sqrt(a +
b*x**2) + 315*a*b**5*x**4*sqrt(a + b*x**2) + 105*b**6*x**6*sqrt(a + b*x**2
)), Ne(b, 0)), (x**6/(6*a**(9/2)), True)) + C*x**7/(7*a**(9/2)*sqrt(1 + b*
x**2/a) + 21*a**(7/2)*b*x**2*sqrt(1 + b*x**2/a) + 21*a**(5/2)*b**2*x**4*sq
rt(1 + b*x**2/a) + 7*a**(3/2)*b**3*x**6*sqrt(1 + b*x**2/a))
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(90) = 180$.

Time = 0.04 (sec) , antiderivative size = 253, normalized size of antiderivative = 2.46

$$\int \frac{x^4(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = -\frac{Cx^5}{2\,(bx^2+a)^{\frac{7}{2}}b} - \frac{Bx^4}{3\,(bx^2+a)^{\frac{7}{2}}b} - \frac{5\,Cax^3}{8\,(bx^2+a)^{\frac{7}{2}}b^2} - \frac{Ax^3}{4\,(bx^2+a)^{\frac{7}{2}}b} - \frac{4\,Bax^2}{15\,(bx^2+a)^{\frac{7}{2}}b^2} + \frac{Cx}{14\,(bx^2+a)^{\frac{3}{2}}b^3} + \frac{Cx}{7\,\sqrt{bx^2+a}ab^3} + \frac{3\,Cax}{56\,(bx^2+a)^{\frac{5}{2}}b^3} - \frac{15\,Ca^2x}{56\,(bx^2+a)^{\frac{7}{2}}b^3} + \frac{3\,Ax}{140\,(bx^2+a)^{\frac{5}{2}}b^2} + \frac{2\,Ax}{35\,\sqrt{bx^2+a}a^2b^2} + \frac{Ax}{35\,(bx^2+a)^{\frac{3}{2}}ab^2} - \frac{3\,Aax}{28\,(bx^2+a)^{\frac{7}{2}}b^2} - \frac{8\,Ba^2}{105\,(bx^2+a)^{\frac{7}{2}}b^3}$$

input
```
integrate(x^4*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

output
```
-1/2*C*x^5/((b*x^2 + a)^(7/2)*b) - 1/3*B*x^4/((b*x^2 + a)^(7/2)*b) - 5/8*C
*a*x^3/((b*x^2 + a)^(7/2)*b^2) - 1/4*A*x^3/((b*x^2 + a)^(7/2)*b) - 4/15*B*
a*x^2/((b*x^2 + a)^(7/2)*b^2) + 1/14*C*x/((b*x^2 + a)^(3/2)*b^3) + 1/7*C*x
/(sqrt(b*x^2 + a)*a*b^3) + 3/56*C*a*x/((b*x^2 + a)^(5/2)*b^3) - 15/56*C*a^
2*x/((b*x^2 + a)^(7/2)*b^3) + 3/140*A*x/((b*x^2 + a)^(5/2)*b^2) + 2/35*A*x
/(sqrt(b*x^2 + a)*a^2*b^2) + 1/35*A*x/((b*x^2 + a)^(3/2)*a*b^2) - 3/28*A*a
*x/((b*x^2 + a)^(7/2)*b^2) - 8/105*B*a^2/((b*x^2 + a)^(7/2)*b^3)
```

## Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.79

$$\int \frac{x^4(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = \frac{\left(\left(3\,x\left(\frac{7\,A}{a}+\frac{(5\,Ca^2b^3+2\,Aab^4)x^2}{a^3b^3}\right)-\frac{35\,B}{b}\right)x^2-\frac{28\,Ba}{b^2}\right)x^2-\frac{8\,Ba^2}{b^3}}{105\,(bx^2+a)^{\frac{7}{2}}}$$

input
```
integrate(x^4*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="giac")
```

output
```
1/105*(((3*x*(7*A/a + (5*C*a^2*b^3 + 2*A*a*b^4)*x^2/(a^3*b^3)) - 35*B/b)*x
^2 - 28*B*a/b^2)*x^2 - 8*B*a^2/b^3)/(b*x^2 + a)^(7/2)
```

## Mupad [B] (verification not implemented)

Time = 2.43 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.81

$$\int \frac{x^4(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = \frac{x\left(\frac{C\,a^2-A\,a\,b}{35\,a\,b^3}+\frac{a\left(\frac{C}{5\,b^2}-\frac{7\,A\,b^2-7\,C\,a\,b}{35\,a\,b^3}\right)}{b}\right)+\frac{2\,B\,a}{5\,b^3}}{(b\,x^2+a)^{5/2}} - \frac{\frac{B}{3\,b^3}+x\left(\frac{C}{3\,b^3}-\frac{3\,A\,b-10\,C\,a}{105\,a\,b^3}\right)}{(b\,x^2+a)^{3/2}} - \frac{\frac{B\,a^2}{7\,b^3}-\frac{a\,x\left(\frac{A}{7\,b}-\frac{C\,a}{7\,b^2}\right)}{b}}{(b\,x^2+a)^{7/2}} + \frac{x\,(2\,A\,b+5\,C\,a)}{35\,a^2\,b^3\,\sqrt{b\,x^2+a}}$$

input
```
int((x^4*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x)
```

output
```
(x*((C*a^2 - A*a*b)/(35*a*b^3) + (a*(C/(5*b^2) - (7*A*b^2 - 7*C*a*b)/(35*a
*b^3)))/b) + (2*B*a)/(5*b^3))/(a + b*x^2)^(5/2) - (B/(3*b^3) + x*(C/(3*b^3
) - (3*A*b - 10*C*a)/(105*a*b^3)))/(a + b*x^2)^(3/2) - ((B*a^2)/(7*b^3) -
(a*x*(A/(7*b) - (C*a)/(7*b^2)))/b)/(a + b*x^2)^(7/2) + (x*(2*A*b + 5*C*a))
/(35*a^2*b^3*(a + b*x^2)^(1/2))
```

## Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 265, normalized size of antiderivative = 2.57

$$\int \frac{x^4(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = \frac{-8\sqrt{b\,x^2+a}\,a^3b^2-28\sqrt{b\,x^2+a}\,a^2b^3x^2+21\sqrt{b\,x^2+a}\,a\,b^4x^5-35\sqrt{b\,x^2+a}\,a}{}$$

input
```
int(x^4*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x)
```

output
```
( - 8*sqrt(a + b*x**2)*a**3*b**2 - 28*sqrt(a + b*x**2)*a**2*b**3*x**2 + 21
*sqrt(a + b*x**2)*a*b**4*x**5 - 35*sqrt(a + b*x**2)*a*b**4*x**4 + 6*sqrt(a
 + b*x**2)*b**5*x**7 + 15*sqrt(a + b*x**2)*b**4*c*x**7 - 6*sqrt(b)*a**4*b
+ 15*sqrt(b)*a**4*c - 24*sqrt(b)*a**3*b**2*x**2 + 60*sqrt(b)*a**3*b*c*x**2
 - 36*sqrt(b)*a**2*b**3*x**4 + 90*sqrt(b)*a**2*b**2*c*x**4 - 24*sqrt(b)*a*
b**4*x**6 + 60*sqrt(b)*a*b**3*c*x**6 - 6*sqrt(b)*b**5*x**8 + 15*sqrt(b)*b*
*4*c*x**8)/(105*a*b**4*(a**4 + 4*a**3*b*x**2 + 6*a**2*b**2*x**4 + 4*a*b**3
*x**6 + b**4*x**8))
```

# 3.5 $\int \frac{x^3(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 120

$$\int \frac{x^3(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = \frac{a(Ab-aC+bBx)}{7b^3(a+bx^2)^{7/2}} - \frac{7(Ab-2aC)+8bBx}{35b^3(a+bx^2)^{5/2}} - \frac{35aC-3bBx}{105ab^3(a+bx^2)^{3/2}} + \frac{2Bx}{35a^2b^2\sqrt{a+bx^2}}$$

output
```
1/7*a*(B*b*x+A*b-C*a)/b^3/(b*x^2+a)^(7/2)-1/35*(8*B*b*x+7*A*b-14*C*a)/b^3/
(b*x^2+a)^(5/2)-1/105*(-3*B*b*x+35*C*a)/a/b^3/(b*x^2+a)^(3/2)+2/35*B*x/a^2
/b^2/(b*x^2+a)^(1/2)
```

## Mathematica [A] (verified)

Time = 1.24 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.70

$$\int \frac{x^3(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = \frac{-8a^4C+21ab^3Bx^5+6b^4Bx^7-7a^2b^2x^2(3A+5Cx^2)-2a^3b(3A+14Cx^2)}{105a^2b^3(a+bx^2)^{7/2}}$$

input
```
Integrate[(x^3*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

output
```
(-8*a^4*C + 21*a*b^3*B*x^5 + 6*b^4*B*x^7 - 7*a^2*b^2*x^2*(3*A + 5*C*x^2) -
 2*a^3*b*(3*A + 14*C*x^2))/(105*a^2*b^3*(a + b*x^2)^(7/2))
```

## Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.21, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {2335, 25, 530, 25, 27, 454, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\left(A+Bx+Cx^2\right)}{\left(a+bx^2\right)^{9/2}}dx$$

$$\Big\downarrow \text{ 2335}$$

$$-\frac{\int -\frac{x^2(3aB+(3Ab+4aC)x)}{(bx^2+a)^{7/2}}dx}{7ab}-\frac{x^3(aB-x(Ab-aC))}{7ab\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{x^2(3aB+(3Ab+4aC)x)}{(bx^2+a)^{7/2}}dx}{7ab}-\frac{x^3(aB-x(Ab-aC))}{7ab\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \text{ 530}$$

$$\frac{\frac{a(4aC+3Ab-3bBx)}{5b^2(a+bx^2)^{5/2}}-\frac{\int -\frac{a\left(3aB+5b\left(3A+\frac{4aC}{b}\right)x\right)}{b\left(bx^2+a\right)^{5/2}}dx}{5a}}{7ab}-\frac{x^3(aB-x(Ab-aC))}{7ab\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\frac{\int \frac{a(3aB+5(3Ab+4aC)x)}{b\left(bx^2+a\right)^{5/2}}dx}{5a}+\frac{a(4aC+3Ab-3bBx)}{5b^2(a+bx^2)^{5/2}}}{7ab}-\frac{x^3(aB-x(Ab-aC))}{7ab\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{\int \frac{3aB+5(3Ab+4aC)x}{(bx^2+a)^{5/2}}dx}{5b}+\frac{a(4aC+3Ab-3bBx)}{5b^2(a+bx^2)^{5/2}}}{7ab}-\frac{x^3(aB-x(Ab-aC))}{7ab\,(a+bx^2)^{7/2}}$$

$$\Big\downarrow \text{ 454}$$

$$\frac{\frac{2B\int \frac{1}{(bx^2+a)^{3/2}}dx-\frac{5(4aC+3Ab)-3bBx}{3b\left(a+bx^2\right)^{3/2}}}{5b}+\frac{a(4aC+3Ab-3bBx)}{5b^2(a+bx^2)^{5/2}}}{7ab}-\frac{x^3(aB-x(Ab-aC))}{7ab\,(a+bx^2)^{7/2}}$$

$$\Big\downarrow \text{ 208}$$

$$\frac{\frac{a(4aC+3Ab-3bBx)}{5b^2(a+bx^2)^{5/2}}+\frac{\frac{2Bx}{a\sqrt{a+bx^2}}-\frac{5(4aC+3Ab)-3bBx}{3b\left(a+bx^2\right)^{3/2}}}{5b}}{7ab}-\frac{x^3(aB-x(Ab-aC))}{7ab\,(a+bx^2)^{7/2}}$$

input
```
Int[(x^3*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

output
```
-1/7*(x^3*(a*B - (A*b - a*C)*x))/(a*b*(a + b*x^2)^(7/2)) + ((a*(3*A*b + 4*
a*C - 3*b*B*x))/(5*b^2*(a + b*x^2)^(5/2)) + (-1/3*(5*(3*A*b + 4*a*C) - 3*b
*B*x)/(b*(a + b*x^2)^(3/2)) + (2*B*x)/(a*Sqrt[a + b*x^2]))/(5*b))/(7*a*b)
```

**Defintions of rubi rules used**

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 454
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*d
 - b*c*x)/(2*a*b*(p + 1)))*(a + b*x^2)^(p + 1), x] + Simp[c*((2*p + 3)/(2*a
*(p + 1)))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d}, x] && L
tQ[p, -1] && NeQ[p, -3/2]
```

rule 530
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symb
ol] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Co
eff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Po
lynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x
)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(a
 + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Qx + e*(2*p + 3), x], x], x]] /;
FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && IGtQ[m, 0] && LtQ[p, -1] && EqQ[n,
1] && IntegerQ[2*p]
```

rule 2335
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Simp[c/(2*a*b*(p + 1))   Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSu
m[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
 b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

## Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.71

| method | result |
|---|---|
| gosper | $-\frac{-6B\,x^{7} b^{4}-21B\,x^{5} a\,b^{3}+35C\,a^{2} b^{2} x^{4}+21A\,a^{2} b^{2} x^{2}+28C\,a^{3} b\,x^{2}+6A\,a^{3} b+8C\,a^{4}}{105\left(b\,x^{2}+a\right)^{\frac{7}{2}} a^{2} b^{3}}$ |
| trager | $-\frac{-6B\,x^{7} b^{4}-21B\,x^{5} a\,b^{3}+35C\,a^{2} b^{2} x^{4}+21A\,a^{2} b^{2} x^{2}+28C\,a^{3} b\,x^{2}+6A\,a^{3} b+8C\,a^{4}}{105\left(b\,x^{2}+a\right)^{\frac{7}{2}} a^{2} b^{3}}$ |
| orering | $-\frac{-6B\,x^{7} b^{4}-21B\,x^{5} a\,b^{3}+35C\,a^{2} b^{2} x^{4}+21A\,a^{2} b^{2} x^{2}+28C\,a^{3} b\,x^{2}+6A\,a^{3} b+8C\,a^{4}}{105\left(b\,x^{2}+a\right)^{\frac{7}{2}} a^{2} b^{3}}$ |
| default | $A\left(-\frac{x^{2}}{5b\left(b\,x^{2}+a\right)^{\frac{7}{2}}}-\frac{2a}{35b^{2}\left(b\,x^{2}+a\right)^{\frac{7}{2}}}\right)+B\left(-\frac{x^{3}}{4b\left(b\,x^{2}+a\right)^{\frac{7}{2}}}+\frac{3a\left(-\frac{x}{6b\left(b\,x^{2}+a\right)^{\frac{7}{2}}}+\frac{a\left(\frac{x}{7a\left(b\,x^{2}+a\right)^{\frac{7}{2}}}+\frac{\frac{6x}{35a\left(b\,x^{2}+a\right)^{\frac{5}{2}}}+\cdots}{\cdots}\right)}{6b\cdots}\right)}{4b}\cdots\right.$ |

input
```
int(x^3*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-1/105*(-6*B*b^4*x^7-21*B*a*b^3*x^5+35*C*a^2*b^2*x^4+21*A*a^2*b^2*x^2+28*C
*a^3*b*x^2+6*A*a^3*b+8*C*a^4)/(b*x^2+a)^(7/2)/a^2/b^3
```

## Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.09

$$\int \frac{x^3(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = \frac{(6\,Bb^4x^7+21\,Bab^3x^5-35\,Ca^2b^2x^4-8\,Ca^4-6\,Aa^3b-7\,(4\,Ca^3b+3\,Aa^2b^2)x}{105\,(a^2b^7x^8+4\,a^3b^6x^6+6\,a^4b^5x^4+4\,a^5b^4x^2+a^6b^3)}$$

input

```
integrate(x^3*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

output

```
1/105*(6*B*b^4*x^7 + 21*B*a*b^3*x^5 - 35*C*a^2*b^2*x^4 - 8*C*a^4 - 6*A*a^3
*b - 7*(4*C*a^3*b + 3*A*a^2*b^2)*x^2)*sqrt(b*x^2 + a)/(a^2*b^7*x^8 + 4*a^3
*b^6*x^6 + 6*a^4*b^5*x^4 + 4*a^5*b^4*x^2 + a^6*b^3)
```

## Sympy [A] (verification not implemented)

Time = 23.24 (sec) , antiderivative size = 660, normalized size of antiderivative = 5.50

$$\int \frac{x^3(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = A\left(\left\{\begin{array}{l} -\frac{2a}{35a^3b^2\sqrt{a+bx^2}+105a^2b^3x^2\sqrt{a+bx^2}+105ab^4x^4\sqrt{a+bx^2}+35b^5x^6\sqrt{a+bx^2}} - \frac{}{35a^3b^2\sqrt{a+bx^2}+1} \\ \frac{x^4}{4a^{\frac{9}{2}}} \end{array}\right.\right.$$
$$+B\left(\frac{7ax^5}{35a^{\frac{11}{2}}\sqrt{1+\frac{bx^2}{a}}+105a^{\frac{9}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}+105a^{\frac{7}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}}+35a^{\frac{5}{2}}b^3x^6\sqrt{1+\frac{bx^2}{a}}}\right.$$
$$\left.+\frac{2bx^7}{35a^{\frac{11}{2}}\sqrt{1+\frac{bx^2}{a}}+105a^{\frac{9}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}+105a^{\frac{7}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}}+35a^{\frac{5}{2}}b^3x^6\sqrt{1+\frac{bx^2}{a}}}\right)$$
$$+C\left(\left\{\begin{array}{l} -\frac{8a^2}{105a^3b^3\sqrt{a+bx^2}+315a^2b^4x^2\sqrt{a+bx^2}+315ab^5x^4\sqrt{a+bx^2}+105b^6x^6\sqrt{a+bx^2}} - \frac{28abx^2}{105a^3b^3\sqrt{a+bx^2}+315a^2b^4x^2\sqrt{a+bx^2}+315ab^5x^4\sqrt{a}} \\ \frac{x^6}{6a^{\frac{9}{2}}} \end{array}\right.\right.$$

input

```
integrate(x**3*(C*x**2+B*x+A)/(b*x**2+a)**(9/2),x)
```

output
```
A*Piecewise((-2*a/(35*a**3*b**2*sqrt(a + b*x**2) + 105*a**2*b**3*x**2*sqrt
(a + b*x**2) + 105*a*b**4*x**4*sqrt(a + b*x**2) + 35*b**5*x**6*sqrt(a + b*
x**2)) - 7*b*x**2/(35*a**3*b**2*sqrt(a + b*x**2) + 105*a**2*b**3*x**2*sqrt
(a + b*x**2) + 105*a*b**4*x**4*sqrt(a + b*x**2) + 35*b**5*x**6*sqrt(a + b*
x**2)), Ne(b, 0)), (x**4/(4*a**(9/2)), True)) + B*(7*a*x**5/(35*a**(11/2)*
sqrt(1 + b*x**2/a) + 105*a**(9/2)*b*x**2*sqrt(1 + b*x**2/a) + 105*a**(7/2)
*b**2*x**4*sqrt(1 + b*x**2/a) + 35*a**(5/2)*b**3*x**6*sqrt(1 + b*x**2/a))
+ 2*b*x**7/(35*a**(11/2)*sqrt(1 + b*x**2/a) + 105*a**(9/2)*b*x**2*sqrt(1 +
 b*x**2/a) + 105*a**(7/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 35*a**(5/2)*b**3*
x**6*sqrt(1 + b*x**2/a))) + C*Piecewise((-8*a**2/(105*a**3*b**3*sqrt(a + b
*x**2) + 315*a**2*b**4*x**2*sqrt(a + b*x**2) + 315*a*b**5*x**4*sqrt(a + b*
x**2) + 105*b**6*x**6*sqrt(a + b*x**2)) - 28*a*b*x**2/(105*a**3*b**3*sqrt(
a + b*x**2) + 315*a**2*b**4*x**2*sqrt(a + b*x**2) + 315*a*b**5*x**4*sqrt(a
 + b*x**2) + 105*b**6*x**6*sqrt(a + b*x**2)) - 35*b**2*x**4/(105*a**3*b**3
*sqrt(a + b*x**2) + 315*a**2*b**4*x**2*sqrt(a + b*x**2) + 315*a*b**5*x**4*
sqrt(a + b*x**2) + 105*b**6*x**6*sqrt(a + b*x**2)), Ne(b, 0)), (x**6/(6*a*
*(9/2)), True))
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.49

$$\int \frac{x^3(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = -\frac{Cx^4}{3\,(bx^2+a)^{\frac{7}{2}}b} - \frac{Bx^3}{4\,(bx^2+a)^{\frac{7}{2}}b} - \frac{4\,Cax^2}{15\,(bx^2+a)^{\frac{7}{2}}b^2} - \frac{Ax^2}{5\,(bx^2+a)^{\frac{7}{2}}b} + \frac{3\,Bx}{140\,(bx^2+a)^{\frac{5}{2}}b^2} + \frac{2\,Bx}{35\,\sqrt{bx^2+a}a^2b^2} + \frac{Bx}{35\,(bx^2+a)^{\frac{3}{2}}ab^2} - \frac{3\,Bax}{28\,(bx^2+a)^{\frac{7}{2}}b^2} - \frac{8\,Ca^2}{105\,(bx^2+a)^{\frac{7}{2}}b^3} - \frac{2\,Aa}{35\,(bx^2+a)^{\frac{7}{2}}b^2}$$

input
```
integrate(x^3*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

output
```
-1/3*C*x^4/((b*x^2 + a)^(7/2)*b) - 1/4*B*x^3/((b*x^2 + a)^(7/2)*b) - 4/15*
C*a*x^2/((b*x^2 + a)^(7/2)*b^2) - 1/5*A*x^2/((b*x^2 + a)^(7/2)*b) + 3/140*
B*x/((b*x^2 + a)^(5/2)*b^2) + 2/35*B*x/(sqrt(b*x^2 + a)*a^2*b^2) + 1/35*B*
x/((b*x^2 + a)^(3/2)*a*b^2) - 3/28*B*a*x/((b*x^2 + a)^(7/2)*b^2) - 8/105*C
*a^2/((b*x^2 + a)^(7/2)*b^3) - 2/35*A*a/((b*x^2 + a)^(7/2)*b^2)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.79

$$\int \frac{x^3(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = \frac{\left(\left(3\left(\frac{2\,Bbx^2}{a^2}+\frac{7\,B}{a}\right)x-\frac{35\,C}{b}\right)x^2-\frac{7\,(4\,Ca^4b+3\,Aa^3b^2)}{a^3b^3}\right)x^2-\frac{2\,(4\,Ca^5+3\,Aa^4b)}{a^3b^3}}{105\,(bx^2+a)^{\frac{7}{2}}}$$

input
```
integrate(x^3*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="giac")
```

output
```
1/105*(((3*(2*B*b*x^2/a^2 + 7*B/a)*x - 35*C/b)*x^2 - 7*(4*C*a^4*b + 3*A*a^
3*b^2)/(a^3*b^3))*x^2 - 2*(4*C*a^5 + 3*A*a^4*b)/(a^3*b^3))/(b*x^2 + a)^(7/
2)
```

## Mupad [B] (verification not implemented)

Time = 2.29 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.11

$$\int \frac{x^3(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = \frac{\frac{a\left(\frac{A}{7\,b}-\frac{C\,a}{7\,b^2}\right)}{b}+\frac{B\,a\,x}{7\,b^2}}{(b\,x^2+a)^{7/2}}-\frac{\frac{C}{3\,b^3}-\frac{B\,x}{35\,a\,b^2}}{(b\,x^2+a)^{3/2}} + \frac{\frac{a\left(\frac{C}{5\,b^2}-\frac{7\,A\,b-7\,C\,a}{35\,a\,b^2}\right)}{b}-\frac{8\,B\,x}{35\,b^2}}{(b\,x^2+a)^{5/2}}+\frac{2\,B\,x}{35\,a^2\,b^2\,\sqrt{b\,x^2+a}}$$

input
```
int((x^3*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x)
```

output
```
((a*(A/(7*b) - (C*a)/(7*b^2)))/b + (B*a*x)/(7*b^2))/(a + b*x^2)^(7/2) - (C
/(3*b^3) - (B*x)/(35*a*b^2))/(a + b*x^2)^(3/2) + ((a*(C/(5*b^2) - (7*A*b -
 7*C*a)/(35*a*b^2)))/b - (8*B*x)/(35*b^2))/(a + b*x^2)^(5/2) + (2*B*x)/(35
*a^2*b^2*(a + b*x^2)^(1/2))
```

## Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.87

$$\int \frac{x^3(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = \frac{-6\sqrt{b\,x^2+a}\,a^4b-8\sqrt{b\,x^2+a}\,a^4c-21\sqrt{b\,x^2+a}\,a^3b^2x^2-28\sqrt{b\,x^2+a}\,a^3bc\,x^2}{}$$

input
```
int(x^3*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x)
```

output
```
( - 6*sqrt(a + b*x**2)*a**4*b - 8*sqrt(a + b*x**2)*a**4*c - 21*sqrt(a + b*
x**2)*a**3*b**2*x**2 - 28*sqrt(a + b*x**2)*a**3*b*c*x**2 - 35*sqrt(a + b*x
**2)*a**2*b**2*c*x**4 + 21*sqrt(a + b*x**2)*a*b**4*x**5 + 6*sqrt(a + b*x**
2)*b**5*x**7 - 6*sqrt(b)*a**4*b - 24*sqrt(b)*a**3*b**2*x**2 - 36*sqrt(b)*a
**2*b**3*x**4 - 24*sqrt(b)*a*b**4*x**6 - 6*sqrt(b)*b**5*x**8)/(105*a**2*b*
*3*(a**4 + 4*a**3*b*x**2 + 6*a**2*b**2*x**4 + 4*a*b**3*x**6 + b**4*x**8))
```

# 3.6 $\int \frac{x^2(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 133

$$\int \frac{x^2(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = \frac{aB-(Ab-aC)x}{7b^2(a+bx^2)^{7/2}} - \frac{7aB-(Ab-8aC)x}{35ab^2(a+bx^2)^{5/2}} + \frac{(4Ab+3aC)x}{105a^2b^2(a+bx^2)^{3/2}} + \frac{2(4Ab+3aC)x}{105a^3b^2\sqrt{a+bx^2}}$$

output
```
1/7*(B*a-(A*b-C*a)*x)/b^2/(b*x^2+a)^(7/2)-1/35*(7*B*a-(A*b-8*C*a)*x)/a/b^2
/(b*x^2+a)^(5/2)+1/105*(4*A*b+3*C*a)*x/a^2/b^2/(b*x^2+a)^(3/2)+2/105*(4*A*
b+3*C*a)*x/a^3/b^2/(b*x^2+a)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.93 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.65

$$\int \frac{x^2(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = \frac{-6a^4B-21a^3bBx^2+8Ab^4x^7+7a^2b^2x^3(5A+3Cx^2)+2ab^3x^5(14A+3Cx^2)}{105a^3b^2(a+bx^2)^{7/2}}$$

input
```
Integrate[(x^2*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

output
```
(-6*a^4*B - 21*a^3*b*B*x^2 + 8*A*b^4*x^7 + 7*a^2*b^2*x^3*(5*A + 3*C*x^2) +
 2*a*b^3*x^5*(14*A + 3*C*x^2))/(105*a^3*b^2*(a + b*x^2)^(7/2))
```

## Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {2335, 25, 530, 25, 27, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx$$

$$\Big\downarrow \text{2335}$$

$$-\frac{\int -\frac{x(2aB+(4Ab+3aC)x)}{(bx^2+a)^{7/2}}dx}{7ab}-\frac{x^2(aB-x(Ab-aC))}{7ab\,(a+bx^2)^{7/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{x(2aB+(4Ab+3aC)x)}{(bx^2+a)^{7/2}}dx}{7ab}-\frac{x^2(aB-x(Ab-aC))}{7ab\,(a+bx^2)^{7/2}}$$

$$\Big\downarrow \text{530}$$

$$\frac{-\frac{\int -\frac{a\left(4A+\frac{3aC}{b}\right)}{\left(bx^2+a\right)^{5/2}}dx}{5a}-\frac{x(3aC+4Ab)+2aB}{5b(a+bx^2)^{5/2}}}{7ab}-\frac{x^2(aB-x(Ab-aC))}{7ab\,(a+bx^2)^{7/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{a\left(4A+\frac{3aC}{b}\right)}{\left(bx^2+a\right)^{5/2}}dx}{5a}-\frac{x(3aC+4Ab)+2aB}{5b(a+bx^2)^{5/2}}}{7ab}-\frac{x^2(aB-x(Ab-aC))}{7ab\,(a+bx^2)^{7/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{5}\left(\frac{3aC}{b}+4A\right)\int \frac{1}{(bx^2+a)^{5/2}}dx-\frac{x(3aC+4Ab)+2aB}{5b(a+bx^2)^{5/2}}}{7ab}-\frac{x^2(aB-x(Ab-aC))}{7ab\,(a+bx^2)^{7/2}}$$

$$\Big\downarrow \text{209}$$

$$\frac{\frac{1}{5}\left(\frac{3aC}{b}+4A\right)\left(\frac{2\int \frac{1}{\left(bx^2+a\right)^{3/2}}dx}{3a}+\frac{x}{3a(a+bx^2)^{3/2}}\right)-\frac{x(3aC+4Ab)+2aB}{5b(a+bx^2)^{5/2}}}{7ab}-\frac{x^2(aB-x(Ab-aC))}{7ab\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \text{208}$$

$$\frac{\frac{1}{5}\left(\frac{2x}{3a^2\sqrt{a+bx^2}}+\frac{x}{3a(a+bx^2)^{3/2}}\right)\left(\frac{3aC}{b}+4A\right)-\frac{x(3aC+4Ab)+2aB}{5b(a+bx^2)^{5/2}}}{7ab}-\frac{x^2(aB-x(Ab-aC))}{7ab\left(a+bx^2\right)^{7/2}}$$

input
```
Int[(x^2*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

output
```
-1/7*(x^2*(a*B - (A*b - a*C)*x))/(a*b*(a + b*x^2)^(7/2)) + (-1/5*(2*a*B +
(4*A*b + 3*a*C)*x)/(b*(a + b*x^2)^(5/2)) + ((4*A + (3*a*C)/b)*(x/(3*a*(a +
 b*x^2)^(3/2)) + (2*x)/(3*a^2*Sqrt[a + b*x^2])))/5)/(7*a*b)
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 530
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symb
ol] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Co
eff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Po
lynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x
)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(a
 + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Qx + e*(2*p + 3), x], x], x]] /;
FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && IGtQ[m, 0] && LtQ[p, -1] && EqQ[n,
1] && IntegerQ[2*p]
```

rule 2335
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Simp[c/(2*a*b*(p + 1))   Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSu
m[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
 b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

## Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.66

| method | result |
|---|---|
| gosper | $\frac{8A\,b^4x^7+6Ca\,x^7b^3+28Aa\,b^3x^5+21C\,a^2b^2x^5+35A\,a^2b^2x^3-21B\,a^3b\,x^2-6B\,a^4}{105(b\,x^2+a)^{\frac{7}{2}}a^3b^2}$ |
| trager | $\frac{8A\,b^4x^7+6Ca\,x^7b^3+28Aa\,b^3x^5+21C\,a^2b^2x^5+35A\,a^2b^2x^3-21B\,a^3b\,x^2-6B\,a^4}{105(b\,x^2+a)^{\frac{7}{2}}a^3b^2}$ |
| orering | $\frac{8A\,b^4x^7+6Ca\,x^7b^3+28Aa\,b^3x^5+21C\,a^2b^2x^5+35A\,a^2b^2x^3-21B\,a^3b\,x^2-6B\,a^4}{105(b\,x^2+a)^{\frac{7}{2}}a^3b^2}$ |
| default | $A\left(-\frac{x}{6b(b\,x^2+a)^{\frac{7}{2}}}+\frac{a\left(\frac{x}{7a\left(b\,x^2+a\right)^{\frac{7}{2}}}+\frac{\frac{6x}{35a\left(b\,x^2+a\right)^{\frac{5}{2}}}+\frac{6\left(\frac{4x}{15a\left(b\,x^2+a\right)^{\frac{3}{2}}}+\frac{8x}{15a^2\sqrt{b\,x^2+a}}\right)}{7a}}{a}\right)}{6b}\right)+B\left(-\frac{x^2}{5b(b\,x^2+a)^{\frac{7}{2}}}-\frac{3}{}\right.$ |

input
```
int(x^2*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
1/105*(8*A*b^4*x^7+6*C*a*b^3*x^7+28*A*a*b^3*x^5+21*C*a^2*b^2*x^5+35*A*a^2*
b^2*x^3-21*B*a^3*b*x^2-6*B*a^4)/(b*x^2+a)^(7/2)/a^3/b^2
```

## Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.01

$$\int \frac{x^2(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = \frac{(35\,Aa^2b^2x^3+2\,(3\,Cab^3+4\,Ab^4)x^7-21\,Ba^3bx^2+7\,(3\,Ca^2b^2+4\,Aab^3)x^5-6}{105\,(a^3b^6x^8+4\,a^4b^5x^6+6\,a^5b^4x^4+4\,a^6b^3x^2+a^7b^2)}$$

input
```
integrate(x^2*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

output
```
1/105*(35*A*a^2*b^2*x^3 + 2*(3*C*a*b^3 + 4*A*b^4)*x^7 - 21*B*a^3*b*x^2 + 7
*(3*C*a^2*b^2 + 4*A*a*b^3)*x^5 - 6*B*a^4)*sqrt(b*x^2 + a)/(a^3*b^6*x^8 + 4
*a^4*b^5*x^6 + 6*a^5*b^4*x^4 + 4*a^6*b^3*x^2 + a^7*b^2)
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 518 vs. 2(122) = 244.

Time = 37.44 (sec) , antiderivative size = 904, normalized size of antiderivative = 6.80

$$\int \frac{x^2(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x**2*(C*x**2+B*x+A)/(b*x**2+a)**(9/2),x)
```

output
```
A*(35*a**5*x**3/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a**(17/2)*b*x**2*s
qrt(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 420*a**(1
3/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*sqrt(1 + b*x**
2/a)) + 63*a**4*b*x**5/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a**(17/2)*b
*x**2*sqrt(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 42
0*a**(13/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*sqrt(1
+ b*x**2/a)) + 36*a**3*b**2*x**7/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a
**(17/2)*b*x**2*sqrt(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x*
*2/a) + 420*a**(13/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x*
*8*sqrt(1 + b*x**2/a)) + 8*a**2*b**3*x**9/(105*a**(19/2)*sqrt(1 + b*x**2/a
) + 420*a**(17/2)*b*x**2*sqrt(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt
(1 + b*x**2/a) + 420*a**(13/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2
)*b**4*x**8*sqrt(1 + b*x**2/a))) + B*Piecewise((-2*a/(35*a**3*b**2*sqrt(a
+ b*x**2) + 105*a**2*b**3*x**2*sqrt(a + b*x**2) + 105*a*b**4*x**4*sqrt(a +
 b*x**2) + 35*b**5*x**6*sqrt(a + b*x**2)) - 7*b*x**2/(35*a**3*b**2*sqrt(a
+ b*x**2) + 105*a**2*b**3*x**2*sqrt(a + b*x**2) + 105*a*b**4*x**4*sqrt(a +
 b*x**2) + 35*b**5*x**6*sqrt(a + b*x**2)), Ne(b, 0)), (x**4/(4*a**(9/2)),
True)) + C*(7*a*x**5/(35*a**(11/2)*sqrt(1 + b*x**2/a) + 105*a**(9/2)*b*x**
2*sqrt(1 + b*x**2/a) + 105*a**(7/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 35*a**(
5/2)*b**3*x**6*sqrt(1 + b*x**2/a)) + 2*b*x**7/(35*a**(11/2)*sqrt(1 + b*...
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.48

$$\int \frac{x^2(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = -\frac{Cx^3}{4\,(bx^2+a)^{\frac{7}{2}}b} - \frac{Bx^2}{5\,(bx^2+a)^{\frac{7}{2}}b} + \frac{3\,Cx}{140\,(bx^2+a)^{\frac{5}{2}}b^2} + \frac{2\,Cx}{35\,\sqrt{bx^2+a}a^2b^2} + \frac{Cx}{35\,(bx^2+a)^{\frac{3}{2}}ab^2} - \frac{3\,Cax}{28\,(bx^2+a)^{\frac{7}{2}}b^2} - \frac{Ax}{7\,(bx^2+a)^{\frac{7}{2}}b} + \frac{8\,Ax}{105\,\sqrt{bx^2+a}a^3b} + \frac{4\,Ax}{105\,(bx^2+a)^{\frac{3}{2}}a^2b} + \frac{Ax}{35\,(bx^2+a)^{\frac{5}{2}}ab} - \frac{2\,Ba}{35\,(bx^2+a)^{\frac{7}{2}}b^2}$$

input
```
integrate(x^2*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

output
```
-1/4*C*x^3/((b*x^2 + a)^(7/2)*b) - 1/5*B*x^2/((b*x^2 + a)^(7/2)*b) + 3/140
*C*x/((b*x^2 + a)^(5/2)*b^2) + 2/35*C*x/(sqrt(b*x^2 + a)*a^2*b^2) + 1/35*C
*x/((b*x^2 + a)^(3/2)*a*b^2) - 3/28*C*a*x/((b*x^2 + a)^(7/2)*b^2) - 1/7*A*
x/((b*x^2 + a)^(7/2)*b) + 8/105*A*x/(sqrt(b*x^2 + a)*a^3*b) + 4/105*A*x/((
b*x^2 + a)^(3/2)*a^2*b) + 1/35*A*x/((b*x^2 + a)^(5/2)*a*b) - 2/35*B*a/((b*
x^2 + a)^(7/2)*b^2)
```

## Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.71

$$\int \frac{x^2(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = \frac{\left(\left(x^2\left(\frac{2\,(3\,Cab^4+4\,Ab^5)x^2}{a^3b^3}+\frac{7\,(3\,Ca^2b^3+4\,Aab^4)}{a^3b^3}\right)+\frac{35\,A}{a}\right)x-\frac{21\,B}{b}\right)x^2-\frac{6\,Ba}{b^2}}{105\,(bx^2+a)^{\frac{7}{2}}}$$

input
```
integrate(x^2*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="giac")
```

output
```
1/105*(((x^2*(2*(3*C*a*b^4 + 4*A*b^5)*x^2/(a^3*b^3) + 7*(3*C*a^2*b^3 + 4*A
*a*b^4)/(a^3*b^3)) + 35*A/a)*x - 21*B/b)*x^2 - 6*B*a/b^2)/(b*x^2 + a)^(7/2
)
```

## Mupad [B] (verification not implemented)

Time = 2.24 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00

$$\int \frac{x^2(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = \frac{x\,(4\,A\,b+3\,C\,a)}{105\,a^2\,b^2\,(b\,x^2+a)^{3/2}} - \frac{\frac{B}{5\,b^2}+x\left(\frac{C}{5\,b^2}-\frac{A\,b-C\,a}{35\,a\,b^2}\right)}{(b\,x^2+a)^{5/2}} - \frac{x\left(\frac{A}{7\,b}-\frac{C\,a}{7\,b^2}\right)-\frac{B\,a}{7\,b^2}}{(b\,x^2+a)^{7/2}} + \frac{x\,(8\,A\,b+6\,C\,a)}{105\,a^3\,b^2\,\sqrt{b\,x^2+a}}$$

input
```
int((x^2*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x)
```

output
```
(x*(4*A*b + 3*C*a))/(105*a^2*b^2*(a + b*x^2)^(3/2)) - (B/(5*b^2) + x*(C/(5
*b^2) - (A*b - C*a)/(35*a*b^2)))/(a + b*x^2)^(5/2) - (x*(A/(7*b) - (C*a)/(
7*b^2)) - (B*a)/(7*b^2))/(a + b*x^2)^(7/2) + (x*(8*A*b + 6*C*a))/(105*a^3*
b^2*(a + b*x^2)^(1/2))
```

## Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 285, normalized size of antiderivative = 2.14

$$\int \frac{x^2(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = \frac{-6\sqrt{b\,x^2+a}\,a^3b^2+35\sqrt{b\,x^2+a}\,a^2b^3x^3-21\sqrt{b\,x^2+a}\,a^2b^3x^2+28\sqrt{b\,x^2+a}\,a}{}$$

input
```
int(x^2*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x)
```

output
```
( - 6*sqrt(a + b*x**2)*a**3*b**2 + 35*sqrt(a + b*x**2)*a**2*b**3*x**3 - 21
*sqrt(a + b*x**2)*a**2*b**3*x**2 + 28*sqrt(a + b*x**2)*a*b**4*x**5 + 21*sq
rt(a + b*x**2)*a*b**3*c*x**5 + 8*sqrt(a + b*x**2)*b**5*x**7 + 6*sqrt(a + b
*x**2)*b**4*c*x**7 - 8*sqrt(b)*a**4*b - 6*sqrt(b)*a**4*c - 32*sqrt(b)*a**3
*b**2*x**2 - 24*sqrt(b)*a**3*b*c*x**2 - 48*sqrt(b)*a**2*b**3*x**4 - 36*sqr
t(b)*a**2*b**2*c*x**4 - 32*sqrt(b)*a*b**4*x**6 - 24*sqrt(b)*a*b**3*c*x**6
- 8*sqrt(b)*b**5*x**8 - 6*sqrt(b)*b**4*c*x**8)/(105*a**2*b**3*(a**4 + 4*a*
*3*b*x**2 + 6*a**2*b**2*x**4 + 4*a*b**3*x**6 + b**4*x**8))
```

# 3.7 $\int \frac{x(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 108

$$\int \frac{x(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = -\frac{Ab-aC+bBx}{7b^2(a+bx^2)^{7/2}} - \frac{7aC-bBx}{35ab^2(a+bx^2)^{5/2}} + \frac{4Bx}{105a^2b(a+bx^2)^{3/2}} + \frac{8Bx}{105a^3b\sqrt{a+bx^2}}$$

output
```
-1/7*(B*b*x+A*b-C*a)/b^2/(b*x^2+a)^(7/2)-1/35*(-B*b*x+7*C*a)/a/b^2/(b*x^2+
a)^(5/2)+4/105*B*x/a^2/b/(b*x^2+a)^(3/2)+8/105*B*x/a^3/b/(b*x^2+a)^(1/2)
```

## Mathematica [A] (verified)

Time = 1.11 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.70

$$\int \frac{x(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = \frac{-15a^3Ab-6a^4C-21a^3bCx^2+35a^2b^2Bx^3+28ab^3Bx^5+8b^4Bx^7}{105a^3b^2(a+bx^2)^{7/2}}$$

input
```
Integrate[(x*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

output
```
(-15*a^3*A*b - 6*a^4*C - 21*a^3*b*C*x^2 + 35*a^2*b^2*B*x^3 + 28*a*b^3*B*x^
5 + 8*b^4*B*x^7)/(105*a^3*b^2*(a + b*x^2)^(7/2))
```

## Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.16, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2335, 25, 454, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}dx$$

$$\Big\downarrow \text{2335}$$

$$-\frac{\int -\frac{aB+(5Ab+2aC)x}{(bx^2+a)^{7/2}}dx}{7ab}-\frac{x(aB-x(Ab-aC))}{7ab\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{aB+(5Ab+2aC)x}{(bx^2+a)^{7/2}}dx}{7ab}-\frac{x(aB-x(Ab-aC))}{7ab\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \text{454}$$

$$\frac{\frac{4}{5}B\int \frac{1}{(bx^2+a)^{5/2}}dx-\frac{2aC+5Ab-bBx}{5b(a+bx^2)^{5/2}}}{7ab}-\frac{x(aB-x(Ab-aC))}{7ab\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \text{209}$$

$$\frac{\frac{4}{5}B\left(\frac{2\int \frac{1}{\left(bx^2+a\right)^{3/2}}dx}{3a}+\frac{x}{3a(a+bx^2)^{3/2}}\right)-\frac{2aC+5Ab-bBx}{5b(a+bx^2)^{5/2}}}{7ab}-\frac{x(aB-x(Ab-aC))}{7ab\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \text{208}$$

$$\frac{\frac{4}{5}B\left(\frac{2x}{3a^2\sqrt{a+bx^2}}+\frac{x}{3a(a+bx^2)^{3/2}}\right)-\frac{2aC+5Ab-bBx}{5b(a+bx^2)^{5/2}}}{7ab}-\frac{x(aB-x(Ab-aC))}{7ab\left(a+bx^2\right)^{7/2}}$$

input
```
Int[(x*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

output
```
-1/7*(x*(a*B - (A*b - a*C)*x))/(a*b*(a + b*x^2)^(7/2)) + (-1/5*(5*A*b + 2*
a*C - b*B*x)/(b*(a + b*x^2)^(5/2)) + (4*B*(x/(3*a*(a + b*x^2)^(3/2)) + (2*
x)/(3*a^2*Sqrt[a + b*x^2])))/5)/(7*a*b)
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 454
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*d
 - b*c*x)/(2*a*b*(p + 1)))*(a + b*x^2)^(p + 1), x] + Simp[c*((2*p + 3)/(2*a
*(p + 1)))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d}, x] && L
tQ[p, -1] && NeQ[p, -3/2]
```

rule 2335
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Simp[c/(2*a*b*(p + 1))   Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSu
m[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
 b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

## Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.68

| method | result |
|---|---|
| gosper | $-\frac{-8B\,x^{7} b^{4}-28B\,x^{5} a\,b^{3}-35B\,a^{2} b^{2} x^{3}+21C\,a^{3} b\,x^{2}+15A\,a^{3} b+6C\,a^{4}}{105\left(b\,x^{2}+a\right)^{\frac{7}{2}} a^{3} b^{2}}$ |
| trager | $-\frac{-8B\,x^{7} b^{4}-28B\,x^{5} a\,b^{3}-35B\,a^{2} b^{2} x^{3}+21C\,a^{3} b\,x^{2}+15A\,a^{3} b+6C\,a^{4}}{105\left(b\,x^{2}+a\right)^{\frac{7}{2}} a^{3} b^{2}}$ |
| orering | $-\frac{-8B\,x^{7} b^{4}-28B\,x^{5} a\,b^{3}-35B\,a^{2} b^{2} x^{3}+21C\,a^{3} b\,x^{2}+15A\,a^{3} b+6C\,a^{4}}{105\left(b\,x^{2}+a\right)^{\frac{7}{2}} a^{3} b^{2}}$ |
| default | $C\left(-\frac{x^{2}}{5b\left(b\,x^{2}+a\right)^{\frac{7}{2}}}-\frac{2a}{35b^{2}\left(b\,x^{2}+a\right)^{\frac{7}{2}}}\right)-\frac{A}{7b\left(b\,x^{2}+a\right)^{\frac{7}{2}}}+B\left(-\frac{x}{6b\left(b\,x^{2}+a\right)^{\frac{7}{2}}}+\frac{a\left(\frac{x}{7a\left(b\,x^{2}+a\right)^{\frac{7}{2}}}+\frac{\frac{6x}{35a\left(b\,x^{2}+a\right)^{\frac{5}{2}}}+\frac{6\left(-\ldots\right.}{15\ldots}}{\ldots}\right.}{6b}\right.$ |

input
```
int(x*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-1/105*(-8*B*b^4*x^7-28*B*a*b^3*x^5-35*B*a^2*b^2*x^3+21*C*a^3*b*x^2+15*A*a
^3*b+6*C*a^4)/(b*x^2+a)^(7/2)/a^3/b^2
```

## Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.10

$$\int \frac{x(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = \frac{(8\,Bb^4x^7+28\,Bab^3x^5+35\,Ba^2b^2x^3-21\,Ca^3bx^2-6\,Ca^4-15\,Aa^3b)\sqrt{bx^2+a}}{105\,(a^3b^6x^8+4\,a^4b^5x^6+6\,a^5b^4x^4+4\,a^6b^3x^2+a^7b^2)}$$

input
```
integrate(x*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

output
```
1/105*(8*B*b^4*x^7 + 28*B*a*b^3*x^5 + 35*B*a^2*b^2*x^3 - 21*C*a^3*b*x^2 -
6*C*a^4 - 15*A*a^3*b)*sqrt(b*x^2 + a)/(a^3*b^6*x^8 + 4*a^4*b^5*x^6 + 6*a^5
*b^4*x^4 + 4*a^6*b^3*x^2 + a^7*b^2)
```

## Sympy [A] (verification not implemented)

Time = 24.77 (sec) , antiderivative size = 796, normalized size of antiderivative = 7.37

$$\int \frac{x(A + Bx + Cx^2)}{(a + bx^2)^{9/2}}\, dx = \text{Too large to display}$$

input
```
integrate(x*(C*x**2+B*x+A)/(b*x**2+a)**(9/2),x)
```

output
```
A*Piecewise((-1/(7*a**3*b*sqrt(a + b*x**2) + 21*a**2*b**2*x**2*sqrt(a + b*
x**2) + 21*a*b**3*x**4*sqrt(a + b*x**2) + 7*b**4*x**6*sqrt(a + b*x**2)), N
e(b, 0)), (x**2/(2*a**(9/2)), True)) + B*(35*a**5*x**3/(105*a**(19/2)*sqrt
(1 + b*x**2/a) + 420*a**(17/2)*b*x**2*sqrt(1 + b*x**2/a) + 630*a**(15/2)*b
**2*x**4*sqrt(1 + b*x**2/a) + 420*a**(13/2)*b**3*x**6*sqrt(1 + b*x**2/a) +
 105*a**(11/2)*b**4*x**8*sqrt(1 + b*x**2/a)) + 63*a**4*b*x**5/(105*a**(19/
2)*sqrt(1 + b*x**2/a) + 420*a**(17/2)*b*x**2*sqrt(1 + b*x**2/a) + 630*a**(
15/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 420*a**(13/2)*b**3*x**6*sqrt(1 + b*x*
*2/a) + 105*a**(11/2)*b**4*x**8*sqrt(1 + b*x**2/a)) + 36*a**3*b**2*x**7/(1
05*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a**(17/2)*b*x**2*sqrt(1 + b*x**2/a)
+ 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 420*a**(13/2)*b**3*x**6*sqr
t(1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*sqrt(1 + b*x**2/a)) + 8*a**2*b**
3*x**9/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a**(17/2)*b*x**2*sqrt(1 + b
*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 420*a**(13/2)*b**3
*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*sqrt(1 + b*x**2/a))) +
C*Piecewise((-2*a/(35*a**3*b**2*sqrt(a + b*x**2) + 105*a**2*b**3*x**2*sqrt
(a + b*x**2) + 105*a*b**4*x**4*sqrt(a + b*x**2) + 35*b**5*x**6*sqrt(a + b*
x**2)) - 7*b*x**2/(35*a**3*b**2*sqrt(a + b*x**2) + 105*a**2*b**3*x**2*sqrt
(a + b*x**2) + 105*a*b**4*x**4*sqrt(a + b*x**2) + 35*b**5*x**6*sqrt(a + b*
x**2)), Ne(b, 0)), (x**4/(4*a**(9/2)), True))
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.14

$$\int \frac{x(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = -\frac{Cx^2}{5\,(bx^2+a)^{\frac{7}{2}}b} - \frac{Bx}{7\,(bx^2+a)^{\frac{7}{2}}b} + \frac{8\,Bx}{105\,\sqrt{bx^2+a}a^3b} + \frac{4\,Bx}{105\,(bx^2+a)^{\frac{3}{2}}a^2b} + \frac{Bx}{35\,(bx^2+a)^{\frac{5}{2}}ab} - \frac{2\,Ca}{35\,(bx^2+a)^{\frac{7}{2}}b^2} - \frac{A}{7\,(bx^2+a)^{\frac{7}{2}}b}$$

input
```
integrate(x*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

output
```
-1/5*C*x^2/((b*x^2 + a)^(7/2)*b) - 1/7*B*x/((b*x^2 + a)^(7/2)*b) + 8/105*B
*x/(sqrt(b*x^2 + a)*a^3*b) + 4/105*B*x/((b*x^2 + a)^(3/2)*a^2*b) + 1/35*B*
x/((b*x^2 + a)^(5/2)*a*b) - 2/35*C*a/((b*x^2 + a)^(7/2)*b^2) - 1/7*A/((b*x
^2 + a)^(7/2)*b)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.76

$$\int \frac{x(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = \frac{\left(\left(4\left(\frac{2\,Bb^2x^2}{a^3}+\frac{7\,Bb}{a^2}\right)x^2+\frac{35\,B}{a}\right)x-\frac{21\,C}{b}\right)x^2-\frac{3\,(2\,Ca^4b+5\,Aa^3b^2)}{a^3b^3}}{105\,(bx^2+a)^{\frac{7}{2}}}$$

input
```
integrate(x*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="giac")
```

output
```
1/105*(((4*(2*B*b^2*x^2/a^3 + 7*B*b/a^2)*x^2 + 35*B/a)*x - 21*C/b)*x^2 - 3
*(2*C*a^4*b + 5*A*a^3*b^2)/(a^3*b^3))/(b*x^2 + a)^(7/2)
```

## Mupad [B] (verification not implemented)

Time = 2.22 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.92

$$\int \frac{x(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = \frac{8\,B\,x}{105\,a^3\,b\,\sqrt{b\,x^2+a}} - \frac{\frac{A}{7\,b}-\frac{C\,a}{7\,b^2}+\frac{B\,x}{7\,b}}{\left(b\,x^2+a\right)^{7/2}} - \frac{\frac{C}{5\,b^2}-\frac{B\,x}{35\,a\,b}}{\left(b\,x^2+a\right)^{5/2}} + \frac{4\,B\,x}{105\,a^2\,b\,\left(b\,x^2+a\right)^{3/2}}$$

input
```
int((x*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x)
```

output
```
(8*B*x)/(105*a^3*b*(a + b*x^2)^(1/2)) - (A/(7*b) - (C*a)/(7*b^2) + (B*x)/(
7*b))/(a + b*x^2)^(7/2) - (C/(5*b^2) - (B*x)/(35*a*b))/(a + b*x^2)^(5/2) +
 (4*B*x)/(105*a^2*b*(a + b*x^2)^(3/2))
```

## Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.89

$$\int \frac{x(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = \frac{-15\sqrt{b\,x^2+a}\,a^4b-6\sqrt{b\,x^2+a}\,a^4c-21\sqrt{b\,x^2+a}\,a^3bc\,x^2+35\sqrt{b\,x^2+a}\,a^2b^3x}{105a^3b^2\,(b}$$

input
```
int(x*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x)
```

output
```
( - 15*sqrt(a + b*x**2)*a**4*b - 6*sqrt(a + b*x**2)*a**4*c - 21*sqrt(a + b
*x**2)*a**3*b*c*x**2 + 35*sqrt(a + b*x**2)*a**2*b**3*x**3 + 28*sqrt(a + b*
x**2)*a*b**4*x**5 + 8*sqrt(a + b*x**2)*b**5*x**7 - 8*sqrt(b)*a**4*b - 32*s
qrt(b)*a**3*b**2*x**2 - 48*sqrt(b)*a**2*b**3*x**4 - 32*sqrt(b)*a*b**4*x**6
 - 8*sqrt(b)*b**5*x**8)/(105*a**3*b**2*(a**4 + 4*a**3*b*x**2 + 6*a**2*b**2
*x**4 + 4*a*b**3*x**6 + b**4*x**8))
```

# 3.8 $\int \frac{A+Bx+Cx^2}{(a+bx^2)^{9/2}}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 127

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^{9/2}}\,dx = -\frac{aB-(Ab-aC)x}{7ab\,(a+bx^2)^{7/2}} + \frac{(6Ab+aC)x}{35a^2b\,(a+bx^2)^{5/2}} + \frac{4(6Ab+aC)x}{105a^3b\,(a+bx^2)^{3/2}} + \frac{8(6Ab+aC)x}{105a^4b\sqrt{a+bx^2}}$$

output
```
-1/7*(B*a-(A*b-C*a)*x)/a/b/(b*x^2+a)^(7/2)+1/35*(6*A*b+C*a)*x/a^2/b/(b*x^2
+a)^(5/2)+4/105*(6*A*b+C*a)*x/a^3/b/(b*x^2+a)^(3/2)+8/105*(6*A*b+C*a)*x/a^
4/b/(b*x^2+a)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.72

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^{9/2}}\,dx = \frac{-15a^4B+48Ab^4x^7+35a^3bx(3A+Cx^2)+8ab^3x^5(21A+Cx^2)+14a^2b^2x^3(15A -}{105a^4b\,(a+bx^2)^{7/2}}$$

input
```
Integrate[(A + B*x + C*x^2)/(a + b*x^2)^(9/2),x]
```

output
```
(-15*a^4*B + 48*A*b^4*x^7 + 35*a^3*b*x*(3*A + C*x^2) + 8*a*b^3*x^5*(21*A +
 C*x^2) + 14*a^2*b^2*x^3*(15*A + 2*C*x^2))/(105*a^4*b*(a + b*x^2)^(7/2))
```

## Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {2345, 25, 27, 209, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^{9/2}}dx$$

$$\Big\downarrow \text{2345}$$

$$-\frac{\int -\frac{6Ab+aC}{b(bx^2+a)^{7/2}}dx}{7a}-\frac{aB-x(Ab-aC)}{7ab\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{6Ab+aC}{b(bx^2+a)^{7/2}}dx}{7a}-\frac{aB-x(Ab-aC)}{7ab\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(aC+6Ab)\int \frac{1}{(bx^2+a)^{7/2}}dx}{7ab}-\frac{aB-x(Ab-aC)}{7ab\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \text{209}$$

$$\frac{(aC+6Ab)\left(\frac{4\int \frac{1}{\left(bx^2+a\right)^{5/2}}dx}{5a}+\frac{x}{5a(a+bx^2)^{5/2}}\right)}{7ab}-\frac{aB-x(Ab-aC)}{7ab\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \text{209}$$

$$\frac{(aC+6Ab)\left(\frac{4\left(\frac{2\int\frac{1}{\left(bx^2+a\right)^{3/2}}dx}{3a}+\frac{x}{3a\left(a+bx^2\right)^{3/2}}\right)}{5a}+\frac{x}{5a(a+bx^2)^{5/2}}\right)}{7ab}-\frac{aB-x(Ab-aC)}{7ab\left(a+bx^2\right)^{7/2}}$$

$\big\downarrow$ 208

$$\frac{\left(\frac{4\left(\frac{2x}{3a^2\sqrt{a+bx^2}}+\frac{x}{3a\left(a+bx^2\right)^{3/2}}\right)}{5a}+\frac{x}{5a(a+bx^2)^{5/2}}\right)(aC+6Ab)}{7ab}-\frac{aB-x(Ab-aC)}{7ab\left(a+bx^2\right)^{7/2}}$$

input
```
Int[(A + B*x + C*x^2)/(a + b*x^2)^(9/2),x]
```

output
```
-1/7*(a*B - (A*b - a*C)*x)/(a*b*(a + b*x^2)^(7/2)) + ((6*A*b + a*C)*(x/(5*
a*(a + b*x^2)^(5/2)) + (4*(x/(3*a*(a + b*x^2)^(3/2)) + (2*x)/(3*a^2*Sqrt[a
 + b*x^2])))/(5*a)))/(7*a*b)
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

## Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.76

| method | result |
|---|---|
| gosper | $\frac{48A\,b^4x^7+8Ca\,x^7b^3+168Aa\,b^3x^5+28C\,a^2b^2x^5+210A\,a^2b^2x^3+35C\,a^3x^3b+105A\,a^3bx-15B\,a^4}{105(b\,x^2+a)^{\frac{7}{2}}a^4b}$ |
| trager | $\frac{48A\,b^4x^7+8Ca\,x^7b^3+168Aa\,b^3x^5+28C\,a^2b^2x^5+210A\,a^2b^2x^3+35C\,a^3x^3b+105A\,a^3bx-15B\,a^4}{105(b\,x^2+a)^{\frac{7}{2}}a^4b}$ |
| orering | $\frac{48A\,b^4x^7+8Ca\,x^7b^3+168Aa\,b^3x^5+28C\,a^2b^2x^5+210A\,a^2b^2x^3+35C\,a^3x^3b+105A\,a^3bx-15B\,a^4}{105(b\,x^2+a)^{\frac{7}{2}}a^4b}$ |
| default | $A\left(\frac{x}{7a(b\,x^2+a)^{\frac{7}{2}}}+\frac{\frac{6x}{35a\left(b\,x^2+a\right)^{\frac{5}{2}}}+\frac{6\left(\frac{4x}{15a\left(b\,x^2+a\right)^{\frac{3}{2}}}+\frac{8x}{15a^2\sqrt{b\,x^2+a}}\right)}{7a}}{a}\right)-\frac{B}{7b(b\,x^2+a)^{\frac{7}{2}}}+C\left(-\frac{x}{6b(b\,x^2+a)^{\frac{7}{2}}}+\frac{a\left(\frac{}{7a(}\right.}{}\right.$ |

input
```
int((C*x^2+B*x+A)/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
1/105*(48*A*b^4*x^7+8*C*a*b^3*x^7+168*A*a*b^3*x^5+28*C*a^2*b^2*x^5+210*A*a
^2*b^2*x^3+35*C*a^3*b*x^3+105*A*a^3*b*x-15*B*a^4)/(b*x^2+a)^(7/2)/a^4/b
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.08

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^{9/2}}\,dx = \frac{(8\,(Cab^3+6\,Ab^4)x^7+105\,Aa^3bx+28\,(Ca^2b^2+6\,Aab^3)x^5-15\,Ba^4+35\,(Ca^3b+}{105\,(a^4b^5x^8+4\,a^5b^4x^6+6\,a^6b^3x^4+4\,a^7b^2x^2+a^8b)}$$

input
```
integrate((C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

output
```
1/105*(8*(C*a*b^3 + 6*A*b^4)*x^7 + 105*A*a^3*b*x + 28*(C*a^2*b^2 + 6*A*a*b
^3)*x^5 - 15*B*a^4 + 35*(C*a^3*b + 6*A*a^2*b^2)*x^3)*sqrt(b*x^2 + a)/(a^4*
b^5*x^8 + 4*a^5*b^4*x^6 + 6*a^6*b^3*x^4 + 4*a^7*b^2*x^2 + a^8*b)
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1266 vs. 2(117) = 234.

Time = 31.19 (sec) , antiderivative size = 1880, normalized size of antiderivative = 14.80

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^{9/2}}\,dx = \text{Too large to display}$$

input
```
integrate((C*x**2+B*x+A)/(b*x**2+a)**(9/2),x)
```

output
```
A*(35*a**14*x/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt
(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2
)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a
) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*
sqrt(1 + b*x**2/a)) + 175*a**13*b*x**3/(35*a**(37/2)*sqrt(1 + b*x**2/a) +
210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 +
 b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b*
*4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) +
 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a)) + 371*a**12*b**2*x**5/(35*a**
(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*
a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 +
b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**
5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a)) +
 429*a**11*b**3*x**7/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x*
*2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a
**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b
*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6
*x**12*sqrt(1 + b*x**2/a)) + 286*a**10*b**4*x**9/(35*a**(37/2)*sqrt(1 + b*
x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**
4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525...
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.20

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^{9/2}}\,dx = \frac{16\,Ax}{35\,\sqrt{bx^2+a}a^4} + \frac{8\,Ax}{35\,(bx^2+a)^{\frac{3}{2}}a^3} + \frac{6\,Ax}{35\,(bx^2+a)^{\frac{5}{2}}a^2} + \frac{Ax}{7\,(bx^2+a)^{\frac{7}{2}}a} - \frac{Cx}{7\,(bx^2+a)^{\frac{7}{2}}b} + \frac{8\,Cx}{105\,\sqrt{bx^2+a}a^3b} + \frac{4\,Cx}{105\,(bx^2+a)^{\frac{3}{2}}a^2b} + \frac{Cx}{35\,(bx^2+a)^{\frac{5}{2}}ab} - \frac{B}{7\,(bx^2+a)^{\frac{7}{2}}b}$$

input
```
integrate((C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

output
```
16/35*A*x/(sqrt(b*x^2 + a)*a^4) + 8/35*A*x/((b*x^2 + a)^(3/2)*a^3) + 6/35*
A*x/((b*x^2 + a)^(5/2)*a^2) + 1/7*A*x/((b*x^2 + a)^(7/2)*a) - 1/7*C*x/((b*
x^2 + a)^(7/2)*b) + 8/105*C*x/(sqrt(b*x^2 + a)*a^3*b) + 4/105*C*x/((b*x^2
+ a)^(3/2)*a^2*b) + 1/35*C*x/((b*x^2 + a)^(5/2)*a*b) - 1/7*B/((b*x^2 + a)^
(7/2)*b)
```

## Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.88

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^{9/2}}\,dx = \frac{\left(\left(4\,x^2\left(\frac{2\,(Cab^5+6\,Ab^6)x^2}{a^4b^3}+\frac{7\,(Ca^2b^4+6\,Aab^5)}{a^4b^3}\right)+\frac{35\,(Ca^3b^3+6\,Aa^2b^4)}{a^4b^3}\right)x^2+\frac{105\,A}{a}\right)x-\frac{15\,B}{b}}{105\,(bx^2+a)^{\frac{7}{2}}}$$

input
```
integrate((C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="giac")
```

output
```
1/105*(((4*x^2*(2*(C*a*b^5 + 6*A*b^6)*x^2/(a^4*b^3) + 7*(C*a^2*b^4 + 6*A*a
*b^5)/(a^4*b^3)) + 35*(C*a^3*b^3 + 6*A*a^2*b^4)/(a^4*b^3))*x^2 + 105*A/a)*
x - 15*B/b)/(b*x^2 + a)^(7/2)
```

## Mupad [B] (verification not implemented)

Time = 2.23 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.91

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^{9/2}}\,dx = \frac{x\,(6\,A\,b+C\,a)}{35\,a^2\,b\,(b\,x^2+a)^{5/2}} - \frac{\frac{B}{7\,b}-x\left(\frac{A}{7\,a}-\frac{C}{7\,b}\right)}{(b\,x^2+a)^{7/2}} + \frac{x\,(24\,A\,b+4\,C\,a)}{105\,a^3\,b\,(b\,x^2+a)^{3/2}} + \frac{x\,(48\,A\,b+8\,C\,a)}{105\,a^4\,b\,\sqrt{b\,x^2+a}}$$

input
```
int((A + B*x + C*x^2)/(a + b*x^2)^(9/2),x)
```

output
```
(x*(6*A*b + C*a))/(35*a^2*b*(a + b*x^2)^(5/2)) - (B/(7*b) - x*(A/(7*a) - C
/(7*b)))/(a + b*x^2)^(7/2) + (x*(24*A*b + 4*C*a))/(105*a^3*b*(a + b*x^2)^(
3/2)) + (x*(48*A*b + 8*C*a))/(105*a^4*b*(a + b*x^2)^(1/2))
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 303, normalized size of antiderivative = 2.39

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^{9/2}}\,dx = \frac{105\sqrt{b\,x^2+a}\,a^3b^2x - 15\sqrt{b\,x^2+a}\,a^3b^2 + 210\sqrt{b\,x^2+a}\,a^2b^3x^3 + 35\sqrt{b\,x^2+a}\,a^2b^2}{}$$

input
```
int((C*x^2+B*x+A)/(b*x^2+a)^(9/2),x)
```

output
```
(105*sqrt(a + b*x**2)*a**3*b**2*x - 15*sqrt(a + b*x**2)*a**3*b**2 + 210*sq
rt(a + b*x**2)*a**2*b**3*x**3 + 35*sqrt(a + b*x**2)*a**2*b**2*c*x**3 + 168
*sqrt(a + b*x**2)*a*b**4*x**5 + 28*sqrt(a + b*x**2)*a*b**3*c*x**5 + 48*sqr
t(a + b*x**2)*b**5*x**7 + 8*sqrt(a + b*x**2)*b**4*c*x**7 - 48*sqrt(b)*a**4
*b - 8*sqrt(b)*a**4*c - 192*sqrt(b)*a**3*b**2*x**2 - 32*sqrt(b)*a**3*b*c*x
**2 - 288*sqrt(b)*a**2*b**3*x**4 - 48*sqrt(b)*a**2*b**2*c*x**4 - 192*sqrt(
b)*a*b**4*x**6 - 32*sqrt(b)*a*b**3*c*x**6 - 48*sqrt(b)*b**5*x**8 - 8*sqrt(
b)*b**4*c*x**8)/(105*a**3*b**2*(a**4 + 4*a**3*b*x**2 + 6*a**2*b**2*x**4 +
4*a*b**3*x**6 + b**4*x**8))
```

## 3.9 $\int \frac{A+Bx+Cx^2}{x\left(a+bx^2\right)^{9/2}}\,dx$



### Optimal result

Integrand size = 25, antiderivative size = 138

$$\int \frac{A+Bx+Cx^2}{x\left(a+bx^2\right)^{9/2}}\,dx = \frac{Ab-aC+bBx}{7ab\left(a+bx^2\right)^{7/2}}+\frac{7A+6Bx}{35a^2\left(a+bx^2\right)^{5/2}} + \frac{35A+24Bx}{105a^3\left(a+bx^2\right)^{3/2}}+\frac{35A+16Bx}{35a^4\sqrt{a+bx^2}}-\frac{A\,\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{9/2}}$$

output
```
1/7*(B*b*x+A*b-C*a)/a/b/(b*x^2+a)^(7/2)+1/35*(6*B*x+7*A)/a^2/(b*x^2+a)^(5/
2)+1/105*(24*B*x+35*A)/a^3/(b*x^2+a)^(3/2)+1/35*(16*B*x+35*A)/a^4/(b*x^2+a
)^(1/2)-A*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(9/2)
```

### Mathematica [A] (verified)

Time = 1.33 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.94

$$\int \frac{A+Bx+Cx^2}{x\left(a+bx^2\right)^{9/2}}\,dx = \frac{-15a^4C+14ab^3x^4(25A+12Bx)+14a^2b^2x^2(29A+15Bx)+3b^4x^6(35A+16Bx}{105a^4b\left(a+bx^2\right)^{7/2}} + \frac{2A\,\text{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{9/2}}$$

input
```
Integrate[(A + B*x + C*x^2)/(x*(a + b*x^2)^(9/2)),x]
```

output
```
(-15*a^4*C + 14*a*b^3*x^4*(25*A + 12*B*x) + 14*a^2*b^2*x^2*(29*A + 15*B*x)
 + 3*b^4*x^6*(35*A + 16*B*x) + a^3*b*(176*A + 105*B*x))/(105*a^4*b*(a + b*
x^2)^(7/2)) + (2*A*ArcTanh[(Sqrt[b]*x - Sqrt[a + b*x^2])/Sqrt[a]])/a^(9/2)
```

## Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.13, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {2336, 25, 532, 25, 532, 27, 532, 27, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{x\left(a+bx^2\right)^{9/2}}dx$$

$$\Big\downarrow \text{2336}$$

$$\frac{-aC+Ab+bBx}{7ab\left(a+bx^2\right)^{7/2}}-\frac{\int -\frac{7A+6Bx}{x\left(bx^2+a\right)^{7/2}}dx}{7a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{7A+6Bx}{x\left(bx^2+a\right)^{7/2}}dx}{7a}+\frac{-aC+Ab+bBx}{7ab\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \text{532}$$

$$\frac{\frac{7A+6Bx}{5a\left(a+bx^2\right)^{5/2}}-\frac{\int -\frac{35A+24Bx}{x\left(bx^2+a\right)^{5/2}}dx}{5a}}{7a}+\frac{-aC+Ab+bBx}{7ab\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{35A+24Bx}{x\left(bx^2+a\right)^{5/2}}dx}{5a}+\frac{7A+6Bx}{5a\left(a+bx^2\right)^{5/2}}}{7a}+\frac{-aC+Ab+bBx}{7ab\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \text{532}$$

$$\frac{\frac{\frac{35A+24Bx}{3a\left(a+bx^2\right)^{3/2}}-\frac{\int -\frac{3(35A+16Bx)}{x\left(bx^2+a\right)^{3/2}}dx}{3a}}{5a}+\frac{7A+6Bx}{5a(a+bx^2)^{5/2}}}{7a}+\frac{-aC+Ab+bBx}{7ab\left(a+bx^2\right)^{7/2}}$$

$\Big\downarrow$ 27

$$\frac{\frac{\frac{\int \frac{35A+16Bx}{x\left(bx^2+a\right)^{3/2}}dx}{a}+\frac{35A+24Bx}{3a\left(a+bx^2\right)^{3/2}}}{5a}+\frac{7A+6Bx}{5a(a+bx^2)^{5/2}}}{7a}+\frac{-aC+Ab+bBx}{7ab\left(a+bx^2\right)^{7/2}}$$

$\Big\downarrow$ 532

$$\frac{\frac{\frac{\frac{35A+16Bx}{a\sqrt{a+bx^2}}-\frac{\int -\frac{35A}{x\sqrt{bx^2+a}}dx}{a}}{a}+\frac{35A+24Bx}{3a\left(a+bx^2\right)^{3/2}}}{5a}+\frac{7A+6Bx}{5a(a+bx^2)^{5/2}}}{7a}+\frac{-aC+Ab+bBx}{7ab\left(a+bx^2\right)^{7/2}}$$

$\Big\downarrow$ 27

$$\frac{\frac{\frac{\frac{35A\int \frac{1}{x\sqrt{bx^2+a}}dx}{a}+\frac{35A+16Bx}{a\sqrt{a+bx^2}}}{a}+\frac{35A+24Bx}{3a\left(a+bx^2\right)^{3/2}}}{5a}+\frac{7A+6Bx}{5a(a+bx^2)^{5/2}}}{7a}+\frac{-aC+Ab+bBx}{7ab\left(a+bx^2\right)^{7/2}}$$

$\Big\downarrow$ 243

$$\frac{\frac{\frac{\frac{35A\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2}{2a}+\frac{35A+16Bx}{a\sqrt{a+bx^2}}}{a}+\frac{35A+24Bx}{3a\left(a+bx^2\right)^{3/2}}}{5a}+\frac{7A+6Bx}{5a(a+bx^2)^{5/2}}}{7a}+\frac{-aC+Ab+bBx}{7ab\left(a+bx^2\right)^{7/2}}$$

$\Big\downarrow$ 73

$$\frac{\frac{\frac{\frac{35A\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{ab}+\frac{35A+16Bx}{a\sqrt{a+bx^2}}}{a}+\frac{35A+24Bx}{3a\left(a+bx^2\right)^{3/2}}}{5a}+\frac{7A+6Bx}{5a(a+bx^2)^{5/2}}}{7a}+\frac{-aC+Ab+bBx}{7ab\left(a+bx^2\right)^{7/2}}$$

$\Big\downarrow$ 221

$$\frac{\frac{\frac{\frac{35A+16Bx}{a\sqrt{a+bx^2}}-\frac{35A\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}}{a}+\frac{35A+24Bx}{3a\left(a+bx^2\right)^{3/2}}}{5a}+\frac{7A+6Bx}{5a(a+bx^2)^{5/2}}}{7a}+\frac{-aC+Ab+bBx}{7ab\left(a+bx^2\right)^{7/2}}$$

input
```
Int[(A + B*x + C*x^2)/(x*(a + b*x^2)^(9/2)),x]
```

output
```
(A*b - a*C + b*B*x)/(7*a*b*(a + b*x^2)^(7/2)) + ((7*A + 6*B*x)/(5*a*(a + b
*x^2)^(5/2)) + ((35*A + 24*B*x)/(3*a*(a + b*x^2)^(3/2)) + ((35*A + 16*B*x)
/(a*Sqrt[a + b*x^2]) - (35*A*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/a^(3/2))/a)
/(5*a))/(7*a)
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 532
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[x^m
*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*(Qx/x^m) + e*((2*p + 3)/x^m),
x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && ILtQ[m, 0] && LtQ[p,
 -1] && IntegerQ[2*p]
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

## Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.40

| method | result |
|---|---|
| default | $B\left(\frac{x}{7a(b\,x^2+a)^{\frac{7}{2}}}+\frac{\frac{6x}{35a\left(b\,x^2+a\right)^{\frac{5}{2}}}+\frac{6\left(\frac{4x}{15a\left(b\,x^2+a\right)^{\frac{3}{2}}}+\frac{8x}{15a^2\sqrt{b\,x^2+a}}\right)}{7a}}{a}\right)+A\left(\frac{1}{7a(b\,x^2+a)^{\frac{7}{2}}}+\frac{\frac{1}{5a\left(b\,x^2+a\right)^{\frac{5}{2}}}+\frac{1}{3a\left(b\,x^2+a\right)\right.$ |

input
```
int((C*x^2+B*x+A)/x/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
B*(1/7*x/a/(b*x^2+a)^(7/2)+6/7/a*(1/5*x/a/(b*x^2+a)^(5/2)+4/5/a*(1/3*x/a/(
b*x^2+a)^(3/2)+2/3*x/a^2/(b*x^2+a)^(1/2))))+A*(1/7/a/(b*x^2+a)^(7/2)+1/a*(
1/5/a/(b*x^2+a)^(5/2)+1/a*(1/3/a/(b*x^2+a)^(3/2)+1/a*(1/a/(b*x^2+a)^(1/2)-
1/a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)))))-1/7*C/b/(b*x^2+a)^(7/2
)
```

## Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 468, normalized size of antiderivative = 3.39

$$\int \frac{A+Bx+Cx^2}{x\left(a+bx^2\right)^{9/2}}\,dx = \left[\frac{105\left(Ab^5x^8+4\,Aab^4x^6+6\,Aa^2b^3x^4+4\,Aa^3b^2x^2+Aa^4b\right)\sqrt{a}\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{}}{x^2}\right.}{}\right.$$

input
```
integrate((C*x^2+B*x+A)/x/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

output
```
[1/210*(105*(A*b^5*x^8 + 4*A*a*b^4*x^6 + 6*A*a^2*b^3*x^4 + 4*A*a^3*b^2*x^2
 + A*a^4*b)*sqrt(a)*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) +
2*(48*B*a*b^4*x^7 + 105*A*a*b^4*x^6 + 168*B*a^2*b^3*x^5 + 350*A*a^2*b^3*x^
4 + 210*B*a^3*b^2*x^3 + 406*A*a^3*b^2*x^2 + 105*B*a^4*b*x - 15*C*a^5 + 176
*A*a^4*b)*sqrt(b*x^2 + a))/(a^5*b^5*x^8 + 4*a^6*b^4*x^6 + 6*a^7*b^3*x^4 +
4*a^8*b^2*x^2 + a^9*b), 1/105*(105*(A*b^5*x^8 + 4*A*a*b^4*x^6 + 6*A*a^2*b^
3*x^4 + 4*A*a^3*b^2*x^2 + A*a^4*b)*sqrt(-a)*arctan(sqrt(b*x^2 + a)*sqrt(-a
)/a) + (48*B*a*b^4*x^7 + 105*A*a*b^4*x^6 + 168*B*a^2*b^3*x^5 + 350*A*a^2*b
^3*x^4 + 210*B*a^3*b^2*x^3 + 406*A*a^3*b^2*x^2 + 105*B*a^4*b*x - 15*C*a^5
+ 176*A*a^4*b)*sqrt(b*x^2 + a))/(a^5*b^5*x^8 + 4*a^6*b^4*x^6 + 6*a^7*b^3*x
^4 + 4*a^8*b^2*x^2 + a^9*b)]
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5251 vs. 2(122) = 244.

Time = 39.15 (sec) , antiderivative size = 6613, normalized size of antiderivative = 47.92

$$\int \frac{A+Bx+Cx^2}{x\left(a+bx^2\right)^{9/2}}\,dx = \text{Too large to display}$$

input
```
integrate((C*x**2+B*x+A)/x/(b*x**2+a)**(9/2),x)
```

output
```
A*(352*a**32*sqrt(1 + b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9
450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4
*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a*
*(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18
 + 210*a**(53/2)*b**10*x**20) + 105*a**32*log(b*x**2/a)/(210*a**(73/2) + 2
100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**
6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/
2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2
100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) - 210*a**32*log(sqrt
(1 + b*x**2/a) + 1)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2
)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 5292
0*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7
*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(5
3/2)*b**10*x**20) + 2924*a**31*b*x**2*sqrt(1 + b*x**2/a)/(210*a**(73/2) +
2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x*
*6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61
/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 +
2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) + 1050*a**31*b*x**2
*log(b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**
2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*...
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.14

$$\int \frac{A+Bx+Cx^2}{x\left(a+bx^2\right)^{9/2}}\,dx = \frac{16\,Bx}{35\,\sqrt{bx^2+a}a^4} + \frac{8\,Bx}{35\,(bx^2+a)^{\frac{3}{2}}a^3} + \frac{6\,Bx}{35\,(bx^2+a)^{\frac{5}{2}}a^2} + \frac{Bx}{7\,(bx^2+a)^{\frac{7}{2}}a} - \frac{A\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{a^{\frac{9}{2}}} + \frac{A}{\sqrt{bx^2+a}a^4} + \frac{A}{3\,(bx^2+a)^{\frac{3}{2}}a^3} + \frac{A}{5\,(bx^2+a)^{\frac{5}{2}}a^2} + \frac{A}{7\,(bx^2+a)^{\frac{7}{2}}a} - \frac{C}{7\,(bx^2+a)^{\frac{7}{2}}b}$$

input
```
integrate((C*x^2+B*x+A)/x/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

output
```
16/35*B*x/(sqrt(b*x^2 + a)*a^4) + 8/35*B*x/((b*x^2 + a)^(3/2)*a^3) + 6/35*
B*x/((b*x^2 + a)^(5/2)*a^2) + 1/7*B*x/((b*x^2 + a)^(7/2)*a) - A*arcsinh(a/
(sqrt(a*b)*abs(x)))/a^(9/2) + A/(sqrt(b*x^2 + a)*a^4) + 1/3*A/((b*x^2 + a)
^(3/2)*a^3) + 1/5*A/((b*x^2 + a)^(5/2)*a^2) + 1/7*A/((b*x^2 + a)^(7/2)*a)
- 1/7*C/((b*x^2 + a)^(7/2)*b)
```

## Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.10

$$\int \frac{A+Bx+Cx^2}{x\left(a+bx^2\right)^{9/2}}\,dx=\frac{\left(\left(\left(\left(\left(3\left(\left(\frac{16\,Bb^3x}{a^4}+\frac{35\,Ab^3}{a^4}\right)x+\frac{56\,Bb^2}{a^3}\right)x+\frac{350\,Ab^2}{a^3}\right)x+\frac{210\,Bb}{a^2}\right)x+\frac{406\,Ab}{a^2}\right)x+\frac{105\,B}{a}\right)\cdots}{105\,(bx^2+a)^{\frac{7}{2}}}+\frac{2\,A\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^4}$$

input
```
integrate((C*x^2+B*x+A)/x/(b*x^2+a)^(9/2),x, algorithm="giac")
```

output
```
1/105*((((((3*((16*B*b^3*x/a^4 + 35*A*b^3/a^4)*x + 56*B*b^2/a^3)*x + 350*A*
b^2/a^3)*x + 210*B*b/a^2)*x + 406*A*b/a^2)*x + 105*B/a)*x - (15*C*a^14*b^2
 - 176*A*a^13*b^3)/(a^14*b^3))/(b*x^2 + a)^(7/2) + 2*A*arctan(-(sqrt(b)*x
- sqrt(b*x^2 + a))/sqrt(-a))/(sqrt(-a)*a^4)
```

## Mupad [B] (verification not implemented)

Time = 2.83 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.15

$$\int \frac{A+Bx+Cx^2}{x\left(a+bx^2\right)^{9/2}}\,dx=\frac{\frac{A}{7\,a}+\frac{A\,(b\,x^2+a)^2}{3\,a^3}+\frac{A\,(b\,x^2+a)^3}{a^4}+\frac{A\,(b\,x^2+a)}{5\,a^2}}{(b\,x^2+a)^{7/2}}-\frac{C}{7\,b\,(b\,x^2+a)^{7/2}}-\frac{A\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{a^{9/2}}+\frac{16\,B\,x}{35\,a^4\,\sqrt{b\,x^2+a}}+\frac{8\,B\,x}{35\,a^3\,(b\,x^2+a)^{3/2}}+\frac{6\,B\,x}{35\,a^2\,(b\,x^2+a)^{5/2}}+\frac{B\,x}{7\,a\,(b\,x^2+a)^{7/2}}$$

input
```
int((A + B*x + C*x^2)/(x*(a + b*x^2)^(9/2)),x)
```

output
```
(A/(7*a) + (A*(a + b*x^2)^2)/(3*a^3) + (A*(a + b*x^2)^3)/a^4 + (A*(a + b*x
^2))/(5*a^2))/(a + b*x^2)^(7/2) - C/(7*b*(a + b*x^2)^(7/2)) - (A*atanh((a
+ b*x^2)^(1/2)/a^(1/2)))/a^(9/2) + (16*B*x)/(35*a^4*(a + b*x^2)^(1/2)) + (
8*B*x)/(35*a^3*(a + b*x^2)^(3/2)) + (6*B*x)/(35*a^2*(a + b*x^2)^(5/2)) + (
B*x)/(7*a*(a + b*x^2)^(7/2))
```

## Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 588, normalized size of antiderivative = 4.26

$$\int \frac{A+Bx+Cx^2}{x\left(a+bx^2\right)^{9/2}}\,dx = \frac{176\sqrt{b\,x^2+a}\,a^4b-15\sqrt{b\,x^2+a}\,a^4c+406\sqrt{b\,x^2+a}\,a^3b^2x^2+105\sqrt{b\,x^2+a}\,a^3b^2x}{}$$

input
```
int((C*x^2+B*x+A)/x/(b*x^2+a)^(9/2),x)
```

output
```
(176*sqrt(a + b*x**2)*a**4*b - 15*sqrt(a + b*x**2)*a**4*c + 406*sqrt(a + b
*x**2)*a**3*b**2*x**2 + 105*sqrt(a + b*x**2)*a**3*b**2*x + 350*sqrt(a + b*
x**2)*a**2*b**3*x**4 + 210*sqrt(a + b*x**2)*a**2*b**3*x**3 + 105*sqrt(a +
b*x**2)*a*b**4*x**6 + 168*sqrt(a + b*x**2)*a*b**4*x**5 + 48*sqrt(a + b*x**
2)*b**5*x**7 + 105*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sq
rt(a))*a**4*b + 420*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/s
qrt(a))*a**3*b**2*x**2 + 630*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqr
t(b)*x)/sqrt(a))*a**2*b**3*x**4 + 420*sqrt(a)*log((sqrt(a + b*x**2) - sqrt
(a) + sqrt(b)*x)/sqrt(a))*a*b**4*x**6 + 105*sqrt(a)*log((sqrt(a + b*x**2)
- sqrt(a) + sqrt(b)*x)/sqrt(a))*b**5*x**8 - 105*sqrt(a)*log((sqrt(a + b*x*
*2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*a**4*b - 420*sqrt(a)*log((sqrt(a + b*x
**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*a**3*b**2*x**2 - 630*sqrt(a)*log((sqr
t(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*a**2*b**3*x**4 - 420*sqrt(a)
*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b**4*x**6 - 105*s
qrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*b**5*x**8 - 4
8*sqrt(b)*a**4*b - 192*sqrt(b)*a**3*b**2*x**2 - 288*sqrt(b)*a**2*b**3*x**4
 - 192*sqrt(b)*a*b**4*x**6 - 48*sqrt(b)*b**5*x**8)/(105*a**4*b*(a**4 + 4*a
**3*b*x**2 + 6*a**2*b**2*x**4 + 4*a*b**3*x**6 + b**4*x**8))
```

# 3.10 $\int \frac{A+Bx+Cx^2}{x^2\left(a+bx^2\right)^{9/2}}\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 207

$$\int \frac{A+Bx+Cx^2}{x^2\left(a+bx^2\right)^{9/2}}\,dx=\frac{Ab-aC+bBx}{7abx\left(a+bx^2\right)^{7/2}}+\frac{7aB-(8Ab-aC)x}{35a^3\left(a+bx^2\right)^{5/2}}+\frac{9(8Ab-aC)+35bBx}{105a^3bx\left(a+bx^2\right)^{3/2}}+\frac{35aB-8(8Ab-aC)x}{35a^5\sqrt{a+bx^2}}-\frac{8(8Ab-aC)\sqrt{a+bx^2}}{35a^5bx}-\frac{B\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{9/2}}$$

output
```
1/7*(B*b*x+A*b-C*a)/a/b/x/(b*x^2+a)^(7/2)+1/35*(7*B*a-(8*A*b-C*a)*x)/a^3/(
b*x^2+a)^(5/2)+1/105*(35*B*b*x+72*A*b-9*C*a)/a^3/b/x/(b*x^2+a)^(3/2)+1/35*
(35*B*a-8*(8*A*b-C*a)*x)/a^5/(b*x^2+a)^(1/2)-8/35*(8*A*b-C*a)*(b*x^2+a)^(1
/2)/a^5/b/x-B*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(9/2)
```

## Mathematica [A] (verified)

Time = 1.30 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.81

$$\int \frac{A+Bx+Cx^2}{x^2\left(a+bx^2\right)^{9/2}}\,dx = \frac{-384Ab^4x^8+14a^2b^2x^4(-120A+x(25B+12Cx))+14a^3bx^2(-60A+x(29B+1}{}$$

input
```
Integrate[(A + B*x + C*x^2)/(x^2*(a + b*x^2)^(9/2)),x]
```

output
```
(-384*A*b^4*x^8 + 14*a^2*b^2*x^4*(-120*A + x*(25*B + 12*C*x)) + 14*a^3*b*x
^2*(-60*A + x*(29*B + 15*C*x)) + 3*a*b^3*x^6*(-448*A + x*(35*B + 16*C*x))
+ a^4*(-105*A + x*(176*B + 105*C*x)) + 210*Sqrt[a]*B*x*(a + b*x^2)^(7/2)*A
rcTanh[(Sqrt[b]*x - Sqrt[a + b*x^2])/Sqrt[a]])/(105*a^5*x*(a + b*x^2)^(7/2
))
```

## Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.02, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {2336, 25, 2336, 25, 2336, 27, 2336, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{x^2\left(a+bx^2\right)^{9/2}}\,dx$$

$$\Big\downarrow \text{2336}$$

$$\frac{B-x\left(\frac{Ab}{a}-C\right)}{7a\left(a+bx^2\right)^{7/2}}-\frac{\int -\frac{-6\left(\frac{Ab}{a}-C\right)x^2+7Bx+7A}{x^2\left(bx^2+a\right)^{7/2}}dx}{7a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{-6\left(\frac{Ab}{a}-C\right)x^2+7Bx+7A}{x^2\left(bx^2+a\right)^{7/2}}dx}{7a}+\frac{B-x\left(\frac{Ab}{a}-C\right)}{7a\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \ 2336$$

$$\frac{\frac{7B-x\left(\frac{13Ab}{a}-6C\right)}{5a(a+bx^2)^{5/2}}-\frac{\int -\frac{-4\left(\frac{13Ab}{a}-6C\right)x^2+35Bx+35A}{x^2\left(bx^2+a\right)^{5/2}}dx}{5a}}{7a}+\frac{B-x\left(\frac{Ab}{a}-C\right)}{7a\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{\int \frac{-4\left(\frac{13Ab}{a}-6C\right)x^2+35Bx+35A}{x^2\left(bx^2+a\right)^{5/2}}dx}{5a}+\frac{7B-x\left(\frac{13Ab}{a}-6C\right)}{5a(a+bx^2)^{5/2}}}{7a}+\frac{B-x\left(\frac{Ab}{a}-C\right)}{7a\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \ 2336$$

$$\frac{\frac{\frac{35B-3x\left(\frac{29Ab}{a}-8C\right)}{3a\left(a+bx^2\right)^{3/2}}-\frac{\int -\frac{3\left(-2\left(\frac{29Ab}{a}-8C\right)x^2+35Bx+35A\right)}{x^2\left(bx^2+a\right)^{3/2}}dx}{3a}}{5a}+\frac{7B-x\left(\frac{13Ab}{a}-6C\right)}{5a(a+bx^2)^{5/2}}}{7a}+\frac{B-x\left(\frac{Ab}{a}-C\right)}{7a\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\frac{\int \frac{-2\left(\frac{29Ab}{a}-8C\right)x^2+35Bx+35A}{x^2\left(bx^2+a\right)^{3/2}}dx}{a}+\frac{35B-3x\left(\frac{29Ab}{a}-8C\right)}{3a\left(a+bx^2\right)^{3/2}}}{5a}+\frac{7B-x\left(\frac{13Ab}{a}-6C\right)}{5a(a+bx^2)^{5/2}}}{7a}+\frac{B-x\left(\frac{Ab}{a}-C\right)}{7a\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \ 2336$$

$$\frac{\frac{\frac{\frac{35B-x\left(\frac{93Ab}{a}-16C\right)}{a\sqrt{a+bx^2}}-\frac{\int -\frac{35(A+Bx)}{x^2\sqrt{bx^2+a}}dx}{a}}{a}+\frac{35B-3x\left(\frac{29Ab}{a}-8C\right)}{3a\left(a+bx^2\right)^{3/2}}}{5a}+\frac{7B-x\left(\frac{13Ab}{a}-6C\right)}{5a(a+bx^2)^{5/2}}}{7a}+\frac{B-x\left(\frac{Ab}{a}-C\right)}{7a\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\frac{\frac{35\int \frac{A+Bx}{x^2\sqrt{bx^2+a}}dx}{a}+\frac{35B-x\left(\frac{93Ab}{a}-16C\right)}{a\sqrt{a+bx^2}}}{a}+\frac{35B-3x\left(\frac{29Ab}{a}-8C\right)}{3a\left(a+bx^2\right)^{3/2}}}{5a}+\frac{7B-x\left(\frac{13Ab}{a}-6C\right)}{5a(a+bx^2)^{5/2}}}{7a}+\frac{B-x\left(\frac{Ab}{a}-C\right)}{7a\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \ 534$$

$$\frac{\frac{\frac{\frac{35\left(B\int \frac{1}{x\sqrt{bx^2+a}}dx-\frac{A\sqrt{a+bx^2}}{ax}\right)}{a}+\frac{35B-x\left(\frac{93Ab}{a}-16C\right)}{a\sqrt{a+bx^2}}}{a}+\frac{35B-3x\left(\frac{29Ab}{a}-8C\right)}{3a\left(a+bx^2\right)^{3/2}}}{5a}+\frac{7B-x\left(\frac{13Ab}{a}-6C\right)}{5a(a+bx^2)^{5/2}}}{7a}+\frac{B-x\left(\frac{Ab}{a}-C\right)}{7a\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \; 243$$

$$\frac{\frac{\frac{\frac{35\left(\frac{1}{2}B\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2-\frac{A\sqrt{a+bx^2}}{ax}\right)}{a}+\frac{35B-x\left(\frac{93Ab}{a}-16C\right)}{a\sqrt{a+bx^2}}}{a}+\frac{35B-3x\left(\frac{29Ab}{a}-8C\right)}{3a\left(a+bx^2\right)^{3/2}}}{5a}+\frac{7B-x\left(\frac{13Ab}{a}-6C\right)}{5a(a+bx^2)^{5/2}}}{7a}+\frac{B-x\left(\frac{Ab}{a}-C\right)}{7a\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \; 73$$

$$\frac{\frac{\frac{\frac{35\left(\frac{B\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{b}-\frac{A\sqrt{a+bx^2}}{ax}\right)}{a}+\frac{35B-x\left(\frac{93Ab}{a}-16C\right)}{a\sqrt{a+bx^2}}}{a}+\frac{35B-3x\left(\frac{29Ab}{a}-8C\right)}{3a\left(a+bx^2\right)^{3/2}}}{5a}+\frac{7B-x\left(\frac{13Ab}{a}-6C\right)}{5a(a+bx^2)^{5/2}}}{7a}+\frac{B-x\left(\frac{Ab}{a}-C\right)}{7a\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{\frac{\frac{35\left(-\frac{A\sqrt{a+bx^2}}{ax}-\frac{B\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}\right)}{a}+\frac{35B-x\left(\frac{93Ab}{a}-16C\right)}{a\sqrt{a+bx^2}}}{a}+\frac{35B-3x\left(\frac{29Ab}{a}-8C\right)}{3a\left(a+bx^2\right)^{3/2}}}{5a}+\frac{7B-x\left(\frac{13Ab}{a}-6C\right)}{5a(a+bx^2)^{5/2}}}{7a}+\frac{B-x\left(\frac{Ab}{a}-C\right)}{7a\left(a+bx^2\right)^{7/2}}$$

input
```
Int[(A + B*x + C*x^2)/(x^2*(a + b*x^2)^(9/2)),x]
```

output
```
(B - ((A*b)/a - C)*x)/(7*a*(a + b*x^2)^(7/2)) + ((7*B - ((13*A*b)/a - 6*C)
*x)/(5*a*(a + b*x^2)^(5/2)) + ((35*B - 3*((29*A*b)/a - 8*C)*x)/(3*a*(a + b
*x^2)^(3/2)) + ((35*B - ((93*A*b)/a - 16*C)*x)/(a*Sqrt[a + b*x^2]) + (35*(
-((A*Sqrt[a + b*x^2])/(a*x)) - (B*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a
]))/a)/a)/(5*a))/(7*a)
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

## Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.34

| method | result |
|---|---|
| default | $C\left(\frac{x}{7a(b\,x^{2}+a)^{\frac{7}{2}}}+\frac{\frac{6x}{35a\left(b\,x^{2}+a\right)^{\frac{5}{2}}}+\frac{6\left(\frac{4x}{15a\left(b\,x^{2}+a\right)^{\frac{3}{2}}}+\frac{8x}{15a^{2}\sqrt{b\,x^{2}+a}}\right)}{7a}}{a}\right)+A\left(-\frac{1}{ax(b\,x^{2}+a)^{\frac{7}{2}}}-\frac{8b\left(\frac{x}{7a\left(b\,x^{2}+a\right)^{\frac{7}{2}}}+\frac{\cdots}{35a\cdots}\right.}{\cdots}\right.$ |
| risch | Expression too large to display |

input
```
int((C*x^2+B*x+A)/x^2/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
C*(1/7*x/a/(b*x^2+a)^(7/2)+6/7/a*(1/5*x/a/(b*x^2+a)^(5/2)+4/5/a*(1/3*x/a/(
b*x^2+a)^(3/2)+2/3*x/a^2/(b*x^2+a)^(1/2))))+A*(-1/a/x/(b*x^2+a)^(7/2)-8*b/
a*(1/7*x/a/(b*x^2+a)^(7/2)+6/7/a*(1/5*x/a/(b*x^2+a)^(5/2)+4/5/a*(1/3*x/a/(
b*x^2+a)^(3/2)+2/3*x/a^2/(b*x^2+a)^(1/2)))))+B*(1/7/a/(b*x^2+a)^(7/2)+1/a*
(1/5/a/(b*x^2+a)^(5/2)+1/a*(1/3/a/(b*x^2+a)^(3/2)+1/a*(1/a/(b*x^2+a)^(1/2)
-1/a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)))))
```

## Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 528, normalized size of antiderivative = 2.55

$$\int \frac{A+Bx+Cx^2}{x^2\left(a+bx^2\right)^{9/2}}\,dx = \left[\frac{105\left(Bb^4x^9+4\,Bab^3x^7+6\,Ba^2b^2x^5+4\,Ba^3bx^3+Ba^4x\right)\sqrt{a}\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{}}{x^2}\right.}{}\right.$$

input
```
integrate((C*x^2+B*x+A)/x^2/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

output
```
[1/210*(105*(B*b^4*x^9 + 4*B*a*b^3*x^7 + 6*B*a^2*b^2*x^5 + 4*B*a^3*b*x^3 +
 B*a^4*x)*sqrt(a)*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*
(105*B*a*b^3*x^7 + 350*B*a^2*b^2*x^5 + 48*(C*a*b^3 - 8*A*b^4)*x^8 + 406*B*
a^3*b*x^3 + 168*(C*a^2*b^2 - 8*A*a*b^3)*x^6 + 176*B*a^4*x - 105*A*a^4 + 21
0*(C*a^3*b - 8*A*a^2*b^2)*x^4 + 105*(C*a^4 - 8*A*a^3*b)*x^2)*sqrt(b*x^2 +
a))/(a^5*b^4*x^9 + 4*a^6*b^3*x^7 + 6*a^7*b^2*x^5 + 4*a^8*b*x^3 + a^9*x), 1
/105*(105*(B*b^4*x^9 + 4*B*a*b^3*x^7 + 6*B*a^2*b^2*x^5 + 4*B*a^3*b*x^3 + B
*a^4*x)*sqrt(-a)*arctan(sqrt(b*x^2 + a)*sqrt(-a)/a) + (105*B*a*b^3*x^7 + 3
50*B*a^2*b^2*x^5 + 48*(C*a*b^3 - 8*A*b^4)*x^8 + 406*B*a^3*b*x^3 + 168*(C*a
^2*b^2 - 8*A*a*b^3)*x^6 + 176*B*a^4*x - 105*A*a^4 + 210*(C*a^3*b - 8*A*a^2
*b^2)*x^4 + 105*(C*a^4 - 8*A*a^3*b)*x^2)*sqrt(b*x^2 + a))/(a^5*b^4*x^9 + 4
*a^6*b^3*x^7 + 6*a^7*b^2*x^5 + 4*a^8*b*x^3 + a^9*x)]
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6922 vs. 2(184) = 368.

Time = 54.80 (sec) , antiderivative size = 6922, normalized size of antiderivative = 33.44

$$\int \frac{A+Bx+Cx^2}{x^2\left(a+bx^2\right)^{9/2}}\,dx = \text{Too large to display}$$

input
```
integrate((C*x**2+B*x+A)/x**2/(b*x**2+a)**(9/2),x)
```

output
```
A*(-35*a**4*b**(33/2)*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b**17
*x**2 + 210*a**7*b**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8) -
280*a**3*b**(35/2)*x**2*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b**
17*x**2 + 210*a**7*b**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8)
- 560*a**2*b**(37/2)*x**4*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b
**17*x**2 + 210*a**7*b**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8
) - 448*a*b**(39/2)*x**6*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b*
*17*x**2 + 210*a**7*b**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8)
 - 128*b**(41/2)*x**8*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b**17
*x**2 + 210*a**7*b**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8)) +
 B*(352*a**32*sqrt(1 + b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 +
9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**
4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a
**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**1
8 + 210*a**(53/2)*b**10*x**20) + 105*a**32*log(b*x**2/a)/(210*a**(73/2) +
2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x*
*6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61
/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 +
2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) - 210*a**32*log(sqr
t(1 + b*x**2/a) + 1)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(...
```

## Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.10

$$\int \frac{A+Bx+Cx^2}{x^2\left(a+bx^2\right)^{9/2}}\,dx = \frac{16\,Cx}{35\,\sqrt{bx^2+a}a^4} + \frac{8\,Cx}{35\,(bx^2+a)^{\frac{3}{2}}a^3} + \frac{6\,Cx}{35\,(bx^2+a)^{\frac{5}{2}}a^2} + \frac{Cx}{7\,(bx^2+a)^{\frac{7}{2}}a} - \frac{128\,Abx}{35\,\sqrt{bx^2+a}a^5} - \frac{64\,Abx}{35\,(bx^2+a)^{\frac{3}{2}}a^4} - \frac{48\,Abx}{35\,(bx^2+a)^{\frac{5}{2}}a^3} - \frac{8\,Abx}{7\,(bx^2+a)^{\frac{7}{2}}a^2} - \frac{B\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{a^{\frac{9}{2}}} + \frac{B}{\sqrt{bx^2+a}a^4} + \frac{B}{3\,(bx^2+a)^{\frac{3}{2}}a^3} + \frac{B}{5\,(bx^2+a)^{\frac{5}{2}}a^2} + \frac{B}{7\,(bx^2+a)^{\frac{7}{2}}a} - \frac{A}{(bx^2+a)^{\frac{7}{2}}ax}$$

input
```
integrate((C*x^2+B*x+A)/x^2/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

output
```
16/35*C*x/(sqrt(b*x^2 + a)*a^4) + 8/35*C*x/((b*x^2 + a)^(3/2)*a^3) + 6/35*
C*x/((b*x^2 + a)^(5/2)*a^2) + 1/7*C*x/((b*x^2 + a)^(7/2)*a) - 128/35*A*b*x
/(sqrt(b*x^2 + a)*a^5) - 64/35*A*b*x/((b*x^2 + a)^(3/2)*a^4) - 48/35*A*b*x
/((b*x^2 + a)^(5/2)*a^3) - 8/7*A*b*x/((b*x^2 + a)^(7/2)*a^2) - B*arcsinh(a
/(sqrt(a*b)*abs(x)))/a^(9/2) + B/(sqrt(b*x^2 + a)*a^4) + 1/3*B/((b*x^2 + a
)^(3/2)*a^3) + 1/5*B/((b*x^2 + a)^(5/2)*a^2) + 1/7*B/((b*x^2 + a)^(7/2)*a)
 - A/((b*x^2 + a)^(7/2)*a*x)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.15

$$\int \frac{A+Bx+Cx^2}{x^2\left(a+bx^2\right)^{9/2}}\,dx = \frac{\left(\left(\left(\left(3\left(x\left(\frac{35\,Bb^3}{a^4}+\frac{\left(16\,Ca^{20}b^6-93\,Aa^{19}b^7\right)x}{a^{24}b^3}\right)+\frac{28\left(2\,Ca^{21}b^5-11\,Aa^{20}b^6\right)}{a^{24}b^3}\right)x+\frac{350\,Bb^2}{a^3}\right)x+21\cdots}{105\left(bx^2+a\right)^{\frac{7}{2}}} + \frac{2\,B\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^4} + \frac{2\,A\sqrt{b}}{\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)a^4}$$

input
```
integrate((C*x^2+B*x+A)/x^2/(b*x^2+a)^(9/2),x, algorithm="giac")
```

output
```
1/105*(((((3*(x*(35*B*b^3/a^4 + (16*C*a^20*b^6 - 93*A*a^19*b^7)*x/(a^24*b^
3)) + 28*(2*C*a^21*b^5 - 11*A*a^20*b^6)/(a^24*b^3))*x + 350*B*b^2/a^3)*x +
 210*(C*a^22*b^4 - 5*A*a^21*b^5)/(a^24*b^3))*x + 406*B*b/a^2)*x + 105*(C*a
^23*b^3 - 4*A*a^22*b^4)/(a^24*b^3))*x + 176*B/a)/(b*x^2 + a)^(7/2) + 2*B*a
rctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/(sqrt(-a)*a^4) + 2*A*sqrt(b
)/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)*a^4)
```

## Mupad [B] (verification not implemented)

Time = 3.31 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.09

$$\int \frac{A+Bx+Cx^2}{x^2 (a+bx^2)^{9/2}} \, dx = \frac{\frac{B}{7a}+\frac{B\,(b\,x^2+a)^2}{3\,a^3}+\frac{B\,(b\,x^2+a)^3}{a^4}+\frac{B\,(b\,x^2+a)}{5\,a^2}}{(b\,x^2+a)^{7/2}}-\frac{\frac{A}{a^4}+\frac{128\,A\,b\,x^2}{35\,a^5}}{x\,\sqrt{b\,x^2+a}} - \frac{B\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{a^{9/2}}+\frac{16\,C\,x}{35\,a^4\,\sqrt{b\,x^2+a}}+\frac{8\,C\,x}{35\,a^3\,(b\,x^2+a)^{3/2}}+\frac{6\,C\,x}{35\,a^2\,(b\,x^2+a)^{5/2}} + \frac{C\,x}{7\,a\,(b\,x^2+a)^{7/2}}-\frac{29\,A\,b\,x}{35\,a^4\,(b\,x^2+a)^{3/2}}-\frac{13\,A\,b\,x}{35\,a^3\,(b\,x^2+a)^{5/2}}-\frac{A\,b\,x}{7\,a^2\,(b\,x^2+a)^{7/2}}$$

input
```
int((A + B*x + C*x^2)/(x^2*(a + b*x^2)^(9/2)),x)
```

output
```
(B/(7*a) + (B*(a + b*x^2)^2)/(3*a^3) + (B*(a + b*x^2)^3)/a^4 + (B*(a + b*x
^2))/(5*a^2))/(a + b*x^2)^(7/2) - (A/a^4 + (128*A*b*x^2)/(35*a^5))/(x*(a +
 b*x^2)^(1/2)) - (B*atanh((a + b*x^2)^(1/2)/a^(1/2)))/a^(9/2) + (16*C*x)/(
35*a^4*(a + b*x^2)^(1/2)) + (8*C*x)/(35*a^3*(a + b*x^2)^(3/2)) + (6*C*x)/(
35*a^2*(a + b*x^2)^(5/2)) + (C*x)/(7*a*(a + b*x^2)^(7/2)) - (29*A*b*x)/(35
*a^4*(a + b*x^2)^(3/2)) - (13*A*b*x)/(35*a^3*(a + b*x^2)^(5/2)) - (A*b*x)/
(7*a^2*(a + b*x^2)^(7/2))
```

## Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 746, normalized size of antiderivative = 3.60

$$\int \frac{A+Bx+Cx^2}{x^2 (a+bx^2)^{9/2}} \, dx = \text{Too large to display}$$

input
```
int((C*x^2+B*x+A)/x^2/(b*x^2+a)^(9/2),x)
```

output

```
( - 105*sqrt(a + b*x**2)*a**5*b - 840*sqrt(a + b*x**2)*a**4*b**2*x**2 + 17
6*sqrt(a + b*x**2)*a**4*b**2*x + 105*sqrt(a + b*x**2)*a**4*b*c*x**2 - 1680
*sqrt(a + b*x**2)*a**3*b**3*x**4 + 406*sqrt(a + b*x**2)*a**3*b**3*x**3 + 2
10*sqrt(a + b*x**2)*a**3*b**2*c*x**4 - 1344*sqrt(a + b*x**2)*a**2*b**4*x**
6 + 350*sqrt(a + b*x**2)*a**2*b**4*x**5 + 168*sqrt(a + b*x**2)*a**2*b**3*c
*x**6 - 384*sqrt(a + b*x**2)*a*b**5*x**8 + 105*sqrt(a + b*x**2)*a*b**5*x**
7 + 48*sqrt(a + b*x**2)*a*b**4*c*x**8 + 105*sqrt(a)*log((sqrt(a + b*x**2)
- sqrt(a) + sqrt(b)*x)/sqrt(a))*a**4*b**2*x + 420*sqrt(a)*log((sqrt(a + b*
x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*a**3*b**3*x**3 + 630*sqrt(a)*log((sq
rt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*a**2*b**4*x**5 + 420*sqrt(a
)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b**5*x**7 + 105*
sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*b**6*x**9 -
105*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*a**4*b**
2*x - 420*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*a*
*3*b**3*x**3 - 630*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sq
rt(a))*a**2*b**4*x**5 - 420*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt
(b)*x)/sqrt(a))*a*b**5*x**7 - 105*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a)
+ sqrt(b)*x)/sqrt(a))*b**6*x**9 + 384*sqrt(b)*a**5*b*x - 48*sqrt(b)*a**5*c
*x + 1536*sqrt(b)*a**4*b**2*x**3 - 192*sqrt(b)*a**4*b*c*x**3 + 2304*sqrt(b
)*a**3*b**3*x**5 - 288*sqrt(b)*a**3*b**2*c*x**5 + 1536*sqrt(b)*a**2*b**...
```

# 3.11 $\int \frac{A+Bx+Cx^2}{x^3(a+bx^2)^{9/2}}\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 230

$$\int \frac{A+Bx+Cx^2}{x^3(a+bx^2)^{9/2}}\,dx = \frac{Ab-aC+bBx}{7abx^2(a+bx^2)^{7/2}} - \frac{9Ab-2aC+8bBx}{35a^3(a+bx^2)^{5/2}} - \frac{2(5(9Ab-2aC)+36bBx)}{105a^4(a+bx^2)^{3/2}} - \frac{15(9Ab-2aC)+88bBx}{35a^5\sqrt{a+bx^2}} - \frac{(9Ab-2aC)\sqrt{a+bx^2}}{14a^5bx^2} - \frac{8B\sqrt{a+bx^2}}{7a^5x} + \frac{(9Ab-2aC)\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{11/2}}$$

output
```
1/7*(B*b*x+A*b-C*a)/a/b/x^2/(b*x^2+a)^(7/2)-1/35*(8*B*b*x+9*A*b-2*C*a)/a^3
/(b*x^2+a)^(5/2)-2/105*(36*B*b*x+45*A*b-10*C*a)/a^4/(b*x^2+a)^(3/2)-1/35*(
88*B*b*x+135*A*b-30*C*a)/a^5/(b*x^2+a)^(1/2)-1/14*(9*A*b-2*C*a)*(b*x^2+a)^
(1/2)/a^5/b/x^2-8/7*B*(b*x^2+a)^(1/2)/a^5/x+1/2*(9*A*b-2*C*a)*arctanh((b*x
^2+a)^(1/2)/a^(1/2))/a^(11/2)
```

## Mathematica [A] (verified)

Time = 1.66 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.75

$$\int \frac{A+Bx+Cx^2}{x^3\left(a+bx^2\right)^{9/2}}\,dx = \frac{-3b^4x^8(315A+256Bx)+a^4(-105A-210Bx+352Cx^2)-4a^3bx^2(396A+7x(6\ldots}{210a\ldots} + \frac{(-9Ab+2aC)\operatorname{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{11/2}}$$

input
```
Integrate[(A + B*x + C*x^2)/(x^3*(a + b*x^2)^(9/2)),x]
```

output
```
(-3*b^4*x^8*(315*A + 256*B*x) + a^4*(-105*A - 210*B*x + 352*C*x^2) - 4*a^3
*b*x^2*(396*A + 7*x*(60*B - 29*C*x)) + 42*a*b^3*x^6*(-75*A + x*(-64*B + 5*
C*x)) + 14*a^2*b^2*x^4*(-261*A + 10*x*(-24*B + 5*C*x)))/(210*a^5*x^2*(a +
b*x^2)^(7/2)) + ((-9*A*b + 2*a*C)*ArcTanh[(Sqrt[b]*x - Sqrt[a + b*x^2])/Sq
rt[a]])/a^(11/2)
```

## Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.07, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.560$, Rules used = {2336, 25, 2336, 25, 2336, 27, 2336, 27, 2338, 25, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{x^3\left(a+bx^2\right)^{9/2}}\,dx$$

$$\Big\downarrow \text{2336}$$

$$-\frac{\int -\frac{-\frac{6bBx^3}{a}-7\left(\frac{Ab}{a}-C\right)x^2+7Bx+7A}{x^3\left(bx^2+a\right)^{7/2}}dx}{7a}-\frac{a\left(\frac{Ab}{a}-C\right)+bBx}{7a^2\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{-\frac{6bBx^3}{a}-7\left(\frac{Ab}{a}-C\right)x^2+7Bx+7A}{x^3\left(bx^2+a\right)^{7/2}}dx}{7a}-\frac{a\left(\frac{Ab}{a}-C\right)+bBx}{7a^2\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \text{2336}$$

$$\frac{-\frac{\int -\frac{-\frac{52bBx^3}{a}-35\left(\frac{2Ab}{a}-C\right)x^2+35Bx+35A}{x^3\left(bx^2+a\right)^{5/2}}dx}{5a}-\frac{7(2Ab-aC)+13bBx}{5a^2\left(a+bx^2\right)^{5/2}}}{7a}-\frac{a\left(\frac{Ab}{a}-C\right)+bBx}{7a^2\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{-\frac{52bBx^3}{a}-35\left(\frac{2Ab}{a}-C\right)x^2+35Bx+35A}{x^3\left(bx^2+a\right)^{5/2}}dx}{5a}-\frac{7(2Ab-aC)+13bBx}{5a^2\left(a+bx^2\right)^{5/2}}}{7a}-\frac{a\left(\frac{Ab}{a}-C\right)+bBx}{7a^2\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \text{2336}$$

$$\frac{\frac{-\frac{\int -\frac{3\left(-\frac{58bBx^3}{a}-35\left(\frac{3Ab}{a}-C\right)x^2+35Bx+35A\right)}{x^3\left(bx^2+a\right)^{3/2}}dx}{3a}-\frac{35(3Ab-aC)+87bBx}{3a^2\left(a+bx^2\right)^{3/2}}}{5a}-\frac{7(2Ab-aC)+13bBx}{5a^2\left(a+bx^2\right)^{5/2}}}{7a}-\frac{a\left(\frac{Ab}{a}-C\right)+bBx}{7a^2\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{\int \frac{-\frac{58bBx^3}{a}-35\left(\frac{3Ab}{a}-C\right)x^2+35Bx+35A}{x^3\left(bx^2+a\right)^{3/2}}dx}{a}-\frac{35(3Ab-aC)+87bBx}{3a^2\left(a+bx^2\right)^{3/2}}}{5a}-\frac{7(2Ab-aC)+13bBx}{5a^2\left(a+bx^2\right)^{5/2}}}{7a}-\frac{a\left(\frac{Ab}{a}-C\right)+bBx}{7a^2\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \text{2336}$$

$$\frac{\frac{\frac{-\frac{\int -\frac{35\left(-\left(\left(\frac{4Ab}{a}-C\right)x^2\right)+Bx+A\right)}{x^3\sqrt{bx^2+a}}dx}{a}-\frac{35(4Ab-aC)+93bBx}{a^2\sqrt{a+bx^2}}}{a}-\frac{35(3Ab-aC)+87bBx}{3a^2\left(a+bx^2\right)^{3/2}}}{5a}-\frac{7(2Ab-aC)+13bBx}{5a^2\left(a+bx^2\right)^{5/2}}}{7a}-\frac{a\left(\frac{Ab}{a}-C\right)+bBx}{7a^2\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{\frac{35\int \frac{-\left(\left(\frac{4Ab}{a}-C\right)x^2\right)+Bx+A}{x^3\sqrt{bx^2+a}}dx}{a}-\frac{35(4Ab-aC)+93bBx}{a^2\sqrt{a+bx^2}}}{a}-\frac{35(3Ab-aC)+87bBx}{3a^2\left(a+bx^2\right)^{3/2}}}{5a}-\frac{7(2Ab-aC)+13bBx}{5a^2(a+bx^2)^{5/2}}}{7a}-\frac{a\left(\frac{Ab}{a}-C\right)+bBx}{7a^2\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \; 2338$$

$$\frac{\frac{\frac{35\left(-\frac{\int -\frac{2aB-(9Ab-2aC)x}{x^2\sqrt{bx^2+a}}dx}{2a}-\frac{A\sqrt{a+bx^2}}{2ax^2}\right)}{a}-\frac{35(4Ab-aC)+93bBx}{a^2\sqrt{a+bx^2}}}{a}-\frac{35(3Ab-aC)+87bBx}{3a^2\left(a+bx^2\right)^{3/2}}}{5a}-\frac{7(2Ab-aC)+13bBx}{5a^2(a+bx^2)^{5/2}}}{7a}-\frac{a\left(\frac{Ab}{a}-C\right)+bBx}{7a^2\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\frac{35\left(\frac{\int \frac{2aB-(9Ab-2aC)x}{x^2\sqrt{bx^2+a}}dx}{2a}-\frac{A\sqrt{a+bx^2}}{2ax^2}\right)}{a}-\frac{35(4Ab-aC)+93bBx}{a^2\sqrt{a+bx^2}}}{a}-\frac{35(3Ab-aC)+87bBx}{3a^2\left(a+bx^2\right)^{3/2}}}{5a}-\frac{7(2Ab-aC)+13bBx}{5a^2(a+bx^2)^{5/2}}}{7a}-\frac{a\left(\frac{Ab}{a}-C\right)+bBx}{7a^2\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \; 534$$

$$\frac{\frac{\frac{35\left(\frac{-(9Ab-2aC)\int \frac{1}{x\sqrt{bx^2+a}}dx-\frac{2B\sqrt{a+bx^2}}{x}}{2a}-\frac{A\sqrt{a+bx^2}}{2ax^2}\right)}{a}-\frac{35(4Ab-aC)+93bBx}{a^2\sqrt{a+bx^2}}}{a}-\frac{35(3Ab-aC)+87bBx}{3a^2\left(a+bx^2\right)^{3/2}}}{5a}-\frac{7(2Ab-aC)+13bBx}{5a^2(a+bx^2)^{5/2}}}{7a}-\frac{a\left(\frac{Ab}{a}-C\right)+bBx}{7a^2\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \; 243$$

$$\frac{\frac{\frac{35\left(\frac{-\frac{1}{2}(9Ab-2aC)\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2-\frac{2B\sqrt{a+bx^2}}{x}}{2a}-\frac{A\sqrt{a+bx^2}}{2ax^2}\right)}{a}-\frac{35(4Ab-aC)+93bBx}{a^2\sqrt{a+bx^2}}}{a}-\frac{35(3Ab-aC)+87bBx}{3a^2\left(a+bx^2\right)^{3/2}}}{5a}-\frac{7(2Ab-aC)+13bBx}{5a^2(a+bx^2)^{5/2}}}{7a}-\frac{a\left(\frac{Ab}{a}-C\right)+bBx}{7a^2\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \; 73$$

$$\frac{\frac{\frac{35\left(\frac{-\frac{(9Ab-2aC)\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{b}-\frac{2B\sqrt{a+bx^2}}{x}}{2a}-\frac{A\sqrt{a+bx^2}}{2ax^2}\right)}{a}-\frac{35(4Ab-aC)+93bBx}{a^2\sqrt{a+bx^2}}}{a}-\frac{35(3Ab-aC)+87bBx}{3a^2\left(a+bx^2\right)^{3/2}}}{5a}-\frac{7(2Ab-aC)+13bBx}{5a^2(a+bx^2)^{5/2}}}{7a}-\frac{a\left(\frac{Ab}{a}-C\right)+bBx}{7a^2\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{\frac{35\left(\frac{\frac{(9Ab-2aC)\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{2B\sqrt{a+bx^2}}{x}}{2a}-\frac{A\sqrt{a+bx^2}}{2ax^2}\right)}{a}-\frac{35(4Ab-aC)+93bBx}{a^2\sqrt{a+bx^2}}}{a}-\frac{35(3Ab-aC)+87bBx}{3a^2\left(a+bx^2\right)^{3/2}}}{5a}-\frac{7(2Ab-aC)+13bBx}{5a^2(a+bx^2)^{5/2}}}{7a}-\frac{a\left(\frac{Ab}{a}-C\right)+bBx}{7a^2\left(a+bx^2\right)^{7/2}}$$

input
```
Int[(A + B*x + C*x^2)/(x^3*(a + b*x^2)^(9/2)),x]
```

output
```
-1/7*(a*((A*b)/a - C) + b*B*x)/(a^2*(a + b*x^2)^(7/2)) + (-1/5*(7*(2*A*b -
 a*C) + 13*b*B*x)/(a^2*(a + b*x^2)^(5/2)) + (-1/3*(35*(3*A*b - a*C) + 87*b
*B*x)/(a^2*(a + b*x^2)^(3/2)) + (-((35*(4*A*b - a*C) + 93*b*B*x)/(a^2*Sqrt
[a + b*x^2])) + (35*(-1/2*(A*Sqrt[a + b*x^2])/(a*x^2) + ((-2*B*Sqrt[a + b*
x^2])/x + ((9*A*b - 2*a*C)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a])/(2*a
)))/a)/a)/(5*a))/(7*a)
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

## Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.42

| method | result |
|---|---|
| default | $A\left(-\frac{1}{2a\,x^{2}\left(b\,x^{2}+a\right)^{\frac{7}{2}}}-\frac{9b\left(\frac{1}{7a\left(b\,x^{2}+a\right)^{\frac{7}{2}}}+\frac{\frac{1}{5a\left(b\,x^{2}+a\right)^{\frac{5}{2}}}+\frac{\frac{1}{3a\left(b\,x^{2}+a\right)^{\frac{3}{2}}}+\frac{\frac{1}{a\sqrt{b\,x^{2}+a}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{a^{\frac{3}{2}}}}{a}}{a}}{a}\right)}{2a}\right)+B\left(-\right.$ |
| risch | Expression too large to display |

input
```
int((C*x^2+B*x+A)/x^3/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
A*(-1/2/a/x^2/(b*x^2+a)^(7/2)-9/2*b/a*(1/7/a/(b*x^2+a)^(7/2)+1/a*(1/5/a/(b
*x^2+a)^(5/2)+1/a*(1/3/a/(b*x^2+a)^(3/2)+1/a*(1/a/(b*x^2+a)^(1/2)-1/a^(3/2
)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x))))))+B*(-1/a/x/(b*x^2+a)^(7/2)-8*b
/a*(1/7*x/a/(b*x^2+a)^(7/2)+6/7/a*(1/5*x/a/(b*x^2+a)^(5/2)+4/5/a*(1/3*x/a/
(b*x^2+a)^(3/2)+2/3*x/a^2/(b*x^2+a)^(1/2)))))+C*(1/7/a/(b*x^2+a)^(7/2)+1/a
*(1/5/a/(b*x^2+a)^(5/2)+1/a*(1/3/a/(b*x^2+a)^(3/2)+1/a*(1/a/(b*x^2+a)^(1/2
)-1/a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)))))
```

## Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 691, normalized size of antiderivative = 3.00

$$\int \frac{A+Bx+Cx^2}{x^3(a+bx^2)^{9/2}}\,dx = \left[-\frac{105\left((2\,Cab^4-9\,Ab^5)x^{10}+4\,(2\,Ca^2b^3-9\,Aab^4)x^8+6\,(2\,Ca^3b^2-9\,Aa^2b^3)x^6\right.}{}\right.$$

input
```
integrate((C*x^2+B*x+A)/x^3/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

output
```
[-1/420*(105*((2*C*a*b^4 - 9*A*b^5)*x^10 + 4*(2*C*a^2*b^3 - 9*A*a*b^4)*x^8
 + 6*(2*C*a^3*b^2 - 9*A*a^2*b^3)*x^6 + 4*(2*C*a^4*b - 9*A*a^3*b^2)*x^4 + (
2*C*a^5 - 9*A*a^4*b)*x^2)*sqrt(a)*log(-(b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(a)
+ 2*a)/x^2) + 2*(768*B*a*b^4*x^9 + 2688*B*a^2*b^3*x^7 + 3360*B*a^3*b^2*x^5
 + 1680*B*a^4*b*x^3 - 105*(2*C*a^2*b^3 - 9*A*a*b^4)*x^8 + 210*B*a^5*x - 35
0*(2*C*a^3*b^2 - 9*A*a^2*b^3)*x^6 + 105*A*a^5 - 406*(2*C*a^4*b - 9*A*a^3*b
^2)*x^4 - 176*(2*C*a^5 - 9*A*a^4*b)*x^2)*sqrt(b*x^2 + a))/(a^6*b^4*x^10 +
4*a^7*b^3*x^8 + 6*a^8*b^2*x^6 + 4*a^9*b*x^4 + a^10*x^2), 1/210*(105*((2*C*
a*b^4 - 9*A*b^5)*x^10 + 4*(2*C*a^2*b^3 - 9*A*a*b^4)*x^8 + 6*(2*C*a^3*b^2 -
 9*A*a^2*b^3)*x^6 + 4*(2*C*a^4*b - 9*A*a^3*b^2)*x^4 + (2*C*a^5 - 9*A*a^4*b
)*x^2)*sqrt(-a)*arctan(sqrt(b*x^2 + a)*sqrt(-a)/a) - (768*B*a*b^4*x^9 + 26
88*B*a^2*b^3*x^7 + 3360*B*a^3*b^2*x^5 + 1680*B*a^4*b*x^3 - 105*(2*C*a^2*b^
3 - 9*A*a*b^4)*x^8 + 210*B*a^5*x - 350*(2*C*a^3*b^2 - 9*A*a^2*b^3)*x^6 + 1
05*A*a^5 - 406*(2*C*a^4*b - 9*A*a^3*b^2)*x^4 - 176*(2*C*a^5 - 9*A*a^4*b)*x
^2)*sqrt(b*x^2 + a))/(a^6*b^4*x^10 + 4*a^7*b^3*x^8 + 6*a^8*b^2*x^6 + 4*a^9
*b*x^4 + a^10*x^2)]
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11198 vs. $2(223) = 446$.

Time = 87.98 (sec) , antiderivative size = 11198, normalized size of antiderivative = 48.69

$$\int \frac{A + Bx + Cx^2}{x^3 (a + bx^2)^{9/2}} \, dx = \text{Too large to display}$$

input
```
integrate((C*x**2+B*x+A)/x**3/(b*x**2+a)**(9/2),x)
```

output
```
A*(-70*a**49*sqrt(1 + b*x**2/a)/(140*a**(107/2)*x**2 + 1400*a**(105/2)*b*x
**4 + 6300*a**(103/2)*b**2*x**6 + 16800*a**(101/2)*b**3*x**8 + 29400*a**(9
9/2)*b**4*x**10 + 35280*a**(97/2)*b**5*x**12 + 29400*a**(95/2)*b**6*x**14
+ 16800*a**(93/2)*b**7*x**16 + 6300*a**(91/2)*b**8*x**18 + 1400*a**(89/2)*
b**9*x**20 + 140*a**(87/2)*b**10*x**22) - 1476*a**48*b*x**2*sqrt(1 + b*x**
2/a)/(140*a**(107/2)*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2)*b**2*
x**6 + 16800*a**(101/2)*b**3*x**8 + 29400*a**(99/2)*b**4*x**10 + 35280*a**
(97/2)*b**5*x**12 + 29400*a**(95/2)*b**6*x**14 + 16800*a**(93/2)*b**7*x**1
6 + 6300*a**(91/2)*b**8*x**18 + 1400*a**(89/2)*b**9*x**20 + 140*a**(87/2)*
b**10*x**22) - 315*a**48*b*x**2*log(b*x**2/a)/(140*a**(107/2)*x**2 + 1400*
a**(105/2)*b*x**4 + 6300*a**(103/2)*b**2*x**6 + 16800*a**(101/2)*b**3*x**8
 + 29400*a**(99/2)*b**4*x**10 + 35280*a**(97/2)*b**5*x**12 + 29400*a**(95/
2)*b**6*x**14 + 16800*a**(93/2)*b**7*x**16 + 6300*a**(91/2)*b**8*x**18 + 1
400*a**(89/2)*b**9*x**20 + 140*a**(87/2)*b**10*x**22) + 630*a**48*b*x**2*l
og(sqrt(1 + b*x**2/a) + 1)/(140*a**(107/2)*x**2 + 1400*a**(105/2)*b*x**4 +
 6300*a**(103/2)*b**2*x**6 + 16800*a**(101/2)*b**3*x**8 + 29400*a**(99/2)*
b**4*x**10 + 35280*a**(97/2)*b**5*x**12 + 29400*a**(95/2)*b**6*x**14 + 168
00*a**(93/2)*b**7*x**16 + 6300*a**(91/2)*b**8*x**18 + 1400*a**(89/2)*b**9*
x**20 + 140*a**(87/2)*b**10*x**22) - 9822*a**47*b**2*x**4*sqrt(1 + b*x**2/
a)/(140*a**(107/2)*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2)*b**2...
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.15

$$\int \frac{A+Bx+Cx^2}{x^3(a+bx^2)^{9/2}}\,dx = -\frac{128\,Bbx}{35\,\sqrt{bx^2+a}a^5} - \frac{64\,Bbx}{35\,(bx^2+a)^{\frac{3}{2}}a^4} - \frac{48\,Bbx}{35\,(bx^2+a)^{\frac{5}{2}}a^3} - \frac{8\,Bbx}{7\,(bx^2+a)^{\frac{7}{2}}a^2} - \frac{C\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{a^{\frac{9}{2}}} + \frac{9\,Ab\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{2\,a^{\frac{11}{2}}} + \frac{C}{\sqrt{bx^2+a}a^4} + \frac{C}{3\,(bx^2+a)^{\frac{3}{2}}a^3} + \frac{C}{5\,(bx^2+a)^{\frac{5}{2}}a^2} + \frac{C}{7\,(bx^2+a)^{\frac{7}{2}}a} - \frac{9\,Ab}{2\,\sqrt{bx^2+a}a^5} - \frac{3\,Ab}{2\,(bx^2+a)^{\frac{3}{2}}a^4} - \frac{9\,Ab}{10\,(bx^2+a)^{\frac{5}{2}}a^3} - \frac{9\,Ab}{14\,(bx^2+a)^{\frac{7}{2}}a^2} - \frac{B}{(bx^2+a)^{\frac{7}{2}}ax} - \frac{A}{2\,(bx^2+a)^{\frac{7}{2}}ax^2}$$

input
```
integrate((C*x^2+B*x+A)/x^3/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

output
```
-128/35*B*b*x/(sqrt(b*x^2 + a)*a^5) - 64/35*B*b*x/((b*x^2 + a)^(3/2)*a^4)
- 48/35*B*b*x/((b*x^2 + a)^(5/2)*a^3) - 8/7*B*b*x/((b*x^2 + a)^(7/2)*a^2)
- C*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(9/2) + 9/2*A*b*arcsinh(a/(sqrt(a*b)*a
bs(x)))/a^(11/2) + C/(sqrt(b*x^2 + a)*a^4) + 1/3*C/((b*x^2 + a)^(3/2)*a^3)
 + 1/5*C/((b*x^2 + a)^(5/2)*a^2) + 1/7*C/((b*x^2 + a)^(7/2)*a) - 9/2*A*b/(
sqrt(b*x^2 + a)*a^5) - 3/2*A*b/((b*x^2 + a)^(3/2)*a^4) - 9/10*A*b/((b*x^2
+ a)^(5/2)*a^3) - 9/14*A*b/((b*x^2 + a)^(7/2)*a^2) - B/((b*x^2 + a)^(7/2)*
a*x) - 1/2*A/((b*x^2 + a)^(7/2)*a*x^2)
```

## Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.41

$$\int \frac{A+Bx+Cx^2}{x^3\left(a+bx^2\right)^{9/2}}\,dx =$$

$$-\frac{\left(\left(\left(\left(3\left(\left(\frac{93\,Bb^4x}{a^5}-\frac{35\,(Ca^{24}b^6-4\,Aa^{23}b^7)}{a^{28}b^3}\right)x+\frac{308\,Bb^3}{a^4}\right)x-\frac{35\,(10\,Ca^{25}b^5-39\,Aa^{24}b^6)}{a^{28}b^3}\right)x+\frac{1050\,Bb^2}{a^3}\right)x-\frac{14\,(29\,Ca^{26}b^4\cdots}{a^{28}\cdots}}{105\,(bx^2+a)^{\frac{7}{2}}}$$

$$+\frac{(2\,Ca-9\,Ab)\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^5}$$

$$+\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^3Ab+2\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2Ba\sqrt{b}+\left(\sqrt{b}x-\sqrt{bx^2+a}\right)Aab-2\,Ba^2\sqrt{b}}{\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^2a^5}$$

input
```
integrate((C*x^2+B*x+A)/x^3/(b*x^2+a)^(9/2),x, algorithm="giac")
```

output
```
-1/105*(((((3*((93*B*b^4*x/a^5 - 35*(C*a^24*b^6 - 4*A*a^23*b^7)/(a^28*b^3)
)*x + 308*B*b^3/a^4)*x - 35*(10*C*a^25*b^5 - 39*A*a^24*b^6)/(a^28*b^3))*x
+ 1050*B*b^2/a^3)*x - 14*(29*C*a^26*b^4 - 108*A*a^25*b^5)/(a^28*b^3))*x +
420*B*b/a^2)*x - 2*(88*C*a^27*b^3 - 291*A*a^26*b^4)/(a^28*b^3))/(b*x^2 + a
)^(7/2) + (2*C*a - 9*A*b)*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/
(sqrt(-a)*a^5) + ((sqrt(b)*x - sqrt(b*x^2 + a))^3*A*b + 2*(sqrt(b)*x - sqr
t(b*x^2 + a))^2*B*a*sqrt(b) + (sqrt(b)*x - sqrt(b*x^2 + a))*A*a*b - 2*B*a^
2*sqrt(b))/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^2*a^5)
```

## Mupad [B] (verification not implemented)

Time = 3.60 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.21

$$\int \frac{A+Bx+Cx^2}{x^3\left(a+bx^2\right)^{9/2}}\,dx = \frac{\frac{C}{7\,a}+\frac{C\,\left(b\,x^2+a\right)^2}{3\,a^3}+\frac{C\,\left(b\,x^2+a\right)^3}{a^4}+\frac{C\,\left(b\,x^2+a\right)}{5\,a^2}}{\left(b\,x^2+a\right)^{7/2}} - \frac{\frac{A\,b}{7\,a}+\frac{9\,A\,b\,\left(b\,x^2+a\right)}{35\,a^2}+\frac{3\,A\,b\,\left(b\,x^2+a\right)^2}{5\,a^3}+\frac{3\,A\,b\,\left(b\,x^2+a\right)^3}{a^4}-\frac{9\,A\,b\,\left(b\,x^2+a\right)^4}{2\,a^5}}{a\,\left(b\,x^2+a\right)^{7/2}-\left(b\,x^2+a\right)^{9/2}} - \frac{\frac{B}{a^4}+\frac{128\,B\,b\,x^2}{35\,a^5}}{x\,\sqrt{b\,x^2+a}} - \frac{C\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{a^{9/2}} + \frac{9\,A\,b\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2\,a^{11/2}} - \frac{29\,B\,b\,x}{35\,a^4\,\left(b\,x^2+a\right)^{3/2}} - \frac{13\,B\,b\,x}{35\,a^3\,\left(b\,x^2+a\right)^{5/2}} - \frac{B\,b\,x}{7\,a^2\,\left(b\,x^2+a\right)^{7/2}}$$

input
```
int((A + B*x + C*x^2)/(x^3*(a + b*x^2)^(9/2)),x)
```

output
```
(C/(7*a) + (C*(a + b*x^2)^2)/(3*a^3) + (C*(a + b*x^2)^3)/a^4 + (C*(a + b*x
^2))/(5*a^2))/(a + b*x^2)^(7/2) - ((A*b)/(7*a) + (9*A*b*(a + b*x^2))/(35*a
^2) + (3*A*b*(a + b*x^2)^2)/(5*a^3) + (3*A*b*(a + b*x^2)^3)/a^4 - (9*A*b*(
a + b*x^2)^4)/(2*a^5))/(a*(a + b*x^2)^(7/2) - (a + b*x^2)^(9/2)) - (B/a^4
+ (128*B*b*x^2)/(35*a^5))/(x*(a + b*x^2)^(1/2)) - (C*atanh((a + b*x^2)^(1/
2)/a^(1/2)))/a^(9/2) + (9*A*b*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(2*a^(11/2
)) - (29*B*b*x)/(35*a^4*(a + b*x^2)^(3/2)) - (13*B*b*x)/(35*a^3*(a + b*x^2
)^(5/2)) - (B*b*x)/(7*a^2*(a + b*x^2)^(7/2))
```

## Reduce [B] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 1029, normalized size of antiderivative = 4.47

$$\int \frac{A+Bx+Cx^2}{x^3\left(a+bx^2\right)^{9/2}}\,dx = \text{Too large to display}$$

input
```
int((C*x^2+B*x+A)/x^3/(b*x^2+a)^(9/2),x)
```

output
```
( - 105*sqrt(a + b*x**2)*a**5 - 1584*sqrt(a + b*x**2)*a**4*b*x**2 - 210*sq
rt(a + b*x**2)*a**4*b*x + 352*sqrt(a + b*x**2)*a**4*c*x**2 - 3654*sqrt(a +
 b*x**2)*a**3*b**2*x**4 - 1680*sqrt(a + b*x**2)*a**3*b**2*x**3 + 812*sqrt(
a + b*x**2)*a**3*b*c*x**4 - 3150*sqrt(a + b*x**2)*a**2*b**3*x**6 - 3360*sq
rt(a + b*x**2)*a**2*b**3*x**5 + 700*sqrt(a + b*x**2)*a**2*b**2*c*x**6 - 94
5*sqrt(a + b*x**2)*a*b**4*x**8 - 2688*sqrt(a + b*x**2)*a*b**4*x**7 + 210*s
qrt(a + b*x**2)*a*b**3*c*x**8 - 768*sqrt(a + b*x**2)*b**5*x**9 - 945*sqrt(
a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*a**4*b*x**2 + 210
*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*a**4*c*x**2
 - 3780*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*a**3
*b**2*x**4 + 840*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt
(a))*a**3*b*c*x**4 - 5670*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b
)*x)/sqrt(a))*a**2*b**3*x**6 + 1260*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a
) + sqrt(b)*x)/sqrt(a))*a**2*b**2*c*x**6 - 3780*sqrt(a)*log((sqrt(a + b*x*
*2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b**4*x**8 + 840*sqrt(a)*log((sqrt(a
+ b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b**3*c*x**8 - 945*sqrt(a)*log(
(sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*b**5*x**10 + 210*sqrt(a)
*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*b**4*c*x**10 + 945*
sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*a**4*b*x**2
- 210*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*a**...
```

# 3.12 $\int x^3(a+bx^2)(A+Bx+Cx^2+Dx^3)\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 65

$$\int x^3(a+bx^2)\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{1}{4}aAx^4+\frac{1}{5}aBx^5+\frac{1}{6}(Ab+aC)x^6+\frac{1}{7}(bB+aD)x^7+\frac{1}{8}bCx^8+\frac{1}{9}bDx^9$$

output
```
1/4*a*A*x^4+1/5*a*B*x^5+1/6*(A*b+C*a)*x^6+1/7*(B*b+D*a)*x^7+1/8*b*C*x^8+1/
9*b*D*x^9
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00

$$\int x^3(a+bx^2)\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{1}{4}aAx^4+\frac{1}{5}aBx^5+\frac{1}{6}(Ab+aC)x^6+\frac{1}{7}(bB+aD)x^7+\frac{1}{8}bCx^8+\frac{1}{9}bDx^9$$

input
```
Integrate[x^3*(a + b*x^2)*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(a*A*x^4)/4 + (a*B*x^5)/5 + ((A*b + a*C)*x^6)/6 + ((b*B + a*D)*x^7)/7 + (b
*C*x^8)/8 + (b*D*x^9)/9
```

## Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(a+bx^2\right) \left(A+Bx+Cx^2+Dx^3\right) \, dx$$

$$\Big\downarrow \text{2333}$$

$$\int \left(x^5(aC+Ab)+aAx^3+x^6(aD+bB)+aBx^4+bCx^7+bDx^8\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{6}x^6(aC+Ab)+\frac{1}{4}aAx^4+\frac{1}{7}x^7(aD+bB)+\frac{1}{5}aBx^5+\frac{1}{8}bCx^8+\frac{1}{9}bDx^9$$

input
```
Int[x^3*(a + b*x^2)*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(a*A*x^4)/4 + (a*B*x^5)/5 + ((A*b + a*C)*x^6)/6 + ((b*B + a*D)*x^7)/7 + (b
*C*x^8)/8 + (b*D*x^9)/9
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

## Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.83

| method | result | size |
|---|---|---|
| default | $\frac{aA\,x^{4}}{4}+\frac{aB\,x^{5}}{5}+\frac{(Ab+Ca)x^{6}}{6}+\frac{(Bb+Da)x^{7}}{7}+\frac{bC\,x^{8}}{8}+\frac{bDx^{9}}{9}$ | 54 |
| norman | $\frac{bDx^{9}}{9}+\frac{bC\,x^{8}}{8}+\left(\frac{Bb}{7}+\frac{Da}{7}\right)x^{7}+\left(\frac{Ab}{6}+\frac{Ca}{6}\right)x^{6}+\frac{aB\,x^{5}}{5}+\frac{aA\,x^{4}}{4}$ | 56 |
| gosper | $\frac{1}{9}bDx^{9}+\frac{1}{8}bC\,x^{8}+\frac{1}{7}bB\,x^{7}+\frac{1}{7}x^{7}Da+\frac{1}{6}x^{6}Ab+\frac{1}{6}x^{6}Ca+\frac{1}{5}aB\,x^{5}+\frac{1}{4}aA\,x^{4}$ | 58 |
| parallelrisch | $\frac{1}{9}bDx^{9}+\frac{1}{8}bC\,x^{8}+\frac{1}{7}bB\,x^{7}+\frac{1}{7}x^{7}Da+\frac{1}{6}x^{6}Ab+\frac{1}{6}x^{6}Ca+\frac{1}{5}aB\,x^{5}+\frac{1}{4}aA\,x^{4}$ | 58 |
| orering | $\frac{x^{4}\left(280Db\,x^{5}+315Cb\,x^{4}+360bB\,x^{3}+360Da\,x^{3}+420Ab\,x^{2}+420Ca\,x^{2}+504Bax+630Aa\right)}{2520}$ | 58 |

input
```
int(x^3*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
1/4*a*A*x^4+1/5*a*B*x^5+1/6*(A*b+C*a)*x^6+1/7*(B*b+D*a)*x^7+1/8*b*C*x^8+1/
9*b*D*x^9
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.82

$$\int x^{3}\left(a+bx^{2}\right)\left(A+Bx+Cx^{2}+Dx^{3}\right)\,dx=\frac{1}{9}\,Dbx^{9}+\frac{1}{8}\,Cbx^{8}+\frac{1}{7}\,(Da+Bb)x^{7}+\frac{1}{5}\,Bax^{5}+\frac{1}{6}\,(Ca+Ab)x^{6}+\frac{1}{4}\,Aax^{4}$$

input
```
integrate(x^3*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
1/9*D*b*x^9 + 1/8*C*b*x^8 + 1/7*(D*a + B*b)*x^7 + 1/5*B*a*x^5 + 1/6*(C*a +
 A*b)*x^6 + 1/4*A*a*x^4
```

## Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.92

$$\int x^3(a+bx^2)\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{Aax^4}{4}+\frac{Bax^5}{5}+\frac{Cbx^8}{8}+\frac{Dbx^9}{9}+x^7\left(\frac{Bb}{7}+\frac{Da}{7}\right)+x^6\left(\frac{Ab}{6}+\frac{Ca}{6}\right)$$

input
```
integrate(x**3*(b*x**2+a)*(D*x**3+C*x**2+B*x+A),x)
```

output
```
A*a*x**4/4 + B*a*x**5/5 + C*b*x**8/8 + D*b*x**9/9 + x**7*(B*b/7 + D*a/7) +
 x**6*(A*b/6 + C*a/6)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.82

$$\int x^3(a+bx^2)\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{1}{9}\,Dbx^9+\frac{1}{8}\,Cbx^8+\frac{1}{7}\,(Da+Bb)x^7+\frac{1}{5}\,Bax^5+\frac{1}{6}\,(Ca+Ab)x^6+\frac{1}{4}\,Aax^4$$

input
```
integrate(x^3*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
1/9*D*b*x^9 + 1/8*C*b*x^8 + 1/7*(D*a + B*b)*x^7 + 1/5*B*a*x^5 + 1/6*(C*a +
 A*b)*x^6 + 1/4*A*a*x^4
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.88

$$\int x^3(a+bx^2)\left(A+Bx+Cx^2+Dx^3\right)\,dx = \frac{1}{9}Dbx^9+\frac{1}{8}Cbx^8+\frac{1}{7}Dax^7+\frac{1}{7}Bbx^7+\frac{1}{6}Cax^6+\frac{1}{6}Abx^6+\frac{1}{5}Bax^5+\frac{1}{4}Aax^4$$

input
```
integrate(x^3*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

output
```
1/9*D*b*x^9 + 1/8*C*b*x^8 + 1/7*D*a*x^7 + 1/7*B*b*x^7 + 1/6*C*a*x^6 + 1/6*
A*b*x^6 + 1/5*B*a*x^5 + 1/4*A*a*x^4
```

## Mupad [B] (verification not implemented)

Time = 2.38 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.88

$$\int x^3(a+bx^2)\left(A+Bx+Cx^2+Dx^3\right)\,dx = \frac{a\,x^7\,D}{7}+\frac{b\,x^9\,D}{9}+\frac{A\,a\,x^4}{4}+\frac{B\,a\,x^5}{5}+\frac{A\,b\,x^6}{6}+\frac{C\,a\,x^6}{6}+\frac{B\,b\,x^7}{7}+\frac{C\,b\,x^8}{8}$$

input
```
int(x^3*(a + b*x^2)*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
(a*x^7*D)/7 + (b*x^9*D)/9 + (A*a*x^4)/4 + (B*a*x^5)/5 + (A*b*x^6)/6 + (C*a
*x^6)/6 + (B*b*x^7)/7 + (C*b*x^8)/8
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.91

$$\int x^3(a+bx^2)\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$=\frac{x^4(280bd\,x^5+315bc\,x^4+360ad\,x^3+360b^2x^3+420ab\,x^2+420ac\,x^2+504abx+630a^2)}{2520}$$

input
```
int(x^3*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x)
```

output
```
(x**4*(630*a**2 + 420*a*b*x**2 + 504*a*b*x + 420*a*c*x**2 + 360*a*d*x**3 +
 360*b**2*x**3 + 315*b*c*x**4 + 280*b*d*x**5))/2520
```

# 3.13 $\int x^2(a+bx^2)\left(A+Bx+Cx^2+Dx^3\right)\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 65

$$\int x^2\left(a+bx^2\right)\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{1}{3}aAx^3+\frac{1}{4}aBx^4+\frac{1}{5}(Ab+aC)x^5+\frac{1}{6}(bB+aD)x^6+\frac{1}{7}bCx^7+\frac{1}{8}bDx^8$$

output
```
1/3*a*A*x^3+1/4*a*B*x^4+1/5*(A*b+C*a)*x^5+1/6*(B*b+D*a)*x^6+1/7*b*C*x^7+1/
8*b*D*x^8
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00

$$\int x^2\left(a+bx^2\right)\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{1}{3}aAx^3+\frac{1}{4}aBx^4+\frac{1}{5}(Ab+aC)x^5+\frac{1}{6}(bB+aD)x^6+\frac{1}{7}bCx^7+\frac{1}{8}bDx^8$$

input
```
Integrate[x^2*(a + b*x^2)*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(a*A*x^3)/3 + (a*B*x^4)/4 + ((A*b + a*C)*x^5)/5 + ((b*B + a*D)*x^6)/6 + (b
*C*x^7)/7 + (b*D*x^8)/8
```

## Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+bx^2\right) \left(A+Bx+Cx^2+Dx^3\right) \, dx$$

$$\Big\downarrow \text{2333}$$

$$\int \left(x^4(aC+Ab)+aAx^2+x^5(aD+bB)+aBx^3+bCx^6+bDx^7\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{5}x^5(aC+Ab)+\frac{1}{3}aAx^3+\frac{1}{6}x^6(aD+bB)+\frac{1}{4}aBx^4+\frac{1}{7}bCx^7+\frac{1}{8}bDx^8$$

input
```
Int[x^2*(a + b*x^2)*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(a*A*x^3)/3 + (a*B*x^4)/4 + ((A*b + a*C)*x^5)/5 + ((b*B + a*D)*x^6)/6 + (b
*C*x^7)/7 + (b*D*x^8)/8
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

## Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.83

| method | result | size |
|---|---|---|
| default | $\frac{aA\,x^3}{3}+\frac{aB\,x^4}{4}+\frac{(Ab+Ca)x^5}{5}+\frac{(Bb+Da)x^6}{6}+\frac{bC\,x^7}{7}+\frac{Db\,x^8}{8}$ | 54 |
| norman | $\frac{Db\,x^8}{8}+\frac{bC\,x^7}{7}+\left(\frac{Bb}{6}+\frac{Da}{6}\right)x^6+\left(\frac{Ab}{5}+\frac{Ca}{5}\right)x^5+\frac{aB\,x^4}{4}+\frac{aA\,x^3}{3}$ | 56 |
| gosper | $\frac{1}{8}Db\,x^8+\frac{1}{7}bC\,x^7+\frac{1}{6}bB\,x^6+\frac{1}{6}Da\,x^6+\frac{1}{5}x^5Ab+\frac{1}{5}x^5Ca+\frac{1}{4}aB\,x^4+\frac{1}{3}aA\,x^3$ | 58 |
| parallelrisch | $\frac{1}{8}Db\,x^8+\frac{1}{7}bC\,x^7+\frac{1}{6}bB\,x^6+\frac{1}{6}Da\,x^6+\frac{1}{5}x^5Ab+\frac{1}{5}x^5Ca+\frac{1}{4}aB\,x^4+\frac{1}{3}aA\,x^3$ | 58 |
| orering | $\frac{x^3(105Db\,x^5+120Cb\,x^4+140bB\,x^3+140Da\,x^3+168Ab\,x^2+168Ca\,x^2+210Bax+280Aa)}{840}$ | 58 |

input
```
int(x^2*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
1/3*a*A*x^3+1/4*a*B*x^4+1/5*(A*b+C*a)*x^5+1/6*(B*b+D*a)*x^6+1/7*b*C*x^7+1/
8*D*b*x^8
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.82

$$\int x^2(a+bx^2)\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{1}{8}\,Dbx^8+\frac{1}{7}\,Cbx^7+\frac{1}{6}\,(Da+Bb)x^6+\frac{1}{4}\,Bax^4+\frac{1}{5}\,(Ca+Ab)x^5+\frac{1}{3}\,Aax^3$$

input
```
integrate(x^2*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
1/8*D*b*x^8 + 1/7*C*b*x^7 + 1/6*(D*a + B*b)*x^6 + 1/4*B*a*x^4 + 1/5*(C*a +
 A*b)*x^5 + 1/3*A*a*x^3
```

## Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.92

$$\int x^2(a+bx^2)\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{Aax^3}{3}+\frac{Bax^4}{4}+\frac{Cbx^7}{7}+\frac{Dbx^8}{8}+x^6\left(\frac{Bb}{6}+\frac{Da}{6}\right)+x^5\left(\frac{Ab}{5}+\frac{Ca}{5}\right)$$

input
```
integrate(x**2*(b*x**2+a)*(D*x**3+C*x**2+B*x+A),x)
```

output
```
A*a*x**3/3 + B*a*x**4/4 + C*b*x**7/7 + D*b*x**8/8 + x**6*(B*b/6 + D*a/6) +
 x**5*(A*b/5 + C*a/5)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.82

$$\int x^2(a+bx^2)\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{1}{8}\,Dbx^8+\frac{1}{7}\,Cbx^7+\frac{1}{6}\,(Da+Bb)x^6+\frac{1}{4}\,Bax^4+\frac{1}{5}\,(Ca+Ab)x^5+\frac{1}{3}\,Aax^3$$

input
```
integrate(x^2*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
1/8*D*b*x^8 + 1/7*C*b*x^7 + 1/6*(D*a + B*b)*x^6 + 1/4*B*a*x^4 + 1/5*(C*a +
 A*b)*x^5 + 1/3*A*a*x^3
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.88

$$\int x^2(a+bx^2)\left(A+Bx+Cx^2+Dx^3\right)\,dx = \frac{1}{8}Dbx^8+\frac{1}{7}Cbx^7+\frac{1}{6}Dax^6+\frac{1}{6}Bbx^6+\frac{1}{5}Cax^5+\frac{1}{5}Abx^5+\frac{1}{4}Bax^4+\frac{1}{3}Aax^3$$

input
```
integrate(x^2*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

output
```
1/8*D*b*x^8 + 1/7*C*b*x^7 + 1/6*D*a*x^6 + 1/6*B*b*x^6 + 1/5*C*a*x^5 + 1/5*
A*b*x^5 + 1/4*B*a*x^4 + 1/3*A*a*x^3
```

## Mupad [B] (verification not implemented)

Time = 2.36 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.88

$$\int x^2(a+bx^2)\left(A+Bx+Cx^2+Dx^3\right)\,dx = \frac{a\,x^6\,D}{6}+\frac{b\,x^8\,D}{8}+\frac{A\,a\,x^3}{3}+\frac{B\,a\,x^4}{4}+\frac{A\,b\,x^5}{5}+\frac{C\,a\,x^5}{5}+\frac{B\,b\,x^6}{6}+\frac{C\,b\,x^7}{7}$$

input
```
int(x^2*(a + b*x^2)*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
(a*x^6*D)/6 + (b*x^8*D)/8 + (A*a*x^3)/3 + (B*a*x^4)/4 + (A*b*x^5)/5 + (C*a
*x^5)/5 + (B*b*x^6)/6 + (C*b*x^7)/7
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.91

$$\int x^2(a+bx^2)\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$=\frac{x^3(105bd\,x^5+120bc\,x^4+140ad\,x^3+140b^2x^3+168ab\,x^2+168ac\,x^2+210abx+280a^2)}{840}$$

input
```
int(x^2*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x)
```

output
```
(x**3*(280*a**2 + 168*a*b*x**2 + 210*a*b*x + 168*a*c*x**2 + 140*a*d*x**3 +
 140*b**2*x**3 + 120*b*c*x**4 + 105*b*d*x**5))/840
```

# 3.14 $\int x(a+bx^2)\left(A+Bx+Cx^2+Dx^3\right)\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 65

$$\int x\left(a+bx^2\right)\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{1}{2}aAx^2+\frac{1}{3}aBx^3+\frac{1}{4}(Ab+aC)x^4+\frac{1}{5}(bB+aD)x^5+\frac{1}{6}bCx^6+\frac{1}{7}bDx^7$$

output
```
1/2*a*A*x^2+1/3*a*B*x^3+1/4*(A*b+C*a)*x^4+1/5*(B*b+D*a)*x^5+1/6*b*C*x^6+1/
7*b*D*x^7
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00

$$\int x\left(a+bx^2\right)\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{1}{2}aAx^2+\frac{1}{3}aBx^3+\frac{1}{4}(Ab+aC)x^4+\frac{1}{5}(bB+aD)x^5+\frac{1}{6}bCx^6+\frac{1}{7}bDx^7$$

input
```
Integrate[x*(a + b*x^2)*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(a*A*x^2)/2 + (a*B*x^3)/3 + ((A*b + a*C)*x^4)/4 + ((b*B + a*D)*x^5)/5 + (b
*C*x^6)/6 + (b*D*x^7)/7
```

## Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+bx^2\right)\left(A+Bx+Cx^2+Dx^3\right)\,dx$$

$$\Big\downarrow \text{2333}$$

$$\int \left(x^3(aC+Ab)+aAx+x^4(aD+bB)+aBx^2+bCx^5+bDx^6\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}x^4(aC+Ab)+\frac{1}{2}aAx^2+\frac{1}{5}x^5(aD+bB)+\frac{1}{3}aBx^3+\frac{1}{6}bCx^6+\frac{1}{7}bDx^7$$

input
```
Int[x*(a + b*x^2)*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(a*A*x^2)/2 + (a*B*x^3)/3 + ((A*b + a*C)*x^4)/4 + ((b*B + a*D)*x^5)/5 + (b
*C*x^6)/6 + (b*D*x^7)/7
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

## Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.83

| method | result | size |
|---|---|---|
| default | $\frac{aA\,x^{2}}{2}+\frac{x^{3}Ba}{3}+\frac{(Ab+Ca)x^{4}}{4}+\frac{(Bb+Da)x^{5}}{5}+\frac{Cb\,x^{6}}{6}+\frac{x^{7}bD}{7}$ | 54 |
| norman | $\frac{x^{7}bD}{7}+\frac{Cb\,x^{6}}{6}+\left(\frac{Bb}{5}+\frac{Da}{5}\right)x^{5}+\left(\frac{Ab}{4}+\frac{Ca}{4}\right)x^{4}+\frac{x^{3}Ba}{3}+\frac{aA\,x^{2}}{2}$ | 56 |
| gosper | $\frac{1}{7}x^{7}bD+\frac{1}{6}Cb\,x^{6}+\frac{1}{5}bB\,x^{5}+\frac{1}{5}x^{5}Da+\frac{1}{4}x^{4}Ab+\frac{1}{4}Ca\,x^{4}+\frac{1}{3}x^{3}Ba+\frac{1}{2}aA\,x^{2}$ | 58 |
| parallelrisch | $\frac{1}{7}x^{7}bD+\frac{1}{6}Cb\,x^{6}+\frac{1}{5}bB\,x^{5}+\frac{1}{5}x^{5}Da+\frac{1}{4}x^{4}Ab+\frac{1}{4}Ca\,x^{4}+\frac{1}{3}x^{3}Ba+\frac{1}{2}aA\,x^{2}$ | 58 |
| orering | $\frac{x^{2}\left(60Db\,x^{5}+70Cb\,x^{4}+84bB\,x^{3}+84Da\,x^{3}+105Ab\,x^{2}+105Ca\,x^{2}+140Bax+210Aa\right)}{420}$ | 58 |

input
```
int(x*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
1/2*a*A*x^2+1/3*x^3*B*a+1/4*(A*b+C*a)*x^4+1/5*(B*b+D*a)*x^5+1/6*C*b*x^6+1/
7*x^7*b*D
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.82

$$\int x\left(a+bx^2\right)\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{1}{7}\,Dbx^7+\frac{1}{6}\,Cbx^6+\frac{1}{5}\,(Da+Bb)x^5+\frac{1}{3}\,Bax^3+\frac{1}{4}\,(Ca+Ab)x^4+\frac{1}{2}\,Aax^2$$

input
```
integrate(x*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
1/7*D*b*x^7 + 1/6*C*b*x^6 + 1/5*(D*a + B*b)*x^5 + 1/3*B*a*x^3 + 1/4*(C*a +
 A*b)*x^4 + 1/2*A*a*x^2
```

## Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.92

$$\int x(a+bx^2)\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{Aax^2}{2}+\frac{Bax^3}{3}+\frac{Cbx^6}{6}+\frac{Dbx^7}{7}+x^5\left(\frac{Bb}{5}+\frac{Da}{5}\right)+x^4\left(\frac{Ab}{4}+\frac{Ca}{4}\right)$$

input
```
integrate(x*(b*x**2+a)*(D*x**3+C*x**2+B*x+A),x)
```

output
```
A*a*x**2/2 + B*a*x**3/3 + C*b*x**6/6 + D*b*x**7/7 + x**5*(B*b/5 + D*a/5) +
 x**4*(A*b/4 + C*a/4)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.82

$$\int x(a+bx^2)\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{1}{7}Dbx^7+\frac{1}{6}Cbx^6+\frac{1}{5}(Da+Bb)x^5+\frac{1}{3}Bax^3+\frac{1}{4}(Ca+Ab)x^4+\frac{1}{2}Aax^2$$

input
```
integrate(x*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
1/7*D*b*x^7 + 1/6*C*b*x^6 + 1/5*(D*a + B*b)*x^5 + 1/3*B*a*x^3 + 1/4*(C*a +
 A*b)*x^4 + 1/2*A*a*x^2
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.88

$$\int x(a+bx^2)\left(A+Bx+Cx^2+Dx^3\right)\,dx = \frac{1}{7}\,Dbx^7 + \frac{1}{6}\,Cbx^6 + \frac{1}{5}\,Dax^5 + \frac{1}{5}\,Bbx^5 + \frac{1}{4}\,Cax^4 + \frac{1}{4}\,Abx^4 + \frac{1}{3}\,Bax^3 + \frac{1}{2}\,Aax^2$$

input
```
integrate(x*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

output
```
1/7*D*b*x^7 + 1/6*C*b*x^6 + 1/5*D*a*x^5 + 1/5*B*b*x^5 + 1/4*C*a*x^4 + 1/4*
A*b*x^4 + 1/3*B*a*x^3 + 1/2*A*a*x^2
```

## Mupad [B] (verification not implemented)

Time = 2.29 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.88

$$\int x(a+bx^2)\left(A+Bx+Cx^2+Dx^3\right)\,dx = \frac{a\,x^5\,D}{5} + \frac{b\,x^7\,D}{7} + \frac{A\,a\,x^2}{2} + \frac{B\,a\,x^3}{3} + \frac{A\,b\,x^4}{4} + \frac{C\,a\,x^4}{4} + \frac{B\,b\,x^5}{5} + \frac{C\,b\,x^6}{6}$$

input
```
int(x*(a + b*x^2)*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
(a*x^5*D)/5 + (b*x^7*D)/7 + (A*a*x^2)/2 + (B*a*x^3)/3 + (A*b*x^4)/4 + (C*a
*x^4)/4 + (B*b*x^5)/5 + (C*b*x^6)/6
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.91

$$\int x(a+bx^2)\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$=\frac{x^2(60bd\,x^5+70bc\,x^4+84ad\,x^3+84b^2x^3+105ab\,x^2+105ac\,x^2+140abx+210a^2)}{420}$$

input
```
int(x*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x)
```

output
```
(x**2*(210*a**2 + 105*a*b*x**2 + 140*a*b*x + 105*a*c*x**2 + 84*a*d*x**3 +
84*b**2*x**3 + 70*b*c*x**4 + 60*b*d*x**5))/420
```

# 3.15 $\int (a+bx^2)(A+Bx+Cx^2+Dx^3)\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 60

$$\int (a+bx^2)(A+Bx+Cx^2+Dx^3)\,dx = aAx+\frac{1}{2}aBx^2+\frac{1}{3}(Ab+aC)x^3+\frac{1}{4}(bB+aD)x^4+\frac{1}{5}bCx^5+\frac{1}{6}bDx^6$$

output
```
a*A*x+1/2*a*B*x^2+1/3*(A*b+C*a)*x^3+1/4*(B*b+D*a)*x^4+1/5*b*C*x^5+1/6*b*D*
x^6
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$\int (a+bx^2)(A+Bx+Cx^2+Dx^3)\,dx = aAx+\frac{1}{2}aBx^2+\frac{1}{3}(Ab+aC)x^3+\frac{1}{4}(bB+aD)x^4+\frac{1}{5}bCx^5+\frac{1}{6}bDx^6$$

input
```
Integrate[(a + b*x^2)*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
a*A*x + (a*B*x^2)/2 + ((A*b + a*C)*x^3)/3 + ((b*B + a*D)*x^4)/4 + (b*C*x^5
)/5 + (b*D*x^6)/6
```

## Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2341, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^2)(A+Bx+Cx^2+Dx^3)\,dx$$
$$\Big\downarrow \text{2341}$$
$$\int \left(x^2(aC+Ab)+aA+x^3(aD+bB)+aBx+bCx^4+bDx^5\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{1}{3}x^3(aC+Ab)+aAx+\frac{1}{4}x^4(aD+bB)+\frac{1}{2}aBx^2+\frac{1}{5}bCx^5+\frac{1}{6}bDx^6$$

input
```
Int[(a + b*x^2)*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
a*A*x + (a*B*x^2)/2 + ((A*b + a*C)*x^3)/3 + ((b*B + a*D)*x^4)/4 + (b*C*x^5
)/5 + (b*D*x^6)/6
```


### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2341
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

## Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.85

| method | result | size |
|---|---|---|
| default | $aAx+\frac{Ba\,x^{2}}{2}+\frac{(Ab+Ca)x^{3}}{3}+\frac{(Bb+Da)x^{4}}{4}+\frac{bC\,x^{5}}{5}+\frac{bDx^{6}}{6}$ | 51 |
| norman | $\frac{bDx^{6}}{6}+\frac{bC\,x^{5}}{5}+\left(\frac{Bb}{4}+\frac{Da}{4}\right)x^{4}+\left(\frac{Ab}{3}+\frac{Ca}{3}\right)x^{3}+\frac{Ba\,x^{2}}{2}+aAx$ | 53 |
| gosper | $\frac{1}{6}bDx^{6}+\frac{1}{5}bC\,x^{5}+\frac{1}{4}bB\,x^{4}+\frac{1}{4}x^{4}Da+\frac{1}{3}Ab\,x^{3}+\frac{1}{3}x^{3}Ca+\frac{1}{2}Ba\,x^{2}+aAx$ | 55 |
| parallelrisch | $\frac{1}{6}bDx^{6}+\frac{1}{5}bC\,x^{5}+\frac{1}{4}bB\,x^{4}+\frac{1}{4}x^{4}Da+\frac{1}{3}Ab\,x^{3}+\frac{1}{3}x^{3}Ca+\frac{1}{2}Ba\,x^{2}+aAx$ | 55 |
| orering | $\frac{x\left(10Db\,x^{5}+12Cb\,x^{4}+15bB\,x^{3}+15Da\,x^{3}+20Ab\,x^{2}+20Ca\,x^{2}+30Bax+60Aa\right)}{60}$ | 56 |

input
```
int((b*x^2+a)*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
a*A*x+1/2*B*a*x^2+1/3*(A*b+C*a)*x^3+1/4*(B*b+D*a)*x^4+1/5*b*C*x^5+1/6*b*D*
x^6
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.83

$$\int \left(a+bx^{2}\right)\left(A+Bx+Cx^{2}+Dx^{3}\right)\,dx=\frac{1}{6}\,Dbx^{6}+\frac{1}{5}\,Cbx^{5}+\frac{1}{4}\,(Da+Bb)x^{4}+\frac{1}{2}\,Bax^{2}+\frac{1}{3}\,(Ca+Ab)x^{3}+Aax$$

input
```
integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
1/6*D*b*x^6 + 1/5*C*b*x^5 + 1/4*(D*a + B*b)*x^4 + 1/2*B*a*x^2 + 1/3*(C*a +
 A*b)*x^3 + A*a*x
```

## Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.93

$$\int (a+bx^2)\left(A+Bx+Cx^2+Dx^3\right)\,dx = Aax+\frac{Bax^2}{2}+\frac{Cbx^5}{5}+\frac{Dbx^6}{6} + x^4\left(\frac{Bb}{4}+\frac{Da}{4}\right)+x^3\left(\frac{Ab}{3}+\frac{Ca}{3}\right)$$

input
```
integrate((b*x**2+a)*(D*x**3+C*x**2+B*x+A),x)
```

output
```
A*a*x + B*a*x**2/2 + C*b*x**5/5 + D*b*x**6/6 + x**4*(B*b/4 + D*a/4) + x**3
*(A*b/3 + C*a/3)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.83

$$\int (a+bx^2)\left(A+Bx+Cx^2+Dx^3\right)\,dx = \frac{1}{6}\,Dbx^6+\frac{1}{5}\,Cbx^5+\frac{1}{4}\,(Da+Bb)x^4 + \frac{1}{2}\,Bax^2+\frac{1}{3}\,(Ca+Ab)x^3+Aax$$

input
```
integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
1/6*D*b*x^6 + 1/5*C*b*x^5 + 1/4*(D*a + B*b)*x^4 + 1/2*B*a*x^2 + 1/3*(C*a +
 A*b)*x^3 + A*a*x
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.90

$$\int (a+bx^2) (A+Bx+Cx^2+Dx^3) \, dx = \frac{1}{6} Dbx^6 + \frac{1}{5} Cbx^5 + \frac{1}{4} Dax^4 + \frac{1}{4} Bbx^4 + \frac{1}{3} Cax^3 + \frac{1}{3} Abx^3 + \frac{1}{2} Bax^2 + Aax$$

input
```
integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

output
```
1/6*D*b*x^6 + 1/5*C*b*x^5 + 1/4*D*a*x^4 + 1/4*B*b*x^4 + 1/3*C*a*x^3 + 1/3*
A*b*x^3 + 1/2*B*a*x^2 + A*a*x
```

## Mupad [B] (verification not implemented)

Time = 2.35 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.90

$$\int (a+bx^2) (A+Bx+Cx^2+Dx^3) \, dx = \frac{a x^4 D}{4} + \frac{b x^6 D}{6} + A a x + \frac{B a x^2}{2} + \frac{A b x^3}{3} + \frac{C a x^3}{3} + \frac{B b x^4}{4} + \frac{C b x^5}{5}$$

input
```
int((a + b*x^2)*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
(a*x^4*D)/4 + (b*x^6*D)/6 + A*a*x + (B*a*x^2)/2 + (A*b*x^3)/3 + (C*a*x^3)/
3 + (B*b*x^4)/4 + (C*b*x^5)/5
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.95

$$\int (a+bx^2) (A+Bx+Cx^2+Dx^3) \, dx = \frac{x(10bd\,x^5+12bc\,x^4+15ad\,x^3+15b^2x^3+20ab\,x^2+20ac\,x^2+30abx+60a^2)}{60}$$

input
```
int((b*x^2+a)*(D*x^3+C*x^2+B*x+A),x)
```

output
```
(x*(60*a**2 + 20*a*b*x**2 + 30*a*b*x + 20*a*c*x**2 + 15*a*d*x**3 + 15*b**2
*x**3 + 12*b*c*x**4 + 10*b*d*x**5))/60
```

# 3.16 $\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 56

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x}\,dx = aBx+\frac{1}{2}(Ab+aC)x^2+\frac{1}{3}(bB+aD)x^3 + \frac{1}{4}bCx^4+\frac{1}{5}bDx^5+aA\log(x)$$

output
```
a*B*x+1/2*(A*b+C*a)*x^2+1/3*(B*b+D*a)*x^3+1/4*b*C*x^4+1/5*b*D*x^5+a*A*ln(x
)
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x}\,dx = aBx+\frac{1}{2}(Ab+aC)x^2+\frac{1}{3}(bB+aD)x^3 + \frac{1}{4}bCx^4+\frac{1}{5}bDx^5+aA\log(x)$$

input
```
Integrate[((a + b*x^2)*(A + B*x + C*x^2 + D*x^3))/x,x]
```

output
```
a*B*x + ((A*b + a*C)*x^2)/2 + ((b*B + a*D)*x^3)/3 + (b*C*x^4)/4 + (b*D*x^5
)/5 + a*A*Log[x]
```

## Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x} \, dx$$

$$\Big\downarrow \textbf{2333}$$

$$\int \left(x(aC+Ab)+\frac{aA}{x}+x^2(aD+bB)+aB+bCx^3+bDx^4\right)dx$$

$$\Big\downarrow \textbf{2009}$$

$$\frac{1}{2}x^2(aC+Ab)+aA\log(x)+\frac{1}{3}x^3(aD+bB)+aBx+\frac{1}{4}bCx^4+\frac{1}{5}bDx^5$$

input
```
Int[((a + b*x^2)*(A + B*x + C*x^2 + D*x^3))/x,x]
```

output
```
a*B*x + ((A*b + a*C)*x^2)/2 + ((b*B + a*D)*x^3)/3 + (b*C*x^4)/4 + (b*D*x^5
)/5 + a*A*Log[x]
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

## Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.91

| method | result | size |
|---|---|---|
| norman | $\left(\frac{Ab}{2}+\frac{Ca}{2}\right)x^{2}+\left(\frac{Bb}{3}+\frac{Da}{3}\right)x^{3}+Bax+\frac{Cb\,x^{4}}{4}+\frac{Db\,x^{5}}{5}+aA\ln\left(x\right)$ | 51 |
| default | $\frac{Db\,x^{5}}{5}+\frac{Cb\,x^{4}}{4}+\frac{bB\,x^{3}}{3}+\frac{Da\,x^{3}}{3}+\frac{Ab\,x^{2}}{2}+\frac{Ca\,x^{2}}{2}+Bax+aA\ln\left(x\right)$ | 53 |
| parallelrisch | $\frac{Db\,x^{5}}{5}+\frac{Cb\,x^{4}}{4}+\frac{bB\,x^{3}}{3}+\frac{Da\,x^{3}}{3}+\frac{Ab\,x^{2}}{2}+\frac{Ca\,x^{2}}{2}+Bax+aA\ln\left(x\right)$ | 53 |

input
```
int((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x,x,method=_RETURNVERBOSE)
```

output
```
(1/2*A*b+1/2*C*a)*x^2+(1/3*B*b+1/3*D*a)*x^3+B*a*x+1/4*C*b*x^4+1/5*D*b*x^5+
a*A*ln(x)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^2)\left(A+Bx+Cx^2+Dx^3\right)}{x}\,dx = \frac{1}{5}\,Dbx^5+\frac{1}{4}\,Cbx^4+\frac{1}{3}\,(Da+Bb)x^3 + Bax+\frac{1}{2}\,(Ca+Ab)x^2+Aa\log\left(x\right)$$

input
```
integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x,x, algorithm="fricas")
```

output
```
1/5*D*b*x^5 + 1/4*C*b*x^4 + 1/3*(D*a + B*b)*x^3 + B*a*x + 1/2*(C*a + A*b)*
x^2 + A*a*log(x)
```

## Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x}\,dx = Aa\log(x) + Bax + \frac{Cbx^4}{4} + \frac{Dbx^5}{5} + x^3\left(\frac{Bb}{3}+\frac{Da}{3}\right) + x^2\left(\frac{Ab}{2}+\frac{Ca}{2}\right)$$

input
```
integrate((b*x**2+a)*(D*x**3+C*x**2+B*x+A)/x,x)
```

output
```
A*a*log(x) + B*a*x + C*b*x**4/4 + D*b*x**5/5 + x**3*(B*b/3 + D*a/3) + x**2
*(A*b/2 + C*a/2)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x}\,dx = \frac{1}{5}Dbx^5 + \frac{1}{4}Cbx^4 + \frac{1}{3}(Da+Bb)x^3 + Bax + \frac{1}{2}(Ca+Ab)x^2 + Aa\log(x)$$

input
```
integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x,x, algorithm="maxima")
```

output
```
1/5*D*b*x^5 + 1/4*C*b*x^4 + 1/3*(D*a + B*b)*x^3 + B*a*x + 1/2*(C*a + A*b)*
x^2 + A*a*log(x)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x}\,dx = \frac{1}{5}Dbx^5+\frac{1}{4}Cbx^4+\frac{1}{3}Dax^3+\frac{1}{3}Bbx^3+\frac{1}{2}Cax^2+\frac{1}{2}Abx^2+Bax+Aa\log(|x|)$$

input
```
integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x,x, algorithm="giac")
```

output
```
1/5*D*b*x^5 + 1/4*C*b*x^4 + 1/3*D*a*x^3 + 1/3*B*b*x^3 + 1/2*C*a*x^2 + 1/2*
A*b*x^2 + B*a*x + A*a*log(abs(x))
```

## Mupad [B] (verification not implemented)

Time = 2.30 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x}\,dx = \frac{a\,x^3\,D}{3}+\frac{b\,x^5\,D}{5}+B\,a\,x+\frac{A\,b\,x^2}{2}+\frac{C\,a\,x^2}{2}+\frac{B\,b\,x^3}{3}+\frac{C\,b\,x^4}{4}+A\,a\,\ln(x)$$

input
```
int(((a + b*x^2)*(A + B*x + C*x^2 + x^3*D))/x,x)
```

output
```
(a*x^3*D)/3 + (b*x^5*D)/5 + B*a*x + (A*b*x^2)/2 + (C*a*x^2)/2 + (B*b*x^3)/
3 + (C*b*x^4)/4 + A*a*log(x)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x}\,dx = \log(x)\,a^2 + \frac{ab\,x^2}{2} + abx + \frac{ac\,x^2}{2} + \frac{ad\,x^3}{3} + \frac{b^2x^3}{3} + \frac{bc\,x^4}{4} + \frac{bd\,x^5}{5}$$

input
```
int((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x,x)
```

output
```
(60*log(x)*a**2 + 30*a*b*x**2 + 60*a*b*x + 30*a*c*x**2 + 20*a*d*x**3 + 20*
b**2*x**3 + 15*b*c*x**4 + 12*b*d*x**5)/60
```

# 3.17 $\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x^2}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 54

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x^2}\,dx = -\frac{aA}{x}+(Ab+aC)x+\frac{1}{2}(bB+aD)x^2 + \frac{1}{3}bCx^3+\frac{1}{4}bDx^4+aB\log(x)$$

output
```
-a*A/x+(A*b+C*a)*x+1/2*(B*b+D*a)*x^2+1/3*b*C*x^3+1/4*b*D*x^4+a*B*ln(x)
```

## Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x^2}\,dx = -\frac{aA}{x}+(Ab+aC)x+\frac{1}{2}(bB+aD)x^2 + \frac{1}{3}bCx^3+\frac{1}{4}bDx^4+aB\log(x)$$

input
```
Integrate[((a + b*x^2)*(A + B*x + C*x^2 + D*x^3))/x^2,x]
```

output
```
-((a*A)/x) + (A*b + a*C)*x + ((b*B + a*D)*x^2)/2 + (b*C*x^3)/3 + (b*D*x^4)
/4 + a*B*Log[x]
```

## Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x^2}dx$$

$$\Big\downarrow \text{2333}$$

$$\int \left(Ab\left(\frac{aC}{Ab}+1\right)+\frac{aA}{x^2}+x(aD+bB)+\frac{aB}{x}+bCx^2+bDx^3\right)dx$$

$$\Big\downarrow \text{2009}$$

$$x(aC+Ab)-\frac{aA}{x}+\frac{1}{2}x^2(aD+bB)+aB\log(x)+\frac{1}{3}bCx^3+\frac{1}{4}bDx^4$$

input
```
Int[((a + b*x^2)*(A + B*x + C*x^2 + D*x^3))/x^2,x]
```

output
```
-((a*A)/x) + (A*b + a*C)*x + ((b*B + a*D)*x^2)/2 + (b*C*x^3)/3 + (b*D*x^4)
/4 + a*B*Log[x]
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

## Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.93

| method | result | size |
|---|---|---|
| default | $\frac{Db\,x^4}{4}+\frac{Cb\,x^3}{3}+\frac{bB\,x^2}{2}+\frac{Da\,x^2}{2}+Abx+Cax+aB\ln\left(x\right)-\frac{aA}{x}$ | 50 |
| norman | $\frac{\left(\frac{Bb}{2}+\frac{Da}{2}\right)x^3+(Ab+Ca)x^2-Aa+\frac{Cb\,x^4}{3}+\frac{Db\,x^5}{4}}{x}+aB\ln\left(x\right)$ | 54 |
| parallelrisch | $\frac{3Db\,x^5+4Cb\,x^4+6bB\,x^3+6Da\,x^3+12Ab\,x^2+12Ba\ln(x)x+12Ca\,x^2-12Aa}{12x}$ | 60 |

input
```
int((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*D*b*x^4+1/3*C*b*x^3+1/2*b*B*x^2+1/2*D*a*x^2+A*b*x+C*a*x+a*B*ln(x)-a*A/
x
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^2)\left(A+Bx+Cx^2+Dx^3\right)}{x^2}\,dx = \frac{3\,Dbx^5+4\,Cbx^4+6\,(Da+Bb)x^3+12\,Bax\log\left(x\right)+12\,(Ca+Ab)x^2-12\,Aa}{12\,x}$$

input
```
integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x^2,x, algorithm="fricas")
```

output
```
1/12*(3*D*b*x^5 + 4*C*b*x^4 + 6*(D*a + B*b)*x^3 + 12*B*a*x*log(x) + 12*(C*
a + A*b)*x^2 - 12*A*a)/x
```

## Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x^2}\,dx = -\frac{Aa}{x}+Ba\log(x)+\frac{Cbx^3}{3}+\frac{Dbx^4}{4}+x^2\left(\frac{Bb}{2}+\frac{Da}{2}\right)+x(Ab+Ca)$$

input
```
integrate((b*x**2+a)*(D*x**3+C*x**2+B*x+A)/x**2,x)
```

output
```
-A*a/x + B*a*log(x) + C*b*x**3/3 + D*b*x**4/4 + x**2*(B*b/2 + D*a/2) + x*(
A*b + C*a)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x^2}\,dx = \frac{1}{4}Dbx^4+\frac{1}{3}Cbx^3+\frac{1}{2}(Da+Bb)x^2+Ba\log(x)+(Ca+Ab)x-\frac{Aa}{x}$$

input
```
integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x^2,x, algorithm="maxima")
```

output
```
1/4*D*b*x^4 + 1/3*C*b*x^3 + 1/2*(D*a + B*b)*x^2 + B*a*log(x) + (C*a + A*b)
*x - A*a/x
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^2)\,(A+Bx+Cx^2+Dx^3)}{x^2}\,dx = \frac{1}{4}\,Dbx^4+\frac{1}{3}\,Cbx^3+\frac{1}{2}\,Dax^2+\frac{1}{2}\,Bbx^2+Cax+Abx+Ba\log\left(|x|\right)-\frac{Aa}{x}$$

input
```
integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x^2,x, algorithm="giac")
```

output
```
1/4*D*b*x^4 + 1/3*C*b*x^3 + 1/2*D*a*x^2 + 1/2*B*b*x^2 + C*a*x + A*b*x + B*
a*log(abs(x)) - A*a/x
```

## Mupad [B] (verification not implemented)

Time = 2.40 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^2)\,(A+Bx+Cx^2+Dx^3)}{x^2}\,dx = \frac{a\,x^2\,D}{2}+\frac{b\,x^4\,D}{4}+A\,b\,x+C\,a\,x-\frac{A\,a}{x}+\frac{B\,b\,x^2}{2}+\frac{C\,b\,x^3}{3}+B\,a\,\ln\left(x\right)$$

input
```
int(((a + b*x^2)*(A + B*x + C*x^2 + x^3*D))/x^2,x)
```

output
```
(a*x^2*D)/2 + (b*x^4*D)/4 + A*b*x + C*a*x - (A*a)/x + (B*b*x^2)/2 + (C*b*x
^3)/3 + B*a*log(x)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.13

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x^2}\,dx = \frac{12\log(x)\,abx-12a^2+12ab\,x^2+12ac\,x^2+6ad\,x^3+6b^2x^3+4bc\,x^4+3bd\,x^5}{12x}$$

input
```
int((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x^2,x)
```

output
```
(12*log(x)*a*b*x - 12*a**2 + 12*a*b*x**2 + 12*a*c*x**2 + 6*a*d*x**3 + 6*b*
*2*x**3 + 4*b*c*x**4 + 3*b*d*x**5)/(12*x)
```

# 3.18 $\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x^3}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 54

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x^3}\,dx = -\frac{aA}{2x^2}-\frac{aB}{x}+(bB+aD)x+\frac{1}{2}bCx^2+\frac{1}{3}bDx^3+(Ab+aC)\log(x)$$

output
```
-1/2*a*A/x^2-a*B/x+(B*b+D*a)*x+1/2*b*C*x^2+1/3*b*D*x^3+(A*b+C*a)*ln(x)
```

## Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x^3}\,dx = \frac{1}{6}bx(6B+3Cx+2Dx^2)-\frac{a(A+2Bx-2Dx^3)}{2x^2}+(Ab+aC)\log(x)$$

input
```
Integrate[((a + b*x^2)*(A + B*x + C*x^2 + D*x^3))/x^3,x]
```

output
```
(b*x*(6*B + 3*C*x + 2*D*x^2))/6 - (a*(A + 2*B*x - 2*D*x^3))/(2*x^2) + (A*b
 + a*C)*Log[x]
```

## Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x^3}dx$$

$$\Big\downarrow \text{ 2333}$$

$$\int \left(\frac{aC+Ab}{x}+\frac{aA}{x^3}+bB\left(\frac{aD}{bB}+1\right)+\frac{aB}{x^2}+bCx+bDx^2\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\log(x)(aC+Ab)-\frac{aA}{2x^2}+x(aD+bB)-\frac{aB}{x}+\frac{1}{2}bCx^2+\frac{1}{3}bDx^3$$

input
```
Int[((a + b*x^2)*(A + B*x + C*x^2 + D*x^3))/x^3,x]
```

output
```
-1/2*(a*A)/x^2 - (a*B)/x + (b*B + a*D)*x + (b*C*x^2)/2 + (b*D*x^3)/3 + (A*
b + a*C)*Log[x]
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

## Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.89

| method | result | size |
|---|---|---|
| default | $\frac{D x^3 b}{3}+\frac{C b \,x^{2}}{2}+bBx+Dax-\frac{aA}{2x^{2}}+\left(Ab+Ca\right)\ln\left(x\right)-\frac{aB}{x}$ | 48 |
| norman | $\frac{\left(Bb+Da\right)x^{3}-\frac{Aa}{2}-Bax+\frac{C b \,x^{4}}{2}+\frac{D b \,x^{5}}{3}}{x^{2}}+\left(Ab+Ca\right)\ln\left(x\right)$ | 51 |
| parallelrisch | $\frac{2D b \,x^{5}+3C b \,x^{4}+6A\ln\left(x\right)x^{2}b+6bB \,x^{3}+6C\ln\left(x\right)x^{2}a+6D a \,x^{3}-6Bax-3Aa}{6x^{2}}$ | 62 |

input
```
int((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/3*D*x^3*b+1/2*C*b*x^2+b*B*x+D*a*x-1/2*a*A/x^2+(A*b+C*a)*ln(x)-a*B/x
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^2)\left(A+Bx+Cx^2+Dx^3\right)}{x^3}\,dx = \frac{2\,Dbx^{5}+3\,Cbx^{4}+6\,(Da+Bb)x^{3}+6\,(Ca+Ab)x^{2}\log\left(x\right)-6\,Bax-3\,Aa}{6\,x^{2}}$$

input
```
integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x^3,x, algorithm="fricas")
```

output
```
1/6*(2*D*b*x^5 + 3*C*b*x^4 + 6*(D*a + B*b)*x^3 + 6*(C*a + A*b)*x^2*log(x)
- 6*B*a*x - 3*A*a)/x^2
```

## Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x^3}\,dx = \frac{Cbx^2}{2}+\frac{Dbx^3}{3}+x(Bb+Da)+(Ab+Ca)\log(x)+\frac{-Aa-2Bax}{2x^2}$$

input
```
integrate((b*x**2+a)*(D*x**3+C*x**2+B*x+A)/x**3,x)
```

output
```
C*b*x**2/2 + D*b*x**3/3 + x*(B*b + D*a) + (A*b + C*a)*log(x) + (-A*a - 2*B
*a*x)/(2*x**2)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x^3}\,dx = \frac{1}{3}\,Dbx^3+\frac{1}{2}\,Cbx^2+(Da+Bb)x+(Ca+Ab)\log(x)-\frac{2\,Bax+Aa}{2\,x^2}$$

input
```
integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x^3,x, algorithm="maxima")
```

output
```
1/3*D*b*x^3 + 1/2*C*b*x^2 + (D*a + B*b)*x + (C*a + A*b)*log(x) - 1/2*(2*B*
a*x + A*a)/x^2
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x^3}\,dx = \frac{1}{3}\,Dbx^3+\frac{1}{2}\,Cbx^2+Dax+Bbx+(Ca+Ab)\log(|x|)-\frac{2\,Bax+Aa}{2\,x^2}$$

input
```
integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x^3,x, algorithm="giac")
```

output
```
1/3*D*b*x^3 + 1/2*C*b*x^2 + D*a*x + B*b*x + (C*a + A*b)*log(abs(x)) - 1/2*
(2*B*a*x + A*a)/x^2
```

## Mupad [B] (verification not implemented)

Time = 2.25 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x^3}\,dx = \frac{b\,x^3\,D}{3}+B\,b\,x-\frac{A\,a}{2\,x^2}-\frac{B\,a}{x}+\frac{C\,b\,x^2}{2}+A\,b\,\ln(x)+C\,a\,\ln(x)+a\,x\,D$$

input
```
int(((a + b*x^2)*(A + B*x + C*x^2 + x^3*D))/x^3,x)
```

output
```
(b*x^3*D)/3 + B*b*x - (A*a)/(2*x^2) - (B*a)/x + (C*b*x^2)/2 + A*b*log(x) +
 C*a*log(x) + a*x*D
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.17

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x^3}\,dx = \frac{6\log(x)\,ab\,x^2+6\log(x)\,ac\,x^2-3a^2-6abx+6ad\,x^3+6b^2x^3+3bc\,x^4+2bd\,x^5}{6x^2}$$

input
```
int((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x^3,x)
```

output
```
(6*log(x)*a*b*x**2 + 6*log(x)*a*c*x**2 - 3*a**2 - 6*a*b*x + 6*a*d*x**3 + 6
*b**2*x**3 + 3*b*c*x**4 + 2*b*d*x**5)/(6*x**2)
```

# 3.19 $\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x^4}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 54

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x^4}\,dx = -\frac{aA}{3x^3}-\frac{aB}{2x^2}-\frac{Ab+aC}{x}+bCx+\frac{1}{2}bDx^2+(bB+aD)\log(x)$$

output
```
-1/3*a*A/x^3-1/2*a*B/x^2-(A*b+C*a)/x+b*C*x+1/2*b*D*x^2+(B*b+D*a)*ln(x)
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x^4}\,dx = -\frac{aA}{3x^3}-\frac{aB}{2x^2}+\frac{-Ab-aC}{x}+bCx+\frac{1}{2}bDx^2+(bB+aD)\log(x)$$

input
```
Integrate[((a + b*x^2)*(A + B*x + C*x^2 + D*x^3))/x^4,x]
```

output
```
-1/3*(a*A)/x^3 - (a*B)/(2*x^2) + (-(A*b) - a*C)/x + b*C*x + (b*D*x^2)/2 +
(b*B + a*D)*Log[x]
```

## Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x^4}\,dx$$

$$\Big\downarrow \text{2333}$$

$$\int \left(\frac{aC+Ab}{x^2}+\frac{aA}{x^4}+\frac{aD+bB}{x}+\frac{aB}{x^3}+bC+bDx\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{aC+Ab}{x}-\frac{aA}{3x^3}+\log(x)(aD+bB)-\frac{aB}{2x^2}+bCx+\frac{1}{2}bDx^2$$

input
```
Int[((a + b*x^2)*(A + B*x + C*x^2 + D*x^3))/x^4,x]
```

output
```
-1/3*(a*A)/x^3 - (a*B)/(2*x^2) - (A*b + a*C)/x + b*C*x + (b*D*x^2)/2 + (b*
B + a*D)*Log[x]
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

## Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.91

| method | result | size |
|---|---|---|
| default | $-\frac{aA}{3x^{3}}-\frac{aB}{2x^{2}}-\frac{Ab+Ca}{x}+Cbx+\frac{Db\,x^{2}}{2}+\left(Bb+Da\right)\ln\left(x\right)$ | 49 |
| norman | $\frac{\left(-Ab-Ca\right)x^{2}+Cb\,x^{4}-\frac{Aa}{3}-\frac{Bax}{2}+\frac{Db\,x^{5}}{2}}{x^{3}}+\left(Bb+Da\right)\ln\left(x\right)$ | 52 |
| parallelrisch | $-\frac{-3Db\,x^{5}-6B\ln\left(x\right)x^{3}b-6Cb\,x^{4}-6D\ln\left(x\right)x^{3}a+6Ab\,x^{2}+6Ca\,x^{2}+3Bax+2Aa}{6x^{3}}$ | 62 |

input
```
int((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*a*A/x^3-1/2*a*B/x^2-(A*b+C*a)/x+C*b*x+1/2*D*b*x^2+(B*b+D*a)*ln(x)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x^4}\,dx = \frac{3\,Dbx^5+6\,Cbx^4+6\,(Da+Bb)x^3\log{(x)}-3\,Bax-6\,(Ca+Ab)x^2-2\,Aa}{6\,x^3}$$

input
```
integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x^4,x, algorithm="fricas")
```

output
```
1/6*(3*D*b*x^5 + 6*C*b*x^4 + 6*(D*a + B*b)*x^3*log(x) - 3*B*a*x - 6*(C*a +
 A*b)*x^2 - 2*A*a)/x^3
```

## Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x^4}\,dx = Cbx + \frac{Dbx^2}{2} + (Bb+Da)\log(x) + \frac{-2Aa-3Bax+x^2(-6Ab-6Ca)}{6x^3}$$

input
```
integrate((b*x**2+a)*(D*x**3+C*x**2+B*x+A)/x**4,x)
```

output
```
C*b*x + D*b*x**2/2 + (B*b + D*a)*log(x) + (-2*A*a - 3*B*a*x + x**2*(-6*A*b
 - 6*C*a))/(6*x**3)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x^4}\,dx = \frac{1}{2}Dbx^2 + Cbx + (Da+Bb)\log(x) - \frac{3Bax+6(Ca+Ab)x^2+2Aa}{6x^3}$$

input
```
integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x^4,x, algorithm="maxima")
```

output
```
1/2*D*b*x^2 + C*b*x + (D*a + B*b)*log(x) - 1/6*(3*B*a*x + 6*(C*a + A*b)*x^
2 + 2*A*a)/x^3
```

## Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x^4}\,dx = \frac{1}{2}Dbx^2+Cbx+(Da+Bb)\log(|x|) - \frac{3\,Bax+6\,(Ca+Ab)x^2+2\,Aa}{6\,x^3}$$

input
```
integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x^4,x, algorithm="giac")
```

output
```
1/2*D*b*x^2 + C*b*x + (D*a + B*b)*log(abs(x)) - 1/6*(3*B*a*x + 6*(C*a + A*
b)*x^2 + 2*A*a)/x^3
```

## Mupad [B] (verification not implemented)

Time = 2.30 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x^4}\,dx = \frac{b\,x^2\,D}{2}+a\,\ln(x)\,D+C\,b\,x-\frac{A\,a}{3\,x^3} - \frac{A\,b}{x}-\frac{B\,a}{2\,x^2}-\frac{C\,a}{x}+B\,b\,\ln(x)$$

input
```
int(((a + b*x^2)*(A + B*x + C*x^2 + x^3*D))/x^4,x)
```

output
```
(b*x^2*D)/2 + a*log(x)*D + C*b*x - (A*a)/(3*x^3) - (A*b)/x - (B*a)/(2*x^2)
 - (C*a)/x + B*b*log(x)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.17

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x^4}\,dx = \frac{6\log(x)\,ad\,x^3+6\log(x)\,b^2x^3-2a^2-6ab\,x^2-3abx-6ac\,x^2+6bc\,x^4+3bd\,x^5}{6x^3}$$

input
```
int((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x^4,x)
```

output
```
(6*log(x)*a*d*x**3 + 6*log(x)*b**2*x**3 - 2*a**2 - 6*a*b*x**2 - 3*a*b*x -
6*a*c*x**2 + 6*b*c*x**4 + 3*b*d*x**5)/(6*x**3)
```

# 3.20 $\int x^3(a+bx^2)^2 (A+Bx+Cx^2+Dx^3)\,dx$



## Optimal result

Integrand size = 28, antiderivative size = 109

$$\begin{aligned}&\int x^3(a+bx^2)^2 (A+Bx+Cx^2+Dx^3)\,dx\\&=\frac{1}{4}a^2Ax^4+\frac{1}{5}a^2Bx^5+\frac{1}{6}a(2Ab+aC)x^6+\frac{1}{7}a(2bB+aD)x^7\\&\quad+\frac{1}{8}b(Ab+2aC)x^8+\frac{1}{9}b(bB+2aD)x^9+\frac{1}{10}b^2Cx^{10}+\frac{1}{11}b^2Dx^{11}\end{aligned}$$

output
```
1/4*a^2*A*x^4+1/5*a^2*B*x^5+1/6*a*(2*A*b+C*a)*x^6+1/7*a*(2*B*b+D*a)*x^7+1/
8*b*(A*b+2*C*a)*x^8+1/9*b*(B*b+2*D*a)*x^9+1/10*b^2*C*x^10+1/11*b^2*D*x^11
```

## Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.90

$$\begin{aligned}&\int x^3(a+bx^2)^2 (A+Bx+Cx^2+Dx^3)\,dx\\&=a^2\left(\frac{Ax^4}{4}+\frac{Bx^5}{5}+\frac{1}{42}x^6(7C+6Dx)\right)+\frac{b^2x^8(495A+4x(110B+99Cx+90Dx^2))}{3960}\\&\quad+\frac{1}{252}abx^6(84A+x(72B+7x(9C+8Dx)))\end{aligned}$$

input
```
Integrate[x^3*(a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
a^2*((A*x^4)/4 + (B*x^5)/5 + (x^6*(7*C + 6*D*x))/42) + (b^2*x^8*(495*A + 4
*x*(110*B + 99*C*x + 90*D*x^2)))/3960 + (a*b*x^6*(84*A + x*(72*B + 7*x*(9*
C + 8*D*x))))/252
```

## Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 (a+bx^2)^2 (A+Bx+Cx^2+Dx^3)\, dx$$

$$\Big\downarrow \text{ 2333}$$

$$\int (a^2Ax^3+a^2Bx^4+bx^7(2aC+Ab)+ax^5(aC+2Ab)+bx^8(2aD+bB)+ax^6(aD+2bB)+b^2Cx^9+b^2Dx^{10}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{4}a^2Ax^4+\frac{1}{5}a^2Bx^5+\frac{1}{8}bx^8(2aC+Ab)+\frac{1}{6}ax^6(aC+2Ab)+\frac{1}{9}bx^9(2aD+bB)+\frac{1}{7}ax^7(aD+2bB)+\frac{1}{10}b^2Cx^{10}+\frac{1}{11}b^2Dx^{11}$$

input
```
Int[x^3*(a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(a^2*A*x^4)/4 + (a^2*B*x^5)/5 + (a*(2*A*b + a*C)*x^6)/6 + (a*(2*b*B + a*D)
*x^7)/7 + (b*(A*b + 2*a*C)*x^8)/8 + (b*(b*B + 2*a*D)*x^9)/9 + (b^2*C*x^10)
/10 + (b^2*D*x^11)/11
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

## Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.94

| method | result |
|---|---|
| default | $\frac{b^2 D x^{11}}{11}+\frac{b^2 C\,x^{10}}{10}+\frac{\left(B\,b^2+2abD\right)x^9}{9}+\frac{\left(b^2 A+2Cab\right)x^8}{8}+\frac{\left(2abB+Da^2\right)x^7}{7}+\frac{\left(2abA+a^2 C\right)x^6}{6}+\frac{a^2 B\,x^5}{5}+\frac{a^2 A}{4}$ |
| norman | $\frac{b^2 D x^{11}}{11}+\frac{b^2 C\,x^{10}}{10}+\left(\frac{1}{9}B\,b^2+\frac{2}{9}abD\right)x^9+\left(\frac{1}{8}b^2 A+\frac{1}{4}Cab\right)x^8+\left(\frac{2}{7}abB+\frac{1}{7}Da^2\right)x^7+\left(\frac{1}{3}abA\right.$ |
| gosper | $\frac{1}{11}b^2 D x^{11}+\frac{1}{10}b^2 C\,x^{10}+\frac{1}{9}b^2 B\,x^9+\frac{2}{9}x^9 abD+\frac{1}{8}x^8 b^2 A+\frac{1}{4}x^8 Cab+\frac{2}{7}x^7 abB+\frac{1}{7}x^7 Da^2+\frac{1}{3}$ |
| parallelrisch | $\frac{1}{11}b^2 D x^{11}+\frac{1}{10}b^2 C\,x^{10}+\frac{1}{9}b^2 B\,x^9+\frac{2}{9}x^9 abD+\frac{1}{8}x^8 b^2 A+\frac{1}{4}x^8 Cab+\frac{2}{7}x^7 abB+\frac{1}{7}x^7 Da^2+\frac{1}{3}$ |
| orering | $\frac{x^4\left(2520b^2 D x^7+2772b^2 C\,x^6+3080b^2 B\,x^5+6160Dab\,x^5+3465A\,b^2 x^4+6930Cab\,x^4+7920Bab\,x^3+3960Da^2 x^3+9240aAb\,x^2\right.}{27720}$ |

input
```
int(x^3*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
1/11*b^2*D*x^11+1/10*b^2*C*x^10+1/9*(B*b^2+2*D*a*b)*x^9+1/8*(A*b^2+2*C*a*b
)*x^8+1/7*(2*B*a*b+D*a^2)*x^7+1/6*(2*A*a*b+C*a^2)*x^6+1/5*a^2*B*x^5+1/4*a^
2*A*x^4
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.93

$$\int x^3(a+bx^2)^2\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$=\frac{1}{11}Db^2x^{11}+\frac{1}{10}Cb^2x^{10}+\frac{1}{9}\left(2\,Dab+Bb^2\right)x^9+\frac{1}{8}\left(2\,Cab+Ab^2\right)x^8$$
$$+\frac{1}{5}Ba^2x^5+\frac{1}{7}\left(Da^2+2\,Bab\right)x^7+\frac{1}{4}Aa^2x^4+\frac{1}{6}\left(Ca^2+2\,Aab\right)x^6$$

input
```
integrate(x^3*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
1/11*D*b^2*x^11 + 1/10*C*b^2*x^10 + 1/9*(2*D*a*b + B*b^2)*x^9 + 1/8*(2*C*a
*b + A*b^2)*x^8 + 1/5*B*a^2*x^5 + 1/7*(D*a^2 + 2*B*a*b)*x^7 + 1/4*A*a^2*x^
4 + 1/6*(C*a^2 + 2*A*a*b)*x^6
```

## Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.01

$$\int x^3(a+bx^2)^2(A+Bx+Cx^2+Dx^3)\,dx = \frac{Aa^2x^4}{4}+\frac{Ba^2x^5}{5}+\frac{Cb^2x^{10}}{10}+\frac{Db^2x^{11}}{11}+x^9\left(\frac{Bb^2}{9}+\frac{2Dab}{9}\right)+x^8\left(\frac{Ab^2}{8}+\frac{Cab}{4}\right)+x^7\cdot\left(\frac{2Bab}{7}+\frac{Da^2}{7}\right)+x^6\left(\frac{Aab}{3}+\frac{Ca^2}{6}\right)$$

input
```
integrate(x**3*(b*x**2+a)**2*(D*x**3+C*x**2+B*x+A),x)
```

output
```
A*a**2*x**4/4 + B*a**2*x**5/5 + C*b**2*x**10/10 + D*b**2*x**11/11 + x**9*(
B*b**2/9 + 2*D*a*b/9) + x**8*(A*b**2/8 + C*a*b/4) + x**7*(2*B*a*b/7 + D*a*
*2/7) + x**6*(A*a*b/3 + C*a**2/6)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.93

$$\int x^3(a+bx^2)^2(A+Bx+Cx^2+Dx^3)\,dx = \frac{1}{11}\,Db^2x^{11}+\frac{1}{10}\,Cb^2x^{10}+\frac{1}{9}\,(2\,Dab+Bb^2)x^9+\frac{1}{8}\,(2\,Cab+Ab^2)x^8+\frac{1}{5}\,Ba^2x^5+\frac{1}{7}\,(Da^2+2\,Bab)x^7+\frac{1}{4}\,Aa^2x^4+\frac{1}{6}\,(Ca^2+2\,Aab)x^6$$

input
```
integrate(x^3*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
1/11*D*b^2*x^11 + 1/10*C*b^2*x^10 + 1/9*(2*D*a*b + B*b^2)*x^9 + 1/8*(2*C*a
*b + A*b^2)*x^8 + 1/5*B*a^2*x^5 + 1/7*(D*a^2 + 2*B*a*b)*x^7 + 1/4*A*a^2*x^
4 + 1/6*(C*a^2 + 2*A*a*b)*x^6
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.96

$$\int x^3 \left(a+bx^2\right)^2 \left(A+Bx+Cx^2+Dx^3\right) \, dx = \frac{1}{11} Db^2x^{11} + \frac{1}{10} Cb^2x^{10} + \frac{2}{9} Dabx^9 + \frac{1}{9} Bb^2x^9 + \frac{1}{4} Cabx^8 + \frac{1}{8} Ab^2x^8 + \frac{1}{7} Da^2x^7 + \frac{2}{7} Babx^7 + \frac{1}{6} Ca^2x^6 + \frac{1}{3} Aabx^6 + \frac{1}{5} Ba^2x^5 + \frac{1}{4} Aa^2x^4$$

input
```
integrate(x^3*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

output
```
1/11*D*b^2*x^11 + 1/10*C*b^2*x^10 + 2/9*D*a*b*x^9 + 1/9*B*b^2*x^9 + 1/4*C*
a*b*x^8 + 1/8*A*b^2*x^8 + 1/7*D*a^2*x^7 + 2/7*B*a*b*x^7 + 1/6*C*a^2*x^6 +
1/3*A*a*b*x^6 + 1/5*B*a^2*x^5 + 1/4*A*a^2*x^4
```

## Mupad [B] (verification not implemented)

Time = 2.31 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.99

$$\int x^3 \left(a+bx^2\right)^2 \left(A+Bx+Cx^2+Dx^3\right) \, dx = \frac{a^2\,x^7\,D}{7} + \frac{b^2\,x^{11}\,D}{11} + \frac{A\,x^4\,\left(6\,a^2+8\,a\,b\,x^2+3\,b^2\,x^4\right)}{24} + \frac{B\,x^5\,\left(63\,a^2+90\,a\,b\,x^2+35\,b^2\,x^4\right)}{315} + \frac{C\,x^6\,\left(10\,a^2+15\,a\,b\,x^2+6\,b^2\,x^4\right)}{60} + \frac{2\,a\,b\,x^9\,D}{9}$$

input
```
int(x^3*(a + b*x^2)^2*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
(a^2*x^7*D)/7 + (b^2*x^11*D)/11 + (A*x^4*(6*a^2 + 3*b^2*x^4 + 8*a*b*x^2))/
24 + (B*x^5*(63*a^2 + 35*b^2*x^4 + 90*a*b*x^2))/315 + (C*x^6*(10*a^2 + 6*b
^2*x^4 + 15*a*b*x^2))/60 + (2*a*b*x^9*D)/9
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.96

$$\int x^3 (a+bx^2)^2 (A+Bx+Cx^2+Dx^3)\,dx$$
$$= \frac{x^4(2520b^2d\,x^7+2772b^2c\,x^6+6160abd\,x^5+3080b^3x^5+3465a\,b^2x^4+6930abc\,x^4+3960a^2d\,x^3+7920a\,b}{27720}$$

input
```
int(x^3*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x)
```

output
```
(x**4*(6930*a**3 + 9240*a**2*b*x**2 + 5544*a**2*b*x + 4620*a**2*c*x**2 + 3
960*a**2*d*x**3 + 3465*a*b**2*x**4 + 7920*a*b**2*x**3 + 6930*a*b*c*x**4 +
6160*a*b*d*x**5 + 3080*b**3*x**5 + 2772*b**2*c*x**6 + 2520*b**2*d*x**7))/2
7720
```

# 3.21 $\int x^2(a+bx^2)^2 (A+Bx+Cx^2+Dx^3)\,dx$



## Optimal result

Integrand size = 28, antiderivative size = 109

$$\int x^2(a+bx^2)^2 (A+Bx+Cx^2+Dx^3)\,dx = \frac{1}{3}a^2Ax^3+\frac{1}{4}a^2Bx^4+\frac{1}{5}a(2Ab+aC)x^5+\frac{1}{6}a(2bB+aD)x^6 + \frac{1}{7}b(Ab+2aC)x^7+\frac{1}{8}b(bB+2aD)x^8+\frac{1}{9}b^2Cx^9+\frac{1}{10}b^2Dx^{10}$$

output
```
1/3*a^2*A*x^3+1/4*a^2*B*x^4+1/5*a*(2*A*b+C*a)*x^5+1/6*a*(2*B*b+D*a)*x^6+1/
7*b*(A*b+2*C*a)*x^7+1/8*b*(B*b+2*D*a)*x^8+1/9*b^2*C*x^9+1/10*b^2*D*x^10
```

## Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.84

$$\int x^2(a+bx^2)^2 (A+Bx+Cx^2+Dx^3)\,dx = \frac{42a^2x^3(20A+x(15B+2x(6C+5Dx)))+6abx^5(168A+5x(28B+3x(8C+7Dx)))+b^2x^7(360A+7}{2520}$$

input
```
Integrate[x^2*(a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(42*a^2*x^3*(20*A + x*(15*B + 2*x*(6*C + 5*D*x))) + 6*a*b*x^5*(168*A + 5*x
*(28*B + 3*x*(8*C + 7*D*x))) + b^2*x^7*(360*A + 7*x*(45*B + 4*x*(10*C + 9*
D*x))))/2520
```

## Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(a+bx^2\right)^2\left(A+Bx+Cx^2+Dx^3\right)dx$$

$$\Big\downarrow \text{2333}$$

$$\int \left(a^2Ax^2+a^2Bx^3+bx^6(2aC+Ab)+ax^4(aC+2Ab)+bx^7(2aD+bB)+ax^5(aD+2bB)+b^2Cx^8+b^2Dx^9\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}a^2Ax^3+\frac{1}{4}a^2Bx^4+\frac{1}{7}bx^7(2aC+Ab)+\frac{1}{5}ax^5(aC+2Ab)+\frac{1}{8}bx^8(2aD+bB)+\frac{1}{6}ax^6(aD+2bB)+\frac{1}{9}b^2Cx^9+\frac{1}{10}b^2Dx^{10}$$

input
```
Int[x^2*(a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(a^2*A*x^3)/3 + (a^2*B*x^4)/4 + (a*(2*A*b + a*C)*x^5)/5 + (a*(2*b*B + a*D)
*x^6)/6 + (b*(A*b + 2*a*C)*x^7)/7 + (b*(b*B + 2*a*D)*x^8)/8 + (b^2*C*x^9)/
9 + (b^2*D*x^10)/10
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

## Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.94

| method | result |
|---|---|
| default | $\frac{b^2x^{10}D}{10}+\frac{b^2C\,x^9}{9}+\frac{(B\,b^2+2abD)x^8}{8}+\frac{(b^2A+2Cab)x^7}{7}+\frac{(2abB+Da^2)x^6}{6}+\frac{(2abA+a^2C)x^5}{5}+\frac{a^2B\,x^4}{4}+\frac{a^2A}{3}$ |
| norman | $\frac{b^2x^{10}D}{10}+\frac{b^2C\,x^9}{9}+\left(\frac{1}{8}B\,b^2+\frac{1}{4}abD\right)x^8+\left(\frac{1}{7}b^2A+\frac{2}{7}Cab\right)x^7+\left(\frac{1}{3}abB+\frac{1}{6}Da^2\right)x^6+\left(\frac{2}{5}abA\right.$ |
| gosper | $\frac{1}{10}b^2x^{10}D+\frac{1}{9}b^2C\,x^9+\frac{1}{8}b^2B\,x^8+\frac{1}{4}x^8abD+\frac{1}{7}x^7b^2A+\frac{2}{7}x^7Cab+\frac{1}{3}x^6abB+\frac{1}{6}x^6Da^2+\frac{2}{5}x$ |
| parallelrisch | $\frac{1}{10}b^2x^{10}D+\frac{1}{9}b^2C\,x^9+\frac{1}{8}b^2B\,x^8+\frac{1}{4}x^8abD+\frac{1}{7}x^7b^2A+\frac{2}{7}x^7Cab+\frac{1}{3}x^6abB+\frac{1}{6}x^6Da^2+\frac{2}{5}x$ |
| orering | $\frac{x^3(252b^2Dx^7+280b^2C\,x^6+315b^2B\,x^5+630Dab\,x^5+360A\,b^2x^4+720Cab\,x^4+840Bab\,x^3+420Da^2x^3+1008aAb\,x^2+504C\,a^2}{2520}$ |

input
```
int(x^2*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
1/10*b^2*x^10*D+1/9*b^2*C*x^9+1/8*(B*b^2+2*D*a*b)*x^8+1/7*(A*b^2+2*C*a*b)*
x^7+1/6*(2*B*a*b+D*a^2)*x^6+1/5*(2*A*a*b+C*a^2)*x^5+1/4*a^2*B*x^4+1/3*a^2*
A*x^3
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.93

$$\int x^2(a+bx^2)^2\left(A+Bx+Cx^2+Dx^3\right)\,dx =\frac{1}{10}\,Db^2x^{10}+\frac{1}{9}\,Cb^2x^9+\frac{1}{8}\,(2\,Dab+Bb^2)x^8+\frac{1}{7}\,(2\,Cab+Ab^2)x^7+\frac{1}{4}\,Ba^2x^4+\frac{1}{6}\,(Da^2+2\,Bab)x^6+\frac{1}{3}\,Aa^2x^3+\frac{1}{5}\,(Ca^2+2\,Aab)x^5$$

input
```
integrate(x^2*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
1/10*D*b^2*x^10 + 1/9*C*b^2*x^9 + 1/8*(2*D*a*b + B*b^2)*x^8 + 1/7*(2*C*a*b
 + A*b^2)*x^7 + 1/4*B*a^2*x^4 + 1/6*(D*a^2 + 2*B*a*b)*x^6 + 1/3*A*a^2*x^3
+ 1/5*(C*a^2 + 2*A*a*b)*x^5
```

## Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.01

$$\int x^2(a+bx^2)^2(A+Bx+Cx^2+Dx^3)\,dx = \frac{Aa^2x^3}{3}+\frac{Ba^2x^4}{4}+\frac{Cb^2x^9}{9}+\frac{Db^2x^{10}}{10}+x^8\left(\frac{Bb^2}{8}+\frac{Dab}{4}\right)+x^7\left(\frac{Ab^2}{7}+\frac{2Cab}{7}\right)+x^6\left(\frac{Bab}{3}+\frac{Da^2}{6}\right)+x^5\cdot\left(\frac{2Aab}{5}+\frac{Ca^2}{5}\right)$$

input
```
integrate(x**2*(b*x**2+a)**2*(D*x**3+C*x**2+B*x+A),x)
```

output
```
A*a**2*x**3/3 + B*a**2*x**4/4 + C*b**2*x**9/9 + D*b**2*x**10/10 + x**8*(B*
b**2/8 + D*a*b/4) + x**7*(A*b**2/7 + 2*C*a*b/7) + x**6*(B*a*b/3 + D*a**2/6
) + x**5*(2*A*a*b/5 + C*a**2/5)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.93

$$\int x^2(a+bx^2)^2(A+Bx+Cx^2+Dx^3)\,dx = \frac{1}{10}Db^2x^{10}+\frac{1}{9}Cb^2x^9+\frac{1}{8}(2Dab+Bb^2)x^8+\frac{1}{7}(2Cab+Ab^2)x^7+\frac{1}{4}Ba^2x^4+\frac{1}{6}(Da^2+2Bab)x^6+\frac{1}{3}Aa^2x^3+\frac{1}{5}(Ca^2+2Aab)x^5$$

input
```
integrate(x^2*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
1/10*D*b^2*x^10 + 1/9*C*b^2*x^9 + 1/8*(2*D*a*b + B*b^2)*x^8 + 1/7*(2*C*a*b
 + A*b^2)*x^7 + 1/4*B*a^2*x^4 + 1/6*(D*a^2 + 2*B*a*b)*x^6 + 1/3*A*a^2*x^3
+ 1/5*(C*a^2 + 2*A*a*b)*x^5
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.96

$$\int x^2\left(a+bx^2\right)^2\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$=\frac{1}{10}\,Db^2x^{10}+\frac{1}{9}\,Cb^2x^9+\frac{1}{4}\,Dabx^8+\frac{1}{8}\,Bb^2x^8+\frac{2}{7}\,Cabx^7+\frac{1}{7}\,Ab^2x^7+\frac{1}{6}\,Da^2x^6+\frac{1}{3}\,Babx^6+\frac{1}{5}\,Ca^2x^5+\frac{2}{5}\,Aabx^5+\frac{1}{4}\,Ba^2x^4+\frac{1}{3}\,Aa^2x^3$$

input
```
integrate(x^2*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

output
```
1/10*D*b^2*x^10 + 1/9*C*b^2*x^9 + 1/4*D*a*b*x^8 + 1/8*B*b^2*x^8 + 2/7*C*a*
b*x^7 + 1/7*A*b^2*x^7 + 1/6*D*a^2*x^6 + 1/3*B*a*b*x^6 + 1/5*C*a^2*x^5 + 2/
5*A*a*b*x^5 + 1/4*B*a^2*x^4 + 1/3*A*a^2*x^3
```

## Mupad [B] (verification not implemented)

Time = 2.21 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.99

$$\int x^2\left(a+bx^2\right)^2\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$=\frac{a^2\,x^6\,D}{6}+\frac{b^2\,x^{10}\,D}{10}+\frac{A\,x^3\,\left(35\,a^2+42\,a\,b\,x^2+15\,b^2\,x^4\right)}{105}+\frac{B\,x^4\,\left(6\,a^2+8\,a\,b\,x^2+3\,b^2\,x^4\right)}{24}+\frac{C\,x^5\,\left(63\,a^2+90\,a\,b\,x^2+35\,b^2\,x^4\right)}{315}+\frac{a\,b\,x^8\,D}{4}$$

input
```
int(x^2*(a + b*x^2)^2*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
(a^2*x^6*D)/6 + (b^2*x^10*D)/10 + (A*x^3*(35*a^2 + 15*b^2*x^4 + 42*a*b*x^2
))/105 + (B*x^4*(6*a^2 + 3*b^2*x^4 + 8*a*b*x^2))/24 + (C*x^5*(63*a^2 + 35*
b^2*x^4 + 90*a*b*x^2))/315 + (a*b*x^8*D)/4
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.96

$$\int x^2(a+bx^2)^2(A+Bx+Cx^2+Dx^3)\,dx$$
$$=\frac{x^3(252b^2d\,x^7+280b^2c\,x^6+630abd\,x^5+315b^3x^5+360a\,b^2x^4+720abc\,x^4+420a^2d\,x^3+840a\,b^2x^3+100}{2520}$$

input
```
int(x^2*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x)
```

output
```
(x**3*(840*a**3 + 1008*a**2*b*x**2 + 630*a**2*b*x + 504*a**2*c*x**2 + 420*
a**2*d*x**3 + 360*a*b**2*x**4 + 840*a*b**2*x**3 + 720*a*b*c*x**4 + 630*a*b
*d*x**5 + 315*b**3*x**5 + 280*b**2*c*x**6 + 252*b**2*d*x**7))/2520
```

# 3.22 $\int x(a+bx^2)^2 (A+Bx+Cx^2+Dx^3)\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 104

$$\int x(a+bx^2)^2 (A+Bx+Cx^2+Dx^3)\,dx = \frac{1}{3}a^2Bx^3+\frac{1}{4}a^2Cx^4+\frac{1}{5}a(2bB+aD)x^5 + \frac{1}{3}abCx^6+\frac{1}{7}b(bB+2aD)x^7 + \frac{1}{8}b^2Cx^8+\frac{1}{9}b^2Dx^9+\frac{A(a+bx^2)^3}{6b}$$

output
```
1/3*a^2*B*x^3+1/4*a^2*C*x^4+1/5*a*(2*B*b+D*a)*x^5+1/3*a*b*C*x^6+1/7*b*(B*b
+2*D*a)*x^7+1/8*b^2*C*x^8+1/9*b^2*D*x^9+1/6*A*(b*x^2+a)^3/b
```

## Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.88

$$\int x(a+bx^2)^2 (A+Bx+Cx^2+Dx^3)\,dx = \frac{42a^2x^2(30A+x(20B+3x(5C+4Dx)))+12abx^4(105A+2x(42B+5x(7C+6Dx)))+5b^2x^6(84A+}{2520}$$

input
```
Integrate[x*(a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(42*a^2*x^2*(30*A + x*(20*B + 3*x*(5*C + 4*D*x))) + 12*a*b*x^4*(105*A + 2*
x*(42*B + 5*x*(7*C + 6*D*x))) + 5*b^2*x^6*(84*A + x*(72*B + 7*x*(9*C + 8*D
*x))))/2520
```

## Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2017, 2341, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+bx^2)^2(A+Bx+Cx^2+Dx^3)\,dx$$

$$\Big\downarrow \text{2017}$$

$$\int (bx^2+a)^2(x(Dx^3+Cx^2+Bx+A)-Ax)\,dx+\frac{A(a+bx^2)^3}{6b}$$

$$\Big\downarrow \text{2341}$$

$$\int (b^2Dx^8+b^2Cx^7+b(bB+2aD)x^6+2abCx^5+a(2bB+aD)x^4+a^2Cx^3+a^2Bx^2)\,dx+\frac{A(a+bx^2)^3}{6b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}a^2Bx^3+\frac{1}{4}a^2Cx^4+\frac{A(a+bx^2)^3}{6b}+\frac{1}{7}bx^7(2aD+bB)+\frac{1}{5}ax^5(aD+2bB)+\frac{1}{3}abCx^6+\frac{1}{8}b^2Cx^8+\frac{1}{9}b^2Dx^9$$

input
```
Int[x*(a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(a^2*B*x^3)/3 + (a^2*C*x^4)/4 + (a*(2*b*B + a*D)*x^5)/5 + (a*b*C*x^6)/3 +
(b*(b*B + 2*a*D)*x^7)/7 + (b^2*C*x^8)/8 + (b^2*D*x^9)/9 + (A*(a + b*x^2)^3
)/(6*b)
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2017
```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

rule 2341
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

## Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.98

| method | result |
|---|---|
| default | $\frac{Db^2x^9}{9}+\frac{C b^2x^8}{8}+\frac{(B b^2+2abD)x^7}{7}+\frac{(b^2A+2Cab)x^6}{6}+\frac{(2abB+Da^2)x^5}{5}+\frac{(2abA+a^2C)x^4}{4}+\frac{x^3a^2B}{3}+\frac{a^2Ax^2}{2}$ |
| norman | $\frac{Db^2x^9}{9}+\frac{C b^2x^8}{8}+\left(\frac{1}{7}B b^2+\frac{2}{7}abD\right)x^7+\left(\frac{1}{6}b^2A+\frac{1}{3}Cab\right)x^6+\left(\frac{2}{5}abB+\frac{1}{5}Da^2\right)x^5+\left(\frac{1}{2}abA+\right.$ |
| gosper | $\frac{1}{9}Db^2x^9+\frac{1}{8}C b^2x^8+\frac{1}{7}b^2B x^7+\frac{2}{7}x^7abD+\frac{1}{6}x^6b^2A+\frac{1}{3}b x^6Ca+\frac{2}{5}x^5abB+\frac{1}{5}x^5Da^2+\frac{1}{2}x^4$ |
| parallelrisch | $\frac{1}{9}Db^2x^9+\frac{1}{8}C b^2x^8+\frac{1}{7}b^2B x^7+\frac{2}{7}x^7abD+\frac{1}{6}x^6b^2A+\frac{1}{3}b x^6Ca+\frac{2}{5}x^5abB+\frac{1}{5}x^5Da^2+\frac{1}{2}x^4$ |
| orering | $\frac{x^2(280b^2Dx^7+315b^2C x^6+360b^2B x^5+720Dab x^5+420A b^2x^4+840Cab x^4+1008Bab x^3+504Da^2x^3+1260aAb x^2+630C a}{2520}$ |

input
```
int(x*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
1/9*D*b^2*x^9+1/8*C*b^2*x^8+1/7*(B*b^2+2*D*a*b)*x^7+1/6*(A*b^2+2*C*a*b)*x^
6+1/5*(2*B*a*b+D*a^2)*x^5+1/4*(2*A*a*b+C*a^2)*x^4+1/3*x^3*a^2*B+1/2*a^2*A*
x^2
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.97

$$\int x(a+bx^2)^2(A+Bx+Cx^2+Dx^3)\,dx = \frac{1}{9}Db^2x^9+\frac{1}{8}Cb^2x^8+\frac{1}{7}(2Dab+Bb^2)x^7 + \frac{1}{6}(2Cab+Ab^2)x^6+\frac{1}{3}Ba^2x^3 + \frac{1}{5}(Da^2+2Bab)x^5 + \frac{1}{2}Aa^2x^2+\frac{1}{4}(Ca^2+2Aab)x^4$$

input
```
integrate(x*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
1/9*D*b^2*x^9 + 1/8*C*b^2*x^8 + 1/7*(2*D*a*b + B*b^2)*x^7 + 1/6*(2*C*a*b +
 A*b^2)*x^6 + 1/3*B*a^2*x^3 + 1/5*(D*a^2 + 2*B*a*b)*x^5 + 1/2*A*a^2*x^2 +
1/4*(C*a^2 + 2*A*a*b)*x^4
```

## Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.06

$$\int x(a+bx^2)^2(A+Bx+Cx^2+Dx^3)\,dx = \frac{Aa^2x^2}{2}+\frac{Ba^2x^3}{3}+\frac{Cb^2x^8}{8}+\frac{Db^2x^9}{9} + x^7\left(\frac{Bb^2}{7}+\frac{2Dab}{7}\right)+x^6\left(\frac{Ab^2}{6}+\frac{Cab}{3}\right) + x^5\cdot\left(\frac{2Bab}{5}+\frac{Da^2}{5}\right)+x^4\left(\frac{Aab}{2}+\frac{Ca^2}{4}\right)$$

input
```
integrate(x*(b*x**2+a)**2*(D*x**3+C*x**2+B*x+A),x)
```

output
```
A*a**2*x**2/2 + B*a**2*x**3/3 + C*b**2*x**8/8 + D*b**2*x**9/9 + x**7*(B*b*
*2/7 + 2*D*a*b/7) + x**6*(A*b**2/6 + C*a*b/3) + x**5*(2*B*a*b/5 + D*a**2/5
) + x**4*(A*a*b/2 + C*a**2/4)
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.97

$$\int x\left(a+bx^2\right)^2\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{1}{9}\,Db^2x^9+\frac{1}{8}\,Cb^2x^8+\frac{1}{7}\,\left(2\,Dab+Bb^2\right)x^7+\frac{1}{6}\,\left(2\,Cab+Ab^2\right)x^6+\frac{1}{3}\,Ba^2x^3+\frac{1}{5}\,\left(Da^2+2\,Bab\right)x^5+\frac{1}{2}\,Aa^2x^2+\frac{1}{4}\,\left(Ca^2+2\,Aab\right)x^4$$

input
```
integrate(x*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
1/9*D*b^2*x^9 + 1/8*C*b^2*x^8 + 1/7*(2*D*a*b + B*b^2)*x^7 + 1/6*(2*C*a*b +
 A*b^2)*x^6 + 1/3*B*a^2*x^3 + 1/5*(D*a^2 + 2*B*a*b)*x^5 + 1/2*A*a^2*x^2 +
1/4*(C*a^2 + 2*A*a*b)*x^4
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.01

$$\int x\left(a+bx^2\right)^2\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{1}{9}\,Db^2x^9+\frac{1}{8}\,Cb^2x^8+\frac{2}{7}\,Dabx^7+\frac{1}{7}\,Bb^2x^7+\frac{1}{3}\,Cabx^6+\frac{1}{6}\,Ab^2x^6+\frac{1}{5}\,Da^2x^5+\frac{2}{5}\,Babx^5+\frac{1}{4}\,Ca^2x^4+\frac{1}{2}\,Aabx^4+\frac{1}{3}\,Ba^2x^3+\frac{1}{2}\,Aa^2x^2$$

input
```
integrate(x*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

output
```
1/9*D*b^2*x^9 + 1/8*C*b^2*x^8 + 2/7*D*a*b*x^7 + 1/7*B*b^2*x^7 + 1/3*C*a*b*
x^6 + 1/6*A*b^2*x^6 + 1/5*D*a^2*x^5 + 2/5*B*a*b*x^5 + 1/4*C*a^2*x^4 + 1/2*
A*a*b*x^4 + 1/3*B*a^2*x^3 + 1/2*A*a^2*x^2
```

## Mupad [B] (verification not implemented)

Time = 2.29 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.03

$$\int x(a+bx^2)^2\,(A+Bx+Cx^2+Dx^3)\,dx$$
$$=\frac{a^2\,x^5\,D}{5}+\frac{b^2\,x^9\,D}{9}+\frac{A\,x^2\,(3\,a^2+3\,a\,b\,x^2+b^2\,x^4)}{6}+\frac{B\,x^3\,(35\,a^2+42\,a\,b\,x^2+15\,b^2\,x^4)}{105}+\frac{C\,x^4\,(6\,a^2+8\,a\,b\,x^2+3\,b^2\,x^4)}{24}+\frac{2\,a\,b\,x^7\,D}{7}$$

input
```
int(x*(a + b*x^2)^2*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
(a^2*x^5*D)/5 + (b^2*x^9*D)/9 + (A*x^2*(3*a^2 + b^2*x^4 + 3*a*b*x^2))/6 +
(B*x^3*(35*a^2 + 15*b^2*x^4 + 42*a*b*x^2))/105 + (C*x^4*(6*a^2 + 3*b^2*x^4
 + 8*a*b*x^2))/24 + (2*a*b*x^7*D)/7
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.01

$$\int x(a+bx^2)^2\,(A+Bx+Cx^2+Dx^3)\,dx$$
$$=\frac{x^2(280b^2d\,x^7+315b^2c\,x^6+720abd\,x^5+360b^3x^5+420a\,b^2x^4+840abc\,x^4+504a^2d\,x^3+1008a\,b^2x^3+12}{2520}$$

input
```
int(x*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x)
```

output
```
(x**2*(1260*a**3 + 1260*a**2*b*x**2 + 840*a**2*b*x + 630*a**2*c*x**2 + 504
*a**2*d*x**3 + 420*a*b**2*x**4 + 1008*a*b**2*x**3 + 840*a*b*c*x**4 + 720*a
*b*d*x**5 + 360*b**3*x**5 + 315*b**2*c*x**6 + 280*b**2*d*x**7))/2520
```

# 3.23 $\int (a+bx^2)^2 (A+Bx+Cx^2+Dx^3)\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 99

$$\int (a+bx^2)^2 (A+Bx+Cx^2+Dx^3)\,dx = a^2Ax+\frac{1}{3}a(2Ab+aC)x^3+\frac{1}{4}a^2Dx^4 +\frac{1}{5}b(Ab+2aC)x^5+\frac{1}{3}abDx^6 +\frac{1}{7}b^2Cx^7+\frac{1}{8}b^2Dx^8+\frac{B(a+bx^2)^3}{6b}$$

output
```
a^2*A*x+1/3*a*(2*A*b+C*a)*x^3+1/4*a^2*D*x^4+1/5*b*(A*b+2*C*a)*x^5+1/3*a*b*
D*x^6+1/7*b^2*C*x^7+1/8*b^2*D*x^8+1/6*B*(b*x^2+a)^3/b
```

## Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.89

$$\int (a+bx^2)^2 (A+Bx+Cx^2+Dx^3)\,dx = \frac{1}{840}(70a^2x(12A+x(6B+x(4C+3Dx))) +28abx^3(20A+x(15B+2x(6C+5Dx))) +b^2x^5(168A+5x(28B+3x(8C+7Dx))))$$

input
```
Integrate[(a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(70*a^2*x*(12*A + x*(6*B + x*(4*C + 3*D*x))) + 28*a*b*x^3*(20*A + x*(15*B
+ 2*x*(6*C + 5*D*x))) + b^2*x^5*(168*A + 5*x*(28*B + 3*x*(8*C + 7*D*x))))/
840
```

## Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2017, 2341, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^2 \left(A+Bx+Cx^2+Dx^3\right) dx$$

$$\Big\downarrow \text{2017}$$

$$\int \left(bx^2+a\right)^2 \left(Dx^3+Cx^2+A\right) dx+\frac{B\left(a+bx^2\right)^3}{6b}$$

$$\Big\downarrow \text{2341}$$

$$\int \left(b^2Dx^7+b^2Cx^6+2abDx^5+b(Ab+2aC)x^4+a^2Dx^3+a(2Ab+aC)x^2+a^2A\right) dx+\frac{B\left(a+bx^2\right)^3}{6b}$$

$$\Big\downarrow \text{2009}$$

$$a^2Ax+\frac{1}{4}a^2Dx^4+\frac{1}{5}bx^5(2aC+Ab)+\frac{1}{3}ax^3(aC+2Ab)+\frac{B\left(a+bx^2\right)^3}{6b}+\frac{1}{3}abDx^6+\frac{1}{7}b^2Cx^7+\frac{1}{8}b^2Dx^8$$

input
```
Int[(a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
a^2*A*x + (a*(2*A*b + a*C)*x^3)/3 + (a^2*D*x^4)/4 + (b*(A*b + 2*a*C)*x^5)/
5 + (a*b*D*x^6)/3 + (b^2*C*x^7)/7 + (b^2*D*x^8)/8 + (B*(a + b*x^2)^3)/(6*b
)
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2017
```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

rule 2341
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

## Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00

| method | result |
|---|---|
| default | $\frac{b^2Dx^8}{8}+\frac{b^2C\,x^7}{7}+\frac{(B\,b^2+2abD)x^6}{6}+\frac{(b^2A+2Cab)x^5}{5}+\frac{(2abB+Da^2)x^4}{4}+\frac{(2abA+a^2C)x^3}{3}+\frac{B\,a^2x^2}{2}+a^2Ax$ |
| norman | $\frac{b^2Dx^8}{8}+\frac{b^2C\,x^7}{7}+\left(\frac{1}{6}B\,b^2+\frac{1}{3}abD\right)x^6+\left(\frac{1}{5}b^2A+\frac{2}{5}Cab\right)x^5+\left(\frac{1}{2}abB+\frac{1}{4}Da^2\right)x^4+\left(\frac{2}{3}abA+\right.$ |
| gosper | $\frac{1}{8}b^2Dx^8+\frac{1}{7}b^2C\,x^7+\frac{1}{6}b^2B\,x^6+\frac{1}{3}abDx^6+\frac{1}{5}A\,b^2x^5+\frac{2}{5}x^5Cab+\frac{1}{2}Bab\,x^4+\frac{1}{4}a^2Dx^4+\frac{2}{3}aA$ |
| parallelrisch | $\frac{1}{8}b^2Dx^8+\frac{1}{7}b^2C\,x^7+\frac{1}{6}b^2B\,x^6+\frac{1}{3}abDx^6+\frac{1}{5}A\,b^2x^5+\frac{2}{5}x^5Cab+\frac{1}{2}Bab\,x^4+\frac{1}{4}a^2Dx^4+\frac{2}{3}aA$ |
| orering | $\frac{x(105b^2Dx^7+120b^2C\,x^6+140b^2B\,x^5+280Dab\,x^5+168A\,b^2x^4+336Cab\,x^4+420Bab\,x^3+210Da^2x^3+560aAb\,x^2+280C\,a^2x^2}{840}$ |

input
```
int((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
1/8*b^2*D*x^8+1/7*b^2*C*x^7+1/6*(B*b^2+2*D*a*b)*x^6+1/5*(A*b^2+2*C*a*b)*x^
5+1/4*(2*B*a*b+D*a^2)*x^4+1/3*(2*A*a*b+C*a^2)*x^3+1/2*B*a^2*x^2+a^2*A*x
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.99

$$\int \left(a+bx^2\right)^2 \left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{1}{8}\,Db^2x^8+\frac{1}{7}\,Cb^2x^7+\frac{1}{6}\,\left(2\,Dab+Bb^2\right)x^6+\frac{1}{5}\,\left(2\,Cab+Ab^2\right)x^5+\frac{1}{2}\,Ba^2x^2+\frac{1}{4}\,\left(Da^2+2\,Bab\right)x^4+Aa^2x+\frac{1}{3}\,\left(Ca^2+2\,Aab\right)x^3$$

input
```
integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
1/8*D*b^2*x^8 + 1/7*C*b^2*x^7 + 1/6*(2*D*a*b + B*b^2)*x^6 + 1/5*(2*C*a*b +
 A*b^2)*x^5 + 1/2*B*a^2*x^2 + 1/4*(D*a^2 + 2*B*a*b)*x^4 + A*a^2*x + 1/3*(C
*a^2 + 2*A*a*b)*x^3
```

## Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.08

$$\int \left(a+bx^2\right)^2 \left(A+Bx+Cx^2+Dx^3\right)\,dx=Aa^2x+\frac{Ba^2x^2}{2}+\frac{Cb^2x^7}{7}+\frac{Db^2x^8}{8}+x^6\left(\frac{Bb^2}{6}+\frac{Dab}{3}\right)+x^5\left(\frac{Ab^2}{5}+\frac{2Cab}{5}\right)+x^4\left(\frac{Bab}{2}+\frac{Da^2}{4}\right)+x^3\cdot\left(\frac{2Aab}{3}+\frac{Ca^2}{3}\right)$$

input
```
integrate((b*x**2+a)**2*(D*x**3+C*x**2+B*x+A),x)
```

output
```
A*a**2*x + B*a**2*x**2/2 + C*b**2*x**7/7 + D*b**2*x**8/8 + x**6*(B*b**2/6
+ D*a*b/3) + x**5*(A*b**2/5 + 2*C*a*b/5) + x**4*(B*a*b/2 + D*a**2/4) + x**
3*(2*A*a*b/3 + C*a**2/3)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.99

$$\int \left(a+bx^2\right)^2 \left(A+Bx+Cx^2+Dx^3\right)\,dx = \frac{1}{8}\,Db^2x^8+\frac{1}{7}\,Cb^2x^7+\frac{1}{6}\left(2\,Dab+Bb^2\right)x^6+\frac{1}{5}\left(2\,Cab+Ab^2\right)x^5+\frac{1}{2}\,Ba^2x^2+\frac{1}{4}\left(Da^2+2\,Bab\right)x^4+Aa^2x+\frac{1}{3}\left(Ca^2+2\,Aab\right)x^3$$

input
```
integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
1/8*D*b^2*x^8 + 1/7*C*b^2*x^7 + 1/6*(2*D*a*b + B*b^2)*x^6 + 1/5*(2*C*a*b +
 A*b^2)*x^5 + 1/2*B*a^2*x^2 + 1/4*(D*a^2 + 2*B*a*b)*x^4 + A*a^2*x + 1/3*(C
*a^2 + 2*A*a*b)*x^3
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.03

$$\int \left(a+bx^2\right)^2 \left(A+Bx+Cx^2+Dx^3\right)\,dx = \frac{1}{8}\,Db^2x^8+\frac{1}{7}\,Cb^2x^7+\frac{1}{3}\,Dabx^6+\frac{1}{6}\,Bb^2x^6+\frac{2}{5}\,Cabx^5+\frac{1}{5}\,Ab^2x^5+\frac{1}{4}\,Da^2x^4+\frac{1}{2}\,Babx^4+\frac{1}{3}\,Ca^2x^3+\frac{2}{3}\,Aabx^3+\frac{1}{2}\,Ba^2x^2+Aa^2x$$

input
```
integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

output
```
1/8*D*b^2*x^8 + 1/7*C*b^2*x^7 + 1/3*D*a*b*x^6 + 1/6*B*b^2*x^6 + 2/5*C*a*b*
x^5 + 1/5*A*b^2*x^5 + 1/4*D*a^2*x^4 + 1/2*B*a*b*x^4 + 1/3*C*a^2*x^3 + 2/3*
A*a*b*x^3 + 1/2*B*a^2*x^2 + A*a^2*x
```

## Mupad [B] (verification not implemented)

Time = 2.26 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.06

$$\int \left(a+bx^2\right)^2 \left(A+Bx+Cx^2+Dx^3\right)\,dx = \frac{A\,x\,\left(15\,a^2+10\,a\,b\,x^2+3\,b^2\,x^4\right)}{15}+\frac{a^2\,x^4\,D}{4}+\frac{b^2\,x^8\,D}{8}+\frac{B\,x^2\,\left(3\,a^2+3\,a\,b\,x^2+b^2\,x^4\right)}{6}+\frac{C\,x^3\,\left(35\,a^2+42\,a\,b\,x^2+15\,b^2\,x^4\right)}{105}+\frac{a\,b\,x^6\,D}{3}$$

input
```
int((a + b*x^2)^2*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
(A*x*(15*a^2 + 3*b^2*x^4 + 10*a*b*x^2))/15 + (a^2*x^4*D)/4 + (b^2*x^8*D)/8
 + (B*x^2*(3*a^2 + b^2*x^4 + 3*a*b*x^2))/6 + (C*x^3*(35*a^2 + 15*b^2*x^4 +
 42*a*b*x^2))/105 + (a*b*x^6*D)/3
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.04

$$\int \left(a+bx^2\right)^2 \left(A+Bx+Cx^2+Dx^3\right)\,dx = \frac{x(105b^2d\,x^7+120b^2c\,x^6+280abd\,x^5+140b^3x^5+168a\,b^2x^4+336abc\,x^4+210a^2d\,x^3+420a\,b^2x^3+560a$$
$$840$$

input
```
int((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x)
```

output
```
(x*(840*a**3 + 560*a**2*b*x**2 + 420*a**2*b*x + 280*a**2*c*x**2 + 210*a**2
*d*x**3 + 168*a*b**2*x**4 + 420*a*b**2*x**3 + 336*a*b*c*x**4 + 280*a*b*d*x
**5 + 140*b**3*x**5 + 120*b**2*c*x**6 + 105*b**2*d*x**7))/840
```

# 3.24 $\int \frac{(a+bx^2)^2(A+Bx+Cx^2+Dx^3)}{x}\,dx$



## Optimal result

Integrand size = 28, antiderivative size = 92

$$\int \frac{(a+bx^2)^2(A+Bx+Cx^2+Dx^3)}{x}\,dx = a^2Bx+aAbx^2+\frac{1}{3}a(2bB+aD)x^3 + \frac{1}{4}Ab^2x^4+\frac{1}{5}b(bB+2aD)x^5 + \frac{1}{7}b^2Dx^7+\frac{C(a+bx^2)^3}{6b}+a^2A\log(x)$$

output
```
a^2*B*x+a*A*b*x^2+1/3*a*(2*B*b+D*a)*x^3+1/4*A*b^2*x^4+1/5*b*(B*b+2*D*a)*x^
5+1/7*b^2*D*x^7+1/6*C*(b*x^2+a)^3/b+a^2*A*ln(x)
```

## Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^2)^2(A+Bx+Cx^2+Dx^3)}{x}\,dx = \frac{1}{420}x\left(70a^2(6B+x(3C+2Dx)) + 14abx(30A+x(20B+3x(5C+4Dx))) + b^2x^3(105A+2x(42B+5x(7C+6Dx)))\right) + a^2A\log(x)$$

input
```
Integrate[((a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3))/x,x]
```

output
```
(x*(70*a^2*(6*B + x*(3*C + 2*D*x)) + 14*a*b*x*(30*A + x*(20*B + 3*x*(5*C +
 4*D*x))) + b^2*x^3*(105*A + 2*x*(42*B + 5*x*(7*C + 6*D*x)))))/420 + a^2*A
*Log[x]
```

## Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {2018, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2 (A+Bx+Cx^2+Dx^3)}{x} dx$$

$$\Big\downarrow \text{ 2018}$$

$$\int \frac{(bx^2+a)^2 (Dx^3+Bx+A)}{x} dx + \frac{C(a+bx^2)^3}{6b}$$

$$\Big\downarrow \text{ 2333}$$

$$\int \left(b^2Dx^6+b(bB+2aD)x^4+Ab^2x^3+a(2bB+aD)x^2+2aAbx+a^2B+\frac{a^2A}{x}\right)dx+\frac{C(a+bx^2)^3}{6b}$$

$$\Big\downarrow \text{ 2009}$$

$$a^2A\log(x)+a^2Bx+aAbx^2+\frac{1}{5}bx^5(2aD+bB)+\frac{1}{3}ax^3(aD+2bB)+\frac{C(a+bx^2)^3}{6b}+\frac{1}{4}Ab^2x^4+\frac{1}{7}b^2Dx^7$$

input
```
Int[((a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3))/x,x]
```

output
```
a^2*B*x + a*A*b*x^2 + (a*(2*b*B + a*D)*x^3)/3 + (A*b^2*x^4)/4 + (b*(b*B +
2*a*D)*x^5)/5 + (b^2*D*x^7)/7 + (C*(a + b*x^2)^3)/(6*b) + a^2*A*Log[x]
```

**Defintions of rubi rules used**

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2018
```
Int[(Px_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[Coef
f[Px, x, n - m - 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coe
ff[Px, x, n - m - 1]*x^(n - m - 1))*x^m*(a + b*x^n)^p, x] /; FreeQ[{a, b, m
, n}, x] && PolyQ[Px, x] && IGtQ[p, 1] && IGtQ[n - m, 0] && NeQ[Coeff[Px, x
, n - m - 1], 0]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

## Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.04

| method | result |
|---|---|
| norman | $\left(\frac{1}{5}B\,b^2+\frac{2}{5}abD\right)x^5+\left(\frac{1}{4}b^2A+\frac{1}{2}Cab\right)x^4+\left(abA+\frac{1}{2}a^2C\right)x^2+\left(\frac{2}{3}abB+\frac{1}{3}Da^2\right)x^3+B\,a^2$ |
| default | $\frac{b^2Dx^7}{7}+\frac{b^2C\,x^6}{6}+\frac{b^2B\,x^5}{5}+\frac{2Dab\,x^5}{5}+\frac{A\,b^2x^4}{4}+\frac{Cab\,x^4}{2}+\frac{2Bab\,x^3}{3}+\frac{Da^2x^3}{3}+aAb\,x^2+\frac{C\,a^2x^2}{2}+B$ |
| parallelrisch | $\frac{b^2Dx^7}{7}+\frac{b^2C\,x^6}{6}+\frac{b^2B\,x^5}{5}+\frac{2Dab\,x^5}{5}+\frac{A\,b^2x^4}{4}+\frac{Cab\,x^4}{2}+\frac{2Bab\,x^3}{3}+\frac{Da^2x^3}{3}+aAb\,x^2+\frac{C\,a^2x^2}{2}+B$ |

input
```
int((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x,x,method=_RETURNVERBOSE)
```

output
```
(1/5*B*b^2+2/5*a*b*D)*x^5+(1/4*b^2*A+1/2*C*a*b)*x^4+(a*b*A+1/2*a^2*C)*x^2+
(2/3*a*b*B+1/3*D*a^2)*x^3+B*a^2*x+1/6*b^2*C*x^6+1/7*b^2*D*x^7+a^2*A*ln(x)
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx^2)^2 (A+Bx+Cx^2+Dx^3)}{x} dx = \frac{1}{7} Db^2x^7 + \frac{1}{6} Cb^2x^6 + \frac{1}{5} (2 Dab + Bb^2)x^5 + \frac{1}{4} (2 Cab + Ab^2)x^4 + Ba^2x + \frac{1}{3} (Da^2 + 2 Bab)x^3 + Aa^2 \log(x) + \frac{1}{2} (Ca^2 + 2 Aab)x^2$$

input
```
integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x,x, algorithm="fricas")
```

output
```
1/7*D*b^2*x^7 + 1/6*C*b^2*x^6 + 1/5*(2*D*a*b + B*b^2)*x^5 + 1/4*(2*C*a*b +
 A*b^2)*x^4 + B*a^2*x + 1/3*(D*a^2 + 2*B*a*b)*x^3 + A*a^2*log(x) + 1/2*(C*
a^2 + 2*A*a*b)*x^2
```

## Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.13

$$\int \frac{(a+bx^2)^2 (A+Bx+Cx^2+Dx^3)}{x} dx = Aa^2 \log(x) + Ba^2x + \frac{Cb^2x^6}{6} + \frac{Db^2x^7}{7} + x^5 \left(\frac{Bb^2}{5} + \frac{2Dab}{5}\right) + x^4 \left(\frac{Ab^2}{4} + \frac{Cab}{2}\right) + x^3 \cdot \left(\frac{2Bab}{3} + \frac{Da^2}{3}\right) + x^2 \left(Aab + \frac{Ca^2}{2}\right)$$

input
```
integrate((b*x**2+a)**2*(D*x**3+C*x**2+B*x+A)/x,x)
```

output
```
A*a**2*log(x) + B*a**2*x + C*b**2*x**6/6 + D*b**2*x**7/7 + x**5*(B*b**2/5
+ 2*D*a*b/5) + x**4*(A*b**2/4 + C*a*b/2) + x**3*(2*B*a*b/3 + D*a**2/3) + x
**2*(A*a*b + C*a**2/2)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx^2)^2 (A+Bx+Cx^2+Dx^3)}{x}\,dx = \frac{1}{7}\,Db^2x^7 + \frac{1}{6}\,Cb^2x^6 + \frac{1}{5}\,(2\,Dab+Bb^2)x^5 + \frac{1}{4}\,(2\,Cab+Ab^2)x^4 + Ba^2x + \frac{1}{3}\,(Da^2+2\,Bab)x^3 + Aa^2\log(x) + \frac{1}{2}\,(Ca^2+2\,Aab)x^2$$

input
```
integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x,x, algorithm="maxima")
```

output
```
1/7*D*b^2*x^7 + 1/6*C*b^2*x^6 + 1/5*(2*D*a*b + B*b^2)*x^5 + 1/4*(2*C*a*b +
 A*b^2)*x^4 + B*a^2*x + 1/3*(D*a^2 + 2*B*a*b)*x^3 + A*a^2*log(x) + 1/2*(C*
a^2 + 2*A*a*b)*x^2
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx^2)^2 (A+Bx+Cx^2+Dx^3)}{x}\,dx = \frac{1}{7}\,Db^2x^7 + \frac{1}{6}\,Cb^2x^6 + \frac{2}{5}\,Dabx^5 + \frac{1}{5}\,Bb^2x^5 + \frac{1}{2}\,Cabx^4 + \frac{1}{4}\,Ab^2x^4 + \frac{1}{3}\,Da^2x^3 + \frac{2}{3}\,Babx^3 + \frac{1}{2}\,Ca^2x^2 + Aabx^2 + Ba^2x + Aa^2\log(|x|)$$

input
```
integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x,x, algorithm="giac")
```

output
```
1/7*D*b^2*x^7 + 1/6*C*b^2*x^6 + 2/5*D*a*b*x^5 + 1/5*B*b^2*x^5 + 1/2*C*a*b*
x^4 + 1/4*A*b^2*x^4 + 1/3*D*a^2*x^3 + 2/3*B*a*b*x^3 + 1/2*C*a^2*x^2 + A*a*
b*x^2 + B*a^2*x + A*a^2*log(abs(x))
```

## Mupad [B] (verification not implemented)

Time = 2.25 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.12

$$\int \frac{(a+bx^2)^2\,(A+Bx+Cx^2+Dx^3)}{x}\,dx = \frac{A\,(4\,a^2\,\ln(x)+b^2\,x^4+4\,a\,b\,x^2)}{4} + \frac{B\,x\,(15\,a^2+10\,a\,b\,x^2+3\,b^2\,x^4)}{15} + \frac{a^2\,x^3\,D}{3} + \frac{b^2\,x^7\,D}{7} + \frac{C\,x^2\,(3\,a^2+3\,a\,b\,x^2+b^2\,x^4)}{6} + \frac{2\,a\,b\,x^5\,D}{5}$$

input
```
int(((a + b*x^2)^2*(A + B*x + C*x^2 + x^3*D))/x,x)
```

output
```
(A*(4*a^2*log(x) + b^2*x^4 + 4*a*b*x^2))/4 + (B*x*(15*a^2 + 3*b^2*x^4 + 10
*a*b*x^2))/15 + (a^2*x^3*D)/3 + (b^2*x^7*D)/7 + (C*x^2*(3*a^2 + b^2*x^4 +
3*a*b*x^2))/6 + (2*a*b*x^5*D)/5
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^2)^2\,(A+Bx+Cx^2+Dx^3)}{x}\,dx = \log(x)\,a^3 + a^2b\,x^2 + a^2bx + \frac{a^2c\,x^2}{2} + \frac{a^2d\,x^3}{3} + \frac{a\,b^2x^4}{4} + \frac{2a\,b^2x^3}{3} + \frac{abc\,x^4}{2} + \frac{2abd\,x^5}{5} + \frac{b^3x^5}{5} + \frac{b^2c\,x^6}{6} + \frac{b^2d\,x^7}{7}$$

input
```
int((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x,x)
```

output
```
(420*log(x)*a**3 + 420*a**2*b*x**2 + 420*a**2*b*x + 210*a**2*c*x**2 + 140*
a**2*d*x**3 + 105*a*b**2*x**4 + 280*a*b**2*x**3 + 210*a*b*c*x**4 + 168*a*b
*d*x**5 + 84*b**3*x**5 + 70*b**2*c*x**6 + 60*b**2*d*x**7)/420
```

# 3.25 $\int \frac{(a+bx^2)^2(A+Bx+Cx^2+Dx^3)}{x^2}\,dx$



## Optimal result

Integrand size = 28, antiderivative size = 90

$$\int \frac{(a+bx^2)^2(A+Bx+Cx^2+Dx^3)}{x^2}\,dx = -\frac{a^2A}{x}+a(2Ab+aC)x+abBx^2 +\frac{1}{3}b(Ab+2aC)x^3+\frac{1}{4}b^2Bx^4 +\frac{1}{5}b^2Cx^5+\frac{D(a+bx^2)^3}{6b}+a^2B\log(x)$$

output
```
-a^2*A/x+a*(2*A*b+C*a)*x+a*b*B*x^2+1/3*b*(A*b+2*C*a)*x^3+1/4*b^2*B*x^4+1/5
*b^2*C*x^5+1/6*D*(b*x^2+a)^3/b+a^2*B*ln(x)
```

## Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^2)^2(A+Bx+Cx^2+Dx^3)}{x^2}\,dx = a^2\left(-\frac{A}{x}+Cx+\frac{Dx^2}{2}\right) +\frac{1}{6}abx(12A+x(6B+x(4C+3Dx))) +\frac{1}{60}b^2x^3(20A+x(15B+2x(6C+5Dx))) +a^2B\log(x)$$

input
```
Integrate[((a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3))/x^2,x]
```

output
```
a^2*(-(A/x) + C*x + (D*x^2)/2) + (a*b*x*(12*A + x*(6*B + x*(4*C + 3*D*x)))
)/6 + (b^2*x^3*(20*A + x*(15*B + 2*x*(6*C + 5*D*x))))/60 + a^2*B*Log[x]
```

## Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {2018, 2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2 (A+Bx+Cx^2+Dx^3)}{x^2} \, dx$$

$$\Big\downarrow \text{2018}$$

$$\int \frac{(bx^2+a)^2 (Cx^2+Bx+A)}{x^2}dx+\frac{D(a+bx^2)^3}{6b}$$

$$\Big\downarrow \text{2159}$$

$$\int \left(b^2Cx^4+b^2Bx^3+b(Ab+2aC)x^2+2abBx+a(2Ab+aC)+\frac{a^2B}{x}+\frac{a^2A}{x^2}\right)dx+\frac{D(a+bx^2)^3}{6b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2A}{x}+a^2B\log(x)+\frac{1}{3}bx^3(2aC+Ab)+ax(aC+2Ab)+abBx^2+\frac{D(a+bx^2)^3}{6b}+\frac{1}{4}b^2Bx^4+\frac{1}{5}b^2Cx^5$$

input
```
Int[((a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3))/x^2,x]
```

output
```
-((a^2*A)/x) + a*(2*A*b + a*C)*x + a*b*B*x^2 + (b*(A*b + 2*a*C)*x^3)/3 + (
b^2*B*x^4)/4 + (b^2*C*x^5)/5 + (D*(a + b*x^2)^3)/(6*b) + a^2*B*Log[x]
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2018
```
Int[(Px_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[Coef
f[Px, x, n - m - 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coe
ff[Px, x, n - m - 1]*x^(n - m - 1))*x^m*(a + b*x^n)^p, x] /; FreeQ[{a, b, m
, n}, x] && PolyQ[Px, x] && IGtQ[p, 1] && IGtQ[n - m, 0] && NeQ[Coeff[Px, x
, n - m - 1], 0]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

## Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.09

| method | result |
|---|---|
| default | $\frac{D x^6 b^2}{6}+\frac{b^2 C x^5}{5}+\frac{b^2 B x^4}{4}+\frac{D a b x^4}{2}+\frac{A b^2 x^3}{3}+\frac{2 C a b x^3}{3}+B a b x^2+\frac{D a^2 x^2}{2}+2 a A b x+C a^2 x+a^2$ |
| norman | $\frac{\left(\frac{1}{4} B b^2+\frac{1}{2} a b D\right) x^5+\left(\frac{1}{3} b^2 A+\frac{2}{3} C a b\right) x^4+\left(a b B+\frac{1}{2} D a^2\right) x^3+\left(2 a b A+a^2 C\right) x^2-a^2 A+\frac{b^2 C x^6}{5}+\frac{b^2 D x^7}{6}}{x}+a^2 B \ln \left(x\right)$ |
| parallelrisch | $\frac{10 b^2 D x^7+12 b^2 C x^6+15 b^2 B x^5+30 D a b x^5+20 A b^2 x^4+40 C a b x^4+60 B a b x^3+30 D a^2 x^3+120 a A b x^2+60 a^2 B \ln(x) x+60 C a^2}{60 x}$ |

input
```
int((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/6*D*x^6*b^2+1/5*b^2*C*x^5+1/4*b^2*B*x^4+1/2*D*a*b*x^4+1/3*A*b^2*x^3+2/3*
C*a*b*x^3+B*a*b*x^2+1/2*D*a^2*x^2+2*a*A*b*x+C*a^2*x+a^2*B*ln(x)-a^2*A/x
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.14

$$\int \frac{(a+bx^2)^2(A+Bx+Cx^2+Dx^3)}{x^2}\,dx$$
$$= \frac{10\,Db^2x^7+12\,Cb^2x^6+15\,(2\,Dab+Bb^2)x^5+20\,(2\,Cab+Ab^2)x^4+60\,Ba^2x\log(x)+30\,(Da^2+2\,Bab)}{60\,x}$$

input
```
integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x^2,x, algorithm="fricas")
```

output
```
1/60*(10*D*b^2*x^7 + 12*C*b^2*x^6 + 15*(2*D*a*b + B*b^2)*x^5 + 20*(2*C*a*b
 + A*b^2)*x^4 + 60*B*a^2*x*log(x) + 30*(D*a^2 + 2*B*a*b)*x^3 - 60*A*a^2 +
60*(C*a^2 + 2*A*a*b)*x^2)/x
```

## Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.10

$$\int \frac{(a+bx^2)^2(A+Bx+Cx^2+Dx^3)}{x^2}\,dx = -\frac{Aa^2}{x}+Ba^2\log(x)+\frac{Cb^2x^5}{5}+\frac{Db^2x^6}{6}+x^4\left(\frac{Bb^2}{4}+\frac{Dab}{2}\right)+x^3\left(\frac{Ab^2}{3}+\frac{2Cab}{3}\right)+x^2\left(Bab+\frac{Da^2}{2}\right)+x\left(2Aab+Ca^2\right)$$

input
```
integrate((b*x**2+a)**2*(D*x**3+C*x**2+B*x+A)/x**2,x)
```

output
```
-A*a**2/x + B*a**2*log(x) + C*b**2*x**5/5 + D*b**2*x**6/6 + x**4*(B*b**2/4
 + D*a*b/2) + x**3*(A*b**2/3 + 2*C*a*b/3) + x**2*(B*a*b + D*a**2/2) + x*(2
*A*a*b + C*a**2)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^2)^2 (A+Bx+Cx^2+Dx^3)}{x^2}\,dx = \frac{1}{6}\,Db^2x^6 + \frac{1}{5}\,Cb^2x^5 + \frac{1}{4}\,(2\,Dab+Bb^2)x^4 + \frac{1}{3}\,(2\,Cab+Ab^2)x^3 + Ba^2\log(x) + \frac{1}{2}\,(Da^2+2\,Bab)x^2 - \frac{Aa^2}{x} + (Ca^2+2\,Aab)x$$

input
```
integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x^2,x, algorithm="maxima")
```

output
```
1/6*D*b^2*x^6 + 1/5*C*b^2*x^5 + 1/4*(2*D*a*b + B*b^2)*x^4 + 1/3*(2*C*a*b +
 A*b^2)*x^3 + B*a^2*log(x) + 1/2*(D*a^2 + 2*B*a*b)*x^2 - A*a^2/x + (C*a^2
+ 2*A*a*b)*x
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx^2)^2 (A+Bx+Cx^2+Dx^3)}{x^2}\,dx = \frac{1}{6}\,Db^2x^6 + \frac{1}{5}\,Cb^2x^5 + \frac{1}{2}\,Dabx^4 + \frac{1}{4}\,Bb^2x^4 + \frac{2}{3}\,Cabx^3 + \frac{1}{3}\,Ab^2x^3 + \frac{1}{2}\,Da^2x^2 + Babx^2 + Ca^2x + 2\,Aabx + Ba^2\log(|x|) - \frac{Aa^2}{x}$$

input
```
integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x^2,x, algorithm="giac")
```

output
```
1/6*D*b^2*x^6 + 1/5*C*b^2*x^5 + 1/2*D*a*b*x^4 + 1/4*B*b^2*x^4 + 2/3*C*a*b*
x^3 + 1/3*A*b^2*x^3 + 1/2*D*a^2*x^2 + B*a*b*x^2 + C*a^2*x + 2*A*a*b*x + B*
a^2*log(abs(x)) - A*a^2/x
```

## Mupad [B] (verification not implemented)

Time = 2.29 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^2)^2(A+Bx+Cx^2+Dx^3)}{x^2}\,dx = \frac{B\,(4\,a^2\,\ln(x)+b^2\,x^4+4\,a\,b\,x^2)}{4} + \frac{(b\,x^2+a)^3\,D}{6\,b} + \frac{C\,x\,(15\,a^2+10\,a\,b\,x^2+3\,b^2\,x^4)}{15} + \frac{A\,(-3\,a^2+6\,a\,b\,x^2+b^2\,x^4)}{3\,x}$$

input
```
int(((a + b*x^2)^2*(A + B*x + C*x^2 + x^3*D))/x^2,x)
```

output
```
(B*(4*a^2*log(x) + b^2*x^4 + 4*a*b*x^2))/4 + ((a + b*x^2)^3*D)/(6*b) + (C*
x*(15*a^2 + 3*b^2*x^4 + 10*a*b*x^2))/15 + (A*(b^2*x^4 - 3*a^2 + 6*a*b*x^2)
)/(3*x)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.19

$$\int \frac{(a+bx^2)^2(A+Bx+Cx^2+Dx^3)}{x^2}\,dx = \frac{60\log(x)\,a^2bx-60a^3+120a^2b\,x^2+60a^2c\,x^2+30a^2d\,x^3+20a\,b^2x^4+60a\,b^2x^3+40abc\,x^4+30abd\,x^5+}{60x}$$

input
```
int((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x^2,x)
```

output
```
(60*log(x)*a**2*b*x - 60*a**3 + 120*a**2*b*x**2 + 60*a**2*c*x**2 + 30*a**2
*d*x**3 + 20*a*b**2*x**4 + 60*a*b**2*x**3 + 40*a*b*c*x**4 + 30*a*b*d*x**5
+ 15*b**3*x**5 + 12*b**2*c*x**6 + 10*b**2*d*x**7)/(60*x)
```

# 3.26 $\int \frac{(a+bx^2)^2(A+Bx+Cx^2+Dx^3)}{x^3}\,dx$



## Optimal result

Integrand size = 28, antiderivative size = 98

$$\int \frac{(a+bx^2)^2(A+Bx+Cx^2+Dx^3)}{x^3}\,dx = -\frac{a^2A}{2x^2}-\frac{a^2B}{x}+a(2bB+aD)x+\frac{1}{2}b(Ab+2aC)x^2+\frac{1}{3}b(bB+2aD)x^3+\frac{1}{4}b^2Cx^4+\frac{1}{5}b^2Dx^5+a(2Ab+aC)\log(x)$$

output
```
-1/2*a^2*A/x^2-a^2*B/x+a*(2*B*b+D*a)*x+1/2*b*(A*b+2*C*a)*x^2+1/3*b*(B*b+2*
D*a)*x^3+1/4*b^2*C*x^4+1/5*b^2*D*x^5+a*(2*A*b+C*a)*ln(x)
```

## Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx^2)^2(A+Bx+Cx^2+Dx^3)}{x^3}\,dx = -\frac{a^2(A+2Bx-2Dx^3)}{2x^2}+\frac{1}{3}abx(6B+x(3C+2Dx))+\frac{1}{60}b^2x^2(30A+x(20B+3x(5C+4Dx)))+a(2Ab+aC)\log(x)$$

input
```
Integrate[((a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3))/x^3,x]
```

output
```
-1/2*(a^2*(A + 2*B*x - 2*D*x^3))/x^2 + (a*b*x*(6*B + x*(3*C + 2*D*x)))/3 +
 (b^2*x^2*(30*A + x*(20*B + 3*x*(5*C + 4*D*x))))/60 + a*(2*A*b + a*C)*Log[
x]
```

## Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2 (A+Bx+Cx^2+Dx^3)}{x^3} \, dx$$

$$\Big\downarrow \text{2333}$$

$$\int \left(\frac{a^2A}{x^3}+\frac{a^2B}{x^2}+bx(2aC+Ab)+\frac{a(aC+2Ab)}{x}+bx^2(2aD+bB)+a(aD+2bB)+b^2Cx^3+b^2Dx^4\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2A}{2x^2}-\frac{a^2B}{x}+\frac{1}{2}bx^2(2aC+Ab)+a\log(x)(aC+2Ab)+\frac{1}{3}bx^3(2aD+bB)+ax(aD+2bB)+\frac{1}{4}b^2Cx^4+\frac{1}{5}b^2Dx^5$$

input
```
Int[((a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3))/x^3,x]
```

output
```
-1/2*(a^2*A)/x^2 - (a^2*B)/x + a*(2*b*B + a*D)*x + (b*(A*b + 2*a*C)*x^2)/2
 + (b*(b*B + 2*a*D)*x^3)/3 + (b^2*C*x^4)/4 + (b^2*D*x^5)/5 + a*(2*A*b + a*
C)*Log[x]
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

## Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.97

| method | result |
|---|---|
| default | $\frac{Dx^5b^2}{5}+\frac{C\,b^2x^4}{4}+\frac{B\,b^2x^3}{3}+\frac{2Dab\,x^3}{3}+\frac{A\,b^2x^2}{2}+x^2aCb+2Babx+Da^2x-\frac{a^2A}{2x^2}+a(2Ab+Ca)$ |
| norman | $\frac{(\frac{1}{3}B\,b^2+\frac{2}{3}abD)x^5+(\frac{1}{2}b^2A+Cab)x^4+(2abB+Da^2)x^3-\frac{a^2A}{2}-B\,a^2x+\frac{b^2C\,x^6}{4}+\frac{b^2Dx^7}{5}}{x^2}+(2abA+a^2C)\ln(x)$ |
| parallelrisch | $\frac{12b^2Dx^7+15b^2C\,x^6+20b^2B\,x^5+40Dab\,x^5+30A\,b^2x^4+60Cab\,x^4+120A\ln(x)x^2ab+120Bab\,x^3+60C\ln(x)x^2a^2+60Da^2x^3-}{60x^2}$ |

input
```
int((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/5*D*x^5*b^2+1/4*C*b^2*x^4+1/3*B*b^2*x^3+2/3*D*a*b*x^3+1/2*A*b^2*x^2+x^2*
a*C*b+2*B*a*b*x+D*a^2*x-1/2*a^2*A/x^2+a*(2*A*b+C*a)*ln(x)-a^2*B/x
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.05

$$\int\frac{(a+bx^2)^2\,(A+Bx+Cx^2+Dx^3)}{x^3}\,dx$$
$$=\frac{12\,Db^2x^7+15\,Cb^2x^6+20\,(2\,Dab+Bb^2)x^5+30\,(2\,Cab+Ab^2)x^4-60\,Ba^2x+60\,(Da^2+2\,Bab)x^3+6}{60\,x^2}$$

input
```
integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x^3,x, algorithm="fricas")
```

output
```
1/60*(12*D*b^2*x^7 + 15*C*b^2*x^6 + 20*(2*D*a*b + B*b^2)*x^5 + 30*(2*C*a*b
 + A*b^2)*x^4 - 60*B*a^2*x + 60*(D*a^2 + 2*B*a*b)*x^3 + 60*(C*a^2 + 2*A*a*
b)*x^2*log(x) - 30*A*a^2)/x^2
```

## Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^2)^2 (A+Bx+Cx^2+Dx^3)}{x^3} dx = \frac{Cb^2x^4}{4} + \frac{Db^2x^5}{5} + a(2Ab+Ca)\log(x) + x^3\left(\frac{Bb^2}{3} + \frac{2Dab}{3}\right) + x^2\left(\frac{Ab^2}{2} + Cab\right) + x(2Bab + Da^2) + \frac{-Aa^2 - 2Ba^2x}{2x^2}$$

input
```
integrate((b*x**2+a)**2*(D*x**3+C*x**2+B*x+A)/x**3,x)
```

output
```
C*b**2*x**4/4 + D*b**2*x**5/5 + a*(2*A*b + C*a)*log(x) + x**3*(B*b**2/3 +
2*D*a*b/3) + x**2*(A*b**2/2 + C*a*b) + x*(2*B*a*b + D*a**2) + (-A*a**2 - 2
*B*a**2*x)/(2*x**2)
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^2)^2 (A+Bx+Cx^2+Dx^3)}{x^3} dx = \frac{1}{5} Db^2x^5 + \frac{1}{4} Cb^2x^4 + \frac{1}{3} (2Dab + Bb^2)x^3 + \frac{1}{2} (2Cab + Ab^2)x^2 + (Da^2 + 2Bab)x + (Ca^2 + 2Aab)\log(x) - \frac{2Ba^2x + Aa^2}{2x^2}$$

input
```
integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x^3,x, algorithm="maxima")
```

output
```
1/5*D*b^2*x^5 + 1/4*C*b^2*x^4 + 1/3*(2*D*a*b + B*b^2)*x^3 + 1/2*(2*C*a*b +
 A*b^2)*x^2 + (D*a^2 + 2*B*a*b)*x + (C*a^2 + 2*A*a*b)*log(x) - 1/2*(2*B*a^
2*x + A*a^2)/x^2
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx^2)^2(A+Bx+Cx^2+Dx^3)}{x^3}\,dx = \frac{1}{5}\,Db^2x^5+\frac{1}{4}\,Cb^2x^4+\frac{2}{3}\,Dabx^3+\frac{1}{3}\,Bb^2x^3+Cabx^2+\frac{1}{2}\,Ab^2x^2+Da^2x+2\,Babx+\left(Ca^2+2\,Aab\right)\log\left(|x|\right)-\frac{2\,Ba^2x+Aa^2}{2\,x^2}$$

input
```
integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x^3,x, algorithm="giac")
```

output
```
1/5*D*b^2*x^5 + 1/4*C*b^2*x^4 + 2/3*D*a*b*x^3 + 1/3*B*b^2*x^3 + C*a*b*x^2
+ 1/2*A*b^2*x^2 + D*a^2*x + 2*B*a*b*x + (C*a^2 + 2*A*a*b)*log(abs(x)) - 1/
2*(2*B*a^2*x + A*a^2)/x^2
```

## Mupad [B] (verification not implemented)

Time = 2.27 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.05

$$\int \frac{(a+bx^2)^2(A+Bx+Cx^2+Dx^3)}{x^3}\,dx = \frac{C\,(4\,a^2\,\ln(x)+b^2\,x^4+4\,a\,b\,x^2)}{4}+a^2\,x\,D+\frac{b^2\,x^5\,D}{5}+\frac{A\,(b^2\,x^4-a^2+4\,a\,b\,x^2\,\ln(x))}{2\,x^2}+\frac{B\,(-3\,a^2+6\,a\,b\,x^2+b^2\,x^4)}{3\,x}+\frac{2\,a\,b\,x^3\,D}{3}$$

input
```
int(((a + b*x^2)^2*(A + B*x + C*x^2 + x^3*D))/x^3,x)
```

output
```
(C*(4*a^2*log(x) + b^2*x^4 + 4*a*b*x^2))/4 + a^2*x*D + (b^2*x^5*D)/5 + (A*
(b^2*x^4 - a^2 + 4*a*b*x^2*log(x)))/(2*x^2) + (B*(b^2*x^4 - 3*a^2 + 6*a*b*
x^2))/(3*x) + (2*a*b*x^3*D)/3
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.11

$$\int \frac{(a+bx^2)^2 (A+Bx+Cx^2+Dx^3)}{x^3} \, dx$$
$$= \frac{120 \log(x) a^2 b x^2 + 60 \log(x) a^2 c x^2 - 30a^3 - 60a^2 bx + 60a^2 d x^3 + 30a b^2 x^4 + 120a b^2 x^3 + 60abc x^4 + 40}{60x^2}$$

input
```
int((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x^3,x)
```

output
```
(120*log(x)*a**2*b*x**2 + 60*log(x)*a**2*c*x**2 - 30*a**3 - 60*a**2*b*x +
60*a**2*d*x**3 + 30*a*b**2*x**4 + 120*a*b**2*x**3 + 60*a*b*c*x**4 + 40*a*b
*d*x**5 + 20*b**3*x**5 + 15*b**2*c*x**6 + 12*b**2*d*x**7)/(60*x**2)
```

# 3.27 $\int \frac{(a+bx^2)^2(A+Bx+Cx^2+Dx^3)}{x^4}\,dx$



## Optimal result

Integrand size = 28, antiderivative size = 98

$$\int \frac{(a+bx^2)^2(A+Bx+Cx^2+Dx^3)}{x^4}\,dx = -\frac{a^2A}{3x^3}-\frac{a^2B}{2x^2}-\frac{a(2Ab+aC)}{x}+b(Ab+2aC)x+\frac{1}{2}b(bB+2aD)x^2+\frac{1}{3}b^2Cx^3+\frac{1}{4}b^2Dx^4+a(2bB+aD)\log(x)$$

output
```
-1/3*a^2*A/x^3-1/2*a^2*B/x^2-a*(2*A*b+C*a)/x+b*(A*b+2*C*a)*x+1/2*b*(B*b+2*
D*a)*x^2+1/3*b^2*C*x^3+1/4*b^2*D*x^4+a*(2*B*b+D*a)*ln(x)
```

## Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx^2)^2(A+Bx+Cx^2+Dx^3)}{x^4}\,dx = -\frac{2aAb}{x}+abx(2C+Dx)-\frac{a^2(2A+3x(B+2Cx))}{6x^3}+\frac{1}{12}b^2x\left(12A+x\left(6B+4Cx+3Dx^2\right)\right)+a(2bB+aD)\log(x)$$

input
```
Integrate[((a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3))/x^4,x]
```

output
```
(-2*a*A*b)/x + a*b*x*(2*C + D*x) - (a^2*(2*A + 3*x*(B + 2*C*x)))/(6*x^3) +
 (b^2*x*(12*A + x*(6*B + 4*C*x + 3*D*x^2)))/12 + a*(2*b*B + a*D)*Log[x]
```

## Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2 (A+Bx+Cx^2+Dx^3)}{x^4} \, dx$$

$$\Big\downarrow \text{ 2333}$$

$$\int \left(\frac{a^2A}{x^4}+\frac{a^2B}{x^3}+\frac{a(aC+2Ab)}{x^2}+b(2aC+Ab)+bx(2aD+bB)+\frac{a(aD+2bB)}{x}+b^2Cx^2+b^2Dx^3\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a^2A}{3x^3}-\frac{a^2B}{2x^2}+bx(2aC+Ab)-\frac{a(aC+2Ab)}{x}+\frac{1}{2}bx^2(2aD+bB)+a\log(x)(aD+2bB)+\frac{1}{3}b^2Cx^3+\frac{1}{4}b^2Dx^4$$

input
```
Int[((a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3))/x^4,x]
```

output
```
-1/3*(a^2*A)/x^3 - (a^2*B)/(2*x^2) - (a*(2*A*b + a*C))/x + b*(A*b + 2*a*C)
*x + (b*(b*B + 2*a*D)*x^2)/2 + (b^2*C*x^3)/3 + (b^2*D*x^4)/4 + a*(2*b*B +
a*D)*Log[x]
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

## Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.94

| method | result |
|---|---|
| default | $\frac{b^2 D x^4}{4}+\frac{C b^2 x^3}{3}+\frac{b^2 B x^2}{2}+D a b x^2+A b^2 x+2 C a b x-\frac{a^2 A}{3 x^3}-\frac{a^2 B}{2 x^2}+a(2 B b+D a) \ln (x)-\frac{a(}{}$ |
| norman | $\frac{\left(\frac{1}{2} B b^2+a b D\right) x^5+\left(b^2 A+2 C a b\right) x^4+\left(-2 a b A-a^2 C\right) x^2-\frac{a^2 A}{3}-\frac{B a^2 x}{2}+\frac{b^2 C x^6}{3}+\frac{b^2 D x^7}{4}}{x^3}+\left(2 a b B+D a^2\right) \ln (x)$ |
| parallelrisch | $\frac{3 b^2 D x^7+4 b^2 C x^6+6 b^2 B x^5+12 D a b x^5+12 A b^2 x^4+24 B \ln (x) x^3 a b+24 C a b x^4+12 D \ln (x) x^3 a^2-24 a A b x^2-12 C a^2 x^2-6 B a^2}{12 x^3}$ |

input
```
int((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/4*b^2*D*x^4+1/3*C*b^2*x^3+1/2*b^2*B*x^2+D*a*b*x^2+A*b^2*x+2*C*a*b*x-1/3*
a^2*A/x^3-1/2*a^2*B/x^2+a*(2*B*b+D*a)*ln(x)-a*(2*A*b+C*a)/x
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.05

$$\int \frac{(a+bx^2)^2(A+Bx+Cx^2+Dx^3)}{x^4}\,dx$$
$$=\frac{3\,Db^2x^7+4\,Cb^2x^6+6\,(2\,Dab+Bb^2)x^5+12\,(2\,Cab+Ab^2)x^4+12\,(Da^2+2\,Bab)x^3\log(x)-6\,Ba^2x-}{12\,x^3}$$

input
```
integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x^4,x, algorithm="fricas")
```

output
```
1/12*(3*D*b^2*x^7 + 4*C*b^2*x^6 + 6*(2*D*a*b + B*b^2)*x^5 + 12*(2*C*a*b +
A*b^2)*x^4 + 12*(D*a^2 + 2*B*a*b)*x^3*log(x) - 6*B*a^2*x - 4*A*a^2 - 12*(C
*a^2 + 2*A*a*b)*x^2)/x^3
```

## Sympy [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^2)^2(A+Bx+Cx^2+Dx^3)}{x^4}\,dx = \frac{Cb^2x^3}{3}+\frac{Db^2x^4}{4}+a(2Bb+Da)\log(x)+x^2\left(\frac{Bb^2}{2}+Dab\right)+x\left(Ab^2+2Cab\right)+\frac{-2Aa^2-3Ba^2x+x^2(-12Aab-6Ca^2)}{6x^3}$$

input
```
integrate((b*x**2+a)**2*(D*x**3+C*x**2+B*x+A)/x**4,x)
```

output
```
C*b**2*x**3/3 + D*b**2*x**4/4 + a*(2*B*b + D*a)*log(x) + x**2*(B*b**2/2 +
D*a*b) + x*(A*b**2 + 2*C*a*b) + (-2*A*a**2 - 3*B*a**2*x + x**2*(-12*A*a*b
- 6*C*a**2))/(6*x**3)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx^2)^2(A+Bx+Cx^2+Dx^3)}{x^4}\,dx = \frac{1}{4}Db^2x^4+\frac{1}{3}Cb^2x^3+\frac{1}{2}(2Dab+Bb^2)x^2+(2Cab+Ab^2)x+\left(Da^2+2Bab\right)\log(x)-\frac{3Ba^2x+2Aa^2+6(Ca^2+2Aab)x^2}{6x^3}$$

input
```
integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x^4,x, algorithm="maxima")
```

output
```
1/4*D*b^2*x^4 + 1/3*C*b^2*x^3 + 1/2*(2*D*a*b + B*b^2)*x^2 + (2*C*a*b + A*b
^2)*x + (D*a^2 + 2*B*a*b)*log(x) - 1/6*(3*B*a^2*x + 2*A*a^2 + 6*(C*a^2 + 2
*A*a*b)*x^2)/x^3
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx^2)^2 (A+Bx+Cx^2+Dx^3)}{x^4} \, dx = \frac{1}{4} Db^2x^4 + \frac{1}{3} Cb^2x^3 + Dabx^2 + \frac{1}{2} Bb^2x^2 + 2 Cabx + Ab^2x + \left(Da^2 + 2 Bab\right) \log\left(|x|\right) - \frac{3 Ba^2x + 2 Aa^2 + 6 (Ca^2 + 2 Aab)x^2}{6 x^3}$$

input
```
integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x^4,x, algorithm="giac")
```

output
```
1/4*D*b^2*x^4 + 1/3*C*b^2*x^3 + D*a*b*x^2 + 1/2*B*b^2*x^2 + 2*C*a*b*x + A*
b^2*x + (D*a^2 + 2*B*a*b)*log(abs(x)) - 1/6*(3*B*a^2*x + 2*A*a^2 + 6*(C*a^
2 + 2*A*a*b)*x^2)/x^3
```

## Mupad [B] (verification not implemented)

Time = 2.42 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^2)^2 (A+Bx+Cx^2+Dx^3)}{x^4} \, dx = \frac{b^2\,x^4\,D}{4} + \frac{a^2\,\ln\left(x^2\right)\,D}{2} - \frac{A\,\left(a^2+6\,a\,b\,x^2-3\,b^2\,x^4\right)}{3\,x^3} + \frac{B\,\left(b^2\,x^4-a^2+4\,a\,b\,x^2\,\ln\left(x\right)\right)}{2\,x^2} + \frac{C\,\left(-3\,a^2+6\,a\,b\,x^2+b^2\,x^4\right)}{3\,x} + a\,b\,x^2\,D$$

input
```
int(((a + b*x^2)^2*(A + B*x + C*x^2 + x^3*D))/x^4,x)
```

output
```
(b^2*x^4*D)/4 + (a^2*log(x^2)*D)/2 - (A*(a^2 - 3*b^2*x^4 + 6*a*b*x^2))/(3*
x^3) + (B*(b^2*x^4 - a^2 + 4*a*b*x^2*log(x)))/(2*x^2) + (C*(b^2*x^4 - 3*a^
2 + 6*a*b*x^2))/(3*x) + a*b*x^2*D
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.11

$$\int \frac{(a+bx^2)^2 (A+Bx+Cx^2+Dx^3)}{x^4}\,dx$$
$$= \frac{12\log(x)\,a^2d\,x^3+24\log(x)\,a\,b^2x^3-4a^3-24a^2b\,x^2-6a^2bx-12a^2c\,x^2+12a\,b^2x^4+24abc\,x^4+12abd\,x}{12x^3}$$

input
```
int((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x^4,x)
```

output
```
(12*log(x)*a**2*d*x**3 + 24*log(x)*a*b**2*x**3 - 4*a**3 - 24*a**2*b*x**2 -
 6*a**2*b*x - 12*a**2*c*x**2 + 12*a*b**2*x**4 + 24*a*b*c*x**4 + 12*a*b*d*x
**5 + 6*b**3*x**5 + 4*b**2*c*x**6 + 3*b**2*d*x**7)/(12*x**3)
```

# 3.28 $\int x^3(a+bx^2)^3\left(A+Bx+Cx^2+Dx^3\right)\,dx$



## Optimal result

Integrand size = 28, antiderivative size = 149

$$\begin{aligned}\int x^3(a+bx^2)^3\left(A+Bx+Cx^2+Dx^3\right)\,dx &= \frac{1}{4}a^3Ax^4+\frac{1}{5}a^3Bx^5+\frac{1}{6}a^2(3Ab+aC)x^6+\frac{1}{7}a^2(3bB+aD)x^7\\&\quad+\frac{3}{8}ab(Ab+aC)x^8+\frac{1}{3}ab(bB+aD)x^9+\frac{1}{10}b^2(Ab+3aC)x^{10}\\&\quad+\frac{1}{11}b^2(bB+3aD)x^{11}+\frac{1}{12}b^3Cx^{12}+\frac{1}{13}b^3Dx^{13}\end{aligned}$$

output
```
1/4*a^3*A*x^4+1/5*a^3*B*x^5+1/6*a^2*(3*A*b+C*a)*x^6+1/7*a^2*(3*B*b+D*a)*x^
7+3/8*a*b*(A*b+C*a)*x^8+1/3*a*b*(B*b+D*a)*x^9+1/10*b^2*(A*b+3*C*a)*x^10+1/
11*b^2*(B*b+3*D*a)*x^11+1/12*b^3*C*x^12+1/13*b^3*D*x^13
```

## Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00

$$\int x^3\left(a+bx^2\right)^3\left(A+Bx+Cx^2+Dx^3\right)\,dx =\frac{1}{4}a^3Ax^4+\frac{1}{5}a^3Bx^5+\frac{1}{6}a^2(3Ab+aC)x^6+\frac{1}{7}a^2(3bB+aD)x^7 +\frac{3}{8}ab(Ab+aC)x^8+\frac{1}{3}ab(bB+aD)x^9+\frac{1}{10}b^2(Ab+3aC)x^{10} +\frac{1}{11}b^2(bB+3aD)x^{11}+\frac{1}{12}b^3Cx^{12}+\frac{1}{13}b^3Dx^{13}$$

input
```
Integrate[x^3*(a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(a^3*A*x^4)/4 + (a^3*B*x^5)/5 + (a^2*(3*A*b + a*C)*x^6)/6 + (a^2*(3*b*B +
a*D)*x^7)/7 + (3*a*b*(A*b + a*C)*x^8)/8 + (a*b*(b*B + a*D)*x^9)/3 + (b^2*(
A*b + 3*a*C)*x^10)/10 + (b^2*(b*B + 3*a*D)*x^11)/11 + (b^3*C*x^12)/12 + (b
^3*D*x^13)/13
```

## Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(a+bx^2\right)^3\left(A+Bx+Cx^2+Dx^3\right)\,dx$$

$$\Big\downarrow \text{2333}$$

$$\int \left(a^3Ax^3+a^3Bx^4+a^2x^5(aC+3Ab)+a^2x^6(aD+3bB)+b^2x^9(3aC+Ab)+3abx^7(aC+Ab)+b^2x^{10}(3aD+\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}a^3Ax^4+\frac{1}{5}a^3Bx^5+\frac{1}{6}a^2x^6(aC+3Ab)+\frac{1}{7}a^2x^7(aD+3bB)+\frac{1}{10}b^2x^{10}(3aC+Ab)+\frac{3}{8}abx^8(aC+Ab)+\frac{1}{11}b^2x^{11}(3aD+bB)+\frac{1}{3}abx^9(aD+bB)+\frac{1}{12}b^3Cx^{12}+\frac{1}{13}b^3Dx^{13}$$

input
```
Int[x^3*(a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(a^3*A*x^4)/4 + (a^3*B*x^5)/5 + (a^2*(3*A*b + a*C)*x^6)/6 + (a^2*(3*b*B +
a*D)*x^7)/7 + (3*a*b*(A*b + a*C)*x^8)/8 + (a*b*(b*B + a*D)*x^9)/3 + (b^2*(
A*b + 3*a*C)*x^10)/10 + (b^2*(b*B + 3*a*D)*x^11)/11 + (b^3*C*x^12)/12 + (b
^3*D*x^13)/13
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

## Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.99

| method | result |
|---|---|
| norman | $\frac{b^3Dx^{13}}{13}+\frac{b^3Cx^{12}}{12}+\left(\frac{1}{11}Bb^3+\frac{3}{11}ab^2D\right)x^{11}+\left(\frac{1}{10}b^3A+\frac{3}{10}aCb^2\right)x^{10}+\left(\frac{1}{3}ab^2B+\frac{1}{3}a^2bD\right)x^9$ |
| default | $\frac{b^3Dx^{13}}{13}+\frac{b^3Cx^{12}}{12}+\frac{(Bb^3+3ab^2D)x^{11}}{11}+\frac{(b^3A+3aCb^2)x^{10}}{10}+\frac{(3ab^2B+3a^2bD)x^9}{9}+\frac{(3ab^2A+3a^2bC)x^8}{8}+$ |
| gosper | $\frac{1}{13}b^3Dx^{13}+\frac{1}{12}b^3Cx^{12}+\frac{1}{11}x^{11}Bb^3+\frac{3}{11}x^{11}ab^2D+\frac{1}{10}x^{10}b^3A+\frac{3}{10}x^{10}aCb^2+\frac{1}{3}x^9ab^2B+$ |
| parallelrisch | $\frac{1}{13}b^3Dx^{13}+\frac{1}{12}b^3Cx^{12}+\frac{1}{11}x^{11}Bb^3+\frac{3}{11}x^{11}ab^2D+\frac{1}{10}x^{10}b^3A+\frac{3}{10}x^{10}aCb^2+\frac{1}{3}x^9ab^2B+$ |
| orering | $x^4(9240b^3Dx^9+10010Cb^3x^8+10920b^3Bx^7+32760Dab^2x^7+12012Ab^3x^6+36036Cab^2x^6+40040Bab^2x^5+40040Da^2bx^5+$ / 12012 |

input
```
int(x^3*(b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
1/13*b^3*D*x^13+1/12*b^3*C*x^12+(1/11*B*b^3+3/11*a*b^2*D)*x^11+(1/10*b^3*A
+3/10*a*C*b^2)*x^10+(1/3*a*b^2*B+1/3*a^2*b*D)*x^9+(3/8*a*b^2*A+3/8*a^2*b*C
)*x^8+(3/7*a^2*b*B+1/7*a^3*D)*x^7+(1/2*a^2*b*A+1/6*C*a^3)*x^6+1/5*a^3*B*x^
5+1/4*a^3*A*x^4
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.97

$$\begin{aligned}\int x^3(a+bx^2)^3\left(A+Bx+Cx^2+Dx^3\right)\,dx &= \frac{1}{13}\,Db^3x^{13}+\frac{1}{12}\,Cb^3x^{12}+\frac{1}{11}\,\left(3\,Dab^2+Bb^3\right)x^{11}+\frac{1}{10}\,\left(3\,Cab^2+Ab^3\right)x^{10}\\ &\quad+\frac{1}{3}\,\left(Da^2b+Bab^2\right)x^9+\frac{1}{5}\,Ba^3x^5+\frac{3}{8}\,\left(Ca^2b+Aab^2\right)x^8\\ &\quad+\frac{1}{4}\,Aa^3x^4+\frac{1}{7}\,\left(Da^3+3\,Ba^2b\right)x^7+\frac{1}{6}\,\left(Ca^3+3\,Aa^2b\right)x^6\end{aligned}$$

input
```
integrate(x^3*(b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
1/13*D*b^3*x^13 + 1/12*C*b^3*x^12 + 1/11*(3*D*a*b^2 + B*b^3)*x^11 + 1/10*(
3*C*a*b^2 + A*b^3)*x^10 + 1/3*(D*a^2*b + B*a*b^2)*x^9 + 1/5*B*a^3*x^5 + 3/
8*(C*a^2*b + A*a*b^2)*x^8 + 1/4*A*a^3*x^4 + 1/7*(D*a^3 + 3*B*a^2*b)*x^7 +
1/6*(C*a^3 + 3*A*a^2*b)*x^6
```

## Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.09

$$\begin{aligned}\int x^3(a+bx^2)^3\left(A+Bx+Cx^2+Dx^3\right)\,dx &= \frac{Aa^3x^4}{4}+\frac{Ba^3x^5}{5}+\frac{Cb^3x^{12}}{12}+\frac{Db^3x^{13}}{13}+x^{11}\left(\frac{Bb^3}{11}+\frac{3Dab^2}{11}\right)\\ &\quad+x^{10}\left(\frac{Ab^3}{10}+\frac{3Cab^2}{10}\right)+x^9\left(\frac{Bab^2}{3}+\frac{Da^2b}{3}\right)+x^8\\ &\quad\cdot\left(\frac{3Aab^2}{8}+\frac{3Ca^2b}{8}\right)+x^7\cdot\left(\frac{3Ba^2b}{7}+\frac{Da^3}{7}\right)+x^6\left(\frac{Aa^2b}{2}+\frac{Ca^3}{6}\right)\end{aligned}$$

input
```
integrate(x**3*(b*x**2+a)**3*(D*x**3+C*x**2+B*x+A),x)
```

output
```
A*a**3*x**4/4 + B*a**3*x**5/5 + C*b**3*x**12/12 + D*b**3*x**13/13 + x**11*
(B*b**3/11 + 3*D*a*b**2/11) + x**10*(A*b**3/10 + 3*C*a*b**2/10) + x**9*(B*
a*b**2/3 + D*a**2*b/3) + x**8*(3*A*a*b**2/8 + 3*C*a**2*b/8) + x**7*(3*B*a*
*2*b/7 + D*a**3/7) + x**6*(A*a**2*b/2 + C*a**3/6)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.97

$$\int x^3(a+bx^2)^3(A+Bx+Cx^2+Dx^3)\,dx = \frac{1}{13}Db^3x^{13}+\frac{1}{12}Cb^3x^{12}+\frac{1}{11}(3\,Dab^2+Bb^3)x^{11}+\frac{1}{10}(3\,Cab^2+Ab^3)x^{10}+\frac{1}{3}(Da^2b+Bab^2)x^9+\frac{1}{5}Ba^3x^5+\frac{3}{8}(Ca^2b+Aab^2)x^8+\frac{1}{4}Aa^3x^4+\frac{1}{7}(Da^3+3\,Ba^2b)x^7+\frac{1}{6}(Ca^3+3\,Aa^2b)x^6$$

input
```
integrate(x^3*(b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
1/13*D*b^3*x^13 + 1/12*C*b^3*x^12 + 1/11*(3*D*a*b^2 + B*b^3)*x^11 + 1/10*(
3*C*a*b^2 + A*b^3)*x^10 + 1/3*(D*a^2*b + B*a*b^2)*x^9 + 1/5*B*a^3*x^5 + 3/
8*(C*a^2*b + A*a*b^2)*x^8 + 1/4*A*a^3*x^4 + 1/7*(D*a^3 + 3*B*a^2*b)*x^7 +
1/6*(C*a^3 + 3*A*a^2*b)*x^6
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.03

$$\int x^3(a+bx^2)^3(A+Bx+Cx^2+Dx^3)\,dx = \frac{1}{13}Db^3x^{13}+\frac{1}{12}Cb^3x^{12}+\frac{3}{11}Dab^2x^{11}+\frac{1}{11}Bb^3x^{11}+\frac{3}{10}Cab^2x^{10}+\frac{1}{10}Ab^3x^{10}+\frac{1}{3}Da^2bx^9+\frac{1}{3}Bab^2x^9+\frac{3}{8}Ca^2bx^8+\frac{3}{8}Aab^2x^8+\frac{1}{7}Da^3x^7+\frac{3}{7}Ba^2bx^7+\frac{1}{6}Ca^3x^6+\frac{1}{2}Aa^2bx^6+\frac{1}{5}Ba^3x^5+\frac{1}{4}Aa^3x^4$$

input
```
integrate(x^3*(b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

output
```
1/13*D*b^3*x^13 + 1/12*C*b^3*x^12 + 3/11*D*a*b^2*x^11 + 1/11*B*b^3*x^11 +
3/10*C*a*b^2*x^10 + 1/10*A*b^3*x^10 + 1/3*D*a^2*b*x^9 + 1/3*B*a*b^2*x^9 +
3/8*C*a^2*b*x^8 + 3/8*A*a*b^2*x^8 + 1/7*D*a^3*x^7 + 3/7*B*a^2*b*x^7 + 1/6*
C*a^3*x^6 + 1/2*A*a^2*b*x^6 + 1/5*B*a^3*x^5 + 1/4*A*a^3*x^4
```

## Mupad [B] (verification not implemented)

Time = 2.52 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.03

$$\int x^3 (a + bx^2)^3 (A + Bx + Cx^2 + Dx^3) \, dx = \frac{A\,a^3\,x^4}{4} + \frac{B\,a^3\,x^5}{5} + \frac{A\,b^3\,x^{10}}{10} + \frac{C\,a^3\,x^6}{6} + \frac{B\,b^3\,x^{11}}{11} + \frac{C\,b^3\,x^{12}}{12} + \frac{a^3\,x^7\,D}{7} + \frac{b^3\,x^{13}\,D}{13} + \frac{a^2\,b\,x^9\,D}{3} + \frac{3\,a\,b^2\,x^{11}\,D}{11} + \frac{A\,a^2\,b\,x^6}{2} + \frac{3\,A\,a\,b^2\,x^8}{8} + \frac{3\,B\,a^2\,b\,x^7}{7} + \frac{B\,a\,b^2\,x^9}{3} + \frac{3\,C\,a^2\,b\,x^8}{8} + \frac{3\,C\,a\,b^2\,x^{10}}{10}$$

input
```
int(x^3*(a + b*x^2)^3*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
(A*a^3*x^4)/4 + (B*a^3*x^5)/5 + (A*b^3*x^10)/10 + (C*a^3*x^6)/6 + (B*b^3*x
^11)/11 + (C*b^3*x^12)/12 + (a^3*x^7*D)/7 + (b^3*x^13*D)/13 + (a^2*b*x^9*D
)/3 + (3*a*b^2*x^11*D)/11 + (A*a^2*b*x^6)/2 + (3*A*a*b^2*x^8)/8 + (3*B*a^2
*b*x^7)/7 + (B*a*b^2*x^9)/3 + (3*C*a^2*b*x^8)/8 + (3*C*a*b^2*x^10)/10
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.01

$$\int x^3 (a + bx^2)^3 (A + Bx + Cx^2 + Dx^3) \, dx = \frac{x^4(9240b^3d\,x^9 + 10010b^3c\,x^8 + 32760a\,b^2d\,x^7 + 10920b^4x^7 + 12012a\,b^3x^6 + 36036a\,b^2c\,x^6 + 40040a^2bd\,x}{}$$

input
```
int(x^3*(b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x)
```

output
```
(x**4*(30030*a**4 + 60060*a**3*b*x**2 + 24024*a**3*b*x + 20020*a**3*c*x**2
 + 17160*a**3*d*x**3 + 45045*a**2*b**2*x**4 + 51480*a**2*b**2*x**3 + 45045
*a**2*b*c*x**4 + 40040*a**2*b*d*x**5 + 12012*a*b**3*x**6 + 40040*a*b**3*x*
*5 + 36036*a*b**2*c*x**6 + 32760*a*b**2*d*x**7 + 10920*b**4*x**7 + 10010*b
**3*c*x**8 + 9240*b**3*d*x**9))/120120
```

# 3.29 $\int x^2(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)\, dx$



## Optimal result

Integrand size = 28, antiderivative size = 149

$$\int x^2(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)\, dx = \frac{1}{3}a^3Ax^3+\frac{1}{4}a^3Bx^4+\frac{1}{5}a^2(3Ab+aC)x^5+\frac{1}{6}a^2(3bB+aD)x^6+\frac{3}{7}ab(Ab+aC)x^7 + \frac{3}{8}ab(bB+aD)x^8+\frac{1}{9}b^2(Ab+3aC)x^9+\frac{1}{10}b^2(bB+3aD)x^{10}+\frac{1}{11}b^3Cx^{11}+\frac{1}{12}b^3Dx^{12}$$

output
```
1/3*a^3*A*x^3+1/4*a^3*B*x^4+1/5*a^2*(3*A*b+C*a)*x^5+1/6*a^2*(3*B*b+D*a)*x^
6+3/7*a*b*(A*b+C*a)*x^7+3/8*a*b*(B*b+D*a)*x^8+1/9*b^2*(A*b+3*C*a)*x^9+1/10
*b^2*(B*b+3*D*a)*x^10+1/11*b^3*C*x^11+1/12*b^3*D*x^12
```

## Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.84

$$\int x^2(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)\, dx = \frac{14b^3x^9(220A+3x(66B+60Cx+55Dx^2))+462a^3x^3(20A+x(15B+2x(6C+5Dx)))+99a^2bx^5(168}{27720}$$

input
```
Integrate[x^2*(a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(14*b^3*x^9*(220*A + 3*x*(66*B + 60*C*x + 55*D*x^2)) + 462*a^3*x^3*(20*A +
 x*(15*B + 2*x*(6*C + 5*D*x))) + 99*a^2*b*x^5*(168*A + 5*x*(28*B + 3*x*(8*
C + 7*D*x))) + 33*a*b^2*x^7*(360*A + 7*x*(45*B + 4*x*(10*C + 9*D*x))))/277
20
```

## Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+bx^2\right)^3 \left(A+Bx+Cx^2+Dx^3\right) \, dx$$

$$\Big\downarrow \; 2333$$

$$\int \left(a^3Ax^2+a^3Bx^3+a^2x^4(aC+3Ab)+a^2x^5(aD+3bB)+b^2x^8(3aC+Ab)+3abx^6(aC+Ab)+b^2x^9(3aD+\right.$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{3}a^3Ax^3+\frac{1}{4}a^3Bx^4+\frac{1}{5}a^2x^5(aC+3Ab)+\frac{1}{6}a^2x^6(aD+3bB)+\frac{1}{9}b^2x^9(3aC+Ab)+\frac{3}{7}abx^7(aC+Ab)+\frac{1}{10}b^2x^{10}(3aD+bB)+\frac{3}{8}abx^8(aD+bB)+\frac{1}{11}b^3Cx^{11}+\frac{1}{12}b^3Dx^{12}$$

input
```
Int[x^2*(a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(a^3*A*x^3)/3 + (a^3*B*x^4)/4 + (a^2*(3*A*b + a*C)*x^5)/5 + (a^2*(3*b*B +
a*D)*x^6)/6 + (3*a*b*(A*b + a*C)*x^7)/7 + (3*a*b*(b*B + a*D)*x^8)/8 + (b^2
*(A*b + 3*a*C)*x^9)/9 + (b^2*(b*B + 3*a*D)*x^10)/10 + (b^3*C*x^11)/11 + (b
^3*D*x^12)/12
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

## Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.99

| method | result |
|---|---|
| norman | $\frac{b^3 D x^{12}}{12}+\frac{b^3 C x^{11}}{11}+\left(\frac{1}{10} B b^3+\frac{3}{10} a b^2 D\right) x^{10}+\left(\frac{1}{9} b^3 A+\frac{1}{3} a C b^2\right) x^9+\left(\frac{3}{8} a b^2 B+\frac{3}{8} a^2 b D\right) x^8+$ |
| default | $\frac{b^3 D x^{12}}{12}+\frac{b^3 C x^{11}}{11}+\frac{\left(B b^3+3 a b^2 D\right) x^{10}}{10}+\frac{\left(b^3 A+3 a C b^2\right) x^9}{9}+\frac{\left(3 a b^2 B+3 a^2 b D\right) x^8}{8}+\frac{\left(3 a b^2 A+3 a^2 b C\right) x^7}{7}+\frac{(3a^2$ |
| gosper | $\frac{1}{12} b^3 D x^{12}+\frac{1}{11} b^3 C x^{11}+\frac{1}{10} x^{10} B b^3+\frac{3}{10} x^{10} a b^2 D+\frac{1}{9} x^9 b^3 A+\frac{1}{3} x^9 a C b^2+\frac{3}{8} x^8 a b^2 B+\frac{3}{8} D a$ |
| parallelrisch | $\frac{1}{12} b^3 D x^{12}+\frac{1}{11} b^3 C x^{11}+\frac{1}{10} x^{10} B b^3+\frac{3}{10} x^{10} a b^2 D+\frac{1}{9} x^9 b^3 A+\frac{1}{3} x^9 a C b^2+\frac{3}{8} x^8 a b^2 B+\frac{3}{8} D a$ |
| orering | $\frac{x^3 (2310 b^3 D x^9+2520 C b^3 x^8+2772 b^3 B x^7+8316 D a b^2 x^7+3080 A b^3 x^6+9240 C a b^2 x^6+10395 B a b^2 x^5+10395 D a^2 b x^5+11880}{27720}$ |

input
```
int(x^2*(b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
1/12*b^3*D*x^12+1/11*b^3*C*x^11+(1/10*B*b^3+3/10*a*b^2*D)*x^10+(1/9*b^3*A+
1/3*a*C*b^2)*x^9+(3/8*a*b^2*B+3/8*a^2*b*D)*x^8+(3/7*a*b^2*A+3/7*a^2*b*C)*x
^7+(1/2*a^2*b*B+1/6*a^3*D)*x^6+(3/5*a^2*b*A+1/5*C*a^3)*x^5+1/4*a^3*B*x^4+1
/3*a^3*A*x^3
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.97

$$\int x^2(a+bx^2)^3(A+Bx+Cx^2+Dx^3)\,dx = \frac{1}{12}Db^3x^{12}+\frac{1}{11}Cb^3x^{11}+\frac{1}{10}(3Dab^2+Bb^3)x^{10}+\frac{1}{9}(3Cab^2+Ab^3)x^9+\frac{3}{8}(Da^2b+Bab^2)x^8+\frac{1}{4}Ba^3x^4+\frac{3}{7}(Ca^2b+Aab^2)x^7+\frac{1}{3}Aa^3x^3+\frac{1}{6}(Da^3+3Ba^2b)x^6+\frac{1}{5}(Ca^3+3Aa^2b)x^5$$

input
```
integrate(x^2*(b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
1/12*D*b^3*x^12 + 1/11*C*b^3*x^11 + 1/10*(3*D*a*b^2 + B*b^3)*x^10 + 1/9*(3
*C*a*b^2 + A*b^3)*x^9 + 3/8*(D*a^2*b + B*a*b^2)*x^8 + 1/4*B*a^3*x^4 + 3/7*
(C*a^2*b + A*a*b^2)*x^7 + 1/3*A*a^3*x^3 + 1/6*(D*a^3 + 3*B*a^2*b)*x^6 + 1/
5*(C*a^3 + 3*A*a^2*b)*x^5
```

## Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.11

$$\int x^2(a+bx^2)^3(A+Bx+Cx^2+Dx^3)\,dx = \frac{Aa^3x^3}{3}+\frac{Ba^3x^4}{4}+\frac{Cb^3x^{11}}{11}+\frac{Db^3x^{12}}{12}+x^{10}\left(\frac{Bb^3}{10}+\frac{3Dab^2}{10}\right)+x^9\left(\frac{Ab^3}{9}+\frac{Cab^2}{3}\right)+x^8\cdot\left(\frac{3Bab^2}{8}+\frac{3Da^2b}{8}\right)+x^7\cdot\left(\frac{3Aab^2}{7}+\frac{3Ca^2b}{7}\right)+x^6\left(\frac{Ba^2b}{2}+\frac{Da^3}{6}\right)+x^5\cdot\left(\frac{3Aa^2b}{5}+\frac{Ca^3}{5}\right)$$

input
```
integrate(x**2*(b*x**2+a)**3*(D*x**3+C*x**2+B*x+A),x)
```

output
```
A*a**3*x**3/3 + B*a**3*x**4/4 + C*b**3*x**11/11 + D*b**3*x**12/12 + x**10*
(B*b**3/10 + 3*D*a*b**2/10) + x**9*(A*b**3/9 + C*a*b**2/3) + x**8*(3*B*a*b
**2/8 + 3*D*a**2*b/8) + x**7*(3*A*a*b**2/7 + 3*C*a**2*b/7) + x**6*(B*a**2*
b/2 + D*a**3/6) + x**5*(3*A*a**2*b/5 + C*a**3/5)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.97

$$\begin{aligned}\int x^2\left(a+bx^2\right)^3\left(A+Bx+Cx^2+Dx^3\right)\,dx &= \frac{1}{12}\,Db^3x^{12}+\frac{1}{11}\,Cb^3x^{11}+\frac{1}{10}\left(3\,Dab^2+Bb^3\right)x^{10}+\frac{1}{9}\left(3\,Cab^2+Ab^3\right)x^9\\&\quad+\frac{3}{8}\left(Da^2b+Bab^2\right)x^8+\frac{1}{4}\,Ba^3x^4+\frac{3}{7}\left(Ca^2b+Aab^2\right)x^7\\&\quad+\frac{1}{3}\,Aa^3x^3+\frac{1}{6}\left(Da^3+3\,Ba^2b\right)x^6+\frac{1}{5}\left(Ca^3+3\,Aa^2b\right)x^5\end{aligned}$$

input
```
integrate(x^2*(b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
1/12*D*b^3*x^12 + 1/11*C*b^3*x^11 + 1/10*(3*D*a*b^2 + B*b^3)*x^10 + 1/9*(3
*C*a*b^2 + A*b^3)*x^9 + 3/8*(D*a^2*b + B*a*b^2)*x^8 + 1/4*B*a^3*x^4 + 3/7*
(C*a^2*b + A*a*b^2)*x^7 + 1/3*A*a^3*x^3 + 1/6*(D*a^3 + 3*B*a^2*b)*x^6 + 1/
5*(C*a^3 + 3*A*a^2*b)*x^5
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.03

$$\begin{aligned}\int x^2\left(a+bx^2\right)^3\left(A+Bx+Cx^2+Dx^3\right)\,dx &= \frac{1}{12}\,Db^3x^{12}+\frac{1}{11}\,Cb^3x^{11}+\frac{3}{10}\,Dab^2x^{10}+\frac{1}{10}\,Bb^3x^{10}+\frac{1}{3}\,Cab^2x^9\\&\quad+\frac{1}{9}\,Ab^3x^9+\frac{3}{8}\,Da^2bx^8+\frac{3}{8}\,Bab^2x^8+\frac{3}{7}\,Ca^2bx^7+\frac{3}{7}\,Aab^2x^7\\&\quad+\frac{1}{6}\,Da^3x^6+\frac{1}{2}\,Ba^2bx^6+\frac{1}{5}\,Ca^3x^5+\frac{3}{5}\,Aa^2bx^5+\frac{1}{4}\,Ba^3x^4+\frac{1}{3}\,Aa^3x^3\end{aligned}$$

input
```
integrate(x^2*(b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

output
```
1/12*D*b^3*x^12 + 1/11*C*b^3*x^11 + 3/10*D*a*b^2*x^10 + 1/10*B*b^3*x^10 +
1/3*C*a*b^2*x^9 + 1/9*A*b^3*x^9 + 3/8*D*a^2*b*x^8 + 3/8*B*a*b^2*x^8 + 3/7*
C*a^2*b*x^7 + 3/7*A*a*b^2*x^7 + 1/6*D*a^3*x^6 + 1/2*B*a^2*b*x^6 + 1/5*C*a^
3*x^5 + 3/5*A*a^2*b*x^5 + 1/4*B*a^3*x^4 + 1/3*A*a^3*x^3
```

## Mupad [B] (verification not implemented)

Time = 2.54 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.03

$$\int x^2(a+bx^2)^3(A+Bx+Cx^2+Dx^3)\,dx = \frac{A\,a^3\,x^3}{3}+\frac{B\,a^3\,x^4}{4}+\frac{A\,b^3\,x^9}{9}+\frac{C\,a^3\,x^5}{5}+\frac{B\,b^3\,x^{10}}{10}+\frac{C\,b^3\,x^{11}}{11}+\frac{a^3\,x^6\,D}{6}+\frac{b^3\,x^{12}\,D}{12}+\frac{3\,a^2\,b\,x^8\,D}{8}+\frac{3\,a\,b^2\,x^{10}\,D}{10}+\frac{3\,A\,a^2\,b\,x^5}{5}+\frac{3\,A\,a\,b^2\,x^7}{7}+\frac{B\,a^2\,b\,x^6}{2}+\frac{3\,B\,a\,b^2\,x^8}{8}+\frac{3\,C\,a^2\,b\,x^7}{7}+\frac{C\,a\,b^2\,x^9}{3}$$

input
```
int(x^2*(a + b*x^2)^3*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
(A*a^3*x^3)/3 + (B*a^3*x^4)/4 + (A*b^3*x^9)/9 + (C*a^3*x^5)/5 + (B*b^3*x^1
0)/10 + (C*b^3*x^11)/11 + (a^3*x^6*D)/6 + (b^3*x^12*D)/12 + (3*a^2*b*x^8*D
)/8 + (3*a*b^2*x^10*D)/10 + (3*A*a^2*b*x^5)/5 + (3*A*a*b^2*x^7)/7 + (B*a^2
*b*x^6)/2 + (3*B*a*b^2*x^8)/8 + (3*C*a^2*b*x^7)/7 + (C*a*b^2*x^9)/3
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.01

$$\int x^2(a+bx^2)^3(A+Bx+Cx^2+Dx^3)\,dx = \frac{x^3(2310b^3d\,x^9+2520b^3c\,x^8+8316a\,b^2d\,x^7+2772b^4x^7+3080a\,b^3x^6+9240a\,b^2c\,x^6+10395a^2bd\,x^5+10}{}$$

input
```
int(x^2*(b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x)
```

output
```
(x**3*(9240*a**4 + 16632*a**3*b*x**2 + 6930*a**3*b*x + 5544*a**3*c*x**2 +
4620*a**3*d*x**3 + 11880*a**2*b**2*x**4 + 13860*a**2*b**2*x**3 + 11880*a**
2*b*c*x**4 + 10395*a**2*b*d*x**5 + 3080*a*b**3*x**6 + 10395*a*b**3*x**5 +
9240*a*b**2*c*x**6 + 8316*a*b**2*d*x**7 + 2772*b**4*x**7 + 2520*b**3*c*x**
8 + 2310*b**3*d*x**9))/27720
```

# 3.30 $\int x(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 138

$$\begin{aligned}\int x(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)\,dx &= \frac{1}{3}a^3Bx^3+\frac{1}{4}a^3Cx^4+\frac{1}{5}a^2(3bB+aD)x^5 \\ &\quad +\frac{1}{2}a^2bCx^6+\frac{3}{7}ab(bB+aD)x^7 \\ &\quad +\frac{3}{8}ab^2Cx^8+\frac{1}{9}b^2(bB+3aD)x^9 \\ &\quad +\frac{1}{10}b^3Cx^{10}+\frac{1}{11}b^3Dx^{11}+\frac{A(a+bx^2)^4}{8b}\end{aligned}$$

output
```
1/3*a^3*B*x^3+1/4*a^3*C*x^4+1/5*a^2*(3*B*b+D*a)*x^5+1/2*a^2*b*C*x^6+3/7*a*
b*(B*b+D*a)*x^7+3/8*a*b^2*C*x^8+1/9*b^2*(B*b+3*D*a)*x^9+1/10*b^3*C*x^10+1/
11*b^3*D*x^11+1/8*A*(b*x^2+a)^4/b
```

## Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.90

$$\int x\left(a+bx^2\right)^3\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$=\frac{7b^3x^8(495A+4x(110B+99Cx+90Dx^2))+462a^3x^2(30A+x(20B+3x(5C+4Dx)))+198a^2bx^4(10}{27720}$$

input
```
Integrate[x*(a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(7*b^3*x^8*(495*A + 4*x*(110*B + 99*C*x + 90*D*x^2)) + 462*a^3*x^2*(30*A +
 x*(20*B + 3*x*(5*C + 4*D*x))) + 198*a^2*b*x^4*(105*A + 2*x*(42*B + 5*x*(7
*C + 6*D*x))) + 165*a*b^2*x^6*(84*A + x*(72*B + 7*x*(9*C + 8*D*x))))/27720
```

## Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2017, 2341, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+bx^2\right)^3\left(A+Bx+Cx^2+Dx^3\right)\,dx$$

$$\Big\downarrow \text{2017}$$

$$\int \left(bx^2+a\right)^3\left(x\left(Dx^3+Cx^2+Bx+A\right)-Ax\right)dx+\frac{A\left(a+bx^2\right)^4}{8b}$$

$$\Big\downarrow \text{2341}$$

$$\int \left(b^3Dx^{10}+b^3Cx^9+b^2(bB+3aD)x^8+3ab^2Cx^7+3ab(bB+aD)x^6+3a^2bCx^5+a^2(3bB+aD)x^4+a^3Cx^3\right.$$
$$\frac{A\left(a+bx^2\right)^4}{8b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}a^3Bx^3 + \frac{1}{4}a^3Cx^4 + \frac{1}{5}a^2x^5(aD + 3bB) + \frac{1}{2}a^2bCx^6 + \frac{A(a + bx^2)^4}{8b} + \frac{1}{9}b^2x^9(3aD + bB) + \frac{3}{8}ab^2Cx^8 + \frac{3}{7}abx^7(aD + bB) + \frac{1}{10}b^3Cx^{10} + \frac{1}{11}b^3Dx^{11}$$

input
```
Int[x*(a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(a^3*B*x^3)/3 + (a^3*C*x^4)/4 + (a^2*(3*b*B + a*D)*x^5)/5 + (a^2*b*C*x^6)/
2 + (3*a*b*(b*B + a*D)*x^7)/7 + (3*a*b^2*C*x^8)/8 + (b^2*(b*B + 3*a*D)*x^9
)/9 + (b^3*C*x^10)/10 + (b^3*D*x^11)/11 + (A*(a + b*x^2)^4)/(8*b)
```

**Defintions of rubi rules used**

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2017
```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

rule 2341
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

## Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.07

| method | result |
|---|---|
| norman | $\frac{b^3Dx^{11}}{11}+\frac{b^3C\,x^{10}}{10}+\left(\frac{1}{9}B\,b^3+\frac{1}{3}a\,b^2D\right)x^9+\left(\frac{1}{8}b^3A+\frac{3}{8}aC\,b^2\right)x^8+\left(\frac{3}{7}a\,b^2B+\frac{3}{7}a^2bD\right)x^7+\left(\right.$ |
| default | $\frac{b^3Dx^{11}}{11}+\frac{b^3C\,x^{10}}{10}+\frac{(B\,b^3+3a\,b^2D)x^9}{9}+\frac{(b^3A+3aC\,b^2)x^8}{8}+\frac{(3a\,b^2B+3a^2bD)x^7}{7}+\frac{(3a\,b^2A+3a^2bC)x^6}{6}+$ |
| gosper | $\frac{1}{11}b^3Dx^{11}+\frac{1}{10}b^3C\,x^{10}+\frac{1}{9}b^3B\,x^9+\frac{1}{3}x^9a\,b^2D+\frac{1}{8}x^8b^3A+\frac{3}{8}a\,b^2C\,x^8+\frac{3}{7}x^7a\,b^2B+\frac{3}{7}x^7a^2b$ |
| parallelrisch | $\frac{1}{11}b^3Dx^{11}+\frac{1}{10}b^3C\,x^{10}+\frac{1}{9}b^3B\,x^9+\frac{1}{3}x^9a\,b^2D+\frac{1}{8}x^8b^3A+\frac{3}{8}a\,b^2C\,x^8+\frac{3}{7}x^7a\,b^2B+\frac{3}{7}x^7a^2b$ |
| orering | $\frac{x^2(2520b^3Dx^9+2772C\,b^3x^8+3080b^3B\,x^7+9240Da\,b^2x^7+3465A\,b^3x^6+10395Ca\,b^2x^6+11880Ba\,b^2x^5+11880Da^2b\,x^5+1386}{27720}$ |

input
```
int(x*(b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
1/11*b^3*D*x^11+1/10*b^3*C*x^10+(1/9*B*b^3+1/3*a*b^2*D)*x^9+(1/8*b^3*A+3/8
*a*C*b^2)*x^8+(3/7*a*b^2*B+3/7*a^2*b*D)*x^7+(1/2*a*b^2*A+1/2*a^2*b*C)*x^6+
(3/5*a^2*b*B+1/5*a^3*D)*x^5+(3/4*a^2*b*A+1/4*C*a^3)*x^4+1/3*x^3*a^3*B+1/2*
a^3*A*x^2
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.05

$$
\begin{aligned}
&\int x(a+bx^2)^3\left(A+Bx+Cx^2+Dx^3\right)\,dx \\
&=\frac{1}{11}\,Db^3x^{11}+\frac{1}{10}\,Cb^3x^{10}+\frac{1}{9}\left(3\,Dab^2+Bb^3\right)x^9+\frac{1}{8}\left(3\,Cab^2+Ab^3\right)x^8 \\
&\quad+\frac{3}{7}\left(Da^2b+Bab^2\right)x^7+\frac{1}{3}\,Ba^3x^3+\frac{1}{2}\left(Ca^2b+Aab^2\right)x^6 \\
&\quad+\frac{1}{2}\,Aa^3x^2+\frac{1}{5}\left(Da^3+3\,Ba^2b\right)x^5+\frac{1}{4}\left(Ca^3+3\,Aa^2b\right)x^4
\end{aligned}
$$

input
```
integrate(x*(b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
1/11*D*b^3*x^11 + 1/10*C*b^3*x^10 + 1/9*(3*D*a*b^2 + B*b^3)*x^9 + 1/8*(3*C
*a*b^2 + A*b^3)*x^8 + 3/7*(D*a^2*b + B*a*b^2)*x^7 + 1/3*B*a^3*x^3 + 1/2*(C
*a^2*b + A*a*b^2)*x^6 + 1/2*A*a^3*x^2 + 1/5*(D*a^3 + 3*B*a^2*b)*x^5 + 1/4*
(C*a^3 + 3*A*a^2*b)*x^4
```

## Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.18

$$\int x(a+bx^2)^3 \left(A+Bx+Cx^2+Dx^3\right) dx = \frac{Aa^3x^2}{2} + \frac{Ba^3x^3}{3} + \frac{Cb^3x^{10}}{10} + \frac{Db^3x^{11}}{11} + x^9\left(\frac{Bb^3}{9} + \frac{Dab^2}{3}\right) + x^8\left(\frac{Ab^3}{8} + \frac{3Cab^2}{8}\right) + x^7 \cdot \left(\frac{3Bab^2}{7} + \frac{3Da^2b}{7}\right) + x^6\left(\frac{Aab^2}{2} + \frac{Ca^2b}{2}\right) + x^5 \cdot \left(\frac{3Ba^2b}{5} + \frac{Da^3}{5}\right) + x^4 \cdot \left(\frac{3Aa^2b}{4} + \frac{Ca^3}{4}\right)$$

input
```
integrate(x*(b*x**2+a)**3*(D*x**3+C*x**2+B*x+A),x)
```

output
```
A*a**3*x**2/2 + B*a**3*x**3/3 + C*b**3*x**10/10 + D*b**3*x**11/11 + x**9*(
B*b**3/9 + D*a*b**2/3) + x**8*(A*b**3/8 + 3*C*a*b**2/8) + x**7*(3*B*a*b**2
/7 + 3*D*a**2*b/7) + x**6*(A*a*b**2/2 + C*a**2*b/2) + x**5*(3*B*a**2*b/5 +
 D*a**3/5) + x**4*(3*A*a**2*b/4 + C*a**3/4)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.05

$$\int x(a+bx^2)^3 \left(A+Bx+Cx^2+Dx^3\right) dx = \frac{1}{11}\,Db^3x^{11} + \frac{1}{10}\,Cb^3x^{10} + \frac{1}{9}\,(3\,Dab^2+Bb^3)x^9 + \frac{1}{8}\,(3\,Cab^2+Ab^3)x^8 + \frac{3}{7}\,(Da^2b+Bab^2)x^7 + \frac{1}{3}\,Ba^3x^3 + \frac{1}{2}\,(Ca^2b+Aab^2)x^6 + \frac{1}{2}\,Aa^3x^2 + \frac{1}{5}\,(Da^3+3\,Ba^2b)x^5 + \frac{1}{4}\,(Ca^3+3\,Aa^2b)x^4$$

input
```
integrate(x*(b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
1/11*D*b^3*x^11 + 1/10*C*b^3*x^10 + 1/9*(3*D*a*b^2 + B*b^3)*x^9 + 1/8*(3*C
*a*b^2 + A*b^3)*x^8 + 3/7*(D*a^2*b + B*a*b^2)*x^7 + 1/3*B*a^3*x^3 + 1/2*(C
*a^2*b + A*a*b^2)*x^6 + 1/2*A*a^3*x^2 + 1/5*(D*a^3 + 3*B*a^2*b)*x^5 + 1/4*
(C*a^3 + 3*A*a^2*b)*x^4
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.11

$$\int x(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)\, dx = \frac{1}{11}Db^3x^{11}+\frac{1}{10}Cb^3x^{10}+\frac{1}{3}Dab^2x^9+\frac{1}{9}Bb^3x^9 + \frac{3}{8}Cab^2x^8+\frac{1}{8}Ab^3x^8+\frac{3}{7}Da^2bx^7 + \frac{3}{7}Bab^2x^7+\frac{1}{2}Ca^2bx^6+\frac{1}{2}Aab^2x^6 + \frac{1}{5}Da^3x^5+\frac{3}{5}Ba^2bx^5+\frac{1}{4}Ca^3x^4 + \frac{3}{4}Aa^2bx^4+\frac{1}{3}Ba^3x^3+\frac{1}{2}Aa^3x^2$$

input
```
integrate(x*(b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

output
```
1/11*D*b^3*x^11 + 1/10*C*b^3*x^10 + 1/3*D*a*b^2*x^9 + 1/9*B*b^3*x^9 + 3/8*
C*a*b^2*x^8 + 1/8*A*b^3*x^8 + 3/7*D*a^2*b*x^7 + 3/7*B*a*b^2*x^7 + 1/2*C*a^
2*b*x^6 + 1/2*A*a*b^2*x^6 + 1/5*D*a^3*x^5 + 3/5*B*a^2*b*x^5 + 1/4*C*a^3*x^
4 + 3/4*A*a^2*b*x^4 + 1/3*B*a^3*x^3 + 1/2*A*a^3*x^2
```

## Mupad [B] (verification not implemented)

Time = 2.56 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.11

$$\int x(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)\,dx = \frac{A\,a^3\,x^2}{2}+\frac{B\,a^3\,x^3}{3}+\frac{A\,b^3\,x^8}{8}+\frac{C\,a^3\,x^4}{4} + \frac{B\,b^3\,x^9}{9}+\frac{C\,b^3\,x^{10}}{10}+\frac{a^3\,x^5\,D}{5} + \frac{b^3\,x^{11}\,D}{11}+\frac{3\,a^2\,b\,x^7\,D}{7}+\frac{a\,b^2\,x^9\,D}{3} + \frac{3\,A\,a^2\,b\,x^4}{4}+\frac{A\,a\,b^2\,x^6}{2}+\frac{3\,B\,a^2\,b\,x^5}{5} + \frac{3\,B\,a\,b^2\,x^7}{7}+\frac{C\,a^2\,b\,x^6}{2}+\frac{3\,C\,a\,b^2\,x^8}{8}$$

input
```
int(x*(a + b*x^2)^3*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
(A*a^3*x^2)/2 + (B*a^3*x^3)/3 + (A*b^3*x^8)/8 + (C*a^3*x^4)/4 + (B*b^3*x^9
)/9 + (C*b^3*x^10)/10 + (a^3*x^5*D)/5 + (b^3*x^11*D)/11 + (3*a^2*b*x^7*D)/
7 + (a*b^2*x^9*D)/3 + (3*A*a^2*b*x^4)/4 + (A*a*b^2*x^6)/2 + (3*B*a^2*b*x^5
)/5 + (3*B*a*b^2*x^7)/7 + (C*a^2*b*x^6)/2 + (3*C*a*b^2*x^8)/8
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.09

$$\int x(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)\,dx = \frac{x^2(2520b^3d\,x^9+2772b^3c\,x^8+9240a\,b^2d\,x^7+3080b^4x^7+3465a\,b^3x^6+10395a\,b^2c\,x^6+11880a^2bd\,x^5+1}{}$$

input
```
int(x*(b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x)
```

output
```
(x**2*(13860*a**4 + 20790*a**3*b*x**2 + 9240*a**3*b*x + 6930*a**3*c*x**2 +
 5544*a**3*d*x**3 + 13860*a**2*b**2*x**4 + 16632*a**2*b**2*x**3 + 13860*a*
*2*b*c*x**4 + 11880*a**2*b*d*x**5 + 3465*a*b**3*x**6 + 11880*a*b**3*x**5 +
 10395*a*b**2*c*x**6 + 9240*a*b**2*d*x**7 + 3080*b**4*x**7 + 2772*b**3*c*x
**8 + 2520*b**3*d*x**9))/27720
```

# 3.31 $\int (a+bx^2)^3 (A+Bx+Cx^2+Dx^3)\, dx$



## Optimal result

Integrand size = 25, antiderivative size = 133

$$\int (a+bx^2)^3 (A+Bx+Cx^2+Dx^3)\, dx = a^3Ax+\frac{1}{3}a^2(3Ab+aC)x^3+\frac{1}{4}a^3Dx^4 +\frac{3}{5}ab(Ab+aC)x^5+\frac{1}{2}a^2bDx^6 +\frac{1}{7}b^2(Ab+3aC)x^7+\frac{3}{8}ab^2Dx^8 +\frac{1}{9}b^3Cx^9+\frac{1}{10}b^3Dx^{10}+\frac{B(a+bx^2)^4}{8b}$$

output
```
a^3*A*x+1/3*a^2*(3*A*b+C*a)*x^3+1/4*a^3*D*x^4+3/5*a*b*(A*b+C*a)*x^5+1/2*a^
2*b*D*x^6+1/7*b^2*(A*b+3*C*a)*x^7+3/8*a*b^2*D*x^8+1/9*b^3*C*x^9+1/10*b^3*D
*x^10+1/8*B*(b*x^2+a)^4/b
```

## Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.91

$$\int \left(a+bx^2\right)^3 \left(A+Bx+Cx^2+Dx^3\right) \, dx$$
$$= \frac{210a^3x(12A+x(6B+x(4C+3Dx)))+126a^2bx^3(20A+x(15B+2x(6C+5Dx)))+9ab^2x^5(168A+}{2520}$$

input
```
Integrate[(a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(210*a^3*x*(12*A + x*(6*B + x*(4*C + 3*D*x))) + 126*a^2*b*x^3*(20*A + x*(1
5*B + 2*x*(6*C + 5*D*x))) + 9*a*b^2*x^5*(168*A + 5*x*(28*B + 3*x*(8*C + 7*
D*x))) + b^3*x^7*(360*A + 7*x*(45*B + 4*x*(10*C + 9*D*x))))/2520
```

## Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2017, 2341, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^3 \left(A+Bx+Cx^2+Dx^3\right) \, dx$$
$$\Big\downarrow \; 2017$$
$$\int \left(bx^2+a\right)^3 \left(Dx^3+Cx^2+A\right) dx+\frac{B\left(a+bx^2\right)^4}{8b}$$
$$\Big\downarrow \; 2341$$
$$\int \left(b^3Dx^9+b^3Cx^8+3ab^2Dx^7+b^2(Ab+3aC)x^6+3a^2bDx^5+3ab(Ab+aC)x^4+a^3Dx^3+a^2(3Ab+aC)x^2+\right.$$
$$\frac{B\left(a+bx^2\right)^4}{8b}$$
$$\Big\downarrow \; 2009$$

$$a^3Ax+\frac{1}{4}a^3Dx^4+\frac{1}{3}a^2x^3(aC+3Ab)+\frac{1}{2}a^2bDx^6+\frac{1}{7}b^2x^7(3aC+Ab)+\frac{3}{5}abx^5(aC+Ab)+\frac{3}{8}ab^2Dx^8+\frac{B(a+bx^2)^4}{8b}+\frac{1}{9}b^3Cx^9+\frac{1}{10}b^3Dx^{10}$$

input
```
Int[(a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
a^3*A*x + (a^2*(3*A*b + a*C)*x^3)/3 + (a^3*D*x^4)/4 + (3*a*b*(A*b + a*C)*x
^5)/5 + (a^2*b*D*x^6)/2 + (b^2*(A*b + 3*a*C)*x^7)/7 + (3*a*b^2*D*x^8)/8 +
(b^3*C*x^9)/9 + (b^3*D*x^10)/10 + (B*(a + b*x^2)^4)/(8*b)
```

**Defintions of rubi rules used**

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2017
```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

rule 2341
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

## Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.08

| method | result |
|---|---|
| norman | $\frac{b^3 D x^{10}}{10}+\frac{b^3 C x^9}{9}+\left(\frac{1}{8} B b^3+\frac{3}{8} a b^2 D\right) x^8+\left(\frac{1}{7} b^3 A+\frac{3}{7} a C b^2\right) x^7+\left(\frac{1}{2} a b^2 B+\frac{1}{2} a^2 b D\right) x^6+\left(\frac{3}{5}\right.$ |
| default | $\frac{b^3 D x^{10}}{10}+\frac{b^3 C x^9}{9}+\frac{(B b^3+3 a b^2 D) x^8}{8}+\frac{(b^3 A+3 a C b^2) x^7}{7}+\frac{(3 a b^2 B+3 a^2 b D) x^6}{6}+\frac{(3 a b^2 A+3 a^2 b C) x^5}{5}+\frac{(3 a^2 b}{}$ |
| gosper | $\frac{1}{10} b^3 D x^{10}+\frac{1}{9} b^3 C x^9+\frac{1}{8} b^3 B x^8+\frac{3}{8} a b^2 D x^8+\frac{1}{7} A b^3 x^7+\frac{3}{7} x^7 a C b^2+\frac{1}{2} B a b^2 x^6+\frac{1}{2} a^2 b D x^6$ |
| parallelrisch | $\frac{1}{10} b^3 D x^{10}+\frac{1}{9} b^3 C x^9+\frac{1}{8} b^3 B x^8+\frac{3}{8} a b^2 D x^8+\frac{1}{7} A b^3 x^7+\frac{3}{7} x^7 a C b^2+\frac{1}{2} B a b^2 x^6+\frac{1}{2} a^2 b D x^6$ |
| orering | $\frac{x(252 b^3 D x^9+280 C b^3 x^8+315 b^3 B x^7+945 D a b^2 x^7+360 A b^3 x^6+1080 C a b^2 x^6+1260 B a b^2 x^5+1260 D a^2 b x^5+1512 a A b^2 x^4+}{2520}$ |

input
```
int((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
1/10*b^3*D*x^10+1/9*b^3*C*x^9+(1/8*B*b^3+3/8*a*b^2*D)*x^8+(1/7*b^3*A+3/7*a
*C*b^2)*x^7+(1/2*a*b^2*B+1/2*a^2*b*D)*x^6+(3/5*a*b^2*A+3/5*a^2*b*C)*x^5+(3
/4*a^2*b*B+1/4*a^3*D)*x^4+(a^2*b*A+1/3*C*a^3)*x^3+1/2*B*a^3*x^2+a^3*A*x
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.07

$$\int (a+bx^2)^3 (A+Bx+Cx^2+Dx^3)\,dx = \frac{1}{10}\,Db^3x^{10}+\frac{1}{9}\,Cb^3x^9+\frac{1}{8}\,(3\,Dab^2+Bb^3)x^8 + \frac{1}{7}\,(3\,Cab^2+Ab^3)x^7+\frac{1}{2}\,(Da^2b+Bab^2)x^6 + \frac{1}{2}\,Ba^3x^2+\frac{3}{5}\,(Ca^2b+Aab^2)x^5+Aa^3x + \frac{1}{4}\,(Da^3+3\,Ba^2b)x^4+\frac{1}{3}\,(Ca^3+3\,Aa^2b)x^3$$

input
```
integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
1/10*D*b^3*x^10 + 1/9*C*b^3*x^9 + 1/8*(3*D*a*b^2 + B*b^3)*x^8 + 1/7*(3*C*a
*b^2 + A*b^3)*x^7 + 1/2*(D*a^2*b + B*a*b^2)*x^6 + 1/2*B*a^3*x^2 + 3/5*(C*a
^2*b + A*a*b^2)*x^5 + A*a^3*x + 1/4*(D*a^3 + 3*B*a^2*b)*x^4 + 1/3*(C*a^3 +
 3*A*a^2*b)*x^3
```

## Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.19

$$\int \left(a+bx^2\right)^3 \left(A+Bx+Cx^2+Dx^3\right)\,dx = Aa^3x+\frac{Ba^3x^2}{2}+\frac{Cb^3x^9}{9}+\frac{Db^3x^{10}}{10}+x^8\left(\frac{Bb^3}{8}+\frac{3Dab^2}{8}\right)+x^7\left(\frac{Ab^3}{7}+\frac{3Cab^2}{7}\right)+x^6\left(\frac{Bab^2}{2}+\frac{Da^2b}{2}\right)+x^5\cdot\left(\frac{3Aab^2}{5}+\frac{3Ca^2b}{5}\right)+x^4\cdot\left(\frac{3Ba^2b}{4}+\frac{Da^3}{4}\right)+x^3\left(Aa^2b+\frac{Ca^3}{3}\right)$$

input
```
integrate((b*x**2+a)**3*(D*x**3+C*x**2+B*x+A),x)
```

output
```
A*a**3*x + B*a**3*x**2/2 + C*b**3*x**9/9 + D*b**3*x**10/10 + x**8*(B*b**3/
8 + 3*D*a*b**2/8) + x**7*(A*b**3/7 + 3*C*a*b**2/7) + x**6*(B*a*b**2/2 + D*
a**2*b/2) + x**5*(3*A*a*b**2/5 + 3*C*a**2*b/5) + x**4*(3*B*a**2*b/4 + D*a*
*3/4) + x**3*(A*a**2*b + C*a**3/3)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.07

$$\int \left(a+bx^2\right)^3 \left(A+Bx+Cx^2+Dx^3\right)\,dx = \frac{1}{10}\,Db^3x^{10}+\frac{1}{9}\,Cb^3x^9+\frac{1}{8}\,(3\,Dab^2+Bb^3)x^8+\frac{1}{7}\,(3\,Cab^2+Ab^3)x^7+\frac{1}{2}\,(Da^2b+Bab^2)x^6+\frac{1}{2}\,Ba^3x^2+\frac{3}{5}\,(Ca^2b+Aab^2)x^5+Aa^3x+\frac{1}{4}\,(Da^3+3\,Ba^2b)x^4+\frac{1}{3}\,(Ca^3+3\,Aa^2b)x^3$$

input
```
integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
1/10*D*b^3*x^10 + 1/9*C*b^3*x^9 + 1/8*(3*D*a*b^2 + B*b^3)*x^8 + 1/7*(3*C*a
*b^2 + A*b^3)*x^7 + 1/2*(D*a^2*b + B*a*b^2)*x^6 + 1/2*B*a^3*x^2 + 3/5*(C*a
^2*b + A*a*b^2)*x^5 + A*a^3*x + 1/4*(D*a^3 + 3*B*a^2*b)*x^4 + 1/3*(C*a^3 +
 3*A*a^2*b)*x^3
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.12

$$\int (a+bx^2)^3 (A+Bx+Cx^2+Dx^3) \, dx = \frac{1}{10}Db^3x^{10} + \frac{1}{9}Cb^3x^9 + \frac{3}{8}Dab^2x^8 + \frac{1}{8}Bb^3x^8 + \frac{3}{7}Cab^2x^7 + \frac{1}{7}Ab^3x^7 + \frac{1}{2}Da^2bx^6 + \frac{1}{2}Bab^2x^6 + \frac{3}{5}Ca^2bx^5 + \frac{3}{5}Aab^2x^5 + \frac{1}{4}Da^3x^4 + \frac{3}{4}Ba^2bx^4 + \frac{1}{3}Ca^3x^3 + Aa^2bx^3 + \frac{1}{2}Ba^3x^2 + Aa^3x$$

input
```
integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

output
```
1/10*D*b^3*x^10 + 1/9*C*b^3*x^9 + 3/8*D*a*b^2*x^8 + 1/8*B*b^3*x^8 + 3/7*C*
a*b^2*x^7 + 1/7*A*b^3*x^7 + 1/2*D*a^2*b*x^6 + 1/2*B*a*b^2*x^6 + 3/5*C*a^2*
b*x^5 + 3/5*A*a*b^2*x^5 + 1/4*D*a^3*x^4 + 3/4*B*a^2*b*x^4 + 1/3*C*a^3*x^3
+ A*a^2*b*x^3 + 1/2*B*a^3*x^2 + A*a^3*x
```

## Mupad [B] (verification not implemented)

Time = 2.49 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.12

$$\int \left(a+bx^2\right)^3 \left(A+Bx+Cx^2+Dx^3\right)\,dx = \frac{B\,a^3\,x^2}{2}+\frac{A\,b^3\,x^7}{7}+\frac{C\,a^3\,x^3}{3}+\frac{B\,b^3\,x^8}{8}+\frac{C\,b^3\,x^9}{9}+\frac{a^3\,x^4\,D}{4}+\frac{b^3\,x^{10}\,D}{10}+A\,a^3\,x+\frac{a^2\,b\,x^6\,D}{2}+\frac{3\,a\,b^2\,x^8\,D}{8}+A\,a^2\,b\,x^3+\frac{3\,A\,a\,b^2\,x^5}{5}+\frac{3\,B\,a^2\,b\,x^4}{4}+\frac{B\,a\,b^2\,x^6}{2}+\frac{3\,C\,a^2\,b\,x^5}{5}+\frac{3\,C\,a\,b^2\,x^7}{7}$$

input
```
int((a + b*x^2)^3*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
(B*a^3*x^2)/2 + (A*b^3*x^7)/7 + (C*a^3*x^3)/3 + (B*b^3*x^8)/8 + (C*b^3*x^9
)/9 + (a^3*x^4*D)/4 + (b^3*x^10*D)/10 + A*a^3*x + (a^2*b*x^6*D)/2 + (3*a*b
^2*x^8*D)/8 + A*a^2*b*x^3 + (3*A*a*b^2*x^5)/5 + (3*B*a^2*b*x^4)/4 + (B*a*b
^2*x^6)/2 + (3*C*a^2*b*x^5)/5 + (3*C*a*b^2*x^7)/7
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.12

$$\int \left(a+bx^2\right)^3 \left(A+Bx+Cx^2+Dx^3\right)\,dx = \frac{x(252b^3d\,x^9+280b^3c\,x^8+945a\,b^2d\,x^7+315b^4x^7+360a\,b^3x^6+1080a\,b^2c\,x^6+1260a^2bd\,x^5+1260a\,b^3x^5}{25}$$

input
```
int((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x)
```

output
```
(x*(2520*a**4 + 2520*a**3*b*x**2 + 1260*a**3*b*x + 840*a**3*c*x**2 + 630*a
**3*d*x**3 + 1512*a**2*b**2*x**4 + 1890*a**2*b**2*x**3 + 1512*a**2*b*c*x**
4 + 1260*a**2*b*d*x**5 + 360*a*b**3*x**6 + 1260*a*b**3*x**5 + 1080*a*b**2*
c*x**6 + 945*a*b**2*d*x**7 + 315*b**4*x**7 + 280*b**3*c*x**8 + 252*b**3*d*
x**9))/2520
```

# 3.32 $\int \frac{(a+bx^2)^3(A+Bx+Cx^2+Dx^3)}{x} \, dx$



## Optimal result

Integrand size = 28, antiderivative size = 129

$$\begin{aligned}\int \frac{(a+bx^2)^3(A+Bx+Cx^2+Dx^3)}{x} \, dx = {} & a^3Bx + \frac{3}{2}a^2Abx^2 + \frac{1}{3}a^2(3bB+aD)x^3 \\ & + \frac{3}{4}aAb^2x^4 + \frac{3}{5}ab(bB+aD)x^5 \\ & + \frac{1}{6}Ab^3x^6 + \frac{1}{7}b^2(bB+3aD)x^7 \\ & + \frac{1}{9}b^3Dx^9 + \frac{C(a+bx^2)^4}{8b} + a^3A\log(x)\end{aligned}$$

output
```
a^3*B*x+3/2*a^2*A*b*x^2+1/3*a^2*(3*B*b+D*a)*x^3+3/4*a*A*b^2*x^4+3/5*a*b*(B
*b+D*a)*x^5+1/6*A*b^3*x^6+1/7*b^2*(B*b+3*D*a)*x^7+1/9*b^3*D*x^9+1/8*C*(b*x
^2+a)^4/b+a^3*A*ln(x)
```

## Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)}{x}\,dx = \frac{x(420a^3(6B+x(3C+2Dx))+126a^2bx(30A+x(20B+3x(5C+4Dx)))+18ab^2x^3(105A+2x(42B+\cdots}{2520} + a^3A\log(x)$$

input
```
Integrate[((a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3))/x,x]
```

output
```
(x*(420*a^3*(6*B + x*(3*C + 2*D*x)) + 126*a^2*b*x*(30*A + x*(20*B + 3*x*(5
*C + 4*D*x))) + 18*a*b^2*x^3*(105*A + 2*x*(42*B + 5*x*(7*C + 6*D*x))) + 5*
b^3*x^5*(84*A + x*(72*B + 7*x*(9*C + 8*D*x)))))/2520 + a^3*A*Log[x]
```

## Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {2018, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)}{x}\,dx$$

$$\Big\downarrow \text{2018}$$

$$\int \frac{(bx^2+a)^3 (Dx^3+Bx+A)}{x}dx + \frac{C(a+bx^2)^4}{8b}$$

$$\Big\downarrow \text{2333}$$

$$\int \Big(b^3Dx^8+b^2(bB+3aD)x^6+Ab^3x^5+3ab(bB+aD)x^4+3aAb^2x^3+a^2(3bB+aD)x^2+3a^2Abx+a^3B+\cdots$$

$$\frac{C(a+bx^2)^4}{8b}$$

$$\downarrow \text{2009}$$

$$a^3A\log(x)+a^3Bx+\frac{3}{2}a^2Abx^2+\frac{1}{3}a^2x^3(aD+3bB)+\frac{3}{4}aAb^2x^4+\frac{1}{7}b^2x^7(3aD+bB)+\frac{3}{5}abx^5(aD+bB)+\frac{C\left(a+bx^2\right)^4}{8b}+\frac{1}{6}Ab^3x^6+\frac{1}{9}b^3Dx^9$$

input
```
Int[((a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3))/x,x]
```

output
```
a^3*B*x + (3*a^2*A*b*x^2)/2 + (a^2*(3*b*B + a*D)*x^3)/3 + (3*a*A*b^2*x^4)/
4 + (3*a*b*(b*B + a*D)*x^5)/5 + (A*b^3*x^6)/6 + (b^2*(b*B + 3*a*D)*x^7)/7
+ (b^3*D*x^9)/9 + (C*(a + b*x^2)^4)/(8*b) + a^3*A*Log[x]
```

## Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2018
```
Int[(Px_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[Coef
f[Px, x, n - m - 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coe
ff[Px, x, n - m - 1]*x^(n - m - 1))*x^m*(a + b*x^n)^p, x] /; FreeQ[{a, b, m
, n}, x] && PolyQ[Px, x] && IGtQ[p, 1] && IGtQ[n - m, 0] && NeQ[Coeff[Px, x
, n - m - 1], 0]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

## Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.10

| method | result |
|---|---|
| norman | $\left(\frac{1}{7}B\,b^{3}+\frac{3}{7}a\,b^{2}D\right)x^{7}+\left(\frac{1}{6}b^{3}A+\frac{1}{2}aC\,b^{2}\right)x^{6}+\left(\frac{3}{4}a\,b^{2}A+\frac{3}{4}a^{2}bC\right)x^{4}+\left(\frac{3}{5}a\,b^{2}B+\frac{3}{5}a^{2}bD\right)x$ |
| default | $\frac{b^{3}Dx^{9}}{9}+\frac{C\,b^{3}x^{8}}{8}+\frac{b^{3}B\,x^{7}}{7}+\frac{3Da\,b^{2}x^{7}}{7}+\frac{A\,b^{3}x^{6}}{6}+\frac{Ca\,b^{2}x^{6}}{2}+\frac{3Ba\,b^{2}x^{5}}{5}+\frac{3Da^{2}b\,x^{5}}{5}+\frac{3aA\,b^{2}x^{4}}{4}+\frac{3C\,a^{2}b\,x}{4}$ |
| parallelrisch | $\frac{b^{3}Dx^{9}}{9}+\frac{C\,b^{3}x^{8}}{8}+\frac{b^{3}B\,x^{7}}{7}+\frac{3Da\,b^{2}x^{7}}{7}+\frac{A\,b^{3}x^{6}}{6}+\frac{Ca\,b^{2}x^{6}}{2}+\frac{3Ba\,b^{2}x^{5}}{5}+\frac{3Da^{2}b\,x^{5}}{5}+\frac{3aA\,b^{2}x^{4}}{4}+\frac{3C\,a^{2}b\,x}{4}$ |

input
```
int((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x,x,method=_RETURNVERBOSE)
```

output
```
(1/7*B*b^3+3/7*a*b^2*D)*x^7+(1/6*b^3*A+1/2*a*C*b^2)*x^6+(3/4*a*b^2*A+3/4*a
^2*b*C)*x^4+(3/5*a*b^2*B+3/5*a^2*b*D)*x^5+(3/2*a^2*b*A+1/2*C*a^3)*x^2+(a^2
*b*B+1/3*a^3*D)*x^3+B*a^3*x+1/8*C*b^3*x^8+1/9*b^3*D*x^9+a^3*A*ln(x)
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx^2)^3\,(A+Bx+Cx^2+Dx^3)}{x}\,dx = \frac{1}{9}\,Db^3x^9+\frac{1}{8}\,Cb^3x^8+\frac{1}{7}\,(3\,Dab^2+Bb^3)x^7 + \frac{1}{6}\,(3\,Cab^2+Ab^3)x^6+\frac{3}{5}\,(Da^2b+Bab^2)x^5 + Ba^3x+\frac{3}{4}\,(Ca^2b+Aab^2)x^4+Aa^3\log(x) + \frac{1}{3}\,(Da^3+3\,Ba^2b)x^3+\frac{1}{2}\,(Ca^3+3\,Aa^2b)x^2$$

input
```
integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x,x, algorithm="fricas")
```

output
```
1/9*D*b^3*x^9 + 1/8*C*b^3*x^8 + 1/7*(3*D*a*b^2 + B*b^3)*x^7 + 1/6*(3*C*a*b
^2 + A*b^3)*x^6 + 3/5*(D*a^2*b + B*a*b^2)*x^5 + B*a^3*x + 3/4*(C*a^2*b + A
*a*b^2)*x^4 + A*a^3*log(x) + 1/3*(D*a^3 + 3*B*a^2*b)*x^3 + 1/2*(C*a^3 + 3*
A*a^2*b)*x^2
```

## Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.22

$$\int \frac{(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)}{x} dx = Aa^3 \log(x) + Ba^3x + \frac{Cb^3x^8}{8} + \frac{Db^3x^9}{9} + x^7\left(\frac{Bb^3}{7} + \frac{3Dab^2}{7}\right) + x^6\left(\frac{Ab^3}{6} + \frac{Cab^2}{2}\right) + x^5 \cdot \left(\frac{3Bab^2}{5} + \frac{3Da^2b}{5}\right) + x^4 \cdot \left(\frac{3Aab^2}{4} + \frac{3Ca^2b}{4}\right) + x^3\left(Ba^2b + \frac{Da^3}{3}\right) + x^2 \cdot \left(\frac{3Aa^2b}{2} + \frac{Ca^3}{2}\right)$$

input
```
integrate((b*x**2+a)**3*(D*x**3+C*x**2+B*x+A)/x,x)
```

output
```
A*a**3*log(x) + B*a**3*x + C*b**3*x**8/8 + D*b**3*x**9/9 + x**7*(B*b**3/7
+ 3*D*a*b**2/7) + x**6*(A*b**3/6 + C*a*b**2/2) + x**5*(3*B*a*b**2/5 + 3*D*
a**2*b/5) + x**4*(3*A*a*b**2/4 + 3*C*a**2*b/4) + x**3*(B*a**2*b + D*a**3/3
) + x**2*(3*A*a**2*b/2 + C*a**3/2)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)}{x} dx = \frac{1}{9} Db^3x^9 + \frac{1}{8} Cb^3x^8 + \frac{1}{7} (3 Dab^2 + Bb^3)x^7 + \frac{1}{6} (3 Cab^2 + Ab^3)x^6 + \frac{3}{5} (Da^2b + Bab^2)x^5 + Ba^3x + \frac{3}{4} (Ca^2b + Aab^2)x^4 + Aa^3 \log(x) + \frac{1}{3} (Da^3 + 3 Ba^2b)x^3 + \frac{1}{2} (Ca^3 + 3 Aa^2b)x^2$$

input
```
integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x,x, algorithm="maxima")
```

output
```
1/9*D*b^3*x^9 + 1/8*C*b^3*x^8 + 1/7*(3*D*a*b^2 + B*b^3)*x^7 + 1/6*(3*C*a*b
^2 + A*b^3)*x^6 + 3/5*(D*a^2*b + B*a*b^2)*x^5 + B*a^3*x + 3/4*(C*a^2*b + A
*a*b^2)*x^4 + A*a^3*log(x) + 1/3*(D*a^3 + 3*B*a^2*b)*x^3 + 1/2*(C*a^3 + 3*
A*a^2*b)*x^2
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.15

$$\int \frac{(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)}{x} dx = \frac{1}{9} Db^3x^9 + \frac{1}{8} Cb^3x^8 + \frac{3}{7} Dab^2x^7 + \frac{1}{7} Bb^3x^7 + \frac{1}{2} Cab^2x^6 + \frac{1}{6} Ab^3x^6 + \frac{3}{5} Da^2bx^5 + \frac{3}{5} Bab^2x^5 + \frac{3}{4} Ca^2bx^4 + \frac{3}{4} Aab^2x^4 + \frac{1}{3} Da^3x^3 + Ba^2bx^3 + \frac{1}{2} Ca^3x^2 + \frac{3}{2} Aa^2bx^2 + Ba^3x + Aa^3 \log(|x|)$$

input
```
integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x,x, algorithm="giac")
```

output
```
1/9*D*b^3*x^9 + 1/8*C*b^3*x^8 + 3/7*D*a*b^2*x^7 + 1/7*B*b^3*x^7 + 1/2*C*a*
b^2*x^6 + 1/6*A*b^3*x^6 + 3/5*D*a^2*b*x^5 + 3/5*B*a*b^2*x^5 + 3/4*C*a^2*b*
x^4 + 3/4*A*a*b^2*x^4 + 1/3*D*a^3*x^3 + B*a^2*b*x^3 + 1/2*C*a^3*x^2 + 3/2*
A*a^2*b*x^2 + B*a^3*x + A*a^3*log(abs(x))
```

## Mupad [B] (verification not implemented)

Time = 2.44 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.14

$$\int \frac{(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)}{x} dx = \frac{A\,b^3\,x^6}{6} + \frac{C\,a^3\,x^2}{2} + \frac{B\,b^3\,x^7}{7} + \frac{C\,b^3\,x^8}{8} + A\,a^3\,\ln(x) + \frac{a^3\,x^3\,D}{3} + \frac{b^3\,x^9\,D}{9} + B\,a^3\,x + \frac{3\,a^2\,b\,x^5\,D}{5} + \frac{3\,a\,b^2\,x^7\,D}{7} + \frac{3\,A\,a^2\,b\,x^2}{2} + \frac{3\,A\,a\,b^2\,x^4}{4} + B\,a^2\,b\,x^3 + \frac{3\,B\,a\,b^2\,x^5}{5} + \frac{3\,C\,a^2\,b\,x^4}{4} + \frac{C\,a\,b^2\,x^6}{2}$$

input
```
int(((a + b*x^2)^3*(A + B*x + C*x^2 + x^3*D))/x,x)
```

output
```
(A*b^3*x^6)/6 + (C*a^3*x^2)/2 + (B*b^3*x^7)/7 + (C*b^3*x^8)/8 + A*a^3*log(
x) + (a^3*x^3*D)/3 + (b^3*x^9*D)/9 + B*a^3*x + (3*a^2*b*x^5*D)/5 + (3*a*b^
2*x^7*D)/7 + (3*A*a^2*b*x^2)/2 + (3*A*a*b^2*x^4)/4 + B*a^2*b*x^3 + (3*B*a*
b^2*x^5)/5 + (3*C*a^2*b*x^4)/4 + (C*a*b^2*x^6)/2
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.12

$$\int \frac{(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)}{x}\,dx = \log(x)\,a^4 + \frac{3a^3b\,x^2}{2} + a^3bx + \frac{a^3c\,x^2}{2} + \frac{a^3d\,x^3}{3} + \frac{3a^2b^2x^4}{4} + a^2b^2x^3 + \frac{3a^2bc\,x^4}{4} + \frac{3a^2bd\,x^5}{5} + \frac{a\,b^3x^6}{6} + \frac{3a\,b^3x^5}{5} + \frac{a\,b^2c\,x^6}{2} + \frac{3a\,b^2d\,x^7}{7} + \frac{b^4x^7}{7} + \frac{b^3c\,x^8}{8} + \frac{b^3d\,x^9}{9}$$

input
```
int((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x,x)
```

output
```
(2520*log(x)*a**4 + 3780*a**3*b*x**2 + 2520*a**3*b*x + 1260*a**3*c*x**2 +
840*a**3*d*x**3 + 1890*a**2*b**2*x**4 + 2520*a**2*b**2*x**3 + 1890*a**2*b*
c*x**4 + 1512*a**2*b*d*x**5 + 420*a*b**3*x**6 + 1512*a*b**3*x**5 + 1260*a*
b**2*c*x**6 + 1080*a*b**2*d*x**7 + 360*b**4*x**7 + 315*b**3*c*x**8 + 280*b
**3*d*x**9)/2520
```

# 3.33 $\int \frac{(a+bx^2)^3(A+Bx+Cx^2+Dx^3)}{x^2} dx$



## Optimal result

Integrand size = 28, antiderivative size = 124

$$\begin{aligned}\int \frac{(a+bx^2)^3\,(A+Bx+Cx^2+Dx^3)}{x^2}\,dx &= -\frac{a^3A}{x}+a^2(3Ab+aC)x+\frac{3}{2}a^2bBx^2 \\ &\quad +ab(Ab+aC)x^3+\frac{3}{4}ab^2Bx^4 \\ &\quad +\frac{1}{5}b^2(Ab+3aC)x^5+\frac{1}{6}b^3Bx^6 \\ &\quad +\frac{1}{7}b^3Cx^7+\frac{D(a+bx^2)^4}{8b}+a^3B\log(x)\end{aligned}$$

output
```
-a^3*A/x+a^2*(3*A*b+C*a)*x+3/2*a^2*b*B*x^2+a*b*(A*b+C*a)*x^3+3/4*a*b^2*B*x
^4+1/5*b^2*(A*b+3*C*a)*x^5+1/6*b^3*B*x^6+1/7*b^3*C*x^7+1/8*D*(b*x^2+a)^4/b
+a^3*B*ln(x)
```

## Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)}{x^2} dx = a^3\left(-\frac{A}{x}+Cx+\frac{Dx^2}{2}\right) + \frac{1}{4}a^2bx(12A+x(6B+x(4C+3Dx))) + \frac{1}{20}ab^2x^3(20A+x(15B+2x(6C+5Dx))) + \frac{1}{840}b^3x^5(168A+5x(28B+3x(8C+7Dx))) + a^3B\log(x)$$

input
```
Integrate[((a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3))/x^2,x]
```

output
```
a^3*(-(A/x) + C*x + (D*x^2)/2) + (a^2*b*x*(12*A + x*(6*B + x*(4*C + 3*D*x)
)))/4 + (a*b^2*x^3*(20*A + x*(15*B + 2*x*(6*C + 5*D*x))))/20 + (b^3*x^5*(1
68*A + 5*x*(28*B + 3*x*(8*C + 7*D*x))))/840 + a^3*B*Log[x]
```

## Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {2018, 2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)}{x^2} dx$$

$$\Big\downarrow \ 2018$$

$$\int \frac{(bx^2+a)^3 (Cx^2+Bx+A)}{x^2}dx + \frac{D(a+bx^2)^4}{8b}$$

$$\Big\downarrow \ 2159$$

$$\int \left(b^3Cx^6 + b^3Bx^5 + b^2(Ab+3aC)x^4 + 3ab^2Bx^3 + 3ab(Ab+aC)x^2 + 3a^2bBx + a^2(3Ab+aC) + \frac{a^3B}{x} + \frac{a^3A}{x^2}\right.$$
$$\frac{D(a+bx^2)^4}{8b}$$
$$\Big\downarrow \text{2009}$$
$$-\frac{a^3A}{x} + a^3B\log(x) + a^2x(aC+3Ab) + \frac{3}{2}a^2bBx^2 + \frac{1}{5}b^2x^5(3aC+Ab) + abx^3(aC+Ab) + \frac{3}{4}ab^2Bx^4 + \frac{D(a+bx^2)^4}{8b} + \frac{1}{6}b^3Bx^6 + \frac{1}{7}b^3Cx^7$$

input
```
Int[((a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3))/x^2,x]
```

output
```
-((a^3*A)/x) + a^2*(3*A*b + a*C)*x + (3*a^2*b*B*x^2)/2 + a*b*(A*b + a*C)*x
^3 + (3*a*b^2*B*x^4)/4 + (b^2*(A*b + 3*a*C)*x^5)/5 + (b^3*B*x^6)/6 + (b^3*
C*x^7)/7 + (D*(a + b*x^2)^4)/(8*b) + a^3*B*Log[x]
```

## Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2018
```
Int[(Px_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[Coef
f[Px, x, n - m - 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coe
ff[Px, x, n - m - 1]*x^(n - m - 1))*x^m*(a + b*x^n)^p, x] /; FreeQ[{a, b, m
, n}, x] && PolyQ[Px, x] && IGtQ[p, 1] && IGtQ[n - m, 0] && NeQ[Coeff[Px, x
, n - m - 1], 0]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

## Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.17

| method | result |
|---|---|
| default | $\frac{b^3 D x^8}{8}+\frac{b^3 C x^7}{7}+\frac{b^3 B x^6}{6}+\frac{D a b^2 x^6}{2}+\frac{A b^3 x^5}{5}+\frac{3 C a b^2 x^5}{5}+\frac{3 B x^4 a b^2}{4}+\frac{3 D x^4 b a^2}{4}+a A b^2 x^3+C a^2 b$ |
| norman | $\frac{\left(\frac{1}{6} B b^3+\frac{1}{2} a b^2 D\right) x^7+\left(\frac{1}{5} b^3 A+\frac{3}{5} a C b^2\right) x^6+\left(\frac{3}{4} a b^2 B+\frac{3}{4} a^2 b D\right) x^5+\left(\frac{3}{2} a^2 b B+\frac{1}{2} a^3 D\right) x^3+\left(a b^2 A+a^2 b C\right) x^4+\left(3 a^2 b A+C a^3\right) x^2}{x}$ |
| parallelrisch | $\frac{105 b^3 D x^9+120 C b^3 x^8+140 b^3 B x^7+420 D a b^2 x^7+168 A b^3 x^6+504 C a b^2 x^6+630 B a b^2 x^5+630 D a^2 b x^5+840 a A b^2 x^4+840 C a}{840 x}$ |

input
```
int((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/8*b^3*D*x^8+1/7*b^3*C*x^7+1/6*b^3*B*x^6+1/2*D*a*b^2*x^6+1/5*A*b^3*x^5+3/
5*C*a*b^2*x^5+3/4*B*x^4*a*b^2+3/4*D*x^4*b*a^2+a*A*b^2*x^3+C*a^2*b*x^3+3/2*
B*a^2*b*x^2+1/2*D*a^3*x^2+3*a^2*A*b*x+C*a^3*x+a^3*B*ln(x)-a^3*A/x
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.19

$$\int \frac{(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)}{x^2}\,dx$$
$$=\frac{105\,Db^3x^9+120\,Cb^3x^8+140\,(3\,Dab^2+Bb^3)x^7+168\,(3\,Cab^2+Ab^3)x^6+630\,(Da^2b+Bab^2)x^5+840}{840\,x}$$

input
```
integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x^2,x, algorithm="fricas")
```

output
```
1/840*(105*D*b^3*x^9 + 120*C*b^3*x^8 + 140*(3*D*a*b^2 + B*b^3)*x^7 + 168*(
3*C*a*b^2 + A*b^3)*x^6 + 630*(D*a^2*b + B*a*b^2)*x^5 + 840*B*a^3*x*log(x)
+ 840*(C*a^2*b + A*a*b^2)*x^4 - 840*A*a^3 + 420*(D*a^3 + 3*B*a^2*b)*x^3 +
840*(C*a^3 + 3*A*a^2*b)*x^2)/x
```

## Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.21

$$\int \frac{(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)}{x^2}\,dx = -\frac{Aa^3}{x} + Ba^3\log{(x)} + \frac{Cb^3x^7}{7} + \frac{Db^3x^8}{8} + x^6\left(\frac{Bb^3}{6} + \frac{Dab^2}{2}\right) + x^5\left(\frac{Ab^3}{5} + \frac{3Cab^2}{5}\right) + x^4\cdot\left(\frac{3Bab^2}{4} + \frac{3Da^2b}{4}\right) + x^3\left(Aab^2 + Ca^2b\right) + x^2\cdot\left(\frac{3Ba^2b}{2} + \frac{Da^3}{2}\right) + x\left(3Aa^2b + Ca^3\right)$$

input
```
integrate((b*x**2+a)**3*(D*x**3+C*x**2+B*x+A)/x**2,x)
```

output
```
-A*a**3/x + B*a**3*log(x) + C*b**3*x**7/7 + D*b**3*x**8/8 + x**6*(B*b**3/6
 + D*a*b**2/2) + x**5*(A*b**3/5 + 3*C*a*b**2/5) + x**4*(3*B*a*b**2/4 + 3*D
*a**2*b/4) + x**3*(A*a*b**2 + C*a**2*b) + x**2*(3*B*a**2*b/2 + D*a**3/2) +
 x*(3*A*a**2*b + C*a**3)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.12

$$\int \frac{(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)}{x^2}\,dx = \frac{1}{8}\,Db^3x^8 + \frac{1}{7}\,Cb^3x^7 + \frac{1}{6}\,(3\,Dab^2 + Bb^3)x^6 + \frac{1}{5}\,(3\,Cab^2 + Ab^3)x^5 + \frac{3}{4}\,(Da^2b + Bab^2)x^4 + Ba^3\log{(x)} + (Ca^2b + Aab^2)x^3 - \frac{Aa^3}{x} + \frac{1}{2}\,(Da^3 + 3\,Ba^2b)x^2 + (Ca^3 + 3\,Aa^2b)x$$

input
```
integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x^2,x, algorithm="maxima")
```

output
```
1/8*D*b^3*x^8 + 1/7*C*b^3*x^7 + 1/6*(3*D*a*b^2 + B*b^3)*x^6 + 1/5*(3*C*a*b
^2 + A*b^3)*x^5 + 3/4*(D*a^2*b + B*a*b^2)*x^4 + B*a^3*log(x) + (C*a^2*b +
A*a*b^2)*x^3 - A*a^3/x + 1/2*(D*a^3 + 3*B*a^2*b)*x^2 + (C*a^3 + 3*A*a^2*b)
*x
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.17

$$\int \frac{(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)}{x^2}\,dx = \frac{1}{8}Db^3x^8 + \frac{1}{7}Cb^3x^7 + \frac{1}{2}Dab^2x^6 + \frac{1}{6}Bb^3x^6 + \frac{3}{5}Cab^2x^5 + \frac{1}{5}Ab^3x^5 + \frac{3}{4}Da^2bx^4 + \frac{3}{4}Bab^2x^4 + Ca^2bx^3 + Aab^2x^3 + \frac{1}{2}Da^3x^2 + \frac{3}{2}Ba^2bx^2 + Ca^3x + 3Aa^2bx + Ba^3\log(|x|) - \frac{Aa^3}{x}$$

input
```
integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x^2,x, algorithm="giac")
```

output
```
1/8*D*b^3*x^8 + 1/7*C*b^3*x^7 + 1/2*D*a*b^2*x^6 + 1/6*B*b^3*x^6 + 3/5*C*a*
b^2*x^5 + 1/5*A*b^3*x^5 + 3/4*D*a^2*b*x^4 + 3/4*B*a*b^2*x^4 + C*a^2*b*x^3
+ A*a*b^2*x^3 + 1/2*D*a^3*x^2 + 3/2*B*a^2*b*x^2 + C*a^3*x + 3*A*a^2*b*x +
B*a^3*log(abs(x)) - A*a^3/x
```

## Mupad [B] (verification not implemented)

Time = 2.30 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)}{x^2}\,dx = \frac{(b\,x^2+a)^4\,D}{8\,b} - \frac{A\,a^3}{x} + \frac{A\,b^3\,x^5}{5} + \frac{B\,b^3\,x^6}{6} + \frac{C\,b^3\,x^7}{7} + B\,a^3\,\ln(x) + C\,a^3\,x + 3\,A\,a^2\,b\,x + A\,a\,b^2\,x^3 + \frac{3\,B\,a^2\,b\,x^2}{2} + \frac{3\,B\,a\,b^2\,x^4}{4} + C\,a^2\,b\,x^3 + \frac{3\,C\,a\,b^2\,x^5}{5}$$

input
```
int(((a + b*x^2)^3*(A + B*x + C*x^2 + x^3*D))/x^2,x)
```

output
```
((a + b*x^2)^4*D)/(8*b) - (A*a^3)/x + (A*b^3*x^5)/5 + (B*b^3*x^6)/6 + (C*b
^3*x^7)/7 + B*a^3*log(x) + C*a^3*x + 3*A*a^2*b*x + A*a*b^2*x^3 + (3*B*a^2*
b*x^2)/2 + (3*B*a*b^2*x^4)/4 + C*a^2*b*x^3 + (3*C*a*b^2*x^5)/5
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.23

$$\int \frac{(a+bx^2)^3(A+Bx+Cx^2+Dx^3)}{x^2}\,dx$$
$$= \frac{840\log(x)\,a^3bx - 840a^4 + 2520a^3b\,x^2 + 840a^3c\,x^2 + 420a^3d\,x^3 + 840a^2b^2x^4 + 1260a^2b^2x^3 + 840a^2bc\,x^4 \cdots}{840x}$$

input
```
int((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x^2,x)
```

output
```
(840*log(x)*a**3*b*x - 840*a**4 + 2520*a**3*b*x**2 + 840*a**3*c*x**2 + 420
*a**3*d*x**3 + 840*a**2*b**2*x**4 + 1260*a**2*b**2*x**3 + 840*a**2*b*c*x**
4 + 630*a**2*b*d*x**5 + 168*a*b**3*x**6 + 630*a*b**3*x**5 + 504*a*b**2*c*x
**6 + 420*a*b**2*d*x**7 + 140*b**4*x**7 + 120*b**3*c*x**8 + 105*b**3*d*x**
9)/(840*x)
```

# 3.34 $\int \frac{(a+bx^2)^3(A+Bx+Cx^2+Dx^3)}{x^3}\,dx$



## Optimal result

Integrand size = 28, antiderivative size = 135

$$\int \frac{(a+bx^2)^3(A+Bx+Cx^2+Dx^3)}{x^3}\,dx = -\frac{a^3A}{2x^2} - \frac{a^3B}{x} + a^2(3bB+aD)x + \frac{3}{2}ab(Ab+aC)x^2 + ab(bB+aD)x^3 + \frac{1}{4}b^2(Ab+3aC)x^4 + \frac{1}{5}b^2(bB+3aD)x^5 + \frac{1}{6}b^3Cx^6 + \frac{1}{7}b^3Dx^7 + a^2(3Ab+aC)\log(x)$$

output
```
-1/2*a^3*A/x^2-a^3*B/x+a^2*(3*B*b+D*a)*x+3/2*a*b*(A*b+C*a)*x^2+a*b*(B*b+D*
a)*x^3+1/4*b^2*(A*b+3*C*a)*x^4+1/5*b^2*(B*b+3*D*a)*x^5+1/6*b^3*C*x^6+1/7*b
^3*D*x^7+a^2*(3*A*b+C*a)*ln(x)
```

## Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)}{x^3} dx = -\frac{a^3(A+2Bx-2Dx^3)}{2x^2} + \frac{1}{2}a^2bx(6B+x(3C+2Dx)) + \frac{1}{20}ab^2x^2(30A+x(20B+3x(5C+4Dx))) + \frac{1}{420}b^3x^4(105A+2x(42B+5x(7C+6Dx))) + a^2(3Ab+aC)\log(x)$$

input
```
Integrate[((a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3))/x^3,x]
```

output
```
-1/2*(a^3*(A + 2*B*x - 2*D*x^3))/x^2 + (a^2*b*x*(6*B + x*(3*C + 2*D*x)))/2
 + (a*b^2*x^2*(30*A + x*(20*B + 3*x*(5*C + 4*D*x))))/20 + (b^3*x^4*(105*A
+ 2*x*(42*B + 5*x*(7*C + 6*D*x))))/420 + a^2*(3*A*b + a*C)*Log[x]
```

## Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)}{x^3} dx$$

$$\Big\downarrow \text{ 2333}$$

$$\int \left(\frac{a^3A}{x^3} + \frac{a^3B}{x^2} + \frac{a^2(aC+3Ab)}{x} + a^2(aD+3bB) + b^2x^3(3aC+Ab) + 3abx(aC+Ab) + b^2x^4(3aD+bB) + 3a\right.$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a^3A}{2x^2}-\frac{a^3B}{x}+a^2\log(x)(aC+3Ab)+a^2x(aD+3bB)+\frac{1}{4}b^2x^4(3aC+Ab)+\frac{3}{2}abx^2(aC+Ab)+\frac{1}{5}b^2x^5(3aD+bB)+abx^3(aD+bB)+\frac{1}{6}b^3Cx^6+\frac{1}{7}b^3Dx^7$$

input
```
Int[((a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3))/x^3,x]
```

output
```
-1/2*(a^3*A)/x^2 - (a^3*B)/x + a^2*(3*b*B + a*D)*x + (3*a*b*(A*b + a*C)*x^
2)/2 + a*b*(b*B + a*D)*x^3 + (b^2*(A*b + 3*a*C)*x^4)/4 + (b^2*(b*B + 3*a*D
)*x^5)/5 + (b^3*C*x^6)/6 + (b^3*D*x^7)/7 + a^2*(3*A*b + a*C)*Log[x]
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

## Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.05

| method | result |
|---|---|
| default | $\frac{b^3Dx^7}{7}+\frac{b^3Cx^6}{6}+\frac{b^3Bx^5}{5}+\frac{3Dab^2x^5}{5}+\frac{Ax^4b^3}{4}+\frac{3Cab^2x^4}{4}+Bab^2x^3+Dx^3ba^2+\frac{3aAb^2x^2}{2}+\frac{3Ca^2}{2}$ |
| norman | $\frac{\left(\frac{1}{5}Bb^3+\frac{3}{5}ab^2D\right)x^7+\left(\frac{1}{4}b^3A+\frac{3}{4}aCb^2\right)x^6+\left(\frac{3}{2}ab^2A+\frac{3}{2}a^2bC\right)x^4+\left(ab^2B+a^2bD\right)x^5+\left(3a^2bB+a^3D\right)x^3-\frac{a^3A}{2}-Ba^3x+\frac{Cb^3x}{6}}{x^2}$ |
| parallelrisch | $\frac{60b^3Dx^9+70Cb^3x^8+84b^3Bx^7+252Dab^2x^7+105Ab^3x^6+315Cab^2x^6+420Bab^2x^5+420Da^2bx^5+630aAb^2x^4+630Ca^2bx}{420x^2}$ |

input
```
int((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/7*b^3*D*x^7+1/6*b^3*C*x^6+1/5*b^3*B*x^5+3/5*D*a*b^2*x^5+1/4*A*x^4*b^3+3/
4*C*a*b^2*x^4+B*a*b^2*x^3+D*x^3*b*a^2+3/2*a*A*b^2*x^2+3/2*C*a^2*b*x^2+3*B*
x*a^2*b+D*a^3*x-1/2*a^3*A/x^2+a^2*(3*A*b+C*a)*ln(x)-a^3*B/x
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)}{x^3}\,dx$$
$$= \frac{60\,Db^3x^9 + 70\,Cb^3x^8 + 84\,(3\,Dab^2 + Bb^3)x^7 + 105\,(3\,Cab^2 + Ab^3)x^6 + 420\,(Da^2b + Bab^2)x^5 - 420\,Ba^3}{420\,x^2}$$

input
```
integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x^3,x, algorithm="fricas")
```

output
```
1/420*(60*D*b^3*x^9 + 70*C*b^3*x^8 + 84*(3*D*a*b^2 + B*b^3)*x^7 + 105*(3*C
*a*b^2 + A*b^3)*x^6 + 420*(D*a^2*b + B*a*b^2)*x^5 - 420*B*a^3*x + 630*(C*a
^2*b + A*a*b^2)*x^4 - 210*A*a^3 + 420*(D*a^3 + 3*B*a^2*b)*x^3 + 420*(C*a^3
 + 3*A*a^2*b)*x^2*log(x))/x^2
```

## Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.12

$$\int \frac{(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)}{x^3}\,dx = \frac{Cb^3x^6}{6} + \frac{Db^3x^7}{7} + a^2\cdot(3Ab+Ca)\log(x) + x^5\left(\frac{Bb^3}{5} + \frac{3Dab^2}{5}\right) + x^4\left(\frac{Ab^3}{4} + \frac{3Cab^2}{4}\right) + x^3(Bab^2 + Da^2b) + x^2\cdot\left(\frac{3Aab^2}{2} + \frac{3Ca^2b}{2}\right) + x(3Ba^2b + Da^3) + \frac{-Aa^3 - 2Ba^3x}{2x^2}$$

input
```
integrate((b*x**2+a)**3*(D*x**3+C*x**2+B*x+A)/x**3,x)
```

output
```
C*b**3*x**6/6 + D*b**3*x**7/7 + a**2*(3*A*b + C*a)*log(x) + x**5*(B*b**3/5
 + 3*D*a*b**2/5) + x**4*(A*b**3/4 + 3*C*a*b**2/4) + x**3*(B*a*b**2 + D*a**
2*b) + x**2*(3*A*a*b**2/2 + 3*C*a**2*b/2) + x*(3*B*a**2*b + D*a**3) + (-A*
a**3 - 2*B*a**3*x)/(2*x**2)
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)}{x^3}\,dx = \frac{1}{7}\,Db^3x^7 + \frac{1}{6}\,Cb^3x^6 + \frac{1}{5}\,(3\,Dab^2+Bb^3)x^5 + \frac{1}{4}\,(3\,Cab^2+Ab^3)x^4 + (Da^2b+Bab^2)x^3 + \frac{3}{2}\,(Ca^2b+Aab^2)x^2 + (Da^3+3\,Ba^2b)x + (Ca^3+3\,Aa^2b)\log(x) - \frac{2\,Ba^3x+Aa^3}{2\,x^2}$$

input
```
integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x^3,x, algorithm="maxima")
```

output
```
1/7*D*b^3*x^7 + 1/6*C*b^3*x^6 + 1/5*(3*D*a*b^2 + B*b^3)*x^5 + 1/4*(3*C*a*b
^2 + A*b^3)*x^4 + (D*a^2*b + B*a*b^2)*x^3 + 3/2*(C*a^2*b + A*a*b^2)*x^2 +
(D*a^3 + 3*B*a^2*b)*x + (C*a^3 + 3*A*a^2*b)*log(x) - 1/2*(2*B*a^3*x + A*a^
3)/x^2
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)}{x^3}\,dx = \frac{1}{7}\,Db^3x^7 + \frac{1}{6}\,Cb^3x^6 + \frac{3}{5}\,Dab^2x^5 + \frac{1}{5}\,Bb^3x^5 + \frac{3}{4}\,Cab^2x^4 + \frac{1}{4}\,Ab^3x^4 + Da^2bx^3 + Bab^2x^3 + \frac{3}{2}\,Ca^2bx^2 + \frac{3}{2}\,Aab^2x^2 + Da^3x + 3\,Ba^2bx + (Ca^3+3\,Aa^2b)\log(|x|) - \frac{2\,Ba^3x+Aa^3}{2\,x^2}$$

input
```
integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x^3,x, algorithm="giac")
```

output
```
1/7*D*b^3*x^7 + 1/6*C*b^3*x^6 + 3/5*D*a*b^2*x^5 + 1/5*B*b^3*x^5 + 3/4*C*a*
b^2*x^4 + 1/4*A*b^3*x^4 + D*a^2*b*x^3 + B*a*b^2*x^3 + 3/2*C*a^2*b*x^2 + 3/
2*A*a*b^2*x^2 + D*a^3*x + 3*B*a^2*b*x + (C*a^3 + 3*A*a^2*b)*log(abs(x)) -
1/2*(2*B*a^3*x + A*a^3)/x^2
```

## Mupad [B] (verification not implemented)

Time = 2.39 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)}{x^3}\,dx = \frac{A\,b^3\,x^4}{4}-\frac{B\,a^3}{x}-\frac{A\,a^3}{2\,x^2}+\frac{B\,b^3\,x^5}{5}+\frac{C\,b^3\,x^6}{6}+C\,a^3\,\ln(x)+a^3\,x\,D+\frac{b^3\,x^7\,D}{7}+a^2\,b\,x^3\,D+\frac{3\,a\,b^2\,x^5\,D}{5}+3\,B\,a^2\,b\,x+\frac{3\,A\,a\,b^2\,x^2}{2}+B\,a\,b^2\,x^3+\frac{3\,C\,a^2\,b\,x^2}{2}+\frac{3\,C\,a\,b^2\,x^4}{4}+3\,A\,a^2\,b\,\ln(x)$$

input
```
int(((a + b*x^2)^3*(A + B*x + C*x^2 + x^3*D))/x^3,x)
```

output
```
(A*b^3*x^4)/4 - (B*a^3)/x - (A*a^3)/(2*x^2) + (B*b^3*x^5)/5 + (C*b^3*x^6)/
6 + C*a^3*log(x) + a^3*x*D + (b^3*x^7*D)/7 + a^2*b*x^3*D + (3*a*b^2*x^5*D)
/5 + 3*B*a^2*b*x + (3*A*a*b^2*x^2)/2 + B*a*b^2*x^3 + (3*C*a^2*b*x^2)/2 + (
3*C*a*b^2*x^4)/4 + 3*A*a^2*b*log(x)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.15

$$\int \frac{(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)}{x^3}\,dx$$
$$= \frac{1260\log(x)\,a^3b\,x^2+420\log(x)\,a^3c\,x^2-210a^4-420a^3bx+420a^3d\,x^3+630a^2b^2x^4+1260a^2b^2x^3+630a}{420}$$

input
```
int((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x^3,x)
```

output
```
(1260*log(x)*a**3*b*x**2 + 420*log(x)*a**3*c*x**2 - 210*a**4 - 420*a**3*b*
x + 420*a**3*d*x**3 + 630*a**2*b**2*x**4 + 1260*a**2*b**2*x**3 + 630*a**2*
b*c*x**4 + 420*a**2*b*d*x**5 + 105*a*b**3*x**6 + 420*a*b**3*x**5 + 315*a*b
**2*c*x**6 + 252*a*b**2*d*x**7 + 84*b**4*x**7 + 70*b**3*c*x**8 + 60*b**3*d
*x**9)/(420*x**2)
```

# 3.35 $\int \frac{(a+bx^2)^3(A+Bx+Cx^2+Dx^3)}{x^4}\,dx$



## Optimal result

Integrand size = 28, antiderivative size = 139

$$\begin{aligned}\int \frac{(a+bx^2)^3(A+Bx+Cx^2+Dx^3)}{x^4}\,dx &= -\frac{a^3A}{3x^3}-\frac{a^3B}{2x^2}-\frac{a^2(3Ab+aC)}{x} \\ &\quad +3ab(Ab+aC)x+\frac{3}{2}ab(bB+aD)x^2 \\ &\quad +\frac{1}{3}b^2(Ab+3aC)x^3+\frac{1}{4}b^2(bB+3aD)x^4 \\ &\quad +\frac{1}{5}b^3Cx^5+\frac{1}{6}b^3Dx^6+a^2(3bB+aD)\log(x)\end{aligned}$$

output
```
-1/3*a^3*A/x^3-1/2*a^3*B/x^2-a^2*(3*A*b+C*a)/x+3*a*b*(A*b+C*a)*x+3/2*a*b*(
B*b+D*a)*x^2+1/3*b^2*(A*b+3*C*a)*x^3+1/4*b^2*(B*b+3*D*a)*x^4+1/5*b^3*C*x^5
+1/6*b^3*D*x^6+a^2*(3*B*b+D*a)*ln(x)
```

## Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)}{x^4} dx = -\frac{a^3(2A+3x(B+2Cx))}{6x^3} + \frac{3a^2b(-2A+x^2(2C+Dx))}{2x} + \frac{1}{4}ab^2x(12A+x(6B+x(4C+3Dx))) + \frac{1}{60}b^3x^3(20A+x(15B+2x(6C+5Dx))) + a^2(3bB+aD)\log(x)$$

input
```
Integrate[((a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3))/x^4,x]
```

output
```
-1/6*(a^3*(2*A + 3*x*(B + 2*C*x)))/x^3 + (3*a^2*b*(-2*A + x^2*(2*C + D*x))
)/(2*x) + (a*b^2*x*(12*A + x*(6*B + x*(4*C + 3*D*x))))/4 + (b^3*x^3*(20*A
+ x*(15*B + 2*x*(6*C + 5*D*x))))/60 + a^2*(3*b*B + a*D)*Log[x]
```

## Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)}{x^4} dx$$

$$\Big\downarrow \text{2333}$$

$$\int \left(\frac{a^3A}{x^4} + \frac{a^3B}{x^3} + \frac{a^2(aC+3Ab)}{x^2} + \frac{a^2(aD+3bB)}{x} + b^2x^2(3aC+Ab) + 3ab(aC+Ab) + b^2x^3(3aD+bB) + 3a$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^3A}{3x^3}-\frac{a^3B}{2x^2}-\frac{a^2(aC+3Ab)}{x}+a^2\log(x)(aD+3bB)+\frac{1}{3}b^2x^3(3aC+Ab)+3abx(aC+Ab)+\frac{1}{4}b^2x^4(3aD+bB)+\frac{3}{2}abx^2(aD+bB)+\frac{1}{5}b^3Cx^5+\frac{1}{6}b^3Dx^6$$

input
```
Int[((a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3))/x^4,x]
```

output
```
-1/3*(a^3*A)/x^3 - (a^3*B)/(2*x^2) - (a^2*(3*A*b + a*C))/x + 3*a*b*(A*b +
a*C)*x + (3*a*b*(b*B + a*D)*x^2)/2 + (b^2*(A*b + 3*a*C)*x^3)/3 + (b^2*(b*B
 + 3*a*D)*x^4)/4 + (b^3*C*x^5)/5 + (b^3*D*x^6)/6 + a^2*(3*b*B + a*D)*Log[x
]
```

**Defintions of rubi rules used**

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

## Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.01

| method | result |
|---|---|
| default | $\frac{b^3Dx^6}{6}+\frac{b^3Cx^5}{5}+\frac{Bb^3x^4}{4}+\frac{3Dab^2x^4}{4}+\frac{Ab^3x^3}{3}+Cab^2x^3+\frac{3Bx^2ab^2}{2}+\frac{3Da^2bx^2}{2}+3Axab^2+3C$ |
| norman | $\frac{\left(\frac{1}{4}Bb^3+\frac{3}{4}ab^2D\right)x^7+\left(\frac{1}{3}b^3A+aCb^2\right)x^6+\left(\frac{3}{2}ab^2B+\frac{3}{2}a^2bD\right)x^5+\left(3ab^2A+3a^2bC\right)x^4+\left(-3a^2bA-Ca^3\right)x^2-\frac{a^3A}{3}-\frac{Ba^3x}{2}+}{x^3}$ |
| parallelrisch | $\frac{10b^3Dx^9+12Cb^3x^8+15b^3Bx^7+45Dab^2x^7+20Ab^3x^6+60Cab^2x^6+90Bab^2x^5+90Da^2bx^5+180aAb^2x^4+180B\ln(x)x^3a^2}{60x^3}$ |

input
```
int((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/6*b^3*D*x^6+1/5*b^3*C*x^5+1/4*B*b^3*x^4+3/4*D*a*b^2*x^4+1/3*A*b^3*x^3+C*
a*b^2*x^3+3/2*B*x^2*a*b^2+3/2*D*a^2*b*x^2+3*A*x*a*b^2+3*C*a^2*b*x-1/3*a^3*
A/x^3-1/2*a^3*B/x^2+a^2*(3*B*b+D*a)*ln(x)-a^2*(3*A*b+C*a)/x
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)}{x^4}\,dx$$
$$= \frac{10\,Db^3x^9 + 12\,Cb^3x^8 + 15\,(3\,Dab^2 + Bb^3)x^7 + 20\,(3\,Cab^2 + Ab^3)x^6 + 90\,(Da^2b + Bab^2)x^5 - 30\,Ba^3x +}{60\,x^3}$$

input
```
integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x^4,x, algorithm="fricas")
```

output
```
1/60*(10*D*b^3*x^9 + 12*C*b^3*x^8 + 15*(3*D*a*b^2 + B*b^3)*x^7 + 20*(3*C*a
*b^2 + A*b^3)*x^6 + 90*(D*a^2*b + B*a*b^2)*x^5 - 30*B*a^3*x + 180*(C*a^2*b
 + A*a*b^2)*x^4 + 60*(D*a^3 + 3*B*a^2*b)*x^3*log(x) - 20*A*a^3 - 60*(C*a^3
 + 3*A*a^2*b)*x^2)/x^3
```

## Sympy [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.12

$$\int \frac{(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)}{x^4}\,dx = \frac{Cb^3x^5}{5} + \frac{Db^3x^6}{6} + a^2 \cdot (3Bb + Da)\log(x) + x^4\left(\frac{Bb^3}{4} + \frac{3Dab^2}{4}\right) + x^3\left(\frac{Ab^3}{3} + Cab^2\right) + x^2 \cdot \left(\frac{3Bab^2}{2} + \frac{3Da^2b}{2}\right) + x\left(3Aab^2 + 3Ca^2b\right) + \frac{-2Aa^3 - 3Ba^3x + x^2(-18Aa^2b - 6Ca^3)}{6x^3}$$

input
```
integrate((b*x**2+a)**3*(D*x**3+C*x**2+B*x+A)/x**4,x)
```

output
```
C*b**3*x**5/5 + D*b**3*x**6/6 + a**2*(3*B*b + D*a)*log(x) + x**4*(B*b**3/4
 + 3*D*a*b**2/4) + x**3*(A*b**3/3 + C*a*b**2) + x**2*(3*B*a*b**2/2 + 3*D*a
**2*b/2) + x*(3*A*a*b**2 + 3*C*a**2*b) + (-2*A*a**3 - 3*B*a**3*x + x**2*(-
18*A*a**2*b - 6*C*a**3))/(6*x**3)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^2)^3(A+Bx+Cx^2+Dx^3)}{x^4}\,dx = \frac{1}{6}\,Db^3x^6 + \frac{1}{5}\,Cb^3x^5 + \frac{1}{4}\left(3\,Dab^2 + Bb^3\right)x^4 + \frac{1}{3}\left(3\,Cab^2 + Ab^3\right)x^3 + \frac{3}{2}\left(Da^2b + Bab^2\right)x^2 + 3\left(Ca^2b + Aab^2\right)x + \left(Da^3 + 3\,Ba^2b\right)\log(x) - \frac{3\,Ba^3x + 2\,Aa^3 + 6\,(Ca^3 + 3\,Aa^2b)x^2}{6\,x^3}$$

input
```
integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x^4,x, algorithm="maxima")
```

output
```
1/6*D*b^3*x^6 + 1/5*C*b^3*x^5 + 1/4*(3*D*a*b^2 + B*b^3)*x^4 + 1/3*(3*C*a*b
^2 + A*b^3)*x^3 + 3/2*(D*a^2*b + B*a*b^2)*x^2 + 3*(C*a^2*b + A*a*b^2)*x +
(D*a^3 + 3*B*a^2*b)*log(x) - 1/6*(3*B*a^3*x + 2*A*a^3 + 6*(C*a^3 + 3*A*a^2
*b)*x^2)/x^3
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.05

$$\int \frac{(a+bx^2)^3(A+Bx+Cx^2+Dx^3)}{x^4}\,dx = \frac{1}{6}\,Db^3x^6 + \frac{1}{5}\,Cb^3x^5 + \frac{3}{4}\,Dab^2x^4 + \frac{1}{4}\,Bb^3x^4 + Cab^2x^3 + \frac{1}{3}\,Ab^3x^3 + \frac{3}{2}\,Da^2bx^2 + \frac{3}{2}\,Bab^2x^2 + 3\,Ca^2bx + 3\,Aab^2x + \left(Da^3 + 3\,Ba^2b\right)\log(|x|) - \frac{3\,Ba^3x + 2\,Aa^3 + 6\,(Ca^3 + 3\,Aa^2b)x^2}{6\,x^3}$$

input
```
integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x^4,x, algorithm="giac")
```

output
```
1/6*D*b^3*x^6 + 1/5*C*b^3*x^5 + 3/4*D*a*b^2*x^4 + 1/4*B*b^3*x^4 + C*a*b^2*
x^3 + 1/3*A*b^3*x^3 + 3/2*D*a^2*b*x^2 + 3/2*B*a*b^2*x^2 + 3*C*a^2*b*x + 3*
A*a*b^2*x + (D*a^3 + 3*B*a^2*b)*log(abs(x)) - 1/6*(3*B*a^3*x + 2*A*a^3 + 6
*(C*a^3 + 3*A*a^2*b)*x^2)/x^3
```

## Mupad [B] (verification not implemented)

Time = 2.53 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx^2)^3(A+Bx+Cx^2+Dx^3)}{x^4}\,dx = \frac{B\,b^3\,x^4}{4}-\frac{C\,a^3}{x}-\frac{B\,a^3}{2\,x^2}+\frac{C\,b^3\,x^5}{5}+\frac{b^3\,x^6\,D}{6} - \frac{A\,(a^3+9\,a^2\,b\,x^2-9\,a\,b^2\,x^4-b^3\,x^6)}{3\,x^3} + \frac{a^3\,\ln(x^2)\,D}{2}+\frac{3\,a^2\,b\,x^2\,D}{2} + 3\,C\,a^2\,b\,x+\frac{3\,a\,b^2\,x^4\,D}{4}+\frac{3\,B\,a\,b^2\,x^2}{2} + C\,a\,b^2\,x^3+3\,B\,a^2\,b\,\ln(x)$$

input
```
int(((a + b*x^2)^3*(A + B*x + C*x^2 + x^3*D))/x^4,x)
```

output
```
(B*b^3*x^4)/4 - (C*a^3)/x - (B*a^3)/(2*x^2) + (C*b^3*x^5)/5 + (b^3*x^6*D)/
6 - (A*(a^3 - b^3*x^6 + 9*a^2*b*x^2 - 9*a*b^2*x^4))/(3*x^3) + (a^3*log(x^2
)*D)/2 + (3*a^2*b*x^2*D)/2 + 3*C*a^2*b*x + (3*a*b^2*x^4*D)/4 + (3*B*a*b^2*
x^2)/2 + C*a*b^2*x^3 + 3*B*a^2*b*log(x)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.12

$$\int \frac{(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)}{x^4}\,dx$$
$$= \frac{60\log(x)\,a^3 d\,x^3 + 180\log(x)\,a^2 b^2 x^3 - 20a^4 - 180a^3 b\,x^2 - 30a^3 bx - 60a^3 c\,x^2 + 180a^2 b^2 x^4 + 180a^2 bc\,x^4}{60x^3}$$

input
```
int((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x^4,x)
```

output
```
(60*log(x)*a**3*d*x**3 + 180*log(x)*a**2*b**2*x**3 - 20*a**4 - 180*a**3*b*
x**2 - 30*a**3*b*x - 60*a**3*c*x**2 + 180*a**2*b**2*x**4 + 180*a**2*b*c*x*
*4 + 90*a**2*b*d*x**5 + 20*a*b**3*x**6 + 90*a*b**3*x**5 + 60*a*b**2*c*x**6
 + 45*a*b**2*d*x**7 + 15*b**4*x**7 + 12*b**3*c*x**8 + 10*b**3*d*x**9)/(60*
x**3)
```

# 3.36 $\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx$



## Optimal result

Integrand size = 28, antiderivative size = 130

$$\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx = -\frac{a(bB-aD)x}{b^3}+\frac{(Ab-aC)x^2}{2b^2}+\frac{(bB-aD)x^3}{3b^2}+\frac{Cx^4}{4b}+\frac{Dx^5}{5b}+\frac{a^{3/2}(bB-aD)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{7/2}}-\frac{a(Ab-aC)\log(a+bx^2)}{2b^3}$$

output
```
-a*(B*b-D*a)*x/b^3+1/2*(A*b-C*a)*x^2/b^2+1/3*(B*b-D*a)*x^3/b^2+1/4*C*x^4/b
+1/5*D*x^5/b+a^(3/2)*(B*b-D*a)*arctan(b^(1/2)*x/a^(1/2))/b^(7/2)-1/2*a*(A*
b-C*a)*ln(b*x^2+a)/b^3
```

## Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.88

$$\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx = -\frac{a^{3/2}(-bB+aD)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{7/2}}+\frac{x(60a^2D-10ab(6B+x(3C+2Dx))+b^2x(30A+x(20B+3x(5C+4Dx))))+30a(-Ab+aC)\log}{60b^3}$$

input
```
Integrate[(x^3*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2),x]
```

output
```
-((a^(3/2)*(-(b*B) + a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(7/2)) + (x*(60*a
^2*D - 10*a*b*(6*B + x*(3*C + 2*D*x)) + b^2*x*(30*A + x*(20*B + 3*x*(5*C +
 4*D*x)))) + 30*a*(-(A*b) + a*C)*Log[a + b*x^2])/(60*b^3)
```

## Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 (A+Bx+Cx^2+Dx^3)}{a+bx^2} \, dx$$

$$\Big\downarrow \text{2333}$$

$$\int \left(\frac{a^2(bB-aD)-abx(Ab-aC)}{b^3 (a+bx^2)}+\frac{x(Ab-aC)}{b^2}-\frac{a(bB-aD)}{b^3}+\frac{x^2(bB-aD)}{b^2}+\frac{Cx^3}{b}+\frac{Dx^4}{b}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^{3/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(bB-aD)}{b^{7/2}}-\frac{a(Ab-aC)\log(a+bx^2)}{2b^3}+\frac{x^2(Ab-aC)}{2b^2}-\frac{ax(bB-aD)}{b^3}+\frac{x^3(bB-aD)}{3b^2}+\frac{Cx^4}{4b}+\frac{Dx^5}{5b}$$

input
```
Int[(x^3*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2),x]
```

output
```
-((a*(b*B - a*D)*x)/b^3) + ((A*b - a*C)*x^2)/(2*b^2) + ((b*B - a*D)*x^3)/(
3*b^2) + (C*x^4)/(4*b) + (D*x^5)/(5*b) + (a^(3/2)*(b*B - a*D)*ArcTan[(Sqrt
[b]*x)/Sqrt[a]])/b^(7/2) - (a*(A*b - a*C)*Log[a + b*x^2])/(2*b^3)
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

## Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.98

| method | result |
|---|---|
| default | $\frac{\frac{1}{5}Dx^5b^2+\frac{1}{4}Cb^2x^4+\frac{1}{3}Bb^2x^3-\frac{1}{3}Dabx^3+\frac{1}{2}Ab^2x^2-\frac{1}{2}x^2aCb-Babx+Da^2x}{b^3}-\frac{a\left(\frac{\left(b^2A-Cab\right)\ln\left(bx^2+a\right)}{2b}+\frac{\left(-abB+Da^2\right)\arctan\left(\frac{b}{\sqrt{}}\right.}{\sqrt{ab}}\right.}{b^3}$ |

input
```
int(x^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/b^3*(1/5*D*x^5*b^2+1/4*C*b^2*x^4+1/3*B*b^2*x^3-1/3*D*a*b*x^3+1/2*A*b^2*x
^2-1/2*x^2*a*C*b-B*a*b*x+D*a^2*x)-a/b^3*(1/2*(A*b^2-C*a*b)/b*ln(b*x^2+a)+(
-B*a*b+D*a^2)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

## Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 270, normalized size of antiderivative = 2.08

$$\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx$$
$$= \left[\frac{12\,Db^2x^5+15\,Cb^2x^4-20\,(Dab-Bb^2)x^3-30\,(Cab-Ab^2)x^2-30\,(Da^2-Bab)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2+2\,bx\sqrt{-}}{bx^2+a}\right.}{60\,b^3}\right.$$

input
```
integrate(x^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="fricas")
```

output
```
[1/60*(12*D*b^2*x^5 + 15*C*b^2*x^4 - 20*(D*a*b - B*b^2)*x^3 - 30*(C*a*b -
A*b^2)*x^2 - 30*(D*a^2 - B*a*b)*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) -
 a)/(b*x^2 + a)) + 60*(D*a^2 - B*a*b)*x + 30*(C*a^2 - A*a*b)*log(b*x^2 + a
))/b^3, 1/60*(12*D*b^2*x^5 + 15*C*b^2*x^4 - 20*(D*a*b - B*b^2)*x^3 - 30*(C
*a*b - A*b^2)*x^2 - 60*(D*a^2 - B*a*b)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) +
 60*(D*a^2 - B*a*b)*x + 30*(C*a^2 - A*a*b)*log(b*x^2 + a))/b^3]
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 274 vs. $2(116) = 232$.

Time = 0.69 (sec) , antiderivative size = 274, normalized size of antiderivative = 2.11

$$\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx = \frac{Cx^4}{4b}+\frac{Dx^5}{5b}+x^3\left(\frac{B}{3b}-\frac{Da}{3b^2}\right)+x^2\left(\frac{A}{2b}-\frac{Ca}{2b^2}\right)+x\left(-\frac{Ba}{b^2}+\frac{Da^2}{b^3}\right)+\left(\frac{a(-Ab+Ca)}{2b^3}-\frac{\sqrt{-a^3b^7}(-Bb+Da)}{2b^7}\right)\log\left(x+\frac{-Aab+Ca^2-2b^3\left(\frac{a(-Ab+Ca)}{2b^3}-\frac{\sqrt{-a^3b^7}(-Bb+Da)}{2b^7}\right)}{-Bab+Da^2}\right)+\left(\frac{a(-Ab+Ca)}{2b^3}+\frac{\sqrt{-a^3b^7}(-Bb+Da)}{2b^7}\right)\log\left(x+\frac{-Aab+Ca^2-2b^3\left(\frac{a(-Ab+Ca)}{2b^3}+\frac{\sqrt{-a^3b^7}(-Bb+Da)}{2b^7}\right)}{-Bab+Da^2}\right)$$

input
```
integrate(x**3*(D*x**3+C*x**2+B*x+A)/(b*x**2+a),x)
```

output
```
C*x**4/(4*b) + D*x**5/(5*b) + x**3*(B/(3*b) - D*a/(3*b**2)) + x**2*(A/(2*b
) - C*a/(2*b**2)) + x*(-B*a/b**2 + D*a**2/b**3) + (a*(-A*b + C*a)/(2*b**3)
 - sqrt(-a**3*b**7)*(-B*b + D*a)/(2*b**7))*log(x + (-A*a*b + C*a**2 - 2*b*
*3*(a*(-A*b + C*a)/(2*b**3) - sqrt(-a**3*b**7)*(-B*b + D*a)/(2*b**7)))/(-B
*a*b + D*a**2)) + (a*(-A*b + C*a)/(2*b**3) + sqrt(-a**3*b**7)*(-B*b + D*a)
/(2*b**7))*log(x + (-A*a*b + C*a**2 - 2*b**3*(a*(-A*b + C*a)/(2*b**3) + sq
rt(-a**3*b**7)*(-B*b + D*a)/(2*b**7)))/(-B*a*b + D*a**2))
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.98

$$\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx$$
$$= \frac{(Ca^2-Aab)\log(bx^2+a)}{2\,b^3} - \frac{(Da^3-Ba^2b)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^3} + \frac{12\,Db^2x^5+15\,Cb^2x^4-20\,(Dab-Bb^2)x^3-30\,(Cab-Ab^2)x^2+60\,(Da^2-Bab)x}{60\,b^3}$$

input
```
integrate(x^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="maxima")
```

output
```
1/2*(C*a^2 - A*a*b)*log(b*x^2 + a)/b^3 - (D*a^3 - B*a^2*b)*arctan(b*x/sqrt
(a*b))/(sqrt(a*b)*b^3) + 1/60*(12*D*b^2*x^5 + 15*C*b^2*x^4 - 20*(D*a*b - B
*b^2)*x^3 - 30*(C*a*b - A*b^2)*x^2 + 60*(D*a^2 - B*a*b)*x)/b^3
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.05

$$\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx$$
$$= \frac{(Ca^2-Aab)\log(bx^2+a)}{2\,b^3} - \frac{(Da^3-Ba^2b)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^3} + \frac{12\,Db^4x^5+15\,Cb^4x^4-20\,Dab^3x^3+20\,Bb^4x^3-30\,Cab^3x^2+30\,Ab^4x^2+60\,Da^2b^2x-60\,Bab^3x}{60\,b^5}$$

input
```
integrate(x^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="giac")
```

output
```
1/2*(C*a^2 - A*a*b)*log(b*x^2 + a)/b^3 - (D*a^3 - B*a^2*b)*arctan(b*x/sqrt
(a*b))/(sqrt(a*b)*b^3) + 1/60*(12*D*b^4*x^5 + 15*C*b^4*x^4 - 20*D*a*b^3*x^
3 + 20*B*b^4*x^3 - 30*C*a*b^3*x^2 + 30*A*b^4*x^2 + 60*D*a^2*b^2*x - 60*B*a
*b^3*x)/b^5
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx=\int \frac{x^3\,\left(A+B\,x+C\,x^2+x^3\,D\right)}{b\,x^2+a}\,dx$$

input
```
int((x^3*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2),x)
```

output
```
int((x^3*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2), x)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.16

$$\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx$$
$$=\frac{-60\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^2d+60\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a\,b^2-30\log(b\,x^2+a)\,a^2b^2+30\log(b\,x^2+a)\,a^2bc}{60b^4}$$

input
```
int(x^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x)
```

output
```
( - 60*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2*d + 60*sqrt(b)*s
qrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*b**2 - 30*log(a + b*x**2)*a**2*b**2
 + 30*log(a + b*x**2)*a**2*b*c + 60*a**2*b*d*x + 30*a*b**3*x**2 - 60*a*b**
3*x - 30*a*b**2*c*x**2 - 20*a*b**2*d*x**3 + 20*b**4*x**3 + 15*b**3*c*x**4
+ 12*b**3*d*x**5)/(60*b**4)
```

# 3.37 $\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{a+bx^2} dx$



## Optimal result

Integrand size = 28, antiderivative size = 111

$$\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{a+bx^2} dx = \frac{(Ab-aC)x}{b^2}+\frac{(bB-aD)x^2}{2b^2}+\frac{Cx^3}{3b}+\frac{Dx^4}{4b}-\frac{\sqrt{a}(Ab-aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{5/2}}-\frac{a(bB-aD)\log(a+bx^2)}{2b^3}$$

output
```
(A*b-C*a)*x/b^2+1/2*(B*b-D*a)*x^2/b^2+1/3*C*x^3/b+1/4*D*x^4/b-a^(1/2)*(A*b
-C*a)*arctan(b^(1/2)*x/a^(1/2))/b^(5/2)-1/2*a*(B*b-D*a)*ln(b*x^2+a)/b^3
```

## Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.86

$$\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{a+bx^2} dx = \frac{bx(12Ab-6a(2C+Dx)+bx(6B+4Cx+3Dx^2))+12\sqrt{a}\sqrt{b}(-Ab+aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)+6a(-bB+}{12b^3}$$

input
```
Integrate[(x^2*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2),x]
```

output
```
(b*x*(12*A*b - 6*a*(2*C + D*x) + b*x*(6*B + 4*C*x + 3*D*x^2)) + 12*Sqrt[a]
*Sqrt[b]*(-(A*b) + a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]] + 6*a*(-(b*B) + a*D)*L
og[a + b*x^2])/(12*b^3)
```

## Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 (A+Bx+Cx^2+Dx^3)}{a+bx^2} \, dx$$

$$\Big\downarrow \text{2333}$$

$$\int \left(-\frac{a(Ab-aC)+ax(bB-aD)}{b^2(a+bx^2)}+\frac{Ab-aC}{b^2}+\frac{x(bB-aD)}{b^2}+\frac{Cx^2}{b}+\frac{Dx^3}{b}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{a}(Ab-aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{5/2}}+\frac{x(Ab-aC)}{b^2}-\frac{a(bB-aD)\log(a+bx^2)}{2b^3}+\frac{x^2(bB-aD)}{2b^2}+\frac{Cx^3}{3b}+\frac{Dx^4}{4b}$$

input
```
Int[(x^2*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2),x]
```

output
```
((A*b - a*C)*x)/b^2 + ((b*B - a*D)*x^2)/(2*b^2) + (C*x^3)/(3*b) + (D*x^4)/
(4*b) - (Sqrt[a]*(A*b - a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(5/2) - (a*(b*
B - a*D)*Log[a + b*x^2])/(2*b^3)
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

## Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.86

| method | result | size |
|---|---|---|
| default | $\frac{\frac{1}{4}Db\,x^4+\frac{1}{3}Cb\,x^3+\frac{1}{2}bB\,x^2-\frac{1}{2}Da\,x^2+Abx-Cax}{b^2}-\frac{a\left(\frac{(Bb-Da)\ln\left(b\,x^2+a\right)}{2b}+\frac{(Ab-Ca)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}\right)}{b^2}$ | 95 |

input
```
int(x^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/b^2*(1/4*D*b*x^4+1/3*C*b*x^3+1/2*b*B*x^2-1/2*D*a*x^2+A*b*x-C*a*x)-a/b^2*
(1/2*(B*b-D*a)/b*ln(b*x^2+a)+(A*b-C*a)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
)
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 238, normalized size of antiderivative = 2.14

$$\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx$$
$$= \left[\frac{3\,Db^2x^4+4\,Cb^2x^3-6\,(Dab-Bb^2)x^2-6\,(Cab-Ab^2)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)-12\,(Cab-Ab^2)x}{12\,b^3}\right.$$

input
```
integrate(x^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="fricas")
```

output
```
[1/12*(3*D*b^2*x^4 + 4*C*b^2*x^3 - 6*(D*a*b - B*b^2)*x^2 - 6*(C*a*b - A*b^
2)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) - 12*(C*a*b
- A*b^2)*x + 6*(D*a^2 - B*a*b)*log(b*x^2 + a))/b^3, 1/12*(3*D*b^2*x^4 + 4*
C*b^2*x^3 - 6*(D*a*b - B*b^2)*x^2 + 12*(C*a*b - A*b^2)*sqrt(a/b)*arctan(b*
x*sqrt(a/b)/a) - 12*(C*a*b - A*b^2)*x + 6*(D*a^2 - B*a*b)*log(b*x^2 + a))/
b^3]
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(97) = 194$.

Time = 0.65 (sec) , antiderivative size = 245, normalized size of antiderivative = 2.21

$$\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx = \frac{Cx^3}{3b}+\frac{Dx^4}{4b}+x^2\left(\frac{B}{2b}-\frac{Da}{2b^2}\right)+x\left(\frac{A}{b}-\frac{Ca}{b^2}\right)+\left(\frac{a(-Bb+Da)}{2b^3}-\frac{\sqrt{-ab^7}(-Ab+Ca)}{2b^6}\right)\log\left(x+\frac{Bab-Da^2+2b^3\left(\frac{a(-Bb+Da)}{2b^3}-\frac{\sqrt{-ab^7}(-Ab+Ca)}{2b^6}\right)}{-Ab^2+Cab}\right)+\left(\frac{a(-Bb+Da)}{2b^3}+\frac{\sqrt{-ab^7}(-Ab+Ca)}{2b^6}\right)\log\left(x+\frac{Bab-Da^2+2b^3\left(\frac{a(-Bb+Da)}{2b^3}+\frac{\sqrt{-ab^7}(-Ab+Ca)}{2b^6}\right)}{-Ab^2+Cab}\right)$$

input
```
integrate(x**2*(D*x**3+C*x**2+B*x+A)/(b*x**2+a),x)
```

output
```
C*x**3/(3*b) + D*x**4/(4*b) + x**2*(B/(2*b) - D*a/(2*b**2)) + x*(A/b - C*a
/b**2) + (a*(-B*b + D*a)/(2*b**3) - sqrt(-a*b**7)*(-A*b + C*a)/(2*b**6))*l
og(x + (B*a*b - D*a**2 + 2*b**3*(a*(-B*b + D*a)/(2*b**3) - sqrt(-a*b**7)*(
-A*b + C*a)/(2*b**6)))/(-A*b**2 + C*a*b)) + (a*(-B*b + D*a)/(2*b**3) + sqr
t(-a*b**7)*(-A*b + C*a)/(2*b**6))*log(x + (B*a*b - D*a**2 + 2*b**3*(a*(-B*
b + D*a)/(2*b**3) + sqrt(-a*b**7)*(-A*b + C*a)/(2*b**6)))/(-A*b**2 + C*a*b
))
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.88

$$\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx = \frac{(Ca^2-Aab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^2} + \frac{3\,Dbx^4+4\,Cbx^3-6\,(Da-Bb)x^2-12\,(Ca-Ab)x}{12\,b^2} + \frac{(Da^2-Bab)\log(bx^2+a)}{2\,b^3}$$

input
```
integrate(x^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="maxima")
```

output
```
(C*a^2 - A*a*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/12*(3*D*b*x^4 +
4*C*b*x^3 - 6*(D*a - B*b)*x^2 - 12*(C*a - A*b)*x)/b^2 + 1/2*(D*a^2 - B*a*b
)*log(b*x^2 + a)/b^3
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.01

$$\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx = \frac{(Ca^2-Aab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^2} + \frac{(Da^2-Bab)\log(bx^2+a)}{2\,b^3} + \frac{3\,Db^3x^4+4\,Cb^3x^3-6\,Dab^2x^2+6\,Bb^3x^2-12\,Cab^2x+12\,Ab^3x}{12\,b^4}$$

input
```
integrate(x^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="giac")
```

output
```
(C*a^2 - A*a*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/2*(D*a^2 - B*a*b
)*log(b*x^2 + a)/b^3 + 1/12*(3*D*b^3*x^4 + 4*C*b^3*x^3 - 6*D*a*b^2*x^2 + 6
*B*b^3*x^2 - 12*C*a*b^2*x + 12*A*b^3*x)/b^4
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx = \int \frac{x^2\,(A+B\,x+C\,x^2+x^3\,D)}{b\,x^2+a}\,dx$$

input
```
int((x^2*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2),x)
```

output
```
int((x^2*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2), x)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.09

$$\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx = \frac{-12\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)ab+12\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)ac+6\log(b\,x^2+a)\,a^2d-6\log(b\,x^2+a)\,a\,b^2+12a}{12b^3}$$

input
```
int(x^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x)
```

output
```
( - 12*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*b + 12*sqrt(b)*sqrt
(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*c + 6*log(a + b*x**2)*a**2*d - 6*log(a
 + b*x**2)*a*b**2 + 12*a*b**2*x - 12*a*b*c*x - 6*a*b*d*x**2 + 6*b**3*x**2
+ 4*b**2*c*x**3 + 3*b**2*d*x**4)/(12*b**3)
```

# 3.38 $\int \frac{x(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 92

$$\int \frac{x(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx = \frac{(bB-aD)x}{b^2}+\frac{Cx^2}{2b}+\frac{Dx^3}{3b}-\frac{\sqrt{a}(bB-aD)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{5/2}}+\frac{(Ab-aC)\log(a+bx^2)}{2b^2}$$

output
```
(B*b-D*a)*x/b^2+1/2*C*x^2/b+1/3*D*x^3/b-a^(1/2)*(B*b-D*a)*arctan(b^(1/2)*x
/a^(1/2))/b^(5/2)+1/2*(A*b-C*a)*ln(b*x^2+a)/b^2
```

## Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.88

$$\int \frac{x(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx = \frac{\sqrt{a}(-bB+aD)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{5/2}}+\frac{x(6bB-6aD+bx(3C+2Dx))+3(Ab-aC)\log(a+bx^2)}{6b^2}$$

input
```
Integrate[(x*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2),x]
```

output
```
(Sqrt[a]*(-(b*B) + a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(5/2) + (x*(6*b*B -
 6*a*D + b*x*(3*C + 2*D*x)) + 3*(A*b - a*C)*Log[a + b*x^2])/(6*b^2)
```

## Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx$$

$$\Big\downarrow \text{2333}$$

$$\int \left(-\frac{a(bB-aD)-bx(Ab-aC)}{b^2(a+bx^2)}+\frac{bB-aD}{b^2}+\frac{Cx}{b}+\frac{Dx^2}{b}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{(Ab-aC)\log(a+bx^2)}{2b^2}-\frac{\sqrt{a}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(bB-aD)}{b^{5/2}}+\frac{x(bB-aD)}{b^2}+\frac{Cx^2}{2b}+\frac{Dx^3}{3b}$$

input
```
Int[(x*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2),x]
```

output
```
((b*B - a*D)*x)/b^2 + (C*x^2)/(2*b) + (D*x^3)/(3*b) - (Sqrt[a]*(b*B - a*D)
*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(5/2) + ((A*b - a*C)*Log[a + b*x^2])/(2*b^
2)
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

## Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.92

| method | result | size |
|---|---|---|
| default | $\frac{\frac{1}{3}Dx^3b+\frac{1}{2}Cb\,x^2+bBx-Dax}{b^2}+\frac{\frac{\left(b^2A-Cab\right)\ln\left(b\,x^2+a\right)}{2b}+\frac{\left(-abB+Da^2\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}}{b^2}$ | 85 |

input
```
int(x*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/b^2*(1/3*D*x^3*b+1/2*C*b*x^2+b*B*x-D*a*x)+1/b^2*(1/2*(A*b^2-C*a*b)/b*ln(
b*x^2+a)+(-B*a*b+D*a^2)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.96

$$\int \frac{x(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx$$
$$= \left[\frac{2\,Dbx^3+3\,Cbx^2-3\,(Da-Bb)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)-6\,(Da-Bb)x-3\,(Ca-Ab)\log\left(bx^2+\right.}{6\,b^2}\right.$$

input
```
integrate(x*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="fricas")
```

output
```
[1/6*(2*D*b*x^3 + 3*C*b*x^2 - 3*(D*a - B*b)*sqrt(-a/b)*log((b*x^2 - 2*b*x*
sqrt(-a/b) - a)/(b*x^2 + a)) - 6*(D*a - B*b)*x - 3*(C*a - A*b)*log(b*x^2 +
 a))/b^2, 1/6*(2*D*b*x^3 + 3*C*b*x^2 + 6*(D*a - B*b)*sqrt(a/b)*arctan(b*x*
sqrt(a/b)/a) - 6*(D*a - B*b)*x - 3*(C*a - A*b)*log(b*x^2 + a))/b^2]
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 211 vs. $2(80) = 160$.

Time = 0.56 (sec) , antiderivative size = 211, normalized size of antiderivative = 2.29

$$\int \frac{x(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx$$
$$= \frac{Cx^2}{2b}+\frac{Dx^3}{3b}+x\left(\frac{B}{b}-\frac{Da}{b^2}\right)+\left(-\frac{-Ab+Ca}{2b^2}-\frac{\sqrt{-ab^5}(-Bb+Da)}{2b^5}\right)\log\left(x+\frac{-Ab+Ca+2b^2\left(-\frac{-Ab+Ca}{2b^2}-\frac{\sqrt{-ab^5}(-Bb+Da)}{2b^5}\right)}{-Bb+Da}\right)+\left(-\frac{-Ab+Ca}{2b^2}+\frac{\sqrt{-ab^5}(-Bb+Da)}{2b^5}\right)\log\left(x+\frac{-Ab+Ca+2b^2\left(-\frac{-Ab+Ca}{2b^2}+\frac{\sqrt{-ab^5}(-Bb+Da)}{2b^5}\right)}{-Bb+Da}\right)$$

input
```
integrate(x*(D*x**3+C*x**2+B*x+A)/(b*x**2+a),x)
```

output
```
C*x**2/(2*b) + D*x**3/(3*b) + x*(B/b - D*a/b**2) + (-(-A*b + C*a)/(2*b**2)
 - sqrt(-a*b**5)*(-B*b + D*a)/(2*b**5))*log(x + (-A*b + C*a + 2*b**2*(-(-A
*b + C*a)/(2*b**2) - sqrt(-a*b**5)*(-B*b + D*a)/(2*b**5)))/(-B*b + D*a)) +
 (-(-A*b + C*a)/(2*b**2) + sqrt(-a*b**5)*(-B*b + D*a)/(2*b**5))*log(x + (-
A*b + C*a + 2*b**2*(-(-A*b + C*a)/(2*b**2) + sqrt(-a*b**5)*(-B*b + D*a)/(2
*b**5)))/(-B*b + D*a))
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.89

$$\int \frac{x(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx = -\frac{(Ca-Ab)\log(bx^2+a)}{2\,b^2} + \frac{(Da^2-Bab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^2} + \frac{2\,Dbx^3+3\,Cbx^2-6\,(Da-Bb)x}{6\,b^2}$$

input
```
integrate(x*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="maxima")
```

output
```
-1/2*(C*a - A*b)*log(b*x^2 + a)/b^2 + (D*a^2 - B*a*b)*arctan(b*x/sqrt(a*b)
)/(sqrt(a*b)*b^2) + 1/6*(2*D*b*x^3 + 3*C*b*x^2 - 6*(D*a - B*b)*x)/b^2
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.96

$$\int \frac{x(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx = -\frac{(Ca-Ab)\log(bx^2+a)}{2\,b^2} + \frac{(Da^2-Bab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^2} + \frac{2\,Db^2x^3+3\,Cb^2x^2-6\,Dabx+6\,Bb^2x}{6\,b^3}$$

input
```
integrate(x*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="giac")
```

output
```
-1/2*(C*a - A*b)*log(b*x^2 + a)/b^2 + (D*a^2 - B*a*b)*arctan(b*x/sqrt(a*b)
)/(sqrt(a*b)*b^2) + 1/6*(2*D*b^2*x^3 + 3*C*b^2*x^2 - 6*D*a*b*x + 6*B*b^2*x
)/b^3
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx=\int \frac{x\,\left(A+B\,x+C\,x^2+x^3\,D\right)}{b\,x^2+a}\,dx$$

input
```
int((x*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2),x)
```

output
```
int((x*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2), x)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.13

$$\int \frac{x(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx = \frac{6\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)ad-6\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)b^2+3\log(b\,x^2+a)\,a\,b^2-3\log(b\,x^2+a)\,abc-6abdx+}{6b^3}$$

input
```
int(x*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x)
```

output
```
(6*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*d - 6*sqrt(b)*sqrt(a)*a
tan((b*x)/(sqrt(b)*sqrt(a)))*b**2 + 3*log(a + b*x**2)*a*b**2 - 3*log(a + b
*x**2)*a*b*c - 6*a*b*d*x + 6*b**3*x + 3*b**2*c*x**2 + 2*b**2*d*x**3)/(6*b*
*3)
```

# 3.39 $\int \frac{A+Bx+Cx^2+Dx^3}{a+bx^2}\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 73

$$\int \frac{A+Bx+Cx^2+Dx^3}{a+bx^2}\,dx=\frac{Cx}{b}+\frac{Dx^2}{2b}+\frac{(Ab-aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}+\frac{(bB-aD)\log(a+bx^2)}{2b^2}$$

output
```
C*x/b+1/2*D*x^2/b+(A*b-C*a)*arctan(b^(1/2)*x/a^(1/2))/a^(1/2)/b^(3/2)+1/2*
(B*b-D*a)*ln(b*x^2+a)/b^2
```

## Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.93

$$\int \frac{A+Bx+Cx^2+Dx^3}{a+bx^2}\,dx=\frac{bx(2C+Dx)+\frac{2\sqrt{b}(Ab-aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}}+(bB-aD)\log(a+bx^2)}{2b^2}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/(a + b*x^2),x]
```

output
```
(b*x*(2*C + D*x) + (2*Sqrt[b]*(A*b - a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqr
t[a] + (b*B - a*D)*Log[a + b*x^2])/(2*b^2)
```

## Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2341, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{a+bx^2} \, dx$$

$$\Big\downarrow \text{ 2341}$$

$$\int \left(\frac{x(bB-aD)-aC+Ab}{b\left(a+bx^2\right)}+\frac{C}{b}+\frac{Dx}{b}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{(Ab-aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}+\frac{(bB-aD)\log\left(a+bx^2\right)}{2b^2}+\frac{Cx}{b}+\frac{Dx^2}{2b}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/(a + b*x^2),x]
```

output
```
(C*x)/b + (D*x^2)/(2*b) + ((A*b - a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[
a]*b^(3/2)) + ((b*B - a*D)*Log[a + b*x^2])/(2*b^2)
```

**Defintions of rubi rules used**

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2341
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

## Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.89

| method | result | size |
|---|---|---|
| default | $\frac{\frac{1}{2}Dx^2+Cx}{b}+\frac{\frac{(Bb-Da)\ln\left(b\,x^2+a\right)}{2b}+\frac{(Ab-Ca)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}}{b}$ | 65 |

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/2*D*x^2+C*x)+1/b*(1/2*(B*b-D*a)/b*ln(b*x^2+a)+(A*b-C*a)/(a*b)^(1/2)
*arctan(b*x/(a*b)^(1/2)))
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 157, normalized size of antiderivative = 2.15

$$\int \frac{A+Bx+Cx^2+Dx^3}{a+bx^2}\,dx$$
$$=\left[\frac{Dabx^2+2\,Cabx+(Ca-Ab)\sqrt{-ab}\log\left(\frac{bx^2-2\,\sqrt{-ab}x-a}{bx^2+a}\right)-(Da^2-Bab)\log\left(bx^2+a\right)}{2\,ab^2},\frac{Dabx^2+2\,Cab}{}\right.$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="fricas")
```

output
```
[1/2*(D*a*b*x^2 + 2*C*a*b*x + (C*a - A*b)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-
a*b)*x - a)/(b*x^2 + a)) - (D*a^2 - B*a*b)*log(b*x^2 + a))/(a*b^2), 1/2*(D
*a*b*x^2 + 2*C*a*b*x - 2*(C*a - A*b)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) - (D*
a^2 - B*a*b)*log(b*x^2 + a))/(a*b^2)]
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 219 vs. $2(65) = 130$.

Time = 0.52 (sec) , antiderivative size = 219, normalized size of antiderivative = 3.00

$$\int \frac{A+Bx+Cx^2+Dx^3}{a+bx^2}\,dx = \frac{Cx}{b}+\frac{Dx^2}{2b}+\left(-\frac{-Bb+Da}{2b^2}-\frac{\sqrt{-ab^5}(-Ab+Ca)}{2ab^4}\right)\log\left(x+\frac{Bab-Da^2-2ab^2\left(-\frac{-Bb+Da}{2b^2}-\frac{\sqrt{-ab^5}(-Ab+Ca)}{2ab^4}\right)}{-Ab^2+Cab}\right)+\left(-\frac{-Bb+Da}{2b^2}+\frac{\sqrt{-ab^5}(-Ab+Ca)}{2ab^4}\right)\log\left(x+\frac{Bab-Da^2-2ab^2\left(-\frac{-Bb+Da}{2b^2}+\frac{\sqrt{-ab^5}(-Ab+Ca)}{2ab^4}\right)}{-Ab^2+Cab}\right)$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(b*x**2+a),x)
```

output
```
C*x/b + D*x**2/(2*b) + (-(-B*b + D*a)/(2*b**2) - sqrt(-a*b**5)*(-A*b + C*a
)/(2*a*b**4))*log(x + (B*a*b - D*a**2 - 2*a*b**2*(-(-B*b + D*a)/(2*b**2) -
 sqrt(-a*b**5)*(-A*b + C*a)/(2*a*b**4)))/(-A*b**2 + C*a*b)) + (-(-B*b + D*
a)/(2*b**2) + sqrt(-a*b**5)*(-A*b + C*a)/(2*a*b**4))*log(x + (B*a*b - D*a*
*2 - 2*a*b**2*(-(-B*b + D*a)/(2*b**2) + sqrt(-a*b**5)*(-A*b + C*a)/(2*a*b*
*4)))/(-A*b**2 + C*a*b))
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.88

$$\int \frac{A+Bx+Cx^2+Dx^3}{a+bx^2}\,dx = -\frac{(Ca-Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b} + \frac{Dx^2+2\,Cx}{2\,b} - \frac{(Da-Bb)\log\left(bx^2+a\right)}{2\,b^2}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="maxima")
```

output
```
-(C*a - A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b) + 1/2*(D*x^2 + 2*C*x)/b -
 1/2*(D*a - B*b)*log(b*x^2 + a)/b^2
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.90

$$\int \frac{A+Bx+Cx^2+Dx^3}{a+bx^2}\,dx = -\frac{(Ca-Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b} - \frac{(Da-Bb)\log\left(bx^2+a\right)}{2\,b^2} + \frac{Dbx^2+2\,Cbx}{2\,b^2}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="giac")
```

output
```
-(C*a - A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b) - 1/2*(D*a - B*b)*log(b*x
^2 + a)/b^2 + 1/2*(D*b*x^2 + 2*C*b*x)/b^2
```

## Mupad [B] (verification not implemented)

Time = 2.30 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.08

$$\int \frac{A+Bx+Cx^2+Dx^3}{a+bx^2}\,dx = \frac{B\,\ln\left(b\,x^2+a\right)}{2\,b}-\frac{\left(a\,\ln\left(b\,x^2+a\right)-b\,x^2\right)\,D}{2\,b^2}+\frac{C\,x}{b}+\frac{A\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}-\frac{C\,\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{3/2}}$$

input
```
int((A + B*x + C*x^2 + x^3*D)/(a + b*x^2),x)
```

output
```
(B*log(a + b*x^2))/(2*b) - ((a*log(a + b*x^2) - b*x^2)*D)/(2*b^2) + (C*x)/
b + (A*atan((b^(1/2)*x)/a^(1/2)))/(a^(1/2)*b^(1/2)) - (C*a^(1/2)*atan((b^(
1/2)*x)/a^(1/2)))/b^(3/2)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.08

$$\int \frac{A+Bx+Cx^2+Dx^3}{a+bx^2}\,dx = \frac{2\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)b-2\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)c-\log(b\,x^2+a)\,ad+\log(b\,x^2+a)\,b^2+2bcx+bd\,x^2}{2b^2}$$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x^2+a),x)
```

output
```
(2*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b - 2*sqrt(b)*sqrt(a)*ata
n((b*x)/(sqrt(b)*sqrt(a)))*c - log(a + b*x**2)*a*d + log(a + b*x**2)*b**2
+ 2*b*c*x + b*d*x**2)/(2*b**2)
```

# 3.40 $\int \frac{A+Bx+Cx^2+Dx^3}{x(a+bx^2)}\,dx$



## Optimal result

Integrand size = 28, antiderivative size = 72

$$\int \frac{A+Bx+Cx^2+Dx^3}{x(a+bx^2)}\,dx=\frac{Dx}{b}+\frac{(bB-aD)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}+\frac{A\log(x)}{a}-\frac{(Ab-aC)\log(a+bx^2)}{2ab}$$

output
```
D*x/b+(B*b-D*a)*arctan(b^(1/2)*x/a^(1/2))/a^(1/2)/b^(3/2)+A*ln(x)/a-1/2*(A
*b-C*a)*ln(b*x^2+a)/a/b
```

## Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.01

$$\int \frac{A+Bx+Cx^2+Dx^3}{x(a+bx^2)}\,dx=\frac{Dx}{b}-\frac{(-bB+aD)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}+\frac{A\log(x)}{a}+\frac{(-Ab+aC)\log(a+bx^2)}{2ab}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/(x*(a + b*x^2)),x]
```

output
```
(D*x)/b - ((-(b*B) + a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(3/2)) +
 (A*Log[x])/a + ((-(A*b) + a*C)*Log[a + b*x^2])/(2*a*b)
```

## Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{x\left(a+bx^2\right)}\,dx$$

$$\Big\downarrow \text{2333}$$

$$\int \left(\frac{a(bB-aD)-bx(Ab-aC)}{ab\left(a+bx^2\right)}+\frac{A}{ax}+\frac{D}{b}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{(Ab-aC)\log\left(a+bx^2\right)}{2ab}+\frac{A\log(x)}{a}+\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(bB-aD)}{\sqrt{a}b^{3/2}}+\frac{Dx}{b}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/(x*(a + b*x^2)),x]
```

output
```
(D*x)/b + ((b*B - a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(3/2)) + (A
*Log[x])/a - ((A*b - a*C)*Log[a + b*x^2])/(2*a*b)
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

## Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.01

| method | result | size |
|---|---|---|
| default | $\frac{Dx}{b}+\frac{\frac{\left(-b^{2}A+Cab\right)\ln\left(b\,x^{2}+a\right)}{2b}+\frac{\left(abB-Da^{2}\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}}{ab}+\frac{A\ln(x)}{a}$ | 73 |

input
```
int((D*x^3+C*x^2+B*x+A)/x/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
D*x/b+1/a/b*(1/2*(-A*b^2+C*a*b)/b*ln(b*x^2+a)+(B*a*b-D*a^2)/(a*b)^(1/2)*ar
ctan(b*x/(a*b)^(1/2)))+A*ln(x)/a
```

## Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 157, normalized size of antiderivative = 2.18

$$\int \frac{A+Bx+Cx^2+Dx^3}{x\left(a+bx^2\right)}\,dx = \left[\frac{2\,Dabx+2\,Ab^2\log\left(x\right)+\left(Da-Bb\right)\sqrt{-ab}\log\left(\frac{bx^2-2\,\sqrt{-ab}x-a}{bx^2+a}\right)+\left(Cab-Ab^2\right)\log\left(bx^2+a\right)}{2\,ab^2}, \frac{2\,Dabx+\right.$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/x/(b*x^2+a),x, algorithm="fricas")
```

output
```
[1/2*(2*D*a*b*x + 2*A*b^2*log(x) + (D*a - B*b)*sqrt(-a*b)*log((b*x^2 - 2*s
qrt(-a*b)*x - a)/(b*x^2 + a)) + (C*a*b - A*b^2)*log(b*x^2 + a))/(a*b^2), 1
/2*(2*D*a*b*x + 2*A*b^2*log(x) - 2*(D*a - B*b)*sqrt(a*b)*arctan(sqrt(a*b)*
x/a) + (C*a*b - A*b^2)*log(b*x^2 + a))/(a*b^2)]
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{A + Bx + Cx^2 + Dx^3}{x\left(a + bx^2\right)}\,dx = \text{Timed out}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/x/(b*x**2+a),x)
```

output
```
Timed out
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.90

$$\int \frac{A + Bx + Cx^2 + Dx^3}{x\left(a + bx^2\right)}\,dx = \frac{Dx}{b} + \frac{A\log\left(x\right)}{a} - \frac{(Da - Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b} + \frac{(Ca - Ab)\log\left(bx^2 + a\right)}{2\,ab}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/x/(b*x^2+a),x, algorithm="maxima")
```

output
```
D*x/b + A*log(x)/a - (D*a - B*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b) + 1/2
*(C*a - A*b)*log(b*x^2 + a)/(a*b)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.92

$$\int \frac{A+Bx+Cx^2+Dx^3}{x(a+bx^2)}\,dx = \frac{Dx}{b}+\frac{A\log(|x|)}{a}-\frac{(Da-Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b}+\frac{(Ca-Ab)\log(bx^2+a)}{2ab}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/x/(b*x^2+a),x, algorithm="giac")
```

output
```
D*x/b + A*log(abs(x))/a - (D*a - B*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b)
+ 1/2*(C*a - A*b)*log(b*x^2 + a)/(a*b)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{x(a+bx^2)}\,dx = \int \frac{A+B\,x+C\,x^2+x^3\,D}{x\,(b\,x^2+a)}\,dx$$

input
```
int((A + B*x + C*x^2 + x^3*D)/(x*(a + b*x^2)),x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/(x*(a + b*x^2)), x)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.25

$$\int \frac{A+Bx+Cx^2+Dx^3}{x(a+bx^2)}\,dx = \frac{-2\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)ad+2\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)b^2-\log(b\,x^2+a)\,a\,b^2+\log(b\,x^2+a)\,abc+2\log(x)\,a}{2a\,b^2}$$

input
```
int((D*x^3+C*x^2+B*x+A)/x/(b*x^2+a),x)
```

output
```
( - 2*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*d + 2*sqrt(b)*sqrt(a
)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**2 - log(a + b*x**2)*a*b**2 + log(a + b*
x**2)*a*b*c + 2*log(x)*a*b**2 + 2*a*b*d*x)/(2*a*b**2)
```

# 3.41 $\int \frac{A+Bx+Cx^2+Dx^3}{x^2(a+bx^2)}\,dx$



## Optimal result

Integrand size = 28, antiderivative size = 76

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^2(a+bx^2)}\,dx=-\frac{A}{ax}-\frac{(Ab-aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}+\frac{B\log(x)}{a}-\frac{(bB-aD)\log(a+bx^2)}{2ab}$$

output
```
-A/a/x-(A*b-C*a)*arctan(b^(1/2)*x/a^(1/2))/a^(3/2)/b^(1/2)+B*ln(x)/a-1/2*(
B*b-D*a)*ln(b*x^2+a)/a/b
```

## Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.99

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^2(a+bx^2)}\,dx=-\frac{A}{ax}+\frac{(-Ab+aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}+\frac{B\log(x)}{a}+\frac{(-bB+aD)\log(a+bx^2)}{2ab}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/(x^2*(a + b*x^2)),x]
```

output
```
-(A/(a*x)) + ((-(A*b) + a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*Sqrt[b]
) + (B*Log[x])/a + ((-(b*B) + a*D)*Log[a + b*x^2])/(2*a*b)
```

## Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A + Bx + Cx^2 + Dx^3}{x^2 (a + bx^2)} \, dx$$

$$\Big\downarrow \; 2333$$

$$\int \left( \frac{-x(bB - aD) + aC - Ab}{a (a + bx^2)} + \frac{A}{ax^2} + \frac{B}{ax} \right) dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{(Ab - aC) \arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}} - \frac{A}{ax} - \frac{(bB - aD) \log\left(a + bx^2\right)}{2ab} + \frac{B \log(x)}{a}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/(x^2*(a + b*x^2)),x]
```

output
```
-(A/(a*x)) - ((A*b - a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*Sqrt[b]) +
 (B*Log[x])/a - ((b*B - a*D)*Log[a + b*x^2])/(2*a*b)
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

## Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.88

| method | result | size |
|---|---|---|
| default | $\frac{\frac{(-Bb+Da)\ln\left(b\,x^{2}+a\right)}{2b}+\frac{(-Ab+Ca)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}}{a}-\frac{A}{ax}+\frac{B\ln(x)}{a}$ | 67 |

input
```
int((D*x^3+C*x^2+B*x+A)/x^2/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/a*(1/2*(-B*b+D*a)/b*ln(b*x^2+a)+(-A*b+C*a)/(a*b)^(1/2)*arctan(b*x/(a*b)^
(1/2)))-A/a/x+B*ln(x)/a
```

## Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 165, normalized size of antiderivative = 2.17

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^2\,(a+bx^2)}\,dx$$
$$=\left[\frac{2\,Babx\log\left(x\right)+(Ca-Ab)\sqrt{-ab}x\log\left(\frac{bx^2+2\,\sqrt{-ab}x-a}{bx^2+a}\right)-2\,Aab+(Da^2-Bab)x\log\left(bx^2+a\right)}{2\,a^2bx},\ \frac{2\,Babx}{}\right.$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/x^2/(b*x^2+a),x, algorithm="fricas")
```

output
```
[1/2*(2*B*a*b*x*log(x) + (C*a - A*b)*sqrt(-a*b)*x*log((b*x^2 + 2*sqrt(-a*b
)*x - a)/(b*x^2 + a)) - 2*A*a*b + (D*a^2 - B*a*b)*x*log(b*x^2 + a))/(a^2*b
*x), 1/2*(2*B*a*b*x*log(x) + 2*(C*a - A*b)*sqrt(a*b)*x*arctan(sqrt(a*b)*x/
a) - 2*A*a*b + (D*a^2 - B*a*b)*x*log(b*x^2 + a))/(a^2*b*x)]
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^2\left(a+bx^2\right)}\,dx=\text{Timed out}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/x**2/(b*x**2+a),x)
```

output
```
Timed out
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.88

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^2\left(a+bx^2\right)}\,dx=\frac{B\log\left(x\right)}{a}+\frac{\left(Ca-Ab\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a}+\frac{\left(Da-Bb\right)\log\left(bx^2+a\right)}{2\,ab}-\frac{A}{ax}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/x^2/(b*x^2+a),x, algorithm="maxima")
```

output
```
B*log(x)/a + (C*a - A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) + 1/2*(D*a -
B*b)*log(b*x^2 + a)/(a*b) - A/(a*x)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.89

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^2(a+bx^2)}\,dx = \frac{B\log(|x|)}{a} + \frac{(Ca-Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a} + \frac{(Da-Bb)\log(bx^2+a)}{2ab} - \frac{A}{ax}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/x^2/(b*x^2+a),x, algorithm="giac")
```

output
```
B*log(abs(x))/a + (C*a - A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) + 1/2*(D
*a - B*b)*log(b*x^2 + a)/(a*b) - A/(a*x)
```

## Mupad [B] (verification not implemented)

Time = 2.34 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.03

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^2(a+bx^2)}\,dx = \frac{\ln(b\,x^2+a)\ D}{2\,b} - \frac{A}{a\,x} - \frac{B\,(\ln(b\,x^2+a)-2\,\ln(x))}{2\,a} - \frac{A\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}} + \frac{C\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

input
```
int((A + B*x + C*x^2 + x^3*D)/(x^2*(a + b*x^2)),x)
```

output
```
(log(a + b*x^2)*D)/(2*b) - A/(a*x) - (B*(log(a + b*x^2) - 2*log(x)))/(2*a)
 - (A*b^(1/2)*atan((b^(1/2)*x)/a^(1/2)))/a^(3/2) + (C*atan((b^(1/2)*x)/a^(
1/2)))/(a^(1/2)*b^(1/2))
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.18

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^2\left(a+bx^2\right)}\,dx$$
$$=\frac{-2\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)bx+2\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)cx+\log(b\,x^2+a)\,adx-\log(b\,x^2+a)\,b^2x+2\log(x)\,b}{2abx}$$

input
```
int((D*x^3+C*x^2+B*x+A)/x^2/(b*x^2+a),x)
```

output
```
( - 2*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b*x + 2*sqrt(b)*sqrt(a
)*atan((b*x)/(sqrt(b)*sqrt(a)))*c*x + log(a + b*x**2)*a*d*x - log(a + b*x*
*2)*b**2*x + 2*log(x)*b**2*x - 2*a*b)/(2*a*b*x)
```

# 3.42 $\int \frac{A+Bx+Cx^2+Dx^3}{x^3(a+bx^2)}\,dx$



## Optimal result

Integrand size = 28, antiderivative size = 92

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^3(a+bx^2)}\,dx = -\frac{A}{2ax^2}-\frac{B}{ax}-\frac{(bB-aD)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}} - \frac{(Ab-aC)\log(x)}{a^2}+\frac{(Ab-aC)\log(a+bx^2)}{2a^2}$$

output
```
-1/2*A/a/x^2-B/a/x-(B*b-D*a)*arctan(b^(1/2)*x/a^(1/2))/a^(3/2)/b^(1/2)-(A*
b-C*a)*ln(x)/a^2+1/2*(A*b-C*a)*ln(b*x^2+a)/a^2
```

## Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.91

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^3(a+bx^2)}\,dx = \frac{-\frac{aA}{x^2}-\frac{2aB}{x}+\frac{2\sqrt{a}(-bB+aD)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}+2(-Ab+aC)\log(x)+(Ab-aC)\log(a+bx^2)}{2a^2}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/(x^3*(a + b*x^2)),x]
```

output
```
(-((a*A)/x^2) - (2*a*B)/x + (2*Sqrt[a]*(-(b*B) + a*D)*ArcTan[(Sqrt[b]*x)/S
qrt[a]])/Sqrt[b] + 2*(-(A*b) + a*C)*Log[x] + (A*b - a*C)*Log[a + b*x^2])/(
2*a^2)
```

## Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^3\left(a+bx^2\right)}dx$$

$$\Big\downarrow \text{2333}$$

$$\int \left(\frac{bx(Ab-aC)-a(bB-aD)}{a^2\left(a+bx^2\right)}+\frac{aC-Ab}{a^2x}+\frac{A}{ax^3}+\frac{B}{ax^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(bB-aD)}{a^{3/2}\sqrt{b}}+\frac{(Ab-aC)\log\left(a+bx^2\right)}{2a^2}-\frac{\log(x)(Ab-aC)}{a^2}-\frac{A}{2ax^2}-\frac{B}{ax}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/(x^3*(a + b*x^2)),x]
```

output
```
-1/2*A/(a*x^2) - B/(a*x) - ((b*B - a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3
/2)*Sqrt[b]) - ((A*b - a*C)*Log[x])/a^2 + ((A*b - a*C)*Log[a + b*x^2])/(2*
a^2)
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

## Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.97

| method | result | size |
|---|---|---|
| default | $\frac{\frac{\left(b^2A-Cab\right)\ln\left(b\,x^2+a\right)}{2b}+\frac{\left(-abB+Da^2\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}}{a^2}-\frac{A}{2a\,x^2}-\frac{B}{ax}+\frac{\left(-Ab+Ca\right)\ln\left(x\right)}{a^2}$ | 89 |

input
```
int((D*x^3+C*x^2+B*x+A)/x^3/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/a^2*(1/2*(A*b^2-C*a*b)/b*ln(b*x^2+a)+(-B*a*b+D*a^2)/(a*b)^(1/2)*arctan(b
*x/(a*b)^(1/2)))-1/2*A/a/x^2-B/a/x+1/a^2*(-A*b+C*a)*ln(x)
```

## Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 207, normalized size of antiderivative = 2.25

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^3\left(a+bx^2\right)}\,dx$$
$$=\left[\frac{(Da-Bb)\sqrt{-ab}x^2\log\left(\frac{bx^2+2\sqrt{-ab}x-a}{bx^2+a}\right)-2\,Babx-(Cab-Ab^2)x^2\log\left(bx^2+a\right)+2\,(Cab-Ab^2)x^2\log\right.}{2\,a^2bx^2}\right.$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/x^3/(b*x^2+a),x, algorithm="fricas")
```

output
```
[1/2*((D*a - B*b)*sqrt(-a*b)*x^2*log((b*x^2 + 2*sqrt(-a*b)*x - a)/(b*x^2 +
 a)) - 2*B*a*b*x - (C*a*b - A*b^2)*x^2*log(b*x^2 + a) + 2*(C*a*b - A*b^2)*
x^2*log(x) - A*a*b)/(a^2*b*x^2), 1/2*(2*(D*a - B*b)*sqrt(a*b)*x^2*arctan(s
qrt(a*b)*x/a) - 2*B*a*b*x - (C*a*b - A*b^2)*x^2*log(b*x^2 + a) + 2*(C*a*b
- A*b^2)*x^2*log(x) - A*a*b)/(a^2*b*x^2)]
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^3(a+bx^2)}\,dx = \text{Timed out}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/x**3/(b*x**2+a),x)
```

output
```
Timed out
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.83

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^3(a+bx^2)}\,dx = \frac{(Da-Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a} - \frac{(Ca-Ab)\log(bx^2+a)}{2\,a^2} + \frac{(Ca-Ab)\log(x)}{a^2} - \frac{2\,Bx+A}{2\,ax^2}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/x^3/(b*x^2+a),x, algorithm="maxima")
```

output
```
(D*a - B*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) - 1/2*(C*a - A*b)*log(b*x^
2 + a)/a^2 + (C*a - A*b)*log(x)/a^2 - 1/2*(2*B*x + A)/(a*x^2)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.87

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^3(a+bx^2)}\,dx = \frac{(Da-Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a} - \frac{(Ca-Ab)\log(bx^2+a)}{2\,a^2} + \frac{(Ca-Ab)\log(|x|)}{a^2} - \frac{2\,Bax+Aa}{2\,a^2x^2}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/x^3/(b*x^2+a),x, algorithm="giac")
```

output
```
(D*a - B*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) - 1/2*(C*a - A*b)*log(b*x^
2 + a)/a^2 + (C*a - A*b)*log(abs(x))/a^2 - 1/2*(2*B*a*x + A*a)/(a^2*x^2)
```

## Mupad [B] (verification not implemented)

Time = 2.43 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.05

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^3(a+bx^2)}\,dx = \frac{\operatorname{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,D}{\sqrt{a}\,\sqrt{b}} - \frac{B}{a\,x} - \frac{C\,(\ln(b\,x^2+a)-2\,\ln(x))}{2\,a} - \frac{A}{2\,a\,x^2} + \frac{A\,b\,\ln(b\,x^2+a)}{2\,a^2} - \frac{A\,b\,\ln(x)}{a^2} - \frac{B\,\sqrt{b}\,\operatorname{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}}$$

input
```
int((A + B*x + C*x^2 + x^3*D)/(x^3*(a + b*x^2)),x)
```

output
```
(atan((b^(1/2)*x)/a^(1/2))*D)/(a^(1/2)*b^(1/2)) - B/(a*x) - (C*(log(a + b*
x^2) - 2*log(x)))/(2*a) - A/(2*a*x^2) + (A*b*log(a + b*x^2))/(2*a^2) - (A*
b*log(x))/a^2 - (B*b^(1/2)*atan((b^(1/2)*x)/a^(1/2)))/a^(3/2)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.36

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^3\,(a+bx^2)}\,dx$$
$$= \frac{2\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)ad\,x^2-2\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)b^2x^2+\log(b\,x^2+a)\,a\,b^2x^2-\log(b\,x^2+a)\,abc\,x^2-2}{2a^2b\,x^2}$$

input
```
int((D*x^3+C*x^2+B*x+A)/x^3/(b*x^2+a),x)
```

output
```
(2*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*d*x**2 - 2*sqrt(b)*sqrt
(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**2*x**2 + log(a + b*x**2)*a*b**2*x**2
- log(a + b*x**2)*a*b*c*x**2 - 2*log(x)*a*b**2*x**2 + 2*log(x)*a*b*c*x**2
- a**2*b - 2*a*b**2*x)/(2*a**2*b*x**2)
```

# 3.43 $\int \frac{A+Bx+Cx^2+Dx^3}{x^4(a+bx^2)}\,dx$



## Optimal result

Integrand size = 28, antiderivative size = 108

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^4(a+bx^2)}\,dx=-\frac{A}{3ax^3}-\frac{B}{2ax^2}+\frac{Ab-aC}{a^2x}+\frac{\sqrt{b}(Ab-aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}}-\frac{(bB-aD)\log(x)}{a^2}+\frac{(bB-aD)\log(a+bx^2)}{2a^2}$$

output
```
-1/3*A/a/x^3-1/2*B/a/x^2+(A*b-C*a)/a^2/x+b^(1/2)*(A*b-C*a)*arctan(b^(1/2)*
x/a^(1/2))/a^(5/2)-(B*b-D*a)*ln(x)/a^2+1/2*(B*b-D*a)*ln(b*x^2+a)/a^2
```

## Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.95

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^4(a+bx^2)}\,dx=\frac{\sqrt{b}(Ab-aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}}-\frac{2aA+3aBx-6Abx^2+6aCx^2+6(bB-aD)x^3\log(x)+3(-bB+aD)x^3\log(a+bx^2)}{6a^2x^3}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/(x^4*(a + b*x^2)),x]
```

output
```
(Sqrt[b]*(A*b - a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(5/2) - (2*a*A + 3*a*B
*x - 6*A*b*x^2 + 6*a*C*x^2 + 6*(b*B - a*D)*x^3*Log[x] + 3*(-(b*B) + a*D)*x
^3*Log[a + b*x^2])/(6*a^2*x^3)
```

## Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^4\left(a+bx^2\right)}dx$$

$$\Big\downarrow \text{ 2333}$$

$$\int \left(\frac{b(x(bB-aD)-aC+Ab)}{a^2\left(a+bx^2\right)}+\frac{aC-Ab}{a^2x^2}+\frac{aD-bB}{a^2x}+\frac{A}{ax^4}+\frac{B}{ax^3}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{b}(Ab-aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}}+\frac{Ab-aC}{a^2x}+\frac{(bB-aD)\log\left(a+bx^2\right)}{2a^2}-\frac{\log(x)(bB-aD)}{a^2}-\frac{A}{3ax^3}-\frac{B}{2ax^2}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/(x^4*(a + b*x^2)),x]
```

output
```
-1/3*A/(a*x^3) - B/(2*a*x^2) + (A*b - a*C)/(a^2*x) + (Sqrt[b]*(A*b - a*C)*
ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(5/2) - ((b*B - a*D)*Log[x])/a^2 + ((b*B -
a*D)*Log[a + b*x^2])/(2*a^2)
```

**Defintions of rubi rules used**

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

## Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.93

| method | result | size |
|---|---|---|
| default | $\frac{b\left(\frac{(Bb-Da)\ln\left(b\,x^{2}+a\right)}{2b}+\frac{(Ab-Ca)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}\right)}{a^{2}}-\frac{A}{3a\,x^{3}}-\frac{B}{2a\,x^{2}}-\frac{-Ab+Ca}{x\,a^{2}}+\frac{(-Bb+Da)\ln(x)}{a^{2}}$ | 100 |

input
```
int((D*x^3+C*x^2+B*x+A)/x^4/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
b/a^2*(1/2*(B*b-D*a)/b*ln(b*x^2+a)+(A*b-C*a)/(a*b)^(1/2)*arctan(b*x/(a*b)^
(1/2)))-1/3*A/a/x^3-1/2*B/a/x^2-(-A*b+C*a)/x/a^2+1/a^2*(-B*b+D*a)*ln(x)
```

## Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 218, normalized size of antiderivative = 2.02

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^4\left(a+bx^2\right)}\,dx$$
$$= \left[-\frac{3\,(Ca-Ab)x^3\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)+3\,(Da-Bb)x^3\log\left(bx^2+a\right)-6\,(Da-Bb)x^3\log\left(x\right)+}{6\,a^2x^3}\right.$$

$$-\frac{6\,(Ca-Ab)x^3\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)+3\,(Da-Bb)x^3\log\left(bx^2+a\right)-6\,(Da-Bb)x^3\log\left(x\right)+3\,Bax+}{6\,a^2x^3}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/x^4/(b*x^2+a),x, algorithm="fricas")
```

output
```
[-1/6*(3*(C*a - A*b)*x^3*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*
x^2 + a)) + 3*(D*a - B*b)*x^3*log(b*x^2 + a) - 6*(D*a - B*b)*x^3*log(x) +
3*B*a*x + 6*(C*a - A*b)*x^2 + 2*A*a)/(a^2*x^3), -1/6*(6*(C*a - A*b)*x^3*sq
rt(b/a)*arctan(x*sqrt(b/a)) + 3*(D*a - B*b)*x^3*log(b*x^2 + a) - 6*(D*a -
B*b)*x^3*log(x) + 3*B*a*x + 6*(C*a - A*b)*x^2 + 2*A*a)/(a^2*x^3)]
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{A + Bx + Cx^2 + Dx^3}{x^4 (a + bx^2)}\, dx = \text{Timed out}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/x**4/(b*x**2+a),x)
```

output
```
Timed out
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.90

$$\int \frac{A + Bx + Cx^2 + Dx^3}{x^4 (a + bx^2)}\, dx = -\frac{(Da - Bb)\log(bx^2 + a)}{2a^2} + \frac{(Da - Bb)\log(x)}{a^2} - \frac{(Cab - Ab^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a^2} - \frac{3Bax + 6(Ca - Ab)x^2 + 2Aa}{6a^2x^3}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/x^4/(b*x^2+a),x, algorithm="maxima")
```

output
```
-1/2*(D*a - B*b)*log(b*x^2 + a)/a^2 + (D*a - B*b)*log(x)/a^2 - (C*a*b - A*
b^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) - 1/6*(3*B*a*x + 6*(C*a - A*b)*
x^2 + 2*A*a)/(a^2*x^3)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.91

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^4(a+bx^2)}\,dx = -\frac{(Da-Bb)\log(bx^2+a)}{2a^2} + \frac{(Da-Bb)\log(|x|)}{a^2} - \frac{(Cab-Ab^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a^2} - \frac{3Bax+6(Ca-Ab)x^2+2Aa}{6a^2x^3}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/x^4/(b*x^2+a),x, algorithm="giac")
```

output
```
-1/2*(D*a - B*b)*log(b*x^2 + a)/a^2 + (D*a - B*b)*log(abs(x))/a^2 - (C*a*b
 - A*b^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) - 1/6*(3*B*a*x + 6*(C*a -
A*b)*x^2 + 2*A*a)/(a^2*x^3)
```

## Mupad [B] (verification not implemented)

Time = 2.30 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.09

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^4(a+bx^2)}\,dx = \frac{B\,b\,\ln(b\,x^2+a)}{2\,a^2} - \frac{B}{2\,a\,x^2} - \frac{C}{a\,x} - \frac{\ln\left(\frac{b\,x^2+a}{x^2}\right)\,D}{2\,a} - \frac{\frac{A}{3\,a} - \frac{A\,b\,x^2}{a^2}}{x^3} - \frac{B\,b\,\ln(x)}{a^2} + \frac{A\,b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}} - \frac{C\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}}$$

input
```
int((A + B*x + C*x^2 + x^3*D)/(x^4*(a + b*x^2)),x)
```

output
```
(B*b*log(a + b*x^2))/(2*a^2) - B/(2*a*x^2) - C/(a*x) - (log((a + b*x^2)/x^
2)*D)/(2*a) - (A/(3*a) - (A*b*x^2)/a^2)/x^3 - (B*b*log(x))/a^2 + (A*b^(3/2
)*atan((b^(1/2)*x)/a^(1/2)))/a^(5/2) - (C*b^(1/2)*atan((b^(1/2)*x)/a^(1/2)
))/a^(3/2)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.18

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^4\left(a+bx^2\right)}\,dx$$
$$=\frac{6\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)b\,x^3-6\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)c\,x^3-3\log(b\,x^2+a)\,ad\,x^3+3\log(b\,x^2+a)\,b^2x^3+6\,\mathrm{lo}}{6a^2x^3}$$

input
```
int((D*x^3+C*x^2+B*x+A)/x^4/(b*x^2+a),x)
```

output
```
(6*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b*x**3 - 6*sqrt(b)*sqrt(a
)*atan((b*x)/(sqrt(b)*sqrt(a)))*c*x**3 - 3*log(a + b*x**2)*a*d*x**3 + 3*lo
g(a + b*x**2)*b**2*x**3 + 6*log(x)*a*d*x**3 - 6*log(x)*b**2*x**3 - 2*a**2
+ 6*a*b*x**2 - 3*a*b*x - 6*a*c*x**2)/(6*a**2*x**3)
```

# 3.44 $\int \frac{A+Bx+Cx^2+Dx^3}{x^5(a+bx^2)}\,dx$



## Optimal result

Integrand size = 28, antiderivative size = 127

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^5(a+bx^2)}\,dx = -\frac{A}{4ax^4}-\frac{B}{3ax^3}+\frac{Ab-aC}{2a^2x^2}+\frac{bB-aD}{a^2x} + \frac{\sqrt{b}(bB-aD)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}} + \frac{b(Ab-aC)\log(x)}{a^3}-\frac{b(Ab-aC)\log(a+bx^2)}{2a^3}$$

output
```
-1/4*A/a/x^4-1/3*B/a/x^3+1/2*(A*b-C*a)/a^2/x^2+(B*b-D*a)/a^2/x+b^(1/2)*(B*
b-D*a)*arctan(b^(1/2)*x/a^(1/2))/a^(5/2)+b*(A*b-C*a)*ln(x)/a^3-1/2*b*(A*b-
C*a)*ln(b*x^2+a)/a^3
```

## Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.89

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^5(a+bx^2)}\,dx = \frac{\frac{6ab(A+2Bx)}{x^2}-\frac{a^2(3A+4Bx+6x^2(C+2Dx))}{x^4}-12\sqrt{a}\sqrt{b}(-bB+aD)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)+12b(Ab-aC)\log(x)+6b(-}{12a^3}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/(x^5*(a + b*x^2)),x]
```

output
```
((6*a*b*(A + 2*B*x))/x^2 - (a^2*(3*A + 4*B*x + 6*x^2*(C + 2*D*x)))/x^4 - 1
2*Sqrt[a]*Sqrt[b]*(-(b*B) + a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]] + 12*b*(A*b -
 a*C)*Log[x] + 6*b*(-(A*b) + a*C)*Log[a + b*x^2])/(12*a^3)
```

## Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^5\left(a+bx^2\right)}\,dx$$

$$\Big\downarrow \text{2333}$$

$$\int \left(\frac{b(a(bB-aD)-bx(Ab-aC))}{a^3\left(a+bx^2\right)}-\frac{b(aC-Ab)}{a^3x}+\frac{aC-Ab}{a^2x^3}+\frac{aD-bB}{a^2x^2}+\frac{A}{ax^5}+\frac{B}{ax^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(bB-aD)}{a^{5/2}}-\frac{b(Ab-aC)\log\left(a+bx^2\right)}{2a^3}+\frac{b\log(x)(Ab-aC)}{a^3}+\frac{Ab-aC}{2a^2x^2}+\frac{bB-aD}{a^2x}-\frac{A}{4ax^4}-\frac{B}{3ax^3}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/(x^5*(a + b*x^2)),x]
```

output
```
-1/4*A/(a*x^4) - B/(3*a*x^3) + (A*b - a*C)/(2*a^2*x^2) + (b*B - a*D)/(a^2*
x) + (Sqrt[b]*(b*B - a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(5/2) + (b*(A*b -
 a*C)*Log[x])/a^3 - (b*(A*b - a*C)*Log[a + b*x^2])/(2*a^3)
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

## Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.98

| method | result |
|---|---|
| default | $-\frac{b\left(\frac{\left(b^{2}A-Cab\right)\ln\left(b\,x^{2}+a\right)}{2b}+\frac{\left(-abB+D a^{2}\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}\right)}{a^{3}}-\frac{A}{4a\,x^{4}}-\frac{B}{3a\,x^{3}}-\frac{-Ab+Ca}{2x^{2}a^{2}}-\frac{-Bb+Da}{x\,a^{2}}+\frac{b(Ab-Ca)\ln(x)}{a^{3}}$ |

input
```
int((D*x^3+C*x^2+B*x+A)/x^5/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/a^3*b*(1/2*(A*b^2-C*a*b)/b*ln(b*x^2+a)+(-B*a*b+D*a^2)/(a*b)^(1/2)*arcta
n(b*x/(a*b)^(1/2)))-1/4*A/a/x^4-1/3*B/a/x^3-1/2*(-A*b+C*a)/x^2/a^2-(-B*b+D
*a)/x/a^2+b*(A*b-C*a)*ln(x)/a^3
```

## Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 282, normalized size of antiderivative = 2.22

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^5(a+bx^2)}\,dx$$
$$= \left[-\frac{6\,(Da^2-Bab)x^4\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)-6\,(Cab-Ab^2)x^4\log\left(bx^2+a\right)+12\,(Cab-Ab^2)x^4\log}{12\,a^3x^4}\right.$$
$$-\frac{12\,(Da^2-Bab)x^4\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)-6\,(Cab-Ab^2)x^4\log\left(bx^2+a\right)+12\,(Cab-Ab^2)x^4\log\left(x\right)+}{12\,a^3x^4}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/x^5/(b*x^2+a),x, algorithm="fricas")
```

output
```
[-1/12*(6*(D*a^2 - B*a*b)*x^4*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a
)/(b*x^2 + a)) - 6*(C*a*b - A*b^2)*x^4*log(b*x^2 + a) + 12*(C*a*b - A*b^2)
*x^4*log(x) + 4*B*a^2*x + 12*(D*a^2 - B*a*b)*x^3 + 3*A*a^2 + 6*(C*a^2 - A*
a*b)*x^2)/(a^3*x^4), -1/12*(12*(D*a^2 - B*a*b)*x^4*sqrt(b/a)*arctan(x*sqrt
(b/a)) - 6*(C*a*b - A*b^2)*x^4*log(b*x^2 + a) + 12*(C*a*b - A*b^2)*x^4*log
(x) + 4*B*a^2*x + 12*(D*a^2 - B*a*b)*x^3 + 3*A*a^2 + 6*(C*a^2 - A*a*b)*x^2
)/(a^3*x^4)]
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^5(a+bx^2)}\,dx = \text{Timed out}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/x**5/(b*x**2+a),x)
```

output
```
Timed out
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.92

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^5(a+bx^2)}\,dx = -\frac{(Dab-Bb^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a^2} + \frac{(Cab-Ab^2)\log(bx^2+a)}{2a^3} - \frac{(Cab-Ab^2)\log(x)}{a^3} - \frac{12(Da-Bb)x^3+4Bax+6(Ca-Ab)x^2+3Aa}{12a^2x^4}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/x^5/(b*x^2+a),x, algorithm="maxima")
```

output
```
-(D*a*b - B*b^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) + 1/2*(C*a*b - A*b^
2)*log(b*x^2 + a)/a^3 - (C*a*b - A*b^2)*log(x)/a^3 - 1/12*(12*(D*a - B*b)*
x^3 + 4*B*a*x + 6*(C*a - A*b)*x^2 + 3*A*a)/(a^2*x^4)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.01

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^5(a+bx^2)}\,dx = -\frac{(Dab-Bb^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a^2} + \frac{(Cab-Ab^2)\log(bx^2+a)}{2a^3} - \frac{(Cab-Ab^2)\log(|x|)}{a^3} - \frac{4Ba^2x+12(Da^2-Bab)x^3+3Aa^2+6(Ca^2-Aab)x^2}{12a^3x^4}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/x^5/(b*x^2+a),x, algorithm="giac")
```

output
```
-(D*a*b - B*b^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) + 1/2*(C*a*b - A*b^
2)*log(b*x^2 + a)/a^3 - (C*a*b - A*b^2)*log(abs(x))/a^3 - 1/12*(4*B*a^2*x
+ 12*(D*a^2 - B*a*b)*x^3 + 3*A*a^2 + 6*(C*a^2 - A*a*b)*x^2)/(a^3*x^4)
```

## Mupad [B] (verification not implemented)

Time = 2.39 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.14

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^5\,(a+bx^2)}\,dx = \frac{A\,b^2\,\ln(x)}{a^3}-\frac{\frac{B}{3\,a}-\frac{B\,b\,x^2}{a^2}}{x^3}-\frac{C}{2\,a\,x^2}-\frac{\frac{A}{4\,a}-\frac{A\,b\,x^2}{2\,a^2}}{x^4}-\frac{D\,{}_2\mathrm{F}_1\left(1,\frac{3}{2};\ \frac{5}{2};\ -\frac{a}{b\,x^2}\right)}{3\,b\,x^3}+\frac{C\,b\,\ln\left(b\,x^2+a\right)}{2\,a^2}-\frac{C\,b\,\ln(x)}{a^2}+\frac{B\,b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}}-\frac{A\,b^2\,\ln\left(b\,x^2+a\right)}{2\,a^3}$$

input
```
int((A + B*x + C*x^2 + x^3*D)/(x^5*(a + b*x^2)),x)
```

output
```
(A*b^2*log(x))/a^3 - (B/(3*a) - (B*b*x^2)/a^2)/x^3 - C/(2*a*x^2) - (A/(4*a
) - (A*b*x^2)/(2*a^2))/x^4 - (D*hypergeom([1, 3/2], 5/2, -a/(b*x^2)))/(3*b
*x^3) + (C*b*log(a + b*x^2))/(2*a^2) - (C*b*log(x))/a^2 + (B*b^(3/2)*atan(
(b^(1/2)*x)/a^(1/2)))/a^(5/2) - (A*b^2*log(a + b*x^2))/(2*a^3)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.24

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^5\,(a+bx^2)}\,dx = \frac{-12\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)ad\,x^4+12\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)b^2x^4-6\log(b\,x^2+a)\,a\,b^2x^4+6\log(b\,x^2+a)\,abc}{12a^3x^4}$$

input
```
int((D*x^3+C*x^2+B*x+A)/x^5/(b*x^2+a),x)
```

output
```
( - 12*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*d*x**4 + 12*sqrt(b)
*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**2*x**4 - 6*log(a + b*x**2)*a*b**
2*x**4 + 6*log(a + b*x**2)*a*b*c*x**4 + 12*log(x)*a*b**2*x**4 - 12*log(x)*
a*b*c*x**4 - 3*a**3 + 6*a**2*b*x**2 - 4*a**2*b*x - 6*a**2*c*x**2 - 12*a**2
*d*x**3 + 12*a*b**2*x**3)/(12*a**3*x**4)
```

# 3.45 $\int \frac{x^4(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx$



## Optimal result

Integrand size = 28, antiderivative size = 155

$$\int \frac{x^4(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx = \frac{(Ab-2aC)x}{b^3}+\frac{(bB-2aD)x^2}{2b^3}+\frac{Cx^3}{3b^2}+\frac{Dx^4}{4b^2}-\frac{a(a(bB-aD)-b(Ab-aC)x)}{2b^4(a+bx^2)}-\frac{\sqrt{a}(3Ab-5aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{7/2}}-\frac{a(2bB-3aD)\log(a+bx^2)}{2b^4}$$

output
```
(A*b-2*C*a)*x/b^3+1/2*(B*b-2*D*a)*x^2/b^3+1/3*C*x^3/b^2+1/4*D*x^4/b^2-1/2*
a*(a*(B*b-D*a)-b*(A*b-C*a)*x)/b^4/(b*x^2+a)-1/2*a^(1/2)*(3*A*b-5*C*a)*arct
an(b^(1/2)*x/a^(1/2))/b^(7/2)-1/2*a*(2*B*b-3*D*a)*ln(b*x^2+a)/b^4
```

## Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.90

$$\int \frac{x^4(A + Bx + Cx^2 + Dx^3)}{(a + bx^2)^2}\, dx$$
$$= \frac{12b(Ab - 2aC)x + 6b(bB - 2aD)x^2 + 4b^2Cx^3 + 3b^2Dx^4 + \frac{6a(a^2D+Ab^2x-ab(B+Cx))}{a+bx^2} + 6\sqrt{a}\sqrt{b}(-3Ab + 5a}{12b^4}$$

input
```
Integrate[(x^4*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^2,x]
```

output
```
(12*b*(A*b - 2*a*C)*x + 6*b*(b*B - 2*a*D)*x^2 + 4*b^2*C*x^3 + 3*b^2*D*x^4
+ (6*a*(a^2*D + A*b^2*x - a*b*(B + C*x)))/(a + b*x^2) + 6*Sqrt[a]*Sqrt[b]*
(-3*A*b + 5*a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]] + 6*a*(-2*b*B + 3*a*D)*Log[a
+ b*x^2])/(12*b^4)
```

## Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.15, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2335, 25, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(A + Bx + Cx^2 + Dx^3)}{(a + bx^2)^2}\, dx$$

$$\Big\downarrow \ 2335$$

$$-\frac{\int -\frac{x^3\left(2aDx^2-(3Ab-5aC)x+4a\left(B-\frac{aD}{b}\right)\right)}{bx^2+a}dx}{2ab} - \frac{x^4\left(a\left(B - \frac{aD}{b}\right) - x(Ab - aC)\right)}{2ab\,(a + bx^2)}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{x^3\left(2aDx^2-(3Ab-5aC)x+4a\left(B-\frac{aD}{b}\right)\right)}{bx^2+a}dx}{2ab} - \frac{x^4\left(a\left(B - \frac{aD}{b}\right) - x(Ab - aC)\right)}{2ab\,(a + bx^2)}$$

$$\Big\downarrow \textcolor{blue}{2333}$$

$$\frac{\int \left(\frac{2aDx^3}{b}-\frac{(3Ab-5aC)x^2}{b}+\frac{2a(2bB-3aD)x}{b^2}+\frac{a(3Ab-5aC)}{b^2}-\frac{(3Ab-5aC)a^2+2(2bB-3aD)xa^2}{b^2(bx^2+a)}\right)dx}{2ab}-\frac{x^4\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{2ab\left(a+bx^2\right)}$$

$$\Big\downarrow \textcolor{blue}{2009}$$

$$\frac{-\frac{a^{3/2}(3Ab-5aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{5/2}}-\frac{a^2(2bB-3aD)\log\left(a+bx^2\right)}{b^3}+\frac{ax(3Ab-5aC)}{b^2}-\frac{x^3(3Ab-5aC)}{3b}+\frac{ax^2(2bB-3aD)}{b^2}+\frac{aDx^4}{2b}}{2ab}-\frac{x^4\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{2ab\left(a+bx^2\right)}$$

input
```
Int[(x^4*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^2,x]
```

output
```
-1/2*(x^4*(a*(B - (a*D)/b) - (A*b - a*C)*x))/(a*b*(a + b*x^2)) + ((a*(3*A*
b - 5*a*C)*x)/b^2 + (a*(2*b*B - 3*a*D)*x^2)/b^2 - ((3*A*b - 5*a*C)*x^3)/(3
*b) + (a*D*x^4)/(2*b) - (a^(3/2)*(3*A*b - 5*a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a
]])/b^(5/2) - (a^2*(2*b*B - 3*a*D)*Log[a + b*x^2])/b^3)/(2*a*b)
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2335
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Simp[c/(2*a*b*(p + 1))   Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSu
m[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
 b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

## Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.86

| method | result |
|---|---|
| default | $\frac{\frac{1}{4}D b\,x^{4}+\frac{1}{3}C b\,x^{3}+\frac{1}{2}b B\,x^{2}-D a\,x^{2}+A b x-2 C a x}{b^{3}}-\frac{a\left(\frac{\left(-\frac{A b}{2}+\frac{C a}{2}\right)x+\frac{a(B b-D a)}{2 b}}{b\,x^{2}+a}+\frac{(4 B b-6 D a)\ln\left(b\,x^{2}+a\right)}{4 b}+\frac{(3 A b-5 C a)\arctan\left(\frac{b x}{\sqrt{a b}}\right)}{2\sqrt{a b}}\right)}{b^{3}}$ |

input
```
int(x^4*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b^3*(1/4*D*b*x^4+1/3*C*b*x^3+1/2*b*B*x^2-D*a*x^2+A*b*x-2*C*a*x)-a/b^3*((
(-1/2*A*b+1/2*C*a)*x+1/2*a*(B*b-D*a)/b)/(b*x^2+a)+1/4*(4*B*b-6*D*a)/b*ln(b
*x^2+a)+1/2*(3*A*b-5*C*a)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 468, normalized size of antiderivative = 3.02

$$\int \frac{x^4(A+Bx+Cx^2+Dx^3)}{\left(a+bx^2\right)^2}\,dx$$

$$= \left[\frac{3\,Db^3x^6+4\,Cb^3x^5-3\,(3\,Dab^2-2\,Bb^3)x^4+6\,Da^3-6\,Ba^2b-4\,(5\,Cab^2-3\,Ab^3)x^3-6\,(2\,Da^2b-Ba\right.$$

input
```
integrate(x^4*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[1/12*(3*D*b^3*x^6 + 4*C*b^3*x^5 - 3*(3*D*a*b^2 - 2*B*b^3)*x^4 + 6*D*a^3 -
 6*B*a^2*b - 4*(5*C*a*b^2 - 3*A*b^3)*x^3 - 6*(2*D*a^2*b - B*a*b^2)*x^2 - 3
*(5*C*a^2*b - 3*A*a*b^2 + (5*C*a*b^2 - 3*A*b^3)*x^2)*sqrt(-a/b)*log((b*x^2
 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) - 6*(5*C*a^2*b - 3*A*a*b^2)*x + 6*(3
*D*a^3 - 2*B*a^2*b + (3*D*a^2*b - 2*B*a*b^2)*x^2)*log(b*x^2 + a))/(b^5*x^2
 + a*b^4), 1/12*(3*D*b^3*x^6 + 4*C*b^3*x^5 - 3*(3*D*a*b^2 - 2*B*b^3)*x^4 +
 6*D*a^3 - 6*B*a^2*b - 4*(5*C*a*b^2 - 3*A*b^3)*x^3 - 6*(2*D*a^2*b - B*a*b^
2)*x^2 + 6*(5*C*a^2*b - 3*A*a*b^2 + (5*C*a*b^2 - 3*A*b^3)*x^2)*sqrt(a/b)*a
rctan(b*x*sqrt(a/b)/a) - 6*(5*C*a^2*b - 3*A*a*b^2)*x + 6*(3*D*a^3 - 2*B*a^
2*b + (3*D*a^2*b - 2*B*a*b^2)*x^2)*log(b*x^2 + a))/(b^5*x^2 + a*b^4)]
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 335 vs. $2(141) = 282$.

Time = 2.42 (sec) , antiderivative size = 335, normalized size of antiderivative = 2.16

$$\int \frac{x^4(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx$$
$$= \frac{Cx^3}{3b^2} + \frac{Dx^4}{4b^2} + x^2\left(\frac{B}{2b^2} - \frac{Da}{b^3}\right) + x\left(\frac{A}{b^2} - \frac{2Ca}{b^3}\right) + \left(\frac{a(-2Bb+3Da)}{2b^4} - \frac{\sqrt{-ab^9}(-3Ab+5Ca)}{4b^8}\right)\log\left(x + \frac{4Bab - 6Da^2 + 4b^4\left(\frac{a(-2Bb+3Da)}{2b^4} - \frac{\sqrt{-ab^9}(-3Ab+5Ca)}{4b^8}\right)}{-3Ab^2+5Cab}\right) + \left(\frac{a(-2Bb+3Da)}{2b^4} + \frac{\sqrt{-ab^9}(-3Ab+5Ca)}{4b^8}\right)\log\left(x + \frac{4Bab - 6Da^2 + 4b^4\left(\frac{a(-2Bb+3Da)}{2b^4} + \frac{\sqrt{-ab^9}(-3Ab+5Ca)}{4b^8}\right)}{-3Ab^2+5Cab}\right) + \frac{-Ba^2b + Da^3 + x(Aab^2 - Ca^2b)}{2ab^4 + 2b^5x^2}$$

input
```
integrate(x**4*(D*x**3+C*x**2+B*x+A)/(b*x**2+a)**2,x)
```

output
```
C*x**3/(3*b**2) + D*x**4/(4*b**2) + x**2*(B/(2*b**2) - D*a/b**3) + x*(A/b*
*2 - 2*C*a/b**3) + (a*(-2*B*b + 3*D*a)/(2*b**4) - sqrt(-a*b**9)*(-3*A*b +
5*C*a)/(4*b**8))*log(x + (4*B*a*b - 6*D*a**2 + 4*b**4*(a*(-2*B*b + 3*D*a)/
(2*b**4) - sqrt(-a*b**9)*(-3*A*b + 5*C*a)/(4*b**8)))/(-3*A*b**2 + 5*C*a*b)
) + (a*(-2*B*b + 3*D*a)/(2*b**4) + sqrt(-a*b**9)*(-3*A*b + 5*C*a)/(4*b**8)
)*log(x + (4*B*a*b - 6*D*a**2 + 4*b**4*(a*(-2*B*b + 3*D*a)/(2*b**4) + sqrt
(-a*b**9)*(-3*A*b + 5*C*a)/(4*b**8)))/(-3*A*b**2 + 5*C*a*b)) + (-B*a**2*b
+ D*a**3 + x*(A*a*b**2 - C*a**2*b))/(2*a*b**4 + 2*b**5*x**2)
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.97

$$\int \frac{x^4(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx$$
$$= \frac{Da^3-Ba^2b-(Ca^2b-Aab^2)x}{2\,(b^5x^2+ab^4)} + \frac{(5\,Ca^2-3\,Aab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^3}$$
$$+ \frac{3\,Dbx^4+4\,Cbx^3-6\,(2\,Da-Bb)x^2-12\,(2\,Ca-Ab)x}{12\,b^3}$$
$$+ \frac{(3\,Da^2-2\,Bab)\log(bx^2+a)}{2\,b^4}$$

input
```
integrate(x^4*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*(D*a^3 - B*a^2*b - (C*a^2*b - A*a*b^2)*x)/(b^5*x^2 + a*b^4) + 1/2*(5*C
*a^2 - 3*A*a*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) + 1/12*(3*D*b*x^4 +
4*C*b*x^3 - 6*(2*D*a - B*b)*x^2 - 12*(2*C*a - A*b)*x)/b^3 + 1/2*(3*D*a^2 -
 2*B*a*b)*log(b*x^2 + a)/b^4
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.03

$$\int \frac{x^4(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx$$
$$=\frac{(5\,Ca^2-3\,Aab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^3}+\frac{(3\,Da^2-2\,Bab)\log\left(bx^2+a\right)}{2\,b^4}$$
$$+\frac{Da^3-Ba^2b-(Ca^2b-Aab^2)x}{2\,(bx^2+a)b^4}$$
$$+\frac{3\,Db^6x^4+4\,Cb^6x^3-12\,Dab^5x^2+6\,Bb^6x^2-24\,Cab^5x+12\,Ab^6x}{12\,b^8}$$

input
```
integrate(x^4*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*(5*C*a^2 - 3*A*a*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) + 1/2*(3*D*a
^2 - 2*B*a*b)*log(b*x^2 + a)/b^4 + 1/2*(D*a^3 - B*a^2*b - (C*a^2*b - A*a*b
^2)*x)/((b*x^2 + a)*b^4) + 1/12*(3*D*b^6*x^4 + 4*C*b^6*x^3 - 12*D*a*b^5*x^
2 + 6*B*b^6*x^2 - 24*C*a*b^5*x + 12*A*b^6*x)/b^8
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx=\int \frac{x^4\,\left(A+B\,x+C\,x^2+x^3\,D\right)}{{\left(b\,x^2+a\right)}^2}\,dx$$

input
```
int((x^4*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^2,x)
```

output
```
int((x^4*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^2, x)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.70

$$\int \frac{x^4(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx$$
$$= \frac{-18\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^2b+30\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^2c-18\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a\,b^2x^2+30\sqrt{b}\,\sqrt{a}\,ata}{}$$

input
```
int(x^4*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x)
```

output
```
( - 18*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2*b + 30*sqrt(b)*s
qrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2*c - 18*sqrt(b)*sqrt(a)*atan((b*x
)/(sqrt(b)*sqrt(a)))*a*b**2*x**2 + 30*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*
sqrt(a)))*a*b*c*x**2 + 18*log(a + b*x**2)*a**3*d - 12*log(a + b*x**2)*a**2
*b**2 + 18*log(a + b*x**2)*a**2*b*d*x**2 - 12*log(a + b*x**2)*a*b**3*x**2
+ 18*a**2*b**2*x - 30*a**2*b*c*x - 18*a**2*b*d*x**2 + 12*a*b**3*x**3 + 12*
a*b**3*x**2 - 20*a*b**2*c*x**3 - 9*a*b**2*d*x**4 + 6*b**4*x**4 + 4*b**3*c*
x**5 + 3*b**3*d*x**6)/(12*b**4*(a + b*x**2))
```

# 3.46 $\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx$



## Optimal result

Integrand size = 28, antiderivative size = 130

$$\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx = \frac{(bB-2aD)x}{b^3}+\frac{Cx^2}{2b^2}+\frac{Dx^3}{3b^2}+\frac{a(Ab-aC+(bB-aD)x)}{2b^3(a+bx^2)}-\frac{\sqrt{a}(3bB-5aD)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{7/2}}+\frac{(Ab-2aC)\log(a+bx^2)}{2b^3}$$

output
```
(B*b-2*D*a)*x/b^3+1/2*C*x^2/b^2+1/3*D*x^3/b^2+1/2*a*(A*b-C*a+(B*b-D*a)*x)/
b^3/(b*x^2+a)-1/2*a^(1/2)*(3*B*b-5*D*a)*arctan(b^(1/2)*x/a^(1/2))/b^(7/2)+
1/2*(A*b-2*C*a)*ln(b*x^2+a)/b^3
```

## Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.98

$$\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx = \frac{(bB-2aD)x}{b^3}+\frac{Cx^2}{2b^2}+\frac{Dx^3}{3b^2} + \frac{a(Ab+bBx-a(C+Dx))}{2b^3(a+bx^2)} + \frac{\sqrt{a}(-3bB+5aD)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{7/2}} + \frac{(Ab-2aC)\log(a+bx^2)}{2b^3}$$

input
```
Integrate[(x^3*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^2,x]
```

output
```
((b*B - 2*a*D)*x)/b^3 + (C*x^2)/(2*b^2) + (D*x^3)/(3*b^2) + (a*(A*b + b*B*
x - a*(C + D*x)))/(2*b^3*(a + b*x^2)) + (Sqrt[a]*(-3*b*B + 5*a*D)*ArcTan[(
Sqrt[b]*x)/Sqrt[a]])/(2*b^(7/2)) + ((A*b - 2*a*C)*Log[a + b*x^2])/(2*b^3)
```

## Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.19, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2335, 25, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx$$

$$\Big\downarrow \text{2335}$$

$$-\frac{\int -\frac{x^2\left(2aDx^2-2(Ab-2aC)x+3a\left(B-\frac{aD}{b}\right)\right)}{bx^2+a}dx}{2ab}-\frac{x^3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{2ab(a+bx^2)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{x^2\left(2aDx^2-2(Ab-2aC)x+3a\left(B-\frac{aD}{b}\right)\right)}{bx^2+a}dx}{2ab}-\frac{x^3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{2ab\left(a+bx^2\right)}$$

$$\Big\downarrow \ 2333$$

$$\frac{\int \left(\frac{2aDx^2}{b}-\frac{2(Ab-2aC)x}{b}+\frac{a(3bB-5aD)}{b^2}-\frac{a^2(3bB-5aD)-2ab(Ab-2aC)x}{b^2\left(bx^2+a\right)}\right)dx}{2ab}-\frac{x^3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{2ab\left(a+bx^2\right)}$$

$$\Big\downarrow \ 2009$$

$$\frac{-\frac{a^{3/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(3bB-5aD)}{b^{5/2}}+\frac{a(Ab-2aC)\log\left(a+bx^2\right)}{b^2}-\frac{x^2(Ab-2aC)}{b}+\frac{ax(3bB-5aD)}{b^2}+\frac{2aDx^3}{3b}}{2ab}-\frac{x^3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{2ab\left(a+bx^2\right)}$$

input
```
Int[(x^3*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^2,x]
```

output
```
-1/2*(x^3*(a*(B - (a*D)/b) - (A*b - a*C)*x))/(a*b*(a + b*x^2)) + ((a*(3*b*
B - 5*a*D)*x)/b^2 - ((A*b - 2*a*C)*x^2)/b + (2*a*D*x^3)/(3*b) - (a^(3/2)*(
3*b*B - 5*a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(5/2) + (a*(A*b - 2*a*C)*Log
[a + b*x^2])/b^2)/(2*a*b)
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2335
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Simp[c/(2*a*b*(p + 1))   Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSu
m[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
 b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

## Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.95

| method | result | size |
|---|---|---|
| default | $\frac{\frac{1}{3}Dx^3b+\frac{1}{2}Cb\,x^2+bBx-2Dax}{b^3}+\frac{\frac{\left(\frac{1}{2}abB-\frac{1}{2}Da^2\right)x+\frac{a(Ab-Ca)}{2}}{b\,x^2+a}+\frac{\left(2b^2A-4Cab\right)\ln\left(b\,x^2+a\right)}{4b}+\frac{\left(-3abB+5Da^2\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}}{b^3}$ | 124 |

input
```
int(x^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b^3*(1/3*D*x^3*b+1/2*C*b*x^2+b*B*x-2*D*a*x)+1/b^3*(((1/2*a*b*B-1/2*D*a^2
)*x+1/2*a*(A*b-C*a))/(b*x^2+a)+1/4*(2*A*b^2-4*C*a*b)/b*ln(b*x^2+a)+1/2*(-3
*B*a*b+5*D*a^2)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 372, normalized size of antiderivative = 2.86

$$\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx$$

$$= \left[\frac{4\,Db^2x^5+6\,Cb^2x^4+6\,Cabx^2-4\,(5\,Dab-3\,Bb^2)x^3-6\,Ca^2+6\,Aab-3\,(5\,Da^2-3\,Bab+(5\,Dab-}{12\,(}\right.$$

input
```
integrate(x^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[1/12*(4*D*b^2*x^5 + 6*C*b^2*x^4 + 6*C*a*b*x^2 - 4*(5*D*a*b - 3*B*b^2)*x^3
 - 6*C*a^2 + 6*A*a*b - 3*(5*D*a^2 - 3*B*a*b + (5*D*a*b - 3*B*b^2)*x^2)*sqr
t(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) - 6*(5*D*a^2 - 3*B
*a*b)*x - 6*(2*C*a^2 - A*a*b + (2*C*a*b - A*b^2)*x^2)*log(b*x^2 + a))/(b^4
*x^2 + a*b^3), 1/6*(2*D*b^2*x^5 + 3*C*b^2*x^4 + 3*C*a*b*x^2 - 2*(5*D*a*b -
 3*B*b^2)*x^3 - 3*C*a^2 + 3*A*a*b + 3*(5*D*a^2 - 3*B*a*b + (5*D*a*b - 3*B*
b^2)*x^2)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) - 3*(5*D*a^2 - 3*B*a*b)*x - 3*
(2*C*a^2 - A*a*b + (2*C*a*b - A*b^2)*x^2)*log(b*x^2 + a))/(b^4*x^2 + a*b^3
)]
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 289 vs. $2(117) = 234$.

Time = 2.22 (sec) , antiderivative size = 289, normalized size of antiderivative = 2.22

$$\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx = \frac{Cx^2}{2b^2} + \frac{Dx^3}{3b^2} + x\left(\frac{B}{b^2} - \frac{2Da}{b^3}\right) + \left(-\frac{-Ab+2Ca}{2b^3} - \frac{\sqrt{-ab^7}(-3Bb+5Da)}{4b^7}\right)\log\left(x + \frac{-2Ab+4Ca+4b^3\left(-\frac{-Ab+2Ca}{2b^3} - \frac{\sqrt{-ab^7}(-3Bb+5Da)}{4b^7}\right)}{-3Bb+5Da}\right) + \left(-\frac{-Ab+2Ca}{2b^3} + \frac{\sqrt{-ab^7}(-3Bb+5Da)}{4b^7}\right)\log\left(x + \frac{-2Ab+4Ca+4b^3\left(-\frac{-Ab+2Ca}{2b^3} + \frac{\sqrt{-ab^7}(-3Bb+5Da)}{4b^7}\right)}{-3Bb+5Da}\right) + \frac{Aab-Ca^2+x(Bab-Da^2)}{2ab^3+2b^4x^2}$$

input
```
integrate(x**3*(D*x**3+C*x**2+B*x+A)/(b*x**2+a)**2,x)
```

output
```
C*x**2/(2*b**2) + D*x**3/(3*b**2) + x*(B/b**2 - 2*D*a/b**3) + (-(-A*b + 2*
C*a)/(2*b**3) - sqrt(-a*b**7)*(-3*B*b + 5*D*a)/(4*b**7))*log(x + (-2*A*b +
 4*C*a + 4*b**3*(-(-A*b + 2*C*a)/(2*b**3) - sqrt(-a*b**7)*(-3*B*b + 5*D*a)
/(4*b**7)))/(-3*B*b + 5*D*a)) + (-(-A*b + 2*C*a)/(2*b**3) + sqrt(-a*b**7)*
(-3*B*b + 5*D*a)/(4*b**7))*log(x + (-2*A*b + 4*C*a + 4*b**3*(-(-A*b + 2*C*
a)/(2*b**3) + sqrt(-a*b**7)*(-3*B*b + 5*D*a)/(4*b**7)))/(-3*B*b + 5*D*a))
+ (A*a*b - C*a**2 + x*(B*a*b - D*a**2))/(2*a*b**3 + 2*b**4*x**2)
```

## Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.98

$$\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx = -\frac{Ca^2-Aab+(Da^2-Bab)x}{2\,(b^4x^2+ab^3)} - \frac{(2\,Ca-Ab)\log(bx^2+a)}{2\,b^3} + \frac{(5\,Da^2-3\,Bab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^3} + \frac{2\,Dbx^3+3\,Cbx^2-6\,(2\,Da-Bb)x}{6\,b^3}$$

input
```
integrate(x^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
-1/2*(C*a^2 - A*a*b + (D*a^2 - B*a*b)*x)/(b^4*x^2 + a*b^3) - 1/2*(2*C*a -
A*b)*log(b*x^2 + a)/b^3 + 1/2*(5*D*a^2 - 3*B*a*b)*arctan(b*x/sqrt(a*b))/(s
qrt(a*b)*b^3) + 1/6*(2*D*b*x^3 + 3*C*b*x^2 - 6*(2*D*a - B*b)*x)/b^3
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.01

$$\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx = -\frac{(2\,Ca-Ab)\log(bx^2+a)}{2\,b^3} + \frac{(5\,Da^2-3\,Bab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^3} - \frac{Ca^2-Aab+(Da^2-Bab)x}{2\,(bx^2+a)b^3} + \frac{2\,Db^4x^3+3\,Cb^4x^2-12\,Dab^3x+6\,Bb^4x}{6\,b^6}$$

input
```
integrate(x^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
-1/2*(2*C*a - A*b)*log(b*x^2 + a)/b^3 + 1/2*(5*D*a^2 - 3*B*a*b)*arctan(b*x
/sqrt(a*b))/(sqrt(a*b)*b^3) - 1/2*(C*a^2 - A*a*b + (D*a^2 - B*a*b)*x)/((b*
x^2 + a)*b^3) + 1/6*(2*D*b^4*x^3 + 3*C*b^4*x^2 - 12*D*a*b^3*x + 6*B*b^4*x)
/b^6
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx = \int \frac{x^3\,(A+B\,x+C\,x^2+x^3\,D)}{(b\,x^2+a)^2}\,dx$$

input
```
int((x^3*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^2,x)
```

output
```
int((x^3*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^2, x)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.87

$$\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx$$
$$= \frac{15\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^2d-9\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a\,b^2+15\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)abd\,x^2-9\sqrt{b}\,\sqrt{a}\,atan\left(\right.}{}$$

input
```
int(x^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x)
```

output
```
(15*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2*d - 9*sqrt(b)*sqrt(
a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*b**2 + 15*sqrt(b)*sqrt(a)*atan((b*x)/(s
qrt(b)*sqrt(a)))*a*b*d*x**2 - 9*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a
)))*b**3*x**2 + 3*log(a + b*x**2)*a**2*b**2 - 6*log(a + b*x**2)*a**2*b*c +
 3*log(a + b*x**2)*a*b**3*x**2 - 6*log(a + b*x**2)*a*b**2*c*x**2 - 15*a**2
*b*d*x - 3*a*b**3*x**2 + 9*a*b**3*x + 6*a*b**2*c*x**2 - 10*a*b**2*d*x**3 +
 6*b**4*x**3 + 3*b**3*c*x**4 + 2*b**3*d*x**5)/(6*b**4*(a + b*x**2))
```

# 3.47 $\int \frac{x^2\left(A+Bx+Cx^2+Dx^3\right)}{\left(a+bx^2\right)^2} \, dx$



## Optimal result

Integrand size = 28, antiderivative size = 115

$$\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2} \, dx = \frac{Cx}{b^2}+\frac{Dx^2}{2b^2}+\frac{a(bB-aD)-b(Ab-aC)x}{2b^3\left(a+bx^2\right)} + \frac{(Ab-3aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{5/2}} + \frac{(bB-2aD)\log\left(a+bx^2\right)}{2b^3}$$

output
```
C*x/b^2+1/2*D*x^2/b^2+1/2*(a*(B*b-D*a)-b*(A*b-C*a)*x)/b^3/(b*x^2+a)+1/2*(A
*b-3*C*a)*arctan(b^(1/2)*x/a^(1/2))/a^(1/2)/b^(5/2)+1/2*(B*b-2*D*a)*ln(b*x
^2+a)/b^3
```

## Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.87

$$\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx = \frac{2bCx+bDx^2+\frac{-a^2D-Ab^2x+ab(B+Cx)}{a+bx^2}+\frac{\sqrt{b}(Ab-3aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}}+(bB-2aD)\log(a+bx^2)}{2b^3}$$

input
```
Integrate[(x^2*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^2,x]
```

output
```
(2*b*C*x + b*D*x^2 + (-(a^2*D) - A*b^2*x + a*b*(B + C*x))/(a + b*x^2) + (S
qrt[b]*(A*b - 3*a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[a] + (b*B - 2*a*D)*
Log[a + b*x^2])/(2*b^3)
```

## Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.14, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2335, 25, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx$$

$$\Big\downarrow \text{2335}$$

$$-\frac{\int -\frac{x\left(2aDx^2-(Ab-3aC)x+2a\left(B-\frac{aD}{b}\right)\right)}{bx^2+a}dx}{2ab}-\frac{x^2\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{2ab(a+bx^2)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{x\left(2aDx^2-(Ab-3aC)x+2a\left(B-\frac{aD}{b}\right)\right)}{bx^2+a}dx}{2ab}-\frac{x^2\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{2ab(a+bx^2)}$$

$$\Big\downarrow \text{2333}$$

$$\frac{\int \left(-A+\frac{3aC}{b}+\frac{2aDx}{b}+\frac{a(Ab-3aC)+2a(bB-2aD)x}{b(bx^2+a)}\right)dx}{2ab}-\frac{x^2\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{2ab\left(a+bx^2\right)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{\sqrt{a}(Ab-3aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}}-x\left(A-\frac{3aC}{b}\right)+\frac{a(bB-2aD)\log\left(a+bx^2\right)}{b^2}+\frac{aDx^2}{b}}{2ab}-\frac{x^2\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{2ab\left(a+bx^2\right)}$$

input
```
Int[(x^2*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^2,x]
```

output
```
-1/2*(x^2*(a*(B - (a*D)/b) - (A*b - a*C)*x))/(a*b*(a + b*x^2)) + (-((A - (
3*a*C)/b)*x) + (a*D*x^2)/b + (Sqrt[a]*(A*b - 3*a*C)*ArcTan[(Sqrt[b]*x)/Sqr
t[a]])/b^(3/2) + (a*(b*B - 2*a*D)*Log[a + b*x^2])/b^2)/(2*a*b)
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2335
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Simp[c/(2*a*b*(p + 1))   Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSu
m[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
 b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

## Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.90

| method | result | size |
|---|---|---|
| default | $\frac{\frac{1}{2}Dx^2+Cx}{b^2}+\frac{\frac{\left(-\frac{Ab}{2}+\frac{Ca}{2}\right)x+\frac{a(Bb-Da)}{2b}}{b\,x^2+a}+\frac{(2Bb-4Da)\ln\left(b\,x^2+a\right)}{4b}+\frac{(Ab-3Ca)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}}{b^2}$ | 103 |

input
```
int(x^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b^2*(1/2*D*x^2+C*x)+1/b^2*(((-1/2*A*b+1/2*C*a)*x+1/2*a*(B*b-D*a)/b)/(b*x
^2+a)+1/4*(2*B*b-4*D*a)/b*ln(b*x^2+a)+1/2*(A*b-3*C*a)/(a*b)^(1/2)*arctan(b
*x/(a*b)^(1/2)))
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 357, normalized size of antiderivative = 3.10

$$\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx$$
$$=\left[\frac{2\,Dab^2x^4+4\,Cab^2x^3+2\,Da^2bx^2-2\,Da^3+2\,Ba^2b+(3\,Ca^2-Aab+(3\,Cab-Ab^2)x^2)\sqrt{-ab}\log\left(\frac{bx^2}{\cdots}\right.}{4\,(ab^4x^2+a^2b\cdots}\right.$$

input
```
integrate(x^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[1/4*(2*D*a*b^2*x^4 + 4*C*a*b^2*x^3 + 2*D*a^2*b*x^2 - 2*D*a^3 + 2*B*a^2*b
+ (3*C*a^2 - A*a*b + (3*C*a*b - A*b^2)*x^2)*sqrt(-a*b)*log((b*x^2 - 2*sqrt
(-a*b)*x - a)/(b*x^2 + a)) + 2*(3*C*a^2*b - A*a*b^2)*x - 2*(2*D*a^3 - B*a^
2*b + (2*D*a^2*b - B*a*b^2)*x^2)*log(b*x^2 + a))/(a*b^4*x^2 + a^2*b^3), 1/
2*(D*a*b^2*x^4 + 2*C*a*b^2*x^3 + D*a^2*b*x^2 - D*a^3 + B*a^2*b - (3*C*a^2
- A*a*b + (3*C*a*b - A*b^2)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) + (3*C*a^
2*b - A*a*b^2)*x - (2*D*a^3 - B*a^2*b + (2*D*a^2*b - B*a*b^2)*x^2)*log(b*x
^2 + a))/(a*b^4*x^2 + a^2*b^3)]
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 284 vs. $2(104) = 208$.

Time = 2.18 (sec) , antiderivative size = 284, normalized size of antiderivative = 2.47

$$\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx = \frac{Cx}{b^2}+\frac{Dx^2}{2b^2}+\left(-\frac{-Bb+2Da}{2b^3} - \frac{\sqrt{-ab^7}(-Ab+3Ca)}{4ab^6}\right)\log\left(x+\frac{2Bab-4Da^2-4ab^3\left(-\frac{-Bb+2Da}{2b^3}-\frac{\sqrt{-ab^7}(-Ab+3Ca)}{4ab^6}\right)}{-Ab^2+3Cab}\right) + \left(-\frac{-Bb+2Da}{2b^3} + \frac{\sqrt{-ab^7}(-Ab+3Ca)}{4ab^6}\right)\log\left(x+\frac{2Bab-4Da^2-4ab^3\left(-\frac{-Bb+2Da}{2b^3}+\frac{\sqrt{-ab^7}(-Ab+3Ca)}{4ab^6}\right)}{-Ab^2+3Cab}\right) + \frac{Bab-Da^2+x(-Ab^2+Cab)}{2ab^3+2b^4x^2}$$

input
```
integrate(x**2*(D*x**3+C*x**2+B*x+A)/(b*x**2+a)**2,x)
```

output
```
C*x/b**2 + D*x**2/(2*b**2) + (-(-B*b + 2*D*a)/(2*b**3) - sqrt(-a*b**7)*(-A
*b + 3*C*a)/(4*a*b**6))*log(x + (2*B*a*b - 4*D*a**2 - 4*a*b**3*(-(-B*b + 2
*D*a)/(2*b**3) - sqrt(-a*b**7)*(-A*b + 3*C*a)/(4*a*b**6)))/(-A*b**2 + 3*C*
a*b)) + (-(-B*b + 2*D*a)/(2*b**3) + sqrt(-a*b**7)*(-A*b + 3*C*a)/(4*a*b**6
))*log(x + (2*B*a*b - 4*D*a**2 - 4*a*b**3*(-(-B*b + 2*D*a)/(2*b**3) + sqrt
(-a*b**7)*(-A*b + 3*C*a)/(4*a*b**6)))/(-A*b**2 + 3*C*a*b)) + (B*a*b - D*a*
*2 + x*(-A*b**2 + C*a*b))/(2*a*b**3 + 2*b**4*x**2)
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.94

$$\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx = -\frac{Da^2-Bab-(Cab-Ab^2)x}{2\,(b^4x^2+ab^3)} - \frac{(3\,Ca-Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^2} + \frac{Dx^2+2\,Cx}{2\,b^2} - \frac{(2\,Da-Bb)\log\left(bx^2+a\right)}{2\,b^3}$$

input
```
integrate(x^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
-1/2*(D*a^2 - B*a*b - (C*a*b - A*b^2)*x)/(b^4*x^2 + a*b^3) - 1/2*(3*C*a -
A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/2*(D*x^2 + 2*C*x)/b^2 - 1/2
*(2*D*a - B*b)*log(b*x^2 + a)/b^3
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.97

$$\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx = -\frac{(3\,Ca-Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^2} - \frac{(2\,Da-Bb)\log\left(bx^2+a\right)}{2\,b^3} + \frac{Db^2x^2+2\,Cb^2x}{2\,b^4} - \frac{Da^2-Bab-(Cab-Ab^2)x}{2\,(bx^2+a)b^3}$$

input
```
integrate(x^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
-1/2*(3*C*a - A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) - 1/2*(2*D*a - B*
b)*log(b*x^2 + a)/b^3 + 1/2*(D*b^2*x^2 + 2*C*b^2*x)/b^4 - 1/2*(D*a^2 - B*a
*b - (C*a*b - A*b^2)*x)/((b*x^2 + a)*b^3)
```

## Mupad [B] (verification not implemented)

Time = 2.35 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.32

$$\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx = \frac{B\,\ln\left(b\,x^2+a\right)}{2\,b^2}+\frac{x^2\,D}{2\,b^2}+\frac{C\,x}{b^2}-\frac{a^2\,D}{2\,b^3\,\left(b\,x^2+a\right)} + \frac{B\,a}{2\,b^2\,\left(b\,x^2+a\right)}-\frac{A\,x}{2\,b\,\left(b\,x^2+a\right)} + \frac{C\,a\,x}{2\,\left(b^3\,x^2+a\,b^2\right)}+\frac{A\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,\sqrt{a}\,b^{3/2}} - \frac{3\,C\,\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,b^{5/2}}-\frac{a\,\ln\left(b\,x^2+a\right)\,D}{b^3}$$

input

```
int((x^2*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^2,x)
```

output

```
(B*log(a + b*x^2))/(2*b^2) + (x^2*D)/(2*b^2) + (C*x)/b^2 - (a^2*D)/(2*b^3*
(a + b*x^2)) + (B*a)/(2*b^2*(a + b*x^2)) - (A*x)/(2*b*(a + b*x^2)) + (C*a*
x)/(2*(a*b^2 + b^3*x^2)) + (A*atan((b^(1/2)*x)/a^(1/2)))/(2*a^(1/2)*b^(3/2
)) - (3*C*a^(1/2)*atan((b^(1/2)*x)/a^(1/2)))/(2*b^(5/2)) - (a*log(a + b*x^
2)*D)/b^3
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.77

$$\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx = \frac{\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)ab-3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)ac+\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)b^2x^2-3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)bc}{}$$

input

```
int(x^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x)
```

output
```
(sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*b - 3*sqrt(b)*sqrt(a)*ata
n((b*x)/(sqrt(b)*sqrt(a)))*a*c + sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(
a)))*b**2*x**2 - 3*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b*c*x**2
- 2*log(a + b*x**2)*a**2*d + log(a + b*x**2)*a*b**2 - 2*log(a + b*x**2)*a*
b*d*x**2 + log(a + b*x**2)*b**3*x**2 - a*b**2*x + 3*a*b*c*x + 2*a*b*d*x**2
 - b**3*x**2 + 2*b**2*c*x**3 + b**2*d*x**4)/(2*b**3*(a + b*x**2))
```

# 3.48 $\int \frac{x\left(A+Bx+Cx^2+Dx^3\right)}{\left(a+bx^2\right)^2}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 92

$$\int \frac{x(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx = \frac{Dx}{b^2}-\frac{Ab-aC+(bB-aD)x}{2b^2(a+bx^2)}+\frac{(bB-3aD)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{5/2}}+\frac{C\log(a+bx^2)}{2b^2}$$

output
```
D*x/b^2-1/2*(A*b-C*a+(B*b-D*a)*x)/b^2/(b*x^2+a)+1/2*(B*b-3*D*a)*arctan(b^(
1/2)*x/a^(1/2))/a^(1/2)/b^(5/2)+1/2*C*ln(b*x^2+a)/b^2
```

## Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00

$$\int \frac{x(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx = \frac{Dx}{b^2}+\frac{-Ab+aC-bBx+aDx}{2b^2(a+bx^2)}-\frac{(-bB+3aD)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{5/2}}+\frac{C\log(a+bx^2)}{2b^2}$$

input
```
Integrate[(x*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^2,x]
```

output
```
(D*x)/b^2 + (-(A*b) + a*C - b*B*x + a*D*x)/(2*b^2*(a + b*x^2)) - ((-(b*B)
+ 3*a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*Sqrt[a]*b^(5/2)) + (C*Log[a + b*x
^2])/(2*b^2)
```

## Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.18, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2335, 25, 2341, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx$$

$$\Big\downarrow \text{2335}$$

$$-\frac{\int -\frac{2aDx^2+2aCx+\frac{a(bB-aD)}{b}}{bx^2+a}dx}{2ab}-\frac{x\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{2ab\left(a+bx^2\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{2aDx^2+2aCx+\frac{a(bB-aD)}{b}}{bx^2+a}dx}{2ab}-\frac{x\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{2ab\left(a+bx^2\right)}$$

$$\Big\downarrow \text{2341}$$

$$\frac{\int \left(\frac{2aD}{b}+\frac{a(bB-3aD)+2abCx}{b(bx^2+a)}\right)dx}{2ab}-\frac{x\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{2ab\left(a+bx^2\right)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{\sqrt{a}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(bB-3aD)}{b^{3/2}}+\frac{aC\log\left(a+bx^2\right)}{b}+\frac{2aDx}{b}}{2ab}-\frac{x\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{2ab\left(a+bx^2\right)}$$

input
```
Int[(x*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^2,x]
```

output
```
-1/2*(x*(a*(B - (a*D)/b) - (A*b - a*C)*x))/(a*b*(a + b*x^2)) + ((2*a*D*x)/
b + (Sqrt[a]*(b*B - 3*a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(3/2) + (a*C*Log
[a + b*x^2])/b)/(2*a*b)
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2335
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Simp[c/(2*a*b*(p + 1))   Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSu
m[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
 b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

rule 2341
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

## Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.85

| method | result | size |
|---|---|---|
| default | $\frac{Dx}{b^2}+\frac{\frac{\left(-\frac{Bb}{2}+\frac{Da}{2}\right)x-\frac{Ab}{2}+\frac{Ca}{2}}{b\,x^2+a}+\frac{C\ln\left(b\,x^2+a\right)}{2}+\frac{(Bb-3Da)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}}{b^2}$ | 78 |

input
```
int(x*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
D*x/b^2+1/b^2*(((-1/2*B*b+1/2*D*a)*x-1/2*A*b+1/2*C*a)/(b*x^2+a)+1/2*C*ln(b
*x^2+a)+1/2*(B*b-3*D*a)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 286, normalized size of antiderivative = 3.11

$$\int \frac{x(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx$$
$$= \left[\frac{4\,Dab^2x^3+2\,Ca^2b-2\,Aab^2+(3\,Da^2-Bab+(3\,Dab-Bb^2)x^2)\sqrt{-ab}\log\left(\frac{bx^2-2\sqrt{-ab}x-a}{bx^2+a}\right)+2\,(3\,Da^2}{4\,(ab^4x^2+a^2b^3)}\right.$$

input
```
integrate(x*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[1/4*(4*D*a*b^2*x^3 + 2*C*a^2*b - 2*A*a*b^2 + (3*D*a^2 - B*a*b + (3*D*a*b
- B*b^2)*x^2)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 2
*(3*D*a^2*b - B*a*b^2)*x + 2*(C*a*b^2*x^2 + C*a^2*b)*log(b*x^2 + a))/(a*b^
4*x^2 + a^2*b^3), 1/2*(2*D*a*b^2*x^3 + C*a^2*b - A*a*b^2 - (3*D*a^2 - B*a*
b + (3*D*a*b - B*b^2)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) + (3*D*a^2*b -
B*a*b^2)*x + (C*a*b^2*x^2 + C*a^2*b)*log(b*x^2 + a))/(a*b^4*x^2 + a^2*b^3)
]
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 212 vs. $2(87) = 174$.

Time = 1.66 (sec) , antiderivative size = 212, normalized size of antiderivative = 2.30

$$\int \frac{x(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx = \frac{Dx}{b^2} + \left(\frac{C}{2b^2} - \frac{\sqrt{-ab^5}(-Bb+3Da)}{4ab^5}\right)\log\left(x + \frac{2Ca - 4ab^2\left(\frac{C}{2b^2} - \frac{\sqrt{-ab^5}(-Bb+3Da)}{4ab^5}\right)}{-Bb+3Da}\right) + \left(\frac{C}{2b^2} + \frac{\sqrt{-ab^5}(-Bb+3Da)}{4ab^5}\right)\log\left(x + \frac{2Ca - 4ab^2\left(\frac{C}{2b^2} + \frac{\sqrt{-ab^5}(-Bb+3Da)}{4ab^5}\right)}{-Bb+3Da}\right) + \frac{-Ab+Ca+x(-Bb+Da)}{2ab^2+2b^3x^2}$$

input
```
integrate(x*(D*x**3+C*x**2+B*x+A)/(b*x**2+a)**2,x)
```

output
```
D*x/b**2 + (C/(2*b**2) - sqrt(-a*b**5)*(-B*b + 3*D*a)/(4*a*b**5))*log(x +
(2*C*a - 4*a*b**2*(C/(2*b**2) - sqrt(-a*b**5)*(-B*b + 3*D*a)/(4*a*b**5)))/
(-B*b + 3*D*a)) + (C/(2*b**2) + sqrt(-a*b**5)*(-B*b + 3*D*a)/(4*a*b**5))*l
og(x + (2*C*a - 4*a*b**2*(C/(2*b**2) + sqrt(-a*b**5)*(-B*b + 3*D*a)/(4*a*b
**5)))/(-B*b + 3*D*a)) + (-A*b + C*a + x*(-B*b + D*a))/(2*a*b**2 + 2*b**3*
x**2)
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.91

$$\int \frac{x(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx = \frac{Ca-Ab+(Da-Bb)x}{2\,(b^3x^2+ab^2)} + \frac{Dx}{b^2} + \frac{C\log(bx^2+a)}{2\,b^2} - \frac{(3\,Da-Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^2}$$

input
```
integrate(x*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*(C*a - A*b + (D*a - B*b)*x)/(b^3*x^2 + a*b^2) + D*x/b^2 + 1/2*C*log(b*
x^2 + a)/b^2 - 1/2*(3*D*a - B*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.88

$$\int \frac{x(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx = \frac{Dx}{b^2}+\frac{C\log(bx^2+a)}{2\,b^2}-\frac{(3\,Da-Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^2}+\frac{Ca-Ab+(Da-Bb)x}{2\,(bx^2+a)b^2}$$

input
```
integrate(x*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
D*x/b^2 + 1/2*C*log(b*x^2 + a)/b^2 - 1/2*(3*D*a - B*b)*arctan(b*x/sqrt(a*b
))/(sqrt(a*b)*b^2) + 1/2*(C*a - A*b + (D*a - B*b)*x)/((b*x^2 + a)*b^2)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx = \int \frac{x\,(A+B\,x+C\,x^2+x^3\,D)}{(b\,x^2+a)^2}\,dx$$

input
```
int((x*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^2,x)
```

output
```
int((x*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^2, x)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.98

$$\int \frac{x(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx$$
$$= \frac{-3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^2d+\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a\,b^2-3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)abd\,x^2+\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{\ }}\right.}{2a\,b^3\,(b\,x^2}$$

input
```
int(x*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x)
```

output
```
( - 3*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2*d + sqrt(b)*sqrt(
a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*b**2 - 3*sqrt(b)*sqrt(a)*atan((b*x)/(sq
rt(b)*sqrt(a)))*a*b*d*x**2 + sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))
*b**3*x**2 + log(a + b*x**2)*a**2*b*c + log(a + b*x**2)*a*b**2*c*x**2 + 3*
a**2*b*d*x + a*b**3*x**2 - a*b**3*x - a*b**2*c*x**2 + 2*a*b**2*d*x**3)/(2*
a*b**3*(a + b*x**2))
```

# 3.49 $\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^2} \, dx$



## Optimal result

Integrand size = 25, antiderivative size = 93

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^2} \, dx = -\frac{a(bB-aD)-b(Ab-aC)x}{2ab^2(a+bx^2)} + \frac{(Ab+aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}} + \frac{D\log(a+bx^2)}{2b^2}$$

output
```
-1/2*(a*(B*b-D*a)-b*(A*b-C*a)*x)/a/b^2/(b*x^2+a)+1/2*(A*b+C*a)*arctan(b^(1
/2)*x/a^(1/2))/a^(3/2)/b^(3/2)+1/2*D*ln(b*x^2+a)/b^2
```

## Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.89

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^2} \, dx = \frac{\frac{a^2D+Ab^2x-ab(B+Cx)}{a(a+bx^2)} + \frac{\sqrt{b}(Ab+aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}} + D\log(a+bx^2)}{2b^2}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/(a + b*x^2)^2,x]
```

output
```
((a^2*D + A*b^2*x - a*b*(B + C*x))/(a*(a + b*x^2)) + (Sqrt[b]*(A*b + a*C)*
ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(3/2) + D*Log[a + b*x^2])/(2*b^2)
```

## Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2345, 25, 27, 452, 218, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^2}dx$$

$$\Big\downarrow \text{2345}$$

$$-\frac{\int -\frac{Ab+aC+2aDx}{b(bx^2+a)}dx}{2a}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{2ab\left(a+bx^2\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{Ab+aC+2aDx}{b(bx^2+a)}dx}{2a}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{2ab\left(a+bx^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{Ab+aC+2aDx}{bx^2+a}dx}{2ab}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{2ab\left(a+bx^2\right)}$$

$$\Big\downarrow \text{452}$$

$$\frac{(aC+Ab)\int \frac{1}{bx^2+a}dx+2aD\int \frac{x}{bx^2+a}dx}{2ab}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{2ab\left(a+bx^2\right)}$$

$$\Big\downarrow \text{218}$$

$$\frac{2aD\int \frac{x}{bx^2+a}dx+\frac{(aC+Ab)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}}{2ab}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{2ab\left(a+bx^2\right)}$$

$$\Big\downarrow \text{240}$$

$$\frac{\frac{(aC+Ab)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}+\frac{aD\log(a+bx^2)}{b}}{2ab}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{2ab\,(a+bx^2)}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/(a + b*x^2)^2,x]
```

output
```
-1/2*(a*(B - (a*D)/b) - (A*b - a*C)*x)/(a*b*(a + b*x^2)) + (((A*b + a*C)*A
rcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*Sqrt[b]) + (a*D*Log[a + b*x^2])/b)/(2
*a*b)
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 452
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[c   Int[1/
(a + b*x^2), x], x] + Simp[d   Int[x/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c^2 + a*d^2, 0]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

## Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.95

| method | result | size |
|---|---|---|
| default | $\frac{\frac{(Ab-Ca)x}{2ab}-\frac{Bb-Da}{2b^2}}{b\,x^2+a}+\frac{\frac{Da\ln\left(b\,x^2+a\right)}{b}+\frac{(Ab+Ca)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}}{2ab}$ | 88 |

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/2*(A*b-C*a)/a/b*x-1/2*(B*b-D*a)/b^2)/(b*x^2+a)+1/2/a/b*(D*a/b*ln(b*x^2+
a)+(A*b+C*a)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 257, normalized size of antiderivative = 2.76

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^2}\,dx = \left[\frac{2\,Da^3-2\,Ba^2b-(Ca^2+Aab+(Cab+Ab^2)x^2)\sqrt{-ab}\log\left(\frac{bx^2-2\,\sqrt{-ab}x-a}{bx^2+a}\right)-2\,(Ca^2b-Aab^2)x+2\,(I}{4\,(a^2b^3x^2+a^3b^2)}\right.$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[1/4*(2*D*a^3 - 2*B*a^2*b - (C*a^2 + A*a*b + (C*a*b + A*b^2)*x^2)*sqrt(-a*
b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) - 2*(C*a^2*b - A*a*b^2)*x
 + 2*(D*a^2*b*x^2 + D*a^3)*log(b*x^2 + a))/(a^2*b^3*x^2 + a^3*b^2), 1/2*(D
*a^3 - B*a^2*b + (C*a^2 + A*a*b + (C*a*b + A*b^2)*x^2)*sqrt(a*b)*arctan(sq
rt(a*b)*x/a) - (C*a^2*b - A*a*b^2)*x + (D*a^2*b*x^2 + D*a^3)*log(b*x^2 + a
))/(a^2*b^3*x^2 + a^3*b^2)]
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 233 vs. $2(82) = 164$.

Time = 1.80 (sec) , antiderivative size = 233, normalized size of antiderivative = 2.51

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(a + bx^2)^2}\,dx$$
$$= \left(\frac{D}{2b^2} - \frac{\sqrt{-a^3b^5}(Ab + Ca)}{4a^3b^4}\right)\log\left(x + \frac{-2Da^2 + 4a^2b^2\left(\frac{D}{2b^2} - \frac{\sqrt{-a^3b^5}(Ab+Ca)}{4a^3b^4}\right)}{Ab^2 + Cab}\right)$$
$$+ \left(\frac{D}{2b^2} + \frac{\sqrt{-a^3b^5}(Ab + Ca)}{4a^3b^4}\right)\log\left(x + \frac{-2Da^2 + 4a^2b^2\left(\frac{D}{2b^2} + \frac{\sqrt{-a^3b^5}(Ab+Ca)}{4a^3b^4}\right)}{Ab^2 + Cab}\right)$$
$$+ \frac{-Bab + Da^2 + x(Ab^2 - Cab)}{2a^2b^2 + 2ab^3x^2}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(b*x**2+a)**2,x)
```

output
```
(D/(2*b**2) - sqrt(-a**3*b**5)*(A*b + C*a)/(4*a**3*b**4))*log(x + (-2*D*a*
*2 + 4*a**2*b**2*(D/(2*b**2) - sqrt(-a**3*b**5)*(A*b + C*a)/(4*a**3*b**4))
)/(A*b**2 + C*a*b)) + (D/(2*b**2) + sqrt(-a**3*b**5)*(A*b + C*a)/(4*a**3*b
**4))*log(x + (-2*D*a**2 + 4*a**2*b**2*(D/(2*b**2) + sqrt(-a**3*b**5)*(A*b
 + C*a)/(4*a**3*b**4)))/(A*b**2 + C*a*b)) + (-B*a*b + D*a**2 + x*(A*b**2 -
 C*a*b))/(2*a**2*b**2 + 2*a*b**3*x**2)
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.96

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^2}\,dx = \frac{Da^2-Bab-(Cab-Ab^2)x}{2\,(ab^3x^2+a^2b^2)} + \frac{D\log\left(bx^2+a\right)}{2\,b^2} + \frac{(Ca+Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}ab}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*(D*a^2 - B*a*b - (C*a*b - A*b^2)*x)/(a*b^3*x^2 + a^2*b^2) + 1/2*D*log(
b*x^2 + a)/b^2 + 1/2*(C*a + A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.95

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^2}\,dx = \frac{D\log\left(bx^2+a\right)}{2\,b^2} + \frac{(Ca+Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}ab} - \frac{(Ca-Ab)x-\frac{Da^2-Bab}{b}}{2\,(bx^2+a)ab}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*D*log(b*x^2 + a)/b^2 + 1/2*(C*a + A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b
)*a*b) - 1/2*((C*a - A*b)*x - (D*a^2 - B*a*b)/b)/((b*x^2 + a)*a*b)
```

## Mupad [B] (verification not implemented)

Time = 2.26 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.18

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^2}\,dx = \frac{\left(\ln\left(b\,x^2+a\right)+\frac{a}{b\,x^2+a}\right)\,D}{2\,b^2}-\frac{B}{2\,b\,\left(b\,x^2+a\right)}+\frac{A\,x}{2\,a\,\left(b\,x^2+a\right)}-\frac{C\,x}{2\,b\,\left(b\,x^2+a\right)}+\frac{A\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,a^{3/2}\,\sqrt{b}}+\frac{C\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,\sqrt{a}\,b^{3/2}}$$

input
```
int((A + B*x + C*x^2 + x^3*D)/(a + b*x^2)^2,x)
```

output
```
((log(a + b*x^2) + a/(a + b*x^2))*D)/(2*b^2) - B/(2*b*(a + b*x^2)) + (A*x)
/(2*a*(a + b*x^2)) - (C*x)/(2*b*(a + b*x^2)) + (A*atan((b^(1/2)*x)/a^(1/2)
))/(2*a^(3/2)*b^(1/2)) + (C*atan((b^(1/2)*x)/a^(1/2)))/(2*a^(1/2)*b^(3/2))
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.68

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^2}\,dx = \frac{\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)ab+\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)ac+\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)b^2x^2+\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)bc\,x^2}{2a\,b^2\left(b\,x^2+a\right)}$$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x)
```

output
```
(sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*b + sqrt(b)*sqrt(a)*atan(
(b*x)/(sqrt(b)*sqrt(a)))*a*c + sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)
))*b**2*x**2 + sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b*c*x**2 + lo
g(a + b*x**2)*a**2*d + log(a + b*x**2)*a*b*d*x**2 + a*b**2*x - a*b*c*x - a
*b*d*x**2 + b**3*x**2)/(2*a*b**2*(a + b*x**2))
```

# 3.50 $\int \frac{A+Bx+Cx^2+Dx^3}{x(a+bx^2)^2}\,dx$



## Optimal result

Integrand size = 28, antiderivative size = 95

$$\int \frac{A+Bx+Cx^2+Dx^3}{x\left(a+bx^2\right)^2}\,dx=\frac{Ab-aC+(bB-aD)x}{2ab\left(a+bx^2\right)}+\frac{(bB+aD)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}+\frac{A\log(x)}{a^2}-\frac{A\log\left(a+bx^2\right)}{2a^2}$$

output
```
1/2*(A*b-C*a+(B*b-D*a)*x)/a/b/(b*x^2+a)+1/2*(B*b+D*a)*arctan(b^(1/2)*x/a^(
1/2))/a^(3/2)/b^(3/2)+A*ln(x)/a^2-1/2*A*ln(b*x^2+a)/a^2
```

## Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.89

$$\int \frac{A+Bx+Cx^2+Dx^3}{x\left(a+bx^2\right)^2}\,dx=\frac{\frac{a(Ab+bBx-a(C+Dx))}{b(a+bx^2)}+\frac{\sqrt{a}(bB+aD)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}}+2A\log(x)-A\log\left(a+bx^2\right)}{2a^2}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/(x*(a + b*x^2)^2),x]
```

output
```
((a*(A*b + b*B*x - a*(C + D*x)))/(b*(a + b*x^2)) + (Sqrt[a]*(b*B + a*D)*Ar
cTan[(Sqrt[b]*x)/Sqrt[a]])/b^(3/2) + 2*A*Log[x] - A*Log[a + b*x^2])/(2*a^2
)
```

## Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {2336, 25, 27, 523, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{x\left(a+bx^2\right)^2}dx$$

$$\Big\downarrow \text{2336}$$

$$\frac{x(bB-aD)-aC+Ab}{2ab\left(a+bx^2\right)}-\frac{\int -\frac{2Ab+(bB+aD)x}{bx(bx^2+a)}dx}{2a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{2Ab+(bB+aD)x}{bx(bx^2+a)}dx}{2a}+\frac{x(bB-aD)-aC+Ab}{2ab\left(a+bx^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{2Ab+(bB+aD)x}{x(bx^2+a)}dx}{2ab}+\frac{x(bB-aD)-aC+Ab}{2ab\left(a+bx^2\right)}$$

$$\Big\downarrow \text{523}$$

$$\frac{\int \left(\frac{2Ab}{ax}+\frac{Da^2+bBa-2Ab^2x}{a(bx^2+a)}\right)dx}{2ab}+\frac{x(bB-aD)-aC+Ab}{2ab\left(a+bx^2\right)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{Ab\log(a+bx^2)}{a}+\frac{2Ab\log(x)}{a}+\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(aD+bB)}{\sqrt{a}\sqrt{b}}}{2ab}+\frac{x(bB-aD)-aC+Ab}{2ab\left(a+bx^2\right)}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/(x*(a + b*x^2)^2),x]
```

output
```
(A*b - a*C + (b*B - a*D)*x)/(2*a*b*(a + b*x^2)) + (((b*B + a*D)*ArcTan[(Sq
rt[b]*x)/Sqrt[a]])/(Sqrt[a]*Sqrt[b]) + (2*A*b*Log[x])/a - (A*b*Log[a + b*x
^2])/a)/(2*a*b)
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 523
```
Int[((x_)^(m_.)*((c_) + (d_.)*(x_)))/((a_) + (b_.)*(x_)^2), x_Symbol] :> In
t[ExpandIntegrand[x^m*((c + d*x)/(a + b*x^2)), x], x] /; FreeQ[{a, b, c, d}
, x] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

## Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.04

| method | result | size |
|---|---|---|
| default | $-\frac{\frac{-\frac{a(Bb-Da)x}{2b}-\frac{a(Ab-Ca)}{2b}}{b\,x^2+a}+\frac{bA\ln\left(b\,x^2+a\right)+\frac{\left(-abB-Da^2\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}}{2b}}{a^2}+\frac{A\ln(x)}{a^2}$ | 99 |

input
```
int((D*x^3+C*x^2+B*x+A)/x/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/a^2*((-1/2*a*(B*b-D*a)/b*x-1/2*a*(A*b-C*a)/b)/(b*x^2+a)+1/2/b*(b*A*ln(b
*x^2+a)+(-B*a*b-D*a^2)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))))+A*ln(x)/a^2
```

## Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 296, normalized size of antiderivative = 3.12

$$\int \frac{A+Bx+Cx^2+Dx^3}{x\left(a+bx^2\right)^2}\,dx$$

$$= \left[-\frac{2\,Ca^2b-2\,Aab^2+\left(Da^2+Bab+\left(Dab+Bb^2\right)x^2\right)\sqrt{-ab}\log\left(\frac{bx^2-2\sqrt{-ab}x-a}{bx^2+a}\right)+2\left(Da^2b-Bab^2\right)x+}{4\left(a^2b^3x^2+a^3b^2\right)}\right.$$

$$-\frac{Ca^2b-Aab^2-\left(Da^2+Bab+\left(Dab+Bb^2\right)x^2\right)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}x}{a}\right)+\left(Da^2b-Bab^2\right)x+\left(Ab^3x^2+Aa\right.}{2\left(a^2b^3x^2+a^3b^2\right)}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/x/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[-1/4*(2*C*a^2*b - 2*A*a*b^2 + (D*a^2 + B*a*b + (D*a*b + B*b^2)*x^2)*sqrt(
-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 2*(D*a^2*b - B*a*b^2
)*x + 2*(A*b^3*x^2 + A*a*b^2)*log(b*x^2 + a) - 4*(A*b^3*x^2 + A*a*b^2)*log
(x))/(a^2*b^3*x^2 + a^3*b^2), -1/2*(C*a^2*b - A*a*b^2 - (D*a^2 + B*a*b + (
D*a*b + B*b^2)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) + (D*a^2*b - B*a*b^2)*
x + (A*b^3*x^2 + A*a*b^2)*log(b*x^2 + a) - 2*(A*b^3*x^2 + A*a*b^2)*log(x))
/(a^2*b^3*x^2 + a^3*b^2)]
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{x\left(a+bx^2\right)^2}\,dx = \text{Timed out}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/x/(b*x**2+a)**2,x)
```

output
```
Timed out
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.92

$$\int \frac{A+Bx+Cx^2+Dx^3}{x\left(a+bx^2\right)^2}\,dx = -\frac{Ca-Ab+(Da-Bb)x}{2\left(ab^2x^2+a^2b\right)} - \frac{A\log\left(bx^2+a\right)}{2\,a^2} + \frac{A\log\left(x\right)}{a^2} + \frac{(Da+Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}ab}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/x/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
-1/2*(C*a - A*b + (D*a - B*b)*x)/(a*b^2*x^2 + a^2*b) - 1/2*A*log(b*x^2 + a
)/a^2 + A*log(x)/a^2 + 1/2*(D*a + B*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*
b)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.98

$$\int \frac{A+Bx+Cx^2+Dx^3}{x\left(a+bx^2\right)^2}\,dx = -\frac{A\log\left(bx^2+a\right)}{2\,a^2} + \frac{A\log\left(|x|\right)}{a^2} + \frac{(Da+Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}ab} - \frac{Ca^2-Aab+(Da^2-Bab)x}{2\,(bx^2+a)a^2b}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/x/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
-1/2*A*log(b*x^2 + a)/a^2 + A*log(abs(x))/a^2 + 1/2*(D*a + B*b)*arctan(b*x
/sqrt(a*b))/(sqrt(a*b)*a*b) - 1/2*(C*a^2 - A*a*b + (D*a^2 - B*a*b)*x)/((b*
x^2 + a)*a^2*b)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{x\,(a+bx^2)^2}\,dx=\int \frac{A+B\,x+C\,x^2+x^3\,D}{x\,(b\,x^2+a)^2}\,dx$$

input
```
int((A + B*x + C*x^2 + x^3*D)/(x*(a + b*x^2)^2),x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/(x*(a + b*x^2)^2), x)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 192, normalized size of antiderivative = 2.02

$$\int \frac{A+Bx+Cx^2+Dx^3}{x\,(a+bx^2)^2}\,dx$$
$$=\frac{\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^2d+\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a\,b^2+\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)abd\,x^2+\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)b}{2a}$$

input
```
int((D*x^3+C*x^2+B*x+A)/x/(b*x^2+a)^2,x)
```

output
```
(sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2*d + sqrt(b)*sqrt(a)*at
an((b*x)/(sqrt(b)*sqrt(a)))*a*b**2 + sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*s
qrt(a)))*a*b*d*x**2 + sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**3*x
**2 - log(a + b*x**2)*a**2*b**2 - log(a + b*x**2)*a*b**3*x**2 + 2*log(x)*a
**2*b**2 + 2*log(x)*a*b**3*x**2 - a**2*b*d*x - a*b**3*x**2 + a*b**3*x + a*
b**2*c*x**2)/(2*a**2*b**2*(a + b*x**2))
```

# 3.51 $\int \frac{A+Bx+Cx^2+Dx^3}{x^2(a+bx^2)^2} dx$



## Optimal result

Integrand size = 28, antiderivative size = 111

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^2(a+bx^2)^2} dx = -\frac{A}{a^2x} + \frac{a(bB-aD)-b(Ab-aC)x}{2a^2b(a+bx^2)} - \frac{(3Ab-aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}\sqrt{b}} + \frac{B\log(x)}{a^2} - \frac{B\log(a+bx^2)}{2a^2}$$

output
```
-A/a^2/x+1/2*(a*(B*b-D*a)-b*(A*b-C*a)*x)/a^2/b/(b*x^2+a)-1/2*(3*A*b-C*a)*a
rctan(b^(1/2)*x/a^(1/2))/a^(5/2)/b^(1/2)+B*ln(x)/a^2-1/2*B*ln(b*x^2+a)/a^2
```

## Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.99

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^2(a+bx^2)^2} dx = -\frac{A}{a^2x} + \frac{abB-a^2D-Ab^2x+abCx}{2a^2b(a+bx^2)} + \frac{(-3Ab+aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}\sqrt{b}} + \frac{B\log(x)}{a^2} - \frac{B\log(a+bx^2)}{2a^2}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/(x^2*(a + b*x^2)^2),x]
```

output
```
-(A/(a^2*x)) + (a*b*B - a^2*D - A*b^2*x + a*b*C*x)/(2*a^2*b*(a + b*x^2)) +
 ((-3*A*b + a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(5/2)*Sqrt[b]) + (B*Log
[x])/a^2 - (B*Log[a + b*x^2])/(2*a^2)
```

## Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2336, 25, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^2\left(a+bx^2\right)^2}dx$$

$$\Big\downarrow \text{2336}$$

$$\frac{-bx\left(\frac{Ab}{a}-C\right)-aD+bB}{2ab\left(a+bx^2\right)}-\frac{\int -\frac{-\left(\left(\frac{Ab}{a}-C\right)x^2\right)+2Bx+2A}{x^2\left(bx^2+a\right)}dx}{2a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{-\left(\left(\frac{Ab}{a}-C\right)x^2\right)+2Bx+2A}{x^2\left(bx^2+a\right)}dx}{2a}+\frac{-bx\left(\frac{Ab}{a}-C\right)-aD+bB}{2ab\left(a+bx^2\right)}$$

$$\Big\downarrow \text{2333}$$

$$\frac{\int \left(\frac{2A}{ax^2}+\frac{2B}{ax}+\frac{-3Ab-2Bxb+aC}{a\left(bx^2+a\right)}\right)dx}{2a}+\frac{-bx\left(\frac{Ab}{a}-C\right)-aD+bB}{2ab\left(a+bx^2\right)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{(3Ab-aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}-\frac{2A}{ax}-\frac{B\log\left(a+bx^2\right)}{a}+\frac{2B\log(x)}{a}}{2a}+\frac{-bx\left(\frac{Ab}{a}-C\right)-aD+bB}{2ab\left(a+bx^2\right)}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/(x^2*(a + b*x^2)^2),x]
```

output
```
(b*B - a*D - b*((A*b)/a - C)*x)/(2*a*b*(a + b*x^2)) + ((-2*A)/(a*x) - ((3*
A*b - a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*Sqrt[b]) + (2*B*Log[x])/a
 - (B*Log[a + b*x^2])/a)/(2*a)
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

## Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.86

| method | result | size |
|---|---|---|
| default | $-\frac{\frac{\left(\frac{Ab}{2}-\frac{Ca}{2}\right)x-\frac{a(Bb-Da)}{2b}}{b\,x^{2}+a}+\frac{B\ln\left(b\,x^{2}+a\right)}{2}+\frac{(3Ab-Ca)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}}{a^{2}}-\frac{A}{a^{2}x}+\frac{B\ln(x)}{a^{2}}$ | 96 |

input
```
int((D*x^3+C*x^2+B*x+A)/x^2/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/a^2*(((1/2*A*b-1/2*C*a)*x-1/2*a*(B*b-D*a)/b)/(b*x^2+a)+1/2*B*ln(b*x^2+a
)+1/2*(3*A*b-C*a)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))-A/a^2/x+B*ln(x)/a^2
```

## Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 336, normalized size of antiderivative = 3.03

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^2(a+bx^2)^2}\,dx$$
$$= \left[-\frac{4\,Aa^2b-2\,(Ca^2b-3\,Aab^2)x^2-((Cab-3\,Ab^2)x^3+(Ca^2-3\,Aab)x)\sqrt{-ab}\log\left(\frac{bx^2+2\sqrt{-ab}x-a}{bx^2+a}\right)+2}{4\,(a^3b^2x^3+a^4bx)}\right.$$
$$-\frac{2\,Aa^2b-(Ca^2b-3\,Aab^2)x^2-((Cab-3\,Ab^2)x^3+(Ca^2-3\,Aab)x)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}x}{a}\right)+(Da^3-Ba}{2\,(a^3b^2x^3+a^4bx)}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/x^2/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[-1/4*(4*A*a^2*b - 2*(C*a^2*b - 3*A*a*b^2)*x^2 - ((C*a*b - 3*A*b^2)*x^3 +
(C*a^2 - 3*A*a*b)*x)*sqrt(-a*b)*log((b*x^2 + 2*sqrt(-a*b)*x - a)/(b*x^2 +
a)) + 2*(D*a^3 - B*a^2*b)*x + 2*(B*a*b^2*x^3 + B*a^2*b*x)*log(b*x^2 + a) -
 4*(B*a*b^2*x^3 + B*a^2*b*x)*log(x))/(a^3*b^2*x^3 + a^4*b*x), -1/2*(2*A*a^
2*b - (C*a^2*b - 3*A*a*b^2)*x^2 - ((C*a*b - 3*A*b^2)*x^3 + (C*a^2 - 3*A*a*
b)*x)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) + (D*a^3 - B*a^2*b)*x + (B*a*b^2*x^3
 + B*a^2*b*x)*log(b*x^2 + a) - 2*(B*a*b^2*x^3 + B*a^2*b*x)*log(x))/(a^3*b^
2*x^3 + a^4*b*x)]
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^2\left(a+bx^2\right)^2}\,dx=\text{Timed out}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/x**2/(b*x**2+a)**2,x)
```

output
```
Timed out
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.95

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^2\left(a+bx^2\right)^2}\,dx=-\frac{2\,Aab-(Cab-3\,Ab^2)x^2+(Da^2-Bab)x}{2\,(a^2b^2x^3+a^3bx)}-\frac{B\log\left(bx^2+a\right)}{2\,a^2}+\frac{B\log\left(x\right)}{a^2}+\frac{(Ca-3\,Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^2}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/x^2/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
-1/2*(2*A*a*b - (C*a*b - 3*A*b^2)*x^2 + (D*a^2 - B*a*b)*x)/(a^2*b^2*x^3 +
a^3*b*x) - 1/2*B*log(b*x^2 + a)/a^2 + B*log(x)/a^2 + 1/2*(C*a - 3*A*b)*arc
tan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.93

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^2(a+bx^2)^2}\,dx = -\frac{B\log(bx^2+a)}{2a^2}+\frac{B\log(|x|)}{a^2}+\frac{(Ca-3Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}a^2}+\frac{Cabx^2-3Ab^2x^2-Da^2x+Babx-2Aab}{2(bx^3+ax)a^2b}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/x^2/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
-1/2*B*log(b*x^2 + a)/a^2 + B*log(abs(x))/a^2 + 1/2*(C*a - 3*A*b)*arctan(b
*x/sqrt(a*b))/(sqrt(a*b)*a^2) + 1/2*(C*a*b*x^2 - 3*A*b^2*x^2 - D*a^2*x + B
*a*b*x - 2*A*a*b)/((b*x^3 + a*x)*a^2*b)
```

## Mupad [B] (verification not implemented)

Time = 2.43 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.20

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^2(a+bx^2)^2}\,dx = \frac{B}{2a(bx^2+a)}-\frac{\frac{A}{a}+\frac{3Abx^2}{2a^2}}{bx^3+ax}-\frac{B\ln(bx^2+a)}{2a^2}+\frac{B\ln(x)}{a^2}-\frac{D}{2b(bx^2+a)}+\frac{Cx}{2a(bx^2+a)}-\frac{3A\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}}+\frac{C\,\mathrm{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}$$

input
```
int((A + B*x + C*x^2 + x^3*D)/(x^2*(a + b*x^2)^2),x)
```

output
```
B/(2*a*(a + b*x^2)) - (A/a + (3*A*b*x^2)/(2*a^2))/(a*x + b*x^3) - (B*log(a
 + b*x^2))/(2*a^2) + (B*log(x))/a^2 - D/(2*b*(a + b*x^2)) + (C*x)/(2*a*(a
+ b*x^2)) - (3*A*b^(1/2)*atan((b^(1/2)*x)/a^(1/2)))/(2*a^(5/2)) + (C*atan(
(b^(1/2)*x)/a^(1/2)))/(2*a^(3/2)*b^(1/2))
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.76

$$\int \frac{A + Bx + Cx^2 + Dx^3}{x^2\left(a + bx^2\right)^2}\,dx$$
$$= \frac{-3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)abx + \sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)acx - 3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)b^2x^3 + \sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right.}{}$$

input
```
int((D*x^3+C*x^2+B*x+A)/x^2/(b*x^2+a)^2,x)
```

output
```
( - 3*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*b*x + sqrt(b)*sqrt(a
)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*c*x - 3*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt
(b)*sqrt(a)))*b**2*x**3 + sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b*
c*x**3 - log(a + b*x**2)*a*b**2*x - log(a + b*x**2)*b**3*x**3 + 2*log(x)*a
*b**2*x + 2*log(x)*b**3*x**3 - 2*a**2*b - 3*a*b**2*x**2 + a*b*c*x**2 + a*b
*d*x**3 - b**3*x**3)/(2*a**2*b*x*(a + b*x**2))
```

# 3.52 $\int \frac{A+Bx+Cx^2+Dx^3}{x^3(a+bx^2)^2} dx$



## Optimal result

Integrand size = 28, antiderivative size = 131

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^3(a+bx^2)^2} dx = -\frac{A}{2a^2x^2} - \frac{B}{a^2x} - \frac{Ab-aC+(bB-aD)x}{2a^2(a+bx^2)} - \frac{(3bB-aD)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}\sqrt{b}} - \frac{(2Ab-aC)\log(x)}{a^3} + \frac{(2Ab-aC)\log(a+bx^2)}{2a^3}$$

output
```
-1/2*A/a^2/x^2-B/a^2/x-1/2*(A*b-C*a+(B*b-D*a)*x)/a^2/(b*x^2+a)-1/2*(3*B*b-
D*a)*arctan(b^(1/2)*x/a^(1/2))/a^(5/2)/b^(1/2)-(2*A*b-C*a)*ln(x)/a^3+1/2*(
2*A*b-C*a)*ln(b*x^2+a)/a^3
```

## Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.85

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^3\left(a+bx^2\right)^2}\,dx$$
$$= \frac{-\frac{aA}{x^2}-\frac{2aB}{x}+\frac{a(-Ab-bBx+a(C+Dx))}{a+bx^2}+\frac{\sqrt{a}(-3bB+aD)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}+2(-2Ab+aC)\log(x)+(2Ab-aC)\log(a}{2a^3}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/(x^3*(a + b*x^2)^2),x]
```

output
```
(-((a*A)/x^2) - (2*a*B)/x + (a*(-(A*b) - b*B*x + a*(C + D*x)))/(a + b*x^2)
 + (Sqrt[a]*(-3*b*B + a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[b] + 2*(-2*A*
b + a*C)*Log[x] + (2*A*b - a*C)*Log[a + b*x^2])/(2*a^3)
```

## Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2336, 25, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^3\left(a+bx^2\right)^2}\,dx$$
$$\Big\downarrow \text{2336}$$
$$-\frac{\int -\frac{-\left(\left(\frac{bB}{a}-D\right)x^3\right)-2\left(\frac{Ab}{a}-C\right)x^2+2Bx+2A}{x^3(bx^2+a)}dx}{2a}-\frac{\frac{Ab}{a}+x\left(\frac{bB}{a}-D\right)-C}{2a\left(a+bx^2\right)}$$
$$\Big\downarrow \text{25}$$
$$\frac{\int \frac{-\left(\left(\frac{bB}{a}-D\right)x^3\right)-2\left(\frac{Ab}{a}-C\right)x^2+2Bx+2A}{x^3(bx^2+a)}dx}{2a}-\frac{\frac{Ab}{a}+x\left(\frac{bB}{a}-D\right)-C}{2a\left(a+bx^2\right)}$$
$$\Big\downarrow \text{2333}$$

$$\frac{\int \left(\frac{2A}{ax^3}+\frac{2(aC-2Ab)}{a^2x}+\frac{2b(2Ab-aC)x-a(3bB-aD)}{a^2(bx^2+a)}+\frac{2B}{ax^2}\right)dx}{2a}-\frac{\frac{Ab}{a}+x\left(\frac{bB}{a}-D\right)-C}{2a\left(a+bx^2\right)}$$

$$\Big\downarrow \ 2009$$

$$\frac{-\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(3bB-aD)}{a^{3/2}\sqrt{b}}+\frac{(2Ab-aC)\log\left(a+bx^2\right)}{a^2}-\frac{2\log(x)(2Ab-aC)}{a^2}-\frac{A}{ax^2}-\frac{2B}{ax}}{2a}-\frac{\frac{Ab}{a}+x\left(\frac{bB}{a}-D\right)-C}{2a\left(a+bx^2\right)}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/(x^3*(a + b*x^2)^2),x]
```

output
```
-1/2*((A*b)/a - C + ((b*B)/a - D)*x)/(a*(a + b*x^2)) + (-(A/(a*x^2)) - (2*
B)/(a*x) - ((3*b*B - a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*Sqrt[b]) -
 (2*(2*A*b - a*C)*Log[x])/a^2 + ((2*A*b - a*C)*Log[a + b*x^2])/a^2)/(2*a)
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

## Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.97

| method | result |
|---|---|
| default | $\frac{\frac{\left(-\frac{1}{2}abB+\frac{1}{2}Da^2\right)x-\frac{a(Ab-Ca)}{2}}{b\,x^2+a}+\frac{\left(4b^2A-2Cab\right)\ln\left(b\,x^2+a\right)}{4b}+\frac{\left(-3abB+Da^2\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}}{a^3}-\frac{A}{2a^2x^2}-\frac{B}{a^2x}+\frac{(-2Ab+Ca)\ln(x)}{a^3}$ |

input
```
int((D*x^3+C*x^2+B*x+A)/x^3/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a^3*(((-1/2*a*b*B+1/2*D*a^2)*x-1/2*a*(A*b-C*a))/(b*x^2+a)+1/4*(4*A*b^2-2
*C*a*b)/b*ln(b*x^2+a)+1/2*(-3*B*a*b+D*a^2)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1
/2)))-1/2*A/a^2/x^2-B/a^2/x+(-2*A*b+C*a)/a^3*ln(x)
```

## Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 442, normalized size of antiderivative = 3.37

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^3\left(a+bx^2\right)^2}\,dx$$

$$= \Bigg[-\frac{4\,Ba^2bx+2\,Aa^2b-2\,(Da^2b-3\,Bab^2)x^3-2\,(Ca^2b-2\,Aab^2)x^2-((Dab-3\,Bb^2)x^4+(Da^2-3\,Ba}{}$$

$$-\frac{2\,Ba^2bx+Aa^2b-(Da^2b-3\,Bab^2)x^3-(Ca^2b-2\,Aab^2)x^2-((Dab-3\,Bb^2)x^4+(Da^2-3\,Bab)x^2)}{}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/x^3/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[-1/4*(4*B*a^2*b*x + 2*A*a^2*b - 2*(D*a^2*b - 3*B*a*b^2)*x^3 - 2*(C*a^2*b
- 2*A*a*b^2)*x^2 - ((D*a*b - 3*B*b^2)*x^4 + (D*a^2 - 3*B*a*b)*x^2)*sqrt(-a
*b)*log((b*x^2 + 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 2*((C*a*b^2 - 2*A*b^3)
*x^4 + (C*a^2*b - 2*A*a*b^2)*x^2)*log(b*x^2 + a) - 4*((C*a*b^2 - 2*A*b^3)*
x^4 + (C*a^2*b - 2*A*a*b^2)*x^2)*log(x))/(a^3*b^2*x^4 + a^4*b*x^2), -1/2*(
2*B*a^2*b*x + A*a^2*b - (D*a^2*b - 3*B*a*b^2)*x^3 - (C*a^2*b - 2*A*a*b^2)*
x^2 - ((D*a*b - 3*B*b^2)*x^4 + (D*a^2 - 3*B*a*b)*x^2)*sqrt(a*b)*arctan(sqr
t(a*b)*x/a) + ((C*a*b^2 - 2*A*b^3)*x^4 + (C*a^2*b - 2*A*a*b^2)*x^2)*log(b*
x^2 + a) - 2*((C*a*b^2 - 2*A*b^3)*x^4 + (C*a^2*b - 2*A*a*b^2)*x^2)*log(x))
/(a^3*b^2*x^4 + a^4*b*x^2)]
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{A + Bx + Cx^2 + Dx^3}{x^3 (a + bx^2)^2} \, dx = \text{Timed out}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/x**3/(b*x**2+a)**2,x)
```

output
```
Timed out
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.89

$$\int \frac{A + Bx + Cx^2 + Dx^3}{x^3 (a + bx^2)^2} \, dx = \frac{(Da - 3\,Bb)x^3 - 2\,Bax + (Ca - 2\,Ab)x^2 - Aa}{2\,(a^2bx^4 + a^3x^2)} + \frac{(Da - 3\,Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^2} - \frac{(Ca - 2\,Ab)\log\left(bx^2 + a\right)}{2\,a^3} + \frac{(Ca - 2\,Ab)\log\left(x\right)}{a^3}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/x^3/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*((D*a - 3*B*b)*x^3 - 2*B*a*x + (C*a - 2*A*b)*x^2 - A*a)/(a^2*b*x^4 + a
^3*x^2) + 1/2*(D*a - 3*B*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) - 1/2*(C
*a - 2*A*b)*log(b*x^2 + a)/a^3 + (C*a - 2*A*b)*log(x)/a^3
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.96

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^3\left(a+bx^2\right)^2}\,dx = \frac{(Da-3\,Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^2} - \frac{(Ca-2\,Ab)\log\left(bx^2+a\right)}{2\,a^3} + \frac{(Ca-2\,Ab)\log\left(|x|\right)}{a^3} - \frac{2\,Ba^2x-(Da^2-3\,Bab)x^3+Aa^2-(Ca^2-2\,Aab)x^2}{2\,(bx^2+a)a^3x^2}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/x^3/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*(D*a - 3*B*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) - 1/2*(C*a - 2*A*b
)*log(b*x^2 + a)/a^3 + (C*a - 2*A*b)*log(abs(x))/a^3 - 1/2*(2*B*a^2*x - (D
*a^2 - 3*B*a*b)*x^3 + A*a^2 - (C*a^2 - 2*A*a*b)*x^2)/((b*x^2 + a)*a^3*x^2)
```

## Mupad [B] (verification not implemented)

Time = 2.41 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.21

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^3\left(a+bx^2\right)^2}\,dx = \frac{C}{2\,a\,\left(b\,x^2+a\right)} - \frac{\frac{A}{2\,a}+\frac{A\,b\,x^2}{a^2}}{b\,x^4+a\,x^2} - \frac{\frac{B}{a}+\frac{3\,B\,b\,x^2}{2\,a^2}}{b\,x^3+a\,x} - \frac{C\,\ln\left(b\,x^2+a\right)}{2\,a^2} + \frac{C\,\ln\left(x\right)}{a^2} + \frac{A\,b\,\ln\left(b\,x^2+a\right)}{a^3} - \frac{2\,A\,b\,\ln\left(x\right)}{a^3} + \frac{x\,D\,{}_2\mathrm{F}_1\left(\frac{1}{2},2;\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{a^2} - \frac{3\,B\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,a^{5/2}}$$

input
```
int((A + B*x + C*x^2 + x^3*D)/(x^3*(a + b*x^2)^2),x)
```

output
```
C/(2*a*(a + b*x^2)) - (A/(2*a) + (A*b*x^2)/a^2)/(a*x^2 + b*x^4) - (B/a + (
3*B*b*x^2)/(2*a^2))/(a*x + b*x^3) - (C*log(a + b*x^2))/(2*a^2) + (C*log(x)
)/a^2 + (A*b*log(a + b*x^2))/a^3 - (2*A*b*log(x))/a^3 + (x*D*hypergeom([1/
2, 2], 3/2, -(b*x^2)/a))/a^2 - (3*B*b^(1/2)*atan((b^(1/2)*x)/a^(1/2)))/(2*
a^(5/2))
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 289, normalized size of antiderivative = 2.21

$$\int \frac{A + Bx + Cx^2 + Dx^3}{x^3 (a + bx^2)^2} \, dx$$
$$= \frac{\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right) a^2 d\,x^2 - 3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right) a\,b^2 x^2 + \sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right) abd\,x^4 - 3\sqrt{b}\,\sqrt{a}\,atan\left(\right.}{}$$

input
```
int((D*x^3+C*x^2+B*x+A)/x^3/(b*x^2+a)^2,x)
```

output
```
(sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2*d*x**2 - 3*sqrt(b)*sqr
t(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*b**2*x**2 + sqrt(b)*sqrt(a)*atan((b*x
)/(sqrt(b)*sqrt(a)))*a*b*d*x**4 - 3*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sq
rt(a)))*b**3*x**4 + 2*log(a + b*x**2)*a**2*b**2*x**2 - log(a + b*x**2)*a**
2*b*c*x**2 + 2*log(a + b*x**2)*a*b**3*x**4 - log(a + b*x**2)*a*b**2*c*x**4
 - 4*log(x)*a**2*b**2*x**2 + 2*log(x)*a**2*b*c*x**2 - 4*log(x)*a*b**3*x**4
 + 2*log(x)*a*b**2*c*x**4 - a**3*b - 2*a**2*b**2*x + a**2*b*d*x**3 + 2*a*b
**3*x**4 - 3*a*b**3*x**3 - a*b**2*c*x**4)/(2*a**3*b*x**2*(a + b*x**2))
```

# 3.53 $\int \frac{x^4(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx$



## Optimal result

Integrand size = 28, antiderivative size = 158

$$\int \frac{x^4(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx = \frac{Cx}{b^3}+\frac{Dx^2}{2b^3}-\frac{a(a(bB-aD)-b(Ab-aC)x)}{4b^4(a+bx^2)^2} + \frac{4a(2bB-3aD)-b(5Ab-9aC)x}{8b^4(a+bx^2)} + \frac{3(Ab-5aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8\sqrt{a}b^{7/2}} + \frac{(bB-3aD)\log(a+bx^2)}{2b^4}$$

output
```
C*x/b^3+1/2*D*x^2/b^3-1/4*a*(a*(B*b-D*a)-b*(A*b-C*a)*x)/b^4/(b*x^2+a)^2+1/
8*(4*a*(2*B*b-3*D*a)-b*(5*A*b-9*C*a)*x)/b^4/(b*x^2+a)+3/8*(A*b-5*C*a)*arct
an(b^(1/2)*x/a^(1/2))/a^(1/2)/b^(7/2)+1/2*(B*b-3*D*a)*ln(b*x^2+a)/b^4
```

## Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.88

$$\int \frac{x^4(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx$$

$$= \frac{8bCx+4bDx^2+\frac{8abB-12a^2D-5Ab^2x+9abCx}{a+bx^2}+\frac{2a(a^2D+Ab^2x-ab(B+Cx))}{(a+bx^2)^2}+\frac{3\sqrt{b}(Ab-5aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}}+4(bB-3a}{8b^4}$$

input
```
Integrate[(x^4*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^3,x]
```

output
```
(8*b*C*x + 4*b*D*x^2 + (8*a*b*B - 12*a^2*D - 5*A*b^2*x + 9*a*b*C*x)/(a + b
*x^2) + (2*a*(a^2*D + A*b^2*x - a*b*(B + C*x)))/(a + b*x^2)^2 + (3*Sqrt[b]
*(A*b - 5*a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[a] + 4*(b*B - 3*a*D)*Log[
a + b*x^2])/(8*b^4)
```

## Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.18, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2335, 25, 2335, 27, 523, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx$$

$$\Big\downarrow \ 2335$$

$$-\frac{\int -\frac{x^3\left(4aDx^2-(Ab-5aC)x+4a\left(B-\frac{aD}{b}\right)\right)}{(bx^2+a)^2}dx}{4ab}-\frac{x^4\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab\,(a+bx^2)^2}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{x^3\left(4aDx^2-(Ab-5aC)x+4a\left(B-\frac{aD}{b}\right)\right)}{(bx^2+a)^2}dx}{4ab}-\frac{x^4\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab\,(a+bx^2)^2}$$

$$\Big\downarrow \; 2335$$

$$\frac{\frac{x^3(4x(bB-2aD)-5aC+Ab)}{2b(a+bx^2)}-\frac{\int \frac{ax^2(3(Ab-5aC)+8(bB-3aD)x)}{bx^2+a}dx}{2ab}}{4ab}-\frac{x^4\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab\left(a+bx^2\right)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{x^3(4x(bB-2aD)-5aC+Ab)}{2b(a+bx^2)}-\frac{\int \frac{x^2(3(Ab-5aC)+8(bB-3aD)x)}{bx^2+a}dx}{2b}}{4ab}-\frac{x^4\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab\left(a+bx^2\right)^2}$$

$$\Big\downarrow \; 523$$

$$\frac{\frac{x^3(4x(bB-2aD)-5aC+Ab)}{2b(a+bx^2)}-\frac{\int \left(3\left(A-\frac{5aC}{b}\right)+\frac{8(bB-3aD)x}{b}-\frac{3a(Ab-5aC)+8a(bB-3aD)x}{b\left(bx^2+a\right)}\right)dx}{2b}}{4ab}-\frac{x^4\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab\left(a+bx^2\right)^2}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{x^3(4x(bB-2aD)-5aC+Ab)}{2b(a+bx^2)}-\frac{-\frac{3\sqrt{a}(Ab-5aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}}+3x\left(A-\frac{5aC}{b}\right)-\frac{4a(bB-3aD)\log\left(a+bx^2\right)}{b^2}+\frac{4x^2(bB-3aD)}{b}}{2b}}{4ab}-\frac{x^4\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab\left(a+bx^2\right)^2}$$

input
```
Int[(x^4*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^3,x]
```

output
```
-1/4*(x^4*(a*(B - (a*D)/b) - (A*b - a*C)*x))/(a*b*(a + b*x^2)^2) + ((x^3*(
A*b - 5*a*C + 4*(b*B - 2*a*D)*x))/(2*b*(a + b*x^2)) - (3*(A - (5*a*C)/b)*x
 + (4*(b*B - 3*a*D)*x^2)/b - (3*Sqrt[a]*(A*b - 5*a*C)*ArcTan[(Sqrt[b]*x)/S
qrt[a]])/b^(3/2) - (4*a*(b*B - 3*a*D)*Log[a + b*x^2])/b^2)/(2*b))/(4*a*b)
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 523
```
Int[((x_)^(m_.)*((c_) + (d_.)*(x_)))/((a_) + (b_.)*(x_)^2), x_Symbol] :> In
t[ExpandIntegrand[x^m*((c + d*x)/(a + b*x^2)), x], x] /; FreeQ[{a, b, c, d}
, x] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2335
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Simp[c/(2*a*b*(p + 1))   Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSu
m[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
 b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

## Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.89

| method | result |
|---|---|
| default | $\frac{\frac{1}{2} D x^{2}+C x}{b^{3}}+\frac{\frac{\left(-\frac{5}{8} b^{2} A+\frac{9}{8} C a b\right) x^{3}+\left(a b B-\frac{3}{2} D a^{2}\right) x^{2}-\frac{a \left(3 A b-7 C a\right) x}{8}+\frac{a^{2} \left(3 B b-5 D a\right)}{4 b}}{\left(b \,x^{2}+a\right)^{2}}+\frac{\left(8 B b-24 D a\right) \ln \left(b \,x^{2}+a\right)}{16 b}+\frac{\left(3 A b-15 C a\right) \arctan \left(\frac{b}{\sqrt{}}\right.}{8 \sqrt{a b}}}{b^{3}}$ |

input
```
int(x^4*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b^3*(1/2*D*x^2+C*x)+1/b^3*(((-5/8*b^2*A+9/8*C*a*b)*x^3+(a*b*B-3/2*D*a^2)
*x^2-1/8*a*(3*A*b-7*C*a)*x+1/4*a^2*(3*B*b-5*D*a)/b)/(b*x^2+a)^2+1/16*(8*B*
b-24*D*a)/b*ln(b*x^2+a)+1/8*(3*A*b-15*C*a)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1
/2)))
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 574, normalized size of antiderivative = 3.63

$$\int \frac{x^4(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx$$
$$= \left[\frac{8\,Dab^3x^6+16\,Cab^3x^5+16\,Da^2b^2x^4-20\,Da^4+12\,Ba^3b+10\,(5\,Ca^2b^2-Aab^3)x^3-16\,(Da^3b-Ba^2b\right.$$

input
```
integrate(x^4*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
[1/16*(8*D*a*b^3*x^6 + 16*C*a*b^3*x^5 + 16*D*a^2*b^2*x^4 - 20*D*a^4 + 12*B
*a^3*b + 10*(5*C*a^2*b^2 - A*a*b^3)*x^3 - 16*(D*a^3*b - B*a^2*b^2)*x^2 + 3
*((5*C*a*b^2 - A*b^3)*x^4 + 5*C*a^3 - A*a^2*b + 2*(5*C*a^2*b - A*a*b^2)*x^
2)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 6*(5*C*a^3*b
 - A*a^2*b^2)*x - 8*(3*D*a^4 - B*a^3*b + (3*D*a^2*b^2 - B*a*b^3)*x^4 + 2*(
3*D*a^3*b - B*a^2*b^2)*x^2)*log(b*x^2 + a))/(a*b^6*x^4 + 2*a^2*b^5*x^2 + a
^3*b^4), 1/8*(4*D*a*b^3*x^6 + 8*C*a*b^3*x^5 + 8*D*a^2*b^2*x^4 - 10*D*a^4 +
 6*B*a^3*b + 5*(5*C*a^2*b^2 - A*a*b^3)*x^3 - 8*(D*a^3*b - B*a^2*b^2)*x^2 -
 3*((5*C*a*b^2 - A*b^3)*x^4 + 5*C*a^3 - A*a^2*b + 2*(5*C*a^2*b - A*a*b^2)*
x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) + 3*(5*C*a^3*b - A*a^2*b^2)*x - 4*(3*
D*a^4 - B*a^3*b + (3*D*a^2*b^2 - B*a*b^3)*x^4 + 2*(3*D*a^3*b - B*a^2*b^2)*
x^2)*log(b*x^2 + a))/(a*b^6*x^4 + 2*a^2*b^5*x^2 + a^3*b^4)]
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 357 vs. $2(150) = 300$.

Time = 12.23 (sec) , antiderivative size = 357, normalized size of antiderivative = 2.26

$$\int \frac{x^4(A + Bx + Cx^2 + Dx^3)}{(a + bx^2)^3}\,dx = \frac{Cx}{b^3} + \frac{Dx^2}{2b^3} + \left(-\frac{-Bb + 3Da}{2b^4} - \frac{3\sqrt{-ab^9}(-Ab + 5Ca)}{16ab^8}\right)\log\left(x + \frac{8Bab - 24Da^2 - 16ab^4\left(-\frac{-Bb+3Da}{2b^4} - \frac{3\sqrt{-ab^9}(-Ab+5Ca)}{16ab^8}\right)}{-3Ab^2 + 15Cab}\right) + \left(-\frac{-Bb + 3Da}{2b^4} + \frac{3\sqrt{-ab^9}(-Ab + 5Ca)}{16ab^8}\right)\log\left(x + \frac{8Bab - 24Da^2 - 16ab^4\left(-\frac{-Bb+3Da}{2b^4} + \frac{3\sqrt{-ab^9}(-Ab+5Ca)}{16ab^8}\right)}{-3Ab^2 + 15Cab}\right) + \frac{6Ba^2b - 10Da^3 + x^3(-5Ab^3 + 9Cab^2) + x^2\cdot(8Bab^2 - 12Da^2b) + x(-3Aab^2 + 7Ca^2b)}{8a^2b^4 + 16ab^5x^2 + 8b^6x^4}$$

input
```
integrate(x**4*(D*x**3+C*x**2+B*x+A)/(b*x**2+a)**3,x)
```

output
```
C*x/b**3 + D*x**2/(2*b**3) + (-(-B*b + 3*D*a)/(2*b**4) - 3*sqrt(-a*b**9)*(
-A*b + 5*C*a)/(16*a*b**8))*log(x + (8*B*a*b - 24*D*a**2 - 16*a*b**4*(-(-B*
b + 3*D*a)/(2*b**4) - 3*sqrt(-a*b**9)*(-A*b + 5*C*a)/(16*a*b**8)))/(-3*A*b
**2 + 15*C*a*b)) + (-(-B*b + 3*D*a)/(2*b**4) + 3*sqrt(-a*b**9)*(-A*b + 5*C
*a)/(16*a*b**8))*log(x + (8*B*a*b - 24*D*a**2 - 16*a*b**4*(-(-B*b + 3*D*a)
/(2*b**4) + 3*sqrt(-a*b**9)*(-A*b + 5*C*a)/(16*a*b**8)))/(-3*A*b**2 + 15*C
*a*b)) + (6*B*a**2*b - 10*D*a**3 + x**3*(-5*A*b**3 + 9*C*a*b**2) + x**2*(8
*B*a*b**2 - 12*D*a**2*b) + x*(-3*A*a*b**2 + 7*C*a**2*b))/(8*a**2*b**4 + 16
*a*b**5*x**2 + 8*b**6*x**4)
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.04

$$\int \frac{x^4(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx = -\frac{10\,Da^3-6\,Ba^2b-(9\,Cab^2-5\,Ab^3)x^3+4\,(3\,Da^2b-2\,Bab^2)x^2-(7\,Ca^2b-3\,Aab^2)x}{8\,(b^6x^4+2\,ab^5x^2+a^2b^4)} - \frac{3\,(5\,Ca-Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}b^3} + \frac{Dx^2+2\,Cx}{2\,b^3} - \frac{(3\,Da-Bb)\log\left(bx^2+a\right)}{2\,b^4}$$

input
```
integrate(x^4*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
-1/8*(10*D*a^3 - 6*B*a^2*b - (9*C*a*b^2 - 5*A*b^3)*x^3 + 4*(3*D*a^2*b - 2*
B*a*b^2)*x^2 - (7*C*a^2*b - 3*A*a*b^2)*x)/(b^6*x^4 + 2*a*b^5*x^2 + a^2*b^4
) - 3/8*(5*C*a - A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) + 1/2*(D*x^2 +
 2*C*x)/b^3 - 1/2*(3*D*a - B*b)*log(b*x^2 + a)/b^4
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.99

$$\int \frac{x^4(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx = -\frac{3\,(5\,Ca-Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}b^3} - \frac{(3\,Da-Bb)\log\left(bx^2+a\right)}{2\,b^4} + \frac{Db^3x^2+2\,Cb^3x}{2\,b^6} - \frac{10\,Da^3-6\,Ba^2b-(9\,Cab^2-5\,Ab^3)x^3+4\,(3\,Da^2b-2\,Bab^2)x^2-(7\,Ca^2b-3\,Aab^2)x}{8\,(bx^2+a)^2b^4}$$

input
```
integrate(x^4*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
-3/8*(5*C*a - A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) - 1/2*(3*D*a - B*
b)*log(b*x^2 + a)/b^4 + 1/2*(D*b^3*x^2 + 2*C*b^3*x)/b^6 - 1/8*(10*D*a^3 -
6*B*a^2*b - (9*C*a*b^2 - 5*A*b^3)*x^3 + 4*(3*D*a^2*b - 2*B*a*b^2)*x^2 - (7
*C*a^2*b - 3*A*a*b^2)*x)/((b*x^2 + a)^2*b^4)
```

## Mupad [B] (verification not implemented)

Time = 2.52 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.47

$$\int \frac{x^4(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx = \frac{\frac{7\,C\,a^2\,x}{8}+\frac{9\,C\,b\,a\,x^3}{8}}{a^2\,b^3+2\,a\,b^4\,x^2+b^5\,x^4} - \frac{\frac{5\,A\,x^3}{8\,b}+\frac{3\,A\,a\,x}{8\,b^2}}{a^2+2\,a\,b\,x^2+b^2\,x^4} + \frac{\frac{3\,B\,a^2}{4\,b^3}+\frac{B\,a\,x^2}{b^2}}{a^2+2\,a\,b\,x^2+b^2\,x^4} - \frac{D\left(3\,a\,\ln\left(b\,x^2+a\right)-b\,x^2+\frac{3\,a^2}{b\,x^2+a}-\frac{a^3}{2\,(b\,x^2+a)^2}\right)}{2\,b^4} + \frac{B\,\ln\left(b\,x^2+a\right)}{2\,b^3} + \frac{C\,x}{b^3} + \frac{3\,A\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,\sqrt{a}\,b^{5/2}} - \frac{15\,C\,\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,b^{7/2}}$$

input
```
int((x^4*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^3,x)
```

output
```
((7*C*a^2*x)/8 + (9*C*a*b*x^3)/8)/(a^2*b^3 + b^5*x^4 + 2*a*b^4*x^2) - ((5*
A*x^3)/(8*b) + (3*A*a*x)/(8*b^2))/(a^2 + b^2*x^4 + 2*a*b*x^2) + ((3*B*a^2)
/(4*b^3) + (B*a*x^2)/b^2)/(a^2 + b^2*x^4 + 2*a*b*x^2) - (D*(3*a*log(a + b*
x^2) - b*x^2 + (3*a^2)/(a + b*x^2) - a^3/(2*(a + b*x^2)^2)))/(2*b^4) + (B*
log(a + b*x^2))/(2*b^3) + (C*x)/b^3 + (3*A*atan((b^(1/2)*x)/a^(1/2)))/(8*a
^(1/2)*b^(5/2)) - (15*C*a^(1/2)*atan((b^(1/2)*x)/a^(1/2)))/(8*b^(7/2))
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 353, normalized size of antiderivative = 2.23

$$\int \frac{x^4(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx = \frac{3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^2b-15\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^2c+6\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a\,b^2x^2-30\sqrt{b}\,\sqrt{a}\,atan\left(\right.}{}$$

input
```
int(x^4*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x)
```

output
```
(3*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2*b - 15*sqrt(b)*sqrt(
a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2*c + 6*sqrt(b)*sqrt(a)*atan((b*x)/(sq
rt(b)*sqrt(a)))*a*b**2*x**2 - 30*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(
a)))*a*b*c*x**2 + 3*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**3*x**
4 - 15*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**2*c*x**4 - 12*log(
a + b*x**2)*a**3*d + 4*log(a + b*x**2)*a**2*b**2 - 24*log(a + b*x**2)*a**2
*b*d*x**2 + 8*log(a + b*x**2)*a*b**3*x**2 - 12*log(a + b*x**2)*a*b**2*d*x*
*4 + 4*log(a + b*x**2)*b**4*x**4 - 6*a**3*d - 3*a**2*b**2*x + 2*a**2*b**2
+ 15*a**2*b*c*x - 5*a*b**3*x**3 + 25*a*b**2*c*x**3 + 12*a*b**2*d*x**4 - 4*
b**4*x**4 + 8*b**3*c*x**5 + 4*b**3*d*x**6)/(8*b**4*(a**2 + 2*a*b*x**2 + b*
*2*x**4))
```

# 3.54 $\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx$



## Optimal result

Integrand size = 28, antiderivative size = 131

$$\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx = \frac{Dx}{b^3}+\frac{a(Ab-aC+(bB-aD)x)}{4b^3(a+bx^2)^2} - \frac{4(Ab-2aC)+(5bB-9aD)x}{8b^3(a+bx^2)} + \frac{3(bB-5aD)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8\sqrt{a}b^{7/2}}+\frac{C\log(a+bx^2)}{2b^3}$$

output
```
D*x/b^3+1/4*a*(A*b-C*a+(B*b-D*a)*x)/b^3/(b*x^2+a)^2-1/8*(4*A*b-8*C*a+(5*B*
b-9*D*a)*x)/b^3/(b*x^2+a)+3/8*(B*b-5*D*a)*arctan(b^(1/2)*x/a^(1/2))/a^(1/2
)/b^(7/2)+1/2*C*ln(b*x^2+a)/b^3
```

## Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.96

$$\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx = \frac{Dx}{b^3}+\frac{-4Ab+8aC-5bBx+9aDx}{8b^3(a+bx^2)}+\frac{a(Ab+bBx-a(C+Dx))}{4b^3(a+bx^2)^2}+\frac{3(bB-5aD)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8\sqrt{a}b^{7/2}}+\frac{C\log(a+bx^2)}{2b^3}$$

input
```
Integrate[(x^3*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^3,x]
```

output
```
(D*x)/b^3 + (-4*A*b + 8*a*C - 5*b*B*x + 9*a*D*x)/(8*b^3*(a + b*x^2)) + (a*
(A*b + b*B*x - a*(C + D*x)))/(4*b^3*(a + b*x^2)^2) + (3*(b*B - 5*a*D)*ArcT
an[(Sqrt[b]*x)/Sqrt[a]])/(8*Sqrt[a]*b^(7/2)) + (C*Log[a + b*x^2])/(2*b^3)
```

## Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.23, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2335, 25, 2335, 25, 27, 523, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx^2)^3}dx$$

$$\Big\downarrow \text{2335}$$

$$-\frac{\int -\frac{x^2\left(4aDx^2+4aCx+3a\left(B-\frac{aD}{b}\right)\right)}{(bx^2+a)^2}dx}{4ab}-\frac{x^3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab(a+bx^2)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{x^2\left(4aDx^2+4aCx+3a\left(B-\frac{aD}{b}\right)\right)}{(bx^2+a)^2}dx}{4ab}-\frac{x^3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab\left(a+bx^2\right)^2}$$

$$\Big\downarrow \; 2335$$

$$\frac{-\frac{\int -\frac{ax(8aC-3(bB-5aD)x)}{bx^2+a}dx}{2ab}-\frac{x^2(4aC-x(3bB-7aD))}{2b(a+bx^2)}}{4ab}-\frac{x^3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab\left(a+bx^2\right)^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{ax(8aC-3(bB-5aD)x)}{bx^2+a}dx}{2ab}-\frac{x^2(4aC-x(3bB-7aD))}{2b(a+bx^2)}}{4ab}-\frac{x^3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab\left(a+bx^2\right)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{x(8aC-3(bB-5aD)x)}{bx^2+a}dx}{2b}-\frac{x^2(4aC-x(3bB-7aD))}{2b(a+bx^2)}}{4ab}-\frac{x^3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab\left(a+bx^2\right)^2}$$

$$\Big\downarrow \; 523$$

$$\frac{\frac{\int \left(\frac{3a(bB-5aD)+8abCx}{b\left(bx^2+a\right)}-3\left(B-\frac{5aD}{b}\right)\right)dx}{2b}-\frac{x^2(4aC-x(3bB-7aD))}{2b(a+bx^2)}}{4ab}-\frac{x^3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab\left(a+bx^2\right)^2}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{\frac{3\sqrt{a}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(bB-5aD)}{b^{3/2}}-3x\left(B-\frac{5aD}{b}\right)+\frac{4aC\log\left(a+bx^2\right)}{b}}{2b}-\frac{x^2(4aC-x(3bB-7aD))}{2b(a+bx^2)}}{4ab}-\frac{x^3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab\left(a+bx^2\right)^2}$$

input
```
Int[(x^3*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^3,x]
```

output
```
-1/4*(x^3*(a*(B - (a*D)/b) - (A*b - a*C)*x))/(a*b*(a + b*x^2)^2) + (-1/2*(
x^2*(4*a*C - (3*b*B - 7*a*D)*x))/(b*(a + b*x^2)) + (-3*(B - (5*a*D)/b)*x +
 (3*Sqrt[a]*(b*B - 5*a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(3/2) + (4*a*C*Lo
g[a + b*x^2])/b)/(2*b))/(4*a*b)
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 523
```
Int[((x_)^(m_.)*((c_) + (d_.)*(x_)))/((a_) + (b_.)*(x_)^2), x_Symbol] :> In
t[ExpandIntegrand[x^m*((c + d*x)/(a + b*x^2)), x], x] /; FreeQ[{a, b, c, d}
, x] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2335
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Simp[c/(2*a*b*(p + 1))   Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSu
m[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
 b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

## Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.88

| method | result | size |
|---|---|---|
| default | $\frac{Dx}{b^{3}}+\frac{\frac{\left(-\frac{5}{8}B\,b^{2}+\frac{9}{8}abD\right)x^{3}+\left(-\frac{1}{2}b^{2}A+Cab\right)x^{2}-\frac{a(3Bb-7Da)x}{8}-\frac{abA}{4}+\frac{3a^{2}C}{4}}{\left(b\,x^{2}+a\right)^{2}}+\frac{C\ln\left(b\,x^{2}+a\right)}{2}+\frac{(3Bb-15Da)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}}}{b^{3}}$ | 115 |

input
```
int(x^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
D*x/b^3+1/b^3*(((-5/8*B*b^2+9/8*a*b*D)*x^3+(-1/2*b^2*A+C*a*b)*x^2-1/8*a*(3
*B*b-7*D*a)*x-1/4*a*b*A+3/4*a^2*C)/(b*x^2+a)^2+1/2*C*ln(b*x^2+a)+1/8*(3*B*
b-15*D*a)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 230 vs. $2(114) = 228$.

Time = 0.08 (sec) , antiderivative size = 480, normalized size of antiderivative = 3.66

$$\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx$$
$$= \left[\frac{16\,Dab^3x^5+12\,Ca^3b-4\,Aa^2b^2+10\,(5\,Da^2b^2-Bab^3)x^3+8\,(2\,Ca^2b^2-Aab^3)x^2+3\,((5\,Dab^2-Bb^3)\right.$$

input
```
integrate(x^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
[1/16*(16*D*a*b^3*x^5 + 12*C*a^3*b - 4*A*a^2*b^2 + 10*(5*D*a^2*b^2 - B*a*b
^3)*x^3 + 8*(2*C*a^2*b^2 - A*a*b^3)*x^2 + 3*((5*D*a*b^2 - B*b^3)*x^4 + 5*D
*a^3 - B*a^2*b + 2*(5*D*a^2*b - B*a*b^2)*x^2)*sqrt(-a*b)*log((b*x^2 - 2*sq
rt(-a*b)*x - a)/(b*x^2 + a)) + 6*(5*D*a^3*b - B*a^2*b^2)*x + 8*(C*a*b^3*x^
4 + 2*C*a^2*b^2*x^2 + C*a^3*b)*log(b*x^2 + a))/(a*b^6*x^4 + 2*a^2*b^5*x^2
+ a^3*b^4), 1/8*(8*D*a*b^3*x^5 + 6*C*a^3*b - 2*A*a^2*b^2 + 5*(5*D*a^2*b^2
- B*a*b^3)*x^3 + 4*(2*C*a^2*b^2 - A*a*b^3)*x^2 - 3*((5*D*a*b^2 - B*b^3)*x^
4 + 5*D*a^3 - B*a^2*b + 2*(5*D*a^2*b - B*a*b^2)*x^2)*sqrt(a*b)*arctan(sqrt
(a*b)*x/a) + 3*(5*D*a^3*b - B*a^2*b^2)*x + 4*(C*a*b^3*x^4 + 2*C*a^2*b^2*x^
2 + C*a^3*b)*log(b*x^2 + a))/(a*b^6*x^4 + 2*a^2*b^5*x^2 + a^3*b^4)]
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 282 vs. $2(121) = 242$.

Time = 11.33 (sec) , antiderivative size = 282, normalized size of antiderivative = 2.15

$$\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx$$
$$= \frac{Dx}{b^3} + \left(\frac{C}{2b^3} - \frac{3\sqrt{-ab^7}(-Bb+5Da)}{16ab^7}\right)\log\left(x + \frac{8Ca - 16ab^3\left(\frac{C}{2b^3} - \frac{3\sqrt{-ab^7}(-Bb+5Da)}{16ab^7}\right)}{-3Bb+15Da}\right) + \left(\frac{C}{2b^3} + \frac{3\sqrt{-ab^7}(-Bb+5Da)}{16ab^7}\right)\log\left(x + \frac{8Ca - 16ab^3\left(\frac{C}{2b^3} + \frac{3\sqrt{-ab^7}(-Bb+5Da)}{16ab^7}\right)}{-3Bb+15Da}\right) + \frac{-2Aab + 6Ca^2 + x^3(-5Bb^2+9Dab) + x^2(-4Ab^2+8Cab) + x(-3Bab+7Da^2)}{8a^2b^3 + 16ab^4x^2 + 8b^5x^4}$$

input
```
integrate(x**3*(D*x**3+C*x**2+B*x+A)/(b*x**2+a)**3,x)
```

output
```
D*x/b**3 + (C/(2*b**3) - 3*sqrt(-a*b**7)*(-B*b + 5*D*a)/(16*a*b**7))*log(x
 + (8*C*a - 16*a*b**3*(C/(2*b**3) - 3*sqrt(-a*b**7)*(-B*b + 5*D*a)/(16*a*b
**7)))/(-3*B*b + 15*D*a)) + (C/(2*b**3) + 3*sqrt(-a*b**7)*(-B*b + 5*D*a)/(
16*a*b**7))*log(x + (8*C*a - 16*a*b**3*(C/(2*b**3) + 3*sqrt(-a*b**7)*(-B*b
 + 5*D*a)/(16*a*b**7)))/(-3*B*b + 15*D*a)) + (-2*A*a*b + 6*C*a**2 + x**3*(
-5*B*b**2 + 9*D*a*b) + x**2*(-4*A*b**2 + 8*C*a*b) + x*(-3*B*a*b + 7*D*a**2
))/(8*a**2*b**3 + 16*a*b**4*x**2 + 8*b**5*x**4)
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.04

$$\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx = \frac{(9\,Dab-5\,Bb^2)x^3+6\,Ca^2-2\,Aab+4\,(2\,Cab-Ab^2)x^2+(7\,Da^2-3\,Bab)x}{8\,(b^5x^4+2\,ab^4x^2+a^2b^3)} + \frac{Dx}{b^3} + \frac{C\log(bx^2+a)}{2\,b^3} - \frac{3\,(5\,Da-Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}b^3}$$

input
```
integrate(x^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/8*((9*D*a*b - 5*B*b^2)*x^3 + 6*C*a^2 - 2*A*a*b + 4*(2*C*a*b - A*b^2)*x^2
 + (7*D*a^2 - 3*B*a*b)*x)/(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3) + D*x/b^3 + 1/
2*C*log(b*x^2 + a)/b^3 - 3/8*(5*D*a - B*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b
)*b^3)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.93

$$\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx = \frac{Dx}{b^3} + \frac{C\log(bx^2+a)}{2\,b^3} - \frac{3\,(5\,Da-Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}b^3} + \frac{(9\,Dab-5\,Bb^2)x^3+6\,Ca^2-2\,Aab+4\,(2\,Cab-Ab^2)x^2+(7\,Da^2-3\,Bab)x}{8\,(bx^2+a)^2b^3}$$

input
```
integrate(x^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
D*x/b^3 + 1/2*C*log(b*x^2 + a)/b^3 - 3/8*(5*D*a - B*b)*arctan(b*x/sqrt(a*b
))/(sqrt(a*b)*b^3) + 1/8*((9*D*a*b - 5*B*b^2)*x^3 + 6*C*a^2 - 2*A*a*b + 4*
(2*C*a*b - A*b^2)*x^2 + (7*D*a^2 - 3*B*a*b)*x)/((b*x^2 + a)^2*b^3)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx=\int \frac{x^3\,(A+B\,x+C\,x^2+x^3\,D)}{(b\,x^2+a)^3}\,dx$$

input
```
int((x^3*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^3,x)
```

output
```
int((x^3*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^3, x)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 303, normalized size of antiderivative = 2.31

$$\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx$$
$$=\frac{-15\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^3d+3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^2b^2-30\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^2bd\,x^2+6\sqrt{b}\,\sqrt{a}\,ata}{}$$

input
```
int(x^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x)
```

output
```
( - 15*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**3*d + 3*sqrt(b)*sq
rt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2*b**2 - 30*sqrt(b)*sqrt(a)*atan((b
*x)/(sqrt(b)*sqrt(a)))*a**2*b*d*x**2 + 6*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(
b)*sqrt(a)))*a*b**3*x**2 - 15*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a))
)*a*b**2*d*x**4 + 3*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**4*x**
4 + 4*log(a + b*x**2)*a**3*b*c + 8*log(a + b*x**2)*a**2*b**2*c*x**2 + 4*lo
g(a + b*x**2)*a*b**3*c*x**4 + 2*a**3*b*c + 15*a**3*b*d*x - 3*a**2*b**3*x +
 25*a**2*b**2*d*x**3 + 2*a*b**4*x**4 - 5*a*b**4*x**3 - 4*a*b**3*c*x**4 + 8
*a*b**3*d*x**5)/(8*a*b**4*(a**2 + 2*a*b*x**2 + b**2*x**4))
```

# 3.55 $\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx$



## Optimal result

Integrand size = 28, antiderivative size = 134

$$\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx = \frac{a(bB-aD)-b(Ab-aC)x}{4b^3(a+bx^2)^2} - \frac{4a(bB-2aD)-b(Ab-5aC)x}{8ab^3(a+bx^2)} + \frac{(Ab+3aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{3/2}b^{5/2}} + \frac{D\log(a+bx^2)}{2b^3}$$

output
```
1/4*(a*(B*b-D*a)-b*(A*b-C*a)*x)/b^3/(b*x^2+a)^2-1/8*(4*a*(B*b-2*D*a)-b*(A*
b-5*C*a)*x)/a/b^3/(b*x^2+a)+1/8*(A*b+3*C*a)*arctan(b^(1/2)*x/a^(1/2))/a^(3
/2)/b^(5/2)+1/2*D*ln(b*x^2+a)/b^3
```

## Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.91

$$\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx = \frac{\frac{-2a^2D-2Ab^2x+2ab(B+Cx)}{(a+bx^2)^2}+\frac{8a^2D+Ab^2x-ab(4B+5Cx)}{a(a+bx^2)}+\frac{\sqrt{b}(Ab+3aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}}+4D\log(a+bx^2)}{8b^3}$$

input
```
Integrate[(x^2*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^3,x]
```

output
```
((-2*a^2*D - 2*A*b^2*x + 2*a*b*(B + C*x))/(a + b*x^2)^2 + (8*a^2*D + A*b^2
*x - a*b*(4*B + 5*C*x))/(a*(a + b*x^2)) + (Sqrt[b]*(A*b + 3*a*C)*ArcTan[(S
qrt[b]*x)/Sqrt[a]])/a^(3/2) + 4*D*Log[a + b*x^2])/(8*b^3)
```

## Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2335, 25, 2335, 25, 27, 452, 218, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx$$

$$\Big\downarrow \text{2335}$$

$$-\frac{\int -\frac{x\left(4aDx^2+(Ab+3aC)x+2a\left(B-\frac{aD}{b}\right)\right)}{(bx^2+a)^2}dx}{4ab}-\frac{x^2\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab\,(a+bx^2)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{x\left(4aDx^2+(Ab+3aC)x+2a\left(B-\frac{aD}{b}\right)\right)}{(bx^2+a)^2}dx}{4ab}-\frac{x^2\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab\,(a+bx^2)^2}$$

$$\Big\downarrow \; 2335$$

$$\frac{-\frac{\int -\frac{a(Ab+3aC+8aDx)}{bx^2+a}dx}{2ab}-\frac{x(-2x(bB-3aD)+3aC+Ab)}{2b(a+bx^2)}}{4ab}-\frac{x^2\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab\left(a+bx^2\right)^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{a(Ab+3aC+8aDx)}{bx^2+a}dx}{2ab}-\frac{x(-2x(bB-3aD)+3aC+Ab)}{2b(a+bx^2)}}{4ab}-\frac{x^2\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab\left(a+bx^2\right)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{Ab+3aC+8aDx}{bx^2+a}dx}{2b}-\frac{x(-2x(bB-3aD)+3aC+Ab)}{2b(a+bx^2)}}{4ab}-\frac{x^2\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab\left(a+bx^2\right)^2}$$

$$\Big\downarrow \; 452$$

$$\frac{\frac{(3aC+Ab)\int \frac{1}{bx^2+a}dx+8aD\int \frac{x}{bx^2+a}dx}{2b}-\frac{x(-2x(bB-3aD)+3aC+Ab)}{2b(a+bx^2)}}{4ab}-\frac{x^2\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab\left(a+bx^2\right)^2}$$

$$\Big\downarrow \; 218$$

$$\frac{\frac{8aD\int \frac{x}{bx^2+a}dx+\frac{(3aC+Ab)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}}{2b}-\frac{x(-2x(bB-3aD)+3aC+Ab)}{2b(a+bx^2)}}{4ab}-\frac{x^2\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab\left(a+bx^2\right)^2}$$

$$\Big\downarrow \; 240$$

$$\frac{\frac{\frac{(3aC+Ab)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}+\frac{4aD\log\left(a+bx^2\right)}{b}}{2b}-\frac{x(-2x(bB-3aD)+3aC+Ab)}{2b(a+bx^2)}}{4ab}-\frac{x^2\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab\left(a+bx^2\right)^2}$$

input
```
Int[(x^2*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^3,x]
```

output
```
-1/4*(x^2*(a*(B - (a*D)/b) - (A*b - a*C)*x))/(a*b*(a + b*x^2)^2) + (-1/2*(
x*(A*b + 3*a*C - 2*(b*B - 3*a*D)*x))/(b*(a + b*x^2)) + (((A*b + 3*a*C)*Arc
Tan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*Sqrt[b]) + (4*a*D*Log[a + b*x^2])/b)/(2
*b))/(4*a*b)
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 452
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[c   Int[1/
(a + b*x^2), x], x] + Simp[d   Int[x/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c^2 + a*d^2, 0]
```

rule 2335
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Simp[c/(2*a*b*(p + 1))   Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSu
m[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
 b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

## Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.92

| method | result | size |
|---|---|---|
| default | $\frac{\frac{(Ab-5Ca)x^3}{8ab}-\frac{(Bb-2Da)x^2}{2b^2}-\frac{(Ab+3Ca)x}{8b^2}-\frac{a(Bb-3Da)}{4b^3}}{(b\,x^2+a)^2}+\frac{\frac{4Da\ln\left(b\,x^2+a\right)}{b}+\frac{(Ab+3Ca)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}}{8a\,b^2}$ | 123 |

input
```
int(x^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/8*(A*b-5*C*a)/a/b*x^3-1/2*(B*b-2*D*a)/b^2*x^2-1/8*(A*b+3*C*a)/b^2*x-1/4
*a*(B*b-3*D*a)/b^3)/(b*x^2+a)^2+1/8/a/b^2*(4*D*a/b*ln(b*x^2+a)+(A*b+3*C*a)
/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 447, normalized size of antiderivative = 3.34

$$\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx$$
$$= \left[\frac{12\,Da^4 - 4\,Ba^3b - 2\,(5\,Ca^2b^2 - Aab^3)x^3 + 8\,(2\,Da^3b - Ba^2b^2)x^2 - ((3\,Cab^2 + Ab^3)x^4 + 3\,Ca^3 + Aa^2}{16\,(a}\right.$$

input
```
integrate(x^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
[1/16*(12*D*a^4 - 4*B*a^3*b - 2*(5*C*a^2*b^2 - A*a*b^3)*x^3 + 8*(2*D*a^3*b
 - B*a^2*b^2)*x^2 - ((3*C*a*b^2 + A*b^3)*x^4 + 3*C*a^3 + A*a^2*b + 2*(3*C*
a^2*b + A*a*b^2)*x^2)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 +
 a)) - 2*(3*C*a^3*b + A*a^2*b^2)*x + 8*(D*a^2*b^2*x^4 + 2*D*a^3*b*x^2 + D*
a^4)*log(b*x^2 + a))/(a^2*b^5*x^4 + 2*a^3*b^4*x^2 + a^4*b^3), 1/8*(6*D*a^4
 - 2*B*a^3*b - (5*C*a^2*b^2 - A*a*b^3)*x^3 + 4*(2*D*a^3*b - B*a^2*b^2)*x^2
 + ((3*C*a*b^2 + A*b^3)*x^4 + 3*C*a^3 + A*a^2*b + 2*(3*C*a^2*b + A*a*b^2)*
x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) - (3*C*a^3*b + A*a^2*b^2)*x + 4*(D*a^
2*b^2*x^4 + 2*D*a^3*b*x^2 + D*a^4)*log(b*x^2 + a))/(a^2*b^5*x^4 + 2*a^3*b^
4*x^2 + a^4*b^3)]
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 304 vs. $2(121) = 242$.

Time = 10.08 (sec) , antiderivative size = 304, normalized size of antiderivative = 2.27

$$\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx$$
$$= \left(\frac{D}{2b^3} - \frac{\sqrt{-a^3b^7}(Ab+3Ca)}{16a^3b^6}\right)\log\left(x + \frac{-8Da^2+16a^2b^3\left(\frac{D}{2b^3} - \frac{\sqrt{-a^3b^7}(Ab+3Ca)}{16a^3b^6}\right)}{Ab^2+3Cab}\right)$$
$$+ \left(\frac{D}{2b^3} + \frac{\sqrt{-a^3b^7}(Ab+3Ca)}{16a^3b^6}\right)\log\left(x + \frac{-8Da^2+16a^2b^3\left(\frac{D}{2b^3} + \frac{\sqrt{-a^3b^7}(Ab+3Ca)}{16a^3b^6}\right)}{Ab^2+3Cab}\right)$$
$$+ \frac{-2Ba^2b+6Da^3+x^3(Ab^3-5Cab^2)+x^2(-4Bab^2+8Da^2b)+x(-Aab^2-3Ca^2b)}{8a^3b^3+16a^2b^4x^2+8ab^5x^4}$$

input
```
integrate(x**2*(D*x**3+C*x**2+B*x+A)/(b*x**2+a)**3,x)
```

output
```
(D/(2*b**3) - sqrt(-a**3*b**7)*(A*b + 3*C*a)/(16*a**3*b**6))*log(x + (-8*D
*a**2 + 16*a**2*b**3*(D/(2*b**3) - sqrt(-a**3*b**7)*(A*b + 3*C*a)/(16*a**3
*b**6)))/(A*b**2 + 3*C*a*b)) + (D/(2*b**3) + sqrt(-a**3*b**7)*(A*b + 3*C*a
)/(16*a**3*b**6))*log(x + (-8*D*a**2 + 16*a**2*b**3*(D/(2*b**3) + sqrt(-a*
*3*b**7)*(A*b + 3*C*a)/(16*a**3*b**6)))/(A*b**2 + 3*C*a*b)) + (-2*B*a**2*b
 + 6*D*a**3 + x**3*(A*b**3 - 5*C*a*b**2) + x**2*(-4*B*a*b**2 + 8*D*a**2*b)
 + x*(-A*a*b**2 - 3*C*a**2*b))/(8*a**3*b**3 + 16*a**2*b**4*x**2 + 8*a*b**5
*x**4)
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.09

$$\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx$$
$$= \frac{6\,Da^3 - 2\,Ba^2b - (5\,Cab^2 - Ab^3)x^3 + 4\,(2\,Da^2b - Bab^2)x^2 - (3\,Ca^2b + Aab^2)x}{8\,(ab^5x^4 + 2\,a^2b^4x^2 + a^3b^3)}$$
$$+ \frac{D\log(bx^2+a)}{2\,b^3} + \frac{(3\,Ca + Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}ab^2}$$

input
```
integrate(x^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/8*(6*D*a^3 - 2*B*a^2*b - (5*C*a*b^2 - A*b^3)*x^3 + 4*(2*D*a^2*b - B*a*b^
2)*x^2 - (3*C*a^2*b + A*a*b^2)*x)/(a*b^5*x^4 + 2*a^2*b^4*x^2 + a^3*b^3) +
1/2*D*log(b*x^2 + a)/b^3 + 1/8*(3*C*a + A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a
*b)*a*b^2)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.96

$$\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx$$
$$= \frac{D\log(bx^2+a)}{2\,b^3} + \frac{(3\,Ca+Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}ab^2} - \frac{(5\,Cab-Ab^2)x^3 - 4\,(2\,Da^2-Bab)x^2 + (3\,Ca^2+Aab)x - \frac{2\,(3\,Da^3-Ba^2b)}{b}}{8\,(bx^2+a)^2ab^2}$$

input
```
integrate(x^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
1/2*D*log(b*x^2 + a)/b^3 + 1/8*(3*C*a + A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a
*b)*a*b^2) - 1/8*((5*C*a*b - A*b^2)*x^3 - 4*(2*D*a^2 - B*a*b)*x^2 + (3*C*a
^2 + A*a*b)*x - 2*(3*D*a^3 - B*a^2*b)/b)/((b*x^2 + a)^2*a*b^2)
```

## Mupad [B] (verification not implemented)

Time = 2.44 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.46

$$\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx = \frac{\frac{A\,x^3}{8\,a}-\frac{A\,x}{8\,b}}{a^2+2\,a\,b\,x^2+b^2\,x^4}-\frac{\frac{B\,x^2}{2\,b}+\frac{B\,a}{4\,b^2}}{a^2+2\,a\,b\,x^2+b^2\,x^4} - \frac{\frac{5\,C\,x^3}{8\,b}+\frac{3\,C\,a\,x}{8\,b^2}}{a^2+2\,a\,b\,x^2+b^2\,x^4} + \frac{D\left(\ln\left(b\,x^2+a\right)+\frac{2\,a}{b\,x^2+a}-\frac{a^2}{2\,\left(b\,x^2+a\right)^2}\right)}{2\,b^3} + \frac{A\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,a^{3/2}\,b^{3/2}}+\frac{3\,C\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,\sqrt{a}\,b^{5/2}}$$

input
```
int((x^2*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^3,x)
```

output
```
((A*x^3)/(8*a) - (A*x)/(8*b))/(a^2 + b^2*x^4 + 2*a*b*x^2) - ((B*x^2)/(2*b)
 + (B*a)/(4*b^2))/(a^2 + b^2*x^4 + 2*a*b*x^2) - ((5*C*x^3)/(8*b) + (3*C*a*
x)/(8*b^2))/(a^2 + b^2*x^4 + 2*a*b*x^2) + (D*(log(a + b*x^2) + (2*a)/(a +
b*x^2) - a^2/(2*(a + b*x^2)^2)))/(2*b^3) + (A*atan((b^(1/2)*x)/a^(1/2)))/(
8*a^(3/2)*b^(3/2)) + (3*C*atan((b^(1/2)*x)/a^(1/2)))/(8*a^(1/2)*b^(5/2))
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 278, normalized size of antiderivative = 2.07

$$\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx = \frac{\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^2b+3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^2c+2\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a\,b^2x^2+6\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right.}{}$$

input
```
int(x^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x)
```

output
```
(sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2*b + 3*sqrt(b)*sqrt(a)*
atan((b*x)/(sqrt(b)*sqrt(a)))*a**2*c + 2*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(
b)*sqrt(a)))*a*b**2*x**2 + 6*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))
*a*b*c*x**2 + sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**3*x**4 + 3*
sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**2*c*x**4 + 4*log(a + b*x*
*2)*a**3*d + 8*log(a + b*x**2)*a**2*b*d*x**2 + 4*log(a + b*x**2)*a*b**2*d*
x**4 + 2*a**3*d - a**2*b**2*x - 3*a**2*b*c*x + a*b**3*x**3 - 5*a*b**2*c*x*
*3 - 4*a*b**2*d*x**4 + 2*b**4*x**4)/(8*a*b**3*(a**2 + 2*a*b*x**2 + b**2*x*
*4))
```

# 3.56 $\int \frac{x(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3} dx$



## Optimal result

Integrand size = 26, antiderivative size = 105

$$\int \frac{x(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3} dx = -\frac{Ab-aC+(bB-aD)x}{4b^2(a+bx^2)^2} - \frac{4aC-(bB-5aD)x}{8ab^2(a+bx^2)} + \frac{(bB+3aD)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{3/2}b^{5/2}}$$

output
```
-1/4*(A*b-C*a+(B*b-D*a)*x)/b^2/(b*x^2+a)^2-1/8*(4*C*a-(B*b-5*D*a)*x)/a/b^2
/(b*x^2+a)+1/8*(B*b+3*D*a)*arctan(b^(1/2)*x/a^(1/2))/a^(3/2)/b^(5/2)
```

## Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.94

$$\int \frac{x(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3} dx = \frac{\frac{\sqrt{b}(b^2Bx^3-a^2(2C+3Dx)-ab(2A+x(B+4Cx+5Dx^2)))}{a(a+bx^2)^2} + \frac{(bB+3aD)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}}}{8b^{5/2}}$$

input
```
Integrate[(x*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^3,x]
```

output
```
((Sqrt[b]*(b^2*B*x^3 - a^2*(2*C + 3*D*x) - a*b*(2*A + x*(B + 4*C*x + 5*D*x
^2))))/(a*(a + b*x^2)^2) + ((b*B + 3*a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(
3/2))/(8*b^(5/2))
```

## Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.21, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2335, 25, 2345, 25, 27, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx$$

$$\Big\downarrow \ 2335$$

$$-\frac{\int -\frac{4aDx^2+2(Ab+aC)x+\frac{a(bB-aD)}{b}}{(bx^2+a)^2}dx}{4ab}-\frac{x\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab\,(a+bx^2)^2}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{4aDx^2+2(Ab+aC)x+\frac{a(bB-aD)}{b}}{(bx^2+a)^2}dx}{4ab}-\frac{x\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab\,(a+bx^2)^2}$$

$$\Big\downarrow \ 2345$$

$$\frac{-\frac{\int -\frac{a(bB+3aD)}{b(bx^2+a)}dx}{2a}-\frac{2(aC+Ab)-x(bB-5aD)}{2b(a+bx^2)}}{4ab}-\frac{x\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab\,(a+bx^2)^2}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{\int \frac{a(bB+3aD)}{b(bx^2+a)}dx}{2a}-\frac{2(aC+Ab)-x(bB-5aD)}{2b(a+bx^2)}}{4ab}-\frac{x\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab\,(a+bx^2)^2}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{(3aD+bB)\int \frac{1}{bx^2+a}dx}{2b}-\frac{2(aC+Ab)-x(bB-5aD)}{2b(a+bx^2)}}{4ab}-\frac{x\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab\,(a+bx^2)^2}$$

$$\Big\downarrow \; 218$$

$$\frac{\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(3aD+bB)}{2\sqrt{a}b^{3/2}}-\frac{2(aC+Ab)-x(bB-5aD)}{2b(a+bx^2)}}{4ab}-\frac{x\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab\left(a+bx^2\right)^2}$$

input
```
Int[(x*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^3,x]
```

output
```
-1/4*(x*(a*(B - (a*D)/b) - (A*b - a*C)*x))/(a*b*(a + b*x^2)^2) + (-1/2*(2*
(A*b + a*C) - (b*B - 5*a*D)*x)/(b*(a + b*x^2)) + ((b*B + 3*a*D)*ArcTan[(Sq
rt[b]*x)/Sqrt[a]])/(2*Sqrt[a]*b^(3/2)))/(4*a*b)
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 2335
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Simp[c/(2*a*b*(p + 1))   Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSu
m[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
 b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

## Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.92

| method | result | size |
|---|---|---|
| default | $\frac{\frac{(Bb-5Da)x^3}{8ab}-\frac{C x^2}{2b}-\frac{(Bb+3Da)x}{8b^2}-\frac{Ab+Ca}{4b^2}}{(b x^2+a)^2}+\frac{(Bb+3Da)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8b^2 a\sqrt{ab}}$ | 97 |

input
```
int(x*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/8*(B*b-5*D*a)/a/b*x^3-1/2*C*x^2/b-1/8*(B*b+3*D*a)/b^2*x-1/4*(A*b+C*a)/b
^2)/(b*x^2+a)^2+1/8*(B*b+3*D*a)/b^2/a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 357, normalized size of antiderivative = 3.40

$$\int \frac{x(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx$$
$$= \left[-\frac{8\,Ca^2b^2x^2+4\,Ca^3b+4\,Aa^2b^2+2\,(5\,Da^2b^2-Bab^3)x^3+((3\,Dab^2+Bb^3)x^4+3\,Da^3+Ba^2b+2\,(3\,}{16\,(a^2b^5x^4+2\,a^3b^4x^2+a^4b^3)}\right.$$
$$-\frac{4\,Ca^2b^2x^2+2\,Ca^3b+2\,Aa^2b^2+(5\,Da^2b^2-Bab^3)x^3-((3\,Dab^2+Bb^3)x^4+3\,Da^3+Ba^2b+2\,(3\,Da}{8\,(a^2b^5x^4+2\,a^3b^4x^2+a^4b^3)}$$

input
```
integrate(x*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
[-1/16*(8*C*a^2*b^2*x^2 + 4*C*a^3*b + 4*A*a^2*b^2 + 2*(5*D*a^2*b^2 - B*a*b
^3)*x^3 + ((3*D*a*b^2 + B*b^3)*x^4 + 3*D*a^3 + B*a^2*b + 2*(3*D*a^2*b + B*
a*b^2)*x^2)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 2*(
3*D*a^3*b + B*a^2*b^2)*x)/(a^2*b^5*x^4 + 2*a^3*b^4*x^2 + a^4*b^3), -1/8*(4
*C*a^2*b^2*x^2 + 2*C*a^3*b + 2*A*a^2*b^2 + (5*D*a^2*b^2 - B*a*b^3)*x^3 - (
(3*D*a*b^2 + B*b^3)*x^4 + 3*D*a^3 + B*a^2*b + 2*(3*D*a^2*b + B*a*b^2)*x^2)
*sqrt(a*b)*arctan(sqrt(a*b)*x/a) + (3*D*a^3*b + B*a^2*b^2)*x)/(a^2*b^5*x^4
 + 2*a^3*b^4*x^2 + a^4*b^3)]
```

## Sympy [A] (verification not implemented)

Time = 6.73 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.70

$$\int \frac{x(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx = -\frac{\sqrt{-\frac{1}{a^3b^5}}(Bb+3Da)\log\left(-a^2b^2\sqrt{-\frac{1}{a^3b^5}}+x\right)}{16} + \frac{\sqrt{-\frac{1}{a^3b^5}}(Bb+3Da)\log\left(a^2b^2\sqrt{-\frac{1}{a^3b^5}}+x\right)}{16} + \frac{-2Aab-2Ca^2-4Cabx^2+x^3(Bb^2-5Dab)+x(-Bab-3Da^2)}{8a^3b^2+16a^2b^3x^2+8ab^4x^4}$$

input
```
integrate(x*(D*x**3+C*x**2+B*x+A)/(b*x**2+a)**3,x)
```

output
```
-sqrt(-1/(a**3*b**5))*(B*b + 3*D*a)*log(-a**2*b**2*sqrt(-1/(a**3*b**5)) +
x)/16 + sqrt(-1/(a**3*b**5))*(B*b + 3*D*a)*log(a**2*b**2*sqrt(-1/(a**3*b**
5)) + x)/16 + (-2*A*a*b - 2*C*a**2 - 4*C*a*b*x**2 + x**3*(B*b**2 - 5*D*a*b
) + x*(-B*a*b - 3*D*a**2))/(8*a**3*b**2 + 16*a**2*b**3*x**2 + 8*a*b**4*x**
4)
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.06

$$\int \frac{x(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx = -\frac{4\,Cabx^2+(5\,Dab-Bb^2)x^3+2\,Ca^2+2\,Aab+(3\,Da^2+Bab)x}{8\,(ab^4x^4+2\,a^2b^3x^2+a^3b^2)} + \frac{(3\,Da+Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}ab^2}$$

input
```
integrate(x*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
-1/8*(4*C*a*b*x^2 + (5*D*a*b - B*b^2)*x^3 + 2*C*a^2 + 2*A*a*b + (3*D*a^2 +
 B*a*b)*x)/(a*b^4*x^4 + 2*a^2*b^3*x^2 + a^3*b^2) + 1/8*(3*D*a + B*b)*arcta
n(b*x/sqrt(a*b))/(sqrt(a*b)*a*b^2)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.92

$$\int \frac{x(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx = \frac{(3\,Da+Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}ab^2} - \frac{5\,Dabx^3-Bb^2x^3+4\,Cabx^2+3\,Da^2x+Babx+2\,Ca^2+2\,Aab}{8\,(bx^2+a)^2ab^2}$$

input
```
integrate(x*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
1/8*(3*D*a + B*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b^2) - 1/8*(5*D*a*b*x
^3 - B*b^2*x^3 + 4*C*a*b*x^2 + 3*D*a^2*x + B*a*b*x + 2*C*a^2 + 2*A*a*b)/((
b*x^2 + a)^2*a*b^2)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx = \int \frac{x\,(A+B\,x+C\,x^2+x^3\,D)}{(b\,x^2+a)^3}\,dx$$

input
```
int((x*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^3,x)
```

output
```
int((x*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^3, x)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 229, normalized size of antiderivative = 2.18

$$\int \frac{x(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx$$
$$= \frac{3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^3d+\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^2b^2+6\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^2bd\,x^2+2\sqrt{b}\,\sqrt{a}\,atan\left(\frac{b}{\sqrt{b}}\right.}{8}$$

input
```
int(x*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x)
```

output
```
(3*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**3*d + sqrt(b)*sqrt(a)*
atan((b*x)/(sqrt(b)*sqrt(a)))*a**2*b**2 + 6*sqrt(b)*sqrt(a)*atan((b*x)/(sq
rt(b)*sqrt(a)))*a**2*b*d*x**2 + 2*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt
(a)))*a*b**3*x**2 + 3*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*b**2
*d*x**4 + sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**4*x**4 - 2*a**3
*b**2 - 3*a**3*b*d*x - a**2*b**3*x - 5*a**2*b**2*d*x**3 + a*b**4*x**3 + 2*
a*b**3*c*x**4)/(8*a**2*b**3*(a**2 + 2*a*b*x**2 + b**2*x**4))
```

# 3.57 $\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^3}\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 116

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^3}\,dx = -\frac{a(bB-aD)-b(Ab-aC)x}{4ab^2(a+bx^2)^2} - \frac{4a^2D-b(3Ab+aC)x}{8a^2b^2(a+bx^2)} + \frac{(3Ab+aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}}$$

output
```
-1/4*(a*(B*b-D*a)-b*(A*b-C*a)*x)/a/b^2/(b*x^2+a)^2-1/8*(4*a^2*D-b*(3*A*b+C
*a)*x)/a^2/b^2/(b*x^2+a)+1/8*(3*A*b+C*a)*arctan(b^(1/2)*x/a^(1/2))/a^(5/2)
/b^(3/2)
```

## Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.90

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^3}\,dx = \frac{\frac{\sqrt{a}(-2a^3D+3Ab^3x^3+ab^2x(5A+Cx^2)-a^2b(2B+x(C+4Dx)))}{(a+bx^2)^2}+\sqrt{b}(3Ab+aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^2}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/(a + b*x^2)^3,x]
```

output
```
((Sqrt[a]*(-2*a^3*D + 3*A*b^3*x^3 + a*b^2*x*(5*A + C*x^2) - a^2*b*(2*B + x
*(C + 4*D*x))))/(a + b*x^2)^2 + Sqrt[b]*(3*A*b + a*C)*ArcTan[(Sqrt[b]*x)/S
qrt[a]])/(8*a^(5/2)*b^2)
```

## Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2345, 25, 27, 454, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^3}dx$$

$$\Big\downarrow \text{ 2345}$$

$$-\frac{\int -\frac{b\left(3A+\frac{aC}{b}\right)+4aDx}{b(bx^2+a)^2}dx}{4a}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{4ab\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{3Ab+aC+4aDx}{b(bx^2+a)^2}dx}{4a}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{4ab\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{3Ab+aC+4aDx}{(bx^2+a)^2}dx}{4ab}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{4ab\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{ 454}$$

$$\frac{\frac{(aC+3Ab)\int \frac{1}{bx^2+a}dx}{2a}-\frac{4a^2D-bx(aC+3Ab)}{2ab(a+bx^2)}}{4ab}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{4ab\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{\frac{(aC+3Ab)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}-\frac{4a^2D-bx(aC+3Ab)}{2ab(a+bx^2)}}{4ab}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{4ab\left(a+bx^2\right)^2}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/(a + b*x^2)^3,x]
```

output
```
-1/4*(a*(B - (a*D)/b) - (A*b - a*C)*x)/(a*b*(a + b*x^2)^2) + (-1/2*(4*a^2*
D - b*(3*A*b + a*C)*x)/(a*b*(a + b*x^2)) + ((3*A*b + a*C)*ArcTan[(Sqrt[b]*
x)/Sqrt[a]])/(2*a^(3/2)*Sqrt[b]))/(4*a*b)
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 454
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*d
 - b*c*x)/(2*a*b*(p + 1)))*(a + b*x^2)^(p + 1), x] + Simp[c*((2*p + 3)/(2*a
*(p + 1)))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d}, x] && L
tQ[p, -1] && NeQ[p, -3/2]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

## Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.84

| method | result | size |
|---|---|---|
| default | $\frac{\frac{(3Ab+Ca)x^3}{8a^2}-\frac{Dx^2}{2b}+\frac{(5Ab-Ca)x}{8ab}-\frac{Bb+Da}{4b^2}}{(b\,x^2+a)^2}+\frac{(3Ab+Ca)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8a^2b\sqrt{ab}}$ | 98 |

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/8*(3*A*b+C*a)/a^2*x^3-1/2*D*x^2/b+1/8*(5*A*b-C*a)/a/b*x-1/4*(B*b+D*a)/b
^2)/(b*x^2+a)^2+1/8*(3*A*b+C*a)/a^2/b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 346, normalized size of antiderivative = 2.98

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^3}\,dx$$
$$= \left[-\frac{8\,Da^3bx^2+4\,Da^4+4\,Ba^3b-2\,(Ca^2b^2+3\,Aab^3)x^3+((Cab^2+3\,Ab^3)x^4+Ca^3+3\,Aa^2b+2\,(Ca^2b}{16\,(a^3b^4x^4+2\,a^4b^3x^2+a^5b^2)}\right.$$
$$-\frac{4\,Da^3bx^2+2\,Da^4+2\,Ba^3b-(Ca^2b^2+3\,Aab^3)x^3-((Cab^2+3\,Ab^3)x^4+Ca^3+3\,Aa^2b+2\,(Ca^2b+}{8\,(a^3b^4x^4+2\,a^4b^3x^2+a^5b^2)}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
[-1/16*(8*D*a^3*b*x^2 + 4*D*a^4 + 4*B*a^3*b - 2*(C*a^2*b^2 + 3*A*a*b^3)*x^
3 + ((C*a*b^2 + 3*A*b^3)*x^4 + C*a^3 + 3*A*a^2*b + 2*(C*a^2*b + 3*A*a*b^2)
*x^2)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 2*(C*a^3*
b - 5*A*a^2*b^2)*x)/(a^3*b^4*x^4 + 2*a^4*b^3*x^2 + a^5*b^2), -1/8*(4*D*a^3
*b*x^2 + 2*D*a^4 + 2*B*a^3*b - (C*a^2*b^2 + 3*A*a*b^3)*x^3 - ((C*a*b^2 + 3
*A*b^3)*x^4 + C*a^3 + 3*A*a^2*b + 2*(C*a^2*b + 3*A*a*b^2)*x^2)*sqrt(a*b)*a
rctan(sqrt(a*b)*x/a) + (C*a^3*b - 5*A*a^2*b^2)*x)/(a^3*b^4*x^4 + 2*a^4*b^3
*x^2 + a^5*b^2)]
```

## Sympy [A] (verification not implemented)

Time = 3.77 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.59

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^3}\,dx = -\frac{\sqrt{-\frac{1}{a^5b^3}}\cdot(3Ab+Ca)\log\left(-a^3b\sqrt{-\frac{1}{a^5b^3}}+x\right)}{16} + \frac{\sqrt{-\frac{1}{a^5b^3}}\cdot(3Ab+Ca)\log\left(a^3b\sqrt{-\frac{1}{a^5b^3}}+x\right)}{16} + \frac{-2Ba^2b-2Da^3-4Da^2bx^2+x^3\cdot(3Ab^3+Cab^2)+x(5Aab^2-Ca^2b)}{8a^4b^2+16a^3b^3x^2+8a^2b^4x^4}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(b*x**2+a)**3,x)
```

output
```
-sqrt(-1/(a**5*b**3))*(3*A*b + C*a)*log(-a**3*b*sqrt(-1/(a**5*b**3)) + x)/
16 + sqrt(-1/(a**5*b**3))*(3*A*b + C*a)*log(a**3*b*sqrt(-1/(a**5*b**3)) +
x)/16 + (-2*B*a**2*b - 2*D*a**3 - 4*D*a**2*b*x**2 + x**3*(3*A*b**3 + C*a*b
**2) + x*(5*A*a*b**2 - C*a**2*b))/(8*a**4*b**2 + 16*a**3*b**3*x**2 + 8*a**
2*b**4*x**4)
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.05

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^3}\,dx = -\frac{4\,Da^2bx^2+2\,Da^3+2\,Ba^2b-(Cab^2+3\,Ab^3)x^3+(Ca^2b-5\,Aab^2)x}{8\,(a^2b^4x^4+2\,a^3b^3x^2+a^4b^2)} + \frac{(Ca+3\,Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^2b}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
-1/8*(4*D*a^2*b*x^2 + 2*D*a^3 + 2*B*a^2*b - (C*a*b^2 + 3*A*b^3)*x^3 + (C*a
^2*b - 5*A*a*b^2)*x)/(a^2*b^4*x^4 + 2*a^3*b^3*x^2 + a^4*b^2) + 1/8*(C*a +
3*A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.91

$$\int \frac{A+Bx+Cx^2+Dx^3}{\left(a+bx^2\right)^3}\,dx = \frac{(Ca+3\,Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^2b} + \frac{Cab^2x^3+3\,Ab^3x^3-4\,Da^2bx^2-Ca^2bx+5\,Aab^2x-2\,Da^3-2\,Ba^2b}{8\left(bx^2+a\right)^2a^2b^2}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
1/8*(C*a + 3*A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b) + 1/8*(C*a*b^2*x
^3 + 3*A*b^3*x^3 - 4*D*a^2*b*x^2 - C*a^2*b*x + 5*A*a*b^2*x - 2*D*a^3 - 2*B
*a^2*b)/((b*x^2 + a)^2*a^2*b^2)
```

## Mupad [B] (verification not implemented)

Time = 2.45 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.41

$$\int \frac{A+Bx+Cx^2+Dx^3}{\left(a+bx^2\right)^3}\,dx = \frac{\frac{C\,x^3}{8\,a}-\frac{C\,x}{8\,b}}{a^2+2\,a\,b\,x^2+b^2\,x^4} + \frac{\frac{5\,A\,x}{8\,a}+\frac{3\,A\,b\,x^3}{8\,a^2}}{a^2+2\,a\,b\,x^2+b^2\,x^4} - \frac{B}{4\,b\,\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)} - \frac{\left(2\,b\,x^2+a\right)\,D}{4\,b^2\,{\left(b\,x^2+a\right)}^2} + \frac{3\,A\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,a^{5/2}\,\sqrt{b}} + \frac{C\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,a^{3/2}\,b^{3/2}}$$

input
```
int((A + B*x + C*x^2 + x^3*D)/(a + b*x^2)^3,x)
```

output
```
((C*x^3)/(8*a) - (C*x)/(8*b))/(a^2 + b^2*x^4 + 2*a*b*x^2) + ((5*A*x)/(8*a)
 + (3*A*b*x^3)/(8*a^2))/(a^2 + b^2*x^4 + 2*a*b*x^2) - B/(4*b*(a^2 + b^2*x^
4 + 2*a*b*x^2)) - ((a + 2*b*x^2)*D)/(4*b^2*(a + b*x^2)^2) + (3*A*atan((b^(
1/2)*x)/a^(1/2)))/(8*a^(5/2)*b^(1/2)) + (C*atan((b^(1/2)*x)/a^(1/2)))/(8*a
^(3/2)*b^(3/2))
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.91

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(a + bx^2)^3} \, dx$$
$$= \frac{3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right) a^2 b + \sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right) a^2 c + 6\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right) a\, b^2 x^2 + 2\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right.}{8a^2}$$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x)
```

output
```
(3*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2*b + sqrt(b)*sqrt(a)*
atan((b*x)/(sqrt(b)*sqrt(a)))*a**2*c + 6*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(
b)*sqrt(a)))*a*b**2*x**2 + 2*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))
*a*b*c*x**2 + 3*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**3*x**4 +
sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**2*c*x**4 + 5*a**2*b**2*x
- 2*a**2*b**2 - a**2*b*c*x + 3*a*b**3*x**3 + a*b**2*c*x**3 + 2*a*b**2*d*x*
*4)/(8*a**2*b**2*(a**2 + 2*a*b*x**2 + b**2*x**4))
```

# 3.58 $\int \frac{A+Bx+Cx^2+Dx^3}{x(a+bx^2)^3}\,dx$



## Optimal result

Integrand size = 28, antiderivative size = 130

$$\int \frac{A+Bx+Cx^2+Dx^3}{x(a+bx^2)^3}\,dx = \frac{Ab-aC+(bB-aD)x}{4ab(a+bx^2)^2}+\frac{4Ab+(3bB+aD)x}{8a^2b(a+bx^2)} + \frac{(3bB+aD)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}} + \frac{A\log(x)}{a^3}-\frac{A\log(a+bx^2)}{2a^3}$$

output
```
1/4*(A*b-C*a+(B*b-D*a)*x)/a/b/(b*x^2+a)^2+1/8*(4*A*b+(3*B*b+D*a)*x)/a^2/b/
(b*x^2+a)+1/8*(3*B*b+D*a)*arctan(b^(1/2)*x/a^(1/2))/a^(5/2)/b^(3/2)+A*ln(x
)/a^3-1/2*A*ln(b*x^2+a)/a^3
```

## Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.90

$$\int \frac{A+Bx+Cx^2+Dx^3}{x(a+bx^2)^3}\,dx = \frac{\frac{a(4Ab+3bBx+aDx)}{b(a+bx^2)}+\frac{2a^2(Ab+bBx-a(C+Dx))}{b(a+bx^2)^2}+\frac{\sqrt{a}(3bB+aD)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}}+8A\log(x)-4A\log(a+bx^2)}{8a^3}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/(x*(a + b*x^2)^3),x]
```

output
```
((a*(4*A*b + 3*b*B*x + a*D*x))/(b*(a + b*x^2)) + (2*a^2*(A*b + b*B*x - a*(
C + D*x)))/(b*(a + b*x^2)^2) + (Sqrt[a]*(3*b*B + a*D)*ArcTan[(Sqrt[b]*x)/S
qrt[a]])/b^(3/2) + 8*A*Log[x] - 4*A*Log[a + b*x^2])/(8*a^3)
```

## Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2336, 25, 27, 532, 25, 523, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{x\left(a+bx^2\right)^3}dx$$

$$\Big\downarrow \text{ 2336}$$

$$\frac{x(bB-aD)-aC+Ab}{4ab\left(a+bx^2\right)^2}-\frac{\int -\frac{4Ab+(3bB+aD)x}{bx\left(bx^2+a\right)^2}dx}{4a}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{4Ab+(3bB+aD)x}{bx\left(bx^2+a\right)^2}dx}{4a}+\frac{x(bB-aD)-aC+Ab}{4ab\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{4Ab+(3bB+aD)x}{x\left(bx^2+a\right)^2}dx}{4ab}+\frac{x(bB-aD)-aC+Ab}{4ab\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{ 532}$$

$$\frac{\frac{x(aD+3bB)+4Ab}{2a\left(a+bx^2\right)}-\frac{\int -\frac{8Ab+(3bB+aD)x}{x\left(bx^2+a\right)}dx}{2a}}{4ab}+\frac{x(bB-aD)-aC+Ab}{4ab\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\frac{\int \frac{8Ab+(3bB+aD)x}{x(bx^2+a)}dx}{2a}+\frac{x(aD+3bB)+4Ab}{2a(a+bx^2)}}{4ab}+\frac{x(bB-aD)-aC+Ab}{4ab\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{523}$$

$$\frac{\frac{\int \left(\frac{8Ab}{ax}+\frac{Da^2+3bBa-8Ab^2x}{a(bx^2+a)}\right)dx}{2a}+\frac{x(aD+3bB)+4Ab}{2a(a+bx^2)}}{4ab}+\frac{x(bB-aD)-aC+Ab}{4ab\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{-\frac{4Ab\log(a+bx^2)}{a}+\frac{8Ab\log(x)}{a}+\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(aD+3bB)}{\sqrt{a}\sqrt{b}}}{2a}+\frac{x(aD+3bB)+4Ab}{2a(a+bx^2)}}{4ab}+\frac{x(bB-aD)-aC+Ab}{4ab\left(a+bx^2\right)^2}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/(x*(a + b*x^2)^3),x]
```

output
```
(A*b - a*C + (b*B - a*D)*x)/(4*a*b*(a + b*x^2)^2) + ((4*A*b + (3*b*B + a*D
)*x)/(2*a*(a + b*x^2)) + (((3*b*B + a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqr
t[a]*Sqrt[b]) + (8*A*b*Log[x])/a - (4*A*b*Log[a + b*x^2])/a)/(2*a))/(4*a*b
)
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 523
```
Int[((x_)^(m_.)*((c_) + (d_.)*(x_)))/((a_) + (b_.)*(x_)^2), x_Symbol] :> In
t[ExpandIntegrand[x^m*((c + d*x)/(a + b*x^2)), x], x] /; FreeQ[{a, b, c, d}
, x] && IntegerQ[m]
```

rule 532
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[x^m
*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*(Qx/x^m) + e*((2*p + 3)/x^m),
x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && ILtQ[m, 0] && LtQ[p,
 -1] && IntegerQ[2*p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

## Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00

| method | result | size |
|---|---|---|
| default | $-\frac{\frac{\left(-\frac{3}{8}abB-\frac{1}{8}Da^{2}\right)x^{3}-\frac{aAb\,x^{2}}{2}-\frac{a^{2}(5Bb-Da)x}{8b}-\frac{a^{2}(3Ab-Ca)}{4b}}{\left(b\,x^{2}+a\right)^{2}}+\frac{4bA\ln\left(b\,x^{2}+a\right)+\frac{\left(-3abB-Da^{2}\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}}{8b}}{a^{3}}+\frac{A\ln(x)}{a^{3}}$ | 130 |

input
```
int((D*x^3+C*x^2+B*x+A)/x/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/a^3*(((-3/8*a*b*B-1/8*D*a^2)*x^3-1/2*a*A*b*x^2-1/8*a^2*(5*B*b-D*a)/b*x-
1/4*a^2*(3*A*b-C*a)/b)/(b*x^2+a)^2+1/8/b*(4*b*A*ln(b*x^2+a)+(-3*B*a*b-D*a^
2)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))))+A*ln(x)/a^3
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 233 vs. 2(114) = 228.

Time = 0.10 (sec) , antiderivative size = 488, normalized size of antiderivative = 3.75

$$\int \frac{A+Bx+Cx^2+Dx^3}{x\left(a+bx^2\right)^3}\,dx$$
$$= \left[\frac{8\,Aab^3x^2 - 4\,Ca^3b + 12\,Aa^2b^2 + 2\,(Da^2b^2 + 3\,Bab^3)x^3 - ((Dab^2 + 3\,Bb^3)x^4 + Da^3 + 3\,Ba^2b + 2\,(Da^2\right.$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/x/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
[1/16*(8*A*a*b^3*x^2 - 4*C*a^3*b + 12*A*a^2*b^2 + 2*(D*a^2*b^2 + 3*B*a*b^3
)*x^3 - ((D*a*b^2 + 3*B*b^3)*x^4 + D*a^3 + 3*B*a^2*b + 2*(D*a^2*b + 3*B*a*
b^2)*x^2)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) - 2*(D*
a^3*b - 5*B*a^2*b^2)*x - 8*(A*b^4*x^4 + 2*A*a*b^3*x^2 + A*a^2*b^2)*log(b*x
^2 + a) + 16*(A*b^4*x^4 + 2*A*a*b^3*x^2 + A*a^2*b^2)*log(x))/(a^3*b^4*x^4
+ 2*a^4*b^3*x^2 + a^5*b^2), 1/8*(4*A*a*b^3*x^2 - 2*C*a^3*b + 6*A*a^2*b^2 +
 (D*a^2*b^2 + 3*B*a*b^3)*x^3 + ((D*a*b^2 + 3*B*b^3)*x^4 + D*a^3 + 3*B*a^2*
b + 2*(D*a^2*b + 3*B*a*b^2)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) - (D*a^3*
b - 5*B*a^2*b^2)*x - 4*(A*b^4*x^4 + 2*A*a*b^3*x^2 + A*a^2*b^2)*log(b*x^2 +
 a) + 8*(A*b^4*x^4 + 2*A*a*b^3*x^2 + A*a^2*b^2)*log(x))/(a^3*b^4*x^4 + 2*a
^4*b^3*x^2 + a^5*b^2)]
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{x\left(a+bx^2\right)^3}\,dx = \text{Timed out}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/x/(b*x**2+a)**3,x)
```

output
```
Timed out
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.02

$$\int \frac{A+Bx+Cx^2+Dx^3}{x\left(a+bx^2\right)^3}\,dx = \frac{4\,Ab^2x^2+(Dab+3\,Bb^2)x^3-2\,Ca^2+6\,Aab-(Da^2-5\,Bab)x}{8\,(a^2b^3x^4+2\,a^3b^2x^2+a^4b)} - \frac{A\log\left(bx^2+a\right)}{2\,a^3} + \frac{A\log\left(x\right)}{a^3} + \frac{(Da+3\,Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^2b}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/x/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/8*(4*A*b^2*x^2 + (D*a*b + 3*B*b^2)*x^3 - 2*C*a^2 + 6*A*a*b - (D*a^2 - 5*
B*a*b)*x)/(a^2*b^3*x^4 + 2*a^3*b^2*x^2 + a^4*b) - 1/2*A*log(b*x^2 + a)/a^3
 + A*log(x)/a^3 + 1/8*(D*a + 3*B*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b
)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.98

$$\int \frac{A+Bx+Cx^2+Dx^3}{x\left(a+bx^2\right)^3}\,dx = -\frac{A\log\left(bx^2+a\right)}{2\,a^3} + \frac{A\log\left(|x|\right)}{a^3} + \frac{(Da+3\,Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^2b} + \frac{4\,Aab^2x^2-2\,Ca^3+6\,Aa^2b+(Da^2b+3\,Bab^2)x^3-(Da^3-5\,Ba^2b)x}{8\,\left(bx^2+a\right)^2a^3b}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/x/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
-1/2*A*log(b*x^2 + a)/a^3 + A*log(abs(x))/a^3 + 1/8*(D*a + 3*B*b)*arctan(b
*x/sqrt(a*b))/(sqrt(a*b)*a^2*b) + 1/8*(4*A*a*b^2*x^2 - 2*C*a^3 + 6*A*a^2*b
 + (D*a^2*b + 3*B*a*b^2)*x^3 - (D*a^3 - 5*B*a^2*b)*x)/((b*x^2 + a)^2*a^3*b
)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{x\left(a+bx^2\right)^3}\,dx=\int \frac{A+B\,x+C\,x^2+x^3\,D}{x\,\left(b\,x^2+a\right)^3}\,dx$$

input
```
int((A + B*x + C*x^2 + x^3*D)/(x*(a + b*x^2)^3),x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/(x*(a + b*x^2)^3), x)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 321, normalized size of antiderivative = 2.47

$$\int \frac{A+Bx+Cx^2+Dx^3}{x\left(a+bx^2\right)^3}\,dx = \frac{\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^3d+3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^2b^2+2\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^2bd\,x^2+6\sqrt{b}\,\sqrt{a}\,atan\left(\frac{b}{\sqrt{b}}\right.}{}$$

input
```
int((D*x^3+C*x^2+B*x+A)/x/(b*x^2+a)^3,x)
```

output
```
(sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**3*d + 3*sqrt(b)*sqrt(a)*
atan((b*x)/(sqrt(b)*sqrt(a)))*a**2*b**2 + 2*sqrt(b)*sqrt(a)*atan((b*x)/(sq
rt(b)*sqrt(a)))*a**2*b*d*x**2 + 6*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt
(a)))*a*b**3*x**2 + sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*b**2*d
*x**4 + 3*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**4*x**4 - 4*log(
a + b*x**2)*a**3*b**2 - 8*log(a + b*x**2)*a**2*b**3*x**2 - 4*log(a + b*x**
2)*a*b**4*x**4 + 8*log(x)*a**3*b**2 + 16*log(x)*a**2*b**3*x**2 + 8*log(x)*
a*b**4*x**4 + 4*a**3*b**2 - 2*a**3*b*c - a**3*b*d*x + 5*a**2*b**3*x + a**2
*b**2*d*x**3 - 2*a*b**4*x**4 + 3*a*b**4*x**3)/(8*a**3*b**2*(a**2 + 2*a*b*x
**2 + b**2*x**4))
```

# 3.59 $\int \frac{A+Bx+Cx^2+Dx^3}{x^2(a+bx^2)^3} dx$



## Optimal result

Integrand size = 28, antiderivative size = 144

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^2(a+bx^2)^3} dx = -\frac{A}{a^3x} + \frac{a(bB-aD)-b(Ab-aC)x}{4a^2b(a+bx^2)^2} + \frac{4aB-(7Ab-3aC)x}{8a^3(a+bx^2)} - \frac{3(5Ab-aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{7/2}\sqrt{b}} + \frac{B\log(x)}{a^3} - \frac{B\log(a+bx^2)}{2a^3}$$

output
```
-A/a^3/x+1/4*(a*(B*b-D*a)-b*(A*b-C*a)*x)/a^2/b/(b*x^2+a)^2+1/8*(4*B*a-(7*A
*b-3*C*a)*x)/a^3/(b*x^2+a)-3/8*(5*A*b-C*a)*arctan(b^(1/2)*x/a^(1/2))/a^(7/
2)/b^(1/2)+B*ln(x)/a^3-1/2*B*ln(b*x^2+a)/a^3
```

## Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.98

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^2(a+bx^2)^3}\,dx=-\frac{A}{a^3x}+\frac{abB-a^2D-Ab^2x+abCx}{4a^2b(a+bx^2)^2}+\frac{4aB-7Abx+3aCx}{8a^3(a+bx^2)}+\frac{3(-5Ab+aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{7/2}\sqrt{b}}+\frac{B\log(x)}{a^3}-\frac{B\log(a+bx^2)}{2a^3}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/(x^2*(a + b*x^2)^3),x]
```

output
```
-(A/(a^3*x)) + (a*b*B - a^2*D - A*b^2*x + a*b*C*x)/(4*a^2*b*(a + b*x^2)^2)
 + (4*a*B - 7*A*b*x + 3*a*C*x)/(8*a^3*(a + b*x^2)) + (3*(-5*A*b + a*C)*Arc
Tan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(7/2)*Sqrt[b]) + (B*Log[x])/a^3 - (B*Log[a
+ b*x^2])/(2*a^3)
```

## Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2336, 25, 2336, 25, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^2(a+bx^2)^3}\,dx$$

$$\Big\downarrow \text{2336}$$

$$\frac{-bx\left(\frac{Ab}{a}-C\right)-aD+bB}{4ab(a+bx^2)^2}-\frac{\int -\frac{-3\left(\frac{Ab}{a}-C\right)x^2+4Bx+4A}{x^2(bx^2+a)^2}dx}{4a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{-3\left(\frac{Ab}{a}-C\right)x^2+4Bx+4A}{x^2(bx^2+a)^2}dx}{4a}+\frac{-bx\left(\frac{Ab}{a}-C\right)-aD+bB}{4ab\left(a+bx^2\right)^2}$$

$$\Big\downarrow \ 2336$$

$$\frac{\frac{4B-x\left(\frac{7Ab}{a}-3C\right)}{2a(a+bx^2)}-\frac{\int -\frac{-\left(\left(\frac{7Ab}{a}-3C\right)x^2\right)+8Bx+8A}{x^2\left(bx^2+a\right)}dx}{2a}}{4a}+\frac{-bx\left(\frac{Ab}{a}-C\right)-aD+bB}{4ab\left(a+bx^2\right)^2}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{\int \frac{-\left(\left(\frac{7Ab}{a}-3C\right)x^2\right)+8Bx+8A}{x^2\left(bx^2+a\right)}dx}{2a}+\frac{4B-x\left(\frac{7Ab}{a}-3C\right)}{2a(a+bx^2)}}{4a}+\frac{-bx\left(\frac{Ab}{a}-C\right)-aD+bB}{4ab\left(a+bx^2\right)^2}$$

$$\Big\downarrow \ 2333$$

$$\frac{\frac{\int \left(\frac{8A}{ax^2}+\frac{8B}{ax}+\frac{-15Ab-8Bxb+3aC}{a\left(bx^2+a\right)}\right)dx}{2a}+\frac{4B-x\left(\frac{7Ab}{a}-3C\right)}{2a(a+bx^2)}}{4a}+\frac{-bx\left(\frac{Ab}{a}-C\right)-aD+bB}{4ab\left(a+bx^2\right)^2}$$

$$\Big\downarrow \ 2009$$

$$\frac{\frac{-\frac{3(5Ab-aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}-\frac{8A}{ax}-\frac{4B\log\left(a+bx^2\right)}{a}+\frac{8B\log(x)}{a}}{2a}+\frac{4B-x\left(\frac{7Ab}{a}-3C\right)}{2a(a+bx^2)}}{4a}+\frac{-bx\left(\frac{Ab}{a}-C\right)-aD+bB}{4ab\left(a+bx^2\right)^2}$$

input

```
Int[(A + B*x + C*x^2 + D*x^3)/(x^2*(a + b*x^2)^3),x]
```

output

```
(b*B - a*D - b*((A*b)/a - C)*x)/(4*a*b*(a + b*x^2)^2) + ((4*B - ((7*A*b)/a
 - 3*C)*x)/(2*a*(a + b*x^2)) + ((-8*A)/(a*x) - (3*(5*A*b - a*C)*ArcTan[(Sq
rt[b]*x)/Sqrt[a]])/(a^(3/2)*Sqrt[b]) + (8*B*Log[x])/a - (4*B*Log[a + b*x^2
])/a)/(2*a))/(4*a)
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

## Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.87

| method | result | size |
|---|---|---|
| default | $-\frac{\frac{\left(\frac{7}{8}b^{2}A-\frac{3}{8}Cab\right)x^{3}-\frac{Bab\,x^{2}}{2}+\frac{a(9Ab-5Ca)x}{8}-\frac{a^{2}(3Bb-Da)}{4b}}{\left(b\,x^{2}+a\right)^{2}}+\frac{B\ln\left(b\,x^{2}+a\right)}{2}+\frac{(15Ab-3Ca)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}}}{a^{3}}-\frac{A}{a^{3}x}+\frac{B\ln(x)}{a^{3}}$ | 125 |

input
```
int((D*x^3+C*x^2+B*x+A)/x^2/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/a^3*(((7/8*b^2*A-3/8*C*a*b)*x^3-1/2*B*a*b*x^2+1/8*a*(9*A*b-5*C*a)*x-1/4
*a^2*(3*B*b-D*a)/b)/(b*x^2+a)^2+1/2*B*ln(b*x^2+a)+1/8*(15*A*b-3*C*a)/(a*b)
^(1/2)*arctan(b*x/(a*b)^(1/2)))-A/a^3/x+B*ln(x)/a^3
```

## Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 524, normalized size of antiderivative = 3.64

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^2\left(a+bx^2\right)^3}\,dx = \left[\frac{8\,Ba^2b^2x^3-16\,Aa^3b+6\,(Ca^2b^2-5\,Aab^3)x^4+10\,(Ca^3b-5\,Aa^2b^2)x^2+3\,((Cab^2-5\,Ab^3)x^5+2\,(Ca^2\right.$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/x^2/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
[1/16*(8*B*a^2*b^2*x^3 - 16*A*a^3*b + 6*(C*a^2*b^2 - 5*A*a*b^3)*x^4 + 10*(
C*a^3*b - 5*A*a^2*b^2)*x^2 + 3*((C*a*b^2 - 5*A*b^3)*x^5 + 2*(C*a^2*b - 5*A
*a*b^2)*x^3 + (C*a^3 - 5*A*a^2*b)*x)*sqrt(-a*b)*log((b*x^2 + 2*sqrt(-a*b)*
x - a)/(b*x^2 + a)) - 4*(D*a^4 - 3*B*a^3*b)*x - 8*(B*a*b^3*x^5 + 2*B*a^2*b
^2*x^3 + B*a^3*b*x)*log(b*x^2 + a) + 16*(B*a*b^3*x^5 + 2*B*a^2*b^2*x^3 + B
*a^3*b*x)*log(x))/(a^4*b^3*x^5 + 2*a^5*b^2*x^3 + a^6*b*x), 1/8*(4*B*a^2*b^
2*x^3 - 8*A*a^3*b + 3*(C*a^2*b^2 - 5*A*a*b^3)*x^4 + 5*(C*a^3*b - 5*A*a^2*b
^2)*x^2 + 3*((C*a*b^2 - 5*A*b^3)*x^5 + 2*(C*a^2*b - 5*A*a*b^2)*x^3 + (C*a^
3 - 5*A*a^2*b)*x)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) - 2*(D*a^4 - 3*B*a^3*b)*
x - 4*(B*a*b^3*x^5 + 2*B*a^2*b^2*x^3 + B*a^3*b*x)*log(b*x^2 + a) + 8*(B*a*
b^3*x^5 + 2*B*a^2*b^2*x^3 + B*a^3*b*x)*log(x))/(a^4*b^3*x^5 + 2*a^5*b^2*x^
3 + a^6*b*x)]
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^2\left(a+bx^2\right)^3}\,dx = \text{Timed out}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/x**2/(b*x**2+a)**3,x)
```

output
```
Timed out
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.06

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^2\left(a+bx^2\right)^3}\,dx$$
$$=\frac{4\,Bab^2x^3+3\,(Cab^2-5\,Ab^3)x^4-8\,Aa^2b+5\,(Ca^2b-5\,Aab^2)x^2-2\,(Da^3-3\,Ba^2b)x}{8\,(a^3b^3x^5+2\,a^4b^2x^3+a^5bx)}$$
$$-\frac{B\log\left(bx^2+a\right)}{2\,a^3}+\frac{B\log\left(x\right)}{a^3}+\frac{3\,(Ca-5\,Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^3}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/x^2/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/8*(4*B*a*b^2*x^3 + 3*(C*a*b^2 - 5*A*b^3)*x^4 - 8*A*a^2*b + 5*(C*a^2*b -
5*A*a*b^2)*x^2 - 2*(D*a^3 - 3*B*a^2*b)*x)/(a^3*b^3*x^5 + 2*a^4*b^2*x^3 + a
^5*b*x) - 1/2*B*log(b*x^2 + a)/a^3 + B*log(x)/a^3 + 3/8*(C*a - 5*A*b)*arct
an(b*x/sqrt(a*b))/(sqrt(a*b)*a^3)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.98

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^2\left(a+bx^2\right)^3}\,dx$$
$$=-\frac{B\log\left(bx^2+a\right)}{2\,a^3}+\frac{B\log\left(|x|\right)}{a^3}+\frac{3\,(Ca-5\,Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^3}$$
$$+\frac{4\,Bab^2x^3+3\,(Cab^2-5\,Ab^3)x^4-8\,Aa^2b+5\,(Ca^2b-5\,Aab^2)x^2-2\,(Da^3-3\,Ba^2b)x}{8\,(bx^2+a)^2a^3bx}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/x^2/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
-1/2*B*log(b*x^2 + a)/a^3 + B*log(abs(x))/a^3 + 3/8*(C*a - 5*A*b)*arctan(b
*x/sqrt(a*b))/(sqrt(a*b)*a^3) + 1/8*(4*B*a*b^2*x^3 + 3*(C*a*b^2 - 5*A*b^3)
*x^4 - 8*A*a^2*b + 5*(C*a^2*b - 5*A*a*b^2)*x^2 - 2*(D*a^3 - 3*B*a^2*b)*x)/
((b*x^2 + a)^2*a^3*b*x)
```

## Mupad [B] (verification not implemented)

Time = 2.56 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.40

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^2\left(a+bx^2\right)^3}\,dx = \frac{\frac{3\,B}{4\,a}+\frac{B\,b\,x^2}{2\,a^2}}{a^2+2\,a\,b\,x^2+b^2\,x^4}+\frac{\frac{5\,C\,x}{8\,a}+\frac{3\,C\,b\,x^3}{8\,a^2}}{a^2+2\,a\,b\,x^2+b^2\,x^4} - \frac{\frac{A}{a}+\frac{25\,A\,b\,x^2}{8\,a^2}+\frac{15\,A\,b^2\,x^4}{8\,a^3}}{a^2\,x+2\,a\,b\,x^3+b^2\,x^5} - \frac{D}{4\,b\,(b\,x^2+a)^2} - \frac{B\,\ln\left(b\,x^2+a\right)}{2\,a^3}+\frac{B\,\ln\left(x\right)}{a^3} - \frac{15\,A\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,a^{7/2}}+\frac{3\,C\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,a^{5/2}\,\sqrt{b}}$$

input
```
int((A + B*x + C*x^2 + x^3*D)/(x^2*(a + b*x^2)^3),x)
```

output
```
((3*B)/(4*a) + (B*b*x^2)/(2*a^2))/(a^2 + b^2*x^4 + 2*a*b*x^2) + ((5*C*x)/(
8*a) + (3*C*b*x^3)/(8*a^2))/(a^2 + b^2*x^4 + 2*a*b*x^2) - (A/a + (25*A*b*x
^2)/(8*a^2) + (15*A*b^2*x^4)/(8*a^3))/(a^2*x + b^2*x^5 + 2*a*b*x^3) - D/(4
*b*(a + b*x^2)^2) - (B*log(a + b*x^2))/(2*a^3) + (B*log(x))/a^3 - (15*A*b^
(1/2)*atan((b^(1/2)*x)/a^(1/2)))/(8*a^(7/2)) + (3*C*atan((b^(1/2)*x)/a^(1/
2)))/(8*a^(5/2)*b^(1/2))
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 328, normalized size of antiderivative = 2.28

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^2\left(a+bx^2\right)^3}\,dx = \frac{-15\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^2bx+3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^2cx-30\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a\,b^2x^3+6\sqrt{b}\,\sqrt{a}\,ata}{}$$

input
```
int((D*x^3+C*x^2+B*x+A)/x^2/(b*x^2+a)^3,x)
```

output
```
( - 15*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2*b*x + 3*sqrt(b)*
sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2*c*x - 30*sqrt(b)*sqrt(a)*atan((
b*x)/(sqrt(b)*sqrt(a)))*a*b**2*x**3 + 6*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b
)*sqrt(a)))*a*b*c*x**3 - 15*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*
b**3*x**5 + 3*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**2*c*x**5 -
4*log(a + b*x**2)*a**2*b**2*x - 8*log(a + b*x**2)*a*b**3*x**3 - 4*log(a +
b*x**2)*b**4*x**5 + 8*log(x)*a**2*b**2*x + 16*log(x)*a*b**3*x**3 + 8*log(x
)*b**4*x**5 - 8*a**3*b - 2*a**3*d*x - 25*a**2*b**2*x**2 + 4*a**2*b**2*x +
5*a**2*b*c*x**2 - 15*a*b**3*x**4 + 3*a*b**2*c*x**4 - 2*b**4*x**5)/(8*a**3*
b*x*(a**2 + 2*a*b*x**2 + b**2*x**4))
```

## 3.60 $\int \frac{A+Bx+Cx^2+Dx^3}{x^3\left(a+bx^2\right)^3}\,dx$



## Optimal result

Integrand size = 28, antiderivative size = 170

$$\begin{aligned}\int \frac{A+Bx+Cx^2+Dx^3}{x^3\left(a+bx^2\right)^3}\,dx = &-\frac{A}{2a^3x^2}-\frac{B}{a^3x}-\frac{Ab-aC+(bB-aD)x}{4a^2\left(a+bx^2\right)^2}\\&-\frac{4(2Ab-aC)+(7bB-3aD)x}{8a^3\left(a+bx^2\right)}\\&-\frac{3(5bB-aD)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{7/2}\sqrt{b}}\\&-\frac{(3Ab-aC)\log(x)}{a^4}+\frac{(3Ab-aC)\log\left(a+bx^2\right)}{2a^4}\end{aligned}$$

output
```
-1/2*A/a^3/x^2-B/a^3/x-1/4*(A*b-C*a+(B*b-D*a)*x)/a^2/(b*x^2+a)^2-1/8*(8*A*
b-4*C*a+(7*B*b-3*D*a)*x)/a^3/(b*x^2+a)-3/8*(5*B*b-D*a)*arctan(b^(1/2)*x/a^
(1/2))/a^(7/2)/b^(1/2)-(3*A*b-C*a)*ln(x)/a^4+1/2*(3*A*b-C*a)*ln(b*x^2+a)/a
^4
```

## Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.86

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^3\left(a+bx^2\right)^3}\,dx$$
$$=\frac{-\frac{4aA}{x^2}-\frac{8aB}{x}+\frac{a(-8Ab+4aC-7bBx+3aDx)}{a+bx^2}+\frac{2a^2(-Ab-bBx+a(C+Dx))}{(a+bx^2)^2}+\frac{3\sqrt{a}(-5bB+aD)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}+8(-3Ab+aC}{8a^4}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/(x^3*(a + b*x^2)^3),x]
```

output
```
((-4*a*A)/x^2 - (8*a*B)/x + (a*(-8*A*b + 4*a*C - 7*b*B*x + 3*a*D*x))/(a +
b*x^2) + (2*a^2*(-(A*b) - b*B*x + a*(C + D*x)))/(a + b*x^2)^2 + (3*Sqrt[a]
*(-5*b*B + a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[b] + 8*(-3*A*b + a*C)*Lo
g[x] + 4*(3*A*b - a*C)*Log[a + b*x^2])/(8*a^4)
```

## Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2336, 25, 2336, 25, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^3\left(a+bx^2\right)^3}\,dx$$

$$\Big\downarrow \text{ 2336}$$

$$-\frac{\int -\frac{-3\left(\frac{bB}{a}-D\right)x^3-4\left(\frac{Ab}{a}-C\right)x^2+4Bx+4A}{x^3(bx^2+a)^2}dx}{4a}-\frac{\frac{Ab}{a}+x\left(\frac{bB}{a}-D\right)-C}{4a\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{-3\left(\frac{bB}{a}-D\right)x^3-4\left(\frac{Ab}{a}-C\right)x^2+4Bx+4A}{x^3(bx^2+a)^2}dx}{4a}-\frac{\frac{Ab}{a}+x\left(\frac{bB}{a}-D\right)-C}{4a\left(a+bx^2\right)^2}$$

$$\Big\downarrow \ 2336$$

$$-\frac{-\frac{\int -\frac{-\left(\left(\frac{7bB}{a}-3D\right)x^3\right)-8\left(\frac{2Ab}{a}-C\right)x^2+8Bx+8A}{x^3\left(bx^2+a\right)}dx}{2a}-\frac{4\left(\frac{2Ab}{a}-C\right)+x\left(\frac{7bB}{a}-3D\right)}{2a(a+bx^2)}}{4a}-\frac{\frac{Ab}{a}+x\left(\frac{bB}{a}-D\right)-C}{4a\left(a+bx^2\right)^2}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{\int \frac{-\left(\left(\frac{7bB}{a}-3D\right)x^3\right)-8\left(\frac{2Ab}{a}-C\right)x^2+8Bx+8A}{x^3\left(bx^2+a\right)}dx}{2a}-\frac{4\left(\frac{2Ab}{a}-C\right)+x\left(\frac{7bB}{a}-3D\right)}{2a(a+bx^2)}}{4a}-\frac{\frac{Ab}{a}+x\left(\frac{bB}{a}-D\right)-C}{4a\left(a+bx^2\right)^2}$$

$$\Big\downarrow \ 2333$$

$$\frac{\frac{\int \left(\frac{8A}{ax^3}+\frac{8(aC-3Ab)}{a^2x}+\frac{8b(3Ab-aC)x-3a(5bB-aD)}{a^2\left(bx^2+a\right)}+\frac{8B}{ax^2}\right)dx}{2a}-\frac{4\left(\frac{2Ab}{a}-C\right)+x\left(\frac{7bB}{a}-3D\right)}{2a(a+bx^2)}}{4a}-\frac{\frac{Ab}{a}+x\left(\frac{bB}{a}-D\right)-C}{4a\left(a+bx^2\right)^2}$$

$$\Big\downarrow \ 2009$$

$$\frac{\frac{-\frac{3\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(5bB-aD)}{a^{3/2}\sqrt{b}}+\frac{4(3Ab-aC)\log\left(a+bx^2\right)}{a^2}-\frac{8\log(x)(3Ab-aC)}{a^2}-\frac{4A}{ax^2}-\frac{8B}{ax}}{2a}-\frac{4\left(\frac{2Ab}{a}-C\right)+x\left(\frac{7bB}{a}-3D\right)}{2a(a+bx^2)}}{4a}-\frac{\frac{Ab}{a}+x\left(\frac{bB}{a}-D\right)-C}{4a\left(a+bx^2\right)^2}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/(x^3*(a + b*x^2)^3),x]
```

output
```
-1/4*((A*b)/a - C + ((b*B)/a - D)*x)/(a*(a + b*x^2)^2) + (-1/2*(4*((2*A*b)
/a - C) + ((7*b*B)/a - 3*D)*x)/(a*(a + b*x^2)) + ((-4*A)/(a*x^2) - (8*B)/(
a*x) - (3*(5*b*B - a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*Sqrt[b]) - (
8*(3*A*b - a*C)*Log[x])/a^2 + (4*(3*A*b - a*C)*Log[a + b*x^2])/a^2)/(2*a))
/(4*a)
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

## Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.99

| method | result |
|---|---|
| default | $\frac{\frac{\left(-\frac{7}{8} a\,b^2 B+\frac{3}{8} a^2 b D\right) x^3+\left(-a\,b^2 A+\frac{1}{2} a^2 b C\right) x^2-\frac{a^2 (9Bb-5Da) x}{8}-\frac{5a^2 bA}{4}+\frac{3C\,a^3}{4}}{\left(b\,x^2+a\right)^2}+\frac{\left(24b^2 A-8Cab\right) \ln\left(b\,x^2+a\right)}{16b}+\frac{\left(-15abB+3Da^2\right) \arctan\left(\frac{b}{\sqrt{}}\right)}{8\sqrt{ab}}}{a^4}$ |

input
```
int((D*x^3+C*x^2+B*x+A)/x^3/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/a^4*(((-7/8*a*b^2*B+3/8*a^2*b*D)*x^3+(-a*b^2*A+1/2*a^2*b*C)*x^2-1/8*a^2*
(9*B*b-5*D*a)*x-5/4*a^2*b*A+3/4*C*a^3)/(b*x^2+a)^2+1/16*(24*A*b^2-8*C*a*b)
/b*ln(b*x^2+a)+1/8*(-15*B*a*b+3*D*a^2)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
)-1/2*A/a^3/x^2-B/a^3/x+(-3*A*b+C*a)/a^4*ln(x)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 338 vs. $2(145) = 290$.

Time = 0.13 (sec) , antiderivative size = 696, normalized size of antiderivative = 4.09

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^3 (a+bx^2)^3} \, dx = \text{Too large to display}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/x^3/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
[-1/16*(16*B*a^3*b*x - 6*(D*a^2*b^2 - 5*B*a*b^3)*x^5 + 8*A*a^3*b - 8*(C*a^
2*b^2 - 3*A*a*b^3)*x^4 - 10*(D*a^3*b - 5*B*a^2*b^2)*x^3 - 12*(C*a^3*b - 3*
A*a^2*b^2)*x^2 - 3*((D*a*b^2 - 5*B*b^3)*x^6 + 2*(D*a^2*b - 5*B*a*b^2)*x^4
+ (D*a^3 - 5*B*a^2*b)*x^2)*sqrt(-a*b)*log((b*x^2 + 2*sqrt(-a*b)*x - a)/(b*
x^2 + a)) + 8*((C*a*b^3 - 3*A*b^4)*x^6 + 2*(C*a^2*b^2 - 3*A*a*b^3)*x^4 + (
C*a^3*b - 3*A*a^2*b^2)*x^2)*log(b*x^2 + a) - 16*((C*a*b^3 - 3*A*b^4)*x^6 +
 2*(C*a^2*b^2 - 3*A*a*b^3)*x^4 + (C*a^3*b - 3*A*a^2*b^2)*x^2)*log(x))/(a^4
*b^3*x^6 + 2*a^5*b^2*x^4 + a^6*b*x^2), -1/8*(8*B*a^3*b*x - 3*(D*a^2*b^2 -
5*B*a*b^3)*x^5 + 4*A*a^3*b - 4*(C*a^2*b^2 - 3*A*a*b^3)*x^4 - 5*(D*a^3*b -
5*B*a^2*b^2)*x^3 - 6*(C*a^3*b - 3*A*a^2*b^2)*x^2 - 3*((D*a*b^2 - 5*B*b^3)*
x^6 + 2*(D*a^2*b - 5*B*a*b^2)*x^4 + (D*a^3 - 5*B*a^2*b)*x^2)*sqrt(a*b)*arc
tan(sqrt(a*b)*x/a) + 4*((C*a*b^3 - 3*A*b^4)*x^6 + 2*(C*a^2*b^2 - 3*A*a*b^3
)*x^4 + (C*a^3*b - 3*A*a^2*b^2)*x^2)*log(b*x^2 + a) - 8*((C*a*b^3 - 3*A*b^
4)*x^6 + 2*(C*a^2*b^2 - 3*A*a*b^3)*x^4 + (C*a^3*b - 3*A*a^2*b^2)*x^2)*log(
x))/(a^4*b^3*x^6 + 2*a^5*b^2*x^4 + a^6*b*x^2)]
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^3 (a+bx^2)^3} \, dx = \text{Timed out}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/x**3/(b*x**2+a)**3,x)
```

output
```
Timed out
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.01

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^3\left(a+bx^2\right)^3}\,dx$$
$$=\frac{3\,(Dab-5\,Bb^2)x^5+4\,(Cab-3\,Ab^2)x^4-8\,Ba^2x+5\,(Da^2-5\,Bab)x^3-4\,Aa^2+6\,(Ca^2-3\,Aab)x^2}{8\,(a^3b^2x^6+2\,a^4bx^4+a^5x^2)}$$
$$+\frac{3\,(Da-5\,Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^3}-\frac{(Ca-3\,Ab)\log\left(bx^2+a\right)}{2\,a^4}+\frac{(Ca-3\,Ab)\log\left(x\right)}{a^4}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/x^3/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/8*(3*(D*a*b - 5*B*b^2)*x^5 + 4*(C*a*b - 3*A*b^2)*x^4 - 8*B*a^2*x + 5*(D*
a^2 - 5*B*a*b)*x^3 - 4*A*a^2 + 6*(C*a^2 - 3*A*a*b)*x^2)/(a^3*b^2*x^6 + 2*a
^4*b*x^4 + a^5*x^2) + 3/8*(D*a - 5*B*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a
^3) - 1/2*(C*a - 3*A*b)*log(b*x^2 + a)/a^4 + (C*a - 3*A*b)*log(x)/a^4
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.95

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^3\left(a+bx^2\right)^3}\,dx$$
$$=\frac{3\,(Da-5\,Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^3}-\frac{(Ca-3\,Ab)\log\left(bx^2+a\right)}{2\,a^4}+\frac{(Ca-3\,Ab)\log\left(|x|\right)}{a^4}$$
$$+\frac{3\,Dabx^5-15\,Bb^2x^5+4\,Cabx^4-12\,Ab^2x^4+5\,Da^2x^3-25\,Babx^3+6\,Ca^2x^2-18\,Aabx^2-8\,Ba^2x-}{8\,\left(bx^3+ax\right)^2a^3}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/x^3/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
3/8*(D*a - 5*B*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3) - 1/2*(C*a - 3*A*b
)*log(b*x^2 + a)/a^4 + (C*a - 3*A*b)*log(abs(x))/a^4 + 1/8*(3*D*a*b*x^5 -
15*B*b^2*x^5 + 4*C*a*b*x^4 - 12*A*b^2*x^4 + 5*D*a^2*x^3 - 25*B*a*b*x^3 + 6
*C*a^2*x^2 - 18*A*a*b*x^2 - 8*B*a^2*x - 4*A*a^2)/((b*x^3 + a*x)^2*a^3)
```

## Mupad [B] (verification not implemented)

Time = 2.59 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.35

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^3\left(a+bx^2\right)^3}\,dx=\frac{\frac{3\,C}{4\,a}+\frac{C\,b\,x^2}{2\,a^2}}{a^2+2\,a\,b\,x^2+b^2\,x^4}-\frac{\frac{A}{2\,a}+\frac{9\,A\,b\,x^2}{4\,a^2}+\frac{3\,A\,b^2\,x^4}{2\,a^3}}{a^2\,x^2+2\,a\,b\,x^4+b^2\,x^6}-\frac{\frac{B}{a}+\frac{25\,B\,b\,x^2}{8\,a^2}+\frac{15\,B\,b^2\,x^4}{8\,a^3}}{a^2\,x+2\,a\,b\,x^3+b^2\,x^5}-\frac{C\,\ln\left(b\,x^2+a\right)}{2\,a^3}+\frac{C\,\ln\left(x\right)}{a^3}+\frac{3\,A\,b\,\ln\left(b\,x^2+a\right)}{2\,a^4}-\frac{3\,A\,b\,\ln\left(x\right)}{a^4}+\frac{x\,D\,{}_2\mathrm{F}_1\left(\frac{1}{2},3;\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{a^3}-\frac{15\,B\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,a^{7/2}}$$

input
```
int((A + B*x + C*x^2 + x^3*D)/(x^3*(a + b*x^2)^3),x)
```

output
```
((3*C)/(4*a) + (C*b*x^2)/(2*a^2))/(a^2 + b^2*x^4 + 2*a*b*x^2) - (A/(2*a) +
 (9*A*b*x^2)/(4*a^2) + (3*A*b^2*x^4)/(2*a^3))/(a^2*x^2 + b^2*x^6 + 2*a*b*x
^4) - (B/a + (25*B*b*x^2)/(8*a^2) + (15*B*b^2*x^4)/(8*a^3))/(a^2*x + b^2*x
^5 + 2*a*b*x^3) - (C*log(a + b*x^2))/(2*a^3) + (C*log(x))/a^3 + (3*A*b*log
(a + b*x^2))/(2*a^4) - (3*A*b*log(x))/a^4 + (x*D*hypergeom([1/2, 3], 3/2,
-(b*x^2)/a))/a^3 - (15*B*b^(1/2)*atan((b^(1/2)*x)/a^(1/2)))/(8*a^(7/2))
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 468, normalized size of antiderivative = 2.75

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^3\left(a+bx^2\right)^3}\,dx$$
$$=\frac{6\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^2bd\,x^4+3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a\,b^2d\,x^6-8a^3b^2x-25a^2b^3x^3-15a\,b^4x^5-4a^4b+\cdots}{}$$

input
```
int((D*x^3+C*x^2+B*x+A)/x^3/(b*x^2+a)^3,x)
```

output
```
(3*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**3*d*x**2 - 15*sqrt(b)*
sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2*b**2*x**2 + 6*sqrt(b)*sqrt(a)*a
tan((b*x)/(sqrt(b)*sqrt(a)))*a**2*b*d*x**4 - 30*sqrt(b)*sqrt(a)*atan((b*x)
/(sqrt(b)*sqrt(a)))*a*b**3*x**4 + 3*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sq
rt(a)))*a*b**2*d*x**6 - 15*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b
**4*x**6 + 12*log(a + b*x**2)*a**3*b**2*x**2 - 4*log(a + b*x**2)*a**3*b*c*
x**2 + 24*log(a + b*x**2)*a**2*b**3*x**4 - 8*log(a + b*x**2)*a**2*b**2*c*x
**4 + 12*log(a + b*x**2)*a*b**4*x**6 - 4*log(a + b*x**2)*a*b**3*c*x**6 - 2
4*log(x)*a**3*b**2*x**2 + 8*log(x)*a**3*b*c*x**2 - 48*log(x)*a**2*b**3*x**
4 + 16*log(x)*a**2*b**2*c*x**4 - 24*log(x)*a*b**4*x**6 + 8*log(x)*a*b**3*c
*x**6 - 4*a**4*b - 12*a**3*b**2*x**2 - 8*a**3*b**2*x + 4*a**3*b*c*x**2 + 5
*a**3*b*d*x**3 - 25*a**2*b**3*x**3 + 3*a**2*b**2*d*x**5 + 6*a*b**4*x**6 -
15*a*b**4*x**5 - 2*a*b**3*c*x**6)/(8*a**4*b*x**2*(a**2 + 2*a*b*x**2 + b**2
*x**4))
```

# 3.61 $\int x^3\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 227

$$\begin{aligned}\int x^3\sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx &= -\frac{a^2(8bB-5aD)x\sqrt{a+bx^2}}{128b^3}+\frac{a(8bB-5aD)x^3\sqrt{a+bx^2}}{192b^2}\\ &\quad+\frac{(8bB-5aD)x^5\sqrt{a+bx^2}}{48b}-\frac{a(Ab-aC)\left(a+bx^2\right)^{3/2}}{3b^3}+\frac{Dx^5\left(a+bx^2\right)^{3/2}}{8b}\\ &\quad+\frac{(Ab-2aC)\left(a+bx^2\right)^{5/2}}{5b^3}+\frac{C\left(a+bx^2\right)^{7/2}}{7b^3}+\frac{a^3(8bB-5aD)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{7/2}}\end{aligned}$$

output
```
-1/128*a^2*(8*B*b-5*D*a)*x*(b*x^2+a)^(1/2)/b^3+1/192*a*(8*B*b-5*D*a)*x^3*(
b*x^2+a)^(1/2)/b^2+1/48*(8*B*b-5*D*a)*x^5*(b*x^2+a)^(1/2)/b-1/3*a*(A*b-C*a
)*(b*x^2+a)^(3/2)/b^3+1/8*D*x^5*(b*x^2+a)^(3/2)/b+1/5*(A*b-2*C*a)*(b*x^2+a
)^(5/2)/b^3+1/7*C*(b*x^2+a)^(7/2)/b^3+1/128*a^3*(8*B*b-5*D*a)*arctanh(b^(1
/2)*x/(b*x^2+a)^(1/2))/b^(7/2)
```

## Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.70

$$\int x^3\sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$=\frac{\sqrt{b}\sqrt{a+bx^2}(a^3(1024C+525Dx)+8ab^2x^2(112A+x(70B+48Cx+35Dx^2))-2a^2b(896A+x(420B$$

input
```
Integrate[x^3*Sqrt[a + b*x^2]*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(Sqrt[b]*Sqrt[a + b*x^2]*(a^3*(1024*C + 525*D*x) + 8*a*b^2*x^2*(112*A + x*
(70*B + 48*C*x + 35*D*x^2)) - 2*a^2*b*(896*A + x*(420*B + 256*C*x + 175*D*
x^2)) + 16*b^3*x^4*(168*A + 5*x*(28*B + 3*x*(8*C + 7*D*x)))) + 105*a^3*(-8
*b*B + 5*a*D)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(13440*b^(7/2))
```

## Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.07, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.433$, Rules used = {2340, 2340, 27, 533, 27, 533, 25, 27, 533, 455, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$

$$\Big\downarrow \text{2340}$$

$$\frac{\int x^3\sqrt{bx^2+a}\left(8bCx^2+(8bB-5aD)x+8Ab\right)dx}{8b}+\frac{Dx^5\left(a+bx^2\right)^{3/2}}{8b}$$

$$\Big\downarrow \text{2340}$$

$$\frac{\frac{\int bx^3(8(7Ab-4aC)+7(8bB-5aD)x)\sqrt{bx^2+a}dx}{7b}+\frac{8}{7}Cx^4\left(a+bx^2\right)^{3/2}}{8b}+\frac{Dx^5\left(a+bx^2\right)^{3/2}}{8b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{7}\int x^3(8(7Ab-4aC)+7(8bB-5aD)x)\sqrt{bx^2+a}dx+\frac{8}{7}Cx^4\left(a+bx^2\right)^{3/2}}{8b}+\frac{Dx^5\left(a+bx^2\right)^{3/2}}{8b}$$

$\Big\downarrow$ 533

$$\frac{\frac{1}{7}\left(\frac{7x^3\left(a+bx^2\right)^{3/2}(8bB-5aD)}{6b}-\frac{\int 3x^2(7a(8bB-5aD)-16b(7Ab-4aC)x)\sqrt{bx^2+a}dx}{6b}\right)+\frac{8}{7}Cx^4\left(a+bx^2\right)^{3/2}}{8b}+\frac{Dx^5\left(a+bx^2\right)^{3/2}}{8b}$$

$\Big\downarrow$ 27

$$\frac{\frac{1}{7}\left(\frac{7x^3\left(a+bx^2\right)^{3/2}(8bB-5aD)}{6b}-\frac{\int x^2(7a(8bB-5aD)-16b(7Ab-4aC)x)\sqrt{bx^2+a}dx}{2b}\right)+\frac{8}{7}Cx^4\left(a+bx^2\right)^{3/2}}{8b}+\frac{Dx^5\left(a+bx^2\right)^{3/2}}{8b}$$

$\Big\downarrow$ 533

$$\frac{\frac{1}{7}\left(\frac{7x^3\left(a+bx^2\right)^{3/2}(8bB-5aD)}{6b}-\frac{-\frac{\int -abx(32(7Ab-4aC)+35(8bB-5aD)x)\sqrt{bx^2+a}dx}{5b}-\frac{16}{5}x^2\left(a+bx^2\right)^{3/2}(7Ab-4aC)}{2b}\right)+\frac{8}{7}Cx^4\left(a+bx^2\right)^{3/2}}{8b}+\frac{Dx^5\left(a+bx^2\right)^{3/2}}{8b}$$

$\Big\downarrow$ 25

$$\frac{\frac{1}{7}\left(\frac{7x^3\left(a+bx^2\right)^{3/2}(8bB-5aD)}{6b}-\frac{\frac{\int abx(32(7Ab-4aC)+35(8bB-5aD)x)\sqrt{bx^2+a}dx}{5b}-\frac{16}{5}x^2\left(a+bx^2\right)^{3/2}(7Ab-4aC)}{2b}\right)+\frac{8}{7}Cx^4\left(a+bx^2\right)^{3/2}}{8b}+\frac{Dx^5\left(a+bx^2\right)^{3/2}}{8b}$$

$\Big\downarrow$ 27

$$\frac{\frac{1}{7}\left(\frac{7x^3\left(a+bx^2\right)^{3/2}(8bB-5aD)}{6b}-\frac{\frac{1}{5}a\int x(32(7Ab-4aC)+35(8bB-5aD)x)\sqrt{bx^2+a}dx-\frac{16}{5}x^2\left(a+bx^2\right)^{3/2}(7Ab-4aC)}{2b}\right)+\frac{8}{7}Cx^4\left(a+bx^2\right)^{3/2}}{8b}+\frac{Dx^5\left(a+bx^2\right)^{3/2}}{8b}$$

$\Big\downarrow$ 533

$$\frac{\frac{1}{7}\left(\frac{7x^3(a+bx^2)^{3/2}(8bB-5aD)}{6b}-\frac{\frac{1}{5}a\left(\frac{35x\left(a+bx^2\right)^{3/2}(8bB-5aD)}{4b}-\frac{\int(35a(8bB-5aD)-128b(7Ab-4aC)x)\sqrt{bx^2+a}dx}{4b}\right)-\frac{16}{5}x^2\left(a+bx^2\right)^{3/2}(7Ab-4a}{2b}}{8b}$$

$$\frac{Dx^5\left(a+bx^2\right)^{3/2}}{8b}$$

$\big\downarrow$ 455

$$\frac{\frac{1}{7}\left(\frac{7x^3(a+bx^2)^{3/2}(8bB-5aD)}{6b}-\frac{\frac{1}{5}a\left(\frac{35x\left(a+bx^2\right)^{3/2}(8bB-5aD)}{4b}-\frac{35a(8bB-5aD)\int\sqrt{bx^2+a}dx-\frac{128}{3}\left(a+bx^2\right)^{3/2}(7Ab-4aC)}{4b}\right)-\frac{16}{5}x^2\left(a+bx^2\right)^{3/2}}{2b}}{8b}$$

$$\frac{Dx^5\left(a+bx^2\right)^{3/2}}{8b}$$

$\big\downarrow$ 211

$$\frac{\frac{1}{7}\left(\frac{7x^3(a+bx^2)^{3/2}(8bB-5aD)}{6b}-\frac{\frac{1}{5}a\left(\frac{35x\left(a+bx^2\right)^{3/2}(8bB-5aD)}{4b}-\frac{35a(8bB-5aD)\left(\frac{1}{2}a\int\frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)-\frac{128}{3}\left(a+bx^2\right)^{3/2}(7Ab-4aC)}{4b}\right)}{2b}}{8b}$$

$$\frac{Dx^5\left(a+bx^2\right)^{3/2}}{8b}$$

$\big\downarrow$ 224

$$\frac{\frac{1}{7}\left(\frac{7x^3(a+bx^2)^{3/2}(8bB-5aD)}{6b}-\frac{\frac{1}{5}a\left(\frac{35x\left(a+bx^2\right)^{3/2}(8bB-5aD)}{4b}-\frac{35a(8bB-5aD)\left(\frac{1}{2}a\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)-\frac{128}{3}\left(a+bx^2\right)^{3/2}(7Ab}{4b}\right.}{2b}}{8b}$$

$$\frac{Dx^5\left(a+bx^2\right)^{3/2}}{8b}$$

$\big\downarrow$ 219

$$\frac{\frac{1}{7}\left(\frac{7x^3\left(a+bx^2\right)^{3/2}(8bB-5aD)}{6b}-\frac{\frac{1}{5}a\left(\frac{35x\left(a+bx^2\right)^{3/2}(8bB-5aD)}{4b}-\frac{35a\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)(8bB-5aD)-\frac{128}{3}\left(a+bx^2\right)^{3/2}(7Ab-}{4b}\right.}{2b}\right.}{8b}$$

$$\frac{Dx^5\left(a+bx^2\right)^{3/2}}{8b}$$

input
```
Int[x^3*Sqrt[a + b*x^2]*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(D*x^5*(a + b*x^2)^(3/2))/(8*b) + ((8*C*x^4*(a + b*x^2)^(3/2))/7 + ((7*(8*
b*B - 5*a*D)*x^3*(a + b*x^2)^(3/2))/(6*b) - ((-16*(7*A*b - 4*a*C)*x^2*(a +
 b*x^2)^(3/2))/5 + (a*((35*(8*b*B - 5*a*D)*x*(a + b*x^2)^(3/2))/(4*b) - ((
-128*(7*A*b - 4*a*C)*(a + b*x^2)^(3/2))/3 + 35*a*(8*b*B - 5*a*D)*((x*Sqrt[
a + b*x^2])/2 + (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b])))/(4*
b)))/5)/(2*b))/7)/(8*b)
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

rule 2340
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Simp[1/(b*(m
+ q + 2*p + 1))   Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)
*Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /;
GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ
[Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

## Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.26

| method | result |
|---|---|
| default | $A\left(\frac{x^{2}\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{5b}-\frac{2a\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{15b^{2}}\right)+B\left(\frac{x^{3}\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{6b}-\frac{a\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{4b}-\frac{a\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)}{4b}\right)}{2b}\right)+$ |

input
```
int(x^3*(b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
A*(1/5*x^2*(b*x^2+a)^(3/2)/b-2/15*a/b^2*(b*x^2+a)^(3/2))+B*(1/6*x^3*(b*x^2
+a)^(3/2)/b-1/2*a/b*(1/4*x*(b*x^2+a)^(3/2)/b-1/4*a/b*(1/2*x*(b*x^2+a)^(1/2
)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))))+C*(1/7*x^4*(b*x^2+a)^(3/2
)/b-4/7*a/b*(1/5*x^2*(b*x^2+a)^(3/2)/b-2/15*a/b^2*(b*x^2+a)^(3/2)))+D*(1/8
*x^5*(b*x^2+a)^(3/2)/b-5/8*a/b*(1/6*x^3*(b*x^2+a)^(3/2)/b-1/2*a/b*(1/4*x*(
b*x^2+a)^(3/2)/b-1/4*a/b*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x
+(b*x^2+a)^(1/2))))))
```

## Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 387, normalized size of antiderivative = 1.70

$$\int x^3\sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$=\left[-\frac{105\,(5\,Da^4-8\,Ba^3b)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\,(1680\,Db^4x^7+1920\,Cb^4x^6+280\,(I\right.$$

input
```
integrate(x^3*(b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
[-1/26880*(105*(5*D*a^4 - 8*B*a^3*b)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 +
 a)*sqrt(b)*x - a) - 2*(1680*D*b^4*x^7 + 1920*C*b^4*x^6 + 280*(D*a*b^3 + 8
*B*b^4)*x^5 + 1024*C*a^3*b - 1792*A*a^2*b^2 + 384*(C*a*b^3 + 7*A*b^4)*x^4
- 70*(5*D*a^2*b^2 - 8*B*a*b^3)*x^3 - 128*(4*C*a^2*b^2 - 7*A*a*b^3)*x^2 + 1
05*(5*D*a^3*b - 8*B*a^2*b^2)*x)*sqrt(b*x^2 + a))/b^4, 1/13440*(105*(5*D*a^
4 - 8*B*a^3*b)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (1680*D*b^4*x
^7 + 1920*C*b^4*x^6 + 280*(D*a*b^3 + 8*B*b^4)*x^5 + 1024*C*a^3*b - 1792*A*
a^2*b^2 + 384*(C*a*b^3 + 7*A*b^4)*x^4 - 70*(5*D*a^2*b^2 - 8*B*a*b^3)*x^3 -
 128*(4*C*a^2*b^2 - 7*A*a*b^3)*x^2 + 105*(5*D*a^3*b - 8*B*a^2*b^2)*x)*sqrt
(b*x^2 + a))/b^4]
```

## Sympy [A] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.16

$$\int x^3\sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$=\begin{cases}\frac{3a^2\left(Ba-\frac{5a\left(Bb+\frac{Da}{8}\right)}{6b}\right)\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right)}{8b^2}+\sqrt{a+bx^2}\left(\frac{Cx^6}{7}+\frac{Dx^7}{8}-\frac{3ax\left(Ba-\frac{5a\left(Bb+\frac{Da}{8}\right)}{6b}\right)}{8b^2}-\right. \\ \sqrt{a}\left(\frac{Ax^4}{4}+\frac{Bx^5}{5}+\frac{Cx^6}{6}+\frac{Dx^7}{7}\right)\end{cases}$$

input
```
integrate(x**3*(b*x**2+a)**(1/2)*(D*x**3+C*x**2+B*x+A),x)
```

output
```
Piecewise((3*a**2*(B*a - 5*a*(B*b + D*a/8)/(6*b))*Piecewise((log(2*sqrt(b)
*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), Tru
e))/(8*b**2) + sqrt(a + b*x**2)*(C*x**6/7 + D*x**7/8 - 3*a*x*(B*a - 5*a*(B
*b + D*a/8)/(6*b))/(8*b**2) - 2*a*(A*a - 4*a*(A*b + C*a/7)/(5*b))/(3*b**2)
 + x**5*(B*b + D*a/8)/(6*b) + x**4*(A*b + C*a/7)/(5*b) + x**3*(B*a - 5*a*(
B*b + D*a/8)/(6*b))/(4*b) + x**2*(A*a - 4*a*(A*b + C*a/7)/(5*b))/(3*b)), N
e(b, 0)), (sqrt(a)*(A*x**4/4 + B*x**5/5 + C*x**6/6 + D*x**7/7), True))
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.12

$$\int x^3\sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx = \frac{(bx^2+a)^{\frac{3}{2}}Dx^5}{8\,b} + \frac{(bx^2+a)^{\frac{3}{2}}Cx^4}{7\,b} - \frac{5\,(bx^2+a)^{\frac{3}{2}}Dax^3}{48\,b^2} + \frac{(bx^2+a)^{\frac{3}{2}}Bx^3}{6\,b} - \frac{4\,(bx^2+a)^{\frac{3}{2}}Cax^2}{35\,b^2} + \frac{(bx^2+a)^{\frac{3}{2}}Ax^2}{5\,b} + \frac{5\,(bx^2+a)^{\frac{3}{2}}Da^2x}{64\,b^3} - \frac{5\,\sqrt{bx^2+a}Da^3x}{128\,b^3} - \frac{(bx^2+a)^{\frac{3}{2}}Bax}{8\,b^2} + \frac{\sqrt{bx^2+a}Ba^2x}{16\,b^2} - \frac{5\,Da^4\,\text{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\,b^{\frac{7}{2}}} + \frac{Ba^3\,\text{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{5}{2}}} + \frac{8\,(bx^2+a)^{\frac{3}{2}}Ca^2}{105\,b^3} - \frac{2\,(bx^2+a)^{\frac{3}{2}}Aa}{15\,b^2}$$

input
```
integrate(x^3*(b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
1/8*(b*x^2 + a)^(3/2)*D*x^5/b + 1/7*(b*x^2 + a)^(3/2)*C*x^4/b - 5/48*(b*x^
2 + a)^(3/2)*D*a*x^3/b^2 + 1/6*(b*x^2 + a)^(3/2)*B*x^3/b - 4/35*(b*x^2 + a
)^(3/2)*C*a*x^2/b^2 + 1/5*(b*x^2 + a)^(3/2)*A*x^2/b + 5/64*(b*x^2 + a)^(3/
2)*D*a^2*x/b^3 - 5/128*sqrt(b*x^2 + a)*D*a^3*x/b^3 - 1/8*(b*x^2 + a)^(3/2)
*B*a*x/b^2 + 1/16*sqrt(b*x^2 + a)*B*a^2*x/b^2 - 5/128*D*a^4*arcsinh(b*x/sq
rt(a*b))/b^(7/2) + 1/16*B*a^3*arcsinh(b*x/sqrt(a*b))/b^(5/2) + 8/105*(b*x^
2 + a)^(3/2)*C*a^2/b^3 - 2/15*(b*x^2 + a)^(3/2)*A*a/b^2
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.91

$$\int x^3\sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$=\frac{1}{13440}\sqrt{bx^2+a}\left(\left(2\left(\left(\left(4\left(5\left(6\,(7\,Dx+8\,C)x+\frac{7\,(Dab^5+8\,Bb^6)}{b^6}\right)x+\frac{48\,(Cab^5+7\,Ab^6)}{b^6}\right)x-\frac{35\,(5\right.\right.\right.\right.\right.\right.$$
$$+\frac{(5\,Da^4-8\,Ba^3b)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{128\,b^{\frac{7}{2}}}$$

input
```
integrate(x^3*(b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

output
```
1/13440*sqrt(b*x^2 + a)*((2*((4*(5*(6*(7*D*x + 8*C)*x + 7*(D*a*b^5 + 8*B*b
^6)/b^6)*x + 48*(C*a*b^5 + 7*A*b^6)/b^6)*x - 35*(5*D*a^2*b^4 - 8*B*a*b^5)/
b^6)*x - 64*(4*C*a^2*b^4 - 7*A*a*b^5)/b^6)*x + 105*(5*D*a^3*b^3 - 8*B*a^2*
b^4)/b^6)*x + 256*(4*C*a^3*b^3 - 7*A*a^2*b^4)/b^6) + 1/128*(5*D*a^4 - 8*B*
a^3*b)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(7/2)
```

## Mupad [F(-1)]

Timed out.

$$\int x^3\sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx=\int x^3\,\sqrt{b\,x^2+a}\,\left(A+B\,x+C\,x^2+x^3\,D\right)\,dx$$

input
```
int(x^3*(a + b*x^2)^(1/2)*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
int(x^3*(a + b*x^2)^(1/2)*(A + B*x + C*x^2 + x^3*D), x)
```

## Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.34

$$\int x^3\sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$

$$= \frac{-1792\sqrt{b\,x^2+a}\,a^3b^2+1024\sqrt{b\,x^2+a}\,a^3bc+525\sqrt{b\,x^2+a}\,a^3bdx+896\sqrt{b\,x^2+a}\,a^2b^3x^2-840\sqrt{b\,x^2+}}{}$$

input
```
int(x^3*(b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A),x)
```

output
```
( - 1792*sqrt(a + b*x**2)*a**3*b**2 + 1024*sqrt(a + b*x**2)*a**3*b*c + 525
*sqrt(a + b*x**2)*a**3*b*d*x + 896*sqrt(a + b*x**2)*a**2*b**3*x**2 - 840*s
qrt(a + b*x**2)*a**2*b**3*x - 512*sqrt(a + b*x**2)*a**2*b**2*c*x**2 - 350*
sqrt(a + b*x**2)*a**2*b**2*d*x**3 + 2688*sqrt(a + b*x**2)*a*b**4*x**4 + 56
0*sqrt(a + b*x**2)*a*b**4*x**3 + 384*sqrt(a + b*x**2)*a*b**3*c*x**4 + 280*
sqrt(a + b*x**2)*a*b**3*d*x**5 + 2240*sqrt(a + b*x**2)*b**5*x**5 + 1920*sq
rt(a + b*x**2)*b**4*c*x**6 + 1680*sqrt(a + b*x**2)*b**4*d*x**7 - 525*sqrt(
b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**4*d + 840*sqrt(b)*log((s
qrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**3*b**2)/(13440*b**4)
```

# 3.62 $\int x^2\sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 194

$$\int x^2\sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx = \frac{a(2Ab-aC)x\sqrt{a+bx^2}}{16b^2}+\frac{(2Ab-aC)x^3\sqrt{a+bx^2}}{8b} - \frac{a(bB-aD)\left(a+bx^2\right)^{3/2}}{3b^3}+\frac{Cx^3\left(a+bx^2\right)^{3/2}}{6b}+\frac{(bB-2aD)\left(a+bx^2\right)^{5/2}}{5b^3} + \frac{D\left(a+bx^2\right)^{7/2}}{7b^3}-\frac{a^2(2Ab-aC)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{5/2}}$$

output
```
1/16*a*(2*A*b-C*a)*x*(b*x^2+a)^(1/2)/b^2+1/8*(2*A*b-C*a)*x^3*(b*x^2+a)^(1/
2)/b-1/3*a*(B*b-D*a)*(b*x^2+a)^(3/2)/b^3+1/6*C*x^3*(b*x^2+a)^(3/2)/b+1/5*(
B*b-2*D*a)*(b*x^2+a)^(5/2)/b^3+1/7*D*(b*x^2+a)^(7/2)/b^3-1/16*a^2*(2*A*b-C
*a)*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(5/2)
```

## Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.73

$$\int x^2\sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$=\frac{\sqrt{a+bx^2}(128a^3D+4b^3x^3(105A+84Bx+70Cx^2+60Dx^3)-a^2b(224B+x(105C+64Dx))+2ab^2x}{1680b^3}$$

input
```
Integrate[x^2*Sqrt[a + b*x^2]*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(Sqrt[a + b*x^2]*(128*a^3*D + 4*b^3*x^3*(105*A + 84*B*x + 70*C*x^2 + 60*D*
x^3) - a^2*b*(224*B + x*(105*C + 64*D*x)) + 2*a*b^2*x*(105*A + x*(56*B + 3
5*C*x + 24*D*x^2))) - 105*a^2*Sqrt[b]*(-2*A*b + a*C)*Log[-(Sqrt[b]*x) + Sq
rt[a + b*x^2]])/(1680*b^3)
```

## Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.07, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.367, Rules used = {2340, 2340, 27, 533, 533, 25, 27, 455, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$

$$\Big\downarrow \text{2340}$$

$$\frac{\int x^2\sqrt{bx^2+a}\left(7bCx^2+(7bB-4aD)x+7Ab\right)dx}{7b}+\frac{Dx^4\left(a+bx^2\right)^{3/2}}{7b}$$

$$\Big\downarrow \text{2340}$$

$$\frac{\frac{\int 3bx^2(7(2Ab-aC)+2(7bB-4aD)x)\sqrt{bx^2+a}dx}{6b}+\frac{7}{6}Cx^3\left(a+bx^2\right)^{3/2}}{7b}+\frac{Dx^4\left(a+bx^2\right)^{3/2}}{7b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{2}\int x^2(7(2Ab-aC)+2(7bB-4aD)x)\sqrt{bx^2+a}dx+\frac{7}{6}Cx^3\left(a+bx^2\right)^{3/2}}{7b}+\frac{Dx^4\left(a+bx^2\right)^{3/2}}{7b}$$

$$\Big\downarrow \; 533$$

$$\frac{\frac{1}{2}\left(\frac{2x^2\left(a+bx^2\right)^{3/2}(7bB-4aD)}{5b}-\frac{\int x(4a(7bB-4aD)-35b(2Ab-aC)x)\sqrt{bx^2+a}dx}{5b}\right)+\frac{7}{6}Cx^3\left(a+bx^2\right)^{3/2}}{7b}+\frac{Dx^4\left(a+bx^2\right)^{3/2}}{7b}$$

$$\Big\downarrow \; 533$$

$$\frac{\frac{1}{2}\left(\frac{2x^2\left(a+bx^2\right)^{3/2}(7bB-4aD)}{5b}-\frac{-\frac{\int -ab(35(2Ab-aC)+16(7bB-4aD)x)\sqrt{bx^2+a}dx}{4b}-\frac{35}{4}x\left(a+bx^2\right)^{3/2}(2Ab-aC)}{5b}\right)+\frac{7}{6}Cx^3\left(a+bx^2\right)^{3/2}}{7b}+\frac{Dx^4\left(a+bx^2\right)^{3/2}}{7b}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{1}{2}\left(\frac{2x^2\left(a+bx^2\right)^{3/2}(7bB-4aD)}{5b}-\frac{\frac{\int ab(35(2Ab-aC)+16(7bB-4aD)x)\sqrt{bx^2+a}dx}{4b}-\frac{35}{4}x\left(a+bx^2\right)^{3/2}(2Ab-aC)}{5b}\right)+\frac{7}{6}Cx^3\left(a+bx^2\right)^{3/2}}{7b}+\frac{Dx^4\left(a+bx^2\right)^{3/2}}{7b}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{1}{2}\left(\frac{2x^2\left(a+bx^2\right)^{3/2}(7bB-4aD)}{5b}-\frac{\frac{1}{4}a\int(35(2Ab-aC)+16(7bB-4aD)x)\sqrt{bx^2+a}dx-\frac{35}{4}x\left(a+bx^2\right)^{3/2}(2Ab-aC)}{5b}\right)+\frac{7}{6}Cx^3\left(a+bx^2\right)^{3/2}}{7b}+\frac{Dx^4\left(a+bx^2\right)^{3/2}}{7b}$$

$$\Big\downarrow \; 455$$

$$\frac{\frac{1}{2}\left(\frac{2x^2\left(a+bx^2\right)^{3/2}(7bB-4aD)}{5b}-\frac{\frac{1}{4}a\left(35(2Ab-aC)\int\sqrt{bx^2+a}dx+\frac{16\left(a+bx^2\right)^{3/2}(7bB-4aD)}{3b}\right)-\frac{35}{4}x\left(a+bx^2\right)^{3/2}(2Ab-aC)}{5b}\right)+\frac{7}{6}Cx^3\left(a-\right.}{7b}$$

$$\frac{Dx^4\left(a+bx^2\right)^{3/2}}{7b}$$

$$\Big\downarrow \; 211$$

$$\frac{\frac{1}{2}\left(\frac{2x^2\left(a+bx^2\right)^{3/2}(7bB-4aD)}{5b}-\frac{\frac{1}{4}a\left(35(2Ab-aC)\left(\frac{1}{2}a\int\frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{16\left(a+bx^2\right)^{3/2}(7bB-4aD)}{3b}\right)-\frac{35}{4}x\left(a+bx^2\right)^{3/2}(2Ab-aC}{5b}\right.}{7b}$$

$$\frac{Dx^4\left(a+bx^2\right)^{3/2}}{7b}$$

$$\Big\downarrow \; 224$$

$$\frac{\frac{1}{2}\left(\frac{2x^2\left(a+bx^2\right)^{3/2}(7bB-4aD)}{5b}-\frac{\frac{1}{4}a\left(35(2Ab-aC)\left(\frac{1}{2}a\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{16\left(a+bx^2\right)^{3/2}(7bB-4aD)}{3b}\right)-\frac{35}{4}x\left(a+bx^2\right)^{3/2}(}{5b}\right.}{7b}$$

$$\frac{Dx^4\left(a+bx^2\right)^{3/2}}{7b}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{1}{2}\left(\frac{2x^2\left(a+bx^2\right)^{3/2}(7bB-4aD)}{5b}-\frac{\frac{1}{4}a\left(35(2Ab-aC)\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{16\left(a+bx^2\right)^{3/2}(7bB-4aD)}{3b}\right)-\frac{35}{4}x\left(a+bx^2\right)^{3/2}(2}{5b}\right.}{7b}$$

$$\frac{Dx^4\left(a+bx^2\right)^{3/2}}{7b}$$

input
```
Int[x^2*Sqrt[a + b*x^2]*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(D*x^4*(a + b*x^2)^(3/2))/(7*b) + ((7*C*x^3*(a + b*x^2)^(3/2))/6 + ((2*(7*
b*B - 4*a*D)*x^2*(a + b*x^2)^(3/2))/(5*b) - ((-35*(2*A*b - a*C)*x*(a + b*x
^2)^(3/2))/4 + (a*((16*(7*b*B - 4*a*D)*(a + b*x^2)^(3/2))/(3*b) + 35*(2*A*
b - a*C)*((x*Sqrt[a + b*x^2])/2 + (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])
/(2*Sqrt[b]))))/4)/(5*b))/2)/(7*b)
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

rule 2340
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Simp[1/(b*(m
+ q + 2*p + 1))   Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)
*Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /;
GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ
[Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

## Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.23

| method | result |
|---|---|
| default | $A\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{4b}-\frac{a\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)}{4b}\right)+B\left(\frac{x^{2}\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{5b}-\frac{2a\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{15b^{2}}\right)+C\left(\frac{x^{3}\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{6b}\right.$ |

input
```
int(x^2*(b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
A*(1/4*x*(b*x^2+a)^(3/2)/b-1/4*a/b*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln
(b^(1/2)*x+(b*x^2+a)^(1/2))))+B*(1/5*x^2*(b*x^2+a)^(3/2)/b-2/15*a/b^2*(b*x
^2+a)^(3/2))+C*(1/6*x^3*(b*x^2+a)^(3/2)/b-1/2*a/b*(1/4*x*(b*x^2+a)^(3/2)/b
-1/4*a/b*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)
))))+D*(1/7*x^4*(b*x^2+a)^(3/2)/b-4/7*a/b*(1/5*x^2*(b*x^2+a)^(3/2)/b-2/15*
a/b^2*(b*x^2+a)^(3/2)))
```

## Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.68

$$\int x^2\sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$

$$= \left[-\frac{105\,(Ca^3-2\,Aa^2b)\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\,(240\,Db^3x^6+280\,Cb^3x^5+48\,(Dab^2 -}{}\right.$$

$$-\frac{105\,(Ca^3-2\,Aa^2b)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(240\,Db^3x^6+280\,Cb^3x^5+48\,(Dab^2+7\,Bb^3)x^4+128\,Da}{1680}$$

input
```
integrate(x^2*(b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
[-1/3360*(105*(C*a^3 - 2*A*a^2*b)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)
*sqrt(b)*x - a) - 2*(240*D*b^3*x^6 + 280*C*b^3*x^5 + 48*(D*a*b^2 + 7*B*b^3
)*x^4 + 128*D*a^3 - 224*B*a^2*b + 70*(C*a*b^2 + 6*A*b^3)*x^3 - 16*(4*D*a^2
*b - 7*B*a*b^2)*x^2 - 105*(C*a^2*b - 2*A*a*b^2)*x)*sqrt(b*x^2 + a))/b^3, -
1/1680*(105*(C*a^3 - 2*A*a^2*b)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)
) - (240*D*b^3*x^6 + 280*C*b^3*x^5 + 48*(D*a*b^2 + 7*B*b^3)*x^4 + 128*D*a^
3 - 224*B*a^2*b + 70*(C*a*b^2 + 6*A*b^3)*x^3 - 16*(4*D*a^2*b - 7*B*a*b^2)*
x^2 - 105*(C*a^2*b - 2*A*a*b^2)*x)*sqrt(b*x^2 + a))/b^3]
```

## Sympy [A] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.18

$$\int x^2\sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$

$$= \begin{cases} -\frac{a\left(Aa-\frac{3a\left(Ab+\frac{Ca}{6}\right)}{4b}\right)\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for}\ a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right)}{2b}+\sqrt{a+bx^2}\left(\frac{Cx^5}{6}+\frac{Dx^6}{7}-\frac{2a\left(Ba-\frac{4a\left(Bb+\frac{Da}{7}\right)}{5b}\right)}{3b^2}+\right. \\ \sqrt{a}\left(\frac{Ax^3}{3}+\frac{Bx^4}{4}+\frac{Cx^5}{5}+\frac{Dx^6}{6}\right) \end{cases}$$

input
```
integrate(x**2*(b*x**2+a)**(1/2)*(D*x**3+C*x**2+B*x+A),x)
```

output
```
Piecewise((-a*(A*a - 3*a*(A*b + C*a/6)/(4*b))*Piecewise((log(2*sqrt(b)*sqr
t(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True))/
(2*b) + sqrt(a + b*x**2)*(C*x**5/6 + D*x**6/7 - 2*a*(B*a - 4*a*(B*b + D*a/
7)/(5*b))/(3*b**2) + x**4*(B*b + D*a/7)/(5*b) + x**3*(A*b + C*a/6)/(4*b) +
 x**2*(B*a - 4*a*(B*b + D*a/7)/(5*b))/(3*b) + x*(A*a - 3*a*(A*b + C*a/6)/(
4*b))/(2*b)), Ne(b, 0)), (sqrt(a)*(A*x**3/3 + B*x**4/4 + C*x**5/5 + D*x**6
/6), True))
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.10

$$\begin{aligned}\int x^2\sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx &= \frac{(bx^2+a)^{\frac{3}{2}}Dx^4}{7\,b}+\frac{(bx^2+a)^{\frac{3}{2}}Cx^3}{6\,b}\\&\quad-\frac{4\,(bx^2+a)^{\frac{3}{2}}Dax^2}{35\,b^2}+\frac{(bx^2+a)^{\frac{3}{2}}Bx^2}{5\,b}\\&\quad-\frac{(bx^2+a)^{\frac{3}{2}}Cax}{8\,b^2}+\frac{\sqrt{bx^2+a}Ca^2x}{16\,b^2}\\&\quad+\frac{(bx^2+a)^{\frac{3}{2}}Ax}{4\,b}-\frac{\sqrt{bx^2+a}Aax}{8\,b}\\&\quad+\frac{Ca^3\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{5}{2}}}-\frac{Aa^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{3}{2}}}\\&\quad+\frac{8\,(bx^2+a)^{\frac{3}{2}}Da^2}{105\,b^3}-\frac{2\,(bx^2+a)^{\frac{3}{2}}Ba}{15\,b^2}\end{aligned}$$

input
```
integrate(x^2*(b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
1/7*(b*x^2 + a)^(3/2)*D*x^4/b + 1/6*(b*x^2 + a)^(3/2)*C*x^3/b - 4/35*(b*x^
2 + a)^(3/2)*D*a*x^2/b^2 + 1/5*(b*x^2 + a)^(3/2)*B*x^2/b - 1/8*(b*x^2 + a)
^(3/2)*C*a*x/b^2 + 1/16*sqrt(b*x^2 + a)*C*a^2*x/b^2 + 1/4*(b*x^2 + a)^(3/2
)*A*x/b - 1/8*sqrt(b*x^2 + a)*A*a*x/b + 1/16*C*a^3*arcsinh(b*x/sqrt(a*b))/
b^(5/2) - 1/8*A*a^2*arcsinh(b*x/sqrt(a*b))/b^(3/2) + 8/105*(b*x^2 + a)^(3/
2)*D*a^2/b^3 - 2/15*(b*x^2 + a)^(3/2)*B*a/b^2
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.91

$$\int x^2\sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$=\frac{1}{1680}\sqrt{bx^2+a}\left(\left(2\left(\left(4\left(5\,(6\,Dx+7\,C)x+\frac{6\,(Dab^4+7\,Bb^5)}{b^5}\right)x+\frac{35\,(Cab^4+6\,Ab^5)}{b^5}\right)x-\frac{8\,(4\,Da^2b^3\right.\right.\right.$$
$$-\frac{(Ca^3-2\,Aa^2b)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{16\,b^{\frac{5}{2}}}$$

input

```
integrate(x^2*(b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

output

```
1/1680*sqrt(b*x^2 + a)*((2*((4*(5*(6*D*x + 7*C)*x + 6*(D*a*b^4 + 7*B*b^5)/
b^5)*x + 35*(C*a*b^4 + 6*A*b^5)/b^5)*x - 8*(4*D*a^2*b^3 - 7*B*a*b^4)/b^5)*
x - 105*(C*a^2*b^3 - 2*A*a*b^4)/b^5)*x + 32*(4*D*a^3*b^2 - 7*B*a^2*b^3)/b^
5) - 1/16*(C*a^3 - 2*A*a^2*b)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/
2)
```

## Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx=\int x^2\,\sqrt{b\,x^2+a}\,\left(A+B\,x+C\,x^2+x^3\,D\right)\,dx$$

input

```
int(x^2*(a + b*x^2)^(1/2)*(A + B*x + C*x^2 + x^3*D),x)
```

output

```
int(x^2*(a + b*x^2)^(1/2)*(A + B*x + C*x^2 + x^3*D), x)
```

## Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.35

$$\int x^2\sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$= \frac{128\sqrt{b\,x^2+a}\,a^3d+210\sqrt{b\,x^2+a}\,a^2b^2x-224\sqrt{b\,x^2+a}\,a^2b^2-105\sqrt{b\,x^2+a}\,a^2bcx-64\sqrt{b\,x^2+a}\,a^2bd\,x}{}$$

input
```
int(x^2*(b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A),x)
```

output
```
(128*sqrt(a + b*x**2)*a**3*d + 210*sqrt(a + b*x**2)*a**2*b**2*x - 224*sqrt
(a + b*x**2)*a**2*b**2 - 105*sqrt(a + b*x**2)*a**2*b*c*x - 64*sqrt(a + b*x
**2)*a**2*b*d*x**2 + 420*sqrt(a + b*x**2)*a*b**3*x**3 + 112*sqrt(a + b*x**
2)*a*b**3*x**2 + 70*sqrt(a + b*x**2)*a*b**2*c*x**3 + 48*sqrt(a + b*x**2)*a
*b**2*d*x**4 + 336*sqrt(a + b*x**2)*b**4*x**4 + 280*sqrt(a + b*x**2)*b**3*
c*x**5 + 240*sqrt(a + b*x**2)*b**3*d*x**6 - 210*sqrt(b)*log((sqrt(a + b*x*
*2) + sqrt(b)*x)/sqrt(a))*a**3*b + 105*sqrt(b)*log((sqrt(a + b*x**2) + sqr
t(b)*x)/sqrt(a))*a**3*c)/(1680*b**3)
```

# 3.63 $\int x\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)\,dx$



## Optimal result

Integrand size = 28, antiderivative size = 167

$$\int x\sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{a(2bB-aD)x\sqrt{a+bx^2}}{16b^2}+\frac{(2bB-aD)x^3\sqrt{a+bx^2}}{8b}+\frac{(Ab-aC)\left(a+bx^2\right)^{3/2}}{3b^2}+\frac{Dx^3(a+bx^2)^{3/2}}{6b}+\frac{C(a+bx^2)^{5/2}}{5b^2}-\frac{a^2(2bB-aD)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{5/2}}$$

output
```
1/16*a*(2*B*b-D*a)*x*(b*x^2+a)^(1/2)/b^2+1/8*(2*B*b-D*a)*x^3*(b*x^2+a)^(1/
2)/b+1/3*(A*b-C*a)*(b*x^2+a)^(3/2)/b^2+1/6*D*x^3*(b*x^2+a)^(3/2)/b+1/5*C*(
b*x^2+a)^(5/2)/b^2-1/16*a^2*(2*B*b-D*a)*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))
/b^(5/2)
```

## Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.77

$$\int x\sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$=\frac{\sqrt{b}\sqrt{a+bx^2}(-a^2(32C+15Dx)+4b^2x^2(20A+x(15B+2x(6C+5Dx)))+2ab(40A+x(15B+x(8C}{240b^{5/2}}$$

input
```
Integrate[x*Sqrt[a + b*x^2]*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(Sqrt[b]*Sqrt[a + b*x^2]*(-(a^2*(32*C + 15*D*x)) + 4*b^2*x^2*(20*A + x*(15
*B + 2*x*(6*C + 5*D*x))) + 2*a*b*(40*A + x*(15*B + x*(8*C + 5*D*x)))) - 15
*a^2*(-2*b*B + a*D)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(240*b^(5/2))
```

## Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {2340, 27, 2340, 27, 533, 455, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$

$$\Big\downarrow \; 2340$$

$$\frac{\int 3x\sqrt{bx^2+a}\left(2bCx^2+(2bB-aD)x+2Ab\right)dx}{6b}+\frac{Dx^3\left(a+bx^2\right)^{3/2}}{6b}$$

$$\Big\downarrow \; 27$$

$$\frac{\int x\sqrt{bx^2+a}\left(2bCx^2+(2bB-aD)x+2Ab\right)dx}{2b}+\frac{Dx^3\left(a+bx^2\right)^{3/2}}{6b}$$

$$\Big\downarrow \; 2340$$

$$\frac{\frac{\int bx(2(5Ab-2aC)+5(2bB-aD)x)\sqrt{bx^2+a}dx}{5b}+\frac{2}{5}Cx^2\left(a+bx^2\right)^{3/2}}{2b}+\frac{Dx^3\left(a+bx^2\right)^{3/2}}{6b}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{1}{5}\int x(2(5Ab-2aC)+5(2bB-aD)x)\sqrt{bx^2+a}dx+\frac{2}{5}Cx^2\left(a+bx^2\right)^{3/2}}{2b}+\frac{Dx^3\left(a+bx^2\right)^{3/2}}{6b}$$

$$\Big\downarrow \; 533$$

$$\frac{\frac{1}{5}\left(\frac{5x\left(a+bx^2\right)^{3/2}(2bB-aD)}{4b}-\frac{\int (5a(2bB-aD)-8b(5Ab-2aC)x)\sqrt{bx^2+a}dx}{4b}\right)+\frac{2}{5}Cx^2\left(a+bx^2\right)^{3/2}}{2b}+\frac{Dx^3\left(a+bx^2\right)^{3/2}}{6b}$$

$$\Big\downarrow \; 455$$

$$\frac{\frac{1}{5}\left(\frac{5x\left(a+bx^2\right)^{3/2}(2bB-aD)}{4b}-\frac{5a(2bB-aD)\int \sqrt{bx^2+a}dx-\frac{8}{3}\left(a+bx^2\right)^{3/2}(5Ab-2aC)}{4b}\right)+\frac{2}{5}Cx^2\left(a+bx^2\right)^{3/2}}{2b}+\frac{Dx^3\left(a+bx^2\right)^{3/2}}{6b}$$

$$\Big\downarrow \; 211$$

$$\frac{\frac{1}{5}\left(\frac{5x\left(a+bx^2\right)^{3/2}(2bB-aD)}{4b}-\frac{5a(2bB-aD)\left(\frac{1}{2}a\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)-\frac{8}{3}\left(a+bx^2\right)^{3/2}(5Ab-2aC)}{4b}\right)+\frac{2}{5}Cx^2\left(a+bx^2\right)^{3/2}}{2b}+\frac{Dx^3\left(a+bx^2\right)^{3/2}}{6b}$$

$$\Big\downarrow \; 224$$

$$\frac{\frac{1}{5}\left(\frac{5x\left(a+bx^2\right)^{3/2}(2bB-aD)}{4b}-\frac{5a(2bB-aD)\left(\frac{1}{2}a\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)-\frac{8}{3}\left(a+bx^2\right)^{3/2}(5Ab-2aC)}{4b}\right)+\frac{2}{5}Cx^2\left(a+bx^2\right)}{2b}$$

$$\frac{Dx^3\left(a+bx^2\right)^{3/2}}{6b}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{1}{5}\left(\frac{5x(a+bx^2)^{3/2}(2bB-aD)}{4b}-\frac{5a\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)(2bB-aD)-\frac{8}{3}(a+bx^2)^{3/2}(5Ab-2aC)}{4b}\right)+\frac{2}{5}Cx^2(a+bx^2)^{3/}}{2b}$$

$$+\frac{Dx^3(a+bx^2)^{3/2}}{6b}$$

input
```
Int[x*Sqrt[a + b*x^2]*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(D*x^3*(a + b*x^2)^(3/2))/(6*b) + ((2*C*x^2*(a + b*x^2)^(3/2))/5 + ((5*(2*
b*B - a*D)*x*(a + b*x^2)^(3/2))/(4*b) - ((-8*(5*A*b - 2*a*C)*(a + b*x^2)^(
3/2))/3 + 5*a*(2*b*B - a*D)*((x*Sqrt[a + b*x^2])/2 + (a*ArcTanh[(Sqrt[b]*x
)/Sqrt[a + b*x^2]])/(2*Sqrt[b])))/(4*b))/5)/(2*b)
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

rule 2340
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Simp[1/(b*(m
+ q + 2*p + 1))   Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)
*Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /;
GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ
[Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

## Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.16

| method | result |
|---|---|
| default | $\frac{A(b\,x^2+a)^{\frac{3}{2}}}{3b}+C\left(\frac{x^2(b\,x^2+a)^{\frac{3}{2}}}{5b}-\frac{2a(b\,x^2+a)^{\frac{3}{2}}}{15b^2}\right)+D\left(\frac{x^3(b\,x^2+a)^{\frac{3}{2}}}{6b}-\frac{a\left(\frac{x\left(b\,x^2+a\right)^{\frac{3}{2}}}{4b}-\frac{a\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b}\right.}{2\sqrt{b}}\right.}{4b}\right.}{2b}\right.$ |

input
```
int(x*(b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
1/3*A*(b*x^2+a)^(3/2)/b+C*(1/5*x^2*(b*x^2+a)^(3/2)/b-2/15*a/b^2*(b*x^2+a)^
(3/2))+D*(1/6*x^3*(b*x^2+a)^(3/2)/b-1/2*a/b*(1/4*x*(b*x^2+a)^(3/2)/b-1/4*a
/b*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))))+B
*(1/4*x*(b*x^2+a)^(3/2)/b-1/4*a/b*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(
b^(1/2)*x+(b*x^2+a)^(1/2))))
```

## Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.72

$$\int x\sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$

$$= \left[-\frac{15\,(Da^3-2\,Ba^2b)\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\,(40\,Db^3x^5+48\,Cb^3x^4-32\,Ca^2b+80}{480\,b^3}\right.$$

$$-\frac{15\,(Da^3-2\,Ba^2b)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(40\,Db^3x^5+48\,Cb^3x^4-32\,Ca^2b+80\,Aab^2+10\,(Dab^2+}{240\,b^3}$$

input
```
integrate(x*(b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
[-1/480*(15*(D*a^3 - 2*B*a^2*b)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*s
qrt(b)*x - a) - 2*(40*D*b^3*x^5 + 48*C*b^3*x^4 - 32*C*a^2*b + 80*A*a*b^2 +
 10*(D*a*b^2 + 6*B*b^3)*x^3 + 16*(C*a*b^2 + 5*A*b^3)*x^2 - 15*(D*a^2*b - 2
*B*a*b^2)*x)*sqrt(b*x^2 + a))/b^3, -1/240*(15*(D*a^3 - 2*B*a^2*b)*sqrt(-b)
*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (40*D*b^3*x^5 + 48*C*b^3*x^4 - 32*C*
a^2*b + 80*A*a*b^2 + 10*(D*a*b^2 + 6*B*b^3)*x^3 + 16*(C*a*b^2 + 5*A*b^3)*x
^2 - 15*(D*a^2*b - 2*B*a*b^2)*x)*sqrt(b*x^2 + a))/b^3]
```

## Sympy [A] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.17

$$\int x\sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$= \begin{cases} -\frac{a\left(Ba-\frac{3a\left(Bb+\frac{Da}{6}\right)}{4b}\right)\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right)}{2b}+\sqrt{a+bx^2}\left(\frac{Cx^4}{5}+\frac{Dx^5}{6}+\frac{x^3\left(Bb+\frac{Da}{6}\right)}{4b}+\frac{x^2\left(Ab+\right.}{3b}\right. \\ \sqrt{a}\left(\frac{Ax^2}{2}+\frac{Bx^3}{3}+\frac{Cx^4}{4}+\frac{Dx^5}{5}\right) \end{cases}$$

input
```
integrate(x*(b*x**2+a)**(1/2)*(D*x**3+C*x**2+B*x+A),x)
```

output
```
Piecewise((-a*(B*a - 3*a*(B*b + D*a/6)/(4*b))*Piecewise((log(2*sqrt(b)*sqr
t(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True))/
(2*b) + sqrt(a + b*x**2)*(C*x**4/5 + D*x**5/6 + x**3*(B*b + D*a/6)/(4*b) +
 x**2*(A*b + C*a/5)/(3*b) + x*(B*a - 3*a*(B*b + D*a/6)/(4*b))/(2*b) + (A*a
 - 2*a*(A*b + C*a/5)/(3*b))/b), Ne(b, 0)), (sqrt(a)*(A*x**2/2 + B*x**3/3 +
 C*x**4/4 + D*x**5/5), True))
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.04

$$\begin{aligned}\int x\sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx &= \frac{\left(bx^2+a\right)^{\frac{3}{2}}Dx^3}{6\,b}+\frac{\left(bx^2+a\right)^{\frac{3}{2}}Cx^2}{5\,b}\\ &\quad-\frac{\left(bx^2+a\right)^{\frac{3}{2}}Dax}{8\,b^2}+\frac{\sqrt{bx^2+a}Da^2x}{16\,b^2}\\ &\quad+\frac{\left(bx^2+a\right)^{\frac{3}{2}}Bx}{4\,b}-\frac{\sqrt{bx^2+a}Bax}{8\,b}\\ &\quad+\frac{Da^3\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{5}{2}}}-\frac{Ba^2\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{3}{2}}}\\ &\quad-\frac{2\left(bx^2+a\right)^{\frac{3}{2}}Ca}{15\,b^2}+\frac{\left(bx^2+a\right)^{\frac{3}{2}}A}{3\,b}\end{aligned}$$

input
```
integrate(x*(b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
1/6*(b*x^2 + a)^(3/2)*D*x^3/b + 1/5*(b*x^2 + a)^(3/2)*C*x^2/b - 1/8*(b*x^2
 + a)^(3/2)*D*a*x/b^2 + 1/16*sqrt(b*x^2 + a)*D*a^2*x/b^2 + 1/4*(b*x^2 + a)
^(3/2)*B*x/b - 1/8*sqrt(b*x^2 + a)*B*a*x/b + 1/16*D*a^3*arcsinh(b*x/sqrt(a
*b))/b^(5/2) - 1/8*B*a^2*arcsinh(b*x/sqrt(a*b))/b^(3/2) - 2/15*(b*x^2 + a)
^(3/2)*C*a/b^2 + 1/3*(b*x^2 + a)^(3/2)*A/b
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.89

$$\int x\sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$=\frac{1}{240}\sqrt{bx^2+a}\left(\left(2\left(\left(4\,(5\,Dx+6\,C)x+\frac{5\,(Dab^3+6\,Bb^4)}{b^4}\right)x+\frac{8\,(Cab^3+5\,Ab^4)}{b^4}\right)x-\frac{15\,(Da^2b^2-2\,B\right.\right.$$
$$-\frac{(Da^3-2\,Ba^2b)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{16\,b^{\frac{5}{2}}}$$

input
```
integrate(x*(b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

output
```
1/240*sqrt(b*x^2 + a)*((2*((4*(5*D*x + 6*C)*x + 5*(D*a*b^3 + 6*B*b^4)/b^4)
*x + 8*(C*a*b^3 + 5*A*b^4)/b^4)*x - 15*(D*a^2*b^2 - 2*B*a*b^3)/b^4)*x - 16
*(2*C*a^2*b^2 - 5*A*a*b^3)/b^4) - 1/16*(D*a^3 - 2*B*a^2*b)*log(abs(-sqrt(b
)*x + sqrt(b*x^2 + a)))/b^(5/2)
```

## Mupad [F(-1)]

Timed out.

$$\int x\sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx=\int x\,\sqrt{b\,x^2+a}\,\left(A+B\,x+C\,x^2+x^3\,D\right)\,dx$$

input
```
int(x*(a + b*x^2)^(1/2)*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
int(x*(a + b*x^2)^(1/2)*(A + B*x + C*x^2 + x^3*D), x)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.36

$$\int x\sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$=\frac{80\sqrt{b\,x^2+a}\,a^2b^2-32\sqrt{b\,x^2+a}\,a^2bc-15\sqrt{b\,x^2+a}\,a^2bdx+80\sqrt{b\,x^2+a}\,a\,b^3x^2+30\sqrt{b\,x^2+a}\,a\,b^3x+1}{}$$

input
```
int(x*(b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A),x)
```

output
```
(80*sqrt(a + b*x**2)*a**2*b**2 - 32*sqrt(a + b*x**2)*a**2*b*c - 15*sqrt(a
+ b*x**2)*a**2*b*d*x + 80*sqrt(a + b*x**2)*a*b**3*x**2 + 30*sqrt(a + b*x**
2)*a*b**3*x + 16*sqrt(a + b*x**2)*a*b**2*c*x**2 + 10*sqrt(a + b*x**2)*a*b*
*2*d*x**3 + 60*sqrt(a + b*x**2)*b**4*x**3 + 48*sqrt(a + b*x**2)*b**3*c*x**
4 + 40*sqrt(a + b*x**2)*b**3*d*x**5 + 15*sqrt(b)*log((sqrt(a + b*x**2) + s
qrt(b)*x)/sqrt(a))*a**3*d - 30*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/
sqrt(a))*a**2*b**2)/(240*b**3)
```

# 3.64 $\int \sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)\ dx$



## Optimal result

Integrand size = 27, antiderivative size = 132

$$\int \sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)\ dx = \frac{(4Ab-aC)x\sqrt{a+bx^2}}{8b} + \frac{(bB-aD)\,(a+bx^2)^{3/2}}{3b^2} + \frac{Cx(a+bx^2)^{3/2}}{4b} + \frac{D(a+bx^2)^{5/2}}{5b^2} + \frac{a(4Ab-aC)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{3/2}}$$

output
```
1/8*(4*A*b-C*a)*x*(b*x^2+a)^(1/2)/b+1/3*(B*b-D*a)*(b*x^2+a)^(3/2)/b^2+1/4*
C*x*(b*x^2+a)^(3/2)/b+1/5*D*(b*x^2+a)^(5/2)/b^2+1/8*a*(4*A*b-C*a)*arctanh(
b^(1/2)*x/(b*x^2+a)^(1/2))/b^(3/2)
```

## Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.82

$$\int \sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$=\frac{\sqrt{a+bx^2}(-16a^2D+ab(40B+x(15C+8Dx))+2b^2x(30A+x(20B+3x(5C+4Dx))))+15a\sqrt{b}(-4}{120b^2}$$

input
```
Integrate[Sqrt[a + b*x^2]*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(Sqrt[a + b*x^2]*(-16*a^2*D + a*b*(40*B + x*(15*C + 8*D*x)) + 2*b^2*x*(30*
A + x*(20*B + 3*x*(5*C + 4*D*x)))) + 15*a*Sqrt[b]*(-4*A*b + a*C)*Log[-(Sqr
t[b]*x) + Sqrt[a + b*x^2]])/(120*b^2)
```

## Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2346, 2346, 27, 455, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$\Big\downarrow \text{ 2346}$$
$$\frac{\int \sqrt{bx^2+a}\left(5bCx^2+(5bB-2aD)x+5Ab\right)dx}{5b}+\frac{Dx^2\left(a+bx^2\right)^{3/2}}{5b}$$
$$\Big\downarrow \text{ 2346}$$
$$\frac{\frac{\int b(5(4Ab-aC)+4(5bB-2aD)x)\sqrt{bx^2+a}dx}{4b}+\frac{5}{4}Cx\left(a+bx^2\right)^{3/2}}{5b}+\frac{Dx^2\left(a+bx^2\right)^{3/2}}{5b}$$
$$\Big\downarrow \text{ 27}$$
$$\frac{\frac{1}{4}\int (5(4Ab-aC)+4(5bB-2aD)x)\sqrt{bx^2+a}dx+\frac{5}{4}Cx\left(a+bx^2\right)^{3/2}}{5b}+\frac{Dx^2\left(a+bx^2\right)^{3/2}}{5b}$$

$$\Big\downarrow \ 455$$

$$\frac{\frac{1}{4}\left(5(4Ab-aC)\int \sqrt{bx^2+a}dx+\frac{4(a+bx^2)^{3/2}(5bB-2aD)}{3b}\right)+\frac{5}{4}Cx(a+bx^2)^{3/2}}{5b}+\frac{Dx^2(a+bx^2)^{3/2}}{5b}$$

$$\Big\downarrow \ 211$$

$$\frac{\frac{1}{4}\left(5(4Ab-aC)\left(\frac{1}{2}a\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{4(a+bx^2)^{3/2}(5bB-2aD)}{3b}\right)+\frac{5}{4}Cx(a+bx^2)^{3/2}}{5b}+\frac{Dx^2(a+bx^2)^{3/2}}{5b}$$

$$\Big\downarrow \ 224$$

$$\frac{\frac{1}{4}\left(5(4Ab-aC)\left(\frac{1}{2}a\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{4(a+bx^2)^{3/2}(5bB-2aD)}{3b}\right)+\frac{5}{4}Cx(a+bx^2)^{3/2}}{5b}+\frac{Dx^2(a+bx^2)^{3/2}}{5b}$$

$$\Big\downarrow \ 219$$

$$\frac{\frac{1}{4}\left(5(4Ab-aC)\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{4(a+bx^2)^{3/2}(5bB-2aD)}{3b}\right)+\frac{5}{4}Cx(a+bx^2)^{3/2}}{5b}+\frac{Dx^2(a+bx^2)^{3/2}}{5b}$$

input
```
Int[Sqrt[a + b*x^2]*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(D*x^2*(a + b*x^2)^(3/2))/(5*b) + ((5*C*x*(a + b*x^2)^(3/2))/4 + ((4*(5*b*
B - 2*a*D)*(a + b*x^2)^(3/2))/(3*b) + 5*(4*A*b - a*C)*((x*Sqrt[a + b*x^2])
/2 + (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b])))/4)/(5*b)
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

## Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.12

| method | result |
|---|---|
| default | $A\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)+\frac{B\left(b\,x^2+a\right)^{\frac{3}{2}}}{3b}+C\left(\frac{x\left(b\,x^2+a\right)^{\frac{3}{2}}}{4b}-\frac{a\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)}{4b}\right)$ |

input
```
int((b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
A*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))+1/3*
B*(b*x^2+a)^(3/2)/b+C*(1/4*x*(b*x^2+a)^(3/2)/b-1/4*a/b*(1/2*x*(b*x^2+a)^(1
/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))))+D*(1/5*x^2*(b*x^2+a)^(3/
2)/b-2/15*a/b^2*(b*x^2+a)^(3/2))
```

## Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.75

$$\int \sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$= \left[-\frac{15\,(Ca^2-4\,Aab)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\,(24\,Db^2x^4+30\,Cb^2x^3-16\,Da^2+40\,B}{240\,b^2}\right.$$

input
```
integrate((b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
[-1/240*(15*(C*a^2 - 4*A*a*b)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqr
t(b)*x - a) - 2*(24*D*b^2*x^4 + 30*C*b^2*x^3 - 16*D*a^2 + 40*B*a*b + 8*(D*
a*b + 5*B*b^2)*x^2 + 15*(C*a*b + 4*A*b^2)*x)*sqrt(b*x^2 + a))/b^2, 1/120*(
15*(C*a^2 - 4*A*a*b)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (24*D*b
^2*x^4 + 30*C*b^2*x^3 - 16*D*a^2 + 40*B*a*b + 8*(D*a*b + 5*B*b^2)*x^2 + 15
*(C*a*b + 4*A*b^2)*x)*sqrt(b*x^2 + a))/b^2]
```

## Sympy [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.21

$$\int \sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$= \begin{cases} \sqrt{a+bx^2}\left(\frac{Cx^3}{4}+\frac{Dx^4}{5}+\frac{x^2\left(Bb+\frac{Da}{5}\right)}{3b}+\frac{x\left(Ab+\frac{Ca}{4}\right)}{2b}+\frac{Ba-\frac{2a\left(Bb+\frac{Da}{5}\right)}{3b}}{b}\right)+\left(Aa-\frac{a\left(Ab+\frac{Ca}{4}\right)}{2b}\right)\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+\right.}{\sqrt{b}} \\ \frac{x\log(x)}{\sqrt{bx^2}}\end{cases}\right. \\ \sqrt{a}\left(Ax+\frac{Bx^2}{2}+\frac{Cx^3}{3}+\frac{Dx^4}{4}\right) \end{cases}$$

input
```
integrate((b*x**2+a)**(1/2)*(D*x**3+C*x**2+B*x+A),x)
```

output
```
Piecewise((sqrt(a + b*x**2)*(C*x**3/4 + D*x**4/5 + x**2*(B*b + D*a/5)/(3*b
) + x*(A*b + C*a/4)/(2*b) + (B*a - 2*a*(B*b + D*a/5)/(3*b))/b) + (A*a - a*
(A*b + C*a/4)/(2*b))*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sq
rt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True)), Ne(b, 0)), (sqrt(a)*(A*x
 + B*x**2/2 + C*x**3/3 + D*x**4/4), True))
```

## Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.98

$$\int \sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx = \frac{(bx^2+a)^{\frac{3}{2}}Dx^2}{5\,b}+\frac{1}{2}\sqrt{bx^2+a}Ax+\frac{(bx^2+a)^{\frac{3}{2}}Cx}{4\,b}-\frac{\sqrt{bx^2+a}Cax}{8\,b}-\frac{Ca^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{3}{2}}}+\frac{Aa\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{b}}-\frac{2\,(bx^2+a)^{\frac{3}{2}}Da}{15\,b^2}+\frac{(bx^2+a)^{\frac{3}{2}}B}{3\,b}$$

input
```
integrate((b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
1/5*(b*x^2 + a)^(3/2)*D*x^2/b + 1/2*sqrt(b*x^2 + a)*A*x + 1/4*(b*x^2 + a)^
(3/2)*C*x/b - 1/8*sqrt(b*x^2 + a)*C*a*x/b - 1/8*C*a^2*arcsinh(b*x/sqrt(a*b
))/b^(3/2) + 1/2*A*a*arcsinh(b*x/sqrt(a*b))/sqrt(b) - 2/15*(b*x^2 + a)^(3/
2)*D*a/b^2 + 1/3*(b*x^2 + a)^(3/2)*B/b
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.92

$$\int \sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$= \frac{1}{120}\sqrt{bx^2+a}\left(\left(2\left(3\,(4\,Dx+5\,C)x+\frac{4\,(Dab^2+5\,Bb^3)}{b^3}\right)x+\frac{15\,(Cab^2+4\,Ab^3)}{b^3}\right)x-\frac{8\,(2\,Da^2b-5\,B}{b^3}\right.$$
$$+\frac{(Ca^2-4\,Aab)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{8\,b^{\frac{3}{2}}}$$

input
```
integrate((b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

output
```
1/120*sqrt(b*x^2 + a)*((2*(3*(4*D*x + 5*C)*x + 4*(D*a*b^2 + 5*B*b^3)/b^3)*
x + 15*(C*a*b^2 + 4*A*b^3)/b^3)*x - 8*(2*D*a^2*b - 5*B*a*b^2)/b^3) + 1/8*(
C*a^2 - 4*A*a*b)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2)
```

## Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx=\int \sqrt{b\,x^2+a}\,\left(A+B\,x+C\,x^2+x^3\,D\right)\,dx$$

input
```
int((a + b*x^2)^(1/2)*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
int((a + b*x^2)^(1/2)*(A + B*x + C*x^2 + x^3*D), x)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.39

$$\int \sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$= \frac{-16\sqrt{b\,x^2+a}\,a^2d+60\sqrt{b\,x^2+a}\,a\,b^2x+40\sqrt{b\,x^2+a}\,a\,b^2+15\sqrt{b\,x^2+a}\,abcx+8\sqrt{b\,x^2+a}\,abd\,x^2+40}{}$$

input
```
int((b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A),x)
```

output
```
( - 16*sqrt(a + b*x**2)*a**2*d + 60*sqrt(a + b*x**2)*a*b**2*x + 40*sqrt(a
+ b*x**2)*a*b**2 + 15*sqrt(a + b*x**2)*a*b*c*x + 8*sqrt(a + b*x**2)*a*b*d*
x**2 + 40*sqrt(a + b*x**2)*b**3*x**2 + 30*sqrt(a + b*x**2)*b**2*c*x**3 + 2
4*sqrt(a + b*x**2)*b**2*d*x**4 + 60*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b
)*x)/sqrt(a))*a**2*b - 15*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(
a))*a**2*c)/(120*b**2)
```

# 3.65 $\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{x}\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 138

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{x}\,dx = \frac{(8Ab+(4bB-aD)x)\sqrt{a+bx^2}}{8b} + \frac{C(a+bx^2)^{3/2}}{3b} + \frac{Dx(a+bx^2)^{3/2}}{4b} + \frac{a(4bB-aD)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{3/2}} - \sqrt{a}A\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

output
```
1/8*(8*A*b+(4*B*b-D*a)*x)*(b*x^2+a)^(1/2)/b+1/3*C*(b*x^2+a)^(3/2)/b+1/4*D*
x*(b*x^2+a)^(3/2)/b+1/8*a*(4*B*b-D*a)*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b
^(3/2)-a^(1/2)*A*arctanh((b*x^2+a)^(1/2)/a^(1/2))
```

## Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{x}\,dx$$
$$=\frac{\sqrt{a+bx^2}(24Ab+a(8C+3Dx)+2bx(6B+4Cx+3Dx^2))}{24b}$$
$$+2\sqrt{a}A\operatorname{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{a(-4bB+aD)\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{8b^{3/2}}$$

input
```
Integrate[(Sqrt[a + b*x^2]*(A + B*x + C*x^2 + D*x^3))/x,x]
```

output
```
(Sqrt[a + b*x^2]*(24*A*b + a*(8*C + 3*D*x) + 2*b*x*(6*B + 4*C*x + 3*D*x^2)
))/(24*b) + 2*Sqrt[a]*A*ArcTanh[(Sqrt[b]*x - Sqrt[a + b*x^2])/Sqrt[a]] + (
a*(-4*b*B + a*D)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(8*b^(3/2))
```

## Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.04, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2340, 2340, 27, 535, 538, 224, 219, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)}{x}\,dx$$

$$\Big\downarrow \text{ 2340}$$

$$\frac{\int \frac{\sqrt{bx^2+a}\left(4bCx^2+(4bB-aD)x+4Ab\right)}{x}dx}{4b}+\frac{Dx\left(a+bx^2\right)^{3/2}}{4b}$$

$$\Big\downarrow \text{ 2340}$$

$$\frac{\frac{\int \frac{3b(4Ab+(4bB-aD)x)\sqrt{bx^2+a}}{x}dx}{3b}+\frac{4}{3}C\left(a+bx^2\right)^{3/2}}{4b}+\frac{Dx\left(a+bx^2\right)^{3/2}}{4b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(4Ab+(4bB-aD)x)\sqrt{bx^2+a}}{x}dx+\frac{4}{3}C\left(a+bx^2\right)^{3/2}}{4b}+\frac{Dx\left(a+bx^2\right)^{3/2}}{4b}$$

$$\Big\downarrow \text{535}$$

$$\frac{\frac{1}{2}a\int \frac{8Ab+(4bB-aD)x}{x\sqrt{bx^2+a}}dx+\frac{1}{2}\sqrt{a+bx^2}(x(4bB-aD)+8Ab)+\frac{4}{3}C\left(a+bx^2\right)^{3/2}}{4b}+\frac{Dx\left(a+bx^2\right)^{3/2}}{4b}$$

$$\Big\downarrow \text{538}$$

$$\frac{\frac{1}{2}a\left(8Ab\int \frac{1}{x\sqrt{bx^2+a}}dx+(4bB-aD)\int \frac{1}{\sqrt{bx^2+a}}dx\right)+\frac{1}{2}\sqrt{a+bx^2}(x(4bB-aD)+8Ab)+\frac{4}{3}C\left(a+bx^2\right)^{3/2}}{4b}+\frac{Dx\left(a+bx^2\right)^{3/2}}{4b}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{1}{2}a\left(8Ab\int \frac{1}{x\sqrt{bx^2+a}}dx+(4bB-aD)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}\right)+\frac{1}{2}\sqrt{a+bx^2}(x(4bB-aD)+8Ab)+\frac{4}{3}C\left(a+bx^2\right)^{3/2}}{4b}+\frac{Dx\left(a+bx^2\right)^{3/2}}{4b}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{1}{2}a\left(8Ab\int \frac{1}{x\sqrt{bx^2+a}}dx+\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(4bB-aD)}{\sqrt{b}}\right)+\frac{1}{2}\sqrt{a+bx^2}(x(4bB-aD)+8Ab)+\frac{4}{3}C\left(a+bx^2\right)^{3/2}}{4b}+\frac{Dx\left(a+bx^2\right)^{3/2}}{4b}$$

$$\Big\downarrow \text{243}$$

$$\frac{\frac{1}{2}a\left(4Ab\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2+\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(4bB-aD)}{\sqrt{b}}\right)+\frac{1}{2}\sqrt{a+bx^2}(x(4bB-aD)+8Ab)+\frac{4}{3}C\left(a+bx^2\right)^{3/2}}{4b}+\frac{Dx\left(a+bx^2\right)^{3/2}}{4b}$$

$$\Big\downarrow \; 73$$

$$\frac{\frac{\frac{1}{2}a\left(8A\int\frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}+\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(4bB-aD)}{\sqrt{b}}\right)+\frac{1}{2}\sqrt{a+bx^2}(x(4bB-aD)+8Ab)+\frac{4}{3}C\left(a+bx^2\right)^{3/2}}{4b}}{}$$

$$\frac{Dx\left(a+bx^2\right)^{3/2}}{4b}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{1}{2}a\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(4bB-aD)}{\sqrt{b}}-\frac{8Ab\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}\right)+\frac{1}{2}\sqrt{a+bx^2}(x(4bB-aD)+8Ab)+\frac{4}{3}C\left(a+bx^2\right)^{3/2}}{4b}$$

$$\frac{Dx\left(a+bx^2\right)^{3/2}}{4b}$$

input
```
Int[(Sqrt[a + b*x^2]*(A + B*x + C*x^2 + D*x^3))/x,x]
```

output
```
(D*x*(a + b*x^2)^(3/2))/(4*b) + (((8*A*b + (4*b*B - a*D)*x)*Sqrt[a + b*x^2
])/2 + (4*C*(a + b*x^2)^(3/2))/3 + (a*(((4*b*B - a*D)*ArcTanh[(Sqrt[b]*x)/
Sqrt[a + b*x^2]])/Sqrt[b] - (8*A*b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[
a]))/2)/(4*b)
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 535
```
Int[(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_))/(x_), x_Symbol] :> Sim
p[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^p/(2*p*(2*p + 1))), x] + Simp[a/(2*p
 + 1)   Int[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^(p - 1)/x), x], x] /; Free
Q[{a, b, c, d}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 2340
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Simp[1/(b*(m
+ q + 2*p + 1))   Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)
*Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /;
GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ
[Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

## Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.11

| method | result |
|---|---|
| default | $B\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)+A\left(\sqrt{b\,x^{2}+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)\right)+D\left(\frac{x(b\,x^{2}+a)^{\frac{3}{2}}}{4b}-\right.$ |

input
```
int((b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/x,x,method=_RETURNVERBOSE)
```

output
```
B*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))+A*((
b*x^2+a)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x))+D*(1/4*x*(b*
x^2+a)^(3/2)/b-1/4*a/b*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(
b*x^2+a)^(1/2))))+1/3*C*(b*x^2+a)^(3/2)/b
```

## Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 533, normalized size of antiderivative = 3.86

$$\int\frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{x}\,dx$$

$$=\left[\frac{24\,A\sqrt{a}b^{2}\log\left(-\frac{bx^{2}-2\,\sqrt{bx^{2}+a}\sqrt{a}+2\,a}{x^{2}}\right)-3\,(Da^{2}-4\,Bab)\sqrt{b}\log\left(-2\,bx^{2}-2\,\sqrt{bx^{2}+a}\sqrt{b}x-a\right)+2\,(6\,I}{48\,b^{2}}\right.$$

input
```
integrate((b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/x,x, algorithm="fricas")
```

output
```
[1/48*(24*A*sqrt(a)*b^2*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2
) - 3*(D*a^2 - 4*B*a*b)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x
 - a) + 2*(6*D*b^2*x^3 + 8*C*b^2*x^2 + 8*C*a*b + 24*A*b^2 + 3*(D*a*b + 4*B
*b^2)*x)*sqrt(b*x^2 + a))/b^2, 1/24*(12*A*sqrt(a)*b^2*log(-(b*x^2 - 2*sqrt
(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 3*(D*a^2 - 4*B*a*b)*sqrt(-b)*arctan(sqrt
(-b)*x/sqrt(b*x^2 + a)) + (6*D*b^2*x^3 + 8*C*b^2*x^2 + 8*C*a*b + 24*A*b^2
+ 3*(D*a*b + 4*B*b^2)*x)*sqrt(b*x^2 + a))/b^2, 1/48*(48*A*sqrt(-a)*b^2*arc
tan(sqrt(b*x^2 + a)*sqrt(-a)/a) - 3*(D*a^2 - 4*B*a*b)*sqrt(b)*log(-2*b*x^2
 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(6*D*b^2*x^3 + 8*C*b^2*x^2 + 8*C*a
*b + 24*A*b^2 + 3*(D*a*b + 4*B*b^2)*x)*sqrt(b*x^2 + a))/b^2, 1/24*(24*A*sq
rt(-a)*b^2*arctan(sqrt(b*x^2 + a)*sqrt(-a)/a) + 3*(D*a^2 - 4*B*a*b)*sqrt(-
b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (6*D*b^2*x^3 + 8*C*b^2*x^2 + 8*C*a
*b + 24*A*b^2 + 3*(D*a*b + 4*B*b^2)*x)*sqrt(b*x^2 + a))/b^2]
```

## Sympy [A] (verification not implemented)

Time = 3.42 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.92

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{x}\,dx$$

$$= -A\sqrt{a}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right) + \frac{Aa}{\sqrt{b}x\sqrt{\frac{a}{bx^2}+1}} + \frac{A\sqrt{b}x}{\sqrt{\frac{a}{bx^2}+1}}$$
$$+ B\left(\begin{cases} \frac{a\left(\begin{cases} \frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a \neq 0 \\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise} \end{cases}\right)}{2} + \frac{x\sqrt{a+bx^2}}{2} & \text{for } b \neq 0 \\ \sqrt{a}x & \text{otherwise} \end{cases}\right)$$
$$+ C\left(\begin{cases} \frac{a\sqrt{a+bx^2}}{3b} + \frac{x^2\sqrt{a+bx^2}}{3} & \text{for } b \neq 0 \\ \frac{\sqrt{a}x^2}{2} & \text{otherwise} \end{cases}\right)$$
$$+ D\left(\begin{cases} -\frac{a^2\left(\begin{cases} \frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a \neq 0 \\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise} \end{cases}\right)}{8b} + \frac{ax\sqrt{a+bx^2}}{8b} + \frac{x^3\sqrt{a+bx^2}}{4} & \text{for } b \neq 0 \\ \frac{\sqrt{a}x^3}{3} & \text{otherwise} \end{cases}\right)$$

input
```
integrate((b*x**2+a)**(1/2)*(D*x**3+C*x**2+B*x+A)/x,x)
```

output
```
-A*sqrt(a)*asinh(sqrt(a)/(sqrt(b)*x)) + A*a/(sqrt(b)*x*sqrt(a/(b*x**2) + 1
)) + A*sqrt(b)*x/sqrt(a/(b*x**2) + 1) + B*Piecewise((a*Piecewise((log(2*sq
rt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2)
, True))/2 + x*sqrt(a + b*x**2)/2, Ne(b, 0)), (sqrt(a)*x, True)) + C*Piece
wise((a*sqrt(a + b*x**2)/(3*b) + x**2*sqrt(a + b*x**2)/3, Ne(b, 0)), (sqrt
(a)*x**2/2, True)) + D*Piecewise((-a**2*Piecewise((log(2*sqrt(b)*sqrt(a +
b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True))/(8*b)
+ a*x*sqrt(a + b*x**2)/(8*b) + x**3*sqrt(a + b*x**2)/4, Ne(b, 0)), (sqrt(a
)*x**3/3, True))
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{x}\,dx = \frac{1}{2}\sqrt{bx^2+a}Bx + \frac{(bx^2+a)^{\frac{3}{2}}Dx}{4\,b} - \frac{\sqrt{bx^2+a}Dax}{8\,b} - \frac{Da^2\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{3}{2}}} + \frac{Ba\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{b}} - A\sqrt{a}\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right) + \sqrt{bx^2+a}A + \frac{(bx^2+a)^{\frac{3}{2}}C}{3\,b}$$

input
```
integrate((b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/x,x, algorithm="maxima")
```

output
```
1/2*sqrt(b*x^2 + a)*B*x + 1/4*(b*x^2 + a)^(3/2)*D*x/b - 1/8*sqrt(b*x^2 + a
)*D*a*x/b - 1/8*D*a^2*arcsinh(b*x/sqrt(a*b))/b^(3/2) + 1/2*B*a*arcsinh(b*x
/sqrt(a*b))/sqrt(b) - A*sqrt(a)*arcsinh(a/(sqrt(a*b)*abs(x))) + sqrt(b*x^2
 + a)*A + 1/3*(b*x^2 + a)^(3/2)*C/b
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{x}\,dx = \int \frac{\sqrt{b\,x^2+a}\,(A+B\,x+C\,x^2+x^3\,D)}{x}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(A + B*x + C*x^2 + x^3*D))/x,x)
```

output
```
int(((a + b*x^2)^(1/2)*(A + B*x + C*x^2 + x^3*D))/x, x)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.51

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{x}\,dx$$
$$= \frac{24\sqrt{b\,x^2+a}\,a\,b^2+8\sqrt{b\,x^2+a}\,abc+3\sqrt{b\,x^2+a}\,abdx+12\sqrt{b\,x^2+a}\,b^3x+8\sqrt{b\,x^2+a}\,b^2c\,x^2+6\sqrt{b\,x^2+\cdots}}{\cdots}$$

input
```
int((b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/x,x)
```

output

```
(24*sqrt(a + b*x**2)*a*b**2 + 8*sqrt(a + b*x**2)*a*b*c + 3*sqrt(a + b*x**2
)*a*b*d*x + 12*sqrt(a + b*x**2)*b**3*x + 8*sqrt(a + b*x**2)*b**2*c*x**2 +
6*sqrt(a + b*x**2)*b**2*d*x**3 + 24*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a
) + sqrt(b)*x)/sqrt(a))*a*b**2 - 24*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a
) + sqrt(b)*x)/sqrt(a))*a*b**2 - 3*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)
*x)/sqrt(a))*a**2*d + 12*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a
))*a*b**2)/(24*b**2)
```

# 3.66 $\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{x^2}\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 135

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{x^2}\,dx = \frac{(2aB+(2Ab+aC)x)\sqrt{a+bx^2}}{2a} + \frac{D(a+bx^2)^{3/2}}{3b} - \frac{A(a+bx^2)^{3/2}}{ax} + \frac{(2Ab+aC)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}} - \sqrt{a}B\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

output
```
1/2*(2*B*a+(2*A*b+C*a)*x)*(b*x^2+a)^(1/2)/a+1/3*D*(b*x^2+a)^(3/2)/b-A*(b*x
^2+a)^(3/2)/a/x+1/2*(2*A*b+C*a)*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(1/2)
-a^(1/2)*B*arctanh((b*x^2+a)^(1/2)/a^(1/2))
```

## Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.97

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{x^2}\,dx$$
$$= \frac{\sqrt{a+bx^2}(-6Ab+6bBx+2aDx+3bCx^2+2bDx^3)}{6bx}$$
$$+ \frac{(2Ab+aC)\text{arctanh}\left(\frac{\sqrt{b}x}{-\sqrt{a}+\sqrt{a+bx^2}}\right)}{\sqrt{b}} - \sqrt{a}B\log(x) + \sqrt{a}B\log\left(-\sqrt{a}+\sqrt{a+bx^2}\right)$$

input
```
Integrate[(Sqrt[a + b*x^2]*(A + B*x + C*x^2 + D*x^3))/x^2,x]
```

output
```
(Sqrt[a + b*x^2]*(-6*A*b + 6*b*B*x + 2*a*D*x + 3*b*C*x^2 + 2*b*D*x^3))/(6*
b*x) + ((2*A*b + a*C)*ArcTanh[(Sqrt[b]*x)/(-Sqrt[a] + Sqrt[a + b*x^2]])]/S
qrt[b] - Sqrt[a]*B*Log[x] + Sqrt[a]*B*Log[-Sqrt[a] + Sqrt[a + b*x^2]]
```

## Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.04, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.367$, Rules used = {2338, 25, 2340, 27, 535, 538, 224, 219, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)}{x^2}\,dx$$

$$\Big\downarrow \; 2338$$

$$-\frac{\int -\frac{\sqrt{bx^2+a}\left(aDx^2+(2Ab+aC)x+aB\right)}{x}dx}{a} - \frac{A\left(a+bx^2\right)^{3/2}}{ax}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{\sqrt{bx^2+a}\left(aDx^2+(2Ab+aC)x+aB\right)}{x}dx}{a} - \frac{A\left(a+bx^2\right)^{3/2}}{ax}$$

$$\Big\downarrow \text{ 2340}$$

$$\frac{\frac{\int \frac{3b(aB+(2Ab+aC)x)\sqrt{bx^2+a}}{x}dx}{3b}+\frac{aD(a+bx^2)^{3/2}}{3b}}{a}-\frac{A(a+bx^2)^{3/2}}{ax}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{(aB+(2Ab+aC)x)\sqrt{bx^2+a}}{x}dx+\frac{aD(a+bx^2)^{3/2}}{3b}}{a}-\frac{A(a+bx^2)^{3/2}}{ax}$$

$$\Big\downarrow \text{ 535}$$

$$\frac{\frac{1}{2}a\int \frac{2aB+(2Ab+aC)x}{x\sqrt{bx^2+a}}dx+\frac{1}{2}\sqrt{a+bx^2}(x(aC+2Ab)+2aB)+\frac{aD(a+bx^2)^{3/2}}{3b}}{a}-\frac{A(a+bx^2)^{3/2}}{ax}$$

$$\Big\downarrow \text{ 538}$$

$$\frac{\frac{1}{2}a\left((aC+2Ab)\int \frac{1}{\sqrt{bx^2+a}}dx+2aB\int \frac{1}{x\sqrt{bx^2+a}}dx\right)+\frac{1}{2}\sqrt{a+bx^2}(x(aC+2Ab)+2aB)+\frac{aD(a+bx^2)^{3/2}}{3b}}{a}-\frac{A(a+bx^2)^{3/2}}{ax}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{\frac{1}{2}a\left((aC+2Ab)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+2aB\int \frac{1}{x\sqrt{bx^2+a}}dx\right)+\frac{1}{2}\sqrt{a+bx^2}(x(aC+2Ab)+2aB)+\frac{aD(a+bx^2)^{3/2}}{3b}}{a}-\frac{A(a+bx^2)^{3/2}}{ax}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\frac{1}{2}a\left(2aB\int \frac{1}{x\sqrt{bx^2+a}}dx+\frac{(aC+2Ab)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}\right)+\frac{1}{2}\sqrt{a+bx^2}(x(aC+2Ab)+2aB)+\frac{aD(a+bx^2)^{3/2}}{3b}}{a}-\frac{A(a+bx^2)^{3/2}}{ax}$$

$$\Big\downarrow \text{ 243}$$

$$\frac{\frac{\frac{1}{2}a\left(aB\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2+\frac{(aC+2Ab)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}\right)+\frac{1}{2}\sqrt{a+bx^2}(x(aC+2Ab)+2aB)+\frac{aD\left(a+bx^2\right)^{3/2}}{3b}}{a}-\frac{A\left(a+bx^2\right)^{3/2}}{ax}}{}$$

$$\Big\downarrow \ 73$$

$$\frac{\frac{1}{2}a\left(\frac{2aB\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{b}+\frac{(aC+2Ab)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}\right)+\frac{1}{2}\sqrt{a+bx^2}(x(aC+2Ab)+2aB)+\frac{aD\left(a+bx^2\right)^{3/2}}{3b}}{a}-\frac{A\left(a+bx^2\right)^{3/2}}{ax}$$

$$\Big\downarrow \ 221$$

$$\frac{\frac{1}{2}a\left(\frac{(aC+2Ab)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}-2\sqrt{a}B\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\right)+\frac{1}{2}\sqrt{a+bx^2}(x(aC+2Ab)+2aB)+\frac{aD\left(a+bx^2\right)^{3/2}}{3b}}{a}-\frac{A\left(a+bx^2\right)^{3/2}}{ax}$$

input
```
Int[(Sqrt[a + b*x^2]*(A + B*x + C*x^2 + D*x^3))/x^2,x]
```

output
```
-((A*(a + b*x^2)^(3/2))/(a*x)) + (((2*a*B + (2*A*b + a*C)*x)*Sqrt[a + b*x^
2])/2 + (a*D*(a + b*x^2)^(3/2))/(3*b) + (a*(((2*A*b + a*C)*ArcTanh[(Sqrt[b
]*x)/Sqrt[a + b*x^2]])/Sqrt[b] - 2*Sqrt[a]*B*ArcTanh[Sqrt[a + b*x^2]/Sqrt[
a]]))/2)/a
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 535
```
Int[(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_))/(x_), x_Symbol] :> Sim
p[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^p/(2*p*(2*p + 1))), x] + Simp[a/(2*p
 + 1)   Int[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^(p - 1)/x), x], x] /; Free
Q[{a, b, c, d}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

rule 2340
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Simp[1/(b*(m
+ q + 2*p + 1))   Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)
*Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /;
GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ
[Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

## Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.15

| method | result |
|---|---|
| default | $C\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)+A\left(-\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}}{ax}+\frac{2b\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)}{a}\right)+B\Big(\sqrt{b\,x^2}$ |

input
```
int((b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/x^2,x,method=_RETURNVERBOSE)
```

output
```
C*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))+A*(-
1/a/x*(b*x^2+a)^(3/2)+2*b/a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2
)*x+(b*x^2+a)^(1/2))))+B*((b*x^2+a)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2
+a)^(1/2))/x))+1/3*D*(b*x^2+a)^(3/2)/b
```

## Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 477, normalized size of antiderivative = 3.53

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{x^2}\,dx$$
$$= \left[\frac{6\,B\sqrt{a}bx\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+3\,(Ca+2\,Ab)\sqrt{b}x\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,(2\,Db}{12\,bx}\right.$$

input
```
integrate((b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/x^2,x, algorithm="fricas")
```

output
```
[1/12*(6*B*sqrt(a)*b*x*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2)
 + 3*(C*a + 2*A*b)*sqrt(b)*x*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x -
a) + 2*(2*D*b*x^3 + 3*C*b*x^2 - 6*A*b + 2*(D*a + 3*B*b)*x)*sqrt(b*x^2 + a)
)/(b*x), 1/6*(3*B*sqrt(a)*b*x*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*
a)/x^2) - 3*(C*a + 2*A*b)*sqrt(-b)*x*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) +
(2*D*b*x^3 + 3*C*b*x^2 - 6*A*b + 2*(D*a + 3*B*b)*x)*sqrt(b*x^2 + a))/(b*x)
, 1/12*(12*B*sqrt(-a)*b*x*arctan(sqrt(b*x^2 + a)*sqrt(-a)/a) + 3*(C*a + 2*
A*b)*sqrt(b)*x*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(2*D*b*
x^3 + 3*C*b*x^2 - 6*A*b + 2*(D*a + 3*B*b)*x)*sqrt(b*x^2 + a))/(b*x), 1/6*(
6*B*sqrt(-a)*b*x*arctan(sqrt(b*x^2 + a)*sqrt(-a)/a) - 3*(C*a + 2*A*b)*sqrt
(-b)*x*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (2*D*b*x^3 + 3*C*b*x^2 - 6*A*b
 + 2*(D*a + 3*B*b)*x)*sqrt(b*x^2 + a))/(b*x)]
```

## Sympy [A] (verification not implemented)

Time = 2.66 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.74

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{x^2}\,dx$$
$$= -\frac{A\sqrt{a}}{x\sqrt{1+\frac{bx^2}{a}}} + A\sqrt{b}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right) - \frac{Abx}{\sqrt{a}\sqrt{1+\frac{bx^2}{a}}} - B\sqrt{a}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right) + \frac{Ba}{\sqrt{b}x\sqrt{\frac{a}{bx^2}+1}} + \frac{B\sqrt{b}x}{\sqrt{\frac{a}{bx^2}+1}} + C\left(\begin{cases} \frac{a\left(\begin{cases} \frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a \neq 0 \\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise} \end{cases}\right)}{2} + \frac{x\sqrt{a+bx^2}}{2} & \text{for } b \neq 0 \\ \sqrt{a}x & \text{otherwise} \end{cases}\right) + D\left(\begin{cases} \frac{a\sqrt{a+bx^2}}{3b} + \frac{x^2\sqrt{a+bx^2}}{3} & \text{for } b \neq 0 \\ \frac{\sqrt{a}x^2}{2} & \text{otherwise} \end{cases}\right)$$

input
```
integrate((b*x**2+a)**(1/2)*(D*x**3+C*x**2+B*x+A)/x**2,x)
```

output
```
-A*sqrt(a)/(x*sqrt(1 + b*x**2/a)) + A*sqrt(b)*asinh(sqrt(b)*x/sqrt(a)) - A
*b*x/(sqrt(a)*sqrt(1 + b*x**2/a)) - B*sqrt(a)*asinh(sqrt(a)/(sqrt(b)*x)) +
 B*a/(sqrt(b)*x*sqrt(a/(b*x**2) + 1)) + B*sqrt(b)*x/sqrt(a/(b*x**2) + 1) +
 C*Piecewise((a*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b)
, Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True))/2 + x*sqrt(a + b*x**2)/2, Ne(b
, 0)), (sqrt(a)*x, True)) + D*Piecewise((a*sqrt(a + b*x**2)/(3*b) + x**2*s
qrt(a + b*x**2)/3, Ne(b, 0)), (sqrt(a)*x**2/2, True))
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.76

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{x^2}\,dx = \frac{1}{2}\sqrt{bx^2+a}Cx + \frac{Ca\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{b}} + A\sqrt{b}\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right) - B\sqrt{a}\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right) + \sqrt{bx^2+a}B + \frac{(bx^2+a)^{\frac{3}{2}}D}{3\,b} - \frac{\sqrt{bx^2+a}A}{x}$$

input
```
integrate((b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/x^2,x, algorithm="maxima")
```

output
```
1/2*sqrt(b*x^2 + a)*C*x + 1/2*C*a*arcsinh(b*x/sqrt(a*b))/sqrt(b) + A*sqrt(
b)*arcsinh(b*x/sqrt(a*b)) - B*sqrt(a)*arcsinh(a/(sqrt(a*b)*abs(x))) + sqrt
(b*x^2 + a)*B + 1/3*(b*x^2 + a)^(3/2)*D/b - sqrt(b*x^2 + a)*A/x
```

## Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.99

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{x^2}\,dx = \frac{2\,Ba\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} + \frac{1}{6}\sqrt{bx^2+a}\left((2\,Dx+3\,C)x + \frac{2\,(Da+3\,Bb)}{b}\right) + \frac{2\,Aa\sqrt{b}}{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2 - a} - \frac{(Ca+2\,Ab)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2\sqrt{b}}$$

input
```
integrate((b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/x^2,x, algorithm="giac")
```

output
```
2*B*a*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/sqrt(-a) + 1/6*sqrt(
b*x^2 + a)*((2*D*x + 3*C)*x + 2*(D*a + 3*B*b)/b) + 2*A*a*sqrt(b)/((sqrt(b)
*x - sqrt(b*x^2 + a))^2 - a) - 1/2*(C*a + 2*A*b)*log(abs(-sqrt(b)*x + sqrt
(b*x^2 + a)))/sqrt(b)
```

## Mupad [B] (verification not implemented)

Time = 3.10 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.23

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{x^2}\,dx = B\sqrt{b\,x^2+a} - \frac{A\sqrt{b\,x^2+a}}{x} - B\sqrt{a}\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right) + \frac{C\,x\sqrt{b\,x^2+a}}{2} + \frac{C\,a\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}} + \frac{x^2\sqrt{b\,x^2+a}\,D\,{}_2\mathrm{F}_1\left(-\frac{1}{2},1;\;2;\;-\frac{b\,x^2}{a}\right)}{2\sqrt{\frac{b\,x^2}{a}+1}} - \frac{A\sqrt{b}\,\mathrm{asin}\left(\frac{\sqrt{b}\,x\,1\mathrm{i}}{\sqrt{a}}\right)\sqrt{b\,x^2+a}\,1\mathrm{i}}{\sqrt{a}\sqrt{\frac{b\,x^2}{a}+1}}$$

input
```
int(((a + b*x^2)^(1/2)*(A + B*x + C*x^2 + x^3*D))/x^2,x)
```

output
```
B*(a + b*x^2)^(1/2) - (A*(a + b*x^2)^(1/2))/x - B*a^(1/2)*atanh((a + b*x^2
)^(1/2)/a^(1/2)) + (C*x*(a + b*x^2)^(1/2))/2 + (C*a*log(b^(1/2)*x + (a + b
*x^2)^(1/2)))/(2*b^(1/2)) + (x^2*(a + b*x^2)^(1/2)*D*hypergeom([-1/2, 1],
2, -(b*x^2)/a))/(2*((b*x^2)/a + 1)^(1/2)) - (A*b^(1/2)*asin((b^(1/2)*x*1i)
/a^(1/2))*(a + b*x^2)^(1/2)*1i)/(a^(1/2)*((b*x^2)/a + 1)^(1/2))
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.51

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{x^2}\,dx = \frac{-24\sqrt{b\,x^2+a}\,ab+8\sqrt{b\,x^2+a}\,adx+24\sqrt{b\,x^2+a}\,b^2x+12\sqrt{b\,x^2+a}\,bc\,x^2+8\sqrt{b\,x^2+a}\,bd\,x^3+24\sqrt{a}\,\mathrm{lo}}{}$$

input
```
int((b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/x^2,x)
```

output
```
( - 24*sqrt(a + b*x**2)*a*b + 8*sqrt(a + b*x**2)*a*d*x + 24*sqrt(a + b*x**
2)*b**2*x + 12*sqrt(a + b*x**2)*b*c*x**2 + 8*sqrt(a + b*x**2)*b*d*x**3 + 2
4*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*b**2*x - 2
4*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*b**2*x + 2
4*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a*b*x + 12*sqrt(b)*l
og((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a*c*x - 24*sqrt(b)*a*b*x - 3*sq
rt(b)*a*c*x)/(24*b*x)
```

# 3.67 $\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{x^3}\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 150

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{x^3}\,dx = \frac{(Ab+2aC+(2bB+aD)x)\sqrt{a+bx^2}}{2a} - \frac{A(a+bx^2)^{3/2}}{2ax^2} - \frac{B(a+bx^2)^{3/2}}{ax} + \frac{(2bB+aD)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}} - \frac{(Ab+2aC)\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

output
```
1/2*(A*b+2*C*a+(2*B*b+D*a)*x)*(b*x^2+a)^(1/2)/a-1/2*A*(b*x^2+a)^(3/2)/a/x^
2-B*(b*x^2+a)^(3/2)/a/x+1/2*(2*B*b+D*a)*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))
/b^(1/2)-1/2*(A*b+2*C*a)*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(1/2)
```

## Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{x^3}\,dx = \frac{\sqrt{a+bx^2}(-A-2Bx+2Cx^2+Dx^3)}{2x^2} + \frac{(Ab+2aC)\text{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}} + \frac{(-2bB-aD)\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{2\sqrt{b}}$$

input
```
Integrate[(Sqrt[a + b*x^2]*(A + B*x + C*x^2 + D*x^3))/x^3,x]
```

output
```
(Sqrt[a + b*x^2]*(-A - 2*B*x + 2*C*x^2 + D*x^3))/(2*x^2) + ((A*b + 2*a*C)*
ArcTanh[(Sqrt[b]*x - Sqrt[a + b*x^2])/Sqrt[a]])/Sqrt[a] + ((-2*b*B - a*D)*
Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(2*Sqrt[b])
```

## Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.98, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.433$, Rules used = {2338, 25, 2338, 25, 27, 535, 27, 538, 224, 219, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{x^3}\,dx$$

$$\Big\downarrow \text{2338}$$

$$-\frac{\int -\frac{\sqrt{bx^2+a}(2aDx^2+(Ab+2aC)x+2aB)}{x^2}dx}{2a} - \frac{A(a+bx^2)^{3/2}}{2ax^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\sqrt{bx^2+a}(2aDx^2+(Ab+2aC)x+2aB)}{x^2}dx}{2a} - \frac{A(a+bx^2)^{3/2}}{2ax^2}$$

$$\Big\downarrow \text{ 2338}$$

$$\frac{-\frac{\int -\frac{a(Ab+2aC+2(2bB+aD)x)\sqrt{bx^2+a}}{x}dx}{a}-\frac{2B(a+bx^2)^{3/2}}{x}}{2a}-\frac{A(a+bx^2)^{3/2}}{2ax^2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\frac{\int \frac{a(Ab+2aC+2(2bB+aD)x)\sqrt{bx^2+a}}{x}dx}{a}-\frac{2B(a+bx^2)^{3/2}}{x}}{2a}-\frac{A(a+bx^2)^{3/2}}{2ax^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{(Ab+2aC+2(2bB+aD)x)\sqrt{bx^2+a}}{x}dx-\frac{2B(a+bx^2)^{3/2}}{x}}{2a}-\frac{A(a+bx^2)^{3/2}}{2ax^2}$$

$$\Big\downarrow \text{ 535}$$

$$\frac{\frac{1}{2}a\int \frac{2(Ab+2aC+(2bB+aD)x)}{x\sqrt{bx^2+a}}dx+\sqrt{a+bx^2}(x(aD+2bB)+2aC+Ab)-\frac{2B(a+bx^2)^{3/2}}{x}}{2a}-\frac{A(a+bx^2)^{3/2}}{2ax^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{a\int \frac{Ab+2aC+(2bB+aD)x}{x\sqrt{bx^2+a}}dx+\sqrt{a+bx^2}(x(aD+2bB)+2aC+Ab)-\frac{2B(a+bx^2)^{3/2}}{x}}{2a}-\frac{A(a+bx^2)^{3/2}}{2ax^2}$$

$$\Big\downarrow \text{ 538}$$

$$\frac{a\left((2aC+Ab)\int \frac{1}{x\sqrt{bx^2+a}}dx+(aD+2bB)\int \frac{1}{\sqrt{bx^2+a}}dx\right)+\sqrt{a+bx^2}(x(aD+2bB)+2aC+Ab)-\frac{2B(a+bx^2)^{3/2}}{x}}{2a}\ \frac{A(a+bx^2)^{3/2}}{2ax^2}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{a\left((2aC+Ab)\int \frac{1}{x\sqrt{bx^2+a}}dx+(aD+2bB)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}\right)+\sqrt{a+bx^2}(x(aD+2bB)+2aC+Ab)-\frac{2B(a+}{}}{2a}\ \frac{A(a+bx^2)^{3/2}}{2ax^2}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{a\left((2aC+Ab)\int \frac{1}{x\sqrt{bx^2+a}}dx+\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(aD+2bB)}{\sqrt{b}}\right)+\sqrt{a+bx^2}(x(aD+2bB)+2aC+Ab)-\frac{2B(a+bx^2)}{x}}{2a}$$

$$\frac{A\left(a+bx^2\right)^{3/2}}{2ax^2}$$

$\Big\downarrow$ 243

$$\frac{a\left(\frac{1}{2}(2aC+Ab)\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2+\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(aD+2bB)}{\sqrt{b}}\right)+\sqrt{a+bx^2}(x(aD+2bB)+2aC+Ab)-\frac{2B(a+}{x}}{2a}$$

$$\frac{A\left(a+bx^2\right)^{3/2}}{2ax^2}$$

$\Big\downarrow$ 73

$$\frac{a\left(\frac{(2aC+Ab)\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{b}+\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(aD+2bB)}{\sqrt{b}}\right)+\sqrt{a+bx^2}(x(aD+2bB)+2aC+Ab)-\frac{2B(a+bx^2)^{3/}}{x}}{2a}$$

$$\frac{A\left(a+bx^2\right)^{3/2}}{2ax^2}$$

$\Big\downarrow$ 221

$$\frac{a\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(aD+2bB)}{\sqrt{b}}-\frac{(2aC+Ab)\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}\right)+\sqrt{a+bx^2}(x(aD+2bB)+2aC+Ab)-\frac{2B(a+bx^2}{x}}{2a}$$

$$\frac{A\left(a+bx^2\right)^{3/2}}{2ax^2}$$

input
```
Int[(Sqrt[a + b*x^2]*(A + B*x + C*x^2 + D*x^3))/x^3,x]
```

output
```
-1/2*(A*(a + b*x^2)^(3/2))/(a*x^2) + ((A*b + 2*a*C + (2*b*B + a*D)*x)*Sqrt
[a + b*x^2] - (2*B*(a + b*x^2)^(3/2))/x + a*(((2*b*B + a*D)*ArcTanh[(Sqrt[
b]*x)/Sqrt[a + b*x^2]])/Sqrt[b] - ((A*b + 2*a*C)*ArcTanh[Sqrt[a + b*x^2]/S
qrt[a]])/Sqrt[a]))/(2*a)
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 535
```
Int[(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_))/(x_), x_Symbol] :> Sim
p[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^p/(2*p*(2*p + 1))), x] + Simp[a/(2*p
 + 1)   Int[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^(p - 1)/x), x], x] /; Free
Q[{a, b, c, d}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

## Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.36

| method | result |
|---|---|
| default | $D\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)+A\left(-\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{2a\,x^{2}}+\frac{b\left(\sqrt{b\,x^{2}+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)\right)}{2a}\right)+B\Bigg(-$ |

input
```
int((b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/x^3,x,method=_RETURNVERBOSE)
```

output
```
D*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))+A*(-
1/2/a/x^2*(b*x^2+a)^(3/2)+1/2*b/a*((b*x^2+a)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/
2)*(b*x^2+a)^(1/2))/x)))+B*(-1/a/x*(b*x^2+a)^(3/2)+2*b/a*(1/2*x*(b*x^2+a)^
(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))))+C*((b*x^2+a)^(1/2)-a^(
1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x))
```

## Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 539, normalized size of antiderivative = 3.59

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{x^3}\,dx$$
$$= \left[\frac{(Da^2+2\,Bab)\sqrt{b}x^2\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+(2\,Cab+Ab^2)\sqrt{a}x^2\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2}{x^2}\right.}{4\,abx^2}\right.$$
$$-\frac{2\,(Da^2+2\,Bab)\sqrt{-b}x^2\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(2\,Cab+Ab^2)\sqrt{a}x^2\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)-2\,(Dabx^3}{4\,abx^2}$$
$$-\frac{(Da^2+2\,Bab)\sqrt{-b}x^2\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(2\,Cab+Ab^2)\sqrt{-a}x^2\arctan\left(\frac{\sqrt{bx^2+a}\sqrt{-a}}{a}\right)-(Dabx^3+2\,C}{2\,abx^2}$$

input
```
integrate((b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/x^3,x, algorithm="fricas")
```

output
```
[1/4*((D*a^2 + 2*B*a*b)*sqrt(b)*x^2*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(
b)*x - a) + (2*C*a*b + A*b^2)*sqrt(a)*x^2*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*
sqrt(a) + 2*a)/x^2) + 2*(D*a*b*x^3 + 2*C*a*b*x^2 - 2*B*a*b*x - A*a*b)*sqrt
(b*x^2 + a))/(a*b*x^2), -1/4*(2*(D*a^2 + 2*B*a*b)*sqrt(-b)*x^2*arctan(sqrt
(-b)*x/sqrt(b*x^2 + a)) - (2*C*a*b + A*b^2)*sqrt(a)*x^2*log(-(b*x^2 - 2*sq
rt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(D*a*b*x^3 + 2*C*a*b*x^2 - 2*B*a*b*x
 - A*a*b)*sqrt(b*x^2 + a))/(a*b*x^2), 1/4*(2*(2*C*a*b + A*b^2)*sqrt(-a)*x^
2*arctan(sqrt(b*x^2 + a)*sqrt(-a)/a) + (D*a^2 + 2*B*a*b)*sqrt(b)*x^2*log(-
2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(D*a*b*x^3 + 2*C*a*b*x^2 -
2*B*a*b*x - A*a*b)*sqrt(b*x^2 + a))/(a*b*x^2), -1/2*((D*a^2 + 2*B*a*b)*sqr
t(-b)*x^2*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (2*C*a*b + A*b^2)*sqrt(-a)*
x^2*arctan(sqrt(b*x^2 + a)*sqrt(-a)/a) - (D*a*b*x^3 + 2*C*a*b*x^2 - 2*B*a*
b*x - A*a*b)*sqrt(b*x^2 + a))/(a*b*x^2)]
```

## Sympy [A] (verification not implemented)

Time = 3.40 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.59

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{x^3}\,dx = -\frac{A\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{2x} - \frac{Ab\,\text{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2\sqrt{a}} - \frac{B\sqrt{a}}{x\sqrt{1+\frac{bx^2}{a}}} + B\sqrt{b}\,\text{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right) - \frac{Bbx}{\sqrt{a}\sqrt{1+\frac{bx^2}{a}}} - C\sqrt{a}\,\text{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right) + \frac{Ca}{\sqrt{b}x\sqrt{\frac{a}{bx^2}+1}} + \frac{C\sqrt{b}x}{\sqrt{\frac{a}{bx^2}+1}} + D\left(\begin{cases} \frac{a\left(\begin{cases} \frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0 \\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise} \end{cases}\right)}{2} + \frac{x\sqrt{a+bx^2}}{2} & \text{for } b\neq 0 \\ \sqrt{a}x & \text{otherwise} \end{cases}\right)$$

input

```
integrate((b*x**2+a)**(1/2)*(D*x**3+C*x**2+B*x+A)/x**3,x)
```

output

```
-A*sqrt(b)*sqrt(a/(b*x**2) + 1)/(2*x) - A*b*asinh(sqrt(a)/(sqrt(b)*x))/(2*
sqrt(a)) - B*sqrt(a)/(x*sqrt(1 + b*x**2/a)) + B*sqrt(b)*asinh(sqrt(b)*x/sq
rt(a)) - B*b*x/(sqrt(a)*sqrt(1 + b*x**2/a)) - C*sqrt(a)*asinh(sqrt(a)/(sqr
t(b)*x)) + C*a/(sqrt(b)*x*sqrt(a/(b*x**2) + 1)) + C*sqrt(b)*x/sqrt(a/(b*x*
*2) + 1) + D*Piecewise((a*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*
x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True))/2 + x*sqrt(a + b*x**
2)/2, Ne(b, 0)), (sqrt(a)*x, True))
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{x^3}\,dx = \frac{1}{2}\sqrt{bx^2+a}Dx + \frac{Da\,\text{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{b}} + B\sqrt{b}\,\text{arsinh}\left(\frac{bx}{\sqrt{ab}}\right) - C\sqrt{a}\,\text{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right) - \frac{Ab\,\text{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{2\sqrt{a}} + \sqrt{bx^2+a}C + \frac{\sqrt{bx^2+a}Ab}{2a} - \frac{\sqrt{bx^2+a}B}{x} - \frac{(bx^2+a)^{\frac{3}{2}}A}{2ax^2}$$

input
```
integrate((b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/x^3,x, algorithm="maxima")
```

output
```
1/2*sqrt(b*x^2 + a)*D*x + 1/2*D*a*arcsinh(b*x/sqrt(a*b))/sqrt(b) + B*sqrt(
b)*arcsinh(b*x/sqrt(a*b)) - C*sqrt(a)*arcsinh(a/(sqrt(a*b)*abs(x))) - 1/2*
A*b*arcsinh(a/(sqrt(a*b)*abs(x)))/sqrt(a) + sqrt(b*x^2 + a)*C + 1/2*sqrt(b
*x^2 + a)*A*b/a - sqrt(b*x^2 + a)*B/x - 1/2*(b*x^2 + a)^(3/2)*A/(a*x^2)
```

## Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.29

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{x^3}\,dx = \frac{1}{2}\sqrt{bx^2+a}(Dx+2C) + \frac{(2Ca+Ab)\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{(Da+2Bb)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2\sqrt{b}} + \frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^3Ab+2\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2Ba\sqrt{b}+\left(\sqrt{b}x-\sqrt{bx^2+a}\right)Aab-2Ba^2\sqrt{b}}{\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^2}$$

input
```
integrate((b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/x^3,x, algorithm="giac")
```

output
```
1/2*sqrt(b*x^2 + a)*(D*x + 2*C) + (2*C*a + A*b)*arctan(-(sqrt(b)*x - sqrt(
b*x^2 + a))/sqrt(-a))/sqrt(-a) - 1/2*(D*a + 2*B*b)*log(abs(-sqrt(b)*x + sq
rt(b*x^2 + a)))/sqrt(b) + ((sqrt(b)*x - sqrt(b*x^2 + a))^3*A*b + 2*(sqrt(b
)*x - sqrt(b*x^2 + a))^2*B*a*sqrt(b) + (sqrt(b)*x - sqrt(b*x^2 + a))*A*a*b
 - 2*B*a^2*sqrt(b))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^2
```

## Mupad [B] (verification not implemented)

Time = 3.18 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{x^3}\,dx = C\,\sqrt{b\,x^2+a}-\frac{A\,\sqrt{b\,x^2+a}}{2\,x^2}-\frac{B\,\sqrt{b\,x^2+a}}{x}+\frac{x\,\sqrt{b\,x^2+a}\,D}{2}-C\,\sqrt{a}\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)+\frac{a\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)\,D}{2\,\sqrt{b}}-\frac{A\,b\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2\,\sqrt{a}}-\frac{B\,\sqrt{b}\,\mathrm{asin}\left(\frac{\sqrt{b}\,x\,1\mathrm{i}}{\sqrt{a}}\right)\,\sqrt{b\,x^2+a}\,1\mathrm{i}}{\sqrt{a}\,\sqrt{\frac{b\,x^2}{a}+1}}$$

input
```
int(((a + b*x^2)^(1/2)*(A + B*x + C*x^2 + x^3*D))/x^3,x)
```

output
```
C*(a + b*x^2)^(1/2) - (A*(a + b*x^2)^(1/2))/(2*x^2) - (B*(a + b*x^2)^(1/2)
)/x + (x*(a + b*x^2)^(1/2)*D)/2 - C*a^(1/2)*atanh((a + b*x^2)^(1/2)/a^(1/2
)) + (a*log(b^(1/2)*x + (a + b*x^2)^(1/2))*D)/(2*b^(1/2)) - (A*b*atanh((a
+ b*x^2)^(1/2)/a^(1/2)))/(2*a^(1/2)) - (B*b^(1/2)*asin((b^(1/2)*x*1i)/a^(1
/2))*(a + b*x^2)^(1/2)*1i)/(a^(1/2)*((b*x^2)/a + 1)^(1/2))
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.63

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{x^3}\,dx$$
$$= \frac{-\sqrt{b\,x^2+a}\,ab-2\sqrt{b\,x^2+a}\,b^2x+2\sqrt{b\,x^2+a}\,bc\,x^2+\sqrt{b\,x^2+a}\,bd\,x^3+\sqrt{a}\,\log\left(\frac{\sqrt{b\,x^2+a}-\sqrt{a}+\sqrt{b}\,x}{\sqrt{a}}\right)b^2x^2+$$

input
```
int((b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/x^3,x)
```

output
```
( - sqrt(a + b*x**2)*a*b - 2*sqrt(a + b*x**2)*b**2*x + 2*sqrt(a + b*x**2)*
b*c*x**2 + sqrt(a + b*x**2)*b*d*x**3 + sqrt(a)*log((sqrt(a + b*x**2) - sqr
t(a) + sqrt(b)*x)/sqrt(a))*b**2*x**2 + 2*sqrt(a)*log((sqrt(a + b*x**2) - s
qrt(a) + sqrt(b)*x)/sqrt(a))*b*c*x**2 - sqrt(a)*log((sqrt(a + b*x**2) + sq
rt(a) + sqrt(b)*x)/sqrt(a))*b**2*x**2 - 2*sqrt(a)*log((sqrt(a + b*x**2) +
sqrt(a) + sqrt(b)*x)/sqrt(a))*b*c*x**2 + sqrt(b)*log((sqrt(a + b*x**2) + s
qrt(b)*x)/sqrt(a))*a*d*x**2 + 2*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)
/sqrt(a))*b**2*x**2)/(2*b*x**2)
```

# 3.68 $\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{x^4}\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 143

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{x^4}\,dx = -\frac{(2aC-(bB+2aD)x)\sqrt{a+bx^2}}{2ax} - \frac{A(a+bx^2)^{3/2}}{3ax^3} - \frac{B(a+bx^2)^{3/2}}{2ax^2} + \sqrt{b}C\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right) - \frac{(bB+2aD)\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

output
```
-1/2*(2*C*a-(B*b+2*D*a)*x)*(b*x^2+a)^(1/2)/a/x-1/3*A*(b*x^2+a)^(3/2)/a/x^3
-1/2*B*(b*x^2+a)^(3/2)/a/x^2+b^(1/2)*C*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))-
1/2*(B*b+2*D*a)*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(1/2)
```

## Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.84

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{x^4}\,dx = \frac{\sqrt{a+bx^2}(-2aA-2Abx^2-3ax(B+2x(C-Dx)))}{6ax^3} + \frac{(bB+2aD)\text{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}} - \sqrt{b}C\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)$$

input
```
Integrate[(Sqrt[a + b*x^2]*(A + B*x + C*x^2 + D*x^3))/x^4,x]
```

output
```
(Sqrt[a + b*x^2]*(-2*a*A - 2*A*b*x^2 - 3*a*x*(B + 2*x*(C - D*x))))/(6*a*x^
3) + ((b*B + 2*a*D)*ArcTanh[(Sqrt[b]*x - Sqrt[a + b*x^2])/Sqrt[a]])/Sqrt[a
] - Sqrt[b]*C*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]]
```

## Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.01, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2338, 27, 2338, 25, 27, 536, 538, 224, 219, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{x^4}\,dx$$

$$\Big\downarrow \text{2338}$$

$$-\frac{\int -\frac{3\sqrt{bx^2+a}(aDx^2+aCx+aB)}{x^3}dx}{3a} - \frac{A(a+bx^2)^{3/2}}{3ax^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sqrt{bx^2+a}(aDx^2+aCx+aB)}{x^3}dx}{a} - \frac{A(a+bx^2)^{3/2}}{3ax^3}$$

$$\Big\downarrow \ 2338$$

$$\frac{-\frac{\int -\frac{a(2aC+(bB+2aD)x)\sqrt{bx^2+a}}{x^2}dx}{2a}-\frac{B\left(a+bx^2\right)^{3/2}}{2x^2}}{a}-\frac{A\left(a+bx^2\right)^{3/2}}{3ax^3}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{\int \frac{a(2aC+(bB+2aD)x)\sqrt{bx^2+a}}{x^2}dx}{2a}-\frac{B\left(a+bx^2\right)^{3/2}}{2x^2}}{a}-\frac{A\left(a+bx^2\right)^{3/2}}{3ax^3}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{1}{2}\int \frac{(2aC+(bB+2aD)x)\sqrt{bx^2+a}}{x^2}dx-\frac{B\left(a+bx^2\right)^{3/2}}{2x^2}}{a}-\frac{A\left(a+bx^2\right)^{3/2}}{3ax^3}$$

$$\Big\downarrow \ 536$$

$$\frac{\frac{1}{2}\left(\int \frac{a(bB+2aD)+2abCx}{x\sqrt{bx^2+a}}dx-\frac{\sqrt{a+bx^2}(2aC-x(2aD+bB))}{x}\right)-\frac{B\left(a+bx^2\right)^{3/2}}{2x^2}}{a}-\frac{A\left(a+bx^2\right)^{3/2}}{3ax^3}$$

$$\Big\downarrow \ 538$$

$$\frac{\frac{1}{2}\left(a(2aD+bB)\int \frac{1}{x\sqrt{bx^2+a}}dx+2abC\int \frac{1}{\sqrt{bx^2+a}}dx-\frac{\sqrt{a+bx^2}(2aC-x(2aD+bB))}{x}\right)-\frac{B\left(a+bx^2\right)^{3/2}}{2x^2}}{a}-\frac{A\left(a+bx^2\right)^{3/2}}{3ax^3}$$

$$\Big\downarrow \ 224$$

$$\frac{\frac{1}{2}\left(a(2aD+bB)\int \frac{1}{x\sqrt{bx^2+a}}dx+2abC\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}-\frac{\sqrt{a+bx^2}(2aC-x(2aD+bB))}{x}\right)-\frac{B\left(a+bx^2\right)^{3/2}}{2x^2}}{a}-\frac{A\left(a+bx^2\right)^{3/2}}{3ax^3}$$

$$\Big\downarrow \ 219$$

$$\frac{\frac{1}{2}\left(a(2aD+bB)\int \frac{1}{x\sqrt{bx^2+a}}dx+2a\sqrt{b}C\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)-\frac{\sqrt{a+bx^2}(2aC-x(2aD+bB))}{x}\right)-\frac{B\left(a+bx^2\right)^{3/2}}{2x^2}}{a}-\frac{A\left(a+bx^2\right)^{3/2}}{3ax^3}$$

$$\Big\downarrow \ 243$$

$$\frac{\frac{1}{2}\left(\frac{1}{2}a(2aD+bB)\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2+2a\sqrt{b}C\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)-\frac{\sqrt{a+bx^2}(2aC-x(2aD+bB))}{x}\right)-\frac{B\left(a+bx^2\right)^{3/2}}{2x^2}}{a}-\frac{A\left(a+bx^2\right)^{3/2}}{3ax^3}$$

$\downarrow$ 73

$$\frac{\frac{1}{2}\left(\frac{a(2aD+bB)\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{b}+2a\sqrt{b}C\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)-\frac{\sqrt{a+bx^2}(2aC-x(2aD+bB))}{x}\right)-\frac{B\left(a+bx^2\right)^{3/2}}{2x^2}}{a}-\frac{A\left(a+bx^2\right)^{3/2}}{3ax^3}$$

$\downarrow$ 221

$$\frac{\frac{1}{2}\left(-\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)(2aD+bB)+2a\sqrt{b}C\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)-\frac{\sqrt{a+bx^2}(2aC-x(2aD+bB))}{x}\right)-\frac{B\left(a+bx^2\right)^{3/2}}{2x^2}}{a}-\frac{A\left(a+bx^2\right)^{3/2}}{3ax^3}$$

input
```
Int[(Sqrt[a + b*x^2]*(A + B*x + C*x^2 + D*x^3))/x^4,x]
```

output
```
-1/3*(A*(a + b*x^2)^(3/2))/(a*x^3) + (-1/2*(B*(a + b*x^2)^(3/2))/x^2 + (-(
((2*a*C - (b*B + 2*a*D)*x)*Sqrt[a + b*x^2])/x) + 2*a*Sqrt[b]*C*ArcTanh[(Sq
rt[b]*x)/Sqrt[a + b*x^2]] - Sqrt[a]*(b*B + 2*a*D)*ArcTanh[Sqrt[a + b*x^2]/
Sqrt[a]])/2)/a
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 536
```
Int[(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_))/(x_)^2, x_Symbol] :> S
imp[(-(2*c*p - d*x))*((a + b*x^2)^p/(2*p*x)), x] + Int[(a*d + 2*b*c*p*x)*((
a + b*x^2)^(p - 1)/x), x] /; FreeQ[{a, b, c, d}, x] && GtQ[p, 0] && Integer
Q[2*p]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

## Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.29

| method | result |
|---|---|
| default | $-\frac{A\left(b\,x^2+a\right)^{\frac{3}{2}}}{3a\,x^3}+B\left(-\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}}{2a\,x^2}+\frac{b\left(\sqrt{b\,x^2+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)\right)}{2a}\right)+C\left(-\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}}{ax}+\frac{2b\left(\frac{x\sqrt{b\,x^2+a}}{2}\right.}{}\right.$ |

input
```
int((b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*A*(b*x^2+a)^(3/2)/a/x^3+B*(-1/2/a/x^2*(b*x^2+a)^(3/2)+1/2*b/a*((b*x^2
+a)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)))+C*(-1/a/x*(b*x^2
+a)^(3/2)+2*b/a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a
)^(1/2))))+D*((b*x^2+a)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x
))
```

## Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 494, normalized size of antiderivative = 3.45

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{x^4}\,dx$$

$$= \left[\frac{6\,Ca\sqrt{b}x^3\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+3\,(2\,Da+Bb)\sqrt{a}x^3\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\,(6\,\ldots}{12\,ax^3}\right.$$

$$-\frac{12\,Ca\sqrt{-b}x^3\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-3\,(2\,Da+Bb)\sqrt{a}x^3\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)-2\,(6\,Dax^3-3\,Bax\ldots}{12\,ax^3}$$

$$-\frac{6\,Ca\sqrt{-b}x^3\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-3\,(2\,Da+Bb)\sqrt{-a}x^3\arctan\left(\frac{\sqrt{bx^2+a}\sqrt{-a}}{a}\right)-(6\,Dax^3-3\,Bax-2\,(3\ldots}{6\,ax^3}$$

input
```
integrate((b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/x^4,x, algorithm="fricas")
```

output
```
[1/12*(6*C*a*sqrt(b)*x^3*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) +
 3*(2*D*a + B*b)*sqrt(a)*x^3*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a
)/x^2) + 2*(6*D*a*x^3 - 3*B*a*x - 2*(3*C*a + A*b)*x^2 - 2*A*a)*sqrt(b*x^2
+ a))/(a*x^3), -1/12*(12*C*a*sqrt(-b)*x^3*arctan(sqrt(-b)*x/sqrt(b*x^2 + a
)) - 3*(2*D*a + B*b)*sqrt(a)*x^3*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) +
 2*a)/x^2) - 2*(6*D*a*x^3 - 3*B*a*x - 2*(3*C*a + A*b)*x^2 - 2*A*a)*sqrt(b*
x^2 + a))/(a*x^3), 1/6*(3*C*a*sqrt(b)*x^3*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)
*sqrt(b)*x - a) + 3*(2*D*a + B*b)*sqrt(-a)*x^3*arctan(sqrt(b*x^2 + a)*sqrt
(-a)/a) + (6*D*a*x^3 - 3*B*a*x - 2*(3*C*a + A*b)*x^2 - 2*A*a)*sqrt(b*x^2 +
 a))/(a*x^3), -1/6*(6*C*a*sqrt(-b)*x^3*arctan(sqrt(-b)*x/sqrt(b*x^2 + a))
- 3*(2*D*a + B*b)*sqrt(-a)*x^3*arctan(sqrt(b*x^2 + a)*sqrt(-a)/a) - (6*D*a
*x^3 - 3*B*a*x - 2*(3*C*a + A*b)*x^2 - 2*A*a)*sqrt(b*x^2 + a))/(a*x^3)]
```

## Sympy [A] (verification not implemented)

Time = 3.61 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.51

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{x^4}\,dx = -\frac{A\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{3x^2} - \frac{Ab^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{3a} - \frac{B\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{2x} - \frac{Bb\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2\sqrt{a}} - \frac{C\sqrt{a}}{x\sqrt{1+\frac{bx^2}{a}}} + C\sqrt{b}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right) - \frac{Cbx}{\sqrt{a}\sqrt{1+\frac{bx^2}{a}}} - D\sqrt{a}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right) + \frac{Da}{\sqrt{b}x\sqrt{\frac{a}{bx^2}+1}} + \frac{D\sqrt{b}x}{\sqrt{\frac{a}{bx^2}+1}}$$

input
```
integrate((b*x**2+a)**(1/2)*(D*x**3+C*x**2+B*x+A)/x**4,x)
```

output
```
-A*sqrt(b)*sqrt(a/(b*x**2) + 1)/(3*x**2) - A*b**(3/2)*sqrt(a/(b*x**2) + 1)
/(3*a) - B*sqrt(b)*sqrt(a/(b*x**2) + 1)/(2*x) - B*b*asinh(sqrt(a)/(sqrt(b)
*x))/(2*sqrt(a)) - C*sqrt(a)/(x*sqrt(1 + b*x**2/a)) + C*sqrt(b)*asinh(sqrt
(b)*x/sqrt(a)) - C*b*x/(sqrt(a)*sqrt(1 + b*x**2/a)) - D*sqrt(a)*asinh(sqrt
(a)/(sqrt(b)*x)) + D*a/(sqrt(b)*x*sqrt(a/(b*x**2) + 1)) + D*sqrt(b)*x/sqrt
(a/(b*x**2) + 1)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{x^4}\,dx = C\sqrt{b}\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right) - D\sqrt{a}\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right) - \frac{Bb\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{2\sqrt{a}} + \sqrt{bx^2+a}D + \frac{\sqrt{bx^2+a}Bb}{2a} - \frac{\sqrt{bx^2+a}C}{x} - \frac{(bx^2+a)^{\frac{3}{2}}B}{2ax^2} - \frac{(bx^2+a)^{\frac{3}{2}}A}{3ax^3}$$

input
```
integrate((b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/x^4,x, algorithm="maxima")
```

output
```
C*sqrt(b)*arcsinh(b*x/sqrt(a*b)) - D*sqrt(a)*arcsinh(a/(sqrt(a*b)*abs(x)))
 - 1/2*B*b*arcsinh(a/(sqrt(a*b)*abs(x)))/sqrt(a) + sqrt(b*x^2 + a)*D + 1/2
*sqrt(b*x^2 + a)*B*b/a - sqrt(b*x^2 + a)*C/x - 1/2*(b*x^2 + a)^(3/2)*B/(a*
x^2) - 1/3*(b*x^2 + a)^(3/2)*A/(a*x^3)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 247 vs. $2(117) = 234$.

Time = 0.14 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.73

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{x^4}\,dx$$

$$= -C\sqrt{b}\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)+\sqrt{bx^2+a}D+\frac{(2\,Da+Bb)\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$

$$+\frac{3\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^5Bb+6\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4Ca\sqrt{b}+6\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4Ab^{\frac{3}{2}}-12\left(\sqrt{b}x-\sqrt{\right.}}{3\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^3}$$

input
```
integrate((b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/x^4,x, algorithm="giac")
```

output
```
-C*sqrt(b)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a))) + sqrt(b*x^2 + a)*D + (2
*D*a + B*b)*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/sqrt(-a) + 1/3
*(3*(sqrt(b)*x - sqrt(b*x^2 + a))^5*B*b + 6*(sqrt(b)*x - sqrt(b*x^2 + a))^
4*C*a*sqrt(b) + 6*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*b^(3/2) - 12*(sqrt(b)*
x - sqrt(b*x^2 + a))^2*C*a^2*sqrt(b) - 3*(sqrt(b)*x - sqrt(b*x^2 + a))*B*a
^2*b + 6*C*a^3*sqrt(b) + 2*A*a^2*b^(3/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2
 - a)^3
```

## Mupad [B] (verification not implemented)

Time = 3.35 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{x^4}\,dx = \sqrt{b\,x^2+a}\,D-\sqrt{a}\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)\,D - \frac{B\,\sqrt{b\,x^2+a}}{2\,x^2}-\frac{C\,\sqrt{b\,x^2+a}}{x} - \frac{A\,{\left(b\,x^2+a\right)}^{3/2}}{3\,a\,x^3}-\frac{B\,b\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2\,\sqrt{a}} - \frac{C\,\sqrt{b}\,\mathrm{asin}\left(\frac{\sqrt{b}\,x\,1{}\mathrm{i}}{\sqrt{a}}\right)\,\sqrt{b\,x^2+a}\,1{}\mathrm{i}}{\sqrt{a}\,\sqrt{\frac{b\,x^2}{a}+1}}$$

input
```
int(((a + b*x^2)^(1/2)*(A + B*x + C*x^2 + x^3*D))/x^4,x)
```

output
```
(a + b*x^2)^(1/2)*D - a^(1/2)*atanh((a + b*x^2)^(1/2)/a^(1/2))*D - (B*(a +
 b*x^2)^(1/2))/(2*x^2) - (C*(a + b*x^2)^(1/2))/x - (A*(a + b*x^2)^(3/2))/(
3*a*x^3) - (B*b*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(2*a^(1/2)) - (C*b^(1/2)
*asin((b^(1/2)*x*1i)/a^(1/2))*(a + b*x^2)^(1/2)*1i)/(a^(1/2)*((b*x^2)/a +
1)^(1/2))
```

## Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 400, normalized size of antiderivative = 2.80

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{x^4}\,dx$$

$$= \frac{-4\sqrt{b\,x^2+a}\,a^2-4\sqrt{b\,x^2+a}\,ab\,x^2-6\sqrt{b\,x^2+a}\,abx-12\sqrt{b\,x^2+a}\,ac\,x^2+12\sqrt{b\,x^2+a}\,ad\,x^3+6\sqrt{a}\,\mathrm{lo}}{}$$

input
```
int((b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/x^4,x)
```

output

```
( - 4*sqrt(a + b*x**2)*a**2 - 4*sqrt(a + b*x**2)*a*b*x**2 - 6*sqrt(a + b*x
**2)*a*b*x - 12*sqrt(a + b*x**2)*a*c*x**2 + 12*sqrt(a + b*x**2)*a*d*x**3 +
 6*sqrt(a)*log(( - sqrt(a)*sqrt(a + b*x**2) + sqrt(b)*sqrt(a + b*x**2)*x -
 sqrt(b)*sqrt(a)*x + a + b*x**2)/(sqrt(a)*sqrt(a + b*x**2) + sqrt(b)*sqrt(
a)*x))*a*d*x**3 + 3*sqrt(a)*log(( - sqrt(a)*sqrt(a + b*x**2) + sqrt(b)*sqr
t(a + b*x**2)*x - sqrt(b)*sqrt(a)*x + a + b*x**2)/(sqrt(a)*sqrt(a + b*x**2
) + sqrt(b)*sqrt(a)*x))*b**2*x**3 - 6*sqrt(a)*log((sqrt(a)*sqrt(a + b*x**2
) + sqrt(b)*sqrt(a + b*x**2)*x + sqrt(b)*sqrt(a)*x + a + b*x**2)/(sqrt(a)*
sqrt(a + b*x**2) + sqrt(b)*sqrt(a)*x))*a*d*x**3 - 3*sqrt(a)*log((sqrt(a)*s
qrt(a + b*x**2) + sqrt(b)*sqrt(a + b*x**2)*x + sqrt(b)*sqrt(a)*x + a + b*x
**2)/(sqrt(a)*sqrt(a + b*x**2) + sqrt(b)*sqrt(a)*x))*b**2*x**3 + 12*sqrt(b
)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a*c*x**3 - 4*sqrt(b)*a*b*x**
3 + 4*sqrt(b)*a*c*x**3)/(12*a*x**3)
```

# 3.69 $\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{x^5} dx$



## Optimal result

Integrand size = 30, antiderivative size = 141

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{x^5} dx = \frac{(Ab-4aC-8aDx)\sqrt{a+bx^2}}{8ax^2} - \frac{A(a+bx^2)^{3/2}}{4ax^4} - \frac{B(a+bx^2)^{3/2}}{3ax^3} + \sqrt{b}D\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right) + \frac{b(Ab-4aC)\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{3/2}}$$

output
```
1/8*(-8*D*a*x+A*b-4*C*a)*(b*x^2+a)^(1/2)/a/x^2-1/4*A*(b*x^2+a)^(3/2)/a/x^4
-1/3*B*(b*x^2+a)^(3/2)/a/x^3+b^(1/2)*D*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))+
1/8*b*(A*b-4*C*a)*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(3/2)
```

## Mathematica [A] (verified)

Time = 1.10 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{x^5}\,dx$$
$$= -\frac{\sqrt{a+bx^2}(6aA+bx^2(3A+8Bx)+4ax(2B+3x(C+2Dx)))}{24ax^4} + \frac{b(-Ab+4aC)\operatorname{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right)}{4a^{3/2}} - \sqrt{b}D\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)$$

input
```
Integrate[(Sqrt[a + b*x^2]*(A + B*x + C*x^2 + D*x^3))/x^5,x]
```

output
```
-1/24*(Sqrt[a + b*x^2]*(6*a*A + b*x^2*(3*A + 8*B*x) + 4*a*x*(2*B + 3*x*(C
+ 2*D*x))))/(a*x^4) + (b*(-(A*b) + 4*a*C)*ArcTanh[(Sqrt[b]*x - Sqrt[a + b*
x^2])/Sqrt[a]])/(4*a^(3/2)) - Sqrt[b]*D*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]
]
```

## Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.05, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2338, 25, 2338, 27, 537, 25, 538, 224, 219, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)}{x^5}\,dx$$

$$\Big\downarrow \text{2338}$$

$$-\frac{\int -\frac{\sqrt{bx^2+a}(4aDx^2-(Ab-4aC)x+4aB)}{x^4}dx}{4a} - \frac{A\left(a+bx^2\right)^{3/2}}{4ax^4}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\sqrt{bx^2+a}(4aDx^2-(Ab-4aC)x+4aB)}{x^4}dx}{4a} - \frac{A\left(a+bx^2\right)^{3/2}}{4ax^4}$$

$$\Big\downarrow \text{2338}$$

$$\frac{-\frac{\int \frac{3a(Ab-4aC-4aDx)\sqrt{bx^2+a}}{x^3}dx}{3a}-\frac{4B\left(a+bx^2\right)^{3/2}}{3x^3}}{4a}-\frac{A\left(a+bx^2\right)^{3/2}}{4ax^4}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\int \frac{(Ab-4aC-4aDx)\sqrt{bx^2+a}}{x^3}dx-\frac{4B\left(a+bx^2\right)^{3/2}}{3x^3}}{4a}-\frac{A\left(a+bx^2\right)^{3/2}}{4ax^4}$$

$$\Big\downarrow \text{537}$$

$$\frac{\frac{1}{2}b\int -\frac{Ab-4aC-8aDx}{x\sqrt{bx^2+a}}dx+\frac{\sqrt{a+bx^2}(-4aC-8aDx+Ab)}{2x^2}-\frac{4B\left(a+bx^2\right)^{3/2}}{3x^3}}{4a}-\frac{A\left(a+bx^2\right)^{3/2}}{4ax^4}$$

$$\Big\downarrow \text{25}$$

$$\frac{-\frac{1}{2}b\int \frac{Ab-4aC-8aDx}{x\sqrt{bx^2+a}}dx+\frac{\sqrt{a+bx^2}(-4aC-8aDx+Ab)}{2x^2}-\frac{4B\left(a+bx^2\right)^{3/2}}{3x^3}}{4a}-\frac{A\left(a+bx^2\right)^{3/2}}{4ax^4}$$

$$\Big\downarrow \text{538}$$

$$\frac{-\frac{1}{2}b\left((Ab-4aC)\int \frac{1}{x\sqrt{bx^2+a}}dx-8aD\int \frac{1}{\sqrt{bx^2+a}}dx\right)+\frac{\sqrt{a+bx^2}(-4aC-8aDx+Ab)}{2x^2}-\frac{4B\left(a+bx^2\right)^{3/2}}{3x^3}}{4a}-\frac{A\left(a+bx^2\right)^{3/2}}{4ax^4}$$

$$\Big\downarrow \text{224}$$

$$\frac{-\frac{1}{2}b\left((Ab-4aC)\int \frac{1}{x\sqrt{bx^2+a}}dx-8aD\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}\right)+\frac{\sqrt{a+bx^2}(-4aC-8aDx+Ab)}{2x^2}-\frac{4B\left(a+bx^2\right)^{3/2}}{3x^3}}{4a}-\frac{A\left(a+bx^2\right)^{3/2}}{4ax^4}$$

$$\Big\downarrow \text{219}$$

$$\frac{-\frac{1}{2}b\left((Ab-4aC)\int \frac{1}{x\sqrt{bx^2+a}}dx-\frac{8aD\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}\right)+\frac{\sqrt{a+bx^2}(-4aC-8aDx+Ab)}{2x^2}-\frac{4B\left(a+bx^2\right)^{3/2}}{3x^3}}{4a}-\frac{A\left(a+bx^2\right)^{3/2}}{4ax^4}$$

$$\Big\downarrow \text{243}$$

$$\frac{-\frac{1}{2}b\left(\frac{1}{2}(Ab-4aC)\int\frac{1}{x^2\sqrt{bx^2+a}}dx^2-\frac{8aD\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}\right)+\frac{\sqrt{a+bx^2}(-4aC-8aDx+Ab)}{2x^2}-\frac{4B(a+bx^2)^{3/2}}{3x^3}}{4a}-\frac{A\left(a+bx^2\right)^{3/2}}{4ax^4}$$

$$\Big\downarrow \ 73$$

$$\frac{-\frac{1}{2}b\left(\frac{(Ab-4aC)\int\frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{b}-\frac{8aD\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}\right)+\frac{\sqrt{a+bx^2}(-4aC-8aDx+Ab)}{2x^2}-\frac{4B(a+bx^2)^{3/2}}{3x^3}}{4a}-\frac{A\left(a+bx^2\right)^{3/2}}{4ax^4}$$

$$\Big\downarrow \ 221$$

$$\frac{-\frac{1}{2}b\left(-\frac{(Ab-4aC)\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{8aD\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}\right)+\frac{\sqrt{a+bx^2}(-4aC-8aDx+Ab)}{2x^2}-\frac{4B(a+bx^2)^{3/2}}{3x^3}}{4a}-\frac{A\left(a+bx^2\right)^{3/2}}{4ax^4}$$

input
```
Int[(Sqrt[a + b*x^2]*(A + B*x + C*x^2 + D*x^3))/x^5,x]
```

output
```
-1/4*(A*(a + b*x^2)^(3/2))/(a*x^4) + (((A*b - 4*a*C - 8*a*D*x)*Sqrt[a + b*
x^2])/(2*x^2) - (4*B*(a + b*x^2)^(3/2))/(3*x^3) - (b*((-8*a*D*ArcTanh[(Sqr
t[b]*x)/Sqrt[a + b*x^2]])/Sqrt[b] - ((A*b - 4*a*C)*ArcTanh[Sqrt[a + b*x^2]
/Sqrt[a]])/Sqrt[a]))/2)/(4*a)
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 537
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[x^(m + 1)*(c*(m + 2) + d*(m + 1)*x)*((a + b*x^2)^p/((m + 1)*(m + 2))),
 x] - Simp[2*b*(p/((m + 1)*(m + 2)))   Int[x^(m + 2)*(c*(m + 2) + d*(m + 1)
*x)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[m, -2] &&
 GtQ[p, 0] &&  !ILtQ[m + 2*p + 3, 0] && IntegerQ[2*p]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 232 vs. $2(115) = 230$.

Time = 0.57 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.65

| method | result |
|---|---|
| default | $A\left(-\frac{(b\,x^2+a)^{\frac{3}{2}}}{4a\,x^4}-\frac{b\left(-\frac{(b\,x^2+a)^{\frac{3}{2}}}{2a\,x^2}+\frac{b\left(\sqrt{b\,x^2+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)\right)}{2a}\right)}{4a}\right)-\frac{B(b\,x^2+a)^{\frac{3}{2}}}{3a\,x^3}+C\left(-\frac{(b\,x^2+a)^{\frac{3}{2}}}{2a\,x^2}+\right.$ |

input
```
int((b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/x^5,x,method=_RETURNVERBOSE)
```

output
```
A*(-1/4/a/x^4*(b*x^2+a)^(3/2)-1/4*b/a*(-1/2/a/x^2*(b*x^2+a)^(3/2)+1/2*b/a*
((b*x^2+a)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x))))-1/3*B*(b
*x^2+a)^(3/2)/a/x^3+C*(-1/2/a/x^2*(b*x^2+a)^(3/2)+1/2*b/a*((b*x^2+a)^(1/2)
-a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)))+D*(-1/a/x*(b*x^2+a)^(3/2)
+2*b/a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))
)
```

## Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 582, normalized size of antiderivative = 4.13

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{x^5}\,dx$$

$$= \left[\frac{24\,Da^2\sqrt{b}x^4\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-3\,(4\,Cab-Ab^2)\sqrt{a}x^4\log\left(-\frac{bx^2+2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)-2\cdots}{48\,a^2x^4}\right.$$

$$-\frac{48\,Da^2\sqrt{-b}x^4\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)+3\,(4\,Cab-Ab^2)\sqrt{a}x^4\log\left(-\frac{bx^2+2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\,(8\,Ba^2x+8\,(3\cdots}{48\,a^2x^4}$$

$$-\frac{24\,Da^2\sqrt{-b}x^4\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-3\,(4\,Cab-Ab^2)\sqrt{-a}x^4\arctan\left(\frac{\sqrt{bx^2+a}\sqrt{-a}}{a}\right)+(8\,Ba^2x+8\,(3\,Da^2\cdots}{24\,a^2x^4}$$

input
```
integrate((b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/x^5,x, algorithm="fricas")
```

output
```
[1/48*(24*D*a^2*sqrt(b)*x^4*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a
) - 3*(4*C*a*b - A*b^2)*sqrt(a)*x^4*log(-(b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(a
) + 2*a)/x^2) - 2*(8*B*a^2*x + 8*(3*D*a^2 + B*a*b)*x^3 + 6*A*a^2 + 3*(4*C*
a^2 + A*a*b)*x^2)*sqrt(b*x^2 + a))/(a^2*x^4), -1/48*(48*D*a^2*sqrt(-b)*x^4
*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + 3*(4*C*a*b - A*b^2)*sqrt(a)*x^4*log(
-(b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(8*B*a^2*x + 8*(3*D*a^
2 + B*a*b)*x^3 + 6*A*a^2 + 3*(4*C*a^2 + A*a*b)*x^2)*sqrt(b*x^2 + a))/(a^2*
x^4), 1/24*(12*D*a^2*sqrt(b)*x^4*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*
x - a) + 3*(4*C*a*b - A*b^2)*sqrt(-a)*x^4*arctan(sqrt(b*x^2 + a)*sqrt(-a)/
a) - (8*B*a^2*x + 8*(3*D*a^2 + B*a*b)*x^3 + 6*A*a^2 + 3*(4*C*a^2 + A*a*b)*
x^2)*sqrt(b*x^2 + a))/(a^2*x^4), -1/24*(24*D*a^2*sqrt(-b)*x^4*arctan(sqrt(
-b)*x/sqrt(b*x^2 + a)) - 3*(4*C*a*b - A*b^2)*sqrt(-a)*x^4*arctan(sqrt(b*x^
2 + a)*sqrt(-a)/a) + (8*B*a^2*x + 8*(3*D*a^2 + B*a*b)*x^3 + 6*A*a^2 + 3*(4
*C*a^2 + A*a*b)*x^2)*sqrt(b*x^2 + a))/(a^2*x^4)]
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(126) = 252$.

Time = 4.81 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.79

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{x^5}\,dx = -\frac{Aa}{4\sqrt{b}x^5\sqrt{\frac{a}{bx^2}+1}} - \frac{3A\sqrt{b}}{8x^3\sqrt{\frac{a}{bx^2}+1}} - \frac{Ab^{\frac{3}{2}}}{8ax\sqrt{\frac{a}{bx^2}+1}} + \frac{Ab^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{8a^{\frac{3}{2}}} - \frac{B\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{3x^2} - \frac{Bb^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{3a} - \frac{C\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{2x} - \frac{Cb\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2\sqrt{a}} - \frac{D\sqrt{a}}{x\sqrt{1+\frac{bx^2}{a}}} + D\sqrt{b}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right) - \frac{Dbx}{\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

input
```
integrate((b*x**2+a)**(1/2)*(D*x**3+C*x**2+B*x+A)/x**5,x)
```

output
```
-A*a/(4*sqrt(b)*x**5*sqrt(a/(b*x**2) + 1)) - 3*A*sqrt(b)/(8*x**3*sqrt(a/(b
*x**2) + 1)) - A*b**(3/2)/(8*a*x*sqrt(a/(b*x**2) + 1)) + A*b**2*asinh(sqrt
(a)/(sqrt(b)*x))/(8*a**(3/2)) - B*sqrt(b)*sqrt(a/(b*x**2) + 1)/(3*x**2) -
B*b**(3/2)*sqrt(a/(b*x**2) + 1)/(3*a) - C*sqrt(b)*sqrt(a/(b*x**2) + 1)/(2*
x) - C*b*asinh(sqrt(a)/(sqrt(b)*x))/(2*sqrt(a)) - D*sqrt(a)/(x*sqrt(1 + b*
x**2/a)) + D*sqrt(b)*asinh(sqrt(b)*x/sqrt(a)) - D*b*x/(sqrt(a)*sqrt(1 + b*
x**2/a))
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.26

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{x^5}\,dx = D\sqrt{b}\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right) - \frac{Cb\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{2\sqrt{a}} + \frac{Ab^2\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{8a^{\frac{3}{2}}} + \frac{\sqrt{bx^2+a}Cb}{2a} - \frac{\sqrt{bx^2+a}Ab^2}{8a^2} - \frac{\sqrt{bx^2+a}D}{x} - \frac{(bx^2+a)^{\frac{3}{2}}C}{2ax^2} + \frac{(bx^2+a)^{\frac{3}{2}}Ab}{8a^2x^2} - \frac{(bx^2+a)^{\frac{3}{2}}B}{3ax^3} - \frac{(bx^2+a)^{\frac{3}{2}}A}{4ax^4}$$

input
```
integrate((b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/x^5,x, algorithm="maxima")
```

output
```
D*sqrt(b)*arcsinh(b*x/sqrt(a*b)) - 1/2*C*b*arcsinh(a/(sqrt(a*b)*abs(x)))/s
qrt(a) + 1/8*A*b^2*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2) + 1/2*sqrt(b*x^2
+ a)*C*b/a - 1/8*sqrt(b*x^2 + a)*A*b^2/a^2 - sqrt(b*x^2 + a)*D/x - 1/2*(b*
x^2 + a)^(3/2)*C/(a*x^2) + 1/8*(b*x^2 + a)^(3/2)*A*b/(a^2*x^2) - 1/3*(b*x^
2 + a)^(3/2)*B/(a*x^3) - 1/4*(b*x^2 + a)^(3/2)*A/(a*x^4)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 492 vs. 2(117) = 234.

Time = 0.14 (sec) , antiderivative size = 492, normalized size of antiderivative = 3.49

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{x^5}\,dx$$

$$= -D\sqrt{b}\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right) + \frac{(4Cab-Ab^2)\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{4\sqrt{-a}a} + \frac{12\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^7Cab+3\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^7Ab^2+24\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6Da^2\sqrt{b}+24\left(\sqrt{b}x\right.}{}$$

input
```
integrate((b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/x^5,x, algorithm="giac")
```

output
```
-D*sqrt(b)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a))) + 1/4*(4*C*a*b - A*b^2)*
arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/(sqrt(-a)*a) + 1/12*(12*(s
qrt(b)*x - sqrt(b*x^2 + a))^7*C*a*b + 3*(sqrt(b)*x - sqrt(b*x^2 + a))^7*A*
b^2 + 24*(sqrt(b)*x - sqrt(b*x^2 + a))^6*D*a^2*sqrt(b) + 24*(sqrt(b)*x - s
qrt(b*x^2 + a))^6*B*a*b^(3/2) - 12*(sqrt(b)*x - sqrt(b*x^2 + a))^5*C*a^2*b
 + 21*(sqrt(b)*x - sqrt(b*x^2 + a))^5*A*a*b^2 - 72*(sqrt(b)*x - sqrt(b*x^2
 + a))^4*D*a^3*sqrt(b) - 24*(sqrt(b)*x - sqrt(b*x^2 + a))^4*B*a^2*b^(3/2)
- 12*(sqrt(b)*x - sqrt(b*x^2 + a))^3*C*a^3*b + 21*(sqrt(b)*x - sqrt(b*x^2
+ a))^3*A*a^2*b^2 + 72*(sqrt(b)*x - sqrt(b*x^2 + a))^2*D*a^4*sqrt(b) + 8*(
sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a^3*b^(3/2) + 12*(sqrt(b)*x - sqrt(b*x^2
+ a))*C*a^4*b + 3*(sqrt(b)*x - sqrt(b*x^2 + a))*A*a^3*b^2 - 24*D*a^5*sqrt(
b) - 8*B*a^4*b^(3/2))/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^4*a)
```

## Mupad [B] (verification not implemented)

Time = 3.52 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.19

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{x^5}\,dx = \frac{A\,b^2\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{8\,a^{3/2}} - \frac{C\,\sqrt{b\,x^2+a}}{2\,x^2} - \frac{\sqrt{b\,x^2+a}\,D}{x} - \frac{A\,\sqrt{b\,x^2+a}}{8\,x^4} - \frac{A\,{\left(b\,x^2+a\right)}^{3/2}}{8\,a\,x^4} - \frac{B\,{\left(b\,x^2+a\right)}^{3/2}}{3\,a\,x^3} - \frac{C\,b\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2\,\sqrt{a}} - \frac{\sqrt{b}\,\mathrm{asin}\left(\frac{\sqrt{b}\,x\,1{}\mathrm{i}}{\sqrt{a}}\right)\,\sqrt{b\,x^2+a}\,D\,1{}\mathrm{i}}{\sqrt{a}\,\sqrt{\frac{b\,x^2}{a}+1}}$$

input
```
int(((a + b*x^2)^(1/2)*(A + B*x + C*x^2 + x^3*D))/x^5,x)
```

output
```
(A*b^2*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(8*a^(3/2)) - (C*(a + b*x^2)^(1/2
))/(2*x^2) - ((a + b*x^2)^(1/2)*D)/x - (A*(a + b*x^2)^(1/2))/(8*x^4) - (A*
(a + b*x^2)^(3/2))/(8*a*x^4) - (B*(a + b*x^2)^(3/2))/(3*a*x^3) - (C*b*atan
h((a + b*x^2)^(1/2)/a^(1/2)))/(2*a^(1/2)) - (b^(1/2)*asin((b^(1/2)*x*1i)/a
^(1/2))*(a + b*x^2)^(1/2)*D*1i)/(a^(1/2)*((b*x^2)/a + 1)^(1/2))
```

## Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.91

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{x^5}\,dx$$
$$= \frac{-6\sqrt{b\,x^2+a}\,a^2-3\sqrt{b\,x^2+a}\,ab\,x^2-8\sqrt{b\,x^2+a}\,abx-12\sqrt{b\,x^2+a}\,ac\,x^2-24\sqrt{b\,x^2+a}\,ad\,x^3-8\sqrt{b\,x^2}}{}$$

input
```
int((b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/x^5,x)
```

output
```
( - 6*sqrt(a + b*x**2)*a**2 - 3*sqrt(a + b*x**2)*a*b*x**2 - 8*sqrt(a + b*x
**2)*a*b*x - 12*sqrt(a + b*x**2)*a*c*x**2 - 24*sqrt(a + b*x**2)*a*d*x**3 -
 8*sqrt(a + b*x**2)*b**2*x**3 - 3*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a)
+ sqrt(b)*x)/sqrt(a))*b**2*x**4 + 12*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(
a) + sqrt(b)*x)/sqrt(a))*b*c*x**4 + 3*sqrt(a)*log((sqrt(a + b*x**2) + sqrt
(a) + sqrt(b)*x)/sqrt(a))*b**2*x**4 - 12*sqrt(a)*log((sqrt(a + b*x**2) + s
qrt(a) + sqrt(b)*x)/sqrt(a))*b*c*x**4 + 24*sqrt(b)*log((sqrt(a + b*x**2) +
 sqrt(b)*x)/sqrt(a))*a*d*x**4 + 12*sqrt(b)*a*d*x**4 - 4*sqrt(b)*b**2*x**4)
/(24*a*x**4)
```

# 3.70 $\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{x^6} dx$



## Optimal result

Integrand size = 30, antiderivative size = 140

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{x^6} dx = \frac{(bB-4aD)\sqrt{a+bx^2}}{8ax^2} - \frac{A(a+bx^2)^{3/2}}{5ax^5} - \frac{B(a+bx^2)^{3/2}}{4ax^4} + \frac{(2Ab-5aC)(a+bx^2)^{3/2}}{15a^2x^3} + \frac{b(bB-4aD)\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{3/2}}$$

output

```
1/8*(B*b-4*D*a)*(b*x^2+a)^(1/2)/a/x^2-1/5*A*(b*x^2+a)^(3/2)/a/x^5-1/4*B*(b
*x^2+a)^(3/2)/a/x^4+1/15*(2*A*b-5*C*a)*(b*x^2+a)^(3/2)/a^2/x^3+1/8*b*(B*b-
4*D*a)*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(3/2)
```

## Mathematica [A] (verified)

Time = 1.09 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{x^6}\,dx = -\frac{\sqrt{a+bx^2}(-16Ab^2x^4+abx^2(8A+5x(3B+8Cx))+a^2(24A+10x(3B+4Cx+6Dx^2)))}{120a^2x^5} + \frac{b(-bB+4aD)\text{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right)}{4a^{3/2}}$$

input
```
Integrate[(Sqrt[a + b*x^2]*(A + B*x + C*x^2 + D*x^3))/x^6,x]
```

output
```
-1/120*(Sqrt[a + b*x^2]*(-16*A*b^2*x^4 + a*b*x^2*(8*A + 5*x*(3*B + 8*C*x))
 + a^2*(24*A + 10*x*(3*B + 4*C*x + 6*D*x^2))))/(a^2*x^5) + (b*(-(b*B) + 4*
a*D)*ArcTanh[(Sqrt[b]*x - Sqrt[a + b*x^2])/Sqrt[a]])/(4*a^(3/2))
```

## Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {2338, 25, 2338, 27, 534, 243, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{x^6}\,dx$$

$$\Big\downarrow \text{2338}$$

$$-\frac{\int -\frac{\sqrt{bx^2+a}(5aDx^2-(2Ab-5aC)x+5aB)}{x^5}dx}{5a}-\frac{A(a+bx^2)^{3/2}}{5ax^5}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\sqrt{bx^2+a}(5aDx^2-(2Ab-5aC)x+5aB)}{x^5}dx}{5a}-\frac{A(a+bx^2)^{3/2}}{5ax^5}$$

$$\Big\downarrow \text{2338}$$

$$\frac{-\frac{\int \frac{a(4(2Ab-5aC)+5(bB-4aD)x)\sqrt{bx^2+a}}{x^4}dx}{4a}-\frac{5B(a+bx^2)^{3/2}}{4x^4}}{5a}-\frac{A(a+bx^2)^{3/2}}{5ax^5}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{1}{4}\int \frac{(4(2Ab-5aC)+5(bB-4aD)x)\sqrt{bx^2+a}}{x^4}dx-\frac{5B(a+bx^2)^{3/2}}{4x^4}}{5a}-\frac{A(a+bx^2)^{3/2}}{5ax^5}$$

$$\Big\downarrow \text{534}$$

$$\frac{\frac{1}{4}\left(\frac{4(a+bx^2)^{3/2}(2Ab-5aC)}{3ax^3}-5(bB-4aD)\int \frac{\sqrt{bx^2+a}}{x^3}dx\right)-\frac{5B(a+bx^2)^{3/2}}{4x^4}}{5a}-\frac{A(a+bx^2)^{3/2}}{5ax^5}$$

$$\Big\downarrow \text{243}$$

$$\frac{\frac{1}{4}\left(\frac{4(a+bx^2)^{3/2}(2Ab-5aC)}{3ax^3}-\frac{5}{2}(bB-4aD)\int \frac{\sqrt{bx^2+a}}{x^4}dx^2\right)-\frac{5B(a+bx^2)^{3/2}}{4x^4}}{5a}-\frac{A(a+bx^2)^{3/2}}{5ax^5}$$

$$\Big\downarrow \text{51}$$

$$\frac{\frac{1}{4}\left(\frac{4(a+bx^2)^{3/2}(2Ab-5aC)}{3ax^3}-\frac{5}{2}(bB-4aD)\left(\frac{1}{2}b\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2-\frac{\sqrt{a+bx^2}}{x^2}\right)\right)-\frac{5B(a+bx^2)^{3/2}}{4x^4}}{5a}-\frac{A(a+bx^2)^{3/2}}{5ax^5}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{1}{4}\left(\frac{4(a+bx^2)^{3/2}(2Ab-5aC)}{3ax^3}-\frac{5}{2}(bB-4aD)\left(\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}-\frac{\sqrt{a+bx^2}}{x^2}\right)\right)-\frac{5B(a+bx^2)^{3/2}}{4x^4}}{5a}-\frac{A(a+bx^2)^{3/2}}{5ax^5}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{1}{4}\left(\frac{4(a+bx^2)^{3/2}(2Ab-5aC)}{3ax^3}-\frac{5}{2}\left(-\frac{b\,\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\sqrt{a+bx^2}}{x^2}\right)(bB-4aD)\right)-\frac{5B(a+bx^2)^{3/2}}{4x^4}}{5a}-\frac{A(a+bx^2)^{3/2}}{5ax^5}$$

input
```
Int[(Sqrt[a + b*x^2]*(A + B*x + C*x^2 + D*x^3))/x^6,x]
```

output
```
-1/5*(A*(a + b*x^2)^(3/2))/(a*x^5) + ((-5*B*(a + b*x^2)^(3/2))/(4*x^4) + (
(4*(2*A*b - 5*a*C)*(a + b*x^2)^(3/2))/(3*a*x^3) - (5*(b*B - 4*a*D)*(-(Sqrt
[a + b*x^2]/x^2) - (b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a]))/2)/4)/(5
*a)
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

## Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.50

| method | result |
|---|---|
| default | $A\left(-\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{5a\,x^{5}}+\frac{2b\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{15a^{2}x^{3}}\right)+B\left(-\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{4a\,x^{4}}-\frac{b\left(-\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{2a\,x^{2}}+\frac{b\left(\sqrt{b\,x^{2}+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)\right)}{2a}\right)}{4a}\right)$ |

input
```
int((b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/x^6,x,method=_RETURNVERBOSE)
```

output
```
A*(-1/5/a/x^5*(b*x^2+a)^(3/2)+2/15*b/a^2*(b*x^2+a)^(3/2)/x^3)+B*(-1/4/a/x^
4*(b*x^2+a)^(3/2)-1/4*b/a*(-1/2/a/x^2*(b*x^2+a)^(3/2)+1/2*b/a*((b*x^2+a)^(
1/2)-a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x))))-1/3*C*(b*x^2+a)^(3/2
)/a/x^3+D*(-1/2/a/x^2*(b*x^2+a)^(3/2)+1/2*b/a*((b*x^2+a)^(1/2)-a^(1/2)*ln(
(2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)))
```

## Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.84

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{x^6}\,dx$$
$$= \left[-\frac{15\,(4\,Dab-Bb^2)\sqrt{a}x^5\log\left(-\frac{bx^2+2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\,(8\,(5\,Cab-2\,Ab^2)x^4+30\,Ba^2x+15\,(4\,Da^2+\right.}{240\,a^2x^5}$$

input
```
integrate((b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/x^6,x, algorithm="fricas")
```

output
```
[-1/240*(15*(4*D*a*b - B*b^2)*sqrt(a)*x^5*log(-(b*x^2 + 2*sqrt(b*x^2 + a)*
sqrt(a) + 2*a)/x^2) + 2*(8*(5*C*a*b - 2*A*b^2)*x^4 + 30*B*a^2*x + 15*(4*D*
a^2 + B*a*b)*x^3 + 24*A*a^2 + 8*(5*C*a^2 + A*a*b)*x^2)*sqrt(b*x^2 + a))/(a
^2*x^5), 1/120*(15*(4*D*a*b - B*b^2)*sqrt(-a)*x^5*arctan(sqrt(b*x^2 + a)*s
qrt(-a)/a) - (8*(5*C*a*b - 2*A*b^2)*x^4 + 30*B*a^2*x + 15*(4*D*a^2 + B*a*b
)*x^3 + 24*A*a^2 + 8*(5*C*a^2 + A*a*b)*x^2)*sqrt(b*x^2 + a))/(a^2*x^5)]
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 265 vs. 2(124) = 248.

Time = 4.46 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.89

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{x^6}\,dx = -\frac{A\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{5x^4} - \frac{Ab^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{15ax^2} + \frac{2Ab^{\frac{5}{2}}\sqrt{\frac{a}{bx^2}+1}}{15a^2} - \frac{Ba}{4\sqrt{b}x^5\sqrt{\frac{a}{bx^2}+1}} - \frac{3B\sqrt{b}}{8x^3\sqrt{\frac{a}{bx^2}+1}} - \frac{Bb^{\frac{3}{2}}}{8ax\sqrt{\frac{a}{bx^2}+1}} + \frac{Bb^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{8a^{\frac{3}{2}}} - \frac{C\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{3x^2} - \frac{Cb^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{3a} - \frac{D\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{2x} - \frac{Db\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2\sqrt{a}}$$

input
```
integrate((b*x**2+a)**(1/2)*(D*x**3+C*x**2+B*x+A)/x**6,x)
```

output
```
-A*sqrt(b)*sqrt(a/(b*x**2) + 1)/(5*x**4) - A*b**(3/2)*sqrt(a/(b*x**2) + 1)
/(15*a*x**2) + 2*A*b**(5/2)*sqrt(a/(b*x**2) + 1)/(15*a**2) - B*a/(4*sqrt(b
)*x**5*sqrt(a/(b*x**2) + 1)) - 3*B*sqrt(b)/(8*x**3*sqrt(a/(b*x**2) + 1)) -
 B*b**(3/2)/(8*a*x*sqrt(a/(b*x**2) + 1)) + B*b**2*asinh(sqrt(a)/(sqrt(b)*x
))/(8*a**(3/2)) - C*sqrt(b)*sqrt(a/(b*x**2) + 1)/(3*x**2) - C*b**(3/2)*sqr
t(a/(b*x**2) + 1)/(3*a) - D*sqrt(b)*sqrt(a/(b*x**2) + 1)/(2*x) - D*b*asinh
(sqrt(a)/(sqrt(b)*x))/(2*sqrt(a))
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.32

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{x^6}\,dx = -\frac{Db\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{2\sqrt{a}} + \frac{Bb^2\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{8a^{\frac{3}{2}}} + \frac{\sqrt{bx^2+a}Db}{2a} - \frac{\sqrt{bx^2+a}Bb^2}{8a^2} - \frac{(bx^2+a)^{\frac{3}{2}}D}{2ax^2} + \frac{(bx^2+a)^{\frac{3}{2}}Bb}{8a^2x^2} - \frac{(bx^2+a)^{\frac{3}{2}}C}{3ax^3} + \frac{2(bx^2+a)^{\frac{3}{2}}Ab}{15a^2x^3} - \frac{(bx^2+a)^{\frac{3}{2}}B}{4ax^4} - \frac{(bx^2+a)^{\frac{3}{2}}A}{5ax^5}$$

input
```
integrate((b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/x^6,x, algorithm="maxima")
```

output
```
-1/2*D*b*arcsinh(a/(sqrt(a*b)*abs(x)))/sqrt(a) + 1/8*B*b^2*arcsinh(a/(sqrt
(a*b)*abs(x)))/a^(3/2) + 1/2*sqrt(b*x^2 + a)*D*b/a - 1/8*sqrt(b*x^2 + a)*B
*b^2/a^2 - 1/2*(b*x^2 + a)^(3/2)*D/(a*x^2) + 1/8*(b*x^2 + a)^(3/2)*B*b/(a^
2*x^2) - 1/3*(b*x^2 + a)^(3/2)*C/(a*x^3) + 2/15*(b*x^2 + a)^(3/2)*A*b/(a^2
*x^3) - 1/4*(b*x^2 + a)^(3/2)*B/(a*x^4) - 1/5*(b*x^2 + a)^(3/2)*A/(a*x^5)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 494 vs. $2(118) = 236$.

Time = 0.12 (sec) , antiderivative size = 494, normalized size of antiderivative = 3.53

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{x^6}\,dx = \frac{(4\,Dab-Bb^2)\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{4\sqrt{-a}a} + \frac{60\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^9 Dab + 15\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^9 Bb^2 + 120\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^8 Cab^{\frac{3}{2}} - 120\left(\sqrt{b}\right.}{}$$

input
```
integrate((b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/x^6,x, algorithm="giac")
```

output
```
1/4*(4*D*a*b - B*b^2)*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/(sqr
t(-a)*a) + 1/60*(60*(sqrt(b)*x - sqrt(b*x^2 + a))^9*D*a*b + 15*(sqrt(b)*x
- sqrt(b*x^2 + a))^9*B*b^2 + 120*(sqrt(b)*x - sqrt(b*x^2 + a))^8*C*a*b^(3/
2) - 120*(sqrt(b)*x - sqrt(b*x^2 + a))^7*D*a^2*b + 90*(sqrt(b)*x - sqrt(b*
x^2 + a))^7*B*a*b^2 - 240*(sqrt(b)*x - sqrt(b*x^2 + a))^6*C*a^2*b^(3/2) +
240*(sqrt(b)*x - sqrt(b*x^2 + a))^6*A*a*b^(5/2) + 160*(sqrt(b)*x - sqrt(b*
x^2 + a))^4*C*a^3*b^(3/2) + 80*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*a^2*b^(5/
2) + 120*(sqrt(b)*x - sqrt(b*x^2 + a))^3*D*a^4*b - 90*(sqrt(b)*x - sqrt(b*
x^2 + a))^3*B*a^3*b^2 - 80*(sqrt(b)*x - sqrt(b*x^2 + a))^2*C*a^4*b^(3/2) +
 80*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A*a^3*b^(5/2) - 60*(sqrt(b)*x - sqrt(b
*x^2 + a))*D*a^5*b - 15*(sqrt(b)*x - sqrt(b*x^2 + a))*B*a^4*b^2 + 40*C*a^5
*b^(3/2) - 16*A*a^4*b^(5/2))/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^5*a)
```

## Mupad [B] (verification not implemented)

Time = 3.40 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.06

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{x^6}\,dx = \frac{B\,b^2\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{8\,a^{3/2}} - \frac{\sqrt{b\,x^2+a}\,D}{2\,x^2} - \frac{B\,\sqrt{b\,x^2+a}}{8\,x^4} - \frac{b\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)\,D}{2\,\sqrt{a}} - \frac{B\,(b\,x^2+a)^{3/2}}{8\,a\,x^4} - \frac{C\,(b\,x^2+a)^{3/2}}{3\,a\,x^3} - \frac{A\,\sqrt{b\,x^2+a}\,(3\,a^2+a\,b\,x^2-2\,b^2\,x^4)}{15\,a^2\,x^5}$$

input
```
int(((a + b*x^2)^(1/2)*(A + B*x + C*x^2 + x^3*D))/x^6,x)
```

output
```
(B*b^2*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(8*a^(3/2)) - ((a + b*x^2)^(1/2)*
D)/(2*x^2) - (B*(a + b*x^2)^(1/2))/(8*x^4) - (b*atanh((a + b*x^2)^(1/2)/a^
(1/2))*D)/(2*a^(1/2)) - (B*(a + b*x^2)^(3/2))/(8*a*x^4) - (C*(a + b*x^2)^(
3/2))/(3*a*x^3) - (A*(a + b*x^2)^(1/2)*(3*a^2 - 2*b^2*x^4 + a*b*x^2))/(15*
a^2*x^5)
```

## Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 287, normalized size of antiderivative = 2.05

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{x^6}\,dx = \frac{-24\sqrt{b\,x^2+a}\,a^3 - 8\sqrt{b\,x^2+a}\,a^2b\,x^2 - 30\sqrt{b\,x^2+a}\,a^2bx - 40\sqrt{b\,x^2+a}\,a^2c\,x^2 - 60\sqrt{b\,x^2+a}\,a^2d\,x^3 + \cdots}{\cdots}$$

input
```
int((b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/x^6,x)
```

output
```
( - 24*sqrt(a + b*x**2)*a**3 - 8*sqrt(a + b*x**2)*a**2*b*x**2 - 30*sqrt(a
+ b*x**2)*a**2*b*x - 40*sqrt(a + b*x**2)*a**2*c*x**2 - 60*sqrt(a + b*x**2)
*a**2*d*x**3 + 16*sqrt(a + b*x**2)*a*b**2*x**4 - 15*sqrt(a + b*x**2)*a*b**
2*x**3 - 40*sqrt(a + b*x**2)*a*b*c*x**4 + 60*sqrt(a)*log((sqrt(a + b*x**2)
 - sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b*d*x**5 - 15*sqrt(a)*log((sqrt(a + b*x
**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*b**3*x**5 - 60*sqrt(a)*log((sqrt(a +
b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b*d*x**5 + 15*sqrt(a)*log((sqrt(
a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*b**3*x**5 - 16*sqrt(b)*a*b**2*
x**5 - 8*sqrt(b)*a*b*c*x**5)/(120*a**2*x**5)
```

# 3.71 $\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{x^7} dx$



## Optimal result

Integrand size = 30, antiderivative size = 172

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{x^7} dx = \frac{(Ab-2aC)\sqrt{a+bx^2}}{8ax^4} + \frac{b(Ab-2aC)\sqrt{a+bx^2}}{16a^2x^2} - \frac{A(a+bx^2)^{3/2}}{6ax^6} - \frac{B(a+bx^2)^{3/2}}{5ax^5} + \frac{(2bB-5aD)(a+bx^2)^{3/2}}{15a^2x^3} - \frac{b^2(Ab-2aC)\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{5/2}}$$

output
```
1/8*(A*b-2*C*a)*(b*x^2+a)^(1/2)/a/x^4+1/16*b*(A*b-2*C*a)*(b*x^2+a)^(1/2)/a
^2/x^2-1/6*A*(b*x^2+a)^(3/2)/a/x^6-1/5*B*(b*x^2+a)^(3/2)/a/x^5+1/15*(2*B*b
-5*D*a)*(b*x^2+a)^(3/2)/a^2/x^3-1/16*b^2*(A*b-2*C*a)*arctanh((b*x^2+a)^(1/
2)/a^(1/2))/a^(5/2)
```

## Mathematica [A] (verified)

Time = 1.47 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{x^7}\,dx$$
$$= \frac{\sqrt{a+bx^2}(b^2x^4(15A+32Bx)-4a^2(10A+x(12B+5x(3C+4Dx)))-2abx^2(5A+x(8B+5x(3C+8Dx))))}{240a^2x^6}$$
$$+\frac{b^2(Ab-2aC)\mathrm{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{5/2}}$$

input
```
Integrate[(Sqrt[a + b*x^2]*(A + B*x + C*x^2 + D*x^3))/x^7,x]
```

output
```
(Sqrt[a + b*x^2]*(b^2*x^4*(15*A + 32*B*x) - 4*a^2*(10*A + x*(12*B + 5*x*(3
*C + 4*D*x))) - 2*a*b*x^2*(5*A + x*(8*B + 5*x*(3*C + 8*D*x)))))/(240*a^2*x
^6) + (b^2*(A*b - 2*a*C)*ArcTanh[(Sqrt[b]*x - Sqrt[a + b*x^2])/Sqrt[a]])/(
8*a^(5/2))
```

## Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.02, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.367, Rules used = {2338, 27, 2338, 27, 539, 25, 534, 243, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)}{x^7}\,dx$$

$$\Big\downarrow \text{2338}$$

$$-\frac{\int -\frac{3\sqrt{bx^2+a}\left(2aDx^2-(Ab-2aC)x+2aB\right)}{x^6}dx}{6a}-\frac{A\left(a+bx^2\right)^{3/2}}{6ax^6}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sqrt{bx^2+a}\left(2aDx^2-(Ab-2aC)x+2aB\right)}{x^6}dx}{2a}-\frac{A\left(a+bx^2\right)^{3/2}}{6ax^6}$$

$$\Big\downarrow \text{2338}$$

$$\frac{-\frac{\int \frac{a(5(Ab-2aC)+2(2bB-5aD)x)\sqrt{bx^2+a}}{x^5}dx}{5a}-\frac{2B\left(a+bx^2\right)^{3/2}}{5x^5}}{2a}-\frac{A\left(a+bx^2\right)^{3/2}}{6ax^6}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{1}{5}\int \frac{(5(Ab-2aC)+2(2bB-5aD)x)\sqrt{bx^2+a}}{x^5}dx-\frac{2B\left(a+bx^2\right)^{3/2}}{5x^5}}{2a}-\frac{A\left(a+bx^2\right)^{3/2}}{6ax^6}$$

$$\Big\downarrow \text{539}$$

$$\frac{\frac{1}{5}\left(\frac{\int -\frac{(8a(2bB-5aD)-5b(Ab-2aC)x)\sqrt{bx^2+a}}{x^4}dx}{4a}+\frac{5\left(a+bx^2\right)^{3/2}(Ab-2aC)}{4ax^4}\right)-\frac{2B\left(a+bx^2\right)^{3/2}}{5x^5}}{2a}-\frac{A\left(a+bx^2\right)^{3/2}}{6ax^6}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{1}{5}\left(\frac{5\left(a+bx^2\right)^{3/2}(Ab-2aC)}{4ax^4}-\frac{\int \frac{(8a(2bB-5aD)-5b(Ab-2aC)x)\sqrt{bx^2+a}}{x^4}dx}{4a}\right)-\frac{2B\left(a+bx^2\right)^{3/2}}{5x^5}}{2a}-\frac{A\left(a+bx^2\right)^{3/2}}{6ax^6}$$

$$\Big\downarrow \text{534}$$

$$\frac{\frac{1}{5}\left(\frac{5\left(a+bx^2\right)^{3/2}(Ab-2aC)}{4ax^4}-\frac{-5b(Ab-2aC)\int \frac{\sqrt{bx^2+a}}{x^3}dx-\frac{8\left(a+bx^2\right)^{3/2}(2bB-5aD)}{3x^3}}{4a}\right)-\frac{2B\left(a+bx^2\right)^{3/2}}{5x^5}}{2a}-\frac{A\left(a+bx^2\right)^{3/2}}{6ax^6}$$

$$\Big\downarrow \text{243}$$

$$\frac{\frac{1}{5}\left(\frac{5\left(a+bx^2\right)^{3/2}(Ab-2aC)}{4ax^4}-\frac{-\frac{5}{2}b(Ab-2aC)\int \frac{\sqrt{bx^2+a}}{x^4}dx^2-\frac{8\left(a+bx^2\right)^{3/2}(2bB-5aD)}{3x^3}}{4a}\right)-\frac{2B\left(a+bx^2\right)^{3/2}}{5x^5}}{2a}-\frac{A\left(a+bx^2\right)^{3/2}}{6ax^6}$$

$$\Big\downarrow \text{51}$$

$$\frac{\frac{1}{5}\left(\frac{5(a+bx^2)^{3/2}(Ab-2aC)}{4ax^4}-\frac{-\frac{5}{2}b(Ab-2aC)\left(\frac{1}{2}b\int\frac{1}{x^2\sqrt{bx^2+a}}dx^2-\frac{\sqrt{a+bx^2}}{x^2}\right)-\frac{8(a+bx^2)^{3/2}(2bB-5aD)}{3x^3}}{4a}\right)-\frac{2B(a+bx^2)^{3/2}}{5x^5}}{2a}-\frac{A(a+bx^2)^{3/2}}{6ax^6}$$

$\Big\downarrow$ 73

$$\frac{\frac{1}{5}\left(\frac{5(a+bx^2)^{3/2}(Ab-2aC)}{4ax^4}-\frac{-\frac{5}{2}b(Ab-2aC)\left(\int\frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}-\frac{\sqrt{a+bx^2}}{x^2}\right)-\frac{8(a+bx^2)^{3/2}(2bB-5aD)}{3x^3}}{4a}\right)-\frac{2B(a+bx^2)^{3/2}}{5x^5}}{2a}-\frac{A(a+bx^2)^{3/2}}{6ax^6}$$

$\Big\downarrow$ 221

$$\frac{\frac{1}{5}\left(\frac{5(a+bx^2)^{3/2}(Ab-2aC)}{4ax^4}-\frac{-\frac{5}{2}b(Ab-2aC)\left(-\frac{b\,\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\sqrt{a+bx^2}}{x^2}\right)-\frac{8(a+bx^2)^{3/2}(2bB-5aD)}{3x^3}}{4a}\right)-\frac{2B(a+bx^2)^{3/2}}{5x^5}}{2a}-\frac{A(a+bx^2)^{3/2}}{6ax^6}$$

input
```
Int[(Sqrt[a + b*x^2]*(A + B*x + C*x^2 + D*x^3))/x^7,x]
```

output
```
-1/6*(A*(a + b*x^2)^(3/2))/(a*x^6) + ((-2*B*(a + b*x^2)^(3/2))/(5*x^5) + (
(5*(A*b - 2*a*C)*(a + b*x^2)^(3/2))/(4*a*x^4) - ((-8*(2*b*B - 5*a*D)*(a +
b*x^2)^(3/2))/(3*x^3) - (5*b*(A*b - 2*a*C)*(-(Sqrt[a + b*x^2]/x^2) - (b*Ar
cTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a]))/2)/(4*a))/5)/(2*a)
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 539
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[c*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))
   Int[x^(m + 1)*(a + b*x^2)^p*(a*d*(m + 1) - b*c*(m + 2*p + 3)*x), x], x]
/; FreeQ[{a, b, c, d, p}, x] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

## Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.50

| method | result |
|---|---|
| default | $A\left(-\frac{(b\,x^2+a)^{\frac{3}{2}}}{6a\,x^6}-\frac{b\left(-\frac{(b\,x^2+a)^{\frac{3}{2}}}{4a\,x^4}-\frac{b\left(-\frac{(b\,x^2+a)^{\frac{3}{2}}}{2a\,x^2}+\frac{b\left(\sqrt{b\,x^2+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)\right)}{2a}\right)}{4a}\right)}{2a}\right)+B\left(-\frac{(b\,x^2+a)^{\frac{3}{2}}}{5a\,x^5}\right.$ |

input
```
int((b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/x^7,x,method=_RETURNVERBOSE)
```

output
```
A*(-1/6/a/x^6*(b*x^2+a)^(3/2)-1/2*b/a*(-1/4/a/x^4*(b*x^2+a)^(3/2)-1/4*b/a*
(-1/2/a/x^2*(b*x^2+a)^(3/2)+1/2*b/a*((b*x^2+a)^(1/2)-a^(1/2)*ln((2*a+2*a^(
1/2)*(b*x^2+a)^(1/2))/x)))))+B*(-1/5/a/x^5*(b*x^2+a)^(3/2)+2/15*b/a^2*(b*x
^2+a)^(3/2)/x^3)+C*(-1/4/a/x^4*(b*x^2+a)^(3/2)-1/4*b/a*(-1/2/a/x^2*(b*x^2+
a)^(3/2)+1/2*b/a*((b*x^2+a)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2
))/x))))-1/3*D*(b*x^2+a)^(3/2)/a/x^3
```

## Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.83

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{x^7}\,dx$$

$$= \left[-\frac{15\,(2\,Cab^2-Ab^3)\sqrt{a}x^6\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\,(16\,(5\,Da^2b-2\,Bab^2)x^5+48\,Ba^3x+15\,(2\,Ca}{480\,a^3x^6}\right.$$

$$-\frac{15\,(2\,Cab^2-Ab^3)\sqrt{-a}x^6\arctan\left(\frac{\sqrt{bx^2+a}\sqrt{-a}}{a}\right)+(16\,(5\,Da^2b-2\,Bab^2)x^5+48\,Ba^3x+15\,(2\,Ca^2b-}{240\,a^3x^6}$$

input
```
integrate((b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/x^7,x, algorithm="fricas")
```

output
```
[-1/480*(15*(2*C*a*b^2 - A*b^3)*sqrt(a)*x^6*log(-(b*x^2 - 2*sqrt(b*x^2 + a
)*sqrt(a) + 2*a)/x^2) + 2*(16*(5*D*a^2*b - 2*B*a*b^2)*x^5 + 48*B*a^3*x + 1
5*(2*C*a^2*b - A*a*b^2)*x^4 + 40*A*a^3 + 16*(5*D*a^3 + B*a^2*b)*x^3 + 10*(
6*C*a^3 + A*a^2*b)*x^2)*sqrt(b*x^2 + a))/(a^3*x^6), -1/240*(15*(2*C*a*b^2
- A*b^3)*sqrt(-a)*x^6*arctan(sqrt(b*x^2 + a)*sqrt(-a)/a) + (16*(5*D*a^2*b
- 2*B*a*b^2)*x^5 + 48*B*a^3*x + 15*(2*C*a^2*b - A*a*b^2)*x^4 + 40*A*a^3 +
16*(5*D*a^3 + B*a^2*b)*x^3 + 10*(6*C*a^3 + A*a^2*b)*x^2)*sqrt(b*x^2 + a))/
(a^3*x^6)]
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 347 vs. $2(155) = 310$.

Time = 9.08 (sec) , antiderivative size = 347, normalized size of antiderivative = 2.02

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{x^7}\,dx = -\frac{Aa}{6\sqrt{b}x^7\sqrt{\frac{a}{bx^2}+1}} - \frac{5A\sqrt{b}}{24x^5\sqrt{\frac{a}{bx^2}+1}} + \frac{Ab^{\frac{3}{2}}}{48ax^3\sqrt{\frac{a}{bx^2}+1}} + \frac{Ab^{\frac{5}{2}}}{16a^2x\sqrt{\frac{a}{bx^2}+1}} - \frac{Ab^3\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{16a^{\frac{5}{2}}} - \frac{B\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{5x^4} - \frac{Bb^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{15ax^2} + \frac{2Bb^{\frac{5}{2}}\sqrt{\frac{a}{bx^2}+1}}{15a^2} - \frac{Ca}{4\sqrt{b}x^5\sqrt{\frac{a}{bx^2}+1}} - \frac{3C\sqrt{b}}{8x^3\sqrt{\frac{a}{bx^2}+1}} - \frac{Cb^{\frac{3}{2}}}{8ax\sqrt{\frac{a}{bx^2}+1}} + \frac{Cb^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{8a^{\frac{3}{2}}} - \frac{D\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{3x^2} - \frac{Db^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{3a}$$

input
```
integrate((b*x**2+a)**(1/2)*(D*x**3+C*x**2+B*x+A)/x**7,x)
```

output
```
-A*a/(6*sqrt(b)*x**7*sqrt(a/(b*x**2) + 1)) - 5*A*sqrt(b)/(24*x**5*sqrt(a/(
b*x**2) + 1)) + A*b**(3/2)/(48*a*x**3*sqrt(a/(b*x**2) + 1)) + A*b**(5/2)/(
16*a**2*x*sqrt(a/(b*x**2) + 1)) - A*b**3*asinh(sqrt(a)/(sqrt(b)*x))/(16*a*
*(5/2)) - B*sqrt(b)*sqrt(a/(b*x**2) + 1)/(5*x**4) - B*b**(3/2)*sqrt(a/(b*x
**2) + 1)/(15*a*x**2) + 2*B*b**(5/2)*sqrt(a/(b*x**2) + 1)/(15*a**2) - C*a/
(4*sqrt(b)*x**5*sqrt(a/(b*x**2) + 1)) - 3*C*sqrt(b)/(8*x**3*sqrt(a/(b*x**2
) + 1)) - C*b**(3/2)/(8*a*x*sqrt(a/(b*x**2) + 1)) + C*b**2*asinh(sqrt(a)/(
sqrt(b)*x))/(8*a**(3/2)) - D*sqrt(b)*sqrt(a/(b*x**2) + 1)/(3*x**2) - D*b**
(3/2)*sqrt(a/(b*x**2) + 1)/(3*a)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.33

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{x^7}\,dx = \frac{Cb^2 \operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{8\,a^{\frac{3}{2}}} - \frac{Ab^3 \operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{16\,a^{\frac{5}{2}}} - \frac{\sqrt{bx^2+a}Cb^2}{8\,a^2} + \frac{\sqrt{bx^2+a}Ab^3}{16\,a^3} + \frac{(bx^2+a)^{\frac{3}{2}}Cb}{8\,a^2x^2} - \frac{(bx^2+a)^{\frac{3}{2}}Ab^2}{16\,a^3x^2} - \frac{(bx^2+a)^{\frac{3}{2}}D}{3\,ax^3} + \frac{2\,(bx^2+a)^{\frac{3}{2}}Bb}{15\,a^2x^3} - \frac{(bx^2+a)^{\frac{3}{2}}C}{4\,ax^4} + \frac{(bx^2+a)^{\frac{3}{2}}Ab}{8\,a^2x^4} - \frac{(bx^2+a)^{\frac{3}{2}}B}{5\,ax^5} - \frac{(bx^2+a)^{\frac{3}{2}}A}{6\,ax^6}$$

input
```
integrate((b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/x^7,x, algorithm="maxima")
```

output
```
1/8*C*b^2*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2) - 1/16*A*b^3*arcsinh(a/(sq
rt(a*b)*abs(x)))/a^(5/2) - 1/8*sqrt(b*x^2 + a)*C*b^2/a^2 + 1/16*sqrt(b*x^2
 + a)*A*b^3/a^3 + 1/8*(b*x^2 + a)^(3/2)*C*b/(a^2*x^2) - 1/16*(b*x^2 + a)^(
3/2)*A*b^2/(a^3*x^2) - 1/3*(b*x^2 + a)^(3/2)*D/(a*x^3) + 2/15*(b*x^2 + a)^
(3/2)*B*b/(a^2*x^3) - 1/4*(b*x^2 + a)^(3/2)*C/(a*x^4) + 1/8*(b*x^2 + a)^(3
/2)*A*b/(a^2*x^4) - 1/5*(b*x^2 + a)^(3/2)*B/(a*x^5) - 1/6*(b*x^2 + a)^(3/2
)*A/(a*x^6)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 648 vs. $2(147) = 294$.

Time = 0.13 (sec) , antiderivative size = 648, normalized size of antiderivative = 3.77

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{x^7}\,dx = -\frac{(2\,Cab^2-Ab^3)\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{8\,\sqrt{-a}a^2} + \frac{30\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{11}Cab^2-15\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{11}Ab^3+240\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{10}Da^2b^{\frac{3}{2}}+150}{}$$

input
```
integrate((b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/x^7,x, algorithm="giac")
```

output
```
-1/8*(2*C*a*b^2 - A*b^3)*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/(
sqrt(-a)*a^2) + 1/120*(30*(sqrt(b)*x - sqrt(b*x^2 + a))^11*C*a*b^2 - 15*(s
qrt(b)*x - sqrt(b*x^2 + a))^11*A*b^3 + 240*(sqrt(b)*x - sqrt(b*x^2 + a))^1
0*D*a^2*b^(3/2) + 150*(sqrt(b)*x - sqrt(b*x^2 + a))^9*C*a^2*b^2 + 85*(sqrt
(b)*x - sqrt(b*x^2 + a))^9*A*a*b^3 - 720*(sqrt(b)*x - sqrt(b*x^2 + a))^8*D
*a^3*b^(3/2) + 480*(sqrt(b)*x - sqrt(b*x^2 + a))^8*B*a^2*b^(5/2) - 180*(sq
rt(b)*x - sqrt(b*x^2 + a))^7*C*a^3*b^2 + 570*(sqrt(b)*x - sqrt(b*x^2 + a))
^7*A*a^2*b^3 + 800*(sqrt(b)*x - sqrt(b*x^2 + a))^6*D*a^4*b^(3/2) - 320*(sq
rt(b)*x - sqrt(b*x^2 + a))^6*B*a^3*b^(5/2) - 180*(sqrt(b)*x - sqrt(b*x^2 +
 a))^5*C*a^4*b^2 + 570*(sqrt(b)*x - sqrt(b*x^2 + a))^5*A*a^3*b^3 - 480*(sq
rt(b)*x - sqrt(b*x^2 + a))^4*D*a^5*b^(3/2) + 150*(sqrt(b)*x - sqrt(b*x^2 +
 a))^3*C*a^5*b^2 + 85*(sqrt(b)*x - sqrt(b*x^2 + a))^3*A*a^4*b^3 + 240*(sqr
t(b)*x - sqrt(b*x^2 + a))^2*D*a^6*b^(3/2) - 192*(sqrt(b)*x - sqrt(b*x^2 +
a))^2*B*a^5*b^(5/2) + 30*(sqrt(b)*x - sqrt(b*x^2 + a))*C*a^6*b^2 - 15*(sqr
t(b)*x - sqrt(b*x^2 + a))*A*a^5*b^3 - 80*D*a^7*b^(3/2) + 32*B*a^6*b^(5/2))
/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^6*a^2)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{x^7}\,dx = \int \frac{\sqrt{b\,x^2+a}\,\left(A+B\,x+C\,x^2+x^3\,D\right)}{x^7}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(A + B*x + C*x^2 + x^3*D))/x^7,x)
```

output
```
int(((a + b*x^2)^(1/2)*(A + B*x + C*x^2 + x^3*D))/x^7, x)
```

## Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.80

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{x^7}\,dx$$
$$= \frac{-40\sqrt{b\,x^2+a}\,a^3-10\sqrt{b\,x^2+a}\,a^2b\,x^2-48\sqrt{b\,x^2+a}\,a^2bx-60\sqrt{b\,x^2+a}\,a^2c\,x^2-80\sqrt{b\,x^2+a}\,a^2d\,x^3+}{}$$

input
```
int((b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/x^7,x)
```

output
```
( - 40*sqrt(a + b*x**2)*a**3 - 10*sqrt(a + b*x**2)*a**2*b*x**2 - 48*sqrt(a
 + b*x**2)*a**2*b*x - 60*sqrt(a + b*x**2)*a**2*c*x**2 - 80*sqrt(a + b*x**2
)*a**2*d*x**3 + 15*sqrt(a + b*x**2)*a*b**2*x**4 - 16*sqrt(a + b*x**2)*a*b*
*2*x**3 - 30*sqrt(a + b*x**2)*a*b*c*x**4 - 80*sqrt(a + b*x**2)*a*b*d*x**5
+ 32*sqrt(a + b*x**2)*b**3*x**5 + 15*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(
a) + sqrt(b)*x)/sqrt(a))*b**3*x**6 - 30*sqrt(a)*log((sqrt(a + b*x**2) - sq
rt(a) + sqrt(b)*x)/sqrt(a))*b**2*c*x**6 - 15*sqrt(a)*log((sqrt(a + b*x**2)
 + sqrt(a) + sqrt(b)*x)/sqrt(a))*b**3*x**6 + 30*sqrt(a)*log((sqrt(a + b*x*
*2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*b**2*c*x**6 - 32*sqrt(b)*b**3*x**6)/(2
40*a**2*x**6)
```

# 3.72 $\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{x^8}\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 200

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{x^8}\,dx = -\frac{B\sqrt{a+bx^2}}{6x^6} - \frac{(bB+6aD)\sqrt{a+bx^2}}{24ax^4} + \frac{b(bB-2aD)\sqrt{a+bx^2}}{16a^2x^2} - \frac{A(a+bx^2)^{3/2}}{7ax^7} + \frac{(4Ab-7aC)\,(a+bx^2)^{3/2}}{35a^2x^5} - \frac{2b(4Ab-7aC)\,(a+bx^2)^{3/2}}{105a^3x^3} - \frac{b^2(bB-2aD)\mathrm{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{5/2}}$$

output
```
-1/6*B*(b*x^2+a)^(1/2)/x^6-1/24*(B*b+6*D*a)*(b*x^2+a)^(1/2)/a/x^4+1/16*b*(
B*b-2*D*a)*(b*x^2+a)^(1/2)/a^2/x^2-1/7*A*(b*x^2+a)^(3/2)/a/x^7+1/35*(4*A*b
-7*C*a)*(b*x^2+a)^(3/2)/a^2/x^5-2/105*b*(4*A*b-7*C*a)*(b*x^2+a)^(3/2)/a^3/
x^3-1/16*b^2*(B*b-2*D*a)*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(5/2)
```

## Mathematica [A] (verified)

Time = 1.48 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.80

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{x^8}\,dx$$
$$= \frac{\sqrt{a+bx^2}(-128Ab^3x^6+ab^2x^4(64A+7x(15B+32Cx))-4a^3(60A+7x(10B+3x(4C+5Dx)))-2a^2\dots}{1680a^3x^7}$$
$$+\frac{b^2(bB-2aD)\operatorname{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{5/2}}$$

input
```
Integrate[(Sqrt[a + b*x^2]*(A + B*x + C*x^2 + D*x^3))/x^8,x]
```

output
```
(Sqrt[a + b*x^2]*(-128*A*b^3*x^6 + a*b^2*x^4*(64*A + 7*x*(15*B + 32*C*x))
- 4*a^3*(60*A + 7*x*(10*B + 3*x*(4*C + 5*D*x))) - 2*a^2*b*x^2*(24*A + 7*x*
(5*B + x*(8*C + 15*D*x)))))/(1680*a^3*x^7) + (b^2*(b*B - 2*a*D)*ArcTanh[(S
qrt[b]*x - Sqrt[a + b*x^2])/Sqrt[a]])/(8*a^(5/2))
```

## Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.05, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.433$, Rules used = {2338, 25, 2338, 27, 539, 25, 539, 27, 534, 243, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)}{x^8}\,dx$$
$$\Big\downarrow \text{2338}$$
$$-\frac{\int -\frac{\sqrt{bx^2+a}(7aDx^2-(4Ab-7aC)x+7aB)}{x^7}dx}{7a}-\frac{A\left(a+bx^2\right)^{3/2}}{7ax^7}$$
$$\Big\downarrow \text{25}$$
$$\frac{\int \frac{\sqrt{bx^2+a}(7aDx^2-(4Ab-7aC)x+7aB)}{x^7}dx}{7a}-\frac{A\left(a+bx^2\right)^{3/2}}{7ax^7}$$

$$\Big\downarrow \text{2338}$$

$$\frac{-\frac{\int \frac{3a(2(4Ab-7aC)+7(bB-2aD)x)\sqrt{bx^2+a}}{x^6}dx}{6a}-\frac{7B\left(a+bx^2\right)^{3/2}}{6x^6}}{7a}-\frac{A\left(a+bx^2\right)^{3/2}}{7ax^7}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{1}{2}\int \frac{(2(4Ab-7aC)+7(bB-2aD)x)\sqrt{bx^2+a}}{x^6}dx-\frac{7B\left(a+bx^2\right)^{3/2}}{6x^6}}{7a}-\frac{A\left(a+bx^2\right)^{3/2}}{7ax^7}$$

$$\Big\downarrow \text{539}$$

$$\frac{\frac{1}{2}\left(\frac{\int -\frac{(35a(bB-2aD)-4b(4Ab-7aC)x)\sqrt{bx^2+a}}{x^5}dx}{5a}+\frac{2\left(a+bx^2\right)^{3/2}(4Ab-7aC)}{5ax^5}\right)-\frac{7B\left(a+bx^2\right)^{3/2}}{6x^6}}{7a}-\frac{A\left(a+bx^2\right)^{3/2}}{7ax^7}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{1}{2}\left(\frac{2\left(a+bx^2\right)^{3/2}(4Ab-7aC)}{5ax^5}-\frac{\int \frac{(35a(bB-2aD)-4b(4Ab-7aC)x)\sqrt{bx^2+a}}{x^5}dx}{5a}\right)-\frac{7B\left(a+bx^2\right)^{3/2}}{6x^6}}{7a}-\frac{A\left(a+bx^2\right)^{3/2}}{7ax^7}$$

$$\Big\downarrow \text{539}$$

$$\frac{\frac{1}{2}\left(\frac{2\left(a+bx^2\right)^{3/2}(4Ab-7aC)}{5ax^5}-\frac{-\frac{\int \frac{ab(16(4Ab-7aC)+35(bB-2aD)x)\sqrt{bx^2+a}}{x^4}dx}{4a}-\frac{35\left(a+bx^2\right)^{3/2}(bB-2aD)}{4x^4}}{5a}\right)-\frac{7B\left(a+bx^2\right)^{3/2}}{6x^6}}{7a}-\frac{A\left(a+bx^2\right)^{3/2}}{7ax^7}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{2}\left(\frac{2\left(a+bx^2\right)^{3/2}(4Ab-7aC)}{5ax^5}-\frac{-\frac{1}{4}b\int \frac{(16(4Ab-7aC)+35(bB-2aD)x)\sqrt{bx^2+a}}{x^4}dx-\frac{35\left(a+bx^2\right)^{3/2}(bB-2aD)}{4x^4}}{5a}\right)-\frac{7B\left(a+bx^2\right)^{3/2}}{6x^6}}{7a}-\frac{A\left(a+bx^2\right)^{3/2}}{7ax^7}$$

$$\Big\downarrow \text{534}$$

$$\frac{\frac{1}{2}\left(\frac{2\left(a+bx^2\right)^{3/2}(4Ab-7aC)}{5ax^5}-\frac{-\frac{1}{4}b\left(35(bB-2aD)\int\frac{\sqrt{bx^2+a}}{x^3}dx-\frac{16\left(a+bx^2\right)^{3/2}(4Ab-7aC)}{3ax^3}\right)-\frac{35\left(a+bx^2\right)^{3/2}(bB-2aD)}{4x^4}}{5a}\right)-\frac{7B\left(a+bx^2\right)^{3/2}}{6x^6}}{7a}$$

$$\frac{A\left(a+bx^2\right)^{3/2}}{7ax^7}$$

$\downarrow$ 243

$$\frac{\frac{1}{2}\left(\frac{2\left(a+bx^2\right)^{3/2}(4Ab-7aC)}{5ax^5}-\frac{-\frac{1}{4}b\left(\frac{35}{2}(bB-2aD)\int\frac{\sqrt{bx^2+a}}{x^4}dx^2-\frac{16\left(a+bx^2\right)^{3/2}(4Ab-7aC)}{3ax^3}\right)-\frac{35\left(a+bx^2\right)^{3/2}(bB-2aD)}{4x^4}}{5a}\right)-\frac{7B\left(a+bx^2\right)^{3/2}}{6x^6}}{7a}$$

$$\frac{A\left(a+bx^2\right)^{3/2}}{7ax^7}$$

$\downarrow$ 51

$$\frac{\frac{1}{2}\left(\frac{2\left(a+bx^2\right)^{3/2}(4Ab-7aC)}{5ax^5}-\frac{-\frac{1}{4}b\left(\frac{35}{2}(bB-2aD)\left(\frac{1}{2}b\int\frac{1}{x^2\sqrt{bx^2+a}}dx^2-\frac{\sqrt{a+bx^2}}{x^2}\right)-\frac{16\left(a+bx^2\right)^{3/2}(4Ab-7aC)}{3ax^3}\right)-\frac{35\left(a+bx^2\right)^{3/2}(bB-2aD)}{4x^4}}{5a}\right)}{7a}$$

$$\frac{A\left(a+bx^2\right)^{3/2}}{7ax^7}$$

$\downarrow$ 73

$$\frac{\frac{1}{2}\left(\frac{2\left(a+bx^2\right)^{3/2}(4Ab-7aC)}{5ax^5}-\frac{-\frac{1}{4}b\left(\frac{35}{2}(bB-2aD)\left(\int\frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}-\frac{\sqrt{a+bx^2}}{x^2}\right)-\frac{16\left(a+bx^2\right)^{3/2}(4Ab-7aC)}{3ax^3}\right)-\frac{35\left(a+bx^2\right)^{3/2}(bB-2aD)}{4x^4}}{5a}\right)}{7a}$$

$$\frac{A\left(a+bx^2\right)^{3/2}}{7ax^7}$$

$\downarrow$ 221

$$\frac{\frac{1}{2}\left(\frac{2(a+bx^2)^{3/2}(4Ab-7aC)}{5ax^5}-\frac{-\frac{1}{4}b\left(\frac{35}{2}\left(-\frac{b\,\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\sqrt{a+bx^2}}{x^2}\right)(bB-2aD)-\frac{16(a+bx^2)^{3/2}(4Ab-7aC)}{3ax^3}\right)-\frac{35(a+bx^2)^{3/2}(bB-2a}{4x^4}}{5a}\right.}{7a}$$

$$\frac{A(a+bx^2)^{3/2}}{7ax^7}$$

input
```
Int[(Sqrt[a + b*x^2]*(A + B*x + C*x^2 + D*x^3))/x^8,x]
```

output
```
-1/7*(A*(a + b*x^2)^(3/2))/(a*x^7) + ((-7*B*(a + b*x^2)^(3/2))/(6*x^6) + (
(2*(4*A*b - 7*a*C)*(a + b*x^2)^(3/2))/(5*a*x^5) - ((-35*(b*B - 2*a*D)*(a +
 b*x^2)^(3/2))/(4*x^4) - (b*((-16*(4*A*b - 7*a*C)*(a + b*x^2)^(3/2))/(3*a*
x^3) + (35*(b*B - 2*a*D)*(-(Sqrt[a + b*x^2]/x^2) - (b*ArcTanh[Sqrt[a + b*x
^2]/Sqrt[a]])/Sqrt[a]))/2))/4)/(5*a))/2)/(7*a)
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 539
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[c*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))
   Int[x^(m + 1)*(a + b*x^2)^p*(a*d*(m + 1) - b*c*(m + 2*p + 3)*x), x], x]
/; FreeQ[{a, b, c, d, p}, x] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

## Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.51

| method | result |
|---|---|
| default | $A\left(-\frac{(b\,x^2+a)^{\frac{3}{2}}}{7a\,x^7}-\frac{4b\left(-\frac{(b\,x^2+a)^{\frac{3}{2}}}{5a\,x^5}+\frac{2b(b\,x^2+a)^{\frac{3}{2}}}{15a^2x^3}\right)}{7a}\right)+B\left(-\frac{(b\,x^2+a)^{\frac{3}{2}}}{6a\,x^6}-\frac{b\left(-\frac{(b\,x^2+a)^{\frac{3}{2}}}{4a\,x^4}-\frac{b\left(-\frac{(b\,x^2+a)^{\frac{3}{2}}}{2a\,x^2}+\frac{b(\sqrt{b\ldots}}{}\right)}{}\right)}{2a}\right.$ |

input
```
int((b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/x^8,x,method=_RETURNVERBOSE)
```

output
```
A*(-1/7/a/x^7*(b*x^2+a)^(3/2)-4/7*b/a*(-1/5/a/x^5*(b*x^2+a)^(3/2)+2/15*b/a
^2*(b*x^2+a)^(3/2)/x^3))+B*(-1/6/a/x^6*(b*x^2+a)^(3/2)-1/2*b/a*(-1/4/a/x^4
*(b*x^2+a)^(3/2)-1/4*b/a*(-1/2/a/x^2*(b*x^2+a)^(3/2)+1/2*b/a*((b*x^2+a)^(1
/2)-a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)))))+C*(-1/5/a/x^5*(b*x^2
+a)^(3/2)+2/15*b/a^2*(b*x^2+a)^(3/2)/x^3)+D*(-1/4/a/x^4*(b*x^2+a)^(3/2)-1/
4*b/a*(-1/2/a/x^2*(b*x^2+a)^(3/2)+1/2*b/a*((b*x^2+a)^(1/2)-a^(1/2)*ln((2*a
+2*a^(1/2)*(b*x^2+a)^(1/2))/x))))
```

## Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.76

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{x^8}\,dx$$

$$=\left[-\frac{105\,(2\,Dab^2-Bb^3)\sqrt{a}x^7\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)-2\,(32\,(7\,Cab^2-4\,Ab^3)x^6-105\,(2\,Da^2b-Bab^2\ldots}{}\right.$$

$$-\frac{105\,(2\,Dab^2-Bb^3)\sqrt{-a}x^7\arctan\left(\frac{\sqrt{bx^2+a}\sqrt{-a}}{a}\right)-(32\,(7\,Cab^2-4\,Ab^3)x^6-105\,(2\,Da^2b-Bab^2)x^5-\ldots}{168\ldots}$$

input
```
integrate((b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/x^8,x, algorithm="fricas")
```

output
```
[-1/3360*(105*(2*D*a*b^2 - B*b^3)*sqrt(a)*x^7*log(-(b*x^2 - 2*sqrt(b*x^2 +
 a)*sqrt(a) + 2*a)/x^2) - 2*(32*(7*C*a*b^2 - 4*A*b^3)*x^6 - 105*(2*D*a^2*b
 - B*a*b^2)*x^5 - 280*B*a^3*x - 16*(7*C*a^2*b - 4*A*a*b^2)*x^4 - 240*A*a^3
 - 70*(6*D*a^3 + B*a^2*b)*x^3 - 48*(7*C*a^3 + A*a^2*b)*x^2)*sqrt(b*x^2 + a
))/(a^3*x^7), -1/1680*(105*(2*D*a*b^2 - B*b^3)*sqrt(-a)*x^7*arctan(sqrt(b*
x^2 + a)*sqrt(-a)/a) - (32*(7*C*a*b^2 - 4*A*b^3)*x^6 - 105*(2*D*a^2*b - B*
a*b^2)*x^5 - 280*B*a^3*x - 16*(7*C*a^2*b - 4*A*a*b^2)*x^4 - 240*A*a^3 - 70
*(6*D*a^3 + B*a^2*b)*x^3 - 48*(7*C*a^3 + A*a^2*b)*x^2)*sqrt(b*x^2 + a))/(a
^3*x^7)]
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 670 vs. $2(185) = 370$.

Time = 9.74 (sec) , antiderivative size = 670, normalized size of antiderivative = 3.35

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{x^8}\,dx = -\frac{15Aa^5b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}} - \frac{33Aa^4b^{\frac{11}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}} - \frac{17Aa^3b^{\frac{13}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}} - \frac{3Aa^2b^{\frac{15}{2}}x^6\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}} - \frac{12Aab^{\frac{17}{2}}x^8\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}} - \frac{8Ab^{\frac{19}{2}}x^{10}\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}} - \frac{Ba}{6\sqrt{b}x^7\sqrt{\frac{a}{bx^2}+1}} - \frac{5B\sqrt{b}}{24x^5\sqrt{\frac{a}{bx^2}+1}} + \frac{Bb^{\frac{3}{2}}}{48ax^3\sqrt{\frac{a}{bx^2}+1}} + \frac{Bb^{\frac{5}{2}}}{16a^2x\sqrt{\frac{a}{bx^2}+1}} - \frac{Bb^3\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{16a^{\frac{5}{2}}} - \frac{C\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{5x^4} - \frac{Cb^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{15ax^2} + \frac{2Cb^{\frac{5}{2}}\sqrt{\frac{a}{bx^2}+1}}{15a^2} - \frac{Da}{4\sqrt{b}x^5\sqrt{\frac{a}{bx^2}+1}} - \frac{3D\sqrt{b}}{8x^3\sqrt{\frac{a}{bx^2}+1}} - \frac{Db^{\frac{3}{2}}}{8ax\sqrt{\frac{a}{bx^2}+1}} + \frac{Db^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{8a^{\frac{3}{2}}}$$

input
```
integrate((b*x**2+a)**(1/2)*(D*x**3+C*x**2+B*x+A)/x**8,x)
```

output
```
-15*A*a**5*b**(9/2)*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b*
*5*x**8 + 105*a**3*b**6*x**10) - 33*A*a**4*b**(11/2)*x**2*sqrt(a/(b*x**2)
+ 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 17*
A*a**3*b**(13/2)*x**4*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*
b**5*x**8 + 105*a**3*b**6*x**10) - 3*A*a**2*b**(15/2)*x**6*sqrt(a/(b*x**2)
 + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 12
*A*a*b**(17/2)*x**8*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b*
*5*x**8 + 105*a**3*b**6*x**10) - 8*A*b**(19/2)*x**10*sqrt(a/(b*x**2) + 1)/
(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - B*a/(6*s
qrt(b)*x**7*sqrt(a/(b*x**2) + 1)) - 5*B*sqrt(b)/(24*x**5*sqrt(a/(b*x**2) +
 1)) + B*b**(3/2)/(48*a*x**3*sqrt(a/(b*x**2) + 1)) + B*b**(5/2)/(16*a**2*x
*sqrt(a/(b*x**2) + 1)) - B*b**3*asinh(sqrt(a)/(sqrt(b)*x))/(16*a**(5/2)) -
 C*sqrt(b)*sqrt(a/(b*x**2) + 1)/(5*x**4) - C*b**(3/2)*sqrt(a/(b*x**2) + 1)
/(15*a*x**2) + 2*C*b**(5/2)*sqrt(a/(b*x**2) + 1)/(15*a**2) - D*a/(4*sqrt(b
)*x**5*sqrt(a/(b*x**2) + 1)) - 3*D*sqrt(b)/(8*x**3*sqrt(a/(b*x**2) + 1)) -
 D*b**(3/2)/(8*a*x*sqrt(a/(b*x**2) + 1)) + D*b**2*asinh(sqrt(a)/(sqrt(b)*x
))/(8*a**(3/2))
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.34

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{x^8}\,dx = \frac{Db^2\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{8\,a^{\frac{3}{2}}} - \frac{Bb^3\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{16\,a^{\frac{5}{2}}} - \frac{\sqrt{bx^2+a}Db^2}{8\,a^2} + \frac{\sqrt{bx^2+a}Bb^3}{16\,a^3} + \frac{(bx^2+a)^{\frac{3}{2}}Db}{8\,a^2x^2} - \frac{(bx^2+a)^{\frac{3}{2}}Bb^2}{16\,a^3x^2} + \frac{2\,(bx^2+a)^{\frac{3}{2}}Cb}{15\,a^2x^3} - \frac{8\,(bx^2+a)^{\frac{3}{2}}Ab^2}{105\,a^3x^3} - \frac{(bx^2+a)^{\frac{3}{2}}D}{4\,ax^4} + \frac{(bx^2+a)^{\frac{3}{2}}Bb}{8\,a^2x^4} - \frac{(bx^2+a)^{\frac{3}{2}}C}{5\,ax^5} + \frac{4\,(bx^2+a)^{\frac{3}{2}}Ab}{35\,a^2x^5} - \frac{(bx^2+a)^{\frac{3}{2}}B}{6\,ax^6} - \frac{(bx^2+a)^{\frac{3}{2}}A}{7\,ax^7}$$

input
```
integrate((b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/x^8,x, algorithm="maxima")
```

output
```
1/8*D*b^2*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2) - 1/16*B*b^3*arcsinh(a/(sq
rt(a*b)*abs(x)))/a^(5/2) - 1/8*sqrt(b*x^2 + a)*D*b^2/a^2 + 1/16*sqrt(b*x^2
 + a)*B*b^3/a^3 + 1/8*(b*x^2 + a)^(3/2)*D*b/(a^2*x^2) - 1/16*(b*x^2 + a)^(
3/2)*B*b^2/(a^3*x^2) + 2/15*(b*x^2 + a)^(3/2)*C*b/(a^2*x^3) - 8/105*(b*x^2
 + a)^(3/2)*A*b^2/(a^3*x^3) - 1/4*(b*x^2 + a)^(3/2)*D/(a*x^4) + 1/8*(b*x^2
 + a)^(3/2)*B*b/(a^2*x^4) - 1/5*(b*x^2 + a)^(3/2)*C/(a*x^5) + 4/35*(b*x^2
+ a)^(3/2)*A*b/(a^2*x^5) - 1/6*(b*x^2 + a)^(3/2)*B/(a*x^6) - 1/7*(b*x^2 +
a)^(3/2)*A/(a*x^7)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 676 vs. $2(170) = 340$.

Time = 0.14 (sec) , antiderivative size = 676, normalized size of antiderivative = 3.38

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{x^8}\,dx = -\frac{(2\,Dab^2-Bb^3)\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{8\sqrt{-a}a^2} + \frac{210\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{13}Dab^2 - 105\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{13}Bb^3 + 840\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{11}Da^2b^2 + 7}{}$$

input
```
integrate((b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/x^8,x, algorithm="giac")
```

output
```
-1/8*(2*D*a*b^2 - B*b^3)*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/(
sqrt(-a)*a^2) + 1/840*(210*(sqrt(b)*x - sqrt(b*x^2 + a))^13*D*a*b^2 - 105*
(sqrt(b)*x - sqrt(b*x^2 + a))^13*B*b^3 + 840*(sqrt(b)*x - sqrt(b*x^2 + a))
^11*D*a^2*b^2 + 700*(sqrt(b)*x - sqrt(b*x^2 + a))^11*B*a*b^3 + 3360*(sqrt(
b)*x - sqrt(b*x^2 + a))^10*C*a^2*b^(5/2) - 2310*(sqrt(b)*x - sqrt(b*x^2 +
a))^9*D*a^3*b^2 + 3395*(sqrt(b)*x - sqrt(b*x^2 + a))^9*B*a^2*b^3 - 5600*(s
qrt(b)*x - sqrt(b*x^2 + a))^8*C*a^3*b^(5/2) + 8960*(sqrt(b)*x - sqrt(b*x^2
 + a))^8*A*a^2*b^(7/2) + 2240*(sqrt(b)*x - sqrt(b*x^2 + a))^6*C*a^4*b^(5/2
) + 4480*(sqrt(b)*x - sqrt(b*x^2 + a))^6*A*a^3*b^(7/2) + 2310*(sqrt(b)*x -
 sqrt(b*x^2 + a))^5*D*a^5*b^2 - 3395*(sqrt(b)*x - sqrt(b*x^2 + a))^5*B*a^4
*b^3 - 1344*(sqrt(b)*x - sqrt(b*x^2 + a))^4*C*a^5*b^(5/2) + 2688*(sqrt(b)*
x - sqrt(b*x^2 + a))^4*A*a^4*b^(7/2) - 840*(sqrt(b)*x - sqrt(b*x^2 + a))^3
*D*a^6*b^2 - 700*(sqrt(b)*x - sqrt(b*x^2 + a))^3*B*a^5*b^3 + 1568*(sqrt(b)
*x - sqrt(b*x^2 + a))^2*C*a^6*b^(5/2) - 896*(sqrt(b)*x - sqrt(b*x^2 + a))^
2*A*a^5*b^(7/2) - 210*(sqrt(b)*x - sqrt(b*x^2 + a))*D*a^7*b^2 + 105*(sqrt(
b)*x - sqrt(b*x^2 + a))*B*a^6*b^3 - 224*C*a^7*b^(5/2) + 128*A*a^6*b^(7/2))
/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^7*a^2)
```

## Mupad [B] (verification not implemented)

Time = 3.74 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.24

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{x^8}\,dx = \frac{B\,{\left(b\,x^2+a\right)}^{5/2}}{16\,a^2\,x^6} - \frac{B\,\sqrt{b\,x^2+a}}{16\,x^6} - \frac{b\,\left(\frac{\sqrt{b\,x^2+a}}{a\,x^2}-\frac{b\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{a^{3/2}}\right)\,D}{8} - \frac{\sqrt{b\,x^2+a}\,D}{4\,x^4} - \frac{B\,{\left(b\,x^2+a\right)}^{3/2}}{6\,a\,x^6} - \frac{A\,\sqrt{b\,x^2+a}}{7\,x^7} - \frac{A\,b\,\sqrt{b\,x^2+a}}{35\,a\,x^5} - \frac{C\,\sqrt{b\,x^2+a}\,\left(3\,a^2+a\,b\,x^2-2\,b^2\,x^4\right)}{15\,a^2\,x^5} + \frac{4\,A\,b^2\,\sqrt{b\,x^2+a}}{105\,a^2\,x^3} - \frac{8\,A\,b^3\,\sqrt{b\,x^2+a}}{105\,a^3\,x} + \frac{B\,b^3\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,1{}\mathrm{i}}{16\,a^{5/2}}$$

input
```
int(((a + b*x^2)^(1/2)*(A + B*x + C*x^2 + x^3*D))/x^8,x)
```

output
```
(B*b^3*atan(((a + b*x^2)^(1/2)*1i)/a^(1/2))*1i)/(16*a^(5/2)) - (B*(a + b*x
^2)^(1/2))/(16*x^6) - (b*((a + b*x^2)^(1/2)/(a*x^2) - (b*atanh((a + b*x^2)
^(1/2)/a^(1/2)))/a^(3/2))*D)/8 - ((a + b*x^2)^(1/2)*D)/(4*x^4) - (A*(a + b
*x^2)^(1/2))/(7*x^7) - (B*(a + b*x^2)^(3/2))/(6*a*x^6) + (B*(a + b*x^2)^(5
/2))/(16*a^2*x^6) - (A*b*(a + b*x^2)^(1/2))/(35*a*x^5) - (C*(a + b*x^2)^(1
/2)*(3*a^2 - 2*b^2*x^4 + a*b*x^2))/(15*a^2*x^5) + (4*A*b^2*(a + b*x^2)^(1/
2))/(105*a^2*x^3) - (8*A*b^3*(a + b*x^2)^(1/2))/(105*a^3*x)
```

## Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.84

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{x^8}\,dx$$
$$= \frac{-240\sqrt{b\,x^2+a}\,a^4-48\sqrt{b\,x^2+a}\,a^3b\,x^2-280\sqrt{b\,x^2+a}\,a^3bx-336\sqrt{b\,x^2+a}\,a^3c\,x^2-420\sqrt{b\,x^2+a}\,a^3d}{}$$

input
```
int((b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/x^8,x)
```

output
```
( - 240*sqrt(a + b*x**2)*a**4 - 48*sqrt(a + b*x**2)*a**3*b*x**2 - 280*sqrt
(a + b*x**2)*a**3*b*x - 336*sqrt(a + b*x**2)*a**3*c*x**2 - 420*sqrt(a + b*
x**2)*a**3*d*x**3 + 64*sqrt(a + b*x**2)*a**2*b**2*x**4 - 70*sqrt(a + b*x**
2)*a**2*b**2*x**3 - 112*sqrt(a + b*x**2)*a**2*b*c*x**4 - 210*sqrt(a + b*x*
*2)*a**2*b*d*x**5 - 128*sqrt(a + b*x**2)*a*b**3*x**6 + 105*sqrt(a + b*x**2
)*a*b**3*x**5 + 224*sqrt(a + b*x**2)*a*b**2*c*x**6 - 210*sqrt(a)*log((sqrt
(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b**2*d*x**7 + 105*sqrt(a)*l
og((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*b**4*x**7 + 210*sqrt(
a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b**2*d*x**7 - 1
05*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*b**4*x**7
 + 128*sqrt(b)*a*b**3*x**7 - 224*sqrt(b)*a*b**2*c*x**7)/(1680*a**3*x**7)
```

# 3.73 $\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{x^9} dx$



## Optimal result

Integrand size = 30, antiderivative size = 235

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{x^9} dx = -\frac{A\sqrt{a+bx^2}}{8x^8} - \frac{(Ab+8aC)\sqrt{a+bx^2}}{48ax^6} + \frac{b(5Ab-8aC)\sqrt{a+bx^2}}{192a^2x^4} - \frac{b^2(5Ab-8aC)\sqrt{a+bx^2}}{128a^3x^2} - \frac{B(a+bx^2)^{3/2}}{7ax^7} + \frac{(4bB-7aD)(a+bx^2)^{3/2}}{35a^2x^5} - \frac{2b(4bB-7aD)(a+bx^2)^{3/2}}{105a^3x^3} + \frac{b^3(5Ab-8aC)\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{128a^{7/2}}$$

output
```
-1/8*A*(b*x^2+a)^(1/2)/x^8-1/48*(A*b+8*C*a)*(b*x^2+a)^(1/2)/a/x^6+1/192*b*
(5*A*b-8*C*a)*(b*x^2+a)^(1/2)/a^2/x^4-1/128*b^2*(5*A*b-8*C*a)*(b*x^2+a)^(1
/2)/a^3/x^2-1/7*B*(b*x^2+a)^(3/2)/a/x^7+1/35*(4*B*b-7*D*a)*(b*x^2+a)^(3/2)
/a^2/x^5-2/105*b*(4*B*b-7*D*a)*(b*x^2+a)^(3/2)/a^3/x^3+1/128*b^3*(5*A*b-8*
C*a)*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(7/2)
```

## Mathematica [A] (verified)

Time = 1.99 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.75

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{x^9}\,dx = -\frac{\sqrt{a+bx^2}(b^3x^6(525A+1024Bx)+16a^3(105A+4x(30B+7x(5C+6Dx)))+8a^2bx^2(35A+2x(24B}{13440a^3x^8} + \frac{b^3(-5Ab+8aC)\operatorname{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right)}{64a^{7/2}}$$

input
```
Integrate[(Sqrt[a + b*x^2]*(A + B*x + C*x^2 + D*x^3))/x^9,x]
```

output
```
-1/13440*(Sqrt[a + b*x^2]*(b^3*x^6*(525*A + 1024*B*x) + 16*a^3*(105*A + 4*
x*(30*B + 7*x*(5*C + 6*D*x))) + 8*a^2*b*x^2*(35*A + 2*x*(24*B + 7*x*(5*C +
 8*D*x))) - 2*a*b^2*x^4*(175*A + 4*x*(64*B + 7*x*(15*C + 32*D*x)))))/(a^3*
x^8) + (b^3*(-5*A*b + 8*a*C)*ArcTanh[(Sqrt[b]*x - Sqrt[a + b*x^2])/Sqrt[a]
])/(64*a^(7/2))
```

## Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.06, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2338, 25, 2338, 27, 539, 27, 539, 27, 539, 25, 534, 243, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)}{x^9}dx$$

$$\Big\downarrow \text{2338}$$

$$-\frac{\int -\frac{\sqrt{bx^2+a}\left(8aDx^2-(5Ab-8aC)x+8aB\right)}{x^8}dx}{8a}-\frac{A\left(a+bx^2\right)^{3/2}}{8ax^8}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\sqrt{bx^2+a}(8aDx^2-(5Ab-8aC)x+8aB)}{x^8}dx}{8a}-\frac{A\left(a+bx^2\right)^{3/2}}{8ax^8}$$

$\Big\downarrow$ 2338

$$\frac{-\frac{\int \frac{a(7(5Ab-8aC)+8(4bB-7aD)x)\sqrt{bx^2+a}}{x^7}dx}{7a}-\frac{8B\left(a+bx^2\right)^{3/2}}{7x^7}}{8a}-\frac{A\left(a+bx^2\right)^{3/2}}{8ax^8}$$

$\Big\downarrow$ 27

$$\frac{-\frac{1}{7}\int \frac{(7(5Ab-8aC)+8(4bB-7aD)x)\sqrt{bx^2+a}}{x^7}dx-\frac{8B\left(a+bx^2\right)^{3/2}}{7x^7}}{8a}-\frac{A\left(a+bx^2\right)^{3/2}}{8ax^8}$$

$\Big\downarrow$ 539

$$\frac{\frac{1}{7}\left(\frac{\int -\frac{3(16a(4bB-7aD)-7b(5Ab-8aC)x)\sqrt{bx^2+a}}{x^6}dx}{6a}+\frac{7\left(a+bx^2\right)^{3/2}(5Ab-8aC)}{6ax^6}\right)-\frac{8B\left(a+bx^2\right)^{3/2}}{7x^7}}{8a}-\frac{A\left(a+bx^2\right)^{3/2}}{8ax^8}$$

$\Big\downarrow$ 27

$$\frac{\frac{1}{7}\left(\frac{7\left(a+bx^2\right)^{3/2}(5Ab-8aC)}{6ax^6}-\frac{\int \frac{(16a(4bB-7aD)-7b(5Ab-8aC)x)\sqrt{bx^2+a}}{x^6}dx}{2a}\right)-\frac{8B\left(a+bx^2\right)^{3/2}}{7x^7}}{8a}-\frac{A\left(a+bx^2\right)^{3/2}}{8ax^8}$$

$\Big\downarrow$ 539

$$\frac{\frac{1}{7}\left(\frac{7\left(a+bx^2\right)^{3/2}(5Ab-8aC)}{6ax^6}-\frac{-\frac{\int \frac{ab(35(5Ab-8aC)+32(4bB-7aD)x)\sqrt{bx^2+a}}{x^5}dx}{5a}-\frac{16\left(a+bx^2\right)^{3/2}(4bB-7aD)}{5x^5}}{2a}\right)-\frac{8B\left(a+bx^2\right)^{3/2}}{7x^7}}{8a}-\frac{A\left(a+bx^2\right)^{3/2}}{8ax^8}$$

$\Big\downarrow$ 27

$$\frac{\frac{1}{7}\left(\frac{7\left(a+bx^2\right)^{3/2}(5Ab-8aC)}{6ax^6}-\frac{-\frac{1}{5}b\int \frac{(35(5Ab-8aC)+32(4bB-7aD)x)\sqrt{bx^2+a}}{x^5}dx-\frac{16\left(a+bx^2\right)^{3/2}(4bB-7aD)}{5x^5}}{2a}\right)-\frac{8B\left(a+bx^2\right)^{3/2}}{7x^7}}{8a}-\frac{A\left(a+bx^2\right)^{3/2}}{8ax^8}$$

$\Big\downarrow$ 539

$$\frac{\frac{1}{7}\left(\frac{7\left(a+bx^2\right)^{3/2}(5Ab-8aC)}{6ax^6}-\frac{-\frac{1}{5}b\left(-\frac{\int -\frac{(128a(4bB-7aD)-35b(5Ab-8aC)x)\sqrt{bx^2+a}}{x^4}dx}{4a}-\frac{35\left(a+bx^2\right)^{3/2}(5Ab-8aC)}{4ax^4}\right)-\frac{16\left(a+bx^2\right)^{3/2}(4bB-7aD)}{5x^5}}{2a}\right.}{8a}$$

$$\frac{A\left(a+bx^2\right)^{3/2}}{8ax^8}$$

$\Big\downarrow$ 25

$$\frac{\frac{1}{7}\left(\frac{7\left(a+bx^2\right)^{3/2}(5Ab-8aC)}{6ax^6}-\frac{-\frac{1}{5}b\left(\frac{\int \frac{(128a(4bB-7aD)-35b(5Ab-8aC)x)\sqrt{bx^2+a}}{x^4}dx}{4a}-\frac{35\left(a+bx^2\right)^{3/2}(5Ab-8aC)}{4ax^4}\right)-\frac{16\left(a+bx^2\right)^{3/2}(4bB-7aD)}{5x^5}}{2a}\right)}{8a}$$

$$\frac{A\left(a+bx^2\right)^{3/2}}{8ax^8}$$

$\Big\downarrow$ 534

$$\frac{\frac{1}{7}\left(\frac{7\left(a+bx^2\right)^{3/2}(5Ab-8aC)}{6ax^6}-\frac{-\frac{1}{5}b\left(\frac{-35b(5Ab-8aC)\int \frac{\sqrt{bx^2+a}}{x^3}dx-\frac{128\left(a+bx^2\right)^{3/2}(4bB-7aD)}{3x^3}}{4a}-\frac{35\left(a+bx^2\right)^{3/2}(5Ab-8aC)}{4ax^4}\right)-\frac{16\left(a+bx^2\right)^{3/2}(4bB}{5x^5}}{2a}\right.}{8a}$$

$$\frac{A\left(a+bx^2\right)^{3/2}}{8ax^8}$$

$\Big\downarrow$ 243

$$\frac{\frac{1}{7}\left(\frac{7\left(a+bx^2\right)^{3/2}(5Ab-8aC)}{6ax^6}-\frac{-\frac{1}{5}b\left(\frac{-\frac{35}{2}b(5Ab-8aC)\int \frac{\sqrt{bx^2+a}}{x^4}dx^2-\frac{128\left(a+bx^2\right)^{3/2}(4bB-7aD)}{3x^3}}{4a}-\frac{35\left(a+bx^2\right)^{3/2}(5Ab-8aC)}{4ax^4}\right)-\frac{16\left(a+bx^2\right)^{3/2}(4}{5x^5}}{2a}\right.}{8a}$$

$$\frac{A\left(a+bx^2\right)^{3/2}}{8ax^8}$$

$\Big\downarrow$ 51

$$\frac{\frac{1}{7}\left(\frac{7(a+bx^2)^{3/2}(5Ab-8aC)}{6ax^6}-\frac{-\frac{1}{5}b\left(\frac{-\frac{35}{2}b(5Ab-8aC)\left(\frac{1}{2}b\int\frac{1}{x^2\sqrt{bx^2+a}}dx^2-\frac{\sqrt{a+bx^2}}{x^2}\right)-\frac{128(a+bx^2)^{3/2}(4bB-7aD)}{3x^3}}{4a}-\frac{35(a+bx^2)^{3/2}(5Ab-8aC)}{4ax^4}\right)}{2a}\right)}{8a}$$

$$\frac{A(a+bx^2)^{3/2}}{8ax^8}$$

$\downarrow$ 73

$$\frac{\frac{1}{7}\left(\frac{7(a+bx^2)^{3/2}(5Ab-8aC)}{6ax^6}-\frac{-\frac{1}{5}b\left(\frac{-\frac{35}{2}b(5Ab-8aC)\left(\int\frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}-\frac{\sqrt{a+bx^2}}{x^2}\right)-\frac{128(a+bx^2)^{3/2}(4bB-7aD)}{3x^3}}{4a}-\frac{35(a+bx^2)^{3/2}(5Ab-8aC)}{4ax^4}\right)}{2a}\right)}{8a}$$

$$\frac{A(a+bx^2)^{3/2}}{8ax^8}$$

$\downarrow$ 221

$$\frac{\frac{1}{7}\left(\frac{7(a+bx^2)^{3/2}(5Ab-8aC)}{6ax^6}-\frac{-\frac{1}{5}b\left(\frac{-\frac{35}{2}b(5Ab-8aC)\left(-\frac{b\,\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\sqrt{a+bx^2}}{x^2}\right)-\frac{128(a+bx^2)^{3/2}(4bB-7aD)}{3x^3}}{4a}-\frac{35(a+bx^2)^{3/2}(5Ab-8}{4ax^4}\right)}{2a}\right)}{8a}$$

$$\frac{A(a+bx^2)^{3/2}}{8ax^8}$$

input
```
Int[(Sqrt[a + b*x^2]*(A + B*x + C*x^2 + D*x^3))/x^9,x]
```

output
```
-1/8*(A*(a + b*x^2)^(3/2))/(a*x^8) + ((-8*B*(a + b*x^2)^(3/2))/(7*x^7) + (
(7*(5*A*b - 8*a*C)*(a + b*x^2)^(3/2))/(6*a*x^6) - ((-16*(4*b*B - 7*a*D)*(a
 + b*x^2)^(3/2))/(5*x^5) - (b*((-35*(5*A*b - 8*a*C)*(a + b*x^2)^(3/2))/(4*
a*x^4) + ((-128*(4*b*B - 7*a*D)*(a + b*x^2)^(3/2))/(3*x^3) - (35*b*(5*A*b
- 8*a*C)*(-(Sqrt[a + b*x^2]/x^2) - (b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sq
rt[a]))/2)/(4*a)))/5)/(2*a))/7)/(8*a)
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 539
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[c*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))
   Int[x^(m + 1)*(a + b*x^2)^p*(a*d*(m + 1) - b*c*(m + 2*p + 3)*x), x], x]
/; FreeQ[{a, b, c, d, p}, x] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

## Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.49

| method | result |
|---|---|
| default | $A\left(-\frac{(b\,x^2+a)^{\frac{3}{2}}}{8a\,x^8}-\frac{5b\left(-\frac{(b\,x^2+a)^{\frac{3}{2}}}{6a\,x^6}-\frac{b\left(-\frac{(b\,x^2+a)^{\frac{3}{2}}}{4a\,x^4}-\frac{b\left(-\frac{(b\,x^2+a)^{\frac{3}{2}}}{2a\,x^2}+\frac{b\left(\sqrt{b\,x^2+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)\right)}{2a}\right)}{4a}\right)}{2a}\right)}{8a}\right)+$ |

input

```
int((b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/x^9,x,method=_RETURNVERBOSE)
```

output

```
A*(-1/8/a/x^8*(b*x^2+a)^(3/2)-5/8*b/a*(-1/6/a/x^6*(b*x^2+a)^(3/2)-1/2*b/a*
(-1/4/a/x^4*(b*x^2+a)^(3/2)-1/4*b/a*(-1/2/a/x^2*(b*x^2+a)^(3/2)+1/2*b/a*((
b*x^2+a)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x))))))+B*(-1/7/
a/x^7*(b*x^2+a)^(3/2)-4/7*b/a*(-1/5/a/x^5*(b*x^2+a)^(3/2)+2/15*b/a^2*(b*x^
2+a)^(3/2)/x^3))+C*(-1/6/a/x^6*(b*x^2+a)^(3/2)-1/2*b/a*(-1/4/a/x^4*(b*x^2+
a)^(3/2)-1/4*b/a*(-1/2/a/x^2*(b*x^2+a)^(3/2)+1/2*b/a*((b*x^2+a)^(1/2)-a^(1
/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)))))+D*(-1/5/a/x^5*(b*x^2+a)^(3/2
)+2/15*b/a^2*(b*x^2+a)^(3/2)/x^3)
```

## Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 410, normalized size of antiderivative = 1.74

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{x^9}\,dx$$
$$= \left[-\frac{105\,(8\,Cab^3-5\,Ab^4)\sqrt{a}x^8\log\left(-\frac{bx^2+2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)-2\,(256\,(7\,Da^2b^2-4\,Bab^3)x^7+105\,(8\,Ca^2b^2-\right.$$

input
```
integrate((b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/x^9,x, algorithm="fricas")
```

output
```
[-1/26880*(105*(8*C*a*b^3 - 5*A*b^4)*sqrt(a)*x^8*log(-(b*x^2 + 2*sqrt(b*x^
2 + a)*sqrt(a) + 2*a)/x^2) - 2*(256*(7*D*a^2*b^2 - 4*B*a*b^3)*x^7 + 105*(8
*C*a^2*b^2 - 5*A*a*b^3)*x^6 - 1920*B*a^4*x - 128*(7*D*a^3*b - 4*B*a^2*b^2)
*x^5 - 1680*A*a^4 - 70*(8*C*a^3*b - 5*A*a^2*b^2)*x^4 - 384*(7*D*a^4 + B*a^
3*b)*x^3 - 280*(8*C*a^4 + A*a^3*b)*x^2)*sqrt(b*x^2 + a))/(a^4*x^8), 1/1344
0*(105*(8*C*a*b^3 - 5*A*b^4)*sqrt(-a)*x^8*arctan(sqrt(b*x^2 + a)*sqrt(-a)/
a) + (256*(7*D*a^2*b^2 - 4*B*a*b^3)*x^7 + 105*(8*C*a^2*b^2 - 5*A*a*b^3)*x^
6 - 1920*B*a^4*x - 128*(7*D*a^3*b - 4*B*a^2*b^2)*x^5 - 1680*A*a^4 - 70*(8*
C*a^3*b - 5*A*a^2*b^2)*x^4 - 384*(7*D*a^4 + B*a^3*b)*x^3 - 280*(8*C*a^4 +
A*a^3*b)*x^2)*sqrt(b*x^2 + a))/(a^4*x^8)]
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 729 vs. $2(221) = 442$.

Time = 30.78 (sec) , antiderivative size = 729, normalized size of antiderivative = 3.10

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{x^9}\,dx = -\frac{Aa}{8\sqrt{b}x^9\sqrt{\frac{a}{bx^2}+1}} - \frac{7A\sqrt{b}}{48x^7\sqrt{\frac{a}{bx^2}+1}} + \frac{Ab^{\frac{3}{2}}}{192ax^5\sqrt{\frac{a}{bx^2}+1}} - \frac{5Ab^{\frac{5}{2}}}{384a^2x^3\sqrt{\frac{a}{bx^2}+1}} - \frac{5Ab^{\frac{7}{2}}}{128a^3x\sqrt{\frac{a}{bx^2}+1}} + \frac{5Ab^4\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{128a^{\frac{7}{2}}} - \frac{15Ba^5b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}} - \frac{33Ba^4b^{\frac{11}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}} - \frac{17Ba^3b^{\frac{13}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}} - \frac{3Ba^2b^{\frac{15}{2}}x^6\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}} - \frac{12Bab^{\frac{17}{2}}x^8\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}} - \frac{8Bb^{\frac{19}{2}}x^{10}\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}} - \frac{Ca}{6\sqrt{b}x^7\sqrt{\frac{a}{bx^2}+1}} - \frac{5C\sqrt{b}}{24x^5\sqrt{\frac{a}{bx^2}+1}} + \frac{Cb^{\frac{3}{2}}}{48ax^3\sqrt{\frac{a}{bx^2}+1}} + \frac{Cb^{\frac{5}{2}}}{16a^2x\sqrt{\frac{a}{bx^2}+1}} - \frac{Cb^3\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{16a^{\frac{5}{2}}} - \frac{D\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{5x^4} - \frac{Db^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{15ax^2} + \frac{2Db^{\frac{5}{2}}\sqrt{\frac{a}{bx^2}+1}}{15a^2}$$

input
```
integrate((b*x**2+a)**(1/2)*(D*x**3+C*x**2+B*x+A)/x**9,x)
```

output
```
-A*a/(8*sqrt(b)*x**9*sqrt(a/(b*x**2) + 1)) - 7*A*sqrt(b)/(48*x**7*sqrt(a/(
b*x**2) + 1)) + A*b**(3/2)/(192*a*x**5*sqrt(a/(b*x**2) + 1)) - 5*A*b**(5/2
)/(384*a**2*x**3*sqrt(a/(b*x**2) + 1)) - 5*A*b**(7/2)/(128*a**3*x*sqrt(a/(
b*x**2) + 1)) + 5*A*b**4*asinh(sqrt(a)/(sqrt(b)*x))/(128*a**(7/2)) - 15*B*
a**5*b**(9/2)*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**
8 + 105*a**3*b**6*x**10) - 33*B*a**4*b**(11/2)*x**2*sqrt(a/(b*x**2) + 1)/(
105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 17*B*a**3
*b**(13/2)*x**4*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x
**8 + 105*a**3*b**6*x**10) - 3*B*a**2*b**(15/2)*x**6*sqrt(a/(b*x**2) + 1)/
(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 12*B*a*b
**(17/2)*x**8*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**
8 + 105*a**3*b**6*x**10) - 8*B*b**(19/2)*x**10*sqrt(a/(b*x**2) + 1)/(105*a
**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - C*a/(6*sqrt(b)
*x**7*sqrt(a/(b*x**2) + 1)) - 5*C*sqrt(b)/(24*x**5*sqrt(a/(b*x**2) + 1)) +
 C*b**(3/2)/(48*a*x**3*sqrt(a/(b*x**2) + 1)) + C*b**(5/2)/(16*a**2*x*sqrt(
a/(b*x**2) + 1)) - C*b**3*asinh(sqrt(a)/(sqrt(b)*x))/(16*a**(5/2)) - D*sqr
t(b)*sqrt(a/(b*x**2) + 1)/(5*x**4) - D*b**(3/2)*sqrt(a/(b*x**2) + 1)/(15*a
*x**2) + 2*D*b**(5/2)*sqrt(a/(b*x**2) + 1)/(15*a**2)
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.32

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{x^9}\,dx = -\frac{Cb^3\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{16\,a^{\frac{5}{2}}} + \frac{5\,Ab^4\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{128\,a^{\frac{7}{2}}} + \frac{\sqrt{bx^2+a}Cb^3}{16\,a^3} - \frac{5\,\sqrt{bx^2+a}Ab^4}{128\,a^4} - \frac{(bx^2+a)^{\frac{3}{2}}Cb^2}{16\,a^3x^2} + \frac{5\,(bx^2+a)^{\frac{3}{2}}Ab^3}{128\,a^4x^2} + \frac{2\,(bx^2+a)^{\frac{3}{2}}Db}{15\,a^2x^3} - \frac{8\,(bx^2+a)^{\frac{3}{2}}Bb^2}{105\,a^3x^3} + \frac{(bx^2+a)^{\frac{3}{2}}Cb}{8\,a^2x^4} - \frac{5\,(bx^2+a)^{\frac{3}{2}}Ab^2}{64\,a^3x^4} - \frac{(bx^2+a)^{\frac{3}{2}}D}{5\,ax^5} + \frac{4\,(bx^2+a)^{\frac{3}{2}}Bb}{35\,a^2x^5} - \frac{(bx^2+a)^{\frac{3}{2}}C}{6\,ax^6} + \frac{5\,(bx^2+a)^{\frac{3}{2}}Ab}{48\,a^2x^6} - \frac{(bx^2+a)^{\frac{3}{2}}B}{7\,ax^7} - \frac{(bx^2+a)^{\frac{3}{2}}A}{8\,ax^8}$$

input
```
integrate((b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/x^9,x, algorithm="maxima")
```

output
```
-1/16*C*b^3*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(5/2) + 5/128*A*b^4*arcsinh(a/
(sqrt(a*b)*abs(x)))/a^(7/2) + 1/16*sqrt(b*x^2 + a)*C*b^3/a^3 - 5/128*sqrt(
b*x^2 + a)*A*b^4/a^4 - 1/16*(b*x^2 + a)^(3/2)*C*b^2/(a^3*x^2) + 5/128*(b*x
^2 + a)^(3/2)*A*b^3/(a^4*x^2) + 2/15*(b*x^2 + a)^(3/2)*D*b/(a^2*x^3) - 8/1
05*(b*x^2 + a)^(3/2)*B*b^2/(a^3*x^3) + 1/8*(b*x^2 + a)^(3/2)*C*b/(a^2*x^4)
 - 5/64*(b*x^2 + a)^(3/2)*A*b^2/(a^3*x^4) - 1/5*(b*x^2 + a)^(3/2)*D/(a*x^5
) + 4/35*(b*x^2 + a)^(3/2)*B*b/(a^2*x^5) - 1/6*(b*x^2 + a)^(3/2)*C/(a*x^6)
 + 5/48*(b*x^2 + a)^(3/2)*A*b/(a^2*x^6) - 1/7*(b*x^2 + a)^(3/2)*B/(a*x^7)
- 1/8*(b*x^2 + a)^(3/2)*A/(a*x^8)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 844 vs. $2(199) = 398$.

Time = 0.13 (sec) , antiderivative size = 844, normalized size of antiderivative = 3.59

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{x^9}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/x^9,x, algorithm="giac")
```

output
```
1/64*(8*C*a*b^3 - 5*A*b^4)*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))
/(sqrt(-a)*a^3) - 1/6720*(840*(sqrt(b)*x - sqrt(b*x^2 + a))^15*C*a*b^3 - 5
25*(sqrt(b)*x - sqrt(b*x^2 + a))^15*A*b^4 - 6440*(sqrt(b)*x - sqrt(b*x^2 +
 a))^13*C*a^2*b^3 + 4025*(sqrt(b)*x - sqrt(b*x^2 + a))^13*A*a*b^4 - 26880*
(sqrt(b)*x - sqrt(b*x^2 + a))^12*D*a^3*b^(5/2) - 21560*(sqrt(b)*x - sqrt(b
*x^2 + a))^11*C*a^3*b^3 - 13405*(sqrt(b)*x - sqrt(b*x^2 + a))^11*A*a^2*b^4
 + 71680*(sqrt(b)*x - sqrt(b*x^2 + a))^10*D*a^4*b^(5/2) - 71680*(sqrt(b)*x
 - sqrt(b*x^2 + a))^10*B*a^3*b^(7/2) + 27160*(sqrt(b)*x - sqrt(b*x^2 + a))
^9*C*a^4*b^3 - 97615*(sqrt(b)*x - sqrt(b*x^2 + a))^9*A*a^3*b^4 - 62720*(sq
rt(b)*x - sqrt(b*x^2 + a))^8*D*a^5*b^(5/2) + 35840*(sqrt(b)*x - sqrt(b*x^2
 + a))^8*B*a^4*b^(7/2) + 27160*(sqrt(b)*x - sqrt(b*x^2 + a))^7*C*a^5*b^3 -
 97615*(sqrt(b)*x - sqrt(b*x^2 + a))^7*A*a^4*b^4 + 28672*(sqrt(b)*x - sqrt
(b*x^2 + a))^6*D*a^6*b^(5/2) + 14336*(sqrt(b)*x - sqrt(b*x^2 + a))^6*B*a^5
*b^(7/2) - 21560*(sqrt(b)*x - sqrt(b*x^2 + a))^5*C*a^6*b^3 - 13405*(sqrt(b
)*x - sqrt(b*x^2 + a))^5*A*a^5*b^4 - 23296*(sqrt(b)*x - sqrt(b*x^2 + a))^4
*D*a^7*b^(5/2) + 28672*(sqrt(b)*x - sqrt(b*x^2 + a))^4*B*a^6*b^(7/2) - 644
0*(sqrt(b)*x - sqrt(b*x^2 + a))^3*C*a^7*b^3 + 4025*(sqrt(b)*x - sqrt(b*x^2
 + a))^3*A*a^6*b^4 + 14336*(sqrt(b)*x - sqrt(b*x^2 + a))^2*D*a^8*b^(5/2) -
 8192*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a^7*b^(7/2) + 840*(sqrt(b)*x - sqr
t(b*x^2 + a))*C*a^8*b^3 - 525*(sqrt(b)*x - sqrt(b*x^2 + a))*A*a^7*b^4 -...
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{x^9}\,dx = \int \frac{\sqrt{b\,x^2+a}\,(A+B\,x+C\,x^2+x^3\,D)}{x^9}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(A + B*x + C*x^2 + x^3*D))/x^9,x)
```

output
```
int(((a + b*x^2)^(1/2)*(A + B*x + C*x^2 + x^3*D))/x^9, x)
```

## Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 400, normalized size of antiderivative = 1.70

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2+Dx^3)}{x^9}\,dx$$
$$= \frac{-1680\sqrt{b\,x^2+a}\,a^4-280\sqrt{b\,x^2+a}\,a^3b\,x^2-1920\sqrt{b\,x^2+a}\,a^3bx-2240\sqrt{b\,x^2+a}\,a^3c\,x^2-2688\sqrt{b\,x^2+}}{}$$

input
```
int((b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/x^9,x)
```

output
```
( - 1680*sqrt(a + b*x**2)*a**4 - 280*sqrt(a + b*x**2)*a**3*b*x**2 - 1920*s
qrt(a + b*x**2)*a**3*b*x - 2240*sqrt(a + b*x**2)*a**3*c*x**2 - 2688*sqrt(a
 + b*x**2)*a**3*d*x**3 + 350*sqrt(a + b*x**2)*a**2*b**2*x**4 - 384*sqrt(a
+ b*x**2)*a**2*b**2*x**3 - 560*sqrt(a + b*x**2)*a**2*b*c*x**4 - 896*sqrt(a
 + b*x**2)*a**2*b*d*x**5 - 525*sqrt(a + b*x**2)*a*b**3*x**6 + 512*sqrt(a +
 b*x**2)*a*b**3*x**5 + 840*sqrt(a + b*x**2)*a*b**2*c*x**6 + 1792*sqrt(a +
b*x**2)*a*b**2*d*x**7 - 1024*sqrt(a + b*x**2)*b**4*x**7 - 525*sqrt(a)*log(
(sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*b**4*x**8 + 840*sqrt(a)*
log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*b**3*c*x**8 + 525*sq
rt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*b**4*x**8 - 84
0*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*b**3*c*x**
8 - 1792*sqrt(b)*a*b**2*d*x**8 + 1024*sqrt(b)*b**4*x**8)/(13440*a**3*x**8)
```

# 3.74 $\int x^3(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 260

$$\int x^3\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx=-\frac{3a^3(2bB-aD)x\sqrt{a+bx^2}}{256b^3}+\frac{a^2(2bB-aD)x^3\sqrt{a+bx^2}}{128b^2}+\frac{a(2bB-aD)x^5\sqrt{a+bx^2}}{32b}+\frac{(2bB-aD)x^5(a+bx^2)^{3/2}}{16b}-\frac{a(Ab-aC)\left(a+bx^2\right)^{5/2}}{5b^3}+\frac{Dx^5(a+bx^2)^{5/2}}{10b}+\frac{(Ab-2aC)\left(a+bx^2\right)^{7/2}}{7b^3}+\frac{C(a+bx^2)^{9/2}}{9b^3}+\frac{3a^4(2bB-aD)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{256b^{7/2}}$$

output

```
-3/256*a^3*(2*B*b-D*a)*x*(b*x^2+a)^(1/2)/b^3+1/128*a^2*(2*B*b-D*a)*x^3*(b*
x^2+a)^(1/2)/b^2+1/32*a*(2*B*b-D*a)*x^5*(b*x^2+a)^(1/2)/b+1/16*(2*B*b-D*a)
*x^5*(b*x^2+a)^(3/2)/b-1/5*a*(A*b-C*a)*(b*x^2+a)^(5/2)/b^3+1/10*D*x^5*(b*x
^2+a)^(5/2)/b+1/7*(A*b-2*C*a)*(b*x^2+a)^(7/2)/b^3+1/9*C*(b*x^2+a)^(9/2)/b^
3+3/256*a^4*(2*B*b-D*a)*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(7/2)
```

## Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.73

$$\int x^3 (a+bx^2)^{3/2} (A+Bx+Cx^2+Dx^3)\,dx = \frac{\sqrt{b}\sqrt{a+bx^2}(a^4(2048C+945Dx)+32b^4x^6(360A+7x(45B+40Cx+36Dx^2))+12a^2b^2x^2(\cdots}{\cdots}$$

input
```
Integrate[x^3*(a + b*x^2)^(3/2)*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(Sqrt[b]*Sqrt[a + b*x^2]*(a^4*(2048*C + 945*D*x) + 32*b^4*x^6*(360*A + 7*x
*(45*B + 40*C*x + 36*D*x^2)) + 12*a^2*b^2*x^2*(192*A + x*(105*B + 64*C*x +
 42*D*x^2)) - 2*a^3*b*(2304*A + x*(945*B + 512*C*x + 315*D*x^2)) + 16*a*b^
3*x^4*(1152*A + x*(945*B + 800*C*x + 693*D*x^2))) + 945*a^4*(-2*b*B + a*D)
*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(80640*b^(7/2))
```

## Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.02, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2340, 27, 2340, 27, 533, 533, 25, 27, 533, 27, 455, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 (a+bx^2)^{3/2} (A+Bx+Cx^2+Dx^3)\,dx$$

$$\Big\downarrow \text{2340}$$

$$\frac{\int 5x^3 (bx^2+a)^{3/2} (2bCx^2+(2bB-aD)x+2Ab)\,dx}{10b}+\frac{Dx^5(a+bx^2)^{5/2}}{10b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int x^3 (bx^2+a)^{3/2} (2bCx^2+(2bB-aD)x+2Ab)\,dx}{2b}+\frac{Dx^5(a+bx^2)^{5/2}}{10b}$$

$$\Big\downarrow \text{2340}$$

$$\frac{\frac{\int bx^3(2(9Ab-4aC)+9(2bB-aD)x)\left(bx^2+a\right)^{3/2}dx}{9b}+\frac{2}{9}Cx^4\left(a+bx^2\right)^{5/2}}{2b}+\frac{Dx^5\left(a+bx^2\right)^{5/2}}{10b}$$

$\Big\downarrow$ 27

$$\frac{\frac{1}{9}\int x^3(2(9Ab-4aC)+9(2bB-aD)x)\left(bx^2+a\right)^{3/2}dx+\frac{2}{9}Cx^4\left(a+bx^2\right)^{5/2}}{2b}+\frac{Dx^5\left(a+bx^2\right)^{5/2}}{10b}$$

$\Big\downarrow$ 533

$$\frac{\frac{1}{9}\left(\frac{9x^3\left(a+bx^2\right)^{5/2}(2bB-aD)}{8b}-\frac{\int x^2(27a(2bB-aD)-16b(9Ab-4aC)x)\left(bx^2+a\right)^{3/2}dx}{8b}\right)+\frac{2}{9}Cx^4\left(a+bx^2\right)^{5/2}}{2b}+\frac{Dx^5\left(a+bx^2\right)^{5/2}}{10b}$$

$\Big\downarrow$ 533

$$\frac{\frac{1}{9}\left(\frac{9x^3\left(a+bx^2\right)^{5/2}(2bB-aD)}{8b}-\frac{-\frac{\int -abx(32(9Ab-4aC)+189(2bB-aD)x)\left(bx^2+a\right)^{3/2}dx}{7b}-\frac{16}{7}x^2\left(a+bx^2\right)^{5/2}(9Ab-4aC)}{8b}\right)+\frac{2}{9}Cx^4\left(a+bx^2\right)^{5/2}}{2b}+\frac{Dx^5\left(a+bx^2\right)^{5/2}}{10b}$$

$\Big\downarrow$ 25

$$\frac{\frac{1}{9}\left(\frac{9x^3\left(a+bx^2\right)^{5/2}(2bB-aD)}{8b}-\frac{\frac{\int abx(32(9Ab-4aC)+189(2bB-aD)x)\left(bx^2+a\right)^{3/2}dx}{7b}-\frac{16}{7}x^2\left(a+bx^2\right)^{5/2}(9Ab-4aC)}{8b}\right)+\frac{2}{9}Cx^4\left(a+bx^2\right)^{5/2}}{2b}+\frac{Dx^5\left(a+bx^2\right)^{5/2}}{10b}$$

$\Big\downarrow$ 27

$$\frac{\frac{1}{9}\left(\frac{9x^3\left(a+bx^2\right)^{5/2}(2bB-aD)}{8b}-\frac{\frac{1}{7}a\int x(32(9Ab-4aC)+189(2bB-aD)x)\left(bx^2+a\right)^{3/2}dx-\frac{16}{7}x^2\left(a+bx^2\right)^{5/2}(9Ab-4aC)}{8b}\right)+\frac{2}{9}Cx^4\left(a+bx^2\right)^{5/2}}{2b}+\frac{Dx^5\left(a+bx^2\right)^{5/2}}{10b}$$

$\Big\downarrow$ 533

$$\frac{\frac{1}{9}\left(\frac{9x^3\left(a+bx^2\right)^{5/2}(2bB-aD)}{8b}-\frac{\frac{1}{7}a\left(\frac{63x\left(a+bx^2\right)^{5/2}(2bB-aD)}{2b}-\frac{\int 3(63a(2bB-aD)-64b(9Ab-4aC)x)\left(bx^2+a\right)^{3/2}dx}{6b}\right)-\frac{16}{7}x^2\left(a+bx^2\right)^{5/2}(9Ab-4 \cdots}{8b}\cdots}{2b}$$

$$\frac{Dx^5\left(a+bx^2\right)^{5/2}}{10b}$$

$\Big\downarrow$ 27

$$\frac{\frac{1}{9}\left(\frac{9x^3\left(a+bx^2\right)^{5/2}(2bB-aD)}{8b}-\frac{\frac{1}{7}a\left(\frac{63x\left(a+bx^2\right)^{5/2}(2bB-aD)}{2b}-\frac{\int (63a(2bB-aD)-64b(9Ab-4aC)x)\left(bx^2+a\right)^{3/2}dx}{2b}\right)-\frac{16}{7}x^2\left(a+bx^2\right)^{5/2}(9Ab-4aC \cdots}{8b}\cdots}{2b}$$

$$\frac{Dx^5\left(a+bx^2\right)^{5/2}}{10b}$$

$\Big\downarrow$ 455

$$\frac{\frac{1}{9}\left(\frac{9x^3\left(a+bx^2\right)^{5/2}(2bB-aD)}{8b}-\frac{\frac{1}{7}a\left(\frac{63x\left(a+bx^2\right)^{5/2}(2bB-aD)}{2b}-\frac{63a(2bB-aD)\int\left(bx^2+a\right)^{3/2}dx-\frac{64}{5}\left(a+bx^2\right)^{5/2}(9Ab-4aC)}{2b}\right)-\frac{16}{7}x^2\left(a+bx^2\right)^{5/2}( \cdots}{8b}\cdots}{2b}$$

$$\frac{Dx^5\left(a+bx^2\right)^{5/2}}{10b}$$

$\Big\downarrow$ 211

$$\frac{\frac{1}{9}\left(\frac{9x^3\left(a+bx^2\right)^{5/2}(2bB-aD)}{8b}-\frac{\frac{1}{7}a\left(\frac{63x\left(a+bx^2\right)^{5/2}(2bB-aD)}{2b}-\frac{63a(2bB-aD)\left(\frac{3}{4}a\int\sqrt{bx^2+a}dx+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)-\frac{64}{5}\left(a+bx^2\right)^{5/2}(9Ab-4aC)}{2b}\right)- \cdots}{8b}\cdots}{2b}$$

$$\frac{Dx^5\left(a+bx^2\right)^{5/2}}{10b}$$

$\Big\downarrow$ 211

$$\frac{\frac{1}{9}\left(\frac{9x^3\left(a+bx^2\right)^{5/2}(2bB-aD)}{8b}-\frac{\frac{1}{7}a\left(\frac{63x\left(a+bx^2\right)^{5/2}(2bB-aD)}{2b}-\frac{63a(2bB-aD)\left(\frac{3}{4}a\left(\frac{1}{2}a\int\frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)-\frac{64}{5}\left(a+bx^2\right.}{2b}\right.}{8b}\right.}{2b}$$

$$\frac{Dx^5\left(a+bx^2\right)^{5/2}}{10b}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{1}{9}\left(\frac{9x^3\left(a+bx^2\right)^{5/2}(2bB-aD)}{8b}-\frac{\frac{1}{7}a\left(\frac{63x\left(a+bx^2\right)^{5/2}(2bB-aD)}{2b}-\frac{63a(2bB-aD)\left(\frac{3}{4}a\left(\frac{1}{2}a\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)-}{2b}\right.}{8b}\right.}{2b}$$

$$\frac{Dx^5\left(a+bx^2\right)^{5/2}}{10b}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{1}{9}\left(\frac{9x^3\left(a+bx^2\right)^{5/2}(2bB-aD)}{8b}-\frac{\frac{1}{7}a\left(\frac{63x\left(a+bx^2\right)^{5/2}(2bB-aD)}{2b}-\frac{63a\left(\frac{3}{4}a\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)(2bB-aD)-\frac{6}{5}}{2b}\right.}{8b}\right.}{2b}$$

$$\frac{Dx^5\left(a+bx^2\right)^{5/2}}{10b}$$

input
```
Int[x^3*(a + b*x^2)^(3/2)*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(D*x^5*(a + b*x^2)^(5/2))/(10*b) + ((2*C*x^4*(a + b*x^2)^(5/2))/9 + ((9*(2
*b*B - a*D)*x^3*(a + b*x^2)^(5/2))/(8*b) - ((-16*(9*A*b - 4*a*C)*x^2*(a +
b*x^2)^(5/2))/7 + (a*((63*(2*b*B - a*D)*x*(a + b*x^2)^(5/2))/(2*b) - ((-64
*(9*A*b - 4*a*C)*(a + b*x^2)^(5/2))/5 + 63*a*(2*b*B - a*D)*((x*(a + b*x^2)
^(3/2))/4 + (3*a*((x*Sqrt[a + b*x^2])/2 + (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a +
b*x^2]])/(2*Sqrt[b])))/4))/(2*b)))/7)/(8*b))/9)/(2*b)
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

rule 2340
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Simp[1/(b*(m
+ q + 2*p + 1))   Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)
*Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /;
GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ
[Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

## Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.22

| method | result |
|---|---|
| default | $A\left(\frac{x^{2}\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{7b}-\frac{2a\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{35b^{2}}\right)+B\left(\frac{x^{3}\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{8b}-\frac{3a\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{6b}-\frac{a\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{\right.}}{2\sqrt{b}}\right.}{4}\right.}{6b}\right.}{8b}\right.$ |

input
```
int(x^3*(b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
A*(1/7*x^2*(b*x^2+a)^(5/2)/b-2/35*a/b^2*(b*x^2+a)^(5/2))+B*(1/8*x^3*(b*x^2
+a)^(5/2)/b-3/8*a/b*(1/6*x*(b*x^2+a)^(5/2)/b-1/6*a/b*(1/4*x*(b*x^2+a)^(3/2
)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
))))+C*(1/9*x^4*(b*x^2+a)^(5/2)/b-4/9*a/b*(1/7*x^2*(b*x^2+a)^(5/2)/b-2/35*
a/b^2*(b*x^2+a)^(5/2)))+D*(1/10*x^5*(b*x^2+a)^(5/2)/b-1/2*a/b*(1/8*x^3*(b*
x^2+a)^(5/2)/b-3/8*a/b*(1/6*x*(b*x^2+a)^(5/2)/b-1/6*a/b*(1/4*x*(b*x^2+a)^(
3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/
2)))))))
```

## Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 477, normalized size of antiderivative = 1.83

$$\int x^3(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)\,dx = \left[-\frac{945\,(Da^5-2\,Ba^4b)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\,(8064\,Db^5x^9+8960\,Cb^5x^8}{}\right.$$

input
```
integrate(x^3*(b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
[-1/161280*(945*(D*a^5 - 2*B*a^4*b)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 +
a)*sqrt(b)*x - a) - 2*(8064*D*b^5*x^9 + 8960*C*b^5*x^8 + 1008*(11*D*a*b^4
+ 10*B*b^5)*x^7 + 1280*(10*C*a*b^4 + 9*A*b^5)*x^6 + 2048*C*a^4*b - 4608*A*
a^3*b^2 + 504*(D*a^2*b^3 + 30*B*a*b^4)*x^5 + 768*(C*a^2*b^3 + 24*A*a*b^4)*
x^4 - 630*(D*a^3*b^2 - 2*B*a^2*b^3)*x^3 - 256*(4*C*a^3*b^2 - 9*A*a^2*b^3)*
x^2 + 945*(D*a^4*b - 2*B*a^3*b^2)*x)*sqrt(b*x^2 + a))/b^4, 1/80640*(945*(D
*a^5 - 2*B*a^4*b)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (8064*D*b^
5*x^9 + 8960*C*b^5*x^8 + 1008*(11*D*a*b^4 + 10*B*b^5)*x^7 + 1280*(10*C*a*b
^4 + 9*A*b^5)*x^6 + 2048*C*a^4*b - 4608*A*a^3*b^2 + 504*(D*a^2*b^3 + 30*B*
a*b^4)*x^5 + 768*(C*a^2*b^3 + 24*A*a*b^4)*x^4 - 630*(D*a^3*b^2 - 2*B*a^2*b
^3)*x^3 - 256*(4*C*a^3*b^2 - 9*A*a^2*b^3)*x^2 + 945*(D*a^4*b - 2*B*a^3*b^2
)*x)*sqrt(b*x^2 + a))/b^4]
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 483 vs. $2(236) = 472$.

Time = 0.88 (sec) , antiderivative size = 483, normalized size of antiderivative = 1.86

$$\int x^3(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)\,dx = \begin{cases} \frac{3a^2\left(Ba^2-\frac{5a\left(2Bab+Da^2-\frac{7a\left(Bb^2+\frac{11Dab}{10}\right)}{8b}\right)}{6b}\right)\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right)}{8b^2}+\sqrt{a+bx^2}\left(\frac{Cbx^8}{9}+\right. \\ a^{\frac{3}{2}}\left(\frac{Ax^4}{4}+\frac{Bx^5}{5}+\frac{Cx^6}{6}+\frac{Dx^7}{7}\right) \end{cases}$$

input

```
integrate(x**3*(b*x**2+a)**(3/2)*(D*x**3+C*x**2+B*x+A),x)
```

output

```
Piecewise((3*a**2*(B*a**2 - 5*a*(2*B*a*b + D*a**2 - 7*a*(B*b**2 + 11*D*a*b
/10)/(8*b))/(6*b))*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt
(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True))/(8*b**2) + sqrt(a + b*x**2)
*(C*b*x**8/9 + D*b*x**9/10 - 3*a*x*(B*a**2 - 5*a*(2*B*a*b + D*a**2 - 7*a*(
B*b**2 + 11*D*a*b/10)/(8*b))/(6*b))/(8*b**2) - 2*a*(A*a**2 - 4*a*(2*A*a*b
+ C*a**2 - 6*a*(A*b**2 + 10*C*a*b/9)/(7*b))/(5*b))/(3*b**2) + x**7*(B*b**2
 + 11*D*a*b/10)/(8*b) + x**6*(A*b**2 + 10*C*a*b/9)/(7*b) + x**5*(2*B*a*b +
 D*a**2 - 7*a*(B*b**2 + 11*D*a*b/10)/(8*b))/(6*b) + x**4*(2*A*a*b + C*a**2
 - 6*a*(A*b**2 + 10*C*a*b/9)/(7*b))/(5*b) + x**3*(B*a**2 - 5*a*(2*B*a*b +
D*a**2 - 7*a*(B*b**2 + 11*D*a*b/10)/(8*b))/(6*b))/(4*b) + x**2*(A*a**2 - 4
*a*(2*A*a*b + C*a**2 - 6*a*(A*b**2 + 10*C*a*b/9)/(7*b))/(5*b))/(3*b)), Ne(
b, 0)), (a**(3/2)*(A*x**4/4 + B*x**5/5 + C*x**6/6 + D*x**7/7), True))
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.13

$$\int x^3 \left(a+bx^2\right)^{3/2} \left(A+Bx+Cx^2+Dx^3\right)\,dx = \frac{\left(bx^2+a\right)^{\frac{5}{2}} D x^5}{10\,b} + \frac{\left(bx^2+a\right)^{\frac{5}{2}} C x^4}{9\,b} - \frac{\left(bx^2+a\right)^{\frac{5}{2}} D a x^3}{16\,b^2} + \frac{\left(bx^2+a\right)^{\frac{5}{2}} B x^3}{8\,b} - \frac{4\left(bx^2+a\right)^{\frac{5}{2}} C a x^2}{63\,b^2} + \frac{\left(bx^2+a\right)^{\frac{5}{2}} A x^2}{7\,b} + \frac{\left(bx^2+a\right)^{\frac{5}{2}} D a^2 x}{32\,b^3} - \frac{\left(bx^2+a\right)^{\frac{3}{2}} D a^3 x}{128\,b^3} - \frac{3\sqrt{bx^2+a}\,D a^4 x}{256\,b^3} - \frac{\left(bx^2+a\right)^{\frac{5}{2}} B a x}{16\,b^2} + \frac{\left(bx^2+a\right)^{\frac{3}{2}} B a^2 x}{64\,b^2} + \frac{3\sqrt{bx^2+a}\,B a^3 x}{128\,b^2} - \frac{3\,D a^5 \operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{256\,b^{\frac{7}{2}}} + \frac{3\,B a^4 \operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\,b^{\frac{5}{2}}} + \frac{8\left(bx^2+a\right)^{\frac{5}{2}} C a^2}{315\,b^3} - \frac{2\left(bx^2+a\right)^{\frac{5}{2}} A a}{35\,b^2}$$

input
```
integrate(x^3*(b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
1/10*(b*x^2 + a)^(5/2)*D*x^5/b + 1/9*(b*x^2 + a)^(5/2)*C*x^4/b - 1/16*(b*x
^2 + a)^(5/2)*D*a*x^3/b^2 + 1/8*(b*x^2 + a)^(5/2)*B*x^3/b - 4/63*(b*x^2 +
a)^(5/2)*C*a*x^2/b^2 + 1/7*(b*x^2 + a)^(5/2)*A*x^2/b + 1/32*(b*x^2 + a)^(5
/2)*D*a^2*x/b^3 - 1/128*(b*x^2 + a)^(3/2)*D*a^3*x/b^3 - 3/256*sqrt(b*x^2 +
 a)*D*a^4*x/b^3 - 1/16*(b*x^2 + a)^(5/2)*B*a*x/b^2 + 1/64*(b*x^2 + a)^(3/2
)*B*a^2*x/b^2 + 3/128*sqrt(b*x^2 + a)*B*a^3*x/b^2 - 3/256*D*a^5*arcsinh(b*
x/sqrt(a*b))/b^(7/2) + 3/128*B*a^4*arcsinh(b*x/sqrt(a*b))/b^(5/2) + 8/315*
(b*x^2 + a)^(5/2)*C*a^2/b^3 - 2/35*(b*x^2 + a)^(5/2)*A*a/b^2
```

## Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.00

$$\int x^3 \left(a+bx^2\right)^{3/2} \left(A+Bx+Cx^2+Dx^3\right)\,dx = \frac{1}{80640}\sqrt{bx^2+a}\left(\left(2\left(\left(4\left(\left(2\left(7\left(8\left(9\,Dbx+10\,Cb\right)x+\frac{9\left(11\,Dab^8+10\,Bb^9\right)}{b^8}\right)x+\frac{80\,(10}{}\right.\right.\right.\right.\right.\right.\right.$$

$$+\frac{3\left(Da^5-2\,Ba^4b\right)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{256\,b^{\frac{7}{2}}}$$

input
```
integrate(x^3*(b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

output
```
1/80640*sqrt(b*x^2 + a)*((2*((4*((2*(7*(8*(9*D*b*x + 10*C*b)*x + 9*(11*D*a
*b^8 + 10*B*b^9)/b^8)*x + 80*(10*C*a*b^8 + 9*A*b^9)/b^8)*x + 63*(D*a^2*b^7
 + 30*B*a*b^8)/b^8)*x + 96*(C*a^2*b^7 + 24*A*a*b^8)/b^8)*x - 315*(D*a^3*b^
6 - 2*B*a^2*b^7)/b^8)*x - 128*(4*C*a^3*b^6 - 9*A*a^2*b^7)/b^8)*x + 945*(D*
a^4*b^5 - 2*B*a^3*b^6)/b^8)*x + 512*(4*C*a^4*b^5 - 9*A*a^3*b^6)/b^8) + 3/2
56*(D*a^5 - 2*B*a^4*b)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(7/2)
```

## Mupad [F(-1)]

Timed out.

$$\int x^3(a+bx^2)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx=\int x^3\,\left(b\,x^2+a\right)^{3/2}\,\left(A+B\,x+C\,x^2+x^3\,D\right)\,dx$$

input
```
int(x^3*(a + b*x^2)^(3/2)*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
int(x^3*(a + b*x^2)^(3/2)*(A + B*x + C*x^2 + x^3*D), x)
```

## Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.47

$$\int x^3(a+bx^2)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{-4608\sqrt{b\,x^2+a}\,a^4b^2+2048\sqrt{b\,x^2+a}\,a^4bc+945\sqrt{b\,x^2+a}\,a^4bdx+2304\sqrt{b\,x^2+a}\,a^3b^3x^2-}{}$$

input
```
int(x^3*(b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A),x)
```

output
```
( - 4608*sqrt(a + b*x**2)*a**4*b**2 + 2048*sqrt(a + b*x**2)*a**4*b*c + 945
*sqrt(a + b*x**2)*a**4*b*d*x + 2304*sqrt(a + b*x**2)*a**3*b**3*x**2 - 1890
*sqrt(a + b*x**2)*a**3*b**3*x - 1024*sqrt(a + b*x**2)*a**3*b**2*c*x**2 - 6
30*sqrt(a + b*x**2)*a**3*b**2*d*x**3 + 18432*sqrt(a + b*x**2)*a**2*b**4*x*
*4 + 1260*sqrt(a + b*x**2)*a**2*b**4*x**3 + 768*sqrt(a + b*x**2)*a**2*b**3
*c*x**4 + 504*sqrt(a + b*x**2)*a**2*b**3*d*x**5 + 11520*sqrt(a + b*x**2)*a
*b**5*x**6 + 15120*sqrt(a + b*x**2)*a*b**5*x**5 + 12800*sqrt(a + b*x**2)*a
*b**4*c*x**6 + 11088*sqrt(a + b*x**2)*a*b**4*d*x**7 + 10080*sqrt(a + b*x**
2)*b**6*x**7 + 8960*sqrt(a + b*x**2)*b**5*c*x**8 + 8064*sqrt(a + b*x**2)*b
**5*d*x**9 - 945*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**5*
d + 1890*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**4*b**2)/(8
0640*b**4)
```

# 3.75 $\int x^2(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 227

$$\int x^2(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)\,dx=\frac{a^2(8Ab-3aC)x\sqrt{a+bx^2}}{128b^2}+\frac{a(8Ab-3aC)x^3\sqrt{a+bx^2}}{64b}+\frac{(8Ab-3aC)x^3(a+bx^2)^{3/2}}{48b}-\frac{a(bB-aD)(a+bx^2)^{5/2}}{5b^3}+\frac{Cx^3(a+bx^2)^{5/2}}{8b}+\frac{(bB-2aD)(a+bx^2)^{7/2}}{7b^3}+\frac{D(a+bx^2)^{9/2}}{9b^3}-\frac{a^3(8Ab-3aC)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{5/2}}$$

output
```
1/128*a^2*(8*A*b-3*C*a)*x*(b*x^2+a)^(1/2)/b^2+1/64*a*(8*A*b-3*C*a)*x^3*(b*
x^2+a)^(1/2)/b+1/48*(8*A*b-3*C*a)*x^3*(b*x^2+a)^(3/2)/b-1/5*a*(B*b-D*a)*(b
*x^2+a)^(5/2)/b^3+1/8*C*x^3*(b*x^2+a)^(5/2)/b+1/7*(B*b-2*D*a)*(b*x^2+a)^(7
/2)/b^3+1/9*D*(b*x^2+a)^(9/2)/b^3-1/128*a^3*(8*A*b-3*C*a)*arctanh(b^(1/2)*
x/(b*x^2+a)^(1/2))/b^(5/2)
```

## Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.77

$$\int x^2 \left(a+bx^2\right)^{3/2} \left(A+Bx+Cx^2+Dx^3\right)\,dx = \frac{\sqrt{a+bx^2}(1024a^4D-a^3b(2304B+x(945C+512Dx))+8ab^3x^3(1470A+x(1152B+945Cx}{}$$

input
```
Integrate[x^2*(a + b*x^2)^(3/2)*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(Sqrt[a + b*x^2]*(1024*a^4*D - a^3*b*(2304*B + x*(945*C + 512*D*x)) + 8*a*
b^3*x^3*(1470*A + x*(1152*B + 945*C*x + 800*D*x^2)) + 80*b^4*x^5*(84*A + x
*(72*B + 7*x*(9*C + 8*D*x))) + 6*a^2*b^2*x*(420*A + x*(192*B + x*(105*C +
64*D*x)))) - 315*a^3*Sqrt[b]*(-8*A*b + 3*a*C)*Log[-(Sqrt[b]*x) + Sqrt[a +
b*x^2]])/(40320*b^3)
```

## Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.01, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.367$, Rules used = {2340, 2340, 27, 533, 533, 27, 455, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+bx^2\right)^{3/2} \left(A+Bx+Cx^2+Dx^3\right)\,dx$$

$$\Big\downarrow \text{2340}$$

$$\frac{\int x^2 \left(bx^2+a\right)^{3/2} \left(9bCx^2+(9bB-4aD)x+9Ab\right)dx}{9b}+\frac{Dx^4\left(a+bx^2\right)^{5/2}}{9b}$$

$$\Big\downarrow \text{2340}$$

$$\frac{\frac{\int bx^2(9(8Ab-3aC)+8(9bB-4aD)x)\left(bx^2+a\right)^{3/2}dx}{8b}+\frac{9}{8}Cx^3\left(a+bx^2\right)^{5/2}}{9b}+\frac{Dx^4\left(a+bx^2\right)^{5/2}}{9b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{8}\int x^2(9(8Ab-3aC)+8(9bB-4aD)x)\left(bx^2+a\right)^{3/2}dx+\frac{9}{8}Cx^3\left(a+bx^2\right)^{5/2}}{9b}+\frac{Dx^4\left(a+bx^2\right)^{5/2}}{9b}$$

$\Big\downarrow$ 533

$$\frac{\frac{1}{8}\left(\frac{8x^2\left(a+bx^2\right)^{5/2}(9bB-4aD)}{7b}-\frac{\int x(16a(9bB-4aD)-63b(8Ab-3aC)x)\left(bx^2+a\right)^{3/2}dx}{7b}\right)+\frac{9}{8}Cx^3\left(a+bx^2\right)^{5/2}}{9b}+\frac{Dx^4\left(a+bx^2\right)^{5/2}}{9b}$$

$\Big\downarrow$ 533

$$\frac{\frac{1}{8}\left(\frac{8x^2\left(a+bx^2\right)^{5/2}(9bB-4aD)}{7b}-\frac{-\frac{\int -3ab(21(8Ab-3aC)+32(9bB-4aD)x)\left(bx^2+a\right)^{3/2}dx}{6b}-\frac{21}{2}x\left(a+bx^2\right)^{5/2}(8Ab-3aC)}{7b}\right)+\frac{9}{8}Cx^3\left(a+bx^2\right)}{9b}\frac{Dx^4\left(a+bx^2\right)^{5/2}}{9b}$$

$\Big\downarrow$ 27

$$\frac{\frac{1}{8}\left(\frac{8x^2\left(a+bx^2\right)^{5/2}(9bB-4aD)}{7b}-\frac{\frac{1}{2}a\int(21(8Ab-3aC)+32(9bB-4aD)x)\left(bx^2+a\right)^{3/2}dx-\frac{21}{2}x\left(a+bx^2\right)^{5/2}(8Ab-3aC)}{7b}\right)+\frac{9}{8}Cx^3\left(a+bx^2\right.}{9b}\frac{Dx^4\left(a+bx^2\right)^{5/2}}{9b}$$

$\Big\downarrow$ 455

$$\frac{\frac{1}{8}\left(\frac{8x^2\left(a+bx^2\right)^{5/2}(9bB-4aD)}{7b}-\frac{\frac{1}{2}a\left(21(8Ab-3aC)\int\left(bx^2+a\right)^{3/2}dx+\frac{32\left(a+bx^2\right)^{5/2}(9bB-4aD)}{5b}\right)-\frac{21}{2}x\left(a+bx^2\right)^{5/2}(8Ab-3aC)}{7b}\right)+\frac{9}{8}Cx^{3}}{9b}\frac{Dx^4\left(a+bx^2\right)^{5/2}}{9b}$$

$\Big\downarrow$ 211

$$\frac{\frac{1}{8}\left(\frac{8x^2(a+bx^2)^{5/2}(9bB-4aD)}{7b}-\frac{\frac{1}{2}a\left(21(8Ab-3aC)\left(\frac{3}{4}a\int\sqrt{bx^2+a}dx+\frac{1}{4}x(a+bx^2)^{3/2}\right)+\frac{32(a+bx^2)^{5/2}(9bB-4aD)}{5b}\right)-\frac{21}{2}x(a+bx^2)^{5/2}(8Ab-}{7b}\right.}{9b}$$

$$\frac{Dx^4(a+bx^2)^{5/2}}{9b}$$

$\Big\downarrow$ 211

$$\frac{\frac{1}{8}\left(\frac{8x^2(a+bx^2)^{5/2}(9bB-4aD)}{7b}-\frac{\frac{1}{2}a\left(21(8Ab-3aC)\left(\frac{3}{4}a\left(\frac{1}{2}a\int\frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x(a+bx^2)^{3/2}\right)+\frac{32(a+bx^2)^{5/2}(9bB-4aD)}{5b}\right)-\frac{2}{}}{7b}\right.}{9b}$$

$$\frac{Dx^4(a+bx^2)^{5/2}}{9b}$$

$\Big\downarrow$ 224

$$\frac{\frac{1}{8}\left(\frac{8x^2(a+bx^2)^{5/2}(9bB-4aD)}{7b}-\frac{\frac{1}{2}a\left(21(8Ab-3aC)\left(\frac{3}{4}a\left(\frac{1}{2}a\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x(a+bx^2)^{3/2}\right)+\frac{32(a+bx^2)^{5/2}(9bB-}{5b}}{7b}\right.}{9b}$$

$$\frac{Dx^4(a+bx^2)^{5/2}}{9b}$$

$\Big\downarrow$ 219

$$\frac{\frac{1}{8}\left(\frac{8x^2(a+bx^2)^{5/2}(9bB-4aD)}{7b}-\frac{\frac{1}{2}a\left(21(8Ab-3aC)\left(\frac{3}{4}a\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x(a+bx^2)^{3/2}\right)+\frac{32(a+bx^2)^{5/2}(9bB-4a}{5b}}{7b}\right.}{9b}$$

$$\frac{Dx^4(a+bx^2)^{5/2}}{9b}$$

input
```
Int[x^2*(a + b*x^2)^(3/2)*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(D*x^4*(a + b*x^2)^(5/2))/(9*b) + ((9*C*x^3*(a + b*x^2)^(5/2))/8 + ((8*(9*
b*B - 4*a*D)*x^2*(a + b*x^2)^(5/2))/(7*b) - ((-21*(8*A*b - 3*a*C)*x*(a + b
*x^2)^(5/2))/2 + (a*((32*(9*b*B - 4*a*D)*(a + b*x^2)^(5/2))/(5*b) + 21*(8*
A*b - 3*a*C)*((x*(a + b*x^2)^(3/2))/4 + (3*a*((x*Sqrt[a + b*x^2])/2 + (a*A
rcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b])))/4)))/2)/(7*b))/8)/(9*b)
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

rule 2340
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Simp[1/(b*(m
+ q + 2*p + 1))   Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)
*Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /;
GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ
[Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

## Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.19

| method | result |
|---|---|
| default | $A\left(\frac{x(b\,x^2+a)^{\frac{5}{2}}}{6b}-\frac{a\left(\frac{x\left(b\,x^2+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)}{4}\right)}{6b}\right)+B\left(\frac{x^2(b\,x^2+a)^{\frac{5}{2}}}{7b}-\frac{2a(b\,x^2+a)^{\frac{5}{2}}}{35b^2}\right)+$ |

input
```
int(x^2*(b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
A*(1/6*x*(b*x^2+a)^(5/2)/b-1/6*a/b*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*
x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))))+B*(1/7*x^2*(b*
x^2+a)^(5/2)/b-2/35*a/b^2*(b*x^2+a)^(5/2))+C*(1/8*x^3*(b*x^2+a)^(5/2)/b-3/
8*a/b*(1/6*x*(b*x^2+a)^(5/2)/b-1/6*a/b*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x
*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))))))+D*(1/9*x^
4*(b*x^2+a)^(5/2)/b-4/9*a/b*(1/7*x^2*(b*x^2+a)^(5/2)/b-2/35*a/b^2*(b*x^2+a
)^(5/2)))
```

## Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 428, normalized size of antiderivative = 1.89

$$\int x^2(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)\,dx = \left[-\frac{315\,(3\,Ca^4-8\,Aa^3b)\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\,(4480\,Db^4x^8+5040\,Cb^4x}{}\right.$$

$$-\frac{315\,(3\,Ca^4-8\,Aa^3b)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(4480\,Db^4x^8+5040\,Cb^4x^7+640\,(10\,Dab^3+9\,Bb^4)x^6+84}{}$$

input
```
integrate(x^2*(b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
[-1/80640*(315*(3*C*a^4 - 8*A*a^3*b)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 +
 a)*sqrt(b)*x - a) - 2*(4480*D*b^4*x^8 + 5040*C*b^4*x^7 + 640*(10*D*a*b^3
+ 9*B*b^4)*x^6 + 840*(9*C*a*b^3 + 8*A*b^4)*x^5 + 1024*D*a^4 - 2304*B*a^3*b
 + 384*(D*a^2*b^2 + 24*B*a*b^3)*x^4 + 210*(3*C*a^2*b^2 + 56*A*a*b^3)*x^3 -
 128*(4*D*a^3*b - 9*B*a^2*b^2)*x^2 - 315*(3*C*a^3*b - 8*A*a^2*b^2)*x)*sqrt
(b*x^2 + a))/b^3, -1/40320*(315*(3*C*a^4 - 8*A*a^3*b)*sqrt(-b)*arctan(sqrt
(-b)*x/sqrt(b*x^2 + a)) - (4480*D*b^4*x^8 + 5040*C*b^4*x^7 + 640*(10*D*a*b
^3 + 9*B*b^4)*x^6 + 840*(9*C*a*b^3 + 8*A*b^4)*x^5 + 1024*D*a^4 - 2304*B*a^
3*b + 384*(D*a^2*b^2 + 24*B*a*b^3)*x^4 + 210*(3*C*a^2*b^2 + 56*A*a*b^3)*x^
3 - 128*(4*D*a^3*b - 9*B*a^2*b^2)*x^2 - 315*(3*C*a^3*b - 8*A*a^2*b^2)*x)*s
qrt(b*x^2 + a))/b^3]
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 422 vs. $2(207) = 414$.

Time = 0.83 (sec) , antiderivative size = 422, normalized size of antiderivative = 1.86

$$\int x^2(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)\,dx = \begin{cases} -\frac{a\left(Aa^2-\frac{3a\left(2Aab+Ca^2-\frac{5a\left(Ab^2+\frac{9Cab}{8}\right)}{6b}\right)}{4b}\right)\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right)}{2b}+\sqrt{a+bx^2}\left(\frac{Cbx^7}{8}+\right. \\ a^{\frac{3}{2}}\left(\frac{Ax^3}{3}+\frac{Bx^4}{4}+\frac{Cx^5}{5}+\frac{Dx^6}{6}\right) \end{cases}$$

input
```
integrate(x**2*(b*x**2+a)**(3/2)*(D*x**3+C*x**2+B*x+A),x)
```

output
```
Piecewise((-a*(A*a**2 - 3*a*(2*A*a*b + C*a**2 - 5*a*(A*b**2 + 9*C*a*b/8)/(
6*b))/(4*b))*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), N
e(a, 0)), (x*log(x)/sqrt(b*x**2), True))/(2*b) + sqrt(a + b*x**2)*(C*b*x**
7/8 + D*b*x**8/9 - 2*a*(B*a**2 - 4*a*(2*B*a*b + D*a**2 - 6*a*(B*b**2 + 10*
D*a*b/9)/(7*b))/(5*b))/(3*b**2) + x**6*(B*b**2 + 10*D*a*b/9)/(7*b) + x**5*
(A*b**2 + 9*C*a*b/8)/(6*b) + x**4*(2*B*a*b + D*a**2 - 6*a*(B*b**2 + 10*D*a
*b/9)/(7*b))/(5*b) + x**3*(2*A*a*b + C*a**2 - 5*a*(A*b**2 + 9*C*a*b/8)/(6*
b))/(4*b) + x**2*(B*a**2 - 4*a*(2*B*a*b + D*a**2 - 6*a*(B*b**2 + 10*D*a*b/
9)/(7*b))/(5*b))/(3*b) + x*(A*a**2 - 3*a*(2*A*a*b + C*a**2 - 5*a*(A*b**2 +
 9*C*a*b/8)/(6*b))/(4*b))/(2*b)), Ne(b, 0)), (a**(3/2)*(A*x**3/3 + B*x**4/
4 + C*x**5/5 + D*x**6/6), True))
```

## Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.11

$$\int x^2(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)\,dx = \frac{(bx^2+a)^{\frac{5}{2}}Dx^4}{9\,b}+\frac{(bx^2+a)^{\frac{5}{2}}Cx^3}{8\,b} - \frac{4\,(bx^2+a)^{\frac{5}{2}}Dax^2}{63\,b^2}+\frac{(bx^2+a)^{\frac{5}{2}}Bx^2}{7\,b}-\frac{(bx^2+a)^{\frac{5}{2}}Cax}{16\,b^2}+\frac{(bx^2+a)^{\frac{3}{2}}Ca^2x}{64\,b^2} + \frac{3\sqrt{bx^2+a}Ca^3x}{128\,b^2}+\frac{(bx^2+a)^{\frac{5}{2}}Ax}{6\,b}-\frac{(bx^2+a)^{\frac{3}{2}}Aax}{24\,b}-\frac{\sqrt{bx^2+a}Aa^2x}{16\,b} + \frac{3\,Ca^4\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\,b^{\frac{5}{2}}}-\frac{Aa^3\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{3}{2}}}+\frac{8\,(bx^2+a)^{\frac{5}{2}}Da^2}{315\,b^3}-\frac{2\,(bx^2+a)^{\frac{5}{2}}Ba}{35\,b^2}$$

input
```
integrate(x^2*(b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
1/9*(b*x^2 + a)^(5/2)*D*x^4/b + 1/8*(b*x^2 + a)^(5/2)*C*x^3/b - 4/63*(b*x^
2 + a)^(5/2)*D*a*x^2/b^2 + 1/7*(b*x^2 + a)^(5/2)*B*x^2/b - 1/16*(b*x^2 + a
)^(5/2)*C*a*x/b^2 + 1/64*(b*x^2 + a)^(3/2)*C*a^2*x/b^2 + 3/128*sqrt(b*x^2
+ a)*C*a^3*x/b^2 + 1/6*(b*x^2 + a)^(5/2)*A*x/b - 1/24*(b*x^2 + a)^(3/2)*A*
a*x/b - 1/16*sqrt(b*x^2 + a)*A*a^2*x/b + 3/128*C*a^4*arcsinh(b*x/sqrt(a*b)
)/b^(5/2) - 1/16*A*a^3*arcsinh(b*x/sqrt(a*b))/b^(3/2) + 8/315*(b*x^2 + a)^
(5/2)*D*a^2/b^3 - 2/35*(b*x^2 + a)^(5/2)*B*a/b^2
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.05

$$\int x^2(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)\,dx = \frac{1}{40320}\sqrt{bx^2+a}\Bigg(\Bigg(2\Bigg(\Bigg(\Bigg(4\Bigg(5\Bigg(2\Bigg(7\,(8\,Dbx+9\,Cb)x+\frac{8\,(10\,Dab^7+9\,Bb^8)}{b^7}\Bigg)x+\frac{21\,(9\,Cab^7}{b^7} - \frac{(3\,Ca^4-8\,Aa^3b)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{128\,b^{\frac{5}{2}}}$$

input
```
integrate(x^2*(b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

output
```
1/40320*sqrt(b*x^2 + a)*((2*((4*(5*(2*(7*(8*D*b*x + 9*C*b)*x + 8*(10*D*a*b
^7 + 9*B*b^8)/b^7)*x + 21*(9*C*a*b^7 + 8*A*b^8)/b^7)*x + 48*(D*a^2*b^6 + 2
4*B*a*b^7)/b^7)*x + 105*(3*C*a^2*b^6 + 56*A*a*b^7)/b^7)*x - 64*(4*D*a^3*b^
5 - 9*B*a^2*b^6)/b^7)*x - 315*(3*C*a^3*b^5 - 8*A*a^2*b^6)/b^7)*x + 256*(4*
D*a^4*b^4 - 9*B*a^3*b^5)/b^7) - 1/128*(3*C*a^4 - 8*A*a^3*b)*log(abs(-sqrt(
b)*x + sqrt(b*x^2 + a)))/b^(5/2)
```

## Mupad [F(-1)]

Timed out.

$$\int x^2(a+bx^2)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx=\int x^2\left(b\,x^2+a\right)^{3/2}\left(A+B\,x+C\,x^2+x^3\,D\right)\,dx$$

input
```
int(x^2*(a + b*x^2)^(3/2)*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
int(x^2*(a + b*x^2)^(3/2)*(A + B*x + C*x^2 + x^3*D), x)
```

## Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.49

$$\int x^2(a+bx^2)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{1024\sqrt{b\,x^2+a}\,a^4d+2520\sqrt{b\,x^2+a}\,a^3b^2x-2304\sqrt{b\,x^2+a}\,a^3b^2-945\sqrt{b\,x^2+a}\,a^3bcx-512}{}$$

input
```
int(x^2*(b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A),x)
```

output
```
(1024*sqrt(a + b*x**2)*a**4*d + 2520*sqrt(a + b*x**2)*a**3*b**2*x - 2304*s
qrt(a + b*x**2)*a**3*b**2 - 945*sqrt(a + b*x**2)*a**3*b*c*x - 512*sqrt(a +
 b*x**2)*a**3*b*d*x**2 + 11760*sqrt(a + b*x**2)*a**2*b**3*x**3 + 1152*sqrt
(a + b*x**2)*a**2*b**3*x**2 + 630*sqrt(a + b*x**2)*a**2*b**2*c*x**3 + 384*
sqrt(a + b*x**2)*a**2*b**2*d*x**4 + 6720*sqrt(a + b*x**2)*a*b**4*x**5 + 92
16*sqrt(a + b*x**2)*a*b**4*x**4 + 7560*sqrt(a + b*x**2)*a*b**3*c*x**5 + 64
00*sqrt(a + b*x**2)*a*b**3*d*x**6 + 5760*sqrt(a + b*x**2)*b**5*x**6 + 5040
*sqrt(a + b*x**2)*b**4*c*x**7 + 4480*sqrt(a + b*x**2)*b**4*d*x**8 - 2520*s
qrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**4*b + 945*sqrt(b)*lo
g((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**4*c)/(40320*b**3)
```

# 3.76 $\int x(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)\,dx$



## Optimal result

Integrand size = 28, antiderivative size = 200

$$\int x(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)\,dx = \frac{a^2(8bB-3aD)x\sqrt{a+bx^2}}{128b^2} + \frac{a(8bB-3aD)x^3\sqrt{a+bx^2}}{64b} + \frac{(8bB-3aD)x^3(a+bx^2)^{3/2}}{48b} + \frac{(Ab-aC)(a+bx^2)^{5/2}}{5b^2} + \frac{Dx^3(a+bx^2)^{5/2}}{8b} + \frac{C(a+bx^2)^{7/2}}{7b^2} - \frac{a^3(8bB-3aD)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{5/2}}$$

output
```
1/128*a^2*(8*B*b-3*D*a)*x*(b*x^2+a)^(1/2)/b^2+1/64*a*(8*B*b-3*D*a)*x^3*(b*
x^2+a)^(1/2)/b+1/48*(8*B*b-3*D*a)*x^3*(b*x^2+a)^(3/2)/b+1/5*(A*b-C*a)*(b*x
^2+a)^(5/2)/b^2+1/8*D*x^3*(b*x^2+a)^(5/2)/b+1/7*C*(b*x^2+a)^(7/2)/b^2-1/12
8*a^3*(8*B*b-3*D*a)*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(5/2)
```

## Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.80

$$\int x(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)\,dx = \frac{\sqrt{b}\sqrt{a+bx^2}(-3a^3(256C+105Dx)+6a^2b(448A+x(140B+64Cx+35Dx^2))+8ab^2x^2(67}{}$$

input
```
Integrate[x*(a + b*x^2)^(3/2)*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(Sqrt[b]*Sqrt[a + b*x^2]*(-3*a^3*(256*C + 105*D*x) + 6*a^2*b*(448*A + x*(1
40*B + 64*C*x + 35*D*x^2)) + 8*a*b^2*x^2*(672*A + x*(490*B + 384*C*x + 315
*D*x^2)) + 16*b^3*x^4*(168*A + 5*x*(28*B + 3*x*(8*C + 7*D*x)))) - 105*a^3*
(-8*b*B + 3*a*D)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(13440*b^(5/2))
```

## Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.98, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {2340, 2340, 27, 533, 455, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)\,dx$$

$$\Big\downarrow \text{ 2340}$$

$$\frac{\int x(bx^2+a)^{3/2}(8bCx^2+(8bB-3aD)x+8Ab)\,dx}{8b}+\frac{Dx^3(a+bx^2)^{5/2}}{8b}$$

$$\Big\downarrow \text{ 2340}$$

$$\frac{\frac{\int bx(8(7Ab-2aC)+7(8bB-3aD)x)(bx^2+a)^{3/2}dx}{7b}+\frac{8}{7}Cx^2(a+bx^2)^{5/2}}{8b}+\frac{Dx^3(a+bx^2)^{5/2}}{8b}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{1}{7}\int x(8(7Ab-2aC)+7(8bB-3aD)x)\left(bx^2+a\right)^{3/2}dx+\frac{8}{7}Cx^2\left(a+bx^2\right)^{5/2}}{8b}+\frac{Dx^3\left(a+bx^2\right)^{5/2}}{8b}$$

$$\Big\downarrow \ 533$$

$$\frac{\frac{1}{7}\left(\frac{7x\left(a+bx^2\right)^{5/2}(8bB-3aD)}{6b}-\frac{\int (7a(8bB-3aD)-48b(7Ab-2aC)x)\left(bx^2+a\right)^{3/2}dx}{6b}\right)+\frac{8}{7}Cx^2\left(a+bx^2\right)^{5/2}}{8b}+\frac{Dx^3\left(a+bx^2\right)^{5/2}}{8b}$$

$$\Big\downarrow \ 455$$

$$\frac{\frac{1}{7}\left(\frac{7x\left(a+bx^2\right)^{5/2}(8bB-3aD)}{6b}-\frac{7a(8bB-3aD)\int \left(bx^2+a\right)^{3/2}dx-\frac{48}{5}\left(a+bx^2\right)^{5/2}(7Ab-2aC)}{6b}\right)+\frac{8}{7}Cx^2\left(a+bx^2\right)^{5/2}}{8b}+\frac{Dx^3\left(a+bx^2\right)^{5/2}}{8b}$$

$$\Big\downarrow \ 211$$

$$\frac{\frac{1}{7}\left(\frac{7x\left(a+bx^2\right)^{5/2}(8bB-3aD)}{6b}-\frac{7a(8bB-3aD)\left(\frac{3}{4}a\int \sqrt{bx^2+a}dx+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)-\frac{48}{5}\left(a+bx^2\right)^{5/2}(7Ab-2aC)}{6b}\right)+\frac{8}{7}Cx^2\left(a+bx^2\right)^{5/2}}{8b}$$

$$\frac{Dx^3\left(a+bx^2\right)^{5/2}}{8b}$$

$$\Big\downarrow \ 211$$

$$\frac{\frac{1}{7}\left(\frac{7x\left(a+bx^2\right)^{5/2}(8bB-3aD)}{6b}-\frac{7a(8bB-3aD)\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)-\frac{48}{5}\left(a+bx^2\right)^{5/2}(7Ab-2aC)}{6b}\right)+\frac{8}{7}}{8b}$$

$$\frac{Dx^3\left(a+bx^2\right)^{5/2}}{8b}$$

$$\Big\downarrow \ 224$$

$$\frac{\frac{1}{7}\left(\frac{7x\left(a+bx^2\right)^{5/2}(8bB-3aD)}{6b}-\frac{7a(8bB-3aD)\left(\frac{3}{4}a\left(\frac{1}{2}a\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)-\frac{48}{5}\left(a+bx^2\right)^{5/2}(7Ab-2aC}{6b}\right.}{8b}$$

$$\frac{Dx^3\left(a+bx^2\right)^{5/2}}{8b}$$

$\Big\downarrow$ 219

$$\frac{\frac{1}{7}\left(\frac{7x\left(a+bx^2\right)^{5/2}(8bB-3aD)}{6b}-\frac{7a\left(\frac{3}{4}a\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)(8bB-3aD)-\frac{48}{5}\left(a+bx^2\right)^{5/2}(7Ab-2aC)}{6b}\right.}{8b}$$

$$\frac{Dx^3\left(a+bx^2\right)^{5/2}}{8b}$$

input
```
Int[x*(a + b*x^2)^(3/2)*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(D*x^3*(a + b*x^2)^(5/2))/(8*b) + ((8*C*x^2*(a + b*x^2)^(5/2))/7 + ((7*(8*
b*B - 3*a*D)*x*(a + b*x^2)^(5/2))/(6*b) - ((-48*(7*A*b - 2*a*C)*(a + b*x^2
)^(5/2))/5 + 7*a*(8*b*B - 3*a*D)*((x*(a + b*x^2)^(3/2))/4 + (3*a*((x*Sqrt[
a + b*x^2])/2 + (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b])))/4))
/(6*b))/7)/(8*b)
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

rule 2340
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Simp[1/(b*(m
+ q + 2*p + 1))   Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)
*Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /;
GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ
[Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

## Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.13

| method | result |
|---|---|
| default | $\frac{A(b\,x^2+a)^{\frac{5}{2}}}{5b}+C\left(\frac{x^2(b\,x^2+a)^{\frac{5}{2}}}{7b}-\frac{2a(b\,x^2+a)^{\frac{5}{2}}}{35b^2}\right)+D\left(\frac{x^3(b\,x^2+a)^{\frac{5}{2}}}{8b}-\frac{3a\left(\frac{x(b\,x^2+a)^{\frac{5}{2}}}{6b}-\frac{a\left(\frac{x(b\,x^2+a)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x}}{2}\right.}{}\right.}{}\right)}{8b}\right.$ |

input
```
int(x*(b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
1/5*A/b*(b*x^2+a)^(5/2)+C*(1/7*x^2*(b*x^2+a)^(5/2)/b-2/35*a/b^2*(b*x^2+a)^
(5/2))+D*(1/8*x^3*(b*x^2+a)^(5/2)/b-3/8*a/b*(1/6*x*(b*x^2+a)^(5/2)/b-1/6*a
/b*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^
(1/2)*x+(b*x^2+a)^(1/2))))))+B*(1/6*x*(b*x^2+a)^(5/2)/b-1/6*a/b*(1/4*x*(b*
x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^
2+a)^(1/2)))))
```

## Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 390, normalized size of antiderivative = 1.95

$$\int x\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx=\left[-\frac{105\,(3\,Da^4-8\,Ba^3b)\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\,(1680\,Db^4x^7+1920\,Cb^4x^{\cdots}}{\cdots}\right.$$

$$-\frac{105\,(3\,Da^4-8\,Ba^3b)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(1680\,Db^4x^7+1920\,Cb^4x^6+280\,(9\,Dab^3+8\,Bb^4)x^5-76\cdots}{\cdots}$$

input
```
integrate(x*(b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
[-1/26880*(105*(3*D*a^4 - 8*B*a^3*b)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 +
 a)*sqrt(b)*x - a) - 2*(1680*D*b^4*x^7 + 1920*C*b^4*x^6 + 280*(9*D*a*b^3 +
 8*B*b^4)*x^5 - 768*C*a^3*b + 2688*A*a^2*b^2 + 384*(8*C*a*b^3 + 7*A*b^4)*x
^4 + 70*(3*D*a^2*b^2 + 56*B*a*b^3)*x^3 + 384*(C*a^2*b^2 + 14*A*a*b^3)*x^2
- 105*(3*D*a^3*b - 8*B*a^2*b^2)*x)*sqrt(b*x^2 + a))/b^3, -1/13440*(105*(3*
D*a^4 - 8*B*a^3*b)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (1680*D*b
^4*x^7 + 1920*C*b^4*x^6 + 280*(9*D*a*b^3 + 8*B*b^4)*x^5 - 768*C*a^3*b + 26
88*A*a^2*b^2 + 384*(8*C*a*b^3 + 7*A*b^4)*x^4 + 70*(3*D*a^2*b^2 + 56*B*a*b^
3)*x^3 + 384*(C*a^2*b^2 + 14*A*a*b^3)*x^2 - 105*(3*D*a^3*b - 8*B*a^2*b^2)*
x)*sqrt(b*x^2 + a))/b^3]
```

## Sympy [A] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.82

$$\int x\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx=\begin{cases} -\frac{a\left(Ba^2-\frac{3a\left(2Bab+Da^2-\frac{5a\left(Bb^2+\frac{9Dab}{8}\right)}{6b}\right)}{4b}\right)\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right)}{2b}+\sqrt{a+bx^2}\left(\frac{Cbx^6}{7}+\right. \\ a^{\frac{3}{2}}\left(\frac{Ax^2}{2}+\frac{Bx^3}{3}+\frac{Cx^4}{4}+\frac{Dx^5}{5}\right) \end{cases}$$

input
```
integrate(x*(b*x**2+a)**(3/2)*(D*x**3+C*x**2+B*x+A),x)
```

output
```
Piecewise((-a*(B*a**2 - 3*a*(2*B*a*b + D*a**2 - 5*a*(B*b**2 + 9*D*a*b/8)/(
6*b))/(4*b))*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), N
e(a, 0)), (x*log(x)/sqrt(b*x**2), True))/(2*b) + sqrt(a + b*x**2)*(C*b*x**
6/7 + D*b*x**7/8 + x**5*(B*b**2 + 9*D*a*b/8)/(6*b) + x**4*(A*b**2 + 8*C*a*
b/7)/(5*b) + x**3*(2*B*a*b + D*a**2 - 5*a*(B*b**2 + 9*D*a*b/8)/(6*b))/(4*b
) + x**2*(2*A*a*b + C*a**2 - 4*a*(A*b**2 + 8*C*a*b/7)/(5*b))/(3*b) + x*(B*
a**2 - 3*a*(2*B*a*b + D*a**2 - 5*a*(B*b**2 + 9*D*a*b/8)/(6*b))/(4*b))/(2*b
) + (A*a**2 - 2*a*(2*A*a*b + C*a**2 - 4*a*(A*b**2 + 8*C*a*b/7)/(5*b))/(3*b
))/b), Ne(b, 0)), (a**(3/2)*(A*x**2/2 + B*x**3/3 + C*x**4/4 + D*x**5/5), T
rue))
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.06

$$\int x(a+bx^2)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx = \frac{(bx^2+a)^{\frac{5}{2}}Dx^3}{8\,b} + \frac{(bx^2+a)^{\frac{5}{2}}Cx^2}{7\,b} - \frac{(bx^2+a)^{\frac{5}{2}}Dax}{16\,b^2} + \frac{(bx^2+a)^{\frac{3}{2}}Da^2x}{64\,b^2} + \frac{3\sqrt{bx^2+a}Da^3x}{128\,b^2} + \frac{(bx^2+a)^{\frac{5}{2}}Bx}{6\,b} - \frac{(bx^2+a)^{\frac{3}{2}}Bax}{24\,b} - \frac{\sqrt{bx^2+a}Ba^2x}{16\,b} + \frac{3\,Da^4\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\,b^{\frac{5}{2}}} - \frac{Ba^3\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{3}{2}}} - \frac{2\,(bx^2+a)^{\frac{5}{2}}Ca}{35\,b^2} + \frac{(bx^2+a)^{\frac{5}{2}}A}{5\,b}$$

input
```
integrate(x*(b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
1/8*(b*x^2 + a)^(5/2)*D*x^3/b + 1/7*(b*x^2 + a)^(5/2)*C*x^2/b - 1/16*(b*x^
2 + a)^(5/2)*D*a*x/b^2 + 1/64*(b*x^2 + a)^(3/2)*D*a^2*x/b^2 + 3/128*sqrt(b
*x^2 + a)*D*a^3*x/b^2 + 1/6*(b*x^2 + a)^(5/2)*B*x/b - 1/24*(b*x^2 + a)^(3/
2)*B*a*x/b - 1/16*sqrt(b*x^2 + a)*B*a^2*x/b + 3/128*D*a^4*arcsinh(b*x/sqrt
(a*b))/b^(5/2) - 1/16*B*a^3*arcsinh(b*x/sqrt(a*b))/b^(3/2) - 2/35*(b*x^2 +
 a)^(5/2)*C*a/b^2 + 1/5*(b*x^2 + a)^(5/2)*A/b
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.05

$$\int x(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)\,dx=\frac{1}{13440}\sqrt{bx^2+a}\left(\left(2\left(\left(\left(4\left(5\left(6(7Dbx+8Cb)x+\frac{7(9Dab^6+8Bb^7)}{b^6}\right)x+\frac{48(8Cab^6+7\ldots}{b^6}\right.\right.\right.\right.\right.\right.$$
$$-\frac{(3Da^4-8Ba^3b)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{128\,b^{\frac{5}{2}}}$$

input
```
integrate(x*(b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

output
```
1/13440*sqrt(b*x^2 + a)*((2*((4*(5*(6*(7*D*b*x + 8*C*b)*x + 7*(9*D*a*b^6 +
 8*B*b^7)/b^6)*x + 48*(8*C*a*b^6 + 7*A*b^7)/b^6)*x + 35*(3*D*a^2*b^5 + 56*
B*a*b^6)/b^6)*x + 192*(C*a^2*b^5 + 14*A*a*b^6)/b^6)*x - 105*(3*D*a^3*b^4 -
 8*B*a^2*b^5)/b^6)*x - 384*(2*C*a^3*b^4 - 7*A*a^2*b^5)/b^6) - 1/128*(3*D*a
^4 - 8*B*a^3*b)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2)
```

## Mupad [F(-1)]

Timed out.

$$\int x(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)\,dx=\int x\,(b\,x^2+a)^{3/2}\,(A+B\,x+C\,x^2+x^3\,D)\,dx$$

input
```
int(x*(a + b*x^2)^(3/2)*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
int(x*(a + b*x^2)^(3/2)*(A + B*x + C*x^2 + x^3*D), x)
```

## Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.52

$$\int x(a+bx^2)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx = \frac{2688\sqrt{b\,x^2+a}\,a^3b^2-768\sqrt{b\,x^2+a}\,a^3bc-315\sqrt{b\,x^2+a}\,a^3bdx+5376\sqrt{b\,x^2+a}\,a^2b^3x^2+840}{}$$

input
```
int(x*(b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A),x)
```

output
```
(2688*sqrt(a + b*x**2)*a**3*b**2 - 768*sqrt(a + b*x**2)*a**3*b*c - 315*sqr
t(a + b*x**2)*a**3*b*d*x + 5376*sqrt(a + b*x**2)*a**2*b**3*x**2 + 840*sqrt
(a + b*x**2)*a**2*b**3*x + 384*sqrt(a + b*x**2)*a**2*b**2*c*x**2 + 210*sqr
t(a + b*x**2)*a**2*b**2*d*x**3 + 2688*sqrt(a + b*x**2)*a*b**4*x**4 + 3920*
sqrt(a + b*x**2)*a*b**4*x**3 + 3072*sqrt(a + b*x**2)*a*b**3*c*x**4 + 2520*
sqrt(a + b*x**2)*a*b**3*d*x**5 + 2240*sqrt(a + b*x**2)*b**5*x**5 + 1920*sq
rt(a + b*x**2)*b**4*c*x**6 + 1680*sqrt(a + b*x**2)*b**4*d*x**7 + 315*sqrt(
b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**4*d - 840*sqrt(b)*log((s
qrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**3*b**2)/(13440*b**3)
```

# 3.77 $\int (a+bx^2)^{3/2} (A+Bx+Cx^2+Dx^3)\, dx$



## Optimal result

Integrand size = 27, antiderivative size = 163

$$\int (a+bx^2)^{3/2} (A+Bx+Cx^2+Dx^3)\, dx = \frac{a(6Ab-aC)x\sqrt{a+bx^2}}{16b} + \frac{(6Ab-aC)x(a+bx^2)^{3/2}}{24b} + \frac{(bB-aD)(a+bx^2)^{5/2}}{5b^2} + \frac{Cx(a+bx^2)^{5/2}}{6b} + \frac{D(a+bx^2)^{7/2}}{7b^2} + \frac{a^2(6Ab-aC)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{3/2}}$$

output
```
1/16*a*(6*A*b-C*a)*x*(b*x^2+a)^(1/2)/b+1/24*(6*A*b-C*a)*x*(b*x^2+a)^(3/2)/
b+1/5*(B*b-D*a)*(b*x^2+a)^(5/2)/b^2+1/6*C*x*(b*x^2+a)^(5/2)/b+1/7*D*(b*x^2
+a)^(7/2)/b^2+1/16*a^2*(6*A*b-C*a)*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(3
/2)
```

## Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.87

$$\int \left(a+bx^2\right)^{3/2} \left(A+Bx+Cx^2+Dx^3\right) \, dx = \frac{\sqrt{a+bx^2}(-96a^3D+4b^3x^3(105A+84Bx+70Cx^2+60Dx^3)+3a^2b(112B+x(35C+16Dx$$

input
```
Integrate[(a + b*x^2)^(3/2)*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(Sqrt[a + b*x^2]*(-96*a^3*D + 4*b^3*x^3*(105*A + 84*B*x + 70*C*x^2 + 60*D*
x^3) + 3*a^2*b*(112*B + x*(35*C + 16*D*x)) + 2*a*b^2*x*(525*A + x*(336*B +
 245*C*x + 192*D*x^2))) + 105*a^2*Sqrt[b]*(-6*A*b + a*C)*Log[-(Sqrt[b]*x)
+ Sqrt[a + b*x^2]])/(1680*b^2)
```

## Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.98, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {2346, 2346, 27, 455, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^{3/2} \left(A+Bx+Cx^2+Dx^3\right) \, dx$$

$$\Big\downarrow \; 2346$$

$$\frac{\int \left(bx^2+a\right)^{3/2} \left(7bCx^2+(7bB-2aD)x+7Ab\right)dx}{7b}+\frac{Dx^2\left(a+bx^2\right)^{5/2}}{7b}$$

$$\Big\downarrow \; 2346$$

$$\frac{\frac{\int b(7(6Ab-aC)+6(7bB-2aD)x)\left(bx^2+a\right)^{3/2}dx}{6b}+\frac{7}{6}Cx\left(a+bx^2\right)^{5/2}}{7b}+\frac{Dx^2\left(a+bx^2\right)^{5/2}}{7b}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{1}{6}\int (7(6Ab-aC)+6(7bB-2aD)x)\left(bx^2+a\right)^{3/2}dx+\frac{7}{6}Cx\left(a+bx^2\right)^{5/2}}{7b}+\frac{Dx^2\left(a+bx^2\right)^{5/2}}{7b}$$

$\Big\downarrow$ 455

$$\frac{\frac{1}{6}\left(7(6Ab-aC)\int \left(bx^2+a\right)^{3/2}dx+\frac{6\left(a+bx^2\right)^{5/2}(7bB-2aD)}{5b}\right)+\frac{7}{6}Cx\left(a+bx^2\right)^{5/2}}{7b}+\frac{Dx^2\left(a+bx^2\right)^{5/2}}{7b}$$

$\Big\downarrow$ 211

$$\frac{\frac{1}{6}\left(7(6Ab-aC)\left(\frac{3}{4}a\int \sqrt{bx^2+a}dx+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{6\left(a+bx^2\right)^{5/2}(7bB-2aD)}{5b}\right)+\frac{7}{6}Cx\left(a+bx^2\right)^{5/2}}{7b}+\frac{Dx^2\left(a+bx^2\right)^{5/2}}{7b}$$

$\Big\downarrow$ 211

$$\frac{\frac{1}{6}\left(7(6Ab-aC)\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{6\left(a+bx^2\right)^{5/2}(7bB-2aD)}{5b}\right)+\frac{7}{6}Cx\left(a+bx\right.}{7b}$$
$$\frac{Dx^2\left(a+bx^2\right)^{5/2}}{7b}$$

$\Big\downarrow$ 224

$$\frac{\frac{1}{6}\left(7(6Ab-aC)\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{6\left(a+bx^2\right)^{5/2}(7bB-2aD)}{5b}\right)+\frac{7}{6}C}{7b}$$
$$\frac{Dx^2\left(a+bx^2\right)^{5/2}}{7b}$$

$\Big\downarrow$ 219

$$\frac{\frac{1}{6}\left(7(6Ab-aC)\left(\frac{3}{4}a\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{6\left(a+bx^2\right)^{5/2}(7bB-2aD)}{5b}\right)+\frac{7}{6}Cx}{7b}$$
$$\frac{Dx^2\left(a+bx^2\right)^{5/2}}{7b}$$

input
```
Int[(a + b*x^2)^(3/2)*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(D*x^2*(a + b*x^2)^(5/2))/(7*b) + ((7*C*x*(a + b*x^2)^(5/2))/6 + ((6*(7*b*
B - 2*a*D)*(a + b*x^2)^(5/2))/(5*b) + 7*(6*A*b - a*C)*((x*(a + b*x^2)^(3/2
))/4 + (3*a*((x*Sqrt[a + b*x^2])/2 + (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2
]])/(2*Sqrt[b])))/4))/6)/(7*b)
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

## Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.10

| method | result |
|---|---|
| default | $A\left(\frac{x(bx^2+a)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{bx^2+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{bx^2+a}\right)}{2\sqrt{b}}\right)}{4}\right)+\frac{B(bx^2+a)^{\frac{5}{2}}}{5b}+C\left(\frac{x(bx^2+a)^{\frac{5}{2}}}{6b}-\frac{a\left(\frac{x(bx^2+a)^{\frac{3}{2}}}{4}+\frac{3a(\ldots}{\ldots}\right.}{\ldots}\right.$ |

input
```
int((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
A*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(
1/2)*x+(b*x^2+a)^(1/2))))+1/5*B*(b*x^2+a)^(5/2)/b+C*(1/6*x*(b*x^2+a)^(5/2)
/b-1/6*a/b*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/
2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))))+D*(1/7*x^2*(b*x^2+a)^(5/2)/b-2/35*a/b^
2*(b*x^2+a)^(5/2))
```

## Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 327, normalized size of antiderivative = 2.01

$$\int (a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)\,dx = \left[-\frac{105\,(Ca^3-6\,Aa^2b)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\,(240\,Db^3x^6+280\,Cb^3x^5+\ldots}{\ldots}\right.$$

input
```
integrate((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
[-1/3360*(105*(C*a^3 - 6*A*a^2*b)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)
*sqrt(b)*x - a) - 2*(240*D*b^3*x^6 + 280*C*b^3*x^5 + 48*(8*D*a*b^2 + 7*B*b
^3)*x^4 - 96*D*a^3 + 336*B*a^2*b + 70*(7*C*a*b^2 + 6*A*b^3)*x^3 + 48*(D*a^
2*b + 14*B*a*b^2)*x^2 + 105*(C*a^2*b + 10*A*a*b^2)*x)*sqrt(b*x^2 + a))/b^2
, 1/1680*(105*(C*a^3 - 6*A*a^2*b)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 +
a)) + (240*D*b^3*x^6 + 280*C*b^3*x^5 + 48*(8*D*a*b^2 + 7*B*b^3)*x^4 - 96*D
*a^3 + 336*B*a^2*b + 70*(7*C*a*b^2 + 6*A*b^3)*x^3 + 48*(D*a^2*b + 14*B*a*b
^2)*x^2 + 105*(C*a^2*b + 10*A*a*b^2)*x)*sqrt(b*x^2 + a))/b^2]
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 303 vs. $2(143) = 286$.

Time = 0.61 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.86

$$\int \left(a+bx^2\right)^{3/2} \left(A+Bx+Cx^2+Dx^3\right)\,dx = \begin{cases} \sqrt{a+bx^2}\left(\frac{Cbx^5}{6}+\frac{Dbx^6}{7}+\frac{x^4\left(Bb^2+\frac{8Dab}{7}\right)}{5b}+\frac{x^3\left(Ab^2+\frac{7Cab}{6}\right)}{4b}+\frac{x^2\cdot\left(2Bab+Da^2-\frac{4a\left(Bb^2+\frac{8Dab}{7}\right)}{5b}\right)}{3b}+\frac{x\left(2A\right.}{} \right. \\ a^{\frac{3}{2}}\left(Ax+\frac{Bx^2}{2}+\frac{Cx^3}{3}+\frac{Dx^4}{4}\right) \end{cases}$$

input
```
integrate((b*x**2+a)**(3/2)*(D*x**3+C*x**2+B*x+A),x)
```

output
```
Piecewise((sqrt(a + b*x**2)*(C*b*x**5/6 + D*b*x**6/7 + x**4*(B*b**2 + 8*D*
a*b/7)/(5*b) + x**3*(A*b**2 + 7*C*a*b/6)/(4*b) + x**2*(2*B*a*b + D*a**2 -
4*a*(B*b**2 + 8*D*a*b/7)/(5*b))/(3*b) + x*(2*A*a*b + C*a**2 - 3*a*(A*b**2
+ 7*C*a*b/6)/(4*b))/(2*b) + (B*a**2 - 2*a*(2*B*a*b + D*a**2 - 4*a*(B*b**2
+ 8*D*a*b/7)/(5*b))/(3*b))/b) + (A*a**2 - a*(2*A*a*b + C*a**2 - 3*a*(A*b**
2 + 7*C*a*b/6)/(4*b))/(2*b))*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2
*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True)), Ne(b, 0)), (a**(
3/2)*(A*x + B*x**2/2 + C*x**3/3 + D*x**4/4), True))
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.01

$$\int \left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{\left(bx^2+a\right)^{\frac{5}{2}}Dx^2}{7\,b}+\frac{1}{4}\left(bx^2+a\right)^{\frac{3}{2}}Ax+\frac{3}{8}\sqrt{bx^2+a}Aax+\frac{\left(bx^2+a\right)^{\frac{5}{2}}Cx}{6\,b}-\frac{\left(bx^2+a\right)^{\frac{3}{2}}Cax}{24\,b}-\frac{\sqrt{bx^2+a}Ca^2x}{16\,b}-\frac{Ca^3\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{3}{2}}}+\frac{3\,Aa^2\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{b}}-\frac{2\left(bx^2+a\right)^{\frac{5}{2}}Da}{35\,b^2}+\frac{\left(bx^2+a\right)^{\frac{5}{2}}B}{5\,b}$$

input
```
integrate((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
1/7*(b*x^2 + a)^(5/2)*D*x^2/b + 1/4*(b*x^2 + a)^(3/2)*A*x + 3/8*sqrt(b*x^2
 + a)*A*a*x + 1/6*(b*x^2 + a)^(5/2)*C*x/b - 1/24*(b*x^2 + a)^(3/2)*C*a*x/b
 - 1/16*sqrt(b*x^2 + a)*C*a^2*x/b - 1/16*C*a^3*arcsinh(b*x/sqrt(a*b))/b^(3
/2) + 3/8*A*a^2*arcsinh(b*x/sqrt(a*b))/sqrt(b) - 2/35*(b*x^2 + a)^(5/2)*D*
a/b^2 + 1/5*(b*x^2 + a)^(5/2)*B/b
```

## Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.10

$$\int \left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{1}{1680}\sqrt{bx^2+a}\Bigg(\bigg(2\bigg(\bigg(4\left(5\,(6\,Dbx+7\,Cb)x+\frac{6\,(8\,Dab^5+7\,Bb^6)}{b^5}\right)x+\frac{35\,(7\,Cab^5+6\,Ab^6)}{b^5}+\frac{(Ca^3-6\,Aa^2b)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{16\,b^{\frac{3}{2}}}$$

input
```
integrate((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

output
```
1/1680*sqrt(b*x^2 + a)*((2*((4*(5*(6*D*b*x + 7*C*b)*x + 6*(8*D*a*b^5 + 7*B
*b^6)/b^5)*x + 35*(7*C*a*b^5 + 6*A*b^6)/b^5)*x + 24*(D*a^2*b^4 + 14*B*a*b^
5)/b^5)*x + 105*(C*a^2*b^4 + 10*A*a*b^5)/b^5)*x - 48*(2*D*a^3*b^3 - 7*B*a^
2*b^4)/b^5) + 1/16*(C*a^3 - 6*A*a^2*b)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a
)))/b^(3/2)
```

## Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^2\right)^{3/2} \left(A+Bx+Cx^2+Dx^3\right) \, dx = \int \left(b\,x^2+a\right)^{3/2}\,\left(A+B\,x+C\,x^2+x^3\,D\right) \,d x$$

input
```
int((a + b*x^2)^(3/2)*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
int((a + b*x^2)^(3/2)*(A + B*x + C*x^2 + x^3*D), x)
```

## Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.60

$$\int \left(a+bx^2\right)^{3/2} \left(A+Bx+Cx^2+Dx^3\right) \, dx = \frac{-96\sqrt{b\,x^2+a}\,a^3d+1050\sqrt{b\,x^2+a}\,a^2b^2x+336\sqrt{b\,x^2+a}\,a^2b^2+105\sqrt{b\,x^2+a}\,a^2bcx+48\sqrt{b}}{}$$

input
```
int((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A),x)
```

output
```
( - 96*sqrt(a + b*x**2)*a**3*d + 1050*sqrt(a + b*x**2)*a**2*b**2*x + 336*s
qrt(a + b*x**2)*a**2*b**2 + 105*sqrt(a + b*x**2)*a**2*b*c*x + 48*sqrt(a +
b*x**2)*a**2*b*d*x**2 + 420*sqrt(a + b*x**2)*a*b**3*x**3 + 672*sqrt(a + b*
x**2)*a*b**3*x**2 + 490*sqrt(a + b*x**2)*a*b**2*c*x**3 + 384*sqrt(a + b*x*
*2)*a*b**2*d*x**4 + 336*sqrt(a + b*x**2)*b**4*x**4 + 280*sqrt(a + b*x**2)*
b**3*c*x**5 + 240*sqrt(a + b*x**2)*b**3*d*x**6 + 630*sqrt(b)*log((sqrt(a +
 b*x**2) + sqrt(b)*x)/sqrt(a))*a**3*b - 105*sqrt(b)*log((sqrt(a + b*x**2)
+ sqrt(b)*x)/sqrt(a))*a**3*c)/(1680*b**2)
```

# 3.78 $\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{x}\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 175

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{x}\,dx = \frac{a(16Ab+(6bB-aD)x)\sqrt{a+bx^2}}{16b} + \frac{(8Ab+(6bB-aD)x)(a+bx^2)^{3/2}}{24b} + \frac{C(a+bx^2)^{5/2}}{5b} + \frac{Dx(a+bx^2)^{5/2}}{6b} + \frac{a^2(6bB-aD)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{3/2}} - a^{3/2}A\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

output
```
1/16*a*(16*A*b+(6*B*b-D*a)*x)*(b*x^2+a)^(1/2)/b+1/24*(8*A*b+(6*B*b-D*a)*x)
*(b*x^2+a)^(3/2)/b+1/5*C*(b*x^2+a)^(5/2)/b+1/6*D*x*(b*x^2+a)^(5/2)/b+1/16*
a^2*(6*B*b-D*a)*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(3/2)-a^(3/2)*A*arcta
nh((b*x^2+a)^(1/2)/a^(1/2))
```

## Mathematica [A] (verified)

Time = 0.90 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{x}\,dx = \frac{\sqrt{a+bx^2}(3a^2(16C+5Dx)+4b^2x^2(20A+x(15B+2x(6C}{240b}$$

$$+2a^{3/2}A\,\text{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{a^2(-6bB+aD)\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{16b^{3/2}}$$

input
```
Integrate[((a + b*x^2)^(3/2)*(A + B*x + C*x^2 + D*x^3))/x,x]
```

output
```
(Sqrt[a + b*x^2]*(3*a^2*(16*C + 5*D*x) + 4*b^2*x^2*(20*A + x*(15*B + 2*x*(
6*C + 5*D*x))) + 2*a*b*(160*A + x*(75*B + x*(48*C + 35*D*x)))))/(240*b) +
2*a^(3/2)*A*ArcTanh[(Sqrt[b]*x - Sqrt[a + b*x^2])/Sqrt[a]] + (a^2*(-6*b*B
+ a*D)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(16*b^(3/2))
```

## Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.03, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2340, 2340, 27, 535, 27, 535, 538, 224, 219, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{x}\,dx$$

$$\Big\downarrow \text{2340}$$

$$\frac{\int \frac{(bx^2+a)^{3/2}(6bCx^2+(6bB-aD)x+6Ab)}{x}dx}{6b}+\frac{Dx(a+bx^2)^{5/2}}{6b}$$

$$\Big\downarrow \text{2340}$$

$$\frac{\frac{\int \frac{5b(6Ab+(6bB-aD)x)(bx^2+a)^{3/2}}{x}dx}{5b}+\frac{6}{5}C(a+bx^2)^{5/2}}{6b}+\frac{Dx(a+bx^2)^{5/2}}{6b}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{(6Ab+(6bB-aD)x)\left(bx^2+a\right)^{3/2}}{x}dx+\frac{6}{5}C\left(a+bx^2\right)^{5/2}}{6b}+\frac{Dx\left(a+bx^2\right)^{5/2}}{6b}$$

$$\Big\downarrow \ 535$$

$$\frac{\frac{1}{4}a\int \frac{3(8Ab+(6bB-aD)x)\sqrt{bx^2+a}}{x}dx+\frac{1}{4}\left(a+bx^2\right)^{3/2}(x(6bB-aD)+8Ab)+\frac{6}{5}C\left(a+bx^2\right)^{5/2}}{6b}+\frac{Dx\left(a+bx^2\right)^{5/2}}{6b}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{3}{4}a\int \frac{(8Ab+(6bB-aD)x)\sqrt{bx^2+a}}{x}dx+\frac{1}{4}\left(a+bx^2\right)^{3/2}(x(6bB-aD)+8Ab)+\frac{6}{5}C\left(a+bx^2\right)^{5/2}}{6b}+\frac{Dx\left(a+bx^2\right)^{5/2}}{6b}$$

$$\Big\downarrow \ 535$$

$$\frac{\frac{3}{4}a\left(\frac{1}{2}a\int \frac{16Ab+(6bB-aD)x}{x\sqrt{bx^2+a}}dx+\frac{1}{2}\sqrt{a+bx^2}(x(6bB-aD)+16Ab)\right)+\frac{1}{4}\left(a+bx^2\right)^{3/2}(x(6bB-aD)+8Ab)+\frac{6}{5}C(}{6b}$$

$$\frac{Dx\left(a+bx^2\right)^{5/2}}{6b}$$

$$\Big\downarrow \ 538$$

$$\frac{\frac{3}{4}a\left(\frac{1}{2}a\left(16Ab\int \frac{1}{x\sqrt{bx^2+a}}dx+(6bB-aD)\int \frac{1}{\sqrt{bx^2+a}}dx\right)+\frac{1}{2}\sqrt{a+bx^2}(x(6bB-aD)+16Ab)\right)+\frac{1}{4}\left(a+bx^2\right)^{3/2}(x}{6b}$$

$$\frac{Dx\left(a+bx^2\right)^{5/2}}{6b}$$

$$\Big\downarrow \ 224$$

$$\frac{\frac{3}{4}a\left(\frac{1}{2}a\left(16Ab\int \frac{1}{x\sqrt{bx^2+a}}dx+(6bB-aD)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}\right)+\frac{1}{2}\sqrt{a+bx^2}(x(6bB-aD)+16Ab)\right)+\frac{1}{4}(a+b}{6b}$$

$$\frac{Dx\left(a+bx^2\right)^{5/2}}{6b}$$

$$\Big\downarrow \ 219$$

$$\frac{\frac{3}{4}a\left(\frac{1}{2}a\left(16Ab\int \frac{1}{x\sqrt{bx^2+a}}dx+\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(6bB-aD)}{\sqrt{b}}\right)+\frac{1}{2}\sqrt{a+bx^2}(x(6bB-aD)+16Ab)\right)+\frac{1}{4}\left(a+bx^2\right)}{6b}$$

$$\frac{Dx\left(a+bx^2\right)^{5/2}}{6b}$$

$$\Big\downarrow \ 243$$

$$\frac{\frac{3}{4}a\left(\frac{1}{2}a\left(8Ab\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2+\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(6bB-aD)}{\sqrt{b}}\right)+\frac{1}{2}\sqrt{a+bx^2}(x(6bB-aD)+16Ab)\right)+\frac{1}{4}\left(a+bx^2\right)}{6b}$$

$$\frac{Dx\left(a+bx^2\right)^{5/2}}{6b}$$

$$\Big\downarrow \ 73$$

$$\frac{\frac{3}{4}a\left(\frac{1}{2}a\left(16A\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}+\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(6bB-aD)}{\sqrt{b}}\right)+\frac{1}{2}\sqrt{a+bx^2}(x(6bB-aD)+16Ab)\right)+\frac{1}{4}(a+}{6b}$$

$$\frac{Dx\left(a+bx^2\right)^{5/2}}{6b}$$

$$\Big\downarrow \ 221$$

$$\frac{\frac{3}{4}a\left(\frac{1}{2}a\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(6bB-aD)}{\sqrt{b}}-\frac{16Ab\,\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}\right)+\frac{1}{2}\sqrt{a+bx^2}(x(6bB-aD)+16Ab)\right)+\frac{1}{4}(a+b}{6b}$$

$$\frac{Dx\left(a+bx^2\right)^{5/2}}{6b}$$

input
```
Int[((a + b*x^2)^(3/2)*(A + B*x + C*x^2 + D*x^3))/x,x]
```

output
```
(D*x*(a + b*x^2)^(5/2))/(6*b) + (((8*A*b + (6*b*B - a*D)*x)*(a + b*x^2)^(3
/2))/4 + (6*C*(a + b*x^2)^(5/2))/5 + (3*a*(((16*A*b + (6*b*B - a*D)*x)*Sqr
t[a + b*x^2])/2 + (a*(((6*b*B - a*D)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])
/Sqrt[b] - (16*A*b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a]))/2))/4)/(6*b
)
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 535
```
Int[(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_))/(x_), x_Symbol] :> Sim
p[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^p/(2*p*(2*p + 1))), x] + Simp[a/(2*p
 + 1)   Int[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^(p - 1)/x), x], x] /; Free
Q[{a, b, c, d}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 2340
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Simp[1/(b*(m
+ q + 2*p + 1))   Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)
*Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /;
GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ
[Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

## Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.14

| method | result |
|---|---|
| default | $B\left(\frac{x(b\,x^2+a)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)}{4}\right)+A\left(\frac{(b\,x^2+a)^{\frac{3}{2}}}{3}+a\left(\sqrt{b\,x^2+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\cdots}{x}\right.\right.\right.$ |

input
```
int((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/x,x,method=_RETURNVERBOSE)
```

output
```
B*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(
1/2)*x+(b*x^2+a)^(1/2))))+A*(1/3*(b*x^2+a)^(3/2)+a*((b*x^2+a)^(1/2)-a^(1/2
)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)))+1/5*C*(b*x^2+a)^(5/2)/b+D*(1/6*x
*(b*x^2+a)^(5/2)/b-1/6*a/b*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(
1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))))
```

## Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 719, normalized size of antiderivative = 4.11

$$\int \frac{(a+bx^2)^{3/2}\,(A+Bx+Cx^2+Dx^3)}{x}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/x,x, algorithm="fricas")
```

output
```
[1/480*(240*A*a^(3/2)*b^2*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x
^2) - 15*(D*a^3 - 6*B*a^2*b)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt
(b)*x - a) + 2*(40*D*b^3*x^5 + 48*C*b^3*x^4 + 48*C*a^2*b + 320*A*a*b^2 + 1
0*(7*D*a*b^2 + 6*B*b^3)*x^3 + 16*(6*C*a*b^2 + 5*A*b^3)*x^2 + 15*(D*a^2*b +
 10*B*a*b^2)*x)*sqrt(b*x^2 + a))/b^2, 1/240*(120*A*a^(3/2)*b^2*log(-(b*x^2
 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 15*(D*a^3 - 6*B*a^2*b)*sqrt(-b)
*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (40*D*b^3*x^5 + 48*C*b^3*x^4 + 48*C*
a^2*b + 320*A*a*b^2 + 10*(7*D*a*b^2 + 6*B*b^3)*x^3 + 16*(6*C*a*b^2 + 5*A*b
^3)*x^2 + 15*(D*a^2*b + 10*B*a*b^2)*x)*sqrt(b*x^2 + a))/b^2, 1/480*(480*A*
sqrt(-a)*a*b^2*arctan(sqrt(b*x^2 + a)*sqrt(-a)/a) - 15*(D*a^3 - 6*B*a^2*b)
*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(40*D*b^3*x^5
 + 48*C*b^3*x^4 + 48*C*a^2*b + 320*A*a*b^2 + 10*(7*D*a*b^2 + 6*B*b^3)*x^3
+ 16*(6*C*a*b^2 + 5*A*b^3)*x^2 + 15*(D*a^2*b + 10*B*a*b^2)*x)*sqrt(b*x^2 +
 a))/b^2, 1/240*(240*A*sqrt(-a)*a*b^2*arctan(sqrt(b*x^2 + a)*sqrt(-a)/a) +
 15*(D*a^3 - 6*B*a^2*b)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (40*
D*b^3*x^5 + 48*C*b^3*x^4 + 48*C*a^2*b + 320*A*a*b^2 + 10*(7*D*a*b^2 + 6*B*
b^3)*x^3 + 16*(6*C*a*b^2 + 5*A*b^3)*x^2 + 15*(D*a^2*b + 10*B*a*b^2)*x)*sqr
t(b*x^2 + a))/b^2]
```

## Sympy [A] (verification not implemented)

Time = 6.10 (sec) , antiderivative size = 600, normalized size of antiderivative = 3.43

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{x}\,dx = \text{Too large to display}$$

input
```
integrate((b*x**2+a)**(3/2)*(D*x**3+C*x**2+B*x+A)/x,x)
```

output
```
-A*a**(3/2)*asinh(sqrt(a)/(sqrt(b)*x)) + A*a**2/(sqrt(b)*x*sqrt(a/(b*x**2)
 + 1)) + A*a*sqrt(b)*x/sqrt(a/(b*x**2) + 1) + A*b*Piecewise((a*sqrt(a + b*
x**2)/(3*b) + x**2*sqrt(a + b*x**2)/3, Ne(b, 0)), (sqrt(a)*x**2/2, True))
+ B*a*Piecewise((a*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt
(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True))/2 + x*sqrt(a + b*x**2)/2, N
e(b, 0)), (sqrt(a)*x, True)) + B*b*Piecewise((-a**2*Piecewise((log(2*sqrt(
b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), T
rue))/(8*b) + a*x*sqrt(a + b*x**2)/(8*b) + x**3*sqrt(a + b*x**2)/4, Ne(b,
0)), (sqrt(a)*x**3/3, True)) + C*a*Piecewise((a*sqrt(a + b*x**2)/(3*b) + x
**2*sqrt(a + b*x**2)/3, Ne(b, 0)), (sqrt(a)*x**2/2, True)) + C*b*Piecewise
((-2*a**2*sqrt(a + b*x**2)/(15*b**2) + a*x**2*sqrt(a + b*x**2)/(15*b) + x*
*4*sqrt(a + b*x**2)/5, Ne(b, 0)), (sqrt(a)*x**4/4, True)) + D*a*Piecewise(
(-a**2*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0
)), (x*log(x)/sqrt(b*x**2), True))/(8*b) + a*x*sqrt(a + b*x**2)/(8*b) + x*
*3*sqrt(a + b*x**2)/4, Ne(b, 0)), (sqrt(a)*x**3/3, True)) + D*b*Piecewise(
(a**3*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)
), (x*log(x)/sqrt(b*x**2), True))/(16*b**2) - a**2*x*sqrt(a + b*x**2)/(16*
b**2) + a*x**3*sqrt(a + b*x**2)/(24*b) + x**5*sqrt(a + b*x**2)/6, Ne(b, 0)
), (sqrt(a)*x**5/5, True))
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{x}\,dx = \frac{1}{4}\left(bx^2+a\right)^{\frac{3}{2}}Bx + \frac{3}{8}\sqrt{bx^2+a}Bax + \frac{(bx^2+a)^{\frac{5}{2}}Dx}{6\,b} - \frac{(bx^2+a)^{\frac{3}{2}}Dax}{24\,b} - \frac{\sqrt{bx^2+a}Da^2x}{16\,b} - \frac{Da^3\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{3}{2}}} + \frac{3\,Ba^2\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{b}} - Aa^{\frac{3}{2}}\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right) + \frac{1}{3}\left(bx^2+a\right)^{\frac{3}{2}}A + \sqrt{bx^2+a}Aa + \frac{(bx^2+a)^{\frac{5}{2}}C}{5\,b}$$

input
```
integrate((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/x,x, algorithm="maxima")
```

output
```
1/4*(b*x^2 + a)^(3/2)*B*x + 3/8*sqrt(b*x^2 + a)*B*a*x + 1/6*(b*x^2 + a)^(5
/2)*D*x/b - 1/24*(b*x^2 + a)^(3/2)*D*a*x/b - 1/16*sqrt(b*x^2 + a)*D*a^2*x/
b - 1/16*D*a^3*arcsinh(b*x/sqrt(a*b))/b^(3/2) + 3/8*B*a^2*arcsinh(b*x/sqrt
(a*b))/sqrt(b) - A*a^(3/2)*arcsinh(a/(sqrt(a*b)*abs(x))) + 1/3*(b*x^2 + a)
^(3/2)*A + sqrt(b*x^2 + a)*A*a + 1/5*(b*x^2 + a)^(5/2)*C/b
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{x}\,dx = \int \frac{(b\,x^2+a)^{3/2}\,(A+B\,x+C\,x^2+x^3\,D)}{x}\,dx$$

input
```
int(((a + b*x^2)^(3/2)*(A + B*x + C*x^2 + x^3*D))/x,x)
```

output
```
int(((a + b*x^2)^(3/2)*(A + B*x + C*x^2 + x^3*D))/x, x)
```

## Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.66

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{x}\,dx = \frac{320\sqrt{b\,x^2+a}\,a^2b^2+48\sqrt{b\,x^2+a}\,a^2bc+15\sqrt{b\,x^2+a}\,a^2bdx+}{}$$

input
```
int((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/x,x)
```

output
```
(320*sqrt(a + b*x**2)*a**2*b**2 + 48*sqrt(a + b*x**2)*a**2*b*c + 15*sqrt(a
 + b*x**2)*a**2*b*d*x + 80*sqrt(a + b*x**2)*a*b**3*x**2 + 150*sqrt(a + b*x
**2)*a*b**3*x + 96*sqrt(a + b*x**2)*a*b**2*c*x**2 + 70*sqrt(a + b*x**2)*a*
b**2*d*x**3 + 60*sqrt(a + b*x**2)*b**4*x**3 + 48*sqrt(a + b*x**2)*b**3*c*x
**4 + 40*sqrt(a + b*x**2)*b**3*d*x**5 + 240*sqrt(a)*log((sqrt(a + b*x**2)
- sqrt(a) + sqrt(b)*x)/sqrt(a))*a**2*b**2 - 240*sqrt(a)*log((sqrt(a + b*x*
*2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*a**2*b**2 - 15*sqrt(b)*log((sqrt(a + b
*x**2) + sqrt(b)*x)/sqrt(a))*a**3*d + 90*sqrt(b)*log((sqrt(a + b*x**2) + s
qrt(b)*x)/sqrt(a))*a**2*b**2)/(240*b**2)
```

# 3.79 $\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{x^2} \, dx$



## Optimal result

Integrand size = 30, antiderivative size = 168

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{x^2} \, dx = \frac{1}{8}(8aB+3(4Ab+aC)x)\sqrt{a+bx^2} + \frac{(4aB+3(4Ab+aC)x)(a+bx^2)^{3/2}}{12a} + \frac{D(a+bx^2)^{5/2}}{5b} - \frac{A(a+bx^2)^{5/2}}{ax} + \frac{3a(4Ab+aC)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}} - a^{3/2}B\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

output
```
1/8*(8*B*a+3*(4*A*b+C*a)*x)*(b*x^2+a)^(1/2)+1/12*(4*B*a+3*(4*A*b+C*a)*x)*(
b*x^2+a)^(3/2)/a+1/5*D*(b*x^2+a)^(5/2)/b-A*(b*x^2+a)^(5/2)/a/x+3/8*a*(4*A*
b+C*a)*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(1/2)-a^(3/2)*B*arctanh((b*x^2
+a)^(1/2)/a^(1/2))
```

## Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{x^2}\,dx = \frac{\sqrt{a+bx^2}(24a^2Dx+ab(-120A+x(160B+75Cx+48Dx^2}{120bx}$$

$$+2a^{3/2}B\operatorname{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right)-\frac{3a(4Ab+aC)\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{8\sqrt{b}}$$

input
```
Integrate[((a + b*x^2)^(3/2)*(A + B*x + C*x^2 + D*x^3))/x^2,x]
```

output
```
(Sqrt[a + b*x^2]*(24*a^2*D*x + a*b*(-120*A + x*(160*B + 75*C*x + 48*D*x^2)
) + 2*b^2*x^2*(30*A + x*(20*B + 3*x*(5*C + 4*D*x)))))/(120*b*x) + 2*a^(3/2
)*B*ArcTanh[(Sqrt[b]*x - Sqrt[a + b*x^2])/Sqrt[a]] - (3*a*(4*A*b + a*C)*Lo
g[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(8*Sqrt[b])
```

## Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.07, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2338, 25, 2340, 27, 535, 535, 538, 224, 219, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{x^2}dx$$

$$\Big\downarrow \text{ 2338}$$

$$-\frac{\int -\frac{(bx^2+a)^{3/2}(aDx^2+(4Ab+aC)x+aB)}{x}dx}{a}-\frac{A(a+bx^2)^{5/2}}{ax}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{(bx^2+a)^{3/2}(aDx^2+(4Ab+aC)x+aB)}{x}dx}{a}-\frac{A(a+bx^2)^{5/2}}{ax}$$

$$\Big\downarrow \text{ 2340}$$

$$\frac{\frac{\int \frac{5b(aB+(4Ab+aC)x)\left(bx^2+a\right)^{3/2}}{x}dx}{5b}+\frac{aD\left(a+bx^2\right)^{5/2}}{5b}}{a}-\frac{A\left(a+bx^2\right)^{5/2}}{ax}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{(aB+(4Ab+aC)x)\left(bx^2+a\right)^{3/2}}{x}dx+\frac{aD\left(a+bx^2\right)^{5/2}}{5b}}{a}-\frac{A\left(a+bx^2\right)^{5/2}}{ax}$$

$\Big\downarrow$ 535

$$\frac{\frac{1}{4}a\int \frac{(4aB+3(4Ab+aC)x)\sqrt{bx^2+a}}{x}dx+\frac{1}{12}\left(a+bx^2\right)^{3/2}(3x(aC+4Ab)+4aB)+\frac{aD\left(a+bx^2\right)^{5/2}}{5b}}{a}-\frac{A\left(a+bx^2\right)^{5/2}}{ax}$$

$\Big\downarrow$ 535

$$\frac{\frac{1}{4}a\left(\frac{1}{2}a\int \frac{8aB+3(4Ab+aC)x}{x\sqrt{bx^2+a}}dx+\frac{1}{2}\sqrt{a+bx^2}(3x(aC+4Ab)+8aB)\right)+\frac{1}{12}\left(a+bx^2\right)^{3/2}(3x(aC+4Ab)+4aB)+\frac{aI}{}}{a}$$

$$\frac{A\left(a+bx^2\right)^{5/2}}{ax}$$

$\Big\downarrow$ 538

$$\frac{\frac{1}{4}a\left(\frac{1}{2}a\left(3(aC+4Ab)\int \frac{1}{\sqrt{bx^2+a}}dx+8aB\int \frac{1}{x\sqrt{bx^2+a}}dx\right)+\frac{1}{2}\sqrt{a+bx^2}(3x(aC+4Ab)+8aB)\right)+\frac{1}{12}\left(a+bx^2\right)^{3/2}(}{a}$$

$$\frac{A\left(a+bx^2\right)^{5/2}}{ax}$$

$\Big\downarrow$ 224

$$\frac{\frac{1}{4}a\left(\frac{1}{2}a\left(3(aC+4Ab)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+8aB\int \frac{1}{x\sqrt{bx^2+a}}dx\right)+\frac{1}{2}\sqrt{a+bx^2}(3x(aC+4Ab)+8aB)\right)+\frac{1}{12}(a+}{a}$$

$$\frac{A\left(a+bx^2\right)^{5/2}}{ax}$$

$\Big\downarrow$ 219

$$\frac{\frac{1}{4}a\left(\frac{1}{2}a\left(8aB\int\frac{1}{x\sqrt{bx^2+a}}dx+\frac{3(aC+4Ab)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}\right)+\frac{1}{2}\sqrt{a+bx^2}(3x(aC+4Ab)+8aB)\right)+\frac{1}{12}(a+bx^2)}{a}$$

$$\frac{A(a+bx^2)^{5/2}}{ax}$$

$\big\downarrow$ 243

$$\frac{\frac{1}{4}a\left(\frac{1}{2}a\left(4aB\int\frac{1}{x^2\sqrt{bx^2+a}}dx^2+\frac{3(aC+4Ab)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}\right)+\frac{1}{2}\sqrt{a+bx^2}(3x(aC+4Ab)+8aB)\right)+\frac{1}{12}(a+bx}{a}$$

$$\frac{A(a+bx^2)^{5/2}}{ax}$$

$\big\downarrow$ 73

$$\frac{\frac{1}{4}a\left(\frac{1}{2}a\left(\frac{8aB\int\frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{b}+\frac{3(aC+4Ab)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}\right)+\frac{1}{2}\sqrt{a+bx^2}(3x(aC+4Ab)+8aB)\right)+\frac{1}{12}(a+bx}{a}$$

$$\frac{A(a+bx^2)^{5/2}}{ax}$$

$\big\downarrow$ 221

$$\frac{\frac{1}{4}a\left(\frac{1}{2}a\left(\frac{3(aC+4Ab)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}-8\sqrt{a}B\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\right)+\frac{1}{2}\sqrt{a+bx^2}(3x(aC+4Ab)+8aB)\right)+\frac{1}{12}(a}{a}$$

$$\frac{A(a+bx^2)^{5/2}}{ax}$$

input
```
Int[((a + b*x^2)^(3/2)*(A + B*x + C*x^2 + D*x^3))/x^2,x]
```

output
```
-((A*(a + b*x^2)^(5/2))/(a*x)) + (((4*a*B + 3*(4*A*b + a*C)*x)*(a + b*x^2)
^(3/2))/12 + (a*D*(a + b*x^2)^(5/2))/(5*b) + (a*(((8*a*B + 3*(4*A*b + a*C)
*x)*Sqrt[a + b*x^2])/2 + (a*((3*(4*A*b + a*C)*ArcTanh[(Sqrt[b]*x)/Sqrt[a +
 b*x^2]])/Sqrt[b] - 8*Sqrt[a]*B*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]))/2))/4)/
a
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 535
```
Int[(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_))/(x_), x_Symbol] :> Sim
p[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^p/(2*p*(2*p + 1))), x] + Simp[a/(2*p
 + 1)   Int[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^(p - 1)/x), x], x] /; Free
Q[{a, b, c, d}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

rule 2340
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Simp[1/(b*(m
+ q + 2*p + 1))   Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)
*Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /;
GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ
[Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

## Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.20

| method | result |
|---|---|
| default | $C\left(\frac{x(b\,x^2+a)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)}{4}\right)+A\left(-\frac{(b\,x^2+a)^{\frac{5}{2}}}{ax}+\frac{4b\left(\frac{x(b\,x^2+a)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln(}{}\right.}{4}\right.}{a}\right.$ |

input
```
int((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/x^2,x,method=_RETURNVERBOSE)
```

output
```
C*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(
1/2)*x+(b*x^2+a)^(1/2))))+A*(-1/a/x*(b*x^2+a)^(5/2)+4*b/a*(1/4*x*(b*x^2+a)
^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(
1/2)))))+B*(1/3*(b*x^2+a)^(3/2)+a*((b*x^2+a)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/
2)*(b*x^2+a)^(1/2))/x)))+1/5*D*(b*x^2+a)^(5/2)/b
```

## Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 663, normalized size of antiderivative = 3.95

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{x^2}\,dx = \left[\frac{120\,Ba^{\frac{3}{2}}bx\log\left(-\frac{bx^2-2\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+45\,(Ca^2+4\,Aab)\sqrt{}\right.$$

input
```
integrate((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/x^2,x, algorithm="fricas")
```

output
```
[1/240*(120*B*a^(3/2)*b*x*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x
^2) + 45*(C*a^2 + 4*A*a*b)*sqrt(b)*x*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt
(b)*x - a) + 2*(24*D*b^2*x^5 + 30*C*b^2*x^4 + 8*(6*D*a*b + 5*B*b^2)*x^3 -
120*A*a*b + 15*(5*C*a*b + 4*A*b^2)*x^2 + 8*(3*D*a^2 + 20*B*a*b)*x)*sqrt(b*
x^2 + a))/(b*x), 1/120*(60*B*a^(3/2)*b*x*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*s
qrt(a) + 2*a)/x^2) - 45*(C*a^2 + 4*A*a*b)*sqrt(-b)*x*arctan(sqrt(-b)*x/sqr
t(b*x^2 + a)) + (24*D*b^2*x^5 + 30*C*b^2*x^4 + 8*(6*D*a*b + 5*B*b^2)*x^3 -
 120*A*a*b + 15*(5*C*a*b + 4*A*b^2)*x^2 + 8*(3*D*a^2 + 20*B*a*b)*x)*sqrt(b
*x^2 + a))/(b*x), 1/240*(240*B*sqrt(-a)*a*b*x*arctan(sqrt(b*x^2 + a)*sqrt(
-a)/a) + 45*(C*a^2 + 4*A*a*b)*sqrt(b)*x*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*s
qrt(b)*x - a) + 2*(24*D*b^2*x^5 + 30*C*b^2*x^4 + 8*(6*D*a*b + 5*B*b^2)*x^3
 - 120*A*a*b + 15*(5*C*a*b + 4*A*b^2)*x^2 + 8*(3*D*a^2 + 20*B*a*b)*x)*sqrt
(b*x^2 + a))/(b*x), 1/120*(120*B*sqrt(-a)*a*b*x*arctan(sqrt(b*x^2 + a)*sqr
t(-a)/a) - 45*(C*a^2 + 4*A*a*b)*sqrt(-b)*x*arctan(sqrt(-b)*x/sqrt(b*x^2 +
a)) + (24*D*b^2*x^5 + 30*C*b^2*x^4 + 8*(6*D*a*b + 5*B*b^2)*x^3 - 120*A*a*b
 + 15*(5*C*a*b + 4*A*b^2)*x^2 + 8*(3*D*a^2 + 20*B*a*b)*x)*sqrt(b*x^2 + a))
/(b*x)]
```

## Sympy [A] (verification not implemented)

Time = 3.95 (sec) , antiderivative size = 520, normalized size of antiderivative = 3.10

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{x^2}\,dx = \text{Too large to display}$$

input
```
integrate((b*x**2+a)**(3/2)*(D*x**3+C*x**2+B*x+A)/x**2,x)
```

output
```
-A*a**(3/2)/(x*sqrt(1 + b*x**2/a)) - A*sqrt(a)*b*x/sqrt(1 + b*x**2/a) + A*
a*sqrt(b)*asinh(sqrt(b)*x/sqrt(a)) + A*b*Piecewise((a*Piecewise((log(2*sqr
t(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2),
 True))/2 + x*sqrt(a + b*x**2)/2, Ne(b, 0)), (sqrt(a)*x, True)) - B*a**(3/
2)*asinh(sqrt(a)/(sqrt(b)*x)) + B*a**2/(sqrt(b)*x*sqrt(a/(b*x**2) + 1)) +
B*a*sqrt(b)*x/sqrt(a/(b*x**2) + 1) + B*b*Piecewise((a*sqrt(a + b*x**2)/(3*
b) + x**2*sqrt(a + b*x**2)/3, Ne(b, 0)), (sqrt(a)*x**2/2, True)) + C*a*Pie
cewise((a*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a
, 0)), (x*log(x)/sqrt(b*x**2), True))/2 + x*sqrt(a + b*x**2)/2, Ne(b, 0)),
 (sqrt(a)*x, True)) + C*b*Piecewise((-a**2*Piecewise((log(2*sqrt(b)*sqrt(a
 + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True))/(8*
b) + a*x*sqrt(a + b*x**2)/(8*b) + x**3*sqrt(a + b*x**2)/4, Ne(b, 0)), (sqr
t(a)*x**3/3, True)) + D*a*Piecewise((a*sqrt(a + b*x**2)/(3*b) + x**2*sqrt(
a + b*x**2)/3, Ne(b, 0)), (sqrt(a)*x**2/2, True)) + D*b*Piecewise((-2*a**2
*sqrt(a + b*x**2)/(15*b**2) + a*x**2*sqrt(a + b*x**2)/(15*b) + x**4*sqrt(a
 + b*x**2)/5, Ne(b, 0)), (sqrt(a)*x**4/4, True))
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.88

$$\begin{aligned}\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{x^2}\,dx &= \frac{1}{4}\left(bx^2+a\right)^{\frac{3}{2}}Cx \\ &+ \frac{3}{8}\sqrt{bx^2+a}Cax + \frac{3}{2}\sqrt{bx^2+a}Abx + \frac{3\,Ca^2\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{b}} \\ &+ \frac{3}{2}Aa\sqrt{b}\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right) - Ba^{\frac{3}{2}}\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right) \\ &+ \frac{1}{3}\left(bx^2+a\right)^{\frac{3}{2}}B + \sqrt{bx^2+a}Ba + \frac{(bx^2+a)^{\frac{5}{2}}D}{5\,b} - \frac{(bx^2+a)^{\frac{3}{2}}A}{x}\end{aligned}$$

input
```
integrate((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/x^2,x, algorithm="maxima")
```

output
```
1/4*(b*x^2 + a)^(3/2)*C*x + 3/8*sqrt(b*x^2 + a)*C*a*x + 3/2*sqrt(b*x^2 + a
)*A*b*x + 3/8*C*a^2*arcsinh(b*x/sqrt(a*b))/sqrt(b) + 3/2*A*a*sqrt(b)*arcsi
nh(b*x/sqrt(a*b)) - B*a^(3/2)*arcsinh(a/(sqrt(a*b)*abs(x))) + 1/3*(b*x^2 +
 a)^(3/2)*B + sqrt(b*x^2 + a)*B*a + 1/5*(b*x^2 + a)^(5/2)*D/b - (b*x^2 + a
)^(3/2)*A/x
```

## Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.17

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{x^2}\,dx = \frac{2\,Ba^2\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$
$$+\frac{2\,Aa^2\sqrt{b}}{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a}$$
$$+\frac{1}{120}\sqrt{bx^2+a}\left(\left(2\left(3\left(4\,Dbx+5\,Cb\right)x+\frac{4\left(6\,Dab^3+5\,Bb^4\right)}{b^3}\right)x+\frac{15\left(5\,Cab^3+4\,Ab^4\right)}{b^3}\right)x+\frac{8\left(3\,Da^2b^2-\right.}{b^3}\right.$$
$$-\frac{3\left(Ca^2+4\,Aab\right)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{8\,\sqrt{b}}$$

input
```
integrate((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/x^2,x, algorithm="giac")
```

output
```
2*B*a^2*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/sqrt(-a) + 2*A*a^2
*sqrt(b)/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a) + 1/120*sqrt(b*x^2 + a)*((2
*(3*(4*D*b*x + 5*C*b)*x + 4*(6*D*a*b^3 + 5*B*b^4)/b^3)*x + 15*(5*C*a*b^3 +
 4*A*b^4)/b^3)*x + 8*(3*D*a^2*b^2 + 20*B*a*b^3)/b^3) - 3/8*(C*a^2 + 4*A*a*
b)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b)
```

## Mupad [B] (verification not implemented)

Time = 3.04 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{x^2}\,dx = \frac{B\,(b\,x^2+a)^{3/2}}{3}$$
$$-B\,a^{3/2}\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)+B\,a\,\sqrt{b\,x^2+a}-\frac{A\,(b\,x^2+a)^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},-\frac{1}{2};\ \frac{1}{2};\ -\frac{b\,x^2}{a}\right)}{x\,\left(\frac{b\,x^2}{a}+1\right)^{3/2}}+\frac{x^2\,(b\,x^2+a)^{3/2}\,D}{2\,\left(\frac{b\,x}{a}\right.}$$

input
```
int(((a + b*x^2)^(3/2)*(A + B*x + C*x^2 + x^3*D))/x^2,x)
```

output
```
(B*(a + b*x^2)^(3/2))/3 - B*a^(3/2)*atanh((a + b*x^2)^(1/2)/a^(1/2)) + B*a
*(a + b*x^2)^(1/2) - (A*(a + b*x^2)^(3/2)*hypergeom([-3/2, -1/2], 1/2, -(b
*x^2)/a))/(x*((b*x^2)/a + 1)^(3/2)) + (x^2*(a + b*x^2)^(3/2)*D*hypergeom([
-3/2, 1], 2, -(b*x^2)/a))/(2*((b*x^2)/a + 1)^(3/2)) + (C*x*(a + b*x^2)^(3/
2)*hypergeom([-3/2, 1/2], 3/2, -(b*x^2)/a))/((b*x^2)/a + 1)^(3/2)
```

## Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.71

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{x^2}\,dx = \frac{-120\sqrt{b\,x^2+a}\,a^2b+24\sqrt{b\,x^2+a}\,a^2dx+60\sqrt{b\,x^2+a}\,a\,b^2x^2}{}$$

input
```
int((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/x^2,x)
```

output
```
( - 120*sqrt(a + b*x**2)*a**2*b + 24*sqrt(a + b*x**2)*a**2*d*x + 60*sqrt(a
 + b*x**2)*a*b**2*x**2 + 160*sqrt(a + b*x**2)*a*b**2*x + 75*sqrt(a + b*x**
2)*a*b*c*x**2 + 48*sqrt(a + b*x**2)*a*b*d*x**3 + 40*sqrt(a + b*x**2)*b**3*
x**3 + 30*sqrt(a + b*x**2)*b**2*c*x**4 + 24*sqrt(a + b*x**2)*b**2*d*x**5 +
 120*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b**2*
x - 120*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b*
*2*x + 180*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2*b*x +
45*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2*c*x - 135*sqrt
(b)*a**2*b*x - 15*sqrt(b)*a**2*c*x)/(120*b*x)
```

# 3.80 $\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{x^3}\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 195

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{x^3}\,dx = \frac{1}{8}(4(3Ab+2aC) + 3(4bB+aD)x)\sqrt{a+bx^2} + \frac{(2(3Ab+2aC)+3(4bB+aD)x)(a+bx^2)^{3/2}}{12a} - \frac{A(a+bx^2)^{5/2}}{2ax^2} - \frac{B(a+bx^2)^{5/2}}{ax} + \frac{3a(4bB+aD)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}} - \frac{1}{2}\sqrt{a}(3Ab+2aC)\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

output

```
1/8*(12*A*b+8*C*a+3*(4*B*b+D*a)*x)*(b*x^2+a)^(1/2)+1/12*(6*A*b+4*C*a+3*(4*
B*b+D*a)*x)*(b*x^2+a)^(3/2)/a-1/2*A*(b*x^2+a)^(5/2)/a/x^2-B*(b*x^2+a)^(5/2
)/a/x+3/8*a*(4*B*b+D*a)*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(1/2)-1/2*a^(
1/2)*(3*A*b+2*C*a)*arctanh((b*x^2+a)^(1/2)/a^(1/2))
```

## Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.77

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{x^3}\,dx = \sqrt{a}(3Ab+2aC)\operatorname{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{1}{24}\left(\frac{\sqrt{a+bx^2}(2bx^2(12A+6Bx+4Cx^2+3Dx^3)+a(-12A-24Bx+32Cx^2+15Dx^3))}{x^2}-\frac{9a(4bB+aD)\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{\sqrt{b}}\right)$$

input
```
Integrate[((a + b*x^2)^(3/2)*(A + B*x + C*x^2 + D*x^3))/x^3,x]
```

output
```
Sqrt[a]*(3*A*b + 2*a*C)*ArcTanh[(Sqrt[b]*x - Sqrt[a + b*x^2])/Sqrt[a]] + (
(Sqrt[a + b*x^2]*(2*b*x^2*(12*A + 6*B*x + 4*C*x^2 + 3*D*x^3) + a*(-12*A -
24*B*x + 32*C*x^2 + 15*D*x^3)))/x^2 - (9*a*(4*b*B + a*D)*Log[-(Sqrt[b]*x)
+ Sqrt[a + b*x^2]])/Sqrt[b])/24
```

## Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.05, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {2338, 25, 2338, 25, 27, 535, 27, 535, 538, 224, 219, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{x^3}\,dx$$

$$\Big\downarrow \; 2338$$

$$-\frac{\int -\frac{(bx^2+a)^{3/2}(2aDx^2+(3Ab+2aC)x+2aB)}{x^2}dx}{2a}-\frac{A(a+bx^2)^{5/2}}{2ax^2}$$

$\downarrow$ 25

$$\frac{\int \frac{(bx^2+a)^{3/2}(2aDx^2+(3Ab+2aC)x+2aB)}{x^2}dx}{2a}-\frac{A(a+bx^2)^{5/2}}{2ax^2}$$

$\downarrow$ 2338

$$\frac{-\frac{\int -\frac{a(3Ab+2aC+2(4bB+aD)x)(bx^2+a)^{3/2}}{x}dx}{a}-\frac{2B(a+bx^2)^{5/2}}{x}}{2a}-\frac{A(a+bx^2)^{5/2}}{2ax^2}$$

$\downarrow$ 25

$$\frac{\frac{\int \frac{a(3Ab+2aC+2(4bB+aD)x)(bx^2+a)^{3/2}}{x}dx}{a}-\frac{2B(a+bx^2)^{5/2}}{x}}{2a}-\frac{A(a+bx^2)^{5/2}}{2ax^2}$$

$\downarrow$ 27

$$\frac{\int \frac{(3Ab+2aC+2(4bB+aD)x)(bx^2+a)^{3/2}}{x}dx-\frac{2B(a+bx^2)^{5/2}}{x}}{2a}-\frac{A(a+bx^2)^{5/2}}{2ax^2}$$

$\downarrow$ 535

$$\frac{\frac{1}{4}a\int \frac{2(2(3Ab+2aC)+3(4bB+aD)x)\sqrt{bx^2+a}}{x}dx+\frac{1}{6}(a+bx^2)^{3/2}(2(2aC+3Ab)+3x(aD+4bB))-\frac{2B(a+bx^2)^{5/2}}{x}}{2a}-\frac{A(a+bx^2)^{5/2}}{2ax^2}$$

$\downarrow$ 27

$$\frac{\frac{1}{2}a\int \frac{(2(3Ab+2aC)+3(4bB+aD)x)\sqrt{bx^2+a}}{x}dx+\frac{1}{6}(a+bx^2)^{3/2}(2(2aC+3Ab)+3x(aD+4bB))-\frac{2B(a+bx^2)^{5/2}}{x}}{2a}-\frac{A(a+bx^2)^{5/2}}{2ax^2}$$

$\downarrow$ 535

$$\frac{\frac{1}{2}a\left(\frac{1}{2}a\int \frac{4(3Ab+2aC)+3(4bB+aD)x}{x\sqrt{bx^2+a}}dx+\frac{1}{2}\sqrt{a+bx^2}(4(2aC+3Ab)+3x(aD+4bB))\right)+\frac{1}{6}(a+bx^2)^{3/2}(2(2aC+3A}{2a}$$
$$\frac{A(a+bx^2)^{5/2}}{2ax^2}$$

$$\Big\downarrow \text{538}$$

$$\frac{\frac{1}{2}a\left(\frac{1}{2}a\left(4(2aC+3Ab)\int \frac{1}{x\sqrt{bx^2+a}}dx+3(aD+4bB)\int \frac{1}{\sqrt{bx^2+a}}dx\right)+\frac{1}{2}\sqrt{a+bx^2}(4(2aC+3Ab)+3x(aD+4bB))}{2a}$$

$$\frac{A\left(a+bx^2\right)^{5/2}}{2ax^2}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{1}{2}a\left(\frac{1}{2}a\left(4(2aC+3Ab)\int \frac{1}{x\sqrt{bx^2+a}}dx+3(aD+4bB)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}\right)+\frac{1}{2}\sqrt{a+bx^2}(4(2aC+3Ab)+3x(aD}{2a}$$

$$\frac{A\left(a+bx^2\right)^{5/2}}{2ax^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{1}{2}a\left(\frac{1}{2}a\left(4(2aC+3Ab)\int \frac{1}{x\sqrt{bx^2+a}}dx+\frac{3\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(aD+4bB)}{\sqrt{b}}\right)+\frac{1}{2}\sqrt{a+bx^2}(4(2aC+3Ab)+3x(aD+4b}{2a}$$

$$\frac{A\left(a+bx^2\right)^{5/2}}{2ax^2}$$

$$\Big\downarrow \text{243}$$

$$\frac{\frac{1}{2}a\left(\frac{1}{2}a\left(2(2aC+3Ab)\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2+\frac{3\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(aD+4bB)}{\sqrt{b}}\right)+\frac{1}{2}\sqrt{a+bx^2}(4(2aC+3Ab)+3x(aD+}{2a}$$

$$\frac{A\left(a+bx^2\right)^{5/2}}{2ax^2}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{1}{2}a\left(\frac{1}{2}a\left(\frac{4(2aC+3Ab)\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{b}+\frac{3\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(aD+4bB)}{\sqrt{b}}\right)+\frac{1}{2}\sqrt{a+bx^2}(4(2aC+3Ab)+3x(aD+4bB}{2a}$$

$$\frac{A\left(a+bx^2\right)^{5/2}}{2ax^2}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{1}{2}a\left(\frac{1}{2}a\left(\frac{3\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(aD+4bB)}{\sqrt{b}}-\frac{4(2aC+3Ab)\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}\right)+\frac{1}{2}\sqrt{a+bx^2}(4(2aC+3Ab)+3x(aD+4\ldots}{2a}$$

$$\frac{A\left(a+bx^2\right)^{5/2}}{2ax^2}$$

input
```
Int[((a + b*x^2)^(3/2)*(A + B*x + C*x^2 + D*x^3))/x^3,x]
```

output
```
-1/2*(A*(a + b*x^2)^(5/2))/(a*x^2) + (((2*(3*A*b + 2*a*C) + 3*(4*b*B + a*D
)*x)*(a + b*x^2)^(3/2))/6 - (2*B*(a + b*x^2)^(5/2))/x + (a*(((4*(3*A*b + 2
*a*C) + 3*(4*b*B + a*D)*x)*Sqrt[a + b*x^2])/2 + (a*((3*(4*b*B + a*D)*ArcTa
nh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/Sqrt[b] - (4*(3*A*b + 2*a*C)*ArcTanh[Sqrt
[a + b*x^2]/Sqrt[a]])/Sqrt[a]))/2))/2)/(2*a)
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 535
```
Int[(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_))/(x_), x_Symbol] :> Sim
p[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^p/(2*p*(2*p + 1))), x] + Simp[a/(2*p
 + 1)   Int[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^(p - 1)/x), x], x] /; Free
Q[{a, b, c, d}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

## Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.35

| method | result |
|---|---|
| default | $D\left(\frac{x(b\,x^{2}+a)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)}{4}\right)+A\left(-\frac{(b\,x^{2}+a)^{\frac{5}{2}}}{2a\,x^{2}}+\frac{3b\left(\frac{(b\,x^{2}+a)^{\frac{3}{2}}}{3}+a\left(\sqrt{b\,x^{2}+a}-\sqrt{a}\,\ln\left(\right.\right.\right.}{2a}\right.$ |

input
```
int((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/x^3,x,method=_RETURNVERBOSE)
```

output
```
D*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(
1/2)*x+(b*x^2+a)^(1/2))))+A*(-1/2/a/x^2*(b*x^2+a)^(5/2)+3/2*b/a*(1/3*(b*x^
2+a)^(3/2)+a*((b*x^2+a)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x
))))+B*(-1/a/x*(b*x^2+a)^(5/2)+4*b/a*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(
b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))))+C*(1/3*(b*x^
2+a)^(3/2)+a*((b*x^2+a)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x
)))
```

## Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 683, normalized size of antiderivative = 3.50

$$\int \frac{(a+bx^2)^{3/2}\,(A+Bx+Cx^2+Dx^3)}{x^3}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/x^3,x, algorithm="fricas")
```

output

```
[1/48*(9*(D*a^2 + 4*B*a*b)*sqrt(b)*x^2*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sq
rt(b)*x - a) + 12*(2*C*a*b + 3*A*b^2)*sqrt(a)*x^2*log(-(b*x^2 - 2*sqrt(b*x
^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(6*D*b^2*x^5 + 8*C*b^2*x^4 - 24*B*a*b*x +
3*(5*D*a*b + 4*B*b^2)*x^3 - 12*A*a*b + 8*(4*C*a*b + 3*A*b^2)*x^2)*sqrt(b*x
^2 + a))/(b*x^2), -1/24*(9*(D*a^2 + 4*B*a*b)*sqrt(-b)*x^2*arctan(sqrt(-b)*
x/sqrt(b*x^2 + a)) - 6*(2*C*a*b + 3*A*b^2)*sqrt(a)*x^2*log(-(b*x^2 - 2*sqr
t(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - (6*D*b^2*x^5 + 8*C*b^2*x^4 - 24*B*a*b*x
 + 3*(5*D*a*b + 4*B*b^2)*x^3 - 12*A*a*b + 8*(4*C*a*b + 3*A*b^2)*x^2)*sqrt(
b*x^2 + a))/(b*x^2), 1/48*(24*(2*C*a*b + 3*A*b^2)*sqrt(-a)*x^2*arctan(sqrt
(b*x^2 + a)*sqrt(-a)/a) + 9*(D*a^2 + 4*B*a*b)*sqrt(b)*x^2*log(-2*b*x^2 - 2
*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(6*D*b^2*x^5 + 8*C*b^2*x^4 - 24*B*a*b*
x + 3*(5*D*a*b + 4*B*b^2)*x^3 - 12*A*a*b + 8*(4*C*a*b + 3*A*b^2)*x^2)*sqrt
(b*x^2 + a))/(b*x^2), -1/24*(9*(D*a^2 + 4*B*a*b)*sqrt(-b)*x^2*arctan(sqrt(
-b)*x/sqrt(b*x^2 + a)) - 12*(2*C*a*b + 3*A*b^2)*sqrt(-a)*x^2*arctan(sqrt(b
*x^2 + a)*sqrt(-a)/a) - (6*D*b^2*x^5 + 8*C*b^2*x^4 - 24*B*a*b*x + 3*(5*D*a
*b + 4*B*b^2)*x^3 - 12*A*a*b + 8*(4*C*a*b + 3*A*b^2)*x^2)*sqrt(b*x^2 + a))
/(b*x^2)]
```

## Sympy [A] (verification not implemented)

Time = 5.50 (sec) , antiderivative size = 500, normalized size of antiderivative = 2.56

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{x^3}\,dx = -\frac{3A\sqrt{a}b\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2} - \frac{Aa\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{2x} + \frac{Aa\sqrt{b}}{x\sqrt{\frac{a}{bx^2}+1}} + \frac{Ab^{\frac{3}{2}}x}{\sqrt{\frac{a}{bx^2}+1}} - \frac{Ba^{\frac{3}{2}}}{x\sqrt{1+\frac{bx^2}{a}}} - \frac{B\sqrt{a}bx}{\sqrt{1+\frac{bx^2}{a}}} + Ba\sqrt{b}\,\mathrm{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right) + Bb\left(\begin{cases}\frac{a\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for}\ a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right)}{2} + \frac{x\sqrt{a+bx^2}}{2} & \text{for}\ b\neq 0\\ \sqrt{a}x & \text{otherwise}\end{cases}\right) - Ca^{\frac{3}{2}}\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right) + \frac{Ca^2}{\sqrt{b}x\sqrt{\frac{a}{bx^2}+1}} + \frac{Ca\sqrt{b}x}{\sqrt{\frac{a}{bx^2}+1}} + Cb\left(\begin{cases}\frac{a\sqrt{a+bx^2}}{3b} + \frac{x^2\sqrt{a+bx^2}}{3} & \text{for}\ b\neq 0\\ \frac{\sqrt{a}x^2}{2} & \text{otherwise}\end{cases}\right) + Da\left(\begin{cases}\frac{a\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for}\ a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right)}{2} + \frac{x\sqrt{a+bx^2}}{2} & \text{for}\ b\neq 0\\ \sqrt{a}x & \text{otherwise}\end{cases}\right) + Db\left(\begin{cases}-\frac{a^2\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for}\ a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right)}{8b} + \frac{ax\sqrt{a+bx^2}}{8b} + \frac{x^3\sqrt{a+bx^2}}{4} & \text{for}\ b\neq 0\\ \frac{\sqrt{a}x^3}{3} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((b*x**2+a)**(3/2)*(D*x**3+C*x**2+B*x+A)/x**3,x)
```

output
```
-3*A*sqrt(a)*b*asinh(sqrt(a)/(sqrt(b)*x))/2 - A*a*sqrt(b)*sqrt(a/(b*x**2)
+ 1)/(2*x) + A*a*sqrt(b)/(x*sqrt(a/(b*x**2) + 1)) + A*b**(3/2)*x/sqrt(a/(b
*x**2) + 1) - B*a**(3/2)/(x*sqrt(1 + b*x**2/a)) - B*sqrt(a)*b*x/sqrt(1 + b
*x**2/a) + B*a*sqrt(b)*asinh(sqrt(b)*x/sqrt(a)) + B*b*Piecewise((a*Piecewi
se((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/
sqrt(b*x**2), True))/2 + x*sqrt(a + b*x**2)/2, Ne(b, 0)), (sqrt(a)*x, True
)) - C*a**(3/2)*asinh(sqrt(a)/(sqrt(b)*x)) + C*a**2/(sqrt(b)*x*sqrt(a/(b*x
**2) + 1)) + C*a*sqrt(b)*x/sqrt(a/(b*x**2) + 1) + C*b*Piecewise((a*sqrt(a
+ b*x**2)/(3*b) + x**2*sqrt(a + b*x**2)/3, Ne(b, 0)), (sqrt(a)*x**2/2, Tru
e)) + D*a*Piecewise((a*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/
sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True))/2 + x*sqrt(a + b*x**2)/
2, Ne(b, 0)), (sqrt(a)*x, True)) + D*b*Piecewise((-a**2*Piecewise((log(2*s
qrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2
), True))/(8*b) + a*x*sqrt(a + b*x**2)/(8*b) + x**3*sqrt(a + b*x**2)/4, Ne
(b, 0)), (sqrt(a)*x**3/3, True))
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{x^3}\,dx = \frac{1}{4}\left(bx^2+a\right)^{\frac{3}{2}}Dx + \frac{3}{8}\sqrt{bx^2+a}Dax + \frac{3}{2}\sqrt{bx^2+a}Bbx + \frac{3\,Da^2\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{b}} + \frac{3}{2}Ba\sqrt{b}\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right) - Ca^{\frac{3}{2}}\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right) - \frac{3}{2}A\sqrt{a}b\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right) + \frac{1}{3}\left(bx^2+a\right)^{\frac{3}{2}}C + \sqrt{bx^2+a}Ca + \frac{3}{2}\sqrt{bx^2+a}Ab + \frac{(bx^2+a)^{\frac{3}{2}}Ab}{2a} - \frac{(bx^2+a)^{\frac{3}{2}}B}{x} - \frac{(bx^2+a)^{\frac{5}{2}}A}{2ax^2}$$

input
```
integrate((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/x^3,x, algorithm="maxima")
```

output
```
1/4*(b*x^2 + a)^(3/2)*D*x + 3/8*sqrt(b*x^2 + a)*D*a*x + 3/2*sqrt(b*x^2 + a
)*B*b*x + 3/8*D*a^2*arcsinh(b*x/sqrt(a*b))/sqrt(b) + 3/2*B*a*sqrt(b)*arcsi
nh(b*x/sqrt(a*b)) - C*a^(3/2)*arcsinh(a/(sqrt(a*b)*abs(x))) - 3/2*A*sqrt(a
)*b*arcsinh(a/(sqrt(a*b)*abs(x))) + 1/3*(b*x^2 + a)^(3/2)*C + sqrt(b*x^2 +
 a)*C*a + 3/2*sqrt(b*x^2 + a)*A*b + 1/2*(b*x^2 + a)^(3/2)*A*b/a - (b*x^2 +
 a)^(3/2)*B/x - 1/2*(b*x^2 + a)^(5/2)*A/(a*x^2)
```

## Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.30

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{x^3}\,dx = \frac{1}{24}\sqrt{bx^2+a}\left(\left(2\,(3\,Dbx+4\,Cb)x+\frac{3\,(5\,Dab^2+4\,Bb^3)}{b^2}\right)x\right.$$

$$+\frac{(2\,Ca^2+3\,Aab)\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{3\,(Da^2+4\,Bab)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{8\,\sqrt{b}}+\frac{\left(\sqrt{b}x-\sqrt{bx^2+\right.}{}$$

input
```
integrate((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/x^3,x, algorithm="giac")
```

output
```
1/24*sqrt(b*x^2 + a)*((2*(3*D*b*x + 4*C*b)*x + 3*(5*D*a*b^2 + 4*B*b^3)/b^2
)*x + 8*(4*C*a*b^2 + 3*A*b^3)/b^2) + (2*C*a^2 + 3*A*a*b)*arctan(-(sqrt(b)*
x - sqrt(b*x^2 + a))/sqrt(-a))/sqrt(-a) - 3/8*(D*a^2 + 4*B*a*b)*log(abs(-s
qrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b) + ((sqrt(b)*x - sqrt(b*x^2 + a))^3*A*
a*b + 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a^2*sqrt(b) + (sqrt(b)*x - sqrt(
b*x^2 + a))*A*a^2*b - 2*B*a^3*sqrt(b))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 -
a)^2
```

## Mupad [B] (verification not implemented)

Time = 3.21 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{x^3}\,dx = \frac{C\,(b\,x^2+a)^{3/2}}{3}$$

$$-C\,a^{3/2}\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)+A\,b\,\sqrt{b\,x^2+a}+C\,a\,\sqrt{b\,x^2+a}-\frac{A\,a\,\sqrt{b\,x^2+a}}{2\,x^2}-\frac{3\,A\,\sqrt{a}\,b\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2}-$$

input
```
int(((a + b*x^2)^(3/2)*(A + B*x + C*x^2 + x^3*D))/x^3,x)
```

output
```
(C*(a + b*x^2)^(3/2))/3 - C*a^(3/2)*atanh((a + b*x^2)^(1/2)/a^(1/2)) + A*b
*(a + b*x^2)^(1/2) + C*a*(a + b*x^2)^(1/2) - (A*a*(a + b*x^2)^(1/2))/(2*x^
2) - (3*A*a^(1/2)*b*atanh((a + b*x^2)^(1/2)/a^(1/2)))/2 - (B*(a + b*x^2)^(
3/2)*hypergeom([-3/2, -1/2], 1/2, -(b*x^2)/a))/(x*((b*x^2)/a + 1)^(3/2)) +
 (x*(a + b*x^2)^(3/2)*D*hypergeom([-3/2, 1/2], 3/2, -(b*x^2)/a))/((b*x^2)/
a + 1)^(3/2)
```

## Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.68

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{x^3}\,dx = \frac{-12\sqrt{b\,x^2+a}\,a^2b+24\sqrt{b\,x^2+a}\,a\,b^2x^2-24\sqrt{b\,x^2+a}\,a\,b^2x\,}{}$$

input
```
int((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/x^3,x)
```

output
```
( - 12*sqrt(a + b*x**2)*a**2*b + 24*sqrt(a + b*x**2)*a*b**2*x**2 - 24*sqrt
(a + b*x**2)*a*b**2*x + 32*sqrt(a + b*x**2)*a*b*c*x**2 + 15*sqrt(a + b*x**
2)*a*b*d*x**3 + 12*sqrt(a + b*x**2)*b**3*x**3 + 8*sqrt(a + b*x**2)*b**2*c*
x**4 + 6*sqrt(a + b*x**2)*b**2*d*x**5 + 36*sqrt(a)*log((sqrt(a + b*x**2) -
 sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b**2*x**2 + 24*sqrt(a)*log((sqrt(a + b*x*
*2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b*c*x**2 - 36*sqrt(a)*log((sqrt(a +
b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b**2*x**2 - 24*sqrt(a)*log((sqrt
(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b*c*x**2 + 9*sqrt(b)*log((s
qrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2*d*x**2 + 36*sqrt(b)*log((sqrt(a
 + b*x**2) + sqrt(b)*x)/sqrt(a))*a*b**2*x**2)/(24*b*x**2)
```

# 3.81 $\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{x^4}\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 205

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{x^4}\,dx = \frac{(a(3bB+2aD)+b(2Ab+3aC)x)\sqrt{a+bx^2}}{2a} - \frac{(2(2Ab+3aC)-(3bB+2aD)x)(a+bx^2)^{3/2}}{6ax} - \frac{A(a+bx^2)^{5/2}}{3ax^3} - \frac{B(a+bx^2)^{5/2}}{2ax^2} + \frac{1}{2}\sqrt{b}(2Ab+3aC)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right) - \frac{1}{2}\sqrt{a}(3bB+2aD)\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

output

```
1/2*(a*(3*B*b+2*D*a)+b*(2*A*b+3*C*a)*x)*(b*x^2+a)^(1/2)/a-1/6*(4*A*b+6*C*a
-(3*B*b+2*D*a)*x)*(b*x^2+a)^(3/2)/a/x-1/3*A*(b*x^2+a)^(5/2)/a/x^3-1/2*B*(b
*x^2+a)^(5/2)/a/x^2+1/2*b^(1/2)*(2*A*b+3*C*a)*arctanh(b^(1/2)*x/(b*x^2+a)^
(1/2))-1/2*a^(1/2)*(3*B*b+2*D*a)*arctanh((b*x^2+a)^(1/2)/a^(1/2))
```

## Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.73

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{x^4}\,dx = \sqrt{a}(3bB + 2aD)\text{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right) + \frac{1}{6}\left(\frac{\sqrt{a+bx^2}(-a(2A+3Bx+6Cx^2-8Dx^3)+bx^2(-8A+6Bx+3Cx^2+2Dx^3))}{x^3} - 3\sqrt{b}(2Ab+3aC)\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)\right)$$

input
```
Integrate[((a + b*x^2)^(3/2)*(A + B*x + C*x^2 + D*x^3))/x^4,x]
```

output
```
Sqrt[a]*(3*b*B + 2*a*D)*ArcTanh[(Sqrt[b]*x - Sqrt[a + b*x^2])/Sqrt[a]] + (
(Sqrt[a + b*x^2]*(-(a*(2*A + 3*B*x + 6*C*x^2 - 8*D*x^3)) + b*x^2*(-8*A + 6
*B*x + 3*C*x^2 + 2*D*x^3)))/x^3 - 3*Sqrt[b]*(2*A*b + 3*a*C)*Log[-(Sqrt[b]*
x) + Sqrt[a + b*x^2]])/6
```

## Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {2338, 25, 2338, 25, 27, 536, 535, 27, 538, 224, 219, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{x^4}\,dx$$

$$\Big\downarrow \text{2338}$$

$$-\frac{\int -\frac{(bx^2+a)^{3/2}(3aDx^2+(2Ab+3aC)x+3aB)}{x^3}dx}{3a} - \frac{A(a+bx^2)^{5/2}}{3ax^3}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{(bx^2+a)^{3/2}(3aDx^2+(2Ab+3aC)x+3aB)}{x^3}dx}{3a}-\frac{A(a+bx^2)^{5/2}}{3ax^3}$$

$$\Big\downarrow \ 2338$$

$$\frac{-\frac{\int -\frac{a(2(2Ab+3aC)+3(3bB+2aD)x)(bx^2+a)^{3/2}}{x^2}dx}{2a}-\frac{3B(a+bx^2)^{5/2}}{2x^2}}{3a}-\frac{A(a+bx^2)^{5/2}}{3ax^3}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{\int \frac{a(2(2Ab+3aC)+3(3bB+2aD)x)(bx^2+a)^{3/2}}{x^2}dx}{2a}-\frac{3B(a+bx^2)^{5/2}}{2x^2}}{3a}-\frac{A(a+bx^2)^{5/2}}{3ax^3}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{1}{2}\int \frac{(2(2Ab+3aC)+3(3bB+2aD)x)(bx^2+a)^{3/2}}{x^2}dx-\frac{3B(a+bx^2)^{5/2}}{2x^2}}{3a}-\frac{A(a+bx^2)^{5/2}}{3ax^3}$$

$$\Big\downarrow \ 536$$

$$\frac{\frac{1}{2}\left(\int \frac{(3a(3bB+2aD)+6b(2Ab+3aC)x)\sqrt{bx^2+a}}{x}dx-\frac{(a+bx^2)^{3/2}(2(3aC+2Ab)-x(2aD+3bB))}{x}\right)-\frac{3B(a+bx^2)^{5/2}}{2x^2}}{3a}-\frac{A(a+bx^2)^{5/2}}{3ax^3}$$

$$\Big\downarrow \ 535$$

$$\frac{\frac{1}{2}\left(\frac{1}{2}a\int \frac{6(a(3bB+2aD)+b(2Ab+3aC)x)}{x\sqrt{bx^2+a}}dx-\frac{(a+bx^2)^{3/2}(2(3aC+2Ab)-x(2aD+3bB))}{x}+3\sqrt{a+bx^2}(bx(3aC+2Ab)+a(2aD\right.}{3a}$$
$$\frac{A(a+bx^2)^{5/2}}{3ax^3}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{1}{2}\left(3a\int \frac{a(3bB+2aD)+b(2Ab+3aC)x}{x\sqrt{bx^2+a}}dx-\frac{(a+bx^2)^{3/2}(2(3aC+2Ab)-x(2aD+3bB))}{x}+3\sqrt{a+bx^2}(bx(3aC+2Ab)+a(2aD+\right.}{3a}$$
$$\frac{A(a+bx^2)^{5/2}}{3ax^3}$$

$$\Big\downarrow \ 538$$

$$\frac{\frac{1}{2}\left(3a\left(b(3aC+2Ab)\int\frac{1}{\sqrt{bx^2+a}}dx+a(2aD+3bB)\int\frac{1}{x\sqrt{bx^2+a}}dx\right)-\frac{(a+bx^2)^{3/2}(2(3aC+2Ab)-x(2aD+3bB))}{x}+3\sqrt{a+b}\right.}{3a}$$

$$\frac{A(a+bx^2)^{5/2}}{3ax^3}$$

$\Big\downarrow$ 224

$$\frac{\frac{1}{2}\left(3a\left(b(3aC+2Ab)\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+a(2aD+3bB)\int\frac{1}{x\sqrt{bx^2+a}}dx\right)-\frac{(a+bx^2)^{3/2}(2(3aC+2Ab)-x(2aD+3bB))}{x}+3\right.}{3a}$$

$$\frac{A(a+bx^2)^{5/2}}{3ax^3}$$

$\Big\downarrow$ 219

$$\frac{\frac{1}{2}\left(3a\left(a(2aD+3bB)\int\frac{1}{x\sqrt{bx^2+a}}dx+\sqrt{b}(3aC+2Ab)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\right)-\frac{(a+bx^2)^{3/2}(2(3aC+2Ab)-x(2aD+3bB))}{x}+3\right.}{3a}$$

$$\frac{A(a+bx^2)^{5/2}}{3ax^3}$$

$\Big\downarrow$ 243

$$\frac{\frac{1}{2}\left(3a\left(\frac{1}{2}a(2aD+3bB)\int\frac{1}{x^2\sqrt{bx^2+a}}dx^2+\sqrt{b}(3aC+2Ab)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\right)-\frac{(a+bx^2)^{3/2}(2(3aC+2Ab)-x(2aD+3bB))}{x}\right.}{3a}$$

$$\frac{A(a+bx^2)^{5/2}}{3ax^3}$$

$\Big\downarrow$ 73

$$\frac{\frac{1}{2}\left(3a\left(\frac{a(2aD+3bB)\int\frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{b}+\sqrt{b}(3aC+2Ab)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\right)-\frac{(a+bx^2)^{3/2}(2(3aC+2Ab)-x(2aD+3bB))}{x}+3\right.}{3a}$$

$$\frac{A(a+bx^2)^{5/2}}{3ax^3}$$

$\Big\downarrow$ 221

$$\frac{\frac{1}{2}\left(3a\left(\sqrt{b}(3aC+2Ab)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)-\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)(2aD+3bB)\right)-\frac{(a+bx^2)^{3/2}(2(3aC+2Ab)-x(2aD+3}{x}\right.}{3a}$$

$$\frac{A(a+bx^2)^{5/2}}{3ax^3}$$

input
```
Int[((a + b*x^2)^(3/2)*(A + B*x + C*x^2 + D*x^3))/x^4,x]
```

output
```
-1/3*(A*(a + b*x^2)^(5/2))/(a*x^3) + ((-3*B*(a + b*x^2)^(5/2))/(2*x^2) + (
3*(a*(3*b*B + 2*a*D) + b*(2*A*b + 3*a*C)*x)*Sqrt[a + b*x^2] - ((2*(2*A*b +
 3*a*C) - (3*b*B + 2*a*D)*x)*(a + b*x^2)^(3/2))/x + 3*a*(Sqrt[b]*(2*A*b +
3*a*C)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]] - Sqrt[a]*(3*b*B + 2*a*D)*ArcT
anh[Sqrt[a + b*x^2]/Sqrt[a]]))/2)/(3*a)
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 535
```
Int[(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_))/(x_), x_Symbol] :> Sim
p[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^p/(2*p*(2*p + 1))), x] + Simp[a/(2*p
 + 1)   Int[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^(p - 1)/x), x], x] /; Free
Q[{a, b, c, d}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 536
```
Int[(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_))/(x_)^2, x_Symbol] :> S
imp[(-(2*c*p - d*x))*((a + b*x^2)^p/(2*p*x)), x] + Int[(a*d + 2*b*c*p*x)*((
a + b*x^2)^(p - 1)/x), x] /; FreeQ[{a, b, c, d}, x] && GtQ[p, 0] && Integer
Q[2*p]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

## Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.52

| method | result |
|---|---|
| default | $A\left(-\frac{(b\,x^2+a)^{\frac{5}{2}}}{3a\,x^3}+\frac{2b\left(-\frac{(b\,x^2+a)^{\frac{5}{2}}}{ax}+\frac{4b\left(\frac{x(b\,x^2+a)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)}{4}\right)}{a}\right)}{3a}\right)+B\left(-\frac{(b\,x^2+a)^{\frac{5}{2}}}{2a\,x^2}\right.$ |

input
```
int((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/x^4,x,method=_RETURNVERBOSE)
```

output
```
A*(-1/3/a/x^3*(b*x^2+a)^(5/2)+2/3*b/a*(-1/a/x*(b*x^2+a)^(5/2)+4*b/a*(1/4*x
*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(
b*x^2+a)^(1/2))))))+B*(-1/2/a/x^2*(b*x^2+a)^(5/2)+3/2*b/a*(1/3*(b*x^2+a)^(
3/2)+a*((b*x^2+a)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x))))+C
*(-1/a/x*(b*x^2+a)^(5/2)+4*b/a*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+
a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))))+D*(1/3*(b*x^2+a)^(
3/2)+a*((b*x^2+a)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)))
```

## Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 603, normalized size of antiderivative = 2.94

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{x^4}\,dx = \left[\frac{3\,(3\,Ca+2\,Ab)\sqrt{b}x^3\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+\cdots}{\cdots}\right.$$

$$-\frac{6\,(3\,Ca+2\,Ab)\sqrt{-b}x^3\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-3\,(2\,Da+3\,Bb)\sqrt{a}x^3\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)-2\,(2\,Dbx^5+\cdots}{12\,x^3}$$

$$-\frac{3\,(3\,Ca+2\,Ab)\sqrt{-b}x^3\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-3\,(2\,Da+3\,Bb)\sqrt{-a}x^3\arctan\left(\frac{\sqrt{bx^2+a}\sqrt{-a}}{a}\right)-(2\,Dbx^5+3\,Cb\cdots}{6\,x^3}$$

input
```
integrate((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/x^4,x, algorithm="fricas")
```

output
```
[1/12*(3*(3*C*a + 2*A*b)*sqrt(b)*x^3*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt
(b)*x - a) + 3*(2*D*a + 3*B*b)*sqrt(a)*x^3*log(-(b*x^2 - 2*sqrt(b*x^2 + a)
*sqrt(a) + 2*a)/x^2) + 2*(2*D*b*x^5 + 3*C*b*x^4 + 2*(4*D*a + 3*B*b)*x^3 -
3*B*a*x - 2*(3*C*a + 4*A*b)*x^2 - 2*A*a)*sqrt(b*x^2 + a))/x^3, -1/12*(6*(3
*C*a + 2*A*b)*sqrt(-b)*x^3*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - 3*(2*D*a +
 3*B*b)*sqrt(a)*x^3*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) -
2*(2*D*b*x^5 + 3*C*b*x^4 + 2*(4*D*a + 3*B*b)*x^3 - 3*B*a*x - 2*(3*C*a + 4*
A*b)*x^2 - 2*A*a)*sqrt(b*x^2 + a))/x^3, 1/12*(6*(2*D*a + 3*B*b)*sqrt(-a)*x
^3*arctan(sqrt(b*x^2 + a)*sqrt(-a)/a) + 3*(3*C*a + 2*A*b)*sqrt(b)*x^3*log(
-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(2*D*b*x^5 + 3*C*b*x^4 + 2
*(4*D*a + 3*B*b)*x^3 - 3*B*a*x - 2*(3*C*a + 4*A*b)*x^2 - 2*A*a)*sqrt(b*x^2
 + a))/x^3, -1/6*(3*(3*C*a + 2*A*b)*sqrt(-b)*x^3*arctan(sqrt(-b)*x/sqrt(b*
x^2 + a)) - 3*(2*D*a + 3*B*b)*sqrt(-a)*x^3*arctan(sqrt(b*x^2 + a)*sqrt(-a)
/a) - (2*D*b*x^5 + 3*C*b*x^4 + 2*(4*D*a + 3*B*b)*x^3 - 3*B*a*x - 2*(3*C*a
+ 4*A*b)*x^2 - 2*A*a)*sqrt(b*x^2 + a))/x^3]
```

## Sympy [A] (verification not implemented)

Time = 6.26 (sec) , antiderivative size = 447, normalized size of antiderivative = 2.18

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{x^4}\,dx = -\frac{A\sqrt{a}b}{x\sqrt{1+\frac{bx^2}{a}}} - \frac{Aa\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{3x^2} - \frac{Ab^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{3} + Ab^{\frac{3}{2}}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right) - \frac{Ab^2x}{\sqrt{a}\sqrt{1+\frac{bx^2}{a}}} - \frac{3B\sqrt{a}b\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2} - \frac{Ba\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{2x} + \frac{Ba\sqrt{b}}{x\sqrt{\frac{a}{bx^2}+1}} + \frac{Bb^{\frac{3}{2}}x}{\sqrt{\frac{a}{bx^2}+1}} - \frac{Ca^{\frac{3}{2}}}{x\sqrt{1+\frac{bx^2}{a}}} - \frac{C\sqrt{a}bx}{\sqrt{1+\frac{bx^2}{a}}} + Ca\sqrt{b}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right) + Cb\left(\begin{cases}\frac{a\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for}\ a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right)}{2} + \frac{x\sqrt{a+bx^2}}{2} & \text{for}\ b\neq 0\\ \sqrt{a}x & \text{otherwise}\end{cases}\right) - Da^{\frac{3}{2}}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right) + \frac{Da^2}{\sqrt{b}x\sqrt{\frac{a}{bx^2}+1}} + \frac{Da\sqrt{b}x}{\sqrt{\frac{a}{bx^2}+1}} + Db\left(\begin{cases}\frac{a\sqrt{a+bx^2}}{3b} + \frac{x^2\sqrt{a+bx^2}}{3} & \text{for}\ b\neq 0\\ \frac{\sqrt{a}x^2}{2} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((b*x**2+a)**(3/2)*(D*x**3+C*x**2+B*x+A)/x**4,x)
```

output
```
-A*sqrt(a)*b/(x*sqrt(1 + b*x**2/a)) - A*a*sqrt(b)*sqrt(a/(b*x**2) + 1)/(3*
x**2) - A*b**(3/2)*sqrt(a/(b*x**2) + 1)/3 + A*b**(3/2)*asinh(sqrt(b)*x/sqr
t(a)) - A*b**2*x/(sqrt(a)*sqrt(1 + b*x**2/a)) - 3*B*sqrt(a)*b*asinh(sqrt(a
)/(sqrt(b)*x))/2 - B*a*sqrt(b)*sqrt(a/(b*x**2) + 1)/(2*x) + B*a*sqrt(b)/(x
*sqrt(a/(b*x**2) + 1)) + B*b**(3/2)*x/sqrt(a/(b*x**2) + 1) - C*a**(3/2)/(x
*sqrt(1 + b*x**2/a)) - C*sqrt(a)*b*x/sqrt(1 + b*x**2/a) + C*a*sqrt(b)*asin
h(sqrt(b)*x/sqrt(a)) + C*b*Piecewise((a*Piecewise((log(2*sqrt(b)*sqrt(a +
b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True))/2 + x*
sqrt(a + b*x**2)/2, Ne(b, 0)), (sqrt(a)*x, True)) - D*a**(3/2)*asinh(sqrt(
a)/(sqrt(b)*x)) + D*a**2/(sqrt(b)*x*sqrt(a/(b*x**2) + 1)) + D*a*sqrt(b)*x/
sqrt(a/(b*x**2) + 1) + D*b*Piecewise((a*sqrt(a + b*x**2)/(3*b) + x**2*sqrt
(a + b*x**2)/3, Ne(b, 0)), (sqrt(a)*x**2/2, True))
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{x^4}\,dx = \frac{3}{2}\sqrt{bx^2+a}Cbx + \frac{\sqrt{bx^2+a}Ab^2x}{a} + \frac{3}{2}Ca\sqrt{b}\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right) + Ab^{\frac{3}{2}}\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right) - Da^{\frac{3}{2}}\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right) - \frac{3}{2}B\sqrt{a}b\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right) + \frac{1}{3}(bx^2+a)^{\frac{3}{2}}D + \sqrt{bx^2+a}Da + \frac{3}{2}\sqrt{bx^2+a}Bb + \frac{(bx^2+a)^{\frac{3}{2}}Bb}{2a} - \frac{(bx^2+a)^{\frac{3}{2}}C}{x} - \frac{2(bx^2+a)^{\frac{3}{2}}Ab}{3ax} - \frac{(bx^2+a)^{\frac{5}{2}}B}{2ax^2} - \frac{(bx^2+a)^{\frac{5}{2}}A}{3ax^3}$$

input
```
integrate((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/x^4,x, algorithm="maxima")
```

output
```
3/2*sqrt(b*x^2 + a)*C*b*x + sqrt(b*x^2 + a)*A*b^2*x/a + 3/2*C*a*sqrt(b)*ar
csinh(b*x/sqrt(a*b)) + A*b^(3/2)*arcsinh(b*x/sqrt(a*b)) - D*a^(3/2)*arcsin
h(a/(sqrt(a*b)*abs(x))) - 3/2*B*sqrt(a)*b*arcsinh(a/(sqrt(a*b)*abs(x))) +
1/3*(b*x^2 + a)^(3/2)*D + sqrt(b*x^2 + a)*D*a + 3/2*sqrt(b*x^2 + a)*B*b +
1/2*(b*x^2 + a)^(3/2)*B*b/a - (b*x^2 + a)^(3/2)*C/x - 2/3*(b*x^2 + a)^(3/2
)*A*b/(a*x) - 1/2*(b*x^2 + a)^(5/2)*B/(a*x^2) - 1/3*(b*x^2 + a)^(5/2)*A/(a
*x^3)
```

## Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.58

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{x^4}\,dx =$$
$$-\frac{1}{2}\left(3\,Ca\sqrt{b}+2\,Ab^{\frac{3}{2}}\right)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)$$
$$+\frac{1}{6}\sqrt{bx^2+a}\left((2\,Dbx+3\,Cb)x+\frac{2\,(4\,Dab+3\,Bb^2)}{b}\right)$$
$$+\frac{(2\,Da^2+3\,Bab)\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$
$$+\frac{3\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^5 Bab+6\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4 Ca^2\sqrt{b}+12\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4 Aab^{\frac{3}{2}}-12\left(\sqrt{b}x-\right.}{3\left(\left(\sqrt{b}x-\sqrt{bx\right.\right.}}$$

input
```
integrate((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/x^4,x, algorithm="giac")
```

output
```
-1/2*(3*C*a*sqrt(b) + 2*A*b^(3/2))*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))
+ 1/6*sqrt(b*x^2 + a)*((2*D*b*x + 3*C*b)*x + 2*(4*D*a*b + 3*B*b^2)/b) + (2
*D*a^2 + 3*B*a*b)*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/sqrt(-a)
 + 1/3*(3*(sqrt(b)*x - sqrt(b*x^2 + a))^5*B*a*b + 6*(sqrt(b)*x - sqrt(b*x^
2 + a))^4*C*a^2*sqrt(b) + 12*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*a*b^(3/2) -
 12*(sqrt(b)*x - sqrt(b*x^2 + a))^2*C*a^3*sqrt(b) - 12*(sqrt(b)*x - sqrt(b
*x^2 + a))^2*A*a^2*b^(3/2) - 3*(sqrt(b)*x - sqrt(b*x^2 + a))*B*a^3*b + 6*C
*a^4*sqrt(b) + 8*A*a^3*b^(3/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^3
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{x^4}\,dx = \int \frac{(b\,x^2+a)^{3/2}\,(A+B\,x+C\,x^2+x^3\,D)}{x^4}\,dx$$

input
```
int(((a + b*x^2)^(3/2)*(A + B*x + C*x^2 + x^3*D))/x^4,x)
```

output
```
int(((a + b*x^2)^(3/2)*(A + B*x + C*x^2 + x^3*D))/x^4, x)
```

## Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.53

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{x^4}\,dx = \frac{-4\sqrt{b\,x^2+a}\,a^2-16\sqrt{b\,x^2+a}\,ab\,x^2-6\sqrt{b\,x^2+a}\,abx-12\sqrt{}}{}$$

input
```
int((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/x^4,x)
```

output
```
( - 4*sqrt(a + b*x**2)*a**2 - 16*sqrt(a + b*x**2)*a*b*x**2 - 6*sqrt(a + b*
x**2)*a*b*x - 12*sqrt(a + b*x**2)*a*c*x**2 + 16*sqrt(a + b*x**2)*a*d*x**3
+ 12*sqrt(a + b*x**2)*b**2*x**3 + 6*sqrt(a + b*x**2)*b*c*x**4 + 4*sqrt(a +
 b*x**2)*b*d*x**5 + 12*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x
)/sqrt(a))*a*d*x**3 + 18*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)
*x)/sqrt(a))*b**2*x**3 - 12*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt
(b)*x)/sqrt(a))*a*d*x**3 - 18*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sq
rt(b)*x)/sqrt(a))*b**2*x**3 + 12*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x
)/sqrt(a))*a*b*x**3 + 18*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a
))*a*c*x**3 + 5*sqrt(b)*a*c*x**3)/(12*x**3)
```

# 3.82 $\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{x^5} dx$



## Optimal result

Integrand size = 30, antiderivative size = 204

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{x^5} dx = \frac{b(3(Ab+4aC)+4(2bB+3aD)x)\sqrt{a+bx^2}}{8a} - \frac{(3(Ab+4aC)+8(2bB+3aD)x)(a+bx^2)^{3/2}}{24ax^2} - \frac{A(a+bx^2)^{5/2}}{4ax^4} - \frac{B(a+bx^2)^{5/2}}{3ax^3} + \frac{1}{2}\sqrt{b}(2bB+3aD)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right) - \frac{3b(Ab+4aC)\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8\sqrt{a}}$$

output
```
1/8*b*(3*A*b+12*C*a+4*(2*B*b+3*D*a)*x)*(b*x^2+a)^(1/2)/a-1/24*(3*A*b+12*C*
a+8*(2*B*b+3*D*a)*x)*(b*x^2+a)^(3/2)/a/x^2-1/4*A*(b*x^2+a)^(5/2)/a/x^4-1/3
*B*(b*x^2+a)^(5/2)/a/x^3+1/2*b^(1/2)*(2*B*b+3*D*a)*arctanh(b^(1/2)*x/(b*x^
2+a)^(1/2))-3/8*b*(A*b+4*C*a)*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(1/2)
```

## Mathematica [A] (verified)

Time = 1.17 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.74

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{x^5}\,dx = \frac{1}{24}\left(-\frac{\sqrt{a+bx^2}(6aA+4ax(2B+3x(C+2Dx))+bx^2(15A}{x^4}\right.$$

$$\left.+\frac{18b(Ab+4aC)\text{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}-12\sqrt{b}(2bB+3aD)\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)\right)$$

input
```
Integrate[((a + b*x^2)^(3/2)*(A + B*x + C*x^2 + D*x^3))/x^5,x]
```

output
```
(-((Sqrt[a + b*x^2]*(6*a*A + 4*a*x*(2*B + 3*x*(C + 2*D*x)) + b*x^2*(15*A +
 4*x*(8*B - 3*x*(2*C + D*x)))))/x^4) + (18*b*(A*b + 4*a*C)*ArcTanh[(Sqrt[b
]*x - Sqrt[a + b*x^2])/Sqrt[a]])/Sqrt[a] - 12*Sqrt[b]*(2*b*B + 3*a*D)*Log[
-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/24
```

## Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.02, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2338, 25, 2338, 25, 27, 537, 25, 535, 27, 538, 224, 219, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{x^5}\,dx$$

$$\Big\downarrow \text{2338}$$

$$-\frac{\int -\frac{(bx^2+a)^{3/2}(4aDx^2+(Ab+4aC)x+4aB)}{x^4}dx}{4a}-\frac{A(a+bx^2)^{5/2}}{4ax^4}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{(bx^2+a)^{3/2}(4aDx^2+(Ab+4aC)x+4aB)}{x^4}dx}{4a}-\frac{A(a+bx^2)^{5/2}}{4ax^4}$$

$$\Big\downarrow \ 2338$$

$$\frac{-\frac{\int -\frac{a(3(Ab+4aC)+4(2bB+3aD)x)\left(bx^2+a\right)^{3/2}}{x^3}dx}{3a}-\frac{4B\left(a+bx^2\right)^{5/2}}{3x^3}}{4a}-\frac{A\left(a+bx^2\right)^{5/2}}{4ax^4}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{\int \frac{a(3(Ab+4aC)+4(2bB+3aD)x)\left(bx^2+a\right)^{3/2}}{x^3}dx}{3a}-\frac{4B\left(a+bx^2\right)^{5/2}}{3x^3}}{4a}-\frac{A\left(a+bx^2\right)^{5/2}}{4ax^4}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{1}{3}\int \frac{(3(Ab+4aC)+4(2bB+3aD)x)\left(bx^2+a\right)^{3/2}}{x^3}dx-\frac{4B\left(a+bx^2\right)^{5/2}}{3x^3}}{4a}-\frac{A\left(a+bx^2\right)^{5/2}}{4ax^4}$$

$$\Big\downarrow \ 537$$

$$\frac{\frac{1}{3}\left(-\frac{3}{2}b\int -\frac{(3(Ab+4aC)+8(2bB+3aD)x)\sqrt{bx^2+a}}{x}dx-\frac{\left(a+bx^2\right)^{3/2}(3(4aC+Ab)+8x(3aD+2bB))}{2x^2}\right)-\frac{4B\left(a+bx^2\right)^{5/2}}{3x^3}}{4a}-\frac{A\left(a+bx^2\right)^{5/2}}{4ax^4}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{1}{3}\left(\frac{3}{2}b\int \frac{(3(Ab+4aC)+8(2bB+3aD)x)\sqrt{bx^2+a}}{x}dx-\frac{\left(a+bx^2\right)^{3/2}(3(4aC+Ab)+8x(3aD+2bB))}{2x^2}\right)-\frac{4B\left(a+bx^2\right)^{5/2}}{3x^3}}{4a}-\frac{A\left(a+bx^2\right)^{5/2}}{4ax^4}$$

$$\Big\downarrow \ 535$$

$$\frac{\frac{1}{3}\left(\frac{3}{2}b\left(\frac{1}{2}a\int \frac{2(3(Ab+4aC)+4(2bB+3aD)x)}{x\sqrt{bx^2+a}}dx+\sqrt{a+bx^2}(3(4aC+Ab)+4x(3aD+2bB))\right)-\frac{\left(a+bx^2\right)^{3/2}(3(4aC+Ab)+8a}{2x^2}\right.}{4a}-\frac{A\left(a+bx^2\right)^{5/2}}{4ax^4}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{1}{3}\left(\frac{3}{2}b\left(a\int\frac{3(Ab+4aC)+4(2bB+3aD)x}{x\sqrt{bx^2+a}}dx+\sqrt{a+bx^2}(3(4aC+Ab)+4x(3aD+2bB))\right)-\frac{\left(a+bx^2\right)^{3/2}(3(4aC+Ab)+8x(3aI}{2x^2}}{4a}$$

$$\frac{A\left(a+bx^2\right)^{5/2}}{4ax^4}$$

$\downarrow$ 538

$$\frac{\frac{1}{3}\left(\frac{3}{2}b\left(a\left(3(4aC+Ab)\int\frac{1}{x\sqrt{bx^2+a}}dx+4(3aD+2bB)\int\frac{1}{\sqrt{bx^2+a}}dx\right)+\sqrt{a+bx^2}(3(4aC+Ab)+4x(3aD+2bB)\right)}{4a}$$

$$\frac{A\left(a+bx^2\right)^{5/2}}{4ax^4}$$

$\downarrow$ 224

$$\frac{\frac{1}{3}\left(\frac{3}{2}b\left(a\left(3(4aC+Ab)\int\frac{1}{x\sqrt{bx^2+a}}dx+4(3aD+2bB)\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}\right)+\sqrt{a+bx^2}(3(4aC+Ab)+4x(3aD\right.}{4a}$$

$$\frac{A\left(a+bx^2\right)^{5/2}}{4ax^4}$$

$\downarrow$ 219

$$\frac{\frac{1}{3}\left(\frac{3}{2}b\left(a\left(3(4aC+Ab)\int\frac{1}{x\sqrt{bx^2+a}}dx+\frac{4\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(3aD+2bB)}{\sqrt{b}}\right)+\sqrt{a+bx^2}(3(4aC+Ab)+4x(3aD+2b\right.}{4a}$$

$$\frac{A\left(a+bx^2\right)^{5/2}}{4ax^4}$$

$\downarrow$ 243

$$\frac{\frac{1}{3}\left(\frac{3}{2}b\left(a\left(\frac{3}{2}(4aC+Ab)\int\frac{1}{x^2\sqrt{bx^2+a}}dx^2+\frac{4\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(3aD+2bB)}{\sqrt{b}}\right)+\sqrt{a+bx^2}(3(4aC+Ab)+4x(3aD+\right.}{4a}$$

$$\frac{A\left(a+bx^2\right)^{5/2}}{4ax^4}$$

$\downarrow$ 73

$$\frac{\frac{1}{3}\left(\frac{3}{2}b\left(a\left(\frac{3(4aC+Ab)\int\frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{b}+\frac{4\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(3aD+2bB)}{\sqrt{b}}\right)+\sqrt{a+bx^2}(3(4aC+Ab)+4x(3aD+2bB\right.\right.}{4a}$$

$$\frac{A(a+bx^2)^{5/2}}{4ax^4}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{1}{3}\left(\frac{3}{2}b\left(a\left(\frac{4\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(3aD+2bB)}{\sqrt{b}}-\frac{3(4aC+Ab)\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}\right)+\sqrt{a+bx^2}(3(4aC+Ab)+4x(3aD+2\right.\right.}{4a}$$

$$\frac{A(a+bx^2)^{5/2}}{4ax^4}$$

input
```
Int[((a + b*x^2)^(3/2)*(A + B*x + C*x^2 + D*x^3))/x^5,x]
```

output
```
-1/4*(A*(a + b*x^2)^(5/2))/(a*x^4) + ((-4*B*(a + b*x^2)^(5/2))/(3*x^3) + (
-1/2*((3*(A*b + 4*a*C) + 8*(2*b*B + 3*a*D)*x)*(a + b*x^2)^(3/2))/x^2 + (3*
b*((3*(A*b + 4*a*C) + 4*(2*b*B + 3*a*D)*x)*Sqrt[a + b*x^2] + a*((4*(2*b*B
+ 3*a*D)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/Sqrt[b] - (3*(A*b + 4*a*C)*
ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a])))/2)/3)/(4*a)
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 535
```
Int[(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_))/(x_), x_Symbol] :> Sim
p[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^p/(2*p*(2*p + 1))), x] + Simp[a/(2*p
 + 1)   Int[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^(p - 1)/x), x], x] /; Free
Q[{a, b, c, d}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 537
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[x^(m + 1)*(c*(m + 2) + d*(m + 1)*x)*((a + b*x^2)^p/((m + 1)*(m + 2))),
 x] - Simp[2*b*(p/((m + 1)*(m + 2)))   Int[x^(m + 2)*(c*(m + 2) + d*(m + 1)
*x)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[m, -2] &&
 GtQ[p, 0] &&  !ILtQ[m + 2*p + 3, 0] && IntegerQ[2*p]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 359 vs. $2(168) = 336$.

Time = 0.60 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.76

| method | result |
|---|---|
| default | $A\left(-\frac{(b\,x^{2}+a)^{\frac{5}{2}}}{4a\,x^{4}}+\frac{b\left(-\frac{(b\,x^{2}+a)^{\frac{5}{2}}}{2a\,x^{2}}+\frac{3b\left(\frac{(b\,x^{2}+a)^{\frac{3}{2}}}{3}+a\left(\sqrt{b\,x^{2}+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)\right)\right)}{2a}\right)}{4a}\right)+B\left(-\frac{(b\,x^{2}+a)^{\frac{5}{2}}}{3a\,x^{3}}+\right.$ |

input
```
int((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/x^5,x,method=_RETURNVERBOSE)
```

output
```
A*(-1/4/a/x^4*(b*x^2+a)^(5/2)+1/4*b/a*(-1/2/a/x^2*(b*x^2+a)^(5/2)+3/2*b/a*
(1/3*(b*x^2+a)^(3/2)+a*((b*x^2+a)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a
)^(1/2))/x)))))+B*(-1/3/a/x^3*(b*x^2+a)^(5/2)+2/3*b/a*(-1/a/x*(b*x^2+a)^(5
/2)+4*b/a*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2
)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))))))+C*(-1/2/a/x^2*(b*x^2+a)^(5/2)+3/2*b/a*
(1/3*(b*x^2+a)^(3/2)+a*((b*x^2+a)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a
)^(1/2))/x))))+D*(-1/a/x*(b*x^2+a)^(5/2)+4*b/a*(1/4*x*(b*x^2+a)^(3/2)+3/4*
a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))))
```

## Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 682, normalized size of antiderivative = 3.34

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{x^5}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/x^5,x, algorithm="fricas")
```

output
```
[1/48*(12*(3*D*a^2 + 2*B*a*b)*sqrt(b)*x^4*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)
*sqrt(b)*x - a) + 9*(4*C*a*b + A*b^2)*sqrt(a)*x^4*log(-(b*x^2 - 2*sqrt(b*x
^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(12*D*a*b*x^5 + 24*C*a*b*x^4 - 8*B*a^2*x -
 8*(3*D*a^2 + 4*B*a*b)*x^3 - 6*A*a^2 - 3*(4*C*a^2 + 5*A*a*b)*x^2)*sqrt(b*x
^2 + a))/(a*x^4), -1/48*(24*(3*D*a^2 + 2*B*a*b)*sqrt(-b)*x^4*arctan(sqrt(-
b)*x/sqrt(b*x^2 + a)) - 9*(4*C*a*b + A*b^2)*sqrt(a)*x^4*log(-(b*x^2 - 2*sq
rt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(12*D*a*b*x^5 + 24*C*a*b*x^4 - 8*B*a
^2*x - 8*(3*D*a^2 + 4*B*a*b)*x^3 - 6*A*a^2 - 3*(4*C*a^2 + 5*A*a*b)*x^2)*sq
rt(b*x^2 + a))/(a*x^4), 1/24*(9*(4*C*a*b + A*b^2)*sqrt(-a)*x^4*arctan(sqrt
(b*x^2 + a)*sqrt(-a)/a) + 6*(3*D*a^2 + 2*B*a*b)*sqrt(b)*x^4*log(-2*b*x^2 -
 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + (12*D*a*b*x^5 + 24*C*a*b*x^4 - 8*B*a^2
*x - 8*(3*D*a^2 + 4*B*a*b)*x^3 - 6*A*a^2 - 3*(4*C*a^2 + 5*A*a*b)*x^2)*sqrt
(b*x^2 + a))/(a*x^4), -1/24*(12*(3*D*a^2 + 2*B*a*b)*sqrt(-b)*x^4*arctan(sq
rt(-b)*x/sqrt(b*x^2 + a)) - 9*(4*C*a*b + A*b^2)*sqrt(-a)*x^4*arctan(sqrt(b
*x^2 + a)*sqrt(-a)/a) - (12*D*a*b*x^5 + 24*C*a*b*x^4 - 8*B*a^2*x - 8*(3*D*
a^2 + 4*B*a*b)*x^3 - 6*A*a^2 - 3*(4*C*a^2 + 5*A*a*b)*x^2)*sqrt(b*x^2 + a))
/(a*x^4)]
```

## Sympy [A] (verification not implemented)

Time = 8.36 (sec) , antiderivative size = 462, normalized size of antiderivative = 2.26

$$
\begin{aligned}
\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{x^5}\,dx &= -\frac{Aa^2}{4\sqrt{b}x^5\sqrt{\frac{a}{bx^2}+1}} \\
&- \frac{3Aa\sqrt{b}}{8x^3\sqrt{\frac{a}{bx^2}+1}} - \frac{Ab^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{2x} - \frac{Ab^{\frac{3}{2}}}{8x\sqrt{\frac{a}{bx^2}+1}} - \frac{3Ab^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{8\sqrt{a}} \\
&- \frac{B\sqrt{a}b}{x\sqrt{1+\frac{bx^2}{a}}} - \frac{Ba\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{3x^2} - \frac{Bb^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{3} + Bb^{\frac{3}{2}}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right) \\
&- \frac{Bb^2x}{\sqrt{a}\sqrt{1+\frac{bx^2}{a}}} - \frac{3C\sqrt{a}b\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2} - \frac{Ca\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{2x} + \frac{Ca\sqrt{b}}{x\sqrt{\frac{a}{bx^2}+1}} \\
&+ \frac{Cb^{\frac{3}{2}}x}{\sqrt{\frac{a}{bx^2}+1}} - \frac{Da^{\frac{3}{2}}}{x\sqrt{1+\frac{bx^2}{a}}} - \frac{D\sqrt{a}bx}{\sqrt{1+\frac{bx^2}{a}}} + Da\sqrt{b}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right) \\
&+ Db\left(\begin{cases} \frac{a\left(\begin{cases} \frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0 \\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise} \end{cases}\right)}{2} + \frac{x\sqrt{a+bx^2}}{2} & \text{for } b\neq 0 \\ \sqrt{a}x & \text{otherwise} \end{cases}\right)
\end{aligned}
$$

input
```
integrate((b*x**2+a)**(3/2)*(D*x**3+C*x**2+B*x+A)/x**5,x)
```

output
```
-A*a**2/(4*sqrt(b)*x**5*sqrt(a/(b*x**2) + 1)) - 3*A*a*sqrt(b)/(8*x**3*sqrt
(a/(b*x**2) + 1)) - A*b**(3/2)*sqrt(a/(b*x**2) + 1)/(2*x) - A*b**(3/2)/(8*
x*sqrt(a/(b*x**2) + 1)) - 3*A*b**2*asinh(sqrt(a)/(sqrt(b)*x))/(8*sqrt(a))
- B*sqrt(a)*b/(x*sqrt(1 + b*x**2/a)) - B*a*sqrt(b)*sqrt(a/(b*x**2) + 1)/(3
*x**2) - B*b**(3/2)*sqrt(a/(b*x**2) + 1)/3 + B*b**(3/2)*asinh(sqrt(b)*x/sq
rt(a)) - B*b**2*x/(sqrt(a)*sqrt(1 + b*x**2/a)) - 3*C*sqrt(a)*b*asinh(sqrt(
a)/(sqrt(b)*x))/2 - C*a*sqrt(b)*sqrt(a/(b*x**2) + 1)/(2*x) + C*a*sqrt(b)/(
x*sqrt(a/(b*x**2) + 1)) + C*b**(3/2)*x/sqrt(a/(b*x**2) + 1) - D*a**(3/2)/(
x*sqrt(1 + b*x**2/a)) - D*sqrt(a)*b*x/sqrt(1 + b*x**2/a) + D*a*sqrt(b)*asi
nh(sqrt(b)*x/sqrt(a)) + D*b*Piecewise((a*Piecewise((log(2*sqrt(b)*sqrt(a +
 b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True))/2 + x
*sqrt(a + b*x**2)/2, Ne(b, 0)), (sqrt(a)*x, True))
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.35

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{x^5}\,dx = \frac{3}{2}\sqrt{bx^2+a}Dbx + \frac{\sqrt{bx^2+a}Bb^2x}{a} + \frac{3}{2}Da\sqrt{b}\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right) + Bb^{\frac{3}{2}}\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right) - \frac{3}{2}C\sqrt{a}b\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right) - \frac{3Ab^2\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{8\sqrt{a}} + \frac{3}{2}\sqrt{bx^2+a}Cb + \frac{(bx^2+a)^{\frac{3}{2}}Cb}{2a} + \frac{(bx^2+a)^{\frac{3}{2}}Ab^2}{8a^2} + \frac{3\sqrt{bx^2+a}Ab^2}{8a} - \frac{(bx^2+a)^{\frac{3}{2}}D}{x} - \frac{2(bx^2+a)^{\frac{3}{2}}Bb}{3ax} - \frac{(bx^2+a)^{\frac{5}{2}}C}{2ax^2} - \frac{(bx^2+a)^{\frac{5}{2}}Ab}{8a^2x^2} - \frac{(bx^2+a)^{\frac{5}{2}}B}{3ax^3} - \frac{(bx^2+a)^{\frac{5}{2}}A}{4ax^4}$$

input
```
integrate((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/x^5,x, algorithm="maxima")
```

output
```
3/2*sqrt(b*x^2 + a)*D*b*x + sqrt(b*x^2 + a)*B*b^2*x/a + 3/2*D*a*sqrt(b)*ar
csinh(b*x/sqrt(a*b)) + B*b^(3/2)*arcsinh(b*x/sqrt(a*b)) - 3/2*C*sqrt(a)*b*
arcsinh(a/(sqrt(a*b)*abs(x))) - 3/8*A*b^2*arcsinh(a/(sqrt(a*b)*abs(x)))/sq
rt(a) + 3/2*sqrt(b*x^2 + a)*C*b + 1/2*(b*x^2 + a)^(3/2)*C*b/a + 1/8*(b*x^2
 + a)^(3/2)*A*b^2/a^2 + 3/8*sqrt(b*x^2 + a)*A*b^2/a - (b*x^2 + a)^(3/2)*D/
x - 2/3*(b*x^2 + a)^(3/2)*B*b/(a*x) - 1/2*(b*x^2 + a)^(5/2)*C/(a*x^2) - 1/
8*(b*x^2 + a)^(5/2)*A*b/(a^2*x^2) - 1/3*(b*x^2 + a)^(5/2)*B/(a*x^3) - 1/4*
(b*x^2 + a)^(5/2)*A/(a*x^4)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 515 vs. $2(168) = 336$.

Time = 0.15 (sec) , antiderivative size = 515, normalized size of antiderivative = 2.52

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{x^5}\,dx =$$
$$-\frac{1}{2}\left(3\,Da\sqrt{b}+2\,Bb^{\frac{3}{2}}\right)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)$$
$$+\frac{1}{2}(Dbx+2\,Cb)\sqrt{bx^2+a}+\frac{3\,(4\,Cab+Ab^2)\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{4\sqrt{-a}}$$
$$+\frac{12\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^7Cab+15\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^7Ab^2+24\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6Da^2\sqrt{b}+48\left(\sqrt{b}x-\right.}{}$$

input
```
integrate((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/x^5,x, algorithm="giac")
```

output
```
-1/2*(3*D*a*sqrt(b) + 2*B*b^(3/2))*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))
+ 1/2*(D*b*x + 2*C*b)*sqrt(b*x^2 + a) + 3/4*(4*C*a*b + A*b^2)*arctan(-(sqr
t(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/sqrt(-a) + 1/12*(12*(sqrt(b)*x - sqrt(
b*x^2 + a))^7*C*a*b + 15*(sqrt(b)*x - sqrt(b*x^2 + a))^7*A*b^2 + 24*(sqrt(
b)*x - sqrt(b*x^2 + a))^6*D*a^2*sqrt(b) + 48*(sqrt(b)*x - sqrt(b*x^2 + a))
^6*B*a*b^(3/2) - 12*(sqrt(b)*x - sqrt(b*x^2 + a))^5*C*a^2*b + 9*(sqrt(b)*x
 - sqrt(b*x^2 + a))^5*A*a*b^2 - 72*(sqrt(b)*x - sqrt(b*x^2 + a))^4*D*a^3*s
qrt(b) - 96*(sqrt(b)*x - sqrt(b*x^2 + a))^4*B*a^2*b^(3/2) - 12*(sqrt(b)*x
- sqrt(b*x^2 + a))^3*C*a^3*b + 9*(sqrt(b)*x - sqrt(b*x^2 + a))^3*A*a^2*b^2
 + 72*(sqrt(b)*x - sqrt(b*x^2 + a))^2*D*a^4*sqrt(b) + 80*(sqrt(b)*x - sqrt
(b*x^2 + a))^2*B*a^3*b^(3/2) + 12*(sqrt(b)*x - sqrt(b*x^2 + a))*C*a^4*b +
15*(sqrt(b)*x - sqrt(b*x^2 + a))*A*a^3*b^2 - 24*D*a^5*sqrt(b) - 32*B*a^4*b
^(3/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^4
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{x^5}\,dx=\int \frac{(b\,x^2+a)^{3/2}\,(A+B\,x+C\,x^2+x^3\,D)}{x^5}\,dx$$

input
```
int(((a + b*x^2)^(3/2)*(A + B*x + C*x^2 + x^3*D))/x^5,x)
```

output
```
int(((a + b*x^2)^(3/2)*(A + B*x + C*x^2 + x^3*D))/x^5, x)
```

## Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.59

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{x^5}\,dx=\frac{-24\sqrt{b\,x^2+a}\,a^2-60\sqrt{b\,x^2+a}\,ab\,x^2-32\sqrt{b\,x^2+a}\,abx-4$$

input
```
int((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/x^5,x)
```

output
```
( - 24*sqrt(a + b*x**2)*a**2 - 60*sqrt(a + b*x**2)*a*b*x**2 - 32*sqrt(a +
b*x**2)*a*b*x - 48*sqrt(a + b*x**2)*a*c*x**2 - 96*sqrt(a + b*x**2)*a*d*x**
3 - 128*sqrt(a + b*x**2)*b**2*x**3 + 96*sqrt(a + b*x**2)*b*c*x**4 + 48*sqr
t(a + b*x**2)*b*d*x**5 + 36*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt
(b)*x)/sqrt(a))*b**2*x**4 + 144*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) +
sqrt(b)*x)/sqrt(a))*b*c*x**4 - 36*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a)
+ sqrt(b)*x)/sqrt(a))*b**2*x**4 - 144*sqrt(a)*log((sqrt(a + b*x**2) + sqrt
(a) + sqrt(b)*x)/sqrt(a))*b*c*x**4 + 144*sqrt(b)*log((sqrt(a + b*x**2) + s
qrt(b)*x)/sqrt(a))*a*d*x**4 + 96*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x
)/sqrt(a))*b**2*x**4 + 63*sqrt(b)*a*d*x**4 + 32*sqrt(b)*b**2*x**4)/(96*x**
4)
```

# 3.83 $\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{x^6}\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 182

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{x^6}\,dx = -\frac{b(8aC-3(bB+4aD)x)\sqrt{a+bx^2}}{8ax} - \frac{(8aC+3(bB+4aD)x)(a+bx^2)^{3/2}}{24ax^3} - \frac{A(a+bx^2)^{5/2}}{5ax^5} - \frac{B(a+bx^2)^{5/2}}{4ax^4} + b^{3/2}C\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right) - \frac{3b(bB+4aD)\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8\sqrt{a}}$$

output

```
-1/8*b*(8*C*a-3*(B*b+4*D*a)*x)*(b*x^2+a)^(1/2)/a/x-1/24*(8*C*a+3*(B*b+4*D*
a)*x)*(b*x^2+a)^(3/2)/a/x^3-1/5*A*(b*x^2+a)^(5/2)/a/x^5-1/4*B*(b*x^2+a)^(5
/2)/a/x^4+b^(3/2)*C*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))-3/8*b*(B*b+4*D*a)*a
rctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(1/2)
```

## Mathematica [A] (verified)

Time = 1.24 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{x^6}\,dx = -\frac{\sqrt{a+bx^2}(24Ab^2x^4+a^2(24A+10x(3B+4Cx+6Dx^2))+abx^2(48A+5x(15B+8x(4C-3Dx))))}{120ax^5} + \frac{3b(bB+4aD)\operatorname{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right)}{4\sqrt{a}} - b^{3/2}C\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)$$

input
```
Integrate[((a + b*x^2)^(3/2)*(A + B*x + C*x^2 + D*x^3))/x^6,x]
```

output
```
-1/120*(Sqrt[a + b*x^2]*(24*A*b^2*x^4 + a^2*(24*A + 10*x*(3*B + 4*C*x + 6*
D*x^2)) + a*b*x^2*(48*A + 5*x*(15*B + 8*x*(4*C - 3*D*x)))))/(a*x^5) + (3*b
*(b*B + 4*a*D)*ArcTanh[(Sqrt[b]*x - Sqrt[a + b*x^2])/Sqrt[a]])/(4*Sqrt[a])
 - b^(3/2)*C*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]]
```

## Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.02, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {2338, 27, 2338, 25, 27, 537, 25, 536, 538, 224, 219, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{x^6}\,dx$$

$$\Big\downarrow \text{2338}$$

$$-\frac{\int -\frac{5(bx^2+a)^{3/2}(aDx^2+aCx+aB)}{x^5}dx}{5a} - \frac{A(a+bx^2)^{5/2}}{5ax^5}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(bx^2+a)^{3/2}(aDx^2+aCx+aB)}{x^5}dx}{a}-\frac{A(a+bx^2)^{5/2}}{5ax^5}$$

$\Big\downarrow$ 2338

$$\frac{-\frac{\int -\frac{a(4aC+(bB+4aD)x)\left(bx^2+a\right)^{3/2}}{x^4}dx}{4a}-\frac{B(a+bx^2)^{5/2}}{4x^4}}{a}-\frac{A(a+bx^2)^{5/2}}{5ax^5}$$

$\Big\downarrow$ 25

$$\frac{\frac{\int \frac{a(4aC+(bB+4aD)x)\left(bx^2+a\right)^{3/2}}{x^4}dx}{4a}-\frac{B(a+bx^2)^{5/2}}{4x^4}}{a}-\frac{A(a+bx^2)^{5/2}}{5ax^5}$$

$\Big\downarrow$ 27

$$\frac{\frac{1}{4}\int \frac{(4aC+(bB+4aD)x)(bx^2+a)^{3/2}}{x^4}dx-\frac{B(a+bx^2)^{5/2}}{4x^4}}{a}-\frac{A(a+bx^2)^{5/2}}{5ax^5}$$

$\Big\downarrow$ 537

$$\frac{\frac{1}{4}\left(-\frac{1}{2}b\int -\frac{(8aC+3(bB+4aD)x)\sqrt{bx^2+a}}{x^2}dx-\frac{(a+bx^2)^{3/2}(3x(4aD+bB)+8aC)}{6x^3}\right)-\frac{B(a+bx^2)^{5/2}}{4x^4}}{a}-\frac{A(a+bx^2)^{5/2}}{5ax^5}$$

$\Big\downarrow$ 25

$$\frac{\frac{1}{4}\left(\frac{1}{2}b\int \frac{(8aC+3(bB+4aD)x)\sqrt{bx^2+a}}{x^2}dx-\frac{(a+bx^2)^{3/2}(3x(4aD+bB)+8aC)}{6x^3}\right)-\frac{B(a+bx^2)^{5/2}}{4x^4}}{a}-\frac{A(a+bx^2)^{5/2}}{5ax^5}$$

$\Big\downarrow$ 536

$$\frac{\frac{1}{4}\left(\frac{1}{2}b\left(\int \frac{3a(bB+4aD)+8abCx}{x\sqrt{bx^2+a}}dx-\frac{\sqrt{a+bx^2}(8aC-3x(4aD+bB))}{x}\right)-\frac{(a+bx^2)^{3/2}(3x(4aD+bB)+8aC)}{6x^3}\right)-\frac{B(a+bx^2)^{5/2}}{4x^4}}{a}-\frac{A(a+bx^2)^{5/2}}{5ax^5}$$

$\Big\downarrow$ 538

$$\frac{\frac{1}{4}\left(\frac{1}{2}b\left(3a(4aD+bB)\int\frac{1}{x\sqrt{bx^2+a}}dx+8abC\int\frac{1}{\sqrt{bx^2+a}}dx-\frac{\sqrt{a+bx^2}(8aC-3x(4aD+bB))}{x}\right)-\frac{\left(a+bx^2\right)^{3/2}(3x(4aD+bB)+8aC)}{6x^3}\right.}{a}$$

$$\frac{A\left(a+bx^2\right)^{5/2}}{5ax^5}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}b\left(3a(4aD+bB)\int\frac{1}{x\sqrt{bx^2+a}}dx+8abC\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}-\frac{\sqrt{a+bx^2}(8aC-3x(4aD+bB))}{x}\right)-\frac{\left(a+bx^2\right)^{3/2}(3x(4aD+bB}{6x^3}\right.}{a}$$

$$\frac{A\left(a+bx^2\right)^{5/2}}{5ax^5}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}b\left(3a(4aD+bB)\int\frac{1}{x\sqrt{bx^2+a}}dx+8a\sqrt{b}C\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)-\frac{\sqrt{a+bx^2}(8aC-3x(4aD+bB))}{x}\right)-\frac{\left(a+bx^2\right)^{3/2}(3x(4aD+b}{6x^3}\right.}{a}$$

$$\frac{A\left(a+bx^2\right)^{5/2}}{5ax^5}$$

$$\Big\downarrow \text{ 243}$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}b\left(\frac{3}{2}a(4aD+bB)\int\frac{1}{x^2\sqrt{bx^2+a}}dx^2+8a\sqrt{b}C\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)-\frac{\sqrt{a+bx^2}(8aC-3x(4aD+bB))}{x}\right)-\frac{\left(a+bx^2\right)^{3/2}(3x(4aD}{6x^3}\right.}{a}$$

$$\frac{A\left(a+bx^2\right)^{5/2}}{5ax^5}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}b\left(\frac{3a(4aD+bB)\int\frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{b}+8a\sqrt{b}C\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)-\frac{\sqrt{a+bx^2}(8aC-3x(4aD+bB))}{x}\right)-\frac{\left(a+bx^2\right)^{3/2}(3x(4aD+bB}{6x^3}\right.}{a}$$

$$\frac{A\left(a+bx^2\right)^{5/2}}{5ax^5}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}b\left(-3\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)(4aD+bB)+8a\sqrt{b}C\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)-\frac{\sqrt{a+bx^2}(8aC-3x(4aD+bB))}{x}\right)-\frac{(a+bx^2)^{3/2}}{}\right.}{a}$$

$$\frac{A(a+bx^2)^{5/2}}{5ax^5}$$

input
```
Int[((a + b*x^2)^(3/2)*(A + B*x + C*x^2 + D*x^3))/x^6,x]
```

output
```
-1/5*(A*(a + b*x^2)^(5/2))/(a*x^5) + (-1/4*(B*(a + b*x^2)^(5/2))/x^4 + (-1
/6*((8*a*C + 3*(b*B + 4*a*D)*x)*(a + b*x^2)^(3/2))/x^3 + (b*(-(((8*a*C - 3
*(b*B + 4*a*D)*x)*Sqrt[a + b*x^2])/x) + 8*a*Sqrt[b]*C*ArcTanh[(Sqrt[b]*x)/
Sqrt[a + b*x^2]] - 3*Sqrt[a]*(b*B + 4*a*D)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]
]))/2)/4)/a
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 536
```
Int[(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_))/(x_)^2, x_Symbol] :> S
imp[(-(2*c*p - d*x))*((a + b*x^2)^p/(2*p*x)), x] + Int[(a*d + 2*b*c*p*x)*((
a + b*x^2)^(p - 1)/x), x] /; FreeQ[{a, b, c, d}, x] && GtQ[p, 0] && Integer
Q[2*p]
```

rule 537
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[x^(m + 1)*(c*(m + 2) + d*(m + 1)*x)*((a + b*x^2)^p/((m + 1)*(m + 2))),
 x] - Simp[2*b*(p/((m + 1)*(m + 2)))   Int[x^(m + 2)*(c*(m + 2) + d*(m + 1)
*x)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[m, -2] &&
 GtQ[p, 0] &&  !ILtQ[m + 2*p + 3, 0] && IntegerQ[2*p]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

## Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.65

| method | result |
|---|---|
| default | $-\frac{A(b\,x^2+a)^{\frac{5}{2}}}{5a\,x^5}+B\left(-\frac{(b\,x^2+a)^{\frac{5}{2}}}{4a\,x^4}+\frac{b\left(-\frac{(b\,x^2+a)^{\frac{5}{2}}}{2a\,x^2}+\frac{3b\left(\frac{(b\,x^2+a)^{\frac{3}{2}}}{3}+a\left(\sqrt{b\,x^2+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)\right)\right)}{2a}\right)}{4a}\right)+C$ |

input
```
int((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/5*A*(b*x^2+a)^(5/2)/a/x^5+B*(-1/4/a/x^4*(b*x^2+a)^(5/2)+1/4*b/a*(-1/2/a
/x^2*(b*x^2+a)^(5/2)+3/2*b/a*(1/3*(b*x^2+a)^(3/2)+a*((b*x^2+a)^(1/2)-a^(1/
2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)))))+C*(-1/3/a/x^3*(b*x^2+a)^(5/2)
+2/3*b/a*(-1/a/x*(b*x^2+a)^(5/2)+4*b/a*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x
*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))))))+D*(-1/2/a
/x^2*(b*x^2+a)^(5/2)+3/2*b/a*(1/3*(b*x^2+a)^(3/2)+a*((b*x^2+a)^(1/2)-a^(1/
2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x))))
```

## Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 680, normalized size of antiderivative = 3.74

$$\int \frac{(a+bx^2)^{3/2}\,(A+Bx+Cx^2+Dx^3)}{x^6}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/x^6,x, algorithm="fricas")
```

output
```
[1/240*(120*C*a*b^(3/2)*x^5*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a
) + 45*(4*D*a*b + B*b^2)*sqrt(a)*x^5*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(
a) + 2*a)/x^2) + 2*(120*D*a*b*x^5 - 8*(20*C*a*b + 3*A*b^2)*x^4 - 30*B*a^2*
x - 15*(4*D*a^2 + 5*B*a*b)*x^3 - 24*A*a^2 - 8*(5*C*a^2 + 6*A*a*b)*x^2)*sqr
t(b*x^2 + a))/(a*x^5), -1/240*(240*C*a*sqrt(-b)*b*x^5*arctan(sqrt(-b)*x/sq
rt(b*x^2 + a)) - 45*(4*D*a*b + B*b^2)*sqrt(a)*x^5*log(-(b*x^2 - 2*sqrt(b*x
^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(120*D*a*b*x^5 - 8*(20*C*a*b + 3*A*b^2)*x^
4 - 30*B*a^2*x - 15*(4*D*a^2 + 5*B*a*b)*x^3 - 24*A*a^2 - 8*(5*C*a^2 + 6*A*
a*b)*x^2)*sqrt(b*x^2 + a))/(a*x^5), 1/120*(60*C*a*b^(3/2)*x^5*log(-2*b*x^2
 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 45*(4*D*a*b + B*b^2)*sqrt(-a)*x^5*ar
ctan(sqrt(b*x^2 + a)*sqrt(-a)/a) + (120*D*a*b*x^5 - 8*(20*C*a*b + 3*A*b^2)
*x^4 - 30*B*a^2*x - 15*(4*D*a^2 + 5*B*a*b)*x^3 - 24*A*a^2 - 8*(5*C*a^2 + 6
*A*a*b)*x^2)*sqrt(b*x^2 + a))/(a*x^5), -1/120*(120*C*a*sqrt(-b)*b*x^5*arct
an(sqrt(-b)*x/sqrt(b*x^2 + a)) - 45*(4*D*a*b + B*b^2)*sqrt(-a)*x^5*arctan(
sqrt(b*x^2 + a)*sqrt(-a)/a) - (120*D*a*b*x^5 - 8*(20*C*a*b + 3*A*b^2)*x^4
- 30*B*a^2*x - 15*(4*D*a^2 + 5*B*a*b)*x^3 - 24*A*a^2 - 8*(5*C*a^2 + 6*A*a*
b)*x^2)*sqrt(b*x^2 + a))/(a*x^5)]
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 401 vs. $2(165) = 330$.

Time = 8.43 (sec) , antiderivative size = 401, normalized size of antiderivative = 2.20

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{x^6}\,dx = -\frac{Aa\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{5x^4} - \frac{2Ab^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{5x^2} - \frac{Ab^{\frac{5}{2}}\sqrt{\frac{a}{bx^2}+1}}{5a} - \frac{Ba^2}{4\sqrt{b}x^5\sqrt{\frac{a}{bx^2}+1}} - \frac{3Ba\sqrt{b}}{8x^3\sqrt{\frac{a}{bx^2}+1}} - \frac{Bb^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{2x} - \frac{Bb^{\frac{3}{2}}}{8x\sqrt{\frac{a}{bx^2}+1}} - \frac{3Bb^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{8\sqrt{a}} - \frac{C\sqrt{a}b}{x\sqrt{1+\frac{bx^2}{a}}} - \frac{Ca\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{3x^2} - \frac{Cb^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{3} + Cb^{\frac{3}{2}}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right) - \frac{Cb^2x}{\sqrt{a}\sqrt{1+\frac{bx^2}{a}}} - \frac{3D\sqrt{a}b\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2} - \frac{Da\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{2x} + \frac{Da\sqrt{b}}{x\sqrt{\frac{a}{bx^2}+1}} + \frac{Db^{\frac{3}{2}}x}{\sqrt{\frac{a}{bx^2}+1}}$$

input
```
integrate((b*x**2+a)**(3/2)*(D*x**3+C*x**2+B*x+A)/x**6,x)
```

output
```
-A*a*sqrt(b)*sqrt(a/(b*x**2) + 1)/(5*x**4) - 2*A*b**(3/2)*sqrt(a/(b*x**2)
+ 1)/(5*x**2) - A*b**(5/2)*sqrt(a/(b*x**2) + 1)/(5*a) - B*a**2/(4*sqrt(b)*
x**5*sqrt(a/(b*x**2) + 1)) - 3*B*a*sqrt(b)/(8*x**3*sqrt(a/(b*x**2) + 1)) -
 B*b**(3/2)*sqrt(a/(b*x**2) + 1)/(2*x) - B*b**(3/2)/(8*x*sqrt(a/(b*x**2) +
 1)) - 3*B*b**2*asinh(sqrt(a)/(sqrt(b)*x))/(8*sqrt(a)) - C*sqrt(a)*b/(x*sq
rt(1 + b*x**2/a)) - C*a*sqrt(b)*sqrt(a/(b*x**2) + 1)/(3*x**2) - C*b**(3/2)
*sqrt(a/(b*x**2) + 1)/3 + C*b**(3/2)*asinh(sqrt(b)*x/sqrt(a)) - C*b**2*x/(
sqrt(a)*sqrt(1 + b*x**2/a)) - 3*D*sqrt(a)*b*asinh(sqrt(a)/(sqrt(b)*x))/2 -
 D*a*sqrt(b)*sqrt(a/(b*x**2) + 1)/(2*x) + D*a*sqrt(b)/(x*sqrt(a/(b*x**2) +
 1)) + D*b**(3/2)*x/sqrt(a/(b*x**2) + 1)
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.36

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{x^6}\,dx = \frac{\sqrt{bx^2+a}Cb^2x}{a} + Cb^{\frac{3}{2}}\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right) - \frac{3}{2}D\sqrt{a}b\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right) - \frac{3\,Bb^2\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{8\sqrt{a}} + \frac{3}{2}\sqrt{bx^2+a}Db + \frac{(bx^2+a)^{\frac{3}{2}}Db}{2\,a} + \frac{(bx^2+a)^{\frac{3}{2}}Bb^2}{8\,a^2} + \frac{3\sqrt{bx^2+a}Bb^2}{8\,a} - \frac{2\,(bx^2+a)^{\frac{3}{2}}Cb}{3\,ax} - \frac{(bx^2+a)^{\frac{5}{2}}D}{2\,ax^2} - \frac{(bx^2+a)^{\frac{5}{2}}Bb}{8\,a^2x^2} - \frac{(bx^2+a)^{\frac{5}{2}}C}{3\,ax^3} - \frac{(bx^2+a)^{\frac{5}{2}}B}{4\,ax^4} - \frac{(bx^2+a)^{\frac{5}{2}}A}{5\,ax^5}$$

input
```
integrate((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/x^6,x, algorithm="maxima")
```

output
```
sqrt(b*x^2 + a)*C*b^2*x/a + C*b^(3/2)*arcsinh(b*x/sqrt(a*b)) - 3/2*D*sqrt(
a)*b*arcsinh(a/(sqrt(a*b)*abs(x))) - 3/8*B*b^2*arcsinh(a/(sqrt(a*b)*abs(x)
))/sqrt(a) + 3/2*sqrt(b*x^2 + a)*D*b + 1/2*(b*x^2 + a)^(3/2)*D*b/a + 1/8*(
b*x^2 + a)^(3/2)*B*b^2/a^2 + 3/8*sqrt(b*x^2 + a)*B*b^2/a - 2/3*(b*x^2 + a)
^(3/2)*C*b/(a*x) - 1/2*(b*x^2 + a)^(5/2)*D/(a*x^2) - 1/8*(b*x^2 + a)^(5/2)
*B*b/(a^2*x^2) - 1/3*(b*x^2 + a)^(5/2)*C/(a*x^3) - 1/4*(b*x^2 + a)^(5/2)*B
/(a*x^4) - 1/5*(b*x^2 + a)^(5/2)*A/(a*x^5)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 494 vs. $2(152) = 304$.

Time = 0.15 (sec) , antiderivative size = 494, normalized size of antiderivative = 2.71

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{x^6}\,dx = -Cb^{\frac{3}{2}}\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)$$
$$+\sqrt{bx^2+a}Db+\frac{3\,(4\,Dab+Bb^2)\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{4\sqrt{-a}}$$
$$+\frac{60\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^9Dab+75\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^9Bb^2+240\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^8Cab^{\frac{3}{2}}+120\left(\sqrt{b}x\right.}{}$$

input
```
integrate((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/x^6,x, algorithm="giac")
```

output
```
-C*b^(3/2)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a))) + sqrt(b*x^2 + a)*D*b +
3/4*(4*D*a*b + B*b^2)*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/sqrt
(-a) + 1/60*(60*(sqrt(b)*x - sqrt(b*x^2 + a))^9*D*a*b + 75*(sqrt(b)*x - sq
rt(b*x^2 + a))^9*B*b^2 + 240*(sqrt(b)*x - sqrt(b*x^2 + a))^8*C*a*b^(3/2) +
 120*(sqrt(b)*x - sqrt(b*x^2 + a))^8*A*b^(5/2) - 120*(sqrt(b)*x - sqrt(b*x
^2 + a))^7*D*a^2*b - 30*(sqrt(b)*x - sqrt(b*x^2 + a))^7*B*a*b^2 - 720*(sqr
t(b)*x - sqrt(b*x^2 + a))^6*C*a^2*b^(3/2) + 880*(sqrt(b)*x - sqrt(b*x^2 +
a))^4*C*a^3*b^(3/2) + 240*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*a^2*b^(5/2) +
120*(sqrt(b)*x - sqrt(b*x^2 + a))^3*D*a^4*b + 30*(sqrt(b)*x - sqrt(b*x^2 +
 a))^3*B*a^3*b^2 - 560*(sqrt(b)*x - sqrt(b*x^2 + a))^2*C*a^4*b^(3/2) - 60*
(sqrt(b)*x - sqrt(b*x^2 + a))*D*a^5*b - 75*(sqrt(b)*x - sqrt(b*x^2 + a))*B
*a^4*b^2 + 160*C*a^5*b^(3/2) + 24*A*a^4*b^(5/2))/((sqrt(b)*x - sqrt(b*x^2
+ a))^2 - a)^5
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{x^6}\,dx = \int \frac{(b\,x^2+a)^{3/2}\,(A+B\,x+C\,x^2+x^3\,D)}{x^6}\,dx$$

input
```
int(((a + b*x^2)^(3/2)*(A + B*x + C*x^2 + x^3*D))/x^6,x)
```

output
```
int(((a + b*x^2)^(3/2)*(A + B*x + C*x^2 + x^3*D))/x^6, x)
```

## Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.82

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{x^6}\,dx = \frac{-24\sqrt{b\,x^2+a}\,a^3-48\sqrt{b\,x^2+a}\,a^2b\,x^2-30\sqrt{b\,x^2+a}\,a^2bx-$$

input
```
int((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/x^6,x)
```

output
```
( - 24*sqrt(a + b*x**2)*a**3 - 48*sqrt(a + b*x**2)*a**2*b*x**2 - 30*sqrt(a
 + b*x**2)*a**2*b*x - 40*sqrt(a + b*x**2)*a**2*c*x**2 - 60*sqrt(a + b*x**2
)*a**2*d*x**3 - 24*sqrt(a + b*x**2)*a*b**2*x**4 - 75*sqrt(a + b*x**2)*a*b*
*2*x**3 - 160*sqrt(a + b*x**2)*a*b*c*x**4 + 120*sqrt(a + b*x**2)*a*b*d*x**
5 + 180*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b*
d*x**5 + 45*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*
b**3*x**5 - 180*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(
a))*a*b*d*x**5 - 45*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/s
qrt(a))*b**3*x**5 + 120*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a)
)*a*b*c*x**5 - 24*sqrt(b)*a*b**2*x**5 + 64*sqrt(b)*a*b*c*x**5)/(120*a*x**5
)
```

# 3.84 $\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{x^7}\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 178

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{x^7}\,dx = \frac{b(Ab-6aC-16aDx)\sqrt{a+bx^2}}{16ax^2} + \frac{(Ab-6aC-8aDx)(a+bx^2)^{3/2}}{24ax^4} - \frac{A(a+bx^2)^{5/2}}{6ax^6} - \frac{B(a+bx^2)^{5/2}}{5ax^5} + b^{3/2}D\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right) + \frac{b^2(Ab-6aC)\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{3/2}}$$

output
```
1/16*b*(-16*D*a*x+A*b-6*C*a)*(b*x^2+a)^(1/2)/a/x^2+1/24*(-8*D*a*x+A*b-6*C*
a)*(b*x^2+a)^(3/2)/a/x^4-1/6*A*(b*x^2+a)^(5/2)/a/x^6-1/5*B*(b*x^2+a)^(5/2)
/a/x^5+b^(3/2)*D*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))+1/16*b^2*(A*b-6*C*a)*a
rctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(3/2)
```

## Mathematica [A] (verified)

Time = 1.61 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{x^7}\,dx =$$
$$-\frac{\sqrt{a+bx^2}(3b^2x^4(5A+16Bx)+a^2(40A+48Bx+60Cx^2+80Dx^3)+2abx^2(35A+48Bx+75Cx^2+16}{240ax^6}$$
$$+\frac{b^2(-Ab+6aC)\text{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{3/2}}-b^{3/2}D\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)$$

input
```
Integrate[((a + b*x^2)^(3/2)*(A + B*x + C*x^2 + D*x^3))/x^7,x]
```

output
```
-1/240*(Sqrt[a + b*x^2]*(3*b^2*x^4*(5*A + 16*B*x) + a^2*(40*A + 48*B*x + 6
0*C*x^2 + 80*D*x^3) + 2*a*b*x^2*(35*A + 48*B*x + 75*C*x^2 + 160*D*x^3)))/(
a*x^6) + (b^2*(-(A*b) + 6*a*C)*ArcTanh[(Sqrt[b]*x - Sqrt[a + b*x^2])/Sqrt[
a]])/(8*a^(3/2)) - b^(3/2)*D*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]]
```

## Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.04, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {2338, 25, 2338, 27, 537, 27, 537, 25, 538, 224, 219, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{x^7}\,dx$$

$$\Big\downarrow \text{ 2338}$$

$$-\frac{\int -\frac{(bx^2+a)^{3/2}(6aDx^2-(Ab-6aC)x+6aB)}{x^6}dx}{6a}-\frac{A(a+bx^2)^{5/2}}{6ax^6}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{(bx^2+a)^{3/2}(6aDx^2-(Ab-6aC)x+6aB)}{x^6}dx}{6a}-\frac{A(a+bx^2)^{5/2}}{6ax^6}$$

$\Big\downarrow$ 2338

$$\frac{-\frac{\int \frac{5a(Ab-6aC-6aDx)\left(bx^2+a\right)^{3/2}}{x^5}dx}{5a}-\frac{6B\left(a+bx^2\right)^{5/2}}{5x^5}}{6a}-\frac{A\left(a+bx^2\right)^{5/2}}{6ax^6}$$

$\Big\downarrow$ 27

$$\frac{-\int \frac{(Ab-6aC-6aDx)\left(bx^2+a\right)^{3/2}}{x^5}dx-\frac{6B\left(a+bx^2\right)^{5/2}}{5x^5}}{6a}-\frac{A\left(a+bx^2\right)^{5/2}}{6ax^6}$$

$\Big\downarrow$ 537

$$\frac{\frac{1}{4}b\int -\frac{3(Ab-6aC-8aDx)\sqrt{bx^2+a}}{x^3}dx+\frac{\left(a+bx^2\right)^{3/2}(-6aC-8aDx+Ab)}{4x^4}-\frac{6B\left(a+bx^2\right)^{5/2}}{5x^5}}{6a}-\frac{A\left(a+bx^2\right)^{5/2}}{6ax^6}$$

$\Big\downarrow$ 27

$$\frac{-\frac{3}{4}b\int \frac{(Ab-6aC-8aDx)\sqrt{bx^2+a}}{x^3}dx+\frac{\left(a+bx^2\right)^{3/2}(-6aC-8aDx+Ab)}{4x^4}-\frac{6B\left(a+bx^2\right)^{5/2}}{5x^5}}{6a}-\frac{A\left(a+bx^2\right)^{5/2}}{6ax^6}$$

$\Big\downarrow$ 537

$$\frac{-\frac{3}{4}b\left(-\frac{1}{2}b\int -\frac{Ab-6aC-16aDx}{x\sqrt{bx^2+a}}dx-\frac{\sqrt{a+bx^2}(-6aC-16aDx+Ab)}{2x^2}\right)+\frac{\left(a+bx^2\right)^{3/2}(-6aC-8aDx+Ab)}{4x^4}-\frac{6B\left(a+bx^2\right)^{5/2}}{5x^5}}{6a}-\frac{A\left(a+bx^2\right)^{5/2}}{6ax^6}$$

$\Big\downarrow$ 25

$$\frac{-\frac{3}{4}b\left(\frac{1}{2}b\int \frac{Ab-6aC-16aDx}{x\sqrt{bx^2+a}}dx-\frac{\sqrt{a+bx^2}(-6aC-16aDx+Ab)}{2x^2}\right)+\frac{\left(a+bx^2\right)^{3/2}(-6aC-8aDx+Ab)}{4x^4}-\frac{6B\left(a+bx^2\right)^{5/2}}{5x^5}}{6a}-\frac{A\left(a+bx^2\right)^{5/2}}{6ax^6}$$

$\Big\downarrow$ 538

$$\frac{-\frac{3}{4}b\left(\frac{1}{2}b\left((Ab-6aC)\int \frac{1}{x\sqrt{bx^2+a}}dx-16aD\int \frac{1}{\sqrt{bx^2+a}}dx\right)-\frac{\sqrt{a+bx^2}(-6aC-16aDx+Ab)}{2x^2}\right)+\frac{\left(a+bx^2\right)^{3/2}(-6aC-8aDx+Ab)}{4x^4}}{6a}$$

$$\frac{A\left(a+bx^2\right)^{5/2}}{6ax^6}$$

$\Big\downarrow$ 224

$$\frac{-\frac{3}{4}b\left(\frac{1}{2}b\left((Ab-6aC)\int \frac{1}{x\sqrt{bx^2+a}}dx-16aD\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}\right)-\frac{\sqrt{a+bx^2}(-6aC-16aDx+Ab)}{2x^2}\right)+\frac{(a+bx^2)^{3/2}(-6aC-8}{4x^4}}{6a}$$

$$\frac{A(a+bx^2)^{5/2}}{6ax^6}$$

$\downarrow$ 219

$$\frac{-\frac{3}{4}b\left(\frac{1}{2}b\left((Ab-6aC)\int \frac{1}{x\sqrt{bx^2+a}}dx-\frac{16aD\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}\right)-\frac{\sqrt{a+bx^2}(-6aC-16aDx+Ab)}{2x^2}\right)+\frac{(a+bx^2)^{3/2}(-6aC-8a}{4x^4}}{6a}$$

$$\frac{A(a+bx^2)^{5/2}}{6ax^6}$$

$\downarrow$ 243

$$\frac{-\frac{3}{4}b\left(\frac{1}{2}b\left(\frac{1}{2}(Ab-6aC)\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2-\frac{16aD\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}\right)-\frac{\sqrt{a+bx^2}(-6aC-16aDx+Ab)}{2x^2}\right)+\frac{(a+bx^2)^{3/2}(-6aC}{4x^4}}{6a}$$

$$\frac{A(a+bx^2)^{5/2}}{6ax^6}$$

$\downarrow$ 73

$$\frac{-\frac{3}{4}b\left(\frac{1}{2}b\left(\frac{(Ab-6aC)\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{b}-\frac{16aD\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}\right)-\frac{\sqrt{a+bx^2}(-6aC-16aDx+Ab)}{2x^2}\right)+\frac{(a+bx^2)^{3/2}(-6aC-8aL}{4x^4}}{6a}$$

$$\frac{A(a+bx^2)^{5/2}}{6ax^6}$$

$\downarrow$ 221

$$\frac{-\frac{3}{4}b\left(\frac{1}{2}b\left(-\frac{(Ab-6aC)\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{16aD\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}\right)-\frac{\sqrt{a+bx^2}(-6aC-16aDx+Ab)}{2x^2}\right)+\frac{(a+bx^2)^{3/2}(-6aC}{4x^4}}{6a}$$

$$\frac{A(a+bx^2)^{5/2}}{6ax^6}$$

input
```
Int[((a + b*x^2)^(3/2)*(A + B*x + C*x^2 + D*x^3))/x^7,x]
```

output
```
-1/6*(A*(a + b*x^2)^(5/2))/(a*x^6) + (((A*b - 6*a*C - 8*a*D*x)*(a + b*x^2)
^(3/2))/(4*x^4) - (6*B*(a + b*x^2)^(5/2))/(5*x^5) - (3*b*(-1/2*((A*b - 6*a
*C - 16*a*D*x)*Sqrt[a + b*x^2])/x^2 + (b*((-16*a*D*ArcTanh[(Sqrt[b]*x)/Sqr
t[a + b*x^2]])/Sqrt[b] - ((A*b - 6*a*C)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/
Sqrt[a]))/2))/4)/(6*a)
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 537
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[x^(m + 1)*(c*(m + 2) + d*(m + 1)*x)*((a + b*x^2)^p/((m + 1)*(m + 2))),
 x] - Simp[2*b*(p/((m + 1)*(m + 2)))   Int[x^(m + 2)*(c*(m + 2) + d*(m + 1)
*x)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[m, -2] &&
 GtQ[p, 0] &&  !ILtQ[m + 2*p + 3, 0] && IntegerQ[2*p]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 348 vs. $2(148) = 296$.

Time = 0.69 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.96

| method | result |
|---|---|
| default | $A\left(-\frac{(b\,x^2+a)^{\frac{5}{2}}}{6a\,x^6}-\frac{b\left(-\frac{(b\,x^2+a)^{\frac{5}{2}}}{4a\,x^4}+\frac{b\left(-\frac{(b\,x^2+a)^{\frac{5}{2}}}{2a\,x^2}+\frac{3b\left(\frac{(b\,x^2+a)^{\frac{3}{2}}}{3}+a\left(\sqrt{b\,x^2+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)\right)\right)}{2a}\right)}{4a}\right)}{6a}\right)-\frac{B(}{}$ |

input
```
int((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/x^7,x,method=_RETURNVERBOSE)
```

output
```
A*(-1/6/a/x^6*(b*x^2+a)^(5/2)-1/6*b/a*(-1/4/a/x^4*(b*x^2+a)^(5/2)+1/4*b/a*
(-1/2/a/x^2*(b*x^2+a)^(5/2)+3/2*b/a*(1/3*(b*x^2+a)^(3/2)+a*((b*x^2+a)^(1/2
)-a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x))))))-1/5*B*(b*x^2+a)^(5/2)
/a/x^5+C*(-1/4/a/x^4*(b*x^2+a)^(5/2)+1/4*b/a*(-1/2/a/x^2*(b*x^2+a)^(5/2)+3
/2*b/a*(1/3*(b*x^2+a)^(3/2)+a*((b*x^2+a)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/2)*(
b*x^2+a)^(1/2))/x)))))+D*(-1/3/a/x^3*(b*x^2+a)^(5/2)+2/3*b/a*(-1/a/x*(b*x^
2+a)^(5/2)+4*b/a*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a
/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))))))
```

## Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 772, normalized size of antiderivative = 4.34

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{x^7}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/x^7,x, algorithm="fricas")
```

output

```
[1/480*(240*D*a^2*b^(3/2)*x^6*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x -
 a) - 15*(6*C*a*b^2 - A*b^3)*sqrt(a)*x^6*log(-(b*x^2 + 2*sqrt(b*x^2 + a)*s
qrt(a) + 2*a)/x^2) - 2*(16*(20*D*a^2*b + 3*B*a*b^2)*x^5 + 48*B*a^3*x + 15*
(10*C*a^2*b + A*a*b^2)*x^4 + 40*A*a^3 + 16*(5*D*a^3 + 6*B*a^2*b)*x^3 + 10*
(6*C*a^3 + 7*A*a^2*b)*x^2)*sqrt(b*x^2 + a))/(a^2*x^6), -1/480*(480*D*a^2*s
qrt(-b)*b*x^6*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + 15*(6*C*a*b^2 - A*b^3)*
sqrt(a)*x^6*log(-(b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(16*(2
0*D*a^2*b + 3*B*a*b^2)*x^5 + 48*B*a^3*x + 15*(10*C*a^2*b + A*a*b^2)*x^4 +
40*A*a^3 + 16*(5*D*a^3 + 6*B*a^2*b)*x^3 + 10*(6*C*a^3 + 7*A*a^2*b)*x^2)*sq
rt(b*x^2 + a))/(a^2*x^6), 1/240*(120*D*a^2*b^(3/2)*x^6*log(-2*b*x^2 - 2*sq
rt(b*x^2 + a)*sqrt(b)*x - a) + 15*(6*C*a*b^2 - A*b^3)*sqrt(-a)*x^6*arctan(
sqrt(b*x^2 + a)*sqrt(-a)/a) - (16*(20*D*a^2*b + 3*B*a*b^2)*x^5 + 48*B*a^3*
x + 15*(10*C*a^2*b + A*a*b^2)*x^4 + 40*A*a^3 + 16*(5*D*a^3 + 6*B*a^2*b)*x^
3 + 10*(6*C*a^3 + 7*A*a^2*b)*x^2)*sqrt(b*x^2 + a))/(a^2*x^6), -1/240*(240*
D*a^2*sqrt(-b)*b*x^6*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - 15*(6*C*a*b^2 -
A*b^3)*sqrt(-a)*x^6*arctan(sqrt(b*x^2 + a)*sqrt(-a)/a) + (16*(20*D*a^2*b +
 3*B*a*b^2)*x^5 + 48*B*a^3*x + 15*(10*C*a^2*b + A*a*b^2)*x^4 + 40*A*a^3 +
16*(5*D*a^3 + 6*B*a^2*b)*x^3 + 10*(6*C*a^3 + 7*A*a^2*b)*x^2)*sqrt(b*x^2 +
a))/(a^2*x^6)]
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 439 vs. $2(163) = 326$.

Time = 14.39 (sec) , antiderivative size = 439, normalized size of antiderivative = 2.47

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{x^7}\,dx = -\frac{Aa^2}{6\sqrt{b}x^7\sqrt{\frac{a}{bx^2}+1}} - \frac{11Aa\sqrt{b}}{24x^5\sqrt{\frac{a}{bx^2}+1}} - \frac{17Ab^{\frac{3}{2}}}{48x^3\sqrt{\frac{a}{bx^2}+1}} - \frac{Ab^{\frac{5}{2}}}{16ax\sqrt{\frac{a}{bx^2}+1}} + \frac{Ab^3\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{16a^{\frac{3}{2}}} - \frac{Ba\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{5x^4} - \frac{2Bb^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{5x^2} - \frac{Bb^{\frac{5}{2}}\sqrt{\frac{a}{bx^2}+1}}{5a} - \frac{Ca^2}{4\sqrt{b}x^5\sqrt{\frac{a}{bx^2}+1}} - \frac{3Ca\sqrt{b}}{8x^3\sqrt{\frac{a}{bx^2}+1}} - \frac{Cb^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{2x} - \frac{Cb^{\frac{3}{2}}}{8x\sqrt{\frac{a}{bx^2}+1}} - \frac{3Cb^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{8\sqrt{a}} - \frac{D\sqrt{a}b}{x\sqrt{1+\frac{bx^2}{a}}} - \frac{Da\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{3x^2} - \frac{Db^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{3} + Db^{\frac{3}{2}}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right) - \frac{Db^2x}{\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

input
```
integrate((b*x**2+a)**(3/2)*(D*x**3+C*x**2+B*x+A)/x**7,x)
```

output
```
-A*a**2/(6*sqrt(b)*x**7*sqrt(a/(b*x**2) + 1)) - 11*A*a*sqrt(b)/(24*x**5*sq
rt(a/(b*x**2) + 1)) - 17*A*b**(3/2)/(48*x**3*sqrt(a/(b*x**2) + 1)) - A*b**
(5/2)/(16*a*x*sqrt(a/(b*x**2) + 1)) + A*b**3*asinh(sqrt(a)/(sqrt(b)*x))/(1
6*a**(3/2)) - B*a*sqrt(b)*sqrt(a/(b*x**2) + 1)/(5*x**4) - 2*B*b**(3/2)*sqr
t(a/(b*x**2) + 1)/(5*x**2) - B*b**(5/2)*sqrt(a/(b*x**2) + 1)/(5*a) - C*a**
2/(4*sqrt(b)*x**5*sqrt(a/(b*x**2) + 1)) - 3*C*a*sqrt(b)/(8*x**3*sqrt(a/(b*
x**2) + 1)) - C*b**(3/2)*sqrt(a/(b*x**2) + 1)/(2*x) - C*b**(3/2)/(8*x*sqrt
(a/(b*x**2) + 1)) - 3*C*b**2*asinh(sqrt(a)/(sqrt(b)*x))/(8*sqrt(a)) - D*sq
rt(a)*b/(x*sqrt(1 + b*x**2/a)) - D*a*sqrt(b)*sqrt(a/(b*x**2) + 1)/(3*x**2)
 - D*b**(3/2)*sqrt(a/(b*x**2) + 1)/3 + D*b**(3/2)*asinh(sqrt(b)*x/sqrt(a))
 - D*b**2*x/(sqrt(a)*sqrt(1 + b*x**2/a))
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.67

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{x^7}\,dx = \frac{\sqrt{bx^2+a}Db^2x}{a} + Db^{\frac{3}{2}}\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right) - \frac{3\,Cb^2\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{8\sqrt{a}} + \frac{Ab^3\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{16\,a^{\frac{3}{2}}} + \frac{(bx^2+a)^{\frac{3}{2}}Cb^2}{8\,a^2} + \frac{3\sqrt{bx^2+a}Cb^2}{8\,a} - \frac{(bx^2+a)^{\frac{3}{2}}Ab^3}{48\,a^3} - \frac{\sqrt{bx^2+a}Ab^3}{16\,a^2} - \frac{2\,(bx^2+a)^{\frac{3}{2}}Db}{3\,ax} - \frac{(bx^2+a)^{\frac{5}{2}}Cb}{8\,a^2x^2} + \frac{(bx^2+a)^{\frac{5}{2}}Ab^2}{48\,a^3x^2} - \frac{(bx^2+a)^{\frac{5}{2}}D}{3\,ax^3} - \frac{(bx^2+a)^{\frac{5}{2}}C}{4\,ax^4} + \frac{(bx^2+a)^{\frac{5}{2}}Ab}{24\,a^2x^4} - \frac{(bx^2+a)^{\frac{5}{2}}B}{5\,ax^5} - \frac{(bx^2+a)^{\frac{5}{2}}A}{6\,ax^6}$$

input
```
integrate((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/x^7,x, algorithm="maxima")
```

output
```
sqrt(b*x^2 + a)*D*b^2*x/a + D*b^(3/2)*arcsinh(b*x/sqrt(a*b)) - 3/8*C*b^2*a
rcsinh(a/(sqrt(a*b)*abs(x)))/sqrt(a) + 1/16*A*b^3*arcsinh(a/(sqrt(a*b)*abs
(x)))/a^(3/2) + 1/8*(b*x^2 + a)^(3/2)*C*b^2/a^2 + 3/8*sqrt(b*x^2 + a)*C*b^
2/a - 1/48*(b*x^2 + a)^(3/2)*A*b^3/a^3 - 1/16*sqrt(b*x^2 + a)*A*b^3/a^2 -
2/3*(b*x^2 + a)^(3/2)*D*b/(a*x) - 1/8*(b*x^2 + a)^(5/2)*C*b/(a^2*x^2) + 1/
48*(b*x^2 + a)^(5/2)*A*b^2/(a^3*x^2) - 1/3*(b*x^2 + a)^(5/2)*D/(a*x^3) - 1
/4*(b*x^2 + a)^(5/2)*C/(a*x^4) + 1/24*(b*x^2 + a)^(5/2)*A*b/(a^2*x^4) - 1/
5*(b*x^2 + a)^(5/2)*B/(a*x^5) - 1/6*(b*x^2 + a)^(5/2)*A/(a*x^6)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 726 vs. 2(151) = 302.

Time = 0.16 (sec) , antiderivative size = 726, normalized size of antiderivative = 4.08

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{x^7}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/x^7,x, algorithm="giac")
```

output
```
-D*b^(3/2)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a))) + 1/8*(6*C*a*b^2 - A*b^3
)*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/(sqrt(-a)*a) + 1/120*(15
0*(sqrt(b)*x - sqrt(b*x^2 + a))^11*C*a*b^2 + 15*(sqrt(b)*x - sqrt(b*x^2 +
a))^11*A*b^3 + 480*(sqrt(b)*x - sqrt(b*x^2 + a))^10*D*a^2*b^(3/2) + 240*(s
qrt(b)*x - sqrt(b*x^2 + a))^10*B*a*b^(5/2) - 210*(sqrt(b)*x - sqrt(b*x^2 +
 a))^9*C*a^2*b^2 + 235*(sqrt(b)*x - sqrt(b*x^2 + a))^9*A*a*b^3 - 1920*(sqr
t(b)*x - sqrt(b*x^2 + a))^8*D*a^3*b^(3/2) - 240*(sqrt(b)*x - sqrt(b*x^2 +
a))^8*B*a^2*b^(5/2) + 60*(sqrt(b)*x - sqrt(b*x^2 + a))^7*C*a^3*b^2 + 390*(
sqrt(b)*x - sqrt(b*x^2 + a))^7*A*a^2*b^3 + 3200*(sqrt(b)*x - sqrt(b*x^2 +
a))^6*D*a^4*b^(3/2) + 480*(sqrt(b)*x - sqrt(b*x^2 + a))^6*B*a^3*b^(5/2) +
60*(sqrt(b)*x - sqrt(b*x^2 + a))^5*C*a^4*b^2 + 390*(sqrt(b)*x - sqrt(b*x^2
 + a))^5*A*a^3*b^3 - 2880*(sqrt(b)*x - sqrt(b*x^2 + a))^4*D*a^5*b^(3/2) -
480*(sqrt(b)*x - sqrt(b*x^2 + a))^4*B*a^4*b^(5/2) - 210*(sqrt(b)*x - sqrt(
b*x^2 + a))^3*C*a^5*b^2 + 235*(sqrt(b)*x - sqrt(b*x^2 + a))^3*A*a^4*b^3 +
1440*(sqrt(b)*x - sqrt(b*x^2 + a))^2*D*a^6*b^(3/2) + 48*(sqrt(b)*x - sqrt(
b*x^2 + a))^2*B*a^5*b^(5/2) + 150*(sqrt(b)*x - sqrt(b*x^2 + a))*C*a^6*b^2
+ 15*(sqrt(b)*x - sqrt(b*x^2 + a))*A*a^5*b^3 - 320*D*a^7*b^(3/2) - 48*B*a^
6*b^(5/2))/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^6*a)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}\,(A+Bx+Cx^2+Dx^3)}{x^7}\,dx=\int \frac{(b\,x^2+a)^{3/2}\,(A+B\,x+C\,x^2+x^3\,D)}{x^7}\,dx$$

input
```
int(((a + b*x^2)^(3/2)*(A + B*x + C*x^2 + x^3*D))/x^7,x)
```

output
```
int(((a + b*x^2)^(3/2)*(A + B*x + C*x^2 + x^3*D))/x^7, x)
```

## Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.96

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{x^7}\,dx = \frac{-40\sqrt{b\,x^2+a}\,a^3-70\sqrt{b\,x^2+a}\,a^2b\,x^2-48\sqrt{b\,x^2+a}\,a^2bx-}{}$$

input
```
int((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/x^7,x)
```

output
```
( - 40*sqrt(a + b*x**2)*a**3 - 70*sqrt(a + b*x**2)*a**2*b*x**2 - 48*sqrt(a
 + b*x**2)*a**2*b*x - 60*sqrt(a + b*x**2)*a**2*c*x**2 - 80*sqrt(a + b*x**2
)*a**2*d*x**3 - 15*sqrt(a + b*x**2)*a*b**2*x**4 - 96*sqrt(a + b*x**2)*a*b*
*2*x**3 - 150*sqrt(a + b*x**2)*a*b*c*x**4 - 320*sqrt(a + b*x**2)*a*b*d*x**
5 - 48*sqrt(a + b*x**2)*b**3*x**5 - 15*sqrt(a)*log((sqrt(a + b*x**2) - sqr
t(a) + sqrt(b)*x)/sqrt(a))*b**3*x**6 + 90*sqrt(a)*log((sqrt(a + b*x**2) -
sqrt(a) + sqrt(b)*x)/sqrt(a))*b**2*c*x**6 + 15*sqrt(a)*log((sqrt(a + b*x**
2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*b**3*x**6 - 90*sqrt(a)*log((sqrt(a + b*
x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*b**2*c*x**6 + 240*sqrt(b)*log((sqrt(
a + b*x**2) + sqrt(b)*x)/sqrt(a))*a*b*d*x**6 + 160*sqrt(b)*a*b*d*x**6 - 32
*sqrt(b)*b**3*x**6)/(240*a*x**6)
```

# 3.85 $\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{x^8}\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 169

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{x^8}\,dx = \frac{(bB-6aD)\sqrt{a+bx^2}}{24x^4} + \frac{5b(bB-6aD)\sqrt{a+bx^2}}{48ax^2} - \frac{A(a+bx^2)^{5/2}}{7ax^7} - \frac{B(a+bx^2)^{5/2}}{6ax^6} + \frac{(2Ab-7aC)(a+bx^2)^{5/2}}{35a^2x^5} + \frac{b^2(bB-6aD)\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{3/2}}$$

output
```
1/24*(B*b-6*D*a)*(b*x^2+a)^(1/2)/x^4+5/48*b*(B*b-6*D*a)*(b*x^2+a)^(1/2)/a/
x^2-1/7*A*(b*x^2+a)^(5/2)/a/x^7-1/6*B*(b*x^2+a)^(5/2)/a/x^6+1/35*(2*A*b-7*
C*a)*(b*x^2+a)^(5/2)/a^2/x^5+1/16*b^2*(B*b-6*D*a)*arctanh((b*x^2+a)^(1/2)/
a^(1/2))/a^(3/2)
```

## Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{x^8}\,dx = -\frac{\frac{\sqrt{a+bx^2}(-96Ab^3x^6+3ab^2x^4(16A+7x(5B+16Cx))+2a^2bx^2(192A+7x(35B+48Cx+75Dx^2))+4a^3(60A+7x(10B+3x(4C+5Dx))))}{x^7}+105}{1680a^2}$$

input
```
Integrate[((a + b*x^2)^(3/2)*(A + B*x + C*x^2 + D*x^3))/x^8,x]
```

output
```
-1/1680*((Sqrt[a + b*x^2]*(-96*A*b^3*x^6 + 3*a*b^2*x^4*(16*A + 7*x*(5*B +
16*C*x)) + 2*a^2*b*x^2*(192*A + 7*x*(35*B + 48*C*x + 75*D*x^2)) + 4*a^3*(6
0*A + 7*x*(10*B + 3*x*(4*C + 5*D*x)))))/x^7 + 105*Sqrt[a]*b^2*(-(b*B) + 6*
a*D)*Log[x] - 105*Sqrt[a]*b^2*(-(b*B) + 6*a*D)*Log[-Sqrt[a] + Sqrt[a + b*x
^2]])/a^2
```

## Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.97, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2338, 25, 2338, 27, 534, 243, 51, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{x^8}dx$$

$$\Big\downarrow \; 2338$$

$$-\frac{\int -\frac{(bx^2+a)^{3/2}(7aDx^2-(2Ab-7aC)x+7aB)}{x^7}dx}{7a}-\frac{A(a+bx^2)^{5/2}}{7ax^7}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{(bx^2+a)^{3/2}(7aDx^2-(2Ab-7aC)x+7aB)}{x^7}dx}{7a}-\frac{A(a+bx^2)^{5/2}}{7ax^7}$$

$$\Big\downarrow \text{2338}$$

$$\frac{-\frac{\int \frac{a(6(2Ab-7aC)+7(bB-6aD)x)\left(bx^2+a\right)^{3/2}}{x^6}dx}{6a}-\frac{7B\left(a+bx^2\right)^{5/2}}{6x^6}}{7a}-\frac{A\left(a+bx^2\right)^{5/2}}{7ax^7}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{1}{6}\int \frac{(6(2Ab-7aC)+7(bB-6aD)x)\left(bx^2+a\right)^{3/2}}{x^6}dx-\frac{7B\left(a+bx^2\right)^{5/2}}{6x^6}}{7a}-\frac{A\left(a+bx^2\right)^{5/2}}{7ax^7}$$

$$\Big\downarrow \text{534}$$

$$\frac{\frac{1}{6}\left(\frac{6\left(a+bx^2\right)^{5/2}(2Ab-7aC)}{5ax^5}-7(bB-6aD)\int \frac{\left(bx^2+a\right)^{3/2}}{x^5}dx\right)-\frac{7B\left(a+bx^2\right)^{5/2}}{6x^6}}{7a}-\frac{A\left(a+bx^2\right)^{5/2}}{7ax^7}$$

$$\Big\downarrow \text{243}$$

$$\frac{\frac{1}{6}\left(\frac{6\left(a+bx^2\right)^{5/2}(2Ab-7aC)}{5ax^5}-\frac{7}{2}(bB-6aD)\int \frac{\left(bx^2+a\right)^{3/2}}{x^6}dx^2\right)-\frac{7B\left(a+bx^2\right)^{5/2}}{6x^6}}{7a}-\frac{A\left(a+bx^2\right)^{5/2}}{7ax^7}$$

$$\Big\downarrow \text{51}$$

$$\frac{\frac{1}{6}\left(\frac{6\left(a+bx^2\right)^{5/2}(2Ab-7aC)}{5ax^5}-\frac{7}{2}(bB-6aD)\left(\frac{3}{4}b\int \frac{\sqrt{bx^2+a}}{x^4}dx^2-\frac{\left(a+bx^2\right)^{3/2}}{2x^4}\right)\right)-\frac{7B\left(a+bx^2\right)^{5/2}}{6x^6}}{7a}-\frac{A\left(a+bx^2\right)^{5/2}}{7ax^7}$$

$$\Big\downarrow \text{51}$$

$$\frac{\frac{1}{6}\left(\frac{6\left(a+bx^2\right)^{5/2}(2Ab-7aC)}{5ax^5}-\frac{7}{2}(bB-6aD)\left(\frac{3}{4}b\left(\frac{1}{2}b\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2-\frac{\sqrt{a+bx^2}}{x^2}\right)-\frac{\left(a+bx^2\right)^{3/2}}{2x^4}\right)\right)-\frac{7B\left(a+bx^2\right)^{5/2}}{6x^6}}{7a}-\frac{A\left(a+bx^2\right)^{5/2}}{7ax^7}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{1}{6}\left(\frac{6\left(a+bx^2\right)^{5/2}(2Ab-7aC)}{5ax^5}-\frac{7}{2}(bB-6aD)\left(\frac{3}{4}b\left(\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}-\frac{\sqrt{a+bx^2}}{x^2}\right)-\frac{\left(a+bx^2\right)^{3/2}}{2x^4}\right)\right)-\frac{7B\left(a+bx^2\right)^{5/2}}{6x^6}}{7a}-\frac{A\left(a+bx^2\right)^{5/2}}{7ax^7}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{1}{6}\left(\frac{6(a+bx^2)^{5/2}(2Ab-7aC)}{5ax^5}-\frac{7}{2}\left(\frac{3}{4}b\left(-\frac{b\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\sqrt{a+bx^2}}{x^2}\right)-\frac{(a+bx^2)^{3/2}}{2x^4}\right)(bB-6aD)\right)-\frac{7B(a+bx^2)^{5/2}}{6x^6}}{7a}-\frac{A(a+bx^2)^{5/2}}{7ax^7}$$

input
```
Int[((a + b*x^2)^(3/2)*(A + B*x + C*x^2 + D*x^3))/x^8,x]
```

output
```
-1/7*(A*(a + b*x^2)^(5/2))/(a*x^7) + ((-7*B*(a + b*x^2)^(5/2))/(6*x^6) + (
(6*(2*A*b - 7*a*C)*(a + b*x^2)^(5/2))/(5*a*x^5) - (7*(b*B - 6*a*D)*(-1/2*(
a + b*x^2)^(3/2)/x^4 + (3*b*(-(Sqrt[a + b*x^2]/x^2) - (b*ArcTanh[Sqrt[a +
b*x^2]/Sqrt[a]])/Sqrt[a]))/4))/2)/6)/(7*a)
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 285 vs. $2(141) = 282$.

Time = 0.62 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.69

| method | result |
|---|---|
| default | $A\left(-\frac{(b\,x^2+a)^{\frac{5}{2}}}{7a\,x^7}+\frac{2b(b\,x^2+a)^{\frac{5}{2}}}{35a^2x^5}\right)+B\left(-\frac{(b\,x^2+a)^{\frac{5}{2}}}{6a\,x^6}-\frac{b\left(-\frac{(b\,x^2+a)^{\frac{5}{2}}}{4a\,x^4}+\frac{b\left(-\frac{(b\,x^2+a)^{\frac{5}{2}}}{2a\,x^2}+\frac{3b\left(\frac{(b\,x^2+a)^{\frac{3}{2}}}{3}+a\left(\sqrt{b\,x^2+a}\right.\right.}{}\right.}{4a}\right.}{6a}\right.$ |

input
```
int((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/x^8,x,method=_RETURNVERBOSE)
```

output
```
A*(-1/7/a/x^7*(b*x^2+a)^(5/2)+2/35*b/a^2/x^5*(b*x^2+a)^(5/2))+B*(-1/6/a/x^
6*(b*x^2+a)^(5/2)-1/6*b/a*(-1/4/a/x^4*(b*x^2+a)^(5/2)+1/4*b/a*(-1/2/a/x^2*
(b*x^2+a)^(5/2)+3/2*b/a*(1/3*(b*x^2+a)^(3/2)+a*((b*x^2+a)^(1/2)-a^(1/2)*ln
((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x))))))-1/5*C/a/x^5*(b*x^2+a)^(5/2)+D*(-1
/4/a/x^4*(b*x^2+a)^(5/2)+1/4*b/a*(-1/2/a/x^2*(b*x^2+a)^(5/2)+3/2*b/a*(1/3*
(b*x^2+a)^(3/2)+a*((b*x^2+a)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/
2))/x)))))
```

## Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 353, normalized size of antiderivative = 2.09

$$\int \frac{(a+bx^2)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{x^8}\,dx=\left[-\frac{105\left(6\,Dab^2-Bb^3\right)\sqrt{a}x^7\log\left(-\frac{bx^2+2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\,(4\right.}{}$$

input
```
integrate((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/x^8,x, algorithm="fricas")
```

output
```
[-1/3360*(105*(6*D*a*b^2 - B*b^3)*sqrt(a)*x^7*log(-(b*x^2 + 2*sqrt(b*x^2 +
 a)*sqrt(a) + 2*a)/x^2) + 2*(48*(7*C*a*b^2 - 2*A*b^3)*x^6 + 105*(10*D*a^2*
b + B*a*b^2)*x^5 + 280*B*a^3*x + 48*(14*C*a^2*b + A*a*b^2)*x^4 + 240*A*a^3
 + 70*(6*D*a^3 + 7*B*a^2*b)*x^3 + 48*(7*C*a^3 + 8*A*a^2*b)*x^2)*sqrt(b*x^2
 + a))/(a^2*x^7), 1/1680*(105*(6*D*a*b^2 - B*b^3)*sqrt(-a)*x^7*arctan(sqrt
(b*x^2 + a)*sqrt(-a)/a) - (48*(7*C*a*b^2 - 2*A*b^3)*x^6 + 105*(10*D*a^2*b
+ B*a*b^2)*x^5 + 280*B*a^3*x + 48*(14*C*a^2*b + A*a*b^2)*x^4 + 240*A*a^3 +
 70*(6*D*a^3 + 7*B*a^2*b)*x^3 + 48*(7*C*a^3 + 8*A*a^2*b)*x^2)*sqrt(b*x^2 +
 a))/(a^2*x^7)]
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 774 vs. $2(153) = 306$.

Time = 14.98 (sec) , antiderivative size = 774, normalized size of antiderivative = 4.58

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{x^8}\,dx = -\frac{15Aa^6b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}} - \frac{33Aa^5b^{\frac{11}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}} - \frac{17Aa^4b^{\frac{13}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}} - \frac{3Aa^3b^{\frac{15}{2}}x^6\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}} - \frac{12Aa^2b^{\frac{17}{2}}x^8\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}} - \frac{8Aab^{\frac{19}{2}}x^{10}\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}} - \frac{Ab^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{5x^4} - \frac{Ab^{\frac{5}{2}}\sqrt{\frac{a}{bx^2}+1}}{15ax^2} + \frac{2Ab^{\frac{7}{2}}\sqrt{\frac{a}{bx^2}+1}}{15a^2} - \frac{Ba^2}{6\sqrt{b}x^7\sqrt{\frac{a}{bx^2}+1}} - \frac{11Ba\sqrt{b}}{24x^5\sqrt{\frac{a}{bx^2}+1}} - \frac{17Bb^{\frac{3}{2}}}{48x^3\sqrt{\frac{a}{bx^2}+1}} - \frac{Bb^{\frac{5}{2}}}{16ax\sqrt{\frac{a}{bx^2}+1}} + \frac{Bb^3\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{16a^{\frac{3}{2}}} - \frac{Ca\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{5x^4} - \frac{2Cb^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{5x^2} - \frac{Cb^{\frac{5}{2}}\sqrt{\frac{a}{bx^2}+1}}{5a} - \frac{Da^2}{4\sqrt{b}x^5\sqrt{\frac{a}{bx^2}+1}} - \frac{3Da\sqrt{b}}{8x^3\sqrt{\frac{a}{bx^2}+1}} - \frac{Db^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{2x} - \frac{Db^{\frac{3}{2}}}{8x\sqrt{\frac{a}{bx^2}+1}} - \frac{3Db^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{8\sqrt{a}}$$

input
```
integrate((b*x**2+a)**(3/2)*(D*x**3+C*x**2+B*x+A)/x**8,x)
```

output
```
-15*A*a**6*b**(9/2)*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b*
*5*x**8 + 105*a**3*b**6*x**10) - 33*A*a**5*b**(11/2)*x**2*sqrt(a/(b*x**2)
+ 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 17*
A*a**4*b**(13/2)*x**4*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*
b**5*x**8 + 105*a**3*b**6*x**10) - 3*A*a**3*b**(15/2)*x**6*sqrt(a/(b*x**2)
 + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 12
*A*a**2*b**(17/2)*x**8*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4
*b**5*x**8 + 105*a**3*b**6*x**10) - 8*A*a*b**(19/2)*x**10*sqrt(a/(b*x**2)
+ 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - A*b
**(3/2)*sqrt(a/(b*x**2) + 1)/(5*x**4) - A*b**(5/2)*sqrt(a/(b*x**2) + 1)/(1
5*a*x**2) + 2*A*b**(7/2)*sqrt(a/(b*x**2) + 1)/(15*a**2) - B*a**2/(6*sqrt(b
)*x**7*sqrt(a/(b*x**2) + 1)) - 11*B*a*sqrt(b)/(24*x**5*sqrt(a/(b*x**2) + 1
)) - 17*B*b**(3/2)/(48*x**3*sqrt(a/(b*x**2) + 1)) - B*b**(5/2)/(16*a*x*sqr
t(a/(b*x**2) + 1)) + B*b**3*asinh(sqrt(a)/(sqrt(b)*x))/(16*a**(3/2)) - C*a
*sqrt(b)*sqrt(a/(b*x**2) + 1)/(5*x**4) - 2*C*b**(3/2)*sqrt(a/(b*x**2) + 1)
/(5*x**2) - C*b**(5/2)*sqrt(a/(b*x**2) + 1)/(5*a) - D*a**2/(4*sqrt(b)*x**5
*sqrt(a/(b*x**2) + 1)) - 3*D*a*sqrt(b)/(8*x**3*sqrt(a/(b*x**2) + 1)) - D*b
**(3/2)*sqrt(a/(b*x**2) + 1)/(2*x) - D*b**(3/2)/(8*x*sqrt(a/(b*x**2) + 1))
 - 3*D*b**2*asinh(sqrt(a)/(sqrt(b)*x))/(8*sqrt(a))
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.57

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{x^8}\,dx = -\frac{3\,Db^2\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{8\,\sqrt{a}} + \frac{Bb^3\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{16\,a^{\frac{3}{2}}} + \frac{(bx^2+a)^{\frac{3}{2}}Db^2}{8\,a^2} + \frac{3\,\sqrt{bx^2+a}Db^2}{8\,a} - \frac{(bx^2+a)^{\frac{3}{2}}Bb^3}{48\,a^3} - \frac{\sqrt{bx^2+a}Bb^3}{16\,a^2} - \frac{(bx^2+a)^{\frac{5}{2}}Db}{8\,a^2x^2} + \frac{(bx^2+a)^{\frac{5}{2}}Bb^2}{48\,a^3x^2} - \frac{(bx^2+a)^{\frac{5}{2}}D}{4\,ax^4} + \frac{(bx^2+a)^{\frac{5}{2}}Bb}{24\,a^2x^4} - \frac{(bx^2+a)^{\frac{5}{2}}C}{5\,ax^5} + \frac{2\,(bx^2+a)^{\frac{5}{2}}Ab}{35\,a^2x^5} - \frac{(bx^2+a)^{\frac{5}{2}}B}{6\,ax^6} - \frac{(bx^2+a)^{\frac{5}{2}}A}{7\,ax^7}$$

input
```
integrate((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/x^8,x, algorithm="maxima")
```

output
```
-3/8*D*b^2*arcsinh(a/(sqrt(a*b)*abs(x)))/sqrt(a) + 1/16*B*b^3*arcsinh(a/(s
qrt(a*b)*abs(x)))/a^(3/2) + 1/8*(b*x^2 + a)^(3/2)*D*b^2/a^2 + 3/8*sqrt(b*x
^2 + a)*D*b^2/a - 1/48*(b*x^2 + a)^(3/2)*B*b^3/a^3 - 1/16*sqrt(b*x^2 + a)*
B*b^3/a^2 - 1/8*(b*x^2 + a)^(5/2)*D*b/(a^2*x^2) + 1/48*(b*x^2 + a)^(5/2)*B
*b^2/(a^3*x^2) - 1/4*(b*x^2 + a)^(5/2)*D/(a*x^4) + 1/24*(b*x^2 + a)^(5/2)*
B*b/(a^2*x^4) - 1/5*(b*x^2 + a)^(5/2)*C/(a*x^5) + 2/35*(b*x^2 + a)^(5/2)*A
*b/(a^2*x^5) - 1/6*(b*x^2 + a)^(5/2)*B/(a*x^6) - 1/7*(b*x^2 + a)^(5/2)*A/(
a*x^7)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 728 vs. $2(144) = 288$.

Time = 0.15 (sec) , antiderivative size = 728, normalized size of antiderivative = 4.31

$$\int \frac{(a+bx^2)^{3/2}\,(A+Bx+Cx^2+Dx^3)}{x^8}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/x^8,x, algorithm="giac")
```

output
```
1/8*(6*D*a*b^2 - B*b^3)*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/(s
qrt(-a)*a) + 1/840*(1050*(sqrt(b)*x - sqrt(b*x^2 + a))^13*D*a*b^2 + 105*(s
qrt(b)*x - sqrt(b*x^2 + a))^13*B*b^3 + 1680*(sqrt(b)*x - sqrt(b*x^2 + a))^
12*C*a*b^(5/2) - 2520*(sqrt(b)*x - sqrt(b*x^2 + a))^11*D*a^2*b^2 + 1540*(s
qrt(b)*x - sqrt(b*x^2 + a))^11*B*a*b^3 - 3360*(sqrt(b)*x - sqrt(b*x^2 + a)
)^10*C*a^2*b^(5/2) + 3360*(sqrt(b)*x - sqrt(b*x^2 + a))^10*A*a*b^(7/2) + 1
890*(sqrt(b)*x - sqrt(b*x^2 + a))^9*D*a^3*b^2 + 1085*(sqrt(b)*x - sqrt(b*x
^2 + a))^9*B*a^2*b^3 + 5040*(sqrt(b)*x - sqrt(b*x^2 + a))^8*C*a^3*b^(5/2)
+ 3360*(sqrt(b)*x - sqrt(b*x^2 + a))^8*A*a^2*b^(7/2) - 6720*(sqrt(b)*x - s
qrt(b*x^2 + a))^6*C*a^4*b^(5/2) + 6720*(sqrt(b)*x - sqrt(b*x^2 + a))^6*A*a
^3*b^(7/2) - 1890*(sqrt(b)*x - sqrt(b*x^2 + a))^5*D*a^5*b^2 - 1085*(sqrt(b
)*x - sqrt(b*x^2 + a))^5*B*a^4*b^3 + 3696*(sqrt(b)*x - sqrt(b*x^2 + a))^4*
C*a^5*b^(5/2) + 1344*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*a^4*b^(7/2) + 2520*
(sqrt(b)*x - sqrt(b*x^2 + a))^3*D*a^6*b^2 - 1540*(sqrt(b)*x - sqrt(b*x^2 +
 a))^3*B*a^5*b^3 - 672*(sqrt(b)*x - sqrt(b*x^2 + a))^2*C*a^6*b^(5/2) + 672
*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A*a^5*b^(7/2) - 1050*(sqrt(b)*x - sqrt(b*
x^2 + a))*D*a^7*b^2 - 105*(sqrt(b)*x - sqrt(b*x^2 + a))*B*a^6*b^3 + 336*C*
a^7*b^(5/2) - 96*A*a^6*b^(7/2))/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^7*a
)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{x^8}\,dx = \int \frac{(b\,x^2+a)^{3/2}\,(A+B\,x+C\,x^2+x^3\,D)}{x^8}\,dx$$

input
```
int(((a + b*x^2)^(3/2)*(A + B*x + C*x^2 + x^3*D))/x^8,x)
```

output
```
int(((a + b*x^2)^(3/2)*(A + B*x + C*x^2 + x^3*D))/x^8, x)
```

## Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 369, normalized size of antiderivative = 2.18

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{x^8}\,dx = \frac{-240\sqrt{b\,x^2+a}\,a^4-384\sqrt{b\,x^2+a}\,a^3b\,x^2-280\sqrt{b\,x^2+a}\,a^3b}{}$$

input
```
int((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/x^8,x)
```

output
```
( - 240*sqrt(a + b*x**2)*a**4 - 384*sqrt(a + b*x**2)*a**3*b*x**2 - 280*sqr
t(a + b*x**2)*a**3*b*x - 336*sqrt(a + b*x**2)*a**3*c*x**2 - 420*sqrt(a + b
*x**2)*a**3*d*x**3 - 48*sqrt(a + b*x**2)*a**2*b**2*x**4 - 490*sqrt(a + b*x
**2)*a**2*b**2*x**3 - 672*sqrt(a + b*x**2)*a**2*b*c*x**4 - 1050*sqrt(a + b
*x**2)*a**2*b*d*x**5 + 96*sqrt(a + b*x**2)*a*b**3*x**6 - 105*sqrt(a + b*x*
*2)*a*b**3*x**5 - 336*sqrt(a + b*x**2)*a*b**2*c*x**6 + 630*sqrt(a)*log((sq
rt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b**2*d*x**7 - 105*sqrt(a)
*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*b**4*x**7 - 630*sqr
t(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b**2*d*x**7 +
 105*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*b**4*x*
*7 - 96*sqrt(b)*a*b**3*x**7 - 144*sqrt(b)*a*b**2*c*x**7)/(1680*a**2*x**7)
```

# 3.86 $\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{x^9}\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 205

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{x^9}\,dx = \frac{(3Ab-8aC)\sqrt{a+bx^2}}{48x^6} + \frac{7b(3Ab-8aC)\sqrt{a+bx^2}}{192ax^4} + \frac{b^2(3Ab-8aC)\sqrt{a+bx^2}}{128a^2x^2} - \frac{A(a+bx^2)^{5/2}}{8ax^8} - \frac{B(a+bx^2)^{5/2}}{7ax^7} + \frac{(2bB-7aD)(a+bx^2)^{5/2}}{35a^2x^5} - \frac{b^3(3Ab-8aC)\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{128a^{5/2}}$$

output
```
1/48*(3*A*b-8*C*a)*(b*x^2+a)^(1/2)/x^6+7/192*b*(3*A*b-8*C*a)*(b*x^2+a)^(1/
2)/a/x^4+1/128*b^2*(3*A*b-8*C*a)*(b*x^2+a)^(1/2)/a^2/x^2-1/8*A*(b*x^2+a)^(
5/2)/a/x^8-1/7*B*(b*x^2+a)^(5/2)/a/x^7+1/35*(2*B*b-7*D*a)*(b*x^2+a)^(5/2)/
a^2/x^5-1/128*b^3*(3*A*b-8*C*a)*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(5/2)
```

## Mathematica [A] (verified)

Time = 2.10 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{x^9}\,dx =$$
$$-\frac{\sqrt{a+bx^2}(-3b^3x^6(105A+256Bx)+16a^3(105A+4x(30B+7x(5C+6Dx)))+6ab^2x^4(35A+4x(16B}{13440a^2x^8}$$
$$+\frac{b^3(3Ab-8aC)\text{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right)}{64a^{5/2}}$$

input
```
Integrate[((a + b*x^2)^(3/2)*(A + B*x + C*x^2 + D*x^3))/x^9,x]
```

output
```
-1/13440*(Sqrt[a + b*x^2]*(-3*b^3*x^6*(105*A + 256*B*x) + 16*a^3*(105*A +
4*x*(30*B + 7*x*(5*C + 6*D*x))) + 6*a*b^2*x^4*(35*A + 4*x*(16*B + 7*x*(5*C
 + 16*D*x))) + 8*a^2*b*x^2*(315*A + 2*x*(192*B + 7*x*(35*C + 48*D*x)))))/(
a^2*x^8) + (b^3*(3*A*b - 8*a*C)*ArcTanh[(Sqrt[b]*x - Sqrt[a + b*x^2])/Sqrt
[a]])/(64*a^(5/2))
```

## Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.98, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2338, 25, 2338, 27, 539, 25, 534, 243, 51, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{x^9}\,dx$$

$$\Big\downarrow \; 2338$$

$$-\frac{\int -\frac{(bx^2+a)^{3/2}(8aDx^2-(3Ab-8aC)x+8aB)}{x^8}dx}{8a}-\frac{A(a+bx^2)^{5/2}}{8ax^8}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{\left(bx^2+a\right)^{3/2}\left(8aDx^2-(3Ab-8aC)x+8aB\right)}{x^8}dx}{8a}-\frac{A\left(a+bx^2\right)^{5/2}}{8ax^8}$$

$\Big\downarrow$ 2338

$$\frac{-\frac{\int \frac{a(7(3Ab-8aC)+8(2bB-7aD)x)\left(bx^2+a\right)^{3/2}}{x^7}dx}{7a}-\frac{8B\left(a+bx^2\right)^{5/2}}{7x^7}}{8a}-\frac{A\left(a+bx^2\right)^{5/2}}{8ax^8}$$

$\Big\downarrow$ 27

$$\frac{-\frac{1}{7}\int \frac{(7(3Ab-8aC)+8(2bB-7aD)x)\left(bx^2+a\right)^{3/2}}{x^7}dx-\frac{8B\left(a+bx^2\right)^{5/2}}{7x^7}}{8a}-\frac{A\left(a+bx^2\right)^{5/2}}{8ax^8}$$

$\Big\downarrow$ 539

$$\frac{\frac{1}{7}\left(\frac{\int -\frac{(48a(2bB-7aD)-7b(3Ab-8aC)x)\left(bx^2+a\right)^{3/2}}{x^6}dx}{6a}+\frac{7\left(a+bx^2\right)^{5/2}(3Ab-8aC)}{6ax^6}\right)-\frac{8B\left(a+bx^2\right)^{5/2}}{7x^7}}{8a}-\frac{A\left(a+bx^2\right)^{5/2}}{8ax^8}$$

$\Big\downarrow$ 25

$$\frac{\frac{1}{7}\left(\frac{7\left(a+bx^2\right)^{5/2}(3Ab-8aC)}{6ax^6}-\frac{\int \frac{(48a(2bB-7aD)-7b(3Ab-8aC)x)\left(bx^2+a\right)^{3/2}}{x^6}dx}{6a}\right)-\frac{8B\left(a+bx^2\right)^{5/2}}{7x^7}}{8a}-\frac{A\left(a+bx^2\right)^{5/2}}{8ax^8}$$

$\Big\downarrow$ 534

$$\frac{\frac{1}{7}\left(\frac{7\left(a+bx^2\right)^{5/2}(3Ab-8aC)}{6ax^6}-\frac{-7b(3Ab-8aC)\int \frac{\left(bx^2+a\right)^{3/2}}{x^5}dx-\frac{48\left(a+bx^2\right)^{5/2}(2bB-7aD)}{5x^5}}{6a}\right)-\frac{8B\left(a+bx^2\right)^{5/2}}{7x^7}}{8a}-\frac{A\left(a+bx^2\right)^{5/2}}{8ax^8}$$

$\Big\downarrow$ 243

$$\frac{\frac{1}{7}\left(\frac{7\left(a+bx^2\right)^{5/2}(3Ab-8aC)}{6ax^6}-\frac{-\frac{7}{2}b(3Ab-8aC)\int \frac{\left(bx^2+a\right)^{3/2}}{x^6}dx^2-\frac{48\left(a+bx^2\right)^{5/2}(2bB-7aD)}{5x^5}}{6a}\right)-\frac{8B\left(a+bx^2\right)^{5/2}}{7x^7}}{8a}-\frac{A\left(a+bx^2\right)^{5/2}}{8ax^8}$$

$$\Big\downarrow \; 51$$

$$\frac{\frac{1}{7}\left(\frac{7(a+bx^2)^{5/2}(3Ab-8aC)}{6ax^6}-\frac{-\frac{7}{2}b(3Ab-8aC)\left(\frac{3}{4}b\int\frac{\sqrt{bx^2+a}}{x^4}dx^2-\frac{(a+bx^2)^{3/2}}{2x^4}\right)-\frac{48(a+bx^2)^{5/2}(2bB-7aD)}{5x^5}}{6a}\right)-\frac{8B(a+bx^2)^{5/2}}{7x^7}}{8a}-\frac{A(a+bx^2)^{5/2}}{8ax^8}$$

$$\Big\downarrow \; 51$$

$$\frac{\frac{1}{7}\left(\frac{7(a+bx^2)^{5/2}(3Ab-8aC)}{6ax^6}-\frac{-\frac{7}{2}b(3Ab-8aC)\left(\frac{3}{4}b\left(\frac{1}{2}b\int\frac{1}{x^2\sqrt{bx^2+a}}dx^2-\frac{\sqrt{a+bx^2}}{x^2}\right)-\frac{(a+bx^2)^{3/2}}{2x^4}\right)-\frac{48(a+bx^2)^{5/2}(2bB-7aD)}{5x^5}}{6a}\right)-\frac{8B(a+bx^2)^{5/2}}{7x^7}}{8a}-\frac{A(a+bx^2)^{5/2}}{8ax^8}$$

$$\Big\downarrow \; 73$$

$$\frac{\frac{1}{7}\left(\frac{7(a+bx^2)^{5/2}(3Ab-8aC)}{6ax^6}-\frac{-\frac{7}{2}b(3Ab-8aC)\left(\frac{3}{4}b\left(\int\frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}-\frac{\sqrt{a+bx^2}}{x^2}\right)-\frac{(a+bx^2)^{3/2}}{2x^4}\right)-\frac{48(a+bx^2)^{5/2}(2bB-7aD)}{5x^5}}{6a}\right)-\frac{8B(a+bx^2)^{5/2}}{7x^7}}{8a}-\frac{A(a+bx^2)^{5/2}}{8ax^8}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{1}{7}\left(\frac{7(a+bx^2)^{5/2}(3Ab-8aC)}{6ax^6}-\frac{-\frac{7}{2}b(3Ab-8aC)\left(\frac{3}{4}b\left(-\frac{b\,\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\sqrt{a+bx^2}}{x^2}\right)-\frac{(a+bx^2)^{3/2}}{2x^4}\right)-\frac{48(a+bx^2)^{5/2}(2bB-7aD)}{5x^5}}{6a}\right)-\frac{8B(a+bx^2)^{5/2}}{7x^7}}{8a}-\frac{A(a+bx^2)^{5/2}}{8ax^8}$$

input
```
Int[((a + b*x^2)^(3/2)*(A + B*x + C*x^2 + D*x^3))/x^9,x]
```

output
```
-1/8*(A*(a + b*x^2)^(5/2))/(a*x^8) + ((-8*B*(a + b*x^2)^(5/2))/(7*x^7) + (
(7*(3*A*b - 8*a*C)*(a + b*x^2)^(5/2))/(6*a*x^6) - ((-48*(2*b*B - 7*a*D)*(a
 + b*x^2)^(5/2))/(5*x^5) - (7*b*(3*A*b - 8*a*C)*(-1/2*(a + b*x^2)^(3/2)/x^
4 + (3*b*(-(Sqrt[a + b*x^2]/x^2) - (b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sq
rt[a]))/4))/2)/(6*a))/7)/(8*a)
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 539
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[c*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))
   Int[x^(m + 1)*(a + b*x^2)^p*(a*d*(m + 1) - b*c*(m + 2*p + 3)*x), x], x]
/; FreeQ[{a, b, c, d, p}, x] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

## Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.63

| method | result |
|---|---|
| default | $A\left(-\frac{(b\,x^2+a)^{\frac{5}{2}}}{8a\,x^8}-\frac{3b\left(-\frac{(b\,x^2+a)^{\frac{5}{2}}}{6a\,x^6}-\frac{b\left(-\frac{(b\,x^2+a)^{\frac{5}{2}}}{4a\,x^4}+\frac{b\left(-\frac{(b\,x^2+a)^{\frac{5}{2}}}{2a\,x^2}+\frac{3b\left(\frac{(b\,x^2+a)^{\frac{3}{2}}}{3}+a\left(\sqrt{b\,x^2+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)\right.\right.}{2a}\right.}{4a}\right.}{6a}\right.}{8a}\right.$ |

input
```
int((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/x^9,x,method=_RETURNVERBOSE)
```

output
```
A*(-1/8/a/x^8*(b*x^2+a)^(5/2)-3/8*b/a*(-1/6/a/x^6*(b*x^2+a)^(5/2)-1/6*b/a*
(-1/4/a/x^4*(b*x^2+a)^(5/2)+1/4*b/a*(-1/2/a/x^2*(b*x^2+a)^(5/2)+3/2*b/a*(1
/3*(b*x^2+a)^(3/2)+a*((b*x^2+a)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^
(1/2))/x))))))+B*(-1/7/a/x^7*(b*x^2+a)^(5/2)+2/35*b/a^2/x^5*(b*x^2+a)^(5/
2))+C*(-1/6/a/x^6*(b*x^2+a)^(5/2)-1/6*b/a*(-1/4/a/x^4*(b*x^2+a)^(5/2)+1/4*
b/a*(-1/2/a/x^2*(b*x^2+a)^(5/2)+3/2*b/a*(1/3*(b*x^2+a)^(3/2)+a*((b*x^2+a)^
(1/2)-a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x))))))-1/5*D/a/x^5*(b*x^
2+a)^(5/2)
```

## Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 412, normalized size of antiderivative = 2.01

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{x^9}\,dx = \left[-\frac{105\,(8\,Cab^3-3\,Ab^4)\sqrt{a}x^8\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\,(\cdots}{\cdots}\right.$$

$$-\frac{105\,(8\,Cab^3-3\,Ab^4)\sqrt{-a}x^8\arctan\left(\frac{\sqrt{bx^2+a}\sqrt{-a}}{a}\right)+(384\,(7\,Da^2b^2-2\,Bab^3)x^7+105\,(8\,Ca^2b^2-3\,Aab^3)\cdots}{\cdots}$$

input
```
integrate((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/x^9,x, algorithm="fricas")
```

output
```
[-1/26880*(105*(8*C*a*b^3 - 3*A*b^4)*sqrt(a)*x^8*log(-(b*x^2 - 2*sqrt(b*x^
2 + a)*sqrt(a) + 2*a)/x^2) + 2*(384*(7*D*a^2*b^2 - 2*B*a*b^3)*x^7 + 105*(8
*C*a^2*b^2 - 3*A*a*b^3)*x^6 + 1920*B*a^4*x + 384*(14*D*a^3*b + B*a^2*b^2)*
x^5 + 1680*A*a^4 + 70*(56*C*a^3*b + 3*A*a^2*b^2)*x^4 + 384*(7*D*a^4 + 8*B*
a^3*b)*x^3 + 280*(8*C*a^4 + 9*A*a^3*b)*x^2)*sqrt(b*x^2 + a))/(a^3*x^8), -1
/13440*(105*(8*C*a*b^3 - 3*A*b^4)*sqrt(-a)*x^8*arctan(sqrt(b*x^2 + a)*sqrt
(-a)/a) + (384*(7*D*a^2*b^2 - 2*B*a*b^3)*x^7 + 105*(8*C*a^2*b^2 - 3*A*a*b^
3)*x^6 + 1920*B*a^4*x + 384*(14*D*a^3*b + B*a^2*b^2)*x^5 + 1680*A*a^4 + 70
*(56*C*a^3*b + 3*A*a^2*b^2)*x^4 + 384*(7*D*a^4 + 8*B*a^3*b)*x^3 + 280*(8*C
*a^4 + 9*A*a^3*b)*x^2)*sqrt(b*x^2 + a))/(a^3*x^8)]
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 808 vs. $2(189) = 378$.

Time = 39.96 (sec) , antiderivative size = 808, normalized size of antiderivative = 3.94

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{x^9}\,dx = \text{Too large to display}$$

input
```
integrate((b*x**2+a)**(3/2)*(D*x**3+C*x**2+B*x+A)/x**9,x)
```

output
```
-A*a**2/(8*sqrt(b)*x**9*sqrt(a/(b*x**2) + 1)) - 5*A*a*sqrt(b)/(16*x**7*sqr
t(a/(b*x**2) + 1)) - 13*A*b**(3/2)/(64*x**5*sqrt(a/(b*x**2) + 1)) + A*b**(
5/2)/(128*a*x**3*sqrt(a/(b*x**2) + 1)) + 3*A*b**(7/2)/(128*a**2*x*sqrt(a/(
b*x**2) + 1)) - 3*A*b**4*asinh(sqrt(a)/(sqrt(b)*x))/(128*a**(5/2)) - 15*B*
a**6*b**(9/2)*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**
8 + 105*a**3*b**6*x**10) - 33*B*a**5*b**(11/2)*x**2*sqrt(a/(b*x**2) + 1)/(
105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 17*B*a**4
*b**(13/2)*x**4*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x
**8 + 105*a**3*b**6*x**10) - 3*B*a**3*b**(15/2)*x**6*sqrt(a/(b*x**2) + 1)/
(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 12*B*a**
2*b**(17/2)*x**8*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*
x**8 + 105*a**3*b**6*x**10) - 8*B*a*b**(19/2)*x**10*sqrt(a/(b*x**2) + 1)/(
105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - B*b**(3/2
)*sqrt(a/(b*x**2) + 1)/(5*x**4) - B*b**(5/2)*sqrt(a/(b*x**2) + 1)/(15*a*x*
*2) + 2*B*b**(7/2)*sqrt(a/(b*x**2) + 1)/(15*a**2) - C*a**2/(6*sqrt(b)*x**7
*sqrt(a/(b*x**2) + 1)) - 11*C*a*sqrt(b)/(24*x**5*sqrt(a/(b*x**2) + 1)) - 1
7*C*b**(3/2)/(48*x**3*sqrt(a/(b*x**2) + 1)) - C*b**(5/2)/(16*a*x*sqrt(a/(b
*x**2) + 1)) + C*b**3*asinh(sqrt(a)/(sqrt(b)*x))/(16*a**(3/2)) - D*a*sqrt(
b)*sqrt(a/(b*x**2) + 1)/(5*x**4) - 2*D*b**(3/2)*sqrt(a/(b*x**2) + 1)/(5*x*
*2) - D*b**(5/2)*sqrt(a/(b*x**2) + 1)/(5*a)
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.50

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{x^9}\,dx = \frac{Cb^3\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{16\,a^{\frac{3}{2}}} - \frac{3\,Ab^4\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{128\,a^{\frac{5}{2}}} - \frac{(bx^2+a)^{\frac{3}{2}}Cb^3}{48\,a^3} - \frac{\sqrt{bx^2+a}Cb^3}{16\,a^2} + \frac{(bx^2+a)^{\frac{3}{2}}Ab^4}{128\,a^4} + \frac{3\sqrt{bx^2+a}Ab^4}{128\,a^3} + \frac{(bx^2+a)^{\frac{5}{2}}Cb^2}{48\,a^3x^2} - \frac{(bx^2+a)^{\frac{5}{2}}Ab^3}{128\,a^4x^2} + \frac{(bx^2+a)^{\frac{5}{2}}Cb}{24\,a^2x^4} - \frac{(bx^2+a)^{\frac{5}{2}}Ab^2}{64\,a^3x^4} - \frac{(bx^2+a)^{\frac{5}{2}}D}{5\,ax^5} + \frac{2\,(bx^2+a)^{\frac{5}{2}}Bb}{35\,a^2x^5} - \frac{(bx^2+a)^{\frac{5}{2}}C}{6\,ax^6} + \frac{(bx^2+a)^{\frac{5}{2}}Ab}{16\,a^2x^6} - \frac{(bx^2+a)^{\frac{5}{2}}B}{7\,ax^7} - \frac{(bx^2+a)^{\frac{5}{2}}A}{8\,ax^8}$$

input
```
integrate((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/x^9,x, algorithm="maxima")
```

output
```
1/16*C*b^3*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2) - 3/128*A*b^4*arcsinh(a/(
sqrt(a*b)*abs(x)))/a^(5/2) - 1/48*(b*x^2 + a)^(3/2)*C*b^3/a^3 - 1/16*sqrt(
b*x^2 + a)*C*b^3/a^2 + 1/128*(b*x^2 + a)^(3/2)*A*b^4/a^4 + 3/128*sqrt(b*x^
2 + a)*A*b^4/a^3 + 1/48*(b*x^2 + a)^(5/2)*C*b^2/(a^3*x^2) - 1/128*(b*x^2 +
 a)^(5/2)*A*b^3/(a^4*x^2) + 1/24*(b*x^2 + a)^(5/2)*C*b/(a^2*x^4) - 1/64*(b
*x^2 + a)^(5/2)*A*b^2/(a^3*x^4) - 1/5*(b*x^2 + a)^(5/2)*D/(a*x^5) + 2/35*(
b*x^2 + a)^(5/2)*B*b/(a^2*x^5) - 1/6*(b*x^2 + a)^(5/2)*C/(a*x^6) + 1/16*(b
*x^2 + a)^(5/2)*A*b/(a^2*x^6) - 1/7*(b*x^2 + a)^(5/2)*B/(a*x^7) - 1/8*(b*x
^2 + a)^(5/2)*A/(a*x^8)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 872 vs. $2(173) = 346$.

Time = 0.14 (sec) , antiderivative size = 872, normalized size of antiderivative = 4.25

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{x^9}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/x^9,x, algorithm="giac")
```

output
```
-1/64*(8*C*a*b^3 - 3*A*b^4)*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a)
)/(sqrt(-a)*a^2) + 1/6720*(840*(sqrt(b)*x - sqrt(b*x^2 + a))^15*C*a*b^3 -
315*(sqrt(b)*x - sqrt(b*x^2 + a))^15*A*b^4 + 13440*(sqrt(b)*x - sqrt(b*x^2
 + a))^14*D*a^2*b^(5/2) + 11480*(sqrt(b)*x - sqrt(b*x^2 + a))^13*C*a^2*b^3
 + 2415*(sqrt(b)*x - sqrt(b*x^2 + a))^13*A*a*b^4 - 40320*(sqrt(b)*x - sqrt
(b*x^2 + a))^12*D*a^3*b^(5/2) + 26880*(sqrt(b)*x - sqrt(b*x^2 + a))^12*B*a
^2*b^(7/2) - 3640*(sqrt(b)*x - sqrt(b*x^2 + a))^11*C*a^3*b^3 + 34965*(sqrt
(b)*x - sqrt(b*x^2 + a))^11*A*a^2*b^4 + 67200*(sqrt(b)*x - sqrt(b*x^2 + a)
)^10*D*a^4*b^(5/2) - 8680*(sqrt(b)*x - sqrt(b*x^2 + a))^9*C*a^4*b^3 + 7045
5*(sqrt(b)*x - sqrt(b*x^2 + a))^9*A*a^3*b^4 - 94080*(sqrt(b)*x - sqrt(b*x^
2 + a))^8*D*a^5*b^(5/2) + 26880*(sqrt(b)*x - sqrt(b*x^2 + a))^8*B*a^4*b^(7
/2) - 8680*(sqrt(b)*x - sqrt(b*x^2 + a))^7*C*a^5*b^3 + 70455*(sqrt(b)*x -
sqrt(b*x^2 + a))^7*A*a^4*b^4 + 83328*(sqrt(b)*x - sqrt(b*x^2 + a))^6*D*a^6
*b^(5/2) - 43008*(sqrt(b)*x - sqrt(b*x^2 + a))^6*B*a^5*b^(7/2) - 3640*(sqr
t(b)*x - sqrt(b*x^2 + a))^5*C*a^6*b^3 + 34965*(sqrt(b)*x - sqrt(b*x^2 + a)
)^5*A*a^5*b^4 - 34944*(sqrt(b)*x - sqrt(b*x^2 + a))^4*D*a^7*b^(5/2) - 5376
*(sqrt(b)*x - sqrt(b*x^2 + a))^4*B*a^6*b^(7/2) + 11480*(sqrt(b)*x - sqrt(b
*x^2 + a))^3*C*a^7*b^3 + 2415*(sqrt(b)*x - sqrt(b*x^2 + a))^3*A*a^6*b^4 +
8064*(sqrt(b)*x - sqrt(b*x^2 + a))^2*D*a^8*b^(5/2) - 6144*(sqrt(b)*x - sqr
t(b*x^2 + a))^2*B*a^7*b^(7/2) + 840*(sqrt(b)*x - sqrt(b*x^2 + a))*C*a^8...
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{x^9}\,dx = \int \frac{(b\,x^2+a)^{3/2}\,(A+B\,x+C\,x^2+x^3\,D)}{x^9}\,dx$$

input
```
int(((a + b*x^2)^(3/2)*(A + B*x + C*x^2 + x^3*D))/x^9,x)
```

output
```
int(((a + b*x^2)^(3/2)*(A + B*x + C*x^2 + x^3*D))/x^9, x)
```

## Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 400, normalized size of antiderivative = 1.95

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{x^9}\,dx = \frac{-1680\sqrt{b\,x^2+a}\,a^4-2520\sqrt{b\,x^2+a}\,a^3b\,x^2-1920\sqrt{b\,x^2+a}}{}$$

input
```
int((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/x^9,x)
```

output
```
( - 1680*sqrt(a + b*x**2)*a**4 - 2520*sqrt(a + b*x**2)*a**3*b*x**2 - 1920*
sqrt(a + b*x**2)*a**3*b*x - 2240*sqrt(a + b*x**2)*a**3*c*x**2 - 2688*sqrt(
a + b*x**2)*a**3*d*x**3 - 210*sqrt(a + b*x**2)*a**2*b**2*x**4 - 3072*sqrt(
a + b*x**2)*a**2*b**2*x**3 - 3920*sqrt(a + b*x**2)*a**2*b*c*x**4 - 5376*sq
rt(a + b*x**2)*a**2*b*d*x**5 + 315*sqrt(a + b*x**2)*a*b**3*x**6 - 384*sqrt
(a + b*x**2)*a*b**3*x**5 - 840*sqrt(a + b*x**2)*a*b**2*c*x**6 - 2688*sqrt(
a + b*x**2)*a*b**2*d*x**7 + 768*sqrt(a + b*x**2)*b**4*x**7 + 315*sqrt(a)*l
og((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*b**4*x**8 - 840*sqrt(
a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*b**3*c*x**8 - 315
*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*b**4*x**8 +
 840*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*b**3*c*
x**8 - 672*sqrt(b)*a*b**2*d*x**8 - 768*sqrt(b)*b**4*x**8)/(13440*a**2*x**8
)
```

# 3.87 $\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{x^{10}}\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 233

$$\begin{aligned}\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{x^{10}}\,dx &= -\frac{(3bB+8aD)\sqrt{a+bx^2}}{48x^6} \\ &- \frac{b(3bB+56aD)\sqrt{a+bx^2}}{192ax^4} + \frac{b^2(3bB-8aD)\sqrt{a+bx^2}}{128a^2x^2} \\ &- \frac{B(a+bx^2)^{3/2}}{8x^8} - \frac{A(a+bx^2)^{5/2}}{9ax^9} + \frac{(4Ab-9aC)(a+bx^2)^{5/2}}{63a^2x^7} \\ &- \frac{2b(4Ab-9aC)(a+bx^2)^{5/2}}{315a^3x^5} - \frac{b^3(3bB-8aD)\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{128a^{5/2}}\end{aligned}$$

output
```
-1/48*(3*B*b+8*D*a)*(b*x^2+a)^(1/2)/x^6-1/192*b*(3*B*b+56*D*a)*(b*x^2+a)^(
1/2)/a/x^4+1/128*b^2*(3*B*b-8*D*a)*(b*x^2+a)^(1/2)/a^2/x^2-1/8*B*(b*x^2+a)
^(3/2)/x^8-1/9*A*(b*x^2+a)^(5/2)/a/x^9+1/63*(4*A*b-9*C*a)*(b*x^2+a)^(5/2)/
a^2/x^7-2/315*b*(4*A*b-9*C*a)*(b*x^2+a)^(5/2)/a^3/x^5-1/128*b^3*(3*B*b-8*D
*a)*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(5/2)
```

## Mathematica [A] (verified)

Time = 2.00 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.82

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{x^{10}}\,dx =$$
$$-\frac{\sqrt{a+bx^2}(1024Ab^4x^8+80a^4(56A+63Bx+72Cx^2+84Dx^3)-ab^3x^6(512A+9x(105B+256Cx))+6}{40320a^3x^9}$$
$$+\frac{b^3(3bB-8aD)\operatorname{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right)}{64a^{5/2}}$$

input
```
Integrate[((a + b*x^2)^(3/2)*(A + B*x + C*x^2 + D*x^3))/x^10,x]
```

output
```
-1/40320*(Sqrt[a + b*x^2]*(1024*A*b^4*x^8 + 80*a^4*(56*A + 63*B*x + 72*C*x
^2 + 84*D*x^3) - a*b^3*x^6*(512*A + 9*x*(105*B + 256*C*x)) + 6*a^2*b^2*x^4
*(64*A + 3*x*(35*B + 64*C*x + 140*D*x^2)) + 8*a^3*b*x^2*(800*A + 3*x*(315*
B + 384*C*x + 490*D*x^2))))/(a^3*x^9) + (b^3*(3*b*B - 8*a*D)*ArcTanh[(Sqrt
[b]*x - Sqrt[a + b*x^2])/Sqrt[a]])/(64*a^(5/2))
```

## Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.01, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {2338, 25, 2338, 27, 539, 25, 539, 27, 534, 243, 51, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{x^{10}}dx$$

$$\Big\downarrow \text{2338}$$

$$-\frac{\int -\frac{(bx^2+a)^{3/2}(9aDx^2-(4Ab-9aC)x+9aB)}{x^9}dx}{9a}-\frac{A(a+bx^2)^{5/2}}{9ax^9}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{(bx^2+a)^{3/2}(9aDx^2-(4Ab-9aC)x+9aB)}{x^9}dx}{9a}-\frac{A(a+bx^2)^{5/2}}{9ax^9}$$

$\Big\downarrow$ 2338

$$\frac{-\frac{\int \frac{a(8(4Ab-9aC)+9(3bB-8aD)x)\left(bx^2+a\right)^{3/2}}{x^8}dx}{8a}-\frac{9B(a+bx^2)^{5/2}}{8x^8}}{9a}-\frac{A(a+bx^2)^{5/2}}{9ax^9}$$

$\Big\downarrow$ 27

$$\frac{-\frac{1}{8}\int \frac{(8(4Ab-9aC)+9(3bB-8aD)x)(bx^2+a)^{3/2}}{x^8}dx-\frac{9B(a+bx^2)^{5/2}}{8x^8}}{9a}-\frac{A(a+bx^2)^{5/2}}{9ax^9}$$

$\Big\downarrow$ 539

$$\frac{\frac{1}{8}\left(\frac{\int -\frac{(63a(3bB-8aD)-16b(4Ab-9aC)x)\left(bx^2+a\right)^{3/2}}{x^7}dx}{7a}+\frac{8(a+bx^2)^{5/2}(4Ab-9aC)}{7ax^7}\right)-\frac{9B(a+bx^2)^{5/2}}{8x^8}}{9a}-\frac{A(a+bx^2)^{5/2}}{9ax^9}$$

$\Big\downarrow$ 25

$$\frac{\frac{1}{8}\left(\frac{8(a+bx^2)^{5/2}(4Ab-9aC)}{7ax^7}-\frac{\int \frac{(63a(3bB-8aD)-16b(4Ab-9aC)x)\left(bx^2+a\right)^{3/2}}{x^7}dx}{7a}\right)-\frac{9B(a+bx^2)^{5/2}}{8x^8}}{9a}-\frac{A(a+bx^2)^{5/2}}{9ax^9}$$

$\Big\downarrow$ 539

$$\frac{\frac{1}{8}\left(\frac{8(a+bx^2)^{5/2}(4Ab-9aC)}{7ax^7}-\frac{-\frac{\int \frac{3ab(32(4Ab-9aC)+21(3bB-8aD)x)\left(bx^2+a\right)^{3/2}}{x^6}dx}{6a}-\frac{21\left(a+bx^2\right)^{5/2}(3bB-8aD)}{2x^6}}{7a}\right)-\frac{9B(a+bx^2)^{5/2}}{8x^8}}{9a}-\frac{A(a+bx^2)^{5/2}}{9ax^9}$$

$\Big\downarrow$ 27

$$\frac{\frac{1}{8}\left(\frac{8(a+bx^2)^{5/2}(4Ab-9aC)}{7ax^7}-\frac{-\frac{1}{2}b\int\frac{(32(4Ab-9aC)+21(3bB-8aD)x)\left(bx^2+a\right)^{3/2}}{x^6}dx-\frac{21\left(a+bx^2\right)^{5/2}(3bB-8aD)}{2x^6}}{7a}\right)-\frac{9B(a+bx^2)^{5/2}}{8x^8}}{9a}-\frac{A\left(a+bx^2\right)^{5/2}}{9ax^9}$$

$\Big\downarrow$ 534

$$\frac{\frac{1}{8}\left(\frac{8(a+bx^2)^{5/2}(4Ab-9aC)}{7ax^7}-\frac{-\frac{1}{2}b\left(21(3bB-8aD)\int\frac{\left(bx^2+a\right)^{3/2}}{x^5}dx-\frac{32\left(a+bx^2\right)^{5/2}(4Ab-9aC)}{5ax^5}\right)-\frac{21\left(a+bx^2\right)^{5/2}(3bB-8aD)}{2x^6}}{7a}\right)-\frac{9B(a+bx^2)^{5/2}}{8x^8}}{9a}-\frac{A\left(a+bx^2\right)^{5/2}}{9ax^9}$$

$\Big\downarrow$ 243

$$\frac{\frac{1}{8}\left(\frac{8(a+bx^2)^{5/2}(4Ab-9aC)}{7ax^7}-\frac{-\frac{1}{2}b\left(\frac{21}{2}(3bB-8aD)\int\frac{\left(bx^2+a\right)^{3/2}}{x^6}dx^2-\frac{32\left(a+bx^2\right)^{5/2}(4Ab-9aC)}{5ax^5}\right)-\frac{21\left(a+bx^2\right)^{5/2}(3bB-8aD)}{2x^6}}{7a}\right)-\frac{9B(a+bx^2)^{5/2}}{8x^8}}{9a}-\frac{A\left(a+bx^2\right)^{5/2}}{9ax^9}$$

$\Big\downarrow$ 51

$$\frac{\frac{1}{8}\left(\frac{8(a+bx^2)^{5/2}(4Ab-9aC)}{7ax^7}-\frac{-\frac{1}{2}b\left(\frac{21}{2}(3bB-8aD)\left(\frac{3}{4}b\int\frac{\sqrt{bx^2+a}}{x^4}dx^2-\frac{\left(a+bx^2\right)^{3/2}}{2x^4}\right)-\frac{32\left(a+bx^2\right)^{5/2}(4Ab-9aC)}{5ax^5}\right)-\frac{21\left(a+bx^2\right)^{5/2}(3bB-8aD)}{2x^6}}{7a}\right)-\frac{9B(a+bx^2)^{5/2}}{8x^8}}{9a}-\frac{A\left(a+bx^2\right)^{5/2}}{9ax^9}$$

$\Big\downarrow$ 51

$$\frac{\frac{1}{8}\left(\frac{8(a+bx^2)^{5/2}(4Ab-9aC)}{7ax^7}-\frac{-\frac{1}{2}b\left(\frac{21}{2}(3bB-8aD)\left(\frac{3}{4}b\left(\frac{1}{2}b\int\frac{1}{x^2\sqrt{bx^2+a}}dx^2-\frac{\sqrt{a+bx^2}}{x^2}\right)-\frac{(a+bx^2)^{3/2}}{2x^4}\right)-\frac{32(a+bx^2)^{5/2}(4Ab-9aC)}{5ax^5}\right)-\frac{21(a}{}}{7a}\right.}{9a}$$

$$\frac{A(a+bx^2)^{5/2}}{9ax^9}$$

$\downarrow$ 73

$$\frac{\frac{1}{8}\left(\frac{8(a+bx^2)^{5/2}(4Ab-9aC)}{7ax^7}-\frac{-\frac{1}{2}b\left(\frac{21}{2}(3bB-8aD)\left(\frac{3}{4}b\left(\int\frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}-\frac{\sqrt{a+bx^2}}{x^2}\right)-\frac{(a+bx^2)^{3/2}}{2x^4}\right)-\frac{32(a+bx^2)^{5/2}(4Ab-9aC)}{5ax^5}\right)-\frac{21(a}{}}{7a}\right.}{9a}$$

$$\frac{A(a+bx^2)^{5/2}}{9ax^9}$$

$\downarrow$ 221

$$\frac{\frac{1}{8}\left(\frac{8(a+bx^2)^{5/2}(4Ab-9aC)}{7ax^7}-\frac{-\frac{1}{2}b\left(\frac{21}{2}\left(\frac{3}{4}b\left(-\frac{b\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\sqrt{a+bx^2}}{x^2}\right)-\frac{(a+bx^2)^{3/2}}{2x^4}\right)(3bB-8aD)-\frac{32(a+bx^2)^{5/2}(4Ab-9aC)}{5ax^5}\right)}{7a}\right.}{9a}$$

$$\frac{A(a+bx^2)^{5/2}}{9ax^9}$$

input
```
Int[((a + b*x^2)^(3/2)*(A + B*x + C*x^2 + D*x^3))/x^10,x]
```

output
```
-1/9*(A*(a + b*x^2)^(5/2))/(a*x^9) + ((-9*B*(a + b*x^2)^(5/2))/(8*x^8) + (
(8*(4*A*b - 9*a*C)*(a + b*x^2)^(5/2))/(7*a*x^7) - ((-21*(3*b*B - 8*a*D)*(a
 + b*x^2)^(5/2))/(2*x^6) - (b*((-32*(4*A*b - 9*a*C)*(a + b*x^2)^(5/2))/(5*
a*x^5) + (21*(3*b*B - 8*a*D)*(-1/2*(a + b*x^2)^(3/2)/x^4 + (3*b*(-(Sqrt[a
+ b*x^2]/x^2) - (b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a]))/4))/2))/2)/
(7*a))/8)/(9*a)
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 539
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[c*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))
   Int[x^(m + 1)*(a + b*x^2)^p*(a*d*(m + 1) - b*c*(m + 2*p + 3)*x), x], x]
/; FreeQ[{a, b, c, d, p}, x] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

## Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.62

| method | result |
|---|---|
| default | $A\left(-\frac{(b\,x^{2}+a)^{\frac{5}{2}}}{9a\,x^{9}}-\frac{4b\left(-\frac{(b\,x^{2}+a)^{\frac{5}{2}}}{7a\,x^{7}}+\frac{2b(b\,x^{2}+a)^{\frac{5}{2}}}{35a^{2}x^{5}}\right)}{9a}\right)+B\left(-\frac{(b\,x^{2}+a)^{\frac{5}{2}}}{8a\,x^{8}}-\frac{3b\left(-\frac{(b\,x^{2}+a)^{\frac{5}{2}}}{6a\,x^{6}}-\frac{b\left(-\frac{(b\,x^{2}+a)^{\frac{5}{2}}}{4a\,x^{4}}+\frac{b\left(-\right.}{}\right.}{}\right.}{}\right.$ |

input
```
int((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/x^10,x,method=_RETURNVERBOSE)
```

output
```
A*(-1/9/a/x^9*(b*x^2+a)^(5/2)-4/9*b/a*(-1/7/a/x^7*(b*x^2+a)^(5/2)+2/35*b/a
^2/x^5*(b*x^2+a)^(5/2)))+B*(-1/8/a/x^8*(b*x^2+a)^(5/2)-3/8*b/a*(-1/6/a/x^6
*(b*x^2+a)^(5/2)-1/6*b/a*(-1/4/a/x^4*(b*x^2+a)^(5/2)+1/4*b/a*(-1/2/a/x^2*(
b*x^2+a)^(5/2)+3/2*b/a*(1/3*(b*x^2+a)^(3/2)+a*((b*x^2+a)^(1/2)-a^(1/2)*ln(
(2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)))))))+C*(-1/7/a/x^7*(b*x^2+a)^(5/2)+2/3
5*b/a^2/x^5*(b*x^2+a)^(5/2))+D*(-1/6/a/x^6*(b*x^2+a)^(5/2)-1/6*b/a*(-1/4/a
/x^4*(b*x^2+a)^(5/2)+1/4*b/a*(-1/2/a/x^2*(b*x^2+a)^(5/2)+3/2*b/a*(1/3*(b*x
^2+a)^(3/2)+a*((b*x^2+a)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/
x)))))
```

## Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 451, normalized size of antiderivative = 1.94

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{x^{10}}\,dx = \left[-\frac{315\,(8\,Dab^3-3\,Bb^4)\sqrt{a}x^9\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)-2\,\cdots}{\cdots}\right.$$

$$-\frac{315\,(8\,Dab^3-3\,Bb^4)\sqrt{-a}x^9\arctan\left(\frac{\sqrt{bx^2+a}\sqrt{-a}}{a}\right)-(256\,(9\,Cab^3-4\,Ab^4)x^8-315\,(8\,Da^2b^2-3\,Bab^3)x^7\cdots}{\cdots}$$

input
```
integrate((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/x^10,x, algorithm="fricas")
```

output
```
[-1/80640*(315*(8*D*a*b^3 - 3*B*b^4)*sqrt(a)*x^9*log(-(b*x^2 - 2*sqrt(b*x^
2 + a)*sqrt(a) + 2*a)/x^2) - 2*(256*(9*C*a*b^3 - 4*A*b^4)*x^8 - 315*(8*D*a
^2*b^2 - 3*B*a*b^3)*x^7 - 128*(9*C*a^2*b^2 - 4*A*a*b^3)*x^6 - 5040*B*a^4*x
 - 210*(56*D*a^3*b + 3*B*a^2*b^2)*x^5 - 4480*A*a^4 - 384*(24*C*a^3*b + A*a
^2*b^2)*x^4 - 840*(8*D*a^4 + 9*B*a^3*b)*x^3 - 640*(9*C*a^4 + 10*A*a^3*b)*x
^2)*sqrt(b*x^2 + a))/(a^3*x^9), -1/40320*(315*(8*D*a*b^3 - 3*B*b^4)*sqrt(-
a)*x^9*arctan(sqrt(b*x^2 + a)*sqrt(-a)/a) - (256*(9*C*a*b^3 - 4*A*b^4)*x^8
 - 315*(8*D*a^2*b^2 - 3*B*a*b^3)*x^7 - 128*(9*C*a^2*b^2 - 4*A*a*b^3)*x^6 -
 5040*B*a^4*x - 210*(56*D*a^3*b + 3*B*a^2*b^2)*x^5 - 4480*A*a^4 - 384*(24*
C*a^3*b + A*a^2*b^2)*x^4 - 840*(8*D*a^4 + 9*B*a^3*b)*x^3 - 640*(9*C*a^4 +
10*A*a^3*b)*x^2)*sqrt(b*x^2 + a))/(a^3*x^9)]
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1697 vs. $2(219) = 438$.

Time = 39.19 (sec) , antiderivative size = 1697, normalized size of antiderivative = 7.28

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{x^{10}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x**2+a)**(3/2)*(D*x**3+C*x**2+B*x+A)/x**10,x)
```

output
```
-35*A*a**8*b**(19/2)*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b
**10*x**10 + 945*a**5*b**11*x**12 + 315*a**4*b**12*x**14) - 110*A*a**7*b**
(21/2)*x**2*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**1
0 + 945*a**5*b**11*x**12 + 315*a**4*b**12*x**14) - 114*A*a**6*b**(23/2)*x*
*4*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**10 + 945*a
**5*b**11*x**12 + 315*a**4*b**12*x**14) - 40*A*a**5*b**(25/2)*x**6*sqrt(a/
(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**10 + 945*a**5*b**11*
x**12 + 315*a**4*b**12*x**14) - 15*A*a**5*b**(11/2)*sqrt(a/(b*x**2) + 1)/(
105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) + 5*A*a**4*
b**(27/2)*x**8*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x
**10 + 945*a**5*b**11*x**12 + 315*a**4*b**12*x**14) - 33*A*a**4*b**(13/2)*
x**2*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a
**3*b**6*x**10) + 30*A*a**3*b**(29/2)*x**10*sqrt(a/(b*x**2) + 1)/(315*a**7
*b**9*x**8 + 945*a**6*b**10*x**10 + 945*a**5*b**11*x**12 + 315*a**4*b**12*
x**14) - 17*A*a**3*b**(15/2)*x**4*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6
 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) + 40*A*a**2*b**(31/2)*x**12*s
qrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**10 + 945*a**5*
b**11*x**12 + 315*a**4*b**12*x**14) - 3*A*a**2*b**(17/2)*x**6*sqrt(a/(b*x*
*2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) +
 16*A*a*b**(33/2)*x**14*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*...
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.49

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{x^{10}}\,dx = \frac{Db^3\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{16\,a^{\frac{3}{2}}} - \frac{3\,Bb^4\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{128\,a^{\frac{5}{2}}} - \frac{(bx^2+a)^{\frac{3}{2}}Db^3}{48\,a^3} - \frac{\sqrt{bx^2+a}Db^3}{16\,a^2} + \frac{(bx^2+a)^{\frac{3}{2}}Bb^4}{128\,a^4} + \frac{3\sqrt{bx^2+a}Bb^4}{128\,a^3} + \frac{(bx^2+a)^{\frac{5}{2}}Db^2}{48\,a^3x^2} - \frac{(bx^2+a)^{\frac{5}{2}}Bb^3}{128\,a^4x^2} + \frac{(bx^2+a)^{\frac{5}{2}}Db}{24\,a^2x^4} - \frac{(bx^2+a)^{\frac{5}{2}}Bb^2}{64\,a^3x^4} + \frac{2\,(bx^2+a)^{\frac{5}{2}}Cb}{35\,a^2x^5} - \frac{8\,(bx^2+a)^{\frac{5}{2}}Ab^2}{315\,a^3x^5} - \frac{(bx^2+a)^{\frac{5}{2}}D}{6\,ax^6} + \frac{(bx^2+a)^{\frac{5}{2}}Bb}{16\,a^2x^6} - \frac{(bx^2+a)^{\frac{5}{2}}C}{7\,ax^7} + \frac{4\,(bx^2+a)^{\frac{5}{2}}Ab}{63\,a^2x^7} - \frac{(bx^2+a)^{\frac{5}{2}}B}{8\,ax^8} - \frac{(bx^2+a)^{\frac{5}{2}}A}{9\,ax^9}$$

input
```
integrate((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/x^10,x, algorithm="maxima")
```

output
```
1/16*D*b^3*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2) - 3/128*B*b^4*arcsinh(a/(
sqrt(a*b)*abs(x)))/a^(5/2) - 1/48*(b*x^2 + a)^(3/2)*D*b^3/a^3 - 1/16*sqrt(
b*x^2 + a)*D*b^3/a^2 + 1/128*(b*x^2 + a)^(3/2)*B*b^4/a^4 + 3/128*sqrt(b*x^
2 + a)*B*b^4/a^3 + 1/48*(b*x^2 + a)^(5/2)*D*b^2/(a^3*x^2) - 1/128*(b*x^2 +
 a)^(5/2)*B*b^3/(a^4*x^2) + 1/24*(b*x^2 + a)^(5/2)*D*b/(a^2*x^4) - 1/64*(b
*x^2 + a)^(5/2)*B*b^2/(a^3*x^4) + 2/35*(b*x^2 + a)^(5/2)*C*b/(a^2*x^5) - 8
/315*(b*x^2 + a)^(5/2)*A*b^2/(a^3*x^5) - 1/6*(b*x^2 + a)^(5/2)*D/(a*x^6) +
 1/16*(b*x^2 + a)^(5/2)*B*b/(a^2*x^6) - 1/7*(b*x^2 + a)^(5/2)*C/(a*x^7) +
4/63*(b*x^2 + a)^(5/2)*A*b/(a^2*x^7) - 1/8*(b*x^2 + a)^(5/2)*B/(a*x^8) - 1
/9*(b*x^2 + a)^(5/2)*A/(a*x^9)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 900 vs. $2(197) = 394$.

Time = 0.14 (sec) , antiderivative size = 900, normalized size of antiderivative = 3.86

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{x^{10}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/x^10,x, algorithm="giac")
```

output
```
-1/64*(8*D*a*b^3 - 3*B*b^4)*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a)
)/(sqrt(-a)*a^2) + 1/20160*(2520*(sqrt(b)*x - sqrt(b*x^2 + a))^17*D*a*b^3
- 945*(sqrt(b)*x - sqrt(b*x^2 + a))^17*B*b^4 + 31920*(sqrt(b)*x - sqrt(b*x
^2 + a))^15*D*a^2*b^3 + 8190*(sqrt(b)*x - sqrt(b*x^2 + a))^15*B*a*b^4 + 80
640*(sqrt(b)*x - sqrt(b*x^2 + a))^14*C*a^2*b^(7/2) - 45360*(sqrt(b)*x - sq
rt(b*x^2 + a))^13*D*a^3*b^3 + 97650*(sqrt(b)*x - sqrt(b*x^2 + a))^13*B*a^2
*b^4 - 80640*(sqrt(b)*x - sqrt(b*x^2 + a))^12*C*a^3*b^(7/2) + 215040*(sqrt
(b)*x - sqrt(b*x^2 + a))^12*A*a^2*b^(9/2) - 15120*(sqrt(b)*x - sqrt(b*x^2
+ a))^11*D*a^4*b^3 + 106470*(sqrt(b)*x - sqrt(b*x^2 + a))^11*B*a^3*b^4 + 8
0640*(sqrt(b)*x - sqrt(b*x^2 + a))^10*C*a^4*b^(7/2) + 322560*(sqrt(b)*x -
sqrt(b*x^2 + a))^10*A*a^3*b^(9/2) - 209664*(sqrt(b)*x - sqrt(b*x^2 + a))^8
*C*a^5*b^(7/2) + 451584*(sqrt(b)*x - sqrt(b*x^2 + a))^8*A*a^4*b^(9/2) + 15
120*(sqrt(b)*x - sqrt(b*x^2 + a))^7*D*a^6*b^3 - 106470*(sqrt(b)*x - sqrt(b
*x^2 + a))^7*B*a^5*b^4 + 112896*(sqrt(b)*x - sqrt(b*x^2 + a))^6*C*a^6*b^(7
/2) + 129024*(sqrt(b)*x - sqrt(b*x^2 + a))^6*A*a^5*b^(9/2) + 45360*(sqrt(b
)*x - sqrt(b*x^2 + a))^5*D*a^7*b^3 - 97650*(sqrt(b)*x - sqrt(b*x^2 + a))^5
*B*a^6*b^4 - 2304*(sqrt(b)*x - sqrt(b*x^2 + a))^4*C*a^7*b^(7/2) + 36864*(s
qrt(b)*x - sqrt(b*x^2 + a))^4*A*a^6*b^(9/2) - 31920*(sqrt(b)*x - sqrt(b*x^
2 + a))^3*D*a^8*b^3 - 8190*(sqrt(b)*x - sqrt(b*x^2 + a))^3*B*a^7*b^4 + 207
36*(sqrt(b)*x - sqrt(b*x^2 + a))^2*C*a^8*b^(7/2) - 9216*(sqrt(b)*x - sq...
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{x^{10}}\,dx=\int \frac{(b\,x^2+a)^{3/2}\,(A+B\,x+C\,x^2+x^3\,D)}{x^{10}}\,dx$$

input
```
int(((a + b*x^2)^(3/2)*(A + B*x + C*x^2 + x^3*D))/x^10,x)
```

output
```
int(((a + b*x^2)^(3/2)*(A + B*x + C*x^2 + x^3*D))/x^10, x)
```

## Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 447, normalized size of antiderivative = 1.92

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{x^{10}}\,dx = \frac{-4480\sqrt{b\,x^2+a}\,a^5-6400\sqrt{b\,x^2+a}\,a^4b\,x^2-5040\sqrt{b\,x^2+a}}{}$$

input
```
int((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/x^10,x)
```

output
```
( - 4480*sqrt(a + b*x**2)*a**5 - 6400*sqrt(a + b*x**2)*a**4*b*x**2 - 5040*
sqrt(a + b*x**2)*a**4*b*x - 5760*sqrt(a + b*x**2)*a**4*c*x**2 - 6720*sqrt(
a + b*x**2)*a**4*d*x**3 - 384*sqrt(a + b*x**2)*a**3*b**2*x**4 - 7560*sqrt(
a + b*x**2)*a**3*b**2*x**3 - 9216*sqrt(a + b*x**2)*a**3*b*c*x**4 - 11760*s
qrt(a + b*x**2)*a**3*b*d*x**5 + 512*sqrt(a + b*x**2)*a**2*b**3*x**6 - 630*
sqrt(a + b*x**2)*a**2*b**3*x**5 - 1152*sqrt(a + b*x**2)*a**2*b**2*c*x**6 -
 2520*sqrt(a + b*x**2)*a**2*b**2*d*x**7 - 1024*sqrt(a + b*x**2)*a*b**4*x**
8 + 945*sqrt(a + b*x**2)*a*b**4*x**7 + 2304*sqrt(a + b*x**2)*a*b**3*c*x**8
 - 2520*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b*
*3*d*x**9 + 945*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(
a))*b**5*x**9 + 2520*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/
sqrt(a))*a*b**3*d*x**9 - 945*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqr
t(b)*x)/sqrt(a))*b**5*x**9 + 1024*sqrt(b)*a*b**4*x**9 - 2304*sqrt(b)*a*b**
3*c*x**9)/(40320*a**3*x**9)
```

# 3.88 $\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{x^{11}}\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 262

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{x^{11}}\,dx = -\frac{(3Ab+10aC)\sqrt{a+bx^2}}{80x^8} - \frac{b(Ab+30aC)\sqrt{a+bx^2}}{160ax^6} + \frac{b^2(Ab-2aC)\sqrt{a+bx^2}}{128a^2x^4} - \frac{3b^3(Ab-2aC)\sqrt{a+bx^2}}{256a^3x^2} - \frac{A(a+bx^2)^{3/2}}{10x^{10}} - \frac{B(a+bx^2)^{5/2}}{9ax^9} + \frac{(4bB-9aD)(a+bx^2)^{5/2}}{63a^2x^7} - \frac{2b(4bB-9aD)(a+bx^2)^{5/2}}{315a^3x^5} + \frac{3b^4(Ab-2aC)\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{256a^{7/2}}$$

output
```
-1/80*(3*A*b+10*C*a)*(b*x^2+a)^(1/2)/x^8-1/160*b*(A*b+30*C*a)*(b*x^2+a)^(1
/2)/a/x^6+1/128*b^2*(A*b-2*C*a)*(b*x^2+a)^(1/2)/a^2/x^4-3/256*b^3*(A*b-2*C
*a)*(b*x^2+a)^(1/2)/a^3/x^2-1/10*A*(b*x^2+a)^(3/2)/x^10-1/9*B*(b*x^2+a)^(5
/2)/a/x^9+1/63*(4*B*b-9*D*a)*(b*x^2+a)^(5/2)/a^2/x^7-2/315*b*(4*B*b-9*D*a)
*(b*x^2+a)^(5/2)/a^3/x^5+3/256*b^4*(A*b-2*C*a)*arctanh((b*x^2+a)^(1/2)/a^(
1/2))/a^(7/2)
```

## Mathematica [A] (verified)

Time = 2.72 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.79

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{x^{11}}\,dx =$$
$$-\frac{\sqrt{a+bx^2}(b^4x^8(945A+2048Bx)+32a^4(252A+5x(56B+9x(7C+8Dx)))+12a^2b^2x^4(42A+x(64B\ldots}{}$$
$$+\frac{3b^4(-Ab+2aC)\text{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right)}{128a^{7/2}}$$

input
```
Integrate[((a + b*x^2)^(3/2)*(A + B*x + C*x^2 + D*x^3))/x^11,x]
```

output
```
-1/80640*(Sqrt[a + b*x^2]*(b^4*x^8*(945*A + 2048*B*x) + 32*a^4*(252*A + 5*
x*(56*B + 9*x*(7*C + 8*D*x))) + 12*a^2*b^2*x^4*(42*A + x*(64*B + 3*x*(35*C
 + 64*D*x))) + 16*a^3*b*x^2*(693*A + x*(800*B + 9*x*(105*C + 128*D*x))) -
2*a*b^3*x^6*(315*A + x*(512*B + 9*x*(105*C + 256*D*x)))))/(a^3*x^10) + (3*
b^4*(-(A*b) + 2*a*C)*ArcTanh[(Sqrt[b]*x - Sqrt[a + b*x^2])/Sqrt[a]])/(128*
a^(7/2))
```

## Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.03, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {2338, 27, 2338, 27, 539, 25, 539, 27, 539, 27, 534, 243, 51, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{x^{11}}dx$$

$$\Big\downarrow \text{2338}$$

$$-\frac{\int -\frac{5(bx^2+a)^{3/2}(2aDx^2-(Ab-2aC)x+2aB)}{x^{10}}dx}{10a}-\frac{A(a+bx^2)^{5/2}}{10ax^{10}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(bx^2+a)^{3/2}(2aDx^2-(Ab-2aC)x+2aB)}{x^{10}}dx}{2a}-\frac{A(a+bx^2)^{5/2}}{10ax^{10}}$$

$\Big\downarrow$ 2338

$$\frac{-\frac{\int \frac{a(9(Ab-2aC)+2(4bB-9aD)x)(bx^2+a)^{3/2}}{x^9}dx}{9a}-\frac{2B(a+bx^2)^{5/2}}{9x^9}}{2a}-\frac{A(a+bx^2)^{5/2}}{10ax^{10}}$$

$\Big\downarrow$ 27

$$\frac{-\frac{1}{9}\int \frac{(9(Ab-2aC)+2(4bB-9aD)x)(bx^2+a)^{3/2}}{x^9}dx-\frac{2B(a+bx^2)^{5/2}}{9x^9}}{2a}-\frac{A(a+bx^2)^{5/2}}{10ax^{10}}$$

$\Big\downarrow$ 539

$$\frac{\frac{1}{9}\left(\frac{\int -\frac{(16a(4bB-9aD)-27b(Ab-2aC)x)(bx^2+a)^{3/2}}{x^8}dx}{8a}+\frac{9(a+bx^2)^{5/2}(Ab-2aC)}{8ax^8}\right)-\frac{2B(a+bx^2)^{5/2}}{9x^9}}{2a}-\frac{A(a+bx^2)^{5/2}}{10ax^{10}}$$

$\Big\downarrow$ 25

$$\frac{\frac{1}{9}\left(\frac{9(a+bx^2)^{5/2}(Ab-2aC)}{8ax^8}-\frac{\int \frac{(16a(4bB-9aD)-27b(Ab-2aC)x)(bx^2+a)^{3/2}}{x^8}dx}{8a}\right)-\frac{2B(a+bx^2)^{5/2}}{9x^9}}{2a}-\frac{A(a+bx^2)^{5/2}}{10ax^{10}}$$

$\Big\downarrow$ 539

$$\frac{\frac{1}{9}\left(\frac{9(a+bx^2)^{5/2}(Ab-2aC)}{8ax^8}-\frac{-\frac{\int \frac{ab(189(Ab-2aC)+32(4bB-9aD)x)(bx^2+a)^{3/2}}{x^7}dx}{7a}-\frac{16(a+bx^2)^{5/2}(4bB-9aD)}{7x^7}}{8a}\right)-\frac{2B(a+bx^2)^{5/2}}{9x^9}}{2a}-\frac{A(a+bx^2)^{5/2}}{10ax^{10}}$$

$\Big\downarrow$ 27

$$\frac{\frac{1}{9}\left(\frac{9(a+bx^2)^{5/2}(Ab-2aC)}{8ax^8}-\frac{-\frac{1}{7}b\int\frac{(189(Ab-2aC)+32(4bB-9aD)x)\left(bx^2+a\right)^{3/2}}{x^7}dx-\frac{16\left(a+bx^2\right)^{5/2}(4bB-9aD)}{7x^7}}{8a}\right)-\frac{2B(a+bx^2)^{5/2}}{9x^9}}{2a}-\frac{A\left(a+bx^2\right)^{5/2}}{10ax^{10}}$$

$\Big\downarrow$ 539

$$\frac{\frac{1}{9}\left(\frac{9(a+bx^2)^{5/2}(Ab-2aC)}{8ax^8}-\frac{-\frac{1}{7}b\left(-\frac{\int-\frac{3(64a(4bB-9aD)-63b(Ab-2aC)x)\left(bx^2+a\right)^{3/2}}{x^6}dx}{6a}-\frac{63\left(a+bx^2\right)^{5/2}(Ab-2aC)}{2ax^6}\right)-\frac{16\left(a+bx^2\right)^{5/2}(4bB-9aD)}{7x^7}}{8a}\right.}{2a}$$

$$\frac{A\left(a+bx^2\right)^{5/2}}{10ax^{10}}$$

$\Big\downarrow$ 27

$$\frac{\frac{1}{9}\left(\frac{9(a+bx^2)^{5/2}(Ab-2aC)}{8ax^8}-\frac{-\frac{1}{7}b\left(\frac{\int\frac{(64a(4bB-9aD)-63b(Ab-2aC)x)\left(bx^2+a\right)^{3/2}}{x^6}dx}{2a}-\frac{63\left(a+bx^2\right)^{5/2}(Ab-2aC)}{2ax^6}\right)-\frac{16\left(a+bx^2\right)^{5/2}(4bB-9aD)}{7x^7}}{8a}\right)}{2a}$$

$$\frac{A\left(a+bx^2\right)^{5/2}}{10ax^{10}}$$

$\Big\downarrow$ 534

$$\frac{\frac{1}{9}\left(\frac{9(a+bx^2)^{5/2}(Ab-2aC)}{8ax^8}-\frac{-\frac{1}{7}b\left(\frac{-63b(Ab-2aC)\int\frac{\left(bx^2+a\right)^{3/2}}{x^5}dx-\frac{64\left(a+bx^2\right)^{5/2}(4bB-9aD)}{5x^5}}{2a}-\frac{63\left(a+bx^2\right)^{5/2}(Ab-2aC)}{2ax^6}\right)-\frac{16\left(a+bx^2\right)^{5/2}(4bB}{7x^7}}{8a}\right.}{2a}$$

$$\frac{A\left(a+bx^2\right)^{5/2}}{10ax^{10}}$$

$\Big\downarrow$ 243

$$\frac{\frac{1}{9}\left(\frac{9\left(a+bx^2\right)^{5/2}(Ab-2aC)}{8ax^8}-\frac{-\frac{1}{7}b\left(\frac{-\frac{63}{2}b(Ab-2aC)\int\frac{\left(bx^2+a\right)^{3/2}}{x^6}dx^2-\frac{64\left(a+bx^2\right)^{5/2}(4bB-9aD)}{5x^5}}{2a}-\frac{63\left(a+bx^2\right)^{5/2}(Ab-2aC)}{2ax^6}\right)-\frac{16\left(a+bx^2\right)^{5/2}(4b\cdots}{7x^7}\cdots}{8a}\cdots\right.}{2a}\cdots$$

$$\frac{A\left(a+bx^2\right)^{5/2}}{10ax^{10}}$$

$\Big\downarrow$ 51

$$\frac{\frac{1}{9}\left(\frac{9\left(a+bx^2\right)^{5/2}(Ab-2aC)}{8ax^8}-\frac{-\frac{1}{7}b\left(\frac{-\frac{63}{2}b(Ab-2aC)\left(\frac{3}{4}b\int\frac{\sqrt{bx^2+a}}{x^4}dx^2-\frac{\left(a+bx^2\right)^{3/2}}{2x^4}\right)-\frac{64\left(a+bx^2\right)^{5/2}(4bB-9aD)}{5x^5}}{2a}-\frac{63\left(a+bx^2\right)^{5/2}(Ab-2aC)}{2ax^6}\right)-\cdots}{8a}\cdots\right.}{2a}\cdots$$

$$\frac{A\left(a+bx^2\right)^{5/2}}{10ax^{10}}$$

$\Big\downarrow$ 51

$$\frac{\frac{1}{9}\left(\frac{9\left(a+bx^2\right)^{5/2}(Ab-2aC)}{8ax^8}-\frac{-\frac{1}{7}b\left(\frac{-\frac{63}{2}b(Ab-2aC)\left(\frac{3}{4}b\left(\frac{1}{2}b\int\frac{1}{x^2\sqrt{bx^2+a}}dx^2-\frac{\sqrt{a+bx^2}}{x^2}\right)-\frac{\left(a+bx^2\right)^{3/2}}{2x^4}\right)-\frac{64\left(a+bx^2\right)^{5/2}(4bB-9aD)}{5x^5}}{2a}-\frac{63\left(a+bx^2\right)\cdots}{2\cdots}\cdots\right)\cdots}{8a}\cdots\right.}{2a}\cdots$$

$$\frac{A\left(a+bx^2\right)^{5/2}}{10ax^{10}}$$

$\Big\downarrow$ 73

$$\frac{\frac{1}{9}\left(\frac{9\left(a+bx^2\right)^{5/2}(Ab-2aC)}{8ax^8}-\frac{-\frac{1}{7}b\left(\frac{-\frac{63}{2}b(Ab-2aC)\left(\frac{3}{4}b\left(\int\frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}-\frac{\sqrt{a+bx^2}}{x^2}\right)-\frac{\left(a+bx^2\right)^{3/2}}{2x^4}\right)-\frac{64\left(a+bx^2\right)^{5/2}(4bB-9aD)}{5x^5}}{2a}-\frac{63\left(a+bx^2\right)}{2}\cdots}{8a}\cdots}{2a}$$

$$\frac{A\left(a+bx^2\right)^{5/2}}{10ax^{10}}$$

$$\Big\downarrow \ 221$$

$$\frac{\frac{1}{9}\left(\frac{9\left(a+bx^2\right)^{5/2}(Ab-2aC)}{8ax^8}-\frac{-\frac{1}{7}b\left(\frac{-\frac{63}{2}b(Ab-2aC)\left(\frac{3}{4}b\left(-\frac{b\,\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\sqrt{a+bx^2}}{x^2}\right)-\frac{\left(a+bx^2\right)^{3/2}}{2x^4}\right)-\frac{64\left(a+bx^2\right)^{5/2}(4bB-9aD)}{5x^5}}{2a}-\frac{63(\cdots}{\cdots}\cdots}{8a}\cdots}{2a}$$

$$\frac{A\left(a+bx^2\right)^{5/2}}{10ax^{10}}$$

input
```
Int[((a + b*x^2)^(3/2)*(A + B*x + C*x^2 + D*x^3))/x^11,x]
```

output
```
-1/10*(A*(a + b*x^2)^(5/2))/(a*x^10) + ((-2*B*(a + b*x^2)^(5/2))/(9*x^9) +
 ((9*(A*b - 2*a*C)*(a + b*x^2)^(5/2))/(8*a*x^8) - ((-16*(4*b*B - 9*a*D)*(a
 + b*x^2)^(5/2))/(7*x^7) - (b*((-63*(A*b - 2*a*C)*(a + b*x^2)^(5/2))/(2*a*
x^6) + ((-64*(4*b*B - 9*a*D)*(a + b*x^2)^(5/2))/(5*x^5) - (63*b*(A*b - 2*a
*C)*(-1/2*(a + b*x^2)^(3/2)/x^4 + (3*b*(-(Sqrt[a + b*x^2]/x^2) - (b*ArcTan
h[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a]))/4))/2)/(2*a)))/7)/(8*a))/9)/(2*a)
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 539
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[c*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))
   Int[x^(m + 1)*(a + b*x^2)^p*(a*d*(m + 1) - b*c*(m + 2*p + 3)*x), x], x]
/; FreeQ[{a, b, c, d, p}, x] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

## Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 426, normalized size of antiderivative = 1.63

| method | result |
|---|---|
| default | $A\left(-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{10a\,x^{10}}-\frac{b\left(-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{8a\,x^{8}}-\frac{3b\left(-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{6a\,x^{6}}-\frac{b\left(-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{4a\,x^{4}}+\frac{b\left(-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{2a\,x^{2}}+\frac{3b\left(\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{3}+a\left(\sqrt{b\,x^{2}+a}-\sqrt{a}\,\ln\left(\cdots\right.\right.\right.}{2a}\cdots}{4a}\cdots}{6a}\cdots}{8a}\cdots}{2a}\right.$ |

input
```
int((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/x^11,x,method=_RETURNVERBOSE)
```

output
```
A*(-1/10/a/x^10*(b*x^2+a)^(5/2)-1/2*b/a*(-1/8/a/x^8*(b*x^2+a)^(5/2)-3/8*b/
a*(-1/6/a/x^6*(b*x^2+a)^(5/2)-1/6*b/a*(-1/4/a/x^4*(b*x^2+a)^(5/2)+1/4*b/a*
(-1/2/a/x^2*(b*x^2+a)^(5/2)+3/2*b/a*(1/3*(b*x^2+a)^(3/2)+a*((b*x^2+a)^(1/2
)-a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x))))))))+B*(-1/9/a/x^9*(b*x^
2+a)^(5/2)-4/9*b/a*(-1/7/a/x^7*(b*x^2+a)^(5/2)+2/35*b/a^2/x^5*(b*x^2+a)^(5
/2)))+C*(-1/8/a/x^8*(b*x^2+a)^(5/2)-3/8*b/a*(-1/6/a/x^6*(b*x^2+a)^(5/2)-1/
6*b/a*(-1/4/a/x^4*(b*x^2+a)^(5/2)+1/4*b/a*(-1/2/a/x^2*(b*x^2+a)^(5/2)+3/2*
b/a*(1/3*(b*x^2+a)^(3/2)+a*((b*x^2+a)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/2)*(b*x
^2+a)^(1/2))/x)))))))+D*(-1/7/a/x^7*(b*x^2+a)^(5/2)+2/35*b/a^2/x^5*(b*x^2+
a)^(5/2))
```

## Fricas [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 506, normalized size of antiderivative = 1.93

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{x^{11}}\,dx = \left[-\frac{945\,(2\,Cab^4-Ab^5)\sqrt{a}x^{10}\log\left(-\frac{bx^2+2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)-2\,(\cdots}{\cdots}\right.$$

input
```
integrate((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/x^11,x, algorithm="fricas")
```

output
```
[-1/161280*(945*(2*C*a*b^4 - A*b^5)*sqrt(a)*x^10*log(-(b*x^2 + 2*sqrt(b*x^
2 + a)*sqrt(a) + 2*a)/x^2) - 2*(512*(9*D*a^2*b^3 - 4*B*a*b^4)*x^9 + 945*(2
*C*a^2*b^3 - A*a*b^4)*x^8 - 256*(9*D*a^3*b^2 - 4*B*a^2*b^3)*x^7 - 8960*B*a
^5*x - 630*(2*C*a^3*b^2 - A*a^2*b^3)*x^6 - 8064*A*a^5 - 768*(24*D*a^4*b +
B*a^3*b^2)*x^5 - 504*(30*C*a^4*b + A*a^3*b^2)*x^4 - 1280*(9*D*a^5 + 10*B*a
^4*b)*x^3 - 1008*(10*C*a^5 + 11*A*a^4*b)*x^2)*sqrt(b*x^2 + a))/(a^4*x^10),
 1/80640*(945*(2*C*a*b^4 - A*b^5)*sqrt(-a)*x^10*arctan(sqrt(b*x^2 + a)*sqr
t(-a)/a) + (512*(9*D*a^2*b^3 - 4*B*a*b^4)*x^9 + 945*(2*C*a^2*b^3 - A*a*b^4
)*x^8 - 256*(9*D*a^3*b^2 - 4*B*a^2*b^3)*x^7 - 8960*B*a^5*x - 630*(2*C*a^3*
b^2 - A*a^2*b^3)*x^6 - 8064*A*a^5 - 768*(24*D*a^4*b + B*a^3*b^2)*x^5 - 504
*(30*C*a^4*b + A*a^3*b^2)*x^4 - 1280*(9*D*a^5 + 10*B*a^4*b)*x^3 - 1008*(10
*C*a^5 + 11*A*a^4*b)*x^2)*sqrt(b*x^2 + a))/(a^4*x^10)]
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{x^{11}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(3/2)*(D*x**3+C*x**2+B*x+A)/x**11,x)
```

output
```
Timed out
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.48

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{x^{11}}\,dx = -\frac{3\,Cb^4\,\text{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{128\,a^{\frac{5}{2}}} + \frac{3\,Ab^5\,\text{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{256\,a^{\frac{7}{2}}} + \frac{(bx^2+a)^{\frac{3}{2}}Cb^4}{128\,a^4} + \frac{3\,\sqrt{bx^2+a}Cb^4}{128\,a^3} - \frac{(bx^2+a)^{\frac{3}{2}}Ab^5}{256\,a^5} - \frac{3\,\sqrt{bx^2+a}Ab^5}{256\,a^4} - \frac{(bx^2+a)^{\frac{5}{2}}Cb^3}{128\,a^4x^2} + \frac{(bx^2+a)^{\frac{5}{2}}Ab^4}{256\,a^5x^2} - \frac{(bx^2+a)^{\frac{5}{2}}Cb^2}{64\,a^3x^4} + \frac{(bx^2+a)^{\frac{5}{2}}Ab^3}{128\,a^4x^4} + \frac{2\,(bx^2+a)^{\frac{5}{2}}Db}{35\,a^2x^5} - \frac{8\,(bx^2+a)^{\frac{5}{2}}Bb^2}{315\,a^3x^5} + \frac{(bx^2+a)^{\frac{5}{2}}Cb}{16\,a^2x^6} - \frac{(bx^2+a)^{\frac{5}{2}}Ab^2}{32\,a^3x^6} - \frac{(bx^2+a)^{\frac{5}{2}}D}{7\,ax^7} + \frac{4\,(bx^2+a)^{\frac{5}{2}}Bb}{63\,a^2x^7} - \frac{(bx^2+a)^{\frac{5}{2}}C}{8\,ax^8} + \frac{(bx^2+a)^{\frac{5}{2}}Ab}{16\,a^2x^8} - \frac{(bx^2+a)^{\frac{5}{2}}B}{9\,ax^9} - \frac{(bx^2+a)^{\frac{5}{2}}A}{10\,ax^{10}}$$

input
```
integrate((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/x^11,x, algorithm="maxima")
```

output
```
-3/128*C*b^4*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(5/2) + 3/256*A*b^5*arcsinh(a
/(sqrt(a*b)*abs(x)))/a^(7/2) + 1/128*(b*x^2 + a)^(3/2)*C*b^4/a^4 + 3/128*s
qrt(b*x^2 + a)*C*b^4/a^3 - 1/256*(b*x^2 + a)^(3/2)*A*b^5/a^5 - 3/256*sqrt(
b*x^2 + a)*A*b^5/a^4 - 1/128*(b*x^2 + a)^(5/2)*C*b^3/(a^4*x^2) + 1/256*(b*
x^2 + a)^(5/2)*A*b^4/(a^5*x^2) - 1/64*(b*x^2 + a)^(5/2)*C*b^2/(a^3*x^4) +
1/128*(b*x^2 + a)^(5/2)*A*b^3/(a^4*x^4) + 2/35*(b*x^2 + a)^(5/2)*D*b/(a^2*
x^5) - 8/315*(b*x^2 + a)^(5/2)*B*b^2/(a^3*x^5) + 1/16*(b*x^2 + a)^(5/2)*C*
b/(a^2*x^6) - 1/32*(b*x^2 + a)^(5/2)*A*b^2/(a^3*x^6) - 1/7*(b*x^2 + a)^(5/
2)*D/(a*x^7) + 4/63*(b*x^2 + a)^(5/2)*B*b/(a^2*x^7) - 1/8*(b*x^2 + a)^(5/2
)*C/(a*x^8) + 1/16*(b*x^2 + a)^(5/2)*A*b/(a^2*x^8) - 1/9*(b*x^2 + a)^(5/2)
*B/(a*x^9) - 1/10*(b*x^2 + a)^(5/2)*A/(a*x^10)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1068 vs. $2(225) = 450$.

Time = 0.14 (sec) , antiderivative size = 1068, normalized size of antiderivative = 4.08

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{x^{11}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/x^11,x, algorithm="giac")
```

output
```
3/128*(2*C*a*b^4 - A*b^5)*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/
(sqrt(-a)*a^3) - 1/40320*(1890*(sqrt(b)*x - sqrt(b*x^2 + a))^19*C*a*b^4 -
945*(sqrt(b)*x - sqrt(b*x^2 + a))^19*A*b^5 - 18270*(sqrt(b)*x - sqrt(b*x^2
 + a))^17*C*a^2*b^4 + 9135*(sqrt(b)*x - sqrt(b*x^2 + a))^17*A*a*b^5 - 1612
80*(sqrt(b)*x - sqrt(b*x^2 + a))^16*D*a^3*b^(7/2) - 178920*(sqrt(b)*x - sq
rt(b*x^2 + a))^15*C*a^3*b^4 - 39564*(sqrt(b)*x - sqrt(b*x^2 + a))^15*A*a^2
*b^5 + 322560*(sqrt(b)*x - sqrt(b*x^2 + a))^14*D*a^4*b^(7/2) - 430080*(sqr
t(b)*x - sqrt(b*x^2 + a))^14*B*a^3*b^(9/2) - 17640*(sqrt(b)*x - sqrt(b*x^2
 + a))^13*C*a^4*b^4 - 636300*(sqrt(b)*x - sqrt(b*x^2 + a))^13*A*a^3*b^5 -
322560*(sqrt(b)*x - sqrt(b*x^2 + a))^12*D*a^5*b^(7/2) - 215040*(sqrt(b)*x
- sqrt(b*x^2 + a))^12*B*a^4*b^(9/2) + 212940*(sqrt(b)*x - sqrt(b*x^2 + a))
^11*C*a^5*b^4 - 1396710*(sqrt(b)*x - sqrt(b*x^2 + a))^11*A*a^4*b^5 + 58060
8*(sqrt(b)*x - sqrt(b*x^2 + a))^10*D*a^6*b^(7/2) - 258048*(sqrt(b)*x - sqr
t(b*x^2 + a))^10*B*a^5*b^(9/2) + 212940*(sqrt(b)*x - sqrt(b*x^2 + a))^9*C*
a^6*b^4 - 1396710*(sqrt(b)*x - sqrt(b*x^2 + a))^9*A*a^5*b^5 - 645120*(sqrt
(b)*x - sqrt(b*x^2 + a))^8*D*a^7*b^(7/2) + 645120*(sqrt(b)*x - sqrt(b*x^2
+ a))^8*B*a^6*b^(9/2) - 17640*(sqrt(b)*x - sqrt(b*x^2 + a))^7*C*a^7*b^4 -
636300*(sqrt(b)*x - sqrt(b*x^2 + a))^7*A*a^6*b^5 + 230400*(sqrt(b)*x - sqr
t(b*x^2 + a))^6*D*a^8*b^(7/2) + 184320*(sqrt(b)*x - sqrt(b*x^2 + a))^6*B*a
^7*b^(9/2) - 178920*(sqrt(b)*x - sqrt(b*x^2 + a))^5*C*a^8*b^4 - 39564*(...
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2+Dx^3)}{x^{11}}\,dx = \int \frac{(b\,x^2+a)^{3/2}\,(A+B\,x+C\,x^2+x^3\,D)}{x^{11}}\,dx$$

input
```
int(((a + b*x^2)^(3/2)*(A + B*x + C*x^2 + x^3*D))/x^11,x)
```

output
```
int(((a + b*x^2)^(3/2)*(A + B*x + C*x^2 + x^3*D))/x^11, x)
```

## Reduce [B] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 478, normalized size of antiderivative = 1.82

$$\int \frac{(a+bx^2)^{3/2}\,(A+Bx+Cx^2+Dx^3)}{x^{11}}\,dx = \frac{-8064\sqrt{b\,x^2+a}\,a^5-11088\sqrt{b\,x^2+a}\,a^4b\,x^2-8960\sqrt{b\,x^2+}$$

input
```
int((b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/x^11,x)
```

output
```
( - 8064*sqrt(a + b*x**2)*a**5 - 11088*sqrt(a + b*x**2)*a**4*b*x**2 - 8960
*sqrt(a + b*x**2)*a**4*b*x - 10080*sqrt(a + b*x**2)*a**4*c*x**2 - 11520*sq
rt(a + b*x**2)*a**4*d*x**3 - 504*sqrt(a + b*x**2)*a**3*b**2*x**4 - 12800*s
qrt(a + b*x**2)*a**3*b**2*x**3 - 15120*sqrt(a + b*x**2)*a**3*b*c*x**4 - 18
432*sqrt(a + b*x**2)*a**3*b*d*x**5 + 630*sqrt(a + b*x**2)*a**2*b**3*x**6 -
 768*sqrt(a + b*x**2)*a**2*b**3*x**5 - 1260*sqrt(a + b*x**2)*a**2*b**2*c*x
**6 - 2304*sqrt(a + b*x**2)*a**2*b**2*d*x**7 - 945*sqrt(a + b*x**2)*a*b**4
*x**8 + 1024*sqrt(a + b*x**2)*a*b**4*x**7 + 1890*sqrt(a + b*x**2)*a*b**3*c
*x**8 + 4608*sqrt(a + b*x**2)*a*b**3*d*x**9 - 2048*sqrt(a + b*x**2)*b**5*x
**9 - 945*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*b*
*5*x**10 + 1890*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(
a))*b**4*c*x**10 + 945*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x
)/sqrt(a))*b**5*x**10 - 1890*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqr
t(b)*x)/sqrt(a))*b**4*c*x**10 - 4608*sqrt(b)*a*b**3*d*x**10 + 2048*sqrt(b)
*b**5*x**10)/(80640*a**3*x**10)
```

# 3.89 $\int \frac{x^4(A+Bx+Cx^2+Dx^3)}{\sqrt{a+bx^2}}\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 221

$$\int \frac{x^4(A+Bx+Cx^2+Dx^3)}{\sqrt{a+bx^2}}\,dx = \frac{a^2(bB-aD)\sqrt{a+bx^2}}{b^4} - \frac{a(6Ab-5aC)x\sqrt{a+bx^2}}{16b^3} + \frac{(6Ab-5aC)x^3\sqrt{a+bx^2}}{24b^2} + \frac{Cx^5\sqrt{a+bx^2}}{6b} - \frac{a(2bB-3aD)\left(a+bx^2\right)^{3/2}}{3b^4} + \frac{(bB-3aD)\left(a+bx^2\right)^{5/2}}{5b^4} + \frac{D\left(a+bx^2\right)^{7/2}}{7b^4} + \frac{a^2(6Ab-5aC)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{7/2}}$$

output
```
a^2*(B*b-D*a)*(b*x^2+a)^(1/2)/b^4-1/16*a*(6*A*b-5*C*a)*x*(b*x^2+a)^(1/2)/b
^3+1/24*(6*A*b-5*C*a)*x^3*(b*x^2+a)^(1/2)/b^2+1/6*C*x^5*(b*x^2+a)^(1/2)/b-
1/3*a*(2*B*b-3*D*a)*(b*x^2+a)^(3/2)/b^4+1/5*(B*b-3*D*a)*(b*x^2+a)^(5/2)/b^
4+1/7*D*(b*x^2+a)^(7/2)/b^4+1/16*a^2*(6*A*b-5*C*a)*arctanh(b^(1/2)*x/(b*x^
2+a)^(1/2))/b^(7/2)
```

## Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.64

$$\int \frac{x^4(A+Bx+Cx^2+Dx^3)}{\sqrt{a+bx^2}}\,dx$$
$$= \frac{\sqrt{a+bx^2}(-768a^3D+a^2b(896B+525Cx+384Dx^2)+4b^3x^3(105A+84Bx+70Cx^2+60Dx^3)-2ab^2}{1680b^4}$$

input
```
Integrate[(x^4*(A + B*x + C*x^2 + D*x^3))/Sqrt[a + b*x^2],x]
```

output
```
(Sqrt[a + b*x^2]*(-768*a^3*D + a^2*b*(896*B + 525*C*x + 384*D*x^2) + 4*b^3
*x^3*(105*A + 84*B*x + 70*C*x^2 + 60*D*x^3) - 2*a*b^2*x*(315*A + x*(224*B
+ 175*C*x + 144*D*x^2))) + 105*a^2*Sqrt[b]*(-6*A*b + 5*a*C)*Log[-(Sqrt[b]*
x) + Sqrt[a + b*x^2]])/(1680*b^4)
```

## Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.15, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.433$, Rules used = {2340, 2340, 27, 533, 533, 27, 533, 533, 25, 27, 455, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(A+Bx+Cx^2+Dx^3)}{\sqrt{a+bx^2}}\,dx$$
$$\Big\downarrow \text{2340}$$
$$\frac{\int \frac{x^4(7bCx^2+(7bB-6aD)x+7Ab)}{\sqrt{bx^2+a}}dx}{7b}+\frac{Dx^6\sqrt{a+bx^2}}{7b}$$
$$\Big\downarrow \text{2340}$$
$$\frac{\frac{\int \frac{bx^4(7(6Ab-5aC)+6(7bB-6aD)x)}{\sqrt{bx^2+a}}dx}{6b}+\frac{7}{6}Cx^5\sqrt{a+bx^2}}{7b}+\frac{Dx^6\sqrt{a+bx^2}}{7b}$$
$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{6}\int \frac{x^4(7(6Ab-5aC)+6(7bB-6aD)x)}{\sqrt{bx^2+a}}dx+\frac{7}{6}Cx^5\sqrt{a+bx^2}}{7b}+\frac{Dx^6\sqrt{a+bx^2}}{7b}$$

$\Big\downarrow$ 533

$$\frac{\frac{1}{6}\left(\frac{6x^4\sqrt{a+bx^2}(7bB-6aD)}{5b}-\frac{\int \frac{x^3(24a(7bB-6aD)-35b(6Ab-5aC)x)}{\sqrt{bx^2+a}}dx}{5b}\right)+\frac{7}{6}Cx^5\sqrt{a+bx^2}}{7b}+\frac{Dx^6\sqrt{a+bx^2}}{7b}$$

$\Big\downarrow$ 533

$$\frac{\frac{1}{6}\left(\frac{6x^4\sqrt{a+bx^2}(7bB-6aD)}{5b}-\frac{-\frac{\int -\frac{3abx^2(35(6Ab-5aC)+32(7bB-6aD)x)}{\sqrt{bx^2+a}}dx}{4b}-\frac{35}{4}x^3\sqrt{a+bx^2}(6Ab-5aC)}{5b}\right)+\frac{7}{6}Cx^5\sqrt{a+bx^2}}{7b}+\frac{Dx^6\sqrt{a+bx^2}}{7b}$$

$\Big\downarrow$ 27

$$\frac{\frac{1}{6}\left(\frac{6x^4\sqrt{a+bx^2}(7bB-6aD)}{5b}-\frac{\frac{3}{4}a\int \frac{x^2(35(6Ab-5aC)+32(7bB-6aD)x)}{\sqrt{bx^2+a}}dx-\frac{35}{4}x^3\sqrt{a+bx^2}(6Ab-5aC)}{5b}\right)+\frac{7}{6}Cx^5\sqrt{a+bx^2}}{7b}+\frac{Dx^6\sqrt{a+bx^2}}{7b}$$

$\Big\downarrow$ 533

$$\frac{\frac{1}{6}\left(\frac{6x^4\sqrt{a+bx^2}(7bB-6aD)}{5b}-\frac{\frac{3}{4}a\left(\frac{32x^2\sqrt{a+bx^2}(7bB-6aD)}{3b}-\frac{\int \frac{x(64a(7bB-6aD)-105b(6Ab-5aC)x)}{\sqrt{bx^2+a}}dx}{3b}\right)-\frac{35}{4}x^3\sqrt{a+bx^2}(6Ab-5aC)}{5b}\right)+\frac{7}{6}Cx}{7b}$$

$$\frac{Dx^6\sqrt{a+bx^2}}{7b}$$

$\Big\downarrow$ 533

$$\frac{\frac{1}{6}\left(\frac{6x^4\sqrt{a+bx^2}(7bB-6aD)}{5b}-\frac{\frac{3}{4}a\left(\frac{32x^2\sqrt{a+bx^2}(7bB-6aD)}{3b}-\frac{-\frac{\int -\frac{ab(105(6Ab-5aC)+128(7bB-6aD)x)}{\sqrt{bx^2+a}}dx}{2b}-\frac{105}{2}x\sqrt{a+bx^2}(6Ab-5aC)}{3b}\right)-\frac{35}{4}x^3\sqrt{a+\cdots}}{5b}\cdots\right)}{7b}$$

$$\frac{Dx^6\sqrt{a+bx^2}}{7b}$$

$\Big\downarrow$ 25

$$\frac{\frac{1}{6}\left(\frac{6x^4\sqrt{a+bx^2}(7bB-6aD)}{5b}-\frac{\frac{3}{4}a\left(\frac{32x^2\sqrt{a+bx^2}(7bB-6aD)}{3b}-\frac{\frac{\int \frac{ab(105(6Ab-5aC)+128(7bB-6aD)x)}{\sqrt{bx^2+a}}dx}{2b}-\frac{105}{2}x\sqrt{a+bx^2}(6Ab-5aC)}{3b}\right)-\frac{35}{4}x^3\sqrt{a+bx\cdots}}{5b}\cdots\right)}{7b}$$

$$\frac{Dx^6\sqrt{a+bx^2}}{7b}$$

$\Big\downarrow$ 27

$$\frac{\frac{1}{6}\left(\frac{6x^4\sqrt{a+bx^2}(7bB-6aD)}{5b}-\frac{\frac{3}{4}a\left(\frac{32x^2\sqrt{a+bx^2}(7bB-6aD)}{3b}-\frac{\frac{1}{2}a\int \frac{105(6Ab-5aC)+128(7bB-6aD)x}{\sqrt{bx^2+a}}dx-\frac{105}{2}x\sqrt{a+bx^2}(6Ab-5aC)}{3b}\right)-\frac{35}{4}x^3\sqrt{a+bx^2}\cdots}{5b}\cdots\right)}{7b}$$

$$\frac{Dx^6\sqrt{a+bx^2}}{7b}$$

$\Big\downarrow$ 455

$$\frac{\frac{1}{6}\left(\frac{6x^4\sqrt{a+bx^2}(7bB-6aD)}{5b}-\frac{\frac{3}{4}a\left(\frac{32x^2\sqrt{a+bx^2}(7bB-6aD)}{3b}-\frac{\frac{1}{2}a\left(105(6Ab-5aC)\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{128\sqrt{a+bx^2}(7bB-6aD)}{b}\right)-\frac{105}{2}x\sqrt{a+bx^2}(6Ab-5aC)}{3b}\cdots\right)}{5b}\cdots\right)}{7b}$$

$$\frac{Dx^6\sqrt{a+bx^2}}{7b}$$

$\Big\downarrow$ 224

$$\frac{\frac{1}{6}\left(\frac{6x^4\sqrt{a+bx^2}(7bB-6aD)}{5b}-\frac{\frac{3}{4}a\left(\frac{32x^2\sqrt{a+bx^2}(7bB-6aD)}{3b}-\frac{\frac{1}{2}a\left(105(6Ab-5aC)\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{128\sqrt{a+bx^2}(7bB-6aD)}{b}\right)-\frac{105}{2}x\sqrt{a+bx^2}(6}{3b}\right.}{5b}\right.}{7b}$$

$$\frac{Dx^6\sqrt{a+bx^2}}{7b}$$

$\Big\downarrow$ 219

$$\frac{\frac{1}{6}\left(\frac{6x^4\sqrt{a+bx^2}(7bB-6aD)}{5b}-\frac{\frac{3}{4}a\left(\frac{32x^2\sqrt{a+bx^2}(7bB-6aD)}{3b}-\frac{\frac{1}{2}a\left(\frac{105(6Ab-5aC)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}+\frac{128\sqrt{a+bx^2}(7bB-6aD)}{b}\right)-\frac{105}{2}x\sqrt{a+bx^2}(}{3b}\right.}{5b}\right.}{7b}$$

$$\frac{Dx^6\sqrt{a+bx^2}}{7b}$$

input
```
Int[(x^4*(A + B*x + C*x^2 + D*x^3))/Sqrt[a + b*x^2],x]
```

output
```
(D*x^6*Sqrt[a + b*x^2])/(7*b) + ((7*C*x^5*Sqrt[a + b*x^2])/6 + ((6*(7*b*B
 - 6*a*D)*x^4*Sqrt[a + b*x^2])/(5*b) - ((-35*(6*A*b - 5*a*C)*x^3*Sqrt[a + b
*x^2])/4 + (3*a*((32*(7*b*B - 6*a*D)*x^2*Sqrt[a + b*x^2])/(3*b) - ((-105*(
6*A*b - 5*a*C)*x*Sqrt[a + b*x^2])/2 + (a*((128*(7*b*B - 6*a*D)*Sqrt[a + b*
x^2])/b + (105*(6*A*b - 5*a*C)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/Sqrt[
b]))/2)/(3*b)))/4)/(5*b))/6)/(7*b)
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

rule 2340
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Simp[1/(b*(m
+ q + 2*p + 1))   Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)
*Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /;
GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ
[Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

## Maple [A] (verified)

Time = 0.61 (sec), antiderivative size = 296, normalized size of antiderivative = 1.34

| method | result |
|---|---|
| default | $A\left(\frac{x^{3}\sqrt{b\,x^{2}+a}}{4b}-\frac{3a\left(\frac{x\sqrt{b\,x^{2}+a}}{2b}-\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{2b^{\frac{3}{2}}}\right)}{4b}\right)+B\left(\frac{x^{4}\sqrt{b\,x^{2}+a}}{5b}-\frac{4a\left(\frac{x^{2}\sqrt{b\,x^{2}+a}}{3b}-\frac{2a\sqrt{b\,x^{2}+a}}{3b^{2}}\right)}{5b}\right)+C$ |

input
```
int(x^4*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
A*(1/4*x^3/b*(b*x^2+a)^(1/2)-3/4*a/b*(1/2*x*(b*x^2+a)^(1/2)/b-1/2*a/b^(3/2
)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))))+B*(1/5*x^4/b*(b*x^2+a)^(1/2)-4/5*a/b*(1/
3*x^2/b*(b*x^2+a)^(1/2)-2/3*a/b^2*(b*x^2+a)^(1/2)))+C*(1/6*x^5/b*(b*x^2+a)
^(1/2)-5/6*a/b*(1/4*x^3/b*(b*x^2+a)^(1/2)-3/4*a/b*(1/2*x*(b*x^2+a)^(1/2)/b
-1/2*a/b^(3/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))))+D*(1/7*x^6/b*(b*x^2+a)^(1/
2)-6/7*a/b*(1/5*x^4/b*(b*x^2+a)^(1/2)-4/5*a/b*(1/3*x^2/b*(b*x^2+a)^(1/2)-2
/3*a/b^2*(b*x^2+a)^(1/2))))
```

## Fricas [A] (verification not implemented)

Time = 0.09 (sec), antiderivative size = 333, normalized size of antiderivative = 1.51

$$\int \frac{x^4(A+Bx+Cx^2+Dx^3)}{\sqrt{a+bx^2}}\,dx$$
$$=\left[-\frac{105\,(5\,Ca^3-6\,Aa^2b)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\,(240\,Db^3x^6+280\,Cb^3x^5-48\,(6\,Da\right.$$

input
```
integrate(x^4*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/3360*(105*(5*C*a^3 - 6*A*a^2*b)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 +
a)*sqrt(b)*x - a) - 2*(240*D*b^3*x^6 + 280*C*b^3*x^5 - 48*(6*D*a*b^2 - 7*B
*b^3)*x^4 - 768*D*a^3 + 896*B*a^2*b - 70*(5*C*a*b^2 - 6*A*b^3)*x^3 + 64*(6
*D*a^2*b - 7*B*a*b^2)*x^2 + 105*(5*C*a^2*b - 6*A*a*b^2)*x)*sqrt(b*x^2 + a)
)/b^4, 1/1680*(105*(5*C*a^3 - 6*A*a^2*b)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b
*x^2 + a)) + (240*D*b^3*x^6 + 280*C*b^3*x^5 - 48*(6*D*a*b^2 - 7*B*b^3)*x^4
 - 768*D*a^3 + 896*B*a^2*b - 70*(5*C*a*b^2 - 6*A*b^3)*x^3 + 64*(6*D*a^2*b
- 7*B*a*b^2)*x^2 + 105*(5*C*a^2*b - 6*A*a*b^2)*x)*sqrt(b*x^2 + a))/b^4]
```

## Sympy [A] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.99

$$\int \frac{x^4(A+Bx+Cx^2+Dx^3)}{\sqrt{a+bx^2}}\,dx = \begin{cases} \frac{3a^2\left(A-\frac{5Ca}{6b}\right)\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right)}{8b^2} + \sqrt{a+bx^2}\left(\frac{Cx^5}{6b}+\frac{Dx^6}{7b}+\frac{8a^2\left(B-\frac{6Da}{7b}\right)}{15b^3}-\frac{4ax^2\left(B-\frac{6Da}{7b}\right)}{15b^2}-\right. \\ \frac{\frac{Ax^5}{5}+\frac{Bx^6}{6}+\frac{Cx^7}{7}+\frac{Dx^8}{8}}{\sqrt{a}} \end{cases}$$

input
```
integrate(x**4*(D*x**3+C*x**2+B*x+A)/(b*x**2+a)**(1/2),x)
```

output
```
Piecewise(((3*a**2*(A - 5*C*a/(6*b))*Piecewise((log(2*sqrt(b)*sqrt(a + b*x*
*2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True))/(8*b**2) +
 sqrt(a + b*x**2)*(C*x**5/(6*b) + D*x**6/(7*b) + 8*a**2*(B - 6*D*a/(7*b))/
(15*b**3) - 4*a*x**2*(B - 6*D*a/(7*b))/(15*b**2) - 3*a*x*(A - 5*C*a/(6*b))
/(8*b**2) + x**4*(B - 6*D*a/(7*b))/(5*b) + x**3*(A - 5*C*a/(6*b))/(4*b)),
Ne(b, 0)), ((A*x**5/5 + B*x**6/6 + C*x**7/7 + D*x**8/8)/sqrt(a), True))
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.17

$$\int \frac{x^4(A+Bx+Cx^2+Dx^3)}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{bx^2+a}Dx^6}{7\,b} + \frac{\sqrt{bx^2+a}Cx^5}{6\,b} - \frac{6\,\sqrt{bx^2+a}Dax^4}{35\,b^2} + \frac{\sqrt{bx^2+a}Bx^4}{5\,b} - \frac{5\,\sqrt{bx^2+a}Cax^3}{24\,b^2} + \frac{\sqrt{bx^2+a}Ax^3}{4\,b} + \frac{8\,\sqrt{bx^2+a}Da^2x^2}{35\,b^3} - \frac{4\,\sqrt{bx^2+a}Bax^2}{15\,b^2} + \frac{5\,\sqrt{bx^2+a}Ca^2x}{16\,b^3} - \frac{3\,\sqrt{bx^2+a}Aax}{8\,b^2} - \frac{5\,Ca^3\,\text{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{7}{2}}} + \frac{3\,Aa^2\,\text{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{5}{2}}} - \frac{16\,\sqrt{bx^2+a}Da^3}{35\,b^4} + \frac{8\,\sqrt{bx^2+a}Ba^2}{15\,b^3}$$

input
```
integrate(x^4*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/7*sqrt(b*x^2 + a)*D*x^6/b + 1/6*sqrt(b*x^2 + a)*C*x^5/b - 6/35*sqrt(b*x^
2 + a)*D*a*x^4/b^2 + 1/5*sqrt(b*x^2 + a)*B*x^4/b - 5/24*sqrt(b*x^2 + a)*C*
a*x^3/b^2 + 1/4*sqrt(b*x^2 + a)*A*x^3/b + 8/35*sqrt(b*x^2 + a)*D*a^2*x^2/b
^3 - 4/15*sqrt(b*x^2 + a)*B*a*x^2/b^2 + 5/16*sqrt(b*x^2 + a)*C*a^2*x/b^3 -
 3/8*sqrt(b*x^2 + a)*A*a*x/b^2 - 5/16*C*a^3*arcsinh(b*x/sqrt(a*b))/b^(7/2)
 + 3/8*A*a^2*arcsinh(b*x/sqrt(a*b))/b^(5/2) - 16/35*sqrt(b*x^2 + a)*D*a^3/
b^4 + 8/15*sqrt(b*x^2 + a)*B*a^2/b^3
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.85

$$\int \frac{x^4(A+Bx+Cx^2+Dx^3)}{\sqrt{a+bx^2}}\,dx = \frac{1}{1680}\sqrt{bx^2+a}\left(\left(2\left(\left(4\left(5\left(\frac{6\,Dx}{b}+\frac{7\,C}{b}\right)x-\frac{6\,(6\,Dab^5-7\,Bb^6)}{b^7}\right)x-\frac{35\,(5\,Cab^5-6\,Ab^6)}{b^7}\right)x+\frac{32\,(\right.\right.\right. + \frac{(5\,Ca^3-6\,Aa^2b)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{16\,b^{\frac{7}{2}}}$$

input
```
integrate(x^4*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/1680*sqrt(b*x^2 + a)*((2*((4*(5*(6*D*x/b + 7*C/b)*x - 6*(6*D*a*b^5 - 7*B
*b^6)/b^7)*x - 35*(5*C*a*b^5 - 6*A*b^6)/b^7)*x + 32*(6*D*a^2*b^4 - 7*B*a*b
^5)/b^7)*x + 105*(5*C*a^2*b^4 - 6*A*a*b^5)/b^7)*x - 128*(6*D*a^3*b^3 - 7*B
*a^2*b^4)/b^7) + 1/16*(5*C*a^3 - 6*A*a^2*b)*log(abs(-sqrt(b)*x + sqrt(b*x^
2 + a)))/b^(7/2)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(A+Bx+Cx^2+Dx^3)}{\sqrt{a+bx^2}}\,dx=\int \frac{x^4\,\left(A+B\,x+C\,x^2+x^3\,D\right)}{\sqrt{b\,x^2+a}}\,dx$$

input
```
int((x^4*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^(1/2),x)
```

output
```
int((x^4*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^(1/2), x)
```

## Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.18

$$\int \frac{x^4(A+Bx+Cx^2+Dx^3)}{\sqrt{a+bx^2}}\,dx = \frac{-768\sqrt{b\,x^2+a}\,a^3d-630\sqrt{b\,x^2+a}\,a^2b^2x+896\sqrt{b\,x^2+a}\,a^2b^2+525\sqrt{b\,x^2+a}\,a^2bcx+384\sqrt{b\,x^2+a}\,a^2}{}$$

input
```
int(x^4*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(1/2),x)
```

output
```
( - 768*sqrt(a + b*x**2)*a**3*d - 630*sqrt(a + b*x**2)*a**2*b**2*x + 896*s
qrt(a + b*x**2)*a**2*b**2 + 525*sqrt(a + b*x**2)*a**2*b*c*x + 384*sqrt(a +
 b*x**2)*a**2*b*d*x**2 + 420*sqrt(a + b*x**2)*a*b**3*x**3 - 448*sqrt(a + b
*x**2)*a*b**3*x**2 - 350*sqrt(a + b*x**2)*a*b**2*c*x**3 - 288*sqrt(a + b*x
**2)*a*b**2*d*x**4 + 336*sqrt(a + b*x**2)*b**4*x**4 + 280*sqrt(a + b*x**2)
*b**3*c*x**5 + 240*sqrt(a + b*x**2)*b**3*d*x**6 + 630*sqrt(b)*log((sqrt(a
+ b*x**2) + sqrt(b)*x)/sqrt(a))*a**3*b - 525*sqrt(b)*log((sqrt(a + b*x**2)
 + sqrt(b)*x)/sqrt(a))*a**3*c)/(1680*b**4)
```

# 3.90 $\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{\sqrt{a+bx^2}}\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 192

$$\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{\sqrt{a+bx^2}}\,dx = -\frac{a(Ab-aC)\sqrt{a+bx^2}}{b^3} - \frac{a(6bB-5aD)x\sqrt{a+bx^2}}{16b^3} + \frac{(6bB-5aD)x^3\sqrt{a+bx^2}}{24b^2} + \frac{Dx^5\sqrt{a+bx^2}}{6b} + \frac{(Ab-2aC)\left(a+bx^2\right)^{3/2}}{3b^3} + \frac{C\left(a+bx^2\right)^{5/2}}{5b^3} + \frac{a^2(6bB-5aD)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{7/2}}$$

output
```
-a*(A*b-C*a)*(b*x^2+a)^(1/2)/b^3-1/16*a*(6*B*b-5*D*a)*x*(b*x^2+a)^(1/2)/b^
3+1/24*(6*B*b-5*D*a)*x^3*(b*x^2+a)^(1/2)/b^2+1/6*D*x^5*(b*x^2+a)^(1/2)/b+1
/3*(A*b-2*C*a)*(b*x^2+a)^(3/2)/b^3+1/5*C*(b*x^2+a)^(5/2)/b^3+1/16*a^2*(6*B
*b-5*D*a)*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(7/2)
```

## Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.67

$$\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{\sqrt{a+bx^2}}\,dx$$
$$= \frac{\sqrt{b}\sqrt{a+bx^2}(a^2(128C+75Dx)+4b^2x^2(20A+x(15B+2x(6C+5Dx)))-2ab(80A+x(45B+x(32C}{240b^{7/2}}$$

input
```
Integrate[(x^3*(A + B*x + C*x^2 + D*x^3))/Sqrt[a + b*x^2],x]
```

output
```
(Sqrt[b]*Sqrt[a + b*x^2]*(a^2*(128*C + 75*D*x) + 4*b^2*x^2*(20*A + x*(15*B
 + 2*x*(6*C + 5*D*x))) - 2*a*b*(80*A + x*(45*B + x*(32*C + 25*D*x)))) + 15
*a^2*(-6*b*B + 5*a*D)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(240*b^(7/2))
```

## Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.15, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2340, 2340, 27, 533, 27, 533, 25, 27, 533, 455, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\left(A+Bx+Cx^2+Dx^3\right)}{\sqrt{a+bx^2}}\,dx$$
$$\Big\downarrow \ 2340$$
$$\frac{\int \frac{x^3\left(6bCx^2+(6bB-5aD)x+6Ab\right)}{\sqrt{bx^2+a}}dx}{6b}+\frac{Dx^5\sqrt{a+bx^2}}{6b}$$
$$\Big\downarrow \ 2340$$
$$\frac{\frac{\int \frac{bx^3(6(5Ab-4aC)+5(6bB-5aD)x)}{\sqrt{bx^2+a}}dx}{5b}+\frac{6}{5}Cx^4\sqrt{a+bx^2}}{6b}+\frac{Dx^5\sqrt{a+bx^2}}{6b}$$
$$\Big\downarrow \ 27$$

$$\frac{\frac{1}{5}\int \frac{x^3(6(5Ab-4aC)+5(6bB-5aD)x)}{\sqrt{bx^2+a}}dx+\frac{6}{5}Cx^4\sqrt{a+bx^2}}{6b}+\frac{Dx^5\sqrt{a+bx^2}}{6b}$$

$$\Big\downarrow \text{533}$$

$$\frac{\frac{1}{5}\left(\frac{5x^3\sqrt{a+bx^2}(6bB-5aD)}{4b}-\frac{\int \frac{3x^2(5a(6bB-5aD)-8b(5Ab-4aC)x)}{\sqrt{bx^2+a}}dx}{4b}\right)+\frac{6}{5}Cx^4\sqrt{a+bx^2}}{6b}+\frac{Dx^5\sqrt{a+bx^2}}{6b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{5}\left(\frac{5x^3\sqrt{a+bx^2}(6bB-5aD)}{4b}-\frac{3\int \frac{x^2(5a(6bB-5aD)-8b(5Ab-4aC)x)}{\sqrt{bx^2+a}}dx}{4b}\right)+\frac{6}{5}Cx^4\sqrt{a+bx^2}}{6b}+\frac{Dx^5\sqrt{a+bx^2}}{6b}$$

$$\Big\downarrow \text{533}$$

$$\frac{\frac{1}{5}\left(\frac{5x^3\sqrt{a+bx^2}(6bB-5aD)}{4b}-\frac{3\left(-\frac{\int -\frac{abx(16(5Ab-4aC)+15(6bB-5aD)x)}{\sqrt{bx^2+a}}dx}{3b}-\frac{8}{3}x^2\sqrt{a+bx^2}(5Ab-4aC)\right)}{4b}\right)+\frac{6}{5}Cx^4\sqrt{a+bx^2}}{6b}+\frac{Dx^5\sqrt{a+bx^2}}{6b}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{1}{5}\left(\frac{5x^3\sqrt{a+bx^2}(6bB-5aD)}{4b}-\frac{3\left(\frac{\int \frac{abx(16(5Ab-4aC)+15(6bB-5aD)x)}{\sqrt{bx^2+a}}dx}{3b}-\frac{8}{3}x^2\sqrt{a+bx^2}(5Ab-4aC)\right)}{4b}\right)+\frac{6}{5}Cx^4\sqrt{a+bx^2}}{6b}+\frac{Dx^5\sqrt{a+bx^2}}{6b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{5}\left(\frac{5x^3\sqrt{a+bx^2}(6bB-5aD)}{4b}-\frac{3\left(\frac{1}{3}a\int \frac{x(16(5Ab-4aC)+15(6bB-5aD)x)}{\sqrt{bx^2+a}}dx-\frac{8}{3}x^2\sqrt{a+bx^2}(5Ab-4aC)\right)}{4b}\right)+\frac{6}{5}Cx^4\sqrt{a+bx^2}}{6b}+\frac{Dx^5\sqrt{a+bx^2}}{6b}$$

$$\Big\downarrow \text{533}$$

$$\frac{\frac{1}{5}\left(\frac{5x^3\sqrt{a+bx^2}(6bB-5aD)}{4b}-\frac{3\left(\frac{1}{3}a\left(\frac{15x\sqrt{a+bx^2}(6bB-5aD)}{2b}-\frac{\int\frac{15a(6bB-5aD)-32b(5Ab-4aC)x}{\sqrt{bx^2+a}}dx}{2b}\right)-\frac{8}{3}x^2\sqrt{a+bx^2}(5Ab-4aC)\right)}{4b}\right)+\frac{6}{5}Cx}{6b}$$

$$\frac{Dx^5\sqrt{a+bx^2}}{6b}$$

$$\Big\downarrow \; 455$$

$$\frac{\frac{1}{5}\left(\frac{5x^3\sqrt{a+bx^2}(6bB-5aD)}{4b}-\frac{3\left(\frac{1}{3}a\left(\frac{15x\sqrt{a+bx^2}(6bB-5aD)}{2b}-\frac{15a(6bB-5aD)\int\frac{1}{\sqrt{bx^2+a}}dx-32\sqrt{a+bx^2}(5Ab-4aC)}{2b}\right)-\frac{8}{3}x^2\sqrt{a+bx^2}(5Ab-4aC)\right)}{4b}\right.}{6b}$$

$$\frac{Dx^5\sqrt{a+bx^2}}{6b}$$

$$\Big\downarrow \; 224$$

$$\frac{\frac{1}{5}\left(\frac{5x^3\sqrt{a+bx^2}(6bB-5aD)}{4b}-\frac{3\left(\frac{1}{3}a\left(\frac{15x\sqrt{a+bx^2}(6bB-5aD)}{2b}-\frac{15a(6bB-5aD)\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}-32\sqrt{a+bx^2}(5Ab-4aC)}{2b}\right)-\frac{8}{3}x^2\sqrt{a+bx^2}(5Ab\right.}{4b}\right.}{6b}$$

$$\frac{Dx^5\sqrt{a+bx^2}}{6b}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{1}{5}\left(\frac{5x^3\sqrt{a+bx^2}(6bB-5aD)}{4b}-\frac{3\left(\frac{1}{3}a\left(\frac{15x\sqrt{a+bx^2}(6bB-5aD)}{2b}-\frac{\frac{15a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(6bB-5aD)}{\sqrt{b}}-32\sqrt{a+bx^2}(5Ab-4aC)}{2b}\right)-\frac{8}{3}x^2\sqrt{a+bx^2}(5A\right.}{4b}\right.}{6b}$$

$$\frac{Dx^5\sqrt{a+bx^2}}{6b}$$

input
```
Int[(x^3*(A + B*x + C*x^2 + D*x^3))/Sqrt[a + b*x^2],x]
```

output
```
(D*x^5*Sqrt[a + b*x^2])/(6*b) + ((6*C*x^4*Sqrt[a + b*x^2])/5 + ((5*(6*b*B
- 5*a*D)*x^3*Sqrt[a + b*x^2])/(4*b) - (3*((-8*(5*A*b - 4*a*C)*x^2*Sqrt[a +
 b*x^2])/3 + (a*((15*(6*b*B - 5*a*D)*x*Sqrt[a + b*x^2])/(2*b) - (-32*(5*A*
b - 4*a*C)*Sqrt[a + b*x^2] + (15*a*(6*b*B - 5*a*D)*ArcTanh[(Sqrt[b]*x)/Sqr
t[a + b*x^2]])/Sqrt[b])/(2*b)))/3))/(4*b))/5)/(6*b)
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

rule 2340
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Simp[1/(b*(m
+ q + 2*p + 1))   Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)
*Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /;
GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ
[Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

## Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.29

| method | result |
|---|---|
| default | $A\left(\frac{x^{2}\sqrt{b\,x^{2}+a}}{3b}-\frac{2a\sqrt{b\,x^{2}+a}}{3b^{2}}\right)+B\left(\frac{x^{3}\sqrt{b\,x^{2}+a}}{4b}-\frac{3a\left(\frac{x\sqrt{b\,x^{2}+a}}{2b}-\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{2b^{\frac{3}{2}}}\right)}{4b}\right)+C\left(\frac{x^{4}\sqrt{b\,x^{2}+a}}{5b}-\frac{4a}{}\right.$ |

input
```
int(x^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
A*(1/3*x^2/b*(b*x^2+a)^(1/2)-2/3*a/b^2*(b*x^2+a)^(1/2))+B*(1/4*x^3/b*(b*x^
2+a)^(1/2)-3/4*a/b*(1/2*x*(b*x^2+a)^(1/2)/b-1/2*a/b^(3/2)*ln(b^(1/2)*x+(b*
x^2+a)^(1/2))))+C*(1/5*x^4/b*(b*x^2+a)^(1/2)-4/5*a/b*(1/3*x^2/b*(b*x^2+a)^
(1/2)-2/3*a/b^2*(b*x^2+a)^(1/2)))+D*(1/6*x^5/b*(b*x^2+a)^(1/2)-5/6*a/b*(1/
4*x^3/b*(b*x^2+a)^(1/2)-3/4*a/b*(1/2*x*(b*x^2+a)^(1/2)/b-1/2*a/b^(3/2)*ln(
b^(1/2)*x+(b*x^2+a)^(1/2)))))
```

## Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.54

$$\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{\sqrt{a+bx^2}}\,dx$$
$$= \left[ -\frac{15\,(5\,Da^3-6\,Ba^2b)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\,(40\,Db^3x^5+48\,Cb^3x^4+128\,Ca^2b-}{480\,b^4}\right.$$

input
```
integrate(x^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/480*(15*(5*D*a^3 - 6*B*a^2*b)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)
*sqrt(b)*x - a) - 2*(40*D*b^3*x^5 + 48*C*b^3*x^4 + 128*C*a^2*b - 160*A*a*b
^2 - 10*(5*D*a*b^2 - 6*B*b^3)*x^3 - 16*(4*C*a*b^2 - 5*A*b^3)*x^2 + 15*(5*D
*a^2*b - 6*B*a*b^2)*x)*sqrt(b*x^2 + a))/b^4, 1/240*(15*(5*D*a^3 - 6*B*a^2*
b)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (40*D*b^3*x^5 + 48*C*b^3*
x^4 + 128*C*a^2*b - 160*A*a*b^2 - 10*(5*D*a*b^2 - 6*B*b^3)*x^3 - 16*(4*C*a
*b^2 - 5*A*b^3)*x^2 + 15*(5*D*a^2*b - 6*B*a*b^2)*x)*sqrt(b*x^2 + a))/b^4]
```

## Sympy [A] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.01

$$\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{\sqrt{a+bx^2}}\,dx$$
$$= \begin{cases} \frac{3a^2\left(B-\frac{5Da}{6b}\right)\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right)}{8b^2}+\sqrt{a+bx^2}\left(\frac{Cx^4}{5b}+\frac{Dx^5}{6b}-\frac{3ax\left(B-\frac{5Da}{6b}\right)}{8b^2}-\frac{2a\left(A-\frac{4Ca}{5b}\right)}{3b^2}+\frac{x}{}\right. \\ \frac{\frac{Ax^4}{4}+\frac{Bx^5}{5}+\frac{Cx^6}{6}+\frac{Dx^7}{7}}{\sqrt{a}} \end{cases}$$

input
```
integrate(x**3*(D*x**3+C*x**2+B*x+A)/(b*x**2+a)**(1/2),x)
```

output
```
Piecewise((3*a**2*(B - 5*D*a/(6*b))*Piecewise((log(2*sqrt(b)*sqrt(a + b*x*
*2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True))/(8*b**2) +
 sqrt(a + b*x**2)*(C*x**4/(5*b) + D*x**5/(6*b) - 3*a*x*(B - 5*D*a/(6*b))/(
8*b**2) - 2*a*(A - 4*C*a/(5*b))/(3*b**2) + x**3*(B - 5*D*a/(6*b))/(4*b) +
x**2*(A - 4*C*a/(5*b))/(3*b)), Ne(b, 0)), ((A*x**4/4 + B*x**5/5 + C*x**6/6
 + D*x**7/7)/sqrt(a), True))
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.13

$$\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{bx^2+a}Dx^5}{6\,b} + \frac{\sqrt{bx^2+a}Cx^4}{5\,b} - \frac{5\,\sqrt{bx^2+a}Dax^3}{24\,b^2} + \frac{\sqrt{bx^2+a}Bx^3}{4\,b} - \frac{4\,\sqrt{bx^2+a}Cax^2}{15\,b^2} + \frac{\sqrt{bx^2+a}Ax^2}{3\,b} + \frac{5\,\sqrt{bx^2+a}Da^2x}{16\,b^3} - \frac{3\,\sqrt{bx^2+a}Bax}{8\,b^2} - \frac{5\,Da^3\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{7}{2}}} + \frac{3\,Ba^2\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{5}{2}}} + \frac{8\,\sqrt{bx^2+a}Ca^2}{15\,b^3} - \frac{2\,\sqrt{bx^2+a}Aa}{3\,b^2}$$

input
```
integrate(x^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/6*sqrt(b*x^2 + a)*D*x^5/b + 1/5*sqrt(b*x^2 + a)*C*x^4/b - 5/24*sqrt(b*x^
2 + a)*D*a*x^3/b^2 + 1/4*sqrt(b*x^2 + a)*B*x^3/b - 4/15*sqrt(b*x^2 + a)*C*
a*x^2/b^2 + 1/3*sqrt(b*x^2 + a)*A*x^2/b + 5/16*sqrt(b*x^2 + a)*D*a^2*x/b^3
 - 3/8*sqrt(b*x^2 + a)*B*a*x/b^2 - 5/16*D*a^3*arcsinh(b*x/sqrt(a*b))/b^(7/
2) + 3/8*B*a^2*arcsinh(b*x/sqrt(a*b))/b^(5/2) + 8/15*sqrt(b*x^2 + a)*C*a^2
/b^3 - 2/3*sqrt(b*x^2 + a)*A*a/b^2
```

## Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.83

$$\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{\sqrt{a+bx^2}}\,dx$$
$$= \frac{1}{240}\sqrt{bx^2+a}\left(\left(2\left(\left(4\left(\frac{5\,Dx}{b}+\frac{6\,C}{b}\right)x-\frac{5\,(5\,Dab^4-6\,Bb^5)}{b^6}\right)x-\frac{8\,(4\,Cab^4-5\,Ab^5)}{b^6}\right)x+\frac{15\,(5\,Da^2\right.\right.$$
$$+\frac{(5\,Da^3-6\,Ba^2b)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{16\,b^{\frac{7}{2}}}$$

input
```
integrate(x^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/240*sqrt(b*x^2 + a)*((2*((4*(5*D*x/b + 6*C/b)*x - 5*(5*D*a*b^4 - 6*B*b^5
)/b^6)*x - 8*(4*C*a*b^4 - 5*A*b^5)/b^6)*x + 15*(5*D*a^2*b^3 - 6*B*a*b^4)/b
^6)*x + 32*(4*C*a^2*b^3 - 5*A*a*b^4)/b^6) + 1/16*(5*D*a^3 - 6*B*a^2*b)*log
(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(7/2)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{\sqrt{a+bx^2}}\,dx=\int \frac{x^3\,\left(A+B\,x+C\,x^2+x^3\,D\right)}{\sqrt{b\,x^2+a}}\,dx$$

input
```
int((x^3*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^(1/2),x)
```

output
```
int((x^3*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^(1/2), x)
```

## Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.18

$$\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{\sqrt{a+bx^2}}\,dx$$
$$= \frac{-160\sqrt{b\,x^2+a}\,a^2b^2+128\sqrt{b\,x^2+a}\,a^2bc+75\sqrt{b\,x^2+a}\,a^2bdx+80\sqrt{b\,x^2+a}\,a\,b^3x^2-90\sqrt{b\,x^2+a}\,a\,b^3x}{}$$

input
```
int(x^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(1/2),x)
```

output
```
( - 160*sqrt(a + b*x**2)*a**2*b**2 + 128*sqrt(a + b*x**2)*a**2*b*c + 75*sq
rt(a + b*x**2)*a**2*b*d*x + 80*sqrt(a + b*x**2)*a*b**3*x**2 - 90*sqrt(a +
b*x**2)*a*b**3*x - 64*sqrt(a + b*x**2)*a*b**2*c*x**2 - 50*sqrt(a + b*x**2)
*a*b**2*d*x**3 + 60*sqrt(a + b*x**2)*b**4*x**3 + 48*sqrt(a + b*x**2)*b**3*
c*x**4 + 40*sqrt(a + b*x**2)*b**3*d*x**5 - 75*sqrt(b)*log((sqrt(a + b*x**2
) + sqrt(b)*x)/sqrt(a))*a**3*d + 90*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b
)*x)/sqrt(a))*a**2*b**2)/(240*b**4)
```

# 3.91 $\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{\sqrt{a+bx^2}}\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 159

$$\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{\sqrt{a+bx^2}}\,dx = -\frac{a(bB-aD)\sqrt{a+bx^2}}{b^3}+\frac{(4Ab-3aC)x\sqrt{a+bx^2}}{8b^2} +\frac{Cx^3\sqrt{a+bx^2}}{4b}+\frac{(bB-2aD)\left(a+bx^2\right)^{3/2}}{3b^3} +\frac{D(a+bx^2)^{5/2}}{5b^3}-\frac{a(4Ab-3aC)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{5/2}}$$

output
```
-a*(B*b-D*a)*(b*x^2+a)^(1/2)/b^3+1/8*(4*A*b-3*C*a)*x*(b*x^2+a)^(1/2)/b^2+1
/4*C*x^3*(b*x^2+a)^(1/2)/b+1/3*(B*b-2*D*a)*(b*x^2+a)^(3/2)/b^3+1/5*D*(b*x^
2+a)^(5/2)/b^3-1/8*a*(4*A*b-3*C*a)*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(5
/2)
```

## Mathematica [A] (verified)

Time = 0.93 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.74

$$\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{\sqrt{a+bx^2}}\,dx$$
$$= \frac{\sqrt{a+bx^2}(64a^2D-ab(80B+x(45C+32Dx))+2b^2x(30A+x(20B+3x(5C+4Dx))))+30a\sqrt{b}(-4}{120b^3}$$

input
```
Integrate[(x^2*(A + B*x + C*x^2 + D*x^3))/Sqrt[a + b*x^2],x]
```

output
```
(Sqrt[a + b*x^2]*(64*a^2*D - a*b*(80*B + x*(45*C + 32*D*x)) + 2*b^2*x*(30*
A + x*(20*B + 3*x*(5*C + 4*D*x)))) + 30*a*Sqrt[b]*(-4*A*b + 3*a*C)*ArcTanh
[(Sqrt[b]*x)/(-Sqrt[a] + Sqrt[a + b*x^2])])/(120*b^3)
```

## Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.16, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2340, 2340, 27, 533, 533, 25, 27, 455, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\left(A+Bx+Cx^2+Dx^3\right)}{\sqrt{a+bx^2}}\,dx$$

$$\Big\downarrow \ 2340$$

$$\frac{\int \frac{x^2\left(5bCx^2+(5bB-4aD)x+5Ab\right)}{\sqrt{bx^2+a}}dx}{5b}+\frac{Dx^4\sqrt{a+bx^2}}{5b}$$

$$\Big\downarrow \ 2340$$

$$\frac{\frac{\int \frac{bx^2(5(4Ab-3aC)+4(5bB-4aD)x)}{\sqrt{bx^2+a}}dx}{4b}+\frac{5}{4}Cx^3\sqrt{a+bx^2}}{5b}+\frac{Dx^4\sqrt{a+bx^2}}{5b}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{1}{4}\int \frac{x^2(5(4Ab-3aC)+4(5bB-4aD)x)}{\sqrt{bx^2+a}}dx+\frac{5}{4}Cx^3\sqrt{a+bx^2}}{5b}+\frac{Dx^4\sqrt{a+bx^2}}{5b}$$

$\Big\downarrow$ 533

$$\frac{\frac{1}{4}\left(\frac{4x^2\sqrt{a+bx^2}(5bB-4aD)}{3b}-\frac{\int \frac{x(8a(5bB-4aD)-15b(4Ab-3aC)x)}{\sqrt{bx^2+a}}dx}{3b}\right)+\frac{5}{4}Cx^3\sqrt{a+bx^2}}{5b}+\frac{Dx^4\sqrt{a+bx^2}}{5b}$$

$\Big\downarrow$ 533

$$\frac{\frac{1}{4}\left(\frac{4x^2\sqrt{a+bx^2}(5bB-4aD)}{3b}-\frac{-\frac{\int -\frac{ab(15(4Ab-3aC)+16(5bB-4aD)x)}{\sqrt{bx^2+a}}dx}{2b}-\frac{15}{2}x\sqrt{a+bx^2}(4Ab-3aC)}{3b}\right)+\frac{5}{4}Cx^3\sqrt{a+bx^2}}{5b}+\frac{Dx^4\sqrt{a+bx^2}}{5b}$$

$\Big\downarrow$ 25

$$\frac{\frac{1}{4}\left(\frac{4x^2\sqrt{a+bx^2}(5bB-4aD)}{3b}-\frac{\frac{\int \frac{ab(15(4Ab-3aC)+16(5bB-4aD)x)}{\sqrt{bx^2+a}}dx}{2b}-\frac{15}{2}x\sqrt{a+bx^2}(4Ab-3aC)}{3b}\right)+\frac{5}{4}Cx^3\sqrt{a+bx^2}}{5b}+\frac{Dx^4\sqrt{a+bx^2}}{5b}$$

$\Big\downarrow$ 27

$$\frac{\frac{1}{4}\left(\frac{4x^2\sqrt{a+bx^2}(5bB-4aD)}{3b}-\frac{\frac{1}{2}a\int \frac{15(4Ab-3aC)+16(5bB-4aD)x}{\sqrt{bx^2+a}}dx-\frac{15}{2}x\sqrt{a+bx^2}(4Ab-3aC)}{3b}\right)+\frac{5}{4}Cx^3\sqrt{a+bx^2}}{5b}+\frac{Dx^4\sqrt{a+bx^2}}{5b}$$

$\Big\downarrow$ 455

$$\frac{\frac{1}{4}\left(\frac{4x^2\sqrt{a+bx^2}(5bB-4aD)}{3b}-\frac{\frac{1}{2}a\left(15(4Ab-3aC)\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{16\sqrt{a+bx^2}(5bB-4aD)}{b}\right)-\frac{15}{2}x\sqrt{a+bx^2}(4Ab-3aC)}{3b}\right)+\frac{5}{4}Cx^3\sqrt{a+bx^2}}{5b}+\frac{Dx^4\sqrt{a+bx^2}}{5b}$$

$\Big\downarrow$ 224

$$\frac{\frac{1}{4}\left(\frac{4x^2\sqrt{a+bx^2}(5bB-4aD)}{3b}-\frac{\frac{1}{2}a\left(15(4Ab-3aC)\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{16\sqrt{a+bx^2}(5bB-4aD)}{b}\right)-\frac{15}{2}x\sqrt{a+bx^2}(4Ab-3aC)}{3b}\right)+\frac{5}{4}Cx^3\sqrt{a}}{5b}$$

$$\frac{Dx^4\sqrt{a+bx^2}}{5b}$$

$\Big\downarrow$ 219

$$\frac{\frac{1}{4}\left(\frac{4x^2\sqrt{a+bx^2}(5bB-4aD)}{3b}-\frac{\frac{1}{2}a\left(\frac{15(4Ab-3aC)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}+\frac{16\sqrt{a+bx^2}(5bB-4aD)}{b}\right)-\frac{15}{2}x\sqrt{a+bx^2}(4Ab-3aC)}{3b}\right)+\frac{5}{4}Cx^3\sqrt{a+}}{5b}$$

$$\frac{Dx^4\sqrt{a+bx^2}}{5b}$$

input
```
Int[(x^2*(A + B*x + C*x^2 + D*x^3))/Sqrt[a + b*x^2],x]
```

output
```
(D*x^4*Sqrt[a + b*x^2])/(5*b) + ((5*C*x^3*Sqrt[a + b*x^2])/4 + ((4*(5*b*B
- 4*a*D)*x^2*Sqrt[a + b*x^2])/(3*b) - ((-15*(4*A*b - 3*a*C)*x*Sqrt[a + b*x
^2])/2 + (a*((16*(5*b*B - 4*a*D)*Sqrt[a + b*x^2])/b + (15*(4*A*b - 3*a*C)*
ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/Sqrt[b]))/2)/(3*b))/4)/(5*b)
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] && !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

rule 2340
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Simp[1/(b*(m
+ q + 2*p + 1))   Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)
*Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /;
GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ
[Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

## Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.26

| method | result |
|---|---|
| default | $A\left(\frac{x\sqrt{b\,x^{2}+a}}{2b}-\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{2b^{\frac{3}{2}}}\right)+B\left(\frac{x^{2}\sqrt{b\,x^{2}+a}}{3b}-\frac{2a\sqrt{b\,x^{2}+a}}{3b^{2}}\right)+C\left(\frac{x^{3}\sqrt{b\,x^{2}+a}}{4b}-\frac{3a\left(\frac{x\sqrt{b\,x^{2}+a}}{2b}-\frac{a\ln(}{}\right.}{4b}\right.$ |

input
```
int(x^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
A*(1/2*x*(b*x^2+a)^(1/2)/b-1/2*a/b^(3/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))+B*
(1/3*x^2/b*(b*x^2+a)^(1/2)-2/3*a/b^2*(b*x^2+a)^(1/2))+C*(1/4*x^3/b*(b*x^2+
a)^(1/2)-3/4*a/b*(1/2*x*(b*x^2+a)^(1/2)/b-1/2*a/b^(3/2)*ln(b^(1/2)*x+(b*x^
2+a)^(1/2))))+D*(1/5*x^4/b*(b*x^2+a)^(1/2)-4/5*a/b*(1/3*x^2/b*(b*x^2+a)^(1
/2)-2/3*a/b^2*(b*x^2+a)^(1/2)))
```

## Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.50

$$\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{\sqrt{a+bx^2}}\,dx$$
$$= \left[-\frac{15\,(3\,Ca^2-4\,Aab)\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\,(24\,Db^2x^4+30\,Cb^2x^3+64\,Da^2-80\,\ldots}{240\,b^3}\right.$$
$$-\frac{15\,(3\,Ca^2-4\,Aab)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(24\,Db^2x^4+30\,Cb^2x^3+64\,Da^2-80\,Bab-8\,(4\,Dab-5\ldots}{120\,b^3}$$

input
```
integrate(x^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/240*(15*(3*C*a^2 - 4*A*a*b)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*s
qrt(b)*x - a) - 2*(24*D*b^2*x^4 + 30*C*b^2*x^3 + 64*D*a^2 - 80*B*a*b - 8*(
4*D*a*b - 5*B*b^2)*x^2 - 15*(3*C*a*b - 4*A*b^2)*x)*sqrt(b*x^2 + a))/b^3, -
1/120*(15*(3*C*a^2 - 4*A*a*b)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a))
- (24*D*b^2*x^4 + 30*C*b^2*x^3 + 64*D*a^2 - 80*B*a*b - 8*(4*D*a*b - 5*B*b^
2)*x^2 - 15*(3*C*a*b - 4*A*b^2)*x)*sqrt(b*x^2 + a))/b^3]
```

## Sympy [A] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.05

$$\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{\sqrt{a+bx^2}}\,dx = \begin{cases} -\frac{a\left(A-\frac{3Ca}{4b}\right)\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right)}{2b} + \sqrt{a+bx^2}\left(\frac{Cx^3}{4b}+\frac{Dx^4}{5b}-\frac{2a\left(B-\frac{4Da}{5b}\right)}{3b^2}+\frac{x^2\left(B-\frac{4Da}{5b}\right)}{3b}+\frac{x(\right. \\ \frac{\frac{Ax^3}{3}+\frac{Bx^4}{4}+\frac{Cx^5}{5}+\frac{Dx^6}{6}}{\sqrt{a}} \end{cases}$$

input
```
integrate(x**2*(D*x**3+C*x**2+B*x+A)/(b*x**2+a)**(1/2),x)
```

output
```
Piecewise((-a*(A - 3*C*a/(4*b))*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2)
+ 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True))/(2*b) + sqrt(a
 + b*x**2)*(C*x**3/(4*b) + D*x**4/(5*b) - 2*a*(B - 4*D*a/(5*b))/(3*b**2) +
 x**2*(B - 4*D*a/(5*b))/(3*b) + x*(A - 3*C*a/(4*b))/(2*b)), Ne(b, 0)), ((A
*x**3/3 + B*x**4/4 + C*x**5/5 + D*x**6/6)/sqrt(a), True))
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.10

$$\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{bx^2+a}Dx^4}{5\,b}+\frac{\sqrt{bx^2+a}Cx^3}{4\,b}-\frac{4\,\sqrt{bx^2+a}Dax^2}{15\,b^2}+\frac{\sqrt{bx^2+a}Bx^2}{3\,b}-\frac{3\,\sqrt{bx^2+a}Cax}{8\,b^2}+\frac{\sqrt{bx^2+a}Ax}{2\,b}+\frac{3\,Ca^2\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{5}{2}}}-\frac{Aa\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{3}{2}}}+\frac{8\,\sqrt{bx^2+a}Da^2}{15\,b^3}-\frac{2\,\sqrt{bx^2+a}Ba}{3\,b^2}$$

input
```
integrate(x^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/5*sqrt(b*x^2 + a)*D*x^4/b + 1/4*sqrt(b*x^2 + a)*C*x^3/b - 4/15*sqrt(b*x^
2 + a)*D*a*x^2/b^2 + 1/3*sqrt(b*x^2 + a)*B*x^2/b - 3/8*sqrt(b*x^2 + a)*C*a
*x/b^2 + 1/2*sqrt(b*x^2 + a)*A*x/b + 3/8*C*a^2*arcsinh(b*x/sqrt(a*b))/b^(5
/2) - 1/2*A*a*arcsinh(b*x/sqrt(a*b))/b^(3/2) + 8/15*sqrt(b*x^2 + a)*D*a^2/
b^3 - 2/3*sqrt(b*x^2 + a)*B*a/b^2
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.83

$$\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{\sqrt{a+bx^2}}\,dx = \frac{1}{120}\sqrt{bx^2+a}\left(\left(2\left(3\left(\frac{4Dx}{b}+\frac{5C}{b}\right)x-\frac{4(4Dab^3-5Bb^4)}{b^5}\right)x-\frac{15(3Cab^3-4Ab^4)}{b^5}\right)x+\frac{16(4Da^2b^2-5Bab^3)}{b^5}\right) - \frac{(3Ca^2-4Aab)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{8b^{\frac{5}{2}}}$$

input
```
integrate(x^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/120*sqrt(b*x^2 + a)*((2*(3*(4*D*x/b + 5*C/b)*x - 4*(4*D*a*b^3 - 5*B*b^4)
/b^5)*x - 15*(3*C*a*b^3 - 4*A*b^4)/b^5)*x + 16*(4*D*a^2*b^2 - 5*B*a*b^3)/b
^5) - 1/8*(3*C*a^2 - 4*A*a*b)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/
2)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{\sqrt{a+bx^2}}\,dx = \int \frac{x^2\,(A+B\,x+C\,x^2+x^3\,D)}{\sqrt{b\,x^2+a}}\,dx$$

input
```
int((x^2*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^(1/2),x)
```

output
```
int((x^2*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^(1/2), x)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.15

$$\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{\sqrt{a+bx^2}}\,dx$$
$$= \frac{64\sqrt{b\,x^2+a}\,a^2d+60\sqrt{b\,x^2+a}\,a\,b^2x-80\sqrt{b\,x^2+a}\,a\,b^2-45\sqrt{b\,x^2+a}\,abcx-32\sqrt{b\,x^2+a}\,abd\,x^2+40\sqrt{}}{}$$

input
```
int(x^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(1/2),x)
```

output
```
(64*sqrt(a + b*x**2)*a**2*d + 60*sqrt(a + b*x**2)*a*b**2*x - 80*sqrt(a + b
*x**2)*a*b**2 - 45*sqrt(a + b*x**2)*a*b*c*x - 32*sqrt(a + b*x**2)*a*b*d*x*
*2 + 40*sqrt(a + b*x**2)*b**3*x**2 + 30*sqrt(a + b*x**2)*b**2*c*x**3 + 24*
sqrt(a + b*x**2)*b**2*d*x**4 - 60*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*
x)/sqrt(a))*a**2*b + 45*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a)
)*a**2*c)/(120*b**3)
```

# 3.92 $\int \frac{x(A+Bx+Cx^2+Dx^3)}{\sqrt{a+bx^2}}\,dx$



## Optimal result

Integrand size = 28, antiderivative size = 131

$$\int \frac{x(A+Bx+Cx^2+Dx^3)}{\sqrt{a+bx^2}}\,dx = \frac{(Ab-aC)\sqrt{a+bx^2}}{b^2}+\frac{(4bB-3aD)x\sqrt{a+bx^2}}{8b^2}+\frac{Dx^3\sqrt{a+bx^2}}{4b}+\frac{C(a+bx^2)^{3/2}}{3b^2}-\frac{a(4bB-3aD)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{5/2}}$$

output
```
(A*b-C*a)*(b*x^2+a)^(1/2)/b^2+1/8*(4*B*b-3*D*a)*x*(b*x^2+a)^(1/2)/b^2+1/4*
D*x^3*(b*x^2+a)^(1/2)/b+1/3*C*(b*x^2+a)^(3/2)/b^2-1/8*a*(4*B*b-3*D*a)*arct
anh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(5/2)
```

## Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.70

$$\int \frac{x(A+Bx+Cx^2+Dx^3)}{\sqrt{a+bx^2}}\,dx$$
$$=\frac{\sqrt{a+bx^2}(24Ab-16aC+12bBx-9aDx+8bCx^2+6bDx^3)}{24b^2}$$
$$-\frac{a(-4bB+3aD)\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{8b^{5/2}}$$

input
```
Integrate[(x*(A + B*x + C*x^2 + D*x^3))/Sqrt[a + b*x^2],x]
```

output
```
(Sqrt[a + b*x^2]*(24*A*b - 16*a*C + 12*b*B*x - 9*a*D*x + 8*b*C*x^2 + 6*b*D
*x^3))/(24*b^2) - (a*(-4*b*B + 3*a*D)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])
/(8*b^(5/2))
```

## Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.14, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2340, 2340, 27, 533, 455, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(A+Bx+Cx^2+Dx^3\right)}{\sqrt{a+bx^2}}\,dx$$
$$\Big\downarrow \text{2340}$$
$$\frac{\int \frac{x\left(4bCx^2+(4bB-3aD)x+4Ab\right)}{\sqrt{bx^2+a}}dx}{4b}+\frac{Dx^3\sqrt{a+bx^2}}{4b}$$
$$\Big\downarrow \text{2340}$$
$$\frac{\frac{\int \frac{bx(4(3Ab-2aC)+3(4bB-3aD)x)}{\sqrt{bx^2+a}}dx}{3b}+\frac{4}{3}Cx^2\sqrt{a+bx^2}}{4b}+\frac{Dx^3\sqrt{a+bx^2}}{4b}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{1}{3}\int \frac{x(4(3Ab-2aC)+3(4bB-3aD)x)}{\sqrt{bx^2+a}}dx+\frac{4}{3}Cx^2\sqrt{a+bx^2}}{4b}+\frac{Dx^3\sqrt{a+bx^2}}{4b}$$

$$\Big\downarrow \; 533$$

$$\frac{\frac{1}{3}\left(\frac{3x\sqrt{a+bx^2}(4bB-3aD)}{2b}-\frac{\int \frac{3a(4bB-3aD)-8b(3Ab-2aC)x}{\sqrt{bx^2+a}}dx}{2b}\right)+\frac{4}{3}Cx^2\sqrt{a+bx^2}}{4b}+\frac{Dx^3\sqrt{a+bx^2}}{4b}$$

$$\Big\downarrow \; 455$$

$$\frac{\frac{1}{3}\left(\frac{3x\sqrt{a+bx^2}(4bB-3aD)}{2b}-\frac{3a(4bB-3aD)\int \frac{1}{\sqrt{bx^2+a}}dx-8\sqrt{a+bx^2}(3Ab-2aC)}{2b}\right)+\frac{4}{3}Cx^2\sqrt{a+bx^2}}{4b}+\frac{Dx^3\sqrt{a+bx^2}}{4b}$$

$$\Big\downarrow \; 224$$

$$\frac{\frac{1}{3}\left(\frac{3x\sqrt{a+bx^2}(4bB-3aD)}{2b}-\frac{3a(4bB-3aD)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}-8\sqrt{a+bx^2}(3Ab-2aC)}{2b}\right)+\frac{4}{3}Cx^2\sqrt{a+bx^2}}{4b}+\frac{Dx^3\sqrt{a+bx^2}}{4b}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{1}{3}\left(\frac{3x\sqrt{a+bx^2}(4bB-3aD)}{2b}-\frac{\frac{3a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(4bB-3aD)}{\sqrt{b}}-8\sqrt{a+bx^2}(3Ab-2aC)}{2b}\right)+\frac{4}{3}Cx^2\sqrt{a+bx^2}}{4b}+\frac{Dx^3\sqrt{a+bx^2}}{4b}$$

input
```
Int[(x*(A + B*x + C*x^2 + D*x^3))/Sqrt[a + b*x^2],x]
```

output
```
(D*x^3*Sqrt[a + b*x^2])/(4*b) + ((4*C*x^2*Sqrt[a + b*x^2])/3 + ((3*(4*b*B
 - 3*a*D)*x*Sqrt[a + b*x^2])/(2*b) - (-8*(3*A*b - 2*a*C)*Sqrt[a + b*x^2] +
(3*a*(4*b*B - 3*a*D)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/Sqrt[b])/(2*b))
/3)/(4*b)
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

rule 2340
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Simp[1/(b*(m
+ q + 2*p + 1))   Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)
*Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /;
GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ
[Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

## Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.18

| method | result |
|---|---|
| default | $C\left(\frac{x^{2}\sqrt{b\,x^{2}+a}}{3b}-\frac{2a\sqrt{b\,x^{2}+a}}{3b^{2}}\right)+D\left(\frac{x^{3}\sqrt{b\,x^{2}+a}}{4b}-\frac{3a\left(\frac{x\sqrt{b\,x^{2}+a}}{2b}-\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{2b^{\frac{3}{2}}}\right)}{4b}\right)+\frac{A\sqrt{b\,x^{2}+a}}{b}+B\Bigg(\frac{x\sqrt{}}{}$ |

input
```
int(x*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
C*(1/3*x^2/b*(b*x^2+a)^(1/2)-2/3*a/b^2*(b*x^2+a)^(1/2))+D*(1/4*x^3/b*(b*x^
2+a)^(1/2)-3/4*a/b*(1/2*x*(b*x^2+a)^(1/2)/b-1/2*a/b^(3/2)*ln(b^(1/2)*x+(b*
x^2+a)^(1/2))))+A*(b*x^2+a)^(1/2)/b+B*(1/2*x*(b*x^2+a)^(1/2)/b-1/2*a/b^(3/
2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))
```

## Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.56

$$\int \frac{x(A+Bx+Cx^2+Dx^3)}{\sqrt{a+bx^2}}\,dx$$
$$=\Bigg[-\frac{3\,(3\,Da^2-4\,Bab)\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\,(6\,Db^2x^3+8\,Cb^2x^2-16\,Cab+24\,Ab^2}{48\,b^3}$$
$$-\frac{3\,(3\,Da^2-4\,Bab)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(6\,Db^2x^3+8\,Cb^2x^2-16\,Cab+24\,Ab^2-3\,(3\,Dab-4\,Bb^2}{24\,b^3}$$

input
```
integrate(x*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/48*(3*(3*D*a^2 - 4*B*a*b)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqr
t(b)*x - a) - 2*(6*D*b^2*x^3 + 8*C*b^2*x^2 - 16*C*a*b + 24*A*b^2 - 3*(3*D*
a*b - 4*B*b^2)*x)*sqrt(b*x^2 + a))/b^3, -1/24*(3*(3*D*a^2 - 4*B*a*b)*sqrt(
-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (6*D*b^2*x^3 + 8*C*b^2*x^2 - 16*C
*a*b + 24*A*b^2 - 3*(3*D*a*b - 4*B*b^2)*x)*sqrt(b*x^2 + a))/b^3]
```

## Sympy [A] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.09

$$\int \frac{x(A+Bx+Cx^2+Dx^3)}{\sqrt{a+bx^2}}\,dx = \begin{cases} -\frac{a\left(B-\frac{3Da}{4b}\right)\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right)}{2b} + \sqrt{a+bx^2}\left(\frac{Cx^2}{3b}+\frac{Dx^3}{4b}+\frac{x\left(B-\frac{3Da}{4b}\right)}{2b}+\frac{A-\frac{2Ca}{3b}}{b}\right) & \text{for } b \neq \\ \frac{\frac{Ax^2}{2}+\frac{Bx^3}{3}+\frac{Cx^4}{4}+\frac{Dx^5}{5}}{\sqrt{a}} & \text{other} \end{cases}$$

input
```
integrate(x*(D*x**3+C*x**2+B*x+A)/(b*x**2+a)**(1/2),x)
```

output
```
Piecewise((-a*(B - 3*D*a/(4*b))*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2)
+ 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True))/(2*b) + sqrt(a
 + b*x**2)*(C*x**2/(3*b) + D*x**3/(4*b) + x*(B - 3*D*a/(4*b))/(2*b) + (A -
 2*C*a/(3*b))/b), Ne(b, 0)), ((A*x**2/2 + B*x**3/3 + C*x**4/4 + D*x**5/5)/
sqrt(a), True))
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.02

$$\int \frac{x(A+Bx+Cx^2+Dx^3)}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{bx^2+a}Dx^3}{4\,b} + \frac{\sqrt{bx^2+a}Cx^2}{3\,b} - \frac{3\,\sqrt{bx^2+a}Dax}{8\,b^2} + \frac{\sqrt{bx^2+a}Bx}{2\,b} + \frac{3\,Da^2\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{5}{2}}} - \frac{Ba\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{3}{2}}} - \frac{2\,\sqrt{bx^2+a}Ca}{3\,b^2} + \frac{\sqrt{bx^2+a}A}{b}$$

input
```
integrate(x*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/4*sqrt(b*x^2 + a)*D*x^3/b + 1/3*sqrt(b*x^2 + a)*C*x^2/b - 3/8*sqrt(b*x^2
 + a)*D*a*x/b^2 + 1/2*sqrt(b*x^2 + a)*B*x/b + 3/8*D*a^2*arcsinh(b*x/sqrt(a
*b))/b^(5/2) - 1/2*B*a*arcsinh(b*x/sqrt(a*b))/b^(3/2) - 2/3*sqrt(b*x^2 + a
)*C*a/b^2 + sqrt(b*x^2 + a)*A/b
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.81

$$\int \frac{x(A+Bx+Cx^2+Dx^3)}{\sqrt{a+bx^2}}\,dx = \frac{1}{24}\sqrt{bx^2+a}\left(\left(2\left(\frac{3\,Dx}{b}+\frac{4\,C}{b}\right)x-\frac{3\,(3\,Dab^2-4\,Bb^3)}{b^4}\right)x-\frac{8\,(2\,Cab^2-3\,Ab^3)}{b^4}\right) - \frac{(3\,Da^2-4\,Bab)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{8\,b^{\frac{5}{2}}}$$

input
```
integrate(x*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/24*sqrt(b*x^2 + a)*((2*(3*D*x/b + 4*C/b)*x - 3*(3*D*a*b^2 - 4*B*b^3)/b^4
)*x - 8*(2*C*a*b^2 - 3*A*b^3)/b^4) - 1/8*(3*D*a^2 - 4*B*a*b)*log(abs(-sqrt
(b)*x + sqrt(b*x^2 + a)))/b^(5/2)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x(A+Bx+Cx^2+Dx^3)}{\sqrt{a+bx^2}}\,dx = \int \frac{x\,(A+B\,x+C\,x^2+x^3\,D)}{\sqrt{b\,x^2+a}}\,dx$$

input
```
int((x*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^(1/2),x)
```

output
```
int((x*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^(1/2), x)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.14

$$\int \frac{x(A+Bx+Cx^2+Dx^3)}{\sqrt{a+bx^2}}\,dx$$
$$= \frac{24\sqrt{b\,x^2+a}\,a\,b^2-16\sqrt{b\,x^2+a}\,abc-9\sqrt{b\,x^2+a}\,abdx+12\sqrt{b\,x^2+a}\,b^3x+8\sqrt{b\,x^2+a}\,b^2c\,x^2+6\sqrt{b\,x^2}}{24b^3}$$

input
```
int(x*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(1/2),x)
```

output
```
(24*sqrt(a + b*x**2)*a*b**2 - 16*sqrt(a + b*x**2)*a*b*c - 9*sqrt(a + b*x**
2)*a*b*d*x + 12*sqrt(a + b*x**2)*b**3*x + 8*sqrt(a + b*x**2)*b**2*c*x**2 +
 6*sqrt(a + b*x**2)*b**2*d*x**3 + 9*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b
)*x)/sqrt(a))*a**2*d - 12*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(
a))*a*b**2)/(24*b**3)
```

# 3.93 $\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{a+bx^2}}\,dx$



## Optimal result

Integrand size = 27, antiderivative size = 100

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{a+bx^2}}\,dx = \frac{(bB-aD)\sqrt{a+bx^2}}{b^2}+\frac{Cx\sqrt{a+bx^2}}{2b}+\frac{D(a+bx^2)^{3/2}}{3b^2}+\frac{(2Ab-aC)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}$$

output
```
(B*b-D*a)*(b*x^2+a)^(1/2)/b^2+1/2*C*x*(b*x^2+a)^(1/2)/b+1/3*D*(b*x^2+a)^(3
/2)/b^2+1/2*(2*A*b-C*a)*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(3/2)
```

## Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.78

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{a+bx^2}(6bB-4aD+3bCx+2bDx^2)}{6b^2}+\frac{(-2Ab+aC)\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{2b^{3/2}}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/Sqrt[a + b*x^2],x]
```

output
```
(Sqrt[a + b*x^2]*(6*b*B - 4*a*D + 3*b*C*x + 2*b*D*x^2))/(6*b^2) + ((-2*A*b
 + a*C)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(2*b^(3/2))
```

## Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.13, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2346, 2346, 27, 455, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{a+bx^2}}\,dx$$

$$\Big\downarrow \text{ 2346}$$

$$\frac{\int \frac{3bCx^2+(3bB-2aD)x+3Ab}{\sqrt{bx^2+a}}dx}{3b}+\frac{Dx^2\sqrt{a+bx^2}}{3b}$$

$$\Big\downarrow \text{ 2346}$$

$$\frac{\frac{\int \frac{b(3(2Ab-aC)+2(3bB-2aD)x)}{\sqrt{bx^2+a}}dx}{2b}+\frac{3}{2}Cx\sqrt{a+bx^2}}{3b}+\frac{Dx^2\sqrt{a+bx^2}}{3b}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{1}{2}\int \frac{3(2Ab-aC)+2(3bB-2aD)x}{\sqrt{bx^2+a}}dx+\frac{3}{2}Cx\sqrt{a+bx^2}}{3b}+\frac{Dx^2\sqrt{a+bx^2}}{3b}$$

$$\Big\downarrow \text{ 455}$$

$$\frac{\frac{1}{2}\left(3(2Ab-aC)\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{2\sqrt{a+bx^2}(3bB-2aD)}{b}\right)+\frac{3}{2}Cx\sqrt{a+bx^2}}{3b}+\frac{Dx^2\sqrt{a+bx^2}}{3b}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{\frac{1}{2}\left(3(2Ab-aC)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{2\sqrt{a+bx^2}(3bB-2aD)}{b}\right)+\frac{3}{2}Cx\sqrt{a+bx^2}}{3b}+\frac{Dx^2\sqrt{a+bx^2}}{3b}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\frac{1}{2}\left(\frac{3(2Ab-aC)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}+\frac{2\sqrt{a+bx^2}(3bB-2aD)}{b}\right)+\frac{3}{2}Cx\sqrt{a+bx^2}}{3b}+\frac{Dx^2\sqrt{a+bx^2}}{3b}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/Sqrt[a + b*x^2],x]
```

output
```
(D*x^2*Sqrt[a + b*x^2])/(3*b) + ((3*C*x*Sqrt[a + b*x^2])/2 + ((2*(3*b*B -
2*a*D)*Sqrt[a + b*x^2])/b + (3*(2*A*b - a*C)*ArcTanh[(Sqrt[b]*x)/Sqrt[a +
b*x^2]])/Sqrt[b])/2)/(3*b)
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

## Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.12

| method | result |
|---|---|
| default | $\frac{A\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{\sqrt{b}}+\frac{B\sqrt{b\,x^{2}+a}}{b}+C\left(\frac{x\sqrt{b\,x^{2}+a}}{2b}-\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{2b^{\frac{3}{2}}}\right)+D\left(\frac{x^{2}\sqrt{b\,x^{2}+a}}{3b}-\frac{2a\sqrt{b\,x^{2}+a}}{3b^{2}}\right)$ |

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
A*ln(b^(1/2)*x+(b*x^2+a)^(1/2))/b^(1/2)+B*(b*x^2+a)^(1/2)/b+C*(1/2*x*(b*x^
2+a)^(1/2)/b-1/2*a/b^(3/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))+D*(1/3*x^2/b*(b*
x^2+a)^(1/2)-2/3*a/b^2*(b*x^2+a)^(1/2))
```

## Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.47

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{a+bx^2}}\,dx = \left[-\frac{3\,(Ca-2\,Ab)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\,(2\,Dbx^2+3\,Cbx-4\,Da+6\,Bb)\sqrt{bx^2+a}}{12\,b^2},\right.$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/12*(3*(C*a - 2*A*b)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x
 - a) - 2*(2*D*b*x^2 + 3*C*b*x - 4*D*a + 6*B*b)*sqrt(b*x^2 + a))/b^2, 1/6*
(3*(C*a - 2*A*b)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (2*D*b*x^2
+ 3*C*b*x - 4*D*a + 6*B*b)*sqrt(b*x^2 + a))/b^2]
```

## Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.16

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{a+bx^2}}\,dx = \begin{cases} \left(A-\frac{Ca}{2b}\right)\left(\begin{cases} \frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for}\ a\neq 0 \\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise} \end{cases}\right)+\sqrt{a+bx^2}\left(\frac{Cx}{2b}+\frac{Dx^2}{3b}+\frac{B-\frac{2Da}{3b}}{b}\right) & \text{for}\ b\neq 0 \\ \frac{Ax+\frac{Bx^2}{2}+\frac{Cx^3}{3}+\frac{Dx^4}{4}}{\sqrt{a}} & \text{otherwise} \end{cases}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(b*x**2+a)**(1/2),x)
```

output
```
Piecewise(((A - C*a/(2*b))*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b
*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True)) + sqrt(a + b*x**2)*
(C*x/(2*b) + D*x**2/(3*b) + (B - 2*D*a/(3*b))/b), Ne(b, 0)), ((A*x + B*x**
2/2 + C*x**3/3 + D*x**4/4)/sqrt(a), True))
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.95

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{bx^2+a}Dx^2}{3\,b}+\frac{\sqrt{bx^2+a}Cx}{2\,b}-\frac{Ca\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{3}{2}}}+\frac{A\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}}-\frac{2\sqrt{bx^2+a}Da}{3\,b^2}+\frac{\sqrt{bx^2+a}B}{b}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/3*sqrt(b*x^2 + a)*D*x^2/b + 1/2*sqrt(b*x^2 + a)*C*x/b - 1/2*C*a*arcsinh(
b*x/sqrt(a*b))/b^(3/2) + A*arcsinh(b*x/sqrt(a*b))/sqrt(b) - 2/3*sqrt(b*x^2
 + a)*D*a/b^2 + sqrt(b*x^2 + a)*B/b
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.77

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{a+bx^2}}\,dx = \frac{1}{6}\sqrt{bx^2+a}\left(\left(\frac{2\,Dx}{b}+\frac{3\,C}{b}\right)x-\frac{2\,(2\,Dab-3\,Bb^2)}{b^3}\right) + \frac{(Ca-2\,Ab)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2\,b^{\frac{3}{2}}}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/6*sqrt(b*x^2 + a)*((2*D*x/b + 3*C/b)*x - 2*(2*D*a*b - 3*B*b^2)/b^3) + 1/
2*(C*a - 2*A*b)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2)
```

## Mupad [B] (verification not implemented)

Time = 3.13 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.49

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{a+bx^2}}\,dx = \begin{cases} \frac{B\,x^2}{2\,\sqrt{a}}+\frac{C\,x^3}{3\,\sqrt{a}}+\frac{x^4\,D}{4\,\sqrt{a}}+\frac{A\,x}{\sqrt{a}} & \text{if } b= \\ \frac{{\left(b\,x^2+a\right)}^{3/2}\,D-3\,a\,\sqrt{b\,x^2+a}\,D}{3\,b^2}+\frac{B\,\sqrt{b\,x^2+a}}{b}+\frac{A\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}-\frac{C\,a\,\ln\left(2\,\sqrt{b}\,x+2\,\sqrt{b\,x^2+a}\right)}{2\,b^{3/2}}+\frac{C\,x\,\sqrt{b\,x^2+a}}{2\,b} & \text{if } b\neq \end{cases}$$

input
```
int((A + B*x + C*x^2 + x^3*D)/(a + b*x^2)^(1/2),x)
```

output
```
piecewise(b == 0, (B*x^2)/(2*a^(1/2)) + (C*x^3)/(3*a^(1/2)) + (x^4*D)/(4*a
^(1/2)) + (A*x)/a^(1/2), b ~= 0, ((a + b*x^2)^(3/2)*D - 3*a*(a + b*x^2)^(1
/2)*D)/(3*b^2) + (B*(a + b*x^2)^(1/2))/b + (A*log(b^(1/2)*x + (a + b*x^2)^
(1/2)))/b^(1/2) - (C*a*log(2*b^(1/2)*x + 2*(a + b*x^2)^(1/2)))/(2*b^(3/2))
 + (C*x*(a + b*x^2)^(1/2))/(2*b))
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.09

$$\int \frac{A + Bx + Cx^2 + Dx^3}{\sqrt{a + bx^2}}\,dx$$
$$= \frac{-4\sqrt{b\,x^2 + a}\,ad + 6\sqrt{b\,x^2 + a}\,b^2 + 3\sqrt{b\,x^2 + a}\,bcx + 2\sqrt{b\,x^2 + a}\,bd\,x^2 + 6\sqrt{b}\log\left(\frac{\sqrt{b\,x^2+a}+\sqrt{b}\,x}{\sqrt{a}}\right)ab - 3\sqrt{b}}{6b^2}$$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(1/2),x)
```

output
```
( - 4*sqrt(a + b*x**2)*a*d + 6*sqrt(a + b*x**2)*b**2 + 3*sqrt(a + b*x**2)*
b*c*x + 2*sqrt(a + b*x**2)*b*d*x**2 + 6*sqrt(b)*log((sqrt(a + b*x**2) + sq
rt(b)*x)/sqrt(a))*a*b - 3*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(
a))*a*c)/(6*b**2)
```

# 3.94 $\int \frac{A+Bx+Cx^2+Dx^3}{x\sqrt{a+bx^2}}\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 100

$$\int \frac{A+Bx+Cx^2+Dx^3}{x\sqrt{a+bx^2}}\,dx=\frac{C\sqrt{a+bx^2}}{b}+\frac{Dx\sqrt{a+bx^2}}{2b}+\frac{(2bB-aD)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}-\frac{A\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

output
```
C*(b*x^2+a)^(1/2)/b+1/2*D*x*(b*x^2+a)^(1/2)/b+1/2*(2*B*b-D*a)*arctanh(b^(1
/2)*x/(b*x^2+a)^(1/2))/b^(3/2)-A*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(1/2)
```

## Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.07

$$\int \frac{A+Bx+Cx^2+Dx^3}{x\sqrt{a+bx^2}}\,dx=\frac{(2C+Dx)\sqrt{a+bx^2}}{2b}+\frac{(2bB-aD)\text{arctanh}\left(\frac{\sqrt{b}x}{-\sqrt{a}+\sqrt{a+bx^2}}\right)}{b^{3/2}}-\frac{A\log(x)}{\sqrt{a}}+\frac{A\log\left(-\sqrt{a}+\sqrt{a+bx^2}\right)}{\sqrt{a}}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/(x*Sqrt[a + b*x^2]),x]
```

output
```
((2*C + D*x)*Sqrt[a + b*x^2])/(2*b) + ((2*b*B - a*D)*ArcTanh[(Sqrt[b]*x)/(
-Sqrt[a] + Sqrt[a + b*x^2])])/b^(3/2) - (A*Log[x])/Sqrt[a] + (A*Log[-Sqrt[
a] + Sqrt[a + b*x^2]])/Sqrt[a]
```

## Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {2340, 2340, 27, 538, 224, 219, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{x\sqrt{a+bx^2}}dx$$

$$\Big\downarrow \text{2340}$$

$$\frac{\int \frac{2bCx^2+(2bB-aD)x+2Ab}{x\sqrt{bx^2+a}}dx}{2b}+\frac{Dx\sqrt{a+bx^2}}{2b}$$

$$\Big\downarrow \text{2340}$$

$$\frac{\frac{\int \frac{b(2Ab+(2bB-aD)x)}{x\sqrt{bx^2+a}}dx}{b}+2C\sqrt{a+bx^2}}{2b}+\frac{Dx\sqrt{a+bx^2}}{2b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{2Ab+(2bB-aD)x}{x\sqrt{bx^2+a}}dx+2C\sqrt{a+bx^2}}{2b}+\frac{Dx\sqrt{a+bx^2}}{2b}$$

$$\Big\downarrow \text{538}$$

$$\frac{2Ab\int \frac{1}{x\sqrt{bx^2+a}}dx+(2bB-aD)\int \frac{1}{\sqrt{bx^2+a}}dx+2C\sqrt{a+bx^2}}{2b}+\frac{Dx\sqrt{a+bx^2}}{2b}$$

$$\Big\downarrow \text{224}$$

$$\frac{2Ab\int \frac{1}{x\sqrt{bx^2+a}}dx+(2bB-aD)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+2C\sqrt{a+bx^2}}{2b}+\frac{Dx\sqrt{a+bx^2}}{2b}$$

$$\Big\downarrow \text{219}$$

$$\frac{2Ab\int \frac{1}{x\sqrt{bx^2+a}}dx + \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bB-aD)}{\sqrt{b}} + 2C\sqrt{a+bx^2}}{2b} + \frac{Dx\sqrt{a+bx^2}}{2b}$$

$$\Big\downarrow \text{243}$$

$$\frac{Ab\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2 + \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bB-aD)}{\sqrt{b}} + 2C\sqrt{a+bx^2}}{2b} + \frac{Dx\sqrt{a+bx^2}}{2b}$$

$$\Big\downarrow \text{73}$$

$$\frac{2A\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bB-aD)}{\sqrt{b}} + 2C\sqrt{a+bx^2}}{2b} + \frac{Dx\sqrt{a+bx^2}}{2b}$$

$$\Big\downarrow \text{221}$$

$$\frac{-\frac{2Ab\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bB-aD)}{\sqrt{b}} + 2C\sqrt{a+bx^2}}{2b} + \frac{Dx\sqrt{a+bx^2}}{2b}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/(x*Sqrt[a + b*x^2]),x]
```

output
```
(D*x*Sqrt[a + b*x^2])/(2*b) + (2*C*Sqrt[a + b*x^2] + ((2*b*B - a*D)*ArcTan
h[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/Sqrt[b] - (2*A*b*ArcTanh[Sqrt[a + b*x^2]/S
qrt[a]])/Sqrt[a])/(2*b)
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 2340
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Simp[1/(b*(m
+ q + 2*p + 1))   Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)
*Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /;
GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ
[Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

## Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.06

| method | result | size |
|---|---|---|
| default | $\frac{B\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{\sqrt{b}}-\frac{A\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{\sqrt{a}}+\frac{C\sqrt{b\,x^{2}+a}}{b}+D\left(\frac{x\sqrt{b\,x^{2}+a}}{2b}-\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{2b^{\frac{3}{2}}}\right)$ | 106 |

input
```
int((D*x^3+C*x^2+B*x+A)/x/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
B*ln(b^(1/2)*x+(b*x^2+a)^(1/2))/b^(1/2)-A/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2
+a)^(1/2))/x)+C/b*(b*x^2+a)^(1/2)+D*(1/2*x*(b*x^2+a)^(1/2)/b-1/2*a/b^(3/2)
*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))
```

## Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 410, normalized size of antiderivative = 4.10

$$\int \frac{A+Bx+Cx^2+Dx^3}{x\sqrt{a+bx^2}}\,dx$$
$$= \left[\frac{2\,A\sqrt{a}b^2\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)-(Da^2-2\,Bab)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,(Dabx}{4\,ab^2}\right.$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/x/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(2*A*sqrt(a)*b^2*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2)
- (D*a^2 - 2*B*a*b)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a
) + 2*(D*a*b*x + 2*C*a*b)*sqrt(b*x^2 + a))/(a*b^2), 1/2*(A*sqrt(a)*b^2*log
(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + (D*a^2 - 2*B*a*b)*sqrt(
-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (D*a*b*x + 2*C*a*b)*sqrt(b*x^2 +
a))/(a*b^2), 1/4*(4*A*sqrt(-a)*b^2*arctan(sqrt(b*x^2 + a)*sqrt(-a)/a) - (D
*a^2 - 2*B*a*b)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) +
2*(D*a*b*x + 2*C*a*b)*sqrt(b*x^2 + a))/(a*b^2), 1/2*(2*A*sqrt(-a)*b^2*arct
an(sqrt(b*x^2 + a)*sqrt(-a)/a) + (D*a^2 - 2*B*a*b)*sqrt(-b)*arctan(sqrt(-b
)*x/sqrt(b*x^2 + a)) + (D*a*b*x + 2*C*a*b)*sqrt(b*x^2 + a))/(a*b^2)]
```

## Sympy [A] (verification not implemented)

Time = 1.97 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.68

$$\int \frac{A+Bx+Cx^2+Dx^3}{x\sqrt{a+bx^2}}\,dx$$

$$= -\frac{A\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{\sqrt{a}} + B\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0 \wedge b\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{for } b\neq 0\\ \frac{x}{\sqrt{a}} & \text{otherwise}\end{cases}\right) + C\left(\begin{cases}\frac{\sqrt{a+bx^2}}{b} & \text{for } b\neq 0\\ \frac{x^2}{2\sqrt{a}} & \text{otherwise}\end{cases}\right) + D\left(\begin{cases}-\frac{a\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right)}{2b}+\frac{x\sqrt{a+bx^2}}{2b} & \text{for } b\neq 0\\ \frac{x^3}{3\sqrt{a}} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/x/(b*x**2+a)**(1/2),x)
```

output
```
-A*asinh(sqrt(a)/(sqrt(b)*x))/sqrt(a) + B*Piecewise((log(2*sqrt(b)*sqrt(a
+ b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0) & Ne(b, 0)), (x*log(x)/sqrt(b*x**2),
Ne(b, 0)), (x/sqrt(a), True)) + C*Piecewise((sqrt(a + b*x**2)/b, Ne(b, 0))
, (x**2/(2*sqrt(a)), True)) + D*Piecewise((-a*Piecewise((log(2*sqrt(b)*sqr
t(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True))/
(2*b) + x*sqrt(a + b*x**2)/(2*b), Ne(b, 0)), (x**3/(3*sqrt(a)), True))
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.79

$$\int \frac{A+Bx+Cx^2+Dx^3}{x\sqrt{a+bx^2}}\,dx = \frac{\sqrt{bx^2+a}Dx}{2b} - \frac{Da\,\text{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2b^{\frac{3}{2}}} + \frac{B\,\text{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}} - \frac{A\,\text{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{\sqrt{a}} + \frac{\sqrt{bx^2+a}C}{b}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/x/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(b*x^2 + a)*D*x/b - 1/2*D*a*arcsinh(b*x/sqrt(a*b))/b^(3/2) + B*arc
sinh(b*x/sqrt(a*b))/sqrt(b) - A*arcsinh(a/(sqrt(a*b)*abs(x)))/sqrt(a) + sq
rt(b*x^2 + a)*C/b
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{A+Bx+Cx^2+Dx^3}{x\sqrt{a+bx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/x/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

## Mupad [B] (verification not implemented)

Time = 3.15 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.37

$$\int \frac{A+Bx+Cx^2+Dx^3}{x\sqrt{a+bx^2}}\,dx = \begin{cases} \frac{C\,x^2}{2\,\sqrt{a}}+\frac{x^3\,D}{3\,\sqrt{a}}+\frac{A\,\ln(x)}{\sqrt{a}}+\frac{B\,x}{\sqrt{a}} & \text{if } b=0 \\ \frac{C\,\sqrt{b\,x^2+a}}{b}+\frac{B\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}-\frac{A\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{x\,\sqrt{b\,x^2+a}\,D}{2\,b}-\frac{a\,\ln\left(2\,\sqrt{b}\,x+2\,\sqrt{b\,x^2+a}\right)\,D}{2\,b^{3/2}} & \text{if } b\neq 0 \end{cases}$$

input
```
int((A + B*x + C*x^2 + x^3*D)/(x*(a + b*x^2)^(1/2)),x)
```

output
```
piecewise(b == 0, (C*x^2)/(2*a^(1/2)) + (x^3*D)/(3*a^(1/2)) + (A*log(x))/a
^(1/2) + (B*x)/a^(1/2), b ~= 0, (C*(a + b*x^2)^(1/2))/b + (B*log(b^(1/2)*x
 + (a + b*x^2)^(1/2)))/b^(1/2) - (A*atanh((a + b*x^2)^(1/2)/a^(1/2)))/a^(1
/2) + (x*(a + b*x^2)^(1/2)*D)/(2*b) - (a*log(2*b^(1/2)*x + 2*(a + b*x^2)^(
1/2))*D)/(2*b^(3/2)))
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.39

$$\int \frac{A+Bx+Cx^2+Dx^3}{x\sqrt{a+bx^2}}\,dx = \frac{2\sqrt{b\,x^2+a}\,bc+\sqrt{b\,x^2+a}\,bdx+2\sqrt{a}\log\left(\frac{\sqrt{b\,x^2+a}-\sqrt{a}+\sqrt{b}\,x}{\sqrt{a}}\right)b^2-2\sqrt{a}\log\left(\frac{\sqrt{b\,x^2+a}+\sqrt{a}+\sqrt{b}\,x}{\sqrt{a}}\right)b^2-\sqrt{b}\log\left(\cdots\right.}{2b^2}$$

input
```
int((D*x^3+C*x^2+B*x+A)/x/(b*x^2+a)^(1/2),x)
```

output
```
(2*sqrt(a + b*x**2)*b*c + sqrt(a + b*x**2)*b*d*x + 2*sqrt(a)*log((sqrt(a +
 b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*b**2 - 2*sqrt(a)*log((sqrt(a + b*
x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*b**2 - sqrt(b)*log((sqrt(a + b*x**2)
 + sqrt(b)*x)/sqrt(a))*a*d + 2*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/
sqrt(a))*b**2)/(2*b**2)
```

# 3.95 $\int \frac{A+Bx+Cx^2+Dx^3}{x^2\sqrt{a+bx^2}}\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 89

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^2\sqrt{a+bx^2}}\,dx = \frac{D\sqrt{a+bx^2}}{b} - \frac{A\sqrt{a+bx^2}}{ax} + \frac{C\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}} - \frac{B\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

output
```
D*(b*x^2+a)^(1/2)/b-A*(b*x^2+a)^(1/2)/a/x+C*arctanh(b^(1/2)*x/(b*x^2+a)^(1
/2))/b^(1/2)-B*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(1/2)
```

## Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.08

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^2\sqrt{a+bx^2}}\,dx = \frac{(-Ab+aDx)\sqrt{a+bx^2}}{abx} + \frac{2B\text{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}} - \frac{C\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{\sqrt{b}}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/(x^2*Sqrt[a + b*x^2]),x]
```

output
```
((-(A*b) + a*D*x)*Sqrt[a + b*x^2])/(a*b*x) + (2*B*ArcTanh[(Sqrt[b]*x - Sqr
t[a + b*x^2])/Sqrt[a]])/Sqrt[a] - (C*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/
Sqrt[b]
```

## Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.10, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2338, 25, 2340, 27, 538, 224, 219, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^2\sqrt{a+bx^2}}\,dx$$

$$\Big\downarrow \text{2338}$$

$$-\frac{\int -\frac{aDx^2+aCx+aB}{x\sqrt{bx^2+a}}dx}{a}-\frac{A\sqrt{a+bx^2}}{ax}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{aDx^2+aCx+aB}{x\sqrt{bx^2+a}}dx}{a}-\frac{A\sqrt{a+bx^2}}{ax}$$

$$\Big\downarrow \text{2340}$$

$$\frac{\frac{\int \frac{ab(B+Cx)}{x\sqrt{bx^2+a}}dx}{b}+\frac{aD\sqrt{a+bx^2}}{b}}{a}-\frac{A\sqrt{a+bx^2}}{ax}$$

$$\Big\downarrow \text{27}$$

$$\frac{a\int \frac{B+Cx}{x\sqrt{bx^2+a}}dx+\frac{aD\sqrt{a+bx^2}}{b}}{a}-\frac{A\sqrt{a+bx^2}}{ax}$$

$$\Big\downarrow \text{538}$$

$$\frac{a\left(B\int \frac{1}{x\sqrt{bx^2+a}}dx+C\int \frac{1}{\sqrt{bx^2+a}}dx\right)+\frac{aD\sqrt{a+bx^2}}{b}}{a}-\frac{A\sqrt{a+bx^2}}{ax}$$

$$\Big\downarrow \text{224}$$

$$\frac{a\left(B\int \frac{1}{x\sqrt{bx^2+a}}dx+C\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}\right)+\frac{aD\sqrt{a+bx^2}}{b}}{a}-\frac{A\sqrt{a+bx^2}}{ax}$$

$$\Big\downarrow \textbf{219}$$

$$\frac{a\left(B\int \frac{1}{x\sqrt{bx^2+a}}dx+\frac{C\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}\right)+\frac{aD\sqrt{a+bx^2}}{b}}{a}-\frac{A\sqrt{a+bx^2}}{ax}$$

$$\Big\downarrow \textbf{243}$$

$$\frac{a\left(\frac{1}{2}B\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2+\frac{C\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}\right)+\frac{aD\sqrt{a+bx^2}}{b}}{a}-\frac{A\sqrt{a+bx^2}}{ax}$$

$$\Big\downarrow \textbf{73}$$

$$\frac{a\left(\frac{B\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{b}+\frac{C\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}\right)+\frac{aD\sqrt{a+bx^2}}{b}}{a}-\frac{A\sqrt{a+bx^2}}{ax}$$

$$\Big\downarrow \textbf{221}$$

$$\frac{a\left(\frac{C\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}-\frac{B\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}\right)+\frac{aD\sqrt{a+bx^2}}{b}}{a}-\frac{A\sqrt{a+bx^2}}{ax}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/(x^2*Sqrt[a + b*x^2]),x]
```

output
```
-((A*Sqrt[a + b*x^2])/(a*x)) + ((a*D*Sqrt[a + b*x^2])/b + a*((C*ArcTanh[(S
qrt[b]*x)/Sqrt[a + b*x^2]])/Sqrt[b] - (B*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])
/Sqrt[a]))/a
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

rule 2340
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Simp[1/(b*(m
+ q + 2*p + 1))   Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)
*Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /;
GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ
[Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

## Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.94

| method | result | size |
|---|---|---|
| default | $\frac{C \ln\left(\sqrt{b}\, x+\sqrt{b\,x^{2}+a}\right)}{\sqrt{b}}-\frac{A\sqrt{b\,x^{2}+a}}{a x}-\frac{B \ln\left(\frac{2 a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{\sqrt{a}}+\frac{D\sqrt{b\,x^{2}+a}}{b}$ | 84 |

input
```
int((D*x^3+C*x^2+B*x+A)/x^2/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
C*ln(b^(1/2)*x+(b*x^2+a)^(1/2))/b^(1/2)-A*(b*x^2+a)^(1/2)/a/x-B/a^(1/2)*ln
((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)+D*(b*x^2+a)^(1/2)/b
```

## Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 379, normalized size of antiderivative = 4.26

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^2\sqrt{a+bx^2}}\,dx$$
$$= \left[\frac{Ca\sqrt{b}x\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+B\sqrt{a}bx\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\,(Dax-Ab)\sqrt{bx^2}}{2\,abx}\right.$$
$$-\frac{2\,Ca\sqrt{-b}x\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-B\sqrt{a}bx\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)-2\,(Dax-Ab)\sqrt{bx^2+a}}{2\,abx},\ \frac{2\,B\sqrt{-a}}{}$$
$$\left.-\frac{Ca\sqrt{-b}x\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-B\sqrt{-a}bx\arctan\left(\frac{\sqrt{bx^2+a}\sqrt{-a}}{a}\right)-(Dax-Ab)\sqrt{bx^2+a}}{abx}\right]$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/x^2/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(C*a*sqrt(b)*x*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + B*sq
rt(a)*b*x*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(D*a*x -
 A*b)*sqrt(b*x^2 + a))/(a*b*x), -1/2*(2*C*a*sqrt(-b)*x*arctan(sqrt(-b)*x/s
qrt(b*x^2 + a)) - B*sqrt(a)*b*x*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) +
2*a)/x^2) - 2*(D*a*x - A*b)*sqrt(b*x^2 + a))/(a*b*x), 1/2*(2*B*sqrt(-a)*b*
x*arctan(sqrt(b*x^2 + a)*sqrt(-a)/a) + C*a*sqrt(b)*x*log(-2*b*x^2 - 2*sqrt
(b*x^2 + a)*sqrt(b)*x - a) + 2*(D*a*x - A*b)*sqrt(b*x^2 + a))/(a*b*x), -(C
*a*sqrt(-b)*x*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - B*sqrt(-a)*b*x*arctan(s
qrt(b*x^2 + a)*sqrt(-a)/a) - (D*a*x - A*b)*sqrt(b*x^2 + a))/(a*b*x)]
```

## Sympy [A] (verification not implemented)

Time = 1.40 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.28

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^2\sqrt{a+bx^2}}\,dx = -\frac{A\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{a} - \frac{B\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{\sqrt{a}} + C\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0\wedge b\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{for } b\neq 0\\ \frac{x}{\sqrt{a}} & \text{otherwise}\end{cases}\right) + D\left(\begin{cases}\frac{\sqrt{a+bx^2}}{b} & \text{for } b\neq 0\\ \frac{x^2}{2\sqrt{a}} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/x**2/(b*x**2+a)**(1/2),x)
```

output
```
-A*sqrt(b)*sqrt(a/(b*x**2) + 1)/a - B*asinh(sqrt(a)/(sqrt(b)*x))/sqrt(a) +
 C*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0) &
Ne(b, 0)), (x*log(x)/sqrt(b*x**2), Ne(b, 0)), (x/sqrt(a), True)) + D*Piece
wise((sqrt(a + b*x**2)/b, Ne(b, 0)), (x**2/(2*sqrt(a)), True))
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.73

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^2\sqrt{a+bx^2}}\,dx = \frac{C\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}} - \frac{B\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{\sqrt{a}} + \frac{\sqrt{bx^2+a}D}{b} - \frac{\sqrt{bx^2+a}A}{ax}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/x^2/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
C*arcsinh(b*x/sqrt(a*b))/sqrt(b) - B*arcsinh(a/(sqrt(a*b)*abs(x)))/sqrt(a)
 + sqrt(b*x^2 + a)*D/b - sqrt(b*x^2 + a)*A/(a*x)
```

## Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.16

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^2\sqrt{a+bx^2}}\,dx = \frac{2\,B\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{C\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{\sqrt{b}} + \frac{\sqrt{bx^2+a}D}{b} + \frac{2\,A\sqrt{b}}{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/x^2/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
2*B*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/sqrt(-a) - C*log(abs(-
sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b) + sqrt(b*x^2 + a)*D/b + 2*A*sqrt(b)/
((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)
```

## Mupad [B] (verification not implemented)

Time = 2.56 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.16

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^2\sqrt{a+bx^2}}\,dx = \frac{C\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{\sqrt{b}} - \frac{B\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{\sqrt{a}} + \frac{a\left(\frac{b\,x^2}{a}+1\right)\,D-a\,\sqrt{\frac{b\,x^2}{a}+1}\,D}{b\,\sqrt{b\,x^2+a}} - \frac{A\,\sqrt{b\,x^2+a}}{a\,x}$$

input
```
int((A + B*x + C*x^2 + x^3*D)/(x^2*(a + b*x^2)^(1/2)),x)
```

output
```
(C*log(b^(1/2)*x + (a + b*x^2)^(1/2)))/b^(1/2) - (B*atanh((a + b*x^2)^(1/2
)/a^(1/2)))/a^(1/2) + (a*((b*x^2)/a + 1)*D - a*((b*x^2)/a + 1)^(1/2)*D)/(b
*(a + b*x^2)^(1/2)) - (A*(a + b*x^2)^(1/2))/(a*x)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 203, normalized size of antiderivative = 2.28

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^2\sqrt{a+bx^2}}\,dx$$
$$= \frac{-2\sqrt{b\,x^2+a}\,ab+2\sqrt{b\,x^2+a}\,adx+\sqrt{a}\log\left(\frac{-\sqrt{a}\,\sqrt{b\,x^2+a}+\sqrt{b}\,\sqrt{b\,x^2+a}\,x-\sqrt{b}\,\sqrt{a}\,x+a+b\,x^2}{\sqrt{a}\,\sqrt{b\,x^2+a}+\sqrt{b}\,\sqrt{a}\,x}\right)b^2x-\sqrt{a}\log\left(\frac{\sqrt{a}\,\sqrt{b\,x^2}\cdots}{}\right.}{2abx}$$

input
```
int((D*x^3+C*x^2+B*x+A)/x^2/(b*x^2+a)^(1/2),x)
```

output
```
( - 2*sqrt(a + b*x**2)*a*b + 2*sqrt(a + b*x**2)*a*d*x + sqrt(a)*log(( - sq
rt(a)*sqrt(a + b*x**2) + sqrt(b)*sqrt(a + b*x**2)*x - sqrt(b)*sqrt(a)*x +
a + b*x**2)/(sqrt(a)*sqrt(a + b*x**2) + sqrt(b)*sqrt(a)*x))*b**2*x - sqrt(
a)*log((sqrt(a)*sqrt(a + b*x**2) + sqrt(b)*sqrt(a + b*x**2)*x + sqrt(b)*sq
rt(a)*x + a + b*x**2)/(sqrt(a)*sqrt(a + b*x**2) + sqrt(b)*sqrt(a)*x))*b**2
*x + 2*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a*c*x - 2*sqrt(
b)*a*b*x)/(2*a*b*x)
```

# 3.96 $\int \frac{A+Bx+Cx^2+Dx^3}{x^3\sqrt{a+bx^2}}\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 104

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^3\sqrt{a+bx^2}}\,dx=-\frac{A\sqrt{a+bx^2}}{2ax^2}-\frac{B\sqrt{a+bx^2}}{ax}+\frac{D\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}+\frac{(Ab-2aC)\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}}$$

output
```
-1/2*A*(b*x^2+a)^(1/2)/a/x^2-B*(b*x^2+a)^(1/2)/a/x+D*arctanh(b^(1/2)*x/(b*
x^2+a)^(1/2))/b^(1/2)+1/2*(A*b-2*C*a)*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(
3/2)
```

## Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.98

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^3\sqrt{a+bx^2}}\,dx=\frac{(-A-2Bx)\sqrt{a+bx^2}}{2ax^2}+\frac{(-Ab+2aC)\text{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{D\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{\sqrt{b}}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/(x^3*Sqrt[a + b*x^2]),x]
```

output
```
((-A - 2*B*x)*Sqrt[a + b*x^2])/(2*a*x^2) + ((-(A*b) + 2*a*C)*ArcTanh[(Sqrt
[b]*x - Sqrt[a + b*x^2])/Sqrt[a]])/a^(3/2) - (D*Log[-(Sqrt[b]*x) + Sqrt[a
+ b*x^2]])/Sqrt[b]
```

## Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.04, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2338, 25, 2338, 27, 538, 224, 219, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^3\sqrt{a+bx^2}}dx$$

$$\Big\downarrow \text{ 2338}$$

$$-\frac{\int -\frac{2aDx^2-(Ab-2aC)x+2aB}{x^2\sqrt{bx^2+a}}dx}{2a}-\frac{A\sqrt{a+bx^2}}{2ax^2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{2aDx^2-(Ab-2aC)x+2aB}{x^2\sqrt{bx^2+a}}dx}{2a}-\frac{A\sqrt{a+bx^2}}{2ax^2}$$

$$\Big\downarrow \text{ 2338}$$

$$\frac{-\frac{\int \frac{a(Ab-2aC-2aDx)}{x\sqrt{bx^2+a}}dx}{a}-\frac{2B\sqrt{a+bx^2}}{x}}{2a}-\frac{A\sqrt{a+bx^2}}{2ax^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{-\int \frac{Ab-2aC-2aDx}{x\sqrt{bx^2+a}}dx-\frac{2B\sqrt{a+bx^2}}{x}}{2a}-\frac{A\sqrt{a+bx^2}}{2ax^2}$$

$$\Big\downarrow \text{ 538}$$

$$\frac{-(Ab-2aC)\int \frac{1}{x\sqrt{bx^2+a}}dx+2aD\int \frac{1}{\sqrt{bx^2+a}}dx-\frac{2B\sqrt{a+bx^2}}{x}}{2a}-\frac{A\sqrt{a+bx^2}}{2ax^2}$$

$$\Big\downarrow \; 224$$

$$\frac{-(Ab-2aC)\int \frac{1}{x\sqrt{bx^2+a}}dx+2aD\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}-\frac{2B\sqrt{a+bx^2}}{x}}{2a}-\frac{A\sqrt{a+bx^2}}{2ax^2}$$

$$\Big\downarrow \; 219$$

$$\frac{-(Ab-2aC)\int \frac{1}{x\sqrt{bx^2+a}}dx+\frac{2aD\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}-\frac{2B\sqrt{a+bx^2}}{x}}{2a}-\frac{A\sqrt{a+bx^2}}{2ax^2}$$

$$\Big\downarrow \; 243$$

$$\frac{-\frac{1}{2}(Ab-2aC)\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2+\frac{2aD\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}-\frac{2B\sqrt{a+bx^2}}{x}}{2a}-\frac{A\sqrt{a+bx^2}}{2ax^2}$$

$$\Big\downarrow \; 73$$

$$\frac{-\frac{(Ab-2aC)\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{b}+\frac{2aD\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}-\frac{2B\sqrt{a+bx^2}}{x}}{2a}-\frac{A\sqrt{a+bx^2}}{2ax^2}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{(Ab-2aC)\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{2aD\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}-\frac{2B\sqrt{a+bx^2}}{x}}{2a}-\frac{A\sqrt{a+bx^2}}{2ax^2}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/(x^3*Sqrt[a + b*x^2]),x]
```

output
```
-1/2*(A*Sqrt[a + b*x^2])/(a*x^2) + ((-2*B*Sqrt[a + b*x^2])/x + (2*a*D*ArcT
anh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/Sqrt[b] + ((A*b - 2*a*C)*ArcTanh[Sqrt[a
+ b*x^2]/Sqrt[a]])/Sqrt[a])/(2*a)
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

## Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.14

| method | result | size |
|---|---|---|
| default | $\frac{D\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{\sqrt{b}}+A\left(-\frac{\sqrt{b\,x^{2}+a}}{2a\,x^{2}}+\frac{b\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{2a^{\frac{3}{2}}}\right)-\frac{B\sqrt{b\,x^{2}+a}}{ax}-\frac{C\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{\sqrt{a}}$ | 119 |

input
```
int((D*x^3+C*x^2+B*x+A)/x^3/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
D*ln(b^(1/2)*x+(b*x^2+a)^(1/2))/b^(1/2)+A*(-1/2*(b*x^2+a)^(1/2)/a/x^2+1/2*
b/a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x))-B*(b*x^2+a)^(1/2)/a/x-C/a
^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)
```

## Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 451, normalized size of antiderivative = 4.34

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^3\sqrt{a+bx^2}}\,dx$$
$$= \left[\frac{2\,Da^2\sqrt{b}x^2\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-(2\,Cab-Ab^2)\sqrt{a}x^2\log\left(-\frac{bx^2+2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)-2\,(2\right.$$
$$-\frac{4\,Da^2\sqrt{-b}x^2\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)+(2\,Cab-Ab^2)\sqrt{a}x^2\log\left(-\frac{bx^2+2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\,(2\,Babx+Aab)\sqrt{}}{4\,a^2bx^2}$$
$$-\frac{2\,Da^2\sqrt{-b}x^2\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(2\,Cab-Ab^2)\sqrt{-a}x^2\arctan\left(\frac{\sqrt{bx^2+a}\sqrt{-a}}{a}\right)+(2\,Babx+Aab)\sqrt{bx^2+}}{2\,a^2bx^2}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/x^3/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(2*D*a^2*sqrt(b)*x^2*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a)
- (2*C*a*b - A*b^2)*sqrt(a)*x^2*log(-(b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(a) +
2*a)/x^2) - 2*(2*B*a*b*x + A*a*b)*sqrt(b*x^2 + a))/(a^2*b*x^2), -1/4*(4*D*
a^2*sqrt(-b)*x^2*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (2*C*a*b - A*b^2)*sq
rt(a)*x^2*log(-(b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(2*B*a*b
*x + A*a*b)*sqrt(b*x^2 + a))/(a^2*b*x^2), 1/2*(D*a^2*sqrt(b)*x^2*log(-2*b*
x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + (2*C*a*b - A*b^2)*sqrt(-a)*x^2*ar
ctan(sqrt(b*x^2 + a)*sqrt(-a)/a) - (2*B*a*b*x + A*a*b)*sqrt(b*x^2 + a))/(a
^2*b*x^2), -1/2*(2*D*a^2*sqrt(-b)*x^2*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) -
 (2*C*a*b - A*b^2)*sqrt(-a)*x^2*arctan(sqrt(b*x^2 + a)*sqrt(-a)/a) + (2*B*
a*b*x + A*a*b)*sqrt(b*x^2 + a))/(a^2*b*x^2)]
```

## Sympy [A] (verification not implemented)

Time = 2.49 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.33

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^3\sqrt{a+bx^2}}\,dx = -\frac{A\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{2ax} + \frac{Ab\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2a^{\frac{3}{2}}} - \frac{B\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{a} - \frac{C\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{\sqrt{a}} + D\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0\wedge b\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{for } b\neq 0\\ \frac{x}{\sqrt{a}} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/x**3/(b*x**2+a)**(1/2),x)
```

output
```
-A*sqrt(b)*sqrt(a/(b*x**2) + 1)/(2*a*x) + A*b*asinh(sqrt(a)/(sqrt(b)*x))/(
2*a**(3/2)) - B*sqrt(b)*sqrt(a/(b*x**2) + 1)/a - C*asinh(sqrt(a)/(sqrt(b)*
x))/sqrt(a) + D*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b)
, Ne(a, 0) & Ne(b, 0)), (x*log(x)/sqrt(b*x**2), Ne(b, 0)), (x/sqrt(a), Tru
e))
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.85

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^3\sqrt{a+bx^2}}\,dx = \frac{D\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}} - \frac{C\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{\sqrt{a}} + \frac{Ab\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{2\,a^{\frac{3}{2}}} - \frac{\sqrt{bx^2+a}B}{ax} - \frac{\sqrt{bx^2+a}A}{2\,ax^2}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/x^3/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
D*arcsinh(b*x/sqrt(a*b))/sqrt(b) - C*arcsinh(a/(sqrt(a*b)*abs(x)))/sqrt(a)
 + 1/2*A*b*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2) - sqrt(b*x^2 + a)*B/(a*x)
 - 1/2*sqrt(b*x^2 + a)*A/(a*x^2)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 176 vs. 2(85) = 170.

Time = 0.14 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.69

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^3\sqrt{a+bx^2}}\,dx = -\frac{D\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{\sqrt{b}} + \frac{(2\,Ca-Ab)\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a} + \frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^3Ab+2\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2Ba\sqrt{b}+\left(\sqrt{b}x-\sqrt{bx^2+a}\right)Aab-2\,Ba^2\sqrt{b}}{\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^2a}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/x^3/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
-D*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b) + (2*C*a - A*b)*arctan(-
(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/(sqrt(-a)*a) + ((sqrt(b)*x - sqrt(
b*x^2 + a))^3*A*b + 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a*sqrt(b) + (sqrt(
b)*x - sqrt(b*x^2 + a))*A*a*b - 2*B*a^2*sqrt(b))/(((sqrt(b)*x - sqrt(b*x^2
 + a))^2 - a)^2*a)
```

## Mupad [B] (verification not implemented)

Time = 2.72 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.95

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^3\sqrt{a+bx^2}}\,dx = \frac{\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)\,D}{\sqrt{b}} - \frac{C\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{\sqrt{a}} - \frac{A\,\sqrt{b\,x^2+a}}{2\,a\,x^2} - \frac{B\,\sqrt{b\,x^2+a}}{a\,x} + \frac{A\,b\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2\,a^{3/2}}$$

input
```
int((A + B*x + C*x^2 + x^3*D)/(x^3*(a + b*x^2)^(1/2)),x)
```

output
```
(log(b^(1/2)*x + (a + b*x^2)^(1/2))*D)/b^(1/2) - (C*atanh((a + b*x^2)^(1/2
)/a^(1/2)))/a^(1/2) - (A*(a + b*x^2)^(1/2))/(2*a*x^2) - (B*(a + b*x^2)^(1/
2))/(a*x) + (A*b*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(2*a^(3/2))
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.84

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^3\sqrt{a+bx^2}}\,dx = \frac{-\sqrt{b\,x^2+a}\,ab - 2\sqrt{b\,x^2+a}\,b^2x - \sqrt{a}\log\left(\frac{\sqrt{b\,x^2+a}-\sqrt{a}+\sqrt{b}\,x}{\sqrt{a}}\right)b^2x^2 + 2\sqrt{a}\log\left(\frac{\sqrt{b\,x^2+a}-\sqrt{a}+\sqrt{b}\,x}{\sqrt{a}}\right)bc\,x^2 + \sqrt{}}{2ab\,x^2}$$

input
```
int((D*x^3+C*x^2+B*x+A)/x^3/(b*x^2+a)^(1/2),x)
```

output
```
( - sqrt(a + b*x**2)*a*b - 2*sqrt(a + b*x**2)*b**2*x - sqrt(a)*log((sqrt(a
 + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*b**2*x**2 + 2*sqrt(a)*log((sqrt
(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*b*c*x**2 + sqrt(a)*log((sqrt(
a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*b**2*x**2 - 2*sqrt(a)*log((sqr
t(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*b*c*x**2 + 2*sqrt(b)*log((sq
rt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a*d*x**2)/(2*a*b*x**2)
```

# 3.97 $\int \frac{A+Bx+Cx^2+Dx^3}{x^4\sqrt{a+bx^2}}\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 110

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^4\sqrt{a+bx^2}}\,dx = -\frac{A\sqrt{a+bx^2}}{3ax^3}-\frac{B\sqrt{a+bx^2}}{2ax^2}+\frac{(2Ab-3aC)\sqrt{a+bx^2}}{3a^2x}+\frac{(bB-2aD)\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}}$$

output
```
-1/3*A*(b*x^2+a)^(1/2)/a/x^3-1/2*B*(b*x^2+a)^(1/2)/a/x^2+1/3*(2*A*b-3*C*a)
*(b*x^2+a)^(1/2)/a^2/x+1/2*(B*b-2*D*a)*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^
(3/2)
```

## Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.81

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^4\sqrt{a+bx^2}}\,dx = \frac{\sqrt{a+bx^2}(-2aA-3aBx+4Abx^2-6aCx^2)}{6a^2x^3}+\frac{(-bB+2aD)\operatorname{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/(x^4*Sqrt[a + b*x^2]),x]
```

output
```
(Sqrt[a + b*x^2]*(-2*a*A - 3*a*B*x + 4*A*b*x^2 - 6*a*C*x^2))/(6*a^2*x^3) +
 ((-(b*B) + 2*a*D)*ArcTanh[(Sqrt[b]*x - Sqrt[a + b*x^2])/Sqrt[a]])/a^(3/2)
```

## Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {2338, 25, 2338, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^4\sqrt{a+bx^2}}dx$$

$$\Big\downarrow \text{2338}$$

$$-\frac{\int -\frac{3aDx^2-(2Ab-3aC)x+3aB}{x^3\sqrt{bx^2+a}}dx}{3a}-\frac{A\sqrt{a+bx^2}}{3ax^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{3aDx^2-(2Ab-3aC)x+3aB}{x^3\sqrt{bx^2+a}}dx}{3a}-\frac{A\sqrt{a+bx^2}}{3ax^3}$$

$$\Big\downarrow \text{2338}$$

$$\frac{-\frac{\int \frac{a(2(2Ab-3aC)+3(bB-2aD)x)}{x^2\sqrt{bx^2+a}}dx}{2a}-\frac{3B\sqrt{a+bx^2}}{2x^2}}{3a}-\frac{A\sqrt{a+bx^2}}{3ax^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{1}{2}\int \frac{2(2Ab-3aC)+3(bB-2aD)x}{x^2\sqrt{bx^2+a}}dx-\frac{3B\sqrt{a+bx^2}}{2x^2}}{3a}-\frac{A\sqrt{a+bx^2}}{3ax^3}$$

$$\Big\downarrow \text{534}$$

$$\frac{\frac{1}{2}\left(\frac{2\sqrt{a+bx^2}(2Ab-3aC)}{ax}-3(bB-2aD)\int \frac{1}{x\sqrt{bx^2+a}}dx\right)-\frac{3B\sqrt{a+bx^2}}{2x^2}}{3a}-\frac{A\sqrt{a+bx^2}}{3ax^3}$$

$$\Big\downarrow \text{243}$$

$$\frac{\frac{1}{2}\left(\frac{2\sqrt{a+bx^2}(2Ab-3aC)}{ax}-\frac{3}{2}(bB-2aD)\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2\right)-\frac{3B\sqrt{a+bx^2}}{2x^2}}{3a}-\frac{A\sqrt{a+bx^2}}{3ax^3}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{1}{2}\left(\frac{2\sqrt{a+bx^2}(2Ab-3aC)}{ax}-\frac{3(bB-2aD)\int\frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{b}\right)-\frac{3B\sqrt{a+bx^2}}{2x^2}}{3a}-\frac{A\sqrt{a+bx^2}}{3ax^3}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{1}{2}\left(\frac{2\sqrt{a+bx^2}(2Ab-3aC)}{ax}+\frac{3\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)(bB-2aD)}{\sqrt{a}}\right)-\frac{3B\sqrt{a+bx^2}}{2x^2}}{3a}-\frac{A\sqrt{a+bx^2}}{3ax^3}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/(x^4*Sqrt[a + b*x^2]),x]
```

output
```
-1/3*(A*Sqrt[a + b*x^2])/(a*x^3) + ((-3*B*Sqrt[a + b*x^2])/(2*x^2) + ((2*(
2*A*b - 3*a*C)*Sqrt[a + b*x^2])/(a*x) + (3*(b*B - 2*a*D)*ArcTanh[Sqrt[a +
b*x^2]/Sqrt[a]])/Sqrt[a])/2)/(3*a)
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

## Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.24

| method | result |
|---|---|
| default | $A\left(-\frac{\sqrt{b\,x^2+a}}{3a\,x^3}+\frac{2b\sqrt{b\,x^2+a}}{3a^2x}\right)+B\left(-\frac{\sqrt{b\,x^2+a}}{2a\,x^2}+\frac{b\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)}{2a^{\frac{3}{2}}}\right)-\frac{C\sqrt{b\,x^2+a}}{ax}-\frac{D\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)}{\sqrt{a}}$ |

input
```
int((D*x^3+C*x^2+B*x+A)/x^4/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
A*(-1/3*(b*x^2+a)^(1/2)/a/x^3+2/3*b/a^2*(b*x^2+a)^(1/2)/x)+B*(-1/2*(b*x^2+
a)^(1/2)/a/x^2+1/2*b/a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x))-C*(b*x
^2+a)^(1/2)/a/x-D/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)
```

## Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.57

$$\int \frac{A + Bx + Cx^2 + Dx^3}{x^4\sqrt{a + bx^2}}\, dx$$
$$= \left[-\frac{3\,(2\,Da - Bb)\sqrt{a}x^3 \log\left(-\frac{bx^2+2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right) + 2\,(3\,Bax + 2\,(3\,Ca - 2\,Ab)x^2 + 2\,Aa)\sqrt{bx^2 + a}}{12\,a^2x^3}, \frac{3\,(2\right.$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/x^4/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/12*(3*(2*D*a - B*b)*sqrt(a)*x^3*log(-(b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(a
) + 2*a)/x^2) + 2*(3*B*a*x + 2*(3*C*a - 2*A*b)*x^2 + 2*A*a)*sqrt(b*x^2 + a
))/(a^2*x^3), 1/6*(3*(2*D*a - B*b)*sqrt(-a)*x^3*arctan(sqrt(b*x^2 + a)*sqr
t(-a)/a) - (3*B*a*x + 2*(3*C*a - 2*A*b)*x^2 + 2*A*a)*sqrt(b*x^2 + a))/(a^2
*x^3)]
```

## Sympy [A] (verification not implemented)

Time = 2.75 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.25

$$\int \frac{A + Bx + Cx^2 + Dx^3}{x^4\sqrt{a + bx^2}}\, dx = -\frac{A\sqrt{b}\sqrt{\frac{a}{bx^2} + 1}}{3ax^2} + \frac{2Ab^{\frac{3}{2}}\sqrt{\frac{a}{bx^2} + 1}}{3a^2} - \frac{B\sqrt{b}\sqrt{\frac{a}{bx^2} + 1}}{2ax} + \frac{Bb\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2a^{\frac{3}{2}}} - \frac{C\sqrt{b}\sqrt{\frac{a}{bx^2} + 1}}{a} - \frac{D\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{\sqrt{a}}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/x**4/(b*x**2+a)**(1/2),x)
```

output
```
-A*sqrt(b)*sqrt(a/(b*x**2) + 1)/(3*a*x**2) + 2*A*b**(3/2)*sqrt(a/(b*x**2)
+ 1)/(3*a**2) - B*sqrt(b)*sqrt(a/(b*x**2) + 1)/(2*a*x) + B*b*asinh(sqrt(a)
/(sqrt(b)*x))/(2*a**(3/2)) - C*sqrt(b)*sqrt(a/(b*x**2) + 1)/a - D*asinh(sq
rt(a)/(sqrt(b)*x))/sqrt(a)
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.01

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^4\sqrt{a+bx^2}}\,dx = -\frac{D\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{\sqrt{a}} + \frac{Bb\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{2\,a^{\frac{3}{2}}} - \frac{\sqrt{bx^2+a}C}{ax} + \frac{2\sqrt{bx^2+a}Ab}{3\,a^2x} - \frac{\sqrt{bx^2+a}B}{2\,ax^2} - \frac{\sqrt{bx^2+a}A}{3\,ax^3}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/x^4/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
-D*arcsinh(a/(sqrt(a*b)*abs(x)))/sqrt(a) + 1/2*B*b*arcsinh(a/(sqrt(a*b)*ab
s(x)))/a^(3/2) - sqrt(b*x^2 + a)*C/(a*x) + 2/3*sqrt(b*x^2 + a)*A*b/(a^2*x)
 - 1/2*sqrt(b*x^2 + a)*B/(a*x^2) - 1/3*sqrt(b*x^2 + a)*A/(a*x^3)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 220 vs. $2(91) = 182$.

Time = 0.14 (sec) , antiderivative size = 220, normalized size of antiderivative = 2.00

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^4\sqrt{a+bx^2}}\,dx = \frac{(2\,Da-Bb)\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a} + \frac{3\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^5Bb+6\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4Ca\sqrt{b}-12\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2Ca^2\sqrt{b}+12\left(\sqrt{b}x\right.}{3\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^3}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/x^4/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
(2*D*a - B*b)*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/(sqrt(-a)*a)
 + 1/3*(3*(sqrt(b)*x - sqrt(b*x^2 + a))^5*B*b + 6*(sqrt(b)*x - sqrt(b*x^2
+ a))^4*C*a*sqrt(b) - 12*(sqrt(b)*x - sqrt(b*x^2 + a))^2*C*a^2*sqrt(b) + 1
2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A*a*b^(3/2) - 3*(sqrt(b)*x - sqrt(b*x^2
+ a))*B*a^2*b + 6*C*a^3*sqrt(b) - 4*A*a^2*b^(3/2))/(((sqrt(b)*x - sqrt(b*x
^2 + a))^2 - a)^3*a)
```

## Mupad [B] (verification not implemented)

Time = 3.06 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.97

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^4\sqrt{a+bx^2}}\,dx = \frac{\operatorname{atan}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{-a}}\right)\,D}{\sqrt{-a}} - \frac{B\,\sqrt{b\,x^2+a}}{2\,a\,x^2} - \frac{C\,\sqrt{b\,x^2+a}}{a\,x} + \frac{B\,b\,\operatorname{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2\,a^{3/2}} - \frac{A\,\sqrt{b\,x^2+a}\,(a-2\,b\,x^2)}{3\,a^2\,x^3}$$

input
```
int((A + B*x + C*x^2 + x^3*D)/(x^4*(a + b*x^2)^(1/2)),x)
```

output
```
(atan((a + b*x^2)^(1/2)/(-a)^(1/2))*D)/(-a)^(1/2) - (B*(a + b*x^2)^(1/2))/
(2*a*x^2) - (C*(a + b*x^2)^(1/2))/(a*x) + (B*b*atanh((a + b*x^2)^(1/2)/a^(
1/2)))/(2*a^(3/2)) - (A*(a + b*x^2)^(1/2)*(a - 2*b*x^2))/(3*a^2*x^3)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.90

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^4\sqrt{a+bx^2}}\,dx = \frac{-2\sqrt{b\,x^2+a}\,a^2+4\sqrt{b\,x^2+a}\,ab\,x^2-3\sqrt{b\,x^2+a}\,abx-6\sqrt{b\,x^2+a}\,ac\,x^2+6\sqrt{a}\log\left(\frac{\sqrt{b\,x^2+a}-\sqrt{a}+\sqrt{b}\,x}{\sqrt{a}}\right)ad}{}$$

input
```
int((D*x^3+C*x^2+B*x+A)/x^4/(b*x^2+a)^(1/2),x)
```

output
```
( - 2*sqrt(a + b*x**2)*a**2 + 4*sqrt(a + b*x**2)*a*b*x**2 - 3*sqrt(a + b*x
**2)*a*b*x - 6*sqrt(a + b*x**2)*a*c*x**2 + 6*sqrt(a)*log((sqrt(a + b*x**2)
 - sqrt(a) + sqrt(b)*x)/sqrt(a))*a*d*x**3 - 3*sqrt(a)*log((sqrt(a + b*x**2
) - sqrt(a) + sqrt(b)*x)/sqrt(a))*b**2*x**3 - 6*sqrt(a)*log((sqrt(a + b*x*
*2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*a*d*x**3 + 3*sqrt(a)*log((sqrt(a + b*x
**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*b**2*x**3 - 4*sqrt(b)*a*b*x**3 + 2*sq
rt(b)*a*c*x**3)/(6*a**2*x**3)
```

# 3.98 $\int \frac{A+Bx+Cx^2+Dx^3}{x^5\sqrt{a+bx^2}}\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 142

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^5\sqrt{a+bx^2}}\,dx = -\frac{A\sqrt{a+bx^2}}{4ax^4}-\frac{B\sqrt{a+bx^2}}{3ax^3}+\frac{(3Ab-4aC)\sqrt{a+bx^2}}{8a^2x^2}+\frac{(2bB-3aD)\sqrt{a+bx^2}}{3a^2x}-\frac{b(3Ab-4aC)\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{5/2}}$$

output
```
-1/4*A*(b*x^2+a)^(1/2)/a/x^4-1/3*B*(b*x^2+a)^(1/2)/a/x^3+1/8*(3*A*b-4*C*a)
*(b*x^2+a)^(1/2)/a^2/x^2+1/3*(2*B*b-3*D*a)*(b*x^2+a)^(1/2)/a^2/x-1/8*b*(3*
A*b-4*C*a)*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(5/2)
```

## Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.73

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^5\sqrt{a+bx^2}}\,dx = \frac{\sqrt{a+bx^2}(-6aA+bx^2(9A+16Bx)-4ax(2B+3x(C+2Dx)))}{24a^2x^4} + \frac{b(3Ab-4aC)\text{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right)}{4a^{5/2}}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/(x^5*Sqrt[a + b*x^2]),x]
```

output
```
(Sqrt[a + b*x^2]*(-6*a*A + b*x^2*(9*A + 16*B*x) - 4*a*x*(2*B + 3*x*(C + 2*
D*x))))/(24*a^2*x^4) + (b*(3*A*b - 4*a*C)*ArcTanh[(Sqrt[b]*x - Sqrt[a + b*
x^2])/Sqrt[a]])/(4*a^(5/2))
```

## Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.08, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2338, 25, 2338, 27, 539, 25, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^5\sqrt{a+bx^2}}\,dx$$

$$\Big\downarrow \text{2338}$$

$$-\frac{\int -\frac{4aDx^2-(3Ab-4aC)x+4aB}{x^4\sqrt{bx^2+a}}dx}{4a}-\frac{A\sqrt{a+bx^2}}{4ax^4}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{4aDx^2-(3Ab-4aC)x+4aB}{x^4\sqrt{bx^2+a}}dx}{4a}-\frac{A\sqrt{a+bx^2}}{4ax^4}$$

$$\Big\downarrow \ 2338$$

$$\frac{-\frac{\int \frac{a(3(3Ab-4aC)+4(2bB-3aD)x)}{x^3\sqrt{bx^2+a}}dx}{3a}-\frac{4B\sqrt{a+bx^2}}{3x^3}}{4a}-\frac{A\sqrt{a+bx^2}}{4ax^4}$$

$$\Big\downarrow \ 27$$

$$\frac{-\frac{1}{3}\int \frac{3(3Ab-4aC)+4(2bB-3aD)x}{x^3\sqrt{bx^2+a}}dx-\frac{4B\sqrt{a+bx^2}}{3x^3}}{4a}-\frac{A\sqrt{a+bx^2}}{4ax^4}$$

$$\Big\downarrow \ 539$$

$$\frac{\frac{1}{3}\left(\frac{\int -\frac{8a(2bB-3aD)-3b(3Ab-4aC)x}{x^2\sqrt{bx^2+a}}dx}{2a}+\frac{3\sqrt{a+bx^2}(3Ab-4aC)}{2ax^2}\right)-\frac{4B\sqrt{a+bx^2}}{3x^3}}{4a}-\frac{A\sqrt{a+bx^2}}{4ax^4}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{1}{3}\left(\frac{3\sqrt{a+bx^2}(3Ab-4aC)}{2ax^2}-\frac{\int \frac{8a(2bB-3aD)-3b(3Ab-4aC)x}{x^2\sqrt{bx^2+a}}dx}{2a}\right)-\frac{4B\sqrt{a+bx^2}}{3x^3}}{4a}-\frac{A\sqrt{a+bx^2}}{4ax^4}$$

$$\Big\downarrow \ 534$$

$$\frac{\frac{1}{3}\left(\frac{3\sqrt{a+bx^2}(3Ab-4aC)}{2ax^2}-\frac{-3b(3Ab-4aC)\int \frac{1}{x\sqrt{bx^2+a}}dx-\frac{8\sqrt{a+bx^2}(2bB-3aD)}{x}}{2a}\right)-\frac{4B\sqrt{a+bx^2}}{3x^3}}{4a}-\frac{A\sqrt{a+bx^2}}{4ax^4}$$

$$\Big\downarrow \ 243$$

$$\frac{\frac{1}{3}\left(\frac{3\sqrt{a+bx^2}(3Ab-4aC)}{2ax^2}-\frac{-\frac{3}{2}b(3Ab-4aC)\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2-\frac{8\sqrt{a+bx^2}(2bB-3aD)}{x}}{2a}\right)-\frac{4B\sqrt{a+bx^2}}{3x^3}}{4a}-\frac{A\sqrt{a+bx^2}}{4ax^4}$$

$$\Big\downarrow \ 73$$

$$\frac{\frac{1}{3}\left(\frac{3\sqrt{a+bx^2}(3Ab-4aC)}{2ax^2}-\frac{-3(3Ab-4aC)\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}-\frac{8\sqrt{a+bx^2}(2bB-3aD)}{x}}{2a}\right)-\frac{4B\sqrt{a+bx^2}}{3x^3}}{4a}-\frac{A\sqrt{a+bx^2}}{4ax^4}$$

$$\Big\downarrow \ 221$$

$$\frac{\frac{1}{3}\left(\frac{3\sqrt{a+bx^2}(3Ab-4aC)}{2ax^2}-\frac{\frac{3b(3Ab-4aC)\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{8\sqrt{a+bx^2}(2bB-3aD)}{x}}{2a}\right)-\frac{4B\sqrt{a+bx^2}}{3x^3}}{4a}-\frac{A\sqrt{a+bx^2}}{4ax^4}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/(x^5*Sqrt[a + b*x^2]),x]
```

output
```
-1/4*(A*Sqrt[a + b*x^2])/(a*x^4) + ((-4*B*Sqrt[a + b*x^2])/(3*x^3) + ((3*(
3*A*b - 4*a*C)*Sqrt[a + b*x^2])/(2*a*x^2) - ((-8*(2*b*B - 3*a*D)*Sqrt[a +
b*x^2])/x + (3*b*(3*A*b - 4*a*C)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a]
)/(2*a))/3)/(4*a)
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 539
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[c*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))
   Int[x^(m + 1)*(a + b*x^2)^p*(a*d*(m + 1) - b*c*(m + 2*p + 3)*x), x], x]
/; FreeQ[{a, b, c, d, p}, x] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

## Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.27

| method | result |
|---|---|
| default | $A\left(-\frac{\sqrt{b\,x^{2}+a}}{4a\,x^{4}}-\frac{3b\left(-\frac{\sqrt{b\,x^{2}+a}}{2a\,x^{2}}+\frac{b\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{2a^{\frac{3}{2}}}\right)}{4a}\right)+B\left(-\frac{\sqrt{b\,x^{2}+a}}{3a\,x^{3}}+\frac{2b\sqrt{b\,x^{2}+a}}{3a^{2}x}\right)+C\left(-\frac{\sqrt{b\,x^{2}+a}}{2a\,x^{2}}+\right.$ |

input
```
int((D*x^3+C*x^2+B*x+A)/x^5/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
A*(-1/4*(b*x^2+a)^(1/2)/a/x^4-3/4*b/a*(-1/2*(b*x^2+a)^(1/2)/a/x^2+1/2*b/a^
(3/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)))+B*(-1/3*(b*x^2+a)^(1/2)/a/x^
3+2/3*b/a^2*(b*x^2+a)^(1/2)/x)+C*(-1/2*(b*x^2+a)^(1/2)/a/x^2+1/2*b/a^(3/2)
*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x))-D*(b*x^2+a)^(1/2)/a/x
```

## Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.59

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^5\sqrt{a+bx^2}}\,dx = \left[-\frac{3\,(4\,Cab-3\,Ab^2)\sqrt{a}x^4\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\,(8\,Ba^2x+8\,(3\,Da^2-2\,Bab)x^3+6\,Aa^2+3\,(4\right.}{48\,a^3x^4}$$

$$-\frac{3\,(4\,Cab-3\,Ab^2)\sqrt{-a}x^4\arctan\left(\frac{\sqrt{bx^2+a}\sqrt{-a}}{a}\right)+(8\,Ba^2x+8\,(3\,Da^2-2\,Bab)x^3+6\,Aa^2+3\,(4\,Ca^2-}{24\,a^3x^4}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/x^5/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/48*(3*(4*C*a*b - 3*A*b^2)*sqrt(a)*x^4*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*
sqrt(a) + 2*a)/x^2) + 2*(8*B*a^2*x + 8*(3*D*a^2 - 2*B*a*b)*x^3 + 6*A*a^2 +
 3*(4*C*a^2 - 3*A*a*b)*x^2)*sqrt(b*x^2 + a))/(a^3*x^4), -1/24*(3*(4*C*a*b
- 3*A*b^2)*sqrt(-a)*x^4*arctan(sqrt(b*x^2 + a)*sqrt(-a)/a) + (8*B*a^2*x +
8*(3*D*a^2 - 2*B*a*b)*x^3 + 6*A*a^2 + 3*(4*C*a^2 - 3*A*a*b)*x^2)*sqrt(b*x^
2 + a))/(a^3*x^4)]
```

## Sympy [A] (verification not implemented)

Time = 5.09 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.56

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^5\sqrt{a+bx^2}}\,dx = -\frac{A}{4\sqrt{b}x^5\sqrt{\frac{a}{bx^2}+1}}+\frac{A\sqrt{b}}{8ax^3\sqrt{\frac{a}{bx^2}+1}}+\frac{3Ab^{\frac{3}{2}}}{8a^2x\sqrt{\frac{a}{bx^2}+1}} - \frac{3Ab^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{8a^{\frac{5}{2}}}-\frac{B\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{3ax^2}+\frac{2Bb^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{3a^2} - \frac{C\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{2ax}+\frac{Cb\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2a^{\frac{3}{2}}}-\frac{D\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{a}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/x**5/(b*x**2+a)**(1/2),x)
```

output
```
-A/(4*sqrt(b)*x**5*sqrt(a/(b*x**2) + 1)) + A*sqrt(b)/(8*a*x**3*sqrt(a/(b*x
**2) + 1)) + 3*A*b**(3/2)/(8*a**2*x*sqrt(a/(b*x**2) + 1)) - 3*A*b**2*asinh
(sqrt(a)/(sqrt(b)*x))/(8*a**(5/2)) - B*sqrt(b)*sqrt(a/(b*x**2) + 1)/(3*a*x
**2) + 2*B*b**(3/2)*sqrt(a/(b*x**2) + 1)/(3*a**2) - C*sqrt(b)*sqrt(a/(b*x*
*2) + 1)/(2*a*x) + C*b*asinh(sqrt(a)/(sqrt(b)*x))/(2*a**(3/2)) - D*sqrt(b)
*sqrt(a/(b*x**2) + 1)/a
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.06

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^5\sqrt{a+bx^2}}\,dx = \frac{Cb\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{2\,a^{\frac{3}{2}}} - \frac{3\,Ab^2\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{8\,a^{\frac{5}{2}}} - \frac{\sqrt{bx^2+a}D}{ax} + \frac{2\sqrt{bx^2+a}Bb}{3\,a^2x} - \frac{\sqrt{bx^2+a}C}{2\,ax^2} + \frac{3\sqrt{bx^2+a}Ab}{8\,a^2x^2} - \frac{\sqrt{bx^2+a}B}{3\,ax^3} - \frac{\sqrt{bx^2+a}A}{4\,ax^4}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/x^5/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/2*C*b*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2) - 3/8*A*b^2*arcsinh(a/(sqrt(
a*b)*abs(x)))/a^(5/2) - sqrt(b*x^2 + a)*D/(a*x) + 2/3*sqrt(b*x^2 + a)*B*b/
(a^2*x) - 1/2*sqrt(b*x^2 + a)*C/(a*x^2) + 3/8*sqrt(b*x^2 + a)*A*b/(a^2*x^2
) - 1/3*sqrt(b*x^2 + a)*B/(a*x^3) - 1/4*sqrt(b*x^2 + a)*A/(a*x^4)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 442 vs. 2(118) = 236.

Time = 0.13 (sec) , antiderivative size = 442, normalized size of antiderivative = 3.11

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^5\sqrt{a+bx^2}}\,dx = -\frac{(4\,Cab-3\,Ab^2)\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{4\sqrt{-a}a^2} + \frac{12\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^7Cab-9\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^7Ab^2+24\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6Da^2\sqrt{b}-12\left(\sqrt{b}x\right.}{}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/x^5/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
-1/4*(4*C*a*b - 3*A*b^2)*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/(
sqrt(-a)*a^2) + 1/12*(12*(sqrt(b)*x - sqrt(b*x^2 + a))^7*C*a*b - 9*(sqrt(b
)*x - sqrt(b*x^2 + a))^7*A*b^2 + 24*(sqrt(b)*x - sqrt(b*x^2 + a))^6*D*a^2*
sqrt(b) - 12*(sqrt(b)*x - sqrt(b*x^2 + a))^5*C*a^2*b + 33*(sqrt(b)*x - sqr
t(b*x^2 + a))^5*A*a*b^2 - 72*(sqrt(b)*x - sqrt(b*x^2 + a))^4*D*a^3*sqrt(b)
 + 48*(sqrt(b)*x - sqrt(b*x^2 + a))^4*B*a^2*b^(3/2) - 12*(sqrt(b)*x - sqrt
(b*x^2 + a))^3*C*a^3*b + 33*(sqrt(b)*x - sqrt(b*x^2 + a))^3*A*a^2*b^2 + 72
*(sqrt(b)*x - sqrt(b*x^2 + a))^2*D*a^4*sqrt(b) - 64*(sqrt(b)*x - sqrt(b*x^
2 + a))^2*B*a^3*b^(3/2) + 12*(sqrt(b)*x - sqrt(b*x^2 + a))*C*a^4*b - 9*(sq
rt(b)*x - sqrt(b*x^2 + a))*A*a^3*b^2 - 24*D*a^5*sqrt(b) + 16*B*a^4*b^(3/2)
)/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^4*a^2)
```

## Mupad [B] (verification not implemented)

Time = 2.96 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.01

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^5\sqrt{a+bx^2}}\,dx = \frac{3\,A\,{\left(b\,x^2+a\right)}^{3/2}}{8\,a^2\,x^4}-\frac{\sqrt{b\,x^2+a}\,D}{a\,x}-\frac{5\,A\,\sqrt{b\,x^2+a}}{8\,a\,x^4}-\frac{3\,A\,b^2\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{8\,a^{5/2}}-\frac{C\,\sqrt{b\,x^2+a}}{2\,a\,x^2}+\frac{C\,b\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2\,a^{3/2}}-\frac{B\,\sqrt{b\,x^2+a}\,\left(a-2\,b\,x^2\right)}{3\,a^2\,x^3}$$

input
```
int((A + B*x + C*x^2 + x^3*D)/(x^5*(a + b*x^2)^(1/2)),x)
```

output
```
(3*A*(a + b*x^2)^(3/2))/(8*a^2*x^4) - ((a + b*x^2)^(1/2)*D)/(a*x) - (5*A*(
a + b*x^2)^(1/2))/(8*a*x^4) - (3*A*b^2*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(
8*a^(5/2)) - (C*(a + b*x^2)^(1/2))/(2*a*x^2) + (C*b*atanh((a + b*x^2)^(1/2
)/a^(1/2)))/(2*a^(3/2)) - (B*(a + b*x^2)^(1/2)*(a - 2*b*x^2))/(3*a^2*x^3)
```

## Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.70

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^5\sqrt{a+bx^2}}\,dx$$
$$= \frac{-6\sqrt{b\,x^2+a}\,a^2+9\sqrt{b\,x^2+a}\,ab\,x^2-8\sqrt{b\,x^2+a}\,abx-12\sqrt{b\,x^2+a}\,ac\,x^2-24\sqrt{b\,x^2+a}\,ad\,x^3+16\sqrt{b\,x}}{}$$

input
```
int((D*x^3+C*x^2+B*x+A)/x^5/(b*x^2+a)^(1/2),x)
```

output
```
( - 6*sqrt(a + b*x**2)*a**2 + 9*sqrt(a + b*x**2)*a*b*x**2 - 8*sqrt(a + b*x
**2)*a*b*x - 12*sqrt(a + b*x**2)*a*c*x**2 - 24*sqrt(a + b*x**2)*a*d*x**3 +
 16*sqrt(a + b*x**2)*b**2*x**3 + 9*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a)
 + sqrt(b)*x)/sqrt(a))*b**2*x**4 - 12*sqrt(a)*log((sqrt(a + b*x**2) - sqrt
(a) + sqrt(b)*x)/sqrt(a))*b*c*x**4 - 9*sqrt(a)*log((sqrt(a + b*x**2) + sqr
t(a) + sqrt(b)*x)/sqrt(a))*b**2*x**4 + 12*sqrt(a)*log((sqrt(a + b*x**2) +
sqrt(a) + sqrt(b)*x)/sqrt(a))*b*c*x**4 + 12*sqrt(b)*a*d*x**4 - 16*sqrt(b)*
b**2*x**4)/(24*a**2*x**4)
```

# 3.99 $\int \frac{A+Bx+Cx^2+Dx^3}{x^6\sqrt{a+bx^2}}\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 173

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^6\sqrt{a+bx^2}}\,dx = -\frac{A\sqrt{a+bx^2}}{5ax^5}-\frac{B\sqrt{a+bx^2}}{4ax^4}+\frac{(4Ab-5aC)\sqrt{a+bx^2}}{15a^2x^3}+\frac{(3bB-4aD)\sqrt{a+bx^2}}{8a^2x^2}-\frac{2b(4Ab-5aC)\sqrt{a+bx^2}}{15a^3x}-\frac{b(3bB-4aD)\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{5/2}}$$

output
```
-1/5*A*(b*x^2+a)^(1/2)/a/x^5-1/4*B*(b*x^2+a)^(1/2)/a/x^4+1/15*(4*A*b-5*C*a
)*(b*x^2+a)^(1/2)/a^2/x^3+1/8*(3*B*b-4*D*a)*(b*x^2+a)^(1/2)/a^2/x^2-2/15*b
*(4*A*b-5*C*a)*(b*x^2+a)^(1/2)/a^3/x-1/8*b*(3*B*b-4*D*a)*arctanh((b*x^2+a)
^(1/2)/a^(1/2))/a^(5/2)
```

## Mathematica [A] (verified)

Time = 0.93 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.72

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^6\sqrt{a+bx^2}}\,dx = \frac{\frac{\sqrt{a+bx^2}(-64Ab^2x^4+abx^2(32A+5x(9B+16Cx))-2a^2(12A+5x(3B+4Cx+6Dx^2)))}{x^5}-30\sqrt{a}b(-3bB+4aD)\text{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right)}{120a^3}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/(x^6*Sqrt[a + b*x^2]),x]
```

output
```
((Sqrt[a + b*x^2]*(-64*A*b^2*x^4 + a*b*x^2*(32*A + 5*x*(9*B + 16*C*x)) - 2
*a^2*(12*A + 5*x*(3*B + 4*C*x + 6*D*x^2))))/x^5 - 30*Sqrt[a]*b*(-3*b*B + 4
*a*D)*ArcTanh[(Sqrt[b]*x - Sqrt[a + b*x^2])/Sqrt[a]])/(120*a^3)
```

## Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.09, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2338, 25, 2338, 27, 539, 25, 539, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^6\sqrt{a+bx^2}}\,dx$$

$$\Big\downarrow \text{2338}$$

$$-\frac{\int -\frac{5aDx^2-(4Ab-5aC)x+5aB}{x^5\sqrt{bx^2+a}}dx}{5a}-\frac{A\sqrt{a+bx^2}}{5ax^5}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{5aDx^2-(4Ab-5aC)x+5aB}{x^5\sqrt{bx^2+a}}dx}{5a}-\frac{A\sqrt{a+bx^2}}{5ax^5}$$

$$\Big\downarrow \text{2338}$$

$$\frac{-\frac{\int \frac{a(4(4Ab-5aC)+5(3bB-4aD)x)}{x^4\sqrt{bx^2+a}}dx}{4a}-\frac{5B\sqrt{a+bx^2}}{4x^4}}{5a}-\frac{A\sqrt{a+bx^2}}{5ax^5}$$

$$\Big\downarrow \ 27$$

$$\frac{-\frac{1}{4}\int \frac{4(4Ab-5aC)+5(3bB-4aD)x}{x^4\sqrt{bx^2+a}}dx-\frac{5B\sqrt{a+bx^2}}{4x^4}}{5a}-\frac{A\sqrt{a+bx^2}}{5ax^5}$$

$$\Big\downarrow \ 539$$

$$\frac{\frac{1}{4}\left(\frac{\int -\frac{15a(3bB-4aD)-8b(4Ab-5aC)x}{x^3\sqrt{bx^2+a}}dx}{3a}+\frac{4\sqrt{a+bx^2}(4Ab-5aC)}{3ax^3}\right)-\frac{5B\sqrt{a+bx^2}}{4x^4}}{5a}-\frac{A\sqrt{a+bx^2}}{5ax^5}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{1}{4}\left(\frac{4\sqrt{a+bx^2}(4Ab-5aC)}{3ax^3}-\frac{\int \frac{15a(3bB-4aD)-8b(4Ab-5aC)x}{x^3\sqrt{bx^2+a}}dx}{3a}\right)-\frac{5B\sqrt{a+bx^2}}{4x^4}}{5a}-\frac{A\sqrt{a+bx^2}}{5ax^5}$$

$$\Big\downarrow \ 539$$

$$\frac{\frac{1}{4}\left(\frac{4\sqrt{a+bx^2}(4Ab-5aC)}{3ax^3}-\frac{-\frac{\int \frac{ab(16(4Ab-5aC)+15(3bB-4aD)x)}{x^2\sqrt{bx^2+a}}dx}{2a}-\frac{15\sqrt{a+bx^2}(3bB-4aD)}{2x^2}}{3a}\right)-\frac{5B\sqrt{a+bx^2}}{4x^4}}{5a}-\frac{A\sqrt{a+bx^2}}{5ax^5}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{1}{4}\left(\frac{4\sqrt{a+bx^2}(4Ab-5aC)}{3ax^3}-\frac{-\frac{1}{2}b\int \frac{16(4Ab-5aC)+15(3bB-4aD)x}{x^2\sqrt{bx^2+a}}dx-\frac{15\sqrt{a+bx^2}(3bB-4aD)}{2x^2}}{3a}\right)-\frac{5B\sqrt{a+bx^2}}{4x^4}}{5a}-\frac{A\sqrt{a+bx^2}}{5ax^5}$$

$$\Big\downarrow \ 534$$

$$\frac{\frac{1}{4}\left(\frac{4\sqrt{a+bx^2}(4Ab-5aC)}{3ax^3}-\frac{-\frac{1}{2}b\left(15(3bB-4aD)\int \frac{1}{x\sqrt{bx^2+a}}dx-\frac{16\sqrt{a+bx^2}(4Ab-5aC)}{ax}\right)-\frac{15\sqrt{a+bx^2}(3bB-4aD)}{2x^2}}{3a}\right)-\frac{5B\sqrt{a+bx^2}}{4x^4}}{5a}-\frac{A\sqrt{a+bx^2}}{5ax^5}$$

$$\Big\downarrow \ 243$$

$$\frac{\frac{1}{4}\left(\frac{4\sqrt{a+bx^2}(4Ab-5aC)}{3ax^3}-\frac{-\frac{1}{2}b\left(\frac{15}{2}(3bB-4aD)\int\frac{1}{x^2\sqrt{bx^2+a}}dx^2-\frac{16\sqrt{a+bx^2}(4Ab-5aC)}{ax}\right)-\frac{15\sqrt{a+bx^2}(3bB-4aD)}{2x^2}}{3a}\right)-\frac{5B\sqrt{a+bx^2}}{4x^4}}{5a}-\frac{A\sqrt{a+bx^2}}{5ax^5}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{1}{4}\left(\frac{4\sqrt{a+bx^2}(4Ab-5aC)}{3ax^3}-\frac{-\frac{1}{2}b\left(\frac{15(3bB-4aD)\int\frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{b}-\frac{16\sqrt{a+bx^2}(4Ab-5aC)}{ax}\right)-\frac{15\sqrt{a+bx^2}(3bB-4aD)}{2x^2}}{3a}\right)-\frac{5B\sqrt{a+bx^2}}{4x^4}}{5a}-\frac{A\sqrt{a+bx^2}}{5ax^5}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{1}{4}\left(\frac{4\sqrt{a+bx^2}(4Ab-5aC)}{3ax^3}-\frac{-\frac{1}{2}b\left(-\frac{16\sqrt{a+bx^2}(4Ab-5aC)}{ax}-\frac{15\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)(3bB-4aD)}{\sqrt{a}}\right)-\frac{15\sqrt{a+bx^2}(3bB-4aD)}{2x^2}}{3a}\right)-\frac{5B\sqrt{a+bx^2}}{4x^4}}{5a}-\frac{A\sqrt{a+bx^2}}{5ax^5}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/(x^6*Sqrt[a + b*x^2]),x]
```

output
```
-1/5*(A*Sqrt[a + b*x^2])/(a*x^5) + ((-5*B*Sqrt[a + b*x^2])/(4*x^4) + ((4*(
4*A*b - 5*a*C)*Sqrt[a + b*x^2])/(3*a*x^3) - ((-15*(3*b*B - 4*a*D)*Sqrt[a +
 b*x^2])/(2*x^2) - (b*((-16*(4*A*b - 5*a*C)*Sqrt[a + b*x^2])/(a*x) - (15*(
3*b*B - 4*a*D)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a]))/2)/(3*a))/4)/(5
*a)
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 539
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[c*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))
   Int[x^(m + 1)*(a + b*x^2)^p*(a*d*(m + 1) - b*c*(m + 2*p + 3)*x), x], x]
/; FreeQ[{a, b, c, d, p}, x] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

## Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.29

| method | result |
|---|---|
| default | $A\left(-\frac{\sqrt{b\,x^{2}+a}}{5a\,x^{5}}-\frac{4b\left(-\frac{\sqrt{b\,x^{2}+a}}{3a\,x^{3}}+\frac{2b\sqrt{b\,x^{2}+a}}{3a^{2}x}\right)}{5a}\right)+B\left(-\frac{\sqrt{b\,x^{2}+a}}{4a\,x^{4}}-\frac{3b\left(-\frac{\sqrt{b\,x^{2}+a}}{2a\,x^{2}}+\frac{b\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{2a^{\frac{3}{2}}}\right)}{4a}\right)+C$ |

input
```
int((D*x^3+C*x^2+B*x+A)/x^6/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
A*(-1/5*(b*x^2+a)^(1/2)/a/x^5-4/5*b/a*(-1/3*(b*x^2+a)^(1/2)/a/x^3+2/3*b/a^
2*(b*x^2+a)^(1/2)/x))+B*(-1/4*(b*x^2+a)^(1/2)/a/x^4-3/4*b/a*(-1/2*(b*x^2+a
)^(1/2)/a/x^2+1/2*b/a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)))+C*(-1/
3*(b*x^2+a)^(1/2)/a/x^3+2/3*b/a^2*(b*x^2+a)^(1/2)/x)+D*(-1/2*(b*x^2+a)^(1/
2)/a/x^2+1/2*b/a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x))
```

## Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.51

$$\int \frac{A + Bx + Cx^2 + Dx^3}{x^6\sqrt{a + bx^2}}\,dx$$
$$= \left[-\frac{15\,(4\,Dab - 3\,Bb^2)\sqrt{a}x^5\log\left(-\frac{bx^2 - 2\,\sqrt{bx^2+a}\sqrt{a} + 2\,a}{x^2}\right) - 2\,(16\,(5\,Cab - 4\,Ab^2)x^4 - 30\,Ba^2x - 15\,(4\,Da^2}{240\,a^3x^5}\right.$$
$$-\frac{15\,(4\,Dab - 3\,Bb^2)\sqrt{-a}x^5\arctan\left(\frac{\sqrt{bx^2+a}\sqrt{-a}}{a}\right) - (16\,(5\,Cab - 4\,Ab^2)x^4 - 30\,Ba^2x - 15\,(4\,Da^2 - 3\,B}{120\,a^3x^5}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/x^6/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/240*(15*(4*D*a*b - 3*B*b^2)*sqrt(a)*x^5*log(-(b*x^2 - 2*sqrt(b*x^2 + a
)*sqrt(a) + 2*a)/x^2) - 2*(16*(5*C*a*b - 4*A*b^2)*x^4 - 30*B*a^2*x - 15*(4
*D*a^2 - 3*B*a*b)*x^3 - 24*A*a^2 - 8*(5*C*a^2 - 4*A*a*b)*x^2)*sqrt(b*x^2 +
 a))/(a^3*x^5), -1/120*(15*(4*D*a*b - 3*B*b^2)*sqrt(-a)*x^5*arctan(sqrt(b*
x^2 + a)*sqrt(-a)/a) - (16*(5*C*a*b - 4*A*b^2)*x^4 - 30*B*a^2*x - 15*(4*D*
a^2 - 3*B*a*b)*x^3 - 24*A*a^2 - 8*(5*C*a^2 - 4*A*a*b)*x^2)*sqrt(b*x^2 + a)
)/(a^3*x^5)]
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 507 vs. $2(160) = 320$.

Time = 5.15 (sec) , antiderivative size = 507, normalized size of antiderivative = 2.93

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^6\sqrt{a+bx^2}}\,dx = -\frac{3Aa^4b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2}+1}}{15a^5b^4x^4+30a^4b^5x^6+15a^3b^6x^8} - \frac{2Aa^3b^{\frac{11}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{15a^5b^4x^4+30a^4b^5x^6+15a^3b^6x^8} - \frac{3Aa^2b^{\frac{13}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{15a^5b^4x^4+30a^4b^5x^6+15a^3b^6x^8} - \frac{12Aab^{\frac{15}{2}}x^6\sqrt{\frac{a}{bx^2}+1}}{15a^5b^4x^4+30a^4b^5x^6+15a^3b^6x^8} - \frac{8Ab^{\frac{17}{2}}x^8\sqrt{\frac{a}{bx^2}+1}}{15a^5b^4x^4+30a^4b^5x^6+15a^3b^6x^8} - \frac{B}{4\sqrt{b}x^5\sqrt{\frac{a}{bx^2}+1}} + \frac{B\sqrt{b}}{8ax^3\sqrt{\frac{a}{bx^2}+1}} + \frac{3Bb^{\frac{3}{2}}}{8a^2x\sqrt{\frac{a}{bx^2}+1}} - \frac{3Bb^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{8a^{\frac{5}{2}}} - \frac{C\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{3ax^2} + \frac{2Cb^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{3a^2} - \frac{D\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{2ax} + \frac{Db\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2a^{\frac{3}{2}}}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/x**6/(b*x**2+a)**(1/2),x)
```

output
```
-3*A*a**4*b**(9/2)*sqrt(a/(b*x**2) + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*
x**6 + 15*a**3*b**6*x**8) - 2*A*a**3*b**(11/2)*x**2*sqrt(a/(b*x**2) + 1)/(
15*a**5*b**4*x**4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x**8) - 3*A*a**2*b**(
13/2)*x**4*sqrt(a/(b*x**2) + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*x**6 + 1
5*a**3*b**6*x**8) - 12*A*a*b**(15/2)*x**6*sqrt(a/(b*x**2) + 1)/(15*a**5*b*
*4*x**4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x**8) - 8*A*b**(17/2)*x**8*sqrt
(a/(b*x**2) + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x**
8) - B/(4*sqrt(b)*x**5*sqrt(a/(b*x**2) + 1)) + B*sqrt(b)/(8*a*x**3*sqrt(a/
(b*x**2) + 1)) + 3*B*b**(3/2)/(8*a**2*x*sqrt(a/(b*x**2) + 1)) - 3*B*b**2*a
sinh(sqrt(a)/(sqrt(b)*x))/(8*a**(5/2)) - C*sqrt(b)*sqrt(a/(b*x**2) + 1)/(3
*a*x**2) + 2*C*b**(3/2)*sqrt(a/(b*x**2) + 1)/(3*a**2) - D*sqrt(b)*sqrt(a/(
b*x**2) + 1)/(2*a*x) + D*b*asinh(sqrt(a)/(sqrt(b)*x))/(2*a**(3/2))
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.10

$$\int \frac{A + Bx + Cx^2 + Dx^3}{x^6\sqrt{a + bx^2}}\,dx = \frac{Db\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{2\,a^{\frac{3}{2}}} - \frac{3\,Bb^2\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{8\,a^{\frac{5}{2}}} + \frac{2\,\sqrt{bx^2 + a}Cb}{3\,a^2x} - \frac{8\,\sqrt{bx^2 + a}Ab^2}{15\,a^3x} - \frac{\sqrt{bx^2 + a}D}{2\,ax^2} + \frac{3\,\sqrt{bx^2 + a}Bb}{8\,a^2x^2} - \frac{\sqrt{bx^2 + a}C}{3\,ax^3} + \frac{4\,\sqrt{bx^2 + a}Ab}{15\,a^2x^3} - \frac{\sqrt{bx^2 + a}B}{4\,ax^4} - \frac{\sqrt{bx^2 + a}A}{5\,ax^5}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/x^6/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/2*D*b*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2) - 3/8*B*b^2*arcsinh(a/(sqrt(
a*b)*abs(x)))/a^(5/2) + 2/3*sqrt(b*x^2 + a)*C*b/(a^2*x) - 8/15*sqrt(b*x^2
+ a)*A*b^2/(a^3*x) - 1/2*sqrt(b*x^2 + a)*D/(a*x^2) + 3/8*sqrt(b*x^2 + a)*B
*b/(a^2*x^2) - 1/3*sqrt(b*x^2 + a)*C/(a*x^3) + 4/15*sqrt(b*x^2 + a)*A*b/(a
^2*x^3) - 1/4*sqrt(b*x^2 + a)*B/(a*x^4) - 1/5*sqrt(b*x^2 + a)*A/(a*x^5)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 442 vs. 2(145) = 290.

Time = 0.13 (sec) , antiderivative size = 442, normalized size of antiderivative = 2.55

$$\int \frac{A + Bx + Cx^2 + Dx^3}{x^6\sqrt{a + bx^2}}\,dx = -\frac{(4\,Dab - 3\,Bb^2)\arctan\left(-\frac{\sqrt{b}x - \sqrt{bx^2 + a}}{\sqrt{-a}}\right)}{4\,\sqrt{-a}a^2} + \frac{60\left(\sqrt{b}x - \sqrt{bx^2 + a}\right)^9 Dab - 45\left(\sqrt{b}x - \sqrt{bx^2 + a}\right)^9 Bb^2 - 120\left(\sqrt{b}x - \sqrt{bx^2 + a}\right)^7 Da^2b + 210\,(\sqrt{b}}{}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/x^6/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
-1/4*(4*D*a*b - 3*B*b^2)*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/(
sqrt(-a)*a^2) + 1/60*(60*(sqrt(b)*x - sqrt(b*x^2 + a))^9*D*a*b - 45*(sqrt(
b)*x - sqrt(b*x^2 + a))^9*B*b^2 - 120*(sqrt(b)*x - sqrt(b*x^2 + a))^7*D*a^
2*b + 210*(sqrt(b)*x - sqrt(b*x^2 + a))^7*B*a*b^2 + 240*(sqrt(b)*x - sqrt(
b*x^2 + a))^6*C*a^2*b^(3/2) - 560*(sqrt(b)*x - sqrt(b*x^2 + a))^4*C*a^3*b^
(3/2) + 640*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*a^2*b^(5/2) + 120*(sqrt(b)*x
 - sqrt(b*x^2 + a))^3*D*a^4*b - 210*(sqrt(b)*x - sqrt(b*x^2 + a))^3*B*a^3*
b^2 + 400*(sqrt(b)*x - sqrt(b*x^2 + a))^2*C*a^4*b^(3/2) - 320*(sqrt(b)*x -
 sqrt(b*x^2 + a))^2*A*a^3*b^(5/2) - 60*(sqrt(b)*x - sqrt(b*x^2 + a))*D*a^5
*b + 45*(sqrt(b)*x - sqrt(b*x^2 + a))*B*a^4*b^2 - 80*C*a^5*b^(3/2) + 64*A*
a^4*b^(5/2))/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^5*a^2)
```

## Mupad [B] (verification not implemented)

Time = 3.05 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.95

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^6\sqrt{a+bx^2}}\,dx = \frac{b\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)\,D}{2\,a^{3/2}} - \frac{\sqrt{b\,x^2+a}\,D}{2\,a\,x^2} - \frac{3\,B\,b^2\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{8\,a^{5/2}} - \frac{5\,B\,\sqrt{b\,x^2+a}}{8\,a\,x^4} + \frac{3\,B\,{\left(b\,x^2+a\right)}^{3/2}}{8\,a^2\,x^4} - \frac{C\,\sqrt{b\,x^2+a}\,\left(a-2\,b\,x^2\right)}{3\,a^2\,x^3} - \frac{A\,\sqrt{b\,x^2+a}\,\left(3\,a^2-4\,a\,b\,x^2+8\,b^2\,x^4\right)}{15\,a^3\,x^5}$$

input
```
int((A + B*x + C*x^2 + x^3*D)/(x^6*(a + b*x^2)^(1/2)),x)
```

output
```
(b*atanh((a + b*x^2)^(1/2)/a^(1/2))*D)/(2*a^(3/2)) - ((a + b*x^2)^(1/2)*D)
/(2*a*x^2) - (3*B*b^2*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(8*a^(5/2)) - (5*B
*(a + b*x^2)^(1/2))/(8*a*x^4) + (3*B*(a + b*x^2)^(3/2))/(8*a^2*x^4) - (C*(
a + b*x^2)^(1/2)*(a - 2*b*x^2))/(3*a^2*x^3) - (A*(a + b*x^2)^(1/2)*(3*a^2
+ 8*b^2*x^4 - 4*a*b*x^2))/(15*a^3*x^5)
```

## Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.66

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^6\sqrt{a+bx^2}}\,dx = \frac{-24\sqrt{b\,x^2+a}\,a^3+32\sqrt{b\,x^2+a}\,a^2b\,x^2-30\sqrt{b\,x^2+a}\,a^2bx-40\sqrt{b\,x^2+a}\,a^2c\,x^2-60\sqrt{b\,x^2+a}\,a^2d\,x^3-}{}$$

input

```
int((D*x^3+C*x^2+B*x+A)/x^6/(b*x^2+a)^(1/2),x)
```

output

```
( - 24*sqrt(a + b*x**2)*a**3 + 32*sqrt(a + b*x**2)*a**2*b*x**2 - 30*sqrt(a
 + b*x**2)*a**2*b*x - 40*sqrt(a + b*x**2)*a**2*c*x**2 - 60*sqrt(a + b*x**2
)*a**2*d*x**3 - 64*sqrt(a + b*x**2)*a*b**2*x**4 + 45*sqrt(a + b*x**2)*a*b*
*2*x**3 + 80*sqrt(a + b*x**2)*a*b*c*x**4 - 60*sqrt(a)*log((sqrt(a + b*x**2
) - sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b*d*x**5 + 45*sqrt(a)*log((sqrt(a + b*
x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*b**3*x**5 + 60*sqrt(a)*log((sqrt(a +
 b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b*d*x**5 - 45*sqrt(a)*log((sqrt
(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*b**3*x**5 + 64*sqrt(b)*a*b**2
*x**5 - 80*sqrt(b)*a*b*c*x**5)/(120*a**3*x**5)
```

# 3.100 $\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^{3/2}}\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 165

$$\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^{3/2}}\,dx = \frac{a(Ab-aC+(bB-aD)x)}{b^3\sqrt{a+bx^2}} + \frac{(Ab-2aC)\sqrt{a+bx^2}}{b^3} + \frac{(4bB-7aD)x\sqrt{a+bx^2}}{8b^3} + \frac{Dx^3\sqrt{a+bx^2}}{4b^2} + \frac{C(a+bx^2)^{3/2}}{3b^3} - \frac{3a(4bB-5aD)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{7/2}}$$

output
```
a*(A*b-C*a+(B*b-D*a)*x)/b^3/(b*x^2+a)^(1/2)+(A*b-2*C*a)*(b*x^2+a)^(1/2)/b^
3+1/8*(4*B*b-7*D*a)*x*(b*x^2+a)^(1/2)/b^3+1/4*D*x^3*(b*x^2+a)^(1/2)/b^2+1/
3*C*(b*x^2+a)^(3/2)/b^3-3/8*a*(4*B*b-5*D*a)*arctanh(b^(1/2)*x/(b*x^2+a)^(1
/2))/b^(7/2)
```

## Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.75

$$\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^{3/2}}\,dx = \frac{-a^2(64C+45Dx)+ab(48A+x(36B-32Cx-15Dx^2))+2b^2x^2(12A}{24b^3\sqrt{a+bx^2}}$$
$$-\frac{3a(-4bB+5aD)\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{8b^{7/2}}$$

input
```
Integrate[(x^3*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^(3/2),x]
```

output
```
(-(a^2*(64*C + 45*D*x)) + a*b*(48*A + x*(36*B - 32*C*x - 15*D*x^2)) + 2*b^
2*x^2*(12*A + x*(6*B + 4*C*x + 3*D*x^2)))/(24*b^3*Sqrt[a + b*x^2]) - (3*a*
(-4*b*B + 5*a*D)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(8*b^(7/2))
```

## Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.28, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2335, 25, 2340, 533, 25, 27, 533, 455, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx^2)^{3/2}}dx$$

$$\Big\downarrow \ 2335$$

$$-\frac{\int -\frac{x^2\left(aDx^2-(3Ab-4aC)x+3a\left(B-\frac{aD}{b}\right)\right)}{\sqrt{bx^2+a}}dx}{ab}-\frac{x^3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{x^2\left(aDx^2-(3Ab-4aC)x+3a\left(B-\frac{aD}{b}\right)\right)}{\sqrt{bx^2+a}}dx}{ab}-\frac{x^3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \ 2340$$

$$\frac{\frac{\int \frac{x^2(3a(4bB-5aD)-4b(3Ab-4aC)x)}{\sqrt{bx^2+a}}dx}{4b}+\frac{aDx^3\sqrt{a+bx^2}}{4b}}{ab}-\frac{x^3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{533}$$

$$\frac{\frac{-\frac{\int -\frac{abx(8(3Ab-4aC)+9(4bB-5aD)x)}{\sqrt{bx^2+a}}dx}{3b}-\frac{4}{3}x^2\sqrt{a+bx^2}(3Ab-4aC)}{4b}+\frac{aDx^3\sqrt{a+bx^2}}{4b}}{ab}-\frac{x^3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\frac{\int \frac{abx(8(3Ab-4aC)+9(4bB-5aD)x)}{\sqrt{bx^2+a}}dx}{3b}-\frac{4}{3}x^2\sqrt{a+bx^2}(3Ab-4aC)}{4b}+\frac{aDx^3\sqrt{a+bx^2}}{4b}}{ab}-\frac{x^3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{1}{3}a\int \frac{x(8(3Ab-4aC)+9(4bB-5aD)x)}{\sqrt{bx^2+a}}dx-\frac{4}{3}x^2\sqrt{a+bx^2}(3Ab-4aC)}{4b}+\frac{aDx^3\sqrt{a+bx^2}}{4b}}{ab}-\frac{x^3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{533}$$

$$\frac{\frac{\frac{1}{3}a\left(\frac{9x\sqrt{a+bx^2}(4bB-5aD)}{2b}-\frac{\int \frac{9a(4bB-5aD)-16b(3Ab-4aC)x}{\sqrt{bx^2+a}}dx}{2b}\right)-\frac{4}{3}x^2\sqrt{a+bx^2}(3Ab-4aC)}{4b}+\frac{aDx^3\sqrt{a+bx^2}}{4b}}{ab}-\frac{x^3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{455}$$

$$\frac{\frac{\frac{1}{3}a\left(\frac{9x\sqrt{a+bx^2}(4bB-5aD)}{2b}-\frac{9a(4bB-5aD)\int \frac{1}{\sqrt{bx^2+a}}dx-16\sqrt{a+bx^2}(3Ab-4aC)}{2b}\right)-\frac{4}{3}x^2\sqrt{a+bx^2}(3Ab-4aC)}{4b}+\frac{aDx^3\sqrt{a+bx^2}}{4b}}{ab}-\frac{x^3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{\frac{1}{3}a\left(\frac{9x\sqrt{a+bx^2}(4bB-5aD)}{2b}-\frac{9a(4bB-5aD)\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}-16\sqrt{a+bx^2}(3Ab-4aC)}{2b}\right)-\frac{4}{3}x^2\sqrt{a+bx^2}(3Ab-4aC)}{4b}+\frac{aDx^3\sqrt{a+bx^2}}{4b}}{ab}-\frac{x^3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{\frac{1}{3}a\left(\frac{9x\sqrt{a+bx^2}(4bB-5aD)}{2b}-\frac{\frac{9a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(4bB-5aD)}{\sqrt{b}}-16\sqrt{a+bx^2}(3Ab-4aC)}{2b}\right)-\frac{4}{3}x^2\sqrt{a+bx^2}(3Ab-4aC)}{4b}+\frac{aDx^3\sqrt{a+bx^2}}{4b}}{ab}-\frac{x^3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{ab\sqrt{a+bx^2}}$$

input
```
Int[(x^3*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^(3/2),x]
```

output
```
-((x^3*(a*(B - (a*D)/b) - (A*b - a*C)*x))/(a*b*Sqrt[a + b*x^2])) + ((a*D*x
^3*Sqrt[a + b*x^2])/(4*b) + ((-4*(3*A*b - 4*a*C)*x^2*Sqrt[a + b*x^2])/3 +
(a*((9*(4*b*B - 5*a*D)*x*Sqrt[a + b*x^2])/(2*b) - (-16*(3*A*b - 4*a*C)*Sqr
t[a + b*x^2] + (9*a*(4*b*B - 5*a*D)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/
Sqrt[b])/(2*b)))/3)/(4*b))/(a*b)
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

rule 2335
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Simp[c/(2*a*b*(p + 1))   Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSu
m[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
 b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

rule 2340
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Simp[1/(b*(m
+ q + 2*p + 1))   Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)
*Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /;
GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ
[Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

## Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.47

| method | result |
|---|---|
| default | $A\left(\frac{x^2}{b\sqrt{b\,x^2+a}}+\frac{2a}{b^2\sqrt{b\,x^2+a}}\right)+B\left(\frac{x^3}{2b\sqrt{b\,x^2+a}}-\frac{3a\left(-\frac{x}{b\sqrt{b\,x^2+a}}+\frac{\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{b^{\frac{3}{2}}}\right)}{2b}\right)+C\left(\frac{x^4}{3b\sqrt{b\,x^2+a}}-\frac{4a\left(\right.}{}\right.$ |

input
```
int(x^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
A*(x^2/b/(b*x^2+a)^(1/2)+2*a/b^2/(b*x^2+a)^(1/2))+B*(1/2*x^3/b/(b*x^2+a)^(
1/2)-3/2*a/b*(-x/b/(b*x^2+a)^(1/2)+1/b^(3/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
))+C*(1/3*x^4/b/(b*x^2+a)^(1/2)-4/3*a/b*(x^2/b/(b*x^2+a)^(1/2)+2*a/b^2/(b*
x^2+a)^(1/2)))+D*(1/4*x^5/b/(b*x^2+a)^(1/2)-5/4*a/b*(1/2*x^3/b/(b*x^2+a)^(
1/2)-3/2*a/b*(-x/b/(b*x^2+a)^(1/2)+1/b^(3/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
)))
```

## Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 358, normalized size of antiderivative = 2.17

$$\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^{3/2}}\,dx = \left[-\frac{9\,(5\,Da^3-4\,Ba^2b+(5\,Da^2b-4\,Bab^2)x^2)\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2}\right.}{}\right.$$

$$-\frac{9\,(5\,Da^3-4\,Ba^2b+(5\,Da^2b-4\,Bab^2)x^2)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(6\,Db^3x^5+8\,Cb^3x^4-64\,Ca^2b+48\,A}{24\,(b^5x^2+ab^4)}$$

input
```
integrate(x^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[-1/48*(9*(5*D*a^3 - 4*B*a^2*b + (5*D*a^2*b - 4*B*a*b^2)*x^2)*sqrt(b)*log(
-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(6*D*b^3*x^5 + 8*C*b^3*x^4
 - 64*C*a^2*b + 48*A*a*b^2 - 3*(5*D*a*b^2 - 4*B*b^3)*x^3 - 8*(4*C*a*b^2 -
3*A*b^3)*x^2 - 9*(5*D*a^2*b - 4*B*a*b^2)*x)*sqrt(b*x^2 + a))/(b^5*x^2 + a*
b^4), -1/24*(9*(5*D*a^3 - 4*B*a^2*b + (5*D*a^2*b - 4*B*a*b^2)*x^2)*sqrt(-b
)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (6*D*b^3*x^5 + 8*C*b^3*x^4 - 64*C*a
^2*b + 48*A*a*b^2 - 3*(5*D*a*b^2 - 4*B*b^3)*x^3 - 8*(4*C*a*b^2 - 3*A*b^3)*
x^2 - 9*(5*D*a^2*b - 4*B*a*b^2)*x)*sqrt(b*x^2 + a))/(b^5*x^2 + a*b^4)]
```

## Sympy [A] (verification not implemented)

Time = 11.46 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.77

$$\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^{3/2}}\,dx = A\left(\begin{cases} \frac{2a}{b^2\sqrt{a+bx^2}}+\frac{x^2}{b\sqrt{a+bx^2}} & \text{for } b\neq 0\\ \frac{x^4}{4a^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right) + B\left(\frac{3\sqrt{a}x}{2b^2\sqrt{1+\frac{bx^2}{a}}}-\frac{3a\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{\frac{5}{2}}}+\frac{x^3}{2\sqrt{a}b\sqrt{1+\frac{bx^2}{a}}}\right) + C\left(\begin{cases} -\frac{8a^2}{3b^3\sqrt{a+bx^2}}-\frac{4ax^2}{3b^2\sqrt{a+bx^2}}+\frac{x^4}{3b\sqrt{a+bx^2}} & \text{for } b\neq 0\\ \frac{x^6}{6a^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right) + D\left(-\frac{15a^{\frac{3}{2}}x}{8b^3\sqrt{1+\frac{bx^2}{a}}}-\frac{5\sqrt{a}x^3}{8b^2\sqrt{1+\frac{bx^2}{a}}}+\frac{15a^2\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{\frac{7}{2}}}+\frac{x^5}{4\sqrt{a}b\sqrt{1+\frac{bx^2}{a}}}\right)$$

input
```
integrate(x**3*(D*x**3+C*x**2+B*x+A)/(b*x**2+a)**(3/2),x)
```

output
```
A*Piecewise((2*a/(b**2*sqrt(a + b*x**2)) + x**2/(b*sqrt(a + b*x**2)), Ne(b
, 0)), (x**4/(4*a**(3/2)), True)) + B*(3*sqrt(a)*x/(2*b**2*sqrt(1 + b*x**2
/a)) - 3*a*asinh(sqrt(b)*x/sqrt(a))/(2*b**(5/2)) + x**3/(2*sqrt(a)*b*sqrt(
1 + b*x**2/a))) + C*Piecewise((-8*a**2/(3*b**3*sqrt(a + b*x**2)) - 4*a*x**
2/(3*b**2*sqrt(a + b*x**2)) + x**4/(3*b*sqrt(a + b*x**2)), Ne(b, 0)), (x**
6/(6*a**(3/2)), True)) + D*(-15*a**(3/2)*x/(8*b**3*sqrt(1 + b*x**2/a)) - 5
*sqrt(a)*x**3/(8*b**2*sqrt(1 + b*x**2/a)) + 15*a**2*asinh(sqrt(b)*x/sqrt(a
))/(8*b**(7/2)) + x**5/(4*sqrt(a)*b*sqrt(1 + b*x**2/a)))
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.30

$$\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^{3/2}}\,dx = \frac{Dx^5}{4\sqrt{bx^2+a}b} + \frac{Cx^4}{3\sqrt{bx^2+a}b} - \frac{5\,Dax^3}{8\sqrt{bx^2+a}b^2} + \frac{Bx^3}{2\sqrt{bx^2+a}b} - \frac{4\,Cax^2}{3\sqrt{bx^2+a}b^2} + \frac{Ax^2}{\sqrt{bx^2+a}b} - \frac{15\,Da^2x}{8\sqrt{bx^2+a}b^3} + \frac{3\,Bax}{2\sqrt{bx^2+a}b^2} + \frac{15\,Da^2\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{7}{2}}} - \frac{3\,Ba\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{5}{2}}} - \frac{8\,Ca^2}{3\sqrt{bx^2+a}b^3} + \frac{2\,Aa}{\sqrt{bx^2+a}b^2}$$

input
```
integrate(x^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
1/4*D*x^5/(sqrt(b*x^2 + a)*b) + 1/3*C*x^4/(sqrt(b*x^2 + a)*b) - 5/8*D*a*x^
3/(sqrt(b*x^2 + a)*b^2) + 1/2*B*x^3/(sqrt(b*x^2 + a)*b) - 4/3*C*a*x^2/(sqr
t(b*x^2 + a)*b^2) + A*x^2/(sqrt(b*x^2 + a)*b) - 15/8*D*a^2*x/(sqrt(b*x^2 +
 a)*b^3) + 3/2*B*a*x/(sqrt(b*x^2 + a)*b^2) + 15/8*D*a^2*arcsinh(b*x/sqrt(a
*b))/b^(7/2) - 3/2*B*a*arcsinh(b*x/sqrt(a*b))/b^(5/2) - 8/3*C*a^2/(sqrt(b*
x^2 + a)*b^3) + 2*A*a/(sqrt(b*x^2 + a)*b^2)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.95

$$\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^{3/2}}\,dx = \frac{\left(\left(\left(2\left(\frac{3\,Dx}{b} + \frac{4\,C}{b}\right)x - \frac{3\,(5\,Dab^4-4\,Bb^5)}{b^6}\right)x - \frac{8\,(4\,Cab^4-3\,Ab^5)}{b^6}\right)x - \frac{9\,(5\,Da^2b^3-4\,}{b^6}}{24\sqrt{bx^2+a}} - \frac{3\,(5\,Da^2-4\,Bab)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{8\,b^{\frac{7}{2}}}$$

input
```
integrate(x^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
1/24*((((2*(3*D*x/b + 4*C/b)*x - 3*(5*D*a*b^4 - 4*B*b^5)/b^6)*x - 8*(4*C*a
*b^4 - 3*A*b^5)/b^6)*x - 9*(5*D*a^2*b^3 - 4*B*a*b^4)/b^6)*x - 16*(4*C*a^2*
b^3 - 3*A*a*b^4)/b^6)/sqrt(b*x^2 + a) - 3/8*(5*D*a^2 - 4*B*a*b)*log(abs(-s
qrt(b)*x + sqrt(b*x^2 + a)))/b^(7/2)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^{3/2}}\,dx = \int \frac{x^3\,(A+B\,x+C\,x^2+x^3\,D)}{(b\,x^2+a)^{3/2}}\,dx$$

input
```
int((x^3*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^(3/2),x)
```

output
```
int((x^3*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^(3/2), x)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 338, normalized size of antiderivative = 2.05

$$\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^{3/2}}\,dx = \frac{48\sqrt{b\,x^2+a}\,a^2b^2-64\sqrt{b\,x^2+a}\,a^2bc-45\sqrt{b\,x^2+a}\,a^2bdx+24\sqrt{b\,x^2+}$$

input
```
int(x^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(3/2),x)
```

output
```
(48*sqrt(a + b*x**2)*a**2*b**2 - 64*sqrt(a + b*x**2)*a**2*b*c - 45*sqrt(a
+ b*x**2)*a**2*b*d*x + 24*sqrt(a + b*x**2)*a*b**3*x**2 + 36*sqrt(a + b*x**
2)*a*b**3*x - 32*sqrt(a + b*x**2)*a*b**2*c*x**2 - 15*sqrt(a + b*x**2)*a*b*
*2*d*x**3 + 12*sqrt(a + b*x**2)*b**4*x**3 + 8*sqrt(a + b*x**2)*b**3*c*x**4
 + 6*sqrt(a + b*x**2)*b**3*d*x**5 + 45*sqrt(b)*log((sqrt(a + b*x**2) + sqr
t(b)*x)/sqrt(a))*a**3*d - 36*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sq
rt(a))*a**2*b**2 + 45*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*
a**2*b*d*x**2 - 36*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a*b
**3*x**2 - 30*sqrt(b)*a**3*d + 27*sqrt(b)*a**2*b**2 - 30*sqrt(b)*a**2*b*d*
x**2 + 27*sqrt(b)*a*b**3*x**2)/(24*b**4*(a + b*x**2))
```

# 3.101 $\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^{3/2}}\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 138

$$\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^{3/2}}\,dx = \frac{a(bB-aD)-b(Ab-aC)x}{b^3\sqrt{a+bx^2}} + \frac{(bB-2aD)\sqrt{a+bx^2}}{b^3} + \frac{Cx\sqrt{a+bx^2}}{2b^2} + \frac{D(a+bx^2)^{3/2}}{3b^3} + \frac{(2Ab-3aC)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{5/2}}$$

output

```
(a*(B*b-D*a)-b*(A*b-C*a)*x)/b^3/(b*x^2+a)^(1/2)+(B*b-2*D*a)*(b*x^2+a)^(1/2
)/b^3+1/2*C*x*(b*x^2+a)^(1/2)/b^2+1/3*D*(b*x^2+a)^(3/2)/b^3+1/2*(2*A*b-3*C
*a)*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(5/2)
```

## Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.81

$$\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^{3/2}}\,dx = \frac{-16a^2D+ab(12B+x(9C-8Dx))+b^2x(-6A+x(6B+3Cx+2Dx}{6b^3\sqrt{a+bx^2}}$$
$$+\frac{(2Ab-3aC)\text{arctanh}\left(\frac{\sqrt{b}x}{-\sqrt{a}+\sqrt{a+bx^2}}\right)}{b^{5/2}}$$

input
```
Integrate[(x^2*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^(3/2),x]
```

output
```
(-16*a^2*D + a*b*(12*B + x*(9*C - 8*D*x)) + b^2*x*(-6*A + x*(6*B + 3*C*x +
 2*D*x^2)))/(6*b^3*Sqrt[a + b*x^2]) + ((2*A*b - 3*a*C)*ArcTanh[(Sqrt[b]*x)
/(-Sqrt[a] + Sqrt[a + b*x^2])])/b^(5/2)
```

## Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.28, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {2335, 25, 2340, 533, 25, 27, 455, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^{3/2}}dx$$

$$\Big\downarrow \text{2335}$$

$$-\frac{\int -\frac{x\left(aDx^2-(2Ab-3aC)x+2a\left(B-\frac{aD}{b}\right)\right)}{\sqrt{bx^2+a}}dx}{ab}-\frac{x^2\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{x\left(aDx^2-(2Ab-3aC)x+2a\left(B-\frac{aD}{b}\right)\right)}{\sqrt{bx^2+a}}dx}{ab}-\frac{x^2\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{2340}$$

$$\frac{\frac{\int \frac{x(2a(3bB-4aD)-3b(2Ab-3aC)x)}{\sqrt{bx^2+a}}dx}{3b}+\frac{aDx^2\sqrt{a+bx^2}}{3b}}{ab}-\frac{x^2\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{ 533}$$

$$\frac{\frac{-\frac{\int -\frac{ab(3(2Ab-3aC)+4(3bB-4aD)x)}{\sqrt{bx^2+a}}dx}{2b}-\frac{3}{2}x\sqrt{a+bx^2}(2Ab-3aC)}{3b}+\frac{aDx^2\sqrt{a+bx^2}}{3b}}{ab}-\frac{x^2\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\frac{\frac{\int \frac{ab(3(2Ab-3aC)+4(3bB-4aD)x)}{\sqrt{bx^2+a}}dx}{2b}-\frac{3}{2}x\sqrt{a+bx^2}(2Ab-3aC)}{3b}+\frac{aDx^2\sqrt{a+bx^2}}{3b}}{ab}-\frac{x^2\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{\frac{1}{2}a\int \frac{3(2Ab-3aC)+4(3bB-4aD)x}{\sqrt{bx^2+a}}dx-\frac{3}{2}x\sqrt{a+bx^2}(2Ab-3aC)}{3b}+\frac{aDx^2\sqrt{a+bx^2}}{3b}}{ab}-\frac{x^2\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{ 455}$$

$$\frac{\frac{\frac{1}{2}a\left(3(2Ab-3aC)\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{4\sqrt{a+bx^2}(3bB-4aD)}{b}\right)-\frac{3}{2}x\sqrt{a+bx^2}(2Ab-3aC)}{3b}+\frac{aDx^2\sqrt{a+bx^2}}{3b}}{ab}-\frac{x^2\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{\frac{\frac{1}{2}a\left(3(2Ab-3aC)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{4\sqrt{a+bx^2}(3bB-4aD)}{b}\right)-\frac{3}{2}x\sqrt{a+bx^2}(2Ab-3aC)}{3b}+\frac{aDx^2\sqrt{a+bx^2}}{3b}}{ab}-\frac{x^2\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\frac{\frac{1}{2}a\left(\frac{3(2Ab-3aC)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}+\frac{4\sqrt{a+bx^2}(3bB-4aD)}{b}\right)-\frac{3}{2}x\sqrt{a+bx^2}(2Ab-3aC)}{3b}+\frac{aDx^2\sqrt{a+bx^2}}{3b}}{ab}-\frac{x^2\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{ab\sqrt{a+bx^2}}$$

input
```
Int[(x^2*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^(3/2),x]
```

output
```
-((x^2*(a*(B - (a*D)/b) - (A*b - a*C)*x))/(a*b*Sqrt[a + b*x^2])) + ((a*D*x
^2*Sqrt[a + b*x^2])/(3*b) + ((-3*(2*A*b - 3*a*C)*x*Sqrt[a + b*x^2])/2 + (a
*((4*(3*b*B - 4*a*D)*Sqrt[a + b*x^2])/b + (3*(2*A*b - 3*a*C)*ArcTanh[(Sqrt
[b]*x)/Sqrt[a + b*x^2]])/Sqrt[b]))/2)/(3*b))/(a*b)
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

rule 2335
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Simp[c/(2*a*b*(p + 1))   Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSu
m[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
 b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

rule 2340
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Simp[1/(b*(m
+ q + 2*p + 1))   Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)
*Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /;
GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ
[Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

## Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.41

| method | result |
|---|---|
| default | $A\left(-\frac{x}{b\sqrt{b\,x^{2}+a}}+\frac{\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{b^{\frac{3}{2}}}\right)+B\left(\frac{x^{2}}{b\sqrt{b\,x^{2}+a}}+\frac{2a}{b^{2}\sqrt{b\,x^{2}+a}}\right)+C\left(\frac{x^{3}}{2b\sqrt{b\,x^{2}+a}}-\frac{3a\left(-\frac{x}{b\sqrt{b\,x^{2}+a}}+\frac{\ln\left(\sqrt{\right.}\right.}{}\right.}{2b}\right.$ |

input
```
int(x^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
A*(-x/b/(b*x^2+a)^(1/2)+1/b^(3/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))+B*(x^2/b/
(b*x^2+a)^(1/2)+2*a/b^2/(b*x^2+a)^(1/2))+C*(1/2*x^3/b/(b*x^2+a)^(1/2)-3/2*
a/b*(-x/b/(b*x^2+a)^(1/2)+1/b^(3/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))))+D*(1/3
*x^4/b/(b*x^2+a)^(1/2)-4/3*a/b*(x^2/b/(b*x^2+a)^(1/2)+2*a/b^2/(b*x^2+a)^(1
/2)))
```

## Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 293, normalized size of antiderivative = 2.12

$$\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^{3/2}}\,dx = \left[-\frac{3\,(3\,Ca^2-2\,Aab+(3\,Cab-2\,Ab^2)x^2)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\right.}{}\right.$$

input
```
integrate(x^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[-1/12*(3*(3*C*a^2 - 2*A*a*b + (3*C*a*b - 2*A*b^2)*x^2)*sqrt(b)*log(-2*b*x
^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(2*D*b^2*x^4 + 3*C*b^2*x^3 - 16*
D*a^2 + 12*B*a*b - 2*(4*D*a*b - 3*B*b^2)*x^2 + 3*(3*C*a*b - 2*A*b^2)*x)*sq
rt(b*x^2 + a))/(b^4*x^2 + a*b^3), 1/6*(3*(3*C*a^2 - 2*A*a*b + (3*C*a*b - 2
*A*b^2)*x^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (2*D*b^2*x^4 +
3*C*b^2*x^3 - 16*D*a^2 + 12*B*a*b - 2*(4*D*a*b - 3*B*b^2)*x^2 + 3*(3*C*a*b
 - 2*A*b^2)*x)*sqrt(b*x^2 + a))/(b^4*x^2 + a*b^3)]
```

## Sympy [A] (verification not implemented)

Time = 7.39 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.67

$$\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^{3/2}}\,dx = A\left(\frac{\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{\frac{3}{2}}} - \frac{x}{\sqrt{a}b\sqrt{1+\frac{bx^2}{a}}}\right) + B\left(\begin{cases}\frac{2a}{b^2\sqrt{a+bx^2}} + \frac{x^2}{b\sqrt{a+bx^2}} & \text{for}\ b\neq 0\\ \frac{x^4}{4a^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right) + C\left(\frac{3\sqrt{a}x}{2b^2\sqrt{1+\frac{bx^2}{a}}} - \frac{3a\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{\frac{5}{2}}} + \frac{x^3}{2\sqrt{a}b\sqrt{1+\frac{bx^2}{a}}}\right) + D\left(\begin{cases}-\frac{8a^2}{3b^3\sqrt{a+bx^2}} - \frac{4ax^2}{3b^2\sqrt{a+bx^2}} + \frac{x^4}{3b\sqrt{a+bx^2}} & \text{for}\ b\neq 0\\ \frac{x^6}{6a^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right)$$

input
```
integrate(x**2*(D*x**3+C*x**2+B*x+A)/(b*x**2+a)**(3/2),x)
```

output
```
A*(asinh(sqrt(b)*x/sqrt(a))/b**(3/2) - x/(sqrt(a)*b*sqrt(1 + b*x**2/a))) +
 B*Piecewise((2*a/(b**2*sqrt(a + b*x**2)) + x**2/(b*sqrt(a + b*x**2)), Ne(
b, 0)), (x**4/(4*a**(3/2)), True)) + C*(3*sqrt(a)*x/(2*b**2*sqrt(1 + b*x**
2/a)) - 3*a*asinh(sqrt(b)*x/sqrt(a))/(2*b**(5/2)) + x**3/(2*sqrt(a)*b*sqrt
(1 + b*x**2/a))) + D*Piecewise((-8*a**2/(3*b**3*sqrt(a + b*x**2)) - 4*a*x*
*2/(3*b**2*sqrt(a + b*x**2)) + x**4/(3*b*sqrt(a + b*x**2)), Ne(b, 0)), (x*
*6/(6*a**(3/2)), True))
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.23

$$\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^{3/2}}\,dx = \frac{Dx^4}{3\sqrt{bx^2+a}b} + \frac{Cx^3}{2\sqrt{bx^2+a}b} - \frac{4\,Dax^2}{3\sqrt{bx^2+a}b^2} + \frac{Bx^2}{\sqrt{bx^2+a}b} + \frac{3\,Cax}{2\sqrt{bx^2+a}b^2} - \frac{Ax}{\sqrt{bx^2+a}b} - \frac{3\,Ca\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{5}{2}}} + \frac{A\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{3}{2}}} - \frac{8\,Da^2}{3\sqrt{bx^2+a}b^3} + \frac{2\,Ba}{\sqrt{bx^2+a}b^2}$$

input
```
integrate(x^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
1/3*D*x^4/(sqrt(b*x^2 + a)*b) + 1/2*C*x^3/(sqrt(b*x^2 + a)*b) - 4/3*D*a*x^
2/(sqrt(b*x^2 + a)*b^2) + B*x^2/(sqrt(b*x^2 + a)*b) + 3/2*C*a*x/(sqrt(b*x^
2 + a)*b^2) - A*x/(sqrt(b*x^2 + a)*b) - 3/2*C*a*arcsinh(b*x/sqrt(a*b))/b^(
5/2) + A*arcsinh(b*x/sqrt(a*b))/b^(3/2) - 8/3*D*a^2/(sqrt(b*x^2 + a)*b^3)
+ 2*B*a/(sqrt(b*x^2 + a)*b^2)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.92

$$\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^{3/2}}\,dx = \frac{\left(\left(\left(\frac{2\,Dx}{b}+\frac{3\,C}{b}\right)x-\frac{2\,(4\,Dab^3-3\,Bb^4)}{b^5}\right)x+\frac{3\,(3\,Cab^3-2\,Ab^4)}{b^5}\right)x-\frac{4\,(4\,Da^2b^2-3\,Ba}{b^5}}{6\,\sqrt{bx^2+a}} + \frac{(3\,Ca-2\,Ab)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2\,b^{\frac{5}{2}}}$$

input
```
integrate(x^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
1/6*((((2*D*x/b + 3*C/b)*x - 2*(4*D*a*b^3 - 3*B*b^4)/b^5)*x + 3*(3*C*a*b^3
 - 2*A*b^4)/b^5)*x - 4*(4*D*a^2*b^2 - 3*B*a*b^3)/b^5)/sqrt(b*x^2 + a) + 1/
2*(3*C*a - 2*A*b)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^{3/2}}\,dx = \int \frac{x^2\,\left(A+B\,x+C\,x^2+x^3\,D\right)}{{\left(b\,x^2+a\right)}^{3/2}}\,dx$$

input
```
int((x^2*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^(3/2),x)
```

output
```
int((x^2*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^(3/2), x)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 288, normalized size of antiderivative = 2.09

$$\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^{3/2}}\,dx = \frac{-64\sqrt{b\,x^2+a}\,a^2d-24\sqrt{b\,x^2+a}\,a\,b^2x+48\sqrt{b\,x^2+a}\,a\,b^2+36\sqrt{b\,x^2+}}{}$$

input
```
int(x^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(3/2),x)
```

output
```
( - 64*sqrt(a + b*x**2)*a**2*d - 24*sqrt(a + b*x**2)*a*b**2*x + 48*sqrt(a
+ b*x**2)*a*b**2 + 36*sqrt(a + b*x**2)*a*b*c*x - 32*sqrt(a + b*x**2)*a*b*d
*x**2 + 24*sqrt(a + b*x**2)*b**3*x**2 + 12*sqrt(a + b*x**2)*b**2*c*x**3 +
8*sqrt(a + b*x**2)*b**2*d*x**4 + 24*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b
)*x)/sqrt(a))*a**2*b - 36*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(
a))*a**2*c + 24*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a*b**2
*x**2 - 36*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a*b*c*x**2
- 24*sqrt(b)*a**2*b + 27*sqrt(b)*a**2*c - 24*sqrt(b)*a*b**2*x**2 + 27*sqrt
(b)*a*b*c*x**2)/(24*b**3*(a + b*x**2))
```

# 3.102 $\int \frac{x\left(A+Bx+Cx^2+Dx^3\right)}{\left(a+bx^2\right)^{3/2}}\,dx$



## Optimal result

Integrand size = 28, antiderivative size = 108

$$\int \frac{x(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^{3/2}}\,dx=-\frac{Ab-aC+(bB-aD)x}{b^2\sqrt{a+bx^2}}+\frac{C\sqrt{a+bx^2}}{b^2}+\frac{Dx\sqrt{a+bx^2}}{2b^2}+\frac{(2bB-3aD)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{5/2}}$$

output
```
-(A*b-C*a+(B*b-D*a)*x)/b^2/(b*x^2+a)^(1/2)+C*(b*x^2+a)^(1/2)/b^2+1/2*D*x*(
b*x^2+a)^(1/2)/b^2+1/2*(2*B*b-3*D*a)*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^
(5/2)
```

## Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.83

$$\int \frac{x(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^{3/2}}\,dx=\frac{-2Ab+4aC-2bBx+3aDx+2bCx^2+bDx^3}{2b^2\sqrt{a+bx^2}}+\frac{(-2bB+3aD)\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{2b^{5/2}}$$

input
```
Integrate[(x*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^(3/2),x]
```

output
```
(-2*A*b + 4*a*C - 2*b*B*x + 3*a*D*x + 2*b*C*x^2 + b*D*x^3)/(2*b^2*Sqrt[a +
 b*x^2]) + ((-2*b*B + 3*a*D)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(2*b^(5/
2))
```

## Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.27, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {2335, 25, 2346, 455, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx^2)^{3/2}}dx$$

$$\Big\downarrow \text{2335}$$

$$-\frac{\int -\frac{aDx^2-(Ab-2aC)x+\frac{a(bB-aD)}{b}}{\sqrt{bx^2+a}}dx}{ab}-\frac{x\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{aDx^2-(Ab-2aC)x+\frac{a(bB-aD)}{b}}{\sqrt{bx^2+a}}dx}{ab}-\frac{x\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{2346}$$

$$\frac{\frac{\int \frac{a(2bB-3aD)-2b(Ab-2aC)x}{\sqrt{bx^2+a}}dx}{2b}+\frac{aDx\sqrt{a+bx^2}}{2b}}{ab}-\frac{x\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{455}$$

$$\frac{\frac{a(2bB-3aD)\int \frac{1}{\sqrt{bx^2+a}}dx-2\sqrt{a+bx^2}(Ab-2aC)}{2b}+\frac{aDx\sqrt{a+bx^2}}{2b}}{ab}-\frac{x\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{a(2bB-3aD)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}-2\sqrt{a+bx^2}(Ab-2aC)}{2b}+\frac{aDx\sqrt{a+bx^2}}{2b}}{ab}-\frac{x\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bB-3aD)}{\sqrt{b}}-2\sqrt{a+bx^2}(Ab-2aC)}{2b}+\frac{aDx\sqrt{a+bx^2}}{2b}}{ab}-\frac{x\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{ab\sqrt{a+bx^2}}$$

input
```
Int[(x*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^(3/2),x]
```

output
```
-((x*(a*(B - (a*D)/b) - (A*b - a*C)*x))/(a*b*Sqrt[a + b*x^2])) + ((a*D*x*S
qrt[a + b*x^2])/(2*b) + (-2*(A*b - 2*a*C)*Sqrt[a + b*x^2] + (a*(2*b*B - 3*
a*D)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/Sqrt[b])/(2*b))/(a*b)
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 2335
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Simp[c/(2*a*b*(p + 1))   Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSu
m[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
 b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

## Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.40

| method | result |
|---|---|
| default | $C\left(\frac{x^2}{b\sqrt{b\,x^2+a}}+\frac{2a}{b^2\sqrt{b\,x^2+a}}\right)+D\left(\frac{x^3}{2b\sqrt{b\,x^2+a}}-\frac{3a\left(-\frac{x}{b\sqrt{b\,x^2+a}}+\frac{\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{b^{\frac{3}{2}}}\right)}{2b}\right)-\frac{A}{b\sqrt{b\,x^2+a}}+B\left(-\frac{}{b\sqrt{b}}\right.$ |

input
```
int(x*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
C*(x^2/b/(b*x^2+a)^(1/2)+2*a/b^2/(b*x^2+a)^(1/2))+D*(1/2*x^3/b/(b*x^2+a)^(
1/2)-3/2*a/b*(-x/b/(b*x^2+a)^(1/2)+1/b^(3/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
))-A/b/(b*x^2+a)^(1/2)+B*(-x/b/(b*x^2+a)^(1/2)+1/b^(3/2)*ln(b^(1/2)*x+(b*x
^2+a)^(1/2)))
```

## Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 253, normalized size of antiderivative = 2.34

$$\int \frac{x(A + Bx + Cx^2 + Dx^3)}{(a + bx^2)^{3/2}} dx = \left[ -\frac{(3\,Da^2 - 2\,Bab + (3\,Dab - 2\,Bb^2)x^2)\sqrt{b}\log\left(-2\,bx^2 - 2\,\sqrt{bx^2 + a}\sqrt{\phantom{b}}\right.}{4} \right.$$

input
```
integrate(x*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[-1/4*((3*D*a^2 - 2*B*a*b + (3*D*a*b - 2*B*b^2)*x^2)*sqrt(b)*log(-2*b*x^2
- 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(D*b^2*x^3 + 2*C*b^2*x^2 + 4*C*a*b
- 2*A*b^2 + (3*D*a*b - 2*B*b^2)*x)*sqrt(b*x^2 + a))/(b^4*x^2 + a*b^3), 1/2
*((3*D*a^2 - 2*B*a*b + (3*D*a*b - 2*B*b^2)*x^2)*sqrt(-b)*arctan(sqrt(-b)*x
/sqrt(b*x^2 + a)) + (D*b^2*x^3 + 2*C*b^2*x^2 + 4*C*a*b - 2*A*b^2 + (3*D*a*
b - 2*B*b^2)*x)*sqrt(b*x^2 + a))/(b^4*x^2 + a*b^3)]
```

## Sympy [A] (verification not implemented)

Time = 7.00 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.71

$$\int \frac{x(A + Bx + Cx^2 + Dx^3)}{(a + bx^2)^{3/2}} dx = A \left( \begin{cases} -\frac{1}{b\sqrt{a+bx^2}} & \text{for } b \neq 0 \\ \frac{x^2}{2a^{\frac{3}{2}}} & \text{otherwise} \end{cases} \right) + B \left( \frac{\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{\frac{3}{2}}} - \frac{x}{\sqrt{a}b\sqrt{1 + \frac{bx^2}{a}}} \right) + C \left( \begin{cases} \frac{2a}{b^2\sqrt{a+bx^2}} + \frac{x^2}{b\sqrt{a+bx^2}} & \text{for } b \neq 0 \\ \frac{x^4}{4a^{\frac{3}{2}}} & \text{otherwise} \end{cases} \right) + D \left( \frac{3\sqrt{a}x}{2b^2\sqrt{1 + \frac{bx^2}{a}}} - \frac{3a\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{\frac{5}{2}}} + \frac{x^3}{2\sqrt{a}b\sqrt{1 + \frac{bx^2}{a}}} \right)$$

input
```
integrate(x*(D*x**3+C*x**2+B*x+A)/(b*x**2+a)**(3/2),x)
```

output
```
A*Piecewise((-1/(b*sqrt(a + b*x**2)), Ne(b, 0)), (x**2/(2*a**(3/2)), True)
) + B*(asinh(sqrt(b)*x/sqrt(a))/b**(3/2) - x/(sqrt(a)*b*sqrt(1 + b*x**2/a)
)) + C*Piecewise((2*a/(b**2*sqrt(a + b*x**2)) + x**2/(b*sqrt(a + b*x**2)),
 Ne(b, 0)), (x**4/(4*a**(3/2)), True)) + D*(3*sqrt(a)*x/(2*b**2*sqrt(1 + b
*x**2/a)) - 3*a*asinh(sqrt(b)*x/sqrt(a))/(2*b**(5/2)) + x**3/(2*sqrt(a)*b*
sqrt(1 + b*x**2/a)))
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.20

$$\int \frac{x(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^{3/2}}\,dx = \frac{Dx^3}{2\sqrt{bx^2+a}b} + \frac{Cx^2}{\sqrt{bx^2+a}b} + \frac{3\,Dax}{2\sqrt{bx^2+a}b^2} - \frac{Bx}{\sqrt{bx^2+a}b} - \frac{3\,Da\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{5}{2}}} + \frac{B\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{3}{2}}} + \frac{2\,Ca}{\sqrt{bx^2+a}b^2} - \frac{A}{\sqrt{bx^2+a}b}$$

input
```
integrate(x*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
1/2*D*x^3/(sqrt(b*x^2 + a)*b) + C*x^2/(sqrt(b*x^2 + a)*b) + 3/2*D*a*x/(sqr
t(b*x^2 + a)*b^2) - B*x/(sqrt(b*x^2 + a)*b) - 3/2*D*a*arcsinh(b*x/sqrt(a*b
))/b^(5/2) + B*arcsinh(b*x/sqrt(a*b))/b^(3/2) + 2*C*a/(sqrt(b*x^2 + a)*b^2
) - A/(sqrt(b*x^2 + a)*b)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.93

$$\int \frac{x(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^{3/2}}\,dx = \frac{\left(\left(\frac{Dx}{b}+\frac{2\,C}{b}\right)x+\frac{3\,Dab^2-2\,Bb^3}{b^4}\right)x+\frac{2\,(2\,Cab^2-Ab^3)}{b^4}}{2\sqrt{bx^2+a}} + \frac{(3\,Da-2\,Bb)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2\,b^{\frac{5}{2}}}$$

input
```
integrate(x*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
1/2*(((D*x/b + 2*C/b)*x + (3*D*a*b^2 - 2*B*b^3)/b^4)*x + 2*(2*C*a*b^2 - A*
b^3)/b^4)/sqrt(b*x^2 + a) + 1/2*(3*D*a - 2*B*b)*log(abs(-sqrt(b)*x + sqrt(
b*x^2 + a)))/b^(5/2)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^{3/2}}\,dx=\int \frac{x\,(A+B\,x+C\,x^2+x^3\,D)}{(b\,x^2+a)^{3/2}}\,dx$$

input
```
int((x*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^(3/2),x)
```

output
```
int((x*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^(3/2), x)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 251, normalized size of antiderivative = 2.32

$$\int \frac{x(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^{3/2}}\,dx=\frac{-2\sqrt{b\,x^2+a}\,a\,b^2+4\sqrt{b\,x^2+a}\,abc+3\sqrt{b\,x^2+a}\,abdx-2\sqrt{b\,x^2+a}\,b^3x}{}$$

input
```
int(x*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(3/2),x)
```

output
```
( - 2*sqrt(a + b*x**2)*a*b**2 + 4*sqrt(a + b*x**2)*a*b*c + 3*sqrt(a + b*x*
*2)*a*b*d*x - 2*sqrt(a + b*x**2)*b**3*x + 2*sqrt(a + b*x**2)*b**2*c*x**2 +
 sqrt(a + b*x**2)*b**2*d*x**3 - 3*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*
x)/sqrt(a))*a**2*d + 2*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))
*a*b**2 - 3*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a*b*d*x**2
 + 2*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*b**3*x**2 + 2*sqr
t(b)*a**2*d - 2*sqrt(b)*a*b**2 + 2*sqrt(b)*a*b*d*x**2 - 2*sqrt(b)*b**3*x**
2)/(2*b**3*(a + b*x**2))
```

# 3.103 $\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{3/2}}\,dx$



## Optimal result

Integrand size = 27, antiderivative size = 85

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{3/2}}\,dx = -\frac{a(bB-aD)-b(Ab-aC)x}{ab^2\sqrt{a+bx^2}} + \frac{D\sqrt{a+bx^2}}{b^2} + \frac{C\,\mathrm{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}$$

output
```
-(a*(B*b-D*a)-b*(A*b-C*a)*x)/a/b^2/(b*x^2+a)^(1/2)+D*(b*x^2+a)^(1/2)/b^2+C
*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(3/2)
```

## Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.88

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{3/2}}\,dx = \frac{2a^2D+Ab^2x-ab(B+x(C-Dx))}{ab^2\sqrt{a+bx^2}} - \frac{C\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{b^{3/2}}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/(a + b*x^2)^(3/2),x]
```

output
```
(2*a^2*D + A*b^2*x - a*b*(B + x*(C - D*x)))/(a*b^2*Sqrt[a + b*x^2]) - (C*L
og[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/b^(3/2)
```

## Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2345, 25, 27, 455, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{3/2}}dx$$

$$\Big\downarrow \text{2345}$$

$$-\frac{\int -\frac{a(C+Dx)}{b\sqrt{bx^2+a}}dx}{a}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{a(C+Dx)}{b\sqrt{bx^2+a}}dx}{a}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{C+Dx}{\sqrt{bx^2+a}}dx}{b}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{455}$$

$$\frac{C\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{D\sqrt{a+bx^2}}{b}}{b}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{224}$$

$$\frac{C\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{D\sqrt{a+bx^2}}{b}}{b}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{C\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}+\frac{D\sqrt{a+bx^2}}{b}}{b}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{ab\sqrt{a+bx^2}}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/(a + b*x^2)^(3/2),x]
```

output
```
-((a*(B - (a*D)/b) - (A*b - a*C)*x)/(a*b*Sqrt[a + b*x^2])) + ((D*Sqrt[a +
b*x^2])/b + (C*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/Sqrt[b])/b
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

## Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.22

| method | result | size |
|---|---|---|
| default | $\frac{Ax}{a\sqrt{b\,x^2+a}}-\frac{B}{b\sqrt{b\,x^2+a}}+C\left(-\frac{x}{b\sqrt{b\,x^2+a}}+\frac{\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{b^{\frac{3}{2}}}\right)+D\left(\frac{x^2}{b\sqrt{b\,x^2+a}}+\frac{2a}{b^2\sqrt{b\,x^2+a}}\right)$ | 104 |

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
A*x/a/(b*x^2+a)^(1/2)-B/b/(b*x^2+a)^(1/2)+C*(-x/b/(b*x^2+a)^(1/2)+1/b^(3/2
)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))+D*(x^2/b/(b*x^2+a)^(1/2)+2*a/b^2/(b*x^2+a
)^(1/2))
```

## Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 210, normalized size of antiderivative = 2.47

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{3/2}}\,dx=\left[\frac{(Cabx^2+Ca^2)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,(Dabx^2+2\,Da\ldots}{2\,(ab^3x^2+a^2b^2)}\right.$$
$$\left.-\frac{(Cabx^2+Ca^2)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(Dabx^2+2\,Da^2-Bab-(Cab-Ab^2)x)\sqrt{bx^2+a}}{ab^3x^2+a^2b^2}\right]$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/2*((C*a*b*x^2 + C*a^2)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)
*x - a) + 2*(D*a*b*x^2 + 2*D*a^2 - B*a*b - (C*a*b - A*b^2)*x)*sqrt(b*x^2 +
 a))/(a*b^3*x^2 + a^2*b^2), -((C*a*b*x^2 + C*a^2)*sqrt(-b)*arctan(sqrt(-b)
*x/sqrt(b*x^2 + a)) - (D*a*b*x^2 + 2*D*a^2 - B*a*b - (C*a*b - A*b^2)*x)*sq
rt(b*x^2 + a))/(a*b^3*x^2 + a^2*b^2)]
```

## Sympy [A] (verification not implemented)

Time = 4.94 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.54

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{3/2}}\,dx = \frac{Ax}{a^{\frac{3}{2}}\sqrt{1+\frac{bx^2}{a}}} + B\left(\begin{cases} -\frac{1}{b\sqrt{a+bx^2}} & \text{for } b\neq 0 \\ \frac{x^2}{2a^{\frac{3}{2}}} & \text{otherwise} \end{cases}\right) + C\left(\frac{\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{\frac{3}{2}}} - \frac{x}{\sqrt{a}b\sqrt{1+\frac{bx^2}{a}}}\right) + D\left(\begin{cases} \frac{2a}{b^2\sqrt{a+bx^2}} + \frac{x^2}{b\sqrt{a+bx^2}} & \text{for } b\neq 0 \\ \frac{x^4}{4a^{\frac{3}{2}}} & \text{otherwise} \end{cases}\right)$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(b*x**2+a)**(3/2),x)
```

output
```
A*x/(a**(3/2)*sqrt(1 + b*x**2/a)) + B*Piecewise((-1/(b*sqrt(a + b*x**2)),
Ne(b, 0)), (x**2/(2*a**(3/2)), True)) + C*(asinh(sqrt(b)*x/sqrt(a))/b**(3/
2) - x/(sqrt(a)*b*sqrt(1 + b*x**2/a))) + D*Piecewise((2*a/(b**2*sqrt(a + b
*x**2)) + x**2/(b*sqrt(a + b*x**2)), Ne(b, 0)), (x**4/(4*a**(3/2)), True))
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.11

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{3/2}}\,dx = \frac{Dx^2}{\sqrt{bx^2+a}b} + \frac{Ax}{\sqrt{bx^2+a}a} - \frac{Cx}{\sqrt{bx^2+a}b} + \frac{C\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{3}{2}}} + \frac{2\,Da}{\sqrt{bx^2+a}b^2} - \frac{B}{\sqrt{bx^2+a}b}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
D*x^2/(sqrt(b*x^2 + a)*b) + A*x/(sqrt(b*x^2 + a)*a) - C*x/(sqrt(b*x^2 + a)
*b) + C*arcsinh(b*x/sqrt(a*b))/b^(3/2) + 2*D*a/(sqrt(b*x^2 + a)*b^2) - B/(
sqrt(b*x^2 + a)*b)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.04

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(a + bx^2)^{3/2}}\,dx = \frac{\left(\frac{Dx}{b} - \frac{Cab^2 - Ab^3}{ab^3}\right)x + \frac{2\,Da^2b - Bab^2}{ab^3}}{\sqrt{bx^2 + a}} - \frac{C\log\left(\left|-\sqrt{b}x + \sqrt{bx^2 + a}\right|\right)}{b^{\frac{3}{2}}}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
((D*x/b - (C*a*b^2 - A*b^3)/(a*b^3))*x + (2*D*a^2*b - B*a*b^2)/(a*b^3))/sq
rt(b*x^2 + a) - C*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2)
```

## Mupad [B] (verification not implemented)

Time = 2.37 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.07

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(a + bx^2)^{3/2}}\,dx = \frac{C\,\ln\left(\sqrt{b}\,x + \sqrt{b\,x^2 + a}\right)}{b^{3/2}} - \frac{B}{b\,\sqrt{b\,x^2 + a}} + \frac{(b\,x^2 + 2\,a)\,D}{b^2\,\sqrt{b\,x^2 + a}} + \frac{A\,x}{a\,\sqrt{b\,x^2 + a}} - \frac{C\,x}{b\,\sqrt{b\,x^2 + a}}$$

input
```
int((A + B*x + C*x^2 + x^3*D)/(a + b*x^2)^(3/2),x)
```

output
```
(C*log(b^(1/2)*x + (a + b*x^2)^(1/2)))/b^(3/2) - B/(b*(a + b*x^2)^(1/2)) +
 ((2*a + b*x^2)*D)/(b^2*(a + b*x^2)^(1/2)) + (A*x)/(a*(a + b*x^2)^(1/2)) -
 (C*x)/(b*(a + b*x^2)^(1/2))
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.87

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{3/2}}\,dx = \frac{2\sqrt{b\,x^2+a}\,ad+\sqrt{b\,x^2+a}\,b^2x-\sqrt{b\,x^2+a}\,b^2-\sqrt{b\,x^2+a}\,bcx+\sqrt{b\,x^2+a}$$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(3/2),x)
```

output
```
(2*sqrt(a + b*x**2)*a*d + sqrt(a + b*x**2)*b**2*x - sqrt(a + b*x**2)*b**2
- sqrt(a + b*x**2)*b*c*x + sqrt(a + b*x**2)*b*d*x**2 + sqrt(b)*log((sqrt(a
 + b*x**2) + sqrt(b)*x)/sqrt(a))*a*c + sqrt(b)*log((sqrt(a + b*x**2) + sqr
t(b)*x)/sqrt(a))*b*c*x**2 + sqrt(b)*a*b - sqrt(b)*a*c + sqrt(b)*b**2*x**2
- sqrt(b)*b*c*x**2)/(b**2*(a + b*x**2))
```

# 3.104 $\int \frac{A+Bx+Cx^2+Dx^3}{x(a+bx^2)^{3/2}}\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 89

$$\int \frac{A+Bx+Cx^2+Dx^3}{x(a+bx^2)^{3/2}}\,dx = \frac{Ab-aC+(bB-aD)x}{ab\sqrt{a+bx^2}} + \frac{D\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{3/2}} - \frac{A\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}$$

output
```
(A*b-C*a+(B*b-D*a)*x)/a/b/(b*x^2+a)^(1/2)+D*arctanh(b^(1/2)*x/(b*x^2+a)^(1
/2))/b^(3/2)-A*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(3/2)
```

## Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.12

$$\int \frac{A+Bx+Cx^2+Dx^3}{x(a+bx^2)^{3/2}}\,dx = \frac{Ab+bBx-a(C+Dx)}{ab\sqrt{a+bx^2}} + \frac{2A\operatorname{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}} - \frac{D\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{b^{3/2}}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/(x*(a + b*x^2)^(3/2)),x]
```

output
```
(A*b + b*B*x - a*(C + D*x))/(a*b*Sqrt[a + b*x^2]) + (2*A*ArcTanh[(Sqrt[b]*
x - Sqrt[a + b*x^2])/Sqrt[a]])/a^(3/2) - (D*Log[-(Sqrt[b]*x) + Sqrt[a + b*
x^2]])/b^(3/2)
```

## Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.11, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {2336, 25, 27, 538, 224, 219, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{x\left(a+bx^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{2336}$$

$$\frac{x(bB-aD)-aC+Ab}{ab\sqrt{a+bx^2}}-\frac{\int -\frac{Ab+aDx}{bx\sqrt{bx^2+a}}dx}{a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{Ab+aDx}{bx\sqrt{bx^2+a}}dx}{a}+\frac{x(bB-aD)-aC+Ab}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{Ab+aDx}{x\sqrt{bx^2+a}}dx}{ab}+\frac{x(bB-aD)-aC+Ab}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{538}$$

$$\frac{Ab\int \frac{1}{x\sqrt{bx^2+a}}dx+aD\int \frac{1}{\sqrt{bx^2+a}}dx}{ab}+\frac{x(bB-aD)-aC+Ab}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{224}$$

$$\frac{Ab\int \frac{1}{x\sqrt{bx^2+a}}dx+aD\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{ab}+\frac{x(bB-aD)-aC+Ab}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{Ab\int \frac{1}{x\sqrt{bx^2+a}}dx+\frac{aD\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}}{ab}+\frac{x(bB-aD)-aC+Ab}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \ 243$$

$$\frac{\frac{1}{2}Ab\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2+\frac{aD\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}}{ab}+\frac{x(bB-aD)-aC+Ab}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \ 73$$

$$\frac{A\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}+\frac{aD\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}}{ab}+\frac{x(bB-aD)-aC+Ab}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \ 221$$

$$\frac{\frac{aD\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}-\frac{Ab\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}}{ab}+\frac{x(bB-aD)-aC+Ab}{ab\sqrt{a+bx^2}}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/(x*(a + b*x^2)^(3/2)),x]
```

output
```
(A*b - a*C + (b*B - a*D)*x)/(a*b*Sqrt[a + b*x^2]) + ((a*D*ArcTanh[(Sqrt[b]
*x)/Sqrt[a + b*x^2]])/Sqrt[b] - (A*b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqr
t[a])/(a*b)
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

## Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.28

| method | result | si |
|---|---|---|
| default | $\frac{Bx}{a\sqrt{b\,x^{2}+a}}+A\left(\frac{1}{a\sqrt{b\,x^{2}+a}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{a^{\frac{3}{2}}}\right)-\frac{C}{b\sqrt{b\,x^{2}+a}}+D\left(-\frac{x}{b\sqrt{b\,x^{2}+a}}+\frac{\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{b^{\frac{3}{2}}}\right)$ | 1 |

input
```
int((D*x^3+C*x^2+B*x+A)/x/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
B*x/a/(b*x^2+a)^(1/2)+A*(1/a/(b*x^2+a)^(1/2)-1/a^(3/2)*ln((2*a+2*a^(1/2)*(
b*x^2+a)^(1/2))/x))-C/b/(b*x^2+a)^(1/2)+D*(-x/b/(b*x^2+a)^(1/2)+1/b^(3/2)*
ln(b^(1/2)*x+(b*x^2+a)^(1/2)))
```

## Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 602, normalized size of antiderivative = 6.76

$$\int \frac{A+Bx+Cx^2+Dx^3}{x\left(a+bx^2\right)^{3/2}}\,dx=\left[\frac{(Da^2bx^2+Da^3)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+(Ab^3x^2+Aab^2}{2\,(a^2b}\right.$$

$$-\frac{2\,(Da^2bx^2+Da^3)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(Ab^3x^2+Aab^2)\sqrt{a}\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\,(Ca^2b-Aa}{2\,(a^2b^3x^2+a^3b^2)}$$

$$-\frac{(Da^2bx^2+Da^3)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(Ab^3x^2+Aab^2)\sqrt{-a}\arctan\left(\frac{\sqrt{bx^2+a}\sqrt{-a}}{a}\right)+(Ca^2b-Aab^2+(}{a^2b^3x^2+a^3b^2}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/x/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/2*((D*a^2*b*x^2 + D*a^3)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(
b)*x - a) + (A*b^3*x^2 + A*a*b^2)*sqrt(a)*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*
sqrt(a) + 2*a)/x^2) - 2*(C*a^2*b - A*a*b^2 + (D*a^2*b - B*a*b^2)*x)*sqrt(b
*x^2 + a))/(a^2*b^3*x^2 + a^3*b^2), -1/2*(2*(D*a^2*b*x^2 + D*a^3)*sqrt(-b)
*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (A*b^3*x^2 + A*a*b^2)*sqrt(a)*log(-(
b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(C*a^2*b - A*a*b^2 + (D*
a^2*b - B*a*b^2)*x)*sqrt(b*x^2 + a))/(a^2*b^3*x^2 + a^3*b^2), 1/2*(2*(A*b^
3*x^2 + A*a*b^2)*sqrt(-a)*arctan(sqrt(b*x^2 + a)*sqrt(-a)/a) + (D*a^2*b*x^
2 + D*a^3)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(C*
a^2*b - A*a*b^2 + (D*a^2*b - B*a*b^2)*x)*sqrt(b*x^2 + a))/(a^2*b^3*x^2 + a
^3*b^2), -((D*a^2*b*x^2 + D*a^3)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a
)) - (A*b^3*x^2 + A*a*b^2)*sqrt(-a)*arctan(sqrt(b*x^2 + a)*sqrt(-a)/a) + (
C*a^2*b - A*a*b^2 + (D*a^2*b - B*a*b^2)*x)*sqrt(b*x^2 + a))/(a^2*b^3*x^2 +
 a^3*b^2)]
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 185 vs. $2(76) = 152$.

Time = 8.46 (sec) , antiderivative size = 274, normalized size of antiderivative = 3.08

$$\int \frac{A+Bx+Cx^2+Dx^3}{x\left(a+bx^2\right)^{3/2}}\,dx = A\left(\frac{2a^3\sqrt{1+\frac{bx^2}{a}}}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}+\frac{a^3\log\left(\frac{bx^2}{a}\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2} - \frac{2a^3\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}+\frac{a^2bx^2\log\left(\frac{bx^2}{a}\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}-\frac{2a^2bx^2\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}\right) + \frac{Bx}{a^{\frac{3}{2}}\sqrt{1+\frac{bx^2}{a}}}+C\left(\begin{cases}-\frac{1}{b\sqrt{a+bx^2}} & \text{for } b\neq 0\\ \frac{x^2}{2a^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right) + D\left(\frac{\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{\frac{3}{2}}}-\frac{x}{\sqrt{a}b\sqrt{1+\frac{bx^2}{a}}}\right)$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/x/(b*x**2+a)**(3/2),x)
```

output
```
A*(2*a**3*sqrt(1 + b*x**2/a)/(2*a**(9/2) + 2*a**(7/2)*b*x**2) + a**3*log(b
*x**2/a)/(2*a**(9/2) + 2*a**(7/2)*b*x**2) - 2*a**3*log(sqrt(1 + b*x**2/a)
+ 1)/(2*a**(9/2) + 2*a**(7/2)*b*x**2) + a**2*b*x**2*log(b*x**2/a)/(2*a**(9
/2) + 2*a**(7/2)*b*x**2) - 2*a**2*b*x**2*log(sqrt(1 + b*x**2/a) + 1)/(2*a*
*(9/2) + 2*a**(7/2)*b*x**2)) + B*x/(a**(3/2)*sqrt(1 + b*x**2/a)) + C*Piece
wise((-1/(b*sqrt(a + b*x**2)), Ne(b, 0)), (x**2/(2*a**(3/2)), True)) + D*(
asinh(sqrt(b)*x/sqrt(a))/b**(3/2) - x/(sqrt(a)*b*sqrt(1 + b*x**2/a)))
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.04

$$\int \frac{A+Bx+Cx^2+Dx^3}{x\left(a+bx^2\right)^{3/2}}\,dx = \frac{Bx}{\sqrt{bx^2+a}a} - \frac{Dx}{\sqrt{bx^2+a}b} + \frac{D\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{3}{2}}} - \frac{A\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{a^{\frac{3}{2}}} + \frac{A}{\sqrt{bx^2+a}a} - \frac{C}{\sqrt{bx^2+a}b}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/x/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
B*x/(sqrt(b*x^2 + a)*a) - D*x/(sqrt(b*x^2 + a)*b) + D*arcsinh(b*x/sqrt(a*b
))/b^(3/2) - A*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2) + A/(sqrt(b*x^2 + a)*
a) - C/(sqrt(b*x^2 + a)*b)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{A+Bx+Cx^2+Dx^3}{x\left(a+bx^2\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/x/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

## Mupad [B] (verification not implemented)

Time = 2.48 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.15

$$\int \frac{A+Bx+Cx^2+Dx^3}{x\left(a+bx^2\right)^{3/2}}\,dx = \frac{A}{a\sqrt{b\,x^2+a}} - \frac{C}{b\sqrt{b\,x^2+a}} - \frac{A\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{a^{3/2}} + \frac{\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)\,D}{b^{3/2}} - \frac{x\,D}{b\sqrt{b\,x^2+a}} + \frac{B\,x}{a\sqrt{b\,x^2+a}}$$

input
```
int((A + B*x + C*x^2 + x^3*D)/(x*(a + b*x^2)^(3/2)),x)
```

output
```
A/(a*(a + b*x^2)^(1/2)) - C/(b*(a + b*x^2)^(1/2)) - (A*atanh((a + b*x^2)^(
1/2)/a^(1/2)))/a^(3/2) + (log(b^(1/2)*x + (a + b*x^2)^(1/2))*D)/b^(3/2) -
(x*D)/(b*(a + b*x^2)^(1/2)) + (B*x)/(a*(a + b*x^2)^(1/2))
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 280, normalized size of antiderivative = 3.15

$$\int \frac{A+Bx+Cx^2+Dx^3}{x\left(a+bx^2\right)^{3/2}}\,dx = \frac{\sqrt{b\,x^2+a}\,a\,b^2 - \sqrt{b\,x^2+a}\,abc - \sqrt{b\,x^2+a}\,abdx + \sqrt{b\,x^2+a}\,b^3x + \sqrt{a}\log}{}$$

input
```
int((D*x^3+C*x^2+B*x+A)/x/(b*x^2+a)^(3/2),x)
```

output
```
(sqrt(a + b*x**2)*a*b**2 - sqrt(a + b*x**2)*a*b*c - sqrt(a + b*x**2)*a*b*d
*x + sqrt(a + b*x**2)*b**3*x + sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + s
qrt(b)*x)/sqrt(a))*a*b**2 + sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt
(b)*x)/sqrt(a))*b**3*x**2 - sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt
(b)*x)/sqrt(a))*a*b**2 - sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)
*x)/sqrt(a))*b**3*x**2 + sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a
))*a**2*d + sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a*b*d*x**2
 - sqrt(b)*a**2*d + sqrt(b)*a*b**2 - sqrt(b)*a*b*d*x**2 + sqrt(b)*b**3*x**
2)/(a*b**2*(a + b*x**2))
```

# 3.105 $\int \frac{A+Bx+Cx^2+Dx^3}{x^2(a+bx^2)^{3/2}}\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 88

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^2(a+bx^2)^{3/2}}\,dx = \frac{a(bB-aD)-b(Ab-aC)x}{a^2b\sqrt{a+bx^2}} - \frac{A\sqrt{a+bx^2}}{a^2x} - \frac{B\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}$$

output
```
(a*(B*b-D*a)-b*(A*b-C*a)*x)/a^2/b/(b*x^2+a)^(1/2)-A*(b*x^2+a)^(1/2)/a^2/x-
B*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(3/2)
```

## Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.06

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^2(a+bx^2)^{3/2}}\,dx = \frac{-aAb+abBx-a^2Dx-2Ab^2x^2+abCx^2}{a^2bx\sqrt{a+bx^2}} - \frac{B\log(x)}{a^{3/2}} + \frac{B\log\left(-\sqrt{a}+\sqrt{a+bx^2}\right)}{a^{3/2}}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/(x^2*(a + b*x^2)^(3/2)),x]
```

output
```
(-(a*A*b) + a*b*B*x - a^2*D*x - 2*A*b^2*x^2 + a*b*C*x^2)/(a^2*b*x*Sqrt[a +
 b*x^2]) - (B*Log[x])/a^(3/2) + (B*Log[-Sqrt[a] + Sqrt[a + b*x^2]])/a^(3/2
)
```

## Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2336, 25, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^2 \left(a+bx^2\right)^{3/2}} \, dx$$

$$\Big\downarrow \text{2336}$$

$$\frac{-bx\left(\frac{Ab}{a}-C\right)-aD+bB}{ab\sqrt{a+bx^2}}-\frac{\int -\frac{A+Bx}{x^2\sqrt{bx^2+a}}dx}{a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{A+Bx}{x^2\sqrt{bx^2+a}}dx}{a}+\frac{-bx\left(\frac{Ab}{a}-C\right)-aD+bB}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{534}$$

$$\frac{B\int \frac{1}{x\sqrt{bx^2+a}}dx-\frac{A\sqrt{a+bx^2}}{ax}}{a}+\frac{-bx\left(\frac{Ab}{a}-C\right)-aD+bB}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{243}$$

$$\frac{\frac{1}{2}B\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2-\frac{A\sqrt{a+bx^2}}{ax}}{a}+\frac{-bx\left(\frac{Ab}{a}-C\right)-aD+bB}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{B\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{b}-\frac{A\sqrt{a+bx^2}}{ax}}{a}+\frac{-bx\left(\frac{Ab}{a}-C\right)-aD+bB}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{221}$$

$$\frac{-\frac{A\sqrt{a+bx^2}}{ax}-\frac{B\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}}{a}+\frac{-bx\left(\frac{Ab}{a}-C\right)-aD+bB}{ab\sqrt{a+bx^2}}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/(x^2*(a + b*x^2)^(3/2)),x]
```

output
```
(b*B - a*D - b*((A*b)/a - C)*x)/(a*b*Sqrt[a + b*x^2]) + (-((A*Sqrt[a + b*x
^2])/(a*x)) - (B*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a])/a
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

## Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.27

| method | result | size |
|---|---|---|
| default | $\frac{Cx}{a\sqrt{b\,x^2+a}}+A\left(-\frac{1}{ax\sqrt{b\,x^2+a}}-\frac{2bx}{a^2\sqrt{b\,x^2+a}}\right)+B\left(\frac{1}{a\sqrt{b\,x^2+a}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)}{a^{\frac{3}{2}}}\right)-\frac{D}{b\sqrt{b\,x^2+a}}$ | 112 |

input
```
int((D*x^3+C*x^2+B*x+A)/x^2/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
C*x/a/(b*x^2+a)^(1/2)+A*(-1/a/x/(b*x^2+a)^(1/2)-2*b/a^2*x/(b*x^2+a)^(1/2))
+B*(1/a/(b*x^2+a)^(1/2)-1/a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x))-D
/b/(b*x^2+a)^(1/2)
```

## Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 220, normalized size of antiderivative = 2.50

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^2\left(a+bx^2\right)^{3/2}}\,dx = \left[\frac{(Bb^2x^3+Babx)\sqrt{a}\log\left(-\frac{bx^2-2\sqrt{bx^2+a}\sqrt{a}+2a}{x^2}\right)-2\left(Aab-(Cab-2Ab^2)x\right.}{2\left(a^2b^2x^3+a^3bx\right)}\right.$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/x^2/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/2*((B*b^2*x^3 + B*a*b*x)*sqrt(a)*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a
) + 2*a)/x^2) - 2*(A*a*b - (C*a*b - 2*A*b^2)*x^2 + (D*a^2 - B*a*b)*x)*sqrt
(b*x^2 + a))/(a^2*b^2*x^3 + a^3*b*x), ((B*b^2*x^3 + B*a*b*x)*sqrt(-a)*arct
an(sqrt(b*x^2 + a)*sqrt(-a)/a) - (A*a*b - (C*a*b - 2*A*b^2)*x^2 + (D*a^2 -
 B*a*b)*x)*sqrt(b*x^2 + a))/(a^2*b^2*x^3 + a^3*b*x)]
```

## Sympy [A] (verification not implemented)

Time = 8.50 (sec) , antiderivative size = 282, normalized size of antiderivative = 3.20

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^2(a+bx^2)^{3/2}}\,dx = A\left(-\frac{1}{a\sqrt{b}x^2\sqrt{\frac{a}{bx^2}+1}}-\frac{2\sqrt{b}}{a^2\sqrt{\frac{a}{bx^2}+1}}\right) + B\left(\frac{2a^3\sqrt{1+\frac{bx^2}{a}}}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}+\frac{a^3\log\left(\frac{bx^2}{a}\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}-\frac{2a^3\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}+\frac{a^2bx^2\log\left(\frac{bx^2}{a}\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}-\frac{2a^2bx^2\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}\right)+\frac{Cx}{a^{\frac{3}{2}}\sqrt{1+\frac{bx^2}{a}}}+D\left(\begin{cases}-\frac{1}{b\sqrt{a+bx^2}} & \text{for } b\neq 0\\ \frac{x^2}{2a^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/x**2/(b*x**2+a)**(3/2),x)
```

output
```
A*(-1/(a*sqrt(b)*x**2*sqrt(a/(b*x**2) + 1)) - 2*sqrt(b)/(a**2*sqrt(a/(b*x*
*2) + 1))) + B*(2*a**3*sqrt(1 + b*x**2/a)/(2*a**(9/2) + 2*a**(7/2)*b*x**2)
 + a**3*log(b*x**2/a)/(2*a**(9/2) + 2*a**(7/2)*b*x**2) - 2*a**3*log(sqrt(1
 + b*x**2/a) + 1)/(2*a**(9/2) + 2*a**(7/2)*b*x**2) + a**2*b*x**2*log(b*x**
2/a)/(2*a**(9/2) + 2*a**(7/2)*b*x**2) - 2*a**2*b*x**2*log(sqrt(1 + b*x**2/
a) + 1)/(2*a**(9/2) + 2*a**(7/2)*b*x**2)) + C*x/(a**(3/2)*sqrt(1 + b*x**2/
a)) + D*Piecewise((-1/(b*sqrt(a + b*x**2)), Ne(b, 0)), (x**2/(2*a**(3/2)),
 True))
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.11

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^2(a+bx^2)^{3/2}}\,dx = \frac{Cx}{\sqrt{bx^2+a}a} - \frac{2\,Abx}{\sqrt{bx^2+a}a^2} - \frac{B\,\text{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{a^{\frac{3}{2}}} + \frac{B}{\sqrt{bx^2+a}a} - \frac{D}{\sqrt{bx^2+a}b} - \frac{A}{\sqrt{bx^2+a}ax}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/x^2/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
C*x/(sqrt(b*x^2 + a)*a) - 2*A*b*x/(sqrt(b*x^2 + a)*a^2) - B*arcsinh(a/(sqr
t(a*b)*abs(x)))/a^(3/2) + B/(sqrt(b*x^2 + a)*a) - D/(sqrt(b*x^2 + a)*b) -
A/(sqrt(b*x^2 + a)*a*x)
```

## Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.42

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^2(a+bx^2)^{3/2}}\,dx = \frac{\frac{(Ca^2b-Aab^2)x}{a^3b} - \frac{Da^3-Ba^2b}{a^3b}}{\sqrt{bx^2+a}} + \frac{2\,B\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a} + \frac{2\,A\sqrt{b}}{\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)a}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/x^2/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
((C*a^2*b - A*a*b^2)*x/(a^3*b) - (D*a^3 - B*a^2*b)/(a^3*b))/sqrt(b*x^2 + a
) + 2*B*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/(sqrt(-a)*a) + 2*A
*sqrt(b)/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)*a)
```

## Mupad [B] (verification not implemented)

Time = 2.60 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.17

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^2\left(a+bx^2\right)^{3/2}}\,dx=\frac{B}{a\,\sqrt{b\,x^2+a}}-\frac{\sqrt{b\,x^2+a}\,\left(\frac{A}{a}+\frac{2\,A\,b\,x^2}{a^2}\right)}{b\,x^3+a\,x}-\frac{D}{b\,\sqrt{b\,x^2+a}}-\frac{B\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{a^{3/2}}+\frac{C\,x}{a\,\sqrt{b\,x^2+a}}$$

input
```
int((A + B*x + C*x^2 + x^3*D)/(x^2*(a + b*x^2)^(3/2)),x)
```

output
```
B/(a*(a + b*x^2)^(1/2)) - ((a + b*x^2)^(1/2)*(A/a + (2*A*b*x^2)/a^2))/(a*x
 + b*x^3) - D/(b*(a + b*x^2)^(1/2)) - (B*atanh((a + b*x^2)^(1/2)/a^(1/2)))
/a^(3/2) + (C*x)/(a*(a + b*x^2)^(1/2))
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 256, normalized size of antiderivative = 2.91

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^2\left(a+bx^2\right)^{3/2}}\,dx=\frac{-\sqrt{b\,x^2+a}\,a^2b-\sqrt{b\,x^2+a}\,a^2dx-2\sqrt{b\,x^2+a}\,a\,b^2x^2+\sqrt{b\,x^2+a}\,a\,b^2x+}{}$$

input
```
int((D*x^3+C*x^2+B*x+A)/x^2/(b*x^2+a)^(3/2),x)
```

output
```
( - sqrt(a + b*x**2)*a**2*b - sqrt(a + b*x**2)*a**2*d*x - 2*sqrt(a + b*x**
2)*a*b**2*x**2 + sqrt(a + b*x**2)*a*b**2*x + sqrt(a + b*x**2)*a*b*c*x**2 +
 sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b**2*x +
sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*b**3*x**3 -
sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b**2*x - s
qrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*b**3*x**3 - 2
*sqrt(b)*a**2*b*x + sqrt(b)*a**2*c*x - 2*sqrt(b)*a*b**2*x**3 + sqrt(b)*a*b
*c*x**3)/(a**2*b*x*(a + b*x**2))
```

# 3.106 $\int \frac{A+Bx+Cx^2+Dx^3}{x^3(a+bx^2)^{3/2}} dx$



## Optimal result

Integrand size = 30, antiderivative size = 113

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^3(a+bx^2)^{3/2}} dx = -\frac{Ab-aC+(bB-aD)x}{a^2\sqrt{a+bx^2}} - \frac{A\sqrt{a+bx^2}}{2a^2x^2} - \frac{B\sqrt{a+bx^2}}{a^2x} + \frac{(3Ab-2aC)\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{5/2}}$$

output
```
-(A*b-C*a+(B*b-D*a)*x)/a^2/(b*x^2+a)^(1/2)-1/2*A*(b*x^2+a)^(1/2)/a^2/x^2-B
*(b*x^2+a)^(1/2)/a^2/x+1/2*(3*A*b-2*C*a)*arctanh((b*x^2+a)^(1/2)/a^(1/2))/
a^(5/2)
```

## Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.88

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^3(a+bx^2)^{3/2}} dx = \frac{-bx^2(3A+4Bx)-a(A+2Bx-2x^2(C+Dx))}{2a^2x^2\sqrt{a+bx^2}} + \frac{(-3Ab+2aC)\text{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{5/2}}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/(x^3*(a + b*x^2)^(3/2)),x]
```

output
```
(-(b*x^2*(3*A + 4*B*x)) - a*(A + 2*B*x - 2*x^2*(C + D*x)))/(2*a^2*x^2*Sqrt
[a + b*x^2]) + ((-3*A*b + 2*a*C)*ArcTanh[(Sqrt[b]*x - Sqrt[a + b*x^2])/Sqr
t[a]])/a^(5/2)
```

## Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.10, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2336, 25, 2338, 25, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^3\left(a+bx^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 2336}$$

$$-\frac{\int -\frac{-\left(\left(\frac{Ab}{a}-C\right)x^2\right)+Bx+A}{x^3\sqrt{bx^2+a}}dx}{a}-\frac{\frac{Ab}{a}+x\left(\frac{bB}{a}-D\right)-C}{a\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{-\left(\left(\frac{Ab}{a}-C\right)x^2\right)+Bx+A}{x^3\sqrt{bx^2+a}}dx}{a}-\frac{\frac{Ab}{a}+x\left(\frac{bB}{a}-D\right)-C}{a\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{ 2338}$$

$$\frac{-\frac{\int -\frac{2aB-(3Ab-2aC)x}{x^2\sqrt{bx^2+a}}dx}{2a}-\frac{A\sqrt{a+bx^2}}{2ax^2}}{a}-\frac{\frac{Ab}{a}+x\left(\frac{bB}{a}-D\right)-C}{a\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\frac{\int \frac{2aB-(3Ab-2aC)x}{x^2\sqrt{bx^2+a}}dx}{2a}-\frac{A\sqrt{a+bx^2}}{2ax^2}}{a}-\frac{\frac{Ab}{a}+x\left(\frac{bB}{a}-D\right)-C}{a\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{ 534}$$

$$\frac{\frac{-(3Ab-2aC)\int \frac{1}{x\sqrt{bx^2+a}}dx-\frac{2B\sqrt{a+bx^2}}{x}}{2a}-\frac{A\sqrt{a+bx^2}}{2ax^2}}{a}-\frac{\frac{Ab}{a}+x\left(\frac{bB}{a}-D\right)-C}{a\sqrt{a+bx^2}}$$

$\Big\downarrow$ 243

$$\frac{\frac{-\frac{1}{2}(3Ab-2aC)\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2-\frac{2B\sqrt{a+bx^2}}{x}}{2a}-\frac{A\sqrt{a+bx^2}}{2ax^2}}{a}-\frac{\frac{Ab}{a}+x\left(\frac{bB}{a}-D\right)-C}{a\sqrt{a+bx^2}}$$

$\Big\downarrow$ 73

$$\frac{\frac{-\frac{(3Ab-2aC)\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{b}-\frac{2B\sqrt{a+bx^2}}{x}}{2a}-\frac{A\sqrt{a+bx^2}}{2ax^2}}{a}-\frac{\frac{Ab}{a}+x\left(\frac{bB}{a}-D\right)-C}{a\sqrt{a+bx^2}}$$

$\Big\downarrow$ 221

$$\frac{\frac{\frac{(3Ab-2aC)\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{2B\sqrt{a+bx^2}}{x}}{2a}-\frac{A\sqrt{a+bx^2}}{2ax^2}}{a}-\frac{\frac{Ab}{a}+x\left(\frac{bB}{a}-D\right)-C}{a\sqrt{a+bx^2}}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/(x^3*(a + b*x^2)^(3/2)),x]
```

output
```
-((((A*b)/a - C + ((b*B)/a - D)*x)/(a*Sqrt[a + b*x^2])) + (-1/2*(A*Sqrt[a +
 b*x^2])/(a*x^2) + ((-2*B*Sqrt[a + b*x^2])/x + ((3*A*b - 2*a*C)*ArcTanh[Sq
rt[a + b*x^2]/Sqrt[a]])/Sqrt[a])/(2*a))/a
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

## Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.46

| method | result |
|---|---|
| default | $\frac{Dx}{a\sqrt{b\,x^2+a}}+A\left(-\frac{1}{2a\,x^2\sqrt{b\,x^2+a}}-\frac{3b\left(\frac{1}{a\sqrt{b\,x^2+a}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)}{a^{\frac{3}{2}}}\right)}{2a}\right)+B\left(-\frac{1}{ax\sqrt{b\,x^2+a}}-\frac{2bx}{a^2\sqrt{b\,x^2+a}}\right)+$ |

input
```
int((D*x^3+C*x^2+B*x+A)/x^3/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
D*x/a/(b*x^2+a)^(1/2)+A*(-1/2/a/x^2/(b*x^2+a)^(1/2)-3/2*b/a*(1/a/(b*x^2+a)
^(1/2)-1/a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)))+B*(-1/a/x/(b*x^2+
a)^(1/2)-2*b/a^2*x/(b*x^2+a)^(1/2))+C*(1/a/(b*x^2+a)^(1/2)-1/a^(3/2)*ln((2
*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x))
```

## Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 281, normalized size of antiderivative = 2.49

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^3\,(a+bx^2)^{3/2}}\,dx=\left[-\frac{((2\,Cab-3\,Ab^2)x^4+(2\,Ca^2-3\,Aab)x^2)\sqrt{a}\log\left(-\frac{bx^2+2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)}{4\,(a^3bx^4}\right.$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/x^3/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[-1/4*(((2*C*a*b - 3*A*b^2)*x^4 + (2*C*a^2 - 3*A*a*b)*x^2)*sqrt(a)*log(-(b
*x^2 + 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(2*B*a^2*x - 2*(D*a^2 - 2
*B*a*b)*x^3 + A*a^2 - (2*C*a^2 - 3*A*a*b)*x^2)*sqrt(b*x^2 + a))/(a^3*b*x^4
 + a^4*x^2), 1/2*(((2*C*a*b - 3*A*b^2)*x^4 + (2*C*a^2 - 3*A*a*b)*x^2)*sqrt
(-a)*arctan(sqrt(b*x^2 + a)*sqrt(-a)/a) - (2*B*a^2*x - 2*(D*a^2 - 2*B*a*b)
*x^3 + A*a^2 - (2*C*a^2 - 3*A*a*b)*x^2)*sqrt(b*x^2 + a))/(a^3*b*x^4 + a^4*
x^2)]
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 332 vs. $2(97) = 194$.

Time = 7.87 (sec) , antiderivative size = 332, normalized size of antiderivative = 2.94

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^3\left(a+bx^2\right)^{3/2}}\,dx = A\left(-\frac{1}{2a\sqrt{b}x^3\sqrt{\frac{a}{bx^2}+1}}-\frac{3\sqrt{b}}{2a^2x\sqrt{\frac{a}{bx^2}+1}}+\frac{3b\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2a^{\frac{5}{2}}}\right)+B\left(-\frac{1}{a\sqrt{b}x^2\sqrt{\frac{a}{bx^2}+1}}-\frac{2\sqrt{b}}{a^2\sqrt{\frac{a}{bx^2}+1}}\right)+C\left(\frac{2a^3\sqrt{1+\frac{bx^2}{a}}}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}+\frac{a^3\log\left(\frac{bx^2}{a}\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}-\frac{2a^3\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}+\frac{a^2bx^2\log\left(\frac{bx^2}{a}\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}-\frac{2a^2bx^2\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}\right)+\frac{Dx}{a^{\frac{3}{2}}\sqrt{1+\frac{bx^2}{a}}}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/x**3/(b*x**2+a)**(3/2),x)
```

output
```
A*(-1/(2*a*sqrt(b)*x**3*sqrt(a/(b*x**2) + 1)) - 3*sqrt(b)/(2*a**2*x*sqrt(a
/(b*x**2) + 1)) + 3*b*asinh(sqrt(a)/(sqrt(b)*x))/(2*a**(5/2))) + B*(-1/(a*
sqrt(b)*x**2*sqrt(a/(b*x**2) + 1)) - 2*sqrt(b)/(a**2*sqrt(a/(b*x**2) + 1))
) + C*(2*a**3*sqrt(1 + b*x**2/a)/(2*a**(9/2) + 2*a**(7/2)*b*x**2) + a**3*l
og(b*x**2/a)/(2*a**(9/2) + 2*a**(7/2)*b*x**2) - 2*a**3*log(sqrt(1 + b*x**2
/a) + 1)/(2*a**(9/2) + 2*a**(7/2)*b*x**2) + a**2*b*x**2*log(b*x**2/a)/(2*a
**(9/2) + 2*a**(7/2)*b*x**2) - 2*a**2*b*x**2*log(sqrt(1 + b*x**2/a) + 1)/(
2*a**(9/2) + 2*a**(7/2)*b*x**2)) + D*x/(a**(3/2)*sqrt(1 + b*x**2/a))
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.20

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^3\left(a+bx^2\right)^{3/2}}\,dx = \frac{Dx}{\sqrt{bx^2+a}a}-\frac{2\,Bbx}{\sqrt{bx^2+a}a^2}-\frac{C\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{a^{\frac{3}{2}}}+\frac{3\,Ab\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{2\,a^{\frac{5}{2}}}+\frac{C}{\sqrt{bx^2+a}a}-\frac{3\,Ab}{2\,\sqrt{bx^2+a}a^2}-\frac{B}{\sqrt{bx^2+a}ax}-\frac{A}{2\,\sqrt{bx^2+a}ax^2}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/x^3/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
D*x/(sqrt(b*x^2 + a)*a) - 2*B*b*x/(sqrt(b*x^2 + a)*a^2) - C*arcsinh(a/(sqr
t(a*b)*abs(x)))/a^(3/2) + 3/2*A*b*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(5/2) +
C/(sqrt(b*x^2 + a)*a) - 3/2*A*b/(sqrt(b*x^2 + a)*a^2) - B/(sqrt(b*x^2 + a)
*a*x) - 1/2*A/(sqrt(b*x^2 + a)*a*x^2)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 198 vs. 2(96) = 192.

Time = 0.13 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.75

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^3\left(a+bx^2\right)^{3/2}}\,dx = \frac{\frac{\left(Da^3-Ba^2b\right)x}{a^4}+\frac{Ca^3-Aa^2b}{a^4}}{\sqrt{bx^2+a}}+\frac{\left(2\,Ca-3\,Ab\right)\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2}+\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^3Ab+2\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2Ba\sqrt{b}+\left(\sqrt{b}x-\sqrt{bx^2+a}\right)Aab-2\,Ba^2\sqrt{b}}{\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^2a^2}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/x^3/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
((D*a^3 - B*a^2*b)*x/a^4 + (C*a^3 - A*a^2*b)/a^4)/sqrt(b*x^2 + a) + (2*C*a
 - 3*A*b)*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/(sqrt(-a)*a^2) +
 ((sqrt(b)*x - sqrt(b*x^2 + a))^3*A*b + 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*
B*a*sqrt(b) + (sqrt(b)*x - sqrt(b*x^2 + a))*A*a*b - 2*B*a^2*sqrt(b))/(((sq
rt(b)*x - sqrt(b*x^2 + a))^2 - a)^2*a^2)
```

## Mupad [B] (verification not implemented)

Time = 2.93 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.27

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^3 (a+bx^2)^{3/2}} dx = \frac{C}{a\sqrt{b\,x^2+a}} - \frac{\sqrt{b\,x^2+a}\left(\frac{B}{a}+\frac{2\,B\,b\,x^2}{a^2}\right)}{b\,x^3+a\,x} - \frac{C\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{a^{3/2}} - \frac{3\,A\,b}{2\,a^2\sqrt{b\,x^2+a}} + \frac{x\,D}{a\sqrt{b\,x^2+a}} - \frac{A}{2\,a\,x^2\sqrt{b\,x^2+a}} + \frac{3\,A\,b\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2\,a^{5/2}}$$

input
```
int((A + B*x + C*x^2 + x^3*D)/(x^3*(a + b*x^2)^(3/2)),x)
```

output
```
C/(a*(a + b*x^2)^(1/2)) - ((a + b*x^2)^(1/2)*(B/a + (2*B*b*x^2)/a^2))/(a*x
 + b*x^3) - (C*atanh((a + b*x^2)^(1/2)/a^(1/2)))/a^(3/2) - (3*A*b)/(2*a^2*
(a + b*x^2)^(1/2)) + (x*D)/(a*(a + b*x^2)^(1/2)) - A/(2*a*x^2*(a + b*x^2)^
(1/2)) + (3*A*b*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(2*a^(5/2))
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 417, normalized size of antiderivative = 3.69

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^3 (a+bx^2)^{3/2}} dx = \frac{-\sqrt{b\,x^2+a}\,a^2b-3\sqrt{b\,x^2+a}\,a\,b^2x^2-2\sqrt{b\,x^2+a}\,a\,b^2x+2\sqrt{b\,x^2+a}\,abc\,x^2}{}$$

input
```
int((D*x^3+C*x^2+B*x+A)/x^3/(b*x^2+a)^(3/2),x)
```

output

```
( - sqrt(a + b*x**2)*a**2*b - 3*sqrt(a + b*x**2)*a*b**2*x**2 - 2*sqrt(a +
b*x**2)*a*b**2*x + 2*sqrt(a + b*x**2)*a*b*c*x**2 + 2*sqrt(a + b*x**2)*a*b*
d*x**3 - 4*sqrt(a + b*x**2)*b**3*x**3 - 3*sqrt(a)*log((sqrt(a + b*x**2) -
sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b**2*x**2 + 2*sqrt(a)*log((sqrt(a + b*x**2
) - sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b*c*x**2 - 3*sqrt(a)*log((sqrt(a + b*x
**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*b**3*x**4 + 2*sqrt(a)*log((sqrt(a + b
*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*b**2*c*x**4 + 3*sqrt(a)*log((sqrt(a
 + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b**2*x**2 - 2*sqrt(a)*log((sq
rt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b*c*x**2 + 3*sqrt(a)*log(
(sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*b**3*x**4 - 2*sqrt(a)*lo
g((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*b**2*c*x**4 - 6*sqrt(b
)*a**2*d*x**2 + 4*sqrt(b)*a*b**2*x**2 - 6*sqrt(b)*a*b*d*x**4 + 4*sqrt(b)*b
**3*x**4)/(2*a**2*b*x**2*(a + b*x**2))
```

# 3.107 $\int \frac{A+Bx+Cx^2+Dx^3}{x^4(a+bx^2)^{3/2}}\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 150

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^4\left(a+bx^2\right)^{3/2}}\,dx = -\frac{a(bB-aD)-b(Ab-aC)x}{a^3\sqrt{a+bx^2}} - \frac{A\sqrt{a+bx^2}}{3a^2x^3} - \frac{B\sqrt{a+bx^2}}{2a^2x^2} + \frac{(5Ab-3aC)\sqrt{a+bx^2}}{3a^3x} + \frac{(3bB-2aD)\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{5/2}}$$

output
```
-(a*(B*b-D*a)-b*(A*b-C*a)*x)/a^3/(b*x^2+a)^(1/2)-1/3*A*(b*x^2+a)^(1/2)/a^2
/x^3-1/2*B*(b*x^2+a)^(1/2)/a^2/x^2+1/3*(5*A*b-3*C*a)*(b*x^2+a)^(1/2)/a^3/x
+1/2*(3*B*b-2*D*a)*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(5/2)
```

## Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.87

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^4\left(a+bx^2\right)^{3/2}}\,dx = \frac{-2a^2A-3a^2Bx+8aAbx^2-6a^2Cx^2-9abBx^3+6a^2Dx^3+16Ab^2x^4-1\ldots}{6a^3x^3\sqrt{a+bx^2}} + \frac{(-3bB+2aD)\operatorname{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{5/2}}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/(x^4*(a + b*x^2)^(3/2)),x]
```

output
```
(-2*a^2*A - 3*a^2*B*x + 8*a*A*b*x^2 - 6*a^2*C*x^2 - 9*a*b*B*x^3 + 6*a^2*D*
x^3 + 16*A*b^2*x^4 - 12*a*b*C*x^4)/(6*a^3*x^3*Sqrt[a + b*x^2]) + ((-3*b*B
+ 2*a*D)*ArcTanh[(Sqrt[b]*x - Sqrt[a + b*x^2])/Sqrt[a]])/a^(5/2)
```

## Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.10, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2336, 25, 2338, 25, 2338, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^4\left(a+bx^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{2336}$$

$$-\frac{\int -\frac{-\left(\left(\frac{bB}{a}-D\right)x^3\right)-\left(\frac{Ab}{a}-C\right)x^2+Bx+A}{x^4\sqrt{bx^2+a}}dx}{a}-\frac{a^2\left(\frac{bB}{a}-D\right)-bx(Ab-aC)}{a^3\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{-\left(\left(\frac{bB}{a}-D\right)x^3\right)-\left(\frac{Ab}{a}-C\right)x^2+Bx+A}{x^4\sqrt{bx^2+a}}dx}{a}-\frac{a^2\left(\frac{bB}{a}-D\right)-bx(Ab-aC)}{a^3\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{2338}$$

$$-\frac{-\frac{\int -\frac{-3(bB-aD)x^2-(5Ab-3aC)x+3aB}{x^3\sqrt{bx^2+a}}dx}{3a}-\frac{A\sqrt{a+bx^2}}{3ax^3}}{a}-\frac{a^2\left(\frac{bB}{a}-D\right)-bx(Ab-aC)}{a^3\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{-3(bB-aD)x^2-(5Ab-3aC)x+3aB}{x^3\sqrt{bx^2+a}}dx}{3a}-\frac{A\sqrt{a+bx^2}}{3ax^3}}{a}-\frac{a^2\left(\frac{bB}{a}-D\right)-bx(Ab-aC)}{a^3\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{2338}$$

$$\frac{\frac{-\frac{\int \frac{a(2(5Ab-3aC)+3(3bB-2aD)x)}{x^2\sqrt{bx^2+a}}dx}{2a}-\frac{3B\sqrt{a+bx^2}}{2x^2}}{3a}-\frac{A\sqrt{a+bx^2}}{3ax^3}}{a}-\frac{a^2\left(\frac{bB}{a}-D\right)-bx(Ab-aC)}{a^3\sqrt{a+bx^2}}$$

$\Big\downarrow$ 27

$$\frac{\frac{-\frac{1}{2}\int \frac{2(5Ab-3aC)+3(3bB-2aD)x}{x^2\sqrt{bx^2+a}}dx-\frac{3B\sqrt{a+bx^2}}{2x^2}}{3a}-\frac{A\sqrt{a+bx^2}}{3ax^3}}{a}-\frac{a^2\left(\frac{bB}{a}-D\right)-bx(Ab-aC)}{a^3\sqrt{a+bx^2}}$$

$\Big\downarrow$ 534

$$\frac{\frac{\frac{1}{2}\left(\frac{2\sqrt{a+bx^2}(5Ab-3aC)}{ax}-3(3bB-2aD)\int \frac{1}{x\sqrt{bx^2+a}}dx\right)-\frac{3B\sqrt{a+bx^2}}{2x^2}}{3a}-\frac{A\sqrt{a+bx^2}}{3ax^3}}{a}-\frac{a^2\left(\frac{bB}{a}-D\right)-bx(Ab-aC)}{a^3\sqrt{a+bx^2}}$$

$\Big\downarrow$ 243

$$\frac{\frac{\frac{1}{2}\left(\frac{2\sqrt{a+bx^2}(5Ab-3aC)}{ax}-\frac{3}{2}(3bB-2aD)\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2\right)-\frac{3B\sqrt{a+bx^2}}{2x^2}}{3a}-\frac{A\sqrt{a+bx^2}}{3ax^3}}{a}-\frac{a^2\left(\frac{bB}{a}-D\right)-bx(Ab-aC)}{a^3\sqrt{a+bx^2}}$$

$\Big\downarrow$ 73

$$\frac{\frac{\frac{1}{2}\left(\frac{2\sqrt{a+bx^2}(5Ab-3aC)}{ax}-\frac{3(3bB-2aD)\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{b}\right)-\frac{3B\sqrt{a+bx^2}}{2x^2}}{3a}-\frac{A\sqrt{a+bx^2}}{3ax^3}}{a}-\frac{a^2\left(\frac{bB}{a}-D\right)-bx(Ab-aC)}{a^3\sqrt{a+bx^2}}$$

$\Big\downarrow$ 221

$$\frac{\frac{\frac{1}{2}\left(\frac{2\sqrt{a+bx^2}(5Ab-3aC)}{ax}+\frac{3\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)(3bB-2aD)}{\sqrt{a}}\right)-\frac{3B\sqrt{a+bx^2}}{2x^2}}{3a}-\frac{A\sqrt{a+bx^2}}{3ax^3}}{a}-\frac{a^2\left(\frac{bB}{a}-D\right)-bx(Ab-aC)}{a^3\sqrt{a+bx^2}}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/(x^4*(a + b*x^2)^(3/2)),x]
```

output
```
-((a^2*((b*B)/a - D) - b*(A*b - a*C)*x)/(a^3*Sqrt[a + b*x^2])) + (-1/3*(A*
Sqrt[a + b*x^2])/(a*x^3) + ((-3*B*Sqrt[a + b*x^2])/(2*x^2) + ((2*(5*A*b -
3*a*C)*Sqrt[a + b*x^2])/(a*x) + (3*(3*b*B - 2*a*D)*ArcTanh[Sqrt[a + b*x^2]
/Sqrt[a]])/Sqrt[a])/2)/(3*a))/a
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

## Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.40

| method | result |
|---|---|
| default | $A\left(-\frac{1}{3a\,x^{3}\sqrt{b\,x^{2}+a}}-\frac{4b\left(-\frac{1}{ax\sqrt{b\,x^{2}+a}}-\frac{2bx}{a^{2}\sqrt{b\,x^{2}+a}}\right)}{3a}\right)+B\left(-\frac{1}{2a\,x^{2}\sqrt{b\,x^{2}+a}}-\frac{3b\left(\frac{1}{a\sqrt{b\,x^{2}+a}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{a^{\frac{3}{2}}}\right)}{2a}\right.$ |

input
```
int((D*x^3+C*x^2+B*x+A)/x^4/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
A*(-1/3/a/x^3/(b*x^2+a)^(1/2)-4/3*b/a*(-1/a/x/(b*x^2+a)^(1/2)-2*b/a^2*x/(b
*x^2+a)^(1/2)))+B*(-1/2/a/x^2/(b*x^2+a)^(1/2)-3/2*b/a*(1/a/(b*x^2+a)^(1/2)
-1/a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)))+C*(-1/a/x/(b*x^2+a)^(1/
2)-2*b/a^2*x/(b*x^2+a)^(1/2))+D*(1/a/(b*x^2+a)^(1/2)-1/a^(3/2)*ln((2*a+2*a
^(1/2)*(b*x^2+a)^(1/2))/x))
```

## Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 321, normalized size of antiderivative = 2.14

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^4\left(a+bx^2\right)^{3/2}}\,dx = \left[-\frac{3\,((2\,Dab-3\,Bb^2)x^5+(2\,Da^2-3\,Bab)x^3)\sqrt{a}\log\left(-\frac{bx^2+2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right.}{}\right.$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/x^4/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[-1/12*(3*((2*D*a*b - 3*B*b^2)*x^5 + (2*D*a^2 - 3*B*a*b)*x^3)*sqrt(a)*log(
-(b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(4*(3*C*a*b - 4*A*b^2)
*x^4 + 3*B*a^2*x - 3*(2*D*a^2 - 3*B*a*b)*x^3 + 2*A*a^2 + 2*(3*C*a^2 - 4*A*
a*b)*x^2)*sqrt(b*x^2 + a))/(a^3*b*x^5 + a^4*x^3), 1/6*(3*((2*D*a*b - 3*B*b
^2)*x^5 + (2*D*a^2 - 3*B*a*b)*x^3)*sqrt(-a)*arctan(sqrt(b*x^2 + a)*sqrt(-a
)/a) - (4*(3*C*a*b - 4*A*b^2)*x^4 + 3*B*a^2*x - 3*(2*D*a^2 - 3*B*a*b)*x^3
+ 2*A*a^2 + 2*(3*C*a^2 - 4*A*a*b)*x^2)*sqrt(b*x^2 + a))/(a^3*b*x^5 + a^4*x
^3)]
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 547 vs. $2(129) = 258$.

Time = 16.00 (sec) , antiderivative size = 547, normalized size of antiderivative = 3.65

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^4\left(a+bx^2\right)^{3/2}}\,dx = A\left(-\frac{a^3b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4x^2+6a^4b^5x^4+3a^3b^6x^6} + \frac{3a^2b^{\frac{11}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4x^2+6a^4b^5x^4+3a^3b^6x^6} + \frac{12ab^{\frac{13}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4x^2+6a^4b^5x^4+3a^3b^6x^6} + \frac{8b^{\frac{15}{2}}x^6\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4x^2+6a^4b^5x^4+3a^3b^6x^6}\right) + B\left(-\frac{1}{2a\sqrt{b}x^3\sqrt{\frac{a}{bx^2}+1}} - \frac{3\sqrt{b}}{2a^2x\sqrt{\frac{a}{bx^2}+1}} + \frac{3b\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2a^{\frac{5}{2}}}\right) + C\left(-\frac{1}{a\sqrt{b}x^2\sqrt{\frac{a}{bx^2}+1}} - \frac{2\sqrt{b}}{a^2\sqrt{\frac{a}{bx^2}+1}}\right) + D\left(\frac{2a^3\sqrt{1+\frac{bx^2}{a}}}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2} + \frac{a^3\log\left(\frac{bx^2}{a}\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2} - \frac{2a^3\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2} + \frac{a^2bx^2\log\left(\frac{bx^2}{a}\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2} - \frac{2a^2bx^2\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}\right)$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/x**4/(b*x**2+a)**(3/2),x)
```

output
```
A*(-a**3*b**(9/2)*sqrt(a/(b*x**2) + 1)/(3*a**5*b**4*x**2 + 6*a**4*b**5*x**
4 + 3*a**3*b**6*x**6) + 3*a**2*b**(11/2)*x**2*sqrt(a/(b*x**2) + 1)/(3*a**5
*b**4*x**2 + 6*a**4*b**5*x**4 + 3*a**3*b**6*x**6) + 12*a*b**(13/2)*x**4*sq
rt(a/(b*x**2) + 1)/(3*a**5*b**4*x**2 + 6*a**4*b**5*x**4 + 3*a**3*b**6*x**6
) + 8*b**(15/2)*x**6*sqrt(a/(b*x**2) + 1)/(3*a**5*b**4*x**2 + 6*a**4*b**5*
x**4 + 3*a**3*b**6*x**6)) + B*(-1/(2*a*sqrt(b)*x**3*sqrt(a/(b*x**2) + 1))
- 3*sqrt(b)/(2*a**2*x*sqrt(a/(b*x**2) + 1)) + 3*b*asinh(sqrt(a)/(sqrt(b)*x
))/(2*a**(5/2))) + C*(-1/(a*sqrt(b)*x**2*sqrt(a/(b*x**2) + 1)) - 2*sqrt(b)
/(a**2*sqrt(a/(b*x**2) + 1))) + D*(2*a**3*sqrt(1 + b*x**2/a)/(2*a**(9/2) +
 2*a**(7/2)*b*x**2) + a**3*log(b*x**2/a)/(2*a**(9/2) + 2*a**(7/2)*b*x**2)
- 2*a**3*log(sqrt(1 + b*x**2/a) + 1)/(2*a**(9/2) + 2*a**(7/2)*b*x**2) + a*
*2*b*x**2*log(b*x**2/a)/(2*a**(9/2) + 2*a**(7/2)*b*x**2) - 2*a**2*b*x**2*l
og(sqrt(1 + b*x**2/a) + 1)/(2*a**(9/2) + 2*a**(7/2)*b*x**2))
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.18

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^4\left(a+bx^2\right)^{3/2}}\,dx = -\frac{2\,Cbx}{\sqrt{bx^2+a}a^2} + \frac{8\,Ab^2x}{3\,\sqrt{bx^2+a}a^3} - \frac{D\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{a^{\frac{3}{2}}} + \frac{3\,Bb\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{2\,a^{\frac{5}{2}}} + \frac{D}{\sqrt{bx^2+a}a} - \frac{3\,Bb}{2\,\sqrt{bx^2+a}a^2} - \frac{C}{\sqrt{bx^2+a}ax} + \frac{4\,Ab}{3\,\sqrt{bx^2+a}a^2x} - \frac{B}{2\,\sqrt{bx^2+a}ax^2} - \frac{A}{3\,\sqrt{bx^2+a}ax^3}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/x^4/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
-2*C*b*x/(sqrt(b*x^2 + a)*a^2) + 8/3*A*b^2*x/(sqrt(b*x^2 + a)*a^3) - D*arc
sinh(a/(sqrt(a*b)*abs(x)))/a^(3/2) + 3/2*B*b*arcsinh(a/(sqrt(a*b)*abs(x)))
/a^(5/2) + D/(sqrt(b*x^2 + a)*a) - 3/2*B*b/(sqrt(b*x^2 + a)*a^2) - C/(sqrt
(b*x^2 + a)*a*x) + 4/3*A*b/(sqrt(b*x^2 + a)*a^2*x) - 1/2*B/(sqrt(b*x^2 + a
)*a*x^2) - 1/3*A/(sqrt(b*x^2 + a)*a*x^3)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 296 vs. 2(128) = 256.

Time = 0.15 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.97

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^4\left(a+bx^2\right)^{3/2}}\,dx = -\frac{\frac{(Ca^3b-Aa^2b^2)x}{a^5} - \frac{Da^4-Ba^3b}{a^5}}{\sqrt{bx^2+a}} + \frac{(2\,Da-3\,Bb)\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2} + \frac{3\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^5Bb+6\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4Ca\sqrt{b}-6\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4Ab^{\frac{3}{2}}-12\left(\sqrt{b}x-\sqrt{bx\right.}}{3\left(\left(\sqrt{b}x-\sqrt{bx^2+\right.\right.}}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/x^4/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
-((C*a^3*b - A*a^2*b^2)*x/a^5 - (D*a^4 - B*a^3*b)/a^5)/sqrt(b*x^2 + a) + (
2*D*a - 3*B*b)*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/(sqrt(-a)*a
^2) + 1/3*(3*(sqrt(b)*x - sqrt(b*x^2 + a))^5*B*b + 6*(sqrt(b)*x - sqrt(b*x
^2 + a))^4*C*a*sqrt(b) - 6*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*b^(3/2) - 12*
(sqrt(b)*x - sqrt(b*x^2 + a))^2*C*a^2*sqrt(b) + 24*(sqrt(b)*x - sqrt(b*x^2
 + a))^2*A*a*b^(3/2) - 3*(sqrt(b)*x - sqrt(b*x^2 + a))*B*a^2*b + 6*C*a^3*s
qrt(b) - 10*A*a^2*b^(3/2))/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^3*a^2)
```

## Mupad [B] (verification not implemented)

Time = 3.16 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.11

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^4\left(a+bx^2\right)^{3/2}}\,dx=\frac{D}{a\,\sqrt{b\,x^2+a}}-\frac{\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)\,D}{a^{3/2}}-\frac{\sqrt{b\,x^2+a}\,\left(\frac{C}{a}+\frac{2\,C\,b\,x^2}{a^2}\right)}{b\,x^3+a\,x}-\frac{3\,B\,b}{2\,a^2\,\sqrt{b\,x^2+a}}-\frac{B}{2\,a\,x^2\,\sqrt{b\,x^2+a}}+\frac{3\,B\,b\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2\,a^{5/2}}+\frac{A\,\left(-a^2+4\,a\,b\,x^2+8\,b^2\,x^4\right)}{3\,a^3\,x^3\,\sqrt{b\,x^2+a}}$$

input
```
int((A + B*x + C*x^2 + x^3*D)/(x^4*(a + b*x^2)^(3/2)),x)
```

output
```
D/(a*(a + b*x^2)^(1/2)) - (atanh((a + b*x^2)^(1/2)/a^(1/2))*D)/a^(3/2) - (
(a + b*x^2)^(1/2)*(C/a + (2*C*b*x^2)/a^2))/(a*x + b*x^3) - (3*B*b)/(2*a^2*
(a + b*x^2)^(1/2)) - B/(2*a*x^2*(a + b*x^2)^(1/2)) + (3*B*b*atanh((a + b*x
^2)^(1/2)/a^(1/2)))/(2*a^(5/2)) + (A*(8*b^2*x^4 - a^2 + 4*a*b*x^2))/(3*a^3
*x^3*(a + b*x^2)^(1/2))
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 450, normalized size of antiderivative = 3.00

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^4\left(a+bx^2\right)^{3/2}}\,dx = \frac{-2\sqrt{b\,x^2+a}\,a^3+8\sqrt{b\,x^2+a}\,a^2b\,x^2-3\sqrt{b\,x^2+a}\,a^2bx-6\sqrt{b\,x^2+a}\,a^2c\,x^2}{}$$

input
```
int((D*x^3+C*x^2+B*x+A)/x^4/(b*x^2+a)^(3/2),x)
```

output
```
( - 2*sqrt(a + b*x**2)*a**3 + 8*sqrt(a + b*x**2)*a**2*b*x**2 - 3*sqrt(a +
b*x**2)*a**2*b*x - 6*sqrt(a + b*x**2)*a**2*c*x**2 + 6*sqrt(a + b*x**2)*a**
2*d*x**3 + 16*sqrt(a + b*x**2)*a*b**2*x**4 - 9*sqrt(a + b*x**2)*a*b**2*x**
3 - 12*sqrt(a + b*x**2)*a*b*c*x**4 + 6*sqrt(a)*log((sqrt(a + b*x**2) - sqr
t(a) + sqrt(b)*x)/sqrt(a))*a**2*d*x**3 - 9*sqrt(a)*log((sqrt(a + b*x**2) -
 sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b**2*x**3 + 6*sqrt(a)*log((sqrt(a + b*x**
2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b*d*x**5 - 9*sqrt(a)*log((sqrt(a + b*
x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*b**3*x**5 - 6*sqrt(a)*log((sqrt(a +
b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*a**2*d*x**3 + 9*sqrt(a)*log((sqrt(
a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b**2*x**3 - 6*sqrt(a)*log((s
qrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b*d*x**5 + 9*sqrt(a)*log
((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*b**3*x**5 - 16*sqrt(b)*
a**2*b*x**3 + 12*sqrt(b)*a**2*c*x**3 - 16*sqrt(b)*a*b**2*x**5 + 12*sqrt(b)
*a*b*c*x**5)/(6*a**3*x**3*(a + b*x**2))
```

# 3.108 $\int \frac{A+Bx+Cx^2+Dx^3}{x^5(a+bx^2)^{3/2}}\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 176

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^5(a+bx^2)^{3/2}}\,dx = \frac{b(Ab-aC+(bB-aD)x)}{a^3\sqrt{a+bx^2}} - \frac{A\sqrt{a+bx^2}}{4a^2x^4} - \frac{B\sqrt{a+bx^2}}{3a^2x^3} + \frac{(7Ab-4aC)\sqrt{a+bx^2}}{8a^3x^2} + \frac{(5bB-3aD)\sqrt{a+bx^2}}{3a^3x} - \frac{3b(5Ab-4aC)\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{7/2}}$$

output
```
b*(A*b-C*a+(B*b-D*a)*x)/a^3/(b*x^2+a)^(1/2)-1/4*A*(b*x^2+a)^(1/2)/a^2/x^4-
1/3*B*(b*x^2+a)^(1/2)/a^2/x^3+1/8*(7*A*b-4*C*a)*(b*x^2+a)^(1/2)/a^3/x^2+1/
3*(5*B*b-3*D*a)*(b*x^2+a)^(1/2)/a^3/x-3/8*b*(5*A*b-4*C*a)*arctanh((b*x^2+a
)^(1/2)/a^(1/2))/a^(7/2)
```

## Mathematica [A] (verified)

Time = 1.09 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.78

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^5\left(a+bx^2\right)^{3/2}}\,dx = \frac{b^2x^4(45A+64Bx)-2a^2(3A+4Bx+6x^2(C+2Dx))+abx^2(15A-4x(-}{24a^3x^4\sqrt{a+bx^2}}$$
$$+\frac{3b(5Ab-4aC)\operatorname{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right)}{4a^{7/2}}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/(x^5*(a + b*x^2)^(3/2)),x]
```

output
```
(b^2*x^4*(45*A + 64*B*x) - 2*a^2*(3*A + 4*B*x + 6*x^2*(C + 2*D*x)) + a*b*x
^2*(15*A - 4*x*(-8*B + 3*x*(3*C + 4*D*x))))/(24*a^3*x^4*Sqrt[a + b*x^2]) +
 (3*b*(5*A*b - 4*a*C)*ArcTanh[(Sqrt[b]*x - Sqrt[a + b*x^2])/Sqrt[a]])/(4*a
^(7/2))
```

## Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.07, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2336, 25, 2338, 25, 2338, 2338, 25, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^5\left(a+bx^2\right)^{3/2}}\,dx$$

$$\Big\downarrow \; 2336$$

$$\frac{b(x(bB-aD)-aC+Ab)}{a^3\sqrt{a+bx^2}}-\frac{\int -\frac{\frac{b(Ab-aC)x^4}{a^2}-\left(\frac{bB}{a}-D\right)x^3-\left(\frac{Ab}{a}-C\right)x^2+Bx+A}{x^5\sqrt{bx^2+a}}dx}{a}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{\frac{b(Ab-aC)x^4}{a^2}-\left(\frac{bB}{a}-D\right)x^3-\left(\frac{Ab}{a}-C\right)x^2+Bx+A}{x^5\sqrt{bx^2+a}}dx}{a}+\frac{b(x(bB-aD)-aC+Ab)}{a^3\sqrt{a+bx^2}}$$

$\downarrow$ 2338

$$\frac{-\frac{\int -\frac{4b\left(\frac{Ab}{a}-C\right)x^3-4(bB-aD)x^2-(7Ab-4aC)x+4aB}{x^4\sqrt{bx^2+a}}dx}{4a}-\frac{A\sqrt{a+bx^2}}{4ax^4}}{a}+\frac{b(x(bB-aD)-aC+Ab)}{a^3\sqrt{a+bx^2}}$$

$\downarrow$ 25

$$\frac{\frac{\int \frac{4b\left(\frac{Ab}{a}-C\right)x^3-4(bB-aD)x^2-(7Ab-4aC)x+4aB}{x^4\sqrt{bx^2+a}}dx}{4a}-\frac{A\sqrt{a+bx^2}}{4ax^4}}{a}+\frac{b(x(bB-aD)-aC+Ab)}{a^3\sqrt{a+bx^2}}$$

$\downarrow$ 2338

$$\frac{\frac{-\frac{\int \frac{-12b(Ab-aC)x^2+4a(5bB-3aD)x+3a(7Ab-4aC)}{x^3\sqrt{bx^2+a}}dx}{3a}-\frac{4B\sqrt{a+bx^2}}{3x^3}}{4a}-\frac{A\sqrt{a+bx^2}}{4ax^4}}{a}+\frac{b(x(bB-aD)-aC+Ab)}{a^3\sqrt{a+bx^2}}$$

$\downarrow$ 2338

$$\frac{\frac{-\frac{-\frac{\int -\frac{a(8a(5bB-3aD)-9b(5Ab-4aC)x)}{x^2\sqrt{bx^2+a}}dx}{2a}-\frac{3\sqrt{a+bx^2}(7Ab-4aC)}{2x^2}}{3a}-\frac{4B\sqrt{a+bx^2}}{3x^3}}{4a}-\frac{A\sqrt{a+bx^2}}{4ax^4}}{a}+\frac{b(x(bB-aD)-aC+Ab)}{a^3\sqrt{a+bx^2}}$$

$\downarrow$ 25

$$\frac{\frac{-\frac{\frac{\int \frac{a(8a(5bB-3aD)-9b(5Ab-4aC)x)}{x^2\sqrt{bx^2+a}}dx}{2a}-\frac{3\sqrt{a+bx^2}(7Ab-4aC)}{2x^2}}{3a}-\frac{4B\sqrt{a+bx^2}}{3x^3}}{4a}-\frac{A\sqrt{a+bx^2}}{4ax^4}}{a}+\frac{b(x(bB-aD)-aC+Ab)}{a^3\sqrt{a+bx^2}}$$

$\downarrow$ 27

$$\frac{\frac{-\frac{\frac{1}{2}\int \frac{8a(5bB-3aD)-9b(5Ab-4aC)x}{x^2\sqrt{bx^2+a}}dx-\frac{3\sqrt{a+bx^2}(7Ab-4aC)}{2x^2}}{3a}-\frac{4B\sqrt{a+bx^2}}{3x^3}}{4a}-\frac{A\sqrt{a+bx^2}}{4ax^4}}{a}+\frac{b(x(bB-aD)-aC+Ab)}{a^3\sqrt{a+bx^2}}$$

$\downarrow$ 534

$$\frac{-\frac{\frac{\frac{1}{2}\left(-9b(5Ab-4aC)\int\frac{1}{x\sqrt{bx^2+a}}dx-\frac{8\sqrt{a+bx^2}(5bB-3aD)}{x}\right)-\frac{3\sqrt{a+bx^2}(7Ab-4aC)}{2x^2}}{3a}-\frac{4B\sqrt{a+bx^2}}{3x^3}}{4a}-\frac{A\sqrt{a+bx^2}}{4ax^4}}{a}+\frac{b(x(bB-aD)-aC+Ab)}{a^3\sqrt{a+bx^2}}$$

$\Big\downarrow$ 243

$$\frac{-\frac{\frac{\frac{1}{2}\left(-\frac{9}{2}b(5Ab-4aC)\int\frac{1}{x^2\sqrt{bx^2+a}}dx^2-\frac{8\sqrt{a+bx^2}(5bB-3aD)}{x}\right)-\frac{3\sqrt{a+bx^2}(7Ab-4aC)}{2x^2}}{3a}-\frac{4B\sqrt{a+bx^2}}{3x^3}}{4a}-\frac{A\sqrt{a+bx^2}}{4ax^4}}{a}+\frac{b(x(bB-aD)-aC+Ab)}{a^3\sqrt{a+bx^2}}$$

$\Big\downarrow$ 73

$$\frac{-\frac{\frac{\frac{1}{2}\left(-9(5Ab-4aC)\int\frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}-\frac{8\sqrt{a+bx^2}(5bB-3aD)}{x}\right)-\frac{3\sqrt{a+bx^2}(7Ab-4aC)}{2x^2}}{3a}-\frac{4B\sqrt{a+bx^2}}{3x^3}}{4a}-\frac{A\sqrt{a+bx^2}}{4ax^4}}{a}+\frac{b(x(bB-aD)-aC+Ab)}{a^3\sqrt{a+bx^2}}$$

$\Big\downarrow$ 221

$$\frac{b(x(bB-aD)-aC+Ab)}{a^3\sqrt{a+bx^2}}+\frac{-\frac{\frac{\frac{1}{2}\left(\frac{9b(5Ab-4aC)\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{8\sqrt{a+bx^2}(5bB-3aD)}{x}\right)-\frac{3\sqrt{a+bx^2}(7Ab-4aC)}{2x^2}}{3a}-\frac{4B\sqrt{a+bx^2}}{3x^3}}{4a}-\frac{A\sqrt{a+bx^2}}{4ax^4}}{a}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/(x^5*(a + b*x^2)^(3/2)),x]
```

output
```
(b*(A*b - a*C + (b*B - a*D)*x))/(a^3*Sqrt[a + b*x^2]) + (-1/4*(A*Sqrt[a +
b*x^2])/(a*x^4) + ((-4*B*Sqrt[a + b*x^2])/(3*x^3) - ((-3*(7*A*b - 4*a*C)*S
qrt[a + b*x^2])/(2*x^2) + ((-8*(5*b*B - 3*a*D)*Sqrt[a + b*x^2])/x + (9*b*(
5*A*b - 4*a*C)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a])/2)/(3*a))/(4*a))
/a
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

## Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.47

| method | result |
|---|---|
| default | $A\left(-\frac{1}{4a\,x^{4}\sqrt{b\,x^{2}+a}}-\frac{5b\left(-\frac{1}{2a\,x^{2}\sqrt{b\,x^{2}+a}}-\frac{3b\left(\frac{1}{a\sqrt{b\,x^{2}+a}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{a^{\frac{3}{2}}}\right)}{2a}\right)}{4a}\right)+B\left(-\frac{1}{3a\,x^{3}\sqrt{b\,x^{2}+a}}-\frac{4b\left(-\right.}{}\right.$ |

input
```
int((D*x^3+C*x^2+B*x+A)/x^5/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
A*(-1/4/a/x^4/(b*x^2+a)^(1/2)-5/4*b/a*(-1/2/a/x^2/(b*x^2+a)^(1/2)-3/2*b/a*
(1/a/(b*x^2+a)^(1/2)-1/a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x))))+B*
(-1/3/a/x^3/(b*x^2+a)^(1/2)-4/3*b/a*(-1/a/x/(b*x^2+a)^(1/2)-2*b/a^2*x/(b*x
^2+a)^(1/2)))+C*(-1/2/a/x^2/(b*x^2+a)^(1/2)-3/2*b/a*(1/a/(b*x^2+a)^(1/2)-1
/a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)))+D*(-1/a/x/(b*x^2+a)^(1/2)
-2*b/a^2*x/(b*x^2+a)^(1/2))
```

## Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 384, normalized size of antiderivative = 2.18

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^5\left(a+bx^2\right)^{3/2}}\,dx = \left[-\frac{9\left((4\,Cab^2-5\,Ab^3)x^6+(4\,Ca^2b-5\,Aab^2)x^4\right)\sqrt{a}\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+}{x^2}\right.}{}\right.$$

$$-\frac{9\left((4\,Cab^2-5\,Ab^3)x^6+(4\,Ca^2b-5\,Aab^2)x^4\right)\sqrt{-a}\arctan\left(\frac{\sqrt{bx^2+a}\sqrt{-a}}{a}\right)+(16\,(3\,Da^2b-4\,Bab^2)x^5+8\,B}{24\,(a^4bx^6+a}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/x^5/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[-1/48*(9*((4*C*a*b^2 - 5*A*b^3)*x^6 + (4*C*a^2*b - 5*A*a*b^2)*x^4)*sqrt(a
)*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(16*(3*D*a^2*b -
 4*B*a*b^2)*x^5 + 8*B*a^3*x + 9*(4*C*a^2*b - 5*A*a*b^2)*x^4 + 6*A*a^3 + 8*
(3*D*a^3 - 4*B*a^2*b)*x^3 + 3*(4*C*a^3 - 5*A*a^2*b)*x^2)*sqrt(b*x^2 + a))/
(a^4*b*x^6 + a^5*x^4), -1/24*(9*((4*C*a*b^2 - 5*A*b^3)*x^6 + (4*C*a^2*b -
5*A*a*b^2)*x^4)*sqrt(-a)*arctan(sqrt(b*x^2 + a)*sqrt(-a)/a) + (16*(3*D*a^2
*b - 4*B*a*b^2)*x^5 + 8*B*a^3*x + 9*(4*C*a^2*b - 5*A*a*b^2)*x^4 + 6*A*a^3
+ 8*(3*D*a^3 - 4*B*a^2*b)*x^3 + 3*(4*C*a^3 - 5*A*a^2*b)*x^2)*sqrt(b*x^2 +
a))/(a^4*b*x^6 + a^5*x^4)]
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 466 vs. $2(162) = 324$.

Time = 13.38 (sec) , antiderivative size = 466, normalized size of antiderivative = 2.65

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^5\left(a+bx^2\right)^{3/2}}\,dx = A\left(-\frac{1}{4a\sqrt{b}x^5\sqrt{\frac{a}{bx^2}+1}} + \frac{5\sqrt{b}}{8a^2x^3\sqrt{\frac{a}{bx^2}+1}} + \frac{15b^{\frac{3}{2}}}{8a^3x\sqrt{\frac{a}{bx^2}+1}} - \frac{15b^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{8a^{\frac{7}{2}}}\right) + B\left(-\frac{a^3b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4x^2+6a^4b^5x^4+3a^3b^6x^6} + \frac{3a^2b^{\frac{11}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4x^2+6a^4b^5x^4+3a^3b^6x^6} + \frac{12ab^{\frac{13}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4x^2+6a^4b^5x^4+3a^3b^6x^6} + \frac{8b^{\frac{15}{2}}x^6\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4x^2+6a^4b^5x^4+3a^3b^6x^6}\right) + C\left(-\frac{1}{2a\sqrt{b}x^3\sqrt{\frac{a}{bx^2}+1}} - \frac{3\sqrt{b}}{2a^2x\sqrt{\frac{a}{bx^2}+1}} + \frac{3b\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2a^{\frac{5}{2}}}\right) + D\left(-\frac{1}{a\sqrt{b}x^2\sqrt{\frac{a}{bx^2}+1}} - \frac{2\sqrt{b}}{a^2\sqrt{\frac{a}{bx^2}+1}}\right)$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/x**5/(b*x**2+a)**(3/2),x)
```

output
```
A*(-1/(4*a*sqrt(b)*x**5*sqrt(a/(b*x**2) + 1)) + 5*sqrt(b)/(8*a**2*x**3*sqr
t(a/(b*x**2) + 1)) + 15*b**(3/2)/(8*a**3*x*sqrt(a/(b*x**2) + 1)) - 15*b**2
*asinh(sqrt(a)/(sqrt(b)*x))/(8*a**(7/2))) + B*(-a**3*b**(9/2)*sqrt(a/(b*x*
*2) + 1)/(3*a**5*b**4*x**2 + 6*a**4*b**5*x**4 + 3*a**3*b**6*x**6) + 3*a**2
*b**(11/2)*x**2*sqrt(a/(b*x**2) + 1)/(3*a**5*b**4*x**2 + 6*a**4*b**5*x**4
+ 3*a**3*b**6*x**6) + 12*a*b**(13/2)*x**4*sqrt(a/(b*x**2) + 1)/(3*a**5*b**
4*x**2 + 6*a**4*b**5*x**4 + 3*a**3*b**6*x**6) + 8*b**(15/2)*x**6*sqrt(a/(b
*x**2) + 1)/(3*a**5*b**4*x**2 + 6*a**4*b**5*x**4 + 3*a**3*b**6*x**6)) + C*
(-1/(2*a*sqrt(b)*x**3*sqrt(a/(b*x**2) + 1)) - 3*sqrt(b)/(2*a**2*x*sqrt(a/(
b*x**2) + 1)) + 3*b*asinh(sqrt(a)/(sqrt(b)*x))/(2*a**(5/2))) + D*(-1/(a*sq
rt(b)*x**2*sqrt(a/(b*x**2) + 1)) - 2*sqrt(b)/(a**2*sqrt(a/(b*x**2) + 1)))
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.26

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^5(a+bx^2)^{3/2}}\,dx = -\frac{2\,Dbx}{\sqrt{bx^2+a}a^2}+\frac{8\,Bb^2x}{3\,\sqrt{bx^2+a}a^3}+\frac{3\,Cb\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{2\,a^{\frac{5}{2}}}-\frac{15\,Ab^2\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{8\,a^{\frac{7}{2}}}-\frac{3\,Cb}{2\,\sqrt{bx^2+a}a^2}+\frac{15\,Ab^2}{8\,\sqrt{bx^2+a}a^3}-\frac{D}{\sqrt{bx^2+a}ax}+\frac{4\,Bb}{3\,\sqrt{bx^2+a}a^2x}-\frac{C}{2\,\sqrt{bx^2+a}ax^2}+\frac{5\,Ab}{8\,\sqrt{bx^2+a}a^2x^2}-\frac{B}{3\,\sqrt{bx^2+a}ax^3}-\frac{A}{4\,\sqrt{bx^2+a}ax^4}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/x^5/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
-2*D*b*x/(sqrt(b*x^2 + a)*a^2) + 8/3*B*b^2*x/(sqrt(b*x^2 + a)*a^3) + 3/2*C
*b*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(5/2) - 15/8*A*b^2*arcsinh(a/(sqrt(a*b)
*abs(x)))/a^(7/2) - 3/2*C*b/(sqrt(b*x^2 + a)*a^2) + 15/8*A*b^2/(sqrt(b*x^2
 + a)*a^3) - D/(sqrt(b*x^2 + a)*a*x) + 4/3*B*b/(sqrt(b*x^2 + a)*a^2*x) - 1
/2*C/(sqrt(b*x^2 + a)*a*x^2) + 5/8*A*b/(sqrt(b*x^2 + a)*a^2*x^2) - 1/3*B/(
sqrt(b*x^2 + a)*a*x^3) - 1/4*A/(sqrt(b*x^2 + a)*a*x^4)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 521 vs. 2(151) = 302.

Time = 0.15 (sec) , antiderivative size = 521, normalized size of antiderivative = 2.96

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^5(a+bx^2)^{3/2}}\,dx = -\frac{\frac{(Da^4b-Ba^3b^2)x}{a^6}+\frac{Ca^4b-Aa^3b^2}{a^6}}{\sqrt{bx^2+a}} - \frac{3\,(4\,Cab-5\,Ab^2)\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{4\,\sqrt{-a}a^3} + \frac{12\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^7Cab-21\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^7Ab^2+24\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6Da^2\sqrt{b}-24\left(\sqrt{b}x-\right.}{}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/x^5/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
-((D*a^4*b - B*a^3*b^2)*x/a^6 + (C*a^4*b - A*a^3*b^2)/a^6)/sqrt(b*x^2 + a)
 - 3/4*(4*C*a*b - 5*A*b^2)*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))
/(sqrt(-a)*a^3) + 1/12*(12*(sqrt(b)*x - sqrt(b*x^2 + a))^7*C*a*b - 21*(sqr
t(b)*x - sqrt(b*x^2 + a))^7*A*b^2 + 24*(sqrt(b)*x - sqrt(b*x^2 + a))^6*D*a
^2*sqrt(b) - 24*(sqrt(b)*x - sqrt(b*x^2 + a))^6*B*a*b^(3/2) - 12*(sqrt(b)*
x - sqrt(b*x^2 + a))^5*C*a^2*b + 45*(sqrt(b)*x - sqrt(b*x^2 + a))^5*A*a*b^
2 - 72*(sqrt(b)*x - sqrt(b*x^2 + a))^4*D*a^3*sqrt(b) + 120*(sqrt(b)*x - sq
rt(b*x^2 + a))^4*B*a^2*b^(3/2) - 12*(sqrt(b)*x - sqrt(b*x^2 + a))^3*C*a^3*
b + 45*(sqrt(b)*x - sqrt(b*x^2 + a))^3*A*a^2*b^2 + 72*(sqrt(b)*x - sqrt(b*
x^2 + a))^2*D*a^4*sqrt(b) - 136*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a^3*b^(3
/2) + 12*(sqrt(b)*x - sqrt(b*x^2 + a))*C*a^4*b - 21*(sqrt(b)*x - sqrt(b*x^
2 + a))*A*a^3*b^2 - 24*D*a^5*sqrt(b) + 40*B*a^4*b^(3/2))/(((sqrt(b)*x - sq
rt(b*x^2 + a))^2 - a)^4*a^3)
```

## Mupad [B] (verification not implemented)

Time = 3.42 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.21

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^5\left(a+bx^2\right)^{3/2}}\,dx = \frac{15\,A\,b^2}{8\,a^3\,\sqrt{b\,x^2+a}} - \frac{15\,A\,b^2\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{8\,a^{7/2}} - \frac{3\,C\,b}{2\,a^2\,\sqrt{b\,x^2+a}} - \frac{a^2\,D+2\,b^2\,x^4\,D+3\,a\,b\,x^2\,D}{a^2\,x\,\left(b\,x^2+a\right)^{3/2}} - \frac{A}{4\,a\,x^4\,\sqrt{b\,x^2+a}} - \frac{C}{2\,a\,x^2\,\sqrt{b\,x^2+a}} + \frac{3\,C\,b\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2\,a^{5/2}} + \frac{5\,A\,b}{8\,a^2\,x^2\,\sqrt{b\,x^2+a}} + \frac{B\,\left(-a^2+4\,a\,b\,x^2+8\,b^2\,x^4\right)}{3\,a^3\,x^3\,\sqrt{b\,x^2+a}}$$

input
```
int((A + B*x + C*x^2 + x^3*D)/(x^5*(a + b*x^2)^(3/2)),x)
```

output
```
(15*A*b^2)/(8*a^3*(a + b*x^2)^(1/2)) - (15*A*b^2*atanh((a + b*x^2)^(1/2)/a
^(1/2)))/(8*a^(7/2)) - (3*C*b)/(2*a^2*(a + b*x^2)^(1/2)) - (a^2*D + 2*b^2*
x^4*D + 3*a*b*x^2*D)/(a^2*x*(a + b*x^2)^(3/2)) - A/(4*a*x^4*(a + b*x^2)^(1
/2)) - C/(2*a*x^2*(a + b*x^2)^(1/2)) + (3*C*b*atanh((a + b*x^2)^(1/2)/a^(1
/2)))/(2*a^(5/2)) + (5*A*b)/(8*a^2*x^2*(a + b*x^2)^(1/2)) + (B*(8*b^2*x^4
- a^2 + 4*a*b*x^2))/(3*a^3*x^3*(a + b*x^2)^(1/2))
```

## Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 481, normalized size of antiderivative = 2.73

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^5\left(a+bx^2\right)^{3/2}}\,dx = \frac{-6\sqrt{b\,x^2+a}\,a^3+15\sqrt{b\,x^2+a}\,a^2b\,x^2-8\sqrt{b\,x^2+a}\,a^2bx-12\sqrt{b\,x^2+a}\,a^2c}{}$$

input
```
int((D*x^3+C*x^2+B*x+A)/x^5/(b*x^2+a)^(3/2),x)
```

output
```
( - 6*sqrt(a + b*x**2)*a**3 + 15*sqrt(a + b*x**2)*a**2*b*x**2 - 8*sqrt(a +
 b*x**2)*a**2*b*x - 12*sqrt(a + b*x**2)*a**2*c*x**2 - 24*sqrt(a + b*x**2)*
a**2*d*x**3 + 45*sqrt(a + b*x**2)*a*b**2*x**4 + 32*sqrt(a + b*x**2)*a*b**2
*x**3 - 36*sqrt(a + b*x**2)*a*b*c*x**4 - 48*sqrt(a + b*x**2)*a*b*d*x**5 +
64*sqrt(a + b*x**2)*b**3*x**5 + 45*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a)
 + sqrt(b)*x)/sqrt(a))*a*b**2*x**4 - 36*sqrt(a)*log((sqrt(a + b*x**2) - sq
rt(a) + sqrt(b)*x)/sqrt(a))*a*b*c*x**4 + 45*sqrt(a)*log((sqrt(a + b*x**2)
- sqrt(a) + sqrt(b)*x)/sqrt(a))*b**3*x**6 - 36*sqrt(a)*log((sqrt(a + b*x**
2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*b**2*c*x**6 - 45*sqrt(a)*log((sqrt(a +
b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b**2*x**4 + 36*sqrt(a)*log((sqrt
(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b*c*x**4 - 45*sqrt(a)*log((
sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*b**3*x**6 + 36*sqrt(a)*lo
g((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*b**2*c*x**6 + 48*sqrt(
b)*a**2*d*x**4 - 64*sqrt(b)*a*b**2*x**4 + 48*sqrt(b)*a*b*d*x**6 - 64*sqrt(
b)*b**3*x**6)/(24*a**3*x**4*(a + b*x**2))
```

# 3.109 $\int \frac{x^4(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^{5/2}}\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 186

$$\int \frac{x^4(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^{5/2}}\,dx = -\frac{a(a(bB-aD)-b(Ab-aC)x)}{3b^4(a+bx^2)^{3/2}} + \frac{3a(2bB-3aD)-b(4Ab-7aC)x}{3b^4\sqrt{a+bx^2}} + \frac{(bB-3aD)\sqrt{a+bx^2}}{b^4} + \frac{Cx\sqrt{a+bx^2}}{2b^3} + \frac{D(a+bx^2)^{3/2}}{3b^4} + \frac{(2Ab-5aC)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{7/2}}$$

output
```
-1/3*a*(a*(B*b-D*a)-b*(A*b-C*a)*x)/b^4/(b*x^2+a)^(3/2)+1/3*(3*a*(2*B*b-3*D
*a)-b*(4*A*b-7*C*a)*x)/b^4/(b*x^2+a)^(1/2)+(B*b-3*D*a)*(b*x^2+a)^(1/2)/b^4
+1/2*C*x*(b*x^2+a)^(1/2)/b^3+1/3*D*(b*x^2+a)^(3/2)/b^4+1/2*(2*A*b-5*C*a)*a
rctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(7/2)
```

## Mathematica [A] (verified)

Time = 1.10 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.78

$$\int \frac{x^4(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^{5/2}}\,dx = \frac{-32a^3D+a^2b(16B+3x(5C-16Dx))+2ab^2x(-3A+2x(6B+5Cx}{6b^4\left(a+bx^2\right)^{3/2}}$$
$$+\frac{(2Ab-5aC)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{-\sqrt{a}+\sqrt{a+bx^2}}\right)}{b^{7/2}}$$

input
```
Integrate[(x^4*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^(5/2),x]
```

output
```
(-32*a^3*D + a^2*b*(16*B + 3*x*(5*C - 16*D*x)) + 2*a*b^2*x*(-3*A + 2*x*(6*
B + 5*C*x - 3*D*x^2)) + b^3*x^3*(-8*A + x*(6*B + 3*C*x + 2*D*x^2)))/(6*b^4
*(a + b*x^2)^(3/2)) + ((2*A*b - 5*a*C)*ArcTanh[(Sqrt[b]*x)/(-Sqrt[a] + Sqr
t[a + b*x^2])])/b^(7/2)
```

## Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.24, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.367$, Rules used = {2335, 25, 2335, 27, 533, 533, 25, 27, 455, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^{5/2}}\,dx$$

$$\Big\downarrow \ 2335$$

$$-\frac{\int -\frac{x^3\left(3aDx^2-(2Ab-5aC)x+4a\left(B-\frac{aD}{b}\right)\right)}{(bx^2+a)^{3/2}}dx}{3ab}-\frac{x^4\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{3ab\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{x^3\left(3aDx^2-(2Ab-5aC)x+4a\left(B-\frac{aD}{b}\right)\right)}{(bx^2+a)^{3/2}}dx}{3ab}-\frac{x^4\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{3ab\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \; 2335$$

$$\frac{\frac{x^3(x(4bB-7aD)-5aC+2Ab)}{b\sqrt{a+bx^2}}-\frac{\int \frac{3ax^2(2Ab-5aC+4(bB-2aD)x)}{\sqrt{bx^2+a}}dx}{ab}}{3ab}-\frac{x^4\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{3ab\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{x^3(x(4bB-7aD)-5aC+2Ab)}{b\sqrt{a+bx^2}}-\frac{3\int \frac{x^2(2Ab-5aC+4(bB-2aD)x)}{\sqrt{bx^2+a}}dx}{b}}{3ab}-\frac{x^4\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{3ab\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \; 533$$

$$\frac{\frac{x^3(x(4bB-7aD)-5aC+2Ab)}{b\sqrt{a+bx^2}}-\frac{3\left(\frac{4x^2\sqrt{a+bx^2}(bB-2aD)}{3b}-\frac{\int \frac{x(8a(bB-2aD)-3b(2Ab-5aC)x)}{\sqrt{bx^2+a}}dx}{3b}\right)}{b}}{3ab}-\frac{x^4\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{3ab\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \; 533$$

$$\frac{\frac{x^3(x(4bB-7aD)-5aC+2Ab)}{b\sqrt{a+bx^2}}-\frac{3\left(\frac{4x^2\sqrt{a+bx^2}(bB-2aD)}{3b}-\frac{-\frac{\int -\frac{ab(3(2Ab-5aC)+16(bB-2aD)x)}{\sqrt{bx^2+a}}dx}{2b}-\frac{3}{2}x\sqrt{a+bx^2}(2Ab-5aC)}{3b}\right)}{b}}{3ab}-\frac{x^4\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{3ab\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{x^3(x(4bB-7aD)-5aC+2Ab)}{b\sqrt{a+bx^2}}-\frac{3\left(\frac{4x^2\sqrt{a+bx^2}(bB-2aD)}{3b}-\frac{\frac{\int \frac{ab(3(2Ab-5aC)+16(bB-2aD)x)}{\sqrt{bx^2+a}}dx}{2b}-\frac{3}{2}x\sqrt{a+bx^2}(2Ab-5aC)}{3b}\right)}{b}}{3ab}-\frac{x^4\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{3ab\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{x^3(x(4bB-7aD)-5aC+2Ab)}{b\sqrt{a+bx^2}}-\frac{3\left(\frac{4x^2\sqrt{a+bx^2}(bB-2aD)}{3b}-\frac{\frac{1}{2}a\int\frac{3(2Ab-5aC)+16(bB-2aD)x}{\sqrt{bx^2+a}}dx-\frac{3}{2}x\sqrt{a+bx^2}(2Ab-5aC)}{3b}\right)}{b}}{3ab}-\frac{x^4\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{3ab\left(a+bx^2\right)^{3/2}}$$

$\downarrow$ 455

$$\frac{\frac{x^3(x(4bB-7aD)-5aC+2Ab)}{b\sqrt{a+bx^2}}-\frac{3\left(\frac{4x^2\sqrt{a+bx^2}(bB-2aD)}{3b}-\frac{\frac{1}{2}a\left(3(2Ab-5aC)\int\frac{1}{\sqrt{bx^2+a}}dx+\frac{16\sqrt{a+bx^2}(bB-2aD)}{b}\right)-\frac{3}{2}x\sqrt{a+bx^2}(2Ab-5aC)}{3b}\right)}{b}}{3ab}-\frac{x^4\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{3ab\left(a+bx^2\right)^{3/2}}$$

$\downarrow$ 224

$$\frac{\frac{x^3(x(4bB-7aD)-5aC+2Ab)}{b\sqrt{a+bx^2}}-\frac{3\left(\frac{4x^2\sqrt{a+bx^2}(bB-2aD)}{3b}-\frac{\frac{1}{2}a\left(3(2Ab-5aC)\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{16\sqrt{a+bx^2}(bB-2aD)}{b}\right)-\frac{3}{2}x\sqrt{a+bx^2}(2Ab-5aC)}{3b}\right)}{b}}{3ab}-\frac{x^4\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{3ab\left(a+bx^2\right)^{3/2}}$$

$\downarrow$ 219

$$\frac{\frac{x^3(x(4bB-7aD)-5aC+2Ab)}{b\sqrt{a+bx^2}}-\frac{3\left(\frac{4x^2\sqrt{a+bx^2}(bB-2aD)}{3b}-\frac{\frac{1}{2}a\left(\frac{3(2Ab-5aC)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}+\frac{16\sqrt{a+bx^2}(bB-2aD)}{b}\right)-\frac{3}{2}x\sqrt{a+bx^2}(2Ab-5aC)}{3b}\right)}{b}}{3ab}-\frac{x^4\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{3ab\left(a+bx^2\right)^{3/2}}$$

input
```
Int[(x^4*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^(5/2),x]
```

output
```
-1/3*(x^4*(a*(B - (a*D)/b) - (A*b - a*C)*x))/(a*b*(a + b*x^2)^(3/2)) + ((x
^3*(2*A*b - 5*a*C + (4*b*B - 7*a*D)*x))/(b*Sqrt[a + b*x^2]) - (3*((4*(b*B
- 2*a*D)*x^2*Sqrt[a + b*x^2])/(3*b) - ((-3*(2*A*b - 5*a*C)*x*Sqrt[a + b*x^
2])/2 + (a*((16*(b*B - 2*a*D)*Sqrt[a + b*x^2])/b + (3*(2*A*b - 5*a*C)*ArcT
anh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/Sqrt[b]))/2)/(3*b)))/b)/(3*a*b)
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

rule 2335
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Simp[c/(2*a*b*(p + 1))   Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSu
m[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
 b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

## Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.54

| method | result |
|---|---|
| default | $A\left(-\frac{x^{3}}{3b(b\,x^{2}+a)^{\frac{3}{2}}}+\frac{-\frac{x}{b\sqrt{b\,x^{2}+a}}+\frac{\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{b^{\frac{3}{2}}}}{b}\right)+B\left(\frac{x^{4}}{b(b\,x^{2}+a)^{\frac{3}{2}}}-\frac{4a\left(-\frac{x^{2}}{b\left(b\,x^{2}+a\right)^{\frac{3}{2}}}-\frac{2a}{3b^{2}\left(b\,x^{2}+a\right)^{\frac{3}{2}}}\right)}{b}\right)+$ |

input
```
int(x^4*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
A*(-1/3*x^3/b/(b*x^2+a)^(3/2)+1/b*(-x/b/(b*x^2+a)^(1/2)+1/b^(3/2)*ln(b^(1/
2)*x+(b*x^2+a)^(1/2))))+B*(x^4/b/(b*x^2+a)^(3/2)-4*a/b*(-x^2/b/(b*x^2+a)^(
3/2)-2/3*a/b^2/(b*x^2+a)^(3/2)))+C*(1/2*x^5/b/(b*x^2+a)^(3/2)-5/2*a/b*(-1/
3*x^3/b/(b*x^2+a)^(3/2)+1/b*(-x/b/(b*x^2+a)^(1/2)+1/b^(3/2)*ln(b^(1/2)*x+(
b*x^2+a)^(1/2)))))+D*(1/3*x^6/b/(b*x^2+a)^(3/2)-2*a/b*(x^4/b/(b*x^2+a)^(3/
2)-4*a/b*(-x^2/b/(b*x^2+a)^(3/2)-2/3*a/b^2/(b*x^2+a)^(3/2))))
```

## Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 455, normalized size of antiderivative = 2.45

$$\int \frac{x^4(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^{5/2}}\,dx = \left[-\frac{3\left((5\,Cab^2-2\,Ab^3)x^4+5\,Ca^3-2\,Aa^2b+2\,(5\,Ca^2b-2\,Aab^2)x^2\right)\sqrt{}\right.$$

input
```
integrate(x^4*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
[-1/12*(3*((5*C*a*b^2 - 2*A*b^3)*x^4 + 5*C*a^3 - 2*A*a^2*b + 2*(5*C*a^2*b
- 2*A*a*b^2)*x^2)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a)
- 2*(2*D*b^3*x^6 + 3*C*b^3*x^5 - 6*(2*D*a*b^2 - B*b^3)*x^4 - 32*D*a^3 + 16
*B*a^2*b + 4*(5*C*a*b^2 - 2*A*b^3)*x^3 - 24*(2*D*a^2*b - B*a*b^2)*x^2 + 3*
(5*C*a^2*b - 2*A*a*b^2)*x)*sqrt(b*x^2 + a))/(b^6*x^4 + 2*a*b^5*x^2 + a^2*b
^4), 1/6*(3*((5*C*a*b^2 - 2*A*b^3)*x^4 + 5*C*a^3 - 2*A*a^2*b + 2*(5*C*a^2*
b - 2*A*a*b^2)*x^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (2*D*b^3
*x^6 + 3*C*b^3*x^5 - 6*(2*D*a*b^2 - B*b^3)*x^4 - 32*D*a^3 + 16*B*a^2*b + 4
*(5*C*a*b^2 - 2*A*b^3)*x^3 - 24*(2*D*a^2*b - B*a*b^2)*x^2 + 3*(5*C*a^2*b -
 2*A*a*b^2)*x)*sqrt(b*x^2 + a))/(b^6*x^4 + 2*a*b^5*x^2 + a^2*b^4)]
```

## Sympy [A] (verification not implemented)

Time = 14.92 (sec) , antiderivative size = 1003, normalized size of antiderivative = 5.39

$$\int \frac{x^4(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x**4*(D*x**3+C*x**2+B*x+A)/(b*x**2+a)**(5/2),x)
```

output
```
A*(3*a**(39/2)*b**11*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(3*a**(39
/2)*b**(27/2)*sqrt(1 + b*x**2/a) + 3*a**(37/2)*b**(29/2)*x**2*sqrt(1 + b*x
**2/a)) + 3*a**(37/2)*b**12*x**2*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a
))/(3*a**(39/2)*b**(27/2)*sqrt(1 + b*x**2/a) + 3*a**(37/2)*b**(29/2)*x**2*
sqrt(1 + b*x**2/a)) - 3*a**19*b**(23/2)*x/(3*a**(39/2)*b**(27/2)*sqrt(1 +
b*x**2/a) + 3*a**(37/2)*b**(29/2)*x**2*sqrt(1 + b*x**2/a)) - 4*a**18*b**(2
5/2)*x**3/(3*a**(39/2)*b**(27/2)*sqrt(1 + b*x**2/a) + 3*a**(37/2)*b**(29/2
)*x**2*sqrt(1 + b*x**2/a))) + B*Piecewise((8*a**2/(3*a*b**3*sqrt(a + b*x**
2) + 3*b**4*x**2*sqrt(a + b*x**2)) + 12*a*b*x**2/(3*a*b**3*sqrt(a + b*x**2
) + 3*b**4*x**2*sqrt(a + b*x**2)) + 3*b**2*x**4/(3*a*b**3*sqrt(a + b*x**2)
 + 3*b**4*x**2*sqrt(a + b*x**2)), Ne(b, 0)), (x**6/(6*a**(5/2)), True)) +
C*(-15*a**(81/2)*b**22*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(6*a**(
79/2)*b**(51/2)*sqrt(1 + b*x**2/a) + 6*a**(77/2)*b**(53/2)*x**2*sqrt(1 + b
*x**2/a)) - 15*a**(79/2)*b**23*x**2*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqr
t(a))/(6*a**(79/2)*b**(51/2)*sqrt(1 + b*x**2/a) + 6*a**(77/2)*b**(53/2)*x*
*2*sqrt(1 + b*x**2/a)) + 15*a**40*b**(45/2)*x/(6*a**(79/2)*b**(51/2)*sqrt(
1 + b*x**2/a) + 6*a**(77/2)*b**(53/2)*x**2*sqrt(1 + b*x**2/a)) + 20*a**39*
b**(47/2)*x**3/(6*a**(79/2)*b**(51/2)*sqrt(1 + b*x**2/a) + 6*a**(77/2)*b**
(53/2)*x**2*sqrt(1 + b*x**2/a)) + 3*a**38*b**(49/2)*x**5/(6*a**(79/2)*b**(
51/2)*sqrt(1 + b*x**2/a) + 6*a**(77/2)*b**(53/2)*x**2*sqrt(1 + b*x**2/a...
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.56

$$\int \frac{x^4(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^{5/2}}\,dx = \frac{Dx^6}{3\,(bx^2+a)^{\frac{3}{2}}b} + \frac{Cx^5}{2\,(bx^2+a)^{\frac{3}{2}}b} - \frac{1}{3}\,Ax\left(\frac{3\,x^2}{(bx^2+a)^{\frac{3}{2}}b} + \frac{2\,a}{(bx^2+a)^{\frac{3}{2}}b^2}\right) + \frac{5\,Cax\left(\frac{3\,x^2}{(bx^2+a)^{\frac{3}{2}}b} + \frac{2\,a}{(bx^2+a)^{\frac{3}{2}}b^2}\right)}{6\,b} - \frac{2\,Dax^4}{(bx^2+a)^{\frac{3}{2}}b^2} + \frac{Bx^4}{(bx^2+a)^{\frac{3}{2}}b} - \frac{8\,Da^2x^2}{(bx^2+a)^{\frac{3}{2}}b^3} + \frac{4\,Bax^2}{(bx^2+a)^{\frac{3}{2}}b^2} + \frac{5\,Cax}{6\,\sqrt{bx^2+a}b^3} - \frac{Ax}{3\,\sqrt{bx^2+a}b^2} - \frac{5\,Ca\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{7}{2}}} + \frac{A\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{5}{2}}} - \frac{16\,Da^3}{3\,(bx^2+a)^{\frac{3}{2}}b^4} + \frac{8\,Ba^2}{3\,(bx^2+a)^{\frac{3}{2}}b^3}$$

input
```
integrate(x^4*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
1/3*D*x^6/((b*x^2 + a)^(3/2)*b) + 1/2*C*x^5/((b*x^2 + a)^(3/2)*b) - 1/3*A*
x*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2)) + 5/6*C*a*x*
(3*x^2/((b*x^2 + a)^(3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2))/b - 2*D*a*x^4/
((b*x^2 + a)^(3/2)*b^2) + B*x^4/((b*x^2 + a)^(3/2)*b) - 8*D*a^2*x^2/((b*x^
2 + a)^(3/2)*b^3) + 4*B*a*x^2/((b*x^2 + a)^(3/2)*b^2) + 5/6*C*a*x/(sqrt(b*
x^2 + a)*b^3) - 1/3*A*x/(sqrt(b*x^2 + a)*b^2) - 5/2*C*a*arcsinh(b*x/sqrt(a
*b))/b^(7/2) + A*arcsinh(b*x/sqrt(a*b))/b^(5/2) - 16/3*D*a^3/((b*x^2 + a)^
(3/2)*b^4) + 8/3*B*a^2/((b*x^2 + a)^(3/2)*b^3)
```

## Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.10

$$\int \frac{x^4(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^{5/2}}\,dx = \frac{\left(\left(\left(\left(\left(\left(\frac{2\,Dx}{b}+\frac{3\,C}{b}\right)x-\frac{6\,(2\,Da^2b^5-Bab^6)}{ab^7}\right)x+\frac{4\,(5\,Ca^2b^5-2\,Aab^6)}{ab^7}\right)x-\frac{24\,(2\,Da^3\cdots}{a\cdots}\cdots}{6\,(bx^2+a)^{\frac{3}{2}}} + \frac{(5\,Ca-2\,Ab)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2\,b^{\frac{7}{2}}}$$

input
```
integrate(x^4*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
1/6*((((((2*D*x/b + 3*C/b)*x - 6*(2*D*a^2*b^5 - B*a*b^6)/(a*b^7))*x + 4*(5
*C*a^2*b^5 - 2*A*a*b^6)/(a*b^7))*x - 24*(2*D*a^3*b^4 - B*a^2*b^5)/(a*b^7))
*x + 3*(5*C*a^3*b^4 - 2*A*a^2*b^5)/(a*b^7))*x - 16*(2*D*a^4*b^3 - B*a^3*b^
4)/(a*b^7))/(b*x^2 + a)^(3/2) + 1/2*(5*C*a - 2*A*b)*log(abs(-sqrt(b)*x + s
qrt(b*x^2 + a)))/b^(7/2)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^{5/2}}\,dx = \int \frac{x^4\,(A+B\,x+C\,x^2+x^3\,D)}{(b\,x^2+a)^{5/2}}\,dx$$

input
```
int((x^4*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^(5/2),x)
```

output
```
int((x^4*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^(5/2), x)
```

## Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 437, normalized size of antiderivative = 2.35

$$\int \frac{x^4(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^{5/2}}\,dx = \frac{-64\sqrt{b\,x^2+a}\,a^3d-12\sqrt{b\,x^2+a}\,a^2b^2x+32\sqrt{b\,x^2+a}\,a^2b^2+30\sqrt{b\,x^2+}}{}$$

input
```
int(x^4*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(5/2),x)
```

output
```
( - 64*sqrt(a + b*x**2)*a**3*d - 12*sqrt(a + b*x**2)*a**2*b**2*x + 32*sqrt
(a + b*x**2)*a**2*b**2 + 30*sqrt(a + b*x**2)*a**2*b*c*x - 96*sqrt(a + b*x*
*2)*a**2*b*d*x**2 - 16*sqrt(a + b*x**2)*a*b**3*x**3 + 48*sqrt(a + b*x**2)*
a*b**3*x**2 + 40*sqrt(a + b*x**2)*a*b**2*c*x**3 - 24*sqrt(a + b*x**2)*a*b*
*2*d*x**4 + 12*sqrt(a + b*x**2)*b**4*x**4 + 6*sqrt(a + b*x**2)*b**3*c*x**5
 + 4*sqrt(a + b*x**2)*b**3*d*x**6 + 12*sqrt(b)*log((sqrt(a + b*x**2) + sqr
t(b)*x)/sqrt(a))*a**3*b - 30*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sq
rt(a))*a**3*c + 24*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**
2*b**2*x**2 - 60*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2*
b*c*x**2 + 12*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a*b**3*x
**4 - 30*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a*b**2*c*x**4
 - 5*sqrt(b)*a**3*c - 10*sqrt(b)*a**2*b*c*x**2 - 5*sqrt(b)*a*b**2*c*x**4)/
(12*b**4*(a**2 + 2*a*b*x**2 + b**2*x**4))
```

# 3.110 $\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^{5/2}}\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 151

$$\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^{5/2}}\,dx = \frac{a(Ab-aC+(bB-aD)x)}{3b^3(a+bx^2)^{3/2}} - \frac{3(Ab-2aC)+(4bB-7aD)x}{3b^3\sqrt{a+bx^2}} + \frac{C\sqrt{a+bx^2}}{b^3} + \frac{Dx\sqrt{a+bx^2}}{2b^3} + \frac{(2bB-5aD)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{7/2}}$$

output
```
1/3*a*(A*b-C*a+(B*b-D*a)*x)/b^3/(b*x^2+a)^(3/2)-1/3*(3*A*b-6*C*a+(4*B*b-7*
D*a)*x)/b^3/(b*x^2+a)^(1/2)+C*(b*x^2+a)^(1/2)/b^3+1/2*D*x*(b*x^2+a)^(1/2)/
b^3+1/2*(2*B*b-5*D*a)*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(7/2)
```

## Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.81

$$\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^{5/2}}\,dx = \frac{a^2(16C+15Dx)+b^2x^2(-6A+x(-8B+3x(2C+Dx)))+2ab(-2A}{6b^3\left(a+bx^2\right)^{3/2}}$$
$$+\frac{(-2bB+5aD)\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{2b^{7/2}}$$

input
```
Integrate[(x^3*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^(5/2),x]
```

output
```
(a^2*(16*C + 15*D*x) + b^2*x^2*(-6*A + x*(-8*B + 3*x*(2*C + D*x))) + 2*a*b
*(-2*A + x*(-3*B + 2*x*(6*C + 5*D*x))))/(6*b^3*(a + b*x^2)^(3/2)) + ((-2*b
*B + 5*a*D)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(2*b^(7/2))
```

## Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.29, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {2335, 25, 2335, 27, 533, 455, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\left(A+Bx+Cx^2+Dx^3\right)}{\left(a+bx^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 2335}$$

$$-\frac{\int -\frac{x^2\left(3aDx^2-(Ab-4aC)x+3a\left(B-\frac{aD}{b}\right)\right)}{\left(bx^2+a\right)^{3/2}}dx}{3ab}-\frac{x^3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{3ab\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{x^2\left(3aDx^2-(Ab-4aC)x+3a\left(B-\frac{aD}{b}\right)\right)}{\left(bx^2+a\right)^{3/2}}dx}{3ab}-\frac{x^3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{3ab\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \ 2335$$

$$\frac{\frac{x^2(3x(bB-2aD)-4aC+Ab)}{b\sqrt{a+bx^2}}-\frac{\int\frac{ax(2(Ab-4aC)+3(2bB-5aD)x)}{\sqrt{bx^2+a}}dx}{ab}}{3ab}-\frac{x^3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{3ab\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{x^2(3x(bB-2aD)-4aC+Ab)}{b\sqrt{a+bx^2}}-\frac{\int\frac{x(2(Ab-4aC)+3(2bB-5aD)x)}{\sqrt{bx^2+a}}dx}{b}}{3ab}-\frac{x^3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{3ab\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \ 533$$

$$\frac{\frac{x^2(3x(bB-2aD)-4aC+Ab)}{b\sqrt{a+bx^2}}-\frac{\frac{3x\sqrt{a+bx^2}(2bB-5aD)}{2b}-\frac{\int\frac{3a(2bB-5aD)-4b(Ab-4aC)x}{\sqrt{bx^2+a}}dx}{2b}}{b}}{3ab}-\frac{x^3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{3ab\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \ 455$$

$$\frac{\frac{x^2(3x(bB-2aD)-4aC+Ab)}{b\sqrt{a+bx^2}}-\frac{\frac{3x\sqrt{a+bx^2}(2bB-5aD)}{2b}-\frac{3a(2bB-5aD)\int\frac{1}{\sqrt{bx^2+a}}dx-4\sqrt{a+bx^2}(Ab-4aC)}{2b}}{b}}{3ab}-\frac{x^3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{3ab\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \ 224$$

$$\frac{\frac{x^2(3x(bB-2aD)-4aC+Ab)}{b\sqrt{a+bx^2}}-\frac{\frac{3x\sqrt{a+bx^2}(2bB-5aD)}{2b}-\frac{3a(2bB-5aD)\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}-4\sqrt{a+bx^2}(Ab-4aC)}{2b}}{b}}{3ab}-\frac{x^3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{3ab\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \ 219$$

$$\frac{\frac{x^2(3x(bB-2aD)-4aC+Ab)}{b\sqrt{a+bx^2}}-\frac{\frac{3x\sqrt{a+bx^2}(2bB-5aD)}{2b}-\frac{\frac{3a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bB-5aD)}{\sqrt{b}}-4\sqrt{a+bx^2}(Ab-4aC)}{2b}}{b}}{3ab}-\frac{x^3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{3ab\left(a+bx^2\right)^{3/2}}$$

input
```
Int[(x^3*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^(5/2),x]
```

output
```
-1/3*(x^3*(a*(B - (a*D)/b) - (A*b - a*C)*x))/(a*b*(a + b*x^2)^(3/2)) + ((x
^2*(A*b - 4*a*C + 3*(b*B - 2*a*D)*x))/(b*Sqrt[a + b*x^2]) - ((3*(2*b*B - 5
*a*D)*x*Sqrt[a + b*x^2])/(2*b) - (-4*(A*b - 4*a*C)*Sqrt[a + b*x^2] + (3*a*
(2*b*B - 5*a*D)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/Sqrt[b])/(2*b))/b)/(
3*a*b)
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

rule 2335
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Simp[c/(2*a*b*(p + 1))   Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSu
m[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
 b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

## Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.58

| method | result |
|---|---|
| default | $A\left(-\frac{x^{2}}{b\left(b\,x^{2}+a\right)^{\frac{3}{2}}}-\frac{2a}{3b^{2}\left(b\,x^{2}+a\right)^{\frac{3}{2}}}\right)+B\left(-\frac{x^{3}}{3b\left(b\,x^{2}+a\right)^{\frac{3}{2}}}+\frac{-\frac{x}{b\sqrt{b\,x^{2}+a}}+\frac{\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{b^{\frac{3}{2}}}}{b}\right)+C\left(\frac{x^{4}}{b\left(b\,x^{2}+a\right)^{\frac{3}{2}}}-\right.$ |

input
```
int(x^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
A*(-x^2/b/(b*x^2+a)^(3/2)-2/3*a/b^2/(b*x^2+a)^(3/2))+B*(-1/3*x^3/b/(b*x^2+
a)^(3/2)+1/b*(-x/b/(b*x^2+a)^(1/2)+1/b^(3/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
))+C*(x^4/b/(b*x^2+a)^(3/2)-4*a/b*(-x^2/b/(b*x^2+a)^(3/2)-2/3*a/b^2/(b*x^2
+a)^(3/2)))+D*(1/2*x^5/b/(b*x^2+a)^(3/2)-5/2*a/b*(-1/3*x^3/b/(b*x^2+a)^(3/
2)+1/b*(-x/b/(b*x^2+a)^(1/2)+1/b^(3/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))))
```

## Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 417, normalized size of antiderivative = 2.76

$$\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^{5/2}}\,dx=\left[-\frac{3\left((5\,Dab^2-2\,Bb^3)x^4+5\,Da^3-2\,Ba^2b+2\,(5\,Da^2b-2\,Bab^2)x^2\right)\sqrt{}\right.$$

input
```
integrate(x^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
[-1/12*(3*((5*D*a*b^2 - 2*B*b^3)*x^4 + 5*D*a^3 - 2*B*a^2*b + 2*(5*D*a^2*b
- 2*B*a*b^2)*x^2)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a)
- 2*(3*D*b^3*x^5 + 6*C*b^3*x^4 + 16*C*a^2*b - 4*A*a*b^2 + 4*(5*D*a*b^2 - 2
*B*b^3)*x^3 + 6*(4*C*a*b^2 - A*b^3)*x^2 + 3*(5*D*a^2*b - 2*B*a*b^2)*x)*sqr
t(b*x^2 + a))/(b^6*x^4 + 2*a*b^5*x^2 + a^2*b^4), 1/6*(3*((5*D*a*b^2 - 2*B*
b^3)*x^4 + 5*D*a^3 - 2*B*a^2*b + 2*(5*D*a^2*b - 2*B*a*b^2)*x^2)*sqrt(-b)*a
rctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (3*D*b^3*x^5 + 6*C*b^3*x^4 + 16*C*a^2*
b - 4*A*a*b^2 + 4*(5*D*a*b^2 - 2*B*b^3)*x^3 + 6*(4*C*a*b^2 - A*b^3)*x^2 +
3*(5*D*a^2*b - 2*B*a*b^2)*x)*sqrt(b*x^2 + a))/(b^6*x^4 + 2*a*b^5*x^2 + a^2
*b^4)]
```

## Sympy [A] (verification not implemented)

Time = 14.29 (sec) , antiderivative size = 911, normalized size of antiderivative = 6.03

$$\int \frac{x^3(A + Bx + Cx^2 + Dx^3)}{(a + bx^2)^{5/2}}\, dx = \text{Too large to display}$$

input
```
integrate(x**3*(D*x**3+C*x**2+B*x+A)/(b*x**2+a)**(5/2),x)
```

output
```
A*Piecewise((-2*a/(3*a*b**2*sqrt(a + b*x**2) + 3*b**3*x**2*sqrt(a + b*x**2
)) - 3*b*x**2/(3*a*b**2*sqrt(a + b*x**2) + 3*b**3*x**2*sqrt(a + b*x**2)),
Ne(b, 0)), (x**4/(4*a**(5/2)), True)) + B*(3*a**(39/2)*b**11*sqrt(1 + b*x*
*2/a)*asinh(sqrt(b)*x/sqrt(a))/(3*a**(39/2)*b**(27/2)*sqrt(1 + b*x**2/a) +
 3*a**(37/2)*b**(29/2)*x**2*sqrt(1 + b*x**2/a)) + 3*a**(37/2)*b**12*x**2*s
qrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(3*a**(39/2)*b**(27/2)*sqrt(1 +
 b*x**2/a) + 3*a**(37/2)*b**(29/2)*x**2*sqrt(1 + b*x**2/a)) - 3*a**19*b**(
23/2)*x/(3*a**(39/2)*b**(27/2)*sqrt(1 + b*x**2/a) + 3*a**(37/2)*b**(29/2)*
x**2*sqrt(1 + b*x**2/a)) - 4*a**18*b**(25/2)*x**3/(3*a**(39/2)*b**(27/2)*s
qrt(1 + b*x**2/a) + 3*a**(37/2)*b**(29/2)*x**2*sqrt(1 + b*x**2/a))) + C*Pi
ecewise((8*a**2/(3*a*b**3*sqrt(a + b*x**2) + 3*b**4*x**2*sqrt(a + b*x**2))
 + 12*a*b*x**2/(3*a*b**3*sqrt(a + b*x**2) + 3*b**4*x**2*sqrt(a + b*x**2))
+ 3*b**2*x**4/(3*a*b**3*sqrt(a + b*x**2) + 3*b**4*x**2*sqrt(a + b*x**2)),
Ne(b, 0)), (x**6/(6*a**(5/2)), True)) + D*(-15*a**(81/2)*b**22*sqrt(1 + b*
x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(6*a**(79/2)*b**(51/2)*sqrt(1 + b*x**2/a)
 + 6*a**(77/2)*b**(53/2)*x**2*sqrt(1 + b*x**2/a)) - 15*a**(79/2)*b**23*x**
2*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(6*a**(79/2)*b**(51/2)*sqrt(
1 + b*x**2/a) + 6*a**(77/2)*b**(53/2)*x**2*sqrt(1 + b*x**2/a)) + 15*a**40*
b**(45/2)*x/(6*a**(79/2)*b**(51/2)*sqrt(1 + b*x**2/a) + 6*a**(77/2)*b**(53
/2)*x**2*sqrt(1 + b*x**2/a)) + 20*a**39*b**(47/2)*x**3/(6*a**(79/2)*b**...
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.64

$$\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^{5/2}}\,dx = \frac{Dx^5}{2\,(bx^2+a)^{\frac{3}{2}}b} - \frac{1}{3}\,Bx\left(\frac{3\,x^2}{(bx^2+a)^{\frac{3}{2}}b} + \frac{2\,a}{(bx^2+a)^{\frac{3}{2}}b^2}\right) + \frac{5\,Dax\left(\frac{3\,x^2}{(bx^2+a)^{\frac{3}{2}}b} + \frac{2\,a}{(bx^2+a)^{\frac{3}{2}}b^2}\right)}{6\,b} + \frac{Cx^4}{(bx^2+a)^{\frac{3}{2}}b} + \frac{4\,Cax^2}{(bx^2+a)^{\frac{3}{2}}b^2} - \frac{Ax^2}{(bx^2+a)^{\frac{3}{2}}b} + \frac{5\,Dax}{6\,\sqrt{bx^2+a}b^3} - \frac{Bx}{3\,\sqrt{bx^2+a}b^2} - \frac{5\,Da\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{7}{2}}} + \frac{B\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{5}{2}}} + \frac{8\,Ca^2}{3\,(bx^2+a)^{\frac{3}{2}}b^3} - \frac{2\,Aa}{3\,(bx^2+a)^{\frac{3}{2}}b^2}$$

input
```
integrate(x^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
1/2*D*x^5/((b*x^2 + a)^(3/2)*b) - 1/3*B*x*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2
*a/((b*x^2 + a)^(3/2)*b^2)) + 5/6*D*a*x*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2*a
/((b*x^2 + a)^(3/2)*b^2))/b + C*x^4/((b*x^2 + a)^(3/2)*b) + 4*C*a*x^2/((b*
x^2 + a)^(3/2)*b^2) - A*x^2/((b*x^2 + a)^(3/2)*b) + 5/6*D*a*x/(sqrt(b*x^2
+ a)*b^3) - 1/3*B*x/(sqrt(b*x^2 + a)*b^2) - 5/2*D*a*arcsinh(b*x/sqrt(a*b))
/b^(7/2) + B*arcsinh(b*x/sqrt(a*b))/b^(5/2) + 8/3*C*a^2/((b*x^2 + a)^(3/2)
*b^3) - 2/3*A*a/((b*x^2 + a)^(3/2)*b^2)
```

## Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.15

$$\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^{5/2}}\,dx = \frac{\left(\left(\left(3\left(\frac{Dx}{b}+\frac{2C}{b}\right)x+\frac{4\left(5Da^2b^4-2Bab^5\right)}{ab^6}\right)x+\frac{6\left(4Ca^2b^4-Aab^5\right)}{ab^6}\right)x+\frac{3\left(5Da^3b^3-\cdots\right)}{ab\cdots}\cdots}{6\left(bx^2+a\right)^{\frac{3}{2}}} + \frac{\left(5Da-2Bb\right)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2b^{\frac{7}{2}}}$$

input
```
integrate(x^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
1/6*((((3*(D*x/b + 2*C/b)*x + 4*(5*D*a^2*b^4 - 2*B*a*b^5)/(a*b^6))*x + 6*(
4*C*a^2*b^4 - A*a*b^5)/(a*b^6))*x + 3*(5*D*a^3*b^3 - 2*B*a^2*b^4)/(a*b^6))
*x + 4*(4*C*a^3*b^3 - A*a^2*b^4)/(a*b^6))/(b*x^2 + a)^(3/2) + 1/2*(5*D*a -
 2*B*b)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(7/2)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^{5/2}}\,dx = \int \frac{x^3\left(A+B\,x+C\,x^2+x^3\,D\right)}{{\left(b\,x^2+a\right)}^{5/2}}\,dx$$

input
```
int((x^3*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^(5/2),x)
```

output
```
int((x^3*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^(5/2), x)
```

## Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 400, normalized size of antiderivative = 2.65

$$\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^{5/2}}\,dx = \frac{-8\sqrt{b\,x^2+a}\,a^2b^2+32\sqrt{b\,x^2+a}\,a^2bc+30\sqrt{b\,x^2+a}\,a^2bdx-12\sqrt{b\,x^2}}{}$$

input
```
int(x^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(5/2),x)
```

output
```
( - 8*sqrt(a + b*x**2)*a**2*b**2 + 32*sqrt(a + b*x**2)*a**2*b*c + 30*sqrt(
a + b*x**2)*a**2*b*d*x - 12*sqrt(a + b*x**2)*a*b**3*x**2 - 12*sqrt(a + b*x
**2)*a*b**3*x + 48*sqrt(a + b*x**2)*a*b**2*c*x**2 + 40*sqrt(a + b*x**2)*a*
b**2*d*x**3 - 16*sqrt(a + b*x**2)*b**4*x**3 + 12*sqrt(a + b*x**2)*b**3*c*x
**4 + 6*sqrt(a + b*x**2)*b**3*d*x**5 - 30*sqrt(b)*log((sqrt(a + b*x**2) +
sqrt(b)*x)/sqrt(a))*a**3*d + 12*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)
/sqrt(a))*a**2*b**2 - 60*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a
))*a**2*b*d*x**2 + 24*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*
a*b**3*x**2 - 30*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a*b**
2*d*x**4 + 12*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*b**4*x**
4 - 5*sqrt(b)*a**3*d - 10*sqrt(b)*a**2*b*d*x**2 - 5*sqrt(b)*a*b**2*d*x**4)
/(12*b**4*(a**2 + 2*a*b*x**2 + b**2*x**4))
```

# 3.111 $\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^{5/2}}\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 129

$$\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^{5/2}}\,dx = \frac{a(bB-aD)-b(Ab-aC)x}{3b^3(a+bx^2)^{3/2}} - \frac{3a(bB-2aD)-b(Ab-4aC)x}{3ab^3\sqrt{a+bx^2}} + \frac{D\sqrt{a+bx^2}}{b^3} + \frac{C\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{5/2}}$$

output
```
1/3*(a*(B*b-D*a)-b*(A*b-C*a)*x)/b^3/(b*x^2+a)^(3/2)-1/3*(3*a*(B*b-2*D*a)-b
*(A*b-4*C*a)*x)/a/b^3/(b*x^2+a)^(1/2)+D*(b*x^2+a)^(1/2)/b^3+C*arctanh(b^(1
/2)*x/(b*x^2+a)^(1/2))/b^(5/2)
```

## Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.91

$$\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^{5/2}}\,dx = \frac{8a^3D+Ab^3x^3+ab^2x^2(-3B-4Cx+3Dx^2)+a^2b(-2B-3Cx+12I}{3ab^3(a+bx^2)^{3/2}}$$

input
```
Integrate[(x^2*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^(5/2),x]
```

output
```
(8*a^3*D + A*b^3*x^3 + a*b^2*x^2*(-3*B - 4*C*x + 3*D*x^2) + a^2*b*(-2*B -
3*C*x + 12*D*x^2) - 3*a*Sqrt[b]*C*(a + b*x^2)^(3/2)*Log[-(Sqrt[b]*x) + Sqr
t[a + b*x^2]])/(3*a*b^3*(a + b*x^2)^(3/2))
```

## Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.16, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {2335, 25, 2335, 25, 27, 455, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 (A+Bx+Cx^2+Dx^3)}{(a+bx^2)^{5/2}} \, dx$$

$$\Big\downarrow \text{2335}$$

$$-\frac{\int -\frac{x\left(3aDx^2+3aCx+2a\left(B-\frac{aD}{b}\right)\right)}{(bx^2+a)^{3/2}}dx}{3ab}-\frac{x^2\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{3ab\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{x\left(3aDx^2+3aCx+2a\left(B-\frac{aD}{b}\right)\right)}{(bx^2+a)^{3/2}}dx}{3ab}-\frac{x^2\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{3ab\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{2335}$$

$$\frac{-\frac{\int -\frac{a(3aC-2(bB-4aD)x)}{\sqrt{bx^2+a}}dx}{ab}-\frac{x(3aC-x(2bB-5aD))}{b\sqrt{a+bx^2}}}{3ab}-\frac{x^2\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{3ab\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{a(3aC-2(bB-4aD)x)}{\sqrt{bx^2+a}}dx}{ab}-\frac{x(3aC-x(2bB-5aD))}{b\sqrt{a+bx^2}}}{3ab}-\frac{x^2\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{3ab\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{3aC-2(bB-4aD)x}{\sqrt{bx^2+a}}dx}{b}-\frac{x(3aC-x(2bB-5aD))}{b\sqrt{a+bx^2}}}{3ab}-\frac{x^2\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{3ab\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{455}$$

$$\frac{\frac{3aC\int \frac{1}{\sqrt{bx^2+a}}dx-\frac{2\sqrt{a+bx^2}(bB-4aD)}{b}}{b}-\frac{x(3aC-x(2bB-5aD))}{b\sqrt{a+bx^2}}}{3ab}-\frac{x^2\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{3ab\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{3aC\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}-\frac{2\sqrt{a+bx^2}(bB-4aD)}{b}}{b}-\frac{x(3aC-x(2bB-5aD))}{b\sqrt{a+bx^2}}}{3ab}-\frac{x^2\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{3ab\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{\frac{3aC\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}-\frac{2\sqrt{a+bx^2}(bB-4aD)}{b}}{b}-\frac{x(3aC-x(2bB-5aD))}{b\sqrt{a+bx^2}}}{3ab}-\frac{x^2\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{3ab\left(a+bx^2\right)^{3/2}}$$

input
```
Int[(x^2*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^(5/2),x]
```

output
```
-1/3*(x^2*(a*(B - (a*D)/b) - (A*b - a*C)*x))/(a*b*(a + b*x^2)^(3/2)) + (-(
(x*(3*a*C - (2*b*B - 5*a*D)*x))/(b*Sqrt[a + b*x^2])) + ((-2*(b*B - 4*a*D)*
Sqrt[a + b*x^2])/b + (3*a*C*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/Sqrt[b])
/b)/(3*a*b)
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 2335
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Simp[c/(2*a*b*(p + 1))   Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSu
m[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
 b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

## Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.63

| method | result |
|---|---|
| default | $A\left(-\frac{x}{2b(b\,x^2+a)^{\frac{3}{2}}}+\frac{a\left(\frac{x}{3a\left(b\,x^2+a\right)^{\frac{3}{2}}}+\frac{2x}{3a^2\sqrt{b\,x^2+a}}\right)}{2b}\right)+B\left(-\frac{x^2}{b(b\,x^2+a)^{\frac{3}{2}}}-\frac{2a}{3b^2(b\,x^2+a)^{\frac{3}{2}}}\right)+C\left(-\frac{x^3}{3b(b\,x^2+a)}\right.$ |

input
```
int(x^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
A*(-1/2*x/b/(b*x^2+a)^(3/2)+1/2*a/b*(1/3*x/a/(b*x^2+a)^(3/2)+2/3*x/a^2/(b*
x^2+a)^(1/2)))+B*(-x^2/b/(b*x^2+a)^(3/2)-2/3*a/b^2/(b*x^2+a)^(3/2))+C*(-1/
3*x^3/b/(b*x^2+a)^(3/2)+1/b*(-x/b/(b*x^2+a)^(1/2)+1/b^(3/2)*ln(b^(1/2)*x+(
b*x^2+a)^(1/2))))+D*(x^4/b/(b*x^2+a)^(3/2)-4*a/b*(-x^2/b/(b*x^2+a)^(3/2)-2
/3*a/b^2/(b*x^2+a)^(3/2)))
```

## Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 334, normalized size of antiderivative = 2.59

$$\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^{5/2}}\,dx = \left[\frac{3\left(Cab^2x^4+2\,Ca^2bx^2+Ca^3\right)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)}{\quad}\right.$$

$$-\frac{3\left(Cab^2x^4+2\,Ca^2bx^2+Ca^3\right)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-\left(3\,Dab^2x^4-3\,Ca^2bx+8\,Da^3-2\,Ba^2b-\left(4\,Cab^2\right.\right.}{3\left(ab^5x^4+2\,a^2b^4x^2+a^3b^3\right)}$$

input
```
integrate(x^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
[1/6*(3*(C*a*b^2*x^4 + 2*C*a^2*b*x^2 + C*a^3)*sqrt(b)*log(-2*b*x^2 - 2*sqr
t(b*x^2 + a)*sqrt(b)*x - a) + 2*(3*D*a*b^2*x^4 - 3*C*a^2*b*x + 8*D*a^3 - 2
*B*a^2*b - (4*C*a*b^2 - A*b^3)*x^3 + 3*(4*D*a^2*b - B*a*b^2)*x^2)*sqrt(b*x
^2 + a))/(a*b^5*x^4 + 2*a^2*b^4*x^2 + a^3*b^3), -1/3*(3*(C*a*b^2*x^4 + 2*C
*a^2*b*x^2 + C*a^3)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (3*D*a*b
^2*x^4 - 3*C*a^2*b*x + 8*D*a^3 - 2*B*a^2*b - (4*C*a*b^2 - A*b^3)*x^3 + 3*(
4*D*a^2*b - B*a*b^2)*x^2)*sqrt(b*x^2 + a))/(a*b^5*x^4 + 2*a^2*b^4*x^2 + a^
3*b^3)]
```

## Sympy [A] (verification not implemented)

Time = 11.11 (sec) , antiderivative size = 588, normalized size of antiderivative = 4.56

$$\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^{5/2}}\,dx = \frac{Ax^3}{3a^{\frac{5}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{3}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}}$$
$$+B\left(\begin{cases}-\frac{2a}{3ab^2\sqrt{a+bx^2}+3b^3x^2\sqrt{a+bx^2}}-\frac{3bx^2}{3ab^2\sqrt{a+bx^2}+3b^3x^2\sqrt{a+bx^2}} & \text{for } b\neq 0\\ \frac{x^4}{4a^{\frac{5}{2}}} & \text{otherwise}\end{cases}\right)$$
$$+C\left(\frac{3a^{\frac{39}{2}}b^{11}\sqrt{1+\frac{bx^2}{a}}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3a^{\frac{39}{2}}b^{\frac{27}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{37}{2}}b^{\frac{29}{2}}x^2\sqrt{1+\frac{bx^2}{a}}}+\frac{3a^{\frac{37}{2}}b^{12}x^2\sqrt{1+\frac{bx^2}{a}}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3a^{\frac{39}{2}}b^{\frac{27}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{37}{2}}b^{\frac{29}{2}}x^2\sqrt{1+\frac{bx^2}{a}}}\right.$$
$$\left.-\frac{3a^{19}b^{\frac{23}{2}}x}{3a^{\frac{39}{2}}b^{\frac{27}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{37}{2}}b^{\frac{29}{2}}x^2\sqrt{1+\frac{bx^2}{a}}}-\frac{4a^{18}b^{\frac{25}{2}}x^3}{3a^{\frac{39}{2}}b^{\frac{27}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{37}{2}}b^{\frac{29}{2}}x^2\sqrt{1+\frac{bx^2}{a}}}\right)$$
$$+D\left(\begin{cases}\frac{8a^2}{3ab^3\sqrt{a+bx^2}+3b^4x^2\sqrt{a+bx^2}}+\frac{12abx^2}{3ab^3\sqrt{a+bx^2}+3b^4x^2\sqrt{a+bx^2}}+\frac{3b^2x^4}{3ab^3\sqrt{a+bx^2}+3b^4x^2\sqrt{a+bx^2}} & \text{for } b\neq 0\\ \frac{x^6}{6a^{\frac{5}{2}}} & \text{otherwise}\end{cases}\right)$$

input
```
integrate(x**2*(D*x**3+C*x**2+B*x+A)/(b*x**2+a)**(5/2),x)
```

output
```
A*x**3/(3*a**(5/2)*sqrt(1 + b*x**2/a) + 3*a**(3/2)*b*x**2*sqrt(1 + b*x**2/
a)) + B*Piecewise((-2*a/(3*a*b**2*sqrt(a + b*x**2) + 3*b**3*x**2*sqrt(a +
b*x**2)) - 3*b*x**2/(3*a*b**2*sqrt(a + b*x**2) + 3*b**3*x**2*sqrt(a + b*x*
*2)), Ne(b, 0)), (x**4/(4*a**(5/2)), True)) + C*(3*a**(39/2)*b**11*sqrt(1
+ b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(3*a**(39/2)*b**(27/2)*sqrt(1 + b*x**
2/a) + 3*a**(37/2)*b**(29/2)*x**2*sqrt(1 + b*x**2/a)) + 3*a**(37/2)*b**12*
x**2*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(3*a**(39/2)*b**(27/2)*sq
rt(1 + b*x**2/a) + 3*a**(37/2)*b**(29/2)*x**2*sqrt(1 + b*x**2/a)) - 3*a**1
9*b**(23/2)*x/(3*a**(39/2)*b**(27/2)*sqrt(1 + b*x**2/a) + 3*a**(37/2)*b**(
29/2)*x**2*sqrt(1 + b*x**2/a)) - 4*a**18*b**(25/2)*x**3/(3*a**(39/2)*b**(2
7/2)*sqrt(1 + b*x**2/a) + 3*a**(37/2)*b**(29/2)*x**2*sqrt(1 + b*x**2/a)))
+ D*Piecewise((8*a**2/(3*a*b**3*sqrt(a + b*x**2) + 3*b**4*x**2*sqrt(a + b*
x**2)) + 12*a*b*x**2/(3*a*b**3*sqrt(a + b*x**2) + 3*b**4*x**2*sqrt(a + b*x
**2)) + 3*b**2*x**4/(3*a*b**3*sqrt(a + b*x**2) + 3*b**4*x**2*sqrt(a + b*x*
*2)), Ne(b, 0)), (x**6/(6*a**(5/2)), True))
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.48

$$\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^{5/2}}\,dx = -\frac{1}{3}\,Cx\left(\frac{3\,x^2}{(bx^2+a)^{\frac{3}{2}}b}+\frac{2\,a}{(bx^2+a)^{\frac{3}{2}}b^2}\right) + \frac{Dx^4}{(bx^2+a)^{\frac{3}{2}}b}+\frac{4\,Dax^2}{(bx^2+a)^{\frac{3}{2}}b^2}-\frac{Bx^2}{(bx^2+a)^{\frac{3}{2}}b}-\frac{Cx}{3\,\sqrt{bx^2+a}b^2}-\frac{Ax}{3\,(bx^2+a)^{\frac{3}{2}}b} + \frac{Ax}{3\,\sqrt{bx^2+a}ab}+\frac{C\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{5}{2}}}+\frac{8\,Da^2}{3\,(bx^2+a)^{\frac{3}{2}}b^3}-\frac{2\,Ba}{3\,(bx^2+a)^{\frac{3}{2}}b^2}$$

input
```
integrate(x^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
-1/3*C*x*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2)) + D*x
^4/((b*x^2 + a)^(3/2)*b) + 4*D*a*x^2/((b*x^2 + a)^(3/2)*b^2) - B*x^2/((b*x
^2 + a)^(3/2)*b) - 1/3*C*x/(sqrt(b*x^2 + a)*b^2) - 1/3*A*x/((b*x^2 + a)^(3
/2)*b) + 1/3*A*x/(sqrt(b*x^2 + a)*a*b) + C*arcsinh(b*x/sqrt(a*b))/b^(5/2)
+ 8/3*D*a^2/((b*x^2 + a)^(3/2)*b^3) - 2/3*B*a/((b*x^2 + a)^(3/2)*b^2)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.04

$$\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^{5/2}}\,dx = \frac{\left(\left(\left(\frac{3\,Dx}{b}-\frac{4\,Cab^4-Ab^5}{ab^5}\right)x+\frac{3\,(4\,Da^2b^3-Bab^4)}{ab^5}\right)x-\frac{3\,Ca}{b^2}\right)x+\frac{2\,(4\,Da^3b^2-Ba^2b^3)}{ab^5}}{3\,(bx^2+a)^{\frac{3}{2}}} - \frac{C\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{b^{\frac{5}{2}}}$$

input
```
integrate(x^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
1/3*((((3*D*x/b - (4*C*a*b^4 - A*b^5)/(a*b^5))*x + 3*(4*D*a^2*b^3 - B*a*b^
4)/(a*b^5))*x - 3*C*a/b^2)*x + 2*(4*D*a^3*b^2 - B*a^2*b^3)/(a*b^5))/(b*x^2
 + a)^(3/2) - C*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^{5/2}}\,dx=\int \frac{x^2\,\left(A+B\,x+C\,x^2+x^3\,D\right)}{{\left(b\,x^2+a\right)}^{5/2}}\,dx$$

input
```
int((x^2*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^(5/2),x)
```

output
```
int((x^2*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^(5/2), x)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 262, normalized size of antiderivative = 2.03

$$\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^{5/2}}\,dx=\frac{8\sqrt{b\,x^2+a}\,a^2d-2\sqrt{b\,x^2+a}\,a\,b^2-3\sqrt{b\,x^2+a}\,abcx+12\sqrt{b\,x^2+a}\,abd$$

input
```
int(x^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(5/2),x)
```

output
```
(8*sqrt(a + b*x**2)*a**2*d - 2*sqrt(a + b*x**2)*a*b**2 - 3*sqrt(a + b*x**2
)*a*b*c*x + 12*sqrt(a + b*x**2)*a*b*d*x**2 + sqrt(a + b*x**2)*b**3*x**3 -
3*sqrt(a + b*x**2)*b**3*x**2 - 4*sqrt(a + b*x**2)*b**2*c*x**3 + 3*sqrt(a +
 b*x**2)*b**2*d*x**4 + 3*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a
))*a**2*c + 6*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a*b*c*x*
*2 + 3*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*b**2*c*x**4 + s
qrt(b)*a**2*b + 2*sqrt(b)*a*b**2*x**2 + sqrt(b)*b**3*x**4)/(3*b**3*(a**2 +
 2*a*b*x**2 + b**2*x**4))
```

# 3.112 $\int \frac{x(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^{5/2}}\,dx$



## Optimal result

Integrand size = 28, antiderivative size = 100

$$\int \frac{x(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^{5/2}}\,dx = -\frac{Ab-aC+(bB-aD)x}{3b^2(a+bx^2)^{3/2}} - \frac{3aC-(bB-4aD)x}{3ab^2\sqrt{a+bx^2}} + \frac{D\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{5/2}}$$

output
```
-1/3*(A*b-C*a+(B*b-D*a)*x)/b^2/(b*x^2+a)^(3/2)-1/3*(3*C*a-(B*b-4*D*a)*x)/a
/b^2/(b*x^2+a)^(1/2)+D*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(5/2)
```

## Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.91

$$\int \frac{x(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^{5/2}}\,dx = \frac{b^2Bx^3-a^2(2C+3Dx)-ab(A+3Cx^2+4Dx^3)}{3ab^2(a+bx^2)^{3/2}} - \frac{D\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{b^{5/2}}$$

input
```
Integrate[(x*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^(5/2),x]
```

output
```
(b^2*B*x^3 - a^2*(2*C + 3*D*x) - a*b*(A + 3*C*x^2 + 4*D*x^3))/(3*a*b^2*(a
+ b*x^2)^(3/2)) - (D*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/b^(5/2)
```

## Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.20, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {2335, 25, 2345, 27, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{2335}$$

$$-\frac{\int -\frac{3aDx^2+(Ab+2aC)x+\frac{a(bB-aD)}{b}}{(bx^2+a)^{3/2}}dx}{3ab}-\frac{x\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{3ab\,(a+bx^2)^{3/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{3aDx^2+(Ab+2aC)x+\frac{a(bB-aD)}{b}}{(bx^2+a)^{3/2}}dx}{3ab}-\frac{x\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{3ab\,(a+bx^2)^{3/2}}$$

$$\Big\downarrow \text{2345}$$

$$\frac{-\frac{\int -\frac{3a^2D}{b\sqrt{bx^2+a}}dx}{a}-\frac{-x(bB-4aD)+2aC+Ab}{b\sqrt{a+bx^2}}}{3ab}-\frac{x\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{3ab\,(a+bx^2)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{3aD\int \frac{1}{\sqrt{bx^2+a}}dx}{b}-\frac{-x(bB-4aD)+2aC+Ab}{b\sqrt{a+bx^2}}}{3ab}-\frac{x\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{3ab\,(a+bx^2)^{3/2}}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{3aD\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{b}-\frac{-x(bB-4aD)+2aC+Ab}{b\sqrt{a+bx^2}}}{3ab}-\frac{x\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{3ab\,(a+bx^2)^{3/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{3aD\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}-\frac{-x(bB-4aD)+2aC+Ab}{b\sqrt{a+bx^2}}}{3ab}-\frac{x\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{3ab\left(a+bx^2\right)^{3/2}}$$

input
```
Int[(x*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^(5/2),x]
```

output
```
-1/3*(x*(a*(B - (a*D)/b) - (A*b - a*C)*x))/(a*b*(a + b*x^2)^(3/2)) + (-((A
*b + 2*a*C - (b*B - 4*a*D)*x)/(b*Sqrt[a + b*x^2])) + (3*a*D*ArcTanh[(Sqrt[
b]*x)/Sqrt[a + b*x^2]])/b^(3/2))/(3*a*b)
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 2335
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Simp[c/(2*a*b*(p + 1))   Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSu
m[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
 b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

## Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.67

| method | result |
|---|---|
| default | $C\left(-\frac{x^{2}}{b\left(b\,x^{2}+a\right)^{\frac{3}{2}}}-\frac{2a}{3b^{2}\left(b\,x^{2}+a\right)^{\frac{3}{2}}}\right)+D\left(-\frac{x^{3}}{3b\left(b\,x^{2}+a\right)^{\frac{3}{2}}}+\frac{-\frac{x}{b\sqrt{b\,x^{2}+a}}+\frac{\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{b^{\frac{3}{2}}}}{b}\right)-\frac{A}{3b\left(b\,x^{2}+a\right)^{\frac{3}{2}}}+B$ |

input
```
int(x*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
C*(-x^2/b/(b*x^2+a)^(3/2)-2/3*a/b^2/(b*x^2+a)^(3/2))+D*(-1/3*x^3/b/(b*x^2+
a)^(3/2)+1/b*(-x/b/(b*x^2+a)^(1/2)+1/b^(3/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
))-1/3*A/b/(b*x^2+a)^(3/2)+B*(-1/2*x/b/(b*x^2+a)^(3/2)+1/2*a/b*(1/3*x/a/(b
*x^2+a)^(3/2)+2/3*x/a^2/(b*x^2+a)^(1/2)))
```

## Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 291, normalized size of antiderivative = 2.91

$$\int\frac{x(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^{5/2}}\,dx=\left[\frac{3\left(Dab^2x^4+2\,Da^2bx^2+Da^3\right)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)}{6\left(ab^5x^4+2\right.}\right.$$

$$-\frac{3\left(Dab^2x^4+2\,Da^2bx^2+Da^3\right)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)+\left(3\,Cab^2x^2+3\,Da^2bx+2\,Ca^2b+Aab^2+\left(4\,Dab^2\right.\right.}{3\left(ab^5x^4+2\,a^2b^4x^2+a^3b^3\right)}$$

input
```
integrate(x*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
[1/6*(3*(D*a*b^2*x^4 + 2*D*a^2*b*x^2 + D*a^3)*sqrt(b)*log(-2*b*x^2 - 2*sqr
t(b*x^2 + a)*sqrt(b)*x - a) - 2*(3*C*a*b^2*x^2 + 3*D*a^2*b*x + 2*C*a^2*b +
 A*a*b^2 + (4*D*a*b^2 - B*b^3)*x^3)*sqrt(b*x^2 + a))/(a*b^5*x^4 + 2*a^2*b^
4*x^2 + a^3*b^3), -1/3*(3*(D*a*b^2*x^4 + 2*D*a^2*b*x^2 + D*a^3)*sqrt(-b)*a
rctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (3*C*a*b^2*x^2 + 3*D*a^2*b*x + 2*C*a^2
*b + A*a*b^2 + (4*D*a*b^2 - B*b^3)*x^3)*sqrt(b*x^2 + a))/(a*b^5*x^4 + 2*a^
2*b^4*x^2 + a^3*b^3)]
```

## Sympy [A] (verification not implemented)

Time = 10.41 (sec) , antiderivative size = 496, normalized size of antiderivative = 4.96

$$
\begin{aligned}
\int \frac{x(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^{5/2}}\,dx = {} & A\left(\begin{cases} -\frac{1}{3ab\sqrt{a+bx^2}+3b^2x^2\sqrt{a+bx^2}} & \text{for } b\neq 0 \\ \frac{x^2}{2a^{\frac{5}{2}}} & \text{otherwise} \end{cases}\right) \\
& + \frac{Bx^3}{3a^{\frac{5}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{3}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}} \\
& + C\left(\begin{cases} -\frac{2a}{3ab^2\sqrt{a+bx^2}+3b^3x^2\sqrt{a+bx^2}} - \frac{3bx^2}{3ab^2\sqrt{a+bx^2}+3b^3x^2\sqrt{a+bx^2}} & \text{for } b\neq 0 \\ \frac{x^4}{4a^{\frac{5}{2}}} & \text{otherwise} \end{cases}\right) \\
& + D\left(\frac{3a^{\frac{39}{2}}b^{11}\sqrt{1+\frac{bx^2}{a}}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3a^{\frac{39}{2}}b^{\frac{27}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{37}{2}}b^{\frac{29}{2}}x^2\sqrt{1+\frac{bx^2}{a}}}\right. \\
& + \frac{3a^{\frac{37}{2}}b^{12}x^2\sqrt{1+\frac{bx^2}{a}}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3a^{\frac{39}{2}}b^{\frac{27}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{37}{2}}b^{\frac{29}{2}}x^2\sqrt{1+\frac{bx^2}{a}}} \\
& - \frac{3a^{19}b^{\frac{23}{2}}x}{3a^{\frac{39}{2}}b^{\frac{27}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{37}{2}}b^{\frac{29}{2}}x^2\sqrt{1+\frac{bx^2}{a}}} \\
& \left.- \frac{4a^{18}b^{\frac{25}{2}}x^3}{3a^{\frac{39}{2}}b^{\frac{27}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{37}{2}}b^{\frac{29}{2}}x^2\sqrt{1+\frac{bx^2}{a}}}\right)
\end{aligned}
$$

input
```
integrate(x*(D*x**3+C*x**2+B*x+A)/(b*x**2+a)**(5/2),x)
```

output
```
A*Piecewise((-1/(3*a*b*sqrt(a + b*x**2) + 3*b**2*x**2*sqrt(a + b*x**2)), N
e(b, 0)), (x**2/(2*a**(5/2)), True)) + B*x**3/(3*a**(5/2)*sqrt(1 + b*x**2/
a) + 3*a**(3/2)*b*x**2*sqrt(1 + b*x**2/a)) + C*Piecewise((-2*a/(3*a*b**2*s
qrt(a + b*x**2) + 3*b**3*x**2*sqrt(a + b*x**2)) - 3*b*x**2/(3*a*b**2*sqrt(
a + b*x**2) + 3*b**3*x**2*sqrt(a + b*x**2)), Ne(b, 0)), (x**4/(4*a**(5/2))
, True)) + D*(3*a**(39/2)*b**11*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a)
)/(3*a**(39/2)*b**(27/2)*sqrt(1 + b*x**2/a) + 3*a**(37/2)*b**(29/2)*x**2*s
qrt(1 + b*x**2/a)) + 3*a**(37/2)*b**12*x**2*sqrt(1 + b*x**2/a)*asinh(sqrt(
b)*x/sqrt(a))/(3*a**(39/2)*b**(27/2)*sqrt(1 + b*x**2/a) + 3*a**(37/2)*b**(
29/2)*x**2*sqrt(1 + b*x**2/a)) - 3*a**19*b**(23/2)*x/(3*a**(39/2)*b**(27/2
)*sqrt(1 + b*x**2/a) + 3*a**(37/2)*b**(29/2)*x**2*sqrt(1 + b*x**2/a)) - 4*
a**18*b**(25/2)*x**3/(3*a**(39/2)*b**(27/2)*sqrt(1 + b*x**2/a) + 3*a**(37/
2)*b**(29/2)*x**2*sqrt(1 + b*x**2/a)))
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.52

$$\int \frac{x(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^{5/2}}\,dx = -\frac{1}{3}\,Dx\left(\frac{3\,x^2}{(bx^2+a)^{\frac{3}{2}}b}+\frac{2\,a}{(bx^2+a)^{\frac{3}{2}}b^2}\right) - \frac{Cx^2}{(bx^2+a)^{\frac{3}{2}}b} - \frac{Dx}{3\,\sqrt{bx^2+a}b^2} - \frac{Bx}{3\,(bx^2+a)^{\frac{3}{2}}b} + \frac{Bx}{3\,\sqrt{bx^2+a}ab} + \frac{D\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{5}{2}}} - \frac{2\,Ca}{3\,(bx^2+a)^{\frac{3}{2}}b^2} - \frac{A}{3\,(bx^2+a)^{\frac{3}{2}}b}$$

input
```
integrate(x*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
-1/3*D*x*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2)) - C*x
^2/((b*x^2 + a)^(3/2)*b) - 1/3*D*x/(sqrt(b*x^2 + a)*b^2) - 1/3*B*x/((b*x^2
 + a)^(3/2)*b) + 1/3*B*x/(sqrt(b*x^2 + a)*a*b) + D*arcsinh(b*x/sqrt(a*b))/
b^(5/2) - 2/3*C*a/((b*x^2 + a)^(3/2)*b^2) - 1/3*A/((b*x^2 + a)^(3/2)*b)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.01

$$\int \frac{x(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^{5/2}}\,dx = -\frac{\left(x\left(\frac{3\,C}{b}+\frac{(4\,Dab^3-Bb^4)x}{ab^4}\right)+\frac{3\,Da}{b^2}\right)x+\frac{2\,Ca^2b^2+Aab^3}{ab^4}}{3\,(bx^2+a)^{\frac{3}{2}}}-\frac{D\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{b^{\frac{5}{2}}}$$

input
```
integrate(x*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
-1/3*((x*(3*C/b + (4*D*a*b^3 - B*b^4)*x/(a*b^4)) + 3*D*a/b^2)*x + (2*C*a^2
*b^2 + A*a*b^3)/(a*b^4))/(b*x^2 + a)^(3/2) - D*log(abs(-sqrt(b)*x + sqrt(b
*x^2 + a)))/b^(5/2)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^{5/2}}\,dx = \int \frac{x\,(A+B\,x+C\,x^2+x^3\,D)}{(b\,x^2+a)^{5/2}}\,dx$$

input
```
int((x*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^(5/2),x)
```

output
```
int((x*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^(5/2), x)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 245, normalized size of antiderivative = 2.45

$$\int \frac{x(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^{5/2}}\,dx = \frac{-\sqrt{b\,x^2+a}\,a^2b^2-2\sqrt{b\,x^2+a}\,a^2bc-3\sqrt{b\,x^2+a}\,a^2bdx-3\sqrt{b\,x^2+a}\,a\,b}{}$$

input
```
int(x*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(5/2),x)
```

output
```
( - sqrt(a + b*x**2)*a**2*b**2 - 2*sqrt(a + b*x**2)*a**2*b*c - 3*sqrt(a +
b*x**2)*a**2*b*d*x - 3*sqrt(a + b*x**2)*a*b**2*c*x**2 - 4*sqrt(a + b*x**2)
*a*b**2*d*x**3 + sqrt(a + b*x**2)*b**4*x**3 + 3*sqrt(b)*log((sqrt(a + b*x*
*2) + sqrt(b)*x)/sqrt(a))*a**3*d + 6*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(
b)*x)/sqrt(a))*a**2*b*d*x**2 + 3*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x
)/sqrt(a))*a*b**2*d*x**4 + sqrt(b)*a**2*b**2 + 2*sqrt(b)*a*b**3*x**2 + sqr
t(b)*b**4*x**4)/(3*a*b**3*(a**2 + 2*a*b*x**2 + b**2*x**4))
```

# 3.113 $\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{5/2}}\,dx$



## Optimal result

Integrand size = 27, antiderivative size = 85

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{5/2}}\,dx = -\frac{a(bB-aD)-b(Ab-aC)x}{3ab^2(a+bx^2)^{3/2}} - \frac{3a^2D-b(2Ab+aC)x}{3a^2b^2\sqrt{a+bx^2}}$$

output
```
-1/3*(a*(B*b-D*a)-b*(A*b-C*a)*x)/a/b^2/(b*x^2+a)^(3/2)-1/3*(3*a^2*D-b*(2*A
*b+C*a)*x)/a^2/b^2/(b*x^2+a)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.78

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{5/2}}\,dx = \frac{-2a^3D+2Ab^3x^3+ab^2x(3A+Cx^2)-a^2b(B+3Dx^2)}{3a^2b^2(a+bx^2)^{3/2}}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/(a + b*x^2)^(5/2),x]
```

output
```
(-2*a^3*D + 2*A*b^3*x^3 + a*b^2*x*(3*A + C*x^2) - a^2*b*(B + 3*D*x^2))/(3*
a^2*b^2*(a + b*x^2)^(3/2))
```

## Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2345, 25, 27, 453}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{5/2}}\,dx$$

$$\Big\downarrow \; 2345$$

$$-\frac{\int -\frac{b\left(2A+\frac{aC}{b}\right)+3aDx}{b(bx^2+a)^{3/2}}dx}{3a}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{3ab\,(a+bx^2)^{3/2}}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{2Ab+aC+3aDx}{b(bx^2+a)^{3/2}}dx}{3a}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{3ab\,(a+bx^2)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{2Ab+aC+3aDx}{(bx^2+a)^{3/2}}dx}{3ab}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{3ab\,(a+bx^2)^{3/2}}$$

$$\Big\downarrow \; 453$$

$$-\frac{3a^2D-bx(aC+2Ab)}{3a^2b^2\sqrt{a+bx^2}}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{3ab\,(a+bx^2)^{3/2}}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/(a + b*x^2)^(5/2),x]
```

output
```
-1/3*(a*(B - (a*D)/b) - (A*b - a*C)*x)/(a*b*(a + b*x^2)^(3/2)) - (3*a^2*D
- b*(2*A*b + a*C)*x)/(3*a^2*b^2*Sqrt[a + b*x^2])
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 453
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[-(a*
d - b*c*x)/(a*b*Sqrt[a + b*x^2]), x] /; FreeQ[{a, b, c, d}, x]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

## Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.80

| method | result |
|---|---|
| gosper | $\frac{2A\,b^{3}x^{3}+C a\,b^{2}x^{3}-3D a^{2}b\,x^{2}+3A x a\,b^{2}-a^{2}b B-2a^{3}D}{3\left(b\,x^{2}+a\right)^{\frac{3}{2}}a^{2}b^{2}}$ |
| trager | $\frac{2A\,b^{3}x^{3}+C a\,b^{2}x^{3}-3D a^{2}b\,x^{2}+3A x a\,b^{2}-a^{2}b B-2a^{3}D}{3\left(b\,x^{2}+a\right)^{\frac{3}{2}}a^{2}b^{2}}$ |
| orering | $\frac{2A\,b^{3}x^{3}+C a\,b^{2}x^{3}-3D a^{2}b\,x^{2}+3A x a\,b^{2}-a^{2}b B-2a^{3}D}{3\left(b\,x^{2}+a\right)^{\frac{3}{2}}a^{2}b^{2}}$ |
| default | $A\left(\frac{x}{3a\left(b\,x^{2}+a\right)^{\frac{3}{2}}}+\frac{2x}{3a^{2}\sqrt{b\,x^{2}+a}}\right)-\frac{B}{3b\left(b\,x^{2}+a\right)^{\frac{3}{2}}}+C\left(-\frac{x}{2b\left(b\,x^{2}+a\right)^{\frac{3}{2}}}+\frac{a\left(\frac{x}{3a\left(b\,x^{2}+a\right)^{\frac{3}{2}}}+\frac{2x}{3a^{2}\sqrt{b\,x^{2}+a}}\right)}{2b}\right)+D\Bigg($ |

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(2*A*b^3*x^3+C*a*b^2*x^3-3*D*a^2*b*x^2+3*A*a*b^2*x-B*a^2*b-2*D*a^3)/(b
*x^2+a)^(3/2)/a^2/b^2
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.07

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{5/2}}\,dx = -\frac{(3\,Da^2bx^2 - 3\,Aab^2x + 2\,Da^3 + Ba^2b - (Cab^2 + 2\,Ab^3)x^3)\sqrt{bx^2+a}}{3\,(a^2b^4x^4 + 2\,a^3b^3x^2 + a^4b^2)}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
-1/3*(3*D*a^2*b*x^2 - 3*A*a*b^2*x + 2*D*a^3 + B*a^2*b - (C*a*b^2 + 2*A*b^3
)*x^3)*sqrt(b*x^2 + a)/(a^2*b^4*x^4 + 2*a^3*b^3*x^2 + a^4*b^2)
```

## Sympy [A] (verification not implemented)

Time = 8.96 (sec) , antiderivative size = 289, normalized size of antiderivative = 3.40

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{5/2}}\,dx = A\left(\frac{3ax}{3a^{\frac{7}{2}}\sqrt{1+\frac{bx^2}{a}} + 3a^{\frac{5}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}} + \frac{2bx^3}{3a^{\frac{7}{2}}\sqrt{1+\frac{bx^2}{a}} + 3a^{\frac{5}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}}\right) + B\left(\begin{cases} -\frac{1}{3ab\sqrt{a+bx^2}+3b^2x^2\sqrt{a+bx^2}} & \text{for } b \neq 0 \\ \frac{x^2}{2a^{\frac{5}{2}}} & \text{otherwise} \end{cases}\right) + \frac{Cx^3}{3a^{\frac{5}{2}}\sqrt{1+\frac{bx^2}{a}} + 3a^{\frac{3}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}} + D\left(\begin{cases} -\frac{2a}{3ab^2\sqrt{a+bx^2}+3b^3x^2\sqrt{a+bx^2}} - \frac{3bx^2}{3ab^2\sqrt{a+bx^2}+3b^3x^2\sqrt{a+bx^2}} & \text{for } b \neq 0 \\ \frac{x^4}{4a^{\frac{5}{2}}} & \text{otherwise} \end{cases}\right)$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(b*x**2+a)**(5/2),x)
```

output
```
A*(3*a*x/(3*a**(7/2)*sqrt(1 + b*x**2/a) + 3*a**(5/2)*b*x**2*sqrt(1 + b*x**
2/a)) + 2*b*x**3/(3*a**(7/2)*sqrt(1 + b*x**2/a) + 3*a**(5/2)*b*x**2*sqrt(1
 + b*x**2/a))) + B*Piecewise((-1/(3*a*b*sqrt(a + b*x**2) + 3*b**2*x**2*sqr
t(a + b*x**2)), Ne(b, 0)), (x**2/(2*a**(5/2)), True)) + C*x**3/(3*a**(5/2)
*sqrt(1 + b*x**2/a) + 3*a**(3/2)*b*x**2*sqrt(1 + b*x**2/a)) + D*Piecewise(
(-2*a/(3*a*b**2*sqrt(a + b*x**2) + 3*b**3*x**2*sqrt(a + b*x**2)) - 3*b*x**
2/(3*a*b**2*sqrt(a + b*x**2) + 3*b**3*x**2*sqrt(a + b*x**2)), Ne(b, 0)), (
x**4/(4*a**(5/2)), True))
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.38

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{5/2}}\,dx = -\frac{Dx^2}{(bx^2+a)^{\frac{3}{2}}b} + \frac{2\,Ax}{3\,\sqrt{bx^2+a}a^2} + \frac{Ax}{3\,(bx^2+a)^{\frac{3}{2}}a} - \frac{Cx}{3\,(bx^2+a)^{\frac{3}{2}}b} + \frac{Cx}{3\,\sqrt{bx^2+a}ab} - \frac{2\,Da}{3\,(bx^2+a)^{\frac{3}{2}}b^2} - \frac{B}{3\,(bx^2+a)^{\frac{3}{2}}b}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
-D*x^2/((b*x^2 + a)^(3/2)*b) + 2/3*A*x/(sqrt(b*x^2 + a)*a^2) + 1/3*A*x/((b
*x^2 + a)^(3/2)*a) - 1/3*C*x/((b*x^2 + a)^(3/2)*b) + 1/3*C*x/(sqrt(b*x^2 +
 a)*a*b) - 2/3*D*a/((b*x^2 + a)^(3/2)*b^2) - 1/3*B/((b*x^2 + a)^(3/2)*b)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.85

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{5/2}}\,dx = -\frac{\left(x\left(\frac{3\,D}{b} - \frac{(Cab^2+2\,Ab^3)x}{a^2b^2}\right) - \frac{3\,A}{a}\right)x + \frac{2\,Da^3+Ba^2b}{a^2b^2}}{3\,(bx^2+a)^{\frac{3}{2}}}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
-1/3*((x*(3*D/b - (C*a*b^2 + 2*A*b^3)*x/(a^2*b^2)) - 3*A/a)*x + (2*D*a^3 +
 B*a^2*b)/(a^2*b^2))/(b*x^2 + a)^(3/2)
```

## Mupad [B] (verification not implemented)

Time = 2.71 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.05

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{5/2}}\,dx = \frac{2\,A\,x\,(b\,x^2+a)+A\,a\,x}{3\,a^2\,(b\,x^2+a)^{3/2}} - \frac{B}{3\,b\,(b\,x^2+a)^{3/2}} - \frac{(3\,b\,x^2+2\,a)\,D}{3\,b^2\,(b\,x^2+a)^{3/2}} + \frac{C\,x^3}{3\,a\,(b\,x^2+a)^{3/2}}$$

input
```
int((A + B*x + C*x^2 + x^3*D)/(a + b*x^2)^(5/2),x)
```

output
```
(2*A*x*(a + b*x^2) + A*a*x)/(3*a^2*(a + b*x^2)^(3/2)) - B/(3*b*(a + b*x^2)
^(3/2)) - ((2*a + 3*b*x^2)*D)/(3*b^2*(a + b*x^2)^(3/2)) + (C*x^3)/(3*a*(a
+ b*x^2)^(3/2))
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 176, normalized size of antiderivative = 2.07

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^{5/2}}\,dx = \frac{-2\sqrt{b\,x^2+a}\,a^2d+3\sqrt{b\,x^2+a}\,a\,b^2x-\sqrt{b\,x^2+a}\,a\,b^2-3\sqrt{b\,x^2+a}\,abd\,x^2 -}{}$$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(5/2),x)
```

output
```
( - 2*sqrt(a + b*x**2)*a**2*d + 3*sqrt(a + b*x**2)*a*b**2*x - sqrt(a + b*x
**2)*a*b**2 - 3*sqrt(a + b*x**2)*a*b*d*x**2 + 2*sqrt(a + b*x**2)*b**3*x**3
 + sqrt(a + b*x**2)*b**2*c*x**3 - 2*sqrt(b)*a**2*b + sqrt(b)*a**2*c - 4*sq
rt(b)*a*b**2*x**2 + 2*sqrt(b)*a*b*c*x**2 - 2*sqrt(b)*b**3*x**4 + sqrt(b)*b
**2*c*x**4)/(3*a*b**2*(a**2 + 2*a*b*x**2 + b**2*x**4))
```

# 3.114 $\int \frac{A+Bx+Cx^2+Dx^3}{x\left(a+bx^2\right)^{5/2}}\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 102

$$\int \frac{A+Bx+Cx^2+Dx^3}{x\left(a+bx^2\right)^{5/2}}\,dx=\frac{Ab-aC+(bB-aD)x}{3ab\left(a+bx^2\right)^{3/2}}+\frac{3Ab+(2bB+aD)x}{3a^2b\sqrt{a+bx^2}}-\frac{A\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{5/2}}$$

output

```
1/3*(A*b-C*a+(B*b-D*a)*x)/a/b/(b*x^2+a)^(3/2)+1/3*(3*A*b+(2*B*b+D*a)*x)/a^
2/b/(b*x^2+a)^(1/2)-A*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(5/2)
```

## Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.94

$$\int \frac{A+Bx+Cx^2+Dx^3}{x\left(a+bx^2\right)^{5/2}}\,dx=\frac{-a^2C+b^2x^2(3A+2Bx)+ab(4A+3Bx+Dx^3)}{3a^2b\left(a+bx^2\right)^{3/2}}+\frac{2A\operatorname{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{5/2}}$$

input

```
Integrate[(A + B*x + C*x^2 + D*x^3)/(x*(a + b*x^2)^(5/2)),x]
```

output
```
(-(a^2*C) + b^2*x^2*(3*A + 2*B*x) + a*b*(4*A + 3*B*x + D*x^3))/(3*a^2*b*(a
 + b*x^2)^(3/2)) + (2*A*ArcTanh[(Sqrt[b]*x - Sqrt[a + b*x^2])/Sqrt[a]])/a^
(5/2)
```

## Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2336, 25, 27, 532, 27, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{x\left(a+bx^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{2336}$$

$$\frac{x(bB-aD)-aC+Ab}{3ab\left(a+bx^2\right)^{3/2}}-\frac{\int -\frac{3Ab+(2bB+aD)x}{bx(bx^2+a)^{3/2}}dx}{3a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{3Ab+(2bB+aD)x}{bx(bx^2+a)^{3/2}}dx}{3a}+\frac{x(bB-aD)-aC+Ab}{3ab\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{3Ab+(2bB+aD)x}{x(bx^2+a)^{3/2}}dx}{3ab}+\frac{x(bB-aD)-aC+Ab}{3ab\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{532}$$

$$\frac{\frac{x(aD+2bB)+3Ab}{a\sqrt{a+bx^2}}-\frac{\int -\frac{3Ab}{x\sqrt{bx^2+a}}dx}{a}}{3ab}+\frac{x(bB-aD)-aC+Ab}{3ab\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{3Ab\int \frac{1}{x\sqrt{bx^2+a}}dx}{a}+\frac{x(aD+2bB)+3Ab}{a\sqrt{a+bx^2}}}{3ab}+\frac{x(bB-aD)-aC+Ab}{3ab\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{ 243}$$

$$\frac{\frac{3Ab\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2}{2a}+\frac{x(aD+2bB)+3Ab}{a\sqrt{a+bx^2}}}{3ab}+\frac{x(bB-aD)-aC+Ab}{3ab\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{\frac{3A\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{a}+\frac{x(aD+2bB)+3Ab}{a\sqrt{a+bx^2}}}{3ab}+\frac{x(bB-aD)-aC+Ab}{3ab\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{\frac{x(aD+2bB)+3Ab}{a\sqrt{a+bx^2}}-\frac{3Ab\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}}{3ab}+\frac{x(bB-aD)-aC+Ab}{3ab\left(a+bx^2\right)^{3/2}}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/(x*(a + b*x^2)^(5/2)),x]
```

output
```
(A*b - a*C + (b*B - a*D)*x)/(3*a*b*(a + b*x^2)^(3/2)) + ((3*A*b + (2*b*B +
 a*D)*x)/(a*Sqrt[a + b*x^2]) - (3*A*b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/a^
(3/2))/(3*a*b)
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 532
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[x^m
*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*(Qx/x^m) + e*((2*p + 3)/x^m),
x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && ILtQ[m, 0] && LtQ[p,
 -1] && IntegerQ[2*p]
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

## Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.65

| method | result |
|---|---|
| default | $B\left(\frac{x}{3a(b\,x^2+a)^{\frac{3}{2}}}+\frac{2x}{3a^2\sqrt{b\,x^2+a}}\right)+A\left(\frac{1}{3a(b\,x^2+a)^{\frac{3}{2}}}+\frac{\frac{1}{a\sqrt{b\,x^2+a}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)}{a^{\frac{3}{2}}}}{a}\right)-\frac{C}{3b(b\,x^2+a)^{\frac{3}{2}}}+D\Bigg($ |

input
```
int((D*x^3+C*x^2+B*x+A)/x/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
B*(1/3*x/a/(b*x^2+a)^(3/2)+2/3*x/a^2/(b*x^2+a)^(1/2))+A*(1/3/a/(b*x^2+a)^(
3/2)+1/a*(1/a/(b*x^2+a)^(1/2)-1/a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))
/x)))-1/3*C/b/(b*x^2+a)^(3/2)+D*(-1/2*x/b/(b*x^2+a)^(3/2)+1/2*a/b*(1/3*x/a
/(b*x^2+a)^(3/2)+2/3*x/a^2/(b*x^2+a)^(1/2)))
```

## Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 296, normalized size of antiderivative = 2.90

$$\int \frac{A+Bx+Cx^2+Dx^3}{x\left(a+bx^2\right)^{5/2}}\,dx = \left[\frac{3\left(Ab^3x^4+2\,Aab^2x^2+Aa^2b\right)\sqrt{a}\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\left(3\,Aab^2x^2\right.}{6\left(a^3b^3x^4+2\,a^4b^2x^2\right.}\right.$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/x/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
[1/6*(3*(A*b^3*x^4 + 2*A*a*b^2*x^2 + A*a^2*b)*sqrt(a)*log(-(b*x^2 - 2*sqrt
(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(3*A*a*b^2*x^2 + 3*B*a^2*b*x - C*a^3 +
 4*A*a^2*b + (D*a^2*b + 2*B*a*b^2)*x^3)*sqrt(b*x^2 + a))/(a^3*b^3*x^4 + 2*
a^4*b^2*x^2 + a^5*b), 1/3*(3*(A*b^3*x^4 + 2*A*a*b^2*x^2 + A*a^2*b)*sqrt(-a
)*arctan(sqrt(b*x^2 + a)*sqrt(-a)/a) + (3*A*a*b^2*x^2 + 3*B*a^2*b*x - C*a^
3 + 4*A*a^2*b + (D*a^2*b + 2*B*a*b^2)*x^3)*sqrt(b*x^2 + a))/(a^3*b^3*x^4 +
 2*a^4*b^2*x^2 + a^5*b)]
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 741 vs. 2(87) = 174.

Time = 18.48 (sec) , antiderivative size = 937, normalized size of antiderivative = 9.19

$$\int \frac{A+Bx+Cx^2+Dx^3}{x\left(a+bx^2\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/x/(b*x**2+a)**(5/2),x)
```

output
```
A*(8*a**7*sqrt(1 + b*x**2/a)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(1
5/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) + 3*a**7*log(b*x**2/a)/(6*a**(19/2
) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6)
- 6*a**7*log(sqrt(1 + b*x**2/a) + 1)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 +
18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) + 14*a**6*b*x**2*sqrt(1 +
b*x**2/a)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*
a**(13/2)*b**3*x**6) + 9*a**6*b*x**2*log(b*x**2/a)/(6*a**(19/2) + 18*a**(1
7/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) - 18*a**6*b*
x**2*log(sqrt(1 + b*x**2/a) + 1)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a
**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) + 6*a**5*b**2*x**4*sqrt(1 + b*
x**2/a)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a*
*(13/2)*b**3*x**6) + 9*a**5*b**2*x**4*log(b*x**2/a)/(6*a**(19/2) + 18*a**(
17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) - 18*a**5*b
**2*x**4*log(sqrt(1 + b*x**2/a) + 1)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 +
18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) + 3*a**4*b**3*x**6*log(b*x
**2/a)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**
(13/2)*b**3*x**6) - 6*a**4*b**3*x**6*log(sqrt(1 + b*x**2/a) + 1)/(6*a**(19
/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6
)) + B*(3*a*x/(3*a**(7/2)*sqrt(1 + b*x**2/a) + 3*a**(5/2)*b*x**2*sqrt(1 +
b*x**2/a)) + 2*b*x**3/(3*a**(7/2)*sqrt(1 + b*x**2/a) + 3*a**(5/2)*b*x**...
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.27

$$\int \frac{A+Bx+Cx^2+Dx^3}{x\,(a+bx^2)^{5/2}}\,dx = \frac{2\,Bx}{3\,\sqrt{bx^2+a}a^2}+\frac{Bx}{3\,(bx^2+a)^{\frac{3}{2}}a}-\frac{Dx}{3\,(bx^2+a)^{\frac{3}{2}}b}+\frac{Dx}{3\,\sqrt{bx^2+a}ab}-\frac{A\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{a^{\frac{5}{2}}}+\frac{A}{\sqrt{bx^2+a}a^2}+\frac{A}{3\,(bx^2+a)^{\frac{3}{2}}a}-\frac{C}{3\,(bx^2+a)^{\frac{3}{2}}b}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/x/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
2/3*B*x/(sqrt(b*x^2 + a)*a^2) + 1/3*B*x/((b*x^2 + a)^(3/2)*a) - 1/3*D*x/((
b*x^2 + a)^(3/2)*b) + 1/3*D*x/(sqrt(b*x^2 + a)*a*b) - A*arcsinh(a/(sqrt(a*
b)*abs(x)))/a^(5/2) + A/(sqrt(b*x^2 + a)*a^2) + 1/3*A/((b*x^2 + a)^(3/2)*a
) - 1/3*C/((b*x^2 + a)^(3/2)*b)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.11

$$\int \frac{A+Bx+Cx^2+Dx^3}{x(a+bx^2)^{5/2}}\,dx = \frac{\left(x\left(\frac{3\,Ab}{a^2}+\frac{(Da^4b+2\,Ba^3b^2)x}{a^5b}\right)+\frac{3\,B}{a}\right)x-\frac{Ca^5-4\,Aa^4b}{a^5b}}{3\,(bx^2+a)^{\frac{3}{2}}} + \frac{2\,A\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/x/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
1/3*((x*(3*A*b/a^2 + (D*a^4*b + 2*B*a^3*b^2)*x/(a^5*b)) + 3*B/a)*x - (C*a^
5 - 4*A*a^4*b)/(a^5*b))/(b*x^2 + a)^(3/2) + 2*A*arctan(-(sqrt(b)*x - sqrt(
b*x^2 + a))/sqrt(-a))/(sqrt(-a)*a^2)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{x(a+bx^2)^{5/2}}\,dx = \int \frac{A+B\,x+C\,x^2+x^3\,D}{x\,(b\,x^2+a)^{5/2}}\,dx$$

input
```
int((A + B*x + C*x^2 + x^3*D)/(x*(a + b*x^2)^(5/2)),x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/(x*(a + b*x^2)^(5/2)), x)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 380, normalized size of antiderivative = 3.73

$$\int \frac{A+Bx+Cx^2+Dx^3}{x(a+bx^2)^{5/2}}\,dx = \frac{4\sqrt{b\,x^2+a}\,a^2b^2-\sqrt{b\,x^2+a}\,a^2bc+3\sqrt{b\,x^2+a}\,a\,b^3x^2+3\sqrt{b\,x^2+a}\,a\,b^3x+}{}$$

input
```
int((D*x^3+C*x^2+B*x+A)/x/(b*x^2+a)^(5/2),x)
```

output
```
(4*sqrt(a + b*x**2)*a**2*b**2 - sqrt(a + b*x**2)*a**2*b*c + 3*sqrt(a + b*x
**2)*a*b**3*x**2 + 3*sqrt(a + b*x**2)*a*b**3*x + sqrt(a + b*x**2)*a*b**2*d
*x**3 + 2*sqrt(a + b*x**2)*b**4*x**3 + 3*sqrt(a)*log((sqrt(a + b*x**2) - s
qrt(a) + sqrt(b)*x)/sqrt(a))*a**2*b**2 + 6*sqrt(a)*log((sqrt(a + b*x**2) -
 sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b**3*x**2 + 3*sqrt(a)*log((sqrt(a + b*x**
2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*b**4*x**4 - 3*sqrt(a)*log((sqrt(a + b*x
**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*a**2*b**2 - 6*sqrt(a)*log((sqrt(a + b
*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b**3*x**2 - 3*sqrt(a)*log((sqrt(a
 + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*b**4*x**4 + sqrt(b)*a**3*d - 2*
sqrt(b)*a**2*b**2 + 2*sqrt(b)*a**2*b*d*x**2 - 4*sqrt(b)*a*b**3*x**2 + sqrt
(b)*a*b**2*d*x**4 - 2*sqrt(b)*b**4*x**4)/(3*a**2*b**2*(a**2 + 2*a*b*x**2 +
 b**2*x**4))
```

# 3.115 $\int \frac{A+Bx+Cx^2+Dx^3}{x^2\left(a+bx^2\right)^{5/2}}\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 142

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^2\left(a+bx^2\right)^{5/2}}\,dx=\frac{a(bB-aD)-b(Ab-aC)x}{3a^2b\left(a+bx^2\right)^{3/2}}+\frac{5Ab-2aC+3bBx}{3a^2bx\sqrt{a+bx^2}}-\frac{2(4Ab-aC)\sqrt{a+bx^2}}{3a^3bx}-\frac{B\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{5/2}}$$

output
```
1/3*(a*(B*b-D*a)-b*(A*b-C*a)*x)/a^2/b/(b*x^2+a)^(3/2)+1/3*(3*B*b*x+5*A*b-2
*C*a)/a^2/b/x/(b*x^2+a)^(1/2)-2/3*(4*A*b-C*a)*(b*x^2+a)^(1/2)/a^3/b/x-B*ar
ctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(5/2)
```

## Mathematica [A] (verified)

Time = 0.92 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.92

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^2\left(a+bx^2\right)^{5/2}}\,dx=\frac{-a^3Dx-8Ab^3x^4+ab^2x^2(-12A+3Bx+2Cx^2)+a^2b(-3A+4Bx+3C}{3a^3bx\left(a+bx^2\right)^{3/2}}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/(x^2*(a + b*x^2)^(5/2)),x]
```

output
```
(-(a^3*D*x) - 8*A*b^3*x^4 + a*b^2*x^2*(-12*A + 3*B*x + 2*C*x^2) + a^2*b*(-
3*A + 4*B*x + 3*C*x^2) + 6*Sqrt[a]*b*B*x*(a + b*x^2)^(3/2)*ArcTanh[(Sqrt[b
]*x - Sqrt[a + b*x^2])/Sqrt[a]])/(3*a^3*b*x*(a + b*x^2)^(3/2))
```

## Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.96, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2336, 25, 2336, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^2\left(a+bx^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{2336}$$

$$\frac{-bx\left(\frac{Ab}{a}-C\right)-aD+bB}{3ab\left(a+bx^2\right)^{3/2}}-\frac{\int -\frac{-2\left(\frac{Ab}{a}-C\right)x^2+3Bx+3A}{x^2\left(bx^2+a\right)^{3/2}}dx}{3a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{-2\left(\frac{Ab}{a}-C\right)x^2+3Bx+3A}{x^2\left(bx^2+a\right)^{3/2}}dx}{3a}+\frac{-bx\left(\frac{Ab}{a}-C\right)-aD+bB}{3ab\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{2336}$$

$$\frac{\frac{3B-x\left(\frac{5Ab}{a}-2C\right)}{a\sqrt{a+bx^2}}-\frac{\int -\frac{3(A+Bx)}{x^2\sqrt{bx^2+a}}dx}{a}}{3a}+\frac{-bx\left(\frac{Ab}{a}-C\right)-aD+bB}{3ab\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{3\int \frac{A+Bx}{x^2\sqrt{bx^2+a}}dx}{a}+\frac{3B-x\left(\frac{5Ab}{a}-2C\right)}{a\sqrt{a+bx^2}}}{3a}+\frac{-bx\left(\frac{Ab}{a}-C\right)-aD+bB}{3ab\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{534}$$

$$\frac{\frac{3\left(B\int\frac{1}{x\sqrt{bx^2+a}}dx-\frac{A\sqrt{a+bx^2}}{ax}\right)}{a}+\frac{3B-x\left(\frac{5Ab}{a}-2C\right)}{a\sqrt{a+bx^2}}}{3a}+\frac{-bx\left(\frac{Ab}{a}-C\right)-aD+bB}{3ab\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{243}$$

$$\frac{\frac{3\left(\frac{1}{2}B\int\frac{1}{x^2\sqrt{bx^2+a}}dx^2-\frac{A\sqrt{a+bx^2}}{ax}\right)}{a}+\frac{3B-x\left(\frac{5Ab}{a}-2C\right)}{a\sqrt{a+bx^2}}}{3a}+\frac{-bx\left(\frac{Ab}{a}-C\right)-aD+bB}{3ab\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{3\left(\frac{B\int\frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{b}-\frac{A\sqrt{a+bx^2}}{ax}\right)}{a}+\frac{3B-x\left(\frac{5Ab}{a}-2C\right)}{a\sqrt{a+bx^2}}}{3a}+\frac{-bx\left(\frac{Ab}{a}-C\right)-aD+bB}{3ab\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{3\left(-\frac{A\sqrt{a+bx^2}}{ax}-\frac{B\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}\right)}{a}+\frac{3B-x\left(\frac{5Ab}{a}-2C\right)}{a\sqrt{a+bx^2}}}{3a}+\frac{-bx\left(\frac{Ab}{a}-C\right)-aD+bB}{3ab\left(a+bx^2\right)^{3/2}}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/(x^2*(a + b*x^2)^(5/2)),x]
```

output
```
(b*B - a*D - b*((A*b)/a - C)*x)/(3*a*b*(a + b*x^2)^(3/2)) + ((3*B - ((5*A*
b)/a - 2*C)*x)/(a*Sqrt[a + b*x^2]) + (3*(-((A*Sqrt[a + b*x^2])/(a*x)) - (B
*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a]))/a)/(3*a)
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

## Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.20

| method | result |
|---|---|
| default | $C\left(\frac{x}{3a(b\,x^2+a)^{\frac{3}{2}}}+\frac{2x}{3a^2\sqrt{b\,x^2+a}}\right)+A\left(-\frac{1}{ax(b\,x^2+a)^{\frac{3}{2}}}-\frac{4b\left(\frac{x}{3a\left(b\,x^2+a\right)^{\frac{3}{2}}}+\frac{2x}{3a^2\sqrt{b\,x^2+a}}\right)}{a}\right)+B\left(\frac{1}{3a(b\,x^2+a)^{\frac{3}{2}}}\right.$ |

input
```
int((D*x^3+C*x^2+B*x+A)/x^2/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
C*(1/3*x/a/(b*x^2+a)^(3/2)+2/3*x/a^2/(b*x^2+a)^(1/2))+A*(-1/a/x/(b*x^2+a)^
(3/2)-4*b/a*(1/3*x/a/(b*x^2+a)^(3/2)+2/3*x/a^2/(b*x^2+a)^(1/2)))+B*(1/3/a/
(b*x^2+a)^(3/2)+1/a*(1/a/(b*x^2+a)^(1/2)-1/a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^
2+a)^(1/2))/x)))-1/3*D/b/(b*x^2+a)^(3/2)
```

## Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 342, normalized size of antiderivative = 2.41

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^2\left(a+bx^2\right)^{5/2}}\,dx = \left[\frac{3\left(Bb^3x^5+2\,Bab^2x^3+Ba^2bx\right)\sqrt{a}\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\,(3\,Bab^2x}{6\,(a^3b^3x}\right.$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/x^2/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
[1/6*(3*(B*b^3*x^5 + 2*B*a*b^2*x^3 + B*a^2*b*x)*sqrt(a)*log(-(b*x^2 - 2*sq
rt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(3*B*a*b^2*x^3 + 2*(C*a*b^2 - 4*A*b^
3)*x^4 - 3*A*a^2*b + 3*(C*a^2*b - 4*A*a*b^2)*x^2 - (D*a^3 - 4*B*a^2*b)*x)*
sqrt(b*x^2 + a))/(a^3*b^3*x^5 + 2*a^4*b^2*x^3 + a^5*b*x), 1/3*(3*(B*b^3*x^
5 + 2*B*a*b^2*x^3 + B*a^2*b*x)*sqrt(-a)*arctan(sqrt(b*x^2 + a)*sqrt(-a)/a)
 + (3*B*a*b^2*x^3 + 2*(C*a*b^2 - 4*A*b^3)*x^4 - 3*A*a^2*b + 3*(C*a^2*b - 4
*A*a*b^2)*x^2 - (D*a^3 - 4*B*a^2*b)*x)*sqrt(b*x^2 + a))/(a^3*b^3*x^5 + 2*a
^4*b^2*x^3 + a^5*b*x)]
```

## Sympy [A] (verification not implemented)

Time = 16.11 (sec) , antiderivative size = 1057, normalized size of antiderivative = 7.44

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^2\left(a+bx^2\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/x**2/(b*x**2+a)**(5/2),x)
```

output
```
A*(-3*a**2*b**(9/2)*sqrt(a/(b*x**2) + 1)/(3*a**5*b**4 + 6*a**4*b**5*x**2 +
 3*a**3*b**6*x**4) - 12*a*b**(11/2)*x**2*sqrt(a/(b*x**2) + 1)/(3*a**5*b**4
 + 6*a**4*b**5*x**2 + 3*a**3*b**6*x**4) - 8*b**(13/2)*x**4*sqrt(a/(b*x**2)
 + 1)/(3*a**5*b**4 + 6*a**4*b**5*x**2 + 3*a**3*b**6*x**4)) + B*(8*a**7*sqr
t(1 + b*x**2/a)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**
4 + 6*a**(13/2)*b**3*x**6) + 3*a**7*log(b*x**2/a)/(6*a**(19/2) + 18*a**(17
/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) - 6*a**7*log(
sqrt(1 + b*x**2/a) + 1)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*
b**2*x**4 + 6*a**(13/2)*b**3*x**6) + 14*a**6*b*x**2*sqrt(1 + b*x**2/a)/(6*
a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**
3*x**6) + 9*a**6*b*x**2*log(b*x**2/a)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 +
 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) - 18*a**6*b*x**2*log(sqrt
(1 + b*x**2/a) + 1)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2
*x**4 + 6*a**(13/2)*b**3*x**6) + 6*a**5*b**2*x**4*sqrt(1 + b*x**2/a)/(6*a*
*(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*
x**6) + 9*a**5*b**2*x**4*log(b*x**2/a)/(6*a**(19/2) + 18*a**(17/2)*b*x**2
+ 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) - 18*a**5*b**2*x**4*log(
sqrt(1 + b*x**2/a) + 1)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*
b**2*x**4 + 6*a**(13/2)*b**3*x**6) + 3*a**4*b**3*x**6*log(b*x**2/a)/(6*a**
(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**...
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.04

$$\int \frac{A + Bx + Cx^2 + Dx^3}{x^2 (a + bx^2)^{5/2}} dx = \frac{2 Cx}{3 \sqrt{bx^2 + a}a^2} + \frac{Cx}{3 (bx^2 + a)^{\frac{3}{2}} a} - \frac{8 Abx}{3 \sqrt{bx^2 + a}a^3} - \frac{4 Abx}{3 (bx^2 + a)^{\frac{3}{2}} a^2} - \frac{B \operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{a^{\frac{5}{2}}} + \frac{B}{\sqrt{bx^2 + a}a^2} + \frac{B}{3 (bx^2 + a)^{\frac{3}{2}} a} - \frac{D}{3 (bx^2 + a)^{\frac{3}{2}} b} - \frac{A}{(bx^2 + a)^{\frac{3}{2}} ax}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/x^2/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
2/3*C*x/(sqrt(b*x^2 + a)*a^2) + 1/3*C*x/((b*x^2 + a)^(3/2)*a) - 8/3*A*b*x/
(sqrt(b*x^2 + a)*a^3) - 4/3*A*b*x/((b*x^2 + a)^(3/2)*a^2) - B*arcsinh(a/(s
qrt(a*b)*abs(x)))/a^(5/2) + B/(sqrt(b*x^2 + a)*a^2) + 1/3*B/((b*x^2 + a)^(
3/2)*a) - 1/3*D/((b*x^2 + a)^(3/2)*b) - A/((b*x^2 + a)^(3/2)*a*x)
```

## Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.18

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^2\left(a+bx^2\right)^{5/2}}\,dx = \frac{\left(x\left(\frac{3\,Bb}{a^2}+\frac{(2\,Ca^6b^2-5\,Aa^5b^3)x}{a^8b}\right)+\frac{3\,(Ca^7b-2\,Aa^6b^2)}{a^8b}\right)x-\frac{Da^8-4\,Ba^7b}{a^8b}}{3\,(bx^2+a)^{\frac{3}{2}}} + \frac{2\,B\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2} + \frac{2\,A\sqrt{b}}{\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)a^2}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/x^2/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
1/3*((x*(3*B*b/a^2 + (2*C*a^6*b^2 - 5*A*a^5*b^3)*x/(a^8*b)) + 3*(C*a^7*b -
 2*A*a^6*b^2)/(a^8*b))*x - (D*a^8 - 4*B*a^7*b)/(a^8*b))/(b*x^2 + a)^(3/2)
+ 2*B*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/(sqrt(-a)*a^2) + 2*A
*sqrt(b)/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)*a^2)
```

## Mupad [B] (verification not implemented)

Time = 2.92 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.17

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^2\left(a+bx^2\right)^{5/2}}\,dx = \frac{\frac{B}{3\,a}+\frac{B\,(b\,x^2+a)}{a^2}}{(b\,x^2+a)^{3/2}} + \frac{2\,C\,x\,(b\,x^2+a)+C\,a\,x}{3\,a^2\,(b\,x^2+a)^{3/2}} - \frac{B\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{a^{5/2}} + \frac{A\,a^2-8\,A\,(b\,x^2+a)^2+4\,A\,a\,(b\,x^2+a)}{3\,a^3\,x\,(b\,x^2+a)^{3/2}} - \frac{a\,D\,\left(\frac{b\,x^2}{a}-\left(\frac{b\,x^2}{a}+1\right)^{5/2}+1\right)}{3\,b\,(b\,x^2+a)^{5/2}}$$

input
```
int((A + B*x + C*x^2 + x^3*D)/(x^2*(a + b*x^2)^(5/2)),x)
```

output
```
(B/(3*a) + (B*(a + b*x^2))/a^2)/(a + b*x^2)^(3/2) + (2*C*x*(a + b*x^2) + C
*a*x)/(3*a^2*(a + b*x^2)^(3/2)) - (B*atanh((a + b*x^2)^(1/2)/a^(1/2)))/a^(
5/2) + (A*a^2 - 8*A*(a + b*x^2)^2 + 4*A*a*(a + b*x^2))/(3*a^3*x*(a + b*x^2
)^(3/2)) - (a*D*((b*x^2)/a - ((b*x^2)/a + 1)^(5/2) + 1))/(3*b*(a + b*x^2)^
(5/2))
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 429, normalized size of antiderivative = 3.02

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^2\left(a+bx^2\right)^{5/2}}\,dx = \frac{-3\sqrt{b\,x^2+a}\,a^3b-\sqrt{b\,x^2+a}\,a^3dx-12\sqrt{b\,x^2+a}\,a^2b^2x^2+4\sqrt{b\,x^2+a}\,a^2b^2}{}$$

input
```
int((D*x^3+C*x^2+B*x+A)/x^2/(b*x^2+a)^(5/2),x)
```

output
```
( - 3*sqrt(a + b*x**2)*a**3*b - sqrt(a + b*x**2)*a**3*d*x - 12*sqrt(a + b*
x**2)*a**2*b**2*x**2 + 4*sqrt(a + b*x**2)*a**2*b**2*x + 3*sqrt(a + b*x**2)
*a**2*b*c*x**2 - 8*sqrt(a + b*x**2)*a*b**3*x**4 + 3*sqrt(a + b*x**2)*a*b**
3*x**3 + 2*sqrt(a + b*x**2)*a*b**2*c*x**4 + 3*sqrt(a)*log((sqrt(a + b*x**2
) - sqrt(a) + sqrt(b)*x)/sqrt(a))*a**2*b**2*x + 6*sqrt(a)*log((sqrt(a + b*
x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b**3*x**3 + 3*sqrt(a)*log((sqrt(a
+ b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*b**4*x**5 - 3*sqrt(a)*log((sqrt(
a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*a**2*b**2*x - 6*sqrt(a)*log((s
qrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b**3*x**3 - 3*sqrt(a)*lo
g((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*b**4*x**5 + 8*sqrt(b)*
a**3*b*x - 2*sqrt(b)*a**3*c*x + 16*sqrt(b)*a**2*b**2*x**3 - 4*sqrt(b)*a**2
*b*c*x**3 + 8*sqrt(b)*a*b**3*x**5 - 2*sqrt(b)*a*b**2*c*x**5)/(3*a**3*b*x*(
a**2 + 2*a*b*x**2 + b**2*x**4))
```

# 3.116 $\int \frac{A+Bx+Cx^2+Dx^3}{x^3\left(a+bx^2\right)^{5/2}}\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 156

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^3\left(a+bx^2\right)^{5/2}}\,dx = -\frac{Ab-aC+(bB-aD)x}{3a^2\left(a+bx^2\right)^{3/2}} - \frac{3(2Ab-aC)+(5bB-2aD)x}{3a^3\sqrt{a+bx^2}} - \frac{A\sqrt{a+bx^2}}{2a^3x^2} - \frac{B\sqrt{a+bx^2}}{a^3x} + \frac{(5Ab-2aC)\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{7/2}}$$

output
```
-1/3*(A*b-C*a+(B*b-D*a)*x)/a^2/(b*x^2+a)^(3/2)-1/3*(6*A*b-3*C*a+(5*B*b-2*D
*a)*x)/a^3/(b*x^2+a)^(1/2)-1/2*A*(b*x^2+a)^(1/2)/a^3/x^2-B*(b*x^2+a)^(1/2)
/a^3/x+1/2*(5*A*b-2*C*a)*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(7/2)
```

## Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.85

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^3(a+bx^2)^{5/2}}\,dx = \frac{-b^2x^4(15A+16Bx)+a^2(-3A-6Bx+8Cx^2+6Dx^3)+2abx^2(-10A+}{6a^3x^2(a+bx^2)^{3/2}}$$
$$+\frac{(-5Ab+2aC)\operatorname{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{7/2}}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/(x^3*(a + b*x^2)^(5/2)),x]
```

output
```
(-(b^2*x^4*(15*A + 16*B*x)) + a^2*(-3*A - 6*B*x + 8*C*x^2 + 6*D*x^3) + 2*a
*b*x^2*(-10*A + x*(-12*B + 3*C*x + 2*D*x^2)))/(6*a^3*x^2*(a + b*x^2)^(3/2)
) + ((-5*A*b + 2*a*C)*ArcTanh[(Sqrt[b]*x - Sqrt[a + b*x^2])/Sqrt[a]])/a^(7
/2)
```

## Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.14, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2336, 25, 2336, 27, 2338, 25, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^3(a+bx^2)^{5/2}}\,dx$$
$$\Big\downarrow \text{2336}$$
$$-\frac{\int -\frac{-2\left(\frac{bB}{a}-D\right)x^3-3\left(\frac{Ab}{a}-C\right)x^2+3Bx+3A}{x^3(bx^2+a)^{3/2}}dx}{3a}-\frac{\frac{Ab}{a}+x\left(\frac{bB}{a}-D\right)-C}{3a(a+bx^2)^{3/2}}$$
$$\Big\downarrow \text{25}$$
$$\frac{\int \frac{-2\left(\frac{bB}{a}-D\right)x^3-3\left(\frac{Ab}{a}-C\right)x^2+3Bx+3A}{x^3(bx^2+a)^{3/2}}dx}{3a}-\frac{\frac{Ab}{a}+x\left(\frac{bB}{a}-D\right)-C}{3a(a+bx^2)^{3/2}}$$

$$\Big\downarrow \ 2336$$

$$\frac{-\frac{\int -\frac{3\left(-\left(\left(\frac{2Ab}{a}-C\right)x^2\right)+Bx+A\right)}{x^3\sqrt{bx^2+a}}dx}{a}-\frac{3\left(\frac{2Ab}{a}-C\right)+x\left(\frac{5bB}{a}-2D\right)}{a\sqrt{a+bx^2}}}{3a}-\frac{\frac{Ab}{a}+x\left(\frac{bB}{a}-D\right)-C}{3a\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{3\int \frac{-\left(\left(\frac{2Ab}{a}-C\right)x^2\right)+Bx+A}{x^3\sqrt{bx^2+a}}dx}{a}-\frac{3\left(\frac{2Ab}{a}-C\right)+x\left(\frac{5bB}{a}-2D\right)}{a\sqrt{a+bx^2}}}{3a}-\frac{\frac{Ab}{a}+x\left(\frac{bB}{a}-D\right)-C}{3a\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \ 2338$$

$$\frac{\frac{3\left(-\frac{\int -\frac{2aB-(5Ab-2aC)x}{x^2\sqrt{bx^2+a}}dx}{2a}-\frac{A\sqrt{a+bx^2}}{2ax^2}\right)}{a}-\frac{3\left(\frac{2Ab}{a}-C\right)+x\left(\frac{5bB}{a}-2D\right)}{a\sqrt{a+bx^2}}}{3a}-\frac{\frac{Ab}{a}+x\left(\frac{bB}{a}-D\right)-C}{3a\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{3\left(\frac{\int \frac{2aB-(5Ab-2aC)x}{x^2\sqrt{bx^2+a}}dx}{2a}-\frac{A\sqrt{a+bx^2}}{2ax^2}\right)}{a}-\frac{3\left(\frac{2Ab}{a}-C\right)+x\left(\frac{5bB}{a}-2D\right)}{a\sqrt{a+bx^2}}}{3a}-\frac{\frac{Ab}{a}+x\left(\frac{bB}{a}-D\right)-C}{3a\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \ 534$$

$$\frac{\frac{3\left(\frac{-(5Ab-2aC)\int \frac{1}{x\sqrt{bx^2+a}}dx-\frac{2B\sqrt{a+bx^2}}{x}}{2a}-\frac{A\sqrt{a+bx^2}}{2ax^2}\right)}{a}-\frac{3\left(\frac{2Ab}{a}-C\right)+x\left(\frac{5bB}{a}-2D\right)}{a\sqrt{a+bx^2}}}{3a}-\frac{\frac{Ab}{a}+x\left(\frac{bB}{a}-D\right)-C}{3a\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \ 243$$

$$\frac{\frac{3\left(\frac{-\frac{1}{2}(5Ab-2aC)\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2-\frac{2B\sqrt{a+bx^2}}{x}}{2a}-\frac{A\sqrt{a+bx^2}}{2ax^2}\right)}{a}-\frac{3\left(\frac{2Ab}{a}-C\right)+x\left(\frac{5bB}{a}-2D\right)}{a\sqrt{a+bx^2}}}{3a}-\frac{\frac{Ab}{a}+x\left(\frac{bB}{a}-D\right)-C}{3a\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \ 73$$

$$\frac{\frac{3\left(\frac{-\frac{(5Ab-2aC)\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{b}-\frac{2B\sqrt{a+bx^2}}{x}}{2a}-\frac{A\sqrt{a+bx^2}}{2ax^2}\right)}{a}-\frac{3\left(\frac{2Ab}{a}-C\right)+x\left(\frac{5bB}{a}-2D\right)}{a\sqrt{a+bx^2}}}{3a}-\frac{\frac{Ab}{a}+x\left(\frac{bB}{a}-D\right)-C}{3a\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{3\left(\frac{\frac{(5Ab-2aC)\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{2B\sqrt{a+bx^2}}{x}}{2a}-\frac{A\sqrt{a+bx^2}}{2ax^2}\right)}{a}-\frac{3\left(\frac{2Ab}{a}-C\right)+x\left(\frac{5bB}{a}-2D\right)}{a\sqrt{a+bx^2}}}{3a}-\frac{\frac{Ab}{a}+x\left(\frac{bB}{a}-D\right)-C}{3a\left(a+bx^2\right)^{3/2}}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/(x^3*(a + b*x^2)^(5/2)),x]
```

output
```
-1/3*((A*b)/a - C + ((b*B)/a - D)*x)/(a*(a + b*x^2)^(3/2)) + (-((3*((2*A*b
)/a - C) + ((5*b*B)/a - 2*D)*x)/(a*Sqrt[a + b*x^2])) + (3*(-1/2*(A*Sqrt[a
+ b*x^2])/(a*x^2) + ((-2*B*Sqrt[a + b*x^2])/x + ((5*A*b - 2*a*C)*ArcTanh[S
qrt[a + b*x^2]/Sqrt[a]])/Sqrt[a])/(2*a)))/a)/(3*a)
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

## Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.55

| method | result |
|---|---|
| default | $D\left(\frac{x}{3a(b\,x^2+a)^{\frac{3}{2}}}+\frac{2x}{3a^2\sqrt{b\,x^2+a}}\right)+A\left(-\frac{1}{2a\,x^2(b\,x^2+a)^{\frac{3}{2}}}-\frac{5b\left(\frac{1}{3a\left(b\,x^2+a\right)^{\frac{3}{2}}}+\frac{\frac{1}{a\sqrt{b\,x^2+a}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)}{a^{\frac{3}{2}}}}{a}\right)}{2a}\right)$ |

input
```
int((D*x^3+C*x^2+B*x+A)/x^3/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
D*(1/3*x/a/(b*x^2+a)^(3/2)+2/3*x/a^2/(b*x^2+a)^(1/2))+A*(-1/2/a/x^2/(b*x^2
+a)^(3/2)-5/2*b/a*(1/3/a/(b*x^2+a)^(3/2)+1/a*(1/a/(b*x^2+a)^(1/2)-1/a^(3/2
)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x))))+B*(-1/a/x/(b*x^2+a)^(3/2)-4*b/a
*(1/3*x/a/(b*x^2+a)^(3/2)+2/3*x/a^2/(b*x^2+a)^(1/2)))+C*(1/3/a/(b*x^2+a)^(
3/2)+1/a*(1/a/(b*x^2+a)^(1/2)-1/a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))
/x)))
```

## Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 440, normalized size of antiderivative = 2.82

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^3\left(a+bx^2\right)^{5/2}}\,dx=\left[-\frac{3\left((2\,Cab^2-5\,Ab^3)x^6+2\,(2\,Ca^2b-5\,Aab^2)x^4+(2\,Ca^3-5\,Aa^2b)x^2\right)\sqrt{}}{}\right.$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/x^3/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
[-1/12*(3*((2*C*a*b^2 - 5*A*b^3)*x^6 + 2*(2*C*a^2*b - 5*A*a*b^2)*x^4 + (2*
C*a^3 - 5*A*a^2*b)*x^2)*sqrt(a)*log(-(b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(a) +
2*a)/x^2) - 2*(4*(D*a^2*b - 4*B*a*b^2)*x^5 - 6*B*a^3*x + 3*(2*C*a^2*b - 5*
A*a*b^2)*x^4 - 3*A*a^3 + 6*(D*a^3 - 4*B*a^2*b)*x^3 + 4*(2*C*a^3 - 5*A*a^2*
b)*x^2)*sqrt(b*x^2 + a))/(a^4*b^2*x^6 + 2*a^5*b*x^4 + a^6*x^2), 1/6*(3*((2
*C*a*b^2 - 5*A*b^3)*x^6 + 2*(2*C*a^2*b - 5*A*a*b^2)*x^4 + (2*C*a^3 - 5*A*a
^2*b)*x^2)*sqrt(-a)*arctan(sqrt(b*x^2 + a)*sqrt(-a)/a) + (4*(D*a^2*b - 4*B
*a*b^2)*x^5 - 6*B*a^3*x + 3*(2*C*a^2*b - 5*A*a*b^2)*x^4 - 3*A*a^3 + 6*(D*a
^3 - 4*B*a^2*b)*x^3 + 4*(2*C*a^3 - 5*A*a^2*b)*x^2)*sqrt(b*x^2 + a))/(a^4*b
^2*x^6 + 2*a^5*b*x^4 + a^6*x^2)]
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1875 vs. $2(138) = 276$.

Time = 28.33 (sec) , antiderivative size = 1875, normalized size of antiderivative = 12.02

$$\int \frac{A + Bx + Cx^2 + Dx^3}{x^3 (a + bx^2)^{5/2}} \, dx = \text{Too large to display}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/x**3/(b*x**2+a)**(5/2),x)
```

output

```
A*(-6*a**17*sqrt(1 + b*x**2/a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 +
36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) - 46*a**16*b*x**2*sqrt(1
+ b*x**2/a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x
**6 + 12*a**(33/2)*b**3*x**8) - 15*a**16*b*x**2*log(b*x**2/a)/(12*a**(39/2
)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*
x**8) + 30*a**16*b*x**2*log(sqrt(1 + b*x**2/a) + 1)/(12*a**(39/2)*x**2 + 3
6*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) - 70
*a**15*b**2*x**4*sqrt(1 + b*x**2/a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x*
*4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) - 45*a**15*b**2*x**4
*log(b*x**2/a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**
2*x**6 + 12*a**(33/2)*b**3*x**8) + 90*a**15*b**2*x**4*log(sqrt(1 + b*x**2/
a) + 1)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6
+ 12*a**(33/2)*b**3*x**8) - 30*a**14*b**3*x**6*sqrt(1 + b*x**2/a)/(12*a**(
39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b
**3*x**8) - 45*a**14*b**3*x**6*log(b*x**2/a)/(12*a**(39/2)*x**2 + 36*a**(3
7/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) + 90*a**14*
b**3*x**6*log(sqrt(1 + b*x**2/a) + 1)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*
x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) - 15*a**13*b**4*x*
*8*log(b*x**2/a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b
**2*x**6 + 12*a**(33/2)*b**3*x**8) + 30*a**13*b**4*x**8*log(sqrt(1 + b*...
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.29

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^3\left(a+bx^2\right)^{5/2}}\,dx = \frac{2\,Dx}{3\,\sqrt{bx^2+a}a^2} + \frac{Dx}{3\,(bx^2+a)^{\frac{3}{2}}a} - \frac{8\,Bbx}{3\,\sqrt{bx^2+a}a^3} - \frac{4\,Bbx}{3\,(bx^2+a)^{\frac{3}{2}}a^2} - \frac{C\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{a^{\frac{5}{2}}} + \frac{5\,Ab\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{2\,a^{\frac{7}{2}}} + \frac{C}{\sqrt{bx^2+a}a^2} + \frac{C}{3\,(bx^2+a)^{\frac{3}{2}}a} - \frac{5\,Ab}{2\,\sqrt{bx^2+a}a^3} - \frac{5\,Ab}{6\,(bx^2+a)^{\frac{3}{2}}a^2} - \frac{B}{(bx^2+a)^{\frac{3}{2}}ax} - \frac{A}{2\,(bx^2+a)^{\frac{3}{2}}ax^2}$$

input

```
integrate((D*x^3+C*x^2+B*x+A)/x^3/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
2/3*D*x/(sqrt(b*x^2 + a)*a^2) + 1/3*D*x/((b*x^2 + a)^(3/2)*a) - 8/3*B*b*x/
(sqrt(b*x^2 + a)*a^3) - 4/3*B*b*x/((b*x^2 + a)^(3/2)*a^2) - C*arcsinh(a/(s
qrt(a*b)*abs(x)))/a^(5/2) + 5/2*A*b*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(7/2)
+ C/(sqrt(b*x^2 + a)*a^2) + 1/3*C/((b*x^2 + a)^(3/2)*a) - 5/2*A*b/(sqrt(b*
x^2 + a)*a^3) - 5/6*A*b/((b*x^2 + a)^(3/2)*a^2) - B/((b*x^2 + a)^(3/2)*a*x
) - 1/2*A/((b*x^2 + a)^(3/2)*a*x^2)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 271 vs. $2(131) = 262$.

Time = 0.14 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.74

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^3(a+bx^2)^{5/2}}\,dx = \frac{\left(x\left(\frac{(2\,Da^8b^2-5\,Ba^7b^3)x}{a^{10}b}+\frac{3\,(Ca^8b^2-2\,Aa^7b^3)}{a^{10}b}\right)+\frac{3\,(Da^9b-2\,Ba^8b^2)}{a^{10}b}\right)x+\frac{4\,Ca^9b-7\,Aa^8b^2}{a^{10}b}}{3\,(bx^2+a)^{\frac{3}{2}}}$$
$$+\frac{(2\,Ca-5\,Ab)\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^3}$$
$$+\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^3Ab+2\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2Ba\sqrt{b}+\left(\sqrt{b}x-\sqrt{bx^2+a}\right)Aab-2\,Ba^2\sqrt{b}}{\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^2a^3}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/x^3/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
1/3*((x*((2*D*a^8*b^2 - 5*B*a^7*b^3)*x/(a^10*b) + 3*(C*a^8*b^2 - 2*A*a^7*b
^3)/(a^10*b)) + 3*(D*a^9*b - 2*B*a^8*b^2)/(a^10*b))*x + (4*C*a^9*b - 7*A*a
^8*b^2)/(a^10*b))/(b*x^2 + a)^(3/2) + (2*C*a - 5*A*b)*arctan(-(sqrt(b)*x -
 sqrt(b*x^2 + a))/sqrt(-a))/(sqrt(-a)*a^3) + ((sqrt(b)*x - sqrt(b*x^2 + a)
)^3*A*b + 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a*sqrt(b) + (sqrt(b)*x - sqr
t(b*x^2 + a))*A*a*b - 2*B*a^2*sqrt(b))/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 -
 a)^2*a^3)
```

## Mupad [B] (verification not implemented)

Time = 3.15 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.44

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^3(a+bx^2)^{5/2}}\,dx = \frac{\frac{C}{3\,a}+\frac{C\,(b\,x^2+a)}{a^2}}{(b\,x^2+a)^{3/2}}+\frac{x\,D}{(b\,x^2+a)^{5/2}}-\frac{C\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{B\,a^2-8\,B\,(b\,x^2+a)^2+4\,B\,a\,(b\,x^2+a)}{3\,a^3\,x\,(b\,x^2+a)^{3/2}}-\frac{10\,A\,b}{3\,a^2\,(b\,x^2+a)^{3/2}}-\frac{A}{2\,a\,x^2\,(b\,x^2+a)^{3/2}}+\frac{5\,A\,b\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2\,a^{7/2}}+\frac{2\,b^2\,x^5\,D}{3\,a^2\,(b\,x^2+a)^{5/2}}-\frac{5\,A\,b^2\,x^2}{2\,a^3\,(b\,x^2+a)^{3/2}}+\frac{5\,b\,x^3\,D}{3\,a\,(b\,x^2+a)^{5/2}}$$

input
```
int((A + B*x + C*x^2 + x^3*D)/(x^3*(a + b*x^2)^(5/2)),x)
```

output
```
(C/(3*a) + (C*(a + b*x^2))/a^2)/(a + b*x^2)^(3/2) + (x*D)/(a + b*x^2)^(5/2
) - (C*atanh((a + b*x^2)^(1/2)/a^(1/2)))/a^(5/2) + (B*a^2 - 8*B*(a + b*x^2
)^2 + 4*B*a*(a + b*x^2))/(3*a^3*x*(a + b*x^2)^(3/2)) - (10*A*b)/(3*a^2*(a
+ b*x^2)^(3/2)) - A/(2*a*x^2*(a + b*x^2)^(3/2)) + (5*A*b*atanh((a + b*x^2)
^(1/2)/a^(1/2)))/(2*a^(7/2)) + (2*b^2*x^5*D)/(3*a^2*(a + b*x^2)^(5/2)) - (
5*A*b^2*x^2)/(2*a^3*(a + b*x^2)^(3/2)) + (5*b*x^3*D)/(3*a*(a + b*x^2)^(5/2
))
```

## Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 675, normalized size of antiderivative = 4.33

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^3(a+bx^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
int((D*x^3+C*x^2+B*x+A)/x^3/(b*x^2+a)^(5/2),x)
```

output

```
( - 3*sqrt(a + b*x**2)*a**3*b - 20*sqrt(a + b*x**2)*a**2*b**2*x**2 - 6*sqr
t(a + b*x**2)*a**2*b**2*x + 8*sqrt(a + b*x**2)*a**2*b*c*x**2 + 6*sqrt(a +
b*x**2)*a**2*b*d*x**3 - 15*sqrt(a + b*x**2)*a*b**3*x**4 - 24*sqrt(a + b*x*
*2)*a*b**3*x**3 + 6*sqrt(a + b*x**2)*a*b**2*c*x**4 + 4*sqrt(a + b*x**2)*a*
b**2*d*x**5 - 16*sqrt(a + b*x**2)*b**4*x**5 - 15*sqrt(a)*log((sqrt(a + b*x
**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*a**2*b**2*x**2 + 6*sqrt(a)*log((sqrt(
a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*a**2*b*c*x**2 - 30*sqrt(a)*log
((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b**3*x**4 + 12*sqrt(a
)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b**2*c*x**4 - 15
*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*b**4*x**6 +
 6*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*b**3*c*x*
*6 + 15*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*a**2
*b**2*x**2 - 6*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a
))*a**2*b*c*x**2 + 30*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)
/sqrt(a))*a*b**3*x**4 - 12*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(
b)*x)/sqrt(a))*a*b**2*c*x**4 + 15*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a)
+ sqrt(b)*x)/sqrt(a))*b**4*x**6 - 6*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a
) + sqrt(b)*x)/sqrt(a))*b**3*c*x**6 - 4*sqrt(b)*a**3*d*x**2 + 16*sqrt(b)*a
**2*b**2*x**2 - 8*sqrt(b)*a**2*b*d*x**4 + 32*sqrt(b)*a*b**3*x**4 - 4*sqrt(
b)*a*b**2*d*x**6 + 16*sqrt(b)*b**4*x**6)/(6*a**3*b*x**2*(a**2 + 2*a*b*x...
```

# 3.117 $\int (cx)^{3/2} \left(a+bx^2\right) \left(A+Bx+Cx^2+Dx^3\right) \, dx$



## Optimal result

Integrand size = 30, antiderivative size = 107

$$\int (cx)^{3/2} \left(a+bx^2\right) \left(A+Bx+Cx^2+Dx^3\right) \, dx = \frac{2aA(cx)^{5/2}}{5c}+\frac{2aB(cx)^{7/2}}{7c^2} +\frac{2(Ab+aC)(cx)^{9/2}}{9c^3}+\frac{2(bB+aD)(cx)^{11/2}}{11c^4}+\frac{2bC(cx)^{13/2}}{13c^5}+\frac{2bD(cx)^{15/2}}{15c^6}$$

output
```
2/5*a*A*(c*x)^(5/2)/c+2/7*a*B*(c*x)^(7/2)/c^2+2/9*(A*b+C*a)*(c*x)^(9/2)/c^
3+2/11*(B*b+D*a)*(c*x)^(11/2)/c^4+2/13*b*C*(c*x)^(13/2)/c^5+2/15*b*D*(c*x)
^(15/2)/c^6
```

## Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.58

$$\int (cx)^{3/2} \left(a+bx^2\right) \left(A+Bx+Cx^2 +Dx^3\right) \, dx = \frac{2x(cx)^{3/2} (9009aA+7bx^2(715A+585Bx+495Cx^2+429Dx^3)+65ax(99B+7x(11C+9D}{45045}$$

input
```
Integrate[(c*x)^(3/2)*(a + b*x^2)*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(2*x*(c*x)^(3/2)*(9009*a*A + 7*b*x^2*(715*A + 585*B*x + 495*C*x^2 + 429*D*
x^3) + 65*a*x*(99*B + 7*x*(11*C + 9*D*x))))/45045
```

## Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^{3/2} \left(a+bx^2\right) \left(A+Bx+Cx^2+Dx^3\right) dx$$

$$\Big\downarrow \ 2333$$

$$\int \left(\frac{(cx)^{7/2}(aC+Ab)}{c^2}+aA(cx)^{3/2}+\frac{(cx)^{9/2}(aD+bB)}{c^3}+\frac{aB(cx)^{5/2}}{c}+\frac{bD(cx)^{13/2}}{c^5}+\frac{bC(cx)^{11/2}}{c^4}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{2(cx)^{9/2}(aC+Ab)}{9c^3}+\frac{2aA(cx)^{5/2}}{5c}+\frac{2(cx)^{11/2}(aD+bB)}{11c^4}+\frac{2aB(cx)^{7/2}}{7c^2}+\frac{2bD(cx)^{15/2}}{15c^6}+\frac{2bC(cx)^{13/2}}{13c^5}$$

input
```
Int[(c*x)^(3/2)*(a + b*x^2)*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(2*a*A*(c*x)^(5/2))/(5*c) + (2*a*B*(c*x)^(7/2))/(7*c^2) + (2*(A*b + a*C)*(
c*x)^(9/2))/(9*c^3) + (2*(b*B + a*D)*(c*x)^(11/2))/(11*c^4) + (2*b*C*(c*x)
^(13/2))/(13*c^5) + (2*b*D*(c*x)^(15/2))/(15*c^6)
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

## Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.55

| method | result | size |
|---|---|---|
| pseudoelliptic | $\frac{2\sqrt{cx}\left(\frac{Db\,x^{5}}{3}+\frac{5Cb\,x^{4}}{13}+\frac{5(Bb+Da)x^{3}}{11}+\frac{5(Ab+Ca)x^{2}}{9}+\frac{5Bax}{7}+Aa\right)x^{2}c}{5}$ | 59 |
| gosper | $\frac{2x\left(3003Db\,x^{5}+3465Cb\,x^{4}+4095bB\,x^{3}+4095Da\,x^{3}+5005Ab\,x^{2}+5005Ca\,x^{2}+6435Bax+9009Aa\right)(cx)^{\frac{3}{2}}}{45045}$ | 61 |
| orering | $\frac{2x\left(3003Db\,x^{5}+3465Cb\,x^{4}+4095bB\,x^{3}+4095Da\,x^{3}+5005Ab\,x^{2}+5005Ca\,x^{2}+6435Bax+9009Aa\right)(cx)^{\frac{3}{2}}}{45045}$ | 61 |
| trager | $\frac{2c\,x^{2}\left(3003Db\,x^{5}+3465Cb\,x^{4}+4095bB\,x^{3}+4095Da\,x^{3}+5005Ab\,x^{2}+5005Ca\,x^{2}+6435Bax+9009Aa\right)\sqrt{cx}}{45045}$ | 64 |
| derivativedivides | $\frac{\frac{2Db(cx)^{\frac{15}{2}}}{15}+\frac{2bCc(cx)^{\frac{13}{2}}}{13}+\frac{2\left(Bb\,c^{2}+a\,c^{2}D\right)(cx)^{\frac{11}{2}}}{11}+\frac{2\left(Ab\,c^{3}+Ca\,c^{3}\right)(cx)^{\frac{9}{2}}}{9}+\frac{2a\,c^{4}B(cx)^{\frac{7}{2}}}{7}+\frac{2a\,c^{5}A(cx)^{\frac{5}{2}}}{5}}{c^{6}}$ | 90 |
| default | $\frac{\frac{2Db(cx)^{\frac{15}{2}}}{15}+\frac{2bCc(cx)^{\frac{13}{2}}}{13}+\frac{2\left(Bb\,c^{2}+a\,c^{2}D\right)(cx)^{\frac{11}{2}}}{11}+\frac{2\left(Ab\,c^{3}+Ca\,c^{3}\right)(cx)^{\frac{9}{2}}}{9}+\frac{2a\,c^{4}B(cx)^{\frac{7}{2}}}{7}+\frac{2a\,c^{5}A(cx)^{\frac{5}{2}}}{5}}{c^{6}}$ | 90 |

input
```
int((c*x)^(3/2)*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
2/5*(c*x)^(1/2)*(1/3*D*b*x^5+5/13*C*b*x^4+5/11*(B*b+D*a)*x^3+5/9*(A*b+C*a)
*x^2+5/7*B*a*x+A*a)*x^2*c
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.62

$$\int (cx)^{3/2}\left(a+bx^2\right)\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{2}{45045}\left(3003\,Dbcx^7+3465\,Cbcx^6+4095\,(Da+Bb)cx^5+6435\,Bacx^3+5005\,(Ca+Ab)cx^4\right.$$

input
```
integrate((c*x)^(3/2)*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
2/45045*(3003*D*b*c*x^7 + 3465*C*b*c*x^6 + 4095*(D*a + B*b)*c*x^5 + 6435*B
*a*c*x^3 + 5005*(C*a + A*b)*c*x^4 + 9009*A*a*c*x^2)*sqrt(c*x)
```

## Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.24

$$\int (cx)^{3/2}\left(a+bx^2\right)\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{2Aax(cx)^{\frac{3}{2}}}{5}+\frac{2Abx^3(cx)^{\frac{3}{2}}}{9}+\frac{2Bax^2(cx)^{\frac{3}{2}}}{7}+\frac{2Bbx^4(cx)^{\frac{3}{2}}}{11}+\frac{2Cax^3(cx)^{\frac{3}{2}}}{9}+\frac{2Cbx^5(cx)^{\frac{3}{2}}}{13}+\frac{2Dax^4(cx)^{\frac{3}{2}}}{11}+\frac{2Dbx^6(cx)^{\frac{3}{2}}}{15}$$

input
```
integrate((c*x)**(3/2)*(b*x**2+a)*(D*x**3+C*x**2+B*x+A),x)
```

output
```
2*A*a*x*(c*x)**(3/2)/5 + 2*A*b*x**3*(c*x)**(3/2)/9 + 2*B*a*x**2*(c*x)**(3/
2)/7 + 2*B*b*x**4*(c*x)**(3/2)/11 + 2*C*a*x**3*(c*x)**(3/2)/9 + 2*C*b*x**5
*(c*x)**(3/2)/13 + 2*D*a*x**4*(c*x)**(3/2)/11 + 2*D*b*x**6*(c*x)**(3/2)/15
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.78

$$\int (cx)^{3/2} (a+bx^2) (A+Bx+Cx^2+Dx^3) \, dx = \frac{2 \left(3003 (cx)^{\frac{15}{2}} Db + 3465 (cx)^{\frac{13}{2}} Cbc + 6435 (cx)^{\frac{7}{2}} Bac^4 + 9009 (cx)^{\frac{5}{2}} Aac^5 + 4095 (Da+Bb)\right.}{45045 c^6}$$

input
```
integrate((c*x)^(3/2)*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
2/45045*(3003*(c*x)^(15/2)*D*b + 3465*(c*x)^(13/2)*C*b*c + 6435*(c*x)^(7/2
)*B*a*c^4 + 9009*(c*x)^(5/2)*A*a*c^5 + 4095*(D*a + B*b)*(c*x)^(11/2)*c^2 +
 5005*(C*a + A*b)*(c*x)^(9/2)*c^3)/c^6
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.93

$$\int (cx)^{3/2} (a+bx^2) (A+Bx+Cx^2+Dx^3) \, dx = \frac{2}{45045} \left(3003 \sqrt{cx} Dbx^7 + 3465 \sqrt{cx} Cbx^6 + 4095 \sqrt{cx} Dax^5 + 4095 \sqrt{cx} Bbx^5 + 5005 \sqrt{cx} Cax^4\right.$$

input
```
integrate((c*x)^(3/2)*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

output
```
2/45045*(3003*sqrt(c*x)*D*b*x^7 + 3465*sqrt(c*x)*C*b*x^6 + 4095*sqrt(c*x)*
D*a*x^5 + 4095*sqrt(c*x)*B*b*x^5 + 5005*sqrt(c*x)*C*a*x^4 + 5005*sqrt(c*x)
*A*b*x^4 + 6435*sqrt(c*x)*B*a*x^3 + 9009*sqrt(c*x)*A*a*x^2)*c
```

## Mupad [F(-1)]

Timed out.

$$\int (cx)^{3/2}\left(a+bx^2\right)\left(A+Bx+Cx^2+Dx^3\right)\,dx=\int (c\,x)^{3/2}\left(b\,x^2+a\right)\left(A+B\,x+C\,x^2+x^3\,D\right)\,dx$$

input
```
int((c*x)^(3/2)*(a + b*x^2)*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
int((c*x)^(3/2)*(a + b*x^2)*(A + B*x + C*x^2 + x^3*D), x)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.60

$$\int (cx)^{3/2}\left(a+bx^2\right)\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{2\sqrt{x}\,\sqrt{c}\,c\,x^2(3003bd\,x^5+3465bc\,x^4+4095ad\,x^3+4095b^2x^3+5005ab\,x^2+5005ac\,x^2+6435a}{45045}$$

input
```
int((c*x)^(3/2)*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x)
```

output
```
(2*sqrt(x)*sqrt(c)*c*x**2*(9009*a**2 + 5005*a*b*x**2 + 6435*a*b*x + 5005*a
*c*x**2 + 4095*a*d*x**3 + 4095*b**2*x**3 + 3465*b*c*x**4 + 3003*b*d*x**5))
/45045
```

# 3.118 $\int \sqrt{cx}(a+bx^2)\left(A+Bx+Cx^2+Dx^3\right)\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 107

$$\int \sqrt{cx}\left(a+bx^2\right)\left(A+Bx+Cx^2+Dx^3\right)\,dx = \frac{2aA(cx)^{3/2}}{3c}+\frac{2aB(cx)^{5/2}}{5c^2}+\frac{2(Ab+aC)(cx)^{7/2}}{7c^3}+\frac{2(bB+aD)(cx)^{9/2}}{9c^4}+\frac{2bC(cx)^{11/2}}{11c^5}+\frac{2bD(cx)^{13/2}}{13c^6}$$

output
```
2/3*a*A*(c*x)^(3/2)/c+2/5*a*B*(c*x)^(5/2)/c^2+2/7*(A*b+C*a)*(c*x)^(7/2)/c^
3+2/9*(B*b+D*a)*(c*x)^(9/2)/c^4+2/11*b*C*(c*x)^(11/2)/c^5+2/13*b*D*(c*x)^(
13/2)/c^6
```

## Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.61

$$\int \sqrt{cx}\left(a+bx^2\right)\left(A+Bx+Cx^2+Dx^3\right)\,dx = \frac{2x\sqrt{cx}(143a(105A+x(63B+5x(9C+7Dx)))+5bx^2(1287A+7x(143B+9x(13C+11Dx))))}{45045}$$

input
```
Integrate[Sqrt[c*x]*(a + b*x^2)*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(2*x*Sqrt[c*x]*(143*a*(105*A + x*(63*B + 5*x*(9*C + 7*D*x))) + 5*b*x^2*(12
87*A + 7*x*(143*B + 9*x*(13*C + 11*D*x)))))/45045
```

## Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{cx}\left(a+bx^2\right)\left(A+Bx+Cx^2+Dx^3\right)\,dx$$

$$\Big\downarrow \text{2333}$$

$$\int \left(\frac{(cx)^{5/2}(aC+Ab)}{c^2}+aA\sqrt{cx}+\frac{(cx)^{7/2}(aD+bB)}{c^3}+\frac{aB(cx)^{3/2}}{c}+\frac{bD(cx)^{11/2}}{c^5}+\frac{bC(cx)^{9/2}}{c^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2(cx)^{7/2}(aC+Ab)}{7c^3}+\frac{2aA(cx)^{3/2}}{3c}+\frac{2(cx)^{9/2}(aD+bB)}{9c^4}+\frac{2aB(cx)^{5/2}}{5c^2}+\frac{2bD(cx)^{13/2}}{13c^6}+\frac{2bC(cx)^{11/2}}{11c^5}$$

input
```
Int[Sqrt[c*x]*(a + b*x^2)*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(2*a*A*(c*x)^(3/2))/(3*c) + (2*a*B*(c*x)^(5/2))/(5*c^2) + (2*(A*b + a*C)*(
c*x)^(7/2))/(7*c^3) + (2*(b*B + a*D)*(c*x)^(9/2))/(9*c^4) + (2*b*C*(c*x)^(
11/2))/(11*c^5) + (2*b*D*(c*x)^(13/2))/(13*c^6)
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

## Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.52

| method | result | size |
|---|---|---|
| pseudoelliptic | $\frac{2\left(\frac{3 D b \,x^{5}}{13}+\frac{3 C b \,x^{4}}{11}+\frac{\left(B b +D a \right) x^{3}}{3}+\frac{3 \left(A b +C a \right) x^{2}}{7}+\frac{3 B a x}{5}+A a \right) \sqrt{c x}\, x}{3}$ | 56 |
| gosper | $\frac{2 x \left(3465 D b \,x^{5}+4095 C b \,x^{4}+5005 b B \,x^{3}+5005 D a \,x^{3}+6435 A b \,x^{2}+6435 C a \,x^{2}+9009 B a x +15015 A a \right) \sqrt{c x}}{45045}$ | 61 |
| trager | $\frac{2 x \left(3465 D b \,x^{5}+4095 C b \,x^{4}+5005 b B \,x^{3}+5005 D a \,x^{3}+6435 A b \,x^{2}+6435 C a \,x^{2}+9009 B a x +15015 A a \right) \sqrt{c x}}{45045}$ | 61 |
| orering | $\frac{2 x \left(3465 D b \,x^{5}+4095 C b \,x^{4}+5005 b B \,x^{3}+5005 D a \,x^{3}+6435 A b \,x^{2}+6435 C a \,x^{2}+9009 B a x +15015 A a \right) \sqrt{c x}}{45045}$ | 61 |
| derivativedivides | $\frac{\frac{2 D b \left(c x \right)^{\frac{13}{2}}}{13}+\frac{2 b C c \left(c x \right)^{\frac{11}{2}}}{11}+\frac{2 \left(B b \,c^{2}+a \,c^{2} D\right) \left(c x \right)^{\frac{9}{2}}}{9}+\frac{2 \left(A b \,c^{3}+C a \,c^{3}\right) \left(c x \right)^{\frac{7}{2}}}{7}+\frac{2 a \,c^{4} B \left(c x \right)^{\frac{5}{2}}}{5}+\frac{2 a \,c^{5} A \left(c x \right)^{\frac{3}{2}}}{3}}{c^{6}}$ | 90 |
| default | $\frac{\frac{2 D b \left(c x \right)^{\frac{13}{2}}}{13}+\frac{2 b C c \left(c x \right)^{\frac{11}{2}}}{11}+\frac{2 \left(B b \,c^{2}+a \,c^{2} D\right) \left(c x \right)^{\frac{9}{2}}}{9}+\frac{2 \left(A b \,c^{3}+C a \,c^{3}\right) \left(c x \right)^{\frac{7}{2}}}{7}+\frac{2 a \,c^{4} B \left(c x \right)^{\frac{5}{2}}}{5}+\frac{2 a \,c^{5} A \left(c x \right)^{\frac{3}{2}}}{3}}{c^{6}}$ | 90 |

input
```
int((c*x)^(1/2)*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
2/3*(3/13*D*b*x^5+3/11*C*b*x^4+1/3*(B*b+D*a)*x^3+3/7*(A*b+C*a)*x^2+3/5*B*a
*x+A*a)*(c*x)^(1/2)*x
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.54

$$\int \sqrt{cx}(a+bx^2)\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$=\frac{2}{45045}\left(3465\,Dbx^6+4095\,Cbx^5+5005\,(Da+Bb)x^4+9009\,Bax^2+6435\,(Ca+Ab)x^3+15015\,Aax\right)\sqrt{}$$

input
```
integrate((c*x)^(1/2)*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
2/45045*(3465*D*b*x^6 + 4095*C*b*x^5 + 5005*(D*a + B*b)*x^4 + 9009*B*a*x^2
 + 6435*(C*a + A*b)*x^3 + 15015*A*a*x)*sqrt(c*x)
```

## Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.24

$$\int \sqrt{cx}(a+bx^2)\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$=\frac{2Aax\sqrt{cx}}{3}+\frac{2Abx^3\sqrt{cx}}{7}+\frac{2Bax^2\sqrt{cx}}{5}+\frac{2Bbx^4\sqrt{cx}}{9}+\frac{2Cax^3\sqrt{cx}}{7}+\frac{2Cbx^5\sqrt{cx}}{11}+\frac{2Dax^4\sqrt{cx}}{9}+\frac{2Dbx^6\sqrt{cx}}{13}$$

input
```
integrate((c*x)**(1/2)*(b*x**2+a)*(D*x**3+C*x**2+B*x+A),x)
```

output
```
2*A*a*x*sqrt(c*x)/3 + 2*A*b*x**3*sqrt(c*x)/7 + 2*B*a*x**2*sqrt(c*x)/5 + 2*
B*b*x**4*sqrt(c*x)/9 + 2*C*a*x**3*sqrt(c*x)/7 + 2*C*b*x**5*sqrt(c*x)/11 +
2*D*a*x**4*sqrt(c*x)/9 + 2*D*b*x**6*sqrt(c*x)/13
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.78

$$\int \sqrt{cx}(a+bx^2)\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$=\frac{2\left(3465\,(cx)^{\frac{13}{2}}\,Db+4095\,(cx)^{\frac{11}{2}}\,Cbc+9009\,(cx)^{\frac{5}{2}}\,Bac^4+15015\,(cx)^{\frac{3}{2}}\,Aac^5+5005\,(Da+Bb)(cx)^{\frac{9}{2}}\,c^2\right.}{45045\,c^6}$$

input
```
integrate((c*x)^(1/2)*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
2/45045*(3465*(c*x)^(13/2)*D*b + 4095*(c*x)^(11/2)*C*b*c + 9009*(c*x)^(5/2
)*B*a*c^4 + 15015*(c*x)^(3/2)*A*a*c^5 + 5005*(D*a + B*b)*(c*x)^(9/2)*c^2 +
 6435*(C*a + A*b)*(c*x)^(7/2)*c^3)/c^6
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.89

$$\int \sqrt{cx}(a+bx^2)\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$=\frac{2}{13}\sqrt{cx}Dbx^6+\frac{2}{11}\sqrt{cx}Cbx^5+\frac{2}{9}\sqrt{cx}Dax^4+\frac{2}{9}\sqrt{cx}Bbx^4$$
$$+\frac{2}{7}\sqrt{cx}Cax^3+\frac{2}{7}\sqrt{cx}Abx^3+\frac{2}{5}\sqrt{cx}Bax^2+\frac{2}{3}\sqrt{cx}Aax$$

input
```
integrate((c*x)^(1/2)*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

output
```
2/13*sqrt(c*x)*D*b*x^6 + 2/11*sqrt(c*x)*C*b*x^5 + 2/9*sqrt(c*x)*D*a*x^4 +
2/9*sqrt(c*x)*B*b*x^4 + 2/7*sqrt(c*x)*C*a*x^3 + 2/7*sqrt(c*x)*A*b*x^3 + 2/
5*sqrt(c*x)*B*a*x^2 + 2/3*sqrt(c*x)*A*a*x
```

## Mupad [F(-1)]

Timed out.

$$\int \sqrt{cx}(a+bx^2)\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$=\int \sqrt{c\,x}\,\left(b\,x^2+a\right)\,\left(A+B\,x+C\,x^2+x^3\,D\right)\,dx$$

input
```
int((c*x)^(1/2)*(a + b*x^2)*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
int((c*x)^(1/2)*(a + b*x^2)*(A + B*x + C*x^2 + x^3*D), x)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.57

$$\int \sqrt{cx}(a+bx^2)\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$=\frac{2\sqrt{x}\,\sqrt{c}\,x(3465bd\,x^5+4095bc\,x^4+5005ad\,x^3+5005b^2x^3+6435ab\,x^2+6435ac\,x^2+9009abx+15015a}{45045}$$

input
```
int((c*x)^(1/2)*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x)
```

output
```
(2*sqrt(x)*sqrt(c)*x*(15015*a**2 + 6435*a*b*x**2 + 9009*a*b*x + 6435*a*c*x
**2 + 5005*a*d*x**3 + 5005*b**2*x**3 + 4095*b*c*x**4 + 3465*b*d*x**5))/450
45
```

# 3.119 $\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{\sqrt{cx}}\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 105

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{\sqrt{cx}}\,dx = \frac{2aA\sqrt{cx}}{c}+\frac{2aB(cx)^{3/2}}{3c^2}+\frac{2(Ab+aC)(cx)^{5/2}}{5c^3}+\frac{2(bB+aD)(cx)^{7/2}}{7c^4}+\frac{2bC(cx)^{9/2}}{9c^5}+\frac{2bD(cx)^{11/2}}{11c^6}$$

output
```
2*a*A*(c*x)^(1/2)/c+2/3*a*B*(c*x)^(3/2)/c^2+2/5*(A*b+C*a)*(c*x)^(5/2)/c^3+
2/7*(B*b+D*a)*(c*x)^(7/2)/c^4+2/9*b*C*(c*x)^(9/2)/c^5+2/11*b*D*(c*x)^(11/2
)/c^6
```

## Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.62

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{\sqrt{cx}}\,dx = \frac{66ax(105A+x(35B+3x(7C+5Dx)))+2bx^3(693A+5x(99B+7x(11C+9Dx)))}{3465\sqrt{cx}}$$

input
```
Integrate[((a + b*x^2)*(A + B*x + C*x^2 + D*x^3))/Sqrt[c*x],x]
```

output
```
(66*a*x*(105*A + x*(35*B + 3*x*(7*C + 5*D*x))) + 2*b*x^3*(693*A + 5*x*(99*
B + 7*x*(11*C + 9*D*x))))/(3465*Sqrt[c*x])
```

## Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{\sqrt{cx}}dx$$

$$\Big\downarrow \text{2333}$$

$$\int \left(\frac{(cx)^{3/2}(aC+Ab)}{c^2}+\frac{aA}{\sqrt{cx}}+\frac{(cx)^{5/2}(aD+bB)}{c^3}+\frac{aB\sqrt{cx}}{c}+\frac{bD(cx)^{9/2}}{c^5}+\frac{bC(cx)^{7/2}}{c^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2(cx)^{5/2}(aC+Ab)}{5c^3}+\frac{2aA\sqrt{cx}}{c}+\frac{2(cx)^{7/2}(aD+bB)}{7c^4}+\frac{2aB(cx)^{3/2}}{3c^2}+\frac{2bD(cx)^{11/2}}{11c^6}+\frac{2bC(cx)^{9/2}}{9c^5}$$

input
```
Int[((a + b*x^2)*(A + B*x + C*x^2 + D*x^3))/Sqrt[c*x],x]
```

output
```
(2*a*A*Sqrt[c*x])/c + (2*a*B*(c*x)^(3/2))/(3*c^2) + (2*(A*b + a*C)*(c*x)^(
5/2))/(5*c^3) + (2*(b*B + a*D)*(c*x)^(7/2))/(7*c^4) + (2*b*C*(c*x)^(9/2))/
(9*c^5) + (2*b*D*(c*x)^(11/2))/(11*c^6)
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

## Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.58

| method | result | size |
|---|---|---|
| gosper | $\frac{2x\left(315Db\,x^{5}+385Cb\,x^{4}+495bB\,x^{3}+495Da\,x^{3}+693Ab\,x^{2}+693Ca\,x^{2}+1155Bax+3465Aa\right)}{3465\sqrt{cx}}$ | 61 |
| orering | $\frac{2x\left(315Db\,x^{5}+385Cb\,x^{4}+495bB\,x^{3}+495Da\,x^{3}+693Ab\,x^{2}+693Ca\,x^{2}+1155Bax+3465Aa\right)}{3465\sqrt{cx}}$ | 61 |
| trager | $\frac{\left(\frac{2}{11}Db\,x^{5}+\frac{2}{9}Cb\,x^{4}+\frac{2}{7}bB\,x^{3}+\frac{2}{7}Da\,x^{3}+\frac{2}{5}Ab\,x^{2}+\frac{2}{5}Ca\,x^{2}+\frac{2}{3}Bax+2Aa\right)\sqrt{cx}}{c}$ | 62 |
| pseudoelliptic | $\frac{2\sqrt{cx}\,\left(315Db\,x^{5}+385Cb\,x^{4}+495bB\,x^{3}+495Da\,x^{3}+693Ab\,x^{2}+693Ca\,x^{2}+1155Bax+3465Aa\right)}{3465c}$ | 63 |
| derivativedivides | $\frac{\frac{2Db(cx)^{\frac{11}{2}}}{11}+\frac{2bCc(cx)^{\frac{9}{2}}}{9}+\frac{2\left(Bb\,c^{2}+a\,c^{2}D\right)(cx)^{\frac{7}{2}}}{7}+\frac{2\left(Ab\,c^{3}+Ca\,c^{3}\right)(cx)^{\frac{5}{2}}}{5}+\frac{2a\,c^{4}B(cx)^{\frac{3}{2}}}{3}+2a\,c^{5}A\sqrt{cx}}{c^{6}}$ | 89 |
| default | $\frac{\frac{2Db(cx)^{\frac{11}{2}}}{11}+\frac{2bCc(cx)^{\frac{9}{2}}}{9}+\frac{2\left(Bb\,c^{2}+a\,c^{2}D\right)(cx)^{\frac{7}{2}}}{7}+\frac{2\left(Ab\,c^{3}+Ca\,c^{3}\right)(cx)^{\frac{5}{2}}}{5}+\frac{2a\,c^{4}B(cx)^{\frac{3}{2}}}{3}+2a\,c^{5}A\sqrt{cx}}{c^{6}}$ | 89 |

input
```
int((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/(c*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3465*x*(315*D*b*x^5+385*C*b*x^4+495*B*b*x^3+495*D*a*x^3+693*A*b*x^2+693*
C*a*x^2+1155*B*a*x+3465*A*a)/(c*x)^(1/2)
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.55

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{\sqrt{cx}}\,dx = \frac{2\,(315\,Dbx^5+385\,Cbx^4+495\,(Da+Bb)x^3+1155\,Bax+693\,(Ca+Ab)x^2+3465\,Aa)\sqrt{cx}}{3465\,c}$$

input
```
integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/(c*x)^(1/2),x, algorithm="fricas")
```

output
```
2/3465*(315*D*b*x^5 + 385*C*b*x^4 + 495*(D*a + B*b)*x^3 + 1155*B*a*x + 693
*(C*a + A*b)*x^2 + 3465*A*a)*sqrt(c*x)/c
```

## Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.25

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{\sqrt{cx}}\,dx = \frac{2Aax}{\sqrt{cx}} + \frac{2Abx^3}{5\sqrt{cx}} + \frac{2Bax^2}{3\sqrt{cx}} + \frac{2Bbx^4}{7\sqrt{cx}} + \frac{2Cax^3}{5\sqrt{cx}} + \frac{2Cbx^5}{9\sqrt{cx}} + \frac{2Dax^4}{7\sqrt{cx}} + \frac{2Dbx^6}{11\sqrt{cx}}$$

input
```
integrate((b*x**2+a)*(D*x**3+C*x**2+B*x+A)/(c*x)**(1/2),x)
```

output
```
2*A*a*x/sqrt(c*x) + 2*A*b*x**3/(5*sqrt(c*x)) + 2*B*a*x**2/(3*sqrt(c*x)) +
2*B*b*x**4/(7*sqrt(c*x)) + 2*C*a*x**3/(5*sqrt(c*x)) + 2*C*b*x**5/(9*sqrt(c
*x)) + 2*D*a*x**4/(7*sqrt(c*x)) + 2*D*b*x**6/(11*sqrt(c*x))
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.79

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{\sqrt{cx}}\,dx$$
$$= \frac{2\left(315\,(cx)^{\frac{11}{2}}\,Db+385\,(cx)^{\frac{9}{2}}\,Cbc+1155\,(cx)^{\frac{3}{2}}\,Bac^4+3465\,\sqrt{cx}Aac^5+495\,(Da+Bb)(cx)^{\frac{7}{2}}\,c^2+693\,(\right.}{3465\,c^6}$$

input
```
integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/(c*x)^(1/2),x, algorithm="maxima")
```

output
```
2/3465*(315*(c*x)^(11/2)*D*b + 385*(c*x)^(9/2)*C*b*c + 1155*(c*x)^(3/2)*B*
a*c^4 + 3465*sqrt(c*x)*A*a*c^5 + 495*(D*a + B*b)*(c*x)^(7/2)*c^2 + 693*(C*
a + A*b)*(c*x)^(5/2)*c^3)/c^6
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{\sqrt{cx}}\,dx$$
$$= \frac{2\left(315\,\sqrt{cx}Dbx^5+385\,\sqrt{cx}Cbx^4+495\,\sqrt{cx}Dax^3+495\,\sqrt{cx}Bbx^3+693\,\sqrt{cx}Cax^2+693\,\sqrt{cx}Abx^2+11\right.}{3465\,c}$$

input
```
integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/(c*x)^(1/2),x, algorithm="giac")
```

output
```
2/3465*(315*sqrt(c*x)*D*b*x^5 + 385*sqrt(c*x)*C*b*x^4 + 495*sqrt(c*x)*D*a*
x^3 + 495*sqrt(c*x)*B*b*x^3 + 693*sqrt(c*x)*C*a*x^2 + 693*sqrt(c*x)*A*b*x^
2 + 1155*sqrt(c*x)*B*a*x + 3465*sqrt(c*x)*A*a)/c
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{\sqrt{cx}}\,dx=\int \frac{(b\,x^2+a)\,(A+B\,x+C\,x^2+x^3\,D)}{\sqrt{c\,x}}\,dx$$

input
```
int(((a + b*x^2)*(A + B*x + C*x^2 + x^3*D))/(c*x)^(1/2),x)
```

output
```
int(((a + b*x^2)*(A + B*x + C*x^2 + x^3*D))/(c*x)^(1/2), x)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.60

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{\sqrt{cx}}\,dx$$
$$=\frac{2\sqrt{x}\,\sqrt{c}\,(315bd\,x^5+385bc\,x^4+495ad\,x^3+495b^2x^3+693ab\,x^2+693ac\,x^2+1155abx+3465a^2)}{3465c}$$

input
```
int((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/(c*x)^(1/2),x)
```

output
```
(2*sqrt(x)*sqrt(c)*(3465*a**2 + 693*a*b*x**2 + 1155*a*b*x + 693*a*c*x**2 +
 495*a*d*x**3 + 495*b**2*x**3 + 385*b*c*x**4 + 315*b*d*x**5))/(3465*c)
```

# 3.120 $\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{(cx)^{3/2}}\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 103

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{(cx)^{3/2}}\,dx = -\frac{2aA}{c\sqrt{cx}}+\frac{2aB\sqrt{cx}}{c^2}+\frac{2(Ab+aC)(cx)^{3/2}}{3c^3}+\frac{2(bB+aD)(cx)^{5/2}}{5c^4}+\frac{2bC(cx)^{7/2}}{7c^5}+\frac{2bD(cx)^{9/2}}{9c^6}$$

output
```
-2*a*A/c/(c*x)^(1/2)+2*a*B*(c*x)^(1/2)/c^2+2/3*(A*b+C*a)*(c*x)^(3/2)/c^3+2
/5*(B*b+D*a)*(c*x)^(5/2)/c^4+2/7*b*C*(c*x)^(7/2)/c^5+2/9*b*D*(c*x)^(9/2)/c
^6
```

## Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.62

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{(cx)^{3/2}}\,dx = -\frac{2x(315aA-315aBx-105Abx^2-105aCx^2-63bBx^3-63aDx^3-45bCx^4-35bDx^5)}{315(cx)^{3/2}}$$

input
```
Integrate[((a + b*x^2)*(A + B*x + C*x^2 + D*x^3))/(c*x)^(3/2),x]
```

output
```
(-2*x*(315*a*A - 315*a*B*x - 105*A*b*x^2 - 105*a*C*x^2 - 63*b*B*x^3 - 63*a
*D*x^3 - 45*b*C*x^4 - 35*b*D*x^5))/(315*(c*x)^(3/2))
```

## Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)\left(A+Bx+Cx^2+Dx^3\right)}{(cx)^{3/2}}dx$$

$$\Big\downarrow \text{ 2333}$$

$$\int \left(\frac{\sqrt{cx}(aC+Ab)}{c^2}+\frac{aA}{(cx)^{3/2}}+\frac{(cx)^{3/2}(aD+bB)}{c^3}+\frac{aB}{c\sqrt{cx}}+\frac{bD(cx)^{7/2}}{c^5}+\frac{bC(cx)^{5/2}}{c^4}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2(cx)^{3/2}(aC+Ab)}{3c^3}-\frac{2aA}{c\sqrt{cx}}+\frac{2(cx)^{5/2}(aD+bB)}{5c^4}+\frac{2aB\sqrt{cx}}{c^2}+\frac{2bD(cx)^{9/2}}{9c^6}+\frac{2bC(cx)^{7/2}}{7c^5}$$

input
```
Int[((a + b*x^2)*(A + B*x + C*x^2 + D*x^3))/(c*x)^(3/2),x]
```

output
```
(-2*a*A)/(c*Sqrt[c*x]) + (2*a*B*Sqrt[c*x])/c^2 + (2*(A*b + a*C)*(c*x)^(3/2
))/(3*c^3) + (2*(b*B + a*D)*(c*x)^(5/2))/(5*c^4) + (2*b*C*(c*x)^(7/2))/(7*
c^5) + (2*b*D*(c*x)^(9/2))/(9*c^6)
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

## Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.59

| method | result | size |
|---|---|---|
| gosper | $-\frac{2x\left(-35Db\,x^{5}-45Cb\,x^{4}-63bB\,x^{3}-63Da\,x^{3}-105Ab\,x^{2}-105Ca\,x^{2}-315Bax+315Aa\right)}{315\left(cx\right)^{\frac{3}{2}}}$ | 61 |
| orering | $-\frac{2x\left(-35Db\,x^{5}-45Cb\,x^{4}-63bB\,x^{3}-63Da\,x^{3}-105Ab\,x^{2}-105Ca\,x^{2}-315Bax+315Aa\right)}{315\left(cx\right)^{\frac{3}{2}}}$ | 61 |
| pseudoelliptic | $-\frac{2\left(-35Db\,x^{5}-45Cb\,x^{4}-63bB\,x^{3}-63Da\,x^{3}-105Ab\,x^{2}-105Ca\,x^{2}-315Bax+315Aa\right)}{315c\sqrt{cx}}$ | 63 |
| trager | $-\frac{2\left(-35Db\,x^{5}-45Cb\,x^{4}-63bB\,x^{3}-63Da\,x^{3}-105Ab\,x^{2}-105Ca\,x^{2}-315Bax+315Aa\right)\sqrt{cx}}{315c^{2}x}$ | 66 |
| derivativedivides | $\frac{\frac{2Db\left(cx\right)^{\frac{9}{2}}}{9}+\frac{2Cbc\left(cx\right)^{\frac{7}{2}}}{7}+\frac{2Bb\,c^{2}\left(cx\right)^{\frac{5}{2}}}{5}+\frac{2Da\,c^{2}\left(cx\right)^{\frac{5}{2}}}{5}+\frac{2Ab\,c^{3}\left(cx\right)^{\frac{3}{2}}}{3}+\frac{2Ca\,c^{3}\left(cx\right)^{\frac{3}{2}}}{3}+2Ba\,c^{4}\sqrt{cx}-\frac{2Aa\,c^{5}}{\sqrt{cx}}}{c^{6}}$ | 97 |
| default | $\frac{\frac{2Db\left(cx\right)^{\frac{9}{2}}}{9}+\frac{2Cbc\left(cx\right)^{\frac{7}{2}}}{7}+\frac{2Bb\,c^{2}\left(cx\right)^{\frac{5}{2}}}{5}+\frac{2Da\,c^{2}\left(cx\right)^{\frac{5}{2}}}{5}+\frac{2Ab\,c^{3}\left(cx\right)^{\frac{3}{2}}}{3}+\frac{2Ca\,c^{3}\left(cx\right)^{\frac{3}{2}}}{3}+2Ba\,c^{4}\sqrt{cx}-\frac{2Aa\,c^{5}}{\sqrt{cx}}}{c^{6}}$ | 97 |

input
```
int((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/(c*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/315*x*(-35*D*b*x^5-45*C*b*x^4-63*B*b*x^3-63*D*a*x^3-105*A*b*x^2-105*C*a
*x^2-315*B*a*x+315*A*a)/(c*x)^(3/2)
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.59

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{(cx)^{3/2}}\,dx = \frac{2\,(35\,Dbx^5+45\,Cbx^4+63\,(Da+Bb)x^3+315\,Bax+105\,(Ca}{315\,c^2x}$$

input
```
integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/(c*x)^(3/2),x, algorithm="fricas")
```

output
```
2/315*(35*D*b*x^5 + 45*C*b*x^4 + 63*(D*a + B*b)*x^3 + 315*B*a*x + 105*(C*a
 + A*b)*x^2 - 315*A*a)*sqrt(c*x)/(c^2*x)
```

## Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.25

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{(cx)^{3/2}}\,dx = -\frac{2Aax}{(cx)^{\frac{3}{2}}}+\frac{2Abx^3}{3\,(cx)^{\frac{3}{2}}}+\frac{2Bax^2}{(cx)^{\frac{3}{2}}}+\frac{2Bbx^4}{5\,(cx)^{\frac{3}{2}}}+\frac{2Cax^3}{3\,(cx)^{\frac{3}{2}}}+\frac{2Cbx^5}{7\,(cx)^{\frac{3}{2}}}+\frac{2Dax^4}{5\,(cx)^{\frac{3}{2}}}+\frac{2Dbx^6}{9\,(cx)^{\frac{3}{2}}}$$

input
```
integrate((b*x**2+a)*(D*x**3+C*x**2+B*x+A)/(c*x)**(3/2),x)
```

output
```
-2*A*a*x/(c*x)**(3/2) + 2*A*b*x**3/(3*(c*x)**(3/2)) + 2*B*a*x**2/(c*x)**(3
/2) + 2*B*b*x**4/(5*(c*x)**(3/2)) + 2*C*a*x**3/(3*(c*x)**(3/2)) + 2*C*b*x*
*5/(7*(c*x)**(3/2)) + 2*D*a*x**4/(5*(c*x)**(3/2)) + 2*D*b*x**6/(9*(c*x)**(
3/2))
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.83

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{(cx)^{3/2}}\,dx = -\frac{2\left(\frac{315\,Aa}{\sqrt{cx}}-\frac{35\,(cx)^{\frac{9}{2}}Db+45\,(cx)^{\frac{7}{2}}Cbc+315\,\sqrt{cx}Bac^4+63\,(Da+Bb)(cx)^{\frac{5}{2}}c^2+105\,(Ca+Ab)(cx)^{\frac{3}{2}}c^3}{c^5}\right)}{315\,c}$$

input
```
integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/(c*x)^(3/2),x, algorithm="maxima")
```

output
```
-2/315*(315*A*a/sqrt(c*x) - (35*(c*x)^(9/2)*D*b + 45*(c*x)^(7/2)*C*b*c + 3
15*sqrt(c*x)*B*a*c^4 + 63*(D*a + B*b)*(c*x)^(5/2)*c^2 + 105*(C*a + A*b)*(c
*x)^(3/2)*c^3)/c^5)/c
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.16

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{(cx)^{3/2}}\,dx = -\frac{2\left(\frac{315\,Aa}{\sqrt{cx}}-\frac{35\,\sqrt{cx}Dbc^{44}x^4+45\,\sqrt{cx}Cbc^{44}x^3+63\,\sqrt{cx}Dac^{44}x^2+63\,\sqrt{cx}Bbc^{44}x^2+105\,\sqrt{cx}Cac^{44}x+105\,\sqrt{cx}Abc^{44}x+315\,\sqrt{cx}Bac^{44}}{c^{45}}\right)}{315\,c}$$

input
```
integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/(c*x)^(3/2),x, algorithm="giac")
```

output
```
-2/315*(315*A*a/sqrt(c*x) - (35*sqrt(c*x)*D*b*c^44*x^4 + 45*sqrt(c*x)*C*b*
c^44*x^3 + 63*sqrt(c*x)*D*a*c^44*x^2 + 63*sqrt(c*x)*B*b*c^44*x^2 + 105*sqr
t(c*x)*C*a*c^44*x + 105*sqrt(c*x)*A*b*c^44*x + 315*sqrt(c*x)*B*a*c^44)/c^4
5)/c
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{(cx)^{3/2}}\,dx=\int \frac{(b\,x^2+a)\,\left(A+B\,x+C\,x^2+x^3\,D\right)}{{\left(c\,x\right)}^{3/2}}\,dx$$

input
```
int(((a + b*x^2)*(A + B*x + C*x^2 + x^3*D))/(c*x)^(3/2),x)
```

output
```
int(((a + b*x^2)*(A + B*x + C*x^2 + x^3*D))/(c*x)^(3/2), x)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.63

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{(cx)^{3/2}}\,dx=\frac{2\sqrt{c}\,(35bd\,x^5+45bc\,x^4+63ad\,x^3+63b^2x^3+105ab\,x^2+105ac}{315\sqrt{x}\,c^2}$$

input
```
int((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/(c*x)^(3/2),x)
```

output
```
(2*sqrt(c)*( - 315*a**2 + 105*a*b*x**2 + 315*a*b*x + 105*a*c*x**2 + 63*a*d
*x**3 + 63*b**2*x**3 + 45*b*c*x**4 + 35*b*d*x**5))/(315*sqrt(x)*c**2)
```

# 3.121 $\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{(cx)^{5/2}}\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 103

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{(cx)^{5/2}}\,dx = -\frac{2aA}{3c(cx)^{3/2}} - \frac{2aB}{c^2\sqrt{cx}} + \frac{2(Ab+aC)\sqrt{cx}}{c^3} + \frac{2(bB+aD)(cx)^{3/2}}{3c^4} + \frac{2bC(cx)^{5/2}}{5c^5} + \frac{2bD(cx)^{7/2}}{7c^6}$$

output
```
-2/3*a*A/c/(c*x)^(3/2)-2*a*B/c^2/(c*x)^(1/2)+2*(A*b+C*a)*(c*x)^(1/2)/c^3+2
/3*(B*b+D*a)*(c*x)^(3/2)/c^4+2/5*b*C*(c*x)^(5/2)/c^5+2/7*b*D*(c*x)^(7/2)/c
^6
```

## Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.59

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{(cx)^{5/2}}\,dx = \frac{x(-70a(A+3Bx-x^2(3C+Dx))+2bx^2(105A+x(35B+3x}{105(cx)^{5/2}}$$

input
```
Integrate[((a + b*x^2)*(A + B*x + C*x^2 + D*x^3))/(c*x)^(5/2),x]
```

output
```
(x*(-70*a*(A + 3*B*x - x^2*(3*C + D*x)) + 2*b*x^2*(105*A + x*(35*B + 3*x*(
7*C + 5*D*x)))))/(105*(c*x)^(5/2))
```

## Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{(cx)^{5/2}}dx$$

$$\Big\downarrow \text{2333}$$

$$\int \left(\frac{aC+Ab}{c^2\sqrt{cx}}+\frac{aA}{(cx)^{5/2}}+\frac{\sqrt{cx}(aD+bB)}{c^3}+\frac{aB}{c(cx)^{3/2}}+\frac{bD(cx)^{5/2}}{c^5}+\frac{bC(cx)^{3/2}}{c^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\sqrt{cx}(aC+Ab)}{c^3}-\frac{2aA}{3c(cx)^{3/2}}+\frac{2(cx)^{3/2}(aD+bB)}{3c^4}-\frac{2aB}{c^2\sqrt{cx}}+\frac{2bD(cx)^{7/2}}{7c^6}+\frac{2bC(cx)^{5/2}}{5c^5}$$

input
```
Int[((a + b*x^2)*(A + B*x + C*x^2 + D*x^3))/(c*x)^(5/2),x]
```

output
```
(-2*a*A)/(3*c*(c*x)^(3/2)) - (2*a*B)/(c^2*Sqrt[c*x]) + (2*(A*b + a*C)*Sqrt
[c*x])/c^3 + (2*(b*B + a*D)*(c*x)^(3/2))/(3*c^4) + (2*b*C*(c*x)^(5/2))/(5*
c^5) + (2*b*D*(c*x)^(7/2))/(7*c^6)
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

## Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.59

| method | result | size |
|---|---|---|
| gosper | $-\frac{2x\left(-15Db\,x^{5}-21Cb\,x^{4}-35bB\,x^{3}-35Da\,x^{3}-105Ab\,x^{2}-105Ca\,x^{2}+105Bax+35Aa\right)}{105\left(cx\right)^{\frac{5}{2}}}$ | 61 |
| orering | $-\frac{2x\left(-15Db\,x^{5}-21Cb\,x^{4}-35bB\,x^{3}-35Da\,x^{3}-105Ab\,x^{2}-105Ca\,x^{2}+105Bax+35Aa\right)}{105\left(cx\right)^{\frac{5}{2}}}$ | 61 |
| pseudoelliptic | $-\frac{2\left(-\frac{3Db\,x^{5}}{7}-\frac{3Cb\,x^{4}}{5}+\left(-Bb-Da\right)x^{3}+\left(-3Ab-3Ca\right)x^{2}+3Bax+Aa\right)}{3\sqrt{cx}\,c^{2}x}$ | 63 |
| trager | $-\frac{2\left(-15Db\,x^{5}-21Cb\,x^{4}-35bB\,x^{3}-35Da\,x^{3}-105Ab\,x^{2}-105Ca\,x^{2}+105Bax+35Aa\right)\sqrt{cx}}{105c^{3}x^{2}}$ | 66 |
| derivativedivides | $\frac{\frac{2Db\left(cx\right)^{\frac{7}{2}}}{7}+\frac{2Cbc\left(cx\right)^{\frac{5}{2}}}{5}+\frac{2Bb\,c^{2}\left(cx\right)^{\frac{3}{2}}}{3}+\frac{2Da\,c^{2}\left(cx\right)^{\frac{3}{2}}}{3}+2Ab\,c^{3}\sqrt{cx}+2Ca\,c^{3}\sqrt{cx}-\frac{2Ba\,c^{4}}{\sqrt{cx}}-\frac{2Aa\,c^{5}}{3\left(cx\right)^{\frac{3}{2}}}}{c^{6}}$ | 96 |
| default | $\frac{\frac{2Db\left(cx\right)^{\frac{7}{2}}}{7}+\frac{2Cbc\left(cx\right)^{\frac{5}{2}}}{5}+\frac{2Bb\,c^{2}\left(cx\right)^{\frac{3}{2}}}{3}+\frac{2Da\,c^{2}\left(cx\right)^{\frac{3}{2}}}{3}+2Ab\,c^{3}\sqrt{cx}+2Ca\,c^{3}\sqrt{cx}-\frac{2Ba\,c^{4}}{\sqrt{cx}}-\frac{2Aa\,c^{5}}{3\left(cx\right)^{\frac{3}{2}}}}{c^{6}}$ | 96 |

input
```
int((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/(c*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/105*x*(-15*D*b*x^5-21*C*b*x^4-35*B*b*x^3-35*D*a*x^3-105*A*b*x^2-105*C*a
*x^2+105*B*a*x+35*A*a)/(c*x)^(5/2)
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.59

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{(cx)^{5/2}}\,dx = \frac{2\,(15\,Dbx^5+21\,Cbx^4+35\,(Da+Bb)x^3-105\,Bax+105\,(Ca}{105\,c^3x^2}$$

input
```
integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/(c*x)^(5/2),x, algorithm="fricas")
```

output
```
2/105*(15*D*b*x^5 + 21*C*b*x^4 + 35*(D*a + B*b)*x^3 - 105*B*a*x + 105*(C*a
 + A*b)*x^2 - 35*A*a)*sqrt(c*x)/(c^3*x^2)
```

## Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.24

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{(cx)^{5/2}}\,dx = -\frac{2Aax}{3\,(cx)^{\frac{5}{2}}}+\frac{2Abx^3}{(cx)^{\frac{5}{2}}} - \frac{2Bax^2}{(cx)^{\frac{5}{2}}}+\frac{2Bbx^4}{3\,(cx)^{\frac{5}{2}}}+\frac{2Cax^3}{(cx)^{\frac{5}{2}}}+\frac{2Cbx^5}{5\,(cx)^{\frac{5}{2}}}+\frac{2Dax^4}{3\,(cx)^{\frac{5}{2}}}+\frac{2Dbx^6}{7\,(cx)^{\frac{5}{2}}}$$

input
```
integrate((b*x**2+a)*(D*x**3+C*x**2+B*x+A)/(c*x)**(5/2),x)
```

output
```
-2*A*a*x/(3*(c*x)**(5/2)) + 2*A*b*x**3/(c*x)**(5/2) - 2*B*a*x**2/(c*x)**(5
/2) + 2*B*b*x**4/(3*(c*x)**(5/2)) + 2*C*a*x**3/(c*x)**(5/2) + 2*C*b*x**5/(
5*(c*x)**(5/2)) + 2*D*a*x**4/(3*(c*x)**(5/2)) + 2*D*b*x**6/(7*(c*x)**(5/2)
)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.83

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{(cx)^{5/2}}\,dx = -\frac{2\left(\frac{35\,(3\,Bacx+Aac)}{(cx)^{\frac{3}{2}}c}-\frac{15\,(cx)^{\frac{7}{2}}Db+21\,(cx)^{\frac{5}{2}}Cbc+35\,(Da+Bb)(cx)^{\frac{3}{2}}c^2+105\,(Ca+Ab)\sqrt{cx}c^3}{c^5}\right)}{105\,c}$$

input
```
integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/(c*x)^(5/2),x, algorithm="maxima")
```

output
```
-2/105*(35*(3*B*a*c*x + A*a*c)/((c*x)^(3/2)*c) - (15*(c*x)^(7/2)*D*b + 21*
(c*x)^(5/2)*C*b*c + 35*(D*a + B*b)*(c*x)^(3/2)*c^2 + 105*(C*a + A*b)*sqrt(
c*x)*c^3)/c^5)/c
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.13

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{(cx)^{5/2}}\,dx = -\frac{2\left(\frac{35\,(3\,Bacx+Aac)}{\sqrt{cx}c^2x}-\frac{15\,\sqrt{cx}Dbc^{33}x^3+21\,\sqrt{cx}Cbc^{33}x^2+35\,\sqrt{cx}Dac^{33}x+35\,\sqrt{cx}Bbc^{33}x+105\,\sqrt{cx}Cac^{33}+105\,\sqrt{cx}Abc^{33}}{c^{35}}\right)}{105\,c}$$

input
```
integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/(c*x)^(5/2),x, algorithm="giac")
```

output
```
-2/105*(35*(3*B*a*c*x + A*a*c)/(sqrt(c*x)*c^2*x) - (15*sqrt(c*x)*D*b*c^33*
x^3 + 21*sqrt(c*x)*C*b*c^33*x^2 + 35*sqrt(c*x)*D*a*c^33*x + 35*sqrt(c*x)*B
*b*c^33*x + 105*sqrt(c*x)*C*a*c^33 + 105*sqrt(c*x)*A*b*c^33)/c^35)/c
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{(cx)^{5/2}}\,dx=\int \frac{\left(b\,x^2+a\right)\,\left(A+B\,x+C\,x^2+x^3\,D\right)}{{\left(c\,x\right)}^{5/2}}\,dx$$

input
```
int(((a + b*x^2)*(A + B*x + C*x^2 + x^3*D))/(c*x)^(5/2),x)
```

output
```
int(((a + b*x^2)*(A + B*x + C*x^2 + x^3*D))/(c*x)^(5/2), x)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.66

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{(cx)^{5/2}}\,dx=\frac{2\sqrt{c}\,(15bd\,x^5+21bc\,x^4+35ad\,x^3+35b^2x^3+105ab\,x^2+105ac}{105\sqrt{x}\,c^3x}$$

input
```
int((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/(c*x)^(5/2),x)
```

output
```
(2*sqrt(c)*( - 35*a**2 + 105*a*b*x**2 - 105*a*b*x + 105*a*c*x**2 + 35*a*d*
x**3 + 35*b**2*x**3 + 21*b*c*x**4 + 15*b*d*x**5))/(105*sqrt(x)*c**3*x)
```

# 3.122 $\int \frac{(cx)^{5/2}(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx$



## Optimal result

Integrand size = 32, antiderivative size = 366

$$\begin{aligned}\int \frac{(cx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{a+bx^2}\,dx &= -\frac{2ac^2(bB-aD)\sqrt{cx}}{b^3} \\
&+\frac{2c(Ab-aC)(cx)^{3/2}}{3b^2}+\frac{2(bB-aD)(cx)^{5/2}}{5b^2}+\frac{2C(cx)^{7/2}}{7bc}+\frac{2D(cx)^{9/2}}{9bc^2} \\
&+\frac{a^{3/4}c^{5/2}\left(\sqrt{b}(Ab-aC)-\sqrt{a}(bB-aD)\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)}{\sqrt{2}b^{13/4}} \\
&-\frac{a^{3/4}c^{5/2}\left(\sqrt{b}(Ab-aC)-\sqrt{a}(bB-aD)\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)}{\sqrt{2}b^{13/4}} \\
&+\frac{a^{3/4}c^{5/2}\left(\sqrt{b}(Ab-aC)+\sqrt{a}(bB-aD)\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\left(\sqrt{a}+\sqrt{b}x\right)}\right)}{\sqrt{2}b^{13/4}}\end{aligned}$$

output
```
-2*a*c^2*(B*b-D*a)*(c*x)^(1/2)/b^3+2/3*c*(A*b-C*a)*(c*x)^(3/2)/b^2+2/5*(B*
b-D*a)*(c*x)^(5/2)/b^2+2/7*C*(c*x)^(7/2)/b/c+2/9*D*(c*x)^(9/2)/b/c^2+1/2*a
^(3/4)*c^(5/2)*(b^(1/2)*(A*b-C*a)-a^(1/2)*(B*b-D*a))*arctan(1-2^(1/2)*b^(1
/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))*2^(1/2)/b^(13/4)-1/2*a^(3/4)*c^(5/2)*(b^(
1/2)*(A*b-C*a)-a^(1/2)*(B*b-D*a))*arctan(1+2^(1/2)*b^(1/4)*(c*x)^(1/2)/a^(
1/4)/c^(1/2))*2^(1/2)/b^(13/4)+1/2*a^(3/4)*c^(5/2)*(b^(1/2)*(A*b-C*a)+a^(1
/2)*(B*b-D*a))*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*(c*x)^(1/2)/c^(1/2)/(a^(1/2
)+b^(1/2)*x))*2^(1/2)/b^(13/4)
```

## Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.68

$$\int \frac{(cx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{a+bx^2}\,dx = \frac{(cx)^{5/2}\left(4\sqrt[4]{b}\sqrt{x}(315a^2D-21ab(15B+x(5C+3Dx))+b^2x(105\right.}{}$$

input
```
Integrate[((c*x)^(5/2)*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2),x]
```

output
```
((c*x)^(5/2)*(4*b^(1/4)*Sqrt[x]*(315*a^2*D - 21*a*b*(15*B + x*(5*C + 3*D*x
)) + b^2*x*(105*A + x*(63*B + 5*x*(9*C + 7*D*x)))) + 315*Sqrt[2]*a^(3/4)*(
A*b^(3/2) - Sqrt[a]*b*B - a*Sqrt[b]*C + a^(3/2)*D)*ArcTan[(Sqrt[a] - Sqrt[
b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] - 315*Sqrt[2]*a^(3/4)*(-(A*b^(3/2
)) - Sqrt[a]*b*B + a*Sqrt[b]*C + a^(3/2)*D)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/
4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)]))/(630*b^(13/4)*x^(5/2))
```

## Rubi [A] (verified)

Time = 1.18 (sec) , antiderivative size = 475, normalized size of antiderivative = 1.30, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{a+bx^2}\,dx$$

$$\Big\downarrow \text{2333}$$

$$\int \left(\frac{(cx)^{5/2}(x(bB-aD)-aC+Ab)}{b\left(a+bx^2\right)}+\frac{C(cx)^{5/2}}{b}+\frac{D(cx)^{7/2}}{bc}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^{3/4}c^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)\left(\sqrt{b}(Ab-aC)-\sqrt{a}(bB-aD)\right)}{\sqrt{2}b^{13/4}}-\frac{a^{3/4}c^{5/2}\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}+1\right)\left(\sqrt{b}(Ab-aC)-\sqrt{a}(bB-aD)\right)}{\sqrt{2}b^{13/4}}-\frac{a^{3/4}c^{5/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{cx}+\sqrt{a}\sqrt{c}+\sqrt{b}\sqrt{cx}\right)\left(\sqrt{b}(Ab-aC)+\sqrt{a}(bB-aD)\right)}{2\sqrt{2}b^{13/4}}+\frac{a^{3/4}c^{5/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{cx}+\sqrt{a}\sqrt{c}+\sqrt{b}\sqrt{cx}\right)\left(\sqrt{b}(Ab-aC)+\sqrt{a}(bB-aD)\right)}{2\sqrt{2}b^{13/4}}+\frac{2c(cx)^{3/2}(Ab-aC)}{3b^2}-\frac{2ac^2\sqrt{cx}(bB-aD)}{b^3}+\frac{2(cx)^{5/2}(bB-aD)}{5b^2}+\frac{2D(cx)^{9/2}}{9bc^2}+\frac{2C(cx)^{7/2}}{7bc}$$

input
```
Int[((c*x)^(5/2)*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2),x]
```

output
```
(-2*a*c^2*(b*B - a*D)*Sqrt[c*x])/b^3 + (2*c*(A*b - a*C)*(c*x)^(3/2))/(3*b^
2) + (2*(b*B - a*D)*(c*x)^(5/2))/(5*b^2) + (2*C*(c*x)^(7/2))/(7*b*c) + (2*
D*(c*x)^(9/2))/(9*b*c^2) + (a^(3/4)*c^(5/2)*(Sqrt[b]*(A*b - a*C) - Sqrt[a]
*(b*B - a*D))*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])]]/(
Sqrt[2]*b^(13/4)) - (a^(3/4)*c^(5/2)*(Sqrt[b]*(A*b - a*C) - Sqrt[a]*(b*B -
 a*D))*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])]]/(Sqrt[2]
*b^(13/4)) - (a^(3/4)*c^(5/2)*(Sqrt[b]*(A*b - a*C) + Sqrt[a]*(b*B - a*D))*
Log[Sqrt[a]*Sqrt[c] + Sqrt[b]*Sqrt[c]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[c*x
]])/(2*Sqrt[2]*b^(13/4)) + (a^(3/4)*c^(5/2)*(Sqrt[b]*(A*b - a*C) + Sqrt[a]
*(b*B - a*D))*Log[Sqrt[a]*Sqrt[c] + Sqrt[b]*Sqrt[c]*x + Sqrt[2]*a^(1/4)*b^
(1/4)*Sqrt[c*x]])/(2*Sqrt[2]*b^(13/4))
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

## Maple [A] (verified)

Time = 3.43 (sec) , antiderivative size = 405, normalized size of antiderivative = 1.11

| method | result |
|---|---|
| pseudoelliptic | $-\frac{\left(-\sqrt{2}\left(\ln\left(\frac{cx+\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{4}}\sqrt{cx}\,\sqrt{2}+\sqrt{\frac{a\,c^{2}}{b}}}{cx-\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{4}}\sqrt{cx}\,\sqrt{2}+\sqrt{\frac{a\,c^{2}}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{cx}-\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{4}}}{\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{cx}+\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{4}}}{\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{4}}}\right)\right)a(Bb}{}$ |
| derivativedivides | $\frac{2\left(\frac{D(cx)^{\frac{9}{2}}b^{2}}{9}+\frac{Cc(cx)^{\frac{7}{2}}b^{2}}{7}+\frac{B\,b^{2}c^{2}(cx)^{\frac{5}{2}}}{5}-\frac{Dab\,c^{2}(cx)^{\frac{5}{2}}}{5}+\frac{A\,b^{2}c^{3}(cx)^{\frac{3}{2}}}{3}-\frac{Cab\,c^{3}(cx)^{\frac{3}{2}}}{3}-Bab\,c^{4}\sqrt{cx}+Da^{2}c^{4}\sqrt{cx}\right)}{b^{3}}-\frac{2a\,c^{5}\left(\frac{(-}{}\right.}{}$ |
| default | $\frac{2\left(\frac{D(cx)^{\frac{9}{2}}b^{2}}{9}+\frac{Cc(cx)^{\frac{7}{2}}b^{2}}{7}+\frac{B\,b^{2}c^{2}(cx)^{\frac{5}{2}}}{5}-\frac{Dab\,c^{2}(cx)^{\frac{5}{2}}}{5}+\frac{A\,b^{2}c^{3}(cx)^{\frac{3}{2}}}{3}-\frac{Cab\,c^{3}(cx)^{\frac{3}{2}}}{3}-Bab\,c^{4}\sqrt{cx}+Da^{2}c^{4}\sqrt{cx}\right)}{b^{3}}-\frac{2a\,c^{5}\left(\frac{(-}{}\right.}{}$ |

input
```
int((c*x)^(5/2)*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(-2^(1/2)*(ln((c*x+(a*c^2/b)^(1/4)*(c*x)^(1/2)*2^(1/2)+(a*c^2/b)^(1/2
))/(c*x-(a*c^2/b)^(1/4)*(c*x)^(1/2)*2^(1/2)+(a*c^2/b)^(1/2)))+2*arctan((2^
(1/2)*(c*x)^(1/2)-(a*c^2/b)^(1/4))/(a*c^2/b)^(1/4))+2*arctan((2^(1/2)*(c*x
)^(1/2)+(a*c^2/b)^(1/4))/(a*c^2/b)^(1/4)))*a*(B*b-D*a)*(a*c^2/b)^(1/2)-8/3
*(c*x)^(1/2)*(a*c^2/b)^(1/4)*(3*D*a^2-3*(1/5*D*x^2+1/3*C*x+B)*b*a+b^2*x*(1
/3*D*x^3+3/7*C*x^2+3/5*B*x+A))+2^(1/2)*(A*b-C*a)*(ln((c*x-(a*c^2/b)^(1/4)*
(c*x)^(1/2)*2^(1/2)+(a*c^2/b)^(1/2))/(c*x+(a*c^2/b)^(1/4)*(c*x)^(1/2)*2^(1
/2)+(a*c^2/b)^(1/2)))+2*arctan((2^(1/2)*(c*x)^(1/2)-(a*c^2/b)^(1/4))/(a*c^
2/b)^(1/4))+2*arctan((2^(1/2)*(c*x)^(1/2)+(a*c^2/b)^(1/4))/(a*c^2/b)^(1/4)
))*c*a)/(a*c^2/b)^(1/4)*c^2/b^3
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3486 vs. $2(280) = 560$.

Time = 0.23 (sec) , antiderivative size = 3486, normalized size of antiderivative = 9.52

$$\int \frac{(cx)^{5/2}(A + Bx + Cx^2 + Dx^3)}{a + bx^2}\,dx = \text{Too large to display}$$

input
```
integrate((c*x)^(5/2)*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="fricas")
```

output
```
Too large to include
```

## Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 28.35 (sec) , antiderivative size = 1285, normalized size of antiderivative = 3.51

$$\int \frac{(cx)^{5/2}(A + Bx + Cx^2 + Dx^3)}{a + bx^2}\,dx = \text{Too large to display}$$

input
```
integrate((c*x)**(5/2)*(D*x**3+C*x**2+B*x+A)/(b*x**2+a),x)
```

output
```
7*A*a**(3/4)*c**(5/2)*exp(-3*I*pi/4)*log(1 - b**(1/4)*sqrt(x)*exp_polar(I*
pi/4)/a**(1/4))*gamma(7/4)/(8*b**(7/4)*gamma(11/4)) + 7*I*A*a**(3/4)*c**(5
/2)*exp(-3*I*pi/4)*log(1 - b**(1/4)*sqrt(x)*exp_polar(3*I*pi/4)/a**(1/4))*
gamma(7/4)/(8*b**(7/4)*gamma(11/4)) - 7*A*a**(3/4)*c**(5/2)*exp(-3*I*pi/4)
*log(1 - b**(1/4)*sqrt(x)*exp_polar(5*I*pi/4)/a**(1/4))*gamma(7/4)/(8*b**(
7/4)*gamma(11/4)) - 7*I*A*a**(3/4)*c**(5/2)*exp(-3*I*pi/4)*log(1 - b**(1/4
)*sqrt(x)*exp_polar(7*I*pi/4)/a**(1/4))*gamma(7/4)/(8*b**(7/4)*gamma(11/4)
) + 7*A*c**(5/2)*x**(3/2)*gamma(7/4)/(6*b*gamma(11/4)) - 9*B*a**(5/4)*c**(
5/2)*exp(-I*pi/4)*log(1 - b**(1/4)*sqrt(x)*exp_polar(I*pi/4)/a**(1/4))*gam
ma(9/4)/(8*b**(9/4)*gamma(13/4)) + 9*I*B*a**(5/4)*c**(5/2)*exp(-I*pi/4)*lo
g(1 - b**(1/4)*sqrt(x)*exp_polar(3*I*pi/4)/a**(1/4))*gamma(9/4)/(8*b**(9/4
)*gamma(13/4)) + 9*B*a**(5/4)*c**(5/2)*exp(-I*pi/4)*log(1 - b**(1/4)*sqrt(
x)*exp_polar(5*I*pi/4)/a**(1/4))*gamma(9/4)/(8*b**(9/4)*gamma(13/4)) - 9*I
*B*a**(5/4)*c**(5/2)*exp(-I*pi/4)*log(1 - b**(1/4)*sqrt(x)*exp_polar(7*I*p
i/4)/a**(1/4))*gamma(9/4)/(8*b**(9/4)*gamma(13/4)) - 9*B*a*c**(5/2)*sqrt(x
)*gamma(9/4)/(2*b**2*gamma(13/4)) + 9*B*c**(5/2)*x**(5/2)*gamma(9/4)/(10*b
*gamma(13/4)) - 11*C*a**(7/4)*c**(5/2)*exp(-3*I*pi/4)*log(1 - b**(1/4)*sqr
t(x)*exp_polar(I*pi/4)/a**(1/4))*gamma(11/4)/(8*b**(11/4)*gamma(15/4)) - 1
1*I*C*a**(7/4)*c**(5/2)*exp(-3*I*pi/4)*log(1 - b**(1/4)*sqrt(x)*exp_polar(
3*I*pi/4)/a**(1/4))*gamma(11/4)/(8*b**(11/4)*gamma(15/4)) + 11*C*a**(7/...
```

## Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 477, normalized size of antiderivative = 1.30

$$\int \frac{(cx)^{5/2}(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx =$$

$$-\frac{315\,ac^4\left(\frac{\sqrt{2}\left(\left(Cab-Ab^2\right)\sqrt{a}c+\left(Da^2\sqrt{b}-Bab^{\frac{3}{2}}\right)c\right)\log\left(\sqrt{b}cx+\sqrt{2}\left(ac^2\right)^{\frac{1}{4}}\sqrt{cx}b^{\frac{1}{4}}+\sqrt{a}c\right)}{\left(ac^2\right)^{\frac{3}{4}}b^{\frac{3}{4}}}-\frac{\sqrt{2}\left(\left(Cab-Ab^2\right)\sqrt{a}c+\left(Da^2\sqrt{b}-Bab^{\frac{3}{2}}\right)c\right)\log\left(\sqrt{b}cx-\sqrt{2}\left(ac^2\right.\right.}{\left(ac^2\right)^{\frac{3}{4}}b^{\frac{3}{4}}}\right.}{}$$

input
```
integrate((c*x)^(5/2)*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="maxima")
```

output
```
-1/1260*(315*a*c^4*(sqrt(2)*((C*a*b - A*b^2)*sqrt(a)*c + (D*a^2*sqrt(b) -
B*a*b^(3/2))*c)*log(sqrt(b)*c*x + sqrt(2)*(a*c^2)^(1/4)*sqrt(c*x)*b^(1/4)
+ sqrt(a)*c)/((a*c^2)^(3/4)*b^(3/4)) - sqrt(2)*((C*a*b - A*b^2)*sqrt(a)*c
+ (D*a^2*sqrt(b) - B*a*b^(3/2))*c)*log(sqrt(b)*c*x - sqrt(2)*(a*c^2)^(1/4)
*sqrt(c*x)*b^(1/4) + sqrt(a)*c)/((a*c^2)^(3/4)*b^(3/4)) - 2*sqrt(2)*((C*a*
b - A*b^2)*sqrt(a)*c - (D*a^2*sqrt(b) - B*a*b^(3/2))*c)*arctan(1/2*sqrt(2)
*(sqrt(2)*(a*c^2)^(1/4)*b^(1/4) + 2*sqrt(c*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b
)*c))/(sqrt(sqrt(a)*sqrt(b)*c)*sqrt(a)*sqrt(b)*c) - 2*sqrt(2)*((C*a*b - A*
b^2)*sqrt(a)*c - (D*a^2*sqrt(b) - B*a*b^(3/2))*c)*arctan(-1/2*sqrt(2)*(sqr
t(2)*(a*c^2)^(1/4)*b^(1/4) - 2*sqrt(c*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*c))
/(sqrt(sqrt(a)*sqrt(b)*c)*sqrt(a)*sqrt(b)*c))/b^3 - 8*(35*(c*x)^(9/2)*D*b^
2 + 45*(c*x)^(7/2)*C*b^2*c - 63*(D*a*b - B*b^2)*(c*x)^(5/2)*c^2 - 105*(C*a
*b - A*b^2)*(c*x)^(3/2)*c^3 + 315*(D*a^2 - B*a*b)*sqrt(c*x)*c^4)/(b^3*c))/
c
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 595 vs. $2(280) = 560$.

Time = 0.13 (sec) , antiderivative size = 595, normalized size of antiderivative = 1.63

$$\int \frac{(cx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{a+bx^2}\,dx =$$

$$-\frac{\sqrt{2}\left((ab^3c^2)^{\frac{1}{4}}Da^2bc^2-(ab^3c^2)^{\frac{1}{4}}Bab^2c^2-(ab^3c^2)^{\frac{3}{4}}Cac+(ab^3c^2)^{\frac{3}{4}}Abc\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ac^2}{b}\right)^{\frac{1}{4}}+2\sqrt{cx}\right)}{2\left(\frac{ac^2}{b}\right)^{\frac{1}{4}}}\right)}{2b^5}$$

$$-\frac{\sqrt{2}\left((ab^3c^2)^{\frac{1}{4}}Da^2bc^2-(ab^3c^2)^{\frac{1}{4}}Bab^2c^2-(ab^3c^2)^{\frac{3}{4}}Cac+(ab^3c^2)^{\frac{3}{4}}Abc\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ac^2}{b}\right)^{\frac{1}{4}}-2\sqrt{cx}\right)}{2\left(\frac{ac^2}{b}\right)^{\frac{1}{4}}}\right.}{2b^5}$$

$$-\frac{\sqrt{2}\left((ab^3c^2)^{\frac{1}{4}}Da^2bc^2-(ab^3c^2)^{\frac{1}{4}}Bab^2c^2+(ab^3c^2)^{\frac{3}{4}}Cac-(ab^3c^2)^{\frac{3}{4}}Abc\right)\log\left(cx+\sqrt{2}\left(\frac{ac^2}{b}\right)^{\frac{1}{4}}\sqrt{cx}+\sqrt{\right.}{4b^5}$$

$$+\frac{\sqrt{2}\left((ab^3c^2)^{\frac{1}{4}}Da^2bc^2-(ab^3c^2)^{\frac{1}{4}}Bab^2c^2+(ab^3c^2)^{\frac{3}{4}}Cac-(ab^3c^2)^{\frac{3}{4}}Abc\right)\log\left(cx-\sqrt{2}\left(\frac{ac^2}{b}\right)^{\frac{1}{4}}\sqrt{cx}+\sqrt{\right.}{4b^5}$$

$$+\frac{2\left(35\sqrt{cx}Db^8c^{20}x^4+45\sqrt{cx}Cb^8c^{20}x^3-63\sqrt{cx}Dab^7c^{20}x^2+63\sqrt{cx}Bb^8c^{20}x^2-105\sqrt{cx}Cab^7c^{20}x+105\right.}{315b^9c^{18}}$$

input
```
integrate((c*x)^(5/2)*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="giac")
```

output
```
-1/2*sqrt(2)*((a*b^3*c^2)^(1/4)*D*a^2*b*c^2 - (a*b^3*c^2)^(1/4)*B*a*b^2*c^
2 - (a*b^3*c^2)^(3/4)*C*a*c + (a*b^3*c^2)^(3/4)*A*b*c)*arctan(1/2*sqrt(2)*
(sqrt(2)*(a*c^2/b)^(1/4) + 2*sqrt(c*x))/(a*c^2/b)^(1/4))/b^5 - 1/2*sqrt(2)
*((a*b^3*c^2)^(1/4)*D*a^2*b*c^2 - (a*b^3*c^2)^(1/4)*B*a*b^2*c^2 - (a*b^3*c
^2)^(3/4)*C*a*c + (a*b^3*c^2)^(3/4)*A*b*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a
*c^2/b)^(1/4) - 2*sqrt(c*x))/(a*c^2/b)^(1/4))/b^5 - 1/4*sqrt(2)*((a*b^3*c^
2)^(1/4)*D*a^2*b*c^2 - (a*b^3*c^2)^(1/4)*B*a*b^2*c^2 + (a*b^3*c^2)^(3/4)*C
*a*c - (a*b^3*c^2)^(3/4)*A*b*c)*log(c*x + sqrt(2)*(a*c^2/b)^(1/4)*sqrt(c*x
) + sqrt(a*c^2/b))/b^5 + 1/4*sqrt(2)*((a*b^3*c^2)^(1/4)*D*a^2*b*c^2 - (a*b
^3*c^2)^(1/4)*B*a*b^2*c^2 + (a*b^3*c^2)^(3/4)*C*a*c - (a*b^3*c^2)^(3/4)*A*
b*c)*log(c*x - sqrt(2)*(a*c^2/b)^(1/4)*sqrt(c*x) + sqrt(a*c^2/b))/b^5 + 2/
315*(35*sqrt(c*x)*D*b^8*c^20*x^4 + 45*sqrt(c*x)*C*b^8*c^20*x^3 - 63*sqrt(c
*x)*D*a*b^7*c^20*x^2 + 63*sqrt(c*x)*B*b^8*c^20*x^2 - 105*sqrt(c*x)*C*a*b^7
*c^20*x + 105*sqrt(c*x)*A*b^8*c^20*x + 315*sqrt(c*x)*D*a^2*b^6*c^20 - 315*
sqrt(c*x)*B*a*b^7*c^20)/(b^9*c^18)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{a+bx^2}\,dx=\int \frac{{\left(c\,x\right)}^{5/2}\,\left(A+B\,x+C\,x^2+x^3\,D\right)}{b\,x^2+a}\,dx$$

input
```
int(((c*x)^(5/2)*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2),x)
```

output
```
int(((c*x)^(5/2)*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2), x)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 639, normalized size of antiderivative = 1.75

$$\int \frac{(cx)^{5/2} (A + Bx + Cx^2 + Dx^3)}{a + bx^2} \, dx = \text{Too large to display}$$

input
```
int((c*x)^(5/2)*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x)
```

output
```
(sqrt(c)*c**2*(630*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(
2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b**2 - 630*b**(1/4)
*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b*
*(1/4)*a**(1/4)*sqrt(2)))*a*b*c + 630*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(
1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*
*2*d - 630*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*s
qrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b**2 - 630*b**(1/4)*a**(3/4
)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a
**(1/4)*sqrt(2)))*a*b**2 + 630*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a*
*(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b*c - 6
30*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*s
qrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2*d + 630*b**(3/4)*a**(1/4)*sqrt(2
)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*
sqrt(2)))*a*b**2 - 315*b**(1/4)*a**(3/4)*sqrt(2)*log( - sqrt(x)*b**(1/4)*a
**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a*b**2 + 315*b**(1/4)*a**(3/4)*sqrt
(2)*log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a*b*c
+ 315*b**(1/4)*a**(3/4)*sqrt(2)*log(sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sq
rt(a) + sqrt(b)*x)*a*b**2 - 315*b**(1/4)*a**(3/4)*sqrt(2)*log(sqrt(x)*b**(
1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a*b*c + 315*b**(3/4)*a**(1/4)
*sqrt(2)*log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x...
```

# 3.123 $\int \frac{(cx)^{3/2}(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx$



## Optimal result

Integrand size = 32, antiderivative size = 340

$$
\begin{aligned}
\int \frac{(cx)^{3/2}(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx &= \frac{2c(Ab-aC)\sqrt{cx}}{b^2} \\
&+ \frac{2(bB-aD)(cx)^{3/2}}{3b^2} + \frac{2C(cx)^{5/2}}{5bc} + \frac{2D(cx)^{7/2}}{7bc^2} \\
&+ \frac{\sqrt[4]{a}c^{3/2}\left(\sqrt{b}(Ab-aC)+\sqrt{a}(bB-aD)\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)}{\sqrt{2}b^{11/4}} \\
&- \frac{\sqrt[4]{a}c^{3/2}\left(\sqrt{b}(Ab-aC)+\sqrt{a}(bB-aD)\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)}{\sqrt{2}b^{11/4}} \\
&- \frac{\sqrt[4]{a}c^{3/2}\left(\sqrt{b}(Ab-aC)-\sqrt{a}(bB-aD)\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\left(\sqrt{a}+\sqrt{b}x\right)}\right)}{\sqrt{2}b^{11/4}}
\end{aligned}
$$

output
```
2*c*(A*b-C*a)*(c*x)^(1/2)/b^2+2/3*(B*b-D*a)*(c*x)^(3/2)/b^2+2/5*C*(c*x)^(5
/2)/b/c+2/7*D*(c*x)^(7/2)/b/c^2+1/2*a^(1/4)*c^(3/2)*(b^(1/2)*(A*b-C*a)+a^(
1/2)*(B*b-D*a))*arctan(1-2^(1/2)*b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))*2^(1
/2)/b^(11/4)-1/2*a^(1/4)*c^(3/2)*(b^(1/2)*(A*b-C*a)+a^(1/2)*(B*b-D*a))*arc
tan(1+2^(1/2)*b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))*2^(1/2)/b^(11/4)-1/2*a^
(1/4)*c^(3/2)*(b^(1/2)*(A*b-C*a)-a^(1/2)*(B*b-D*a))*arctanh(2^(1/2)*a^(1/4
)*b^(1/4)*(c*x)^(1/2)/c^(1/2)/(a^(1/2)+b^(1/2)*x))*2^(1/2)/b^(11/4)
```

## Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.68

$$\int \frac{(cx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{a+bx^2}\,dx = \frac{(cx)^{3/2}\left(4b^{3/4}\sqrt{x}(105Ab-35a(3C+Dx)+bx(35B+3x(7C+5$$

input
```
Integrate[((c*x)^(3/2)*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2),x]
```

output
```
((c*x)^(3/2)*(4*b^(3/4)*Sqrt[x]*(105*A*b - 35*a*(3*C + D*x) + b*x*(35*B +
3*x*(7*C + 5*D*x))) - 105*Sqrt[2]*a^(1/4)*(-(A*b^(3/2)) - Sqrt[a]*b*B + a*
Sqrt[b]*C + a^(3/2)*D)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/
4)*Sqrt[x])] - 105*Sqrt[2]*a^(1/4)*(A*b^(3/2) - Sqrt[a]*b*B - a*Sqrt[b]*C
+ a^(3/2)*D)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*
x)]))/(210*b^(11/4)*x^(3/2))
```

## Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 449, normalized size of antiderivative = 1.32, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{a+bx^2}\,dx$$

$$\Big\downarrow \text{2333}$$

$$\int \left(\frac{(cx)^{3/2}(x(bB-aD)-aC+Ab)}{b\,(a+bx^2)}+\frac{C(cx)^{3/2}}{b}+\frac{D(cx)^{5/2}}{bc}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt[4]{a}c^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)\left(\sqrt{b}(Ab-aC)+\sqrt{a}(bB-aD)\right)}{\sqrt{2}b^{11/4}}-\frac{\sqrt[4]{a}c^{3/2}\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}+1\right)\left(\sqrt{b}(Ab-aC)+\sqrt{a}(bB-aD)\right)}{\sqrt{2}b^{11/4}}+\frac{\sqrt[4]{a}c^{3/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{cx}+\sqrt{a}\sqrt{c}+\sqrt{b}\sqrt{cx}\right)\left(\sqrt{b}(Ab-aC)-\sqrt{a}(bB-aD)\right)}{2\sqrt{2}b^{11/4}}-\frac{\sqrt[4]{a}c^{3/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{cx}+\sqrt{a}\sqrt{c}+\sqrt{b}\sqrt{cx}\right)\left(\sqrt{b}(Ab-aC)-\sqrt{a}(bB-aD)\right)}{2\sqrt{2}b^{11/4}}+\frac{2c\sqrt{cx}(Ab-aC)}{b^2}+\frac{2(cx)^{3/2}(bB-aD)}{3b^2}+\frac{2D(cx)^{7/2}}{7bc^2}+\frac{2C(cx)^{5/2}}{5bc}$$

input
```
Int[((c*x)^(3/2)*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2),x]
```

output
```
(2*c*(A*b - a*C)*Sqrt[c*x])/b^2 + (2*(b*B - a*D)*(c*x)^(3/2))/(3*b^2) + (2
*C*(c*x)^(5/2))/(5*b*c) + (2*D*(c*x)^(7/2))/(7*b*c^2) + (a^(1/4)*c^(3/2)*(
Sqrt[b]*(A*b - a*C) + Sqrt[a]*(b*B - a*D))*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqr
t[c*x])/(a^(1/4)*Sqrt[c])]])/(Sqrt[2]*b^(11/4)) - (a^(1/4)*c^(3/2)*(Sqrt[b]
*(A*b - a*C) + Sqrt[a]*(b*B - a*D))*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[c*x])
/(a^(1/4)*Sqrt[c])]])/(Sqrt[2]*b^(11/4)) + (a^(1/4)*c^(3/2)*(Sqrt[b]*(A*b -
 a*C) - Sqrt[a]*(b*B - a*D))*Log[Sqrt[a]*Sqrt[c] + Sqrt[b]*Sqrt[c]*x - Sqr
t[2]*a^(1/4)*b^(1/4)*Sqrt[c*x]])/(2*Sqrt[2]*b^(11/4)) - (a^(1/4)*c^(3/2)*(
Sqrt[b]*(A*b - a*C) - Sqrt[a]*(b*B - a*D))*Log[Sqrt[a]*Sqrt[c] + Sqrt[b]*S
qrt[c]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[c*x]])/(2*Sqrt[2]*b^(11/4))
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

## Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.13

| method | result |
|---|---|
| pseudoelliptic | $2\left(-\frac{\left(\arctan\left(\frac{\sqrt{2}\,\sqrt{cx}-\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{4}}}{\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{4}}}\right)+\arctan\left(\frac{\sqrt{2}\,\sqrt{cx}+\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{4}}}{\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{4}}}\right)+\frac{\ln\left(\frac{cx+\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{4}}\sqrt{cx}\,\sqrt{2}+\sqrt{\frac{a\,c^{2}}{b}}}{cx-\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{4}}\sqrt{cx}\,\sqrt{2}+\sqrt{\frac{a\,c^{2}}{b}}}\right)}{2}\right)\sqrt{2}\,(Ab-Ca)b\sqrt{\frac{a\,c^{2}}{b}}}{4}\right.$ |
| derivativedivides | $\frac{2\left(\frac{Db(cx)^{\frac{7}{2}}}{7}+\frac{Cbc(cx)^{\frac{5}{2}}}{5}+\frac{Bb\,c^{2}(cx)^{\frac{3}{2}}}{3}-\frac{Da\,c^{2}(cx)^{\frac{3}{2}}}{3}+Ab\,c^{3}\sqrt{cx}-Ca\,c^{3}\sqrt{cx}\right)}{b^{2}}-2a\,c^{4}\left(\frac{(Abc-Cac)\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{cx+\left(\frac{a\,c^{2}}{b}\right.}{cx-\left(\frac{a\,c^{2}}{b}\right.}\right.\right.}{}\right.$ |
| default | $\frac{2\left(\frac{Db(cx)^{\frac{7}{2}}}{7}+\frac{Cbc(cx)^{\frac{5}{2}}}{5}+\frac{Bb\,c^{2}(cx)^{\frac{3}{2}}}{3}-\frac{Da\,c^{2}(cx)^{\frac{3}{2}}}{3}+Ab\,c^{3}\sqrt{cx}-Ca\,c^{3}\sqrt{cx}\right)}{b^{2}}-2a\,c^{4}\left(\frac{(Abc-Cac)\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{cx+\left(\frac{a\,c^{2}}{b}\right.}{cx-\left(\frac{a\,c^{2}}{b}\right.}\right.\right.}{}\right.$ |

input
```
int((c*x)^(3/2)*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
2*(-1/4*(arctan((2^(1/2)*(c*x)^(1/2)-(a*c^2/b)^(1/4))/(a*c^2/b)^(1/4))+arc
tan((2^(1/2)*(c*x)^(1/2)+(a*c^2/b)^(1/4))/(a*c^2/b)^(1/4))+1/2*ln((c*x+(a*
c^2/b)^(1/4)*(c*x)^(1/2)*2^(1/2)+(a*c^2/b)^(1/2))/(c*x-(a*c^2/b)^(1/4)*(c*
x)^(1/2)*2^(1/2)+(a*c^2/b)^(1/2))))*2^(1/2)*(A*b-C*a)*b*(a*c^2/b)^(1/2)+(c
*x)^(1/2)*((1/5*C*x^2+1/3*B*x+1/7*D*x^3+A)*b-(1/3*D*x+C)*a)*(a*c^2/b)^(1/4
)*b-1/4*(arctan((2^(1/2)*(c*x)^(1/2)-(a*c^2/b)^(1/4))/(a*c^2/b)^(1/4))+arc
tan((2^(1/2)*(c*x)^(1/2)+(a*c^2/b)^(1/4))/(a*c^2/b)^(1/4))+1/2*ln((c*x-(a*
c^2/b)^(1/4)*(c*x)^(1/2)*2^(1/2)+(a*c^2/b)^(1/2))/(c*x+(a*c^2/b)^(1/4)*(c*
x)^(1/2)*2^(1/2)+(a*c^2/b)^(1/2))))*2^(1/2)*c*a*(B*b-D*a))/(a*c^2/b)^(1/4)
*c/b^3
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3365 vs. $2(257) = 514$.

Time = 0.23 (sec) , antiderivative size = 3365, normalized size of antiderivative = 9.90

$$\int \frac{(cx)^{3/2}(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx = \text{Too large to display}$$

input
```
integrate((c*x)^(3/2)*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="fricas")
```

output
```
Too large to include
```

## Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 22.33 (sec) , antiderivative size = 1222, normalized size of antiderivative = 3.59

$$\int \frac{(cx)^{3/2}(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx = \text{Too large to display}$$

input
```
integrate((c*x)**(3/2)*(D*x**3+C*x**2+B*x+A)/(b*x**2+a),x)
```

output
```
5*A*a**(1/4)*c**(3/2)*exp(-I*pi/4)*log(1 - b**(1/4)*sqrt(x)*exp_polar(I*pi
/4)/a**(1/4))*gamma(5/4)/(8*b**(5/4)*gamma(9/4)) - 5*I*A*a**(1/4)*c**(3/2)
*exp(-I*pi/4)*log(1 - b**(1/4)*sqrt(x)*exp_polar(3*I*pi/4)/a**(1/4))*gamma
(5/4)/(8*b**(5/4)*gamma(9/4)) - 5*A*a**(1/4)*c**(3/2)*exp(-I*pi/4)*log(1 -
 b**(1/4)*sqrt(x)*exp_polar(5*I*pi/4)/a**(1/4))*gamma(5/4)/(8*b**(5/4)*gam
ma(9/4)) + 5*I*A*a**(1/4)*c**(3/2)*exp(-I*pi/4)*log(1 - b**(1/4)*sqrt(x)*e
xp_polar(7*I*pi/4)/a**(1/4))*gamma(5/4)/(8*b**(5/4)*gamma(9/4)) + 5*A*c**(
3/2)*sqrt(x)*gamma(5/4)/(2*b*gamma(9/4)) + 7*B*a**(3/4)*c**(3/2)*exp(-3*I*
pi/4)*log(1 - b**(1/4)*sqrt(x)*exp_polar(I*pi/4)/a**(1/4))*gamma(7/4)/(8*b
**(7/4)*gamma(11/4)) + 7*I*B*a**(3/4)*c**(3/2)*exp(-3*I*pi/4)*log(1 - b**(
1/4)*sqrt(x)*exp_polar(3*I*pi/4)/a**(1/4))*gamma(7/4)/(8*b**(7/4)*gamma(11
/4)) - 7*B*a**(3/4)*c**(3/2)*exp(-3*I*pi/4)*log(1 - b**(1/4)*sqrt(x)*exp_p
olar(5*I*pi/4)/a**(1/4))*gamma(7/4)/(8*b**(7/4)*gamma(11/4)) - 7*I*B*a**(3
/4)*c**(3/2)*exp(-3*I*pi/4)*log(1 - b**(1/4)*sqrt(x)*exp_polar(7*I*pi/4)/a
**(1/4))*gamma(7/4)/(8*b**(7/4)*gamma(11/4)) + 7*B*c**(3/2)*x**(3/2)*gamma
(7/4)/(6*b*gamma(11/4)) - 9*C*a**(5/4)*c**(3/2)*exp(-I*pi/4)*log(1 - b**(1
/4)*sqrt(x)*exp_polar(I*pi/4)/a**(1/4))*gamma(9/4)/(8*b**(9/4)*gamma(13/4)
) + 9*I*C*a**(5/4)*c**(3/2)*exp(-I*pi/4)*log(1 - b**(1/4)*sqrt(x)*exp_pola
r(3*I*pi/4)/a**(1/4))*gamma(9/4)/(8*b**(9/4)*gamma(13/4)) + 9*C*a**(5/4)*c
**(3/2)*exp(-I*pi/4)*log(1 - b**(1/4)*sqrt(x)*exp_polar(5*I*pi/4)/a**(1...
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 422, normalized size of antiderivative = 1.24

$$\int \frac{(cx)^{3/2}(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx =$$

$$-\frac{105\,ac^3\left(\frac{\sqrt{2}\left((Da-Bb)\sqrt{a}c-\left(Ca\sqrt{b}-Ab^{\frac{3}{2}}\right)c\right)\log\left(\sqrt{b}cx+\sqrt{2}\left(ac^2\right)^{\frac{1}{4}}\sqrt{cx}b^{\frac{1}{4}}+\sqrt{a}c\right)}{\left(ac^2\right)^{\frac{3}{4}}b^{\frac{3}{4}}}-\frac{\sqrt{2}\left((Da-Bb)\sqrt{a}c-\left(Ca\sqrt{b}-Ab^{\frac{3}{2}}\right)c\right)\log\left(\sqrt{b}cx-\sqrt{2}\left(ac^2\right)^{\frac{1}{4}}\sqrt{cx}b^{\frac{1}{4}}\right.}{\left(ac^2\right)^{\frac{3}{4}}b^{\frac{3}{4}}}\right.}{}$$

input
```
integrate((c*x)^(3/2)*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="maxima")
```

output
```
-1/420*(105*a*c^3*(sqrt(2)*((D*a - B*b)*sqrt(a)*c - (C*a*sqrt(b) - A*b^(3/
2))*c)*log(sqrt(b)*c*x + sqrt(2)*(a*c^2)^(1/4)*sqrt(c*x)*b^(1/4) + sqrt(a)
*c)/((a*c^2)^(3/4)*b^(3/4)) - sqrt(2)*((D*a - B*b)*sqrt(a)*c - (C*a*sqrt(b
) - A*b^(3/2))*c)*log(sqrt(b)*c*x - sqrt(2)*(a*c^2)^(1/4)*sqrt(c*x)*b^(1/4
) + sqrt(a)*c)/((a*c^2)^(3/4)*b^(3/4)) - 2*sqrt(2)*((D*a - B*b)*sqrt(a)*c
+ (C*a*sqrt(b) - A*b^(3/2))*c)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*c^2)^(1/4)*b
^(1/4) + 2*sqrt(c*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*c))/(sqrt(sqrt(a)*sqrt(
b)*c)*sqrt(a)*sqrt(b)*c) - 2*sqrt(2)*((D*a - B*b)*sqrt(a)*c + (C*a*sqrt(b)
 - A*b^(3/2))*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*c^2)^(1/4)*b^(1/4) - 2*sq
rt(c*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*c))/(sqrt(sqrt(a)*sqrt(b)*c)*sqrt(a)
*sqrt(b)*c))/b^2 - 8*(15*(c*x)^(7/2)*D*b + 21*(c*x)^(5/2)*C*b*c - 35*(D*a
- B*b)*(c*x)^(3/2)*c^2 - 105*(C*a - A*b)*sqrt(c*x)*c^3)/(b^2*c))/c
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 544 vs. $2(257) = 514$.

Time = 0.14 (sec) , antiderivative size = 544, normalized size of antiderivative = 1.60

$$\int \frac{(cx)^{3/2}(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx = \frac{1}{420}c\left(\frac{210\sqrt{2}\left((ab^3c^2)^{\frac{1}{4}}Cab^2c-(ab^3c^2)^{\frac{1}{4}}Ab^3c+(ab^3c^2)^{\frac{3}{4}}Da-\left(\right.\right.}{b^5c}\right.$$

input
```
integrate((c*x)^(3/2)*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="giac")
```

output
```
1/420*c*(210*sqrt(2)*((a*b^3*c^2)^(1/4)*C*a*b^2*c - (a*b^3*c^2)^(1/4)*A*b^
3*c + (a*b^3*c^2)^(3/4)*D*a - (a*b^3*c^2)^(3/4)*B*b)*arctan(1/2*sqrt(2)*(s
qrt(2)*(a*c^2/b)^(1/4) + 2*sqrt(c*x))/(a*c^2/b)^(1/4))/(b^5*c) + 210*sqrt(
2)*((a*b^3*c^2)^(1/4)*C*a*b^2*c - (a*b^3*c^2)^(1/4)*A*b^3*c + (a*b^3*c^2)^
(3/4)*D*a - (a*b^3*c^2)^(3/4)*B*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*c^2/b)^
(1/4) - 2*sqrt(c*x))/(a*c^2/b)^(1/4))/(b^5*c) + 105*sqrt(2)*((a*b^3*c^2)^(
1/4)*C*a*b^2*c - (a*b^3*c^2)^(1/4)*A*b^3*c - (a*b^3*c^2)^(3/4)*D*a + (a*b^
3*c^2)^(3/4)*B*b)*log(c*x + sqrt(2)*(a*c^2/b)^(1/4)*sqrt(c*x) + sqrt(a*c^2
/b))/(b^5*c) - 105*sqrt(2)*((a*b^3*c^2)^(1/4)*C*a*b^2*c - (a*b^3*c^2)^(1/4
)*A*b^3*c - (a*b^3*c^2)^(3/4)*D*a + (a*b^3*c^2)^(3/4)*B*b)*log(c*x - sqrt(
2)*(a*c^2/b)^(1/4)*sqrt(c*x) + sqrt(a*c^2/b))/(b^5*c) + 8*(15*sqrt(c*x)*D*
b^6*c^21*x^3 + 21*sqrt(c*x)*C*b^6*c^21*x^2 - 35*sqrt(c*x)*D*a*b^5*c^21*x +
 35*sqrt(c*x)*B*b^6*c^21*x - 105*sqrt(c*x)*C*a*b^5*c^21 + 105*sqrt(c*x)*A*
b^6*c^21)/(b^7*c^21))
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{3/2}(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx = \int \frac{(c\,x)^{3/2}\,(A+B\,x+C\,x^2+x^3\,D)}{b\,x^2+a}\,dx$$

input
```
int(((c*x)^(3/2)*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2),x)
```

output
```
int(((c*x)^(3/2)*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2), x)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 610, normalized size of antiderivative = 1.79

$$\int \frac{(cx)^{3/2}(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx = \text{Too large to display}$$

input
```
int((c*x)^(3/2)*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x)
```

output
```
(sqrt(c)*c*( - 210*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(
2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*d + 210*b**(1/4)*a*
*(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1
/4)*a**(1/4)*sqrt(2)))*b**2 + 210*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)
*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b -
210*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*
sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*c + 210*b**(1/4)*a**(3/4)*sqrt(2)*
atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sq
rt(2)))*a*d - 210*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2
) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*b**2 - 210*b**(3/4)*a*
*(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1
/4)*a**(1/4)*sqrt(2)))*a*b + 210*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*
a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*c + 1
05*b**(1/4)*a**(3/4)*sqrt(2)*log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sq
rt(a) + sqrt(b)*x)*a*d - 105*b**(1/4)*a**(3/4)*sqrt(2)*log( - sqrt(x)*b**(
1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*b**2 - 105*b**(1/4)*a**(3/4)*
sqrt(2)*log(sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a*d +
 105*b**(1/4)*a**(3/4)*sqrt(2)*log(sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqr
t(a) + sqrt(b)*x)*b**2 + 105*b**(3/4)*a**(1/4)*sqrt(2)*log( - sqrt(x)*b**(
1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a*b - 105*b**(3/4)*a**(1/4...
```

# 3.124 $\int \frac{\sqrt{cx}(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx$



## Optimal result

Integrand size = 32, antiderivative size = 318

$$\begin{aligned}\int \frac{\sqrt{cx}(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx &= \frac{2(bB-aD)\sqrt{cx}}{b^2}+\frac{2C(cx)^{3/2}}{3bc}+\frac{2D(cx)^{5/2}}{5bc^2}\\
&\quad-\frac{\sqrt{c}\left(\sqrt{b}(Ab-aC)-\sqrt{a}(bB-aD)\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)}{\sqrt{2}\sqrt[4]{a}b^{9/4}}\\
&\quad+\frac{\sqrt{c}\left(\sqrt{b}(Ab-aC)-\sqrt{a}(bB-aD)\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)}{\sqrt{2}\sqrt[4]{a}b^{9/4}}\\
&\quad-\frac{\sqrt{c}\left(\sqrt{b}(Ab-aC)+\sqrt{a}(bB-aD)\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\left(\sqrt{a}+\sqrt{b}x\right)}\right)}{\sqrt{2}\sqrt[4]{a}b^{9/4}}\end{aligned}$$

output
```
2*(B*b-D*a)*(c*x)^(1/2)/b^2+2/3*C*(c*x)^(3/2)/b/c+2/5*D*(c*x)^(5/2)/b/c^2-
1/2*c^(1/2)*(b^(1/2)*(A*b-C*a)-a^(1/2)*(B*b-D*a))*arctan(1-2^(1/2)*b^(1/4)
*(c*x)^(1/2)/a^(1/4)/c^(1/2))*2^(1/2)/a^(1/4)/b^(9/4)+1/2*c^(1/2)*(b^(1/2)
*(A*b-C*a)-a^(1/2)*(B*b-D*a))*arctan(1+2^(1/2)*b^(1/4)*(c*x)^(1/2)/a^(1/4)
/c^(1/2))*2^(1/2)/a^(1/4)/b^(9/4)-1/2*c^(1/2)*(b^(1/2)*(A*b-C*a)+a^(1/2)*(
B*b-D*a))*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*(c*x)^(1/2)/c^(1/2)/(a^(1/2)+b^(
1/2)*x))*2^(1/2)/a^(1/4)/b^(9/4)
```

## Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.68

$$\int \frac{\sqrt{cx}(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx$$

$$= \frac{\sqrt{cx}\left(4\sqrt[4]{b}(15bB-15aD+bx(5C+3Dx)) - \frac{15\sqrt{2}\left(Ab^{3/2}-\sqrt{a}bB-a\sqrt{b}C+a^{3/2}D\right)\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{\sqrt[4]{a}\sqrt{x}} + \frac{15\sqrt{2}\left(-\right.}{}\right.}{30b^{9/4}}$$

input
```
Integrate[(Sqrt[c*x]*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2),x]
```

output
```
(Sqrt[c*x]*(4*b^(1/4)*(15*b*B - 15*a*D + b*x*(5*C + 3*D*x)) - (15*Sqrt[2]*
(A*b^(3/2) - Sqrt[a]*b*B - a*Sqrt[b]*C + a^(3/2)*D)*ArcTan[(Sqrt[a] - Sqrt
[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/(a^(1/4)*Sqrt[x]) + (15*Sqrt[2]
*(-(A*b^(3/2)) - Sqrt[a]*b*B + a*Sqrt[b]*C + a^(3/2)*D)*ArcTanh[(Sqrt[2]*a
^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(a^(1/4)*Sqrt[x])))/(30*b^
(9/4))
```

## Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 426, normalized size of antiderivative = 1.34, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{cx}(A+Bx+Cx^2+Dx^3)}{a+bx^2}dx$$

$$\Big\downarrow \ 2333$$

$$\int \left(\frac{\sqrt{cx}(x(bB-aD)-aC+Ab)}{b(a+bx^2)}+\frac{C\sqrt{cx}}{b}+\frac{D(cx)^{3/2}}{bc}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{\sqrt{c}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)\left(\sqrt{b}(Ab-aC)-\sqrt{a}(bB-aD)\right)}{\sqrt{2}\sqrt[4]{a}b^{9/4}}+\frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}+1\right)\left(\sqrt{b}(Ab-aC)-\sqrt{a}(bB-aD)\right)}{\sqrt{2}\sqrt[4]{a}b^{9/4}}+\frac{\sqrt{c}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{cx}+\sqrt{a}\sqrt{c}+\sqrt{b}\sqrt{cx}\right)\left(\sqrt{b}(Ab-aC)+\sqrt{a}(bB-aD)\right)}{2\sqrt{2}\sqrt[4]{a}b^{9/4}}-\frac{\sqrt{c}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{cx}+\sqrt{a}\sqrt{c}+\sqrt{b}\sqrt{cx}\right)\left(\sqrt{b}(Ab-aC)+\sqrt{a}(bB-aD)\right)}{2\sqrt{2}\sqrt[4]{a}b^{9/4}}+\frac{2\sqrt{cx}(bB-aD)}{b^2}+\frac{2D(cx)^{5/2}}{5bc^2}+\frac{2C(cx)^{3/2}}{3bc}$$

input
```
Int[(Sqrt[c*x]*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2),x]
```

output
```
(2*(b*B - a*D)*Sqrt[c*x])/b^2 + (2*C*(c*x)^(3/2))/(3*b*c) + (2*D*(c*x)^(5/
2))/(5*b*c^2) - (Sqrt[c]*(Sqrt[b]*(A*b - a*C) - Sqrt[a]*(b*B - a*D))*ArcTa
n[1 - (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])])/(Sqrt[2]*a^(1/4)*b^(
9/4)) + (Sqrt[c]*(Sqrt[b]*(A*b - a*C) - Sqrt[a]*(b*B - a*D))*ArcTan[1 + (S
qrt[2]*b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])])/(Sqrt[2]*a^(1/4)*b^(9/4)) +
(Sqrt[c]*(Sqrt[b]*(A*b - a*C) + Sqrt[a]*(b*B - a*D))*Log[Sqrt[a]*Sqrt[c] +
 Sqrt[b]*Sqrt[c]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[c*x]])/(2*Sqrt[2]*a^(1/4
)*b^(9/4)) - (Sqrt[c]*(Sqrt[b]*(A*b - a*C) + Sqrt[a]*(b*B - a*D))*Log[Sqrt
[a]*Sqrt[c] + Sqrt[b]*Sqrt[c]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[c*x]])/(2*S
qrt[2]*a^(1/4)*b^(9/4))
```

**Defintions of rubi rules used**

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

## Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.15

| method | result |
|---|---|
| derivativedivides | $\frac{2\left(\frac{D(cx)^{\frac{5}{2}}b}{5}+\frac{Cc(cx)^{\frac{3}{2}}b}{3}+Bb\,c^{2}\sqrt{cx}-Da\,c^{2}\sqrt{cx}\right)}{b^{2}}+\frac{2c^{3}\left(\frac{\left(-abBc+Da^{2}c\right)\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{cx+\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{4}}\sqrt{cx}\,\sqrt{2}+\sqrt{\frac{a\,c^{2}}{b}}}{cx-\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{4}}\sqrt{cx}\,\sqrt{2}+\sqrt{\frac{a\,c^{2}}{b}}}\right)+2}{8a\,c^{2}}\right.}{}$ |
| default | $\frac{2\left(\frac{D(cx)^{\frac{5}{2}}b}{5}+\frac{Cc(cx)^{\frac{3}{2}}b}{3}+Bb\,c^{2}\sqrt{cx}-Da\,c^{2}\sqrt{cx}\right)}{b^{2}}+\frac{2c^{3}\left(\frac{\left(-abBc+Da^{2}c\right)\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{cx+\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{4}}\sqrt{cx}\,\sqrt{2}+\sqrt{\frac{a\,c^{2}}{b}}}{cx-\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{4}}\sqrt{cx}\,\sqrt{2}+\sqrt{\frac{a\,c^{2}}{b}}}\right)+2}{8a\,c^{2}}\right.}{}$ |
| pseudoelliptic | $-\frac{\sqrt{2}\left(\ln\left(\frac{cx+\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{4}}\sqrt{cx}\,\sqrt{2}+\sqrt{\frac{a\,c^{2}}{b}}}{cx-\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{4}}\sqrt{cx}\,\sqrt{2}+\sqrt{\frac{a\,c^{2}}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{cx}-\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{4}}}{\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{cx}+\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{4}}}{\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{4}}}\right)\right)(Bb-Da)\sqrt{\frac{a\,c^{2}}{b}}}{4}+$ |

input
```
int((c*x)^(1/2)*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
2/c^2*(1/b^2*(1/5*D*(c*x)^(5/2)*b+1/3*C*c*(c*x)^(3/2)*b+B*b*c^2*(c*x)^(1/2
)-D*a*c^2*(c*x)^(1/2))+c^3/b^2*(1/8*(-B*a*b*c+D*a^2*c)*(a*c^2/b)^(1/4)/a/c
^2*2^(1/2)*(ln((c*x+(a*c^2/b)^(1/4)*(c*x)^(1/2)*2^(1/2)+(a*c^2/b)^(1/2))/(
c*x-(a*c^2/b)^(1/4)*(c*x)^(1/2)*2^(1/2)+(a*c^2/b)^(1/2)))+2*arctan(2^(1/2)
/(a*c^2/b)^(1/4)*(c*x)^(1/2)+1)+2*arctan(2^(1/2)/(a*c^2/b)^(1/4)*(c*x)^(1/
2)-1))+1/8*(A*b^2-C*a*b)/b/(a*c^2/b)^(1/4)*2^(1/2)*(ln((c*x-(a*c^2/b)^(1/4
)*(c*x)^(1/2)*2^(1/2)+(a*c^2/b)^(1/2))/(c*x+(a*c^2/b)^(1/4)*(c*x)^(1/2)*2^
(1/2)+(a*c^2/b)^(1/2)))+2*arctan(2^(1/2)/(a*c^2/b)^(1/4)*(c*x)^(1/2)+1)+2*
arctan(2^(1/2)/(a*c^2/b)^(1/4)*(c*x)^(1/2)-1))))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3318 vs. $2(239) = 478$.

Time = 0.22 (sec) , antiderivative size = 3318, normalized size of antiderivative = 10.43

$$\int \frac{\sqrt{cx}(A + Bx + Cx^2 + Dx^3)}{a + bx^2} \, dx = \text{Too large to display}$$

input
```
integrate((c*x)^(1/2)*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="fricas")
```

output
```
Too large to include
```

## Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 24.68 (sec) , antiderivative size = 1164, normalized size of antiderivative = 3.66

$$\int \frac{\sqrt{cx}(A + Bx + Cx^2 + Dx^3)}{a + bx^2} \, dx = \text{Too large to display}$$

input
```
integrate((c*x)**(1/2)*(D*x**3+C*x**2+B*x+A)/(b*x**2+a),x)
```

output
```
-3*A*sqrt(c)*exp(-3*I*pi/4)*log(1 - b**(1/4)*sqrt(x)*exp_polar(I*pi/4)/a**
(1/4))*gamma(3/4)/(8*a**(1/4)*b**(3/4)*gamma(7/4)) - 3*I*A*sqrt(c)*exp(-3*
I*pi/4)*log(1 - b**(1/4)*sqrt(x)*exp_polar(3*I*pi/4)/a**(1/4))*gamma(3/4)/
(8*a**(1/4)*b**(3/4)*gamma(7/4)) + 3*A*sqrt(c)*exp(-3*I*pi/4)*log(1 - b**(
1/4)*sqrt(x)*exp_polar(5*I*pi/4)/a**(1/4))*gamma(3/4)/(8*a**(1/4)*b**(3/4)
*gamma(7/4)) + 3*I*A*sqrt(c)*exp(-3*I*pi/4)*log(1 - b**(1/4)*sqrt(x)*exp_p
olar(7*I*pi/4)/a**(1/4))*gamma(3/4)/(8*a**(1/4)*b**(3/4)*gamma(7/4)) + 5*B
*a**(1/4)*sqrt(c)*exp(-I*pi/4)*log(1 - b**(1/4)*sqrt(x)*exp_polar(I*pi/4)/
a**(1/4))*gamma(5/4)/(8*b**(5/4)*gamma(9/4)) - 5*I*B*a**(1/4)*sqrt(c)*exp(
-I*pi/4)*log(1 - b**(1/4)*sqrt(x)*exp_polar(3*I*pi/4)/a**(1/4))*gamma(5/4)
/(8*b**(5/4)*gamma(9/4)) - 5*B*a**(1/4)*sqrt(c)*exp(-I*pi/4)*log(1 - b**(1
/4)*sqrt(x)*exp_polar(5*I*pi/4)/a**(1/4))*gamma(5/4)/(8*b**(5/4)*gamma(9/4
)) + 5*I*B*a**(1/4)*sqrt(c)*exp(-I*pi/4)*log(1 - b**(1/4)*sqrt(x)*exp_pola
r(7*I*pi/4)/a**(1/4))*gamma(5/4)/(8*b**(5/4)*gamma(9/4)) + 5*B*sqrt(c)*sqr
t(x)*gamma(5/4)/(2*b*gamma(9/4)) + 7*C*a**(3/4)*sqrt(c)*exp(-3*I*pi/4)*log
(1 - b**(1/4)*sqrt(x)*exp_polar(I*pi/4)/a**(1/4))*gamma(7/4)/(8*b**(7/4)*g
amma(11/4)) + 7*I*C*a**(3/4)*sqrt(c)*exp(-3*I*pi/4)*log(1 - b**(1/4)*sqrt(
x)*exp_polar(3*I*pi/4)/a**(1/4))*gamma(7/4)/(8*b**(7/4)*gamma(11/4)) - 7*C
*a**(3/4)*sqrt(c)*exp(-3*I*pi/4)*log(1 - b**(1/4)*sqrt(x)*exp_polar(5*I*pi
/4)/a**(1/4))*gamma(7/4)/(8*b**(7/4)*gamma(11/4)) - 7*I*C*a**(3/4)*sqrt...
```

## Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 427, normalized size of antiderivative = 1.34

$$\int \frac{\sqrt{cx}(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx$$

$$= \frac{15\,c^2 \left( \frac{\sqrt{2}\left(\left(Cab-Ab^2\right)\sqrt{a}c+\left(Da^2\sqrt{b}-Bab^{\frac{3}{2}}\right)c\right)\log\left(\sqrt{b}cx+\sqrt{2}\left(ac^2\right)^{\frac{1}{4}}\sqrt{cx}b^{\frac{1}{4}}+\sqrt{a}c\right)}{\left(ac^2\right)^{\frac{3}{4}}b^{\frac{3}{4}}} - \frac{\sqrt{2}\left(\left(Cab-Ab^2\right)\sqrt{a}c+\left(Da^2\sqrt{b}-Bab^{\frac{3}{2}}\right)c\right)\log\left(\sqrt{b}cx-\sqrt{2}\left(ac^2\right)\right.}{\left(ac^2\right)^{\frac{3}{4}}b^{\frac{3}{4}}}\right.}{}$$

input
```
integrate((c*x)^(1/2)*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="maxima")
```

output
```
1/60*(15*c^2*(sqrt(2)*((C*a*b - A*b^2)*sqrt(a)*c + (D*a^2*sqrt(b) - B*a*b^
(3/2))*c)*log(sqrt(b)*c*x + sqrt(2)*(a*c^2)^(1/4)*sqrt(c*x)*b^(1/4) + sqrt
(a)*c)/((a*c^2)^(3/4)*b^(3/4)) - sqrt(2)*((C*a*b - A*b^2)*sqrt(a)*c + (D*a
^2*sqrt(b) - B*a*b^(3/2))*c)*log(sqrt(b)*c*x - sqrt(2)*(a*c^2)^(1/4)*sqrt(
c*x)*b^(1/4) + sqrt(a)*c)/((a*c^2)^(3/4)*b^(3/4)) - 2*sqrt(2)*((C*a*b - A*
b^2)*sqrt(a)*c - (D*a^2*sqrt(b) - B*a*b^(3/2))*c)*arctan(1/2*sqrt(2)*(sqrt
(2)*(a*c^2)^(1/4)*b^(1/4) + 2*sqrt(c*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*c))/
(sqrt(sqrt(a)*sqrt(b)*c)*sqrt(a)*sqrt(b)*c) - 2*sqrt(2)*((C*a*b - A*b^2)*s
qrt(a)*c - (D*a^2*sqrt(b) - B*a*b^(3/2))*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(
a*c^2)^(1/4)*b^(1/4) - 2*sqrt(c*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*c))/(sqrt
(sqrt(a)*sqrt(b)*c)*sqrt(a)*sqrt(b)*c))/b^2 + 8*(3*(c*x)^(5/2)*D*b + 5*(c*
x)^(3/2)*C*b*c - 15*(D*a - B*b)*sqrt(c*x)*c^2)/(b^2*c))/c
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 524 vs. $2(239) = 478$.

Time = 0.14 (sec) , antiderivative size = 524, normalized size of antiderivative = 1.65

$$\int \frac{\sqrt{cx}(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx$$
$$= \frac{\sqrt{2}\left((ab^3c^2)^{\frac{1}{4}}Da^2bc-(ab^3c^2)^{\frac{1}{4}}Bab^2c-(ab^3c^2)^{\frac{3}{4}}Ca+(ab^3c^2)^{\frac{3}{4}}Ab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ac^2}{b}\right)^{\frac{1}{4}}+2\sqrt{cx}\right)}{2\left(\frac{ac^2}{b}\right)^{\frac{1}{4}}}\right)}{2\,ab^4c}$$
$$+\frac{\sqrt{2}\left((ab^3c^2)^{\frac{1}{4}}Da^2bc-(ab^3c^2)^{\frac{1}{4}}Bab^2c-(ab^3c^2)^{\frac{3}{4}}Ca+(ab^3c^2)^{\frac{3}{4}}Ab\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ac^2}{b}\right)^{\frac{1}{4}}-2\sqrt{cx}\right)}{2\left(\frac{ac^2}{b}\right)^{\frac{1}{4}}}\right)}{2\,ab^4c}$$
$$+\frac{\sqrt{2}\left((ab^3c^2)^{\frac{1}{4}}Da^2bc-(ab^3c^2)^{\frac{1}{4}}Bab^2c+(ab^3c^2)^{\frac{3}{4}}Ca-(ab^3c^2)^{\frac{3}{4}}Ab\right)\log\left(cx+\sqrt{2}\left(\frac{ac^2}{b}\right)^{\frac{1}{4}}\sqrt{cx}+\sqrt{\frac{ac^2}{b}}\right)}{4\,ab^4c}$$
$$-\frac{\sqrt{2}\left((ab^3c^2)^{\frac{1}{4}}Da^2bc-(ab^3c^2)^{\frac{1}{4}}Bab^2c+(ab^3c^2)^{\frac{3}{4}}Ca-(ab^3c^2)^{\frac{3}{4}}Ab\right)\log\left(cx-\sqrt{2}\left(\frac{ac^2}{b}\right)^{\frac{1}{4}}\sqrt{cx}+\sqrt{\frac{ac^2}{b}}\right)}{4\,ab^4c}$$
$$+\frac{2\left(3\sqrt{cx}Db^4c^{10}x^2+5\sqrt{cx}Cb^4c^{10}x-15\sqrt{cx}Dab^3c^{10}+15\sqrt{cx}Bb^4c^{10}\right)}{15\,b^5c^{10}}$$

input
```
integrate((c*x)^(1/2)*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="giac")
```

output
```
1/2*sqrt(2)*((a*b^3*c^2)^(1/4)*D*a^2*b*c - (a*b^3*c^2)^(1/4)*B*a*b^2*c - (
a*b^3*c^2)^(3/4)*C*a + (a*b^3*c^2)^(3/4)*A*b)*arctan(1/2*sqrt(2)*(sqrt(2)*
(a*c^2/b)^(1/4) + 2*sqrt(c*x))/(a*c^2/b)^(1/4))/(a*b^4*c) + 1/2*sqrt(2)*((
a*b^3*c^2)^(1/4)*D*a^2*b*c - (a*b^3*c^2)^(1/4)*B*a*b^2*c - (a*b^3*c^2)^(3/
4)*C*a + (a*b^3*c^2)^(3/4)*A*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*c^2/b)^(1/
4) - 2*sqrt(c*x))/(a*c^2/b)^(1/4))/(a*b^4*c) + 1/4*sqrt(2)*((a*b^3*c^2)^(1
/4)*D*a^2*b*c - (a*b^3*c^2)^(1/4)*B*a*b^2*c + (a*b^3*c^2)^(3/4)*C*a - (a*b
^3*c^2)^(3/4)*A*b)*log(c*x + sqrt(2)*(a*c^2/b)^(1/4)*sqrt(c*x) + sqrt(a*c^
2/b))/(a*b^4*c) - 1/4*sqrt(2)*((a*b^3*c^2)^(1/4)*D*a^2*b*c - (a*b^3*c^2)^(
1/4)*B*a*b^2*c + (a*b^3*c^2)^(3/4)*C*a - (a*b^3*c^2)^(3/4)*A*b)*log(c*x -
sqrt(2)*(a*c^2/b)^(1/4)*sqrt(c*x) + sqrt(a*c^2/b))/(a*b^4*c) + 2/15*(3*sqr
t(c*x)*D*b^4*c^10*x^2 + 5*sqrt(c*x)*C*b^4*c^10*x - 15*sqrt(c*x)*D*a*b^3*c^
10 + 15*sqrt(c*x)*B*b^4*c^10)/(b^5*c^10)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{cx}(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx=\int \frac{\sqrt{c\,x}\,\left(A+B\,x+C\,x^2+x^3\,D\right)}{b\,x^2+a}\,dx$$

input
```
int(((c*x)^(1/2)*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2),x)
```

output
```
int(((c*x)^(1/2)*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2), x)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 590, normalized size of antiderivative = 1.86

$$\int \frac{\sqrt{cx}(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx=\text{Too large to display}$$

input
```
int((c*x)^(1/2)*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x)
```

output

```
(sqrt(c)*( - 30*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2)
- 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*b**2 + 30*b**(1/4)*a**(3
/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)
*a**(1/4)*sqrt(2)))*b*c - 30*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(
1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*d + 30*b*
*(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b
))/(b**(1/4)*a**(1/4)*sqrt(2)))*b**2 + 30*b**(1/4)*a**(3/4)*sqrt(2)*atan((
b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2))
)*b**2 - 30*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*
sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*b*c + 30*b**(3/4)*a**(1/4)*s
qrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(
1/4)*sqrt(2)))*a*d - 30*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*
sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*b**2 + 15*b**(1/
4)*a**(3/4)*sqrt(2)*log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + s
qrt(b)*x)*b**2 - 15*b**(1/4)*a**(3/4)*sqrt(2)*log( - sqrt(x)*b**(1/4)*a**(
1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*b*c - 15*b**(1/4)*a**(3/4)*sqrt(2)*log
(sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*b**2 + 15*b**(1/
4)*a**(3/4)*sqrt(2)*log(sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt
(b)*x)*b*c - 15*b**(3/4)*a**(1/4)*sqrt(2)*log( - sqrt(x)*b**(1/4)*a**(1/4)
*sqrt(2) + sqrt(a) + sqrt(b)*x)*a*d + 15*b**(3/4)*a**(1/4)*sqrt(2)*log(...
```

# 3.125 $\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{cx}(a+bx^2)}\,dx$



## Optimal result

Integrand size = 32, antiderivative size = 294

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{cx}\,(a+bx^2)}\,dx = \frac{2C\sqrt{cx}}{bc}+\frac{2D(cx)^{3/2}}{3bc^2}-\frac{\left(\sqrt{b}(Ab-aC)+\sqrt{a}(bB-aD)\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)}{\sqrt{2}a^{3/4}b^{7/4}\sqrt{c}} + \frac{\left(\sqrt{b}(Ab-aC)+\sqrt{a}(bB-aD)\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)}{\sqrt{2}a^{3/4}b^{7/4}\sqrt{c}} + \frac{\left(\sqrt{b}(Ab-aC)-\sqrt{a}(bB-aD)\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\left(\sqrt{a}+\sqrt{b}x\right)}\right)}{\sqrt{2}a^{3/4}b^{7/4}\sqrt{c}}$$

output
```
2*C*(c*x)^(1/2)/b/c+2/3*D*(c*x)^(3/2)/b/c^2-1/2*(b^(1/2)*(A*b-C*a)+a^(1/2)
*(B*b-D*a))*arctan(1-2^(1/2)*b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))*2^(1/2)/
a^(3/4)/b^(7/4)/c^(1/2)+1/2*(b^(1/2)*(A*b-C*a)+a^(1/2)*(B*b-D*a))*arctan(1
+2^(1/2)*b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))*2^(1/2)/a^(3/4)/b^(7/4)/c^(1
/2)+1/2*(b^(1/2)*(A*b-C*a)-a^(1/2)*(B*b-D*a))*arctanh(2^(1/2)*a^(1/4)*b^(1
/4)*(c*x)^(1/2)/c^(1/2)/(a^(1/2)+b^(1/2)*x))*2^(1/2)/a^(3/4)/b^(7/4)/c^(1/
2)
```

## Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.69

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{cx}\,(a+bx^2)}\,dx$$
$$= \frac{\sqrt{x}\left(4a^{3/4}b^{3/4}\sqrt{x}(3C+Dx)-3\sqrt{2}\left(Ab^{3/2}+\sqrt{a}bB-a\sqrt{b}C-a^{3/2}D\right)\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)+3\sqrt{2}\left(A\right.\right.}{6a^{3/4}b^{7/4}\sqrt{cx}}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/(Sqrt[c*x]*(a + b*x^2)),x]
```

output
```
(Sqrt[x]*(4*a^(3/4)*b^(3/4)*Sqrt[x]*(3*C + D*x) - 3*Sqrt[2]*(A*b^(3/2) + S
qrt[a]*b*B - a*Sqrt[b]*C - a^(3/2)*D)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2
]*a^(1/4)*b^(1/4)*Sqrt[x])] + 3*Sqrt[2]*(A*b^(3/2) - Sqrt[a]*b*B - a*Sqrt[
b]*C + a^(3/2)*D)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqr
t[b]*x)]))/(6*a^(3/4)*b^(7/4)*Sqrt[c*x])
```

## Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 404, normalized size of antiderivative = 1.37, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{cx}\,(a+bx^2)}\,dx$$
$$\Big\downarrow \text{2333}$$
$$\int \left(\frac{x(bB-aD)-aC+Ab}{b\sqrt{cx}\,(a+bx^2)}+\frac{C}{b\sqrt{cx}}+\frac{D\sqrt{cx}}{bc}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)\left(\sqrt{b}(Ab-aC)+\sqrt{a}(bB-aD)\right)}{\sqrt{2}a^{3/4}b^{7/4}\sqrt{c}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}+1\right)\left(\sqrt{b}(Ab-aC)+\sqrt{a}(bB-aD)\right)}{\sqrt{2}a^{3/4}b^{7/4}\sqrt{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{cx}+\sqrt{a}\sqrt{c}+\sqrt{b}\sqrt{c}x\right)\left(\sqrt{b}(Ab-aC)-\sqrt{a}(bB-aD)\right)}{2\sqrt{2}a^{3/4}b^{7/4}\sqrt{c}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{cx}+\sqrt{a}\sqrt{c}+\sqrt{b}\sqrt{c}x\right)\left(\sqrt{b}(Ab-aC)-\sqrt{a}(bB-aD)\right)}{2\sqrt{2}a^{3/4}b^{7/4}\sqrt{c}}+\frac{2D(cx)^{3/2}}{3bc^2}+\frac{2C\sqrt{cx}}{bc}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/(Sqrt[c*x]*(a + b*x^2)),x]
```

output
```
(2*C*Sqrt[c*x])/(b*c) + (2*D*(c*x)^(3/2))/(3*b*c^2) - ((Sqrt[b]*(A*b - a*C
) + Sqrt[a]*(b*B - a*D))*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(a^(1/4)*S
qrt[c])]])/(Sqrt[2]*a^(3/4)*b^(7/4)*Sqrt[c]) + ((Sqrt[b]*(A*b - a*C) + Sqrt
[a]*(b*B - a*D))*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])]
)/(Sqrt[2]*a^(3/4)*b^(7/4)*Sqrt[c]) - ((Sqrt[b]*(A*b - a*C) - Sqrt[a]*(b*B
 - a*D))*Log[Sqrt[a]*Sqrt[c] + Sqrt[b]*Sqrt[c]*x - Sqrt[2]*a^(1/4)*b^(1/4)
*Sqrt[c*x]])/(2*Sqrt[2]*a^(3/4)*b^(7/4)*Sqrt[c]) + ((Sqrt[b]*(A*b - a*C) -
 Sqrt[a]*(b*B - a*D))*Log[Sqrt[a]*Sqrt[c] + Sqrt[b]*Sqrt[c]*x + Sqrt[2]*a^
(1/4)*b^(1/4)*Sqrt[c*x]])/(2*Sqrt[2]*a^(3/4)*b^(7/4)*Sqrt[c])
```

## Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

## Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.10

| method | result |
|---|---|
| pseudoelliptic | $\frac{\frac{\sqrt{2}\,(Ab-Ca)\left(\arctan\left(\frac{\sqrt{2}\,\sqrt{cx}}{\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{4}}}-1\right)+\arctan\left(\frac{\sqrt{2}\,\sqrt{cx}}{\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{4}}}+1\right)+\frac{\ln\left(\frac{cx+\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{4}}\sqrt{cx}\,\sqrt{2}+\sqrt{\frac{a\,c^{2}}{b}}}{cx-\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{4}}\sqrt{cx}\,\sqrt{2}+\sqrt{\frac{a\,c^{2}}{b}}}\right)}{2}\right)b\sqrt{\frac{a\,c^{2}}{b}}}{2}+2\left(\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{4}}\left(\cdots\right.\right.}{c\,b^{2}a\left(\frac{a\,c^{2}}{b}\cdots\right.}$ |
| derivativedivides | $\frac{\frac{2\left(\frac{D(cx)^{\frac{3}{2}}}{3}+Cc\sqrt{cx}\right)}{b}+\frac{2c^{2}\left(\frac{(Abc-Cac)\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{cx+\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{4}}\sqrt{cx}\,\sqrt{2}+\sqrt{\frac{a\,c^{2}}{b}}}{cx-\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{4}}\sqrt{cx}\,\sqrt{2}+\sqrt{\frac{a\,c^{2}}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{cx}}{\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\cdots\right.}{8a\,c^{2}}\cdots\right.}{\cdots}}{c\cdots}$ |
| default | $\frac{\frac{2\left(\frac{D(cx)^{\frac{3}{2}}}{3}+Cc\sqrt{cx}\right)}{b}+\frac{2c^{2}\left(\frac{(Abc-Cac)\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{cx+\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{4}}\sqrt{cx}\,\sqrt{2}+\sqrt{\frac{a\,c^{2}}{b}}}{cx-\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{4}}\sqrt{cx}\,\sqrt{2}+\sqrt{\frac{a\,c^{2}}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{cx}}{\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\cdots\right.}{8a\,c^{2}}\cdots\right.}{\cdots}}{c\cdots}$ |

input
```
int((D*x^3+C*x^2+B*x+A)/(c*x)^(1/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
2*(1/4*2^(1/2)*(A*b-C*a)*(arctan(2^(1/2)/(a*c^2/b)^(1/4)*(c*x)^(1/2)-1)+ar
ctan(2^(1/2)/(a*c^2/b)^(1/4)*(c*x)^(1/2)+1)+1/2*ln((c*x+(a*c^2/b)^(1/4)*(c
*x)^(1/2)*2^(1/2)+(a*c^2/b)^(1/2))/(c*x-(a*c^2/b)^(1/4)*(c*x)^(1/2)*2^(1/2
)+(a*c^2/b)^(1/2))))*b*(a*c^2/b)^(1/2)+((a*c^2/b)^(1/4)*(1/3*D*x+C)*b*(c*x
)^(1/2)+1/4*2^(1/2)*(arctan(2^(1/2)/(a*c^2/b)^(1/4)*(c*x)^(1/2)-1)+arctan(
2^(1/2)/(a*c^2/b)^(1/4)*(c*x)^(1/2)+1)+1/2*ln((c*x-(a*c^2/b)^(1/4)*(c*x)^(
1/2)*2^(1/2)+(a*c^2/b)^(1/2))/(c*x+(a*c^2/b)^(1/4)*(c*x)^(1/2)*2^(1/2)+(a*
c^2/b)^(1/2))))*c*(B*b-D*a))*a)/(a*c^2/b)^(1/4)/c/b^2/a
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3363 vs. $2(220) = 440$.

Time = 0.34 (sec) , antiderivative size = 3363, normalized size of antiderivative = 11.44

$$\int \frac{A + Bx + Cx^2 + Dx^3}{\sqrt{cx}\,(a + bx^2)}\,dx = \text{Too large to display}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(c*x)^(1/2)/(b*x^2+a),x, algorithm="fricas")
```

output
```
Too large to include
```

## Sympy [F]

$$\int \frac{A + Bx + Cx^2 + Dx^3}{\sqrt{cx}\,(a + bx^2)}\,dx = \int \frac{A + Bx + Cx^2 + Dx^3}{\sqrt{cx}\,(a + bx^2)}\,dx$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(c*x)**(1/2)/(b*x**2+a),x)
```

output
```
Integral((A + B*x + C*x**2 + D*x**3)/(sqrt(c*x)*(a + b*x**2)), x)
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 380, normalized size of antiderivative = 1.29

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{cx}\,(a+bx^2)}\,dx$$

$$= \frac{3\,c\left(\frac{\sqrt{2}\left((Da-Bb)\sqrt{a}c-\left(Ca\sqrt{b}-Ab^{\frac{3}{2}}\right)c\right)\log\left(\sqrt{b}cx+\sqrt{2}\left(ac^2\right)^{\frac{1}{4}}\sqrt{cx}b^{\frac{1}{4}}+\sqrt{a}c\right)}{\left(ac^2\right)^{\frac{3}{4}}b^{\frac{3}{4}}}-\frac{\sqrt{2}\left((Da-Bb)\sqrt{a}c-\left(Ca\sqrt{b}-Ab^{\frac{3}{2}}\right)c\right)\log\left(\sqrt{b}cx-\sqrt{2}\left(ac^2\right)^{\frac{1}{4}}\sqrt{cx}b^{\frac{1}{4}}+\sqrt{a}\right.}{\left(ac^2\right)^{\frac{3}{4}}b^{\frac{3}{4}}}\right.}{}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(c*x)^(1/2)/(b*x^2+a),x, algorithm="maxima")
```

output
```
1/12*(3*c*(sqrt(2)*((D*a - B*b)*sqrt(a)*c - (C*a*sqrt(b) - A*b^(3/2))*c)*l
og(sqrt(b)*c*x + sqrt(2)*(a*c^2)^(1/4)*sqrt(c*x)*b^(1/4) + sqrt(a)*c)/((a*
c^2)^(3/4)*b^(3/4)) - sqrt(2)*((D*a - B*b)*sqrt(a)*c - (C*a*sqrt(b) - A*b^
(3/2))*c)*log(sqrt(b)*c*x - sqrt(2)*(a*c^2)^(1/4)*sqrt(c*x)*b^(1/4) + sqrt
(a)*c)/((a*c^2)^(3/4)*b^(3/4)) - 2*sqrt(2)*((D*a - B*b)*sqrt(a)*c + (C*a*s
qrt(b) - A*b^(3/2))*c)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*c^2)^(1/4)*b^(1/4) +
 2*sqrt(c*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*c))/(sqrt(sqrt(a)*sqrt(b)*c)*sq
rt(a)*sqrt(b)*c) - 2*sqrt(2)*((D*a - B*b)*sqrt(a)*c + (C*a*sqrt(b) - A*b^(
3/2))*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*c^2)^(1/4)*b^(1/4) - 2*sqrt(c*x)*
sqrt(b))/sqrt(sqrt(a)*sqrt(b)*c))/(sqrt(sqrt(a)*sqrt(b)*c)*sqrt(a)*sqrt(b)
*c))/b + 8*((c*x)^(3/2)*D + 3*sqrt(c*x)*C*c)/(b*c))/c
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 487 vs. $2(220) = 440$.

Time = 0.13 (sec) , antiderivative size = 487, normalized size of antiderivative = 1.66

$$\int \frac{A + Bx + Cx^2 + Dx^3}{\sqrt{cx}\,(a + bx^2)}\,dx =$$

$$-\frac{\sqrt{2}\left((ab^3c^2)^{\frac{1}{4}}Cab^2c - (ab^3c^2)^{\frac{1}{4}}Ab^3c + (ab^3c^2)^{\frac{3}{4}}Da - (ab^3c^2)^{\frac{3}{4}}Bb\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ac^2}{b}\right)^{\frac{1}{4}} + 2\sqrt{cx}\right)}{2\left(\frac{ac^2}{b}\right)^{\frac{1}{4}}}\right)}{2\,ab^4c^2}$$
$$-\frac{\sqrt{2}\left((ab^3c^2)^{\frac{1}{4}}Cab^2c - (ab^3c^2)^{\frac{1}{4}}Ab^3c + (ab^3c^2)^{\frac{3}{4}}Da - (ab^3c^2)^{\frac{3}{4}}Bb\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ac^2}{b}\right)^{\frac{1}{4}} - 2\sqrt{cx}\right)}{2\left(\frac{ac^2}{b}\right)^{\frac{1}{4}}}\right)}{2\,ab^4c^2}$$
$$-\frac{\sqrt{2}\left((ab^3c^2)^{\frac{1}{4}}Cab^2c - (ab^3c^2)^{\frac{1}{4}}Ab^3c - (ab^3c^2)^{\frac{3}{4}}Da + (ab^3c^2)^{\frac{3}{4}}Bb\right)\log\left(cx + \sqrt{2}\left(\frac{ac^2}{b}\right)^{\frac{1}{4}}\sqrt{cx} + \sqrt{\frac{ac^2}{b}}\right)}{4\,ab^4c^2}$$
$$+\frac{\sqrt{2}\left((ab^3c^2)^{\frac{1}{4}}Cab^2c - (ab^3c^2)^{\frac{1}{4}}Ab^3c - (ab^3c^2)^{\frac{3}{4}}Da + (ab^3c^2)^{\frac{3}{4}}Bb\right)\log\left(cx - \sqrt{2}\left(\frac{ac^2}{b}\right)^{\frac{1}{4}}\sqrt{cx} + \sqrt{\frac{ac^2}{b}}\right)}{4\,ab^4c^2}$$
$$+\frac{2\left(\sqrt{cx}Db^2c^5x + 3\sqrt{cx}Cb^2c^5\right)}{3\,b^3c^6}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(c*x)^(1/2)/(b*x^2+a),x, algorithm="giac")
```

output
```
-1/2*sqrt(2)*((a*b^3*c^2)^(1/4)*C*a*b^2*c - (a*b^3*c^2)^(1/4)*A*b^3*c + (a
*b^3*c^2)^(3/4)*D*a - (a*b^3*c^2)^(3/4)*B*b)*arctan(1/2*sqrt(2)*(sqrt(2)*(
a*c^2/b)^(1/4) + 2*sqrt(c*x))/(a*c^2/b)^(1/4))/(a*b^4*c^2) - 1/2*sqrt(2)*(
(a*b^3*c^2)^(1/4)*C*a*b^2*c - (a*b^3*c^2)^(1/4)*A*b^3*c + (a*b^3*c^2)^(3/4
)*D*a - (a*b^3*c^2)^(3/4)*B*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*c^2/b)^(1/4
) - 2*sqrt(c*x))/(a*c^2/b)^(1/4))/(a*b^4*c^2) - 1/4*sqrt(2)*((a*b^3*c^2)^(
1/4)*C*a*b^2*c - (a*b^3*c^2)^(1/4)*A*b^3*c - (a*b^3*c^2)^(3/4)*D*a + (a*b^
3*c^2)^(3/4)*B*b)*log(c*x + sqrt(2)*(a*c^2/b)^(1/4)*sqrt(c*x) + sqrt(a*c^2
/b))/(a*b^4*c^2) + 1/4*sqrt(2)*((a*b^3*c^2)^(1/4)*C*a*b^2*c - (a*b^3*c^2)^
(1/4)*A*b^3*c - (a*b^3*c^2)^(3/4)*D*a + (a*b^3*c^2)^(3/4)*B*b)*log(c*x - s
qrt(2)*(a*c^2/b)^(1/4)*sqrt(c*x) + sqrt(a*c^2/b))/(a*b^4*c^2) + 2/3*(sqrt(
c*x)*D*b^2*c^5*x + 3*sqrt(c*x)*C*b^2*c^5)/(b^3*c^6)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{cx}\,(a+bx^2)}\,dx=\int \frac{A+B\,x+C\,x^2+x^3\,D}{\sqrt{c\,x}\,\left(b\,x^2+a\right)}\,dx$$

input
```
int((A + B*x + C*x^2 + x^3*D)/((c*x)^(1/2)*(a + b*x^2)),x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/((c*x)^(1/2)*(a + b*x^2)), x)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 577, normalized size of antiderivative = 1.96

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{cx}\,(a+bx^2)}\,dx=\text{Too large to display}$$

input
```
int((D*x^3+C*x^2+B*x+A)/(c*x)^(1/2)/(b*x^2+a),x)
```

output
```
(sqrt(c)*(6*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*
sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*d - 6*b**(1/4)*a**(3/4)*sq
rt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1
/4)*sqrt(2)))*b**2 - 6*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*s
qrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b + 6*b**(3/4)*
a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**
(1/4)*a**(1/4)*sqrt(2)))*a*c - 6*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*
a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*d + 6
*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqr
t(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*b**2 + 6*b**(3/4)*a**(1/4)*sqrt(2)*atan
((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2
)))*a*b - 6*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*
sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*c - 3*b**(1/4)*a**(3/4)*sq
rt(2)*log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a*d
+ 3*b**(1/4)*a**(3/4)*sqrt(2)*log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + s
qrt(a) + sqrt(b)*x)*b**2 + 3*b**(1/4)*a**(3/4)*sqrt(2)*log(sqrt(x)*b**(1/4
)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a*d - 3*b**(1/4)*a**(3/4)*sqrt(2
)*log(sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*b**2 - 3*b*
*(3/4)*a**(1/4)*sqrt(2)*log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a)
 + sqrt(b)*x)*a*b + 3*b**(3/4)*a**(1/4)*sqrt(2)*log( - sqrt(x)*b**(1/4)...
```

# 3.126 $\int \frac{A+Bx+Cx^2+Dx^3}{(cx)^{3/2}(a+bx^2)}\,dx$



## Optimal result

Integrand size = 32, antiderivative size = 293

$$\begin{aligned}\int \frac{A+Bx+Cx^2+Dx^3}{(cx)^{3/2}\left(a+bx^2\right)}\,dx &= -\frac{2A}{ac\sqrt{cx}}+\frac{2D\sqrt{cx}}{bc^2}\\
&+\frac{\left(\sqrt{b}(Ab-aC)-\sqrt{a}(bB-aD)\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)}{\sqrt{2}a^{5/4}b^{5/4}c^{3/2}}\\
&-\frac{\left(\sqrt{b}(Ab-aC)-\sqrt{a}(bB-aD)\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)}{\sqrt{2}a^{5/4}b^{5/4}c^{3/2}}\\
&+\frac{\left(\sqrt{b}(Ab-aC)+\sqrt{a}(bB-aD)\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\left(\sqrt{a}+\sqrt{b}x\right)}\right)}{\sqrt{2}a^{5/4}b^{5/4}c^{3/2}}\end{aligned}$$

output
```
-2*A/a/c/(c*x)^(1/2)+2*D*(c*x)^(1/2)/b/c^2+1/2*(b^(1/2)*(A*b-C*a)-a^(1/2)*
(B*b-D*a))*arctan(1-2^(1/2)*b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))*2^(1/2)/a
^(5/4)/b^(5/4)/c^(3/2)-1/2*(b^(1/2)*(A*b-C*a)-a^(1/2)*(B*b-D*a))*arctan(1+
2^(1/2)*b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))*2^(1/2)/a^(5/4)/b^(5/4)/c^(3/
2)+1/2*(b^(1/2)*(A*b-C*a)+a^(1/2)*(B*b-D*a))*arctanh(2^(1/2)*a^(1/4)*b^(1/
4)*(c*x)^(1/2)/c^(1/2)/(a^(1/2)+b^(1/2)*x))*2^(1/2)/a^(5/4)/b^(5/4)/c^(3/2
)
```

## Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.70

$$\int \frac{A+Bx+Cx^2+Dx^3}{(cx)^{3/2}(a+bx^2)}\,dx = \frac{x\left(4\sqrt[4]{a}\sqrt[4]{b}(-Ab+aDx)+\sqrt{2}\left(Ab^{3/2}-\sqrt{a}bB-a\sqrt{b}C+a^{3/2}D\right)\sqrt{x}\,\text{arcta}\right.}{2}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/((c*x)^(3/2)*(a + b*x^2)),x]
```

output
```
(x*(4*a^(1/4)*b^(1/4)*(-(A*b) + a*D*x) + Sqrt[2]*(A*b^(3/2) - Sqrt[a]*b*B
- a*Sqrt[b]*C + a^(3/2)*D)*Sqrt[x]*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a
^(1/4)*b^(1/4)*Sqrt[x])] + Sqrt[2]*(A*b^(3/2) + Sqrt[a]*b*B - a*Sqrt[b]*C
- a^(3/2)*D)*Sqrt[x]*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] +
Sqrt[b]*x)]))/(2*a^(5/4)*b^(5/4)*(c*x)^(3/2))
```

## Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 428, normalized size of antiderivative = 1.46, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(cx)^{3/2}(a+bx^2)}\,dx$$

$$\Big\downarrow \text{ 2333}$$

$$\int \left(\frac{x(bB-aD)-aC+Ab}{b(cx)^{3/2}(a+bx^2)}+\frac{C}{b(cx)^{3/2}}+\frac{D}{bc\sqrt{cx}}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)\left(\sqrt{b}(Ab-aC)-\sqrt{a}(bB-aD)\right)}{\sqrt{2}a^{5/4}b^{5/4}c^{3/2}}-\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}+1\right)\left(\sqrt{b}(Ab-aC)-\sqrt{a}(bB-aD)\right)}{\sqrt{2}a^{5/4}b^{5/4}c^{3/2}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{cx}+\sqrt{a}\sqrt{c}+\sqrt{b}\sqrt{cx}\right)\left(\sqrt{b}(Ab-aC)+\sqrt{a}(bB-aD)\right)}{2\sqrt{2}a^{5/4}b^{5/4}c^{3/2}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{cx}+\sqrt{a}\sqrt{c}+\sqrt{b}\sqrt{cx}\right)\left(\sqrt{b}(Ab-aC)+\sqrt{a}(bB-aD)\right)}{2\sqrt{2}a^{5/4}b^{5/4}c^{3/2}}-\frac{2(Ab-aC)}{abc\sqrt{cx}}+\frac{2D\sqrt{cx}}{bc^2}-\frac{2C}{bc\sqrt{cx}}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/((c*x)^(3/2)*(a + b*x^2)),x]
```

output
```
(-2*C)/(b*c*Sqrt[c*x]) - (2*(A*b - a*C))/(a*b*c*Sqrt[c*x]) + (2*D*Sqrt[c*x
])/(b*c^2) + ((Sqrt[b]*(A*b - a*C) - Sqrt[a]*(b*B - a*D))*ArcTan[1 - (Sqrt
[2]*b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])]])/(Sqrt[2]*a^(5/4)*b^(5/4)*c^(3/2
)) - ((Sqrt[b]*(A*b - a*C) - Sqrt[a]*(b*B - a*D))*ArcTan[1 + (Sqrt[2]*b^(1
/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])]])/(Sqrt[2]*a^(5/4)*b^(5/4)*c^(3/2)) - ((S
qrt[b]*(A*b - a*C) + Sqrt[a]*(b*B - a*D))*Log[Sqrt[a]*Sqrt[c] + Sqrt[b]*Sq
rt[c]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[c*x]])/(2*Sqrt[2]*a^(5/4)*b^(5/4)*c
^(3/2)) + ((Sqrt[b]*(A*b - a*C) + Sqrt[a]*(b*B - a*D))*Log[Sqrt[a]*Sqrt[c]
 + Sqrt[b]*Sqrt[c]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[c*x]])/(2*Sqrt[2]*a^(5
/4)*b^(5/4)*c^(3/2))
```

## Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

## Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.14

| method | result |
|---|---|
| pseudoelliptic | $-\frac{2\left(-\frac{\left(\ln\left(\frac{cx+\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{4}}\sqrt{cx}\,\sqrt{2}+\sqrt{\frac{a\,c^{2}}{b}}}{cx-\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{4}}\sqrt{cx}\,\sqrt{2}+\sqrt{\frac{a\,c^{2}}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{cx}}{\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{cx}}{\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{4}}}-1\right)\right)\sqrt{2}\,\sqrt{cx}\,(Bb-Da)\sqrt{\frac{a\,c^{2}}{b}}}{8}+c \cdots\right.}{\sqrt{c \cdots}}$ |
| derivativedivides | $\frac{\frac{2D\sqrt{cx}}{b}-\frac{2Ac}{a\sqrt{cx}}-\frac{2c\left(\frac{\left(-abBc+Da^{2}c\right)\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{cx+\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{4}}\sqrt{cx}\,\sqrt{2}+\sqrt{\frac{a\,c^{2}}{b}}}{cx-\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{4}}\sqrt{cx}\,\sqrt{2}+\sqrt{\frac{a\,c^{2}}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{cx}}{\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\cdots\right.\right.}{8a\,c^{2}}\cdots\right.}{\cdots}}{c \cdots}$ |
| default | $\frac{\frac{2D\sqrt{cx}}{b}-\frac{2Ac}{a\sqrt{cx}}-\frac{2c\left(\frac{\left(-abBc+Da^{2}c\right)\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{cx+\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{4}}\sqrt{cx}\,\sqrt{2}+\sqrt{\frac{a\,c^{2}}{b}}}{cx-\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{4}}\sqrt{cx}\,\sqrt{2}+\sqrt{\frac{a\,c^{2}}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{cx}}{\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\cdots\right.\right.}{8a\,c^{2}}\cdots\right.}{\cdots}}{c \cdots}$ |

input
```
int((D*x^3+C*x^2+B*x+A)/(c*x)^(3/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-2/(c*x)^(1/2)/(a*c^2/b)^(1/4)*(-1/8*(ln((c*x+(a*c^2/b)^(1/4)*(c*x)^(1/2)*
2^(1/2)+(a*c^2/b)^(1/2))/(c*x-(a*c^2/b)^(1/4)*(c*x)^(1/2)*2^(1/2)+(a*c^2/b
)^(1/2)))+2*arctan(2^(1/2)/(a*c^2/b)^(1/4)*(c*x)^(1/2)+1)+2*arctan(2^(1/2)
/(a*c^2/b)^(1/4)*(c*x)^(1/2)-1))*2^(1/2)*(c*x)^(1/2)*(B*b-D*a)*(a*c^2/b)^(
1/2)+c*(1/4*2^(1/2)*(arctan(2^(1/2)/(a*c^2/b)^(1/4)*(c*x)^(1/2)-1)+arctan(
2^(1/2)/(a*c^2/b)^(1/4)*(c*x)^(1/2)+1)+1/2*ln((c*x-(a*c^2/b)^(1/4)*(c*x)^(
1/2)*2^(1/2)+(a*c^2/b)^(1/2))/(c*x+(a*c^2/b)^(1/4)*(c*x)^(1/2)*2^(1/2)+(a*
c^2/b)^(1/2))))*(A*b-C*a)*(c*x)^(1/2)+(-D*a*x+A*b)*(a*c^2/b)^(1/4)))/b/a/c
^2
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3429 vs. $2(221) = 442$.

Time = 0.48 (sec) , antiderivative size = 3429, normalized size of antiderivative = 11.70

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(cx)^{3/2}(a + bx^2)}\,dx = \text{Too large to display}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(c*x)^(3/2)/(b*x^2+a),x, algorithm="fricas")
```

output
```
Too large to include
```

## Sympy [F]

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(cx)^{3/2}(a + bx^2)}\,dx = \int \frac{A + Bx + Cx^2 + Dx^3}{(cx)^{\frac{3}{2}}(a + bx^2)}\,dx$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(c*x)**(3/2)/(b*x**2+a),x)
```

output
```
Integral((A + B*x + C*x**2 + D*x**3)/((c*x)**(3/2)*(a + b*x**2)), x)
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 405, normalized size of antiderivative = 1.38

$$\int \frac{A+Bx+Cx^2+Dx^3}{(cx)^{3/2}\,(a+bx^2)}\,dx =$$

$$-\frac{\frac{8A}{\sqrt{cx}a}-\frac{8\sqrt{cx}D}{bc}+\frac{\frac{\sqrt{2}\left(\left(Cab-Ab^2\right)\sqrt{a}c+\left(Da^2\sqrt{b}-Bab^{\frac{3}{2}}\right)c\right)\log\left(\sqrt{b}cx+\sqrt{2}\left(ac^2\right)^{\frac{1}{4}}\sqrt{cx}b^{\frac{1}{4}}+\sqrt{a}c\right)}{\left(ac^2\right)^{\frac{3}{4}}b^{\frac{3}{4}}}-\frac{\sqrt{2}\left(\left(Cab-Ab^2\right)\sqrt{a}c+\left(Da^2\sqrt{b}-Bab^{\frac{3}{2}}\right)c\right)\log\left(\cdots\right.}{\left(ac^2\right)^{\frac{3}{4}}b^{\frac{3}{4}}}\cdots}{\cdots}}{\cdots}$$

input

```
integrate((D*x^3+C*x^2+B*x+A)/(c*x)^(3/2)/(b*x^2+a),x, algorithm="maxima")
```

output

```
-1/4*(8*A/(sqrt(c*x)*a) - 8*sqrt(c*x)*D/(b*c) + (sqrt(2)*((C*a*b - A*b^2)*
sqrt(a)*c + (D*a^2*sqrt(b) - B*a*b^(3/2))*c)*log(sqrt(b)*c*x + sqrt(2)*(a*
c^2)^(1/4)*sqrt(c*x)*b^(1/4) + sqrt(a)*c)/((a*c^2)^(3/4)*b^(3/4)) - sqrt(2
)*((C*a*b - A*b^2)*sqrt(a)*c + (D*a^2*sqrt(b) - B*a*b^(3/2))*c)*log(sqrt(b
)*c*x - sqrt(2)*(a*c^2)^(1/4)*sqrt(c*x)*b^(1/4) + sqrt(a)*c)/((a*c^2)^(3/4
)*b^(3/4)) - 2*sqrt(2)*((C*a*b - A*b^2)*sqrt(a)*c - (D*a^2*sqrt(b) - B*a*b
^(3/2))*c)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*c^2)^(1/4)*b^(1/4) + 2*sqrt(c*x)
*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*c))/(sqrt(sqrt(a)*sqrt(b)*c)*sqrt(a)*sqrt(b
)*c) - 2*sqrt(2)*((C*a*b - A*b^2)*sqrt(a)*c - (D*a^2*sqrt(b) - B*a*b^(3/2)
)*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*c^2)^(1/4)*b^(1/4) - 2*sqrt(c*x)*sqrt
(b))/sqrt(sqrt(a)*sqrt(b)*c))/(sqrt(sqrt(a)*sqrt(b)*c)*sqrt(a)*sqrt(b)*c))
/(a*b))/c
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 483 vs. 2(221) = 442.

Time = 0.14 (sec) , antiderivative size = 483, normalized size of antiderivative = 1.65

$$\int \frac{A+Bx+Cx^2+Dx^3}{(cx)^{3/2}\,(a+bx^2)}\,dx =$$

$$-\frac{\frac{8A}{\sqrt{cx}a}-\frac{8\sqrt{cx}D}{bc}+\frac{2\sqrt{2}\left(\left(ab^3c^2\right)^{\frac{1}{4}}Da^2bc-\left(ab^3c^2\right)^{\frac{1}{4}}Bab^2c-\left(ab^3c^2\right)^{\frac{3}{4}}Ca+\left(ab^3c^2\right)^{\frac{3}{4}}Ab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ac^2}{b}\right)^{\frac{1}{4}}+2\sqrt{cx}\right)}{2\left(\frac{ac^2}{b}\right)^{\frac{1}{4}}}\right)}{a^2b^3c^2}+\frac{2\sqrt{2}\left(\left(ab\cdots\right.\right.}{\cdots}\cdots}{\cdots}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(c*x)^(3/2)/(b*x^2+a),x, algorithm="giac")
```

output
```
-1/4*(8*A/(sqrt(c*x)*a) - 8*sqrt(c*x)*D/(b*c) + 2*sqrt(2)*((a*b^3*c^2)^(1/
4)*D*a^2*b*c - (a*b^3*c^2)^(1/4)*B*a*b^2*c - (a*b^3*c^2)^(3/4)*C*a + (a*b^
3*c^2)^(3/4)*A*b)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*c^2/b)^(1/4) + 2*sqrt(c*x
))/(a*c^2/b)^(1/4))/(a^2*b^3*c^2) + 2*sqrt(2)*((a*b^3*c^2)^(1/4)*D*a^2*b*c
 - (a*b^3*c^2)^(1/4)*B*a*b^2*c - (a*b^3*c^2)^(3/4)*C*a + (a*b^3*c^2)^(3/4)
*A*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*c^2/b)^(1/4) - 2*sqrt(c*x))/(a*c^2/b
)^(1/4))/(a^2*b^3*c^2) + sqrt(2)*((a*b^3*c^2)^(1/4)*D*a^2*b*c - (a*b^3*c^2
)^(1/4)*B*a*b^2*c + (a*b^3*c^2)^(3/4)*C*a - (a*b^3*c^2)^(3/4)*A*b)*log(c*x
 + sqrt(2)*(a*c^2/b)^(1/4)*sqrt(c*x) + sqrt(a*c^2/b))/(a^2*b^3*c^2) - sqrt
(2)*((a*b^3*c^2)^(1/4)*D*a^2*b*c - (a*b^3*c^2)^(1/4)*B*a*b^2*c + (a*b^3*c^
2)^(3/4)*C*a - (a*b^3*c^2)^(3/4)*A*b)*log(c*x - sqrt(2)*(a*c^2/b)^(1/4)*sq
rt(c*x) + sqrt(a*c^2/b))/(a^2*b^3*c^2))/c
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(cx)^{3/2}\,(a+bx^2)}\,dx = \int \frac{A+B\,x+C\,x^2+x^3\,D}{(c\,x)^{3/2}\,\left(b\,x^2+a\right)}\,dx$$

input
```
int((A + B*x + C*x^2 + x^3*D)/((c*x)^(3/2)*(a + b*x^2)),x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/((c*x)^(3/2)*(a + b*x^2)), x)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 606, normalized size of antiderivative = 2.07

$$\int \frac{A+Bx+Cx^2+Dx^3}{(cx)^{3/2}\,(a+bx^2)}\,dx = \text{Too large to display}$$

input
```
int((D*x^3+C*x^2+B*x+A)/(c*x)^(3/2)/(b*x^2+a),x)
```

output

```
(sqrt(c)*(2*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt
(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*b**2 - 2*sqrt(x)*b**
(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b)
)/(b**(1/4)*a**(1/4)*sqrt(2)))*b*c + 2*sqrt(x)*b**(3/4)*a**(1/4)*sqrt(2)*a
tan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqr
t(2)))*a*d - 2*sqrt(x)*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*s
qrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*b**2 - 2*sqrt(x)*
b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt
(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*b**2 + 2*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(
2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)
*sqrt(2)))*b*c - 2*sqrt(x)*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/
4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*d + 2*sqrt(
x)*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*s
qrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*b**2 - sqrt(x)*b**(1/4)*a**(3/4)*sqrt
(2)*log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*b**2 +
 sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2
) + sqrt(a) + sqrt(b)*x)*b*c + sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*log(sqrt(
x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*b**2 - sqrt(x)*b**(1/4
)*a**(3/4)*sqrt(2)*log(sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(
b)*x)*b*c + sqrt(x)*b**(3/4)*a**(1/4)*sqrt(2)*log( - sqrt(x)*b**(1/4)*a...
```

# 3.127 $\int \frac{A+Bx+Cx^2+Dx^3}{(cx)^{5/2}(a+bx^2)}\,dx$



## Optimal result

Integrand size = 32, antiderivative size = 295

$$\int \frac{A+Bx+Cx^2+Dx^3}{(cx)^{5/2}\,(a+bx^2)}\,dx = -\frac{2A}{3ac(cx)^{3/2}}-\frac{2B}{ac^2\sqrt{cx}} + \frac{\left(\sqrt{b}(Ab-aC)+\sqrt{a}(bB-aD)\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)}{\sqrt{2}a^{7/4}b^{3/4}c^{5/2}} - \frac{\left(\sqrt{b}(Ab-aC)+\sqrt{a}(bB-aD)\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)}{\sqrt{2}a^{7/4}b^{3/4}c^{5/2}} - \frac{\left(\sqrt{b}(Ab-aC)-\sqrt{a}(bB-aD)\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\left(\sqrt{a}+\sqrt{b}x\right)}\right)}{\sqrt{2}a^{7/4}b^{3/4}c^{5/2}}$$

output
```
-2/3*A/a/c/(c*x)^(3/2)-2*B/a/c^2/(c*x)^(1/2)+1/2*(b^(1/2)*(A*b-C*a)+a^(1/2
)*(B*b-D*a))*arctan(1-2^(1/2)*b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))*2^(1/2)
/a^(7/4)/b^(3/4)/c^(5/2)-1/2*(b^(1/2)*(A*b-C*a)+a^(1/2)*(B*b-D*a))*arctan(
1+2^(1/2)*b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))*2^(1/2)/a^(7/4)/b^(3/4)/c^(
5/2)-1/2*(b^(1/2)*(A*b-C*a)-a^(1/2)*(B*b-D*a))*arctanh(2^(1/2)*a^(1/4)*b^(
1/4)*(c*x)^(1/2)/c^(1/2)/(a^(1/2)+b^(1/2)*x))*2^(1/2)/a^(7/4)/b^(3/4)/c^(5
/2)
```

## Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.69

$$\int \frac{A+Bx+Cx^2+Dx^3}{(cx)^{5/2}\,(a+bx^2)}\,dx = \frac{x\left(-4a^{3/4}b^{3/4}(A+3Bx)+3\sqrt{2}\left(Ab^{3/2}+\sqrt{a}bB-a\sqrt{b}C-a^{3/2}D\right)x^{3/2}\,\mathrm{arc}\right.}{}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/((c*x)^(5/2)*(a + b*x^2)),x]
```

output
```
(x*(-4*a^(3/4)*b^(3/4)*(A + 3*B*x) + 3*Sqrt[2]*(A*b^(3/2) + Sqrt[a]*b*B -
a*Sqrt[b]*C - a^(3/2)*D)*x^(3/2)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(
1/4)*b^(1/4)*Sqrt[x])] - 3*Sqrt[2]*(A*b^(3/2) - Sqrt[a]*b*B - a*Sqrt[b]*C
+ a^(3/2)*D)*x^(3/2)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] +
Sqrt[b]*x)]))/(6*a^(7/4)*b^(3/4)*(c*x)^(5/2))
```

## Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 458, normalized size of antiderivative = 1.55, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(cx)^{5/2}\,(a+bx^2)}\,dx$$

$$\Big\downarrow \; 2333$$

$$\int \left(\frac{x(bB-aD)-aC+Ab}{b(cx)^{5/2}\,(a+bx^2)}+\frac{C}{b(cx)^{5/2}}+\frac{D}{bc(cx)^{3/2}}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)\left(\sqrt{b}(Ab-aC)+\sqrt{a}(bB-aD)\right)}{\sqrt{2}a^{7/4}b^{3/4}c^{5/2}}-\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}+1\right)\left(\sqrt{b}(Ab-aC)+\sqrt{a}(bB-aD)\right)}{\sqrt{2}a^{7/4}b^{3/4}c^{5/2}}+\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{cx}+\sqrt{a}\sqrt{c}+\sqrt{b}\sqrt{cx}\right)\left(\sqrt{b}(Ab-aC)-\sqrt{a}(bB-aD)\right)}{2\sqrt{2}a^{7/4}b^{3/4}c^{5/2}}-\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{cx}+\sqrt{a}\sqrt{c}+\sqrt{b}\sqrt{cx}\right)\left(\sqrt{b}(Ab-aC)-\sqrt{a}(bB-aD)\right)}{2\sqrt{2}a^{7/4}b^{3/4}c^{5/2}}-\frac{2(Ab-aC)}{3abc(cx)^{3/2}}-\frac{2(bB-aD)}{abc^2\sqrt{cx}}-\frac{2D}{bc^2\sqrt{cx}}-\frac{2C}{3bc(cx)^{3/2}}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/((c*x)^(5/2)*(a + b*x^2)),x]
```

output
```
(-2*C)/(3*b*c*(c*x)^(3/2)) - (2*(A*b - a*C))/(3*a*b*c*(c*x)^(3/2)) - (2*D)
/(b*c^2*Sqrt[c*x]) - (2*(b*B - a*D))/(a*b*c^2*Sqrt[c*x]) + ((Sqrt[b]*(A*b
- a*C) + Sqrt[a]*(b*B - a*D))*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(a^(1
/4)*Sqrt[c])])/(Sqrt[2]*a^(7/4)*b^(3/4)*c^(5/2)) - ((Sqrt[b]*(A*b - a*C) +
 Sqrt[a]*(b*B - a*D))*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt
[c])])/(Sqrt[2]*a^(7/4)*b^(3/4)*c^(5/2)) + ((Sqrt[b]*(A*b - a*C) - Sqrt[a]
*(b*B - a*D))*Log[Sqrt[a]*Sqrt[c] + Sqrt[b]*Sqrt[c]*x - Sqrt[2]*a^(1/4)*b^
(1/4)*Sqrt[c*x]])/(2*Sqrt[2]*a^(7/4)*b^(3/4)*c^(5/2)) - ((Sqrt[b]*(A*b - a
*C) - Sqrt[a]*(b*B - a*D))*Log[Sqrt[a]*Sqrt[c] + Sqrt[b]*Sqrt[c]*x + Sqrt[
2]*a^(1/4)*b^(1/4)*Sqrt[c*x]])/(2*Sqrt[2]*a^(7/4)*b^(3/4)*c^(5/2))
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

## Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.13

| method | result |
|---|---|
| derivativedivides | $\frac{-\frac{2B}{a\sqrt{cx}}-\frac{2Ac}{3a(cx)^{\frac{3}{2}}}+\frac{2\left(\frac{(-Abc+Cac)\left(\frac{a\,c^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{cx+\left(\frac{a\,c^2}{b}\right)^{\frac{1}{4}}\sqrt{cx}\,\sqrt{2}+\sqrt{\frac{a\,c^2}{b}}}{cx-\left(\frac{a\,c^2}{b}\right)^{\frac{1}{4}}\sqrt{cx}\,\sqrt{2}+\sqrt{\frac{a\,c^2}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{cx}}{\left(\frac{a\,c^2}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(-\ldots\right.\right.}{8a\,c^2}\ldots\right)}{\ldots}}{c^2\ldots}$ |
| default | $\frac{-\frac{2B}{a\sqrt{cx}}-\frac{2Ac}{3a(cx)^{\frac{3}{2}}}+\frac{2\left(\frac{(-Abc+Cac)\left(\frac{a\,c^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{cx+\left(\frac{a\,c^2}{b}\right)^{\frac{1}{4}}\sqrt{cx}\,\sqrt{2}+\sqrt{\frac{a\,c^2}{b}}}{cx-\left(\frac{a\,c^2}{b}\right)^{\frac{1}{4}}\sqrt{cx}\,\sqrt{2}+\sqrt{\frac{a\,c^2}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{cx}}{\left(\frac{a\,c^2}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(-\ldots\right.\right.}{8a\,c^2}\ldots\right)}{\ldots}}{c^2\ldots}$ |
| pseudoelliptic | $-\frac{\sqrt{2}\,(cx)^{\frac{3}{2}}(Ab-Ca)b\left(\ln\left(\frac{cx+\left(\frac{a\,c^2}{b}\right)^{\frac{1}{4}}\sqrt{cx}\,\sqrt{2}+\sqrt{\frac{a\,c^2}{b}}}{cx-\left(\frac{a\,c^2}{b}\right)^{\frac{1}{4}}\sqrt{cx}\,\sqrt{2}+\sqrt{\frac{a\,c^2}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{cx}}{\left(\frac{a\,c^2}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{cx}}{\left(\frac{a\,c^2}{b}\right)^{\frac{1}{4}}}-1\right)\right)\sqrt{\ldots}}{\ldots}$ |

input

```
int((D*x^3+C*x^2+B*x+A)/(c*x)^(5/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output

```
2/c^2*(-1/a*B/(c*x)^(1/2)-1/3*A*c/a/(c*x)^(3/2)+1/a*(1/8*(-A*b*c+C*a*c)*(a
*c^2/b)^(1/4)/a/c^2*2^(1/2)*(ln((c*x+(a*c^2/b)^(1/4)*(c*x)^(1/2)*2^(1/2)+(
a*c^2/b)^(1/2))/(c*x-(a*c^2/b)^(1/4)*(c*x)^(1/2)*2^(1/2)+(a*c^2/b)^(1/2)))
+2*arctan(2^(1/2)/(a*c^2/b)^(1/4)*(c*x)^(1/2)+1)+2*arctan(2^(1/2)/(a*c^2/b
)^(1/4)*(c*x)^(1/2)-1))+1/8*(-B*b+D*a)/b/(a*c^2/b)^(1/4)*2^(1/2)*(ln((c*x-
(a*c^2/b)^(1/4)*(c*x)^(1/2)*2^(1/2)+(a*c^2/b)^(1/2))/(c*x+(a*c^2/b)^(1/4)*
(c*x)^(1/2)*2^(1/2)+(a*c^2/b)^(1/2)))+2*arctan(2^(1/2)/(a*c^2/b)^(1/4)*(c*
x)^(1/2)+1)+2*arctan(2^(1/2)/(a*c^2/b)^(1/4)*(c*x)^(1/2)-1))))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3409 vs. 2(220) = 440.

Time = 0.36 (sec) , antiderivative size = 3409, normalized size of antiderivative = 11.56

$$\int \frac{A+Bx+Cx^2+Dx^3}{(cx)^{5/2}\,(a+bx^2)}\,dx = \text{Too large to display}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(c*x)^(5/2)/(b*x^2+a),x, algorithm="fricas")
```

output
```
Too large to include
```

## Sympy [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{(cx)^{5/2}\,(a+bx^2)}\,dx = \int \frac{A+Bx+Cx^2+Dx^3}{(cx)^{\frac{5}{2}}\,(a+bx^2)}\,dx$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(c*x)**(5/2)/(b*x**2+a),x)
```

output
```
Integral((A + B*x + C*x**2 + D*x**3)/((c*x)**(5/2)*(a + b*x**2)), x)
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 379, normalized size of antiderivative = 1.28

$$\int \frac{A+Bx+Cx^2+Dx^3}{(cx)^{5/2}\,(a+bx^2)}\,dx =$$

$$-\frac{3\left(\frac{\sqrt{2}\left((Da-Bb)\sqrt{a}c-\left(Ca\sqrt{b}-Ab^{\frac{3}{2}}\right)c\right)\log\left(\sqrt{b}cx+\sqrt{2}\left(ac^2\right)^{\frac{1}{4}}\sqrt{cx}b^{\frac{1}{4}}+\sqrt{a}c\right)}{\left(ac^2\right)^{\frac{3}{4}}b^{\frac{3}{4}}}-\frac{\sqrt{2}\left((Da-Bb)\sqrt{a}c-\left(Ca\sqrt{b}-Ab^{\frac{3}{2}}\right)c\right)\log\left(\sqrt{b}cx-\sqrt{2}\left(ac^2\right)^{\frac{1}{4}}\sqrt{cx}b^{\frac{1}{4}}+\sqrt{a}c\right)}{\left(ac^2\right)^{\frac{3}{4}}b^{\frac{3}{4}}}\right.}{}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(c*x)^(5/2)/(b*x^2+a),x, algorithm="maxima")
```

output
```
-1/12*(3*(sqrt(2)*((D*a - B*b)*sqrt(a)*c - (C*a*sqrt(b) - A*b^(3/2))*c)*lo
g(sqrt(b)*c*x + sqrt(2)*(a*c^2)^(1/4)*sqrt(c*x)*b^(1/4) + sqrt(a)*c)/((a*c
^2)^(3/4)*b^(3/4)) - sqrt(2)*((D*a - B*b)*sqrt(a)*c - (C*a*sqrt(b) - A*b^(
3/2))*c)*log(sqrt(b)*c*x - sqrt(2)*(a*c^2)^(1/4)*sqrt(c*x)*b^(1/4) + sqrt(
a)*c)/((a*c^2)^(3/4)*b^(3/4)) - 2*sqrt(2)*((D*a - B*b)*sqrt(a)*c + (C*a*sq
rt(b) - A*b^(3/2))*c)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*c^2)^(1/4)*b^(1/4) +
2*sqrt(c*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*c))/(sqrt(sqrt(a)*sqrt(b)*c)*sqr
t(a)*sqrt(b)*c) - 2*sqrt(2)*((D*a - B*b)*sqrt(a)*c + (C*a*sqrt(b) - A*b^(3
/2))*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*c^2)^(1/4)*b^(1/4) - 2*sqrt(c*x)*s
qrt(b))/sqrt(sqrt(a)*sqrt(b)*c))/(sqrt(sqrt(a)*sqrt(b)*c)*sqrt(a)*sqrt(b)*
c))/(a*c) + 8*(3*B*c*x + A*c)/((c*x)^(3/2)*a*c))/c
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 475 vs. $2(220) = 440$.

Time = 0.13 (sec) , antiderivative size = 475, normalized size of antiderivative = 1.61

$$\int \frac{A+Bx+Cx^2+Dx^3}{(cx)^{5/2}(a+bx^2)}\,dx = -\frac{2\,(3\,Bcx+Ac)}{3\,\sqrt{cx}ac^3x} + \frac{\sqrt{2}\left((ab^3c^2)^{\frac14}\,Cab^2c-(ab^3c^2)^{\frac14}\,Ab^3c+(ab^3c^2)^{\frac34}\,Da-(ab^3c^2)^{\frac34}\,Bb\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ac^2}{b}\right)^{\frac14}+2\sqrt{cx}\right)}{2\left(\frac{ac^2}{b}\right)^{\frac14}}\right)}{2\,a^2b^3c^4} + \frac{\sqrt{2}\left((ab^3c^2)^{\frac14}\,Cab^2c-(ab^3c^2)^{\frac14}\,Ab^3c+(ab^3c^2)^{\frac34}\,Da-(ab^3c^2)^{\frac34}\,Bb\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ac^2}{b}\right)^{\frac14}-2\sqrt{cx}\right)}{2\left(\frac{ac^2}{b}\right)^{\frac14}}\right)}{2\,a^2b^3c^4} + \frac{\sqrt{2}\left((ab^3c^2)^{\frac14}\,Cab^2c-(ab^3c^2)^{\frac14}\,Ab^3c-(ab^3c^2)^{\frac34}\,Da+(ab^3c^2)^{\frac34}\,Bb\right)\log\left(cx+\sqrt{2}\left(\frac{ac^2}{b}\right)^{\frac14}\sqrt{cx}+\sqrt{\frac{ac^2}{b}}\right)}{4\,a^2b^3c^4} - \frac{\sqrt{2}\left((ab^3c^2)^{\frac14}\,Cab^2c-(ab^3c^2)^{\frac14}\,Ab^3c-(ab^3c^2)^{\frac34}\,Da+(ab^3c^2)^{\frac34}\,Bb\right)\log\left(cx-\sqrt{2}\left(\frac{ac^2}{b}\right)^{\frac14}\sqrt{cx}+\sqrt{\frac{ac^2}{b}}\right)}{4\,a^2b^3c^4}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(c*x)^(5/2)/(b*x^2+a),x, algorithm="giac")
```

output
```
-2/3*(3*B*c*x + A*c)/(sqrt(c*x)*a*c^3*x) + 1/2*sqrt(2)*((a*b^3*c^2)^(1/4)*
C*a*b^2*c - (a*b^3*c^2)^(1/4)*A*b^3*c + (a*b^3*c^2)^(3/4)*D*a - (a*b^3*c^2
)^(3/4)*B*b)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*c^2/b)^(1/4) + 2*sqrt(c*x))/(a
*c^2/b)^(1/4))/(a^2*b^3*c^4) + 1/2*sqrt(2)*((a*b^3*c^2)^(1/4)*C*a*b^2*c -
(a*b^3*c^2)^(1/4)*A*b^3*c + (a*b^3*c^2)^(3/4)*D*a - (a*b^3*c^2)^(3/4)*B*b)
*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*c^2/b)^(1/4) - 2*sqrt(c*x))/(a*c^2/b)^(1/
4))/(a^2*b^3*c^4) + 1/4*sqrt(2)*((a*b^3*c^2)^(1/4)*C*a*b^2*c - (a*b^3*c^2)
^(1/4)*A*b^3*c - (a*b^3*c^2)^(3/4)*D*a + (a*b^3*c^2)^(3/4)*B*b)*log(c*x +
sqrt(2)*(a*c^2/b)^(1/4)*sqrt(c*x) + sqrt(a*c^2/b))/(a^2*b^3*c^4) - 1/4*sqr
t(2)*((a*b^3*c^2)^(1/4)*C*a*b^2*c - (a*b^3*c^2)^(1/4)*A*b^3*c - (a*b^3*c^2
)^(3/4)*D*a + (a*b^3*c^2)^(3/4)*B*b)*log(c*x - sqrt(2)*(a*c^2/b)^(1/4)*sqr
t(c*x) + sqrt(a*c^2/b))/(a^2*b^3*c^4)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(cx)^{5/2}\,(a+bx^2)}\,dx=\int \frac{A+B\,x+C\,x^2+x^3\,D}{{\left(c\,x\right)}^{5/2}\,\left(b\,x^2+a\right)} \,d x$$

input
```
int((A + B*x + C*x^2 + x^3*D)/((c*x)^(5/2)*(a + b*x^2)),x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/((c*x)^(5/2)*(a + b*x^2)), x)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 630, normalized size of antiderivative = 2.14

$$\int \frac{A+Bx+Cx^2+Dx^3}{(cx)^{5/2}\,(a+bx^2)}\,dx=\text{Too large to display}$$

input
```
int((D*x^3+C*x^2+B*x+A)/(c*x)^(5/2)/(b*x^2+a),x)
```

output

```
(sqrt(c)*( - 6*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*s
qrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*d*x + 6*sqrt(x)
*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqr
t(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*b**2*x + 6*sqrt(x)*b**(3/4)*a**(1/4)*sq
rt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1
/4)*sqrt(2)))*a*b*x - 6*sqrt(x)*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a
**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*c*x +
6*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sq
rt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*d*x - 6*sqrt(x)*b**(1/4)*a**
(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/
4)*a**(1/4)*sqrt(2)))*b**2*x - 6*sqrt(x)*b**(3/4)*a**(1/4)*sqrt(2)*atan((b
**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))
*a*b*x + 6*sqrt(x)*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(
2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*c*x + 3*sqrt(x)*b**
(1/4)*a**(3/4)*sqrt(2)*log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a)
+ sqrt(b)*x)*a*d*x - 3*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*log( - sqrt(x)*b*
*(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*b**2*x - 3*sqrt(x)*b**(1/4)
*a**(3/4)*sqrt(2)*log(sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b
)*x)*a*d*x + 3*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*log(sqrt(x)*b**(1/4)*a**(
1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*b**2*x + 3*sqrt(x)*b**(3/4)*a**(1/4...
```

# 3.128 $\int \frac{A+Bx+Cx^2+Dx^3}{(cx)^{7/2}(a+bx^2)}\,dx$



## Optimal result

Integrand size = 32, antiderivative size = 321

$$\begin{aligned}\int \frac{A+Bx+Cx^2+Dx^3}{(cx)^{7/2}(a+bx^2)}\,dx &= -\frac{2A}{5ac(cx)^{5/2}}-\frac{2B}{3ac^2(cx)^{3/2}}+\frac{2(Ab-aC)}{a^2c^3\sqrt{cx}} \\ &- \frac{\left(\sqrt{b}(Ab-aC)-\sqrt{a}(bB-aD)\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)}{\sqrt{2}a^{9/4}\sqrt[4]{b}c^{7/2}} \\ &+ \frac{\left(\sqrt{b}(Ab-aC)-\sqrt{a}(bB-aD)\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)}{\sqrt{2}a^{9/4}\sqrt[4]{b}c^{7/2}} \\ &- \frac{\left(\sqrt{b}(Ab-aC)+\sqrt{a}(bB-aD)\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\left(\sqrt{a}+\sqrt{b}x\right)}\right)}{\sqrt{2}a^{9/4}\sqrt[4]{b}c^{7/2}}\end{aligned}$$

output
```
-2/5*A/a/c/(c*x)^(5/2)-2/3*B/a/c^2/(c*x)^(3/2)+2*(A*b-C*a)/a^2/c^3/(c*x)^(
1/2)-1/2*(b^(1/2)*(A*b-C*a)-a^(1/2)*(B*b-D*a))*arctan(1-2^(1/2)*b^(1/4)*(c
*x)^(1/2)/a^(1/4)/c^(1/2))*2^(1/2)/a^(9/4)/b^(1/4)/c^(7/2)+1/2*(b^(1/2)*(A
*b-C*a)-a^(1/2)*(B*b-D*a))*arctan(1+2^(1/2)*b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^
(1/2))*2^(1/2)/a^(9/4)/b^(1/4)/c^(7/2)-1/2*(b^(1/2)*(A*b-C*a)+a^(1/2)*(B*b
-D*a))*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*(c*x)^(1/2)/c^(1/2)/(a^(1/2)+b^(1/2
)*x))*2^(1/2)/a^(9/4)/b^(1/4)/c^(7/2)
```

## Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.69

$$\int \frac{A+Bx+Cx^2+Dx^3}{(cx)^{7/2}\left(a+bx^2\right)}\,dx = \frac{x\left(-4\sqrt[4]{a}(3aA-15Abx^2+5ax(B+3Cx)) - \frac{15\sqrt{2}\left(Ab^{3/2}-\sqrt{a}bB-a\sqrt{b}C+a^{3/2}D\right)}{\sqrt[4]{b}}\right.}{30}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/((c*x)^(7/2)*(a + b*x^2)),x]
```

output
```
(x*(-4*a^(1/4)*(3*a*A - 15*A*b*x^2 + 5*a*x*(B + 3*C*x)) - (15*Sqrt[2]*(A*b
^(3/2) - Sqrt[a]*b*B - a*Sqrt[b]*C + a^(3/2)*D)*x^(5/2)*ArcTan[(Sqrt[a] -
Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/b^(1/4) + (15*Sqrt[2]*(-(A*
b^(3/2)) - Sqrt[a]*b*B + a*Sqrt[b]*C + a^(3/2)*D)*x^(5/2)*ArcTanh[(Sqrt[2]
*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/b^(1/4)))/(30*a^(9/4)*(c
*x)^(7/2))
```

## Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 485, normalized size of antiderivative = 1.51, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(cx)^{7/2}\left(a+bx^2\right)}\,dx$$

$$\Big\downarrow \ 2333$$

$$\int \left(\frac{x(bB-aD)-aC+Ab}{b(cx)^{7/2}\left(a+bx^2\right)}+\frac{C}{b(cx)^{7/2}}+\frac{D}{bc(cx)^{5/2}}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)\left(\sqrt{b}(Ab-aC)-\sqrt{a}(bB-aD)\right)}{\sqrt{2}a^{9/4}\sqrt[4]{b}c^{7/2}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}+1\right)\left(\sqrt{b}(Ab-aC)-\sqrt{a}(bB-aD)\right)}{\sqrt{2}a^{9/4}\sqrt[4]{b}c^{7/2}}+\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{cx}+\sqrt{a}\sqrt{c}+\sqrt{b}\sqrt{cx}\right)\left(\sqrt{b}(Ab-aC)+\sqrt{a}(bB-aD)\right)}{2\sqrt{2}a^{9/4}\sqrt[4]{b}c^{7/2}}-\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{cx}+\sqrt{a}\sqrt{c}+\sqrt{b}\sqrt{cx}\right)\left(\sqrt{b}(Ab-aC)+\sqrt{a}(bB-aD)\right)}{2\sqrt{2}a^{9/4}\sqrt[4]{b}c^{7/2}}+\frac{2(Ab-aC)}{a^2c^3\sqrt{cx}}-\frac{2(Ab-aC)}{5abc(cx)^{5/2}}-\frac{2(bB-aD)}{3abc^2(cx)^{3/2}}-\frac{2D}{3bc^2(cx)^{3/2}}-\frac{2C}{5bc(cx)^{5/2}}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/((c*x)^(7/2)*(a + b*x^2)),x]
```

output
```
(-2*C)/(5*b*c*(c*x)^(5/2)) - (2*(A*b - a*C))/(5*a*b*c*(c*x)^(5/2)) - (2*D)
/(3*b*c^2*(c*x)^(3/2)) - (2*(b*B - a*D))/(3*a*b*c^2*(c*x)^(3/2)) + (2*(A*b
 - a*C))/(a^2*c^3*Sqrt[c*x]) - ((Sqrt[b]*(A*b - a*C) - Sqrt[a]*(b*B - a*D)
)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])])/(Sqrt[2]*a^(9
/4)*b^(1/4)*c^(7/2)) + ((Sqrt[b]*(A*b - a*C) - Sqrt[a]*(b*B - a*D))*ArcTan
[1 + (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])])/(Sqrt[2]*a^(9/4)*b^(1
/4)*c^(7/2)) + ((Sqrt[b]*(A*b - a*C) + Sqrt[a]*(b*B - a*D))*Log[Sqrt[a]*Sq
rt[c] + Sqrt[b]*Sqrt[c]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[c*x]])/(2*Sqrt[2]
*a^(9/4)*b^(1/4)*c^(7/2)) - ((Sqrt[b]*(A*b - a*C) + Sqrt[a]*(b*B - a*D))*L
og[Sqrt[a]*Sqrt[c] + Sqrt[b]*Sqrt[c]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[c*x]
])/(2*Sqrt[2]*a^(9/4)*b^(1/4)*c^(7/2))
```

## Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

## Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.13

| method | result |
|---|---|
| derivativedivides | $-\frac{2B}{3a(cx)^{\frac{3}{2}}}-\frac{2Ac}{5a(cx)^{\frac{5}{2}}}-\frac{2(-Ab+Ca)}{c\,a^{2}\sqrt{cx}}+2\left(\frac{\left(-abBc+Da^{2}c\right)\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{cx+\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{4}}\sqrt{cx}\,\sqrt{2}+\sqrt{\frac{a\,c^{2}}{b}}}{cx-\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{4}}\sqrt{cx}\,\sqrt{2}+\sqrt{\frac{a\,c^{2}}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{c}\ldots}{\left(\frac{a\,c^{2}}{b}\right)\ldots}\right)\ldots\right.}{8a\,c^{2}}\ldots\right.$ … |
| default | $-\frac{2B}{3a(cx)^{\frac{3}{2}}}-\frac{2Ac}{5a(cx)^{\frac{5}{2}}}-\frac{2(-Ab+Ca)}{c\,a^{2}\sqrt{cx}}+2\left(\frac{\left(-abBc+Da^{2}c\right)\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{cx+\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{4}}\sqrt{cx}\,\sqrt{2}+\sqrt{\frac{a\,c^{2}}{b}}}{cx-\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{4}}\sqrt{cx}\,\sqrt{2}+\sqrt{\frac{a\,c^{2}}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{c}\ldots}{\left(\frac{a\,c^{2}}{b}\right)\ldots}\right)\ldots\right.}{8a\,c^{2}}\ldots\right.$ … |
| pseudoelliptic | $-\sqrt{2}\,\sqrt{cx}\left(\arctan\left(\frac{\sqrt{2}\,\sqrt{cx}-\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{4}}}{\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{4}}}\right)+\arctan\left(\frac{\sqrt{2}\,\sqrt{cx}+\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{4}}}{\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{4}}}\right)+\frac{\ln\left(\frac{cx+\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{4}}\sqrt{cx}\,\sqrt{2}+\sqrt{\frac{a\,c^{2}}{b}}}{cx-\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{4}}\sqrt{cx}\,\sqrt{2}+\sqrt{\frac{a\,c^{2}}{b}}}\right)}{2}\right)x^{2}(Bb-\ldots$ … |

input
```
int((D*x^3+C*x^2+B*x+A)/(c*x)^(7/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
2/c^2*(-1/3/a*B/(c*x)^(3/2)-1/5*A*c/a/(c*x)^(5/2)-1/c/a^2*(-A*b+C*a)/(c*x)
^(1/2)+1/c/a^2*(1/8*(-B*a*b*c+D*a^2*c)*(a*c^2/b)^(1/4)/a/c^2*2^(1/2)*(ln((
c*x+(a*c^2/b)^(1/4)*(c*x)^(1/2)*2^(1/2)+(a*c^2/b)^(1/2))/(c*x-(a*c^2/b)^(1
/4)*(c*x)^(1/2)*2^(1/2)+(a*c^2/b)^(1/2)))+2*arctan(2^(1/2)/(a*c^2/b)^(1/4)
*(c*x)^(1/2)+1)+2*arctan(2^(1/2)/(a*c^2/b)^(1/4)*(c*x)^(1/2)-1))+1/8*(A*b^
2-C*a*b)/b/(a*c^2/b)^(1/4)*2^(1/2)*(ln((c*x-(a*c^2/b)^(1/4)*(c*x)^(1/2)*2^
(1/2)+(a*c^2/b)^(1/2))/(c*x+(a*c^2/b)^(1/4)*(c*x)^(1/2)*2^(1/2)+(a*c^2/b)^
(1/2)))+2*arctan(2^(1/2)/(a*c^2/b)^(1/4)*(c*x)^(1/2)+1)+2*arctan(2^(1/2)/(
a*c^2/b)^(1/4)*(c*x)^(1/2)-1))))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3386 vs. $2(242) = 484$.

Time = 0.50 (sec) , antiderivative size = 3386, normalized size of antiderivative = 10.55

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(cx)^{7/2}(a + bx^2)}\,dx = \text{Too large to display}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(c*x)^(7/2)/(b*x^2+a),x, algorithm="fricas")
```

output
```
Too large to include
```

## Sympy [F]

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(cx)^{7/2}(a + bx^2)}\,dx = \int \frac{A + Bx + Cx^2 + Dx^3}{(cx)^{\frac{7}{2}}(a + bx^2)}\,dx$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(c*x)**(7/2)/(b*x**2+a),x)
```

output
```
Integral((A + B*x + C*x**2 + D*x**3)/((c*x)**(7/2)*(a + b*x**2)), x)
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 426, normalized size of antiderivative = 1.33

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(cx)^{7/2}(a + bx^2)}\,dx = \frac{15\left(\frac{\sqrt{2}\left(\left(Cab - Ab^2\right)\sqrt{a}c + \left(Da^2\sqrt{b} - Bab^{\frac{3}{2}}\right)c\right)\log\left(\sqrt{b}cx + \sqrt{2}\left(ac^2\right)^{\frac{1}{4}}\sqrt{cx}b^{\frac{1}{4}} + \sqrt{a}c\right)}{\left(ac^2\right)^{\frac{3}{4}}b^{\frac{3}{4}}} - \frac{\sqrt{2}\left(\left(Cab - Ab^2\right)\sqrt{a}c + \left(\cdots\right.\right.}{\cdots}\right.}{\cdots}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(c*x)^(7/2)/(b*x^2+a),x, algorithm="maxima")
```

output
```
1/60*(15*(sqrt(2)*((C*a*b - A*b^2)*sqrt(a)*c + (D*a^2*sqrt(b) - B*a*b^(3/2
))*c)*log(sqrt(b)*c*x + sqrt(2)*(a*c^2)^(1/4)*sqrt(c*x)*b^(1/4) + sqrt(a)*
c)/((a*c^2)^(3/4)*b^(3/4)) - sqrt(2)*((C*a*b - A*b^2)*sqrt(a)*c + (D*a^2*s
qrt(b) - B*a*b^(3/2))*c)*log(sqrt(b)*c*x - sqrt(2)*(a*c^2)^(1/4)*sqrt(c*x)
*b^(1/4) + sqrt(a)*c)/((a*c^2)^(3/4)*b^(3/4)) - 2*sqrt(2)*((C*a*b - A*b^2)
*sqrt(a)*c - (D*a^2*sqrt(b) - B*a*b^(3/2))*c)*arctan(1/2*sqrt(2)*(sqrt(2)*
(a*c^2)^(1/4)*b^(1/4) + 2*sqrt(c*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*c))/(sqr
t(sqrt(a)*sqrt(b)*c)*sqrt(a)*sqrt(b)*c) - 2*sqrt(2)*((C*a*b - A*b^2)*sqrt(
a)*c - (D*a^2*sqrt(b) - B*a*b^(3/2))*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*c^
2)^(1/4)*b^(1/4) - 2*sqrt(c*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*c))/(sqrt(sqr
t(a)*sqrt(b)*c)*sqrt(a)*sqrt(b)*c))/(a^2*c^2) - 8*(5*B*a*c^2*x + 15*(C*a -
 A*b)*c^2*x^2 + 3*A*a*c^2)/((c*x)^(5/2)*a^2*c^2))/c
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 506 vs. $2(242) = 484$.

Time = 0.14 (sec) , antiderivative size = 506, normalized size of antiderivative = 1.58

$$\int \frac{A+Bx+Cx^2+Dx^3}{(cx)^{7/2}(a+bx^2)}\,dx = \frac{\sqrt{2}\left((ab^3c^2)^{\frac{1}{4}}Da^2bc-(ab^3c^2)^{\frac{1}{4}}Bab^2c-(ab^3c^2)^{\frac{3}{4}}Ca+(ab^3c^2)^{\frac{3}{4}}Ab\right)\arctan}{2\,a^3b^2c^5}$$
$$+\frac{\sqrt{2}\left((ab^3c^2)^{\frac{1}{4}}Da^2bc-(ab^3c^2)^{\frac{1}{4}}Bab^2c-(ab^3c^2)^{\frac{3}{4}}Ca+(ab^3c^2)^{\frac{3}{4}}Ab\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ac^2}{b}\right)^{\frac{1}{4}}-2\sqrt{cx}\right)}{2\left(\frac{ac^2}{b}\right)^{\frac{1}{4}}}\right)}{2\,a^3b^2c^5}$$
$$+\frac{\sqrt{2}\left((ab^3c^2)^{\frac{1}{4}}Da^2bc-(ab^3c^2)^{\frac{1}{4}}Bab^2c+(ab^3c^2)^{\frac{3}{4}}Ca-(ab^3c^2)^{\frac{3}{4}}Ab\right)\log\left(cx+\sqrt{2}\left(\frac{ac^2}{b}\right)^{\frac{1}{4}}\sqrt{cx}+\sqrt{\frac{ac^2}{b}}\right)}{4\,a^3b^2c^5}$$
$$-\frac{\sqrt{2}\left((ab^3c^2)^{\frac{1}{4}}Da^2bc-(ab^3c^2)^{\frac{1}{4}}Bab^2c+(ab^3c^2)^{\frac{3}{4}}Ca-(ab^3c^2)^{\frac{3}{4}}Ab\right)\log\left(cx-\sqrt{2}\left(\frac{ac^2}{b}\right)^{\frac{1}{4}}\sqrt{cx}+\sqrt{\frac{ac^2}{b}}\right)}{4\,a^3b^2c^5}$$
$$-\frac{2\,(15\,Cac^2x^2-15\,Abc^2x^2+5\,Bac^2x+3\,Aac^2)}{15\,\sqrt{cx}a^2c^5x^2}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(c*x)^(7/2)/(b*x^2+a),x, algorithm="giac")
```

output
```
1/2*sqrt(2)*((a*b^3*c^2)^(1/4)*D*a^2*b*c - (a*b^3*c^2)^(1/4)*B*a*b^2*c - (
a*b^3*c^2)^(3/4)*C*a + (a*b^3*c^2)^(3/4)*A*b)*arctan(1/2*sqrt(2)*(sqrt(2)*
(a*c^2/b)^(1/4) + 2*sqrt(c*x))/(a*c^2/b)^(1/4))/(a^3*b^2*c^5) + 1/2*sqrt(2
)*((a*b^3*c^2)^(1/4)*D*a^2*b*c - (a*b^3*c^2)^(1/4)*B*a*b^2*c - (a*b^3*c^2)
^(3/4)*C*a + (a*b^3*c^2)^(3/4)*A*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*c^2/b)
^(1/4) - 2*sqrt(c*x))/(a*c^2/b)^(1/4))/(a^3*b^2*c^5) + 1/4*sqrt(2)*((a*b^3
*c^2)^(1/4)*D*a^2*b*c - (a*b^3*c^2)^(1/4)*B*a*b^2*c + (a*b^3*c^2)^(3/4)*C*
a - (a*b^3*c^2)^(3/4)*A*b)*log(c*x + sqrt(2)*(a*c^2/b)^(1/4)*sqrt(c*x) + s
qrt(a*c^2/b))/(a^3*b^2*c^5) - 1/4*sqrt(2)*((a*b^3*c^2)^(1/4)*D*a^2*b*c - (
a*b^3*c^2)^(1/4)*B*a*b^2*c + (a*b^3*c^2)^(3/4)*C*a - (a*b^3*c^2)^(3/4)*A*b
)*log(c*x - sqrt(2)*(a*c^2/b)^(1/4)*sqrt(c*x) + sqrt(a*c^2/b))/(a^3*b^2*c^
5) - 2/15*(15*C*a*c^2*x^2 - 15*A*b*c^2*x^2 + 5*B*a*c^2*x + 3*A*a*c^2)/(sqr
t(c*x)*a^2*c^5*x^2)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(cx)^{7/2}\,(a+bx^2)}\,dx=\int \frac{A+B\,x+C\,x^2+x^3\,D}{{\left(c\,x\right)}^{7/2}\,\left(b\,x^2+a\right)}\,dx$$

input
```
int((A + B*x + C*x^2 + x^3*D)/((c*x)^(7/2)*(a + b*x^2)),x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/((c*x)^(7/2)*(a + b*x^2)), x)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 679, normalized size of antiderivative = 2.12

$$\int \frac{A+Bx+Cx^2+Dx^3}{(cx)^{7/2}\,(a+bx^2)}\,dx=\text{Too large to display}$$

input
```
int((D*x^3+C*x^2+B*x+A)/(c*x)^(7/2)/(b*x^2+a),x)
```

output
```
(sqrt(c)*( - 30*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*
sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*b**2*x**2 + 30*s
qrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(
x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*b*c*x**2 - 30*sqrt(x)*b**(3/4)*a*
*(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1
/4)*a**(1/4)*sqrt(2)))*a*d*x**2 + 30*sqrt(x)*b**(3/4)*a**(1/4)*sqrt(2)*ata
n((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(
2)))*b**2*x**2 + 30*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1
/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*b**2*x**2 -
30*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*s
qrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*b*c*x**2 + 30*sqrt(x)*b**(3/4
)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b
**(1/4)*a**(1/4)*sqrt(2)))*a*d*x**2 - 30*sqrt(x)*b**(3/4)*a**(1/4)*sqrt(2)
*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*s
qrt(2)))*b**2*x**2 + 15*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*log( - sqrt(x)*b
**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*b**2*x**2 - 15*sqrt(x)*b**
(1/4)*a**(3/4)*sqrt(2)*log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a)
+ sqrt(b)*x)*b*c*x**2 - 15*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*log(sqrt(x)*b
**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*b**2*x**2 + 15*sqrt(x)*b**
(1/4)*a**(3/4)*sqrt(2)*log(sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) ...
```

# 3.129 $\int \frac{(cx)^{5/2}(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx$



## Optimal result

Integrand size = 32, antiderivative size = 405

$$
\begin{aligned}
\int \frac{(cx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{\left(a+bx^2\right)^2}\,dx &= \frac{c^2(5bB-9aD)\sqrt{cx}}{2b^3} \\
&- \frac{c(3Ab-7aC)(cx)^{3/2}}{6ab^2} + \frac{2D(cx)^{9/2}}{5bc^2\left(a+bx^2\right)} \\
&- \frac{(cx)^{5/2}(a(5bB-9aD)-5b(Ab-aC)x)}{10ab^2\left(a+bx^2\right)} \\
&- \frac{c^{5/2}\left(\sqrt{b}(3Ab-7aC)-\sqrt{a}(5bB-9aD)\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)}{4\sqrt{2}\sqrt[4]{a}b^{13/4}} \\
&+ \frac{c^{5/2}\left(\sqrt{b}(3Ab-7aC)-\sqrt{a}(5bB-9aD)\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)}{4\sqrt{2}\sqrt[4]{a}b^{13/4}} \\
&- \frac{c^{5/2}\left(\sqrt{b}(3Ab-7aC)+\sqrt{a}(5bB-9aD)\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\left(\sqrt{a}+\sqrt{b}x\right)}\right)}{4\sqrt{2}\sqrt[4]{a}b^{13/4}}
\end{aligned}
$$

output
```
1/2*c^2*(5*B*b-9*D*a)*(c*x)^(1/2)/b^3-1/6*c*(3*A*b-7*C*a)*(c*x)^(3/2)/a/b^
2+2/5*D*(c*x)^(9/2)/b/c^2/(b*x^2+a)-1/10*(c*x)^(5/2)*(a*(5*B*b-9*D*a)-5*b*
(A*b-C*a)*x)/a/b^2/(b*x^2+a)-1/8*c^(5/2)*(b^(1/2)*(3*A*b-7*C*a)-a^(1/2)*(5
*B*b-9*D*a))*arctan(1-2^(1/2)*b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))*2^(1/2)
/a^(1/4)/b^(13/4)+1/8*c^(5/2)*(b^(1/2)*(3*A*b-7*C*a)-a^(1/2)*(5*B*b-9*D*a)
)*arctan(1+2^(1/2)*b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))*2^(1/2)/a^(1/4)/b^
(13/4)-1/8*c^(5/2)*(b^(1/2)*(3*A*b-7*C*a)+a^(1/2)*(5*B*b-9*D*a))*arctanh(2
^(1/2)*a^(1/4)*b^(1/4)*(c*x)^(1/2)/c^(1/2)/(a^(1/2)+b^(1/2)*x))*2^(1/2)/a^
(1/4)/b^(13/4)
```

## Mathematica [A] (verified)

Time = 1.28 (sec) , antiderivative size = 260, normalized size of antiderivative = 0.64

$$\int \frac{(cx)^{5/2} (A + Bx + Cx^2 + Dx^3)}{(a + bx^2)^2} dx = \frac{(cx)^{5/2} \left( \frac{4\sqrt[4]{b}\sqrt{x}(-135a^2D+ab(75B+x(35C-108Dx))+b^2x(-15A+4x(15B+5Cx+\right.}{a+bx^2}}{}$$

input
```
Integrate[((c*x)^(5/2)*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^2,x]
```

output
```
((c*x)^(5/2)*((4*b^(1/4)*Sqrt[x]*(-135*a^2*D + a*b*(75*B + x*(35*C - 108*D
*x)) + b^2*x*(-15*A + 4*x*(15*B + 5*C*x + 3*D*x^2))))/(a + b*x^2) - (15*Sq
rt[2]*(3*A*b^(3/2) - 5*Sqrt[a]*b*B - 7*a*Sqrt[b]*C + 9*a^(3/2)*D)*ArcTan[(
Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])]/a^(1/4) + (15*Sqr
t[2]*(-3*A*b^(3/2) - 5*Sqrt[a]*b*B + 7*a*Sqrt[b]*C + 9*a^(3/2)*D)*ArcTanh[
(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/a^(1/4)))/(120*b
^(13/4)*x^(5/2))
```

## Rubi [A] (verified)

Time = 1.24 (sec) , antiderivative size = 504, normalized size of antiderivative = 1.24, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2335, 27, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{5/2} \left(A+Bx+Cx^2+Dx^3\right)}{\left(a+bx^2\right)^2} \, dx$$

$$\Big\downarrow \ 2335$$

$$-\frac{c\int -\frac{(cx)^{3/2}\left(4aDx^2-(3Ab-7aC)x+\frac{5a(bB-aD)}{b}\right)}{2(bx^2+a)}dx}{2ab}-\frac{(cx)^{5/2}\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{2ab\left(a+bx^2\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{c\int \frac{(cx)^{3/2}\left(4aDx^2-(3Ab-7aC)x+\frac{5a(bB-aD)}{b}\right)}{bx^2+a}dx}{4ab}-\frac{(cx)^{5/2}\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{2ab\left(a+bx^2\right)}$$

$$\Big\downarrow \ 2333$$

$$\frac{c\int \left(\frac{4aD(cx)^{3/2}}{b}+\frac{(a(5bB-9aD)-b(3Ab-7aC)x)(cx)^{3/2}}{b(bx^2+a)}\right)dx}{4ab}-\frac{(cx)^{5/2}\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{2ab\left(a+bx^2\right)}$$

$$\Big\downarrow \ 2009$$

$$\frac{c\left(-\frac{a^{3/4}c^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)\left(\sqrt{b}(3Ab-7aC)-\sqrt{a}(5bB-9aD)\right)}{\sqrt{2}b^{9/4}}+\frac{a^{3/4}c^{3/2}\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}+1\right)\left(\sqrt{b}(3Ab-7aC)-\sqrt{a}(5bB-9a\right.}{\sqrt{2}b^{9/4}}\right.}{}$$

$$\frac{(cx)^{5/2}\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{2ab\left(a+bx^2\right)}$$

input
```
Int[((c*x)^(5/2)*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^2,x]
```

output
```
-1/2*((c*x)^(5/2)*(a*(B - (a*D)/b) - (A*b - a*C)*x))/(a*b*(a + b*x^2)) + (
c*((2*a*c*(5*b*B - 9*a*D)*Sqrt[c*x])/b^2 - (2*(3*A*b - 7*a*C)*(c*x)^(3/2))
/(3*b) + (8*a*D*(c*x)^(5/2))/(5*b*c) - (a^(3/4)*c^(3/2)*(Sqrt[b]*(3*A*b -
7*a*C) - Sqrt[a]*(5*b*B - 9*a*D))*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(
a^(1/4)*Sqrt[c])])/(Sqrt[2]*b^(9/4)) + (a^(3/4)*c^(3/2)*(Sqrt[b]*(3*A*b -
7*a*C) - Sqrt[a]*(5*b*B - 9*a*D))*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(
a^(1/4)*Sqrt[c])])/(Sqrt[2]*b^(9/4)) + (a^(3/4)*c^(3/2)*(Sqrt[b]*(3*A*b -
7*a*C) + Sqrt[a]*(5*b*B - 9*a*D))*Log[Sqrt[a]*Sqrt[c] + Sqrt[b]*Sqrt[c]*x
- Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[c*x]])/(2*Sqrt[2]*b^(9/4)) - (a^(3/4)*c^(3/
2)*(Sqrt[b]*(3*A*b - 7*a*C) + Sqrt[a]*(5*b*B - 9*a*D))*Log[Sqrt[a]*Sqrt[c]
 + Sqrt[b]*Sqrt[c]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[c*x]])/(2*Sqrt[2]*b^(9
/4))))/(4*a*b)
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2335
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Simp[c/(2*a*b*(p + 1))   Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSu
m[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
 b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

## Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 402, normalized size of antiderivative = 0.99

| method | result |
|---|---|
| pseudoelliptic | $3\left(-\frac{5\left(b\,x^{2}+a\right)\left(Bb-\frac{9Da}{5}\right)\sqrt{2}\,c\left(\arctan\left(\frac{\sqrt{2}\,\sqrt{cx}}{\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{4}}}-1\right)+\arctan\left(\frac{\sqrt{2}\,\sqrt{cx}}{\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{4}}}+1\right)+\frac{\ln\left(\frac{cx+\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{4}}\sqrt{cx}\,\sqrt{2}+\sqrt{\frac{a\,c^{2}}{b}}}{cx-\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{4}}\sqrt{cx}\,\sqrt{2}+\sqrt{\frac{a\,c^{2}}{b}}}\right)}{2}\right)\sqrt{\frac{a\,c^{2}}{b}}}{3}\right.$ |
| derivativedivides | $\frac{\frac{2D(cx)^{\frac{5}{2}}b}{5}+\frac{2Cc(cx)^{\frac{3}{2}}b}{3}+2Bb\,c^{2}\sqrt{cx}-4Da\,c^{2}\sqrt{cx}}{b^{3}}+\frac{2c^{3}\left(\frac{\left(-\frac{1}{4}b^{2}A+\frac{1}{4}Cab\right)(cx)^{\frac{3}{2}}+\left(\frac{1}{4}abBc-\frac{1}{4}Da^{2}c\right)\sqrt{cx}}{b\,c^{2}x^{2}+a\,c^{2}}+\frac{\left(-5abBc+9D\right.}{}\right.}{}$ |
| default | $\frac{\frac{2D(cx)^{\frac{5}{2}}b}{5}+\frac{2Cc(cx)^{\frac{3}{2}}b}{3}+2Bb\,c^{2}\sqrt{cx}-4Da\,c^{2}\sqrt{cx}}{b^{3}}+\frac{2c^{3}\left(\frac{\left(-\frac{1}{4}b^{2}A+\frac{1}{4}Cab\right)(cx)^{\frac{3}{2}}+\left(\frac{1}{4}abBc-\frac{1}{4}Da^{2}c\right)\sqrt{cx}}{b\,c^{2}x^{2}+a\,c^{2}}+\frac{\left(-5abBc+9D\right.}{}\right.}{}$ |

input
```
int((c*x)^(5/2)*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
3/8/(a*c^2/b)^(1/4)*(-5/3*(b*x^2+a)*(B*b-9/5*D*a)*2^(1/2)*c*(arctan(2^(1/2
)/(a*c^2/b)^(1/4)*(c*x)^(1/2)-1)+arctan(2^(1/2)/(a*c^2/b)^(1/4)*(c*x)^(1/2
)+1)+1/2*ln((c*x+(a*c^2/b)^(1/4)*(c*x)^(1/2)*2^(1/2)+(a*c^2/b)^(1/2))/(c*x
-(a*c^2/b)^(1/4)*(c*x)^(1/2)*2^(1/2)+(a*c^2/b)^(1/2))))*(a*c^2/b)^(1/2)-4/
3*(A*b-C*a)*b*(a*c^2/b)^(1/4)*(c*x)^(3/2)+(20/3*(4/5*(1/5*D*x^2+1/3*C*x+B)
*x^2*b^2+a*(-36/25*D*x^2+4/15*C*x+B)*b-9/5*D*a^2)*(a*c^2/b)^(1/4)*(c*x)^(1
/2)+(b*x^2+a)*2^(1/2)*(arctan(2^(1/2)/(a*c^2/b)^(1/4)*(c*x)^(1/2)-1)+arcta
n(2^(1/2)/(a*c^2/b)^(1/4)*(c*x)^(1/2)+1)+1/2*ln((c*x-(a*c^2/b)^(1/4)*(c*x)
^(1/2)*2^(1/2)+(a*c^2/b)^(1/2))/(c*x+(a*c^2/b)^(1/4)*(c*x)^(1/2)*2^(1/2)+(
a*c^2/b)^(1/2))))*c*(A*b-7/3*C*a))*c)*c/b^3/(b*x^2+a)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3697 vs. $2(314) = 628$.

Time = 0.28 (sec) , antiderivative size = 3697, normalized size of antiderivative = 9.13

$$\int \frac{(cx)^{5/2}(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate((c*x)^(5/2)*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="fricas
")
```

output
```
Too large to include
```

## Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 107.11 (sec) , antiderivative size = 4189, normalized size of antiderivative = 10.34

$$\int \frac{(cx)^{5/2}(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate((c*x)**(5/2)*(D*x**3+C*x**2+B*x+A)/(b*x**2+a)**2,x)
```

output
```
A*(-21*a**(11/4)*b*c**(5/2)*x**(11/2)*log(1 - b**(1/4)*sqrt(x)*exp_polar(I
*pi/4)/a**(1/4))*gamma(7/4)/(32*a**3*b**(11/4)*x**(11/2)*exp(3*I*pi/4)*gam
ma(11/4) + 32*a**2*b**(15/4)*x**(15/2)*exp(3*I*pi/4)*gamma(11/4)) - 21*I*a
**(11/4)*b*c**(5/2)*x**(11/2)*log(1 - b**(1/4)*sqrt(x)*exp_polar(3*I*pi/4)
/a**(1/4))*gamma(7/4)/(32*a**3*b**(11/4)*x**(11/2)*exp(3*I*pi/4)*gamma(11/
4) + 32*a**2*b**(15/4)*x**(15/2)*exp(3*I*pi/4)*gamma(11/4)) + 21*a**(11/4)
*b*c**(5/2)*x**(11/2)*log(1 - b**(1/4)*sqrt(x)*exp_polar(5*I*pi/4)/a**(1/4
))*gamma(7/4)/(32*a**3*b**(11/4)*x**(11/2)*exp(3*I*pi/4)*gamma(11/4) + 32*
a**2*b**(15/4)*x**(15/2)*exp(3*I*pi/4)*gamma(11/4)) + 21*I*a**(11/4)*b*c**
(5/2)*x**(11/2)*log(1 - b**(1/4)*sqrt(x)*exp_polar(7*I*pi/4)/a**(1/4))*gam
ma(7/4)/(32*a**3*b**(11/4)*x**(11/2)*exp(3*I*pi/4)*gamma(11/4) + 32*a**2*b
**(15/4)*x**(15/2)*exp(3*I*pi/4)*gamma(11/4)) - 21*a**(7/4)*b**2*c**(5/2)*
x**(15/2)*log(1 - b**(1/4)*sqrt(x)*exp_polar(I*pi/4)/a**(1/4))*gamma(7/4)/
(32*a**3*b**(11/4)*x**(11/2)*exp(3*I*pi/4)*gamma(11/4) + 32*a**2*b**(15/4)
*x**(15/2)*exp(3*I*pi/4)*gamma(11/4)) - 21*I*a**(7/4)*b**2*c**(5/2)*x**(15
/2)*log(1 - b**(1/4)*sqrt(x)*exp_polar(3*I*pi/4)/a**(1/4))*gamma(7/4)/(32*
a**3*b**(11/4)*x**(11/2)*exp(3*I*pi/4)*gamma(11/4) + 32*a**2*b**(15/4)*x**
(15/2)*exp(3*I*pi/4)*gamma(11/4)) + 21*a**(7/4)*b**2*c**(5/2)*x**(15/2)*lo
g(1 - b**(1/4)*sqrt(x)*exp_polar(5*I*pi/4)/a**(1/4))*gamma(7/4)/(32*a**3*b
**(11/4)*x**(11/2)*exp(3*I*pi/4)*gamma(11/4) + 32*a**2*b**(15/4)*x**(15...
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 501, normalized size of antiderivative = 1.24

$$\int \frac{(cx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{\left(a+bx^2\right)^2}\,dx = \frac{15\,c^4\left(\frac{\sqrt{2}\left(\left(7\,Cab-3\,Ab^2\right)\sqrt{a}c+\left(9\,Da^2\sqrt{b}-5\,Bab^{\frac{3}{2}}\right)c\right)\log\left(\sqrt{b}cx+\sqrt{2}\left(ac^2\right)^{\frac{1}{4}}\sqrt{cx}b^{\frac{1}{4}}+\sqrt{a}c\right)}{\left(ac^2\right)^{\frac{3}{4}}b^{\frac{3}{4}}} - \cdots\right.}{\cdots}$$

input
```
integrate((c*x)^(5/2)*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="maxima
")
```

output
```
1/240*(15*c^4*(sqrt(2)*((7*C*a*b - 3*A*b^2)*sqrt(a)*c + (9*D*a^2*sqrt(b) -
 5*B*a*b^(3/2))*c)*log(sqrt(b)*c*x + sqrt(2)*(a*c^2)^(1/4)*sqrt(c*x)*b^(1/
4) + sqrt(a)*c)/((a*c^2)^(3/4)*b^(3/4)) - sqrt(2)*((7*C*a*b - 3*A*b^2)*sqr
t(a)*c + (9*D*a^2*sqrt(b) - 5*B*a*b^(3/2))*c)*log(sqrt(b)*c*x - sqrt(2)*(a
*c^2)^(1/4)*sqrt(c*x)*b^(1/4) + sqrt(a)*c)/((a*c^2)^(3/4)*b^(3/4)) - 2*sqr
t(2)*((7*C*a*b - 3*A*b^2)*sqrt(a)*c - (9*D*a^2*sqrt(b) - 5*B*a*b^(3/2))*c)
*arctan(1/2*sqrt(2)*(sqrt(2)*(a*c^2)^(1/4)*b^(1/4) + 2*sqrt(c*x)*sqrt(b))/
sqrt(sqrt(a)*sqrt(b)*c))/(sqrt(sqrt(a)*sqrt(b)*c)*sqrt(a)*sqrt(b)*c) - 2*s
qrt(2)*((7*C*a*b - 3*A*b^2)*sqrt(a)*c - (9*D*a^2*sqrt(b) - 5*B*a*b^(3/2))*
c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*c^2)^(1/4)*b^(1/4) - 2*sqrt(c*x)*sqrt(b
))/sqrt(sqrt(a)*sqrt(b)*c))/(sqrt(sqrt(a)*sqrt(b)*c)*sqrt(a)*sqrt(b)*c))/b
^3 + 120*((C*a*b - A*b^2)*(c*x)^(3/2)*c^4 - (D*a^2 - B*a*b)*sqrt(c*x)*c^5)
/(b^4*c^2*x^2 + a*b^3*c^2) + 32*(3*(c*x)^(5/2)*D*b*c + 5*(c*x)^(3/2)*C*b*c
^2 - 15*(2*D*a - B*b)*sqrt(c*x)*c^3)/b^3)/c
```

## Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 617, normalized size of antiderivative = 1.52

$$\int \frac{(cx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{\left(a+bx^2\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate((c*x)^(5/2)*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*(sqrt(c*x)*C*a*b*c^4*x - sqrt(c*x)*A*b^2*c^4*x - sqrt(c*x)*D*a^2*c^4 +
 sqrt(c*x)*B*a*b*c^4)/((b*c^2*x^2 + a*c^2)*b^3) + 1/8*sqrt(2)*(9*(a*b^3*c^
2)^(1/4)*D*a^2*b*c^2 - 5*(a*b^3*c^2)^(1/4)*B*a*b^2*c^2 - 7*(a*b^3*c^2)^(3/
4)*C*a*c + 3*(a*b^3*c^2)^(3/4)*A*b*c)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*c^2/b
)^(1/4) + 2*sqrt(c*x))/(a*c^2/b)^(1/4))/(a*b^5) + 1/8*sqrt(2)*(9*(a*b^3*c^
2)^(1/4)*D*a^2*b*c^2 - 5*(a*b^3*c^2)^(1/4)*B*a*b^2*c^2 - 7*(a*b^3*c^2)^(3/
4)*C*a*c + 3*(a*b^3*c^2)^(3/4)*A*b*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*c^2/
b)^(1/4) - 2*sqrt(c*x))/(a*c^2/b)^(1/4))/(a*b^5) + 1/16*sqrt(2)*(9*(a*b^3*
c^2)^(1/4)*D*a^2*b*c^2 - 5*(a*b^3*c^2)^(1/4)*B*a*b^2*c^2 + 7*(a*b^3*c^2)^(
3/4)*C*a*c - 3*(a*b^3*c^2)^(3/4)*A*b*c)*log(c*x + sqrt(2)*(a*c^2/b)^(1/4)*
sqrt(c*x) + sqrt(a*c^2/b))/(a*b^5) - 1/16*sqrt(2)*(9*(a*b^3*c^2)^(1/4)*D*a
^2*b*c^2 - 5*(a*b^3*c^2)^(1/4)*B*a*b^2*c^2 + 7*(a*b^3*c^2)^(3/4)*C*a*c - 3
*(a*b^3*c^2)^(3/4)*A*b*c)*log(c*x - sqrt(2)*(a*c^2/b)^(1/4)*sqrt(c*x) + sq
rt(a*c^2/b))/(a*b^5) + 2/15*(3*sqrt(c*x)*D*b^8*c^2*x^2 + 5*sqrt(c*x)*C*b^8
*c^2*x - 30*sqrt(c*x)*D*a*b^7*c^2 + 15*sqrt(c*x)*B*b^8*c^2)/b^10
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{\left(a+bx^2\right)^2}\,dx=\int \frac{{\left(c\,x\right)}^{5/2}\,\left(A+B\,x+C\,x^2+x^3\,D\right)}{{\left(b\,x^2+a\right)}^2}\,dx$$

input
```
int(((c*x)^(5/2)*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^2,x)
```

output
```
int(((c*x)^(5/2)*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^2, x)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 1244, normalized size of antiderivative = 3.07

$$\int \frac{(cx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{\left(a+bx^2\right)^2}\,dx=\text{Too large to display}$$

input
```
int((c*x)^(5/2)*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x)
```

output

```
(sqrt(c)*c**2*( - 90*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqr
t(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b**2 + 210*b**(1/
4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(
b**(1/4)*a**(1/4)*sqrt(2)))*a*b*c - 90*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**
(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*b
**3*x**2 + 210*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) -
 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*b**2*c*x**2 - 270*b**(3/4
)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b
**(1/4)*a**(1/4)*sqrt(2)))*a**2*d + 150*b**(3/4)*a**(1/4)*sqrt(2)*atan((b*
*(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*
a*b**2 - 270*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2
*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b*d*x**2 + 150*b**(3/4)*a
**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(
1/4)*a**(1/4)*sqrt(2)))*b**3*x**2 + 90*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**
(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a
*b**2 - 210*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*
sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b*c + 90*b**(1/4)*a**(3/4)
*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a*
*(1/4)*sqrt(2)))*b**3*x**2 - 210*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*
a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*b**2...
```

# 3.130 $\int \frac{(cx)^{3/2}(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx$



## Optimal result

Integrand size = 32, antiderivative size = 374

$$\begin{aligned}\int \frac{(cx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{\left(a+bx^2\right)^2}\,dx &= -\frac{c(Ab-5aC)\sqrt{cx}}{2ab^2} \\ &+ \frac{2D(cx)^{7/2}}{3bc^2\left(a+bx^2\right)} - \frac{(cx)^{3/2}(a(3bB-7aD)-3b(Ab-aC)x)}{6ab^2\left(a+bx^2\right)} \\ &- \frac{c^{3/2}\left(\sqrt{b}(Ab-5aC)+\sqrt{a}(3bB-7aD)\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)}{4\sqrt{2}a^{3/4}b^{11/4}} \\ &+ \frac{c^{3/2}\left(\sqrt{b}(Ab-5aC)+\sqrt{a}(3bB-7aD)\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)}{4\sqrt{2}a^{3/4}b^{11/4}} \\ &+ \frac{c^{3/2}\left(\sqrt{b}(Ab-5aC)-\sqrt{a}(3bB-7aD)\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\left(\sqrt{a}+\sqrt{b}x\right)}\right)}{4\sqrt{2}a^{3/4}b^{11/4}}\end{aligned}$$

output
```
-1/2*c*(A*b-5*C*a)*(c*x)^(1/2)/a/b^2+2/3*D*(c*x)^(7/2)/b/c^2/(b*x^2+a)-1/6
*(c*x)^(3/2)*(a*(3*B*b-7*D*a)-3*b*(A*b-C*a)*x)/a/b^2/(b*x^2+a)-1/8*c^(3/2)
*(b^(1/2)*(A*b-5*C*a)+a^(1/2)*(3*B*b-7*D*a))*arctan(1-2^(1/2)*b^(1/4)*(c*x
)^(1/2)/a^(1/4)/c^(1/2))*2^(1/2)/a^(3/4)/b^(11/4)+1/8*c^(3/2)*(b^(1/2)*(A*
b-5*C*a)+a^(1/2)*(3*B*b-7*D*a))*arctan(1+2^(1/2)*b^(1/4)*(c*x)^(1/2)/a^(1/
4)/c^(1/2))*2^(1/2)/a^(3/4)/b^(11/4)+1/8*c^(3/2)*(b^(1/2)*(A*b-5*C*a)-a^(1
/2)*(3*B*b-7*D*a))*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*(c*x)^(1/2)/c^(1/2)/(a^
(1/2)+b^(1/2)*x))*2^(1/2)/a^(3/4)/b^(11/4)
```

## Mathematica [A] (verified)

Time = 1.26 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.64

$$\int \frac{(cx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{\left(a+bx^2\right)^2}\,dx = \frac{(cx)^{3/2}\left(\frac{4b^{3/4}\sqrt{x}(-3Ab+a(15C+7Dx)+bx(-3B+4x(3C+Dx)))}{a+bx^2}+\frac{3\sqrt{2}\left(-Ab^{3/2}-\right.}{}\right.}{}$$

input
```
Integrate[((c*x)^(3/2)*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^2,x]
```

output
```
((c*x)^(3/2)*((4*b^(3/4)*Sqrt[x]*(-3*A*b + a*(15*C + 7*D*x) + b*x*(-3*B +
4*x*(3*C + D*x))))/(a + b*x^2) + (3*Sqrt[2]*(-(A*b^(3/2)) - 3*Sqrt[a]*b*B
+ 5*a*Sqrt[b]*C + 7*a^(3/2)*D)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/
4)*b^(1/4)*Sqrt[x])])/a^(3/4) + (3*Sqrt[2]*(A*b^(3/2) - 3*Sqrt[a]*b*B - 5*
a*Sqrt[b]*C + 7*a^(3/2)*D)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt
[a] + Sqrt[b]*x)])/a^(3/4)))/(24*b^(11/4)*x^(3/2))
```

## Rubi [A] (verified)

Time = 1.13 (sec) , antiderivative size = 474, normalized size of antiderivative = 1.27, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2335, 27, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{\left(a+bx^2\right)^2}dx$$

$\Big\downarrow$ 2335

$$-\frac{c\int -\frac{\sqrt{cx}\left(4aDx^2-(Ab-5aC)x+\frac{3a(bB-aD)}{b}\right)}{2(bx^2+a)}dx}{2ab}-\frac{(cx)^{3/2}\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{2ab\left(a+bx^2\right)}$$

$\Big\downarrow$ 27

$$\frac{c\int \frac{\sqrt{cx}\left(4aDx^2-(Ab-5aC)x+\frac{3a(bB-aD)}{b}\right)}{bx^2+a}dx}{4ab}-\frac{(cx)^{3/2}\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{2ab\left(a+bx^2\right)}$$

$\Big\downarrow$ 2333

$$\frac{c\int \left(\frac{4a\sqrt{cx}D}{b}+\frac{\sqrt{cx}(a(3bB-7aD)-b(Ab-5aC)x)}{b(bx^2+a)}\right)dx}{4ab}-\frac{(cx)^{3/2}\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{2ab\left(a+bx^2\right)}$$

$\Big\downarrow$ 2009

$$\frac{c\left(-\frac{\sqrt[4]{a}\sqrt{c}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)\left(\sqrt{b}(Ab-5aC)+\sqrt{a}(3bB-7aD)\right)}{\sqrt{2}b^{7/4}}+\frac{\sqrt[4]{a}\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}+1\right)\left(\sqrt{b}(Ab-5aC)+\sqrt{a}(3bB-7aD)\right)}{\sqrt{2}b^{7/4}}\right.}{}$$

$$\frac{(cx)^{3/2}\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{2ab\left(a+bx^2\right)}$$

input
```
Int[((c*x)^(3/2)*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^2,x]
```

output
```
-1/2*((c*x)^(3/2)*(a*(B - (a*D)/b) - (A*b - a*C)*x))/(a*b*(a + b*x^2)) + (
c*((-2*(A*b - 5*a*C)*Sqrt[c*x])/b + (8*a*D*(c*x)^(3/2))/(3*b*c) - (a^(1/4)
*Sqrt[c]*(Sqrt[b]*(A*b - 5*a*C) + Sqrt[a]*(3*b*B - 7*a*D))*ArcTan[1 - (Sqr
t[2]*b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])])/(Sqrt[2]*b^(7/4)) + (a^(1/4)*S
qrt[c]*(Sqrt[b]*(A*b - 5*a*C) + Sqrt[a]*(3*b*B - 7*a*D))*ArcTan[1 + (Sqrt[
2]*b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])])/(Sqrt[2]*b^(7/4)) - (a^(1/4)*Sqr
t[c]*(Sqrt[b]*(A*b - 5*a*C) - Sqrt[a]*(3*b*B - 7*a*D))*Log[Sqrt[a]*Sqrt[c]
 + Sqrt[b]*Sqrt[c]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[c*x]])/(2*Sqrt[2]*b^(7
/4)) + (a^(1/4)*Sqrt[c]*(Sqrt[b]*(A*b - 5*a*C) - Sqrt[a]*(3*b*B - 7*a*D))*
Log[Sqrt[a]*Sqrt[c] + Sqrt[b]*Sqrt[c]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[c*x
]])/(2*Sqrt[2]*b^(7/4))))/(4*a*b)
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2335
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Simp[c/(2*a*b*(p + 1))   Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSu
m[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
 b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

## Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 381, normalized size of antiderivative = 1.02

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{2D(cx)^{\frac{3}{2}}}{3}+2Cc\sqrt{cx}}{b^2}+\frac{2c^2\left(\frac{\left(-\frac{Bb}{4}+\frac{Da}{4}\right)(cx)^{\frac{3}{2}}+\left(-\frac{1}{4}Abc+\frac{1}{4}Cac\right)\sqrt{cx}}{b\,c^2x^2+a\,c^2}+\frac{(Abc-5Cac)\left(\frac{a\,c^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{cx+\left(\frac{a\,c^2}{b}\right)^{\frac{1}{4}}\sqrt{cx}\,\sqrt{}}{cx-\left(\frac{a\,c^2}{b}\right)^{\frac{1}{4}}\sqrt{cx}\,\sqrt{}}\right.\right.}{}\right.}{}$ |
| default | $\frac{\frac{2D(cx)^{\frac{3}{2}}}{3}+2Cc\sqrt{cx}}{b^2}+\frac{2c^2\left(\frac{\left(-\frac{Bb}{4}+\frac{Da}{4}\right)(cx)^{\frac{3}{2}}+\left(-\frac{1}{4}Abc+\frac{1}{4}Cac\right)\sqrt{cx}}{b\,c^2x^2+a\,c^2}+\frac{(Abc-5Cac)\left(\frac{a\,c^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{cx+\left(\frac{a\,c^2}{b}\right)^{\frac{1}{4}}\sqrt{cx}\,\sqrt{}}{cx-\left(\frac{a\,c^2}{b}\right)^{\frac{1}{4}}\sqrt{cx}\,\sqrt{}}\right.\right.}{}\right.}{}$ |
| pseudoelliptic | $-\frac{\left(b\,x^2+a\right)\sqrt{2}\left(\arctan\left(\frac{\sqrt{2}\,\sqrt{cx}}{\left(\frac{a\,c^2}{b}\right)^{\frac{1}{4}}}-1\right)+\arctan\left(\frac{\sqrt{2}\,\sqrt{cx}}{\left(\frac{a\,c^2}{b}\right)^{\frac{1}{4}}}+1\right)+\frac{\ln\left(\frac{cx+\left(\frac{a\,c^2}{b}\right)^{\frac{1}{4}}\sqrt{cx}\,\sqrt{2}+\sqrt{\frac{a\,c^2}{b}}}{cx-\left(\frac{a\,c^2}{b}\right)^{\frac{1}{4}}\sqrt{cx}\,\sqrt{2}+\sqrt{\frac{a\,c^2}{b}}}\right)}{2}\right)(Ab-5Ca)cb\sqrt{\frac{a\,c^2}{b}}}{4}+$ <br> $-\frac{}{}$ |

input

```
int((c*x)^(3/2)*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output

```
2/b^2*(1/3*D*(c*x)^(3/2)+C*c*(c*x)^(1/2))+2*c^2/b^2*(((-1/4*B*b+1/4*D*a)*(
c*x)^(3/2)+(-1/4*A*b*c+1/4*C*a*c)*(c*x)^(1/2))/(b*c^2*x^2+a*c^2)+1/32*(A*b
*c-5*C*a*c)*(a*c^2/b)^(1/4)/a/c^2*2^(1/2)*(ln((c*x+(a*c^2/b)^(1/4)*(c*x)^(
1/2)*2^(1/2)+(a*c^2/b)^(1/2))/(c*x-(a*c^2/b)^(1/4)*(c*x)^(1/2)*2^(1/2)+(a*
c^2/b)^(1/2)))+2*arctan(2^(1/2)/(a*c^2/b)^(1/4)*(c*x)^(1/2)+1)+2*arctan(2^
(1/2)/(a*c^2/b)^(1/4)*(c*x)^(1/2)-1))+1/32*(3*B*b-7*D*a)/b/(a*c^2/b)^(1/4)
*2^(1/2)*(ln((c*x-(a*c^2/b)^(1/4)*(c*x)^(1/2)*2^(1/2)+(a*c^2/b)^(1/2))/(c*
x+(a*c^2/b)^(1/4)*(c*x)^(1/2)*2^(1/2)+(a*c^2/b)^(1/2)))+2*arctan(2^(1/2)/(
a*c^2/b)^(1/4)*(c*x)^(1/2)+1)+2*arctan(2^(1/2)/(a*c^2/b)^(1/4)*(c*x)^(1/2)
-1)))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3694 vs. $2(291) = 582$.

Time = 0.27 (sec) , antiderivative size = 3694, normalized size of antiderivative = 9.88

$$\int \frac{(cx)^{3/2} (A + Bx + Cx^2 + Dx^3)}{(a + bx^2)^2} dx = \text{Too large to display}$$

input
```
integrate((c*x)^(3/2)*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="fricas
")
```

output
```
Too large to include
```

## Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 83.21 (sec) , antiderivative size = 4012, normalized size of antiderivative = 10.73

$$\int \frac{(cx)^{3/2} (A + Bx + Cx^2 + Dx^3)}{(a + bx^2)^2} dx = \text{Too large to display}$$

input
```
integrate((c*x)**(3/2)*(D*x**3+C*x**2+B*x+A)/(b*x**2+a)**2,x)
```

output
```
A*(-5*a**(9/4)*b*c**(3/2)*x**(9/2)*log(1 - b**(1/4)*sqrt(x)*exp_polar(I*pi
/4)/a**(1/4))*gamma(5/4)/(32*a**3*b**(9/4)*x**(9/2)*exp(I*pi/4)*gamma(9/4)
 + 32*a**2*b**(13/4)*x**(13/2)*exp(I*pi/4)*gamma(9/4)) + 5*I*a**(9/4)*b*c*
*(3/2)*x**(9/2)*log(1 - b**(1/4)*sqrt(x)*exp_polar(3*I*pi/4)/a**(1/4))*gam
ma(5/4)/(32*a**3*b**(9/4)*x**(9/2)*exp(I*pi/4)*gamma(9/4) + 32*a**2*b**(13
/4)*x**(13/2)*exp(I*pi/4)*gamma(9/4)) + 5*a**(9/4)*b*c**(3/2)*x**(9/2)*log
(1 - b**(1/4)*sqrt(x)*exp_polar(5*I*pi/4)/a**(1/4))*gamma(5/4)/(32*a**3*b*
*(9/4)*x**(9/2)*exp(I*pi/4)*gamma(9/4) + 32*a**2*b**(13/4)*x**(13/2)*exp(I
*pi/4)*gamma(9/4)) - 5*I*a**(9/4)*b*c**(3/2)*x**(9/2)*log(1 - b**(1/4)*sqr
t(x)*exp_polar(7*I*pi/4)/a**(1/4))*gamma(5/4)/(32*a**3*b**(9/4)*x**(9/2)*e
xp(I*pi/4)*gamma(9/4) + 32*a**2*b**(13/4)*x**(13/2)*exp(I*pi/4)*gamma(9/4)
) - 5*a**(5/4)*b**2*c**(3/2)*x**(13/2)*log(1 - b**(1/4)*sqrt(x)*exp_polar(
I*pi/4)/a**(1/4))*gamma(5/4)/(32*a**3*b**(9/4)*x**(9/2)*exp(I*pi/4)*gamma(
9/4) + 32*a**2*b**(13/4)*x**(13/2)*exp(I*pi/4)*gamma(9/4)) + 5*I*a**(5/4)*
b**2*c**(3/2)*x**(13/2)*log(1 - b**(1/4)*sqrt(x)*exp_polar(3*I*pi/4)/a**(1
/4))*gamma(5/4)/(32*a**3*b**(9/4)*x**(9/2)*exp(I*pi/4)*gamma(9/4) + 32*a**
2*b**(13/4)*x**(13/2)*exp(I*pi/4)*gamma(9/4)) + 5*a**(5/4)*b**2*c**(3/2)*x
**(13/2)*log(1 - b**(1/4)*sqrt(x)*exp_polar(5*I*pi/4)/a**(1/4))*gamma(5/4)
/(32*a**3*b**(9/4)*x**(9/2)*exp(I*pi/4)*gamma(9/4) + 32*a**2*b**(13/4)*x**
(13/2)*exp(I*pi/4)*gamma(9/4)) - 5*I*a**(5/4)*b**2*c**(3/2)*x**(13/2)*l...
```

## Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 448, normalized size of antiderivative = 1.20

$$\int \frac{(cx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{\left(a+bx^2\right)^2}\,dx = \frac{3c^3\left(\frac{\sqrt{2}\left((7\,Da-3\,Bb)\sqrt{a}c-\left(5\,Ca\sqrt{b}-Ab^{\frac{3}{2}}\right)c\right)\log\left(\sqrt{b}cx+\sqrt{2}\left(ac^2\right)^{\frac{1}{4}}\sqrt{cx}b^{\frac{1}{4}}+\sqrt{a}c\right)}{\left(ac^2\right)^{\frac{3}{4}}b^{\frac{3}{4}}}-\frac{\sqrt{2}\left((7\,Da\cdots\right.}{\cdots}\right.}{\cdots}$$

input
```
integrate((c*x)^(3/2)*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="maxima
")
```

output
```
1/48*(3*c^3*(sqrt(2)*((7*D*a - 3*B*b)*sqrt(a)*c - (5*C*a*sqrt(b) - A*b^(3/
2))*c)*log(sqrt(b)*c*x + sqrt(2)*(a*c^2)^(1/4)*sqrt(c*x)*b^(1/4) + sqrt(a)
*c)/((a*c^2)^(3/4)*b^(3/4)) - sqrt(2)*((7*D*a - 3*B*b)*sqrt(a)*c - (5*C*a*
sqrt(b) - A*b^(3/2))*c)*log(sqrt(b)*c*x - sqrt(2)*(a*c^2)^(1/4)*sqrt(c*x)*
b^(1/4) + sqrt(a)*c)/((a*c^2)^(3/4)*b^(3/4)) - 2*sqrt(2)*((7*D*a - 3*B*b)*
sqrt(a)*c + (5*C*a*sqrt(b) - A*b^(3/2))*c)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*
c^2)^(1/4)*b^(1/4) + 2*sqrt(c*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*c))/(sqrt(s
qrt(a)*sqrt(b)*c)*sqrt(a)*sqrt(b)*c) - 2*sqrt(2)*((7*D*a - 3*B*b)*sqrt(a)*
c + (5*C*a*sqrt(b) - A*b^(3/2))*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*c^2)^(1
/4)*b^(1/4) - 2*sqrt(c*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*c))/(sqrt(sqrt(a)*
sqrt(b)*c)*sqrt(a)*sqrt(b)*c))/b^2 + 24*((D*a - B*b)*(c*x)^(3/2)*c^3 + (C*
a - A*b)*sqrt(c*x)*c^4)/(b^3*c^2*x^2 + a*b^2*c^2) + 32*((c*x)^(3/2)*D*c +
3*sqrt(c*x)*C*c^2)/b^2)/c
```

## Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 568, normalized size of antiderivative = 1.52

$$\int \frac{(cx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{\left(a+bx^2\right)^2}\,dx=\frac{1}{48}\,c\left(\frac{24\left(\sqrt{cx}Dac^2x-\sqrt{cx}Bbc^2x+\sqrt{cx}Cac^2-\sqrt{cx}Abc^2\right)}{(bc^2x^2+ac^2)b^2}-\frac{6\cdots}{\cdots}\right.$$

input
```
integrate((c*x)^(3/2)*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/48*c*(24*(sqrt(c*x)*D*a*c^2*x - sqrt(c*x)*B*b*c^2*x + sqrt(c*x)*C*a*c^2
- sqrt(c*x)*A*b*c^2)/((b*c^2*x^2 + a*c^2)*b^2) - 6*sqrt(2)*(5*(a*b^3*c^2)^
(1/4)*C*a*b^2*c - (a*b^3*c^2)^(1/4)*A*b^3*c + 7*(a*b^3*c^2)^(3/4)*D*a - 3*
(a*b^3*c^2)^(3/4)*B*b)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*c^2/b)^(1/4) + 2*sqr
t(c*x))/(a*c^2/b)^(1/4))/(a*b^5*c) - 6*sqrt(2)*(5*(a*b^3*c^2)^(1/4)*C*a*b^
2*c - (a*b^3*c^2)^(1/4)*A*b^3*c + 7*(a*b^3*c^2)^(3/4)*D*a - 3*(a*b^3*c^2)^
(3/4)*B*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*c^2/b)^(1/4) - 2*sqrt(c*x))/(a*
c^2/b)^(1/4))/(a*b^5*c) - 3*sqrt(2)*(5*(a*b^3*c^2)^(1/4)*C*a*b^2*c - (a*b^
3*c^2)^(1/4)*A*b^3*c - 7*(a*b^3*c^2)^(3/4)*D*a + 3*(a*b^3*c^2)^(3/4)*B*b)*
log(c*x + sqrt(2)*(a*c^2/b)^(1/4)*sqrt(c*x) + sqrt(a*c^2/b))/(a*b^5*c) + 3
*sqrt(2)*(5*(a*b^3*c^2)^(1/4)*C*a*b^2*c - (a*b^3*c^2)^(1/4)*A*b^3*c - 7*(a
*b^3*c^2)^(3/4)*D*a + 3*(a*b^3*c^2)^(3/4)*B*b)*log(c*x - sqrt(2)*(a*c^2/b)
^(1/4)*sqrt(c*x) + sqrt(a*c^2/b))/(a*b^5*c) + 32*(sqrt(c*x)*D*b^4*c^3*x +
3*sqrt(c*x)*C*b^4*c^3)/(b^6*c^3))
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{3/2}(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx=\int \frac{(c\,x)^{3/2}\,(A+B\,x+C\,x^2+x^3\,D)}{(b\,x^2+a)^2}\,dx$$

input
```
int(((c*x)^(3/2)*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^2,x)
```

output
```
int(((c*x)^(3/2)*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^2, x)
```

## Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 1227, normalized size of antiderivative = 3.28

$$\int \frac{(cx)^{3/2}(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx=\text{Too large to display}$$

input
```
int((c*x)^(3/2)*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x)
```

output
```
(sqrt(c)*c*(42*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) -
 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2*d - 18*b**(1/4)*a**(
3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4
)*a**(1/4)*sqrt(2)))*a*b**2 + 42*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*
a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b*d*x
**2 - 18*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqr
t(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*b**3*x**2 - 6*b**(3/4)*a**(1/4)
*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a*
*(1/4)*sqrt(2)))*a**2*b + 30*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(
1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2*c - 6*
b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt
(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b**2*x**2 + 30*b**(3/4)*a**(1/4)*sqrt(
2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)
*sqrt(2)))*a*b*c*x**2 - 42*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/
4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2*d + 18*b
**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(
b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b**2 - 42*b**(1/4)*a**(3/4)*sqrt(2)*ata
n((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(
2)))*a*b*d*x**2 + 18*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqr
t(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*b**3*x**2 + 6*b*...
```

# 3.131 $\int \frac{\sqrt{cx}(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx$



## Optimal result

Integrand size = 32, antiderivative size = 342

$$\begin{aligned}
&\int \frac{\sqrt{cx}(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx \\
&= \frac{2D(cx)^{5/2}}{bc^2(a+bx^2)} - \frac{\sqrt{cx}(a(bB-5aD)-b(Ab-aC)x)}{2ab^2(a+bx^2)} \\
&\quad - \frac{\sqrt{c}\left(\sqrt{b}(Ab+3aC)+\sqrt{a}(bB-5aD)\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)}{4\sqrt{2}a^{5/4}b^{9/4}} \\
&\quad + \frac{\sqrt{c}\left(\sqrt{b}(Ab+3aC)+\sqrt{a}(bB-5aD)\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)}{4\sqrt{2}a^{5/4}b^{9/4}} \\
&\quad - \frac{\sqrt{c}\left(\sqrt{b}(Ab+3aC)-\sqrt{a}(bB-5aD)\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\left(\sqrt{a}+\sqrt{b}x\right)}\right)}{4\sqrt{2}a^{5/4}b^{9/4}}
\end{aligned}$$

output
```
2*D*(c*x)^(5/2)/b/c^2/(b*x^2+a)-1/2*(c*x)^(1/2)*(a*(B*b-5*D*a)-b*(A*b-C*a)
*x)/a/b^2/(b*x^2+a)-1/8*c^(1/2)*(b^(1/2)*(A*b+3*C*a)+a^(1/2)*(B*b-5*D*a))*
arctan(1-2^(1/2)*b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))*2^(1/2)/a^(5/4)/b^(9
/4)+1/8*c^(1/2)*(b^(1/2)*(A*b+3*C*a)+a^(1/2)*(B*b-5*D*a))*arctan(1+2^(1/2)
*b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))*2^(1/2)/a^(5/4)/b^(9/4)-1/8*c^(1/2)*
(b^(1/2)*(A*b+3*C*a)-a^(1/2)*(B*b-5*D*a))*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*
(c*x)^(1/2)/c^(1/2)/(a^(1/2)+b^(1/2)*x))*2^(1/2)/a^(5/4)/b^(9/4)
```

## Mathematica [A] (verified)

Time = 1.25 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.68

$$\int \frac{\sqrt{cx}(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx$$
$$= \frac{\sqrt{cx}\left(\frac{4\sqrt[4]{a}\sqrt[4]{b}(5a^2D+Ab^2x-ab(B+x(C-4Dx)))}{a+bx^2} - \frac{\sqrt{2}\left(Ab^{3/2}+\sqrt{a}bB+3a\sqrt{b}C-5a^{3/2}D\right)\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{\sqrt{x}} - \frac{\sqrt{2}\left(Ab^{3/2}-\right.}{}\right.}{8a^{5/4}b^{9/4}}$$

input
```
Integrate[(Sqrt[c*x]*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^2,x]
```

output
```
(Sqrt[c*x]*((4*a^(1/4)*b^(1/4)*(5*a^2*D + A*b^2*x - a*b*(B + x*(C - 4*D*x)
)))/(a + b*x^2) - (Sqrt[2]*(A*b^(3/2) + Sqrt[a]*b*B + 3*a*Sqrt[b]*C - 5*a^
(3/2)*D)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/
Sqrt[x] - (Sqrt[2]*(A*b^(3/2) - Sqrt[a]*b*B + 3*a*Sqrt[b]*C + 5*a^(3/2)*D)
*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/Sqrt[x]
))/(8*a^(5/4)*b^(9/4))
```

## Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 448, normalized size of antiderivative = 1.31, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2335, 27, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{cx}\left(A+Bx+Cx^2+Dx^3\right)}{\left(a+bx^2\right)^2}dx$$

$$\Big\downarrow \text{2335}$$

$$-\frac{c\int -\frac{4aDx^2+(Ab+3aC)x+\frac{a(bB-aD)}{b}}{2\sqrt{cx}(bx^2+a)}dx}{2ab}-\frac{\sqrt{cx}\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{2ab\left(a+bx^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{c\int \frac{4aDx^2+(Ab+3aC)x+\frac{a(bB-aD)}{b}}{\sqrt{cx}(bx^2+a)}dx}{4ab}-\frac{\sqrt{cx}\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{2ab\left(a+bx^2\right)}$$

$$\Big\downarrow \text{2333}$$

$$\frac{c\int \left(\frac{4aD}{b\sqrt{cx}}+\frac{a(bB-5aD)+b(Ab+3aC)x}{b\sqrt{cx}(bx^2+a)}\right)dx}{4ab}-\frac{\sqrt{cx}\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{2ab\left(a+bx^2\right)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{c\left(-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)\left(\sqrt{b}(3aC+Ab)+\sqrt{a}(bB-5aD)\right)}{\sqrt{2}\sqrt[4]{a}b^{5/4}\sqrt{c}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}+1\right)\left(\sqrt{b}(3aC+Ab)+\sqrt{a}(bB-5aD)\right)}{\sqrt{2}\sqrt[4]{a}b^{5/4}\sqrt{c}}+\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\right.}{}\right.}{}$$

$$\frac{\sqrt{cx}\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{2ab\left(a+bx^2\right)}$$

input
```
Int[(Sqrt[c*x]*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^2,x]
```

output
```
-1/2*(Sqrt[c*x]*(a*(B - (a*D)/b) - (A*b - a*C)*x))/(a*b*(a + b*x^2)) + (c*
((8*a*D*Sqrt[c*x])/(b*c) - ((Sqrt[b]*(A*b + 3*a*C) + Sqrt[a]*(b*B - 5*a*D)
)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])])/(Sqrt[2]*a^(1
/4)*b^(5/4)*Sqrt[c]) + ((Sqrt[b]*(A*b + 3*a*C) + Sqrt[a]*(b*B - 5*a*D))*Ar
cTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])])/(Sqrt[2]*a^(1/4)*
b^(5/4)*Sqrt[c]) + ((Sqrt[b]*(A*b + 3*a*C) - Sqrt[a]*(b*B - 5*a*D))*Log[Sq
rt[a]*Sqrt[c] + Sqrt[b]*Sqrt[c]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[c*x]])/(2
*Sqrt[2]*a^(1/4)*b^(5/4)*Sqrt[c]) - ((Sqrt[b]*(A*b + 3*a*C) - Sqrt[a]*(b*B
 - 5*a*D))*Log[Sqrt[a]*Sqrt[c] + Sqrt[b]*Sqrt[c]*x + Sqrt[2]*a^(1/4)*b^(1/
4)*Sqrt[c*x]])/(2*Sqrt[2]*a^(1/4)*b^(5/4)*Sqrt[c])))/(4*a*b)
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2335
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Simp[c/(2*a*b*(p + 1))   Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSu
m[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
 b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

## Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 380, normalized size of antiderivative = 1.11

| method | result |
|---|---|
| pseudoelliptic | $\frac{(b x^2+a)\sqrt{2}\,(Bb-5Da)\left(\arctan\left(\frac{\sqrt{2}\,\sqrt{cx}}{\left(\frac{a c^2}{b}\right)^{\frac{1}{4}}}-1\right)+\arctan\left(\frac{\sqrt{2}\,\sqrt{cx}}{\left(\frac{a c^2}{b}\right)^{\frac{1}{4}}}+1\right)+\frac{\ln\left(\frac{cx+\left(\frac{a c^2}{b}\right)^{\frac{1}{4}}\sqrt{cx}\,\sqrt{2}+\sqrt{\frac{a c^2}{b}}}{cx-\left(\frac{a c^2}{b}\right)^{\frac{1}{4}}\sqrt{cx}\,\sqrt{2}+\sqrt{\frac{a c^2}{b}}}\right)}{2}\right) c\sqrt{\frac{a c}{b}}}{}$ |
| derivativedivides | $\frac{2D\sqrt{cx}}{b^2}+\frac{2c\left(\frac{\frac{b(Ab-Ca)(cx)^{\frac{3}{2}}}{4a}+\left(-\frac{1}{4}Bbc+\frac{1}{4}Dca\right)\sqrt{cx}}{b\,c^2x^2+a\,c^2}+\frac{\frac{\left(abBc-5Da^2c\right)\left(\frac{a c^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{cx+\left(\frac{a c^2}{b}\right)^{\frac{1}{4}}\sqrt{cx}\,\sqrt{2}+\sqrt{\frac{a c^2}{b}}}{cx-\left(\frac{a c^2}{b}\right)^{\frac{1}{4}}\sqrt{cx}\,\sqrt{2}+\sqrt{\frac{a c^2}{b}}}\right)\right.}{8a\,c^2}}{}\right.}{}$ |
| default | $\frac{2D\sqrt{cx}}{b^2}+\frac{2c\left(\frac{\frac{b(Ab-Ca)(cx)^{\frac{3}{2}}}{4a}+\left(-\frac{1}{4}Bbc+\frac{1}{4}Dca\right)\sqrt{cx}}{b\,c^2x^2+a\,c^2}+\frac{\frac{\left(abBc-5Da^2c\right)\left(\frac{a c^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{cx+\left(\frac{a c^2}{b}\right)^{\frac{1}{4}}\sqrt{cx}\,\sqrt{2}+\sqrt{\frac{a c^2}{b}}}{cx-\left(\frac{a c^2}{b}\right)^{\frac{1}{4}}\sqrt{cx}\,\sqrt{2}+\sqrt{\frac{a c^2}{b}}}\right)\right.}{8a\,c^2}}{}\right.}{}$ |

input
```
int((c*x)^(1/2)*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/8/(a*c^2/b)^(1/4)*((b*x^2+a)*2^(1/2)*(B*b-5*D*a)*(arctan(2^(1/2)/(a*c^2/
b)^(1/4)*(c*x)^(1/2)-1)+arctan(2^(1/2)/(a*c^2/b)^(1/4)*(c*x)^(1/2)+1)+1/2*
ln((c*x+(a*c^2/b)^(1/4)*(c*x)^(1/2)*2^(1/2)+(a*c^2/b)^(1/2))/(c*x-(a*c^2/b
)^(1/4)*(c*x)^(1/2)*2^(1/2)+(a*c^2/b)^(1/2))))*c*(a*c^2/b)^(1/2)+4*(A*b-C*
a)*b*(a*c^2/b)^(1/4)*(c*x)^(3/2)+(-4*(a*c^2/b)^(1/4)*a*(-5*D*a+b*(-4*D*x^2
+B))*(c*x)^(1/2)+(A*b+3*C*a)*(b*x^2+a)*2^(1/2)*(arctan(2^(1/2)/(a*c^2/b)^(
1/4)*(c*x)^(1/2)-1)+arctan(2^(1/2)/(a*c^2/b)^(1/4)*(c*x)^(1/2)+1)+1/2*ln((
c*x-(a*c^2/b)^(1/4)*(c*x)^(1/2)*2^(1/2)+(a*c^2/b)^(1/2))/(c*x+(a*c^2/b)^(1
/4)*(c*x)^(1/2)*2^(1/2)+(a*c^2/b)^(1/2))))*c)*c)/c/b^2/(b*x^2+a)/a
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3584 vs. $2(265) = 530$.

Time = 0.26 (sec) , antiderivative size = 3584, normalized size of antiderivative = 10.48

$$\int \frac{\sqrt{cx}(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate((c*x)^(1/2)*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="fricas
")
```

output
```
Too large to include
```

## Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 70.35 (sec) , antiderivative size = 3900, normalized size of antiderivative = 11.40

$$\int \frac{\sqrt{cx}(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate((c*x)**(1/2)*(D*x**3+C*x**2+B*x+A)/(b*x**2+a)**2,x)
```

output
```
A*(-3*a**(7/4)*sqrt(c)*x**(3/2)*log(1 - b**(1/4)*sqrt(x)*exp_polar(I*pi/4)
/a**(1/4))*gamma(3/4)/(32*a**3*b**(3/4)*x**(3/2)*exp(3*I*pi/4)*gamma(7/4)
+ 32*a**2*b**(7/4)*x**(7/2)*exp(3*I*pi/4)*gamma(7/4)) - 3*I*a**(7/4)*sqrt(
c)*x**(3/2)*log(1 - b**(1/4)*sqrt(x)*exp_polar(3*I*pi/4)/a**(1/4))*gamma(3
/4)/(32*a**3*b**(3/4)*x**(3/2)*exp(3*I*pi/4)*gamma(7/4) + 32*a**2*b**(7/4)
*x**(7/2)*exp(3*I*pi/4)*gamma(7/4)) + 3*a**(7/4)*sqrt(c)*x**(3/2)*log(1 -
b**(1/4)*sqrt(x)*exp_polar(5*I*pi/4)/a**(1/4))*gamma(3/4)/(32*a**3*b**(3/4
)*x**(3/2)*exp(3*I*pi/4)*gamma(7/4) + 32*a**2*b**(7/4)*x**(7/2)*exp(3*I*pi
/4)*gamma(7/4)) + 3*I*a**(7/4)*sqrt(c)*x**(3/2)*log(1 - b**(1/4)*sqrt(x)*e
xp_polar(7*I*pi/4)/a**(1/4))*gamma(3/4)/(32*a**3*b**(3/4)*x**(3/2)*exp(3*I
*pi/4)*gamma(7/4) + 32*a**2*b**(7/4)*x**(7/2)*exp(3*I*pi/4)*gamma(7/4)) -
3*a**(3/4)*b*sqrt(c)*x**(7/2)*log(1 - b**(1/4)*sqrt(x)*exp_polar(I*pi/4)/a
**(1/4))*gamma(3/4)/(32*a**3*b**(3/4)*x**(3/2)*exp(3*I*pi/4)*gamma(7/4) +
32*a**2*b**(7/4)*x**(7/2)*exp(3*I*pi/4)*gamma(7/4)) - 3*I*a**(3/4)*b*sqrt(
c)*x**(7/2)*log(1 - b**(1/4)*sqrt(x)*exp_polar(3*I*pi/4)/a**(1/4))*gamma(3
/4)/(32*a**3*b**(3/4)*x**(3/2)*exp(3*I*pi/4)*gamma(7/4) + 32*a**2*b**(7/4)
*x**(7/2)*exp(3*I*pi/4)*gamma(7/4)) + 3*a**(3/4)*b*sqrt(c)*x**(7/2)*log(1
- b**(1/4)*sqrt(x)*exp_polar(5*I*pi/4)/a**(1/4))*gamma(3/4)/(32*a**3*b**(3
/4)*x**(3/2)*exp(3*I*pi/4)*gamma(7/4) + 32*a**2*b**(7/4)*x**(7/2)*exp(3*I*
pi/4)*gamma(7/4)) + 3*I*a**(3/4)*b*sqrt(c)*x**(7/2)*log(1 - b**(1/4)*sq...
```

## Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 468, normalized size of antiderivative = 1.37

$$\int \frac{\sqrt{cx}(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx$$

$$= \frac{\frac{32\sqrt{cx}Dc}{b^2} - \frac{8\left((Cab-Ab^2)(cx)^{\frac{3}{2}}c^2-(Da^2-Bab)\sqrt{cx}c^3\right)}{ab^3c^2x^2+a^2b^2c^2} - \frac{c^2\left(\frac{\sqrt{2}\left(\left(3\,Cab+Ab^2\right)\sqrt{a}c+\left(5\,Da^2\sqrt{b}-Bab^{\frac{3}{2}}\right)c\right)\log\left(\sqrt{b}cx+\sqrt{2}\left(ac^2\right)^{\frac{1}{4}}\sqrt{cx}b^{\frac{1}{4}}+\sqrt{\cdots}\right.}{\left(ac^2\right)^{\frac{3}{4}}b^{\frac{3}{4}}}\cdots\right)}{\cdots}}{\cdots}$$

input
```
integrate((c*x)^(1/2)*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="maxima
")
```

output
```
1/16*(32*sqrt(c*x)*D*c/b^2 - 8*((C*a*b - A*b^2)*(c*x)^(3/2)*c^2 - (D*a^2 -
 B*a*b)*sqrt(c*x)*c^3)/(a*b^3*c^2*x^2 + a^2*b^2*c^2) - c^2*(sqrt(2)*((3*C*
a*b + A*b^2)*sqrt(a)*c + (5*D*a^2*sqrt(b) - B*a*b^(3/2))*c)*log(sqrt(b)*c*
x + sqrt(2)*(a*c^2)^(1/4)*sqrt(c*x)*b^(1/4) + sqrt(a)*c)/((a*c^2)^(3/4)*b^
(3/4)) - sqrt(2)*((3*C*a*b + A*b^2)*sqrt(a)*c + (5*D*a^2*sqrt(b) - B*a*b^(
3/2))*c)*log(sqrt(b)*c*x - sqrt(2)*(a*c^2)^(1/4)*sqrt(c*x)*b^(1/4) + sqrt(
a)*c)/((a*c^2)^(3/4)*b^(3/4)) - 2*sqrt(2)*((3*C*a*b + A*b^2)*sqrt(a)*c - (
5*D*a^2*sqrt(b) - B*a*b^(3/2))*c)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*c^2)^(1/4
)*b^(1/4) + 2*sqrt(c*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*c))/(sqrt(sqrt(a)*sq
rt(b)*c)*sqrt(a)*sqrt(b)*c) - 2*sqrt(2)*((3*C*a*b + A*b^2)*sqrt(a)*c - (5*
D*a^2*sqrt(b) - B*a*b^(3/2))*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*c^2)^(1/4)
*b^(1/4) - 2*sqrt(c*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*c))/(sqrt(sqrt(a)*sqr
t(b)*c)*sqrt(a)*sqrt(b)*c))/(a*b^2))/c
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 550 vs. $2(265) = 530$.

Time = 0.14 (sec) , antiderivative size = 550, normalized size of antiderivative = 1.61

$$\int \frac{\sqrt{cx}(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx$$
$$= \frac{2\sqrt{cx}D}{b^2} - \frac{\sqrt{cx}Cabc^2x - \sqrt{cx}Ab^2c^2x - \sqrt{cx}Da^2c^2 + \sqrt{cx}Babc^2}{2\,(bc^2x^2+ac^2)ab^2}$$
$$- \frac{\sqrt{2}\Big(5\,(ab^3c^2)^{\frac{1}{4}}\,Da^2bc - (ab^3c^2)^{\frac{1}{4}}\,Bab^2c - 3\,(ab^3c^2)^{\frac{3}{4}}\,Ca - (ab^3c^2)^{\frac{3}{4}}\,Ab\Big)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ac^2}{b}\right)^{\frac{1}{4}}+2\sqrt{cx}\right)}{2\left(\frac{ac^2}{b}\right)^{\frac{1}{4}}}\right.}{8\,a^2b^4c}$$
$$- \frac{\sqrt{2}\Big(5\,(ab^3c^2)^{\frac{1}{4}}\,Da^2bc - (ab^3c^2)^{\frac{1}{4}}\,Bab^2c - 3\,(ab^3c^2)^{\frac{3}{4}}\,Ca - (ab^3c^2)^{\frac{3}{4}}\,Ab\Big)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ac^2}{b}\right)^{\frac{1}{4}}-2\sqrt{cx}\right)}{2\left(\frac{ac^2}{b}\right)^{\frac{1}{4}}}\right.}{8\,a^2b^4c}$$
$$- \frac{\sqrt{2}\Big(5\,(ab^3c^2)^{\frac{1}{4}}\,Da^2bc - (ab^3c^2)^{\frac{1}{4}}\,Bab^2c + 3\,(ab^3c^2)^{\frac{3}{4}}\,Ca + (ab^3c^2)^{\frac{3}{4}}\,Ab\Big)\log\left(cx + \sqrt{2}\left(\frac{ac^2}{b}\right)^{\frac{1}{4}}\sqrt{cx} + \right.}{16\,a^2b^4c}$$
$$+ \frac{\sqrt{2}\Big(5\,(ab^3c^2)^{\frac{1}{4}}\,Da^2bc - (ab^3c^2)^{\frac{1}{4}}\,Bab^2c + 3\,(ab^3c^2)^{\frac{3}{4}}\,Ca + (ab^3c^2)^{\frac{3}{4}}\,Ab\Big)\log\left(cx - \sqrt{2}\left(\frac{ac^2}{b}\right)^{\frac{1}{4}}\sqrt{cx} + \right.}{16\,a^2b^4c}$$

input
```
integrate((c*x)^(1/2)*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
2*sqrt(c*x)*D/b^2 - 1/2*(sqrt(c*x)*C*a*b*c^2*x - sqrt(c*x)*A*b^2*c^2*x - s
qrt(c*x)*D*a^2*c^2 + sqrt(c*x)*B*a*b*c^2)/((b*c^2*x^2 + a*c^2)*a*b^2) - 1/
8*sqrt(2)*(5*(a*b^3*c^2)^(1/4)*D*a^2*b*c - (a*b^3*c^2)^(1/4)*B*a*b^2*c - 3
*(a*b^3*c^2)^(3/4)*C*a - (a*b^3*c^2)^(3/4)*A*b)*arctan(1/2*sqrt(2)*(sqrt(2
)*(a*c^2/b)^(1/4) + 2*sqrt(c*x))/(a*c^2/b)^(1/4))/(a^2*b^4*c) - 1/8*sqrt(2
)*(5*(a*b^3*c^2)^(1/4)*D*a^2*b*c - (a*b^3*c^2)^(1/4)*B*a*b^2*c - 3*(a*b^3*
c^2)^(3/4)*C*a - (a*b^3*c^2)^(3/4)*A*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*c^
2/b)^(1/4) - 2*sqrt(c*x))/(a*c^2/b)^(1/4))/(a^2*b^4*c) - 1/16*sqrt(2)*(5*(
a*b^3*c^2)^(1/4)*D*a^2*b*c - (a*b^3*c^2)^(1/4)*B*a*b^2*c + 3*(a*b^3*c^2)^(
3/4)*C*a + (a*b^3*c^2)^(3/4)*A*b)*log(c*x + sqrt(2)*(a*c^2/b)^(1/4)*sqrt(c
*x) + sqrt(a*c^2/b))/(a^2*b^4*c) + 1/16*sqrt(2)*(5*(a*b^3*c^2)^(1/4)*D*a^2
*b*c - (a*b^3*c^2)^(1/4)*B*a*b^2*c + 3*(a*b^3*c^2)^(3/4)*C*a + (a*b^3*c^2)
^(3/4)*A*b)*log(c*x - sqrt(2)*(a*c^2/b)^(1/4)*sqrt(c*x) + sqrt(a*c^2/b))/(
a^2*b^4*c)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{cx}(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx=\int \frac{\sqrt{c\,x}\,(A+B\,x+C\,x^2+x^3\,D)}{(b\,x^2+a)^2}\,dx$$

input
```
int(((c*x)^(1/2)*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^2,x)
```

output
```
int(((c*x)^(1/2)*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^2, x)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 1208, normalized size of antiderivative = 3.53

$$\int \frac{\sqrt{cx}(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx = \text{Too large to display}$$

input
```
int((c*x)^(1/2)*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x)
```

output
```
(sqrt(c)*( - 2*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) -
 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b**2 - 6*b**(1/4)*a**(3
/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)
*a**(1/4)*sqrt(2)))*a*b*c - 2*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**
(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*b**3*x**2
- 6*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*
sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*b**2*c*x**2 + 10*b**(3/4)*a**(1/4)*s
qrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(
1/4)*sqrt(2)))*a**2*d - 2*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4
)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b**2 + 10*b*
*(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b
))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b*d*x**2 - 2*b**(3/4)*a**(1/4)*sqrt(2)*a
tan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqr
t(2)))*b**3*x**2 + 2*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqr
t(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b**2 + 6*b**(1/4)
*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b*
*(1/4)*a**(1/4)*sqrt(2)))*a*b*c + 2*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/
4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*b**3
*x**2 + 6*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sq
rt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*b**2*c*x**2 - 10*b**(3/4)*a...
```

# 3.132 $\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{cx}\left(a+bx^2\right)^2} dx$



## Optimal result

Integrand size = 32, antiderivative size = 340

$$\begin{aligned}
&\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{cx}\left(a+bx^2\right)^2} dx \\
&= -\frac{2D(cx)^{3/2}}{bc^2\left(a+bx^2\right)}+\frac{\sqrt{cx}(Ab-aC+(bB+3aD)x)}{2abc\left(a+bx^2\right)} \\
&\quad -\frac{\left(\sqrt{b}(3Ab+aC)+\sqrt{a}(bB+3aD)\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)}{4\sqrt{2}a^{7/4}b^{7/4}\sqrt{c}} \\
&\quad +\frac{\left(\sqrt{b}(3Ab+aC)+\sqrt{a}(bB+3aD)\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)}{4\sqrt{2}a^{7/4}b^{7/4}\sqrt{c}} \\
&\quad +\frac{\left(\sqrt{b}(3Ab+aC)-\sqrt{a}(bB+3aD)\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\left(\sqrt{a}+\sqrt{b}x\right)}\right)}{4\sqrt{2}a^{7/4}b^{7/4}\sqrt{c}}
\end{aligned}$$

output

```
-2*D*(c*x)^(3/2)/b/c^2/(b*x^2+a)+1/2*(c*x)^(1/2)*(A*b-C*a+(B*b+3*D*a)*x)/a
/b/c/(b*x^2+a)-1/8*(b^(1/2)*(3*A*b+C*a)+a^(1/2)*(B*b+3*D*a))*arctan(1-2^(1
/2)*b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))*2^(1/2)/a^(7/4)/b^(7/4)/c^(1/2)+1
/8*(b^(1/2)*(3*A*b+C*a)+a^(1/2)*(B*b+3*D*a))*arctan(1+2^(1/2)*b^(1/4)*(c*x
)^(1/2)/a^(1/4)/c^(1/2))*2^(1/2)/a^(7/4)/b^(7/4)/c^(1/2)+1/8*(b^(1/2)*(3*A
*b+C*a)-a^(1/2)*(B*b+3*D*a))*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*(c*x)^(1/2)/c
^(1/2)/(a^(1/2)+b^(1/2)*x))*2^(1/2)/a^(7/4)/b^(7/4)/c^(1/2)
```

## Mathematica [A] (verified)

Time = 1.13 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.66

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{cx}\,(a+bx^2)^2}\,dx$$
$$= \frac{\sqrt{x}\left(-\frac{4a^{3/4}b^{3/4}\sqrt{x}(-Ab-bBx+a(C+Dx))}{a+bx^2}-\sqrt{2}\left(3Ab^{3/2}+\sqrt{a}bB+a\sqrt{b}C+3a^{3/2}D\right)\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)+\right.}{8a^{7/4}b^{7/4}\sqrt{cx}}$$

input

```
Integrate[(A + B*x + C*x^2 + D*x^3)/(Sqrt[c*x]*(a + b*x^2)^2),x]
```

output

```
(Sqrt[x]*((-4*a^(3/4)*b^(3/4)*Sqrt[x]*(-(A*b) - b*B*x + a*(C + D*x)))/(a +
 b*x^2) - Sqrt[2]*(3*A*b^(3/2) + Sqrt[a]*b*B + a*Sqrt[b]*C + 3*a^(3/2)*D)*
ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] + Sqrt[2]*
(3*A*b^(3/2) - Sqrt[a]*b*B + a*Sqrt[b]*C - 3*a^(3/2)*D)*ArcTanh[(Sqrt[2]*a
^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)]))/(8*a^(7/4)*b^(7/4)*Sqrt[c
*x])
```

## Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.11, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {2337, 27, 554, 1482, 27, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{cx}\left(a+bx^2\right)^2}dx$$

$$\Big\downarrow \ 2337$$

$$\frac{\sqrt{cx}\left(x\left(B-\frac{aD}{b}\right)-\frac{aC}{b}+A\right)}{2ac\left(a+bx^2\right)}-\frac{\int -\frac{3Ab+aC+(bB+3aD)x}{2b\sqrt{cx}\left(bx^2+a\right)}dx}{2a}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{3Ab+aC+(bB+3aD)x}{\sqrt{cx}\left(bx^2+a\right)}dx}{4ab}+\frac{\sqrt{cx}\left(x\left(B-\frac{aD}{b}\right)-\frac{aC}{b}+A\right)}{2ac\left(a+bx^2\right)}$$

$$\Big\downarrow \ 554$$

$$\frac{\int \frac{c(3Ab+aC)+c(bB+3aD)x}{bx^2c^2+ac^2}d\sqrt{cx}}{2ab}+\frac{\sqrt{cx}\left(x\left(B-\frac{aD}{b}\right)-\frac{aC}{b}+A\right)}{2ac\left(a+bx^2\right)}$$

$$\Big\downarrow \ 1482$$

$$\frac{\frac{\left(\frac{\sqrt{b}(aC+3Ab)}{\sqrt{a}}+3aD+bB\right)\int \frac{\sqrt{b}\left(\sqrt{b}xc+\sqrt{a}c\right)}{bx^2c^2+ac^2}d\sqrt{cx}}{2b}-\frac{\left(-\frac{\sqrt{b}(aC+3Ab)}{\sqrt{a}}+3aD+bB\right)\int \frac{\sqrt{b}\left(\sqrt{a}c-\sqrt{b}cx\right)}{bx^2c^2+ac^2}d\sqrt{cx}}{2b}}{2ab}+\frac{\sqrt{cx}\left(x\left(B-\frac{aD}{b}\right)-\frac{aC}{b}+A\right)}{2ac\left(a+bx^2\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\left(\frac{\sqrt{b}(aC+3Ab)}{\sqrt{a}}+3aD+bB\right)\int \frac{\sqrt{b}xc+\sqrt{a}c}{bx^2c^2+ac^2}d\sqrt{cx}}{2\sqrt{b}}-\frac{\left(-\frac{\sqrt{b}(aC+3Ab)}{\sqrt{a}}+3aD+bB\right)\int \frac{\sqrt{a}c-\sqrt{b}cx}{bx^2c^2+ac^2}d\sqrt{cx}}{2\sqrt{b}}}{2ab}+\frac{\sqrt{cx}\left(x\left(B-\frac{aD}{b}\right)-\frac{aC}{b}+A\right)}{2ac\left(a+bx^2\right)}$$

$$\Big\downarrow \ 1476$$

$$\frac{\frac{\left(\frac{\sqrt{b}(aC+3Ab)}{\sqrt{a}}+3aD+bB\right)\left(\frac{\int \frac{1}{xc+\frac{\sqrt{a}c}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{cx}\sqrt{c}}{\sqrt[4]{b}}}d\sqrt{cx}}{2\sqrt{b}}+\frac{\int \frac{1}{xc+\frac{\sqrt{a}c}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{cx}\sqrt{c}}{\sqrt[4]{b}}}d\sqrt{cx}}{2\sqrt{b}}\right)}{2\sqrt{b}}-\frac{\left(-\frac{\sqrt{b}(aC+3Ab)}{\sqrt{a}}+3aD+bB\right)\int \frac{\sqrt{a}c-\sqrt{b}cx}{bx^2c^2+ac^2}d\surd}{2\sqrt{b}}}{2ab}+\frac{\sqrt{cx}\left(x\left(B-\frac{aD}{b}\right)-\frac{aC}{b}+A\right)}{2ac\left(a+bx^2\right)}$$

$$\Big\downarrow \ 1082$$

$$\frac{\frac{\left(\frac{\sqrt{b}(aC+3Ab)}{\sqrt{a}}+3aD+bB\right)\left(\frac{\int \frac{1}{-cx-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}}-\frac{\int \frac{1}{-cx-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}}\right)}{2\sqrt{b}}-\frac{\left(-\frac{\sqrt{b}(aC+3Ab)}{\sqrt{a}}+3aD+bB\right)\int \frac{\sqrt{a}c-\sqrt{b}cx}{bx^2c^2+ac^2}d\sqrt{cx}}{2\sqrt{b}}}{2ab}+$$

$$\frac{\sqrt{cx}\left(x\left(B-\frac{aD}{b}\right)-\frac{aC}{b}+A\right)}{2ac\left(a+bx^2\right)}$$

$$\Big\downarrow \; 217$$

$$\frac{\frac{\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}}\right)\left(\frac{\sqrt{b}(aC+3Ab)}{\sqrt{a}}+3aD+bB\right)}{2\sqrt{b}}-\frac{\left(-\frac{\sqrt{b}(aC+3Ab)}{\sqrt{a}}+3aD+bB\right)\int \frac{\sqrt{a}c-\sqrt{b}cx}{bx^2c^2+ac^2}d\sqrt{cx}}{2\sqrt{b}}}{2ab}+$$

$$\frac{\sqrt{cx}\left(x\left(B-\frac{aD}{b}\right)-\frac{aC}{b}+A\right)}{2ac\left(a+bx^2\right)}$$

$$\Big\downarrow \; 1479$$

$$\frac{\frac{\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}}\right)\left(\frac{\sqrt{b}(aC+3Ab)}{\sqrt{a}}+3aD+bB\right)}{2\sqrt{b}}-\frac{\left(-\frac{\sqrt{b}(aC+3Ab)}{\sqrt{a}}+3aD+bB\right)\left(-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}\sqrt{c}-2\sqrt[4]{b}\sqrt{\cdots}}{\sqrt[4]{b}\left(xc+\frac{\sqrt{a}c}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\cdots}{\sqrt[4]{\cdots}}\right)}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}\cdots}\cdots\right.}{2\sqrt{\cdots}}}{2ab}$$

$$\frac{\sqrt{cx}\left(x\left(B-\frac{aD}{b}\right)-\frac{aC}{b}+A\right)}{2ac\left(a+bx^2\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}}\right)\left(\frac{\sqrt{b}(aC+3Ab)}{\sqrt{a}}+3aD+bB\right)}{2\sqrt{b}}-\frac{\left(-\frac{\sqrt{b}(aC+3Ab)}{\sqrt{a}}+3aD+bB\right)\left(\frac{\int \frac{\sqrt{2}\sqrt[4]{a}\sqrt{c}-2\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{b}\left(xc+\frac{\sqrt{a}c}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{cx}\cdots}{\sqrt[4]{b}}\right)}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}}\cdots\right.}{2\sqrt{b}}}{2ab}$$

$$\frac{\sqrt{cx}\left(x\left(B-\frac{aD}{b}\right)-\frac{aC}{b}+A\right)}{2ac\left(a+bx^2\right)}$$

$\Big\downarrow$ 27

$$\frac{\frac{\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}}\right)\left(\frac{\sqrt{b}(aC+3Ab)}{\sqrt{a}}+3aD+bB\right)}{2\sqrt{b}}-\frac{\left(-\frac{\sqrt{b}(aC+3Ab)}{\sqrt{a}}+3aD+bB\right)\left(\frac{\int\frac{\sqrt{2}\sqrt[4]{a}\sqrt{c}-2\sqrt[4]{b}\sqrt{cx}}{xc+\frac{\sqrt{ac}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{cx}\sqrt{c}}{\sqrt[4]{b}}}d\sqrt{c}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}\sqrt{c}}\right.}{2\sqrt{b}}}{2ab}$$

$$\frac{\sqrt{cx}\left(x\left(B-\frac{aD}{b}\right)-\frac{aC}{b}+A\right)}{2ac\left(a+bx^2\right)}$$

$\Big\downarrow$ 1103

$$\frac{\frac{\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}}\right)\left(\frac{\sqrt{b}(aC+3Ab)}{\sqrt{a}}+3aD+bB\right)}{2\sqrt{b}}-\frac{\left(\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}\sqrt{cx}+\sqrt{a}c+\sqrt{b}cx\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}\right.}{2\sqrt{2}\sqrt[4]{a}}\right.}{2\sqrt{b}}}{2ab}$$

$$\frac{\sqrt{cx}\left(x\left(B-\frac{aD}{b}\right)-\frac{aC}{b}+A\right)}{2ac\left(a+bx^2\right)}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/(Sqrt[c*x]*(a + b*x^2)^2),x]
```

output
```
(Sqrt[c*x]*(A - (a*C)/b + (B - (a*D)/b)*x))/(2*a*c*(a + b*x^2)) + (((b*B +
 (Sqrt[b]*(3*A*b + a*C))/Sqrt[a] + 3*a*D)*(-(ArcTan[1 - (Sqrt[2]*b^(1/4)*S
qrt[c*x])/(a^(1/4)*Sqrt[c])]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[c])) + ArcTan[1
 + (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])]/(Sqrt[2]*a^(1/4)*b^(1/4)
*Sqrt[c])))/(2*Sqrt[b]) - ((b*B - (Sqrt[b]*(3*A*b + a*C))/Sqrt[a] + 3*a*D)
*(-1/2*Log[Sqrt[a]*c + Sqrt[b]*c*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[c]*Sqrt[
c*x]]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[c]) + Log[Sqrt[a]*c + Sqrt[b]*c*x + Sq
rt[2]*a^(1/4)*b^(1/4)*Sqrt[c]*Sqrt[c*x]]/(2*Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[c
])))/(2*Sqrt[b]))/(2*a*b)
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 554
```
Int[((c_) + (d_.)*(x_))/(Sqrt[(e_.)*(x_)]*((a_) + (b_.)*(x_)^2)), x_Symbol]
 :> Simp[2   Subst[Int[(e*c + d*x^2)/(a*e^2 + b*x^4), x], x, Sqrt[e*x]], x]
 /; FreeQ[{a, b, c, d, e}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 2337
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(-(c*x)^(m + 1))*(f + g*x)*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1)))
, x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*ExpandToSum[2
*a*(p + 1)*Q + f*(m + 2*p + 3) + g*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a
, b, c, m}, x] && PolyQ[Pq, x] && LtQ[p, -1] && !GtQ[m, 0]
```

## Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.06

| method | result |
|---|---|
| pseudoelliptic | $\frac{\frac{3\left(b\,x^{2}+a\right)\left(Ab+\frac{Ca}{3}\right)\sqrt{2}\left(\ln\left(\frac{cx+\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{4}}\sqrt{cx}\,\sqrt{2}+\sqrt{\frac{a\,c^{2}}{b}}}{cx-\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{4}}\sqrt{cx}\,\sqrt{2}+\sqrt{\frac{a\,c^{2}}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{cx}}{\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{cx}}{\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{4}}}-1\right)\right)b\sqrt{\frac{a\,c^{2}}{b}}}{8}+}{}$ |
| derivativedivides | $\frac{\frac{(Bb-Da)(cx)^{\frac{3}{2}}}{2ab}+\frac{c(Ab-Ca)\sqrt{cx}}{2ab}}{b\,c^{2}x^{2}+a\,c^{2}}+\frac{\frac{(3Abc+Cac)\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{cx+\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{4}}\sqrt{cx}\,\sqrt{2}+\sqrt{\frac{a\,c^{2}}{b}}}{cx-\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{4}}\sqrt{cx}\,\sqrt{2}+\sqrt{\frac{a\,c^{2}}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{cx}}{\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{4}}}+1\right)}{8a\,c^{2}}}{}$ |
| default | $\frac{\frac{(Bb-Da)(cx)^{\frac{3}{2}}}{2ab}+\frac{c(Ab-Ca)\sqrt{cx}}{2ab}}{b\,c^{2}x^{2}+a\,c^{2}}+\frac{\frac{(3Abc+Cac)\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{cx+\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{4}}\sqrt{cx}\,\sqrt{2}+\sqrt{\frac{a\,c^{2}}{b}}}{cx-\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{4}}\sqrt{cx}\,\sqrt{2}+\sqrt{\frac{a\,c^{2}}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{cx}}{\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{4}}}+1\right)}{8a\,c^{2}}}{}$ |

input
```
int((D*x^3+C*x^2+B*x+A)/(c*x)^(1/2)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2/(a*c^2/b)^(1/4)*(3/8*(b*x^2+a)*(A*b+1/3*C*a)*2^(1/2)*(ln((c*x+(a*c^2/b
)^(1/4)*(c*x)^(1/2)*2^(1/2)+(a*c^2/b)^(1/2))/(c*x-(a*c^2/b)^(1/4)*(c*x)^(1
/2)*2^(1/2)+(a*c^2/b)^(1/2)))+2*arctan(2^(1/2)/(a*c^2/b)^(1/4)*(c*x)^(1/2)
+1)+2*arctan(2^(1/2)/(a*c^2/b)^(1/4)*(c*x)^(1/2)-1))*b*(a*c^2/b)^(1/2)+((a
*c^2/b)^(1/4)*((-D*x-C)*a+b*(B*x+A))*b*(c*x)^(1/2)+1/8*(b*x^2+a)*2^(1/2)*(
ln((c*x-(a*c^2/b)^(1/4)*(c*x)^(1/2)*2^(1/2)+(a*c^2/b)^(1/2))/(c*x+(a*c^2/b
)^(1/4)*(c*x)^(1/2)*2^(1/2)+(a*c^2/b)^(1/2)))+2*arctan(2^(1/2)/(a*c^2/b)^(
1/4)*(c*x)^(1/2)+1)+2*arctan(2^(1/2)/(a*c^2/b)^(1/4)*(c*x)^(1/2)-1))*c*(B*
b+3*D*a))*a)/c/a^2/b^2/(b*x^2+a)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3500 vs. $2(259) = 518$.

Time = 0.40 (sec) , antiderivative size = 3500, normalized size of antiderivative = 10.29

$$\int \frac{A + Bx + Cx^2 + Dx^3}{\sqrt{cx}\,(a + bx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(c*x)^(1/2)/(b*x^2+a)^2,x, algorithm="fricas
")
```

output
```
Too large to include
```

## Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 75.23 (sec) , antiderivative size = 4184, normalized size of antiderivative = 12.31

$$\int \frac{A + Bx + Cx^2 + Dx^3}{\sqrt{cx}\,(a + bx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(c*x)**(1/2)/(b*x**2+a)**2,x)
```

output

```
A*(-3*a**(5/4)*sqrt(x)*log(1 - b**(1/4)*sqrt(x)*exp_polar(I*pi/4)/a**(1/4)
)*gamma(1/4)/(32*a**3*b**(1/4)*sqrt(c)*sqrt(x)*exp(I*pi/4)*gamma(5/4) + 32
*a**2*b**(5/4)*sqrt(c)*x**(5/2)*exp(I*pi/4)*gamma(5/4)) + 3*I*a**(5/4)*sqr
t(x)*log(1 - b**(1/4)*sqrt(x)*exp_polar(3*I*pi/4)/a**(1/4))*gamma(1/4)/(32
*a**3*b**(1/4)*sqrt(c)*sqrt(x)*exp(I*pi/4)*gamma(5/4) + 32*a**2*b**(5/4)*s
qrt(c)*x**(5/2)*exp(I*pi/4)*gamma(5/4)) + 3*a**(5/4)*sqrt(x)*log(1 - b**(1
/4)*sqrt(x)*exp_polar(5*I*pi/4)/a**(1/4))*gamma(1/4)/(32*a**3*b**(1/4)*sqr
t(c)*sqrt(x)*exp(I*pi/4)*gamma(5/4) + 32*a**2*b**(5/4)*sqrt(c)*x**(5/2)*ex
p(I*pi/4)*gamma(5/4)) - 3*I*a**(5/4)*sqrt(x)*log(1 - b**(1/4)*sqrt(x)*exp_
polar(7*I*pi/4)/a**(1/4))*gamma(1/4)/(32*a**3*b**(1/4)*sqrt(c)*sqrt(x)*exp
(I*pi/4)*gamma(5/4) + 32*a**2*b**(5/4)*sqrt(c)*x**(5/2)*exp(I*pi/4)*gamma(
5/4)) - 3*a**(1/4)*b*x**(5/2)*log(1 - b**(1/4)*sqrt(x)*exp_polar(I*pi/4)/a
**(1/4))*gamma(1/4)/(32*a**3*b**(1/4)*sqrt(c)*sqrt(x)*exp(I*pi/4)*gamma(5/
4) + 32*a**2*b**(5/4)*sqrt(c)*x**(5/2)*exp(I*pi/4)*gamma(5/4)) + 3*I*a**(1
/4)*b*x**(5/2)*log(1 - b**(1/4)*sqrt(x)*exp_polar(3*I*pi/4)/a**(1/4))*gamm
a(1/4)/(32*a**3*b**(1/4)*sqrt(c)*sqrt(x)*exp(I*pi/4)*gamma(5/4) + 32*a**2*
b**(5/4)*sqrt(c)*x**(5/2)*exp(I*pi/4)*gamma(5/4)) + 3*a**(1/4)*b*x**(5/2)*
log(1 - b**(1/4)*sqrt(x)*exp_polar(5*I*pi/4)/a**(1/4))*gamma(1/4)/(32*a**3
*b**(1/4)*sqrt(c)*sqrt(x)*exp(I*pi/4)*gamma(5/4) + 32*a**2*b**(5/4)*sqrt(c
)*x**(5/2)*exp(I*pi/4)*gamma(5/4)) - 3*I*a**(1/4)*b*x**(5/2)*log(1 - b*...
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 414, normalized size of antiderivative = 1.22

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{cx}\,(a+bx^2)^2}\,dx =$$

$$-\frac{8\left((Da-Bb)(cx)^{\frac{3}{2}}c+(Ca-Ab)\sqrt{cx}c^2\right)}{ab^2c^2x^2+a^2bc^2}+\frac{c\left(\frac{\sqrt{2}\left((3\,Da+Bb)\sqrt{a}c-\left(Ca\sqrt{b}+3\,Ab^{\frac{3}{2}}\right)c\right)\log\left(\sqrt{b}cx+\sqrt{2}\left(ac^2\right)^{\frac{1}{4}}\sqrt{cx}b^{\frac{1}{4}}+\sqrt{a}c\right)}{\left(ac^2\right)^{\frac{3}{4}}b^{\frac{3}{4}}}-\frac{\sqrt{2}\left((3\,Da+Bb)\sqrt{a}c\cdots\right.}{\cdots}\right.}{\cdots}$$

input

```
integrate((D*x^3+C*x^2+B*x+A)/(c*x)^(1/2)/(b*x^2+a)^2,x, algorithm="maxima
")
```

output
```
-1/16*(8*((D*a - B*b)*(c*x)^(3/2)*c + (C*a - A*b)*sqrt(c*x)*c^2)/(a*b^2*c^
2*x^2 + a^2*b*c^2) + c*(sqrt(2)*((3*D*a + B*b)*sqrt(a)*c - (C*a*sqrt(b) +
3*A*b^(3/2))*c)*log(sqrt(b)*c*x + sqrt(2)*(a*c^2)^(1/4)*sqrt(c*x)*b^(1/4)
+ sqrt(a)*c)/((a*c^2)^(3/4)*b^(3/4)) - sqrt(2)*((3*D*a + B*b)*sqrt(a)*c -
(C*a*sqrt(b) + 3*A*b^(3/2))*c)*log(sqrt(b)*c*x - sqrt(2)*(a*c^2)^(1/4)*sqr
t(c*x)*b^(1/4) + sqrt(a)*c)/((a*c^2)^(3/4)*b^(3/4)) - 2*sqrt(2)*((3*D*a +
B*b)*sqrt(a)*c + (C*a*sqrt(b) + 3*A*b^(3/2))*c)*arctan(1/2*sqrt(2)*(sqrt(2
)*(a*c^2)^(1/4)*b^(1/4) + 2*sqrt(c*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*c))/(s
qrt(sqrt(a)*sqrt(b)*c)*sqrt(a)*sqrt(b)*c) - 2*sqrt(2)*((3*D*a + B*b)*sqrt(
a)*c + (C*a*sqrt(b) + 3*A*b^(3/2))*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*c^2)
^(1/4)*b^(1/4) - 2*sqrt(c*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*c))/(sqrt(sqrt(
a)*sqrt(b)*c)*sqrt(a)*sqrt(b)*c))/(a*b))/c
```

## Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 517, normalized size of antiderivative = 1.52

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{cx}\,(a+bx^2)^2}\,dx = -\frac{\sqrt{cx}Dacx-\sqrt{cx}Bbcx+\sqrt{cx}Cac-\sqrt{cx}Abc}{2\,(bc^2x^2+ac^2)ab}$$
$$+\frac{\sqrt{2}\Big((ab^3c^2)^{\frac{1}{4}}\,Cab^2c+3\,(ab^3c^2)^{\frac{1}{4}}\,Ab^3c+3\,(ab^3c^2)^{\frac{3}{4}}\,Da+(ab^3c^2)^{\frac{3}{4}}\,Bb\Big)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ac^2}{b}\right)^{\frac{1}{4}}+2\sqrt{cx}\right)}{2\left(\frac{ac^2}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^2b^4c^2}$$
$$+\frac{\sqrt{2}\Big((ab^3c^2)^{\frac{1}{4}}\,Cab^2c+3\,(ab^3c^2)^{\frac{1}{4}}\,Ab^3c+3\,(ab^3c^2)^{\frac{3}{4}}\,Da+(ab^3c^2)^{\frac{3}{4}}\,Bb\Big)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ac^2}{b}\right)^{\frac{1}{4}}-2\sqrt{cx}\right)}{2\left(\frac{ac^2}{b}\right)^{\frac{1}{4}}}\right.}{8\,a^2b^4c^2}$$
$$+\frac{\sqrt{2}\Big((ab^3c^2)^{\frac{1}{4}}\,Cab^2c+3\,(ab^3c^2)^{\frac{1}{4}}\,Ab^3c-3\,(ab^3c^2)^{\frac{3}{4}}\,Da-(ab^3c^2)^{\frac{3}{4}}\,Bb\Big)\log\left(cx+\sqrt{2}\left(\frac{ac^2}{b}\right)^{\frac{1}{4}}\sqrt{cx}+\sqrt{\right.}{16\,a^2b^4c^2}$$
$$-\frac{\sqrt{2}\Big((ab^3c^2)^{\frac{1}{4}}\,Cab^2c+3\,(ab^3c^2)^{\frac{1}{4}}\,Ab^3c-3\,(ab^3c^2)^{\frac{3}{4}}\,Da-(ab^3c^2)^{\frac{3}{4}}\,Bb\Big)\log\left(cx-\sqrt{2}\left(\frac{ac^2}{b}\right)^{\frac{1}{4}}\sqrt{cx}+\sqrt{\right.}{16\,a^2b^4c^2}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(c*x)^(1/2)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
-1/2*(sqrt(c*x)*D*a*c*x - sqrt(c*x)*B*b*c*x + sqrt(c*x)*C*a*c - sqrt(c*x)*
A*b*c)/((b*c^2*x^2 + a*c^2)*a*b) + 1/8*sqrt(2)*((a*b^3*c^2)^(1/4)*C*a*b^2*
c + 3*(a*b^3*c^2)^(1/4)*A*b^3*c + 3*(a*b^3*c^2)^(3/4)*D*a + (a*b^3*c^2)^(3
/4)*B*b)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*c^2/b)^(1/4) + 2*sqrt(c*x))/(a*c^2
/b)^(1/4))/(a^2*b^4*c^2) + 1/8*sqrt(2)*((a*b^3*c^2)^(1/4)*C*a*b^2*c + 3*(a
*b^3*c^2)^(1/4)*A*b^3*c + 3*(a*b^3*c^2)^(3/4)*D*a + (a*b^3*c^2)^(3/4)*B*b)
*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*c^2/b)^(1/4) - 2*sqrt(c*x))/(a*c^2/b)^(1/
4))/(a^2*b^4*c^2) + 1/16*sqrt(2)*((a*b^3*c^2)^(1/4)*C*a*b^2*c + 3*(a*b^3*c
^2)^(1/4)*A*b^3*c - 3*(a*b^3*c^2)^(3/4)*D*a - (a*b^3*c^2)^(3/4)*B*b)*log(c
*x + sqrt(2)*(a*c^2/b)^(1/4)*sqrt(c*x) + sqrt(a*c^2/b))/(a^2*b^4*c^2) - 1/
16*sqrt(2)*((a*b^3*c^2)^(1/4)*C*a*b^2*c + 3*(a*b^3*c^2)^(1/4)*A*b^3*c - 3*
(a*b^3*c^2)^(3/4)*D*a - (a*b^3*c^2)^(3/4)*B*b)*log(c*x - sqrt(2)*(a*c^2/b)
^(1/4)*sqrt(c*x) + sqrt(a*c^2/b))/(a^2*b^4*c^2)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{cx}\,(a+bx^2)^2}\,dx=\int \frac{A+B\,x+C\,x^2+x^3\,D}{\sqrt{c\,x}\,(b\,x^2+a)^2}\,dx$$

input
```
int((A + B*x + C*x^2 + x^3*D)/((c*x)^(1/2)*(a + b*x^2)^2),x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/((c*x)^(1/2)*(a + b*x^2)^2), x)
```

## Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 1201, normalized size of antiderivative = 3.53

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{cx}\,(a+bx^2)^2}\,dx=\text{Too large to display}$$

input
```
int((D*x^3+C*x^2+B*x+A)/(c*x)^(1/2)/(b*x^2+a)^2,x)
```

output

```
(sqrt(c)*( - 6*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) -
 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2*d - 2*b**(1/4)*a**(3
/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)
*a**(1/4)*sqrt(2)))*a*b**2 - 6*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a*
*(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b*d*x**
2 - 2*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x
)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*b**3*x**2 - 6*b**(3/4)*a**(1/4)*sq
rt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1
/4)*sqrt(2)))*a**2*b - 2*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)
*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2*c - 6*b**(
3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))
/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b**2*x**2 - 2*b**(3/4)*a**(1/4)*sqrt(2)*at
an((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt
(2)))*a*b*c*x**2 + 6*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqr
t(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2*d + 2*b**(1/4)
*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b*
*(1/4)*a**(1/4)*sqrt(2)))*a*b**2 + 6*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1
/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b
*d*x**2 + 2*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*
sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*b**3*x**2 + 6*b**(3/4)*a*...
```

# 3.133 $\int \frac{A+Bx+Cx^2+Dx^3}{(cx)^{3/2}(a+bx^2)^2}\,dx$



## Optimal result

Integrand size = 32, antiderivative size = 350

$$\begin{aligned}\int \frac{A+Bx+Cx^2+Dx^3}{(cx)^{3/2}(a+bx^2)^2}\,dx &= -\frac{2A}{ac\sqrt{cx}\,(a+bx^2)} \\ &+ \frac{\sqrt{cx}(a(bB-aD)-b(5Ab-aC)x)}{2a^2bc^2\,(a+bx^2)} \\ &+ \frac{\left(\sqrt{b}(5Ab-aC)-\sqrt{a}(3bB+aD)\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)}{4\sqrt{2}a^{9/4}b^{5/4}c^{3/2}} \\ &- \frac{\left(\sqrt{b}(5Ab-aC)-\sqrt{a}(3bB+aD)\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)}{4\sqrt{2}a^{9/4}b^{5/4}c^{3/2}} \\ &+ \frac{\left(\sqrt{b}(5Ab-aC)+\sqrt{a}(3bB+aD)\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\left(\sqrt{a}+\sqrt{b}x\right)}\right)}{4\sqrt{2}a^{9/4}b^{5/4}c^{3/2}}\end{aligned}$$

output
```
-2*A/a/c/(c*x)^(1/2)/(b*x^2+a)+1/2*(c*x)^(1/2)*(a*(B*b-D*a)-b*(5*A*b-C*a)*
x)/a^2/b/c^2/(b*x^2+a)+1/8*(b^(1/2)*(5*A*b-C*a)-a^(1/2)*(3*B*b+D*a))*arcta
n(1-2^(1/2)*b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))*2^(1/2)/a^(9/4)/b^(5/4)/c
^(3/2)-1/8*(b^(1/2)*(5*A*b-C*a)-a^(1/2)*(3*B*b+D*a))*arctan(1+2^(1/2)*b^(1
/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))*2^(1/2)/a^(9/4)/b^(5/4)/c^(3/2)+1/8*(b^(1
/2)*(5*A*b-C*a)+a^(1/2)*(3*B*b+D*a))*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*(c*x)
^(1/2)/c^(1/2)/(a^(1/2)+b^(1/2)*x))*2^(1/2)/a^(9/4)/b^(5/4)/c^(3/2)
```

## Mathematica [A] (verified)

Time = 1.42 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.68

$$\int \frac{A+Bx+Cx^2+Dx^3}{(cx)^{3/2}\left(a+bx^2\right)^2}\,dx = \frac{x\left(-\frac{4\sqrt[4]{a}\sqrt[4]{b}\left(a^2Dx+5Ab^2x^2+ab(4A-x(B+Cx))\right)}{a+bx^2}-\sqrt{2}\left(-5Ab^{3/2}+3\sqrt{a}bB+a\sqrt{b}C\right.\right.}{}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/((c*x)^(3/2)*(a + b*x^2)^2),x]
```

output
```
(x*((-4*a^(1/4)*b^(1/4)*(a^2*D*x + 5*A*b^2*x^2 + a*b*(4*A - x*(B + C*x))))
/(a + b*x^2) - Sqrt[2]*(-5*A*b^(3/2) + 3*Sqrt[a]*b*B + a*Sqrt[b]*C + a^(3/
2)*D)*Sqrt[x]*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x
])] + Sqrt[2]*(5*A*b^(3/2) + 3*Sqrt[a]*b*B - a*Sqrt[b]*C + a^(3/2)*D)*Sqrt
[x]*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)]))/(8*
a^(9/4)*b^(5/4)*(c*x)^(3/2))
```

## Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 413, normalized size of antiderivative = 1.18, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {2337, 27, 553, 27, 554, 1482, 27, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(cx)^{3/2}\left(a+bx^2\right)^2}dx$$

$$\Big\downarrow \text{2337}$$

$$\frac{x\left(B-\frac{aD}{b}\right)-\frac{aC}{b}+A}{2ac\sqrt{cx}\left(a+bx^2\right)}-\frac{\int -\frac{b\left(5A-\frac{aC}{b}\right)+(3bB+aD)x}{2b(cx)^{3/2}\left(bx^2+a\right)}dx}{2a}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{5Ab-aC+(3bB+aD)x}{(cx)^{3/2}\left(bx^2+a\right)}dx}{4ab}+\frac{x\left(B-\frac{aD}{b}\right)-\frac{aC}{b}+A}{2ac\sqrt{cx}\left(a+bx^2\right)}$$

$$\Big\downarrow \text{553}$$

$$\frac{-\frac{2\int -\frac{a(3bB+aD)-b(5Ab-aC)x}{2\sqrt{cx}\left(bx^2+a\right)}dx}{ac}-\frac{2(5Ab-aC)}{ac\sqrt{cx}}}{4ab}+\frac{x\left(B-\frac{aD}{b}\right)-\frac{aC}{b}+A}{2ac\sqrt{cx}\left(a+bx^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{a(3bB+aD)-b(5Ab-aC)x}{\sqrt{cx}\left(bx^2+a\right)}dx}{ac}-\frac{2(5Ab-aC)}{ac\sqrt{cx}}}{4ab}+\frac{x\left(B-\frac{aD}{b}\right)-\frac{aC}{b}+A}{2ac\sqrt{cx}\left(a+bx^2\right)}$$

$$\Big\downarrow \text{554}$$

$$\frac{\frac{2\int \frac{ac(3bB+aD)-bc(5Ab-aC)x}{bx^2c^2+ac^2}d\sqrt{cx}}{ac}-\frac{2(5Ab-aC)}{ac\sqrt{cx}}}{4ab}+\frac{x\left(B-\frac{aD}{b}\right)-\frac{aC}{b}+A}{2ac\sqrt{cx}\left(a+bx^2\right)}$$

$$\Big\downarrow \text{1482}$$

$$\frac{\frac{2\left(\frac{1}{2}\left(\frac{\sqrt{a}(aD+3bB)}{\sqrt{b}}-aC+5Ab\right)\int \frac{\sqrt{b}\left(\sqrt{a}c-\sqrt{b}cx\right)}{bx^2c^2+ac^2}d\sqrt{cx}-\frac{1}{2}\left(-\frac{\sqrt{a}(aD+3bB)}{\sqrt{b}}-aC+5Ab\right)\int \frac{\sqrt{b}\left(\sqrt{b}xc+\sqrt{a}c\right)}{bx^2c^2+ac^2}d\sqrt{cx}\right)}{ac}-\frac{2(5Ab-aC)}{ac\sqrt{cx}}}{4ab}+\frac{x\left(B-\frac{aD}{b}\right)-\frac{aC}{b}+A}{2ac\sqrt{cx}\left(a+bx^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{2\left(\frac{1}{2}\sqrt{b}\left(\frac{\sqrt{a}(aD+3bB)}{\sqrt{b}}-aC+5Ab\right)\int \frac{\sqrt{a}c-\sqrt{b}cx}{bx^2c^2+ac^2}d\sqrt{cx}-\frac{1}{2}\sqrt{b}\left(-\frac{\sqrt{a}(aD+3bB)}{\sqrt{b}}-aC+5Ab\right)\int \frac{\sqrt{b}xc+\sqrt{a}c}{bx^2c^2+ac^2}d\sqrt{cx}\right)}{ac}-\frac{2(5Ab-aC)}{ac\sqrt{cx}}}{4ab}+\frac{x\left(B-\frac{aD}{b}\right)-\frac{aC}{b}+A}{2ac\sqrt{cx}\left(a+bx^2\right)}$$

$\Big\downarrow$ 1476

$$\frac{2\left(\frac{\frac{1}{2}\sqrt{b}\left(\frac{\sqrt{a}(aD+3bB)}{\sqrt{b}}-aC+5Ab\right)\int\frac{\sqrt{a}c-\sqrt{b}cx}{bx^2c^2+ac^2}d\sqrt{cx}-\frac{1}{2}\sqrt{b}\left(-\frac{\sqrt{a}(aD+3bB)}{\sqrt{b}}-aC+5Ab\right)\left(\frac{\int\frac{1}{xc+\frac{\sqrt{a}c}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{cx}\sqrt{c}}{\sqrt[4]{b}}}d\sqrt{cx}}{2\sqrt{b}}+\frac{\int\frac{1}{xc+\frac{\sqrt{a}c}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{\cdots}}{\sqrt[4]{b}}}\cdots}{2\sqrt{b}}\cdots}{ac}\right.}{4ab}+\frac{x\left(B-\frac{aD}{b}\right)-\frac{aC}{b}+A}{2ac\sqrt{cx}\,(a+bx^2)}$$

$\Big\downarrow$ 1082

$$\frac{2\left(\frac{\frac{1}{2}\sqrt{b}\left(\frac{\sqrt{a}(aD+3bB)}{\sqrt{b}}-aC+5Ab\right)\int\frac{\sqrt{a}c-\sqrt{b}cx}{bx^2c^2+ac^2}d\sqrt{cx}-\frac{1}{2}\sqrt{b}\left(-\frac{\sqrt{a}(aD+3bB)}{\sqrt{b}}-aC+5Ab\right)\left(\frac{\int\frac{1}{-cx-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}}-\frac{\int\frac{1}{-cx-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}+\cdots\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}}\cdots}{ac}\right.}{4ab}+\frac{x\left(B-\frac{aD}{b}\right)-\frac{aC}{b}+A}{2ac\sqrt{cx}\,(a+bx^2)}$$

$\Big\downarrow$ 217

$$\frac{2\left(\frac{\frac{1}{2}\sqrt{b}\left(\frac{\sqrt{a}(aD+3bB)}{\sqrt{b}}-aC+5Ab\right)\int\frac{\sqrt{a}c-\sqrt{b}cx}{bx^2c^2+ac^2}d\sqrt{cx}-\frac{1}{2}\sqrt{b}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}}\right)\left(-\frac{\sqrt{a}(aD+3bB)}{\sqrt{b}}-aC+5Ab\right)}{ac}\right)\cdots}{4ab}+\frac{x\left(B-\frac{aD}{b}\right)-\frac{aC}{b}+A}{2ac\sqrt{cx}\,(a+bx^2)}$$

$\Big\downarrow$ 1479

$$\frac{2\left(\frac{\frac{1}{2}\sqrt{b}\left(\frac{\sqrt{a}(aD+3bB)}{\sqrt{b}}-aC+5Ab\right)\left(-\frac{\int-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{c}-2\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{b}\left(xc+\frac{\sqrt{a}c}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{cx}\sqrt{c}}{\sqrt[4]{b}}\right)}d\sqrt{cx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{c}+\sqrt{2}\sqrt[4]{b}\sqrt{cx}\right)}{\sqrt[4]{b}\left(xc+\frac{\sqrt{a}c}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{cx}\sqrt{c}}{\sqrt[4]{b}}\right)}d\sqrt{cx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}}\right)-\frac{1}{2}\sqrt{b}\left(\frac{\arctan\left(\frac{\sqrt{2}\cdots}{\cdots}\right)}{\sqrt{2}\sqrt[4]{a}\cdots}\cdots\right.}{ac}\cdots}{4ab}+\frac{x\left(B-\frac{aD}{b}\right)-\frac{aC}{b}+A}{2ac\sqrt{cx}\,(a+bx^2)}$$

$\big\downarrow$ 25

$$\frac{2\left(\frac{\frac{1}{2}\sqrt{b}\left(\frac{\sqrt{a}(aD+3bB)}{\sqrt{b}}-aC+5Ab\right)\left(\frac{\int\frac{\sqrt{2}\sqrt[4]{a}\sqrt{c}-2\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{b}\left(xc+\frac{\sqrt{a}c}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{cx}\sqrt{c}}{\sqrt[4]{b}}\right)}d\sqrt{cx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}}+\frac{\int\frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{c}+\sqrt{2}\sqrt[4]{b}\sqrt{cx}\right)}{\sqrt[4]{b}\left(xc+\frac{\sqrt{a}c}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{cx}\sqrt{c}}{\sqrt[4]{b}}\right)}d\sqrt{cx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}}\right)-\frac{1}{2}\sqrt{b}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{\ldots}}{\sqrt[4]{a}\sqrt{\ldots}}\right.}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\ldots}\right.}{ac}\right.}{4ab}$$

$$\frac{x\left(B-\frac{aD}{b}\right)-\frac{aC}{b}+A}{2ac\sqrt{cx}\,(a+bx^2)}$$

$\big\downarrow$ 27

$$\frac{2\left(\frac{\frac{1}{2}\sqrt{b}\left(\frac{\sqrt{a}(aD+3bB)}{\sqrt{b}}-aC+5Ab\right)\left(\frac{\int\frac{\sqrt{2}\sqrt[4]{a}\sqrt{c}-2\sqrt[4]{b}\sqrt{cx}}{xc+\frac{\sqrt{a}c}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{cx}\sqrt{c}}{\sqrt[4]{b}}}d\sqrt{cx}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}\sqrt{c}}+\frac{\int\frac{\sqrt[4]{a}\sqrt{c}+\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{xc+\frac{\sqrt{a}c}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{cx}\sqrt{c}}{\sqrt[4]{b}}}d\sqrt{cx}}{2\sqrt[4]{a}\sqrt{b}\sqrt{c}}\right)-\frac{1}{2}\sqrt{b}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}}-\frac{\arctan\ldots}{\sqrt{\ldots}}\right.}{ac}\right.}{4ab}$$

$$\frac{x\left(B-\frac{aD}{b}\right)-\frac{aC}{b}+A}{2ac\sqrt{cx}\,(a+bx^2)}$$

$\big\downarrow$ 1103

$$\frac{2\left(\frac{\frac{1}{2}\sqrt{b}\left(\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}\sqrt{cx}+\sqrt{a}c+\sqrt{b}cx\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}\sqrt{cx}+\sqrt{a}c+\sqrt{b}cx\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}}\right)\left(\frac{\sqrt{a}(aD+3bB)}{\sqrt{b}}-aC+5Ab\right)-\frac{1}{2}\sqrt{b}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}}\ldots\right.}{ac}\right.}{4ab}$$

$$\frac{x\left(B-\frac{aD}{b}\right)-\frac{aC}{b}+A}{2ac\sqrt{cx}\,(a+bx^2)}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/((c*x)^(3/2)*(a + b*x^2)^2),x]
```

output
```
(A - (a*C)/b + (B - (a*D)/b)*x)/(2*a*c*Sqrt[c*x]*(a + b*x^2)) + ((-2*(5*A*
b - a*C))/(a*c*Sqrt[c*x]) + (2*(-1/2*(Sqrt[b]*(5*A*b - a*C - (Sqrt[a]*(3*b
*B + a*D))/Sqrt[b])*(-(ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqr
t[c])]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[c])) + ArcTan[1 + (Sqrt[2]*b^(1/4)*Sq
rt[c*x])/(a^(1/4)*Sqrt[c])]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[c]))) + (Sqrt[b]
*(5*A*b - a*C + (Sqrt[a]*(3*b*B + a*D))/Sqrt[b])*(-1/2*Log[Sqrt[a]*c + Sqr
t[b]*c*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[c]*Sqrt[c*x]]/(Sqrt[2]*a^(1/4)*b^(
1/4)*Sqrt[c]) + Log[Sqrt[a]*c + Sqrt[b]*c*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt
[c]*Sqrt[c*x]]/(2*Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[c])))/2))/(a*c))/(4*a*b)
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 553
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Sym
bol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] + Simp
[1/(a*e*(m + 1))   Int[(e*x)^(m + 1)*(a + b*x^2)^p*(a*d*(m + 1) - b*c*(m +
2*p + 3)*x), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && LtQ[m, -1]
```

rule 554
```
Int[((c_) + (d_.)*(x_))/(Sqrt[(e_.)*(x_)]*((a_) + (b_.)*(x_)^2)), x_Symbol]
 :> Simp[2   Subst[Int[(e*c + d*x^2)/(a*e^2 + b*x^4), x], x, Sqrt[e*x]], x]
 /; FreeQ[{a, b, c, d, e}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 2337
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(-(c*x)^(m + 1))*(f + g*x)*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1)))
, x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*ExpandToSum[2
*a*(p + 1)*Q + f*(m + 2*p + 3) + g*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a
, b, c, m}, x] && PolyQ[Pq, x] && LtQ[p, -1] &&  !GtQ[m, 0]
```

## Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.11

| method | result |
|---|---|
| derivativedivides | $-\frac{2\left(\frac{\left(\frac{Ab}{4}-\frac{Ca}{4}\right)(cx)^{\frac{3}{2}}-\frac{ca(Bb-Da)\sqrt{cx}}{4b}}{b c^2 x^2+a c^2}+\frac{\frac{\left(-3abBc-Da^2c\right)\left(\frac{ac^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{cx+\left(\frac{ac^2}{b}\right)^{\frac{1}{4}}\sqrt{cx}\sqrt{2}+\sqrt{\frac{ac^2}{b}}}{cx-\left(\frac{ac^2}{b}\right)^{\frac{1}{4}}\sqrt{cx}\sqrt{2}+\sqrt{\frac{ac^2}{b}}}\right)+2\arctan\left(\frac{\sqrt{\cdots}}{\left(\cdots\right)}\right)\cdots\right)}{8ac^2}}{\cdots}\right)}{\cdots}$ |
| default | $-\frac{2\left(\frac{\left(\frac{Ab}{4}-\frac{Ca}{4}\right)(cx)^{\frac{3}{2}}-\frac{ca(Bb-Da)\sqrt{cx}}{4b}}{b c^2 x^2+a c^2}+\frac{\frac{\left(-3abBc-Da^2c\right)\left(\frac{ac^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{cx+\left(\frac{ac^2}{b}\right)^{\frac{1}{4}}\sqrt{cx}\sqrt{2}+\sqrt{\frac{ac^2}{b}}}{cx-\left(\frac{ac^2}{b}\right)^{\frac{1}{4}}\sqrt{cx}\sqrt{2}+\sqrt{\frac{ac^2}{b}}}\right)+2\arctan\left(\frac{\sqrt{\cdots}}{\left(\cdots\right)}\right)\cdots\right)}{8ac^2}}{\cdots}\right)}{\cdots}$ |
| pseudoelliptic | $-\frac{5\left(-\frac{3\left(bx^2+a\right)\sqrt{2}\left(Bb+\frac{Da}{3}\right)\left(\arctan\left(\frac{\sqrt{2}\sqrt{cx}-\left(\frac{ac^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{ac^2}{b}\right)^{\frac{1}{4}}}\right)+\arctan\left(\frac{\sqrt{2}\sqrt{cx}+\left(\frac{ac^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{ac^2}{b}\right)^{\frac{1}{4}}}\right)+\frac{\ln\left(\frac{cx+\left(\frac{ac^2}{b}\right)^{\frac{1}{4}}\sqrt{cx}\sqrt{2}+\sqrt{\cdots}}{cx-\left(\frac{ac^2}{b}\right)^{\frac{1}{4}}\sqrt{cx}\sqrt{2}+\sqrt{\cdots}}\right)}{2}\cdots\right)}{5}\cdots\right)}{\cdots}$ |

input
```
int((D*x^3+C*x^2+B*x+A)/(c*x)^(3/2)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-2/c/a^2*(((1/4*A*b-1/4*C*a)*(c*x)^(3/2)-1/4*c*a*(B*b-D*a)/b*(c*x)^(1/2))/
(b*c^2*x^2+a*c^2)+1/4/b*(1/8*(-3*B*a*b*c-D*a^2*c)*(a*c^2/b)^(1/4)/a/c^2*2^
(1/2)*(ln((c*x+(a*c^2/b)^(1/4)*(c*x)^(1/2)*2^(1/2)+(a*c^2/b)^(1/2))/(c*x-(
a*c^2/b)^(1/4)*(c*x)^(1/2)*2^(1/2)+(a*c^2/b)^(1/2)))+2*arctan(2^(1/2)/(a*c
^2/b)^(1/4)*(c*x)^(1/2)+1)+2*arctan(2^(1/2)/(a*c^2/b)^(1/4)*(c*x)^(1/2)-1)
)+1/8*(5*A*b^2-C*a*b)/b/(a*c^2/b)^(1/4)*2^(1/2)*(ln((c*x-(a*c^2/b)^(1/4)*(
c*x)^(1/2)*2^(1/2)+(a*c^2/b)^(1/2))/(c*x+(a*c^2/b)^(1/4)*(c*x)^(1/2)*2^(1/
2)+(a*c^2/b)^(1/2)))+2*arctan(2^(1/2)/(a*c^2/b)^(1/4)*(c*x)^(1/2)+1)+2*arc
tan(2^(1/2)/(a*c^2/b)^(1/4)*(c*x)^(1/2)-1))))-2*A/c/a^2/(c*x)^(1/2)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3700 vs. $2(263) = 526$.

Time = 0.51 (sec) , antiderivative size = 3700, normalized size of antiderivative = 10.57

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(cx)^{3/2} (a + bx^2)^2} dx = \text{Too large to display}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(c*x)^(3/2)/(b*x^2+a)^2,x, algorithm="fricas
")
```

output
```
Too large to include
```

## Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 82.54 (sec) , antiderivative size = 4014, normalized size of antiderivative = 11.47

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(cx)^{3/2} (a + bx^2)^2} dx = \text{Too large to display}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(c*x)**(3/2)/(b*x**2+a)**2,x)
```

output
```
A*(16*a**(5/4)*exp(3*I*pi/4)*gamma(-1/4)/(32*a**(13/4)*c**(3/2)*sqrt(x)*ex
p(3*I*pi/4)*gamma(3/4) + 32*a**(9/4)*b*c**(3/2)*x**(5/2)*exp(3*I*pi/4)*gam
ma(3/4)) + 20*a**(1/4)*b*x**2*exp(3*I*pi/4)*gamma(-1/4)/(32*a**(13/4)*c**(
3/2)*sqrt(x)*exp(3*I*pi/4)*gamma(3/4) + 32*a**(9/4)*b*c**(3/2)*x**(5/2)*ex
p(3*I*pi/4)*gamma(3/4)) - 5*a*b**(1/4)*sqrt(x)*log(1 - b**(1/4)*sqrt(x)*ex
p_polar(I*pi/4)/a**(1/4))*gamma(-1/4)/(32*a**(13/4)*c**(3/2)*sqrt(x)*exp(3
*I*pi/4)*gamma(3/4) + 32*a**(9/4)*b*c**(3/2)*x**(5/2)*exp(3*I*pi/4)*gamma(
3/4)) - 5*I*a*b**(1/4)*sqrt(x)*log(1 - b**(1/4)*sqrt(x)*exp_polar(3*I*pi/4
)/a**(1/4))*gamma(-1/4)/(32*a**(13/4)*c**(3/2)*sqrt(x)*exp(3*I*pi/4)*gamma
(3/4) + 32*a**(9/4)*b*c**(3/2)*x**(5/2)*exp(3*I*pi/4)*gamma(3/4)) + 5*a*b*
*(1/4)*sqrt(x)*log(1 - b**(1/4)*sqrt(x)*exp_polar(5*I*pi/4)/a**(1/4))*gamm
a(-1/4)/(32*a**(13/4)*c**(3/2)*sqrt(x)*exp(3*I*pi/4)*gamma(3/4) + 32*a**(9
/4)*b*c**(3/2)*x**(5/2)*exp(3*I*pi/4)*gamma(3/4)) + 5*I*a*b**(1/4)*sqrt(x)
*log(1 - b**(1/4)*sqrt(x)*exp_polar(7*I*pi/4)/a**(1/4))*gamma(-1/4)/(32*a*
*(13/4)*c**(3/2)*sqrt(x)*exp(3*I*pi/4)*gamma(3/4) + 32*a**(9/4)*b*c**(3/2)
*x**(5/2)*exp(3*I*pi/4)*gamma(3/4)) - 5*b**(5/4)*x**(5/2)*log(1 - b**(1/4)
*sqrt(x)*exp_polar(I*pi/4)/a**(1/4))*gamma(-1/4)/(32*a**(13/4)*c**(3/2)*sq
rt(x)*exp(3*I*pi/4)*gamma(3/4) + 32*a**(9/4)*b*c**(3/2)*x**(5/2)*exp(3*I*p
i/4)*gamma(3/4)) - 5*I*b**(5/4)*x**(5/2)*log(1 - b**(1/4)*sqrt(x)*exp_pola
r(3*I*pi/4)/a**(1/4))*gamma(-1/4)/(32*a**(13/4)*c**(3/2)*sqrt(x)*exp(3*...
```

## Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 454, normalized size of antiderivative = 1.30

$$\int \frac{A+Bx+Cx^2+Dx^3}{(cx)^{3/2}\left(a+bx^2\right)^2}\,dx =$$

$$-\frac{\frac{8\left(4\,Aabc^2-\left(Cab-5\,Ab^2\right)c^2x^2+\left(Da^2-Bab\right)c^2x\right)}{(cx)^{\frac{5}{2}}a^2b^2+\sqrt{cx}a^3bc^2}+\frac{\frac{\sqrt{2}\left(\left(Cab-5\,Ab^2\right)\sqrt{a}c-\left(Da^2\sqrt{b}+3\,Bab^{\frac{3}{2}}\right)c\right)\log\left(\sqrt{b}cx+\sqrt{2}\left(ac^2\right)^{\frac{1}{4}}\sqrt{cx}b^{\frac{1}{4}}+\sqrt{a}c\right)}{\left(ac^2\right)^{\frac{3}{4}}b^{\frac{3}{4}}}-\frac{\sqrt{2}\left(\left(Cab-\right.\right.}{}}{}}{}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(c*x)^(3/2)/(b*x^2+a)^2,x, algorithm="maxima
")
```

output
```
-1/16*(8*(4*A*a*b*c^2 - (C*a*b - 5*A*b^2)*c^2*x^2 + (D*a^2 - B*a*b)*c^2*x)
/((c*x)^(5/2)*a^2*b^2 + sqrt(c*x)*a^3*b*c^2) + (sqrt(2)*((C*a*b - 5*A*b^2)
*sqrt(a)*c - (D*a^2*sqrt(b) + 3*B*a*b^(3/2))*c)*log(sqrt(b)*c*x + sqrt(2)*
(a*c^2)^(1/4)*sqrt(c*x)*b^(1/4) + sqrt(a)*c)/((a*c^2)^(3/4)*b^(3/4)) - sqr
t(2)*((C*a*b - 5*A*b^2)*sqrt(a)*c - (D*a^2*sqrt(b) + 3*B*a*b^(3/2))*c)*log
(sqrt(b)*c*x - sqrt(2)*(a*c^2)^(1/4)*sqrt(c*x)*b^(1/4) + sqrt(a)*c)/((a*c^
2)^(3/4)*b^(3/4)) - 2*sqrt(2)*((C*a*b - 5*A*b^2)*sqrt(a)*c + (D*a^2*sqrt(b
) + 3*B*a*b^(3/2))*c)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*c^2)^(1/4)*b^(1/4) +
2*sqrt(c*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*c))/(sqrt(sqrt(a)*sqrt(b)*c)*sqr
t(a)*sqrt(b)*c) - 2*sqrt(2)*((C*a*b - 5*A*b^2)*sqrt(a)*c + (D*a^2*sqrt(b)
+ 3*B*a*b^(3/2))*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*c^2)^(1/4)*b^(1/4) - 2
*sqrt(c*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*c))/(sqrt(sqrt(a)*sqrt(b)*c)*sqrt
(a)*sqrt(b)*c))/(a^2*b))/c
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 543 vs. $2(263) = 526$.

Time = 0.14 (sec) , antiderivative size = 543, normalized size of antiderivative = 1.55

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(cx)^{3/2}\left(a + bx^2\right)^2}\,dx = \frac{\frac{8\left(Cabc^2x^2 - 5\,Ab^2c^2x^2 - Da^2c^2x + Babc^2x - 4\,Aabc^2\right)}{\left(\sqrt{cx}bc^2x^2 + \sqrt{cx}ac^2\right)a^2b} + \frac{2\sqrt{2}\left(\left(ab^3c^2\right)^{\frac{1}{4}}Da^2bc + 3\left(ab^3c^2\right)^{\frac{1}{4}}Bab^2c + (\right.}{}}{}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(c*x)^(3/2)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/16*(8*(C*a*b*c^2*x^2 - 5*A*b^2*c^2*x^2 - D*a^2*c^2*x + B*a*b*c^2*x - 4*A
*a*b*c^2)/((sqrt(c*x)*b*c^2*x^2 + sqrt(c*x)*a*c^2)*a^2*b) + 2*sqrt(2)*((a*
b^3*c^2)^(1/4)*D*a^2*b*c + 3*(a*b^3*c^2)^(1/4)*B*a*b^2*c + (a*b^3*c^2)^(3/
4)*C*a - 5*(a*b^3*c^2)^(3/4)*A*b)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*c^2/b)^(1
/4) + 2*sqrt(c*x))/(a*c^2/b)^(1/4))/(a^3*b^3*c^2) + 2*sqrt(2)*((a*b^3*c^2)
^(1/4)*D*a^2*b*c + 3*(a*b^3*c^2)^(1/4)*B*a*b^2*c + (a*b^3*c^2)^(3/4)*C*a -
 5*(a*b^3*c^2)^(3/4)*A*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*c^2/b)^(1/4) - 2
*sqrt(c*x))/(a*c^2/b)^(1/4))/(a^3*b^3*c^2) + sqrt(2)*((a*b^3*c^2)^(1/4)*D*
a^2*b*c + 3*(a*b^3*c^2)^(1/4)*B*a*b^2*c - (a*b^3*c^2)^(3/4)*C*a + 5*(a*b^3
*c^2)^(3/4)*A*b)*log(c*x + sqrt(2)*(a*c^2/b)^(1/4)*sqrt(c*x) + sqrt(a*c^2/
b))/(a^3*b^3*c^2) - sqrt(2)*((a*b^3*c^2)^(1/4)*D*a^2*b*c + 3*(a*b^3*c^2)^(
1/4)*B*a*b^2*c - (a*b^3*c^2)^(3/4)*C*a + 5*(a*b^3*c^2)^(3/4)*A*b)*log(c*x
- sqrt(2)*(a*c^2/b)^(1/4)*sqrt(c*x) + sqrt(a*c^2/b))/(a^3*b^3*c^2))/c
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(cx)^{3/2}\left(a+bx^2\right)^2}\,dx=\int \frac{A+B\,x+C\,x^2+x^3\,D}{\left(c\,x\right)^{3/2}\,\left(b\,x^2+a\right)^2}\,dx$$

input
```
int((A + B*x + C*x^2 + x^3*D)/((c*x)^(3/2)*(a + b*x^2)^2),x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/((c*x)^(3/2)*(a + b*x^2)^2), x)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 1273, normalized size of antiderivative = 3.64

$$\int \frac{A+Bx+Cx^2+Dx^3}{(cx)^{3/2}\left(a+bx^2\right)^2}\,dx=\text{Too large to display}$$

input
```
int((D*x^3+C*x^2+B*x+A)/(c*x)^(3/2)/(b*x^2+a)^2,x)
```

output
```
(sqrt(c)*(10*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqr
t(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b**2 - 2*sqrt(x)*
b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt
(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b*c + 10*sqrt(x)*b**(1/4)*a**(3/4)*sqr
t(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/
4)*sqrt(2)))*b**3*x**2 - 2*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4
)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*b**2*
c*x**2 - 2*sqrt(x)*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(
2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2*d - 6*sqrt(x)*b*
*(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b
))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b**2 - 2*sqrt(x)*b**(3/4)*a**(1/4)*sqrt(
2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)
*sqrt(2)))*a*b*d*x**2 - 6*sqrt(x)*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)
*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*b**3*x
**2 - 10*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2)
 + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b**2 + 2*sqrt(x)*b**(
1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))
/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b*c - 10*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)
*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*s
qrt(2)))*b**3*x**2 + 2*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)...
```

# 3.134 $\int \frac{A+Bx+Cx^2+Dx^3}{(cx)^{5/2}\left(a+bx^2\right)^2}\,dx$



## Optimal result

Integrand size = 32, antiderivative size = 384

$$\begin{aligned}\int \frac{A+Bx+Cx^2+Dx^3}{(cx)^{5/2}\left(a+bx^2\right)^2}\,dx &= -\frac{5bB-aD}{2a^2bc^2\sqrt{cx}} \\ &- \frac{2A}{3ac(cx)^{3/2}\left(a+bx^2\right)} + \frac{3a(bB-aD)-b(7Ab-3aC)x}{6a^2bc^2\sqrt{cx}\left(a+bx^2\right)} \\ &+ \frac{\left(\sqrt{b}(7Ab-3aC)+\sqrt{a}(5bB-aD)\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)}{4\sqrt{2}a^{11/4}b^{3/4}c^{5/2}} \\ &- \frac{\left(\sqrt{b}(7Ab-3aC)+\sqrt{a}(5bB-aD)\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)}{4\sqrt{2}a^{11/4}b^{3/4}c^{5/2}} \\ &- \frac{\left(\sqrt{b}(7Ab-3aC)-\sqrt{a}(5bB-aD)\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\left(\sqrt{a}+\sqrt{b}x\right)}\right)}{4\sqrt{2}a^{11/4}b^{3/4}c^{5/2}}\end{aligned}$$

output
```
-1/2*(5*B*b-D*a)/a^2/b/c^2/(c*x)^(1/2)-2/3*A/a/c/(c*x)^(3/2)/(b*x^2+a)+1/6
*(3*a*(B*b-D*a)-b*(7*A*b-3*C*a)*x)/a^2/b/c^2/(c*x)^(1/2)/(b*x^2+a)+1/8*(b^
(1/2)*(7*A*b-3*C*a)+a^(1/2)*(5*B*b-D*a))*arctan(1-2^(1/2)*b^(1/4)*(c*x)^(1
/2)/a^(1/4)/c^(1/2))*2^(1/2)/a^(11/4)/b^(3/4)/c^(5/2)-1/8*(b^(1/2)*(7*A*b-
3*C*a)+a^(1/2)*(5*B*b-D*a))*arctan(1+2^(1/2)*b^(1/4)*(c*x)^(1/2)/a^(1/4)/c
^(1/2))*2^(1/2)/a^(11/4)/b^(3/4)/c^(5/2)-1/8*(b^(1/2)*(7*A*b-3*C*a)-a^(1/2
)*(5*B*b-D*a))*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*(c*x)^(1/2)/c^(1/2)/(a^(1/2
)+b^(1/2)*x))*2^(1/2)/a^(11/4)/b^(3/4)/c^(5/2)
```

## Mathematica [A] (verified)

Time = 1.36 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.63

$$\int \frac{A+Bx+Cx^2+Dx^3}{(cx)^{5/2}\left(a+bx^2\right)^2}\,dx=\frac{x\left(-\frac{4a^{3/4}(4aA+bx^2(7A+15Bx)-3ax(-4B+x(C+Dx)))}{a+bx^2}+\frac{3\sqrt{2}\left(7Ab^{3/2}+5\sqrt{a}bB-3a\sqrt{b}C-a^{3/2}\cdots\right.}{b^{3/\cdots}}\cdots\right.}{2\cdots}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/((c*x)^(5/2)*(a + b*x^2)^2),x]
```

output
```
(x*((-4*a^(3/4)*(4*a*A + b*x^2*(7*A + 15*B*x) - 3*a*x*(-4*B + x*(C + D*x))
))/(a + b*x^2) + (3*Sqrt[2]*(7*A*b^(3/2) + 5*Sqrt[a]*b*B - 3*a*Sqrt[b]*C -
 a^(3/2)*D)*x^(3/2)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*
Sqrt[x])])/b^(3/4) - (3*Sqrt[2]*(7*A*b^(3/2) - 5*Sqrt[a]*b*B - 3*a*Sqrt[b]
*C + a^(3/2)*D)*x^(3/2)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a]
 + Sqrt[b]*x)])/b^(3/4)))/(24*a^(11/4)*(c*x)^(5/2))
```

## Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 446, normalized size of antiderivative = 1.16, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2337, 27, 553, 27, 553, 27, 554, 1482, 27, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(cx)^{5/2}\left(a+bx^2\right)^2}dx$$

$$\Big\downarrow \ 2337$$

$$\frac{x\left(B-\frac{aD}{b}\right)-\frac{aC}{b}+A}{2ac(cx)^{3/2}\left(a+bx^2\right)}-\frac{\int -\frac{7A-\frac{3aC}{b}+\left(5B-\frac{aD}{b}\right)x}{2(cx)^{5/2}(bx^2+a)}dx}{2a}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{7A-\frac{3aC}{b}+\left(5B-\frac{aD}{b}\right)x}{(cx)^{5/2}(bx^2+a)}dx}{4a}+\frac{x\left(B-\frac{aD}{b}\right)-\frac{aC}{b}+A}{2ac(cx)^{3/2}\left(a+bx^2\right)}$$

$$\Big\downarrow \ 553$$

$$\frac{-\frac{2\int -\frac{3(a(5bB-aD)-b(7Ab-3aC)x)}{2b(cx)^{3/2}\left(bx^2+a\right)}dx}{3ac}-\frac{2\left(7A-\frac{3aC}{b}\right)}{3ac(cx)^{3/2}}}{4a}+\frac{x\left(B-\frac{aD}{b}\right)-\frac{aC}{b}+A}{2ac(cx)^{3/2}\left(a+bx^2\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\int \frac{a(5bB-aD)-b(7Ab-3aC)x}{(cx)^{3/2}\left(bx^2+a\right)}dx}{abc}-\frac{2\left(7A-\frac{3aC}{b}\right)}{3ac(cx)^{3/2}}}{4a}+\frac{x\left(B-\frac{aD}{b}\right)-\frac{aC}{b}+A}{2ac(cx)^{3/2}\left(a+bx^2\right)}$$

$$\Big\downarrow \ 553$$

$$\frac{\frac{-\frac{2\int \frac{ab(7Ab-3aC+(5bB-aD)x)}{2\sqrt{cx}\left(bx^2+a\right)}dx}{ac}-\frac{2(5bB-aD)}{c\sqrt{cx}}}{abc}-\frac{2\left(7A-\frac{3aC}{b}\right)}{3ac(cx)^{3/2}}}{4a}+\frac{x\left(B-\frac{aD}{b}\right)-\frac{aC}{b}+A}{2ac(cx)^{3/2}\left(a+bx^2\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{-\frac{b\int \frac{7Ab-3aC+(5bB-aD)x}{\sqrt{cx}\left(bx^2+a\right)}dx}{c}-\frac{2(5bB-aD)}{c\sqrt{cx}}}{abc}-\frac{2\left(7A-\frac{3aC}{b}\right)}{3ac(cx)^{3/2}}}{4a}+\frac{x\left(B-\frac{aD}{b}\right)-\frac{aC}{b}+A}{2ac(cx)^{3/2}\left(a+bx^2\right)}$$

$$\Big\downarrow \ 554$$

$$\frac{\frac{-\frac{2b\int \frac{c(7Ab-3aC)+c(5bB-aD)x}{bx^2c^2+ac^2}d\sqrt{cx}}{c}-\frac{2(5bB-aD)}{c\sqrt{cx}}}{abc}-\frac{2\left(7A-\frac{3aC}{b}\right)}{3ac(cx)^{3/2}}}{4a}+\frac{x\left(B-\frac{aD}{b}\right)-\frac{aC}{b}+A}{2ac(cx)^{3/2}\left(a+bx^2\right)}$$

$$\Big\downarrow \ 1482$$

$$\frac{-\frac{\frac{2b\left(\frac{\left(\frac{\sqrt{b}(7Ab-3aC)}{\sqrt{a}}-aD+5bB\right)\int\frac{\sqrt{b}\left(\sqrt{b}xc+\sqrt{ac}\right)}{bx^2c^2+ac^2}d\sqrt{cx}}{2b}-\frac{\left(-\frac{\sqrt{b}(7Ab-3aC)}{\sqrt{a}}-aD+5bB\right)\int\frac{\sqrt{b}\left(\sqrt{a}c-\sqrt{b}cx\right)}{bx^2c^2+ac^2}d\sqrt{cx}}{2b}\right)}{c}-\frac{2(5bB-aD)}{c\sqrt{cx}}}{abc}-\frac{2\left(7A-\frac{3aC}{b}\right)}{3ac(cx)^{3/2}}}{4a}+\frac{x\left(B-\frac{aD}{b}\right)-\frac{aC}{b}+A}{2ac(cx)^{3/2}\left(a+bx^2\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{-\frac{\frac{2b\left(\frac{\left(\frac{\sqrt{b}(7Ab-3aC)}{\sqrt{a}}-aD+5bB\right)\int\frac{\sqrt{b}xc+\sqrt{ac}}{bx^2c^2+ac^2}d\sqrt{cx}}{2\sqrt{b}}-\frac{\left(-\frac{\sqrt{b}(7Ab-3aC)}{\sqrt{a}}-aD+5bB\right)\int\frac{\sqrt{a}c-\sqrt{b}cx}{bx^2c^2+ac^2}d\sqrt{cx}}{2\sqrt{b}}\right)}{c}-\frac{2(5bB-aD)}{c\sqrt{cx}}}{abc}-\frac{2\left(7A-\frac{3aC}{b}\right)}{3ac(cx)^{3/2}}}{4a}+\frac{x\left(B-\frac{aD}{b}\right)-\frac{aC}{b}+A}{2ac(cx)^{3/2}\left(a+bx^2\right)}$$

$$\Big\downarrow \ 1476$$

$$\frac{-\frac{\frac{2b\left(\frac{\left(\frac{\sqrt{b}(7Ab-3aC)}{\sqrt{a}}-aD+5bB\right)\left(\frac{\int\frac{1}{xc+\frac{\sqrt{ac}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{cx}\sqrt{c}}{\sqrt[4]{b}}}d\sqrt{cx}}{2\sqrt{b}}+\frac{\int\frac{1}{xc+\frac{\sqrt{ac}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{cx}\sqrt{c}}{\sqrt[4]{b}}}d\sqrt{cx}}{2\sqrt{b}}\right)}{2\sqrt{b}}-\frac{\left(-\frac{\sqrt{b}(7Ab-3aC)}{\sqrt{a}}-aD+5bB\right)\int\frac{\sqrt{a}c-\sqrt{b}cx}{bx^2c^2+ac^2}d}{2\sqrt{b}}}{c}}{abc}}{4a}+\frac{x\left(B-\frac{aD}{b}\right)-\frac{aC}{b}+A}{2ac(cx)^{3/2}\left(a+bx^2\right)}$$

$$\Big\downarrow \ 1082$$

$$-\frac{-\frac{\frac{2b\left(\frac{\left(\frac{\sqrt{b}(7Ab-3aC)}{\sqrt{a}}-aD+5bB\right)\left(\frac{\int\frac{1}{-cx-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}}-\frac{\int\frac{1}{-cx-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}}\right)}{2\sqrt{b}}-\frac{\left(-\frac{\sqrt{b}(7Ab-3aC)}{\sqrt{a}}-aD+5bB\right)\int\frac{\sqrt{a}c-\sqrt{b}cx}{bx^2c^2+ac^2}d\sqrt{cx}}{2\sqrt{b}}\right)}{c}-}{abc}}{4a}$$

$$\frac{x\left(B-\frac{aD}{b}\right)-\frac{aC}{b}+A}{2ac(cx)^{3/2}\left(a+bx^2\right)}$$

$\Big\downarrow$ 217

$$-\frac{-\frac{\frac{2b\left(\frac{\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}}\right)\left(\frac{\sqrt{b}(7Ab-3aC)}{\sqrt{a}}-aD+5bB\right)}{2\sqrt{b}}-\frac{\left(-\frac{\sqrt{b}(7Ab-3aC)}{\sqrt{a}}-aD+5bB\right)\int\frac{\sqrt{a}c-\sqrt{b}cx}{bx^2c^2+ac^2}d\sqrt{cx}}{2\sqrt{b}}\right)}{c}-\frac{2(5bB-a}{c\sqrt{cx}}}{abc}}{4a}$$

$$\frac{x\left(B-\frac{aD}{b}\right)-\frac{aC}{b}+A}{2ac(cx)^{3/2}\left(a+bx^2\right)}$$

$\Big\downarrow$ 1479

$$-\frac{2b\left(\frac{\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}}\right)\left(\frac{\sqrt{b}(7Ab-3aC)}{\sqrt{a}}-aD+5bB\right)}{2\sqrt{b}}-\frac{\left(-\frac{\sqrt{b}(7Ab-3aC)}{\sqrt{a}}-aD+5bB\right)\left(\int-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{c}-2\sqrt[4]{b}\cdots}{\sqrt[4]{b}\left(xc+\frac{\sqrt{ac}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\cdots}{\sqrt[4]{\cdots}}\right.}-\frac{\cdots}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{\cdots}}\right.}{2\cdots}\cdots}{\frac{c}{\frac{abc}{4a}}}\cdots$$

$$\frac{x\left(B-\frac{aD}{b}\right)-\frac{aC}{b}+A}{2ac(cx)^{3/2}\left(a+bx^2\right)}$$

$\downarrow$ 25

$$-\frac{2b\left(\frac{\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}}\right)\left(\frac{\sqrt{b}(7Ab-3aC)}{\sqrt{a}}-aD+5bB\right)}{2\sqrt{b}}-\frac{\left(-\frac{\sqrt{b}(7Ab-3aC)}{\sqrt{a}}-aD+5bB\right)\left(\frac{\int\frac{\sqrt{2}\sqrt[4]{a}\sqrt{c}-2\sqrt[4]{b}\sqrt{cx}\cdots}{\sqrt[4]{b}\left(xc+\frac{\sqrt{ac}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{c}\cdots}{\sqrt[4]{b}}\right.}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}}\cdots\right.}{2\sqrt{b}\cdots}\cdots}{\frac{c}{\frac{abc}{4a}}}\cdots$$

$$\frac{x\left(B-\frac{aD}{b}\right)-\frac{aC}{b}+A}{2ac(cx)^{3/2}\left(a+bx^2\right)}$$

$\downarrow$ 27

$$\frac{-\frac{2b\left(\frac{\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}}\right)\left(\frac{\sqrt{b}(7Ab-3aC)}{\sqrt{a}}-aD+5bB\right)}{2\sqrt{b}}-\frac{\left(-\frac{\sqrt{b}(7Ab-3aC)}{\sqrt{a}}-aD+5bB\right)\left(\frac{\int\frac{\sqrt{2}\sqrt[4]{a}\sqrt{c}-2\sqrt[4]{b}\sqrt{cx}}{xc+\frac{\sqrt{ac}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{cx}\sqrt{c}}{\sqrt[4]{b}}}d\sqrt{cx}\cdots}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}\sqrt{c}}\cdots\right)}{2\sqrt{b}}\cdots\right)}{c}\cdots}{abc}\cdots}{4a}\cdots$$

$$\frac{x\left(B-\frac{aD}{b}\right)-\frac{aC}{b}+A}{2ac(cx)^{3/2}\left(a+bx^2\right)}$$

$\downarrow$ 1103

$$\frac{-\frac{2b\left(\frac{\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}}\right)\left(\frac{\sqrt{b}(7Ab-3aC)}{\sqrt{a}}-aD+5bB\right)}{2\sqrt{b}}-\frac{\left(\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}\sqrt{cx}+\sqrt{a}c+\sqrt{b}cx\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\cdots\right)}{2\sqrt{2}\sqrt[4]{a}\cdots}\cdots\right)\cdots}{2\sqrt{b}}\cdots\right)}{c}\cdots}{abc}\cdots}{4a}\cdots$$

$$\frac{x\left(B-\frac{aD}{b}\right)-\frac{aC}{b}+A}{2ac(cx)^{3/2}\left(a+bx^2\right)}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/((c*x)^(5/2)*(a + b*x^2)^2),x]
```

output
```
(A - (a*C)/b + (B - (a*D)/b)*x)/(2*a*c*(c*x)^(3/2)*(a + b*x^2)) + ((-2*(7*
A - (3*a*C)/b))/(3*a*c*(c*x)^(3/2)) + ((-2*(5*b*B - a*D))/(c*Sqrt[c*x]) -
(2*b*(((5*b*B + (Sqrt[b]*(7*A*b - 3*a*C))/Sqrt[a] - a*D)*(-(ArcTan[1 - (Sq
rt[2]*b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[
c])) + ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])]/(Sqrt[2]*
a^(1/4)*b^(1/4)*Sqrt[c])))/(2*Sqrt[b]) - ((5*b*B - (Sqrt[b]*(7*A*b - 3*a*C
))/Sqrt[a] - a*D)*(-1/2*Log[Sqrt[a]*c + Sqrt[b]*c*x - Sqrt[2]*a^(1/4)*b^(1
/4)*Sqrt[c]*Sqrt[c*x]]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[c]) + Log[Sqrt[a]*c +
 Sqrt[b]*c*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[c]*Sqrt[c*x]]/(2*Sqrt[2]*a^(1/
4)*b^(1/4)*Sqrt[c])))/(2*Sqrt[b])))/c)/(a*b*c))/(4*a)
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 553
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Sym
bol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] + Simp
[1/(a*e*(m + 1))   Int[(e*x)^(m + 1)*(a + b*x^2)^p*(a*d*(m + 1) - b*c*(m +
2*p + 3)*x), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && LtQ[m, -1]
```

rule 554
```
Int[((c_) + (d_.)*(x_))/(Sqrt[(e_.)*(x_)]*((a_) + (b_.)*(x_)^2)), x_Symbol]
 :> Simp[2   Subst[Int[(e*c + d*x^2)/(a*e^2 + b*x^4), x], x, Sqrt[e*x]], x]
 /; FreeQ[{a, b, c, d, e}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 2337
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(-(c*x)^(m + 1))*(f + g*x)*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1)))
, x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*ExpandToSum[2
*a*(p + 1)*Q + f*(m + 2*p + 3) + g*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a
, b, c, m}, x] && PolyQ[Pq, x] && LtQ[p, -1] &&  !GtQ[m, 0]
```

## Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.01

| method | result |
|---|---|
| derivativedivides | $-\frac{2A}{3c\,a^{2}(cx)^{\frac{3}{2}}}-\frac{2B}{c^{2}a^{2}\sqrt{cx}}-\frac{2\left(\frac{\left(\frac{Bb}{4}-\frac{Da}{4}\right)(cx)^{\frac{3}{2}}+\left(\frac{1}{4}Abc-\frac{1}{4}Cac\right)\sqrt{cx}}{b\,c^{2}x^{2}+a\,c^{2}}+\frac{(7Abc-3Cac)\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{cx+\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{4}}\sqrt{\cdots}}{cx-\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{4}}\sqrt{\cdots}}\right)\cdots\right.}{\cdots}\cdots\right)}{\cdots}$ |
| default | $-\frac{2A}{3c\,a^{2}(cx)^{\frac{3}{2}}}-\frac{2B}{c^{2}a^{2}\sqrt{cx}}-\frac{2\left(\frac{\left(\frac{Bb}{4}-\frac{Da}{4}\right)(cx)^{\frac{3}{2}}+\left(\frac{1}{4}Abc-\frac{1}{4}Cac\right)\sqrt{cx}}{b\,c^{2}x^{2}+a\,c^{2}}+\frac{(7Abc-3Cac)\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{cx+\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{4}}\sqrt{\cdots}}{cx-\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{4}}\sqrt{\cdots}}\right)\cdots\right.}{\cdots}\cdots\right)}{\cdots}$ |
| pseudoelliptic | $-\frac{2\left(\frac{21\left(b\,x^{2}+a\right)\left(\ln\left(\frac{cx+\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{4}}\sqrt{cx}\,\sqrt{2}+\sqrt{\frac{a\,c^{2}}{b}}}{cx-\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{4}}\sqrt{cx}\,\sqrt{2}+\sqrt{\frac{a\,c^{2}}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{cx}-\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{4}}}{\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{cx}+\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{4}}}{\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{4}}}\right)\right)(cx)^{\frac{3}{2}}\cdots}{32}\cdots\right)}{\cdots}$ |

input
```
int((D*x^3+C*x^2+B*x+A)/(c*x)^(5/2)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-2/3*A/c/a^2/(c*x)^(3/2)-2*B/c^2/a^2/(c*x)^(1/2)-2/a^2/c^2*(((1/4*B*b-1/4*
D*a)*(c*x)^(3/2)+(1/4*A*b*c-1/4*C*a*c)*(c*x)^(1/2))/(b*c^2*x^2+a*c^2)+1/32
*(7*A*b*c-3*C*a*c)*(a*c^2/b)^(1/4)/a/c^2*2^(1/2)*(ln((c*x+(a*c^2/b)^(1/4)*
(c*x)^(1/2)*2^(1/2)+(a*c^2/b)^(1/2))/(c*x-(a*c^2/b)^(1/4)*(c*x)^(1/2)*2^(1
/2)+(a*c^2/b)^(1/2)))+2*arctan(2^(1/2)/(a*c^2/b)^(1/4)*(c*x)^(1/2)+1)+2*ar
ctan(2^(1/2)/(a*c^2/b)^(1/4)*(c*x)^(1/2)-1))+1/32*(5*B*b-D*a)/b/(a*c^2/b)^
(1/4)*2^(1/2)*(ln((c*x-(a*c^2/b)^(1/4)*(c*x)^(1/2)*2^(1/2)+(a*c^2/b)^(1/2)
)/(c*x+(a*c^2/b)^(1/4)*(c*x)^(1/2)*2^(1/2)+(a*c^2/b)^(1/2)))+2*arctan(2^(1
/2)/(a*c^2/b)^(1/4)*(c*x)^(1/2)+1)+2*arctan(2^(1/2)/(a*c^2/b)^(1/4)*(c*x)^
(1/2)-1)))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3722 vs. $2(291) = 582$.

Time = 0.42 (sec) , antiderivative size = 3722, normalized size of antiderivative = 9.69

$$\int \frac{A+Bx+Cx^2+Dx^3}{(cx)^{5/2}\left(a+bx^2\right)^2}\,dx=\text{Too large to display}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(c*x)^(5/2)/(b*x^2+a)^2,x, algorithm="fricas
")
```

output
```
Too large to include
```

## Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 106.93 (sec) , antiderivative size = 4048, normalized size of antiderivative = 10.54

$$\int \frac{A+Bx+Cx^2+Dx^3}{(cx)^{5/2}\left(a+bx^2\right)^2}\,dx=\text{Too large to display}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(c*x)**(5/2)/(b*x**2+a)**2,x)
```

output
```
A*(16*a**(7/4)*exp(I*pi/4)*gamma(-3/4)/(32*a**(15/4)*c**(5/2)*x**(3/2)*exp
(I*pi/4)*gamma(1/4) + 32*a**(11/4)*b*c**(5/2)*x**(7/2)*exp(I*pi/4)*gamma(1
/4)) + 28*a**(3/4)*b*x**2*exp(I*pi/4)*gamma(-3/4)/(32*a**(15/4)*c**(5/2)*x
**(3/2)*exp(I*pi/4)*gamma(1/4) + 32*a**(11/4)*b*c**(5/2)*x**(7/2)*exp(I*pi
/4)*gamma(1/4)) - 21*a*b**(3/4)*x**(3/2)*log(1 - b**(1/4)*sqrt(x)*exp_pola
r(I*pi/4)/a**(1/4))*gamma(-3/4)/(32*a**(15/4)*c**(5/2)*x**(3/2)*exp(I*pi/4
)*gamma(1/4) + 32*a**(11/4)*b*c**(5/2)*x**(7/2)*exp(I*pi/4)*gamma(1/4)) +
21*I*a*b**(3/4)*x**(3/2)*log(1 - b**(1/4)*sqrt(x)*exp_polar(3*I*pi/4)/a**(
1/4))*gamma(-3/4)/(32*a**(15/4)*c**(5/2)*x**(3/2)*exp(I*pi/4)*gamma(1/4) +
 32*a**(11/4)*b*c**(5/2)*x**(7/2)*exp(I*pi/4)*gamma(1/4)) + 21*a*b**(3/4)*
x**(3/2)*log(1 - b**(1/4)*sqrt(x)*exp_polar(5*I*pi/4)/a**(1/4))*gamma(-3/4
)/(32*a**(15/4)*c**(5/2)*x**(3/2)*exp(I*pi/4)*gamma(1/4) + 32*a**(11/4)*b*
c**(5/2)*x**(7/2)*exp(I*pi/4)*gamma(1/4)) - 21*I*a*b**(3/4)*x**(3/2)*log(1
 - b**(1/4)*sqrt(x)*exp_polar(7*I*pi/4)/a**(1/4))*gamma(-3/4)/(32*a**(15/4
)*c**(5/2)*x**(3/2)*exp(I*pi/4)*gamma(1/4) + 32*a**(11/4)*b*c**(5/2)*x**(7
/2)*exp(I*pi/4)*gamma(1/4)) - 21*b**(7/4)*x**(7/2)*log(1 - b**(1/4)*sqrt(x
)*exp_polar(I*pi/4)/a**(1/4))*gamma(-3/4)/(32*a**(15/4)*c**(5/2)*x**(3/2)*
exp(I*pi/4)*gamma(1/4) + 32*a**(11/4)*b*c**(5/2)*x**(7/2)*exp(I*pi/4)*gamm
a(1/4)) + 21*I*b**(7/4)*x**(7/2)*log(1 - b**(1/4)*sqrt(x)*exp_polar(3*I*pi
/4)/a**(1/4))*gamma(-3/4)/(32*a**(15/4)*c**(5/2)*x**(3/2)*exp(I*pi/4)*g...
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 437, normalized size of antiderivative = 1.14

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(cx)^{5/2} (a + bx^2)^2} \, dx = \frac{\frac{8 \left(3 \, (Da - 5 \, Bb) c^3 x^3 - 12 \, Bac^3 x + (3 \, Ca - 7 \, Ab) c^3 x^2 - 4 \, Aac^3\right)}{(cx)^{\frac{7}{2}} a^2 bc + (cx)^{\frac{3}{2}} a^3 c^3} - \frac{3 \left(\frac{\sqrt{2} \left((Da - 5 \, Bb) \sqrt{a} c - \left(3 \, Ca \sqrt{b} - 7 \, Ab^{\cdots}\right.\right.}{\left(a \cdots\right.}\right.}{\cdots}}{\cdots}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(c*x)^(5/2)/(b*x^2+a)^2,x, algorithm="maxima
")
```

output
```
1/48*(8*(3*(D*a - 5*B*b)*c^3*x^3 - 12*B*a*c^3*x + (3*C*a - 7*A*b)*c^3*x^2
- 4*A*a*c^3)/((c*x)^(7/2)*a^2*b*c + (c*x)^(3/2)*a^3*c^3) - 3*(sqrt(2)*((D*
a - 5*B*b)*sqrt(a)*c - (3*C*a*sqrt(b) - 7*A*b^(3/2))*c)*log(sqrt(b)*c*x +
sqrt(2)*(a*c^2)^(1/4)*sqrt(c*x)*b^(1/4) + sqrt(a)*c)/((a*c^2)^(3/4)*b^(3/4
)) - sqrt(2)*((D*a - 5*B*b)*sqrt(a)*c - (3*C*a*sqrt(b) - 7*A*b^(3/2))*c)*l
og(sqrt(b)*c*x - sqrt(2)*(a*c^2)^(1/4)*sqrt(c*x)*b^(1/4) + sqrt(a)*c)/((a*
c^2)^(3/4)*b^(3/4)) - 2*sqrt(2)*((D*a - 5*B*b)*sqrt(a)*c + (3*C*a*sqrt(b)
- 7*A*b^(3/2))*c)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*c^2)^(1/4)*b^(1/4) + 2*sq
rt(c*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*c))/(sqrt(sqrt(a)*sqrt(b)*c)*sqrt(a)
*sqrt(b)*c) - 2*sqrt(2)*((D*a - 5*B*b)*sqrt(a)*c + (3*C*a*sqrt(b) - 7*A*b^
(3/2))*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*c^2)^(1/4)*b^(1/4) - 2*sqrt(c*x)
*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*c))/(sqrt(sqrt(a)*sqrt(b)*c)*sqrt(a)*sqrt(b
)*c))/(a^2*c))/c
```

## Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 546, normalized size of antiderivative = 1.42

$$\int \frac{A+Bx+Cx^2+Dx^3}{(cx)^{5/2}\left(a+bx^2\right)^2}\,dx = \frac{\sqrt{cx}Dacx-\sqrt{cx}Bbcx+\sqrt{cx}Cac-\sqrt{cx}Abc}{2\,(bc^2x^2+ac^2)a^2c^2} - \frac{2\,(3\,Bcx+Ac)}{3\,\sqrt{cx}a^2c^3x}$$
$$+\frac{\sqrt{2}\left(3\,(ab^3c^2)^{\frac14}\,Cab^2c-7\,(ab^3c^2)^{\frac14}\,Ab^3c+(ab^3c^2)^{\frac34}\,Da-5\,(ab^3c^2)^{\frac34}\,Bb\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ac^2}{b}\right)^{\frac14}+2\,\sqrt{cx}\right)}{2\left(\frac{ac^2}{b}\right)^{\frac14}}\right)}{8\,a^3b^3c^4}$$
$$+\frac{\sqrt{2}\left(3\,(ab^3c^2)^{\frac14}\,Cab^2c-7\,(ab^3c^2)^{\frac14}\,Ab^3c+(ab^3c^2)^{\frac34}\,Da-5\,(ab^3c^2)^{\frac34}\,Bb\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ac^2}{b}\right)^{\frac14}-2\,\sqrt{cx}\right)}{2\left(\frac{ac^2}{b}\right)^{\frac14}}\right)}{8\,a^3b^3c^4}$$
$$+\frac{\sqrt{2}\left(3\,(ab^3c^2)^{\frac14}\,Cab^2c-7\,(ab^3c^2)^{\frac14}\,Ab^3c-(ab^3c^2)^{\frac34}\,Da+5\,(ab^3c^2)^{\frac34}\,Bb\right)\log\left(cx+\sqrt{2}\left(\frac{ac^2}{b}\right)^{\frac14}\sqrt{cx}+\sqrt{\frac{ac^2}{b}}\right)}{16\,a^3b^3c^4}$$
$$-\frac{\sqrt{2}\left(3\,(ab^3c^2)^{\frac14}\,Cab^2c-7\,(ab^3c^2)^{\frac14}\,Ab^3c-(ab^3c^2)^{\frac34}\,Da+5\,(ab^3c^2)^{\frac34}\,Bb\right)\log\left(cx-\sqrt{2}\left(\frac{ac^2}{b}\right)^{\frac14}\sqrt{cx}+\sqrt{\frac{ac^2}{b}}\right)}{16\,a^3b^3c^4}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(c*x)^(5/2)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*(sqrt(c*x)*D*a*c*x - sqrt(c*x)*B*b*c*x + sqrt(c*x)*C*a*c - sqrt(c*x)*A
*b*c)/((b*c^2*x^2 + a*c^2)*a^2*c^2) - 2/3*(3*B*c*x + A*c)/(sqrt(c*x)*a^2*c
^3*x) + 1/8*sqrt(2)*(3*(a*b^3*c^2)^(1/4)*C*a*b^2*c - 7*(a*b^3*c^2)^(1/4)*A
*b^3*c + (a*b^3*c^2)^(3/4)*D*a - 5*(a*b^3*c^2)^(3/4)*B*b)*arctan(1/2*sqrt(
2)*(sqrt(2)*(a*c^2/b)^(1/4) + 2*sqrt(c*x))/(a*c^2/b)^(1/4))/(a^3*b^3*c^4)
+ 1/8*sqrt(2)*(3*(a*b^3*c^2)^(1/4)*C*a*b^2*c - 7*(a*b^3*c^2)^(1/4)*A*b^3*c
 + (a*b^3*c^2)^(3/4)*D*a - 5*(a*b^3*c^2)^(3/4)*B*b)*arctan(-1/2*sqrt(2)*(s
qrt(2)*(a*c^2/b)^(1/4) - 2*sqrt(c*x))/(a*c^2/b)^(1/4))/(a^3*b^3*c^4) + 1/1
6*sqrt(2)*(3*(a*b^3*c^2)^(1/4)*C*a*b^2*c - 7*(a*b^3*c^2)^(1/4)*A*b^3*c - (
a*b^3*c^2)^(3/4)*D*a + 5*(a*b^3*c^2)^(3/4)*B*b)*log(c*x + sqrt(2)*(a*c^2/b
)^(1/4)*sqrt(c*x) + sqrt(a*c^2/b))/(a^3*b^3*c^4) - 1/16*sqrt(2)*(3*(a*b^3*
c^2)^(1/4)*C*a*b^2*c - 7*(a*b^3*c^2)^(1/4)*A*b^3*c - (a*b^3*c^2)^(3/4)*D*a
 + 5*(a*b^3*c^2)^(3/4)*B*b)*log(c*x - sqrt(2)*(a*c^2/b)^(1/4)*sqrt(c*x) +
sqrt(a*c^2/b))/(a^3*b^3*c^4)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(cx)^{5/2}\,(a+bx^2)^2}\,dx=\int \frac{A+B\,x+C\,x^2+x^3\,D}{(c\,x)^{5/2}\,(b\,x^2+a)^2}\,dx$$

input
```
int((A + B*x + C*x^2 + x^3*D)/((c*x)^(5/2)*(a + b*x^2)^2),x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/((c*x)^(5/2)*(a + b*x^2)^2), x)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 1309, normalized size of antiderivative = 3.41

$$\int \frac{A+Bx+Cx^2+Dx^3}{(cx)^{5/2}\,(a+bx^2)^2}\,dx=\text{Too large to display}$$

input
```
int((D*x^3+C*x^2+B*x+A)/(c*x)^(5/2)/(b*x^2+a)^2,x)
```

output
```
(sqrt(c)*( - 6*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*s
qrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2*d*x + 30*sqr
t(x)*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)
*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b**2*x - 6*sqrt(x)*b**(1/4)*a**(3
/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)
*a**(1/4)*sqrt(2)))*a*b*d*x**3 + 30*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*atan
((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2
)))*b**3*x**3 + 42*sqrt(x)*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/
4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2*b*x - 18
*sqrt(x)*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqr
t(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2*c*x + 42*sqrt(x)*b**(3/4)*
a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**
(1/4)*a**(1/4)*sqrt(2)))*a*b**2*x**3 - 18*sqrt(x)*b**(3/4)*a**(1/4)*sqrt(2
)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*
sqrt(2)))*a*b*c*x**3 + 6*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*
a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2*d*
x - 30*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) +
 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b**2*x + 6*sqrt(x)*b**(
1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))
/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b*d*x**3 - 30*sqrt(x)*b**(1/4)*a**(3/4)...
```

# 3.135 $\int \frac{A+Bx+Cx^2+Dx^3}{(cx)^{7/2}(a+bx^2)^2}\,dx$



## Optimal result

Integrand size = 32, antiderivative size = 411

$$
\begin{aligned}
\int \frac{A+Bx+Cx^2+Dx^3}{(cx)^{7/2}\left(a+bx^2\right)^2}\,dx &= -\frac{7bB-3aD}{6a^2bc^2(cx)^{3/2}}+\frac{9Ab-5aC}{2a^3c^3\sqrt{cx}} \\
&- \frac{2A}{5ac(cx)^{5/2}\left(a+bx^2\right)}+\frac{5a(bB-aD)-b(9Ab-5aC)x}{10a^2bc^2(cx)^{3/2}\left(a+bx^2\right)} \\
&- \frac{\left(\sqrt{b}(9Ab-5aC)-\sqrt{a}(7bB-3aD)\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)}{4\sqrt{2}a^{13/4}\sqrt[4]{b}c^{7/2}} \\
&+ \frac{\left(\sqrt{b}(9Ab-5aC)-\sqrt{a}(7bB-3aD)\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)}{4\sqrt{2}a^{13/4}\sqrt[4]{b}c^{7/2}} \\
&- \frac{\left(\sqrt{b}(9Ab-5aC)+\sqrt{a}(7bB-3aD)\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\left(\sqrt{a}+\sqrt{b}x\right)}\right)}{4\sqrt{2}a^{13/4}\sqrt[4]{b}c^{7/2}}
\end{aligned}
$$

output
```
-1/6*(7*B*b-3*D*a)/a^2/b/c^2/(c*x)^(3/2)+1/2*(9*A*b-5*C*a)/a^3/c^3/(c*x)^(
1/2)-2/5*A/a/c/(c*x)^(5/2)/(b*x^2+a)+1/10*(5*a*(B*b-D*a)-b*(9*A*b-5*C*a)*x
)/a^2/b/c^2/(c*x)^(3/2)/(b*x^2+a)-1/8*(b^(1/2)*(9*A*b-5*C*a)-a^(1/2)*(7*B*
b-3*D*a))*arctan(1-2^(1/2)*b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))*2^(1/2)/a^
(13/4)/b^(1/4)/c^(7/2)+1/8*(b^(1/2)*(9*A*b-5*C*a)-a^(1/2)*(7*B*b-3*D*a))*a
rctan(1+2^(1/2)*b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))*2^(1/2)/a^(13/4)/b^(1
/4)/c^(7/2)-1/8*(b^(1/2)*(9*A*b-5*C*a)+a^(1/2)*(7*B*b-3*D*a))*arctanh(2^(1
/2)*a^(1/4)*b^(1/4)*(c*x)^(1/2)/c^(1/2)/(a^(1/2)+b^(1/2)*x))*2^(1/2)/a^(13
/4)/b^(1/4)/c^(7/2)
```

## Mathematica [A] (verified)

Time = 1.36 (sec) , antiderivative size = 267, normalized size of antiderivative = 0.65

$$\int \frac{A+Bx+Cx^2+Dx^3}{(cx)^{7/2}(a+bx^2)^2}\,dx = \frac{x\left(-\frac{4\sqrt[4]{a}(-135Ab^2x^4+abx^2(-108A+5x(7B+15Cx))+a^2(12A+5x(4B+12Cx-3Dx^2)))}{a+bx^2} - \frac{15\sqrt{\phantom{x}}}{}\right.}{}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/((c*x)^(7/2)*(a + b*x^2)^2),x]
```

output
```
(x*((-4*a^(1/4)*(-135*A*b^2*x^4 + a*b*x^2*(-108*A + 5*x*(7*B + 15*C*x)) +
a^2*(12*A + 5*x*(4*B + 12*C*x - 3*D*x^2))))/(a + b*x^2) - (15*Sqrt[2]*(9*A
*b^(3/2) - 7*Sqrt[a]*b*B - 5*a*Sqrt[b]*C + 3*a^(3/2)*D)*x^(5/2)*ArcTan[(Sq
rt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])]])/b^(1/4) + (15*Sqrt[
2]*(-9*A*b^(3/2) - 7*Sqrt[a]*b*B + 5*a*Sqrt[b]*C + 3*a^(3/2)*D)*x^(5/2)*Ar
cTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/b^(1/4)))/
(120*a^(13/4)*(c*x)^(7/2))
```

## Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 481, normalized size of antiderivative = 1.17, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {2337, 27, 553, 27, 553, 27, 553, 27, 554, 1482, 27, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(cx)^{7/2}\left(a+bx^2\right)^2}dx$$

$$\Big\downarrow \text{2337}$$

$$\frac{x\left(B-\frac{aD}{b}\right)-\frac{aC}{b}+A}{2ac(cx)^{5/2}\left(a+bx^2\right)}-\frac{\int -\frac{9A-\frac{5aC}{b}+\left(7B-\frac{3aD}{b}\right)x}{2(cx)^{7/2}(bx^2+a)}dx}{2a}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{9A-\frac{5aC}{b}+\left(7B-\frac{3aD}{b}\right)x}{(cx)^{7/2}(bx^2+a)}dx}{4a}+\frac{x\left(B-\frac{aD}{b}\right)-\frac{aC}{b}+A}{2ac(cx)^{5/2}\left(a+bx^2\right)}$$

$$\Big\downarrow \text{553}$$

$$\frac{-\frac{2\int -\frac{5(a(7bB-3aD)-b(9Ab-5aC)x)}{2b(cx)^{5/2}\left(bx^2+a\right)}dx}{5ac}-\frac{2\left(9A-\frac{5aC}{b}\right)}{5ac(cx)^{5/2}}}{4a}+\frac{x\left(B-\frac{aD}{b}\right)-\frac{aC}{b}+A}{2ac(cx)^{5/2}\left(a+bx^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{a(7bB-3aD)-b(9Ab-5aC)x}{(cx)^{5/2}\left(bx^2+a\right)}dx}{abc}-\frac{2\left(9A-\frac{5aC}{b}\right)}{5ac(cx)^{5/2}}}{4a}+\frac{x\left(B-\frac{aD}{b}\right)-\frac{aC}{b}+A}{2ac(cx)^{5/2}\left(a+bx^2\right)}$$

$$\Big\downarrow \text{553}$$

$$\frac{\frac{-\frac{2\int \frac{3ab(9Ab-5aC+(7bB-3aD)x)}{2(cx)^{3/2}\left(bx^2+a\right)}dx}{3ac}-\frac{2(7bB-3aD)}{3c(cx)^{3/2}}}{abc}-\frac{2\left(9A-\frac{5aC}{b}\right)}{5ac(cx)^{5/2}}}{4a}+\frac{x\left(B-\frac{aD}{b}\right)-\frac{aC}{b}+A}{2ac(cx)^{5/2}\left(a+bx^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{-\frac{b\int \frac{9Ab-5aC+(7bB-3aD)x}{(cx)^{3/2}\left(bx^2+a\right)}dx}{c}-\frac{2(7bB-3aD)}{3c(cx)^{3/2}}}{abc}-\frac{2\left(9A-\frac{5aC}{b}\right)}{5ac(cx)^{5/2}}}{4a}+\frac{x\left(B-\frac{aD}{b}\right)-\frac{aC}{b}+A}{2ac(cx)^{5/2}\left(a+bx^2\right)}$$

$$\Big\downarrow \; 553$$

$$\frac{\frac{-\frac{b\left(-\frac{2\int -\frac{a(7bB-3aD)-b(9Ab-5aC)x}{2\sqrt{cx}\left(bx^2+a\right)}dx}{ac}-\frac{2(9Ab-5aC)}{ac\sqrt{cx}}\right)}{c}-\frac{2(7bB-3aD)}{3c(cx)^{3/2}}}{abc}-\frac{2\left(9A-\frac{5aC}{b}\right)}{5ac(cx)^{5/2}}}{4a}+\frac{x\left(B-\frac{aD}{b}\right)-\frac{aC}{b}+A}{2ac(cx)^{5/2}\left(a+bx^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{-\frac{b\left(\frac{\int \frac{a(7bB-3aD)-b(9Ab-5aC)x}{\sqrt{cx}\left(bx^2+a\right)}dx}{ac}-\frac{2(9Ab-5aC)}{ac\sqrt{cx}}\right)}{c}-\frac{2(7bB-3aD)}{3c(cx)^{3/2}}}{abc}-\frac{2\left(9A-\frac{5aC}{b}\right)}{5ac(cx)^{5/2}}}{4a}+\frac{x\left(B-\frac{aD}{b}\right)-\frac{aC}{b}+A}{2ac(cx)^{5/2}\left(a+bx^2\right)}$$

$$\Big\downarrow \; 554$$

$$\frac{\frac{-\frac{b\left(\frac{2\int \frac{ac(7bB-3aD)-bc(9Ab-5aC)x}{bx^2c^2+ac^2}d\sqrt{cx}}{ac}-\frac{2(9Ab-5aC)}{ac\sqrt{cx}}\right)}{c}-\frac{2(7bB-3aD)}{3c(cx)^{3/2}}}{abc}-\frac{2\left(9A-\frac{5aC}{b}\right)}{5ac(cx)^{5/2}}}{4a}+\frac{x\left(B-\frac{aD}{b}\right)-\frac{aC}{b}+A}{2ac(cx)^{5/2}\left(a+bx^2\right)}$$

$$\Big\downarrow \; 1482$$

$$\frac{\frac{-\frac{b\left(\frac{2\left(\frac{1}{2}\left(\frac{\sqrt{a}(7bB-3aD)}{\sqrt{b}}-5aC+9Ab\right)\int \frac{\sqrt{b}\left(\sqrt{a}c-\sqrt{b}cx\right)}{bx^2c^2+ac^2}d\sqrt{cx}-\frac{1}{2}\left(-\frac{\sqrt{a}(7bB-3aD)}{\sqrt{b}}-5aC+9Ab\right)\int \frac{\sqrt{b}\left(\sqrt{b}xc+\sqrt{a}c\right)}{bx^2c^2+ac^2}d\sqrt{cx}\right)}{ac}-\frac{2(9Ab-5aC)}{ac\sqrt{cx}}\right)}{c}-\frac{2(7bB-3aD}{3c(cx)^{3/2}}}{abc}}{4a}$$

$$\frac{x\left(B-\frac{aD}{b}\right)-\frac{aC}{b}+A}{2ac(cx)^{5/2}\left(a+bx^2\right)}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{b\left(\frac{2\left(\frac{1}{2}\sqrt{b}\left(\frac{\sqrt{a}(7bB-3aD)}{\sqrt{b}}-5aC+9Ab\right)\int\frac{\sqrt{a}c-\sqrt{b}cx}{bx^2c^2+ac^2}d\sqrt{cx}-\frac{1}{2}\sqrt{b}\left(-\frac{\sqrt{a}(7bB-3aD)}{\sqrt{b}}-5aC+9Ab\right)\int\frac{\sqrt{b}xc+\sqrt{a}c}{bx^2c^2+ac^2}d\sqrt{cx}\right)}{ac}-\frac{2(9Ab-5aC)}{ac\sqrt{cx}}\right)}{c}-\frac{2(7bB-3aD)}{3c(cx)^{3/2}}}{abc}-}{4a}$$

$$\frac{x\left(B-\frac{aD}{b}\right)-\frac{aC}{b}+A}{2ac(cx)^{5/2}\left(a+bx^2\right)}$$

$$\Big\downarrow \; 1476$$

$$-\frac{\frac{b\left(\frac{2\left(\frac{1}{2}\sqrt{b}\left(\frac{\sqrt{a}(7bB-3aD)}{\sqrt{b}}-5aC+9Ab\right)\int\frac{\sqrt{a}c-\sqrt{b}cx}{bx^2c^2+ac^2}d\sqrt{cx}-\frac{1}{2}\sqrt{b}\left(-\frac{\sqrt{a}(7bB-3aD)}{\sqrt{b}}-5aC+9Ab\right)\left(\frac{\int\frac{1}{xc+\frac{\sqrt{a}c}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{cx}\sqrt{c}}{\sqrt[4]{b}}}d\sqrt{cx}}{2\sqrt{b}}+\frac{\int\frac{1}{xc+\frac{\sqrt{a}c}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{}}{}}}{2\sqrt{b}}\right.}{ac}}{c}}{abc}}{4a}$$

$$\frac{x\left(B-\frac{aD}{b}\right)-\frac{aC}{b}+A}{2ac(cx)^{5/2}\left(a+bx^2\right)}$$

$$\Big\downarrow \; 1082$$

$$-\frac{\frac{b\left(\frac{2\left(\frac{1}{2}\sqrt{b}\left(\frac{\sqrt{a}(7bB-3aD)}{\sqrt{b}}-5aC+9Ab\right)\int\frac{\sqrt{a}c-\sqrt{b}cx}{bx^2c^2+ac^2}d\sqrt{cx}-\frac{1}{2}\sqrt{b}\left(-\frac{\sqrt{a}(7bB-3aD)}{\sqrt{b}}-5aC+9Ab\right)\left(\frac{\int\frac{1}{-cx-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}}-\frac{\int\frac{1}{-cx-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{c}}{\sqrt[4]{a}\sqrt{c}}\right.}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}}\right.}{ac}}{c}}{abc}}{4a}$$

$$\frac{x\left(B-\frac{aD}{b}\right)-\frac{aC}{b}+A}{2ac(cx)^{5/2}\left(a+bx^2\right)}$$

$$\Big\downarrow \; 217$$

$$-\frac{\frac{b\left(\frac{2\left(\frac{1}{2}\sqrt{b}\left(\frac{\sqrt{a}(7bB-3aD)}{\sqrt{b}}-5aC+9Ab\right)\int \frac{\sqrt{a}c-\sqrt{b}cx}{bx^2c^2+ac^2}d\sqrt{cx}-\frac{1}{2}\sqrt{b}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}}\right)\left(-\frac{\sqrt{a}(7bB-3aD)}{\sqrt{b}}-5aC+9Ab\right)\cdots\right)}{ac}\cdots\right)}{c}\cdots}{abc}\cdots}{4a}\cdots$$

$$\frac{x\left(B-\frac{aD}{b}\right)-\frac{aC}{b}+A}{2ac(cx)^{5/2}\left(a+bx^2\right)}$$

$\Big\downarrow$ 1479

$$-\frac{\frac{b\left(\frac{2\left(\frac{1}{2}\sqrt{b}\left(\frac{\sqrt{a}(7bB-3aD)}{\sqrt{b}}-5aC+9Ab\right)\left(-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}\sqrt{c}-2\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{b}\left(xc+\frac{\sqrt{a}c}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{cx}\sqrt{c}}{\sqrt[4]{b}}\right)}d\sqrt{cx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{c}+\sqrt{2}\sqrt[4]{b}\sqrt{cx}\right)}{\sqrt[4]{b}\left(xc+\frac{\sqrt{a}c}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{cx}\sqrt{c}}{\sqrt[4]{b}}\right)}d\sqrt{cx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}}\right)-\frac{1}{2}\sqrt{b}\left(\frac{\arctan\left(\cdots\right)}{\sqrt{2}\sqrt[4]{\cdots}}\cdots\right)\cdots\right)}{ac}\cdots\right)}{c}\cdots}{abc}\cdots}{4a}\cdots$$

$$\frac{x\left(B-\frac{aD}{b}\right)-\frac{aC}{b}+A}{2ac(cx)^{5/2}\left(a+bx^2\right)}$$

$\Big\downarrow$ 25

$$-\frac{\frac{b\left(\frac{2\left(\frac{1}{2}\sqrt{b}\left(\frac{\sqrt{a}(7bB-3aD)}{\sqrt{b}}-5aC+9Ab\right)\left(\frac{\int\frac{\sqrt{2}\sqrt[4]{a}\sqrt{c}-2\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{b}\left(xc+\frac{\sqrt{ac}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{cx}\sqrt{c}}{\sqrt[4]{b}}\right)}d\sqrt{cx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}}+\frac{\int\frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{c}+\sqrt{2}\sqrt[4]{b}\sqrt{cx}\right)}{\sqrt[4]{b}\left(xc+\frac{\sqrt{ac}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{cx}\sqrt{c}}{\sqrt[4]{b}}\right)}d\sqrt{cx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}}\right)-\frac{1}{2}\sqrt{b}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{\cdots}}{\sqrt[4]{a}\cdots}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{\cdots}}\cdots\right.}{ac}\cdots\right)}{c}\cdots}{abc}\cdots}{4a}\cdots$$

$$\frac{x\left(B-\frac{aD}{b}\right)-\frac{aC}{b}+A}{2ac(cx)^{5/2}\left(a+bx^2\right)}$$

$\Big\downarrow$ 27

$$-\frac{\frac{b\left(\frac{2\left(\frac{1}{2}\sqrt{b}\left(\frac{\sqrt{a}(7bB-3aD)}{\sqrt{b}}-5aC+9Ab\right)\left(\frac{\int\frac{\sqrt{2}\sqrt[4]{a}\sqrt{c}-2\sqrt[4]{b}\sqrt{cx}}{xc+\frac{\sqrt{ac}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{cx}\sqrt{c}}{\sqrt[4]{b}}}d\sqrt{cx}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}\sqrt{c}}+\frac{\int\frac{\sqrt[4]{a}\sqrt{c}+\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{xc+\frac{\sqrt{ac}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{cx}\sqrt{c}}{\sqrt[4]{b}}}d\sqrt{cx}}{2\sqrt[4]{a}\sqrt{b}\sqrt{c}}\right)-\frac{1}{2}\sqrt{b}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}}-\frac{\text{arct}\cdots}{\cdots}\right.}{ac}\cdots\right)}{c}\cdots}{abc}\cdots}{4a}\cdots$$

$$\frac{x\left(B-\frac{aD}{b}\right)-\frac{aC}{b}+A}{2ac(cx)^{5/2}\left(a+bx^2\right)}$$

$\Big\downarrow$ 1103

$$-\frac{\frac{b\left(\frac{2\left(\frac{1}{2}\sqrt{b}\left(\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}\sqrt{cx}+\sqrt{a}c+\sqrt{b}cx\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}\sqrt{cx}+\sqrt{a}c+\sqrt{b}cx\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}}\right)\left(\frac{\sqrt{a}(7bB-3aD)}{\sqrt{b}}-5aC+9Ab\right)-\frac{1}{2}\sqrt{b}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\ldots\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}}\ldots\right.}{ac}\ldots\right)}{c}}{abc}}{4a}$$

$$\frac{x\left(B-\frac{aD}{b}\right)-\frac{aC}{b}+A}{2ac(cx)^{5/2}\left(a+bx^2\right)}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/((c*x)^(7/2)*(a + b*x^2)^2),x]
```

output
```
(A - (a*C)/b + (B - (a*D)/b)*x)/(2*a*c*(c*x)^(5/2)*(a + b*x^2)) + ((-2*(9*
A - (5*a*C)/b))/(5*a*c*(c*x)^(5/2)) + ((-2*(7*b*B - 3*a*D))/(3*c*(c*x)^(3/
2)) - (b*((-2*(9*A*b - 5*a*C))/(a*c*Sqrt[c*x]) + (2*(-1/2*(Sqrt[b]*(9*A*b
- 5*a*C - (Sqrt[a]*(7*b*B - 3*a*D))/Sqrt[b])*(-(ArcTan[1 - (Sqrt[2]*b^(1/4
)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[c])) + ArcTa
n[1 + (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])]/(Sqrt[2]*a^(1/4)*b^(1
/4)*Sqrt[c]))) + (Sqrt[b]*(9*A*b - 5*a*C + (Sqrt[a]*(7*b*B - 3*a*D))/Sqrt[
b])*(-1/2*Log[Sqrt[a]*c + Sqrt[b]*c*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[c]*Sq
rt[c*x]]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[c]) + Log[Sqrt[a]*c + Sqrt[b]*c*x +
 Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[c]*Sqrt[c*x]]/(2*Sqrt[2]*a^(1/4)*b^(1/4)*Sqr
t[c])))/2))/(a*c)))/c)/(a*b*c))/(4*a)
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 553
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Sym
bol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] + Simp
[1/(a*e*(m + 1))   Int[(e*x)^(m + 1)*(a + b*x^2)^p*(a*d*(m + 1) - b*c*(m +
2*p + 3)*x), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && LtQ[m, -1]
```

rule 554
```
Int[((c_) + (d_.)*(x_))/(Sqrt[(e_.)*(x_)]*((a_) + (b_.)*(x_)^2)), x_Symbol]
 :> Simp[2   Subst[Int[(e*c + d*x^2)/(a*e^2 + b*x^4), x], x, Sqrt[e*x]], x]
 /; FreeQ[{a, b, c, d, e}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 2337
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(-(c*x)^(m + 1))*(f + g*x)*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1)))
, x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*ExpandToSum[2
*a*(p + 1)*Q + f*(m + 2*p + 3) + g*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a
, b, c, m}, x] && PolyQ[Pq, x] && LtQ[p, -1] &&  !GtQ[m, 0]
```

## Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 421, normalized size of antiderivative = 1.02

| method | result |
|---|---|
| derivativedivides | $\frac{2\left(\left(\frac{1}{4}b^2A-\frac{1}{4}Cab\right)(cx)^{\frac{3}{2}}+\left(-\frac{1}{4}abBc+\frac{1}{4}Da^2c\right)\sqrt{cx}\right)}{b\,c^2x^2+a\,c^2}+\frac{\left(-7abBc+3Da^2c\right)\left(\frac{a\,c^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{cx+\left(\frac{a\,c^2}{b}\right)^{\frac{1}{4}}\sqrt{cx}\,\sqrt{2}+\sqrt{\frac{a\,c^2}{b}}}{cx-\left(\frac{a\,c^2}{b}\right)^{\frac{1}{4}}\sqrt{cx}\,\sqrt{2}+\sqrt{\frac{a\,c^2}{b}}}\right)+2\right.}{16a\,c^2}$ |
| default | $\frac{2\left(\left(\frac{1}{4}b^2A-\frac{1}{4}Cab\right)(cx)^{\frac{3}{2}}+\left(-\frac{1}{4}abBc+\frac{1}{4}Da^2c\right)\sqrt{cx}\right)}{b\,c^2x^2+a\,c^2}+\frac{\left(-7abBc+3Da^2c\right)\left(\frac{a\,c^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{cx+\left(\frac{a\,c^2}{b}\right)^{\frac{1}{4}}\sqrt{cx}\,\sqrt{2}+\sqrt{\frac{a\,c^2}{b}}}{cx-\left(\frac{a\,c^2}{b}\right)^{\frac{1}{4}}\sqrt{cx}\,\sqrt{2}+\sqrt{\frac{a\,c^2}{b}}}\right)+2\right.}{16a\,c^2}$ |
| pseudoelliptic | $-\frac{\left(7\left(b\,x^2+a\right)\sqrt{2}\left(\ln\left(\frac{cx+\left(\frac{a\,c^2}{b}\right)^{\frac{1}{4}}\sqrt{cx}\,\sqrt{2}+\sqrt{\frac{a\,c^2}{b}}}{cx-\left(\frac{a\,c^2}{b}\right)^{\frac{1}{4}}\sqrt{cx}\,\sqrt{2}+\sqrt{\frac{a\,c^2}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{cx}-\left(\frac{a\,c^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{a\,c^2}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{cx}+\left(\frac{a\,c^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{a\,c^2}{b}\right)^{\frac{1}{4}}}\right.\right.\right.}{}$ |

input
```
int((D*x^3+C*x^2+B*x+A)/(c*x)^(7/2)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
2/a^3/c^3*(((1/4*b^2*A-1/4*C*a*b)*(c*x)^(3/2)+(-1/4*a*b*B*c+1/4*D*a^2*c)*(
c*x)^(1/2))/(b*c^2*x^2+a*c^2)+1/32*(-7*B*a*b*c+3*D*a^2*c)*(a*c^2/b)^(1/4)/
a/c^2*2^(1/2)*(ln((c*x+(a*c^2/b)^(1/4)*(c*x)^(1/2)*2^(1/2)+(a*c^2/b)^(1/2)
)/(c*x-(a*c^2/b)^(1/4)*(c*x)^(1/2)*2^(1/2)+(a*c^2/b)^(1/2)))+2*arctan(2^(1
/2)/(a*c^2/b)^(1/4)*(c*x)^(1/2)+1)+2*arctan(2^(1/2)/(a*c^2/b)^(1/4)*(c*x)^
(1/2)-1))+1/32*(9*A*b^2-5*C*a*b)/b/(a*c^2/b)^(1/4)*2^(1/2)*(ln((c*x-(a*c^2
/b)^(1/4)*(c*x)^(1/2)*2^(1/2)+(a*c^2/b)^(1/2))/(c*x+(a*c^2/b)^(1/4)*(c*x)^
(1/2)*2^(1/2)+(a*c^2/b)^(1/2)))+2*arctan(2^(1/2)/(a*c^2/b)^(1/4)*(c*x)^(1/
2)+1)+2*arctan(2^(1/2)/(a*c^2/b)^(1/4)*(c*x)^(1/2)-1)))-2/3/a^2/c^2*B/(c*x
)^(3/2)-2/5/a^2/c*A/(c*x)^(5/2)-2*(-2*A*b+C*a)/a^3/c^3/(c*x)^(1/2)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3715 vs. 2(318) = 636.

Time = 0.61 (sec) , antiderivative size = 3715, normalized size of antiderivative = 9.04

$$\int \frac{A+Bx+Cx^2+Dx^3}{(cx)^{7/2}\left(a+bx^2\right)^2}\,dx=\text{Too large to display}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(c*x)^(7/2)/(b*x^2+a)^2,x, algorithm="fricas
")
```

output
```
Too large to include
```

## Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 135.63 (sec) , antiderivative size = 4245, normalized size of antiderivative = 10.33

$$\int \frac{A+Bx+Cx^2+Dx^3}{(cx)^{7/2}\left(a+bx^2\right)^2}\,dx=\text{Too large to display}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(c*x)**(7/2)/(b*x**2+a)**2,x)
```

output
```
A*(16*a**(13/4)*exp(3*I*pi/4)*gamma(-5/4)/(32*a**(21/4)*c**(7/2)*x**(5/2)*
exp(3*I*pi/4)*gamma(-1/4) + 32*a**(17/4)*b*c**(7/2)*x**(9/2)*exp(3*I*pi/4)
*gamma(-1/4)) - 144*a**(9/4)*b*x**2*exp(3*I*pi/4)*gamma(-5/4)/(32*a**(21/4
)*c**(7/2)*x**(5/2)*exp(3*I*pi/4)*gamma(-1/4) + 32*a**(17/4)*b*c**(7/2)*x*
*(9/2)*exp(3*I*pi/4)*gamma(-1/4)) - 180*a**(5/4)*b**2*x**4*exp(3*I*pi/4)*g
amma(-5/4)/(32*a**(21/4)*c**(7/2)*x**(5/2)*exp(3*I*pi/4)*gamma(-1/4) + 32*
a**(17/4)*b*c**(7/2)*x**(9/2)*exp(3*I*pi/4)*gamma(-1/4)) + 45*a**2*b**(5/4
)*x**(5/2)*log(1 - b**(1/4)*sqrt(x)*exp_polar(I*pi/4)/a**(1/4))*gamma(-5/4
)/(32*a**(21/4)*c**(7/2)*x**(5/2)*exp(3*I*pi/4)*gamma(-1/4) + 32*a**(17/4)
*b*c**(7/2)*x**(9/2)*exp(3*I*pi/4)*gamma(-1/4)) + 45*I*a**2*b**(5/4)*x**(5
/2)*log(1 - b**(1/4)*sqrt(x)*exp_polar(3*I*pi/4)/a**(1/4))*gamma(-5/4)/(32
*a**(21/4)*c**(7/2)*x**(5/2)*exp(3*I*pi/4)*gamma(-1/4) + 32*a**(17/4)*b*c*
*(7/2)*x**(9/2)*exp(3*I*pi/4)*gamma(-1/4)) - 45*a**2*b**(5/4)*x**(5/2)*log
(1 - b**(1/4)*sqrt(x)*exp_polar(5*I*pi/4)/a**(1/4))*gamma(-5/4)/(32*a**(21
/4)*c**(7/2)*x**(5/2)*exp(3*I*pi/4)*gamma(-1/4) + 32*a**(17/4)*b*c**(7/2)*
x**(9/2)*exp(3*I*pi/4)*gamma(-1/4)) - 45*I*a**2*b**(5/4)*x**(5/2)*log(1 -
b**(1/4)*sqrt(x)*exp_polar(7*I*pi/4)/a**(1/4))*gamma(-5/4)/(32*a**(21/4)*c
**(7/2)*x**(5/2)*exp(3*I*pi/4)*gamma(-1/4) + 32*a**(17/4)*b*c**(7/2)*x**(9
/2)*exp(3*I*pi/4)*gamma(-1/4)) + 45*a*b**(9/4)*x**(9/2)*log(1 - b**(1/4)*s
qrt(x)*exp_polar(I*pi/4)/a**(1/4))*gamma(-5/4)/(32*a**(21/4)*c**(7/2)*x...
```

## Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 499, normalized size of antiderivative = 1.21

$$\int \frac{A+Bx+Cx^2+Dx^3}{(cx)^{7/2}\,(a+bx^2)^2}\,dx =$$

$$-\frac{\frac{8\,(15\,(5\,Cab-9\,Ab^2)c^4x^4+20\,Ba^2c^4x-5\,(3\,Da^2-7\,Bab)c^4x^3+12\,Aa^2c^4+12\,(5\,Ca^2-9\,Aab)c^4x^2)}{(cx)^{\frac{9}{2}}a^3bc^2+(cx)^{\frac{5}{2}}a^4c^4} - \frac{15\left(\frac{\sqrt{2}\left(\left(5\,Cab-9\,Ab^2\right)\sqrt{a}c+\left(3\,Da^2\sqrt{b}-7\ldots\right.\right.}{\ldots}\right.}{\ldots}}{\ldots}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(c*x)^(7/2)/(b*x^2+a)^2,x, algorithm="maxima
")
```

output
```
-1/240*(8*(15*(5*C*a*b - 9*A*b^2)*c^4*x^4 + 20*B*a^2*c^4*x - 5*(3*D*a^2 -
7*B*a*b)*c^4*x^3 + 12*A*a^2*c^4 + 12*(5*C*a^2 - 9*A*a*b)*c^4*x^2)/((c*x)^(
9/2)*a^3*b*c^2 + (c*x)^(5/2)*a^4*c^4) - 15*(sqrt(2)*((5*C*a*b - 9*A*b^2)*s
qrt(a)*c + (3*D*a^2*sqrt(b) - 7*B*a*b^(3/2))*c)*log(sqrt(b)*c*x + sqrt(2)*
(a*c^2)^(1/4)*sqrt(c*x)*b^(1/4) + sqrt(a)*c)/((a*c^2)^(3/4)*b^(3/4)) - sqr
t(2)*((5*C*a*b - 9*A*b^2)*sqrt(a)*c + (3*D*a^2*sqrt(b) - 7*B*a*b^(3/2))*c)
*log(sqrt(b)*c*x - sqrt(2)*(a*c^2)^(1/4)*sqrt(c*x)*b^(1/4) + sqrt(a)*c)/((
a*c^2)^(3/4)*b^(3/4)) - 2*sqrt(2)*((5*C*a*b - 9*A*b^2)*sqrt(a)*c - (3*D*a^
2*sqrt(b) - 7*B*a*b^(3/2))*c)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*c^2)^(1/4)*b^
(1/4) + 2*sqrt(c*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*c))/(sqrt(sqrt(a)*sqrt(b
)*c)*sqrt(a)*sqrt(b)*c) - 2*sqrt(2)*((5*C*a*b - 9*A*b^2)*sqrt(a)*c - (3*D*
a^2*sqrt(b) - 7*B*a*b^(3/2))*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*c^2)^(1/4)
*b^(1/4) - 2*sqrt(c*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*c))/(sqrt(sqrt(a)*sqr
t(b)*c)*sqrt(a)*sqrt(b)*c))/(a^3*c^2))/c
```

## Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 585, normalized size of antiderivative = 1.42

$$\int \frac{A+Bx+Cx^2+Dx^3}{(cx)^{7/2}\,(a+bx^2)^2}\,dx = -\frac{\sqrt{cx}Cabcx-\sqrt{cx}Ab^2cx-\sqrt{cx}Da^2c+\sqrt{cx}Babc}{2\,(bc^2x^2+ac^2)a^3c^3}$$

$$+\frac{\sqrt{2}\Big(3\,(ab^3c^2)^{\frac14}\,Da^2bc-7\,(ab^3c^2)^{\frac14}\,Bab^2c-5\,(ab^3c^2)^{\frac34}\,Ca+9\,(ab^3c^2)^{\frac34}\,Ab\Big)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ac^2}{b}\right)^{\frac14}+2\sqrt{cx}\right)}{2\left(\frac{ac^2}{b}\right)^{\frac14}}\right.}{8\,a^4b^2c^5}$$

$$+\frac{\sqrt{2}\Big(3\,(ab^3c^2)^{\frac14}\,Da^2bc-7\,(ab^3c^2)^{\frac14}\,Bab^2c-5\,(ab^3c^2)^{\frac34}\,Ca+9\,(ab^3c^2)^{\frac34}\,Ab\Big)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ac^2}{b}\right)^{\frac14}-2\sqrt{cx}\right)}{2\left(\frac{ac^2}{b}\right)^{\frac14}}\right.}{8\,a^4b^2c^5}$$

$$+\frac{\sqrt{2}\Big(3\,(ab^3c^2)^{\frac14}\,Da^2bc-7\,(ab^3c^2)^{\frac14}\,Bab^2c+5\,(ab^3c^2)^{\frac34}\,Ca-9\,(ab^3c^2)^{\frac34}\,Ab\Big)\log\left(cx+\sqrt{2}\left(\frac{ac^2}{b}\right)^{\frac14}\sqrt{cx}+\right.}{16\,a^4b^2c^5}$$

$$-\frac{\sqrt{2}\Big(3\,(ab^3c^2)^{\frac14}\,Da^2bc-7\,(ab^3c^2)^{\frac14}\,Bab^2c+5\,(ab^3c^2)^{\frac34}\,Ca-9\,(ab^3c^2)^{\frac34}\,Ab\Big)\log\left(cx-\sqrt{2}\left(\frac{ac^2}{b}\right)^{\frac14}\sqrt{cx}+\right.}{16\,a^4b^2c^5}$$

$$-\frac{2\,(15\,Cac^2x^2-30\,Abc^2x^2+5\,Bac^2x+3\,Aac^2)}{15\sqrt{cx}a^3c^5x^2}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(c*x)^(7/2)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
-1/2*(sqrt(c*x)*C*a*b*c*x - sqrt(c*x)*A*b^2*c*x - sqrt(c*x)*D*a^2*c + sqrt
(c*x)*B*a*b*c)/((b*c^2*x^2 + a*c^2)*a^3*c^3) + 1/8*sqrt(2)*(3*(a*b^3*c^2)^
(1/4)*D*a^2*b*c - 7*(a*b^3*c^2)^(1/4)*B*a*b^2*c - 5*(a*b^3*c^2)^(3/4)*C*a
+ 9*(a*b^3*c^2)^(3/4)*A*b)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*c^2/b)^(1/4) + 2
*sqrt(c*x))/(a*c^2/b)^(1/4))/(a^4*b^2*c^5) + 1/8*sqrt(2)*(3*(a*b^3*c^2)^(1
/4)*D*a^2*b*c - 7*(a*b^3*c^2)^(1/4)*B*a*b^2*c - 5*(a*b^3*c^2)^(3/4)*C*a +
9*(a*b^3*c^2)^(3/4)*A*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*c^2/b)^(1/4) - 2*
sqrt(c*x))/(a*c^2/b)^(1/4))/(a^4*b^2*c^5) + 1/16*sqrt(2)*(3*(a*b^3*c^2)^(1
/4)*D*a^2*b*c - 7*(a*b^3*c^2)^(1/4)*B*a*b^2*c + 5*(a*b^3*c^2)^(3/4)*C*a -
9*(a*b^3*c^2)^(3/4)*A*b)*log(c*x + sqrt(2)*(a*c^2/b)^(1/4)*sqrt(c*x) + sqr
t(a*c^2/b))/(a^4*b^2*c^5) - 1/16*sqrt(2)*(3*(a*b^3*c^2)^(1/4)*D*a^2*b*c -
7*(a*b^3*c^2)^(1/4)*B*a*b^2*c + 5*(a*b^3*c^2)^(3/4)*C*a - 9*(a*b^3*c^2)^(3
/4)*A*b)*log(c*x - sqrt(2)*(a*c^2/b)^(1/4)*sqrt(c*x) + sqrt(a*c^2/b))/(a^4
*b^2*c^5) - 2/15*(15*C*a*c^2*x^2 - 30*A*b*c^2*x^2 + 5*B*a*c^2*x + 3*A*a*c^
2)/(sqrt(c*x)*a^3*c^5*x^2)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(cx)^{7/2}\left(a+bx^2\right)^2}\,dx=\int \frac{A+B\,x+C\,x^2+x^3\,D}{{\left(c\,x\right)}^{7/2}\,{\left(b\,x^2+a\right)}^2}\,dx$$

input
```
int((A + B*x + C*x^2 + x^3*D)/((c*x)^(7/2)*(a + b*x^2)^2),x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/((c*x)^(7/2)*(a + b*x^2)^2), x)
```

## Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 1360, normalized size of antiderivative = 3.31

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(cx)^{7/2} (a + bx^2)^2} \, dx = \text{Too large to display}$$

input
```
int((D*x^3+C*x^2+B*x+A)/(c*x)^(7/2)/(b*x^2+a)^2,x)
```

output
```
(sqrt(c)*( - 270*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)
*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b**2*x**2 + 1
50*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*s
qrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b*c*x**2 - 270*sqrt(x)*b**(
1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))
/(b**(1/4)*a**(1/4)*sqrt(2)))*b**3*x**4 + 150*sqrt(x)*b**(1/4)*a**(3/4)*sq
rt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1
/4)*sqrt(2)))*b**2*c*x**4 - 90*sqrt(x)*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**
(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a
**2*d*x**2 + 210*sqrt(x)*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)
*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b**2*x**2 - 9
0*sqrt(x)*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sq
rt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b*d*x**4 + 210*sqrt(x)*b**(3
/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/
(b**(1/4)*a**(1/4)*sqrt(2)))*b**3*x**4 + 270*sqrt(x)*b**(1/4)*a**(3/4)*sqr
t(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/
4)*sqrt(2)))*a*b**2*x**2 - 150*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**
(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a
*b*c*x**2 + 270*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*
sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*b**3*x**4 - 1...
```

# 3.136 $\int (cx)^m \left(a+bx^2\right)^2 \left(A+Bx+Cx^2+Dx^3\right) \, dx$



## Optimal result

Integrand size = 30, antiderivative size = 181

$$\int (cx)^m \left(a+bx^2\right)^2 \left(A+Bx+Cx^2+Dx^3\right) \, dx = \frac{a^2A(cx)^{1+m}}{c(1+m)}+\frac{a^2B(cx)^{2+m}}{c^2(2+m)}+\frac{a(2Ab+aC)(cx)^{3+m}}{c^3(3+m)}+\frac{a(2bB+aD)(cx)^{4+m}}{c^4(4+m)}+\frac{b(Ab+2aC)(cx)^{5+m}}{c^5(5+m)}+\frac{b(bB+2aD)(cx)^{6+m}}{c^6(6+m)}+\frac{b^2C(cx)^{7+m}}{c^7(7+m)}+\frac{b^2D(cx)^{8+m}}{c^8(8+m)}$$

output
```
a^2*A*(c*x)^(1+m)/c/(1+m)+a^2*B*(c*x)^(2+m)/c^2/(2+m)+a*(2*A*b+C*a)*(c*x)^
(3+m)/c^3/(3+m)+a*(2*B*b+D*a)*(c*x)^(4+m)/c^4/(4+m)+b*(A*b+2*C*a)*(c*x)^(5
+m)/c^5/(5+m)+b*(B*b+2*D*a)*(c*x)^(6+m)/c^6/(6+m)+b^2*C*(c*x)^(7+m)/c^7/(7
+m)+b^2*D*(c*x)^(8+m)/c^8/(8+m)
```

## Mathematica [A] (verified)

Time = 1.66 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.70

$$\int (cx)^m \left(a+bx^2\right)^2 \left(A+Bx+Cx^2+Dx^3\right) \, dx = x(cx)^m \left(\frac{a^2A}{1+m}+\frac{a^2Bx}{2+m}+\frac{a(2Ab+aC)x^2}{3+m}+\frac{a(2bB+aD)x^3}{4+m}+\frac{b(Ab+2aC)x^4}{5+m}+\frac{b(bB+2aD)x^5}{6+m}+\frac{b^2Cx^6}{7+m}+\frac{b^2Dx^7}{8+m}\right)$$

input
```
Integrate[(c*x)^m*(a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
x*(c*x)^m*((a^2*A)/(1 + m) + (a^2*B*x)/(2 + m) + (a*(2*A*b + a*C)*x^2)/(3
+ m) + (a*(2*b*B + a*D)*x^3)/(4 + m) + (b*(A*b + 2*a*C)*x^4)/(5 + m) + (b*
(b*B + 2*a*D)*x^5)/(6 + m) + (b^2*C*x^6)/(7 + m) + (b^2*D*x^7)/(8 + m))
```

## Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^2 (cx)^m \left(A+Bx+Cx^2+Dx^3\right) \, dx$$

$$\Big\downarrow \text{2333}$$

$$\int \left(a^2A(cx)^m+\frac{a^2B(cx)^{m+1}}{c}+\frac{b(cx)^{m+4}(2aC+Ab)}{c^4}+\frac{a(cx)^{m+2}(aC+2Ab)}{c^2}+\frac{b(cx)^{m+5}(2aD+bB)}{c^5}+\frac{a(cx)\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2A(cx)^{m+1}}{c(m+1)}+\frac{a^2B(cx)^{m+2}}{c^2(m+2)}+\frac{b(cx)^{m+5}(2aC+Ab)}{c^5(m+5)}+\frac{a(cx)^{m+3}(aC+2Ab)}{c^3(m+3)}+\frac{b(cx)^{m+6}(2aD+bB)}{c^6(m+6)}+\frac{a(cx)^{m+4}(aD+2bB)}{c^4(m+4)}+\frac{b^2D(cx)^{m+8}}{c^8(m+8)}+\frac{b^2C(cx)^{m+7}}{c^7(m+7)}$$

input
```
Int[(c*x)^m*(a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(a^2*A*(c*x)^(1 + m))/(c*(1 + m)) + (a^2*B*(c*x)^(2 + m))/(c^2*(2 + m)) +
(a*(2*A*b + a*C)*(c*x)^(3 + m))/(c^3*(3 + m)) + (a*(2*b*B + a*D)*(c*x)^(4
+ m))/(c^4*(4 + m)) + (b*(A*b + 2*a*C)*(c*x)^(5 + m))/(c^5*(5 + m)) + (b*(
b*B + 2*a*D)*(c*x)^(6 + m))/(c^6*(6 + m)) + (b^2*C*(c*x)^(7 + m))/(c^7*(7
+ m)) + (b^2*D*(c*x)^(8 + m))/(c^8*(8 + m))
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

## Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.99

| method | result |
|---|---|
| norman | $\frac{a(2Ab+Ca)x^3 e^{m\ln(cx)}}{3+m}+\frac{a(2Bb+Da)x^4 e^{m\ln(cx)}}{4+m}+\frac{a^2 A x\, e^{m\ln(cx)}}{1+m}+\frac{a^2 B\, x^2 e^{m\ln(cx)}}{2+m}+\frac{b(Ab+2Ca)x^5 e^{m\ln(cx)}}{5+m}+$ |
| gosper | Expression too large to display |
| orering | Expression too large to display |
| parallelrisch | Expression too large to display |

input
```
int((c*x)^m*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
a*(2*A*b+C*a)/(3+m)*x^3*exp(m*ln(c*x))+a*(2*B*b+D*a)/(4+m)*x^4*exp(m*ln(c*
x))+a^2*A/(1+m)*x*exp(m*ln(c*x))+a^2*B/(2+m)*x^2*exp(m*ln(c*x))+b*(A*b+2*C
*a)/(5+m)*x^5*exp(m*ln(c*x))+b*(B*b+2*D*a)/(6+m)*x^6*exp(m*ln(c*x))+b^2*C/
(7+m)*x^7*exp(m*ln(c*x))+b^2*D/(8+m)*x^8*exp(m*ln(c*x))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 829 vs. 2(181) = 362.

Time = 0.09 (sec) , antiderivative size = 829, normalized size of antiderivative = 4.58

$$\int (cx)^m \left(a+bx^2\right)^2 \left(A+Bx+Cx^2+Dx^3\right)\,dx = \text{Too large to display}$$

input
```
integrate((c*x)^m*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
((D*b^2*m^7 + 28*D*b^2*m^6 + 322*D*b^2*m^5 + 1960*D*b^2*m^4 + 6769*D*b^2*m
^3 + 13132*D*b^2*m^2 + 13068*D*b^2*m + 5040*D*b^2)*x^8 + (C*b^2*m^7 + 29*C
*b^2*m^6 + 343*C*b^2*m^5 + 2135*C*b^2*m^4 + 7504*C*b^2*m^3 + 14756*C*b^2*m
^2 + 14832*C*b^2*m + 5760*C*b^2)*x^7 + ((2*D*a*b + B*b^2)*m^7 + 30*(2*D*a*
b + B*b^2)*m^6 + 366*(2*D*a*b + B*b^2)*m^5 + 2340*(2*D*a*b + B*b^2)*m^4 +
8409*(2*D*a*b + B*b^2)*m^3 + 13440*D*a*b + 6720*B*b^2 + 16830*(2*D*a*b + B
*b^2)*m^2 + 17144*(2*D*a*b + B*b^2)*m)*x^6 + ((2*C*a*b + A*b^2)*m^7 + 31*(
2*C*a*b + A*b^2)*m^6 + 391*(2*C*a*b + A*b^2)*m^5 + 2581*(2*C*a*b + A*b^2)*
m^4 + 9544*(2*C*a*b + A*b^2)*m^3 + 16128*C*a*b + 8064*A*b^2 + 19564*(2*C*a
*b + A*b^2)*m^2 + 20304*(2*C*a*b + A*b^2)*m)*x^5 + ((D*a^2 + 2*B*a*b)*m^7
+ 32*(D*a^2 + 2*B*a*b)*m^6 + 418*(D*a^2 + 2*B*a*b)*m^5 + 2864*(D*a^2 + 2*B
*a*b)*m^4 + 10993*(D*a^2 + 2*B*a*b)*m^3 + 10080*D*a^2 + 20160*B*a*b + 2331
2*(D*a^2 + 2*B*a*b)*m^2 + 24876*(D*a^2 + 2*B*a*b)*m)*x^4 + ((C*a^2 + 2*A*a
*b)*m^7 + 33*(C*a^2 + 2*A*a*b)*m^6 + 447*(C*a^2 + 2*A*a*b)*m^5 + 3195*(C*a
^2 + 2*A*a*b)*m^4 + 12864*(C*a^2 + 2*A*a*b)*m^3 + 13440*C*a^2 + 26880*A*a*
b + 28692*(C*a^2 + 2*A*a*b)*m^2 + 32048*(C*a^2 + 2*A*a*b)*m)*x^3 + (B*a^2*
m^7 + 34*B*a^2*m^6 + 478*B*a^2*m^5 + 3580*B*a^2*m^4 + 15289*B*a^2*m^3 + 36
706*B*a^2*m^2 + 44712*B*a^2*m + 20160*B*a^2)*x^2 + (A*a^2*m^7 + 35*A*a^2*m
^6 + 511*A*a^2*m^5 + 4025*A*a^2*m^4 + 18424*A*a^2*m^3 + 48860*A*a^2*m^2 +
69264*A*a^2*m + 40320*A*a^2)*x)*(c*x)^m/(m^8 + 36*m^7 + 546*m^6 + 4536*...
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6390 vs. $2(167) = 334$.

Time = 1.04 (sec) , antiderivative size = 6390, normalized size of antiderivative = 35.30

$$\int (cx)^m \left(a+bx^2\right)^2 \left(A+Bx+Cx^2+Dx^3\right)\,dx = \text{Too large to display}$$

input
```
integrate((c*x)**m*(b*x**2+a)**2*(D*x**3+C*x**2+B*x+A),x)
```

output
```
Piecewise(((-A*a**2/(7*x**7) - 2*A*a*b/(5*x**5) - A*b**2/(3*x**3) - B*a**2
/(6*x**6) - B*a*b/(2*x**4) - B*b**2/(2*x**2) - C*a**2/(5*x**5) - 2*C*a*b/(
3*x**3) - C*b**2/x - D*a**2/(4*x**4) - D*a*b/x**2 + D*b**2*log(x))/c**8, E
q(m, -8)), ((-A*a**2/(6*x**6) - A*a*b/(2*x**4) - A*b**2/(2*x**2) - B*a**2/
(5*x**5) - 2*B*a*b/(3*x**3) - B*b**2/x - C*a**2/(4*x**4) - C*a*b/x**2 + C*
b**2*log(x) - D*a**2/(3*x**3) - 2*D*a*b/x + D*b**2*x)/c**7, Eq(m, -7)), ((
-A*a**2/(5*x**5) - 2*A*a*b/(3*x**3) - A*b**2/x - B*a**2/(4*x**4) - B*a*b/x
**2 + B*b**2*log(x) - C*a**2/(3*x**3) - 2*C*a*b/x + C*b**2*x - D*a**2/(2*x
**2) + 2*D*a*b*log(x) + D*b**2*x**2/2)/c**6, Eq(m, -6)), ((-A*a**2/(4*x**4
) - A*a*b/x**2 + A*b**2*log(x) - B*a**2/(3*x**3) - 2*B*a*b/x + B*b**2*x -
C*a**2/(2*x**2) + 2*C*a*b*log(x) + C*b**2*x**2/2 - D*a**2/x + 2*D*a*b*x +
D*b**2*x**3/3)/c**5, Eq(m, -5)), ((-A*a**2/(3*x**3) - 2*A*a*b/x + A*b**2*x
 - B*a**2/(2*x**2) + 2*B*a*b*log(x) + B*b**2*x**2/2 - C*a**2/x + 2*C*a*b*x
 + C*b**2*x**3/3 + D*a**2*log(x) + D*a*b*x**2 + D*b**2*x**4/4)/c**4, Eq(m,
 -4)), ((-A*a**2/(2*x**2) + 2*A*a*b*log(x) + A*b**2*x**2/2 - B*a**2/x + 2*
B*a*b*x + B*b**2*x**3/3 + C*a**2*log(x) + C*a*b*x**2 + C*b**2*x**4/4 + D*a
**2*x + 2*D*a*b*x**3/3 + D*b**2*x**5/5)/c**3, Eq(m, -3)), ((-A*a**2/x + 2*
A*a*b*x + A*b**2*x**3/3 + B*a**2*log(x) + B*a*b*x**2 + B*b**2*x**4/4 + C*a
**2*x + 2*C*a*b*x**3/3 + C*b**2*x**5/5 + D*a**2*x**2/2 + D*a*b*x**4/2 + D*
b**2*x**6/6)/c**2, Eq(m, -2)), ((A*a**2*log(x) + A*a*b*x**2 + A*b**2*x*...
```

## Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.27

$$\begin{aligned}\int (cx)^m \left(a+bx^2\right)^2 \left(A+Bx+Cx^2+Dx^3\right)\,dx &= \frac{Db^2c^mx^8x^m}{m+8}+\frac{Cb^2c^mx^7x^m}{m+7}+\frac{2\,Dabc^mx^6x^m}{m+6}+\frac{Bb^2c^mx^6x^m}{m+6}\\&\quad+\frac{2\,Cabc^mx^5x^m}{m+5}+\frac{Ab^2c^mx^5x^m}{m+5}+\frac{Da^2c^mx^4x^m}{m+4}+\frac{2\,Babc^mx^4x^m}{m+4}\\&\quad+\frac{Ca^2c^mx^3x^m}{m+3}+\frac{2\,Aabc^mx^3x^m}{m+3}+\frac{Ba^2c^mx^2x^m}{m+2}+\frac{(cx)^{m+1}\,Aa^2}{c(m+1)}\end{aligned}$$

input
```
integrate((c*x)^m*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
D*b^2*c^m*x^8*x^m/(m + 8) + C*b^2*c^m*x^7*x^m/(m + 7) + 2*D*a*b*c^m*x^6*x^
m/(m + 6) + B*b^2*c^m*x^6*x^m/(m + 6) + 2*C*a*b*c^m*x^5*x^m/(m + 5) + A*b^
2*c^m*x^5*x^m/(m + 5) + D*a^2*c^m*x^4*x^m/(m + 4) + 2*B*a*b*c^m*x^4*x^m/(m
 + 4) + C*a^2*c^m*x^3*x^m/(m + 3) + 2*A*a*b*c^m*x^3*x^m/(m + 3) + B*a^2*c^
m*x^2*x^m/(m + 2) + (c*x)^(m + 1)*A*a^2/(c*(m + 1))
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1558 vs. $2(181) = 362$.

Time = 0.15 (sec) , antiderivative size = 1558, normalized size of antiderivative = 8.61

$$\int (cx)^m \left(a+bx^2\right)^2 \left(A+Bx+Cx^2+Dx^3\right) \, dx = \text{Too large to display}$$

input
```
integrate((c*x)^m*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

output
```
((c*x)^m*D*b^2*m^7*x^8 + (c*x)^m*C*b^2*m^7*x^7 + 28*(c*x)^m*D*b^2*m^6*x^8
+ 2*(c*x)^m*D*a*b*m^7*x^6 + (c*x)^m*B*b^2*m^7*x^6 + 29*(c*x)^m*C*b^2*m^6*x
^7 + 322*(c*x)^m*D*b^2*m^5*x^8 + 2*(c*x)^m*C*a*b*m^7*x^5 + (c*x)^m*A*b^2*m
^7*x^5 + 60*(c*x)^m*D*a*b*m^6*x^6 + 30*(c*x)^m*B*b^2*m^6*x^6 + 343*(c*x)^m
*C*b^2*m^5*x^7 + 1960*(c*x)^m*D*b^2*m^4*x^8 + (c*x)^m*D*a^2*m^7*x^4 + 2*(c
*x)^m*B*a*b*m^7*x^4 + 62*(c*x)^m*C*a*b*m^6*x^5 + 31*(c*x)^m*A*b^2*m^6*x^5
+ 732*(c*x)^m*D*a*b*m^5*x^6 + 366*(c*x)^m*B*b^2*m^5*x^6 + 2135*(c*x)^m*C*b
^2*m^4*x^7 + 6769*(c*x)^m*D*b^2*m^3*x^8 + (c*x)^m*C*a^2*m^7*x^3 + 2*(c*x)^
m*A*a*b*m^7*x^3 + 32*(c*x)^m*D*a^2*m^6*x^4 + 64*(c*x)^m*B*a*b*m^6*x^4 + 78
2*(c*x)^m*C*a*b*m^5*x^5 + 391*(c*x)^m*A*b^2*m^5*x^5 + 4680*(c*x)^m*D*a*b*m
^4*x^6 + 2340*(c*x)^m*B*b^2*m^4*x^6 + 7504*(c*x)^m*C*b^2*m^3*x^7 + 13132*(
c*x)^m*D*b^2*m^2*x^8 + (c*x)^m*B*a^2*m^7*x^2 + 33*(c*x)^m*C*a^2*m^6*x^3 +
66*(c*x)^m*A*a*b*m^6*x^3 + 418*(c*x)^m*D*a^2*m^5*x^4 + 836*(c*x)^m*B*a*b*m
^5*x^4 + 5162*(c*x)^m*C*a*b*m^4*x^5 + 2581*(c*x)^m*A*b^2*m^4*x^5 + 16818*(
c*x)^m*D*a*b*m^3*x^6 + 8409*(c*x)^m*B*b^2*m^3*x^6 + 14756*(c*x)^m*C*b^2*m^
2*x^7 + 13068*(c*x)^m*D*b^2*m*x^8 + (c*x)^m*A*a^2*m^7*x + 34*(c*x)^m*B*a^2
*m^6*x^2 + 447*(c*x)^m*C*a^2*m^5*x^3 + 894*(c*x)^m*A*a*b*m^5*x^3 + 2864*(c
*x)^m*D*a^2*m^4*x^4 + 5728*(c*x)^m*B*a*b*m^4*x^4 + 19088*(c*x)^m*C*a*b*m^3
*x^5 + 9544*(c*x)^m*A*b^2*m^3*x^5 + 33660*(c*x)^m*D*a*b*m^2*x^6 + 16830*(c
*x)^m*B*b^2*m^2*x^6 + 14832*(c*x)^m*C*b^2*m*x^7 + 5040*(c*x)^m*D*b^2*x^...
```

## Mupad [F(-1)]

Timed out.

$$\int (cx)^m \left(a+bx^2\right)^2 \left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$=\int (c\,x)^m \left(b\,x^2+a\right)^2 \left(A+B\,x+C\,x^2+x^3\,D\right)\,dx$$

input
```
int((c*x)^m*(a + b*x^2)^2*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
int((c*x)^m*(a + b*x^2)^2*(A + B*x + C*x^2 + x^3*D), x)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 1061, normalized size of antiderivative = 5.86

$$\int (cx)^m \left(a+bx^2\right)^2 \left(A+Bx+Cx^2+Dx^3\right)\,dx = \text{Too large to display}$$

input
```
int((c*x)^m*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x)
```

output

```
(x**m*c**m*x*(a**3*m**7 + 35*a**3*m**6 + 511*a**3*m**5 + 4025*a**3*m**4 +
18424*a**3*m**3 + 48860*a**3*m**2 + 69264*a**3*m + 40320*a**3 + 2*a**2*b*m
**7*x**2 + a**2*b*m**7*x + 66*a**2*b*m**6*x**2 + 34*a**2*b*m**6*x + 894*a*
*2*b*m**5*x**2 + 478*a**2*b*m**5*x + 6390*a**2*b*m**4*x**2 + 3580*a**2*b*m
**4*x + 25728*a**2*b*m**3*x**2 + 15289*a**2*b*m**3*x + 57384*a**2*b*m**2*x
**2 + 36706*a**2*b*m**2*x + 64096*a**2*b*m*x**2 + 44712*a**2*b*m*x + 26880
*a**2*b*x**2 + 20160*a**2*b*x + a**2*c*m**7*x**2 + 33*a**2*c*m**6*x**2 + 4
47*a**2*c*m**5*x**2 + 3195*a**2*c*m**4*x**2 + 12864*a**2*c*m**3*x**2 + 286
92*a**2*c*m**2*x**2 + 32048*a**2*c*m*x**2 + 13440*a**2*c*x**2 + a**2*d*m**
7*x**3 + 32*a**2*d*m**6*x**3 + 418*a**2*d*m**5*x**3 + 2864*a**2*d*m**4*x**
3 + 10993*a**2*d*m**3*x**3 + 23312*a**2*d*m**2*x**3 + 24876*a**2*d*m*x**3
+ 10080*a**2*d*x**3 + a*b**2*m**7*x**4 + 2*a*b**2*m**7*x**3 + 31*a*b**2*m*
*6*x**4 + 64*a*b**2*m**6*x**3 + 391*a*b**2*m**5*x**4 + 836*a*b**2*m**5*x**
3 + 2581*a*b**2*m**4*x**4 + 5728*a*b**2*m**4*x**3 + 9544*a*b**2*m**3*x**4
+ 21986*a*b**2*m**3*x**3 + 19564*a*b**2*m**2*x**4 + 46624*a*b**2*m**2*x**3
 + 20304*a*b**2*m*x**4 + 49752*a*b**2*m*x**3 + 8064*a*b**2*x**4 + 20160*a*
b**2*x**3 + 2*a*b*c*m**7*x**4 + 62*a*b*c*m**6*x**4 + 782*a*b*c*m**5*x**4 +
 5162*a*b*c*m**4*x**4 + 19088*a*b*c*m**3*x**4 + 39128*a*b*c*m**2*x**4 + 40
608*a*b*c*m*x**4 + 16128*a*b*c*x**4 + 2*a*b*d*m**7*x**5 + 60*a*b*d*m**6*x*
*5 + 732*a*b*d*m**5*x**5 + 4680*a*b*d*m**4*x**5 + 16818*a*b*d*m**3*x**5...
```

# 3.137 $\int (cx)^m (a+bx^2) (A+Bx+Cx^2+Dx^3) \, dx$



## Optimal result

Integrand size = 28, antiderivative size = 119

$$\int (cx)^m (a+bx^2) (A+Bx+Cx^2+Dx^3) \, dx = \frac{aA(cx)^{1+m}}{c(1+m)}+\frac{aB(cx)^{2+m}}{c^2(2+m)}+\frac{(Ab+aC)(cx)^{3+m}}{c^3(3+m)} + \frac{(bB+aD)(cx)^{4+m}}{c^4(4+m)}+\frac{bC(cx)^{5+m}}{c^5(5+m)}+\frac{bD(cx)^{6+m}}{c^6(6+m)}$$

output
```
a*A*(c*x)^(1+m)/c/(1+m)+a*B*(c*x)^(2+m)/c^2/(2+m)+(A*b+C*a)*(c*x)^(3+m)/c^
3/(3+m)+(B*b+D*a)*(c*x)^(4+m)/c^4/(4+m)+b*C*(c*x)^(5+m)/c^5/(5+m)+b*D*(c*x
)^(6+m)/c^6/(6+m)
```

## Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.66

$$\int (cx)^m (a+bx^2) (A+Bx+Cx^2+Dx^3) \, dx = x(cx)^m \left(\frac{aA}{1+m}+\frac{aBx}{2+m}+\frac{(Ab+aC)x^2}{3+m}+\frac{(bB+aD)x^3}{4+m}+\frac{bCx^4}{5+m}+\frac{bDx^5}{6+m}\right)$$

input
```
Integrate[(c*x)^m*(a + b*x^2)*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
x*(c*x)^m*((a*A)/(1 + m) + (a*B*x)/(2 + m) + ((A*b + a*C)*x^2)/(3 + m) + (
(b*B + a*D)*x^3)/(4 + m) + (b*C*x^4)/(5 + m) + (b*D*x^5)/(6 + m))
```

## Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^2)(cx)^m (A+Bx+Cx^2+Dx^3)\,dx$$

$$\Big\downarrow \text{2333}$$

$$\int \left(\frac{(cx)^{m+2}(aC+Ab)}{c^2}+aA(cx)^m+\frac{(cx)^{m+3}(aD+bB)}{c^3}+\frac{aB(cx)^{m+1}}{c}+\frac{bD(cx)^{m+5}}{c^5}+\frac{bC(cx)^{m+4}}{c^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{(cx)^{m+3}(aC+Ab)}{c^3(m+3)}+\frac{aA(cx)^{m+1}}{c(m+1)}+\frac{(cx)^{m+4}(aD+bB)}{c^4(m+4)}+\frac{aB(cx)^{m+2}}{c^2(m+2)}+\frac{bD(cx)^{m+6}}{c^6(m+6)}+\frac{bC(cx)^{m+5}}{c^5(m+5)}$$

input
```
Int[(c*x)^m*(a + b*x^2)*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(a*A*(c*x)^(1 + m))/(c*(1 + m)) + (a*B*(c*x)^(2 + m))/(c^2*(2 + m)) + ((A*
b + a*C)*(c*x)^(3 + m))/(c^3*(3 + m)) + ((b*B + a*D)*(c*x)^(4 + m))/(c^4*(
4 + m)) + (b*C*(c*x)^(5 + m))/(c^5*(5 + m)) + (b*D*(c*x)^(6 + m))/(c^6*(6
+ m))
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

## Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.99

| method | result |
|---|---|
| norman | $\frac{(Ab+Ca)x^3 e^{m\ln(cx)}}{3+m}+\frac{(Bb+Da)x^4 e^{m\ln(cx)}}{4+m}+\frac{Aax\, e^{m\ln(cx)}}{1+m}+\frac{Ba\,x^2 e^{m\ln(cx)}}{2+m}+\frac{Cb\,x^5 e^{m\ln(cx)}}{5+m}+\frac{Db\,x^6 e^{m\ln(cx)}}{6+m}$ |
| gosper | $\frac{x(Dbm^5x^5+Cbm^5x^4+15Dbm^4x^5+Bbm^5x^3+16Cbm^4x^4+Dam^5x^3+85Dbm^3x^5+Abm^5x^2+17Bbm^4x^3+Cam^5x^2+95}{}$ |
| orering | $\frac{x(Dbm^5x^5+Cbm^5x^4+15Dbm^4x^5+Bbm^5x^3+16Cbm^4x^4+Dam^5x^3+85Dbm^3x^5+Abm^5x^2+17Bbm^4x^3+Cam^5x^2+95}{}$ |
| parallelrisch | $\frac{307Dx^4(cx)^m am^2+225Dx^6(cx)^m bm^2+15Dx^6(cx)^m bm^4+Bx^4(cx)^m bm^5+16Cx^5(cx)^m bm^4+85Dx^6(cx)^m bm^3+17Bx}{}$ |

input
```
int((c*x)^m*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
(A*b+C*a)/(3+m)*x^3*exp(m*ln(c*x))+(B*b+D*a)/(4+m)*x^4*exp(m*ln(c*x))+A*a/
(1+m)*x*exp(m*ln(c*x))+B*a/(2+m)*x^2*exp(m*ln(c*x))+C*b/(5+m)*x^5*exp(m*ln
(c*x))+D*b/(6+m)*x^6*exp(m*ln(c*x))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 339 vs. 2(119) = 238.

Time = 0.09 (sec) , antiderivative size = 339, normalized size of antiderivative = 2.85

$$\int (cx)^m\,(a+bx^2)\,(A+Bx+Cx^2+Dx^3)\;dx$$
$$=\frac{((Dbm^5+15\,Dbm^4+85\,Dbm^3+225\,Dbm^2+274\,Dbm+120\,Db)x^6+(Cbm^5+16\,Cbm^4+95\,Cbm^3+}{}$$

input
```
integrate((c*x)^m*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
((D*b*m^5 + 15*D*b*m^4 + 85*D*b*m^3 + 225*D*b*m^2 + 274*D*b*m + 120*D*b)*x
^6 + (C*b*m^5 + 16*C*b*m^4 + 95*C*b*m^3 + 260*C*b*m^2 + 324*C*b*m + 144*C*
b)*x^5 + ((D*a + B*b)*m^5 + 17*(D*a + B*b)*m^4 + 107*(D*a + B*b)*m^3 + 307
*(D*a + B*b)*m^2 + 180*D*a + 180*B*b + 396*(D*a + B*b)*m)*x^4 + ((C*a + A*
b)*m^5 + 18*(C*a + A*b)*m^4 + 121*(C*a + A*b)*m^3 + 372*(C*a + A*b)*m^2 +
240*C*a + 240*A*b + 508*(C*a + A*b)*m)*x^3 + (B*a*m^5 + 19*B*a*m^4 + 137*B
*a*m^3 + 461*B*a*m^2 + 702*B*a*m + 360*B*a)*x^2 + (A*a*m^5 + 20*A*a*m^4 +
155*A*a*m^3 + 580*A*a*m^2 + 1044*A*a*m + 720*A*a)*x)*(c*x)^m/(m^6 + 21*m^5
 + 175*m^4 + 735*m^3 + 1624*m^2 + 1764*m + 720)
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2504 vs. $2(105) = 210$.

Time = 0.61 (sec) , antiderivative size = 2504, normalized size of antiderivative = 21.04

$$\int (cx)^m \left(a+bx^2\right) \left(A+Bx+Cx^2+Dx^3\right) dx = \text{Too large to display}$$

input
```
integrate((c*x)**m*(b*x**2+a)*(D*x**3+C*x**2+B*x+A),x)
```

output
```
Piecewise(((-A*a/(5*x**5) - A*b/(3*x**3) - B*a/(4*x**4) - B*b/(2*x**2) - C
*a/(3*x**3) - C*b/x - D*a/(2*x**2) + D*b*log(x))/c**6, Eq(m, -6)), ((-A*a/
(4*x**4) - A*b/(2*x**2) - B*a/(3*x**3) - B*b/x - C*a/(2*x**2) + C*b*log(x)
 - D*a/x + D*b*x)/c**5, Eq(m, -5)), ((-A*a/(3*x**3) - A*b/x - B*a/(2*x**2)
 + B*b*log(x) - C*a/x + C*b*x + D*a*log(x) + D*b*x**2/2)/c**4, Eq(m, -4)),
 ((-A*a/(2*x**2) + A*b*log(x) - B*a/x + B*b*x + C*a*log(x) + C*b*x**2/2 +
D*a*x + D*b*x**3/3)/c**3, Eq(m, -3)), ((-A*a/x + A*b*x + B*a*log(x) + B*b*
x**2/2 + C*a*x + C*b*x**3/3 + D*a*x**2/2 + D*b*x**4/4)/c**2, Eq(m, -2)), (
(A*a*log(x) + A*b*x**2/2 + B*a*x + B*b*x**3/3 + C*a*x**2/2 + C*b*x**4/4 +
D*a*x**3/3 + D*b*x**5/5)/c, Eq(m, -1)), (A*a*m**5*x*(c*x)**m/(m**6 + 21*m*
*5 + 175*m**4 + 735*m**3 + 1624*m**2 + 1764*m + 720) + 20*A*a*m**4*x*(c*x)
**m/(m**6 + 21*m**5 + 175*m**4 + 735*m**3 + 1624*m**2 + 1764*m + 720) + 15
5*A*a*m**3*x*(c*x)**m/(m**6 + 21*m**5 + 175*m**4 + 735*m**3 + 1624*m**2 +
1764*m + 720) + 580*A*a*m**2*x*(c*x)**m/(m**6 + 21*m**5 + 175*m**4 + 735*m
**3 + 1624*m**2 + 1764*m + 720) + 1044*A*a*m*x*(c*x)**m/(m**6 + 21*m**5 +
175*m**4 + 735*m**3 + 1624*m**2 + 1764*m + 720) + 720*A*a*x*(c*x)**m/(m**6
 + 21*m**5 + 175*m**4 + 735*m**3 + 1624*m**2 + 1764*m + 720) + A*b*m**5*x*
*3*(c*x)**m/(m**6 + 21*m**5 + 175*m**4 + 735*m**3 + 1624*m**2 + 1764*m + 7
20) + 18*A*b*m**4*x**3*(c*x)**m/(m**6 + 21*m**5 + 175*m**4 + 735*m**3 + 16
24*m**2 + 1764*m + 720) + 121*A*b*m**3*x**3*(c*x)**m/(m**6 + 21*m**5 + ...
```

## Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.16

$$\int (cx)^m \left(a+bx^2\right) \left(A+Bx+Cx^2+Dx^3\right) \, dx = \frac{Dbc^m x^6 x^m}{m+6} + \frac{Cbc^m x^5 x^m}{m+5} + \frac{Dac^m x^4 x^m}{m+4} + \frac{Bbc^m x^4 x^m}{m+4} + \frac{Cac^m x^3 x^m}{m+3} + \frac{Abc^m x^3 x^m}{m+3} + \frac{Bac^m x^2 x^m}{m+2} + \frac{(cx)^{m+1} Aa}{c(m+1)}$$

input
```
integrate((c*x)^m*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
D*b*c^m*x^6*x^m/(m + 6) + C*b*c^m*x^5*x^m/(m + 5) + D*a*c^m*x^4*x^m/(m + 4
) + B*b*c^m*x^4*x^m/(m + 4) + C*a*c^m*x^3*x^m/(m + 3) + A*b*c^m*x^3*x^m/(m
 + 3) + B*a*c^m*x^2*x^m/(m + 2) + (c*x)^(m + 1)*A*a/(c*(m + 1))
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 692 vs. $2(119) = 238$.

Time = 0.13 (sec) , antiderivative size = 692, normalized size of antiderivative = 5.82

$$\int (cx)^m \left(a+bx^2\right)\left(A+Bx+Cx^2+Dx^3\right) \, dx = \text{Too large to display}$$

input
```
integrate((c*x)^m*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

output
```
((c*x)^m*D*b*m^5*x^6 + (c*x)^m*C*b*m^5*x^5 + 15*(c*x)^m*D*b*m^4*x^6 + (c*x
)^m*D*a*m^5*x^4 + (c*x)^m*B*b*m^5*x^4 + 16*(c*x)^m*C*b*m^4*x^5 + 85*(c*x)^
m*D*b*m^3*x^6 + (c*x)^m*C*a*m^5*x^3 + (c*x)^m*A*b*m^5*x^3 + 17*(c*x)^m*D*a
*m^4*x^4 + 17*(c*x)^m*B*b*m^4*x^4 + 95*(c*x)^m*C*b*m^3*x^5 + 225*(c*x)^m*D
*b*m^2*x^6 + (c*x)^m*B*a*m^5*x^2 + 18*(c*x)^m*C*a*m^4*x^3 + 18*(c*x)^m*A*b
*m^4*x^3 + 107*(c*x)^m*D*a*m^3*x^4 + 107*(c*x)^m*B*b*m^3*x^4 + 260*(c*x)^m
*C*b*m^2*x^5 + 274*(c*x)^m*D*b*m*x^6 + (c*x)^m*A*a*m^5*x + 19*(c*x)^m*B*a*
m^4*x^2 + 121*(c*x)^m*C*a*m^3*x^3 + 121*(c*x)^m*A*b*m^3*x^3 + 307*(c*x)^m*
D*a*m^2*x^4 + 307*(c*x)^m*B*b*m^2*x^4 + 324*(c*x)^m*C*b*m*x^5 + 120*(c*x)^
m*D*b*x^6 + 20*(c*x)^m*A*a*m^4*x + 137*(c*x)^m*B*a*m^3*x^2 + 372*(c*x)^m*C
*a*m^2*x^3 + 372*(c*x)^m*A*b*m^2*x^3 + 396*(c*x)^m*D*a*m*x^4 + 396*(c*x)^m
*B*b*m*x^4 + 144*(c*x)^m*C*b*x^5 + 155*(c*x)^m*A*a*m^3*x + 461*(c*x)^m*B*a
*m^2*x^2 + 508*(c*x)^m*C*a*m*x^3 + 508*(c*x)^m*A*b*m*x^3 + 180*(c*x)^m*D*a
*x^4 + 180*(c*x)^m*B*b*x^4 + 580*(c*x)^m*A*a*m^2*x + 702*(c*x)^m*B*a*m*x^2
 + 240*(c*x)^m*C*a*x^3 + 240*(c*x)^m*A*b*x^3 + 1044*(c*x)^m*A*a*m*x + 360*
(c*x)^m*B*a*x^2 + 720*(c*x)^m*A*a*x)/(m^6 + 21*m^5 + 175*m^4 + 735*m^3 + 1
624*m^2 + 1764*m + 720)
```

## Mupad [F(-1)]

Timed out.

$$\int (cx)^m \left(a+bx^2\right)\left(A+Bx+Cx^2+Dx^3\right) \, dx = \int (c\,x)^m\,\left(b\,x^2+a\right)\,\left(A+B\,x+C\,x^2+x^3\,D\right) \,d x$$

input
```
int((c*x)^m*(a + b*x^2)*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
int((c*x)^m*(a + b*x^2)*(A + B*x + C*x^2 + x^3*D), x)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 453, normalized size of antiderivative = 3.81

$$\int (cx)^m (a+bx^2) (A+Bx+Cx^2+Dx^3)\, dx$$
$$= \frac{x^m c^m x(bd\, m^5 x^5 + bc\, m^5 x^4 + 15bd\, m^4 x^5 + ad\, m^5 x^3 + b^2 m^5 x^3 + 16bc\, m^4 x^4 + 85bd\, m^3 x^5 + ab\, m^5 x^2 + ac}{}$$

input
```
int((c*x)^m*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x)
```

output
```
(x**m*c**m*x*(a**2*m**5 + 20*a**2*m**4 + 155*a**2*m**3 + 580*a**2*m**2 + 1
044*a**2*m + 720*a**2 + a*b*m**5*x**2 + a*b*m**5*x + 18*a*b*m**4*x**2 + 19
*a*b*m**4*x + 121*a*b*m**3*x**2 + 137*a*b*m**3*x + 372*a*b*m**2*x**2 + 461
*a*b*m**2*x + 508*a*b*m*x**2 + 702*a*b*m*x + 240*a*b*x**2 + 360*a*b*x + a*
c*m**5*x**2 + 18*a*c*m**4*x**2 + 121*a*c*m**3*x**2 + 372*a*c*m**2*x**2 + 5
08*a*c*m*x**2 + 240*a*c*x**2 + a*d*m**5*x**3 + 17*a*d*m**4*x**3 + 107*a*d*
m**3*x**3 + 307*a*d*m**2*x**3 + 396*a*d*m*x**3 + 180*a*d*x**3 + b**2*m**5*
x**3 + 17*b**2*m**4*x**3 + 107*b**2*m**3*x**3 + 307*b**2*m**2*x**3 + 396*b
**2*m*x**3 + 180*b**2*x**3 + b*c*m**5*x**4 + 16*b*c*m**4*x**4 + 95*b*c*m**
3*x**4 + 260*b*c*m**2*x**4 + 324*b*c*m*x**4 + 144*b*c*x**4 + b*d*m**5*x**5
 + 15*b*d*m**4*x**5 + 85*b*d*m**3*x**5 + 225*b*d*m**2*x**5 + 274*b*d*m*x**
5 + 120*b*d*x**5))/(m**6 + 21*m**5 + 175*m**4 + 735*m**3 + 1624*m**2 + 176
4*m + 720)
```

## 3.138 $\int (cx)^m (A+Bx+Cx^2+Dx^3)\,dx$



### Optimal result

Integrand size = 21, antiderivative size = 69

$$\int (cx)^m \left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{A(cx)^{1+m}}{c(1+m)}+\frac{B(cx)^{2+m}}{c^2(2+m)}+\frac{C(cx)^{3+m}}{c^3(3+m)}+\frac{D(cx)^{4+m}}{c^4(4+m)}$$

output
```
A*(c*x)^(1+m)/c/(1+m)+B*(c*x)^(2+m)/c^2/(2+m)+C*(c*x)^(3+m)/c^3/(3+m)+D*(c
*x)^(4+m)/c^4/(4+m)
```

### Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.62

$$\int (cx)^m \left(A+Bx+Cx^2+Dx^3\right)\,dx=x(cx)^m\left(\frac{A}{1+m}+x\left(\frac{B}{2+m}+x\left(\frac{C}{3+m}+\frac{Dx}{4+m}\right)\right)\right)$$

input
```
Integrate[(c*x)^m*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
x*(c*x)^m*(A/(1 + m) + x*(B/(2 + m) + x*(C/(3 + m) + (D*x)/(4 + m))))
```

## Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^m \left(A+Bx+Cx^2+Dx^3\right) \, dx$$

$$\Big\downarrow \text{2010}$$

$$\int \left(A(cx)^m+\frac{B(cx)^{m+1}}{c}+\frac{D(cx)^{m+3}}{c^3}+\frac{C(cx)^{m+2}}{c^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{A(cx)^{m+1}}{c(m+1)}+\frac{B(cx)^{m+2}}{c^2(m+2)}+\frac{D(cx)^{m+4}}{c^4(m+4)}+\frac{C(cx)^{m+3}}{c^3(m+3)}$$

input
```
Int[(c*x)^m*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(A*(c*x)^(1 + m))/(c*(1 + m)) + (B*(c*x)^(2 + m))/(c^2*(2 + m)) + (C*(c*x)
^(3 + m))/(c^3*(3 + m)) + (D*(c*x)^(4 + m))/(c^4*(4 + m))
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

## Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.99

| method | result |
|---|---|
| norman | $\frac{Ax\,\mathrm{e}^{m\ln(cx)}}{1+m}+\frac{B\,x^2\mathrm{e}^{m\ln(cx)}}{2+m}+\frac{C\,x^3\mathrm{e}^{m\ln(cx)}}{3+m}+\frac{Dx^4\mathrm{e}^{m\ln(cx)}}{4+m}$ |
| gosper | $\frac{x(Dm^3x^3+C\,m^3x^2+6Dm^2x^3+B\,m^3x+7C\,m^2x^2+11Dm\,x^3+A\,m^3+8B\,m^2x+14Cm\,x^2+6Dx^3+9A\,m^2+19Bmx+8C\,x^2+}{(4+m)(3+m)(2+m)(1+m)}$ |
| orering | $\frac{x(Dm^3x^3+C\,m^3x^2+6Dm^2x^3+B\,m^3x+7C\,m^2x^2+11Dm\,x^3+A\,m^3+8B\,m^2x+14Cm\,x^2+6Dx^3+9A\,m^2+19Bmx+8C\,x^2+}{(4+m)(3+m)(2+m)(1+m)}$ |
| parallelrisch | $\frac{Dx^4(cx)^m m^3+C\,x^3(cx)^m m^3+6Dx^4(cx)^m m^2+B\,x^2(cx)^m m^3+7C\,x^3(cx)^m m^2+11Dx^4(cx)^m m+Ax(cx)^m m^3+8B\,x^2(cx)^r}{(4+m)(3+m}$ |

input
```
int((c*x)^m*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
A/(1+m)*x*exp(m*ln(c*x))+B/(2+m)*x^2*exp(m*ln(c*x))+C/(3+m)*x^3*exp(m*ln(c
*x))+D/(4+m)*x^4*exp(m*ln(c*x))
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.70

$$\int (cx)^m\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$=\frac{((Dm^3+6\,Dm^2+11\,Dm+6\,D)x^4+(Cm^3+7\,Cm^2+14\,Cm+8\,C)x^3+(Bm^3+8\,Bm^2+19\,Bm+}{m^4+10\,m^3+35\,m^2+50\,m+24}$$

input
```
integrate((c*x)^m*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
((D*m^3 + 6*D*m^2 + 11*D*m + 6*D)*x^4 + (C*m^3 + 7*C*m^2 + 14*C*m + 8*C)*x
^3 + (B*m^3 + 8*B*m^2 + 19*B*m + 12*B)*x^2 + (A*m^3 + 9*A*m^2 + 26*A*m + 2
4*A)*x)*(c*x)^m/(m^4 + 10*m^3 + 35*m^2 + 50*m + 24)
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 604 vs. $2(58) = 116$.

Time = 0.48 (sec) , antiderivative size = 604, normalized size of antiderivative = 8.75

$$\int (cx)^m \left(A + Bx + Cx^2 + Dx^3\right) dx$$

$$= \begin{cases} \frac{-\frac{A}{3x^3} - \frac{B}{2x^2} - \frac{C}{x} + D\log(x)}{c^4} \\ \frac{-\frac{A}{2x^2} - \frac{B}{x} + C\log(x) + Dx}{c^3} \\ \frac{-\frac{A}{x} + B\log(x) + Cx + \frac{Dx^2}{2}}{c^2} \\ \frac{A\log(x) + Bx + \frac{Cx^2}{2} + \frac{Dx^3}{3}}{c} \\ \frac{Am^3x(cx)^m}{m^4+10m^3+35m^2+50m+24} + \frac{9Am^2x(cx)^m}{m^4+10m^3+35m^2+50m+24} + \frac{26Amx(cx)^m}{m^4+10m^3+35m^2+50m+24} + \frac{24Ax(cx)^m}{m^4+10m^3+35m^2+50m+24} + \frac{}{m^4+10} \end{cases}$$

input
```
integrate((c*x)**m*(D*x**3+C*x**2+B*x+A),x)
```

output
```
Piecewise(((-A/(3*x**3) - B/(2*x**2) - C/x + D*log(x))/c**4, Eq(m, -4)), (
(-A/(2*x**2) - B/x + C*log(x) + D*x)/c**3, Eq(m, -3)), ((-A/x + B*log(x) +
 C*x + D*x**2/2)/c**2, Eq(m, -2)), ((A*log(x) + B*x + C*x**2/2 + D*x**3/3)
/c, Eq(m, -1)), (A*m**3*x*(c*x)**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24)
+ 9*A*m**2*x*(c*x)**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + 26*A*m*x*(c
*x)**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + 24*A*x*(c*x)**m/(m**4 + 10
*m**3 + 35*m**2 + 50*m + 24) + B*m**3*x**2*(c*x)**m/(m**4 + 10*m**3 + 35*m
**2 + 50*m + 24) + 8*B*m**2*x**2*(c*x)**m/(m**4 + 10*m**3 + 35*m**2 + 50*m
 + 24) + 19*B*m*x**2*(c*x)**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + 12*
B*x**2*(c*x)**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + C*m**3*x**3*(c*x)
**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + 7*C*m**2*x**3*(c*x)**m/(m**4
+ 10*m**3 + 35*m**2 + 50*m + 24) + 14*C*m*x**3*(c*x)**m/(m**4 + 10*m**3 +
35*m**2 + 50*m + 24) + 8*C*x**3*(c*x)**m/(m**4 + 10*m**3 + 35*m**2 + 50*m
+ 24) + D*m**3*x**4*(c*x)**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + 6*D*
m**2*x**4*(c*x)**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + 11*D*m*x**4*(c
*x)**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + 6*D*x**4*(c*x)**m/(m**4 +
10*m**3 + 35*m**2 + 50*m + 24), True))
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96

$$\int (cx)^m \left(A+Bx+Cx^2+Dx^3\right) dx = \frac{Dc^m x^4 x^m}{m+4} + \frac{Cc^m x^3 x^m}{m+3} + \frac{Bc^m x^2 x^m}{m+2} + \frac{(cx)^{m+1} A}{c(m+1)}$$

input
```
integrate((c*x)^m*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
D*c^m*x^4*x^m/(m + 4) + C*c^m*x^3*x^m/(m + 3) + B*c^m*x^2*x^m/(m + 2) + (c
*x)^(m + 1)*A/(c*(m + 1))
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 214 vs. 2(69) = 138.

Time = 0.12 (sec) , antiderivative size = 214, normalized size of antiderivative = 3.10

$$\int (cx)^m \left(A+Bx+Cx^2+Dx^3\right) dx$$
$$= \frac{(cx)^m Dm^3x^4 + (cx)^m Cm^3x^3 + 6(cx)^m Dm^2x^4 + (cx)^m Bm^3x^2 + 7(cx)^m Cm^2x^3 + 11(cx)^m Dmx^4 + \cdots}{}$$

input
```
integrate((c*x)^m*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

output
```
((c*x)^m*D*m^3*x^4 + (c*x)^m*C*m^3*x^3 + 6*(c*x)^m*D*m^2*x^4 + (c*x)^m*B*m
^3*x^2 + 7*(c*x)^m*C*m^2*x^3 + 11*(c*x)^m*D*m*x^4 + (c*x)^m*A*m^3*x + 8*(c
*x)^m*B*m^2*x^2 + 14*(c*x)^m*C*m*x^3 + 6*(c*x)^m*D*x^4 + 9*(c*x)^m*A*m^2*x
 + 19*(c*x)^m*B*m*x^2 + 8*(c*x)^m*C*x^3 + 26*(c*x)^m*A*m*x + 12*(c*x)^m*B*
x^2 + 24*(c*x)^m*A*x)/(m^4 + 10*m^3 + 35*m^2 + 50*m + 24)
```

## Mupad [F(-1)]

Timed out.

$$\int (cx)^m \left(A+Bx+Cx^2+Dx^3\right) \, dx=\int (c\,x)^m\,\left(A+B\,x+C\,x^2+x^3\,D\right) \, dx$$

input
```
int((c*x)^m*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
int((c*x)^m*(A + B*x + C*x^2 + x^3*D), x)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.87

$$\int (cx)^m \left(A+Bx+Cx^2+Dx^3\right) \, dx$$
$$=\frac{x^m c^m x (d\,m^3x^3+c\,m^3x^2+6d\,m^2x^3+b\,m^3x+7c\,m^2x^2+11dm\,x^3+a\,m^3+8b\,m^2x+14cm\,x^2+6d\,x^3+}{m^4+10m^3+35m^2+50m+24}$$

input
```
int((c*x)^m*(D*x^3+C*x^2+B*x+A),x)
```

output
```
(x**m*c**m*x*(a*m**3 + 9*a*m**2 + 26*a*m + 24*a + b*m**3*x + 8*b*m**2*x +
19*b*m*x + 12*b*x + c*m**3*x**2 + 7*c*m**2*x**2 + 14*c*m*x**2 + 8*c*x**2 +
 d*m**3*x**3 + 6*d*m**2*x**3 + 11*d*m*x**3 + 6*d*x**3))/(m**4 + 10*m**3 +
35*m**2 + 50*m + 24)
```

# 3.139 $\int \frac{(cx)^m(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 151

$$\begin{aligned}\int \frac{(cx)^m\left(A+Bx+Cx^2+Dx^3\right)}{a+bx^2}\,dx &= \frac{C(cx)^{1+m}}{bc(1+m)}+\frac{D(cx)^{2+m}}{bc^2(2+m)} \\ &\quad + \frac{(Ab-aC)(cx)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{bx^2}{a}\right)}{abc(1+m)} \\ &\quad + \frac{(bB-aD)(cx)^{2+m}\,\text{Hypergeometric2F1}\left(1,\frac{2+m}{2},\frac{4+m}{2},-\frac{bx^2}{a}\right)}{abc^2(2+m)}\end{aligned}$$

output
```
C*(c*x)^(1+m)/b/c/(1+m)+D*(c*x)^(2+m)/b/c^2/(2+m)+(A*b-C*a)*(c*x)^(1+m)*hy
pergeom([1, 1/2+1/2*m],[3/2+1/2*m],-b*x^2/a)/a/b/c/(1+m)+(B*b-D*a)*(c*x)^(
2+m)*hypergeom([1, 1+1/2*m],[2+1/2*m],-b*x^2/a)/a/b/c^2/(2+m)
```

## Mathematica [A] (verified)

Time = 0.96 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.74

$$\int \frac{(cx)^m (A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx$$
$$= \frac{x(cx)^m \left(aC(2+m)+aD(1+m)x+(Ab-aC)(2+m)\operatorname{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{bx^2}{a}\right)+(b\right.}{ab(1+m)(2+m)}$$

input
```
Integrate[((c*x)^m*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2),x]
```

output
```
(x*(c*x)^m*(a*C*(2 + m) + a*D*(1 + m)*x + (A*b - a*C)*(2 + m)*Hypergeometr
ic2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)] + (b*B - a*D)*(1 + m)*x*Hyper
geometric2F1[1, (2 + m)/2, (4 + m)/2, -((b*x^2)/a)]))/(a*b*(1 + m)*(2 + m)
)
```

## Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^m (A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx$$
$$\Big\downarrow \ 2333$$
$$\int \left(\frac{(cx)^m(x(bB-aD)-aC+Ab)}{b(a+bx^2)}+\frac{C(cx)^m}{b}+\frac{D(cx)^{m+1}}{bc}\right)dx$$
$$\Big\downarrow \ 2009$$
$$\frac{(cx)^{m+1}(Ab-aC)\operatorname{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\frac{bx^2}{a}\right)}{abc(m+1)}+\frac{(cx)^{m+2}(bB-aD)\operatorname{Hypergeometric2F1}\left(1,\frac{m+2}{2},\frac{m+4}{2},-\frac{bx^2}{a}\right)}{abc^2(m+2)}+\frac{D(cx)^{m+2}}{bc^2(m+2)}+\frac{C(cx)^{m+1}}{bc(m+1)}$$

input
```
Int[((c*x)^m*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2),x]
```

output
```
(C*(c*x)^(1 + m))/(b*c*(1 + m)) + (D*(c*x)^(2 + m))/(b*c^2*(2 + m)) + ((A*
b - a*C)*(c*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2
)/a)])/(a*b*c*(1 + m)) + ((b*B - a*D)*(c*x)^(2 + m)*Hypergeometric2F1[1, (
2 + m)/2, (4 + m)/2, -((b*x^2)/a)])/(a*b*c^2*(2 + m))
```

**Defintions of rubi rules used**

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

## Maple [F]

$$\int \frac{(cx)^m (Dx^3 + C\,x^2 + Bx + A)}{b\,x^2 + a}dx$$

input
```
int((c*x)^m*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x)
```

output
```
int((c*x)^m*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x)
```

## Fricas [F]

$$\int \frac{(cx)^m (A + Bx + Cx^2 + Dx^3)}{a + bx^2}\,dx = \int \frac{(Dx^3 + Cx^2 + Bx + A)(cx)^m}{bx^2 + a}\,dx$$

input
```
integrate((c*x)^m*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="fricas")
```

output
```
integral((D*x^3 + C*x^2 + B*x + A)*(c*x)^m/(b*x^2 + a), x)
```

## Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.97 (sec) , antiderivative size = 379, normalized size of antiderivative = 2.51

$$\int \frac{(cx)^m (A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx = \frac{Ac^m m x^{m+1}\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)} + \frac{Ac^m x^{m+1}\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)} + \frac{Bc^m m x^{m+2}\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+1\right)\Gamma\left(\frac{m}{2}+1\right)}{4a\Gamma\left(\frac{m}{2}+2\right)} + \frac{Bc^m x^{m+2}\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+1\right)\Gamma\left(\frac{m}{2}+1\right)}{2a\Gamma\left(\frac{m}{2}+2\right)} + \frac{Cc^m m x^{m+3}\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{3}{2}\right)\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)} + \frac{3Cc^m x^{m+3}\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{3}{2}\right)\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)} + \frac{Dc^m m x^{m+4}\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+2\right)\Gamma\left(\frac{m}{2}+2\right)}{4a\Gamma\left(\frac{m}{2}+3\right)} + \frac{Dc^m x^{m+4}\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+2\right)\Gamma\left(\frac{m}{2}+2\right)}{a\Gamma\left(\frac{m}{2}+3\right)}$$

input
```
integrate((c*x)**m*(D*x**3+C*x**2+B*x+A)/(b*x**2+a),x)
```

output
```
A*c**m*m*x**(m + 1)*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma
(m/2 + 1/2)/(4*a*gamma(m/2 + 3/2)) + A*c**m*x**(m + 1)*lerchphi(b*x**2*exp
_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(4*a*gamma(m/2 + 3/2)) + B*
c**m*m*x**(m + 2)*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1)*gamma(m/2
 + 1)/(4*a*gamma(m/2 + 2)) + B*c**m*x**(m + 2)*lerchphi(b*x**2*exp_polar(I
*pi)/a, 1, m/2 + 1)*gamma(m/2 + 1)/(2*a*gamma(m/2 + 2)) + C*c**m*m*x**(m +
 3)*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 3/2)*gamma(m/2 + 3/2)/(4*a
*gamma(m/2 + 5/2)) + 3*C*c**m*x**(m + 3)*lerchphi(b*x**2*exp_polar(I*pi)/a
, 1, m/2 + 3/2)*gamma(m/2 + 3/2)/(4*a*gamma(m/2 + 5/2)) + D*c**m*m*x**(m +
 4)*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 2)*gamma(m/2 + 2)/(4*a*gam
ma(m/2 + 3)) + D*c**m*x**(m + 4)*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2
 + 2)*gamma(m/2 + 2)/(a*gamma(m/2 + 3))
```

## Maxima [F]

$$\int \frac{(cx)^m (A + Bx + Cx^2 + Dx^3)}{a + bx^2}\,dx = \int \frac{(Dx^3 + Cx^2 + Bx + A)(cx)^m}{bx^2 + a}\,dx$$

input
```
integrate((c*x)^m*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="maxima")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)*(c*x)^m/(b*x^2 + a), x)
```

## Giac [F]

$$\int \frac{(cx)^m (A + Bx + Cx^2 + Dx^3)}{a + bx^2}\,dx = \int \frac{(Dx^3 + Cx^2 + Bx + A)(cx)^m}{bx^2 + a}\,dx$$

input
```
integrate((c*x)^m*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="giac")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)*(c*x)^m/(b*x^2 + a), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^m (A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx = \int \frac{(c\,x)^m\,(A+B\,x+C\,x^2+x^3\,D)}{b\,x^2+a}\,dx$$

input
```
int(((c*x)^m*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2),x)
```

output
```
int(((c*x)^m*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2), x)
```

## Reduce [F]

$$\int \frac{(cx)^m (A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx = \frac{c^m\left(-x^m ad\,m^2-3x^m adm-2x^m ad+x^m b^2m^2+3x^m b^2m+2x^m b^2+x^m bc\,m^2x+2x^m bcmx+x^m bd\,m^2\right.}{}$$

input
```
int((c*x)^m*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x)
```

output
```
(c**m*( - x**m*a*d*m**2 - 3*x**m*a*d*m - 2*x**m*a*d + x**m*b**2*m**2 + 3*x
**m*b**2*m + 2*x**m*b**2 + x**m*b*c*m**2*x + 2*x**m*b*c*m*x + x**m*b*d*m**
2*x**2 + x**m*b*d*m*x**2 + int(x**m/(a*x + b*x**3),x)*a**2*d*m**3 + 3*int(
x**m/(a*x + b*x**3),x)*a**2*d*m**2 + 2*int(x**m/(a*x + b*x**3),x)*a**2*d*m
 - int(x**m/(a*x + b*x**3),x)*a*b**2*m**3 - 3*int(x**m/(a*x + b*x**3),x)*a
*b**2*m**2 - 2*int(x**m/(a*x + b*x**3),x)*a*b**2*m + int(x**m/(a + b*x**2)
,x)*a*b**2*m**3 + 3*int(x**m/(a + b*x**2),x)*a*b**2*m**2 + 2*int(x**m/(a +
 b*x**2),x)*a*b**2*m - int(x**m/(a + b*x**2),x)*a*b*c*m**3 - 3*int(x**m/(a
 + b*x**2),x)*a*b*c*m**2 - 2*int(x**m/(a + b*x**2),x)*a*b*c*m))/(b**2*m*(m
**2 + 3*m + 2))
```

# 3.140 $\int \frac{(cx)^m(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2} dx$



## Optimal result

Integrand size = 30, antiderivative size = 188

$$\int \frac{(cx)^m (A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2} dx = -\frac{C(cx)^{1+m}}{bc(1-m)(a+bx^2)} + \frac{D(cx)^{2+m}}{bc^2m(a+bx^2)} + \frac{(aC(1+m)+A(b-bm))(cx)^{1+m}\,\text{Hypergeometric2F1}\left(2,\frac{1+m}{2},\frac{3+m}{2},-\frac{bx^2}{a}\right)}{a^2bc(1-m^2)} + \frac{(bBm-aD(2+m))(cx)^{2+m}\,\text{Hypergeometric2F1}\left(2,\frac{2+m}{2},\frac{4+m}{2},-\frac{bx^2}{a}\right)}{a^2bc^2m(2+m)}$$

output
```
-C*(c*x)^(1+m)/b/c/(1-m)/(b*x^2+a)+D*(c*x)^(2+m)/b/c^2/m/(b*x^2+a)+(a*C*(1
+m)+A*(-b*m+b))*(c*x)^(1+m)*hypergeom([2, 1/2+1/2*m],[3/2+1/2*m],-b*x^2/a)
/a^2/b/c/(-m^2+1)+(b*B*m-a*D*(2+m))*(c*x)^(2+m)*hypergeom([2, 1+1/2*m],[2+
1/2*m],-b*x^2/a)/a^2/b/c^2/m/(2+m)
```

## Mathematica [A] (verified)

Time = 1.47 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.85

$$\int \frac{(cx)^m (A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx = \frac{x(cx)^m \left(\frac{aC\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{bx^2}{a}\right)}{1+m}+\frac{aDx\,\text{Hypergeometric2F1}\left(1,\frac{2+m}{2},\frac{4+m}{2},-\frac{bx^2}{a}\right)}{2+m}+\frac{(Ab-aC)\,\text{Hypergeometric2F1}}{1+m}\right.}{a^2b}$$

input
```
Integrate[((c*x)^m*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^2,x]
```

output
```
(x*(c*x)^m*((a*C*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])
/(1 + m) + (a*D*x*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, -((b*x^2)/a)]
)/(2 + m) + ((A*b - a*C)*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, -((b*x
^2)/a)])/(1 + m) + ((b*B - a*D)*x*Hypergeometric2F1[2, (2 + m)/2, (4 + m)/
2, -((b*x^2)/a)])/(2 + m)))/(a^2*b)
```

## Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2337, 25, 27, 557, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^m (A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx$$

$$\Big\downarrow \text{ 2337}$$

$$\frac{(cx)^{m+1}\left(x\left(B-\frac{aD}{b}\right)-\frac{aC}{b}+A\right)}{2ac\,(a+bx^2)}-\frac{\int -\frac{(cx)^m\left(b\left(A(1-m)+\frac{aC(m+1)}{b}\right)-(bBm-aD(m+2))x\right)}{b(bx^2+a)}dx}{2a}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{(cx)^m(aC(m+1)+A(b-bm)-(bBm-aD(m+2))x)}{b(bx^2+a)}dx}{2a}+\frac{(cx)^{m+1}\left(x\left(B-\frac{aD}{b}\right)-\frac{aC}{b}+A\right)}{2ac\,(a+bx^2)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{(cx)^m (aC(m+1)+A(b-bm)-(bBm-aD(m+2))x)}{bx^2+a}dx}{2ab}+\frac{(cx)^{m+1}\left(x\left(B-\frac{aD}{b}\right)-\frac{aC}{b}+A\right)}{2ac\left(a+bx^2\right)}$$

$$\Big\downarrow \; 557$$

$$\frac{(aC(m+1)+A(b-bm))\int \frac{(cx)^m}{bx^2+a}dx-\frac{(bBm-aD(m+2))\int \frac{(cx)^{m+1}}{bx^2+a}dx}{c}}{2ab}+\frac{(cx)^{m+1}\left(x\left(B-\frac{aD}{b}\right)-\frac{aC}{b}+A\right)}{2ac\left(a+bx^2\right)}$$

$$\Big\downarrow \; 278$$

$$\frac{\frac{(cx)^{m+1}(aC(m+1)+A(b-bm))\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\frac{bx^2}{a}\right)}{ac(m+1)}-\frac{(cx)^{m+2}(bBm-aD(m+2))\text{Hypergeometric2F1}\left(1,\frac{m+2}{2},\frac{m+4}{2},-\right.}{ac^2(m+2)}}{2ab}$$

$$\frac{(cx)^{m+1}\left(x\left(B-\frac{aD}{b}\right)-\frac{aC}{b}+A\right)}{2ac\left(a+bx^2\right)}$$

input
```
Int[((c*x)^m*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^2,x]
```

output
```
((c*x)^(1 + m)*(A - (a*C)/b + (B - (a*D)/b)*x))/(2*a*c*(a + b*x^2)) + (((a
*C*(1 + m) + A*(b - b*m))*(c*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3
 + m)/2, -((b*x^2)/a)])/(a*c*(1 + m)) - ((b*B*m - a*D*(2 + m))*(c*x)^(2 +
m)*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, -((b*x^2)/a)])/(a*c^2*(2 + m
)))/(2*a*b)
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 557
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Sym
bol] :> Simp[c   Int[(e*x)^m*(a + b*x^2)^p, x], x] + Simp[d/e   Int[(e*x)^(
m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x]
```

rule 2337
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(-(c*x)^(m + 1))*(f + g*x)*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1)))
, x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*ExpandToSum[2
*a*(p + 1)*Q + f*(m + 2*p + 3) + g*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a
, b, c, m}, x] && PolyQ[Pq, x] && LtQ[p, -1] && !GtQ[m, 0]
```

## Maple [F]

$$\int \frac{(cx)^m (Dx^3 + C\,x^2 + Bx + A)}{(b\,x^2 + a)^2}dx$$

input
```
int((c*x)^m*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x)
```

output
```
int((c*x)^m*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x)
```

## Fricas [F]

$$\int \frac{(cx)^m (A + Bx + Cx^2 + Dx^3)}{(a + bx^2)^2}\,dx = \int \frac{(Dx^3 + Cx^2 + Bx + A)(cx)^m}{(bx^2 + a)^2}\,dx$$

input
```
integrate((c*x)^m*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
integral((D*x^3 + C*x^2 + B*x + A)*(c*x)^m/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

## Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 45.93 (sec) , antiderivative size = 1828, normalized size of antiderivative = 9.72

$$\int \frac{(cx)^m (A + Bx + Cx^2 + Dx^3)}{(a + bx^2)^2} dx = \text{Too large to display}$$

input
```
integrate((c*x)**m*(D*x**3+C*x**2+B*x+A)/(b*x**2+a)**2,x)
```

output
```
A*(-a*c**m*m**2*x**(m + 1)*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2
)*gamma(m/2 + 1/2)/(8*a**3*gamma(m/2 + 3/2) + 8*a**2*b*x**2*gamma(m/2 + 3/
2)) + 2*a*c**m*m*x**(m + 1)*gamma(m/2 + 1/2)/(8*a**3*gamma(m/2 + 3/2) + 8*
a**2*b*x**2*gamma(m/2 + 3/2)) + a*c**m*x**(m + 1)*lerchphi(b*x**2*exp_pola
r(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(8*a**3*gamma(m/2 + 3/2) + 8*a**
2*b*x**2*gamma(m/2 + 3/2)) + 2*a*c**m*x**(m + 1)*gamma(m/2 + 1/2)/(8*a**3*
gamma(m/2 + 3/2) + 8*a**2*b*x**2*gamma(m/2 + 3/2)) - b*c**m*m**2*x**2*x**(
m + 1)*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(
8*a**3*gamma(m/2 + 3/2) + 8*a**2*b*x**2*gamma(m/2 + 3/2)) + b*c**m*x**2*x*
*(m + 1)*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)
/(8*a**3*gamma(m/2 + 3/2) + 8*a**2*b*x**2*gamma(m/2 + 3/2))) + B*(-a*c**m*
m**2*x**(m + 2)*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1)*gamma(m/2 +
 1)/(8*a**3*gamma(m/2 + 2) + 8*a**2*b*x**2*gamma(m/2 + 2)) - 2*a*c**m*m*x*
*(m + 2)*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1)*gamma(m/2 + 1)/(8*
a**3*gamma(m/2 + 2) + 8*a**2*b*x**2*gamma(m/2 + 2)) + 2*a*c**m*m*x**(m + 2
)*gamma(m/2 + 1)/(8*a**3*gamma(m/2 + 2) + 8*a**2*b*x**2*gamma(m/2 + 2)) +
4*a*c**m*x**(m + 2)*gamma(m/2 + 1)/(8*a**3*gamma(m/2 + 2) + 8*a**2*b*x**2*
gamma(m/2 + 2)) - b*c**m*m**2*x**2*x**(m + 2)*lerchphi(b*x**2*exp_polar(I*
pi)/a, 1, m/2 + 1)*gamma(m/2 + 1)/(8*a**3*gamma(m/2 + 2) + 8*a**2*b*x**2*g
amma(m/2 + 2)) - 2*b*c**m*m*x**2*x**(m + 2)*lerchphi(b*x**2*exp_polar(I...
```

## Maxima [F]

$$\int \frac{(cx)^m (A + Bx + Cx^2 + Dx^3)}{(a + bx^2)^2} \, dx = \int \frac{(Dx^3 + Cx^2 + Bx + A)(cx)^m}{(bx^2 + a)^2} \, dx$$

input
```
integrate((c*x)^m*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)*(c*x)^m/(b*x^2 + a)^2, x)
```

## Giac [F]

$$\int \frac{(cx)^m (A + Bx + Cx^2 + Dx^3)}{(a + bx^2)^2} \, dx = \int \frac{(Dx^3 + Cx^2 + Bx + A)(cx)^m}{(bx^2 + a)^2} \, dx$$

input
```
integrate((c*x)^m*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)*(c*x)^m/(b*x^2 + a)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^m (A + Bx + Cx^2 + Dx^3)}{(a + bx^2)^2} \, dx = \int \frac{(c\,x)^m \left(A + B\,x + C\,x^2 + x^3\,D\right)}{(b\,x^2 + a)^2} \, dx$$

input
```
int(((c*x)^m*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^2,x)
```

output
```
int(((c*x)^m*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^2, x)
```

## Reduce [F]

$$\int \frac{(cx)^m (A + Bx + Cx^2 + Dx^3)}{(a + bx^2)^2} \, dx = \text{too large to display}$$

input
```
int((c*x)^m*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x)
```

output
```
(c**m*( - x**m*a*d*m**2 - x**m*a*d*m + 2*x**m*a*d + x**m*b**2*m**2 - x**m*
b**2*m + x**m*b*c*m**2*x - 2*x**m*b*c*m*x + x**m*b*d*m**2*x**2 - 3*x**m*b*
d*m*x**2 + 2*x**m*b*d*x**2 + int(x**m/(a**2*m**2*x - 3*a**2*m*x + 2*a**2*x
 + 2*a*b*m**2*x**3 - 6*a*b*m*x**3 + 4*a*b*x**3 + b**2*m**2*x**5 - 3*b**2*m
*x**5 + 2*b**2*x**5),x)*a**3*d*m**5 - 2*int(x**m/(a**2*m**2*x - 3*a**2*m*x
 + 2*a**2*x + 2*a*b*m**2*x**3 - 6*a*b*m*x**3 + 4*a*b*x**3 + b**2*m**2*x**5
 - 3*b**2*m*x**5 + 2*b**2*x**5),x)*a**3*d*m**4 - 3*int(x**m/(a**2*m**2*x -
 3*a**2*m*x + 2*a**2*x + 2*a*b*m**2*x**3 - 6*a*b*m*x**3 + 4*a*b*x**3 + b**
2*m**2*x**5 - 3*b**2*m*x**5 + 2*b**2*x**5),x)*a**3*d*m**3 + 8*int(x**m/(a*
*2*m**2*x - 3*a**2*m*x + 2*a**2*x + 2*a*b*m**2*x**3 - 6*a*b*m*x**3 + 4*a*b
*x**3 + b**2*m**2*x**5 - 3*b**2*m*x**5 + 2*b**2*x**5),x)*a**3*d*m**2 - 4*i
nt(x**m/(a**2*m**2*x - 3*a**2*m*x + 2*a**2*x + 2*a*b*m**2*x**3 - 6*a*b*m*x
**3 + 4*a*b*x**3 + b**2*m**2*x**5 - 3*b**2*m*x**5 + 2*b**2*x**5),x)*a**3*d
*m - int(x**m/(a**2*m**2*x - 3*a**2*m*x + 2*a**2*x + 2*a*b*m**2*x**3 - 6*a
*b*m*x**3 + 4*a*b*x**3 + b**2*m**2*x**5 - 3*b**2*m*x**5 + 2*b**2*x**5),x)*
a**2*b**2*m**5 + 4*int(x**m/(a**2*m**2*x - 3*a**2*m*x + 2*a**2*x + 2*a*b*m
**2*x**3 - 6*a*b*m*x**3 + 4*a*b*x**3 + b**2*m**2*x**5 - 3*b**2*m*x**5 + 2*
b**2*x**5),x)*a**2*b**2*m**4 - 5*int(x**m/(a**2*m**2*x - 3*a**2*m*x + 2*a*
*2*x + 2*a*b*m**2*x**3 - 6*a*b*m*x**3 + 4*a*b*x**3 + b**2*m**2*x**5 - 3*b*
*2*m*x**5 + 2*b**2*x**5),x)*a**2*b**2*m**3 + 2*int(x**m/(a**2*m**2*x - ...
```

# 3.141 $\int \frac{(cx)^m(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 194

$$\int \frac{(cx)^m (A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx = -\frac{C(cx)^{1+m}}{bc(3-m)(a+bx^2)^2} - \frac{D(cx)^{2+m}}{bc^2(2-m)(a+bx^2)^2} + \frac{\left(\frac{A}{1+m}+\frac{aC}{3b-bm}\right)(cx)^{1+m}\,\text{Hypergeometric2F1}\left(3,\frac{1+m}{2},\frac{3+m}{2},-\frac{bx^2}{a}\right)}{a^3c} + \frac{(bB(2-m)+aD(2+m))(cx)^{2+m}\,\text{Hypergeometric2F1}\left(3,\frac{2+m}{2},\frac{4+m}{2},-\frac{bx^2}{a}\right)}{a^3bc^2(2-m)(2+m)}$$

output

```
-C*(c*x)^(1+m)/b/c/(3-m)/(b*x^2+a)^2-D*(c*x)^(2+m)/b/c^2/(2-m)/(b*x^2+a)^2
+(A/(1+m)+a*C/(-b*m+3*b))*(c*x)^(1+m)*hypergeom([3, 1/2+1/2*m],[3/2+1/2*m]
,-b*x^2/a)/a^3/c+(b*B*(2-m)+a*D*(2+m))*(c*x)^(2+m)*hypergeom([3, 1+1/2*m],
[2+1/2*m],-b*x^2/a)/a^3/b/c^2/(2-m)/(2+m)
```

## Mathematica [A] (verified)

Time = 1.70 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.82

$$\int \frac{(cx)^m (A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx$$
$$= \frac{x(cx)^m \left(\frac{aC\,\text{Hypergeometric2F1}\left(2,\frac{1+m}{2},\frac{3+m}{2},-\frac{bx^2}{a}\right)}{1+m} + \frac{aDx\,\text{Hypergeometric2F1}\left(2,\frac{2+m}{2},\frac{4+m}{2},-\frac{bx^2}{a}\right)}{2+m} + \frac{(Ab-aC)\,\text{Hypergeometric2F1}}{1+m}\right.}{a^3b}$$

input
```
Integrate[((c*x)^m*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^3,x]
```

output
```
(x*(c*x)^m*((a*C*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])
/(1 + m) + (a*D*x*Hypergeometric2F1[2, (2 + m)/2, (4 + m)/2, -((b*x^2)/a)]
)/(2 + m) + ((A*b - a*C)*Hypergeometric2F1[3, (1 + m)/2, (3 + m)/2, -((b*x
^2)/a)])/(1 + m) + ((b*B - a*D)*x*Hypergeometric2F1[3, (2 + m)/2, (4 + m)/
2, -((b*x^2)/a)])/(2 + m)))/(a^3*b)
```

## Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2337, 25, 27, 557, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^m (A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx$$

$$\Big\downarrow \; 2337$$

$$\frac{(cx)^{m+1}\left(x\left(B-\frac{aD}{b}\right)-\frac{aC}{b}+A\right)}{4ac\,(a+bx^2)^2} - \frac{\int -\frac{(cx)^m\left(b\left(A(3-m)+\frac{aC(m+1)}{b}\right)+(bB(2-m)+aD(m+2))x\right)}{b(bx^2+a)^2}dx}{4a}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{(cx)^m(Ab(3-m)+aC(m+1)+(bB(2-m)+aD(m+2))x)}{b(bx^2+a)^2}dx}{4a}+\frac{(cx)^{m+1}\left(x\left(B-\frac{aD}{b}\right)-\frac{aC}{b}+A\right)}{4ac\left(a+bx^2\right)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{(cx)^m(Ab(3-m)+aC(m+1)+(bB(2-m)+aD(m+2))x)}{(bx^2+a)^2}dx}{4ab}+\frac{(cx)^{m+1}\left(x\left(B-\frac{aD}{b}\right)-\frac{aC}{b}+A\right)}{4ac\left(a+bx^2\right)^2}$$

$$\Big\downarrow \; 557$$

$$\frac{(aC(m+1)+Ab(3-m))\int \frac{(cx)^m}{(bx^2+a)^2}dx+\frac{(aD(m+2)+bB(2-m))\int \frac{(cx)^{m+1}}{\left(bx^2+a\right)^2}dx}{c}}{4ab}+\frac{(cx)^{m+1}\left(x\left(B-\frac{aD}{b}\right)-\frac{aC}{b}+A\right)}{4ac\left(a+bx^2\right)^2}$$

$$\Big\downarrow \; 278$$

$$\frac{\frac{(cx)^{m+1}(aC(m+1)+Ab(3-m))\ \text{Hypergeometric2F1}\left(2,\frac{m+1}{2},\frac{m+3}{2},-\frac{bx^2}{a}\right)}{a^2c(m+1)}+\frac{(cx)^{m+2}(aD(m+2)+bB(2-m))\ \text{Hypergeometric2F1}\left(2,\frac{m+2}{2},\frac{m+}{2}\right.}{a^2c^2(m+2)}}{4ab}+\frac{(cx)^{m+1}\left(x\left(B-\frac{aD}{b}\right)-\frac{aC}{b}+A\right)}{4ac\left(a+bx^2\right)^2}$$

input
```
Int[((c*x)^m*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^3,x]
```

output
```
((c*x)^(1 + m)*(A - (a*C)/b + (B - (a*D)/b)*x))/(4*a*c*(a + b*x^2)^2) + ((
(A*b*(3 - m) + a*C*(1 + m))*(c*x)^(1 + m)*Hypergeometric2F1[2, (1 + m)/2,
(3 + m)/2, -((b*x^2)/a)])/(a^2*c*(1 + m)) + ((b*B*(2 - m) + a*D*(2 + m))*(
c*x)^(2 + m)*Hypergeometric2F1[2, (2 + m)/2, (4 + m)/2, -((b*x^2)/a)])/(a^
2*c^2*(2 + m)))/(4*a*b)
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 557
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Sym
bol] :> Simp[c   Int[(e*x)^m*(a + b*x^2)^p, x], x] + Simp[d/e   Int[(e*x)^(
m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x]
```

rule 2337
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(-(c*x)^(m + 1))*(f + g*x)*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1)))
, x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*ExpandToSum[2
*a*(p + 1)*Q + f*(m + 2*p + 3) + g*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a
, b, c, m}, x] && PolyQ[Pq, x] && LtQ[p, -1] &&  !GtQ[m, 0]
```

## Maple [F]

$$\int \frac{(cx)^m (Dx^3 + C\,x^2 + Bx + A)}{(b\,x^2 + a)^3}dx$$

input
```
int((c*x)^m*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x)
```

output
```
int((c*x)^m*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x)
```

## Fricas [F]

$$\int \frac{(cx)^m (A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx=\int \frac{(Dx^3+Cx^2+Bx+A)(cx)^m}{(bx^2+a)^3}\,dx$$

input
```
integrate((c*x)^m*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
integral((D*x^3 + C*x^2 + B*x + A)*(c*x)^m/(b^3*x^6 + 3*a*b^2*x^4 + 3*a^2*
b*x^2 + a^3), x)
```

## Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 127.58 (sec) , antiderivative size = 5258, normalized size of antiderivative = 27.10

$$\int \frac{(cx)^m (A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx=\text{Too large to display}$$

input
```
integrate((c*x)**m*(D*x**3+C*x**2+B*x+A)/(b*x**2+a)**3,x)
```

output
```
A*(a**2*c**m*m**3*x**(m + 1)*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1
/2)*gamma(m/2 + 1/2)/(32*a**5*gamma(m/2 + 3/2) + 64*a**4*b*x**2*gamma(m/2
+ 3/2) + 32*a**3*b**2*x**4*gamma(m/2 + 3/2)) - 3*a**2*c**m*m**2*x**(m + 1)
*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(32*a**
5*gamma(m/2 + 3/2) + 64*a**4*b*x**2*gamma(m/2 + 3/2) + 32*a**3*b**2*x**4*g
amma(m/2 + 3/2)) - 2*a**2*c**m*m**2*x**(m + 1)*gamma(m/2 + 1/2)/(32*a**5*g
amma(m/2 + 3/2) + 64*a**4*b*x**2*gamma(m/2 + 3/2) + 32*a**3*b**2*x**4*gamm
a(m/2 + 3/2)) - a**2*c**m*m*x**(m + 1)*lerchphi(b*x**2*exp_polar(I*pi)/a,
1, m/2 + 1/2)*gamma(m/2 + 1/2)/(32*a**5*gamma(m/2 + 3/2) + 64*a**4*b*x**2*
gamma(m/2 + 3/2) + 32*a**3*b**2*x**4*gamma(m/2 + 3/2)) + 8*a**2*c**m*m*x**
(m + 1)*gamma(m/2 + 1/2)/(32*a**5*gamma(m/2 + 3/2) + 64*a**4*b*x**2*gamma(
m/2 + 3/2) + 32*a**3*b**2*x**4*gamma(m/2 + 3/2)) + 3*a**2*c**m*x**(m + 1)*
lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(32*a**5
*gamma(m/2 + 3/2) + 64*a**4*b*x**2*gamma(m/2 + 3/2) + 32*a**3*b**2*x**4*ga
mma(m/2 + 3/2)) + 10*a**2*c**m*x**(m + 1)*gamma(m/2 + 1/2)/(32*a**5*gamma(
m/2 + 3/2) + 64*a**4*b*x**2*gamma(m/2 + 3/2) + 32*a**3*b**2*x**4*gamma(m/2
 + 3/2)) + 2*a*b*c**m*m**3*x**2*x**(m + 1)*lerchphi(b*x**2*exp_polar(I*pi)
/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(32*a**5*gamma(m/2 + 3/2) + 64*a**4*b*x
**2*gamma(m/2 + 3/2) + 32*a**3*b**2*x**4*gamma(m/2 + 3/2)) - 6*a*b*c**m*m*
*2*x**2*x**(m + 1)*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gam...
```

## Maxima [F]

$$\int \frac{(cx)^m (A + Bx + Cx^2 + Dx^3)}{(a + bx^2)^3} \, dx = \int \frac{(Dx^3 + Cx^2 + Bx + A)(cx)^m}{(bx^2 + a)^3} \, dx$$

input
```
integrate((c*x)^m*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)*(c*x)^m/(b*x^2 + a)^3, x)
```

## Giac [F]

$$\int \frac{(cx)^m (A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx=\int \frac{(Dx^3+Cx^2+Bx+A)(cx)^m}{(bx^2+a)^3}\,dx$$

input
```
integrate((c*x)^m*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)*(c*x)^m/(b*x^2 + a)^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^m (A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx=\int \frac{(c\,x)^m\,(A+B\,x+C\,x^2+x^3\,D)}{(b\,x^2+a)^3}\,dx$$

input
```
int(((c*x)^m*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^3,x)
```

output
```
int(((c*x)^m*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^3, x)
```

## Reduce [F]

$$\int \frac{(cx)^m (A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx=\text{too large to display}$$

input
```
int((c*x)^m*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x)
```

output

```
(c**m*( - x**m*a*d*m**2 + x**m*a*d*m + 6*x**m*a*d + x**m*b**2*m**2 - 5*x**
m*b**2*m + 6*x**m*b**2 + x**m*b*c*m**2*x - 6*x**m*b*c*m*x + 8*x**m*b*c*x +
 x**m*b*d*m**2*x**2 - 7*x**m*b*d*m*x**2 + 12*x**m*b*d*x**2 + int(x**m/(a**
3*m**3*x - 9*a**3*m**2*x + 26*a**3*m*x - 24*a**3*x + 3*a**2*b*m**3*x**3 -
27*a**2*b*m**2*x**3 + 78*a**2*b*m*x**3 - 72*a**2*b*x**3 + 3*a*b**2*m**3*x*
*5 - 27*a*b**2*m**2*x**5 + 78*a*b**2*m*x**5 - 72*a*b**2*x**5 + b**3*m**3*x
**7 - 9*b**3*m**2*x**7 + 26*b**3*m*x**7 - 24*b**3*x**7),x)*a**4*d*m**6 - 1
0*int(x**m/(a**3*m**3*x - 9*a**3*m**2*x + 26*a**3*m*x - 24*a**3*x + 3*a**2
*b*m**3*x**3 - 27*a**2*b*m**2*x**3 + 78*a**2*b*m*x**3 - 72*a**2*b*x**3 + 3
*a*b**2*m**3*x**5 - 27*a*b**2*m**2*x**5 + 78*a*b**2*m*x**5 - 72*a*b**2*x**
5 + b**3*m**3*x**7 - 9*b**3*m**2*x**7 + 26*b**3*m*x**7 - 24*b**3*x**7),x)*
a**4*d*m**5 + 29*int(x**m/(a**3*m**3*x - 9*a**3*m**2*x + 26*a**3*m*x - 24*
a**3*x + 3*a**2*b*m**3*x**3 - 27*a**2*b*m**2*x**3 + 78*a**2*b*m*x**3 - 72*
a**2*b*x**3 + 3*a*b**2*m**3*x**5 - 27*a*b**2*m**2*x**5 + 78*a*b**2*m*x**5
- 72*a*b**2*x**5 + b**3*m**3*x**7 - 9*b**3*m**2*x**7 + 26*b**3*m*x**7 - 24
*b**3*x**7),x)*a**4*d*m**4 + 4*int(x**m/(a**3*m**3*x - 9*a**3*m**2*x + 26*
a**3*m*x - 24*a**3*x + 3*a**2*b*m**3*x**3 - 27*a**2*b*m**2*x**3 + 78*a**2*
b*m*x**3 - 72*a**2*b*x**3 + 3*a*b**2*m**3*x**5 - 27*a*b**2*m**2*x**5 + 78*
a*b**2*m*x**5 - 72*a*b**2*x**5 + b**3*m**3*x**7 - 9*b**3*m**2*x**7 + 26*b*
*3*m*x**7 - 24*b**3*x**7),x)*a**4*d*m**3 - 132*int(x**m/(a**3*m**3*x - ...
```

# 3.142 $\int (cx)^m \left(a+bx^2\right)^{3/2} \left(A+Bx+Cx^2+Dx^3\right) dx$



## Optimal result

Integrand size = 32, antiderivative size = 239

$$\int (cx)^m \left(a+bx^2\right)^{3/2} \left(A+Bx+Cx^2+Dx^3\right) dx = \frac{C(cx)^{1+m}\left(a+bx^2\right)^{5/2}}{bc(6+m)} + \frac{D(cx)^{2+m}\left(a+bx^2\right)^{5/2}}{bc^2(7+m)} + \frac{a\left(\frac{A}{1+m}-\frac{aC}{b(6+m)}\right)(cx)^{1+m}\sqrt{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1+m}{2},\frac{3+m}{2},-\frac{bx^2}{a}\right)}{c\sqrt{1+\frac{bx^2}{a}}} - \frac{a(aD(2+m)-bB(7+m))(cx)^{2+m}\sqrt{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{2+m}{2},\frac{4+m}{2},-\frac{bx^2}{a}\right)}{bc^2(2+m)(7+m)\sqrt{1+\frac{bx^2}{a}}}$$

output
```
C*(c*x)^(1+m)*(b*x^2+a)^(5/2)/b/c/(6+m)+D*(c*x)^(2+m)*(b*x^2+a)^(5/2)/b/c^
2/(7+m)+a*(A/(1+m)-a*C/b/(6+m))*(c*x)^(1+m)*(b*x^2+a)^(1/2)*hypergeom([-3/
2, 1/2+1/2*m],[3/2+1/2*m],-b*x^2/a)/c/(1+b*x^2/a)^(1/2)-a*(a*D*(2+m)-b*B*(
7+m))*(c*x)^(2+m)*(b*x^2+a)^(1/2)*hypergeom([-3/2, 1+1/2*m],[2+1/2*m],-b*x
^2/a)/b/c^2/(2+m)/(7+m)/(1+b*x^2/a)^(1/2)
```

## Mathematica [A] (verified)

Time = 2.12 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.74

$$\int (cx)^m \left(a+bx^2\right)^{3/2} \left(A+Bx+Cx^2+Dx^3\right) dx = \frac{ax(cx)^m\sqrt{a+bx^2}\left(\frac{A\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1+m}{2},\frac{3+m}{2},-\frac{bx^2}{a}\right)}{1+m}+x\left(\frac{B\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{2+m}{2},\frac{4+m}{2},-\frac{bx^2}{a}\right)}{2+m}\right.\right.}{\sqrt{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(c*x)^m*(a + b*x^2)^(3/2)*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(a*x*(c*x)^m*Sqrt[a + b*x^2]*((A*Hypergeometric2F1[-3/2, (1 + m)/2, (3 + m
)/2, -((b*x^2)/a)])/(1 + m) + x*((B*Hypergeometric2F1[-3/2, (2 + m)/2, (4
+ m)/2, -((b*x^2)/a)])/(2 + m) + x*((C*Hypergeometric2F1[-3/2, (3 + m)/2,
(5 + m)/2, -((b*x^2)/a)])/(3 + m) + (D*x*Hypergeometric2F1[-3/2, (4 + m)/2
, (6 + m)/2, -((b*x^2)/a)])/(4 + m)))))/Sqrt[1 + (b*x^2)/a]
```

## Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {2340, 2340, 25, 27, 557, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^{3/2} (cx)^m \left(A+Bx+Cx^2+Dx^3\right) dx$$

$$\Big\downarrow \ 2340$$

$$\frac{\int (cx)^m \left(bx^2+a\right)^{3/2} \left(bC(m+7)x^2-(aD(m+2)-bB(m+7))x+Ab(m+7)\right) dx}{b(m+7)}+\frac{D\left(a+bx^2\right)^{5/2} (cx)^{m+2}}{bc^2(m+7)}$$

$$\Big\downarrow \ 2340$$

$$\frac{\frac{\int -b(cx)^m((m+7)(aC(m+1)-Ab(m+6))+(m+6)(aD(m+2)-bB(m+7))x)\left(bx^2+a\right)^{3/2}dx}{b(m+6)}+\frac{C(m+7)\left(a+bx^2\right)^{5/2}(cx)^{m+1}}{c(m+6)}}{b(m+7)}+\frac{D\left(a+bx^2\right)^{5/2}(cx)^{m+2}}{bc^2(m+7)}$$

$\Big\downarrow$ 25

$$\frac{\frac{C(m+7)\left(a+bx^2\right)^{5/2}(cx)^{m+1}}{c(m+6)}-\frac{\int b(cx)^m((m+7)(aC(m+1)-Ab(m+6))+(m+6)(aD(m+2)-bB(m+7))x)\left(bx^2+a\right)^{3/2}dx}{b(m+6)}}{b(m+7)}+\frac{D\left(a+bx^2\right)^{5/2}(cx)^{m+2}}{bc^2(m+7)}$$

$\Big\downarrow$ 27

$$\frac{\frac{C(m+7)\left(a+bx^2\right)^{5/2}(cx)^{m+1}}{c(m+6)}-\frac{\int (cx)^m((m+7)(aC(m+1)-Ab(m+6))+(m+6)(aD(m+2)-bB(m+7))x)\left(bx^2+a\right)^{3/2}dx}{m+6}}{b(m+7)}+\frac{D\left(a+bx^2\right)^{5/2}(cx)^{m+2}}{bc^2(m+7)}$$

$\Big\downarrow$ 557

$$\frac{\frac{C(m+7)\left(a+bx^2\right)^{5/2}(cx)^{m+1}}{c(m+6)}-\frac{(m+7)(aC(m+1)-Ab(m+6))\int (cx)^m\left(bx^2+a\right)^{3/2}dx+\frac{(m+6)(aD(m+2)-bB(m+7))\int (cx)^{m+1}\left(bx^2+a\right)^{3/2}dx}{c}}{m+6}}{b(m+7)}+\frac{D\left(a+bx^2\right)^{5/2}(cx)^{m+2}}{bc^2(m+7)}$$

$\Big\downarrow$ 279

$$\frac{\frac{C(m+7)\left(a+bx^2\right)^{5/2}(cx)^{m+1}}{c(m+6)}-\frac{\frac{a(m+7)\sqrt{a+bx^2}(aC(m+1)-Ab(m+6))\int (cx)^m\left(\frac{bx^2}{a}+1\right)^{3/2}dx}{\sqrt{\frac{bx^2}{a}+1}}+\frac{a(m+6)\sqrt{a+bx^2}(aD(m+2)-bB(m+7))\int (cx)^{m+1}\left(\frac{bx^2}{a}\right.}{c\sqrt{\frac{bx^2}{a}+1}}}{m+6}}{b(m+7)}+\frac{D\left(a+bx^2\right)^{5/2}(cx)^{m+2}}{bc^2(m+7)}$$

$\Big\downarrow$ 278

$$\frac{\frac{C(m+7)\left(a+bx^2\right)^{5/2}(cx)^{m+1}}{c(m+6)}-\frac{\frac{a(m+7)\sqrt{a+bx^2}(cx)^{m+1}(aC(m+1)-Ab(m+6))\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{m+1}{2},\frac{m+3}{2},-\frac{bx^2}{a}\right)}{c(m+1)\sqrt{\frac{bx^2}{a}+1}}+\frac{a(m+6)\sqrt{a+bx^2}(cx)^{m+2}}{}}{m+6}}{b(m+7)}$$

$$\frac{D\left(a+bx^2\right)^{5/2}(cx)^{m+2}}{bc^2(m+7)}$$

input
```
Int[(c*x)^m*(a + b*x^2)^(3/2)*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(D*(c*x)^(2 + m)*(a + b*x^2)^(5/2))/(b*c^2*(7 + m)) + ((C*(7 + m)*(c*x)^(1
 + m)*(a + b*x^2)^(5/2))/(c*(6 + m)) - ((a*(7 + m)*(a*C*(1 + m) - A*b*(6 +
 m))*(c*x)^(1 + m)*Sqrt[a + b*x^2]*Hypergeometric2F1[-3/2, (1 + m)/2, (3 +
 m)/2, -((b*x^2)/a)])/(c*(1 + m)*Sqrt[1 + (b*x^2)/a]) + (a*(6 + m)*(a*D*(2
 + m) - b*B*(7 + m))*(c*x)^(2 + m)*Sqrt[a + b*x^2]*Hypergeometric2F1[-3/2,
 (2 + m)/2, (4 + m)/2, -((b*x^2)/a)])/(c^2*(2 + m)*Sqrt[1 + (b*x^2)/a]))/(
6 + m))/(b*(7 + m))
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 557
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Sym
bol] :> Simp[c   Int[(e*x)^m*(a + b*x^2)^p, x], x] + Simp[d/e   Int[(e*x)^(
m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x]
```

rule 2340
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Simp[1/(b*(m
+ q + 2*p + 1))   Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)
*Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /;
GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ
[Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

## Maple [F]

$$\int (cx)^m \left(b\,x^2+a\right)^{\frac{3}{2}} \left(D x^3+C\,x^2+Bx+A\right) dx$$

input
```
int((c*x)^m*(b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A),x)
```

output
```
int((c*x)^m*(b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A),x)
```

## Fricas [F]

$$\int (cx)^m (a + bx^2)^{3/2} \left(A+Bx+Cx^2+Dx^3\right)\,dx = \int \left(Dx^3+Cx^2+Bx+A\right) \left(bx^2+a\right)^{\frac{3}{2}} (cx)^m\,dx$$

input
```
integrate((c*x)^m*(b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas
")
```

output
```
integral((D*b*x^5 + C*b*x^4 + (D*a + B*b)*x^3 + B*a*x + (C*a + A*b)*x^2 +
A*a)*sqrt(b*x^2 + a)*(c*x)^m, x)
```

## Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 10.29 (sec) , antiderivative size = 468, normalized size of antiderivative = 1.96

$$\int (cx)^m \left(a+bx^2\right)^{3/2} \left(A+Bx+Cx^2+Dx^3\right) \, dx = \text{Too large to display}$$

input
```
integrate((c*x)**m*(b*x**2+a)**(3/2)*(D*x**3+C*x**2+B*x+A),x)
```

output
```
A*a**(3/2)*c**m*x**(m + 1)*gamma(m/2 + 1/2)*hyper((-1/2, m/2 + 1/2), (m/2
+ 3/2,), b*x**2*exp_polar(I*pi)/a)/(2*gamma(m/2 + 3/2)) + A*sqrt(a)*b*c**m
*x**(m + 3)*gamma(m/2 + 3/2)*hyper((-1/2, m/2 + 3/2), (m/2 + 5/2,), b*x**2
*exp_polar(I*pi)/a)/(2*gamma(m/2 + 5/2)) + B*a**(3/2)*c**m*x**(m + 2)*gamm
a(m/2 + 1)*hyper((-1/2, m/2 + 1), (m/2 + 2,), b*x**2*exp_polar(I*pi)/a)/(2
*gamma(m/2 + 2)) + B*sqrt(a)*b*c**m*x**(m + 4)*gamma(m/2 + 2)*hyper((-1/2,
 m/2 + 2), (m/2 + 3,), b*x**2*exp_polar(I*pi)/a)/(2*gamma(m/2 + 3)) + C*a*
*(3/2)*c**m*x**(m + 3)*gamma(m/2 + 3/2)*hyper((-1/2, m/2 + 3/2), (m/2 + 5/
2,), b*x**2*exp_polar(I*pi)/a)/(2*gamma(m/2 + 5/2)) + C*sqrt(a)*b*c**m*x**
(m + 5)*gamma(m/2 + 5/2)*hyper((-1/2, m/2 + 5/2), (m/2 + 7/2,), b*x**2*exp
_polar(I*pi)/a)/(2*gamma(m/2 + 7/2)) + D*a**(3/2)*c**m*x**(m + 4)*gamma(m/
2 + 2)*hyper((-1/2, m/2 + 2), (m/2 + 3,), b*x**2*exp_polar(I*pi)/a)/(2*gam
ma(m/2 + 3)) + D*sqrt(a)*b*c**m*x**(m + 6)*gamma(m/2 + 3)*hyper((-1/2, m/2
 + 3), (m/2 + 4,), b*x**2*exp_polar(I*pi)/a)/(2*gamma(m/2 + 4))
```

## Maxima [F]

$$\int (cx)^m (a + bx^2)^{3/2} \left(A+Bx+Cx^2+Dx^3\right) \, dx = \int \left(Dx^3+Cx^2+Bx+A\right)\left(bx^2+a\right)^{\frac{3}{2}}(cx)^m \, dx$$

input
```
integrate((c*x)^m*(b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima
")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)*(b*x^2 + a)^(3/2)*(c*x)^m, x)
```

## Giac [F]

$$\int (cx)^m \left(a+bx^2\right)^{3/2} \left(A+Bx+Cx^2+Dx^3\right)\,dx=\int \left(Dx^3+Cx^2+Bx+A\right)\left(bx^2+a\right)^{\frac{3}{2}}(cx)^m\,dx$$

input
```
integrate((c*x)^m*(b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)*(b*x^2 + a)^(3/2)*(c*x)^m, x)
```

## Mupad [F(-1)]

Timed out.

$$\int (cx)^m \left(a+bx^2\right)^{3/2} \left(A+Bx+Cx^2+Dx^3\right)\,dx=\int (c\,x)^m\,\left(b\,x^2+a\right)^{3/2}\,\left(A+B\,x+C\,x^2+x^3\,D\right)\,dx$$

input
```
int((c*x)^m*(a + b*x^2)^(3/2)*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
int((c*x)^m*(a + b*x^2)^(3/2)*(A + B*x + C*x^2 + x^3*D), x)
```

## Reduce [F]

$$\int (cx)^m \left(a+bx^2\right)^{3/2} \left(A+Bx+Cx^2+Dx^3\right)\,dx=c^m\left(\left(\int x^m\sqrt{b\,x^2+a}\,x^5dx\right)bd+\left(\int x^m\sqrt{b\,x^2+a}\,x^4dx\right)bc+\left(\int x^m\sqrt{b\,x^2+a}\,x^3dx\right)ad+\left(\int x^m\sqrt{b\,x^2+a}\,x^3dx\right)b^2+\left(\int x^m\sqrt{b\,x^2+a}\,x^2dx\right)ab+\left(\int x^m\sqrt{b\,x^2+a}\,x^2dx\right)ac+\left(\int x^m\sqrt{b\,x^2+a}\,xdx\right)ab+\left(\int x^m\sqrt{b\,x^2+a}dx\right)a^2\right)$$

input
```
int((c*x)^m*(b*x^2+a)^(3/2)*(D*x^3+C*x^2+B*x+A),x)
```

output
```
c**m*(int(x**m*sqrt(a + b*x**2)*x**5,x)*b*d + int(x**m*sqrt(a + b*x**2)*x*
*4,x)*b*c + int(x**m*sqrt(a + b*x**2)*x**3,x)*a*d + int(x**m*sqrt(a + b*x*
*2)*x**3,x)*b**2 + int(x**m*sqrt(a + b*x**2)*x**2,x)*a*b + int(x**m*sqrt(a
 + b*x**2)*x**2,x)*a*c + int(x**m*sqrt(a + b*x**2)*x,x)*a*b + int(x**m*sqr
t(a + b*x**2),x)*a**2)
```

# 3.143 $\int (cx)^m\sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx$



## Optimal result

Integrand size = 32, antiderivative size = 229

$$\begin{aligned}\int (cx)^m\sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx &= \frac{C(cx)^{1+m}\left(a+bx^2\right)^{3/2}}{bc(4+m)}+\frac{D(cx)^{2+m}\left(a+bx^2\right)^{3/2}}{bc^2(5+m)}\\ &\quad+\frac{\left(\frac{A}{1+m}-\frac{aC}{b(4+m)}\right)(cx)^{1+m}\sqrt{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},-\frac{bx^2}{a}\right)}{c\sqrt{1+\frac{bx^2}{a}}}\\ &\quad+\frac{\left(\frac{B}{2+m}-\frac{aD}{b(5+m)}\right)(cx)^{2+m}\sqrt{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},-\frac{bx^2}{a}\right)}{c^2\sqrt{1+\frac{bx^2}{a}}}\end{aligned}$$

output
```
C*(c*x)^(1+m)*(b*x^2+a)^(3/2)/b/c/(4+m)+D*(c*x)^(2+m)*(b*x^2+a)^(3/2)/b/c^
2/(5+m)+(A/(1+m)-a*C/b/(4+m))*(c*x)^(1+m)*(b*x^2+a)^(1/2)*hypergeom([-1/2,
 1/2+1/2*m],[3/2+1/2*m],-b*x^2/a)/c/(1+b*x^2/a)^(1/2)+(B/(2+m)-a*D/b/(5+m)
)*(c*x)^(2+m)*(b*x^2+a)^(1/2)*hypergeom([-1/2, 1+1/2*m],[2+1/2*m],-b*x^2/a
)/c^2/(1+b*x^2/a)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.93 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.77

$$\int (cx)^m\sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$=\frac{x(cx)^m\sqrt{a+bx^2}\left(\frac{A\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},-\frac{bx^2}{a}\right)}{1+m}+x\left(\frac{B\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},-\frac{bx^2}{a}\right)}{2+m}+x\left(\frac{C\,\text{Hyp}}{}\right.\right.\right.}{\sqrt{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(c*x)^m*Sqrt[a + b*x^2]*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(x*(c*x)^m*Sqrt[a + b*x^2]*((A*Hypergeometric2F1[-1/2, (1 + m)/2, (3 + m)/
2, -((b*x^2)/a)])/(1 + m) + x*((B*Hypergeometric2F1[-1/2, (2 + m)/2, (4 +
m)/2, -((b*x^2)/a)])/(2 + m) + x*((C*Hypergeometric2F1[-1/2, (3 + m)/2, (5
 + m)/2, -((b*x^2)/a)])/(3 + m) + (D*x*Hypergeometric2F1[-1/2, (4 + m)/2,
(6 + m)/2, -((b*x^2)/a)])/(4 + m)))))/Sqrt[1 + (b*x^2)/a]
```

## Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {2340, 2340, 25, 27, 557, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx^2}(cx)^m\left(A+Bx+Cx^2+Dx^3\right)\,dx$$

$$\Big\downarrow \text{2340}$$

$$\frac{\int (cx)^m\sqrt{bx^2+a}\left(bC(m+5)x^2-(aD(m+2)-bB(m+5))x+Ab(m+5)\right)dx}{b(m+5)}+\frac{D\left(a+bx^2\right)^{3/2}(cx)^{m+2}}{bc^2(m+5)}$$

$$\Big\downarrow \text{2340}$$

$$\frac{\frac{\int -b(cx)^m((m+5)(aC(m+1)-Ab(m+4))+(m+4)(aD(m+2)-bB(m+5))x)\sqrt{bx^2+a}dx}{b(m+4)}+\frac{C(m+5)\left(a+bx^2\right)^{3/2}(cx)^{m+1}}{c(m+4)}}{b(m+5)}+\frac{D\left(a+bx^2\right)^{3/2}(cx)^{m+2}}{bc^2(m+5)}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{C(m+5)\left(a+bx^2\right)^{3/2}(cx)^{m+1}}{c(m+4)}-\frac{\int b(cx)^m((m+5)(aC(m+1)-Ab(m+4))+(m+4)(aD(m+2)-bB(m+5))x)\sqrt{bx^2+a}dx}{b(m+4)}}{b(m+5)}+\frac{D\left(a+bx^2\right)^{3/2}(cx)^{m+2}}{bc^2(m+5)}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{C(m+5)\left(a+bx^2\right)^{3/2}(cx)^{m+1}}{c(m+4)}-\frac{\int (cx)^m((m+5)(aC(m+1)-Ab(m+4))+(m+4)(aD(m+2)-bB(m+5))x)\sqrt{bx^2+a}dx}{m+4}}{b(m+5)}+\frac{D\left(a+bx^2\right)^{3/2}(cx)^{m+2}}{bc^2(m+5)}$$

$$\Big\downarrow \ 557$$

$$\frac{\frac{C(m+5)\left(a+bx^2\right)^{3/2}(cx)^{m+1}}{c(m+4)}-\frac{(m+5)(aC(m+1)-Ab(m+4))\int (cx)^m\sqrt{bx^2+a}dx+\frac{(m+4)(aD(m+2)-bB(m+5))\int (cx)^{m+1}\sqrt{bx^2+a}dx}{c}}{m+4}}{b(m+5)}+\frac{D\left(a+bx^2\right)^{3/2}(cx)^{m+2}}{bc^2(m+5)}$$

$$\Big\downarrow \ 279$$

$$\frac{\frac{C(m+5)\left(a+bx^2\right)^{3/2}(cx)^{m+1}}{c(m+4)}-\frac{\frac{(m+5)\sqrt{a+bx^2}(aC(m+1)-Ab(m+4))\int (cx)^m\sqrt{\frac{bx^2}{a}+1}dx}{\sqrt{\frac{bx^2}{a}+1}}+\frac{(m+4)\sqrt{a+bx^2}(aD(m+2)-bB(m+5))\int (cx)^{m+1}\sqrt{\frac{bx^2}{a}+1}dx}{c\sqrt{\frac{bx^2}{a}+1}}}{m+4}}{b(m+5)}+\frac{D\left(a+bx^2\right)^{3/2}(cx)^{m+2}}{bc^2(m+5)}$$

$$\Big\downarrow \ 278$$

$$\frac{\frac{C(m+5)(a+bx^2)^{3/2}(cx)^{m+1}}{c(m+4)} - \frac{\frac{(m+5)\sqrt{a+bx^2}(cx)^{m+1}(aC(m+1)-Ab(m+4))\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},-\frac{bx^2}{a}\right)}{c(m+1)\sqrt{\frac{bx^2}{a}+1}} + \frac{(m+4)\sqrt{a+bx^2}(cx)^{m+2}(a}{}}{m+4}}{b(m+5)}$$

$$\frac{D(a+bx^2)^{3/2}(cx)^{m+2}}{bc^2(m+5)}$$

input
```
Int[(c*x)^m*Sqrt[a + b*x^2]*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(D*(c*x)^(2 + m)*(a + b*x^2)^(3/2))/(b*c^2*(5 + m)) + ((C*(5 + m)*(c*x)^(1
 + m)*(a + b*x^2)^(3/2))/(c*(4 + m)) - (((5 + m)*(a*C*(1 + m) - A*b*(4 + m
))*(c*x)^(1 + m)*Sqrt[a + b*x^2]*Hypergeometric2F1[-1/2, (1 + m)/2, (3 + m
)/2, -((b*x^2)/a)])/(c*(1 + m)*Sqrt[1 + (b*x^2)/a]) + ((4 + m)*(a*D*(2 + m
) - b*B*(5 + m))*(c*x)^(2 + m)*Sqrt[a + b*x^2]*Hypergeometric2F1[-1/2, (2
+ m)/2, (4 + m)/2, -((b*x^2)/a)])/(c^2*(2 + m)*Sqrt[1 + (b*x^2)/a]))/(4 +
m))/(b*(5 + m))
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 557
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Sym
bol] :> Simp[c   Int[(e*x)^m*(a + b*x^2)^p, x], x] + Simp[d/e   Int[(e*x)^(
m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x]
```

rule 2340
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Simp[1/(b*(m
+ q + 2*p + 1))   Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)
*Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /;
GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ
[Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

## Maple [F]

$$\int (cx)^m \sqrt{b\,x^2+a}\,\left(Dx^3+C\,x^2+Bx+A\right) dx$$

input
```
int((c*x)^m*(b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A),x)
```

output
```
int((c*x)^m*(b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A),x)
```

## Fricas [F]

$$\int (cx)^m\sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$=\int \left(Dx^3+Cx^2+Bx+A\right)\sqrt{bx^2+a}(cx)^m\,dx$$

input
```
integrate((c*x)^m*(b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas
")
```

output
```
integral((D*x^3 + C*x^2 + B*x + A)*sqrt(b*x^2 + a)*(c*x)^m, x)
```

## Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.77 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.00

$$\int (cx)^m\sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$=\frac{A\sqrt{a}c^mx^{m+1}\Gamma\left(\frac{m}{2}+\frac{1}{2}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{m}{2}+\frac{1}{2}\\ \frac{m}{2}+\frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}+\frac{B\sqrt{a}c^mx^{m+2}\Gamma\left(\frac{m}{2}+1\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{m}{2}+1\\ \frac{m}{2}+2\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{m}{2}+2\right)}+\frac{C\sqrt{a}c^mx^{m+3}\Gamma\left(\frac{m}{2}+\frac{3}{2}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{m}{2}+\frac{3}{2}\\ \frac{m}{2}+\frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)}+\frac{D\sqrt{a}c^mx^{m+4}\Gamma\left(\frac{m}{2}+2\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{m}{2}+2\\ \frac{m}{2}+3\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{m}{2}+3\right)}$$

input
```
integrate((c*x)**m*(b*x**2+a)**(1/2)*(D*x**3+C*x**2+B*x+A),x)
```

output
```
A*sqrt(a)*c**m*x**(m + 1)*gamma(m/2 + 1/2)*hyper((-1/2, m/2 + 1/2), (m/2 +
 3/2,), b*x**2*exp_polar(I*pi)/a)/(2*gamma(m/2 + 3/2)) + B*sqrt(a)*c**m*x*
*(m + 2)*gamma(m/2 + 1)*hyper((-1/2, m/2 + 1), (m/2 + 2,), b*x**2*exp_pola
r(I*pi)/a)/(2*gamma(m/2 + 2)) + C*sqrt(a)*c**m*x**(m + 3)*gamma(m/2 + 3/2)
*hyper((-1/2, m/2 + 3/2), (m/2 + 5/2,), b*x**2*exp_polar(I*pi)/a)/(2*gamma
(m/2 + 5/2)) + D*sqrt(a)*c**m*x**(m + 4)*gamma(m/2 + 2)*hyper((-1/2, m/2 +
 2), (m/2 + 3,), b*x**2*exp_polar(I*pi)/a)/(2*gamma(m/2 + 3))
```

## Maxima [F]

$$\int (cx)^m\sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx = \int (Dx^3+Cx^2+Bx+A)\sqrt{bx^2+a}(cx)^m\,dx$$

input
```
integrate((c*x)^m*(b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima
")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)*sqrt(b*x^2 + a)*(c*x)^m, x)
```

## Giac [F]

$$\int (cx)^m\sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx = \int (Dx^3+Cx^2+Bx+A)\sqrt{bx^2+a}(cx)^m\,dx$$

input
```
integrate((c*x)^m*(b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)*sqrt(b*x^2 + a)*(c*x)^m, x)
```

## Mupad [F(-1)]

Timed out.

$$\int (cx)^m\sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx = \int (c\,x)^m\,\sqrt{b\,x^2+a}\,\left(A+B\,x+C\,x^2+x^3\,D\right)\,dx$$

input
```
int((c*x)^m*(a + b*x^2)^(1/2)*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
int((c*x)^m*(a + b*x^2)^(1/2)*(A + B*x + C*x^2 + x^3*D), x)
```

## Reduce [F]

$$\int (cx)^m\sqrt{a+bx^2}\left(A+Bx+Cx^2+Dx^3\right)\,dx = c^m\left(\left(\int x^m\sqrt{b\,x^2+a}\,x^3dx\right)d+\left(\int x^m\sqrt{b\,x^2+a}\,x^2dx\right)c + \left(\int x^m\sqrt{b\,x^2+a}\,xdx\right)b+\left(\int x^m\sqrt{b\,x^2+a}dx\right)a\right)$$

input
```
int((c*x)^m*(b*x^2+a)^(1/2)*(D*x^3+C*x^2+B*x+A),x)
```

output
```
c**m*(int(x**m*sqrt(a + b*x**2)*x**3,x)*d + int(x**m*sqrt(a + b*x**2)*x**2
,x)*c + int(x**m*sqrt(a + b*x**2)*x,x)*b + int(x**m*sqrt(a + b*x**2),x)*a)
```

# 3.144 $\int \frac{(cx)^m(A+Bx+Cx^2+Dx^3)}{\sqrt{a+bx^2}}\,dx$



## Optimal result

Integrand size = 32, antiderivative size = 229

$$\int \frac{(cx)^m\left(A+Bx+Cx^2+Dx^3\right)}{\sqrt{a+bx^2}}\,dx = \frac{C(cx)^{1+m}\sqrt{a+bx^2}}{bc(2+m)}+\frac{D(cx)^{2+m}\sqrt{a+bx^2}}{bc^2(3+m)} + \frac{\left(\frac{A}{1+m}-\frac{aC}{b(2+m)}\right)(cx)^{1+m}\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},-\frac{bx^2}{a}\right)}{c\sqrt{a+bx^2}} + \frac{\left(\frac{B}{2+m}-\frac{aD}{b(3+m)}\right)(cx)^{2+m}\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},-\frac{bx^2}{a}\right)}{c^2\sqrt{a+bx^2}}$$

output
```
C*(c*x)^(1+m)*(b*x^2+a)^(1/2)/b/c/(2+m)+D*(c*x)^(2+m)*(b*x^2+a)^(1/2)/b/c^
2/(3+m)+(A/(1+m)-a*C/b/(2+m))*(c*x)^(1+m)*(1+b*x^2/a)^(1/2)*hypergeom([1/2
, 1/2+1/2*m],[3/2+1/2*m],-b*x^2/a)/c/(b*x^2+a)^(1/2)+(B/(2+m)-a*D/b/(3+m))
*(c*x)^(2+m)*(1+b*x^2/a)^(1/2)*hypergeom([1/2, 1+1/2*m],[2+1/2*m],-b*x^2/a
)/c^2/(b*x^2+a)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.91 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.77

$$\int \frac{(cx)^m (A+Bx+Cx^2+Dx^3)}{\sqrt{a+bx^2}}\,dx$$
$$= \frac{x(cx)^m\sqrt{1+\frac{bx^2}{a}}\left(\frac{A\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},-\frac{bx^2}{a}\right)}{1+m}+x\left(\frac{B\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},-\frac{bx^2}{a}\right)}{2+m}+x\left(\frac{C\,\text{Hyperg}}{}\right.\right.\right.}{\sqrt{a+bx^2}}$$

input
```
Integrate[((c*x)^m*(A + B*x + C*x^2 + D*x^3))/Sqrt[a + b*x^2],x]
```

output
```
(x*(c*x)^m*Sqrt[1 + (b*x^2)/a]*((A*Hypergeometric2F1[1/2, (1 + m)/2, (3 +
m)/2, -((b*x^2)/a)])/(1 + m) + x*((B*Hypergeometric2F1[1/2, (2 + m)/2, (4
+ m)/2, -((b*x^2)/a)])/(2 + m) + x*((C*Hypergeometric2F1[1/2, (3 + m)/2, (
5 + m)/2, -((b*x^2)/a)])/(3 + m) + (D*x*Hypergeometric2F1[1/2, (4 + m)/2,
(6 + m)/2, -((b*x^2)/a)])/(4 + m)))))/Sqrt[a + b*x^2]
```

## Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {2340, 2340, 25, 27, 557, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^m (A+Bx+Cx^2+Dx^3)}{\sqrt{a+bx^2}}\,dx$$

$$\Big\downarrow \text{2340}$$

$$\frac{\int \frac{(cx)^m(bC(m+3)x^2-(aD(m+2)-bB(m+3))x+Ab(m+3))}{\sqrt{bx^2+a}}dx}{b(m+3)}+\frac{D\sqrt{a+bx^2}(cx)^{m+2}}{bc^2(m+3)}$$

$$\Big\downarrow \text{2340}$$

$$\frac{\frac{\int -\frac{b(cx)^m((m+3)(aC(m+1)-Ab(m+2))+(m+2)(aD(m+2)-bB(m+3))x)}{\sqrt{bx^2+a}}dx}{b(m+2)}+\frac{C(m+3)\sqrt{a+bx^2}(cx)^{m+1}}{c(m+2)}}{b(m+3)}+\frac{D\sqrt{a+bx^2}(cx)^{m+2}}{bc^2(m+3)}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{C(m+3)\sqrt{a+bx^2}(cx)^{m+1}}{c(m+2)}-\frac{\int \frac{b(cx)^m((m+3)(aC(m+1)-Ab(m+2))+(m+2)(aD(m+2)-bB(m+3))x)}{\sqrt{bx^2+a}}dx}{b(m+2)}}{b(m+3)}+\frac{D\sqrt{a+bx^2}(cx)^{m+2}}{bc^2(m+3)}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{C(m+3)\sqrt{a+bx^2}(cx)^{m+1}}{c(m+2)}-\frac{\int \frac{(cx)^m((m+3)(aC(m+1)-Ab(m+2))+(m+2)(aD(m+2)-bB(m+3))x)}{\sqrt{bx^2+a}}dx}{m+2}}{b(m+3)}+\frac{D\sqrt{a+bx^2}(cx)^{m+2}}{bc^2(m+3)}$$

$$\Big\downarrow \ 557$$

$$\frac{\frac{C(m+3)\sqrt{a+bx^2}(cx)^{m+1}}{c(m+2)}-\frac{(m+3)(aC(m+1)-Ab(m+2))\int \frac{(cx)^m}{\sqrt{bx^2+a}}dx+\frac{(m+2)(aD(m+2)-bB(m+3))\int \frac{(cx)^{m+1}}{\sqrt{bx^2+a}}dx}{c}}{m+2}}{b(m+3)}+\frac{D\sqrt{a+bx^2}(cx)^{m+2}}{bc^2(m+3)}$$

$$\Big\downarrow \ 279$$

$$\frac{\frac{C(m+3)\sqrt{a+bx^2}(cx)^{m+1}}{c(m+2)}-\frac{\frac{(m+3)\sqrt{\frac{bx^2}{a}+1}(aC(m+1)-Ab(m+2))\int \frac{(cx)^m}{\sqrt{\frac{bx^2}{a}+1}}dx}{\sqrt{a+bx^2}}+\frac{(m+2)\sqrt{\frac{bx^2}{a}+1}(aD(m+2)-bB(m+3))\int \frac{(cx)^{m+1}}{\sqrt{\frac{bx^2}{a}+1}}dx}{c\sqrt{a+bx^2}}}{m+2}}{b(m+3)}+\frac{D\sqrt{a+bx^2}(cx)^{m+2}}{bc^2(m+3)}$$

$$\Big\downarrow \ 278$$

$$\frac{\frac{C(m+3)\sqrt{a+bx^2}(cx)^{m+1}}{c(m+2)}-\frac{\frac{(m+3)\sqrt{\frac{bx^2}{a}+1}(cx)^{m+1}(aC(m+1)-Ab(m+2))\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},-\frac{bx^2}{a}\right)}{c(m+1)\sqrt{a+bx^2}}+\frac{\sqrt{\frac{bx^2}{a}+1}(cx)^{m+2}(aD(m+2)-}{}}{m+2}}{b(m+3)}$$

$$\frac{D\sqrt{a+bx^2}(cx)^{m+2}}{bc^2(m+3)}$$

input
```
Int[((c*x)^m*(A + B*x + C*x^2 + D*x^3))/Sqrt[a + b*x^2],x]
```

output
```
(D*(c*x)^(2 + m)*Sqrt[a + b*x^2])/(b*c^2*(3 + m)) + ((C*(3 + m)*(c*x)^(1 +
 m)*Sqrt[a + b*x^2])/(c*(2 + m)) - (((3 + m)*(a*C*(1 + m) - A*b*(2 + m))*(
c*x)^(1 + m)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)
/2, -((b*x^2)/a)])/(c*(1 + m)*Sqrt[a + b*x^2]) + ((a*D*(2 + m) - b*B*(3 +
m))*(c*x)^(2 + m)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/2, (2 + m)/2, (4
 + m)/2, -((b*x^2)/a)])/(c^2*Sqrt[a + b*x^2]))/(2 + m))/(b*(3 + m))
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 557
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Sym
bol] :> Simp[c   Int[(e*x)^m*(a + b*x^2)^p, x], x] + Simp[d/e   Int[(e*x)^(
m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x]
```

rule 2340
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Simp[1/(b*(m
+ q + 2*p + 1))   Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)
*Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /;
GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ
[Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

## Maple [F]

$$\int \frac{(cx)^m (Dx^3 + C\,x^2 + Bx + A)}{\sqrt{b\,x^2 + a}}dx$$

input
```
int((c*x)^m*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(1/2),x)
```

output
```
int((c*x)^m*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(1/2),x)
```

## Fricas [F]

$$\int \frac{(cx)^m (A + Bx + Cx^2 + Dx^3)}{\sqrt{a + bx^2}}\,dx = \int \frac{(Dx^3 + Cx^2 + Bx + A)(cx)^m}{\sqrt{bx^2 + a}}\,dx$$

input
```
integrate((c*x)^m*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(1/2),x, algorithm="fricas
")
```

output
```
integral((D*x^3 + C*x^2 + B*x + A)*(c*x)^m/sqrt(b*x^2 + a), x)
```

## Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.16 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.97

$$\int \frac{(cx)^m (A+Bx+Cx^2+Dx^3)}{\sqrt{a+bx^2}}\,dx = \frac{Ac^m x^{m+1}\Gamma\left(\frac{m}{2}+\frac{1}{2}\right) {}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{m}{2}+\frac{1}{2}\\ \frac{m}{2}+\frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)} + \frac{Bc^m x^{m+2}\Gamma\left(\frac{m}{2}+1\right) {}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{m}{2}+1\\ \frac{m}{2}+2\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}\Gamma\left(\frac{m}{2}+2\right)} + \frac{Cc^m x^{m+3}\Gamma\left(\frac{m}{2}+\frac{3}{2}\right) {}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{m}{2}+\frac{3}{2}\\ \frac{m}{2}+\frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)} + \frac{Dc^m x^{m+4}\Gamma\left(\frac{m}{2}+2\right) {}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{m}{2}+2\\ \frac{m}{2}+3\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}\Gamma\left(\frac{m}{2}+3\right)}$$

input
```
integrate((c*x)**m*(D*x**3+C*x**2+B*x+A)/(b*x**2+a)**(1/2),x)
```

output
```
A*c**m*x**(m + 1)*gamma(m/2 + 1/2)*hyper((1/2, m/2 + 1/2), (m/2 + 3/2,), b
*x**2*exp_polar(I*pi)/a)/(2*sqrt(a)*gamma(m/2 + 3/2)) + B*c**m*x**(m + 2)*
gamma(m/2 + 1)*hyper((1/2, m/2 + 1), (m/2 + 2,), b*x**2*exp_polar(I*pi)/a)
/(2*sqrt(a)*gamma(m/2 + 2)) + C*c**m*x**(m + 3)*gamma(m/2 + 3/2)*hyper((1/
2, m/2 + 3/2), (m/2 + 5/2,), b*x**2*exp_polar(I*pi)/a)/(2*sqrt(a)*gamma(m/
2 + 5/2)) + D*c**m*x**(m + 4)*gamma(m/2 + 2)*hyper((1/2, m/2 + 2), (m/2 +
3,), b*x**2*exp_polar(I*pi)/a)/(2*sqrt(a)*gamma(m/2 + 3))
```

## Maxima [F]

$$\int \frac{(cx)^m (A + Bx + Cx^2 + Dx^3)}{\sqrt{a + bx^2}} dx = \int \frac{(Dx^3 + Cx^2 + Bx + A)(cx)^m}{\sqrt{bx^2 + a}} dx$$

input
```
integrate((c*x)^m*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(1/2),x, algorithm="maxima
")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)*(c*x)^m/sqrt(b*x^2 + a), x)
```

## Giac [F]

$$\int \frac{(cx)^m (A + Bx + Cx^2 + Dx^3)}{\sqrt{a + bx^2}} dx = \int \frac{(Dx^3 + Cx^2 + Bx + A)(cx)^m}{\sqrt{bx^2 + a}} dx$$

input
```
integrate((c*x)^m*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)*(c*x)^m/sqrt(b*x^2 + a), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^m (A + Bx + Cx^2 + Dx^3)}{\sqrt{a + bx^2}} dx = \int \frac{(c\,x)^m\,(A + B\,x + C\,x^2 + x^3\,D)}{\sqrt{b\,x^2 + a}} dx$$

input
```
int(((c*x)^m*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^(1/2),x)
```

output
```
int(((c*x)^m*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^(1/2), x)
```

## Reduce [F]

$$\int \frac{(cx)^m (A + Bx + Cx^2 + Dx^3)}{\sqrt{a + bx^2}} \, dx = c^m \left( \left( \int \frac{x^m}{\sqrt{b\,x^2 + a}} dx \right) a + \left( \int \frac{x^m x^3}{\sqrt{b\,x^2 + a}} dx \right) d + \left( \int \frac{x^m x^2}{\sqrt{b\,x^2 + a}} dx \right) c + \left( \int \frac{x^m x}{\sqrt{b\,x^2 + a}} dx \right) b \right)$$

input
```
int((c*x)^m*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(1/2),x)
```

output
```
c**m*(int(x**m/sqrt(a + b*x**2),x)*a + int((x**m*x**3)/sqrt(a + b*x**2),x)
*d + int((x**m*x**2)/sqrt(a + b*x**2),x)*c + int((x**m*x)/sqrt(a + b*x**2)
,x)*b)
```

# 3.145 $\int \frac{(cx)^m (A+Bx+Cx^2+Dx^3)}{(a+bx^2)^{3/2}}\,dx$



## Optimal result

Integrand size = 32, antiderivative size = 228

$$\int \frac{(cx)^m (A+Bx+Cx^2+Dx^3)}{(a+bx^2)^{3/2}}\,dx = \frac{C(cx)^{1+m}}{bcm\sqrt{a+bx^2}} + \frac{D(cx)^{2+m}}{bc^2(1+m)\sqrt{a+bx^2}} - \frac{\left(\frac{C}{bm}-\frac{A}{a+am}\right)(cx)^{1+m}\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1+m}{2},\frac{3+m}{2},-\frac{bx^2}{a}\right)}{c\sqrt{a+bx^2}} + \frac{\left(\frac{B}{a(2+m)}-\frac{D}{b+bm}\right)(cx)^{2+m}\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{2+m}{2},\frac{4+m}{2},-\frac{bx^2}{a}\right)}{c^2\sqrt{a+bx^2}}$$

output
```
C*(c*x)^(1+m)/b/c/m/(b*x^2+a)^(1/2)+D*(c*x)^(2+m)/b/c^2/(1+m)/(b*x^2+a)^(1
/2)-(C/b/m-A/(a*m+a))*(c*x)^(1+m)*(1+b*x^2/a)^(1/2)*hypergeom([3/2, 1/2+1/
2*m],[3/2+1/2*m],-b*x^2/a)/c/(b*x^2+a)^(1/2)+(B/a/(2+m)-D/(b*m+b))*(c*x)^(
2+m)*(1+b*x^2/a)^(1/2)*hypergeom([3/2, 1+1/2*m],[2+1/2*m],-b*x^2/a)/c^2/(b
*x^2+a)^(1/2)
```

## Mathematica [A] (verified)

Time = 2.33 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.79

$$\int \frac{(cx)^m (A+Bx+Cx^2+Dx^3)}{(a+bx^2)^{3/2}}\,dx = \frac{x(cx)^m\sqrt{1+\frac{bx^2}{a}}\left(\frac{A\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1+m}{2},\frac{3+m}{2},-\frac{bx^2}{a}\right)}{1+m}+x\left(\frac{B\,\text{Hypergeor}}{}\right.\right.}{}$$

input
```
Integrate[((c*x)^m*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^(3/2),x]
```

output
```
(x*(c*x)^m*Sqrt[1 + (b*x^2)/a]*((A*Hypergeometric2F1[3/2, (1 + m)/2, (3 +
m)/2, -((b*x^2)/a)])/(1 + m) + x*((B*Hypergeometric2F1[3/2, (2 + m)/2, (4
+ m)/2, -((b*x^2)/a)])/(2 + m) + x*((C*Hypergeometric2F1[3/2, (3 + m)/2, (
5 + m)/2, -((b*x^2)/a)])/(3 + m) + (D*x*Hypergeometric2F1[3/2, (4 + m)/2,
(6 + m)/2, -((b*x^2)/a)])/(4 + m)))))/(a*Sqrt[a + b*x^2])
```

## Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {2337, 27, 557, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^m (A+Bx+Cx^2+Dx^3)}{(a+bx^2)^{3/2}}dx$$

$$\Big\downarrow \text{2337}$$

$$\frac{(cx)^{m+1}\left(x\left(B-\frac{aD}{b}\right)-\frac{aC}{b}+A\right)}{ac\sqrt{a+bx^2}}-\frac{\int \frac{(cx)^m\left(b\left(Am-\frac{aC(m+1)}{b}\right)+(bB(m+1)-aD(m+2))x\right)}{b\sqrt{bx^2+a}}dx}{a}$$

$$\Big\downarrow \text{27}$$

$$\frac{(cx)^{m+1}\left(x\left(B-\frac{aD}{b}\right)-\frac{aC}{b}+A\right)}{ac\sqrt{a+bx^2}}-\frac{\int \frac{(cx)^m(Abm-aC(m+1)+(bB(m+1)-aD(m+2))x)}{\sqrt{bx^2+a}}dx}{ab}$$

$$\Big\downarrow \text{557}$$

$$\frac{\frac{(cx)^{m+1}\left(x\left(B-\frac{aD}{b}\right)-\frac{aC}{b}+A\right)}{ac\sqrt{a+bx^2}}-\frac{(Abm-aC(m+1))\int\frac{(cx)^m}{\sqrt{bx^2+a}}dx+\frac{(bB(m+1)-aD(m+2))\int\frac{(cx)^{m+1}}{\sqrt{bx^2+a}}dx}{c}}{ab}}{}$$

$\Big\downarrow$ 279

$$\frac{(cx)^{m+1}\left(x\left(B-\frac{aD}{b}\right)-\frac{aC}{b}+A\right)}{ac\sqrt{a+bx^2}}-\frac{\frac{\sqrt{\frac{bx^2}{a}+1}(Abm-aC(m+1))\int\frac{(cx)^m}{\sqrt{\frac{bx^2}{a}+1}}dx}{\sqrt{a+bx^2}}+\frac{\sqrt{\frac{bx^2}{a}+1}(bB(m+1)-aD(m+2))\int\frac{(cx)^{m+1}}{\sqrt{\frac{bx^2}{a}+1}}dx}{c\sqrt{a+bx^2}}}{ab}$$

$\Big\downarrow$ 278

$$\frac{(cx)^{m+1}\left(x\left(B-\frac{aD}{b}\right)-\frac{aC}{b}+A\right)}{ac\sqrt{a+bx^2}}-\frac{\frac{\sqrt{\frac{bx^2}{a}+1}(cx)^{m+1}(Abm-aC(m+1))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},-\frac{bx^2}{a}\right)}{c(m+1)\sqrt{a+bx^2}}+\frac{\sqrt{\frac{bx^2}{a}+1}(cx)^{m+2}(bB(m+1)-aD(m+2))\,\text{Hypergeometric2F}}{c^2(m+2)\sqrt{a+bx^2}}}{ab}$$

input
```
Int[((c*x)^m*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^(3/2),x]
```

output
```
((c*x)^(1 + m)*(A - (a*C)/b + (B - (a*D)/b)*x))/(a*c*Sqrt[a + b*x^2]) - ((
(A*b*m - a*C*(1 + m))*(c*x)^(1 + m)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[
1/2, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(c*(1 + m)*Sqrt[a + b*x^2]) + ((
b*B*(1 + m) - a*D*(2 + m))*(c*x)^(2 + m)*Sqrt[1 + (b*x^2)/a]*Hypergeometri
c2F1[1/2, (2 + m)/2, (4 + m)/2, -((b*x^2)/a)])/(c^2*(2 + m)*Sqrt[a + b*x^2
]))/(a*b)
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 557
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Sym
bol] :> Simp[c   Int[(e*x)^m*(a + b*x^2)^p, x], x] + Simp[d/e   Int[(e*x)^(
m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x]
```

rule 2337
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(-(c*x)^(m + 1))*(f + g*x)*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1)))
, x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*ExpandToSum[2
*a*(p + 1)*Q + f*(m + 2*p + 3) + g*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a
, b, c, m}, x] && PolyQ[Pq, x] && LtQ[p, -1] &&  !GtQ[m, 0]
```

## Maple [F]

$$\int \frac{(cx)^m (Dx^3 + C x^2 + Bx + A)}{(b x^2 + a)^{\frac{3}{2}}} dx$$

input
```
int((c*x)^m*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(3/2),x)
```

output
```
int((c*x)^m*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(3/2),x)
```

## Fricas [F]

$$\int \frac{(cx)^m (A+Bx+Cx^2+Dx^3)}{(a+bx^2)^{3/2}}\,dx = \int \frac{(Dx^3+Cx^2+Bx+A)(cx)^m}{(bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x)^m*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(3/2),x, algorithm="fricas
")
```

output
```
integral((D*x^3 + C*x^2 + B*x + A)*sqrt(b*x^2 + a)*(c*x)^m/(b^2*x^4 + 2*a*
b*x^2 + a^2), x)
```

## Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 11.76 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.98

$$\int \frac{(cx)^m (A+Bx+Cx^2+Dx^3)}{(a+bx^2)^{3/2}}\,dx = \frac{Ac^m x^{m+1}\Gamma\left(\frac{m}{2}+\frac{1}{2}\right) {}_2F_1\left(\begin{matrix}\frac{3}{2}, \frac{m}{2}+\frac{1}{2} \\ \frac{m}{2}+\frac{3}{2}\end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)} + \frac{Bc^m x^{m+2}\Gamma\left(\frac{m}{2}+1\right) {}_2F_1\left(\begin{matrix}\frac{3}{2}, \frac{m}{2}+1 \\ \frac{m}{2}+2\end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}\Gamma\left(\frac{m}{2}+2\right)} + \frac{Cc^m x^{m+3}\Gamma\left(\frac{m}{2}+\frac{3}{2}\right) {}_2F_1\left(\begin{matrix}\frac{3}{2}, \frac{m}{2}+\frac{3}{2} \\ \frac{m}{2}+\frac{5}{2}\end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)} + \frac{Dc^m x^{m+4}\Gamma\left(\frac{m}{2}+2\right) {}_2F_1\left(\begin{matrix}\frac{3}{2}, \frac{m}{2}+2 \\ \frac{m}{2}+3\end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}\Gamma\left(\frac{m}{2}+3\right)}$$

input
```
integrate((c*x)**m*(D*x**3+C*x**2+B*x+A)/(b*x**2+a)**(3/2),x)
```

output
```
A*c**m*x**(m + 1)*gamma(m/2 + 1/2)*hyper((3/2, m/2 + 1/2), (m/2 + 3/2,), b
*x**2*exp_polar(I*pi)/a)/(2*a**(3/2)*gamma(m/2 + 3/2)) + B*c**m*x**(m + 2)
*gamma(m/2 + 1)*hyper((3/2, m/2 + 1), (m/2 + 2,), b*x**2*exp_polar(I*pi)/a
)/(2*a**(3/2)*gamma(m/2 + 2)) + C*c**m*x**(m + 3)*gamma(m/2 + 3/2)*hyper((
3/2, m/2 + 3/2), (m/2 + 5/2,), b*x**2*exp_polar(I*pi)/a)/(2*a**(3/2)*gamma
(m/2 + 5/2)) + D*c**m*x**(m + 4)*gamma(m/2 + 2)*hyper((3/2, m/2 + 2), (m/2
 + 3,), b*x**2*exp_polar(I*pi)/a)/(2*a**(3/2)*gamma(m/2 + 3))
```

## Maxima [F]

$$\int \frac{(cx)^m (A + Bx + Cx^2 + Dx^3)}{(a + bx^2)^{3/2}}\, dx = \int \frac{(Dx^3 + Cx^2 + Bx + A)(cx)^m}{(bx^2 + a)^{\frac{3}{2}}}\, dx$$

input
```
integrate((c*x)^m*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(3/2),x, algorithm="maxima
")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)*(c*x)^m/(b*x^2 + a)^(3/2), x)
```

## Giac [F]

$$\int \frac{(cx)^m (A + Bx + Cx^2 + Dx^3)}{(a + bx^2)^{3/2}}\, dx = \int \frac{(Dx^3 + Cx^2 + Bx + A)(cx)^m}{(bx^2 + a)^{\frac{3}{2}}}\, dx$$

input
```
integrate((c*x)^m*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)*(c*x)^m/(b*x^2 + a)^(3/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^m (A+Bx+Cx^2+Dx^3)}{(a+bx^2)^{3/2}}\,dx=\int \frac{(c\,x)^m\,(A+B\,x+C\,x^2+x^3\,D)}{(b\,x^2+a)^{3/2}}\,dx$$

input
```
int(((c*x)^m*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^(3/2),x)
```

output
```
int(((c*x)^m*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^(3/2), x)
```

## Reduce [F]

$$\int \frac{(cx)^m (A+Bx+Cx^2+Dx^3)}{(a+bx^2)^{3/2}}\,dx=c^m\left(\left(\int \frac{x^m}{\sqrt{b\,x^2+a}\,a+\sqrt{b\,x^2+a}\,b\,x^2}dx\right)a+\left(\int \frac{x^m x^3}{\sqrt{b\,x^2+a}\,a+\sqrt{b\,x^2+a}\,b\,x^2}dx\right)d+\left(\int \frac{x^m x^2}{\sqrt{b\,x^2+a}\,a+\sqrt{b\,x^2+a}\,b\,x^2}dx\right)c+\left(\int \frac{x^m x}{\sqrt{b\,x^2+a}\,a+\sqrt{b\,x^2+a}\,b\,x^2}dx\right)b\right)$$

input
```
int((c*x)^m*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(3/2),x)
```

output
```
c**m*(int(x**m/(sqrt(a + b*x**2)*a + sqrt(a + b*x**2)*b*x**2),x)*a + int((
x**m*x**3)/(sqrt(a + b*x**2)*a + sqrt(a + b*x**2)*b*x**2),x)*d + int((x**m
*x**2)/(sqrt(a + b*x**2)*a + sqrt(a + b*x**2)*b*x**2),x)*c + int((x**m*x)/
(sqrt(a + b*x**2)*a + sqrt(a + b*x**2)*b*x**2),x)*b)
```

# 3.146 $\int \frac{(cx)^m(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^{5/2}}\,dx$



## Optimal result

Integrand size = 32, antiderivative size = 252

$$\int \frac{(cx)^m\left(A+Bx+Cx^2+Dx^3\right)}{\left(a+bx^2\right)^{5/2}}\,dx = -\frac{C(cx)^{1+m}}{bc(2-m)\left(a+bx^2\right)^{3/2}}-\frac{D(cx)^{2+m}}{bc^2(1-m)\left(a+bx^2\right)^{3/2}}+\frac{\left(\frac{A}{1+m}+\frac{aC}{2b-bm}\right)(cx)^{1+m}\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{1+m}{2},\frac{3+m}{2},-\frac{bx^2}{a}\right)}{a^2c\sqrt{a+bx^2}}+\frac{(aD(2+m)+b(B-Bm))(cx)^{2+m}\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{2+m}{2},\frac{4+m}{2},-\frac{bx^2}{a}\right)}{a^2bc^2(1-m)(2+m)\sqrt{a+bx^2}}$$

output
```
-C*(c*x)^(1+m)/b/c/(2-m)/(b*x^2+a)^(3/2)-D*(c*x)^(2+m)/b/c^2/(1-m)/(b*x^2+
a)^(3/2)+(A/(1+m)+a*C/(-b*m+2*b))*(c*x)^(1+m)*(1+b*x^2/a)^(1/2)*hypergeom(
[5/2, 1/2+1/2*m],[3/2+1/2*m],-b*x^2/a)/a^2/c/(b*x^2+a)^(1/2)+(a*D*(2+m)+b*
(-B*m+B))*(c*x)^(2+m)*(1+b*x^2/a)^(1/2)*hypergeom([5/2, 1+1/2*m],[2+1/2*m]
,-b*x^2/a)/a^2/b/c^2/(1-m)/(2+m)/(b*x^2+a)^(1/2)
```

## Mathematica [A] (verified)

Time = 4.06 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.71

$$\int \frac{(cx)^m (A+Bx+Cx^2+Dx^3)}{(a+bx^2)^{5/2}}\,dx = \frac{x(cx)^m\sqrt{1+\frac{bx^2}{a}}\left(\frac{A\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{1+m}{2},\frac{3+m}{2},-\frac{bx^2}{a}\right)}{1+m}+x\left(\frac{B\,\text{Hypergeor}}{}\right.\right.}{}$$

input
```
Integrate[((c*x)^m*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^(5/2),x]
```

output
```
(x*(c*x)^m*Sqrt[1 + (b*x^2)/a]*((A*Hypergeometric2F1[5/2, (1 + m)/2, (3 +
m)/2, -((b*x^2)/a)])/(1 + m) + x*((B*Hypergeometric2F1[5/2, (2 + m)/2, (4
+ m)/2, -((b*x^2)/a)])/(2 + m) + x*((C*Hypergeometric2F1[5/2, (3 + m)/2, (
5 + m)/2, -((b*x^2)/a)])/(3 + m) + (D*x*Hypergeometric2F1[5/2, (4 + m)/2,
(6 + m)/2, -((b*x^2)/a)])/(4 + m)))))/(a^2*Sqrt[a + b*x^2])
```

## Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.92, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2337, 25, 27, 557, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^m (A+Bx+Cx^2+Dx^3)}{(a+bx^2)^{5/2}}dx$$

$$\Big\downarrow \text{2337}$$

$$\frac{(cx)^{m+1}\left(x\left(B-\frac{aD}{b}\right)-\frac{aC}{b}+A\right)}{3ac(a+bx^2)^{3/2}}-\frac{\int -\frac{(cx)^m\left(b\left(A(2-m)+\frac{aC(m+1)}{b}\right)+(aD(m+2)+b(B-Bm))x\right)}{b(bx^2+a)^{3/2}}dx}{3a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{(cx)^m(Ab(2-m)+aC(m+1)+(aD(m+2)+b(B-Bm))x)}{b(bx^2+a)^{3/2}}dx}{3a}+\frac{(cx)^{m+1}\left(x\left(B-\frac{aD}{b}\right)-\frac{aC}{b}+A\right)}{3ac(a+bx^2)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(cx)^m(Ab(2-m)+aC(m+1)+(aD(m+2)+b(B-Bm))x)}{(bx^2+a)^{3/2}}dx}{3ab}+\frac{(cx)^{m+1}\left(x\left(B-\frac{aD}{b}\right)-\frac{aC}{b}+A\right)}{3ac\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{557}$$

$$\frac{(aC(m+1)+Ab(2-m))\int \frac{(cx)^m}{(bx^2+a)^{3/2}}dx+\frac{(aD(m+2)+b(B-Bm))\int \frac{(cx)^{m+1}}{(bx^2+a)^{3/2}}dx}{c}}{3ab}+\frac{(cx)^{m+1}\left(x\left(B-\frac{aD}{b}\right)-\frac{aC}{b}+A\right)}{3ac\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{279}$$

$$\frac{\frac{\sqrt{\frac{bx^2}{a}+1}(aC(m+1)+Ab(2-m))\int \frac{(cx)^m}{\left(\frac{bx^2}{a}+1\right)^{3/2}}dx}{a\sqrt{a+bx^2}}+\frac{\sqrt{\frac{bx^2}{a}+1}(aD(m+2)+b(B-Bm))\int \frac{(cx)^{m+1}}{\left(\frac{bx^2}{a}+1\right)^{3/2}}dx}{ac\sqrt{a+bx^2}}}{3ab}+\frac{(cx)^{m+1}\left(x\left(B-\frac{aD}{b}\right)-\frac{aC}{b}+A\right)}{3ac\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{278}$$

$$\frac{\frac{\sqrt{\frac{bx^2}{a}+1}(cx)^{m+1}(aC(m+1)+Ab(2-m))\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{m+1}{2},\frac{m+3}{2},-\frac{bx^2}{a}\right)}{ac(m+1)\sqrt{a+bx^2}}+\frac{\sqrt{\frac{bx^2}{a}+1}(cx)^{m+2}(aD(m+2)+b(B-Bm))\,\text{Hypergeome}}{ac^2(m+2)\sqrt{a+bx^2}}}{3ab}$$
$$\frac{(cx)^{m+1}\left(x\left(B-\frac{aD}{b}\right)-\frac{aC}{b}+A\right)}{3ac\left(a+bx^2\right)^{3/2}}$$

input
```
Int[((c*x)^m*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^(5/2),x]
```

output
```
((c*x)^(1 + m)*(A - (a*C)/b + (B - (a*D)/b)*x))/(3*a*c*(a + b*x^2)^(3/2))
+ (((A*b*(2 - m) + a*C*(1 + m))*(c*x)^(1 + m)*Sqrt[1 + (b*x^2)/a]*Hypergeo
metric2F1[3/2, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(a*c*(1 + m)*Sqrt[a +
b*x^2]) + ((a*D*(2 + m) + b*(B - B*m))*(c*x)^(2 + m)*Sqrt[1 + (b*x^2)/a]*H
ypergeometric2F1[3/2, (2 + m)/2, (4 + m)/2, -((b*x^2)/a)])/(a*c^2*(2 + m)*
Sqrt[a + b*x^2]))/(3*a*b)
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 557
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Sym
bol] :> Simp[c   Int[(e*x)^m*(a + b*x^2)^p, x], x] + Simp[d/e   Int[(e*x)^(
m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x]
```

rule 2337
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(-(c*x)^(m + 1))*(f + g*x)*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1)))
, x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*ExpandToSum[2
*a*(p + 1)*Q + f*(m + 2*p + 3) + g*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a
, b, c, m}, x] && PolyQ[Pq, x] && LtQ[p, -1] &&  !GtQ[m, 0]
```

## Maple [F]

$$\int \frac{(cx)^m (Dx^3 + C x^2 + Bx + A)}{(b x^2 + a)^{\frac{5}{2}}} dx$$

input
```
int((c*x)^m*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(5/2),x)
```

output
```
int((c*x)^m*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(5/2),x)
```

## Fricas [F]

$$\int \frac{(cx)^m (A + Bx + Cx^2 + Dx^3)}{(a + bx^2)^{5/2}} dx = \int \frac{(Dx^3 + Cx^2 + Bx + A)(cx)^m}{(bx^2 + a)^{\frac{5}{2}}} dx$$

input
```
integrate((c*x)^m*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(5/2),x, algorithm="fricas
")
```

output
```
integral((D*x^3 + C*x^2 + B*x + A)*sqrt(b*x^2 + a)*(c*x)^m/(b^3*x^6 + 3*a*
b^2*x^4 + 3*a^2*b*x^2 + a^3), x)
```

## Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 156.21 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.88

$$\int \frac{(cx)^m (A+Bx+Cx^2+Dx^3)}{(a+bx^2)^{5/2}}\,dx = \frac{Ac^m x^{m+1}\Gamma\left(\frac{m}{2}+\frac{1}{2}\right) {}_2F_1\left(\begin{matrix}\frac{5}{2}, \frac{m}{2}+\frac{1}{2} \\ \frac{m}{2}+\frac{3}{2}\end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{2}}\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)} + \frac{Bc^m x^{m+2}\Gamma\left(\frac{m}{2}+1\right) {}_2F_1\left(\begin{matrix}\frac{5}{2}, \frac{m}{2}+1 \\ \frac{m}{2}+2\end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{2}}\Gamma\left(\frac{m}{2}+2\right)} + \frac{Cc^m x^{m+3}\Gamma\left(\frac{m}{2}+\frac{3}{2}\right) {}_2F_1\left(\begin{matrix}\frac{5}{2}, \frac{m}{2}+\frac{3}{2} \\ \frac{m}{2}+\frac{5}{2}\end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{2}}\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)} + \frac{Dc^m x^{m+4}\Gamma\left(\frac{m}{2}+2\right) {}_2F_1\left(\begin{matrix}\frac{5}{2}, \frac{m}{2}+2 \\ \frac{m}{2}+3\end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{2}}\Gamma\left(\frac{m}{2}+3\right)}$$

input
```
integrate((c*x)**m*(D*x**3+C*x**2+B*x+A)/(b*x**2+a)**(5/2),x)
```

output
```
A*c**m*x**(m + 1)*gamma(m/2 + 1/2)*hyper((5/2, m/2 + 1/2), (m/2 + 3/2,), b
*x**2*exp_polar(I*pi)/a)/(2*a**(5/2)*gamma(m/2 + 3/2)) + B*c**m*x**(m + 2)
*gamma(m/2 + 1)*hyper((5/2, m/2 + 1), (m/2 + 2,), b*x**2*exp_polar(I*pi)/a
)/(2*a**(5/2)*gamma(m/2 + 2)) + C*c**m*x**(m + 3)*gamma(m/2 + 3/2)*hyper((
5/2, m/2 + 3/2), (m/2 + 5/2,), b*x**2*exp_polar(I*pi)/a)/(2*a**(5/2)*gamma
(m/2 + 5/2)) + D*c**m*x**(m + 4)*gamma(m/2 + 2)*hyper((5/2, m/2 + 2), (m/2
 + 3,), b*x**2*exp_polar(I*pi)/a)/(2*a**(5/2)*gamma(m/2 + 3))
```

## Maxima [F]

$$\int \frac{(cx)^m (A+Bx+Cx^2+Dx^3)}{(a+bx^2)^{5/2}}\,dx = \int \frac{(Dx^3+Cx^2+Bx+A)(cx)^m}{(bx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((c*x)^m*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(5/2),x, algorithm="maxima
")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)*(c*x)^m/(b*x^2 + a)^(5/2), x)
```

## Giac [F]

$$\int \frac{(cx)^m (A+Bx+Cx^2+Dx^3)}{(a+bx^2)^{5/2}}\,dx = \int \frac{(Dx^3+Cx^2+Bx+A)(cx)^m}{(bx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((c*x)^m*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)*(c*x)^m/(b*x^2 + a)^(5/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^m (A+Bx+Cx^2+Dx^3)}{(a+bx^2)^{5/2}}\,dx = \int \frac{(c\,x)^m\,(A+B\,x+C\,x^2+x^3\,D)}{(b\,x^2+a)^{5/2}}\,dx$$

input
```
int(((c*x)^m*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^(5/2),x)
```

output
```
int(((c*x)^m*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^(5/2), x)
```

## Reduce [F]

$$\int \frac{(cx)^m (A+Bx+Cx^2+Dx^3)}{(a+bx^2)^{5/2}}\,dx = c^m\left(\left(\int \frac{x^m}{\sqrt{b\,x^2+a}\,a^2+2\sqrt{b\,x^2+a}\,ab\,x^2+\sqrt{b\,x^2+a}\,b^2x^4}dx\right)a + \left(\int \frac{x^m x^3}{\sqrt{b\,x^2+a}\,a^2+2\sqrt{b\,x^2+a}\,ab\,x^2+\sqrt{b\,x^2+a}\,b^2x^4}dx\right)d + \left(\int \frac{x^m x^2}{\sqrt{b\,x^2+a}\,a^2+2\sqrt{b\,x^2+a}\,ab\,x^2+\sqrt{b\,x^2+a}\,b^2x^4}dx\right)c + \left(\int \frac{x^m x}{\sqrt{b\,x^2+a}\,a^2+2\sqrt{b\,x^2+a}\,ab\,x^2+\sqrt{b\,x^2+a}\,b^2x^4}dx\right)b\right)$$

input
```
int((c*x)^m*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(5/2),x)
```

output
```
c**m*(int(x**m/(sqrt(a + b*x**2)*a**2 + 2*sqrt(a + b*x**2)*a*b*x**2 + sqrt
(a + b*x**2)*b**2*x**4),x)*a + int((x**m*x**3)/(sqrt(a + b*x**2)*a**2 + 2*
sqrt(a + b*x**2)*a*b*x**2 + sqrt(a + b*x**2)*b**2*x**4),x)*d + int((x**m*x
**2)/(sqrt(a + b*x**2)*a**2 + 2*sqrt(a + b*x**2)*a*b*x**2 + sqrt(a + b*x**
2)*b**2*x**4),x)*c + int((x**m*x)/(sqrt(a + b*x**2)*a**2 + 2*sqrt(a + b*x*
*2)*a*b*x**2 + sqrt(a + b*x**2)*b**2*x**4),x)*b)
```

# 3.147 $\int \frac{(cx)^m(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^{7/2}}\,dx$



## Optimal result

Integrand size = 32, antiderivative size = 252

$$\int \frac{(cx)^m(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^{7/2}}\,dx = -\frac{C(cx)^{1+m}}{bc(4-m)(a+bx^2)^{5/2}}-\frac{D(cx)^{2+m}}{bc^2(3-m)(a+bx^2)^{5/2}}+\frac{\left(\frac{A}{1+m}+\frac{aC}{4b-bm}\right)(cx)^{1+m}\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{7}{2},\frac{1+m}{2},\frac{3+m}{2},-\frac{bx^2}{a}\right)}{a^3c\sqrt{a+bx^2}}+\frac{(bB(3-m)+aD(2+m))(cx)^{2+m}\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{7}{2},\frac{2+m}{2},\frac{4+m}{2},-\frac{bx^2}{a}\right)}{a^3bc^2(3-m)(2+m)\sqrt{a+bx^2}}$$

output
```
-C*(c*x)^(1+m)/b/c/(4-m)/(b*x^2+a)^(5/2)-D*(c*x)^(2+m)/b/c^2/(3-m)/(b*x^2+
a)^(5/2)+(A/(1+m)+a*C/(-b*m+4*b))*(c*x)^(1+m)*(1+b*x^2/a)^(1/2)*hypergeom(
[7/2, 1/2+1/2*m],[3/2+1/2*m],-b*x^2/a)/a^3/c/(b*x^2+a)^(1/2)+(b*B*(3-m)+a*
D*(2+m))*(c*x)^(2+m)*(1+b*x^2/a)^(1/2)*hypergeom([7/2, 1+1/2*m],[2+1/2*m],
-b*x^2/a)/a^3/b/c^2/(3-m)/(2+m)/(b*x^2+a)^(1/2)
```

## Mathematica [A] (verified)

Time = 5.99 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.71

$$\int \frac{(cx)^m (A+Bx+Cx^2+Dx^3)}{(a+bx^2)^{7/2}}\,dx = \frac{x(cx)^m\sqrt{1+\frac{bx^2}{a}}\left(\frac{A\,\text{Hypergeometric2F1}\left(\frac{7}{2},\frac{1+m}{2},\frac{3+m}{2},-\frac{bx^2}{a}\right)}{1+m}+x\left(\frac{B\,\text{Hypergeor}}{}\right.\right.}{}$$

input
```
Integrate[((c*x)^m*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^(7/2),x]
```

output
```
(x*(c*x)^m*Sqrt[1 + (b*x^2)/a]*((A*Hypergeometric2F1[7/2, (1 + m)/2, (3 +
m)/2, -((b*x^2)/a)])/(1 + m) + x*((B*Hypergeometric2F1[7/2, (2 + m)/2, (4
+ m)/2, -((b*x^2)/a)])/(2 + m) + x*((C*Hypergeometric2F1[7/2, (3 + m)/2, (
5 + m)/2, -((b*x^2)/a)])/(3 + m) + (D*x*Hypergeometric2F1[7/2, (4 + m)/2,
(6 + m)/2, -((b*x^2)/a)])/(4 + m)))))/(a^3*Sqrt[a + b*x^2])
```

## Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.92, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2337, 25, 27, 557, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^m (A+Bx+Cx^2+Dx^3)}{(a+bx^2)^{7/2}}\,dx$$

$$\Big\downarrow \ 2337$$

$$\frac{(cx)^{m+1}\left(x\left(B-\frac{aD}{b}\right)-\frac{aC}{b}+A\right)}{5ac(a+bx^2)^{5/2}}-\frac{\int -\frac{(cx)^m\left(b\left(A(4-m)+\frac{aC(m+1)}{b}\right)+(bB(3-m)+aD(m+2))x\right)}{b(bx^2+a)^{5/2}}dx}{5a}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{(cx)^m(Ab(4-m)+aC(m+1)+(bB(3-m)+aD(m+2))x)}{b(bx^2+a)^{5/2}}dx}{5a}+\frac{(cx)^{m+1}\left(x\left(B-\frac{aD}{b}\right)-\frac{aC}{b}+A\right)}{5ac(a+bx^2)^{5/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{(cx)^m (Ab(4-m)+aC(m+1)+(bB(3-m)+aD(m+2))x)}{(bx^2+a)^{5/2}}dx}{5ab}+\frac{(cx)^{m+1}\left(x\left(B-\frac{aD}{b}\right)-\frac{aC}{b}+A\right)}{5ac\left(a+bx^2\right)^{5/2}}$$

$$\Big\downarrow \; 557$$

$$\frac{(aC(m+1)+Ab(4-m))\int \frac{(cx)^m}{(bx^2+a)^{5/2}}dx+\frac{(aD(m+2)+bB(3-m))\int \frac{(cx)^{m+1}}{(bx^2+a)^{5/2}}dx}{c}}{5ab}+\frac{(cx)^{m+1}\left(x\left(B-\frac{aD}{b}\right)-\frac{aC}{b}+A\right)}{5ac\left(a+bx^2\right)^{5/2}}$$

$$\Big\downarrow \; 279$$

$$\frac{\frac{\sqrt{\frac{bx^2}{a}+1}(aC(m+1)+Ab(4-m))\int \frac{(cx)^m}{\left(\frac{bx^2}{a}+1\right)^{5/2}}dx}{a^2\sqrt{a+bx^2}}+\frac{\sqrt{\frac{bx^2}{a}+1}(aD(m+2)+bB(3-m))\int \frac{(cx)^{m+1}}{\left(\frac{bx^2}{a}+1\right)^{5/2}}dx}{a^2c\sqrt{a+bx^2}}}{5ab}+\frac{(cx)^{m+1}\left(x\left(B-\frac{aD}{b}\right)-\frac{aC}{b}+A\right)}{5ac\left(a+bx^2\right)^{5/2}}$$

$$\Big\downarrow \; 278$$

$$\frac{\frac{\sqrt{\frac{bx^2}{a}+1}(cx)^{m+1}(aC(m+1)+Ab(4-m))\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{m+1}{2},\frac{m+3}{2},-\frac{bx^2}{a}\right)}{a^2c(m+1)\sqrt{a+bx^2}}+\frac{\sqrt{\frac{bx^2}{a}+1}(cx)^{m+2}(aD(m+2)+bB(3-m))\,\text{Hypergeomet}}{a^2c^2(m+2)\sqrt{a+bx^2}}}{5ab}+\frac{(cx)^{m+1}\left(x\left(B-\frac{aD}{b}\right)-\frac{aC}{b}+A\right)}{5ac\left(a+bx^2\right)^{5/2}}$$

input
```
Int[((c*x)^m*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^(7/2),x]
```

output
```
((c*x)^(1 + m)*(A - (a*C)/b + (B - (a*D)/b)*x))/(5*a*c*(a + b*x^2)^(5/2))
+ (((A*b*(4 - m) + a*C*(1 + m))*(c*x)^(1 + m)*Sqrt[1 + (b*x^2)/a]*Hypergeo
metric2F1[5/2, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(a^2*c*(1 + m)*Sqrt[a
+ b*x^2]) + ((b*B*(3 - m) + a*D*(2 + m))*(c*x)^(2 + m)*Sqrt[1 + (b*x^2)/a]
*Hypergeometric2F1[5/2, (2 + m)/2, (4 + m)/2, -((b*x^2)/a)])/(a^2*c^2*(2 +
 m)*Sqrt[a + b*x^2]))/(5*a*b)
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 557
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Sym
bol] :> Simp[c   Int[(e*x)^m*(a + b*x^2)^p, x], x] + Simp[d/e   Int[(e*x)^(
m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x]
```

rule 2337
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(-(c*x)^(m + 1))*(f + g*x)*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1)))
, x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*ExpandToSum[2
*a*(p + 1)*Q + f*(m + 2*p + 3) + g*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a
, b, c, m}, x] && PolyQ[Pq, x] && LtQ[p, -1] &&  !GtQ[m, 0]
```

## Maple [F]

$$\int \frac{(cx)^m \left(D x^3+C\,x^2+B x+A\right)}{\left(b\,x^2+a\right)^{\frac{7}{2}}}dx$$

input
```
int((c*x)^m*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(7/2),x)
```

output
```
int((c*x)^m*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(7/2),x)
```

## Fricas [F]

$$\int \frac{(cx)^m (A+Bx+Cx^2+Dx^3)}{(a+bx^2)^{7/2}}\,dx = \int \frac{(Dx^3+Cx^2+Bx+A)(cx)^m}{(bx^2+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate((c*x)^m*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(7/2),x, algorithm="fricas
")
```

output
```
integral((D*x^3 + C*x^2 + B*x + A)*sqrt(b*x^2 + a)*(c*x)^m/(b^4*x^8 + 4*a*
b^3*x^6 + 6*a^2*b^2*x^4 + 4*a^3*b*x^2 + a^4), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(cx)^m (A+Bx+Cx^2+Dx^3)}{(a+bx^2)^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((c*x)**m*(D*x**3+C*x**2+B*x+A)/(b*x**2+a)**(7/2),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{(cx)^m (A + Bx + Cx^2 + Dx^3)}{(a + bx^2)^{7/2}} \, dx = \int \frac{(Dx^3 + Cx^2 + Bx + A)(cx)^m}{(bx^2 + a)^{\frac{7}{2}}} \, dx$$

input
```
integrate((c*x)^m*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(7/2),x, algorithm="maxima
")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)*(c*x)^m/(b*x^2 + a)^(7/2), x)
```

## Giac [F]

$$\int \frac{(cx)^m (A + Bx + Cx^2 + Dx^3)}{(a + bx^2)^{7/2}} \, dx = \int \frac{(Dx^3 + Cx^2 + Bx + A)(cx)^m}{(bx^2 + a)^{\frac{7}{2}}} \, dx$$

input
```
integrate((c*x)^m*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(7/2),x, algorithm="giac")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)*(c*x)^m/(b*x^2 + a)^(7/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^m (A + Bx + Cx^2 + Dx^3)}{(a + bx^2)^{7/2}} \, dx = \int \frac{(c\,x)^m\,\left(A + B\,x + C\,x^2 + x^3\,D\right)}{\left(b\,x^2 + a\right)^{7/2}} \, dx$$

input
```
int(((c*x)^m*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^(7/2),x)
```

output
```
int(((c*x)^m*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^(7/2), x)
```

## Reduce [F]

$$\int \frac{(cx)^m (A+Bx+Cx^2+Dx^3)}{(a+bx^2)^{7/2}}\,dx = c^m\left(\left(\left(\int \frac{x^m}{\sqrt{b\,x^2+a}\,a^3+3\sqrt{b\,x^2+a}\,a^2b\,x^2+3\sqrt{b\,x^2+a}\,a\,b^2x^4+\sqrt{}\right.\right.\right.$$
$$+\left(\int \frac{x^m x^3}{\sqrt{b\,x^2+a}\,a^3+3\sqrt{b\,x^2+a}\,a^2b\,x^2+3\sqrt{b\,x^2+a}\,a\,b^2x^4+\sqrt{b\,x^2+a}\,b^3x^6}dx\right)d$$
$$+\left(\int \frac{x^m x^2}{\sqrt{b\,x^2+a}\,a^3+3\sqrt{b\,x^2+a}\,a^2b\,x^2+3\sqrt{b\,x^2+a}\,a\,b^2x^4+\sqrt{b\,x^2+a}\,b^3x^6}dx\right)c$$
$$+\left(\int \frac{x^m x}{\sqrt{b\,x^2+a}\,a^3+3\sqrt{b\,x^2+a}\,a^2b\,x^2+3\sqrt{b\,x^2+a}\,a\,b^2x^4+\sqrt{b\,x^2+a}\,b^3x^6}dx\right)b\right)$$

input
```
int((c*x)^m*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^(7/2),x)
```

output
```
c**m*(int(x**m/(sqrt(a + b*x**2)*a**3 + 3*sqrt(a + b*x**2)*a**2*b*x**2 + 3
*sqrt(a + b*x**2)*a*b**2*x**4 + sqrt(a + b*x**2)*b**3*x**6),x)*a + int((x*
*m*x**3)/(sqrt(a + b*x**2)*a**3 + 3*sqrt(a + b*x**2)*a**2*b*x**2 + 3*sqrt(
a + b*x**2)*a*b**2*x**4 + sqrt(a + b*x**2)*b**3*x**6),x)*d + int((x**m*x**
2)/(sqrt(a + b*x**2)*a**3 + 3*sqrt(a + b*x**2)*a**2*b*x**2 + 3*sqrt(a + b*
x**2)*a*b**2*x**4 + sqrt(a + b*x**2)*b**3*x**6),x)*c + int((x**m*x)/(sqrt(
a + b*x**2)*a**3 + 3*sqrt(a + b*x**2)*a**2*b*x**2 + 3*sqrt(a + b*x**2)*a*b
**2*x**4 + sqrt(a + b*x**2)*b**3*x**6),x)*b)
```

# 3.148 $\int x^4(a+bx^2)(A+Bx^2+Cx^4+Dx^6)\,dx$



## Optimal result

Integrand size = 28, antiderivative size = 61

$$\int x^4(a+bx^2)\left(A+Bx^2+Cx^4+Dx^6\right)\,dx = \frac{1}{5}aAx^5+\frac{1}{7}(Ab+aB)x^7+\frac{1}{9}(bB+aC)x^9+\frac{1}{11}(bC+aD)x^{11}+\frac{1}{13}bDx^{13}$$

output
```
1/5*a*A*x^5+1/7*(A*b+B*a)*x^7+1/9*(B*b+C*a)*x^9+1/11*(C*b+D*a)*x^11+1/13*b
*D*x^13
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00

$$\int x^4(a+bx^2)\left(A+Bx^2+Cx^4+Dx^6\right)\,dx = \frac{1}{5}aAx^5+\frac{1}{7}(Ab+aB)x^7+\frac{1}{9}(bB+aC)x^9+\frac{1}{11}(bC+aD)x^{11}+\frac{1}{13}bDx^{13}$$

input
```
Integrate[x^4*(a + b*x^2)*(A + B*x^2 + C*x^4 + D*x^6),x]
```

output
```
(a*A*x^5)/5 + ((A*b + a*B)*x^7)/7 + ((b*B + a*C)*x^9)/9 + ((b*C + a*D)*x^1
1)/11 + (b*D*x^13)/13
```

## Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4(a+bx^2)\left(A+Bx^2+Cx^4+Dx^6\right)\,dx$$

$$\Big\downarrow \text{2333}$$

$$\int \left(x^6(aB+Ab)+aAx^4+x^8(aC+bB)+x^{10}(aD+bC)+bDx^{12}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{7}x^7(aB+Ab)+\frac{1}{5}aAx^5+\frac{1}{9}x^9(aC+bB)+\frac{1}{11}x^{11}(aD+bC)+\frac{1}{13}bDx^{13}$$

input
```
Int[x^4*(a + b*x^2)*(A + B*x^2 + C*x^4 + D*x^6),x]
```

output
```
(a*A*x^5)/5 + ((A*b + a*B)*x^7)/7 + ((b*B + a*C)*x^9)/9 + ((b*C + a*D)*x^1
1)/11 + (b*D*x^13)/13
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

## Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.85

| method | result | size |
|---|---|---|
| default | $\frac{aA\,x^5}{5}+\frac{(Ab+Ba)x^7}{7}+\frac{(Bb+Ca)x^9}{9}+\frac{(Cb+Da)x^{11}}{11}+\frac{bDx^{13}}{13}$ | 52 |
| norman | $\frac{bDx^{13}}{13}+\left(\frac{Cb}{11}+\frac{Da}{11}\right)x^{11}+\left(\frac{Bb}{9}+\frac{Ca}{9}\right)x^9+\left(\frac{Ab}{7}+\frac{Ba}{7}\right)x^7+\frac{aA\,x^5}{5}$ | 55 |
| gosper | $\frac{1}{13}bDx^{13}+\frac{1}{11}x^{11}Cb+\frac{1}{11}x^{11}Da+\frac{1}{9}x^9Bb+\frac{1}{9}x^9Ca+\frac{1}{7}x^7Ab+\frac{1}{7}x^7Ba+\frac{1}{5}aA\,x^5$ | 58 |
| parallelrisch | $\frac{1}{13}bDx^{13}+\frac{1}{11}x^{11}Cb+\frac{1}{11}x^{11}Da+\frac{1}{9}x^9Bb+\frac{1}{9}x^9Ca+\frac{1}{7}x^7Ab+\frac{1}{7}x^7Ba+\frac{1}{5}aA\,x^5$ | 58 |
| orering | $\frac{x^5(3465Db\,x^8+4095Cb\,x^6+4095Da\,x^6+5005bB\,x^4+5005Ca\,x^4+6435Ab\,x^2+6435Ba\,x^2+9009Aa)}{45045}$ | 60 |

input
```
int(x^4*(b*x^2+a)*(D*x^6+C*x^4+B*x^2+A),x,method=_RETURNVERBOSE)
```

output
```
1/5*a*A*x^5+1/7*(A*b+B*a)*x^7+1/9*(B*b+C*a)*x^9+1/11*(C*b+D*a)*x^11+1/13*b
*D*x^13
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.84

$$\int x^4(a+bx^2)\left(A+Bx^2+Cx^4+Dx^6\right)\,dx$$
$$=\frac{1}{13}\,Dbx^{13}+\frac{1}{11}\,(Da+Cb)x^{11}+\frac{1}{9}\,(Ca+Bb)x^9+\frac{1}{7}\,(Ba+Ab)x^7+\frac{1}{5}\,Aax^5$$

input
```
integrate(x^4*(b*x^2+a)*(D*x^6+C*x^4+B*x^2+A),x, algorithm="fricas")
```

output
```
1/13*D*b*x^13 + 1/11*(D*a + C*b)*x^11 + 1/9*(C*a + B*b)*x^9 + 1/7*(B*a + A
*b)*x^7 + 1/5*A*a*x^5
```

## Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.92

$$\int x^4(a+bx^2)\left(A+Bx^2+Cx^4+Dx^6\right)\,dx = \frac{Aax^5}{5}+\frac{Dbx^{13}}{13}+x^{11}\left(\frac{Cb}{11}+\frac{Da}{11}\right)+x^9\left(\frac{Bb}{9}+\frac{Ca}{9}\right)+x^7\left(\frac{Ab}{7}+\frac{Ba}{7}\right)$$

input
```
integrate(x**4*(b*x**2+a)*(D*x**6+C*x**4+B*x**2+A),x)
```

output
```
A*a*x**5/5 + D*b*x**13/13 + x**11*(C*b/11 + D*a/11) + x**9*(B*b/9 + C*a/9)
 + x**7*(A*b/7 + B*a/7)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.84

$$\int x^4(a+bx^2)\left(A+Bx^2+Cx^4+Dx^6\right)\,dx = \frac{1}{13}\,Dbx^{13}+\frac{1}{11}\,(Da+Cb)x^{11}+\frac{1}{9}\,(Ca+Bb)x^9+\frac{1}{7}\,(Ba+Ab)x^7+\frac{1}{5}\,Aax^5$$

input
```
integrate(x^4*(b*x^2+a)*(D*x^6+C*x^4+B*x^2+A),x, algorithm="maxima")
```

output
```
1/13*D*b*x^13 + 1/11*(D*a + C*b)*x^11 + 1/9*(C*a + B*b)*x^9 + 1/7*(B*a + A
*b)*x^7 + 1/5*A*a*x^5
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.93

$$\int x^4(a+bx^2)\left(A+Bx^2+Cx^4+Dx^6\right)\,dx$$
$$=\frac{1}{13}Dbx^{13}+\frac{1}{11}Dax^{11}+\frac{1}{11}Cbx^{11}+\frac{1}{9}Cax^9+\frac{1}{9}Bbx^9+\frac{1}{7}Bax^7+\frac{1}{7}Abx^7+\frac{1}{5}Aax^5$$

input
```
integrate(x^4*(b*x^2+a)*(D*x^6+C*x^4+B*x^2+A),x, algorithm="giac")
```

output
```
1/13*D*b*x^13 + 1/11*D*a*x^11 + 1/11*C*b*x^11 + 1/9*C*a*x^9 + 1/9*B*b*x^9
+ 1/7*B*a*x^7 + 1/7*A*b*x^7 + 1/5*A*a*x^5
```

## Mupad [B] (verification not implemented)

Time = 2.51 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.93

$$\int x^4(a+bx^2)\left(A+Bx^2+Cx^4+Dx^6\right)\,dx$$
$$=\frac{a\,x^{11}\,D}{11}+\frac{b\,x^{13}\,D}{13}+\frac{A\,a\,x^5}{5}+\frac{A\,b\,x^7}{7}+\frac{B\,a\,x^7}{7}+\frac{B\,b\,x^9}{9}+\frac{C\,a\,x^9}{9}+\frac{C\,b\,x^{11}}{11}$$

input
```
int(x^4*(a + b*x^2)*(A + B*x^2 + C*x^4 + x^6*D),x)
```

output
```
(a*x^11*D)/11 + (b*x^13*D)/13 + (A*a*x^5)/5 + (A*b*x^7)/7 + (B*a*x^7)/7 +
(B*b*x^9)/9 + (C*a*x^9)/9 + (C*b*x^11)/11
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.89

$$\int x^4(a+bx^2)\left(A+Bx^2+Cx^4+Dx^6\right)\,dx$$
$$=\frac{x^5(3465bd\,x^8+4095ad\,x^6+4095bc\,x^6+5005ac\,x^4+5005b^2x^4+12870ab\,x^2+9009a^2)}{45045}$$

input
```
int(x^4*(b*x^2+a)*(D*x^6+C*x^4+B*x^2+A),x)
```

output
```
(x**5*(9009*a**2 + 12870*a*b*x**2 + 5005*a*c*x**4 + 4095*a*d*x**6 + 5005*b
**2*x**4 + 4095*b*c*x**6 + 3465*b*d*x**8))/45045
```

# 3.149 $\int x^2(a+bx^2)\left(A+Bx^2+Cx^4+Dx^6\right)\,dx$



## Optimal result

Integrand size = 28, antiderivative size = 61

$$\int x^2\left(a+bx^2\right)\left(A+Bx^2+Cx^4+Dx^6\right)\,dx = \frac{1}{3}aAx^3+\frac{1}{5}(Ab+aB)x^5+\frac{1}{7}(bB+aC)x^7+\frac{1}{9}(bC+aD)x^9+\frac{1}{11}bDx^{11}$$

output
```
1/3*a*A*x^3+1/5*(A*b+B*a)*x^5+1/7*(B*b+C*a)*x^7+1/9*(C*b+D*a)*x^9+1/11*b*D
*x^11
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00

$$\int x^2\left(a+bx^2\right)\left(A+Bx^2+Cx^4+Dx^6\right)\,dx = \frac{1}{3}aAx^3+\frac{1}{5}(Ab+aB)x^5+\frac{1}{7}(bB+aC)x^7+\frac{1}{9}(bC+aD)x^9+\frac{1}{11}bDx^{11}$$

input
```
Integrate[x^2*(a + b*x^2)*(A + B*x^2 + C*x^4 + D*x^6),x]
```

output
```
(a*A*x^3)/3 + ((A*b + a*B)*x^5)/5 + ((b*B + a*C)*x^7)/7 + ((b*C + a*D)*x^9
)/9 + (b*D*x^11)/11
```

## Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 (a+bx^2) \left(A+Bx^2+Cx^4+Dx^6\right) dx$$

$$\Big\downarrow \text{ 2333}$$

$$\int \left(x^4(aB+Ab)+aAx^2+x^6(aC+bB)+x^8(aD+bC)+bDx^{10}\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{5}x^5(aB+Ab)+\frac{1}{3}aAx^3+\frac{1}{7}x^7(aC+bB)+\frac{1}{9}x^9(aD+bC)+\frac{1}{11}bDx^{11}$$

input
```
Int[x^2*(a + b*x^2)*(A + B*x^2 + C*x^4 + D*x^6),x]
```

output
```
(a*A*x^3)/3 + ((A*b + a*B)*x^5)/5 + ((b*B + a*C)*x^7)/7 + ((b*C + a*D)*x^9
)/9 + (b*D*x^11)/11
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

## Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.85

| method | result | size |
|---|---|---|
| default | $\frac{aA\,x^3}{3}+\frac{(Ab+Ba)x^5}{5}+\frac{(Bb+Ca)x^7}{7}+\frac{(Cb+Da)x^9}{9}+\frac{bD x^{11}}{11}$ | 52 |
| norman | $\frac{bD x^{11}}{11}+\left(\frac{Cb}{9}+\frac{Da}{9}\right)x^9+\left(\frac{Bb}{7}+\frac{Ca}{7}\right)x^7+\left(\frac{Ab}{5}+\frac{Ba}{5}\right)x^5+\frac{aA\,x^3}{3}$ | 55 |
| gosper | $\frac{1}{11}bD x^{11}+\frac{1}{9}x^9Cb+\frac{1}{9}x^9Da+\frac{1}{7}bB\,x^7+\frac{1}{7}x^7Ca+\frac{1}{5}x^5Ab+\frac{1}{5}aB\,x^5+\frac{1}{3}aA\,x^3$ | 58 |
| parallelrisch | $\frac{1}{11}bD x^{11}+\frac{1}{9}x^9Cb+\frac{1}{9}x^9Da+\frac{1}{7}bB\,x^7+\frac{1}{7}x^7Ca+\frac{1}{5}x^5Ab+\frac{1}{5}aB\,x^5+\frac{1}{3}aA\,x^3$ | 58 |
| orering | $\frac{x^3\left(315Db\,x^8+385Cb\,x^6+385Da\,x^6+495bB\,x^4+495Ca\,x^4+693Ab\,x^2+693Ba\,x^2+1155Aa\right)}{3465}$ | 60 |

input
```
int(x^2*(b*x^2+a)*(D*x^6+C*x^4+B*x^2+A),x,method=_RETURNVERBOSE)
```

output
```
1/3*a*A*x^3+1/5*(A*b+B*a)*x^5+1/7*(B*b+C*a)*x^7+1/9*(C*b+D*a)*x^9+1/11*b*D
*x^11
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.84

$$\int x^2(a+bx^2)\left(A+Bx^2+Cx^4+Dx^6\right)\,dx$$
$$=\frac{1}{11}\,Dbx^{11}+\frac{1}{9}\,(Da+Cb)x^9+\frac{1}{7}\,(Ca+Bb)x^7+\frac{1}{5}\,(Ba+Ab)x^5+\frac{1}{3}\,Aax^3$$

input
```
integrate(x^2*(b*x^2+a)*(D*x^6+C*x^4+B*x^2+A),x, algorithm="fricas")
```

output
```
1/11*D*b*x^11 + 1/9*(D*a + C*b)*x^9 + 1/7*(C*a + B*b)*x^7 + 1/5*(B*a + A*b
)*x^5 + 1/3*A*a*x^3
```

## Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.92

$$\int x^2(a+bx^2)\left(A+Bx^2+Cx^4+Dx^6\right)\,dx$$
$$=\frac{Aax^3}{3}+\frac{Dbx^{11}}{11}+x^9\left(\frac{Cb}{9}+\frac{Da}{9}\right)+x^7\left(\frac{Bb}{7}+\frac{Ca}{7}\right)+x^5\left(\frac{Ab}{5}+\frac{Ba}{5}\right)$$

input
```
integrate(x**2*(b*x**2+a)*(D*x**6+C*x**4+B*x**2+A),x)
```

output
```
A*a*x**3/3 + D*b*x**11/11 + x**9*(C*b/9 + D*a/9) + x**7*(B*b/7 + C*a/7) +
x**5*(A*b/5 + B*a/5)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.84

$$\int x^2(a+bx^2)\left(A+Bx^2+Cx^4+Dx^6\right)\,dx$$
$$=\frac{1}{11}\,Dbx^{11}+\frac{1}{9}\,(Da+Cb)x^9+\frac{1}{7}\,(Ca+Bb)x^7+\frac{1}{5}\,(Ba+Ab)x^5+\frac{1}{3}\,Aax^3$$

input
```
integrate(x^2*(b*x^2+a)*(D*x^6+C*x^4+B*x^2+A),x, algorithm="maxima")
```

output
```
1/11*D*b*x^11 + 1/9*(D*a + C*b)*x^9 + 1/7*(C*a + B*b)*x^7 + 1/5*(B*a + A*b
)*x^5 + 1/3*A*a*x^3
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.93

$$\int x^2(a+bx^2)\left(A+Bx^2+Cx^4+Dx^6\right)\,dx$$
$$=\frac{1}{11}\,Dbx^{11}+\frac{1}{9}\,Dax^9+\frac{1}{9}\,Cbx^9+\frac{1}{7}\,Cax^7+\frac{1}{7}\,Bbx^7+\frac{1}{5}\,Bax^5+\frac{1}{5}\,Abx^5+\frac{1}{3}\,Aax^3$$

input
```
integrate(x^2*(b*x^2+a)*(D*x^6+C*x^4+B*x^2+A),x, algorithm="giac")
```

output
```
1/11*D*b*x^11 + 1/9*D*a*x^9 + 1/9*C*b*x^9 + 1/7*C*a*x^7 + 1/7*B*b*x^7 + 1/
5*B*a*x^5 + 1/5*A*b*x^5 + 1/3*A*a*x^3
```

## Mupad [B] (verification not implemented)

Time = 2.46 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.93

$$\int x^2(a+bx^2)\left(A+Bx^2+Cx^4+Dx^6\right)\,dx$$
$$=\frac{a\,x^9\,D}{9}+\frac{b\,x^{11}\,D}{11}+\frac{A\,a\,x^3}{3}+\frac{A\,b\,x^5}{5}+\frac{B\,a\,x^5}{5}+\frac{B\,b\,x^7}{7}+\frac{C\,a\,x^7}{7}+\frac{C\,b\,x^9}{9}$$

input
```
int(x^2*(a + b*x^2)*(A + B*x^2 + C*x^4 + x^6*D),x)
```

output
```
(a*x^9*D)/9 + (b*x^11*D)/11 + (A*a*x^3)/3 + (A*b*x^5)/5 + (B*a*x^5)/5 + (B
*b*x^7)/7 + (C*a*x^7)/7 + (C*b*x^9)/9
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.89

$$\int x^2(a+bx^2)\left(A+Bx^2+Cx^4+Dx^6\right)\,dx$$
$$=\frac{x^3(315bd\,x^8+385ad\,x^6+385bc\,x^6+495ac\,x^4+495b^2x^4+1386ab\,x^2+1155a^2)}{3465}$$

input
```
int(x^2*(b*x^2+a)*(D*x^6+C*x^4+B*x^2+A),x)
```

output
```
(x**3*(1155*a**2 + 1386*a*b*x**2 + 495*a*c*x**4 + 385*a*d*x**6 + 495*b**2*
x**4 + 385*b*c*x**6 + 315*b*d*x**8))/3465
```

# 3.150 $\int (a+bx^2)(A+Bx^2+Cx^4+Dx^6)\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 56

$$\int (a+bx^2)(A+Bx^2+Cx^4+Dx^6)\,dx = aAx+\frac{1}{3}(Ab+aB)x^3+\frac{1}{5}(bB+aC)x^5+\frac{1}{7}(bC+aD)x^7+\frac{1}{9}bDx^9$$

output
```
a*A*x+1/3*(A*b+B*a)*x^3+1/5*(B*b+C*a)*x^5+1/7*(C*b+D*a)*x^7+1/9*b*D*x^9
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

$$\int (a+bx^2)(A+Bx^2+Cx^4+Dx^6)\,dx = aAx+\frac{1}{3}(Ab+aB)x^3+\frac{1}{5}(bB+aC)x^5+\frac{1}{7}(bC+aD)x^7+\frac{1}{9}bDx^9$$

input
```
Integrate[(a + b*x^2)*(A + B*x^2 + C*x^4 + D*x^6),x]
```

output
```
a*A*x + ((A*b + a*B)*x^3)/3 + ((b*B + a*C)*x^5)/5 + ((b*C + a*D)*x^7)/7 +
(b*D*x^9)/9
```

## Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2341, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^2)\left(A+Bx^2+Cx^4+Dx^6\right)dx$$
$$\Big\downarrow \text{2341}$$
$$\int \left(x^2(aB+Ab)+aA+x^4(aC+bB)+x^6(aD+bC)+bDx^8\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{1}{3}x^3(aB+Ab)+aAx+\frac{1}{5}x^5(aC+bB)+\frac{1}{7}x^7(aD+bC)+\frac{1}{9}bDx^9$$

input
```
Int[(a + b*x^2)*(A + B*x^2 + C*x^4 + D*x^6),x]
```

output
```
a*A*x + ((A*b + a*B)*x^3)/3 + ((b*B + a*C)*x^5)/5 + ((b*C + a*D)*x^7)/7 +
(b*D*x^9)/9
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2341
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

## Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.88

| method | result | size |
|---|---|---|
| default | $aAx+\frac{(Ab+Ba)x^3}{3}+\frac{(Bb+Ca)x^5}{5}+\frac{(Cb+Da)x^7}{7}+\frac{bDx^9}{9}$ | 49 |
| norman | $\frac{bDx^9}{9}+\left(\frac{Cb}{7}+\frac{Da}{7}\right)x^7+\left(\frac{Bb}{5}+\frac{Ca}{5}\right)x^5+\left(\frac{Ab}{3}+\frac{Ba}{3}\right)x^3+aAx$ | 52 |
| gosper | $\frac{1}{9}bDx^9+\frac{1}{7}bC\,x^7+\frac{1}{7}x^7Da+\frac{1}{5}bB\,x^5+\frac{1}{5}x^5Ca+\frac{1}{3}Ab\,x^3+\frac{1}{3}x^3Ba+aAx$ | 55 |
| parallelrisch | $\frac{1}{9}bDx^9+\frac{1}{7}bC\,x^7+\frac{1}{7}x^7Da+\frac{1}{5}bB\,x^5+\frac{1}{5}x^5Ca+\frac{1}{3}Ab\,x^3+\frac{1}{3}x^3Ba+aAx$ | 55 |
| orering | $\frac{x(35Db\,x^8+45Cb\,x^6+45Da\,x^6+63bB\,x^4+63Ca\,x^4+105Ab\,x^2+105Ba\,x^2+315Aa)}{315}$ | 58 |

input
```
int((b*x^2+a)*(D*x^6+C*x^4+B*x^2+A),x,method=_RETURNVERBOSE)
```

output
```
a*A*x+1/3*(A*b+B*a)*x^3+1/5*(B*b+C*a)*x^5+1/7*(C*b+D*a)*x^7+1/9*b*D*x^9
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

$$\int (a+bx^2)\left(A+Bx^2+Cx^4+Dx^6\right)\,dx=\frac{1}{9}\,Dbx^9+\frac{1}{7}\,(Da+Cb)x^7+\frac{1}{5}\,(Ca+Bb)x^5+\frac{1}{3}\,(Ba+Ab)x^3+Aax$$

input
```
integrate((b*x^2+a)*(D*x^6+C*x^4+B*x^2+A),x, algorithm="fricas")
```

output
```
1/9*D*b*x^9 + 1/7*(D*a + C*b)*x^7 + 1/5*(C*a + B*b)*x^5 + 1/3*(B*a + A*b)*
x^3 + A*a*x
```

## Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.95

$$\int (a+bx^2)(A+Bx^2+Cx^4+Dx^6)\,dx = Aax+\frac{Dbx^9}{9}+x^7\left(\frac{Cb}{7}+\frac{Da}{7}\right)+x^5\left(\frac{Bb}{5}+\frac{Ca}{5}\right)+x^3\left(\frac{Ab}{3}+\frac{Ba}{3}\right)$$

input
```
integrate((b*x**2+a)*(D*x**6+C*x**4+B*x**2+A),x)
```

output
```
A*a*x + D*b*x**9/9 + x**7*(C*b/7 + D*a/7) + x**5*(B*b/5 + C*a/5) + x**3*(A
*b/3 + B*a/3)
```

## Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

$$\int (a+bx^2)(A+Bx^2+Cx^4+Dx^6)\,dx = \frac{1}{9}Dbx^9+\frac{1}{7}(Da+Cb)x^7+\frac{1}{5}(Ca+Bb)x^5+\frac{1}{3}(Ba+Ab)x^3+Aax$$

input
```
integrate((b*x^2+a)*(D*x^6+C*x^4+B*x^2+A),x, algorithm="maxima")
```

output
```
1/9*D*b*x^9 + 1/7*(D*a + C*b)*x^7 + 1/5*(C*a + B*b)*x^5 + 1/3*(B*a + A*b)*
x^3 + A*a*x
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.96

$$\int \left(a+bx^2\right) \left(A+Bx^2+Cx^4+Dx^6\right) \, dx = \frac{1}{9} Dbx^9 + \frac{1}{7} Dax^7 + \frac{1}{7} Cbx^7 + \frac{1}{5} Cax^5 + \frac{1}{5} Bbx^5 + \frac{1}{3} Bax^3 + \frac{1}{3} Abx^3 + Aax$$

input
```
integrate((b*x^2+a)*(D*x^6+C*x^4+B*x^2+A),x, algorithm="giac")
```

output
```
1/9*D*b*x^9 + 1/7*D*a*x^7 + 1/7*C*b*x^7 + 1/5*C*a*x^5 + 1/5*B*b*x^5 + 1/3*
B*a*x^3 + 1/3*A*b*x^3 + A*a*x
```

## Mupad [B] (verification not implemented)

Time = 2.43 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.96

$$\int \left(a+bx^2\right) \left(A+Bx^2+Cx^4+Dx^6\right) \, dx = \frac{a\,x^7\,D}{7} + \frac{b\,x^9\,D}{9} + A\,a\,x + \frac{A\,b\,x^3}{3} + \frac{B\,a\,x^3}{3} + \frac{B\,b\,x^5}{5} + \frac{C\,a\,x^5}{5} + \frac{C\,b\,x^7}{7}$$

input
```
int((a + b*x^2)*(A + B*x^2 + C*x^4 + x^6*D),x)
```

output
```
(a*x^7*D)/7 + (b*x^9*D)/9 + A*a*x + (A*b*x^3)/3 + (B*a*x^3)/3 + (B*b*x^5)/
5 + (C*a*x^5)/5 + (C*b*x^7)/7
```

## Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.93

$$\int (a+bx^2)\left(A+Bx^2+Cx^4+Dx^6\right)\,dx$$
$$=\frac{x(35bd\,x^8+45ad\,x^6+45bc\,x^6+63ac\,x^4+63b^2x^4+210ab\,x^2+315a^2)}{315}$$

input
```
int((b*x^2+a)*(D*x^6+C*x^4+B*x^2+A),x)
```

output
```
(x*(315*a**2 + 210*a*b*x**2 + 63*a*c*x**4 + 45*a*d*x**6 + 63*b**2*x**4 + 4
5*b*c*x**6 + 35*b*d*x**8))/315
```

# 3.151 $\int \frac{(a+bx^2)(A+Bx^2+Cx^4+Dx^6)}{x^2} dx$



## Optimal result

Integrand size = 28, antiderivative size = 54

$$\int \frac{(a+bx^2)(A+Bx^2+Cx^4+Dx^6)}{x^2} dx = -\frac{aA}{x} + (Ab+aB)x + \frac{1}{3}(bB+aC)x^3 + \frac{1}{5}(bC+aD)x^5 + \frac{1}{7}bDx^7$$

output
```
-a*A/x+(A*b+B*a)*x+1/3*(B*b+C*a)*x^3+1/5*(C*b+D*a)*x^5+1/7*b*D*x^7
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)(A+Bx^2+Cx^4+Dx^6)}{x^2} dx = -\frac{aA}{x} + (Ab+aB)x + \frac{1}{3}(bB+aC)x^3 + \frac{1}{5}(bC+aD)x^5 + \frac{1}{7}bDx^7$$

input
```
Integrate[((a + b*x^2)*(A + B*x^2 + C*x^4 + D*x^6))/x^2,x]
```

output
```
-((a*A)/x) + (A*b + a*B)*x + ((b*B + a*C)*x^3)/3 + ((b*C + a*D)*x^5)/5 + (
b*D*x^7)/7
```

## Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)(A+Bx^2+Cx^4+Dx^6)}{x^2}\,dx$$

$$\Big\downarrow \text{2333}$$

$$\int \left(Ab\left(\frac{aB}{Ab}+1\right)+\frac{aA}{x^2}+x^2(aC+bB)+x^4(aD+bC)+bDx^6\right)dx$$

$$\Big\downarrow \text{2009}$$

$$x(aB+Ab)-\frac{aA}{x}+\frac{1}{3}x^3(aC+bB)+\frac{1}{5}x^5(aD+bC)+\frac{1}{7}bDx^7$$

input
```
Int[((a + b*x^2)*(A + B*x^2 + C*x^4 + D*x^6))/x^2,x]
```

output
```
-((a*A)/x) + (A*b + a*B)*x + ((b*B + a*C)*x^3)/3 + ((b*C + a*D)*x^5)/5 + (
b*D*x^7)/7
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

## Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.96

| method | result | size |
|---|---|---|
| default | $\frac{x^{7} b D}{7}+\frac{b C \,x^{5}}{5}+\frac{x^{5} D a}{5}+\frac{b B \,x^{3}}{3}+\frac{x^{3} C a}{3}+A b x +B a x -\frac{a A}{x}$ | 52 |
| norman | $\frac{\frac{D b \,x^{8}}{7}+\left(\frac{C b}{5}+\frac{D a}{5}\right) x^{6}+\left(\frac{B b}{3}+\frac{C a}{3}\right) x^{4}+\left(A b +B a \right) x^{2}-A a}{x}$ | 54 |
| gosper | $-\frac{-15 D b \,x^{8}-21 C b \,x^{6}-21 D a \,x^{6}-35 b B \,x^{4}-35 C a \,x^{4}-105 A b \,x^{2}-105 B a \,x^{2}+105 A a}{105 x}$ | 60 |
| parallelrisch | $\frac{15 D b \,x^{8}+21 C b \,x^{6}+21 D a \,x^{6}+35 b B \,x^{4}+35 C a \,x^{4}+105 A b \,x^{2}+105 B a \,x^{2}-105 A a}{105 x}$ | 60 |
| orering | $-\frac{-15 D b \,x^{8}-21 C b \,x^{6}-21 D a \,x^{6}-35 b B \,x^{4}-35 C a \,x^{4}-105 A b \,x^{2}-105 B a \,x^{2}+105 A a}{105 x}$ | 60 |

input
```
int((b*x^2+a)*(D*x^6+C*x^4+B*x^2+A)/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/7*x^7*b*D+1/5*b*C*x^5+1/5*x^5*D*a+1/3*b*B*x^3+1/3*x^3*C*a+A*b*x+B*a*x-a*
A/x
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^2)(A+Bx^2+Cx^4+Dx^6)}{x^2}\,dx = \frac{15\,Dbx^8+21\,(Da+Cb)x^6+35\,(Ca+Bb)x^4+105\,(Ba+Ab)x^2-105\,Aa}{105\,x}$$

input
```
integrate((b*x^2+a)*(D*x^6+C*x^4+B*x^2+A)/x^2,x, algorithm="fricas")
```

output
```
1/105*(15*D*b*x^8 + 21*(D*a + C*b)*x^6 + 35*(C*a + B*b)*x^4 + 105*(B*a + A
*b)*x^2 - 105*A*a)/x
```

## Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx^2)(A+Bx^2+Cx^4+Dx^6)}{x^2}\,dx = -\frac{Aa}{x}+\frac{Dbx^7}{7}+x^5\left(\frac{Cb}{5}+\frac{Da}{5}\right)+x^3\left(\frac{Bb}{3}+\frac{Ca}{3}\right)+x(Ab+Ba)$$

input
```
integrate((b*x**2+a)*(D*x**6+C*x**4+B*x**2+A)/x**2,x)
```

output
```
-A*a/x + D*b*x**7/7 + x**5*(C*b/5 + D*a/5) + x**3*(B*b/3 + C*a/3) + x*(A*b
 + B*a)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx^2)(A+Bx^2+Cx^4+Dx^6)}{x^2}\,dx = \frac{1}{7}Dbx^7+\frac{1}{5}(Da+Cb)x^5+\frac{1}{3}(Ca+Bb)x^3+(Ba+Ab)x-\frac{Aa}{x}$$

input
```
integrate((b*x^2+a)*(D*x^6+C*x^4+B*x^2+A)/x^2,x, algorithm="maxima")
```

output
```
1/7*D*b*x^7 + 1/5*(D*a + C*b)*x^5 + 1/3*(C*a + B*b)*x^3 + (B*a + A*b)*x -
A*a/x
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^2)(A+Bx^2+Cx^4+Dx^6)}{x^2}\,dx = \frac{1}{7}Dbx^7+\frac{1}{5}Dax^5+\frac{1}{5}Cbx^5+\frac{1}{3}Cax^3+\frac{1}{3}Bbx^3+Bax+Abx-\frac{Aa}{x}$$

input
```
integrate((b*x^2+a)*(D*x^6+C*x^4+B*x^2+A)/x^2,x, algorithm="giac")
```

output
```
1/7*D*b*x^7 + 1/5*D*a*x^5 + 1/5*C*b*x^5 + 1/3*C*a*x^3 + 1/3*B*b*x^3 + B*a*
x + A*b*x - A*a/x
```

## Mupad [B] (verification not implemented)

Time = 2.42 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^2)(A+Bx^2+Cx^4+Dx^6)}{x^2}\,dx = \frac{a\,x^5\,D}{5}+\frac{b\,x^7\,D}{7}+A\,b\,x+B\,a\,x-\frac{A\,a}{x}+\frac{B\,b\,x^3}{3}+\frac{C\,a\,x^3}{3}+\frac{C\,b\,x^5}{5}$$

input
```
int(((a + b*x^2)*(A + B*x^2 + C*x^4 + x^6*D))/x^2,x)
```

output
```
(a*x^5*D)/5 + (b*x^7*D)/7 + A*b*x + B*a*x - (A*a)/x + (B*b*x^3)/3 + (C*a*x
^3)/3 + (C*b*x^5)/5
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)(A+Bx^2+Cx^4+Dx^6)}{x^2}\,dx = \frac{15bd\,x^8+21ad\,x^6+21bc\,x^6+35ac\,x^4+35b^2x^4+210ab\,x^2-105a^2}{105x}$$

input
```
int((b*x^2+a)*(D*x^6+C*x^4+B*x^2+A)/x^2,x)
```

output
```
( - 105*a**2 + 210*a*b*x**2 + 35*a*c*x**4 + 21*a*d*x**6 + 35*b**2*x**4 + 2
1*b*c*x**6 + 15*b*d*x**8)/(105*x)
```

# 3.152 $\int \frac{(a+bx^2)(A+Bx^2+Cx^4+Dx^6)}{x^4}\,dx$



## Optimal result

Integrand size = 28, antiderivative size = 54

$$\int \frac{(a+bx^2)(A+Bx^2+Cx^4+Dx^6)}{x^4}\,dx = -\frac{aA}{3x^3} - \frac{Ab+aB}{x} + (bB+aC)x + \frac{1}{3}(bC+aD)x^3 + \frac{1}{5}bDx^5$$

output
```
-1/3*a*A/x^3-(A*b+B*a)/x+(B*b+C*a)*x+1/3*(C*b+D*a)*x^3+1/5*b*D*x^5
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^2)(A+Bx^2+Cx^4+Dx^6)}{x^4}\,dx = -\frac{aA}{3x^3} + \frac{-Ab-aB}{x} + (bB+aC)x + \frac{1}{3}(bC+aD)x^3 + \frac{1}{5}bDx^5$$

input
```
Integrate[((a + b*x^2)*(A + B*x^2 + C*x^4 + D*x^6))/x^4,x]
```

output
```
-1/3*(a*A)/x^3 + (-(A*b) - a*B)/x + (b*B + a*C)*x + ((b*C + a*D)*x^3)/3 +
(b*D*x^5)/5
```

## Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)(A+Bx^2+Cx^4+Dx^6)}{x^4} dx$$

$$\Big\downarrow \text{2333}$$

$$\int \left(\frac{aB+Ab}{x^2}+\frac{aA}{x^4}+bB\left(\frac{aC}{bB}+1\right)+x^2(aD+bC)+bDx^4\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{aB+Ab}{x}-\frac{aA}{3x^3}+x(aC+bB)+\frac{1}{3}x^3(aD+bC)+\frac{1}{5}bDx^5$$

input
```
Int[((a + b*x^2)*(A + B*x^2 + C*x^4 + D*x^6))/x^4,x]
```

output
```
-1/3*(a*A)/x^3 - (A*b + a*B)/x + (b*B + a*C)*x + ((b*C + a*D)*x^3)/3 + (b*
D*x^5)/5
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

## Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.93

| method | result | size |
|---|---|---|
| default | $\frac{Db\,x^5}{5}+\frac{Cb\,x^3}{3}+\frac{Da\,x^3}{3}+bBx+Cax-\frac{aA}{3x^3}-\frac{Ab+Ba}{x}$ | 50 |
| norman | $\frac{\frac{Db\,x^8}{5}+\left(\frac{Cb}{3}+\frac{Da}{3}\right)x^6+(Bb+Ca)x^4+(-Ab-Ba)x^2-\frac{Aa}{3}}{x^3}$ | 54 |
| gosper | $-\frac{-3Db\,x^8-5Cb\,x^6-5Da\,x^6-15bB\,x^4-15Ca\,x^4+15Ab\,x^2+15Ba\,x^2+5Aa}{15x^3}$ | 60 |
| parallelrisch | $-\frac{-3Db\,x^8-5Cb\,x^6-5Da\,x^6-15bB\,x^4-15Ca\,x^4+15Ab\,x^2+15Ba\,x^2+5Aa}{15x^3}$ | 60 |
| orering | $-\frac{-3Db\,x^8-5Cb\,x^6-5Da\,x^6-15bB\,x^4-15Ca\,x^4+15Ab\,x^2+15Ba\,x^2+5Aa}{15x^3}$ | 60 |

input
```
int((b*x^2+a)*(D*x^6+C*x^4+B*x^2+A)/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/5*D*b*x^5+1/3*C*b*x^3+1/3*D*a*x^3+b*B*x+C*a*x-1/3*a*A/x^3-(A*b+B*a)/x
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^2)(A+Bx^2+Cx^4+Dx^6)}{x^4}\,dx = \frac{3\,Dbx^8+5\,(Da+Cb)x^6+15\,(Ca+Bb)x^4-15\,(Ba+Ab)x^2-5\,Aa}{15\,x^3}$$

input
```
integrate((b*x^2+a)*(D*x^6+C*x^4+B*x^2+A)/x^4,x, algorithm="fricas")
```

output
```
1/15*(3*D*b*x^8 + 5*(D*a + C*b)*x^6 + 15*(C*a + B*b)*x^4 - 15*(B*a + A*b)*
x^2 - 5*A*a)/x^3
```

## Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^2)(A+Bx^2+Cx^4+Dx^6)}{x^4}\,dx = \frac{Dbx^5}{5} + x^3\left(\frac{Cb}{3}+\frac{Da}{3}\right) + x(Bb+Ca) + \frac{-Aa+x^2(-3Ab-3Ba)}{3x^3}$$

input
```
integrate((b*x**2+a)*(D*x**6+C*x**4+B*x**2+A)/x**4,x)
```

output
```
D*b*x**5/5 + x**3*(C*b/3 + D*a/3) + x*(B*b + C*a) + (-A*a + x**2*(-3*A*b -
 3*B*a))/(3*x**3)
```

## Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^2)(A+Bx^2+Cx^4+Dx^6)}{x^4}\,dx = \frac{1}{5}\,Dbx^5 + \frac{1}{3}\,(Da+Cb)x^3 + (Ca+Bb)x - \frac{3\,(Ba+Ab)x^2+Aa}{3\,x^3}$$

input
```
integrate((b*x^2+a)*(D*x^6+C*x^4+B*x^2+A)/x^4,x, algorithm="maxima")
```

output
```
1/5*D*b*x^5 + 1/3*(D*a + C*b)*x^3 + (C*a + B*b)*x - 1/3*(3*(B*a + A*b)*x^2
 + A*a)/x^3
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^2)(A+Bx^2+Cx^4+Dx^6)}{x^4}\,dx = \frac{1}{5}Dbx^5 + \frac{1}{3}Dax^3 + \frac{1}{3}Cbx^3 + Cax + Bbx - \frac{3Bax^2 + 3Abx^2 + Aa}{3x^3}$$

input
```
integrate((b*x^2+a)*(D*x^6+C*x^4+B*x^2+A)/x^4,x, algorithm="giac")
```

output
```
1/5*D*b*x^5 + 1/3*D*a*x^3 + 1/3*C*b*x^3 + C*a*x + B*b*x - 1/3*(3*B*a*x^2 +
 3*A*b*x^2 + A*a)/x^3
```

## Mupad [B] (verification not implemented)

Time = 2.38 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^2)(A+Bx^2+Cx^4+Dx^6)}{x^4}\,dx = \frac{a\,x^3\,D}{3} + \frac{b\,x^5\,D}{5} + B\,b\,x + C\,a\,x - \frac{A\,a}{3\,x^3} - \frac{A\,b}{x} - \frac{B\,a}{x} + \frac{C\,b\,x^3}{3}$$

input
```
int(((a + b*x^2)*(A + B*x^2 + C*x^4 + x^6*D))/x^4,x)
```

output
```
(a*x^3*D)/3 + (b*x^5*D)/5 + B*b*x + C*a*x - (A*a)/(3*x^3) - (A*b)/x - (B*a
)/x + (C*b*x^3)/3
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)(A+Bx^2+Cx^4+Dx^6)}{x^4}\,dx = \frac{3bd\,x^8+5ad\,x^6+5bc\,x^6+15ac\,x^4+15b^2x^4-30ab\,x^2-5a^2}{15x^3}$$

input
```
int((b*x^2+a)*(D*x^6+C*x^4+B*x^2+A)/x^4,x)
```

output
```
( - 5*a**2 - 30*a*b*x**2 + 15*a*c*x**4 + 5*a*d*x**6 + 15*b**2*x**4 + 5*b*c
*x**6 + 3*b*d*x**8)/(15*x**3)
```

# 3.153 $\int \frac{(a+bx^2)(A+Bx^2+Cx^4+Dx^6)}{x^6} dx$



## Optimal result

Integrand size = 28, antiderivative size = 54

$$\int \frac{(a+bx^2)(A+Bx^2+Cx^4+Dx^6)}{x^6} dx = -\frac{aA}{5x^5} - \frac{Ab+aB}{3x^3} - \frac{bB+aC}{x} + (bC+aD)x + \frac{1}{3}bDx^3$$

output
```
-1/5*a*A/x^5-1/3*(A*b+B*a)/x^3-(B*b+C*a)/x+(C*b+D*a)*x+1/3*b*D*x^3
```

## Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^2)(A+Bx^2+Cx^4+Dx^6)}{x^6} dx = -\frac{aA}{5x^5} - \frac{Ab+aB}{3x^3} - \frac{bB+aC}{x} + bCx + aDx + \frac{1}{3}bDx^3$$

input
```
Integrate[((a + b*x^2)*(A + B*x^2 + C*x^4 + D*x^6))/x^6,x]
```

output
```
-1/5*(a*A)/x^5 - (A*b + a*B)/(3*x^3) - (b*B + a*C)/x + b*C*x + a*D*x + (b*
D*x^3)/3
```

## Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)(A+Bx^2+Cx^4+Dx^6)}{x^6}\,dx$$

$$\Big\downarrow \text{2333}$$

$$\int \left(\frac{aB+Ab}{x^4}+\frac{aA}{x^6}+\frac{aC+bB}{x^2}+bC\left(\frac{aD}{bC}+1\right)+bDx^2\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{aB+Ab}{3x^3}-\frac{aA}{5x^5}-\frac{aC+bB}{x}+x(aD+bC)+\frac{1}{3}bDx^3$$

input
```
Int[((a + b*x^2)*(A + B*x^2 + C*x^4 + D*x^6))/x^6,x]
```

output
```
-1/5*(a*A)/x^5 - (A*b + a*B)/(3*x^3) - (b*B + a*C)/x + (b*C + a*D)*x + (b*
D*x^3)/3
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

## Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.89

| method | result | size |
|---|---|---|
| default | $\frac{D x^{3} b}{3}+C b x+D a x-\frac{A b+B a}{3 x^{3}}-\frac{a A}{5 x^{5}}-\frac{B b+C a}{x}$ | 48 |
| norman | $\frac{\frac{D b\, x^{8}}{3}+\left(C b+D a\right) x^{6}+\left(-B b-C a\right) x^{4}+\left(-\frac{A b}{3}-\frac{B a}{3}\right) x^{2}-\frac{A a}{5}}{x^{5}}$ | 54 |
| gosper | $-\frac{-5 D b\, x^{8}-15 C b\, x^{6}-15 D a\, x^{6}+15 b B\, x^{4}+15 C a\, x^{4}+5 A b\, x^{2}+5 B a\, x^{2}+3 A a}{15 x^{5}}$ | 60 |
| parallelrisch | $-\frac{-5 D b\, x^{8}-15 C b\, x^{6}-15 D a\, x^{6}+15 b B\, x^{4}+15 C a\, x^{4}+5 A b\, x^{2}+5 B a\, x^{2}+3 A a}{15 x^{5}}$ | 60 |
| orering | $-\frac{-5 D b\, x^{8}-15 C b\, x^{6}-15 D a\, x^{6}+15 b B\, x^{4}+15 C a\, x^{4}+5 A b\, x^{2}+5 B a\, x^{2}+3 A a}{15 x^{5}}$ | 60 |

input
```
int((b*x^2+a)*(D*x^6+C*x^4+B*x^2+A)/x^6,x,method=_RETURNVERBOSE)
```

output
```
1/3*D*x^3*b+C*b*x+D*a*x-1/3*(A*b+B*a)/x^3-1/5*a*A/x^5-(B*b+C*a)/x
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^2)\,(A+Bx^2+Cx^4+Dx^6)}{x^6}\,dx = \frac{5\,Dbx^8+15\,(Da+Cb)x^6-15\,(Ca+Bb)x^4-5\,(Ba+Ab)x^2-3\,Aa}{15\,x^5}$$

input
```
integrate((b*x^2+a)*(D*x^6+C*x^4+B*x^2+A)/x^6,x, algorithm="fricas")
```

output
```
1/15*(5*D*b*x^8 + 15*(D*a + C*b)*x^6 - 15*(C*a + B*b)*x^4 - 5*(B*a + A*b)*
x^2 - 3*A*a)/x^5
```

## Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx^2)(A+Bx^2+Cx^4+Dx^6)}{x^6}\,dx = \frac{Dbx^3}{3} + x(Cb+Da) + \frac{-3Aa+x^4(-15Bb-15Ca)+x^2(-5Ab-5Ba)}{15x^5}$$

input
```
integrate((b*x**2+a)*(D*x**6+C*x**4+B*x**2+A)/x**6,x)
```

output
```
D*b*x**3/3 + x*(C*b + D*a) + (-3*A*a + x**4*(-15*B*b - 15*C*a) + x**2*(-5*
A*b - 5*B*a))/(15*x**5)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^2)(A+Bx^2+Cx^4+Dx^6)}{x^6}\,dx = \frac{1}{3}\,Dbx^3 + (Da+Cb)x - \frac{15\,(Ca+Bb)x^4 + 5\,(Ba+Ab)x^2 + 3\,Aa}{15\,x^5}$$

input
```
integrate((b*x^2+a)*(D*x^6+C*x^4+B*x^2+A)/x^6,x, algorithm="maxima")
```

output
```
1/3*D*b*x^3 + (D*a + C*b)*x - 1/15*(15*(C*a + B*b)*x^4 + 5*(B*a + A*b)*x^2
 + 3*A*a)/x^5
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)(A+Bx^2+Cx^4+Dx^6)}{x^6}\,dx$$
$$=\frac{1}{3}Dbx^3+Dax+Cbx-\frac{15\,Cax^4+15\,Bbx^4+5\,Bax^2+5\,Abx^2+3\,Aa}{15\,x^5}$$

input
```
integrate((b*x^2+a)*(D*x^6+C*x^4+B*x^2+A)/x^6,x, algorithm="giac")
```

output
```
1/3*D*b*x^3 + D*a*x + C*b*x - 1/15*(15*C*a*x^4 + 15*B*b*x^4 + 5*B*a*x^2 +
5*A*b*x^2 + 3*A*a)/x^5
```

## Mupad [B] (verification not implemented)

Time = 2.41 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^2)(A+Bx^2+Cx^4+Dx^6)}{x^6}\,dx=\frac{b\,x^3\,D}{3}+C\,b\,x-\frac{A\,a}{5\,x^5}-\frac{A\,b}{3\,x^3}-\frac{B\,a}{3\,x^3}-\frac{B\,b}{x}-\frac{C\,a}{x}+a\,x\,D$$

input
```
int(((a + b*x^2)*(A + B*x^2 + C*x^4 + x^6*D))/x^6,x)
```

output
```
(b*x^3*D)/3 + C*b*x - (A*a)/(5*x^5) - (A*b)/(3*x^3) - (B*a)/(3*x^3) - (B*b
)/x - (C*a)/x + a*x*D
```

## Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)(A+Bx^2+Cx^4+Dx^6)}{x^6}\,dx$$
$$=\frac{5bd\,x^8+15ad\,x^6+15bc\,x^6-15ac\,x^4-15b^2x^4-10ab\,x^2-3a^2}{15x^5}$$

input
```
int((b*x^2+a)*(D*x^6+C*x^4+B*x^2+A)/x^6,x)
```

output
```
( - 3*a**2 - 10*a*b*x**2 - 15*a*c*x**4 + 15*a*d*x**6 - 15*b**2*x**4 + 15*b
*c*x**6 + 5*b*d*x**8)/(15*x**5)
```

# 3.154 $\int x^4(a+bx^2)^2(A+Bx^2+Cx^4+Dx^6)\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 101

$$\int x^4(a+bx^2)^2\left(A+Bx^2+Cx^4+Dx^6\right)\,dx = \frac{1}{5}a^2Ax^5+\frac{1}{7}a(2Ab+aB)x^7+\frac{1}{9}\left(Ab^2+a(2bB+aC)\right)x^9 + \frac{1}{11}\left(b^2B+2abC+a^2D\right)x^{11}+\frac{1}{13}b(bC+2aD)x^{13}+\frac{1}{15}b^2Dx^{15}$$

output
```
1/5*a^2*A*x^5+1/7*a*(2*A*b+B*a)*x^7+1/9*(A*b^2+a*(2*B*b+C*a))*x^9+1/11*(B*
b^2+2*C*a*b+D*a^2)*x^11+1/13*b*(C*b+2*D*a)*x^13+1/15*b^2*D*x^15
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00

$$\int x^4(a+bx^2)^2\left(A+Bx^2+Cx^4+Dx^6\right)\,dx = \frac{1}{5}a^2Ax^5+\frac{1}{7}a(2Ab+aB)x^7+\frac{1}{9}\left(Ab^2+2abB+a^2C\right)x^9 + \frac{1}{11}\left(b^2B+2abC+a^2D\right)x^{11}+\frac{1}{13}b(bC+2aD)x^{13}+\frac{1}{15}b^2Dx^{15}$$

input
```
Integrate[x^4*(a + b*x^2)^2*(A + B*x^2 + C*x^4 + D*x^6),x]
```

output
```
(a^2*A*x^5)/5 + (a*(2*A*b + a*B)*x^7)/7 + ((A*b^2 + 2*a*b*B + a^2*C)*x^9)/
9 + ((b^2*B + 2*a*b*C + a^2*D)*x^11)/11 + (b*(b*C + 2*a*D)*x^13)/13 + (b^2
*D*x^15)/15
```

## Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4\left(a+bx^2\right)^2\left(A+Bx^2+Cx^4+Dx^6\right)\,dx$$

$$\Big\downarrow \text{ 2333}$$

$$\int \left(a^2Ax^4+x^{10}\left(a^2D+2abC+b^2B\right)+x^8\left(a(aC+2bB)+Ab^2\right)+ax^6(aB+2Ab)+bx^{12}(2aD+bC)+b^2Dx^{14}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{5}a^2Ax^5+\frac{1}{11}x^{11}\left(a^2D+2abC+b^2B\right)+\frac{1}{9}x^9\left(a(aC+2bB)+Ab^2\right)+\frac{1}{7}ax^7(aB+2Ab)+\frac{1}{13}bx^{13}(2aD+bC)+\frac{1}{15}b^2Dx^{15}$$

input
```
Int[x^4*(a + b*x^2)^2*(A + B*x^2 + C*x^4 + D*x^6),x]
```

output
```
(a^2*A*x^5)/5 + (a*(2*A*b + a*B)*x^7)/7 + ((A*b^2 + a*(2*b*B + a*C))*x^9)/
9 + ((b^2*B + 2*a*b*C + a^2*D)*x^11)/11 + (b*(b*C + 2*a*D)*x^13)/13 + (b^2
*D*x^15)/15
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

## Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.93

| method | result |
|---|---|
| default | $\frac{b^2 D x^{15}}{15}+\frac{(b^2 C+2abD)x^{13}}{13}+\frac{(B\,b^2+2Cab+Da^2)x^{11}}{11}+\frac{(b^2 A+2abB+a^2 C)x^9}{9}+\frac{(2abA+a^2 B)x^7}{7}+\frac{a^2 A\,x^5}{5}$ |
| norman | $\frac{a^2 A\,x^5}{5}+\left(\frac{2}{7}abA+\frac{1}{7}a^2 B\right)x^7+\left(\frac{1}{9}b^2 A+\frac{2}{9}abB+\frac{1}{9}a^2 C\right)x^9+\left(\frac{1}{11}B\,b^2+\frac{2}{11}Cab+\frac{1}{11}Da^2\right)x^{11}$ |
| gosper | $\frac{1}{5}a^2 A\,x^5+\frac{2}{7}x^7 abA+\frac{1}{7}x^7 a^2 B+\frac{1}{9}x^9 b^2 A+\frac{2}{9}x^9 abB+\frac{1}{9}x^9 a^2 C+\frac{1}{11}b^2 B\,x^{11}+\frac{2}{11}x^{11}Cab+$ |
| parallelrisch | $\frac{1}{5}a^2 A\,x^5+\frac{2}{7}x^7 abA+\frac{1}{7}x^7 a^2 B+\frac{1}{9}x^9 b^2 A+\frac{2}{9}x^9 abB+\frac{1}{9}x^9 a^2 C+\frac{1}{11}b^2 B\,x^{11}+\frac{2}{11}x^{11}Cab+$ |
| orering | $\frac{x^5(3003b^2x^{10}D+3465C\,b^2x^8+6930Dab\,x^8+4095b^2B\,x^6+8190b\,x^6Ca+4095Da^2x^6+5005A\,b^2x^4+10010Bab\,x^4+5005C\,a^2}{45045}$ |

input
```
int(x^4*(b*x^2+a)^2*(D*x^6+C*x^4+B*x^2+A),x,method=_RETURNVERBOSE)
```

output
```
1/15*b^2*D*x^15+1/13*(C*b^2+2*D*a*b)*x^13+1/11*(B*b^2+2*C*a*b+D*a^2)*x^11+
1/9*(A*b^2+2*B*a*b+C*a^2)*x^9+1/7*(2*A*a*b+B*a^2)*x^7+1/5*a^2*A*x^5
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.92

$$\int x^4(a+bx^2)^2\left(A+Bx^2+Cx^4+Dx^6\right)\,dx$$
$$=\frac{1}{15}Db^2x^{15}+\frac{1}{13}\left(2\,Dab+Cb^2\right)x^{13}+\frac{1}{11}\left(Da^2+2\,Cab+Bb^2\right)x^{11}$$
$$+\frac{1}{9}\left(Ca^2+2\,Bab+Ab^2\right)x^9+\frac{1}{5}\,Aa^2x^5+\frac{1}{7}\left(Ba^2+2\,Aab\right)x^7$$

input
```
integrate(x^4*(b*x^2+a)^2*(D*x^6+C*x^4+B*x^2+A),x, algorithm="fricas")
```

output
```
1/15*D*b^2*x^15 + 1/13*(2*D*a*b + C*b^2)*x^13 + 1/11*(D*a^2 + 2*C*a*b + B*
b^2)*x^11 + 1/9*(C*a^2 + 2*B*a*b + A*b^2)*x^9 + 1/5*A*a^2*x^5 + 1/7*(B*a^2
 + 2*A*a*b)*x^7
```

## Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.06

$$\int x^4(a+bx^2)^2(A+Bx^2+Cx^4+Dx^6)\,dx$$
$$=\frac{Aa^2x^5}{5}+\frac{Db^2x^{15}}{15}+x^{13}\left(\frac{Cb^2}{13}+\frac{2Dab}{13}\right)+x^{11}\left(\frac{Bb^2}{11}+\frac{2Cab}{11}+\frac{Da^2}{11}\right)$$
$$+x^9\left(\frac{Ab^2}{9}+\frac{2Bab}{9}+\frac{Ca^2}{9}\right)+x^7\cdot\left(\frac{2Aab}{7}+\frac{Ba^2}{7}\right)$$

input
```
integrate(x**4*(b*x**2+a)**2*(D*x**6+C*x**4+B*x**2+A),x)
```

output
```
A*a**2*x**5/5 + D*b**2*x**15/15 + x**13*(C*b**2/13 + 2*D*a*b/13) + x**11*(
B*b**2/11 + 2*C*a*b/11 + D*a**2/11) + x**9*(A*b**2/9 + 2*B*a*b/9 + C*a**2/
9) + x**7*(2*A*a*b/7 + B*a**2/7)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.92

$$\int x^4(a+bx^2)^2(A+Bx^2+Cx^4+Dx^6)\,dx$$
$$=\frac{1}{15}Db^2x^{15}+\frac{1}{13}(2Dab+Cb^2)x^{13}+\frac{1}{11}(Da^2+2Cab+Bb^2)x^{11}$$
$$+\frac{1}{9}(Ca^2+2Bab+Ab^2)x^9+\frac{1}{5}Aa^2x^5+\frac{1}{7}(Ba^2+2Aab)x^7$$

input
```
integrate(x^4*(b*x^2+a)^2*(D*x^6+C*x^4+B*x^2+A),x, algorithm="maxima")
```

output
```
1/15*D*b^2*x^15 + 1/13*(2*D*a*b + C*b^2)*x^13 + 1/11*(D*a^2 + 2*C*a*b + B*
b^2)*x^11 + 1/9*(C*a^2 + 2*B*a*b + A*b^2)*x^9 + 1/5*A*a^2*x^5 + 1/7*(B*a^2
 + 2*A*a*b)*x^7
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.04

$$\int x^4(a+bx^2)^2\left(A+Bx^2+Cx^4+Dx^6\right)\,dx$$
$$=\frac{1}{15}Db^2x^{15}+\frac{2}{13}Dabx^{13}+\frac{1}{13}Cb^2x^{13}+\frac{1}{11}Da^2x^{11}+\frac{2}{11}Cabx^{11}+\frac{1}{11}Bb^2x^{11}+\frac{1}{9}Ca^2x^9+\frac{2}{9}Babx^9+\frac{1}{9}Ab^2x^9+\frac{1}{7}Ba^2x^7+\frac{2}{7}Aabx^7+\frac{1}{5}Aa^2x^5$$

input
```
integrate(x^4*(b*x^2+a)^2*(D*x^6+C*x^4+B*x^2+A),x, algorithm="giac")
```

output
```
1/15*D*b^2*x^15 + 2/13*D*a*b*x^13 + 1/13*C*b^2*x^13 + 1/11*D*a^2*x^11 + 2/
11*C*a*b*x^11 + 1/11*B*b^2*x^11 + 1/9*C*a^2*x^9 + 2/9*B*a*b*x^9 + 1/9*A*b^
2*x^9 + 1/7*B*a^2*x^7 + 2/7*A*a*b*x^7 + 1/5*A*a^2*x^5
```

## Mupad [B] (verification not implemented)

Time = 2.36 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.07

$$\int x^4(a+bx^2)^2\left(A+Bx^2+Cx^4+Dx^6\right)\,dx$$
$$=\frac{a^2\,x^{11}\,D}{11}+\frac{b^2\,x^{15}\,D}{15}+\frac{A\,x^5\,\left(63\,a^2+90\,a\,b\,x^2+35\,b^2\,x^4\right)}{315}+\frac{B\,x^7\,\left(99\,a^2+154\,a\,b\,x^2+63\,b^2\,x^4\right)}{693}+\frac{C\,x^9\,\left(143\,a^2+234\,a\,b\,x^2+99\,b^2\,x^4\right)}{1287}+\frac{2\,a\,b\,x^{13}\,D}{13}$$

input
```
int(x^4*(a + b*x^2)^2*(A + B*x^2 + C*x^4 + x^6*D),x)
```

output
```
(a^2*x^11*D)/11 + (b^2*x^15*D)/15 + (A*x^5*(63*a^2 + 35*b^2*x^4 + 90*a*b*x
^2))/315 + (B*x^7*(99*a^2 + 63*b^2*x^4 + 154*a*b*x^2))/693 + (C*x^9*(143*a
^2 + 99*b^2*x^4 + 234*a*b*x^2))/1287 + (2*a*b*x^13*D)/13
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.88

$$\int x^4(a+bx^2)^2(A+Bx^2+Cx^4+Dx^6)\,dx$$
$$=\frac{x^5(3003b^2d\,x^{10}+6930abd\,x^8+3465b^2c\,x^8+4095a^2d\,x^6+8190abc\,x^6+4095b^3x^6+5005a^2c\,x^4+15015a}{45045}$$

input
```
int(x^4*(b*x^2+a)^2*(D*x^6+C*x^4+B*x^2+A),x)
```

output
```
(x**5*(9009*a**3 + 19305*a**2*b*x**2 + 5005*a**2*c*x**4 + 4095*a**2*d*x**6
 + 15015*a*b**2*x**4 + 8190*a*b*c*x**6 + 6930*a*b*d*x**8 + 4095*b**3*x**6
+ 3465*b**2*c*x**8 + 3003*b**2*d*x**10))/45045
```

# 3.155 $\int x^2(a+bx^2)^2\left(A+Bx^2+Cx^4+Dx^6\right)dx$



## Optimal result

Integrand size = 30, antiderivative size = 101

$$\begin{aligned}\int x^2\left(a+bx^2\right)^2\left(A+Bx^2+Cx^4+Dx^6\right)dx&=\frac{1}{3}a^2Ax^3+\frac{1}{5}a(2Ab+aB)x^5+\frac{1}{7}\left(Ab^2+a(2bB+aC)\right)x^7\\&+\frac{1}{9}\left(b^2B+2abC+a^2D\right)x^9+\frac{1}{11}b(bC+2aD)x^{11}+\frac{1}{13}b^2Dx^{13}\end{aligned}$$

output
```
1/3*a^2*A*x^3+1/5*a*(2*A*b+B*a)*x^5+1/7*(A*b^2+a*(2*B*b+C*a))*x^7+1/9*(B*b
^2+2*C*a*b+D*a^2)*x^9+1/11*b*(C*b+2*D*a)*x^11+1/13*b^2*D*x^13
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00

$$\begin{aligned}\int x^2\left(a+bx^2\right)^2\left(A+Bx^2+Cx^4+Dx^6\right)dx&=\frac{1}{3}a^2Ax^3+\frac{1}{5}a(2Ab+aB)x^5+\frac{1}{7}\left(Ab^2+2abB+a^2C\right)x^7\\&+\frac{1}{9}\left(b^2B+2abC+a^2D\right)x^9+\frac{1}{11}b(bC+2aD)x^{11}+\frac{1}{13}b^2Dx^{13}\end{aligned}$$

input
```
Integrate[x^2*(a + b*x^2)^2*(A + B*x^2 + C*x^4 + D*x^6),x]
```

output
```
(a^2*A*x^3)/3 + (a*(2*A*b + a*B)*x^5)/5 + ((A*b^2 + 2*a*b*B + a^2*C)*x^7)/
7 + ((b^2*B + 2*a*b*C + a^2*D)*x^9)/9 + (b*(b*C + 2*a*D)*x^11)/11 + (b^2*D
*x^13)/13
```

## Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+bx^2\right)^2 \left(A+Bx^2+Cx^4+Dx^6\right) \, dx$$

$$\Big\downarrow \; 2333$$

$$\int \left(a^2Ax^2+x^8\left(a^2D+2abC+b^2B\right)+x^6\left(a(aC+2bB)+Ab^2\right)+ax^4(aB+2Ab)+bx^{10}(2aD+bC)+b^2Dx^{12}\right.$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{3}a^2Ax^3+\frac{1}{9}x^9\left(a^2D+2abC+b^2B\right)+\frac{1}{7}x^7\left(a(aC+2bB)+Ab^2\right)+\frac{1}{5}ax^5(aB+2Ab)+\frac{1}{11}bx^{11}(2aD+bC)+\frac{1}{13}b^2Dx^{13}$$

input
```
Int[x^2*(a + b*x^2)^2*(A + B*x^2 + C*x^4 + D*x^6),x]
```

output
```
(a^2*A*x^3)/3 + (a*(2*A*b + a*B)*x^5)/5 + ((A*b^2 + a*(2*b*B + a*C))*x^7)/
7 + ((b^2*B + 2*a*b*C + a^2*D)*x^9)/9 + (b*(b*C + 2*a*D)*x^11)/11 + (b^2*D
*x^13)/13
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

## Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.93

| method | result |
|---|---|
| default | $\frac{b^2 D x^{13}}{13}+\frac{(b^2 C+2abD)x^{11}}{11}+\frac{(B\,b^2+2Cab+Da^2)x^9}{9}+\frac{(b^2 A+2abB+a^2 C)x^7}{7}+\frac{(2abA+a^2 B)x^5}{5}+\frac{a^2 A\,x^3}{3}$ |
| norman | $\frac{b^2 D x^{13}}{13}+\left(\frac{1}{11}b^2 C+\frac{2}{11}abD\right)x^{11}+\left(\frac{1}{9}B\,b^2+\frac{2}{9}Cab+\frac{1}{9}Da^2\right)x^9+\left(\frac{1}{7}b^2 A+\frac{2}{7}abB+\frac{1}{7}a^2 C\right)x^7$ |
| gosper | $\frac{1}{13}b^2 D x^{13}+\frac{1}{11}x^{11}b^2 C+\frac{2}{11}x^{11}abD+\frac{1}{9}b^2 B\,x^9+\frac{2}{9}x^9 Cab+\frac{1}{9}x^9 Da^2+\frac{1}{7}x^7 b^2 A+\frac{2}{7}x^7 abB+$ |
| parallelrisch | $\frac{1}{13}b^2 D x^{13}+\frac{1}{11}x^{11}b^2 C+\frac{2}{11}x^{11}abD+\frac{1}{9}b^2 B\,x^9+\frac{2}{9}x^9 Cab+\frac{1}{9}x^9 Da^2+\frac{1}{7}x^7 b^2 A+\frac{2}{7}x^7 abB+$ |
| orering | $\frac{x^3(3465b^2x^{10}D+4095C\,b^2x^8+8190Dab\,x^8+5005b^2B\,x^6+10010b\,x^6Ca+5005Da^2x^6+6435A\,b^2x^4+12870Bab\,x^4+6435C\,a^2}{45045}$ |

input
```
int(x^2*(b*x^2+a)^2*(D*x^6+C*x^4+B*x^2+A),x,method=_RETURNVERBOSE)
```

output
```
1/13*b^2*D*x^13+1/11*(C*b^2+2*D*a*b)*x^11+1/9*(B*b^2+2*C*a*b+D*a^2)*x^9+1/
7*(A*b^2+2*B*a*b+C*a^2)*x^7+1/5*(2*A*a*b+B*a^2)*x^5+1/3*a^2*A*x^3
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.92

$$\int x^2\left(a+bx^2\right)^2\left(A+Bx^2+Cx^4+Dx^6\right)\,dx$$
$$=\frac{1}{13}Db^2x^{13}+\frac{1}{11}\left(2\,Dab+Cb^2\right)x^{11}+\frac{1}{9}\left(Da^2+2\,Cab+Bb^2\right)x^9$$
$$+\frac{1}{7}\left(Ca^2+2\,Bab+Ab^2\right)x^7+\frac{1}{3}Aa^2x^3+\frac{1}{5}\left(Ba^2+2\,Aab\right)x^5$$

input
```
integrate(x^2*(b*x^2+a)^2*(D*x^6+C*x^4+B*x^2+A),x, algorithm="fricas")
```

output
```
1/13*D*b^2*x^13 + 1/11*(2*D*a*b + C*b^2)*x^11 + 1/9*(D*a^2 + 2*C*a*b + B*b
^2)*x^9 + 1/7*(C*a^2 + 2*B*a*b + A*b^2)*x^7 + 1/3*A*a^2*x^3 + 1/5*(B*a^2 +
 2*A*a*b)*x^5
```

## Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.06

$$\int x^2(a+bx^2)^2\left(A+Bx^2+Cx^4+Dx^6\right)\,dx = \frac{Aa^2x^3}{3}+\frac{Db^2x^{13}}{13}+x^{11}\left(\frac{Cb^2}{11}+\frac{2Dab}{11}\right)+x^9\left(\frac{Bb^2}{9}+\frac{2Cab}{9}+\frac{Da^2}{9}\right)+x^7\left(\frac{Ab^2}{7}+\frac{2Bab}{7}+\frac{Ca^2}{7}\right)+x^5\cdot\left(\frac{2Aab}{5}+\frac{Ba^2}{5}\right)$$

input
```
integrate(x**2*(b*x**2+a)**2*(D*x**6+C*x**4+B*x**2+A),x)
```

output
```
A*a**2*x**3/3 + D*b**2*x**13/13 + x**11*(C*b**2/11 + 2*D*a*b/11) + x**9*(B
*b**2/9 + 2*C*a*b/9 + D*a**2/9) + x**7*(A*b**2/7 + 2*B*a*b/7 + C*a**2/7) +
 x**5*(2*A*a*b/5 + B*a**2/5)
```

## Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.92

$$\int x^2(a+bx^2)^2\left(A+Bx^2+Cx^4+Dx^6\right)\,dx = \frac{1}{13}\,Db^2x^{13}+\frac{1}{11}\left(2\,Dab+Cb^2\right)x^{11}+\frac{1}{9}\left(Da^2+2\,Cab+Bb^2\right)x^9+\frac{1}{7}\left(Ca^2+2\,Bab+Ab^2\right)x^7+\frac{1}{3}\,Aa^2x^3+\frac{1}{5}\left(Ba^2+2\,Aab\right)x^5$$

input
```
integrate(x^2*(b*x^2+a)^2*(D*x^6+C*x^4+B*x^2+A),x, algorithm="maxima")
```

output
```
1/13*D*b^2*x^13 + 1/11*(2*D*a*b + C*b^2)*x^11 + 1/9*(D*a^2 + 2*C*a*b + B*b
^2)*x^9 + 1/7*(C*a^2 + 2*B*a*b + A*b^2)*x^7 + 1/3*A*a^2*x^3 + 1/5*(B*a^2 +
 2*A*a*b)*x^5
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.04

$$\int x^2 (a + bx^2)^2 (A + Bx^2 + Cx^4 + Dx^6) \, dx = \frac{1}{13} Db^2 x^{13} + \frac{2}{11} Dabx^{11} + \frac{1}{11} Cb^2 x^{11} + \frac{1}{9} Da^2 x^9 + \frac{2}{9} Cabx^9 + \frac{1}{9} Bb^2 x^9 + \frac{1}{7} Ca^2 x^7 + \frac{2}{7} Babx^7 + \frac{1}{7} Ab^2 x^7 + \frac{1}{5} Ba^2 x^5 + \frac{2}{5} Aabx^5 + \frac{1}{3} Aa^2 x^3$$

input
```
integrate(x^2*(b*x^2+a)^2*(D*x^6+C*x^4+B*x^2+A),x, algorithm="giac")
```

output
```
1/13*D*b^2*x^13 + 2/11*D*a*b*x^11 + 1/11*C*b^2*x^11 + 1/9*D*a^2*x^9 + 2/9*
C*a*b*x^9 + 1/9*B*b^2*x^9 + 1/7*C*a^2*x^7 + 2/7*B*a*b*x^7 + 1/7*A*b^2*x^7
+ 1/5*B*a^2*x^5 + 2/5*A*a*b*x^5 + 1/3*A*a^2*x^3
```

## Mupad [B] (verification not implemented)

Time = 2.33 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.07

$$\int x^2 (a + bx^2)^2 (A + Bx^2 + Cx^4 + Dx^6) \, dx = \frac{a^2 x^9 D}{9} + \frac{b^2 x^{13} D}{13} + \frac{A x^3 (35 a^2 + 42 a b x^2 + 15 b^2 x^4)}{105} + \frac{B x^5 (63 a^2 + 90 a b x^2 + 35 b^2 x^4)}{315} + \frac{C x^7 (99 a^2 + 154 a b x^2 + 63 b^2 x^4)}{693} + \frac{2 a b x^{11} D}{11}$$

input
```
int(x^2*(a + b*x^2)^2*(A + B*x^2 + C*x^4 + x^6*D),x)
```

output
```
(a^2*x^9*D)/9 + (b^2*x^13*D)/13 + (A*x^3*(35*a^2 + 15*b^2*x^4 + 42*a*b*x^2
))/105 + (B*x^5*(63*a^2 + 35*b^2*x^4 + 90*a*b*x^2))/315 + (C*x^7*(99*a^2 +
 63*b^2*x^4 + 154*a*b*x^2))/693 + (2*a*b*x^11*D)/11
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.88

$$\int x^2(a+bx^2)^2\left(A+Bx^2+Cx^4+Dx^6\right)\,dx$$
$$=\frac{x^3(3465b^2d\,x^{10}+8190abd\,x^8+4095b^2c\,x^8+5005a^2d\,x^6+10010abc\,x^6+5005b^3x^6+6435a^2c\,x^4+1930}{45045}$$

input
```
int(x^2*(b*x^2+a)^2*(D*x^6+C*x^4+B*x^2+A),x)
```

output
```
(x**3*(15015*a**3 + 27027*a**2*b*x**2 + 6435*a**2*c*x**4 + 5005*a**2*d*x**
6 + 19305*a*b**2*x**4 + 10010*a*b*c*x**6 + 8190*a*b*d*x**8 + 5005*b**3*x**
6 + 4095*b**2*c*x**8 + 3465*b**2*d*x**10))/45045
```

# 3.156 $\int (a+bx^2)^2 (A+Bx^2+Cx^4+Dx^6)\,dx$



## Optimal result

Integrand size = 27, antiderivative size = 96

$$\begin{aligned}\int (a+bx^2)^2 (A+Bx^2+Cx^4+Dx^6)\,dx &= a^2Ax+\frac{1}{3}a(2Ab+aB)x^3 \\ &\quad +\frac{1}{5}\left(Ab^2+a(2bB+aC)\right)x^5 \\ &\quad +\frac{1}{7}\left(b^2B+2abC+a^2D\right)x^7 \\ &\quad +\frac{1}{9}b(bC+2aD)x^9+\frac{1}{11}b^2Dx^{11}\end{aligned}$$

output

```
a^2*A*x+1/3*a*(2*A*b+B*a)*x^3+1/5*(A*b^2+a*(2*B*b+C*a))*x^5+1/7*(B*b^2+2*C
*a*b+D*a^2)*x^7+1/9*b*(C*b+2*D*a)*x^9+1/11*b^2*D*x^11
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00

$$\int (a+bx^2)^2 (A+Bx^2+Cx^4+Dx^6) \, dx = a^2Ax+\frac{1}{3}a(2Ab+aB)x^3 + \frac{1}{5}(Ab^2+2abB+a^2C) x^5 + \frac{1}{7}(b^2B+2abC+a^2D) x^7 + \frac{1}{9}b(bC+2aD)x^9+\frac{1}{11}b^2Dx^{11}$$

input
```
Integrate[(a + b*x^2)^2*(A + B*x^2 + C*x^4 + D*x^6),x]
```

output
```
a^2*A*x + (a*(2*A*b + a*B)*x^3)/3 + ((A*b^2 + 2*a*b*B + a^2*C)*x^5)/5 + ((
b^2*B + 2*a*b*C + a^2*D)*x^7)/7 + (b*(b*C + 2*a*D)*x^9)/9 + (b^2*D*x^11)/1
1
```

## Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2341, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^2)^2 (A+Bx^2+Cx^4+Dx^6) \, dx$$

$$\Big\downarrow \text{2341}$$

$$\int (a^2A+x^6(a^2D+2abC+b^2B)+x^4(a(aC+2bB)+Ab^2)+ax^2(aB+2Ab)+bx^8(2aD+bC)+b^2Dx^{10}) \, dx$$

$$\Big\downarrow \text{2009}$$

$$a^2Ax + \frac{1}{7}x^7\left(a^2D + 2abC + b^2B\right) + \frac{1}{5}x^5\left(a(aC + 2bB) + Ab^2\right) + \frac{1}{3}ax^3(aB + 2Ab) + \frac{1}{9}bx^9(2aD + bC) + \frac{1}{11}b^2Dx^{11}$$

input
```
Int[(a + b*x^2)^2*(A + B*x^2 + C*x^4 + D*x^6),x]
```

output
```
a^2*A*x + (a*(2*A*b + a*B)*x^3)/3 + ((A*b^2 + a*(2*b*B + a*C))*x^5)/5 + ((
b^2*B + 2*a*b*C + a^2*D)*x^7)/7 + (b*(b*C + 2*a*D)*x^9)/9 + (b^2*D*x^11)/1
1
```

**Defintions of rubi rules used**

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2341
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

## Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.95

| method | result |
|---|---|
| default | $\frac{b^2Dx^{11}}{11}+\frac{(b^2C+2abD)x^9}{9}+\frac{(B\,b^2+2Cab+Da^2)x^7}{7}+\frac{(b^2A+2abB+a^2C)x^5}{5}+\frac{(2abA+a^2B)x^3}{3}+a^2Ax$ |
| norman | $\frac{b^2Dx^{11}}{11}+\left(\frac{1}{9}b^2C+\frac{2}{9}abD\right)x^9+\left(\frac{1}{7}B\,b^2+\frac{2}{7}Cab+\frac{1}{7}Da^2\right)x^7+\left(\frac{1}{5}b^2A+\frac{2}{5}abB+\frac{1}{5}a^2C\right)x^5+$ |
| gosper | $\frac{1}{11}b^2Dx^{11}+\frac{1}{9}b^2C\,x^9+\frac{2}{9}x^9abD+\frac{1}{7}b^2B\,x^7+\frac{2}{7}x^7Cab+\frac{1}{7}x^7Da^2+\frac{1}{5}A\,b^2x^5+\frac{2}{5}x^5abB+\frac{1}{5}x$ |
| parallelrisch | $\frac{1}{11}b^2Dx^{11}+\frac{1}{9}b^2C\,x^9+\frac{2}{9}x^9abD+\frac{1}{7}b^2B\,x^7+\frac{2}{7}x^7Cab+\frac{1}{7}x^7Da^2+\frac{1}{5}A\,b^2x^5+\frac{2}{5}x^5abB+\frac{1}{5}x$ |
| orering | $\frac{x(315b^2x^{10}D+385C\,b^2x^8+770Dab\,x^8+495b^2B\,x^6+990b\,x^6Ca+495Da^2x^6+693A\,b^2x^4+1386Bab\,x^4+693C\,a^2x^4+2310aAb}{3465}$ |

input
```
int((b*x^2+a)^2*(D*x^6+C*x^4+B*x^2+A),x,method=_RETURNVERBOSE)
```

output
```
1/11*b^2*D*x^11+1/9*(C*b^2+2*D*a*b)*x^9+1/7*(B*b^2+2*C*a*b+D*a^2)*x^7+1/5*
(A*b^2+2*B*a*b+C*a^2)*x^5+1/3*(2*A*a*b+B*a^2)*x^3+a^2*A*x
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.94

$$\int (a+bx^2)^2 (A+Bx^2+Cx^4+Dx^6)\,dx = \frac{1}{11}\,Db^2x^{11} + \frac{1}{9}\,(2\,Dab+Cb^2)x^9 + \frac{1}{7}\,(Da^2+2\,Cab+Bb^2)x^7 + \frac{1}{5}\,(Ca^2+2\,Bab+Ab^2)x^5 + Aa^2x + \frac{1}{3}\,(Ba^2+2\,Aab)x^3$$

input
```
integrate((b*x^2+a)^2*(D*x^6+C*x^4+B*x^2+A),x, algorithm="fricas")
```

output
```
1/11*D*b^2*x^11 + 1/9*(2*D*a*b + C*b^2)*x^9 + 1/7*(D*a^2 + 2*C*a*b + B*b^2
)*x^7 + 1/5*(C*a^2 + 2*B*a*b + A*b^2)*x^5 + A*a^2*x + 1/3*(B*a^2 + 2*A*a*b
)*x^3
```

## Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.08

$$\int (a+bx^2)^2 (A+Bx^2+Cx^4+Dx^6)\,dx = Aa^2x + \frac{Db^2x^{11}}{11} + x^9\left(\frac{Cb^2}{9}+\frac{2Dab}{9}\right) + x^7\left(\frac{Bb^2}{7}+\frac{2Cab}{7}+\frac{Da^2}{7}\right) + x^5\left(\frac{Ab^2}{5}+\frac{2Bab}{5}+\frac{Ca^2}{5}\right) + x^3\cdot\left(\frac{2Aab}{3}+\frac{Ba^2}{3}\right)$$

input
```
integrate((b*x**2+a)**2*(D*x**6+C*x**4+B*x**2+A),x)
```

output
```
A*a**2*x + D*b**2*x**11/11 + x**9*(C*b**2/9 + 2*D*a*b/9) + x**7*(B*b**2/7
+ 2*C*a*b/7 + D*a**2/7) + x**5*(A*b**2/5 + 2*B*a*b/5 + C*a**2/5) + x**3*(2
*A*a*b/3 + B*a**2/3)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.94

$$\int \left(a+bx^2\right)^2 \left(A+Bx^2+Cx^4+Dx^6\right)\,dx = \frac{1}{11}\,Db^2x^{11}+\frac{1}{9}\,\left(2\,Dab+Cb^2\right)x^9+\frac{1}{7}\,\left(Da^2+2\,Cab+Bb^2\right)x^7+\frac{1}{5}\,\left(Ca^2+2\,Bab+Ab^2\right)x^5+Aa^2x+\frac{1}{3}\,\left(Ba^2+2\,Aab\right)x^3$$

input
```
integrate((b*x^2+a)^2*(D*x^6+C*x^4+B*x^2+A),x, algorithm="maxima")
```

output
```
1/11*D*b^2*x^11 + 1/9*(2*D*a*b + C*b^2)*x^9 + 1/7*(D*a^2 + 2*C*a*b + B*b^2
)*x^7 + 1/5*(C*a^2 + 2*B*a*b + A*b^2)*x^5 + A*a^2*x + 1/3*(B*a^2 + 2*A*a*b
)*x^3
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.06

$$\int \left(a+bx^2\right)^2 \left(A+Bx^2+Cx^4+Dx^6\right)\,dx = \frac{1}{11}\,Db^2x^{11}+\frac{2}{9}\,Dabx^9+\frac{1}{9}\,Cb^2x^9+\frac{1}{7}\,Da^2x^7+\frac{2}{7}\,Cabx^7+\frac{1}{7}\,Bb^2x^7+\frac{1}{5}\,Ca^2x^5+\frac{2}{5}\,Babx^5+\frac{1}{5}\,Ab^2x^5+\frac{1}{3}\,Ba^2x^3+\frac{2}{3}\,Aabx^3+Aa^2x$$

input
```
integrate((b*x^2+a)^2*(D*x^6+C*x^4+B*x^2+A),x, algorithm="giac")
```

output
```
1/11*D*b^2*x^11 + 2/9*D*a*b*x^9 + 1/9*C*b^2*x^9 + 1/7*D*a^2*x^7 + 2/7*C*a*
b*x^7 + 1/7*B*b^2*x^7 + 1/5*C*a^2*x^5 + 2/5*B*a*b*x^5 + 1/5*A*b^2*x^5 + 1/
3*B*a^2*x^3 + 2/3*A*a*b*x^3 + A*a^2*x
```

## Mupad [B] (verification not implemented)

Time = 1.36 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.10

$$\int (a+bx^2)^2 (A+Bx^2+Cx^4+Dx^6)\,dx = \frac{A\,x\,(15\,a^2+10\,a\,b\,x^2+3\,b^2\,x^4)}{15}+\frac{a^2\,x^7\,D}{7}+\frac{b^2\,x^{11}\,D}{11} + \frac{B\,x^3\,(35\,a^2+42\,a\,b\,x^2+15\,b^2\,x^4)}{105}+\frac{C\,x^5\,(63\,a^2+90\,a\,b\,x^2+35\,b^2\,x^4)}{315}+\frac{2\,a\,b\,x^9\,D}{9}$$

input
```
int((a + b*x^2)^2*(A + B*x^2 + C*x^4 + x^6*D),x)
```

output
```
(A*x*(15*a^2 + 3*b^2*x^4 + 10*a*b*x^2))/15 + (a^2*x^7*D)/7 + (b^2*x^11*D)/
11 + (B*x^3*(35*a^2 + 15*b^2*x^4 + 42*a*b*x^2))/105 + (C*x^5*(63*a^2 + 35*
b^2*x^4 + 90*a*b*x^2))/315 + (2*a*b*x^9*D)/9
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.91

$$\int (a+bx^2)^2 (A+Bx^2+Cx^4+Dx^6)\,dx = \frac{x(315b^2d\,x^{10}+770abd\,x^8+385b^2c\,x^8+495a^2d\,x^6+990abc\,x^6+495b^3x^6+693a^2c\,x^4+2079a\,b^2x^4+34}{3465}$$

input
```
int((b*x^2+a)^2*(D*x^6+C*x^4+B*x^2+A),x)
```

output
```
(x*(3465*a**3 + 3465*a**2*b*x**2 + 693*a**2*c*x**4 + 495*a**2*d*x**6 + 207
9*a*b**2*x**4 + 990*a*b*c*x**6 + 770*a*b*d*x**8 + 495*b**3*x**6 + 385*b**2
*c*x**8 + 315*b**2*d*x**10))/3465
```

# 3.157 $\int \frac{(a+bx^2)^2(A+Bx^2+Cx^4+Dx^6)}{x^2}\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 94

$$\int \frac{(a+bx^2)^2(A+Bx^2+Cx^4+Dx^6)}{x^2}\,dx = -\frac{a^2A}{x} + a(2Ab+aB)x + \frac{1}{3}\left(Ab^2+a(2bB+aC)\right)x^3 + \frac{1}{5}\left(b^2B+2abC+a^2D\right)x^5 + \frac{1}{7}b(bC+2aD)x^7 + \frac{1}{9}b^2Dx^9$$

output
```
-a^2*A/x+a*(2*A*b+B*a)*x+1/3*(A*b^2+a*(2*B*b+C*a))*x^3+1/5*(B*b^2+2*C*a*b+
D*a^2)*x^5+1/7*b*(C*b+2*D*a)*x^7+1/9*b^2*D*x^9
```

## Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^2(A+Bx^2+Cx^4+Dx^6)}{x^2}\,dx = -\frac{a^2A}{x}+a(2Ab+aB)x+\frac{1}{3}\left(Ab^2+2abB+a^2C\right)x^3+\frac{1}{5}\left(b^2B+2abC+a^2D\right)x^5+\frac{1}{7}b(bC+2aD)x^7+\frac{1}{9}b^2Dx^9$$

input
```
Integrate[((a + b*x^2)^2*(A + B*x^2 + C*x^4 + D*x^6))/x^2,x]
```

output
```
-((a^2*A)/x) + a*(2*A*b + a*B)*x + ((A*b^2 + 2*a*b*B + a^2*C)*x^3)/3 + ((b
^2*B + 2*a*b*C + a^2*D)*x^5)/5 + (b*(b*C + 2*a*D)*x^7)/7 + (b^2*D*x^9)/9
```

## Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2(A+Bx^2+Cx^4+Dx^6)}{x^2}\,dx$$

$$\Big\downarrow \text{2333}$$

$$\int \left(\frac{a^2A}{x^2}+x^4\left(a^2D+2abC+b^2B\right)+x^2\left(a(aC+2bB)+Ab^2\right)+a(aB+2Ab)+bx^6(2aD+bC)+b^2Dx^8\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2A}{x}+\frac{1}{5}x^5\left(a^2D+2abC+b^2B\right)+\frac{1}{3}x^3\left(a(aC+2bB)+Ab^2\right)+ax(aB+2Ab)+\frac{1}{7}bx^7(2aD+bC)+\frac{1}{9}b^2Dx^9$$

input
```
Int[((a + b*x^2)^2*(A + B*x^2 + C*x^4 + D*x^6))/x^2,x]
```

output
```
-((a^2*A)/x) + a*(2*A*b + a*B)*x + ((A*b^2 + a*(2*b*B + a*C))*x^3)/3 + ((b
^2*B + 2*a*b*C + a^2*D)*x^5)/5 + (b*(b*C + 2*a*D)*x^7)/7 + (b^2*D*x^9)/9
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

## Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.02

| method | result |
|---|---|
| norman | $\frac{\frac{b^2x^{10}D}{9}+\left(\frac{1}{7}b^2C+\frac{2}{7}abD\right)x^8+\left(\frac{1}{5}Bb^2+\frac{2}{5}Cab+\frac{1}{5}Da^2\right)x^6+\left(\frac{1}{3}b^2A+\frac{2}{3}abB+\frac{1}{3}a^2C\right)x^4+\left(2abA+a^2B\right)x^2-a^2A}{x}$ |
| default | $\frac{Db^2x^9}{9}+\frac{b^2Cx^7}{7}+\frac{2x^7abD}{7}+\frac{b^2Bx^5}{5}+\frac{2x^5Cab}{5}+\frac{x^5Da^2}{5}+\frac{Ab^2x^3}{3}+\frac{2Babx^3}{3}+\frac{x^3a^2C}{3}+2aAbx+B$ |
| gosper | $-\frac{-35b^2x^{10}D-45Cb^2x^8-90Dabx^8-63b^2Bx^6-126bx^6Ca-63Da^2x^6-105Ab^2x^4-210Babx^4-105Ca^2x^4-630aAbx^2-3}{315x}$ |
| parallelrisch | $\frac{35b^2x^{10}D+45Cb^2x^8+90Dabx^8+63b^2Bx^6+126bx^6Ca+63Da^2x^6+105Ab^2x^4+210Babx^4+105Ca^2x^4+630aAbx^2+315B}{315x}$ |
| orering | $-\frac{-35b^2x^{10}D-45Cb^2x^8-90Dabx^8-63b^2Bx^6-126bx^6Ca-63Da^2x^6-105Ab^2x^4-210Babx^4-105Ca^2x^4-630aAbx^2-3}{315x}$ |

input
```
int((b*x^2+a)^2*(D*x^6+C*x^4+B*x^2+A)/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/x*(1/9*b^2*x^10*D+(1/7*b^2*C+2/7*a*b*D)*x^8+(1/5*B*b^2+2/5*C*a*b+1/5*D*a
^2)*x^6+(1/3*b^2*A+2/3*a*b*B+1/3*a^2*C)*x^4+(2*A*a*b+B*a^2)*x^2-a^2*A)
```

## Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.01

$$\int \frac{(a+bx^2)^2 (A+Bx^2+Cx^4+Dx^6)}{x^2}\,dx$$
$$= \frac{35\,Db^2x^{10}+45\,(2\,Dab+Cb^2)x^8+63\,(Da^2+2\,Cab+Bb^2)x^6+105\,(Ca^2+2\,Bab+Ab^2)x^4-315\,Aa^2}{315\,x}$$

input
```
integrate((b*x^2+a)^2*(D*x^6+C*x^4+B*x^2+A)/x^2,x, algorithm="fricas")
```

output
```
1/315*(35*D*b^2*x^10 + 45*(2*D*a*b + C*b^2)*x^8 + 63*(D*a^2 + 2*C*a*b + B*
b^2)*x^6 + 105*(C*a^2 + 2*B*a*b + A*b^2)*x^4 - 315*A*a^2 + 315*(B*a^2 + 2*
A*a*b)*x^2)/x
```

## Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.05

$$\int \frac{(a+bx^2)^2 (A+Bx^2+Cx^4+Dx^6)}{x^2}\,dx = -\frac{Aa^2}{x}+\frac{Db^2x^9}{9}+x^7\left(\frac{Cb^2}{7}+\frac{2Dab}{7}\right)+x^5\left(\frac{Bb^2}{5}+\frac{2Cab}{5}+\frac{Da^2}{5}\right)+x^3\left(\frac{Ab^2}{3}+\frac{2Bab}{3}+\frac{Ca^2}{3}\right)+x\left(2Aab+Ba^2\right)$$

input
```
integrate((b*x**2+a)**2*(D*x**6+C*x**4+B*x**2+A)/x**2,x)
```

output
```
-A*a**2/x + D*b**2*x**9/9 + x**7*(C*b**2/7 + 2*D*a*b/7) + x**5*(B*b**2/5 +
 2*C*a*b/5 + D*a**2/5) + x**3*(A*b**2/3 + 2*B*a*b/3 + C*a**2/3) + x*(2*A*a
*b + B*a**2)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^2)^2 (A+Bx^2+Cx^4+Dx^6)}{x^2} \, dx = \frac{1}{9} Db^2x^9 + \frac{1}{7} (2 Dab + Cb^2)x^7 + \frac{1}{5} (Da^2 + 2 Cab + Bb^2)x^5 + \frac{1}{3} (Ca^2 + 2 Bab + Ab^2)x^3 - \frac{Aa^2}{x} + (Ba^2 + 2 Aab)x$$

input
```
integrate((b*x^2+a)^2*(D*x^6+C*x^4+B*x^2+A)/x^2,x, algorithm="maxima")
```

output
```
1/9*D*b^2*x^9 + 1/7*(2*D*a*b + C*b^2)*x^7 + 1/5*(D*a^2 + 2*C*a*b + B*b^2)*
x^5 + 1/3*(C*a^2 + 2*B*a*b + A*b^2)*x^3 - A*a^2/x + (B*a^2 + 2*A*a*b)*x
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx^2)^2 (A+Bx^2+Cx^4+Dx^6)}{x^2} \, dx = \frac{1}{9} Db^2x^9 + \frac{2}{7} Dabx^7 + \frac{1}{7} Cb^2x^7 + \frac{1}{5} Da^2x^5 + \frac{2}{5} Cabx^5 + \frac{1}{5} Bb^2x^5 + \frac{1}{3} Ca^2x^3 + \frac{2}{3} Babx^3 + \frac{1}{3} Ab^2x^3 + Ba^2x + 2 Aabx - \frac{Aa^2}{x}$$

input
```
integrate((b*x^2+a)^2*(D*x^6+C*x^4+B*x^2+A)/x^2,x, algorithm="giac")
```

output
```
1/9*D*b^2*x^9 + 2/7*D*a*b*x^7 + 1/7*C*b^2*x^7 + 1/5*D*a^2*x^5 + 2/5*C*a*b*
x^5 + 1/5*B*b^2*x^5 + 1/3*C*a^2*x^3 + 2/3*B*a*b*x^3 + 1/3*A*b^2*x^3 + B*a^
2*x + 2*A*a*b*x - A*a^2/x
```

## Mupad [B] (verification not implemented)

Time = 1.45 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.12

$$\int \frac{(a+bx^2)^2 (A+Bx^2+Cx^4+Dx^6)}{x^2}\,dx = \frac{B\,x\,(15\,a^2+10\,a\,b\,x^2+3\,b^2\,x^4)}{15}+\frac{a^2\,x^5\,D}{5}+\frac{b^2\,x^9\,D}{9}+\frac{A\,(-3\,a^2+6\,a\,b\,x^2+b^2\,x^4)}{3\,x}+\frac{C\,x^3\,(35\,a^2+42\,a\,b\,x^2+15\,b^2\,x^4)}{105}+\frac{2\,a\,b\,x^7\,D}{7}$$

input
```
int(((a + b*x^2)^2*(A + B*x^2 + C*x^4 + x^6*D))/x^2,x)
```

output
```
(B*x*(15*a^2 + 3*b^2*x^4 + 10*a*b*x^2))/15 + (a^2*x^5*D)/5 + (b^2*x^9*D)/9
 + (A*(b^2*x^4 - 3*a^2 + 6*a*b*x^2))/(3*x) + (C*x^3*(35*a^2 + 15*b^2*x^4 +
 42*a*b*x^2))/105 + (2*a*b*x^7*D)/7
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^2)^2 (A+Bx^2+Cx^4+Dx^6)}{x^2}\,dx = \frac{35b^2d\,x^{10}+90abd\,x^8+45b^2c\,x^8+63a^2d\,x^6+126abc\,x^6+63b^3x^6+105a^2c\,x^4+315a\,b^2x^4+945a^2b\,x^2 -}{315x}$$

input
```
int((b*x^2+a)^2*(D*x^6+C*x^4+B*x^2+A)/x^2,x)
```

output
```
( - 315*a**3 + 945*a**2*b*x**2 + 105*a**2*c*x**4 + 63*a**2*d*x**6 + 315*a*
b**2*x**4 + 126*a*b*c*x**6 + 90*a*b*d*x**8 + 63*b**3*x**6 + 45*b**2*c*x**8
 + 35*b**2*d*x**10)/(315*x)
```

# 3.158 $\int \frac{(a+bx^2)^2(A+Bx^2+Cx^4+Dx^6)}{x^4}\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 94

$$\int \frac{(a+bx^2)^2(A+Bx^2+Cx^4+Dx^6)}{x^4}\,dx = -\frac{a^2A}{3x^3}-\frac{a(2Ab+aB)}{x}+\left(Ab^2+a(2bB+aC)\right)x+\frac{1}{3}\left(b^2B+2abC+a^2D\right)x^3+\frac{1}{5}b(bC+2aD)x^5+\frac{1}{7}b^2Dx^7$$

output
```
-1/3*a^2*A/x^3-a*(2*A*b+B*a)/x+(A*b^2+a*(2*B*b+C*a))*x+1/3*(B*b^2+2*C*a*b+
D*a^2)*x^3+1/5*b*(C*b+2*D*a)*x^5+1/7*b^2*D*x^7
```

## Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx^2)^2(A+Bx^2+Cx^4+Dx^6)}{x^4}\,dx = -\frac{a^2A}{3x^3}-\frac{a(2Ab+aB)}{x}+Ab^2x+a(2bB+aC)x+\frac{1}{3}\left(b^2B+2abC+a^2D\right)x^3+\frac{1}{5}b(bC+2aD)x^5+\frac{1}{7}b^2Dx^7$$

input
```
Integrate[((a + b*x^2)^2*(A + B*x^2 + C*x^4 + D*x^6))/x^4,x]
```

output
```
-1/3*(a^2*A)/x^3 - (a*(2*A*b + a*B))/x + A*b^2*x + a*(2*b*B + a*C)*x + ((b
^2*B + 2*a*b*C + a^2*D)*x^3)/3 + (b*(b*C + 2*a*D)*x^5)/5 + (b^2*D*x^7)/7
```

## Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2 (A+Bx^2+Cx^4+Dx^6)}{x^4} \, dx$$

$$\Big\downarrow \text{2333}$$

$$\int \left(\frac{a^2A}{x^4}+x^2(a^2D+2abC+b^2B)+Ab^2\left(\frac{a(aC+2bB)}{Ab^2}+1\right)+\frac{a(aB+2Ab)}{x^2}+bx^4(2aD+bC)+b^2Dx^6\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2A}{3x^3}+\frac{1}{3}x^3(a^2D+2abC+b^2B)+x(a(aC+2bB)+Ab^2)-\frac{a(aB+2Ab)}{x}+\frac{1}{5}bx^5(2aD+bC)+\frac{1}{7}b^2Dx^7$$

input
```
Int[((a + b*x^2)^2*(A + B*x^2 + C*x^4 + D*x^6))/x^4,x]
```

output
```
-1/3*(a^2*A)/x^3 - (a*(2*A*b + a*B))/x + (A*b^2 + a*(2*b*B + a*C))*x + ((b
^2*B + 2*a*b*C + a^2*D)*x^3)/3 + (b*(b*C + 2*a*D)*x^5)/5 + (b^2*D*x^7)/7
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

## Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.01

| method | result |
|---|---|
| default | $\frac{b^2 D x^7}{7}+\frac{b^2 C x^5}{5}+\frac{2 D a b x^5}{5}+\frac{B b^2 x^3}{3}+\frac{2 C a b x^3}{3}+\frac{D a^2 x^3}{3}+A b^2 x+2 B a b x+C a^2 x-\frac{a^2 A}{3 x^3}-\frac{a(2Ab}{x}$ |
| norman | $\frac{\frac{b^2 x^{10} D}{7}+\left(\frac{1}{5}b^2 C+\frac{2}{5}abD\right)x^8+\left(\frac{1}{3}B b^2+\frac{2}{3}Cab+\frac{1}{3}Da^2\right)x^6+\left(b^2A+2abB+a^2C\right)x^4+\left(-2abA-a^2B\right)x^2-\frac{a^2A}{3}}{x^3}$ |
| gosper | $-\frac{-15b^2x^{10}D-21C b^2x^8-42Dab x^8-35b^2B x^6-70b x^6Ca-35Da^2x^6-105A b^2x^4-210Bab x^4-105C a^2x^4+210aAb x^2+10}{105x^3}$ |
| parallelrisch | $\frac{15b^2x^{10}D+21C b^2x^8+42Dab x^8+35b^2B x^6+70b x^6Ca+35Da^2x^6+105A b^2x^4+210Bab x^4+105C a^2x^4-210aAb x^2-105B a}{105x^3}$ |
| orering | $-\frac{-15b^2x^{10}D-21C b^2x^8-42Dab x^8-35b^2B x^6-70b x^6Ca-35Da^2x^6-105A b^2x^4-210Bab x^4-105C a^2x^4+210aAb x^2+10}{105x^3}$ |

input
```
int((b*x^2+a)^2*(D*x^6+C*x^4+B*x^2+A)/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/7*b^2*D*x^7+1/5*b^2*C*x^5+2/5*D*a*b*x^5+1/3*B*b^2*x^3+2/3*C*a*b*x^3+1/3*
D*a^2*x^3+A*b^2*x+2*B*a*b*x+C*a^2*x-1/3*a^2*A/x^3-a*(2*A*b+B*a)/x
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.01

$$\int \frac{(a+bx^2)^2(A+Bx^2+Cx^4+Dx^6)}{x^4}\,dx$$
$$=\frac{15\,Db^2x^{10}+21\,(2\,Dab+Cb^2)x^8+35\,(Da^2+2\,Cab+Bb^2)x^6+105\,(Ca^2+2\,Bab+Ab^2)x^4-35\,Aa^2-}{105\,x^3}$$

input
```
integrate((b*x^2+a)^2*(D*x^6+C*x^4+B*x^2+A)/x^4,x, algorithm="fricas")
```

output
```
1/105*(15*D*b^2*x^10 + 21*(2*D*a*b + C*b^2)*x^8 + 35*(D*a^2 + 2*C*a*b + B*
b^2)*x^6 + 105*(C*a^2 + 2*B*a*b + A*b^2)*x^4 - 35*A*a^2 - 105*(B*a^2 + 2*A
*a*b)*x^2)/x^3
```

## Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx^2)^2 (A+Bx^2+Cx^4+Dx^6)}{x^4}\,dx = \frac{Db^2x^7}{7} + x^5\left(\frac{Cb^2}{5}+\frac{2Dab}{5}\right) + x^3\left(\frac{Bb^2}{3}+\frac{2Cab}{3}+\frac{Da^2}{3}\right) + x\left(Ab^2+2Bab+Ca^2\right) + \frac{-Aa^2+x^2(-6Aab-3Ba^2)}{3x^3}$$

input
```
integrate((b*x**2+a)**2*(D*x**6+C*x**4+B*x**2+A)/x**4,x)
```

output
```
D*b**2*x**7/7 + x**5*(C*b**2/5 + 2*D*a*b/5) + x**3*(B*b**2/3 + 2*C*a*b/3 +
 D*a**2/3) + x*(A*b**2 + 2*B*a*b + C*a**2) + (-A*a**2 + x**2*(-6*A*a*b - 3
*B*a**2))/(3*x**3)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^2)^2 (A+Bx^2+Cx^4+Dx^6)}{x^4}\,dx = \frac{1}{7}\,Db^2x^7 + \frac{1}{5}\,(2\,Dab+Cb^2)x^5 + \frac{1}{3}\,(Da^2+2\,Cab+Bb^2)x^3 + (Ca^2+2\,Bab+Ab^2)x - \frac{Aa^2+3\,(Ba^2+2\,Aab)x^2}{3\,x^3}$$

input
```
integrate((b*x^2+a)^2*(D*x^6+C*x^4+B*x^2+A)/x^4,x, algorithm="maxima")
```

output
```
1/7*D*b^2*x^7 + 1/5*(2*D*a*b + C*b^2)*x^5 + 1/3*(D*a^2 + 2*C*a*b + B*b^2)*
x^3 + (C*a^2 + 2*B*a*b + A*b^2)*x - 1/3*(A*a^2 + 3*(B*a^2 + 2*A*a*b)*x^2)/
x^3
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.05

$$\int \frac{(a+bx^2)^2 (A+Bx^2+Cx^4+Dx^6)}{x^4} dx = \frac{1}{7} Db^2x^7 + \frac{2}{5} Dabx^5 + \frac{1}{5} Cb^2x^5 + \frac{1}{3} Da^2x^3 + \frac{2}{3} Cabx^3 + \frac{1}{3} Bb^2x^3 + Ca^2x + 2 Babx + Ab^2x - \frac{3 Ba^2x^2 + 6 Aabx^2 + Aa^2}{3 x^3}$$

input
```
integrate((b*x^2+a)^2*(D*x^6+C*x^4+B*x^2+A)/x^4,x, algorithm="giac")
```

output
```
1/7*D*b^2*x^7 + 2/5*D*a*b*x^5 + 1/5*C*b^2*x^5 + 1/3*D*a^2*x^3 + 2/3*C*a*b*
x^3 + 1/3*B*b^2*x^3 + C*a^2*x + 2*B*a*b*x + A*b^2*x - 1/3*(3*B*a^2*x^2 + 6
*A*a*b*x^2 + A*a^2)/x^3
```

## Mupad [B] (verification not implemented)

Time = 1.52 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.10

$$\int \frac{(a+bx^2)^2 (A+Bx^2+Cx^4+Dx^6)}{x^4} dx = \frac{C\,x\,(15\,a^2+10\,a\,b\,x^2+3\,b^2\,x^4)}{15} - \frac{A\,(a^2+6\,a\,b\,x^2-3\,b^2\,x^4)}{3\,x^3} + \frac{a^2\,x^3\,D}{3} + \frac{b^2\,x^7\,D}{7} + \frac{B\,(-3\,a^2+6\,a\,b\,x^2+b^2\,x^4)}{3\,x} + \frac{2\,a\,b\,x^5\,D}{5}$$

input
```
int(((a + b*x^2)^2*(A + B*x^2 + C*x^4 + x^6*D))/x^4,x)
```

output
```
(C*x*(15*a^2 + 3*b^2*x^4 + 10*a*b*x^2))/15 - (A*(a^2 - 3*b^2*x^4 + 6*a*b*x
^2))/(3*x^3) + (a^2*x^3*D)/3 + (b^2*x^7*D)/7 + (B*(b^2*x^4 - 3*a^2 + 6*a*b
*x^2))/(3*x) + (2*a*b*x^5*D)/5
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^2)^2 (A+Bx^2+Cx^4+Dx^6)}{x^4}\,dx = \frac{15b^2d\,x^{10}+42abd\,x^8+21b^2c\,x^8+35a^2d\,x^6+70abc\,x^6+35b^3x^6+105a^2c\,x^4+315a\,b^2x^4-315a^2b\,x^2-}{105x^3}$$

input
```
int((b*x^2+a)^2*(D*x^6+C*x^4+B*x^2+A)/x^4,x)
```

output
```
( - 35*a**3 - 315*a**2*b*x**2 + 105*a**2*c*x**4 + 35*a**2*d*x**6 + 315*a*b
**2*x**4 + 70*a*b*c*x**6 + 42*a*b*d*x**8 + 35*b**3*x**6 + 21*b**2*c*x**8 +
 15*b**2*d*x**10)/(105*x**3)
```

# 3.159 $\int \frac{(a+bx^2)^2(A+Bx^2+Cx^4+Dx^6)}{x^6} dx$



## Optimal result

Integrand size = 30, antiderivative size = 94

$$\int \frac{(a+bx^2)^2(A+Bx^2+Cx^4+Dx^6)}{x^6} dx = -\frac{a^2A}{5x^5} - \frac{a(2Ab+aB)}{3x^3} - \frac{Ab^2+a(2bB+aC)}{x} + (b^2B+2abC+a^2D)x + \frac{1}{3}b(bC+2aD)x^3 + \frac{1}{5}b^2Dx^5$$

output
```
-1/5*a^2*A/x^5-1/3*a*(2*A*b+B*a)/x^3-(A*b^2+a*(2*B*b+C*a))/x+(B*b^2+2*C*a*
b+D*a^2)*x+1/3*b*(C*b+2*D*a)*x^3+1/5*b^2*D*x^5
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx^2)^2(A+Bx^2+Cx^4+Dx^6)}{x^6} dx = \frac{10abx^2(-A-3Bx^2+3Cx^4+Dx^6)+b^2x^4(-15A+15Bx^2+5Cx^4+3Dx^6)-a^2(3A+5x^2(B+3Cx^2}{15x^5}$$

input
```
Integrate[((a + b*x^2)^2*(A + B*x^2 + C*x^4 + D*x^6))/x^6,x]
```

output
```
(10*a*b*x^2*(-A - 3*B*x^2 + 3*C*x^4 + D*x^6) + b^2*x^4*(-15*A + 15*B*x^2 +
 5*C*x^4 + 3*D*x^6) - a^2*(3*A + 5*x^2*(B + 3*C*x^2 - 3*D*x^4)))/(15*x^5)
```

## Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2 (A+Bx^2+Cx^4+Dx^6)}{x^6} \, dx$$

$$\Big\downarrow \text{2333}$$

$$\int \left(\frac{a^2A}{x^6}+\frac{a(aC+2bB)+Ab^2}{x^2}+\frac{a(aB+2Ab)}{x^4}+b^2B\left(\frac{a(aD+2bC)}{b^2B}+1\right)+bx^2(2aD+bC)+b^2Dx^4\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2A}{5x^5}+x\left(a^2D+2abC+b^2B\right)-\frac{a(aC+2bB)+Ab^2}{x}-\frac{a(aB+2Ab)}{3x^3}+\frac{1}{3}bx^3(2aD+bC)+\frac{1}{5}b^2Dx^5$$

input
```
Int[((a + b*x^2)^2*(A + B*x^2 + C*x^4 + D*x^6))/x^6,x]
```

output
```
-1/5*(a^2*A)/x^5 - (a*(2*A*b + a*B))/(3*x^3) - (A*b^2 + a*(2*b*B + a*C))/x
 + (b^2*B + 2*a*b*C + a^2*D)*x + (b*(b*C + 2*a*D)*x^3)/3 + (b^2*D*x^5)/5
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

## Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.96

| method | result |
|---|---|
| default | $\frac{D x^5 b^2}{5}+\frac{C b^2 x^3}{3}+\frac{2 D a b x^3}{3}+b^2 B x+2 C a b x+D a^2 x-\frac{a(2Ab+Ba)}{3x^3}-\frac{a^2 A}{5x^5}-\frac{b^2 A+2abB+a^2 C}{x}$ |
| norman | $\frac{\frac{b^2 x^{10} D}{5}+(\frac{1}{3}b^2 C+\frac{2}{3}abD)x^8+(B b^2+2Cab+Da^2)x^6+(-b^2 A-2abB-a^2 C)x^4+(-\frac{2}{3}abA-\frac{1}{3}a^2 B)x^2-\frac{a^2 A}{5}}{x^5}$ |
| gosper | $-\frac{-3b^2 x^{10} D-5C b^2 x^8-10Dab x^8-15b^2 B x^6-30b x^6 Ca-15Da^2 x^6+15A b^2 x^4+30Bab x^4+15C a^2 x^4+10aAb x^2+5B a^2 x^2+}{15x^5}$ |
| parallelrisch | $-\frac{-3b^2 x^{10} D-5C b^2 x^8-10Dab x^8-15b^2 B x^6-30b x^6 Ca-15Da^2 x^6+15A b^2 x^4+30Bab x^4+15C a^2 x^4+10aAb x^2+5B a^2 x^2+}{15x^5}$ |
| orering | $-\frac{-3b^2 x^{10} D-5C b^2 x^8-10Dab x^8-15b^2 B x^6-30b x^6 Ca-15Da^2 x^6+15A b^2 x^4+30Bab x^4+15C a^2 x^4+10aAb x^2+5B a^2 x^2+}{15x^5}$ |

input
```
int((b*x^2+a)^2*(D*x^6+C*x^4+B*x^2+A)/x^6,x,method=_RETURNVERBOSE)
```

output
```
1/5*D*x^5*b^2+1/3*C*b^2*x^3+2/3*D*a*b*x^3+b^2*B*x+2*C*a*b*x+D*a^2*x-1/3*a*
(2*A*b+B*a)/x^3-1/5*a^2*A/x^5-(A*b^2+2*B*a*b+C*a^2)/x
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.01

$$\int \frac{(a+bx^2)^2 (A+Bx^2+Cx^4+Dx^6)}{x^6}\,dx$$
$$=\frac{3\,Db^2x^{10}+5\,(2\,Dab+Cb^2)x^8+15\,(Da^2+2\,Cab+Bb^2)x^6-15\,(Ca^2+2\,Bab+Ab^2)x^4-3\,Aa^2-5\,(B}{15\,x^5}$$

input
```
integrate((b*x^2+a)^2*(D*x^6+C*x^4+B*x^2+A)/x^6,x, algorithm="fricas")
```

output
```
1/15*(3*D*b^2*x^10 + 5*(2*D*a*b + C*b^2)*x^8 + 15*(D*a^2 + 2*C*a*b + B*b^2
)*x^6 - 15*(C*a^2 + 2*B*a*b + A*b^2)*x^4 - 3*A*a^2 - 5*(B*a^2 + 2*A*a*b)*x
^2)/x^5
```

## Sympy [A] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx^2)^2\,(A+Bx^2+Cx^4+Dx^6)}{x^6}\,dx$$
$$= \frac{Db^2x^5}{5} + x^3\left(\frac{Cb^2}{3}+\frac{2Dab}{3}\right) + x\left(Bb^2+2Cab+Da^2\right) + \frac{-3Aa^2+x^4(-15Ab^2-30Bab-15Ca^2)+x^2(-10Aab-5Ba^2)}{15x^5}$$

input
```
integrate((b*x**2+a)**2*(D*x**6+C*x**4+B*x**2+A)/x**6,x)
```

output
```
D*b**2*x**5/5 + x**3*(C*b**2/3 + 2*D*a*b/3) + x*(B*b**2 + 2*C*a*b + D*a**2
) + (-3*A*a**2 + x**4*(-15*A*b**2 - 30*B*a*b - 15*C*a**2) + x**2*(-10*A*a*
b - 5*B*a**2))/(15*x**5)
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx^2)^2\,(A+Bx^2+Cx^4+Dx^6)}{x^6}\,dx$$
$$= \frac{1}{5}\,Db^2x^5 + \frac{1}{3}\,(2\,Dab+Cb^2)x^3 + \left(Da^2+2\,Cab+Bb^2\right)x - \frac{15\,(Ca^2+2\,Bab+Ab^2)x^4+3\,Aa^2+5\,(Ba^2+2\,Aab)x^2}{15\,x^5}$$

input
```
integrate((b*x^2+a)^2*(D*x^6+C*x^4+B*x^2+A)/x^6,x, algorithm="maxima")
```

output
```
1/5*D*b^2*x^5 + 1/3*(2*D*a*b + C*b^2)*x^3 + (D*a^2 + 2*C*a*b + B*b^2)*x -
1/15*(15*(C*a^2 + 2*B*a*b + A*b^2)*x^4 + 3*A*a^2 + 5*(B*a^2 + 2*A*a*b)*x^2
)/x^5
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx^2)^2 (A+Bx^2+Cx^4+Dx^6)}{x^6}\,dx = \frac{1}{5}Db^2x^5+\frac{2}{3}Dabx^3+\frac{1}{3}Cb^2x^3+Da^2x+2\,Cabx+Bb^2x - \frac{15\,Ca^2x^4+30\,Babx^4+15\,Ab^2x^4+5\,Ba^2x^2+10\,Aabx^2+3\,Aa^2}{15\,x^5}$$

input
```
integrate((b*x^2+a)^2*(D*x^6+C*x^4+B*x^2+A)/x^6,x, algorithm="giac")
```

output
```
1/5*D*b^2*x^5 + 2/3*D*a*b*x^3 + 1/3*C*b^2*x^3 + D*a^2*x + 2*C*a*b*x + B*b^
2*x - 1/15*(15*C*a^2*x^4 + 30*B*a*b*x^4 + 15*A*b^2*x^4 + 5*B*a^2*x^2 + 10*
A*a*b*x^2 + 3*A*a^2)/x^5
```

## Mupad [B] (verification not implemented)

Time = 1.48 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.10

$$\int \frac{(a+bx^2)^2 (A+Bx^2+Cx^4+Dx^6)}{x^6}\,dx = a^2\,x\,D-\frac{\frac{A\,a^2}{5}+\frac{2\,A\,a\,b\,x^2}{3}+A\,b^2\,x^4}{x^5} - \frac{B\,(a^2+6\,a\,b\,x^2-3\,b^2\,x^4)}{3\,x^3}+\frac{b^2\,x^5\,D}{5} + \frac{C\,(-3\,a^2+6\,a\,b\,x^2+b^2\,x^4)}{3\,x}+\frac{2\,a\,b\,x^3\,D}{3}$$

input
```
int(((a + b*x^2)^2*(A + B*x^2 + C*x^4 + x^6*D))/x^6,x)
```

output
```
a^2*x*D - ((A*a^2)/5 + A*b^2*x^4 + (2*A*a*b*x^2)/3)/x^5 - (B*(a^2 - 3*b^2*
x^4 + 6*a*b*x^2))/(3*x^3) + (b^2*x^5*D)/5 + (C*(b^2*x^4 - 3*a^2 + 6*a*b*x^
2))/(3*x) + (2*a*b*x^3*D)/3
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^2)^2 (A+Bx^2+Cx^4+Dx^6)}{x^6} \, dx = \frac{3b^2d\,x^{10}+10abd\,x^8+5b^2c\,x^8+15a^2d\,x^6+30abc\,x^6+15b^3x^6-15a^2c\,x^4-45a\,b^2x^4-15a^2b\,x^2-3a^3}{15x^5}$$

input
```
int((b*x^2+a)^2*(D*x^6+C*x^4+B*x^2+A)/x^6,x)
```

output
```
( - 3*a**3 - 15*a**2*b*x**2 - 15*a**2*c*x**4 + 15*a**2*d*x**6 - 45*a*b**2*
x**4 + 30*a*b*c*x**6 + 10*a*b*d*x**8 + 15*b**3*x**6 + 5*b**2*c*x**8 + 3*b*
*2*d*x**10)/(15*x**5)
```

# 3.160 $\int \frac{x^6(A+Bx^2+Cx^4+Dx^6)}{a+bx^2}\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 210

$$\int \frac{x^6(A+Bx^2+Cx^4+Dx^6)}{a+bx^2}\,dx = \frac{a^2(Ab^3-a(b^2B-abC+a^2D))\,x}{b^6} - \frac{a(Ab^3-a(b^2B-abC+a^2D))\,x^3}{3b^5} + \frac{(Ab^3-a(b^2B-abC+a^2D))\,x^5}{5b^4} + \frac{(b^2B-abC+a^2D)\,x^7}{7b^3} + \frac{(bC-aD)x^9}{9b^2} + \frac{Dx^{11}}{11b} - \frac{a^{5/2}(Ab^3-a(b^2B-abC+a^2D))\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{13/2}}$$

output
```
a^2*(A*b^3-a*(B*b^2-C*a*b+D*a^2))*x/b^6-1/3*a*(A*b^3-a*(B*b^2-C*a*b+D*a^2)
)*x^3/b^5+1/5*(A*b^3-a*(B*b^2-C*a*b+D*a^2))*x^5/b^4+1/7*(B*b^2-C*a*b+D*a^2
)*x^7/b^3+1/9*(C*b-D*a)*x^9/b^2+1/11*D*x^11/b-a^(5/2)*(A*b^3-a*(B*b^2-C*a*
b+D*a^2))*arctan(b^(1/2)*x/a^(1/2))/b^(13/2)
```

## Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.00

$$\int \frac{x^6(A+Bx^2+Cx^4+Dx^6)}{a+bx^2}\,dx = -\frac{a^2(-Ab^3+ab^2B-a^2bC+a^3D)\,x}{b^6} + \frac{a(-Ab^3+ab^2B-a^2bC+a^3D)\,x^3}{3b^5} + \frac{(Ab^3-ab^2B+a^2bC-a^3D)\,x^5}{5b^4} + \frac{(b^2B-abC+a^2D)\,x^7}{7b^3} + \frac{(bC-aD)x^9}{9b^2} + \frac{Dx^{11}}{11b} + \frac{a^{5/2}(-Ab^3+ab^2B-a^2bC+a^3D)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{13/2}}$$

input
```
Integrate[(x^6*(A + B*x^2 + C*x^4 + D*x^6))/(a + b*x^2),x]
```

output
```
-((a^2*(-(A*b^3) + a*b^2*B - a^2*b*C + a^3*D)*x)/b^6) + (a*(-(A*b^3) + a*b
^2*B - a^2*b*C + a^3*D)*x^3)/(3*b^5) + ((A*b^3 - a*b^2*B + a^2*b*C - a^3*D
)*x^5)/(5*b^4) + ((b^2*B - a*b*C + a^2*D)*x^7)/(7*b^3) + ((b*C - a*D)*x^9)
/(9*b^2) + (D*x^11)/(11*b) + (a^(5/2)*(-(A*b^3) + a*b^2*B - a^2*b*C + a^3*
D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(13/2)
```

## Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6\left(A+Bx^2+Cx^4+Dx^6\right)}{a+bx^2}\,dx$$

$$\Big\downarrow \; 2333$$

$$\int \left(\frac{a^2\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{b^6}-\frac{ax^2\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{b^5}+\frac{x^4\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{b^4}\right.$$

$\downarrow$ 2009

$$\frac{a^2x\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{b^6}-\frac{ax^3\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{3b^5}+\frac{x^5\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{5b^4}+\frac{x^7\left(a^2D-abC+b^2B\right)}{7b^3}-\frac{a^{5/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{b^{13/2}}+\frac{x^9(bC-aD)}{9b^2}+\frac{Dx^{11}}{11b}$$

input
```
Int[(x^6*(A + B*x^2 + C*x^4 + D*x^6))/(a + b*x^2),x]
```

output
```
(a^2*(A*b^3 - a*(b^2*B - a*b*C + a^2*D))*x)/b^6 - (a*(A*b^3 - a*(b^2*B - a
*b*C + a^2*D))*x^3)/(3*b^5) + ((A*b^3 - a*(b^2*B - a*b*C + a^2*D))*x^5)/(5
*b^4) + ((b^2*B - a*b*C + a^2*D)*x^7)/(7*b^3) + ((b*C - a*D)*x^9)/(9*b^2)
+ (D*x^11)/(11*b) - (a^(5/2)*(A*b^3 - a*(b^2*B - a*b*C + a^2*D))*ArcTan[(S
qrt[b]*x)/Sqrt[a]])/b^(13/2)
```

## Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

## Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.11

| method | result |
|---|---|
| default | $\frac{\frac{1}{11}Dx^{11}b^5+\frac{1}{9}C\,b^5x^9-\frac{1}{9}Da\,b^4x^9+\frac{1}{7}b^5B\,x^7-\frac{1}{7}Ca\,b^4x^7+\frac{1}{7}Da^2b^3x^7+\frac{1}{5}A\,b^5x^5-\frac{1}{5}Ba\,b^4x^5+\frac{1}{5}C\,a^2b^3x^5-\frac{1}{5}Da^3b^2x^5-\frac{1}{3}Aa\,b^4x^3+\frac{1}{3}}{b^6}$ |

input
```
int(x^6*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/b^6*(1/11*D*x^11*b^5+1/9*C*b^5*x^9-1/9*D*a*b^4*x^9+1/7*b^5*B*x^7-1/7*C*a
*b^4*x^7+1/7*D*a^2*b^3*x^7+1/5*A*b^5*x^5-1/5*B*a*b^4*x^5+1/5*C*a^2*b^3*x^5
-1/5*D*a^3*b^2*x^5-1/3*A*a*b^4*x^3+1/3*B*a^2*b^3*x^3-1/3*C*a^3*b^2*x^3+1/3
*D*a^4*b*x^3+A*a^2*b^3*x-B*a^3*b^2*x+C*a^4*b*x-D*a^5*x)-a^3*(A*b^3-B*a*b^2
+C*a^2*b-D*a^3)/b^6/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 452, normalized size of antiderivative = 2.15

$$\int \frac{x^6(A+Bx^2+Cx^4+Dx^6)}{a+bx^2}\,dx$$
$$=\left[\frac{630\,Db^5x^{11}-770\,(Dab^4-Cb^5)x^9+990\,(Da^2b^3-Cab^4+Bb^5)x^7-1386\,(Da^3b^2-Ca^2b^3+Bab^4-A}{}\right.$$

input
```
integrate(x^6*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a),x, algorithm="fricas")
```

output
```
[1/6930*(630*D*b^5*x^11 - 770*(D*a*b^4 - C*b^5)*x^9 + 990*(D*a^2*b^3 - C*a
*b^4 + B*b^5)*x^7 - 1386*(D*a^3*b^2 - C*a^2*b^3 + B*a*b^4 - A*b^5)*x^5 + 2
310*(D*a^4*b - C*a^3*b^2 + B*a^2*b^3 - A*a*b^4)*x^3 - 3465*(D*a^5 - C*a^4*
b + B*a^3*b^2 - A*a^2*b^3)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(
b*x^2 + a)) - 6930*(D*a^5 - C*a^4*b + B*a^3*b^2 - A*a^2*b^3)*x)/b^6, 1/346
5*(315*D*b^5*x^11 - 385*(D*a*b^4 - C*b^5)*x^9 + 495*(D*a^2*b^3 - C*a*b^4 +
 B*b^5)*x^7 - 693*(D*a^3*b^2 - C*a^2*b^3 + B*a*b^4 - A*b^5)*x^5 + 1155*(D*
a^4*b - C*a^3*b^2 + B*a^2*b^3 - A*a*b^4)*x^3 + 3465*(D*a^5 - C*a^4*b + B*a
^3*b^2 - A*a^2*b^3)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) - 3465*(D*a^5 - C*a^
4*b + B*a^3*b^2 - A*a^2*b^3)*x)/b^6]
```

## Sympy [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.83

$$\int \frac{x^6(A+Bx^2+Cx^4+Dx^6)}{a+bx^2}\,dx = \frac{Dx^{11}}{11b}+x^9\left(\frac{C}{9b}-\frac{Da}{9b^2}\right)+x^7\left(\frac{B}{7b}-\frac{Ca}{7b^2}+\frac{Da^2}{7b^3}\right)+x^5\left(\frac{A}{5b}-\frac{Ba}{5b^2}+\frac{Ca^2}{5b^3}-\frac{Da^3}{5b^4}\right)$$
$$+x^3\left(-\frac{Aa}{3b^2}+\frac{Ba^2}{3b^3}-\frac{Ca^3}{3b^4}+\frac{Da^4}{3b^5}\right)+x\left(\frac{Aa^2}{b^3}-\frac{Ba^3}{b^4}+\frac{Ca^4}{b^5}-\frac{Da^5}{b^6}\right)$$
$$-\frac{\sqrt{-\frac{a^5}{b^{13}}}(-Ab^3+Bab^2-Ca^2b+Da^3)\log\left(-\frac{b^6\sqrt{-\frac{a^5}{b^{13}}}(-Ab^3+Bab^2-Ca^2b+Da^3)}{-Aa^2b^3+Ba^3b^2-Ca^4b+Da^5}+x\right)}{2}$$
$$+\frac{\sqrt{-\frac{a^5}{b^{13}}}(-Ab^3+Bab^2-Ca^2b+Da^3)\log\left(\frac{b^6\sqrt{-\frac{a^5}{b^{13}}}(-Ab^3+Bab^2-Ca^2b+Da^3)}{-Aa^2b^3+Ba^3b^2-Ca^4b+Da^5}+x\right)}{2}$$

input
```
integrate(x**6*(D*x**6+C*x**4+B*x**2+A)/(b*x**2+a),x)
```

output
```
D*x**11/(11*b) + x**9*(C/(9*b) - D*a/(9*b**2)) + x**7*(B/(7*b) - C*a/(7*b*
*2) + D*a**2/(7*b**3)) + x**5*(A/(5*b) - B*a/(5*b**2) + C*a**2/(5*b**3) -
D*a**3/(5*b**4)) + x**3*(-A*a/(3*b**2) + B*a**2/(3*b**3) - C*a**3/(3*b**4)
 + D*a**4/(3*b**5)) + x*(A*a**2/b**3 - B*a**3/b**4 + C*a**4/b**5 - D*a**5/
b**6) - sqrt(-a**5/b**13)*(-A*b**3 + B*a*b**2 - C*a**2*b + D*a**3)*log(-b*
*6*sqrt(-a**5/b**13)*(-A*b**3 + B*a*b**2 - C*a**2*b + D*a**3)/(-A*a**2*b**
3 + B*a**3*b**2 - C*a**4*b + D*a**5) + x)/2 + sqrt(-a**5/b**13)*(-A*b**3 +
 B*a*b**2 - C*a**2*b + D*a**3)*log(b**6*sqrt(-a**5/b**13)*(-A*b**3 + B*a*b
**2 - C*a**2*b + D*a**3)/(-A*a**2*b**3 + B*a**3*b**2 - C*a**4*b + D*a**5)
+ x)/2
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.01

$$\int \frac{x^6(A + Bx^2 + Cx^4 + Dx^6)}{a + bx^2}\,dx = \frac{(Da^6 - Ca^5b + Ba^4b^2 - Aa^3b^3)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^6} + \frac{315\,Db^5x^{11} - 385\,(Dab^4 - Cb^5)x^9 + 495\,(Da^2b^3 - Cab^4 + Bb^5)x^7 - 693\,(Da^3b^2 - Ca^2b^3 + Bab^4 - A}{3465\,b^6}$$

input
```
integrate(x^6*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a),x, algorithm="maxima")
```

output
```
(D*a^6 - C*a^5*b + B*a^4*b^2 - A*a^3*b^3)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)
*b^6) + 1/3465*(315*D*b^5*x^11 - 385*(D*a*b^4 - C*b^5)*x^9 + 495*(D*a^2*b^
3 - C*a*b^4 + B*b^5)*x^7 - 693*(D*a^3*b^2 - C*a^2*b^3 + B*a*b^4 - A*b^5)*x
^5 + 1155*(D*a^4*b - C*a^3*b^2 + B*a^2*b^3 - A*a*b^4)*x^3 - 3465*(D*a^5 -
C*a^4*b + B*a^3*b^2 - A*a^2*b^3)*x)/b^6
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.16

$$\int \frac{x^6(A + Bx^2 + Cx^4 + Dx^6)}{a + bx^2}\,dx = \frac{(Da^6 - Ca^5b + Ba^4b^2 - Aa^3b^3)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^6} + \frac{315\,Db^{10}x^{11} - 385\,Dab^9x^9 + 385\,Cb^{10}x^9 + 495\,Da^2b^8x^7 - 495\,Cab^9x^7 + 495\,Bb^{10}x^7 - 693\,Da^3b^7x^5 +}{}$$

input
```
integrate(x^6*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a),x, algorithm="giac")
```

output
```
(D*a^6 - C*a^5*b + B*a^4*b^2 - A*a^3*b^3)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)
*b^6) + 1/3465*(315*D*b^10*x^11 - 385*D*a*b^9*x^9 + 385*C*b^10*x^9 + 495*D
*a^2*b^8*x^7 - 495*C*a*b^9*x^7 + 495*B*b^10*x^7 - 693*D*a^3*b^7*x^5 + 693*
C*a^2*b^8*x^5 - 693*B*a*b^9*x^5 + 693*A*b^10*x^5 + 1155*D*a^4*b^6*x^3 - 11
55*C*a^3*b^7*x^3 + 1155*B*a^2*b^8*x^3 - 1155*A*a*b^9*x^3 - 3465*D*a^5*b^5*
x + 3465*C*a^4*b^6*x - 3465*B*a^3*b^7*x + 3465*A*a^2*b^8*x)/b^11
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^6(A+Bx^2+Cx^4+Dx^6)}{a+bx^2}\,dx=\int \frac{x^6\,(A+B\,x^2+C\,x^4+x^6\,D)}{b\,x^2+a}\,dx$$

input
```
int((x^6*(A + B*x^2 + C*x^4 + x^6*D))/(a + b*x^2),x)
```

output
```
int((x^6*(A + B*x^2 + C*x^4 + x^6*D))/(a + b*x^2), x)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.83

$$\int \frac{x^6(A+Bx^2+Cx^4+Dx^6)}{a+bx^2}\,dx$$
$$=\frac{3465\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^5d-3465\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^4bc-3465a^5bdx+3465a^4b^2cx+1155a^4b^2d\,x^3}{}$$

input
```
int(x^6*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a),x)
```

output
```
(3465*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**5*d - 3465*sqrt(b)*
sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**4*b*c - 3465*a**5*b*d*x + 3465*a*
*4*b**2*c*x + 1155*a**4*b**2*d*x**3 - 1155*a**3*b**3*c*x**3 - 693*a**3*b**
3*d*x**5 + 693*a**2*b**4*c*x**5 + 495*a**2*b**4*d*x**7 - 495*a*b**5*c*x**7
 - 385*a*b**5*d*x**9 + 495*b**7*x**7 + 385*b**6*c*x**9 + 315*b**6*d*x**11)
/(3465*b**7)
```

# 3.161 $\int \frac{x^4(A+Bx^2+Cx^4+Dx^6)}{a+bx^2}\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 172

$$\begin{aligned}\int \frac{x^4(A+Bx^2+Cx^4+Dx^6)}{a+bx^2}\,dx = &-\frac{a(Ab^3-a(b^2B-abC+a^2D))\,x}{b^5}\\ &+\frac{(Ab^3-a(b^2B-abC+a^2D))\,x^3}{3b^4}\\ &+\frac{(b^2B-abC+a^2D)\,x^5}{5b^3}+\frac{(bC-aD)x^7}{7b^2}+\frac{Dx^9}{9b}\\ &+\frac{a^{3/2}(Ab^3-a(b^2B-abC+a^2D))\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{11/2}}\end{aligned}$$

output
```
-a*(A*b^3-a*(B*b^2-C*a*b+D*a^2))*x/b^5+1/3*(A*b^3-a*(B*b^2-C*a*b+D*a^2))*x
^3/b^4+1/5*(B*b^2-C*a*b+D*a^2)*x^5/b^3+1/7*(C*b-D*a)*x^7/b^2+1/9*D*x^9/b+a
^(3/2)*(A*b^3-a*(B*b^2-C*a*b+D*a^2))*arctan(b^(1/2)*x/a^(1/2))/b^(11/2)
```

## Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.94

$$\int \frac{x^4(A+Bx^2+Cx^4+Dx^6)}{a+bx^2}\,dx$$
$$= \frac{x(315a^4D-105a^3b(3C+Dx^2)+21a^2b^2(15B+5Cx^2+3Dx^4)-3ab^3(105A+35Bx^2+21Cx^4+15D}{315b^5}$$
$$-\frac{a^{3/2}(-Ab^3+a(b^2B-abC+a^2D))\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{11/2}}$$

input
```
Integrate[(x^4*(A + B*x^2 + C*x^4 + D*x^6))/(a + b*x^2),x]
```

output
```
(x*(315*a^4*D - 105*a^3*b*(3*C + D*x^2) + 21*a^2*b^2*(15*B + 5*C*x^2 + 3*D
*x^4) - 3*a*b^3*(105*A + 35*B*x^2 + 21*C*x^4 + 15*D*x^6) + b^4*x^2*(105*A
+ 63*B*x^2 + 45*C*x^4 + 35*D*x^6)))/(315*b^5) - (a^(3/2)*(-(A*b^3) + a*(b^
2*B - a*b*C + a^2*D))*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(11/2)
```

## Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(A+Bx^2+Cx^4+Dx^6)}{a+bx^2}\,dx$$

$$\Big\downarrow \text{2333}$$

$$\int \left(-\frac{a(Ab^3-a(a^2D-abC+b^2B))}{b^5}+\frac{x^2(Ab^3-a(a^2D-abC+b^2B))}{b^4}+\frac{x^4(a^2D-abC+b^2B)}{b^3}+\frac{a^5(-D)}{}\right.$$

$$\Big\downarrow \text{2009}$$

$$
-\frac{ax(Ab^3-a(a^2D-abC+b^2B))}{b^5}+\frac{x^3(Ab^3-a(a^2D-abC+b^2B))}{3b^4}+\frac{x^5(a^2D-abC+b^2B)}{5b^3}+\frac{a^{3/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(Ab^3-a(a^2D-abC+b^2B))}{b^{11/2}}+\frac{x^7(bC-aD)}{7b^2}+\frac{Dx^9}{9b}
$$

input
```
Int[(x^4*(A + B*x^2 + C*x^4 + D*x^6))/(a + b*x^2),x]
```

output
```
-((a*(A*b^3 - a*(b^2*B - a*b*C + a^2*D))*x)/b^5) + ((A*b^3 - a*(b^2*B - a*
b*C + a^2*D))*x^3)/(3*b^4) + ((b^2*B - a*b*C + a^2*D)*x^5)/(5*b^3) + ((b*C
 - a*D)*x^7)/(7*b^2) + (D*x^9)/(9*b) + (a^(3/2)*(A*b^3 - a*(b^2*B - a*b*C
+ a^2*D))*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(11/2)
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

## Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.08

| method | result |
|---|---|
| default | $-\frac{-\frac{1}{9}Dx^9b^4-\frac{1}{7}Cb^4x^7+\frac{1}{7}Dab^3x^7-\frac{1}{5}Bb^4x^5+\frac{1}{5}Cab^3x^5-\frac{1}{5}Da^2b^2x^5-\frac{1}{3}Ax^3b^4+\frac{1}{3}Bx^3ab^3-\frac{1}{3}Ca^2b^2x^3+\frac{1}{3}Da^3bx^3+Aab^3x-B\,a}{b^5}$ |

input
```
int(x^4*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/b^5*(-1/9*D*x^9*b^4-1/7*C*b^4*x^7+1/7*D*a*b^3*x^7-1/5*B*b^4*x^5+1/5*C*a
*b^3*x^5-1/5*D*a^2*b^2*x^5-1/3*A*x^3*b^4+1/3*B*x^3*a*b^3-1/3*C*a^2*b^2*x^3
+1/3*D*a^3*b*x^3+A*a*b^3*x-B*a^2*b^2*x+C*a^3*b*x-D*a^4*x)+a^2*(A*b^3-B*a*b
^2+C*a^2*b-D*a^3)/b^5/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 368, normalized size of antiderivative = 2.14

$$\int \frac{x^4(A+Bx^2+Cx^4+Dx^6)}{a+bx^2}\,dx$$
$$= \left[\frac{70\,Db^4x^9 - 90\,(Dab^3 - Cb^4)x^7 + 126\,(Da^2b^2 - Cab^3 + Bb^4)x^5 - 210\,(Da^3b - Ca^2b^2 + Bab^3 - Ab^4)x^{3}}{63}\right.$$

input
```
integrate(x^4*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a),x, algorithm="fricas")
```

output
```
[1/630*(70*D*b^4*x^9 - 90*(D*a*b^3 - C*b^4)*x^7 + 126*(D*a^2*b^2 - C*a*b^3
 + B*b^4)*x^5 - 210*(D*a^3*b - C*a^2*b^2 + B*a*b^3 - A*b^4)*x^3 - 315*(D*a
^4 - C*a^3*b + B*a^2*b^2 - A*a*b^3)*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/
b) - a)/(b*x^2 + a)) + 630*(D*a^4 - C*a^3*b + B*a^2*b^2 - A*a*b^3)*x)/b^5,
 1/315*(35*D*b^4*x^9 - 45*(D*a*b^3 - C*b^4)*x^7 + 63*(D*a^2*b^2 - C*a*b^3
+ B*b^4)*x^5 - 105*(D*a^3*b - C*a^2*b^2 + B*a*b^3 - A*b^4)*x^3 - 315*(D*a^
4 - C*a^3*b + B*a^2*b^2 - A*a*b^3)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) + 315
*(D*a^4 - C*a^3*b + B*a^2*b^2 - A*a*b^3)*x)/b^5]
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 337 vs. 2(158) = 316.

Time = 0.49 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.96

$$\int \frac{x^4(A+Bx^2+Cx^4+Dx^6)}{a+bx^2}\,dx$$
$$= \frac{Dx^9}{9b} + x^7\left(\frac{C}{7b} - \frac{Da}{7b^2}\right) + x^5\left(\frac{B}{5b} - \frac{Ca}{5b^2} + \frac{Da^2}{5b^3}\right)$$
$$+ x^3\left(\frac{A}{3b} - \frac{Ba}{3b^2} + \frac{Ca^2}{3b^3} - \frac{Da^3}{3b^4}\right) + x\left(-\frac{Aa}{b^2} + \frac{Ba^2}{b^3} - \frac{Ca^3}{b^4} + \frac{Da^4}{b^5}\right)$$
$$+ \frac{\sqrt{-\frac{a^3}{b^{11}}}(-Ab^3+Bab^2-Ca^2b+Da^3)\log\left(-\frac{b^5\sqrt{-\frac{a^3}{b^{11}}}(-Ab^3+Bab^2-Ca^2b+Da^3)}{-Aab^3+Ba^2b^2-Ca^3b+Da^4}+x\right)}{2}$$
$$- \frac{\sqrt{-\frac{a^3}{b^{11}}}(-Ab^3+Bab^2-Ca^2b+Da^3)\log\left(\frac{b^5\sqrt{-\frac{a^3}{b^{11}}}(-Ab^3+Bab^2-Ca^2b+Da^3)}{-Aab^3+Ba^2b^2-Ca^3b+Da^4}+x\right)}{2}$$

input
```
integrate(x**4*(D*x**6+C*x**4+B*x**2+A)/(b*x**2+a),x)
```

output
```
D*x**9/(9*b) + x**7*(C/(7*b) - D*a/(7*b**2)) + x**5*(B/(5*b) - C*a/(5*b**2
) + D*a**2/(5*b**3)) + x**3*(A/(3*b) - B*a/(3*b**2) + C*a**2/(3*b**3) - D*
a**3/(3*b**4)) + x*(-A*a/b**2 + B*a**2/b**3 - C*a**3/b**4 + D*a**4/b**5) +
 sqrt(-a**3/b**11)*(-A*b**3 + B*a*b**2 - C*a**2*b + D*a**3)*log(-b**5*sqrt
(-a**3/b**11)*(-A*b**3 + B*a*b**2 - C*a**2*b + D*a**3)/(-A*a*b**3 + B*a**2
*b**2 - C*a**3*b + D*a**4) + x)/2 - sqrt(-a**3/b**11)*(-A*b**3 + B*a*b**2
- C*a**2*b + D*a**3)*log(b**5*sqrt(-a**3/b**11)*(-A*b**3 + B*a*b**2 - C*a*
*2*b + D*a**3)/(-A*a*b**3 + B*a**2*b**2 - C*a**3*b + D*a**4) + x)/2
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.01

$$\int \frac{x^4(A+Bx^2+Cx^4+Dx^6)}{a+bx^2}\,dx = -\frac{(Da^5-Ca^4b+Ba^3b^2-Aa^2b^3)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^5}$$
$$+ \frac{35\,Db^4x^9 - 45\,(Dab^3-Cb^4)x^7 + 63\,(Da^2b^2-Cab^3+Bb^4)x^5 - 105\,(Da^3b-Ca^2b^2+Bab^3-Ab^4)x^3}{315\,b^5}$$

input
```
integrate(x^4*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a),x, algorithm="maxima")
```

output
```
-(D*a^5 - C*a^4*b + B*a^3*b^2 - A*a^2*b^3)*arctan(b*x/sqrt(a*b))/(sqrt(a*b
)*b^5) + 1/315*(35*D*b^4*x^9 - 45*(D*a*b^3 - C*b^4)*x^7 + 63*(D*a^2*b^2 -
C*a*b^3 + B*b^4)*x^5 - 105*(D*a^3*b - C*a^2*b^2 + B*a*b^3 - A*b^4)*x^3 + 3
15*(D*a^4 - C*a^3*b + B*a^2*b^2 - A*a*b^3)*x)/b^5
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.14

$$\int \frac{x^4(A+Bx^2+Cx^4+Dx^6)}{a+bx^2}\,dx = -\frac{(Da^5-Ca^4b+Ba^3b^2-Aa^2b^3)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^5} + \frac{35\,Db^8x^9-45\,Dab^7x^7+45\,Cb^8x^7+63\,Da^2b^6x^5-63\,Cab^7x^5+63\,Bb^8x^5-105\,Da^3b^5x^3+105\,Ca^2b^6}{315\,b^9}$$

input
```
integrate(x^4*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a),x, algorithm="giac")
```

output
```
-(D*a^5 - C*a^4*b + B*a^3*b^2 - A*a^2*b^3)*arctan(b*x/sqrt(a*b))/(sqrt(a*b
)*b^5) + 1/315*(35*D*b^8*x^9 - 45*D*a*b^7*x^7 + 45*C*b^8*x^7 + 63*D*a^2*b^
6*x^5 - 63*C*a*b^7*x^5 + 63*B*b^8*x^5 - 105*D*a^3*b^5*x^3 + 105*C*a^2*b^6*
x^3 - 105*B*a*b^7*x^3 + 105*A*b^8*x^3 + 315*D*a^4*b^4*x - 315*C*a^3*b^5*x
+ 315*B*a^2*b^6*x - 315*A*a*b^7*x)/b^9
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(A+Bx^2+Cx^4+Dx^6)}{a+bx^2}\,dx = \int \frac{x^4\,\left(A+B\,x^2+C\,x^4+x^6\,D\right)}{b\,x^2+a}\,dx$$

input
```
int((x^4*(A + B*x^2 + C*x^4 + x^6*D))/(a + b*x^2),x)
```

output
```
int((x^4*(A + B*x^2 + C*x^4 + x^6*D))/(a + b*x^2), x)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.88

$$\int \frac{x^4(A+Bx^2+Cx^4+Dx^6)}{a+bx^2}\,dx$$
$$=\frac{-315\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^4d+315\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^3bc+315a^4bdx-315a^3b^2cx-105a^3b^2d\,x^3+1}{315b^6}$$

input
```
int(x^4*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a),x)
```

output
```
( - 315*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**4*d + 315*sqrt(b)
*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**3*b*c + 315*a**4*b*d*x - 315*a**
3*b**2*c*x - 105*a**3*b**2*d*x**3 + 105*a**2*b**3*c*x**3 + 63*a**2*b**3*d*
x**5 - 63*a*b**4*c*x**5 - 45*a*b**4*d*x**7 + 63*b**6*x**5 + 45*b**5*c*x**7
 + 35*b**5*d*x**9)/(315*b**6)
```

# 3.162 $\int \frac{x^2(A+Bx^2+Cx^4+Dx^6)}{a+bx^2} dx$



## Optimal result

Integrand size = 30, antiderivative size = 136

$$\int \frac{x^2(A+Bx^2+Cx^4+Dx^6)}{a+bx^2} dx = \frac{(Ab^3-a(b^2B-abC+a^2D))x}{b^4} + \frac{(b^2B-abC+a^2D)x^3}{3b^3} + \frac{(bC-aD)x^5}{5b^2} + \frac{Dx^7}{7b} - \frac{\sqrt{a}(Ab^3-a(b^2B-abC+a^2D))\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{9/2}}$$

output
```
(A*b^3-a*(B*b^2-C*a*b+D*a^2))*x/b^4+1/3*(B*b^2-C*a*b+D*a^2)*x^3/b^3+1/5*(C
*b-D*a)*x^5/b^2+1/7*D*x^7/b-a^(1/2)*(A*b^3-a*(B*b^2-C*a*b+D*a^2))*arctan(b
^(1/2)*x/a^(1/2))/b^(9/2)
```

## Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.99

$$\int \frac{x^2(A+Bx^2+Cx^4+Dx^6)}{a+bx^2} dx = \frac{(Ab^3-ab^2B+a^2bC-a^3D)x}{b^4} + \frac{(b^2B-abC+a^2D)x^3}{3b^3} + \frac{(bC-aD)x^5}{5b^2} + \frac{Dx^7}{7b} + \frac{\sqrt{a}(-Ab^3+ab^2B-a^2bC+a^3D)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{9/2}}$$

input
```
Integrate[(x^2*(A + B*x^2 + C*x^4 + D*x^6))/(a + b*x^2),x]
```

output
```
((A*b^3 - a*b^2*B + a^2*b*C - a^3*D)*x)/b^4 + ((b^2*B - a*b*C + a^2*D)*x^3
)/(3*b^3) + ((b*C - a*D)*x^5)/(5*b^2) + (D*x^7)/(7*b) + (Sqrt[a]*(-(A*b^3)
 + a*b^2*B - a^2*b*C + a^3*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(9/2)
```

## Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(A+Bx^2+Cx^4+Dx^6)}{a+bx^2}\,dx$$

$$\Big\downarrow \text{ 2333}$$

$$\int \left(\frac{x^2(a^2D-abC+b^2B)}{b^3}+\frac{a^3(-D)+a^2bC-ab^2B+Ab^3}{b^4}+\frac{a^4D-a^3bC+a^2b^2B-aAb^3}{b^4(a+bx^2)}+\frac{x^4(bC-aD)}{b^2}\right.$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\sqrt{a}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(Ab^3-a(a^2D-abC+b^2B))}{b^{9/2}}+\frac{x(Ab^3-a(a^2D-abC+b^2B))}{b^4}+\frac{x^3(a^2D-abC+b^2B)}{3b^3}+\frac{x^5(bC-aD)}{5b^2}+\frac{Dx^7}{7b}$$

input
```
Int[(x^2*(A + B*x^2 + C*x^4 + D*x^6))/(a + b*x^2),x]
```

output
```
((A*b^3 - a*(b^2*B - a*b*C + a^2*D))*x)/b^4 + ((b^2*B - a*b*C + a^2*D)*x^3
)/(3*b^3) + ((b*C - a*D)*x^5)/(5*b^2) + (D*x^7)/(7*b) - (Sqrt[a]*(A*b^3 -
a*(b^2*B - a*b*C + a^2*D))*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(9/2)
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

## Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.01

| method | result |
|---|---|
| default | $\frac{\frac{1}{7}b^3Dx^7+\frac{1}{5}b^3Cx^5-\frac{1}{5}Dab^2x^5+\frac{1}{3}b^3Bx^3-\frac{1}{3}Cab^2x^3+\frac{1}{3}Dx^3ba^2+Ab^3x-Bab^2x+Ca^2bx-Da^3x}{b^4}-\frac{a(b^3A-ab^2B+a^2bC-a^3D)}{b^4\sqrt{ab}}$ |

input
```
int(x^2*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/b^4*(1/7*b^3*D*x^7+1/5*b^3*C*x^5-1/5*D*a*b^2*x^5+1/3*b^3*B*x^3-1/3*C*a*b
^2*x^3+1/3*D*x^3*b*a^2+A*b^3*x-B*a*b^2*x+C*a^2*b*x-D*a^3*x)-a*(A*b^3-B*a*b
^2+C*a^2*b-D*a^3)/b^4/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 286, normalized size of antiderivative = 2.10

$$\int \frac{x^2(A+Bx^2+Cx^4+Dx^6)}{a+bx^2}\,dx$$

$$= \left[\frac{30\,Db^3x^7-42\,(Dab^2-Cb^3)x^5+70\,(Da^2b-Cab^2+Bb^3)x^3-105\,(Da^3-Ca^2b+Bab^2-Ab^3)\sqrt{-\frac{a}{b}}}{210\,b^4}\right.$$

input
```
integrate(x^2*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a),x, algorithm="fricas")
```

output
```
[1/210*(30*D*b^3*x^7 - 42*(D*a*b^2 - C*b^3)*x^5 + 70*(D*a^2*b - C*a*b^2 +
B*b^3)*x^3 - 105*(D*a^3 - C*a^2*b + B*a*b^2 - A*b^3)*sqrt(-a/b)*log((b*x^2
 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) - 210*(D*a^3 - C*a^2*b + B*a*b^2 - A
*b^3)*x)/b^4, 1/105*(15*D*b^3*x^7 - 21*(D*a*b^2 - C*b^3)*x^5 + 35*(D*a^2*b
 - C*a*b^2 + B*b^3)*x^3 + 105*(D*a^3 - C*a^2*b + B*a*b^2 - A*b^3)*sqrt(a/b
)*arctan(b*x*sqrt(a/b)/a) - 105*(D*a^3 - C*a^2*b + B*a*b^2 - A*b^3)*x)/b^4
]
```

## Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.36

$$\int \frac{x^2(A+Bx^2+Cx^4+Dx^6)}{a+bx^2}\,dx$$
$$= \frac{Dx^7}{7b} + x^5\left(\frac{C}{5b} - \frac{Da}{5b^2}\right) + x^3\left(\frac{B}{3b} - \frac{Ca}{3b^2} + \frac{Da^2}{3b^3}\right) + x\left(\frac{A}{b} - \frac{Ba}{b^2} + \frac{Ca^2}{b^3} - \frac{Da^3}{b^4}\right)$$
$$- \frac{\sqrt{-\frac{a}{b^9}}(-Ab^3 + Bab^2 - Ca^2b + Da^3)\log\left(-b^4\sqrt{-\frac{a}{b^9}} + x\right)}{2}$$
$$+ \frac{\sqrt{-\frac{a}{b^9}}(-Ab^3 + Bab^2 - Ca^2b + Da^3)\log\left(b^4\sqrt{-\frac{a}{b^9}} + x\right)}{2}$$

input
```
integrate(x**2*(D*x**6+C*x**4+B*x**2+A)/(b*x**2+a),x)
```

output
```
D*x**7/(7*b) + x**5*(C/(5*b) - D*a/(5*b**2)) + x**3*(B/(3*b) - C*a/(3*b**2
) + D*a**2/(3*b**3)) + x*(A/b - B*a/b**2 + C*a**2/b**3 - D*a**3/b**4) - sq
rt(-a/b**9)*(-A*b**3 + B*a*b**2 - C*a**2*b + D*a**3)*log(-b**4*sqrt(-a/b**
9) + x)/2 + sqrt(-a/b**9)*(-A*b**3 + B*a*b**2 - C*a**2*b + D*a**3)*log(b**
4*sqrt(-a/b**9) + x)/2
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.97

$$\int \frac{x^2(A+Bx^2+Cx^4+Dx^6)}{a+bx^2}\,dx = \frac{(Da^4-Ca^3b+Ba^2b^2-Aab^3)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^4} + \frac{15\,Db^3x^7-21\,(Dab^2-Cb^3)x^5+35\,(Da^2b-Cab^2+Bb^3)x^3-105\,(Da^3-Ca^2b+Bab^2-Ab^3)x}{105\,b^4}$$

input
```
integrate(x^2*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a),x, algorithm="maxima")
```

output
```
(D*a^4 - C*a^3*b + B*a^2*b^2 - A*a*b^3)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b
^4) + 1/105*(15*D*b^3*x^7 - 21*(D*a*b^2 - C*b^3)*x^5 + 35*(D*a^2*b - C*a*b
^2 + B*b^3)*x^3 - 105*(D*a^3 - C*a^2*b + B*a*b^2 - A*b^3)*x)/b^4
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.08

$$\int \frac{x^2(A+Bx^2+Cx^4+Dx^6)}{a+bx^2}\,dx = \frac{(Da^4-Ca^3b+Ba^2b^2-Aab^3)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^4} + \frac{15\,Db^6x^7-21\,Dab^5x^5+21\,Cb^6x^5+35\,Da^2b^4x^3-35\,Cab^5x^3+35\,Bb^6x^3-105\,Da^3b^3x+105\,Ca^2b^4x}{105\,b^7}$$

input
```
integrate(x^2*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a),x, algorithm="giac")
```

output
```
(D*a^4 - C*a^3*b + B*a^2*b^2 - A*a*b^3)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b
^4) + 1/105*(15*D*b^6*x^7 - 21*D*a*b^5*x^5 + 21*C*b^6*x^5 + 35*D*a^2*b^4*x
^3 - 35*C*a*b^5*x^3 + 35*B*b^6*x^3 - 105*D*a^3*b^3*x + 105*C*a^2*b^4*x - 1
05*B*a*b^5*x + 105*A*b^6*x)/b^7
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(A+Bx^2+Cx^4+Dx^6)}{a+bx^2}\,dx = \int \frac{x^2\,(A+B\,x^2+C\,x^4+x^6\,D)}{b\,x^2+a}\,dx$$

input
```
int((x^2*(A + B*x^2 + C*x^4 + x^6*D))/(a + b*x^2),x)
```

output
```
int((x^2*(A + B*x^2 + C*x^4 + x^6*D))/(a + b*x^2), x)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.93

$$\int \frac{x^2(A+Bx^2+Cx^4+Dx^6)}{a+bx^2}\,dx = \frac{105\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^3d - 105\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^2bc - 105a^3bdx + 105a^2b^2cx + 35a^2b^2d\,x^3 - 35a}{105b^5}$$

input
```
int(x^2*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a),x)
```

output
```
(105*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**3*d - 105*sqrt(b)*sq
rt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2*b*c - 105*a**3*b*d*x + 105*a**2*b
**2*c*x + 35*a**2*b**2*d*x**3 - 35*a*b**3*c*x**3 - 21*a*b**3*d*x**5 + 35*b
**5*x**3 + 21*b**4*c*x**5 + 15*b**4*d*x**7)/(105*b**5)
```

# 3.163 $\int \frac{A+Bx^2+Cx^4+Dx^6}{a+bx^2} dx$



## Optimal result

Integrand size = 27, antiderivative size = 100

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{a+bx^2} dx = \frac{(b^2B-abC+a^2D)\,x}{b^3}+\frac{(bC-aD)x^3}{3b^2}+\frac{Dx^5}{5b}+\frac{(Ab^3-a(b^2B-abC+a^2D))\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{7/2}}$$

output
```
(B*b^2-C*a*b+D*a^2)*x/b^3+1/3*(C*b-D*a)*x^3/b^2+1/5*D*x^5/b+(A*b^3-a*(B*b^
2-C*a*b+D*a^2))*arctan(b^(1/2)*x/a^(1/2))/a^(1/2)/b^(7/2)
```

## Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.98

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{a+bx^2} dx = \frac{x(15a^2D-5ab(3C+Dx^2)+b^2(15B+5Cx^2+3Dx^4))}{15b^3}+\frac{(Ab^3-a(b^2B-abC+a^2D))\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{7/2}}$$

input
```
Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(a + b*x^2),x]
```

output
```
(x*(15*a^2*D - 5*a*b*(3*C + D*x^2) + b^2*(15*B + 5*C*x^2 + 3*D*x^4)))/(15*
b^3) + ((A*b^3 - a*(b^2*B - a*b*C + a^2*D))*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(
Sqrt[a]*b^(7/2))
```

## Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2341, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{a+bx^2}\,dx$$

$$\Big\downarrow \text{ 2341}$$

$$\int \left(\frac{a^2D-abC+b^2B}{b^3}+\frac{a^3(-D)+a^2bC-ab^2B+Ab^3}{b^3\left(a+bx^2\right)}+\frac{x^2(bC-aD)}{b^2}+\frac{Dx^4}{b}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{\sqrt{a}b^{7/2}}+\frac{x\left(a^2D-abC+b^2B\right)}{b^3}+\frac{x^3(bC-aD)}{3b^2}+\frac{Dx^5}{5b}$$

input
```
Int[(A + B*x^2 + C*x^4 + D*x^6)/(a + b*x^2),x]
```

output
```
((b^2*B - a*b*C + a^2*D)*x)/b^3 + ((b*C - a*D)*x^3)/(3*b^2) + (D*x^5)/(5*b
) + ((A*b^3 - a*(b^2*B - a*b*C + a^2*D))*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqr
t[a]*b^(7/2))
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2341
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

## Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.94

| method | result | size |
|---|---|---|
| default | $\frac{\frac{1}{5}Dx^5b^2+\frac{1}{3}C\,b^2x^3-\frac{1}{3}Dab\,x^3+b^2Bx-Cabx+Da^2x}{b^3}+\frac{\left(b^3A-a\,b^2B+a^2bC-a^3D\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{b^3\sqrt{ab}}$ | 94 |

input
```
int((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/b^3*(1/5*D*x^5*b^2+1/3*C*b^2*x^3-1/3*D*a*b*x^3+b^2*B*x-C*a*b*x+D*a^2*x)+
(A*b^3-B*a*b^2+C*a^2*b-D*a^3)/b^3/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 236, normalized size of antiderivative = 2.36

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{a+bx^2}\,dx$$
$$= \left[\frac{6\,Dab^3x^5-10\,(Da^2b^2-Cab^3)x^3+15\,(Da^3-Ca^2b+Bab^2-Ab^3)\sqrt{-ab}\log\left(\frac{bx^2-2\sqrt{-ab}x-a}{bx^2+a}\right)+30\,(D}{30\,ab^4}\right.$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a),x, algorithm="fricas")
```

output
```
[1/30*(6*D*a*b^3*x^5 - 10*(D*a^2*b^2 - C*a*b^3)*x^3 + 15*(D*a^3 - C*a^2*b
+ B*a*b^2 - A*b^3)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)
) + 30*(D*a^3*b - C*a^2*b^2 + B*a*b^3)*x)/(a*b^4), 1/15*(3*D*a*b^3*x^5 - 5
*(D*a^2*b^2 - C*a*b^3)*x^3 - 15*(D*a^3 - C*a^2*b + B*a*b^2 - A*b^3)*sqrt(a
*b)*arctan(sqrt(a*b)*x/a) + 15*(D*a^3*b - C*a^2*b^2 + B*a*b^3)*x)/(a*b^4)]
```

## Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.60

$$\begin{aligned}\int \frac{A+Bx^2+Cx^4+Dx^6}{a+bx^2}\,dx &= \frac{Dx^5}{5b}+x^3\left(\frac{C}{3b}-\frac{Da}{3b^2}\right)+x\left(\frac{B}{b}-\frac{Ca}{b^2}+\frac{Da^2}{b^3}\right)\\ &\quad+\frac{\sqrt{-\frac{1}{ab^7}}\left(-Ab^3+Bab^2-Ca^2b+Da^3\right)\log\left(-ab^3\sqrt{-\frac{1}{ab^7}}+x\right)}{2}\\ &\quad-\frac{\sqrt{-\frac{1}{ab^7}}\left(-Ab^3+Bab^2-Ca^2b+Da^3\right)\log\left(ab^3\sqrt{-\frac{1}{ab^7}}+x\right)}{2}\end{aligned}$$

input
```
integrate((D*x**6+C*x**4+B*x**2+A)/(b*x**2+a),x)
```

output
```
D*x**5/(5*b) + x**3*(C/(3*b) - D*a/(3*b**2)) + x*(B/b - C*a/b**2 + D*a**2/
b**3) + sqrt(-1/(a*b**7))*(-A*b**3 + B*a*b**2 - C*a**2*b + D*a**3)*log(-a*
b**3*sqrt(-1/(a*b**7)) + x)/2 - sqrt(-1/(a*b**7))*(-A*b**3 + B*a*b**2 - C*
a**2*b + D*a**3)*log(a*b**3*sqrt(-1/(a*b**7)) + x)/2
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.95

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{a+bx^2}\,dx = -\frac{\left(Da^3-Ca^2b+Bab^2-Ab^3\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^3}+\frac{3\,Db^2x^5-5\,(Dab-Cb^2)x^3+15\,(Da^2-Cab+Bb^2)x}{15\,b^3}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a),x, algorithm="maxima")
```

output
```
-(D*a^3 - C*a^2*b + B*a*b^2 - A*b^3)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3)
 + 1/15*(3*D*b^2*x^5 - 5*(D*a*b - C*b^2)*x^3 + 15*(D*a^2 - C*a*b + B*b^2)*
x)/b^3
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.04

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{a+bx^2}\,dx = -\frac{(Da^3-Ca^2b+Bab^2-Ab^3)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^3} + \frac{3\,Db^4x^5-5\,Dab^3x^3+5\,Cb^4x^3+15\,Da^2b^2x-15\,Cab^3x+15\,Bb^4x}{15\,b^5}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a),x, algorithm="giac")
```

output
```
-(D*a^3 - C*a^2*b + B*a*b^2 - A*b^3)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3)
 + 1/15*(3*D*b^4*x^5 - 5*D*a*b^3*x^3 + 5*C*b^4*x^3 + 15*D*a^2*b^2*x - 15*C
*a*b^3*x + 15*B*b^4*x)/b^5
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{a+bx^2}\,dx = \int \frac{A+B\,x^2+C\,x^4+x^6\,D}{b\,x^2+a}\,dx$$

input
```
int((A + B*x^2 + C*x^4 + x^6*D)/(a + b*x^2),x)
```

output
```
int((A + B*x^2 + C*x^4 + x^6*D)/(a + b*x^2), x)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.99

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{a+bx^2}\,dx$$
$$= \frac{-15\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^2d+15\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)abc+15a^2bdx-15a\,b^2cx-5a\,b^2d\,x^3+15b^4x+5b}{15b^4}$$

input
```
int((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a),x)
```

output
```
( - 15*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2*d + 15*sqrt(b)*s
qrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*b*c + 15*a**2*b*d*x - 15*a*b**2*c*x
 - 5*a*b**2*d*x**3 + 15*b**4*x + 5*b**3*c*x**3 + 3*b**3*d*x**5)/(15*b**4)
```

# 3.164 $\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2(a+bx^2)} dx$



## Optimal result

Integrand size = 30, antiderivative size = 84

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2(a+bx^2)} dx = -\frac{A}{ax}+\frac{(bC-aD)x}{b^2}+\frac{Dx^3}{3b} - \frac{(Ab^3-a(b^2B-abC+a^2D))\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}b^{5/2}}$$

output
```
-A/a/x+(C*b-D*a)*x/b^2+1/3*D*x^3/b-(A*b^3-a*(B*b^2-C*a*b+D*a^2))*arctan(b^
(1/2)*x/a^(1/2))/a^(3/2)/b^(5/2)
```

## Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.99

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2(a+bx^2)} dx = -\frac{A}{ax}+\frac{(bC-aD)x}{b^2}+\frac{Dx^3}{3b} + \frac{(-Ab^3+ab^2B-a^2bC+a^3D)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}b^{5/2}}$$

input
```
Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(x^2*(a + b*x^2)),x]
```

output
```
-(A/(a*x)) + ((b*C - a*D)*x)/b^2 + (D*x^3)/(3*b) + ((-(A*b^3) + a*b^2*B -
a^2*b*C + a^3*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*b^(5/2))
```

## Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2(a+bx^2)}dx$$

$$\Big\downarrow \text{2333}$$

$$\int \left(\frac{a(a^2D-abC+b^2B)-Ab^3}{ab^2(a+bx^2)}+\frac{A}{ax^2}+\frac{bC-aD}{b^2}+\frac{Dx^2}{b}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{a^{3/2}b^{5/2}}-\frac{A}{ax}+\frac{x(bC-aD)}{b^2}+\frac{Dx^3}{3b}$$

input
```
Int[(A + B*x^2 + C*x^4 + D*x^6)/(x^2*(a + b*x^2)),x]
```

output
```
-(A/(a*x)) + ((b*C - a*D)*x)/b^2 + (D*x^3)/(3*b) - ((A*b^3 - a*(b^2*B - a*
b*C + a^2*D))*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*b^(5/2))
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

## Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.93

| method | result | size |
|---|---|---|
| default | $\frac{\frac{1}{3}Dx^3b+Cbx-Dax}{b^2}+\frac{\left(-b^3A+a\,b^2B-a^2bC+a^3D\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{a\,b^2\sqrt{ab}}-\frac{A}{ax}$ | 78 |

input
```
int((D*x^6+C*x^4+B*x^2+A)/x^2/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/b^2*(1/3*D*x^3*b+C*b*x-D*a*x)+1/a/b^2*(-A*b^3+B*a*b^2-C*a^2*b+D*a^3)/(a*
b)^(1/2)*arctan(b*x/(a*b)^(1/2))-A/a/x
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 211, normalized size of antiderivative = 2.51

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2\left(a+bx^2\right)}\,dx$$
$$=\left[\frac{2\,Da^2b^2x^4-6\,Aab^3+3\,(Da^3-Ca^2b+Bab^2-Ab^3)\sqrt{-ab}x\log\left(\frac{bx^2+2\sqrt{-ab}x-a}{bx^2+a}\right)-6\,(Da^3b-Ca^2b^2)x}{6\,a^2b^3x}\right.$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^2/(b*x^2+a),x, algorithm="fricas")
```

output
```
[1/6*(2*D*a^2*b^2*x^4 - 6*A*a*b^3 + 3*(D*a^3 - C*a^2*b + B*a*b^2 - A*b^3)*
sqrt(-a*b)*x*log((b*x^2 + 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) - 6*(D*a^3*b -
C*a^2*b^2)*x^2)/(a^2*b^3*x), 1/3*(D*a^2*b^2*x^4 - 3*A*a*b^3 + 3*(D*a^3 - C
*a^2*b + B*a*b^2 - A*b^3)*sqrt(a*b)*x*arctan(sqrt(a*b)*x/a) - 3*(D*a^3*b -
 C*a^2*b^2)*x^2)/(a^2*b^3*x)]
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 150 vs. $2(70) = 140$.

Time = 0.55 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.79

$$\int \frac{A + Bx^2 + Cx^4 + Dx^6}{x^2\left(a + bx^2\right)}\,dx$$
$$= -\frac{A}{ax} + \frac{Dx^3}{3b} + x\left(\frac{C}{b} - \frac{Da}{b^2}\right) - \frac{\sqrt{-\frac{1}{a^3b^5}}\left(-Ab^3 + Bab^2 - Ca^2b + Da^3\right)\log\left(-a^2b^2\sqrt{-\frac{1}{a^3b^5}} + x\right)}{2} + \frac{\sqrt{-\frac{1}{a^3b^5}}\left(-Ab^3 + Bab^2 - Ca^2b + Da^3\right)\log\left(a^2b^2\sqrt{-\frac{1}{a^3b^5}} + x\right)}{2}$$

input
```
integrate((D*x**6+C*x**4+B*x**2+A)/x**2/(b*x**2+a),x)
```

output
```
-A/(a*x) + D*x**3/(3*b) + x*(C/b - D*a/b**2) - sqrt(-1/(a**3*b**5))*(-A*b*
*3 + B*a*b**2 - C*a**2*b + D*a**3)*log(-a**2*b**2*sqrt(-1/(a**3*b**5)) + x
)/2 + sqrt(-1/(a**3*b**5))*(-A*b**3 + B*a*b**2 - C*a**2*b + D*a**3)*log(a*
*2*b**2*sqrt(-1/(a**3*b**5)) + x)/2
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.94

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2(a+bx^2)}\,dx = \frac{Dbx^3-3(Da-Cb)x}{3\,b^2} - \frac{A}{ax} + \frac{(Da^3-Ca^2b+Bab^2-Ab^3)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}ab^2}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^2/(b*x^2+a),x, algorithm="maxima")
```

output
```
1/3*(D*b*x^3 - 3*(D*a - C*b)*x)/b^2 - A/(a*x) + (D*a^3 - C*a^2*b + B*a*b^2
 - A*b^3)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b^2)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.99

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2(a+bx^2)}\,dx = -\frac{A}{ax} + \frac{(Da^3-Ca^2b+Bab^2-Ab^3)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}ab^2} + \frac{Db^2x^3-3\,Dabx+3\,Cb^2x}{3\,b^3}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^2/(b*x^2+a),x, algorithm="giac")
```

output
```
-A/(a*x) + (D*a^3 - C*a^2*b + B*a*b^2 - A*b^3)*arctan(b*x/sqrt(a*b))/(sqrt
(a*b)*a*b^2) + 1/3*(D*b^2*x^3 - 3*D*a*b*x + 3*C*b^2*x)/b^3
```

## Mupad [B] (verification not implemented)

Time = 1.79 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.15

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2\left(a+bx^2\right)}\,dx = \frac{C\,x}{b}-\frac{A}{a\,x}+\frac{A\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{a^{3/2}}-\frac{B\,\mathrm{atan}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{\sqrt{a}\,\sqrt{b}}+\frac{C\,\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{b^{3/2}}+\frac{x^3\,D\,{}_2\mathrm{F}_1\left(-\frac{3}{2},1;\ -\frac{1}{2};\ -\frac{a}{b\,x^2}\right)}{3\,b}$$

input
```
int((A + B*x^2 + C*x^4 + x^6*D)/(x^2*(a + b*x^2)),x)
```

output
```
(C*x)/b - A/(a*x) + (A*b^(1/2)*atan(a^(1/2)/(b^(1/2)*x)))/a^(3/2) - (B*ata
n(a^(1/2)/(b^(1/2)*x)))/(a^(1/2)*b^(1/2)) + (C*a^(1/2)*atan(a^(1/2)/(b^(1/
2)*x)))/b^(3/2) + (x^3*D*hypergeom([-3/2, 1], -1/2, -a/(b*x^2)))/(3*b)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.96

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2\left(a+bx^2\right)}\,dx = \frac{3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)adx-3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)bcx-3abd\,x^2-3b^3+3b^2c\,x^2+b^2d\,x^4}{3b^3x}$$

input
```
int((D*x^6+C*x^4+B*x^2+A)/x^2/(b*x^2+a),x)
```

output
```
(3*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*d*x - 3*sqrt(b)*sqrt(a)
*atan((b*x)/(sqrt(b)*sqrt(a)))*b*c*x - 3*a*b*d*x**2 - 3*b**3 + 3*b**2*c*x*
*2 + b**2*d*x**4)/(3*b**3*x)
```

# 3.165 $\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4(a+bx^2)}\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 82

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4(a+bx^2)}\,dx = -\frac{A}{3ax^3}+\frac{Ab-aB}{a^2x}+\frac{Dx}{b} + \frac{(Ab^3-a(b^2B-abC+a^2D))\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}b^{3/2}}$$

output
```
-1/3*A/a/x^3+(A*b-B*a)/a^2/x+D*x/b+(A*b^3-a*(B*b^2-C*a*b+D*a^2))*arctan(b^
(1/2)*x/a^(1/2))/a^(5/2)/b^(3/2)
```

## Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.01

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4(a+bx^2)}\,dx = -\frac{A}{3ax^3}+\frac{Ab-aB}{a^2x}+\frac{Dx}{b} - \frac{(-Ab^3+ab^2B-a^2bC+a^3D)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}b^{3/2}}$$

input
```
Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(x^4*(a + b*x^2)),x]
```

output
```
-1/3*A/(a*x^3) + (A*b - a*B)/(a^2*x) + (D*x)/b - ((-(A*b^3) + a*b^2*B - a^
2*b*C + a^3*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(5/2)*b^(3/2))
```

## Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4\left(a+bx^2\right)}dx$$

$$\Big\downarrow \text{2333}$$

$$\int \left(\frac{Ab^3-a\left(a^2D-abC+b^2B\right)}{a^2b\left(a+bx^2\right)}+\frac{aB-Ab}{a^2x^2}+\frac{A}{ax^4}+\frac{D}{b}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{Ab-aB}{a^2x}+\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{a^{5/2}b^{3/2}}-\frac{A}{3ax^3}+\frac{Dx}{b}$$

input
```
Int[(A + B*x^2 + C*x^4 + D*x^6)/(x^4*(a + b*x^2)),x]
```

output
```
-1/3*A/(a*x^3) + (A*b - a*B)/(a^2*x) + (D*x)/b + ((A*b^3 - a*(b^2*B - a*b*
C + a^2*D))*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(5/2)*b^(3/2))
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

## Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.96

| method | result | size |
|---|---|---|
| default | $\frac{Dx}{b}+\frac{\left(b^{3}A-a\,b^{2}B+a^{2}bC-a^{3}D\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{a^{2}b\sqrt{ab}}-\frac{A}{3a\,x^{3}}-\frac{-Ab+Ba}{a^{2}x}$ | 79 |

input
```
int((D*x^6+C*x^4+B*x^2+A)/x^4/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
D*x/b+1/a^2/b*(A*b^3-B*a*b^2+C*a^2*b-D*a^3)/(a*b)^(1/2)*arctan(b*x/(a*b)^(
1/2))-1/3*A/a/x^3-1/a^2*(-A*b+B*a)/x
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 216, normalized size of antiderivative = 2.63

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4\,(a+bx^2)}\,dx$$
$$=\left[\frac{6\,Da^3bx^4+3\,(Da^3-Ca^2b+Bab^2-Ab^3)\sqrt{-ab}x^3\log\left(\frac{bx^2-2\,\sqrt{-ab}x-a}{bx^2+a}\right)-2\,Aa^2b^2-6\,(Ba^2b^2-Aab^3)x}{6\,a^3b^2x^3}\right.$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^4/(b*x^2+a),x, algorithm="fricas")
```

output
```
[1/6*(6*D*a^3*b*x^4 + 3*(D*a^3 - C*a^2*b + B*a*b^2 - A*b^3)*sqrt(-a*b)*x^3
*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) - 2*A*a^2*b^2 - 6*(B*a^2*b^
2 - A*a*b^3)*x^2)/(a^3*b^2*x^3), 1/3*(3*D*a^3*b*x^4 - 3*(D*a^3 - C*a^2*b +
 B*a*b^2 - A*b^3)*sqrt(a*b)*x^3*arctan(sqrt(a*b)*x/a) - A*a^2*b^2 - 3*(B*a
^2*b^2 - A*a*b^3)*x^2)/(a^3*b^2*x^3)]
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 151 vs. $2(70) = 140$.

Time = 1.04 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.84

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4(a+bx^2)}\,dx = \frac{Dx}{b} + \frac{\sqrt{-\frac{1}{a^5b^3}}(-Ab^3+Bab^2-Ca^2b+Da^3)\log\left(-a^3b\sqrt{-\frac{1}{a^5b^3}}+x\right)}{2} - \frac{\sqrt{-\frac{1}{a^5b^3}}(-Ab^3+Bab^2-Ca^2b+Da^3)\log\left(a^3b\sqrt{-\frac{1}{a^5b^3}}+x\right)}{2} + \frac{-Aa+x^2\cdot(3Ab-3Ba)}{3a^2x^3}$$

input
```
integrate((D*x**6+C*x**4+B*x**2+A)/x**4/(b*x**2+a),x)
```

output
```
D*x/b + sqrt(-1/(a**5*b**3))*(-A*b**3 + B*a*b**2 - C*a**2*b + D*a**3)*log(
-a**3*b*sqrt(-1/(a**5*b**3)) + x)/2 - sqrt(-1/(a**5*b**3))*(-A*b**3 + B*a*
b**2 - C*a**2*b + D*a**3)*log(a**3*b*sqrt(-1/(a**5*b**3)) + x)/2 + (-A*a +
 x**2*(3*A*b - 3*B*a))/(3*a**2*x**3)
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.96

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4\left(a+bx^2\right)}\,dx = \frac{Dx}{b} - \frac{\left(Da^3-Ca^2b+Bab^2-Ab^3\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a^2b} - \frac{3\,(Ba-Ab)x^2+Aa}{3\,a^2x^3}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^4/(b*x^2+a),x, algorithm="maxima")
```

output
```
D*x/b - (D*a^3 - C*a^2*b + B*a*b^2 - A*b^3)*arctan(b*x/sqrt(a*b))/(sqrt(a*
b)*a^2*b) - 1/3*(3*(B*a - A*b)*x^2 + A*a)/(a^2*x^3)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.98

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4\left(a+bx^2\right)}\,dx = \frac{Dx}{b} - \frac{\left(Da^3-Ca^2b+Bab^2-Ab^3\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a^2b} - \frac{3\,Bax^2-3\,Abx^2+Aa}{3\,a^2x^3}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^4/(b*x^2+a),x, algorithm="giac")
```

output
```
D*x/b - (D*a^3 - C*a^2*b + B*a*b^2 - A*b^3)*arctan(b*x/sqrt(a*b))/(sqrt(a*
b)*a^2*b) - 1/3*(3*B*a*x^2 - 3*A*b*x^2 + A*a)/(a^2*x^3)
```

## Mupad [B] (verification not implemented)

Time = 1.78 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.30

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4(a+bx^2)}\,dx = \frac{x\,D\,{}_2\mathrm{F}_1\left(-\frac{1}{2},1;\ \frac{1}{2};\ -\frac{a}{b\,x^2}\right)}{b} - \frac{B}{a\,x} - \frac{A}{3\,a\,x^3} - \frac{A\,b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{a^{5/2}} + \frac{B\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{a^{3/2}} - \frac{C\,\mathrm{atan}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{\sqrt{a}\,\sqrt{b}} + \frac{A\,b}{a^2\,x}$$

input
```
int((A + B*x^2 + C*x^4 + x^6*D)/(x^4*(a + b*x^2)),x)
```

output
```
(x*D*hypergeom([-1/2, 1], 1/2, -a/(b*x^2)))/b - B/(a*x) - A/(3*a*x^3) - (A
*b^(3/2)*atan(a^(1/2)/(b^(1/2)*x)))/a^(5/2) + (B*b^(1/2)*atan(a^(1/2)/(b^(
1/2)*x)))/a^(3/2) - (C*atan(a^(1/2)/(b^(1/2)*x)))/(a^(1/2)*b^(1/2)) + (A*b
)/(a^2*x)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.88

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4(a+bx^2)}\,dx = \frac{-3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)ad\,x^3+3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)bc\,x^3-a\,b^2+3abd\,x^4}{3a\,b^2x^3}$$

input
```
int((D*x^6+C*x^4+B*x^2+A)/x^4/(b*x^2+a),x)
```

output
```
( - 3*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*d*x**3 + 3*sqrt(b)*s
qrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b*c*x**3 - a*b**2 + 3*a*b*d*x**4)/(3*
a*b**2*x**3)
```

# 3.166 $\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6(a+bx^2)}\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 105

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6\left(a+bx^2\right)}\,dx=-\frac{A}{5ax^5}+\frac{Ab-aB}{3a^2x^3}-\frac{Ab^2-a(bB-aC)}{a^3x}-\frac{\left(Ab^3-a(b^2B-abC+a^2D)\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{7/2}\sqrt{b}}$$

output
```
-1/5*A/a/x^5+1/3*(A*b-B*a)/a^2/x^3-(A*b^2-a*(B*b-C*a))/a^3/x-(A*b^3-a*(B*b
^2-C*a*b+D*a^2))*arctan(b^(1/2)*x/a^(1/2))/a^(7/2)/b^(1/2)
```

## Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.98

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6\left(a+bx^2\right)}\,dx=-\frac{A}{5ax^5}+\frac{Ab-aB}{3a^2x^3}+\frac{-Ab^2+abB-a^2C}{a^3x}+\frac{\left(-Ab^3+ab^2B-a^2bC+a^3D\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{7/2}\sqrt{b}}$$

input
```
Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(x^6*(a + b*x^2)),x]
```

output
```
-1/5*A/(a*x^5) + (A*b - a*B)/(3*a^2*x^3) + (-(A*b^2) + a*b*B - a^2*C)/(a^3
*x) + ((-(A*b^3) + a*b^2*B - a^2*b*C + a^3*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])
/(a^(7/2)*Sqrt[b])
```

## Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6\left(a+bx^2\right)}dx$$

$$\Big\downarrow \text{2333}$$

$$\int \left(\frac{Ab^2-a(bB-aC)}{a^3x^2}+\frac{aB-Ab}{a^2x^4}+\frac{a\left(a^2D-abC+b^2B\right)-Ab^3}{a^3\left(a+bx^2\right)}+\frac{A}{ax^6}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{Ab^2-a(bB-aC)}{a^3x}+\frac{Ab-aB}{3a^2x^3}-\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{a^{7/2}\sqrt{b}}-\frac{A}{5ax^5}$$

input
```
Int[(A + B*x^2 + C*x^4 + D*x^6)/(x^6*(a + b*x^2)),x]
```

output
```
-1/5*A/(a*x^5) + (A*b - a*B)/(3*a^2*x^3) - (A*b^2 - a*(b*B - a*C))/(a^3*x)
 - ((A*b^3 - a*(b^2*B - a*b*C + a^2*D))*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(7
/2)*Sqrt[b])
```

**Defintions of rubi rules used**

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

## Maple [A] (verified)

Time = 1.89 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.90

| method | result | size |
|---|---|---|
| default | $\frac{(-b^3A+a\,b^2B-a^2bC+a^3D)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{a^3\sqrt{ab}}-\frac{A}{5a\,x^5}-\frac{-Ab+Ba}{3a^2x^3}-\frac{b^2A-abB+a^2C}{a^3x}$ | 94 |

input
```
int((D*x^6+C*x^4+B*x^2+A)/x^6/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/a^3*(-A*b^3+B*a*b^2-C*a^2*b+D*a^3)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))-1
/5*A/a/x^5-1/3*(-A*b+B*a)/a^2/x^3-(A*b^2-B*a*b+C*a^2)/a^3/x
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 246, normalized size of antiderivative = 2.34

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6\,(a+bx^2)}\,dx$$
$$=\left[\frac{15\,(Da^3-Ca^2b+Bab^2-Ab^3)\sqrt{-ab}x^5\log\left(\frac{bx^2+2\sqrt{-ab}x-a}{bx^2+a}\right)-6\,Aa^3b-30\,(Ca^3b-Ba^2b^2+Aab^3)x^4}{30\,a^4bx^5}\right.$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^6/(b*x^2+a),x, algorithm="fricas")
```

output
```
[1/30*(15*(D*a^3 - C*a^2*b + B*a*b^2 - A*b^3)*sqrt(-a*b)*x^5*log((b*x^2 +
2*sqrt(-a*b)*x - a)/(b*x^2 + a)) - 6*A*a^3*b - 30*(C*a^3*b - B*a^2*b^2 + A
*a*b^3)*x^4 - 10*(B*a^3*b - A*a^2*b^2)*x^2)/(a^4*b*x^5), 1/15*(15*(D*a^3 -
 C*a^2*b + B*a*b^2 - A*b^3)*sqrt(a*b)*x^5*arctan(sqrt(a*b)*x/a) - 3*A*a^3*
b - 15*(C*a^3*b - B*a^2*b^2 + A*a*b^3)*x^4 - 5*(B*a^3*b - A*a^2*b^2)*x^2)/
(a^4*b*x^5)]
```

## Sympy [A] (verification not implemented)

Time = 2.52 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.59

$$\begin{aligned}\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6\left(a+bx^2\right)}\,dx &= -\frac{\sqrt{-\frac{1}{a^7b}}(-Ab^3+Bab^2-Ca^2b+Da^3)\log\left(-a^4\sqrt{-\frac{1}{a^7b}}+x\right)}{2} \\ &\quad+\frac{\sqrt{-\frac{1}{a^7b}}(-Ab^3+Bab^2-Ca^2b+Da^3)\log\left(a^4\sqrt{-\frac{1}{a^7b}}+x\right)}{2} \\ &\quad+\frac{-3Aa^2+x^4(-15Ab^2+15Bab-15Ca^2)+x^2\cdot(5Aab-5Ba^2)}{15a^3x^5}\end{aligned}$$

input
```
integrate((D*x**6+C*x**4+B*x**2+A)/x**6/(b*x**2+a),x)
```

output
```
-sqrt(-1/(a**7*b))*(-A*b**3 + B*a*b**2 - C*a**2*b + D*a**3)*log(-a**4*sqrt
(-1/(a**7*b)) + x)/2 + sqrt(-1/(a**7*b))*(-A*b**3 + B*a*b**2 - C*a**2*b +
D*a**3)*log(a**4*sqrt(-1/(a**7*b)) + x)/2 + (-3*A*a**2 + x**4*(-15*A*b**2
+ 15*B*a*b - 15*C*a**2) + x**2*(5*A*a*b - 5*B*a**2))/(15*a**3*x**5)
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.91

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6\left(a+bx^2\right)}\,dx = \frac{(Da^3-Ca^2b+Bab^2-Ab^3)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a^3} - \frac{15\,(Ca^2-Bab+Ab^2)x^4+3\,Aa^2+5\,(Ba^2-Aab)x^2}{15\,a^3x^5}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^6/(b*x^2+a),x, algorithm="maxima")
```

output
```
(D*a^3 - C*a^2*b + B*a*b^2 - A*b^3)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3)
- 1/15*(15*(C*a^2 - B*a*b + A*b^2)*x^4 + 3*A*a^2 + 5*(B*a^2 - A*a*b)*x^2)/
(a^3*x^5)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.97

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6(a+bx^2)}\,dx = \frac{(Da^3-Ca^2b+Bab^2-Ab^3)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a^3} - \frac{15\,Ca^2x^4-15\,Babx^4+15\,Ab^2x^4+5\,Ba^2x^2-5\,Aabx^2+3\,Aa^2}{15\,a^3x^5}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^6/(b*x^2+a),x, algorithm="giac")
```

output
```
(D*a^3 - C*a^2*b + B*a*b^2 - A*b^3)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3)
- 1/15*(15*C*a^2*x^4 - 15*B*a*b*x^4 + 15*A*b^2*x^4 + 5*B*a^2*x^2 - 5*A*a*b
*x^2 + 3*A*a^2)/(a^3*x^5)
```

## Mupad [B] (verification not implemented)

Time = 1.56 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.29

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6(a+bx^2)}\,dx = \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,D}{\sqrt{a}\,\sqrt{b}} - \frac{\frac{B}{3\,a}-\frac{B\,b\,x^2}{a^2}}{x^3} - \frac{C}{a\,x} - \frac{\frac{A}{5\,a}-\frac{A\,b\,x^2}{3\,a^2}+\frac{A\,b^2\,x^4}{a^3}}{x^5} - \frac{A\,b^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{7/2}} + \frac{B\,b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}} - \frac{C\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}}$$

input
```
int((A + B*x^2 + C*x^4 + x^6*D)/(x^6*(a + b*x^2)),x)
```

output
```
(atan((b^(1/2)*x)/a^(1/2))*D)/(a^(1/2)*b^(1/2)) - (B/(3*a) - (B*b*x^2)/a^2
)/x^3 - C/(a*x) - (A/(5*a) - (A*b*x^2)/(3*a^2) + (A*b^2*x^4)/a^3)/x^5 - (A
*b^(5/2)*atan((b^(1/2)*x)/a^(1/2)))/a^(7/2) + (B*b^(3/2)*atan((b^(1/2)*x)/
a^(1/2)))/a^(5/2) - (C*b^(1/2)*atan((b^(1/2)*x)/a^(1/2)))/a^(3/2)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.69

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6(a+bx^2)}\,dx = \frac{5\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)ad\,x^5-5\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)bc\,x^5-a^2b-5abc\,x^4}{5a^2b\,x^5}$$

input
```
int((D*x^6+C*x^4+B*x^2+A)/x^6/(b*x^2+a),x)
```

output
```
(5*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*d*x**5 - 5*sqrt(b)*sqrt
(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b*c*x**5 - a**2*b - 5*a*b*c*x**4)/(5*a**
2*b*x**5)
```

# 3.167 $\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8(a+bx^2)}\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 138

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8\left(a+bx^2\right)}\,dx = -\frac{A}{7ax^7}+\frac{Ab-aB}{5a^2x^5}-\frac{Ab^2-a(bB-aC)}{3a^3x^3} + \frac{Ab^3-a(b^2B-abC+a^2D)}{a^4x} + \frac{\sqrt{b}(Ab^3-a(b^2B-abC+a^2D))\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{9/2}}$$

output
```
-1/7*A/a/x^7+1/5*(A*b-B*a)/a^2/x^5-1/3*(A*b^2-a*(B*b-C*a))/a^3/x^3+(A*b^3-
a*(B*b^2-C*a*b+D*a^2))/a^4/x+b^(1/2)*(A*b^3-a*(B*b^2-C*a*b+D*a^2))*arctan(
b^(1/2)*x/a^(1/2))/a^(9/2)
```

## Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.01

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8\left(a+bx^2\right)}\,dx = -\frac{A}{7ax^7}+\frac{Ab-aB}{5a^2x^5}+\frac{-Ab^2+abB-a^2C}{3a^3x^3} + \frac{Ab^3-ab^2B+a^2bC-a^3D}{a^4x} - \frac{\sqrt{b}(-Ab^3+ab^2B-a^2bC+a^3D)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{9/2}}$$

input
```
Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(x^8*(a + b*x^2)),x]
```

output
```
-1/7*A/(a*x^7) + (A*b - a*B)/(5*a^2*x^5) + (-(A*b^2) + a*b*B - a^2*C)/(3*a
^3*x^3) + (A*b^3 - a*b^2*B + a^2*b*C - a^3*D)/(a^4*x) - (Sqrt[b]*(-(A*b^3)
 + a*b^2*B - a^2*b*C + a^3*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(9/2)
```

## Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8\left(a+bx^2\right)}dx$$

$$\Big\downarrow \text{2333}$$

$$\int \left(\frac{Ab^2-a(bB-aC)}{a^3x^4}+\frac{aB-Ab}{a^2x^6}+\frac{b\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{a^4\left(a+bx^2\right)}+\frac{a\left(a^2D-abC+b^2B\right)-Ab^3}{a^4x^2}+\frac{A}{ax^8}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{Ab^2-a(bB-aC)}{3a^3x^3}+\frac{Ab-aB}{5a^2x^5}+\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{a^{9/2}}+\frac{Ab^3-a\left(a^2D-abC+b^2B\right)}{a^4x}-\frac{A}{7ax^7}$$

input
```
Int[(A + B*x^2 + C*x^4 + D*x^6)/(x^8*(a + b*x^2)),x]
```

output
```
-1/7*A/(a*x^7) + (A*b - a*B)/(5*a^2*x^5) - (A*b^2 - a*(b*B - a*C))/(3*a^3*
x^3) + (A*b^3 - a*(b^2*B - a*b*C + a^2*D))/(a^4*x) + (Sqrt[b]*(A*b^3 - a*(
b^2*B - a*b*C + a^2*D))*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(9/2)
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

## Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.93

| method | result | size |
|---|---|---|
| default | $\frac{b\left(b^{3}A-a\,b^{2}B+a^{2}bC-a^{3}D\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{a^{4}\sqrt{ab}}-\frac{A}{7a\,x^{7}}-\frac{-Ab+Ba}{5a^{2}x^{5}}-\frac{b^{2}A-abB+a^{2}C}{3a^{3}x^{3}}-\frac{-b^{3}A+a\,b^{2}B-a^{2}bC+a^{3}D}{a^{4}x}$ | 128 |

input
```
int((D*x^6+C*x^4+B*x^2+A)/x^8/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
b*(A*b^3-B*a*b^2+C*a^2*b-D*a^3)/a^4/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))-1/
7*A/a/x^7-1/5*(-A*b+B*a)/a^2/x^5-1/3*(A*b^2-B*a*b+C*a^2)/a^3/x^3-1/a^4*(-A
*b^3+B*a*b^2-C*a^2*b+D*a^3)/x
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 292, normalized size of antiderivative = 2.12

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8\left(a+bx^2\right)}\,dx$$

$$=\left[-\frac{105\left(Da^3-Ca^2b+Bab^2-Ab^3\right)x^7\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)+210\left(Da^3-Ca^2b+Bab^2-Ab^3\right)x^6}{210\,a^4x^7}\right.$$

$$-\frac{105\left(Da^3-Ca^2b+Bab^2-Ab^3\right)x^7\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)+105\left(Da^3-Ca^2b+Bab^2-Ab^3\right)x^6+35\left(C\right.}{105\,a^4x^7}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^8/(b*x^2+a),x, algorithm="fricas")
```

output
```
[-1/210*(105*(D*a^3 - C*a^2*b + B*a*b^2 - A*b^3)*x^7*sqrt(-b/a)*log((b*x^2
 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + 210*(D*a^3 - C*a^2*b + B*a*b^2 - A
*b^3)*x^6 + 70*(C*a^3 - B*a^2*b + A*a*b^2)*x^4 + 30*A*a^3 + 42*(B*a^3 - A*
a^2*b)*x^2)/(a^4*x^7), -1/105*(105*(D*a^3 - C*a^2*b + B*a*b^2 - A*b^3)*x^7
*sqrt(b/a)*arctan(x*sqrt(b/a)) + 105*(D*a^3 - C*a^2*b + B*a*b^2 - A*b^3)*x
^6 + 35*(C*a^3 - B*a^2*b + A*a*b^2)*x^4 + 15*A*a^3 + 21*(B*a^3 - A*a^2*b)*
x^2)/(a^4*x^7)]
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 301 vs. $2(121) = 242$.

Time = 7.71 (sec) , antiderivative size = 301, normalized size of antiderivative = 2.18

$$\int \frac{A + Bx^2 + Cx^4 + Dx^6}{x^8 (a + bx^2)}\, dx$$
$$= \frac{\sqrt{-\frac{b}{a^9}}(-Ab^3 + Bab^2 - Ca^2b + Da^3)\log\left(-\frac{a^5\sqrt{-\frac{b}{a^9}}(-Ab^3+Bab^2-Ca^2b+Da^3)}{-Ab^4+Bab^3-Ca^2b^2+Da^3b} + x\right)}{2}$$
$$- \frac{\sqrt{-\frac{b}{a^9}}(-Ab^3 + Bab^2 - Ca^2b + Da^3)\log\left(\frac{a^5\sqrt{-\frac{b}{a^9}}(-Ab^3+Bab^2-Ca^2b+Da^3)}{-Ab^4+Bab^3-Ca^2b^2+Da^3b} + x\right)}{2}$$
$$+ \frac{-15Aa^3 + x^6 \cdot (105Ab^3 - 105Bab^2 + 105Ca^2b - 105Da^3) + x^4(-35Aab^2 + 35Ba^2b - 35Ca^3) + x^2 \cdot}{105a^4x^7}$$

input
```
integrate((D*x**6+C*x**4+B*x**2+A)/x**8/(b*x**2+a),x)
```

output
```
sqrt(-b/a**9)*(-A*b**3 + B*a*b**2 - C*a**2*b + D*a**3)*log(-a**5*sqrt(-b/a
**9)*(-A*b**3 + B*a*b**2 - C*a**2*b + D*a**3)/(-A*b**4 + B*a*b**3 - C*a**2
*b**2 + D*a**3*b) + x)/2 - sqrt(-b/a**9)*(-A*b**3 + B*a*b**2 - C*a**2*b +
D*a**3)*log(a**5*sqrt(-b/a**9)*(-A*b**3 + B*a*b**2 - C*a**2*b + D*a**3)/(-
A*b**4 + B*a*b**3 - C*a**2*b**2 + D*a**3*b) + x)/2 + (-15*A*a**3 + x**6*(1
05*A*b**3 - 105*B*a*b**2 + 105*C*a**2*b - 105*D*a**3) + x**4*(-35*A*a*b**2
 + 35*B*a**2*b - 35*C*a**3) + x**2*(21*A*a**2*b - 21*B*a**3))/(105*a**4*x*
*7)
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.98

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8\,(a+bx^2)}\,dx = -\frac{(Da^3b-Ca^2b^2+Bab^3-Ab^4)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a^4} - \frac{105\,(Da^3-Ca^2b+Bab^2-Ab^3)x^6+35\,(Ca^3-Ba^2b+Aab^2)x^4+15\,Aa^3+21\,(Ba^3-Aa^2b)x^2}{105\,a^4x^7}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^8/(b*x^2+a),x, algorithm="maxima")
```

output
```
-(D*a^3*b - C*a^2*b^2 + B*a*b^3 - A*b^4)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*
a^4) - 1/105*(105*(D*a^3 - C*a^2*b + B*a*b^2 - A*b^3)*x^6 + 35*(C*a^3 - B*
a^2*b + A*a*b^2)*x^4 + 15*A*a^3 + 21*(B*a^3 - A*a^2*b)*x^2)/(a^4*x^7)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.08

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8\,(a+bx^2)}\,dx = -\frac{(Da^3b-Ca^2b^2+Bab^3-Ab^4)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a^4} - \frac{105\,Da^3x^6-105\,Ca^2bx^6+105\,Bab^2x^6-105\,Ab^3x^6+35\,Ca^3x^4-35\,Ba^2bx^4+35\,Aab^2x^4+21\,Ba^3x}{105\,a^4x^7}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^8/(b*x^2+a),x, algorithm="giac")
```

output
```
-(D*a^3*b - C*a^2*b^2 + B*a*b^3 - A*b^4)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*
a^4) - 1/105*(105*D*a^3*x^6 - 105*C*a^2*b*x^6 + 105*B*a*b^2*x^6 - 105*A*b^
3*x^6 + 35*C*a^3*x^4 - 35*B*a^2*b*x^4 + 35*A*a*b^2*x^4 + 21*B*a^3*x^2 - 21
*A*a^2*b*x^2 + 15*A*a^3)/(a^4*x^7)
```

## Mupad [B] (verification not implemented)

Time = 1.51 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.28

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8(a+bx^2)}\,dx = \frac{A\,b^{7/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{9/2}} - \frac{\frac{B}{5\,a}-\frac{B\,b\,x^2}{3\,a^2}+\frac{B\,b^2\,x^4}{a^3}}{x^5} - \frac{\frac{C}{3\,a}-\frac{C\,b\,x^2}{a^2}}{x^3} - \frac{D\,{}_2\mathrm{F}_1\left(1,\frac{3}{2};\ \frac{5}{2};\ -\frac{a}{b\,x^2}\right)}{3\,b\,x^3} - \frac{\frac{A}{7\,a}-\frac{A\,b\,x^2}{5\,a^2}+\frac{A\,b^2\,x^4}{3\,a^3}-\frac{A\,b^3\,x^6}{a^4}}{x^7} - \frac{B\,b^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{7/2}} + \frac{C\,b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}}$$

input
```
int((A + B*x^2 + C*x^4 + x^6*D)/(x^8*(a + b*x^2)),x)
```

output
```
(A*b^(7/2)*atan((b^(1/2)*x)/a^(1/2)))/a^(9/2) - (B/(5*a) - (B*b*x^2)/(3*a^
2) + (B*b^2*x^4)/a^3)/x^5 - (C/(3*a) - (C*b*x^2)/a^2)/x^3 - (D*hypergeom([
1, 3/2], 5/2, -a/(b*x^2)))/(3*b*x^3) - (A/(7*a) - (A*b*x^2)/(5*a^2) + (A*b
^2*x^4)/(3*a^3) - (A*b^3*x^6)/a^4)/x^7 - (B*b^(5/2)*atan((b^(1/2)*x)/a^(1/
2)))/a^(7/2) + (C*b^(3/2)*atan((b^(1/2)*x)/a^(1/2)))/a^(5/2)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.62

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8(a+bx^2)}\,dx = \frac{-21\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)ad\,x^7+21\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)bc\,x^7-3a^3-7a^2c\,x^4-21a^2d\,x^6+21abc\,x^6}{21a^3x^7}$$

input
```
int((D*x^6+C*x^4+B*x^2+A)/x^8/(b*x^2+a),x)
```

output
```
( - 21*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*d*x**7 + 21*sqrt(b)
*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b*c*x**7 - 3*a**3 - 7*a**2*c*x**4 -
 21*a**2*d*x**6 + 21*a*b*c*x**6)/(21*a**3*x**7)
```

# 3.168 $\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{10}(a+bx^2)}\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 176

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{10}(a+bx^2)}\,dx = -\frac{A}{9ax^9}+\frac{Ab-aB}{7a^2x^7}-\frac{Ab^2-a(bB-aC)}{5a^3x^5} + \frac{Ab^3-a(b^2B-abC+a^2D)}{3a^4x^3} - \frac{b(Ab^3-a(b^2B-abC+a^2D))}{a^5x} - \frac{b^{3/2}(Ab^3-a(b^2B-abC+a^2D))\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{11/2}}$$

output
```
-1/9*A/a/x^9+1/7*(A*b-B*a)/a^2/x^7-1/5*(A*b^2-a*(B*b-C*a))/a^3/x^5+1/3*(A*
b^3-a*(B*b^2-C*a*b+D*a^2))/a^4/x^3-b*(A*b^3-a*(B*b^2-C*a*b+D*a^2))/a^5/x-b
^(3/2)*(A*b^3-a*(B*b^2-C*a*b+D*a^2))*arctan(b^(1/2)*x/a^(1/2))/a^(11/2)
```

## Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.99

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{10}(a+bx^2)}\,dx = -\frac{A}{9ax^9}+\frac{Ab-aB}{7a^2x^7}+\frac{-Ab^2+abB-a^2C}{5a^3x^5} + \frac{Ab^3-ab^2B+a^2bC-a^3D}{3a^4x^3} + \frac{b(-Ab^3+ab^2B-a^2bC+a^3D)}{a^5x} + \frac{b^{3/2}(-Ab^3+ab^2B-a^2bC+a^3D)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{11/2}}$$

input
```
Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(x^10*(a + b*x^2)),x]
```

output
```
-1/9*A/(a*x^9) + (A*b - a*B)/(7*a^2*x^7) + (-(A*b^2) + a*b*B - a^2*C)/(5*a
^3*x^5) + (A*b^3 - a*b^2*B + a^2*b*C - a^3*D)/(3*a^4*x^3) + (b*(-(A*b^3) +
 a*b^2*B - a^2*b*C + a^3*D))/(a^5*x) + (b^(3/2)*(-(A*b^3) + a*b^2*B - a^2*
b*C + a^3*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(11/2)
```

## Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{10}(a+bx^2)}dx$$

$$\Big\downarrow \text{2333}$$

$$\int \left(\frac{Ab^2-a(bB-aC)}{a^3x^6}+\frac{aB-Ab}{a^2x^8}+\frac{b^2\left(a\left(a^2D-abC+b^2B\right)-Ab^3\right)}{a^5\left(a+bx^2\right)}+\frac{b\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{a^5x^2}+\frac{a(\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{Ab^2-a(bB-aC)}{5a^3x^5}+\frac{Ab-aB}{7a^2x^7}-\frac{b^{3/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{a^{11/2}}-\frac{b\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{a^5x}+\frac{Ab^3-a\left(a^2D-abC+b^2B\right)}{3a^4x^3}-\frac{A}{9ax^9}$$

input
```
Int[(A + B*x^2 + C*x^4 + D*x^6)/(x^10*(a + b*x^2)),x]
```

output
```
-1/9*A/(a*x^9) + (A*b - a*B)/(7*a^2*x^7) - (A*b^2 - a*(b*B - a*C))/(5*a^3*
x^5) + (A*b^3 - a*(b^2*B - a*b*C + a^2*D))/(3*a^4*x^3) - (b*(A*b^3 - a*(b^
2*B - a*b*C + a^2*D)))/(a^5*x) - (b^(3/2)*(A*b^3 - a*(b^2*B - a*b*C + a^2*
D))*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(11/2)
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

## Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.94

| method | result |
|---|---|
| default | $-\frac{b^2\left(b^3A-a\,b^2B+a^2bC-a^3D\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{a^5\sqrt{ab}}-\frac{A}{9a\,x^9}-\frac{-Ab+Ba}{7a^2x^7}-\frac{b^2A-abB+a^2C}{5a^3x^5}-\frac{-b^3A+a\,b^2B-a^2bC+a^3D}{3a^4x^3}-\frac{(b^3}{}$ |

input
```
int((D*x^6+C*x^4+B*x^2+A)/x^10/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-b^2*(A*b^3-B*a*b^2+C*a^2*b-D*a^3)/a^5/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
-1/9*A/a/x^9-1/7*(-A*b+B*a)/a^2/x^7-1/5*(A*b^2-B*a*b+C*a^2)/a^3/x^5-1/3*(-
A*b^3+B*a*b^2-C*a^2*b+D*a^3)/a^4/x^3-(A*b^3-B*a*b^2+C*a^2*b-D*a^3)/a^5*b/x
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 374, normalized size of antiderivative = 2.12

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{10}\left(a+bx^2\right)}\,dx$$
$$=\left[-\frac{315\left(Da^3b-Ca^2b^2+Bab^3-Ab^4\right)x^9\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)-630\left(Da^3b-Ca^2b^2+Bab^3-Ab^4\right.}{6}\right.$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^10/(b*x^2+a),x, algorithm="fricas")
```

output
```
[-1/630*(315*(D*a^3*b - C*a^2*b^2 + B*a*b^3 - A*b^4)*x^9*sqrt(-b/a)*log((b
*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) - 630*(D*a^3*b - C*a^2*b^2 + B*a
*b^3 - A*b^4)*x^8 + 210*(D*a^4 - C*a^3*b + B*a^2*b^2 - A*a*b^3)*x^6 + 70*A
*a^4 + 126*(C*a^4 - B*a^3*b + A*a^2*b^2)*x^4 + 90*(B*a^4 - A*a^3*b)*x^2)/(
a^5*x^9), 1/315*(315*(D*a^3*b - C*a^2*b^2 + B*a*b^3 - A*b^4)*x^9*sqrt(b/a)
*arctan(x*sqrt(b/a)) + 315*(D*a^3*b - C*a^2*b^2 + B*a*b^3 - A*b^4)*x^8 - 1
05*(D*a^4 - C*a^3*b + B*a^2*b^2 - A*a*b^3)*x^6 - 35*A*a^4 - 63*(C*a^4 - B*
a^3*b + A*a^2*b^2)*x^4 - 45*(B*a^4 - A*a^3*b)*x^2)/(a^5*x^9)]
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 354 vs. $2(155) = 310$.

Time = 12.17 (sec) , antiderivative size = 354, normalized size of antiderivative = 2.01

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{10}\left(a+bx^2\right)}\,dx$$
$$=-\frac{\sqrt{-\frac{b^3}{a^{11}}}\left(-Ab^3+Bab^2-Ca^2b+Da^3\right)\log\left(-\frac{a^6\sqrt{-\frac{b^3}{a^{11}}}\left(-Ab^3+Bab^2-Ca^2b+Da^3\right)}{-Ab^5+Bab^4-Ca^2b^3+Da^3b^2}+x\right)}{2}$$
$$+\frac{\sqrt{-\frac{b^3}{a^{11}}}\left(-Ab^3+Bab^2-Ca^2b+Da^3\right)\log\left(\frac{a^6\sqrt{-\frac{b^3}{a^{11}}}\left(-Ab^3+Bab^2-Ca^2b+Da^3\right)}{-Ab^5+Bab^4-Ca^2b^3+Da^3b^2}+x\right)}{2}$$
$$+\frac{-35Aa^4+x^8\left(-315Ab^4+315Bab^3-315Ca^2b^2+315Da^3b\right)+x^6\cdot\left(105Aab^3-105Ba^2b^2+105Ca^3b\right.}{315a^5x^9}$$

input
```
integrate((D*x**6+C*x**4+B*x**2+A)/x**10/(b*x**2+a),x)
```

output
```
-sqrt(-b**3/a**11)*(-A*b**3 + B*a*b**2 - C*a**2*b + D*a**3)*log(-a**6*sqrt
(-b**3/a**11)*(-A*b**3 + B*a*b**2 - C*a**2*b + D*a**3)/(-A*b**5 + B*a*b**4
 - C*a**2*b**3 + D*a**3*b**2) + x)/2 + sqrt(-b**3/a**11)*(-A*b**3 + B*a*b*
*2 - C*a**2*b + D*a**3)*log(a**6*sqrt(-b**3/a**11)*(-A*b**3 + B*a*b**2 - C
*a**2*b + D*a**3)/(-A*b**5 + B*a*b**4 - C*a**2*b**3 + D*a**3*b**2) + x)/2
+ (-35*A*a**4 + x**8*(-315*A*b**4 + 315*B*a*b**3 - 315*C*a**2*b**2 + 315*D
*a**3*b) + x**6*(105*A*a*b**3 - 105*B*a**2*b**2 + 105*C*a**3*b - 105*D*a**
4) + x**4*(-63*A*a**2*b**2 + 63*B*a**3*b - 63*C*a**4) + x**2*(45*A*a**3*b
- 45*B*a**4))/(315*a**5*x**9)
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.99

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{10}\left(a+bx^2\right)}\,dx=\frac{\left(Da^3b^2-Ca^2b^3+Bab^4-Ab^5\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a^5}+\frac{315\,(Da^3b-Ca^2b^2+Bab^3-Ab^4)x^8-105\,(Da^4-Ca^3b+Ba^2b^2-Aab^3)x^6-35\,Aa^4-63\,(Ca^4-B}{315\,a^5x^9}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^10/(b*x^2+a),x, algorithm="maxima")
```

output
```
(D*a^3*b^2 - C*a^2*b^3 + B*a*b^4 - A*b^5)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)
*a^5) + 1/315*(315*(D*a^3*b - C*a^2*b^2 + B*a*b^3 - A*b^4)*x^8 - 105*(D*a^
4 - C*a^3*b + B*a^2*b^2 - A*a*b^3)*x^6 - 35*A*a^4 - 63*(C*a^4 - B*a^3*b +
A*a^2*b^2)*x^4 - 45*(B*a^4 - A*a^3*b)*x^2)/(a^5*x^9)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.11

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{10}\left(a+bx^2\right)}\,dx = \frac{\left(Da^3b^2-Ca^2b^3+Bab^4-Ab^5\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a^5} + \frac{315\,Da^3bx^8-315\,Ca^2b^2x^8+315\,Bab^3x^8-315\,Ab^4x^8-105\,Da^4x^6+105\,Ca^3bx^6-105\,Ba^2b^2x^6+10}{315\,a^5x^9}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^10/(b*x^2+a),x, algorithm="giac")
```

output
```
(D*a^3*b^2 - C*a^2*b^3 + B*a*b^4 - A*b^5)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)
*a^5) + 1/315*(315*D*a^3*b*x^8 - 315*C*a^2*b^2*x^8 + 315*B*a*b^3*x^8 - 315
*A*b^4*x^8 - 105*D*a^4*x^6 + 105*C*a^3*b*x^6 - 105*B*a^2*b^2*x^6 + 105*A*a
*b^3*x^6 - 63*C*a^4*x^4 + 63*B*a^3*b*x^4 - 63*A*a^2*b^2*x^4 - 45*B*a^4*x^2
 + 45*A*a^3*b*x^2 - 35*A*a^4)/(a^5*x^9)
```

## Mupad [B] (verification not implemented)

Time = 1.57 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.21

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{10}\left(a+bx^2\right)}\,dx = \frac{B\,b^{7/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{9/2}} - \frac{\frac{B}{7\,a}-\frac{B\,b\,x^2}{5\,a^2}+\frac{B\,b^2\,x^4}{3\,a^3}-\frac{B\,b^3\,x^6}{a^4}}{x^7} - \frac{\frac{C}{5\,a}-\frac{C\,b\,x^2}{3\,a^2}+\frac{C\,b^2\,x^4}{a^3}}{x^5} - \frac{D\,{}_2\mathrm{F}_1\left(-\frac{3}{2},1;\ -\frac{1}{2};\ -\frac{b\,x^2}{a}\right)}{3\,a\,x^3} - \frac{A\,b^{9/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{11/2}} - \frac{\frac{A}{9\,a}-\frac{A\,b\,x^2}{7\,a^2}+\frac{A\,b^2\,x^4}{5\,a^3}-\frac{A\,b^3\,x^6}{3\,a^4}+\frac{A\,b^4\,x^8}{a^5}}{x^9} - \frac{C\,b^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{7/2}}$$

input
```
int((A + B*x^2 + C*x^4 + x^6*D)/(x^10*(a + b*x^2)),x)
```

output
```
(B*b^(7/2)*atan((b^(1/2)*x)/a^(1/2)))/a^(9/2) - (B/(7*a) - (B*b*x^2)/(5*a^
2) + (B*b^2*x^4)/(3*a^3) - (B*b^3*x^6)/a^4)/x^7 - (C/(5*a) - (C*b*x^2)/(3*
a^2) + (C*b^2*x^4)/a^3)/x^5 - (D*hypergeom([-3/2, 1], -1/2, -(b*x^2)/a))/(
3*a*x^3) - (A*b^(9/2)*atan((b^(1/2)*x)/a^(1/2)))/a^(11/2) - (A/(9*a) - (A*
b*x^2)/(7*a^2) + (A*b^2*x^4)/(5*a^3) - (A*b^3*x^6)/(3*a^4) + (A*b^4*x^8)/a
^5)/x^9 - (C*b^(5/2)*atan((b^(1/2)*x)/a^(1/2)))/a^(7/2)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.63

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{10}(a+bx^2)}\,dx = \frac{45\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)abd\,x^9-45\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)b^2c\,x^9-5a^4-9a^3c\,x^4-15a^3d\,x^6+15a^2bc\,x^6+4}{45a^4x^9}$$

input
```
int((D*x^6+C*x^4+B*x^2+A)/x^10/(b*x^2+a),x)
```

output
```
(45*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*b*d*x**9 - 45*sqrt(b)*
sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**2*c*x**9 - 5*a**4 - 9*a**3*c*x**4
 - 15*a**3*d*x**6 + 15*a**2*b*c*x**6 + 45*a**2*b*d*x**8 - 45*a*b**2*c*x**8
)/(45*a**4*x**9)
```

# 3.169 $\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{12}(a+bx^2)}\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 212

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{12}(a+bx^2)}\,dx = -\frac{A}{11ax^{11}}+\frac{Ab-aB}{9a^2x^9}-\frac{Ab^2-a(bB-aC)}{7a^3x^7} + \frac{Ab^3-a(b^2B-abC+a^2D)}{5a^4x^5} - \frac{b(Ab^3-a(b^2B-abC+a^2D))}{3a^5x^3} + \frac{b^2(Ab^3-a(b^2B-abC+a^2D))}{a^6x} + \frac{b^{5/2}(Ab^3-a(b^2B-abC+a^2D))\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{13/2}}$$

output
```
-1/11*A/a/x^11+1/9*(A*b-B*a)/a^2/x^9-1/7*(A*b^2-a*(B*b-C*a))/a^3/x^7+1/5*(
A*b^3-a*(B*b^2-C*a*b+D*a^2))/a^4/x^5-1/3*b*(A*b^3-a*(B*b^2-C*a*b+D*a^2))/a
^5/x^3+b^2*(A*b^3-a*(B*b^2-C*a*b+D*a^2))/a^6/x+b^(5/2)*(A*b^3-a*(B*b^2-C*a
*b+D*a^2))*arctan(b^(1/2)*x/a^(1/2))/a^(13/2)
```

## Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{12}(a+bx^2)}\,dx = -\frac{A}{11ax^{11}}+\frac{Ab-aB}{9a^2x^9}-\frac{Ab^2-abB+a^2C}{7a^3x^7} + \frac{Ab^3-ab^2B+a^2bC-a^3D}{5a^4x^5} + \frac{b(-Ab^3+a(b^2B-abC+a^2D))}{3a^5x^3} + \frac{Ab^5-ab^2(b^2B-abC+a^2D)}{a^6x} + \frac{b^{5/2}(Ab^3-a(b^2B-abC+a^2D))\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{13/2}}$$

input
```
Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(x^12*(a + b*x^2)),x]
```

output
```
-1/11*A/(a*x^11) + (A*b - a*B)/(9*a^2*x^9) - (A*b^2 - a*b*B + a^2*C)/(7*a^
3*x^7) + (A*b^3 - a*b^2*B + a^2*b*C - a^3*D)/(5*a^4*x^5) + (b*(-(A*b^3) +
a*(b^2*B - a*b*C + a^2*D)))/(3*a^5*x^3) + (A*b^5 - a*b^2*(b^2*B - a*b*C +
a^2*D))/(a^6*x) + (b^(5/2)*(A*b^3 - a*(b^2*B - a*b*C + a^2*D))*ArcTan[(Sqr
t[b]*x)/Sqrt[a]])/a^(13/2)
```

## Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{12}(a+bx^2)}\,dx$$

$$\Big\downarrow \; 2333$$

$$\int \left(\frac{Ab^2-a(bB-aC)}{a^3x^8}+\frac{aB-Ab}{a^2x^{10}}+\frac{b^3\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{a^6\left(a+bx^2\right)}+\frac{b^2\left(a\left(a^2D-abC+b^2B\right)-Ab^3\right)}{a^6x^2}+\frac{b(\right.$$

$$\Big\downarrow \textcolor{blue}{2009}$$

$$-\frac{Ab^2-a(bB-aC)}{7a^3x^7}+\frac{Ab-aB}{9a^2x^9}+\frac{b^{5/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{a^{13/2}}+\frac{b^2\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{a^6x}-\frac{b\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{3a^5x^3}+\frac{Ab^3-a\left(a^2D-abC+b^2B\right)}{5a^4x^5}-\frac{A}{11ax^{11}}$$

input
```
Int[(A + B*x^2 + C*x^4 + D*x^6)/(x^12*(a + b*x^2)),x]
```

output
```
-1/11*A/(a*x^11) + (A*b - a*B)/(9*a^2*x^9) - (A*b^2 - a*(b*B - a*C))/(7*a^
3*x^7) + (A*b^3 - a*(b^2*B - a*b*C + a^2*D))/(5*a^4*x^5) - (b*(A*b^3 - a*(
b^2*B - a*b*C + a^2*D)))/(3*a^5*x^3) + (b^2*(A*b^3 - a*(b^2*B - a*b*C + a^
2*D)))/(a^6*x) + (b^(5/2)*(A*b^3 - a*(b^2*B - a*b*C + a^2*D))*ArcTan[(Sqrt
[b]*x)/Sqrt[a]])/a^(13/2)
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

## Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.94

| method | result |
|---|---|
| default | $\frac{b^3(b^3A-a\,b^2B+a^2bC-a^3D)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{a^6\sqrt{ab}}-\frac{A}{11a\,x^{11}}-\frac{-Ab+Ba}{9a^2x^9}-\frac{b^2A-abB+a^2C}{7a^3x^7}-\frac{-b^3A+a\,b^2B-a^2bC+a^3D}{5a^4x^5}-\frac{(b^3}{}$ |

input
```
int((D*x^6+C*x^4+B*x^2+A)/x^12/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
b^3*(A*b^3-B*a*b^2+C*a^2*b-D*a^3)/a^6/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))-
1/11*A/a/x^11-1/9*(-A*b+B*a)/a^2/x^9-1/7*(A*b^2-B*a*b+C*a^2)/a^3/x^7-1/5*(
-A*b^3+B*a*b^2-C*a^2*b+D*a^3)/a^4/x^5-1/3*(A*b^3-B*a*b^2+C*a^2*b-D*a^3)/a^
5*b/x^3+(A*b^3-B*a*b^2+C*a^2*b-D*a^3)/a^6*b^2/x
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 458, normalized size of antiderivative = 2.16

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{12}(a+bx^2)}\,dx$$

$$=\left[-\frac{3465\,(Da^3b^2-Ca^2b^3+Bab^4-Ab^5)x^{11}\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)+6930\,(Da^3b^2-Ca^2b^3+Bab^4}{}\right.$$

$$-\frac{3465\,(Da^3b^2-Ca^2b^3+Bab^4-Ab^5)x^{11}\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)+3465\,(Da^3b^2-Ca^2b^3+Bab^4-Ab^5)x^1}{}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^12/(b*x^2+a),x, algorithm="fricas")
```

output
```
[-1/6930*(3465*(D*a^3*b^2 - C*a^2*b^3 + B*a*b^4 - A*b^5)*x^11*sqrt(-b/a)*l
og((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + 6930*(D*a^3*b^2 - C*a^2*b
^3 + B*a*b^4 - A*b^5)*x^10 - 2310*(D*a^4*b - C*a^3*b^2 + B*a^2*b^3 - A*a*b
^4)*x^8 + 1386*(D*a^5 - C*a^4*b + B*a^3*b^2 - A*a^2*b^3)*x^6 + 630*A*a^5 +
 990*(C*a^5 - B*a^4*b + A*a^3*b^2)*x^4 + 770*(B*a^5 - A*a^4*b)*x^2)/(a^6*x
^11), -1/3465*(3465*(D*a^3*b^2 - C*a^2*b^3 + B*a*b^4 - A*b^5)*x^11*sqrt(b/
a)*arctan(x*sqrt(b/a)) + 3465*(D*a^3*b^2 - C*a^2*b^3 + B*a*b^4 - A*b^5)*x^
10 - 1155*(D*a^4*b - C*a^3*b^2 + B*a^2*b^3 - A*a*b^4)*x^8 + 693*(D*a^5 - C
*a^4*b + B*a^3*b^2 - A*a^2*b^3)*x^6 + 315*A*a^5 + 495*(C*a^5 - B*a^4*b + A
*a^3*b^2)*x^4 + 385*(B*a^5 - A*a^4*b)*x^2)/(a^6*x^11)]
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 398 vs. $2(189) = 378$.

Time = 26.17 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.88

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{12}(a+bx^2)}\,dx$$
$$= \frac{\sqrt{-\frac{b^5}{a^{13}}}(-Ab^3+Bab^2-Ca^2b+Da^3)\log\left(-\frac{a^7\sqrt{-\frac{b^5}{a^{13}}}(-Ab^3+Bab^2-Ca^2b+Da^3)}{-Ab^6+Bab^5-Ca^2b^4+Da^3b^3}+x\right)}{2}$$
$$-\frac{\sqrt{-\frac{b^5}{a^{13}}}(-Ab^3+Bab^2-Ca^2b+Da^3)\log\left(\frac{a^7\sqrt{-\frac{b^5}{a^{13}}}(-Ab^3+Bab^2-Ca^2b+Da^3)}{-Ab^6+Bab^5-Ca^2b^4+Da^3b^3}+x\right)}{2}$$
$$+\frac{-315Aa^5+x^{10}\cdot(3465Ab^5-3465Bab^4+3465Ca^2b^3-3465Da^3b^2)+x^8(-1155Aab^4+1155Ba^2b^3-}{}$$

input
```
integrate((D*x**6+C*x**4+B*x**2+A)/x**12/(b*x**2+a),x)
```

output
```
sqrt(-b**5/a**13)*(-A*b**3 + B*a*b**2 - C*a**2*b + D*a**3)*log(-a**7*sqrt(
-b**5/a**13)*(-A*b**3 + B*a*b**2 - C*a**2*b + D*a**3)/(-A*b**6 + B*a*b**5
- C*a**2*b**4 + D*a**3*b**3) + x)/2 - sqrt(-b**5/a**13)*(-A*b**3 + B*a*b**
2 - C*a**2*b + D*a**3)*log(a**7*sqrt(-b**5/a**13)*(-A*b**3 + B*a*b**2 - C*
a**2*b + D*a**3)/(-A*b**6 + B*a*b**5 - C*a**2*b**4 + D*a**3*b**3) + x)/2 +
 (-315*A*a**5 + x**10*(3465*A*b**5 - 3465*B*a*b**4 + 3465*C*a**2*b**3 - 34
65*D*a**3*b**2) + x**8*(-1155*A*a*b**4 + 1155*B*a**2*b**3 - 1155*C*a**3*b*
*2 + 1155*D*a**4*b) + x**6*(693*A*a**2*b**3 - 693*B*a**3*b**2 + 693*C*a**4
*b - 693*D*a**5) + x**4*(-495*A*a**3*b**2 + 495*B*a**4*b - 495*C*a**5) + x
**2*(385*A*a**4*b - 385*B*a**5))/(3465*a**6*x**11)
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.01

$$\int \frac{A + Bx^2 + Cx^4 + Dx^6}{x^{12}(a + bx^2)}\,dx = -\frac{(Da^3b^3 - Ca^2b^4 + Bab^5 - Ab^6)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a^6} - \frac{3465\,(Da^3b^2 - Ca^2b^3 + Bab^4 - Ab^5)x^{10} - 1155\,(Da^4b - Ca^3b^2 + Ba^2b^3 - Aab^4)x^8 + 693\,(Da^5 - Ca}{3465\,a^6x^{11}}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^12/(b*x^2+a),x, algorithm="maxima")
```

output
```
-(D*a^3*b^3 - C*a^2*b^4 + B*a*b^5 - A*b^6)*arctan(b*x/sqrt(a*b))/(sqrt(a*b
)*a^6) - 1/3465*(3465*(D*a^3*b^2 - C*a^2*b^3 + B*a*b^4 - A*b^5)*x^10 - 115
5*(D*a^4*b - C*a^3*b^2 + B*a^2*b^3 - A*a*b^4)*x^8 + 693*(D*a^5 - C*a^4*b +
 B*a^3*b^2 - A*a^2*b^3)*x^6 + 315*A*a^5 + 495*(C*a^5 - B*a^4*b + A*a^3*b^2
)*x^4 + 385*(B*a^5 - A*a^4*b)*x^2)/(a^6*x^11)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.16

$$\int \frac{A + Bx^2 + Cx^4 + Dx^6}{x^{12}(a + bx^2)}\,dx = -\frac{(Da^3b^3 - Ca^2b^4 + Bab^5 - Ab^6)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a^6} - \frac{3465\,Da^3b^2x^{10} - 3465\,Ca^2b^3x^{10} + 3465\,Bab^4x^{10} - 3465\,Ab^5x^{10} - 1155\,Da^4bx^8 + 1155\,Ca^3b^2x^8 - 115}{}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^12/(b*x^2+a),x, algorithm="giac")
```

output
```
-(D*a^3*b^3 - C*a^2*b^4 + B*a*b^5 - A*b^6)*arctan(b*x/sqrt(a*b))/(sqrt(a*b
)*a^6) - 1/3465*(3465*D*a^3*b^2*x^10 - 3465*C*a^2*b^3*x^10 + 3465*B*a*b^4*
x^10 - 3465*A*b^5*x^10 - 1155*D*a^4*b*x^8 + 1155*C*a^3*b^2*x^8 - 1155*B*a^
2*b^3*x^8 + 1155*A*a*b^4*x^8 + 693*D*a^5*x^6 - 693*C*a^4*b*x^6 + 693*B*a^3
*b^2*x^6 - 693*A*a^2*b^3*x^6 + 495*C*a^5*x^4 - 495*B*a^4*b*x^4 + 495*A*a^3
*b^2*x^4 + 385*B*a^5*x^2 - 385*A*a^4*b*x^2 + 315*A*a^5)/(a^6*x^11)
```

## Mupad [B] (verification not implemented)

Time = 1.57 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.17

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{12}(a+bx^2)}\,dx = \frac{A\,b^{11/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{13/2}} - \frac{\frac{B}{9\,a}-\frac{B\,b\,x^2}{7\,a^2}+\frac{B\,b^2\,x^4}{5\,a^3}-\frac{B\,b^3\,x^6}{3\,a^4}+\frac{B\,b^4\,x^8}{a^5}}{x^9} - \frac{\frac{C}{7\,a}-\frac{C\,b\,x^2}{5\,a^2}+\frac{C\,b^2\,x^4}{3\,a^3}-\frac{C\,b^3\,x^6}{a^4}}{x^7} - \frac{D\,{}_2\mathrm{F}_1\left(-\frac{5}{2},1;\ -\frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{5\,a\,x^5} - \frac{\frac{A}{11\,a}-\frac{A\,b\,x^2}{9\,a^2}+\frac{A\,b^2\,x^4}{7\,a^3}-\frac{A\,b^3\,x^6}{5\,a^4}+\frac{A\,b^4\,x^8}{3\,a^5}-\frac{A\,b^5\,x^{10}}{a^6}}{x^{11}} - \frac{B\,b^{9/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{11/2}} + \frac{C\,b^{7/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{9/2}}$$

input
```
int((A + B*x^2 + C*x^4 + x^6*D)/(x^12*(a + b*x^2)),x)
```

output
```
(A*b^(11/2)*atan((b^(1/2)*x)/a^(1/2)))/a^(13/2) - (B/(9*a) - (B*b*x^2)/(7*
a^2) + (B*b^2*x^4)/(5*a^3) - (B*b^3*x^6)/(3*a^4) + (B*b^4*x^8)/a^5)/x^9 -
(C/(7*a) - (C*b*x^2)/(5*a^2) + (C*b^2*x^4)/(3*a^3) - (C*b^3*x^6)/a^4)/x^7
- (D*hypergeom([-5/2, 1], -3/2, -(b*x^2)/a))/(5*a*x^5) - (A/(11*a) - (A*b*
x^2)/(9*a^2) + (A*b^2*x^4)/(7*a^3) - (A*b^3*x^6)/(5*a^4) + (A*b^4*x^8)/(3*
a^5) - (A*b^5*x^10)/a^6)/x^11 - (B*b^(9/2)*atan((b^(1/2)*x)/a^(1/2)))/a^(1
1/2) + (C*b^(7/2)*atan((b^(1/2)*x)/a^(1/2)))/a^(9/2)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.65

$$\int \frac{A + Bx^2 + Cx^4 + Dx^6}{x^{12}(a + bx^2)}\,dx$$
$$= \frac{-1155\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a\,b^2d\,x^{11} + 1155\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)b^3c\,x^{11} - 105a^5 - 165a^4c\,x^4 - 231a^4d\,x^6}{1155a^5x^{11}}$$

input
```
int((D*x^6+C*x^4+B*x^2+A)/x^12/(b*x^2+a),x)
```

output
```
( - 1155*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*b**2*d*x**11 + 11
55*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**3*c*x**11 - 105*a**5 -
 165*a**4*c*x**4 - 231*a**4*d*x**6 + 231*a**3*b*c*x**6 + 385*a**3*b*d*x**8
 - 385*a**2*b**2*c*x**8 - 1155*a**2*b**2*d*x**10 + 1155*a*b**3*c*x**10)/(1
155*a**5*x**11)
```

# 3.170 $\int \frac{x^6(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^2}\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 229

$$\begin{aligned}\int \frac{x^6(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^2}\,dx &= -\frac{a(2Ab^3-a(3b^2B-4abC+5a^2D))\,x}{b^6}\\&+\frac{(Ab^3-a(2b^2B-3abC+4a^2D))\,x^3}{3b^5}+\frac{(b^2B-2abC+3a^2D)\,x^5}{5b^4}\\&+\frac{(bC-2aD)x^7}{7b^3}+\frac{Dx^9}{9b^2}-\frac{a^2(Ab^3-a(b^2B-abC+a^2D))\,x}{2b^6\,(a+bx^2)}\\&+\frac{a^{3/2}(5Ab^3-a(7b^2B-9abC+11a^2D))\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{13/2}}\end{aligned}$$

output
```
-a*(2*A*b^3-a*(3*B*b^2-4*C*a*b+5*D*a^2))*x/b^6+1/3*(A*b^3-a*(2*B*b^2-3*C*a
*b+4*D*a^2))*x^3/b^5+1/5*(B*b^2-2*C*a*b+3*D*a^2)*x^5/b^4+1/7*(C*b-2*D*a)*x
^7/b^3+1/9*D*x^9/b^2-1/2*a^2*(A*b^3-a*(B*b^2-C*a*b+D*a^2))*x/b^6/(b*x^2+a)
+1/2*a^(3/2)*(5*A*b^3-a*(7*B*b^2-9*C*a*b+11*D*a^2))*arctan(b^(1/2)*x/a^(1/
2))/b^(13/2)
```

## Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.99

$$\int \frac{x^6(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^2}\,dx = \frac{a(-2Ab^3+3ab^2B-4a^2bC+5a^3D)\,x}{b^6} + \frac{(Ab^3-2ab^2B+3a^2bC-4a^3D)\,x^3}{3b^5} + \frac{(b^2B-2abC+3a^2D)\,x^5}{5b^4} + \frac{(bC-2aD)x^7}{7b^3} + \frac{Dx^9}{9b^2} - \frac{(a^2Ab^3-a^3b^2B+a^4bC-a^5D)\,x}{2b^6\,(a+bx^2)} - \frac{a^{3/2}(-5Ab^3+7ab^2B-9a^2bC+11a^3D)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{13/2}}$$

input
```
Integrate[(x^6*(A + B*x^2 + C*x^4 + D*x^6))/(a + b*x^2)^2,x]
```

output
```
(a*(-2*A*b^3 + 3*a*b^2*B - 4*a^2*b*C + 5*a^3*D)*x)/b^6 + ((A*b^3 - 2*a*b^2
*B + 3*a^2*b*C - 4*a^3*D)*x^3)/(3*b^5) + ((b^2*B - 2*a*b*C + 3*a^2*D)*x^5)
/(5*b^4) + ((b*C - 2*a*D)*x^7)/(7*b^3) + (D*x^9)/(9*b^2) - ((a^2*A*b^3 - a
^3*b^2*B + a^4*b*C - a^5*D)*x)/(2*b^6*(a + b*x^2)) - (a^(3/2)*(-5*A*b^3 +
7*a*b^2*B - 9*a^2*b*C + 11*a^3*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(13/2)
)
```

## Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2335, 9, 1584, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^2}\,dx$$

$$\Big\downarrow \; 2335$$

$$\frac{x^7\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{2a\left(a+bx^2\right)}-\frac{\int \frac{x^5\left(-2aDx^5-2a\left(C-\frac{aD}{b}\right)x^3+\left(5Ab-\frac{7a\left(Da^2-bCa+b^2B\right)}{b^2}\right)x\right)}{bx^2+a}dx}{2ab}$$

$$\Big\downarrow \; 9$$

$$\frac{x^7\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{2a\left(a+bx^2\right)}-\frac{\int \frac{x^6\left(-2aDx^4-2a\left(C-\frac{aD}{b}\right)x^2+5Ab-\frac{7a\left(Da^2-bCa+b^2B\right)}{b^2}\right)}{bx^2+a}dx}{2ab}$$

$$\Big\downarrow \; 1584$$

$$\frac{x^7\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{2a\left(a+bx^2\right)}-$$

$$\frac{\int \left(-\frac{2aDx^8}{b}-\frac{2a(bC-2aD)x^6}{b^2}+\left(5A-\frac{a(11Da^2-9bCa+7b^2B)}{b^3}\right)x^4-\frac{a(5Ab^3-a(11Da^2-9bCa+7b^2B))x^2}{b^4}+\frac{a^2(5Ab^3-a(11Da^2-}{b^5}\right.}{2ab}$$

$$\Big\downarrow \; 2009$$

$$\frac{x^7\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{2a\left(a+bx^2\right)}-$$

$$\frac{\frac{1}{5}x^5\left(5A-\frac{a(11a^2D-9abC+7b^2B)}{b^3}\right)+\frac{a^2x(5Ab^3-a(11a^2D-9abC+7b^2B))}{b^5}-\frac{ax^3(5Ab^3-a(11a^2D-9abC+7b^2B))}{3b^4}-\frac{a^{5/2}\arctan\left(\frac{\sqrt{b}}{\sqrt{a}}\right.}{}}{2ab}$$

input
```
Int[(x^6*(A + B*x^2 + C*x^4 + D*x^6))/(a + b*x^2)^2,x]
```

output
```
((A - (a*(b^2*B - a*b*C + a^2*D))/b^3)*x^7)/(2*a*(a + b*x^2)) - ((a^2*(5*A
*b^3 - a*(7*b^2*B - 9*a*b*C + 11*a^2*D))*x)/b^5 - (a*(5*A*b^3 - a*(7*b^2*B
 - 9*a*b*C + 11*a^2*D))*x^3)/(3*b^4) + ((5*A - (a*(7*b^2*B - 9*a*b*C + 11*
a^2*D))/b^3)*x^5)/5 - (2*a*(b*C - 2*a*D)*x^7)/(7*b^2) - (2*a*D*x^9)/(9*b)
- (a^(5/2)*(5*A*b^3 - a*(7*b^2*B - 9*a*b*C + 11*a^2*D))*ArcTan[(Sqrt[b]*x)
/Sqrt[a]])/b^(11/2))/(2*a*b)
```

### Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 1584
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2335
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Simp[c/(2*a*b*(p + 1))   Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSu
m[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
 b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

## Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.00

| method | result |
|---|---|
| default | $-\frac{-\frac{1}{9}Dx^{9}b^{4}-\frac{1}{7}C\,b^{4}x^{7}+\frac{2}{7}Da\,b^{3}x^{7}-\frac{1}{5}B\,b^{4}x^{5}+\frac{2}{5}Ca\,b^{3}x^{5}-\frac{3}{5}Da^{2}b^{2}x^{5}-\frac{1}{3}A\,x^{3}b^{4}+\frac{2}{3}B\,x^{3}a\,b^{3}-C\,a^{2}b^{2}x^{3}+\frac{4}{3}Da^{3}b\,x^{3}+2Aa\,b^{3}x-3B}{b^{6}}$ |

input
```
int(x^6*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/b^6*(-1/9*D*x^9*b^4-1/7*C*b^4*x^7+2/7*D*a*b^3*x^7-1/5*B*b^4*x^5+2/5*C*a
*b^3*x^5-3/5*D*a^2*b^2*x^5-1/3*A*x^3*b^4+2/3*B*x^3*a*b^3-C*a^2*b^2*x^3+4/3
*D*a^3*b*x^3+2*A*a*b^3*x-3*B*a^2*b^2*x+4*C*a^3*b*x-5*D*a^4*x)+a^2/b^6*((-1
/2*b^3*A+1/2*a*b^2*B-1/2*a^2*b*C+1/2*a^3*D)*x/(b*x^2+a)+1/2*(5*A*b^3-7*B*a
*b^2+9*C*a^2*b-11*D*a^3)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 572, normalized size of antiderivative = 2.50

$$\int \frac{x^6(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^2}\,dx$$

$$= \left[\frac{140\,Db^5x^{11} - 20\,(11\,Dab^4 - 9\,Cb^5)x^9 + 36\,(11\,Da^2b^3 - 9\,Cab^4 + 7\,Bb^5)x^7 - 84\,(11\,Da^3b^2 - 9\,Ca^2b^3 +}{}\right.$$

input
```
integrate(x^6*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[1/1260*(140*D*b^5*x^11 - 20*(11*D*a*b^4 - 9*C*b^5)*x^9 + 36*(11*D*a^2*b^3
 - 9*C*a*b^4 + 7*B*b^5)*x^7 - 84*(11*D*a^3*b^2 - 9*C*a^2*b^3 + 7*B*a*b^4 -
 5*A*b^5)*x^5 + 420*(11*D*a^4*b - 9*C*a^3*b^2 + 7*B*a^2*b^3 - 5*A*a*b^4)*x
^3 - 315*(11*D*a^5 - 9*C*a^4*b + 7*B*a^3*b^2 - 5*A*a^2*b^3 + (11*D*a^4*b -
 9*C*a^3*b^2 + 7*B*a^2*b^3 - 5*A*a*b^4)*x^2)*sqrt(-a/b)*log((b*x^2 + 2*b*x
*sqrt(-a/b) - a)/(b*x^2 + a)) + 630*(11*D*a^5 - 9*C*a^4*b + 7*B*a^3*b^2 -
5*A*a^2*b^3)*x)/(b^7*x^2 + a*b^6), 1/630*(70*D*b^5*x^11 - 10*(11*D*a*b^4 -
 9*C*b^5)*x^9 + 18*(11*D*a^2*b^3 - 9*C*a*b^4 + 7*B*b^5)*x^7 - 42*(11*D*a^3
*b^2 - 9*C*a^2*b^3 + 7*B*a*b^4 - 5*A*b^5)*x^5 + 210*(11*D*a^4*b - 9*C*a^3*
b^2 + 7*B*a^2*b^3 - 5*A*a*b^4)*x^3 - 315*(11*D*a^5 - 9*C*a^4*b + 7*B*a^3*b
^2 - 5*A*a^2*b^3 + (11*D*a^4*b - 9*C*a^3*b^2 + 7*B*a^2*b^3 - 5*A*a*b^4)*x^
2)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) + 315*(11*D*a^5 - 9*C*a^4*b + 7*B*a^3
*b^2 - 5*A*a^2*b^3)*x)/(b^7*x^2 + a*b^6)]
```

## Sympy [A] (verification not implemented)

Time = 1.08 (sec) , antiderivative size = 444, normalized size of antiderivative = 1.94

$$\int \frac{x^6(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^2}\,dx$$
$$= \frac{Dx^9}{9b^2}+x^7\left(\frac{C}{7b^2}-\frac{2Da}{7b^3}\right)+x^5\left(\frac{B}{5b^2}-\frac{2Ca}{5b^3}+\frac{3Da^2}{5b^4}\right)+x^3\left(\frac{A}{3b^2}-\frac{2Ba}{3b^3}+\frac{Ca^2}{b^4}-\frac{4Da^3}{3b^5}\right)$$
$$+x\left(-\frac{2Aa}{b^3}+\frac{3Ba^2}{b^4}-\frac{4Ca^3}{b^5}+\frac{5Da^4}{b^6}\right)+\frac{x(-Aa^2b^3+Ba^3b^2-Ca^4b+Da^5)}{2ab^6+2b^7x^2}$$
$$+\frac{\sqrt{-\frac{a^3}{b^{13}}}(-5Ab^3+7Bab^2-9Ca^2b+11Da^3)\log\left(-\frac{b^6\sqrt{-\frac{a^3}{b^{13}}}(-5Ab^3+7Bab^2-9Ca^2b+11Da^3)}{-5Aab^3+7Ba^2b^2-9Ca^3b+11Da^4}+x\right)}{4}$$
$$-\frac{\sqrt{-\frac{a^3}{b^{13}}}(-5Ab^3+7Bab^2-9Ca^2b+11Da^3)\log\left(\frac{b^6\sqrt{-\frac{a^3}{b^{13}}}(-5Ab^3+7Bab^2-9Ca^2b+11Da^3)}{-5Aab^3+7Ba^2b^2-9Ca^3b+11Da^4}+x\right)}{4}$$

input
```
integrate(x**6*(D*x**6+C*x**4+B*x**2+A)/(b*x**2+a)**2,x)
```

output
```
D*x**9/(9*b**2) + x**7*(C/(7*b**2) - 2*D*a/(7*b**3)) + x**5*(B/(5*b**2) -
2*C*a/(5*b**3) + 3*D*a**2/(5*b**4)) + x**3*(A/(3*b**2) - 2*B*a/(3*b**3) +
C*a**2/b**4 - 4*D*a**3/(3*b**5)) + x*(-2*A*a/b**3 + 3*B*a**2/b**4 - 4*C*a*
*3/b**5 + 5*D*a**4/b**6) + x*(-A*a**2*b**3 + B*a**3*b**2 - C*a**4*b + D*a*
*5)/(2*a*b**6 + 2*b**7*x**2) + sqrt(-a**3/b**13)*(-5*A*b**3 + 7*B*a*b**2 -
 9*C*a**2*b + 11*D*a**3)*log(-b**6*sqrt(-a**3/b**13)*(-5*A*b**3 + 7*B*a*b*
*2 - 9*C*a**2*b + 11*D*a**3)/(-5*A*a*b**3 + 7*B*a**2*b**2 - 9*C*a**3*b + 1
1*D*a**4) + x)/4 - sqrt(-a**3/b**13)*(-5*A*b**3 + 7*B*a*b**2 - 9*C*a**2*b
+ 11*D*a**3)*log(b**6*sqrt(-a**3/b**13)*(-5*A*b**3 + 7*B*a*b**2 - 9*C*a**2
*b + 11*D*a**3)/(-5*A*a*b**3 + 7*B*a**2*b**2 - 9*C*a**3*b + 11*D*a**4) + x
)/4
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.00

$$\int \frac{x^6(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^2}\,dx = \frac{(Da^5-Ca^4b+Ba^3b^2-Aa^2b^3)x}{2\,(b^7x^2+ab^6)} - \frac{(11\,Da^5-9\,Ca^4b+7\,Ba^3b^2-5\,Aa^2b^3)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^6} + \frac{35\,Db^4x^9-45\,(2\,Dab^3-Cb^4)x^7+63\,(3\,Da^2b^2-2\,Cab^3+Bb^4)x^5-105\,(4\,Da^3b-3\,Ca^2b^2+2\,Bab^3}{315\,b^6}$$

input
```
integrate(x^6*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*(D*a^5 - C*a^4*b + B*a^3*b^2 - A*a^2*b^3)*x/(b^7*x^2 + a*b^6) - 1/2*(1
1*D*a^5 - 9*C*a^4*b + 7*B*a^3*b^2 - 5*A*a^2*b^3)*arctan(b*x/sqrt(a*b))/(sq
rt(a*b)*b^6) + 1/315*(35*D*b^4*x^9 - 45*(2*D*a*b^3 - C*b^4)*x^7 + 63*(3*D*
a^2*b^2 - 2*C*a*b^3 + B*b^4)*x^5 - 105*(4*D*a^3*b - 3*C*a^2*b^2 + 2*B*a*b^
3 - A*b^4)*x^3 + 315*(5*D*a^4 - 4*C*a^3*b + 3*B*a^2*b^2 - 2*A*a*b^3)*x)/b^
6
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.07

$$\int \frac{x^6(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^2}\,dx = -\frac{(11\,Da^5-9\,Ca^4b+7\,Ba^3b^2-5\,Aa^2b^3)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^6} + \frac{Da^5x-Ca^4bx+Ba^3b^2x-Aa^2b^3x}{2\,(bx^2+a)b^6} + \frac{35\,Db^{16}x^9-90\,Dab^{15}x^7+45\,Cb^{16}x^7+189\,Da^2b^{14}x^5-126\,Cab^{15}x^5+63\,Bb^{16}x^5-420\,Da^3b^{13}x^3+31}{315}$$

input
```
integrate(x^6*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
-1/2*(11*D*a^5 - 9*C*a^4*b + 7*B*a^3*b^2 - 5*A*a^2*b^3)*arctan(b*x/sqrt(a*
b))/(sqrt(a*b)*b^6) + 1/2*(D*a^5*x - C*a^4*b*x + B*a^3*b^2*x - A*a^2*b^3*x
)/((b*x^2 + a)*b^6) + 1/315*(35*D*b^16*x^9 - 90*D*a*b^15*x^7 + 45*C*b^16*x
^7 + 189*D*a^2*b^14*x^5 - 126*C*a*b^15*x^5 + 63*B*b^16*x^5 - 420*D*a^3*b^1
3*x^3 + 315*C*a^2*b^14*x^3 - 210*B*a*b^15*x^3 + 105*A*b^16*x^3 + 1575*D*a^
4*b^12*x - 1260*C*a^3*b^13*x + 945*B*a^2*b^14*x - 630*A*a*b^15*x)/b^18
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^6(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^2}\,dx=\int \frac{x^6\,(A+B\,x^2+C\,x^4+x^6\,D)}{(b\,x^2+a)^2}\,dx$$

input
```
int((x^6*(A + B*x^2 + C*x^4 + x^6*D))/(a + b*x^2)^2,x)
```

output
```
int((x^6*(A + B*x^2 + C*x^4 + x^6*D))/(a + b*x^2)^2, x)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.39

$$\int \frac{x^6(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^2}\,dx = \frac{-3465\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^5d+2835\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^4bc-3465\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^4bd\,x^2-630}{}$$

input
```
int(x^6*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^2,x)
```

output
```
( - 3465*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**5*d + 2835*sqrt(
b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**4*b*c - 3465*sqrt(b)*sqrt(a)*a
tan((b*x)/(sqrt(b)*sqrt(a)))*a**4*b*d*x**2 - 630*sqrt(b)*sqrt(a)*atan((b*x
)/(sqrt(b)*sqrt(a)))*a**3*b**3 + 2835*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*
sqrt(a)))*a**3*b**2*c*x**2 - 630*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(
a)))*a**2*b**4*x**2 + 3465*a**5*b*d*x - 2835*a**4*b**2*c*x + 2310*a**4*b**
2*d*x**3 + 630*a**3*b**4*x - 1890*a**3*b**3*c*x**3 - 462*a**3*b**3*d*x**5
+ 420*a**2*b**5*x**3 + 378*a**2*b**4*c*x**5 + 198*a**2*b**4*d*x**7 - 84*a*
b**6*x**5 - 162*a*b**5*c*x**7 - 110*a*b**5*d*x**9 + 126*b**7*x**7 + 90*b**
6*c*x**9 + 70*b**6*d*x**11)/(630*b**7*(a + b*x**2))
```

# 3.171 $\int \frac{x^4(A+Bx^2+Cx^4+Dx^6)}{(d+ex^2)^2}\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 188

$$\int \frac{x^4(A+Bx^2+Cx^4+Dx^6)}{(d+ex^2)^2}\,dx = -\frac{(4d^3D-3Cd^2e+2Bde^2-Ae^3)\,x}{e^5}+\frac{(3d^2D-2Cde+Be^2)\,x^3}{3e^4} - \frac{(2dD-Ce)x^5}{5e^3}+\frac{Dx^7}{7e^2}-\frac{d(d^3D-Cd^2e+Bde^2-Ae^3)\,x}{2e^5\,(d+ex^2)} + \frac{\sqrt{d}(9d^3D-7Cd^2e+5Bde^2-3Ae^3)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2e^{11/2}}$$

output
```
-(-A*e^3+2*B*d*e^2-3*C*d^2*e+4*D*d^3)*x/e^5+1/3*(B*e^2-2*C*d*e+3*D*d^2)*x^
3/e^4-1/5*(-C*e+2*D*d)*x^5/e^3+1/7*D*x^7/e^2-1/2*d*(-A*e^3+B*d*e^2-C*d^2*e
+D*d^3)*x/e^5/(e*x^2+d)+1/2*d^(1/2)*(-3*A*e^3+5*B*d*e^2-7*C*d^2*e+9*D*d^3)
*arctan(e^(1/2)*x/d^(1/2))/e^(11/2)
```

## Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.99

$$\int \frac{x^4(A+Bx^2+Cx^4+Dx^6)}{(d+ex^2)^2}\,dx = \frac{(-4d^3D+3Cd^2e-2Bde^2+Ae^3)\,x}{e^5}+\frac{(3d^2D-2Cde+Be^2)\,x^3}{3e^4} + \frac{(-2dD+Ce)x^5}{5e^3}+\frac{Dx^7}{7e^2}+\frac{(-d^4D+Cd^3e-Bd^2e^2+Ade^3)\,x}{2e^5\,(d+ex^2)} + \frac{\sqrt{d}(9d^3D-7Cd^2e+5Bde^2-3Ae^3)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2e^{11/2}}$$

input
```
Integrate[(x^4*(A + B*x^2 + C*x^4 + D*x^6))/(d + e*x^2)^2,x]
```

output
```
((-4*d^3*D + 3*C*d^2*e - 2*B*d*e^2 + A*e^3)*x)/e^5 + ((3*d^2*D - 2*C*d*e +
 B*e^2)*x^3)/(3*e^4) + ((-2*d*D + C*e)*x^5)/(5*e^3) + (D*x^7)/(7*e^2) + ((
-(d^4*D) + C*d^3*e - B*d^2*e^2 + A*d*e^3)*x)/(2*e^5*(d + e*x^2)) + (Sqrt[d
]*(9*d^3*D - 7*C*d^2*e + 5*B*d*e^2 - 3*A*e^3)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])
/(2*e^(11/2))
```

## Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2335, 9, 25, 1584, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4\left(A+Bx^2+Cx^4+Dx^6\right)}{(d+ex^2)^2}\,dx$$

$$\Big\downarrow \; 2335$$

$$\frac{x^5\left(A-\frac{d(Be^2-Cde+d^2D)}{e^3}\right)}{2d\,(d+ex^2)}-\frac{\int -\frac{x^3\left(2dDx^5+2d\left(C-\frac{dD}{e}\right)x^3+\left(\frac{5Dd^3}{e^2}-\frac{5Cd^2}{e}+5Bd-3Ae\right)x\right)}{ex^2+d}dx}{2de}$$

$$\Big\downarrow \; 9$$

$$\frac{x^5\left(A-\frac{d(Be^2-Cde+d^2D)}{e^3}\right)}{2d(d+ex^2)}-\frac{\int -\frac{x^4\left(2dDx^4+2d\left(C-\frac{dD}{e}\right)x^2+5Bd-3Ae-\frac{5Cd^2}{e}+\frac{5d^3D}{e^2}\right)}{ex^2+d}dx}{2de}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{x^4\left(2dDx^4+2d\left(C-\frac{dD}{e}\right)x^2+5Bd-3Ae-\frac{5Cd^2}{e}+\frac{5d^3D}{e^2}\right)}{ex^2+d}dx}{2de}+\frac{x^5\left(A-\frac{d(Be^2-Cde+d^2D)}{e^3}\right)}{2d(d+ex^2)}$$

$$\Big\downarrow \; 1584$$

$$\frac{\int \left(\frac{2dDx^6}{e}-\frac{2d(2dD-Ce)x^4}{e^2}+\frac{(9Dd^3-7Ced^2+5Be^2d-3Ae^3)x^2}{e^3}-\frac{d(9Dd^3-7Ced^2+5Be^2d-3Ae^3)}{e^4}+\frac{9Dd^5-7Ced^4+5Be^2d^3-3Ae^3d^2}{e^4(ex^2+d)}\right.}{2de}$$
$$\frac{x^5\left(A-\frac{d(Be^2-Cde+d^2D)}{e^3}\right)}{2d(d+ex^2)}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{d^{3/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(-3Ae^3+5Bde^2-7Cd^2e+9d^3D)}{e^{9/2}}+\frac{x^3(-3Ae^3+5Bde^2-7Cd^2e+9d^3D)}{3e^3}-\frac{dx(-3Ae^3+5Bde^2-7Cd^2e+9d^3D)}{e^4}-\frac{2dx^5(2d}{5}}{2de}$$
$$\frac{x^5\left(A-\frac{d(Be^2-Cde+d^2D)}{e^3}\right)}{2d(d+ex^2)}$$

input
```
Int[(x^4*(A + B*x^2 + C*x^4 + D*x^6))/(d + e*x^2)^2,x]
```

output
```
((A - (d*(d^2*D - C*d*e + B*e^2))/e^3)*x^5)/(2*d*(d + e*x^2)) + (-((d*(9*d
^3*D - 7*C*d^2*e + 5*B*d*e^2 - 3*A*e^3)*x)/e^4) + ((9*d^3*D - 7*C*d^2*e +
5*B*d*e^2 - 3*A*e^3)*x^3)/(3*e^3) - (2*d*(2*d*D - C*e)*x^5)/(5*e^2) + (2*d
*D*x^7)/(7*e) + (d^(3/2)*(9*d^3*D - 7*C*d^2*e + 5*B*d*e^2 - 3*A*e^3)*ArcTa
n[(Sqrt[e]*x)/Sqrt[d]])/e^(9/2))/(2*d*e)
```

### Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1584
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2335
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Simp[c/(2*a*b*(p + 1))   Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSu
m[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
 b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

## Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.96

| method | result |
|---|---|
| default | $\frac{\frac{1}{7} D x^{7} e^{3}+\frac{1}{5} C e^{3} x^{5}-\frac{2}{5} D d e^{2} x^{5}+\frac{1}{3} x^{3} B e^{3}-\frac{2}{3} C d e^{2} x^{3}+D d^{2} e x^{3}+A e^{3} x-2 B d e^{2} x+3 C d^{2} e x-4 D d^{3} x}{e^{5}}-\frac{d\left(\frac{\left(-\frac{1}{2} A e^{3}+\frac{1}{2} B d e^{2}-\frac{1}{2} C e \right.}{e x^{2}+d}\right.}{}$ |

input
```
int(x^4*(D*x^6+C*x^4+B*x^2+A)/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/e^5*(1/7*D*x^7*e^3+1/5*C*e^3*x^5-2/5*D*d*e^2*x^5+1/3*x^3*B*e^3-2/3*C*d*e
^2*x^3+D*d^2*e*x^3+A*e^3*x-2*B*d*e^2*x+3*C*d^2*e*x-4*D*d^3*x)-d/e^5*((-1/2
*A*e^3+1/2*B*d*e^2-1/2*C*e*d^2+1/2*D*d^3)*x/(e*x^2+d)+1/2*(3*A*e^3-5*B*d*e
^2+7*C*d^2*e-9*D*d^3)/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2)))
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 478, normalized size of antiderivative = 2.54

$$\int \frac{x^4(A+Bx^2+Cx^4+Dx^6)}{(d+ex^2)^2}\,dx$$

$$= \left[\frac{60\,De^4x^9 - 12\,(9\,Dde^3 - 7\,Ce^4)x^7 + 28\,(9\,Dd^2e^2 - 7\,Cde^3 + 5\,Be^4)x^5 - 140\,(9\,Dd^3e - 7\,Cd^2e^2 + 5\,B\right.$$

input
```
integrate(x^4*(D*x^6+C*x^4+B*x^2+A)/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
[1/420*(60*D*e^4*x^9 - 12*(9*D*d*e^3 - 7*C*e^4)*x^7 + 28*(9*D*d^2*e^2 - 7*
C*d*e^3 + 5*B*e^4)*x^5 - 140*(9*D*d^3*e - 7*C*d^2*e^2 + 5*B*d*e^3 - 3*A*e^
4)*x^3 - 105*(9*D*d^4 - 7*C*d^3*e + 5*B*d^2*e^2 - 3*A*d*e^3 + (9*D*d^3*e -
 7*C*d^2*e^2 + 5*B*d*e^3 - 3*A*e^4)*x^2)*sqrt(-d/e)*log((e*x^2 - 2*e*x*sqr
t(-d/e) - d)/(e*x^2 + d)) - 210*(9*D*d^4 - 7*C*d^3*e + 5*B*d^2*e^2 - 3*A*d
*e^3)*x)/(e^6*x^2 + d*e^5), 1/210*(30*D*e^4*x^9 - 6*(9*D*d*e^3 - 7*C*e^4)*
x^7 + 14*(9*D*d^2*e^2 - 7*C*d*e^3 + 5*B*e^4)*x^5 - 70*(9*D*d^3*e - 7*C*d^2
*e^2 + 5*B*d*e^3 - 3*A*e^4)*x^3 + 105*(9*D*d^4 - 7*C*d^3*e + 5*B*d^2*e^2 -
 3*A*d*e^3 + (9*D*d^3*e - 7*C*d^2*e^2 + 5*B*d*e^3 - 3*A*e^4)*x^2)*sqrt(d/e
)*arctan(e*x*sqrt(d/e)/d) - 105*(9*D*d^4 - 7*C*d^3*e + 5*B*d^2*e^2 - 3*A*d
*e^3)*x)/(e^6*x^2 + d*e^5)]
```

## Sympy [A] (verification not implemented)

Time = 1.14 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.37

$$\int \frac{x^4(A+Bx^2+Cx^4+Dx^6)}{(d+ex^2)^2}\,dx$$
$$= \frac{Dx^7}{7e^2}+x^5\left(\frac{C}{5e^2}-\frac{2Dd}{5e^3}\right)+x^3\left(\frac{B}{3e^2}-\frac{2Cd}{3e^3}+\frac{Dd^2}{e^4}\right)$$
$$+x\left(\frac{A}{e^2}-\frac{2Bd}{e^3}+\frac{3Cd^2}{e^4}-\frac{4Dd^3}{e^5}\right)+\frac{x(Ade^3-Bd^2e^2+Cd^3e-Dd^4)}{2de^5+2e^6x^2}$$
$$-\frac{\sqrt{-\frac{d}{e^{11}}}(-3Ae^3+5Bde^2-7Cd^2e+9Dd^3)\log\left(-e^5\sqrt{-\frac{d}{e^{11}}}+x\right)}{4}$$
$$+\frac{\sqrt{-\frac{d}{e^{11}}}(-3Ae^3+5Bde^2-7Cd^2e+9Dd^3)\log\left(e^5\sqrt{-\frac{d}{e^{11}}}+x\right)}{4}$$

input
```
integrate(x**4*(D*x**6+C*x**4+B*x**2+A)/(e*x**2+d)**2,x)
```

output
```
D*x**7/(7*e**2) + x**5*(C/(5*e**2) - 2*D*d/(5*e**3)) + x**3*(B/(3*e**2) -
2*C*d/(3*e**3) + D*d**2/e**4) + x*(A/e**2 - 2*B*d/e**3 + 3*C*d**2/e**4 - 4
*D*d**3/e**5) + x*(A*d*e**3 - B*d**2*e**2 + C*d**3*e - D*d**4)/(2*d*e**5 +
 2*e**6*x**2) - sqrt(-d/e**11)*(-3*A*e**3 + 5*B*d*e**2 - 7*C*d**2*e + 9*D*
d**3)*log(-e**5*sqrt(-d/e**11) + x)/4 + sqrt(-d/e**11)*(-3*A*e**3 + 5*B*d*
e**2 - 7*C*d**2*e + 9*D*d**3)*log(e**5*sqrt(-d/e**11) + x)/4
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4(A+Bx^2+Cx^4+Dx^6)}{(d+ex^2)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^4*(D*x^6+C*x^4+B*x^2+A)/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.04

$$\int \frac{x^4(A+Bx^2+Cx^4+Dx^6)}{(d+ex^2)^2}\,dx = \frac{(9\,Dd^4-7\,Cd^3e+5\,Bd^2e^2-3\,Ade^3)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\,\sqrt{de}e^5} - \frac{Dd^4x-Cd^3ex+Bd^2e^2x-Ade^3x}{2\,(ex^2+d)e^5} + \frac{15\,De^{12}x^7-42\,Dde^{11}x^5+21\,Ce^{12}x^5+105\,Dd^2e^{10}x^3-70\,Cde^{11}x^3+35\,Be^{12}x^3-420\,Dd^3e^9x+315}{105\,e^{14}}$$

input
```
integrate(x^4*(D*x^6+C*x^4+B*x^2+A)/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
1/2*(9*D*d^4 - 7*C*d^3*e + 5*B*d^2*e^2 - 3*A*d*e^3)*arctan(e*x/sqrt(d*e))/
(sqrt(d*e)*e^5) - 1/2*(D*d^4*x - C*d^3*e*x + B*d^2*e^2*x - A*d*e^3*x)/((e*
x^2 + d)*e^5) + 1/105*(15*D*e^12*x^7 - 42*D*d*e^11*x^5 + 21*C*e^12*x^5 + 1
05*D*d^2*e^10*x^3 - 70*C*d*e^11*x^3 + 35*B*e^12*x^3 - 420*D*d^3*e^9*x + 31
5*C*d^2*e^10*x - 210*B*d*e^11*x + 105*A*e^12*x)/e^14
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(A+Bx^2+Cx^4+Dx^6)}{(d+ex^2)^2}\,dx = \int \frac{x^4\,(A+B\,x^2+C\,x^4+x^6\,D)}{(e\,x^2+d)^2}\,dx$$

input
```
int((x^4*(A + B*x^2 + C*x^4 + x^6*D))/(d + e*x^2)^2,x)
```

output
```
int((x^4*(A + B*x^2 + C*x^4 + x^6*D))/(d + e*x^2)^2, x)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.85

$$\int \frac{x^4(A+Bx^2+Cx^4+Dx^6)}{(d+ex^2)^2}\,dx$$
$$= \frac{-315\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)ad\,e^3-315\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)a\,e^4x^2+525\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)b\,d^2e^2+525\sqrt{}}{}$$

input
```
int(x^4*(D*x^6+C*x^4+B*x^2+A)/(e*x^2+d)^2,x)
```

output
```
( - 315*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*d*e**3 - 315*sqrt(
e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*e**4*x**2 + 525*sqrt(e)*sqrt(d)
*atan((e*x)/(sqrt(e)*sqrt(d)))*b*d**2*e**2 + 525*sqrt(e)*sqrt(d)*atan((e*x
)/(sqrt(e)*sqrt(d)))*b*d*e**3*x**2 - 735*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(
e)*sqrt(d)))*c*d**3*e - 735*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*
c*d**2*e**2*x**2 + 945*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*d**5
+ 945*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*d**4*e*x**2 + 315*a*d*
e**4*x + 210*a*e**5*x**3 - 525*b*d**2*e**3*x - 350*b*d*e**4*x**3 + 70*b*e*
*5*x**5 + 735*c*d**3*e**2*x + 490*c*d**2*e**3*x**3 - 98*c*d*e**4*x**5 + 42
*c*e**5*x**7 - 945*d**5*e*x - 630*d**4*e**2*x**3 + 126*d**3*e**3*x**5 - 54
*d**2*e**4*x**7 + 30*d*e**5*x**9)/(210*e**6*(d + e*x**2))
```

# 3.172 $\int \frac{x^2(A+Bx^2+Cx^4+Dx^6)}{(d+ex^2)^2}\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 149

$$\int \frac{x^2(A+Bx^2+Cx^4+Dx^6)}{(d+ex^2)^2}\,dx = \frac{(3d^2D-2Cde+Be^2)\,x}{e^4}-\frac{(2dD-Ce)x^3}{3e^3}+\frac{Dx^5}{5e^2}+\frac{(d^3D-Cd^2e+Bde^2-Ae^3)\,x}{2e^4\,(d+ex^2)}-\frac{(7d^3D-5Cd^2e+3Bde^2-Ae^3)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2\sqrt{d}e^{9/2}}$$

output

```
(B*e^2-2*C*d*e+3*D*d^2)*x/e^4-1/3*(-C*e+2*D*d)*x^3/e^3+1/5*D*x^5/e^2+1/2*(
-A*e^3+B*d*e^2-C*d^2*e+D*d^3)*x/e^4/(e*x^2+d)-1/2*(-A*e^3+3*B*d*e^2-5*C*d^
2*e+7*D*d^3)*arctan(e^(1/2)*x/d^(1/2))/d^(1/2)/e^(9/2)
```

## Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.99

$$\int \frac{x^2(A+Bx^2+Cx^4+Dx^6)}{(d+ex^2)^2}\,dx = \frac{(3d^2D-2Cde+Be^2)\,x}{e^4} + \frac{(-2dD+Ce)x^3}{3e^3} + \frac{Dx^5}{5e^2} - \frac{(-d^3D+Cd^2e-Bde^2+Ae^3)\,x}{2e^4\,(d+ex^2)} - \frac{(7d^3D-5Cd^2e+3Bde^2-Ae^3)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2\sqrt{d}e^{9/2}}$$

input
```
Integrate[(x^2*(A + B*x^2 + C*x^4 + D*x^6))/(d + e*x^2)^2,x]
```

output
```
((3*d^2*D - 2*C*d*e + B*e^2)*x)/e^4 + ((-2*d*D + C*e)*x^3)/(3*e^3) + (D*x^
5)/(5*e^2) - ((-(d^3*D) + C*d^2*e - B*d*e^2 + A*e^3)*x)/(2*e^4*(d + e*x^2)
) - ((7*d^3*D - 5*C*d^2*e + 3*B*d*e^2 - A*e^3)*ArcTan[(Sqrt[e]*x)/Sqrt[d]]
)/(2*Sqrt[d]*e^(9/2))
```

## Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.15, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2335, 9, 25, 1584, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\left(A+Bx^2+Cx^4+Dx^6\right)}{\left(d+ex^2\right)^2}dx$$

$$\Big\downarrow \text{ 2335}$$

$$\frac{x^3\left(A-\frac{d(Be^2-Cde+d^2D)}{e^3}\right)}{2d\,(d+ex^2)} - \frac{\int -\frac{x\left(2dDx^5+2d\left(C-\frac{dD}{e}\right)x^3+\left(\frac{3Dd^3}{e^2}-\frac{3Cd^2}{e}+3Bd-Ae\right)x\right)}{ex^2+d}dx}{2de}$$

$$\Big\downarrow \text{ 9}$$

$$\frac{x^3\left(A-\frac{d(Be^2-Cde+d^2D)}{e^3}\right)}{2d\left(d+ex^2\right)}-\frac{\int -\frac{x^2\left(2dDx^4+2d\left(C-\frac{dD}{e}\right)x^2+3Bd-Ae-\frac{3Cd^2}{e}+\frac{3d^3D}{e^2}\right)}{ex^2+d}dx}{2de}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{x^2\left(2dDx^4+2d\left(C-\frac{dD}{e}\right)x^2+3Bd-Ae-\frac{3Cd^2}{e}+\frac{3d^3D}{e^2}\right)}{ex^2+d}dx}{2de}+\frac{x^3\left(A-\frac{d(Be^2-Cde+d^2D)}{e^3}\right)}{2d\left(d+ex^2\right)}$$

$$\Big\downarrow \ 1584$$

$$\frac{\int \left(\frac{2dDx^4}{e}-\frac{2d(2dD-Ce)x^2}{e^2}+\frac{7Dd^3-5Ced^2+3Be^2d-Ae^3}{e^3}+\frac{-7Dd^4+5Ced^3-3Be^2d^2+Ae^3d}{e^3(ex^2+d)}\right)dx}{2de}+\frac{x^3\left(A-\frac{d(Be^2-Cde+d^2D)}{e^3}\right)}{2d\left(d+ex^2\right)}$$

$$\Big\downarrow \ 2009$$

$$\frac{-\frac{\sqrt{d}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(-Ae^3+3Bde^2-5Cd^2e+7d^3D)}{e^{7/2}}+\frac{x(-Ae^3+3Bde^2-5Cd^2e+7d^3D)}{e^3}-\frac{2dx^3(2dD-Ce)}{3e^2}+\frac{2dDx^5}{5e}}{2de}+\frac{x^3\left(A-\frac{d(Be^2-Cde+d^2D)}{e^3}\right)}{2d\left(d+ex^2\right)}$$

input

```
Int[(x^2*(A + B*x^2 + C*x^4 + D*x^6))/(d + e*x^2)^2,x]
```

output

```
((A - (d*(d^2*D - C*d*e + B*e^2))/e^3)*x^3)/(2*d*(d + e*x^2)) + (((7*d^3*D
 - 5*C*d^2*e + 3*B*d*e^2 - A*e^3)*x)/e^3 - (2*d*(2*d*D - C*e)*x^3)/(3*e^2)
 + (2*d*D*x^5)/(5*e) - (Sqrt[d]*(7*d^3*D - 5*C*d^2*e + 3*B*d*e^2 - A*e^3)*
ArcTan[(Sqrt[e]*x)/Sqrt[d]])/e^(7/2))/(2*d*e)
```

## Defintions of rubi rules used

rule 9

```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1584
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2335
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Simp[c/(2*a*b*(p + 1))   Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSu
m[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
 b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

## Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.92

| method | result |
|---|---|
| default | $\frac{\frac{1}{5} D x^5 e^2+\frac{1}{3} C e^2 x^3-\frac{2}{3} D d e x^3+B e^2 x-2 C d e x+3 D d^2 x}{e^4}+\frac{\frac{\left(-\frac{1}{2} A e^3+\frac{1}{2} B d e^2-\frac{1}{2} C e d^2+\frac{1}{2} D d^3\right) x}{e x^2+d}+\frac{\left(A e^3-3 B d e^2+5 C e d^2-7 D d^3\right) \text{arcta}}{2\sqrt{d e}}}{e^4}$ |

input
```
int(x^2*(D*x^6+C*x^4+B*x^2+A)/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/e^4*(1/5*D*x^5*e^2+1/3*C*e^2*x^3-2/3*D*d*e*x^3+B*e^2*x-2*C*d*e*x+3*D*d^2
*x)+1/e^4*((-1/2*A*e^3+1/2*B*d*e^2-1/2*C*e*d^2+1/2*D*d^3)*x/(e*x^2+d)+1/2*
(A*e^3-3*B*d*e^2+5*C*d^2*e-7*D*d^3)/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2)))
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 424, normalized size of antiderivative = 2.85

$$\int \frac{x^2(A+Bx^2+Cx^4+Dx^6)}{(d+ex^2)^2}\,dx = \left[\frac{12\,Dde^4x^7-4\,(7\,Dd^2e^3-5\,Cde^4)x^5+20\,(7\,Dd^3e^2-5\,Cd^2e^3+3\,Bde^4)x^3+15\,(7\,Dd^4-5\,Cd^3e+3\,B\right.$$

input
```
integrate(x^2*(D*x^6+C*x^4+B*x^2+A)/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
[1/60*(12*D*d*e^4*x^7 - 4*(7*D*d^2*e^3 - 5*C*d*e^4)*x^5 + 20*(7*D*d^3*e^2
- 5*C*d^2*e^3 + 3*B*d*e^4)*x^3 + 15*(7*D*d^4 - 5*C*d^3*e + 3*B*d^2*e^2 - A
*d*e^3 + (7*D*d^3*e - 5*C*d^2*e^2 + 3*B*d*e^3 - A*e^4)*x^2)*sqrt(-d*e)*log
((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) + 30*(7*D*d^4*e - 5*C*d^3*e^2 +
 3*B*d^2*e^3 - A*d*e^4)*x)/(d*e^6*x^2 + d^2*e^5), 1/30*(6*D*d*e^4*x^7 - 2*
(7*D*d^2*e^3 - 5*C*d*e^4)*x^5 + 10*(7*D*d^3*e^2 - 5*C*d^2*e^3 + 3*B*d*e^4)
*x^3 - 15*(7*D*d^4 - 5*C*d^3*e + 3*B*d^2*e^2 - A*d*e^3 + (7*D*d^3*e - 5*C*
d^2*e^2 + 3*B*d*e^3 - A*e^4)*x^2)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) + 15*(7*
D*d^4*e - 5*C*d^3*e^2 + 3*B*d^2*e^3 - A*d*e^4)*x)/(d*e^6*x^2 + d^2*e^5)]
```

## Sympy [A] (verification not implemented)

Time = 1.01 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.48

$$\int \frac{x^2(A+Bx^2+Cx^4+Dx^6)}{(d+ex^2)^2}\,dx = \frac{Dx^5}{5e^2}+x^3\left(\frac{C}{3e^2}-\frac{2Dd}{3e^3}\right)+x\left(\frac{B}{e^2}-\frac{2Cd}{e^3}+\frac{3Dd^2}{e^4}\right)+\frac{x(-Ae^3+Bde^2-Cd^2e+Dd^3)}{2de^4+2e^5x^2} + \frac{\sqrt{-\frac{1}{de^9}}(-Ae^3+3Bde^2-5Cd^2e+7Dd^3)\log\left(-de^4\sqrt{-\frac{1}{de^9}}+x\right)}{4} - \frac{\sqrt{-\frac{1}{de^9}}(-Ae^3+3Bde^2-5Cd^2e+7Dd^3)\log\left(de^4\sqrt{-\frac{1}{de^9}}+x\right)}{4}$$

input
```
integrate(x**2*(D*x**6+C*x**4+B*x**2+A)/(e*x**2+d)**2,x)
```

output
```
D*x**5/(5*e**2) + x**3*(C/(3*e**2) - 2*D*d/(3*e**3)) + x*(B/e**2 - 2*C*d/e
**3 + 3*D*d**2/e**4) + x*(-A*e**3 + B*d*e**2 - C*d**2*e + D*d**3)/(2*d*e**
4 + 2*e**5*x**2) + sqrt(-1/(d*e**9))*(-A*e**3 + 3*B*d*e**2 - 5*C*d**2*e +
7*D*d**3)*log(-d*e**4*sqrt(-1/(d*e**9)) + x)/4 - sqrt(-1/(d*e**9))*(-A*e**
3 + 3*B*d*e**2 - 5*C*d**2*e + 7*D*d**3)*log(d*e**4*sqrt(-1/(d*e**9)) + x)/
4
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(A+Bx^2+Cx^4+Dx^6)}{(d+ex^2)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*(D*x^6+C*x^4+B*x^2+A)/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00

$$\begin{aligned}
&\int \frac{x^2(A+Bx^2+Cx^4+Dx^6)}{(d+ex^2)^2}\,dx \\
&= -\frac{(7\,Dd^3-5\,Cd^2e+3\,Bde^2-Ae^3)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\,\sqrt{de}e^4} \\
&\quad+\frac{Dd^3x-Cd^2ex+Bde^2x-Ae^3x}{2\,(ex^2+d)e^4} \\
&\quad+\frac{3\,De^8x^5-10\,Dde^7x^3+5\,Ce^8x^3+45\,Dd^2e^6x-30\,Cde^7x+15\,Be^8x}{15\,e^{10}}
\end{aligned}$$

input
```
integrate(x^2*(D*x^6+C*x^4+B*x^2+A)/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
-1/2*(7*D*d^3 - 5*C*d^2*e + 3*B*d*e^2 - A*e^3)*arctan(e*x/sqrt(d*e))/(sqrt
(d*e)*e^4) + 1/2*(D*d^3*x - C*d^2*e*x + B*d*e^2*x - A*e^3*x)/((e*x^2 + d)*
e^4) + 1/15*(3*D*e^8*x^5 - 10*D*d*e^7*x^3 + 5*C*e^8*x^3 + 45*D*d^2*e^6*x -
 30*C*d*e^7*x + 15*B*e^8*x)/e^10
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(A+Bx^2+Cx^4+Dx^6)}{(d+ex^2)^2}\,dx=\int \frac{x^2\,(A+B\,x^2+C\,x^4+x^6\,D)}{(e\,x^2+d)^2}\,dx$$

input
```
int((x^2*(A + B*x^2 + C*x^4 + x^6*D))/(d + e*x^2)^2,x)
```

output
```
int((x^2*(A + B*x^2 + C*x^4 + x^6*D))/(d + e*x^2)^2, x)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 314, normalized size of antiderivative = 2.11

$$\int \frac{x^2(A+Bx^2+Cx^4+Dx^6)}{(d+ex^2)^2}\,dx$$
$$=\frac{15\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)ad\,e^3+15\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)a\,e^4x^2-45\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)b\,d^2e^2-45\sqrt{e}\,\sqrt{d}\,a}{}$$

input
```
int(x^2*(D*x^6+C*x^4+B*x^2+A)/(e*x^2+d)^2,x)
```

output
```
(15*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*d*e**3 + 15*sqrt(e)*sq
rt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*e**4*x**2 - 45*sqrt(e)*sqrt(d)*atan(
(e*x)/(sqrt(e)*sqrt(d)))*b*d**2*e**2 - 45*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt
(e)*sqrt(d)))*b*d*e**3*x**2 + 75*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(
d)))*c*d**3*e + 75*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*c*d**2*e*
*2*x**2 - 105*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*d**5 - 105*sqr
t(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*d**4*e*x**2 - 15*a*d*e**4*x + 4
5*b*d**2*e**3*x + 30*b*d*e**4*x**3 - 75*c*d**3*e**2*x - 50*c*d**2*e**3*x**
3 + 10*c*d*e**4*x**5 + 105*d**5*e*x + 70*d**4*e**2*x**3 - 14*d**3*e**3*x**
5 + 6*d**2*e**4*x**7)/(30*d*e**5*(d + e*x**2))
```

# 3.173 $\int \frac{A+Bx^2+Cx^4+Dx^6}{(d+ex^2)^2} dx$



## Optimal result

Integrand size = 27, antiderivative size = 124

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(d+ex^2)^2} dx = -\frac{(2dD-Ce)x}{e^3}+\frac{Dx^3}{3e^2}-\frac{(d^3D-Cd^2e+Bde^2-Ae^3)\,x}{2de^3\,(d+ex^2)} + \frac{(5d^3D-3Cd^2e+Bde^2+Ae^3)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}e^{7/2}}$$

output
```
-(-C*e+2*D*d)*x/e^3+1/3*D*x^3/e^2-1/2*(-A*e^3+B*d*e^2-C*d^2*e+D*d^3)*x/d/e
^3/(e*x^2+d)+1/2*(A*e^3+B*d*e^2-3*C*d^2*e+5*D*d^3)*arctan(e^(1/2)*x/d^(1/2
))/d^(3/2)/e^(7/2)
```

## Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.98

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(d+ex^2)^2} dx = \frac{(-2dD+Ce)x}{e^3}+\frac{Dx^3}{3e^2}-\frac{(d^3D-Cd^2e+Bde^2-Ae^3)\,x}{2de^3\,(d+ex^2)} + \frac{(5d^3D-3Cd^2e+Bde^2+Ae^3)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}e^{7/2}}$$

input
```
Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(d + e*x^2)^2,x]
```

output
```
((-2*d*D + C*e)*x)/e^3 + (D*x^3)/(3*e^2) - ((d^3*D - C*d^2*e + B*d*e^2 - A
*e^3)*x)/(2*d*e^3*(d + e*x^2)) + ((5*d^3*D - 3*C*d^2*e + B*d*e^2 + A*e^3)*
ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*e^(7/2))
```

## Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {2345, 25, 1467, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(d+ex^2)^2}dx$$

$$\Big\downarrow \text{2345}$$

$$\frac{x\left(A-\frac{d(Be^2-Cde+d^2D)}{e^3}\right)}{2d(d+ex^2)}-\frac{\int -\frac{\frac{2dDx^4}{e}-\frac{2d(dD-Ce)x^2}{e^2}+\frac{Dd^3-Ced^2+Be^2d+Ae^3}{e^3}}{ex^2+d}dx}{2d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\frac{2dDx^4}{e}-\frac{2d(dD-Ce)x^2}{e^2}+A+\frac{d\left(Dd^2-Ced+Be^2\right)}{e^3}}{ex^2+d}dx}{2d}+\frac{x\left(A-\frac{d(Be^2-Cde+d^2D)}{e^3}\right)}{2d(d+ex^2)}$$

$$\Big\downarrow \text{1467}$$

$$\frac{\int \left(\frac{2dDx^2}{e^2}-\frac{2d(2dD-Ce)}{e^3}+\frac{5Dd^3-3Ced^2+Be^2d+Ae^3}{e^3(ex^2+d)}\right)dx}{2d}+\frac{x\left(A-\frac{d(Be^2-Cde+d^2D)}{e^3}\right)}{2d(d+ex^2)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(Ae^3+Bde^2-3Cd^2e+5d^3D)}{\sqrt{d}e^{7/2}}-\frac{2dx(2dD-Ce)}{e^3}+\frac{2dDx^3}{3e^2}}{2d}+\frac{x\left(A-\frac{d(Be^2-Cde+d^2D)}{e^3}\right)}{2d(d+ex^2)}$$

input
```
Int[(A + B*x^2 + C*x^4 + D*x^6)/(d + e*x^2)^2,x]
```

output
```
((A - (d*(d^2*D - C*d*e + B*e^2))/e^3)*x)/(2*d*(d + e*x^2)) + ((-2*d*(2*d*
D - C*e)*x)/e^3 + (2*d*D*x^3)/(3*e^2) + ((5*d^3*D - 3*C*d^2*e + B*d*e^2 +
A*e^3)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*e^(7/2)))/(2*d)
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1467
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

## Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.90

| method | result | size |
|---|---|---|
| default | $\frac{\frac{1}{3} D x^{3} e+C e x-2 D d x}{e^{3}}+\frac{\frac{\left(A \,e^{3}-B d \,e^{2}+C e \,d^{2}-D d^{3}\right) x}{2 d \left(e \,x^{2}+d \right)}+\frac{\left(A \,e^{3}+B d \,e^{2}-3 C e \,d^{2}+5 D d^{3}\right) \arctan \left(\frac{e x}{\sqrt{d e}}\right)}{2 d \sqrt{d e}}}{e^{3}}$ | 112 |

input
```
int((D*x^6+C*x^4+B*x^2+A)/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/e^3*(1/3*D*x^3*e+C*e*x-2*D*d*x)+1/e^3*(1/2*(A*e^3-B*d*e^2+C*d^2*e-D*d^3)
/d*x/(e*x^2+d)+1/2*(A*e^3+B*d*e^2-3*C*d^2*e+5*D*d^3)/d/(d*e)^(1/2)*arctan(
e*x/(d*e)^(1/2)))
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 364, normalized size of antiderivative = 2.94

$$\int \frac{A + Bx^2 + Cx^4 + Dx^6}{(d + ex^2)^2} dx$$

$$= \left[ \frac{4\,Dd^2e^3x^5 - 4\,(5\,Dd^3e^2 - 3\,Cd^2e^3)x^3 - 3\,(5\,Dd^4 - 3\,Cd^3e + Bd^2e^2 + Ade^3 + (5\,Dd^3e - 3\,Cd^2e^2 + Bd}{12\,(d^2e^5x^2 + d^3e^4)} \right.$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
[1/12*(4*D*d^2*e^3*x^5 - 4*(5*D*d^3*e^2 - 3*C*d^2*e^3)*x^3 - 3*(5*D*d^4 -
3*C*d^3*e + B*d^2*e^2 + A*d*e^3 + (5*D*d^3*e - 3*C*d^2*e^2 + B*d*e^3 + A*e
^4)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) - 6*(5*D
*d^4*e - 3*C*d^3*e^2 + B*d^2*e^3 - A*d*e^4)*x)/(d^2*e^5*x^2 + d^3*e^4), 1/
6*(2*D*d^2*e^3*x^5 - 2*(5*D*d^3*e^2 - 3*C*d^2*e^3)*x^3 + 3*(5*D*d^4 - 3*C*
d^3*e + B*d^2*e^2 + A*d*e^3 + (5*D*d^3*e - 3*C*d^2*e^2 + B*d*e^3 + A*e^4)*
x^2)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) - 3*(5*D*d^4*e - 3*C*d^3*e^2 + B*d^2*
e^3 - A*d*e^4)*x)/(d^2*e^5*x^2 + d^3*e^4)]
```

## Sympy [A] (verification not implemented)

Time = 0.78 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.62

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(d+ex^2)^2}\,dx = \frac{Dx^3}{3e^2}+x\left(\frac{C}{e^2}-\frac{2Dd}{e^3}\right)+\frac{x(Ae^3-Bde^2+Cd^2e-Dd^3)}{2d^2e^3+2de^4x^2} - \frac{\sqrt{-\frac{1}{d^3e^7}}(Ae^3+Bde^2-3Cd^2e+5Dd^3)\log\left(-d^2e^3\sqrt{-\frac{1}{d^3e^7}}+x\right)}{4} + \frac{\sqrt{-\frac{1}{d^3e^7}}(Ae^3+Bde^2-3Cd^2e+5Dd^3)\log\left(d^2e^3\sqrt{-\frac{1}{d^3e^7}}+x\right)}{4}$$

input
```
integrate((D*x**6+C*x**4+B*x**2+A)/(e*x**2+d)**2,x)
```

output
```
D*x**3/(3*e**2) + x*(C/e**2 - 2*D*d/e**3) + x*(A*e**3 - B*d*e**2 + C*d**2*
e - D*d**3)/(2*d**2*e**3 + 2*d*e**4*x**2) - sqrt(-1/(d**3*e**7))*(A*e**3 +
 B*d*e**2 - 3*C*d**2*e + 5*D*d**3)*log(-d**2*e**3*sqrt(-1/(d**3*e**7)) + x
)/4 + sqrt(-1/(d**3*e**7))*(A*e**3 + B*d*e**2 - 3*C*d**2*e + 5*D*d**3)*log
(d**2*e**3*sqrt(-1/(d**3*e**7)) + x)/4
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(d+ex^2)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.99

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(d+ex^2)^2}\,dx = \frac{(5\,Dd^3-3\,Cd^2e+Bde^2+Ae^3)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\,\sqrt{de}de^3} - \frac{Dd^3x-Cd^2ex+Bde^2x-Ae^3x}{2\,(ex^2+d)de^3} + \frac{De^4x^3-6\,Dde^3x+3\,Ce^4x}{3\,e^6}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
1/2*(5*D*d^3 - 3*C*d^2*e + B*d*e^2 + A*e^3)*arctan(e*x/sqrt(d*e))/(sqrt(d*
e)*d*e^3) - 1/2*(D*d^3*x - C*d^2*e*x + B*d*e^2*x - A*e^3*x)/((e*x^2 + d)*d
*e^3) + 1/3*(D*e^4*x^3 - 6*D*d*e^3*x + 3*C*e^4*x)/e^6
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(d+ex^2)^2}\,dx = \int \frac{A+B\,x^2+C\,x^4+x^6\,D}{(e\,x^2+d)^2}\,dx$$

input
```
int((A + B*x^2 + C*x^4 + x^6*D)/(d + e*x^2)^2,x)
```

output
```
int((A + B*x^2 + C*x^4 + x^6*D)/(d + e*x^2)^2, x)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 283, normalized size of antiderivative = 2.28

$$\int \frac{A + Bx^2 + Cx^4 + Dx^6}{(d + ex^2)^2}\,dx$$

$$= \frac{3\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right) ad\,e^3 + 3\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right) a\,e^4x^2 + 3\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right) b\,d^2e^2 + 3\sqrt{e}\,\sqrt{d}\,atan(}{}$$

input
```
int((D*x^6+C*x^4+B*x^2+A)/(e*x^2+d)^2,x)
```

output
```
(3*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*d*e**3 + 3*sqrt(e)*sqrt
(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*e**4*x**2 + 3*sqrt(e)*sqrt(d)*atan((e*
x)/(sqrt(e)*sqrt(d)))*b*d**2*e**2 + 3*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*
sqrt(d)))*b*d*e**3*x**2 - 9*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*
c*d**3*e - 9*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*c*d**2*e**2*x**
2 + 15*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*d**5 + 15*sqrt(e)*sqr
t(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*d**4*e*x**2 + 3*a*d*e**4*x - 3*b*d**2*e
**3*x + 9*c*d**3*e**2*x + 6*c*d**2*e**3*x**3 - 15*d**5*e*x - 10*d**4*e**2*
x**3 + 2*d**3*e**3*x**5)/(6*d**2*e**4*(d + e*x**2))
```

# 3.174 $\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2(d+ex^2)^2}\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 115

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2(d+ex^2)^2}\,dx = -\frac{A}{d^2x}+\frac{Dx}{e^2}+\frac{(d^3D-Cd^2e+Bde^2-Ae^3)\,x}{2d^2e^2(d+ex^2)} - \frac{(3d^3D-Cd^2e-Bde^2+3Ae^3)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{5/2}e^{5/2}}$$

output
```
-A/d^2/x+D*x/e^2+1/2*(-A*e^3+B*d*e^2-C*d^2*e+D*d^3)*x/d^2/e^2/(e*x^2+d)-1/
2*(3*A*e^3-B*d*e^2-C*d^2*e+3*D*d^3)*arctan(e^(1/2)*x/d^(1/2))/d^(5/2)/e^(5
/2)
```

## Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2(d+ex^2)^2}\,dx = -\frac{A}{d^2x}+\frac{Dx}{e^2}+\frac{(d^3D-Cd^2e+Bde^2-Ae^3)\,x}{2d^2e^2(d+ex^2)} - \frac{(3d^3D-Cd^2e-Bde^2+3Ae^3)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{5/2}e^{5/2}}$$

input
```
Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(x^2*(d + e*x^2)^2),x]
```

output
```
-(A/(d^2*x)) + (D*x)/e^2 + ((d^3*D - C*d^2*e + B*d*e^2 - A*e^3)*x)/(2*d^2*
e^2*(d + e*x^2)) - ((3*d^3*D - C*d^2*e - B*d*e^2 + 3*A*e^3)*ArcTan[(Sqrt[e
]*x)/Sqrt[d]])/(2*d^(5/2)*e^(5/2))
```

## Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2336, 25, 1584, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2\left(d+ex^2\right)^2}dx$$

$$\Big\downarrow \text{ 2336}$$

$$\frac{x\left(-\frac{Ae}{d}+B-\frac{Cd}{e}+\frac{d^2D}{e^2}\right)}{2d\left(d+ex^2\right)}-\frac{\int -\frac{\frac{2dDx^4}{e}+\left(-\frac{Dd^2}{e^2}+\frac{Cd}{e}+B-\frac{Ae}{d}\right)x^2+2A}{x^2(ex^2+d)}dx}{2d}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{\frac{2dDx^4}{e}+\left(-\frac{Dd^2}{e^2}+\frac{Cd}{e}+B-\frac{Ae}{d}\right)x^2+2A}{x^2(ex^2+d)}dx}{2d}+\frac{x\left(-\frac{Ae}{d}+B-\frac{Cd}{e}+\frac{d^2D}{e^2}\right)}{2d\left(d+ex^2\right)}$$

$$\Big\downarrow \text{ 1584}$$

$$\frac{\int \left(\frac{2A}{dx^2}+\frac{-3Dd^3+Ced^2+Be^2d-3Ae^3}{de^2(ex^2+d)}+\frac{2dD}{e^2}\right)dx}{2d}+\frac{x\left(-\frac{Ae}{d}+B-\frac{Cd}{e}+\frac{d^2D}{e^2}\right)}{2d\left(d+ex^2\right)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{-\frac{\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(3Ae^3-Bde^2-Cd^2e+3d^3D\right)}{d^{3/2}e^{5/2}}-\frac{2A}{dx}+\frac{2dDx}{e^2}}{2d}+\frac{x\left(-\frac{Ae}{d}+B-\frac{Cd}{e}+\frac{d^2D}{e^2}\right)}{2d\left(d+ex^2\right)}$$

input
```
Int[(A + B*x^2 + C*x^4 + D*x^6)/(x^2*(d + e*x^2)^2),x]
```

output
```
((B + (d^2*D)/e^2 - (C*d)/e - (A*e)/d)*x)/(2*d*(d + e*x^2)) + ((-2*A)/(d*x
) + (2*d*D*x)/e^2 - ((3*d^3*D - C*d^2*e - B*d*e^2 + 3*A*e^3)*ArcTan[(Sqrt[
e]*x)/Sqrt[d]])/(d^(3/2)*e^(5/2)))/(2*d)
```

**Defintions of rubi rules used**

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1584
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

## Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.93

| method | result | size |
|---|---|---|
| default | $\frac{Dx}{e^{2}}-\frac{\frac{\left(\frac{1}{2} A\,e^{3}-\frac{1}{2} B d\,e^{2}+\frac{1}{2} C e\,d^{2}-\frac{1}{2} D d^{3}\right) x}{e\,x^{2}+d}+\frac{\left(3 A\,e^{3}-B d\,e^{2}-C e\,d^{2}+3 D d^{3}\right) \arctan\left(\frac{e x}{\sqrt{d e}}\right)}{2 \sqrt{d e}}}{d^{2} e^{2}}-\frac{A}{d^{2} x}$ | 107 |

input
```
int((D*x^6+C*x^4+B*x^2+A)/x^2/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
D*x/e^2-1/d^2/e^2*((1/2*A*e^3-1/2*B*d*e^2+1/2*C*e*d^2-1/2*D*d^3)*x/(e*x^2+
d)+1/2*(3*A*e^3-B*d*e^2-C*d^2*e+3*D*d^3)/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2
)))-A/d^2/x
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 353, normalized size of antiderivative = 3.07

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2\left(d+ex^2\right)^2}\,dx$$
$$= \left[\frac{4\,Dd^3e^2x^4-4\,Ad^2e^3+2\,(3\,Dd^4e-Cd^3e^2+Bd^2e^3-3\,Ade^4)x^2-((3\,Dd^3e-Cd^2e^2-Bde^3+3\,Ae^4)}{4\,(d^3e^4x^3+d^4e^3x)}\right.$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^2/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
[1/4*(4*D*d^3*e^2*x^4 - 4*A*d^2*e^3 + 2*(3*D*d^4*e - C*d^3*e^2 + B*d^2*e^3
 - 3*A*d*e^4)*x^2 - ((3*D*d^3*e - C*d^2*e^2 - B*d*e^3 + 3*A*e^4)*x^3 + (3*
D*d^4 - C*d^3*e - B*d^2*e^2 + 3*A*d*e^3)*x)*sqrt(-d*e)*log((e*x^2 + 2*sqrt
(-d*e)*x - d)/(e*x^2 + d)))/(d^3*e^4*x^3 + d^4*e^3*x), 1/2*(2*D*d^3*e^2*x^
4 - 2*A*d^2*e^3 + (3*D*d^4*e - C*d^3*e^2 + B*d^2*e^3 - 3*A*d*e^4)*x^2 - ((
3*D*d^3*e - C*d^2*e^2 - B*d*e^3 + 3*A*e^4)*x^3 + (3*D*d^4 - C*d^3*e - B*d^
2*e^2 + 3*A*d*e^3)*x)*sqrt(d*e)*arctan(sqrt(d*e)*x/d))/(d^3*e^4*x^3 + d^4*
e^3*x)]
```

## Sympy [A] (verification not implemented)

Time = 2.23 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.71

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2\left(d+ex^2\right)^2}\,dx$$
$$= \frac{Dx}{e^2} + \frac{\sqrt{-\frac{1}{d^5e^5}}\cdot\left(3Ae^3-Bde^2-Cd^2e+3Dd^3\right)\log\left(-d^3e^2\sqrt{-\frac{1}{d^5e^5}}+x\right)}{4} - \frac{\sqrt{-\frac{1}{d^5e^5}}\cdot\left(3Ae^3-Bde^2-Cd^2e+3Dd^3\right)\log\left(d^3e^2\sqrt{-\frac{1}{d^5e^5}}+x\right)}{4} + \frac{-2Ade^2+x^2(-3Ae^3+Bde^2-Cd^2e+Dd^3)}{2d^3e^2x+2d^2e^3x^3}$$

input
```
integrate((D*x**6+C*x**4+B*x**2+A)/x**2/(e*x**2+d)**2,x)
```

output
```
D*x/e**2 + sqrt(-1/(d**5*e**5))*(3*A*e**3 - B*d*e**2 - C*d**2*e + 3*D*d**3
)*log(-d**3*e**2*sqrt(-1/(d**5*e**5)) + x)/4 - sqrt(-1/(d**5*e**5))*(3*A*e
**3 - B*d*e**2 - C*d**2*e + 3*D*d**3)*log(d**3*e**2*sqrt(-1/(d**5*e**5)) +
 x)/4 + (-2*A*d*e**2 + x**2*(-3*A*e**3 + B*d*e**2 - C*d**2*e + D*d**3))/(2
*d**3*e**2*x + 2*d**2*e**3*x**3)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2\left(d+ex^2\right)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^2/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.03

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2(d+ex^2)^2}\,dx = \frac{Dx}{e^2} - \frac{(3\,Dd^3 - Cd^2e - Bde^2 + 3\,Ae^3)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}d^2e^2} + \frac{Dd^3x^2 - Cd^2ex^2 + Bde^2x^2 - 3\,Ae^3x^2 - 2\,Ade^2}{2\,(ex^3+dx)d^2e^2}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^2/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
D*x/e^2 - 1/2*(3*D*d^3 - C*d^2*e - B*d*e^2 + 3*A*e^3)*arctan(e*x/sqrt(d*e)
)/(sqrt(d*e)*d^2*e^2) + 1/2*(D*d^3*x^2 - C*d^2*e*x^2 + B*d*e^2*x^2 - 3*A*e
^3*x^2 - 2*A*d*e^2)/((e*x^3 + d*x)*d^2*e^2)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2(d+ex^2)^2}\,dx = \int \frac{A+B\,x^2+C\,x^4+x^6\,D}{x^2\,(e\,x^2+d)^2}\,dx$$

input
```
int((A + B*x^2 + C*x^4 + x^6*D)/(x^2*(d + e*x^2)^2),x)
```

output
```
int((A + B*x^2 + C*x^4 + x^6*D)/(x^2*(d + e*x^2)^2), x)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 279, normalized size of antiderivative = 2.43

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2(d+ex^2)^2}\,dx = \frac{-3\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)ad\,e^3x - 3\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)a\,e^4x^3 + \sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)b\,d^2e^2x + \sqrt{e}\,\sqrt{d}\,ata}{}$$

input
```
int((D*x^6+C*x^4+B*x^2+A)/x^2/(e*x^2+d)^2,x)
```

output
```
( - 3*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*d*e**3*x - 3*sqrt(e)
*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*e**4*x**3 + sqrt(e)*sqrt(d)*atan(
(e*x)/(sqrt(e)*sqrt(d)))*b*d**2*e**2*x + sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(
e)*sqrt(d)))*b*d*e**3*x**3 + sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))
*c*d**3*e*x + sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*c*d**2*e**2*x*
*3 - 3*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*d**5*x - 3*sqrt(e)*sq
rt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*d**4*e*x**3 - 2*a*d**2*e**3 - 3*a*d*e*
*4*x**2 + b*d**2*e**3*x**2 - c*d**3*e**2*x**2 + 3*d**5*e*x**2 + 2*d**4*e**
2*x**4)/(2*d**3*e**3*x*(d + e*x**2))
```

# 3.175 $\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4(d+ex^2)^2} dx$



## Optimal result

Integrand size = 30, antiderivative size = 125

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4(d+ex^2)^2} dx = -\frac{A}{3d^2x^3} - \frac{Bd-2Ae}{d^3x} - \frac{(d^3D-Cd^2e+Bde^2-Ae^3)x}{2d^3e(d+ex^2)} + \frac{(d^3D+Cd^2e-3Bde^2+5Ae^3)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{7/2}e^{3/2}}$$

output
```
-1/3*A/d^2/x^3-(-2*A*e+B*d)/d^3/x-1/2*(-A*e^3+B*d*e^2-C*d^2*e+D*d^3)*x/d^3
/e/(e*x^2+d)+1/2*(5*A*e^3-3*B*d*e^2+C*d^2*e+D*d^3)*arctan(e^(1/2)*x/d^(1/2
))/d^(7/2)/e^(3/2)
```

## Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4(d+ex^2)^2} dx = -\frac{A}{3d^2x^3} + \frac{-Bd+2Ae}{d^3x} - \frac{(d^3D-Cd^2e+Bde^2-Ae^3)x}{2d^3e(d+ex^2)} + \frac{(d^3D+Cd^2e-3Bde^2+5Ae^3)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{7/2}e^{3/2}}$$

input
```
Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(x^4*(d + e*x^2)^2),x]
```

output
```
-1/3*A/(d^2*x^3) + (-(B*d) + 2*A*e)/(d^3*x) - ((d^3*D - C*d^2*e + B*d*e^2
- A*e^3)*x)/(2*d^3*e*(d + e*x^2)) + ((d^3*D + C*d^2*e - 3*B*d*e^2 + 5*A*e^
3)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(7/2)*e^(3/2))
```

## Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2336, 25, 1584, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4\left(d+ex^2\right)^2}dx$$

$$\Big\downarrow \textbf{2336}$$

$$\frac{x\left(\frac{Ae^2}{d^2}-\frac{Be}{d}+C-\frac{dD}{e}\right)}{2d\left(d+ex^2\right)}-\frac{\int -\frac{\left(\frac{Ae^2}{d^2}-\frac{Be}{d}+C+\frac{dD}{e}\right)x^4+2\left(B-\frac{Ae}{d}\right)x^2+2A}{x^4\left(ex^2+d\right)}dx}{2d}$$

$$\Big\downarrow \textbf{25}$$

$$\frac{\int \frac{\left(\frac{Ae^2}{d^2}-\frac{Be}{d}+C+\frac{dD}{e}\right)x^4+2\left(B-\frac{Ae}{d}\right)x^2+2A}{x^4\left(ex^2+d\right)}dx}{2d}+\frac{x\left(\frac{Ae^2}{d^2}-\frac{Be}{d}+C-\frac{dD}{e}\right)}{2d\left(d+ex^2\right)}$$

$$\Big\downarrow \textbf{1584}$$

$$\frac{\int \left(\frac{2A}{dx^4}+\frac{Dd^3+Ced^2-3Be^2d+5Ae^3}{d^2e\left(ex^2+d\right)}+\frac{2(Bd-2Ae)}{d^2x^2}\right)dx}{2d}+\frac{x\left(\frac{Ae^2}{d^2}-\frac{Be}{d}+C-\frac{dD}{e}\right)}{2d\left(d+ex^2\right)}$$

$$\Big\downarrow \textbf{2009}$$

$$\frac{\frac{\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(5Ae^3-3Bde^2+Cd^2e+d^3D\right)}{d^{5/2}e^{3/2}}-\frac{2(Bd-2Ae)}{d^2x}-\frac{2A}{3dx^3}}{2d}+\frac{x\left(\frac{Ae^2}{d^2}-\frac{Be}{d}+C-\frac{dD}{e}\right)}{2d\left(d+ex^2\right)}$$

input
```
Int[(A + B*x^2 + C*x^4 + D*x^6)/(x^4*(d + e*x^2)^2),x]
```

output
```
((C - (d*D)/e - (B*e)/d + (A*e^2)/d^2)*x)/(2*d*(d + e*x^2)) + ((-2*A)/(3*d
*x^3) - (2*(B*d - 2*A*e))/(d^2*x) + ((d^3*D + C*d^2*e - 3*B*d*e^2 + 5*A*e^
3)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(5/2)*e^(3/2)))/(2*d)
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1584
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

## Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.93

| method | result | size |
|---|---|---|
| default | $\frac{\frac{\left(A\,e^{3}-B d\,e^{2}+C e\,d^{2}-D d^{3}\right)x}{2e\left(e\,x^{2}+d\right)}+\frac{\left(5A\,e^{3}-3B d\,e^{2}+C e\,d^{2}+D d^{3}\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2e\sqrt{de}}}{d^{3}}-\frac{A}{3d^{2}x^{3}}-\frac{-2Ae+Bd}{d^{3}x}$ | 116 |

input
```
int((D*x^6+C*x^4+B*x^2+A)/x^4/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d^3*(1/2*(A*e^3-B*d*e^2+C*d^2*e-D*d^3)/e*x/(e*x^2+d)+1/2*(5*A*e^3-3*B*d*
e^2+C*d^2*e+D*d^3)/e/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2)))-1/3*A/d^2/x^3-(-
2*A*e+B*d)/d^3/x
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 378, normalized size of antiderivative = 3.02

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4(d+ex^2)^2}\,dx$$
$$= \left[-\frac{4\,Ad^3e^2+6\,(Dd^4e-Cd^3e^2+3\,Bd^2e^3-5\,Ade^4)x^4+4\,(3\,Bd^3e^2-5\,Ad^2e^3)x^2+3\,((Dd^3e+Cd^2e^2-}{12\,(d^4e^3x^5+d^5e^2}\right.$$
$$-\frac{2\,Ad^3e^2+3\,(Dd^4e-Cd^3e^2+3\,Bd^2e^3-5\,Ade^4)x^4+2\,(3\,Bd^3e^2-5\,Ad^2e^3)x^2-3\,((Dd^3e+Cd^2e^2-}{6\,(d^4e^3x^5+d^5e^2x^3)}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^4/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
[-1/12*(4*A*d^3*e^2 + 6*(D*d^4*e - C*d^3*e^2 + 3*B*d^2*e^3 - 5*A*d*e^4)*x^
4 + 4*(3*B*d^3*e^2 - 5*A*d^2*e^3)*x^2 + 3*((D*d^3*e + C*d^2*e^2 - 3*B*d*e^
3 + 5*A*e^4)*x^5 + (D*d^4 + C*d^3*e - 3*B*d^2*e^2 + 5*A*d*e^3)*x^3)*sqrt(-
d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)))/(d^4*e^3*x^5 + d^5*e^2
*x^3), -1/6*(2*A*d^3*e^2 + 3*(D*d^4*e - C*d^3*e^2 + 3*B*d^2*e^3 - 5*A*d*e^
4)*x^4 + 2*(3*B*d^3*e^2 - 5*A*d^2*e^3)*x^2 - 3*((D*d^3*e + C*d^2*e^2 - 3*B
*d*e^3 + 5*A*e^4)*x^5 + (D*d^4 + C*d^3*e - 3*B*d^2*e^2 + 5*A*d*e^3)*x^3)*s
qrt(d*e)*arctan(sqrt(d*e)*x/d))/(d^4*e^3*x^5 + d^5*e^2*x^3)]
```

## Sympy [A] (verification not implemented)

Time = 4.81 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.70

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4\left(d+ex^2\right)^2}\,dx$$
$$= -\frac{\sqrt{-\frac{1}{d^7e^3}}\cdot(5Ae^3-3Bde^2+Cd^2e+Dd^3)\log\left(-d^4e\sqrt{-\frac{1}{d^7e^3}}+x\right)}{4}$$
$$+\frac{\sqrt{-\frac{1}{d^7e^3}}\cdot(5Ae^3-3Bde^2+Cd^2e+Dd^3)\log\left(d^4e\sqrt{-\frac{1}{d^7e^3}}+x\right)}{4}$$
$$+\frac{-2Ad^2e+x^4\cdot(15Ae^3-9Bde^2+3Cd^2e-3Dd^3)+x^2\cdot(10Ade^2-6Bd^2e)}{6d^4ex^3+6d^3e^2x^5}$$

input
```
integrate((D*x**6+C*x**4+B*x**2+A)/x**4/(e*x**2+d)**2,x)
```

output
```
-sqrt(-1/(d**7*e**3))*(5*A*e**3 - 3*B*d*e**2 + C*d**2*e + D*d**3)*log(-d**
4*e*sqrt(-1/(d**7*e**3)) + x)/4 + sqrt(-1/(d**7*e**3))*(5*A*e**3 - 3*B*d*e
**2 + C*d**2*e + D*d**3)*log(d**4*e*sqrt(-1/(d**7*e**3)) + x)/4 + (-2*A*d*
*2*e + x**4*(15*A*e**3 - 9*B*d*e**2 + 3*C*d**2*e - 3*D*d**3) + x**2*(10*A*
d*e**2 - 6*B*d**2*e))/(6*d**4*e*x**3 + 6*d**3*e**2*x**5)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4\left(d+ex^2\right)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^4/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.96

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4\left(d+ex^2\right)^2}\,dx = \frac{(Dd^3+Cd^2e-3\,Bde^2+5\,Ae^3)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\,\sqrt{de}d^3e} - \frac{Dd^3x-Cd^2ex+Bde^2x-Ae^3x}{2\,(ex^2+d)d^3e} - \frac{3\,Bdx^2-6\,Aex^2+Ad}{3\,d^3x^3}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^4/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
1/2*(D*d^3 + C*d^2*e - 3*B*d*e^2 + 5*A*e^3)*arctan(e*x/sqrt(d*e))/(sqrt(d*
e)*d^3*e) - 1/2*(D*d^3*x - C*d^2*e*x + B*d*e^2*x - A*e^3*x)/((e*x^2 + d)*d
^3*e) - 1/3*(3*B*d*x^2 - 6*A*e*x^2 + A*d)/(d^3*x^3)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4\left(d+ex^2\right)^2}\,dx = \int \frac{A+B\,x^2+C\,x^4+x^6\,D}{x^4\,\left(e\,x^2+d\right)^2}\,dx$$

input
```
int((A + B*x^2 + C*x^4 + x^6*D)/(x^4*(d + e*x^2)^2),x)
```

output
```
int((A + B*x^2 + C*x^4 + x^6*D)/(x^4*(d + e*x^2)^2), x)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 305, normalized size of antiderivative = 2.44

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4\left(d+ex^2\right)^2}\,dx$$

$$= \frac{15\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right) ad\,e^3x^3 + 15\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right) a\,e^4x^5 - 9\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right) b\,d^2e^2x^3 - 9\sqrt{e}\,\sqrt{}}{}$$

input
```
int((D*x^6+C*x^4+B*x^2+A)/x^4/(e*x^2+d)^2,x)
```

output
```
(15*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*d*e**3*x**3 + 15*sqrt(
e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*e**4*x**5 - 9*sqrt(e)*sqrt(d)*a
tan((e*x)/(sqrt(e)*sqrt(d)))*b*d**2*e**2*x**3 - 9*sqrt(e)*sqrt(d)*atan((e*
x)/(sqrt(e)*sqrt(d)))*b*d*e**3*x**5 + 3*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e
)*sqrt(d)))*c*d**3*e*x**3 + 3*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d))
)*c*d**2*e**2*x**5 + 3*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*d**5*
x**3 + 3*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*d**4*e*x**5 - 2*a*d
**3*e**2 + 10*a*d**2*e**3*x**2 + 15*a*d*e**4*x**4 - 6*b*d**3*e**2*x**2 - 9
*b*d**2*e**3*x**4 + 3*c*d**3*e**2*x**4 - 3*d**5*e*x**4)/(6*d**4*e**2*x**3*
(d + e*x**2))
```

# 3.176 $\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6(d+ex^2)^2}\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 151

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6(d+ex^2)^2}\,dx = -\frac{A}{5d^2x^5}-\frac{Bd-2Ae}{3d^3x^3}-\frac{Cd^2-e(2Bd-3Ae)}{d^4x}+\frac{(d^3D-Cd^2e+Bde^2-Ae^3)\,x}{2d^4(d+ex^2)}+\frac{(d^3D-3Cd^2e+5Bde^2-7Ae^3)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{9/2}\sqrt{e}}$$

output
```
-1/5*A/d^2/x^5-1/3*(-2*A*e+B*d)/d^3/x^3-(C*d^2-e*(-3*A*e+2*B*d))/d^4/x+1/2
*(-A*e^3+B*d*e^2-C*d^2*e+D*d^3)*x/d^4/(e*x^2+d)+1/2*(-7*A*e^3+5*B*d*e^2-3*
C*d^2*e+D*d^3)*arctan(e^(1/2)*x/d^(1/2))/d^(9/2)/e^(1/2)
```

## Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6\left(d+ex^2\right)^2}\,dx=-\frac{A}{5d^2x^5}+\frac{-Bd+2Ae}{3d^3x^3}+\frac{-Cd^2+2Bde-3Ae^2}{d^4x}+\frac{\left(d^3D-Cd^2e+Bde^2-Ae^3\right)x}{2d^4\left(d+ex^2\right)}+\frac{\left(d^3D-3Cd^2e+5Bde^2-7Ae^3\right)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{9/2}\sqrt{e}}$$

input
```
Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(x^6*(d + e*x^2)^2),x]
```

output
```
-1/5*A/(d^2*x^5) + (-(B*d) + 2*A*e)/(3*d^3*x^3) + (-(C*d^2) + 2*B*d*e - 3*
A*e^2)/(d^4*x) + ((d^3*D - C*d^2*e + B*d*e^2 - A*e^3)*x)/(2*d^4*(d + e*x^2
)) + ((d^3*D - 3*C*d^2*e + 5*B*d*e^2 - 7*A*e^3)*ArcTan[(Sqrt[e]*x)/Sqrt[d]
])/(2*d^(9/2)*Sqrt[e])
```

## Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2336, 25, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6\left(d+ex^2\right)^2}\,dx$$

$$\Big\downarrow \text{2336}$$

$$\frac{x\left(-Ae^3+Bde^2-Cd^2e+d^3D\right)}{2d^4\left(d+ex^2\right)}-\frac{\int -\frac{\frac{\left(Dd^3-Ced^2+Be^2d-Ae^3\right)x^6}{d^3}+2\left(C-\frac{e(Bd-Ae)}{d^2}\right)x^4+2\left(B-\frac{Ae}{d}\right)x^2+2A}{x^6\left(ex^2+d\right)}dx}{2d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\frac{\left(Dd^3-Ced^2+Be^2d-Ae^3\right)x^6}{d^3}+2\left(C-\frac{e(Bd-Ae)}{d^2}\right)x^4+2\left(B-\frac{Ae}{d}\right)x^2+2A}{x^6(ex^2+d)}dx}{2d}+\frac{x\left(-Ae^3+Bde^2-Cd^2e+d^3D\right)}{2d^4\left(d+ex^2\right)}$$

$$\Big\downarrow \text{2333}$$

$$\frac{\int \left(\frac{2A}{dx^6}+\frac{Dd^3-3Ced^2+5Be^2d-7Ae^3}{d^3(ex^2+d)}+\frac{2(Cd^2-e(2Bd-3Ae))}{d^3x^2}+\frac{2(Bd-2Ae)}{d^2x^4}\right)dx}{2d}+\frac{x\left(-Ae^3+Bde^2-Cd^2e+d^3D\right)}{2d^4\left(d+ex^2\right)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(-7Ae^3+5Bde^2-3Cd^2e+d^3D\right)}{d^{7/2}\sqrt{e}}-\frac{2(Cd^2-e(2Bd-3Ae))}{d^3x}-\frac{2(Bd-2Ae)}{3d^2x^3}-\frac{2A}{5dx^5}}{2d}+\frac{x\left(-Ae^3+Bde^2-Cd^2e+d^3D\right)}{2d^4\left(d+ex^2\right)}$$

input
```
Int[(A + B*x^2 + C*x^4 + D*x^6)/(x^6*(d + e*x^2)^2),x]
```

output
```
((d^3*D - C*d^2*e + B*d*e^2 - A*e^3)*x)/(2*d^4*(d + e*x^2)) + ((-2*A)/(5*d
*x^5) - (2*(B*d - 2*A*e))/(3*d^2*x^3) - (2*(C*d^2 - e*(2*B*d - 3*A*e)))/(d
^3*x) + ((d^3*D - 3*C*d^2*e + 5*B*d*e^2 - 7*A*e^3)*ArcTan[(Sqrt[e]*x)/Sqrt
[d]])/(d^(7/2)*Sqrt[e]))/(2*d)
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

## Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.92

| method | result |
|---|---|
| default | $-\frac{\frac{\left(\frac{1}{2}A\,e^{3}-\frac{1}{2}B d\,e^{2}+\frac{1}{2}C e\,d^{2}-\frac{1}{2}D d^{3}\right)x}{e\,x^{2}+d}+\frac{\left(7A\,e^{3}-5B d\,e^{2}+3C e\,d^{2}-D d^{3}\right)\arctan\left(\frac{e x}{\sqrt{d e}}\right)}{2\sqrt{d e}}}{d^{4}}-\frac{A}{5d^{2}x^{5}}-\frac{-2Ae+Bd}{3d^{3}x^{3}}-\frac{3A\,e^{2}-2dBe+d^{2}C}{d^{4}x}$ |

input
```
int((D*x^6+C*x^4+B*x^2+A)/x^6/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/d^4*((1/2*A*e^3-1/2*B*d*e^2+1/2*C*e*d^2-1/2*D*d^3)*x/(e*x^2+d)+1/2*(7*A
*e^3-5*B*d*e^2+3*C*d^2*e-D*d^3)/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2)))-1/5*A
/d^2/x^5-1/3*(-2*A*e+B*d)/d^3/x^3-(3*A*e^2-2*B*d*e+C*d^2)/d^4/x
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 432, normalized size of antiderivative = 2.86

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6\left(d+ex^2\right)^2}\,dx$$
$$=\left[\frac{30\left(Dd^4e-3\,Cd^3e^2+5\,Bd^2e^3-7\,Ade^4\right)x^6-12\,Ad^4e-20\left(3\,Cd^4e-5\,Bd^3e^2+7\,Ad^2e^3\right)x^4-4\left(5\,Bd\right.}{}\right.$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^6/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
[1/60*(30*(D*d^4*e - 3*C*d^3*e^2 + 5*B*d^2*e^3 - 7*A*d*e^4)*x^6 - 12*A*d^4
*e - 20*(3*C*d^4*e - 5*B*d^3*e^2 + 7*A*d^2*e^3)*x^4 - 4*(5*B*d^4*e - 7*A*d
^3*e^2)*x^2 + 15*((D*d^3*e - 3*C*d^2*e^2 + 5*B*d*e^3 - 7*A*e^4)*x^7 + (D*d
^4 - 3*C*d^3*e + 5*B*d^2*e^2 - 7*A*d*e^3)*x^5)*sqrt(-d*e)*log((e*x^2 + 2*s
qrt(-d*e)*x - d)/(e*x^2 + d)))/(d^5*e^2*x^7 + d^6*e*x^5), 1/30*(15*(D*d^4*
e - 3*C*d^3*e^2 + 5*B*d^2*e^3 - 7*A*d*e^4)*x^6 - 6*A*d^4*e - 10*(3*C*d^4*e
 - 5*B*d^3*e^2 + 7*A*d^2*e^3)*x^4 - 2*(5*B*d^4*e - 7*A*d^3*e^2)*x^2 + 15*(
(D*d^3*e - 3*C*d^2*e^2 + 5*B*d*e^3 - 7*A*e^4)*x^7 + (D*d^4 - 3*C*d^3*e + 5
*B*d^2*e^2 - 7*A*d*e^3)*x^5)*sqrt(d*e)*arctan(sqrt(d*e)*x/d))/(d^5*e^2*x^7
 + d^6*e*x^5)]
```

## Sympy [A] (verification not implemented)

Time = 11.95 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.50

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6\left(d+ex^2\right)^2}\,dx$$
$$= -\frac{\sqrt{-\frac{1}{d^9e}}\left(-7Ae^3+5Bde^2-3Cd^2e+Dd^3\right)\log\left(-d^5\sqrt{-\frac{1}{d^9e}}+x\right)}{4}$$
$$+\frac{\sqrt{-\frac{1}{d^9e}}\left(-7Ae^3+5Bde^2-3Cd^2e+Dd^3\right)\log\left(d^5\sqrt{-\frac{1}{d^9e}}+x\right)}{4}$$
$$+\frac{-6Ad^3+x^6(-105Ae^3+75Bde^2-45Cd^2e+15Dd^3)+x^4(-70Ade^2+50Bd^2e-30Cd^3)+x^2\cdot(14A}{30d^5x^5+30d^4ex^7}$$

input
```
integrate((D*x**6+C*x**4+B*x**2+A)/x**6/(e*x**2+d)**2,x)
```

output
```
-sqrt(-1/(d**9*e))*(-7*A*e**3 + 5*B*d*e**2 - 3*C*d**2*e + D*d**3)*log(-d**
5*sqrt(-1/(d**9*e)) + x)/4 + sqrt(-1/(d**9*e))*(-7*A*e**3 + 5*B*d*e**2 - 3
*C*d**2*e + D*d**3)*log(d**5*sqrt(-1/(d**9*e)) + x)/4 + (-6*A*d**3 + x**6*
(-105*A*e**3 + 75*B*d*e**2 - 45*C*d**2*e + 15*D*d**3) + x**4*(-70*A*d*e**2
 + 50*B*d**2*e - 30*C*d**3) + x**2*(14*A*d**2*e - 10*B*d**3))/(30*d**5*x**
5 + 30*d**4*e*x**7)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6\left(d+ex^2\right)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^6/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.97

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6\left(d+ex^2\right)^2}\,dx = \frac{\left(Dd^3-3\,Cd^2e+5\,Bde^2-7\,Ae^3\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\,\sqrt{de}d^4} + \frac{Dd^3x-Cd^2ex+Bde^2x-Ae^3x}{2\,(ex^2+d)d^4} - \frac{15\,Cd^2x^4-30\,Bdex^4+45\,Ae^2x^4+5\,Bd^2x^2-10\,Adex^2+3\,Ad^2}{15\,d^4x^5}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^6/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
1/2*(D*d^3 - 3*C*d^2*e + 5*B*d*e^2 - 7*A*e^3)*arctan(e*x/sqrt(d*e))/(sqrt(
d*e)*d^4) + 1/2*(D*d^3*x - C*d^2*e*x + B*d*e^2*x - A*e^3*x)/((e*x^2 + d)*d
^4) - 1/15*(15*C*d^2*x^4 - 30*B*d*e*x^4 + 45*A*e^2*x^4 + 5*B*d^2*x^2 - 10*
A*d*e*x^2 + 3*A*d^2)/(d^4*x^5)
```

## Mupad [B] (verification not implemented)

Time = 1.51 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.34

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6\left(d+ex^2\right)^2}\,dx = \frac{\frac{5\,B\,e^2\,x^4}{2\,d^3}-\frac{B}{3\,d}+\frac{5\,B\,e\,x^2}{3\,d^2}}{e\,x^5+d\,x^3}-\frac{\frac{A}{5\,d}-\frac{7\,A\,e\,x^2}{15\,d^2}+\frac{7\,A\,e^2\,x^4}{3\,d^3}+\frac{7\,A\,e^3\,x^6}{2\,d^4}}{e\,x^7+d\,x^5} - \frac{\frac{C}{d}+\frac{3\,C\,e\,x^2}{2\,d^2}}{e\,x^3+d\,x}+\frac{x\,D\,{}_2\mathrm{F}_1\left(\frac{1}{2},2;\ \frac{3}{2};\ -\frac{e\,x^2}{d}\right)}{d^2} - \frac{7\,A\,e^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2\,d^{9/2}}+\frac{5\,B\,e^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2\,d^{7/2}} - \frac{3\,C\,\sqrt{e}\,\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2\,d^{5/2}}$$

input
```
int((A + B*x^2 + C*x^4 + x^6*D)/(x^6*(d + e*x^2)^2),x)
```

output
```
((5*B*e^2*x^4)/(2*d^3) - B/(3*d) + (5*B*e*x^2)/(3*d^2))/(d*x^3 + e*x^5) -
(A/(5*d) - (7*A*e*x^2)/(15*d^2) + (7*A*e^2*x^4)/(3*d^3) + (7*A*e^3*x^6)/(2
*d^4))/(d*x^5 + e*x^7) - (C/d + (3*C*e*x^2)/(2*d^2))/(d*x + e*x^3) + (x*D*
hypergeom([1/2, 2], 3/2, -(e*x^2)/d))/d^2 - (7*A*e^(5/2)*atan((e^(1/2)*x)/
d^(1/2)))/(2*d^(9/2)) + (5*B*e^(3/2)*atan((e^(1/2)*x)/d^(1/2)))/(2*d^(7/2)
) - (3*C*e^(1/2)*atan((e^(1/2)*x)/d^(1/2)))/(2*d^(5/2))
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 335, normalized size of antiderivative = 2.22

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6\left(d+ex^2\right)^2}\,dx = \frac{-105\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)ad\,e^3x^5-105\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)a\,e^4x^7+75\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)b\,d^2e^2x^5+7}{}$$

input
```
int((D*x^6+C*x^4+B*x^2+A)/x^6/(e*x^2+d)^2,x)
```

output
```
( - 105*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*d*e**3*x**5 - 105*
sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*e**4*x**7 + 75*sqrt(e)*sqr
t(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*b*d**2*e**2*x**5 + 75*sqrt(e)*sqrt(d)*a
tan((e*x)/(sqrt(e)*sqrt(d)))*b*d*e**3*x**7 - 45*sqrt(e)*sqrt(d)*atan((e*x)
/(sqrt(e)*sqrt(d)))*c*d**3*e*x**5 - 45*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)
*sqrt(d)))*c*d**2*e**2*x**7 + 15*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(
d)))*d**5*x**5 + 15*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*d**4*e*x
**7 - 6*a*d**4*e + 14*a*d**3*e**2*x**2 - 70*a*d**2*e**3*x**4 - 105*a*d*e**
4*x**6 - 10*b*d**4*e*x**2 + 50*b*d**3*e**2*x**4 + 75*b*d**2*e**3*x**6 - 30
*c*d**4*e*x**4 - 45*c*d**3*e**2*x**6 + 15*d**5*e*x**6)/(30*d**5*e*x**5*(d
+ e*x**2))
```

# 3.177 $\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8(d+ex^2)^2}\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 189

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8(d+ex^2)^2}\,dx = -\frac{A}{7d^2x^7} - \frac{Bd-2Ae}{5d^3x^5} - \frac{Cd^2-e(2Bd-3Ae)}{3d^4x^3} - \frac{d^3D-2Cd^2e+3Bde^2-4Ae^3}{d^5x} - \frac{e(d^3D-Cd^2e+Bde^2-Ae^3)\,x}{2d^5(d+ex^2)} - \frac{\sqrt{e}(3d^3D-5Cd^2e+7Bde^2-9Ae^3)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{11/2}}$$

output
```
-1/7*A/d^2/x^7-1/5*(-2*A*e+B*d)/d^3/x^5-1/3*(C*d^2-e*(-3*A*e+2*B*d))/d^4/x
^3-(-4*A*e^3+3*B*d*e^2-2*C*d^2*e+D*d^3)/d^5/x-1/2*e*(-A*e^3+B*d*e^2-C*d^2*
e+D*d^3)*x/d^5/(e*x^2+d)-1/2*e^(1/2)*(-9*A*e^3+7*B*d*e^2-5*C*d^2*e+3*D*d^3
)*arctan(e^(1/2)*x/d^(1/2))/d^(11/2)
```

## Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.01

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8\left(d+ex^2\right)^2}\,dx = -\frac{A}{7d^2x^7}+\frac{-Bd+2Ae}{5d^3x^5}+\frac{-Cd^2+2Bde-3Ae^2}{3d^4x^3} + \frac{-d^3D+2Cd^2e-3Bde^2+4Ae^3}{d^5x} - \frac{e(d^3D-Cd^2e+Bde^2-Ae^3)\,x}{2d^5\left(d+ex^2\right)} - \frac{\sqrt{e}(3d^3D-5Cd^2e+7Bde^2-9Ae^3)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{11/2}}$$

input
```
Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(x^8*(d + e*x^2)^2),x]
```

output
```
-1/7*A/(d^2*x^7) + (-(B*d) + 2*A*e)/(5*d^3*x^5) + (-(C*d^2) + 2*B*d*e - 3*
A*e^2)/(3*d^4*x^3) + (-(d^3*D) + 2*C*d^2*e - 3*B*d*e^2 + 4*A*e^3)/(d^5*x)
- (e*(d^3*D - C*d^2*e + B*d*e^2 - A*e^3)*x)/(2*d^5*(d + e*x^2)) - (Sqrt[e]
*(3*d^3*D - 5*C*d^2*e + 7*B*d*e^2 - 9*A*e^3)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/
(2*d^(11/2))
```

## Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2336, 25, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8\left(d+ex^2\right)^2}dx$$

$$\Big\downarrow \text{ 2336}$$

$$-\frac{\int -\frac{-\frac{e\left(Dd^3-Ced^2+Be^2d-Ae^3\right)x^8}{d^4}+\frac{2\left(Dd^3-Ced^2+Be^2d-Ae^3\right)x^6}{d^3}+2\left(C-\frac{e(Bd-Ae)}{d^2}\right)x^4+2\left(B-\frac{Ae}{d}\right)x^2+2A}{x^8(ex^2+d)}dx}{2d}-\frac{ex\left(-Ae^3+Bde^2-Cd^2e+d^3D\right)}{2d^5\left(d+ex^2\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{-\frac{e\left(Dd^3-Ced^2+Be^2d-Ae^3\right)x^8}{d^4}+\frac{2\left(Dd^3-Ced^2+Be^2d-Ae^3\right)x^6}{d^3}+2\left(C-\frac{e(Bd-Ae)}{d^2}\right)x^4+2\left(B-\frac{Ae}{d}\right)x^2+2A}{x^8(ex^2+d)}dx}{2d}-\frac{ex\left(-Ae^3+Bde^2-Cd^2e+d^3D\right)}{2d^5\left(d+ex^2\right)}$$

$$\Big\downarrow \text{2333}$$

$$\frac{\int \left(\frac{2A}{dx^8}-\frac{e(3Dd^3-5Ced^2+7Be^2d-9Ae^3)}{d^4(ex^2+d)}+\frac{2(Dd^3-2Ced^2+3Be^2d-4Ae^3)}{d^4x^2}+\frac{2(Cd^2-e(2Bd-3Ae))}{d^3x^4}+\frac{2(Bd-2Ae)}{d^2x^6}\right)dx}{2d}-\frac{ex\left(-Ae^3+Bde^2-Cd^2e+d^3D\right)}{2d^5\left(d+ex^2\right)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{\sqrt{e}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(-9Ae^3+7Bde^2-5Cd^2e+3d^3D)}{d^{9/2}}-\frac{2(Cd^2-e(2Bd-3Ae))}{3d^3x^3}-\frac{2(-4Ae^3+3Bde^2-2Cd^2e+d^3D)}{d^4x}-\frac{2(Bd-2Ae)}{5d^2x^5}-\frac{2A}{7dx^7}}{2d}-\frac{ex\left(-Ae^3+Bde^2-Cd^2e+d^3D\right)}{2d^5\left(d+ex^2\right)}$$

input
```
Int[(A + B*x^2 + C*x^4 + D*x^6)/(x^8*(d + e*x^2)^2),x]
```

output
```
-1/2*(e*(d^3*D - C*d^2*e + B*d*e^2 - A*e^3)*x)/(d^5*(d + e*x^2)) + ((-2*A)
/(7*d*x^7) - (2*(B*d - 2*A*e))/(5*d^2*x^5) - (2*(C*d^2 - e*(2*B*d - 3*A*e)
))/(3*d^3*x^3) - (2*(d^3*D - 2*C*d^2*e + 3*B*d*e^2 - 4*A*e^3))/(d^4*x) - (
Sqrt[e]*(3*d^3*D - 5*C*d^2*e + 7*B*d*e^2 - 9*A*e^3)*ArcTan[(Sqrt[e]*x)/Sqr
t[d]])/d^(9/2))/(2*d)
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

## Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.92

| method | result |
|---|---|
| default | $\frac{e\left(\frac{\left(\frac{1}{2}A\,e^{3}-\frac{1}{2}Bd\,e^{2}+\frac{1}{2}Ce\,d^{2}-\frac{1}{2}Dd^{3}\right)x}{e\,x^{2}+d}+\frac{\left(9A\,e^{3}-7Bd\,e^{2}+5Ce\,d^{2}-3Dd^{3}\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}}\right)}{d^{5}}-\frac{A}{7d^{2}x^{7}}-\frac{-2Ae+Bd}{5d^{3}x^{5}}-\frac{3A\,e^{2}-2dBe+}{3d^{4}x^{3}}$ |

input
```
int((D*x^6+C*x^4+B*x^2+A)/x^8/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
e/d^5*((1/2*A*e^3-1/2*B*d*e^2+1/2*C*e*d^2-1/2*D*d^3)*x/(e*x^2+d)+1/2*(9*A*
e^3-7*B*d*e^2+5*C*d^2*e-3*D*d^3)/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2)))-1/7*
A/d^2/x^7-1/5*(-2*A*e+B*d)/d^3/x^5-1/3*(3*A*e^2-2*B*d*e+C*d^2)/d^4/x^3-(-4
*A*e^3+3*B*d*e^2-2*C*d^2*e+D*d^3)/d^5/x
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 488, normalized size of antiderivative = 2.58

$$\int \frac{A + Bx^2 + Cx^4 + Dx^6}{x^8 \left(d + ex^2\right)^2}\, dx$$
$$= \left[ -\frac{210\,(3\,Dd^3e - 5\,Cd^2e^2 + 7\,Bde^3 - 9\,Ae^4)x^8 + 140\,(3\,Dd^4 - 5\,Cd^3e + 7\,Bd^2e^2 - 9\,Ade^3)x^6 + 60\,Ad^4}{} \right.$$

$$-\frac{105\,(3\,Dd^3e - 5\,Cd^2e^2 + 7\,Bde^3 - 9\,Ae^4)x^8 + 70\,(3\,Dd^4 - 5\,Cd^3e + 7\,Bd^2e^2 - 9\,Ade^3)x^6 + 30\,Ad^4 +}{}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^8/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
[-1/420*(210*(3*D*d^3*e - 5*C*d^2*e^2 + 7*B*d*e^3 - 9*A*e^4)*x^8 + 140*(3*
D*d^4 - 5*C*d^3*e + 7*B*d^2*e^2 - 9*A*d*e^3)*x^6 + 60*A*d^4 + 28*(5*C*d^4
- 7*B*d^3*e + 9*A*d^2*e^2)*x^4 + 12*(7*B*d^4 - 9*A*d^3*e)*x^2 + 105*((3*D*
d^3*e - 5*C*d^2*e^2 + 7*B*d*e^3 - 9*A*e^4)*x^9 + (3*D*d^4 - 5*C*d^3*e + 7*
B*d^2*e^2 - 9*A*d*e^3)*x^7)*sqrt(-e/d)*log((e*x^2 + 2*d*x*sqrt(-e/d) - d)/
(e*x^2 + d)))/(d^5*e*x^9 + d^6*x^7), -1/210*(105*(3*D*d^3*e - 5*C*d^2*e^2
+ 7*B*d*e^3 - 9*A*e^4)*x^8 + 70*(3*D*d^4 - 5*C*d^3*e + 7*B*d^2*e^2 - 9*A*d
*e^3)*x^6 + 30*A*d^4 + 14*(5*C*d^4 - 7*B*d^3*e + 9*A*d^2*e^2)*x^4 + 6*(7*B
*d^4 - 9*A*d^3*e)*x^2 + 105*((3*D*d^3*e - 5*C*d^2*e^2 + 7*B*d*e^3 - 9*A*e^
4)*x^9 + (3*D*d^4 - 5*C*d^3*e + 7*B*d^2*e^2 - 9*A*d*e^3)*x^7)*sqrt(e/d)*ar
ctan(x*sqrt(e/d)))/(d^5*e*x^9 + d^6*x^7)]
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 394 vs. $2(185) = 370$.

Time = 31.08 (sec) , antiderivative size = 394, normalized size of antiderivative = 2.08

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8\left(d+ex^2\right)^2}\,dx = \frac{\sqrt{-\frac{e}{d^{11}}}\left(-9Ae^3+7Bde^2-5Cd^2e+3Dd^3\right)\log\left(-\frac{d^6\sqrt{-\frac{e}{d^{11}}}\left(-9Ae^3+7Bde^2-5Cd^2e+3Dd^3\right)}{-9Ae^4+7Bde^3-5Cd^2e^2+3Dd^3e}+x\right)}{4} - \frac{\sqrt{-\frac{e}{d^{11}}}\left(-9Ae^3+7Bde^2-5Cd^2e+3Dd^3\right)\log\left(\frac{d^6\sqrt{-\frac{e}{d^{11}}}\left(-9Ae^3+7Bde^2-5Cd^2e+3Dd^3\right)}{-9Ae^4+7Bde^3-5Cd^2e^2+3Dd^3e}+x\right)}{4} + \frac{-30Ad^4+x^8\cdot\left(945Ae^4-735Bde^3+525Cd^2e^2-315Dd^3e\right)+x^6\cdot\left(630Ade^3-490Bd^2e^2+350Cd^3e\right.}{210d^6x^7+210d^5ex^9}$$

input
```
integrate((D*x**6+C*x**4+B*x**2+A)/x**8/(e*x**2+d)**2,x)
```

output
```
sqrt(-e/d**11)*(-9*A*e**3 + 7*B*d*e**2 - 5*C*d**2*e + 3*D*d**3)*log(-d**6*
sqrt(-e/d**11)*(-9*A*e**3 + 7*B*d*e**2 - 5*C*d**2*e + 3*D*d**3)/(-9*A*e**4
 + 7*B*d*e**3 - 5*C*d**2*e**2 + 3*D*d**3*e) + x)/4 - sqrt(-e/d**11)*(-9*A*
e**3 + 7*B*d*e**2 - 5*C*d**2*e + 3*D*d**3)*log(d**6*sqrt(-e/d**11)*(-9*A*e
**3 + 7*B*d*e**2 - 5*C*d**2*e + 3*D*d**3)/(-9*A*e**4 + 7*B*d*e**3 - 5*C*d*
*2*e**2 + 3*D*d**3*e) + x)/4 + (-30*A*d**4 + x**8*(945*A*e**4 - 735*B*d*e*
*3 + 525*C*d**2*e**2 - 315*D*d**3*e) + x**6*(630*A*d*e**3 - 490*B*d**2*e**
2 + 350*C*d**3*e - 210*D*d**4) + x**4*(-126*A*d**2*e**2 + 98*B*d**3*e - 70
*C*d**4) + x**2*(54*A*d**3*e - 42*B*d**4))/(210*d**6*x**7 + 210*d**5*e*x**
9)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8\left(d+ex^2\right)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^8/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.04

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8(d+ex^2)^2}\,dx = -\frac{(3\,Dd^3e-5\,Cd^2e^2+7\,Bde^3-9\,Ae^4)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}d^5} - \frac{Dd^3ex-Cd^2e^2x+Bde^3x-Ae^4x}{2\,(ex^2+d)d^5} - \frac{105\,Dd^3x^6-210\,Cd^2ex^6+315\,Bde^2x^6-420\,Ae^3x^6+35\,Cd^3x^4-70\,Bd^2ex^4+105\,Ade^2x^4+21\,Bd^3}{105\,d^5x^7}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^8/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
-1/2*(3*D*d^3*e - 5*C*d^2*e^2 + 7*B*d*e^3 - 9*A*e^4)*arctan(e*x/sqrt(d*e))
/(sqrt(d*e)*d^5) - 1/2*(D*d^3*e*x - C*d^2*e^2*x + B*d*e^3*x - A*e^4*x)/((e
*x^2 + d)*d^5) - 1/105*(105*D*d^3*x^6 - 210*C*d^2*e*x^6 + 315*B*d*e^2*x^6
- 420*A*e^3*x^6 + 35*C*d^3*x^4 - 70*B*d^2*e*x^4 + 105*A*d*e^2*x^4 + 21*B*d
^3*x^2 - 42*A*d^2*e*x^2 + 15*A*d^3)/(d^5*x^7)
```

## Mupad [B] (verification not implemented)

Time = 1.55 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.28

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8(d+ex^2)^2}\,dx = \frac{\frac{5\,C\,e^2\,x^4}{2\,d^3}-\frac{C}{3\,d}+\frac{5\,C\,e\,x^2}{3\,d^2}}{e\,x^5+d\,x^3} - \frac{\frac{B}{5\,d}+\frac{7\,B\,e^2\,x^4}{3\,d^3}+\frac{7\,B\,e^3\,x^6}{2\,d^4}-\frac{7\,B\,e\,x^2}{15\,d^2}}{e\,x^7+d\,x^5} - \frac{D\,{}_2\mathrm{F}_1\left(2,\frac{5}{2};\ \frac{7}{2};\ -\frac{d}{e\,x^2}\right)}{5\,e^2\,x^5} + \frac{9\,A\,e^{7/2}\,\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2\,d^{11/2}} - \frac{7\,B\,e^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2\,d^{9/2}} + \frac{5\,C\,e^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2\,d^{7/2}} + \frac{-\frac{A\,d^4}{7}+\frac{9\,A\,d^3\,e\,x^2}{35}-\frac{3\,A\,d^2\,e^2\,x^4}{5}+3\,A\,d\,e^3\,x^6+\frac{9\,A\,e^4\,x^8}{2}}{d^5\,x^7\,\left(e\,x^2+d\right)}$$

input
```
int((A + B*x^2 + C*x^4 + x^6*D)/(x^8*(d + e*x^2)^2),x)
```

output
```
((5*C*e^2*x^4)/(2*d^3) - C/(3*d) + (5*C*e*x^2)/(3*d^2))/(d*x^3 + e*x^5) -
(B/(5*d) + (7*B*e^2*x^4)/(3*d^3) + (7*B*e^3*x^6)/(2*d^4) - (7*B*e*x^2)/(15
*d^2))/(d*x^5 + e*x^7) - (D*hypergeom([2, 5/2], 7/2, -d/(e*x^2)))/(5*e^2*x
^5) + (9*A*e^(7/2)*atan((e^(1/2)*x)/d^(1/2)))/(2*d^(11/2)) - (7*B*e^(5/2)*
atan((e^(1/2)*x)/d^(1/2)))/(2*d^(9/2)) + (5*C*e^(3/2)*atan((e^(1/2)*x)/d^(
1/2)))/(2*d^(7/2)) + ((9*A*e^4*x^8)/2 - (A*d^4)/7 + (9*A*d^3*e*x^2)/35 + 3
*A*d*e^3*x^6 - (3*A*d^2*e^2*x^4)/5)/(d^5*x^7*(d + e*x^2))
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.94

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8\left(d+ex^2\right)^2}\,dx$$
$$= \frac{945\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)ad\,e^3x^7+945\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)a\,e^4x^9-735\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)b\,d^2e^2x^7-73}{}$$

input
```
int((D*x^6+C*x^4+B*x^2+A)/x^8/(e*x^2+d)^2,x)
```

output
```
(945*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*d*e**3*x**7 + 945*sqr
t(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*e**4*x**9 - 735*sqrt(e)*sqrt(
d)*atan((e*x)/(sqrt(e)*sqrt(d)))*b*d**2*e**2*x**7 - 735*sqrt(e)*sqrt(d)*at
an((e*x)/(sqrt(e)*sqrt(d)))*b*d*e**3*x**9 + 525*sqrt(e)*sqrt(d)*atan((e*x)
/(sqrt(e)*sqrt(d)))*c*d**3*e*x**7 + 525*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e
)*sqrt(d)))*c*d**2*e**2*x**9 - 315*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqr
t(d)))*d**5*x**7 - 315*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*d**4*
e*x**9 - 30*a*d**5 + 54*a*d**4*e*x**2 - 126*a*d**3*e**2*x**4 + 630*a*d**2*
e**3*x**6 + 945*a*d*e**4*x**8 - 42*b*d**5*x**2 + 98*b*d**4*e*x**4 - 490*b*
d**3*e**2*x**6 - 735*b*d**2*e**3*x**8 - 70*c*d**5*x**4 + 350*c*d**4*e*x**6
 + 525*c*d**3*e**2*x**8 - 210*d**6*x**6 - 315*d**5*e*x**8)/(210*d**6*x**7*
(d + e*x**2))
```

# 3.178 $\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{10}(a+bx^2)^2}\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 234

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{10}(a+bx^2)^2}\,dx = -\frac{A}{9a^2x^9}+\frac{2Ab-aB}{7a^3x^7}-\frac{3Ab^2-a(2bB-aC)}{5a^4x^5} + \frac{4Ab^3-a(3b^2B-2abC+a^2D)}{3a^5x^3} - \frac{b(5Ab^3-a(4b^2B-3abC+2a^2D))}{a^6x} - \frac{b^2(Ab^3-a(b^2B-abC+a^2D))\,x}{2a^6(a+bx^2)} - \frac{b^{3/2}(11Ab^3-a(9b^2B-7abC+5a^2D))\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{13/2}}$$

output
```
-1/9*A/a^2/x^9+1/7*(2*A*b-B*a)/a^3/x^7-1/5*(3*A*b^2-a*(2*B*b-C*a))/a^4/x^5
+1/3*(4*A*b^3-a*(3*B*b^2-2*C*a*b+D*a^2))/a^5/x^3-b*(5*A*b^3-a*(4*B*b^2-3*C
*a*b+2*D*a^2))/a^6/x-1/2*b^2*(A*b^3-a*(B*b^2-C*a*b+D*a^2))*x/a^6/(b*x^2+a)
-1/2*b^(3/2)*(11*A*b^3-a*(9*B*b^2-7*C*a*b+5*D*a^2))*arctan(b^(1/2)*x/a^(1/
2))/a^(13/2)
```

## Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.98

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{10}\left(a+bx^2\right)^2}\,dx = -\frac{A}{9a^2x^9}+\frac{2Ab-aB}{7a^3x^7}+\frac{-3Ab^2+2abB-a^2C}{5a^4x^5}+\frac{4Ab^3-3ab^2B+2a^2bC-a^3D}{3a^5x^3}+\frac{b(-5Ab^3+4ab^2B-3a^2bC+2a^3D)}{a^6x}+\frac{b^2(-Ab^3+ab^2B-a^2bC+a^3D)\,x}{2a^6\left(a+bx^2\right)}+\frac{b^{3/2}(-11Ab^3+9ab^2B-7a^2bC+5a^3D)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{13/2}}$$

input
```
Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(x^10*(a + b*x^2)^2),x]
```

output
```
-1/9*A/(a^2*x^9) + (2*A*b - a*B)/(7*a^3*x^7) + (-3*A*b^2 + 2*a*b*B - a^2*C
)/(5*a^4*x^5) + (4*A*b^3 - 3*a*b^2*B + 2*a^2*b*C - a^3*D)/(3*a^5*x^3) + (b
*(-5*A*b^3 + 4*a*b^2*B - 3*a^2*b*C + 2*a^3*D))/(a^6*x) + (b^2*(-(A*b^3) +
a*b^2*B - a^2*b*C + a^3*D)*x)/(2*a^6*(a + b*x^2)) + (b^(3/2)*(-11*A*b^3 +
9*a*b^2*B - 7*a^2*b*C + 5*a^3*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(13/2))
```

## Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2336, 25, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{10}\left(a+bx^2\right)^2}dx$$

$$\Big\downarrow \; 2336$$

$$-\frac{\int -\frac{-\frac{b^2\left(Ab^3-a\left(Da^2-bCa+b^2B\right)\right)x^{10}}{a^5}+\frac{2b\left(Ab^3-a\left(Da^2-bCa+b^2B\right)\right)x^8}{a^4}-\frac{2\left(Ab^3-a\left(Da^2-bCa+b^2B\right)\right)x^6}{a^3}+\frac{2\left(Ab^2-a(bB-aC)\right)x^4}{a^2}-2\left(\frac{Ab}{a}-B\right)x^2+}{x^{10}(bx^2+a)}}{2a}$$

$$\frac{b^2x\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{2a^6\left(a+bx^2\right)}$$

$\Big\downarrow$ 25

$$\frac{\int \frac{-\frac{b^2\left(Ab^3-a\left(Da^2-bCa+b^2B\right)\right)x^{10}}{a^5}+\frac{2b\left(Ab^3-a\left(Da^2-bCa+b^2B\right)\right)x^8}{a^4}-\frac{2\left(Ab^3-a\left(Da^2-bCa+b^2B\right)\right)x^6}{a^3}+\frac{2\left(Ab^2-a(bB-aC)\right)x^4}{a^2}-2\left(\frac{Ab}{a}-B\right)x^2+2A}{x^{10}(bx^2+a)}d}{2a}$$

$$\frac{b^2x\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{2a^6\left(a+bx^2\right)}$$

$\Big\downarrow$ 2333

$$\frac{\int \left(\frac{\left(a(5Da^2-7bCa+9b^2B)-11Ab^3\right)b^2}{a^5(bx^2+a)}+\frac{2\left(5Ab^3-a(2Da^2-3bCa+4b^2B)\right)b}{a^5x^2}+\frac{2\left(a(Da^2-2bCa+3b^2B)-4Ab^3\right)}{a^4x^4}+\frac{2\left(3Ab^2-a(2bB-aC)\right)}{a^3x^6}\right.}{2a}$$

$$\frac{b^2x\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{2a^6\left(a+bx^2\right)}$$

$\Big\downarrow$ 2009

$$\frac{-\frac{2(3Ab^2-a(2bB-aC))}{5a^3x^5}+\frac{2(2Ab-aB)}{7a^2x^7}-\frac{b^{3/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(11Ab^3-a(5a^2D-7abC+9b^2B)\right)}{a^{11/2}}-\frac{2b(5Ab^3-a(2a^2D-3abC+4b^2B))}{a^5x}+\frac{2(4}{}}{2a}$$

$$\frac{b^2x\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{2a^6\left(a+bx^2\right)}$$

input

```
Int[(A + B*x^2 + C*x^4 + D*x^6)/(x^10*(a + b*x^2)^2),x]
```

output

```
-1/2*(b^2*(A*b^3 - a*(b^2*B - a*b*C + a^2*D))*x)/(a^6*(a + b*x^2)) + ((-2*
A)/(9*a*x^9) + (2*(2*A*b - a*B))/(7*a^2*x^7) - (2*(3*A*b^2 - a*(2*b*B - a*
C)))/(5*a^3*x^5) + (2*(4*A*b^3 - a*(3*b^2*B - 2*a*b*C + a^2*D)))/(3*a^4*x^
3) - (2*b*(5*A*b^3 - a*(4*b^2*B - 3*a*b*C + 2*a^2*D)))/(a^5*x) - (b^(3/2)*
(11*A*b^3 - a*(9*b^2*B - 7*a*b*C + 5*a^2*D))*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/
a^(11/2))/(2*a)
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

## Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.91

| method | result |
|---|---|
| default | $-\frac{b^{2}\left(\frac{\left(\frac{1}{2}b^{3}A-\frac{1}{2}a\,b^{2}B+\frac{1}{2}a^{2}bC-\frac{1}{2}a^{3}D\right)x}{b\,x^{2}+a}+\frac{\left(11b^{3}A-9a\,b^{2}B+7a^{2}bC-5a^{3}D\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^{6}}-\frac{A}{9a^{2}x^{9}}-\frac{-2Ab+Ba}{7a^{3}x^{7}}-\frac{3b^{2}A-2abB}{5a^{4}x^{5}}$ |

input
```
int((D*x^6+C*x^4+B*x^2+A)/x^10/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-b^2/a^6*((1/2*b^3*A-1/2*a*b^2*B+1/2*a^2*b*C-1/2*a^3*D)*x/(b*x^2+a)+1/2*(1
1*A*b^3-9*B*a*b^2+7*C*a^2*b-5*D*a^3)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))-
1/9*A/a^2/x^9-1/7*(-2*A*b+B*a)/a^3/x^7-1/5*(3*A*b^2-2*B*a*b+C*a^2)/a^4/x^5
-1/3*(-4*A*b^3+3*B*a*b^2-2*C*a^2*b+D*a^3)/a^5/x^3-b*(5*A*b^3-4*B*a*b^2+3*C
*a^2*b-2*D*a^3)/a^6/x
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 582, normalized size of antiderivative = 2.49

$$\int \frac{A + Bx^2 + Cx^4 + Dx^6}{x^{10}(a + bx^2)^2} \, dx$$

$$= \left[ \frac{630\,(5\,Da^3b^2 - 7\,Ca^2b^3 + 9\,Bab^4 - 11\,Ab^5)x^{10} + 420\,(5\,Da^4b - 7\,Ca^3b^2 + 9\,Ba^2b^3 - 11\,Aab^4)x^8 - 84\,(}{} \right.$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^10/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[1/1260*(630*(5*D*a^3*b^2 - 7*C*a^2*b^3 + 9*B*a*b^4 - 11*A*b^5)*x^10 + 420
*(5*D*a^4*b - 7*C*a^3*b^2 + 9*B*a^2*b^3 - 11*A*a*b^4)*x^8 - 84*(5*D*a^5 -
7*C*a^4*b + 9*B*a^3*b^2 - 11*A*a^2*b^3)*x^6 - 140*A*a^5 - 36*(7*C*a^5 - 9*
B*a^4*b + 11*A*a^3*b^2)*x^4 - 20*(9*B*a^5 - 11*A*a^4*b)*x^2 - 315*((5*D*a^
3*b^2 - 7*C*a^2*b^3 + 9*B*a*b^4 - 11*A*b^5)*x^11 + (5*D*a^4*b - 7*C*a^3*b^
2 + 9*B*a^2*b^3 - 11*A*a*b^4)*x^9)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a
) - a)/(b*x^2 + a)))/(a^6*b*x^11 + a^7*x^9), 1/630*(315*(5*D*a^3*b^2 - 7*C
*a^2*b^3 + 9*B*a*b^4 - 11*A*b^5)*x^10 + 210*(5*D*a^4*b - 7*C*a^3*b^2 + 9*B
*a^2*b^3 - 11*A*a*b^4)*x^8 - 42*(5*D*a^5 - 7*C*a^4*b + 9*B*a^3*b^2 - 11*A*
a^2*b^3)*x^6 - 70*A*a^5 - 18*(7*C*a^5 - 9*B*a^4*b + 11*A*a^3*b^2)*x^4 - 10
*(9*B*a^5 - 11*A*a^4*b)*x^2 + 315*((5*D*a^3*b^2 - 7*C*a^2*b^3 + 9*B*a*b^4
- 11*A*b^5)*x^11 + (5*D*a^4*b - 7*C*a^3*b^2 + 9*B*a^2*b^3 - 11*A*a*b^4)*x^
9)*sqrt(b/a)*arctan(x*sqrt(b/a)))/(a^6*b*x^11 + a^7*x^9)]
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 449 vs. $2(221) = 442$.

Time = 57.08 (sec) , antiderivative size = 449, normalized size of antiderivative = 1.92

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{10}(a+bx^2)^2}\,dx = -\frac{\sqrt{-\frac{b^3}{a^{13}}}(-11Ab^3+9Bab^2-7Ca^2b+5Da^3)\log\left(-\frac{a^7\sqrt{-\frac{b^3}{a^{13}}}(-11Ab^3+9Bab^2-7Ca^2b+5Da^3)}{-11Ab^5+9Bab^4-7Ca^2b^3+5Da^3b^2}+x\right)}{4} + \frac{\sqrt{-\frac{b^3}{a^{13}}}(-11Ab^3+9Bab^2-7Ca^2b+5Da^3)\log\left(\frac{a^7\sqrt{-\frac{b^3}{a^{13}}}(-11Ab^3+9Bab^2-7Ca^2b+5Da^3)}{-11Ab^5+9Bab^4-7Ca^2b^3+5Da^3b^2}+x\right)}{4} + \frac{-70Aa^5+x^{10}(-3465Ab^5+2835Bab^4-2205Ca^2b^3+1575Da^3b^2)+x^8(-2310Aab^4+1890Ba^2b^3-}{}$$

input
```
integrate((D*x**6+C*x**4+B*x**2+A)/x**10/(b*x**2+a)**2,x)
```

output
```
-sqrt(-b**3/a**13)*(-11*A*b**3 + 9*B*a*b**2 - 7*C*a**2*b + 5*D*a**3)*log(-
a**7*sqrt(-b**3/a**13)*(-11*A*b**3 + 9*B*a*b**2 - 7*C*a**2*b + 5*D*a**3)/(
-11*A*b**5 + 9*B*a*b**4 - 7*C*a**2*b**3 + 5*D*a**3*b**2) + x)/4 + sqrt(-b*
*3/a**13)*(-11*A*b**3 + 9*B*a*b**2 - 7*C*a**2*b + 5*D*a**3)*log(a**7*sqrt(
-b**3/a**13)*(-11*A*b**3 + 9*B*a*b**2 - 7*C*a**2*b + 5*D*a**3)/(-11*A*b**5
 + 9*B*a*b**4 - 7*C*a**2*b**3 + 5*D*a**3*b**2) + x)/4 + (-70*A*a**5 + x**1
0*(-3465*A*b**5 + 2835*B*a*b**4 - 2205*C*a**2*b**3 + 1575*D*a**3*b**2) + x
**8*(-2310*A*a*b**4 + 1890*B*a**2*b**3 - 1470*C*a**3*b**2 + 1050*D*a**4*b)
 + x**6*(462*A*a**2*b**3 - 378*B*a**3*b**2 + 294*C*a**4*b - 210*D*a**5) +
x**4*(-198*A*a**3*b**2 + 162*B*a**4*b - 126*C*a**5) + x**2*(110*A*a**4*b -
 90*B*a**5))/(630*a**7*x**9 + 630*a**6*b*x**11)
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.02

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{10}\left(a+bx^2\right)^2}\,dx = \frac{315\,(5\,Da^3b^2-7\,Ca^2b^3+9\,Bab^4-11\,Ab^5)x^{10}+210\,(5\,Da^4b-7\,Ca^3b^2+9\,Ba^2b^3-11\,Aab^4)x^8-42\,(5}{630\,(a} + \frac{(5\,Da^3b^2-7\,Ca^2b^3+9\,Bab^4-11\,Ab^5)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^6}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^10/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/630*(315*(5*D*a^3*b^2 - 7*C*a^2*b^3 + 9*B*a*b^4 - 11*A*b^5)*x^10 + 210*(
5*D*a^4*b - 7*C*a^3*b^2 + 9*B*a^2*b^3 - 11*A*a*b^4)*x^8 - 42*(5*D*a^5 - 7*
C*a^4*b + 9*B*a^3*b^2 - 11*A*a^2*b^3)*x^6 - 70*A*a^5 - 18*(7*C*a^5 - 9*B*a
^4*b + 11*A*a^3*b^2)*x^4 - 10*(9*B*a^5 - 11*A*a^4*b)*x^2)/(a^6*b*x^11 + a^
7*x^9) + 1/2*(5*D*a^3*b^2 - 7*C*a^2*b^3 + 9*B*a*b^4 - 11*A*b^5)*arctan(b*x
/sqrt(a*b))/(sqrt(a*b)*a^6)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.06

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{10}\left(a+bx^2\right)^2}\,dx = \frac{(5\,Da^3b^2-7\,Ca^2b^3+9\,Bab^4-11\,Ab^5)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^6} + \frac{Da^3b^2x-Ca^2b^3x+Bab^4x-Ab^5x}{2\,(bx^2+a)a^6} + \frac{630\,Da^3bx^8-945\,Ca^2b^2x^8+1260\,Bab^3x^8-1575\,Ab^4x^8-105\,Da^4x^6+210\,Ca^3bx^6-315\,Ba^2b^2x^6+}{315\,a^6x^9}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^10/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*(5*D*a^3*b^2 - 7*C*a^2*b^3 + 9*B*a*b^4 - 11*A*b^5)*arctan(b*x/sqrt(a*b
))/(sqrt(a*b)*a^6) + 1/2*(D*a^3*b^2*x - C*a^2*b^3*x + B*a*b^4*x - A*b^5*x)
/((b*x^2 + a)*a^6) + 1/315*(630*D*a^3*b*x^8 - 945*C*a^2*b^2*x^8 + 1260*B*a
*b^3*x^8 - 1575*A*b^4*x^8 - 105*D*a^4*x^6 + 210*C*a^3*b*x^6 - 315*B*a^2*b^
2*x^6 + 420*A*a*b^3*x^6 - 63*C*a^4*x^4 + 126*B*a^3*b*x^4 - 189*A*a^2*b^2*x
^4 - 45*B*a^4*x^2 + 90*A*a^3*b*x^2 - 35*A*a^4)/(a^6*x^9)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{10}\left(a+bx^2\right)^2}\,dx=\int \frac{A+B\,x^2+C\,x^4+x^6\,D}{x^{10}\,{\left(b\,x^2+a\right)}^2}\,dx$$

input
```
int((A + B*x^2 + C*x^4 + x^6*D)/(x^10*(a + b*x^2)^2),x)
```

output
```
int((A + B*x^2 + C*x^4 + x^6*D)/(x^10*(a + b*x^2)^2), x)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.30

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{10}\left(a+bx^2\right)^2}\,dx$$
$$=\frac{1575\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^3bd\,x^9-2205\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^2b^2c\,x^9+1575\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^2b^2d\,x^1}{}$$

input
```
int((D*x^6+C*x^4+B*x^2+A)/x^10/(b*x^2+a)^2,x)
```

output
```
(1575*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**3*b*d*x**9 - 2205*s
qrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2*b**2*c*x**9 + 1575*sqrt(
b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2*b**2*d*x**11 - 630*sqrt(b)*s
qrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*b**4*x**9 - 2205*sqrt(b)*sqrt(a)*at
an((b*x)/(sqrt(b)*sqrt(a)))*a*b**3*c*x**11 - 630*sqrt(b)*sqrt(a)*atan((b*x
)/(sqrt(b)*sqrt(a)))*b**5*x**11 - 70*a**6 + 20*a**5*b*x**2 - 126*a**5*c*x*
*4 - 210*a**5*d*x**6 - 36*a**4*b**2*x**4 + 294*a**4*b*c*x**6 + 1050*a**4*b
*d*x**8 + 84*a**3*b**3*x**6 - 1470*a**3*b**2*c*x**8 + 1575*a**3*b**2*d*x**
10 - 420*a**2*b**4*x**8 - 2205*a**2*b**3*c*x**10 - 630*a*b**5*x**10)/(630*
a**6*x**9*(a + b*x**2))
```

# 3.179 $\int \frac{x^8\left(A+Bx^2+Cx^4+Dx^6\right)}{\left(a+bx^2\right)^3}\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 277

$$\begin{aligned}\int \frac{x^8(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^3}\,dx &= -\frac{a(3Ab^3-a(6b^2B-10abC+15a^2D))\,x}{b^7}+\frac{(Ab^3-a(3b^2B-6abC+10a^2D))\,x^3}{3b^6}\\ &\quad+\frac{(b^2B-3abC+6a^2D)\,x^5}{5b^5}+\frac{(bC-3aD)x^7}{7b^4}+\frac{Dx^9}{9b^3}\\ &\quad+\frac{a^3(Ab^3-a(b^2B-abC+a^2D))\,x}{4b^7\,(a+bx^2)^2}-\frac{a^2(13Ab^3-a(17b^2B-21abC+25a^2D))\,x}{8b^7\,(a+bx^2)}\\ &\quad+\frac{a^{3/2}(35Ab^3-a(63b^2B-99abC+143a^2D))\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{15/2}}\end{aligned}$$

output
```
-a*(3*A*b^3-a*(6*B*b^2-10*C*a*b+15*D*a^2))*x/b^7+1/3*(A*b^3-a*(3*B*b^2-6*C
*a*b+10*D*a^2))*x^3/b^6+1/5*(B*b^2-3*C*a*b+6*D*a^2)*x^5/b^5+1/7*(C*b-3*D*a
)*x^7/b^4+1/9*D*x^9/b^3+1/4*a^3*(A*b^3-a*(B*b^2-C*a*b+D*a^2))*x/b^7/(b*x^2
+a)^2-1/8*a^2*(13*A*b^3-a*(17*B*b^2-21*C*a*b+25*D*a^2))*x/b^7/(b*x^2+a)+1/
8*a^(3/2)*(35*A*b^3-a*(63*B*b^2-99*C*a*b+143*D*a^2))*arctan(b^(1/2)*x/a^(1
/2))/b^(15/2)
```

## Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 272, normalized size of antiderivative = 0.98

$$\int \frac{x^8(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^3}\,dx$$
$$= \frac{a(-3Ab^3+a(6b^2B-10abC+15a^2D))\,x}{b^7}+\frac{(Ab^3+a(-3b^2B+6abC-10a^2D))\,x^3}{3b^6}$$
$$+\frac{(b^2B-3abC+6a^2D)\,x^5}{5b^5}+\frac{(bC-3aD)x^7}{7b^4}+\frac{Dx^9}{9b^3}$$
$$+\frac{a^3(Ab^3-a(b^2B-abC+a^2D))\,x}{4b^7\,(a+bx^2)^2}+\frac{a^2(-13Ab^3+a(17b^2B-21abC+25a^2D))\,x}{8b^7\,(a+bx^2)}$$
$$-\frac{a^{3/2}(-35Ab^3+a(63b^2B-99abC+143a^2D))\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{15/2}}$$

input
```
Integrate[(x^8*(A + B*x^2 + C*x^4 + D*x^6))/(a + b*x^2)^3,x]
```

output
```
(a*(-3*A*b^3 + a*(6*b^2*B - 10*a*b*C + 15*a^2*D))*x)/b^7 + ((A*b^3 + a*(-3
*b^2*B + 6*a*b*C - 10*a^2*D))*x^3)/(3*b^6) + ((b^2*B - 3*a*b*C + 6*a^2*D)*
x^5)/(5*b^5) + ((b*C - 3*a*D)*x^7)/(7*b^4) + (D*x^9)/(9*b^3) + (a^3*(A*b^3
 - a*(b^2*B - a*b*C + a^2*D))*x)/(4*b^7*(a + b*x^2)^2) + (a^2*(-13*A*b^3 +
 a*(17*b^2*B - 21*a*b*C + 25*a^2*D))*x)/(8*b^7*(a + b*x^2)) - (a^(3/2)*(-3
5*A*b^3 + a*(63*b^2*B - 99*a*b*C + 143*a^2*D))*ArcTan[(Sqrt[b]*x)/Sqrt[a]]
)/(8*b^(15/2))
```

## Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2335, 9, 1580, 2341, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8\left(A+Bx^2+Cx^4+Dx^6\right)}{(a+bx^2)^3}\,dx$$

$\Big\downarrow$ 2335

$$\frac{x^9\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{4a\left(a+bx^2\right)^2}-\frac{\int \frac{x^7\left(-4aDx^5-4a\left(C-\frac{aD}{b}\right)x^3+\left(5Ab-\frac{9a\left(Da^2-bCa+b^2B\right)}{b^2}\right)x\right)}{\left(bx^2+a\right)^2}dx}{4ab}$$

$\Big\downarrow$ 9

$$\frac{x^9\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{4a\left(a+bx^2\right)^2}-\frac{\int \frac{x^8\left(-4aDx^4-4a\left(C-\frac{aD}{b}\right)x^2+5Ab-\frac{9a\left(Da^2-bCa+b^2B\right)}{b^2}\right)}{\left(bx^2+a\right)^2}dx}{4ab}$$

$\Big\downarrow$ 1580

$$\frac{x^9\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{4a\left(a+bx^2\right)^2}-$$

$$\frac{\frac{a^3x\left(5Ab^3-a\left(17a^2D-13abC+9b^2B\right)\right)}{2b^6\left(a+bx^2\right)}-\frac{\int \frac{8ab^5Dx^{10}+8ab^4(bC-2aD)x^8-2b^3\left(5Ab^3-a\left(17Da^2-13bCa+9b^2B\right)\right)x^6+2ab^2\left(5Ab^3-a\left(17Da^2-13bCa+9b^2\right.\right.}{bx^2+a}}{2b^6}}{4ab}$$

$\Big\downarrow$ 2341

$$\frac{x^9\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{4a\left(a+bx^2\right)^2}-$$

$$\frac{\frac{a^3x\left(5Ab^3-a\left(17a^2D-13abC+9b^2B\right)\right)}{2b^6\left(a+bx^2\right)}-\frac{\int \left(8ab^4Dx^8+8ab^3(bC-3aD)x^6-2b^2\left(5Ab^3-a\left(29Da^2-17bCa+9b^2B\right)\right)x^4+4ab\left(5Ab^3-a\left(23Da^2-15b\right.\right.\right.}{2b}}{4ab}$$

$\Big\downarrow$ 2009

$$\frac{x^9\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{4a\left(a+bx^2\right)^2}-$$

$$\frac{\frac{a^3x\left(5Ab^3-a\left(17a^2D-13abC+9b^2B\right)\right)}{2b^6\left(a+bx^2\right)}-\frac{-\frac{2}{5}b^2x^5\left(5Ab^3-a\left(29a^2D-17abC+9b^2B\right)\right)+\frac{4}{3}abx^3\left(5Ab^3-a\left(23a^2D-15abC+9b^2B\right)\right)-2a^2x\left(15Ab^3-\right.}{}}{4ab}$$

input
```
Int[(x^8*(A + B*x^2 + C*x^4 + D*x^6))/(a + b*x^2)^3,x]
```

output
```
((A - (a*(b^2*B - a*b*C + a^2*D))/b^3)*x^9)/(4*a*(a + b*x^2)^2) - ((a^3*(5
*A*b^3 - a*(9*b^2*B - 13*a*b*C + 17*a^2*D))*x)/(2*b^6*(a + b*x^2)) - (-2*a
^2*(15*A*b^3 - a*(27*b^2*B - 43*a*b*C + 63*a^2*D))*x + (4*a*b*(5*A*b^3 - a
*(9*b^2*B - 15*a*b*C + 23*a^2*D))*x^3)/3 - (2*b^2*(5*A*b^3 - a*(9*b^2*B -
17*a*b*C + 29*a^2*D))*x^5)/5 + (8*a*b^3*(b*C - 3*a*D)*x^7)/7 + (8*a*b^4*D*
x^9)/9 + (a^(5/2)*(35*A*b^3 - a*(63*b^2*B - 99*a*b*C + 143*a^2*D))*ArcTan[
(Sqrt[b]*x)/Sqrt[a]])/Sqrt[b])/(2*b^6))/(4*a*b)
```

## Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 1580
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
 + e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Simp[1/(2*e^(2*p + m/2)*
(q + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*x^2))*(2*
e^(2*p + m/2)*(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b
*d*e + a*e^2)^p*(d + e*(2*q + 3)*x^2))], x], x], x] /; FreeQ[{a, b, c, d, e
}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2335
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Simp[c/(2*a*b*(p + 1))   Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSu
m[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
 b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

rule 2341
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

## Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 267, normalized size of antiderivative = 0.96

| method | result |
|---|---|
| default | $-\frac{-\frac{1}{9}Dx^9b^4-\frac{1}{7}Cb^4x^7+\frac{3}{7}Dab^3x^7-\frac{1}{5}Bb^4x^5+\frac{3}{5}Cab^3x^5-\frac{6}{5}Da^2b^2x^5-\frac{1}{3}Ax^3b^4+Bx^3ab^3-2Ca^2b^2x^3+\frac{10}{3}Da^3bx^3+3Aab^3x-6B}{b^7}$ |

input
```
int(x^8*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/b^7*(-1/9*D*x^9*b^4-1/7*C*b^4*x^7+3/7*D*a*b^3*x^7-1/5*B*b^4*x^5+3/5*C*a
*b^3*x^5-6/5*D*a^2*b^2*x^5-1/3*A*x^3*b^4+B*x^3*a*b^3-2*C*a^2*b^2*x^3+10/3*
D*a^3*b*x^3+3*A*a*b^3*x-6*B*a^2*b^2*x+10*C*a^3*b*x-15*D*a^4*x)+a^2/b^7*(((
-13/8*A*b^4+17/8*B*a*b^3-21/8*C*a^2*b^2+25/8*a^3*D*b)*x^3-1/8*a*(11*A*b^3-
15*B*a*b^2+19*C*a^2*b-23*D*a^3)*x)/(b*x^2+a)^2+1/8*(35*A*b^3-63*B*a*b^2+99
*C*a^2*b-143*D*a^3)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 762, normalized size of antiderivative = 2.75

$$\int \frac{x^8(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^8*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
[1/5040*(560*D*b^6*x^13 - 80*(13*D*a*b^5 - 9*C*b^6)*x^11 + 16*(143*D*a^2*b
^4 - 99*C*a*b^5 + 63*B*b^6)*x^9 - 48*(143*D*a^3*b^3 - 99*C*a^2*b^4 + 63*B*
a*b^5 - 35*A*b^6)*x^7 + 336*(143*D*a^4*b^2 - 99*C*a^3*b^3 + 63*B*a^2*b^4 -
 35*A*a*b^5)*x^5 + 1050*(143*D*a^5*b - 99*C*a^4*b^2 + 63*B*a^3*b^3 - 35*A*
a^2*b^4)*x^3 - 315*(143*D*a^6 - 99*C*a^5*b + 63*B*a^4*b^2 - 35*A*a^3*b^3 +
 (143*D*a^4*b^2 - 99*C*a^3*b^3 + 63*B*a^2*b^4 - 35*A*a*b^5)*x^4 + 2*(143*D
*a^5*b - 99*C*a^4*b^2 + 63*B*a^3*b^3 - 35*A*a^2*b^4)*x^2)*sqrt(-a/b)*log((
b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) + 630*(143*D*a^6 - 99*C*a^5*b +
 63*B*a^4*b^2 - 35*A*a^3*b^3)*x)/(b^9*x^4 + 2*a*b^8*x^2 + a^2*b^7), 1/2520
*(280*D*b^6*x^13 - 40*(13*D*a*b^5 - 9*C*b^6)*x^11 + 8*(143*D*a^2*b^4 - 99*
C*a*b^5 + 63*B*b^6)*x^9 - 24*(143*D*a^3*b^3 - 99*C*a^2*b^4 + 63*B*a*b^5 -
35*A*b^6)*x^7 + 168*(143*D*a^4*b^2 - 99*C*a^3*b^3 + 63*B*a^2*b^4 - 35*A*a*
b^5)*x^5 + 525*(143*D*a^5*b - 99*C*a^4*b^2 + 63*B*a^3*b^3 - 35*A*a^2*b^4)*
x^3 - 315*(143*D*a^6 - 99*C*a^5*b + 63*B*a^4*b^2 - 35*A*a^3*b^3 + (143*D*a
^4*b^2 - 99*C*a^3*b^3 + 63*B*a^2*b^4 - 35*A*a*b^5)*x^4 + 2*(143*D*a^5*b -
99*C*a^4*b^2 + 63*B*a^3*b^3 - 35*A*a^2*b^4)*x^2)*sqrt(a/b)*arctan(b*x*sqrt
(a/b)/a) + 315*(143*D*a^6 - 99*C*a^5*b + 63*B*a^4*b^2 - 35*A*a^3*b^3)*x)/(
b^9*x^4 + 2*a*b^8*x^2 + a^2*b^7)]
```

## Sympy [A] (verification not implemented)

Time = 6.72 (sec) , antiderivative size = 503, normalized size of antiderivative = 1.82

$$\int \frac{x^8(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^3}\,dx = \frac{Dx^9}{9b^3}+x^7\left(\frac{C}{7b^3}-\frac{3Da}{7b^4}\right)+x^5\left(\frac{B}{5b^3}-\frac{3Ca}{5b^4}+\frac{6Da^2}{5b^5}\right)+x^3\left(\frac{A}{3b^3}-\frac{Ba}{b^4}+\frac{2Ca^2}{b^5}-\frac{10Da^3}{3b^6}\right)+x\left(-\frac{3Aa}{b^4}+\frac{6Ba^2}{b^5}-\frac{10Ca^3}{b^6}+\frac{15Da^4}{b^7}\right)+\frac{\sqrt{-\frac{a^3}{b^{15}}}(-35Ab^3+63Bab^2-99Ca^2b+143Da^3)\log\left(-\frac{b^7\sqrt{-\frac{a^3}{b^{15}}}(-35Ab^3+63Bab^2-99Ca^2b+143Da^3)}{-35Aab^3+63Ba^2b^2-99Ca^3b+143Da^4}+x\right)}{16}-\frac{\sqrt{-\frac{a^3}{b^{15}}}(-35Ab^3+63Bab^2-99Ca^2b+143Da^3)\log\left(\frac{b^7\sqrt{-\frac{a^3}{b^{15}}}(-35Ab^3+63Bab^2-99Ca^2b+143Da^3)}{-35Aab^3+63Ba^2b^2-99Ca^3b+143Da^4}+x\right)}{16}+\frac{x^3(-13Aa^2b^4+17Ba^3b^3-21Ca^4b^2+25Da^5b)+x(-11Aa^3b^3+15Ba^4b^2-19Ca^5b+23Da^6)}{8a^2b^7+16ab^8x^2+8b^9x^4}$$

input
```
integrate(x**8*(D*x**6+C*x**4+B*x**2+A)/(b*x**2+a)**3,x)
```

output
```
D*x**9/(9*b**3) + x**7*(C/(7*b**3) - 3*D*a/(7*b**4)) + x**5*(B/(5*b**3) -
3*C*a/(5*b**4) + 6*D*a**2/(5*b**5)) + x**3*(A/(3*b**3) - B*a/b**4 + 2*C*a*
*2/b**5 - 10*D*a**3/(3*b**6)) + x*(-3*A*a/b**4 + 6*B*a**2/b**5 - 10*C*a**3
/b**6 + 15*D*a**4/b**7) + sqrt(-a**3/b**15)*(-35*A*b**3 + 63*B*a*b**2 - 99
*C*a**2*b + 143*D*a**3)*log(-b**7*sqrt(-a**3/b**15)*(-35*A*b**3 + 63*B*a*b
**2 - 99*C*a**2*b + 143*D*a**3)/(-35*A*a*b**3 + 63*B*a**2*b**2 - 99*C*a**3
*b + 143*D*a**4) + x)/16 - sqrt(-a**3/b**15)*(-35*A*b**3 + 63*B*a*b**2 - 9
9*C*a**2*b + 143*D*a**3)*log(b**7*sqrt(-a**3/b**15)*(-35*A*b**3 + 63*B*a*b
**2 - 99*C*a**2*b + 143*D*a**3)/(-35*A*a*b**3 + 63*B*a**2*b**2 - 99*C*a**3
*b + 143*D*a**4) + x)/16 + (x**3*(-13*A*a**2*b**4 + 17*B*a**3*b**3 - 21*C*
a**4*b**2 + 25*D*a**5*b) + x*(-11*A*a**3*b**3 + 15*B*a**4*b**2 - 19*C*a**5
*b + 23*D*a**6))/(8*a**2*b**7 + 16*a*b**8*x**2 + 8*b**9*x**4)
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.02

$$\int \frac{x^8(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^3}\,dx$$
$$= \frac{(25\,Da^5b-21\,Ca^4b^2+17\,Ba^3b^3-13\,Aa^2b^4)x^3+(23\,Da^6-19\,Ca^5b+15\,Ba^4b^2-11\,Aa^3b^3)x}{8\,(b^9x^4+2\,ab^8x^2+a^2b^7)}$$
$$-\frac{(143\,Da^5-99\,Ca^4b+63\,Ba^3b^2-35\,Aa^2b^3)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}b^7}$$
$$+\frac{35\,Db^4x^9-45\,(3\,Dab^3-Cb^4)x^7+63\,(6\,Da^2b^2-3\,Cab^3+Bb^4)x^5-105\,(10\,Da^3b-6\,Ca^2b^2+3\,Bab}{315\,b^7}$$

input
```
integrate(x^8*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/8*((25*D*a^5*b - 21*C*a^4*b^2 + 17*B*a^3*b^3 - 13*A*a^2*b^4)*x^3 + (23*D
*a^6 - 19*C*a^5*b + 15*B*a^4*b^2 - 11*A*a^3*b^3)*x)/(b^9*x^4 + 2*a*b^8*x^2
 + a^2*b^7) - 1/8*(143*D*a^5 - 99*C*a^4*b + 63*B*a^3*b^2 - 35*A*a^2*b^3)*a
rctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^7) + 1/315*(35*D*b^4*x^9 - 45*(3*D*a*b^3
 - C*b^4)*x^7 + 63*(6*D*a^2*b^2 - 3*C*a*b^3 + B*b^4)*x^5 - 105*(10*D*a^3*b
 - 6*C*a^2*b^2 + 3*B*a*b^3 - A*b^4)*x^3 + 315*(15*D*a^4 - 10*C*a^3*b + 6*B
*a^2*b^2 - 3*A*a*b^3)*x)/b^7
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.06

$$\int \frac{x^8(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^3}\,dx$$
$$= -\frac{(143\,Da^5 - 99\,Ca^4b + 63\,Ba^3b^2 - 35\,Aa^2b^3)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}b^7}$$
$$+ \frac{25\,Da^5bx^3 - 21\,Ca^4b^2x^3 + 17\,Ba^3b^3x^3 - 13\,Aa^2b^4x^3 + 23\,Da^6x - 19\,Ca^5bx + 15\,Ba^4b^2x - 11\,Aa^3b^3x}{8\,(bx^2+a)^2b^7}$$
$$+ \frac{35\,Db^{24}x^9 - 135\,Dab^{23}x^7 + 45\,Cb^{24}x^7 + 378\,Da^2b^{22}x^5 - 189\,Cab^{23}x^5 + 63\,Bb^{24}x^5 - 1050\,Da^3b^{21}x^3 +}{3}$$

input
```
integrate(x^8*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
-1/8*(143*D*a^5 - 99*C*a^4*b + 63*B*a^3*b^2 - 35*A*a^2*b^3)*arctan(b*x/sqr
t(a*b))/(sqrt(a*b)*b^7) + 1/8*(25*D*a^5*b*x^3 - 21*C*a^4*b^2*x^3 + 17*B*a^
3*b^3*x^3 - 13*A*a^2*b^4*x^3 + 23*D*a^6*x - 19*C*a^5*b*x + 15*B*a^4*b^2*x
- 11*A*a^3*b^3*x)/((b*x^2 + a)^2*b^7) + 1/315*(35*D*b^24*x^9 - 135*D*a*b^2
3*x^7 + 45*C*b^24*x^7 + 378*D*a^2*b^22*x^5 - 189*C*a*b^23*x^5 + 63*B*b^24*
x^5 - 1050*D*a^3*b^21*x^3 + 630*C*a^2*b^22*x^3 - 315*B*a*b^23*x^3 + 105*A*
b^24*x^3 + 4725*D*a^4*b^20*x - 3150*C*a^3*b^21*x + 1890*B*a^2*b^22*x - 945
*A*a*b^23*x)/b^27
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^8(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^3}\,dx = \int \frac{x^8\,(A+B\,x^2+C\,x^4+x^6\,D)}{(b\,x^2+a)^3}\,dx$$

input
```
int((x^8*(A + B*x^2 + C*x^4 + x^6*D))/(a + b*x^2)^3,x)
```

output
```
int((x^8*(A + B*x^2 + C*x^4 + x^6*D))/(a + b*x^2)^3, x)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 447, normalized size of antiderivative = 1.61

$$\int \frac{x^8(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^3}\,dx$$

$$= \frac{8820a^4b^4x + 14700a^3b^5x^3 + 4704a^2b^6x^5 - 672a\,b^7x^7 + 360b^7c\,x^{11} + 280b^7d\,x^{13} + 504b^8x^9 + 31185\sqrt{b}\,\sqrt{a}}{}$$

input
```
int(x^8*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^3,x)
```

output
```
( - 45045*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**6*d + 31185*sqr
t(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**5*b*c - 90090*sqrt(b)*sqrt(a
)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**5*b*d*x**2 - 8820*sqrt(b)*sqrt(a)*atan(
(b*x)/(sqrt(b)*sqrt(a)))*a**4*b**3 + 62370*sqrt(b)*sqrt(a)*atan((b*x)/(sqr
t(b)*sqrt(a)))*a**4*b**2*c*x**2 - 45045*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b
)*sqrt(a)))*a**4*b**2*d*x**4 - 17640*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*s
qrt(a)))*a**3*b**4*x**2 + 31185*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a
)))*a**3*b**3*c*x**4 - 8820*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*
a**2*b**5*x**4 + 45045*a**6*b*d*x - 31185*a**5*b**2*c*x + 75075*a**5*b**2*
d*x**3 + 8820*a**4*b**4*x - 51975*a**4*b**3*c*x**3 + 24024*a**4*b**3*d*x**
5 + 14700*a**3*b**5*x**3 - 16632*a**3*b**4*c*x**5 - 3432*a**3*b**4*d*x**7
 + 4704*a**2*b**6*x**5 + 2376*a**2*b**5*c*x**7 + 1144*a**2*b**5*d*x**9 - 67
2*a*b**7*x**7 - 792*a*b**6*c*x**9 - 520*a*b**6*d*x**11 + 504*b**8*x**9 + 3
60*b**7*c*x**11 + 280*b**7*d*x**13)/(2520*b**8*(a**2 + 2*a*b*x**2 + b**2*x
**4))
```

# 3.180 $\int \frac{x^6(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^3} dx$



## Optimal result

Integrand size = 30, antiderivative size = 235

$$\begin{aligned}\int \frac{x^6(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^3} dx &= \frac{(Ab^3-a(3b^2B-6abC+10a^2D))\,x}{b^6}+\frac{(b^2B-3abC+6a^2D)\,x^3}{3b^5}\\&+\frac{(bC-3aD)x^5}{5b^4}+\frac{Dx^7}{7b^3}-\frac{a^2(Ab^3-a(b^2B-abC+a^2D))\,x}{4b^6\,(a+bx^2)^2}\\&+\frac{a(9Ab^3-a(13b^2B-17abC+21a^2D))\,x}{8b^6\,(a+bx^2)}\\&-\frac{\sqrt{a}(15Ab^3-a(35b^2B-63abC+99a^2D))\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{13/2}}\end{aligned}$$

output
```
(A*b^3-a*(3*B*b^2-6*C*a*b+10*D*a^2))*x/b^6+1/3*(B*b^2-3*C*a*b+6*D*a^2)*x^3
/b^5+1/5*(C*b-3*D*a)*x^5/b^4+1/7*D*x^7/b^3-1/4*a^2*(A*b^3-a*(B*b^2-C*a*b+D
*a^2))*x/b^6/(b*x^2+a)^2+1/8*a*(9*A*b^3-a*(13*B*b^2-17*C*a*b+21*D*a^2))*x/
b^6/(b*x^2+a)-1/8*a^(1/2)*(15*A*b^3-a*(35*B*b^2-63*C*a*b+99*D*a^2))*arctan
(b^(1/2)*x/a^(1/2))/b^(13/2)
```

## Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.99

$$\int \frac{x^6(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^3}\,dx$$
$$= \frac{(Ab^3+a(-3b^2B+6abC-10a^2D))\,x}{b^6}+\frac{(b^2B-3abC+6a^2D)\,x^3}{3b^5}+\frac{(bC-3aD)x^5}{5b^4}+\frac{Dx^7}{7b^3}+\frac{a^2(-Ab^3+a(b^2B-abC+a^2D))\,x}{4b^6\,(a+bx^2)^2}+\frac{a(9Ab^3+a(-13b^2B+17abC-21a^2D))\,x}{8b^6\,(a+bx^2)}+\frac{\sqrt{a}(-15Ab^3+a(35b^2B-63abC+99a^2D))\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{13/2}}$$

input
```
Integrate[(x^6*(A + B*x^2 + C*x^4 + D*x^6))/(a + b*x^2)^3,x]
```

output
```
((A*b^3 + a*(-3*b^2*B + 6*a*b*C - 10*a^2*D))*x)/b^6 + ((b^2*B - 3*a*b*C +
6*a^2*D)*x^3)/(3*b^5) + ((b*C - 3*a*D)*x^5)/(5*b^4) + (D*x^7)/(7*b^3) + (a
^2*(-(A*b^3) + a*(b^2*B - a*b*C + a^2*D))*x)/(4*b^6*(a + b*x^2)^2) + (a*(9
*A*b^3 + a*(-13*b^2*B + 17*a*b*C - 21*a^2*D))*x)/(8*b^6*(a + b*x^2)) + (Sq
rt[a]*(-15*A*b^3 + a*(35*b^2*B - 63*a*b*C + 99*a^2*D))*ArcTan[(Sqrt[b]*x)/
Sqrt[a]])/(8*b^(13/2))
```

## Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2335, 9, 1580, 25, 2341, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6\left(A+Bx^2+Cx^4+Dx^6\right)}{\left(a+bx^2\right)^3}dx$$
$$\Big\downarrow \; 2335$$

$$\frac{x^7\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{4a\left(a+bx^2\right)^2}-\frac{\int\frac{x^5\left(-4aDx^5-4a\left(C-\frac{aD}{b}\right)x^3+\left(3Ab-\frac{7a\left(Da^2-bCa+b^2B\right)}{b^2}\right)x\right)}{\left(bx^2+a\right)^2}dx}{4ab}$$

$\Big\downarrow$ 9

$$\frac{x^7\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{4a\left(a+bx^2\right)^2}-\frac{\int\frac{x^6\left(-4aDx^4-4a\left(C-\frac{aD}{b}\right)x^2+3Ab-\frac{7a\left(Da^2-bCa+b^2B\right)}{b^2}\right)}{\left(bx^2+a\right)^2}dx}{4ab}$$

$\Big\downarrow$ 1580

$$\frac{x^7\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{4a\left(a+bx^2\right)^2}-$$

$$-\frac{\int-\frac{-8ab^4Dx^8-8ab^3(bC-2aD)x^6+2b^2\left(3Ab^3-a\left(15Da^2-11bCa+7b^2B\right)\right)x^4-2ab\left(3Ab^3-a\left(15Da^2-11bCa+7b^2B\right)\right)x^2+a^2\left(3Ab^3-a\left(15Da^2-11bCa+7b^2B\right.\right.}{bx^2+a}}{2b^5}$$

$$4ab$$

$\Big\downarrow$ 25

$$\frac{x^7\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{4a\left(a+bx^2\right)^2}-$$

$$\int\frac{-8ab^4Dx^8-8ab^3(bC-2aD)x^6+2b^2\left(3Ab^3-a\left(15Da^2-11bCa+7b^2B\right)\right)x^4-2ab\left(3Ab^3-a\left(15Da^2-11bCa+7b^2B\right)\right)x^2+a^2\left(3Ab^3-a\left(15Da^2-11bCa+7b^2B\right)\right)}{bx^2+a}d$$

$$2b^5$$

$$4ab$$

$\Big\downarrow$ 2341

$$\frac{x^7\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{4a\left(a+bx^2\right)^2}-$$

$$\int\Big(-8ab^3Dx^6-8ab^2(bC-3aD)x^4+2b\left(3Ab^3-a\left(27Da^2-15bCa+7b^2B\right)\right)x^2-4a\left(3Ab^3-a\left(21Da^2-13bCa+7b^2B\right)\right)+\frac{-99Da^5+63bCa^4-35b^2Ba^3+1}{bx^2+a}$$

$$2b^5$$

$$4ab$$

$\Big\downarrow$ 2009

$$\frac{x^7\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{4a\left(a+bx^2\right)^2}-$$

$$\frac{2}{3}bx^3\left(3Ab^3-a\left(27a^2D-15abC+7b^2B\right)\right)-4ax\left(3Ab^3-a\left(21a^2D-13abC+7b^2B\right)\right)+\frac{a^{3/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(15Ab^3-a\left(99a^2D-63abC+35b^2B\right)\right)}{\sqrt{b}}-\frac{8}{7}ab^3Dx$$

$$2b^5$$

$$4ab$$

input
```
Int[(x^6*(A + B*x^2 + C*x^4 + D*x^6))/(a + b*x^2)^3,x]
```

output
```
((A - (a*(b^2*B - a*b*C + a^2*D))/b^3)*x^7)/(4*a*(a + b*x^2)^2) - (-1/2*(a
^2*(3*A*b^3 - a*(7*b^2*B - 11*a*b*C + 15*a^2*D))*x)/(b^5*(a + b*x^2)) + (-
4*a*(3*A*b^3 - a*(7*b^2*B - 13*a*b*C + 21*a^2*D))*x + (2*b*(3*A*b^3 - a*(7
*b^2*B - 15*a*b*C + 27*a^2*D))*x^3)/3 - (8*a*b^2*(b*C - 3*a*D)*x^5)/5 - (8
*a*b^3*D*x^7)/7 + (a^(3/2)*(15*A*b^3 - a*(35*b^2*B - 63*a*b*C + 99*a^2*D))
*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[b])/(2*b^5))/(4*a*b)
```

### Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1580
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
 + e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Simp[1/(2*e^(2*p + m/2)*
(q + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*x^2))*(2*
e^(2*p + m/2)*(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b
*d*e + a*e^2)^p*(d + e*(2*q + 3)*x^2))], x], x], x] /; FreeQ[{a, b, c, d, e
}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2335
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Simp[c/(2*a*b*(p + 1))   Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSu
m[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
 b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

rule 2341
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

## Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.93

| method | result |
|---|---|
| default | $\frac{\frac{1}{7}b^3Dx^7+\frac{1}{5}b^3Cx^5-\frac{3}{5}Dab^2x^5+\frac{1}{3}b^3Bx^3-Cab^2x^3+2Dx^3ba^2+Ab^3x-3Bab^2x+6Ca^2bx-10Da^3x}{b^6}-\frac{a\left(\frac{\left(-\frac{9}{8}Ab^4+\frac{13}{8}Bab^3-\frac{17}{8}\right.}{}\right.}{}$ |

input
```
int(x^6*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b^6*(1/7*b^3*D*x^7+1/5*b^3*C*x^5-3/5*D*a*b^2*x^5+1/3*b^3*B*x^3-C*a*b^2*x
^3+2*D*x^3*b*a^2+A*b^3*x-3*B*a*b^2*x+6*C*a^2*b*x-10*D*a^3*x)-a/b^6*(((-9/8
*A*b^4+13/8*B*a*b^3-17/8*C*a^2*b^2+21/8*a^3*D*b)*x^3-1/8*a*(7*A*b^3-11*B*a
*b^2+15*C*a^2*b-19*D*a^3)*x)/(b*x^2+a)^2+1/8*(15*A*b^3-35*B*a*b^2+63*C*a^2
*b-99*D*a^3)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

## Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 668, normalized size of antiderivative = 2.84

$$\int \frac{x^6(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^3}\,dx$$

$$=\left[\frac{240\,Db^5x^{11}-48\,(11\,Dab^4-7\,Cb^5)x^9+16\,(99\,Da^2b^3-63\,Cab^4+35\,Bb^5)x^7-112\,(99\,Da^3b^2-63\,Ca\cdots}{\cdots}\right.$$

input
```
integrate(x^6*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
[1/1680*(240*D*b^5*x^11 - 48*(11*D*a*b^4 - 7*C*b^5)*x^9 + 16*(99*D*a^2*b^3
 - 63*C*a*b^4 + 35*B*b^5)*x^7 - 112*(99*D*a^3*b^2 - 63*C*a^2*b^3 + 35*B*a*
b^4 - 15*A*b^5)*x^5 - 350*(99*D*a^4*b - 63*C*a^3*b^2 + 35*B*a^2*b^3 - 15*A
*a*b^4)*x^3 - 105*(99*D*a^5 - 63*C*a^4*b + 35*B*a^3*b^2 - 15*A*a^2*b^3 + (
99*D*a^3*b^2 - 63*C*a^2*b^3 + 35*B*a*b^4 - 15*A*b^5)*x^4 + 2*(99*D*a^4*b -
 63*C*a^3*b^2 + 35*B*a^2*b^3 - 15*A*a*b^4)*x^2)*sqrt(-a/b)*log((b*x^2 - 2*
b*x*sqrt(-a/b) - a)/(b*x^2 + a)) - 210*(99*D*a^5 - 63*C*a^4*b + 35*B*a^3*b
^2 - 15*A*a^2*b^3)*x)/(b^8*x^4 + 2*a*b^7*x^2 + a^2*b^6), 1/840*(120*D*b^5*
x^11 - 24*(11*D*a*b^4 - 7*C*b^5)*x^9 + 8*(99*D*a^2*b^3 - 63*C*a*b^4 + 35*B
*b^5)*x^7 - 56*(99*D*a^3*b^2 - 63*C*a^2*b^3 + 35*B*a*b^4 - 15*A*b^5)*x^5 -
 175*(99*D*a^4*b - 63*C*a^3*b^2 + 35*B*a^2*b^3 - 15*A*a*b^4)*x^3 + 105*(99
*D*a^5 - 63*C*a^4*b + 35*B*a^3*b^2 - 15*A*a^2*b^3 + (99*D*a^3*b^2 - 63*C*a
^2*b^3 + 35*B*a*b^4 - 15*A*b^5)*x^4 + 2*(99*D*a^4*b - 63*C*a^3*b^2 + 35*B*
a^2*b^3 - 15*A*a*b^4)*x^2)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) - 105*(99*D*a
^5 - 63*C*a^4*b + 35*B*a^3*b^2 - 15*A*a^2*b^3)*x)/(b^8*x^4 + 2*a*b^7*x^2 +
 a^2*b^6)]
```

## Sympy [A] (verification not implemented)

Time = 6.97 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.34

$$\int \frac{x^6(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^3}\,dx$$
$$= \frac{Dx^7}{7b^3}+x^5\left(\frac{C}{5b^3}-\frac{3Da}{5b^4}\right)+x^3\left(\frac{B}{3b^3}-\frac{Ca}{b^4}+\frac{2Da^2}{b^5}\right)+x\left(\frac{A}{b^3}-\frac{3Ba}{b^4}+\frac{6Ca^2}{b^5}-\frac{10Da^3}{b^6}\right)$$
$$-\frac{\sqrt{-\frac{a}{b^{13}}}(-15Ab^3+35Bab^2-63Ca^2b+99Da^3)\log\left(-b^6\sqrt{-\frac{a}{b^{13}}}+x\right)}{16}$$
$$+\frac{\sqrt{-\frac{a}{b^{13}}}(-15Ab^3+35Bab^2-63Ca^2b+99Da^3)\log\left(b^6\sqrt{-\frac{a}{b^{13}}}+x\right)}{16}$$
$$+\frac{x^3\cdot(9Aab^4-13Ba^2b^3+17Ca^3b^2-21Da^4b)+x(7Aa^2b^3-11Ba^3b^2+15Ca^4b-19Da^5)}{8a^2b^6+16ab^7x^2+8b^8x^4}$$

input
```
integrate(x**6*(D*x**6+C*x**4+B*x**2+A)/(b*x**2+a)**3,x)
```

output
```
D*x**7/(7*b**3) + x**5*(C/(5*b**3) - 3*D*a/(5*b**4)) + x**3*(B/(3*b**3) -
C*a/b**4 + 2*D*a**2/b**5) + x*(A/b**3 - 3*B*a/b**4 + 6*C*a**2/b**5 - 10*D*
a**3/b**6) - sqrt(-a/b**13)*(-15*A*b**3 + 35*B*a*b**2 - 63*C*a**2*b + 99*D
*a**3)*log(-b**6*sqrt(-a/b**13) + x)/16 + sqrt(-a/b**13)*(-15*A*b**3 + 35*
B*a*b**2 - 63*C*a**2*b + 99*D*a**3)*log(b**6*sqrt(-a/b**13) + x)/16 + (x**
3*(9*A*a*b**4 - 13*B*a**2*b**3 + 17*C*a**3*b**2 - 21*D*a**4*b) + x*(7*A*a*
*2*b**3 - 11*B*a**3*b**2 + 15*C*a**4*b - 19*D*a**5))/(8*a**2*b**6 + 16*a*b
**7*x**2 + 8*b**8*x**4)
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.02

$$\int \frac{x^6(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^3}\,dx = -\frac{(21\,Da^4b-17\,Ca^3b^2+13\,Ba^2b^3-9\,Aab^4)x^3+(19\,Da^5-15\,Ca^4b+11\,Ba^3b^2-7\,Aa^2b^3)x}{8\,(b^8x^4+2\,ab^7x^2+a^2b^6)} + \frac{(99\,Da^4-63\,Ca^3b+35\,Ba^2b^2-15\,Aab^3)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}b^6} + \frac{15\,Db^3x^7-21\,(3\,Dab^2-Cb^3)x^5+35\,(6\,Da^2b-3\,Cab^2+Bb^3)x^3-105\,(10\,Da^3-6\,Ca^2b+3\,Bab^2-}{105\,b^6}$$

input
```
integrate(x^6*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
-1/8*((21*D*a^4*b - 17*C*a^3*b^2 + 13*B*a^2*b^3 - 9*A*a*b^4)*x^3 + (19*D*a
^5 - 15*C*a^4*b + 11*B*a^3*b^2 - 7*A*a^2*b^3)*x)/(b^8*x^4 + 2*a*b^7*x^2 +
a^2*b^6) + 1/8*(99*D*a^4 - 63*C*a^3*b + 35*B*a^2*b^2 - 15*A*a*b^3)*arctan(
b*x/sqrt(a*b))/(sqrt(a*b)*b^6) + 1/105*(15*D*b^3*x^7 - 21*(3*D*a*b^2 - C*b
^3)*x^5 + 35*(6*D*a^2*b - 3*C*a*b^2 + B*b^3)*x^3 - 105*(10*D*a^3 - 6*C*a^2
*b + 3*B*a*b^2 - A*b^3)*x)/b^6
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.04

$$\int \frac{x^6(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^3}\,dx$$
$$= \frac{(99\,Da^4-63\,Ca^3b+35\,Ba^2b^2-15\,Aab^3)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}b^6} - \frac{21\,Da^4bx^3-17\,Ca^3b^2x^3+13\,Ba^2b^3x^3-9\,Aab^4x^3+19\,Da^5x-15\,Ca^4bx+11\,Ba^3b^2x-7\,Aa^2b^3x}{8\,(bx^2+a)^2b^6} + \frac{15\,Db^{18}x^7-63\,Dab^{17}x^5+21\,Cb^{18}x^5+210\,Da^2b^{16}x^3-105\,Cab^{17}x^3+35\,Bb^{18}x^3-1050\,Da^3b^{15}x+6}{105\,b^{21}}$$

input
```
integrate(x^6*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
1/8*(99*D*a^4 - 63*C*a^3*b + 35*B*a^2*b^2 - 15*A*a*b^3)*arctan(b*x/sqrt(a*
b))/(sqrt(a*b)*b^6) - 1/8*(21*D*a^4*b*x^3 - 17*C*a^3*b^2*x^3 + 13*B*a^2*b^
3*x^3 - 9*A*a*b^4*x^3 + 19*D*a^5*x - 15*C*a^4*b*x + 11*B*a^3*b^2*x - 7*A*a
^2*b^3*x)/((b*x^2 + a)^2*b^6) + 1/105*(15*D*b^18*x^7 - 63*D*a*b^17*x^5 + 2
1*C*b^18*x^5 + 210*D*a^2*b^16*x^3 - 105*C*a*b^17*x^3 + 35*B*b^18*x^3 - 105
0*D*a^3*b^15*x + 630*C*a^2*b^16*x - 315*B*a*b^17*x + 105*A*b^18*x)/b^21
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^6(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^3}\,dx = \int \frac{x^6\,(A+B\,x^2+C\,x^4+x^6\,D)}{(b\,x^2+a)^3}\,dx$$

input
```
int((x^6*(A + B*x^2 + C*x^4 + x^6*D))/(a + b*x^2)^3,x)
```

output
```
int((x^6*(A + B*x^2 + C*x^4 + x^6*D))/(a + b*x^2)^3, x)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 410, normalized size of antiderivative = 1.74

$$\int \frac{x^6(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^3}\,dx$$
$$= \frac{10395\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^5d-6615\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^4bc+20790\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^4bd\,x^2+210}{}$$

input
```
int(x^6*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^3,x)
```

output
```
(10395*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**5*d - 6615*sqrt(b)
*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**4*b*c + 20790*sqrt(b)*sqrt(a)*at
an((b*x)/(sqrt(b)*sqrt(a)))*a**4*b*d*x**2 + 2100*sqrt(b)*sqrt(a)*atan((b*x
)/(sqrt(b)*sqrt(a)))*a**3*b**3 - 13230*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)
*sqrt(a)))*a**3*b**2*c*x**2 + 10395*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sq
rt(a)))*a**3*b**2*d*x**4 + 4200*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a
)))*a**2*b**4*x**2 - 6615*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*
*2*b**3*c*x**4 + 2100*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*b**5
*x**4 - 10395*a**5*b*d*x + 6615*a**4*b**2*c*x - 17325*a**4*b**2*d*x**3 - 2
100*a**3*b**4*x + 11025*a**3*b**3*c*x**3 - 5544*a**3*b**3*d*x**5 - 3500*a*
*2*b**5*x**3 + 3528*a**2*b**4*c*x**5 + 792*a**2*b**4*d*x**7 - 1120*a*b**6*
x**5 - 504*a*b**5*c*x**7 - 264*a*b**5*d*x**9 + 280*b**7*x**7 + 168*b**6*c*
x**9 + 120*b**6*d*x**11)/(840*b**7*(a**2 + 2*a*b*x**2 + b**2*x**4))
```

# 3.181 $\int \frac{x^4(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^3}\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 195

$$\begin{aligned}\int \frac{x^4(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^3}\,dx &= \frac{(b^2B-3abC+6a^2D)\,x}{b^5}+\frac{(bC-3aD)x^3}{3b^4}+\frac{Dx^5}{5b^3}\\&\quad+\frac{a(Ab^3-a(b^2B-abC+a^2D))\,x}{4b^5\,(a+bx^2)^2}-\frac{(5Ab^3-a(9b^2B-13abC+17a^2D))\,x}{8b^5\,(a+bx^2)}\\&\quad+\frac{(3Ab^3-a(15b^2B-35abC+63a^2D))\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8\sqrt{a}b^{11/2}}\end{aligned}$$

output
```
(B*b^2-3*C*a*b+6*D*a^2)*x/b^5+1/3*(C*b-3*D*a)*x^3/b^4+1/5*D*x^5/b^3+1/4*a*
(A*b^3-a*(B*b^2-C*a*b+D*a^2))*x/b^5/(b*x^2+a)^2-1/8*(5*A*b^3-a*(9*B*b^2-13
*C*a*b+17*D*a^2))*x/b^5/(b*x^2+a)+1/8*(3*A*b^3-a*(15*B*b^2-35*C*a*b+63*D*a
^2))*arctan(b^(1/2)*x/a^(1/2))/a^(1/2)/b^(11/2)
```

## Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.90

$$\int \frac{x^4(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^3}\,dx$$
$$= \frac{x(945a^4D-525a^3b(C-3Dx^2)+a^2b^2(225B-875Cx^2+504Dx^4)-ab^3(45A-375Bx^2+280Cx^4+7}{120b^5\left(a+bx^2\right)^2}$$
$$+\frac{(3Ab^3+a(-15b^2B+35abC-63a^2D))\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8\sqrt{a}b^{11/2}}$$

input
```
Integrate[(x^4*(A + B*x^2 + C*x^4 + D*x^6))/(a + b*x^2)^3,x]
```

output
```
(x*(945*a^4*D - 525*a^3*b*(C - 3*D*x^2) + a^2*b^2*(225*B - 875*C*x^2 + 504
*D*x^4) - a*b^3*(45*A - 375*B*x^2 + 280*C*x^4 + 72*D*x^6) + b^4*x^2*(-75*A
 + 8*(15*B*x^2 + 5*C*x^4 + 3*D*x^6))))/(120*b^5*(a + b*x^2)^2) + ((3*A*b^3
 + a*(-15*b^2*B + 35*a*b*C - 63*a^2*D))*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*Sq
rt[a]*b^(11/2))
```

## Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.12, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2335, 9, 1580, 2341, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4\left(A+Bx^2+Cx^4+Dx^6\right)}{\left(a+bx^2\right)^3}dx$$
$$\Big\downarrow \text{2335}$$
$$\frac{x^5\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{4a\left(a+bx^2\right)^2}-\frac{\int \frac{x^3\left(-4aDx^5-4a\left(C-\frac{aD}{b}\right)x^3+\left(Ab-\frac{5a\left(Da^2-bCa+b^2B\right)}{b^2}\right)x\right)}{\left(bx^2+a\right)^2}dx}{4ab}$$

$$\Big\downarrow \text{9}$$

$$\frac{x^5\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{4a\left(a+bx^2\right)^2}-\frac{\int \frac{x^4\left(-4aDx^4-4a\left(C-\frac{aD}{b}\right)x^2+Ab-\frac{5a\left(Da^2-bCa+b^2B\right)}{b^2}\right)}{\left(bx^2+a\right)^2}dx}{4ab}$$

$$\Big\downarrow \text{1580}$$

$$\frac{x^5\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{4a\left(a+bx^2\right)^2}-$$
$$\frac{\frac{ax\left(Ab^3-a\left(13a^2D-9abC+5b^2B\right)\right)}{2b^4(a+bx^2)}-\frac{\int \frac{8ab^3Dx^6+8ab^2(bC-2aD)x^4-2b\left(Ab^3-a\left(13Da^2-9bCa+5b^2B\right)\right)x^2+a\left(Ab^3-a\left(13Da^2-9bCa+5b^2B\right)\right)}{bx^2+a}dx}{2b^4}}{4ab}$$

$$\Big\downarrow \text{2341}$$

$$\frac{x^5\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{4a\left(a+bx^2\right)^2}-$$
$$\frac{\frac{ax\left(Ab^3-a\left(13a^2D-9abC+5b^2B\right)\right)}{2b^4(a+bx^2)}-\frac{\int \left(8ab^2Dx^4+8ab(bC-3aD)x^2-2\left(-25Da^3+13bCa^2-5b^2Ba+Ab^3\right)+\frac{-63Da^4+35bCa^3-15b^2Ba^2+3Ab^3a}{bx^2+a}\right)}{2b^4}}{4ab}$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^5\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{4a\left(a+bx^2\right)^2}-$$
$$\frac{\frac{ax\left(Ab^3-a\left(13a^2D-9abC+5b^2B\right)\right)}{2b^4(a+bx^2)}-\frac{\frac{\sqrt{a}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(3Ab^3-a\left(63a^2D-35abC+15b^2B\right)\right)}{\sqrt{b}}-2x\left(Ab^3-a\left(25a^2D-13abC+5b^2B\right)\right)+\frac{8}{5}ab^2Dx^5+\frac{8}{3}ab}{2b^4}}{4ab}$$

input
```
Int[(x^4*(A + B*x^2 + C*x^4 + D*x^6))/(a + b*x^2)^3,x]
```

output
```
((A - (a*(b^2*B - a*b*C + a^2*D))/b^3)*x^5)/(4*a*(a + b*x^2)^2) - ((a*(A*b
^3 - a*(5*b^2*B - 9*a*b*C + 13*a^2*D))*x)/(2*b^4*(a + b*x^2)) - (-2*(A*b^3
 - a*(5*b^2*B - 13*a*b*C + 25*a^2*D))*x + (8*a*b*(b*C - 3*a*D)*x^3)/3 + (8
*a*b^2*D*x^5)/5 + (Sqrt[a]*(3*A*b^3 - a*(15*b^2*B - 35*a*b*C + 63*a^2*D))*
ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[b])/(2*b^4))/(4*a*b)
```

### Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 1580
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
 + e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Simp[1/(2*e^(2*p + m/2)*
(q + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*x^2))*(2*
e^(2*p + m/2)*(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b
*d*e + a*e^2)^p*(d + e*(2*q + 3)*x^2))], x], x], x] /; FreeQ[{a, b, c, d, e
}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2335
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Simp[c/(2*a*b*(p + 1))   Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSu
m[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
 b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

rule 2341
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

## Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.90

| method | result |
|---|---|
| default | $\frac{\frac{1}{5}Dx^5b^2+\frac{1}{3}Cb^2x^3-Dabx^3+b^2Bx-3Cabx+6Da^2x}{b^5}+\frac{\frac{\left(-\frac{5}{8}Ab^4+\frac{9}{8}Bab^3-\frac{13}{8}Ca^2b^2+\frac{17}{8}a^3Db\right)x^3-\frac{a\left(3b^3A-7ab^2B+11a^2bC-15a^3D\right)}{8}}{\left(bx^2+a\right)^2}}{b^5}$ |

input
```
int(x^4*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b^5*(1/5*D*x^5*b^2+1/3*C*b^2*x^3-D*a*b*x^3+b^2*B*x-3*C*a*b*x+6*D*a^2*x)+
1/b^5*(((-5/8*A*b^4+9/8*B*a*b^3-13/8*C*a^2*b^2+17/8*a^3*D*b)*x^3-1/8*a*(3*
A*b^3-7*B*a*b^2+11*C*a^2*b-15*D*a^3)*x)/(b*x^2+a)^2+1/8*(3*A*b^3-15*B*a*b^
2+35*C*a^2*b-63*D*a^3)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 614, normalized size of antiderivative = 3.15

$$\int \frac{x^4(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^3}\,dx$$
$$=\left[\frac{48\,Dab^5x^9-16\,(9\,Da^2b^4-5\,Cab^5)x^7+16\,(63\,Da^3b^3-35\,Ca^2b^4+15\,Bab^5)x^5+50\,(63\,Da^4b^2-35\,C}{}\right.$$

input
```
integrate(x^4*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
[1/240*(48*D*a*b^5*x^9 - 16*(9*D*a^2*b^4 - 5*C*a*b^5)*x^7 + 16*(63*D*a^3*b
^3 - 35*C*a^2*b^4 + 15*B*a*b^5)*x^5 + 50*(63*D*a^4*b^2 - 35*C*a^3*b^3 + 15
*B*a^2*b^4 - 3*A*a*b^5)*x^3 + 15*(63*D*a^5 - 35*C*a^4*b + 15*B*a^3*b^2 - 3
*A*a^2*b^3 + (63*D*a^3*b^2 - 35*C*a^2*b^3 + 15*B*a*b^4 - 3*A*b^5)*x^4 + 2*
(63*D*a^4*b - 35*C*a^3*b^2 + 15*B*a^2*b^3 - 3*A*a*b^4)*x^2)*sqrt(-a*b)*log
((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 30*(63*D*a^5*b - 35*C*a^4*b^2
 + 15*B*a^3*b^3 - 3*A*a^2*b^4)*x)/(a*b^8*x^4 + 2*a^2*b^7*x^2 + a^3*b^6), 1
/120*(24*D*a*b^5*x^9 - 8*(9*D*a^2*b^4 - 5*C*a*b^5)*x^7 + 8*(63*D*a^3*b^3 -
 35*C*a^2*b^4 + 15*B*a*b^5)*x^5 + 25*(63*D*a^4*b^2 - 35*C*a^3*b^3 + 15*B*a
^2*b^4 - 3*A*a*b^5)*x^3 - 15*(63*D*a^5 - 35*C*a^4*b + 15*B*a^3*b^2 - 3*A*a
^2*b^3 + (63*D*a^3*b^2 - 35*C*a^2*b^3 + 15*B*a*b^4 - 3*A*b^5)*x^4 + 2*(63*
D*a^4*b - 35*C*a^3*b^2 + 15*B*a^2*b^3 - 3*A*a*b^4)*x^2)*sqrt(a*b)*arctan(s
qrt(a*b)*x/a) + 15*(63*D*a^5*b - 35*C*a^4*b^2 + 15*B*a^3*b^3 - 3*A*a^2*b^4
)*x)/(a*b^8*x^4 + 2*a^2*b^7*x^2 + a^3*b^6)]
```

## Sympy [A] (verification not implemented)

Time = 6.46 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.44

$$\int \frac{x^4(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^3}\,dx = \frac{Dx^5}{5b^3}+x^3\left(\frac{C}{3b^3}-\frac{Da}{b^4}\right)+x\left(\frac{B}{b^3}-\frac{3Ca}{b^4}+\frac{6Da^2}{b^5}\right) + \frac{\sqrt{-\frac{1}{ab^{11}}}\,(-3Ab^3+15Bab^2-35Ca^2b+63Da^3)\log\left(-ab^5\sqrt{-\frac{1}{ab^{11}}}+x\right)}{16} - \frac{\sqrt{-\frac{1}{ab^{11}}}\,(-3Ab^3+15Bab^2-35Ca^2b+63Da^3)\log\left(ab^5\sqrt{-\frac{1}{ab^{11}}}+x\right)}{16} + \frac{x^3(-5Ab^4+9Bab^3-13Ca^2b^2+17Da^3b)+x(-3Aab^3+7Ba^2b^2-11Ca^3b+15Da^4)}{8a^2b^5+16ab^6x^2+8b^7x^4}$$

input
```
integrate(x**4*(D*x**6+C*x**4+B*x**2+A)/(b*x**2+a)**3,x)
```

output
```
D*x**5/(5*b**3) + x**3*(C/(3*b**3) - D*a/b**4) + x*(B/b**3 - 3*C*a/b**4 +
6*D*a**2/b**5) + sqrt(-1/(a*b**11))*(-3*A*b**3 + 15*B*a*b**2 - 35*C*a**2*b
 + 63*D*a**3)*log(-a*b**5*sqrt(-1/(a*b**11)) + x)/16 - sqrt(-1/(a*b**11))*
(-3*A*b**3 + 15*B*a*b**2 - 35*C*a**2*b + 63*D*a**3)*log(a*b**5*sqrt(-1/(a*
b**11)) + x)/16 + (x**3*(-5*A*b**4 + 9*B*a*b**3 - 13*C*a**2*b**2 + 17*D*a*
*3*b) + x*(-3*A*a*b**3 + 7*B*a**2*b**2 - 11*C*a**3*b + 15*D*a**4))/(8*a**2
*b**5 + 16*a*b**6*x**2 + 8*b**7*x**4)
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.99

$$\int \frac{x^4(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^3}\,dx$$
$$= \frac{(17\,Da^3b - 13\,Ca^2b^2 + 9\,Bab^3 - 5\,Ab^4)x^3 + (15\,Da^4 - 11\,Ca^3b + 7\,Ba^2b^2 - 3\,Aab^3)x}{8\,(b^7x^4 + 2\,ab^6x^2 + a^2b^5)}$$
$$- \frac{(63\,Da^3 - 35\,Ca^2b + 15\,Bab^2 - 3\,Ab^3)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}b^5}$$
$$+ \frac{3\,Db^2x^5 - 5\,(3\,Dab - Cb^2)x^3 + 15\,(6\,Da^2 - 3\,Cab + Bb^2)x}{15\,b^5}$$

input
```
integrate(x^4*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/8*((17*D*a^3*b - 13*C*a^2*b^2 + 9*B*a*b^3 - 5*A*b^4)*x^3 + (15*D*a^4 - 1
1*C*a^3*b + 7*B*a^2*b^2 - 3*A*a*b^3)*x)/(b^7*x^4 + 2*a*b^6*x^2 + a^2*b^5)
- 1/8*(63*D*a^3 - 35*C*a^2*b + 15*B*a*b^2 - 3*A*b^3)*arctan(b*x/sqrt(a*b))
/(sqrt(a*b)*b^5) + 1/15*(3*D*b^2*x^5 - 5*(3*D*a*b - C*b^2)*x^3 + 15*(6*D*a
^2 - 3*C*a*b + B*b^2)*x)/b^5
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.00

$$\int \frac{x^4(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^3}\,dx$$
$$= -\frac{(63\,Da^3 - 35\,Ca^2b + 15\,Bab^2 - 3\,Ab^3)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}b^5}$$
$$+ \frac{17\,Da^3bx^3 - 13\,Ca^2b^2x^3 + 9\,Bab^3x^3 - 5\,Ab^4x^3 + 15\,Da^4x - 11\,Ca^3bx + 7\,Ba^2b^2x - 3\,Aab^3x}{8\,(bx^2+a)^2b^5}$$
$$+ \frac{3\,Db^{12}x^5 - 15\,Dab^{11}x^3 + 5\,Cb^{12}x^3 + 90\,Da^2b^{10}x - 45\,Cab^{11}x + 15\,Bb^{12}x}{15\,b^{15}}$$

input
```
integrate(x^4*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
-1/8*(63*D*a^3 - 35*C*a^2*b + 15*B*a*b^2 - 3*A*b^3)*arctan(b*x/sqrt(a*b))/
(sqrt(a*b)*b^5) + 1/8*(17*D*a^3*b*x^3 - 13*C*a^2*b^2*x^3 + 9*B*a*b^3*x^3 -
 5*A*b^4*x^3 + 15*D*a^4*x - 11*C*a^3*b*x + 7*B*a^2*b^2*x - 3*A*a*b^3*x)/((
b*x^2 + a)^2*b^5) + 1/15*(3*D*b^12*x^5 - 15*D*a*b^11*x^3 + 5*C*b^12*x^3 +
90*D*a^2*b^10*x - 45*C*a*b^11*x + 15*B*b^12*x)/b^15
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^3}\,dx = \int \frac{x^4\,(A+B\,x^2+C\,x^4+x^6\,D)}{(b\,x^2+a)^3}\,dx$$

input
```
int((x^4*(A + B*x^2 + C*x^4 + x^6*D))/(a + b*x^2)^3,x)
```

output
```
int((x^4*(A + B*x^2 + C*x^4 + x^6*D))/(a + b*x^2)^3, x)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.90

$$\int \frac{x^4(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^3}\,dx$$
$$= \frac{-945\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^4d+525\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^3bc-1890\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^3bd\,x^2-180\sqrt{}}{}$$

input
```
int(x^4*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^3,x)
```

output
```
( - 945*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**4*d + 525*sqrt(b)
*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**3*b*c - 1890*sqrt(b)*sqrt(a)*ata
n((b*x)/(sqrt(b)*sqrt(a)))*a**3*b*d*x**2 - 180*sqrt(b)*sqrt(a)*atan((b*x)/
(sqrt(b)*sqrt(a)))*a**2*b**3 + 1050*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sq
rt(a)))*a**2*b**2*c*x**2 - 945*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)
))*a**2*b**2*d*x**4 - 360*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*
b**4*x**2 + 525*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*b**3*c*x**
4 - 180*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**5*x**4 + 945*a**4
*b*d*x - 525*a**3*b**2*c*x + 1575*a**3*b**2*d*x**3 + 180*a**2*b**4*x - 875
*a**2*b**3*c*x**3 + 504*a**2*b**3*d*x**5 + 300*a*b**5*x**3 - 280*a*b**4*c*
x**5 - 72*a*b**4*d*x**7 + 120*b**6*x**5 + 40*b**5*c*x**7 + 24*b**5*d*x**9)
/(120*b**6*(a**2 + 2*a*b*x**2 + b**2*x**4))
```

# 3.182 $\int \frac{x^2(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^3}\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 167

$$\int \frac{x^2(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^3}\,dx = \frac{(bC-3aD)x}{b^4}+\frac{Dx^3}{3b^3} - \frac{(Ab^3-a(b^2B-abC+a^2D))\,x}{4b^4\,(a+bx^2)^2} + \frac{(Ab^3-a(5b^2B-9abC+13a^2D))\,x}{8ab^4\,(a+bx^2)} + \frac{(Ab^3+a(3b^2B-15abC+35a^2D))\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{3/2}b^{9/2}}$$

output
```
(C*b-3*D*a)*x/b^4+1/3*D*x^3/b^3-1/4*(A*b^3-a*(B*b^2-C*a*b+D*a^2))*x/b^4/(b
*x^2+a)^2+1/8*(A*b^3-a*(5*B*b^2-9*C*a*b+13*D*a^2))*x/a/b^4/(b*x^2+a)+1/8*(
A*b^3+a*(3*B*b^2-15*C*a*b+35*D*a^2))*arctan(b^(1/2)*x/a^(1/2))/a^(3/2)/b^(
9/2)
```

## Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.93

$$\int \frac{x^2(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^3}\,dx$$
$$= \frac{x(-105a^4D+3Ab^4x^2+5a^3b(9C-35Dx^2)+a^2b^2(-9B+75Cx^2-56Dx^4)+ab^3(-3A-15Bx^2+24}{24ab^4\left(a+bx^2\right)^2}$$
$$+\frac{(Ab^3+a(3b^2B-15abC+35a^2D))\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{3/2}b^{9/2}}$$

input
```
Integrate[(x^2*(A + B*x^2 + C*x^4 + D*x^6))/(a + b*x^2)^3,x]
```

output
```
(x*(-105*a^4*D + 3*A*b^4*x^2 + 5*a^3*b*(9*C - 35*D*x^2) + a^2*b^2*(-9*B +
75*C*x^2 - 56*D*x^4) + a*b^3*(-3*A - 15*B*x^2 + 24*C*x^4 + 8*D*x^6)))/(24*
a*b^4*(a + b*x^2)^2) + ((A*b^3 + a*(3*b^2*B - 15*a*b*C + 35*a^2*D))*ArcTan
[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(3/2)*b^(9/2))
```

## Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {2335, 9, 25, 1580, 25, 1467, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\left(A+Bx^2+Cx^4+Dx^6\right)}{\left(a+bx^2\right)^3}dx$$

$$\Big\downarrow \ 2335$$

$$\frac{x^3\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{4a\left(a+bx^2\right)^2}-\frac{\int -\frac{x\left(4aDx^5+4a\left(C-\frac{aD}{b}\right)x^3+\left(Ab+\frac{3a\left(Da^2-bCa+b^2B\right)}{b^2}\right)x\right)}{\left(bx^2+a\right)^2}dx}{4ab}$$

$$\Big\downarrow \ 9$$

$$\frac{x^3\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{4a\left(a+bx^2\right)^2}-\frac{\int -\frac{x^2\left(4aDx^4+4a\left(C-\frac{aD}{b}\right)x^2+Ab+\frac{3a\left(Da^2-bCa+b^2B\right)}{b^2}\right)}{(bx^2+a)^2}dx}{4ab}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{x^2\left(4aDx^4+4a\left(C-\frac{aD}{b}\right)x^2+Ab+\frac{3a\left(Da^2-bCa+b^2B\right)}{b^2}\right)}{(bx^2+a)^2}dx}{4ab}+\frac{x^3\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{4a\left(a+bx^2\right)^2}$$

$$\Big\downarrow \; 1580$$

$$\frac{-\frac{\int -\frac{8ab^2Dx^4+8ab(bC-2aD)x^2+Ab^3+a\left(11Da^2-7bCa+3b^2B\right)}{bx^2+a}dx}{2b^3}-\frac{x\left(\frac{a\left(11a^2D-7abC+3b^2B\right)}{b^3}+A\right)}{2(a+bx^2)}}{4ab}+\frac{x^3\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{4a\left(a+bx^2\right)^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{8ab^2Dx^4+8ab(bC-2aD)x^2+Ab^3+a\left(11Da^2-7bCa+3b^2B\right)}{bx^2+a}dx}{2b^3}-\frac{x\left(\frac{a\left(11a^2D-7abC+3b^2B\right)}{b^3}+A\right)}{2(a+bx^2)}}{4ab}+\frac{x^3\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{4a\left(a+bx^2\right)^2}$$

$$\Big\downarrow \; 1467$$

$$\frac{\frac{\int \left(8abDx^2+8a(bC-3aD)+\frac{35Da^3-15bCa^2+3b^2Ba+Ab^3}{bx^2+a}\right)dx}{2b^3}-\frac{x\left(\frac{a\left(11a^2D-7abC+3b^2B\right)}{b^3}+A\right)}{2(a+bx^2)}}{4ab}+\frac{x^3\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{4a\left(a+bx^2\right)^2}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a\left(35a^2D-15abC+3b^2B\right)+Ab^3\right)}{\sqrt{a}\sqrt{b}}+8ax(bC-3aD)+\frac{8}{3}abDx^3}{2b^3}-\frac{x\left(\frac{a\left(11a^2D-7abC+3b^2B\right)}{b^3}+A\right)}{2(a+bx^2)}}{4ab}+\frac{x^3\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{4a\left(a+bx^2\right)^2}$$

input
```
Int[(x^2*(A + B*x^2 + C*x^4 + D*x^6))/(a + b*x^2)^3,x]
```

output
```
((A - (a*(b^2*B - a*b*C + a^2*D))/b^3)*x^3)/(4*a*(a + b*x^2)^2) + (-1/2*((
A + (a*(3*b^2*B - 7*a*b*C + 11*a^2*D))/b^3)*x)/(a + b*x^2) + (8*a*(b*C - 3
*a*D)*x + (8*a*b*D*x^3)/3 + ((A*b^3 + a*(3*b^2*B - 15*a*b*C + 35*a^2*D))*A
rcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*Sqrt[b]))/(2*b^3))/(4*a*b)
```

### Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1467
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 1580
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
 + e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Simp[1/(2*e^(2*p + m/2)*
(q + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*x^2))*(2*
e^(2*p + m/2)*(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b
*d*e + a*e^2)^p*(d + e*(2*q + 3)*x^2))], x], x], x] /; FreeQ[{a, b, c, d, e
}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2335
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Simp[c/(2*a*b*(p + 1))   Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSu
m[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
 b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

## Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.89

| method | result |
|---|---|
| default | $\frac{\frac{1}{3}Dx^3b+Cbx-3Dax}{b^4}+\frac{\frac{\frac{b\left(b^3A-5a\,b^2B+9a^2bC-13a^3D\right)x^3}{8a}+\left(-\frac{1}{8}b^3A-\frac{3}{8}a\,b^2B+\frac{7}{8}a^2bC-\frac{11}{8}a^3D\right)x}{\left(b\,x^2+a\right)^2}+\frac{\left(b^3A+3a\,b^2B-15a^2bC+35a^3D\right)\operatorname{arc}}{8a\sqrt{ab}}}{b^4}$ |

input
```
int(x^2*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b^4*(1/3*D*x^3*b+C*b*x-3*D*a*x)+1/b^4*((1/8*b*(A*b^3-5*B*a*b^2+9*C*a^2*b
-13*D*a^3)/a*x^3+(-1/8*b^3*A-3/8*a*b^2*B+7/8*a^2*b*C-11/8*a^3*D)*x)/(b*x^2
+a)^2+1/8*(A*b^3+3*B*a*b^2-15*C*a^2*b+35*D*a^3)/a/(a*b)^(1/2)*arctan(b*x/(
a*b)^(1/2)))
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 556, normalized size of antiderivative = 3.33

$$\int \frac{x^2(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^3}\,dx$$
$$= \left[\frac{16\,Da^2b^4x^7-16\,(7\,Da^3b^3-3\,Ca^2b^4)x^5-2\,(175\,Da^4b^2-75\,Ca^3b^3+15\,Ba^2b^4-3\,Aab^5)x^3-3\,(35\,Da\right.$$

input
```
integrate(x^2*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
[1/48*(16*D*a^2*b^4*x^7 - 16*(7*D*a^3*b^3 - 3*C*a^2*b^4)*x^5 - 2*(175*D*a^
4*b^2 - 75*C*a^3*b^3 + 15*B*a^2*b^4 - 3*A*a*b^5)*x^3 - 3*(35*D*a^5 - 15*C*
a^4*b + 3*B*a^3*b^2 + A*a^2*b^3 + (35*D*a^3*b^2 - 15*C*a^2*b^3 + 3*B*a*b^4
 + A*b^5)*x^4 + 2*(35*D*a^4*b - 15*C*a^3*b^2 + 3*B*a^2*b^3 + A*a*b^4)*x^2)
*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) - 6*(35*D*a^5*b
- 15*C*a^4*b^2 + 3*B*a^3*b^3 + A*a^2*b^4)*x)/(a^2*b^7*x^4 + 2*a^3*b^6*x^2
+ a^4*b^5), 1/24*(8*D*a^2*b^4*x^7 - 8*(7*D*a^3*b^3 - 3*C*a^2*b^4)*x^5 - (1
75*D*a^4*b^2 - 75*C*a^3*b^3 + 15*B*a^2*b^4 - 3*A*a*b^5)*x^3 + 3*(35*D*a^5
- 15*C*a^4*b + 3*B*a^3*b^2 + A*a^2*b^3 + (35*D*a^3*b^2 - 15*C*a^2*b^3 + 3*
B*a*b^4 + A*b^5)*x^4 + 2*(35*D*a^4*b - 15*C*a^3*b^2 + 3*B*a^2*b^3 + A*a*b^
4)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) - 3*(35*D*a^5*b - 15*C*a^4*b^2 + 3
*B*a^3*b^3 + A*a^2*b^4)*x)/(a^2*b^7*x^4 + 2*a^3*b^6*x^2 + a^4*b^5)]
```

## Sympy [A] (verification not implemented)

Time = 5.33 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.56

$$\int \frac{x^2(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^3}\,dx$$
$$= \frac{Dx^3}{3b^3} + x\left(\frac{C}{b^3} - \frac{3Da}{b^4}\right) - \frac{\sqrt{-\frac{1}{a^3b^9}}(Ab^3+3Bab^2-15Ca^2b+35Da^3)\log\left(-a^2b^4\sqrt{-\frac{1}{a^3b^9}}+x\right)}{16} + \frac{\sqrt{-\frac{1}{a^3b^9}}(Ab^3+3Bab^2-15Ca^2b+35Da^3)\log\left(a^2b^4\sqrt{-\frac{1}{a^3b^9}}+x\right)}{16} + \frac{x^3(Ab^4-5Bab^3+9Ca^2b^2-13Da^3b)+x(-Aab^3-3Ba^2b^2+7Ca^3b-11Da^4)}{8a^3b^4+16a^2b^5x^2+8ab^6x^4}$$

input
```
integrate(x**2*(D*x**6+C*x**4+B*x**2+A)/(b*x**2+a)**3,x)
```

output
```
D*x**3/(3*b**3) + x*(C/b**3 - 3*D*a/b**4) - sqrt(-1/(a**3*b**9))*(A*b**3 +
 3*B*a*b**2 - 15*C*a**2*b + 35*D*a**3)*log(-a**2*b**4*sqrt(-1/(a**3*b**9))
 + x)/16 + sqrt(-1/(a**3*b**9))*(A*b**3 + 3*B*a*b**2 - 15*C*a**2*b + 35*D*
a**3)*log(a**2*b**4*sqrt(-1/(a**3*b**9)) + x)/16 + (x**3*(A*b**4 - 5*B*a*b
**3 + 9*C*a**2*b**2 - 13*D*a**3*b) + x*(-A*a*b**3 - 3*B*a**2*b**2 + 7*C*a*
*3*b - 11*D*a**4))/(8*a**3*b**4 + 16*a**2*b**5*x**2 + 8*a*b**6*x**4)
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.02

$$\int \frac{x^2(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^3}\,dx = -\frac{(13\,Da^3b-9\,Ca^2b^2+5\,Bab^3-Ab^4)x^3+(11\,Da^4-7\,Ca^3b+3\,Ba^2b^2+Aab^3)x}{8\,(ab^6x^4+2\,a^2b^5x^2+a^3b^4)} + \frac{Dbx^3-3\,(3\,Da-Cb)x}{3\,b^4} + \frac{(35\,Da^3-15\,Ca^2b+3\,Bab^2+Ab^3)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}ab^4}$$

input
```
integrate(x^2*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
-1/8*((13*D*a^3*b - 9*C*a^2*b^2 + 5*B*a*b^3 - A*b^4)*x^3 + (11*D*a^4 - 7*C
*a^3*b + 3*B*a^2*b^2 + A*a*b^3)*x)/(a*b^6*x^4 + 2*a^2*b^5*x^2 + a^3*b^4) +
 1/3*(D*b*x^3 - 3*(3*D*a - C*b)*x)/b^4 + 1/8*(35*D*a^3 - 15*C*a^2*b + 3*B*
a*b^2 + A*b^3)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b^4)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.01

$$\int \frac{x^2(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^3}\,dx = \frac{(35\,Da^3-15\,Ca^2b+3\,Bab^2+Ab^3)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}ab^4} - \frac{13\,Da^3bx^3-9\,Ca^2b^2x^3+5\,Bab^3x^3-Ab^4x^3+11\,Da^4x-7\,Ca^3bx+3\,Ba^2b^2x+Aab^3x}{8\,(bx^2+a)^2ab^4} + \frac{Db^6x^3-9\,Dab^5x+3\,Cb^6x}{3\,b^9}$$

input
```
integrate(x^2*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
1/8*(35*D*a^3 - 15*C*a^2*b + 3*B*a*b^2 + A*b^3)*arctan(b*x/sqrt(a*b))/(sqr
t(a*b)*a*b^4) - 1/8*(13*D*a^3*b*x^3 - 9*C*a^2*b^2*x^3 + 5*B*a*b^3*x^3 - A*
b^4*x^3 + 11*D*a^4*x - 7*C*a^3*b*x + 3*B*a^2*b^2*x + A*a*b^3*x)/((b*x^2 +
a)^2*a*b^4) + 1/3*(D*b^6*x^3 - 9*D*a*b^5*x + 3*C*b^6*x)/b^9
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^3}\,dx=\int \frac{x^2\,(A+B\,x^2+C\,x^4+x^6\,D)}{(b\,x^2+a)^3}\,dx$$

input
```
int((x^2*(A + B*x^2 + C*x^4 + x^6*D))/(a + b*x^2)^3,x)
```

output
```
int((x^2*(A + B*x^2 + C*x^4 + x^6*D))/(a + b*x^2)^3, x)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 347, normalized size of antiderivative = 2.08

$$\int \frac{x^2(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^3}\,dx = \frac{105\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^4d-45\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^3bc+210\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^3bd\,x^2+12\sqrt{b}\,\sqrt{a}\,a$$

input
```
int(x^2*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^3,x)
```

output
```
(105*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**4*d - 45*sqrt(b)*sqr
t(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**3*b*c + 210*sqrt(b)*sqrt(a)*atan((b*
x)/(sqrt(b)*sqrt(a)))*a**3*b*d*x**2 + 12*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(
b)*sqrt(a)))*a**2*b**3 - 90*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*
a**2*b**2*c*x**2 + 105*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2*
b**2*d*x**4 + 24*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*b**4*x**2
 - 45*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*b**3*c*x**4 + 12*sqr
t(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**5*x**4 - 105*a**4*b*d*x + 45
*a**3*b**2*c*x - 175*a**3*b**2*d*x**3 - 12*a**2*b**4*x + 75*a**2*b**3*c*x*
*3 - 56*a**2*b**3*d*x**5 - 12*a*b**5*x**3 + 24*a*b**4*c*x**5 + 8*a*b**4*d*
x**7)/(24*a*b**5*(a**2 + 2*a*b*x**2 + b**2*x**4))
```

# 3.183 $\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^3} dx$



## Optimal result

Integrand size = 27, antiderivative size = 147

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^3} dx = \frac{Dx}{b^3} + \frac{\left(\frac{A}{a} - \frac{b^2B-abC+a^2D}{b^3}\right)x}{4(a+bx^2)^2} + \frac{(3Ab^3+a(b^2B-5abC+9a^2D))x}{8a^2b^3(a+bx^2)} + \frac{(3Ab^3+a(b^2B+3abC-15a^2D))\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{7/2}}$$

output
```
D*x/b^3+1/4*(A/a-(B*b^2-C*a*b+D*a^2)/b^3)*x/(b*x^2+a)^2+1/8*(3*A*b^3+a*(B*
b^2-5*C*a*b+9*D*a^2))*x/a^2/b^3/(b*x^2+a)+1/8*(3*A*b^3+a*(B*b^2+3*C*a*b-15
*D*a^2))*arctan(b^(1/2)*x/a^(1/2))/a^(5/2)/b^(7/2)
```

## Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.96

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^3}\,dx = \frac{x(15a^4D+3Ab^4x^2+ab^3(5A+Bx^2)+a^3b(-3C+25Dx^2)-a^2b^2(B+5Cx^2-8Dx^4))}{8a^2b^3(a+bx^2)^2} + \frac{(3Ab^3+a(b^2B+3abC-15a^2D))\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{7/2}}$$

input
```
Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(a + b*x^2)^3,x]
```

output
```
(x*(15*a^4*D + 3*A*b^4*x^2 + a*b^3*(5*A + B*x^2) + a^3*b*(-3*C + 25*D*x^2)
 - a^2*b^2*(B + 5*C*x^2 - 8*D*x^4)))/(8*a^2*b^3*(a + b*x^2)^2) + ((3*A*b^3
 + a*(b^2*B + 3*a*b*C - 15*a^2*D))*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(5/2)
*b^(7/2))
```

## Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2345, 25, 1471, 25, 27, 299, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^3}\,dx$$

$$\Big\downarrow \text{2345}$$

$$\frac{x\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{4a(a+bx^2)^2}-\frac{\int -\frac{\frac{4aDx^4}{b}+\frac{4a(bC-aD)x^2}{b^2}+\frac{Da^3-bCa^2+b^2Ba+3Ab^3}{b^3}}{(bx^2+a)^2}dx}{4a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\frac{4aDx^4}{b}+\frac{4a(bC-aD)x^2}{b^2}+\frac{Da^3-bCa^2+b^2Ba+3Ab^3}{b^3}}{(bx^2+a)^2}dx}{4a}+\frac{x\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{4a\left(a+bx^2\right)^2}$$

$\Big\downarrow$ 1471

$$\frac{\frac{x\left(\frac{9a^2D-5abC+b^2B}{b^3}+\frac{3A}{a}\right)}{2(a+bx^2)}-\frac{\int -\frac{-7Da^3+8bDx^2a^2+3bCa^2+b^2Ba+3Ab^3}{b^3\left(bx^2+a\right)}dx}{2a}}{4a}+\frac{x\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{4a\left(a+bx^2\right)^2}$$

$\Big\downarrow$ 25

$$\frac{\frac{\int \frac{3Ab^3+8a^2Dx^2b+a\left(-7Da^2+3bCa+b^2B\right)}{b^3\left(bx^2+a\right)}dx}{2a}+\frac{x\left(\frac{9a^2D-5abC+b^2B}{b^3}+\frac{3A}{a}\right)}{2(a+bx^2)}}{4a}+\frac{x\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{4a\left(a+bx^2\right)^2}$$

$\Big\downarrow$ 27

$$\frac{\frac{\int \frac{3Ab^3+8a^2Dx^2b+a\left(-7Da^2+3bCa+b^2B\right)}{bx^2+a}dx}{2ab^3}+\frac{x\left(\frac{9a^2D-5abC+b^2B}{b^3}+\frac{3A}{a}\right)}{2(a+bx^2)}}{4a}+\frac{x\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{4a\left(a+bx^2\right)^2}$$

$\Big\downarrow$ 299

$$\frac{\frac{\left(a\left(-15a^2D+3abC+b^2B\right)+3Ab^3\right)\int \frac{1}{bx^2+a}dx+8a^2Dx}{2ab^3}+\frac{x\left(\frac{9a^2D-5abC+b^2B}{b^3}+\frac{3A}{a}\right)}{2(a+bx^2)}}{4a}+\frac{x\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{4a\left(a+bx^2\right)^2}$$

$\Big\downarrow$ 218

$$\frac{\frac{\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a\left(-15a^2D+3abC+b^2B\right)+3Ab^3\right)}{\sqrt{a}\sqrt{b}}+8a^2Dx}{2ab^3}+\frac{x\left(\frac{9a^2D-5abC+b^2B}{b^3}+\frac{3A}{a}\right)}{2(a+bx^2)}}{4a}+\frac{x\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{4a\left(a+bx^2\right)^2}$$

input
```
Int[(A + B*x^2 + C*x^4 + D*x^6)/(a + b*x^2)^3,x]
```

output
```
((A - (a*(b^2*B - a*b*C + a^2*D))/b^3)*x)/(4*a*(a + b*x^2)^2) + ((((3*A)/a
 + (b^2*B - 5*a*b*C + 9*a^2*D)/b^3)*x)/(2*(a + b*x^2)) + (8*a^2*D*x + ((3*
A*b^3 + a*(b^2*B + 3*a*b*C - 15*a^2*D))*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt
[a]*Sqrt[b]))/(2*a*b^3))/(4*a)
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

## Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.93

| method | result | size |
|---|---|---|
| default | $\frac{Dx}{b^3}+\frac{\frac{\frac{b\left(3b^3A+a\,b^2B-5a^2bC+9a^3D\right)x^3}{8a^2}+\frac{\left(5b^3A-a\,b^2B-3a^2bC+7a^3D\right)x}{8a}}{\left(b\,x^2+a\right)^2}+\frac{\left(3b^3A+a\,b^2B+3a^2bC-15a^3D\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8a^2\sqrt{ab}}}{b^3}$ | 137 |

input
```
int((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
D*x/b^3+1/b^3*((1/8*b*(3*A*b^3+B*a*b^2-5*C*a^2*b+9*D*a^3)/a^2*x^3+1/8*(5*A
*b^3-B*a*b^2-3*C*a^2*b+7*D*a^3)/a*x)/(b*x^2+a)^2+1/8*(3*A*b^3+B*a*b^2+3*C*
a^2*b-15*D*a^3)/a^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

## Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 511, normalized size of antiderivative = 3.48

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^3}\,dx$$

$$= \Bigg[\frac{16\,Da^3b^3x^5+2\,(25\,Da^4b^2-5\,Ca^3b^3+Ba^2b^4+3\,Aab^5)x^3+(15\,Da^5-3\,Ca^4b-Ba^3b^2-3\,Aa^2b^3+(1}{}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
[1/16*(16*D*a^3*b^3*x^5 + 2*(25*D*a^4*b^2 - 5*C*a^3*b^3 + B*a^2*b^4 + 3*A*
a*b^5)*x^3 + (15*D*a^5 - 3*C*a^4*b - B*a^3*b^2 - 3*A*a^2*b^3 + (15*D*a^3*b
^2 - 3*C*a^2*b^3 - B*a*b^4 - 3*A*b^5)*x^4 + 2*(15*D*a^4*b - 3*C*a^3*b^2 -
B*a^2*b^3 - 3*A*a*b^4)*x^2)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b
*x^2 + a)) + 2*(15*D*a^5*b - 3*C*a^4*b^2 - B*a^3*b^3 + 5*A*a^2*b^4)*x)/(a^
3*b^6*x^4 + 2*a^4*b^5*x^2 + a^5*b^4), 1/8*(8*D*a^3*b^3*x^5 + (25*D*a^4*b^2
 - 5*C*a^3*b^3 + B*a^2*b^4 + 3*A*a*b^5)*x^3 - (15*D*a^5 - 3*C*a^4*b - B*a^
3*b^2 - 3*A*a^2*b^3 + (15*D*a^3*b^2 - 3*C*a^2*b^3 - B*a*b^4 - 3*A*b^5)*x^4
 + 2*(15*D*a^4*b - 3*C*a^3*b^2 - B*a^2*b^3 - 3*A*a*b^4)*x^2)*sqrt(a*b)*arc
tan(sqrt(a*b)*x/a) + (15*D*a^5*b - 3*C*a^4*b^2 - B*a^3*b^3 + 5*A*a^2*b^4)*
x)/(a^3*b^6*x^4 + 2*a^4*b^5*x^2 + a^5*b^4)]
```

## Sympy [A] (verification not implemented)

Time = 3.40 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.65

$$\begin{aligned}
\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^3}\,dx &= \frac{Dx}{b^3} + \frac{\sqrt{-\frac{1}{a^5b^7}}(-3Ab^3-Bab^2-3Ca^2b+15Da^3)\log\left(-a^3b^3\sqrt{-\frac{1}{a^5b^7}}+x\right)}{16} \\
&\quad - \frac{\sqrt{-\frac{1}{a^5b^7}}(-3Ab^3-Bab^2-3Ca^2b+15Da^3)\log\left(a^3b^3\sqrt{-\frac{1}{a^5b^7}}+x\right)}{16} \\
&\quad + \frac{x^3\cdot(3Ab^4+Bab^3-5Ca^2b^2+9Da^3b)+x(5Aab^3-Ba^2b^2-3Ca^3b+7Da^4)}{8a^4b^3+16a^3b^4x^2+8a^2b^5x^4}
\end{aligned}$$

input
```
integrate((D*x**6+C*x**4+B*x**2+A)/(b*x**2+a)**3,x)
```

output
```
D*x/b**3 + sqrt(-1/(a**5*b**7))*(-3*A*b**3 - B*a*b**2 - 3*C*a**2*b + 15*D*
a**3)*log(-a**3*b**3*sqrt(-1/(a**5*b**7)) + x)/16 - sqrt(-1/(a**5*b**7))*(
-3*A*b**3 - B*a*b**2 - 3*C*a**2*b + 15*D*a**3)*log(a**3*b**3*sqrt(-1/(a**5
*b**7)) + x)/16 + (x**3*(3*A*b**4 + B*a*b**3 - 5*C*a**2*b**2 + 9*D*a**3*b)
 + x*(5*A*a*b**3 - B*a**2*b**2 - 3*C*a**3*b + 7*D*a**4))/(8*a**4*b**3 + 16
*a**3*b**4*x**2 + 8*a**2*b**5*x**4)
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.05

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^3}\,dx = \frac{(9\,Da^3b-5\,Ca^2b^2+Bab^3+3\,Ab^4)x^3+(7\,Da^4-3\,Ca^3b-Ba^2b^2+5\,Aab^3)x}{8\,(a^2b^5x^4+2\,a^3b^4x^2+a^4b^3)} + \frac{Dx}{b^3} - \frac{(15\,Da^3-3\,Ca^2b-Bab^2-3\,Ab^3)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^2b^3}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/8*((9*D*a^3*b - 5*C*a^2*b^2 + B*a*b^3 + 3*A*b^4)*x^3 + (7*D*a^4 - 3*C*a^
3*b - B*a^2*b^2 + 5*A*a*b^3)*x)/(a^2*b^5*x^4 + 2*a^3*b^4*x^2 + a^4*b^3) +
D*x/b^3 - 1/8*(15*D*a^3 - 3*C*a^2*b - B*a*b^2 - 3*A*b^3)*arctan(b*x/sqrt(a
*b))/(sqrt(a*b)*a^2*b^3)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^3}\,dx = \frac{Dx}{b^3} - \frac{(15\,Da^3-3\,Ca^2b-Bab^2-3\,Ab^3)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^2b^3} + \frac{9\,Da^3bx^3-5\,Ca^2b^2x^3+Bab^3x^3+3\,Ab^4x^3+7\,Da^4x-3\,Ca^3bx-Ba^2b^2x+5\,Aab^3x}{8\,(bx^2+a)^2a^2b^3}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
D*x/b^3 - 1/8*(15*D*a^3 - 3*C*a^2*b - B*a*b^2 - 3*A*b^3)*arctan(b*x/sqrt(a
*b))/(sqrt(a*b)*a^2*b^3) + 1/8*(9*D*a^3*b*x^3 - 5*C*a^2*b^2*x^3 + B*a*b^3*
x^3 + 3*A*b^4*x^3 + 7*D*a^4*x - 3*C*a^3*b*x - B*a^2*b^2*x + 5*A*a*b^3*x)/(
(b*x^2 + a)^2*a^2*b^3)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{A + Bx^2 + Cx^4 + Dx^6}{(a + bx^2)^3}\,dx = \int \frac{A + B\,x^2 + C\,x^4 + x^6\,D}{(b\,x^2 + a)^3}\,dx$$

input
```
int((A + B*x^2 + C*x^4 + x^6*D)/(a + b*x^2)^3,x)
```

output
```
int((A + B*x^2 + C*x^4 + x^6*D)/(a + b*x^2)^3, x)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 327, normalized size of antiderivative = 2.22

$$\int \frac{A + Bx^2 + Cx^4 + Dx^6}{(a + bx^2)^3}\,dx$$
$$= \frac{-15\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^4d + 3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^3bc - 30\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^3bd\,x^2 + 4\sqrt{b}\,\sqrt{a}\,ata}{}$$

input
```
int((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^3,x)
```

output
```
( - 15*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**4*d + 3*sqrt(b)*sq
rt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**3*b*c - 30*sqrt(b)*sqrt(a)*atan((b*
x)/(sqrt(b)*sqrt(a)))*a**3*b*d*x**2 + 4*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b
)*sqrt(a)))*a**2*b**3 + 6*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*
*2*b**2*c*x**2 - 15*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2*b**
2*d*x**4 + 8*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*b**4*x**2 + 3
*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*b**3*c*x**4 + 4*sqrt(b)*s
qrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**5*x**4 + 15*a**4*b*d*x - 3*a**3*b*
*2*c*x + 25*a**3*b**2*d*x**3 + 4*a**2*b**4*x - 5*a**2*b**3*c*x**3 + 8*a**2
*b**3*d*x**5 + 4*a*b**5*x**3)/(8*a**2*b**4*(a**2 + 2*a*b*x**2 + b**2*x**4)
)
```

# 3.184 $\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2(a+bx^2)^3}\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 156

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2(a+bx^2)^3}\,dx = -\frac{A}{a^3x} - \frac{(Ab^3-a(b^2B-abC+a^2D))\,x}{4a^2b^2(a+bx^2)^2} - \frac{(7Ab^3-a(3b^2B+abC-5a^2D))\,x}{8a^3b^2(a+bx^2)} - \frac{(15Ab^3-a(3b^2B+abC+3a^2D))\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{7/2}b^{5/2}}$$

output
```
-A/a^3/x-1/4*(A*b^3-a*(B*b^2-C*a*b+D*a^2))*x/a^2/b^2/(b*x^2+a)^2-1/8*(7*A*
b^3-a*(3*B*b^2+C*a*b-5*D*a^2))*x/a^3/b^2/(b*x^2+a)-1/8*(15*A*b^3-a*(3*B*b^
2+C*a*b+3*D*a^2))*arctan(b^(1/2)*x/a^(1/2))/a^(7/2)/b^(5/2)
```

## Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.97

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2(a+bx^2)^3}\,dx = -\frac{3a^4Dx^2+15Ab^4x^4+ab^3x^2(25A-3Bx^2)+a^3bx^2(C+5Dx^2)+a^2b^2(8A-5Bx^2-Cx^4)}{8a^3b^2x(a+bx^2)^2} + \frac{(-15Ab^3+a(3b^2B+abC+3a^2D))\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{7/2}b^{5/2}}$$

input
```
Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(x^2*(a + b*x^2)^3),x]
```

output
```
-1/8*(3*a^4*D*x^2 + 15*A*b^4*x^4 + a*b^3*x^2*(25*A - 3*B*x^2) + a^3*b*x^2*
(C + 5*D*x^2) + a^2*b^2*(8*A - 5*B*x^2 - C*x^4))/(a^3*b^2*x*(a + b*x^2)^2)
 + ((-15*A*b^3 + a*(3*b^2*B + a*b*C + 3*a^2*D))*ArcTan[(Sqrt[b]*x)/Sqrt[a]
])/(8*a^(7/2)*b^(5/2))
```

## Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2336, 25, 1582, 25, 359, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2(a+bx^2)^3}\,dx$$

$$\Big\downarrow \text{2336}$$

$$-\frac{\int -\frac{\frac{4aDx^4}{b}-\left(\frac{Da^2}{b^2}-\frac{Ca}{b}-3B+\frac{3Ab}{a}\right)x^2+4A}{x^2(bx^2+a)^2}dx}{4a}-\frac{x\left(-\frac{a^2D}{b^2}+\frac{Ab}{a}+\frac{aC}{b}-B\right)}{4a(a+bx^2)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\frac{4aDx^4}{b}-\left(\frac{Da^2}{b^2}-\frac{Ca}{b}-3B+\frac{3Ab}{a}\right)x^2+4A}{x^2(bx^2+a)^2}dx}{4a}-\frac{x\left(-\frac{a^2D}{b^2}+\frac{Ab}{a}+\frac{aC}{b}-B\right)}{4a\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{1582}$$

$$\frac{-\frac{\int -\frac{8aAb^2-\left(7Ab^3-a\left(3Da^2+bCa+3b^2B\right)\right)x^2}{x^2\left(bx^2+a\right)}dx}{2a^2b^2}-\frac{x(7Ab^3-a(-5a^2D+abC+3b^2B))}{2a^2b^2(a+bx^2)}}{4a}-\frac{x\left(-\frac{a^2D}{b^2}+\frac{Ab}{a}+\frac{aC}{b}-B\right)}{4a\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{8aAb^2-\left(7Ab^3-a\left(3Da^2+bCa+3b^2B\right)\right)x^2}{x^2\left(bx^2+a\right)}dx}{2a^2b^2}-\frac{x(7Ab^3-a(-5a^2D+abC+3b^2B))}{2a^2b^2(a+bx^2)}}{4a}-\frac{x\left(-\frac{a^2D}{b^2}+\frac{Ab}{a}+\frac{aC}{b}-B\right)}{4a\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{359}$$

$$\frac{\frac{-(15Ab^3-a(3a^2D+abC+3b^2B))\int \frac{1}{bx^2+a}dx-\frac{8Ab^2}{x}}{2a^2b^2}-\frac{x(7Ab^3-a(-5a^2D+abC+3b^2B))}{2a^2b^2(a+bx^2)}}{4a}-\frac{x\left(-\frac{a^2D}{b^2}+\frac{Ab}{a}+\frac{aC}{b}-B\right)}{4a\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{-\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(15Ab^3-a\left(3a^2D+abC+3b^2B\right)\right)}{\sqrt{a}\sqrt{b}}-\frac{8Ab^2}{x}}{2a^2b^2}-\frac{x(7Ab^3-a(-5a^2D+abC+3b^2B))}{2a^2b^2(a+bx^2)}}{4a}-\frac{x\left(-\frac{a^2D}{b^2}+\frac{Ab}{a}+\frac{aC}{b}-B\right)}{4a\left(a+bx^2\right)^2}$$

input
```
Int[(A + B*x^2 + C*x^4 + D*x^6)/(x^2*(a + b*x^2)^3),x]
```

output
```
-1/4*(((A*b)/a - B + (a*C)/b - (a^2*D)/b^2)*x)/(a*(a + b*x^2)^2) + (-1/2*(
(7*A*b^3 - a*(3*b^2*B + a*b*C - 5*a^2*D))*x)/(a^2*b^2*(a + b*x^2)) + ((-8*
A*b^2)/x - ((15*A*b^3 - a*(3*b^2*B + a*b*C + 3*a^2*D))*ArcTan[(Sqrt[b]*x)/
Sqrt[a]])/(Sqrt[a]*Sqrt[b]))/(2*a^2*b^2))/(4*a)
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 1582
```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Simp[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1))   Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e
*x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
 b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
&& ILtQ[m/2, 0]
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

## Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.91

| method | result | size |
|---|---|---|
| default | $-\frac{\frac{\frac{\left(7b^3A-3a\,b^2B-a^2bC+5a^3D\right)x^3}{8b}+\frac{a\left(9b^3A-5a\,b^2B+a^2bC+3a^3D\right)x}{8b^2}}{\left(b\,x^2+a\right)^2}+\frac{\left(15b^3A-3a\,b^2B-a^2bC-3a^3D\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8b^2\sqrt{ab}}}{a^3}-\frac{A}{a^3x}$ | 142 |

input
```
int((D*x^6+C*x^4+B*x^2+A)/x^2/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/a^3*((1/8*(7*A*b^3-3*B*a*b^2-C*a^2*b+5*D*a^3)/b*x^3+1/8*a*(9*A*b^3-5*B*
a*b^2+C*a^2*b+3*D*a^3)/b^2*x)/(b*x^2+a)^2+1/8*(15*A*b^3-3*B*a*b^2-C*a^2*b-
3*D*a^3)/b^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))-A/a^3/x
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 512, normalized size of antiderivative = 3.28

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2\left(a+bx^2\right)^3}\,dx$$

$$= \left[-\frac{16\,Aa^3b^3+2\,(5\,Da^4b^2-Ca^3b^3-3\,Ba^2b^4+15\,Aab^5)x^4+2\,(3\,Da^5b+Ca^4b^2-5\,Ba^3b^3+25\,Aa^2b^4)x}{\phantom{x}}\right.$$

$$-\frac{8\,Aa^3b^3+(5\,Da^4b^2-Ca^3b^3-3\,Ba^2b^4+15\,Aab^5)x^4+(3\,Da^5b+Ca^4b^2-5\,Ba^3b^3+25\,Aa^2b^4)x^2-(}{\phantom{x}}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^2/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
[-1/16*(16*A*a^3*b^3 + 2*(5*D*a^4*b^2 - C*a^3*b^3 - 3*B*a^2*b^4 + 15*A*a*b
^5)*x^4 + 2*(3*D*a^5*b + C*a^4*b^2 - 5*B*a^3*b^3 + 25*A*a^2*b^4)*x^2 - ((3
*D*a^3*b^2 + C*a^2*b^3 + 3*B*a*b^4 - 15*A*b^5)*x^5 + 2*(3*D*a^4*b + C*a^3*
b^2 + 3*B*a^2*b^3 - 15*A*a*b^4)*x^3 + (3*D*a^5 + C*a^4*b + 3*B*a^3*b^2 - 1
5*A*a^2*b^3)*x)*sqrt(-a*b)*log((b*x^2 + 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/
(a^4*b^5*x^5 + 2*a^5*b^4*x^3 + a^6*b^3*x), -1/8*(8*A*a^3*b^3 + (5*D*a^4*b^
2 - C*a^3*b^3 - 3*B*a^2*b^4 + 15*A*a*b^5)*x^4 + (3*D*a^5*b + C*a^4*b^2 - 5
*B*a^3*b^3 + 25*A*a^2*b^4)*x^2 - ((3*D*a^3*b^2 + C*a^2*b^3 + 3*B*a*b^4 - 1
5*A*b^5)*x^5 + 2*(3*D*a^4*b + C*a^3*b^2 + 3*B*a^2*b^3 - 15*A*a*b^4)*x^3 +
(3*D*a^5 + C*a^4*b + 3*B*a^3*b^2 - 15*A*a^2*b^3)*x)*sqrt(a*b)*arctan(sqrt(
a*b)*x/a))/(a^4*b^5*x^5 + 2*a^5*b^4*x^3 + a^6*b^3*x)]
```

## Sympy [A] (verification not implemented)

Time = 8.70 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.60

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2(a+bx^2)^3}\,dx$$
$$= -\frac{\sqrt{-\frac{1}{a^7b^5}}(-15Ab^3+3Bab^2+Ca^2b+3Da^3)\log\left(-a^4b^2\sqrt{-\frac{1}{a^7b^5}}+x\right)}{16} + \frac{\sqrt{-\frac{1}{a^7b^5}}(-15Ab^3+3Bab^2+Ca^2b+3Da^3)\log\left(a^4b^2\sqrt{-\frac{1}{a^7b^5}}+x\right)}{16} + \frac{-8Aa^2b^2+x^4(-15Ab^4+3Bab^3+Ca^2b^2-5Da^3b)+x^2(-25Aab^3+5Ba^2b^2-Ca^3b-3Da^4)}{8a^5b^2x+16a^4b^3x^3+8a^3b^4x^5}$$

input
```
integrate((D*x**6+C*x**4+B*x**2+A)/x**2/(b*x**2+a)**3,x)
```

output
```
-sqrt(-1/(a**7*b**5))*(-15*A*b**3 + 3*B*a*b**2 + C*a**2*b + 3*D*a**3)*log(
-a**4*b**2*sqrt(-1/(a**7*b**5)) + x)/16 + sqrt(-1/(a**7*b**5))*(-15*A*b**3
 + 3*B*a*b**2 + C*a**2*b + 3*D*a**3)*log(a**4*b**2*sqrt(-1/(a**7*b**5)) +
x)/16 + (-8*A*a**2*b**2 + x**4*(-15*A*b**4 + 3*B*a*b**3 + C*a**2*b**2 - 5*
D*a**3*b) + x**2*(-25*A*a*b**3 + 5*B*a**2*b**2 - C*a**3*b - 3*D*a**4))/(8*
a**5*b**2*x + 16*a**4*b**3*x**3 + 8*a**3*b**4*x**5)
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.03

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2(a+bx^2)^3}\,dx = -\frac{8\,Aa^2b^2+(5\,Da^3b-Ca^2b^2-3\,Bab^3+15\,Ab^4)x^4+(3\,Da^4+Ca^3b-5\,Ba^2b^2+25\,Aab^3)x^2}{8\,(a^3b^4x^5+2\,a^4b^3x^3+a^5b^2x)} + \frac{(3\,Da^3+Ca^2b+3\,Bab^2-15\,Ab^3)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^3b^2}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^2/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
-1/8*(8*A*a^2*b^2 + (5*D*a^3*b - C*a^2*b^2 - 3*B*a*b^3 + 15*A*b^4)*x^4 + (
3*D*a^4 + C*a^3*b - 5*B*a^2*b^2 + 25*A*a*b^3)*x^2)/(a^3*b^4*x^5 + 2*a^4*b^
3*x^3 + a^5*b^2*x) + 1/8*(3*D*a^3 + C*a^2*b + 3*B*a*b^2 - 15*A*b^3)*arctan
(b*x/sqrt(a*b))/(sqrt(a*b)*a^3*b^2)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.96

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2(a+bx^2)^3}\,dx = -\frac{A}{a^3x}+\frac{(3\,Da^3+Ca^2b+3\,Bab^2-15\,Ab^3)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^3b^2} - \frac{5\,Da^3bx^3-Ca^2b^2x^3-3\,Bab^3x^3+7\,Ab^4x^3+3\,Da^4x+Ca^3bx-5\,Ba^2b^2x+9\,Aab^3x}{8\,(bx^2+a)^2a^3b^2}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^2/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
-A/(a^3*x) + 1/8*(3*D*a^3 + C*a^2*b + 3*B*a*b^2 - 15*A*b^3)*arctan(b*x/sqr
t(a*b))/(sqrt(a*b)*a^3*b^2) - 1/8*(5*D*a^3*b*x^3 - C*a^2*b^2*x^3 - 3*B*a*b
^3*x^3 + 7*A*b^4*x^3 + 3*D*a^4*x + C*a^3*b*x - 5*B*a^2*b^2*x + 9*A*a*b^3*x
)/((b*x^2 + a)^2*a^3*b^2)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2\left(a+bx^2\right)^3}\,dx=\int \frac{A+B\,x^2+C\,x^4+x^6\,D}{x^2\,\left(b\,x^2+a\right)^3}\,dx$$

input
```
int((A + B*x^2 + C*x^4 + x^6*D)/(x^2*(a + b*x^2)^3),x)
```

output
```
int((A + B*x^2 + C*x^4 + x^6*D)/(x^2*(a + b*x^2)^3), x)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 332, normalized size of antiderivative = 2.13

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2\left(a+bx^2\right)^3}\,dx = \frac{3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^4dx+\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^3bcx+6\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^3bd\,x^3-12\sqrt{b}\,\sqrt{a}\,atan}{}$$

input
```
int((D*x^6+C*x^4+B*x^2+A)/x^2/(b*x^2+a)^3,x)
```

output
```
(3*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**4*d*x + sqrt(b)*sqrt(a
)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**3*b*c*x + 6*sqrt(b)*sqrt(a)*atan((b*x)/
(sqrt(b)*sqrt(a)))*a**3*b*d*x**3 - 12*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*
sqrt(a)))*a**2*b**3*x + 2*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*
*2*b**2*c*x**3 + 3*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2*b**2
*d*x**5 - 24*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*b**4*x**3 + s
qrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*b**3*c*x**5 - 12*sqrt(b)*sq
rt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**5*x**5 - 3*a**4*b*d*x**2 - 8*a**3*b
**3 - a**3*b**2*c*x**2 - 5*a**3*b**2*d*x**4 - 20*a**2*b**4*x**2 + a**2*b**
3*c*x**4 - 12*a*b**5*x**4)/(8*a**3*b**3*x*(a**2 + 2*a*b*x**2 + b**2*x**4))
```

# 3.185 $\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4(a+bx^2)^3}\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 176

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4\left(a+bx^2\right)^3}\,dx = -\frac{A}{3a^3x^3}+\frac{3Ab-aB}{a^4x}+\frac{(Ab^3-a(b^2B-abC+a^2D))\,x}{4a^3b\left(a+bx^2\right)^2}+\frac{(11Ab^3-a(7b^2B-3abC-a^2D))\,x}{8a^4b\left(a+bx^2\right)}+\frac{(35Ab^3-a(15b^2B-3abC-a^2D))\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{9/2}b^{3/2}}$$

output

```
-1/3*A/a^3/x^3+(3*A*b-B*a)/a^4/x+1/4*(A*b^3-a*(B*b^2-C*a*b+D*a^2))*x/a^3/b
/(b*x^2+a)^2+1/8*(11*A*b^3-a*(7*B*b^2-3*C*a*b-D*a^2))*x/a^4/b/(b*x^2+a)+1/
8*(35*A*b^3-a*(15*B*b^2-3*C*a*b-D*a^2))*arctan(b^(1/2)*x/a^(1/2))/a^(9/2)/
b^(3/2)
```

## Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.96

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4(a+bx^2)^3}\,dx$$
$$=\frac{-3a^4Dx^4+105Ab^4x^6+5ab^3x^4(35A-9Bx^2)+a^2b^2x^2(56A-75Bx^2+9Cx^4)+a^3b(-8A+3x^2(-8B}{24a^4bx^3(a+bx^2)^2}$$
$$+\frac{(35Ab^3+a(-15b^2B+3abC+a^2D))\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{9/2}b^{3/2}}$$

input
```
Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(x^4*(a + b*x^2)^3),x]
```

output
```
(-3*a^4*D*x^4 + 105*A*b^4*x^6 + 5*a*b^3*x^4*(35*A - 9*B*x^2) + a^2*b^2*x^2
*(56*A - 75*B*x^2 + 9*C*x^4) + a^3*b*(-8*A + 3*x^2*(-8*B + 5*C*x^2 + D*x^4
)))/(24*a^4*b*x^3*(a + b*x^2)^2) + ((35*A*b^3 + a*(-15*b^2*B + 3*a*b*C + a
^2*D))*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(9/2)*b^(3/2))
```

## Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2336, 25, 1582, 1584, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4(a+bx^2)^3}\,dx$$

$$\Big\downarrow \text{ 2336}$$

$$\frac{x\left(\frac{Ab^2}{a^2}-\frac{bB}{a}-\frac{aD}{b}+C\right)}{4a(a+bx^2)^2}-\frac{\int -\frac{\left(\frac{3Ab^2}{a^2}-\frac{3Bb}{a}+3C+\frac{aD}{b}\right)x^4-4\left(\frac{Ab}{a}-B\right)x^2+4A}{x^4(bx^2+a)^2}dx}{4a}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{\left(\frac{3Ab^2}{a^2}-\frac{3Bb}{a}+3C+\frac{aD}{b}\right)x^4-4\left(\frac{Ab}{a}-B\right)x^2+4A}{x^4(bx^2+a)^2}dx}{4a}+\frac{x\left(\frac{Ab^2}{a^2}-\frac{bB}{a}-\frac{aD}{b}+C\right)}{4a\left(a+bx^2\right)^2}$$

$\Big\downarrow$ 1582

$$\frac{\frac{\int \frac{b\left(11Ab^3-a\left(-Da^2-3bCa+7b^2B\right)\right)x^4-8ab^2(2Ab-aB)x^2+8a^2Ab^2}{x^4\left(bx^2+a\right)}dx}{2a^3b^2}+\frac{x(11Ab^3-a(a^2(-D)-3abC+7b^2B))}{2a^3b(a+bx^2)}}{4a}+\frac{x\left(\frac{Ab^2}{a^2}-\frac{bB}{a}-\frac{aD}{b}+C\right)}{4a\left(a+bx^2\right)^2}$$

$\Big\downarrow$ 1584

$$\frac{\frac{\int \left(-\frac{8(3Ab-aB)b^2}{x^2}+\frac{8aAb^2}{x^4}+\frac{\left(35Ab^3-a\left(-Da^2-3bCa+15b^2B\right)\right)b}{bx^2+a}\right)dx}{2a^3b^2}+\frac{x(11Ab^3-a(a^2(-D)-3abC+7b^2B))}{2a^3b(a+bx^2)}}{4a}+\frac{x\left(\frac{Ab^2}{a^2}-\frac{bB}{a}-\frac{aD}{b}+C\right)}{4a\left(a+bx^2\right)^2}$$

$\Big\downarrow$ 2009

$$\frac{x\left(\frac{Ab^2}{a^2}-\frac{bB}{a}-\frac{aD}{b}+C\right)}{4a\left(a+bx^2\right)^2}+\frac{\frac{\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(35Ab^3-a\left(a^2(-D)-3abC+15b^2B\right)\right)}{\sqrt{a}}+\frac{8b^2(3Ab-aB)}{x}-\frac{8aAb^2}{3x^3}}{2a^3b^2}+\frac{x\left(11Ab^3-a\left(a^2(-D)-3abC+7b^2B\right)\right)}{2a^3b(a+bx^2)}}{4a}$$

input
```
Int[(A + B*x^2 + C*x^4 + D*x^6)/(x^4*(a + b*x^2)^3),x]
```

output
```
(((A*b^2)/a^2 - (b*B)/a + C - (a*D)/b)*x)/(4*a*(a + b*x^2)^2) + ((((11*A*b^
3 - a*(7*b^2*B - 3*a*b*C - a^2*D))*x)/(2*a^3*b*(a + b*x^2)) + ((-8*a*A*b^2
)/(3*x^3) + (8*b^2*(3*A*b - a*B))/x + (Sqrt[b]*(35*A*b^3 - a*(15*b^2*B - 3
*a*b*C - a^2*D))*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[a])/(2*a^3*b^2))/(4*a)
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1582
```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Simp[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1))   Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e
*x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
 b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
&& ILtQ[m/2, 0]
```

rule 1584
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

## Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.87

| method | result |
|---|---|
| default | $\frac{\frac{\left(\frac{11}{8}b^3A-\frac{7}{8}a\,b^2B+\frac{3}{8}a^2bC+\frac{1}{8}a^3D\right)x^3+\frac{a\left(13b^3A-9a\,b^2B+5a^2bC-a^3D\right)x}{8b}}{\left(b\,x^2+a\right)^2}+\frac{\left(35b^3A-15a\,b^2B+3a^2bC+a^3D\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8b\sqrt{ab}}}{a^4}-\frac{A}{3a^3x^3}-$ |

input
```
int((D*x^6+C*x^4+B*x^2+A)/x^4/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/a^4*(((11/8*b^3*A-7/8*a*b^2*B+3/8*a^2*b*C+1/8*a^3*D)*x^3+1/8*a*(13*A*b^3
-9*B*a*b^2+5*C*a^2*b-D*a^3)/b*x)/(b*x^2+a)^2+1/8*(35*A*b^3-15*B*a*b^2+3*C*
a^2*b+D*a^3)/b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))-1/3*A/a^3/x^3-(-3*A*b+
B*a)/a^4/x
```

## Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 569, normalized size of antiderivative = 3.23

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4\left(a+bx^2\right)^3}\,dx$$

$$= \left[-\frac{16\,Aa^4b^2-6\,(Da^4b^2+3\,Ca^3b^3-15\,Ba^2b^4+35\,Aab^5)x^6+2\,(3\,Da^5b-15\,Ca^4b^2+75\,Ba^3b^3-175\,A}{}\right.$$

$$-\frac{8\,Aa^4b^2-3\,(Da^4b^2+3\,Ca^3b^3-15\,Ba^2b^4+35\,Aab^5)x^6+(3\,Da^5b-15\,Ca^4b^2+75\,Ba^3b^3-175\,Aa^2b}{}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^4/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
[-1/48*(16*A*a^4*b^2 - 6*(D*a^4*b^2 + 3*C*a^3*b^3 - 15*B*a^2*b^4 + 35*A*a*
b^5)*x^6 + 2*(3*D*a^5*b - 15*C*a^4*b^2 + 75*B*a^3*b^3 - 175*A*a^2*b^4)*x^4
 + 16*(3*B*a^4*b^2 - 7*A*a^3*b^3)*x^2 + 3*((D*a^3*b^2 + 3*C*a^2*b^3 - 15*B
*a*b^4 + 35*A*b^5)*x^7 + 2*(D*a^4*b + 3*C*a^3*b^2 - 15*B*a^2*b^3 + 35*A*a*
b^4)*x^5 + (D*a^5 + 3*C*a^4*b - 15*B*a^3*b^2 + 35*A*a^2*b^3)*x^3)*sqrt(-a*
b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^5*b^4*x^7 + 2*a^6*b^3
*x^5 + a^7*b^2*x^3), -1/24*(8*A*a^4*b^2 - 3*(D*a^4*b^2 + 3*C*a^3*b^3 - 15*
B*a^2*b^4 + 35*A*a*b^5)*x^6 + (3*D*a^5*b - 15*C*a^4*b^2 + 75*B*a^3*b^3 - 1
75*A*a^2*b^4)*x^4 + 8*(3*B*a^4*b^2 - 7*A*a^3*b^3)*x^2 - 3*((D*a^3*b^2 + 3*
C*a^2*b^3 - 15*B*a*b^4 + 35*A*b^5)*x^7 + 2*(D*a^4*b + 3*C*a^3*b^2 - 15*B*a
^2*b^3 + 35*A*a*b^4)*x^5 + (D*a^5 + 3*C*a^4*b - 15*B*a^3*b^2 + 35*A*a^2*b^
3)*x^3)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a^5*b^4*x^7 + 2*a^6*b^3*x^5 + a^
7*b^2*x^3)]
```

## Sympy [A] (verification not implemented)

Time = 22.48 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.53

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4\left(a+bx^2\right)^3}\,dx$$
$$= -\frac{\sqrt{-\frac{1}{a^9b^3}}\cdot\left(35Ab^3-15Bab^2+3Ca^2b+Da^3\right)\log\left(-a^5b\sqrt{-\frac{1}{a^9b^3}}+x\right)}{16}$$
$$+\frac{\sqrt{-\frac{1}{a^9b^3}}\cdot\left(35Ab^3-15Bab^2+3Ca^2b+Da^3\right)\log\left(a^5b\sqrt{-\frac{1}{a^9b^3}}+x\right)}{16}$$
$$+\frac{-8Aa^3b+x^6\cdot\left(105Ab^4-45Bab^3+9Ca^2b^2+3Da^3b\right)+x^4\cdot\left(175Aab^3-75Ba^2b^2+15Ca^3b-3Da^4\right)}{24a^6bx^3+48a^5b^2x^5+24a^4b^3x^7}$$

input
```
integrate((D*x**6+C*x**4+B*x**2+A)/x**4/(b*x**2+a)**3,x)
```

output
```
-sqrt(-1/(a**9*b**3))*(35*A*b**3 - 15*B*a*b**2 + 3*C*a**2*b + D*a**3)*log(
-a**5*b*sqrt(-1/(a**9*b**3)) + x)/16 + sqrt(-1/(a**9*b**3))*(35*A*b**3 - 1
5*B*a*b**2 + 3*C*a**2*b + D*a**3)*log(a**5*b*sqrt(-1/(a**9*b**3)) + x)/16
+ (-8*A*a**3*b + x**6*(105*A*b**4 - 45*B*a*b**3 + 9*C*a**2*b**2 + 3*D*a**3
*b) + x**4*(175*A*a*b**3 - 75*B*a**2*b**2 + 15*C*a**3*b - 3*D*a**4) + x**2
*(56*A*a**2*b**2 - 24*B*a**3*b))/(24*a**6*b*x**3 + 48*a**5*b**2*x**5 + 24*
a**4*b**3*x**7)
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.03

$$\int \frac{A + Bx^2 + Cx^4 + Dx^6}{x^4 (a + bx^2)^3} \, dx$$
$$= \frac{3 (Da^3b + 3 Ca^2b^2 - 15 Bab^3 + 35 Ab^4)x^6 - 8 Aa^3b - (3 Da^4 - 15 Ca^3b + 75 Ba^2b^2 - 175 Aab^3)x^4 - 8 (}{24 (a^4b^3x^7 + 2 a^5b^2x^5 + a^6bx^3)}$$
$$+ \frac{(Da^3 + 3 Ca^2b - 15 Bab^2 + 35 Ab^3) \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8 \sqrt{ab}a^4b}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^4/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/24*(3*(D*a^3*b + 3*C*a^2*b^2 - 15*B*a*b^3 + 35*A*b^4)*x^6 - 8*A*a^3*b -
(3*D*a^4 - 15*C*a^3*b + 75*B*a^2*b^2 - 175*A*a*b^3)*x^4 - 8*(3*B*a^3*b - 7
*A*a^2*b^2)*x^2)/(a^4*b^3*x^7 + 2*a^5*b^2*x^5 + a^6*b*x^3) + 1/8*(D*a^3 +
3*C*a^2*b - 15*B*a*b^2 + 35*A*b^3)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^4*b)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.94

$$\int \frac{A + Bx^2 + Cx^4 + Dx^6}{x^4 (a + bx^2)^3} \, dx = \frac{(Da^3 + 3 Ca^2b - 15 Bab^2 + 35 Ab^3) \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8 \sqrt{ab}a^4b}$$
$$+ \frac{Da^3bx^3 + 3 Ca^2b^2x^3 - 7 Bab^3x^3 + 11 Ab^4x^3 - Da^4x + 5 Ca^3bx - 9 Ba^2b^2x + 13 Aab^3x}{8 (bx^2 + a)^2 a^4b}$$
$$- \frac{3 Bax^2 - 9 Abx^2 + Aa}{3 a^4x^3}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^4/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
1/8*(D*a^3 + 3*C*a^2*b - 15*B*a*b^2 + 35*A*b^3)*arctan(b*x/sqrt(a*b))/(sqr
t(a*b)*a^4*b) + 1/8*(D*a^3*b*x^3 + 3*C*a^2*b^2*x^3 - 7*B*a*b^3*x^3 + 11*A*
b^4*x^3 - D*a^4*x + 5*C*a^3*b*x - 9*B*a^2*b^2*x + 13*A*a*b^3*x)/((b*x^2 +
a)^2*a^4*b) - 1/3*(3*B*a*x^2 - 9*A*b*x^2 + A*a)/(a^4*x^3)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4\left(a+bx^2\right)^3}\,dx=\int \frac{A+B\,x^2+C\,x^4+x^6\,D}{x^4\,{\left(b\,x^2+a\right)}^3}\,dx$$

input
```
int((A + B*x^2 + C*x^4 + x^6*D)/(x^4*(a + b*x^2)^3),x)
```

output
```
int((A + B*x^2 + C*x^4 + x^6*D)/(x^4*(a + b*x^2)^3), x)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 352, normalized size of antiderivative = 2.00

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4\left(a+bx^2\right)^3}\,dx$$
$$=\frac{3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^4d\,x^3+9\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^3bc\,x^3+6\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^3bd\,x^5+60\sqrt{b}\,\sqrt{a}\,a}{}$$

input
```
int((D*x^6+C*x^4+B*x^2+A)/x^4/(b*x^2+a)^3,x)
```

output
```
(3*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**4*d*x**3 + 9*sqrt(b)*s
qrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**3*b*c*x**3 + 6*sqrt(b)*sqrt(a)*ata
n((b*x)/(sqrt(b)*sqrt(a)))*a**3*b*d*x**5 + 60*sqrt(b)*sqrt(a)*atan((b*x)/(
sqrt(b)*sqrt(a)))*a**2*b**3*x**3 + 18*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*
sqrt(a)))*a**2*b**2*c*x**5 + 3*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)
))*a**2*b**2*d*x**7 + 120*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*
b**4*x**5 + 9*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*b**3*c*x**7
+ 60*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**5*x**7 - 8*a**4*b**2
 - 3*a**4*b*d*x**4 + 32*a**3*b**3*x**2 + 15*a**3*b**2*c*x**4 + 3*a**3*b**2
*d*x**6 + 100*a**2*b**4*x**4 + 9*a**2*b**3*c*x**6 + 60*a*b**5*x**6)/(24*a*
*4*b**2*x**3*(a**2 + 2*a*b*x**2 + b**2*x**4))
```

# 3.186 $\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6(a+bx^2)^3}\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 200

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6(a+bx^2)^3}\,dx = -\frac{A}{5a^3x^5}+\frac{3Ab-aB}{3a^4x^3}-\frac{6Ab^2-a(3bB-aC)}{a^5x} - \frac{(Ab^3-a(b^2B-abC+a^2D))\,x}{4a^4(a+bx^2)^2} - \frac{(15Ab^3-a(11b^2B-7abC+3a^2D))\,x}{8a^5(a+bx^2)} - \frac{(63Ab^3-a(35b^2B-15abC+3a^2D))\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{11/2}\sqrt{b}}$$

output
```
-1/5*A/a^3/x^5+1/3*(3*A*b-B*a)/a^4/x^3-(6*A*b^2-a*(3*B*b-C*a))/a^5/x-1/4*(
A*b^3-a*(B*b^2-C*a*b+D*a^2))*x/a^4/(b*x^2+a)^2-1/8*(15*A*b^3-a*(11*B*b^2-7
*C*a*b+3*D*a^2))*x/a^5/(b*x^2+a)-1/8*(63*A*b^3-a*(35*B*b^2-15*C*a*b+3*D*a^
2))*arctan(b^(1/2)*x/a^(1/2))/a^(11/2)/b^(1/2)
```

## Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.98

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6(a+bx^2)^3}\,dx = -\frac{A}{5a^3x^5}+\frac{3Ab-aB}{3a^4x^3}+\frac{-6Ab^2+3abB-a^2C}{a^5x} + \frac{(-Ab^3+ab^2B-a^2bC+a^3D)\,x}{4a^4(a+bx^2)^2} + \frac{(-15Ab^3+11ab^2B-7a^2bC+3a^3D)\,x}{8a^5(a+bx^2)} + \frac{(-63Ab^3+35ab^2B-15a^2bC+3a^3D)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{11/2}\sqrt{b}}$$

input
```
Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(x^6*(a + b*x^2)^3),x]
```

output
```
-1/5*A/(a^3*x^5) + (3*A*b - a*B)/(3*a^4*x^3) + (-6*A*b^2 + 3*a*b*B - a^2*C
)/(a^5*x) + ((-(A*b^3) + a*b^2*B - a^2*b*C + a^3*D)*x)/(4*a^4*(a + b*x^2)^
2) + ((-15*A*b^3 + 11*a*b^2*B - 7*a^2*b*C + 3*a^3*D)*x)/(8*a^5*(a + b*x^2)
) + ((-63*A*b^3 + 35*a*b^2*B - 15*a^2*b*C + 3*a^3*D)*ArcTan[(Sqrt[b]*x)/Sq
rt[a]])/(8*a^(11/2)*Sqrt[b])
```

## Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2336, 25, 2336, 25, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6(a+bx^2)^3}\,dx$$

$$\Big\downarrow \text{2336}$$

$$-\frac{\int -\frac{-\frac{3\left(Ab^3-a\left(Da^2-bCa+b^2B\right)\right)x^6}{a^3}+\frac{4\left(Ab^2-a(bB-aC)\right)x^4}{a^2}-4\left(\frac{Ab}{a}-B\right)x^2+4A}{x^6\left(bx^2+a\right)^2}dx}{4a}-\frac{x\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{4a^4\left(a+bx^2\right)^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{-\frac{3\left(Ab^3-a\left(Da^2-bCa+b^2B\right)\right)x^6}{a^3}+\frac{4\left(Ab^2-a(bB-aC)\right)x^4}{a^2}-4\left(\frac{Ab}{a}-B\right)x^2+4A}{x^6\left(bx^2+a\right)^2}dx}{4a}-\frac{x\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{4a^4\left(a+bx^2\right)^2}$$

$$\Big\downarrow \; 2336$$

$$\frac{-\frac{\int -\frac{-\frac{\left(15Ab^3-a\left(3Da^2-7bCa+11b^2B\right)\right)x^6}{a^3}+\frac{8\left(3Ab^2-a(2bB-aC)\right)x^4}{a^2}-8\left(\frac{2Ab}{a}-B\right)x^2+8A}{x^6\left(bx^2+a\right)}dx}{2a}-\frac{x\left(15Ab^3-a\left(3a^2D-7abC+11b^2B\right)\right)}{2a^4\left(a+bx^2\right)}}{4a}-\frac{x\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{4a^4\left(a+bx^2\right)^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{-\frac{\left(15Ab^3-a\left(3Da^2-7bCa+11b^2B\right)\right)x^6}{a^3}+\frac{8\left(3Ab^2-a(2bB-aC)\right)x^4}{a^2}-8\left(\frac{2Ab}{a}-B\right)x^2+8A}{x^6\left(bx^2+a\right)}dx}{2a}-\frac{x\left(15Ab^3-a\left(3a^2D-7abC+11b^2B\right)\right)}{2a^4\left(a+bx^2\right)}}{4a}-\frac{x\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{4a^4\left(a+bx^2\right)^2}$$

$$\Big\downarrow \; 2333$$

$$\frac{\frac{\int \left(\frac{8A}{ax^6}+\frac{a\left(3Da^2-15bCa+35b^2B\right)-63Ab^3}{a^3\left(bx^2+a\right)}+\frac{8\left(6Ab^2-a(3bB-aC)\right)}{a^3x^2}+\frac{8(aB-3Ab)}{a^2x^4}\right)dx}{2a}-\frac{x\left(15Ab^3-a\left(3a^2D-7abC+11b^2B\right)\right)}{2a^4\left(a+bx^2\right)}}{4a}-\frac{x\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{4a^4\left(a+bx^2\right)^2}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{-\frac{8\left(6Ab^2-a(3bB-aC)\right)}{a^3x}+\frac{8(3Ab-aB)}{3a^2x^3}-\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(63Ab^3-a\left(3a^2D-15abC+35b^2B\right)\right)}{a^{7/2}\sqrt{b}}-\frac{8A}{5ax^5}}{2a}-\frac{x\left(15Ab^3-a\left(3a^2D-7abC+11b^2B\right)\right)}{2a^4(a+bx^2)}}{4a}-\frac{x\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{4a^4\left(a+bx^2\right)^2}$$

input
```
Int[(A + B*x^2 + C*x^4 + D*x^6)/(x^6*(a + b*x^2)^3),x]
```

output
```
-1/4*((A*b^3 - a*(b^2*B - a*b*C + a^2*D))*x)/(a^4*(a + b*x^2)^2) + (-1/2*(
(15*A*b^3 - a*(11*b^2*B - 7*a*b*C + 3*a^2*D))*x)/(a^4*(a + b*x^2)) + ((-8*
A)/(5*a*x^5) + (8*(3*A*b - a*B))/(3*a^2*x^3) - (8*(6*A*b^2 - a*(3*b*B - a*
C)))/(a^3*x) - ((63*A*b^3 - a*(35*b^2*B - 15*a*b*C + 3*a^2*D))*ArcTan[(Sqr
t[b]*x)/Sqrt[a]])/(a^(7/2)*Sqrt[b]))/(2*a))/(4*a)
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

## Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.88

| method | result |
|---|---|
| default | $-\frac{\frac{\left(\frac{15}{8}A\,b^4-\frac{11}{8}B\,a\,b^3+\frac{7}{8}C\,a^2b^2-\frac{3}{8}a^3Db\right)x^3+\frac{a\left(17b^3A-13a\,b^2B+9a^2bC-5a^3D\right)x}{8}}{\left(b\,x^2+a\right)^2}+\frac{\left(63b^3A-35a\,b^2B+15a^2bC-3a^3D\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}}}{a^5}-\frac{}{5}$ |

input
```
int((D*x^6+C*x^4+B*x^2+A)/x^6/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/a^5*(((15/8*A*b^4-11/8*B*a*b^3+7/8*C*a^2*b^2-3/8*a^3*D*b)*x^3+1/8*a*(17
*A*b^3-13*B*a*b^2+9*C*a^2*b-5*D*a^3)*x)/(b*x^2+a)^2+1/8*(63*A*b^3-35*B*a*b
^2+15*C*a^2*b-3*D*a^3)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))-1/5*A/a^3/x^5-
1/3*(-3*A*b+B*a)/a^4/x^3-(6*A*b^2-3*B*a*b+C*a^2)/a^5/x
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 628, normalized size of antiderivative = 3.14

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6\left(a+bx^2\right)^3}\,dx$$
$$=\left[\frac{30\left(3\,Da^4b^2-15\,Ca^3b^3+35\,Ba^2b^4-63\,Aab^5\right)x^8-48\,Aa^5b+50\left(3\,Da^5b-15\,Ca^4b^2+35\,Ba^3b^3-63\right.}{}\right.$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^6/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
[1/240*(30*(3*D*a^4*b^2 - 15*C*a^3*b^3 + 35*B*a^2*b^4 - 63*A*a*b^5)*x^8 -
48*A*a^5*b + 50*(3*D*a^5*b - 15*C*a^4*b^2 + 35*B*a^3*b^3 - 63*A*a^2*b^4)*x
^6 - 16*(15*C*a^5*b - 35*B*a^4*b^2 + 63*A*a^3*b^3)*x^4 - 16*(5*B*a^5*b - 9
*A*a^4*b^2)*x^2 + 15*((3*D*a^3*b^2 - 15*C*a^2*b^3 + 35*B*a*b^4 - 63*A*b^5)
*x^9 + 2*(3*D*a^4*b - 15*C*a^3*b^2 + 35*B*a^2*b^3 - 63*A*a*b^4)*x^7 + (3*D
*a^5 - 15*C*a^4*b + 35*B*a^3*b^2 - 63*A*a^2*b^3)*x^5)*sqrt(-a*b)*log((b*x^
2 + 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^6*b^3*x^9 + 2*a^7*b^2*x^7 + a^8*b
*x^5), 1/120*(15*(3*D*a^4*b^2 - 15*C*a^3*b^3 + 35*B*a^2*b^4 - 63*A*a*b^5)*
x^8 - 24*A*a^5*b + 25*(3*D*a^5*b - 15*C*a^4*b^2 + 35*B*a^3*b^3 - 63*A*a^2*
b^4)*x^6 - 8*(15*C*a^5*b - 35*B*a^4*b^2 + 63*A*a^3*b^3)*x^4 - 8*(5*B*a^5*b
 - 9*A*a^4*b^2)*x^2 + 15*((3*D*a^3*b^2 - 15*C*a^2*b^3 + 35*B*a*b^4 - 63*A*
b^5)*x^9 + 2*(3*D*a^4*b - 15*C*a^3*b^2 + 35*B*a^2*b^3 - 63*A*a*b^4)*x^7 +
(3*D*a^5 - 15*C*a^4*b + 35*B*a^3*b^2 - 63*A*a^2*b^3)*x^5)*sqrt(a*b)*arctan
(sqrt(a*b)*x/a))/(a^6*b^3*x^9 + 2*a^7*b^2*x^7 + a^8*b*x^5)]
```

## Sympy [A] (verification not implemented)

Time = 52.95 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.42

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6\left(a+bx^2\right)^3}\,dx$$

$$= -\frac{\sqrt{-\frac{1}{a^{11}b}}(-63Ab^3+35Bab^2-15Ca^2b+3Da^3)\log\left(-a^6\sqrt{-\frac{1}{a^{11}b}}+x\right)}{16} + \frac{\sqrt{-\frac{1}{a^{11}b}}(-63Ab^3+35Bab^2-15Ca^2b+3Da^3)\log\left(a^6\sqrt{-\frac{1}{a^{11}b}}+x\right)}{16} + \frac{-24Aa^4+x^8(-945Ab^4+525Bab^3-225Ca^2b^2+45Da^3b)+x^6(-1575Aab^3+875Ba^2b^2-375Ca^3b}{120a^7x^5+240a^6bx^7+120a^5b}$$

input
```
integrate((D*x**6+C*x**4+B*x**2+A)/x**6/(b*x**2+a)**3,x)
```

output
```
-sqrt(-1/(a**11*b))*(-63*A*b**3 + 35*B*a*b**2 - 15*C*a**2*b + 3*D*a**3)*lo
g(-a**6*sqrt(-1/(a**11*b)) + x)/16 + sqrt(-1/(a**11*b))*(-63*A*b**3 + 35*B
*a*b**2 - 15*C*a**2*b + 3*D*a**3)*log(a**6*sqrt(-1/(a**11*b)) + x)/16 + (-
24*A*a**4 + x**8*(-945*A*b**4 + 525*B*a*b**3 - 225*C*a**2*b**2 + 45*D*a**3
*b) + x**6*(-1575*A*a*b**3 + 875*B*a**2*b**2 - 375*C*a**3*b + 75*D*a**4) +
 x**4*(-504*A*a**2*b**2 + 280*B*a**3*b - 120*C*a**4) + x**2*(72*A*a**3*b -
 40*B*a**4))/(120*a**7*x**5 + 240*a**6*b*x**7 + 120*a**5*b**2*x**9)
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.01

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6\left(a+bx^2\right)^3}\,dx$$
$$= \frac{15\,(3\,Da^3b-15\,Ca^2b^2+35\,Bab^3-63\,Ab^4)x^8+25\,(3\,Da^4-15\,Ca^3b+35\,Ba^2b^2-63\,Aab^3)x^6-24\,Aa^4}{120\,(a^5b^2x^9+2\,a^6bx^7+a^7x^5)}$$
$$+\frac{(3\,Da^3-15\,Ca^2b+35\,Bab^2-63\,Ab^3)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^5}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^6/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/120*(15*(3*D*a^3*b - 15*C*a^2*b^2 + 35*B*a*b^3 - 63*A*b^4)*x^8 + 25*(3*D
*a^4 - 15*C*a^3*b + 35*B*a^2*b^2 - 63*A*a*b^3)*x^6 - 24*A*a^4 - 8*(15*C*a^
4 - 35*B*a^3*b + 63*A*a^2*b^2)*x^4 - 8*(5*B*a^4 - 9*A*a^3*b)*x^2)/(a^5*b^2
*x^9 + 2*a^6*b*x^7 + a^7*x^5) + 1/8*(3*D*a^3 - 15*C*a^2*b + 35*B*a*b^2 - 6
3*A*b^3)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^5)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.97

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6\left(a+bx^2\right)^3}\,dx = \frac{(3\,Da^3-15\,Ca^2b+35\,Bab^2-63\,Ab^3)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^5}$$
$$+\frac{3\,Da^3bx^3-7\,Ca^2b^2x^3+11\,Bab^3x^3-15\,Ab^4x^3+5\,Da^4x-9\,Ca^3bx+13\,Ba^2b^2x-17\,Aab^3x}{8\,(bx^2+a)^2a^5}$$
$$-\frac{15\,Ca^2x^4-45\,Babx^4+90\,Ab^2x^4+5\,Ba^2x^2-15\,Aabx^2+3\,Aa^2}{15\,a^5x^5}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^6/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
1/8*(3*D*a^3 - 15*C*a^2*b + 35*B*a*b^2 - 63*A*b^3)*arctan(b*x/sqrt(a*b))/(
sqrt(a*b)*a^5) + 1/8*(3*D*a^3*b*x^3 - 7*C*a^2*b^2*x^3 + 11*B*a*b^3*x^3 - 1
5*A*b^4*x^3 + 5*D*a^4*x - 9*C*a^3*b*x + 13*B*a^2*b^2*x - 17*A*a*b^3*x)/((b
*x^2 + a)^2*a^5) - 1/15*(15*C*a^2*x^4 - 45*B*a*b*x^4 + 90*A*b^2*x^4 + 5*B*
a^2*x^2 - 15*A*a*b*x^2 + 3*A*a^2)/(a^5*x^5)
```

## Mupad [B] (verification not implemented)

Time = 1.58 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.36

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6\left(a+bx^2\right)^3}\,dx = \frac{\frac{7\,B\,b\,x^2}{3\,a^2}-\frac{B}{3\,a}+\frac{175\,B\,b^2\,x^4}{24\,a^3}+\frac{35\,B\,b^3\,x^6}{8\,a^4}}{a^2\,x^3+2\,a\,b\,x^5+b^2\,x^7} - \frac{\frac{A}{5\,a}-\frac{3\,A\,b\,x^2}{5\,a^2}+\frac{21\,A\,b^2\,x^4}{5\,a^3}+\frac{105\,A\,b^3\,x^6}{8\,a^4}+\frac{63\,A\,b^4\,x^8}{8\,a^5}}{a^2\,x^5+2\,a\,b\,x^7+b^2\,x^9} - \frac{\frac{C}{a}+\frac{25\,C\,b\,x^2}{8\,a^2}+\frac{15\,C\,b^2\,x^4}{8\,a^3}}{a^2\,x+2\,a\,b\,x^3+b^2\,x^5} + \frac{x\,D\,{}_2\mathrm{F}_1\left(\frac{1}{2},3;\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{a^3} - \frac{63\,A\,b^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,a^{11/2}} + \frac{35\,B\,b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,a^{9/2}} - \frac{15\,C\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,a^{7/2}}$$

input
```
int((A + B*x^2 + C*x^4 + x^6*D)/(x^6*(a + b*x^2)^3),x)
```

output
```
((7*B*b*x^2)/(3*a^2) - B/(3*a) + (175*B*b^2*x^4)/(24*a^3) + (35*B*b^3*x^6)
/(8*a^4))/(a^2*x^3 + b^2*x^7 + 2*a*b*x^5) - (A/(5*a) - (3*A*b*x^2)/(5*a^2)
 + (21*A*b^2*x^4)/(5*a^3) + (105*A*b^3*x^6)/(8*a^4) + (63*A*b^4*x^8)/(8*a^
5))/(a^2*x^5 + b^2*x^9 + 2*a*b*x^7) - (C/a + (25*C*b*x^2)/(8*a^2) + (15*C*
b^2*x^4)/(8*a^3))/(a^2*x + b^2*x^5 + 2*a*b*x^3) + (x*D*hypergeom([1/2, 3],
 3/2, -(b*x^2)/a))/a^3 - (63*A*b^(5/2)*atan((b^(1/2)*x)/a^(1/2)))/(8*a^(11
/2)) + (35*B*b^(3/2)*atan((b^(1/2)*x)/a^(1/2)))/(8*a^(9/2)) - (15*C*b^(1/2
)*atan((b^(1/2)*x)/a^(1/2)))/(8*a^(7/2))
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.86

$$\int \frac{A + Bx^2 + Cx^4 + Dx^6}{x^6 (a + bx^2)^3} dx$$
$$= \frac{45\sqrt{b}\sqrt{a}\, atan\left(\frac{bx}{\sqrt{b}\sqrt{a}}\right) a^4 d\, x^5 - 225\sqrt{b}\sqrt{a}\, atan\left(\frac{bx}{\sqrt{b}\sqrt{a}}\right) a^3 bc\, x^5 + 90\sqrt{b}\sqrt{a}\, atan\left(\frac{bx}{\sqrt{b}\sqrt{a}}\right) a^3 bd\, x^7 - 420\sqrt{}}{}$$

input
```
int((D*x^6+C*x^4+B*x^2+A)/x^6/(b*x^2+a)^3,x)
```

output
```
(45*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**4*d*x**5 - 225*sqrt(b
)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**3*b*c*x**5 + 90*sqrt(b)*sqrt(a)
*atan((b*x)/(sqrt(b)*sqrt(a)))*a**3*b*d*x**7 - 420*sqrt(b)*sqrt(a)*atan((b
*x)/(sqrt(b)*sqrt(a)))*a**2*b**3*x**5 - 450*sqrt(b)*sqrt(a)*atan((b*x)/(sq
rt(b)*sqrt(a)))*a**2*b**2*c*x**7 + 45*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*
sqrt(a)))*a**2*b**2*d*x**9 - 840*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(
a)))*a*b**4*x**7 - 225*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*b**
3*c*x**9 - 420*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**5*x**9 - 2
4*a**5*b + 32*a**4*b**2*x**2 - 120*a**4*b*c*x**4 + 75*a**4*b*d*x**6 - 224*
a**3*b**3*x**4 - 375*a**3*b**2*c*x**6 + 45*a**3*b**2*d*x**8 - 700*a**2*b**
4*x**6 - 225*a**2*b**3*c*x**8 - 420*a*b**5*x**8)/(120*a**5*b*x**5*(a**2 +
2*a*b*x**2 + b**2*x**4))
```

# 3.187 $\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8(a+bx^2)^3} \, dx$



## Optimal result

Integrand size = 30, antiderivative size = 238

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8(a+bx^2)^3} \, dx = -\frac{A}{7a^3x^7}+\frac{3Ab-aB}{5a^4x^5}-\frac{6Ab^2-a(3bB-aC)}{3a^5x^3}+\frac{10Ab^3-a(6b^2B-3abC+a^2D)}{a^6x} + \frac{b(Ab^3-a(b^2B-abC+a^2D))\,x}{4a^5(a+bx^2)^2}+\frac{b(19Ab^3-a(15b^2B-11abC+7a^2D))\,x}{8a^6(a+bx^2)} + \frac{\sqrt{b}(99Ab^3-a(63b^2B-35abC+15a^2D))\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{13/2}}$$

output
```
-1/7*A/a^3/x^7+1/5*(3*A*b-B*a)/a^4/x^5-1/3*(6*A*b^2-a*(3*B*b-C*a))/a^5/x^3
+(10*A*b^3-a*(6*B*b^2-3*C*a*b+D*a^2))/a^6/x+1/4*b*(A*b^3-a*(B*b^2-C*a*b+D*
a^2))*x/a^5/(b*x^2+a)^2+1/8*b*(19*A*b^3-a*(15*B*b^2-11*C*a*b+7*D*a^2))*x/a
^6/(b*x^2+a)+1/8*b^(1/2)*(99*A*b^3-a*(63*B*b^2-35*C*a*b+15*D*a^2))*arctan(
b^(1/2)*x/a^(1/2))/a^(13/2)
```

## Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.98

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8(a+bx^2)^3}\,dx$$
$$= -\frac{A}{7a^3x^7}+\frac{3Ab-aB}{5a^4x^5}-\frac{6Ab^2-3abB+a^2C}{3a^5x^3}+\frac{10Ab^3-6ab^2B+3a^2bC-a^3D}{a^6x}$$
$$+\frac{b(Ab^3-a(b^2B-abC+a^2D))\,x}{4a^5(a+bx^2)^2}+\frac{(19Ab^4+ab(-15b^2B+11abC-7a^2D))\,x}{8a^6(a+bx^2)}$$
$$+\frac{\sqrt{b}(99Ab^3+a(-63b^2B+35abC-15a^2D))\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{13/2}}$$

input
```
Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(x^8*(a + b*x^2)^3),x]
```

output
```
-1/7*A/(a^3*x^7) + (3*A*b - a*B)/(5*a^4*x^5) - (6*A*b^2 - 3*a*b*B + a^2*C)
/(3*a^5*x^3) + (10*A*b^3 - 6*a*b^2*B + 3*a^2*b*C - a^3*D)/(a^6*x) + (b*(A*
b^3 - a*(b^2*B - a*b*C + a^2*D))*x)/(4*a^5*(a + b*x^2)^2) + ((19*A*b^4 + a
*b*(-15*b^2*B + 11*a*b*C - 7*a^2*D))*x)/(8*a^6*(a + b*x^2)) + (Sqrt[b]*(99
*A*b^3 + a*(-63*b^2*B + 35*a*b*C - 15*a^2*D))*ArcTan[(Sqrt[b]*x)/Sqrt[a]])
/(8*a^(13/2))
```

## Rubi [A] (verified)

Time = 1.16 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2336, 25, 2336, 25, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8(a+bx^2)^3}\,dx$$
$$\Big\downarrow \text{ 2336}$$

$$\frac{bx\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{4a^5\left(a+bx^2\right)^2}-\frac{\int -\frac{\frac{3b\left(Ab^3-a\left(Da^2-bCa+b^2B\right)\right)x^8}{a^4}-\frac{4\left(Ab^3-a\left(Da^2-bCa+b^2B\right)\right)x^6}{a^3}+\frac{4\left(Ab^2-a(bB-aC)\right)x^4}{a^2}-4\left(\frac{Ab}{a}-B\right)x^2+4A}{x^8\left(bx^2+a\right)^2}dx}{4a}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{\frac{3b\left(Ab^3-a\left(Da^2-bCa+b^2B\right)\right)x^8}{a^4}-\frac{4\left(Ab^3-a\left(Da^2-bCa+b^2B\right)\right)x^6}{a^3}+\frac{4\left(Ab^2-a(bB-aC)\right)x^4}{a^2}-4\left(\frac{Ab}{a}-B\right)x^2+4A}{x^8\left(bx^2+a\right)^2}dx}{4a}+\frac{bx\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{4a^5\left(a+bx^2\right)^2}$$

$$\Big\downarrow \; 2336$$

$$\frac{\frac{bx\left(19Ab^3-a\left(7a^2D-11abC+15b^2B\right)\right)}{2a^5\left(a+bx^2\right)}-\frac{\int -\frac{\frac{b\left(19Ab^3-a\left(7Da^2-11bCa+15b^2B\right)\right)x^8}{a^4}-\frac{8\left(4Ab^3-a\left(Da^2-2bCa+3b^2B\right)\right)x^6}{a^3}+\frac{8\left(3Ab^2-a(2bB-aC)\right)x^4}{a^2}-8\cdots}{x^8\left(bx^2+a\right)}\cdots}{2a}}{4a}+\frac{bx\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{4a^5\left(a+bx^2\right)^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{\frac{b\left(19Ab^3-a\left(7Da^2-11bCa+15b^2B\right)\right)x^8}{a^4}-\frac{8\left(4Ab^3-a\left(Da^2-2bCa+3b^2B\right)\right)x^6}{a^3}+\frac{8\left(3Ab^2-a(2bB-aC)\right)x^4}{a^2}-8\left(\frac{2Ab}{a}-B\right)x^2+8A}{x^8\left(bx^2+a\right)}dx}{2a}+\frac{bx\left(19Ab^3-a\left(7a^2D-\cdots\right.\right.}{2a^5\left(a+\cdots\right.}}{4a}+\frac{bx\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{4a^5\left(a+bx^2\right)^2}$$

$$\Big\downarrow \; 2333$$

$$\frac{\frac{\int \left(\frac{8A}{ax^8}+\frac{b\left(99Ab^3-a\left(15Da^2-35bCa+63b^2B\right)\right)}{a^4\left(bx^2+a\right)}+\frac{8\left(a\left(Da^2-3bCa+6b^2B\right)-10Ab^3\right)}{a^4x^2}+\frac{8\left(6Ab^2-a(3bB-aC)\right)}{a^3x^4}+\frac{8(aB-3Ab)}{a^2x^6}\right)dx}{2a}+\frac{bx\left(19Ab^3-a\left(7a^2D-1\cdots\right.\right.}{2a^5\left(a+b\cdots\right.}}{4a}+\frac{bx\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{4a^5\left(a+bx^2\right)^2}$$

$$\Big\downarrow \; 2009$$

$$\frac{bx\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{4a^5\left(a+bx^2\right)^2}+$$
$$\frac{\frac{bx\left(19Ab^3-a\left(7a^2D-11abC+15b^2B\right)\right)}{2a^5(a+bx^2)}+\frac{-\frac{8\left(6Ab^2-a(3bB-aC)\right)}{3a^3x^3}+\frac{8(3Ab-aB)}{5a^2x^5}+\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(99Ab^3-a\left(15a^2D-35abC+63b^2B\right)\right)}{a^{9/2}}+\frac{8\left(10Ab^3-a\left(\right.\right.}{}}{2a}}{4a}$$

input
```
Int[(A + B*x^2 + C*x^4 + D*x^6)/(x^8*(a + b*x^2)^3),x]
```

output
```
(b*(A*b^3 - a*(b^2*B - a*b*C + a^2*D))*x)/(4*a^5*(a + b*x^2)^2) + ((b*(19*
A*b^3 - a*(15*b^2*B - 11*a*b*C + 7*a^2*D))*x)/(2*a^5*(a + b*x^2)) + ((-8*A
)/(7*a*x^7) + (8*(3*A*b - a*B))/(5*a^2*x^5) - (8*(6*A*b^2 - a*(3*b*B - a*C
)))/(3*a^3*x^3) + (8*(10*A*b^3 - a*(6*b^2*B - 3*a*b*C + a^2*D)))/(a^4*x) +
 (Sqrt[b]*(99*A*b^3 - a*(63*b^2*B - 35*a*b*C + 15*a^2*D))*ArcTan[(Sqrt[b]*
x)/Sqrt[a]])/a^(9/2))/(2*a))/(4*a)
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

## Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.89

| method | result |
|---|---|
| default | $\frac{b\left(\frac{\left(\frac{19}{8}A\,b^4-\frac{15}{8}B\,a\,b^3+\frac{11}{8}C\,a^2b^2-\frac{7}{8}a^3Db\right)x^3+\frac{a\left(21b^3A-17a\,b^2B+13a^2bC-9a^3D\right)x}{8}}{\left(b\,x^2+a\right)^2}+\frac{\left(99b^3A-63a\,b^2B+35a^2bC-15a^3D\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{a^6}$ |

input
```
int((D*x^6+C*x^4+B*x^2+A)/x^8/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/a^6*b*(((19/8*A*b^4-15/8*B*a*b^3+11/8*C*a^2*b^2-7/8*a^3*D*b)*x^3+1/8*a*(
21*A*b^3-17*B*a*b^2+13*C*a^2*b-9*D*a^3)*x)/(b*x^2+a)^2+1/8*(99*A*b^3-63*B*
a*b^2+35*C*a^2*b-15*D*a^3)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))-1/7*A/a^3/
x^7-1/5*(-3*A*b+B*a)/a^4/x^5-1/3*(6*A*b^2-3*B*a*b+C*a^2)/a^5/x^3-(-10*A*b^
3+6*B*a*b^2-3*C*a^2*b+D*a^3)/a^6/x
```

## Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 678, normalized size of antiderivative = 2.85

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8\left(a+bx^2\right)^3}\,dx$$

$$= \left[-\frac{210\,(15\,Da^3b^2-35\,Ca^2b^3+63\,Bab^4-99\,Ab^5)x^{10}+350\,(15\,Da^4b-35\,Ca^3b^2+63\,Ba^2b^3-99\,Aab^4)}{}\right.$$

$$-\frac{105\,(15\,Da^3b^2-35\,Ca^2b^3+63\,Bab^4-99\,Ab^5)x^{10}+175\,(15\,Da^4b-35\,Ca^3b^2+63\,Ba^2b^3-99\,Aab^4)}{}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^8/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
[-1/1680*(210*(15*D*a^3*b^2 - 35*C*a^2*b^3 + 63*B*a*b^4 - 99*A*b^5)*x^10 +
 350*(15*D*a^4*b - 35*C*a^3*b^2 + 63*B*a^2*b^3 - 99*A*a*b^4)*x^8 + 112*(15
*D*a^5 - 35*C*a^4*b + 63*B*a^3*b^2 - 99*A*a^2*b^3)*x^6 + 240*A*a^5 + 16*(3
5*C*a^5 - 63*B*a^4*b + 99*A*a^3*b^2)*x^4 + 48*(7*B*a^5 - 11*A*a^4*b)*x^2 +
 105*((15*D*a^3*b^2 - 35*C*a^2*b^3 + 63*B*a*b^4 - 99*A*b^5)*x^11 + 2*(15*D
*a^4*b - 35*C*a^3*b^2 + 63*B*a^2*b^3 - 99*A*a*b^4)*x^9 + (15*D*a^5 - 35*C*
a^4*b + 63*B*a^3*b^2 - 99*A*a^2*b^3)*x^7)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sq
rt(-b/a) - a)/(b*x^2 + a)))/(a^6*b^2*x^11 + 2*a^7*b*x^9 + a^8*x^7), -1/840
*(105*(15*D*a^3*b^2 - 35*C*a^2*b^3 + 63*B*a*b^4 - 99*A*b^5)*x^10 + 175*(15
*D*a^4*b - 35*C*a^3*b^2 + 63*B*a^2*b^3 - 99*A*a*b^4)*x^8 + 56*(15*D*a^5 -
35*C*a^4*b + 63*B*a^3*b^2 - 99*A*a^2*b^3)*x^6 + 120*A*a^5 + 8*(35*C*a^5 -
63*B*a^4*b + 99*A*a^3*b^2)*x^4 + 24*(7*B*a^5 - 11*A*a^4*b)*x^2 + 105*((15*
D*a^3*b^2 - 35*C*a^2*b^3 + 63*B*a*b^4 - 99*A*b^5)*x^11 + 2*(15*D*a^4*b - 3
5*C*a^3*b^2 + 63*B*a^2*b^3 - 99*A*a*b^4)*x^9 + (15*D*a^5 - 35*C*a^4*b + 63
*B*a^3*b^2 - 99*A*a^2*b^3)*x^7)*sqrt(b/a)*arctan(x*sqrt(b/a)))/(a^6*b^2*x^
11 + 2*a^7*b*x^9 + a^8*x^7)]
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 450 vs. $2(223) = 446$.

Time = 119.81 (sec) , antiderivative size = 450, normalized size of antiderivative = 1.89

$$\int \frac{A + Bx^2 + Cx^4 + Dx^6}{x^8 (a + bx^2)^3} dx$$
$$= \frac{\sqrt{-\frac{b}{a^{13}}}(-99Ab^3 + 63Bab^2 - 35Ca^2b + 15Da^3)\log\left(-\frac{a^7\sqrt{-\frac{b}{a^{13}}}(-99Ab^3+63Bab^2-35Ca^2b+15Da^3)}{-99Ab^4+63Bab^3-35Ca^2b^2+15Da^3b} + x\right)}{16}$$
$$- \frac{\sqrt{-\frac{b}{a^{13}}}(-99Ab^3 + 63Bab^2 - 35Ca^2b + 15Da^3)\log\left(\frac{a^7\sqrt{-\frac{b}{a^{13}}}(-99Ab^3+63Bab^2-35Ca^2b+15Da^3)}{-99Ab^4+63Bab^3-35Ca^2b^2+15Da^3b} + x\right)}{16}$$
$$+ \frac{-120Aa^5 + x^{10} \cdot (10395Ab^5 - 6615Bab^4 + 3675Ca^2b^3 - 1575Da^3b^2) + x^8 \cdot (17325Aab^4 - 11025Ba^2b}{}$$

input
```
integrate((D*x**6+C*x**4+B*x**2+A)/x**8/(b*x**2+a)**3,x)
```

output
```
sqrt(-b/a**13)*(-99*A*b**3 + 63*B*a*b**2 - 35*C*a**2*b + 15*D*a**3)*log(-a
**7*sqrt(-b/a**13)*(-99*A*b**3 + 63*B*a*b**2 - 35*C*a**2*b + 15*D*a**3)/(-
99*A*b**4 + 63*B*a*b**3 - 35*C*a**2*b**2 + 15*D*a**3*b) + x)/16 - sqrt(-b/
a**13)*(-99*A*b**3 + 63*B*a*b**2 - 35*C*a**2*b + 15*D*a**3)*log(a**7*sqrt(
-b/a**13)*(-99*A*b**3 + 63*B*a*b**2 - 35*C*a**2*b + 15*D*a**3)/(-99*A*b**4
 + 63*B*a*b**3 - 35*C*a**2*b**2 + 15*D*a**3*b) + x)/16 + (-120*A*a**5 + x*
*10*(10395*A*b**5 - 6615*B*a*b**4 + 3675*C*a**2*b**3 - 1575*D*a**3*b**2) +
 x**8*(17325*A*a*b**4 - 11025*B*a**2*b**3 + 6125*C*a**3*b**2 - 2625*D*a**4
*b) + x**6*(5544*A*a**2*b**3 - 3528*B*a**3*b**2 + 1960*C*a**4*b - 840*D*a*
*5) + x**4*(-792*A*a**3*b**2 + 504*B*a**4*b - 280*C*a**5) + x**2*(264*A*a*
*4*b - 168*B*a**5))/(840*a**8*x**7 + 1680*a**7*b*x**9 + 840*a**6*b**2*x**1
1)
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.04

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8\left(a+bx^2\right)^3}\,dx = -\frac{105\,(15\,Da^3b^2-35\,Ca^2b^3+63\,Bab^4-99\,Ab^5)x^{10}+175\,(15\,Da^4b-35\,Ca^3b^2+63\,Ba^2b^3-99\,Aab^4)x\ldots}{84\ldots} - \frac{(15\,Da^3b-35\,Ca^2b^2+63\,Bab^3-99\,Ab^4)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^6}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^8/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
-1/840*(105*(15*D*a^3*b^2 - 35*C*a^2*b^3 + 63*B*a*b^4 - 99*A*b^5)*x^10 + 1
75*(15*D*a^4*b - 35*C*a^3*b^2 + 63*B*a^2*b^3 - 99*A*a*b^4)*x^8 + 56*(15*D*
a^5 - 35*C*a^4*b + 63*B*a^3*b^2 - 99*A*a^2*b^3)*x^6 + 120*A*a^5 + 8*(35*C*
a^5 - 63*B*a^4*b + 99*A*a^3*b^2)*x^4 + 24*(7*B*a^5 - 11*A*a^4*b)*x^2)/(a^6
*b^2*x^11 + 2*a^7*b*x^9 + a^8*x^7) - 1/8*(15*D*a^3*b - 35*C*a^2*b^2 + 63*B
*a*b^3 - 99*A*b^4)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^6)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.03

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8\left(a+bx^2\right)^3}\,dx$$
$$= -\frac{\left(15\,Da^3b-35\,Ca^2b^2+63\,Bab^3-99\,Ab^4\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^6}$$
$$-\frac{7\,Da^3b^2x^3-11\,Ca^2b^3x^3+15\,Bab^4x^3-19\,Ab^5x^3+9\,Da^4bx-13\,Ca^3b^2x+17\,Ba^2b^3x-21\,Aab^4x}{8\,\left(bx^2+a\right)^2a^6}$$
$$-\frac{105\,Da^3x^6-315\,Ca^2bx^6+630\,Bab^2x^6-1050\,Ab^3x^6+35\,Ca^3x^4-105\,Ba^2bx^4+210\,Aab^2x^4+21\,B}{105\,a^6x^7}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^8/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
-1/8*(15*D*a^3*b - 35*C*a^2*b^2 + 63*B*a*b^3 - 99*A*b^4)*arctan(b*x/sqrt(a
*b))/(sqrt(a*b)*a^6) - 1/8*(7*D*a^3*b^2*x^3 - 11*C*a^2*b^3*x^3 + 15*B*a*b^
4*x^3 - 19*A*b^5*x^3 + 9*D*a^4*b*x - 13*C*a^3*b^2*x + 17*B*a^2*b^3*x - 21*
A*a*b^4*x)/((b*x^2 + a)^2*a^6) - 1/105*(105*D*a^3*x^6 - 315*C*a^2*b*x^6 +
630*B*a*b^2*x^6 - 1050*A*b^3*x^6 + 35*C*a^3*x^4 - 105*B*a^2*b*x^4 + 210*A*
a*b^2*x^4 + 21*B*a^3*x^2 - 63*A*a^2*b*x^2 + 15*A*a^3)/(a^6*x^7)
```

## Mupad [B] (verification not implemented)

Time = 1.63 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.32

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8\left(a+bx^2\right)^3}\,dx$$
$$=\frac{\frac{11\,A\,b\,x^2}{35\,a^2}-\frac{A}{7\,a}-\frac{33\,A\,b^2\,x^4}{35\,a^3}+\frac{33\,A\,b^3\,x^6}{5\,a^4}+\frac{165\,A\,b^4\,x^8}{8\,a^5}+\frac{99\,A\,b^5\,x^{10}}{8\,a^6}}{a^2\,x^7+2\,a\,b\,x^9+b^2\,x^{11}}-\frac{\frac{B}{5\,a}-\frac{3\,B\,b\,x^2}{5\,a^2}+\frac{21\,B\,b^2\,x^4}{5\,a^3}+\frac{105\,B\,b^3\,x^6}{8\,a^4}+\frac{63\,B\,b^4\,x^8}{8\,a^5}}{a^2\,x^5+2\,a\,b\,x^7+b^2\,x^9}+\frac{\frac{7\,C\,b\,x^2}{3\,a^2}-\frac{C}{3\,a}+\frac{175\,C\,b^2\,x^4}{24\,a^3}+\frac{35\,C\,b^3\,x^6}{8\,a^4}}{a^2\,x^3+2\,a\,b\,x^5+b^2\,x^7}-\frac{D\,{}_2\mathrm{F}_1\left(3,\frac{7}{2};\ \frac{9}{2};\ -\frac{a}{b\,x^2}\right)}{7\,b^3\,x^7}+\frac{99\,A\,b^{7/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,a^{13/2}}-\frac{63\,B\,b^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,a^{11/2}}+\frac{35\,C\,b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,a^{9/2}}$$

input
```
int((A + B*x^2 + C*x^4 + x^6*D)/(x^8*(a + b*x^2)^3),x)
```

output
```
((11*A*b*x^2)/(35*a^2) - A/(7*a) - (33*A*b^2*x^4)/(35*a^3) + (33*A*b^3*x^6
)/(5*a^4) + (165*A*b^4*x^8)/(8*a^5) + (99*A*b^5*x^10)/(8*a^6))/(a^2*x^7 +
b^2*x^11 + 2*a*b*x^9) - (B/(5*a) - (3*B*b*x^2)/(5*a^2) + (21*B*b^2*x^4)/(5
*a^3) + (105*B*b^3*x^6)/(8*a^4) + (63*B*b^4*x^8)/(8*a^5))/(a^2*x^5 + b^2*x
^9 + 2*a*b*x^7) + ((7*C*b*x^2)/(3*a^2) - C/(3*a) + (175*C*b^2*x^4)/(24*a^3
) + (35*C*b^3*x^6)/(8*a^4))/(a^2*x^3 + b^2*x^7 + 2*a*b*x^5) - (D*hypergeom
([3, 7/2], 9/2, -a/(b*x^2)))/(7*b^3*x^7) + (99*A*b^(7/2)*atan((b^(1/2)*x)/
a^(1/2)))/(8*a^(13/2)) - (63*B*b^(5/2)*atan((b^(1/2)*x)/a^(1/2)))/(8*a^(11
/2)) + (35*C*b^(3/2)*atan((b^(1/2)*x)/a^(1/2)))/(8*a^(9/2))
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.66

$$\int \frac{A + Bx^2 + Cx^4 + Dx^6}{x^8 (a + bx^2)^3} \, dx$$
$$= \frac{-1575\sqrt{b}\,\sqrt{a}\, atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right) a^4 d\, x^7 + 3675\sqrt{b}\,\sqrt{a}\, atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right) a^3 bc\, x^7 - 3150\sqrt{b}\,\sqrt{a}\, atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right) a^3 bd\, x^9}{}$$

input
```
int((D*x^6+C*x^4+B*x^2+A)/x^8/(b*x^2+a)^3,x)
```

output
```
( - 1575*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**4*d*x**7 + 3675*
sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**3*b*c*x**7 - 3150*sqrt(b)
*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**3*b*d*x**9 + 3780*sqrt(b)*sqrt(a
)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2*b**3*x**7 + 7350*sqrt(b)*sqrt(a)*atan
((b*x)/(sqrt(b)*sqrt(a)))*a**2*b**2*c*x**9 - 1575*sqrt(b)*sqrt(a)*atan((b*
x)/(sqrt(b)*sqrt(a)))*a**2*b**2*d*x**11 + 7560*sqrt(b)*sqrt(a)*atan((b*x)/
(sqrt(b)*sqrt(a)))*a*b**4*x**9 + 3675*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*
sqrt(a)))*a*b**3*c*x**11 + 3780*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a
)))*b**5*x**11 - 120*a**6 + 96*a**5*b*x**2 - 280*a**5*c*x**4 - 840*a**5*d*
x**6 - 288*a**4*b**2*x**4 + 1960*a**4*b*c*x**6 - 2625*a**4*b*d*x**8 + 2016
*a**3*b**3*x**6 + 6125*a**3*b**2*c*x**8 - 1575*a**3*b**2*d*x**10 + 6300*a*
*2*b**4*x**8 + 3675*a**2*b**3*c*x**10 + 3780*a*b**5*x**10)/(840*a**6*x**7*
(a**2 + 2*a*b*x**2 + b**2*x**4))
```

# 3.188 $\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{10}\left(a+bx^2\right)^3}\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 282

$$\begin{aligned}\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{10}\left(a+bx^2\right)^3}\,dx &= -\frac{A}{9a^3x^9}+\frac{3Ab-aB}{7a^4x^7}-\frac{6Ab^2-a(3bB-aC)}{5a^5x^5}\\ &\quad+\frac{10Ab^3-a(6b^2B-3abC+a^2D)}{3a^6x^3}-\frac{b(15Ab^3-a(10b^2B-6abC+3a^2D))}{a^7x}\\ &\quad-\frac{b^2(Ab^3-a(b^2B-abC+a^2D))\,x}{4a^6\left(a+bx^2\right)^2}-\frac{b^2(23Ab^3-a(19b^2B-15abC+11a^2D))\,x}{8a^7\left(a+bx^2\right)}\\ &\quad-\frac{b^{3/2}(143Ab^3-a(99b^2B-63abC+35a^2D))\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{15/2}}\end{aligned}$$

output
```
-1/9*A/a^3/x^9+1/7*(3*A*b-B*a)/a^4/x^7-1/5*(6*A*b^2-a*(3*B*b-C*a))/a^5/x^5
+1/3*(10*A*b^3-a*(6*B*b^2-3*C*a*b+D*a^2))/a^6/x^3-b*(15*A*b^3-a*(10*B*b^2-
6*C*a*b+3*D*a^2))/a^7/x-1/4*b^2*(A*b^3-a*(B*b^2-C*a*b+D*a^2))*x/a^6/(b*x^2
+a)^2-1/8*b^2*(23*A*b^3-a*(19*B*b^2-15*C*a*b+11*D*a^2))*x/a^7/(b*x^2+a)-1/
8*b^(3/2)*(143*A*b^3-a*(99*B*b^2-63*C*a*b+35*D*a^2))*arctan(b^(1/2)*x/a^(1
/2))/a^(15/2)
```

## Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 276, normalized size of antiderivative = 0.98

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{10}(a+bx^2)^3}\,dx$$
$$= -\frac{A}{9a^3x^9}+\frac{3Ab-aB}{7a^4x^7}-\frac{6Ab^2-3abB+a^2C}{5a^5x^5}+\frac{10Ab^3-6ab^2B+3a^2bC-a^3D}{3a^6x^3}$$
$$+\frac{b(-15Ab^3+a(10b^2B-6abC+3a^2D))}{a^7x}+\frac{b^2(-Ab^3+a(b^2B-abC+a^2D))\,x}{4a^6(a+bx^2)^2}$$
$$+\frac{(-23Ab^5+ab^2(19b^2B-15abC+11a^2D))\,x}{8a^7(a+bx^2)}$$
$$+\frac{b^{3/2}(-143Ab^3+a(99b^2B-63abC+35a^2D))\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{15/2}}$$

input
```
Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(x^10*(a + b*x^2)^3),x]
```

output
```
-1/9*A/(a^3*x^9) + (3*A*b - a*B)/(7*a^4*x^7) - (6*A*b^2 - 3*a*b*B + a^2*C)
/(5*a^5*x^5) + (10*A*b^3 - 6*a*b^2*B + 3*a^2*b*C - a^3*D)/(3*a^6*x^3) + (b
*(-15*A*b^3 + a*(10*b^2*B - 6*a*b*C + 3*a^2*D)))/(a^7*x) + (b^2*(-(A*b^3)
+ a*(b^2*B - a*b*C + a^2*D))*x)/(4*a^6*(a + b*x^2)^2) + ((-23*A*b^5 + a*b^
2*(19*b^2*B - 15*a*b*C + 11*a^2*D))*x)/(8*a^7*(a + b*x^2)) + (b^(3/2)*(-14
3*A*b^3 + a*(99*b^2*B - 63*a*b*C + 35*a^2*D))*ArcTan[(Sqrt[b]*x)/Sqrt[a]])
/(8*a^(15/2))
```

## Rubi [A] (verified)

Time = 1.41 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2336, 25, 2336, 25, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{10}(a+bx^2)^3}\,dx$$

$$\Big\downarrow \text{2336}$$

$$-\frac{\int -\frac{-\frac{3b^2\left(Ab^3-a\left(Da^2-bCa+b^2B\right)\right)x^{10}}{a^5}+\frac{4b\left(Ab^3-a\left(Da^2-bCa+b^2B\right)\right)x^8}{a^4}-\frac{4\left(Ab^3-a\left(Da^2-bCa+b^2B\right)\right)x^6}{a^3}+\frac{4\left(Ab^2-a(bB-aC)\right)x^4}{a^2}-4\left(\frac{Ab}{a}-B\right)x^2+}{x^{10}(bx^2+a)^2}}{4a}$$

$$\frac{b^2x\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{4a^6\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{-\frac{3b^2\left(Ab^3-a\left(Da^2-bCa+b^2B\right)\right)x^{10}}{a^5}+\frac{4b\left(Ab^3-a\left(Da^2-bCa+b^2B\right)\right)x^8}{a^4}-\frac{4\left(Ab^3-a\left(Da^2-bCa+b^2B\right)\right)x^6}{a^3}+\frac{4\left(Ab^2-a(bB-aC)\right)x^4}{a^2}-4\left(\frac{Ab}{a}-B\right)x^2+4A}{x^{10}(bx^2+a)^2}}{4a}$$

$$\frac{b^2x\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{4a^6\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{2336}$$

$$-\frac{\frac{\int -\frac{-\frac{b^2\left(23Ab^3-a\left(11Da^2-15bCa+19b^2B\right)\right)x^{10}}{a^5}+\frac{8b\left(5Ab^3-a\left(2Da^2-3bCa+4b^2B\right)\right)x^8}{a^4}-\frac{8\left(4Ab^3-a\left(Da^2-2bCa+3b^2B\right)\right)x^6}{a^3}+\frac{8\left(3Ab^2-a(2bB-aC)\right)x^4}{a^2}-}{x^{10}\left(bx^2+a\right)}}{2a}}{4a}$$

$$\frac{b^2x\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{4a^6\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{-\frac{b^2\left(23Ab^3-a\left(11Da^2-15bCa+19b^2B\right)\right)x^{10}}{a^5}+\frac{8b\left(5Ab^3-a\left(2Da^2-3bCa+4b^2B\right)\right)x^8}{a^4}-\frac{8\left(4Ab^3-a\left(Da^2-2bCa+3b^2B\right)\right)x^6}{a^3}+\frac{8\left(3Ab^2-a(2bB-aC)\right)x^4}{a^2}-8\left(\frac{2A}{a}\right.}{x^{10}\left(bx^2+a\right)}}{2a}}{4a}$$

$$\frac{b^2x\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{4a^6\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{2333}$$

$$\frac{\frac{\int \left(\frac{\left(a\left(35Da^2-63bCa+99b^2B\right)-143Ab^3\right)b^2}{a^5\left(bx^2+a\right)}+\frac{8\left(15Ab^3-a\left(3Da^2-6bCa+10b^2B\right)\right)b}{a^5x^2}+\frac{8\left(a\left(Da^2-3bCa+6b^2B\right)-10Ab^3\right)}{a^4x^4}+\frac{8\left(6Ab^2-a(3bB-aC)\right)}{a^3x^6}+\frac{8(aB-3Ab}{a^2x^8}\right.}{2a}}{4a}$$

$$\frac{b^2x\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{4a^6\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{-\frac{8(6Ab^2-a(3bB-aC))}{5a^3x^5}+\frac{8(3Ab-aB)}{7a^2x^7}-\frac{b^{3/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(143Ab^3-a\left(35a^2D-63abC+99b^2B\right)\right)}{a^{11/2}}-\frac{8b\left(15Ab^3-a\left(3a^2D-6abC+10b^2B\right)\right)}{a^5x}+\frac{8\left(10Ab^3-a\left(a^2D\cdots\right.\right.}{3a^4\cdots}}{2a}\cdots}{4a}$$

$$\frac{b^2x\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{4a^6\left(a+bx^2\right)^2}$$

input
```
Int[(A + B*x^2 + C*x^4 + D*x^6)/(x^10*(a + b*x^2)^3),x]
```

output
```
-1/4*(b^2*(A*b^3 - a*(b^2*B - a*b*C + a^2*D))*x)/(a^6*(a + b*x^2)^2) + (-1
/2*(b^2*(23*A*b^3 - a*(19*b^2*B - 15*a*b*C + 11*a^2*D))*x)/(a^6*(a + b*x^2
)) + ((-8*A)/(9*a*x^9) + (8*(3*A*b - a*B))/(7*a^2*x^7) - (8*(6*A*b^2 - a*(
3*b*B - a*C)))/(5*a^3*x^5) + (8*(10*A*b^3 - a*(6*b^2*B - 3*a*b*C + a^2*D))
)/(3*a^4*x^3) - (8*b*(15*A*b^3 - a*(10*b^2*B - 6*a*b*C + 3*a^2*D)))/(a^5*x
) - (b^(3/2)*(143*A*b^3 - a*(99*b^2*B - 63*a*b*C + 35*a^2*D))*ArcTan[(Sqrt
[b]*x)/Sqrt[a]])/a^(11/2))/(2*a))/(4*a)
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

## Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 250, normalized size of antiderivative = 0.89

| method | result |
|---|---|
| default | $-\frac{b^2\left(\frac{\left(\frac{23}{8}A\,b^4-\frac{19}{8}B a\,b^3+\frac{15}{8}C\,a^2b^2-\frac{11}{8}a^3Db\right)x^3+\frac{a\left(25b^3A-21a\,b^2B+17a^2bC-13a^3D\right)x}{8}}{\left(b\,x^2+a\right)^2}+\frac{\left(143b^3A-99a\,b^2B+63a^2bC-35a^3D\right)\arctan\left(\frac{}{\sqrt{}}\right.}{8\sqrt{ab}}\right.}{a^7}$ |

input
```
int((D*x^6+C*x^4+B*x^2+A)/x^10/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-b^2/a^7*(((23/8*A*b^4-19/8*B*a*b^3+15/8*C*a^2*b^2-11/8*a^3*D*b)*x^3+1/8*a
*(25*A*b^3-21*B*a*b^2+17*C*a^2*b-13*D*a^3)*x)/(b*x^2+a)^2+1/8*(143*A*b^3-9
9*B*a*b^2+63*C*a^2*b-35*D*a^3)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))-1/9*A/
a^3/x^9-1/7*(-3*A*b+B*a)/a^4/x^7-1/5*(6*A*b^2-3*B*a*b+C*a^2)/a^5/x^5-1/3*(
-10*A*b^3+6*B*a*b^2-3*C*a^2*b+D*a^3)/a^6/x^3-b*(15*A*b^3-10*B*a*b^2+6*C*a^
2*b-3*D*a^3)/a^7/x
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 772, normalized size of antiderivative = 2.74

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{10}\left(a+bx^2\right)^3}\,dx=\text{Too large to display}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^10/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
[1/5040*(630*(35*D*a^3*b^3 - 63*C*a^2*b^4 + 99*B*a*b^5 - 143*A*b^6)*x^12 +
 1050*(35*D*a^4*b^2 - 63*C*a^3*b^3 + 99*B*a^2*b^4 - 143*A*a*b^5)*x^10 + 33
6*(35*D*a^5*b - 63*C*a^4*b^2 + 99*B*a^3*b^3 - 143*A*a^2*b^4)*x^8 - 560*A*a
^6 - 48*(35*D*a^6 - 63*C*a^5*b + 99*B*a^4*b^2 - 143*A*a^3*b^3)*x^6 - 16*(6
3*C*a^6 - 99*B*a^5*b + 143*A*a^4*b^2)*x^4 - 80*(9*B*a^6 - 13*A*a^5*b)*x^2
- 315*((35*D*a^3*b^3 - 63*C*a^2*b^4 + 99*B*a*b^5 - 143*A*b^6)*x^13 + 2*(35
*D*a^4*b^2 - 63*C*a^3*b^3 + 99*B*a^2*b^4 - 143*A*a*b^5)*x^11 + (35*D*a^5*b
 - 63*C*a^4*b^2 + 99*B*a^3*b^3 - 143*A*a^2*b^4)*x^9)*sqrt(-b/a)*log((b*x^2
 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)))/(a^7*b^2*x^13 + 2*a^8*b*x^11 + a^9*
x^9), 1/2520*(315*(35*D*a^3*b^3 - 63*C*a^2*b^4 + 99*B*a*b^5 - 143*A*b^6)*x
^12 + 525*(35*D*a^4*b^2 - 63*C*a^3*b^3 + 99*B*a^2*b^4 - 143*A*a*b^5)*x^10
+ 168*(35*D*a^5*b - 63*C*a^4*b^2 + 99*B*a^3*b^3 - 143*A*a^2*b^4)*x^8 - 280
*A*a^6 - 24*(35*D*a^6 - 63*C*a^5*b + 99*B*a^4*b^2 - 143*A*a^3*b^3)*x^6 - 8
*(63*C*a^6 - 99*B*a^5*b + 143*A*a^4*b^2)*x^4 - 40*(9*B*a^6 - 13*A*a^5*b)*x
^2 + 315*((35*D*a^3*b^3 - 63*C*a^2*b^4 + 99*B*a*b^5 - 143*A*b^6)*x^13 + 2*
(35*D*a^4*b^2 - 63*C*a^3*b^3 + 99*B*a^2*b^4 - 143*A*a*b^5)*x^11 + (35*D*a^
5*b - 63*C*a^4*b^2 + 99*B*a^3*b^3 - 143*A*a^2*b^4)*x^9)*sqrt(b/a)*arctan(x
*sqrt(b/a)))/(a^7*b^2*x^13 + 2*a^8*b*x^11 + a^9*x^9)]
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{A + Bx^2 + Cx^4 + Dx^6}{x^{10} (a + bx^2)^3} \, dx = \text{Timed out}$$

input
```
integrate((D*x**6+C*x**4+B*x**2+A)/x**10/(b*x**2+a)**3,x)
```

output
```
Timed out
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.03

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{10}\left(a+bx^2\right)^3}\,dx$$
$$= \frac{315\,(35\,Da^3b^3-63\,Ca^2b^4+99\,Bab^5-143\,Ab^6)x^{12}+525\,(35\,Da^4b^2-63\,Ca^3b^3+99\,Ba^2b^4-143\,Aab^5)}{}$$
$$+\frac{(35\,Da^3b^2-63\,Ca^2b^3+99\,Bab^4-143\,Ab^5)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^7}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^10/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/2520*(315*(35*D*a^3*b^3 - 63*C*a^2*b^4 + 99*B*a*b^5 - 143*A*b^6)*x^12 +
525*(35*D*a^4*b^2 - 63*C*a^3*b^3 + 99*B*a^2*b^4 - 143*A*a*b^5)*x^10 + 168*
(35*D*a^5*b - 63*C*a^4*b^2 + 99*B*a^3*b^3 - 143*A*a^2*b^4)*x^8 - 280*A*a^6
 - 24*(35*D*a^6 - 63*C*a^5*b + 99*B*a^4*b^2 - 143*A*a^3*b^3)*x^6 - 8*(63*C
*a^6 - 99*B*a^5*b + 143*A*a^4*b^2)*x^4 - 40*(9*B*a^6 - 13*A*a^5*b)*x^2)/(a
^7*b^2*x^13 + 2*a^8*b*x^11 + a^9*x^9) + 1/8*(35*D*a^3*b^2 - 63*C*a^2*b^3 +
 99*B*a*b^4 - 143*A*b^5)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^7)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.05

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{10}\left(a+bx^2\right)^3}\,dx$$
$$= \frac{(35\,Da^3b^2-63\,Ca^2b^3+99\,Bab^4-143\,Ab^5)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^7}$$
$$+\frac{11\,Da^3b^3x^3-15\,Ca^2b^4x^3+19\,Bab^5x^3-23\,Ab^6x^3+13\,Da^4b^2x-17\,Ca^3b^3x+21\,Ba^2b^4x-25\,Aab^5x}{8\,(bx^2+a)^2a^7}$$
$$+\frac{945\,Da^3bx^8-1890\,Ca^2b^2x^8+3150\,Bab^3x^8-4725\,Ab^4x^8-105\,Da^4x^6+315\,Ca^3bx^6-630\,Ba^2b^2x^6}{315\,a^7x^9}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^10/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
1/8*(35*D*a^3*b^2 - 63*C*a^2*b^3 + 99*B*a*b^4 - 143*A*b^5)*arctan(b*x/sqrt
(a*b))/(sqrt(a*b)*a^7) + 1/8*(11*D*a^3*b^3*x^3 - 15*C*a^2*b^4*x^3 + 19*B*a
*b^5*x^3 - 23*A*b^6*x^3 + 13*D*a^4*b^2*x - 17*C*a^3*b^3*x + 21*B*a^2*b^4*x
 - 25*A*a*b^5*x)/((b*x^2 + a)^2*a^7) + 1/315*(945*D*a^3*b*x^8 - 1890*C*a^2
*b^2*x^8 + 3150*B*a*b^3*x^8 - 4725*A*b^4*x^8 - 105*D*a^4*x^6 + 315*C*a^3*b
*x^6 - 630*B*a^2*b^2*x^6 + 1050*A*a*b^3*x^6 - 63*C*a^4*x^4 + 189*B*a^3*b*x
^4 - 378*A*a^2*b^2*x^4 - 45*B*a^4*x^2 + 135*A*a^3*b*x^2 - 35*A*a^4)/(a^7*x
^9)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{10}\left(a+bx^2\right)^3}\,dx=\int \frac{A+B\,x^2+C\,x^4+x^6\,D}{x^{10}\,\left(b\,x^2+a\right)^3}\,dx$$

input
```
int((A + B*x^2 + C*x^4 + x^6*D)/(x^10*(a + b*x^2)^3),x)
```

output
```
int((A + B*x^2 + C*x^4 + x^6*D)/(x^10*(a + b*x^2)^3), x)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 434, normalized size of antiderivative = 1.54

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{10}\left(a+bx^2\right)^3}\,dx$$
$$=\frac{160a^6b\,x^2-504a^6c\,x^4-840a^6d\,x^6-352a^5b^2x^4+1056a^4b^3x^6-7392a^3b^4x^8-23100a^2b^5x^{10}-13860a\,b^6}{}$$

input
```
int((D*x^6+C*x^4+B*x^2+A)/x^10/(b*x^2+a)^3,x)
```

output

```
(11025*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**4*b*d*x**9 - 19845
*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**3*b**2*c*x**9 + 22050*sq
rt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**3*b**2*d*x**11 - 13860*sqrt
(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2*b**4*x**9 - 39690*sqrt(b)*s
qrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2*b**3*c*x**11 + 11025*sqrt(b)*sqr
t(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2*b**3*d*x**13 - 27720*sqrt(b)*sqrt(
a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*b**5*x**11 - 19845*sqrt(b)*sqrt(a)*atan
((b*x)/(sqrt(b)*sqrt(a)))*a*b**4*c*x**13 - 13860*sqrt(b)*sqrt(a)*atan((b*x
)/(sqrt(b)*sqrt(a)))*b**6*x**13 - 280*a**7 + 160*a**6*b*x**2 - 504*a**6*c*
x**4 - 840*a**6*d*x**6 - 352*a**5*b**2*x**4 + 1512*a**5*b*c*x**6 + 5880*a*
*5*b*d*x**8 + 1056*a**4*b**3*x**6 - 10584*a**4*b**2*c*x**8 + 18375*a**4*b*
*2*d*x**10 - 7392*a**3*b**4*x**8 - 33075*a**3*b**3*c*x**10 + 11025*a**3*b*
*3*d*x**12 - 23100*a**2*b**5*x**10 - 19845*a**2*b**4*c*x**12 - 13860*a*b**
6*x**12)/(2520*a**7*x**9*(a**2 + 2*a*b*x**2 + b**2*x**4))
```

# 3.189 $\int x^2\sqrt{a+bx^2}\left(A+Bx^2+Cx^4+Dx^6\right)\,dx$



## Optimal result

Integrand size = 32, antiderivative size = 245

$$\begin{aligned}\int x^2\sqrt{a+bx^2}\left(A+Bx^2+Cx^4+Dx^6\right)\,dx &= \frac{a(32Ab^3-a(16b^2B-10abC+7a^2D))\,x\sqrt{a+bx^2}}{256b^4} \\ &+\frac{1}{128}\left(32A-\frac{a(16b^2B-10abC+7a^2D)}{b^3}\right)x^3\sqrt{a+bx^2} \\ &+\frac{(16b^2B-10abC+7a^2D)\,x^3(a+bx^2)^{3/2}}{96b^3}+\frac{(10bC-7aD)x^5(a+bx^2)^{3/2}}{80b^2} \\ &+\frac{Dx^7(a+bx^2)^{3/2}}{10b}-\frac{a^2(32Ab^3-a(16b^2B-10abC+7a^2D))\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{256b^{9/2}}\end{aligned}$$

output
```
1/256*a*(32*A*b^3-a*(16*B*b^2-10*C*a*b+7*D*a^2))*x*(b*x^2+a)^(1/2)/b^4+1/1
28*(32*A-a*(16*B*b^2-10*C*a*b+7*D*a^2)/b^3)*x^3*(b*x^2+a)^(1/2)+1/96*(16*B
*b^2-10*C*a*b+7*D*a^2)*x^3*(b*x^2+a)^(3/2)/b^3+1/80*(10*C*b-7*D*a)*x^5*(b*
x^2+a)^(3/2)/b^2+1/10*D*x^7*(b*x^2+a)^(3/2)/b-1/256*a^2*(32*A*b^3-a*(16*B*
b^2-10*C*a*b+7*D*a^2))*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(9/2)
```

## Mathematica [A] (verified)

Time = 1.44 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.79

$$\int x^2\sqrt{a+bx^2}\left(A+Bx^2+Cx^4+Dx^6\right)\,dx$$
$$=\frac{\sqrt{b}x\sqrt{a+bx^2}(-105a^4D+10a^3b(15C+7Dx^2)-4a^2b^2(60B+25Cx^2+14Dx^4)+16ab^3(30A+10Bx^2}{}$$

input
```
Integrate[x^2*Sqrt[a + b*x^2]*(A + B*x^2 + C*x^4 + D*x^6),x]
```

output
```
(Sqrt[b]*x*Sqrt[a + b*x^2]*(-105*a^4*D + 10*a^3*b*(15*C + 7*D*x^2) - 4*a^2
*b^2*(60*B + 25*C*x^2 + 14*D*x^4) + 16*a*b^3*(30*A + 10*B*x^2 + 5*C*x^4 +
3*D*x^6) + 32*b^4*x^2*(30*A + 20*B*x^2 + 15*C*x^4 + 12*D*x^6)) + 30*a^2*(-
32*A*b^3 + a*(16*b^2*B - 10*a*b*C + 7*a^2*D))*ArcTanh[(Sqrt[b]*x)/(-Sqrt[a
] + Sqrt[a + b*x^2])])/(3840*b^(9/2))
```

## Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.88, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2340, 1590, 27, 363, 248, 262, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{a+bx^2}\left(A+Bx^2+Cx^4+Dx^6\right)\,dx$$

$$\Big\downarrow \text{2340}$$

$$\frac{\int x^2\sqrt{bx^2+a}\left((10bC-7aD)x^4+10bBx^2+10Ab\right)dx}{10b}+\frac{Dx^7\left(a+bx^2\right)^{3/2}}{10b}$$

$$\Big\downarrow \text{1590}$$

$$\frac{\frac{\int 5x^2\sqrt{bx^2+a}\left(16Ab^2+\left(7Da^2-10bCa+16b^2B\right)x^2\right)dx}{8b}+\frac{x^5\left(a+bx^2\right)^{3/2}(10bC-7aD)}{8b}}{10b}+\frac{Dx^7\left(a+bx^2\right)^{3/2}}{10b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{5\int x^2\sqrt{bx^2+a}\left(16Ab^2+\left(7Da^2-10bCa+16b^2B\right)x^2\right)dx}{8b}+\frac{x^5\left(a+bx^2\right)^{3/2}(10bC-7aD)}{8b}}{10b}+\frac{Dx^7\left(a+bx^2\right)^{3/2}}{10b}$$

$\downarrow$ 363

$$\frac{\frac{5\left(\frac{\left(32Ab^3-a\left(7a^2D-10abC+16b^2B\right)\right)\int x^2\sqrt{bx^2+a}dx}{2b}+\frac{x^3\left(a+bx^2\right)^{3/2}\left(7a^2D-10abC+16b^2B\right)}{6b}\right)}{8b}+\frac{x^5\left(a+bx^2\right)^{3/2}(10bC-7aD)}{8b}}{10b}+\frac{Dx^7\left(a+bx^2\right)^{3/2}}{10b}$$

$\downarrow$ 248

$$\frac{\frac{5\left(\frac{\left(32Ab^3-a\left(7a^2D-10abC+16b^2B\right)\right)\left(\frac{1}{4}a\int\frac{x^2}{\sqrt{bx^2+a}}dx+\frac{1}{4}x^3\sqrt{a+bx^2}\right)}{2b}+\frac{x^3\left(a+bx^2\right)^{3/2}\left(7a^2D-10abC+16b^2B\right)}{6b}\right)}{8b}+\frac{x^5\left(a+bx^2\right)^{3/2}(10bC-7aD)}{8b}}{10b}+\frac{Dx^7\left(a+bx^2\right)^{3/2}}{10b}$$

$\downarrow$ 262

$$\frac{\frac{5\left(\frac{\left(32Ab^3-a\left(7a^2D-10abC+16b^2B\right)\right)\left(\frac{1}{4}a\left(\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\int\frac{1}{\sqrt{bx^2+a}}dx}{2b}\right)+\frac{1}{4}x^3\sqrt{a+bx^2}\right)}{2b}+\frac{x^3\left(a+bx^2\right)^{3/2}\left(7a^2D-10abC+16b^2B\right)}{6b}\right)}{8b}+\frac{x^5\left(a+bx^2\right)^{3/2}}{8b}}{10b}+\frac{Dx^7\left(a+bx^2\right)^{3/2}}{10b}$$

$\downarrow$ 224

$$\frac{\frac{5\left(\frac{\left(32Ab^3-a\left(7a^2D-10abC+16b^2B\right)\right)\left(\frac{1}{4}a\left(\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2b}\right)+\frac{1}{4}x^3\sqrt{a+bx^2}\right)}{2b}+\frac{x^3\left(a+bx^2\right)^{3/2}\left(7a^2D-10abC+16b^2B\right)}{6b}\right)}{8b}+\frac{x^5(a+}{}}{10b}+\frac{Dx^7\left(a+bx^2\right)^{3/2}}{10b}$$

$\downarrow$ 219

$$\frac{5\left(\frac{\left(\frac{1}{4}a\left(\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}\right)+\frac{1}{4}x^3\sqrt{a+bx^2}\right)\left(32Ab^3-a\left(7a^2D-10abC+16b^2B\right)\right)}{2b}+\frac{x^3\left(a+bx^2\right)^{3/2}\left(7a^2D-10abC+16b^2B\right)}{6b}\right)}{8b} + \frac{x^5(a+}{}}{10b}$$

$$\frac{Dx^7\left(a+bx^2\right)^{3/2}}{10b}$$

input
```
Int[x^2*Sqrt[a + b*x^2]*(A + B*x^2 + C*x^4 + D*x^6),x]
```

output
```
(D*x^7*(a + b*x^2)^(3/2))/(10*b) + (((10*b*C - 7*a*D)*x^5*(a + b*x^2)^(3/2
))/(8*b) + (5*(((16*b^2*B - 10*a*b*C + 7*a^2*D)*x^3*(a + b*x^2)^(3/2))/(6*
b) + ((32*A*b^3 - a*(16*b^2*B - 10*a*b*C + 7*a^2*D))*((x^3*Sqrt[a + b*x^2]
)/4 + (a*((x*Sqrt[a + b*x^2])/(2*b) - (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^
2]])/(2*b^(3/2))))/4))/(2*b)))/(8*b))/(10*b)
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 1590
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[c^p*(f*x)^(m + 4*p - 1)*((d + e*x^2)^
(q + 1)/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1))), x] + Simp[1/(e*(m + 4*p + 2*q
 + 1))   Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a +
b*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x],
x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p,
0] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

rule 2340
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Simp[1/(b*(m
+ q + 2*p + 1))   Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)
*Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /;
GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ
[Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

## Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.69

| method | result |
|---|---|
| pseudoelliptic | $\frac{-\left(b^3 A-\frac{1}{2} a b^2 B+\frac{5}{16} a^2 b C-\frac{7}{32} a^3 D\right) a^2 \operatorname{arctanh}\left(\frac{\sqrt{b x^2+a}}{x \sqrt{b}}\right)+\sqrt{b x^2+a}\left(a\left(\frac{1}{10} D x^6+\frac{1}{6} C x^4+\frac{1}{3} x^2 B+A\right) b^{\frac{7}{2}}+\left(\frac{4}{5} D x^8+C x^6\right.\right.}{8 b^{\frac{9}{2}}}$ |
| default | $A\left(\frac{x\left(b x^2+a\right)^{\frac{3}{2}}}{4 b}-\frac{a\left(\frac{x \sqrt{b x^2+a}}{2}+\frac{a \ln\left(\sqrt{b}\, x+\sqrt{b x^2+a}\right)}{2 \sqrt{b}}\right)}{4 b}\right)+B\left(\frac{x^3\left(b x^2+a\right)^{\frac{3}{2}}}{6 b}-\frac{a\left(\frac{x\left(b x^2+a\right)^{\frac{3}{2}}}{4 b}-\frac{a\left(\frac{x \sqrt{b x^2+a}}{2}\right.}{}\right.}{2 b}\right.$ |

input
```
int(x^2*(b*x^2+a)^(1/2)*(D*x^6+C*x^4+B*x^2+A),x,method=_RETURNVERBOSE)
```

output
```
1/8/b^(9/2)*(-(b^3*A-1/2*a*b^2*B+5/16*a^2*b*C-7/32*a^3*D)*a^2*arctanh((b*x
^2+a)^(1/2)/x/b^(1/2))+(b*x^2+a)^(1/2)*(a*(1/10*D*x^6+1/6*C*x^4+1/3*x^2*B+
A)*b^(7/2)+(4/5*D*x^8+C*x^6+4/3*B*x^4+2*A*x^2)*b^(9/2)-7/32*((8/15*D*x^4+2
0/21*C*x^2+16/7*B)*b^(5/2)+((-2/3*D*x^2-10/7*C)*b^(3/2)+D*a*b^(1/2))*a)*a^
2)*x)
```

## Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 412, normalized size of antiderivative = 1.68

$$\int x^2\sqrt{a+bx^2}\left(A+Bx^2+Cx^4+Dx^6\right)\,dx$$

$$= \left[-\frac{15\,(7\,Da^5-10\,Ca^4b+16\,Ba^3b^2-32\,Aa^2b^3)\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\,(384\,Db^5x^9\,\cdots}{\cdots}\right.$$

$$-\frac{15\,(7\,Da^5-10\,Ca^4b+16\,Ba^3b^2-32\,Aa^2b^3)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(384\,Db^5x^9+48\,(Dab^4+10\,Cb\,\cdots}{\cdots}$$

input
```
integrate(x^2*(b*x^2+a)^(1/2)*(D*x^6+C*x^4+B*x^2+A),x, algorithm="fricas")
```

output
```
[-1/7680*(15*(7*D*a^5 - 10*C*a^4*b + 16*B*a^3*b^2 - 32*A*a^2*b^3)*sqrt(b)*
log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(384*D*b^5*x^9 + 48*(D
*a*b^4 + 10*C*b^5)*x^7 - 8*(7*D*a^2*b^3 - 10*C*a*b^4 - 80*B*b^5)*x^5 + 10*
(7*D*a^3*b^2 - 10*C*a^2*b^3 + 16*B*a*b^4 + 96*A*b^5)*x^3 - 15*(7*D*a^4*b -
 10*C*a^3*b^2 + 16*B*a^2*b^3 - 32*A*a*b^4)*x)*sqrt(b*x^2 + a))/b^5, -1/384
0*(15*(7*D*a^5 - 10*C*a^4*b + 16*B*a^3*b^2 - 32*A*a^2*b^3)*sqrt(-b)*arctan
(sqrt(-b)*x/sqrt(b*x^2 + a)) - (384*D*b^5*x^9 + 48*(D*a*b^4 + 10*C*b^5)*x^
7 - 8*(7*D*a^2*b^3 - 10*C*a*b^4 - 80*B*b^5)*x^5 + 10*(7*D*a^3*b^2 - 10*C*a
^2*b^3 + 16*B*a*b^4 + 96*A*b^5)*x^3 - 15*(7*D*a^4*b - 10*C*a^3*b^2 + 16*B*
a^2*b^3 - 32*A*a*b^4)*x)*sqrt(b*x^2 + a))/b^5]
```

## Sympy [A] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.14

$$\int x^2\sqrt{a+bx^2}\left(A+Bx^2+Cx^4+Dx^6\right)\,dx$$
$$= \begin{cases} -\frac{a\left(Aa-\frac{3a\left(Ab+Ba-\frac{5a\left(Bb+Ca-\frac{7a\left(Cb+\frac{Da}{10}\right)}{8b}\right)}{6b}\right)}{4b}\right)\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right)}{2b}+\sqrt{a+bx^2}\left(\frac{Dx^9}{10}+\frac{x^7\left(Cb-\right.}{8b}\right. \\ \sqrt{a}\left(\frac{Ax^3}{3}+\frac{Bx^5}{5}+\frac{Cx^7}{7}+\frac{Dx^9}{9}\right) \end{cases}$$

input
```
integrate(x**2*(b*x**2+a)**(1/2)*(D*x**6+C*x**4+B*x**2+A),x)
```

output
```
Piecewise((-a*(A*a - 3*a*(A*b + B*a - 5*a*(B*b + C*a - 7*a*(C*b + D*a/10)/
(8*b))/(6*b))/(4*b))*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sq
rt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True))/(2*b) + sqrt(a + b*x**2)*
(D*x**9/10 + x**7*(C*b + D*a/10)/(8*b) + x**5*(B*b + C*a - 7*a*(C*b + D*a/
10)/(8*b))/(6*b) + x**3*(A*b + B*a - 5*a*(B*b + C*a - 7*a*(C*b + D*a/10)/(
8*b))/(6*b))/(4*b) + x*(A*a - 3*a*(A*b + B*a - 5*a*(B*b + C*a - 7*a*(C*b +
 D*a/10)/(8*b))/(6*b))/(4*b))/(2*b)), Ne(b, 0)), (sqrt(a)*(A*x**3/3 + B*x*
*5/5 + C*x**7/7 + D*x**9/9), True))
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.35

$$\int x^2\sqrt{a+bx^2}\left(A+Bx^2+Cx^4+Dx^6\right)\,dx = \frac{\left(bx^2+a\right)^{\frac{3}{2}}Dx^7}{10\,b} - \frac{7\left(bx^2+a\right)^{\frac{3}{2}}Dax^5}{80\,b^2} + \frac{\left(bx^2+a\right)^{\frac{3}{2}}Cx^5}{8\,b} + \frac{7\left(bx^2+a\right)^{\frac{3}{2}}Da^2x^3}{96\,b^3} - \frac{5\left(bx^2+a\right)^{\frac{3}{2}}Cax^3}{48\,b^2} + \frac{\left(bx^2+a\right)^{\frac{3}{2}}Bx^3}{6\,b} - \frac{7\left(bx^2+a\right)^{\frac{3}{2}}Da^3x}{128\,b^4} + \frac{7\sqrt{bx^2+a}Da^4x}{256\,b^4} + \frac{5\left(bx^2+a\right)^{\frac{3}{2}}Ca^2x}{64\,b^3} - \frac{5\sqrt{bx^2+a}Ca^3x}{128\,b^3} - \frac{\left(bx^2+a\right)^{\frac{3}{2}}Bax}{8\,b^2} + \frac{\sqrt{bx^2+a}Ba^2x}{16\,b^2} + \frac{\left(bx^2+a\right)^{\frac{3}{2}}Ax}{4\,b} - \frac{\sqrt{bx^2+a}Aax}{8\,b} + \frac{7\,Da^5\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{256\,b^{\frac{9}{2}}} - \frac{5\,Ca^4\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\,b^{\frac{7}{2}}} + \frac{Ba^3\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{5}{2}}} - \frac{Aa^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{3}{2}}}$$

input
```
integrate(x^2*(b*x^2+a)^(1/2)*(D*x^6+C*x^4+B*x^2+A),x, algorithm="maxima")
```

output
```
1/10*(b*x^2 + a)^(3/2)*D*x^7/b - 7/80*(b*x^2 + a)^(3/2)*D*a*x^5/b^2 + 1/8*
(b*x^2 + a)^(3/2)*C*x^5/b + 7/96*(b*x^2 + a)^(3/2)*D*a^2*x^3/b^3 - 5/48*(b
*x^2 + a)^(3/2)*C*a*x^3/b^2 + 1/6*(b*x^2 + a)^(3/2)*B*x^3/b - 7/128*(b*x^2
 + a)^(3/2)*D*a^3*x/b^4 + 7/256*sqrt(b*x^2 + a)*D*a^4*x/b^4 + 5/64*(b*x^2
+ a)^(3/2)*C*a^2*x/b^3 - 5/128*sqrt(b*x^2 + a)*C*a^3*x/b^3 - 1/8*(b*x^2 +
a)^(3/2)*B*a*x/b^2 + 1/16*sqrt(b*x^2 + a)*B*a^2*x/b^2 + 1/4*(b*x^2 + a)^(3
/2)*A*x/b - 1/8*sqrt(b*x^2 + a)*A*a*x/b + 7/256*D*a^5*arcsinh(b*x/sqrt(a*b
))/b^(9/2) - 5/128*C*a^4*arcsinh(b*x/sqrt(a*b))/b^(7/2) + 1/16*B*a^3*arcsi
nh(b*x/sqrt(a*b))/b^(5/2) - 1/8*A*a^2*arcsinh(b*x/sqrt(a*b))/b^(3/2)
```

## Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.88

$$\int x^2\sqrt{a+bx^2}\left(A+Bx^2+Cx^4+Dx^6\right)\,dx$$
$$= \frac{1}{3840}\left(2\left(4\left(6\left(8\,Dx^2+\frac{Dab^7+10\,Cb^8}{b^8}\right)x^2-\frac{7\,Da^2b^6-10\,Cab^7-80\,Bb^8}{b^8}\right)x^2+\frac{5\,(7\,Da^3b^5-10\,Ca^2\right.\right.$$
$$-\frac{(7\,Da^5-10\,Ca^4b+16\,Ba^3b^2-32\,Aa^2b^3)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{256\,b^{\frac{9}{2}}}$$

input
```
integrate(x^2*(b*x^2+a)^(1/2)*(D*x^6+C*x^4+B*x^2+A),x, algorithm="giac")
```

output
```
1/3840*(2*(4*(6*(8*D*x^2 + (D*a*b^7 + 10*C*b^8)/b^8)*x^2 - (7*D*a^2*b^6 -
10*C*a*b^7 - 80*B*b^8)/b^8)*x^2 + 5*(7*D*a^3*b^5 - 10*C*a^2*b^6 + 16*B*a*b
^7 + 96*A*b^8)/b^8)*x^2 - 15*(7*D*a^4*b^4 - 10*C*a^3*b^5 + 16*B*a^2*b^6 -
32*A*a*b^7)/b^8)*sqrt(b*x^2 + a)*x - 1/256*(7*D*a^5 - 10*C*a^4*b + 16*B*a^
3*b^2 - 32*A*a^2*b^3)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(9/2)
```

## Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt{a+bx^2}\left(A+Bx^2+Cx^4+Dx^6\right)\,dx=\int x^2\,\sqrt{b\,x^2+a}\,\left(A+B\,x^2+C\,x^4+x^6\,D\right)\,dx$$

input
```
int(x^2*(a + b*x^2)^(1/2)*(A + B*x^2 + C*x^4 + x^6*D),x)
```

output
```
int(x^2*(a + b*x^2)^(1/2)*(A + B*x^2 + C*x^4 + x^6*D), x)
```

## Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.24

$$\int x^2\sqrt{a+bx^2}\left(A+Bx^2+Cx^4+Dx^6\right)\,dx$$
$$=\frac{-105\sqrt{b\,x^2+a}\,a^4bdx+150\sqrt{b\,x^2+a}\,a^3b^2cx+70\sqrt{b\,x^2+a}\,a^3b^2d\,x^3+240\sqrt{b\,x^2+a}\,a^2b^4x-100\sqrt{b\,x^2}}{}$$

input
```
int(x^2*(b*x^2+a)^(1/2)*(D*x^6+C*x^4+B*x^2+A),x)
```

output
```
( - 105*sqrt(a + b*x**2)*a**4*b*d*x + 150*sqrt(a + b*x**2)*a**3*b**2*c*x +
 70*sqrt(a + b*x**2)*a**3*b**2*d*x**3 + 240*sqrt(a + b*x**2)*a**2*b**4*x -
 100*sqrt(a + b*x**2)*a**2*b**3*c*x**3 - 56*sqrt(a + b*x**2)*a**2*b**3*d*x
**5 + 1120*sqrt(a + b*x**2)*a*b**5*x**3 + 80*sqrt(a + b*x**2)*a*b**4*c*x**
5 + 48*sqrt(a + b*x**2)*a*b**4*d*x**7 + 640*sqrt(a + b*x**2)*b**6*x**5 + 4
80*sqrt(a + b*x**2)*b**5*c*x**7 + 384*sqrt(a + b*x**2)*b**5*d*x**9 + 105*s
qrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**5*d - 150*sqrt(b)*lo
g((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**4*b*c - 240*sqrt(b)*log((sqrt
(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**3*b**3)/(3840*b**5)
```

# 3.190 $\int \sqrt{a+bx^2}(A+Bx^2+Cx^4+Dx^6)\,dx$



## Optimal result

Integrand size = 29, antiderivative size = 191

$$\begin{aligned}\int \sqrt{a+bx^2}\left(A+Bx^2+Cx^4+Dx^6\right)\,dx &= \frac{1}{128}\left(64A-\frac{a(16b^2B-8abC+5a^2D)}{b^3}\right)x\sqrt{a+bx^2}\\ &\quad+\frac{(16b^2B-8abC+5a^2D)\,x(a+bx^2)^{3/2}}{64b^3}+\frac{(8bC-5aD)x^3(a+bx^2)^{3/2}}{48b^2}\\ &\quad+\frac{Dx^5(a+bx^2)^{3/2}}{8b}+\frac{a(64Ab^3-a(16b^2B-8abC+5a^2D))\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{7/2}}\end{aligned}$$

output
```
1/128*(64*A-a*(16*B*b^2-8*C*a*b+5*D*a^2)/b^3)*x*(b*x^2+a)^(1/2)+1/64*(16*B
*b^2-8*C*a*b+5*D*a^2)*x*(b*x^2+a)^(3/2)/b^3+1/48*(8*C*b-5*D*a)*x^3*(b*x^2+
a)^(3/2)/b^2+1/8*D*x^5*(b*x^2+a)^(3/2)/b+1/128*a*(64*A*b^3-a*(16*B*b^2-8*C
*a*b+5*D*a^2))*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(7/2)
```

## Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.80

$$\int \sqrt{a+bx^2}\left(A+Bx^2+Cx^4+Dx^6\right) \, dx$$
$$= \frac{\sqrt{b}x\sqrt{a+bx^2}(192Ab^3+15a^3D-2a^2b(12C+5Dx^2)+8ab^2(6B+2Cx^2+Dx^4)+16b^3x^2(6B+4Cx^2}{384b^{7/2}}$$

input
```
Integrate[Sqrt[a + b*x^2]*(A + B*x^2 + C*x^4 + D*x^6),x]
```

output
```
(Sqrt[b]*x*Sqrt[a + b*x^2]*(192*A*b^3 + 15*a^3*D - 2*a^2*b*(12*C + 5*D*x^2
) + 8*a*b^2*(6*B + 2*C*x^2 + D*x^4) + 16*b^3*x^2*(6*B + 4*C*x^2 + 3*D*x^4)
) + 3*a*(-64*A*b^3 + a*(16*b^2*B - 8*a*b*C + 5*a^2*D))*Log[-(Sqrt[b]*x) +
Sqrt[a + b*x^2]])/(384*b^(7/2))
```

## Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2346, 1473, 27, 299, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx^2}\left(A+Bx^2+Cx^4+Dx^6\right) \, dx$$

$$\Big\downarrow \text{2346}$$

$$\frac{\int \sqrt{bx^2+a}\left((8bC-5aD)x^4+8bBx^2+8Ab\right)dx}{8b}+\frac{Dx^5\left(a+bx^2\right)^{3/2}}{8b}$$

$$\Big\downarrow \text{1473}$$

$$\frac{\frac{\int 3\sqrt{bx^2+a}\left(16Ab^2+\left(5Da^2-8bCa+16b^2B\right)x^2\right)dx}{6b}+\frac{x^3\left(a+bx^2\right)^{3/2}(8bC-5aD)}{6b}}{8b}+\frac{Dx^5\left(a+bx^2\right)^{3/2}}{8b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \sqrt{bx^2+a}(16Ab^2+(5Da^2-8bCa+16b^2B)x^2)dx}{2b}+\frac{x^3(a+bx^2)^{3/2}(8bC-5aD)}{6b}}{8b}+\frac{Dx^5(a+bx^2)^{3/2}}{8b}$$

$\Big\downarrow$ 299

$$\frac{\frac{\frac{\left(64Ab^3-a\left(5a^2D-8abC+16b^2B\right)\right)\int \sqrt{bx^2+a}dx}{4b}+\frac{x\left(a+bx^2\right)^{3/2}\left(5a^2D-8abC+16b^2B\right)}{4b}}{2b}+\frac{x^3(a+bx^2)^{3/2}(8bC-5aD)}{6b}}{8b}+\frac{Dx^5(a+bx^2)^{3/2}}{8b}$$

$\Big\downarrow$ 211

$$\frac{\frac{\frac{\left(64Ab^3-a\left(5a^2D-8abC+16b^2B\right)\right)\left(\frac{1}{2}a\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)}{4b}+\frac{x\left(a+bx^2\right)^{3/2}\left(5a^2D-8abC+16b^2B\right)}{4b}}{2b}+\frac{x^3(a+bx^2)^{3/2}(8bC-5aD)}{6b}}{8b}+\frac{Dx^5(a+bx^2)^{3/2}}{8b}$$

$\Big\downarrow$ 224

$$\frac{\frac{\frac{\left(64Ab^3-a\left(5a^2D-8abC+16b^2B\right)\right)\left(\frac{1}{2}a\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)}{4b}+\frac{x\left(a+bx^2\right)^{3/2}\left(5a^2D-8abC+16b^2B\right)}{4b}}{2b}+\frac{x^3(a+bx^2)^{3/2}(8bC-5aD)}{6b}}{8b}+\frac{Dx^5(a+bx^2)^{3/2}}{8b}$$

$\Big\downarrow$ 219

$$\frac{\frac{\frac{\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)\left(64Ab^3-a\left(5a^2D-8abC+16b^2B\right)\right)}{4b}+\frac{x\left(a+bx^2\right)^{3/2}\left(5a^2D-8abC+16b^2B\right)}{4b}}{2b}+\frac{x^3(a+bx^2)^{3/2}(8bC-5aD)}{6b}}{8b}+\frac{Dx^5(a+bx^2)^{3/2}}{8b}$$

input
```
Int[Sqrt[a + b*x^2]*(A + B*x^2 + C*x^4 + D*x^6),x]
```

output
```
(D*x^5*(a + b*x^2)^(3/2))/(8*b) + (((8*b*C - 5*a*D)*x^3*(a + b*x^2)^(3/2))
/(6*b) + (((16*b^2*B - 8*a*b*C + 5*a^2*D)*x*(a + b*x^2)^(3/2))/(4*b) + ((6
4*A*b^3 - a*(16*b^2*B - 8*a*b*C + 5*a^2*D))*((x*Sqrt[a + b*x^2])/2 + (a*Ar
cTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b])))/(4*b))/(2*b))/(8*b)
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1473
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[c^p*x^(4*p - 1)*((d + e*x^2)^(q + 1)/(e*(4*p + 2*q + 1)))
, x] + Simp[1/(e*(4*p + 2*q + 1))   Int[(d + e*x^2)^q*ExpandToSum[e*(4*p +
2*q + 1)*(a + b*x^2 + c*x^4)^p - d*c^p*(4*p - 1)*x^(4*p - 2) - e*c^p*(4*p +
 2*q + 1)*x^(4*p), x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] &&  !LtQ[q, -1]
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

## Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.71

| method | result |
|---|---|
| pseudoelliptic | $\frac{a\left(b^{3}A-\frac{1}{4}a\,b^{2}B+\frac{1}{8}a^{2}bC-\frac{5}{64}a^{3}D\right)\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{x\sqrt{b}}\right)+\sqrt{b\,x^{2}+a}\,x\left(\left(\frac{1}{4}Dx^{6}+\frac{1}{3}C\,x^{4}+\frac{1}{2}x^{2}B+A\right)b^{\frac{7}{2}}+\frac{5\left(\left(\frac{8}{15}Dx^{4}+\frac{16}{15}C\,x^{2}\cdots\right.\right.}{\cdots}\right.}{2b^{\frac{7}{2}}}$ |
| default | $A\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)+C\left(\frac{x^{3}\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{6b}-\frac{a\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{4b}-\frac{a\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+\cdots}\right)}{2\sqrt{b}}\right)}{4b}\cdots\right.}{2b}\cdots\right.$ |

input
```
int((b*x^2+a)^(1/2)*(D*x^6+C*x^4+B*x^2+A),x,method=_RETURNVERBOSE)
```

output
```
1/2*(a*(b^3*A-1/4*a*b^2*B+1/8*a^2*b*C-5/64*a^3*D)*arctanh((b*x^2+a)^(1/2)/
x/b^(1/2))+(b*x^2+a)^(1/2)*x*((1/4*D*x^6+1/3*C*x^4+1/2*x^2*B+A)*b^(7/2)+5/
64*((8/15*D*x^4+16/15*C*x^2+16/5*B)*b^(5/2)+a*((-2/3*D*x^2-8/5*C)*b^(3/2)+
D*a*b^(1/2)))*a))/b^(7/2)
```

## Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.71

$$\int \sqrt{a+bx^2}\left(A+Bx^2+Cx^4+Dx^6\right)\,dx$$
$$=\left[-\frac{3\left(5\,Da^4-8\,Ca^3b+16\,Ba^2b^2-64\,Aab^3\right)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\left(48\,Db^4x^7+8\left(\right.\right.}{}\right.$$

input
```
integrate((b*x^2+a)^(1/2)*(D*x^6+C*x^4+B*x^2+A),x, algorithm="fricas")
```

output
```
[-1/768*(3*(5*D*a^4 - 8*C*a^3*b + 16*B*a^2*b^2 - 64*A*a*b^3)*sqrt(b)*log(-
2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(48*D*b^4*x^7 + 8*(D*a*b^3
+ 8*C*b^4)*x^5 - 2*(5*D*a^2*b^2 - 8*C*a*b^3 - 48*B*b^4)*x^3 + 3*(5*D*a^3*b
 - 8*C*a^2*b^2 + 16*B*a*b^3 + 64*A*b^4)*x)*sqrt(b*x^2 + a))/b^4, 1/384*(3*
(5*D*a^4 - 8*C*a^3*b + 16*B*a^2*b^2 - 64*A*a*b^3)*sqrt(-b)*arctan(sqrt(-b)
*x/sqrt(b*x^2 + a)) + (48*D*b^4*x^7 + 8*(D*a*b^3 + 8*C*b^4)*x^5 - 2*(5*D*a
^2*b^2 - 8*C*a*b^3 - 48*B*b^4)*x^3 + 3*(5*D*a^3*b - 8*C*a^2*b^2 + 16*B*a*b
^3 + 64*A*b^4)*x)*sqrt(b*x^2 + a))/b^4]
```

## Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.13

$$\int \sqrt{a+bx^2}\left(A+Bx^2+Cx^4+Dx^6\right)\,dx$$
$$=\begin{cases}\sqrt{a+bx^2}\left(\frac{Dx^7}{8}+\frac{x^5\left(Cb+\frac{Da}{8}\right)}{6b}+\frac{x^3\left(Bb+Ca-\frac{5a\left(Cb+\frac{Da}{8}\right)}{6b}\right)}{4b}+\frac{x\left(Ab+Ba-\frac{3a\left(Bb+Ca-\frac{5a\left(Cb+\frac{Da}{8}\right)}{6b}\right)}{4b}\right)}{2b}\right)+\left(Aa-\frac{a\left(\right.}{}\right. \\ \sqrt{a}\left(Ax+\frac{Bx^3}{3}+\frac{Cx^5}{5}+\frac{Dx^7}{7}\right)\end{cases}$$

input
```
integrate((b*x**2+a)**(1/2)*(D*x**6+C*x**4+B*x**2+A),x)
```

output
```
Piecewise((sqrt(a + b*x**2)*(D*x**7/8 + x**5*(C*b + D*a/8)/(6*b) + x**3*(B
*b + C*a - 5*a*(C*b + D*a/8)/(6*b))/(4*b) + x*(A*b + B*a - 3*a*(B*b + C*a
- 5*a*(C*b + D*a/8)/(6*b))/(4*b))/(2*b)) + (A*a - a*(A*b + B*a - 3*a*(B*b
+ C*a - 5*a*(C*b + D*a/8)/(6*b))/(4*b))/(2*b))*Piecewise((log(2*sqrt(b)*sq
rt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True))
, Ne(b, 0)), (sqrt(a)*(A*x + B*x**3/3 + C*x**5/5 + D*x**7/7), True))
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.29

$$\int \sqrt{a+bx^2}\left(A+Bx^2+Cx^4+Dx^6\right)\,dx = \frac{(bx^2+a)^{\frac{3}{2}}Dx^5}{8\,b} - \frac{5\,(bx^2+a)^{\frac{3}{2}}Dax^3}{48\,b^2} + \frac{(bx^2+a)^{\frac{3}{2}}Cx^3}{6\,b} + \frac{1}{2}\,\sqrt{bx^2+a}Ax + \frac{5\,(bx^2+a)^{\frac{3}{2}}Da^2x}{64\,b^3} - \frac{5\,\sqrt{bx^2+a}Da^3x}{128\,b^3} - \frac{(bx^2+a)^{\frac{3}{2}}Cax}{8\,b^2} + \frac{\sqrt{bx^2+a}Ca^2x}{16\,b^2} + \frac{(bx^2+a)^{\frac{3}{2}}Bx}{4\,b} - \frac{\sqrt{bx^2+a}Bax}{8\,b} - \frac{5\,Da^4\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\,b^{\frac{7}{2}}} + \frac{Ca^3\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{5}{2}}} - \frac{Ba^2\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{3}{2}}} + \frac{Aa\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{b}}$$

input
```
integrate((b*x^2+a)^(1/2)*(D*x^6+C*x^4+B*x^2+A),x, algorithm="maxima")
```

output
```
1/8*(b*x^2 + a)^(3/2)*D*x^5/b - 5/48*(b*x^2 + a)^(3/2)*D*a*x^3/b^2 + 1/6*(
b*x^2 + a)^(3/2)*C*x^3/b + 1/2*sqrt(b*x^2 + a)*A*x + 5/64*(b*x^2 + a)^(3/2
)*D*a^2*x/b^3 - 5/128*sqrt(b*x^2 + a)*D*a^3*x/b^3 - 1/8*(b*x^2 + a)^(3/2)*
C*a*x/b^2 + 1/16*sqrt(b*x^2 + a)*C*a^2*x/b^2 + 1/4*(b*x^2 + a)^(3/2)*B*x/b
 - 1/8*sqrt(b*x^2 + a)*B*a*x/b - 5/128*D*a^4*arcsinh(b*x/sqrt(a*b))/b^(7/2
) + 1/16*C*a^3*arcsinh(b*x/sqrt(a*b))/b^(5/2) - 1/8*B*a^2*arcsinh(b*x/sqrt
(a*b))/b^(3/2) + 1/2*A*a*arcsinh(b*x/sqrt(a*b))/sqrt(b)
```

## Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.87

$$\int \sqrt{a+bx^2}\left(A+Bx^2+Cx^4+Dx^6\right)\,dx$$
$$=\frac{1}{384}\left(2\left(4\left(6\,Dx^2+\frac{Dab^5+8\,Cb^6}{b^6}\right)x^2-\frac{5\,Da^2b^4-8\,Cab^5-48\,Bb^6}{b^6}\right)x^2+\frac{3\,(5\,Da^3b^3-8\,Ca^2b^4+16}{b^6}\right.$$
$$+\frac{(5\,Da^4-8\,Ca^3b+16\,Ba^2b^2-64\,Aab^3)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{128\,b^{\frac{7}{2}}}$$

input
```
integrate((b*x^2+a)^(1/2)*(D*x^6+C*x^4+B*x^2+A),x, algorithm="giac")
```

output
```
1/384*(2*(4*(6*D*x^2 + (D*a*b^5 + 8*C*b^6)/b^6)*x^2 - (5*D*a^2*b^4 - 8*C*a
*b^5 - 48*B*b^6)/b^6)*x^2 + 3*(5*D*a^3*b^3 - 8*C*a^2*b^4 + 16*B*a*b^5 + 64
*A*b^6)/b^6)*sqrt(b*x^2 + a)*x + 1/128*(5*D*a^4 - 8*C*a^3*b + 16*B*a^2*b^2
 - 64*A*a*b^3)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(7/2)
```

## Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+bx^2}\left(A+Bx^2+Cx^4+Dx^6\right)\,dx=\int \sqrt{b\,x^2+a}\,\left(A+B\,x^2+C\,x^4+x^6\,D\right)\,dx$$

input
```
int((a + b*x^2)^(1/2)*(A + B*x^2 + C*x^4 + x^6*D),x)
```

output
```
int((a + b*x^2)^(1/2)*(A + B*x^2 + C*x^4 + x^6*D), x)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.28

$$\int \sqrt{a+bx^2}\left(A+Bx^2+Cx^4+Dx^6\right)\,dx$$

$$= \frac{15\sqrt{b\,x^2+a}\,a^3bdx-24\sqrt{b\,x^2+a}\,a^2b^2cx-10\sqrt{b\,x^2+a}\,a^2b^2d\,x^3+240\sqrt{b\,x^2+a}\,a\,b^4x+16\sqrt{b\,x^2+a}\,a\,b}{}$$

input
```
int((b*x^2+a)^(1/2)*(D*x^6+C*x^4+B*x^2+A),x)
```

output
```
(15*sqrt(a + b*x**2)*a**3*b*d*x - 24*sqrt(a + b*x**2)*a**2*b**2*c*x - 10*s
qrt(a + b*x**2)*a**2*b**2*d*x**3 + 240*sqrt(a + b*x**2)*a*b**4*x + 16*sqrt
(a + b*x**2)*a*b**3*c*x**3 + 8*sqrt(a + b*x**2)*a*b**3*d*x**5 + 96*sqrt(a
+ b*x**2)*b**5*x**3 + 64*sqrt(a + b*x**2)*b**4*c*x**5 + 48*sqrt(a + b*x**2
)*b**4*d*x**7 - 15*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**
4*d + 24*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**3*b*c + 14
4*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2*b**3)/(384*b**4
)
```

# 3.191 $\int \frac{\sqrt{a+bx^2}(A+Bx^2+Cx^4+Dx^6)}{x^2}\,dx$



## Optimal result

Integrand size = 32, antiderivative size = 170

$$\begin{aligned}\int \frac{\sqrt{a+bx^2}(A+Bx^2+Cx^4+Dx^6)}{x^2}\,dx &= \frac{\left(\frac{16Ab^3}{a}+8b^2B-2abC+a^2D\right)x\sqrt{a+bx^2}}{16b^2}-\frac{A(a+bx^2)^{3/2}}{ax}\\ &+\frac{(2bC-aD)x(a+bx^2)^{3/2}}{8b^2}+\frac{Dx^3(a+bx^2)^{3/2}}{6b}\\ &+\frac{(16Ab^3+a(8b^2B-2abC+a^2D))\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{5/2}}\end{aligned}$$

output
```
1/16*(16*A*b^3/a+8*B*b^2-2*C*a*b+a^2*D)*x*(b*x^2+a)^(1/2)/b^2-A*(b*x^2+a)^
(3/2)/a/x+1/8*(2*C*b-D*a)*x*(b*x^2+a)^(3/2)/b^2+1/6*D*x^3*(b*x^2+a)^(3/2)/
b+1/16*(16*A*b^3+a*(8*B*b^2-2*C*a*b+D*a^2))*arctanh(b^(1/2)*x/(b*x^2+a)^(1
/2))/b^(5/2)
```

## Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2+Cx^4+Dx^6)}{x^2}\,dx = \frac{\sqrt{a+bx^2}(-48Ab^2+x^2(-3a^2D+2ab(3C+Dx^2)+4b^2(6B+3Cx^2+2Dx^4)))}{48b^2x} + \frac{(16Ab^3+a(8b^2B-2abC+a^2D))\,\text{arctanh}\left(\frac{\sqrt{b}x}{-\sqrt{a}+\sqrt{a+bx^2}}\right)}{8b^{5/2}}$$

input
```
Integrate[(Sqrt[a + b*x^2]*(A + B*x^2 + C*x^4 + D*x^6))/x^2,x]
```

output
```
(Sqrt[a + b*x^2]*(-48*A*b^2 + x^2*(-3*a^2*D + 2*a*b*(3*C + D*x^2) + 4*b^2*
(6*B + 3*C*x^2 + 2*D*x^4))))/(48*b^2*x) + ((16*A*b^3 + a*(8*b^2*B - 2*a*b*
C + a^2*D))*ArcTanh[(Sqrt[b]*x)/(-Sqrt[a] + Sqrt[a + b*x^2])])/(8*b^(5/2))
```

## Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.98, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.281$, Rules used = {2338, 9, 25, 1473, 27, 299, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\left(A+Bx^2+Cx^4+Dx^6\right)}{x^2}\,dx$$

$$\Big\downarrow \text{2338}$$

$$-\frac{\int -\frac{\sqrt{bx^2+a}(aDx^5+aCx^3+(2Ab+aB)x)}{x}dx}{a}-\frac{A\left(a+bx^2\right)^{3/2}}{ax}$$

$$\Big\downarrow \text{9}$$

$$-\frac{\int -\sqrt{bx^2+a}\left(aDx^4+aCx^2+2Ab+aB\right)dx}{a}-\frac{A\left(a+bx^2\right)^{3/2}}{ax}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \sqrt{bx^2+a}\left(aDx^4+aCx^2+2Ab+aB\right)dx}{a}-\frac{A\left(a+bx^2\right)^{3/2}}{ax}$$

$$\Big\downarrow \text{1473}$$

$$\frac{\frac{\int 3\sqrt{bx^2+a}\left(a(2bC-aD)x^2+2b(2Ab+aB)\right)dx}{6b}+\frac{aDx^3\left(a+bx^2\right)^{3/2}}{6b}}{a}-\frac{A\left(a+bx^2\right)^{3/2}}{ax}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \sqrt{bx^2+a}\left(a(2bC-aD)x^2+2b(2Ab+aB)\right)dx}{2b}+\frac{aDx^3\left(a+bx^2\right)^{3/2}}{6b}}{a}-\frac{A\left(a+bx^2\right)^{3/2}}{ax}$$

$$\Big\downarrow \text{299}$$

$$\frac{\frac{\frac{\left(8b^2(aB+2Ab)-a^2(2bC-aD)\right)\int \sqrt{bx^2+a}dx}{4b}+\frac{ax\left(a+bx^2\right)^{3/2}(2bC-aD)}{4b}}{2b}+\frac{aDx^3\left(a+bx^2\right)^{3/2}}{6b}}{a}-\frac{A\left(a+bx^2\right)^{3/2}}{ax}$$

$$\Big\downarrow \text{211}$$

$$\frac{\frac{\frac{\left(8b^2(aB+2Ab)-a^2(2bC-aD)\right)\left(\frac{1}{2}a\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)}{4b}+\frac{ax\left(a+bx^2\right)^{3/2}(2bC-aD)}{4b}}{2b}+\frac{aDx^3\left(a+bx^2\right)^{3/2}}{6b}}{a}-\frac{A\left(a+bx^2\right)^{3/2}}{ax}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{\frac{\left(8b^2(aB+2Ab)-a^2(2bC-aD)\right)\left(\frac{1}{2}a\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)}{4b}+\frac{ax\left(a+bx^2\right)^{3/2}(2bC-aD)}{4b}}{2b}+\frac{aDx^3\left(a+bx^2\right)^{3/2}}{6b}}{a}-\frac{A\left(a+bx^2\right)^{3/2}}{ax}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{\frac{\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)\left(8b^2(aB+2Ab)-a^2(2bC-aD)\right)}{4b}+\frac{ax\left(a+bx^2\right)^{3/2}(2bC-aD)}{4b}}{2b}+\frac{aDx^3\left(a+bx^2\right)^{3/2}}{6b}}{a}-\frac{A\left(a+bx^2\right)^{3/2}}{ax}$$

input
```
Int[(Sqrt[a + b*x^2]*(A + B*x^2 + C*x^4 + D*x^6))/x^2,x]
```

output
```
-((A*(a + b*x^2)^(3/2))/(a*x)) + ((a*D*x^3*(a + b*x^2)^(3/2))/(6*b) + ((a*
(2*b*C - a*D)*x*(a + b*x^2)^(3/2))/(4*b) + ((8*b^2*(2*A*b + a*B) - a^2*(2*
b*C - a*D))*((x*Sqrt[a + b*x^2])/2 + (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2
]])/(2*Sqrt[b])))/(4*b))/(2*b))/a
```

## Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1473
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[c^p*x^(4*p - 1)*((d + e*x^2)^(q + 1)/(e*(4*p + 2*q + 1)))
, x] + Simp[1/(e*(4*p + 2*q + 1))   Int[(d + e*x^2)^q*ExpandToSum[e*(4*p +
2*q + 1)*(a + b*x^2 + c*x^4)^p - d*c^p*(4*p - 1)*x^(4*p - 2) - e*c^p*(4*p +
 2*q + 1)*x^(4*p), x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] &&  !LtQ[q, -1]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

## Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.69

| method | result |
|---|---|
| pseudoelliptic | $-\frac{-x\left(b^3A+\frac{1}{2}a\,b^2B-\frac{1}{8}a^2bC+\frac{1}{16}a^3D\right)\operatorname{arctanh}\left(\frac{\sqrt{b\,x^2+a}}{x\sqrt{b}}\right)+\sqrt{b\,x^2+a}\left(\left(-\frac{1}{6}Dx^6-\frac{1}{4}C\,x^4-\frac{1}{2}x^2B+A\right)b^{\frac{5}{2}}+\frac{\left(\left(-\frac{2Dx^2}{3}-2C\right.\right.}{}\right.}{b^{\frac{5}{2}}x}$ |
| default | $B\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)+A\left(-\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}}{ax}+\frac{2b\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)}{a}\right)+C\left(\right.$ |

input
```
int((b*x^2+a)^(1/2)*(D*x^6+C*x^4+B*x^2+A)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-(-x*(b^3*A+1/2*a*b^2*B-1/8*a^2*b*C+1/16*a^3*D)*arctanh((b*x^2+a)^(1/2)/x/
b^(1/2))+(b*x^2+a)^(1/2)*((-1/6*D*x^6-1/4*C*x^4-1/2*x^2*B+A)*b^(5/2)+1/16*
((-2/3*D*x^2-2*C)*b^(3/2)+D*a*b^(1/2))*x^2*a))/b^(5/2)/x
```

## Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.58

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2+Cx^4+Dx^6)}{x^2}\,dx$$
$$= \left[\frac{3\,(Da^3-2\,Ca^2b+8\,Bab^2+16\,Ab^3)\sqrt{b}x\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,(8\,Db^3x^6+2\,(Dab^2+\right.}{96\,b^3x}\right.$$
$$-\frac{3\,(Da^3-2\,Ca^2b+8\,Bab^2+16\,Ab^3)\sqrt{-b}x\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(8\,Db^3x^6+2\,(Dab^2+6\,Cb^3)x^4-48\,A}{48\,b^3x}$$

input
```
integrate((b*x^2+a)^(1/2)*(D*x^6+C*x^4+B*x^2+A)/x^2,x, algorithm="fricas")
```

output
```
[1/96*(3*(D*a^3 - 2*C*a^2*b + 8*B*a*b^2 + 16*A*b^3)*sqrt(b)*x*log(-2*b*x^2
 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(8*D*b^3*x^6 + 2*(D*a*b^2 + 6*C*b^
3)*x^4 - 48*A*b^3 - 3*(D*a^2*b - 2*C*a*b^2 - 8*B*b^3)*x^2)*sqrt(b*x^2 + a)
)/(b^3*x), -1/48*(3*(D*a^3 - 2*C*a^2*b + 8*B*a*b^2 + 16*A*b^3)*sqrt(-b)*x*
arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (8*D*b^3*x^6 + 2*(D*a*b^2 + 6*C*b^3)*
x^4 - 48*A*b^3 - 3*(D*a^2*b - 2*C*a*b^2 - 8*B*b^3)*x^2)*sqrt(b*x^2 + a))/(
b^3*x)]
```

## Sympy [A] (verification not implemented)

Time = 1.55 (sec) , antiderivative size = 340, normalized size of antiderivative = 2.00

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2+Cx^4+Dx^6)}{x^2}\,dx = -\frac{A\sqrt{a}}{x\sqrt{1+\frac{bx^2}{a}}} + A\sqrt{b}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right) - \frac{Abx}{\sqrt{a}\sqrt{1+\frac{bx^2}{a}}} + B\left(\begin{cases} \frac{a\left(\begin{cases} \frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0 \\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise} \end{cases}\right)}{2} + \frac{x\sqrt{a+bx^2}}{2} & \text{for } b\neq 0 \\ \sqrt{a}x & \text{otherwise} \end{cases}\right) + C\left(\begin{cases} -\frac{a^2\left(\begin{cases} \frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0 \\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise} \end{cases}\right)}{8b} + \frac{ax\sqrt{a+bx^2}}{8b} + \frac{x^3\sqrt{a+bx^2}}{4} & \text{for } b\neq 0 \\ \frac{\sqrt{a}x^3}{3} & \text{otherwise} \end{cases}\right) + D\left(\begin{cases} \frac{a^3\left(\begin{cases} \frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0 \\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise} \end{cases}\right)}{16b^2} - \frac{a^2x\sqrt{a+bx^2}}{16b^2} + \frac{ax^3\sqrt{a+bx^2}}{24b} + \frac{x^5\sqrt{a+bx^2}}{6} & \text{for } b\neq 0 \\ \frac{\sqrt{a}x^5}{5} & \text{otherwise} \end{cases}\right)$$

input
```
integrate((b*x**2+a)**(1/2)*(D*x**6+C*x**4+B*x**2+A)/x**2,x)
```

output
```
-A*sqrt(a)/(x*sqrt(1 + b*x**2/a)) + A*sqrt(b)*asinh(sqrt(b)*x/sqrt(a)) - A
*b*x/(sqrt(a)*sqrt(1 + b*x**2/a)) + B*Piecewise((a*Piecewise((log(2*sqrt(b
)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), Tr
ue))/2 + x*sqrt(a + b*x**2)/2, Ne(b, 0)), (sqrt(a)*x, True)) + C*Piecewise
((-a**2*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a,
0)), (x*log(x)/sqrt(b*x**2), True))/(8*b) + a*x*sqrt(a + b*x**2)/(8*b) + x
**3*sqrt(a + b*x**2)/4, Ne(b, 0)), (sqrt(a)*x**3/3, True)) + D*Piecewise((
a**3*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0))
, (x*log(x)/sqrt(b*x**2), True))/(16*b**2) - a**2*x*sqrt(a + b*x**2)/(16*b
**2) + a*x**3*sqrt(a + b*x**2)/(24*b) + x**5*sqrt(a + b*x**2)/6, Ne(b, 0))
, (sqrt(a)*x**5/5, True))
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2+Cx^4+Dx^6)}{x^2}\,dx = \frac{(bx^2+a)^{\frac{3}{2}}Dx^3}{6\,b} + \frac{1}{2}\sqrt{bx^2+a}Bx - \frac{(bx^2+a)^{\frac{3}{2}}Dax}{8\,b^2} + \frac{\sqrt{bx^2+a}Da^2x}{16\,b^2} + \frac{(bx^2+a)^{\frac{3}{2}}Cx}{4\,b} - \frac{\sqrt{bx^2+a}Cax}{8\,b} + \frac{Da^3\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{5}{2}}} - \frac{Ca^2\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{3}{2}}} + \frac{Ba\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{b}} + A\sqrt{b}\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right) - \frac{\sqrt{bx^2+a}A}{x}$$

input
```
integrate((b*x^2+a)^(1/2)*(D*x^6+C*x^4+B*x^2+A)/x^2,x, algorithm="maxima")
```

output
```
1/6*(b*x^2 + a)^(3/2)*D*x^3/b + 1/2*sqrt(b*x^2 + a)*B*x - 1/8*(b*x^2 + a)^
(3/2)*D*a*x/b^2 + 1/16*sqrt(b*x^2 + a)*D*a^2*x/b^2 + 1/4*(b*x^2 + a)^(3/2)
*C*x/b - 1/8*sqrt(b*x^2 + a)*C*a*x/b + 1/16*D*a^3*arcsinh(b*x/sqrt(a*b))/b
^(5/2) - 1/8*C*a^2*arcsinh(b*x/sqrt(a*b))/b^(3/2) + 1/2*B*a*arcsinh(b*x/sq
rt(a*b))/sqrt(b) + A*sqrt(b)*arcsinh(b*x/sqrt(a*b)) - sqrt(b*x^2 + a)*A/x
```

## Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2+Cx^4+Dx^6)}{x^2}\,dx$$
$$= \frac{1}{48}\left(2\left(4\,Dx^2+\frac{Dab^3+6\,Cb^4}{b^4}\right)x^2-\frac{3\,(Da^2b^2-2\,Cab^3-8\,Bb^4)}{b^4}\right)\sqrt{bx^2+a}x$$
$$+\frac{2\,Aa\sqrt{b}}{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a}$$
$$-\frac{(Da^3-2\,Ca^2b+8\,Bab^2+16\,Ab^3)\log\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2\right)}{32\,b^{\frac{5}{2}}}$$

input
```
integrate((b*x^2+a)^(1/2)*(D*x^6+C*x^4+B*x^2+A)/x^2,x, algorithm="giac")
```

output
```
1/48*(2*(4*D*x^2 + (D*a*b^3 + 6*C*b^4)/b^4)*x^2 - 3*(D*a^2*b^2 - 2*C*a*b^3
 - 8*B*b^4)/b^4)*sqrt(b*x^2 + a)*x + 2*A*a*sqrt(b)/((sqrt(b)*x - sqrt(b*x^
2 + a))^2 - a) - 1/32*(D*a^3 - 2*C*a^2*b + 8*B*a*b^2 + 16*A*b^3)*log((sqrt
(b)*x - sqrt(b*x^2 + a))^2)/b^(5/2)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2+Cx^4+Dx^6)}{x^2}\,dx = \int \frac{\sqrt{b\,x^2+a}\,(A+B\,x^2+C\,x^4+x^6\,D)}{x^2}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(A + B*x^2 + C*x^4 + x^6*D))/x^2,x)
```

output
```
int(((a + b*x^2)^(1/2)*(A + B*x^2 + C*x^4 + x^6*D))/x^2, x)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.35

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2+Cx^4+Dx^6)}{x^2}\,dx$$
$$= \frac{-24\sqrt{b\,x^2+a}\,a^2bd\,x^2-384\sqrt{b\,x^2+a}\,a\,b^3+48\sqrt{b\,x^2+a}\,a\,b^2c\,x^2+16\sqrt{b\,x^2+a}\,a\,b^2d\,x^4+192\sqrt{b\,x^2+a}$$

input
```
int((b*x^2+a)^(1/2)*(D*x^6+C*x^4+B*x^2+A)/x^2,x)
```

output
```
( - 24*sqrt(a + b*x**2)*a**2*b*d*x**2 - 384*sqrt(a + b*x**2)*a*b**3 + 48*s
qrt(a + b*x**2)*a*b**2*c*x**2 + 16*sqrt(a + b*x**2)*a*b**2*d*x**4 + 192*sq
rt(a + b*x**2)*b**4*x**2 + 96*sqrt(a + b*x**2)*b**3*c*x**4 + 64*sqrt(a + b
*x**2)*b**3*d*x**6 + 24*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a)
)*a**3*d*x - 48*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2*b
*c*x + 576*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a*b**3*x +
3*sqrt(b)*a**3*d*x - 432*sqrt(b)*a*b**3*x)/(384*b**3*x)
```

# 3.192 $\int \frac{\sqrt{a+bx^2}\left(A+Bx^2+Cx^4+Dx^6\right)}{x^4}\,dx$



## Optimal result

Integrand size = 32, antiderivative size = 143

$$\int \frac{\sqrt{a+bx^2}\left(A+Bx^2+Cx^4+Dx^6\right)}{x^4}\,dx = \frac{1}{8}\left(\frac{8bB}{a}+4C-\frac{aD}{b}\right)x\sqrt{a+bx^2} - \frac{A\left(a+bx^2\right)^{3/2}}{3ax^3} - \frac{B\left(a+bx^2\right)^{3/2}}{ax} + \frac{Dx\left(a+bx^2\right)^{3/2}}{4b} + \frac{\left(8b^2B+4abC-a^2D\right)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{3/2}}$$

output
```
1/8*(8*b*B/a+4*C-a*D/b)*x*(b*x^2+a)^(1/2)-1/3*A*(b*x^2+a)^(3/2)/a/x^3-B*(b
*x^2+a)^(3/2)/a/x+1/4*D*x*(b*x^2+a)^(3/2)/b+1/8*(8*B*b^2+4*C*a*b-D*a^2)*ar
ctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(3/2)
```

## Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.82

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2+Cx^4+Dx^6)}{x^4}\,dx$$
$$=\frac{\sqrt{a+bx^2}(-8Ab^2x^2+3a^2Dx^4+2ab(-4A-12Bx^2+6Cx^4+3Dx^6))}{24abx^3}$$
$$+\frac{(-8b^2B-4abC+a^2D)\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{8b^{3/2}}$$

input
```
Integrate[(Sqrt[a + b*x^2]*(A + B*x^2 + C*x^4 + D*x^6))/x^4,x]
```

output
```
(Sqrt[a + b*x^2]*(-8*A*b^2*x^2 + 3*a^2*D*x^4 + 2*a*b*(-4*A - 12*B*x^2 + 6*
C*x^4 + 3*D*x^6)))/(24*a*b*x^3) + ((-8*b^2*B - 4*a*b*C + a^2*D)*Log[-(Sqrt
[b]*x) + Sqrt[a + b*x^2]])/(8*b^(3/2))
```

## Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.96, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {2338, 9, 27, 1588, 25, 27, 299, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\left(A+Bx^2+Cx^4+Dx^6\right)}{x^4}\,dx$$

$$\Big\downarrow \ 2338$$

$$-\frac{\int -\frac{3\sqrt{bx^2+a}\left(aDx^5+aCx^3+aBx\right)}{x^3}dx}{3a}-\frac{A\left(a+bx^2\right)^{3/2}}{3ax^3}$$

$$\Big\downarrow \ 9$$

$$-\frac{\int -\frac{3\sqrt{bx^2+a}\left(aDx^4+aCx^2+aB\right)}{x^2}dx}{3a}-\frac{A\left(a+bx^2\right)^{3/2}}{3ax^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sqrt{bx^2+a}\left(aDx^4+aCx^2+aB\right)}{x^2}dx}{a}-\frac{A\left(a+bx^2\right)^{3/2}}{3ax^3}$$

$$\Big\downarrow \text{1588}$$

$$\frac{-\frac{\int -a\sqrt{bx^2+a}\left(aDx^2+2bB+aC\right)dx}{a}-\frac{B\left(a+bx^2\right)^{3/2}}{x}}{a}-\frac{A\left(a+bx^2\right)^{3/2}}{3ax^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int a\sqrt{bx^2+a}\left(aDx^2+2bB+aC\right)dx}{a}-\frac{B\left(a+bx^2\right)^{3/2}}{x}}{a}-\frac{A\left(a+bx^2\right)^{3/2}}{3ax^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \sqrt{bx^2+a}\left(aDx^2+2bB+aC\right)dx-\frac{B\left(a+bx^2\right)^{3/2}}{x}}{a}-\frac{A\left(a+bx^2\right)^{3/2}}{3ax^3}$$

$$\Big\downarrow \text{299}$$

$$\frac{\frac{\left(4b(aC+2bB)-a^2D\right)\int \sqrt{bx^2+a}dx}{4b}-\frac{B\left(a+bx^2\right)^{3/2}}{x}+\frac{aDx\left(a+bx^2\right)^{3/2}}{4b}}{a}-\frac{A\left(a+bx^2\right)^{3/2}}{3ax^3}$$

$$\Big\downarrow \text{211}$$

$$\frac{\frac{\left(4b(aC+2bB)-a^2D\right)\left(\frac{1}{2}a\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)}{4b}-\frac{B\left(a+bx^2\right)^{3/2}}{x}+\frac{aDx\left(a+bx^2\right)^{3/2}}{4b}}{a}-\frac{A\left(a+bx^2\right)^{3/2}}{3ax^3}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{\left(4b(aC+2bB)-a^2D\right)\left(\frac{1}{2}a\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)}{4b}-\frac{B\left(a+bx^2\right)^{3/2}}{x}+\frac{aDx\left(a+bx^2\right)^{3/2}}{4b}}{a}-\frac{A\left(a+bx^2\right)^{3/2}}{3ax^3}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)\left(4b(aC+2bB)-a^2D\right)}{4b}-\frac{B\left(a+bx^2\right)^{3/2}}{x}+\frac{aDx\left(a+bx^2\right)^{3/2}}{4b}}{a}-\frac{A\left(a+bx^2\right)^{3/2}}{3ax^3}$$

input
```
Int[(Sqrt[a + b*x^2]*(A + B*x^2 + C*x^4 + D*x^6))/x^4,x]
```

output
```
-1/3*(A*(a + b*x^2)^(3/2))/(a*x^3) + (-((B*(a + b*x^2)^(3/2))/x) + (a*D*x*
(a + b*x^2)^(3/2))/(4*b) + ((4*b*(2*b*B + a*C) - a^2*D)*((x*Sqrt[a + b*x^2
])/2 + (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b])))/(4*b))/a
```

### Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1588
```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Simp[1/(d*f
^2*(m + 1))   Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x
) - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

## Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.77

| method | result |
|---|---|
| pseudoelliptic | $-\frac{-3\left(B\,b^2+\frac{1}{2}Cab-\frac{1}{8}Da^2\right)x^3ba\ \operatorname{arctanh}\left(\frac{\sqrt{b\,x^2+a}}{x\sqrt{b}}\right)+\left(-\frac{3Db^{\frac{3}{2}}a^2x^4}{8}+b^{\frac{5}{2}}\left(a\left(-\frac{3}{4}Dx^6-\frac{3}{2}C\,x^4+3x^2B+A\right)+Ab\,x^2\right)\right)\sqrt{b\,x}}{3b^{\frac{5}{2}}a\,x^3}$ |
| default | $C\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)-\frac{A\left(b\,x^2+a\right)^{\frac{3}{2}}}{3a\,x^3}+B\left(-\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}}{ax}+\frac{2b\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2}\right.}{2\sqrt{b}}\right.}{a}\right.$ |

input
```
int((b*x^2+a)^(1/2)*(D*x^6+C*x^4+B*x^2+A)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(-3*(B*b^2+1/2*C*a*b-1/8*D*a^2)*x^3*b*a*arctanh((b*x^2+a)^(1/2)/x/b^(
1/2))+(-3/8*D*b^(3/2)*a^2*x^4+b^(5/2)*(a*(-3/4*D*x^6-3/2*C*x^4+3*x^2*B+A)+
A*b*x^2))*(b*x^2+a)^(1/2))/b^(5/2)/a/x^3
```

## Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.80

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2+Cx^4+Dx^6)}{x^4}\,dx$$
$$= \Bigg[-\frac{3\,(Da^3-4\,Ca^2b-8\,Bab^2)\sqrt{b}x^3\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\,(6\,Dab^2x^6+3\,(Da^2b+4\,C}{48\,ab^2x^3}$$

input
```
integrate((b*x^2+a)^(1/2)*(D*x^6+C*x^4+B*x^2+A)/x^4,x, algorithm="fricas")
```

output
```
[-1/48*(3*(D*a^3 - 4*C*a^2*b - 8*B*a*b^2)*sqrt(b)*x^3*log(-2*b*x^2 - 2*sqr
t(b*x^2 + a)*sqrt(b)*x - a) - 2*(6*D*a*b^2*x^6 + 3*(D*a^2*b + 4*C*a*b^2)*x
^4 - 8*A*a*b^2 - 8*(3*B*a*b^2 + A*b^3)*x^2)*sqrt(b*x^2 + a))/(a*b^2*x^3),
1/24*(3*(D*a^3 - 4*C*a^2*b - 8*B*a*b^2)*sqrt(-b)*x^3*arctan(sqrt(-b)*x/sqr
t(b*x^2 + a)) + (6*D*a*b^2*x^6 + 3*(D*a^2*b + 4*C*a*b^2)*x^4 - 8*A*a*b^2 -
 8*(3*B*a*b^2 + A*b^3)*x^2)*sqrt(b*x^2 + a))/(a*b^2*x^3)]
```

## Sympy [A] (verification not implemented)

Time = 1.73 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.88

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2+Cx^4+Dx^6)}{x^4}\,dx$$
$$= -\frac{A\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{3x^2} - \frac{Ab^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{3a} - \frac{B\sqrt{a}}{x\sqrt{1+\frac{bx^2}{a}}} + B\sqrt{b}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right) - \frac{Bbx}{\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$
$$+ C\left(\begin{cases} \frac{a\left(\begin{cases} \frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a \neq 0 \\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise} \end{cases}\right)}{2} + \frac{x\sqrt{a+bx^2}}{2} & \text{for } b \neq 0 \\ \sqrt{a}x & \text{otherwise} \end{cases}\right)$$
$$+ D\left(\begin{cases} -\frac{a^2\left(\begin{cases} \frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a \neq 0 \\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise} \end{cases}\right)}{8b} + \frac{ax\sqrt{a+bx^2}}{8b} + \frac{x^3\sqrt{a+bx^2}}{4} & \text{for } b \neq 0 \\ \frac{\sqrt{a}x^3}{3} & \text{otherwise} \end{cases}\right)$$

input
```
integrate((b*x**2+a)**(1/2)*(D*x**6+C*x**4+B*x**2+A)/x**4,x)
```

output
```
-A*sqrt(b)*sqrt(a/(b*x**2) + 1)/(3*x**2) - A*b**(3/2)*sqrt(a/(b*x**2) + 1)
/(3*a) - B*sqrt(a)/(x*sqrt(1 + b*x**2/a)) + B*sqrt(b)*asinh(sqrt(b)*x/sqrt
(a)) - B*b*x/(sqrt(a)*sqrt(1 + b*x**2/a)) + C*Piecewise((a*Piecewise((log(
2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x
**2), True))/2 + x*sqrt(a + b*x**2)/2, Ne(b, 0)), (sqrt(a)*x, True)) + D*P
iecewise((-a**2*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b)
, Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True))/(8*b) + a*x*sqrt(a + b*x**2)/(
8*b) + x**3*sqrt(a + b*x**2)/4, Ne(b, 0)), (sqrt(a)*x**3/3, True))
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2+Cx^4+Dx^6)}{x^4}\,dx = \frac{1}{2}\sqrt{bx^2+a}Cx + \frac{(bx^2+a)^{\frac{3}{2}}Dx}{4\,b} - \frac{\sqrt{bx^2+a}Dax}{8\,b} - \frac{Da^2\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{3}{2}}} + \frac{Ca\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{b}} + B\sqrt{b}\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right) - \frac{\sqrt{bx^2+a}B}{x} - \frac{(bx^2+a)^{\frac{3}{2}}A}{3\,ax^3}$$

input
```
integrate((b*x^2+a)^(1/2)*(D*x^6+C*x^4+B*x^2+A)/x^4,x, algorithm="maxima")
```

output
```
1/2*sqrt(b*x^2 + a)*C*x + 1/4*(b*x^2 + a)^(3/2)*D*x/b - 1/8*sqrt(b*x^2 + a
)*D*a*x/b - 1/8*D*a^2*arcsinh(b*x/sqrt(a*b))/b^(3/2) + 1/2*C*a*arcsinh(b*x
/sqrt(a*b))/sqrt(b) + B*sqrt(b)*arcsinh(b*x/sqrt(a*b)) - sqrt(b*x^2 + a)*B
/x - 1/3*(b*x^2 + a)^(3/2)*A/(a*x^3)
```

## Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.41

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2+Cx^4+Dx^6)}{x^4}\,dx = \frac{1}{8}\left(2\,Dx^2 + \frac{Dab+4\,Cb^2}{b^2}\right)\sqrt{bx^2+a}x + \frac{(Da^2-4\,Cab-8\,Bb^2)\log\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2\right)}{16\,b^{\frac{3}{2}}} + \frac{2\left(3\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4Ba\sqrt{b} + 3\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4Ab^{\frac{3}{2}} - 6\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2Ba^2\sqrt{b} + 3\,Ba^3\right.}{3\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^3}$$

input
```
integrate((b*x^2+a)^(1/2)*(D*x^6+C*x^4+B*x^2+A)/x^4,x, algorithm="giac")
```

output
```
1/8*(2*D*x^2 + (D*a*b + 4*C*b^2)/b^2)*sqrt(b*x^2 + a)*x + 1/16*(D*a^2 - 4*
C*a*b - 8*B*b^2)*log((sqrt(b)*x - sqrt(b*x^2 + a))^2)/b^(3/2) + 2/3*(3*(sq
rt(b)*x - sqrt(b*x^2 + a))^4*B*a*sqrt(b) + 3*(sqrt(b)*x - sqrt(b*x^2 + a))
^4*A*b^(3/2) - 6*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a^2*sqrt(b) + 3*B*a^3*s
qrt(b) + A*a^2*b^(3/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^3
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2+Cx^4+Dx^6)}{x^4}\,dx = \int \frac{\sqrt{b\,x^2+a}\,(A+B\,x^2+C\,x^4+x^6\,D)}{x^4}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(A + B*x^2 + C*x^4 + x^6*D))/x^4,x)
```

output
```
int(((a + b*x^2)^(1/2)*(A + B*x^2 + C*x^4 + x^6*D))/x^4, x)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.38

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2+Cx^4+Dx^6)}{x^4}\,dx = \frac{-64\sqrt{b\,x^2+a}\,a\,b^2+24\sqrt{b\,x^2+a}\,abd\,x^4-256\sqrt{b\,x^2+a}\,b^3x^2+96\sqrt{b\,x^2+a}\,b^2c\,x^4+48\sqrt{b\,x^2+a}\,b^2d\,x^6}{}$$

input
```
int((b*x^2+a)^(1/2)*(D*x^6+C*x^4+B*x^2+A)/x^4,x)
```

output
```
( - 64*sqrt(a + b*x**2)*a*b**2 + 24*sqrt(a + b*x**2)*a*b*d*x**4 - 256*sqrt
(a + b*x**2)*b**3*x**2 + 96*sqrt(a + b*x**2)*b**2*c*x**4 + 48*sqrt(a + b*x
**2)*b**2*d*x**6 - 24*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*
a**2*d*x**3 + 96*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a*b*c
*x**3 + 192*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*b**3*x**3
- sqrt(b)*a**2*d*x**3 + 16*sqrt(b)*a*b*c*x**3)/(192*b**2*x**3)
```

# 3.193 $\int \frac{\sqrt{a+bx^2}(A+Bx^2+Cx^4+Dx^6)}{x^6} dx$



## Optimal result

Integrand size = 32, antiderivative size = 136

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2+Cx^4+Dx^6)}{x^6} dx = \frac{(2bC+aD)x\sqrt{a+bx^2}}{2a} - \frac{A(a+bx^2)^{3/2}}{5ax^5} + \frac{(2Ab-5aB)(a+bx^2)^{3/2}}{15a^2x^3} - \frac{C(a+bx^2)^{3/2}}{ax} + \frac{(2bC+aD)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}$$

output

```
1/2*(2*C*b+D*a)*x*(b*x^2+a)^(1/2)/a-1/5*A*(b*x^2+a)^(3/2)/a/x^5+1/15*(2*A*
b-5*B*a)*(b*x^2+a)^(3/2)/a^2/x^3-C*(b*x^2+a)^(3/2)/a/x+1/2*(2*C*b+D*a)*arc
tanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(1/2)
```

## Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.82

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2+Cx^4+Dx^6)}{x^6}\,dx$$
$$=\frac{\sqrt{a+bx^2}(4Ab^2x^4-2abx^2(A+5Bx^2)-a^2(6A+10Bx^2+30Cx^4-15Dx^6))}{30a^2x^5}$$
$$-\frac{(2bC+aD)\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{2\sqrt{b}}$$

input
```
Integrate[(Sqrt[a + b*x^2]*(A + B*x^2 + C*x^4 + D*x^6))/x^6,x]
```

output
```
(Sqrt[a + b*x^2]*(4*A*b^2*x^4 - 2*a*b*x^2*(A + 5*B*x^2) - a^2*(6*A + 10*B*
x^2 + 30*C*x^4 - 15*D*x^6)))/(30*a^2*x^5) - ((2*b*C + a*D)*Log[-(Sqrt[b]*x
) + Sqrt[a + b*x^2]])/(2*Sqrt[b])
```

## Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2338, 9, 1588, 27, 359, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2+Cx^4+Dx^6)}{x^6}\,dx$$
$$\Big\downarrow \text{2338}$$
$$-\frac{\int \frac{\sqrt{bx^2+a}(-5aDx^5-5aCx^3+(2Ab-5aB)x)}{x^5}dx}{5a}-\frac{A(a+bx^2)^{3/2}}{5ax^5}$$
$$\Big\downarrow \text{9}$$
$$-\frac{\int \frac{\sqrt{bx^2+a}(-5aDx^4-5aCx^2+2Ab-5aB)}{x^4}dx}{5a}-\frac{A(a+bx^2)^{3/2}}{5ax^5}$$

$$\Big\downarrow \; 1588$$

$$-\frac{-\frac{\int \frac{15a^2\sqrt{bx^2+a}\left(Dx^2+C\right)}{x^2}dx}{3a}-\frac{\left(a+bx^2\right)^{3/2}(2Ab-5aB)}{3ax^3}}{5a}-\frac{A\left(a+bx^2\right)^{3/2}}{5ax^5}$$

$$\Big\downarrow \; 27$$

$$-\frac{-5a\int \frac{\sqrt{bx^2+a}\left(Dx^2+C\right)}{x^2}dx-\frac{\left(a+bx^2\right)^{3/2}(2Ab-5aB)}{3ax^3}}{5a}-\frac{A\left(a+bx^2\right)^{3/2}}{5ax^5}$$

$$\Big\downarrow \; 359$$

$$-\frac{-5a\left(\frac{(aD+2bC)\int \sqrt{bx^2+a}dx}{a}-\frac{C\left(a+bx^2\right)^{3/2}}{ax}\right)-\frac{\left(a+bx^2\right)^{3/2}(2Ab-5aB)}{3ax^3}}{5a}-\frac{A\left(a+bx^2\right)^{3/2}}{5ax^5}$$

$$\Big\downarrow \; 211$$

$$-\frac{-5a\left(\frac{(aD+2bC)\left(\frac{1}{2}a\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)}{a}-\frac{C\left(a+bx^2\right)^{3/2}}{ax}\right)-\frac{\left(a+bx^2\right)^{3/2}(2Ab-5aB)}{3ax^3}}{5a}-\frac{A\left(a+bx^2\right)^{3/2}}{5ax^5}$$

$$\Big\downarrow \; 224$$

$$-\frac{-5a\left(\frac{(aD+2bC)\left(\frac{1}{2}a\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)}{a}-\frac{C\left(a+bx^2\right)^{3/2}}{ax}\right)-\frac{\left(a+bx^2\right)^{3/2}(2Ab-5aB)}{3ax^3}}{5a}-\frac{A\left(a+bx^2\right)^{3/2}}{5ax^5}$$

$$\Big\downarrow \; 219$$

$$-\frac{-\frac{\left(a+bx^2\right)^{3/2}(2Ab-5aB)}{3ax^3}-5a\left(\frac{\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)(aD+2bC)}{a}-\frac{C\left(a+bx^2\right)^{3/2}}{ax}\right)}{5a}-\frac{A\left(a+bx^2\right)^{3/2}}{5ax^5}$$

input
```
Int[(Sqrt[a + b*x^2]*(A + B*x^2 + C*x^4 + D*x^6))/x^6,x]
```

output
```
-1/5*(A*(a + b*x^2)^(3/2))/(a*x^5) - (-1/3*((2*A*b - 5*a*B)*(a + b*x^2)^(3
/2))/(a*x^3) - 5*a*(-((C*(a + b*x^2)^(3/2))/(a*x)) + ((2*b*C + a*D)*((x*Sq
rt[a + b*x^2])/2 + (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b])))/
a))/(5*a)
```

### Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 1588
```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Simp[1/(d*f
^2*(m + 1))   Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x
) - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

## Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.81

| method | result |
|---|---|
| pseudoelliptic | $-\frac{-15x^5a^2\left(Cb+\frac{Da}{2}\right)\operatorname{arctanh}\left(\frac{\sqrt{b\,x^2+a}}{x\sqrt{b}}\right)+\sqrt{b\,x^2+a}\,\left(a\,x^2\,(5x^2B+A)b^{\frac{3}{2}}-2A\,b^{\frac{5}{2}}x^4+3(-\frac{5}{2}Dx^6+5C\,x^4+\frac{5}{3}x^2B+A)\sqrt{b}\,a}{15\sqrt{b}\,x^5a^2}$ |
| default | $D\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)+A\left(-\frac{(b\,x^2+a)^{\frac{3}{2}}}{5a\,x^5}+\frac{2b(b\,x^2+a)^{\frac{3}{2}}}{15a^2x^3}\right)-\frac{B(b\,x^2+a)^{\frac{3}{2}}}{3a\,x^3}+C\left(-\frac{(b\,x}{}\right.$ |

input
```
int((b*x^2+a)^(1/2)*(D*x^6+C*x^4+B*x^2+A)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/15/b^(1/2)*(-15*x^5*a^2*(C*b+1/2*D*a)*arctanh((b*x^2+a)^(1/2)/x/b^(1/2)
)+(b*x^2+a)^(1/2)*(a*x^2*(5*B*x^2+A)*b^(3/2)-2*A*b^(5/2)*x^4+3*(-5/2*D*x^6
+5*C*x^4+5/3*x^2*B+A)*b^(1/2)*a^2))/x^5/a^2
```

## Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.91

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2+Cx^4+Dx^6)}{x^6}\,dx$$
$$=\left[\frac{15\,(Da^3+2\,Ca^2b)\sqrt{b}x^5\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,(15\,Da^2bx^6-2\,(15\,Ca^2b+5\,Bab^2-2}{60\,a^2bx^5}\right.$$
$$-\frac{15\,(Da^3+2\,Ca^2b)\sqrt{-b}x^5\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(15\,Da^2bx^6-2\,(15\,Ca^2b+5\,Bab^2-2\,Ab^3)x^4-6\,Aa^2b}{30\,a^2bx^5}$$

input
```
integrate((b*x^2+a)^(1/2)*(D*x^6+C*x^4+B*x^2+A)/x^6,x, algorithm="fricas")
```

output
```
[1/60*(15*(D*a^3 + 2*C*a^2*b)*sqrt(b)*x^5*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)
*sqrt(b)*x - a) + 2*(15*D*a^2*b*x^6 - 2*(15*C*a^2*b + 5*B*a*b^2 - 2*A*b^3)
*x^4 - 6*A*a^2*b - 2*(5*B*a^2*b + A*a*b^2)*x^2)*sqrt(b*x^2 + a))/(a^2*b*x^
5), -1/30*(15*(D*a^3 + 2*C*a^2*b)*sqrt(-b)*x^5*arctan(sqrt(-b)*x/sqrt(b*x^
2 + a)) - (15*D*a^2*b*x^6 - 2*(15*C*a^2*b + 5*B*a*b^2 - 2*A*b^3)*x^4 - 6*A
*a^2*b - 2*(5*B*a^2*b + A*a*b^2)*x^2)*sqrt(b*x^2 + a))/(a^2*b*x^5)]
```

## Sympy [A] (verification not implemented)

Time = 1.92 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.84

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2+Cx^4+Dx^6)}{x^6}\,dx$$
$$= -\frac{A\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{5x^4} - \frac{Ab^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{15ax^2} + \frac{2Ab^{\frac{5}{2}}\sqrt{\frac{a}{bx^2}+1}}{15a^2} - \frac{B\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{3x^2} - \frac{Bb^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{3a} - \frac{C\sqrt{a}}{x\sqrt{1+\frac{bx^2}{a}}} + C\sqrt{b}\,\mathrm{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right) - \frac{Cbx}{\sqrt{a}\sqrt{1+\frac{bx^2}{a}}} + D\left(\begin{cases} \frac{a\left(\begin{cases} \frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for}\ a\neq 0 \\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise} \end{cases}\right)}{2} + \frac{x\sqrt{a+bx^2}}{2} & \text{for}\ b\neq 0 \\ \sqrt{a}x & \text{otherwise} \end{cases}\right)$$

input
```
integrate((b*x**2+a)**(1/2)*(D*x**6+C*x**4+B*x**2+A)/x**6,x)
```

output
```
-A*sqrt(b)*sqrt(a/(b*x**2) + 1)/(5*x**4) - A*b**(3/2)*sqrt(a/(b*x**2) + 1)
/(15*a*x**2) + 2*A*b**(5/2)*sqrt(a/(b*x**2) + 1)/(15*a**2) - B*sqrt(b)*sqr
t(a/(b*x**2) + 1)/(3*x**2) - B*b**(3/2)*sqrt(a/(b*x**2) + 1)/(3*a) - C*sqr
t(a)/(x*sqrt(1 + b*x**2/a)) + C*sqrt(b)*asinh(sqrt(b)*x/sqrt(a)) - C*b*x/(
sqrt(a)*sqrt(1 + b*x**2/a)) + D*Piecewise((a*Piecewise((log(2*sqrt(b)*sqrt
(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True))/2
 + x*sqrt(a + b*x**2)/2, Ne(b, 0)), (sqrt(a)*x, True))
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.84

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2+Cx^4+Dx^6)}{x^6}\,dx = \frac{1}{2}\sqrt{bx^2+a}Dx + \frac{Da\,\text{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{b}} + C\sqrt{b}\,\text{arsinh}\left(\frac{bx}{\sqrt{ab}}\right) - \frac{\sqrt{bx^2+a}C}{x} - \frac{(bx^2+a)^{\frac{3}{2}}B}{3\,ax^3} + \frac{2\,(bx^2+a)^{\frac{3}{2}}Ab}{15\,a^2x^3} - \frac{(bx^2+a)^{\frac{3}{2}}A}{5\,ax^5}$$

input
```
integrate((b*x^2+a)^(1/2)*(D*x^6+C*x^4+B*x^2+A)/x^6,x, algorithm="maxima")
```

output
```
1/2*sqrt(b*x^2 + a)*D*x + 1/2*D*a*arcsinh(b*x/sqrt(a*b))/sqrt(b) + C*sqrt(
b)*arcsinh(b*x/sqrt(a*b)) - sqrt(b*x^2 + a)*C/x - 1/3*(b*x^2 + a)^(3/2)*B/
(a*x^3) + 2/15*(b*x^2 + a)^(3/2)*A*b/(a^2*x^3) - 1/5*(b*x^2 + a)^(3/2)*A/(
a*x^5)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 398 vs. 2(114) = 228.

Time = 0.14 (sec) , antiderivative size = 398, normalized size of antiderivative = 2.93

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2+Cx^4+Dx^6)}{x^6}\,dx$$

$$= \frac{1}{2}\sqrt{bx^2+a}Dx - \frac{(Da+2\,Cb)\log\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2\right)}{4\sqrt{b}}$$

$$+ \frac{2\left(15\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^8Ca\sqrt{b} + 15\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^8Bb^{\frac{3}{2}} - 60\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6Ca^2\sqrt{b} - 30\right.}{}$$

input
```
integrate((b*x^2+a)^(1/2)*(D*x^6+C*x^4+B*x^2+A)/x^6,x, algorithm="giac")
```

output
```
1/2*sqrt(b*x^2 + a)*D*x - 1/4*(D*a + 2*C*b)*log((sqrt(b)*x - sqrt(b*x^2 +
a))^2)/sqrt(b) + 2/15*(15*(sqrt(b)*x - sqrt(b*x^2 + a))^8*C*a*sqrt(b) + 15
*(sqrt(b)*x - sqrt(b*x^2 + a))^8*B*b^(3/2) - 60*(sqrt(b)*x - sqrt(b*x^2 +
a))^6*C*a^2*sqrt(b) - 30*(sqrt(b)*x - sqrt(b*x^2 + a))^6*B*a*b^(3/2) + 30*
(sqrt(b)*x - sqrt(b*x^2 + a))^6*A*b^(5/2) + 90*(sqrt(b)*x - sqrt(b*x^2 + a
))^4*C*a^3*sqrt(b) + 20*(sqrt(b)*x - sqrt(b*x^2 + a))^4*B*a^2*b^(3/2) + 10
*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*a*b^(5/2) - 60*(sqrt(b)*x - sqrt(b*x^2
+ a))^2*C*a^4*sqrt(b) - 10*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a^3*b^(3/2) +
 10*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A*a^2*b^(5/2) + 15*C*a^5*sqrt(b) + 5*B
*a^4*b^(3/2) - 2*A*a^3*b^(5/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^5
```

## Mupad [B] (verification not implemented)

Time = 1.91 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.10

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2+Cx^4+Dx^6)}{x^6}\,dx = \frac{x\sqrt{b\,x^2+a}\,D}{2} - \frac{C\sqrt{b\,x^2+a}}{x} + \frac{a\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)\,D}{2\sqrt{b}} - \frac{B\,\left(b\,x^2+a\right)^{3/2}}{3\,a\,x^3} - \frac{A\sqrt{b\,x^2+a}\,(3\,a^2+a\,b\,x^2-2\,b^2\,x^4)}{15\,a^2\,x^5} - \frac{C\sqrt{b}\,\mathrm{asin}\left(\frac{\sqrt{b}\,x\,1\mathrm{i}}{\sqrt{a}}\right)\,\sqrt{b\,x^2+a}\,1\mathrm{i}}{\sqrt{a}\,\sqrt{\frac{b\,x^2}{a}+1}}$$

input
```
int(((a + b*x^2)^(1/2)*(A + B*x^2 + C*x^4 + x^6*D))/x^6,x)
```

output
```
(x*(a + b*x^2)^(1/2)*D)/2 - (C*(a + b*x^2)^(1/2))/x + (a*log(b^(1/2)*x + (
a + b*x^2)^(1/2))*D)/(2*b^(1/2)) - (B*(a + b*x^2)^(3/2))/(3*a*x^3) - (A*(a
 + b*x^2)^(1/2)*(3*a^2 - 2*b^2*x^4 + a*b*x^2))/(15*a^2*x^5) - (C*b^(1/2)*a
sin((b^(1/2)*x*1i)/a^(1/2))*(a + b*x^2)^(1/2)*1i)/(a^(1/2)*((b*x^2)/a + 1)
^(1/2))
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.33

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2+Cx^4+Dx^6)}{x^6}\,dx$$
$$= \frac{-8\sqrt{b\,x^2+a}\,a^2b-16\sqrt{b\,x^2+a}\,a\,b^2x^2-40\sqrt{b\,x^2+a}\,abc\,x^4+20\sqrt{b\,x^2+a}\,abd\,x^6-8\sqrt{b\,x^2+a}\,b^3x^4+2}{40ab}$$

input
```
int((b*x^2+a)^(1/2)*(D*x^6+C*x^4+B*x^2+A)/x^6,x)
```

output
```
( - 8*sqrt(a + b*x**2)*a**2*b - 16*sqrt(a + b*x**2)*a*b**2*x**2 - 40*sqrt(
a + b*x**2)*a*b*c*x**4 + 20*sqrt(a + b*x**2)*a*b*d*x**6 - 8*sqrt(a + b*x**
2)*b**3*x**4 + 20*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2
*d*x**5 + 40*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a*b*c*x**
5 + 9*sqrt(b)*a**2*d*x**5 + 24*sqrt(b)*a*b*c*x**5 - 8*sqrt(b)*b**3*x**5)/(
40*a*b*x**5)
```

# 3.194 $\int \frac{\sqrt{a+bx^2}(A+Bx^2+Cx^4+Dx^6)}{x^8}\,dx$



## Optimal result

Integrand size = 32, antiderivative size = 135

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2+Cx^4+Dx^6)}{x^8}\,dx = -\frac{D\sqrt{a+bx^2}}{x} - \frac{A(a+bx^2)^{3/2}}{7ax^7} + \frac{(4Ab-7aB)(a+bx^2)^{3/2}}{35a^2x^5} - \frac{(8Ab^2-14abB+35a^2C)(a+bx^2)^{3/2}}{105a^3x^3} + \sqrt{b}D\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)$$

output
```
-D*(b*x^2+a)^(1/2)/x-1/7*A*(b*x^2+a)^(3/2)/a/x^7+1/35*(4*A*b-7*B*a)*(b*x^2
+a)^(3/2)/a^2/x^5-1/105*(8*A*b^2-14*B*a*b+35*C*a^2)*(b*x^2+a)^(3/2)/a^3/x^
3+b^(1/2)*D*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))
```

## Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.98

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2+Cx^4+Dx^6)}{x^8}\,dx =$$
$$-\frac{\sqrt{a+bx^2}(8Ab^3x^6-2ab^2x^4(2A+7Bx^2)+a^2bx^2(3A+7x^2(B+5Cx^2))+a^3(15A+21Bx^2+35x^4(C}{105a^3x^7}$$
$$-\sqrt{b}D\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)$$

input
```
Integrate[(Sqrt[a + b*x^2]*(A + B*x^2 + C*x^4 + D*x^6))/x^8,x]
```

output
```
-1/105*(Sqrt[a + b*x^2]*(8*A*b^3*x^6 - 2*a*b^2*x^4*(2*A + 7*B*x^2) + a^2*b
*x^2*(3*A + 7*x^2*(B + 5*C*x^2)) + a^3*(15*A + 21*B*x^2 + 35*x^4*(C + 3*D*
x^2))))/(a^3*x^7) - Sqrt[b]*D*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]]
```

## Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.16, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {2338, 9, 1588, 358, 247, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2+Cx^4+Dx^6)}{x^8}\,dx$$

$$\Big\downarrow \ 2338$$

$$-\frac{\int \frac{\sqrt{bx^2+a}(-7aDx^5-7aCx^3+(4Ab-7aB)x)}{x^7}dx}{7a}-\frac{A(a+bx^2)^{3/2}}{7ax^7}$$

$$\Big\downarrow \ 9$$

$$-\frac{\int \frac{\sqrt{bx^2+a}(-7aDx^4-7aCx^2+4Ab-7aB)}{x^6}dx}{7a}-\frac{A(a+bx^2)^{3/2}}{7ax^7}$$

$$\Big\downarrow \ 1588$$

$$-\frac{-\frac{\int \frac{\sqrt{bx^2+a}\left(35Dx^2a^2+35Ca^2-14bBa+8Ab^2\right)}{x^4}dx}{5a}-\frac{\left(a+bx^2\right)^{3/2}(4Ab-7aB)}{5ax^5}}{7a}-\frac{A\left(a+bx^2\right)^{3/2}}{7ax^7}$$

$\Big\downarrow$ 358

$$-\frac{-\frac{35a^2D\int \frac{\sqrt{bx^2+a}}{x^2}dx-\frac{\left(a+bx^2\right)^{3/2}\left(35a^2C-14abB+8Ab^2\right)}{3ax^3}}{5a}-\frac{\left(a+bx^2\right)^{3/2}(4Ab-7aB)}{5ax^5}}{7a}-\frac{A\left(a+bx^2\right)^{3/2}}{7ax^7}$$

$\Big\downarrow$ 247

$$-\frac{-\frac{35a^2D\left(b\int \frac{1}{\sqrt{bx^2+a}}dx-\frac{\sqrt{a+bx^2}}{x}\right)-\frac{\left(a+bx^2\right)^{3/2}\left(35a^2C-14abB+8Ab^2\right)}{3ax^3}}{5a}-\frac{\left(a+bx^2\right)^{3/2}(4Ab-7aB)}{5ax^5}}{7a}-\frac{A\left(a+bx^2\right)^{3/2}}{7ax^7}$$

$\Big\downarrow$ 224

$$-\frac{-\frac{35a^2D\left(b\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}-\frac{\sqrt{a+bx^2}}{x}\right)-\frac{\left(a+bx^2\right)^{3/2}\left(35a^2C-14abB+8Ab^2\right)}{3ax^3}}{5a}-\frac{\left(a+bx^2\right)^{3/2}(4Ab-7aB)}{5ax^5}}{7a}-\frac{A\left(a+bx^2\right)^{3/2}}{7ax^7}$$

$\Big\downarrow$ 219

$$-\frac{-\frac{35a^2D\left(\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)-\frac{\sqrt{a+bx^2}}{x}\right)-\frac{\left(a+bx^2\right)^{3/2}\left(35a^2C-14abB+8Ab^2\right)}{3ax^3}}{5a}-\frac{\left(a+bx^2\right)^{3/2}(4Ab-7aB)}{5ax^5}}{7a}-\frac{A\left(a+bx^2\right)^{3/2}}{7ax^7}$$

input
```
Int[(Sqrt[a + b*x^2]*(A + B*x^2 + C*x^4 + D*x^6))/x^8,x]
```

output
```
-1/7*(A*(a + b*x^2)^(3/2))/(a*x^7) - (-1/5*((4*A*b - 7*a*B)*(a + b*x^2)^(3
/2))/(a*x^5) - (-1/3*((8*A*b^2 - 14*a*b*B + 35*a^2*C)*(a + b*x^2)^(3/2))/(
a*x^3) + 35*a^2*D*(-(Sqrt[a + b*x^2]/x) + Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt
[a + b*x^2]]))/(5*a))/(7*a)
```

### Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 358
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x_
Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] + S
imp[d/e^2   Int[(e*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b*c - a*d, 0] && EqQ[Simplify[m + 2*p + 3], 0] && NeQ[m,
 -1]
```

rule 1588
```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Simp[1/(d*f
^2*(m + 1))   Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x
) - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

## Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.92

| method | result |
|---|---|
| pseudoelliptic | $\frac{7a^3 D\sqrt{b}\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^2+a}}{x\sqrt{b}}\right)x^7-\sqrt{b\,x^2+a}\left(\left(7Dx^6+\frac{7}{3}C\,x^4+\frac{7}{5}x^2B+A\right)a^3+\frac{\left(\frac{35}{3}C\,x^4+\frac{7}{3}x^2B+A\right)x^2b\,a^2}{5}-\frac{4\left(\frac{7x^2B}{2}+A\right)x^4b^2}{15}\right.}{7a^3x^7}$ |
| default | $A\left(-\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}}{7a\,x^7}-\frac{4b\left(-\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}}{5a\,x^5}+\frac{2b\left(b\,x^2+a\right)^{\frac{3}{2}}}{15a^2x^3}\right)}{7a}\right)+B\left(-\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}}{5a\,x^5}+\frac{2b\left(b\,x^2+a\right)^{\frac{3}{2}}}{15a^2x^3}\right)-\frac{C\left(b\,x^2+a\right)^{\frac{3}{2}}}{3a\,x^3}$ |

input
```
int((b*x^2+a)^(1/2)*(D*x^6+C*x^4+B*x^2+A)/x^8,x,method=_RETURNVERBOSE)
```

output
```
1/7*(7*a^3*D*b^(1/2)*arctanh((b*x^2+a)^(1/2)/x/b^(1/2))*x^7-(b*x^2+a)^(1/2
)*((7*D*x^6+7/3*C*x^4+7/5*x^2*B+A)*a^3+1/5*(35/3*C*x^4+7/3*x^2*B+A)*x^2*b*
a^2-4/15*(7/2*x^2*B+A)*x^4*b^2*a+8/15*A*b^3*x^6))/a^3/x^7
```

## Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 271, normalized size of antiderivative = 2.01

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2+Cx^4+Dx^6)}{x^8}\,dx$$
$$= \left[\frac{105\,Da^3\sqrt{b}x^7\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\,((105\,Da^3+35\,Ca^2b-14\,Bab^2+8\,Ab^3)x^6+(3}{210\,a^3x^7}\right.$$
$$-\frac{105\,Da^3\sqrt{-b}x^7\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)+((105\,Da^3+35\,Ca^2b-14\,Bab^2+8\,Ab^3)x^6+(35\,Ca^3+7\,Ba^2b-}{105\,a^3x^7}$$

input
```
integrate((b*x^2+a)^(1/2)*(D*x^6+C*x^4+B*x^2+A)/x^8,x, algorithm="fricas")
```

output
```
[1/210*(105*D*a^3*sqrt(b)*x^7*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x -
 a) - 2*((105*D*a^3 + 35*C*a^2*b - 14*B*a*b^2 + 8*A*b^3)*x^6 + (35*C*a^3 +
 7*B*a^2*b - 4*A*a*b^2)*x^4 + 15*A*a^3 + 3*(7*B*a^3 + A*a^2*b)*x^2)*sqrt(b
*x^2 + a))/(a^3*x^7), -1/105*(105*D*a^3*sqrt(-b)*x^7*arctan(sqrt(-b)*x/sqr
t(b*x^2 + a)) + ((105*D*a^3 + 35*C*a^2*b - 14*B*a*b^2 + 8*A*b^3)*x^6 + (35
*C*a^3 + 7*B*a^2*b - 4*A*a*b^2)*x^4 + 15*A*a^3 + 3*(7*B*a^3 + A*a^2*b)*x^2
)*sqrt(b*x^2 + a))/(a^3*x^7)]
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 551 vs. $2(126) = 252$.

Time = 2.34 (sec) , antiderivative size = 551, normalized size of antiderivative = 4.08

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2+Cx^4+Dx^6)}{x^8}\,dx = -\frac{15Aa^5b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}} - \frac{33Aa^4b^{\frac{11}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}} - \frac{17Aa^3b^{\frac{13}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}} - \frac{3Aa^2b^{\frac{15}{2}}x^6\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}} - \frac{12Aab^{\frac{17}{2}}x^8\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}} - \frac{8Ab^{\frac{19}{2}}x^{10}\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}} - \frac{B\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{5x^4} - \frac{Bb^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{15ax^2} + \frac{2Bb^{\frac{5}{2}}\sqrt{\frac{a}{bx^2}+1}}{15a^2} - \frac{C\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{3x^2} - \frac{Cb^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{3a} - \frac{D\sqrt{a}}{x\sqrt{1+\frac{bx^2}{a}}} + D\sqrt{b}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right) - \frac{Dbx}{\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

input
```
integrate((b*x**2+a)**(1/2)*(D*x**6+C*x**4+B*x**2+A)/x**8,x)
```

output
```
-15*A*a**5*b**(9/2)*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b*
*5*x**8 + 105*a**3*b**6*x**10) - 33*A*a**4*b**(11/2)*x**2*sqrt(a/(b*x**2)
+ 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 17*
A*a**3*b**(13/2)*x**4*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*
b**5*x**8 + 105*a**3*b**6*x**10) - 3*A*a**2*b**(15/2)*x**6*sqrt(a/(b*x**2)
 + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 12
*A*a*b**(17/2)*x**8*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b*
*5*x**8 + 105*a**3*b**6*x**10) - 8*A*b**(19/2)*x**10*sqrt(a/(b*x**2) + 1)/
(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - B*sqrt(b
)*sqrt(a/(b*x**2) + 1)/(5*x**4) - B*b**(3/2)*sqrt(a/(b*x**2) + 1)/(15*a*x*
*2) + 2*B*b**(5/2)*sqrt(a/(b*x**2) + 1)/(15*a**2) - C*sqrt(b)*sqrt(a/(b*x*
*2) + 1)/(3*x**2) - C*b**(3/2)*sqrt(a/(b*x**2) + 1)/(3*a) - D*sqrt(a)/(x*s
qrt(1 + b*x**2/a)) + D*sqrt(b)*asinh(sqrt(b)*x/sqrt(a)) - D*b*x/(sqrt(a)*s
qrt(1 + b*x**2/a))
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.06

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2+Cx^4+Dx^6)}{x^8}\,dx = D\sqrt{b}\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)-\frac{\sqrt{bx^2+a}D}{x} - \frac{(bx^2+a)^{\frac{3}{2}}C}{3\,ax^3}+\frac{2\,(bx^2+a)^{\frac{3}{2}}Bb}{15\,a^2x^3} - \frac{8\,(bx^2+a)^{\frac{3}{2}}Ab^2}{105\,a^3x^3}-\frac{(bx^2+a)^{\frac{3}{2}}B}{5\,ax^5} + \frac{4\,(bx^2+a)^{\frac{3}{2}}Ab}{35\,a^2x^5}-\frac{(bx^2+a)^{\frac{3}{2}}A}{7\,ax^7}$$

input
```
integrate((b*x^2+a)^(1/2)*(D*x^6+C*x^4+B*x^2+A)/x^8,x, algorithm="maxima")
```

output
```
D*sqrt(b)*arcsinh(b*x/sqrt(a*b)) - sqrt(b*x^2 + a)*D/x - 1/3*(b*x^2 + a)^(
3/2)*C/(a*x^3) + 2/15*(b*x^2 + a)^(3/2)*B*b/(a^2*x^3) - 8/105*(b*x^2 + a)^
(3/2)*A*b^2/(a^3*x^3) - 1/5*(b*x^2 + a)^(3/2)*B/(a*x^5) + 4/35*(b*x^2 + a)
^(3/2)*A*b/(a^2*x^5) - 1/7*(b*x^2 + a)^(3/2)*A/(a*x^7)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 662 vs. $2(115) = 230$.

Time = 0.15 (sec) , antiderivative size = 662, normalized size of antiderivative = 4.90

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2+Cx^4+Dx^6)}{x^8}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(1/2)*(D*x^6+C*x^4+B*x^2+A)/x^8,x, algorithm="giac")
```

output
```
-1/2*D*sqrt(b)*log((sqrt(b)*x - sqrt(b*x^2 + a))^2) + 2/105*(105*(sqrt(b)*
x - sqrt(b*x^2 + a))^12*D*a*sqrt(b) + 105*(sqrt(b)*x - sqrt(b*x^2 + a))^12
*C*b^(3/2) - 630*(sqrt(b)*x - sqrt(b*x^2 + a))^10*D*a^2*sqrt(b) - 420*(sqr
t(b)*x - sqrt(b*x^2 + a))^10*C*a*b^(3/2) + 210*(sqrt(b)*x - sqrt(b*x^2 + a
))^10*B*b^(5/2) + 1575*(sqrt(b)*x - sqrt(b*x^2 + a))^8*D*a^3*sqrt(b) + 665
*(sqrt(b)*x - sqrt(b*x^2 + a))^8*C*a^2*b^(3/2) - 350*(sqrt(b)*x - sqrt(b*x
^2 + a))^8*B*a*b^(5/2) + 560*(sqrt(b)*x - sqrt(b*x^2 + a))^8*A*b^(7/2) - 2
100*(sqrt(b)*x - sqrt(b*x^2 + a))^6*D*a^4*sqrt(b) - 560*(sqrt(b)*x - sqrt(
b*x^2 + a))^6*C*a^3*b^(3/2) + 140*(sqrt(b)*x - sqrt(b*x^2 + a))^6*B*a^2*b^
(5/2) + 280*(sqrt(b)*x - sqrt(b*x^2 + a))^6*A*a*b^(7/2) + 1575*(sqrt(b)*x
- sqrt(b*x^2 + a))^4*D*a^5*sqrt(b) + 315*(sqrt(b)*x - sqrt(b*x^2 + a))^4*C
*a^4*b^(3/2) - 84*(sqrt(b)*x - sqrt(b*x^2 + a))^4*B*a^3*b^(5/2) + 168*(sqr
t(b)*x - sqrt(b*x^2 + a))^4*A*a^2*b^(7/2) - 630*(sqrt(b)*x - sqrt(b*x^2 +
a))^2*D*a^6*sqrt(b) - 140*(sqrt(b)*x - sqrt(b*x^2 + a))^2*C*a^5*b^(3/2) +
98*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a^4*b^(5/2) - 56*(sqrt(b)*x - sqrt(b*
x^2 + a))^2*A*a^3*b^(7/2) + 105*D*a^7*sqrt(b) + 35*C*a^6*b^(3/2) - 14*B*a^
5*b^(5/2) + 8*A*a^4*b^(7/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^7
```

## Mupad [B] (verification not implemented)

Time = 2.08 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.41

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2+Cx^4+Dx^6)}{x^8}\,dx = \frac{4\,A\,b^2\,\sqrt{b\,x^2+a}}{105\,a^2\,x^3} - \frac{\sqrt{b\,x^2+a}\,D}{x} - \frac{C\,{\left(b\,x^2+a\right)}^{3/2}}{3\,a\,x^3} - \frac{A\,b\,\sqrt{b\,x^2+a}}{35\,a\,x^5} - \frac{B\,\sqrt{b\,x^2+a}\,\left(3\,a^2+a\,b\,x^2-2\,b^2\,x^4\right)}{15\,a^2\,x^5} - \frac{A\,\sqrt{b\,x^2+a}}{7\,x^7} - \frac{8\,A\,b^3\,\sqrt{b\,x^2+a}}{105\,a^3\,x} - \frac{\sqrt{b}\,\mathrm{asin}\left(\frac{\sqrt{b}\,x\,1{}\mathrm{i}}{\sqrt{a}}\right)\,\sqrt{b\,x^2+a}\,D\,1{}\mathrm{i}}{\sqrt{a}\,\sqrt{\frac{b\,x^2}{a}+1}}$$

input
```
int(((a + b*x^2)^(1/2)*(A + B*x^2 + C*x^4 + x^6*D))/x^8,x)
```

output
```
(4*A*b^2*(a + b*x^2)^(1/2))/(105*a^2*x^3) - ((a + b*x^2)^(1/2)*D)/x - (C*(
a + b*x^2)^(3/2))/(3*a*x^3) - (A*b*(a + b*x^2)^(1/2))/(35*a*x^5) - (B*(a +
 b*x^2)^(1/2)*(3*a^2 - 2*b^2*x^4 + a*b*x^2))/(15*a^2*x^5) - (A*(a + b*x^2)
^(1/2))/(7*x^7) - (8*A*b^3*(a + b*x^2)^(1/2))/(105*a^3*x) - (b^(1/2)*asin(
(b^(1/2)*x*1i)/a^(1/2))*(a + b*x^2)^(1/2)*D*1i)/(a^(1/2)*((b*x^2)/a + 1)^(
1/2))
```

## Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.36

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2+Cx^4+Dx^6)}{x^8}\,dx = \frac{-15\sqrt{b\,x^2+a}\,a^3-24\sqrt{b\,x^2+a}\,a^2b\,x^2-35\sqrt{b\,x^2+a}\,a^2c\,x^4-105\sqrt{b\,x^2+a}\,a^2d\,x^6-3\sqrt{b\,x^2+a}\,a\,b^2x^4}{}$$

input
```
int((b*x^2+a)^(1/2)*(D*x^6+C*x^4+B*x^2+A)/x^8,x)
```

output
```
( - 15*sqrt(a + b*x**2)*a**3 - 24*sqrt(a + b*x**2)*a**2*b*x**2 - 35*sqrt(a
 + b*x**2)*a**2*c*x**4 - 105*sqrt(a + b*x**2)*a**2*d*x**6 - 3*sqrt(a + b*x
**2)*a*b**2*x**4 - 35*sqrt(a + b*x**2)*a*b*c*x**6 + 6*sqrt(a + b*x**2)*b**
3*x**6 + 105*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2*d*x*
*7 + 75*sqrt(b)*a**2*d*x**7 + 5*sqrt(b)*a*b*c*x**7 - 6*sqrt(b)*b**3*x**7)/
(105*a**2*x**7)
```

# 3.195 $\int \frac{\sqrt{a+bx^2}(A+Bx^2+Cx^4+Dx^6)}{x^{10}} dx$



## Optimal result

Integrand size = 32, antiderivative size = 141

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2+Cx^4+Dx^6)}{x^{10}}\,dx = -\frac{A(a+bx^2)^{3/2}}{9ax^9}+\frac{(2Ab-3aB)\,(a+bx^2)^{3/2}}{21a^2x^7}-\frac{(8Ab^2-12abB+21a^2C)\,(a+bx^2)^{3/2}}{105a^3x^5}+\frac{(16Ab^3-3a(8b^2B-14abC+35a^2D))\,(a+bx^2)^{3/2}}{315a^4x^3}$$

output
```
-1/9*A*(b*x^2+a)^(3/2)/a/x^9+1/21*(2*A*b-3*B*a)*(b*x^2+a)^(3/2)/a^2/x^7-1/
105*(8*A*b^2-12*B*a*b+21*C*a^2)*(b*x^2+a)^(3/2)/a^3/x^5+1/315*(16*A*b^3-3*
a*(8*B*b^2-14*C*a*b+35*D*a^2))*(b*x^2+a)^(3/2)/a^4/x^3
```

## Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.70

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2+Cx^4+Dx^6)}{x^{10}}\,dx = \frac{(a+bx^2)^{3/2}\,(16Ab^3x^6-24ab^2x^4(A+Bx^2)+6a^2bx^2(5A+6Bx^2+7Cx^4)-a^3(35A+45Bx^2+63Cx^4}{315a^4x^9}$$

input
```
Integrate[(Sqrt[a + b*x^2]*(A + B*x^2 + C*x^4 + D*x^6))/x^10,x]
```

output
```
((a + b*x^2)^(3/2)*(16*A*b^3*x^6 - 24*a*b^2*x^4*(A + B*x^2) + 6*a^2*b*x^2*
(5*A + 6*B*x^2 + 7*C*x^4) - a^3*(35*A + 45*B*x^2 + 63*C*x^4 + 105*D*x^6))
/(315*a^4*x^9)
```

## Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2334, 27, 2089, 1588, 359, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\left(A+Bx^2+Cx^4+Dx^6\right)}{x^{10}}dx$$

$$\Big\downarrow \text{2334}$$

$$-\frac{\int \frac{3\sqrt{bx^2+a}(2Ab-3a(Dx^4+Cx^2+B))}{x^8}dx}{9a}-\frac{A\left(a+bx^2\right)^{3/2}}{9ax^9}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{\sqrt{bx^2+a}(2Ab-3a(Dx^4+Cx^2+B))}{x^8}dx}{3a}-\frac{A\left(a+bx^2\right)^{3/2}}{9ax^9}$$

$$\Big\downarrow \text{2089}$$

$$-\frac{\int \frac{\sqrt{bx^2+a}(-3aDx^4-3aCx^2+2Ab-3aB)}{x^8}dx}{3a}-\frac{A\left(a+bx^2\right)^{3/2}}{9ax^9}$$

$$\Big\downarrow \text{1588}$$

$$-\frac{-\frac{\int \frac{\sqrt{bx^2+a}\left(21Dx^2a^2+21Ca^2-12bBa+8Ab^2\right)}{x^6}dx}{7a}-\frac{(a+bx^2)^{3/2}(2Ab-3aB)}{7ax^7}}{3a}-\frac{A\left(a+bx^2\right)^{3/2}}{9ax^9}$$

$$\Big\downarrow \text{359}$$

$$-\frac{-\frac{-\frac{\left(16Ab^3-3a\left(35a^2D-14abC+8b^2B\right)\right)\int \frac{\sqrt{bx^2+a}}{x^4}dx}{5a}-\frac{\left(a+bx^2\right)^{3/2}\left(21a^2C-12abB+8Ab^2\right)}{5ax^5}}{7a}-\frac{\left(a+bx^2\right)^{3/2}(2Ab-3aB)}{7ax^7}}{3a}-\frac{A\left(a+bx^2\right)^{3/2}}{9ax^9}$$

$\Big\downarrow$ 242

$$-\frac{-\frac{-\frac{\left(a+bx^2\right)^{3/2}\left(16Ab^3-3a\left(35a^2D-14abC+8b^2B\right)\right)}{15a^2x^3}-\frac{\left(a+bx^2\right)^{3/2}\left(21a^2C-12abB+8Ab^2\right)}{5ax^5}}{7a}-\frac{\left(a+bx^2\right)^{3/2}(2Ab-3aB)}{7ax^7}}{3a}-\frac{A\left(a+bx^2\right)^{3/2}}{9ax^9}$$

input
```
Int[(Sqrt[a + b*x^2]*(A + B*x^2 + C*x^4 + D*x^6))/x^10,x]
```

output
```
-1/9*(A*(a + b*x^2)^(3/2))/(a*x^9) - (-1/7*((2*A*b - 3*a*B)*(a + b*x^2)^(3
/2))/(a*x^7) - (-1/5*((8*A*b^2 - 12*a*b*B + 21*a^2*C)*(a + b*x^2)^(3/2))/(
a*x^5) + ((16*A*b^3 - 3*a*(8*b^2*B - 14*a*b*C + 35*a^2*D))*(a + b*x^2)^(3/
2))/(15*a^2*x^3))/(7*a))/(3*a)
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 1588
```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Simp[1/(d*f
^2*(m + 1))   Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x
) - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

rule 2089
```
Int[(u_)^(p_.)*((f_.)*(x_))^(m_.)*(z_)^(q_.), x_Symbol] :> Int[(f*x)^m*Expa
ndToSum[z, x]^q*ExpandToSum[u, x]^p, x] /; FreeQ[{f, m, p, q}, x] && Binomi
alQ[z, x] && TrinomialQ[u, x] &&  !(BinomialMatchQ[z, x] && TrinomialMatchQ
[u, x])
```

rule 2334
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coef
f[Pq, x, 0], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[A*
x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))   Int[
x^(m + 2)*(a + b*x^2)^p*(a*(m + 1)*Q - A*b*(m + 2*(p + 1) + 1)), x], x]] /;
 FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && IntegerQ[m/2] && ILtQ[(m + 1)/2 + p,
 0] && LtQ[m + Expon[Pq, x] + 2*p + 1, 0]
```

## Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.65

| method | result |
|---|---|
| pseudoelliptic | $-\frac{\left(\left(3Dx^{6}+\frac{9}{5}C\,x^{4}+\frac{9}{7}x^{2}B+A\right)a^{3}-\frac{6\left(\frac{7}{5}C\,x^{4}+\frac{6}{5}x^{2}B+A\right)x^{2}b\,a^{2}}{7}+\frac{24b^{2}x^{4}\left(x^{2}B+A\right)a}{35}-\frac{16A\,b^{3}x^{6}}{35}\right)\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{9x^{9}a^{4}}$ |
| gosper | $-\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}\left(-16A\,b^{3}x^{6}+24Ba\,b^{2}x^{6}-42C\,a^{2}b\,x^{6}+105Da^{3}x^{6}+24aA\,b^{2}x^{4}-36B\,a^{2}b\,x^{4}+63C\,a^{3}x^{4}-30a^{2}Ab\,x^{2}+45B\,a^{3}x\right.}{315x^{9}a^{4}}$ |
| orering | $-\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}\left(-16A\,b^{3}x^{6}+24Ba\,b^{2}x^{6}-42C\,a^{2}b\,x^{6}+105Da^{3}x^{6}+24aA\,b^{2}x^{4}-36B\,a^{2}b\,x^{4}+63C\,a^{3}x^{4}-30a^{2}Ab\,x^{2}+45B\,a^{3}x\right.}{315x^{9}a^{4}}$ |
| trager | $-\frac{\left(-16A\,x^{8}b^{4}+24B\,x^{8}a\,b^{3}-42C\,a^{2}b^{2}x^{8}+105Da^{3}b\,x^{8}+8A\,x^{6}a\,b^{3}-12B\,x^{6}a^{2}b^{2}+21C\,a^{3}b\,x^{6}+105Da^{4}x^{6}-6A\,x^{4}a^{2}b^{2}+9B\right.}{315x^{9}a^{4}}$ |
| default | $A\left(-\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{9a\,x^{9}}-\frac{2b\left(-\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{7a\,x^{7}}-\frac{4b\left(-\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{5a\,x^{5}}+\frac{2b\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{15a^{2}x^{3}}\right)}{7a}\right)}{3a}\right)+B\left(-\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{7a\,x^{7}}-\frac{4b\left(-\frac{\left(b\,x^{2}+\right.}{5a\,}\right.}{}\right.$ |

input
```
int((b*x^2+a)^(1/2)*(D*x^6+C*x^4+B*x^2+A)/x^10,x,method=_RETURNVERBOSE)
```

output
```
-1/9*((3*D*x^6+9/5*C*x^4+9/7*x^2*B+A)*a^3-6/7*(7/5*C*x^4+6/5*x^2*B+A)*x^2*
b*a^2+24/35*b^2*x^4*(B*x^2+A)*a-16/35*A*b^3*x^6)*(b*x^2+a)^(3/2)/x^9/a^4
```

## Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.98

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2+Cx^4+Dx^6)}{x^{10}}\,dx =$$
$$-\frac{((105\,Da^3b-42\,Ca^2b^2+24\,Bab^3-16\,Ab^4)x^8+(105\,Da^4+21\,Ca^3b-12\,Ba^2b^2+8\,Aab^3)x^6+35\,A}{315\,a^4x^9}$$

input
```
integrate((b*x^2+a)^(1/2)*(D*x^6+C*x^4+B*x^2+A)/x^10,x, algorithm="fricas"
)
```

output
```
-1/315*((105*D*a^3*b - 42*C*a^2*b^2 + 24*B*a*b^3 - 16*A*b^4)*x^8 + (105*D*
a^4 + 21*C*a^3*b - 12*B*a^2*b^2 + 8*A*a*b^3)*x^6 + 35*A*a^4 + 3*(21*C*a^4
+ 3*B*a^3*b - 2*A*a^2*b^2)*x^4 + 5*(9*B*a^4 + A*a^3*b)*x^2)*sqrt(b*x^2 + a
)/(a^4*x^9)
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1078 vs. $2(136) = 272$.

Time = 2.48 (sec) , antiderivative size = 1078, normalized size of antiderivative = 7.65

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2+Cx^4+Dx^6)}{x^{10}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x**2+a)**(1/2)*(D*x**6+C*x**4+B*x**2+A)/x**10,x)
```

output
```
-35*A*a**7*b**(19/2)*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b
**10*x**10 + 945*a**5*b**11*x**12 + 315*a**4*b**12*x**14) - 110*A*a**6*b**
(21/2)*x**2*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**1
0 + 945*a**5*b**11*x**12 + 315*a**4*b**12*x**14) - 114*A*a**5*b**(23/2)*x*
*4*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**10 + 945*a
**5*b**11*x**12 + 315*a**4*b**12*x**14) - 40*A*a**4*b**(25/2)*x**6*sqrt(a/
(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**10 + 945*a**5*b**11*
x**12 + 315*a**4*b**12*x**14) + 5*A*a**3*b**(27/2)*x**8*sqrt(a/(b*x**2) +
1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**10 + 945*a**5*b**11*x**12 + 315
*a**4*b**12*x**14) + 30*A*a**2*b**(29/2)*x**10*sqrt(a/(b*x**2) + 1)/(315*a
**7*b**9*x**8 + 945*a**6*b**10*x**10 + 945*a**5*b**11*x**12 + 315*a**4*b**
12*x**14) + 40*A*a*b**(31/2)*x**12*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**
8 + 945*a**6*b**10*x**10 + 945*a**5*b**11*x**12 + 315*a**4*b**12*x**14) +
16*A*b**(33/2)*x**14*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b
**10*x**10 + 945*a**5*b**11*x**12 + 315*a**4*b**12*x**14) - 15*B*a**5*b**(
9/2)*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a
**3*b**6*x**10) - 33*B*a**4*b**(11/2)*x**2*sqrt(a/(b*x**2) + 1)/(105*a**5*
b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 17*B*a**3*b**(13/2
)*x**4*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105
*a**3*b**6*x**10) - 3*B*a**2*b**(15/2)*x**6*sqrt(a/(b*x**2) + 1)/(105*a...
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.37

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2+Cx^4+Dx^6)}{x^{10}}\,dx = -\frac{(bx^2+a)^{\frac{3}{2}}D}{3\,ax^3} + \frac{2\,(bx^2+a)^{\frac{3}{2}}Cb}{15\,a^2x^3} - \frac{8\,(bx^2+a)^{\frac{3}{2}}Bb^2}{105\,a^3x^3} + \frac{16\,(bx^2+a)^{\frac{3}{2}}Ab^3}{315\,a^4x^3} - \frac{(bx^2+a)^{\frac{3}{2}}C}{5\,ax^5} + \frac{4\,(bx^2+a)^{\frac{3}{2}}Bb}{35\,a^2x^5} - \frac{8\,(bx^2+a)^{\frac{3}{2}}Ab^2}{105\,a^3x^5} - \frac{(bx^2+a)^{\frac{3}{2}}B}{7\,ax^7} + \frac{2\,(bx^2+a)^{\frac{3}{2}}Ab}{21\,a^2x^7} - \frac{(bx^2+a)^{\frac{3}{2}}A}{9\,ax^9}$$

input
```
integrate((b*x^2+a)^(1/2)*(D*x^6+C*x^4+B*x^2+A)/x^10,x, algorithm="maxima"
)
```

output
```
-1/3*(b*x^2 + a)^(3/2)*D/(a*x^3) + 2/15*(b*x^2 + a)^(3/2)*C*b/(a^2*x^3) -
8/105*(b*x^2 + a)^(3/2)*B*b^2/(a^3*x^3) + 16/315*(b*x^2 + a)^(3/2)*A*b^3/(
a^4*x^3) - 1/5*(b*x^2 + a)^(3/2)*C/(a*x^5) + 4/35*(b*x^2 + a)^(3/2)*B*b/(a
^2*x^5) - 8/105*(b*x^2 + a)^(3/2)*A*b^2/(a^3*x^5) - 1/7*(b*x^2 + a)^(3/2)*
B/(a*x^7) + 2/21*(b*x^2 + a)^(3/2)*A*b/(a^2*x^7) - 1/9*(b*x^2 + a)^(3/2)*A
/(a*x^9)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 772 vs. 2(125) = 250.

Time = 0.14 (sec) , antiderivative size = 772, normalized size of antiderivative = 5.48

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2+Cx^4+Dx^6)}{x^{10}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(1/2)*(D*x^6+C*x^4+B*x^2+A)/x^10,x, algorithm="giac")
```

output
```
2/315*(315*(sqrt(b)*x - sqrt(b*x^2 + a))^16*D*b^(3/2) - 1890*(sqrt(b)*x -
sqrt(b*x^2 + a))^14*D*a*b^(3/2) + 630*(sqrt(b)*x - sqrt(b*x^2 + a))^14*C*b
^(5/2) + 4830*(sqrt(b)*x - sqrt(b*x^2 + a))^12*D*a^2*b^(3/2) - 2310*(sqrt(
b)*x - sqrt(b*x^2 + a))^12*C*a*b^(5/2) + 1680*(sqrt(b)*x - sqrt(b*x^2 + a)
)^12*B*b^(7/2) - 6930*(sqrt(b)*x - sqrt(b*x^2 + a))^10*D*a^3*b^(3/2) + 315
0*(sqrt(b)*x - sqrt(b*x^2 + a))^10*C*a^2*b^(5/2) - 2520*(sqrt(b)*x - sqrt(
b*x^2 + a))^10*B*a*b^(7/2) + 5040*(sqrt(b)*x - sqrt(b*x^2 + a))^10*A*b^(9/
2) + 6300*(sqrt(b)*x - sqrt(b*x^2 + a))^8*D*a^4*b^(3/2) - 2142*(sqrt(b)*x
- sqrt(b*x^2 + a))^8*C*a^3*b^(5/2) + 504*(sqrt(b)*x - sqrt(b*x^2 + a))^8*B
*a^2*b^(7/2) + 3024*(sqrt(b)*x - sqrt(b*x^2 + a))^8*A*a*b^(9/2) - 3990*(sq
rt(b)*x - sqrt(b*x^2 + a))^6*D*a^5*b^(3/2) + 1218*(sqrt(b)*x - sqrt(b*x^2
+ a))^6*C*a^4*b^(5/2) - 336*(sqrt(b)*x - sqrt(b*x^2 + a))^6*B*a^3*b^(7/2)
+ 1344*(sqrt(b)*x - sqrt(b*x^2 + a))^6*A*a^2*b^(9/2) + 1890*(sqrt(b)*x - s
qrt(b*x^2 + a))^4*D*a^6*b^(3/2) - 882*(sqrt(b)*x - sqrt(b*x^2 + a))^4*C*a^
5*b^(5/2) + 864*(sqrt(b)*x - sqrt(b*x^2 + a))^4*B*a^4*b^(7/2) - 576*(sqrt(
b)*x - sqrt(b*x^2 + a))^4*A*a^3*b^(9/2) - 630*(sqrt(b)*x - sqrt(b*x^2 + a)
)^2*D*a^7*b^(3/2) + 378*(sqrt(b)*x - sqrt(b*x^2 + a))^2*C*a^6*b^(5/2) - 21
6*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a^5*b^(7/2) + 144*(sqrt(b)*x - sqrt(b*
x^2 + a))^2*A*a^4*b^(9/2) + 105*D*a^8*b^(3/2) - 42*C*a^7*b^(5/2) + 24*B*a^
6*b^(7/2) - 16*A*a^5*b^(9/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^9
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2+Cx^4+Dx^6)}{x^{10}}\,dx = \int \frac{\sqrt{b\,x^2+a}\,(A+B\,x^2+C\,x^4+x^6\,D)}{x^{10}}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(A + B*x^2 + C*x^4 + x^6*D))/x^10,x)
```

output
```
int(((a + b*x^2)^(1/2)*(A + B*x^2 + C*x^4 + x^6*D))/x^10, x)
```

## Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.51

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2+Cx^4+Dx^6)}{x^{10}}\,dx$$
$$= \frac{-35\sqrt{b\,x^2+a}\,a^4-50\sqrt{b\,x^2+a}\,a^3b\,x^2-63\sqrt{b\,x^2+a}\,a^3c\,x^4-105\sqrt{b\,x^2+a}\,a^3d\,x^6-3\sqrt{b\,x^2+a}\,a^2b^2x^4}{}$$

input
```
int((b*x^2+a)^(1/2)*(D*x^6+C*x^4+B*x^2+A)/x^10,x)
```

output
```
( - 35*sqrt(a + b*x**2)*a**4 - 50*sqrt(a + b*x**2)*a**3*b*x**2 - 63*sqrt(a
 + b*x**2)*a**3*c*x**4 - 105*sqrt(a + b*x**2)*a**3*d*x**6 - 3*sqrt(a + b*x
**2)*a**2*b**2*x**4 - 21*sqrt(a + b*x**2)*a**2*b*c*x**6 - 105*sqrt(a + b*x
**2)*a**2*b*d*x**8 + 4*sqrt(a + b*x**2)*a*b**3*x**6 + 42*sqrt(a + b*x**2)*
a*b**2*c*x**8 - 8*sqrt(a + b*x**2)*b**4*x**8 + 35*sqrt(b)*a**2*b*d*x**9 -
42*sqrt(b)*a*b**2*c*x**9 + 8*sqrt(b)*b**4*x**9)/(315*a**3*x**9)
```

# 3.196 $\int \frac{\sqrt{a+bx^2}(A+Bx^2+Cx^4+Dx^6)}{x^{12}}\,dx$



## Optimal result

Integrand size = 32, antiderivative size = 191

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2+Cx^4+Dx^6)}{x^{12}}\,dx = -\frac{A(a+bx^2)^{3/2}}{11ax^{11}}+\frac{(8Ab-11aB)(a+bx^2)^{3/2}}{99a^2x^9} - \frac{(16Ab^2-22abB+33a^2C)(a+bx^2)^{3/2}}{231a^3x^7} + \frac{(64Ab^3-11a(8b^2B-12abC+21a^2D))(a+bx^2)^{3/2}}{1155a^4x^5} - \frac{2b(64Ab^3-11a(8b^2B-12abC+21a^2D))(a+bx^2)^{3/2}}{3465a^5x^3}$$

output
```
-1/11*A*(b*x^2+a)^(3/2)/a/x^11+1/99*(8*A*b-11*B*a)*(b*x^2+a)^(3/2)/a^2/x^9
-1/231*(16*A*b^2-22*B*a*b+33*C*a^2)*(b*x^2+a)^(3/2)/a^3/x^7+1/1155*(64*A*b
^3-11*a*(8*B*b^2-12*C*a*b+21*D*a^2))*(b*x^2+a)^(3/2)/a^4/x^5-2/3465*b*(64*
A*b^3-11*a*(8*B*b^2-12*C*a*b+21*D*a^2))*(b*x^2+a)^(3/2)/a^5/x^3
```

## Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.72

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2+Cx^4+Dx^6)}{x^{12}}\,dx = -\frac{(a+bx^2)^{3/2}\left(128Ab^4x^8-16ab^3x^6(12A+11Bx^2)+a^4(315A+385Bx^2+495Cx^4+693Dx^6)+24a^2\right.}{3465a^5x^{11}}$$

input
```
Integrate[(Sqrt[a + b*x^2]*(A + B*x^2 + C*x^4 + D*x^6))/x^12,x]
```

output
```
-1/3465*((a + b*x^2)^(3/2)*(128*A*b^4*x^8 - 16*a*b^3*x^6*(12*A + 11*B*x^2)
 + a^4*(315*A + 385*B*x^2 + 495*C*x^4 + 693*D*x^6) + 24*a^2*b^2*x^4*(10*A
+ 11*x^2*(B + C*x^2)) - 2*a^3*b*x^2*(140*A + 33*(5*B*x^2 + 6*C*x^4 + 7*D*x
^6))))/(a^5*x^11)
```

## Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {2334, 2089, 1588, 27, 359, 245, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\left(A+Bx^2+Cx^4+Dx^6\right)}{x^{12}}dx$$

$$\Big\downarrow \text{2334}$$

$$-\frac{\int \frac{\sqrt{bx^2+a}\left(8Ab-11a\left(Dx^4+Cx^2+B\right)\right)}{x^{10}}dx}{11a}-\frac{A\left(a+bx^2\right)^{3/2}}{11ax^{11}}$$

$$\Big\downarrow \text{2089}$$

$$-\frac{\int \frac{\sqrt{bx^2+a}\left(-11aDx^4-11aCx^2+8Ab-11aB\right)}{x^{10}}dx}{11a}-\frac{A\left(a+bx^2\right)^{3/2}}{11ax^{11}}$$

$$\Big\downarrow \text{1588}$$

$$-\frac{-\frac{\int \frac{3\sqrt{bx^2+a}\left(16Ab^2+33a^2Dx^2-11a(2bB-3aC)\right)}{x^8}dx}{9a}-\frac{(a+bx^2)^{3/2}(8Ab-11aB)}{9ax^9}}{11a}-\frac{A(a+bx^2)^{3/2}}{11ax^{11}}$$

$$\Big\downarrow \; 27$$

$$-\frac{-\frac{\int \frac{\sqrt{bx^2+a}\left(33Dx^2a^2+33Ca^2-22bBa+16Ab^2\right)}{x^8}dx}{3a}-\frac{(a+bx^2)^{3/2}(8Ab-11aB)}{9ax^9}}{11a}-\frac{A(a+bx^2)^{3/2}}{11ax^{11}}$$

$$\Big\downarrow \; 359$$

$$-\frac{-\frac{-\frac{\left(64Ab^3-11a\left(21a^2D-12abC+8b^2B\right)\right)\int \frac{\sqrt{bx^2+a}}{x^6}dx}{7a}-\frac{(a+bx^2)^{3/2}\left(33a^2C-22abB+16Ab^2\right)}{7ax^7}}{3a}-\frac{(a+bx^2)^{3/2}(8Ab-11aB)}{9ax^9}}{11a}-\frac{A(a+bx^2)^{3/2}}{11ax^{11}}$$

$$\Big\downarrow \; 245$$

$$-\frac{-\frac{-\frac{\left(64Ab^3-11a\left(21a^2D-12abC+8b^2B\right)\right)\left(-\frac{2b\int \frac{\sqrt{bx^2+a}}{x^4}dx}{5a}-\frac{(a+bx^2)^{3/2}}{5ax^5}\right)}{7a}-\frac{(a+bx^2)^{3/2}\left(33a^2C-22abB+16Ab^2\right)}{7ax^7}}{3a}-\frac{(a+bx^2)^{3/2}(8Ab-11aB)}{9ax^9}}{11a}-\frac{A(a+bx^2)^{3/2}}{11ax^{11}}$$

$$\Big\downarrow \; 242$$

$$-\frac{-\frac{-\frac{(a+bx^2)^{3/2}\left(33a^2C-22abB+16Ab^2\right)}{7ax^7}-\frac{\left(\frac{2b(a+bx^2)^{3/2}}{15a^2x^3}-\frac{(a+bx^2)^{3/2}}{5ax^5}\right)\left(64Ab^3-11a\left(21a^2D-12abC+8b^2B\right)\right)}{7a}}{3a}-\frac{(a+bx^2)^{3/2}(8Ab-11aB)}{9ax^9}}{11a}-\frac{A(a+bx^2)^{3/2}}{11ax^{11}}$$

input
```
Int[(Sqrt[a + b*x^2]*(A + B*x^2 + C*x^4 + D*x^6))/x^12,x]
```

output
```
-1/11*(A*(a + b*x^2)^(3/2))/(a*x^11) - (-1/9*((8*A*b - 11*a*B)*(a + b*x^2)
^(3/2))/(a*x^9) - (-1/7*((16*A*b^2 - 22*a*b*B + 33*a^2*C)*(a + b*x^2)^(3/2
))/(a*x^7) - ((64*A*b^3 - 11*a*(8*b^2*B - 12*a*b*C + 21*a^2*D))*(-1/5*(a +
 b*x^2)^(3/2)/(a*x^5) + (2*b*(a + b*x^2)^(3/2))/(15*a^2*x^3)))/(7*a))/(3*a
))/(11*a)
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 1588
```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Simp[1/(d*f
^2*(m + 1))   Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x
) - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

rule 2089
```
Int[(u_)^(p_.)*((f_.)*(x_))^(m_.)*(z_)^(q_.), x_Symbol] :> Int[(f*x)^m*Expa
ndToSum[z, x]^q*ExpandToSum[u, x]^p, x] /; FreeQ[{f, m, p, q}, x] && Binomi
alQ[z, x] && TrinomialQ[u, x] &&  !(BinomialMatchQ[z, x] && TrinomialMatchQ
[u, x])
```

rule 2334
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coef
f[Pq, x, 0], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[A*
x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))   Int[
x^(m + 2)*(a + b*x^2)^p*(a*(m + 1)*Q - A*b*(m + 2*(p + 1) + 1)), x], x]] /;
 FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && IntegerQ[m/2] && ILtQ[(m + 1)/2 + p,
 0] && LtQ[m + Expon[Pq, x] + 2*p + 1, 0]
```

## Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.64

| method | result |
|---|---|
| pseudoelliptic | $-\frac{\left(\left(\frac{11}{5}Dx^6+\frac{11}{7}Cx^4+\frac{11}{9}x^2B+A\right)a^4-\frac{8x^2b\left(\frac{33}{20}Dx^6+\frac{99}{70}Cx^4+\frac{33}{28}x^2B+A\right)a^3}{9}+\frac{16\left(\frac{11}{10}Cx^4+\frac{11}{10}x^2B+A\right)x^4b^2a^2}{21}-\frac{64\left(\frac{11x^2B}{12}+\right.}{105}\right.}{11x^{11}a^5}$ |
| gosper | $-\frac{\left(bx^2+a\right)^{\frac{3}{2}}\left(128Ax^8b^4-176Bx^8ab^3+264Ca^2b^2x^8-462Da^3bx^8-192Ax^6ab^3+264Bx^6a^2b^2-396Ca^3bx^6+693Da^4x^6\right.}{3465x^{11}a^5}$ |
| orering | $-\frac{\left(bx^2+a\right)^{\frac{3}{2}}\left(128Ax^8b^4-176Bx^8ab^3+264Ca^2b^2x^8-462Da^3bx^8-192Ax^6ab^3+264Bx^6a^2b^2-396Ca^3bx^6+693Da^4x^6\right.}{3465x^{11}a^5}$ |
| trager | $-\frac{\left(128Ab^5x^{10}-176Bab^4x^{10}+264Ca^2b^3x^{10}-462Da^3b^2x^{10}-64aAb^4x^8+88Ba^2b^3x^8-132Ca^3b^2x^8+231Da^4bx^8+48a^2\right.}{3}$ |
| default | $A\left(-\frac{\left(bx^2+a\right)^{\frac{3}{2}}}{11a\,x^{11}}-\frac{8b\left(-\frac{\left(bx^2+a\right)^{\frac{3}{2}}}{9a\,x^9}-\frac{2b\left(-\frac{\left(bx^2+a\right)^{\frac{3}{2}}}{7a\,x^7}-\frac{4b\left(-\frac{\left(bx^2+a\right)^{\frac{3}{2}}}{5a\,x^5}+\frac{2b\left(bx^2+a\right)^{\frac{3}{2}}}{15a^2x^3}\right)}{7a}\right)}{3a}\right)}{11a}\right)+B\left(-\frac{\left(bx^2+a\right.}{9a\,x^9}\right.$ |

input
```
int((b*x^2+a)^(1/2)*(D*x^6+C*x^4+B*x^2+A)/x^12,x,method=_RETURNVERBOSE)
```

output
```
-1/11*((11/5*D*x^6+11/7*C*x^4+11/9*x^2*B+A)*a^4-8/9*x^2*b*(33/20*D*x^6+99/
70*C*x^4+33/28*x^2*B+A)*a^3+16/21*(11/10*C*x^4+11/10*x^2*B+A)*x^4*b^2*a^2-
64/105*(11/12*x^2*B+A)*x^6*b^3*a+128/315*A*x^8*b^4)*(b*x^2+a)^(3/2)/x^11/a
^5
```

## Fricas [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2+Cx^4+Dx^6)}{x^{12}}\,dx$$
$$= \frac{(2\,(231\,Da^3b^2-132\,Ca^2b^3+88\,Bab^4-64\,Ab^5)x^{10}-(231\,Da^4b-132\,Ca^3b^2+88\,Ba^2b^3-64\,Aab^4)x^8}{}$$

input
```
integrate((b*x^2+a)^(1/2)*(D*x^6+C*x^4+B*x^2+A)/x^12,x, algorithm="fricas"
)
```

output
```
1/3465*(2*(231*D*a^3*b^2 - 132*C*a^2*b^3 + 88*B*a*b^4 - 64*A*b^5)*x^10 - (
231*D*a^4*b - 132*C*a^3*b^2 + 88*B*a^2*b^3 - 64*A*a*b^4)*x^8 - 3*(231*D*a^
5 + 33*C*a^4*b - 22*B*a^3*b^2 + 16*A*a^2*b^3)*x^6 - 315*A*a^5 - 5*(99*C*a^
5 + 11*B*a^4*b - 8*A*a^3*b^2)*x^4 - 35*(11*B*a^5 + A*a^4*b)*x^2)*sqrt(b*x^
2 + a)/(a^5*x^11)
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1890 vs. 2(189) = 378.

Time = 3.25 (sec) , antiderivative size = 1890, normalized size of antiderivative = 9.90

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2+Cx^4+Dx^6)}{x^{12}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x**2+a)**(1/2)*(D*x**6+C*x**4+B*x**2+A)/x**12,x)
```

output
```
-315*A*a**9*b**(33/2)*sqrt(a/(b*x**2) + 1)/(3465*a**9*b**16*x**10 + 13860*
a**8*b**17*x**12 + 20790*a**7*b**18*x**14 + 13860*a**6*b**19*x**16 + 3465*
a**5*b**20*x**18) - 1295*A*a**8*b**(35/2)*x**2*sqrt(a/(b*x**2) + 1)/(3465*
a**9*b**16*x**10 + 13860*a**8*b**17*x**12 + 20790*a**7*b**18*x**14 + 13860
*a**6*b**19*x**16 + 3465*a**5*b**20*x**18) - 1990*A*a**7*b**(37/2)*x**4*sq
rt(a/(b*x**2) + 1)/(3465*a**9*b**16*x**10 + 13860*a**8*b**17*x**12 + 20790
*a**7*b**18*x**14 + 13860*a**6*b**19*x**16 + 3465*a**5*b**20*x**18) - 1358
*A*a**6*b**(39/2)*x**6*sqrt(a/(b*x**2) + 1)/(3465*a**9*b**16*x**10 + 13860
*a**8*b**17*x**12 + 20790*a**7*b**18*x**14 + 13860*a**6*b**19*x**16 + 3465
*a**5*b**20*x**18) - 343*A*a**5*b**(41/2)*x**8*sqrt(a/(b*x**2) + 1)/(3465*
a**9*b**16*x**10 + 13860*a**8*b**17*x**12 + 20790*a**7*b**18*x**14 + 13860
*a**6*b**19*x**16 + 3465*a**5*b**20*x**18) - 35*A*a**4*b**(43/2)*x**10*sqr
t(a/(b*x**2) + 1)/(3465*a**9*b**16*x**10 + 13860*a**8*b**17*x**12 + 20790*
a**7*b**18*x**14 + 13860*a**6*b**19*x**16 + 3465*a**5*b**20*x**18) - 280*A
*a**3*b**(45/2)*x**12*sqrt(a/(b*x**2) + 1)/(3465*a**9*b**16*x**10 + 13860*
a**8*b**17*x**12 + 20790*a**7*b**18*x**14 + 13860*a**6*b**19*x**16 + 3465*
a**5*b**20*x**18) - 560*A*a**2*b**(47/2)*x**14*sqrt(a/(b*x**2) + 1)/(3465*
a**9*b**16*x**10 + 13860*a**8*b**17*x**12 + 20790*a**7*b**18*x**14 + 13860
*a**6*b**19*x**16 + 3465*a**5*b**20*x**18) - 448*A*a*b**(49/2)*x**16*sqrt(
a/(b*x**2) + 1)/(3465*a**9*b**16*x**10 + 13860*a**8*b**17*x**12 + 20790...
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.44

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2+Cx^4+Dx^6)}{x^{12}}\,dx = \frac{2\,(bx^2+a)^{\frac{3}{2}}Db}{15\,a^2x^3} - \frac{8\,(bx^2+a)^{\frac{3}{2}}Cb^2}{105\,a^3x^3} + \frac{16\,(bx^2+a)^{\frac{3}{2}}Bb^3}{315\,a^4x^3} - \frac{128\,(bx^2+a)^{\frac{3}{2}}Ab^4}{3465\,a^5x^3} - \frac{(bx^2+a)^{\frac{3}{2}}D}{5\,ax^5} + \frac{4\,(bx^2+a)^{\frac{3}{2}}Cb}{35\,a^2x^5} - \frac{8\,(bx^2+a)^{\frac{3}{2}}Bb^2}{105\,a^3x^5} + \frac{64\,(bx^2+a)^{\frac{3}{2}}Ab^3}{1155\,a^4x^5} - \frac{(bx^2+a)^{\frac{3}{2}}C}{7\,ax^7} + \frac{2\,(bx^2+a)^{\frac{3}{2}}Bb}{21\,a^2x^7} - \frac{16\,(bx^2+a)^{\frac{3}{2}}Ab^2}{231\,a^3x^7} - \frac{(bx^2+a)^{\frac{3}{2}}B}{9\,ax^9} + \frac{8\,(bx^2+a)^{\frac{3}{2}}Ab}{99\,a^2x^9} - \frac{(bx^2+a)^{\frac{3}{2}}A}{11\,ax^{11}}$$

input
```
integrate((b*x^2+a)^(1/2)*(D*x^6+C*x^4+B*x^2+A)/x^12,x, algorithm="maxima"
)
```

output
```
2/15*(b*x^2 + a)^(3/2)*D*b/(a^2*x^3) - 8/105*(b*x^2 + a)^(3/2)*C*b^2/(a^3*
x^3) + 16/315*(b*x^2 + a)^(3/2)*B*b^3/(a^4*x^3) - 128/3465*(b*x^2 + a)^(3/
2)*A*b^4/(a^5*x^3) - 1/5*(b*x^2 + a)^(3/2)*D/(a*x^5) + 4/35*(b*x^2 + a)^(3
/2)*C*b/(a^2*x^5) - 8/105*(b*x^2 + a)^(3/2)*B*b^2/(a^3*x^5) + 64/1155*(b*x
^2 + a)^(3/2)*A*b^3/(a^4*x^5) - 1/7*(b*x^2 + a)^(3/2)*C/(a*x^7) + 2/21*(b*
x^2 + a)^(3/2)*B*b/(a^2*x^7) - 16/231*(b*x^2 + a)^(3/2)*A*b^2/(a^3*x^7) -
1/9*(b*x^2 + a)^(3/2)*B/(a*x^9) + 8/99*(b*x^2 + a)^(3/2)*A*b/(a^2*x^9) - 1
/11*(b*x^2 + a)^(3/2)*A/(a*x^11)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 884 vs. $2(171) = 342$.

Time = 0.15 (sec) , antiderivative size = 884, normalized size of antiderivative = 4.63

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2+Cx^4+Dx^6)}{x^{12}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(1/2)*(D*x^6+C*x^4+B*x^2+A)/x^12,x, algorithm="giac")
```

output
```
4/3465*(3465*(sqrt(b)*x - sqrt(b*x^2 + a))^18*D*b^(5/2) - 19635*(sqrt(b)*x
 - sqrt(b*x^2 + a))^16*D*a*b^(5/2) + 9240*(sqrt(b)*x - sqrt(b*x^2 + a))^16
*C*b^(7/2) + 46200*(sqrt(b)*x - sqrt(b*x^2 + a))^14*D*a^2*b^(5/2) - 32340*
(sqrt(b)*x - sqrt(b*x^2 + a))^14*C*a*b^(7/2) + 27720*(sqrt(b)*x - sqrt(b*x
^2 + a))^14*B*b^(9/2) - 59136*(sqrt(b)*x - sqrt(b*x^2 + a))^12*D*a^3*b^(5/
2) + 39732*(sqrt(b)*x - sqrt(b*x^2 + a))^12*C*a^2*b^(7/2) - 38808*(sqrt(b)
*x - sqrt(b*x^2 + a))^12*B*a*b^(9/2) + 88704*(sqrt(b)*x - sqrt(b*x^2 + a))
^12*A*b^(11/2) + 47586*(sqrt(b)*x - sqrt(b*x^2 + a))^10*D*a^4*b^(5/2) - 21
252*(sqrt(b)*x - sqrt(b*x^2 + a))^10*C*a^3*b^(7/2) + 1848*(sqrt(b)*x - sqr
t(b*x^2 + a))^10*B*a^2*b^(9/2) + 59136*(sqrt(b)*x - sqrt(b*x^2 + a))^10*A*
a*b^(11/2) - 30030*(sqrt(b)*x - sqrt(b*x^2 + a))^8*D*a^5*b^(5/2) + 11220*(
sqrt(b)*x - sqrt(b*x^2 + a))^8*C*a^4*b^(7/2) - 1320*(sqrt(b)*x - sqrt(b*x^
2 + a))^8*B*a^3*b^(9/2) + 21120*(sqrt(b)*x - sqrt(b*x^2 + a))^8*A*a^2*b^(1
1/2) + 18480*(sqrt(b)*x - sqrt(b*x^2 + a))^6*D*a^6*b^(5/2) - 12540*(sqrt(b
)*x - sqrt(b*x^2 + a))^6*C*a^5*b^(7/2) + 14520*(sqrt(b)*x - sqrt(b*x^2 + a
))^6*B*a^4*b^(9/2) - 10560*(sqrt(b)*x - sqrt(b*x^2 + a))^6*A*a^3*b^(11/2)
- 9240*(sqrt(b)*x - sqrt(b*x^2 + a))^4*D*a^7*b^(5/2) + 7260*(sqrt(b)*x - s
qrt(b*x^2 + a))^4*C*a^6*b^(7/2) - 4840*(sqrt(b)*x - sqrt(b*x^2 + a))^4*B*a
^5*b^(9/2) + 3520*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*a^4*b^(11/2) + 2541*(s
qrt(b)*x - sqrt(b*x^2 + a))^2*D*a^8*b^(5/2) - 1452*(sqrt(b)*x - sqrt(b*...
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2+Cx^4+Dx^6)}{x^{12}}\,dx = \int \frac{\sqrt{b\,x^2+a}\,(A+B\,x^2+C\,x^4+x^6\,D)}{x^{12}}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(A + B*x^2 + C*x^4 + x^6*D))/x^12,x)
```

output
```
int(((a + b*x^2)^(1/2)*(A + B*x^2 + C*x^4 + x^6*D))/x^12, x)
```

## Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.43

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2+Cx^4+Dx^6)}{x^{12}}\,dx$$
$$= \frac{-105\sqrt{b\,x^2+a}\,a^5-140\sqrt{b\,x^2+a}\,a^4b\,x^2-165\sqrt{b\,x^2+a}\,a^4c\,x^4-231\sqrt{b\,x^2+a}\,a^4d\,x^6-5\sqrt{b\,x^2+a}\,a^3b}{}$$

input
```
int((b*x^2+a)^(1/2)*(D*x^6+C*x^4+B*x^2+A)/x^12,x)
```

output
```
( - 105*sqrt(a + b*x**2)*a**5 - 140*sqrt(a + b*x**2)*a**4*b*x**2 - 165*sqr
t(a + b*x**2)*a**4*c*x**4 - 231*sqrt(a + b*x**2)*a**4*d*x**6 - 5*sqrt(a +
b*x**2)*a**3*b**2*x**4 - 33*sqrt(a + b*x**2)*a**3*b*c*x**6 - 77*sqrt(a + b
*x**2)*a**3*b*d*x**8 + 6*sqrt(a + b*x**2)*a**2*b**3*x**6 + 44*sqrt(a + b*x
**2)*a**2*b**2*c*x**8 + 154*sqrt(a + b*x**2)*a**2*b**2*d*x**10 - 8*sqrt(a
+ b*x**2)*a*b**4*x**8 - 88*sqrt(a + b*x**2)*a*b**3*c*x**10 + 16*sqrt(a + b
*x**2)*b**5*x**10 - 154*sqrt(b)*a**2*b**2*d*x**11 + 88*sqrt(b)*a*b**3*c*x*
*11 - 16*sqrt(b)*b**5*x**11)/(1155*a**4*x**11)
```

# 3.197 $\int \frac{\sqrt{a+bx^2}(A+Bx^2+Cx^4+Dx^6)}{x^{14}}\,dx$



## Optimal result

Integrand size = 32, antiderivative size = 243

$$\begin{aligned}\int \frac{\sqrt{a+bx^2}(A+Bx^2+Cx^4+Dx^6)}{x^{14}}\,dx
&= -\frac{A(a+bx^2)^{3/2}}{13ax^{13}}+\frac{(10Ab-13aB)\,(a+bx^2)^{3/2}}{143a^2x^{11}}\\
&\quad-\frac{(80Ab^2-104abB+143a^2C)\,(a+bx^2)^{3/2}}{1287a^3x^9}\\
&\quad+\frac{(160Ab^3-13a(16b^2B-22abC+33a^2D))\,(a+bx^2)^{3/2}}{3003a^4x^7}\\
&\quad-\frac{4b(160Ab^3-13a(16b^2B-22abC+33a^2D))\,(a+bx^2)^{3/2}}{15015a^5x^5}\\
&\quad+\frac{8b^2(160Ab^3-13a(16b^2B-22abC+33a^2D))\,(a+bx^2)^{3/2}}{45045a^6x^3}\end{aligned}$$

output

```
-1/13*A*(b*x^2+a)^(3/2)/a/x^13+1/143*(10*A*b-13*B*a)*(b*x^2+a)^(3/2)/a^2/x
^11-1/1287*(80*A*b^2-104*B*a*b+143*C*a^2)*(b*x^2+a)^(3/2)/a^3/x^9+1/3003*(
160*A*b^3-13*a*(16*B*b^2-22*C*a*b+33*D*a^2))*(b*x^2+a)^(3/2)/a^4/x^7-4/150
15*b*(160*A*b^3-13*a*(16*B*b^2-22*C*a*b+33*D*a^2))*(b*x^2+a)^(3/2)/a^5/x^5
+8/45045*b^2*(160*A*b^3-13*a*(16*B*b^2-22*C*a*b+33*D*a^2))*(b*x^2+a)^(3/2)
/a^6/x^3
```

## Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.70

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2+Cx^4+Dx^6)}{x^{14}}\,dx = \frac{(a+bx^2)^{3/2}(1280Ab^5x^{10}-128ab^4x^8(15A+13Bx^2)+16a^2b^3x^6(150A+156Bx^2+143Cx^4)+2a^4bx^2(1$$

input
```
Integrate[(Sqrt[a + b*x^2]*(A + B*x^2 + C*x^4 + D*x^6))/x^14,x]
```

output
```
((a + b*x^2)^(3/2)*(1280*A*b^5*x^10 - 128*a*b^4*x^8*(15*A + 13*B*x^2) + 16
*a^2*b^3*x^6*(150*A + 156*B*x^2 + 143*C*x^4) + 2*a^4*b*x^2*(1575*A + 1820*
B*x^2 + 2145*C*x^4 + 2574*D*x^6) - 8*a^3*b^2*x^4*(350*A + 390*B*x^2 + 429*
x^4*(C + D*x^2)) - 5*a^5*(693*A + 13*(63*B*x^2 + 77*C*x^4 + 99*D*x^6))))/(
45045*a^6*x^13)
```

## Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.89, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {2334, 2089, 1588, 359, 245, 245, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2+Cx^4+Dx^6)}{x^{14}}\,dx$$

$$\Big\downarrow \text{2334}$$

$$-\frac{\int \frac{\sqrt{bx^2+a}(10Ab-13a(Dx^4+Cx^2+B))}{x^{12}}dx}{13a}-\frac{A(a+bx^2)^{3/2}}{13ax^{13}}$$

$$\Big\downarrow \text{2089}$$

$$-\frac{\int \frac{\sqrt{bx^2+a}(-13aDx^4-13aCx^2+10Ab-13aB)}{x^{12}}dx}{13a}-\frac{A(a+bx^2)^{3/2}}{13ax^{13}}$$

$$\Big\downarrow \text{1588}$$

$$-\frac{-\frac{\int \frac{\sqrt{bx^2+a}\left(143Dx^2a^2+143Ca^2-104bBa+80Ab^2\right)}{x^{10}}dx}{11a}-\frac{(a+bx^2)^{3/2}(10Ab-13aB)}{11ax^{11}}}{13a}-\frac{A\left(a+bx^2\right)^{3/2}}{13ax^{13}}$$

$\big\downarrow$ 359

$$-\frac{-\frac{-\frac{\left(160Ab^3-13a\left(33a^2D-22abC+16b^2B\right)\right)\int\frac{\sqrt{bx^2+a}}{x^8}dx}{3a}-\frac{\left(a+bx^2\right)^{3/2}\left(143a^2C-104abB+80Ab^2\right)}{9ax^9}}{11a}-\frac{(a+bx^2)^{3/2}(10Ab-13aB)}{11ax^{11}}}{13a}-\frac{A\left(a+bx^2\right)^{3/2}}{13ax^{13}}$$

$\big\downarrow$ 245

$$-\frac{-\frac{-\frac{\left(160Ab^3-13a\left(33a^2D-22abC+16b^2B\right)\right)\left(-\frac{4b\int\frac{\sqrt{bx^2+a}}{x^6}dx}{7a}-\frac{\left(a+bx^2\right)^{3/2}}{7ax^7}\right)}{3a}-\frac{\left(a+bx^2\right)^{3/2}\left(143a^2C-104abB+80Ab^2\right)}{9ax^9}}{11a}-\frac{(a+bx^2)^{3/2}(10Ab-13a}{11ax^{11}}}{13a}$$

$$\frac{A\left(a+bx^2\right)^{3/2}}{13ax^{13}}$$

$\big\downarrow$ 245

$$-\frac{-\frac{-\frac{\left(160Ab^3-13a\left(33a^2D-22abC+16b^2B\right)\right)\left(-\frac{4b\left(-\frac{2b\int\frac{\sqrt{bx^2+a}}{x^4}dx}{5a}-\frac{\left(a+bx^2\right)^{3/2}}{5ax^5}\right)}{7a}-\frac{\left(a+bx^2\right)^{3/2}}{7ax^7}\right)}{3a}-\frac{\left(a+bx^2\right)^{3/2}\left(143a^2C-104abB+80Ab^2\right)}{9ax^9}}{11a}-\frac{(a}{}}{13a}$$

$$\frac{A\left(a+bx^2\right)^{3/2}}{13ax^{13}}$$

$\big\downarrow$ 242

$$-\frac{-\frac{-\frac{\left(a+bx^2\right)^{3/2}\left(143a^2C-104abB+80Ab^2\right)}{9ax^9}-\frac{\left(-\frac{4b\left(\frac{2b\left(a+bx^2\right)^{3/2}}{15a^2x^3}-\frac{\left(a+bx^2\right)^{3/2}}{5ax^5}\right)}{7a}-\frac{\left(a+bx^2\right)^{3/2}}{7ax^7}\right)\left(160Ab^3-13a\left(33a^2D-22abC+16b^2B\right)\right)}{3a}}{11a}-\frac{(a+b}{}}{13a}$$

$$\frac{A\left(a+bx^2\right)^{3/2}}{13ax^{13}}$$

input
```
Int[(Sqrt[a + b*x^2]*(A + B*x^2 + C*x^4 + D*x^6))/x^14,x]
```

output
```
-1/13*(A*(a + b*x^2)^(3/2))/(a*x^13) - (-1/11*((10*A*b - 13*a*B)*(a + b*x^
2)^(3/2))/(a*x^11) - (-1/9*((80*A*b^2 - 104*a*b*B + 143*a^2*C)*(a + b*x^2)
^(3/2))/(a*x^9) - ((160*A*b^3 - 13*a*(16*b^2*B - 22*a*b*C + 33*a^2*D))*(-1
/7*(a + b*x^2)^(3/2)/(a*x^7) - (4*b*(-1/5*(a + b*x^2)^(3/2)/(a*x^5) + (2*b
*(a + b*x^2)^(3/2))/(15*a^2*x^3)))/(7*a)))/(3*a))/(11*a))/(13*a)
```

### Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 1588
```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Simp[1/(d*f
^2*(m + 1))   Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x
) - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

rule 2089
```
Int[(u_)^(p_.)*((f_.)*(x_))^(m_.)*(z_)^(q_.), x_Symbol] :> Int[(f*x)^m*Expa
ndToSum[z, x]^q*ExpandToSum[u, x]^p, x] /; FreeQ[{f, m, p, q}, x] && Binomi
alQ[z, x] && TrinomialQ[u, x] &&  !(BinomialMatchQ[z, x] && TrinomialMatchQ
[u, x])
```

rule 2334
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coef
f[Pq, x, 0], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[A*
x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))   Int[
x^(m + 2)*(a + b*x^2)^p*(a*(m + 1)*Q - A*b*(m + 2*(p + 1) + 1)), x], x]] /;
 FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && IntegerQ[m/2] && ILtQ[(m + 1)/2 + p,
 0] && LtQ[m + Expon[Pq, x] + 2*p + 1, 0]
```

## Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.63

| method | result |
|---|---|
| pseudoelliptic | $-\frac{\left(\left(\frac{13}{7}Dx^{6}+\frac{13}{9}C\,x^{4}+\frac{13}{11}x^{2}B+A\right)a^{5}-\frac{10\left(\frac{286}{175}Dx^{6}+\frac{143}{105}C\,x^{4}+\frac{52}{45}x^{2}B+A\right)x^{2}b\,a^{4}}{11}+\frac{80\left(\frac{429}{350}Dx^{6}+\frac{429}{350}C\,x^{4}+\frac{39}{35}x^{2}B+A\right)x^{4}b^{2}a^{3}}{99}}{13x^{13}a^{6}}$ |
| gosper | $-\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}\left(-1280A\,b^{5}x^{10}+1664Ba\,b^{4}x^{10}-2288C\,a^{2}b^{3}x^{10}+3432Da^{3}b^{2}x^{10}+1920aA\,b^{4}x^{8}-2496B\,a^{2}b^{3}x^{8}+3432C\,a^{3}b^{2}\right.}{}$ |
| orering | $-\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}\left(-1280A\,b^{5}x^{10}+1664Ba\,b^{4}x^{10}-2288C\,a^{2}b^{3}x^{10}+3432Da^{3}b^{2}x^{10}+1920aA\,b^{4}x^{8}-2496B\,a^{2}b^{3}x^{8}+3432C\,a^{3}b^{2}\right.}{}$ |
| trager | $-\frac{\left(-1280A\,b^{6}x^{12}+1664Ba\,b^{5}x^{12}-2288C\,a^{2}b^{4}x^{12}+3432Da^{3}b^{3}x^{12}+640Aa\,b^{5}x^{10}-832B\,a^{2}b^{4}x^{10}+1144C\,a^{3}b^{3}x^{10}-1716\right.}{}$ |
| default | $A\left(-\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{13a\,x^{13}}-\frac{10b\left(-\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{11a\,x^{11}}-\frac{8b\left(-\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{9a\,x^{9}}-\frac{2b\left(-\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{7a\,x^{7}}-\frac{4b\left(-\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{5a\,x^{5}}+\frac{2b\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{15a^{2}x^{3}}\right)}{7a}\right)}{3a}\right)}{11a}\right)}{13a}\right)$ |

input
```
int((b*x^2+a)^(1/2)*(D*x^6+C*x^4+B*x^2+A)/x^14,x,method=_RETURNVERBOSE)
```

output
```
-1/13*((13/7*D*x^6+13/9*C*x^4+13/11*x^2*B+A)*a^5-10/11*(286/175*D*x^6+143/
105*C*x^4+52/45*x^2*B+A)*x^2*b*a^4+80/99*(429/350*D*x^6+429/350*C*x^4+39/3
5*x^2*B+A)*x^4*b^2*a^3-160/231*(143/150*C*x^4+26/25*x^2*B+A)*x^6*b^3*a^2+1
28/231*(13/15*x^2*B+A)*x^8*b^4*a-256/693*A*b^5*x^10)*(b*x^2+a)^(3/2)/x^13/
a^6
```

## Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2+Cx^4+Dx^6)}{x^{14}}\,dx =$$
$$-\frac{(8\,(429\,Da^3b^3-286\,Ca^2b^4+208\,Bab^5-160\,Ab^6)x^{12}-4\,(429\,Da^4b^2-286\,Ca^3b^3+208\,Ba^2b^4-160}{}$$

input
```
integrate((b*x^2+a)^(1/2)*(D*x^6+C*x^4+B*x^2+A)/x^14,x, algorithm="fricas"
)
```

output
```
-1/45045*(8*(429*D*a^3*b^3 - 286*C*a^2*b^4 + 208*B*a*b^5 - 160*A*b^6)*x^12
 - 4*(429*D*a^4*b^2 - 286*C*a^3*b^3 + 208*B*a^2*b^4 - 160*A*a*b^5)*x^10 +
3*(429*D*a^5*b - 286*C*a^4*b^2 + 208*B*a^3*b^3 - 160*A*a^2*b^4)*x^8 + 3465
*A*a^6 + 5*(1287*D*a^6 + 143*C*a^5*b - 104*B*a^4*b^2 + 80*A*a^3*b^3)*x^6 +
 35*(143*C*a^6 + 13*B*a^5*b - 10*A*a^4*b^2)*x^4 + 315*(13*B*a^6 + A*a^5*b)
*x^2)*sqrt(b*x^2 + a)/(a^6*x^13)
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2990 vs. 2(243) = 486.

Time = 4.32 (sec) , antiderivative size = 2990, normalized size of antiderivative = 12.30

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2+Cx^4+Dx^6)}{x^{14}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x**2+a)**(1/2)*(D*x**6+C*x**4+B*x**2+A)/x**14,x)
```

output
```
-693*A*a**11*b**(51/2)*sqrt(a/(b*x**2) + 1)/(9009*a**11*b**25*x**12 + 4504
5*a**10*b**26*x**14 + 90090*a**9*b**27*x**16 + 90090*a**8*b**28*x**18 + 45
045*a**7*b**29*x**20 + 9009*a**6*b**30*x**22) - 3528*A*a**10*b**(53/2)*x**
2*sqrt(a/(b*x**2) + 1)/(9009*a**11*b**25*x**12 + 45045*a**10*b**26*x**14 +
 90090*a**9*b**27*x**16 + 90090*a**8*b**28*x**18 + 45045*a**7*b**29*x**20
+ 9009*a**6*b**30*x**22) - 7175*A*a**9*b**(55/2)*x**4*sqrt(a/(b*x**2) + 1)
/(9009*a**11*b**25*x**12 + 45045*a**10*b**26*x**14 + 90090*a**9*b**27*x**1
6 + 90090*a**8*b**28*x**18 + 45045*a**7*b**29*x**20 + 9009*a**6*b**30*x**2
2) - 7290*A*a**8*b**(57/2)*x**6*sqrt(a/(b*x**2) + 1)/(9009*a**11*b**25*x**
12 + 45045*a**10*b**26*x**14 + 90090*a**9*b**27*x**16 + 90090*a**8*b**28*x
**18 + 45045*a**7*b**29*x**20 + 9009*a**6*b**30*x**22) - 3699*A*a**7*b**(5
9/2)*x**8*sqrt(a/(b*x**2) + 1)/(9009*a**11*b**25*x**12 + 45045*a**10*b**26
*x**14 + 90090*a**9*b**27*x**16 + 90090*a**8*b**28*x**18 + 45045*a**7*b**2
9*x**20 + 9009*a**6*b**30*x**22) - 756*A*a**6*b**(61/2)*x**10*sqrt(a/(b*x*
*2) + 1)/(9009*a**11*b**25*x**12 + 45045*a**10*b**26*x**14 + 90090*a**9*b*
*27*x**16 + 90090*a**8*b**28*x**18 + 45045*a**7*b**29*x**20 + 9009*a**6*b*
*30*x**22) + 63*A*a**5*b**(63/2)*x**12*sqrt(a/(b*x**2) + 1)/(9009*a**11*b*
*25*x**12 + 45045*a**10*b**26*x**14 + 90090*a**9*b**27*x**16 + 90090*a**8*
b**28*x**18 + 45045*a**7*b**29*x**20 + 9009*a**6*b**30*x**22) + 630*A*a**4
*b**(65/2)*x**14*sqrt(a/(b*x**2) + 1)/(9009*a**11*b**25*x**12 + 45045*a...
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.48

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2+Cx^4+Dx^6)}{x^{14}}\,dx = -\frac{8\,(bx^2+a)^{\frac{3}{2}}Db^2}{105\,a^3x^3} + \frac{16\,(bx^2+a)^{\frac{3}{2}}Cb^3}{315\,a^4x^3} - \frac{128\,(bx^2+a)^{\frac{3}{2}}Bb^4}{3465\,a^5x^3} + \frac{256\,(bx^2+a)^{\frac{3}{2}}Ab^5}{9009\,a^6x^3} + \frac{4\,(bx^2+a)^{\frac{3}{2}}Db}{35\,a^2x^5} - \frac{8\,(bx^2+a)^{\frac{3}{2}}Cb^2}{105\,a^3x^5} + \frac{64\,(bx^2+a)^{\frac{3}{2}}Bb^3}{1155\,a^4x^5} - \frac{128\,(bx^2+a)^{\frac{3}{2}}Ab^4}{3003\,a^5x^5} - \frac{(bx^2+a)^{\frac{3}{2}}D}{7\,ax^7} + \frac{2\,(bx^2+a)^{\frac{3}{2}}Cb}{21\,a^2x^7} - \frac{16\,(bx^2+a)^{\frac{3}{2}}Bb^2}{231\,a^3x^7} + \frac{160\,(bx^2+a)^{\frac{3}{2}}Ab^3}{3003\,a^4x^7} - \frac{(bx^2+a)^{\frac{3}{2}}C}{9\,ax^9} + \frac{8\,(bx^2+a)^{\frac{3}{2}}Bb}{99\,a^2x^9} - \frac{80\,(bx^2+a)^{\frac{3}{2}}Ab^2}{1287\,a^3x^9} - \frac{(bx^2+a)^{\frac{3}{2}}B}{11\,ax^{11}} + \frac{10\,(bx^2+a)^{\frac{3}{2}}Ab}{143\,a^2x^{11}} - \frac{(bx^2+a)^{\frac{3}{2}}A}{13\,ax^{13}}$$

input
```
integrate((b*x^2+a)^(1/2)*(D*x^6+C*x^4+B*x^2+A)/x^14,x, algorithm="maxima"
)
```

output
```
-8/105*(b*x^2 + a)^(3/2)*D*b^2/(a^3*x^3) + 16/315*(b*x^2 + a)^(3/2)*C*b^3/
(a^4*x^3) - 128/3465*(b*x^2 + a)^(3/2)*B*b^4/(a^5*x^3) + 256/9009*(b*x^2 +
 a)^(3/2)*A*b^5/(a^6*x^3) + 4/35*(b*x^2 + a)^(3/2)*D*b/(a^2*x^5) - 8/105*(
b*x^2 + a)^(3/2)*C*b^2/(a^3*x^5) + 64/1155*(b*x^2 + a)^(3/2)*B*b^3/(a^4*x^
5) - 128/3003*(b*x^2 + a)^(3/2)*A*b^4/(a^5*x^5) - 1/7*(b*x^2 + a)^(3/2)*D/
(a*x^7) + 2/21*(b*x^2 + a)^(3/2)*C*b/(a^2*x^7) - 16/231*(b*x^2 + a)^(3/2)*
B*b^2/(a^3*x^7) + 160/3003*(b*x^2 + a)^(3/2)*A*b^3/(a^4*x^7) - 1/9*(b*x^2
+ a)^(3/2)*C/(a*x^9) + 8/99*(b*x^2 + a)^(3/2)*B*b/(a^2*x^9) - 80/1287*(b*x
^2 + a)^(3/2)*A*b^2/(a^3*x^9) - 1/11*(b*x^2 + a)^(3/2)*B/(a*x^11) + 10/143
*(b*x^2 + a)^(3/2)*A*b/(a^2*x^11) - 1/13*(b*x^2 + a)^(3/2)*A/(a*x^13)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 996 vs. $2(219) = 438$.

Time = 0.15 (sec) , antiderivative size = 996, normalized size of antiderivative = 4.10

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2+Cx^4+Dx^6)}{x^{14}}\,dx = \text{Too large to display}$$

input

```
integrate((b*x^2+a)^(1/2)*(D*x^6+C*x^4+B*x^2+A)/x^14,x, algorithm="giac")
```

output

```
16/45045*(30030*(sqrt(b)*x - sqrt(b*x^2 + a))^20*D*b^(7/2) - 165165*(sqrt(
b)*x - sqrt(b*x^2 + a))^18*D*a*b^(7/2) + 90090*(sqrt(b)*x - sqrt(b*x^2 + a
))^18*C*b^(9/2) + 369369*(sqrt(b)*x - sqrt(b*x^2 + a))^16*D*a^2*b^(7/2) -
306306*(sqrt(b)*x - sqrt(b*x^2 + a))^16*C*a*b^(9/2) + 288288*(sqrt(b)*x -
sqrt(b*x^2 + a))^16*B*b^(11/2) - 432432*(sqrt(b)*x - sqrt(b*x^2 + a))^14*D
*a^3*b^(7/2) + 348348*(sqrt(b)*x - sqrt(b*x^2 + a))^14*C*a^2*b^(9/2) - 384
384*(sqrt(b)*x - sqrt(b*x^2 + a))^14*B*a*b^(11/2) + 960960*(sqrt(b)*x - sq
rt(b*x^2 + a))^14*A*b^(13/2) + 303732*(sqrt(b)*x - sqrt(b*x^2 + a))^12*D*a
^4*b^(7/2) - 142428*(sqrt(b)*x - sqrt(b*x^2 + a))^12*C*a^3*b^(9/2) - 27456
*(sqrt(b)*x - sqrt(b*x^2 + a))^12*B*a^2*b^(11/2) + 686400*(sqrt(b)*x - sqr
t(b*x^2 + a))^12*A*a*b^(13/2) - 182754*(sqrt(b)*x - sqrt(b*x^2 + a))^10*D*
a^5*b^(7/2) + 61776*(sqrt(b)*x - sqrt(b*x^2 + a))^10*C*a^4*b^(9/2) + 20592
*(sqrt(b)*x - sqrt(b*x^2 + a))^10*B*a^3*b^(11/2) + 205920*(sqrt(b)*x - sqr
t(b*x^2 + a))^10*A*a^2*b^(13/2) + 141570*(sqrt(b)*x - sqrt(b*x^2 + a))^8*D
*a^6*b^(7/2) - 114400*(sqrt(b)*x - sqrt(b*x^2 + a))^8*C*a^5*b^(9/2) + 1487
20*(sqrt(b)*x - sqrt(b*x^2 + a))^8*B*a^4*b^(11/2) - 114400*(sqrt(b)*x - sq
rt(b*x^2 + a))^8*A*a^3*b^(13/2) - 92664*(sqrt(b)*x - sqrt(b*x^2 + a))^6*D*
a^7*b^(7/2) + 81796*(sqrt(b)*x - sqrt(b*x^2 + a))^6*C*a^6*b^(9/2) - 59488*
(sqrt(b)*x - sqrt(b*x^2 + a))^6*B*a^5*b^(11/2) + 45760*(sqrt(b)*x - sqrt(b
*x^2 + a))^6*A*a^4*b^(13/2) + 33462*(sqrt(b)*x - sqrt(b*x^2 + a))^4*D*a...
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2+Cx^4+Dx^6)}{x^{14}}\,dx=\int \frac{\sqrt{b\,x^2+a}\,(A+B\,x^2+C\,x^4+x^6\,D)}{x^{14}}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(A + B*x^2 + C*x^4 + x^6*D))/x^14,x)
```

output
```
int(((a + b*x^2)^(1/2)*(A + B*x^2 + C*x^4 + x^6*D))/x^14, x)
```

## Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.37

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2+Cx^4+Dx^6)}{x^{14}}\,dx$$
$$=\frac{-3465\sqrt{b\,x^2+a}\,a^6-4410\sqrt{b\,x^2+a}\,a^5b\,x^2-5005\sqrt{b\,x^2+a}\,a^5c\,x^4-6435\sqrt{b\,x^2+a}\,a^5d\,x^6-105\sqrt{b\,x^2}}{}$$

input
```
int((b*x^2+a)^(1/2)*(D*x^6+C*x^4+B*x^2+A)/x^14,x)
```

output
```
( - 3465*sqrt(a + b*x**2)*a**6 - 4410*sqrt(a + b*x**2)*a**5*b*x**2 - 5005*
sqrt(a + b*x**2)*a**5*c*x**4 - 6435*sqrt(a + b*x**2)*a**5*d*x**6 - 105*sqr
t(a + b*x**2)*a**4*b**2*x**4 - 715*sqrt(a + b*x**2)*a**4*b*c*x**6 - 1287*s
qrt(a + b*x**2)*a**4*b*d*x**8 + 120*sqrt(a + b*x**2)*a**3*b**3*x**6 + 858*
sqrt(a + b*x**2)*a**3*b**2*c*x**8 + 1716*sqrt(a + b*x**2)*a**3*b**2*d*x**1
0 - 144*sqrt(a + b*x**2)*a**2*b**4*x**8 - 1144*sqrt(a + b*x**2)*a**2*b**3*
c*x**10 - 3432*sqrt(a + b*x**2)*a**2*b**3*d*x**12 + 192*sqrt(a + b*x**2)*a
*b**5*x**10 + 2288*sqrt(a + b*x**2)*a*b**4*c*x**12 - 384*sqrt(a + b*x**2)*
b**6*x**12 + 3432*sqrt(b)*a**2*b**3*d*x**13 - 2288*sqrt(b)*a*b**4*c*x**13
+ 384*sqrt(b)*b**6*x**13)/(45045*a**5*x**13)
```

# 3.198 $\int x^2(a+bx^2)^{3/2}\left(A+Bx^2+Cx^4+Dx^6\right)\,dx$



## Optimal result

Integrand size = 32, antiderivative size = 297

$$\begin{aligned}\int x^2\left(a+bx^2\right)^{3/2}\left(A+Bx^2+Cx^4\right.&\left.+Dx^6\right)\,dx=\frac{a^2(64Ab^3-a(24b^2B-12abC+7a^2D))\,x\sqrt{a+bx^2}}{1024b^4}\\&+\frac{a(64Ab^3-a(24b^2B-12abC+7a^2D))\,x^3\sqrt{a+bx^2}}{512b^3}\\&+\frac{1}{384}\left(64A-\frac{a(24b^2B-12abC+7a^2D)}{b^3}\right)x^3\left(a+bx^2\right)^{3/2}\\&+\frac{(24b^2B-12abC+7a^2D)\,x^3(a+bx^2)^{5/2}}{192b^3}+\frac{(12bC-7aD)x^5(a+bx^2)^{5/2}}{120b^2}\\&+\frac{Dx^7(a+bx^2)^{5/2}}{12b}-\frac{a^3(64Ab^3-a(24b^2B-12abC+7a^2D))\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{1024b^{9/2}}\end{aligned}$$

output
```
1/1024*a^2*(64*A*b^3-a*(24*B*b^2-12*C*a*b+7*D*a^2))*x*(b*x^2+a)^(1/2)/b^4+
1/512*a*(64*A*b^3-a*(24*B*b^2-12*C*a*b+7*D*a^2))*x^3*(b*x^2+a)^(1/2)/b^3+1
/384*(64*A-a*(24*B*b^2-12*C*a*b+7*D*a^2)/b^3)*x^3*(b*x^2+a)^(3/2)+1/192*(2
4*B*b^2-12*C*a*b+7*D*a^2)*x^3*(b*x^2+a)^(5/2)/b^3+1/120*(12*C*b-7*D*a)*x^5
*(b*x^2+a)^(5/2)/b^2+1/12*D*x^7*(b*x^2+a)^(5/2)/b-1/1024*a^3*(64*A*b^3-a*(
24*B*b^2-12*C*a*b+7*D*a^2))*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(9/2)
```

## Mathematica [A] (verified)

Time = 2.14 (sec) , antiderivative size = 260, normalized size of antiderivative = 0.88

$$\int x^2\left(a+bx^2\right)^{3/2}\left(A+Bx^2+Cx^4+Dx^6\right)\,dx=\frac{\sqrt{b}x\sqrt{a+bx^2}(-105a^5D+10a^4b(18C+7Dx^2)-8a^3b^2(45B+15Cx^2+7Dx^4)+48a^2b^3(20A}{}$$

input
```
Integrate[x^2*(a + b*x^2)^(3/2)*(A + B*x^2 + C*x^4 + D*x^6),x]
```

output
```
(Sqrt[b]*x*Sqrt[a + b*x^2]*(-105*a^5*D + 10*a^4*b*(18*C + 7*D*x^2) - 8*a^3
*b^2*(45*B + 15*C*x^2 + 7*D*x^4) + 48*a^2*b^3*(20*A + 5*B*x^2 + 2*C*x^4 +
D*x^6) + 128*b^5*x^4*(20*A + 15*B*x^2 + 12*C*x^4 + 10*D*x^6) + 64*a*b^4*x^
2*(70*A + 45*B*x^2 + 33*C*x^4 + 26*D*x^6)) + 120*a^3*b*(16*A*b^2 + 3*a^2*C
)*ArcTanh[(Sqrt[b]*x)/(Sqrt[a] - Sqrt[a + b*x^2])] + 30*a^4*(24*b^2*B + 7*
a^2*D)*ArcTanh[(Sqrt[b]*x)/(-Sqrt[a] + Sqrt[a + b*x^2])])/(15360*b^(9/2))
```

## Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.81, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.281, Rules used = {2340, 1590, 27, 363, 248, 248, 262, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(a+bx^2\right)^{3/2}\left(A+Bx^2+Cx^4+Dx^6\right)\,dx$$

$$\Big\downarrow \text{2340}$$

$$\frac{\int x^2\left(bx^2+a\right)^{3/2}\left((12bC-7aD)x^4+12bBx^2+12Ab\right)dx}{12b}+\frac{Dx^7\left(a+bx^2\right)^{5/2}}{12b}$$

$$\Big\downarrow \text{1590}$$

$$\frac{\frac{\int 5x^2\left(bx^2+a\right)^{3/2}\left(24Ab^2+\left(7Da^2-12bCa+24b^2B\right)x^2\right)dx}{10b}+\frac{x^5\left(a+bx^2\right)^{5/2}(12bC-7aD)}{10b}}{12b}+\frac{Dx^7\left(a+bx^2\right)^{5/2}}{12b}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int x^2\left(bx^2+a\right)^{3/2}\left(24Ab^2+\left(7Da^2-12bCa+24b^2B\right)x^2\right)dx}{2b}+\frac{x^5\left(a+bx^2\right)^{5/2}(12bC-7aD)}{10b}}{12b}+\frac{Dx^7\left(a+bx^2\right)^{5/2}}{12b}$$

$$\Big\downarrow \; 363$$

$$\frac{\frac{\frac{3\left(64Ab^3-a\left(7a^2D-12abC+24b^2B\right)\right)\int x^2\left(bx^2+a\right)^{3/2}dx}{8b}+\frac{x^3\left(a+bx^2\right)^{5/2}\left(7a^2D-12abC+24b^2B\right)}{8b}}{2b}+\frac{x^5\left(a+bx^2\right)^{5/2}(12bC-7aD)}{10b}}{12b}+\frac{Dx^7\left(a+bx^2\right)^{5/2}}{12b}$$

$$\Big\downarrow \; 248$$

$$\frac{\frac{\frac{3\left(64Ab^3-a\left(7a^2D-12abC+24b^2B\right)\right)\left(\frac{1}{2}a\int x^2\sqrt{bx^2+a}dx+\frac{1}{6}x^3\left(a+bx^2\right)^{3/2}\right)}{8b}+\frac{x^3\left(a+bx^2\right)^{5/2}\left(7a^2D-12abC+24b^2B\right)}{8b}}{2b}+\frac{x^5\left(a+bx^2\right)^{5/2}(12bC-7aD)}{10b}}{12b}+\frac{Dx^7\left(a+bx^2\right)^{5/2}}{12b}$$

$$\Big\downarrow \; 248$$

$$\frac{\frac{\frac{3\left(64Ab^3-a\left(7a^2D-12abC+24b^2B\right)\right)\left(\frac{1}{2}a\left(\frac{1}{4}a\int\frac{x^2}{\sqrt{bx^2+a}}dx+\frac{1}{4}x^3\sqrt{a+bx^2}\right)+\frac{1}{6}x^3\left(a+bx^2\right)^{3/2}\right)}{8b}+\frac{x^3\left(a+bx^2\right)^{5/2}\left(7a^2D-12abC+24b^2B\right)}{8b}}{2b}+\cdots}{12b}+\frac{Dx^7\left(a+bx^2\right)^{5/2}}{12b}$$

$$\Big\downarrow \; 262$$

$$\frac{\frac{\frac{3\left(64Ab^3-a\left(7a^2D-12abC+24b^2B\right)\right)\left(\frac{1}{2}a\left(\frac{1}{4}a\left(\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\int\frac{1}{\sqrt{bx^2+a}}dx}{2b}\right)+\frac{1}{4}x^3\sqrt{a+bx^2}\right)+\frac{1}{6}x^3\left(a+bx^2\right)^{3/2}\right)}{8b}+\frac{x^3\left(a+bx^2\right)^{5/2}\left(7a^2D-12abC+24b^2B\right)}{8b}}{2b}\cdots}{12b}+\frac{Dx^7\left(a+bx^2\right)^{5/2}}{12b}$$

$$\Big\downarrow \; 224$$

$$\frac{\frac{3\left(64Ab^3-a\left(7a^2D-12abC+24b^2B\right)\right)\left(\frac{1}{2}a\left(\frac{1}{4}a\left(\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2b}\right)+\frac{1}{4}x^3\sqrt{a+bx^2}\right)+\frac{1}{6}x^3\left(a+bx^2\right)^{3/2}\right)}{8b}+\frac{x^3\left(a+bx^2\right)^{5/2}\left(7a^2D-12abC\right.}{8b}}{2b}}{12b}$$

$$\frac{Dx^7\left(a+bx^2\right)^{5/2}}{12b}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{3\left(\frac{1}{2}a\left(\frac{1}{4}a\left(\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}\right)+\frac{1}{4}x^3\sqrt{a+bx^2}\right)+\frac{1}{6}x^3\left(a+bx^2\right)^{3/2}\right)\left(64Ab^3-a\left(7a^2D-12abC+24b^2B\right)\right)}{8b}+\frac{x^3\left(a+bx^2\right)^{5/2}\left(7a^2D-12abC\right.}{8b}}{2b}}{12b}$$

$$\frac{Dx^7\left(a+bx^2\right)^{5/2}}{12b}$$

input
```
Int[x^2*(a + b*x^2)^(3/2)*(A + B*x^2 + C*x^4 + D*x^6),x]
```

output
```
(D*x^7*(a + b*x^2)^(5/2))/(12*b) + (((12*b*C - 7*a*D)*x^5*(a + b*x^2)^(5/2
))/(10*b) + (((24*b^2*B - 12*a*b*C + 7*a^2*D)*x^3*(a + b*x^2)^(5/2))/(8*b)
 + (3*(64*A*b^3 - a*(24*b^2*B - 12*a*b*C + 7*a^2*D))*((x^3*(a + b*x^2)^(3/
2))/6 + (a*((x^3*Sqrt[a + b*x^2])/4 + (a*((x*Sqrt[a + b*x^2])/(2*b) - (a*A
rcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(3/2))))/4))/2))/(8*b))/(2*b))/(
12*b)
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 1590
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[c^p*(f*x)^(m + 4*p - 1)*((d + e*x^2)^
(q + 1)/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1))), x] + Simp[1/(e*(m + 4*p + 2*q
 + 1))   Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a +
b*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x],
x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p,
0] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

rule 2340
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Simp[1/(b*(m
+ q + 2*p + 1))   Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)
*Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /;
GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ
[Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

## Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.68

| method | result |
|---|---|
| pseudoelliptic | $\frac{-\left(b^3A-\frac{3}{8}a\,b^2B+\frac{3}{16}a^2bC-\frac{7}{64}a^3D\right)a^3\operatorname{arctanh}\left(\frac{\sqrt{b\,x^2+a}}{x\sqrt{b}}\right)+x\sqrt{b\,x^2+a}\left(\frac{8\left(\frac{1}{2}Dx^6+\frac{3}{5}C\,x^4+\frac{3}{4}x^2B+A\right)x^4b^{\frac{11}{2}}}{3}+\left(a\left(\frac{1}{20}Dx^6+\right.\right.\right.}{}$ |
| default | $A\left(\frac{x\left(b\,x^2+a\right)^{\frac{5}{2}}}{6b}-\frac{a\left(\frac{x\left(b\,x^2+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)}{4}\right)}{6b}\right)+B\left(\frac{x^3\left(b\,x^2+a\right)^{\frac{5}{2}}}{8b}-\frac{3a\left(\frac{x\left(b\,x^2\right.}{6}\right.}{}\right.$ |

input
```
int(x^2*(b*x^2+a)^(3/2)*(D*x^6+C*x^4+B*x^2+A),x,method=_RETURNVERBOSE)
```

output
```
1/16*(-(b^3*A-3/8*a*b^2*B+3/16*a^2*b*C-7/64*a^3*D)*a^3*arctanh((b*x^2+a)^(
1/2)/x/b^(1/2))+x*(b*x^2+a)^(1/2)*(8/3*(1/2*D*x^6+3/5*C*x^4+3/4*x^2*B+A)*x
^4*b^(11/2)+(a*(1/20*D*x^6+1/10*C*x^4+1/4*x^2*B+A)*b^(7/2)+(26/15*D*x^8+11
/5*C*x^6+3*B*x^4+14/3*A*x^2)*b^(9/2)-7/64*a^2*((8/15*D*x^4+8/7*C*x^2+24/7*
B)*b^(5/2)+((-2/3*D*x^2-12/7*C)*b^(3/2)+D*a*b^(1/2))*a))*a))/b^(9/2)
```

## Fricas [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 496, normalized size of antiderivative = 1.67

$$\int x^2(a+bx^2)^{3/2}(A+Bx^2+Cx^4+Dx^6)\,dx = \left[-\frac{15(7Da^6-12Ca^5b+24Ba^4b^2-64Aa^3b^3)\sqrt{b}\log\left(-2bx^2+2\sqrt{bx^2+a}\sqrt{b}x-a\right)-2(1\ldots}{\ldots}\right.$$

$$-\frac{15(7Da^6-12Ca^5b+24Ba^4b^2-64Aa^3b^3)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(1280Db^6x^{11}+128(13Dab^5+12\ldots}{\ldots}$$

input
```
integrate(x^2*(b*x^2+a)^(3/2)*(D*x^6+C*x^4+B*x^2+A),x, algorithm="fricas")
```

output
```
[-1/30720*(15*(7*D*a^6 - 12*C*a^5*b + 24*B*a^4*b^2 - 64*A*a^3*b^3)*sqrt(b)
*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(1280*D*b^6*x^11 + 12
8*(13*D*a*b^5 + 12*C*b^6)*x^9 + 48*(D*a^2*b^4 + 44*C*a*b^5 + 40*B*b^6)*x^7
 - 8*(7*D*a^3*b^3 - 12*C*a^2*b^4 - 360*B*a*b^5 - 320*A*b^6)*x^5 + 10*(7*D*
a^4*b^2 - 12*C*a^3*b^3 + 24*B*a^2*b^4 + 448*A*a*b^5)*x^3 - 15*(7*D*a^5*b -
 12*C*a^4*b^2 + 24*B*a^3*b^3 - 64*A*a^2*b^4)*x)*sqrt(b*x^2 + a))/b^5, -1/1
5360*(15*(7*D*a^6 - 12*C*a^5*b + 24*B*a^4*b^2 - 64*A*a^3*b^3)*sqrt(-b)*arc
tan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (1280*D*b^6*x^11 + 128*(13*D*a*b^5 + 12*
C*b^6)*x^9 + 48*(D*a^2*b^4 + 44*C*a*b^5 + 40*B*b^6)*x^7 - 8*(7*D*a^3*b^3 -
 12*C*a^2*b^4 - 360*B*a*b^5 - 320*A*b^6)*x^5 + 10*(7*D*a^4*b^2 - 12*C*a^3*
b^3 + 24*B*a^2*b^4 + 448*A*a*b^5)*x^3 - 15*(7*D*a^5*b - 12*C*a^4*b^2 + 24*
B*a^3*b^3 - 64*A*a^2*b^4)*x)*sqrt(b*x^2 + a))/b^5]
```

## Sympy [A] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 505, normalized size of antiderivative = 1.70

$$\int x^2 \left(a+bx^2\right)^{3/2} \left(A+Bx^2+Cx^4+Dx^6\right) dx = \begin{cases} -\frac{a\left(Aa^2-\frac{3a\left(2Aab+Ba^2-\frac{5a\left(Ab^2+2Bab+Ca^2-\frac{7a\left(Bb^2+2Cab+Da^2-\frac{9a\left(Cb^2+\frac{13Dab}{12}\right)}{10b}\right)}{8b}\right)}{6b}\right)}{4b}\right)\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} \\ \frac{x\log(x)}{\sqrt{bx^2}}\end{cases}\right.}{2b} \\ a^{\frac{3}{2}}\left(\frac{Ax^3}{3}+\frac{Bx^5}{5}+\frac{Cx^7}{7}+\frac{Dx^9}{9}\right) \end{cases}$$

input

```
integrate(x**2*(b*x**2+a)**(3/2)*(D*x**6+C*x**4+B*x**2+A),x)
```

output
```
Piecewise((-a*(A*a**2 - 3*a*(2*A*a*b + B*a**2 - 5*a*(A*b**2 + 2*B*a*b + C*
a**2 - 7*a*(B*b**2 + 2*C*a*b + D*a**2 - 9*a*(C*b**2 + 13*D*a*b/12)/(10*b))
/(8*b))/(6*b))/(4*b))*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/s
qrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True))/(2*b) + sqrt(a + b*x**2)
*(D*b*x**11/12 + x**9*(C*b**2 + 13*D*a*b/12)/(10*b) + x**7*(B*b**2 + 2*C*a
*b + D*a**2 - 9*a*(C*b**2 + 13*D*a*b/12)/(10*b))/(8*b) + x**5*(A*b**2 + 2*
B*a*b + C*a**2 - 7*a*(B*b**2 + 2*C*a*b + D*a**2 - 9*a*(C*b**2 + 13*D*a*b/1
2)/(10*b))/(8*b))/(6*b) + x**3*(2*A*a*b + B*a**2 - 5*a*(A*b**2 + 2*B*a*b +
 C*a**2 - 7*a*(B*b**2 + 2*C*a*b + D*a**2 - 9*a*(C*b**2 + 13*D*a*b/12)/(10*
b))/(8*b))/(6*b))/(4*b) + x*(A*a**2 - 3*a*(2*A*a*b + B*a**2 - 5*a*(A*b**2
+ 2*B*a*b + C*a**2 - 7*a*(B*b**2 + 2*C*a*b + D*a**2 - 9*a*(C*b**2 + 13*D*a
*b/12)/(10*b))/(8*b))/(6*b))/(4*b))/(2*b)), Ne(b, 0)), (a**(3/2)*(A*x**3/3
 + B*x**5/5 + C*x**7/7 + D*x**9/9), True))
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 407, normalized size of antiderivative = 1.37

$$\int x^2(a+bx^2)^{3/2}(A+Bx^2+Cx^4+Dx^6)\,dx = \frac{(bx^2+a)^{\frac{5}{2}}Dx^7}{12\,b} - \frac{7\,(bx^2+a)^{\frac{5}{2}}Dax^5}{120\,b^2} + \frac{(bx^2+a)^{\frac{5}{2}}Cx^5}{10\,b} + \frac{7\,(bx^2+a)^{\frac{5}{2}}Da^2x^3}{192\,b^3} - \frac{(bx^2+a)^{\frac{5}{2}}Cax^3}{16\,b^2} + \frac{(bx^2+a)^{\frac{5}{2}}Bx^3}{8\,b} - \frac{7\,(bx^2+a)^{\frac{5}{2}}Da^3x}{384\,b^4} + \frac{7\,(bx^2+a)^{\frac{3}{2}}Da^4x}{1536\,b^4} + \frac{7\,\sqrt{bx^2+a}Da^5x}{1024\,b^4} + \frac{(bx^2+a)^{\frac{5}{2}}Ca^2x}{32\,b^3} - \frac{(bx^2+a)^{\frac{3}{2}}Ca^3x}{128\,b^3} - \frac{3\,\sqrt{bx^2+a}Ca^4x}{256\,b^3} - \frac{(bx^2+a)^{\frac{5}{2}}Bax}{16\,b^2} + \frac{(bx^2+a)^{\frac{3}{2}}Ba^2x}{64\,b^2} + \frac{3\,\sqrt{bx^2+a}Ba^3x}{128\,b^2} + \frac{(bx^2+a)^{\frac{5}{2}}Ax}{6\,b} - \frac{(bx^2+a)^{\frac{3}{2}}Aax}{24\,b} - \frac{\sqrt{bx^2+a}Aa^2x}{16\,b} + \frac{7\,Da^6\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{1024\,b^{\frac{9}{2}}} - \frac{3\,Ca^5\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{256\,b^{\frac{7}{2}}} + \frac{3\,Ba^4\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\,b^{\frac{5}{2}}} - \frac{Aa^3\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{3}{2}}}$$

input
```
integrate(x^2*(b*x^2+a)^(3/2)*(D*x^6+C*x^4+B*x^2+A),x, algorithm="maxima")
```

output
```
1/12*(b*x^2 + a)^(5/2)*D*x^7/b - 7/120*(b*x^2 + a)^(5/2)*D*a*x^5/b^2 + 1/1
0*(b*x^2 + a)^(5/2)*C*x^5/b + 7/192*(b*x^2 + a)^(5/2)*D*a^2*x^3/b^3 - 1/16
*(b*x^2 + a)^(5/2)*C*a*x^3/b^2 + 1/8*(b*x^2 + a)^(5/2)*B*x^3/b - 7/384*(b*
x^2 + a)^(5/2)*D*a^3*x/b^4 + 7/1536*(b*x^2 + a)^(3/2)*D*a^4*x/b^4 + 7/1024
*sqrt(b*x^2 + a)*D*a^5*x/b^4 + 1/32*(b*x^2 + a)^(5/2)*C*a^2*x/b^3 - 1/128*
(b*x^2 + a)^(3/2)*C*a^3*x/b^3 - 3/256*sqrt(b*x^2 + a)*C*a^4*x/b^3 - 1/16*(
b*x^2 + a)^(5/2)*B*a*x/b^2 + 1/64*(b*x^2 + a)^(3/2)*B*a^2*x/b^2 + 3/128*sq
rt(b*x^2 + a)*B*a^3*x/b^2 + 1/6*(b*x^2 + a)^(5/2)*A*x/b - 1/24*(b*x^2 + a)
^(3/2)*A*a*x/b - 1/16*sqrt(b*x^2 + a)*A*a^2*x/b + 7/1024*D*a^6*arcsinh(b*x
/sqrt(a*b))/b^(9/2) - 3/256*C*a^5*arcsinh(b*x/sqrt(a*b))/b^(7/2) + 3/128*B
*a^4*arcsinh(b*x/sqrt(a*b))/b^(5/2) - 1/16*A*a^3*arcsinh(b*x/sqrt(a*b))/b^
(3/2)
```

## Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 264, normalized size of antiderivative = 0.89

$$\int x^2(a+bx^2)^{3/2}(A+Bx^2+Cx^4+Dx^6)\,dx=\frac{1}{15360}\left(2\left(4\left(2\left(8\left(10\,Dbx^2+\frac{13\,Dab^{10}+12\,Cb^{11}}{b^{10}}\right)x^2+\frac{3\,(Da^2b^9+44\,Cab^{10}+40\,Bb^{11})}{b^{10}}\right)\right.\right.\right.$$
$$-\frac{(7\,Da^6-12\,Ca^5b+24\,Ba^4b^2-64\,Aa^3b^3)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{1024\,b^{\frac{9}{2}}}$$

input
```
integrate(x^2*(b*x^2+a)^(3/2)*(D*x^6+C*x^4+B*x^2+A),x, algorithm="giac")
```

output
```
1/15360*(2*(4*(2*(8*(10*D*b*x^2 + (13*D*a*b^10 + 12*C*b^11)/b^10)*x^2 + 3*
(D*a^2*b^9 + 44*C*a*b^10 + 40*B*b^11)/b^10)*x^2 - (7*D*a^3*b^8 - 12*C*a^2*
b^9 - 360*B*a*b^10 - 320*A*b^11)/b^10)*x^2 + 5*(7*D*a^4*b^7 - 12*C*a^3*b^8
 + 24*B*a^2*b^9 + 448*A*a*b^10)/b^10)*x^2 - 15*(7*D*a^5*b^6 - 12*C*a^4*b^7
 + 24*B*a^3*b^8 - 64*A*a^2*b^9)/b^10)*sqrt(b*x^2 + a)*x - 1/1024*(7*D*a^6
- 12*C*a^5*b + 24*B*a^4*b^2 - 64*A*a^3*b^3)*log(abs(-sqrt(b)*x + sqrt(b*x^
2 + a)))/b^(9/2)
```

## Mupad [F(-1)]

Timed out.

$$\int x^2 (a+bx^2)^{3/2} (A+Bx^2+Cx^4+Dx^6) \, dx = \int x^2 \left(b\,x^2+a\right)^{3/2} \left(A+B\,x^2+C\,x^4+x^6\,D\right) \, dx$$

input
```
int(x^2*(a + b*x^2)^(3/2)*(A + B*x^2 + C*x^4 + x^6*D),x)
```

output
```
int(x^2*(a + b*x^2)^(3/2)*(A + B*x^2 + C*x^4 + x^6*D), x)
```

## Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.22

$$\int x^2 (a+bx^2)^{3/2} (A+Bx^2+Cx^4+Dx^6) \, dx = \frac{-105\sqrt{b\,x^2+a}\,a^5bdx+180\sqrt{b\,x^2+a}\,a^4b^2cx+70\sqrt{b\,x^2+a}\,a^4b^2d\,x^3+600\sqrt{b\,x^2+a}\,a^3b^4x-}{}$$

input
```
int(x^2*(b*x^2+a)^(3/2)*(D*x^6+C*x^4+B*x^2+A),x)
```

output
```
( - 105*sqrt(a + b*x**2)*a**5*b*d*x + 180*sqrt(a + b*x**2)*a**4*b**2*c*x +
 70*sqrt(a + b*x**2)*a**4*b**2*d*x**3 + 600*sqrt(a + b*x**2)*a**3*b**4*x -
 120*sqrt(a + b*x**2)*a**3*b**3*c*x**3 - 56*sqrt(a + b*x**2)*a**3*b**3*d*x
**5 + 4720*sqrt(a + b*x**2)*a**2*b**5*x**3 + 96*sqrt(a + b*x**2)*a**2*b**4
*c*x**5 + 48*sqrt(a + b*x**2)*a**2*b**4*d*x**7 + 5440*sqrt(a + b*x**2)*a*b
**6*x**5 + 2112*sqrt(a + b*x**2)*a*b**5*c*x**7 + 1664*sqrt(a + b*x**2)*a*b
**5*d*x**9 + 1920*sqrt(a + b*x**2)*b**7*x**7 + 1536*sqrt(a + b*x**2)*b**6*
c*x**9 + 1280*sqrt(a + b*x**2)*b**6*d*x**11 + 105*sqrt(b)*log((sqrt(a + b*
x**2) + sqrt(b)*x)/sqrt(a))*a**6*d - 180*sqrt(b)*log((sqrt(a + b*x**2) + s
qrt(b)*x)/sqrt(a))*a**5*b*c - 600*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*
x)/sqrt(a))*a**4*b**3)/(15360*b**5)
```

# 3.199 $\int (a+bx^2)^{3/2} (A+Bx^2+Cx^4+Dx^6)\, dx$



## Optimal result

Integrand size = 29, antiderivative size = 241

$$\begin{aligned}\int (a+bx^2)^{3/2} (A+Bx^2+Cx^4 &+ Dx^6)\, dx = \frac{a(96Ab^3-a(16b^2B-6abC+3a^2D))\,x\sqrt{a+bx^2}}{256b^3} \\ &+ \frac{1}{384}\left(96A-\frac{a(16b^2B-6abC+3a^2D)}{b^3}\right)x(a+bx^2)^{3/2} \\ &+ \frac{(16b^2B-6abC+3a^2D)\,x(a+bx^2)^{5/2}}{96b^3}+\frac{(2bC-aD)x^3(a+bx^2)^{5/2}}{16b^2} \\ &+ \frac{Dx^5(a+bx^2)^{5/2}}{10b}+\frac{a^2(96Ab^3-a(16b^2B-6abC+3a^2D))\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{256b^{7/2}}\end{aligned}$$

output
```
1/256*a*(96*A*b^3-a*(16*B*b^2-6*C*a*b+3*D*a^2))*x*(b*x^2+a)^(1/2)/b^3+1/38
4*(96*A-a*(16*B*b^2-6*C*a*b+3*D*a^2)/b^3)*x*(b*x^2+a)^(3/2)+1/96*(16*B*b^2
-6*C*a*b+3*D*a^2)*x*(b*x^2+a)^(5/2)/b^3+1/16*(2*C*b-D*a)*x^3*(b*x^2+a)^(5/
2)/b^2+1/10*D*x^5*(b*x^2+a)^(5/2)/b+1/256*a^2*(96*A*b^3-a*(16*B*b^2-6*C*a*
b+3*D*a^2))*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(7/2)
```

## Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.77

$$\int (a+bx^2)^{3/2} (A+Bx^2+Cx^4+Dx^6) \, dx = \frac{\sqrt{b}x\sqrt{a+bx^2}(45a^4D-30a^3b(3C+Dx^2)+12a^2b^2(20B+5Cx^2+2Dx^4)+32b^4x^2(30A+2\cdots}{\cdots}$$

input
```
Integrate[(a + b*x^2)^(3/2)*(A + B*x^2 + C*x^4 + D*x^6),x]
```

output
```
(Sqrt[b]*x*Sqrt[a + b*x^2]*(45*a^4*D - 30*a^3*b*(3*C + D*x^2) + 12*a^2*b^2
*(20*B + 5*C*x^2 + 2*D*x^4) + 32*b^4*x^2*(30*A + 20*B*x^2 + 15*C*x^4 + 12*
D*x^6) + 16*a*b^3*(150*A + 70*B*x^2 + 45*C*x^4 + 33*D*x^6)) + 15*a^2*(-96*
A*b^3 + a*(16*b^2*B - 6*a*b*C + 3*a^2*D))*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^
2]])/(3840*b^(7/2))
```

## Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.87, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {2346, 27, 1473, 299, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^2)^{3/2} (A+Bx^2+Cx^4+Dx^6) \, dx$$

$$\Big\downarrow \text{2346}$$

$$\frac{\int 5(bx^2+a)^{3/2}((2bC-aD)x^4+2bBx^2+2Ab) \, dx}{10b}+\frac{Dx^5(a+bx^2)^{5/2}}{10b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int (bx^2+a)^{3/2}((2bC-aD)x^4+2bBx^2+2Ab) \, dx}{2b}+\frac{Dx^5(a+bx^2)^{5/2}}{10b}$$

$$\Big\downarrow \text{1473}$$

$$\frac{\frac{\int (bx^2+a)^{3/2}(16Ab^2+(3Da^2-6bCa+16b^2B)x^2)dx}{8b}+\frac{x^3(a+bx^2)^{5/2}(2bC-aD)}{8b}}{2b}+\frac{Dx^5(a+bx^2)^{5/2}}{10b}$$

$$\Big\downarrow \text{ 299}$$

$$\frac{\frac{\frac{\left(96Ab^3-a\left(3a^2D-6abC+16b^2B\right)\right)\int (bx^2+a)^{3/2}dx}{6b}+\frac{x(a+bx^2)^{5/2}\left(3a^2D-6abC+16b^2B\right)}{6b}}{8b}+\frac{x^3(a+bx^2)^{5/2}(2bC-aD)}{8b}}{2b}+\frac{Dx^5(a+bx^2)^{5/2}}{10b}$$

$$\Big\downarrow \text{ 211}$$

$$\frac{\frac{\frac{\left(96Ab^3-a\left(3a^2D-6abC+16b^2B\right)\right)\left(\frac{3}{4}a\int \sqrt{bx^2+a}dx+\frac{1}{4}x(a+bx^2)^{3/2}\right)}{6b}+\frac{x(a+bx^2)^{5/2}\left(3a^2D-6abC+16b^2B\right)}{6b}}{8b}+\frac{x^3(a+bx^2)^{5/2}(2bC-aD)}{8b}}{2b}+\frac{Dx^5(a+bx^2)^{5/2}}{10b}$$

$$\Big\downarrow \text{ 211}$$

$$\frac{\frac{\frac{\left(96Ab^3-a\left(3a^2D-6abC+16b^2B\right)\right)\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x(a+bx^2)^{3/2}\right)}{6b}+\frac{x(a+bx^2)^{5/2}\left(3a^2D-6abC+16b^2B\right)}{6b}}{8b}+\frac{x^3(a+bx^2)^{5/2}(2bC}{8b}}{2b}$$

$$\frac{Dx^5(a+bx^2)^{5/2}}{10b}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{\frac{\frac{\left(96Ab^3-a\left(3a^2D-6abC+16b^2B\right)\right)\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x(a+bx^2)^{3/2}\right)}{6b}+\frac{x(a+bx^2)^{5/2}\left(3a^2D-6abC+16b^2B\right)}{6b}}{8b}+x^3(a+bx^2}{2b}$$

$$\frac{Dx^5(a+bx^2)^{5/2}}{10b}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\frac{\frac{\left(\frac{3}{4}a\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x(a+bx^2)^{3/2}\right)\left(96Ab^3-a\left(3a^2D-6abC+16b^2B\right)\right)}{6b}+\frac{x(a+bx^2)^{5/2}\left(3a^2D-6abC+16b^2B\right)}{6b}}{8b}+x^3(a+bx^2)}{2b}$$

$$\frac{Dx^5(a+bx^2)^{5/2}}{10b}$$

input
```
Int[(a + b*x^2)^(3/2)*(A + B*x^2 + C*x^4 + D*x^6),x]
```

output
```
(D*x^5*(a + b*x^2)^(5/2))/(10*b) + (((2*b*C - a*D)*x^3*(a + b*x^2)^(5/2))/
(8*b) + (((16*b^2*B - 6*a*b*C + 3*a^2*D)*x*(a + b*x^2)^(5/2))/(6*b) + ((96
*A*b^3 - a*(16*b^2*B - 6*a*b*C + 3*a^2*D))*((x*(a + b*x^2)^(3/2))/4 + (3*a
*((x*Sqrt[a + b*x^2])/2 + (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt
[b])))/4))/(6*b))/(8*b))/(2*b)
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1473
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[c^p*x^(4*p - 1)*((d + e*x^2)^(q + 1)/(e*(4*p + 2*q + 1)))
, x] + Simp[1/(e*(4*p + 2*q + 1))   Int[(d + e*x^2)^q*ExpandToSum[e*(4*p +
2*q + 1)*(a + b*x^2 + c*x^4)^p - d*c^p*(4*p - 1)*x^(4*p - 2) - e*c^p*(4*p +
 2*q + 1)*x^(4*p), x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] &&  !LtQ[q, -1]
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

## Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.71

| method | result |
|---|---|
| pseudoelliptic | $\frac{\frac{3\left(b^3 A-\frac{1}{6} a b^2 B+\frac{1}{16} a^2 b C-\frac{1}{32} a^3 D\right) a^2 \operatorname{arctanh}\left(\frac{\sqrt{b x^2+a}}{x \sqrt{b}}\right)}{8}+\frac{5 \sqrt{b x^2+a}\left(a\left(\frac{11}{50} D x^6+\frac{3}{10} C x^4+\frac{7}{15} x^2 B+A\right) b^{\frac{7}{2}}+\frac{2\left(\frac{2}{5} D x^6+\frac{1}{2} C x^4+\frac{2}{3}\right.}{5}\right.}{}}{b^{\frac{7}{2}}}$ |
| default | $A\left(\frac{x\left(b x^2+a\right)^{\frac{3}{2}}}{4}+\frac{3 a\left(\frac{x \sqrt{b x^2+a}}{2}+\frac{a \ln\left(\sqrt{b}\, x+\sqrt{b x^2+a}\right)}{2 \sqrt{b}}\right)}{4}\right)+C\left(\frac{x^3\left(b x^2+a\right)^{\frac{5}{2}}}{8 b}-\frac{3 a\left(\frac{x\left(b x^2+a\right)^{\frac{5}{2}}}{6 b}-\frac{a\left(\frac{x\left(b x^2+a\right.}{4}\right.}{}\right)}{}\right.$ |

input
```
int((b*x^2+a)^(3/2)*(D*x^6+C*x^4+B*x^2+A),x,method=_RETURNVERBOSE)
```

output
```
5/8*(3/5*(b^3*A-1/6*a*b^2*B+1/16*a^2*b*C-1/32*a^3*D)*a^2*arctanh((b*x^2+a)
^(1/2)/x/b^(1/2))+(b*x^2+a)^(1/2)*(a*(11/50*D*x^6+3/10*C*x^4+7/15*x^2*B+A)
*b^(7/2)+2/5*(2/5*D*x^6+1/2*C*x^4+2/3*x^2*B+A)*x^2*b^(9/2)+3/160*(4/3*(2/5
*D*x^4+C*x^2+4*B)*b^(5/2)+(2*(-1/3*D*x^2-C)*b^(3/2)+D*a*b^(1/2))*a)*a^2)*x
)/b^(7/2)
```

## Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 413, normalized size of antiderivative = 1.71

$$\int (a+bx^2)^{3/2} (A+Bx^2+Cx^4+Dx^6)\, dx = \left[-\frac{15\,(3\,Da^5-6\,Ca^4b+16\,Ba^3b^2-96\,Aa^2b^3)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\,(38\right.$$

input
```
integrate((b*x^2+a)^(3/2)*(D*x^6+C*x^4+B*x^2+A),x, algorithm="fricas")
```

output
```
[-1/7680*(15*(3*D*a^5 - 6*C*a^4*b + 16*B*a^3*b^2 - 96*A*a^2*b^3)*sqrt(b)*l
og(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(384*D*b^5*x^9 + 48*(11
*D*a*b^4 + 10*C*b^5)*x^7 + 8*(3*D*a^2*b^3 + 90*C*a*b^4 + 80*B*b^5)*x^5 - 1
0*(3*D*a^3*b^2 - 6*C*a^2*b^3 - 112*B*a*b^4 - 96*A*b^5)*x^3 + 15*(3*D*a^4*b
 - 6*C*a^3*b^2 + 16*B*a^2*b^3 + 160*A*a*b^4)*x)*sqrt(b*x^2 + a))/b^4, 1/38
40*(15*(3*D*a^5 - 6*C*a^4*b + 16*B*a^3*b^2 - 96*A*a^2*b^3)*sqrt(-b)*arctan
(sqrt(-b)*x/sqrt(b*x^2 + a)) + (384*D*b^5*x^9 + 48*(11*D*a*b^4 + 10*C*b^5)
*x^7 + 8*(3*D*a^2*b^3 + 90*C*a*b^4 + 80*B*b^5)*x^5 - 10*(3*D*a^3*b^2 - 6*C
*a^2*b^3 - 112*B*a*b^4 - 96*A*b^5)*x^3 + 15*(3*D*a^4*b - 6*C*a^3*b^2 + 16*
B*a^2*b^3 + 160*A*a*b^4)*x)*sqrt(b*x^2 + a))/b^4]
```

## Sympy [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.64

$$\int \left(a+bx^2\right)^{3/2}\left(A+Bx^2+Cx^4+Dx^6\right)\,dx=\begin{cases}\sqrt{a+bx^2}\left(\frac{Dbx^9}{10}+\frac{x^7\left(Cb^2+\frac{11Dab}{10}\right)}{8b}+\frac{x^5\left(Bb^2+2Cab+Da^2-\frac{7a\left(Cb^2+\frac{11Dab}{10}\right)}{8b}\right)}{6b}+\frac{x^3\left(Ab^2+2Bab+Ca^2-\frac{5a\left(Bb\right.}{}\right.}{}\right. \\ a^{\frac{3}{2}}\left(Ax+\frac{Bx^3}{3}+\frac{Cx^5}{5}+\frac{Dx^7}{7}\right)\end{cases}$$

input
```
integrate((b*x**2+a)**(3/2)*(D*x**6+C*x**4+B*x**2+A),x)
```

output
```
Piecewise((sqrt(a + b*x**2)*(D*b*x**9/10 + x**7*(C*b**2 + 11*D*a*b/10)/(8*
b) + x**5*(B*b**2 + 2*C*a*b + D*a**2 - 7*a*(C*b**2 + 11*D*a*b/10)/(8*b))/(
6*b) + x**3*(A*b**2 + 2*B*a*b + C*a**2 - 5*a*(B*b**2 + 2*C*a*b + D*a**2 -
7*a*(C*b**2 + 11*D*a*b/10)/(8*b))/(6*b))/(4*b) + x*(2*A*a*b + B*a**2 - 3*a
*(A*b**2 + 2*B*a*b + C*a**2 - 5*a*(B*b**2 + 2*C*a*b + D*a**2 - 7*a*(C*b**2
 + 11*D*a*b/10)/(8*b))/(6*b))/(4*b))/(2*b)) + (A*a**2 - a*(2*A*a*b + B*a**
2 - 3*a*(A*b**2 + 2*B*a*b + C*a**2 - 5*a*(B*b**2 + 2*C*a*b + D*a**2 - 7*a*
(C*b**2 + 11*D*a*b/10)/(8*b))/(6*b))/(4*b))/(2*b))*Piecewise((log(2*sqrt(b
)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), Tr
ue)), Ne(b, 0)), (a**(3/2)*(A*x + B*x**3/3 + C*x**5/5 + D*x**7/7), True))
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.32

$$\int \left(a+bx^2\right)^{3/2}\left(A+Bx^2+Cx^4+Dx^6\right)\,dx = \frac{(bx^2+a)^{\frac{5}{2}}Dx^5}{10\,b} - \frac{(bx^2+a)^{\frac{5}{2}}Dax^3}{16\,b^2} + \frac{(bx^2+a)^{\frac{5}{2}}Cx^3}{8\,b} + \frac{1}{4}\,(bx^2+a)^{\frac{3}{2}}Ax + \frac{3}{8}\,\sqrt{bx^2+a}Aax + \frac{(bx^2+a)^{\frac{5}{2}}Da^2x}{32\,b^3} - \frac{(bx^2+a)^{\frac{3}{2}}Da^3x}{128\,b^3} - \frac{3\,\sqrt{bx^2+a}Da^4x}{256\,b^3} - \frac{(bx^2+a)^{\frac{5}{2}}Cax}{16\,b^2} + \frac{(bx^2+a)^{\frac{3}{2}}Ca^2x}{64\,b^2} + \frac{3\,\sqrt{bx^2+a}Ca^3x}{128\,b^2} + \frac{(bx^2+a)^{\frac{5}{2}}Bx}{6\,b} - \frac{(bx^2+a)^{\frac{3}{2}}Bax}{24\,b} - \frac{\sqrt{bx^2+a}Ba^2x}{16\,b} - \frac{3\,Da^5\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{256\,b^{\frac{7}{2}}} + \frac{3\,Ca^4\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\,b^{\frac{5}{2}}} - \frac{Ba^3\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{3}{2}}} + \frac{3\,Aa^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{b}}$$

input
```
integrate((b*x^2+a)^(3/2)*(D*x^6+C*x^4+B*x^2+A),x, algorithm="maxima")
```

output
```
1/10*(b*x^2 + a)^(5/2)*D*x^5/b - 1/16*(b*x^2 + a)^(5/2)*D*a*x^3/b^2 + 1/8*
(b*x^2 + a)^(5/2)*C*x^3/b + 1/4*(b*x^2 + a)^(3/2)*A*x + 3/8*sqrt(b*x^2 + a
)*A*a*x + 1/32*(b*x^2 + a)^(5/2)*D*a^2*x/b^3 - 1/128*(b*x^2 + a)^(3/2)*D*a
^3*x/b^3 - 3/256*sqrt(b*x^2 + a)*D*a^4*x/b^3 - 1/16*(b*x^2 + a)^(5/2)*C*a*
x/b^2 + 1/64*(b*x^2 + a)^(3/2)*C*a^2*x/b^2 + 3/128*sqrt(b*x^2 + a)*C*a^3*x
/b^2 + 1/6*(b*x^2 + a)^(5/2)*B*x/b - 1/24*(b*x^2 + a)^(3/2)*B*a*x/b - 1/16
*sqrt(b*x^2 + a)*B*a^2*x/b - 3/256*D*a^5*arcsinh(b*x/sqrt(a*b))/b^(7/2) +
3/128*C*a^4*arcsinh(b*x/sqrt(a*b))/b^(5/2) - 1/16*B*a^3*arcsinh(b*x/sqrt(a
*b))/b^(3/2) + 3/8*A*a^2*arcsinh(b*x/sqrt(a*b))/sqrt(b)
```

## Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.90

$$\int (a+bx^2)^{3/2} (A+Bx^2+Cx^4+Dx^6)\ dx = \frac{1}{3840}\left(2\left(4\left(6\left(8\,Dbx^2+\frac{11\,Dab^8+10\,Cb^9}{b^8}\right)x^2+\frac{3\,Da^2b^7+90\,Cab^8+80\,Bb^9}{b^8}\right)x^2-\frac{5\,(3\,D\ldots$$
$$+\frac{(3\,Da^5-6\,Ca^4b+16\,Ba^3b^2-96\,Aa^2b^3)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{256\,b^{\frac{7}{2}}}$$

input
```
integrate((b*x^2+a)^(3/2)*(D*x^6+C*x^4+B*x^2+A),x, algorithm="giac")
```

output
```
1/3840*(2*(4*(6*(8*D*b*x^2 + (11*D*a*b^8 + 10*C*b^9)/b^8)*x^2 + (3*D*a^2*b
^7 + 90*C*a*b^8 + 80*B*b^9)/b^8)*x^2 - 5*(3*D*a^3*b^6 - 6*C*a^2*b^7 - 112*
B*a*b^8 - 96*A*b^9)/b^8)*x^2 + 15*(3*D*a^4*b^5 - 6*C*a^3*b^6 + 16*B*a^2*b^
7 + 160*A*a*b^8)/b^8)*sqrt(b*x^2 + a)*x + 1/256*(3*D*a^5 - 6*C*a^4*b + 16*
B*a^3*b^2 - 96*A*a^2*b^3)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(7/2)
```

## Mupad [F(-1)]

Timed out.

$$\int (a+bx^2)^{3/2} (A+Bx^2+Cx^4+Dx^6)\ dx = \int (b\,x^2+a)^{3/2}\,(A+B\,x^2+C\,x^4+x^6\,D)\ dx$$

input
```
int((a + b*x^2)^(3/2)*(A + B*x^2 + C*x^4 + x^6*D),x)
```

output
```
int((a + b*x^2)^(3/2)*(A + B*x^2 + C*x^4 + x^6*D), x)
```

## Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.26

$$\int (a+bx^2)^{3/2} (A+Bx^2+Cx^4+Dx^6)\ dx = \frac{45\sqrt{b\,x^2+a}\,a^4bdx-90\sqrt{b\,x^2+a}\,a^3b^2cx-30\sqrt{b\,x^2+a}\,a^3b^2d\,x^3+2640\sqrt{b\,x^2+a}\,a^2b^4x+60}{}$$

input
```
int((b*x^2+a)^(3/2)*(D*x^6+C*x^4+B*x^2+A),x)
```

output
```
(45*sqrt(a + b*x**2)*a**4*b*d*x - 90*sqrt(a + b*x**2)*a**3*b**2*c*x - 30*s
qrt(a + b*x**2)*a**3*b**2*d*x**3 + 2640*sqrt(a + b*x**2)*a**2*b**4*x + 60*
sqrt(a + b*x**2)*a**2*b**3*c*x**3 + 24*sqrt(a + b*x**2)*a**2*b**3*d*x**5 +
 2080*sqrt(a + b*x**2)*a*b**5*x**3 + 720*sqrt(a + b*x**2)*a*b**4*c*x**5 +
528*sqrt(a + b*x**2)*a*b**4*d*x**7 + 640*sqrt(a + b*x**2)*b**6*x**5 + 480*
sqrt(a + b*x**2)*b**5*c*x**7 + 384*sqrt(a + b*x**2)*b**5*d*x**9 - 45*sqrt(
b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**5*d + 90*sqrt(b)*log((sq
rt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**4*b*c + 1200*sqrt(b)*log((sqrt(a +
 b*x**2) + sqrt(b)*x)/sqrt(a))*a**3*b**3)/(3840*b**4)
```

# 3.200 $\int \frac{(a+bx^2)^{3/2}(A+Bx^2+Cx^4+Dx^6)}{x^2}\,dx$



## Optimal result

Integrand size = 32, antiderivative size = 219

$$\begin{aligned}\int \frac{(a+bx^2)^{3/2}(A+Bx^2+Cx^4+Dx^6)}{x^2}\,dx &= \frac{(192Ab^3+a(48b^2B-8abC+3a^2D))\,x\sqrt{a+bx^2}}{128b^2} \\ &+ \frac{\left(\frac{192Ab^3}{a}+48b^2B-8abC+3a^2D\right)x(a+bx^2)^{3/2}}{192b^2} \\ &- \frac{A(a+bx^2)^{5/2}}{ax}+\frac{(8bC-3aD)x(a+bx^2)^{5/2}}{48b^2}+\frac{Dx^3(a+bx^2)^{5/2}}{8b} \\ &+ \frac{a(192Ab^3+a(48b^2B-8abC+3a^2D))\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{5/2}}\end{aligned}$$

output
```
1/128*(192*A*b^3+a*(48*B*b^2-8*C*a*b+3*D*a^2))*x*(b*x^2+a)^(1/2)/b^2+1/192
*(192*A*b^3/a+48*B*b^2-8*C*a*b+3*a^2*D)*x*(b*x^2+a)^(3/2)/b^2-A*(b*x^2+a)^
(5/2)/a/x+1/48*(8*C*b-3*D*a)*x*(b*x^2+a)^(5/2)/b^2+1/8*D*x^3*(b*x^2+a)^(5/
2)/b+1/128*a*(192*A*b^3+a*(48*B*b^2-8*C*a*b+3*D*a^2))*arctanh(b^(1/2)*x/(b
*x^2+a)^(1/2))/b^(5/2)
```

## Mathematica [A] (verified)

Time = 1.29 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.80

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2+Cx^4+Dx^6)}{x^2}\,dx = \frac{\frac{\sqrt{b}\sqrt{a+bx^2}(-9a^3Dx^2+6a^2bx^2(4C+Dx^2)-8ab^2(48A-30Bx^2-14Cx^4-9Dx^6)+}{x}}{}$$

input
```
Integrate[((a + b*x^2)^(3/2)*(A + B*x^2 + C*x^4 + D*x^6))/x^2,x]
```

output
```
((Sqrt[b]*Sqrt[a + b*x^2]*(-9*a^3*D*x^2 + 6*a^2*b*x^2*(4*C + D*x^2) - 8*a*
b^2*(48*A - 30*B*x^2 - 14*C*x^4 - 9*D*x^6) + 16*b^3*x^2*(12*A + 6*B*x^2 +
4*C*x^4 + 3*D*x^6)))/x + 6*a*(192*A*b^3 + a*(48*b^2*B - 8*a*b*C + 3*a^2*D)
)*ArcTanh[(Sqrt[b]*x)/(-Sqrt[a] + Sqrt[a + b*x^2])])/(384*b^(5/2))
```

## Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.86, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.281, Rules used = {2338, 9, 25, 1473, 299, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2+Cx^4+Dx^6)}{x^2}\,dx$$

$$\Big\downarrow \text{2338}$$

$$-\frac{\int -\frac{(bx^2+a)^{3/2}(aDx^5+aCx^3+(4Ab+aB)x)}{x}dx}{a}-\frac{A(a+bx^2)^{5/2}}{ax}$$

$$\Big\downarrow \text{9}$$

$$-\frac{\int -(bx^2+a)^{3/2}\left(aDx^4+aCx^2+4Ab+aB\right)dx}{a}-\frac{A(a+bx^2)^{5/2}}{ax}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int (bx^2+a)^{3/2}\left(aDx^4+aCx^2+4Ab+aB\right)dx}{a}-\frac{A(a+bx^2)^{5/2}}{ax}$$

$$\Big\downarrow \; 1473$$

$$\frac{\frac{\int \left(bx^2+a\right)^{3/2} \left(a(8bC-3aD)x^2+8b(4Ab+aB)\right)dx}{8b}+\frac{aDx^3\left(a+bx^2\right)^{5/2}}{8b}}{a}-\frac{A\left(a+bx^2\right)^{5/2}}{ax}$$

$$\Big\downarrow \; 299$$

$$\frac{\frac{\frac{\left(48b^2(aB+4Ab)-a^2(8bC-3aD)\right)\int \left(bx^2+a\right)^{3/2}dx}{6b}+\frac{ax\left(a+bx^2\right)^{5/2}(8bC-3aD)}{6b}}{8b}+\frac{aDx^3\left(a+bx^2\right)^{5/2}}{8b}}{a}-\frac{A\left(a+bx^2\right)^{5/2}}{ax}$$

$$\Big\downarrow \; 211$$

$$\frac{\frac{\frac{\left(48b^2(aB+4Ab)-a^2(8bC-3aD)\right)\left(\frac{3}{4}a\int \sqrt{bx^2+a}dx+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)}{6b}+\frac{ax\left(a+bx^2\right)^{5/2}(8bC-3aD)}{6b}}{8b}+\frac{aDx^3\left(a+bx^2\right)^{5/2}}{8b}}{a}-\frac{A\left(a+bx^2\right)^{5/2}}{ax}$$

$$\Big\downarrow \; 211$$

$$\frac{\frac{\frac{\left(48b^2(aB+4Ab)-a^2(8bC-3aD)\right)\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)}{6b}+\frac{ax\left(a+bx^2\right)^{5/2}(8bC-3aD)}{6b}}{8b}+\frac{aDx^3\left(a+bx^2\right)^{5/2}}{8b}}{a}-\frac{A\left(a+bx^2\right)^{5/2}}{ax}$$

$$\Big\downarrow \; 224$$

$$\frac{\frac{\frac{\left(48b^2(aB+4Ab)-a^2(8bC-3aD)\right)\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)}{6b}+\frac{ax\left(a+bx^2\right)^{5/2}(8bC-3aD)}{6b}}{8b}+\frac{aDx^3\left(a+bx^2\right)^{5/2}}{8b}}{a}-\frac{A\left(a+bx^2\right)^{5/2}}{ax}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{\frac{\left(\frac{3}{4}a\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)\left(48b^2(aB+4Ab)-a^2(8bC-3aD)\right)}{6b}+\frac{ax\left(a+bx^2\right)^{5/2}(8bC-3aD)}{6b}}{8b}+\frac{aDx^3\left(a+bx^2\right)^{5/2}}{8b}}{a}-\frac{A\left(a+bx^2\right)^{5/2}}{ax}$$

input
```
Int[((a + b*x^2)^(3/2)*(A + B*x^2 + C*x^4 + D*x^6))/x^2,x]
```

output
```
-((A*(a + b*x^2)^(5/2))/(a*x)) + ((a*D*x^3*(a + b*x^2)^(5/2))/(8*b) + ((a*
(8*b*C - 3*a*D)*x*(a + b*x^2)^(5/2))/(6*b) + ((48*b^2*(4*A*b + a*B) - a^2*
(8*b*C - 3*a*D))*((x*(a + b*x^2)^(3/2))/4 + (3*a*((x*Sqrt[a + b*x^2])/2 +
(a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b])))/4))/(6*b))/(8*b))/a
```

**Defintions of rubi rules used**

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1473
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[c^p*x^(4*p - 1)*((d + e*x^2)^(q + 1)/(e*(4*p + 2*q + 1)))
, x] + Simp[1/(e*(4*p + 2*q + 1))   Int[(d + e*x^2)^q*ExpandToSum[e*(4*p +
2*q + 1)*(a + b*x^2 + c*x^4)^p - d*c^p*(4*p - 1)*x^(4*p - 2) - e*c^p*(4*p +
 2*q + 1)*x^(4*p), x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] &&  !LtQ[q, -1]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

## Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.68

| method | result |
|---|---|
| pseudoelliptic | $-\frac{-\frac{3\left(b^3A+\frac{1}{4}ab^2B-\frac{1}{24}a^2bC+\frac{1}{64}a^3D\right)xa\,\operatorname{arctanh}\left(\frac{\sqrt{bx^2+a}}{x\sqrt{b}}\right)}{2}+\sqrt{bx^2+a}\left(a\left(-\frac{3}{16}Dx^6-\frac{7}{24}Cx^4-\frac{5}{8}x^2B+A\right)b^{\frac{5}{2}}-\frac{\left(\frac{1}{4}Dx^6+\ldots\right.}{\ldots}\right.}{b^{\frac{5}{2}}x}$ |
| default | $B\left(\frac{x\left(bx^2+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{bx^2+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{bx^2+a}\right)}{2\sqrt{b}}\right)}{4}\right)+A\left(-\frac{\left(bx^2+a\right)^{\frac{5}{2}}}{ax}+\frac{4b\left(\frac{x\left(bx^2+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{bx^2\ldots}}{2}\ldots\right.}{\ldots}\right)}{a\ldots}\right.$ |

input
```
int((b*x^2+a)^(3/2)*(D*x^6+C*x^4+B*x^2+A)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/b^(5/2)*(-3/2*(b^3*A+1/4*a*b^2*B-1/24*a^2*b*C+1/64*a^3*D)*x*a*arctanh((
b*x^2+a)^(1/2)/x/b^(1/2))+(b*x^2+a)^(1/2)*(a*(-3/16*D*x^6-7/24*C*x^4-5/8*x
^2*B+A)*b^(5/2)-1/2*((1/4*D*x^6+1/3*C*x^4+1/2*x^2*B+A)*b^(7/2)-3/64*(2/3*(
-D*x^2-4*C)*b^(3/2)+D*a*b^(1/2))*a^2)*x^2))/x
```

## Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.63

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2+Cx^4+Dx^6)}{x^2}\,dx = \left[\frac{3\,(3\,Da^4-8\,Ca^3b+48\,Ba^2b^2+192\,Aab^3)\sqrt{b}x\log\left(-2\,bx\right.}{}\right.$$

$$-\frac{3\,(3\,Da^4-8\,Ca^3b+48\,Ba^2b^2+192\,Aab^3)\sqrt{-b}x\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(48\,Db^4x^8+8\,(9\,Dab^3+8\,Cb^4)x^6-}{384}$$

input
```
integrate((b*x^2+a)^(3/2)*(D*x^6+C*x^4+B*x^2+A)/x^2,x, algorithm="fricas")
```

output
```
[1/768*(3*(3*D*a^4 - 8*C*a^3*b + 48*B*a^2*b^2 + 192*A*a*b^3)*sqrt(b)*x*log
(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(48*D*b^4*x^8 + 8*(9*D*a*
b^3 + 8*C*b^4)*x^6 - 384*A*a*b^3 + 2*(3*D*a^2*b^2 + 56*C*a*b^3 + 48*B*b^4)
*x^4 - 3*(3*D*a^3*b - 8*C*a^2*b^2 - 80*B*a*b^3 - 64*A*b^4)*x^2)*sqrt(b*x^2
 + a))/(b^3*x), -1/384*(3*(3*D*a^4 - 8*C*a^3*b + 48*B*a^2*b^2 + 192*A*a*b^
3)*sqrt(-b)*x*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (48*D*b^4*x^8 + 8*(9*D*
a*b^3 + 8*C*b^4)*x^6 - 384*A*a*b^3 + 2*(3*D*a^2*b^2 + 56*C*a*b^3 + 48*B*b^
4)*x^4 - 3*(3*D*a^3*b - 8*C*a^2*b^2 - 80*B*a*b^3 - 64*A*b^4)*x^2)*sqrt(b*x
^2 + a))/(b^3*x)]
```

## Sympy [A] (verification not implemented)

Time = 2.26 (sec) , antiderivative size = 777, normalized size of antiderivative = 3.55

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2+Cx^4+Dx^6)}{x^2}\,dx = \text{Too large to display}$$

input
```
integrate((b*x**2+a)**(3/2)*(D*x**6+C*x**4+B*x**2+A)/x**2,x)
```

output
```
-A*a**(3/2)/(x*sqrt(1 + b*x**2/a)) - A*sqrt(a)*b*x/sqrt(1 + b*x**2/a) + A*
a*sqrt(b)*asinh(sqrt(b)*x/sqrt(a)) + A*b*Piecewise((a*Piecewise((log(2*sqr
t(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2),
 True))/2 + x*sqrt(a + b*x**2)/2, Ne(b, 0)), (sqrt(a)*x, True)) + B*a*Piec
ewise((a*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a,
 0)), (x*log(x)/sqrt(b*x**2), True))/2 + x*sqrt(a + b*x**2)/2, Ne(b, 0)),
(sqrt(a)*x, True)) + B*b*Piecewise((-a**2*Piecewise((log(2*sqrt(b)*sqrt(a
+ b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True))/(8*b
) + a*x*sqrt(a + b*x**2)/(8*b) + x**3*sqrt(a + b*x**2)/4, Ne(b, 0)), (sqrt
(a)*x**3/3, True)) + C*a*Piecewise((-a**2*Piecewise((log(2*sqrt(b)*sqrt(a
+ b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True))/(8*b
) + a*x*sqrt(a + b*x**2)/(8*b) + x**3*sqrt(a + b*x**2)/4, Ne(b, 0)), (sqrt
(a)*x**3/3, True)) + C*b*Piecewise((a**3*Piecewise((log(2*sqrt(b)*sqrt(a +
 b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True))/(16*b
**2) - a**2*x*sqrt(a + b*x**2)/(16*b**2) + a*x**3*sqrt(a + b*x**2)/(24*b)
+ x**5*sqrt(a + b*x**2)/6, Ne(b, 0)), (sqrt(a)*x**5/5, True)) + D*a*Piecew
ise((a**3*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a
, 0)), (x*log(x)/sqrt(b*x**2), True))/(16*b**2) - a**2*x*sqrt(a + b*x**2)/
(16*b**2) + a*x**3*sqrt(a + b*x**2)/(24*b) + x**5*sqrt(a + b*x**2)/6, Ne(b
, 0)), (sqrt(a)*x**5/5, True)) + D*b*Piecewise((-5*a**4*Piecewise((log(...
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.15

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2+Cx^4+Dx^6)}{x^2}\,dx = \frac{(bx^2+a)^{\frac{5}{2}}Dx^3}{8\,b} + \frac{1}{4}(bx^2+a)^{\frac{3}{2}}Bx + \frac{3}{8}\sqrt{bx^2+a}Bax - \frac{(bx^2+a)^{\frac{5}{2}}Dax}{16\,b^2} + \frac{(bx^2+a)^{\frac{3}{2}}Da^2x}{64\,b^2} + \frac{3\sqrt{bx^2+a}Da^3x}{128\,b^2} + \frac{(bx^2+a)^{\frac{5}{2}}Cx}{6\,b} - \frac{(bx^2+a)^{\frac{3}{2}}Cax}{24\,b} - \frac{\sqrt{bx^2+a}Ca^2x}{16\,b} + \frac{3}{2}\sqrt{bx^2+a}Abx + \frac{3\,Da^4\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\,b^{\frac{5}{2}}} - \frac{Ca^3\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{3}{2}}} + \frac{3\,Ba^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{b}} + \frac{3}{2}Aa\sqrt{b}\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right) - \frac{(bx^2+a)^{\frac{3}{2}}A}{x}$$

input
```
integrate((b*x^2+a)^(3/2)*(D*x^6+C*x^4+B*x^2+A)/x^2,x, algorithm="maxima")
```

output
```
1/8*(b*x^2 + a)^(5/2)*D*x^3/b + 1/4*(b*x^2 + a)^(3/2)*B*x + 3/8*sqrt(b*x^2
 + a)*B*a*x - 1/16*(b*x^2 + a)^(5/2)*D*a*x/b^2 + 1/64*(b*x^2 + a)^(3/2)*D*
a^2*x/b^2 + 3/128*sqrt(b*x^2 + a)*D*a^3*x/b^2 + 1/6*(b*x^2 + a)^(5/2)*C*x/
b - 1/24*(b*x^2 + a)^(3/2)*C*a*x/b - 1/16*sqrt(b*x^2 + a)*C*a^2*x/b + 3/2*
sqrt(b*x^2 + a)*A*b*x + 3/128*D*a^4*arcsinh(b*x/sqrt(a*b))/b^(5/2) - 1/16*
C*a^3*arcsinh(b*x/sqrt(a*b))/b^(3/2) + 3/8*B*a^2*arcsinh(b*x/sqrt(a*b))/sq
rt(b) + 3/2*A*a*sqrt(b)*arcsinh(b*x/sqrt(a*b)) - (b*x^2 + a)^(3/2)*A/x
```

## Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2+Cx^4+Dx^6)}{x^2}\,dx = \frac{2\,Aa^2\sqrt{b}}{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a} + \frac{1}{384}\left(2\left(4\left(6\,Dbx^2+\frac{9\,Dab^6+8\,Cb^7}{b^6}\right)x^2+\frac{3\,Da^2b^5+56\,Cab^6+48\,Bb^7}{b^6}\right)x^2-\frac{3\,(3\,Da^3b^4-8\,Ca^2b^5-}{b^6}\right. - \frac{(3\,Da^4-8\,Ca^3b+48\,Ba^2b^2+192\,Aab^3)\log\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2\right)}{256\,b^{\frac{5}{2}}}$$

input
```
integrate((b*x^2+a)^(3/2)*(D*x^6+C*x^4+B*x^2+A)/x^2,x, algorithm="giac")
```

output
```
2*A*a^2*sqrt(b)/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a) + 1/384*(2*(4*(6*D*b
*x^2 + (9*D*a*b^6 + 8*C*b^7)/b^6)*x^2 + (3*D*a^2*b^5 + 56*C*a*b^6 + 48*B*b
^7)/b^6)*x^2 - 3*(3*D*a^3*b^4 - 8*C*a^2*b^5 - 80*B*a*b^6 - 64*A*b^7)/b^6)*
sqrt(b*x^2 + a)*x - 1/256*(3*D*a^4 - 8*C*a^3*b + 48*B*a^2*b^2 + 192*A*a*b^
3)*log((sqrt(b)*x - sqrt(b*x^2 + a))^2)/b^(5/2)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2+Cx^4+Dx^6)}{x^2}\,dx=\int \frac{(b\,x^2+a)^{3/2}\,(A+B\,x^2+C\,x^4+x^6\,D)}{x^2}\,dx$$

input
```
int(((a + b*x^2)^(3/2)*(A + B*x^2 + C*x^4 + x^6*D))/x^2,x)
```

output
```
int(((a + b*x^2)^(3/2)*(A + B*x^2 + C*x^4 + x^6*D))/x^2, x)
```

## Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.34

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2+Cx^4+Dx^6)}{x^2}\,dx=\frac{-9\sqrt{b\,x^2+a}\,a^3bd\,x^2-384\sqrt{b\,x^2+a}\,a^2b^3+24\sqrt{b\,x^2+a}\,a^2b}{}$$

input
```
int((b*x^2+a)^(3/2)*(D*x^6+C*x^4+B*x^2+A)/x^2,x)
```

output
```
( - 9*sqrt(a + b*x**2)*a**3*b*d*x**2 - 384*sqrt(a + b*x**2)*a**2*b**3 + 24
*sqrt(a + b*x**2)*a**2*b**2*c*x**2 + 6*sqrt(a + b*x**2)*a**2*b**2*d*x**4 +
 432*sqrt(a + b*x**2)*a*b**4*x**2 + 112*sqrt(a + b*x**2)*a*b**3*c*x**4 + 7
2*sqrt(a + b*x**2)*a*b**3*d*x**6 + 96*sqrt(a + b*x**2)*b**5*x**4 + 64*sqrt
(a + b*x**2)*b**4*c*x**6 + 48*sqrt(a + b*x**2)*b**4*d*x**8 + 9*sqrt(b)*log
((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**4*d*x - 24*sqrt(b)*log((sqrt(a
 + b*x**2) + sqrt(b)*x)/sqrt(a))*a**3*b*c*x + 720*sqrt(b)*log((sqrt(a + b*
x**2) + sqrt(b)*x)/sqrt(a))*a**2*b**3*x + 3*sqrt(b)*a**3*b*c*x - 480*sqrt(
b)*a**2*b**3*x)/(384*b**3*x)
```

# 3.201 $\int \frac{(a+bx^2)^{3/2}(A+Bx^2+Cx^4+Dx^6)}{x^4}\,dx$



## Optimal result

Integrand size = 32, antiderivative size = 220

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2+Cx^4+Dx^6)}{x^4}\,dx=\frac{(16Ab^3+6ab(4bB+aC)-a^3D)\,x\sqrt{a+bx^2}}{16ab}$$
$$+\frac{(16Ab^3+6ab(4bB+aC)-a^3D)\,x(a+bx^2)^{3/2}}{24a^2b}$$
$$-\frac{A(a+bx^2)^{5/2}}{3ax^3}-\frac{(2Ab+3aB)\,(a+bx^2)^{5/2}}{3a^2x}+\frac{Dx(a+bx^2)^{5/2}}{6b}$$
$$+\frac{(16Ab^3+6ab(4bB+aC)-a^3D)\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{3/2}}$$

output
```
1/16*(16*A*b^3+6*a*b*(4*B*b+C*a)-a^3*D)*x*(b*x^2+a)^(1/2)/a/b+1/24*(16*A*b
^3+6*a*b*(4*B*b+C*a)-a^3*D)*x*(b*x^2+a)^(3/2)/a^2/b-1/3*A*(b*x^2+a)^(5/2)/
a/x^3-1/3*(2*A*b+3*B*a)*(b*x^2+a)^(5/2)/a^2/x+1/6*D*x*(b*x^2+a)^(5/2)/b+1/
16*(16*A*b^3+6*a*b*(4*B*b+C*a)-a^3*D)*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b
^(3/2)
```

## Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.70

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2+Cx^4+Dx^6)}{x^4}\,dx = \frac{\sqrt{a+bx^2}(3a^2Dx^4-2ab(8A+24Bx^2-15Cx^4-7Dx^6)+}{48bx^3}$$
$$+\frac{(16Ab^3+a(24b^2B+6abC-a^2D))\,\text{arctanh}\left(\frac{\sqrt{b}x}{-\sqrt{a}+\sqrt{a+bx^2}}\right)}{8b^{3/2}}$$

input
```
Integrate[((a + b*x^2)^(3/2)*(A + B*x^2 + C*x^4 + D*x^6))/x^4,x]
```

output
```
(Sqrt[a + b*x^2]*(3*a^2*D*x^4 - 2*a*b*(8*A + 24*B*x^2 - 15*C*x^4 - 7*D*x^6
) + 4*b^2*x^2*(-16*A + 6*B*x^2 + 3*C*x^4 + 2*D*x^6)))/(48*b*x^3) + ((16*A*
b^3 + a*(24*b^2*B + 6*a*b*C - a^2*D))*ArcTanh[(Sqrt[b]*x)/(-Sqrt[a] + Sqrt
[a + b*x^2])])/(8*b^(3/2))
```

## Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.85, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {2338, 9, 25, 1588, 25, 299, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2+Cx^4+Dx^6)}{x^4}\,dx$$

$$\Big\downarrow \text{2338}$$

$$-\frac{\int -\frac{(bx^2+a)^{3/2}(3aDx^5+3aCx^3+(2Ab+3aB)x)}{x^3}dx}{3a}-\frac{A(a+bx^2)^{5/2}}{3ax^3}$$

$$\Big\downarrow \text{9}$$

$$-\frac{\int -\frac{(bx^2+a)^{3/2}(3aDx^4+3aCx^2+2Ab+3aB)}{x^2}dx}{3a}-\frac{A(a+bx^2)^{5/2}}{3ax^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\left(bx^2+a\right)^{3/2}\left(3aDx^4+3aCx^2+2Ab+3aB\right)}{x^2}dx}{3a}-\frac{A\left(a+bx^2\right)^{5/2}}{3ax^3}$$

$\big\downarrow$ 1588

$$\frac{-\frac{\int -\left(bx^2+a\right)^{3/2}\left(8Ab^2+3a^2Dx^2+3a(4bB+aC)\right)dx}{a}-\frac{\left(a+bx^2\right)^{5/2}(3aB+2Ab)}{ax}}{3a}-\frac{A\left(a+bx^2\right)^{5/2}}{3ax^3}$$

$\big\downarrow$ 25

$$\frac{\frac{\int \left(bx^2+a\right)^{3/2}\left(8Ab^2+3a^2Dx^2+3a(4bB+aC)\right)dx}{a}-\frac{\left(a+bx^2\right)^{5/2}(3aB+2Ab)}{ax}}{3a}-\frac{A\left(a+bx^2\right)^{5/2}}{3ax^3}$$

$\big\downarrow$ 299

$$\frac{\frac{\frac{\left(a^3(-D)+6ab(aC+4bB)+16Ab^3\right)\int \left(bx^2+a\right)^{3/2}dx}{2b}+\frac{a^2Dx\left(a+bx^2\right)^{5/2}}{2b}}{a}-\frac{\left(a+bx^2\right)^{5/2}(3aB+2Ab)}{ax}}{3a}-\frac{A\left(a+bx^2\right)^{5/2}}{3ax^3}$$

$\big\downarrow$ 211

$$\frac{\frac{\frac{\left(a^3(-D)+6ab(aC+4bB)+16Ab^3\right)\left(\frac{3}{4}a\int \sqrt{bx^2+a}dx+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)}{2b}+\frac{a^2Dx\left(a+bx^2\right)^{5/2}}{2b}}{a}-\frac{\left(a+bx^2\right)^{5/2}(3aB+2Ab)}{ax}}{3a}-\frac{A\left(a+bx^2\right)^{5/2}}{3ax^3}$$

$\big\downarrow$ 211

$$\frac{\frac{\frac{\left(a^3(-D)+6ab(aC+4bB)+16Ab^3\right)\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)}{2b}+\frac{a^2Dx\left(a+bx^2\right)^{5/2}}{2b}}{a}-\frac{\left(a+bx^2\right)^{5/2}(3aB+2Ab)}{ax}}{3a}-\frac{A\left(a+bx^2\right)^{5/2}}{3ax^3}$$

$\big\downarrow$ 224

$$\frac{\frac{\frac{\left(a^3(-D)+6ab(aC+4bB)+16Ab^3\right)\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)}{2b}+\frac{a^2Dx\left(a+bx^2\right)^{5/2}}{2b}}{a}-\frac{\left(a+bx^2\right)^{5/2}(3aB+2Ab)}{ax}}{3a}-\frac{A\left(a+bx^2\right)^{5/2}}{3ax^3}$$

$\big\downarrow$ 219

$$\frac{\frac{\frac{\left(\frac{3}{4}a\left(\frac{a\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)\left(a^3(-D)+6ab(aC+4bB)+16Ab^3\right)}{2b}+\frac{a^2Dx\left(a+bx^2\right)^{5/2}}{2b}}{a}-\frac{\left(a+bx^2\right)^{5/2}(3aB+2Ab)}{ax}}{3a}-\frac{A\left(a+bx^2\right)^{5/2}}{3ax^3}$$

input
```
Int[((a + b*x^2)^(3/2)*(A + B*x^2 + C*x^4 + D*x^6))/x^4,x]
```

output
```
-1/3*(A*(a + b*x^2)^(5/2))/(a*x^3) + (-(((2*A*b + 3*a*B)*(a + b*x^2)^(5/2)
)/(a*x)) + ((a^2*D*x*(a + b*x^2)^(5/2))/(2*b) + ((16*A*b^3 + 6*a*b*(4*b*B
+ a*C) - a^3*D)*((x*(a + b*x^2)^(3/2))/4 + (3*a*((x*Sqrt[a + b*x^2])/2 + (
a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b])))/4))/(2*b))/a)/(3*a)
```

## Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1588
```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Simp[1/(d*f
^2*(m + 1))   Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x
) - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

## Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.62

| method | result |
|---|---|
| pseudoelliptic | $-\frac{-3x^3\left(b^3A+\frac{3}{2}a\,b^2B+\frac{3}{8}a^2bC-\frac{1}{16}a^3D\right)\operatorname{arctanh}\left(\frac{\sqrt{b\,x^2+a}}{x\sqrt{b}}\right)+\sqrt{b\,x^2+a}\,\left(a\left(-\frac{7}{8}Dx^6-\frac{15}{8}C\,x^4+3x^2B+A\right)b^{\frac{3}{2}}+4\left(\left(-\frac{1}{8}Dx\right.\right.\right.}{3b^{\frac{3}{2}}x^3}$ |
| default | $C\left(\frac{x\left(b\,x^2+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)}{4}\right)+A\left(-\frac{\left(b\,x^2+a\right)^{\frac{5}{2}}}{3a\,x^3}+\frac{2b\left(-\frac{\left(b\,x^2+a\right)^{\frac{5}{2}}}{ax}+\frac{4b\left(\frac{x\left(b\,x^2\right.}{4}\right.}{}\right.}{}\right.$ |

input
```
int((b*x^2+a)^(3/2)*(D*x^6+C*x^4+B*x^2+A)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3/b^(3/2)*(-3*x^3*(b^3*A+3/2*a*b^2*B+3/8*a^2*b*C-1/16*a^3*D)*arctanh((b
*x^2+a)^(1/2)/x/b^(1/2))+(b*x^2+a)^(1/2)*(a*(-7/8*D*x^6-15/8*C*x^4+3*x^2*B
+A)*b^(3/2)+4*((-1/8*D*x^6-3/16*C*x^4-3/8*x^2*B+A)*b^(5/2)-3/64*D*a^2*x^2*
b^(1/2))*x^2))/x^3
```

## Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.42

$$\int \frac{\left(a+bx^2\right)^{3/2}\left(A+Bx^2+Cx^4+Dx^6\right)}{x^4}\,dx = \left[-\frac{3\left(Da^3-6\,Ca^2b-24\,Bab^2-16\,Ab^3\right)\sqrt{b}x^3\log\left(-2\,bx^2\right.}{}\right.$$

input
```
integrate((b*x^2+a)^(3/2)*(D*x^6+C*x^4+B*x^2+A)/x^4,x, algorithm="fricas")
```

output
```
[-1/96*(3*(D*a^3 - 6*C*a^2*b - 24*B*a*b^2 - 16*A*b^3)*sqrt(b)*x^3*log(-2*b
*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(8*D*b^3*x^8 + 2*(7*D*a*b^2 +
6*C*b^3)*x^6 + 3*(D*a^2*b + 10*C*a*b^2 + 8*B*b^3)*x^4 - 16*A*a*b^2 - 16*(3
*B*a*b^2 + 4*A*b^3)*x^2)*sqrt(b*x^2 + a))/(b^2*x^3), 1/48*(3*(D*a^3 - 6*C*
a^2*b - 24*B*a*b^2 - 16*A*b^3)*sqrt(-b)*x^3*arctan(sqrt(-b)*x/sqrt(b*x^2 +
 a)) + (8*D*b^3*x^8 + 2*(7*D*a*b^2 + 6*C*b^3)*x^6 + 3*(D*a^2*b + 10*C*a*b^
2 + 8*B*b^3)*x^4 - 16*A*a*b^2 - 16*(3*B*a*b^2 + 4*A*b^3)*x^2)*sqrt(b*x^2 +
 a))/(b^2*x^3)]
```

## Sympy [A] (verification not implemented)

Time = 2.83 (sec) , antiderivative size = 624, normalized size of antiderivative = 2.84

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2+Cx^4+Dx^6)}{x^4}\,dx = \text{Too large to display}$$

input
```
integrate((b*x**2+a)**(3/2)*(D*x**6+C*x**4+B*x**2+A)/x**4,x)
```

output
```
-A*sqrt(a)*b/(x*sqrt(1 + b*x**2/a)) - A*a*sqrt(b)*sqrt(a/(b*x**2) + 1)/(3*
x**2) - A*b**(3/2)*sqrt(a/(b*x**2) + 1)/3 + A*b**(3/2)*asinh(sqrt(b)*x/sqr
t(a)) - A*b**2*x/(sqrt(a)*sqrt(1 + b*x**2/a)) - B*a**(3/2)/(x*sqrt(1 + b*x
**2/a)) - B*sqrt(a)*b*x/sqrt(1 + b*x**2/a) + B*a*sqrt(b)*asinh(sqrt(b)*x/s
qrt(a)) + B*b*Piecewise((a*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b
*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True))/2 + x*sqrt(a + b*x*
*2)/2, Ne(b, 0)), (sqrt(a)*x, True)) + C*a*Piecewise((a*Piecewise((log(2*s
qrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2
), True))/2 + x*sqrt(a + b*x**2)/2, Ne(b, 0)), (sqrt(a)*x, True)) + C*b*Pi
ecewise((-a**2*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b),
 Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True))/(8*b) + a*x*sqrt(a + b*x**2)/(8
*b) + x**3*sqrt(a + b*x**2)/4, Ne(b, 0)), (sqrt(a)*x**3/3, True)) + D*a*Pi
ecewise((-a**2*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b),
 Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True))/(8*b) + a*x*sqrt(a + b*x**2)/(8
*b) + x**3*sqrt(a + b*x**2)/4, Ne(b, 0)), (sqrt(a)*x**3/3, True)) + D*b*Pi
ecewise((a**3*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b),
Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True))/(16*b**2) - a**2*x*sqrt(a + b*x*
*2)/(16*b**2) + a*x**3*sqrt(a + b*x**2)/(24*b) + x**5*sqrt(a + b*x**2)/6,
Ne(b, 0)), (sqrt(a)*x**5/5, True))
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.05

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2+Cx^4+Dx^6)}{x^4}\,dx = \frac{1}{4}(bx^2+a)^{\frac{3}{2}}Cx + \frac{3}{8}\sqrt{bx^2+a}Cax + \frac{(bx^2+a)^{\frac{5}{2}}Dx}{6\,b} - \frac{(bx^2+a)^{\frac{3}{2}}Dax}{24\,b} - \frac{\sqrt{bx^2+a}Da^2x}{16\,b} + \frac{3}{2}\sqrt{bx^2+a}Bbx + \frac{\sqrt{bx^2+a}Ab^2x}{a} - \frac{Da^3\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{3}{2}}} + \frac{3\,Ca^2\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{b}} + \frac{3}{2}Ba\sqrt{b}\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right) + Ab^{\frac{3}{2}}\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right) - \frac{(bx^2+a)^{\frac{3}{2}}B}{x} - \frac{2\,(bx^2+a)^{\frac{3}{2}}Ab}{3\,ax} - \frac{(bx^2+a)^{\frac{5}{2}}A}{3\,ax^3}$$

input
```
integrate((b*x^2+a)^(3/2)*(D*x^6+C*x^4+B*x^2+A)/x^4,x, algorithm="maxima")
```

output
```
1/4*(b*x^2 + a)^(3/2)*C*x + 3/8*sqrt(b*x^2 + a)*C*a*x + 1/6*(b*x^2 + a)^(5
/2)*D*x/b - 1/24*(b*x^2 + a)^(3/2)*D*a*x/b - 1/16*sqrt(b*x^2 + a)*D*a^2*x/
b + 3/2*sqrt(b*x^2 + a)*B*b*x + sqrt(b*x^2 + a)*A*b^2*x/a - 1/16*D*a^3*arc
sinh(b*x/sqrt(a*b))/b^(3/2) + 3/8*C*a^2*arcsinh(b*x/sqrt(a*b))/sqrt(b) + 3
/2*B*a*sqrt(b)*arcsinh(b*x/sqrt(a*b)) + A*b^(3/2)*arcsinh(b*x/sqrt(a*b)) -
 (b*x^2 + a)^(3/2)*B/x - 2/3*(b*x^2 + a)^(3/2)*A*b/(a*x) - 1/3*(b*x^2 + a)
^(5/2)*A/(a*x^3)
```

## Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.27

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2+Cx^4+Dx^6)}{x^4}\,dx = \frac{1}{48}\left(2\left(4\,Dbx^2 + \frac{7\,Dab^4+6\,Cb^5}{b^4}\right)x^2 + \frac{3\,(Da^2b^3+10\,Cab}{b^4}\right. + \frac{(Da^3-6\,Ca^2b-24\,Bab^2-16\,Ab^3)\log\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2\right)}{32\,b^{\frac{3}{2}}} + \frac{2\left(3\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4Ba^2\sqrt{b}+6\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4Aab^{\frac{3}{2}}-6\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2Ba^3\sqrt{b}-6\left(\sqrt{b}\right.\right.}{3\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^3}$$

input
```
integrate((b*x^2+a)^(3/2)*(D*x^6+C*x^4+B*x^2+A)/x^4,x, algorithm="giac")
```

output
```
1/48*(2*(4*D*b*x^2 + (7*D*a*b^4 + 6*C*b^5)/b^4)*x^2 + 3*(D*a^2*b^3 + 10*C*
a*b^4 + 8*B*b^5)/b^4)*sqrt(b*x^2 + a)*x + 1/32*(D*a^3 - 6*C*a^2*b - 24*B*a
*b^2 - 16*A*b^3)*log((sqrt(b)*x - sqrt(b*x^2 + a))^2)/b^(3/2) + 2/3*(3*(sq
rt(b)*x - sqrt(b*x^2 + a))^4*B*a^2*sqrt(b) + 6*(sqrt(b)*x - sqrt(b*x^2 + a
))^4*A*a*b^(3/2) - 6*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a^3*sqrt(b) - 6*(sq
rt(b)*x - sqrt(b*x^2 + a))^2*A*a^2*b^(3/2) + 3*B*a^4*sqrt(b) + 4*A*a^3*b^(
3/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^3
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2+Cx^4+Dx^6)}{x^4}\,dx = \int \frac{(b\,x^2+a)^{3/2}\,(A+B\,x^2+C\,x^4+x^6\,D)}{x^4}\,dx$$

input
```
int(((a + b*x^2)^(3/2)*(A + B*x^2 + C*x^4 + x^6*D))/x^4,x)
```

output
```
int(((a + b*x^2)^(3/2)*(A + B*x^2 + C*x^4 + x^6*D))/x^4, x)
```

## Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.23

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2+Cx^4+Dx^6)}{x^4}\,dx = \frac{-128\sqrt{b\,x^2+a}\,a^2b^2+24\sqrt{b\,x^2+a}\,a^2bd\,x^4-896\sqrt{b\,x^2+a}\,a}{}$$

input
```
int((b*x^2+a)^(3/2)*(D*x^6+C*x^4+B*x^2+A)/x^4,x)
```

output

```
( - 128*sqrt(a + b*x**2)*a**2*b**2 + 24*sqrt(a + b*x**2)*a**2*b*d*x**4 - 8
96*sqrt(a + b*x**2)*a*b**3*x**2 + 240*sqrt(a + b*x**2)*a*b**2*c*x**4 + 112
*sqrt(a + b*x**2)*a*b**2*d*x**6 + 192*sqrt(a + b*x**2)*b**4*x**4 + 96*sqrt
(a + b*x**2)*b**3*c*x**6 + 64*sqrt(a + b*x**2)*b**3*d*x**8 - 24*sqrt(b)*lo
g((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**3*d*x**3 + 144*sqrt(b)*log((s
qrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2*b*c*x**3 + 960*sqrt(b)*log((sqr
t(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a*b**3*x**3 - 3*sqrt(b)*a**3*d*x**3 +
30*sqrt(b)*a**2*b*c*x**3 + 160*sqrt(b)*a*b**3*x**3)/(384*b**2*x**3)
```

# 3.202 $\int \frac{(a+bx^2)^{3/2}(A+Bx^2+Cx^4+Dx^6)}{x^6}\,dx$



## Optimal result

Integrand size = 32, antiderivative size = 195

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2+Cx^4+Dx^6)}{x^6}\,dx = \frac{(8b^2B+12abC+3a^2D)\,x\sqrt{a+bx^2}}{8a} + \frac{(8b^2B+12abC+3a^2D)\,x(a+bx^2)^{3/2}}{12a^2} - \frac{A(a+bx^2)^{5/2}}{5ax^5} - \frac{B(a+bx^2)^{5/2}}{3ax^3} - \frac{(2bB+3aC)\,(a+bx^2)^{5/2}}{3a^2x} + \frac{(8b^2B+12abC+3a^2D)\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}}$$

output
```
1/8*(8*B*b^2+12*C*a*b+3*D*a^2)*x*(b*x^2+a)^(1/2)/a+1/12*(8*B*b^2+12*C*a*b+
3*D*a^2)*x*(b*x^2+a)^(3/2)/a^2-1/5*A*(b*x^2+a)^(5/2)/a/x^5-1/3*B*(b*x^2+a)
^(5/2)/a/x^3-1/3*(2*B*b+3*C*a)*(b*x^2+a)^(5/2)/a^2/x+1/8*(8*B*b^2+12*C*a*b
+3*D*a^2)*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(1/2)
```

## Mathematica [A] (verified)

Time = 0.90 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.70

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2+Cx^4+Dx^6)}{x^6}\,dx = \frac{\sqrt{a+bx^2}(-24Ab^2x^4-a^2(24A+40Bx^2+120Cx^4-75Dx}{120ax^5}$$
$$-\frac{(8b^2B+12abC+3a^2D)\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{8\sqrt{b}}$$

input
```
Integrate[((a + b*x^2)^(3/2)*(A + B*x^2 + C*x^4 + D*x^6))/x^6,x]
```

output
```
(Sqrt[a + b*x^2]*(-24*A*b^2*x^4 - a^2*(24*A + 40*B*x^2 + 120*C*x^4 - 75*D*
x^6) + 2*a*b*x^2*(-24*A - 80*B*x^2 + 30*C*x^4 + 15*D*x^6)))/(120*a*x^5) -
((8*b^2*B + 12*a*b*C + 3*a^2*D)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(8*Sq
rt[b])
```

## Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.87, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.344, Rules used = {2338, 9, 27, 1588, 25, 27, 359, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2+Cx^4+Dx^6)}{x^6}\,dx$$

$$\Big\downarrow \text{2338}$$

$$-\frac{\int -\frac{5(bx^2+a)^{3/2}(aDx^5+aCx^3+aBx)}{x^5}dx}{5a}-\frac{A(a+bx^2)^{5/2}}{5ax^5}$$

$$\Big\downarrow \text{9}$$

$$-\frac{\int -\frac{5(bx^2+a)^{3/2}(aDx^4+aCx^2+aB)}{x^4}dx}{5a}-\frac{A(a+bx^2)^{5/2}}{5ax^5}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(bx^2+a)^{3/2}(aDx^4+aCx^2+aB)}{x^4}dx}{a} - \frac{A(a+bx^2)^{5/2}}{5ax^5}$$

$\Big\downarrow$ 1588

$$\frac{-\frac{\int -\frac{a(bx^2+a)^{3/2}(3aDx^2+2bB+3aC)}{x^2}dx}{3a} - \frac{B(a+bx^2)^{5/2}}{3x^3}}{a} - \frac{A(a+bx^2)^{5/2}}{5ax^5}$$

$\Big\downarrow$ 25

$$\frac{\frac{\int \frac{a(bx^2+a)^{3/2}(3aDx^2+2bB+3aC)}{x^2}dx}{3a} - \frac{B(a+bx^2)^{5/2}}{3x^3}}{a} - \frac{A(a+bx^2)^{5/2}}{5ax^5}$$

$\Big\downarrow$ 27

$$\frac{\frac{1}{3}\int \frac{(bx^2+a)^{3/2}(3aDx^2+2bB+3aC)}{x^2}dx - \frac{B(a+bx^2)^{5/2}}{3x^3}}{a} - \frac{A(a+bx^2)^{5/2}}{5ax^5}$$

$\Big\downarrow$ 359

$$\frac{\frac{1}{3}\left(\frac{(3a^2D+12abC+8b^2B)\int (bx^2+a)^{3/2}dx}{a} - \frac{(a+bx^2)^{5/2}(3aC+2bB)}{ax}\right) - \frac{B(a+bx^2)^{5/2}}{3x^3}}{a} - \frac{A(a+bx^2)^{5/2}}{5ax^5}$$

$\Big\downarrow$ 211

$$\frac{\frac{1}{3}\left(\frac{(3a^2D+12abC+8b^2B)\left(\frac{3}{4}a\int \sqrt{bx^2+a}dx+\frac{1}{4}x(a+bx^2)^{3/2}\right)}{a} - \frac{(a+bx^2)^{5/2}(3aC+2bB)}{ax}\right) - \frac{B(a+bx^2)^{5/2}}{3x^3}}{a} - \frac{A(a+bx^2)^{5/2}}{5ax^5}$$

$\Big\downarrow$ 211

$$\frac{\frac{1}{3}\left(\frac{(3a^2D+12abC+8b^2B)\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x(a+bx^2)^{3/2}\right)}{a} - \frac{(a+bx^2)^{5/2}(3aC+2bB)}{ax}\right) - \frac{B(a+bx^2)^{5/2}}{3x^3}}{a} - \frac{A(a+bx^2)^{5/2}}{5ax^5}$$

$\Big\downarrow$ 224

$$\frac{\frac{1}{3}\left(\frac{(3a^2D+12abC+8b^2B)\left(\frac{3}{4}a\left(\frac{1}{2}a\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x(a+bx^2)^{3/2}\right)}{a}-\frac{(a+bx^2)^{5/2}(3aC+2bB)}{ax}\right)-\frac{B(a+bx^2)^{5/2}}{3x^3}}{a}$$

$$\frac{A(a+bx^2)^{5/2}}{5ax^5}$$

$\Big\downarrow$ 219

$$\frac{\frac{1}{3}\left(\frac{\left(\frac{3}{4}a\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x(a+bx^2)^{3/2}\right)(3a^2D+12abC+8b^2B)}{a}-\frac{(a+bx^2)^{5/2}(3aC+2bB)}{ax}\right)-\frac{B(a+bx^2)^{5/2}}{3x^3}}{a}-$$

$$\frac{A(a+bx^2)^{5/2}}{5ax^5}$$

input
```
Int[((a + b*x^2)^(3/2)*(A + B*x^2 + C*x^4 + D*x^6))/x^6,x]
```

output
```
-1/5*(A*(a + b*x^2)^(5/2))/(a*x^5) + (-1/3*(B*(a + b*x^2)^(5/2))/x^3 + (-(
((2*b*B + 3*a*C)*(a + b*x^2)^(5/2))/(a*x)) + ((8*b^2*B + 12*a*b*C + 3*a^2*
D)*((x*(a + b*x^2)^(3/2))/4 + (3*a*((x*Sqrt[a + b*x^2])/2 + (a*ArcTanh[(Sq
rt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b])))/4))/a)/3)/a
```

## Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 1588
```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Simp[1/(d*f
^2*(m + 1))   Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x
) - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

## Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.66

| method | result |
|---|---|
| pseudoelliptic | $-\frac{2\left(-\frac{5\left(B\,b^{2}+\frac{3}{2}Cab+\frac{3}{8}Da^{2}\right)x^{5}a\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{x\sqrt{b}}\right)}{2}+\sqrt{b\,x^{2}+a}\left(a\,x^{2}\left(-\frac{5}{8}Dx^{6}-\frac{5}{4}C\,x^{4}+\frac{10}{3}x^{2}B+A\right)b^{\frac{3}{2}}+\frac{A\,b^{\frac{5}{2}}x^{4}}{2}+\frac{\sqrt{b}\left(-\right.}{}\right.\right.}{5\sqrt{b}\,a\,x^{5}}$ |
| default | $D\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)}{4}\right)-\frac{A\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{5a\,x^{5}}+B\left(-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{3a\,x^{3}}+\frac{2b\left(-\frac{\left(b\,x^{2}+\right.}{ax}\right.}{}\right.$ |

input
```
int((b*x^2+a)^(3/2)*(D*x^6+C*x^4+B*x^2+A)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-2/5*(-5/2*(B*b^2+3/2*C*a*b+3/8*D*a^2)*x^5*a*arctanh((b*x^2+a)^(1/2)/x/b^(
1/2))+(b*x^2+a)^(1/2)*(a*x^2*(-5/8*D*x^6-5/4*C*x^4+10/3*x^2*B+A)*b^(3/2)+1
/2*A*b^(5/2)*x^4+1/2*b^(1/2)*(-25/8*D*x^6+5*C*x^4+5/3*x^2*B+A)*a^2))/b^(1/
2)/a/x^5
```

## Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.63

$$\int \frac{(a+bx^2)^{3/2}\left(A+Bx^2+Cx^4+Dx^6\right)}{x^6}\,dx = \left[\frac{15\left(3\,Da^3+12\,Ca^2b+8\,Bab^2\right)\sqrt{b}x^5\log\left(-2\,bx^2-2\,\sqrt{bx^2}\right.}{}\right.$$

$$-\frac{15\left(3\,Da^3+12\,Ca^2b+8\,Bab^2\right)\sqrt{-b}x^5\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-\left(30\,Dab^2x^8+15\left(5\,Da^2b+4\,Cab^2\right)x^6-8\left(15\right.\right.}{120\,abx^5}$$

input
```
integrate((b*x^2+a)^(3/2)*(D*x^6+C*x^4+B*x^2+A)/x^6,x, algorithm="fricas")
```

output
```
[1/240*(15*(3*D*a^3 + 12*C*a^2*b + 8*B*a*b^2)*sqrt(b)*x^5*log(-2*b*x^2 - 2
*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(30*D*a*b^2*x^8 + 15*(5*D*a^2*b + 4*C*
a*b^2)*x^6 - 8*(15*C*a^2*b + 20*B*a*b^2 + 3*A*b^3)*x^4 - 24*A*a^2*b - 8*(5
*B*a^2*b + 6*A*a*b^2)*x^2)*sqrt(b*x^2 + a))/(a*b*x^5), -1/120*(15*(3*D*a^3
 + 12*C*a^2*b + 8*B*a*b^2)*sqrt(-b)*x^5*arctan(sqrt(-b)*x/sqrt(b*x^2 + a))
 - (30*D*a*b^2*x^8 + 15*(5*D*a^2*b + 4*C*a*b^2)*x^6 - 8*(15*C*a^2*b + 20*B
*a*b^2 + 3*A*b^3)*x^4 - 24*A*a^2*b - 8*(5*B*a^2*b + 6*A*a*b^2)*x^2)*sqrt(b
*x^2 + a))/(a*b*x^5)]
```

## Sympy [A] (verification not implemented)

Time = 3.19 (sec) , antiderivative size = 483, normalized size of antiderivative = 2.48

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2+Cx^4+Dx^6)}{x^6}\,dx = -\frac{Aa\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{5x^4} - \frac{2Ab^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{5x^2} - \frac{Ab^{\frac{5}{2}}\sqrt{\frac{a}{bx^2}+1}}{5a} - \frac{B\sqrt{a}b}{x\sqrt{1+\frac{bx^2}{a}}} - \frac{Ba\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{3x^2} - \frac{Bb^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{3} + Bb^{\frac{3}{2}}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right) - \frac{Bb^2x}{\sqrt{a}\sqrt{1+\frac{bx^2}{a}}} - \frac{Ca^{\frac{3}{2}}}{x\sqrt{1+\frac{bx^2}{a}}} - \frac{C\sqrt{a}bx}{\sqrt{1+\frac{bx^2}{a}}} + Ca\sqrt{b}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right) + Cb\left(\begin{cases}\frac{a\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for}\ a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right)}{2} + \frac{x\sqrt{a+bx^2}}{2} & \text{for}\ b\neq 0\\ \sqrt{a}x & \text{otherwise}\end{cases}\right) + Da\left(\begin{cases}\frac{a\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for}\ a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right)}{2} + \frac{x\sqrt{a+bx^2}}{2} & \text{for}\ b\neq 0\\ \sqrt{a}x & \text{otherwise}\end{cases}\right) + Db\left(\begin{cases}-\frac{a^2\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for}\ a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right)}{8b} + \frac{ax\sqrt{a+bx^2}}{8b} + \frac{x^3\sqrt{a+bx^2}}{4} & \text{for}\ b\neq 0\\ \frac{\sqrt{a}x^3}{3} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((b*x**2+a)**(3/2)*(D*x**6+C*x**4+B*x**2+A)/x**6,x)
```

output
```
-A*a*sqrt(b)*sqrt(a/(b*x**2) + 1)/(5*x**4) - 2*A*b**(3/2)*sqrt(a/(b*x**2)
+ 1)/(5*x**2) - A*b**(5/2)*sqrt(a/(b*x**2) + 1)/(5*a) - B*sqrt(a)*b/(x*sqr
t(1 + b*x**2/a)) - B*a*sqrt(b)*sqrt(a/(b*x**2) + 1)/(3*x**2) - B*b**(3/2)*
sqrt(a/(b*x**2) + 1)/3 + B*b**(3/2)*asinh(sqrt(b)*x/sqrt(a)) - B*b**2*x/(s
qrt(a)*sqrt(1 + b*x**2/a)) - C*a**(3/2)/(x*sqrt(1 + b*x**2/a)) - C*sqrt(a)
*b*x/sqrt(1 + b*x**2/a) + C*a*sqrt(b)*asinh(sqrt(b)*x/sqrt(a)) + C*b*Piece
wise((a*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a,
0)), (x*log(x)/sqrt(b*x**2), True))/2 + x*sqrt(a + b*x**2)/2, Ne(b, 0)), (
sqrt(a)*x, True)) + D*a*Piecewise((a*Piecewise((log(2*sqrt(b)*sqrt(a + b*x
**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True))/2 + x*sqr
t(a + b*x**2)/2, Ne(b, 0)), (sqrt(a)*x, True)) + D*b*Piecewise((-a**2*Piec
ewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(
x)/sqrt(b*x**2), True))/(8*b) + a*x*sqrt(a + b*x**2)/(8*b) + x**3*sqrt(a +
 b*x**2)/4, Ne(b, 0)), (sqrt(a)*x**3/3, True))
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2+Cx^4+Dx^6)}{x^6}\,dx = \frac{1}{4}\left(bx^2+a\right)^{\frac{3}{2}}Dx + \frac{3}{8}\sqrt{bx^2+a}Dax + \frac{3}{2}\sqrt{bx^2+a}Cbx + \frac{\sqrt{bx^2+a}Bb^2x}{a} + \frac{3\,Da^2\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{b}} + \frac{3}{2}\,Ca\sqrt{b}\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right) + Bb^{\frac{3}{2}}\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right) - \frac{(bx^2+a)^{\frac{3}{2}}C}{x} - \frac{2\,(bx^2+a)^{\frac{3}{2}}Bb}{3\,ax} - \frac{(bx^2+a)^{\frac{5}{2}}B}{3\,ax^3} - \frac{(bx^2+a)^{\frac{5}{2}}A}{5\,ax^5}$$

input
```
integrate((b*x^2+a)^(3/2)*(D*x^6+C*x^4+B*x^2+A)/x^6,x, algorithm="maxima")
```

output
```
1/4*(b*x^2 + a)^(3/2)*D*x + 3/8*sqrt(b*x^2 + a)*D*a*x + 3/2*sqrt(b*x^2 + a
)*C*b*x + sqrt(b*x^2 + a)*B*b^2*x/a + 3/8*D*a^2*arcsinh(b*x/sqrt(a*b))/sqr
t(b) + 3/2*C*a*sqrt(b)*arcsinh(b*x/sqrt(a*b)) + B*b^(3/2)*arcsinh(b*x/sqrt
(a*b)) - (b*x^2 + a)^(3/2)*C/x - 2/3*(b*x^2 + a)^(3/2)*B*b/(a*x) - 1/3*(b*
x^2 + a)^(5/2)*B/(a*x^3) - 1/5*(b*x^2 + a)^(5/2)*A/(a*x^5)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 412 vs. $2(167) = 334$.

Time = 0.15 (sec) , antiderivative size = 412, normalized size of antiderivative = 2.11

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2+Cx^4+Dx^6)}{x^6}\,dx = \frac{1}{8}\left(2\,Dbx^2+\frac{5\,Dab^2+4\,Cb^3}{b^2}\right)\sqrt{bx^2+a}x$$
$$-\frac{(3\,Da^2+12\,Cab+8\,Bb^2)\log\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2\right)}{16\,\sqrt{b}}$$
$$+\frac{2\left(15\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^8Ca^2\sqrt{b}+30\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^8Bab^{\frac{3}{2}}+15\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^8Ab^{\frac{5}{2}}-60\left(\sqrt{\right.\right.}{}}{}$$

input
```
integrate((b*x^2+a)^(3/2)*(D*x^6+C*x^4+B*x^2+A)/x^6,x, algorithm="giac")
```

output
```
1/8*(2*D*b*x^2 + (5*D*a*b^2 + 4*C*b^3)/b^2)*sqrt(b*x^2 + a)*x - 1/16*(3*D*
a^2 + 12*C*a*b + 8*B*b^2)*log((sqrt(b)*x - sqrt(b*x^2 + a))^2)/sqrt(b) + 2
/15*(15*(sqrt(b)*x - sqrt(b*x^2 + a))^8*C*a^2*sqrt(b) + 30*(sqrt(b)*x - sq
rt(b*x^2 + a))^8*B*a*b^(3/2) + 15*(sqrt(b)*x - sqrt(b*x^2 + a))^8*A*b^(5/2
) - 60*(sqrt(b)*x - sqrt(b*x^2 + a))^6*C*a^3*sqrt(b) - 90*(sqrt(b)*x - sqr
t(b*x^2 + a))^6*B*a^2*b^(3/2) + 90*(sqrt(b)*x - sqrt(b*x^2 + a))^4*C*a^4*s
qrt(b) + 110*(sqrt(b)*x - sqrt(b*x^2 + a))^4*B*a^3*b^(3/2) + 30*(sqrt(b)*x
 - sqrt(b*x^2 + a))^4*A*a^2*b^(5/2) - 60*(sqrt(b)*x - sqrt(b*x^2 + a))^2*C
*a^5*sqrt(b) - 70*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a^4*b^(3/2) + 15*C*a^6
*sqrt(b) + 20*B*a^5*b^(3/2) + 3*A*a^4*b^(5/2))/((sqrt(b)*x - sqrt(b*x^2 +
a))^2 - a)^5
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2+Cx^4+Dx^6)}{x^6}\,dx = \int \frac{(b\,x^2+a)^{3/2}\,(A+B\,x^2+C\,x^4+x^6\,D)}{x^6}\,dx$$

input
```
int(((a + b*x^2)^(3/2)*(A + B*x^2 + C*x^4 + x^6*D))/x^6,x)
```

output
```
int(((a + b*x^2)^(3/2)*(A + B*x^2 + C*x^4 + x^6*D))/x^6, x)
```

## Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.24

$$\int \frac{(a+bx^2)^{3/2}\,(A+Bx^2+Cx^4+Dx^6)}{x^6}\,dx = \frac{-96\sqrt{b\,x^2+a}\,a^2b-352\sqrt{b\,x^2+a}\,a\,b^2x^2-480\sqrt{b\,x^2+a}\,ab}{}$$

input
```
int((b*x^2+a)^(3/2)*(D*x^6+C*x^4+B*x^2+A)/x^6,x)
```

output
```
( - 96*sqrt(a + b*x**2)*a**2*b - 352*sqrt(a + b*x**2)*a*b**2*x**2 - 480*sq
rt(a + b*x**2)*a*b*c*x**4 + 300*sqrt(a + b*x**2)*a*b*d*x**6 - 736*sqrt(a +
 b*x**2)*b**3*x**4 + 240*sqrt(a + b*x**2)*b**2*c*x**6 + 120*sqrt(a + b*x**
2)*b**2*d*x**8 + 180*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a
**2*d*x**5 + 720*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a*b*c
*x**5 + 480*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*b**3*x**5
+ 93*sqrt(b)*a**2*d*x**5 + 396*sqrt(b)*a*b*c*x**5 + 160*sqrt(b)*b**3*x**5)
/(480*b*x**5)
```

# 3.203 $\int \frac{(a+bx^2)^{3/2}(A+Bx^2+Cx^4+Dx^6)}{x^8} \, dx$



## Optimal result

Integrand size = 32, antiderivative size = 168

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2+Cx^4+Dx^6)}{x^8} \, dx = -\frac{(2bC+3aD)\sqrt{a+bx^2}}{3x} + \frac{b(2bC+3aD)x\sqrt{a+bx^2}}{6a} - \frac{A(a+bx^2)^{5/2}}{7ax^7} + \frac{(2Ab-7aB)(a+bx^2)^{5/2}}{35a^2x^5} - \frac{C(a+bx^2)^{5/2}}{3ax^3} + \frac{1}{2}\sqrt{b}(2bC+3aD)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)$$

output

```
-1/3*(2*C*b+3*D*a)*(b*x^2+a)^(1/2)/x+1/6*b*(2*C*b+3*D*a)*x*(b*x^2+a)^(1/2)
/a-1/7*A*(b*x^2+a)^(5/2)/a/x^7+1/35*(2*A*b-7*B*a)*(b*x^2+a)^(5/2)/a^2/x^5-
1/3*C*(b*x^2+a)^(5/2)/a/x^3+1/2*b^(1/2)*(2*C*b+3*D*a)*arctanh(b^(1/2)*x/(b
*x^2+a)^(1/2))
```

## Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2+Cx^4+Dx^6)}{x^8}\,dx =$$
$$-\frac{\sqrt{a+bx^2}(-12Ab^3x^6+6ab^2x^4(A+7Bx^2)+a^2bx^2(48A+84Bx^2+280Cx^4-105Dx^6)+a^3(30A+42B}{210a^2x^7}$$
$$-\frac{1}{2}\sqrt{b}(2bC+3aD)\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)$$

input
```
Integrate[((a + b*x^2)^(3/2)*(A + B*x^2 + C*x^4 + D*x^6))/x^8,x]
```

output
```
-1/210*(Sqrt[a + b*x^2]*(-12*A*b^3*x^6 + 6*a*b^2*x^4*(A + 7*B*x^2) + a^2*b
*x^2*(48*A + 84*B*x^2 + 280*C*x^4 - 105*D*x^6) + a^3*(30*A + 42*B*x^2 + 70
*x^4*(C + 3*D*x^2))))/(a^2*x^7) - (Sqrt[b]*(2*b*C + 3*a*D)*Log[-(Sqrt[b]*x
) + Sqrt[a + b*x^2]])/2
```

## Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.01, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.281$, Rules used = {2338, 9, 1588, 27, 359, 247, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2+Cx^4+Dx^6)}{x^8}\,dx$$
$$\Big\downarrow \text{2338}$$
$$-\frac{\int \frac{(bx^2+a)^{3/2}(-7aDx^5-7aCx^3+(2Ab-7aB)x)}{x^7}dx}{7a}-\frac{A(a+bx^2)^{5/2}}{7ax^7}$$
$$\Big\downarrow \text{9}$$
$$-\frac{\int \frac{(bx^2+a)^{3/2}(-7aDx^4-7aCx^2+2Ab-7aB)}{x^6}dx}{7a}-\frac{A(a+bx^2)^{5/2}}{7ax^7}$$

$$\Big\downarrow \; 1588$$

$$-\frac{-\frac{\int \frac{35a^2\left(bx^2+a\right)^{3/2}\left(Dx^2+C\right)}{x^4}dx}{5a}-\frac{\left(a+bx^2\right)^{5/2}(2Ab-7aB)}{5ax^5}}{7a}-\frac{A\left(a+bx^2\right)^{5/2}}{7ax^7}$$

$$\Big\downarrow \; 27$$

$$-\frac{-7a\int \frac{\left(bx^2+a\right)^{3/2}\left(Dx^2+C\right)}{x^4}dx-\frac{\left(a+bx^2\right)^{5/2}(2Ab-7aB)}{5ax^5}}{7a}-\frac{A\left(a+bx^2\right)^{5/2}}{7ax^7}$$

$$\Big\downarrow \; 359$$

$$-\frac{-7a\left(\frac{(3aD+2bC)\int \frac{\left(bx^2+a\right)^{3/2}}{x^2}dx}{3a}-\frac{C\left(a+bx^2\right)^{5/2}}{3ax^3}\right)-\frac{\left(a+bx^2\right)^{5/2}(2Ab-7aB)}{5ax^5}}{7a}-\frac{A\left(a+bx^2\right)^{5/2}}{7ax^7}$$

$$\Big\downarrow \; 247$$

$$-\frac{-7a\left(\frac{(3aD+2bC)\left(3b\int \sqrt{bx^2+a}dx-\frac{\left(a+bx^2\right)^{3/2}}{x}\right)}{3a}-\frac{C\left(a+bx^2\right)^{5/2}}{3ax^3}\right)-\frac{\left(a+bx^2\right)^{5/2}(2Ab-7aB)}{5ax^5}}{7a}-\frac{A\left(a+bx^2\right)^{5/2}}{7ax^7}$$

$$\Big\downarrow \; 211$$

$$-\frac{-7a\left(\frac{(3aD+2bC)\left(3b\left(\frac{1}{2}a\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)-\frac{\left(a+bx^2\right)^{3/2}}{x}\right)}{3a}-\frac{C\left(a+bx^2\right)^{5/2}}{3ax^3}\right)-\frac{\left(a+bx^2\right)^{5/2}(2Ab-7aB)}{5ax^5}}{7a}-\frac{A\left(a+bx^2\right)^{5/2}}{7ax^7}$$

$$\Big\downarrow \; 224$$

$$-\frac{-7a\left(\frac{(3aD+2bC)\left(3b\left(\frac{1}{2}a\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)-\frac{\left(a+bx^2\right)^{3/2}}{x}\right)}{3a}-\frac{C\left(a+bx^2\right)^{5/2}}{3ax^3}\right)-\frac{\left(a+bx^2\right)^{5/2}(2Ab-7aB)}{5ax^5}}{7a}-\frac{A\left(a+bx^2\right)^{5/2}}{7ax^7}$$

$\downarrow$ 219

$$-\frac{-\frac{\left(a+bx^2\right)^{5/2}(2Ab-7aB)}{5ax^5}-7a\left(\frac{\left(3b\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)-\frac{\left(a+bx^2\right)^{3/2}}{x}\right)(3aD+2bC)}{3a}-\frac{C\left(a+bx^2\right)^{5/2}}{3ax^3}\right)}{7a}-\frac{A\left(a+bx^2\right)^{5/2}}{7ax^7}$$

input
```
Int[((a + b*x^2)^(3/2)*(A + B*x^2 + C*x^4 + D*x^6))/x^8,x]
```

output
```
-1/7*(A*(a + b*x^2)^(5/2))/(a*x^7) - (-1/5*((2*A*b - 7*a*B)*(a + b*x^2)^(5
/2))/(a*x^5) - 7*a*(-1/3*(C*(a + b*x^2)^(5/2))/(a*x^3) + ((2*b*C + 3*a*D)*
(-((a + b*x^2)^(3/2)/x) + 3*b*((x*Sqrt[a + b*x^2])/2 + (a*ArcTanh[(Sqrt[b]
*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b]))))/(3*a)))/(7*a)
```

## Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 1588
```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Simp[1/(d*f
^2*(m + 1))   Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x
) - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

## Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.85

| method | result |
|---|---|
| pseudoelliptic | $-\frac{8\left(-\frac{35\left(Cb+\frac{3Da}{2}\right)x^{7}b\,a^{2}\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{x\sqrt{b}}\right)}{8}+\sqrt{b\,x^{2}+a}\,\left(a^{2}x^{2}\left(-\frac{35}{16}Dx^{6}+\frac{35}{6}C\,x^{4}+\frac{7}{4}x^{2}B+A\right)b^{\frac{3}{2}}+\frac{a\,x^{4}\left(7x^{2}B+A\right)b^{\frac{5}{2}}}{8}-\right.\right.}{35\sqrt{b}\,x^{7}a^{2}}$ |
| default | $A\left(-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{7a\,x^{7}}+\frac{2b\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{35a^{2}x^{5}}\right)-\frac{B\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{5a\,x^{5}}+C\left(-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{3a\,x^{3}}+\frac{2b\left(-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{ax}+\frac{4b\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{4}+\right.}{}\right.}{}\right.$ |

input
```
int((b*x^2+a)^(3/2)*(D*x^6+C*x^4+B*x^2+A)/x^8,x,method=_RETURNVERBOSE)
```

output
```
-8/35*(-35/8*(C*b+3/2*D*a)*x^7*b*a^2*arctanh((b*x^2+a)^(1/2)/x/b^(1/2))+(b
*x^2+a)^(1/2)*(a^2*x^2*(-35/16*D*x^6+35/6*C*x^4+7/4*x^2*B+A)*b^(3/2)+1/8*a
*x^4*(7*B*x^2+A)*b^(5/2)-1/4*A*b^(7/2)*x^6+5/8*b^(1/2)*(7*D*x^6+7/3*C*x^4+
7/5*x^2*B+A)*a^3))/b^(1/2)/x^7/a^2
```

## Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.89

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2+Cx^4+Dx^6)}{x^8}\,dx = \left[\frac{105\,(3\,Da^3+2\,Ca^2b)\sqrt{b}x^7\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x\right.}{}\right.$$

$$-\frac{105\,(3\,Da^3+2\,Ca^2b)\sqrt{-b}x^7\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(105\,Da^2bx^8-2\,(105\,Da^3+140\,Ca^2b+21\,Bab^2-6\,Ab}{210\,a^2x^7}$$

input
```
integrate((b*x^2+a)^(3/2)*(D*x^6+C*x^4+B*x^2+A)/x^8,x, algorithm="fricas")
```

output
```
[1/420*(105*(3*D*a^3 + 2*C*a^2*b)*sqrt(b)*x^7*log(-2*b*x^2 - 2*sqrt(b*x^2
+ a)*sqrt(b)*x - a) + 2*(105*D*a^2*b*x^8 - 2*(105*D*a^3 + 140*C*a^2*b + 21
*B*a*b^2 - 6*A*b^3)*x^6 - 2*(35*C*a^3 + 42*B*a^2*b + 3*A*a*b^2)*x^4 - 30*A
*a^3 - 6*(7*B*a^3 + 8*A*a^2*b)*x^2)*sqrt(b*x^2 + a))/(a^2*x^7), -1/210*(10
5*(3*D*a^3 + 2*C*a^2*b)*sqrt(-b)*x^7*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) -
(105*D*a^2*b*x^8 - 2*(105*D*a^3 + 140*C*a^2*b + 21*B*a*b^2 - 6*A*b^3)*x^6
- 2*(35*C*a^3 + 42*B*a^2*b + 3*A*a*b^2)*x^4 - 30*A*a^3 - 6*(7*B*a^3 + 8*A*
a^2*b)*x^2)*sqrt(b*x^2 + a))/(a^2*x^7)]
```

## Sympy [A] (verification not implemented)

Time = 3.85 (sec) , antiderivative size = 765, normalized size of antiderivative = 4.55

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2+Cx^4+Dx^6)}{x^8}\,dx = \text{Too large to display}$$

input
```
integrate((b*x**2+a)**(3/2)*(D*x**6+C*x**4+B*x**2+A)/x**8,x)
```

output
```
-15*A*a**6*b**(9/2)*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b*
*5*x**8 + 105*a**3*b**6*x**10) - 33*A*a**5*b**(11/2)*x**2*sqrt(a/(b*x**2)
+ 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 17*
A*a**4*b**(13/2)*x**4*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*
b**5*x**8 + 105*a**3*b**6*x**10) - 3*A*a**3*b**(15/2)*x**6*sqrt(a/(b*x**2)
 + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 12
*A*a**2*b**(17/2)*x**8*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4
*b**5*x**8 + 105*a**3*b**6*x**10) - 8*A*a*b**(19/2)*x**10*sqrt(a/(b*x**2)
+ 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - A*b
**(3/2)*sqrt(a/(b*x**2) + 1)/(5*x**4) - A*b**(5/2)*sqrt(a/(b*x**2) + 1)/(1
5*a*x**2) + 2*A*b**(7/2)*sqrt(a/(b*x**2) + 1)/(15*a**2) - B*a*sqrt(b)*sqrt
(a/(b*x**2) + 1)/(5*x**4) - 2*B*b**(3/2)*sqrt(a/(b*x**2) + 1)/(5*x**2) - B
*b**(5/2)*sqrt(a/(b*x**2) + 1)/(5*a) - C*sqrt(a)*b/(x*sqrt(1 + b*x**2/a))
- C*a*sqrt(b)*sqrt(a/(b*x**2) + 1)/(3*x**2) - C*b**(3/2)*sqrt(a/(b*x**2) +
 1)/3 + C*b**(3/2)*asinh(sqrt(b)*x/sqrt(a)) - C*b**2*x/(sqrt(a)*sqrt(1 + b
*x**2/a)) - D*a**(3/2)/(x*sqrt(1 + b*x**2/a)) - D*sqrt(a)*b*x/sqrt(1 + b*x
**2/a) + D*a*sqrt(b)*asinh(sqrt(b)*x/sqrt(a)) + D*b*Piecewise((a*Piecewise
((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sq
rt(b*x**2), True))/2 + x*sqrt(a + b*x**2)/2, Ne(b, 0)), (sqrt(a)*x, True))
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.01

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2+Cx^4+Dx^6)}{x^8}\,dx = \frac{3}{2}\sqrt{bx^2+a}Dbx + \frac{\sqrt{bx^2+a}Cb^2x}{a} + \frac{3}{2}Da\sqrt{b}\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right) + Cb^{\frac{3}{2}}\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right) - \frac{(bx^2+a)^{\frac{3}{2}}D}{x} - \frac{2\,(bx^2+a)^{\frac{3}{2}}Cb}{3\,ax} - \frac{(bx^2+a)^{\frac{5}{2}}C}{3\,ax^3} - \frac{(bx^2+a)^{\frac{5}{2}}B}{5\,ax^5} + \frac{2\,(bx^2+a)^{\frac{5}{2}}Ab}{35\,a^2x^5} - \frac{(bx^2+a)^{\frac{5}{2}}A}{7\,ax^7}$$

input
```
integrate((b*x^2+a)^(3/2)*(D*x^6+C*x^4+B*x^2+A)/x^8,x, algorithm="maxima")
```

output
```
3/2*sqrt(b*x^2 + a)*D*b*x + sqrt(b*x^2 + a)*C*b^2*x/a + 3/2*D*a*sqrt(b)*ar
csinh(b*x/sqrt(a*b)) + C*b^(3/2)*arcsinh(b*x/sqrt(a*b)) - (b*x^2 + a)^(3/2
)*D/x - 2/3*(b*x^2 + a)^(3/2)*C*b/(a*x) - 1/3*(b*x^2 + a)^(5/2)*C/(a*x^3)
- 1/5*(b*x^2 + a)^(5/2)*B/(a*x^5) + 2/35*(b*x^2 + a)^(5/2)*A*b/(a^2*x^5) -
 1/7*(b*x^2 + a)^(5/2)*A/(a*x^7)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 747 vs. $2(140) = 280$.

Time = 0.15 (sec) , antiderivative size = 747, normalized size of antiderivative = 4.45

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2+Cx^4+Dx^6)}{x^8}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(3/2)*(D*x^6+C*x^4+B*x^2+A)/x^8,x, algorithm="giac")
```

output
```
1/2*sqrt(b*x^2 + a)*D*b*x - 1/4*(3*D*a*sqrt(b) + 2*C*b^(3/2))*log((sqrt(b)
*x - sqrt(b*x^2 + a))^2) + 2/105*(105*(sqrt(b)*x - sqrt(b*x^2 + a))^12*D*a
^2*sqrt(b) + 210*(sqrt(b)*x - sqrt(b*x^2 + a))^12*C*a*b^(3/2) + 105*(sqrt(
b)*x - sqrt(b*x^2 + a))^12*B*b^(5/2) - 630*(sqrt(b)*x - sqrt(b*x^2 + a))^1
0*D*a^3*sqrt(b) - 1050*(sqrt(b)*x - sqrt(b*x^2 + a))^10*C*a^2*b^(3/2) - 21
0*(sqrt(b)*x - sqrt(b*x^2 + a))^10*B*a*b^(5/2) + 210*(sqrt(b)*x - sqrt(b*x
^2 + a))^10*A*b^(7/2) + 1575*(sqrt(b)*x - sqrt(b*x^2 + a))^8*D*a^4*sqrt(b)
 + 2240*(sqrt(b)*x - sqrt(b*x^2 + a))^8*C*a^3*b^(3/2) + 315*(sqrt(b)*x - s
qrt(b*x^2 + a))^8*B*a^2*b^(5/2) + 210*(sqrt(b)*x - sqrt(b*x^2 + a))^8*A*a*
b^(7/2) - 2100*(sqrt(b)*x - sqrt(b*x^2 + a))^6*D*a^5*sqrt(b) - 2660*(sqrt(
b)*x - sqrt(b*x^2 + a))^6*C*a^4*b^(3/2) - 420*(sqrt(b)*x - sqrt(b*x^2 + a)
)^6*B*a^3*b^(5/2) + 420*(sqrt(b)*x - sqrt(b*x^2 + a))^6*A*a^2*b^(7/2) + 15
75*(sqrt(b)*x - sqrt(b*x^2 + a))^4*D*a^6*sqrt(b) + 1890*(sqrt(b)*x - sqrt(
b*x^2 + a))^4*C*a^5*b^(3/2) + 231*(sqrt(b)*x - sqrt(b*x^2 + a))^4*B*a^4*b^
(5/2) + 84*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*a^3*b^(7/2) - 630*(sqrt(b)*x
- sqrt(b*x^2 + a))^2*D*a^7*sqrt(b) - 770*(sqrt(b)*x - sqrt(b*x^2 + a))^2*C
*a^6*b^(3/2) - 42*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a^5*b^(5/2) + 42*(sqrt
(b)*x - sqrt(b*x^2 + a))^2*A*a^4*b^(7/2) + 105*D*a^8*sqrt(b) + 140*C*a^7*b
^(3/2) + 21*B*a^6*b^(5/2) - 6*A*a^5*b^(7/2))/((sqrt(b)*x - sqrt(b*x^2 + a)
)^2 - a)^7
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2+Cx^4+Dx^6)}{x^8}\,dx = \int \frac{(b\,x^2+a)^{3/2}\,(A+B\,x^2+C\,x^4+x^6\,D)}{x^8}\,dx$$

input
```
int(((a + b*x^2)^(3/2)*(A + B*x^2 + C*x^4 + x^6*D))/x^8,x)
```

output
```
int(((a + b*x^2)^(3/2)*(A + B*x^2 + C*x^4 + x^6*D))/x^8, x)
```

## Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.36

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2+Cx^4+Dx^6)}{x^8}\,dx = \frac{-6\sqrt{b\,x^2+a}\,a^3-18\sqrt{b\,x^2+a}\,a^2b\,x^2-14\sqrt{b\,x^2+a}\,a^2c\,x^4\ \cdots}{\cdots}$$

input
```
int((b*x^2+a)^(3/2)*(D*x^6+C*x^4+B*x^2+A)/x^8,x)
```

output
```
( - 6*sqrt(a + b*x**2)*a**3 - 18*sqrt(a + b*x**2)*a**2*b*x**2 - 14*sqrt(a
+ b*x**2)*a**2*c*x**4 - 42*sqrt(a + b*x**2)*a**2*d*x**6 - 18*sqrt(a + b*x*
*2)*a*b**2*x**4 - 56*sqrt(a + b*x**2)*a*b*c*x**6 + 21*sqrt(a + b*x**2)*a*b
*d*x**8 - 6*sqrt(a + b*x**2)*b**3*x**6 + 63*sqrt(b)*log((sqrt(a + b*x**2)
+ sqrt(b)*x)/sqrt(a))*a**2*d*x**7 + 42*sqrt(b)*log((sqrt(a + b*x**2) + sqr
t(b)*x)/sqrt(a))*a*b*c*x**7 + 45*sqrt(b)*a**2*d*x**7 + 32*sqrt(b)*a*b*c*x*
*7 - 6*sqrt(b)*b**3*x**7)/(42*a*x**7)
```

# 3.204 $\int \frac{(a+bx^2)^{3/2}(A+Bx^2+Cx^4+Dx^6)}{x^{10}}\,dx$



## Optimal result

Integrand size = 32, antiderivative size = 158

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2+Cx^4+Dx^6)}{x^{10}}\,dx = -\frac{aD\sqrt{a+bx^2}}{3x^3} - \frac{4bD\sqrt{a+bx^2}}{3x} - \frac{A(a+bx^2)^{5/2}}{9ax^9} + \frac{(4Ab-9aB)(a+bx^2)^{5/2}}{63a^2x^7} - \frac{(8Ab^2-18abB+63a^2C)(a+bx^2)^{5/2}}{315a^3x^5} + b^{3/2}D\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)$$

output
```
-1/3*a*D*(b*x^2+a)^(1/2)/x^3-4/3*b*D*(b*x^2+a)^(1/2)/x-1/9*A*(b*x^2+a)^(5/
2)/a/x^9+1/63*(4*A*b-9*B*a)*(b*x^2+a)^(5/2)/a^2/x^7-1/315*(8*A*b^2-18*B*a*
b+63*C*a^2)*(b*x^2+a)^(5/2)/a^3/x^5+b^(3/2)*D*arctanh(b^(1/2)*x/(b*x^2+a)^
(1/2))
```

## Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2+Cx^4+Dx^6)}{x^{10}}\,dx =$$
$$-\frac{\sqrt{a+bx^2}(8Ab^4x^8-2ab^3x^6(2A+9Bx^2)+a^4(35A+45Bx^2+63Cx^4+105Dx^6)+2a^3bx^2(25A+36Bx}{315a^3x^9}$$
$$-b^{3/2}D\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)$$

input
```
Integrate[((a + b*x^2)^(3/2)*(A + B*x^2 + C*x^4 + D*x^6))/x^10,x]
```

output
```
-1/315*(Sqrt[a + b*x^2]*(8*A*b^4*x^8 - 2*a*b^3*x^6*(2*A + 9*B*x^2) + a^4*(
35*A + 45*B*x^2 + 63*C*x^4 + 105*D*x^6) + 2*a^3*b*x^2*(25*A + 36*B*x^2 + 6
3*C*x^4 + 210*D*x^6) + 3*a^2*b^2*x^4*(A + 3*x^2*(B + 7*C*x^2))))/(a^3*x^9)
 - b^(3/2)*D*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]]
```

## Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.12, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2338, 9, 1588, 358, 247, 247, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2+Cx^4+Dx^6)}{x^{10}}\,dx$$
$$\Big\downarrow \text{2338}$$
$$-\frac{\int \frac{(bx^2+a)^{3/2}(-9aDx^5-9aCx^3+(4Ab-9aB)x)}{x^9}dx}{9a}-\frac{A(a+bx^2)^{5/2}}{9ax^9}$$
$$\Big\downarrow \text{9}$$
$$-\frac{\int \frac{(bx^2+a)^{3/2}(-9aDx^4-9aCx^2+4Ab-9aB)}{x^8}dx}{9a}-\frac{A(a+bx^2)^{5/2}}{9ax^9}$$

$$\Big\downarrow \; 1588$$

$$-\frac{-\frac{\int \frac{\left(bx^2+a\right)^{3/2}\left(63Dx^2a^2+63Ca^2-18bBa+8Ab^2\right)}{x^6}dx}{7a}-\frac{\left(a+bx^2\right)^{5/2}(4Ab-9aB)}{7ax^7}}{9a}-\frac{A\left(a+bx^2\right)^{5/2}}{9ax^9}$$

$$\Big\downarrow \; 358$$

$$-\frac{-\frac{63a^2D\int \frac{\left(bx^2+a\right)^{3/2}}{x^4}dx-\frac{\left(a+bx^2\right)^{5/2}\left(63a^2C-18abB+8Ab^2\right)}{5ax^5}}{7a}-\frac{\left(a+bx^2\right)^{5/2}(4Ab-9aB)}{7ax^7}}{9a}-\frac{A\left(a+bx^2\right)^{5/2}}{9ax^9}$$

$$\Big\downarrow \; 247$$

$$-\frac{-\frac{63a^2D\left(b\int \frac{\sqrt{bx^2+a}}{x^2}dx-\frac{\left(a+bx^2\right)^{3/2}}{3x^3}\right)-\frac{\left(a+bx^2\right)^{5/2}\left(63a^2C-18abB+8Ab^2\right)}{5ax^5}}{7a}-\frac{\left(a+bx^2\right)^{5/2}(4Ab-9aB)}{7ax^7}}{9a}-\frac{A\left(a+bx^2\right)^{5/2}}{9ax^9}$$

$$\Big\downarrow \; 247$$

$$-\frac{-\frac{63a^2D\left(b\left(b\int \frac{1}{\sqrt{bx^2+a}}dx-\frac{\sqrt{a+bx^2}}{x}\right)-\frac{\left(a+bx^2\right)^{3/2}}{3x^3}\right)-\frac{\left(a+bx^2\right)^{5/2}\left(63a^2C-18abB+8Ab^2\right)}{5ax^5}}{7a}-\frac{\left(a+bx^2\right)^{5/2}(4Ab-9aB)}{7ax^7}}{9a}-\frac{A\left(a+bx^2\right)^{5/2}}{9ax^9}$$

$$\Big\downarrow \; 224$$

$$-\frac{-\frac{63a^2D\left(b\left(b\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}-\frac{\sqrt{a+bx^2}}{x}\right)-\frac{\left(a+bx^2\right)^{3/2}}{3x^3}\right)-\frac{\left(a+bx^2\right)^{5/2}\left(63a^2C-18abB+8Ab^2\right)}{5ax^5}}{7a}-\frac{\left(a+bx^2\right)^{5/2}(4Ab-9aB)}{7ax^7}}{9a}-\frac{A\left(a+bx^2\right)^{5/2}}{9ax^9}$$

$$\Big\downarrow \; 219$$

$$-\frac{-\frac{63a^2D\left(b\left(\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)-\frac{\sqrt{a+bx^2}}{x}\right)-\frac{\left(a+bx^2\right)^{3/2}}{3x^3}\right)-\frac{\left(a+bx^2\right)^{5/2}\left(63a^2C-18abB+8Ab^2\right)}{5ax^5}}{7a}-\frac{\left(a+bx^2\right)^{5/2}(4Ab-9aB)}{7ax^7}}{9a}-\frac{A\left(a+bx^2\right)^{5/2}}{9ax^9}$$

input
```
Int[((a + b*x^2)^(3/2)*(A + B*x^2 + C*x^4 + D*x^6))/x^10,x]
```

output
```
-1/9*(A*(a + b*x^2)^(5/2))/(a*x^9) - (-1/7*((4*A*b - 9*a*B)*(a + b*x^2)^(5
/2))/(a*x^7) - (-1/5*((8*A*b^2 - 18*a*b*B + 63*a^2*C)*(a + b*x^2)^(5/2))/(
a*x^5) + 63*a^2*D*(-1/3*(a + b*x^2)^(3/2)/x^3 + b*(-(Sqrt[a + b*x^2]/x) +
Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])))/(7*a))/(9*a)
```

### Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 358
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x_
Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] + S
imp[d/e^2   Int[(e*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b*c - a*d, 0] && EqQ[Simplify[m + 2*p + 3], 0] && NeQ[m,
 -1]
```

rule 1588
```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Simp[1/(d*f
^2*(m + 1))   Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x
) - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

## Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.98

| method | result |
|---|---|
| pseudoelliptic | $\frac{9 D a^{3} b^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{x \sqrt{b}}\right) x^{9}-\sqrt{b\,x^{2}+a}\,\left(\left(3 D x^{6}+\frac{9}{5} C\,x^{4}+\frac{9}{7} x^{2} B+A\right) a^{4}+\frac{10\left(\frac{42}{5} D x^{6}+\frac{63}{25} C\,x^{4}+\frac{36}{25} x^{2} B+A\right) x^{2} b\,a^{3}}{7}+\frac{3 b^{2} x^{4}}{}\right.}{9 a^{3} x^{9}}$ |
| default | $A\left(-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{9 a\,x^{9}}-\frac{4 b\left(-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{7 a\,x^{7}}+\frac{2 b\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{35 a^{2} x^{5}}\right)}{9 a}\right)+B\left(-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{7 a\,x^{7}}+\frac{2 b\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{35 a^{2} x^{5}}\right)-\frac{C\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{5 a\,x^{5}}$ |

input
```
int((b*x^2+a)^(3/2)*(D*x^6+C*x^4+B*x^2+A)/x^10,x,method=_RETURNVERBOSE)
```

output
```
1/9*(9*D*a^3*b^(3/2)*arctanh((b*x^2+a)^(1/2)/x/b^(1/2))*x^9-(b*x^2+a)^(1/2
)*((3*D*x^6+9/5*C*x^4+9/7*x^2*B+A)*a^4+10/7*(42/5*D*x^6+63/25*C*x^4+36/25*
x^2*B+A)*x^2*b*a^3+3/35*b^2*x^4*(21*C*x^4+3*B*x^2+A)*a^2-4/35*x^6*b^3*(9/2
*x^2*B+A)*a+8/35*A*x^8*b^4))/a^3/x^9
```

## Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 352, normalized size of antiderivative = 2.23

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2+Cx^4+Dx^6)}{x^{10}}\,dx = \left[\frac{315\,Da^3b^{\frac{3}{2}}x^9\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\,((420\right.$$

$$-\frac{315\,Da^3\sqrt{-b}bx^9\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)+((420\,Da^3b+63\,Ca^2b^2-18\,Bab^3+8\,Ab^4)x^8+(105\,Da^4+126\,Ca^3}{315}$$

input
```
integrate((b*x^2+a)^(3/2)*(D*x^6+C*x^4+B*x^2+A)/x^10,x, algorithm="fricas"
)
```

output
```
[1/630*(315*D*a^3*b^(3/2)*x^9*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x -
 a) - 2*((420*D*a^3*b + 63*C*a^2*b^2 - 18*B*a*b^3 + 8*A*b^4)*x^8 + (105*D*
a^4 + 126*C*a^3*b + 9*B*a^2*b^2 - 4*A*a*b^3)*x^6 + 35*A*a^4 + 3*(21*C*a^4
+ 24*B*a^3*b + A*a^2*b^2)*x^4 + 5*(9*B*a^4 + 10*A*a^3*b)*x^2)*sqrt(b*x^2 +
 a))/(a^3*x^9), -1/315*(315*D*a^3*sqrt(-b)*b*x^9*arctan(sqrt(-b)*x/sqrt(b*
x^2 + a)) + ((420*D*a^3*b + 63*C*a^2*b^2 - 18*B*a*b^3 + 8*A*b^4)*x^8 + (10
5*D*a^4 + 126*C*a^3*b + 9*B*a^2*b^2 - 4*A*a*b^3)*x^6 + 35*A*a^4 + 3*(21*C*
a^4 + 24*B*a^3*b + A*a^2*b^2)*x^4 + 5*(9*B*a^4 + 10*A*a^3*b)*x^2)*sqrt(b*x
^2 + a))/(a^3*x^9)]
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1593 vs. $2(150) = 300$.

Time = 4.35 (sec) , antiderivative size = 1593, normalized size of antiderivative = 10.08

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2+Cx^4+Dx^6)}{x^{10}}\,dx = \text{Too large to display}$$

input

```
integrate((b*x**2+a)**(3/2)*(D*x**6+C*x**4+B*x**2+A)/x**10,x)
```

output

```
-35*A*a**8*b**(19/2)*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b
**10*x**10 + 945*a**5*b**11*x**12 + 315*a**4*b**12*x**14) - 110*A*a**7*b**
(21/2)*x**2*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**1
0 + 945*a**5*b**11*x**12 + 315*a**4*b**12*x**14) - 114*A*a**6*b**(23/2)*x*
*4*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**10 + 945*a
**5*b**11*x**12 + 315*a**4*b**12*x**14) - 40*A*a**5*b**(25/2)*x**6*sqrt(a/
(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**10 + 945*a**5*b**11*
x**12 + 315*a**4*b**12*x**14) - 15*A*a**5*b**(11/2)*sqrt(a/(b*x**2) + 1)/(
105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) + 5*A*a**4*
b**(27/2)*x**8*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x
**10 + 945*a**5*b**11*x**12 + 315*a**4*b**12*x**14) - 33*A*a**4*b**(13/2)*
x**2*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a
**3*b**6*x**10) + 30*A*a**3*b**(29/2)*x**10*sqrt(a/(b*x**2) + 1)/(315*a**7
*b**9*x**8 + 945*a**6*b**10*x**10 + 945*a**5*b**11*x**12 + 315*a**4*b**12*
x**14) - 17*A*a**3*b**(15/2)*x**4*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6
 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) + 40*A*a**2*b**(31/2)*x**12*s
qrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**10 + 945*a**5*
b**11*x**12 + 315*a**4*b**12*x**14) - 3*A*a**2*b**(17/2)*x**6*sqrt(a/(b*x*
*2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) +
 16*A*a*b**(33/2)*x**14*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*...
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.16

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2+Cx^4+Dx^6)}{x^{10}}\,dx = \frac{\sqrt{bx^2+a}Db^2x}{a} + Db^{\frac{3}{2}}\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right) - \frac{2(bx^2+a)^{\frac{3}{2}}Db}{3\,ax} - \frac{(bx^2+a)^{\frac{5}{2}}D}{3\,ax^3} - \frac{(bx^2+a)^{\frac{5}{2}}C}{5\,ax^5} + \frac{2(bx^2+a)^{\frac{5}{2}}Bb}{35\,a^2x^5} - \frac{8(bx^2+a)^{\frac{5}{2}}Ab^2}{315\,a^3x^5} - \frac{(bx^2+a)^{\frac{5}{2}}B}{7\,ax^7} + \frac{4(bx^2+a)^{\frac{5}{2}}Ab}{63\,a^2x^7} - \frac{(bx^2+a)^{\frac{5}{2}}A}{9\,ax^9}$$

input
```
integrate((b*x^2+a)^(3/2)*(D*x^6+C*x^4+B*x^2+A)/x^10,x, algorithm="maxima"
)
```

output
```
sqrt(b*x^2 + a)*D*b^2*x/a + D*b^(3/2)*arcsinh(b*x/sqrt(a*b)) - 2/3*(b*x^2
+ a)^(3/2)*D*b/(a*x) - 1/3*(b*x^2 + a)^(5/2)*D/(a*x^3) - 1/5*(b*x^2 + a)^(
5/2)*C/(a*x^5) + 2/35*(b*x^2 + a)^(5/2)*B*b/(a^2*x^5) - 8/315*(b*x^2 + a)^
(5/2)*A*b^2/(a^3*x^5) - 1/7*(b*x^2 + a)^(5/2)*B/(a*x^7) + 4/63*(b*x^2 + a)
^(5/2)*A*b/(a^2*x^7) - 1/9*(b*x^2 + a)^(5/2)*A/(a*x^9)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 886 vs. 2(132) = 264.

Time = 0.16 (sec) , antiderivative size = 886, normalized size of antiderivative = 5.61

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2+Cx^4+Dx^6)}{x^{10}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(3/2)*(D*x^6+C*x^4+B*x^2+A)/x^10,x, algorithm="giac")
```

output
```
-1/2*D*b^(3/2)*log((sqrt(b)*x - sqrt(b*x^2 + a))^2) + 2/315*(630*(sqrt(b)*
x - sqrt(b*x^2 + a))^16*D*a*b^(3/2) + 315*(sqrt(b)*x - sqrt(b*x^2 + a))^16
*C*b^(5/2) - 4410*(sqrt(b)*x - sqrt(b*x^2 + a))^14*D*a^2*b^(3/2) - 1260*(s
qrt(b)*x - sqrt(b*x^2 + a))^14*C*a*b^(5/2) + 630*(sqrt(b)*x - sqrt(b*x^2 +
 a))^14*B*b^(7/2) + 13650*(sqrt(b)*x - sqrt(b*x^2 + a))^12*D*a^3*b^(3/2) +
 2520*(sqrt(b)*x - sqrt(b*x^2 + a))^12*C*a^2*b^(5/2) - 630*(sqrt(b)*x - sq
rt(b*x^2 + a))^12*B*a*b^(7/2) + 1680*(sqrt(b)*x - sqrt(b*x^2 + a))^12*A*b^
(9/2) - 24570*(sqrt(b)*x - sqrt(b*x^2 + a))^10*D*a^4*b^(3/2) - 3780*(sqrt(
b)*x - sqrt(b*x^2 + a))^10*C*a^3*b^(5/2) + 630*(sqrt(b)*x - sqrt(b*x^2 + a
))^10*B*a^2*b^(7/2) + 2520*(sqrt(b)*x - sqrt(b*x^2 + a))^10*A*a*b^(9/2) +
28350*(sqrt(b)*x - sqrt(b*x^2 + a))^8*D*a^5*b^(3/2) + 4158*(sqrt(b)*x - sq
rt(b*x^2 + a))^8*C*a^4*b^(5/2) - 1638*(sqrt(b)*x - sqrt(b*x^2 + a))^8*B*a^
3*b^(7/2) + 3528*(sqrt(b)*x - sqrt(b*x^2 + a))^8*A*a^2*b^(9/2) - 21630*(sq
rt(b)*x - sqrt(b*x^2 + a))^6*D*a^6*b^(3/2) - 2772*(sqrt(b)*x - sqrt(b*x^2
+ a))^6*C*a^5*b^(5/2) + 882*(sqrt(b)*x - sqrt(b*x^2 + a))^6*B*a^4*b^(7/2)
+ 1008*(sqrt(b)*x - sqrt(b*x^2 + a))^6*A*a^3*b^(9/2) + 10710*(sqrt(b)*x -
sqrt(b*x^2 + a))^4*D*a^7*b^(3/2) + 1008*(sqrt(b)*x - sqrt(b*x^2 + a))^4*C*
a^6*b^(5/2) - 18*(sqrt(b)*x - sqrt(b*x^2 + a))^4*B*a^5*b^(7/2) + 288*(sqrt
(b)*x - sqrt(b*x^2 + a))^4*A*a^4*b^(9/2) - 3150*(sqrt(b)*x - sqrt(b*x^2 +
a))^2*D*a^8*b^(3/2) - 252*(sqrt(b)*x - sqrt(b*x^2 + a))^2*C*a^7*b^(5/2)...
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}\left(A+Bx^2+Cx^4+Dx^6\right)}{x^{10}}\,dx=\int \frac{{\left(b\,x^2+a\right)}^{3/2}\,\left(A+B\,x^2+C\,x^4+x^6\,D\right)}{x^{10}} \,dx$$

input
```
int(((a + b*x^2)^(3/2)*(A + B*x^2 + C*x^4 + x^6*D))/x^10,x)
```

output
```
int(((a + b*x^2)^(3/2)*(A + B*x^2 + C*x^4 + x^6*D))/x^10, x)
```

## Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.54

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2+Cx^4+Dx^6)}{x^{10}}\,dx = \frac{-35\sqrt{b\,x^2+a}\,a^4-95\sqrt{b\,x^2+a}\,a^3b\,x^2-63\sqrt{b\,x^2+a}\,a^3c\,x^4}{}$$

input
```
int((b*x^2+a)^(3/2)*(D*x^6+C*x^4+B*x^2+A)/x^10,x)
```

output
```
( - 35*sqrt(a + b*x**2)*a**4 - 95*sqrt(a + b*x**2)*a**3*b*x**2 - 63*sqrt(a
 + b*x**2)*a**3*c*x**4 - 105*sqrt(a + b*x**2)*a**3*d*x**6 - 75*sqrt(a + b*
x**2)*a**2*b**2*x**4 - 126*sqrt(a + b*x**2)*a**2*b*c*x**6 - 420*sqrt(a + b
*x**2)*a**2*b*d*x**8 - 5*sqrt(a + b*x**2)*a*b**3*x**6 - 63*sqrt(a + b*x**2
)*a*b**2*c*x**8 + 10*sqrt(a + b*x**2)*b**4*x**8 + 315*sqrt(b)*log((sqrt(a
+ b*x**2) + sqrt(b)*x)/sqrt(a))*a**2*b*d*x**9 + 280*sqrt(b)*a**2*b*d*x**9
- 7*sqrt(b)*a*b**2*c*x**9 - 10*sqrt(b)*b**4*x**9)/(315*a**2*x**9)
```

# 3.205 $\int \frac{(a+bx^2)^{3/2}(A+Bx^2+Cx^4+Dx^6)}{x^{12}}\,dx$



## Optimal result

Integrand size = 32, antiderivative size = 141

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2+Cx^4+Dx^6)}{x^{12}}\,dx = -\frac{A(a+bx^2)^{5/2}}{11ax^{11}} + \frac{(6Ab-11aB)(a+bx^2)^{5/2}}{99a^2x^9} - \frac{(24Ab^2-44abB+99a^2C)(a+bx^2)^{5/2}}{693a^3x^7} + \frac{(48Ab^3-11a(8b^2B-18abC+63a^2D))(a+bx^2)^{5/2}}{3465a^4x^5}$$

output
```
-1/11*A*(b*x^2+a)^(5/2)/a/x^11+1/99*(6*A*b-11*B*a)*(b*x^2+a)^(5/2)/a^2/x^9
-1/693*(24*A*b^2-44*B*a*b+99*C*a^2)*(b*x^2+a)^(5/2)/a^3/x^7+1/3465*(48*A*b
^3-11*a*(8*B*b^2-18*C*a*b+63*D*a^2))*(b*x^2+a)^(5/2)/a^4/x^5
```

## Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.72

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2+Cx^4+Dx^6)}{x^{12}}\,dx = \frac{(a+bx^2)^{5/2}(48Ab^3x^6-8ab^2x^4(15A+11Bx^2)+2a^2bx^2(10$$

input
```
Integrate[((a + b*x^2)^(3/2)*(A + B*x^2 + C*x^4 + D*x^6))/x^12,x]
```

output
```
((a + b*x^2)^(5/2)*(48*A*b^3*x^6 - 8*a*b^2*x^4*(15*A + 11*B*x^2) + 2*a^2*b
*x^2*(105*A + 110*B*x^2 + 99*C*x^4) - a^3*(315*A + 385*B*x^2 + 495*C*x^4 +
 693*D*x^6)))/(3465*a^4*x^11)
```

## Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {2334, 2089, 1588, 359, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2+Cx^4+Dx^6)}{x^{12}}dx$$

$$\Big\downarrow \text{2334}$$

$$-\frac{\int \frac{(bx^2+a)^{3/2}(6Ab-11a(Dx^4+Cx^2+B))}{x^{10}}dx}{11a}-\frac{A(a+bx^2)^{5/2}}{11ax^{11}}$$

$$\Big\downarrow \text{2089}$$

$$-\frac{\int \frac{(bx^2+a)^{3/2}(-11aDx^4-11aCx^2+6Ab-11aB)}{x^{10}}dx}{11a}-\frac{A(a+bx^2)^{5/2}}{11ax^{11}}$$

$$\Big\downarrow \text{1588}$$

$$-\frac{-\frac{\int \frac{(bx^2+a)^{3/2}(99Dx^2a^2+99Ca^2-44bBa+24Ab^2)}{x^8}dx}{9a}-\frac{(a+bx^2)^{5/2}(6Ab-11aB)}{9ax^9}}{11a}-\frac{A(a+bx^2)^{5/2}}{11ax^{11}}$$

$$\Big\downarrow \text{359}$$

$$-\frac{-\frac{-\frac{(48Ab^3-11a(63a^2D-18abC+8b^2B))\int \frac{(bx^2+a)^{3/2}}{x^6}dx}{7a}-\frac{(a+bx^2)^{5/2}(99a^2C-44abB+24Ab^2)}{7ax^7}}{9a}-\frac{(a+bx^2)^{5/2}(6Ab-11aB)}{9ax^9}}{11a}-\frac{A(a+bx^2)^{5/2}}{11ax^{11}}$$

$$\Big\downarrow \text{242}$$

$$-\frac{-\frac{\frac{\left(a+bx^2\right)^{5/2}\left(48Ab^3-11a\left(63a^2D-18abC+8b^2B\right)\right)}{35a^2x^5}-\frac{\left(a+bx^2\right)^{5/2}\left(99a^2C-44abB+24Ab^2\right)}{7ax^7}}{9a}-\frac{\left(a+bx^2\right)^{5/2}\left(6Ab-11aB\right)}{9ax^9}}{11a}-\frac{A\left(a+bx^2\right)^{5/2}}{11ax^{11}}$$

input
```
Int[((a + b*x^2)^(3/2)*(A + B*x^2 + C*x^4 + D*x^6))/x^12,x]
```

output
```
-1/11*(A*(a + b*x^2)^(5/2))/(a*x^11) - (-1/9*((6*A*b - 11*a*B)*(a + b*x^2)
^(5/2))/(a*x^9) - (-1/7*((24*A*b^2 - 44*a*b*B + 99*a^2*C)*(a + b*x^2)^(5/2
))/(a*x^7) + ((48*A*b^3 - 11*a*(8*b^2*B - 18*a*b*C + 63*a^2*D))*(a + b*x^2
)^(5/2))/(35*a^2*x^5))/(9*a))/(11*a)
```

## Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 1588
```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Simp[1/(d*f
^2*(m + 1))   Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x
) - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

rule 2089
```
Int[(u_)^(p_.)*((f_.)*(x_))^(m_.)*(z_)^(q_.), x_Symbol] :> Int[(f*x)^m*Expa
ndToSum[z, x]^q*ExpandToSum[u, x]^p, x] /; FreeQ[{f, m, p, q}, x] && Binomi
alQ[z, x] && TrinomialQ[u, x] &&  !(BinomialMatchQ[z, x] && TrinomialMatchQ
[u, x])
```

rule 2334
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coef
f[Pq, x, 0], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[A*
x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))   Int[
x^(m + 2)*(a + b*x^2)^p*(a*(m + 1)*Q - A*b*(m + 2*(p + 1) + 1)), x], x]] /;
 FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && IntegerQ[m/2] && ILtQ[(m + 1)/2 + p,
 0] && LtQ[m + Expon[Pq, x] + 2*p + 1, 0]
```

## Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.65

| method | result |
|---|---|
| pseudoelliptic | $-\frac{\left(\left(\frac{11}{5} D x^{6}+\frac{11}{7} C \,x^{4}+\frac{11}{9} x^{2} B+A\right) a^{3}-\frac{2\left(\frac{33}{35} C \,x^{4}+\frac{22}{21} x^{2} B+A\right) x^{2} b \,a^{2}}{3}+\frac{8 x^{4} b^{2}\left(\frac{11 x^{2} B}{15}+A\right) a}{21}-\frac{16 A \,b^{3} x^{6}}{105}\right)\left(b \,x^{2}+a\right)^{\frac{5}{2}}}{11 x^{11} a^{4}}$ |
| gosper | $-\frac{\left(b \,x^{2}+a\right)^{\frac{5}{2}}\left(-48 A \,b^{3} x^{6}+88 B a \,b^{2} x^{6}-198 C \,a^{2} b \,x^{6}+693 D a^{3} x^{6}+120 a A \,b^{2} x^{4}-220 B \,a^{2} b \,x^{4}+495 C \,a^{3} x^{4}-210 a^{2} A b \,x^{2}+38\right.}{3465 x^{11} a^{4}}$ |
| orering | $-\frac{\left(b \,x^{2}+a\right)^{\frac{5}{2}}\left(-48 A \,b^{3} x^{6}+88 B a \,b^{2} x^{6}-198 C \,a^{2} b \,x^{6}+693 D a^{3} x^{6}+120 a A \,b^{2} x^{4}-220 B \,a^{2} b \,x^{4}+495 C \,a^{3} x^{4}-210 a^{2} A b \,x^{2}+38\right.}{3465 x^{11} a^{4}}$ |
| trager | $-\frac{\left(-48 A \,b^{5} x^{10}+88 B a \,b^{4} x^{10}-198 C \,a^{2} b^{3} x^{10}+693 D a^{3} b^{2} x^{10}+24 a A \,b^{4} x^{8}-44 B \,a^{2} b^{3} x^{8}+99 C \,a^{3} b^{2} x^{8}+1386 D a^{4} b \,x^{8}-18 a^{2}\right.}{}$ |
| default | $A\left(-\frac{\left(b \,x^{2}+a\right)^{\frac{5}{2}}}{11 a \,x^{11}}-\frac{6 b\left(-\frac{\left(b \,x^{2}+a\right)^{\frac{5}{2}}}{9 a \,x^{9}}-\frac{4 b\left(-\frac{\left(b \,x^{2}+a\right)^{\frac{5}{2}}}{7 a \,x^{7}}+\frac{2 b\left(b \,x^{2}+a\right)^{\frac{5}{2}}}{35 a^{2} x^{5}}\right)}{9 a}\right)}{11 a}\right)+B\left(-\frac{\left(b \,x^{2}+a\right)^{\frac{5}{2}}}{9 a \,x^{9}}-\frac{4 b\left(-\frac{\left(b \,x^{2}+\right.}{7 a}\right.}{}\right.$ |

input
```
int((b*x^2+a)^(3/2)*(D*x^6+C*x^4+B*x^2+A)/x^12,x,method=_RETURNVERBOSE)
```

output
```
-1/11*((11/5*D*x^6+11/7*C*x^4+11/9*x^2*B+A)*a^3-2/3*(33/35*C*x^4+22/21*x^2
*B+A)*x^2*b*a^2+8/21*x^4*b^2*(11/15*x^2*B+A)*a-16/105*A*b^3*x^6)*(b*x^2+a)
^(5/2)/x^11/a^4
```

## Fricas [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.28

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2+Cx^4+Dx^6)}{x^{12}}\,dx =$$
$$-\frac{((693\,Da^3b^2-198\,Ca^2b^3+88\,Bab^4-48\,Ab^5)x^{10}+(1386\,Da^4b+99\,Ca^3b^2-44\,Ba^2b^3+24\,Aab^4)x^8+3}{}$$

input
```
integrate((b*x^2+a)^(3/2)*(D*x^6+C*x^4+B*x^2+A)/x^12,x, algorithm="fricas"
)
```

output
```
-1/3465*((693*D*a^3*b^2 - 198*C*a^2*b^3 + 88*B*a*b^4 - 48*A*b^5)*x^10 + (1
386*D*a^4*b + 99*C*a^3*b^2 - 44*B*a^2*b^3 + 24*A*a*b^4)*x^8 + 3*(231*D*a^5
 + 264*C*a^4*b + 11*B*a^3*b^2 - 6*A*a^2*b^3)*x^6 + 315*A*a^5 + 5*(99*C*a^5
 + 110*B*a^4*b + 3*A*a^3*b^2)*x^4 + 35*(11*B*a^5 + 12*A*a^4*b)*x^2)*sqrt(b
*x^2 + a)/(a^4*x^11)
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2932 vs. 2(136) = 272.

Time = 5.27 (sec) , antiderivative size = 2932, normalized size of antiderivative = 20.79

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2+Cx^4+Dx^6)}{x^{12}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x**2+a)**(3/2)*(D*x**6+C*x**4+B*x**2+A)/x**12,x)
```

output
```
-315*A*a**10*b**(33/2)*sqrt(a/(b*x**2) + 1)/(3465*a**9*b**16*x**10 + 13860
*a**8*b**17*x**12 + 20790*a**7*b**18*x**14 + 13860*a**6*b**19*x**16 + 3465
*a**5*b**20*x**18) - 1295*A*a**9*b**(35/2)*x**2*sqrt(a/(b*x**2) + 1)/(3465
*a**9*b**16*x**10 + 13860*a**8*b**17*x**12 + 20790*a**7*b**18*x**14 + 1386
0*a**6*b**19*x**16 + 3465*a**5*b**20*x**18) - 1990*A*a**8*b**(37/2)*x**4*s
qrt(a/(b*x**2) + 1)/(3465*a**9*b**16*x**10 + 13860*a**8*b**17*x**12 + 2079
0*a**7*b**18*x**14 + 13860*a**6*b**19*x**16 + 3465*a**5*b**20*x**18) - 135
8*A*a**7*b**(39/2)*x**6*sqrt(a/(b*x**2) + 1)/(3465*a**9*b**16*x**10 + 1386
0*a**8*b**17*x**12 + 20790*a**7*b**18*x**14 + 13860*a**6*b**19*x**16 + 346
5*a**5*b**20*x**18) - 35*A*a**7*b**(21/2)*sqrt(a/(b*x**2) + 1)/(315*a**7*b
**9*x**8 + 945*a**6*b**10*x**10 + 945*a**5*b**11*x**12 + 315*a**4*b**12*x*
*14) - 343*A*a**6*b**(41/2)*x**8*sqrt(a/(b*x**2) + 1)/(3465*a**9*b**16*x**
10 + 13860*a**8*b**17*x**12 + 20790*a**7*b**18*x**14 + 13860*a**6*b**19*x*
*16 + 3465*a**5*b**20*x**18) - 110*A*a**6*b**(23/2)*x**2*sqrt(a/(b*x**2) +
 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**10 + 945*a**5*b**11*x**12 + 31
5*a**4*b**12*x**14) - 35*A*a**5*b**(43/2)*x**10*sqrt(a/(b*x**2) + 1)/(3465
*a**9*b**16*x**10 + 13860*a**8*b**17*x**12 + 20790*a**7*b**18*x**14 + 1386
0*a**6*b**19*x**16 + 3465*a**5*b**20*x**18) - 114*A*a**5*b**(25/2)*x**4*sq
rt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**10 + 945*a**5*b
**11*x**12 + 315*a**4*b**12*x**14) - 280*A*a**4*b**(45/2)*x**12*sqrt(a/...
```

## Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.37

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2+Cx^4+Dx^6)}{x^{12}}\,dx = -\frac{(bx^2+a)^{\frac{5}{2}}D}{5\,ax^5} + \frac{2\,(bx^2+a)^{\frac{5}{2}}Cb}{35\,a^2x^5} - \frac{8\,(bx^2+a)^{\frac{5}{2}}Bb^2}{315\,a^3x^5} + \frac{16\,(bx^2+a)^{\frac{5}{2}}Ab^3}{1155\,a^4x^5} - \frac{(bx^2+a)^{\frac{5}{2}}C}{7\,ax^7} + \frac{4\,(bx^2+a)^{\frac{5}{2}}Bb}{63\,a^2x^7} - \frac{8\,(bx^2+a)^{\frac{5}{2}}Ab^2}{231\,a^3x^7} - \frac{(bx^2+a)^{\frac{5}{2}}B}{9\,ax^9} + \frac{2\,(bx^2+a)^{\frac{5}{2}}Ab}{33\,a^2x^9} - \frac{(bx^2+a)^{\frac{5}{2}}A}{11\,ax^{11}}$$

input
```
integrate((b*x^2+a)^(3/2)*(D*x^6+C*x^4+B*x^2+A)/x^12,x, algorithm="maxima"
)
```

output
```
-1/5*(b*x^2 + a)^(5/2)*D/(a*x^5) + 2/35*(b*x^2 + a)^(5/2)*C*b/(a^2*x^5) -
8/315*(b*x^2 + a)^(5/2)*B*b^2/(a^3*x^5) + 16/1155*(b*x^2 + a)^(5/2)*A*b^3/
(a^4*x^5) - 1/7*(b*x^2 + a)^(5/2)*C/(a*x^7) + 4/63*(b*x^2 + a)^(5/2)*B*b/(
a^2*x^7) - 8/231*(b*x^2 + a)^(5/2)*A*b^2/(a^3*x^7) - 1/9*(b*x^2 + a)^(5/2)
*B/(a*x^9) + 2/33*(b*x^2 + a)^(5/2)*A*b/(a^2*x^9) - 1/11*(b*x^2 + a)^(5/2)
*A/(a*x^11)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 996 vs. $2(125) = 250$.

Time = 0.16 (sec) , antiderivative size = 996, normalized size of antiderivative = 7.06

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2+Cx^4+Dx^6)}{x^{12}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(3/2)*(D*x^6+C*x^4+B*x^2+A)/x^12,x, algorithm="giac")
```

output
```
2/3465*(3465*(sqrt(b)*x - sqrt(b*x^2 + a))^20*D*b^(5/2) - 20790*(sqrt(b)*x
 - sqrt(b*x^2 + a))^18*D*a*b^(5/2) + 6930*(sqrt(b)*x - sqrt(b*x^2 + a))^18
*C*b^(7/2) + 58905*(sqrt(b)*x - sqrt(b*x^2 + a))^16*D*a^2*b^(5/2) - 20790*
(sqrt(b)*x - sqrt(b*x^2 + a))^16*C*a*b^(7/2) + 18480*(sqrt(b)*x - sqrt(b*x
^2 + a))^16*B*b^(9/2) - 110880*(sqrt(b)*x - sqrt(b*x^2 + a))^14*D*a^3*b^(5
/2) + 27720*(sqrt(b)*x - sqrt(b*x^2 + a))^14*C*a^2*b^(7/2) - 9240*(sqrt(b)
*x - sqrt(b*x^2 + a))^14*B*a*b^(9/2) + 55440*(sqrt(b)*x - sqrt(b*x^2 + a))
^14*A*b^(11/2) + 156618*(sqrt(b)*x - sqrt(b*x^2 + a))^12*D*a^4*b^(5/2) - 3
8808*(sqrt(b)*x - sqrt(b*x^2 + a))^12*C*a^3*b^(7/2) + 1848*(sqrt(b)*x - sq
rt(b*x^2 + a))^12*B*a^2*b^(9/2) + 99792*(sqrt(b)*x - sqrt(b*x^2 + a))^12*A
*a*b^(11/2) - 163548*(sqrt(b)*x - sqrt(b*x^2 + a))^10*D*a^5*b^(5/2) + 5266
8*(sqrt(b)*x - sqrt(b*x^2 + a))^10*C*a^4*b^(7/2) - 38808*(sqrt(b)*x - sqrt
(b*x^2 + a))^10*B*a^3*b^(9/2) + 121968*(sqrt(b)*x - sqrt(b*x^2 + a))^10*A*
a^2*b^(11/2) + 117810*(sqrt(b)*x - sqrt(b*x^2 + a))^8*D*a^6*b^(5/2) - 3762
0*(sqrt(b)*x - sqrt(b*x^2 + a))^8*C*a^5*b^(7/2) + 19800*(sqrt(b)*x - sqrt(
b*x^2 + a))^8*B*a^4*b^(9/2) + 39600*(sqrt(b)*x - sqrt(b*x^2 + a))^8*A*a^3*
b^(11/2) - 55440*(sqrt(b)*x - sqrt(b*x^2 + a))^6*D*a^7*b^(5/2) + 11880*(sq
rt(b)*x - sqrt(b*x^2 + a))^6*C*a^6*b^(7/2) + 3960*(sqrt(b)*x - sqrt(b*x^2
+ a))^6*B*a^5*b^(9/2) + 7920*(sqrt(b)*x - sqrt(b*x^2 + a))^6*A*a^4*b^(11/2
) + 17325*(sqrt(b)*x - sqrt(b*x^2 + a))^4*D*a^8*b^(5/2) - 3960*(sqrt(b)...
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2+Cx^4+Dx^6)}{x^{12}}\,dx=\int \frac{(b\,x^2+a)^{3/2}\,(A+B\,x^2+C\,x^4+x^6\,D)}{x^{12}}\,dx$$

input
```
int(((a + b*x^2)^(3/2)*(A + B*x^2 + C*x^4 + x^6*D))/x^12,x)
```

output
```
int(((a + b*x^2)^(3/2)*(A + B*x^2 + C*x^4 + x^6*D))/x^12, x)
```

## Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.94

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2+Cx^4+Dx^6)}{x^{12}}\,dx=\frac{-315\sqrt{b\,x^2+a}\,a^5-805\sqrt{b\,x^2+a}\,a^4b\,x^2-495\sqrt{b\,x^2+a}\,a^4$$

input
```
int((b*x^2+a)^(3/2)*(D*x^6+C*x^4+B*x^2+A)/x^12,x)
```

output
```
( - 315*sqrt(a + b*x**2)*a**5 - 805*sqrt(a + b*x**2)*a**4*b*x**2 - 495*sqr
t(a + b*x**2)*a**4*c*x**4 - 693*sqrt(a + b*x**2)*a**4*d*x**6 - 565*sqrt(a
+ b*x**2)*a**3*b**2*x**4 - 792*sqrt(a + b*x**2)*a**3*b*c*x**6 - 1386*sqrt(
a + b*x**2)*a**3*b*d*x**8 - 15*sqrt(a + b*x**2)*a**2*b**3*x**6 - 99*sqrt(a
 + b*x**2)*a**2*b**2*c*x**8 - 693*sqrt(a + b*x**2)*a**2*b**2*d*x**10 + 20*
sqrt(a + b*x**2)*a*b**4*x**8 + 198*sqrt(a + b*x**2)*a*b**3*c*x**10 - 40*sq
rt(a + b*x**2)*b**5*x**10 + 63*sqrt(b)*a**2*b**2*d*x**11 - 198*sqrt(b)*a*b
**3*c*x**11 + 40*sqrt(b)*b**5*x**11)/(3465*a**3*x**11)
```

# 3.206 $\int \frac{(a+bx^2)^{3/2}(A+Bx^2+Cx^4+Dx^6)}{x^{14}}\,dx$



## Optimal result

Integrand size = 32, antiderivative size = 191

$$\begin{aligned}\int \frac{(a+bx^2)^{3/2}(A+Bx^2+Cx^4+Dx^6)}{x^{14}}\,dx &= -\frac{A(a+bx^2)^{5/2}}{13ax^{13}} \\ &+\frac{(8Ab-13aB)(a+bx^2)^{5/2}}{143a^2x^{11}}-\frac{(48Ab^2-78abB+143a^2C)(a+bx^2)^{5/2}}{1287a^3x^9} \\ &+\frac{(192Ab^3-13a(24b^2B-44abC+99a^2D))(a+bx^2)^{5/2}}{9009a^4x^7} \\ &-\frac{2b(192Ab^3-13a(24b^2B-44abC+99a^2D))(a+bx^2)^{5/2}}{45045a^5x^5}\end{aligned}$$

output

```
-1/13*A*(b*x^2+a)^(5/2)/a/x^13+1/143*(8*A*b-13*B*a)*(b*x^2+a)^(5/2)/a^2/x^
11-1/1287*(48*A*b^2-78*B*a*b+143*C*a^2)*(b*x^2+a)^(5/2)/a^3/x^9+1/9009*(19
2*A*b^3-13*a*(24*B*b^2-44*C*a*b+99*D*a^2))*(b*x^2+a)^(5/2)/a^4/x^7-2/45045
*b*(192*A*b^3-13*a*(24*B*b^2-44*C*a*b+99*D*a^2))*(b*x^2+a)^(5/2)/a^5/x^5
```

## Mathematica [A] (verified)

Time = 0.95 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.74

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2+Cx^4+Dx^6)}{x^{14}}\,dx = \frac{(a+bx^2)^{5/2}(-384Ab^4x^8+48ab^3x^6(20A+13Bx^2)-8a^2b^2x$$

input
```
Integrate[((a + b*x^2)^(3/2)*(A + B*x^2 + C*x^4 + D*x^6))/x^14,x]
```

output
```
((a + b*x^2)^(5/2)*(-384*A*b^4*x^8 + 48*a*b^3*x^6*(20*A + 13*B*x^2) - 8*a^
2*b^2*x^4*(210*A + 195*B*x^2 + 143*C*x^4) - 5*a^4*(693*A + 13*(63*B*x^2 +
77*C*x^4 + 99*D*x^6)) + 2*a^3*b*x^2*(1260*A + 13*(105*B*x^2 + 110*C*x^4 +
99*D*x^6))))/(45045*a^5*x^13)
```

## Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2334, 2089, 1588, 359, 245, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2+Cx^4+Dx^6)}{x^{14}}dx$$

$$\Big\downarrow \text{2334}$$

$$-\frac{\int \frac{(bx^2+a)^{3/2}(8Ab-13a(Dx^4+Cx^2+B))}{x^{12}}dx}{13a}-\frac{A(a+bx^2)^{5/2}}{13ax^{13}}$$

$$\Big\downarrow \text{2089}$$

$$-\frac{\int \frac{(bx^2+a)^{3/2}(-13aDx^4-13aCx^2+8Ab-13aB)}{x^{12}}dx}{13a}-\frac{A(a+bx^2)^{5/2}}{13ax^{13}}$$

$$\Big\downarrow \text{1588}$$

$$-\frac{-\frac{-\frac{\int \frac{(bx^2+a)^{3/2}(143Dx^2a^2+143Ca^2-78bBa+48Ab^2)}{x^{10}}dx}{11a}-\frac{(a+bx^2)^{5/2}(8Ab-13aB)}{11ax^{11}}}{13a}-\frac{A(a+bx^2)^{5/2}}{13ax^{13}}}{}$$

$\downarrow$ 359

$$-\frac{-\frac{-\frac{(192Ab^3-13a(99a^2D-44abC+24b^2B))\int \frac{(bx^2+a)^{3/2}}{x^8}dx}{9a}-\frac{(a+bx^2)^{5/2}(143a^2C-78abB+48Ab^2)}{9ax^9}}{11a}-\frac{(a+bx^2)^{5/2}(8Ab-13aB)}{11ax^{11}}}{13a}-\frac{A(a+bx^2)^{5/2}}{13ax^{13}}$$

$\downarrow$ 245

$$-\frac{-\frac{-\frac{(192Ab^3-13a(99a^2D-44abC+24b^2B))\left(-\frac{2b\int \frac{(bx^2+a)^{3/2}}{x^6}dx}{7a}-\frac{(a+bx^2)^{5/2}}{7ax^7}\right)}{9a}-\frac{(a+bx^2)^{5/2}(143a^2C-78abB+48Ab^2)}{9ax^9}}{11a}-\frac{(a+bx^2)^{5/2}(8Ab-13aB)}{11ax^{11}}}{13a}-\frac{A(a+bx^2)^{5/2}}{13ax^{13}}$$

$\downarrow$ 242

$$-\frac{-\frac{-\frac{(a+bx^2)^{5/2}(143a^2C-78abB+48Ab^2)}{9ax^9}-\frac{\left(\frac{2b(a+bx^2)^{5/2}}{35a^2x^5}-\frac{(a+bx^2)^{5/2}}{7ax^7}\right)(192Ab^3-13a(99a^2D-44abC+24b^2B))}{9a}}{11a}-\frac{(a+bx^2)^{5/2}(8Ab-13aB)}{11ax^{11}}}{13a}-\frac{A(a+bx^2)^{5/2}}{13ax^{13}}$$

input

```
Int[((a + b*x^2)^(3/2)*(A + B*x^2 + C*x^4 + D*x^6))/x^14,x]
```

output

```
-1/13*(A*(a + b*x^2)^(5/2))/(a*x^13) - (-1/11*((8*A*b - 13*a*B)*(a + b*x^2
)^(5/2))/(a*x^11) - (-1/9*((48*A*b^2 - 78*a*b*B + 143*a^2*C)*(a + b*x^2)^(
5/2))/(a*x^9) - ((192*A*b^3 - 13*a*(24*b^2*B - 44*a*b*C + 99*a^2*D))*(-1/7
*(a + b*x^2)^(5/2)/(a*x^7) + (2*b*(a + b*x^2)^(5/2))/(35*a^2*x^5)))/(9*a))
/(11*a))/(13*a)
```

### Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 1588
```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Simp[1/(d*f
^2*(m + 1))   Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x
) - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

rule 2089
```
Int[(u_)^(p_.)*((f_.)*(x_))^(m_.)*(z_)^(q_.), x_Symbol] :> Int[(f*x)^m*Expa
ndToSum[z, x]^q*ExpandToSum[u, x]^p, x] /; FreeQ[{f, m, p, q}, x] && Binomi
alQ[z, x] && TrinomialQ[u, x] &&  !(BinomialMatchQ[z, x] && TrinomialMatchQ
[u, x])
```

rule 2334

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coef
f[Pq, x, 0], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[A*
x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))   Int[
x^(m + 2)*(a + b*x^2)^p*(a*(m + 1)*Q - A*b*(m + 2*(p + 1) + 1)), x], x]] /;
 FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && IntegerQ[m/2] && ILtQ[(m + 1)/2 + p,
 0] && LtQ[m + Expon[Pq, x] + 2*p + 1, 0]
```

## Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.64

| method | result |
|---|---|
| pseudoelliptic | $-\frac{\left(\left(\frac{13}{7}Dx^6+\frac{13}{9}Cx^4+\frac{13}{11}x^2B+A\right)a^4-\frac{8\left(\frac{143}{140}Dx^6+\frac{143}{126}Cx^4+\frac{13}{12}x^2B+A\right)x^2ba^3}{11}+\frac{16\left(\frac{143}{210}Cx^4+\frac{13}{14}x^2B+A\right)x^4b^2a^2}{33}-\frac{64\left(\frac{13x^2}{20}\cdots\right.}{}\right.}{13x^{13}a^5}$ |
| gosper | $-\frac{(bx^2+a)^{\frac{5}{2}}(384Ax^8b^4-624Bx^8ab^3+1144Ca^2b^2x^8-2574Da^3bx^8-960Ax^6ab^3+1560Bx^6a^2b^2-2860Ca^3bx^6+6435\cdots}{45045x^{13}a^5}$ |
| orering | $-\frac{(bx^2+a)^{\frac{5}{2}}(384Ax^8b^4-624Bx^8ab^3+1144Ca^2b^2x^8-2574Da^3bx^8-960Ax^6ab^3+1560Bx^6a^2b^2-2860Ca^3bx^6+6435\cdots}{45045x^{13}a^5}$ |
| trager | $-\frac{(384Ab^6x^{12}-624Bab^5x^{12}+1144Ca^2b^4x^{12}-2574Da^3b^3x^{12}-192Aab^5x^{10}+312Ba^2b^4x^{10}-572Ca^3b^3x^{10}+1287Da^4b^2\cdots}{}$ |
| default | $A\left(-\frac{(bx^2+a)^{\frac{5}{2}}}{13ax^{13}}-\frac{8b\left(-\frac{(bx^2+a)^{\frac{5}{2}}}{11ax^{11}}-\frac{6b\left(-\frac{(bx^2+a)^{\frac{5}{2}}}{9ax^9}-\frac{4b\left(-\frac{(bx^2+a)^{\frac{5}{2}}}{7ax^7}+\frac{2b(bx^2+a)^{\frac{5}{2}}}{35a^2x^5}\right)}{9a}\right)}{11a}\right)}{13a}\right)+B\left(-\frac{(bx^2+a\cdots}{11ax^{1\cdots}}\right.$ |

input

```
int((b*x^2+a)^(3/2)*(D*x^6+C*x^4+B*x^2+A)/x^14,x,method=_RETURNVERBOSE)
```

output
```
-1/13*((13/7*D*x^6+13/9*C*x^4+13/11*x^2*B+A)*a^4-8/11*(143/140*D*x^6+143/1
26*C*x^4+13/12*x^2*B+A)*x^2*b*a^3+16/33*(143/210*C*x^4+13/14*x^2*B+A)*x^4*
b^2*a^2-64/231*(13/20*x^2*B+A)*x^6*b^3*a+128/1155*A*x^8*b^4)*(b*x^2+a)^(5/
2)/x^13/a^5
```

## Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.18

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2+Cx^4+Dx^6)}{x^{14}}\,dx = \frac{(2\,(1287\,Da^3b^3-572\,Ca^2b^4+312\,Bab^5-192\,Ab^6)x^{12}-(1\ldots}{}$$

input
```
integrate((b*x^2+a)^(3/2)*(D*x^6+C*x^4+B*x^2+A)/x^14,x, algorithm="fricas"
)
```

output
```
1/45045*(2*(1287*D*a^3*b^3 - 572*C*a^2*b^4 + 312*B*a*b^5 - 192*A*b^6)*x^12
 - (1287*D*a^4*b^2 - 572*C*a^3*b^3 + 312*B*a^2*b^4 - 192*A*a*b^5)*x^10 - 3
*(3432*D*a^5*b + 143*C*a^4*b^2 - 78*B*a^3*b^3 + 48*A*a^2*b^4)*x^8 - 3465*A
*a^6 - 5*(1287*D*a^6 + 1430*C*a^5*b + 39*B*a^4*b^2 - 24*A*a^3*b^3)*x^6 - 3
5*(143*C*a^6 + 156*B*a^5*b + 3*A*a^4*b^2)*x^4 - 315*(13*B*a^6 + 14*A*a^5*b
)*x^2)*sqrt(b*x^2 + a)/(a^5*x^13)
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4896 vs. 2(189) = 378.

Time = 6.91 (sec) , antiderivative size = 4896, normalized size of antiderivative = 25.63

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2+Cx^4+Dx^6)}{x^{14}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x**2+a)**(3/2)*(D*x**6+C*x**4+B*x**2+A)/x**14,x)
```

output
```
-693*A*a**12*b**(51/2)*sqrt(a/(b*x**2) + 1)/(9009*a**11*b**25*x**12 + 4504
5*a**10*b**26*x**14 + 90090*a**9*b**27*x**16 + 90090*a**8*b**28*x**18 + 45
045*a**7*b**29*x**20 + 9009*a**6*b**30*x**22) - 3528*A*a**11*b**(53/2)*x**
2*sqrt(a/(b*x**2) + 1)/(9009*a**11*b**25*x**12 + 45045*a**10*b**26*x**14 +
 90090*a**9*b**27*x**16 + 90090*a**8*b**28*x**18 + 45045*a**7*b**29*x**20
+ 9009*a**6*b**30*x**22) - 7175*A*a**10*b**(55/2)*x**4*sqrt(a/(b*x**2) + 1
)/(9009*a**11*b**25*x**12 + 45045*a**10*b**26*x**14 + 90090*a**9*b**27*x**
16 + 90090*a**8*b**28*x**18 + 45045*a**7*b**29*x**20 + 9009*a**6*b**30*x**
22) - 7290*A*a**9*b**(57/2)*x**6*sqrt(a/(b*x**2) + 1)/(9009*a**11*b**25*x*
*12 + 45045*a**10*b**26*x**14 + 90090*a**9*b**27*x**16 + 90090*a**8*b**28*
x**18 + 45045*a**7*b**29*x**20 + 9009*a**6*b**30*x**22) - 315*A*a**9*b**(3
5/2)*sqrt(a/(b*x**2) + 1)/(3465*a**9*b**16*x**10 + 13860*a**8*b**17*x**12
+ 20790*a**7*b**18*x**14 + 13860*a**6*b**19*x**16 + 3465*a**5*b**20*x**18)
 - 3699*A*a**8*b**(59/2)*x**8*sqrt(a/(b*x**2) + 1)/(9009*a**11*b**25*x**12
 + 45045*a**10*b**26*x**14 + 90090*a**9*b**27*x**16 + 90090*a**8*b**28*x**
18 + 45045*a**7*b**29*x**20 + 9009*a**6*b**30*x**22) - 1295*A*a**8*b**(37/
2)*x**2*sqrt(a/(b*x**2) + 1)/(3465*a**9*b**16*x**10 + 13860*a**8*b**17*x**
12 + 20790*a**7*b**18*x**14 + 13860*a**6*b**19*x**16 + 3465*a**5*b**20*x**
18) - 756*A*a**7*b**(61/2)*x**10*sqrt(a/(b*x**2) + 1)/(9009*a**11*b**25*x*
*12 + 45045*a**10*b**26*x**14 + 90090*a**9*b**27*x**16 + 90090*a**8*b**...
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.44

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2+Cx^4+Dx^6)}{x^{14}}\,dx = \frac{2\,(bx^2+a)^{\frac{5}{2}}Db}{35\,a^2x^5} - \frac{8\,(bx^2+a)^{\frac{5}{2}}Cb^2}{315\,a^3x^5} + \frac{16\,(bx^2+a)^{\frac{5}{2}}Bb^3}{1155\,a^4x^5} - \frac{128\,(bx^2+a)^{\frac{5}{2}}Ab^4}{15015\,a^5x^5} - \frac{(bx^2+a)^{\frac{5}{2}}D}{7\,ax^7} + \frac{4\,(bx^2+a)^{\frac{5}{2}}Cb}{63\,a^2x^7} - \frac{8\,(bx^2+a)^{\frac{5}{2}}Bb^2}{231\,a^3x^7} + \frac{64\,(bx^2+a)^{\frac{5}{2}}Ab^3}{3003\,a^4x^7} - \frac{(bx^2+a)^{\frac{5}{2}}C}{9\,ax^9} + \frac{2\,(bx^2+a)^{\frac{5}{2}}Bb}{33\,a^2x^9} - \frac{16\,(bx^2+a)^{\frac{5}{2}}Ab^2}{429\,a^3x^9} - \frac{(bx^2+a)^{\frac{5}{2}}B}{11\,ax^{11}} + \frac{8\,(bx^2+a)^{\frac{5}{2}}Ab}{143\,a^2x^{11}} - \frac{(bx^2+a)^{\frac{5}{2}}A}{13\,ax^{13}}$$

input
```
integrate((b*x^2+a)^(3/2)*(D*x^6+C*x^4+B*x^2+A)/x^14,x, algorithm="maxima"
)
```

output
```
2/35*(b*x^2 + a)^(5/2)*D*b/(a^2*x^5) - 8/315*(b*x^2 + a)^(5/2)*C*b^2/(a^3*
x^5) + 16/1155*(b*x^2 + a)^(5/2)*B*b^3/(a^4*x^5) - 128/15015*(b*x^2 + a)^(
5/2)*A*b^4/(a^5*x^5) - 1/7*(b*x^2 + a)^(5/2)*D/(a*x^7) + 4/63*(b*x^2 + a)^
(5/2)*C*b/(a^2*x^7) - 8/231*(b*x^2 + a)^(5/2)*B*b^2/(a^3*x^7) + 64/3003*(b
*x^2 + a)^(5/2)*A*b^3/(a^4*x^7) - 1/9*(b*x^2 + a)^(5/2)*C/(a*x^9) + 2/33*(
b*x^2 + a)^(5/2)*B*b/(a^2*x^9) - 16/429*(b*x^2 + a)^(5/2)*A*b^2/(a^3*x^9)
- 1/11*(b*x^2 + a)^(5/2)*B/(a*x^11) + 8/143*(b*x^2 + a)^(5/2)*A*b/(a^2*x^1
1) - 1/13*(b*x^2 + a)^(5/2)*A/(a*x^13)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1108 vs. $2(171) = 342$.

Time = 0.15 (sec) , antiderivative size = 1108, normalized size of antiderivative = 5.80

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2+Cx^4+Dx^6)}{x^{14}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(3/2)*(D*x^6+C*x^4+B*x^2+A)/x^14,x, algorithm="giac")
```

output
```
4/45045*(45045*(sqrt(b)*x - sqrt(b*x^2 + a))^22*D*b^(7/2) - 225225*(sqrt(b
)*x - sqrt(b*x^2 + a))^20*D*a*b^(7/2) + 120120*(sqrt(b)*x - sqrt(b*x^2 + a
))^20*C*b^(9/2) + 495495*(sqrt(b)*x - sqrt(b*x^2 + a))^18*D*a^2*b^(7/2) -
300300*(sqrt(b)*x - sqrt(b*x^2 + a))^18*C*a*b^(9/2) + 360360*(sqrt(b)*x -
sqrt(b*x^2 + a))^18*B*b^(11/2) - 747747*(sqrt(b)*x - sqrt(b*x^2 + a))^16*D
*a^3*b^(7/2) + 252252*(sqrt(b)*x - sqrt(b*x^2 + a))^16*C*a^2*b^(9/2) - 720
72*(sqrt(b)*x - sqrt(b*x^2 + a))^16*B*a*b^(11/2) + 1153152*(sqrt(b)*x - sq
rt(b*x^2 + a))^16*A*b^(13/2) + 1027026*(sqrt(b)*x - sqrt(b*x^2 + a))^14*D*
a^4*b^(7/2) - 336336*(sqrt(b)*x - sqrt(b*x^2 + a))^14*C*a^3*b^(9/2) - 1441
44*(sqrt(b)*x - sqrt(b*x^2 + a))^14*B*a^2*b^(11/2) + 2306304*(sqrt(b)*x -
sqrt(b*x^2 + a))^14*A*a*b^(13/2) - 1181466*(sqrt(b)*x - sqrt(b*x^2 + a))^1
2*D*a^5*b^(7/2) + 645216*(sqrt(b)*x - sqrt(b*x^2 + a))^12*C*a^4*b^(9/2) -
679536*(sqrt(b)*x - sqrt(b*x^2 + a))^12*B*a^3*b^(11/2) + 2635776*(sqrt(b)*
x - sqrt(b*x^2 + a))^12*A*a^2*b^(13/2) + 908622*(sqrt(b)*x - sqrt(b*x^2 +
a))^10*D*a^6*b^(7/2) - 483912*(sqrt(b)*x - sqrt(b*x^2 + a))^10*C*a^5*b^(9/
2) + 329472*(sqrt(b)*x - sqrt(b*x^2 + a))^10*B*a^4*b^(11/2) + 906048*(sqrt
(b)*x - sqrt(b*x^2 + a))^10*A*a^3*b^(13/2) - 424710*(sqrt(b)*x - sqrt(b*x^
2 + a))^8*D*a^7*b^(7/2) + 108680*(sqrt(b)*x - sqrt(b*x^2 + a))^8*C*a^6*b^(
9/2) + 137280*(sqrt(b)*x - sqrt(b*x^2 + a))^8*B*a^5*b^(11/2) + 137280*(sqr
t(b)*x - sqrt(b*x^2 + a))^8*A*a^4*b^(13/2) + 142857*(sqrt(b)*x - sqrt(b...
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2+Cx^4+Dx^6)}{x^{14}}\,dx=\int \frac{(b\,x^2+a)^{3/2}\,(A+B\,x^2+C\,x^4+x^6\,D)}{x^{14}}\,dx$$

input
```
int(((a + b*x^2)^(3/2)*(A + B*x^2 + C*x^4 + x^6*D))/x^14,x)
```

output
```
int(((a + b*x^2)^(3/2)*(A + B*x^2 + C*x^4 + x^6*D))/x^14, x)
```

## Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.74

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2+Cx^4+Dx^6)}{x^{14}}\,dx = \frac{-3465\sqrt{b\,x^2+a}\,a^6-8505\sqrt{b\,x^2+a}\,a^5b\,x^2-5005\sqrt{b\,x^2+a}}{}$$

input
```
int((b*x^2+a)^(3/2)*(D*x^6+C*x^4+B*x^2+A)/x^14,x)
```

output
```
( - 3465*sqrt(a + b*x**2)*a**6 - 8505*sqrt(a + b*x**2)*a**5*b*x**2 - 5005*
sqrt(a + b*x**2)*a**5*c*x**4 - 6435*sqrt(a + b*x**2)*a**5*d*x**6 - 5565*sq
rt(a + b*x**2)*a**4*b**2*x**4 - 7150*sqrt(a + b*x**2)*a**4*b*c*x**6 - 1029
6*sqrt(a + b*x**2)*a**4*b*d*x**8 - 75*sqrt(a + b*x**2)*a**3*b**3*x**6 - 42
9*sqrt(a + b*x**2)*a**3*b**2*c*x**8 - 1287*sqrt(a + b*x**2)*a**3*b**2*d*x*
*10 + 90*sqrt(a + b*x**2)*a**2*b**4*x**8 + 572*sqrt(a + b*x**2)*a**2*b**3*
c*x**10 + 2574*sqrt(a + b*x**2)*a**2*b**3*d*x**12 - 120*sqrt(a + b*x**2)*a
*b**5*x**10 - 1144*sqrt(a + b*x**2)*a*b**4*c*x**12 + 240*sqrt(a + b*x**2)*
b**6*x**12 - 2574*sqrt(b)*a**2*b**3*d*x**13 + 1144*sqrt(b)*a*b**4*c*x**13
- 240*sqrt(b)*b**6*x**13)/(45045*a**4*x**13)
```

# 3.207 $\int \frac{(a+bx^2)^{3/2}(A+Bx^2+Cx^4+Dx^6)}{x^{16}}\,dx$



## Optimal result

Integrand size = 32, antiderivative size = 243

$$\begin{aligned}\int \frac{(a+bx^2)^{3/2}(A+Bx^2+Cx^4+Dx^6)}{x^{16}}\,dx &= -\frac{A(a+bx^2)^{5/2}}{15ax^{15}} \\ &+ \frac{(2Ab-3aB)(a+bx^2)^{5/2}}{39a^2x^{13}} - \frac{(16Ab^2-24abB+39a^2C)(a+bx^2)^{5/2}}{429a^3x^{11}} \\ &+ \frac{(32Ab^3-a(48b^2B-78abC+143a^2D))(a+bx^2)^{5/2}}{1287a^4x^9} \\ &- \frac{4b(32Ab^3-a(48b^2B-78abC+143a^2D))(a+bx^2)^{5/2}}{9009a^5x^7} \\ &+ \frac{8b^2(32Ab^3-a(48b^2B-78abC+143a^2D))(a+bx^2)^{5/2}}{45045a^6x^5}\end{aligned}$$

output
```
-1/15*A*(b*x^2+a)^(5/2)/a/x^15+1/39*(2*A*b-3*B*a)*(b*x^2+a)^(5/2)/a^2/x^13
-1/429*(16*A*b^2-24*B*a*b+39*C*a^2)*(b*x^2+a)^(5/2)/a^3/x^11+1/1287*(32*A*
b^3-a*(48*B*b^2-78*C*a*b+143*D*a^2))*(b*x^2+a)^(5/2)/a^4/x^9-4/9009*b*(32*
A*b^3-a*(48*B*b^2-78*C*a*b+143*D*a^2))*(b*x^2+a)^(5/2)/a^5/x^7+8/45045*b^2
*(32*A*b^3-a*(48*B*b^2-78*C*a*b+143*D*a^2))*(b*x^2+a)^(5/2)/a^6/x^5
```

## Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.69

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2+Cx^4+Dx^6)}{x^{16}}\,dx = \frac{(a+bx^2)^{5/2}\left(256Ab^5x^{10}-128ab^4x^8(5A+3Bx^2)+16a^2b^3x\right.}{}$$

input
```
Integrate[((a + b*x^2)^(3/2)*(A + B*x^2 + C*x^4 + D*x^6))/x^16,x]
```

output
```
((a + b*x^2)^(5/2)*(256*A*b^5*x^10 - 128*a*b^4*x^8*(5*A + 3*B*x^2) + 16*a^
2*b^3*x^6*(70*A + 60*B*x^2 + 39*C*x^4) - 8*a^3*b^2*x^4*(210*A + 210*B*x^2
+ 195*C*x^4 + 143*D*x^6) + 10*a^4*b*x^2*(231*A + 252*B*x^2 + 273*C*x^4 + 2
86*D*x^6) - 7*a^5*(429*A + 495*B*x^2 + 585*C*x^4 + 715*D*x^6)))/(45045*a^6
*x^15)
```

## Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.89, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2334, 27, 2089, 1588, 359, 245, 245, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2+Cx^4+Dx^6)}{x^{16}}\,dx$$

$$\Big\downarrow \text{2334}$$

$$-\frac{\int \frac{5(bx^2+a)^{3/2}(2Ab-3a(Dx^4+Cx^2+B))}{x^{14}}dx}{15a}-\frac{A(a+bx^2)^{5/2}}{15ax^{15}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{(bx^2+a)^{3/2}(2Ab-3a(Dx^4+Cx^2+B))}{x^{14}}dx}{3a}-\frac{A(a+bx^2)^{5/2}}{15ax^{15}}$$

$$\Big\downarrow \text{2089}$$

$$-\frac{\int \frac{(bx^2+a)^{3/2}(-3aDx^4-3aCx^2+2Ab-3aB)}{x^{14}}dx}{3a}-\frac{A(a+bx^2)^{5/2}}{15ax^{15}}$$

$\downarrow$ 1588

$$-\frac{-\frac{\int \frac{(bx^2+a)^{3/2}(39Dx^2a^2+39Ca^2-24bBa+16Ab^2)}{x^{12}}dx}{13a}-\frac{(a+bx^2)^{5/2}(2Ab-3aB)}{13ax^{13}}}{3a}-\frac{A(a+bx^2)^{5/2}}{15ax^{15}}$$

$\downarrow$ 359

$$-\frac{-\frac{-\frac{3\left(32Ab^3-a\left(143a^2D-78abC+48b^2B\right)\right)\int \frac{(bx^2+a)^{3/2}}{x^{10}}dx}{11a}-\frac{(a+bx^2)^{5/2}(39a^2C-24abB+16Ab^2)}{11ax^{11}}}{13a}-\frac{(a+bx^2)^{5/2}(2Ab-3aB)}{13ax^{13}}}{3a}-\frac{A(a+bx^2)^{5/2}}{15ax^{15}}$$

$\downarrow$ 245

$$-\frac{-\frac{-\frac{3\left(32Ab^3-a\left(143a^2D-78abC+48b^2B\right)\right)\left(-\frac{4b\int \frac{(bx^2+a)^{3/2}}{x^8}dx}{9a}-\frac{(a+bx^2)^{5/2}}{9ax^9}\right)}{11a}-\frac{(a+bx^2)^{5/2}(39a^2C-24abB+16Ab^2)}{11ax^{11}}}{13a}-\frac{(a+bx^2)^{5/2}(2Ab-3aB}{13ax^{13}}}{3a}$$

$$\frac{A(a+bx^2)^{5/2}}{15ax^{15}}$$

$\downarrow$ 245

$$-\frac{-\frac{-\frac{3\left(32Ab^3-a\left(143a^2D-78abC+48b^2B\right)\right)\left(-\frac{4b\left(-\frac{2b\int \frac{(bx^2+a)^{3/2}}{x^6}dx}{7a}-\frac{(a+bx^2)^{5/2}}{7ax^7}\right)}{9a}-\frac{(a+bx^2)^{5/2}}{9ax^9}\right)}{11a}-\frac{(a+bx^2)^{5/2}(39a^2C-24abB+16Ab^2)}{11ax^{11}}}{13a}-(}{3a}$$

$$\frac{A(a+bx^2)^{5/2}}{15ax^{15}}$$

$\downarrow$ 242

$$-\frac{-\frac{-\frac{\left(a+bx^2\right)^{5/2}\left(39a^2C-24abB+16Ab^2\right)}{11ax^{11}}-\frac{3\left(-\frac{4b\left(\frac{2b\left(a+bx^2\right)^{5/2}}{35a^2x^5}-\frac{\left(a+bx^2\right)^{5/2}}{7ax^7}\right)}{9a}-\frac{\left(a+bx^2\right)^{5/2}}{9ax^9}\right)\left(32Ab^3-a\left(143a^2D-78abC+48b^2B\right)\right)}{11a}}{13a}-\frac{\left(a+bx^2\right.}{}}{3a}$$

$$\frac{A\left(a+bx^2\right)^{5/2}}{15ax^{15}}$$

input
```
Int[((a + b*x^2)^(3/2)*(A + B*x^2 + C*x^4 + D*x^6))/x^16,x]
```

output
```
-1/15*(A*(a + b*x^2)^(5/2))/(a*x^15) - (-1/13*((2*A*b - 3*a*B)*(a + b*x^2)
^(5/2))/(a*x^13) - (-1/11*((16*A*b^2 - 24*a*b*B + 39*a^2*C)*(a + b*x^2)^(5
/2))/(a*x^11) - (3*(32*A*b^3 - a*(48*b^2*B - 78*a*b*C + 143*a^2*D))*(-1/9*
(a + b*x^2)^(5/2)/(a*x^9) - (4*b*(-1/7*(a + b*x^2)^(5/2)/(a*x^7) + (2*b*(a
 + b*x^2)^(5/2))/(35*a^2*x^5)))/(9*a)))/(11*a))/(13*a))/(3*a)
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 1588
```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Simp[1/(d*f
^2*(m + 1))   Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x
) - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

rule 2089
```
Int[(u_)^(p_.)*((f_.)*(x_))^(m_.)*(z_)^(q_.), x_Symbol] :> Int[(f*x)^m*Expa
ndToSum[z, x]^q*ExpandToSum[u, x]^p, x] /; FreeQ[{f, m, p, q}, x] && Binomi
alQ[z, x] && TrinomialQ[u, x] &&  !(BinomialMatchQ[z, x] && TrinomialMatchQ
[u, x])
```

rule 2334
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coef
f[Pq, x, 0], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[A*
x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))   Int[
x^(m + 2)*(a + b*x^2)^p*(a*(m + 1)*Q - A*b*(m + 2*(p + 1) + 1)), x], x]] /;
 FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && IntegerQ[m/2] && ILtQ[(m + 1)/2 + p,
 0] && LtQ[m + Expon[Pq, x] + 2*p + 1, 0]
```

## Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.63

| method | result |
|---|---|
| pseudoelliptic | $-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}\left(\left(\left(\frac{5}{3}Dx^{6}+\frac{15}{11}C\,x^{4}+\frac{15}{13}x^{2}B+A\right)a^{5}-\frac{10\left(\frac{26}{21}Dx^{6}+\frac{13}{11}C\,x^{4}+\frac{12}{11}x^{2}B+A\right)x^{2}b\,a^{4}}{13}+\frac{80x^{4}b^{2}\left(\frac{143}{210}Dx^{6}+\frac{13}{14}C\,x^{4}+x^{2}B+A\right.}{143}\right.}{15x^{15}a^{6}}$ |
| gosper | $-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}\left(-256A\,b^{5}x^{10}+384Ba\,b^{4}x^{10}-624C\,a^{2}b^{3}x^{10}+1144Da^{3}b^{2}x^{10}+640aA\,b^{4}x^{8}-960B\,a^{2}b^{3}x^{8}+1560C\,a^{3}b^{2}x^{8}-2\right.}{}$ |
| orering | $-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}\left(-256A\,b^{5}x^{10}+384Ba\,b^{4}x^{10}-624C\,a^{2}b^{3}x^{10}+1144Da^{3}b^{2}x^{10}+640aA\,b^{4}x^{8}-960B\,a^{2}b^{3}x^{8}+1560C\,a^{3}b^{2}x^{8}-2\right.}{}$ |
| trager | $-\frac{\left(-256A\,b^{7}x^{14}+384Ba\,b^{6}x^{14}-624C\,a^{2}b^{5}x^{14}+1144Da^{3}b^{4}x^{14}+128Aa\,b^{6}x^{12}-192B\,a^{2}b^{5}x^{12}+312C\,a^{3}b^{4}x^{12}-572Da^{4}b^{3}\right.}{}$ |
| default | $A\left(-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{15a\,x^{15}}-\frac{2b\left(-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{13a\,x^{13}}-\frac{8b\left(-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{11a\,x^{11}}-\frac{6b\left(-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{9a\,x^{9}}-\frac{4b\left(-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{7a\,x^{7}}+\frac{2b\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{35a^{2}x^{5}}\right)}{9a}\right)}{11a}\right)}{13a}\right)}{3a}\right)$ |

input
```
int((b*x^2+a)^(3/2)*(D*x^6+C*x^4+B*x^2+A)/x^16,x,method=_RETURNVERBOSE)
```

output
```
-1/15*(b*x^2+a)^(5/2)*((5/3*D*x^6+15/11*C*x^4+15/13*x^2*B+A)*a^5-10/13*(26
/21*D*x^6+13/11*C*x^4+12/11*x^2*B+A)*x^2*b*a^4+80/143*x^4*b^2*(143/210*D*x
^6+13/14*C*x^4+x^2*B+A)*a^3-160/429*(39/70*C*x^4+6/7*x^2*B+A)*x^6*b^3*a^2+
640/3003*(3/5*x^2*B+A)*x^8*b^4*a-256/3003*A*b^5*x^10)/x^15/a^6
```

## Fricas [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2+Cx^4+Dx^6)}{x^{16}}\,dx = -\frac{(8\,(143\,Da^3b^4-78\,Ca^2b^5+48\,Bab^6-32\,Ab^7)x^{14}-4\,(143\,Da^4b^3-78\,Ca^3b^4+48\,Ba^2b^5-32\,Aab^6)x^{12}}{}$$

input
```
integrate((b*x^2+a)^(3/2)*(D*x^6+C*x^4+B*x^2+A)/x^16,x, algorithm="fricas"
)
```

output
```
-1/45045*(8*(143*D*a^3*b^4 - 78*C*a^2*b^5 + 48*B*a*b^6 - 32*A*b^7)*x^14 -
4*(143*D*a^4*b^3 - 78*C*a^3*b^4 + 48*B*a^2*b^5 - 32*A*a*b^6)*x^12 + 3*(143
*D*a^5*b^2 - 78*C*a^4*b^3 + 48*B*a^3*b^4 - 32*A*a^2*b^5)*x^10 + 5*(1430*D*
a^6*b + 39*C*a^5*b^2 - 24*B*a^4*b^3 + 16*A*a^3*b^4)*x^8 + 3003*A*a^7 + 35*
(143*D*a^7 + 156*C*a^6*b + 3*B*a^5*b^2 - 2*A*a^4*b^3)*x^6 + 63*(65*C*a^7 +
 70*B*a^6*b + A*a^5*b^2)*x^4 + 231*(15*B*a^7 + 16*A*a^6*b)*x^2)*sqrt(b*x^2
 + a)/(a^6*x^15)
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7166 vs. 2(240) = 480.

Time = 9.36 (sec) , antiderivative size = 7166, normalized size of antiderivative = 29.49

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2+Cx^4+Dx^6)}{x^{16}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x**2+a)**(3/2)*(D*x**6+C*x**4+B*x**2+A)/x**16,x)
```

output
```
-3003*A*a**14*b**(73/2)*sqrt(a/(b*x**2) + 1)/(45045*a**13*b**36*x**14 + 27
0270*a**12*b**37*x**16 + 675675*a**11*b**38*x**18 + 900900*a**10*b**39*x**
20 + 675675*a**9*b**40*x**22 + 270270*a**8*b**41*x**24 + 45045*a**7*b**42*
x**26) - 18249*A*a**13*b**(75/2)*x**2*sqrt(a/(b*x**2) + 1)/(45045*a**13*b*
*36*x**14 + 270270*a**12*b**37*x**16 + 675675*a**11*b**38*x**18 + 900900*a
**10*b**39*x**20 + 675675*a**9*b**40*x**22 + 270270*a**8*b**41*x**24 + 450
45*a**7*b**42*x**26) - 46179*A*a**12*b**(77/2)*x**4*sqrt(a/(b*x**2) + 1)/(
45045*a**13*b**36*x**14 + 270270*a**12*b**37*x**16 + 675675*a**11*b**38*x*
*18 + 900900*a**10*b**39*x**20 + 675675*a**9*b**40*x**22 + 270270*a**8*b**
41*x**24 + 45045*a**7*b**42*x**26) - 62293*A*a**11*b**(79/2)*x**6*sqrt(a/(
b*x**2) + 1)/(45045*a**13*b**36*x**14 + 270270*a**12*b**37*x**16 + 675675*
a**11*b**38*x**18 + 900900*a**10*b**39*x**20 + 675675*a**9*b**40*x**22 + 2
70270*a**8*b**41*x**24 + 45045*a**7*b**42*x**26) - 693*A*a**11*b**(53/2)*s
qrt(a/(b*x**2) + 1)/(9009*a**11*b**25*x**12 + 45045*a**10*b**26*x**14 + 90
090*a**9*b**27*x**16 + 90090*a**8*b**28*x**18 + 45045*a**7*b**29*x**20 + 9
009*a**6*b**30*x**22) - 47245*A*a**10*b**(81/2)*x**8*sqrt(a/(b*x**2) + 1)/
(45045*a**13*b**36*x**14 + 270270*a**12*b**37*x**16 + 675675*a**11*b**38*x
**18 + 900900*a**10*b**39*x**20 + 675675*a**9*b**40*x**22 + 270270*a**8*b*
*41*x**24 + 45045*a**7*b**42*x**26) - 3528*A*a**10*b**(55/2)*x**2*sqrt(a/(
b*x**2) + 1)/(9009*a**11*b**25*x**12 + 45045*a**10*b**26*x**14 + 90090*...
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.48

$$\int \frac{(a+bx^2)^{3/2}\left(A+Bx^2+Cx^4+Dx^6\right)}{x^{16}}\,dx = -\frac{8\,(bx^2+a)^{\frac{5}{2}}Db^2}{315\,a^3x^5} + \frac{16\,(bx^2+a)^{\frac{5}{2}}Cb^3}{1155\,a^4x^5} - \frac{128\,(bx^2+a)^{\frac{5}{2}}Bb^4}{15015\,a^5x^5} + \frac{256\,(bx^2+a)^{\frac{5}{2}}Ab^5}{45045\,a^6x^5} + \frac{4\,(bx^2+a)^{\frac{5}{2}}Db}{63\,a^2x^7} - \frac{8\,(bx^2+a)^{\frac{5}{2}}Cb^2}{231\,a^3x^7} + \frac{64\,(bx^2+a)^{\frac{5}{2}}Bb^3}{3003\,a^4x^7} - \frac{128\,(bx^2+a)^{\frac{5}{2}}Ab^4}{9009\,a^5x^7} - \frac{(bx^2+a)^{\frac{5}{2}}D}{9\,ax^9} + \frac{2\,(bx^2+a)^{\frac{5}{2}}Cb}{33\,a^2x^9} - \frac{16\,(bx^2+a)^{\frac{5}{2}}Bb^2}{429\,a^3x^9} + \frac{32\,(bx^2+a)^{\frac{5}{2}}Ab^3}{1287\,a^4x^9} - \frac{(bx^2+a)^{\frac{5}{2}}C}{11\,ax^{11}} + \frac{8\,(bx^2+a)^{\frac{5}{2}}Bb}{143\,a^2x^{11}} - \frac{16\,(bx^2+a)^{\frac{5}{2}}Ab^2}{429\,a^3x^{11}} - \frac{(bx^2+a)^{\frac{5}{2}}B}{13\,ax^{13}} + \frac{2\,(bx^2+a)^{\frac{5}{2}}Ab}{39\,a^2x^{13}} - \frac{(bx^2+a)^{\frac{5}{2}}A}{15\,ax^{15}}$$

input
```
integrate((b*x^2+a)^(3/2)*(D*x^6+C*x^4+B*x^2+A)/x^16,x, algorithm="maxima"
)
```

output
```
-8/315*(b*x^2 + a)^(5/2)*D*b^2/(a^3*x^5) + 16/1155*(b*x^2 + a)^(5/2)*C*b^3
/(a^4*x^5) - 128/15015*(b*x^2 + a)^(5/2)*B*b^4/(a^5*x^5) + 256/45045*(b*x^
2 + a)^(5/2)*A*b^5/(a^6*x^5) + 4/63*(b*x^2 + a)^(5/2)*D*b/(a^2*x^7) - 8/23
1*(b*x^2 + a)^(5/2)*C*b^2/(a^3*x^7) + 64/3003*(b*x^2 + a)^(5/2)*B*b^3/(a^4
*x^7) - 128/9009*(b*x^2 + a)^(5/2)*A*b^4/(a^5*x^7) - 1/9*(b*x^2 + a)^(5/2)
*D/(a*x^9) + 2/33*(b*x^2 + a)^(5/2)*C*b/(a^2*x^9) - 16/429*(b*x^2 + a)^(5/
2)*B*b^2/(a^3*x^9) + 32/1287*(b*x^2 + a)^(5/2)*A*b^3/(a^4*x^9) - 1/11*(b*x
^2 + a)^(5/2)*C/(a*x^11) + 8/143*(b*x^2 + a)^(5/2)*B*b/(a^2*x^11) - 16/429
*(b*x^2 + a)^(5/2)*A*b^2/(a^3*x^11) - 1/13*(b*x^2 + a)^(5/2)*B/(a*x^13) +
2/39*(b*x^2 + a)^(5/2)*A*b/(a^2*x^13) - 1/15*(b*x^2 + a)^(5/2)*A/(a*x^15)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1194 vs. $2(219) = 438$.

Time = 0.17 (sec) , antiderivative size = 1194, normalized size of antiderivative = 4.91

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2+Cx^4+Dx^6)}{x^{16}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(3/2)*(D*x^6+C*x^4+B*x^2+A)/x^16,x, algorithm="giac")
```

output
```
16/45045*(30030*(sqrt(b)*x - sqrt(b*x^2 + a))^24*D*b^(9/2) - 135135*(sqrt(
b)*x - sqrt(b*x^2 + a))^22*D*a*b^(9/2) + 90090*(sqrt(b)*x - sqrt(b*x^2 + a
))^22*C*b^(11/2) + 243243*(sqrt(b)*x - sqrt(b*x^2 + a))^20*D*a^2*b^(9/2) -
 198198*(sqrt(b)*x - sqrt(b*x^2 + a))^20*C*a*b^(11/2) + 288288*(sqrt(b)*x
- sqrt(b*x^2 + a))^20*B*b^(13/2) - 285285*(sqrt(b)*x - sqrt(b*x^2 + a))^18
*D*a^3*b^(9/2) + 90090*(sqrt(b)*x - sqrt(b*x^2 + a))^18*C*a^2*b^(11/2) + 9
60960*(sqrt(b)*x - sqrt(b*x^2 + a))^18*A*b^(15/2) + 392535*(sqrt(b)*x - sq
rt(b*x^2 + a))^16*D*a^4*b^(9/2) - 115830*(sqrt(b)*x - sqrt(b*x^2 + a))^16*
C*a^3*b^(11/2) - 205920*(sqrt(b)*x - sqrt(b*x^2 + a))^16*B*a^2*b^(13/2) +
2059200*(sqrt(b)*x - sqrt(b*x^2 + a))^16*A*a*b^(15/2) - 527670*(sqrt(b)*x
- sqrt(b*x^2 + a))^14*D*a^5*b^(9/2) + 386100*(sqrt(b)*x - sqrt(b*x^2 + a))
^14*C*a^4*b^(11/2) - 514800*(sqrt(b)*x - sqrt(b*x^2 + a))^14*B*a^3*b^(13/2
) + 2265120*(sqrt(b)*x - sqrt(b*x^2 + a))^14*A*a^2*b^(15/2) + 430430*(sqrt
(b)*x - sqrt(b*x^2 + a))^12*D*a^6*b^(9/2) - 300300*(sqrt(b)*x - sqrt(b*x^2
 + a))^12*C*a^5*b^(11/2) + 240240*(sqrt(b)*x - sqrt(b*x^2 + a))^12*B*a^4*b
^(13/2) + 800800*(sqrt(b)*x - sqrt(b*x^2 + a))^12*A*a^3*b^(15/2) - 186186*
(sqrt(b)*x - sqrt(b*x^2 + a))^10*D*a^7*b^(9/2) + 36036*(sqrt(b)*x - sqrt(b
*x^2 + a))^10*C*a^6*b^(11/2) + 144144*(sqrt(b)*x - sqrt(b*x^2 + a))^10*B*a
^5*b^(13/2) + 96096*(sqrt(b)*x - sqrt(b*x^2 + a))^10*A*a^4*b^(15/2) + 6006
0*(sqrt(b)*x - sqrt(b*x^2 + a))^8*D*a^8*b^(9/2) - 16380*(sqrt(b)*x - sq...
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2+Cx^4+Dx^6)}{x^{16}}\,dx = \int \frac{(b\,x^2+a)^{3/2}\,(A+B\,x^2+C\,x^4+x^6\,D)}{x^{16}}\,dx$$

input
```
int(((a + b*x^2)^(3/2)*(A + B*x^2 + C*x^4 + x^6*D))/x^16,x)
```

output
```
int(((a + b*x^2)^(3/2)*(A + B*x^2 + C*x^4 + x^6*D))/x^16, x)
```

## Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 392, normalized size of antiderivative = 1.61

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2+Cx^4+Dx^6)}{x^{16}}\,dx = \frac{-3003\sqrt{b\,x^2+a}\,a^7-7161\sqrt{b\,x^2+a}\,a^6b\,x^2-4095\sqrt{b\,x^2+}}{}$$

input
```
int((b*x^2+a)^(3/2)*(D*x^6+C*x^4+B*x^2+A)/x^16,x)
```

output
```
( - 3003*sqrt(a + b*x**2)*a**7 - 7161*sqrt(a + b*x**2)*a**6*b*x**2 - 4095*
sqrt(a + b*x**2)*a**6*c*x**4 - 5005*sqrt(a + b*x**2)*a**6*d*x**6 - 4473*sq
rt(a + b*x**2)*a**5*b**2*x**4 - 5460*sqrt(a + b*x**2)*a**5*b*c*x**6 - 7150
*sqrt(a + b*x**2)*a**5*b*d*x**8 - 35*sqrt(a + b*x**2)*a**4*b**3*x**6 - 195
*sqrt(a + b*x**2)*a**4*b**2*c*x**8 - 429*sqrt(a + b*x**2)*a**4*b**2*d*x**1
0 + 40*sqrt(a + b*x**2)*a**3*b**4*x**8 + 234*sqrt(a + b*x**2)*a**3*b**3*c*
x**10 + 572*sqrt(a + b*x**2)*a**3*b**3*d*x**12 - 48*sqrt(a + b*x**2)*a**2*
b**5*x**10 - 312*sqrt(a + b*x**2)*a**2*b**4*c*x**12 - 1144*sqrt(a + b*x**2
)*a**2*b**4*d*x**14 + 64*sqrt(a + b*x**2)*a*b**6*x**12 + 624*sqrt(a + b*x*
*2)*a*b**5*c*x**14 - 128*sqrt(a + b*x**2)*b**7*x**14 + 1144*sqrt(b)*a**2*b
**4*d*x**15 - 624*sqrt(b)*a*b**5*c*x**15 + 128*sqrt(b)*b**7*x**15)/(45045*
a**5*x**15)
```

# 3.208 $\int \frac{x^5(A+Bx^2+Cx^4+Dx^6)}{\sqrt{a+bx^2}}\,dx$



## Optimal result

Integrand size = 32, antiderivative size = 216

$$\int \frac{x^5(A+Bx^2+Cx^4+Dx^6)}{\sqrt{a+bx^2}}\,dx = \frac{a^2(Ab^3-a(b^2B-abC+a^2D))\sqrt{a+bx^2}}{b^6} - \frac{a(2Ab^3-a(3b^2B-4abC+5a^2D))\left(a+bx^2\right)^{3/2}}{3b^6} + \frac{(Ab^3-a(3b^2B-6abC+10a^2D))\left(a+bx^2\right)^{5/2}}{5b^6} + \frac{(b^2B-4abC+10a^2D)\left(a+bx^2\right)^{7/2}}{7b^6} + \frac{(bC-5aD)\left(a+bx^2\right)^{9/2}}{9b^6} + \frac{D\left(a+bx^2\right)^{11/2}}{11b^6}$$

output
```
a^2*(A*b^3-a*(B*b^2-C*a*b+D*a^2))*(b*x^2+a)^(1/2)/b^6-1/3*a*(2*A*b^3-a*(3*
B*b^2-4*C*a*b+5*D*a^2))*(b*x^2+a)^(3/2)/b^6+1/5*(A*b^3-a*(3*B*b^2-6*C*a*b+
10*D*a^2))*(b*x^2+a)^(5/2)/b^6+1/7*(B*b^2-4*C*a*b+10*D*a^2)*(b*x^2+a)^(7/2
)/b^6+1/9*(C*b-5*D*a)*(b*x^2+a)^(9/2)/b^6+1/11*D*(b*x^2+a)^(11/2)/b^6
```

## Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.73

$$\int \frac{x^5(A+Bx^2+Cx^4+Dx^6)}{\sqrt{a+bx^2}}\,dx$$
$$= \frac{\sqrt{a+bx^2}(-1280a^5D+128a^4b(11C+5Dx^2)-16a^3b^2(99B+44Cx^2+30Dx^4)+8a^2b^3(231A+99Bx^2$$

input
```
Integrate[(x^5*(A + B*x^2 + C*x^4 + D*x^6))/Sqrt[a + b*x^2],x]
```

output
```
(Sqrt[a + b*x^2]*(-1280*a^5*D + 128*a^4*b*(11*C + 5*D*x^2) - 16*a^3*b^2*(9
9*B + 44*C*x^2 + 30*D*x^4) + 8*a^2*b^3*(231*A + 99*B*x^2 + 66*C*x^4 + 50*D
*x^6) - 2*a*b^4*x^2*(462*A + 297*B*x^2 + 220*C*x^4 + 175*D*x^6) + b^5*x^4*
(693*A + 5*(99*B*x^2 + 77*C*x^4 + 63*D*x^6))))/(3465*b^6)
```

## Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {2331, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5\left(A+Bx^2+Cx^4+Dx^6\right)}{\sqrt{a+bx^2}}\,dx$$
$$\Big\downarrow \ 2331$$
$$\frac{1}{2}\int \frac{x^4\left(Dx^6+Cx^4+Bx^2+A\right)}{\sqrt{bx^2+a}}dx^2$$
$$\Big\downarrow \ 2123$$
$$\frac{1}{2}\int \left(\frac{D\left(bx^2+a\right)^{9/2}}{b^5}+\frac{(bC-5aD)\left(bx^2+a\right)^{7/2}}{b^5}+\frac{\left(10Da^2-4bCa+b^2B\right)\left(bx^2+a\right)^{5/2}}{b^5}+\frac{\left(Ab^3-a\left(10Da^2-\right.\right.}{}$$
$$\Big\downarrow \ 2009$$

$$\frac{1}{2}\left(\frac{2\left(a+bx^2\right)^{5/2}\left(Ab^3-a\left(10a^2D-6abC+3b^2B\right)\right)}{5b^6}-\frac{2a\left(a+bx^2\right)^{3/2}\left(2Ab^3-a\left(5a^2D-4abC+3b^2B\right)\right)}{3b^6}+\frac{2a\right.$$

input
```
Int[(x^5*(A + B*x^2 + C*x^4 + D*x^6))/Sqrt[a + b*x^2],x]
```

output
```
((2*a^2*(A*b^3 - a*(b^2*B - a*b*C + a^2*D))*Sqrt[a + b*x^2])/b^6 - (2*a*(2
*A*b^3 - a*(3*b^2*B - 4*a*b*C + 5*a^2*D))*(a + b*x^2)^(3/2))/(3*b^6) + (2*
(A*b^3 - a*(3*b^2*B - 6*a*b*C + 10*a^2*D))*(a + b*x^2)^(5/2))/(5*b^6) + (2
*(b^2*B - 4*a*b*C + 10*a^2*D)*(a + b*x^2)^(7/2))/(7*b^6) + (2*(b*C - 5*a*D
)*(a + b*x^2)^(9/2))/(9*b^6) + (2*D*(a + b*x^2)^(11/2))/(11*b^6))/2
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

rule 2331
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/2   S
ubst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
 FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

## Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.66

| method | result |
|---|---|
| pseudoelliptic | 8( 3(5/11 D x^6 + 5/9 C x^4 + 5/7 x^2 B + A) x^4 b^5 / 8 − (25/66 D x^6 + 10/21 C x^4 + 9/14 x^2 B + A) x^2 a b^4 / 2 + a^2 (50/231 D x^6 + 2/7 C x^4 + 3/7 x^2 B + A) b^3 − 6(10/33 D x^4 … ) / (15 b^6) |
| gosper | sqrt(b x^2 + a) (315 D x^10 b^5 + 385 C b^5 x^8 − 350 D a b^4 x^8 + 495 B b^5 x^6 − 440 C a b^4 x^6 + 400 D a^2 b^3 x^6 + 693 A b^5 x^4 − 594 B a b^4 x^4 + 528 C … |
| trager | sqrt(b x^2 + a) (315 D x^10 b^5 + 385 C b^5 x^8 − 350 D a b^4 x^8 + 495 B b^5 x^6 − 440 C a b^4 x^6 + 400 D a^2 b^3 x^6 + 693 A b^5 x^4 − 594 B a b^4 x^4 + 528 C … |
| orering | sqrt(b x^2 + a) (315 D x^10 b^5 + 385 C b^5 x^8 − 350 D a b^4 x^8 + 495 B b^5 x^6 − 440 C a b^4 x^6 + 400 D a^2 b^3 x^6 + 693 A b^5 x^4 − 594 B a b^4 x^4 + 528 C … |
| default | A( x^4 sqrt(b x^2 + a)/(5b) − 4a( x^2 sqrt(b x^2 + a)/(3b) − 2a sqrt(b x^2 + a)/(3b^2) )/(5b) ) + B( x^6 sqrt(b x^2 + a)/(7b) − 6a( x^4 sqrt(b x^2 + a)/(5b) − 4a( x^2 sqrt(b x^2 + a)/(3b) − 2a sqrt(b x … |

input
```
int(x^5*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
8/15*(3/8*(5/11*D*x^6+5/9*C*x^4+5/7*x^2*B+A)*x^4*b^5-1/2*(25/66*D*x^6+10/2
1*C*x^4+9/14*x^2*B+A)*x^2*a*b^4+a^2*(50/231*D*x^6+2/7*C*x^4+3/7*x^2*B+A)*b
^3-6/7*(10/33*D*x^4+4/9*C*x^2+B)*a^3*b^2+16/21*(5/11*D*x^2+C)*a^4*b-160/23
1*D*a^5)*(b*x^2+a)^(1/2)/b^6
```

## Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.82

$$\int \frac{x^5(A+Bx^2+Cx^4+Dx^6)}{\sqrt{a+bx^2}}\,dx$$
$$= \frac{(315\,Db^5x^{10} - 35\,(10\,Dab^4 - 11\,Cb^5)x^8 + 5\,(80\,Da^2b^3 - 88\,Cab^4 + 99\,Bb^5)x^6 - 1280\,Da^5 + 1408\,Ca^4b$$

input
```
integrate(x^5*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
1/3465*(315*D*b^5*x^10 - 35*(10*D*a*b^4 - 11*C*b^5)*x^8 + 5*(80*D*a^2*b^3
- 88*C*a*b^4 + 99*B*b^5)*x^6 - 1280*D*a^5 + 1408*C*a^4*b - 1584*B*a^3*b^2
+ 1848*A*a^2*b^3 - 3*(160*D*a^3*b^2 - 176*C*a^2*b^3 + 198*B*a*b^4 - 231*A*
b^5)*x^4 + 4*(160*D*a^4*b - 176*C*a^3*b^2 + 198*B*a^2*b^3 - 231*A*a*b^4)*x
^2)*sqrt(b*x^2 + a)/b^6
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 442 vs. 2(207) = 414.

Time = 0.43 (sec) , antiderivative size = 442, normalized size of antiderivative = 2.05

$$\int \frac{x^5(A+Bx^2+Cx^4+Dx^6)}{\sqrt{a+bx^2}}\,dx$$
$$= \begin{cases} \frac{8Aa^2\sqrt{a+bx^2}}{15b^3} - \frac{4Aax^2\sqrt{a+bx^2}}{15b^2} + \frac{Ax^4\sqrt{a+bx^2}}{5b} - \frac{16Ba^3\sqrt{a+bx^2}}{35b^4} + \frac{8Ba^2x^2\sqrt{a+bx^2}}{35b^3} - \frac{6Bax^4\sqrt{a+bx^2}}{35b^2} + \frac{Bx^6\sqrt{a+bx^2}}{7b} + \frac{128C \\ \frac{\frac{Ax^6}{6}+\frac{Bx^8}{8}+\frac{Cx^{10}}{10}+\frac{Dx^{12}}{12}}{\sqrt{a}} \end{cases}$$

input
```
integrate(x**5*(D*x**6+C*x**4+B*x**2+A)/(b*x**2+a)**(1/2),x)
```

output
```
Piecewise((8*A*a**2*sqrt(a + b*x**2)/(15*b**3) - 4*A*a*x**2*sqrt(a + b*x**
2)/(15*b**2) + A*x**4*sqrt(a + b*x**2)/(5*b) - 16*B*a**3*sqrt(a + b*x**2)/
(35*b**4) + 8*B*a**2*x**2*sqrt(a + b*x**2)/(35*b**3) - 6*B*a*x**4*sqrt(a +
 b*x**2)/(35*b**2) + B*x**6*sqrt(a + b*x**2)/(7*b) + 128*C*a**4*sqrt(a + b
*x**2)/(315*b**5) - 64*C*a**3*x**2*sqrt(a + b*x**2)/(315*b**4) + 16*C*a**2
*x**4*sqrt(a + b*x**2)/(105*b**3) - 8*C*a*x**6*sqrt(a + b*x**2)/(63*b**2)
+ C*x**8*sqrt(a + b*x**2)/(9*b) - 256*D*a**5*sqrt(a + b*x**2)/(693*b**6) +
 128*D*a**4*x**2*sqrt(a + b*x**2)/(693*b**5) - 32*D*a**3*x**4*sqrt(a + b*x
**2)/(231*b**4) + 80*D*a**2*x**6*sqrt(a + b*x**2)/(693*b**3) - 10*D*a*x**8
*sqrt(a + b*x**2)/(99*b**2) + D*x**10*sqrt(a + b*x**2)/(11*b), Ne(b, 0)),
((A*x**6/6 + B*x**8/8 + C*x**10/10 + D*x**12/12)/sqrt(a), True))
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.61

$$\begin{aligned}\int \frac{x^5(A+Bx^2+Cx^4+Dx^6)}{\sqrt{a+bx^2}}\,dx &= \frac{\sqrt{bx^2+a}Dx^{10}}{11\,b} - \frac{10\,\sqrt{bx^2+a}Dax^8}{99\,b^2} \\ &\quad + \frac{\sqrt{bx^2+a}Cx^8}{9\,b} + \frac{80\,\sqrt{bx^2+a}Da^2x^6}{693\,b^3} \\ &\quad - \frac{8\,\sqrt{bx^2+a}Cax^6}{63\,b^2} + \frac{\sqrt{bx^2+a}Bx^6}{7\,b} \\ &\quad - \frac{32\,\sqrt{bx^2+a}Da^3x^4}{231\,b^4} + \frac{16\,\sqrt{bx^2+a}Ca^2x^4}{105\,b^3} \\ &\quad - \frac{6\,\sqrt{bx^2+a}Bax^4}{35\,b^2} + \frac{\sqrt{bx^2+a}Ax^4}{5\,b} \\ &\quad + \frac{128\,\sqrt{bx^2+a}Da^4x^2}{693\,b^5} - \frac{64\,\sqrt{bx^2+a}Ca^3x^2}{315\,b^4} \\ &\quad + \frac{8\,\sqrt{bx^2+a}Ba^2x^2}{35\,b^3} - \frac{4\,\sqrt{bx^2+a}Aax^2}{15\,b^2} \\ &\quad - \frac{256\,\sqrt{bx^2+a}Da^5}{693\,b^6} + \frac{128\,\sqrt{bx^2+a}Ca^4}{315\,b^5} \\ &\quad - \frac{16\,\sqrt{bx^2+a}Ba^3}{35\,b^4} + \frac{8\,\sqrt{bx^2+a}Aa^2}{15\,b^3}\end{aligned}$$

input
```
integrate(x^5*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/11*sqrt(b*x^2 + a)*D*x^10/b - 10/99*sqrt(b*x^2 + a)*D*a*x^8/b^2 + 1/9*sq
rt(b*x^2 + a)*C*x^8/b + 80/693*sqrt(b*x^2 + a)*D*a^2*x^6/b^3 - 8/63*sqrt(b
*x^2 + a)*C*a*x^6/b^2 + 1/7*sqrt(b*x^2 + a)*B*x^6/b - 32/231*sqrt(b*x^2 +
a)*D*a^3*x^4/b^4 + 16/105*sqrt(b*x^2 + a)*C*a^2*x^4/b^3 - 6/35*sqrt(b*x^2
+ a)*B*a*x^4/b^2 + 1/5*sqrt(b*x^2 + a)*A*x^4/b + 128/693*sqrt(b*x^2 + a)*D
*a^4*x^2/b^5 - 64/315*sqrt(b*x^2 + a)*C*a^3*x^2/b^4 + 8/35*sqrt(b*x^2 + a)
*B*a^2*x^2/b^3 - 4/15*sqrt(b*x^2 + a)*A*a*x^2/b^2 - 256/693*sqrt(b*x^2 + a
)*D*a^5/b^6 + 128/315*sqrt(b*x^2 + a)*C*a^4/b^5 - 16/35*sqrt(b*x^2 + a)*B*
a^3/b^4 + 8/15*sqrt(b*x^2 + a)*A*a^2/b^3
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.20

$$\int \frac{x^5(A+Bx^2+Cx^4+Dx^6)}{\sqrt{a+bx^2}}\,dx = -\frac{(Da^5-Ca^4b+Ba^3b^2-Aa^2b^3)\sqrt{bx^2+a}}{b^6} + \frac{315\,(bx^2+a)^{\frac{11}{2}}D-1925\,(bx^2+a)^{\frac{9}{2}}Da+4950\,(bx^2+a)^{\frac{7}{2}}Da^2-6930\,(bx^2+a)^{\frac{5}{2}}Da^3+5775\,(bx^2+a}{}$$

input
```
integrate(x^5*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
-(D*a^5 - C*a^4*b + B*a^3*b^2 - A*a^2*b^3)*sqrt(b*x^2 + a)/b^6 + 1/3465*(3
15*(b*x^2 + a)^(11/2)*D - 1925*(b*x^2 + a)^(9/2)*D*a + 4950*(b*x^2 + a)^(7
/2)*D*a^2 - 6930*(b*x^2 + a)^(5/2)*D*a^3 + 5775*(b*x^2 + a)^(3/2)*D*a^4 +
385*(b*x^2 + a)^(9/2)*C*b - 1980*(b*x^2 + a)^(7/2)*C*a*b + 4158*(b*x^2 + a
)^(5/2)*C*a^2*b - 4620*(b*x^2 + a)^(3/2)*C*a^3*b + 495*(b*x^2 + a)^(7/2)*B
*b^2 - 2079*(b*x^2 + a)^(5/2)*B*a*b^2 + 3465*(b*x^2 + a)^(3/2)*B*a^2*b^2 +
 693*(b*x^2 + a)^(5/2)*A*b^3 - 2310*(b*x^2 + a)^(3/2)*A*a*b^3)/b^6
```

## Mupad [B] (verification not implemented)

Time = 2.04 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.13

$$\int \frac{x^5(A+Bx^2+Cx^4+Dx^6)}{\sqrt{a+bx^2}}\,dx$$
$$= \sqrt{b\,x^2+a}\left(\frac{128\,C\,a^4}{315\,b^5}+\frac{C\,x^8}{9\,b}-\frac{8\,C\,a\,x^6}{63\,b^2}+\frac{16\,C\,a^2\,x^4}{105\,b^3}-\frac{64\,C\,a^3\,x^2}{315\,b^4}\right)$$
$$-\sqrt{b\,x^2+a}\left(\frac{16\,B\,a^3}{35\,b^4}-\frac{B\,x^6}{7\,b}+\frac{6\,B\,a\,x^4}{35\,b^2}-\frac{8\,B\,a^2\,x^2}{35\,b^3}\right)$$
$$+\sqrt{b\,x^2+a}\left(\frac{8\,A\,a^2}{15\,b^3}+\frac{A\,x^4}{5\,b}-\frac{4\,A\,a\,x^2}{15\,b^2}\right)$$
$$+\frac{\frac{(b\,x^2+a)^{11/2}\,D}{11}-\frac{5\,a\,(b\,x^2+a)^{9/2}\,D}{9}-a^5\,\sqrt{b\,x^2+a}\,D+\frac{5\,a^4\,(b\,x^2+a)^{3/2}\,D}{3}-2\,a^3\,(b\,x^2+a)^{5/2}\,D+\frac{10\,a^2\,(b\,x^2+a)^{7/2}}{7}}{b^6}$$

input
```
int((x^5*(A + B*x^2 + C*x^4 + x^6*D))/(a + b*x^2)^(1/2),x)
```

output
```
(a + b*x^2)^(1/2)*((128*C*a^4)/(315*b^5) + (C*x^8)/(9*b) - (8*C*a*x^6)/(63
*b^2) + (16*C*a^2*x^4)/(105*b^3) - (64*C*a^3*x^2)/(315*b^4)) - (a + b*x^2)
^(1/2)*((16*B*a^3)/(35*b^4) - (B*x^6)/(7*b) + (6*B*a*x^4)/(35*b^2) - (8*B*
a^2*x^2)/(35*b^3)) + (a + b*x^2)^(1/2)*((8*A*a^2)/(15*b^3) + (A*x^4)/(5*b)
 - (4*A*a*x^2)/(15*b^2)) + (((a + b*x^2)^(11/2)*D)/11 - (5*a*(a + b*x^2)^(
9/2)*D)/9 - a^5*(a + b*x^2)^(1/2)*D + (5*a^4*(a + b*x^2)^(3/2)*D)/3 - 2*a^
3*(a + b*x^2)^(5/2)*D + (10*a^2*(a + b*x^2)^(7/2)*D)/7)/b^6
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.74

$$\int \frac{x^5(A+Bx^2+Cx^4+Dx^6)}{\sqrt{a+bx^2}}\,dx$$
$$= \frac{\sqrt{b\,x^2+a}\,(315b^5d\,x^{10}-350a\,b^4d\,x^8+385b^5c\,x^8+400a^2b^3d\,x^6-440a\,b^4c\,x^6+495b^6x^6-480a^3b^2d\,x^4+}{3465b}$$

input
```
int(x^5*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(1/2),x)
```

output

```
(sqrt(a + b*x**2)*( - 1280*a**5*d + 1408*a**4*b*c + 640*a**4*b*d*x**2 + 26
4*a**3*b**3 - 704*a**3*b**2*c*x**2 - 480*a**3*b**2*d*x**4 - 132*a**2*b**4*
x**2 + 528*a**2*b**3*c*x**4 + 400*a**2*b**3*d*x**6 + 99*a*b**5*x**4 - 440*
a*b**4*c*x**6 - 350*a*b**4*d*x**8 + 495*b**6*x**6 + 385*b**5*c*x**8 + 315*
b**5*d*x**10))/(3465*b**6)
```

# 3.209 $\int \frac{x^3(A+Bx^2+Cx^4+Dx^6)}{\sqrt{a+bx^2}}\,dx$



## Optimal result

Integrand size = 32, antiderivative size = 168

$$\int \frac{x^3(A+Bx^2+Cx^4+Dx^6)}{\sqrt{a+bx^2}}\,dx = -\frac{a(Ab^3-a(b^2B-abC+a^2D))\sqrt{a+bx^2}}{b^5} + \frac{(Ab^3-a(2b^2B-3abC+4a^2D))(a+bx^2)^{3/2}}{3b^5} + \frac{(b^2B-3abC+6a^2D)(a+bx^2)^{5/2}}{5b^5} + \frac{(bC-4aD)(a+bx^2)^{7/2}}{7b^5} + \frac{D(a+bx^2)^{9/2}}{9b^5}$$

output
```
-a*(A*b^3-a*(B*b^2-C*a*b+D*a^2))*(b*x^2+a)^(1/2)/b^5+1/3*(A*b^3-a*(2*B*b^2
-3*C*a*b+4*D*a^2))*(b*x^2+a)^(3/2)/b^5+1/5*(B*b^2-3*C*a*b+6*D*a^2)*(b*x^2+
a)^(5/2)/b^5+1/7*(C*b-4*D*a)*(b*x^2+a)^(7/2)/b^5+1/9*D*(b*x^2+a)^(9/2)/b^5
```

## Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.73

$$\int \frac{x^3(A+Bx^2+Cx^4+Dx^6)}{\sqrt{a+bx^2}}\,dx$$
$$= \frac{\sqrt{a+bx^2}(128a^4D-16a^3b(9C+4Dx^2)+24a^2b^2(7B+3Cx^2+2Dx^4)-2ab^3(105A+42Bx^2+27Cx^4}{315b^5}$$

input
```
Integrate[(x^3*(A + B*x^2 + C*x^4 + D*x^6))/Sqrt[a + b*x^2],x]
```

output
```
(Sqrt[a + b*x^2]*(128*a^4*D - 16*a^3*b*(9*C + 4*D*x^2) + 24*a^2*b^2*(7*B +
 3*C*x^2 + 2*D*x^4) - 2*a*b^3*(105*A + 42*B*x^2 + 27*C*x^4 + 20*D*x^6) + b
^4*x^2*(105*A + 63*B*x^2 + 45*C*x^4 + 35*D*x^6)))/(315*b^5)
```

## Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {2331, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(A+Bx^2+Cx^4+Dx^6)}{\sqrt{a+bx^2}}\,dx$$

$$\Big\downarrow \text{2331}$$

$$\frac{1}{2}\int \frac{x^2(Dx^6+Cx^4+Bx^2+A)}{\sqrt{bx^2+a}}dx^2$$

$$\Big\downarrow \text{2123}$$

$$\frac{1}{2}\int \left(\frac{D(bx^2+a)^{7/2}}{b^4}+\frac{(bC-4aD)(bx^2+a)^{5/2}}{b^4}+\frac{(6Da^2-3bCa+b^2B)(bx^2+a)^{3/2}}{b^4}+\frac{(Ab^3-a(4Da^2-3b\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{2\left(a+bx^2\right)^{3/2}\left(Ab^3-a\left(4a^2D-3abC+2b^2B\right)\right)}{3b^5}-\frac{2a\sqrt{a+bx^2}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{b^5}+\frac{2\left(a+bx^2\right)^5}{}\right.$$

input
```
Int[(x^3*(A + B*x^2 + C*x^4 + D*x^6))/Sqrt[a + b*x^2],x]
```

output
```
((-2*a*(A*b^3 - a*(b^2*B - a*b*C + a^2*D))*Sqrt[a + b*x^2])/b^5 + (2*(A*b^
3 - a*(2*b^2*B - 3*a*b*C + 4*a^2*D))*(a + b*x^2)^(3/2))/(3*b^5) + (2*(b^2*
B - 3*a*b*C + 6*a^2*D)*(a + b*x^2)^(5/2))/(5*b^5) + (2*(b*C - 4*a*D)*(a +
b*x^2)^(7/2))/(7*b^5) + (2*D*(a + b*x^2)^(9/2))/(9*b^5))/2
```

## Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

rule 2331
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/2   S
ubst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
 FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

## Maple [A] (verified)

Time = 1.31 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.66

| method | result |
|---|---|
| pseudoelliptic | $-\frac{2\left(-\frac{\left(\frac{1}{3}Dx^6+\frac{3}{7}C\,x^4+\frac{3}{5}x^2B+A\right)x^2b^4}{2}+a\left(\frac{4}{21}Dx^6+\frac{9}{35}C\,x^4+\frac{2}{5}x^2B+A\right)b^3-\frac{4\left(\frac{2}{7}Dx^4+\frac{3}{7}C\,x^2+B\right)a^2b^2}{5}+\frac{24a^3\left(\frac{4Dx^2}{9}+C\right)b}{35}-\right.}{3b^5}$ |
| gosper | $-\frac{\sqrt{b\,x^2+a}\,(-35Dx^8b^4-45C\,b^4x^6+40Da\,b^3x^6-63B\,x^4b^4+54Ca\,b^3x^4-48Da^2b^2x^4-105A\,b^4x^2+84Ba\,b^3x^2-72C\,a^2b^2x^2}{315b^5}$ |
| trager | $-\frac{\sqrt{b\,x^2+a}\,(-35Dx^8b^4-45C\,b^4x^6+40Da\,b^3x^6-63B\,x^4b^4+54Ca\,b^3x^4-48Da^2b^2x^4-105A\,b^4x^2+84Ba\,b^3x^2-72C\,a^2b^2x^2}{315b^5}$ |
| orering | $-\frac{\sqrt{b\,x^2+a}\,(-35Dx^8b^4-45C\,b^4x^6+40Da\,b^3x^6-63B\,x^4b^4+54Ca\,b^3x^4-48Da^2b^2x^4-105A\,b^4x^2+84Ba\,b^3x^2-72C\,a^2b^2x^2}{315b^5}$ |
| default | $A\left(\frac{x^2\sqrt{b\,x^2+a}}{3b}-\frac{2a\sqrt{b\,x^2+a}}{3b^2}\right)+B\left(\frac{x^4\sqrt{b\,x^2+a}}{5b}-\frac{4a\left(\frac{x^2\sqrt{b\,x^2+a}}{3b}-\frac{2a\sqrt{b\,x^2+a}}{3b^2}\right)}{5b}\right)+C\left(\frac{x^6\sqrt{b\,x^2+a}}{7b}-\frac{6a}{}\right.$ |

input
```
int(x^3*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(-1/2*(1/3*D*x^6+3/7*C*x^4+3/5*x^2*B+A)*x^2*b^4+a*(4/21*D*x^6+9/35*C*
x^4+2/5*x^2*B+A)*b^3-4/5*(2/7*D*x^4+3/7*C*x^2+B)*a^2*b^2+24/35*a^3*(4/9*D*
x^2+C)*b-64/105*D*a^4)*(b*x^2+a)^(1/2)/b^5
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.80

$$\int \frac{x^3(A+Bx^2+Cx^4+Dx^6)}{\sqrt{a+bx^2}}\,dx$$
$$=\frac{(35\,Db^4x^8-5\,(8\,Dab^3-9\,Cb^4)x^6+128\,Da^4-144\,Ca^3b+168\,Ba^2b^2-210\,Aab^3+3\,(16\,Da^2b^2-18\,C}{315\,b^5}$$

input
```
integrate(x^3*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
1/315*(35*D*b^4*x^8 - 5*(8*D*a*b^3 - 9*C*b^4)*x^6 + 128*D*a^4 - 144*C*a^3*
b + 168*B*a^2*b^2 - 210*A*a*b^3 + 3*(16*D*a^2*b^2 - 18*C*a*b^3 + 21*B*b^4)
*x^4 - (64*D*a^3*b - 72*C*a^2*b^2 + 84*B*a*b^3 - 105*A*b^4)*x^2)*sqrt(b*x^
2 + a)/b^5
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 340 vs. $2(160) = 320$.

Time = 0.34 (sec) , antiderivative size = 340, normalized size of antiderivative = 2.02

$$\int \frac{x^3(A + Bx^2 + Cx^4 + Dx^6)}{\sqrt{a + bx^2}}\,dx$$

$$= \begin{cases} -\frac{2Aa\sqrt{a+bx^2}}{3b^2} + \frac{Ax^2\sqrt{a+bx^2}}{3b} + \frac{8Ba^2\sqrt{a+bx^2}}{15b^3} - \frac{4Bax^2\sqrt{a+bx^2}}{15b^2} + \frac{Bx^4\sqrt{a+bx^2}}{5b} - \frac{16Ca^3\sqrt{a+bx^2}}{35b^4} + \frac{8Ca^2x^2\sqrt{a+bx^2}}{35b^3} - 6Cax \\ \frac{\frac{Ax^4}{4} + \frac{Bx^6}{6} + \frac{Cx^8}{8} + \frac{Dx^{10}}{10}}{\sqrt{a}} \end{cases}$$

input
```
integrate(x**3*(D*x**6+C*x**4+B*x**2+A)/(b*x**2+a)**(1/2),x)
```

output
```
Piecewise((-2*A*a*sqrt(a + b*x**2)/(3*b**2) + A*x**2*sqrt(a + b*x**2)/(3*b
) + 8*B*a**2*sqrt(a + b*x**2)/(15*b**3) - 4*B*a*x**2*sqrt(a + b*x**2)/(15*
b**2) + B*x**4*sqrt(a + b*x**2)/(5*b) - 16*C*a**3*sqrt(a + b*x**2)/(35*b**
4) + 8*C*a**2*x**2*sqrt(a + b*x**2)/(35*b**3) - 6*C*a*x**4*sqrt(a + b*x**2
)/(35*b**2) + C*x**6*sqrt(a + b*x**2)/(7*b) + 128*D*a**4*sqrt(a + b*x**2)/
(315*b**5) - 64*D*a**3*x**2*sqrt(a + b*x**2)/(315*b**4) + 16*D*a**2*x**4*s
qrt(a + b*x**2)/(105*b**3) - 8*D*a*x**6*sqrt(a + b*x**2)/(63*b**2) + D*x**
8*sqrt(a + b*x**2)/(9*b), Ne(b, 0)), ((A*x**4/4 + B*x**6/6 + C*x**8/8 + D*
x**10/10)/sqrt(a), True))
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.57

$$\int \frac{x^3(A+Bx^2+Cx^4+Dx^6)}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{bx^2+a}Dx^8}{9\,b} - \frac{8\,\sqrt{bx^2+a}Dax^6}{63\,b^2} + \frac{\sqrt{bx^2+a}Cx^6}{7\,b} + \frac{16\,\sqrt{bx^2+a}Da^2x^4}{105\,b^3} - \frac{6\,\sqrt{bx^2+a}Cax^4}{35\,b^2} + \frac{\sqrt{bx^2+a}Bx^4}{5\,b} - \frac{64\,\sqrt{bx^2+a}Da^3x^2}{315\,b^4} + \frac{8\,\sqrt{bx^2+a}Ca^2x^2}{35\,b^3} - \frac{4\,\sqrt{bx^2+a}Bax^2}{15\,b^2} + \frac{\sqrt{bx^2+a}Ax^2}{3\,b} + \frac{128\,\sqrt{bx^2+a}Da^4}{315\,b^5} - \frac{16\,\sqrt{bx^2+a}Ca^3}{35\,b^4} + \frac{8\,\sqrt{bx^2+a}Ba^2}{15\,b^3} - \frac{2\,\sqrt{bx^2+a}Aa}{3\,b^2}$$

input
```
integrate(x^3*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/9*sqrt(b*x^2 + a)*D*x^8/b - 8/63*sqrt(b*x^2 + a)*D*a*x^6/b^2 + 1/7*sqrt(
b*x^2 + a)*C*x^6/b + 16/105*sqrt(b*x^2 + a)*D*a^2*x^4/b^3 - 6/35*sqrt(b*x^
2 + a)*C*a*x^4/b^2 + 1/5*sqrt(b*x^2 + a)*B*x^4/b - 64/315*sqrt(b*x^2 + a)*
D*a^3*x^2/b^4 + 8/35*sqrt(b*x^2 + a)*C*a^2*x^2/b^3 - 4/15*sqrt(b*x^2 + a)*
B*a*x^2/b^2 + 1/3*sqrt(b*x^2 + a)*A*x^2/b + 128/315*sqrt(b*x^2 + a)*D*a^4/
b^5 - 16/35*sqrt(b*x^2 + a)*C*a^3/b^4 + 8/15*sqrt(b*x^2 + a)*B*a^2/b^3 - 2
/3*sqrt(b*x^2 + a)*A*a/b^2
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.14

$$\int \frac{x^3(A+Bx^2+Cx^4+Dx^6)}{\sqrt{a+bx^2}}\,dx = \frac{(Da^4-Ca^3b+Ba^2b^2-Aab^3)\sqrt{bx^2+a}}{b^5} + \frac{35\,(bx^2+a)^{\frac{9}{2}}D-180\,(bx^2+a)^{\frac{7}{2}}Da+378\,(bx^2+a)^{\frac{5}{2}}Da^2-420\,(bx^2+a)^{\frac{3}{2}}Da^3+45\,(bx^2+a)^{\frac{7}{2}}Cb-}{3}$$

input
```
integrate(x^3*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
(D*a^4 - C*a^3*b + B*a^2*b^2 - A*a*b^3)*sqrt(b*x^2 + a)/b^5 + 1/315*(35*(b
*x^2 + a)^(9/2)*D - 180*(b*x^2 + a)^(7/2)*D*a + 378*(b*x^2 + a)^(5/2)*D*a^
2 - 420*(b*x^2 + a)^(3/2)*D*a^3 + 45*(b*x^2 + a)^(7/2)*C*b - 189*(b*x^2 +
a)^(5/2)*C*a*b + 315*(b*x^2 + a)^(3/2)*C*a^2*b + 63*(b*x^2 + a)^(5/2)*B*b^
2 - 210*(b*x^2 + a)^(3/2)*B*a*b^2 + 105*(b*x^2 + a)^(3/2)*A*b^3)/b^5
```

## Mupad [B] (verification not implemented)

Time = 1.88 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.23

$$\int \frac{x^3(A+Bx^2+Cx^4+Dx^6)}{\sqrt{a+bx^2}}\,dx = \sqrt{b\,x^2+a}\left(\frac{8\,B\,a^2}{15\,b^3}+\frac{B\,x^4}{5\,b}-\frac{4\,B\,a\,x^2}{15\,b^2}\right) - \sqrt{b\,x^2+a}\left(\frac{16\,C\,a^3}{35\,b^4}-\frac{C\,x^6}{7\,b}+\frac{6\,C\,a\,x^4}{35\,b^2}-\frac{8\,C\,a^2\,x^2}{35\,b^3}\right) + \frac{x^8\,\sqrt{b\,x^2+a}\,D}{9\,b}-\frac{A\,\sqrt{b\,x^2+a}\,(2\,a-b\,x^2)}{3\,b^2} - \frac{4\,a\,D\left(\frac{2\,(b\,x^2+a)^{7/2}}{7\,b^4}-\frac{6\,a\,(b\,x^2+a)^{5/2}}{5\,b^4}-\frac{2\,a^3\,\sqrt{b\,x^2+a}}{b^4}+\frac{2\,a^2\,(b\,x^2+a)^{3/2}}{b^4}\right)}{9\,b}$$

input
```
int((x^3*(A + B*x^2 + C*x^4 + x^6*D))/(a + b*x^2)^(1/2),x)
```

output
```
(a + b*x^2)^(1/2)*((8*B*a^2)/(15*b^3) + (B*x^4)/(5*b) - (4*B*a*x^2)/(15*b^
2)) - (a + b*x^2)^(1/2)*((16*C*a^3)/(35*b^4) - (C*x^6)/(7*b) + (6*C*a*x^4)
/(35*b^2) - (8*C*a^2*x^2)/(35*b^3)) + (x^8*(a + b*x^2)^(1/2)*D)/(9*b) - (A
*(a + b*x^2)^(1/2)*(2*a - b*x^2))/(3*b^2) - (4*a*D*((2*(a + b*x^2)^(7/2))/
(7*b^4) - (6*a*(a + b*x^2)^(5/2))/(5*b^4) - (2*a^3*(a + b*x^2)^(1/2))/b^4
+ (2*a^2*(a + b*x^2)^(3/2))/b^4))/(9*b)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.74

$$\int \frac{x^3(A+Bx^2+Cx^4+Dx^6)}{\sqrt{a+bx^2}}\,dx$$
$$= \frac{\sqrt{b\,x^2+a}\,(35b^4d\,x^8-40a\,b^3d\,x^6+45b^4c\,x^6+48a^2b^2d\,x^4-54a\,b^3c\,x^4+63b^5x^4-64a^3bd\,x^2+72a^2b^2c\,x^2}{315b^5}$$

input
```
int(x^3*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(1/2),x)
```

output
```
(sqrt(a + b*x**2)*(128*a**4*d - 144*a**3*b*c - 64*a**3*b*d*x**2 - 42*a**2*
b**3 + 72*a**2*b**2*c*x**2 + 48*a**2*b**2*d*x**4 + 21*a*b**4*x**2 - 54*a*b
**3*c*x**4 - 40*a*b**3*d*x**6 + 63*b**5*x**4 + 45*b**4*c*x**6 + 35*b**4*d*
x**8))/(315*b**5)
```

# 3.210 $\int \frac{x(A+Bx^2+Cx^4+Dx^6)}{\sqrt{a+bx^2}}\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 121

$$\int \frac{x(A+Bx^2+Cx^4+Dx^6)}{\sqrt{a+bx^2}}\,dx = \frac{(Ab^3-a(b^2B-abC+a^2D))\sqrt{a+bx^2}}{b^4} + \frac{(b^2B-2abC+3a^2D)(a+bx^2)^{3/2}}{3b^4} + \frac{(bC-3aD)(a+bx^2)^{5/2}}{5b^4} + \frac{D(a+bx^2)^{7/2}}{7b^4}$$

output
```
(A*b^3-a*(B*b^2-C*a*b+D*a^2))*(b*x^2+a)^(1/2)/b^4+1/3*(B*b^2-2*C*a*b+3*D*a
^2)*(b*x^2+a)^(3/2)/b^4+1/5*(C*b-3*D*a)*(b*x^2+a)^(5/2)/b^4+1/7*D*(b*x^2+a
)^(7/2)/b^4
```

## Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.76

$$\int \frac{x(A+Bx^2+Cx^4+Dx^6)}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{a+bx^2}(105Ab^3-48a^3D+8a^2b(7C+3Dx^2)-2ab^2(35B+14Cx^2+9Dx^4)+b^3x^2(35B+21Cx^2+}{105b^4}$$

input
```
Integrate[(x*(A + B*x^2 + C*x^4 + D*x^6))/Sqrt[a + b*x^2],x]
```

output
```
(Sqrt[a + b*x^2]*(105*A*b^3 - 48*a^3*D + 8*a^2*b*(7*C + 3*D*x^2) - 2*a*b^2
*(35*B + 14*C*x^2 + 9*D*x^4) + b^3*x^2*(35*B + 21*C*x^2 + 15*D*x^4)))/(105
*b^4)
```

## Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2331, 2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(A+Bx^2+Cx^4+Dx^6)}{\sqrt{a+bx^2}}dx$$

$$\Big\downarrow \text{2331}$$

$$\frac{1}{2}\int \frac{Dx^6+Cx^4+Bx^2+A}{\sqrt{bx^2+a}}dx^2$$

$$\Big\downarrow \text{2389}$$

$$\frac{1}{2}\int \left(\frac{D(bx^2+a)^{5/2}}{b^3}+\frac{(bC-3aD)(bx^2+a)^{3/2}}{b^3}+\frac{(3Da^2-2bCa+b^2B)\sqrt{bx^2+a}}{b^3}+\frac{Ab^3-a(Da^2-bCa+$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{2\sqrt{a+bx^2}(Ab^3-a(a^2D-abC+b^2B))}{b^4}+\frac{2(a+bx^2)^{3/2}(3a^2D-2abC+b^2B)}{3b^4}+\frac{2(a+bx^2)^{5/2}(bC-3a$$

input
```
Int[(x*(A + B*x^2 + C*x^4 + D*x^6))/Sqrt[a + b*x^2],x]
```

output
```
((2*(A*b^3 - a*(b^2*B - a*b*C + a^2*D))*Sqrt[a + b*x^2])/b^4 + (2*(b^2*B -
 2*a*b*C + 3*a^2*D)*(a + b*x^2)^(3/2))/(3*b^4) + (2*(b*C - 3*a*D)*(a + b*x
^2)^(5/2))/(5*b^4) + (2*D*(a + b*x^2)^(7/2))/(7*b^4))/2
```

**Defintions of rubi rules used**

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2331
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/2   S
ubst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
 FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

## Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.65

| method | result |
|---|---|
| pseudoelliptic | $\frac{\left(\left(\frac{1}{7}Dx^6+\frac{1}{5}Cx^4+\frac{1}{3}x^2B+A\right)b^3-\frac{2\left(\frac{9}{35}Dx^4+\frac{2}{5}Cx^2+B\right)a\,b^2}{3}+\frac{8\left(\frac{3Dx^2}{7}+C\right)a^2b}{15}-\frac{16a^3D}{35}\right)\sqrt{b\,x^2+a}}{b^4}$ |
| gosper | $\frac{\sqrt{b\,x^2+a}\,(15b^3Dx^6+21b^3C\,x^4-18Da\,b^2x^4+35b^3B\,x^2-28Ca\,b^2x^2+24Da^2b\,x^2+105b^3A-70a\,b^2B+56a^2bC-48a^3D)}{105b^4}$ |
| trager | $\frac{\sqrt{b\,x^2+a}\,(15b^3Dx^6+21b^3C\,x^4-18Da\,b^2x^4+35b^3B\,x^2-28Ca\,b^2x^2+24Da^2b\,x^2+105b^3A-70a\,b^2B+56a^2bC-48a^3D)}{105b^4}$ |
| orering | $\frac{\sqrt{b\,x^2+a}\,(15b^3Dx^6+21b^3C\,x^4-18Da\,b^2x^4+35b^3B\,x^2-28Ca\,b^2x^2+24Da^2b\,x^2+105b^3A-70a\,b^2B+56a^2bC-48a^3D)}{105b^4}$ |
| default | $B\left(\frac{x^2\sqrt{b\,x^2+a}}{3b}-\frac{2a\sqrt{b\,x^2+a}}{3b^2}\right)+C\left(\frac{x^4\sqrt{b\,x^2+a}}{5b}-\frac{4a\left(\frac{x^2\sqrt{b\,x^2+a}}{3b}-\frac{2a\sqrt{b\,x^2+a}}{3b^2}\right)}{5b}\right)+D\left(\frac{x^6\sqrt{b\,x^2+a}}{7b}-\frac{6a}{}\right.$ |

input
```
int(x*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((1/7*D*x^6+1/5*C*x^4+1/3*x^2*B+A)*b^3-2/3*(9/35*D*x^4+2/5*C*x^2+B)*a*b^2+
8/15*(3/7*D*x^2+C)*a^2*b-16/35*a^3*D)*(b*x^2+a)^(1/2)/b^4
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.78

$$\int \frac{x(A+Bx^2+Cx^4+Dx^6)}{\sqrt{a+bx^2}}\,dx$$
$$= \frac{(15\,Db^3x^6 - 3\,(6\,Dab^2 - 7\,Cb^3)x^4 - 48\,Da^3 + 56\,Ca^2b - 70\,Bab^2 + 105\,Ab^3 + (24\,Da^2b - 28\,Cab^2 + 35}{105\,b^4}$$

input
```
integrate(x*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
1/105*(15*D*b^3*x^6 - 3*(6*D*a*b^2 - 7*C*b^3)*x^4 - 48*D*a^3 + 56*C*a^2*b
- 70*B*a*b^2 + 105*A*b^3 + (24*D*a^2*b - 28*C*a*b^2 + 35*B*b^3)*x^2)*sqrt(
b*x^2 + a)/b^4
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 238 vs. $2(110) = 220$.

Time = 0.27 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.97

$$\int \frac{x(A+Bx^2+Cx^4+Dx^6)}{\sqrt{a+bx^2}}\,dx$$
$$= \begin{cases} \frac{A\sqrt{a+bx^2}}{b} - \frac{2Ba\sqrt{a+bx^2}}{3b^2} + \frac{Bx^2\sqrt{a+bx^2}}{3b} + \frac{8Ca^2\sqrt{a+bx^2}}{15b^3} - \frac{4Cax^2\sqrt{a+bx^2}}{15b^2} + \frac{Cx^4\sqrt{a+bx^2}}{5b} - \frac{16Da^3\sqrt{a+bx^2}}{35b^4} + \frac{8Da^2x^2\sqrt{a+b}}{35b^3} \\ \frac{\frac{Ax^2}{2} + \frac{Bx^4}{4} + \frac{Cx^6}{6} + \frac{Dx^8}{8}}{\sqrt{a}} \end{cases}$$

input
```
integrate(x*(D*x**6+C*x**4+B*x**2+A)/(b*x**2+a)**(1/2),x)
```

output
```
Piecewise((A*sqrt(a + b*x**2)/b - 2*B*a*sqrt(a + b*x**2)/(3*b**2) + B*x**2
*sqrt(a + b*x**2)/(3*b) + 8*C*a**2*sqrt(a + b*x**2)/(15*b**3) - 4*C*a*x**2
*sqrt(a + b*x**2)/(15*b**2) + C*x**4*sqrt(a + b*x**2)/(5*b) - 16*D*a**3*sq
rt(a + b*x**2)/(35*b**4) + 8*D*a**2*x**2*sqrt(a + b*x**2)/(35*b**3) - 6*D*
a*x**4*sqrt(a + b*x**2)/(35*b**2) + D*x**6*sqrt(a + b*x**2)/(7*b), Ne(b, 0
)), ((A*x**2/2 + B*x**4/4 + C*x**6/6 + D*x**8/8)/sqrt(a), True))
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.49

$$\int \frac{x(A+Bx^2+Cx^4+Dx^6)}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{bx^2+a}Dx^6}{7\,b} - \frac{6\,\sqrt{bx^2+a}Dax^4}{35\,b^2} + \frac{\sqrt{bx^2+a}Cx^4}{5\,b} + \frac{8\,\sqrt{bx^2+a}Da^2x^2}{35\,b^3} - \frac{4\,\sqrt{bx^2+a}Cax^2}{15\,b^2} + \frac{\sqrt{bx^2+a}Bx^2}{3\,b} - \frac{16\,\sqrt{bx^2+a}Da^3}{35\,b^4} + \frac{8\,\sqrt{bx^2+a}Ca^2}{15\,b^3} - \frac{2\,\sqrt{bx^2+a}Ba}{3\,b^2} + \frac{\sqrt{bx^2+a}A}{b}$$

input
```
integrate(x*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/7*sqrt(b*x^2 + a)*D*x^6/b - 6/35*sqrt(b*x^2 + a)*D*a*x^4/b^2 + 1/5*sqrt(
b*x^2 + a)*C*x^4/b + 8/35*sqrt(b*x^2 + a)*D*a^2*x^2/b^3 - 4/15*sqrt(b*x^2
+ a)*C*a*x^2/b^2 + 1/3*sqrt(b*x^2 + a)*B*x^2/b - 16/35*sqrt(b*x^2 + a)*D*a
^3/b^4 + 8/15*sqrt(b*x^2 + a)*C*a^2/b^3 - 2/3*sqrt(b*x^2 + a)*B*a/b^2 + sq
rt(b*x^2 + a)*A/b
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.06

$$\int \frac{x(A+Bx^2+Cx^4+Dx^6)}{\sqrt{a+bx^2}}\,dx = -\frac{(Da^3-Ca^2b+Bab^2-Ab^3)\sqrt{bx^2+a}}{b^4} + \frac{15\,(bx^2+a)^{\frac{7}{2}}D - 63\,(bx^2+a)^{\frac{5}{2}}Da + 105\,(bx^2+a)^{\frac{3}{2}}Da^2 + 21\,(bx^2+a)^{\frac{5}{2}}Cb - 70\,(bx^2+a)^{\frac{3}{2}}Cab + 35}{105\,b^4}$$

input
```
integrate(x*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
-(D*a^3 - C*a^2*b + B*a*b^2 - A*b^3)*sqrt(b*x^2 + a)/b^4 + 1/105*(15*(b*x^
2 + a)^(7/2)*D - 63*(b*x^2 + a)^(5/2)*D*a + 105*(b*x^2 + a)^(3/2)*D*a^2 +
21*(b*x^2 + a)^(5/2)*C*b - 70*(b*x^2 + a)^(3/2)*C*a*b + 35*(b*x^2 + a)^(3/
2)*B*b^2)/b^4
```

## Mupad [B] (verification not implemented)

Time = 1.83 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.09

$$\int \frac{x(A+Bx^2+Cx^4+Dx^6)}{\sqrt{a+bx^2}}\,dx = \sqrt{b\,x^2+a}\left(\frac{8\,C\,a^2}{15\,b^3}+\frac{C\,x^4}{5\,b}-\frac{4\,C\,a\,x^2}{15\,b^2}\right)+\frac{A\,\sqrt{b\,x^2+a}}{b}-\frac{B\,\sqrt{b\,x^2+a}\,(2\,a-b\,x^2)}{3\,b^2} - \frac{\sqrt{b\,x^2+a}\,D\left(6\,a\,(b\,x^2+a)^2-20\,a^2\,(b\,x^2+a)+30\,a^3-5\,b^3\,x^6\right)}{35\,b^4}$$

input
```
int((x*(A + B*x^2 + C*x^4 + x^6*D))/(a + b*x^2)^(1/2),x)
```

output
```
(a + b*x^2)^(1/2)*((8*C*a^2)/(15*b^3) + (C*x^4)/(5*b) - (4*C*a*x^2)/(15*b^
2)) + (A*(a + b*x^2)^(1/2))/b - (B*(a + b*x^2)^(1/2)*(2*a - b*x^2))/(3*b^2
) - ((a + b*x^2)^(1/2)*D*(6*a*(a + b*x^2)^2 - 20*a^2*(a + b*x^2) + 30*a^3
- 5*b^3*x^6))/(35*b^4)
```

## Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.74

$$\int \frac{x(A+Bx^2+Cx^4+Dx^6)}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{b\,x^2+a}\,(15b^3d\,x^6-18a\,b^2d\,x^4+21b^3c\,x^4+24a^2bd\,x^2-28a\,b^2c\,x^2+35b^4x^2-48a^3d+56a^2bc+35a\,b}{105b^4}$$

input
```
int(x*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(1/2),x)
```

output
```
(sqrt(a + b*x**2)*( - 48*a**3*d + 56*a**2*b*c + 24*a**2*b*d*x**2 + 35*a*b*
*3 - 28*a*b**2*c*x**2 - 18*a*b**2*d*x**4 + 35*b**4*x**2 + 21*b**3*c*x**4 +
 15*b**3*d*x**6))/(105*b**4)
```

# 3.211 $\int \frac{A+Bx^2+Cx^4+Dx^6}{x\sqrt{a+bx^2}}\,dx$



## Optimal result

Integrand size = 32, antiderivative size = 103

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x\sqrt{a+bx^2}}\,dx = \frac{(b^2B-abC+a^2D)\sqrt{a+bx^2}}{b^3} + \frac{(bC-2aD)\,(a+bx^2)^{3/2}}{3b^3} + \frac{D(a+bx^2)^{5/2}}{5b^3} - \frac{A\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

output
```
(B*b^2-C*a*b+D*a^2)*(b*x^2+a)^(1/2)/b^3+1/3*(C*b-2*D*a)*(b*x^2+a)^(3/2)/b^
3+1/5*D*(b*x^2+a)^(5/2)/b^3-A*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(1/2)
```

## Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.83

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x\sqrt{a+bx^2}}\,dx = \frac{\sqrt{a+bx^2}(8a^2D-2ab(5C+2Dx^2)+b^2(15B+5Cx^2+3Dx^4))}{15b^3} - \frac{A\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

input
```
Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(x*Sqrt[a + b*x^2]),x]
```

output
```
(Sqrt[a + b*x^2]*(8*a^2*D - 2*a*b*(5*C + 2*D*x^2) + b^2*(15*B + 5*C*x^2 +
3*D*x^4)))/(15*b^3) - (A*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a]
```

## Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {2331, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x\sqrt{a+bx^2}}dx$$

$$\Big\downarrow \text{2331}$$

$$\frac{1}{2}\int \frac{Dx^6+Cx^4+Bx^2+A}{x^2\sqrt{bx^2+a}}dx^2$$

$$\Big\downarrow \text{2123}$$

$$\frac{1}{2}\int \left(\frac{D\left(bx^2+a\right)^{3/2}}{b^2}+\frac{(bC-2aD)\sqrt{bx^2+a}}{b^2}+\frac{Da^2-bCa+b^2B}{b^2\sqrt{bx^2+a}}+\frac{A}{x^2\sqrt{bx^2+a}}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{2\sqrt{a+bx^2}\left(a^2D-abC+b^2B\right)}{b^3}-\frac{2A\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{2\left(a+bx^2\right)^{3/2}(bC-2aD)}{3b^3}+\frac{2D\left(a+bx^2\right)^{5/2}}{5b^3}\right)$$

input
```
Int[(A + B*x^2 + C*x^4 + D*x^6)/(x*Sqrt[a + b*x^2]),x]
```

output
```
((2*(b^2*B - a*b*C + a^2*D)*Sqrt[a + b*x^2])/b^3 + (2*(b*C - 2*a*D)*(a + b
*x^2)^(3/2))/(3*b^3) + (2*D*(a + b*x^2)^(5/2))/(5*b^3) - (2*A*ArcTanh[Sqrt
[a + b*x^2]/Sqrt[a]])/Sqrt[a])/2
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

rule 2331
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/2   S
ubst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
 FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

## Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.80

| method | result |
|---|---|
| pseudoelliptic | $-\frac{b^3 A \operatorname{arctanh}\left(\frac{\sqrt{b x^2+a}}{\sqrt{a}}\right)+\frac{2\sqrt{b x^2+a}\left(b\left(\frac{2Dx^2}{5}+C\right)a^{\frac{3}{2}}-\frac{4Da^{\frac{5}{2}}}{5}-\frac{3\sqrt{a}\left(\frac{1}{5}Dx^4+\frac{1}{3}Cx^2+B\right)b^2}{2}\right)}{3}}{\sqrt{a}\,b^3}$ |
| default | $-\frac{A\ln\left(\frac{2a+2\sqrt{a}\sqrt{b x^2+a}}{x}\right)}{\sqrt{a}}+\frac{B\sqrt{b x^2+a}}{b}+C\left(\frac{x^2\sqrt{b x^2+a}}{3b}-\frac{2a\sqrt{b x^2+a}}{3b^2}\right)+D\left(\frac{x^4\sqrt{b x^2+a}}{5b}-\frac{4a\left(\frac{x^2\sqrt{b x^2+}}{3b}\right.}{}\right.$ |

input
```
int((D*x^6+C*x^4+B*x^2+A)/x/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(b^3*A*arctanh((b*x^2+a)^(1/2)/a^(1/2))+2/3*(b*x^2+a)^(1/2)*(b*(2/5*D*x^2
+C)*a^(3/2)-4/5*D*a^(5/2)-3/2*a^(1/2)*(1/5*D*x^4+1/3*C*x^2+B)*b^2))/a^(1/2
)/b^3
```

## Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 210, normalized size of antiderivative = 2.04

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x\sqrt{a+bx^2}}\,dx$$
$$= \left[\frac{15\,A\sqrt{a}b^3\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\,(3\,Dab^2x^4+8\,Da^3-10\,Ca^2b+15\,Bab^2-(4\,Da^2b-5\,Cab^2)}{30\,ab^3}\right.$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/30*(15*A*sqrt(a)*b^3*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2
) + 2*(3*D*a*b^2*x^4 + 8*D*a^3 - 10*C*a^2*b + 15*B*a*b^2 - (4*D*a^2*b - 5*
C*a*b^2)*x^2)*sqrt(b*x^2 + a))/(a*b^3), 1/15*(15*A*sqrt(-a)*b^3*arctan(sqr
t(b*x^2 + a)*sqrt(-a)/a) + (3*D*a*b^2*x^4 + 8*D*a^3 - 10*C*a^2*b + 15*B*a*
b^2 - (4*D*a^2*b - 5*C*a*b^2)*x^2)*sqrt(b*x^2 + a))/(a*b^3)]
```

## Sympy [A] (verification not implemented)

Time = 4.90 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.30

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x\sqrt{a+bx^2}}\,dx$$
$$= \frac{\begin{cases} \frac{2A\operatorname{atan}\left(\frac{\sqrt{a+bx^2}}{\sqrt{-a}}\right)}{\sqrt{-a}}+\frac{2D(a+bx^2)^{\frac{5}{2}}}{5b^3}+\frac{2(a+bx^2)^{\frac{3}{2}}(Cb-2Da)}{3b^3}+\frac{2\sqrt{a+bx^2}(Bb^2-Cab+Da^2)}{b^3} & \text{for } b\neq 0 \\ \frac{A\log(x^2)+Bx^2+\frac{Cx^4}{2}+\frac{Dx^6}{3}}{\sqrt{a}} & \text{otherwise} \end{cases}}{2}$$

input
```
integrate((D*x**6+C*x**4+B*x**2+A)/x/(b*x**2+a)**(1/2),x)
```

output
```
Piecewise((2*A*atan(sqrt(a + b*x**2)/sqrt(-a))/sqrt(-a) + 2*D*(a + b*x**2)
**(5/2)/(5*b**3) + 2*(a + b*x**2)**(3/2)*(C*b - 2*D*a)/(3*b**3) + 2*sqrt(a
 + b*x**2)*(B*b**2 - C*a*b + D*a**2)/b**3, Ne(b, 0)), ((A*log(x**2) + B*x*
*2 + C*x**4/2 + D*x**6/3)/sqrt(a), True))/2
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.18

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x\sqrt{a+bx^2}}\,dx = \frac{\sqrt{bx^2+a}Dx^4}{5\,b} - \frac{4\,\sqrt{bx^2+a}Dax^2}{15\,b^2} + \frac{\sqrt{bx^2+a}Cx^2}{3\,b} - \frac{A\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{\sqrt{a}} + \frac{8\,\sqrt{bx^2+a}Da^2}{15\,b^3} - \frac{2\,\sqrt{bx^2+a}Ca}{3\,b^2} + \frac{\sqrt{bx^2+a}B}{b}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/5*sqrt(b*x^2 + a)*D*x^4/b - 4/15*sqrt(b*x^2 + a)*D*a*x^2/b^2 + 1/3*sqrt(
b*x^2 + a)*C*x^2/b - A*arcsinh(a/(sqrt(a*b)*abs(x)))/sqrt(a) + 8/15*sqrt(b
*x^2 + a)*D*a^2/b^3 - 2/3*sqrt(b*x^2 + a)*C*a/b^2 + sqrt(b*x^2 + a)*B/b
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.21

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x\sqrt{a+bx^2}}\,dx = \frac{A\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} + \frac{3\,(bx^2+a)^{\frac{5}{2}}Db^{12} - 10\,(bx^2+a)^{\frac{3}{2}}Dab^{12} + 15\,\sqrt{bx^2+a}Da^2b^{12} + 5\,(bx^2+a)^{\frac{3}{2}}Cb^{13} - 15\,\sqrt{bx^2+a}Cab^{1}}{15\,b^{15}}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
A*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) + 1/15*(3*(b*x^2 + a)^(5/2)*D*
b^12 - 10*(b*x^2 + a)^(3/2)*D*a*b^12 + 15*sqrt(b*x^2 + a)*D*a^2*b^12 + 5*(
b*x^2 + a)^(3/2)*C*b^13 - 15*sqrt(b*x^2 + a)*C*a*b^13 + 15*sqrt(b*x^2 + a)
*B*b^14)/b^15
```

## Mupad [B] (verification not implemented)

Time = 1.89 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.93

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x\sqrt{a+bx^2}}\,dx = \frac{B\sqrt{b\,x^2+a}}{b} - \frac{A\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{\sqrt{a}} + \frac{\sqrt{b\,x^2+a}\,D\,(8\,a^2-4\,a\,b\,x^2+3\,b^2\,x^4)}{15\,b^3} - \frac{C\,\sqrt{b\,x^2+a}\,(2\,a-b\,x^2)}{3\,b^2}$$

input
```
int((A + B*x^2 + C*x^4 + x^6*D)/(x*(a + b*x^2)^(1/2)),x)
```

output
```
(B*(a + b*x^2)^(1/2))/b - (A*atanh((a + b*x^2)^(1/2)/a^(1/2)))/a^(1/2) + (
(a + b*x^2)^(1/2)*D*(8*a^2 + 3*b^2*x^4 - 4*a*b*x^2))/(15*b^3) - (C*(a + b*
x^2)^(1/2)*(2*a - b*x^2))/(3*b^2)
```

## Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.50

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x\sqrt{a+bx^2}}\,dx = \frac{8\sqrt{b\,x^2+a}\,a^2d - 10\sqrt{b\,x^2+a}\,abc - 4\sqrt{b\,x^2+a}\,abd\,x^2 + 15\sqrt{b\,x^2+a}\,b^3 + 5\sqrt{b\,x^2+a}\,b^2c\,x^2 + 3\sqrt{b\,x^2+}}{15b^3}$$

input
```
int((D*x^6+C*x^4+B*x^2+A)/x/(b*x^2+a)^(1/2),x)
```

output
```
(8*sqrt(a + b*x**2)*a**2*d - 10*sqrt(a + b*x**2)*a*b*c - 4*sqrt(a + b*x**2
)*a*b*d*x**2 + 15*sqrt(a + b*x**2)*b**3 + 5*sqrt(a + b*x**2)*b**2*c*x**2 +
 3*sqrt(a + b*x**2)*b**2*d*x**4 + 15*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(
a) + sqrt(b)*x)/sqrt(a))*b**3 - 15*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a)
 + sqrt(b)*x)/sqrt(a))*b**3)/(15*b**3)
```

# 3.212 $\int \frac{A+Bx^2+Cx^4+Dx^6}{x^3\sqrt{a+bx^2}}\,dx$



## Optimal result

Integrand size = 32, antiderivative size = 100

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^3\sqrt{a+bx^2}}\,dx = \frac{(bC-aD)\sqrt{a+bx^2}}{b^2}-\frac{A\sqrt{a+bx^2}}{2ax^2}+\frac{D(a+bx^2)^{3/2}}{3b^2}+\frac{(Ab-2aB)\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}}$$

output
```
(C*b-D*a)*(b*x^2+a)^(1/2)/b^2-1/2*A*(b*x^2+a)^(1/2)/a/x^2+1/3*D*(b*x^2+a)^
(3/2)/b^2+1/2*(A*b-2*B*a)*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(3/2)
```

## Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.88

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^3\sqrt{a+bx^2}}\,dx = \frac{\sqrt{a+bx^2}(-3Ab^2+2ax^2(3bC-2aD+bDx^2))}{6ab^2x^2}+\frac{(Ab-2aB)\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}}$$

input
```
Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(x^3*Sqrt[a + b*x^2]),x]
```

output
```
(Sqrt[a + b*x^2]*(-3*A*b^2 + 2*a*x^2*(3*b*C - 2*a*D + b*D*x^2)))/(6*a*b^2*
x^2) + ((A*b - 2*a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*a^(3/2))
```

## Rubi [A] (warning: unable to verify)

Time = 0.47 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {2331, 2124, 27, 1192, 25, 1467, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^3\sqrt{a+bx^2}}dx$$

$$\Big\downarrow \text{2331}$$

$$\frac{1}{2}\int \frac{Dx^6+Cx^4+Bx^2+A}{x^4\sqrt{bx^2+a}}dx^2$$

$$\Big\downarrow \text{2124}$$

$$\frac{1}{2}\left(-\frac{\int \frac{-2aDx^4-2aCx^2+Ab-2aB}{2x^2\sqrt{bx^2+a}}dx^2}{a}-\frac{A\sqrt{a+bx^2}}{ax^2}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(-\frac{\int \frac{-2aDx^4-2aCx^2+Ab-2aB}{x^2\sqrt{bx^2+a}}dx^2}{2a}-\frac{A\sqrt{a+bx^2}}{ax^2}\right)$$

$$\Big\downarrow \text{1192}$$

$$\frac{1}{2}\left(-\frac{\int -\frac{-2aDx^8-2a(bC-2aD)x^4+Ab^3-2a\left(Da^2-bCa+b^2B\right)}{a-x^4}d\sqrt{bx^2+a}}{ab^2}-\frac{A\sqrt{a+bx^2}}{ax^2}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{\int \frac{-2aDx^8-2a(bC-2aD)x^4+Ab^3-2a\left(Da^2-bCa+b^2B\right)}{a-x^4}d\sqrt{bx^2+a}}{ab^2}-\frac{A\sqrt{a+bx^2}}{ax^2}\right)$$

$$\Big\downarrow \text{1467}$$

$$\frac{1}{2}\left(\frac{\int \left(2aDx^4+2a(bC-aD)+\frac{Ab^3-2ab^2B}{a-x^4}\right)d\sqrt{bx^2+a}}{ab^2}-\frac{A\sqrt{a+bx^2}}{ax^2}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{-\frac{b^2(Ab-2aB)\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}-2a\sqrt{a+bx^2}(bC-aD)-\frac{2}{3}aDx^6}{ab^2}-\frac{A\sqrt{a+bx^2}}{ax^2}\right)$$

input
```
Int[(A + B*x^2 + C*x^4 + D*x^6)/(x^3*Sqrt[a + b*x^2]),x]
```

output
```
(-((A*Sqrt[a + b*x^2])/(a*x^2)) - ((-2*a*D*x^6)/3 - 2*a*(b*C - a*D)*Sqrt[a
 + b*x^2] - (b^2*(A*b - 2*a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a])/
(a*b^2))/2
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1192
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(
2*m + 1)*(e*f - d*g + g*x^2)^n*(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 +
 c*x^4)^p, x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &&
IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m + 1/2]
```

rule 1467
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2124
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
 a*d))), x] + Simp[1/((m + 1)*(b*c - a*d))   Int[(a + b*x)^(m + 1)*(c + d*x
)^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fr
eeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && LtQ[m, -1] && (IntegerQ[m] ||  !
ILtQ[n, -1])
```

rule 2331
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/2   S
ubst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
 FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

## Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.88

| method | result |
|---|---|
| pseudoelliptic | $-\frac{2\left(-\frac{3a\,b^{2}x^{2}\left(Ab-2Ba\right)\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{\sqrt{a}}\right)}{4}+\left(\frac{3A\,a^{\frac{3}{2}}b^{2}}{4}+a^{\frac{5}{2}}\left(\frac{\left(-Dx^{2}-3C\right)b}{2}+Da\right)x^{2}\right)\sqrt{b\,x^{2}+a}\right)}{3a^{\frac{5}{2}}b^{2}x^{2}}$ |
| default | $A\left(-\frac{\sqrt{b\,x^{2}+a}}{2a\,x^{2}}+\frac{b\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{2a^{\frac{3}{2}}}\right)-\frac{B\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{\sqrt{a}}+\frac{C\sqrt{b\,x^{2}+a}}{b}+D\left(\frac{x^{2}\sqrt{b\,x^{2}+a}}{3b}-2a\surd\right.$ |

input
```
int((D*x^6+C*x^4+B*x^2+A)/x^3/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(-3/4*a*b^2*x^2*(A*b-2*B*a)*arctanh((b*x^2+a)^(1/2)/a^(1/2))+(3/4*A*a
^(3/2)*b^2+a^(5/2)*(1/2*(-D*x^2-3*C)*b+D*a)*x^2)*(b*x^2+a)^(1/2))/a^(5/2)/
b^2/x^2
```

## Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 214, normalized size of antiderivative = 2.14

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^3\sqrt{a+bx^2}}\,dx$$
$$= \left[-\frac{3\,(2\,Bab^2-Ab^3)\sqrt{a}x^2\log\left(-\frac{bx^2+2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)-2\,(2\,Da^2bx^4-3\,Aab^2-2\,(2\,Da^3-3\,Ca^2b)x^2)\sqrt{b}}{12\,a^2b^2x^2}\right.$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^3/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/12*(3*(2*B*a*b^2 - A*b^3)*sqrt(a)*x^2*log(-(b*x^2 + 2*sqrt(b*x^2 + a)*
sqrt(a) + 2*a)/x^2) - 2*(2*D*a^2*b*x^4 - 3*A*a*b^2 - 2*(2*D*a^3 - 3*C*a^2*
b)*x^2)*sqrt(b*x^2 + a))/(a^2*b^2*x^2), 1/6*(3*(2*B*a*b^2 - A*b^3)*sqrt(-a
)*x^2*arctan(sqrt(b*x^2 + a)*sqrt(-a)/a) + (2*D*a^2*b*x^4 - 3*A*a*b^2 - 2*
(2*D*a^3 - 3*C*a^2*b)*x^2)*sqrt(b*x^2 + a))/(a^2*b^2*x^2)]
```

## Sympy [A] (verification not implemented)

Time = 14.93 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.38

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^3\sqrt{a+bx^2}}\,dx = -\frac{A\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{2ax}+\frac{Ab\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2a^{\frac{3}{2}}} - \frac{B\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{\sqrt{a}} + C\left(\begin{cases}\frac{\sqrt{a+bx^2}}{b} & \text{for}\ b\neq 0\\ \frac{x^2}{2\sqrt{a}} & \text{otherwise}\end{cases}\right) + D\left(\begin{cases}-\frac{2a\sqrt{a+bx^2}}{3b^2}+\frac{x^2\sqrt{a+bx^2}}{3b} & \text{for}\ b\neq 0\\ \frac{x^4}{4\sqrt{a}} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((D*x**6+C*x**4+B*x**2+A)/x**3/(b*x**2+a)**(1/2),x)
```

output
```
-A*sqrt(b)*sqrt(a/(b*x**2) + 1)/(2*a*x) + A*b*asinh(sqrt(a)/(sqrt(b)*x))/(
2*a**(3/2)) - B*asinh(sqrt(a)/(sqrt(b)*x))/sqrt(a) + C*Piecewise((sqrt(a +
 b*x**2)/b, Ne(b, 0)), (x**2/(2*sqrt(a)), True)) + D*Piecewise((-2*a*sqrt(
a + b*x**2)/(3*b**2) + x**2*sqrt(a + b*x**2)/(3*b), Ne(b, 0)), (x**4/(4*sq
rt(a)), True))
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.04

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^3\sqrt{a+bx^2}}\,dx = \frac{\sqrt{bx^2+a}Dx^2}{3\,b} - \frac{B\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{\sqrt{a}} + \frac{Ab\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{2\,a^{\frac{3}{2}}} - \frac{2\,\sqrt{bx^2+a}Da}{3\,b^2} + \frac{\sqrt{bx^2+a}C}{b} - \frac{\sqrt{bx^2+a}A}{2\,ax^2}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^3/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/3*sqrt(b*x^2 + a)*D*x^2/b - B*arcsinh(a/(sqrt(a*b)*abs(x)))/sqrt(a) + 1/
2*A*b*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2) - 2/3*sqrt(b*x^2 + a)*D*a/b^2
+ sqrt(b*x^2 + a)*C/b - 1/2*sqrt(b*x^2 + a)*A/(a*x^2)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.14

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^3\sqrt{a+bx^2}}\,dx$$
$$= \frac{1}{6}\,b\left(\frac{3\,(2\,Ba-Ab)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}ab} - \frac{3\,\sqrt{bx^2+a}A}{abx^2} + \frac{2\left((bx^2+a)^{\frac{3}{2}}Db^6 - 3\,\sqrt{bx^2+a}Dab^6 + 3\,\sqrt{bx^2-}\right.}{b^9}\right.$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^3/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/6*b*(3*(2*B*a - A*b)*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a*b) - 3
*sqrt(b*x^2 + a)*A/(a*b*x^2) + 2*((b*x^2 + a)^(3/2)*D*b^6 - 3*sqrt(b*x^2 +
 a)*D*a*b^6 + 3*sqrt(b*x^2 + a)*C*b^7)/b^9)
```

## Mupad [B] (verification not implemented)

Time = 1.91 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.04

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^3\sqrt{a+bx^2}}\,dx = \frac{\left(b\,x^2+a\right)^{3/2}D-3\,a\,\sqrt{b\,x^2+a}\,D}{3\,b^2} + \frac{C\,\sqrt{b\,x^2+a}}{b} - \frac{B\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{\sqrt{a}} - \frac{A\,\sqrt{b\,x^2+a}}{2\,a\,x^2} + \frac{A\,b\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2\,a^{3/2}}$$

input
```
int((A + B*x^2 + C*x^4 + x^6*D)/(x^3*(a + b*x^2)^(1/2)),x)
```

output
```
((a + b*x^2)^(3/2)*D - 3*a*(a + b*x^2)^(1/2)*D)/(3*b^2) + (C*(a + b*x^2)^(
1/2))/b - (B*atanh((a + b*x^2)^(1/2)/a^(1/2)))/a^(1/2) - (A*(a + b*x^2)^(1
/2))/(2*a*x^2) + (A*b*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(2*a^(3/2))
```

## Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.39

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^3\sqrt{a+bx^2}}\,dx = \frac{-4\sqrt{b\,x^2+a}\,a^2d\,x^2-3\sqrt{b\,x^2+a}\,a\,b^2+6\sqrt{b\,x^2+a}\,abc\,x^2+2\sqrt{b\,x^2+a}\,abd\,x^4+3\sqrt{a}\log\left(\frac{\sqrt{b\,x^2+a}-\sqrt{a}+\sqrt{}}{\sqrt{a}}\right.}{6a\,b^2x^2}$$

input
```
int((D*x^6+C*x^4+B*x^2+A)/x^3/(b*x^2+a)^(1/2),x)
```

output
```
( - 4*sqrt(a + b*x**2)*a**2*d*x**2 - 3*sqrt(a + b*x**2)*a*b**2 + 6*sqrt(a
+ b*x**2)*a*b*c*x**2 + 2*sqrt(a + b*x**2)*a*b*d*x**4 + 3*sqrt(a)*log((sqrt
(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*b**3*x**2 - 3*sqrt(a)*log((sq
rt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*b**3*x**2)/(6*a*b**2*x**2)
```

# 3.213 $\int \frac{A+Bx^2+Cx^4+Dx^6}{x^5\sqrt{a+bx^2}} \, dx$



## Optimal result

Integrand size = 32, antiderivative size = 114

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^5\sqrt{a+bx^2}} \, dx = \frac{D\sqrt{a+bx^2}}{b} - \frac{A\sqrt{a+bx^2}}{4ax^4} + \frac{(3Ab-4aB)\sqrt{a+bx^2}}{8a^2x^2} - \frac{(3Ab^2-4abB+8a^2C)\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{5/2}}$$

output
```
D*(b*x^2+a)^(1/2)/b-1/4*A*(b*x^2+a)^(1/2)/a/x^4+1/8*(3*A*b-4*B*a)*(b*x^2+a
)^(1/2)/a^2/x^2-1/8*(3*A*b^2-4*B*a*b+8*C*a^2)*arctanh((b*x^2+a)^(1/2)/a^(1
/2))/a^(5/2)
```

## Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.89

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^5\sqrt{a+bx^2}} \, dx = \frac{\sqrt{a+bx^2}(-2aAb+3Ab^2x^2-4abBx^2+8a^2Dx^4)}{8a^2bx^4} + \frac{(-3Ab^2+4abB-8a^2C)\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{5/2}}$$

input
```
Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(x^5*Sqrt[a + b*x^2]),x]
```

output
```
(Sqrt[a + b*x^2]*(-2*a*A*b + 3*A*b^2*x^2 - 4*a*b*B*x^2 + 8*a^2*D*x^4))/(8*
a^2*b*x^4) + ((-3*A*b^2 + 4*a*b*B - 8*a^2*C)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[
a]])/(8*a^(5/2))
```

## Rubi [A] (warning: unable to verify)

Time = 0.48 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.27, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2331, 2124, 27, 1192, 1471, 25, 299, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^5\sqrt{a+bx^2}}dx$$

$$\Big\downarrow \text{2331}$$

$$\frac{1}{2}\int \frac{Dx^6+Cx^4+Bx^2+A}{x^6\sqrt{bx^2+a}}dx^2$$

$$\Big\downarrow \text{2124}$$

$$\frac{1}{2}\left(-\frac{\int \frac{-4aDx^4-4aCx^2+3Ab-4aB}{2x^4\sqrt{bx^2+a}}dx^2}{2a}-\frac{A\sqrt{a+bx^2}}{2ax^4}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(-\frac{\int \frac{-4aDx^4-4aCx^2+3Ab-4aB}{x^4\sqrt{bx^2+a}}dx^2}{4a}-\frac{A\sqrt{a+bx^2}}{2ax^4}\right)$$

$$\Big\downarrow \text{1192}$$

$$\frac{1}{2}\left(-\frac{\int \frac{-4aDx^8-4a(bC-2aD)x^4+3Ab^3-4a\left(Da^2-bCa+b^2B\right)}{(a-x^4)^2}d\sqrt{bx^2+a}}{2ab}-\frac{A\sqrt{a+bx^2}}{2ax^4}\right)$$

$$\Big\downarrow \text{1471}$$

$$\frac{1}{2}\left(-\frac{\frac{b^2\sqrt{a+bx^2}(3Ab-4aB)}{2a(a-x^4)}-\frac{\int -\frac{8a^2Dx^4+3Ab^3-4a\left(2Da^2-2bCa+b^2B\right)}{a-x^4}d\sqrt{bx^2+a}}{2a}}{2ab}-\frac{A\sqrt{a+bx^2}}{2ax^4}\right)$$

$$\Big\downarrow \; 25$$

$$\frac{1}{2}\left(-\frac{\frac{\int \frac{8a^2Dx^4+3Ab^3-4a\left(2Da^2-2bCa+b^2B\right)}{a-x^4}d\sqrt{bx^2+a}}{2a}+\frac{b^2\sqrt{a+bx^2}(3Ab-4aB)}{2a(a-x^4)}}{2ab}-\frac{A\sqrt{a+bx^2}}{2ax^4}\right)$$

$$\Big\downarrow \; 299$$

$$\frac{1}{2}\left(-\frac{\frac{b(8a^2C-4abB+3Ab^2)\int \frac{1}{a-x^4}d\sqrt{bx^2+a}-8a^2D\sqrt{a+bx^2}}{2a}+\frac{b^2\sqrt{a+bx^2}(3Ab-4aB)}{2a(a-x^4)}}{2ab}-\frac{A\sqrt{a+bx^2}}{2ax^4}\right)$$

$$\Big\downarrow \; 219$$

$$\frac{1}{2}\left(-\frac{\frac{\frac{b\,\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)(8a^2C-4abB+3Ab^2)}{\sqrt{a}}-8a^2D\sqrt{a+bx^2}}{2a}+\frac{b^2\sqrt{a+bx^2}(3Ab-4aB)}{2a(a-x^4)}}{2ab}-\frac{A\sqrt{a+bx^2}}{2ax^4}\right)$$

input
```
Int[(A + B*x^2 + C*x^4 + D*x^6)/(x^5*Sqrt[a + b*x^2]),x]
```

output
```
(-1/2*(A*Sqrt[a + b*x^2])/(a*x^4) - ((b^2*(3*A*b - 4*a*B)*Sqrt[a + b*x^2])
/(2*a*(a - x^4)) + (-8*a^2*D*Sqrt[a + b*x^2] + (b*(3*A*b^2 - 4*a*b*B + 8*a
^2*C)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a])/(2*a))/(2*a*b))/2
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1192
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(
2*m + 1)*(e*f - d*g + g*x^2)^n*(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 +
 c*x^4)^p, x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &&
IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m + 1/2]
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2124
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
 a*d))), x] + Simp[1/((m + 1)*(b*c - a*d))   Int[(a + b*x)^(m + 1)*(c + d*x
)^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fr
eeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && LtQ[m, -1] && (IntegerQ[m] ||  !
ILtQ[n, -1])
```

rule 2331
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/2   S
ubst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
 FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

## Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.85

| method | result |
|---|---|
| pseudoelliptic | $\frac{-3\left(b^{2} A-\frac{4}{3} a b B+\frac{8}{3} a^{2} C\right) x^{4} b\, a^{2} \operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{\sqrt{a}}\right)+8 \sqrt{b\,x^{2}+a}\, a^{\frac{5}{2}} \left(a^{2} D x^{4}-\frac{b\left(2 x^{2} B+A\right) a}{4}+\frac{3 A\, b^{2} x^{2}}{8}\right)}{8 b\, a^{\frac{9}{2}} x^{4}}$ |
| default | $A\left(-\frac{\sqrt{b\,x^{2}+a}}{4 a\,x^{4}}-\frac{3 b\left(-\frac{\sqrt{b\,x^{2}+a}}{2 a\,x^{2}}+\frac{b \ln\left(\frac{2 a+2 \sqrt{a}\, \sqrt{b\,x^{2}+a}}{x}\right)}{2 a^{\frac{3}{2}}}\right)}{4 a}\right)+B\left(-\frac{\sqrt{b\,x^{2}+a}}{2 a\,x^{2}}+\frac{b \ln\left(\frac{2 a+2 \sqrt{a}\, \sqrt{b\,x^{2}+a}}{x}\right)}{2 a^{\frac{3}{2}}}\right) -$ |

input
```
int((D*x^6+C*x^4+B*x^2+A)/x^5/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*(-3*(b^2*A-4/3*a*b*B+8/3*a^2*C)*x^4*b*a^2*arctanh((b*x^2+a)^(1/2)/a^(1
/2))+8*(b*x^2+a)^(1/2)*a^(5/2)*(a^2*D*x^4-1/4*b*(2*B*x^2+A)*a+3/8*A*b^2*x^
2))/b/a^(9/2)/x^4
```

## Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.98

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^5\sqrt{a+bx^2}}\,dx$$
$$= \left[\frac{(8\,Ca^2b-4\,Bab^2+3\,Ab^3)\sqrt{a}x^4\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\,(8\,Da^3x^4-2\,Aa^2b-(4\,Ba^2b-3\,Aab^2)}{16\,a^3bx^4}\right.$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^5/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/16*((8*C*a^2*b - 4*B*a*b^2 + 3*A*b^3)*sqrt(a)*x^4*log(-(b*x^2 - 2*sqrt(
b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(8*D*a^3*x^4 - 2*A*a^2*b - (4*B*a^2*b -
 3*A*a*b^2)*x^2)*sqrt(b*x^2 + a))/(a^3*b*x^4), 1/8*((8*C*a^2*b - 4*B*a*b^2
 + 3*A*b^3)*sqrt(-a)*x^4*arctan(sqrt(b*x^2 + a)*sqrt(-a)/a) + (8*D*a^3*x^4
 - 2*A*a^2*b - (4*B*a^2*b - 3*A*a*b^2)*x^2)*sqrt(b*x^2 + a))/(a^3*b*x^4)]
```

## Sympy [A] (verification not implemented)

Time = 37.97 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.70

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^5\sqrt{a+bx^2}}\,dx = -\frac{A}{4\sqrt{b}x^5\sqrt{\frac{a}{bx^2}+1}} + \frac{A\sqrt{b}}{8ax^3\sqrt{\frac{a}{bx^2}+1}} + \frac{3Ab^{\frac{3}{2}}}{8a^2x\sqrt{\frac{a}{bx^2}+1}} - \frac{3Ab^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{8a^{\frac{5}{2}}} - \frac{B\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{2ax} + \frac{Bb\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2a^{\frac{3}{2}}} - \frac{C\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{\sqrt{a}} + D\left(\begin{cases}\frac{\sqrt{a+bx^2}}{b} & \text{for } b\neq 0\\ \frac{x^2}{2\sqrt{a}} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((D*x**6+C*x**4+B*x**2+A)/x**5/(b*x**2+a)**(1/2),x)
```

output
```
-A/(4*sqrt(b)*x**5*sqrt(a/(b*x**2) + 1)) + A*sqrt(b)/(8*a*x**3*sqrt(a/(b*x
**2) + 1)) + 3*A*b**(3/2)/(8*a**2*x*sqrt(a/(b*x**2) + 1)) - 3*A*b**2*asinh
(sqrt(a)/(sqrt(b)*x))/(8*a**(5/2)) - B*sqrt(b)*sqrt(a/(b*x**2) + 1)/(2*a*x
) + B*b*asinh(sqrt(a)/(sqrt(b)*x))/(2*a**(3/2)) - C*asinh(sqrt(a)/(sqrt(b)
*x))/sqrt(a) + D*Piecewise((sqrt(a + b*x**2)/b, Ne(b, 0)), (x**2/(2*sqrt(a
)), True))
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.12

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^5\sqrt{a+bx^2}}\,dx = -\frac{C\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{\sqrt{a}} + \frac{Bb\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{2\,a^{\frac{3}{2}}} - \frac{3\,Ab^2\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{8\,a^{\frac{5}{2}}} + \frac{\sqrt{bx^2+a}D}{b} - \frac{\sqrt{bx^2+a}B}{2\,ax^2} + \frac{3\,\sqrt{bx^2+a}Ab}{8\,a^2x^2} - \frac{\sqrt{bx^2+a}A}{4\,ax^4}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^5/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
-C*arcsinh(a/(sqrt(a*b)*abs(x)))/sqrt(a) + 1/2*B*b*arcsinh(a/(sqrt(a*b)*ab
s(x)))/a^(3/2) - 3/8*A*b^2*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(5/2) + sqrt(b*
x^2 + a)*D/b - 1/2*sqrt(b*x^2 + a)*B/(a*x^2) + 3/8*sqrt(b*x^2 + a)*A*b/(a^
2*x^2) - 1/4*sqrt(b*x^2 + a)*A/(a*x^4)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.24

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^5\sqrt{a+bx^2}}\,dx = \frac{8\sqrt{bx^2+a}D + \frac{(8Ca^2b-4Bab^2+3Ab^3)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2} - \frac{4(bx^2+a)^{\frac{3}{2}}Bab^2-4\sqrt{bx^2+a}Ba^2b^2-3(bx^2+a)^{\frac{3}{2}}Ab^3+5\sqrt{bx^2+a}Aab^3}{a^2b^2x^4}}{8b}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^5/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/8*(8*sqrt(b*x^2 + a)*D + (8*C*a^2*b - 4*B*a*b^2 + 3*A*b^3)*arctan(sqrt(b
*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^2) - (4*(b*x^2 + a)^(3/2)*B*a*b^2 - 4*sqrt
(b*x^2 + a)*B*a^2*b^2 - 3*(b*x^2 + a)^(3/2)*A*b^3 + 5*sqrt(b*x^2 + a)*A*a*
b^3)/(a^2*b^2*x^4))/b
```

## Mupad [B] (verification not implemented)

Time = 1.99 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.42

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^5\sqrt{a+bx^2}}\,dx = \frac{a\left(\frac{b\,x^2}{a}+1\right)D - a\sqrt{\frac{b\,x^2}{a}+1}\,D}{b\sqrt{b\,x^2+a}} - \frac{C\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{\sqrt{a}} - \frac{3\,A\,b^2\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{8\,a^{5/2}} - \frac{5\,A\sqrt{b\,x^2+a}}{8\,a\,x^4} + \frac{3\,A\,(b\,x^2+a)^{3/2}}{8\,a^2\,x^4} - \frac{B\sqrt{b\,x^2+a}}{2\,a\,x^2} + \frac{B\,b\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2\,a^{3/2}}$$

input
```
int((A + B*x^2 + C*x^4 + x^6*D)/(x^5*(a + b*x^2)^(1/2)),x)
```

output
```
(a*((b*x^2)/a + 1)*D - a*((b*x^2)/a + 1)^(1/2)*D)/(b*(a + b*x^2)^(1/2)) -
(C*atanh((a + b*x^2)^(1/2)/a^(1/2)))/a^(1/2) - (3*A*b^2*atanh((a + b*x^2)^
(1/2)/a^(1/2)))/(8*a^(5/2)) - (5*A*(a + b*x^2)^(1/2))/(8*a*x^4) + (3*A*(a
+ b*x^2)^(3/2))/(8*a^2*x^4) - (B*(a + b*x^2)^(1/2))/(2*a*x^2) + (B*b*atanh
((a + b*x^2)^(1/2)/a^(1/2)))/(2*a^(3/2))
```

## Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.64

$$\int \frac{A + Bx^2 + Cx^4 + Dx^6}{x^5\sqrt{a + bx^2}}\,dx$$
$$= \frac{-2\sqrt{b\,x^2+a}\,a^2b + 8\sqrt{b\,x^2+a}\,a^2d\,x^4 - \sqrt{b\,x^2+a}\,a\,b^2x^2 + 8\sqrt{a}\log\left(\frac{\sqrt{b\,x^2+a}-\sqrt{a}+\sqrt{b}\,x}{\sqrt{a}}\right)abc\,x^4 - \sqrt{a}\log\left(\right.}{8a^2b\,x^4}$$

input
```
int((D*x^6+C*x^4+B*x^2+A)/x^5/(b*x^2+a)^(1/2),x)
```

output
```
( - 2*sqrt(a + b*x**2)*a**2*b + 8*sqrt(a + b*x**2)*a**2*d*x**4 - sqrt(a +
b*x**2)*a*b**2*x**2 + 8*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*
x)/sqrt(a))*a*b*c*x**4 - sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)
*x)/sqrt(a))*b**3*x**4 - 8*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(
b)*x)/sqrt(a))*a*b*c*x**4 + sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt
(b)*x)/sqrt(a))*b**3*x**4)/(8*a**2*b*x**4)
```

# 3.214 $\int \frac{A+Bx^2+Cx^4+Dx^6}{x^7\sqrt{a+bx^2}}\,dx$



## Optimal result

Integrand size = 32, antiderivative size = 147

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^7\sqrt{a+bx^2}}\,dx = -\frac{A\sqrt{a+bx^2}}{6ax^6}+\frac{(5Ab-6aB)\sqrt{a+bx^2}}{24a^2x^4} - \frac{(5Ab^2-6abB+8a^2C)\sqrt{a+bx^2}}{16a^3x^2} + \frac{(5Ab^3-2a(3b^2B-4abC+8a^2D))\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{7/2}}$$

output
```
-1/6*A*(b*x^2+a)^(1/2)/a/x^6+1/24*(5*A*b-6*B*a)*(b*x^2+a)^(1/2)/a^2/x^4-1/
16*(5*A*b^2-6*B*a*b+8*C*a^2)*(b*x^2+a)^(1/2)/a^3/x^2+1/16*(5*A*b^3-2*a*(3*
B*b^2-4*C*a*b+8*D*a^2))*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(7/2)
```

## Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.86

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^7\sqrt{a+bx^2}}\,dx = \frac{\sqrt{a+bx^2}(-8a^2A+10aAbx^2-12a^2Bx^2-15Ab^2x^4+18abBx^4-24a^2Cx^4)}{48a^3x^6} + \frac{(5Ab^3-6ab^2B+8a^2bC-16a^3D)\,\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{7/2}}$$

input
```
Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(x^7*Sqrt[a + b*x^2]),x]
```

output
```
(Sqrt[a + b*x^2]*(-8*a^2*A + 10*a*A*b*x^2 - 12*a^2*B*x^2 - 15*A*b^2*x^4 +
18*a*b*B*x^4 - 24*a^2*C*x^4))/(48*a^3*x^6) + ((5*A*b^3 - 6*a*b^2*B + 8*a^2
*b*C - 16*a^3*D)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(16*a^(7/2))
```

## Rubi [A] (warning: unable to verify)

Time = 0.52 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.24, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.281$, Rules used = {2331, 2124, 27, 1192, 25, 1471, 27, 298, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^7\sqrt{a+bx^2}}\,dx$$

$$\Big\downarrow \text{2331}$$

$$\frac{1}{2}\int \frac{Dx^6+Cx^4+Bx^2+A}{x^8\sqrt{bx^2+a}}dx^2$$

$$\Big\downarrow \text{2124}$$

$$\frac{1}{2}\left(-\frac{\int \frac{-6aDx^4-6aCx^2+5Ab-6aB}{2x^6\sqrt{bx^2+a}}dx^2}{3a}-\frac{A\sqrt{a+bx^2}}{3ax^6}\right)$$

$$\Big\downarrow \ 27$$

$$\frac{1}{2}\left(-\frac{\int \frac{-6aDx^4-6aCx^2+5Ab-6aB}{x^6\sqrt{bx^2+a}}dx^2}{6a}-\frac{A\sqrt{a+bx^2}}{3ax^6}\right)$$

$$\Big\downarrow \ 1192$$

$$\frac{1}{2}\left(-\frac{\int -\frac{-6aDx^8-6a(bC-2aD)x^4+5Ab^3-6a\left(Da^2-bCa+b^2B\right)}{(a-x^4)^3}d\sqrt{bx^2+a}}{3a}-\frac{A\sqrt{a+bx^2}}{3ax^6}\right)$$

$$\Big\downarrow \ 25$$

$$\frac{1}{2}\left(\frac{\int \frac{-6aDx^8-6a(bC-2aD)x^4+5Ab^3-6a\left(Da^2-bCa+b^2B\right)}{(a-x^4)^3}d\sqrt{bx^2+a}}{3a}-\frac{A\sqrt{a+bx^2}}{3ax^6}\right)$$

$$\Big\downarrow \ 1471$$

$$\frac{1}{2}\left(-\frac{\frac{\int -\frac{3\left(8a^2Dx^4+5Ab^3-6ab^2B+8a^2bC-8a^3D\right)}{\left(a-x^4\right)^2}d\sqrt{bx^2+a}}{4a}-\frac{b^2\sqrt{a+bx^2}(5Ab-6aB)}{4a(a-x^4)^2}}{3a}-\frac{A\sqrt{a+bx^2}}{3ax^6}\right)$$

$$\Big\downarrow \ 27$$

$$\frac{1}{2}\left(-\frac{-\frac{3\int \frac{8a^2Dx^4+5Ab^3-6ab^2B+8a^2bC-8a^3D}{\left(a-x^4\right)^2}d\sqrt{bx^2+a}}{4a}-\frac{b^2\sqrt{a+bx^2}(5Ab-6aB)}{4a(a-x^4)^2}}{3a}-\frac{A\sqrt{a+bx^2}}{3ax^6}\right)$$

$$\Big\downarrow \ 298$$

$$\frac{1}{2}\left(-\frac{-\frac{3\left(\frac{\left(-16a^3D+8a^2bC-6ab^2B+5Ab^3\right)\int \frac{1}{a-x^4}d\sqrt{bx^2+a}}{2a}+\frac{b\sqrt{a+bx^2}\left(8a^2C-6abB+5Ab^2\right)}{2a\left(a-x^4\right)}\right)}{4a}-\frac{b^2\sqrt{a+bx^2}(5Ab-6aB)}{4a(a-x^4)^2}}{3a}-\frac{A\sqrt{a+bx^2}}{3ax^6}\right)$$

$$\Big\downarrow \ 219$$

$$\frac{1}{2}\left(-\frac{-\frac{3\left(\frac{b\sqrt{a+bx^2}\left(8a^2C-6abB+5Ab^2\right)}{2a\left(a-x^4\right)}+\frac{\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\left(-16a^3D+8a^2bC-6ab^2B+5Ab^3\right)}{2a^{3/2}}\right)}{4a}-\frac{b^2\sqrt{a+bx^2}(5Ab-6aB)}{4a(a-x^4)^2}}{3a}-\frac{A\sqrt{a+bx^2}}{3ax^6}\right.$$

input
```
Int[(A + B*x^2 + C*x^4 + D*x^6)/(x^7*Sqrt[a + b*x^2]),x]
```

output
```
(-1/3*(A*Sqrt[a + b*x^2])/(a*x^6) - (-1/4*(b^2*(5*A*b - 6*a*B)*Sqrt[a + b*
x^2])/(a*(a - x^4)^2) - (3*((b*(5*A*b^2 - 6*a*b*B + 8*a^2*C)*Sqrt[a + b*x^
2])/(2*a*(a - x^4)) + ((5*A*b^3 - 6*a*b^2*B + 8*a^2*b*C - 16*a^3*D)*ArcTan
h[Sqrt[a + b*x^2]/Sqrt[a]])/(2*a^(3/2))))/(4*a))/(3*a))/2
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 1192
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(
2*m + 1)*(e*f - d*g + g*x^2)^n*(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 +
 c*x^4)^p, x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &&
IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m + 1/2]
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2124
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
 a*d))), x] + Simp[1/((m + 1)*(b*c - a*d))   Int[(a + b*x)^(m + 1)*(c + d*x
)^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fr
eeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && LtQ[m, -1] && (IntegerQ[m] ||  !
ILtQ[n, -1])
```

rule 2331
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/2   S
ubst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
 FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

## Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.79

| method | result |
|---|---|
| pseudoelliptic | $-\frac{5\left(-x^{6}\left(b^{3}A-\frac{6}{5}ab^{2}B+\frac{8}{5}a^{2}bC-\frac{16}{5}a^{3}D\right)\operatorname{arctanh}\left(\frac{\sqrt{bx^{2}+a}}{\sqrt{a}}\right)+\left(\frac{4\left(2Cx^{4}+x^{2}B+\frac{2}{3}A\right)a^{\frac{5}{2}}}{5}+\left(2\left(-\frac{3x^{2}B}{5}-\frac{A}{3}\right)a^{\frac{3}{2}}+Abx^{2}\sqrt{\cdots}\right.\right.}{16a^{\frac{7}{2}}x^{6}}$ |
| default | $A\left(-\frac{\sqrt{bx^{2}+a}}{6ax^{6}}-\frac{5b\left(-\frac{\sqrt{bx^{2}+a}}{4ax^{4}}-\frac{3b\left(-\frac{\sqrt{bx^{2}+a}}{2ax^{2}}+\frac{b\ln\left(\frac{2a+2\sqrt{a}\sqrt{bx^{2}+a}}{x}\right)}{2a^{\frac{3}{2}}}\right)}{4a}\right)}{6a}\right)+B\left(-\frac{\sqrt{bx^{2}+a}}{4ax^{4}}-\frac{3b\left(-\frac{\sqrt{bx\cdots}}{2a\cdots}\right.}{\cdots}\right.$ |

input
```
int((D*x^6+C*x^4+B*x^2+A)/x^7/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-5/16*(-x^6*(b^3*A-6/5*a*b^2*B+8/5*a^2*b*C-16/5*a^3*D)*arctanh((b*x^2+a)^(
1/2)/a^(1/2))+(4/5*(2*C*x^4+x^2*B+2/3*A)*a^(5/2)+(2*(-3/5*x^2*B-1/3*A)*a^(
3/2)+A*b*x^2*a^(1/2))*x^2*b)*(b*x^2+a)^(1/2))/a^(7/2)/x^6
```

## Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.80

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^7\sqrt{a+bx^2}}\,dx$$

$$=\left[-\frac{3\left(16\,Da^{3}-8\,Ca^{2}b+6\,Bab^{2}-5\,Ab^{3}\right)\sqrt{a}x^{6}\log\left(-\frac{bx^{2}+2\,\sqrt{bx^{2}+a}\sqrt{a}+2\,a}{x^{2}}\right)+2\left(3\left(8\,Ca^{3}-6\,Ba^{2}b+5\,Aa\cdots\right.\right.}{96\,a^{4}x^{6}}\right.$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^7/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/96*(3*(16*D*a^3 - 8*C*a^2*b + 6*B*a*b^2 - 5*A*b^3)*sqrt(a)*x^6*log(-(b
*x^2 + 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(3*(8*C*a^3 - 6*B*a^2*b +
 5*A*a*b^2)*x^4 + 8*A*a^3 + 2*(6*B*a^3 - 5*A*a^2*b)*x^2)*sqrt(b*x^2 + a))/
(a^4*x^6), 1/48*(3*(16*D*a^3 - 8*C*a^2*b + 6*B*a*b^2 - 5*A*b^3)*sqrt(-a)*x
^6*arctan(sqrt(b*x^2 + a)*sqrt(-a)/a) - (3*(8*C*a^3 - 6*B*a^2*b + 5*A*a*b^
2)*x^4 + 8*A*a^3 + 2*(6*B*a^3 - 5*A*a^2*b)*x^2)*sqrt(b*x^2 + a))/(a^4*x^6)
]
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 303 vs. $2(141) = 282$.

Time = 47.33 (sec) , antiderivative size = 303, normalized size of antiderivative = 2.06

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^7\sqrt{a+bx^2}}\,dx = -\frac{A}{6\sqrt{b}x^7\sqrt{\frac{a}{bx^2}+1}} + \frac{A\sqrt{b}}{24ax^5\sqrt{\frac{a}{bx^2}+1}} - \frac{5Ab^{\frac{3}{2}}}{48a^2x^3\sqrt{\frac{a}{bx^2}+1}} - \frac{5Ab^{\frac{5}{2}}}{16a^3x\sqrt{\frac{a}{bx^2}+1}} + \frac{5Ab^3\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{16a^{\frac{7}{2}}} - \frac{B}{4\sqrt{b}x^5\sqrt{\frac{a}{bx^2}+1}} + \frac{B\sqrt{b}}{8ax^3\sqrt{\frac{a}{bx^2}+1}} + \frac{3Bb^{\frac{3}{2}}}{8a^2x\sqrt{\frac{a}{bx^2}+1}} - \frac{3Bb^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{8a^{\frac{5}{2}}} - \frac{C\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{2ax} + \frac{Cb\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2a^{\frac{3}{2}}} - \frac{D\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{\sqrt{a}}$$

input
```
integrate((D*x**6+C*x**4+B*x**2+A)/x**7/(b*x**2+a)**(1/2),x)
```

output
```
-A/(6*sqrt(b)*x**7*sqrt(a/(b*x**2) + 1)) + A*sqrt(b)/(24*a*x**5*sqrt(a/(b*
x**2) + 1)) - 5*A*b**(3/2)/(48*a**2*x**3*sqrt(a/(b*x**2) + 1)) - 5*A*b**(5
/2)/(16*a**3*x*sqrt(a/(b*x**2) + 1)) + 5*A*b**3*asinh(sqrt(a)/(sqrt(b)*x))
/(16*a**(7/2)) - B/(4*sqrt(b)*x**5*sqrt(a/(b*x**2) + 1)) + B*sqrt(b)/(8*a*
x**3*sqrt(a/(b*x**2) + 1)) + 3*B*b**(3/2)/(8*a**2*x*sqrt(a/(b*x**2) + 1))
- 3*B*b**2*asinh(sqrt(a)/(sqrt(b)*x))/(8*a**(5/2)) - C*sqrt(b)*sqrt(a/(b*x
**2) + 1)/(2*a*x) + C*b*asinh(sqrt(a)/(sqrt(b)*x))/(2*a**(3/2)) - D*asinh(
sqrt(a)/(sqrt(b)*x))/sqrt(a)
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.31

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^7\sqrt{a+bx^2}}\,dx = -\frac{D\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{\sqrt{a}} + \frac{Cb\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{2\,a^{\frac{3}{2}}} - \frac{3\,Bb^2\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{8\,a^{\frac{5}{2}}} + \frac{5\,Ab^3\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{16\,a^{\frac{7}{2}}} - \frac{\sqrt{bx^2+a}C}{2\,ax^2} + \frac{3\,\sqrt{bx^2+a}Bb}{8\,a^2x^2} - \frac{5\,\sqrt{bx^2+a}Ab^2}{16\,a^3x^2} - \frac{\sqrt{bx^2+a}B}{4\,ax^4} + \frac{5\,\sqrt{bx^2+a}Ab}{24\,a^2x^4} - \frac{\sqrt{bx^2+a}A}{6\,ax^6}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^7/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
-D*arcsinh(a/(sqrt(a*b)*abs(x)))/sqrt(a) + 1/2*C*b*arcsinh(a/(sqrt(a*b)*ab
s(x)))/a^(3/2) - 3/8*B*b^2*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(5/2) + 5/16*A*
b^3*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(7/2) - 1/2*sqrt(b*x^2 + a)*C/(a*x^2)
+ 3/8*sqrt(b*x^2 + a)*B*b/(a^2*x^2) - 5/16*sqrt(b*x^2 + a)*A*b^2/(a^3*x^2)
 - 1/4*sqrt(b*x^2 + a)*B/(a*x^4) + 5/24*sqrt(b*x^2 + a)*A*b/(a^2*x^4) - 1/
6*sqrt(b*x^2 + a)*A/(a*x^6)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.46

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^7\sqrt{a+bx^2}}\,dx = \frac{1}{48}\,b^3\left(\frac{3\,(16\,Da^3-8\,Ca^2b+6\,Bab^2-5\,Ab^3)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^3b^3} - \frac{24\,(bx^2+a)^{\frac{5}{2}}Ca^2-48\,(bx^2+a)^{\frac{3}{2}}Ca^3}{}\right.$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^7/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/48*b^3*(3*(16*D*a^3 - 8*C*a^2*b + 6*B*a*b^2 - 5*A*b^3)*arctan(sqrt(b*x^2
 + a)/sqrt(-a))/(sqrt(-a)*a^3*b^3) - (24*(b*x^2 + a)^(5/2)*C*a^2 - 48*(b*x
^2 + a)^(3/2)*C*a^3 + 24*sqrt(b*x^2 + a)*C*a^4 - 18*(b*x^2 + a)^(5/2)*B*a*
b + 48*(b*x^2 + a)^(3/2)*B*a^2*b - 30*sqrt(b*x^2 + a)*B*a^3*b + 15*(b*x^2
+ a)^(5/2)*A*b^2 - 40*(b*x^2 + a)^(3/2)*A*a*b^2 + 33*sqrt(b*x^2 + a)*A*a^2
*b^2)/(a^3*b^5*x^6))
```

## Mupad [B] (verification not implemented)

Time = 2.30 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.37

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^7\sqrt{a+bx^2}}\,dx = \frac{\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{-a}}\right)\,D}{\sqrt{-a}} - \frac{3\,B\,b^2\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{8\,a^{5/2}} - \frac{11\,A\,\sqrt{b\,x^2+a}}{16\,a\,x^6} + \frac{5\,A\,{\left(b\,x^2+a\right)}^{3/2}}{6\,a^2\,x^6} - \frac{5\,A\,{\left(b\,x^2+a\right)}^{5/2}}{16\,a^3\,x^6} - \frac{5\,B\,\sqrt{b\,x^2+a}}{8\,a\,x^4} + \frac{3\,B\,{\left(b\,x^2+a\right)}^{3/2}}{8\,a^2\,x^4} - \frac{C\,\sqrt{b\,x^2+a}}{2\,a\,x^2} + \frac{C\,b\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2\,a^{3/2}} - \frac{A\,b^3\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,5{}\mathrm{i}}{16\,a^{7/2}}$$

input
```
int((A + B*x^2 + C*x^4 + x^6*D)/(x^7*(a + b*x^2)^(1/2)),x)
```

output
```
(atan((a + b*x^2)^(1/2)/(-a)^(1/2))*D)/(-a)^(1/2) - (A*b^3*atan(((a + b*x^
2)^(1/2)*1i)/a^(1/2))*5i)/(16*a^(7/2)) - (3*B*b^2*atanh((a + b*x^2)^(1/2)/
a^(1/2)))/(8*a^(5/2)) - (11*A*(a + b*x^2)^(1/2))/(16*a*x^6) + (5*A*(a + b*
x^2)^(3/2))/(6*a^2*x^6) - (5*A*(a + b*x^2)^(5/2))/(16*a^3*x^6) - (5*B*(a +
 b*x^2)^(1/2))/(8*a*x^4) + (3*B*(a + b*x^2)^(3/2))/(8*a^2*x^4) - (C*(a + b
*x^2)^(1/2))/(2*a*x^2) + (C*b*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(2*a^(3/2)
)
```

## Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.82

$$\int \frac{A + Bx^2 + Cx^4 + Dx^6}{x^7\sqrt{a + bx^2}}\,dx$$
$$= \frac{-8\sqrt{b\,x^2+a}\,a^3 - 2\sqrt{b\,x^2+a}\,a^2 b\,x^2 - 24\sqrt{b\,x^2+a}\,a^2 c\,x^4 + 3\sqrt{b\,x^2+a}\,a\,b^2 x^4 + 48\sqrt{a}\log\left(\frac{\sqrt{b\,x^2+a}-\sqrt{a}+\sqrt{}}{\sqrt{a}}\right.}{}$$

input
```
int((D*x^6+C*x^4+B*x^2+A)/x^7/(b*x^2+a)^(1/2),x)
```

output
```
( - 8*sqrt(a + b*x**2)*a**3 - 2*sqrt(a + b*x**2)*a**2*b*x**2 - 24*sqrt(a +
 b*x**2)*a**2*c*x**4 + 3*sqrt(a + b*x**2)*a*b**2*x**4 + 48*sqrt(a)*log((sq
rt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*a**2*d*x**6 - 24*sqrt(a)*lo
g((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b*c*x**6 + 3*sqrt(a)
*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*b**3*x**6 - 48*sqrt
(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*a**2*d*x**6 + 24
*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b*c*x**6
- 3*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*b**3*x**
6)/(48*a**3*x**6)
```

# 3.215 $\int \frac{A+Bx^2+Cx^4+Dx^6}{x^9\sqrt{a+bx^2}}\,dx$



## Optimal result

Integrand size = 32, antiderivative size = 197

$$\begin{aligned}\int \frac{A+Bx^2+Cx^4+Dx^6}{x^9\sqrt{a+bx^2}}\,dx &= -\frac{A\sqrt{a+bx^2}}{8ax^8}+\frac{(7Ab-8aB)\sqrt{a+bx^2}}{48a^2x^6}-\frac{(35Ab^2-40abB+48a^2C)\sqrt{a+bx^2}}{192a^3x^4}\\ &\quad+\frac{(35Ab^3-8a(5b^2B-6abC+8a^2D))\sqrt{a+bx^2}}{128a^4x^2}\\ &\quad-\frac{b(35Ab^3-8a(5b^2B-6abC+8a^2D))\,\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{128a^{9/2}}\end{aligned}$$

output
```
-1/8*A*(b*x^2+a)^(1/2)/a/x^8+1/48*(7*A*b-8*B*a)*(b*x^2+a)^(1/2)/a^2/x^6-1/
192*(35*A*b^2-40*B*a*b+48*C*a^2)*(b*x^2+a)^(1/2)/a^3/x^4+1/128*(35*A*b^3-8
*a*(5*B*b^2-6*C*a*b+8*D*a^2))*(b*x^2+a)^(1/2)/a^4/x^2-1/128*b*(35*A*b^3-8*
a*(5*B*b^2-6*C*a*b+8*D*a^2))*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(9/2)
```

## Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.81

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^9\sqrt{a+bx^2}}\,dx = \frac{\frac{\sqrt{a}\sqrt{a+bx^2}(105Ab^3x^6-10ab^2x^4(7A+12Bx^2)+8a^2bx^2(7A+10Bx^2+18Cx^4)-16a^3(3A+4Bx^2+6Cx^4+12Dx^6))}{x^8}-3b(35Ab^3-8a(5b}{384a^{9/2}}$$

input
```
Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(x^9*Sqrt[a + b*x^2]),x]
```

output
```
((Sqrt[a]*Sqrt[a + b*x^2]*(105*A*b^3*x^6 - 10*a*b^2*x^4*(7*A + 12*B*x^2) +
 8*a^2*b*x^2*(7*A + 10*B*x^2 + 18*C*x^4) - 16*a^3*(3*A + 4*B*x^2 + 6*C*x^4
 + 12*D*x^6)))/x^8 - 3*b*(35*A*b^3 - 8*a*(5*b^2*B - 6*a*b*C + 8*a^2*D))*Ar
cTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(384*a^(9/2))
```

## Rubi [A] (warning: unable to verify)

Time = 0.52 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.11, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.281, Rules used = {2331, 2124, 27, 1192, 1471, 25, 298, 215, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^9\sqrt{a+bx^2}}dx$$

$$\Big\downarrow \text{ 2331}$$

$$\frac{1}{2}\int \frac{Dx^6+Cx^4+Bx^2+A}{x^{10}\sqrt{bx^2+a}}dx^2$$

$$\Big\downarrow \text{ 2124}$$

$$\frac{1}{2}\left(-\frac{\int \frac{-8aDx^4-8aCx^2+7Ab-8aB}{2x^8\sqrt{bx^2+a}}dx^2}{4a}-\frac{A\sqrt{a+bx^2}}{4ax^8}\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{2}\left(-\frac{\int \frac{-8aDx^4-8aCx^2+7Ab-8aB}{x^8\sqrt{bx^2+a}}dx^2}{8a}-\frac{A\sqrt{a+bx^2}}{4ax^8}\right)$$

$\Big\downarrow$ 1192

$$\frac{1}{2}\left(-\frac{b\int \frac{-8aDx^8-8a(bC-2aD)x^4+7Ab^3-8a\left(Da^2-bCa+b^2B\right)}{(a-x^4)^4}d\sqrt{bx^2+a}}{4a}-\frac{A\sqrt{a+bx^2}}{4ax^8}\right)$$

$\Big\downarrow$ 1471

$$\frac{1}{2}\left(-\frac{b\left(\frac{b^2\sqrt{a+bx^2}(7Ab-8aB)}{6a(a-x^4)^3}-\frac{\int -\frac{48a^2Dx^4+35Ab^3-40ab^2B+48a^2bC-48a^3D}{\left(a-x^4\right)^3}d\sqrt{bx^2+a}}{6a}\right)}{4a}-\frac{A\sqrt{a+bx^2}}{4ax^8}\right)$$

$\Big\downarrow$ 25

$$\frac{1}{2}\left(-\frac{b\left(\frac{\int \frac{48a^2Dx^4+35Ab^3-40ab^2B+48a^2bC-48a^3D}{\left(a-x^4\right)^3}d\sqrt{bx^2+a}}{6a}+\frac{b^2\sqrt{a+bx^2}(7Ab-8aB)}{6a(a-x^4)^3}\right)}{4a}-\frac{A\sqrt{a+bx^2}}{4ax^8}\right)$$

$\Big\downarrow$ 298

$$\frac{1}{2}\left(-\frac{b\left(\frac{\frac{3\left(-64a^3D+48a^2bC-40ab^2B+35Ab^3\right)\int \frac{1}{\left(a-x^4\right)^2}d\sqrt{bx^2+a}}{4a}+\frac{b\sqrt{a+bx^2}\left(48a^2C-40abB+35Ab^2\right)}{4a\left(a-x^4\right)^2}}{6a}+\frac{b^2\sqrt{a+bx^2}(7Ab-8aB)}{6a(a-x^4)^3}\right)}{4a}-\frac{A\sqrt{a+}}{4ax}\right.$$

$\Big\downarrow$ 215

$$\frac{1}{2}\left(-\frac{b\left(\frac{\frac{3\left(-64a^3D+48a^2bC-40ab^2B+35Ab^3\right)\left(\frac{\int\frac{1}{a-x^4}d\sqrt{bx^2+a}}{2a}+\frac{\sqrt{a+bx^2}}{2a\left(a-x^4\right)}\right)}{4a}+\frac{b\sqrt{a+bx^2}\left(48a^2C-40abB+35Ab^2\right)}{4a\left(a-x^4\right)^2}}{6a}+\frac{b^2\sqrt{a+bx^2}(7Ab-8aB)}{6a(a-x^4)^3}\right)}{4a}\right.$$

$\downarrow$ 219

$$\frac{1}{2}\left(-\frac{b\left(\frac{\frac{b\sqrt{a+bx^2}\left(48a^2C-40abB+35Ab^2\right)}{4a\left(a-x^4\right)^2}+\frac{3\left(\frac{\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}}+\frac{\sqrt{a+bx^2}}{2a\left(a-x^4\right)}\right)\left(-64a^3D+48a^2bC-40ab^2B+35Ab^3\right)}{4a}}{6a}+\frac{b^2\sqrt{a+bx^2}(7Ab-8aB}{6a(a-x^4)^3}\right.}{4a}\right.$$

input

```
Int[(A + B*x^2 + C*x^4 + D*x^6)/(x^9*Sqrt[a + b*x^2]),x]
```

output

```
(-1/4*(A*Sqrt[a + b*x^2])/(a*x^8) - (b*((b^2*(7*A*b - 8*a*B)*Sqrt[a + b*x^
2])/(6*a*(a - x^4)^3) + ((b*(35*A*b^2 - 40*a*b*B + 48*a^2*C)*Sqrt[a + b*x^
2])/(4*a*(a - x^4)^2) + (3*(35*A*b^3 - 40*a*b^2*B + 48*a^2*b*C - 64*a^3*D)
*(Sqrt[a + b*x^2]/(2*a*(a - x^4)) + ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]/(2*a^
(3/2))))/(4*a))/(6*a)))/(4*a))/2
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 1192
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(
2*m + 1)*(e*f - d*g + g*x^2)^n*(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 +
 c*x^4)^p, x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &&
IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m + 1/2]
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2124
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
 a*d))), x] + Simp[1/((m + 1)*(b*c - a*d))   Int[(a + b*x)^(m + 1)*(c + d*x
)^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fr
eeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && LtQ[m, -1] && (IntegerQ[m] ||  !
ILtQ[n, -1])
```

rule 2331
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/2   S
ubst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
 FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

## Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.75

| method | result |
|---|---|
| pseudoelliptic | $-\frac{-\frac{35x^8 b\left(b^3 A-\frac{8}{7}a b^2 B+\frac{48}{35}a^2 b C-\frac{64}{35}a^3 D\right)\operatorname{arctanh}\left(\frac{\sqrt{b x^2+a}}{\sqrt{a}}\right)}{128}+\frac{35\sqrt{b x^2+a}\left(\left(-\frac{64}{35}D x^6-\frac{32}{35}C x^4-\frac{64}{105}x^2 B-\frac{16}{35}A\right)a^{\frac{7}{2}}+\left(\left(\frac{48}{35}C x^4 \cdots\right.\right.\right.}{128 \cdots}}{a^{\frac{9}{2}}x^8}$ |
| default | $A\left(-\frac{\sqrt{b x^2+a}}{8a x^8}-\frac{7b\left(-\frac{\sqrt{b x^2+a}}{6a x^6}-\frac{5b\left(-\frac{\sqrt{b x^2+a}}{4a x^4}-\frac{3b\left(-\frac{\sqrt{b x^2+a}}{2a x^2}+\frac{b\ln\left(\frac{2a+2\sqrt{a}\sqrt{b x^2+a}}{x}\right)}{2a^{\frac{3}{2}}}\right)}{4a}\right)}{6a}\right)}{8a}\right)+B\left(-\frac{\sqrt{b x^2 \cdots}}{6a x \cdots}\right.$ |

input
```
int((D*x^6+C*x^4+B*x^2+A)/x^9/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
35/128*(-x^8*b*(b^3*A-8/7*a*b^2*B+48/35*a^2*b*C-64/35*a^3*D)*arctanh((b*x^
2+a)^(1/2)/a^(1/2))+(b*x^2+a)^(1/2)*((-64/35*D*x^6-32/35*C*x^4-64/105*x^2*
B-16/35*A)*a^(7/2)+((48/35*C*x^4+16/21*x^2*B+8/15*A)*a^(5/2)+x^2*b*((-8/7*
x^2*B-2/3*A)*a^(3/2)+A*b*x^2*a^(1/2)))*x^2*b))/a^(9/2)/x^8
```

## Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.75

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^9\sqrt{a+bx^2}}\,dx$$
$$= \left[-\frac{3\,(64\,Da^3b-48\,Ca^2b^2+40\,Bab^3-35\,Ab^4)\sqrt{a}x^8\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\,(3\,(64\,Da^4-48\,Ca^3b}{}\right.$$
$$-\frac{3\,(64\,Da^3b-48\,Ca^2b^2+40\,Bab^3-35\,Ab^4)\sqrt{-a}x^8\arctan\left(\frac{\sqrt{bx^2+a}\sqrt{-a}}{a}\right)+(3\,(64\,Da^4-48\,Ca^3b+40}{384}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^9/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/768*(3*(64*D*a^3*b - 48*C*a^2*b^2 + 40*B*a*b^3 - 35*A*b^4)*sqrt(a)*x^8
*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(3*(64*D*a^4 - 48
*C*a^3*b + 40*B*a^2*b^2 - 35*A*a*b^3)*x^6 + 48*A*a^4 + 2*(48*C*a^4 - 40*B*
a^3*b + 35*A*a^2*b^2)*x^4 + 8*(8*B*a^4 - 7*A*a^3*b)*x^2)*sqrt(b*x^2 + a))/
(a^5*x^8), -1/384*(3*(64*D*a^3*b - 48*C*a^2*b^2 + 40*B*a*b^3 - 35*A*b^4)*s
qrt(-a)*x^8*arctan(sqrt(b*x^2 + a)*sqrt(-a)/a) + (3*(64*D*a^4 - 48*C*a^3*b
 + 40*B*a^2*b^2 - 35*A*a*b^3)*x^6 + 48*A*a^4 + 2*(48*C*a^4 - 40*B*a^3*b +
35*A*a^2*b^2)*x^4 + 8*(8*B*a^4 - 7*A*a^3*b)*x^2)*sqrt(b*x^2 + a))/(a^5*x^8
)]
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 444 vs. $2(192) = 384$.

Time = 116.02 (sec) , antiderivative size = 444, normalized size of antiderivative = 2.25

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^9\sqrt{a+bx^2}}\,dx = -\frac{A}{8\sqrt{b}x^9\sqrt{\frac{a}{bx^2}+1}} + \frac{A\sqrt{b}}{48ax^7\sqrt{\frac{a}{bx^2}+1}} - \frac{7Ab^{\frac{3}{2}}}{192a^2x^5\sqrt{\frac{a}{bx^2}+1}} + \frac{35Ab^{\frac{5}{2}}}{384a^3x^3\sqrt{\frac{a}{bx^2}+1}} + \frac{35Ab^{\frac{7}{2}}}{128a^4x\sqrt{\frac{a}{bx^2}+1}} - \frac{35Ab^4\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{128a^{\frac{9}{2}}} - \frac{B}{6\sqrt{b}x^7\sqrt{\frac{a}{bx^2}+1}} + \frac{B\sqrt{b}}{24ax^5\sqrt{\frac{a}{bx^2}+1}} - \frac{5Bb^{\frac{3}{2}}}{48a^2x^3\sqrt{\frac{a}{bx^2}+1}} - \frac{5Bb^{\frac{5}{2}}}{16a^3x\sqrt{\frac{a}{bx^2}+1}} + \frac{5Bb^3\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{16a^{\frac{7}{2}}} - \frac{C}{4\sqrt{b}x^5\sqrt{\frac{a}{bx^2}+1}} + \frac{C\sqrt{b}}{8ax^3\sqrt{\frac{a}{bx^2}+1}} + \frac{3Cb^{\frac{3}{2}}}{8a^2x\sqrt{\frac{a}{bx^2}+1}} - \frac{3Cb^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{8a^{\frac{5}{2}}} - \frac{D\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{2ax} + \frac{Db\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2a^{\frac{3}{2}}}$$

input
```
integrate((D*x**6+C*x**4+B*x**2+A)/x**9/(b*x**2+a)**(1/2),x)
```

output
```
-A/(8*sqrt(b)*x**9*sqrt(a/(b*x**2) + 1)) + A*sqrt(b)/(48*a*x**7*sqrt(a/(b*
x**2) + 1)) - 7*A*b**(3/2)/(192*a**2*x**5*sqrt(a/(b*x**2) + 1)) + 35*A*b**
(5/2)/(384*a**3*x**3*sqrt(a/(b*x**2) + 1)) + 35*A*b**(7/2)/(128*a**4*x*sqr
t(a/(b*x**2) + 1)) - 35*A*b**4*asinh(sqrt(a)/(sqrt(b)*x))/(128*a**(9/2)) -
 B/(6*sqrt(b)*x**7*sqrt(a/(b*x**2) + 1)) + B*sqrt(b)/(24*a*x**5*sqrt(a/(b*
x**2) + 1)) - 5*B*b**(3/2)/(48*a**2*x**3*sqrt(a/(b*x**2) + 1)) - 5*B*b**(5
/2)/(16*a**3*x*sqrt(a/(b*x**2) + 1)) + 5*B*b**3*asinh(sqrt(a)/(sqrt(b)*x))
/(16*a**(7/2)) - C/(4*sqrt(b)*x**5*sqrt(a/(b*x**2) + 1)) + C*sqrt(b)/(8*a*
x**3*sqrt(a/(b*x**2) + 1)) + 3*C*b**(3/2)/(8*a**2*x*sqrt(a/(b*x**2) + 1))
 - 3*C*b**2*asinh(sqrt(a)/(sqrt(b)*x))/(8*a**(5/2)) - D*sqrt(b)*sqrt(a/(b*x
**2) + 1)/(2*a*x) + D*b*asinh(sqrt(a)/(sqrt(b)*x))/(2*a**(3/2))
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.40

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^9\sqrt{a+bx^2}}\,dx = \frac{Db\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{2\,a^{\frac{3}{2}}} - \frac{3\,Cb^2\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{8\,a^{\frac{5}{2}}} + \frac{5\,Bb^3\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{16\,a^{\frac{7}{2}}} - \frac{35\,Ab^4\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{128\,a^{\frac{9}{2}}} - \frac{\sqrt{bx^2+a}D}{2\,ax^2} + \frac{3\,\sqrt{bx^2+a}Cb}{8\,a^2x^2} - \frac{5\,\sqrt{bx^2+a}Bb^2}{16\,a^3x^2} + \frac{35\,\sqrt{bx^2+a}Ab^3}{128\,a^4x^2} - \frac{\sqrt{bx^2+a}C}{4\,ax^4} + \frac{5\,\sqrt{bx^2+a}Bb}{24\,a^2x^4} - \frac{35\,\sqrt{bx^2+a}Ab^2}{192\,a^3x^4} - \frac{\sqrt{bx^2+a}B}{6\,ax^6} + \frac{7\,\sqrt{bx^2+a}Ab}{48\,a^2x^6} - \frac{\sqrt{bx^2+a}A}{8\,ax^8}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^9/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/2*D*b*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2) - 3/8*C*b^2*arcsinh(a/(sqrt(
a*b)*abs(x)))/a^(5/2) + 5/16*B*b^3*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(7/2) -
 35/128*A*b^4*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(9/2) - 1/2*sqrt(b*x^2 + a)*
D/(a*x^2) + 3/8*sqrt(b*x^2 + a)*C*b/(a^2*x^2) - 5/16*sqrt(b*x^2 + a)*B*b^2
/(a^3*x^2) + 35/128*sqrt(b*x^2 + a)*A*b^3/(a^4*x^2) - 1/4*sqrt(b*x^2 + a)*
C/(a*x^4) + 5/24*sqrt(b*x^2 + a)*B*b/(a^2*x^4) - 35/192*sqrt(b*x^2 + a)*A*
b^2/(a^3*x^4) - 1/6*sqrt(b*x^2 + a)*B/(a*x^6) + 7/48*sqrt(b*x^2 + a)*A*b/(
a^2*x^6) - 1/8*sqrt(b*x^2 + a)*A/(a*x^8)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 356 vs. 2(173) = 346.

Time = 0.13 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.81

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^9\sqrt{a+bx^2}}\,dx = -\frac{\frac{3\,(64\,Da^3b^2-48\,Ca^2b^3+40\,Bab^4-35\,Ab^5)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^4} + \frac{192\,(bx^2+a)^{\frac{7}{2}}Da^3b^2-576\,(bx^2+a)^{\frac{5}{2}}Da^4b^2+576\,(bx^2+a)^{\frac{3}{2}}Da^5b^2-192\cdots}{\cdots}}{\cdots}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^9/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
-1/384*(3*(64*D*a^3*b^2 - 48*C*a^2*b^3 + 40*B*a*b^4 - 35*A*b^5)*arctan(sqr
t(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^4) + (192*(b*x^2 + a)^(7/2)*D*a^3*b^2 -
 576*(b*x^2 + a)^(5/2)*D*a^4*b^2 + 576*(b*x^2 + a)^(3/2)*D*a^5*b^2 - 192*s
qrt(b*x^2 + a)*D*a^6*b^2 - 144*(b*x^2 + a)^(7/2)*C*a^2*b^3 + 528*(b*x^2 +
a)^(5/2)*C*a^3*b^3 - 624*(b*x^2 + a)^(3/2)*C*a^4*b^3 + 240*sqrt(b*x^2 + a)
*C*a^5*b^3 + 120*(b*x^2 + a)^(7/2)*B*a*b^4 - 440*(b*x^2 + a)^(5/2)*B*a^2*b
^4 + 584*(b*x^2 + a)^(3/2)*B*a^3*b^4 - 264*sqrt(b*x^2 + a)*B*a^4*b^4 - 105
*(b*x^2 + a)^(7/2)*A*b^5 + 385*(b*x^2 + a)^(5/2)*A*a*b^5 - 511*(b*x^2 + a)
^(3/2)*A*a^2*b^5 + 279*sqrt(b*x^2 + a)*A*a^3*b^5)/(a^4*b^4*x^8))/b
```

## Mupad [B] (verification not implemented)

Time = 2.39 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.41

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^9\sqrt{a+bx^2}}\,dx = \frac{b\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)\,D}{2\,a^{3/2}}-\frac{\sqrt{b\,x^2+a}\,D}{2\,a\,x^2}-\frac{3\,C\,b^2\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{8\,a^{5/2}}-\frac{93\,A\,\sqrt{b\,x^2+a}}{128\,a\,x^8}+\frac{511\,A\,{\left(b\,x^2+a\right)}^{3/2}}{384\,a^2\,x^8}-\frac{385\,A\,{\left(b\,x^2+a\right)}^{5/2}}{384\,a^3\,x^8}+\frac{35\,A\,{\left(b\,x^2+a\right)}^{7/2}}{128\,a^4\,x^8}-\frac{11\,B\,\sqrt{b\,x^2+a}}{16\,a\,x^6}+\frac{5\,B\,{\left(b\,x^2+a\right)}^{3/2}}{6\,a^2\,x^6}-\frac{5\,B\,{\left(b\,x^2+a\right)}^{5/2}}{16\,a^3\,x^6}-\frac{5\,C\,\sqrt{b\,x^2+a}}{8\,a\,x^4}+\frac{3\,C\,{\left(b\,x^2+a\right)}^{3/2}}{8\,a^2\,x^4}+\frac{A\,b^4\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,35{}\mathrm{i}}{128\,a^{9/2}}-\frac{B\,b^3\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,5{}\mathrm{i}}{16\,a^{7/2}}$$

input
```
int((A + B*x^2 + C*x^4 + x^6*D)/(x^9*(a + b*x^2)^(1/2)),x)
```

output
```
(A*b^4*atan(((a + b*x^2)^(1/2)*1i)/a^(1/2))*35i)/(128*a^(9/2)) - (B*b^3*at
an(((a + b*x^2)^(1/2)*1i)/a^(1/2))*5i)/(16*a^(7/2)) - (3*C*b^2*atanh((a +
b*x^2)^(1/2)/a^(1/2)))/(8*a^(5/2)) - ((a + b*x^2)^(1/2)*D)/(2*a*x^2) + (b*
atanh((a + b*x^2)^(1/2)/a^(1/2))*D)/(2*a^(3/2)) - (93*A*(a + b*x^2)^(1/2))
/(128*a*x^8) + (511*A*(a + b*x^2)^(3/2))/(384*a^2*x^8) - (385*A*(a + b*x^2
)^(5/2))/(384*a^3*x^8) + (35*A*(a + b*x^2)^(7/2))/(128*a^4*x^8) - (11*B*(a
 + b*x^2)^(1/2))/(16*a*x^6) + (5*B*(a + b*x^2)^(3/2))/(6*a^2*x^6) - (5*B*(
a + b*x^2)^(5/2))/(16*a^3*x^6) - (5*C*(a + b*x^2)^(1/2))/(8*a*x^4) + (3*C*
(a + b*x^2)^(3/2))/(8*a^2*x^4)
```

## Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.66

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^9\sqrt{a+bx^2}}\,dx = \frac{-48\sqrt{b\,x^2+a}\,a^4-8\sqrt{b\,x^2+a}\,a^3b\,x^2-96\sqrt{b\,x^2+a}\,a^3c\,x^4-192\sqrt{b\,x^2+a}\,a^3d\,x^6+10\sqrt{b\,x^2+a}\,a^2b^2x^4}{}$$

input
```
int((D*x^6+C*x^4+B*x^2+A)/x^9/(b*x^2+a)^(1/2),x)
```

output
```
( - 48*sqrt(a + b*x**2)*a**4 - 8*sqrt(a + b*x**2)*a**3*b*x**2 - 96*sqrt(a
+ b*x**2)*a**3*c*x**4 - 192*sqrt(a + b*x**2)*a**3*d*x**6 + 10*sqrt(a + b*x
**2)*a**2*b**2*x**4 + 144*sqrt(a + b*x**2)*a**2*b*c*x**6 - 15*sqrt(a + b*x
**2)*a*b**3*x**6 - 192*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x
)/sqrt(a))*a**2*b*d*x**8 + 144*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + s
qrt(b)*x)/sqrt(a))*a*b**2*c*x**8 - 15*sqrt(a)*log((sqrt(a + b*x**2) - sqrt
(a) + sqrt(b)*x)/sqrt(a))*b**4*x**8 + 192*sqrt(a)*log((sqrt(a + b*x**2) +
sqrt(a) + sqrt(b)*x)/sqrt(a))*a**2*b*d*x**8 - 144*sqrt(a)*log((sqrt(a + b*
x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b**2*c*x**8 + 15*sqrt(a)*log((sqrt
(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*b**4*x**8)/(384*a**4*x**8)
```

# 3.216 $\int \frac{x^4\left(A+Bx^2+Cx^4+Dx^6\right)}{\sqrt{a+bx^2}}\,dx$



## Optimal result

Integrand size = 32, antiderivative size = 248

$$\int \frac{x^4(A+Bx^2+Cx^4+Dx^6)}{\sqrt{a+bx^2}}\,dx = -\frac{a(96Ab^3-a(80b^2B-70abC+63a^2D))\,x\sqrt{a+bx^2}}{256b^5} + \frac{(96Ab^3-a(80b^2B-70abC+63a^2D))\,x^3\sqrt{a+bx^2}}{384b^4} + \frac{(80b^2B-70abC+63a^2D)\,x^5\sqrt{a+bx^2}}{480b^3} + \frac{(10bC-9aD)x^7\sqrt{a+bx^2}}{80b^2} + \frac{Dx^9\sqrt{a+bx^2}}{10b} + \frac{a^2(96Ab^3-a(80b^2B-70abC+63a^2D))\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{256b^{11/2}}$$

output
```
-1/256*a*(96*A*b^3-a*(80*B*b^2-70*C*a*b+63*D*a^2))*x*(b*x^2+a)^(1/2)/b^5+1
/384*(96*A*b^3-a*(80*B*b^2-70*C*a*b+63*D*a^2))*x^3*(b*x^2+a)^(1/2)/b^4+1/4
80*(80*B*b^2-70*C*a*b+63*D*a^2)*x^5*(b*x^2+a)^(1/2)/b^3+1/80*(10*C*b-9*D*a
)*x^7*(b*x^2+a)^(1/2)/b^2+1/10*D*x^9*(b*x^2+a)^(1/2)/b+1/256*a^2*(96*A*b^3
-a*(80*B*b^2-70*C*a*b+63*D*a^2))*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(11/
2)
```

## Mathematica [A] (verified)

Time = 1.55 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.88

$$\int \frac{x^4(A+Bx^2+Cx^4+Dx^6)}{\sqrt{a+bx^2}}\,dx$$
$$= \frac{x\sqrt{a+bx^2}(-1440aAb^3+1200a^2b^2B-1050a^3bC+945a^4D+960Ab^4x^2-800ab^3Bx^2+700a^2b^2Cx^2-\cdots}{3840b^5}$$
$$-\frac{a^2(-96Ab^3+80ab^2B-70a^2bC+63a^3D)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{-\sqrt{a}+\sqrt{a+bx^2}}\right)}{128b^{11/2}}$$

input
```
Integrate[(x^4*(A + B*x^2 + C*x^4 + D*x^6))/Sqrt[a + b*x^2],x]
```

output
```
(x*Sqrt[a + b*x^2]*(-1440*a*A*b^3 + 1200*a^2*b^2*B - 1050*a^3*b*C + 945*a^
4*D + 960*A*b^4*x^2 - 800*a*b^3*B*x^2 + 700*a^2*b^2*C*x^2 - 630*a^3*b*D*x^
2 + 640*b^4*B*x^4 - 560*a*b^3*C*x^4 + 504*a^2*b^2*D*x^4 + 480*b^4*C*x^6 -
432*a*b^3*D*x^6 + 384*b^4*D*x^8))/(3840*b^5) - (a^2*(-96*A*b^3 + 80*a*b^2*
B - 70*a^2*b*C + 63*a^3*D)*ArcTanh[(Sqrt[b]*x)/(-Sqrt[a] + Sqrt[a + b*x^2]
)])/(128*b^(11/2))
```

## Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.90, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {2340, 1590, 363, 262, 262, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4\left(A+Bx^2+Cx^4+Dx^6\right)}{\sqrt{a+bx^2}}dx$$
$$\Big\downarrow \text{2340}$$
$$\frac{\int \frac{x^4\left((10bC-9aD)x^4+10bBx^2+10Ab\right)}{\sqrt{bx^2+a}}dx}{10b}+\frac{Dx^9\sqrt{a+bx^2}}{10b}$$
$$\Big\downarrow \text{1590}$$

$$\frac{\frac{\int \frac{x^4\left(80Ab^2+\left(63Da^2-70bCa+80b^2B\right)x^2\right)}{\sqrt{bx^2+a}}dx}{8b}+\frac{x^7\sqrt{a+bx^2}(10bC-9aD)}{8b}}{10b}+\frac{Dx^9\sqrt{a+bx^2}}{10b}$$

$\Big\downarrow$ 363

$$\frac{\frac{\frac{5\left(96Ab^3-a\left(63a^2D-70abC+80b^2B\right)\right)\int \frac{x^4}{\sqrt{bx^2+a}}dx}{6b}+\frac{x^5\sqrt{a+bx^2}\left(63a^2D-70abC+80b^2B\right)}{6b}}{8b}+\frac{x^7\sqrt{a+bx^2}(10bC-9aD)}{8b}}{10b}+\frac{Dx^9\sqrt{a+bx^2}}{10b}$$

$\Big\downarrow$ 262

$$\frac{\frac{\frac{5\left(96Ab^3-a\left(63a^2D-70abC+80b^2B\right)\right)\left(\frac{x^3\sqrt{a+bx^2}}{4b}-\frac{3a\int \frac{x^2}{\sqrt{bx^2+a}}dx}{4b}\right)}{6b}+\frac{x^5\sqrt{a+bx^2}\left(63a^2D-70abC+80b^2B\right)}{6b}}{8b}+\frac{x^7\sqrt{a+bx^2}(10bC-9aD)}{8b}}{10b}+\frac{Dx^9\sqrt{a+bx^2}}{10b}$$

$\Big\downarrow$ 262

$$\frac{\frac{\frac{5\left(96Ab^3-a\left(63a^2D-70abC+80b^2B\right)\right)\left(\frac{x^3\sqrt{a+bx^2}}{4b}-\frac{3a\left(\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\int \frac{1}{\sqrt{bx^2+a}}dx}{2b}\right)}{4b}\right)}{6b}+\frac{x^5\sqrt{a+bx^2}\left(63a^2D-70abC+80b^2B\right)}{6b}}{8b}+\frac{x^7\sqrt{a+bx^2}(10bC-9aD)}{8b}}{10b}+\frac{Dx^9\sqrt{a+bx^2}}{10b}$$

$\Big\downarrow$ 224

$$\frac{\frac{\frac{5\left(96Ab^3-a\left(63a^2D-70abC+80b^2B\right)\right)\left(\frac{x^3\sqrt{a+bx^2}}{4b}-\frac{3a\left(\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2b}\right)}{4b}\right)}{6b}+\frac{x^5\sqrt{a+bx^2}\left(63a^2D-70abC+80b^2B\right)}{6b}}{8b}+\frac{x^7\sqrt{a+bx^2}(10bC-9aD)}{8b}}{10b}+\frac{Dx^9\sqrt{a+bx^2}}{10b}$$

$\Big\downarrow$ 219

$$\frac{\frac{\frac{5\left(\frac{x^3\sqrt{a+bx^2}}{4b}-\frac{3a\left(\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}\right)}{4b}\right)\left(96Ab^3-a\left(63a^2D-70abC+80b^2B\right)\right)}{6b}+\frac{x^5\sqrt{a+bx^2}\left(63a^2D-70abC+80b^2B\right)}{6b}}{8b}+\frac{x^7\sqrt{a+bx^2}(1}{8b}}{10b}$$

$$\frac{Dx^9\sqrt{a+bx^2}}{10b}$$

input
```
Int[(x^4*(A + B*x^2 + C*x^4 + D*x^6))/Sqrt[a + b*x^2],x]
```

output
```
(D*x^9*Sqrt[a + b*x^2])/(10*b) + (((10*b*C - 9*a*D)*x^7*Sqrt[a + b*x^2])/(
8*b) + (((80*b^2*B - 70*a*b*C + 63*a^2*D)*x^5*Sqrt[a + b*x^2])/(6*b) + (5*
(96*A*b^3 - a*(80*b^2*B - 70*a*b*C + 63*a^2*D))*((x^3*Sqrt[a + b*x^2])/(4*
b) - (3*a*((x*Sqrt[a + b*x^2])/(2*b) - (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x
^2]])/(2*b^(3/2))))/(4*b)))/(6*b))/(8*b))/(10*b)
```

## Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 1590
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[c^p*(f*x)^(m + 4*p - 1)*((d + e*x^2)^
(q + 1)/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1))), x] + Simp[1/(e*(m + 4*p + 2*q
 + 1))   Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a +
b*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x],
x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p,
0] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

rule 2340
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Simp[1/(b*(m
+ q + 2*p + 1))   Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)
*Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /;
GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ
[Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

## Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.68

| method | result |
|---|---|
| pseudoelliptic | $-\frac{3\left(-\left(b^3A-\frac{5}{6}a\,b^2B+\frac{35}{48}a^2bC-\frac{21}{32}a^3D\right)a^2\operatorname{arctanh}\left(\frac{\sqrt{b\,x^2+a}}{x\sqrt{b}}\right)+\sqrt{b\,x^2+a}\,x\left(a\left(\frac{3}{10}Dx^6+\frac{7}{18}C\,x^4+\frac{5}{9}x^2B+A\right)b^{\frac{7}{2}}-\frac{2\left(\frac{2}{5}D\cdots\right.}{\cdots}\right.\right.}{8b^{\frac{11}{2}}}$ … |
| default | $A\left(\frac{x^3\sqrt{b\,x^2+a}}{4b}-\frac{3a\left(\frac{x\sqrt{b\,x^2+a}}{2b}-\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2b^{\frac{3}{2}}}\right)}{4b}\right)+B\left(\frac{x^5\sqrt{b\,x^2+a}}{6b}-\frac{5a\left(\frac{x^3\sqrt{b\,x^2+a}}{4b}-\frac{3a\left(\frac{x\sqrt{b\,x^2+a}}{2b}-\cdots\right.}{\cdots}\right.}{6b}\right.$ … |

input
```
int(x^4*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-3/8/b^(11/2)*(-(b^3*A-5/6*a*b^2*B+35/48*a^2*b*C-21/32*a^3*D)*a^2*arctanh(
(b*x^2+a)^(1/2)/x/b^(1/2))+(b*x^2+a)^(1/2)*x*(a*(3/10*D*x^6+7/18*C*x^4+5/9
*x^2*B+A)*b^(7/2)-2/3*(2/5*D*x^6+1/2*C*x^4+2/3*x^2*B+A)*x^2*b^(9/2)+35/48*
((-12/25*D*x^4-2/3*C*x^2-8/7*B)*b^(5/2)+((3/5*D*x^2+C)*b^(3/2)-9/10*D*a*b^
(1/2))*a)*a^2))
```

## Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 413, normalized size of antiderivative = 1.67

$$\int \frac{x^4(A+Bx^2+Cx^4+Dx^6)}{\sqrt{a+bx^2}}\,dx$$
$$=\left[-\frac{15\,(63\,Da^5-70\,Ca^4b+80\,Ba^3b^2-96\,Aa^2b^3)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\,(384\,Db^5x^9}{}\right.$$

input
```
integrate(x^4*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/7680*(15*(63*D*a^5 - 70*C*a^4*b + 80*B*a^3*b^2 - 96*A*a^2*b^3)*sqrt(b)
*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(384*D*b^5*x^9 - 48*(
9*D*a*b^4 - 10*C*b^5)*x^7 + 8*(63*D*a^2*b^3 - 70*C*a*b^4 + 80*B*b^5)*x^5 -
 10*(63*D*a^3*b^2 - 70*C*a^2*b^3 + 80*B*a*b^4 - 96*A*b^5)*x^3 + 15*(63*D*a
^4*b - 70*C*a^3*b^2 + 80*B*a^2*b^3 - 96*A*a*b^4)*x)*sqrt(b*x^2 + a))/b^6,
1/3840*(15*(63*D*a^5 - 70*C*a^4*b + 80*B*a^3*b^2 - 96*A*a^2*b^3)*sqrt(-b)*
arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (384*D*b^5*x^9 - 48*(9*D*a*b^4 - 10*C
*b^5)*x^7 + 8*(63*D*a^2*b^3 - 70*C*a*b^4 + 80*B*b^5)*x^5 - 10*(63*D*a^3*b^
2 - 70*C*a^2*b^3 + 80*B*a*b^4 - 96*A*b^5)*x^3 + 15*(63*D*a^4*b - 70*C*a^3*
b^2 + 80*B*a^2*b^3 - 96*A*a*b^4)*x)*sqrt(b*x^2 + a))/b^6]
```

## Sympy [A] (verification not implemented)

Time = 0.67 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.98

$$\int \frac{x^4(A+Bx^2+Cx^4+Dx^6)}{\sqrt{a+bx^2}}\,dx$$
$$=\begin{cases}\frac{3a^2\left(A-\frac{5a\left(B-\frac{7a\left(C-\frac{9Da}{10b}\right)}{8b}\right)}{6b}\right)\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right)}{8b^2}+\sqrt{a+bx^2}\left(\frac{Dx^9}{10b}-\frac{3ax\left(A-\frac{5a\left(B-\frac{7a\left(C-\frac{9Da}{10b}\right.}{8b}\right.}{6b}\right.}{8b^2}\right.\\ \frac{\frac{Ax^5}{5}+\frac{Bx^7}{7}+\frac{Cx^9}{9}+\frac{Dx^{11}}{11}}{\sqrt{a}}\end{cases}$$

input
```
integrate(x**4*(D*x**6+C*x**4+B*x**2+A)/(b*x**2+a)**(1/2),x)
```

output
```
Piecewise((3*a**2*(A - 5*a*(B - 7*a*(C - 9*D*a/(10*b))/(8*b))/(6*b))*Piece
wise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x
)/sqrt(b*x**2), True))/(8*b**2) + sqrt(a + b*x**2)*(D*x**9/(10*b) - 3*a*x*
(A - 5*a*(B - 7*a*(C - 9*D*a/(10*b))/(8*b))/(6*b))/(8*b**2) + x**7*(C - 9*
D*a/(10*b))/(8*b) + x**5*(B - 7*a*(C - 9*D*a/(10*b))/(8*b))/(6*b) + x**3*(
A - 5*a*(B - 7*a*(C - 9*D*a/(10*b))/(8*b))/(6*b))/(4*b)), Ne(b, 0)), ((A*x
**5/5 + B*x**7/7 + C*x**9/9 + D*x**11/11)/sqrt(a), True))
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.37

$$\int \frac{x^4(A+Bx^2+Cx^4+Dx^6)}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{bx^2+a}Dx^9}{10\,b} - \frac{9\,\sqrt{bx^2+a}Dax^7}{80\,b^2} + \frac{\sqrt{bx^2+a}Cx^7}{8\,b} + \frac{21\,\sqrt{bx^2+a}Da^2x^5}{160\,b^3} - \frac{7\,\sqrt{bx^2+a}Cax^5}{48\,b^2} + \frac{\sqrt{bx^2+a}Bx^5}{6\,b} - \frac{21\,\sqrt{bx^2+a}Da^3x^3}{128\,b^4} + \frac{35\,\sqrt{bx^2+a}Ca^2x^3}{192\,b^3} - \frac{5\,\sqrt{bx^2+a}Bax^3}{24\,b^2} + \frac{\sqrt{bx^2+a}Ax^3}{4\,b} + \frac{63\,\sqrt{bx^2+a}Da^4x}{256\,b^5} - \frac{35\,\sqrt{bx^2+a}Ca^3x}{128\,b^4} + \frac{5\,\sqrt{bx^2+a}Ba^2x}{16\,b^3} - \frac{3\,\sqrt{bx^2+a}Aax}{8\,b^2} - \frac{63\,Da^5\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{256\,b^{\frac{11}{2}}} + \frac{35\,Ca^4\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\,b^{\frac{9}{2}}} - \frac{5\,Ba^3\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{7}{2}}} + \frac{3\,Aa^2\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{5}{2}}}$$

input
```
integrate(x^4*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/10*sqrt(b*x^2 + a)*D*x^9/b - 9/80*sqrt(b*x^2 + a)*D*a*x^7/b^2 + 1/8*sqrt
(b*x^2 + a)*C*x^7/b + 21/160*sqrt(b*x^2 + a)*D*a^2*x^5/b^3 - 7/48*sqrt(b*x
^2 + a)*C*a*x^5/b^2 + 1/6*sqrt(b*x^2 + a)*B*x^5/b - 21/128*sqrt(b*x^2 + a)
*D*a^3*x^3/b^4 + 35/192*sqrt(b*x^2 + a)*C*a^2*x^3/b^3 - 5/24*sqrt(b*x^2 +
a)*B*a*x^3/b^2 + 1/4*sqrt(b*x^2 + a)*A*x^3/b + 63/256*sqrt(b*x^2 + a)*D*a^
4*x/b^5 - 35/128*sqrt(b*x^2 + a)*C*a^3*x/b^4 + 5/16*sqrt(b*x^2 + a)*B*a^2*
x/b^3 - 3/8*sqrt(b*x^2 + a)*A*a*x/b^2 - 63/256*D*a^5*arcsinh(b*x/sqrt(a*b)
)/b^(11/2) + 35/128*C*a^4*arcsinh(b*x/sqrt(a*b))/b^(9/2) - 5/16*B*a^3*arcs
inh(b*x/sqrt(a*b))/b^(7/2) + 3/8*A*a^2*arcsinh(b*x/sqrt(a*b))/b^(5/2)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.88

$$\int \frac{x^4(A+Bx^2+Cx^4+Dx^6)}{\sqrt{a+bx^2}}\,dx$$
$$=\frac{1}{3840}\left(2\left(4\left(6\left(\frac{8\,Dx^2}{b}-\frac{9\,Dab^7-10\,Cb^8}{b^9}\right)x^2+\frac{63\,Da^2b^6-70\,Cab^7+80\,Bb^8}{b^9}\right)x^2-\frac{5\,(63\,Da^3b^5-70\right.\right.$$
$$+\frac{(63\,Da^5-70\,Ca^4b+80\,Ba^3b^2-96\,Aa^2b^3)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{256\,b^{\frac{11}{2}}}$$

input
```
integrate(x^4*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/3840*(2*(4*(6*(8*D*x^2/b - (9*D*a*b^7 - 10*C*b^8)/b^9)*x^2 + (63*D*a^2*b
^6 - 70*C*a*b^7 + 80*B*b^8)/b^9)*x^2 - 5*(63*D*a^3*b^5 - 70*C*a^2*b^6 + 80
*B*a*b^7 - 96*A*b^8)/b^9)*x^2 + 15*(63*D*a^4*b^4 - 70*C*a^3*b^5 + 80*B*a^2
*b^6 - 96*A*a*b^7)/b^9)*sqrt(b*x^2 + a)*x + 1/256*(63*D*a^5 - 70*C*a^4*b +
 80*B*a^3*b^2 - 96*A*a^2*b^3)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(11
/2)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(A+Bx^2+Cx^4+Dx^6)}{\sqrt{a+bx^2}}\,dx=\int \frac{x^4\,\left(A+B\,x^2+C\,x^4+x^6\,D\right)}{\sqrt{b\,x^2+a}}\,dx$$

input
```
int((x^4*(A + B*x^2 + C*x^4 + x^6*D))/(a + b*x^2)^(1/2),x)
```

output
```
int((x^4*(A + B*x^2 + C*x^4 + x^6*D))/(a + b*x^2)^(1/2), x)
```

## Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.23

$$\int \frac{x^4(A+Bx^2+Cx^4+Dx^6)}{\sqrt{a+bx^2}}\,dx$$
$$=\frac{945\sqrt{b\,x^2+a}\,a^4bdx-1050\sqrt{b\,x^2+a}\,a^3b^2cx-630\sqrt{b\,x^2+a}\,a^3b^2d\,x^3-240\sqrt{b\,x^2+a}\,a^2b^4x+700\sqrt{b\,x^2}}{}$$

input
```
int(x^4*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(1/2),x)
```

output
```
(945*sqrt(a + b*x**2)*a**4*b*d*x - 1050*sqrt(a + b*x**2)*a**3*b**2*c*x - 6
30*sqrt(a + b*x**2)*a**3*b**2*d*x**3 - 240*sqrt(a + b*x**2)*a**2*b**4*x +
700*sqrt(a + b*x**2)*a**2*b**3*c*x**3 + 504*sqrt(a + b*x**2)*a**2*b**3*d*x
**5 + 160*sqrt(a + b*x**2)*a*b**5*x**3 - 560*sqrt(a + b*x**2)*a*b**4*c*x**
5 - 432*sqrt(a + b*x**2)*a*b**4*d*x**7 + 640*sqrt(a + b*x**2)*b**6*x**5 +
480*sqrt(a + b*x**2)*b**5*c*x**7 + 384*sqrt(a + b*x**2)*b**5*d*x**9 - 945*
sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**5*d + 1050*sqrt(b)*
log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**4*b*c + 240*sqrt(b)*log((sq
rt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**3*b**3)/(3840*b**6)
```

# 3.217 $\int \frac{x^2(A+Bx^2+Cx^4+Dx^6)}{\sqrt{a+bx^2}}\,dx$



## Optimal result

Integrand size = 32, antiderivative size = 196

$$\begin{aligned}\int \frac{x^2(A+Bx^2+Cx^4+Dx^6)}{\sqrt{a+bx^2}}\,dx &= \frac{(64Ab^3-a(48b^2B-40abC+35a^2D))\,x\sqrt{a+bx^2}}{128b^4} \\ &+ \frac{(48b^2B-40abC+35a^2D)\,x^3\sqrt{a+bx^2}}{192b^3}+\frac{(8bC-7aD)x^5\sqrt{a+bx^2}}{48b^2} \\ &+ \frac{Dx^7\sqrt{a+bx^2}}{8b}-\frac{a(64Ab^3-a(48b^2B-40abC+35a^2D))\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{9/2}}\end{aligned}$$

output
```
1/128*(64*A*b^3-a*(48*B*b^2-40*C*a*b+35*D*a^2))*x*(b*x^2+a)^(1/2)/b^4+1/19
2*(48*B*b^2-40*C*a*b+35*D*a^2)*x^3*(b*x^2+a)^(1/2)/b^3+1/48*(8*C*b-7*D*a)*
x^5*(b*x^2+a)^(1/2)/b^2+1/8*D*x^7*(b*x^2+a)^(1/2)/b-1/128*a*(64*A*b^3-a*(4
8*B*b^2-40*C*a*b+35*D*a^2))*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(9/2)
```

## Mathematica [A] (verified)

Time = 1.05 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.83

$$\int \frac{x^2(A+Bx^2+Cx^4+Dx^6)}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{b}x\sqrt{a+bx^2}(192Ab^3-105a^3D+10a^2b(12C+7Dx^2)+16b^3x^2(6B+4Cx^2+3Dx^4)-8ab^2(18B+1\cdots}{384b^{9/2}}$$

input
```
Integrate[(x^2*(A + B*x^2 + C*x^4 + D*x^6))/Sqrt[a + b*x^2],x]
```

output
```
(Sqrt[b]*x*Sqrt[a + b*x^2]*(192*A*b^3 - 105*a^3*D + 10*a^2*b*(12*C + 7*D*x
^2) + 16*b^3*x^2*(6*B + 4*C*x^2 + 3*D*x^4) - 8*a*b^2*(18*B + 10*C*x^2 + 7*
D*x^4)) + 6*a*(-64*A*b^3 + a*(48*b^2*B - 40*a*b*C + 35*a^2*D))*ArcTanh[(Sq
rt[b]*x)/(-Sqrt[a] + Sqrt[a + b*x^2])])/(384*b^(9/2))
```

## Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2340, 1590, 363, 262, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\left(A+Bx^2+Cx^4+Dx^6\right)}{\sqrt{a+bx^2}}\,dx$$

$$\Big\downarrow \text{2340}$$

$$\frac{\int \frac{x^2\left((8bC-7aD)x^4+8bBx^2+8Ab\right)}{\sqrt{bx^2+a}}dx}{8b}+\frac{Dx^7\sqrt{a+bx^2}}{8b}$$

$$\Big\downarrow \text{1590}$$

$$\frac{\frac{\int \frac{x^2\left(48Ab^2+\left(35Da^2-40bCa+48b^2B\right)x^2\right)}{\sqrt{bx^2+a}}dx}{6b}+\frac{x^5\sqrt{a+bx^2}(8bC-7aD)}{6b}}{8b}+\frac{Dx^7\sqrt{a+bx^2}}{8b}$$

$$\Big\downarrow \text{363}$$

$$\frac{\frac{\frac{3\left(64Ab^3-a\left(35a^2D-40abC+48b^2B\right)\right)\int \frac{x^2}{\sqrt{bx^2+a}}dx}{4b}+\frac{x^3\sqrt{a+bx^2}\left(35a^2D-40abC+48b^2B\right)}{4b}}{6b}+\frac{x^5\sqrt{a+bx^2}(8bC-7aD)}{6b}}{8b}+\frac{Dx^7\sqrt{a+bx^2}}{8b}$$

$$\Big\downarrow \text{262}$$

$$\frac{\frac{\frac{3\left(64Ab^3-a\left(35a^2D-40abC+48b^2B\right)\right)\left(\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\int \frac{1}{\sqrt{bx^2+a}}dx}{2b}\right)}{4b}+\frac{x^3\sqrt{a+bx^2}\left(35a^2D-40abC+48b^2B\right)}{4b}}{6b}+\frac{x^5\sqrt{a+bx^2}(8bC-7aD)}{6b}}{8b}+\frac{Dx^7\sqrt{a+bx^2}}{8b}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{\frac{3\left(64Ab^3-a\left(35a^2D-40abC+48b^2B\right)\right)\left(\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2b}\right)}{4b}+\frac{x^3\sqrt{a+bx^2}\left(35a^2D-40abC+48b^2B\right)}{4b}}{6b}+\frac{x^5\sqrt{a+bx^2}(8bC-7aD)}{6b}}{8b}+\frac{Dx^7\sqrt{a+bx^2}}{8b}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{\frac{3\left(\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}\right)\left(64Ab^3-a\left(35a^2D-40abC+48b^2B\right)\right)}{4b}+\frac{x^3\sqrt{a+bx^2}\left(35a^2D-40abC+48b^2B\right)}{4b}}{6b}+\frac{x^5\sqrt{a+bx^2}(8bC-7aD)}{6b}}{8b}+\frac{Dx^7\sqrt{a+bx^2}}{8b}$$

input

```
Int[(x^2*(A + B*x^2 + C*x^4 + D*x^6))/Sqrt[a + b*x^2],x]
```

output

```
(D*x^7*Sqrt[a + b*x^2])/(8*b) + (((8*b*C - 7*a*D)*x^5*Sqrt[a + b*x^2])/(6*
b) + (((48*b^2*B - 40*a*b*C + 35*a^2*D)*x^3*Sqrt[a + b*x^2])/(4*b) + (3*(6
4*A*b^3 - a*(48*b^2*B - 40*a*b*C + 35*a^2*D))*((x*Sqrt[a + b*x^2])/(2*b) -
 (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(3/2))))/(4*b))/(6*b))/(8*b
)
```

### Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 1590
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[c^p*(f*x)^(m + 4*p - 1)*((d + e*x^2)^
(q + 1)/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1))), x] + Simp[1/(e*(m + 4*p + 2*q
 + 1))   Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a +
b*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x],
x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p,
0] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

rule 2340
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Simp[1/(b*(m
+ q + 2*p + 1))   Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)
*Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /;
GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ
[Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

## Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.69

| method | result |
|---|---|
| pseudoelliptic | $\frac{-(b^3A-\frac{3}{4}ab^2B+\frac{5}{8}a^2bC-\frac{35}{64}a^3D)a\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^2+a}}{x\sqrt{b}}\right)+\sqrt{b\,x^2+a}\left((\frac{1}{4}Dx^6+\frac{1}{3}C\,x^4+\frac{1}{2}x^2B+A)b^{\frac{7}{2}}-\frac{35\left(\left(\frac{8}{15}Dx^4+\frac{16}{21}C\right.\right.}{}\right.}{2b^{\frac{9}{2}}}$ |
| default | $A\left(\frac{x\sqrt{b\,x^2+a}}{2b}-\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2b^{\frac{3}{2}}}\right)+B\left(\frac{x^3\sqrt{b\,x^2+a}}{4b}-\frac{3a\left(\frac{x\sqrt{b\,x^2+a}}{2b}-\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2b^{\frac{3}{2}}}\right)}{4b}\right)+C\left(\right.$ |

input
```
int(x^2*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(-(b^3*A-3/4*a*b^2*B+5/8*a^2*b*C-35/64*a^3*D)*a*arctanh((b*x^2+a)^(1/2
)/x/b^(1/2))+(b*x^2+a)^(1/2)*((1/4*D*x^6+1/3*C*x^4+1/2*x^2*B+A)*b^(7/2)-35
/64*((8/15*D*x^4+16/21*C*x^2+48/35*B)*b^(5/2)+a*((-2/3*D*x^2-8/7*C)*b^(3/2
)+D*a*b^(1/2)))*a)*x)/b^(9/2)
```

## Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.68

$$\int \frac{x^2(A+Bx^2+Cx^4+Dx^6)}{\sqrt{a+bx^2}}\,dx$$

$$= \left[-\frac{3\,(35\,Da^4-40\,Ca^3b+48\,Ba^2b^2-64\,Aab^3)\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\,(48\,Db^4x^7-\right.$$

$$-\frac{3\,(35\,Da^4-40\,Ca^3b+48\,Ba^2b^2-64\,Aab^3)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(48\,Db^4x^7-8\,(7\,Dab^3-8\,Cb^4)x}{384\,b}$$

input
```
integrate(x^2*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/768*(3*(35*D*a^4 - 40*C*a^3*b + 48*B*a^2*b^2 - 64*A*a*b^3)*sqrt(b)*log
(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(48*D*b^4*x^7 - 8*(7*D*a*
b^3 - 8*C*b^4)*x^5 + 2*(35*D*a^2*b^2 - 40*C*a*b^3 + 48*B*b^4)*x^3 - 3*(35*
D*a^3*b - 40*C*a^2*b^2 + 48*B*a*b^3 - 64*A*b^4)*x)*sqrt(b*x^2 + a))/b^5, -
1/384*(3*(35*D*a^4 - 40*C*a^3*b + 48*B*a^2*b^2 - 64*A*a*b^3)*sqrt(-b)*arct
an(sqrt(-b)*x/sqrt(b*x^2 + a)) - (48*D*b^4*x^7 - 8*(7*D*a*b^3 - 8*C*b^4)*x
^5 + 2*(35*D*a^2*b^2 - 40*C*a*b^3 + 48*B*b^4)*x^3 - 3*(35*D*a^3*b - 40*C*a
^2*b^2 + 48*B*a*b^3 - 64*A*b^4)*x)*sqrt(b*x^2 + a))/b^5]
```

## Sympy [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.02

$$\int \frac{x^2(A+Bx^2+Cx^4+Dx^6)}{\sqrt{a+bx^2}}\,dx$$

$$= \begin{cases} -\frac{a\left(A-\frac{3a\left(B-\frac{5a\left(C-\frac{7Da}{8b}\right)}{6b}\right)}{4b}\right)\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right)}{2b}+\sqrt{a+bx^2}\left(\frac{Dx^7}{8b}+\frac{x^5\left(C-\frac{7Da}{8b}\right)}{6b}+\frac{x^3\left(B-\frac{5a}{}\right.}{}\right. \\ \frac{\frac{Ax^3}{3}+\frac{Bx^5}{5}+\frac{Cx^7}{7}+\frac{Dx^9}{9}}{\sqrt{a}} \end{cases}$$

input
```
integrate(x**2*(D*x**6+C*x**4+B*x**2+A)/(b*x**2+a)**(1/2),x)
```

output
```
Piecewise((-a*(A - 3*a*(B - 5*a*(C - 7*D*a/(8*b))/(6*b))/(4*b))*Piecewise(
(log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqr
t(b*x**2), True))/(2*b) + sqrt(a + b*x**2)*(D*x**7/(8*b) + x**5*(C - 7*D*a
/(8*b))/(6*b) + x**3*(B - 5*a*(C - 7*D*a/(8*b))/(6*b))/(4*b) + x*(A - 3*a*
(B - 5*a*(C - 7*D*a/(8*b))/(6*b))/(4*b))/(2*b)), Ne(b, 0)), ((A*x**3/3 + B
*x**5/5 + C*x**7/7 + D*x**9/9)/sqrt(a), True))
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.30

$$\int \frac{x^2(A+Bx^2+Cx^4+Dx^6)}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{bx^2+a}Dx^7}{8\,b} - \frac{7\,\sqrt{bx^2+a}Dax^5}{48\,b^2} + \frac{\sqrt{bx^2+a}Cx^5}{6\,b} + \frac{35\,\sqrt{bx^2+a}Da^2x^3}{192\,b^3} - \frac{5\,\sqrt{bx^2+a}Cax^3}{24\,b^2} + \frac{\sqrt{bx^2+a}Bx^3}{4\,b} - \frac{35\,\sqrt{bx^2+a}Da^3x}{128\,b^4} + \frac{5\,\sqrt{bx^2+a}Ca^2x}{16\,b^3} - \frac{3\,\sqrt{bx^2+a}Bax}{8\,b^2} + \frac{\sqrt{bx^2+a}Ax}{2\,b} + \frac{35\,Da^4\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\,b^{\frac{9}{2}}} - \frac{5\,Ca^3\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{7}{2}}} + \frac{3\,Ba^2\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{5}{2}}} - \frac{Aa\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{3}{2}}}$$

input
```
integrate(x^2*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/8*sqrt(b*x^2 + a)*D*x^7/b - 7/48*sqrt(b*x^2 + a)*D*a*x^5/b^2 + 1/6*sqrt(
b*x^2 + a)*C*x^5/b + 35/192*sqrt(b*x^2 + a)*D*a^2*x^3/b^3 - 5/24*sqrt(b*x^
2 + a)*C*a*x^3/b^2 + 1/4*sqrt(b*x^2 + a)*B*x^3/b - 35/128*sqrt(b*x^2 + a)*
D*a^3*x/b^4 + 5/16*sqrt(b*x^2 + a)*C*a^2*x/b^3 - 3/8*sqrt(b*x^2 + a)*B*a*x
/b^2 + 1/2*sqrt(b*x^2 + a)*A*x/b + 35/128*D*a^4*arcsinh(b*x/sqrt(a*b))/b^(
9/2) - 5/16*C*a^3*arcsinh(b*x/sqrt(a*b))/b^(7/2) + 3/8*B*a^2*arcsinh(b*x/s
qrt(a*b))/b^(5/2) - 1/2*A*a*arcsinh(b*x/sqrt(a*b))/b^(3/2)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.87

$$\int \frac{x^2(A+Bx^2+Cx^4+Dx^6)}{\sqrt{a+bx^2}}\,dx$$
$$= \frac{1}{384}\left(2\left(4\left(\frac{6\,Dx^2}{b}-\frac{7\,Dab^5-8\,Cb^6}{b^7}\right)x^2+\frac{35\,Da^2b^4-40\,Cab^5+48\,Bb^6}{b^7}\right)x^2-\frac{3\,(35\,Da^3b^3-40\,Ca^2b}{b}\right.$$
$$-\frac{(35\,Da^4-40\,Ca^3b+48\,Ba^2b^2-64\,Aab^3)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{128\,b^{\frac{9}{2}}}$$

input
```
integrate(x^2*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/384*(2*(4*(6*D*x^2/b - (7*D*a*b^5 - 8*C*b^6)/b^7)*x^2 + (35*D*a^2*b^4 -
40*C*a*b^5 + 48*B*b^6)/b^7)*x^2 - 3*(35*D*a^3*b^3 - 40*C*a^2*b^4 + 48*B*a*
b^5 - 64*A*b^6)/b^7)*sqrt(b*x^2 + a)*x - 1/128*(35*D*a^4 - 40*C*a^3*b + 48
*B*a^2*b^2 - 64*A*a*b^3)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(9/2)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(A+Bx^2+Cx^4+Dx^6)}{\sqrt{a+bx^2}}\,dx = \int \frac{x^2\,(A+B\,x^2+C\,x^4+x^6\,D)}{\sqrt{b\,x^2+a}}\,dx$$

input
```
int((x^2*(A + B*x^2 + C*x^4 + x^6*D))/(a + b*x^2)^(1/2),x)
```

output
```
int((x^2*(A + B*x^2 + C*x^4 + x^6*D))/(a + b*x^2)^(1/2), x)
```

## Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.25

$$\int \frac{x^2(A + Bx^2 + Cx^4 + Dx^6)}{\sqrt{a + bx^2}}\,dx$$
$$= \frac{-105\sqrt{b\,x^2+a}\,a^3bdx + 120\sqrt{b\,x^2+a}\,a^2b^2cx + 70\sqrt{b\,x^2+a}\,a^2b^2d\,x^3 + 48\sqrt{b\,x^2+a}\,a\,b^4x - 80\sqrt{b\,x^2+a}}{}$$

input
```
int(x^2*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(1/2),x)
```

output
```
( - 105*sqrt(a + b*x**2)*a**3*b*d*x + 120*sqrt(a + b*x**2)*a**2*b**2*c*x +
 70*sqrt(a + b*x**2)*a**2*b**2*d*x**3 + 48*sqrt(a + b*x**2)*a*b**4*x - 80*
sqrt(a + b*x**2)*a*b**3*c*x**3 - 56*sqrt(a + b*x**2)*a*b**3*d*x**5 + 96*sq
rt(a + b*x**2)*b**5*x**3 + 64*sqrt(a + b*x**2)*b**4*c*x**5 + 48*sqrt(a + b
*x**2)*b**4*d*x**7 + 105*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a
))*a**4*d - 120*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**3*b
*c - 48*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2*b**3)/(38
4*b**5)
```

# 3.218 $\int \frac{A+Bx^2+Cx^4+Dx^6}{\sqrt{a+bx^2}}\,dx$



## Optimal result

Integrand size = 29, antiderivative size = 146

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{\sqrt{a+bx^2}}\,dx = \frac{(8b^2B-6abC+5a^2D)\,x\sqrt{a+bx^2}}{16b^3} + \frac{(6bC-5aD)x^3\sqrt{a+bx^2}}{24b^2} + \frac{Dx^5\sqrt{a+bx^2}}{6b} + \frac{(16Ab^3-a(8b^2B-6abC+5a^2D))\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{7/2}}$$

output

```
1/16*(8*B*b^2-6*C*a*b+5*D*a^2)*x*(b*x^2+a)^(1/2)/b^3+1/24*(6*C*b-5*D*a)*x^
3*(b*x^2+a)^(1/2)/b^2+1/6*D*x^5*(b*x^2+a)^(1/2)/b+1/16*(16*A*b^3-a*(8*B*b^
2-6*C*a*b+5*D*a^2))*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(7/2)
```

## Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.88

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{\sqrt{a+bx^2}}\,dx = \frac{x\sqrt{a+bx^2}(24b^2B-18abC+15a^2D+12b^2Cx^2-10abDx^2+8b^2Dx^4)}{48b^3} + \frac{(16Ab^3-8ab^2B+6a^2bC-5a^3D)\,\text{arctanh}\left(\frac{\sqrt{b}x}{-\sqrt{a}+\sqrt{a+bx^2}}\right)}{8b^{7/2}}$$

input
```
Integrate[(A + B*x^2 + C*x^4 + D*x^6)/Sqrt[a + b*x^2],x]
```

output
```
(x*Sqrt[a + b*x^2]*(24*b^2*B - 18*a*b*C + 15*a^2*D + 12*b^2*C*x^2 - 10*a*b
*D*x^2 + 8*b^2*D*x^4))/(48*b^3) + ((16*A*b^3 - 8*a*b^2*B + 6*a^2*b*C - 5*a
^3*D)*ArcTanh[(Sqrt[b]*x)/(-Sqrt[a] + Sqrt[a + b*x^2])])/(8*b^(7/2))
```

## Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2346, 1473, 27, 299, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{\sqrt{a+bx^2}}\,dx$$

$$\Big\downarrow \text{2346}$$

$$\frac{\int \frac{(6bC-5aD)x^4+6bBx^2+6Ab}{\sqrt{bx^2+a}}dx}{6b}+\frac{Dx^5\sqrt{a+bx^2}}{6b}$$

$$\Big\downarrow \text{1473}$$

$$\frac{\frac{\int \frac{3\left(8Ab^2+\left(5Da^2-6bCa+8b^2B\right)x^2\right)}{\sqrt{bx^2+a}}dx}{4b}+\frac{x^3\sqrt{a+bx^2}(6bC-5aD)}{4b}}{6b}+\frac{Dx^5\sqrt{a+bx^2}}{6b}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{3\int \frac{8Ab^2+\left(5Da^2-6bCa+8b^2B\right)x^2}{\sqrt{bx^2+a}}dx}{4b}+\frac{x^3\sqrt{a+bx^2}(6bC-5aD)}{4b}}{6b}+\frac{Dx^5\sqrt{a+bx^2}}{6b}$$

$$\Big\downarrow \; 299$$

$$\frac{\frac{3\left(\frac{\left(16Ab^3-a\left(5a^2D-6abC+8b^2B\right)\right)\int \frac{1}{\sqrt{bx^2+a}}dx}{2b}+\frac{x\sqrt{a+bx^2}\left(5a^2D-6abC+8b^2B\right)}{2b}\right)}{4b}+\frac{x^3\sqrt{a+bx^2}(6bC-5aD)}{4b}}{6b}+\frac{Dx^5\sqrt{a+bx^2}}{6b}$$

$$\Big\downarrow \; 224$$

$$\frac{\frac{3\left(\frac{\left(16Ab^3-a\left(5a^2D-6abC+8b^2B\right)\right)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2b}+\frac{x\sqrt{a+bx^2}\left(5a^2D-6abC+8b^2B\right)}{2b}\right)}{4b}+\frac{x^3\sqrt{a+bx^2}(6bC-5aD)}{4b}}{6b}+\frac{Dx^5\sqrt{a+bx^2}}{6b}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{3\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(16Ab^3-a\left(5a^2D-6abC+8b^2B\right)\right)}{2b^{3/2}}+\frac{x\sqrt{a+bx^2}\left(5a^2D-6abC+8b^2B\right)}{2b}\right)}{4b}+\frac{x^3\sqrt{a+bx^2}(6bC-5aD)}{4b}}{6b}+\frac{Dx^5\sqrt{a+bx^2}}{6b}$$

input
```
Int[(A + B*x^2 + C*x^4 + D*x^6)/Sqrt[a + b*x^2],x]
```

output
```
(D*x^5*Sqrt[a + b*x^2])/(6*b) + (((6*b*C - 5*a*D)*x^3*Sqrt[a + b*x^2])/(4*
b) + (3*(((8*b^2*B - 6*a*b*C + 5*a^2*D)*x*Sqrt[a + b*x^2])/(2*b) + ((16*A*
b^3 - a*(8*b^2*B - 6*a*b*C + 5*a^2*D))*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]
])/(2*b^(3/2))))/(4*b))/(6*b)
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1473
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[c^p*x^(4*p - 1)*((d + e*x^2)^(q + 1)/(e*(4*p + 2*q + 1)))
, x] + Simp[1/(e*(4*p + 2*q + 1))   Int[(d + e*x^2)^q*ExpandToSum[e*(4*p +
2*q + 1)*(a + b*x^2 + c*x^4)^p - d*c^p*(4*p - 1)*x^(4*p - 2) - e*c^p*(4*p +
 2*q + 1)*x^(4*p), x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] &&  !LtQ[q, -1]
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

## Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.73

| method | result |
|---|---|
| pseudoelliptic | $\frac{(b^3A-\frac{1}{2}ab^2B+\frac{3}{8}a^2bC-\frac{5}{16}a^3D)\operatorname{arctanh}\left(\frac{\sqrt{bx^2+a}}{x\sqrt{b}}\right)+\frac{5\sqrt{bx^2+a}\,x\left(\frac{4\left(\frac{2}{3}Dx^4+Cx^2+2B\right)b^{\frac{5}{2}}}{5}+\left(2\left(-\frac{Dx^2}{3}-\frac{3C}{5}\right)b^{\frac{3}{2}}+Da\sqrt{b}\right)a\right.}{16}}{b^{\frac{7}{2}}}$ |
| default | $\frac{A\ln\left(\sqrt{b}\,x+\sqrt{bx^2+a}\right)}{\sqrt{b}}+C\left(\frac{x^3\sqrt{bx^2+a}}{4b}-\frac{3a\left(\frac{x\sqrt{bx^2+a}}{2b}-\frac{a\ln\left(\sqrt{b}\,x+\sqrt{bx^2+a}\right)}{2b^{\frac{3}{2}}}\right)}{4b}\right)+D\left(\frac{x^5\sqrt{bx^2+a}}{6b}-\frac{5a\left(\right.}{}\right.$ |

input
```
int((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((b^3*A-1/2*a*b^2*B+3/8*a^2*b*C-5/16*a^3*D)*arctanh((b*x^2+a)^(1/2)/x/b^(1
/2))+5/16*(b*x^2+a)^(1/2)*x*(4/5*(2/3*D*x^4+C*x^2+2*B)*b^(5/2)+(2*(-1/3*D*
x^2-3/5*C)*b^(3/2)+D*a*b^(1/2))*a))/b^(7/2)
```

## Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.71

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{\sqrt{a+bx^2}}\,dx$$
$$=\left[-\frac{3\,(5\,Da^3-6\,Ca^2b+8\,Bab^2-16\,Ab^3)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\,(8\,Db^3x^5-2\,(5\,Da\right.}{96\,b^4}\right.$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/96*(3*(5*D*a^3 - 6*C*a^2*b + 8*B*a*b^2 - 16*A*b^3)*sqrt(b)*log(-2*b*x^
2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(8*D*b^3*x^5 - 2*(5*D*a*b^2 - 6*C
*b^3)*x^3 + 3*(5*D*a^2*b - 6*C*a*b^2 + 8*B*b^3)*x)*sqrt(b*x^2 + a))/b^4, 1
/48*(3*(5*D*a^3 - 6*C*a^2*b + 8*B*a*b^2 - 16*A*b^3)*sqrt(-b)*arctan(sqrt(-
b)*x/sqrt(b*x^2 + a)) + (8*D*b^3*x^5 - 2*(5*D*a*b^2 - 6*C*b^3)*x^3 + 3*(5*
D*a^2*b - 6*C*a*b^2 + 8*B*b^3)*x)*sqrt(b*x^2 + a))/b^4]
```

## Sympy [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.06

$$\int \frac{A + Bx^2 + Cx^4 + Dx^6}{\sqrt{a + bx^2}}\,dx = \begin{cases} \left(A - \frac{a\left(B - \frac{3a\left(C - \frac{5Da}{6b}\right)}{4b}\right)}{2b}\right)\left(\begin{cases} \frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for}\ a \neq 0 \\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise} \end{cases}\right) + \sqrt{a+bx^2}\left(\frac{Dx^5}{6b} + \frac{x^3\left(C - \frac{5Da}{6b}\right)}{4b} + \frac{x\left(B - \right.}{} \right. \\ \frac{Ax + \frac{Bx^3}{3} + \frac{Cx^5}{5} + \frac{Dx^7}{7}}{\sqrt{a}} \end{cases}$$

input
```
integrate((D*x**6+C*x**4+B*x**2+A)/(b*x**2+a)**(1/2),x)
```

output
```
Piecewise(((A - a*(B - 3*a*(C - 5*D*a/(6*b))/(4*b))/(2*b))*Piecewise((log(
2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x
**2), True)) + sqrt(a + b*x**2)*(D*x**5/(6*b) + x**3*(C - 5*D*a/(6*b))/(4*
b) + x*(B - 3*a*(C - 5*D*a/(6*b))/(4*b))/(2*b)), Ne(b, 0)), ((A*x + B*x**3
/3 + C*x**5/5 + D*x**7/7)/sqrt(a), True))
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.19

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{bx^2+a}Dx^5}{6\,b} - \frac{5\,\sqrt{bx^2+a}Dax^3}{24\,b^2} + \frac{\sqrt{bx^2+a}Cx^3}{4\,b} + \frac{5\,\sqrt{bx^2+a}Da^2x}{16\,b^3} - \frac{3\,\sqrt{bx^2+a}Cax}{8\,b^2} + \frac{\sqrt{bx^2+a}Bx}{2\,b} - \frac{5\,Da^3\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{7}{2}}} + \frac{3\,Ca^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{5}{2}}} - \frac{Ba\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{3}{2}}} + \frac{A\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/6*sqrt(b*x^2 + a)*D*x^5/b - 5/24*sqrt(b*x^2 + a)*D*a*x^3/b^2 + 1/4*sqrt(
b*x^2 + a)*C*x^3/b + 5/16*sqrt(b*x^2 + a)*D*a^2*x/b^3 - 3/8*sqrt(b*x^2 + a
)*C*a*x/b^2 + 1/2*sqrt(b*x^2 + a)*B*x/b - 5/16*D*a^3*arcsinh(b*x/sqrt(a*b)
)/b^(7/2) + 3/8*C*a^2*arcsinh(b*x/sqrt(a*b))/b^(5/2) - 1/2*B*a*arcsinh(b*x
/sqrt(a*b))/b^(3/2) + A*arcsinh(b*x/sqrt(a*b))/sqrt(b)
```

## Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.86

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{\sqrt{a+bx^2}}\,dx = \frac{1}{48}\left(2\left(\frac{4\,Dx^2}{b} - \frac{5\,Dab^3-6\,Cb^4}{b^5}\right)x^2 + \frac{3\,(5\,Da^2b^2-6\,Cab^3+8\,Bb^4)}{b^5}\right)\sqrt{bx^2+a}x + \frac{(5\,Da^3-6\,Ca^2b+8\,Bab^2-16\,Ab^3)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{16\,b^{\frac{7}{2}}}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/48*(2*(4*D*x^2/b - (5*D*a*b^3 - 6*C*b^4)/b^5)*x^2 + 3*(5*D*a^2*b^2 - 6*C
*a*b^3 + 8*B*b^4)/b^5)*sqrt(b*x^2 + a)*x + 1/16*(5*D*a^3 - 6*C*a^2*b + 8*B
*a*b^2 - 16*A*b^3)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(7/2)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{\sqrt{a+bx^2}}\,dx=\int \frac{A+B\,x^2+C\,x^4+x^6\,D}{\sqrt{b\,x^2+a}}\,dx$$

input
```
int((A + B*x^2 + C*x^4 + x^6*D)/(a + b*x^2)^(1/2),x)
```

output
```
int((A + B*x^2 + C*x^4 + x^6*D)/(a + b*x^2)^(1/2), x)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.27

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{\sqrt{a+bx^2}}\,dx$$
$$=\frac{15\sqrt{b\,x^2+a}\,a^2bdx-18\sqrt{b\,x^2+a}\,a\,b^2cx-10\sqrt{b\,x^2+a}\,a\,b^2d\,x^3+24\sqrt{b\,x^2+a}\,b^4x+12\sqrt{b\,x^2+a}\,b^3c\,x^3}{}$$

input
```
int((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(1/2),x)
```

output
```
(15*sqrt(a + b*x**2)*a**2*b*d*x - 18*sqrt(a + b*x**2)*a*b**2*c*x - 10*sqrt
(a + b*x**2)*a*b**2*d*x**3 + 24*sqrt(a + b*x**2)*b**4*x + 12*sqrt(a + b*x*
*2)*b**3*c*x**3 + 8*sqrt(a + b*x**2)*b**3*d*x**5 - 15*sqrt(b)*log((sqrt(a
+ b*x**2) + sqrt(b)*x)/sqrt(a))*a**3*d + 18*sqrt(b)*log((sqrt(a + b*x**2)
+ sqrt(b)*x)/sqrt(a))*a**2*b*c + 24*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b
)*x)/sqrt(a))*a*b**3)/(48*b**4)
```

# 3.219 $\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2\sqrt{a+bx^2}}\,dx$



## Optimal result

Integrand size = 32, antiderivative size = 117

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2\sqrt{a+bx^2}}\,dx = -\frac{A\sqrt{a+bx^2}}{ax}+\frac{(4bC-3aD)x\sqrt{a+bx^2}}{8b^2}+\frac{Dx^3\sqrt{a+bx^2}}{4b}+\frac{(8b^2B-4abC+3a^2D)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{5/2}}$$

output
```
-A*(b*x^2+a)^(1/2)/a/x+1/8*(4*C*b-3*D*a)*x*(b*x^2+a)^(1/2)/b^2+1/4*D*x^3*(
b*x^2+a)^(1/2)/b+1/8*(8*B*b^2-4*C*a*b+3*D*a^2)*arctanh(b^(1/2)*x/(b*x^2+a)
^(1/2))/b^(5/2)
```

## Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.86

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2\sqrt{a+bx^2}}\,dx = \frac{\frac{\sqrt{b}\sqrt{a+bx^2}(-8Ab^2+ax^2(4bC-3aD+2bDx^2))}{ax}+(-8b^2B+4abC-3a^2D)\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{8b^{5/2}}$$

input
```
Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(x^2*Sqrt[a + b*x^2]),x]
```

output
```
((Sqrt[b]*Sqrt[a + b*x^2]*(-8*A*b^2 + a*x^2*(4*b*C - 3*a*D + 2*b*D*x^2)))/
(a*x) + (-8*b^2*B + 4*a*b*C - 3*a^2*D)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]]
)/(8*b^(5/2))
```

## Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.13, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2338, 9, 25, 1473, 27, 299, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2\sqrt{a+bx^2}}dx$$

$$\Big\downarrow \text{2338}$$

$$-\frac{\int -\frac{aDx^5+aCx^3+aBx}{x\sqrt{bx^2+a}}dx}{a}-\frac{A\sqrt{a+bx^2}}{ax}$$

$$\Big\downarrow \text{9}$$

$$-\frac{\int -\frac{aDx^4+aCx^2+aB}{\sqrt{bx^2+a}}dx}{a}-\frac{A\sqrt{a+bx^2}}{ax}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{aDx^4+aCx^2+aB}{\sqrt{bx^2+a}}dx}{a}-\frac{A\sqrt{a+bx^2}}{ax}$$

$$\Big\downarrow \text{1473}$$

$$\frac{\frac{\int \frac{a\left((4bC-3aD)x^2+4bB\right)}{\sqrt{bx^2+a}}dx}{4b}+\frac{aDx^3\sqrt{a+bx^2}}{4b}}{a}-\frac{A\sqrt{a+bx^2}}{ax}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{a\int \frac{(4bC-3aD)x^2+4bB}{\sqrt{bx^2+a}}dx}{4b}+\frac{aDx^3\sqrt{a+bx^2}}{4b}}{a}-\frac{A\sqrt{a+bx^2}}{ax}$$

$$\Big\downarrow \text{ 299}$$

$$\frac{\frac{a\left(\frac{\left(3a^2D-4abC+8b^2B\right)\int \frac{1}{\sqrt{bx^2+a}}dx}{2b}+\frac{x\sqrt{a+bx^2}(4bC-3aD)}{2b}\right)}{4b}+\frac{aDx^3\sqrt{a+bx^2}}{4b}}{a}-\frac{A\sqrt{a+bx^2}}{ax}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{\frac{a\left(\frac{\left(3a^2D-4abC+8b^2B\right)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2b}+\frac{x\sqrt{a+bx^2}(4bC-3aD)}{2b}\right)}{4b}+\frac{aDx^3\sqrt{a+bx^2}}{4b}}{a}-\frac{A\sqrt{a+bx^2}}{ax}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\frac{a\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3a^2D-4abC+8b^2B\right)}{2b^{3/2}}+\frac{x\sqrt{a+bx^2}(4bC-3aD)}{2b}\right)}{4b}+\frac{aDx^3\sqrt{a+bx^2}}{4b}}{a}-\frac{A\sqrt{a+bx^2}}{ax}$$

input
```
Int[(A + B*x^2 + C*x^4 + D*x^6)/(x^2*Sqrt[a + b*x^2]),x]
```

output
```
-((A*Sqrt[a + b*x^2])/(a*x)) + ((a*D*x^3*Sqrt[a + b*x^2])/(4*b) + (a*(((4*
b*C - 3*a*D)*x*Sqrt[a + b*x^2])/(2*b) + ((8*b^2*B - 4*a*b*C + 3*a^2*D)*Arc
Tanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(3/2))))/(4*b))/a
```

## Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1473
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[c^p*x^(4*p - 1)*((d + e*x^2)^(q + 1)/(e*(4*p + 2*q + 1)))
, x] + Simp[1/(e*(4*p + 2*q + 1))   Int[(d + e*x^2)^q*ExpandToSum[e*(4*p +
2*q + 1)*(a + b*x^2 + c*x^4)^p - d*c^p*(4*p - 1)*x^(4*p - 2) - e*c^p*(4*p +
 2*q + 1)*x^(4*p), x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] &&  !LtQ[q, -1]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

## Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.82

| method | result |
|---|---|
| pseudoelliptic | $\frac{x\,b^{2}\left(B\,b^{2}-\frac{1}{2}Cab+\frac{3}{8}Da^{2}\right)a\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{x\sqrt{b}}\right)-b^{\frac{5}{2}}\sqrt{b\,x^{2}+a}\left(b^{2}A-\frac{x^{2}a\left(\frac{D x^{2}}{2}+C\right)b}{2}+\frac{3Da^{2}x^{2}}{8}\right)}{x\,b^{\frac{9}{2}}a}$ |
| default | $\frac{B\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{\sqrt{b}}-\frac{A\sqrt{b\,x^{2}+a}}{ax}+C\left(\frac{x\sqrt{b\,x^{2}+a}}{2b}-\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{2b^{\frac{3}{2}}}\right)+D\left(\frac{x^{3}\sqrt{b\,x^{2}+a}}{4b}-\frac{3a\left(\frac{x\sqrt{}}{}\right.}{}\right.$ |

input
```
int((D*x^6+C*x^4+B*x^2+A)/x^2/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(x*b^2*(B*b^2-1/2*C*a*b+3/8*D*a^2)*a*arctanh((b*x^2+a)^(1/2)/x/b^(1/2))-b^
(5/2)*(b*x^2+a)^(1/2)*(b^2*A-1/2*x^2*a*(1/2*D*x^2+C)*b+3/8*D*a^2*x^2))/x/b
^(9/2)/a
```

## Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.86

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2\sqrt{a+bx^2}}\,dx$$

$$= \left[\frac{(3\,Da^3-4\,Ca^2b+8\,Bab^2)\sqrt{b}x\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,(2\,Dab^2x^4-8\,Ab^3-(3\,Da^2b-}{16\,ab^3x}\right.$$

$$-\frac{(3\,Da^3-4\,Ca^2b+8\,Bab^2)\sqrt{-b}x\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(2\,Dab^2x^4-8\,Ab^3-(3\,Da^2b-4\,Cab^2)x^2)\sqrt{bx^2}}{8\,ab^3x}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^2/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/16*((3*D*a^3 - 4*C*a^2*b + 8*B*a*b^2)*sqrt(b)*x*log(-2*b*x^2 - 2*sqrt(b
*x^2 + a)*sqrt(b)*x - a) + 2*(2*D*a*b^2*x^4 - 8*A*b^3 - (3*D*a^2*b - 4*C*a
*b^2)*x^2)*sqrt(b*x^2 + a))/(a*b^3*x), -1/8*((3*D*a^3 - 4*C*a^2*b + 8*B*a*
b^2)*sqrt(-b)*x*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (2*D*a*b^2*x^4 - 8*A*
b^3 - (3*D*a^2*b - 4*C*a*b^2)*x^2)*sqrt(b*x^2 + a))/(a*b^3*x)]
```

## Sympy [A] (verification not implemented)

Time = 0.86 (sec) , antiderivative size = 246, normalized size of antiderivative = 2.10

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2\sqrt{a+bx^2}}\,dx = -\frac{A\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{a} + B\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0 \wedge b\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{for } b\neq 0\\ \frac{x}{\sqrt{a}} & \text{otherwise}\end{cases}\right) + C\left(\begin{cases}-\frac{a\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right)}{2b} + \frac{x\sqrt{a+bx^2}}{2b} & \text{for } b\neq 0\\ \frac{x^3}{3\sqrt{a}} & \text{otherwise}\end{cases}\right) + D\left(\begin{cases}\frac{3a^2\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right)}{8b^2} - \frac{3ax\sqrt{a+bx^2}}{8b^2} + \frac{x^3\sqrt{a+bx^2}}{4b} & \text{for } b\neq 0\\ \frac{x^5}{5\sqrt{a}} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((D*x**6+C*x**4+B*x**2+A)/x**2/(b*x**2+a)**(1/2),x)
```

output
```
-A*sqrt(b)*sqrt(a/(b*x**2) + 1)/a + B*Piecewise((log(2*sqrt(b)*sqrt(a + b*
x**2) + 2*b*x)/sqrt(b), Ne(a, 0) & Ne(b, 0)), (x*log(x)/sqrt(b*x**2), Ne(b
, 0)), (x/sqrt(a), True)) + C*Piecewise((-a*Piecewise((log(2*sqrt(b)*sqrt(
a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True))/(2
*b) + x*sqrt(a + b*x**2)/(2*b), Ne(b, 0)), (x**3/(3*sqrt(a)), True)) + D*P
iecewise((3*a**2*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b
), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True))/(8*b**2) - 3*a*x*sqrt(a + b*x
**2)/(8*b**2) + x**3*sqrt(a + b*x**2)/(4*b), Ne(b, 0)), (x**5/(5*sqrt(a)),
 True))
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.01

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2\sqrt{a+bx^2}}\,dx = \frac{\sqrt{bx^2+a}Dx^3}{4\,b} - \frac{3\,\sqrt{bx^2+a}Dax}{8\,b^2} + \frac{\sqrt{bx^2+a}Cx}{2\,b} + \frac{3\,Da^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{5}{2}}} - \frac{Ca\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{3}{2}}} + \frac{B\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}} - \frac{\sqrt{bx^2+a}A}{ax}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^2/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/4*sqrt(b*x^2 + a)*D*x^3/b - 3/8*sqrt(b*x^2 + a)*D*a*x/b^2 + 1/2*sqrt(b*x
^2 + a)*C*x/b + 3/8*D*a^2*arcsinh(b*x/sqrt(a*b))/b^(5/2) - 1/2*C*a*arcsinh
(b*x/sqrt(a*b))/b^(3/2) + B*arcsinh(b*x/sqrt(a*b))/sqrt(b) - sqrt(b*x^2 +
a)*A/(a*x)
```

## Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.97

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2\sqrt{a+bx^2}}\,dx = \frac{1}{8}\sqrt{bx^2+a}\left(\frac{2\,Dx^2}{b}-\frac{3\,Dab-4\,Cb^2}{b^3}\right)x + \frac{2\,A\sqrt{b}}{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a} - \frac{(3\,Da^2-4\,Cab+8\,Bb^2)\log\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2\right)}{16\,b^{\frac{5}{2}}}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^2/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/8*sqrt(b*x^2 + a)*(2*D*x^2/b - (3*D*a*b - 4*C*b^2)/b^3)*x + 2*A*sqrt(b)/
((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a) - 1/16*(3*D*a^2 - 4*C*a*b + 8*B*b^2)
*log((sqrt(b)*x - sqrt(b*x^2 + a))^2)/b^(5/2)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2\sqrt{a+bx^2}}\,dx = \int \frac{A+B\,x^2+C\,x^4+x^6\,D}{x^2\,\sqrt{b\,x^2+a}}\,dx$$

input
```
int((A + B*x^2 + C*x^4 + x^6*D)/(x^2*(a + b*x^2)^(1/2)),x)
```

output
```
int((A + B*x^2 + C*x^4 + x^6*D)/(x^2*(a + b*x^2)^(1/2)), x)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.52

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2\sqrt{a+bx^2}}\,dx$$
$$= \frac{-3\sqrt{b\,x^2+a}\,abd\,x^2-8\sqrt{b\,x^2+a}\,b^3+4\sqrt{b\,x^2+a}\,b^2c\,x^2+2\sqrt{b\,x^2+a}\,b^2d\,x^4+3\sqrt{b}\log\left(\frac{\sqrt{b\,x^2+a}+\sqrt{b}\,x}{\sqrt{a}}\right)a^2}{8b^3x}$$

input
```
int((D*x^6+C*x^4+B*x^2+A)/x^2/(b*x^2+a)^(1/2),x)
```

output
```
( - 3*sqrt(a + b*x**2)*a*b*d*x**2 - 8*sqrt(a + b*x**2)*b**3 + 4*sqrt(a + b
*x**2)*b**2*c*x**2 + 2*sqrt(a + b*x**2)*b**2*d*x**4 + 3*sqrt(b)*log((sqrt(
a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2*d*x - 4*sqrt(b)*log((sqrt(a + b*x**
2) + sqrt(b)*x)/sqrt(a))*a*b*c*x + 8*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(
b)*x)/sqrt(a))*b**3*x + sqrt(b)*a**2*d*x - sqrt(b)*a*b*c*x - 8*sqrt(b)*b**
3*x)/(8*b**3*x)
```

# 3.220 $\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4\sqrt{a+bx^2}}\,dx$



## Optimal result

Integrand size = 32, antiderivative size = 110

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4\sqrt{a+bx^2}}\,dx = -\frac{A\sqrt{a+bx^2}}{3ax^3}+\frac{(2Ab-3aB)\sqrt{a+bx^2}}{3a^2x} + \frac{Dx\sqrt{a+bx^2}}{2b}+\frac{(2bC-aD)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}$$

output
```
-1/3*A*(b*x^2+a)^(1/2)/a/x^3+1/3*(2*A*b-3*B*a)*(b*x^2+a)^(1/2)/a^2/x+1/2*D
*x*(b*x^2+a)^(1/2)/b+1/2*(2*C*b-D*a)*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^
(3/2)
```

## Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.86

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4\sqrt{a+bx^2}}\,dx = \frac{\sqrt{a+bx^2}(-2aAb+4Ab^2x^2-6abBx^2+3a^2Dx^4)}{6a^2bx^3} + \frac{(-2bC+aD)\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{2b^{3/2}}$$

input
```
Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(x^4*Sqrt[a + b*x^2]),x]
```

output
```
(Sqrt[a + b*x^2]*(-2*a*A*b + 4*A*b^2*x^2 - 6*a*b*B*x^2 + 3*a^2*D*x^4))/(6*
a^2*b*x^3) + ((-2*b*C + a*D)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(2*b^(3/
2))
```

## Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {2338, 9, 1588, 27, 299, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4\sqrt{a+bx^2}}dx$$

$$\Big\downarrow \text{2338}$$

$$-\frac{\int \frac{-3aDx^5-3aCx^3+(2Ab-3aB)x}{x^3\sqrt{bx^2+a}}dx}{3a}-\frac{A\sqrt{a+bx^2}}{3ax^3}$$

$$\Big\downarrow \text{9}$$

$$-\frac{\int \frac{-3aDx^4-3aCx^2+2Ab-3aB}{x^2\sqrt{bx^2+a}}dx}{3a}-\frac{A\sqrt{a+bx^2}}{3ax^3}$$

$$\Big\downarrow \text{1588}$$

$$-\frac{-\frac{\int \frac{3a^2(Dx^2+C)}{\sqrt{bx^2+a}}dx}{a}-\frac{\sqrt{a+bx^2}(2Ab-3aB)}{ax}}{3a}-\frac{A\sqrt{a+bx^2}}{3ax^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{-3a\int \frac{Dx^2+C}{\sqrt{bx^2+a}}dx-\frac{\sqrt{a+bx^2}(2Ab-3aB)}{ax}}{3a}-\frac{A\sqrt{a+bx^2}}{3ax^3}$$

$$\Big\downarrow \text{299}$$

$$-\frac{-3a\left(\frac{(2bC-aD)\int \frac{1}{\sqrt{bx^2+a}}dx}{2b}+\frac{Dx\sqrt{a+bx^2}}{2b}\right)-\frac{\sqrt{a+bx^2}(2Ab-3aB)}{ax}}{3a}-\frac{A\sqrt{a+bx^2}}{3ax^3}$$

$$\Big\downarrow \text{224}$$

$$-\frac{-3a\left(\frac{(2bC-aD)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2b}+\frac{Dx\sqrt{a+bx^2}}{2b}\right)-\frac{\sqrt{a+bx^2}(2Ab-3aB)}{ax}}{3a}-\frac{A\sqrt{a+bx^2}}{3ax^3}$$

$$\Big\downarrow \text{219}$$

$$-\frac{-\frac{\sqrt{a+bx^2}(2Ab-3aB)}{ax}-3a\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bC-aD)}{2b^{3/2}}+\frac{Dx\sqrt{a+bx^2}}{2b}\right)}{3a}-\frac{A\sqrt{a+bx^2}}{3ax^3}$$

input
```
Int[(A + B*x^2 + C*x^4 + D*x^6)/(x^4*Sqrt[a + b*x^2]),x]
```

output
```
-1/3*(A*Sqrt[a + b*x^2])/(a*x^3) - (-(((2*A*b - 3*a*B)*Sqrt[a + b*x^2])/(a
*x)) - 3*a*((D*x*Sqrt[a + b*x^2])/(2*b) + ((2*b*C - a*D)*ArcTanh[(Sqrt[b]*
x)/Sqrt[a + b*x^2]])/(2*b^(3/2))))/(3*a)
```

## Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1588
```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Simp[1/(d*f
^2*(m + 1))   Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x
) - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

## Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.85

| method | result |
|---|---|
| pseudoelliptic | $\frac{3x^{3}a^{2}b(2Cb-Da)\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{x\sqrt{b}}\right)-\sqrt{b\,x^{2}+a}\,\left(-3a^{2}Dx^{4}-4A\,b^{2}x^{2}+6Bab\,x^{2}+2abA\right)b^{\frac{3}{2}}}{6a^{2}b^{\frac{5}{2}}x^{3}}$ |
| default | $\frac{C\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{\sqrt{b}}+A\left(-\frac{\sqrt{b\,x^{2}+a}}{3a\,x^{3}}+\frac{2b\sqrt{b\,x^{2}+a}}{3a^{2}x}\right)-\frac{B\sqrt{b\,x^{2}+a}}{ax}+D\Bigg(\frac{x\sqrt{b\,x^{2}+a}}{2b}-\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right.}{2b^{\frac{3}{2}}}$ |

input
```
int((D*x^6+C*x^4+B*x^2+A)/x^4/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*(3*x^3*a^2*b*(2*C*b-D*a)*arctanh((b*x^2+a)^(1/2)/x/b^(1/2))-(b*x^2+a)^
(1/2)*(-3*D*a^2*x^4-4*A*b^2*x^2+6*B*a*b*x^2+2*A*a*b)*b^(3/2))/a^2/b^(5/2)/
x^3
```

## Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.88

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4\sqrt{a+bx^2}}\,dx$$
$$= \left[-\frac{3\,(Da^3-2\,Ca^2b)\sqrt{b}x^3\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\,(3\,Da^2bx^4-2\,Aab^2-2\,(3\,Bab^2-2\,A}{12\,a^2b^2x^3}\right.$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^4/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/12*(3*(D*a^3 - 2*C*a^2*b)*sqrt(b)*x^3*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)
*sqrt(b)*x - a) - 2*(3*D*a^2*b*x^4 - 2*A*a*b^2 - 2*(3*B*a*b^2 - 2*A*b^3)*x
^2)*sqrt(b*x^2 + a))/(a^2*b^2*x^3), 1/6*(3*(D*a^3 - 2*C*a^2*b)*sqrt(-b)*x^
3*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (3*D*a^2*b*x^4 - 2*A*a*b^2 - 2*(3*B
*a*b^2 - 2*A*b^3)*x^2)*sqrt(b*x^2 + a))/(a^2*b^2*x^3)]
```

## Sympy [A] (verification not implemented)

Time = 1.16 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.78

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4\sqrt{a+bx^2}}\,dx$$
$$= -\frac{A\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{3ax^2} + \frac{2Ab^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{3a^2} - \frac{B\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{a}$$
$$+ C\left(\begin{cases} \frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0 \wedge b\neq 0 \\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{for } b\neq 0 \\ \frac{x}{\sqrt{a}} & \text{otherwise} \end{cases}\right)$$
$$+ D\left(\begin{cases} -\frac{a\left(\begin{cases} \frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0 \\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise} \end{cases}\right)}{2b} + \frac{x\sqrt{a+bx^2}}{2b} & \text{for } b\neq 0 \\ \frac{x^3}{3\sqrt{a}} & \text{otherwise} \end{cases}\right)$$

input
```
integrate((D*x**6+C*x**4+B*x**2+A)/x**4/(b*x**2+a)**(1/2),x)
```

output
```
-A*sqrt(b)*sqrt(a/(b*x**2) + 1)/(3*a*x**2) + 2*A*b**(3/2)*sqrt(a/(b*x**2)
+ 1)/(3*a**2) - B*sqrt(b)*sqrt(a/(b*x**2) + 1)/a + C*Piecewise((log(2*sqrt
(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0) & Ne(b, 0)), (x*log(x)/sqr
t(b*x**2), Ne(b, 0)), (x/sqrt(a), True)) + D*Piecewise((-a*Piecewise((log(
2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x
**2), True))/(2*b) + x*sqrt(a + b*x**2)/(2*b), Ne(b, 0)), (x**3/(3*sqrt(a)
), True))
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.93

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4\sqrt{a+bx^2}}\,dx = \frac{\sqrt{bx^2+a}Dx}{2\,b} - \frac{Da\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{3}{2}}} + \frac{C\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}} - \frac{\sqrt{bx^2+a}B}{ax} + \frac{2\,\sqrt{bx^2+a}Ab}{3\,a^2x} - \frac{\sqrt{bx^2+a}A}{3\,ax^3}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^4/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(b*x^2 + a)*D*x/b - 1/2*D*a*arcsinh(b*x/sqrt(a*b))/b^(3/2) + C*arc
sinh(b*x/sqrt(a*b))/sqrt(b) - sqrt(b*x^2 + a)*B/(a*x) + 2/3*sqrt(b*x^2 + a
)*A*b/(a^2*x) - 1/3*sqrt(b*x^2 + a)*A/(a*x^3)
```

## Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.55

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4\sqrt{a+bx^2}}\,dx = \frac{\sqrt{bx^2+a}Dx}{2\,b} + \frac{(Da-2\,Cb)\log\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2\right)}{4\,b^{\frac{3}{2}}} + \frac{2\left(3\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4B\sqrt{b}-6\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2Ba\sqrt{b}+6\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2Ab^{\frac{3}{2}}+3\,Ba^2\sqrt{\vphantom{b}}\right.}{3\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^3}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^4/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(b*x^2 + a)*D*x/b + 1/4*(D*a - 2*C*b)*log((sqrt(b)*x - sqrt(b*x^2
+ a))^2)/b^(3/2) + 2/3*(3*(sqrt(b)*x - sqrt(b*x^2 + a))^4*B*sqrt(b) - 6*(s
qrt(b)*x - sqrt(b*x^2 + a))^2*B*a*sqrt(b) + 6*(sqrt(b)*x - sqrt(b*x^2 + a)
)^2*A*b^(3/2) + 3*B*a^2*sqrt(b) - 2*A*a*b^(3/2))/((sqrt(b)*x - sqrt(b*x^2
+ a))^2 - a)^3
```

## Mupad [B] (verification not implemented)

Time = 2.28 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.35

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4\sqrt{a+bx^2}}\,dx$$
$$=\begin{cases} \frac{x^3 D}{3\sqrt{a}}-\frac{B}{\sqrt{a}\,x}-\frac{A}{3\sqrt{a}\,x^3}+\frac{C\,x}{\sqrt{a}} & \text{if } b=0 \\ \frac{C\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}+\frac{x\sqrt{b\,x^2+a}\,D}{2\,b}-\frac{a\,\ln\left(2\sqrt{b}\,x+2\sqrt{b\,x^2+a}\right)D}{2\,b^{3/2}}-\frac{B\sqrt{b\,x^2+a}}{a\,x}-\frac{A\sqrt{b\,x^2+a}\,(a-2\,b\,x^2)}{3\,a^2\,x^3} & \text{if } b\neq 0 \end{cases}$$

input
```
int((A + B*x^2 + C*x^4 + x^6*D)/(x^4*(a + b*x^2)^(1/2)),x)
```

output
```
piecewise(b == 0, - A/(3*a^(1/2)*x^3) - B/(a^(1/2)*x) + (x^3*D)/(3*a^(1/2)
) + (C*x)/a^(1/2), b ~= 0, (C*log(b^(1/2)*x + (a + b*x^2)^(1/2)))/b^(1/2)
+ (x*(a + b*x^2)^(1/2)*D)/(2*b) - (a*log(2*b^(1/2)*x + 2*(a + b*x^2)^(1/2)
)*D)/(2*b^(3/2)) - (B*(a + b*x^2)^(1/2))/(a*x) - (A*(a + b*x^2)^(1/2)*(a -
 2*b*x^2))/(3*a^2*x^3))
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.25

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4\sqrt{a+bx^2}}\,dx$$
$$=\frac{-4\sqrt{b\,x^2+a}\,a\,b^2+6\sqrt{b\,x^2+a}\,abd\,x^4-4\sqrt{b\,x^2+a}\,b^3x^2-6\sqrt{b}\log\left(\frac{\sqrt{b\,x^2+a}+\sqrt{b}\,x}{\sqrt{a}}\right)a^2d\,x^3+12\sqrt{b}\log\left(\frac{\sqrt{b}}{}\right.}{12a\,b^2x^3}$$

input
```
int((D*x^6+C*x^4+B*x^2+A)/x^4/(b*x^2+a)^(1/2),x)
```

output
```
( - 4*sqrt(a + b*x**2)*a*b**2 + 6*sqrt(a + b*x**2)*a*b*d*x**4 - 4*sqrt(a +
 b*x**2)*b**3*x**2 - 6*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))
*a**2*d*x**3 + 12*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a*b*
c*x**3 + sqrt(b)*a**2*d*x**3 - 4*sqrt(b)*b**3*x**3)/(12*a*b**2*x**3)
```

# 3.221 $\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6\sqrt{a+bx^2}}\,dx$



## Optimal result

Integrand size = 32, antiderivative size = 118

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6\sqrt{a+bx^2}}\,dx = -\frac{A\sqrt{a+bx^2}}{5ax^5}+\frac{(4Ab-5aB)\sqrt{a+bx^2}}{15a^2x^3} - \frac{(8Ab^2-10abB+15a^2C)\sqrt{a+bx^2}}{15a^3x} + \frac{D\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

output
```
-1/5*A*(b*x^2+a)^(1/2)/a/x^5+1/15*(4*A*b-5*B*a)*(b*x^2+a)^(1/2)/a^2/x^3-1/
15*(8*A*b^2-10*B*a*b+15*C*a^2)*(b*x^2+a)^(1/2)/a^3/x+D*arctanh(b^(1/2)*x/(
b*x^2+a)^(1/2))/b^(1/2)
```

## Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.83

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6\sqrt{a+bx^2}}\,dx = -\frac{\sqrt{a+bx^2}(8Ab^2x^4-2abx^2(2A+5Bx^2)+a^2(3A+5Bx^2+15Cx^4))}{15a^3x^5} - \frac{D\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{\sqrt{b}}$$

input
```
Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(x^6*Sqrt[a + b*x^2]),x]
```

output
```
-1/15*(Sqrt[a + b*x^2]*(8*A*b^2*x^4 - 2*a*b*x^2*(2*A + 5*B*x^2) + a^2*(3*A
 + 5*B*x^2 + 15*C*x^4)))/(a^3*x^5) - (D*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]
])/Sqrt[b]
```

## Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.15, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2338, 9, 1588, 358, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6\sqrt{a+bx^2}}\,dx$$

$$\Big\downarrow \text{2338}$$

$$-\frac{\int \frac{-5aDx^5-5aCx^3+(4Ab-5aB)x}{x^5\sqrt{bx^2+a}}dx}{5a} - \frac{A\sqrt{a+bx^2}}{5ax^5}$$

$$\Big\downarrow \text{9}$$

$$-\frac{\int \frac{-5aDx^4-5aCx^2+4Ab-5aB}{x^4\sqrt{bx^2+a}}dx}{5a} - \frac{A\sqrt{a+bx^2}}{5ax^5}$$

$$\Big\downarrow \text{1588}$$

$$-\frac{-\frac{\int \frac{15Dx^2a^2+15Ca^2-10bBa+8Ab^2}{x^2\sqrt{bx^2+a}}dx}{3a}-\frac{\sqrt{a+bx^2}(4Ab-5aB)}{3ax^3}}{5a}-\frac{A\sqrt{a+bx^2}}{5ax^5}$$

$$\Big\downarrow \text{358}$$

$$-\frac{-\frac{15a^2D\int \frac{1}{\sqrt{bx^2+a}}dx-\frac{\sqrt{a+bx^2}\left(15a^2C-10abB+8Ab^2\right)}{ax}}{3a}-\frac{\sqrt{a+bx^2}(4Ab-5aB)}{3ax^3}}{5a}-\frac{A\sqrt{a+bx^2}}{5ax^5}$$

$$\Big\downarrow \text{224}$$

$$-\frac{-\frac{15a^2D\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}-\frac{\sqrt{a+bx^2}\left(15a^2C-10abB+8Ab^2\right)}{ax}}{3a}-\frac{\sqrt{a+bx^2}(4Ab-5aB)}{3ax^3}}{5a}-\frac{A\sqrt{a+bx^2}}{5ax^5}$$

$$\Big\downarrow \text{219}$$

$$-\frac{-\frac{\frac{15a^2D\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}-\frac{\sqrt{a+bx^2}\left(15a^2C-10abB+8Ab^2\right)}{ax}}{3a}-\frac{\sqrt{a+bx^2}(4Ab-5aB)}{3ax^3}}{5a}-\frac{A\sqrt{a+bx^2}}{5ax^5}$$

input

```
Int[(A + B*x^2 + C*x^4 + D*x^6)/(x^6*Sqrt[a + b*x^2]),x]
```

output

```
-1/5*(A*Sqrt[a + b*x^2])/(a*x^5) - (-1/3*((4*A*b - 5*a*B)*Sqrt[a + b*x^2])
/(a*x^3) - (-(((8*A*b^2 - 10*a*b*B + 15*a^2*C)*Sqrt[a + b*x^2])/(a*x)) + (
15*a^2*D*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/Sqrt[b])/(3*a))/(5*a)
```

## Defintions of rubi rules used

rule 9

```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 358
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x_
Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] + S
imp[d/e^2   Int[(e*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b*c - a*d, 0] && EqQ[Simplify[m + 2*p + 3], 0] && NeQ[m,
 -1]
```

rule 1588
```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Simp[1/(d*f
^2*(m + 1))   Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x
) - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

## Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.82

| method | result |
|---|---|
| pseudoelliptic | $\frac{D \operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{x\sqrt{b}}\right) a^{3} x^{5}+\frac{4\sqrt{b\,x^{2}+a}\,\left(a\,x^{2}\left(\frac{5x^{2}B}{2}+A\right) b^{\frac{3}{2}}-2A\,b^{\frac{5}{2}}\,x^{4}-\frac{3\sqrt{b}\,\left(5C\,x^{4}+\frac{5}{3}x^{2}B+A\right) a^{2}}{4}\right)}{15}}{x^{5}\sqrt{b}\,a^{3}}$ |
| default | $\frac{D\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{\sqrt{b}}+A\left(-\frac{\sqrt{b\,x^{2}+a}}{5a\,x^{5}}-\frac{4b\left(-\frac{\sqrt{b\,x^{2}+a}}{3a\,x^{3}}+\frac{2b\sqrt{b\,x^{2}+a}}{3a^{2}x}\right)}{5a}\right)+B\left(-\frac{\sqrt{b\,x^{2}+a}}{3a\,x^{3}}+\frac{2b\sqrt{b\,x^{2}+a}}{3a^{2}x}\right)-$ |

input
```
int((D*x^6+C*x^4+B*x^2+A)/x^6/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
4/15*(15/4*D*arctanh((b*x^2+a)^(1/2)/x/b^(1/2))*a^3*x^5+(b*x^2+a)^(1/2)*(a
*x^2*(5/2*x^2*B+A)*b^(3/2)-2*A*b^(5/2)*x^4-3/4*b^(1/2)*(5*C*x^4+5/3*x^2*B+
A)*a^2))/b^(1/2)/x^5/a^3
```

## Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.86

$$\int \frac{A + Bx^2 + Cx^4 + Dx^6}{x^6\sqrt{a + bx^2}}\,dx$$
$$= \left[\frac{15\,Da^3\sqrt{b}x^5\log\left(-2\,bx^2 - 2\,\sqrt{bx^2 + a}\sqrt{b}x - a\right) - 2\,((15\,Ca^2b - 10\,Bab^2 + 8\,Ab^3)x^4 + 3\,Aa^2b + (5\,Ba\right.}{30\,a^3bx^5}$$
$$-\frac{15\,Da^3\sqrt{-b}x^5\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right) + ((15\,Ca^2b - 10\,Bab^2 + 8\,Ab^3)x^4 + 3\,Aa^2b + (5\,Ba^2b - 4\,Aab^2)x^2)\sqrt{}}{15\,a^3bx^5}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^6/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/30*(15*D*a^3*sqrt(b)*x^5*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a
) - 2*((15*C*a^2*b - 10*B*a*b^2 + 8*A*b^3)*x^4 + 3*A*a^2*b + (5*B*a^2*b -
4*A*a*b^2)*x^2)*sqrt(b*x^2 + a))/(a^3*b*x^5), -1/15*(15*D*a^3*sqrt(-b)*x^5
*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + ((15*C*a^2*b - 10*B*a*b^2 + 8*A*b^3)
*x^4 + 3*A*a^2*b + (5*B*a^2*b - 4*A*a*b^2)*x^2)*sqrt(b*x^2 + a))/(a^3*b*x^
5)]
```

## Sympy [A] (verification not implemented)

Time = 1.50 (sec) , antiderivative size = 427, normalized size of antiderivative = 3.62

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6\sqrt{a+bx^2}}\,dx = -\frac{3Aa^4b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2}+1}}{15a^5b^4x^4+30a^4b^5x^6+15a^3b^6x^8} - \frac{2Aa^3b^{\frac{11}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{15a^5b^4x^4+30a^4b^5x^6+15a^3b^6x^8} - \frac{3Aa^2b^{\frac{13}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{15a^5b^4x^4+30a^4b^5x^6+15a^3b^6x^8} - \frac{12Aab^{\frac{15}{2}}x^6\sqrt{\frac{a}{bx^2}+1}}{15a^5b^4x^4+30a^4b^5x^6+15a^3b^6x^8} - \frac{8Ab^{\frac{17}{2}}x^8\sqrt{\frac{a}{bx^2}+1}}{15a^5b^4x^4+30a^4b^5x^6+15a^3b^6x^8} - \frac{B\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{3ax^2} + \frac{2Bb^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{3a^2} - \frac{C\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{a} + D\left(\begin{cases} \frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0 \wedge b\neq 0 \\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{for } b\neq 0 \\ \frac{x}{\sqrt{a}} & \text{otherwise} \end{cases}\right)$$

input
```
integrate((D*x**6+C*x**4+B*x**2+A)/x**6/(b*x**2+a)**(1/2),x)
```

output
```
-3*A*a**4*b**(9/2)*sqrt(a/(b*x**2) + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*
x**6 + 15*a**3*b**6*x**8) - 2*A*a**3*b**(11/2)*x**2*sqrt(a/(b*x**2) + 1)/(
15*a**5*b**4*x**4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x**8) - 3*A*a**2*b**(
13/2)*x**4*sqrt(a/(b*x**2) + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*x**6 + 1
5*a**3*b**6*x**8) - 12*A*a*b**(15/2)*x**6*sqrt(a/(b*x**2) + 1)/(15*a**5*b*
*4*x**4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x**8) - 8*A*b**(17/2)*x**8*sqrt
(a/(b*x**2) + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x**
8) - B*sqrt(b)*sqrt(a/(b*x**2) + 1)/(3*a*x**2) + 2*B*b**(3/2)*sqrt(a/(b*x*
*2) + 1)/(3*a**2) - C*sqrt(b)*sqrt(a/(b*x**2) + 1)/a + D*Piecewise((log(2*
sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0) & Ne(b, 0)), (x*log(x)
/sqrt(b*x**2), Ne(b, 0)), (x/sqrt(a), True))
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.08

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6\sqrt{a+bx^2}}\,dx = \frac{D\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}} - \frac{\sqrt{bx^2+a}C}{ax} + \frac{2\sqrt{bx^2+a}Bb}{3a^2x} - \frac{8\sqrt{bx^2+a}Ab^2}{15a^3x} - \frac{\sqrt{bx^2+a}B}{3ax^3} + \frac{4\sqrt{bx^2+a}Ab}{15a^2x^3} - \frac{\sqrt{bx^2+a}A}{5ax^5}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^6/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
D*arcsinh(b*x/sqrt(a*b))/sqrt(b) - sqrt(b*x^2 + a)*C/(a*x) + 2/3*sqrt(b*x^
2 + a)*B*b/(a^2*x) - 8/15*sqrt(b*x^2 + a)*A*b^2/(a^3*x) - 1/3*sqrt(b*x^2 +
 a)*B/(a*x^3) + 4/15*sqrt(b*x^2 + a)*A*b/(a^2*x^3) - 1/5*sqrt(b*x^2 + a)*A
/(a*x^5)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 319 vs. $2(100) = 200$.

Time = 0.14 (sec) , antiderivative size = 319, normalized size of antiderivative = 2.70

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6\sqrt{a+bx^2}}\,dx = -\frac{D\log\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2\right)}{2\sqrt{b}} + \frac{2\left(15\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^8 C\sqrt{b} - 60\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6 Ca\sqrt{b} + 30\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6 Bb^{\frac{3}{2}} + 90\left(\right.}{}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^6/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
-1/2*D*log((sqrt(b)*x - sqrt(b*x^2 + a))^2)/sqrt(b) + 2/15*(15*(sqrt(b)*x
- sqrt(b*x^2 + a))^8*C*sqrt(b) - 60*(sqrt(b)*x - sqrt(b*x^2 + a))^6*C*a*sq
rt(b) + 30*(sqrt(b)*x - sqrt(b*x^2 + a))^6*B*b^(3/2) + 90*(sqrt(b)*x - sqr
t(b*x^2 + a))^4*C*a^2*sqrt(b) - 70*(sqrt(b)*x - sqrt(b*x^2 + a))^4*B*a*b^(
3/2) + 80*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*b^(5/2) - 60*(sqrt(b)*x - sqrt
(b*x^2 + a))^2*C*a^3*sqrt(b) + 50*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a^2*b^
(3/2) - 40*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A*a*b^(5/2) + 15*C*a^4*sqrt(b)
- 10*B*a^3*b^(3/2) + 8*A*a^2*b^(5/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a
)^5
```

## Mupad [B] (verification not implemented)

Time = 1.86 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.89

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6\sqrt{a+bx^2}}\,dx = \frac{\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)\,D}{\sqrt{b}} - \frac{C\,\sqrt{b\,x^2+a}}{a\,x} - \frac{B\,\sqrt{b\,x^2+a}\,\left(a-2\,b\,x^2\right)}{3\,a^2\,x^3} - \frac{A\,\sqrt{b\,x^2+a}\,\left(3\,a^2-4\,a\,b\,x^2+8\,b^2\,x^4\right)}{15\,a^3\,x^5}$$

input
```
int((A + B*x^2 + C*x^4 + x^6*D)/(x^6*(a + b*x^2)^(1/2)),x)
```

output
```
(log(b^(1/2)*x + (a + b*x^2)^(1/2))*D)/b^(1/2) - (C*(a + b*x^2)^(1/2))/(a*
x) - (B*(a + b*x^2)^(1/2)*(a - 2*b*x^2))/(3*a^2*x^3) - (A*(a + b*x^2)^(1/2
)*(3*a^2 + 8*b^2*x^4 - 4*a*b*x^2))/(15*a^3*x^5)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.06

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6\sqrt{a+bx^2}}\,dx$$

$$= \frac{-3\sqrt{b\,x^2+a}\,a^2b-\sqrt{b\,x^2+a}\,a\,b^2x^2-15\sqrt{b\,x^2+a}\,abc\,x^4+2\sqrt{b\,x^2+a}\,b^3x^4+15\sqrt{b}\log\left(\frac{\sqrt{b\,x^2+a}+\sqrt{b}\,x}{\sqrt{a}}\right)a}{15a^2b\,x^5}$$

input
```
int((D*x^6+C*x^4+B*x^2+A)/x^6/(b*x^2+a)^(1/2),x)
```

output
```
( - 3*sqrt(a + b*x**2)*a**2*b - sqrt(a + b*x**2)*a*b**2*x**2 - 15*sqrt(a +
 b*x**2)*a*b*c*x**4 + 2*sqrt(a + b*x**2)*b**3*x**4 + 15*sqrt(b)*log((sqrt(
a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2*d*x**5 + 9*sqrt(b)*a*b*c*x**5 - 2*s
qrt(b)*b**3*x**5)/(15*a**2*b*x**5)
```

# 3.222 $\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8\sqrt{a+bx^2}}\,dx$



## Optimal result

Integrand size = 32, antiderivative size = 141

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8\sqrt{a+bx^2}}\,dx = -\frac{A\sqrt{a+bx^2}}{7ax^7}+\frac{(6Ab-7aB)\sqrt{a+bx^2}}{35a^2x^5} - \frac{(24Ab^2-28abB+35a^2C)\sqrt{a+bx^2}}{105a^3x^3} + \frac{(48Ab^3-7a(8b^2B-10abC+15a^2D))\sqrt{a+bx^2}}{105a^4x}$$

output
```
-1/7*A*(b*x^2+a)^(1/2)/a/x^7+1/35*(6*A*b-7*B*a)*(b*x^2+a)^(1/2)/a^2/x^5-1/
105*(24*A*b^2-28*B*a*b+35*C*a^2)*(b*x^2+a)^(1/2)/a^3/x^3+1/105*(48*A*b^3-7
*a*(8*B*b^2-10*C*a*b+15*D*a^2))*(b*x^2+a)^(1/2)/a^4/x
```

## Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.73

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8\sqrt{a+bx^2}}\,dx = \frac{\sqrt{a+bx^2}(48Ab^3x^6-8ab^2x^4(3A+7Bx^2)+2a^2bx^2(9A+14Bx^2+35Cx^4)-a^3(15A+21Bx^2+35x^4(C}{105a^4x^7}$$

input
```
Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(x^8*Sqrt[a + b*x^2]),x]
```

output
```
(Sqrt[a + b*x^2]*(48*A*b^3*x^6 - 8*a*b^2*x^4*(3*A + 7*B*x^2) + 2*a^2*b*x^2
*(9*A + 14*B*x^2 + 35*C*x^4) - a^3*(15*A + 21*B*x^2 + 35*x^4*(C + 3*D*x^2)
)))/(105*a^4*x^7)
```

## Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {2334, 2089, 1588, 359, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8\sqrt{a+bx^2}}dx$$

$$\Big\downarrow \text{2334}$$

$$-\frac{\int \frac{6Ab-7a(Dx^4+Cx^2+B)}{x^6\sqrt{bx^2+a}}dx}{7a}-\frac{A\sqrt{a+bx^2}}{7ax^7}$$

$$\Big\downarrow \text{2089}$$

$$-\frac{\int \frac{-7aDx^4-7aCx^2+6Ab-7aB}{x^6\sqrt{bx^2+a}}dx}{7a}-\frac{A\sqrt{a+bx^2}}{7ax^7}$$

$$\Big\downarrow \text{1588}$$

$$-\frac{-\frac{\int \frac{35Dx^2a^2+35Ca^2-28bBa+24Ab^2}{x^4\sqrt{bx^2+a}}dx}{5a}-\frac{\sqrt{a+bx^2}(6Ab-7aB)}{5ax^5}}{7a}-\frac{A\sqrt{a+bx^2}}{7ax^7}$$

$$\Big\downarrow \text{359}$$

$$-\frac{-\frac{-\frac{\left(48Ab^3-7a\left(15a^2D-10abC+8b^2B\right)\right)\int \frac{1}{x^2\sqrt{bx^2+a}}dx}{3a}-\frac{\sqrt{a+bx^2}\left(35a^2C-28abB+24Ab^2\right)}{3ax^3}}{5a}-\frac{\sqrt{a+bx^2}(6Ab-7aB)}{5ax^5}}{7a}-\frac{A\sqrt{a+bx^2}}{7ax^7}$$

$$\Big\downarrow \text{242}$$

$$-\frac{-\frac{\frac{\sqrt{a+bx^2}\left(48Ab^3-7a\left(15a^2D-10abC+8b^2B\right)\right)}{3a^2x}-\frac{\sqrt{a+bx^2}\left(35a^2C-28abB+24Ab^2\right)}{3ax^3}}{5a}-\frac{\sqrt{a+bx^2}(6Ab-7aB)}{5ax^5}}{7a}-\frac{A\sqrt{a+bx^2}}{7ax^7}$$

input
```
Int[(A + B*x^2 + C*x^4 + D*x^6)/(x^8*Sqrt[a + b*x^2]),x]
```

output
```
-1/7*(A*Sqrt[a + b*x^2])/(a*x^7) - (-1/5*((6*A*b - 7*a*B)*Sqrt[a + b*x^2])
/(a*x^5) - (-1/3*((24*A*b^2 - 28*a*b*B + 35*a^2*C)*Sqrt[a + b*x^2])/(a*x^3
) + ((48*A*b^3 - 7*a*(8*b^2*B - 10*a*b*C + 15*a^2*D))*Sqrt[a + b*x^2])/(3*
a^2*x))/(5*a))/(7*a)
```

### Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 1588
```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Simp[1/(d*f
^2*(m + 1))   Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x
) - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

rule 2089
```
Int[(u_)^(p_.)*((f_.)*(x_))^(m_.)*(z_)^(q_.), x_Symbol] :> Int[(f*x)^m*Expa
ndToSum[z, x]^q*ExpandToSum[u, x]^p, x] /; FreeQ[{f, m, p, q}, x] && Binomi
alQ[z, x] && TrinomialQ[u, x] &&  !(BinomialMatchQ[z, x] && TrinomialMatchQ
[u, x])
```

rule 2334
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coef
f[Pq, x, 0], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[A*
x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))   Int[
x^(m + 2)*(a + b*x^2)^p*(a*(m + 1)*Q - A*b*(m + 2*(p + 1) + 1)), x], x]] /;
 FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && IntegerQ[m/2] && ILtQ[(m + 1)/2 + p,
 0] && LtQ[m + Expon[Pq, x] + 2*p + 1, 0]
```

## Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.65

| method | result |
|---|---|
| pseudoelliptic | $-\frac{\left(\left(7D x^{6}+\frac{7}{3}C\,x^{4}+\frac{7}{5}x^{2}B+A\right)a^{3}-\frac{6x^{2}b\left(\frac{35}{9}C\,x^{4}+\frac{14}{9}x^{2}B+A\right)a^{2}}{5}+\frac{8\left(\frac{7x^{2}B}{3}+A\right)x^{4}b^{2}a}{5}-\frac{16A\,b^{3}x^{6}}{5}\right)\sqrt{b\,x^{2}+a}}{7x^{7}a^{4}}$ |
| gosper | $-\frac{\sqrt{b\,x^{2}+a}\,\left(-48A\,b^{3}x^{6}+56Ba\,b^{2}x^{6}-70C\,a^{2}b\,x^{6}+105Da^{3}x^{6}+24aA\,b^{2}x^{4}-28B\,a^{2}b\,x^{4}+35C\,a^{3}x^{4}-18a^{2}Ab\,x^{2}+21B\,a^{3}x^{2}\right.}{105x^{7}a^{4}}$ |
| trager | $-\frac{\sqrt{b\,x^{2}+a}\,\left(-48A\,b^{3}x^{6}+56Ba\,b^{2}x^{6}-70C\,a^{2}b\,x^{6}+105Da^{3}x^{6}+24aA\,b^{2}x^{4}-28B\,a^{2}b\,x^{4}+35C\,a^{3}x^{4}-18a^{2}Ab\,x^{2}+21B\,a^{3}x^{2}\right.}{105x^{7}a^{4}}$ |
| orering | $-\frac{\sqrt{b\,x^{2}+a}\,\left(-48A\,b^{3}x^{6}+56Ba\,b^{2}x^{6}-70C\,a^{2}b\,x^{6}+105Da^{3}x^{6}+24aA\,b^{2}x^{4}-28B\,a^{2}b\,x^{4}+35C\,a^{3}x^{4}-18a^{2}Ab\,x^{2}+21B\,a^{3}x^{2}\right.}{105x^{7}a^{4}}$ |
| default | $A\left(-\frac{\sqrt{b\,x^{2}+a}}{7a\,x^{7}}-\frac{6b\left(-\frac{\sqrt{b\,x^{2}+a}}{5a\,x^{5}}-\frac{4b\left(-\frac{\sqrt{b\,x^{2}+a}}{3a\,x^{3}}+\frac{2b\sqrt{b\,x^{2}+a}}{3a^{2}x}\right)}{5a}\right)}{7a}\right)+B\left(-\frac{\sqrt{b\,x^{2}+a}}{5a\,x^{5}}-\frac{4b\left(-\frac{\sqrt{b\,x^{2}+a}}{3a\,x^{3}}+\frac{2b\sqrt{b\,x^{2}}}{3a^{2}x}\right.}{5a}\right.$ |

input
```
int((D*x^6+C*x^4+B*x^2+A)/x^8/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/7*((7*D*x^6+7/3*C*x^4+7/5*x^2*B+A)*a^3-6/5*x^2*b*(35/9*C*x^4+14/9*x^2*B
+A)*a^2+8/5*(7/3*x^2*B+A)*x^4*b^2*a-16/5*A*b^3*x^6)*(b*x^2+a)^(1/2)/x^7/a^
4
```

## Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.70

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8\sqrt{a+bx^2}}\,dx = -\frac{((105\,Da^3-70\,Ca^2b+56\,Bab^2-48\,Ab^3)x^6+(35\,Ca^3-28\,Ba^2b+24\,Aab^2)x^4+15\,Aa^3+3\,(7\,Ba^3}{105\,a^4x^7}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^8/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
-1/105*((105*D*a^3 - 70*C*a^2*b + 56*B*a*b^2 - 48*A*b^3)*x^6 + (35*C*a^3 -
 28*B*a^2*b + 24*A*a*b^2)*x^4 + 15*A*a^3 + 3*(7*B*a^3 - 6*A*a^2*b)*x^2)*sq
rt(b*x^2 + a)/(a^4*x^7)
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 891 vs. 2(134) = 268.

Time = 2.10 (sec) , antiderivative size = 891, normalized size of antiderivative = 6.32

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8\sqrt{a+bx^2}}\,dx = \text{Too large to display}$$

input
```
integrate((D*x**6+C*x**4+B*x**2+A)/x**8/(b*x**2+a)**(1/2),x)
```

output
```
-5*A*a**6*b**(19/2)*sqrt(a/(b*x**2) + 1)/(35*a**7*b**9*x**6 + 105*a**6*b**
10*x**8 + 105*a**5*b**11*x**10 + 35*a**4*b**12*x**12) - 9*A*a**5*b**(21/2)
*x**2*sqrt(a/(b*x**2) + 1)/(35*a**7*b**9*x**6 + 105*a**6*b**10*x**8 + 105*
a**5*b**11*x**10 + 35*a**4*b**12*x**12) - 5*A*a**4*b**(23/2)*x**4*sqrt(a/(
b*x**2) + 1)/(35*a**7*b**9*x**6 + 105*a**6*b**10*x**8 + 105*a**5*b**11*x**
10 + 35*a**4*b**12*x**12) + 5*A*a**3*b**(25/2)*x**6*sqrt(a/(b*x**2) + 1)/(
35*a**7*b**9*x**6 + 105*a**6*b**10*x**8 + 105*a**5*b**11*x**10 + 35*a**4*b
**12*x**12) + 30*A*a**2*b**(27/2)*x**8*sqrt(a/(b*x**2) + 1)/(35*a**7*b**9*
x**6 + 105*a**6*b**10*x**8 + 105*a**5*b**11*x**10 + 35*a**4*b**12*x**12) +
 40*A*a*b**(29/2)*x**10*sqrt(a/(b*x**2) + 1)/(35*a**7*b**9*x**6 + 105*a**6
*b**10*x**8 + 105*a**5*b**11*x**10 + 35*a**4*b**12*x**12) + 16*A*b**(31/2)
*x**12*sqrt(a/(b*x**2) + 1)/(35*a**7*b**9*x**6 + 105*a**6*b**10*x**8 + 105
*a**5*b**11*x**10 + 35*a**4*b**12*x**12) - 3*B*a**4*b**(9/2)*sqrt(a/(b*x**
2) + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x**8) - 2*B*
a**3*b**(11/2)*x**2*sqrt(a/(b*x**2) + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5
*x**6 + 15*a**3*b**6*x**8) - 3*B*a**2*b**(13/2)*x**4*sqrt(a/(b*x**2) + 1)/
(15*a**5*b**4*x**4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x**8) - 12*B*a*b**(1
5/2)*x**6*sqrt(a/(b*x**2) + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*x**6 + 15
*a**3*b**6*x**8) - 8*B*b**(17/2)*x**8*sqrt(a/(b*x**2) + 1)/(15*a**5*b**4*x
**4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x**8) - C*sqrt(b)*sqrt(a/(b*x**2...
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.37

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8\sqrt{a+bx^2}}\,dx = -\frac{\sqrt{bx^2+a}D}{ax} + \frac{2\sqrt{bx^2+a}Cb}{3\,a^2x} - \frac{8\sqrt{bx^2+a}Bb^2}{15\,a^3x} + \frac{16\sqrt{bx^2+a}Ab^3}{35\,a^4x} - \frac{\sqrt{bx^2+a}C}{3\,ax^3} + \frac{4\sqrt{bx^2+a}Bb}{15\,a^2x^3} - \frac{8\sqrt{bx^2+a}Ab^2}{35\,a^3x^3} - \frac{\sqrt{bx^2+a}B}{5\,ax^5} + \frac{6\sqrt{bx^2+a}Ab}{35\,a^2x^5} - \frac{\sqrt{bx^2+a}A}{7\,ax^7}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^8/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
-sqrt(b*x^2 + a)*D/(a*x) + 2/3*sqrt(b*x^2 + a)*C*b/(a^2*x) - 8/15*sqrt(b*x
^2 + a)*B*b^2/(a^3*x) + 16/35*sqrt(b*x^2 + a)*A*b^3/(a^4*x) - 1/3*sqrt(b*x
^2 + a)*C/(a*x^3) + 4/15*sqrt(b*x^2 + a)*B*b/(a^2*x^3) - 8/35*sqrt(b*x^2 +
 a)*A*b^2/(a^3*x^3) - 1/5*sqrt(b*x^2 + a)*B/(a*x^5) + 6/35*sqrt(b*x^2 + a)
*A*b/(a^2*x^5) - 1/7*sqrt(b*x^2 + a)*A/(a*x^7)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 548 vs. $2(125) = 250$.

Time = 0.14 (sec) , antiderivative size = 548, normalized size of antiderivative = 3.89

$$\int \frac{A + Bx^2 + Cx^4 + Dx^6}{x^8\sqrt{a + bx^2}}\,dx = \frac{2\left(105\left(\sqrt{b}x - \sqrt{bx^2 + a}\right)^{12} D\sqrt{b} - 630\left(\sqrt{b}x - \sqrt{bx^2 + a}\right)^{10} Da\sqrt{b} + 210\left(\sqrt{b}x - \sqrt{bx^2 + a}\right)^{10} Cb^{\frac{3}{2}} + \right.}{}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^8/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
2/105*(105*(sqrt(b)*x - sqrt(b*x^2 + a))^12*D*sqrt(b) - 630*(sqrt(b)*x - s
qrt(b*x^2 + a))^10*D*a*sqrt(b) + 210*(sqrt(b)*x - sqrt(b*x^2 + a))^10*C*b^
(3/2) + 1575*(sqrt(b)*x - sqrt(b*x^2 + a))^8*D*a^2*sqrt(b) - 910*(sqrt(b)*
x - sqrt(b*x^2 + a))^8*C*a*b^(3/2) + 560*(sqrt(b)*x - sqrt(b*x^2 + a))^8*B
*b^(5/2) - 2100*(sqrt(b)*x - sqrt(b*x^2 + a))^6*D*a^3*sqrt(b) + 1540*(sqrt
(b)*x - sqrt(b*x^2 + a))^6*C*a^2*b^(3/2) - 1400*(sqrt(b)*x - sqrt(b*x^2 +
a))^6*B*a*b^(5/2) + 1680*(sqrt(b)*x - sqrt(b*x^2 + a))^6*A*b^(7/2) + 1575*
(sqrt(b)*x - sqrt(b*x^2 + a))^4*D*a^4*sqrt(b) - 1260*(sqrt(b)*x - sqrt(b*x
^2 + a))^4*C*a^3*b^(3/2) + 1176*(sqrt(b)*x - sqrt(b*x^2 + a))^4*B*a^2*b^(5
/2) - 1008*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*a*b^(7/2) - 630*(sqrt(b)*x -
sqrt(b*x^2 + a))^2*D*a^5*sqrt(b) + 490*(sqrt(b)*x - sqrt(b*x^2 + a))^2*C*a
^4*b^(3/2) - 392*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a^3*b^(5/2) + 336*(sqrt
(b)*x - sqrt(b*x^2 + a))^2*A*a^2*b^(7/2) + 105*D*a^6*sqrt(b) - 70*C*a^5*b^
(3/2) + 56*B*a^4*b^(5/2) - 48*A*a^3*b^(7/2))/((sqrt(b)*x - sqrt(b*x^2 + a)
)^2 - a)^7
```

## Mupad [B] (verification not implemented)

Time = 1.88 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.16

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8\sqrt{a+bx^2}}\,dx = \frac{6\,A\,b\,\sqrt{b\,x^2+a}}{35\,a^2\,x^5}-\frac{A\,\sqrt{b\,x^2+a}}{7\,a\,x^7}-\frac{C\,\sqrt{b\,x^2+a}\,\left(a-2\,b\,x^2\right)}{3\,a^2\,x^3}-\frac{\sqrt{b\,x^2+a}\,D}{a\,x}-\frac{B\,\sqrt{b\,x^2+a}\,\left(3\,a^2-4\,a\,b\,x^2+8\,b^2\,x^4\right)}{15\,a^3\,x^5}-\frac{8\,A\,b^2\,\sqrt{b\,x^2+a}}{35\,a^3\,x^3}+\frac{16\,A\,b^3\,\sqrt{b\,x^2+a}}{35\,a^4\,x}$$

input
```
int((A + B*x^2 + C*x^4 + x^6*D)/(x^8*(a + b*x^2)^(1/2)),x)
```

output
```
(6*A*b*(a + b*x^2)^(1/2))/(35*a^2*x^5) - (A*(a + b*x^2)^(1/2))/(7*a*x^7) -
 (C*(a + b*x^2)^(1/2)*(a - 2*b*x^2))/(3*a^2*x^3) - ((a + b*x^2)^(1/2)*D)/(
a*x) - (B*(a + b*x^2)^(1/2)*(3*a^2 + 8*b^2*x^4 - 4*a*b*x^2))/(15*a^3*x^5)
- (8*A*b^2*(a + b*x^2)^(1/2))/(35*a^3*x^3) + (16*A*b^3*(a + b*x^2)^(1/2))/
(35*a^4*x)
```

## Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.09

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8\sqrt{a+bx^2}}\,dx = \frac{-15\sqrt{b\,x^2+a}\,a^3-3\sqrt{b\,x^2+a}\,a^2b\,x^2-35\sqrt{b\,x^2+a}\,a^2c\,x^4-105\sqrt{b\,x^2+a}\,a^2d\,x^6+4\sqrt{b\,x^2+a}\,a\,b^2x^4+}{105a^3x^7}$$

input
```
int((D*x^6+C*x^4+B*x^2+A)/x^8/(b*x^2+a)^(1/2),x)
```

output
```
( - 15*sqrt(a + b*x**2)*a**3 - 3*sqrt(a + b*x**2)*a**2*b*x**2 - 35*sqrt(a
+ b*x**2)*a**2*c*x**4 - 105*sqrt(a + b*x**2)*a**2*d*x**6 + 4*sqrt(a + b*x*
*2)*a*b**2*x**4 + 70*sqrt(a + b*x**2)*a*b*c*x**6 - 8*sqrt(a + b*x**2)*b**3
*x**6 + 75*sqrt(b)*a**2*d*x**7 - 70*sqrt(b)*a*b*c*x**7 + 8*sqrt(b)*b**3*x*
*7)/(105*a**3*x**7)
```

# 3.223 $\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{10}\sqrt{a+bx^2}}\,dx$



## Optimal result

Integrand size = 32, antiderivative size = 191

$$\begin{aligned}\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{10}\sqrt{a+bx^2}}\,dx = &-\frac{A\sqrt{a+bx^2}}{9ax^9}+\frac{(8Ab-9aB)\sqrt{a+bx^2}}{63a^2x^7}\\ &-\frac{(16Ab^2-18abB+21a^2C)\sqrt{a+bx^2}}{105a^3x^5}\\ &+\frac{(64Ab^3-3a(24b^2B-28abC+35a^2D))\sqrt{a+bx^2}}{315a^4x^3}\\ &-\frac{2b(64Ab^3-3a(24b^2B-28abC+35a^2D))\sqrt{a+bx^2}}{315a^5x}\end{aligned}$$

output

```
-1/9*A*(b*x^2+a)^(1/2)/a/x^9+1/63*(8*A*b-9*B*a)*(b*x^2+a)^(1/2)/a^2/x^7-1/
105*(16*A*b^2-18*B*a*b+21*C*a^2)*(b*x^2+a)^(1/2)/a^3/x^5+1/315*(64*A*b^3-3
*a*(24*B*b^2-28*C*a*b+35*D*a^2))*(b*x^2+a)^(1/2)/a^4/x^3-2/315*b*(64*A*b^3
-3*a*(24*B*b^2-28*C*a*b+35*D*a^2))*(b*x^2+a)^(1/2)/a^5/x
```

## Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.70

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{10}\sqrt{a+bx^2}}\,dx = -\frac{\sqrt{a+bx^2}(128Ab^4x^8-16ab^3x^6(4A+9Bx^2)+24a^2b^2x^4(2A+3Bx^2+7Cx^4)-2a^3bx^2(20A+27Bx^2}{315a^5x^9}$$

input
```
Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(x^10*Sqrt[a + b*x^2]),x]
```

output
```
-1/315*(Sqrt[a + b*x^2]*(128*A*b^4*x^8 - 16*a*b^3*x^6*(4*A + 9*B*x^2) + 24
*a^2*b^2*x^4*(2*A + 3*B*x^2 + 7*C*x^4) - 2*a^3*b*x^2*(20*A + 27*B*x^2 + 42
*C*x^4 + 105*D*x^6) + a^4*(35*A + 45*B*x^2 + 63*C*x^4 + 105*D*x^6)))/(a^5*
x^9)
```

## Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {2334, 2089, 1588, 27, 359, 245, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{10}\sqrt{a+bx^2}}dx$$

$$\Big\downarrow \ 2334$$

$$-\frac{\int \frac{8Ab-9a(Dx^4+Cx^2+B)}{x^8\sqrt{bx^2+a}}dx}{9a}-\frac{A\sqrt{a+bx^2}}{9ax^9}$$

$$\Big\downarrow \ 2089$$

$$-\frac{\int \frac{-9aDx^4-9aCx^2+8Ab-9aB}{x^8\sqrt{bx^2+a}}dx}{9a}-\frac{A\sqrt{a+bx^2}}{9ax^9}$$

$$\Big\downarrow \ 1588$$

$$-\frac{-\frac{\int \frac{3\left(16Ab^2+21a^2Dx^2-3a(6bB-7aC)\right)}{x^6\sqrt{bx^2+a}}dx}{7a}-\frac{\sqrt{a+bx^2}(8Ab-9aB)}{7ax^7}}{9a}-\frac{A\sqrt{a+bx^2}}{9ax^9}$$

$\Big\downarrow$ 27

$$-\frac{-\frac{3\int \frac{21Dx^2a^2+21Ca^2-18bBa+16Ab^2}{x^6\sqrt{bx^2+a}}dx}{7a}-\frac{\sqrt{a+bx^2}(8Ab-9aB)}{7ax^7}}{9a}-\frac{A\sqrt{a+bx^2}}{9ax^9}$$

$\Big\downarrow$ 359

$$-\frac{-\frac{3\left(-\frac{\left(64Ab^3-3a\left(35a^2D-28abC+24b^2B\right)\right)\int \frac{1}{x^4\sqrt{bx^2+a}}dx}{5a}-\frac{\sqrt{a+bx^2}\left(21a^2C-18abB+16Ab^2\right)}{5ax^5}\right)}{7a}-\frac{\sqrt{a+bx^2}(8Ab-9aB)}{7ax^7}}{9a}-\frac{A\sqrt{a+bx^2}}{9ax^9}$$

$\Big\downarrow$ 245

$$-\frac{-\frac{3\left(-\frac{\left(64Ab^3-3a\left(35a^2D-28abC+24b^2B\right)\right)\left(-\frac{2b\int \frac{1}{x^2\sqrt{bx^2+a}}dx}{3a}-\frac{\sqrt{a+bx^2}}{3ax^3}\right)}{5a}-\frac{\sqrt{a+bx^2}\left(21a^2C-18abB+16Ab^2\right)}{5ax^5}\right)}{7a}-\frac{\sqrt{a+bx^2}(8Ab-9aB)}{7ax^7}}{9a}-\frac{A\sqrt{a+bx^2}}{9ax^9}$$

$\Big\downarrow$ 242

$$-\frac{-\frac{3\left(-\frac{\sqrt{a+bx^2}\left(21a^2C-18abB+16Ab^2\right)}{5ax^5}-\frac{\left(\frac{2b\sqrt{a+bx^2}}{3a^2x}-\frac{\sqrt{a+bx^2}}{3ax^3}\right)\left(64Ab^3-3a\left(35a^2D-28abC+24b^2B\right)\right)}{5a}\right)}{7a}-\frac{\sqrt{a+bx^2}(8Ab-9aB)}{7ax^7}}{9a}-\frac{A\sqrt{a+bx^2}}{9ax^9}$$

input
```
Int[(A + B*x^2 + C*x^4 + D*x^6)/(x^10*Sqrt[a + b*x^2]),x]
```

output
```
-1/9*(A*Sqrt[a + b*x^2])/(a*x^9) - (-1/7*((8*A*b - 9*a*B)*Sqrt[a + b*x^2])
/(a*x^7) - (3*(-1/5*((16*A*b^2 - 18*a*b*B + 21*a^2*C)*Sqrt[a + b*x^2])/(a*
x^5) - ((64*A*b^3 - 3*a*(24*b^2*B - 28*a*b*C + 35*a^2*D))*(-1/3*Sqrt[a + b
*x^2]/(a*x^3) + (2*b*Sqrt[a + b*x^2])/(3*a^2*x)))/(5*a)))/(7*a))/(9*a)
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 1588
```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Simp[1/(d*f
^2*(m + 1))   Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x
) - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

rule 2089
```
Int[(u_)^(p_.)*((f_.)*(x_))^(m_.)*(z_)^(q_.), x_Symbol] :> Int[(f*x)^m*Expa
ndToSum[z, x]^q*ExpandToSum[u, x]^p, x] /; FreeQ[{f, m, p, q}, x] && Binomi
alQ[z, x] && TrinomialQ[u, x] &&  !(BinomialMatchQ[z, x] && TrinomialMatchQ
[u, x])
```

rule 2334
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coef
f[Pq, x, 0], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[A*
x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))   Int[
x^(m + 2)*(a + b*x^2)^p*(a*(m + 1)*Q - A*b*(m + 2*(p + 1) + 1)), x], x]] /;
 FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && IntegerQ[m/2] && ILtQ[(m + 1)/2 + p,
 0] && LtQ[m + Expon[Pq, x] + 2*p + 1, 0]
```

## Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.64

| method | result |
|---|---|
| pseudoelliptic | $-\frac{\left(\left(3Dx^6+\frac{9}{5}Cx^4+\frac{9}{7}x^2B+A\right)a^4-\frac{8\left(\frac{21}{4}Dx^6+\frac{21}{10}Cx^4+\frac{27}{20}x^2B+A\right)x^2ba^3}{7}+\frac{48\left(\frac{7}{2}Cx^4+\frac{3}{2}x^2B+A\right)x^4b^2a^2}{35}-\frac{64\left(\frac{9x^2B}{4}+A\right)x^6b}{35}\right.}{9a^5x^9}$ |
| gosper | $-\frac{\sqrt{bx^2+a}\,(128Ax^8b^4-144Bx^8ab^3+168Ca^2b^2x^8-210Da^3bx^8-64Ax^6ab^3+72Bx^6a^2b^2-84Ca^3bx^6+105Da^4x^6+48}{315a^5x^9}$ |
| trager | $-\frac{\sqrt{bx^2+a}\,(128Ax^8b^4-144Bx^8ab^3+168Ca^2b^2x^8-210Da^3bx^8-64Ax^6ab^3+72Bx^6a^2b^2-84Ca^3bx^6+105Da^4x^6+48}{315a^5x^9}$ |
| orering | $-\frac{\sqrt{bx^2+a}\,(128Ax^8b^4-144Bx^8ab^3+168Ca^2b^2x^8-210Da^3bx^8-64Ax^6ab^3+72Bx^6a^2b^2-84Ca^3bx^6+105Da^4x^6+48}{315a^5x^9}$ |
| default | $A\left(-\frac{\sqrt{bx^2+a}}{9a\,x^9}-\frac{8b\left(-\frac{\sqrt{bx^2+a}}{7a\,x^7}-\frac{6b\left(-\frac{\sqrt{bx^2+a}}{5a\,x^5}-\frac{4b\left(-\frac{\sqrt{bx^2+a}}{3a\,x^3}+\frac{2b\sqrt{bx^2+a}}{3a^2x}\right)}{5a}\right)}{7a}\right)}{9a}\right)+B\left(-\frac{\sqrt{bx^2+a}}{7a\,x^7}-\frac{6b\left(-\right.}{}\right.$ |

input
```
int((D*x^6+C*x^4+B*x^2+A)/x^10/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/9*((3*D*x^6+9/5*C*x^4+9/7*x^2*B+A)*a^4-8/7*(21/4*D*x^6+21/10*C*x^4+27/2
0*x^2*B+A)*x^2*b*a^3+48/35*(7/2*C*x^4+3/2*x^2*B+A)*x^4*b^2*a^2-64/35*(9/4*
x^2*B+A)*x^6*b^3*a+128/35*A*x^8*b^4)*(b*x^2+a)^(1/2)/a^5/x^9
```

## Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.74

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{10}\sqrt{a+bx^2}}\,dx$$
$$= \frac{(2\,(105\,Da^3b-84\,Ca^2b^2+72\,Bab^3-64\,Ab^4)x^8-(105\,Da^4-84\,Ca^3b+72\,Ba^2b^2-64\,Aab^3)x^6-35\,A}{315\,a^5x^9}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^10/(b*x^2+a)^(1/2),x, algorithm="fricas"
)
```

output
```
1/315*(2*(105*D*a^3*b - 84*C*a^2*b^2 + 72*B*a*b^3 - 64*A*b^4)*x^8 - (105*D
*a^4 - 84*C*a^3*b + 72*B*a^2*b^2 - 64*A*a*b^3)*x^6 - 35*A*a^4 - 3*(21*C*a^
4 - 18*B*a^3*b + 16*A*a^2*b^2)*x^4 - 5*(9*B*a^4 - 8*A*a^3*b)*x^2)*sqrt(b*x
^2 + a)/(a^5*x^9)
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1642 vs. $2(187) = 374$.

Time = 3.23 (sec) , antiderivative size = 1642, normalized size of antiderivative = 8.60

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{10}\sqrt{a+bx^2}}\,dx = \text{Too large to display}$$

input
```
integrate((D*x**6+C*x**4+B*x**2+A)/x**10/(b*x**2+a)**(1/2),x)
```

output
```
-35*A*a**8*b**(33/2)*sqrt(a/(b*x**2) + 1)/(315*a**9*b**16*x**8 + 1260*a**8
*b**17*x**10 + 1890*a**7*b**18*x**12 + 1260*a**6*b**19*x**14 + 315*a**5*b*
*20*x**16) - 100*A*a**7*b**(35/2)*x**2*sqrt(a/(b*x**2) + 1)/(315*a**9*b**1
6*x**8 + 1260*a**8*b**17*x**10 + 1890*a**7*b**18*x**12 + 1260*a**6*b**19*x
**14 + 315*a**5*b**20*x**16) - 98*A*a**6*b**(37/2)*x**4*sqrt(a/(b*x**2) +
1)/(315*a**9*b**16*x**8 + 1260*a**8*b**17*x**10 + 1890*a**7*b**18*x**12 +
1260*a**6*b**19*x**14 + 315*a**5*b**20*x**16) - 28*A*a**5*b**(39/2)*x**6*s
qrt(a/(b*x**2) + 1)/(315*a**9*b**16*x**8 + 1260*a**8*b**17*x**10 + 1890*a*
*7*b**18*x**12 + 1260*a**6*b**19*x**14 + 315*a**5*b**20*x**16) - 35*A*a**4
*b**(41/2)*x**8*sqrt(a/(b*x**2) + 1)/(315*a**9*b**16*x**8 + 1260*a**8*b**1
7*x**10 + 1890*a**7*b**18*x**12 + 1260*a**6*b**19*x**14 + 315*a**5*b**20*x
**16) - 280*A*a**3*b**(43/2)*x**10*sqrt(a/(b*x**2) + 1)/(315*a**9*b**16*x*
*8 + 1260*a**8*b**17*x**10 + 1890*a**7*b**18*x**12 + 1260*a**6*b**19*x**14
 + 315*a**5*b**20*x**16) - 560*A*a**2*b**(45/2)*x**12*sqrt(a/(b*x**2) + 1)
/(315*a**9*b**16*x**8 + 1260*a**8*b**17*x**10 + 1890*a**7*b**18*x**12 + 12
60*a**6*b**19*x**14 + 315*a**5*b**20*x**16) - 448*A*a*b**(47/2)*x**14*sqrt
(a/(b*x**2) + 1)/(315*a**9*b**16*x**8 + 1260*a**8*b**17*x**10 + 1890*a**7*
b**18*x**12 + 1260*a**6*b**19*x**14 + 315*a**5*b**20*x**16) - 128*A*b**(49
/2)*x**16*sqrt(a/(b*x**2) + 1)/(315*a**9*b**16*x**8 + 1260*a**8*b**17*x**1
0 + 1890*a**7*b**18*x**12 + 1260*a**6*b**19*x**14 + 315*a**5*b**20*x**1...
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.44

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{10}\sqrt{a+bx^2}}\,dx = \frac{2\sqrt{bx^2+a}Db}{3a^2x} - \frac{8\sqrt{bx^2+a}Cb^2}{15a^3x} + \frac{16\sqrt{bx^2+a}Bb^3}{35a^4x} - \frac{128\sqrt{bx^2+a}Ab^4}{315a^5x} - \frac{\sqrt{bx^2+a}D}{3ax^3} + \frac{4\sqrt{bx^2+a}Cb}{15a^2x^3} - \frac{8\sqrt{bx^2+a}Bb^2}{35a^3x^3} + \frac{64\sqrt{bx^2+a}Ab^3}{315a^4x^3} - \frac{\sqrt{bx^2+a}C}{5ax^5} + \frac{6\sqrt{bx^2+a}Bb}{35a^2x^5} - \frac{16\sqrt{bx^2+a}Ab^2}{105a^3x^5} - \frac{\sqrt{bx^2+a}B}{7ax^7} + \frac{8\sqrt{bx^2+a}Ab}{63a^2x^7} - \frac{\sqrt{bx^2+a}A}{9ax^9}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^10/(b*x^2+a)^(1/2),x, algorithm="maxima"
)
```

output
```
2/3*sqrt(b*x^2 + a)*D*b/(a^2*x) - 8/15*sqrt(b*x^2 + a)*C*b^2/(a^3*x) + 16/
35*sqrt(b*x^2 + a)*B*b^3/(a^4*x) - 128/315*sqrt(b*x^2 + a)*A*b^4/(a^5*x) -
 1/3*sqrt(b*x^2 + a)*D/(a*x^3) + 4/15*sqrt(b*x^2 + a)*C*b/(a^2*x^3) - 8/35
*sqrt(b*x^2 + a)*B*b^2/(a^3*x^3) + 64/315*sqrt(b*x^2 + a)*A*b^3/(a^4*x^3)
- 1/5*sqrt(b*x^2 + a)*C/(a*x^5) + 6/35*sqrt(b*x^2 + a)*B*b/(a^2*x^5) - 16/
105*sqrt(b*x^2 + a)*A*b^2/(a^3*x^5) - 1/7*sqrt(b*x^2 + a)*B/(a*x^7) + 8/63
*sqrt(b*x^2 + a)*A*b/(a^2*x^7) - 1/9*sqrt(b*x^2 + a)*A/(a*x^9)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 660 vs. $2(171) = 342$.

Time = 0.14 (sec) , antiderivative size = 660, normalized size of antiderivative = 3.46

$$\int \frac{A + Bx^2 + Cx^4 + Dx^6}{x^{10}\sqrt{a + bx^2}}\, dx = \text{Too large to display}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^10/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
4/315*(315*(sqrt(b)*x - sqrt(b*x^2 + a))^14*D*b^(3/2) - 1995*(sqrt(b)*x -
sqrt(b*x^2 + a))^12*D*a*b^(3/2) + 840*(sqrt(b)*x - sqrt(b*x^2 + a))^12*C*b
^(5/2) + 5355*(sqrt(b)*x - sqrt(b*x^2 + a))^10*D*a^2*b^(3/2) - 3780*(sqrt(
b)*x - sqrt(b*x^2 + a))^10*C*a*b^(5/2) + 2520*(sqrt(b)*x - sqrt(b*x^2 + a)
)^10*B*b^(7/2) - 7875*(sqrt(b)*x - sqrt(b*x^2 + a))^8*D*a^3*b^(3/2) + 6804
*(sqrt(b)*x - sqrt(b*x^2 + a))^8*C*a^2*b^(5/2) - 6552*(sqrt(b)*x - sqrt(b*
x^2 + a))^8*B*a*b^(7/2) + 8064*(sqrt(b)*x - sqrt(b*x^2 + a))^8*A*b^(9/2) +
 6825*(sqrt(b)*x - sqrt(b*x^2 + a))^6*D*a^4*b^(3/2) - 6216*(sqrt(b)*x - sq
rt(b*x^2 + a))^6*C*a^3*b^(5/2) + 6048*(sqrt(b)*x - sqrt(b*x^2 + a))^6*B*a^
2*b^(7/2) - 5376*(sqrt(b)*x - sqrt(b*x^2 + a))^6*A*a*b^(9/2) - 3465*(sqrt(
b)*x - sqrt(b*x^2 + a))^4*D*a^5*b^(3/2) + 3024*(sqrt(b)*x - sqrt(b*x^2 + a
))^4*C*a^4*b^(5/2) - 2592*(sqrt(b)*x - sqrt(b*x^2 + a))^4*B*a^3*b^(7/2) +
2304*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*a^2*b^(9/2) + 945*(sqrt(b)*x - sqrt
(b*x^2 + a))^2*D*a^6*b^(3/2) - 756*(sqrt(b)*x - sqrt(b*x^2 + a))^2*C*a^5*b
^(5/2) + 648*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a^4*b^(7/2) - 576*(sqrt(b)*
x - sqrt(b*x^2 + a))^2*A*a^3*b^(9/2) - 105*D*a^7*b^(3/2) + 84*C*a^6*b^(5/2
) - 72*B*a^5*b^(7/2) + 64*A*a^4*b^(9/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2
- a)^9
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{10}\sqrt{a+bx^2}}\,dx = \int \frac{A+B\,x^2+C\,x^4+x^6\,D}{x^{10}\,\sqrt{b\,x^2+a}}\,dx$$

input
```
int((A + B*x^2 + C*x^4 + x^6*D)/(x^10*(a + b*x^2)^(1/2)),x)
```

output
```
int((A + B*x^2 + C*x^4 + x^6*D)/(x^10*(a + b*x^2)^(1/2)), x)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.12

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{10}\sqrt{a+bx^2}}\,dx = \frac{-35\sqrt{b\,x^2+a}\,a^4-5\sqrt{b\,x^2+a}\,a^3b\,x^2-63\sqrt{b\,x^2+a}\,a^3c\,x^4-105\sqrt{b\,x^2+a}\,a^3d\,x^6+6\sqrt{b\,x^2+a}\,a^2b^2x^4+}{}$$

input
```
int((D*x^6+C*x^4+B*x^2+A)/x^10/(b*x^2+a)^(1/2),x)
```

output
```
( - 35*sqrt(a + b*x**2)*a**4 - 5*sqrt(a + b*x**2)*a**3*b*x**2 - 63*sqrt(a
+ b*x**2)*a**3*c*x**4 - 105*sqrt(a + b*x**2)*a**3*d*x**6 + 6*sqrt(a + b*x*
*2)*a**2*b**2*x**4 + 84*sqrt(a + b*x**2)*a**2*b*c*x**6 + 210*sqrt(a + b*x*
*2)*a**2*b*d*x**8 - 8*sqrt(a + b*x**2)*a*b**3*x**6 - 168*sqrt(a + b*x**2)*
a*b**2*c*x**8 + 16*sqrt(a + b*x**2)*b**4*x**8 - 210*sqrt(b)*a**2*b*d*x**9
+ 168*sqrt(b)*a*b**2*c*x**9 - 16*sqrt(b)*b**4*x**9)/(315*a**4*x**9)
```

# 3.224 $\int \frac{x^5(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{3/2}}\,dx$



## Optimal result

Integrand size = 32, antiderivative size = 215

$$\int \frac{x^5(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{3/2}}\,dx = -\frac{a^2(Ab^3-a(b^2B-abC+a^2D))}{b^6\sqrt{a+bx^2}} - \frac{a(2Ab^3-a(3b^2B-4abC+5a^2D))\sqrt{a+bx^2}}{b^6} + \frac{(Ab^3-a(3b^2B-6abC+10a^2D))(a+bx^2)^{3/2}}{3b^6} + \frac{(b^2B-4abC+10a^2D)(a+bx^2)^{5/2}}{5b^6} + \frac{(bC-5aD)(a+bx^2)^{7/2}}{7b^6} + \frac{D(a+bx^2)^{9/2}}{9b^6}$$

output

```
-a^2*(A*b^3-a*(B*b^2-C*a*b+D*a^2))/b^6/(b*x^2+a)^(1/2)-a*(2*A*b^3-a*(3*B*b
^2-4*C*a*b+5*D*a^2))*(b*x^2+a)^(1/2)/b^6+1/3*(A*b^3-a*(3*B*b^2-6*C*a*b+10*
D*a^2))*(b*x^2+a)^(3/2)/b^6+1/5*(B*b^2-4*C*a*b+10*D*a^2)*(b*x^2+a)^(5/2)/b
^6+1/7*(C*b-5*D*a)*(b*x^2+a)^(7/2)/b^6+1/9*D*(b*x^2+a)^(9/2)/b^6
```

## Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.72

$$\int \frac{x^5(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{3/2}}\,dx = \frac{1280a^5D-128a^4b(9C-5Dx^2)+16a^3b^2(63B-36Cx^2-10Dx^4)+}{}$$

input
```
Integrate[(x^5*(A + B*x^2 + C*x^4 + D*x^6))/(a + b*x^2)^(3/2),x]
```

output
```
(1280*a^5*D - 128*a^4*b*(9*C - 5*D*x^2) + 16*a^3*b^2*(63*B - 36*C*x^2 - 10
*D*x^4) + 8*a^2*b^3*(-105*A + 63*B*x^2 + 18*C*x^4 + 10*D*x^6) - 2*a*b^4*x^
2*(210*A + 63*B*x^2 + 36*C*x^4 + 25*D*x^6) + b^5*x^4*(105*A + 63*B*x^2 + 4
5*C*x^4 + 35*D*x^6))/(315*b^6*Sqrt[a + b*x^2])
```

## Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {2331, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5\left(A+Bx^2+Cx^4+Dx^6\right)}{(a+bx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 2331}$$

$$\frac{1}{2}\int \frac{x^4\left(Dx^6+Cx^4+Bx^2+A\right)}{(bx^2+a)^{3/2}}dx^2$$

$$\Big\downarrow \text{ 2123}$$

$$\frac{1}{2}\int \left(\frac{D(bx^2+a)^{7/2}}{b^5}+\frac{(bC-5aD)\left(bx^2+a\right)^{5/2}}{b^5}+\frac{\left(10Da^2-4bCa+b^2B\right)\left(bx^2+a\right)^{3/2}}{b^5}+\frac{\left(Ab^3-a(10Da^2-\right.}{}\right.$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(\frac{2\left(a+bx^2\right)^{3/2}\left(Ab^3-a\left(10a^2D-6abC+3b^2B\right)\right)}{3b^6}-\frac{2a\sqrt{a+bx^2}\left(2Ab^3-a\left(5a^2D-4abC+3b^2B\right)\right)}{b^6}-\frac{2a^2(A$$

input
```
Int[(x^5*(A + B*x^2 + C*x^4 + D*x^6))/(a + b*x^2)^(3/2),x]
```

output
```
((-2*a^2*(A*b^3 - a*(b^2*B - a*b*C + a^2*D)))/(b^6*Sqrt[a + b*x^2]) - (2*a
*(2*A*b^3 - a*(3*b^2*B - 4*a*b*C + 5*a^2*D))*Sqrt[a + b*x^2])/b^6 + (2*(A*
b^3 - a*(3*b^2*B - 6*a*b*C + 10*a^2*D))*(a + b*x^2)^(3/2))/(3*b^6) + (2*(b
^2*B - 4*a*b*C + 10*a^2*D)*(a + b*x^2)^(5/2))/(5*b^6) + (2*(b*C - 5*a*D)*(
a + b*x^2)^(7/2))/(7*b^6) + (2*D*(a + b*x^2)^(9/2))/(9*b^6))/2
```

## Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

rule 2331
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/2   S
ubst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
 FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

## Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.67

| method | result |
|---|---|
| pseudoelliptic | $\frac{(35Dx^{10}+45Cx^8+63Bx^6+105Ax^4)b^5-420(\frac{5}{42}Dx^6+\frac{6}{35}Cx^4+\frac{3}{10}x^2B+A)x^2ab^4-840a^2(-\frac{2}{21}Dx^6-\frac{6}{35}Cx^4-\frac{3}{5}x^2B+A}{315\sqrt{bx^2+a}\,b^6}$ |
| gosper | $-\frac{-35Dx^{10}b^5-45Cb^5x^8+50Dab^4x^8-63Bb^5x^6+72Cab^4x^6-80Da^2b^3x^6-105Ab^5x^4+126Bab^4x^4-144Ca^2b^3x^4+160}{315\sqrt{bx^2+}}$ |
| trager | $-\frac{-35Dx^{10}b^5-45Cb^5x^8+50Dab^4x^8-63Bb^5x^6+72Cab^4x^6-80Da^2b^3x^6-105Ab^5x^4+126Bab^4x^4-144Ca^2b^3x^4+160}{315\sqrt{bx^2+}}$ |
| orering | $-\frac{-35Dx^{10}b^5-45Cb^5x^8+50Dab^4x^8-63Bb^5x^6+72Cab^4x^6-80Da^2b^3x^6-105Ab^5x^4+126Bab^4x^4-144Ca^2b^3x^4+160}{315\sqrt{bx^2+}}$ |
| default | $A\left(\frac{x^4}{3b\sqrt{bx^2+a}}-\frac{4a\left(\frac{x^2}{b\sqrt{bx^2+a}}+\frac{2a}{b^2\sqrt{bx^2+a}}\right)}{3b}\right)+B\left(\frac{x^6}{5b\sqrt{bx^2+a}}-\frac{6a\left(\frac{x^4}{3b\sqrt{bx^2+a}}-\frac{4a\left(\frac{x^2}{b\sqrt{bx^2+a}}+\frac{2a}{b^2\sqrt{bx^2+a}}\right)}{3b}\right)}{5b}\right.$ |

input
```
int(x^5*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/315*((35*D*x^10+45*C*x^8+63*B*x^6+105*A*x^4)*b^5-420*(5/42*D*x^6+6/35*C*
x^4+3/10*x^2*B+A)*x^2*a*b^4-840*a^2*(-2/21*D*x^6-6/35*C*x^4-3/5*x^2*B+A)*b
^3+1008*(-10/63*D*x^4-4/7*C*x^2+B)*a^3*b^2-1152*(-5/9*D*x^2+C)*a^4*b+1280*
D*a^5)/(b*x^2+a)^(1/2)/b^6
```

## Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.87

$$\int \frac{x^5(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{3/2}}\,dx = \frac{(35\,Db^5x^{10}-5\,(10\,Dab^4-9\,Cb^5)x^8+(80\,Da^2b^3-72\,Cab^4+63\,Bb^5)}{}$$

input
```
integrate(x^5*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
1/315*(35*D*b^5*x^10 - 5*(10*D*a*b^4 - 9*C*b^5)*x^8 + (80*D*a^2*b^3 - 72*C
*a*b^4 + 63*B*b^5)*x^6 + 1280*D*a^5 - 1152*C*a^4*b + 1008*B*a^3*b^2 - 840*
A*a^2*b^3 - (160*D*a^3*b^2 - 144*C*a^2*b^3 + 126*B*a*b^4 - 105*A*b^5)*x^4
+ 4*(160*D*a^4*b - 144*C*a^3*b^2 + 126*B*a^2*b^3 - 105*A*a*b^4)*x^2)*sqrt(
b*x^2 + a)/(b^7*x^2 + a*b^6)
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 442 vs. 2(204) = 408.

Time = 0.50 (sec) , antiderivative size = 442, normalized size of antiderivative = 2.06

$$\int \frac{x^5(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{3/2}}\,dx = \begin{cases} -\frac{8Aa^2}{3b^3\sqrt{a+bx^2}}-\frac{4Aax^2}{3b^2\sqrt{a+bx^2}}+\frac{Ax^4}{3b\sqrt{a+bx^2}}+\frac{16Ba^3}{5b^4\sqrt{a+bx^2}}+\frac{8Ba^2x^2}{5b^3\sqrt{a+bx^2}}-\frac{2Bax^4}{5b^2\sqrt{a+}} \\ \frac{\frac{Ax^6}{6}+\frac{Bx^8}{8}+\frac{Cx^{10}}{10}+\frac{Dx^{12}}{12}}{a^{\frac{3}{2}}} \end{cases}$$

input
```
integrate(x**5*(D*x**6+C*x**4+B*x**2+A)/(b*x**2+a)**(3/2),x)
```

output
```
Piecewise((-8*A*a**2/(3*b**3*sqrt(a + b*x**2)) - 4*A*a*x**2/(3*b**2*sqrt(a
 + b*x**2)) + A*x**4/(3*b*sqrt(a + b*x**2)) + 16*B*a**3/(5*b**4*sqrt(a + b
*x**2)) + 8*B*a**2*x**2/(5*b**3*sqrt(a + b*x**2)) - 2*B*a*x**4/(5*b**2*sqr
t(a + b*x**2)) + B*x**6/(5*b*sqrt(a + b*x**2)) - 128*C*a**4/(35*b**5*sqrt(
a + b*x**2)) - 64*C*a**3*x**2/(35*b**4*sqrt(a + b*x**2)) + 16*C*a**2*x**4/
(35*b**3*sqrt(a + b*x**2)) - 8*C*a*x**6/(35*b**2*sqrt(a + b*x**2)) + C*x**
8/(7*b*sqrt(a + b*x**2)) + 256*D*a**5/(63*b**6*sqrt(a + b*x**2)) + 128*D*a
**4*x**2/(63*b**5*sqrt(a + b*x**2)) - 32*D*a**3*x**4/(63*b**4*sqrt(a + b*x
**2)) + 16*D*a**2*x**6/(63*b**3*sqrt(a + b*x**2)) - 10*D*a*x**8/(63*b**2*s
qrt(a + b*x**2)) + D*x**10/(9*b*sqrt(a + b*x**2)), Ne(b, 0)), ((A*x**6/6 +
 B*x**8/8 + C*x**10/10 + D*x**12/12)/a**(3/2), True))
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.61

$$\int \frac{x^5(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{3/2}}\,dx = \frac{Dx^{10}}{9\sqrt{bx^2+a}b} - \frac{10\,Dax^8}{63\sqrt{bx^2+a}b^2} + \frac{Cx^8}{7\sqrt{bx^2+a}b} + \frac{16\,Da^2x^6}{63\sqrt{bx^2+a}b^3} - \frac{8\,Cax^6}{35\sqrt{bx^2+a}b^2} + \frac{Bx^6}{5\sqrt{bx^2+a}b} - \frac{32\,Da^3x^4}{63\sqrt{bx^2+a}b^4} + \frac{16\,Ca^2x^4}{35\sqrt{bx^2+a}b^3} - \frac{2\,Bax^4}{5\sqrt{bx^2+a}b^2} + \frac{Ax^4}{3\sqrt{bx^2+a}b} + \frac{128\,Da^4x^2}{63\sqrt{bx^2+a}b^5} - \frac{64\,Ca^3x^2}{35\sqrt{bx^2+a}b^4} + \frac{8\,Ba^2x^2}{5\sqrt{bx^2+a}b^3} - \frac{4\,Aax^2}{3\sqrt{bx^2+a}b^2} + \frac{256\,Da^5}{63\sqrt{bx^2+a}b^6} - \frac{128\,Ca^4}{35\sqrt{bx^2+a}b^5} + \frac{16\,Ba^3}{5\sqrt{bx^2+a}b^4} - \frac{8\,Aa^2}{3\sqrt{bx^2+a}b^3}$$

input
```
integrate(x^5*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
1/9*D*x^10/(sqrt(b*x^2 + a)*b) - 10/63*D*a*x^8/(sqrt(b*x^2 + a)*b^2) + 1/7
*C*x^8/(sqrt(b*x^2 + a)*b) + 16/63*D*a^2*x^6/(sqrt(b*x^2 + a)*b^3) - 8/35*
C*a*x^6/(sqrt(b*x^2 + a)*b^2) + 1/5*B*x^6/(sqrt(b*x^2 + a)*b) - 32/63*D*a^
3*x^4/(sqrt(b*x^2 + a)*b^4) + 16/35*C*a^2*x^4/(sqrt(b*x^2 + a)*b^3) - 2/5*
B*a*x^4/(sqrt(b*x^2 + a)*b^2) + 1/3*A*x^4/(sqrt(b*x^2 + a)*b) + 128/63*D*a
^4*x^2/(sqrt(b*x^2 + a)*b^5) - 64/35*C*a^3*x^2/(sqrt(b*x^2 + a)*b^4) + 8/5
*B*a^2*x^2/(sqrt(b*x^2 + a)*b^3) - 4/3*A*a*x^2/(sqrt(b*x^2 + a)*b^2) + 256
/63*D*a^5/(sqrt(b*x^2 + a)*b^6) - 128/35*C*a^4/(sqrt(b*x^2 + a)*b^5) + 16/
5*B*a^3/(sqrt(b*x^2 + a)*b^4) - 8/3*A*a^2/(sqrt(b*x^2 + a)*b^3)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.31

$$\int \frac{x^5(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{3/2}}\,dx = \frac{Da^5-Ca^4b+Ba^3b^2-Aa^2b^3}{\sqrt{bx^2+a}b^6} + \frac{35\,(bx^2+a)^{\frac{9}{2}}Db^{48}-225\,(bx^2+a)^{\frac{7}{2}}Dab^{48}+630\,(bx^2+a)^{\frac{5}{2}}Da^2b^{48}-1050\,(bx^2+a)^{\frac{3}{2}}Da^3b^{48}+1575\,\sqrt{bx}$$

input
```
integrate(x^5*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
(D*a^5 - C*a^4*b + B*a^3*b^2 - A*a^2*b^3)/(sqrt(b*x^2 + a)*b^6) + 1/315*(3
5*(b*x^2 + a)^(9/2)*D*b^48 - 225*(b*x^2 + a)^(7/2)*D*a*b^48 + 630*(b*x^2 +
 a)^(5/2)*D*a^2*b^48 - 1050*(b*x^2 + a)^(3/2)*D*a^3*b^48 + 1575*sqrt(b*x^2
 + a)*D*a^4*b^48 + 45*(b*x^2 + a)^(7/2)*C*b^49 - 252*(b*x^2 + a)^(5/2)*C*a
*b^49 + 630*(b*x^2 + a)^(3/2)*C*a^2*b^49 - 1260*sqrt(b*x^2 + a)*C*a^3*b^49
 + 63*(b*x^2 + a)^(5/2)*B*b^50 - 315*(b*x^2 + a)^(3/2)*B*a*b^50 + 945*sqrt
(b*x^2 + a)*B*a^2*b^50 + 105*(b*x^2 + a)^(3/2)*A*b^51 - 630*sqrt(b*x^2 + a
)*A*a*b^51)/b^54
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{3/2}}\,dx = \int \frac{x^5\,(A+B\,x^2+C\,x^4+x^6\,D)}{(b\,x^2+a)^{3/2}}\,dx$$

input
```
int((x^5*(A + B*x^2 + C*x^4 + x^6*D))/(a + b*x^2)^(3/2),x)
```

output
```
int((x^5*(A + B*x^2 + C*x^4 + x^6*D))/(a + b*x^2)^(3/2), x)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.78

$$\int \frac{x^5(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{3/2}}\,dx = \frac{\sqrt{b\,x^2+a}\,(35b^5d\,x^{10}-50a\,b^4d\,x^8+45b^5c\,x^8+80a^2b^3d\,x^6-72a\,b^4c\,x^6$$

input
```
int(x^5*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(3/2),x)
```

output
```
(sqrt(a + b*x**2)*(1280*a**5*d - 1152*a**4*b*c + 640*a**4*b*d*x**2 + 168*a
**3*b**3 - 576*a**3*b**2*c*x**2 - 160*a**3*b**2*d*x**4 + 84*a**2*b**4*x**2
 + 144*a**2*b**3*c*x**4 + 80*a**2*b**3*d*x**6 - 21*a*b**5*x**4 - 72*a*b**4
*c*x**6 - 50*a*b**4*d*x**8 + 63*b**6*x**6 + 45*b**5*c*x**8 + 35*b**5*d*x**
10))/(315*b**6*(a + b*x**2))
```

# 3.225 $\int \frac{x^3(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{3/2}}\,dx$



## Optimal result

Integrand size = 32, antiderivative size = 164

$$\int \frac{x^3(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{3/2}}\,dx = \frac{a(Ab^3-a(b^2B-abC+a^2D))}{b^5\sqrt{a+bx^2}} + \frac{(Ab^3-a(2b^2B-3abC+4a^2D))\sqrt{a+bx^2}}{b^5} + \frac{(b^2B-3abC+6a^2D)(a+bx^2)^{3/2}}{3b^5} + \frac{(bC-4aD)(a+bx^2)^{5/2}}{5b^5} + \frac{D(a+bx^2)^{7/2}}{7b^5}$$

output
```
a*(A*b^3-a*(B*b^2-C*a*b+D*a^2))/b^5/(b*x^2+a)^(1/2)+(A*b^3-a*(2*B*b^2-3*C*
a*b+4*D*a^2))*(b*x^2+a)^(1/2)/b^5+1/3*(B*b^2-3*C*a*b+6*D*a^2)*(b*x^2+a)^(3
/2)/b^5+1/5*(C*b-4*D*a)*(b*x^2+a)^(5/2)/b^5+1/7*D*(b*x^2+a)^(7/2)/b^5
```

## Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.74

$$\int \frac{x^3(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{3/2}}\,dx = \frac{-384a^4D+48a^3b(7C-4Dx^2)-8a^2b^2(35B-21Cx^2-6Dx^4)+2ab}{10}$$

input
```
Integrate[(x^3*(A + B*x^2 + C*x^4 + D*x^6))/(a + b*x^2)^(3/2),x]
```

output
```
(-384*a^4*D + 48*a^3*b*(7*C - 4*D*x^2) - 8*a^2*b^2*(35*B - 21*C*x^2 - 6*D*
x^4) + 2*a*b^3*(105*A - 70*B*x^2 - 21*C*x^4 - 12*D*x^6) + b^4*x^2*(105*A +
 35*B*x^2 + 21*C*x^4 + 15*D*x^6))/(105*b^5*Sqrt[a + b*x^2])
```

## Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {2331, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 2331}$$

$$\frac{1}{2}\int \frac{x^2(Dx^6+Cx^4+Bx^2+A)}{(bx^2+a)^{3/2}}dx^2$$

$$\Big\downarrow \text{ 2123}$$

$$\frac{1}{2}\int \left(\frac{D(bx^2+a)^{5/2}}{b^4}+\frac{(bC-4aD)(bx^2+a)^{3/2}}{b^4}+\frac{(6Da^2-3bCa+b^2B)\sqrt{bx^2+a}}{b^4}+\frac{Ab^3-a(4Da^2-3bCa}{b^4\sqrt{bx^2+a}}\right.$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(\frac{2\sqrt{a+bx^2}(Ab^3-a(4a^2D-3abC+2b^2B))}{b^5}+\frac{2a(Ab^3-a(a^2D-abC+b^2B))}{b^5\sqrt{a+bx^2}}+\frac{2(a+bx^2)^{3/2}(6a^2D-3}{3b^5}\right.$$

input
```
Int[(x^3*(A + B*x^2 + C*x^4 + D*x^6))/(a + b*x^2)^(3/2),x]
```

output
```
((2*a*(A*b^3 - a*(b^2*B - a*b*C + a^2*D)))/(b^5*Sqrt[a + b*x^2]) + (2*(A*b
^3 - a*(2*b^2*B - 3*a*b*C + 4*a^2*D))*Sqrt[a + b*x^2])/b^5 + (2*(b^2*B - 3
*a*b*C + 6*a^2*D)*(a + b*x^2)^(3/2))/(3*b^5) + (2*(b*C - 4*a*D)*(a + b*x^2
)^(5/2))/(5*b^5) + (2*D*(a + b*x^2)^(7/2))/(7*b^5))/2
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

rule 2331
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/2   S
ubst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
 FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

## Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.69

| method | result |
|---|---|
| pseudoelliptic | $\frac{\left(15 D x^{8}+21 C x^{6}+35 B x^{4}+105 A x^{2}\right) b^{4}+210 a \left(-\frac{4}{35} D x^{6}-\frac{1}{5} C x^{4}-\frac{2}{3} x^{2} B+A\right) b^{3}-280 \left(-\frac{6}{35} D x^{4}-\frac{3}{5} C x^{2}+B\right) a^{2} b^{2}+336 a^{3} \left(\right.}{105 \sqrt{b x^{2}+a}\, b^{5}}$ |
| gosper | $\frac{15 D x^{8} b^{4}+21 C b^{4} x^{6}-24 D a b^{3} x^{6}+35 B x^{4} b^{4}-42 C a b^{3} x^{4}+48 D a^{2} b^{2} x^{4}+105 A b^{4} x^{2}-140 B a b^{3} x^{2}+168 C a^{2} b^{2} x^{2}-192 D a^{3} b}{105 \sqrt{b x^{2}+a}\, b^{5}}$ |
| trager | $\frac{15 D x^{8} b^{4}+21 C b^{4} x^{6}-24 D a b^{3} x^{6}+35 B x^{4} b^{4}-42 C a b^{3} x^{4}+48 D a^{2} b^{2} x^{4}+105 A b^{4} x^{2}-140 B a b^{3} x^{2}+168 C a^{2} b^{2} x^{2}-192 D a^{3} b}{105 \sqrt{b x^{2}+a}\, b^{5}}$ |
| orering | $\frac{15 D x^{8} b^{4}+21 C b^{4} x^{6}-24 D a b^{3} x^{6}+35 B x^{4} b^{4}-42 C a b^{3} x^{4}+48 D a^{2} b^{2} x^{4}+105 A b^{4} x^{2}-140 B a b^{3} x^{2}+168 C a^{2} b^{2} x^{2}-192 D a^{3} b}{105 \sqrt{b x^{2}+a}\, b^{5}}$ |
| default | $A \left(\frac{x^{2}}{b \sqrt{b x^{2}+a}}+\frac{2 a}{b^{2} \sqrt{b x^{2}+a}}\right)+B \left(\frac{x^{4}}{3 b \sqrt{b x^{2}+a}}-\frac{4 a \left(\frac{x^{2}}{b \sqrt{b x^{2}+a}}+\frac{2 a}{b^{2} \sqrt{b x^{2}+a}}\right)}{3 b}\right)+C \left(\frac{x^{6}}{5 b \sqrt{b x^{2}+a}}-\frac{6 a \left(\right.}{}\right.$ |

input
```
int(x^3*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/105*((15*D*x^8+21*C*x^6+35*B*x^4+105*A*x^2)*b^4+210*a*(-4/35*D*x^6-1/5*C
*x^4-2/3*x^2*B+A)*b^3-280*(-6/35*D*x^4-3/5*C*x^2+B)*a^2*b^2+336*a^3*(-4/7*
D*x^2+C)*b-384*D*a^4)/(b*x^2+a)^(1/2)/b^5
```

## Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.89

$$\int \frac{x^3(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{3/2}}\,dx = \frac{(15\,Db^4x^8 - 3\,(8\,Dab^3 - 7\,Cb^4)x^6 - 384\,Da^4 + 336\,Ca^3b - 280\,Ba^2b^2}{}$$

input
```
integrate(x^3*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
1/105*(15*D*b^4*x^8 - 3*(8*D*a*b^3 - 7*C*b^4)*x^6 - 384*D*a^4 + 336*C*a^3*
b - 280*B*a^2*b^2 + 210*A*a*b^3 + (48*D*a^2*b^2 - 42*C*a*b^3 + 35*B*b^4)*x
^4 - (192*D*a^3*b - 168*C*a^2*b^2 + 140*B*a*b^3 - 105*A*b^4)*x^2)*sqrt(b*x
^2 + a)/(b^6*x^2 + a*b^5)
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 337 vs. 2(151) = 302.

Time = 0.44 (sec) , antiderivative size = 337, normalized size of antiderivative = 2.05

$$\int \frac{x^3(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{3/2}}\,dx = \begin{cases} \frac{2Aa}{b^2\sqrt{a+bx^2}} + \frac{Ax^2}{b\sqrt{a+bx^2}} - \frac{8Ba^2}{3b^3\sqrt{a+bx^2}} - \frac{4Bax^2}{3b^2\sqrt{a+bx^2}} + \frac{Bx^4}{3b\sqrt{a+bx^2}} + \frac{16Ca^3}{5b^4\sqrt{a+bx^2}} + \\ \frac{\frac{Ax^4}{4} + \frac{Bx^6}{6} + \frac{Cx^8}{8} + \frac{Dx^{10}}{10}}{a^{\frac{3}{2}}} \end{cases}$$

input
```
integrate(x**3*(D*x**6+C*x**4+B*x**2+A)/(b*x**2+a)**(3/2),x)
```

output
```
Piecewise((2*A*a/(b**2*sqrt(a + b*x**2)) + A*x**2/(b*sqrt(a + b*x**2)) - 8
*B*a**2/(3*b**3*sqrt(a + b*x**2)) - 4*B*a*x**2/(3*b**2*sqrt(a + b*x**2)) +
 B*x**4/(3*b*sqrt(a + b*x**2)) + 16*C*a**3/(5*b**4*sqrt(a + b*x**2)) + 8*C
*a**2*x**2/(5*b**3*sqrt(a + b*x**2)) - 2*C*a*x**4/(5*b**2*sqrt(a + b*x**2)
) + C*x**6/(5*b*sqrt(a + b*x**2)) - 128*D*a**4/(35*b**5*sqrt(a + b*x**2))
- 64*D*a**3*x**2/(35*b**4*sqrt(a + b*x**2)) + 16*D*a**2*x**4/(35*b**3*sqrt
(a + b*x**2)) - 8*D*a*x**6/(35*b**2*sqrt(a + b*x**2)) + D*x**8/(7*b*sqrt(a
 + b*x**2)), Ne(b, 0)), ((A*x**4/4 + B*x**6/6 + C*x**8/8 + D*x**10/10)/a**
(3/2), True))
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.60

$$\int \frac{x^3(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{3/2}}\,dx = \frac{Dx^8}{7\sqrt{bx^2+a}b} - \frac{8\,Dax^6}{35\sqrt{bx^2+a}b^2} + \frac{Cx^6}{5\sqrt{bx^2+a}b} + \frac{16\,Da^2x^4}{35\sqrt{bx^2+a}b^3} - \frac{2\,Cax^4}{5\sqrt{bx^2+a}b^2} + \frac{Bx^4}{3\sqrt{bx^2+a}b} - \frac{64\,Da^3x^2}{35\sqrt{bx^2+a}b^4} + \frac{8\,Ca^2x^2}{5\sqrt{bx^2+a}b^3} - \frac{4\,Bax^2}{3\sqrt{bx^2+a}b^2} + \frac{Ax^2}{\sqrt{bx^2+a}b} - \frac{128\,Da^4}{35\sqrt{bx^2+a}b^5} + \frac{16\,Ca^3}{5\sqrt{bx^2+a}b^4} - \frac{8\,Ba^2}{3\sqrt{bx^2+a}b^3} + \frac{2\,Aa}{\sqrt{bx^2+a}b^2}$$

input
```
integrate(x^3*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
1/7*D*x^8/(sqrt(b*x^2 + a)*b) - 8/35*D*a*x^6/(sqrt(b*x^2 + a)*b^2) + 1/5*C
*x^6/(sqrt(b*x^2 + a)*b) + 16/35*D*a^2*x^4/(sqrt(b*x^2 + a)*b^3) - 2/5*C*a
*x^4/(sqrt(b*x^2 + a)*b^2) + 1/3*B*x^4/(sqrt(b*x^2 + a)*b) - 64/35*D*a^3*x
^2/(sqrt(b*x^2 + a)*b^4) + 8/5*C*a^2*x^2/(sqrt(b*x^2 + a)*b^3) - 4/3*B*a*x
^2/(sqrt(b*x^2 + a)*b^2) + A*x^2/(sqrt(b*x^2 + a)*b) - 128/35*D*a^4/(sqrt(
b*x^2 + a)*b^5) + 16/5*C*a^3/(sqrt(b*x^2 + a)*b^4) - 8/3*B*a^2/(sqrt(b*x^2
 + a)*b^3) + 2*A*a/(sqrt(b*x^2 + a)*b^2)
```

## Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.29

$$\int \frac{x^3(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{3/2}}\,dx = -\frac{Da^4-Ca^3b+Ba^2b^2-Aab^3}{\sqrt{bx^2+a}b^5}$$
$$+\frac{15\,(bx^2+a)^{\frac{7}{2}}Db^{30}-84\,(bx^2+a)^{\frac{5}{2}}Dab^{30}+210\,(bx^2+a)^{\frac{3}{2}}Da^2b^{30}-420\,\sqrt{bx^2+a}Da^3b^{30}+21\,(bx^2+a)^{\frac{5}{2}}}{}$$

input
```
integrate(x^3*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
-(D*a^4 - C*a^3*b + B*a^2*b^2 - A*a*b^3)/(sqrt(b*x^2 + a)*b^5) + 1/105*(15
*(b*x^2 + a)^(7/2)*D*b^30 - 84*(b*x^2 + a)^(5/2)*D*a*b^30 + 210*(b*x^2 + a
)^(3/2)*D*a^2*b^30 - 420*sqrt(b*x^2 + a)*D*a^3*b^30 + 21*(b*x^2 + a)^(5/2)
*C*b^31 - 105*(b*x^2 + a)^(3/2)*C*a*b^31 + 315*sqrt(b*x^2 + a)*C*a^2*b^31
+ 35*(b*x^2 + a)^(3/2)*B*b^32 - 210*sqrt(b*x^2 + a)*B*a*b^32 + 105*sqrt(b*
x^2 + a)*A*b^33)/b^35
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{3/2}}\,dx = \int \frac{x^3\,(A+B\,x^2+C\,x^4+x^6\,D)}{(b\,x^2+a)^{3/2}}\,dx$$

input
```
int((x^3*(A + B*x^2 + C*x^4 + x^6*D))/(a + b*x^2)^(3/2),x)
```

output
```
int((x^3*(A + B*x^2 + C*x^4 + x^6*D))/(a + b*x^2)^(3/2), x)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.81

$$\int \frac{x^3(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{3/2}}\,dx = \frac{\sqrt{b\,x^2+a}\,(15b^4d\,x^8-24a\,b^3d\,x^6+21b^4c\,x^6+48a^2b^2d\,x^4-42a\,b^3c\,x^4}{105$$

input
```
int(x^3*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(3/2),x)
```

output
```
(sqrt(a + b*x**2)*( - 384*a**4*d + 336*a**3*b*c - 192*a**3*b*d*x**2 - 70*a
**2*b**3 + 168*a**2*b**2*c*x**2 + 48*a**2*b**2*d*x**4 - 35*a*b**4*x**2 - 4
2*a*b**3*c*x**4 - 24*a*b**3*d*x**6 + 35*b**5*x**4 + 21*b**4*c*x**6 + 15*b*
*4*d*x**8))/(105*b**5*(a + b*x**2))
```

# 3.226 $\int \frac{x(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{3/2}}\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 119

$$\int \frac{x(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{3/2}}\,dx = -\frac{Ab^3-a(b^2B-abC+a^2D)}{b^4\sqrt{a+bx^2}} + \frac{(b^2B-2abC+3a^2D)\sqrt{a+bx^2}}{b^4} + \frac{(bC-3aD)(a+bx^2)^{3/2}}{3b^4} + \frac{D(a+bx^2)^{5/2}}{5b^4}$$

output
```
-(A*b^3-a*(B*b^2-C*a*b+D*a^2))/b^4/(b*x^2+a)^(1/2)+(B*b^2-2*C*a*b+3*D*a^2)
*(b*x^2+a)^(1/2)/b^4+1/3*(C*b-3*D*a)*(b*x^2+a)^(3/2)/b^4+1/5*D*(b*x^2+a)^(
5/2)/b^4
```

## Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.77

$$\int \frac{x(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{3/2}}\,dx = \frac{-15Ab^3+48a^3D-8a^2b(5C-3Dx^2)+2ab^2(15B-10Cx^2-3Dx^4)}{15b^4\sqrt{a+bx^2}}$$

input
```
Integrate[(x*(A + B*x^2 + C*x^4 + D*x^6))/(a + b*x^2)^(3/2),x]
```

output
```
(-15*A*b^3 + 48*a^3*D - 8*a^2*b*(5*C - 3*D*x^2) + 2*a*b^2*(15*B - 10*C*x^2
 - 3*D*x^4) + b^3*x^2*(15*B + 5*C*x^2 + 3*D*x^4))/(15*b^4*Sqrt[a + b*x^2])
```

## Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2331, 2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{3/2}}\,dx$$

$$\Big\downarrow \; 2331$$

$$\frac{1}{2}\int \frac{Dx^6+Cx^4+Bx^2+A}{(bx^2+a)^{3/2}}dx^2$$

$$\Big\downarrow \; 2389$$

$$\frac{1}{2}\int \left(\frac{D(bx^2+a)^{3/2}}{b^3}+\frac{(bC-3aD)\sqrt{bx^2+a}}{b^3}+\frac{3Da^2-2bCa+b^2B}{b^3\sqrt{bx^2+a}}+\frac{Ab^3-a(Da^2-bCa+b^2B)}{b^3(bx^2+a)^{3/2}}\right)dx^2$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\left(-\frac{2(Ab^3-a(a^2D-abC+b^2B))}{b^4\sqrt{a+bx^2}}+\frac{2\sqrt{a+bx^2}(3a^2D-2abC+b^2B)}{b^4}+\frac{2(a+bx^2)^{3/2}(bC-3aD)}{3b^4}+\frac{2D(a$$

input
```
Int[(x*(A + B*x^2 + C*x^4 + D*x^6))/(a + b*x^2)^(3/2),x]
```

output
```
((-2*(A*b^3 - a*(b^2*B - a*b*C + a^2*D)))/(b^4*Sqrt[a + b*x^2]) + (2*(b^2*
B - 2*a*b*C + 3*a^2*D)*Sqrt[a + b*x^2])/b^4 + (2*(b*C - 3*a*D)*(a + b*x^2)
^(3/2))/(3*b^4) + (2*D*(a + b*x^2)^(5/2))/(5*b^4))/2
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2331
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/2   S
ubst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
 FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

## Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.69

| method | result |
|---|---|
| pseudoelliptic | $\frac{(3Dx^6+5C\,x^4+15x^2B-15A)b^3+30\left(-\frac{1}{5}Dx^4-\frac{2}{3}C\,x^2+B\right)a\,b^2-40\left(-\frac{3Dx^2}{5}+C\right)a^2b+48a^3D}{15\sqrt{b\,x^2+a}\,b^4}$ |
| gosper | $-\frac{-3b^3Dx^6-5b^3C\,x^4+6Da\,b^2x^4-15b^3B\,x^2+20Ca\,b^2x^2-24Da^2b\,x^2+15b^3A-30a\,b^2B+40a^2bC-48a^3D}{15\sqrt{b\,x^2+a}\,b^4}$ |
| trager | $-\frac{-3b^3Dx^6-5b^3C\,x^4+6Da\,b^2x^4-15b^3B\,x^2+20Ca\,b^2x^2-24Da^2b\,x^2+15b^3A-30a\,b^2B+40a^2bC-48a^3D}{15\sqrt{b\,x^2+a}\,b^4}$ |
| orering | $-\frac{-3b^3Dx^6-5b^3C\,x^4+6Da\,b^2x^4-15b^3B\,x^2+20Ca\,b^2x^2-24Da^2b\,x^2+15b^3A-30a\,b^2B+40a^2bC-48a^3D}{15\sqrt{b\,x^2+a}\,b^4}$ |
| default | $B\left(\frac{x^2}{b\sqrt{b\,x^2+a}}+\frac{2a}{b^2\sqrt{b\,x^2+a}}\right)+C\left(\frac{x^4}{3b\sqrt{b\,x^2+a}}-\frac{4a\left(\frac{x^2}{b\sqrt{b\,x^2+a}}+\frac{2a}{b^2\sqrt{b\,x^2+a}}\right)}{3b}\right)+D\left(\frac{x^6}{5b\sqrt{b\,x^2+a}}-\frac{6a\left(\cdots\right.}{\cdots}\right.$ |

input
```
int(x*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*((3*D*x^6+5*C*x^4+15*B*x^2-15*A)*b^3+30*(-1/5*D*x^4-2/3*C*x^2+B)*a*b^
2-40*(-3/5*D*x^2+C)*a^2*b+48*a^3*D)/(b*x^2+a)^(1/2)/b^4
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.89

$$\int \frac{x(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{3/2}}\,dx = \frac{(3\,Db^3x^6-(6\,Dab^2-5\,Cb^3)x^4+48\,Da^3-40\,Ca^2b+30\,Bab^2-15\,Ab}{15\,(b^5x^2+ab^4)}$$

input
```
integrate(x*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
1/15*(3*D*b^3*x^6 - (6*D*a*b^2 - 5*C*b^3)*x^4 + 48*D*a^3 - 40*C*a^2*b + 30
*B*a*b^2 - 15*A*b^3 + (24*D*a^2*b - 20*C*a*b^2 + 15*B*b^3)*x^2)*sqrt(b*x^2
 + a)/(b^5*x^2 + a*b^4)
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 235 vs. 2(105) = 210.

Time = 0.31 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.97

$$\int \frac{x(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{3/2}}\,dx = \begin{cases} -\frac{A}{b\sqrt{a+bx^2}}+\frac{2Ba}{b^2\sqrt{a+bx^2}}+\frac{Bx^2}{b\sqrt{a+bx^2}}-\frac{8Ca^2}{3b^3\sqrt{a+bx^2}}-\frac{4Cax^2}{3b^2\sqrt{a+bx^2}}+\frac{Cx^4}{3b\sqrt{a+bx^2}}+ \\ \frac{\frac{Ax^2}{2}+\frac{Bx^4}{4}+\frac{Cx^6}{6}+\frac{Dx^8}{8}}{a^{\frac{3}{2}}} \end{cases}$$

input
```
integrate(x*(D*x**6+C*x**4+B*x**2+A)/(b*x**2+a)**(3/2),x)
```

output
```
Piecewise((-A/(b*sqrt(a + b*x**2)) + 2*B*a/(b**2*sqrt(a + b*x**2)) + B*x**
2/(b*sqrt(a + b*x**2)) - 8*C*a**2/(3*b**3*sqrt(a + b*x**2)) - 4*C*a*x**2/(
3*b**2*sqrt(a + b*x**2)) + C*x**4/(3*b*sqrt(a + b*x**2)) + 16*D*a**3/(5*b*
*4*sqrt(a + b*x**2)) + 8*D*a**2*x**2/(5*b**3*sqrt(a + b*x**2)) - 2*D*a*x**
4/(5*b**2*sqrt(a + b*x**2)) + D*x**6/(5*b*sqrt(a + b*x**2)), Ne(b, 0)), ((
A*x**2/2 + B*x**4/4 + C*x**6/6 + D*x**8/8)/a**(3/2), True))
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.51

$$\int \frac{x(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{3/2}}\,dx = \frac{Dx^6}{5\sqrt{bx^2+a}b} - \frac{2\,Dax^4}{5\sqrt{bx^2+a}b^2} + \frac{Cx^4}{3\sqrt{bx^2+a}b} + \frac{8\,Da^2x^2}{5\sqrt{bx^2+a}b^3} - \frac{4\,Cax^2}{3\sqrt{bx^2+a}b^2} + \frac{Bx^2}{\sqrt{bx^2+a}b} + \frac{16\,Da^3}{5\sqrt{bx^2+a}b^4} - \frac{8\,Ca^2}{3\sqrt{bx^2+a}b^3} + \frac{2\,Ba}{\sqrt{bx^2+a}b^2} - \frac{A}{\sqrt{bx^2+a}b}$$

input
```
integrate(x*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
1/5*D*x^6/(sqrt(b*x^2 + a)*b) - 2/5*D*a*x^4/(sqrt(b*x^2 + a)*b^2) + 1/3*C*
x^4/(sqrt(b*x^2 + a)*b) + 8/5*D*a^2*x^2/(sqrt(b*x^2 + a)*b^3) - 4/3*C*a*x^
2/(sqrt(b*x^2 + a)*b^2) + B*x^2/(sqrt(b*x^2 + a)*b) + 16/5*D*a^3/(sqrt(b*x
^2 + a)*b^4) - 8/3*C*a^2/(sqrt(b*x^2 + a)*b^3) + 2*B*a/(sqrt(b*x^2 + a)*b^
2) - A/(sqrt(b*x^2 + a)*b)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.18

$$\int \frac{x(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{3/2}}\,dx = \frac{Da^3 - Ca^2b + Bab^2 - Ab^3}{\sqrt{bx^2+a}b^4} + \frac{3\,(bx^2+a)^{\frac{5}{2}}Db^{16} - 15\,(bx^2+a)^{\frac{3}{2}}Dab^{16} + 45\sqrt{bx^2+a}Da^2b^{16} + 5\,(bx^2+a)^{\frac{3}{2}}Cb^{17} - 30\sqrt{bx^2+a}Cab^{17} +}{15\,b^{20}}$$

input
```
integrate(x*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
(D*a^3 - C*a^2*b + B*a*b^2 - A*b^3)/(sqrt(b*x^2 + a)*b^4) + 1/15*(3*(b*x^2
 + a)^(5/2)*D*b^16 - 15*(b*x^2 + a)^(3/2)*D*a*b^16 + 45*sqrt(b*x^2 + a)*D*
a^2*b^16 + 5*(b*x^2 + a)^(3/2)*C*b^17 - 30*sqrt(b*x^2 + a)*C*a*b^17 + 15*s
qrt(b*x^2 + a)*B*b^18)/b^20
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{3/2}}\,dx=\int \frac{x\,(A+B\,x^2+C\,x^4+x^6\,D)}{(b\,x^2+a)^{3/2}}\,dx$$

input
```
int((x*(A + B*x^2 + C*x^4 + x^6*D))/(a + b*x^2)^(3/2),x)
```

output
```
int((x*(A + B*x^2 + C*x^4 + x^6*D))/(a + b*x^2)^(3/2), x)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.82

$$\int \frac{x(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{3/2}}\,dx=\frac{\sqrt{b\,x^2+a}\,(3b^3d\,x^6-6a\,b^2d\,x^4+5b^3c\,x^4+24a^2bd\,x^2-20a\,b^2c\,x^2+15b}{15b^4\,(b\,x^2+a)}$$

input
```
int(x*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(3/2),x)
```

output
```
(sqrt(a + b*x**2)*(48*a**3*d - 40*a**2*b*c + 24*a**2*b*d*x**2 + 15*a*b**3
- 20*a*b**2*c*x**2 - 6*a*b**2*d*x**4 + 15*b**4*x**2 + 5*b**3*c*x**4 + 3*b*
*3*d*x**6))/(15*b**4*(a + b*x**2))
```

# 3.227 $\int \frac{A+Bx^2+Cx^4+Dx^6}{x(a+bx^2)^{3/2}}\,dx$



## Optimal result

Integrand size = 32, antiderivative size = 108

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x\left(a+bx^2\right)^{3/2}}\,dx = \frac{\frac{A}{a}-\frac{b^2B-abC+a^2D}{b^3}}{\sqrt{a+bx^2}} + \frac{(bC-2aD)\sqrt{a+bx^2}}{b^3} + \frac{D(a+bx^2)^{3/2}}{3b^3} - \frac{A\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}$$

output
```
(A/a-(B*b^2-C*a*b+D*a^2)/b^3)/(b*x^2+a)^(1/2)+(C*b-2*D*a)*(b*x^2+a)^(1/2)/
b^3+1/3*D*(b*x^2+a)^(3/2)/b^3-A*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(3/2)
```

## Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.89

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x\left(a+bx^2\right)^{3/2}}\,dx = \frac{3Ab^3+a(-8a^2D+ab(6C-4Dx^2)+b^2(-3B+3Cx^2+Dx^4))}{3ab^3\sqrt{a+bx^2}} - \frac{A\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}$$

input
```
Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(x*(a + b*x^2)^(3/2)),x]
```

output
```
(3*A*b^3 + a*(-8*a^2*D + a*b*(6*C - 4*D*x^2) + b^2*(-3*B + 3*C*x^2 + D*x^4
)))/(3*a*b^3*Sqrt[a + b*x^2]) - (A*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/a^(3/
2)
```

## Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.22, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {2331, 2122, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x\left(a+bx^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{2331}$$

$$\frac{1}{2}\int \frac{Dx^6+Cx^4+Bx^2+A}{x^2\left(bx^2+a\right)^{3/2}}dx^2$$

$$\Big\downarrow \text{2122}$$

$$\frac{1}{2}\int \left(\frac{Dx^2}{b\sqrt{bx^2+a}}+\frac{bC-aD}{b^2\sqrt{bx^2+a}}+\frac{a\left(Da^2-bCa+b^2B\right)-Ab^3}{ab^2\left(bx^2+a\right)^{3/2}}+\frac{A}{a\sqrt{bx^2+a}x^2}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{2A\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}+\frac{2\left(\frac{A}{a}-\frac{a^2D-abC+b^2B}{b^3}\right)}{\sqrt{a+bx^2}}+\frac{2\sqrt{a+bx^2}(bC-aD)}{b^3}+\frac{2D\left(a+bx^2\right)^{3/2}}{3b^3}-\frac{2aD\sqrt{a+bx^2}}{b^3}\right)$$

input
```
Int[(A + B*x^2 + C*x^4 + D*x^6)/(x*(a + b*x^2)^(3/2)),x]
```

output
```
((2*(A/a - (b^2*B - a*b*C + a^2*D)/b^3))/Sqrt[a + b*x^2] - (2*a*D*Sqrt[a +
 b*x^2])/b^3 + (2*(b*C - a*D)*Sqrt[a + b*x^2])/b^3 + (2*D*(a + b*x^2)^(3/2
))/(3*b^3) - (2*A*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/a^(3/2))/2
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2122
```
Int[((Px_)*((c_.) + (d_.)*(x_))^(n_))/((a_.) + (b_.)*(x_)), x_Symbol] :> In
t[ExpandIntegrand[1/Sqrt[c + d*x], Px*((c + d*x)^(n + 1/2)/(a + b*x)), x],
x] /; FreeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && ILtQ[n + 1/2, 0]
```

rule 2331
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/2   S
ubst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
 FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

## Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.91

| method | result |
|---|---|
| pseudoelliptic | $\frac{-\frac{8 D a^{\frac{9}{2}}}{3}+2\left(-\frac{2 D x^{2}}{3}+C\right) b\, a^{\frac{7}{2}}-\left(-\frac{1}{3} D x^{4}-C\, x^{2}+B\right) b^{2} a^{\frac{5}{2}}+A\, a^{\frac{3}{2}} b^{3}-A\, b^{3} \operatorname{arctanh}\left(\frac{\sqrt{b\, x^{2}+a}}{\sqrt{a}}\right) a \sqrt{b\, x^{2}+a}}{\sqrt{b\, x^{2}+a}\, a^{\frac{5}{2}} b^{3}}$ |
| default | $A\left(\frac{1}{a\sqrt{b\, x^{2}+a}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\, x^{2}+a}}{x}\right)}{a^{\frac{3}{2}}}\right)-\frac{B}{b\sqrt{b\, x^{2}+a}}+C\left(\frac{x^{2}}{b\sqrt{b\, x^{2}+a}}+\frac{2a}{b^{2}\sqrt{b\, x^{2}+a}}\right)+D\left(\frac{x^{4}}{3b\sqrt{b\, x^{2}+a}}-\right.$ |

input
```
int((D*x^6+C*x^4+B*x^2+A)/x/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/(b*x^2+a)^(1/2)*(-8/3*D*a^(9/2)+2*(-2/3*D*x^2+C)*b*a^(7/2)-(-1/3*D*x^4-C
*x^2+B)*b^2*a^(5/2)+A*a^(3/2)*b^3-A*b^3*arctanh((b*x^2+a)^(1/2)/a^(1/2))*a
*(b*x^2+a)^(1/2))/a^(5/2)/b^3
```

## Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 284, normalized size of antiderivative = 2.63

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x\left(a+bx^2\right)^{3/2}}\,dx = \left[\frac{3\left(Ab^4x^2+Aab^3\right)\sqrt{a}\log\left(-\frac{bx^2-2\sqrt{bx^2+a}\sqrt{a}+2a}{x^2}\right)+2\left(Da^2b^2x^4-8Da^4+\right.}{6\left(a^2b^4x^2+a^3\right.}\right.$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/6*(3*(A*b^4*x^2 + A*a*b^3)*sqrt(a)*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt
(a) + 2*a)/x^2) + 2*(D*a^2*b^2*x^4 - 8*D*a^4 + 6*C*a^3*b - 3*B*a^2*b^2 + 3
*A*a*b^3 - (4*D*a^3*b - 3*C*a^2*b^2)*x^2)*sqrt(b*x^2 + a))/(a^2*b^4*x^2 +
a^3*b^3), 1/3*(3*(A*b^4*x^2 + A*a*b^3)*sqrt(-a)*arctan(sqrt(b*x^2 + a)*sqr
t(-a)/a) + (D*a^2*b^2*x^4 - 8*D*a^4 + 6*C*a^3*b - 3*B*a^2*b^2 + 3*A*a*b^3
- (4*D*a^3*b - 3*C*a^2*b^2)*x^2)*sqrt(b*x^2 + a))/(a^2*b^4*x^2 + a^3*b^3)]
```

## Sympy [A] (verification not implemented)

Time = 12.98 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.31

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x\left(a+bx^2\right)^{3/2}}\,dx = \begin{cases} \frac{2\left(\frac{Ab\operatorname{atan}\left(\frac{\sqrt{a+bx^2}}{\sqrt{-a}}\right)}{2a\sqrt{-a}}+\frac{D\left(a+bx^2\right)^{\frac{3}{2}}}{6b^2}+\frac{\sqrt{a+bx^2}(Cb-2Da)}{2b^2}-\frac{-Ab^3+Bab^2-Ca^2b+Da^3}{2ab^2\sqrt{a+bx^2}}\right)}{b} & \text{for } b \neq \\ \frac{A\log\left(x^2\right)+Bx^2+\frac{Cx^4}{2}+\frac{Dx^6}{3}}{2a^{\frac{3}{2}}} & \text{otherwi} \end{cases}$$

input
```
integrate((D*x**6+C*x**4+B*x**2+A)/x/(b*x**2+a)**(3/2),x)
```

output
```
Piecewise((2*(A*b*atan(sqrt(a + b*x**2)/sqrt(-a))/(2*a*sqrt(-a)) + D*(a +
b*x**2)**(3/2)/(6*b**2) + sqrt(a + b*x**2)*(C*b - 2*D*a)/(2*b**2) - (-A*b*
*3 + B*a*b**2 - C*a**2*b + D*a**3)/(2*a*b**2*sqrt(a + b*x**2)))/b, Ne(b, 0
)), ((A*log(x**2) + B*x**2 + C*x**4/2 + D*x**6/3)/(2*a**(3/2)), True))
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.26

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x\left(a+bx^2\right)^{3/2}}\,dx = \frac{Dx^4}{3\sqrt{bx^2+a}b} - \frac{4\,Dax^2}{3\sqrt{bx^2+a}b^2} + \frac{Cx^2}{\sqrt{bx^2+a}b} - \frac{A\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{a^{\frac{3}{2}}} + \frac{A}{\sqrt{bx^2+a}a} - \frac{8\,Da^2}{3\sqrt{bx^2+a}b^3} + \frac{2\,Ca}{\sqrt{bx^2+a}b^2} - \frac{B}{\sqrt{bx^2+a}b}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
1/3*D*x^4/(sqrt(b*x^2 + a)*b) - 4/3*D*a*x^2/(sqrt(b*x^2 + a)*b^2) + C*x^2/
(sqrt(b*x^2 + a)*b) - A*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2) + A/(sqrt(b*
x^2 + a)*a) - 8/3*D*a^2/(sqrt(b*x^2 + a)*b^3) + 2*C*a/(sqrt(b*x^2 + a)*b^2
) - B/(sqrt(b*x^2 + a)*b)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.11

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x\left(a+bx^2\right)^{3/2}}\,dx = \frac{A\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a} - \frac{Da^3-Ca^2b+Bab^2-Ab^3}{\sqrt{bx^2+a}ab^3} + \frac{\left(bx^2+a\right)^{\frac{3}{2}}Db^6-6\sqrt{bx^2+a}Dab^6+3\sqrt{bx^2+a}Cb^7}{3\,b^9}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
A*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a) - (D*a^3 - C*a^2*b + B*a*b
^2 - A*b^3)/(sqrt(b*x^2 + a)*a*b^3) + 1/3*((b*x^2 + a)^(3/2)*D*b^6 - 6*sqr
t(b*x^2 + a)*D*a*b^6 + 3*sqrt(b*x^2 + a)*C*b^7)/b^9
```

## Mupad [B] (verification not implemented)

Time = 1.94 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.01

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x\left(a+bx^2\right)^{3/2}}\,dx=\frac{A}{a\,\sqrt{b\,x^2+a}}-\frac{B}{b\,\sqrt{b\,x^2+a}}-\frac{A\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{a^{3/2}}+\frac{C\,\left(b\,x^2+2\,a\right)}{b^2\,\sqrt{b\,x^2+a}}-\frac{D\,\left(8\,a^2+4\,a\,b\,x^2-b^2\,x^4\right)}{3\,b^3\,\sqrt{b\,x^2+a}}$$

input

```
int((A + B*x^2 + C*x^4 + x^6*D)/(x*(a + b*x^2)^(3/2)),x)
```

output

```
A/(a*(a + b*x^2)^(1/2)) - B/(b*(a + b*x^2)^(1/2)) - (A*atanh((a + b*x^2)^(
1/2)/a^(1/2)))/a^(3/2) + (C*(2*a + b*x^2))/(b^2*(a + b*x^2)^(1/2)) - (D*(8
*a^2 - b^2*x^4 + 4*a*b*x^2))/(3*b^3*(a + b*x^2)^(1/2))
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 224, normalized size of antiderivative = 2.07

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x\left(a+bx^2\right)^{3/2}}\,dx=\frac{-8\sqrt{b\,x^2+a}\,a^3d+6\sqrt{b\,x^2+a}\,a^2bc-4\sqrt{b\,x^2+a}\,a^2bd\,x^2+3\sqrt{b\,x^2+a}\,a\,b}{}$$

input

```
int((D*x^6+C*x^4+B*x^2+A)/x/(b*x^2+a)^(3/2),x)
```

output

```
( - 8*sqrt(a + b*x**2)*a**3*d + 6*sqrt(a + b*x**2)*a**2*b*c - 4*sqrt(a + b
*x**2)*a**2*b*d*x**2 + 3*sqrt(a + b*x**2)*a*b**2*c*x**2 + sqrt(a + b*x**2)
*a*b**2*d*x**4 + 3*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sq
rt(a))*a*b**3 + 3*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqr
t(a))*b**4*x**2 - 3*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/s
qrt(a))*a*b**3 - 3*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sq
rt(a))*b**4*x**2)/(3*a*b**3*(a + b*x**2))
```

# 3.228 $\int \frac{A+Bx^2+Cx^4+Dx^6}{x^3(a+bx^2)^{3/2}} dx$



## Optimal result

Integrand size = 32, antiderivative size = 119

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^3(a+bx^2)^{3/2}}\,dx = -\frac{Ab^3-a(b^2B-abC+a^2D)}{a^2b^2\sqrt{a+bx^2}} + \frac{D\sqrt{a+bx^2}}{b^2} - \frac{A\sqrt{a+bx^2}}{2a^2x^2} + \frac{(3Ab-2aB)\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{5/2}}$$

output
```
-(A*b^3-a*(B*b^2-C*a*b+D*a^2))/a^2/b^2/(b*x^2+a)^(1/2)+D*(b*x^2+a)^(1/2)/b
^2-1/2*A*(b*x^2+a)^(1/2)/a^2/x^2+1/2*(3*A*b-2*B*a)*arctanh((b*x^2+a)^(1/2)
/a^(1/2))/a^(5/2)
```

## Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.94

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^3(a+bx^2)^{3/2}}\,dx = \frac{\frac{\sqrt{a}(-3Ab^3x^2+4a^3Dx^2-ab^2(A-2Bx^2)+2a^2bx^2(-C+Dx^2))}{b^2x^2\sqrt{a+bx^2}} + (3Ab-2aB)\operatorname{arctanh}\left(\frac{\sqrt{a}}{\sqrt{}}\right.}{2a^{5/2}}$$

input
```
Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(x^3*(a + b*x^2)^(3/2)),x]
```

output
```
((Sqrt[a]*(-3*A*b^3*x^2 + 4*a^3*D*x^2 - a*b^2*(A - 2*B*x^2) + 2*a^2*b*x^2*
(-C + D*x^2)))/(b^2*x^2*Sqrt[a + b*x^2]) + (3*A*b - 2*a*B)*ArcTanh[Sqrt[a
+ b*x^2]/Sqrt[a]])/(2*a^(5/2))
```

## Rubi [A] (warning: unable to verify)

Time = 0.52 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {2331, 2124, 27, 1192, 25, 1584, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^3 \left(a+bx^2\right)^{3/2}} \, dx$$

$$\Big\downarrow \text{2331}$$

$$\frac{1}{2} \int \frac{Dx^6+Cx^4+Bx^2+A}{x^4 \left(bx^2+a\right)^{3/2}}dx^2$$

$$\Big\downarrow \text{2124}$$

$$\frac{1}{2} \left(-\frac{\int \frac{-2aDx^4-2aCx^2+3Ab-2aB}{2x^2 \left(bx^2+a\right)^{3/2}}dx^2}{a}-\frac{A}{ax^2\sqrt{a+bx^2}}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2} \left(-\frac{\int \frac{-2aDx^4-2aCx^2+3Ab-2aB}{x^2 \left(bx^2+a\right)^{3/2}}dx^2}{2a}-\frac{A}{ax^2\sqrt{a+bx^2}}\right)$$

$$\Big\downarrow \text{1192}$$

$$\frac{1}{2} \left(-\frac{\int -\frac{-2aDx^8-2a(bC-2aD)x^4+3Ab^3-2a\left(Da^2-bCa+b^2B\right)}{x^4(a-x^4)}d\sqrt{bx^2+a}}{ab^2}-\frac{A}{ax^2\sqrt{a+bx^2}}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2} \left(\frac{\int \frac{-2aDx^8-2a(bC-2aD)x^4+3Ab^3-2a\left(Da^2-bCa+b^2B\right)}{x^4(a-x^4)}d\sqrt{bx^2+a}}{ab^2}-\frac{A}{ax^2\sqrt{a+bx^2}}\right)$$

$$\Big\downarrow \text{1584}$$

$$\frac{1}{2}\left(\frac{\int \left(-\frac{(2aB-3Ab)b^2}{a(a-x^4)}+2aD+\frac{3Ab^3-2a(Da^2-bCa+b^2B)}{ax^4}\right)d\sqrt{bx^2+a}}{ab^2}-\frac{A}{ax^2\sqrt{a+bx^2}}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{-\frac{b^2(3Ab-2aB)\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}+\frac{3Ab^3-2a(a^2D-abC+b^2B)}{ax^2}-2aD\sqrt{a+bx^2}}{ab^2}-\frac{A}{ax^2\sqrt{a+bx^2}}\right)$$

input
```
Int[(A + B*x^2 + C*x^4 + D*x^6)/(x^3*(a + b*x^2)^(3/2)),x]
```

output
```
(-(A/(a*x^2*Sqrt[a + b*x^2])) - ((3*A*b^3 - 2*a*(b^2*B - a*b*C + a^2*D))/(
a*x^2) - 2*a*D*Sqrt[a + b*x^2] - (b^2*(3*A*b - 2*a*B)*ArcTanh[Sqrt[a + b*x
^2]/Sqrt[a]])/a^(3/2))/(a*b^2))/2
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1192
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(
2*m + 1)*(e*f - d*g + g*x^2)^n*(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 +
 c*x^4)^p, x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &&
IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m + 1/2]
```

rule 1584
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2124
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
 a*d))), x] + Simp[1/((m + 1)*(b*c - a*d))   Int[(a + b*x)^(m + 1)*(c + d*x
)^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fr
eeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && LtQ[m, -1] && (IntegerQ[m] ||  !
ILtQ[n, -1])
```

rule 2331
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/2   S
ubst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
 FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

## Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.96

| method | result |
|---|---|
| pseudoelliptic | $\frac{\frac{3\sqrt{b\,x^{2}+a}\,x^{2}b^{2}\left(Ab-\frac{2Ba}{3}\right)\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{\sqrt{a}}\right)}{2}+2\left(a^{3}x^{2}D-\frac{b\,x^{2}\left(-D x^{2}+C\right)a^{2}}{2}-\frac{b^{2}\left(-2x^{2}B+A\right)a}{4}-\frac{3A\,x^{2}b^{3}}{4}\right)\sqrt{a}}{b^{2}a^{\frac{5}{2}}x^{2}\sqrt{b\,x^{2}+a}}$ |
| default | $A\left(-\frac{1}{2a\,x^{2}\sqrt{b\,x^{2}+a}}-\frac{3b\left(\frac{1}{a\sqrt{b\,x^{2}+a}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{a^{\frac{3}{2}}}\right)}{2a}\right)+B\left(\frac{1}{a\sqrt{b\,x^{2}+a}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{a^{\frac{3}{2}}}\right)+$ |

input
```
int((D*x^6+C*x^4+B*x^2+A)/x^3/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*(3/4*(b*x^2+a)^(1/2)*x^2*b^2*(A*b-2/3*B*a)*arctanh((b*x^2+a)^(1/2)/a^(1/
2))+(a^3*x^2*D-1/2*b*x^2*(-D*x^2+C)*a^2-1/4*b^2*(-2*B*x^2+A)*a-3/4*A*x^2*b
^3)*a^(1/2))/(b*x^2+a)^(1/2)/b^2/a^(5/2)/x^2
```

## Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 320, normalized size of antiderivative = 2.69

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^3\left(a+bx^2\right)^{3/2}}\,dx = \left[-\frac{((2\,Bab^3-3\,Ab^4)x^4+(2\,Ba^2b^2-3\,Aab^3)x^2)\sqrt{a}\log\left(-\frac{bx^2+2\,\sqrt{bx^2+a}\sqrt{a}+}{x^2}\right.}{4\,(a^3}\right.$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^3/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[-1/4*(((2*B*a*b^3 - 3*A*b^4)*x^4 + (2*B*a^2*b^2 - 3*A*a*b^3)*x^2)*sqrt(a)
*log(-(b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(2*D*a^3*b*x^4 -
A*a^2*b^2 + (4*D*a^4 - 2*C*a^3*b + 2*B*a^2*b^2 - 3*A*a*b^3)*x^2)*sqrt(b*x^
2 + a))/(a^3*b^3*x^4 + a^4*b^2*x^2), 1/2*(((2*B*a*b^3 - 3*A*b^4)*x^4 + (2*
B*a^2*b^2 - 3*A*a*b^3)*x^2)*sqrt(-a)*arctan(sqrt(b*x^2 + a)*sqrt(-a)/a) +
(2*D*a^3*b*x^4 - A*a^2*b^2 + (4*D*a^4 - 2*C*a^3*b + 2*B*a^2*b^2 - 3*A*a*b^
3)*x^2)*sqrt(b*x^2 + a))/(a^3*b^3*x^4 + a^4*b^2*x^2)]
```

## Sympy [A] (verification not implemented)

Time = 40.67 (sec) , antiderivative size = 333, normalized size of antiderivative = 2.80

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^3(a+bx^2)^{3/2}}\,dx = A\left(-\frac{1}{2a\sqrt{b}x^3\sqrt{\frac{a}{bx^2}+1}}-\frac{3\sqrt{b}}{2a^2x\sqrt{\frac{a}{bx^2}+1}}+\frac{3b\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2a^{\frac{5}{2}}}\right)+B\left(\frac{2a^3\sqrt{1+\frac{bx^2}{a}}}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}+\frac{a^3\log\left(\frac{bx^2}{a}\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}-\frac{2a^3\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}+\frac{a^2bx^2\log\left(\frac{bx^2}{a}\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}-\frac{2a^2bx^2\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}\right)+C\left(\begin{cases}-\frac{1}{b\sqrt{a+bx^2}} & \text{for } b\neq 0\\ \frac{x^2}{2a^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right)+D\left(\begin{cases}\frac{2a}{b^2\sqrt{a+bx^2}}+\frac{x^2}{b\sqrt{a+bx^2}} & \text{for } b\neq 0\\ \frac{x^4}{4a^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((D*x**6+C*x**4+B*x**2+A)/x**3/(b*x**2+a)**(3/2),x)
```

output
```
A*(-1/(2*a*sqrt(b)*x**3*sqrt(a/(b*x**2) + 1)) - 3*sqrt(b)/(2*a**2*x*sqrt(a
/(b*x**2) + 1)) + 3*b*asinh(sqrt(a)/(sqrt(b)*x))/(2*a**(5/2))) + B*(2*a**3
*sqrt(1 + b*x**2/a)/(2*a**(9/2) + 2*a**(7/2)*b*x**2) + a**3*log(b*x**2/a)/
(2*a**(9/2) + 2*a**(7/2)*b*x**2) - 2*a**3*log(sqrt(1 + b*x**2/a) + 1)/(2*a
**(9/2) + 2*a**(7/2)*b*x**2) + a**2*b*x**2*log(b*x**2/a)/(2*a**(9/2) + 2*a
**(7/2)*b*x**2) - 2*a**2*b*x**2*log(sqrt(1 + b*x**2/a) + 1)/(2*a**(9/2) +
2*a**(7/2)*b*x**2)) + C*Piecewise((-1/(b*sqrt(a + b*x**2)), Ne(b, 0)), (x*
*2/(2*a**(3/2)), True)) + D*Piecewise((2*a/(b**2*sqrt(a + b*x**2)) + x**2/
(b*sqrt(a + b*x**2)), Ne(b, 0)), (x**4/(4*a**(3/2)), True))
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.13

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^3\left(a+bx^2\right)^{3/2}}\,dx = \frac{Dx^2}{\sqrt{bx^2+a}b} - \frac{B\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{a^{\frac{3}{2}}} + \frac{3\,Ab\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{2\,a^{\frac{5}{2}}} + \frac{B}{\sqrt{bx^2+a}a} + \frac{2\,Da}{\sqrt{bx^2+a}b^2} - \frac{C}{\sqrt{bx^2+a}b} - \frac{3\,Ab}{2\,\sqrt{bx^2+a}a^2} - \frac{A}{2\,\sqrt{bx^2+a}ax^2}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^3/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
D*x^2/(sqrt(b*x^2 + a)*b) - B*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2) + 3/2*
A*b*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(5/2) + B/(sqrt(b*x^2 + a)*a) + 2*D*a/
(sqrt(b*x^2 + a)*b^2) - C/(sqrt(b*x^2 + a)*b) - 3/2*A*b/(sqrt(b*x^2 + a)*a
^2) - 1/2*A/(sqrt(b*x^2 + a)*a*x^2)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.39

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^3\left(a+bx^2\right)^{3/2}}\,dx = \frac{\sqrt{bx^2+a}D}{b^2} + \frac{(2\,Ba-3\,Ab)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{2\,\sqrt{-a}a^2} + \frac{2\,(bx^2+a)Da^3-2\,Da^4-2\,(bx^2+a)Ca^2b+2\,Ca^3b+2\,(bx^2+a)Bab^2-2\,Ba^2b^2-3\,(bx^2+a)Ab^3+2\,A}{2\left(\left(bx^2+a\right)^{\frac{3}{2}}-\sqrt{bx^2+a}a\right)a^2b^2}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^3/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
sqrt(b*x^2 + a)*D/b^2 + 1/2*(2*B*a - 3*A*b)*arctan(sqrt(b*x^2 + a)/sqrt(-a
))/(sqrt(-a)*a^2) + 1/2*(2*(b*x^2 + a)*D*a^3 - 2*D*a^4 - 2*(b*x^2 + a)*C*a
^2*b + 2*C*a^3*b + 2*(b*x^2 + a)*B*a*b^2 - 2*B*a^2*b^2 - 3*(b*x^2 + a)*A*b
^3 + 2*A*a*b^3)/(((b*x^2 + a)^(3/2) - sqrt(b*x^2 + a)*a)*a^2*b^2)
```

## Mupad [B] (verification not implemented)

Time = 2.01 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.08

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^3\left(a+bx^2\right)^{3/2}}\,dx=\frac{B}{a\sqrt{b\,x^2+a}}-\frac{C}{b\sqrt{b\,x^2+a}}-\frac{B\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{a^{3/2}}+\frac{\left(b\,x^2+2\,a\right)\,D}{b^2\sqrt{b\,x^2+a}}-\frac{3\,A\,b}{2\,a^2\sqrt{b\,x^2+a}}-\frac{A}{2\,a\,x^2\sqrt{b\,x^2+a}}+\frac{3\,A\,b\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2\,a^{5/2}}$$

input
```
int((A + B*x^2 + C*x^4 + x^6*D)/(x^3*(a + b*x^2)^(3/2)),x)
```

output
```
B/(a*(a + b*x^2)^(1/2)) - C/(b*(a + b*x^2)^(1/2)) - (B*atanh((a + b*x^2)^(
1/2)/a^(1/2)))/a^(3/2) + ((2*a + b*x^2)*D)/(b^2*(a + b*x^2)^(1/2)) - (3*A*
b)/(2*a^2*(a + b*x^2)^(1/2)) - A/(2*a*x^2*(a + b*x^2)^(1/2)) + (3*A*b*atan
h((a + b*x^2)^(1/2)/a^(1/2)))/(2*a^(5/2))
```

## Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.97

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^3\left(a+bx^2\right)^{3/2}}\,dx=\frac{4\sqrt{b\,x^2+a}\,a^3d\,x^2-\sqrt{b\,x^2+a}\,a^2b^2-2\sqrt{b\,x^2+a}\,a^2bc\,x^2+2\sqrt{b\,x^2+a}\,a^2b}{}$$

input
```
int((D*x^6+C*x^4+B*x^2+A)/x^3/(b*x^2+a)^(3/2),x)
```

output
```
(4*sqrt(a + b*x**2)*a**3*d*x**2 - sqrt(a + b*x**2)*a**2*b**2 - 2*sqrt(a +
b*x**2)*a**2*b*c*x**2 + 2*sqrt(a + b*x**2)*a**2*b*d*x**4 - sqrt(a + b*x**2
)*a*b**3*x**2 - sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(
a))*a*b**3*x**2 - sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqr
t(a))*b**4*x**4 + sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqr
t(a))*a*b**3*x**2 + sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/s
qrt(a))*b**4*x**4)/(2*a**2*b**2*x**2*(a + b*x**2))
```

# 3.229 $\int \frac{A+Bx^2+Cx^4+Dx^6}{x^5(a+bx^2)^{3/2}}\,dx$



## Optimal result

Integrand size = 32, antiderivative size = 141

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^5\left(a+bx^2\right)^{3/2}}\,dx=\frac{Ab^3-a(b^2B-abC+a^2D)}{a^3b\sqrt{a+bx^2}}-\frac{A\sqrt{a+bx^2}}{4a^2x^4}+\frac{(7Ab-4aB)\sqrt{a+bx^2}}{8a^3x^2}-\frac{(15Ab^2-12abB+8a^2C)\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{7/2}}$$

output
```
(A*b^3-a*(B*b^2-C*a*b+D*a^2))/a^3/b/(b*x^2+a)^(1/2)-1/4*A*(b*x^2+a)^(1/2)/
a^2/x^4+1/8*(7*A*b-4*B*a)*(b*x^2+a)^(1/2)/a^3/x^2-1/8*(15*A*b^2-12*B*a*b+8
*C*a^2)*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(7/2)
```

## Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.91

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^5\left(a+bx^2\right)^{3/2}}\,dx=\frac{\frac{\sqrt{a}\left(15Ab^3x^4-8a^3Dx^4+ab^2x^2\left(5A-12Bx^2\right)-2a^2b\left(A+2Bx^2-4Cx^4\right)\right)}{bx^4\sqrt{a+bx^2}}+(-15Ab^2+4a(3bB}{8a^{7/2}}$$

input
```
Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(x^5*(a + b*x^2)^(3/2)),x]
```

output
```
((Sqrt[a]*(15*A*b^3*x^4 - 8*a^3*D*x^4 + a*b^2*x^2*(5*A - 12*B*x^2) - 2*a^2
*b*(A + 2*B*x^2 - 4*C*x^4)))/(b*x^4*Sqrt[a + b*x^2]) + (-15*A*b^2 + 4*a*(3
*b*B - 2*a*C))*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(8*a^(7/2))
```

## Rubi [A] (warning: unable to verify)

Time = 0.54 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.13, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2331, 2124, 27, 1192, 1582, 25, 359, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^5 (a+bx^2)^{3/2}} \, dx$$

$$\Big\downarrow \text{2331}$$

$$\frac{1}{2}\int \frac{Dx^6+Cx^4+Bx^2+A}{x^6 (bx^2+a)^{3/2}}dx^2$$

$$\Big\downarrow \text{2124}$$

$$\frac{1}{2}\left(-\frac{\int \frac{-4aDx^4-4aCx^2+5Ab-4aB}{2x^4(bx^2+a)^{3/2}}dx^2}{2a}-\frac{A}{2ax^4\sqrt{a+bx^2}}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(-\frac{\int \frac{-4aDx^4-4aCx^2+5Ab-4aB}{x^4(bx^2+a)^{3/2}}dx^2}{4a}-\frac{A}{2ax^4\sqrt{a+bx^2}}\right)$$

$$\Big\downarrow \text{1192}$$

$$\frac{1}{2}\left(-\frac{\int \frac{-4aDx^8-4a(bC-2aD)x^4+5Ab^3-4a(Da^2-bCa+b^2B)}{x^4(a-x^4)^2}d\sqrt{bx^2+a}}{2ab}-\frac{A}{2ax^4\sqrt{a+bx^2}}\right)$$

$$\Big\downarrow \text{1582}$$

$$\frac{1}{2}\left(-\frac{\frac{b^2\sqrt{a+bx^2}(5Ab-4aB)}{2a^2(a-x^4)}-\frac{\int -\frac{\left(8Da^3-4b^2Ba+5Ab^3\right)x^4+2a\left(5Ab^3-4a\left(Da^2-bCa+b^2B\right)\right)}{x^4\left(a-x^4\right)}d\sqrt{bx^2+a}}{2a^2}}{2ab}-\frac{A}{2ax^4\sqrt{a+bx^2}}\right)$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{2}\left(-\frac{\frac{\int \frac{\left(8Da^3-4b^2Ba+5Ab^3\right)x^4+2a\left(5Ab^3-4a\left(Da^2-bCa+b^2B\right)\right)}{x^4\left(a-x^4\right)}d\sqrt{bx^2+a}}{2a^2}+\frac{b^2\sqrt{a+bx^2}(5Ab-4aB)}{2a^2(a-x^4)}}{2ab}-\frac{A}{2ax^4\sqrt{a+bx^2}}\right)$$

$$\Big\downarrow \text{ 359}$$

$$\frac{1}{2}\left(-\frac{\frac{b\left(8a^2C-12abB+15Ab^2\right)\int \frac{1}{a-x^4}d\sqrt{bx^2+a}-\frac{2\left(5Ab^3-4a\left(a^2D-abC+b^2B\right)\right)}{x^2}}{2a^2}+\frac{b^2\sqrt{a+bx^2}(5Ab-4aB)}{2a^2(a-x^4)}}{2ab}-\frac{A}{2ax^4\sqrt{a+bx^2}}\right)$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{2}\left(-\frac{\frac{\frac{b\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\left(8a^2C-12abB+15Ab^2\right)}{\sqrt{a}}-\frac{2\left(5Ab^3-4a\left(a^2D-abC+b^2B\right)\right)}{x^2}}{2a^2}+\frac{b^2\sqrt{a+bx^2}(5Ab-4aB)}{2a^2(a-x^4)}}{2ab}-\frac{A}{2ax^4\sqrt{a+bx^2}}\right)$$

input
```
Int[(A + B*x^2 + C*x^4 + D*x^6)/(x^5*(a + b*x^2)^(3/2)),x]
```

output
```
(-1/2*A/(a*x^4*Sqrt[a + b*x^2]) - ((b^2*(5*A*b - 4*a*B)*Sqrt[a + b*x^2])/(
2*a^2*(a - x^4)) + ((-2*(5*A*b^3 - 4*a*(b^2*B - a*b*C + a^2*D)))/x^2 + (b*
(15*A*b^2 - 12*a*b*B + 8*a^2*C)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a])
/(2*a^2))/(2*a*b))/2
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 1192
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(
2*m + 1)*(e*f - d*g + g*x^2)^n*(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 +
 c*x^4)^p, x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &&
IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m + 1/2]
```

rule 1582
```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Simp[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1))   Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e
*x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
 b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
&& ILtQ[m/2, 0]
```

rule 2124
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
 a*d))), x] + Simp[1/((m + 1)*(b*c - a*d))   Int[(a + b*x)^(m + 1)*(c + d*x
)^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fr
eeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && LtQ[m, -1] && (IntegerQ[m] ||  !
ILtQ[n, -1])
```

rule 2331
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/2   S
ubst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
 FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

## Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.84

| method | result |
|---|---|
| pseudoelliptic | $\frac{\frac{b\left(-\frac{\sqrt{b\,x^2+a}\,\left(-7Ab\,x^2+4Ba\,x^2+2Aa\right)}{x^4}-\frac{\left(15b^2A-12abB+8a^2C\right)\operatorname{arctanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{\sqrt{a}}\right)}{8}+\frac{b^3A-a\,b^2B+a^2bC-a^3D}{\sqrt{b\,x^2+a}}}{b\,a^3}$ |
| default | $A\left(-\frac{1}{4a\,x^4\sqrt{b\,x^2+a}}-\frac{5b\left(-\frac{1}{2a\,x^2\sqrt{b\,x^2+a}}-\frac{3b\left(\frac{1}{a\sqrt{b\,x^2+a}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)}{a^{\frac{3}{2}}}\right)}{2a}\right)}{4a}\right)+B\left(-\frac{1}{2a\,x^2\sqrt{b\,x^2+a}}\right.$ |

input
```
int((D*x^6+C*x^4+B*x^2+A)/x^5/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/b/a^3*(1/8*b*(-(b*x^2+a)^(1/2)*(-7*A*b*x^2+4*B*a*x^2+2*A*a)/x^4-(15*A*b^
2-12*B*a*b+8*C*a^2)/a^(1/2)*arctanh((b*x^2+a)^(1/2)/a^(1/2)))+(A*b^3-B*a*b
^2+C*a^2*b-D*a^3)/(b*x^2+a)^(1/2))
```

## Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 367, normalized size of antiderivative = 2.60

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^5\left(a+bx^2\right)^{3/2}}\,dx = \left[\frac{((8\,Ca^2b^2-12\,Bab^3+15\,Ab^4)x^6+(8\,Ca^3b-12\,Ba^2b^2+15\,Aab^3)x^4)\sqrt{\phantom{a}}}{}\right.$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^5/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/16*(((8*C*a^2*b^2 - 12*B*a*b^3 + 15*A*b^4)*x^6 + (8*C*a^3*b - 12*B*a^2*
b^2 + 15*A*a*b^3)*x^4)*sqrt(a)*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2
*a)/x^2) - 2*(2*A*a^3*b + (8*D*a^4 - 8*C*a^3*b + 12*B*a^2*b^2 - 15*A*a*b^3
)*x^4 + (4*B*a^3*b - 5*A*a^2*b^2)*x^2)*sqrt(b*x^2 + a))/(a^4*b^2*x^6 + a^5
*b*x^4), 1/8*(((8*C*a^2*b^2 - 12*B*a*b^3 + 15*A*b^4)*x^6 + (8*C*a^3*b - 12
*B*a^2*b^2 + 15*A*a*b^3)*x^4)*sqrt(-a)*arctan(sqrt(b*x^2 + a)*sqrt(-a)/a)
- (2*A*a^3*b + (8*D*a^4 - 8*C*a^3*b + 12*B*a^2*b^2 - 15*A*a*b^3)*x^4 + (4*
B*a^3*b - 5*A*a^2*b^2)*x^2)*sqrt(b*x^2 + a))/(a^4*b^2*x^6 + a^5*b*x^4)]
```

## Sympy [A] (verification not implemented)

Time = 80.29 (sec) , antiderivative size = 394, normalized size of antiderivative = 2.79

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^5\left(a+bx^2\right)^{3/2}}\,dx = A\left(-\frac{1}{4a\sqrt{b}x^5\sqrt{\frac{a}{bx^2}+1}} + \frac{5\sqrt{b}}{8a^2x^3\sqrt{\frac{a}{bx^2}+1}} + \frac{15b^{\frac{3}{2}}}{8a^3x\sqrt{\frac{a}{bx^2}+1}} - \frac{15b^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{8a^{\frac{7}{2}}}\right) + B\left(-\frac{1}{2a\sqrt{b}x^3\sqrt{\frac{a}{bx^2}+1}} - \frac{3\sqrt{b}}{2a^2x\sqrt{\frac{a}{bx^2}+1}} + \frac{3b\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2a^{\frac{5}{2}}}\right) + C\left(\frac{2a^3\sqrt{1+\frac{bx^2}{a}}}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2} + \frac{a^3\log\left(\frac{bx^2}{a}\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2} - \frac{2a^3\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2} + \frac{a^2bx^2\log\left(\frac{bx^2}{a}\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2} - \frac{2a^2bx^2\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}\right) + D\left(\begin{cases} -\frac{1}{b\sqrt{a+bx^2}} & \text{for}\ b\neq 0 \\ \frac{x^2}{2a^{\frac{3}{2}}} & \text{otherwise} \end{cases}\right)$$

input
```
integrate((D*x**6+C*x**4+B*x**2+A)/x**5/(b*x**2+a)**(3/2),x)
```

output
```
A*(-1/(4*a*sqrt(b)*x**5*sqrt(a/(b*x**2) + 1)) + 5*sqrt(b)/(8*a**2*x**3*sqr
t(a/(b*x**2) + 1)) + 15*b**(3/2)/(8*a**3*x*sqrt(a/(b*x**2) + 1)) - 15*b**2
*asinh(sqrt(a)/(sqrt(b)*x))/(8*a**(7/2))) + B*(-1/(2*a*sqrt(b)*x**3*sqrt(a
/(b*x**2) + 1)) - 3*sqrt(b)/(2*a**2*x*sqrt(a/(b*x**2) + 1)) + 3*b*asinh(sq
rt(a)/(sqrt(b)*x))/(2*a**(5/2))) + C*(2*a**3*sqrt(1 + b*x**2/a)/(2*a**(9/2
) + 2*a**(7/2)*b*x**2) + a**3*log(b*x**2/a)/(2*a**(9/2) + 2*a**(7/2)*b*x**
2) - 2*a**3*log(sqrt(1 + b*x**2/a) + 1)/(2*a**(9/2) + 2*a**(7/2)*b*x**2) +
 a**2*b*x**2*log(b*x**2/a)/(2*a**(9/2) + 2*a**(7/2)*b*x**2) - 2*a**2*b*x**
2*log(sqrt(1 + b*x**2/a) + 1)/(2*a**(9/2) + 2*a**(7/2)*b*x**2)) + D*Piecew
ise((-1/(b*sqrt(a + b*x**2)), Ne(b, 0)), (x**2/(2*a**(3/2)), True))
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.26

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^5\left(a+bx^2\right)^{3/2}}\,dx = -\frac{C\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{a^{\frac{3}{2}}} + \frac{3\,Bb\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{2\,a^{\frac{5}{2}}} - \frac{15\,Ab^2\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{8\,a^{\frac{7}{2}}} + \frac{C}{\sqrt{bx^2+a}a} - \frac{D}{\sqrt{bx^2+a}b} - \frac{3\,Bb}{2\,\sqrt{bx^2+a}a^2} + \frac{15\,Ab^2}{8\,\sqrt{bx^2+a}a^3} - \frac{B}{2\,\sqrt{bx^2+a}ax^2} + \frac{5\,Ab}{8\,\sqrt{bx^2+a}a^2x^2} - \frac{A}{4\,\sqrt{bx^2+a}ax^4}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^5/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
-C*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2) + 3/2*B*b*arcsinh(a/(sqrt(a*b)*ab
s(x)))/a^(5/2) - 15/8*A*b^2*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(7/2) + C/(sqr
t(b*x^2 + a)*a) - D/(sqrt(b*x^2 + a)*b) - 3/2*B*b/(sqrt(b*x^2 + a)*a^2) +
15/8*A*b^2/(sqrt(b*x^2 + a)*a^3) - 1/2*B/(sqrt(b*x^2 + a)*a*x^2) + 5/8*A*b
/(sqrt(b*x^2 + a)*a^2*x^2) - 1/4*A/(sqrt(b*x^2 + a)*a*x^4)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.13

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^5\left(a+bx^2\right)^{3/2}}\,dx = \frac{\left(8\,Ca^2-12\,Bab+15\,Ab^2\right)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{8\,\sqrt{-a}a^3} - \frac{Da^3-Ca^2b+Bab^2-Ab^3}{\sqrt{bx^2+a}a^3b} - \frac{4\left(bx^2+a\right)^{\frac{3}{2}}Bab-4\,\sqrt{bx^2+a}Ba^2b-7\left(bx^2+a\right)^{\frac{3}{2}}Ab^2+9\,\sqrt{bx^2+a}Aab^2}{8\,a^3b^2x^4}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^5/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
1/8*(8*C*a^2 - 12*B*a*b + 15*A*b^2)*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt
(-a)*a^3) - (D*a^3 - C*a^2*b + B*a*b^2 - A*b^3)/(sqrt(b*x^2 + a)*a^3*b) -
1/8*(4*(b*x^2 + a)^(3/2)*B*a*b - 4*sqrt(b*x^2 + a)*B*a^2*b - 7*(b*x^2 + a)
^(3/2)*A*b^2 + 9*sqrt(b*x^2 + a)*A*a*b^2)/(a^3*b^2*x^4)
```

## Mupad [B] (verification not implemented)

Time = 2.18 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.30

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^5\left(a+bx^2\right)^{3/2}}\,dx = \frac{C}{a\sqrt{b\,x^2+a}}-\frac{D}{b\sqrt{b\,x^2+a}}-\frac{C\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{a^{3/2}} -\frac{15\,A\,b^2\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{8\,a^{7/2}}-\frac{3\,B\,b}{2\,a^2\sqrt{b\,x^2+a}}+\frac{15\,A\,b^2}{8\,a^3\sqrt{b\,x^2+a}}-\frac{A}{4\,a\,x^4\sqrt{b\,x^2+a}} -\frac{B}{2\,a\,x^2\sqrt{b\,x^2+a}}+\frac{3\,B\,b\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2\,a^{5/2}}+\frac{5\,A\,b}{8\,a^2\,x^2\sqrt{b\,x^2+a}}$$

input
```
int((A + B*x^2 + C*x^4 + x^6*D)/(x^5*(a + b*x^2)^(3/2)),x)
```

output
```
C/(a*(a + b*x^2)^(1/2)) - D/(b*(a + b*x^2)^(1/2)) - (C*atanh((a + b*x^2)^(
1/2)/a^(1/2)))/a^(3/2) - (15*A*b^2*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(8*a^
(7/2)) - (3*B*b)/(2*a^2*(a + b*x^2)^(1/2)) + (15*A*b^2)/(8*a^3*(a + b*x^2)
^(1/2)) - A/(4*a*x^4*(a + b*x^2)^(1/2)) - B/(2*a*x^2*(a + b*x^2)^(1/2)) +
(3*B*b*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(2*a^(5/2)) + (5*A*b)/(8*a^2*x^2*
(a + b*x^2)^(1/2))
```

## Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 371, normalized size of antiderivative = 2.63

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^5\left(a+bx^2\right)^{3/2}}\,dx = \frac{-2\sqrt{b\,x^2+a}\,a^3b-8\sqrt{b\,x^2+a}\,a^3d\,x^4+\sqrt{b\,x^2+a}\,a^2b^2x^2+8\sqrt{b\,x^2+a}\,a^2b}{}$$

input
```
int((D*x^6+C*x^4+B*x^2+A)/x^5/(b*x^2+a)^(3/2),x)
```

output
```
( - 2*sqrt(a + b*x**2)*a**3*b - 8*sqrt(a + b*x**2)*a**3*d*x**4 + sqrt(a +
b*x**2)*a**2*b**2*x**2 + 8*sqrt(a + b*x**2)*a**2*b*c*x**4 + 3*sqrt(a + b*x
**2)*a*b**3*x**4 + 8*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/
sqrt(a))*a**2*b*c*x**4 + 3*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(
b)*x)/sqrt(a))*a*b**3*x**4 + 8*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + s
qrt(b)*x)/sqrt(a))*a*b**2*c*x**6 + 3*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(
a) + sqrt(b)*x)/sqrt(a))*b**4*x**6 - 8*sqrt(a)*log((sqrt(a + b*x**2) + sqr
t(a) + sqrt(b)*x)/sqrt(a))*a**2*b*c*x**4 - 3*sqrt(a)*log((sqrt(a + b*x**2)
 + sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b**3*x**4 - 8*sqrt(a)*log((sqrt(a + b*x
**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b**2*c*x**6 - 3*sqrt(a)*log((sqrt(a
 + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*b**4*x**6)/(8*a**3*b*x**4*(a +
b*x**2))
```

# 3.230 $\int \frac{A+Bx^2+Cx^4+Dx^6}{x^7(a+bx^2)^{3/2}}\,dx$



## Optimal result

Integrand size = 32, antiderivative size = 188

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^7(a+bx^2)^{3/2}}\,dx = -\frac{Ab^3-a(b^2B-abC+a^2D)}{a^4\sqrt{a+bx^2}} - \frac{A\sqrt{a+bx^2}}{6a^2x^6} + \frac{(11Ab-6aB)\sqrt{a+bx^2}}{24a^3x^4} - \frac{(19Ab^2-14abB+8a^2C)\sqrt{a+bx^2}}{16a^4x^2} + \frac{(35Ab^3-2a(15b^2B-12abC+8a^2D))\,\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{9/2}}$$

output
```
-(A*b^3-a*(B*b^2-C*a*b+D*a^2))/a^4/(b*x^2+a)^(1/2)-1/6*A*(b*x^2+a)^(1/2)/a
^2/x^6+1/24*(11*A*b-6*B*a)*(b*x^2+a)^(1/2)/a^3/x^4-1/16*(19*A*b^2-14*B*a*b
+8*C*a^2)*(b*x^2+a)^(1/2)/a^4/x^2+1/16*(35*A*b^3-2*a*(15*B*b^2-12*C*a*b+8*
D*a^2))*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(9/2)
```

## Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.85

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^7(a+bx^2)^{3/2}}\,dx = -\frac{105Ab^3x^6+5ab^2x^4(7A-18Bx^2)+2a^2bx^2(-7A-15Bx^2+36Cx^4)+4a^3(2A+3x^2(B+2Cx^2-4Dx}{48a^4x^6\sqrt{a+bx^2}} + \frac{(35Ab^3-2a(15b^2B-12abC+8a^2D))\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{9/2}}$$

input
```
Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(x^7*(a + b*x^2)^(3/2)),x]
```

output
```
-1/48*(105*A*b^3*x^6 + 5*a*b^2*x^4*(7*A - 18*B*x^2) + 2*a^2*b*x^2*(-7*A -
15*B*x^2 + 36*C*x^4) + 4*a^3*(2*A + 3*x^2*(B + 2*C*x^2 - 4*D*x^4)))/(a^4*x
^6*Sqrt[a + b*x^2]) + ((35*A*b^3 - 2*a*(15*b^2*B - 12*a*b*C + 8*a^2*D))*Ar
cTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(16*a^(9/2))
```

## Rubi [A] (warning: unable to verify)

Time = 0.64 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.16, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.344$, Rules used = {2331, 2124, 27, 1192, 25, 1582, 25, 361, 25, 359, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^7(a+bx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 2331}$$

$$\frac{1}{2}\int \frac{Dx^6+Cx^4+Bx^2+A}{x^8(bx^2+a)^{3/2}}dx^2$$

$$\Big\downarrow \text{ 2124}$$

$$\frac{1}{2}\left(-\frac{\int \frac{-6aDx^4-6aCx^2+7Ab-6aB}{2x^6(bx^2+a)^{3/2}}dx^2}{3a}-\frac{A}{3ax^6\sqrt{a+bx^2}}\right)$$

$\Big\downarrow$ 27

$$\frac{1}{2}\left(-\frac{\int \frac{-6aDx^4-6aCx^2+7Ab-6aB}{x^6(bx^2+a)^{3/2}}dx^2}{6a}-\frac{A}{3ax^6\sqrt{a+bx^2}}\right)$$

$\Big\downarrow$ 1192

$$\frac{1}{2}\left(-\frac{\int -\frac{-6aDx^8-6a(bC-2aD)x^4+7Ab^3-6a\left(Da^2-bCa+b^2B\right)}{x^4(a-x^4)^3}d\sqrt{bx^2+a}}{3a}-\frac{A}{3ax^6\sqrt{a+bx^2}}\right)$$

$\Big\downarrow$ 25

$$\frac{1}{2}\left(\frac{\int \frac{-6aDx^8-6a(bC-2aD)x^4+7Ab^3-6a\left(Da^2-bCa+b^2B\right)}{x^4(a-x^4)^3}d\sqrt{bx^2+a}}{3a}-\frac{A}{3ax^6\sqrt{a+bx^2}}\right)$$

$\Big\downarrow$ 1582

$$\frac{1}{2}\left(-\frac{\frac{\int -\frac{3\left(8Da^3-6b^2Ba+7Ab^3\right)x^4+4a\left(7Ab^3-6a\left(Da^2-bCa+b^2B\right)\right)}{x^4\left(a-x^4\right)^2}d\sqrt{bx^2+a}}{4a^2}-\frac{b^2\sqrt{a+bx^2}(7Ab-6aB)}{4a^2(a-x^4)^2}}{3a}-\frac{A}{3ax^6\sqrt{a+bx^2}}\right)$$

$\Big\downarrow$ 25

$$\frac{1}{2}\left(-\frac{-\frac{\int \frac{3\left(8Da^3-6b^2Ba+7Ab^3\right)x^4+4a\left(7Ab^3-6a\left(Da^2-bCa+b^2B\right)\right)}{x^4\left(a-x^4\right)^2}d\sqrt{bx^2+a}}{4a^2}-\frac{b^2\sqrt{a+bx^2}(7Ab-6aB)}{4a^2(a-x^4)^2}}{3a}-\frac{A}{3ax^6\sqrt{a+bx^2}}\right)$$

$\Big\downarrow$ 361

$$\frac{1}{2}\left(-\frac{-\frac{\frac{b\sqrt{a+bx^2}\left(24a^2C-42abB+49Ab^2\right)}{2a\left(a-x^4\right)}-\frac{1}{2}\int -\frac{b\left(\frac{49Ab^2}{a}-42Bb+24aC\right)x^4+8\left(7Ab^3-6a\left(Da^2-bCa+b^2B\right)\right)}{x^4\left(a-x^4\right)}d\sqrt{bx^2+a}}{4a^2}-\frac{b^2\sqrt{a+bx^2}(7Ab-6aB)}{4a^2(a-x^4)^2}}{3a}-\right.$$

$$\Big\downarrow 25$$

$$\frac{1}{2}\left(-\frac{-\frac{\frac{1}{2}\int \frac{b\left(\frac{49Ab^2}{a}-42Bb+24aC\right)x^4+8\left(7Ab^3-6a\left(Da^2-bCa+b^2B\right)\right)}{x^4\left(a-x^4\right)}d\sqrt{bx^2+a}+\frac{b\sqrt{a+bx^2}\left(24a^2C-42abB+49Ab^2\right)}{2a\left(a-x^4\right)}}{4a^2}-\frac{b^2\sqrt{a+bx^2}(7Ab-6aB)}{4a^2(a-x^4)^2}}{3a}-\right.$$

$$\Big\downarrow 359$$

$$\frac{1}{2}\left(-\frac{-\frac{\frac{1}{2}\left(\frac{3\left(-16a^3D+24a^2bC-30ab^2B+35Ab^3\right)\int \frac{1}{a-x^4}d\sqrt{bx^2+a}}{a}-\frac{8\left(7Ab^3-6a\left(a^2D-abC+b^2B\right)\right)}{ax^2}\right)+\frac{b\sqrt{a+bx^2}\left(24a^2C-42abB+49Ab^2\right)}{2a\left(a-x^4\right)}}{4a^2}-\frac{b^2\sqrt{a+b}}{4a}}{3a}\right.$$

$$\Big\downarrow 219$$

$$\frac{1}{2}\left(-\frac{-\frac{b^2\sqrt{a+bx^2}(7Ab-6aB)}{4a^2(a-x^4)^2}-\frac{\frac{b\sqrt{a+bx^2}\left(24a^2C-42abB+49Ab^2\right)}{2a\left(a-x^4\right)}+\frac{1}{2}\left(\frac{3\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\left(-16a^3D+24a^2bC-30ab^2B+35Ab^3\right)}{a^{3/2}}-\frac{8\left(7Ab^3-6a\right.}{}\right.}{4a^2}}{3a}\right.$$

input
```
Int[(A + B*x^2 + C*x^4 + D*x^6)/(x^7*(a + b*x^2)^(3/2)),x]
```

output
```
(-1/3*A/(a*x^6*Sqrt[a + b*x^2]) - (-1/4*(b^2*(7*A*b - 6*a*B)*Sqrt[a + b*x^
2])/(a^2*(a - x^4)^2) - ((b*(49*A*b^2 - 42*a*b*B + 24*a^2*C)*Sqrt[a + b*x^
2])/(2*a*(a - x^4)) + ((-8*(7*A*b^3 - 6*a*(b^2*B - a*b*C + a^2*D)))/(a*x^2
) + (3*(35*A*b^3 - 30*a*b^2*B + 24*a^2*b*C - 16*a^3*D)*ArcTanh[Sqrt[a + b*
x^2]/Sqrt[a]])/a^(3/2))/2)/(4*a^2))/(3*a))/2
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 361
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[x^m*(a + b*x^2)^(p + 1)*E
xpandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c
- a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/
2, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 1192
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(
2*m + 1)*(e*f - d*g + g*x^2)^n*(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 +
 c*x^4)^p, x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &&
IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m + 1/2]
```

rule 1582
```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Simp[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1))   Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e
*x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
 b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
&& ILtQ[m/2, 0]
```

rule 2124
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
 a*d))), x] + Simp[1/((m + 1)*(b*c - a*d))   Int[(a + b*x)^(m + 1)*(c + d*x
)^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fr
eeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && LtQ[m, -1] && (IntegerQ[m] ||  !
ILtQ[n, -1])
```

rule 2331
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/2   S
ubst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
 FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

## Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.81

| method | result |
|---|---|
| pseudoelliptic | $\frac{-\frac{35b^2\left(-\frac{18x^2B}{7}+A\right)x^4a^{\frac{3}{2}}}{48}+\frac{7\left(-\frac{36}{7}Cx^4+\frac{15}{7}x^2B+A\right)x^2ba^{\frac{5}{2}}}{24}+\frac{\left(4Dx^6-2Cx^4-x^2B-\frac{2}{3}A\right)a^{\frac{7}{2}}}{4}+\frac{35\left(-Ab^3\sqrt{a}+\left(b^3A-\frac{6}{7}ab^2B+\frac{24}{35}\right.\right.}{}}{x^6a^{\frac{9}{2}}\sqrt{bx^2+a}}$ |
| default | $A\left(-\frac{1}{6a\,x^6\sqrt{b\,x^2+a}}-\frac{7b\left(-\frac{1}{4a\,x^4\sqrt{b\,x^2+a}}-\frac{5b\left(-\frac{1}{2a\,x^2\sqrt{b\,x^2+a}}-\frac{3b\left(\frac{1}{a\sqrt{b\,x^2+a}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)}{a^{\frac{3}{2}}}\right)}{2a}\right)}{4a}\right)}{6a}\right)+B$ |

input
```
int((D*x^6+C*x^4+B*x^2+A)/x^7/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
35/16/(b*x^2+a)^(1/2)*(-1/3*b^2*(-18/7*x^2*B+A)*x^4*a^(3/2)+2/15*(-36/7*C*
x^4+15/7*x^2*B+A)*x^2*b*a^(5/2)+4/35*(4*D*x^6-2*C*x^4-x^2*B-2/3*A)*a^(7/2)
+(-A*b^3*a^(1/2)+(b^3*A-6/7*a*b^2*B+24/35*a^2*b*C-16/35*a^3*D)*(b*x^2+a)^(
1/2)*arctanh((b*x^2+a)^(1/2)/a^(1/2)))*x^6)/a^(9/2)/x^6
```

## Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 440, normalized size of antiderivative = 2.34

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^7\left(a+bx^2\right)^{3/2}}\,dx = \left[-\frac{3\left((16\,Da^3b-24\,Ca^2b^2+30\,Bab^3-35\,Ab^4)x^8+(16\,Da^4-24\,Ca^3b+\right.}{}\right.$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^7/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[-1/96*(3*((16*D*a^3*b - 24*C*a^2*b^2 + 30*B*a*b^3 - 35*A*b^4)*x^8 + (16*D
*a^4 - 24*C*a^3*b + 30*B*a^2*b^2 - 35*A*a*b^3)*x^6)*sqrt(a)*log(-(b*x^2 +
2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(3*(16*D*a^4 - 24*C*a^3*b + 30*B
*a^2*b^2 - 35*A*a*b^3)*x^6 - 8*A*a^4 - (24*C*a^4 - 30*B*a^3*b + 35*A*a^2*b
^2)*x^4 - 2*(6*B*a^4 - 7*A*a^3*b)*x^2)*sqrt(b*x^2 + a))/(a^5*b*x^8 + a^6*x
^6), 1/48*(3*((16*D*a^3*b - 24*C*a^2*b^2 + 30*B*a*b^3 - 35*A*b^4)*x^8 + (1
6*D*a^4 - 24*C*a^3*b + 30*B*a^2*b^2 - 35*A*a*b^3)*x^6)*sqrt(-a)*arctan(sqr
t(b*x^2 + a)*sqrt(-a)/a) + (3*(16*D*a^4 - 24*C*a^3*b + 30*B*a^2*b^2 - 35*A
*a*b^3)*x^6 - 8*A*a^4 - (24*C*a^4 - 30*B*a^3*b + 35*A*a^2*b^2)*x^4 - 2*(6*
B*a^4 - 7*A*a^3*b)*x^2)*sqrt(b*x^2 + a))/(a^5*b*x^8 + a^6*x^6)]
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 500 vs. $2(178) = 356$.

Time = 134.28 (sec) , antiderivative size = 500, normalized size of antiderivative = 2.66

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^7(a+bx^2)^{3/2}}\,dx = A\left(-\frac{1}{6a\sqrt{b}x^7\sqrt{\frac{a}{bx^2}+1}}+\frac{7\sqrt{b}}{24a^2x^5\sqrt{\frac{a}{bx^2}+1}}-\frac{35b^{\frac{3}{2}}}{48a^3x^3\sqrt{\frac{a}{bx^2}+1}}-\frac{35b^{\frac{5}{2}}}{16a^4x\sqrt{\frac{a}{bx^2}+1}}+\frac{35b^3\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{16a^{\frac{9}{2}}}\right)+B\left(-\frac{1}{4a\sqrt{b}x^5\sqrt{\frac{a}{bx^2}+1}}+\frac{5\sqrt{b}}{8a^2x^3\sqrt{\frac{a}{bx^2}+1}}+\frac{15b^{\frac{3}{2}}}{8a^3x\sqrt{\frac{a}{bx^2}+1}}-\frac{15b^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{8a^{\frac{7}{2}}}\right)+C\left(-\frac{1}{2a\sqrt{b}x^3\sqrt{\frac{a}{bx^2}+1}}-\frac{3\sqrt{b}}{2a^2x\sqrt{\frac{a}{bx^2}+1}}+\frac{3b\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2a^{\frac{5}{2}}}\right)+D\left(\frac{2a^3\sqrt{1+\frac{bx^2}{a}}}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}+\frac{a^3\log\left(\frac{bx^2}{a}\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}-\frac{2a^3\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}+\frac{a^2bx^2\log\left(\frac{bx^2}{a}\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}-\frac{2a^2bx^2\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}\right)$$

input
```
integrate((D*x**6+C*x**4+B*x**2+A)/x**7/(b*x**2+a)**(3/2),x)
```

output
```
A*(-1/(6*a*sqrt(b)*x**7*sqrt(a/(b*x**2) + 1)) + 7*sqrt(b)/(24*a**2*x**5*sq
rt(a/(b*x**2) + 1)) - 35*b**(3/2)/(48*a**3*x**3*sqrt(a/(b*x**2) + 1)) - 35
*b**(5/2)/(16*a**4*x*sqrt(a/(b*x**2) + 1)) + 35*b**3*asinh(sqrt(a)/(sqrt(b
)*x))/(16*a**(9/2))) + B*(-1/(4*a*sqrt(b)*x**5*sqrt(a/(b*x**2) + 1)) + 5*s
qrt(b)/(8*a**2*x**3*sqrt(a/(b*x**2) + 1)) + 15*b**(3/2)/(8*a**3*x*sqrt(a/(
b*x**2) + 1)) - 15*b**2*asinh(sqrt(a)/(sqrt(b)*x))/(8*a**(7/2))) + C*(-1/(
2*a*sqrt(b)*x**3*sqrt(a/(b*x**2) + 1)) - 3*sqrt(b)/(2*a**2*x*sqrt(a/(b*x**
2) + 1)) + 3*b*asinh(sqrt(a)/(sqrt(b)*x))/(2*a**(5/2))) + D*(2*a**3*sqrt(1
 + b*x**2/a)/(2*a**(9/2) + 2*a**(7/2)*b*x**2) + a**3*log(b*x**2/a)/(2*a**(
9/2) + 2*a**(7/2)*b*x**2) - 2*a**3*log(sqrt(1 + b*x**2/a) + 1)/(2*a**(9/2)
 + 2*a**(7/2)*b*x**2) + a**2*b*x**2*log(b*x**2/a)/(2*a**(9/2) + 2*a**(7/2)
*b*x**2) - 2*a**2*b*x**2*log(sqrt(1 + b*x**2/a) + 1)/(2*a**(9/2) + 2*a**(7
/2)*b*x**2))
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.38

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^7(a+bx^2)^{3/2}}\,dx = -\frac{D\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{a^{\frac{3}{2}}} + \frac{3\,Cb\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{2\,a^{\frac{5}{2}}} - \frac{15\,Bb^2\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{8\,a^{\frac{7}{2}}} + \frac{35\,Ab^3\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{16\,a^{\frac{9}{2}}} + \frac{D}{\sqrt{bx^2+a}a} - \frac{3\,Cb}{2\sqrt{bx^2+a}a^2} + \frac{15\,Bb^2}{8\sqrt{bx^2+a}a^3} - \frac{35\,Ab^3}{16\sqrt{bx^2+a}a^4} - \frac{C}{2\sqrt{bx^2+a}ax^2} + \frac{5\,Bb}{8\sqrt{bx^2+a}a^2x^2} - \frac{35\,Ab^2}{48\sqrt{bx^2+a}a^3x^2} - \frac{B}{4\sqrt{bx^2+a}ax^4} + \frac{7\,Ab}{24\sqrt{bx^2+a}a^2x^4} - \frac{A}{6\sqrt{bx^2+a}ax^6}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^7/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
-D*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2) + 3/2*C*b*arcsinh(a/(sqrt(a*b)*ab
s(x)))/a^(5/2) - 15/8*B*b^2*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(7/2) + 35/16*
A*b^3*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(9/2) + D/(sqrt(b*x^2 + a)*a) - 3/2*
C*b/(sqrt(b*x^2 + a)*a^2) + 15/8*B*b^2/(sqrt(b*x^2 + a)*a^3) - 35/16*A*b^3
/(sqrt(b*x^2 + a)*a^4) - 1/2*C/(sqrt(b*x^2 + a)*a*x^2) + 5/8*B*b/(sqrt(b*x
^2 + a)*a^2*x^2) - 35/48*A*b^2/(sqrt(b*x^2 + a)*a^3*x^2) - 1/4*B/(sqrt(b*x
^2 + a)*a*x^4) + 7/24*A*b/(sqrt(b*x^2 + a)*a^2*x^4) - 1/6*A/(sqrt(b*x^2 +
a)*a*x^6)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.35

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^7\left(a+bx^2\right)^{3/2}}\,dx = \frac{\left(16\,Da^3-24\,Ca^2b+30\,Bab^2-35\,Ab^3\right)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{16\,\sqrt{-a}a^4} + \frac{Da^3-Ca^2b+Bab^2-Ab^3}{\sqrt{bx^2+a}a^4} - \frac{24\,(bx^2+a)^{\frac{5}{2}}Ca^2b-48\,(bx^2+a)^{\frac{3}{2}}Ca^3b+24\,\sqrt{bx^2+a}Ca^4b-42\,(bx^2+a)^{\frac{5}{2}}Bab^2+96\,(bx^2+a)^{\frac{3}{2}}Ba^2b^2}{48\,a^4b^3x^6}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^7/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
1/16*(16*D*a^3 - 24*C*a^2*b + 30*B*a*b^2 - 35*A*b^3)*arctan(sqrt(b*x^2 + a
)/sqrt(-a))/(sqrt(-a)*a^4) + (D*a^3 - C*a^2*b + B*a*b^2 - A*b^3)/(sqrt(b*x
^2 + a)*a^4) - 1/48*(24*(b*x^2 + a)^(5/2)*C*a^2*b - 48*(b*x^2 + a)^(3/2)*C
*a^3*b + 24*sqrt(b*x^2 + a)*C*a^4*b - 42*(b*x^2 + a)^(5/2)*B*a*b^2 + 96*(b
*x^2 + a)^(3/2)*B*a^2*b^2 - 54*sqrt(b*x^2 + a)*B*a^3*b^2 + 57*(b*x^2 + a)^
(5/2)*A*b^3 - 136*(b*x^2 + a)^(3/2)*A*a*b^3 + 87*sqrt(b*x^2 + a)*A*a^2*b^3
)/(a^4*b^3*x^6)
```

## Mupad [B] (verification not implemented)

Time = 2.45 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.42

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^7\left(a+bx^2\right)^{3/2}}\,dx = \frac{D}{a\sqrt{b\,x^2+a}}-\frac{\operatorname{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)\,D}{a^{3/2}} + \frac{35\,A\,b^3\operatorname{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{16\,a^{9/2}}-\frac{15\,B\,b^2\operatorname{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{8\,a^{7/2}}-\frac{3\,C\,b}{2\,a^2\sqrt{b\,x^2+a}} - \frac{35\,A\,b^3}{16\,a^4\sqrt{b\,x^2+a}}+\frac{15\,B\,b^2}{8\,a^3\sqrt{b\,x^2+a}}-\frac{A}{6\,a\,x^6\sqrt{b\,x^2+a}} - \frac{B}{4\,a\,x^4\sqrt{b\,x^2+a}}-\frac{C}{2\,a\,x^2\sqrt{b\,x^2+a}}+\frac{3\,C\,b\operatorname{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2\,a^{5/2}} + \frac{7\,A\,b}{24\,a^2\,x^4\sqrt{b\,x^2+a}}+\frac{5\,B\,b}{8\,a^2\,x^2\sqrt{b\,x^2+a}}-\frac{35\,A\,b^2}{48\,a^3\,x^2\sqrt{b\,x^2+a}}$$

input
```
int((A + B*x^2 + C*x^4 + x^6*D)/(x^7*(a + b*x^2)^(3/2)),x)
```

output
```
D/(a*(a + b*x^2)^(1/2)) - (atanh((a + b*x^2)^(1/2)/a^(1/2))*D)/a^(3/2) + (
35*A*b^3*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(16*a^(9/2)) - (15*B*b^2*atanh(
(a + b*x^2)^(1/2)/a^(1/2)))/(8*a^(7/2)) - (3*C*b)/(2*a^2*(a + b*x^2)^(1/2)
) - (35*A*b^3)/(16*a^4*(a + b*x^2)^(1/2)) + (15*B*b^2)/(8*a^3*(a + b*x^2)^
(1/2)) - A/(6*a*x^6*(a + b*x^2)^(1/2)) - B/(4*a*x^4*(a + b*x^2)^(1/2)) - C
/(2*a*x^2*(a + b*x^2)^(1/2)) + (3*C*b*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(2
*a^(5/2)) + (7*A*b)/(24*a^2*x^4*(a + b*x^2)^(1/2)) + (5*B*b)/(8*a^2*x^2*(a
 + b*x^2)^(1/2)) - (35*A*b^2)/(48*a^3*x^2*(a + b*x^2)^(1/2))
```

## Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 536, normalized size of antiderivative = 2.85

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^7\left(a+bx^2\right)^{3/2}}\,dx = \frac{-8\sqrt{b\,x^2+a}\,a^4+2\sqrt{b\,x^2+a}\,a^3b\,x^2-24\sqrt{b\,x^2+a}\,a^3c\,x^4+48\sqrt{b\,x^2+a}\,a}{}$$

input
```
int((D*x^6+C*x^4+B*x^2+A)/x^7/(b*x^2+a)^(3/2),x)
```

output
```
( - 8*sqrt(a + b*x**2)*a**4 + 2*sqrt(a + b*x**2)*a**3*b*x**2 - 24*sqrt(a +
 b*x**2)*a**3*c*x**4 + 48*sqrt(a + b*x**2)*a**3*d*x**6 - 5*sqrt(a + b*x**2
)*a**2*b**2*x**4 - 72*sqrt(a + b*x**2)*a**2*b*c*x**6 - 15*sqrt(a + b*x**2)
*a*b**3*x**6 + 48*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqr
t(a))*a**3*d*x**6 - 72*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x
)/sqrt(a))*a**2*b*c*x**6 + 48*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sq
rt(b)*x)/sqrt(a))*a**2*b*d*x**8 - 15*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(
a) + sqrt(b)*x)/sqrt(a))*a*b**3*x**6 - 72*sqrt(a)*log((sqrt(a + b*x**2) -
sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b**2*c*x**8 - 15*sqrt(a)*log((sqrt(a + b*x
**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*b**4*x**8 - 48*sqrt(a)*log((sqrt(a +
b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*a**3*d*x**6 + 72*sqrt(a)*log((sqrt
(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*a**2*b*c*x**6 - 48*sqrt(a)*lo
g((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*a**2*b*d*x**8 + 15*sqr
t(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b**3*x**6 + 7
2*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b**2*c*x
**8 + 15*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*b**
4*x**8)/(48*a**4*x**6*(a + b*x**2))
```

# 3.231 $\int \frac{A+Bx^2+Cx^4+Dx^6}{x^9(a+bx^2)^{3/2}}\,dx$



## Optimal result

Integrand size = 32, antiderivative size = 238

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^9\left(a+bx^2\right)^{3/2}}\,dx = \frac{b(Ab^3-a(b^2B-abC+a^2D))}{a^5\sqrt{a+bx^2}} - \frac{A\sqrt{a+bx^2}}{8a^2x^8} + \frac{(15Ab-8aB)\sqrt{a+bx^2}}{48a^3x^6} - \frac{(123Ab^2-88abB+48a^2C)\sqrt{a+bx^2}}{192a^4x^4} + \frac{(187Ab^3-8a(19b^2B-14abC+8a^2D))\sqrt{a+bx^2}}{128a^5x^2} - \frac{b(315Ab^3-8a(35b^2B-30abC+24a^2D))\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{128a^{11/2}}$$

output

```
b*(A*b^3-a*(B*b^2-C*a*b+D*a^2))/a^5/(b*x^2+a)^(1/2)-1/8*A*(b*x^2+a)^(1/2)/
a^2/x^8+1/48*(15*A*b-8*B*a)*(b*x^2+a)^(1/2)/a^3/x^6-1/192*(123*A*b^2-88*B*
a*b+48*C*a^2)*(b*x^2+a)^(1/2)/a^4/x^4+1/128*(187*A*b^3-8*a*(19*B*b^2-14*C*
a*b+8*D*a^2))*(b*x^2+a)^(1/2)/a^5/x^2-1/128*b*(315*A*b^3-8*a*(35*B*b^2-30*
C*a*b+24*D*a^2))*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(11/2)
```

## Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.81

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^9\left(a+bx^2\right)^{3/2}}\,dx = \frac{\frac{\sqrt{a}\left(945Ab^4x^8+105ab^3x^6\left(3A-8Bx^2\right)+2a^2b^2x^4\left(-63A-140Bx^2+360Cx^4\right)+8a^3bx^2\left(9A+14Bx^2+30\right.\right.}{x^8\sqrt{a+bx^2}}}{}$$

input
```
Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(x^9*(a + b*x^2)^(3/2)),x]
```

output
```
((Sqrt[a]*(945*A*b^4*x^8 + 105*a*b^3*x^6*(3*A - 8*B*x^2) + 2*a^2*b^2*x^4*(
-63*A - 140*B*x^2 + 360*C*x^4) + 8*a^3*b*x^2*(9*A + 14*B*x^2 + 30*C*x^4 -
72*D*x^6) - 16*a^4*(3*A + 4*B*x^2 + 6*C*x^4 + 12*D*x^6)))/(x^8*Sqrt[a + b*
x^2]) - 3*b*(315*A*b^3 - 8*a*(35*b^2*B - 30*a*b*C + 24*a^2*D))*ArcTanh[Sqr
t[a + b*x^2]/Sqrt[a]])/(384*a^(11/2))
```

## Rubi [A] (warning: unable to verify)

Time = 0.73 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.17, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.406$, Rules used = {2331, 2124, 27, 1192, 1582, 25, 361, 27, 361, 25, 27, 359, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^9\left(a+bx^2\right)^{3/2}}\,dx$$

$$\Big\downarrow \text{2331}$$

$$\frac{1}{2}\int \frac{Dx^6+Cx^4+Bx^2+A}{x^{10}\left(bx^2+a\right)^{3/2}}dx^2$$

$$\Big\downarrow \text{2124}$$

$$\frac{1}{2}\left(-\frac{\int \frac{-8aDx^4-8aCx^2+9Ab-8aB}{2x^8\left(bx^2+a\right)^{3/2}}dx^2}{4a}-\frac{A}{4ax^8\sqrt{a+bx^2}}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(-\frac{\int \frac{-8aDx^4-8aCx^2+9Ab-8aB}{x^8(bx^2+a)^{3/2}}dx^2}{8a}-\frac{A}{4ax^8\sqrt{a+bx^2}}\right)$$

$\Big\downarrow$ 1192

$$\frac{1}{2}\left(-\frac{b\int \frac{-8aDx^8-8a(bC-2aD)x^4+9Ab^3-8a\left(Da^2-bCa+b^2B\right)}{x^4(a-x^4)^4}d\sqrt{bx^2+a}}{4a}-\frac{A}{4ax^8\sqrt{a+bx^2}}\right)$$

$\Big\downarrow$ 1582

$$\frac{1}{2}\left(-\frac{b\left(\frac{b^2\sqrt{a+bx^2}(9Ab-8aB)}{6a^2(a-x^4)^3}-\frac{\int -\frac{\left(48Da^3-40b^2Ba+45Ab^3\right)x^4+6a\left(9Ab^3-8a\left(Da^2-bCa+b^2B\right)\right)}{x^4\left(a-x^4\right)^3}d\sqrt{bx^2+a}}{6a^2}\right)}{4a}-\frac{A}{4ax^8\sqrt{a+bx^2}}\right)$$

$\Big\downarrow$ 25

$$\frac{1}{2}\left(-\frac{b\left(\frac{\int \frac{\left(48Da^3-40b^2Ba+45Ab^3\right)x^4+6a\left(9Ab^3-8a\left(Da^2-bCa+b^2B\right)\right)}{x^4\left(a-x^4\right)^3}d\sqrt{bx^2+a}}{6a^2}+\frac{b^2\sqrt{a+bx^2}(9Ab-8aB)}{6a^2(a-x^4)^3}\right)}{4a}-\frac{A}{4ax^8\sqrt{a+bx^2}}\right)$$

$\Big\downarrow$ 361

$$\frac{1}{2}\left(-\frac{b\left(\frac{\frac{b\sqrt{a+bx^2}\left(48a^2C-88abB+99Ab^2\right)}{4a\left(a-x^4\right)^2}-\frac{1}{4}\int -\frac{3\left(b\left(\frac{99Ab^2}{a}-88Bb+48aC\right)x^4+8\left(9Ab^3-8a\left(Da^2-bCa+b^2B\right)\right)\right)}{x^4\left(a-x^4\right)^2}d\sqrt{bx^2+a}}{6a^2}+\frac{b^2\sqrt{a+bx^2}(9Ab-8a}{6a^2(a-x^4)^3}\right.}{4a}\right.$$

$\Big\downarrow$ 27

$$\frac{1}{2}\left(-\frac{b\left(\frac{\frac{3}{4}\int \frac{b\left(\frac{99Ab^2}{a}-88Bb+48aC\right)x^4+8\left(9Ab^3-8a\left(Da^2-bCa+b^2B\right)\right)}{x^4\left(a-x^4\right)^2}d\sqrt{bx^2+a}+\frac{b\sqrt{a+bx^2}\left(48a^2C-88abB+99Ab^2\right)}{4a\left(a-x^4\right)^2}}{6a^2}+\frac{b^2\sqrt{a+bx^2}(9Ab-8aB)}{6a^2(a-x^4)^3}\right)}{4a}\right.$$

$$\Big\downarrow \; 361$$

$$\frac{1}{2}\left(-\frac{b\left(\frac{\frac{3}{4}\left(\frac{\sqrt{a+bx^2}\left(171Ab^3-8a\left(8a^2D-14abC+19b^2B\right)\right)}{2a^2\left(a-x^4\right)}-\frac{1}{2}\int -\frac{a\left(\frac{171Ab^3}{a^2}-\frac{152Bb^2}{a}+112Cb-64aD\right)x^4+16\left(9Ab^3-8a\left(Da^2-bCa+b^2B\right)\right)}{ax^4\left(a-x^4\right)}d\sqrt{bx^2+a}\right)}{6a^2}\right.}{4a}\right.$$

$$\Big\downarrow \; 25$$

$$\frac{1}{2}\left(-\frac{b\left(\frac{\frac{3}{4}\left(\frac{1}{2}\int \frac{\left(171Ab^3-8a\left(8Da^2-14bCa+19b^2B\right)\right)x^4+16a\left(9Ab^3-8a\left(Da^2-bCa+b^2B\right)\right)}{a^2x^4\left(a-x^4\right)}d\sqrt{bx^2+a}+\frac{\sqrt{a+bx^2}\left(171Ab^3-8a\left(8a^2D-14abC+19b^2B\right)\right)}{2a^2\left(a-x^4\right)}\right)}{6a^2}\right.}{4a}\right.$$

$$\Big\downarrow \; 27$$

$$\frac{1}{2}\left(-\frac{b\left(\frac{\frac{3}{4}\left(\frac{\int \frac{\left(-64Da^3+112bCa^2-152b^2Ba+171Ab^3\right)x^4+16a\left(9Ab^3-8a\left(Da^2-bCa+b^2B\right)\right)}{x^4\left(a-x^4\right)}d\sqrt{bx^2+a}}{2a^2}+\frac{\sqrt{a+bx^2}\left(171Ab^3-8a\left(8a^2D-14abC+19b^2B\right)\right)}{2a^2\left(a-x^4\right)}\right.}{6a^2}\right.}{4a}\right.$$

$\downarrow$ 359

$$\frac{1}{2}\left(-\frac{b\left(\frac{\frac{3}{4}\left(\frac{\left(-192a^3D+240a^2bC-280ab^2B+315Ab^3\right)\int\frac{1}{a-x^4}d\sqrt{bx^2+a}-\frac{16\left(9Ab^3-8a\left(a^2D-abC+b^2B\right)\right)}{x^2}}{2a^2}+\frac{\sqrt{a+bx^2}\left(171Ab^3-8a\left(8a^2D-14abC+19b^2B\right.\right.}{2a^2\left(a-x^4\right)}\right.}{6a^2}\right.}{4a}\right.$$

$\downarrow$ 219

$$\frac{1}{2}\left(-\frac{b\left(\frac{b^2\sqrt{a+bx^2}(9Ab-8aB)}{6a^2(a-x^4)^3}+\frac{\frac{b\sqrt{a+bx^2}\left(48a^2C-88abB+99Ab^2\right)}{4a\left(a-x^4\right)^2}+\frac{3}{4}\left(\frac{\sqrt{a+bx^2}\left(171Ab^3-8a\left(8a^2D-14abC+19b^2B\right)\right)}{2a^2\left(a-x^4\right)}+\frac{\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\left(\right.}{}\right.}{6a^2}\right.}{4a}\right.$$

input
```
Int[(A + B*x^2 + C*x^4 + D*x^6)/(x^9*(a + b*x^2)^(3/2)),x]
```

output
```
(-1/4*A/(a*x^8*Sqrt[a + b*x^2]) - (b*((b^2*(9*A*b - 8*a*B)*Sqrt[a + b*x^2]
)/(6*a^2*(a - x^4)^3) + ((b*(99*A*b^2 - 88*a*b*B + 48*a^2*C)*Sqrt[a + b*x^
2])/(4*a*(a - x^4)^2) + (3*(((171*A*b^3 - 8*a*(19*b^2*B - 14*a*b*C + 8*a^2
*D))*Sqrt[a + b*x^2])/(2*a^2*(a - x^4)) + ((-16*(9*A*b^3 - 8*a*(b^2*B - a*
b*C + a^2*D)))/x^2 + ((315*A*b^3 - 280*a*b^2*B + 240*a^2*b*C - 192*a^3*D)*
ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a])/(2*a^2)))/4)/(6*a^2)))/(4*a))/2
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 361
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[x^m*(a + b*x^2)^(p + 1)*E
xpandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c
- a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/
2, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 1192
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(
2*m + 1)*(e*f - d*g + g*x^2)^n*(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 +
 c*x^4)^p, x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &&
IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m + 1/2]
```

rule 1582
```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Simp[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1))   Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e
*x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
 b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
&& ILtQ[m/2, 0]
```

rule 2124
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
 a*d))), x] + Simp[1/((m + 1)*(b*c - a*d))   Int[(a + b*x)^(m + 1)*(c + d*x
)^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fr
eeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && LtQ[m, -1] && (IntegerQ[m] ||  !
ILtQ[n, -1])
```

rule 2331
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/2   S
ubst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
 FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

## Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.76

| method | result |
|---|---|
| pseudoelliptic | $-\frac{315\left(-\frac{\left(-\frac{8x^{2}B}{3}+A\right)x^{6}b^{3}a^{\frac{3}{2}}}{3}+\frac{2x^{4}b^{2}\left(-\frac{40}{7}C\,x^{4}+\frac{20}{9}x^{2}B+A\right)a^{\frac{5}{2}}}{15}-\frac{8\left(-8Dx^{6}+\frac{10}{3}C\,x^{4}+\frac{14}{9}x^{2}B+A\right)x^{2}b\,a^{\frac{7}{2}}}{105}+\frac{16\left(4Dx^{6}+2C\,x^{4}+\right.}{315}\right.}{128a^{\frac{11}{2}}\sqrt{b\,x^{2}+a}\,x^{8}}$ |
| default | $A\left(-\frac{1}{8a\,x^{8}\sqrt{b\,x^{2}+a}}-\frac{9b\left(-\frac{1}{6a\,x^{6}\sqrt{b\,x^{2}+a}}-\frac{7b\left(-\frac{1}{4a\,x^{4}\sqrt{b\,x^{2}+a}}-\frac{5b\left(-\frac{1}{2a\,x^{2}\sqrt{b\,x^{2}+a}}-\frac{3b\left(\frac{1}{a\sqrt{b\,x^{2}+a}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x}}{x}\right.}{a^{\frac{3}{2}}}\right.}{2a}\right.}{4a}\right.}{6a}\right.}{8a}\right.$ |

input
```
int((D*x^6+C*x^4+B*x^2+A)/x^9/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-315/128/a^(11/2)/(b*x^2+a)^(1/2)*(-1/3*(-8/3*x^2*B+A)*x^6*b^3*a^(3/2)+2/1
5*x^4*b^2*(-40/7*C*x^4+20/9*x^2*B+A)*a^(5/2)-8/105*(-8*D*x^6+10/3*C*x^4+14
/9*x^2*B+A)*x^2*b*a^(7/2)+16/315*(4*D*x^6+2*C*x^4+4/3*x^2*B+A)*a^(9/2)+(-A
*b^3*a^(1/2)+(b*x^2+a)^(1/2)*(b^3*A-8/9*a*b^2*B+16/21*a^2*b*C-64/105*a^3*D
)*arctanh((b*x^2+a)^(1/2)/a^(1/2)))*x^8*b)/x^8
```

## Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 528, normalized size of antiderivative = 2.22

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^9\left(a+bx^2\right)^{3/2}}\,dx = \left[-\frac{3\left((192\,Da^3b^2-240\,Ca^2b^3+280\,Bab^4-315\,Ab^5)x^{10}+(192\,Da^4b-2\right.}{}\right.$$

$$-\frac{3\left((192\,Da^3b^2-240\,Ca^2b^3+280\,Bab^4-315\,Ab^5)x^{10}+(192\,Da^4b-240\,Ca^3b^2+280\,Ba^2b^3-315\,Aab^4\right.}{}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^9/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[-1/768*(3*((192*D*a^3*b^2 - 240*C*a^2*b^3 + 280*B*a*b^4 - 315*A*b^5)*x^10
 + (192*D*a^4*b - 240*C*a^3*b^2 + 280*B*a^2*b^3 - 315*A*a*b^4)*x^8)*sqrt(a
)*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(3*(192*D*a^4*b
- 240*C*a^3*b^2 + 280*B*a^2*b^3 - 315*A*a*b^4)*x^8 + (192*D*a^5 - 240*C*a^
4*b + 280*B*a^3*b^2 - 315*A*a^2*b^3)*x^6 + 48*A*a^5 + 2*(48*C*a^5 - 56*B*a
^4*b + 63*A*a^3*b^2)*x^4 + 8*(8*B*a^5 - 9*A*a^4*b)*x^2)*sqrt(b*x^2 + a))/(
a^6*b*x^10 + a^7*x^8), -1/384*(3*((192*D*a^3*b^2 - 240*C*a^2*b^3 + 280*B*a
*b^4 - 315*A*b^5)*x^10 + (192*D*a^4*b - 240*C*a^3*b^2 + 280*B*a^2*b^3 - 31
5*A*a*b^4)*x^8)*sqrt(-a)*arctan(sqrt(b*x^2 + a)*sqrt(-a)/a) + (3*(192*D*a^
4*b - 240*C*a^3*b^2 + 280*B*a^2*b^3 - 315*A*a*b^4)*x^8 + (192*D*a^5 - 240*
C*a^4*b + 280*B*a^3*b^2 - 315*A*a^2*b^3)*x^6 + 48*A*a^5 + 2*(48*C*a^5 - 56
*B*a^4*b + 63*A*a^3*b^2)*x^4 + 8*(8*B*a^5 - 9*A*a^4*b)*x^2)*sqrt(b*x^2 + a
))/(a^6*b*x^10 + a^7*x^8)]
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{A + Bx^2 + Cx^4 + Dx^6}{x^9 (a + bx^2)^{3/2}} \, dx = \text{Timed out}$$

input
```
integrate((D*x**6+C*x**4+B*x**2+A)/x**9/(b*x**2+a)**(3/2),x)
```

output
```
Timed out
```

## Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.45

$$\begin{aligned}\int \frac{A + Bx^2 + Cx^4 + Dx^6}{x^9 (a + bx^2)^{3/2}} \, dx &= \frac{3\,Db \operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{2\,a^{\frac{5}{2}}} - \frac{15\,Cb^2 \operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{8\,a^{\frac{7}{2}}} \\ &+ \frac{35\,Bb^3 \operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{16\,a^{\frac{9}{2}}} - \frac{315\,Ab^4 \operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{128\,a^{\frac{11}{2}}} - \frac{3\,Db}{2\,\sqrt{bx^2 + a}a^2} \\ &+ \frac{15\,Cb^2}{8\,\sqrt{bx^2 + a}a^3} - \frac{35\,Bb^3}{16\,\sqrt{bx^2 + a}a^4} + \frac{315\,Ab^4}{128\,\sqrt{bx^2 + a}a^5} - \frac{D}{2\,\sqrt{bx^2 + a}ax^2} \\ &+ \frac{5\,Cb}{8\,\sqrt{bx^2 + a}a^2x^2} - \frac{35\,Bb^2}{48\,\sqrt{bx^2 + a}a^3x^2} + \frac{105\,Ab^3}{128\,\sqrt{bx^2 + a}a^4x^2} \\ &- \frac{C}{4\,\sqrt{bx^2 + a}ax^4} + \frac{7\,Bb}{24\,\sqrt{bx^2 + a}a^2x^4} - \frac{21\,Ab^2}{64\,\sqrt{bx^2 + a}a^3x^4} \\ &- \frac{B}{6\,\sqrt{bx^2 + a}ax^6} + \frac{3\,Ab}{16\,\sqrt{bx^2 + a}a^2x^6} - \frac{A}{8\,\sqrt{bx^2 + a}ax^8}\end{aligned}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^9/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
3/2*D*b*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(5/2) - 15/8*C*b^2*arcsinh(a/(sqrt
(a*b)*abs(x)))/a^(7/2) + 35/16*B*b^3*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(9/2)
 - 315/128*A*b^4*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(11/2) - 3/2*D*b/(sqrt(b*
x^2 + a)*a^2) + 15/8*C*b^2/(sqrt(b*x^2 + a)*a^3) - 35/16*B*b^3/(sqrt(b*x^2
 + a)*a^4) + 315/128*A*b^4/(sqrt(b*x^2 + a)*a^5) - 1/2*D/(sqrt(b*x^2 + a)*
a*x^2) + 5/8*C*b/(sqrt(b*x^2 + a)*a^2*x^2) - 35/48*B*b^2/(sqrt(b*x^2 + a)*
a^3*x^2) + 105/128*A*b^3/(sqrt(b*x^2 + a)*a^4*x^2) - 1/4*C/(sqrt(b*x^2 + a
)*a*x^4) + 7/24*B*b/(sqrt(b*x^2 + a)*a^2*x^4) - 21/64*A*b^2/(sqrt(b*x^2 +
a)*a^3*x^4) - 1/6*B/(sqrt(b*x^2 + a)*a*x^6) + 3/16*A*b/(sqrt(b*x^2 + a)*a^
2*x^6) - 1/8*A/(sqrt(b*x^2 + a)*a*x^8)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.61

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^9\,(a+bx^2)^{3/2}}\,dx =$$

$$-\frac{(192\,Da^3b-240\,Ca^2b^2+280\,Bab^3-315\,Ab^4)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{128\,\sqrt{-a}a^5}$$

$$-\frac{Da^3b-Ca^2b^2+Bab^3-Ab^4}{\sqrt{bx^2+a}a^5}$$

$$-\frac{192\,(bx^2+a)^{\frac{7}{2}}Da^3b-576\,(bx^2+a)^{\frac{5}{2}}Da^4b+576\,(bx^2+a)^{\frac{3}{2}}Da^5b-192\,\sqrt{bx^2+a}Da^6b-336\,(bx^2+a)^{\frac{7}{2}}}{}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^9/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
-1/128*(192*D*a^3*b - 240*C*a^2*b^2 + 280*B*a*b^3 - 315*A*b^4)*arctan(sqrt
(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^5) - (D*a^3*b - C*a^2*b^2 + B*a*b^3 - A*
b^4)/(sqrt(b*x^2 + a)*a^5) - 1/384*(192*(b*x^2 + a)^(7/2)*D*a^3*b - 576*(b
*x^2 + a)^(5/2)*D*a^4*b + 576*(b*x^2 + a)^(3/2)*D*a^5*b - 192*sqrt(b*x^2 +
 a)*D*a^6*b - 336*(b*x^2 + a)^(7/2)*C*a^2*b^2 + 1104*(b*x^2 + a)^(5/2)*C*a
^3*b^2 - 1200*(b*x^2 + a)^(3/2)*C*a^4*b^2 + 432*sqrt(b*x^2 + a)*C*a^5*b^2
+ 456*(b*x^2 + a)^(7/2)*B*a*b^3 - 1544*(b*x^2 + a)^(5/2)*B*a^2*b^3 + 1784*
(b*x^2 + a)^(3/2)*B*a^3*b^3 - 696*sqrt(b*x^2 + a)*B*a^4*b^3 - 561*(b*x^2 +
 a)^(7/2)*A*b^4 + 1929*(b*x^2 + a)^(5/2)*A*a*b^4 - 2295*(b*x^2 + a)^(3/2)*
A*a^2*b^4 + 975*sqrt(b*x^2 + a)*A*a^3*b^4)/(a^5*b^4*x^8)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^9\left(a+bx^2\right)^{3/2}}\,dx=\int \frac{A+B\,x^2+C\,x^4+x^6\,D}{x^9\,{\left(b\,x^2+a\right)}^{3/2}}\,dx$$

input
```
int((A + B*x^2 + C*x^4 + x^6*D)/(x^9*(a + b*x^2)^(3/2)),x)
```

output
```
int((A + B*x^2 + C*x^4 + x^6*D)/(x^9*(a + b*x^2)^(3/2)), x)
```

## Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 603, normalized size of antiderivative = 2.53

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^9\left(a+bx^2\right)^{3/2}}\,dx=\frac{-48\sqrt{b\,x^2+a}\,a^5+8\sqrt{b\,x^2+a}\,a^4b\,x^2-96\sqrt{b\,x^2+a}\,a^4c\,x^4-192\sqrt{b\,x^2+}}{}$$

input
```
int((D*x^6+C*x^4+B*x^2+A)/x^9/(b*x^2+a)^(3/2),x)
```

output
```
( - 48*sqrt(a + b*x**2)*a**5 + 8*sqrt(a + b*x**2)*a**4*b*x**2 - 96*sqrt(a
+ b*x**2)*a**4*c*x**4 - 192*sqrt(a + b*x**2)*a**4*d*x**6 - 14*sqrt(a + b*x
**2)*a**3*b**2*x**4 + 240*sqrt(a + b*x**2)*a**3*b*c*x**6 - 576*sqrt(a + b*
x**2)*a**3*b*d*x**8 + 35*sqrt(a + b*x**2)*a**2*b**3*x**6 + 720*sqrt(a + b*
x**2)*a**2*b**2*c*x**8 + 105*sqrt(a + b*x**2)*a*b**4*x**8 - 576*sqrt(a)*lo
g((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*a**3*b*d*x**8 + 720*sq
rt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*a**2*b**2*c*x*
*8 - 576*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*a**
2*b**2*d*x**10 + 105*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/
sqrt(a))*a*b**4*x**8 + 720*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(
b)*x)/sqrt(a))*a*b**3*c*x**10 + 105*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a
) + sqrt(b)*x)/sqrt(a))*b**5*x**10 + 576*sqrt(a)*log((sqrt(a + b*x**2) + s
qrt(a) + sqrt(b)*x)/sqrt(a))*a**3*b*d*x**8 - 720*sqrt(a)*log((sqrt(a + b*x
**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*a**2*b**2*c*x**8 + 576*sqrt(a)*log((s
qrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*a**2*b**2*d*x**10 - 105*sq
rt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b**4*x**8 -
720*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b**3*c
*x**10 - 105*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))
*b**5*x**10)/(384*a**5*x**8*(a + b*x**2))
```

# 3.232 $\int \frac{x^4(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{3/2}}\,dx$



## Optimal result

Integrand size = 32, antiderivative size = 238

$$\int \frac{x^4(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{3/2}}\,dx = \frac{a(Ab^3-a(b^2B-abC+a^2D))\,x}{b^5\sqrt{a+bx^2}} + \frac{(64Ab^3-a(112b^2B-152abC+187a^2D))\,x\sqrt{a+bx^2}}{128b^5} + \frac{(48b^2B-88abC+123a^2D)\,x^3\sqrt{a+bx^2}}{192b^4} + \frac{(8bC-15aD)x^5\sqrt{a+bx^2}}{48b^3} + \frac{Dx^7\sqrt{a+bx^2}}{8b^2} - \frac{a(192Ab^3-5a(48b^2B-56abC+63a^2D))\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{11/2}}$$

output

```
a*(A*b^3-a*(B*b^2-C*a*b+D*a^2))*x/b^5/(b*x^2+a)^(1/2)+1/128*(64*A*b^3-a*(1
12*B*b^2-152*C*a*b+187*D*a^2))*x*(b*x^2+a)^(1/2)/b^5+1/192*(48*B*b^2-88*C*
a*b+123*D*a^2)*x^3*(b*x^2+a)^(1/2)/b^4+1/48*(8*C*b-15*D*a)*x^5*(b*x^2+a)^(
1/2)/b^3+1/8*D*x^7*(b*x^2+a)^(1/2)/b^2-1/128*a*(192*A*b^3-5*a*(48*B*b^2-56
*C*a*b+63*D*a^2))*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(11/2)
```

## Mathematica [A] (verified)

Time = 1.47 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.81

$$\int \frac{x^4(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{3/2}}\,dx = \frac{\frac{\sqrt{b}x\left(-945a^4D+105a^3b\left(8C-3Dx^2\right)+2a^2b^2\left(-360B+140Cx^2+63Dx^4\right)+8ab^3\left(72A-30Bx^2-14\right.\right.}{\sqrt{a+bx^2}}}{}$$

input
```
Integrate[(x^4*(A + B*x^2 + C*x^4 + D*x^6))/(a + b*x^2)^(3/2),x]
```

output
```
((Sqrt[b]*x*(-945*a^4*D + 105*a^3*b*(8*C - 3*D*x^2) + 2*a^2*b^2*(-360*B +
140*C*x^2 + 63*D*x^4) + 8*a*b^3*(72*A - 30*B*x^2 - 14*C*x^4 - 9*D*x^6) + 1
6*b^4*x^2*(12*A + 6*B*x^2 + 4*C*x^4 + 3*D*x^6)))/Sqrt[a + b*x^2] + 6*a*(-1
92*A*b^3 + 5*a*(48*b^2*B - 56*a*b*C + 63*a^2*D))*ArcTanh[(Sqrt[b]*x)/(-Sqr
t[a] + Sqrt[a + b*x^2])])/(384*b^(11/2))
```

## Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.95, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2335, 9, 1590, 363, 262, 262, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{3/2}}\,dx$$

$$\Big\downarrow \; 2335$$

$$\frac{x^5\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{a\sqrt{a+bx^2}}-\frac{\int \frac{x^3\left(-aDx^5-a\left(C-\frac{aD}{b}\right)x^3+\left(4Ab-\frac{5a\left(Da^2-bCa+b^2B\right)}{b^2}\right)x\right)}{\sqrt{bx^2+a}}dx}{ab}$$

$$\Big\downarrow \; 9$$

$$\frac{x^5\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{a\sqrt{a+bx^2}}-\frac{\int \frac{x^4\left(-aDx^4-a\left(C-\frac{aD}{b}\right)x^2+4Ab-\frac{5a\left(Da^2-bCa+b^2B\right)}{b^2}\right)}{\sqrt{bx^2+a}}dx}{ab}$$

$$\Big\downarrow \text{1590}$$

$$\frac{x^5\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{a\sqrt{a+bx^2}}-\frac{\frac{\int \frac{x^4\left(8\left(4Ab^2-5a\left(\frac{Da^2}{b}-Ca+bB\right)\right)-a(8bC-15aD)x^2\right)}{\sqrt{bx^2+a}}dx}{8b}-\frac{aDx^7\sqrt{a+bx^2}}{8b}}{ab}$$

$$\Big\downarrow \text{363}$$

$$\frac{x^5\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{a\sqrt{a+bx^2}}-\frac{\frac{\frac{\left(-315a^3D+280a^2bC-240ab^2B+192Ab^3\right)\int \frac{x^4}{\sqrt{bx^2+a}}dx}{6b}-\frac{ax^5\sqrt{a+bx^2}(8bC-15aD)}{6b}}{8b}-\frac{aDx^7\sqrt{a+bx^2}}{8b}}{ab}$$

$$\Big\downarrow \text{262}$$

$$\frac{x^5\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{a\sqrt{a+bx^2}}-\frac{\frac{\frac{\left(-315a^3D+280a^2bC-240ab^2B+192Ab^3\right)\left(\frac{x^3\sqrt{a+bx^2}}{4b}-\frac{3a\int \frac{x^2}{\sqrt{bx^2+a}}dx}{4b}\right)}{6b}-\frac{ax^5\sqrt{a+bx^2}(8bC-15aD)}{6b}}{8b}-\frac{aDx^7\sqrt{a+bx^2}}{8b}}{ab}$$

$$\Big\downarrow \text{262}$$

$$\frac{x^5\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{a\sqrt{a+bx^2}}-\frac{\frac{\frac{\left(-315a^3D+280a^2bC-240ab^2B+192Ab^3\right)\left(\frac{x^3\sqrt{a+bx^2}}{4b}-\frac{3a\left(\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\int \frac{1}{\sqrt{bx^2+a}}dx}{2b}\right)}{4b}\right)}{6b}-\frac{ax^5\sqrt{a+bx^2}(8bC-15aD)}{6b}}{8b}-\frac{aDx^7\sqrt{a+bx^2}}{8b}}{ab}$$

$$\Big\downarrow \text{224}$$

$$\frac{x^5\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{a\sqrt{a+bx^2}}-\frac{\frac{\frac{\left(-315a^3D+280a^2bC-240ab^2B+192Ab^3\right)\left(\frac{x^3\sqrt{a+bx^2}}{4b}-\frac{3a\left(\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2b}\right)}{4b}\right)}{6b}-\frac{ax^5\sqrt{a+bx^2}(8bC-15aD)}{6b}}{8b}-\frac{aDx^7\sqrt{a+bx^2}}{8b}}{ab}$$

$$\Big\downarrow \text{219}$$

$$\frac{x^5\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{a\sqrt{a+bx^2}}-\frac{\frac{\frac{\left(\frac{x^3\sqrt{a+bx^2}}{4b}-\frac{3a\left(\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}\right)}{4b}\right)(-315a^3D+280a^2bC-240ab^2B+192Ab^3)}{6b}-\frac{ax^5\sqrt{a+bx^2}(8bC-15aD)}{6b}}{8b}-\frac{aDx^7\sqrt{a+bx^2}}{8b}}{ab}$$

input
```
Int[(x^4*(A + B*x^2 + C*x^4 + D*x^6))/(a + b*x^2)^(3/2),x]
```

output
```
((A - (a*(b^2*B - a*b*C + a^2*D))/b^3)*x^5)/(a*Sqrt[a + b*x^2]) - (-1/8*(a
*D*x^7*Sqrt[a + b*x^2])/b + (-1/6*(a*(8*b*C - 15*a*D)*x^5*Sqrt[a + b*x^2])
/b + ((192*A*b^3 - 240*a*b^2*B + 280*a^2*b*C - 315*a^3*D)*((x^3*Sqrt[a + b
*x^2])/(4*b) - (3*a*((x*Sqrt[a + b*x^2])/(2*b) - (a*ArcTanh[(Sqrt[b]*x)/Sq
rt[a + b*x^2]])/(2*b^(3/2))))/(4*b)))/(6*b))/(8*b))/(a*b)
```

## Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 262

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 363

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 1590

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[c^p*(f*x)^(m + 4*p - 1)*((d + e*x^2)^
(q + 1)/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1))), x] + Simp[1/(e*(m + 4*p + 2*q
 + 1))   Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a +
b*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x],
x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p,
0] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

rule 2335

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Simp[c/(2*a*b*(p + 1))   Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSu
m[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
 b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

## Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.74

| method | result |
|---|---|
| pseudoelliptic | $\frac{-\frac{3\left(b^{3} A-\frac{5}{4} a\, b^{2} B+\frac{35}{24} a^{2} b C-\frac{105}{64} a^{3} D\right) \sqrt{b\, x^{2}+a}\, a \,\operatorname{arctanh}\left(\frac{\sqrt{b\, x^{2}+a}}{x \sqrt{b}}\right)}{2}+\frac{3 x \left(a \left(-\frac{1}{8} D x^{6}-\frac{7}{36} C\, x^{4}-\frac{5}{12} x^{2} B+A\right) b^{\frac{7}{2}}+\frac{x^{2} \left(\frac{1}{4} D x^{6}+\frac{1}{3} C\, x\right.}{}\right.}{}}{b^{\frac{11}{2}} \sqrt{b\, x^{2}+a}}$ |
| default | $A \left(\frac{x^{3}}{2 b \sqrt{b\, x^{2}+a}}-\frac{3 a \left(-\frac{x}{b \sqrt{b\, x^{2}+a}}+\frac{\ln\left(\sqrt{b}\, x+\sqrt{b\, x^{2}+a}\right)}{b^{\frac{3}{2}}}\right)}{2 b}\right)+B \left(\frac{x^{5}}{4 b \sqrt{b\, x^{2}+a}}-\frac{5 a \left(\frac{x^{3}}{2 b \sqrt{b\, x^{2}+a}}-\frac{3 a \left(-\frac{x}{b \sqrt{b\, x^{2}+a}}\right.}{}\right)}{4 b}\right.$ |

input
```
int(x^4*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
3/2/b^(11/2)/(b*x^2+a)^(1/2)*(-(b^3*A-5/4*a*b^2*B+35/24*a^2*b*C-105/64*a^3
*D)*(b*x^2+a)^(1/2)*a*arctanh((b*x^2+a)^(1/2)/x/b^(1/2))+x*(a*(-1/8*D*x^6-
7/36*C*x^4-5/12*x^2*B+A)*b^(7/2)+1/3*x^2*(1/4*D*x^6+1/3*C*x^4+1/2*x^2*B+A)
*b^(9/2)+35/24*((3/20*D*x^4+1/3*C*x^2-6/7*B)*b^(5/2)+((-3/8*D*x^2+C)*b^(3/
2)-9/8*D*a*b^(1/2))*a)*a^2))
```

## Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 512, normalized size of antiderivative = 2.15

$$\int \frac{x^4(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{3/2}}\,dx = \left[-\frac{3\,(315\,Da^5-280\,Ca^4b+240\,Ba^3b^2-192\,Aa^2b^3+(315\,Da^4b-28\cdots}{\cdots}\right.$$

$$-\frac{3\,(315\,Da^5-280\,Ca^4b+240\,Ba^3b^2-192\,Aa^2b^3+(315\,Da^4b-280\,Ca^3b^2+240\,Ba^2b^3-192\,Aab^4)x^2)\sqrt{\cdots}}{\cdots}$$

input
```
integrate(x^4*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[-1/768*(3*(315*D*a^5 - 280*C*a^4*b + 240*B*a^3*b^2 - 192*A*a^2*b^3 + (315
*D*a^4*b - 280*C*a^3*b^2 + 240*B*a^2*b^3 - 192*A*a*b^4)*x^2)*sqrt(b)*log(-
2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(48*D*b^5*x^9 - 8*(9*D*a*b^
4 - 8*C*b^5)*x^7 + 2*(63*D*a^2*b^3 - 56*C*a*b^4 + 48*B*b^5)*x^5 - (315*D*a
^3*b^2 - 280*C*a^2*b^3 + 240*B*a*b^4 - 192*A*b^5)*x^3 - 3*(315*D*a^4*b - 2
80*C*a^3*b^2 + 240*B*a^2*b^3 - 192*A*a*b^4)*x)*sqrt(b*x^2 + a))/(b^7*x^2 +
 a*b^6), -1/384*(3*(315*D*a^5 - 280*C*a^4*b + 240*B*a^3*b^2 - 192*A*a^2*b^
3 + (315*D*a^4*b - 280*C*a^3*b^2 + 240*B*a^2*b^3 - 192*A*a*b^4)*x^2)*sqrt(
-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (48*D*b^5*x^9 - 8*(9*D*a*b^4 - 8*
C*b^5)*x^7 + 2*(63*D*a^2*b^3 - 56*C*a*b^4 + 48*B*b^5)*x^5 - (315*D*a^3*b^2
 - 280*C*a^2*b^3 + 240*B*a*b^4 - 192*A*b^5)*x^3 - 3*(315*D*a^4*b - 280*C*a
^3*b^2 + 240*B*a^2*b^3 - 192*A*a*b^4)*x)*sqrt(b*x^2 + a))/(b^7*x^2 + a*b^6
)]
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 466 vs. 2(231) = 462.

Time = 98.01 (sec) , antiderivative size = 466, normalized size of antiderivative = 1.96

$$\int \frac{x^4(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{3/2}}\,dx = A\left(\frac{3\sqrt{a}x}{2b^2\sqrt{1+\frac{bx^2}{a}}} - \frac{3a\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{\frac{5}{2}}} + \frac{x^3}{2\sqrt{a}b\sqrt{1+\frac{bx^2}{a}}}\right) + B\left(-\frac{15a^{\frac{3}{2}}x}{8b^3\sqrt{1+\frac{bx^2}{a}}} - \frac{5\sqrt{a}x^3}{8b^2\sqrt{1+\frac{bx^2}{a}}} + \frac{15a^2\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{\frac{7}{2}}} + \frac{x^5}{4\sqrt{a}b\sqrt{1+\frac{bx^2}{a}}}\right) + C\left(\frac{35a^{\frac{5}{2}}x}{16b^4\sqrt{1+\frac{bx^2}{a}}} + \frac{35a^{\frac{3}{2}}x^3}{48b^3\sqrt{1+\frac{bx^2}{a}}} - \frac{7\sqrt{a}x^5}{24b^2\sqrt{1+\frac{bx^2}{a}}} - \frac{35a^3\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16b^{\frac{9}{2}}} + \frac{x^7}{6\sqrt{a}b\sqrt{1+\frac{bx^2}{a}}}\right) + D\left(-\frac{315a^{\frac{7}{2}}x}{128b^5\sqrt{1+\frac{bx^2}{a}}} - \frac{105a^{\frac{5}{2}}x^3}{128b^4\sqrt{1+\frac{bx^2}{a}}} + \frac{21a^{\frac{3}{2}}x^5}{64b^3\sqrt{1+\frac{bx^2}{a}}} - \frac{3\sqrt{a}x^7}{16b^2\sqrt{1+\frac{bx^2}{a}}} + \frac{315a^4\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{128b^{\frac{11}{2}}} + \frac{x^9}{8\sqrt{a}b\sqrt{1+\frac{bx^2}{a}}}\right)$$

input

```
integrate(x**4*(D*x**6+C*x**4+B*x**2+A)/(b*x**2+a)**(3/2),x)
```

output
```
A*(3*sqrt(a)*x/(2*b**2*sqrt(1 + b*x**2/a)) - 3*a*asinh(sqrt(b)*x/sqrt(a))/
(2*b**(5/2)) + x**3/(2*sqrt(a)*b*sqrt(1 + b*x**2/a))) + B*(-15*a**(3/2)*x/
(8*b**3*sqrt(1 + b*x**2/a)) - 5*sqrt(a)*x**3/(8*b**2*sqrt(1 + b*x**2/a)) +
 15*a**2*asinh(sqrt(b)*x/sqrt(a))/(8*b**(7/2)) + x**5/(4*sqrt(a)*b*sqrt(1
+ b*x**2/a))) + C*(35*a**(5/2)*x/(16*b**4*sqrt(1 + b*x**2/a)) + 35*a**(3/2
)*x**3/(48*b**3*sqrt(1 + b*x**2/a)) - 7*sqrt(a)*x**5/(24*b**2*sqrt(1 + b*x
**2/a)) - 35*a**3*asinh(sqrt(b)*x/sqrt(a))/(16*b**(9/2)) + x**7/(6*sqrt(a)
*b*sqrt(1 + b*x**2/a))) + D*(-315*a**(7/2)*x/(128*b**5*sqrt(1 + b*x**2/a))
 - 105*a**(5/2)*x**3/(128*b**4*sqrt(1 + b*x**2/a)) + 21*a**(3/2)*x**5/(64*
b**3*sqrt(1 + b*x**2/a)) - 3*sqrt(a)*x**7/(16*b**2*sqrt(1 + b*x**2/a)) + 3
15*a**4*asinh(sqrt(b)*x/sqrt(a))/(128*b**(11/2)) + x**9/(8*sqrt(a)*b*sqrt(
1 + b*x**2/a)))
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.42

$$\int \frac{x^4(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{3/2}}\,dx = \frac{Dx^9}{8\sqrt{bx^2+a}b} - \frac{3\,Dax^7}{16\sqrt{bx^2+a}b^2} + \frac{Cx^7}{6\sqrt{bx^2+a}b} + \frac{21\,Da^2x^5}{64\sqrt{bx^2+a}b^3} - \frac{7\,Cax^5}{24\sqrt{bx^2+a}b^2} + \frac{Bx^5}{4\sqrt{bx^2+a}b} - \frac{105\,Da^3x^3}{128\sqrt{bx^2+a}b^4} + \frac{35\,Ca^2x^3}{48\sqrt{bx^2+a}b^3} - \frac{5\,Bax^3}{8\sqrt{bx^2+a}b^2} + \frac{Ax^3}{2\sqrt{bx^2+a}b} - \frac{315\,Da^4x}{128\sqrt{bx^2+a}b^5} + \frac{35\,Ca^3x}{16\sqrt{bx^2+a}b^4} - \frac{15\,Ba^2x}{8\sqrt{bx^2+a}b^3} + \frac{3\,Aax}{2\sqrt{bx^2+a}b^2} + \frac{315\,Da^4\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\,b^{\frac{11}{2}}} - \frac{35\,Ca^3\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{9}{2}}} + \frac{15\,Ba^2\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{7}{2}}} - \frac{3\,Aa\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{5}{2}}}$$

input
```
integrate(x^4*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
1/8*D*x^9/(sqrt(b*x^2 + a)*b) - 3/16*D*a*x^7/(sqrt(b*x^2 + a)*b^2) + 1/6*C
*x^7/(sqrt(b*x^2 + a)*b) + 21/64*D*a^2*x^5/(sqrt(b*x^2 + a)*b^3) - 7/24*C*
a*x^5/(sqrt(b*x^2 + a)*b^2) + 1/4*B*x^5/(sqrt(b*x^2 + a)*b) - 105/128*D*a^
3*x^3/(sqrt(b*x^2 + a)*b^4) + 35/48*C*a^2*x^3/(sqrt(b*x^2 + a)*b^3) - 5/8*
B*a*x^3/(sqrt(b*x^2 + a)*b^2) + 1/2*A*x^3/(sqrt(b*x^2 + a)*b) - 315/128*D*
a^4*x/(sqrt(b*x^2 + a)*b^5) + 35/16*C*a^3*x/(sqrt(b*x^2 + a)*b^4) - 15/8*B
*a^2*x/(sqrt(b*x^2 + a)*b^3) + 3/2*A*a*x/(sqrt(b*x^2 + a)*b^2) + 315/128*D
*a^4*arcsinh(b*x/sqrt(a*b))/b^(11/2) - 35/16*C*a^3*arcsinh(b*x/sqrt(a*b))/
b^(9/2) + 15/8*B*a^2*arcsinh(b*x/sqrt(a*b))/b^(7/2) - 3/2*A*a*arcsinh(b*x/
sqrt(a*b))/b^(5/2)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.91

$$\int \frac{x^4(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{3/2}}\,dx = \frac{\left(\left(2\left(4\left(\frac{6\,Dx^2}{b}-\frac{9\,Dab^7-8\,Cb^8}{b^9}\right)x^2+\frac{63\,Da^2b^6-56\,Cab^7+48\,Bb^8}{b^9}\right)x^2-\frac{315\,Da^3b^5}{}\right.\right.}{384\,\sqrt{b}} - \frac{(315\,Da^4-280\,Ca^3b+240\,Ba^2b^2-192\,Aab^3)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{128\,b^{\frac{11}{2}}}$$

input
```
integrate(x^4*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
1/384*((2*(4*(6*D*x^2/b - (9*D*a*b^7 - 8*C*b^8)/b^9)*x^2 + (63*D*a^2*b^6 -
 56*C*a*b^7 + 48*B*b^8)/b^9)*x^2 - (315*D*a^3*b^5 - 280*C*a^2*b^6 + 240*B*
a*b^7 - 192*A*b^8)/b^9)*x^2 - 3*(315*D*a^4*b^4 - 280*C*a^3*b^5 + 240*B*a^2
*b^6 - 192*A*a*b^7)/b^9)*x/sqrt(b*x^2 + a) - 1/128*(315*D*a^4 - 280*C*a^3*
b + 240*B*a^2*b^2 - 192*A*a*b^3)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^
(11/2)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{3/2}}\,dx=\int \frac{x^4\,\left(A+B\,x^2+C\,x^4+x^6\,D\right)}{{\left(b\,x^2+a\right)}^{3/2}}\,dx$$

input
```
int((x^4*(A + B*x^2 + C*x^4 + x^6*D))/(a + b*x^2)^(3/2),x)
```

output
```
int((x^4*(A + B*x^2 + C*x^4 + x^6*D))/(a + b*x^2)^(3/2), x)
```

## Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 475, normalized size of antiderivative = 2.00

$$\int \frac{x^4(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{3/2}}\,dx=\frac{-945\sqrt{b\,x^2+a}\,a^4bdx+840\sqrt{b\,x^2+a}\,a^3b^2cx-315\sqrt{b\,x^2+a}\,a^3b^2d\,x^3}{}$$

input
```
int(x^4*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(3/2),x)
```

output
```
( - 945*sqrt(a + b*x**2)*a**4*b*d*x + 840*sqrt(a + b*x**2)*a**3*b**2*c*x -
 315*sqrt(a + b*x**2)*a**3*b**2*d*x**3 - 144*sqrt(a + b*x**2)*a**2*b**4*x
+ 280*sqrt(a + b*x**2)*a**2*b**3*c*x**3 + 126*sqrt(a + b*x**2)*a**2*b**3*d
*x**5 - 48*sqrt(a + b*x**2)*a*b**5*x**3 - 112*sqrt(a + b*x**2)*a*b**4*c*x*
*5 - 72*sqrt(a + b*x**2)*a*b**4*d*x**7 + 96*sqrt(a + b*x**2)*b**6*x**5 + 6
4*sqrt(a + b*x**2)*b**5*c*x**7 + 48*sqrt(a + b*x**2)*b**5*d*x**9 + 945*sqr
t(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**5*d - 840*sqrt(b)*log(
(sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**4*b*c + 945*sqrt(b)*log((sqrt(a
 + b*x**2) + sqrt(b)*x)/sqrt(a))*a**4*b*d*x**2 + 144*sqrt(b)*log((sqrt(a +
 b*x**2) + sqrt(b)*x)/sqrt(a))*a**3*b**3 - 840*sqrt(b)*log((sqrt(a + b*x**
2) + sqrt(b)*x)/sqrt(a))*a**3*b**2*c*x**2 + 144*sqrt(b)*log((sqrt(a + b*x*
*2) + sqrt(b)*x)/sqrt(a))*a**2*b**4*x**2 - 567*sqrt(b)*a**5*d + 525*sqrt(b
)*a**4*b*c - 567*sqrt(b)*a**4*b*d*x**2 - 48*sqrt(b)*a**3*b**3 + 525*sqrt(b
)*a**3*b**2*c*x**2 - 48*sqrt(b)*a**2*b**4*x**2)/(384*b**6*(a + b*x**2))
```

# 3.233 $\int \frac{x^2(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{3/2}}\,dx$



## Optimal result

Integrand size = 32, antiderivative size = 188

$$\int \frac{x^2(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{3/2}}\,dx = -\frac{(Ab^3-a(b^2B-abC+a^2D))\,x}{b^4\sqrt{a+bx^2}} + \frac{(8b^2B-14abC+19a^2D)\,x\sqrt{a+bx^2}}{16b^4} + \frac{(6bC-11aD)x^3\sqrt{a+bx^2}}{24b^3} + \frac{Dx^5\sqrt{a+bx^2}}{6b^2} + \frac{(16Ab^3-a(24b^2B-30abC+35a^2D))\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{9/2}}$$

output
```
-(A*b^3-a*(B*b^2-C*a*b+D*a^2))*x/b^4/(b*x^2+a)^(1/2)+1/16*(8*B*b^2-14*C*a*
b+19*D*a^2)*x*(b*x^2+a)^(1/2)/b^4+1/24*(6*C*b-11*D*a)*x^3*(b*x^2+a)^(1/2)/
b^3+1/6*D*x^5*(b*x^2+a)^(1/2)/b^2+1/16*(16*A*b^3-a*(24*B*b^2-30*C*a*b+35*D
*a^2))*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(9/2)
```

## Mathematica [A] (verified)

Time = 1.00 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.85

$$\int \frac{x^2(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{3/2}}\,dx = \frac{x(-48Ab^3+105a^3D+a^2b(-90C+35Dx^2)+2ab^2(36B-15Cx^2-}{48b^4\sqrt{a+bx^2}}$$
$$+\frac{(16Ab^3+a(-24b^2B+30abC-35a^2D))\,\text{arctanh}\left(\frac{\sqrt{b}x}{-\sqrt{a}+\sqrt{a+bx^2}}\right)}{8b^{9/2}}$$

input
```
Integrate[(x^2*(A + B*x^2 + C*x^4 + D*x^6))/(a + b*x^2)^(3/2),x]
```

output
```
(x*(-48*A*b^3 + 105*a^3*D + a^2*b*(-90*C + 35*D*x^2) + 2*a*b^2*(36*B - 15*
C*x^2 - 7*D*x^4) + 4*b^3*x^2*(6*B + 3*C*x^2 + 2*D*x^4)))/(48*b^4*Sqrt[a +
b*x^2]) + ((16*A*b^3 + a*(-24*b^2*B + 30*a*b*C - 35*a^2*D))*ArcTanh[(Sqrt[
b]*x)/(-Sqrt[a] + Sqrt[a + b*x^2])])/(8*b^(9/2))
```

## Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {2335, 9, 1590, 363, 262, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{2335}$$

$$\frac{x^3\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{a\sqrt{a+bx^2}}-\frac{\int \frac{x\left(-aDx^5-a\left(C-\frac{aD}{b}\right)x^3+\left(2Ab-\frac{3a(Da^2-bCa+b^2B)}{b^2}\right)x\right)}{\sqrt{bx^2+a}}dx}{ab}$$

$$\Big\downarrow \text{9}$$

$$\frac{x^3\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{a\sqrt{a+bx^2}}-\frac{\int\frac{x^2\left(-aDx^4-a\left(C-\frac{aD}{b}\right)x^2+2Ab-\frac{3a\left(Da^2-bCa+b^2B\right)}{b^2}\right)}{\sqrt{bx^2+a}}dx}{ab}$$

$\downarrow$ 1590

$$\frac{x^3\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{a\sqrt{a+bx^2}}-\frac{\frac{\int\frac{x^2\left(6\left(2Ab^2-3a\left(\frac{Da^2}{b}-Ca+bB\right)\right)-a(6bC-11aD)x^2\right)}{\sqrt{bx^2+a}}dx}{6b}-\frac{aDx^5\sqrt{a+bx^2}}{6b}}{ab}$$

$\downarrow$ 363

$$\frac{x^3\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{a\sqrt{a+bx^2}}-\frac{\frac{\frac{3\left(16Ab^3-a\left(35a^2D-30abC+24b^2B\right)\right)\int\frac{x^2}{\sqrt{bx^2+a}}dx}{4b}-\frac{ax^3\sqrt{a+bx^2}(6bC-11aD)}{4b}}{6b}-\frac{aDx^5\sqrt{a+bx^2}}{6b}}{ab}$$

$\downarrow$ 262

$$\frac{x^3\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{a\sqrt{a+bx^2}}-\frac{\frac{\frac{3\left(16Ab^3-a\left(35a^2D-30abC+24b^2B\right)\right)\left(\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\int\frac{1}{\sqrt{bx^2+a}}dx}{2b}\right)}{4b}-\frac{ax^3\sqrt{a+bx^2}(6bC-11aD)}{4b}}{6b}-\frac{aDx^5\sqrt{a+bx^2}}{6b}}{ab}$$

$\downarrow$ 224

$$\frac{x^3\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{a\sqrt{a+bx^2}}-\frac{\frac{\frac{3\left(16Ab^3-a\left(35a^2D-30abC+24b^2B\right)\right)\left(\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2b}\right)}{4b}-\frac{ax^3\sqrt{a+bx^2}(6bC-11aD)}{4b}}{6b}-\frac{aDx^5\sqrt{a+bx^2}}{6b}}{ab}$$

$\downarrow$ 219

$$\frac{x^3\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{a\sqrt{a+bx^2}}-\frac{\frac{\frac{3\left(\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}\right)\left(16Ab^3-a\left(35a^2D-30abC+24b^2B\right)\right)}{4b}-\frac{ax^3\sqrt{a+bx^2}(6bC-11aD)}{4b}}{6b}-\frac{aDx^5\sqrt{a+bx^2}}{6b}}{ab}$$

input
```
Int[(x^2*(A + B*x^2 + C*x^4 + D*x^6))/(a + b*x^2)^(3/2),x]
```

output
```
((A - (a*(b^2*B - a*b*C + a^2*D))/b^3)*x^3)/(a*Sqrt[a + b*x^2]) - (-1/6*(a
*D*x^5*Sqrt[a + b*x^2])/b + (-1/4*(a*(6*b*C - 11*a*D)*x^3*Sqrt[a + b*x^2])
/b + (3*(16*A*b^3 - a*(24*b^2*B - 30*a*b*C + 35*a^2*D))*((x*Sqrt[a + b*x^2
])/(2*b) - (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(3/2))))/(4*b))/(
6*b))/(a*b)
```

## Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 1590
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[c^p*(f*x)^(m + 4*p - 1)*((d + e*x^2)^
(q + 1)/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1))), x] + Simp[1/(e*(m + 4*p + 2*q
 + 1))   Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a +
b*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x],
x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p,
0] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

rule 2335
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Simp[c/(2*a*b*(p + 1))   Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSu
m[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
 b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

## Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.76

| method | result |
|---|---|
| pseudoelliptic | $\frac{-\frac{15\left(-\frac{7Dx^2}{18}+C\right)x\,a^2 b^{\frac{3}{2}}}{8}+\frac{3\left(-\frac{7}{36}Dx^4-\frac{5}{12}C\,x^2+B\right)xa\,b^{\frac{5}{2}}}{2}-(-\frac{1}{6}Dx^6-\frac{1}{4}C\,x^4-\frac{1}{2}x^2B+A)x\,b^{\frac{7}{2}}+\frac{35D\sqrt{b}\,a^3x}{16}+\sqrt{b\,x^2+a}\,(b^3A-}{\sqrt{b\,x^2+a}\,b^{\frac{9}{2}}}$ |
| default | $A\left(-\frac{x}{b\sqrt{b\,x^2+a}}+\frac{\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{b^{\frac{3}{2}}}\right)+B\left(\frac{x^3}{2b\sqrt{b\,x^2+a}}-\frac{3a\left(-\frac{x}{b\sqrt{b\,x^2+a}}+\frac{\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{b^{\frac{3}{2}}}\right)}{2b}\right)+C\left(\right.$ |

input
```
int(x^2*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(-15/8*(-7/18*D*x^2+C)*x*a^2*b^(3/2)+3/2*(-7/36*D*x^4-5/12*C*x^2+B)*x*a*b^
(5/2)-(-1/6*D*x^6-1/4*C*x^4-1/2*x^2*B+A)*x*b^(7/2)+35/16*D*b^(1/2)*a^3*x+(
b*x^2+a)^(1/2)*(b^3*A-3/2*a*b^2*B+15/8*a^2*b*C-35/16*a^3*D)*arctanh((b*x^2
+a)^(1/2)/x/b^(1/2)))/(b*x^2+a)^(1/2)/b^(9/2)
```

## Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 419, normalized size of antiderivative = 2.23

$$\int \frac{x^2(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{3/2}}\,dx = \left[-\frac{3\,(35\,Da^4-30\,Ca^3b+24\,Ba^2b^2-16\,Aab^3+(35\,Da^3b-30\,Ca^2b^2\right.$$

input
```
integrate(x^2*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[-1/96*(3*(35*D*a^4 - 30*C*a^3*b + 24*B*a^2*b^2 - 16*A*a*b^3 + (35*D*a^3*b
 - 30*C*a^2*b^2 + 24*B*a*b^3 - 16*A*b^4)*x^2)*sqrt(b)*log(-2*b*x^2 - 2*sqr
t(b*x^2 + a)*sqrt(b)*x - a) - 2*(8*D*b^4*x^7 - 2*(7*D*a*b^3 - 6*C*b^4)*x^5
 + (35*D*a^2*b^2 - 30*C*a*b^3 + 24*B*b^4)*x^3 + 3*(35*D*a^3*b - 30*C*a^2*b
^2 + 24*B*a*b^3 - 16*A*b^4)*x)*sqrt(b*x^2 + a))/(b^6*x^2 + a*b^5), 1/48*(3
*(35*D*a^4 - 30*C*a^3*b + 24*B*a^2*b^2 - 16*A*a*b^3 + (35*D*a^3*b - 30*C*a
^2*b^2 + 24*B*a*b^3 - 16*A*b^4)*x^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2
 + a)) + (8*D*b^4*x^7 - 2*(7*D*a*b^3 - 6*C*b^4)*x^5 + (35*D*a^2*b^2 - 30*C
*a*b^3 + 24*B*b^4)*x^3 + 3*(35*D*a^3*b - 30*C*a^2*b^2 + 24*B*a*b^3 - 16*A*
b^4)*x)*sqrt(b*x^2 + a))/(b^6*x^2 + a*b^5)]
```

## Sympy [A] (verification not implemented)

Time = 22.48 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.85

$$\int \frac{x^2(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{3/2}}\,dx = A\left(\frac{\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{\frac{3}{2}}}-\frac{x}{\sqrt{a}b\sqrt{1+\frac{bx^2}{a}}}\right) + B\left(\frac{3\sqrt{a}x}{2b^2\sqrt{1+\frac{bx^2}{a}}}-\frac{3a\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{\frac{5}{2}}}+\frac{x^3}{2\sqrt{a}b\sqrt{1+\frac{bx^2}{a}}}\right) + C\left(-\frac{15a^{\frac{3}{2}}x}{8b^3\sqrt{1+\frac{bx^2}{a}}}-\frac{5\sqrt{a}x^3}{8b^2\sqrt{1+\frac{bx^2}{a}}}+\frac{15a^2\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{\frac{7}{2}}}+\frac{x^5}{4\sqrt{a}b\sqrt{1+\frac{bx^2}{a}}}\right) + D\left(\frac{35a^{\frac{5}{2}}x}{16b^4\sqrt{1+\frac{bx^2}{a}}}+\frac{35a^{\frac{3}{2}}x^3}{48b^3\sqrt{1+\frac{bx^2}{a}}}-\frac{7\sqrt{a}x^5}{24b^2\sqrt{1+\frac{bx^2}{a}}} - \frac{35a^3\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16b^{\frac{9}{2}}}+\frac{x^7}{6\sqrt{a}b\sqrt{1+\frac{bx^2}{a}}}\right)$$

input
```
integrate(x**2*(D*x**6+C*x**4+B*x**2+A)/(b*x**2+a)**(3/2),x)
```

output
```
A*(asinh(sqrt(b)*x/sqrt(a))/b**(3/2) - x/(sqrt(a)*b*sqrt(1 + b*x**2/a))) +
 B*(3*sqrt(a)*x/(2*b**2*sqrt(1 + b*x**2/a)) - 3*a*asinh(sqrt(b)*x/sqrt(a))
/(2*b**(5/2)) + x**3/(2*sqrt(a)*b*sqrt(1 + b*x**2/a))) + C*(-15*a**(3/2)*x
/(8*b**3*sqrt(1 + b*x**2/a)) - 5*sqrt(a)*x**3/(8*b**2*sqrt(1 + b*x**2/a))
+ 15*a**2*asinh(sqrt(b)*x/sqrt(a))/(8*b**(7/2)) + x**5/(4*sqrt(a)*b*sqrt(1
 + b*x**2/a))) + D*(35*a**(5/2)*x/(16*b**4*sqrt(1 + b*x**2/a)) + 35*a**(3/
2)*x**3/(48*b**3*sqrt(1 + b*x**2/a)) - 7*sqrt(a)*x**5/(24*b**2*sqrt(1 + b*
x**2/a)) - 35*a**3*asinh(sqrt(b)*x/sqrt(a))/(16*b**(9/2)) + x**7/(6*sqrt(a
)*b*sqrt(1 + b*x**2/a)))
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.34

$$\int \frac{x^2(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{3/2}}\,dx = \frac{Dx^7}{6\sqrt{bx^2+a}b} - \frac{7\,Dax^5}{24\sqrt{bx^2+a}b^2} + \frac{Cx^5}{4\sqrt{bx^2+a}b} + \frac{35\,Da^2x^3}{48\sqrt{bx^2+a}b^3} - \frac{5\,Cax^3}{8\sqrt{bx^2+a}b^2} + \frac{Bx^3}{2\sqrt{bx^2+a}b} + \frac{35\,Da^3x}{16\sqrt{bx^2+a}b^4} - \frac{15\,Ca^2x}{8\sqrt{bx^2+a}b^3} + \frac{3\,Bax}{2\sqrt{bx^2+a}b^2} - \frac{Ax}{\sqrt{bx^2+a}b} - \frac{35\,Da^3\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{9}{2}}} + \frac{15\,Ca^2\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{7}{2}}} - \frac{3\,Ba\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{5}{2}}} + \frac{A\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{3}{2}}}$$

input
```
integrate(x^2*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
1/6*D*x^7/(sqrt(b*x^2 + a)*b) - 7/24*D*a*x^5/(sqrt(b*x^2 + a)*b^2) + 1/4*C
*x^5/(sqrt(b*x^2 + a)*b) + 35/48*D*a^2*x^3/(sqrt(b*x^2 + a)*b^3) - 5/8*C*a
*x^3/(sqrt(b*x^2 + a)*b^2) + 1/2*B*x^3/(sqrt(b*x^2 + a)*b) + 35/16*D*a^3*x
/(sqrt(b*x^2 + a)*b^4) - 15/8*C*a^2*x/(sqrt(b*x^2 + a)*b^3) + 3/2*B*a*x/(s
qrt(b*x^2 + a)*b^2) - A*x/(sqrt(b*x^2 + a)*b) - 35/16*D*a^3*arcsinh(b*x/sq
rt(a*b))/b^(9/2) + 15/8*C*a^2*arcsinh(b*x/sqrt(a*b))/b^(7/2) - 3/2*B*a*arc
sinh(b*x/sqrt(a*b))/b^(5/2) + A*arcsinh(b*x/sqrt(a*b))/b^(3/2)
```

## Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.89

$$\int \frac{x^2(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{3/2}}\,dx = \frac{\left(\left(2\left(\frac{4\,Dx^2}{b} - \frac{7\,Dab^5-6\,Cb^6}{b^7}\right)x^2 + \frac{35\,Da^2b^4-30\,Cab^5+24\,Bb^6}{b^7}\right)x^2 + \frac{3\,(35\,Da^3b^3-3}{}}{48\sqrt{bx^2+a}} + \frac{(35\,Da^3 - 30\,Ca^2b + 24\,Bab^2 - 16\,Ab^3)\log\left(\left|-\sqrt{b}x + \sqrt{bx^2+a}\right|\right)}{16\,b^{\frac{9}{2}}}$$

input
```
integrate(x^2*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
1/48*((2*(4*D*x^2/b - (7*D*a*b^5 - 6*C*b^6)/b^7)*x^2 + (35*D*a^2*b^4 - 30*
C*a*b^5 + 24*B*b^6)/b^7)*x^2 + 3*(35*D*a^3*b^3 - 30*C*a^2*b^4 + 24*B*a*b^5
 - 16*A*b^6)/b^7)*x/sqrt(b*x^2 + a) + 1/16*(35*D*a^3 - 30*C*a^2*b + 24*B*a
*b^2 - 16*A*b^3)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(9/2)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{3/2}}\,dx=\int \frac{x^2\,(A+B\,x^2+C\,x^4+x^6\,D)}{(b\,x^2+a)^{3/2}}\,dx$$

input
```
int((x^2*(A + B*x^2 + C*x^4 + x^6*D))/(a + b*x^2)^(3/2),x)
```

output
```
int((x^2*(A + B*x^2 + C*x^4 + x^6*D))/(a + b*x^2)^(3/2), x)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 412, normalized size of antiderivative = 2.19

$$\int \frac{x^2(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{3/2}}\,dx=\frac{840\sqrt{b\,x^2+a}\,a^3bdx-720\sqrt{b\,x^2+a}\,a^2b^2cx+280\sqrt{b\,x^2+a}\,a^2b^2d\,x^3+}{}$$

input
```
int(x^2*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(3/2),x)
```

output

```
(840*sqrt(a + b*x**2)*a**3*b*d*x - 720*sqrt(a + b*x**2)*a**2*b**2*c*x + 28
0*sqrt(a + b*x**2)*a**2*b**2*d*x**3 + 192*sqrt(a + b*x**2)*a*b**4*x - 240*
sqrt(a + b*x**2)*a*b**3*c*x**3 - 112*sqrt(a + b*x**2)*a*b**3*d*x**5 + 192*
sqrt(a + b*x**2)*b**5*x**3 + 96*sqrt(a + b*x**2)*b**4*c*x**5 + 64*sqrt(a +
 b*x**2)*b**4*d*x**7 - 840*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt
(a))*a**4*d + 720*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**3
*b*c - 840*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**3*b*d*x*
*2 - 192*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2*b**3 + 7
20*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2*b**2*c*x**2 -
192*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a*b**4*x**2 + 525*
sqrt(b)*a**4*d - 480*sqrt(b)*a**3*b*c + 525*sqrt(b)*a**3*b*d*x**2 + 48*sqr
t(b)*a**2*b**3 - 480*sqrt(b)*a**2*b**2*c*x**2 + 48*sqrt(b)*a*b**4*x**2)/(3
84*b**5*(a + b*x**2))
```

# 3.234 $\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{3/2}}\,dx$



## Optimal result

Integrand size = 29, antiderivative size = 137

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{3/2}}\,dx=\frac{\left(\frac{A}{a}-\frac{b^2B-abC+a^2D}{b^3}\right)x}{\sqrt{a+bx^2}}+\frac{(4bC-7aD)x\sqrt{a+bx^2}}{8b^3}+\frac{Dx^3\sqrt{a+bx^2}}{4b^2}+\frac{(8b^2B-12abC+15a^2D)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{7/2}}$$

output
```
(A/a-(B*b^2-C*a*b+D*a^2)/b^3)*x/(b*x^2+a)^(1/2)+1/8*(4*C*b-7*D*a)*x*(b*x^2
+a)^(1/2)/b^3+1/4*D*x^3*(b*x^2+a)^(1/2)/b^2+1/8*(8*B*b^2-12*C*a*b+15*D*a^2
)*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(7/2)
```

## Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.88

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{3/2}}\,dx=\frac{\frac{\sqrt{b}x\left(8Ab^3+a\left(-15a^2D+ab(12C-5Dx^2)+b^2(-8B+4Cx^2+2Dx^4)\right)\right)}{a\sqrt{a+bx^2}}+(-8b^2B+12abC-}{8b^{7/2}}$$

input
```
Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(a + b*x^2)^(3/2),x]
```

output
```
((Sqrt[b]*x*(8*A*b^3 + a*(-15*a^2*D + a*b*(12*C - 5*D*x^2) + b^2*(-8*B + 4
*C*x^2 + 2*D*x^4))))/(a*Sqrt[a + b*x^2]) + (-8*b^2*B + 12*a*b*C - 15*a^2*D
)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(8*b^(7/2))
```

## Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2345, 25, 1473, 27, 299, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{3/2}}dx$$

$$\Big\downarrow \text{2345}$$

$$\frac{x\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{a\sqrt{a+bx^2}}-\frac{\int -\frac{\frac{aDx^4}{b}+\frac{a(bC-aD)x^2}{b^2}+\frac{a\left(Da^2-bCa+b^2B\right)}{b^3}}{\sqrt{bx^2+a}}dx}{a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\frac{aDx^4}{b}+\frac{a(bC-aD)x^2}{b^2}+\frac{a\left(Da^2-bCa+b^2B\right)}{b^3}}{\sqrt{bx^2+a}}dx}{a}+\frac{x\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{a\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{1473}$$

$$\frac{\frac{\int \frac{a\left(b^2\left(4C-\frac{7aD}{b}\right)x^2+4\left(Da^2-bCa+b^2B\right)\right)}{b^2\sqrt{bx^2+a}}dx}{4b}+\frac{aDx^3\sqrt{a+bx^2}}{4b^2}}{a}+\frac{x\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{a\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{a\int \frac{b(4bC-7aD)x^2+4\left(Da^2-bCa+b^2B\right)}{\sqrt{bx^2+a}}dx}{4b^3}+\frac{aDx^3\sqrt{a+bx^2}}{4b^2}}{a}+\frac{x\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{a\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{299}$$

$$\frac{\frac{a\left(\frac{1}{2}(15a^2D-12abC+8b^2B)\int\frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}(4bC-7aD)\right)}{4b^3}+\frac{aDx^3\sqrt{a+bx^2}}{4b^2}}{a}+\frac{x\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{a\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{a\left(\frac{1}{2}(15a^2D-12abC+8b^2B)\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}(4bC-7aD)\right)}{4b^3}+\frac{aDx^3\sqrt{a+bx^2}}{4b^2}}{a}+\frac{x\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{a\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{x\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{a\sqrt{a+bx^2}}+\frac{\frac{a\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(15a^2D-12abC+8b^2B)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}(4bC-7aD)\right)}{4b^3}+\frac{aDx^3\sqrt{a+bx^2}}{4b^2}}{a}$$

input
```
Int[(A + B*x^2 + C*x^4 + D*x^6)/(a + b*x^2)^(3/2),x]
```

output
```
((A - (a*(b^2*B - a*b*C + a^2*D))/b^3)*x)/(a*Sqrt[a + b*x^2]) + ((a*D*x^3*
Sqrt[a + b*x^2])/(4*b^2) + (a*(((4*b*C - 7*a*D)*x*Sqrt[a + b*x^2])/2 + ((8
*b^2*B - 12*a*b*C + 15*a^2*D)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqr
t[b])))/(4*b^3))/a
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1473
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[c^p*x^(4*p - 1)*((d + e*x^2)^(q + 1)/(e*(4*p + 2*q + 1)))
, x] + Simp[1/(e*(4*p + 2*q + 1))   Int[(d + e*x^2)^q*ExpandToSum[e*(4*p +
2*q + 1)*(a + b*x^2 + c*x^4)^p - d*c^p*(4*p - 1)*x^(4*p - 2) - e*c^p*(4*p +
 2*q + 1)*x^(4*p), x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] &&  !LtQ[q, -1]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

## Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.83

| method | result |
|---|---|
| pseudoelliptic | $\frac{\sqrt{b\,x^2+a}\,a\left(B\,b^2-\frac{3}{2}Cab+\frac{15}{8}Da^2\right)\operatorname{arctanh}\left(\frac{\sqrt{b\,x^2+a}}{x\sqrt{b}}\right)+\left(-\frac{15a^3D}{8}+\frac{3\left(-\frac{5Dx^2}{12}+C\right)b\,a^2}{2}-\left(-\frac{1}{4}Dx^4-\frac{1}{2}C\,x^2+B\right)b^2a+b^3A\right.}{\sqrt{b\,x^2+a}\,a\,b^{\frac{7}{2}}}$ |
| default | $\frac{Ax}{a\sqrt{b\,x^2+a}}+C\left(\frac{x^3}{2b\sqrt{b\,x^2+a}}-\frac{3a\left(-\frac{x}{b\sqrt{b\,x^2+a}}+\frac{\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{b^{\frac{3}{2}}}\right)}{2b}\right)+D\left(\frac{x^5}{4b\sqrt{b\,x^2+a}}-\frac{5a\left(\frac{x^3}{2b\sqrt{b\,x^2+a}}-\right.}{}\right.$ |

input
```
int((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/(b*x^2+a)^(1/2)*((b*x^2+a)^(1/2)*a*(B*b^2-3/2*C*a*b+15/8*D*a^2)*arctanh(
(b*x^2+a)^(1/2)/x/b^(1/2))+(-15/8*a^3*D+3/2*(-5/12*D*x^2+C)*b*a^2-(-1/4*D*
x^4-1/2*C*x^2+B)*b^2*a+b^3*A)*b^(1/2)*x)/a/b^(7/2)
```

## Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 352, normalized size of antiderivative = 2.57

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{3/2}}\,dx = \left[\frac{(15\,Da^4-12\,Ca^3b+8\,Ba^2b^2+(15\,Da^3b-12\,Ca^2b^2+8\,Bab^3)x^2)\sqrt{b}\log}{}\right.$$

$$-\frac{(15\,Da^4-12\,Ca^3b+8\,Ba^2b^2+(15\,Da^3b-12\,Ca^2b^2+8\,Bab^3)x^2)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(2\,Dab^3x^5-}{8\,(ab^5x^2+a^2b^4)}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/16*((15*D*a^4 - 12*C*a^3*b + 8*B*a^2*b^2 + (15*D*a^3*b - 12*C*a^2*b^2 +
 8*B*a*b^3)*x^2)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) +
 2*(2*D*a*b^3*x^5 - (5*D*a^2*b^2 - 4*C*a*b^3)*x^3 - (15*D*a^3*b - 12*C*a^2
*b^2 + 8*B*a*b^3 - 8*A*b^4)*x)*sqrt(b*x^2 + a))/(a*b^5*x^2 + a^2*b^4), -1/
8*((15*D*a^4 - 12*C*a^3*b + 8*B*a^2*b^2 + (15*D*a^3*b - 12*C*a^2*b^2 + 8*B
*a*b^3)*x^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (2*D*a*b^3*x^5
- (5*D*a^2*b^2 - 4*C*a*b^3)*x^3 - (15*D*a^3*b - 12*C*a^2*b^2 + 8*B*a*b^3 -
 8*A*b^4)*x)*sqrt(b*x^2 + a))/(a*b^5*x^2 + a^2*b^4)]
```

## Sympy [A] (verification not implemented)

Time = 8.51 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.74

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{3/2}}\,dx = \frac{Ax}{a^{\frac{3}{2}}\sqrt{1+\frac{bx^2}{a}}} + B\left(\frac{\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{\frac{3}{2}}} - \frac{x}{\sqrt{a}b\sqrt{1+\frac{bx^2}{a}}}\right) + C\left(\frac{3\sqrt{a}x}{2b^2\sqrt{1+\frac{bx^2}{a}}} - \frac{3a\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{\frac{5}{2}}} + \frac{x^3}{2\sqrt{a}b\sqrt{1+\frac{bx^2}{a}}}\right) + D\left(-\frac{15a^{\frac{3}{2}}x}{8b^3\sqrt{1+\frac{bx^2}{a}}} - \frac{5\sqrt{a}x^3}{8b^2\sqrt{1+\frac{bx^2}{a}}} + \frac{15a^2\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{\frac{7}{2}}} + \frac{x^5}{4\sqrt{a}b\sqrt{1+\frac{bx^2}{a}}}\right)$$

input
```
integrate((D*x**6+C*x**4+B*x**2+A)/(b*x**2+a)**(3/2),x)
```

output
```
A*x/(a**(3/2)*sqrt(1 + b*x**2/a)) + B*(asinh(sqrt(b)*x/sqrt(a))/b**(3/2) -
 x/(sqrt(a)*b*sqrt(1 + b*x**2/a))) + C*(3*sqrt(a)*x/(2*b**2*sqrt(1 + b*x**
2/a)) - 3*a*asinh(sqrt(b)*x/sqrt(a))/(2*b**(5/2)) + x**3/(2*sqrt(a)*b*sqrt
(1 + b*x**2/a))) + D*(-15*a**(3/2)*x/(8*b**3*sqrt(1 + b*x**2/a)) - 5*sqrt(
a)*x**3/(8*b**2*sqrt(1 + b*x**2/a)) + 15*a**2*asinh(sqrt(b)*x/sqrt(a))/(8*
b**(7/2)) + x**5/(4*sqrt(a)*b*sqrt(1 + b*x**2/a)))
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.25

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{3/2}}\,dx = \frac{Dx^5}{4\sqrt{bx^2+a}b} - \frac{5\,Dax^3}{8\sqrt{bx^2+a}b^2} + \frac{Cx^3}{2\sqrt{bx^2+a}b} + \frac{Ax}{\sqrt{bx^2+a}a} - \frac{15\,Da^2x}{8\sqrt{bx^2+a}b^3} + \frac{3\,Cax}{2\sqrt{bx^2+a}b^2} - \frac{Bx}{\sqrt{bx^2+a}b} + \frac{15\,Da^2\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{7}{2}}} - \frac{3\,Ca\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{5}{2}}} + \frac{B\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{3}{2}}}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
1/4*D*x^5/(sqrt(b*x^2 + a)*b) - 5/8*D*a*x^3/(sqrt(b*x^2 + a)*b^2) + 1/2*C*
x^3/(sqrt(b*x^2 + a)*b) + A*x/(sqrt(b*x^2 + a)*a) - 15/8*D*a^2*x/(sqrt(b*x
^2 + a)*b^3) + 3/2*C*a*x/(sqrt(b*x^2 + a)*b^2) - B*x/(sqrt(b*x^2 + a)*b) +
 15/8*D*a^2*arcsinh(b*x/sqrt(a*b))/b^(7/2) - 3/2*C*a*arcsinh(b*x/sqrt(a*b)
)/b^(5/2) + B*arcsinh(b*x/sqrt(a*b))/b^(3/2)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.98

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{3/2}}\,dx = \frac{\left(\left(\frac{2\,Dx^2}{b} - \frac{5\,Da^2b^3-4\,Cab^4}{ab^5}\right)x^2 - \frac{15\,Da^3b^2-12\,Ca^2b^3+8\,Bab^4-8\,Ab^5}{ab^5}\right)x}{8\sqrt{bx^2+a}} - \frac{(15\,Da^2-12\,Cab+8\,Bb^2)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{8\,b^{\frac{7}{2}}}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
1/8*((2*D*x^2/b - (5*D*a^2*b^3 - 4*C*a*b^4)/(a*b^5))*x^2 - (15*D*a^3*b^2 -
 12*C*a^2*b^3 + 8*B*a*b^4 - 8*A*b^5)/(a*b^5))*x/sqrt(b*x^2 + a) - 1/8*(15*
D*a^2 - 12*C*a*b + 8*B*b^2)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(7/2)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{\left(a+bx^2\right)^{3/2}}\,dx=\int \frac{A+B\,x^2+C\,x^4+x^6\,D}{\left(b\,x^2+a\right)^{3/2}}\,dx$$

input
```
int((A + B*x^2 + C*x^4 + x^6*D)/(a + b*x^2)^(3/2),x)
```

output
```
int((A + B*x^2 + C*x^4 + x^6*D)/(a + b*x^2)^(3/2), x)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 313, normalized size of antiderivative = 2.28

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{\left(a+bx^2\right)^{3/2}}\,dx=\frac{-15\sqrt{b\,x^2+a}\,a^2bdx+12\sqrt{b\,x^2+a}\,a\,b^2cx-5\sqrt{b\,x^2+a}\,a\,b^2d\,x^3+4\sqrt{b\,x^2}}{}$$

input
```
int((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(3/2),x)
```

output
```
( - 15*sqrt(a + b*x**2)*a**2*b*d*x + 12*sqrt(a + b*x**2)*a*b**2*c*x - 5*sq
rt(a + b*x**2)*a*b**2*d*x**3 + 4*sqrt(a + b*x**2)*b**3*c*x**3 + 2*sqrt(a +
 b*x**2)*b**3*d*x**5 + 15*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(
a))*a**3*d - 12*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2*b
*c + 15*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2*b*d*x**2
+ 8*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a*b**3 - 12*sqrt(b
)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a*b**2*c*x**2 + 8*sqrt(b)*lo
g((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*b**4*x**2 - 10*sqrt(b)*a**3*d +
9*sqrt(b)*a**2*b*c - 10*sqrt(b)*a**2*b*d*x**2 + 9*sqrt(b)*a*b**2*c*x**2)/(
8*b**4*(a + b*x**2))
```

# 3.235 $\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2\left(a+bx^2\right)^{3/2}}\,dx$



## Optimal result

Integrand size = 32, antiderivative size = 123

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2\left(a+bx^2\right)^{3/2}}\,dx=-\frac{\left(Ab^3-a\left(b^2B-abC+a^2D\right)\right)x}{a^2b^2\sqrt{a+bx^2}}-\frac{A\sqrt{a+bx^2}}{a^2x}+\frac{Dx\sqrt{a+bx^2}}{2b^2}+\frac{(2bC-3aD)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{5/2}}$$

output
```
-(A*b^3-a*(B*b^2-C*a*b+D*a^2))*x/a^2/b^2/(b*x^2+a)^(1/2)-A*(b*x^2+a)^(1/2)
/a^2/x+1/2*D*x*(b*x^2+a)^(1/2)/b^2+1/2*(2*C*b-3*D*a)*arctanh(b^(1/2)*x/(b*
x^2+a)^(1/2))/b^(5/2)
```

## Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.93

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2\left(a+bx^2\right)^{3/2}}\,dx=\frac{\frac{\sqrt{b}\left(-4Ab^3x^2+3a^3Dx^2-2ab^2\left(A-Bx^2\right)+a^2b\left(-2Cx^2+Dx^4\right)\right)}{a^2x\sqrt{a+bx^2}}+(-2bC+3aD)\log\left(-\sqrt{b}\right.}{2b^{5/2}}$$

input
```
Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(x^2*(a + b*x^2)^(3/2)),x]
```

output
```
((Sqrt[b]*(-4*A*b^3*x^2 + 3*a^3*D*x^2 - 2*a*b^2*(A - B*x^2) + a^2*b*(-2*C*
x^2 + D*x^4)))/(a^2*x*Sqrt[a + b*x^2]) + (-2*b*C + 3*a*D)*Log[-(Sqrt[b]*x)
 + Sqrt[a + b*x^2]])/(2*b^(5/2))
```

## Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2336, 25, 1588, 25, 27, 299, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2\left(a+bx^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{2336}$$

$$-\frac{\int -\frac{\frac{aDx^4}{b}+\frac{a(bC-aD)x^2}{b^2}+A}{x^2\sqrt{bx^2+a}}dx}{a}-\frac{x\left(-\frac{a^2D}{b^2}+\frac{Ab}{a}+\frac{aC}{b}-B\right)}{a\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\frac{aDx^4}{b}+\frac{a(bC-aD)x^2}{b^2}+A}{x^2\sqrt{bx^2+a}}dx}{a}-\frac{x\left(-\frac{a^2D}{b^2}+\frac{Ab}{a}+\frac{aC}{b}-B\right)}{a\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{1588}$$

$$\frac{-\frac{\int -\frac{a^2\left(bDx^2+bC-aD\right)}{b^2\sqrt{bx^2+a}}dx}{a}-\frac{A\sqrt{a+bx^2}}{ax}}{a}-\frac{x\left(-\frac{a^2D}{b^2}+\frac{Ab}{a}+\frac{aC}{b}-B\right)}{a\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{a^2\left(bDx^2+bC-aD\right)}{b^2\sqrt{bx^2+a}}dx}{a}-\frac{A\sqrt{a+bx^2}}{ax}}{a}-\frac{x\left(-\frac{a^2D}{b^2}+\frac{Ab}{a}+\frac{aC}{b}-B\right)}{a\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{a\int \frac{bDx^2+bC-aD}{\sqrt{bx^2+a}}dx}{b^2}-\frac{A\sqrt{a+bx^2}}{ax}}{a}-\frac{x\left(-\frac{a^2D}{b^2}+\frac{Ab}{a}+\frac{aC}{b}-B\right)}{a\sqrt{a+bx^2}}$$

$$\Big\downarrow \ 299$$

$$\frac{\frac{a\left(\frac{1}{2}(2bC-3aD)\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}Dx\sqrt{a+bx^2}\right)}{b^2}-\frac{A\sqrt{a+bx^2}}{ax}}{a}-\frac{x\left(-\frac{a^2D}{b^2}+\frac{Ab}{a}+\frac{aC}{b}-B\right)}{a\sqrt{a+bx^2}}$$

$$\Big\downarrow \ 224$$

$$\frac{\frac{a\left(\frac{1}{2}(2bC-3aD)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}Dx\sqrt{a+bx^2}\right)}{b^2}-\frac{A\sqrt{a+bx^2}}{ax}}{a}-\frac{x\left(-\frac{a^2D}{b^2}+\frac{Ab}{a}+\frac{aC}{b}-B\right)}{a\sqrt{a+bx^2}}$$

$$\Big\downarrow \ 219$$

$$\frac{\frac{a\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bC-3aD)}{2\sqrt{b}}+\frac{1}{2}Dx\sqrt{a+bx^2}\right)}{b^2}-\frac{A\sqrt{a+bx^2}}{ax}}{a}-\frac{x\left(-\frac{a^2D}{b^2}+\frac{Ab}{a}+\frac{aC}{b}-B\right)}{a\sqrt{a+bx^2}}$$

input
```
Int[(A + B*x^2 + C*x^4 + D*x^6)/(x^2*(a + b*x^2)^(3/2)),x]
```

output
```
-((((A*b)/a - B + (a*C)/b - (a^2*D)/b^2)*x)/(a*Sqrt[a + b*x^2])) + (-((A*S
qrt[a + b*x^2])/(a*x)) + (a*((D*x*Sqrt[a + b*x^2])/2 + ((2*b*C - 3*a*D)*Ar
cTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b])))/b^2)/a
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1588
```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Simp[1/(d*f
^2*(m + 1))   Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x
) - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

## Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.93

| method | result |
|---|---|
| pseudoelliptic | $-\frac{2\left(-\frac{\sqrt{b\,x^{2}+a}\,x\,a^{2}\left(Cb-\frac{3Da}{2}\right)\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{x\sqrt{b}}\right)}{2}+\left(A\,x^{2}b^{3}+\frac{a\left(-x^{2}B+A\right)b^{2}}{2}+\frac{\left(-\frac{D x^{2}}{2}+C\right)x^{2}a^{2}b}{2}-\frac{3a^{3}x^{2}D}{4}\right)\sqrt{b}\right)}{\sqrt{b\,x^{2}+a}\,x\,b^{\frac{5}{2}}a^{2}}$ |
| default | $\frac{Bx}{a\sqrt{b\,x^{2}+a}}+A\left(-\frac{1}{ax\sqrt{b\,x^{2}+a}}-\frac{2bx}{a^{2}\sqrt{b\,x^{2}+a}}\right)+C\left(-\frac{x}{b\sqrt{b\,x^{2}+a}}+\frac{\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{b^{\frac{3}{2}}}\right)+D\left(\frac{x^{3}}{2b\sqrt{b\,x^{2}+}\right.$ |

input
```
int((D*x^6+C*x^4+B*x^2+A)/x^2/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/(b*x^2+a)^(1/2)*(-1/2*(b*x^2+a)^(1/2)*x*a^2*(C*b-3/2*D*a)*arctanh((b*x^
2+a)^(1/2)/x/b^(1/2))+(A*x^2*b^3+1/2*a*(-B*x^2+A)*b^2+1/2*(-1/2*D*x^2+C)*x
^2*a^2*b-3/4*a^3*x^2*D)*b^(1/2))/x/b^(5/2)/a^2
```

## Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 305, normalized size of antiderivative = 2.48

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2(a+bx^2)^{3/2}}\,dx = \left[-\frac{((3\,Da^3b-2\,Ca^2b^2)x^3+(3\,Da^4-2\,Ca^3b)x)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2}\right.}{}\right.$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^2/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[-1/4*(((3*D*a^3*b - 2*C*a^2*b^2)*x^3 + (3*D*a^4 - 2*C*a^3*b)*x)*sqrt(b)*l
og(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(D*a^2*b^2*x^4 - 2*A*a*
b^3 + (3*D*a^3*b - 2*C*a^2*b^2 + 2*B*a*b^3 - 4*A*b^4)*x^2)*sqrt(b*x^2 + a)
)/(a^2*b^4*x^3 + a^3*b^3*x), 1/2*(((3*D*a^3*b - 2*C*a^2*b^2)*x^3 + (3*D*a^
4 - 2*C*a^3*b)*x)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (D*a^2*b^2
*x^4 - 2*A*a*b^3 + (3*D*a^3*b - 2*C*a^2*b^2 + 2*B*a*b^3 - 4*A*b^4)*x^2)*sq
rt(b*x^2 + a))/(a^2*b^4*x^3 + a^3*b^3*x)]
```

## Sympy [A] (verification not implemented)

Time = 7.55 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.50

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2(a+bx^2)^{3/2}}\,dx = A\left(-\frac{1}{a\sqrt{b}x^2\sqrt{\frac{a}{bx^2}+1}}-\frac{2\sqrt{b}}{a^2\sqrt{\frac{a}{bx^2}+1}}\right) + \frac{Bx}{a^{\frac{3}{2}}\sqrt{1+\frac{bx^2}{a}}} + C\left(\frac{\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{\frac{3}{2}}}-\frac{x}{\sqrt{a}b\sqrt{1+\frac{bx^2}{a}}}\right) + D\left(\frac{3\sqrt{a}x}{2b^2\sqrt{1+\frac{bx^2}{a}}}-\frac{3a\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{\frac{5}{2}}}+\frac{x^3}{2\sqrt{a}b\sqrt{1+\frac{bx^2}{a}}}\right)$$

input
```
integrate((D*x**6+C*x**4+B*x**2+A)/x**2/(b*x**2+a)**(3/2),x)
```

output
```
A*(-1/(a*sqrt(b)*x**2*sqrt(a/(b*x**2) + 1)) - 2*sqrt(b)/(a**2*sqrt(a/(b*x*
*2) + 1))) + B*x/(a**(3/2)*sqrt(1 + b*x**2/a)) + C*(asinh(sqrt(b)*x/sqrt(a
))/b**(3/2) - x/(sqrt(a)*b*sqrt(1 + b*x**2/a))) + D*(3*sqrt(a)*x/(2*b**2*s
qrt(1 + b*x**2/a)) - 3*a*asinh(sqrt(b)*x/sqrt(a))/(2*b**(5/2)) + x**3/(2*s
qrt(a)*b*sqrt(1 + b*x**2/a)))
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.07

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2(a+bx^2)^{3/2}}\,dx = \frac{Dx^3}{2\sqrt{bx^2+a}b}+\frac{Bx}{\sqrt{bx^2+a}a}+\frac{3\,Dax}{2\sqrt{bx^2+a}b^2} - \frac{Cx}{\sqrt{bx^2+a}b}-\frac{2\,Abx}{\sqrt{bx^2+a}a^2}-\frac{3\,Da\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{5}{2}}}+\frac{C\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{3}{2}}}-\frac{A}{\sqrt{bx^2+a}ax}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^2/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
1/2*D*x^3/(sqrt(b*x^2 + a)*b) + B*x/(sqrt(b*x^2 + a)*a) + 3/2*D*a*x/(sqrt(
b*x^2 + a)*b^2) - C*x/(sqrt(b*x^2 + a)*b) - 2*A*b*x/(sqrt(b*x^2 + a)*a^2)
- 3/2*D*a*arcsinh(b*x/sqrt(a*b))/b^(5/2) + C*arcsinh(b*x/sqrt(a*b))/b^(3/2
) - A/(sqrt(b*x^2 + a)*a*x)
```

## Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.03

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2(a+bx^2)^{3/2}}\,dx = \frac{\left(\frac{Dx^2}{b}+\frac{3\,Da^3b-2\,Ca^2b^2+2\,Bab^3-2\,Ab^4}{a^2b^3}\right)x}{2\sqrt{bx^2+a}} + \frac{2\,A\sqrt{b}}{\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)a}+\frac{(3\,Da-2\,Cb)\log\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2\right)}{4\,b^{\frac{5}{2}}}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^2/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
1/2*(D*x^2/b + (3*D*a^3*b - 2*C*a^2*b^2 + 2*B*a*b^3 - 2*A*b^4)/(a^2*b^3))*
x/sqrt(b*x^2 + a) + 2*A*sqrt(b)/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)*a)
+ 1/4*(3*D*a - 2*C*b)*log((sqrt(b)*x - sqrt(b*x^2 + a))^2)/b^(5/2)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{A + Bx^2 + Cx^4 + Dx^6}{x^2 (a + bx^2)^{3/2}} \, dx = \int \frac{A + B\,x^2 + C\,x^4 + x^6\,D}{x^2\,(b\,x^2 + a)^{3/2}} \, dx$$

input
```
int((A + B*x^2 + C*x^4 + x^6*D)/(x^2*(a + b*x^2)^(3/2)),x)
```

output
```
int((A + B*x^2 + C*x^4 + x^6*D)/(x^2*(a + b*x^2)^(3/2)), x)
```

## Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 285, normalized size of antiderivative = 2.32

$$\int \frac{A + Bx^2 + Cx^4 + Dx^6}{x^2 (a + bx^2)^{3/2}} \, dx = \frac{3\sqrt{b\,x^2+a}\,a^2bd\,x^2 - 2\sqrt{b\,x^2+a}\,a\,b^3 - 2\sqrt{b\,x^2+a}\,a\,b^2c\,x^2 + \sqrt{b\,x^2+a}\,a\,b}{}$$

input
```
int((D*x^6+C*x^4+B*x^2+A)/x^2/(b*x^2+a)^(3/2),x)
```

output
```
(3*sqrt(a + b*x**2)*a**2*b*d*x**2 - 2*sqrt(a + b*x**2)*a*b**3 - 2*sqrt(a +
 b*x**2)*a*b**2*c*x**2 + sqrt(a + b*x**2)*a*b**2*d*x**4 - 2*sqrt(a + b*x**
2)*b**4*x**2 - 3*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**3*
d*x + 2*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2*b*c*x - 3
*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2*b*d*x**3 + 2*sqr
t(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a*b**2*c*x**3 + 2*sqrt(b)
*a**3*d*x - 2*sqrt(b)*a**2*b*c*x + 2*sqrt(b)*a**2*b*d*x**3 - 2*sqrt(b)*a*b
**3*x - 2*sqrt(b)*a*b**2*c*x**3 - 2*sqrt(b)*b**4*x**3)/(2*a*b**3*x*(a + b*
x**2))
```

# 3.236 $\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4(a+bx^2)^{3/2}}\,dx$



## Optimal result

Integrand size = 32, antiderivative size = 123

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4(a+bx^2)^{3/2}}\,dx = \frac{(Ab^3-a(b^2B-abC+a^2D))\,x}{a^3b\sqrt{a+bx^2}} - \frac{A\sqrt{a+bx^2}}{3a^2x^3} + \frac{(5Ab-3aB)\sqrt{a+bx^2}}{3a^3x} + \frac{D\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}$$

output
```
(A*b^3-a*(B*b^2-C*a*b+D*a^2))*x/a^3/b/(b*x^2+a)^(1/2)-1/3*A*(b*x^2+a)^(1/2
)/a^2/x^3+1/3*(5*A*b-3*B*a)*(b*x^2+a)^(1/2)/a^3/x+D*arctanh(b^(1/2)*x/(b*x
^2+a)^(1/2))/b^(3/2)
```

## Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.91

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4(a+bx^2)^{3/2}}\,dx = \frac{8Ab^3x^4-3a^3Dx^4+2ab^2x^2(2A-3Bx^2)-a^2b(A+3Bx^2-3Cx^4)}{3a^3bx^3\sqrt{a+bx^2}} - \frac{D\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{b^{3/2}}$$

input
```
Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(x^4*(a + b*x^2)^(3/2)),x]
```

output
```
(8*A*b^3*x^4 - 3*a^3*D*x^4 + 2*a*b^2*x^2*(2*A - 3*B*x^2) - a^2*b*(A + 3*B*
x^2 - 3*C*x^4))/(3*a^3*b*x^3*Sqrt[a + b*x^2]) - (D*Log[-(Sqrt[b]*x) + Sqrt
[a + b*x^2]])/b^(3/2)
```

## Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.12, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {2336, 25, 1588, 27, 358, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4 (a+bx^2)^{3/2}}dx$$

$$\Big\downarrow \text{2336}$$

$$\frac{x\left(\frac{Ab^2}{a^2}-\frac{bB}{a}-\frac{aD}{b}+C\right)}{a\sqrt{a+bx^2}}-\frac{\int -\frac{\frac{aDx^4}{b}-\left(\frac{Ab}{a}-B\right)x^2+A}{x^4\sqrt{bx^2+a}}dx}{a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\frac{aDx^4}{b}-\left(\frac{Ab}{a}-B\right)x^2+A}{x^4\sqrt{bx^2+a}}dx}{a}+\frac{x\left(\frac{Ab^2}{a^2}-\frac{bB}{a}-\frac{aD}{b}+C\right)}{a\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{1588}$$

$$\frac{-\frac{\int \frac{b(5Ab-3aB)-3a^2Dx^2}{bx^2\sqrt{bx^2+a}}dx}{3a}-\frac{A\sqrt{a+bx^2}}{3ax^3}}{a}+\frac{x\left(\frac{Ab^2}{a^2}-\frac{bB}{a}-\frac{aD}{b}+C\right)}{a\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{\int \frac{b(5Ab-3aB)-3a^2Dx^2}{x^2\sqrt{bx^2+a}}dx}{3ab}-\frac{A\sqrt{a+bx^2}}{3ax^3}}{a}+\frac{x\left(\frac{Ab^2}{a^2}-\frac{bB}{a}-\frac{aD}{b}+C\right)}{a\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{358}$$

$$\frac{-\frac{-3a^2D\int \frac{1}{\sqrt{bx^2+a}}dx-\frac{b\sqrt{a+bx^2}(5Ab-3aB)}{ax}}{3ab}-\frac{A\sqrt{a+bx^2}}{3ax^3}}{a}+\frac{x\left(\frac{Ab^2}{a^2}-\frac{bB}{a}-\frac{aD}{b}+C\right)}{a\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{224}$$

$$\frac{-\frac{-3a^2D\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}-\frac{b\sqrt{a+bx^2}(5Ab-3aB)}{ax}}{3ab}-\frac{A\sqrt{a+bx^2}}{3ax^3}}{a}+\frac{x\left(\frac{Ab^2}{a^2}-\frac{bB}{a}-\frac{aD}{b}+C\right)}{a\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{-\frac{-\frac{3a^2D\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}-\frac{b\sqrt{a+bx^2}(5Ab-3aB)}{ax}}{3ab}-\frac{A\sqrt{a+bx^2}}{3ax^3}}{a}+\frac{x\left(\frac{Ab^2}{a^2}-\frac{bB}{a}-\frac{aD}{b}+C\right)}{a\sqrt{a+bx^2}}$$

input
```
Int[(A + B*x^2 + C*x^4 + D*x^6)/(x^4*(a + b*x^2)^(3/2)),x]
```

output
```
(((A*b^2)/a^2 - (b*B)/a + C - (a*D)/b)*x)/(a*Sqrt[a + b*x^2]) + (-1/3*(A*S
qrt[a + b*x^2])/(a*x^3) - (-((b*(5*A*b - 3*a*B)*Sqrt[a + b*x^2])/(a*x)) -
(3*a^2*D*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/Sqrt[b])/(3*a*b))/a
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 358
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x_
Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] + S
imp[d/e^2   Int[(e*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b*c - a*d, 0] && EqQ[Simplify[m + 2*p + 3], 0] && NeQ[m,
 -1]
```

rule 1588
```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Simp[1/(d*f
^2*(m + 1))   Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x
) - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

## Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.95

| method | result |
|---|---|
| pseudoelliptic | $\frac{a^{3} D \operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{x\sqrt{b}}\right) x^{3} b\sqrt{b\,x^{2}+a}+\frac{8 b^{\frac{3}{2}} \left(A\,x^{4} b^{3}+\frac{\left(-\frac{3 x^{2} B}{2}+A\right) x^{2} a\,b^{2}}{2}-\frac{a^{2} \left(-3 C\,x^{4}+3 x^{2} B+A\right) b}{8}-\frac{3 a^{3} D x^{4}}{8}\right)}{3}}{x^{3} b^{\frac{5}{2}} \sqrt{b\,x^{2}+a}\,a^{3}}$ |
| default | $\frac{C x}{a\sqrt{b\,x^{2}+a}}+A\left(-\frac{1}{3 a\,x^{3}\sqrt{b\,x^{2}+a}}-\frac{4 b\left(-\frac{1}{a x\sqrt{b\,x^{2}+a}}-\frac{2 b x}{a^{2}\sqrt{b\,x^{2}+a}}\right)}{3 a}\right)+B\left(-\frac{1}{a x\sqrt{b\,x^{2}+a}}-\frac{2 b x}{a^{2}\sqrt{b\,x^{2}+a}}\right)+$ |

input
```
int((D*x^6+C*x^4+B*x^2+A)/x^4/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
8/3/(b*x^2+a)^(1/2)*(3/8*a^3*D*arctanh((b*x^2+a)^(1/2)/x/b^(1/2))*x^3*b*(b
*x^2+a)^(1/2)+b^(3/2)*(A*x^4*b^3+1/2*(-3/2*x^2*B+A)*x^2*a*b^2-1/8*a^2*(-3*
C*x^4+3*B*x^2+A)*b-3/8*a^3*D*x^4))/x^3/b^(5/2)/a^3
```

## Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 293, normalized size of antiderivative = 2.38

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4\left(a+bx^2\right)^{3/2}}\,dx = \left[\frac{3\left(Da^3bx^5+Da^4x^3\right)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\left(Aa^2b^2\right.}{6\left(a^3b^3x\right.}\right.$$

$$-\frac{3\left(Da^3bx^5+Da^4x^3\right)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)+\left(Aa^2b^2+\left(3\,Da^3b-3\,Ca^2b^2+6\,Bab^3-8\,Ab^4\right)x^4+\left(3\,Ba^2\right.\right.}{3\left(a^3b^3x^5+a^4b^2x^3\right)}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^4/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/6*(3*(D*a^3*b*x^5 + D*a^4*x^3)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)
*sqrt(b)*x - a) - 2*(A*a^2*b^2 + (3*D*a^3*b - 3*C*a^2*b^2 + 6*B*a*b^3 - 8*
A*b^4)*x^4 + (3*B*a^2*b^2 - 4*A*a*b^3)*x^2)*sqrt(b*x^2 + a))/(a^3*b^3*x^5
+ a^4*b^2*x^3), -1/3*(3*(D*a^3*b*x^5 + D*a^4*x^3)*sqrt(-b)*arctan(sqrt(-b)
*x/sqrt(b*x^2 + a)) + (A*a^2*b^2 + (3*D*a^3*b - 3*C*a^2*b^2 + 6*B*a*b^3 -
8*A*b^4)*x^4 + (3*B*a^2*b^2 - 4*A*a*b^3)*x^2)*sqrt(b*x^2 + a))/(a^3*b^3*x^
5 + a^4*b^2*x^3)]
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 345 vs. $2(109) = 218$.

Time = 6.58 (sec) , antiderivative size = 345, normalized size of antiderivative = 2.80

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4(a+bx^2)^{3/2}}\,dx = A\left(-\frac{a^3b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4x^2+6a^4b^5x^4+3a^3b^6x^6} + \frac{3a^2b^{\frac{11}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4x^2+6a^4b^5x^4+3a^3b^6x^6} + \frac{12ab^{\frac{13}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4x^2+6a^4b^5x^4+3a^3b^6x^6} + \frac{8b^{\frac{15}{2}}x^6\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4x^2+6a^4b^5x^4+3a^3b^6x^6}\right) + B\left(-\frac{1}{a\sqrt{b}x^2\sqrt{\frac{a}{bx^2}+1}} - \frac{2\sqrt{b}}{a^2\sqrt{\frac{a}{bx^2}+1}}\right) + \frac{Cx}{a^{\frac{3}{2}}\sqrt{1+\frac{bx^2}{a}}} + D\left(\frac{\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{\frac{3}{2}}} - \frac{x}{\sqrt{a}b\sqrt{1+\frac{bx^2}{a}}}\right)$$

input
```
integrate((D*x**6+C*x**4+B*x**2+A)/x**4/(b*x**2+a)**(3/2),x)
```

output
```
A*(-a**3*b**(9/2)*sqrt(a/(b*x**2) + 1)/(3*a**5*b**4*x**2 + 6*a**4*b**5*x**
4 + 3*a**3*b**6*x**6) + 3*a**2*b**(11/2)*x**2*sqrt(a/(b*x**2) + 1)/(3*a**5
*b**4*x**2 + 6*a**4*b**5*x**4 + 3*a**3*b**6*x**6) + 12*a*b**(13/2)*x**4*sq
rt(a/(b*x**2) + 1)/(3*a**5*b**4*x**2 + 6*a**4*b**5*x**4 + 3*a**3*b**6*x**6
) + 8*b**(15/2)*x**6*sqrt(a/(b*x**2) + 1)/(3*a**5*b**4*x**2 + 6*a**4*b**5*
x**4 + 3*a**3*b**6*x**6)) + B*(-1/(a*sqrt(b)*x**2*sqrt(a/(b*x**2) + 1)) -
2*sqrt(b)/(a**2*sqrt(a/(b*x**2) + 1))) + C*x/(a**(3/2)*sqrt(1 + b*x**2/a))
 + D*(asinh(sqrt(b)*x/sqrt(a))/b**(3/2) - x/(sqrt(a)*b*sqrt(1 + b*x**2/a))
)
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.11

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4(a+bx^2)^{3/2}}\,dx = \frac{Cx}{\sqrt{bx^2+a}a} - \frac{Dx}{\sqrt{bx^2+a}b} - \frac{2\,Bbx}{\sqrt{bx^2+a}a^2} + \frac{8\,Ab^2x}{3\,\sqrt{bx^2+a}a^3} + \frac{D\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{3}{2}}} - \frac{B}{\sqrt{bx^2+a}ax} + \frac{4\,Ab}{3\,\sqrt{bx^2+a}a^2x} - \frac{A}{3\,\sqrt{bx^2+a}ax^3}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^4/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
C*x/(sqrt(b*x^2 + a)*a) - D*x/(sqrt(b*x^2 + a)*b) - 2*B*b*x/(sqrt(b*x^2 +
a)*a^2) + 8/3*A*b^2*x/(sqrt(b*x^2 + a)*a^3) + D*arcsinh(b*x/sqrt(a*b))/b^(
3/2) - B/(sqrt(b*x^2 + a)*a*x) + 4/3*A*b/(sqrt(b*x^2 + a)*a^2*x) - 1/3*A/(
sqrt(b*x^2 + a)*a*x^3)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 224 vs. $2(107) = 214$.

Time = 0.14 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.82

$$\int \frac{A + Bx^2 + Cx^4 + Dx^6}{x^4 (a + bx^2)^{3/2}} \, dx =$$

$$-\frac{D \log \left( \left( \sqrt{b}x - \sqrt{bx^2 + a} \right)^2 \right)}{2\, b^{\frac{3}{2}}} - \frac{(Da^3 - Ca^2b + Bab^2 - Ab^3)x}{\sqrt{bx^2 + a}a^3b} + \frac{2 \left( 3 \left( \sqrt{b}x - \sqrt{bx^2 + a} \right)^4 Ba\sqrt{b} - 3 \left( \sqrt{b}x - \sqrt{bx^2 + a} \right)^4 Ab^{\frac{3}{2}} - 6 \left( \sqrt{b}x - \sqrt{bx^2 + a} \right)^2 Ba^2\sqrt{b} + 12 \left( \sqrt{b}x \right.\right.}{3 \left( \left( \sqrt{b}x - \sqrt{bx^2 + a} \right)^2 - a \right)^3 a^2}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^4/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
-1/2*D*log((sqrt(b)*x - sqrt(b*x^2 + a))^2)/b^(3/2) - (D*a^3 - C*a^2*b + B
*a*b^2 - A*b^3)*x/(sqrt(b*x^2 + a)*a^3*b) + 2/3*(3*(sqrt(b)*x - sqrt(b*x^2
 + a))^4*B*a*sqrt(b) - 3*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*b^(3/2) - 6*(sq
rt(b)*x - sqrt(b*x^2 + a))^2*B*a^2*sqrt(b) + 12*(sqrt(b)*x - sqrt(b*x^2 +
a))^2*A*a*b^(3/2) + 3*B*a^3*sqrt(b) - 5*A*a^2*b^(3/2))/(((sqrt(b)*x - sqrt
(b*x^2 + a))^2 - a)^3*a^2)
```

## Mupad [B] (verification not implemented)

Time = 1.65 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.06

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4\left(a+bx^2\right)^{3/2}}\,dx = \frac{\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)\,D}{b^{3/2}} - \frac{\sqrt{b\,x^2+a}\,\left(\frac{B}{a}+\frac{2\,B\,b\,x^2}{a^2}\right)}{b\,x^3+a\,x} - \frac{x\,D}{b\,\sqrt{b\,x^2+a}} + \frac{C\,x}{a\,\sqrt{b\,x^2+a}} + \frac{A\,\left(-a^2+4\,a\,b\,x^2+8\,b^2\,x^4\right)}{3\,a^3\,x^3\,\sqrt{b\,x^2+a}}$$

input
```
int((A + B*x^2 + C*x^4 + x^6*D)/(x^4*(a + b*x^2)^(3/2)),x)
```

output
```
(log(b^(1/2)*x + (a + b*x^2)^(1/2))*D)/b^(3/2) - ((a + b*x^2)^(1/2)*(B/a +
 (2*B*b*x^2)/a^2))/(a*x + b*x^3) - (x*D)/(b*(a + b*x^2)^(1/2)) + (C*x)/(a*
(a + b*x^2)^(1/2)) + (A*(8*b^2*x^4 - a^2 + 4*a*b*x^2))/(3*a^3*x^3*(a + b*x
^2)^(1/2))
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.90

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4\left(a+bx^2\right)^{3/2}}\,dx = \frac{-\sqrt{b\,x^2+a}\,a^2b^2-3\sqrt{b\,x^2+a}\,a^2bd\,x^4+\sqrt{b\,x^2+a}\,a\,b^3x^2+3\sqrt{b\,x^2+a}\,a\,b}{}$$

input
```
int((D*x^6+C*x^4+B*x^2+A)/x^4/(b*x^2+a)^(3/2),x)
```

output
```
( - sqrt(a + b*x**2)*a**2*b**2 - 3*sqrt(a + b*x**2)*a**2*b*d*x**4 + sqrt(a
 + b*x**2)*a*b**3*x**2 + 3*sqrt(a + b*x**2)*a*b**2*c*x**4 + 2*sqrt(a + b*x
**2)*b**4*x**4 + 3*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**
3*d*x**3 + 3*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2*b*d*
x**5 + 6*sqrt(b)*a**3*d*x**3 - 6*sqrt(b)*a**2*b*c*x**3 + 6*sqrt(b)*a**2*b*
d*x**5 - 2*sqrt(b)*a*b**3*x**3 - 6*sqrt(b)*a*b**2*c*x**5 - 2*sqrt(b)*b**4*
x**5)/(3*a**2*b**2*x**3*(a + b*x**2))
```

# 3.237 $\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6\left(a+bx^2\right)^{3/2}}\,dx$



## Optimal result

Integrand size = 32, antiderivative size = 134

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6\left(a+bx^2\right)^{3/2}}\,dx = -\frac{\left(Ab^3-a(b^2B-abC+a^2D)\right)x}{a^4\sqrt{a+bx^2}} - \frac{A\sqrt{a+bx^2}}{5a^2x^5} + \frac{(9Ab-5aB)\sqrt{a+bx^2}}{15a^3x^3} - \frac{\left(33Ab^2-25abB+15a^2C\right)\sqrt{a+bx^2}}{15a^4x}$$

output
```
-(A*b^3-a*(B*b^2-C*a*b+D*a^2))*x/a^4/(b*x^2+a)^(1/2)-1/5*A*(b*x^2+a)^(1/2)
/a^2/x^5+1/15*(9*A*b-5*B*a)*(b*x^2+a)^(1/2)/a^3/x^3-1/15*(33*A*b^2-25*B*a*
b+15*C*a^2)*(b*x^2+a)^(1/2)/a^4/x
```

## Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.77

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6\left(a+bx^2\right)^{3/2}}\,dx = \frac{-48Ab^3x^6+8ab^2x^4(-3A+5Bx^2)+2a^2bx^2(3A+10Bx^2-15Cx^4)-a^3(}{15a^4x^5\sqrt{a+bx^2}}$$

input
```
Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(x^6*(a + b*x^2)^(3/2)),x]
```

output
```
(-48*A*b^3*x^6 + 8*a*b^2*x^4*(-3*A + 5*B*x^2) + 2*a^2*b*x^2*(3*A + 10*B*x^
2 - 15*C*x^4) - a^3*(3*A + 5*x^2*(B + 3*C*x^2 - 3*D*x^4)))/(15*a^4*x^5*Sqr
t[a + b*x^2])
```

## Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {2334, 2089, 1588, 359, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6\left(a+bx^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{2334}$$

$$-\frac{\int \frac{6Ab-5a\left(Dx^4+Cx^2+B\right)}{x^4\left(bx^2+a\right)^{3/2}}dx}{5a}-\frac{A}{5ax^5\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{2089}$$

$$-\frac{\int \frac{-5aDx^4-5aCx^2+6Ab-5aB}{x^4\left(bx^2+a\right)^{3/2}}dx}{5a}-\frac{A}{5ax^5\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{1588}$$

$$-\frac{-\frac{\int \frac{15Dx^2a^2+15Ca^2-20bBa+24Ab^2}{x^2\left(bx^2+a\right)^{3/2}}dx}{3a}-\frac{6Ab-5aB}{3ax^3\sqrt{a+bx^2}}}{5a}-\frac{A}{5ax^5\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{359}$$

$$-\frac{-\frac{-\frac{\left(48Ab^3-5a\left(3a^2D-6abC+8b^2B\right)\right)\int \frac{1}{\left(bx^2+a\right)^{3/2}}dx}{a}-\frac{15a^2C-20abB+24Ab^2}{ax\sqrt{a+bx^2}}}{3a}-\frac{6Ab-5aB}{3ax^3\sqrt{a+bx^2}}}{5a}-\frac{A}{5ax^5\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{208}$$

$$-\frac{-\frac{-\frac{15a^2C-20abB+24Ab^2}{ax\sqrt{a+bx^2}}-\frac{x\left(48Ab^3-5a\left(3a^2D-6abC+8b^2B\right)\right)}{a^2\sqrt{a+bx^2}}}{3a}-\frac{6Ab-5aB}{3ax^3\sqrt{a+bx^2}}}{5a}-\frac{A}{5ax^5\sqrt{a+bx^2}}$$

input
```
Int[(A + B*x^2 + C*x^4 + D*x^6)/(x^6*(a + b*x^2)^(3/2)),x]
```

output
```
-1/5*A/(a*x^5*Sqrt[a + b*x^2]) - (-1/3*(6*A*b - 5*a*B)/(a*x^3*Sqrt[a + b*x
^2]) - (-((24*A*b^2 - 20*a*b*B + 15*a^2*C)/(a*x*Sqrt[a + b*x^2])) - ((48*A
*b^3 - 5*a*(8*b^2*B - 6*a*b*C + 3*a^2*D))*x)/(a^2*Sqrt[a + b*x^2]))/(3*a))
/(5*a)
```

## Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 1588
```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Simp[1/(d*f
^2*(m + 1))   Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x
) - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

rule 2089
```
Int[(u_)^(p_.)*((f_.)*(x_))^(m_.)*(z_)^(q_.), x_Symbol] :> Int[(f*x)^m*Expa
ndToSum[z, x]^q*ExpandToSum[u, x]^p, x] /; FreeQ[{f, m, p, q}, x] && Binomi
alQ[z, x] && TrinomialQ[u, x] &&  !(BinomialMatchQ[z, x] && TrinomialMatchQ
[u, x])
```

rule 2334
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coef
f[Pq, x, 0], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[A*
x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))   Int[
x^(m + 2)*(a + b*x^2)^p*(a*(m + 1)*Q - A*b*(m + 2*(p + 1) + 1)), x], x]] /;
 FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && IntegerQ[m/2] && ILtQ[(m + 1)/2 + p,
 0] && LtQ[m + Expon[Pq, x] + 2*p + 1, 0]
```

## Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.69

| method | result |
|---|---|
| pseudoelliptic | $-\frac{\left(-5D x^{6}+5C\,x^{4}+\frac{5}{3}x^{2}B+A\right)a^{3}-2\left(-5C\,x^{4}+\frac{10}{3}x^{2}B+A\right)x^{2}b\,a^{2}+8\left(-\frac{5x^{2}B}{3}+A\right)x^{4}b^{2}a+16A\,b^{3}x^{6}}{5\sqrt{b\,x^{2}+a}\,x^{5}a^{4}}$ |
| gosper | $-\frac{48A\,b^{3}x^{6}-40Ba\,b^{2}x^{6}+30C\,a^{2}b\,x^{6}-15Da^{3}x^{6}+24aA\,b^{2}x^{4}-20B\,a^{2}b\,x^{4}+15C\,a^{3}x^{4}-6a^{2}Ab\,x^{2}+5B\,a^{3}x^{2}+3a^{3}A}{15x^{5}\sqrt{b\,x^{2}+a}\,a^{4}}$ |
| trager | $-\frac{48A\,b^{3}x^{6}-40Ba\,b^{2}x^{6}+30C\,a^{2}b\,x^{6}-15Da^{3}x^{6}+24aA\,b^{2}x^{4}-20B\,a^{2}b\,x^{4}+15C\,a^{3}x^{4}-6a^{2}Ab\,x^{2}+5B\,a^{3}x^{2}+3a^{3}A}{15x^{5}\sqrt{b\,x^{2}+a}\,a^{4}}$ |
| orering | $-\frac{48A\,b^{3}x^{6}-40Ba\,b^{2}x^{6}+30C\,a^{2}b\,x^{6}-15Da^{3}x^{6}+24aA\,b^{2}x^{4}-20B\,a^{2}b\,x^{4}+15C\,a^{3}x^{4}-6a^{2}Ab\,x^{2}+5B\,a^{3}x^{2}+3a^{3}A}{15x^{5}\sqrt{b\,x^{2}+a}\,a^{4}}$ |
| default | $\frac{Dx}{a\sqrt{b\,x^{2}+a}}+A\left(-\frac{1}{5a\,x^{5}\sqrt{b\,x^{2}+a}}-\frac{6b\left(-\frac{1}{3a\,x^{3}\sqrt{b\,x^{2}+a}}-\frac{4b\left(-\frac{1}{ax\sqrt{b\,x^{2}+a}}-\frac{2bx}{a^{2}\sqrt{b\,x^{2}+a}}\right)}{3a}\right)}{5a}\right)+B\left(-\frac{1}{3a\,x^{3}\sqrt{b\,x}}\right.$ |

input
```
int((D*x^6+C*x^4+B*x^2+A)/x^6/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5/(b*x^2+a)^(1/2)*((-5*D*x^6+5*C*x^4+5/3*x^2*B+A)*a^3-2*(-5*C*x^4+10/3*
x^2*B+A)*x^2*b*a^2+8*(-5/3*x^2*B+A)*x^4*b^2*a+16*A*b^3*x^6)/x^5/a^4
```

## Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.84

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6\left(a+bx^2\right)^{3/2}}\,dx = \frac{((15\,Da^3-30\,Ca^2b+40\,Bab^2-48\,Ab^3)x^6-(15\,Ca^3-20\,Ba^2b+24\,Aab}{15\,(a^4bx^7+a^5x^5)}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^6/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
1/15*((15*D*a^3 - 30*C*a^2*b + 40*B*a*b^2 - 48*A*b^3)*x^6 - (15*C*a^3 - 20
*B*a^2*b + 24*A*a*b^2)*x^4 - 3*A*a^3 - (5*B*a^3 - 6*A*a^2*b)*x^2)*sqrt(b*x
^2 + a)/(a^4*b*x^7 + a^5*x^5)
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 663 vs. 2(122) = 244.

Time = 8.61 (sec) , antiderivative size = 663, normalized size of antiderivative = 4.95

$$\begin{aligned}\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6\left(a+bx^2\right)^{3/2}}\,dx = {} & A\left(-\frac{a^5b^{\frac{19}{2}}\sqrt{\frac{a}{bx^2}+1}}{5a^7b^9x^4+15a^6b^{10}x^6+15a^5b^{11}x^8+5a^4b^{12}x^{10}}\right.\\
&-\frac{5a^3b^{\frac{23}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{5a^7b^9x^4+15a^6b^{10}x^6+15a^5b^{11}x^8+5a^4b^{12}x^{10}}\\
&-\frac{30a^2b^{\frac{25}{2}}x^6\sqrt{\frac{a}{bx^2}+1}}{5a^7b^9x^4+15a^6b^{10}x^6+15a^5b^{11}x^8+5a^4b^{12}x^{10}}\\
&-\frac{40ab^{\frac{27}{2}}x^8\sqrt{\frac{a}{bx^2}+1}}{5a^7b^9x^4+15a^6b^{10}x^6+15a^5b^{11}x^8+5a^4b^{12}x^{10}}\\
&\left.-\frac{16b^{\frac{29}{2}}x^{10}\sqrt{\frac{a}{bx^2}+1}}{5a^7b^9x^4+15a^6b^{10}x^6+15a^5b^{11}x^8+5a^4b^{12}x^{10}}\right)\\
&+B\left(-\frac{a^3b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4x^2+6a^4b^5x^4+3a^3b^6x^6}+\frac{3a^2b^{\frac{11}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4x^2+6a^4b^5x^4+3a^3b^6x^6}\right.\\
&\left.+\frac{12ab^{\frac{13}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4x^2+6a^4b^5x^4+3a^3b^6x^6}+\frac{8b^{\frac{15}{2}}x^6\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4x^2+6a^4b^5x^4+3a^3b^6x^6}\right)\\
&+C\left(-\frac{1}{a\sqrt{b}x^2\sqrt{\frac{a}{bx^2}+1}}-\frac{2\sqrt{b}}{a^2\sqrt{\frac{a}{bx^2}+1}}\right)+\frac{Dx}{a^{\frac{3}{2}}\sqrt{1+\frac{bx^2}{a}}}\end{aligned}$$

input
```
integrate((D*x**6+C*x**4+B*x**2+A)/x**6/(b*x**2+a)**(3/2),x)
```

output
```
A*(-a**5*b**(19/2)*sqrt(a/(b*x**2) + 1)/(5*a**7*b**9*x**4 + 15*a**6*b**10*
x**6 + 15*a**5*b**11*x**8 + 5*a**4*b**12*x**10) - 5*a**3*b**(23/2)*x**4*sq
rt(a/(b*x**2) + 1)/(5*a**7*b**9*x**4 + 15*a**6*b**10*x**6 + 15*a**5*b**11*
x**8 + 5*a**4*b**12*x**10) - 30*a**2*b**(25/2)*x**6*sqrt(a/(b*x**2) + 1)/(
5*a**7*b**9*x**4 + 15*a**6*b**10*x**6 + 15*a**5*b**11*x**8 + 5*a**4*b**12*
x**10) - 40*a*b**(27/2)*x**8*sqrt(a/(b*x**2) + 1)/(5*a**7*b**9*x**4 + 15*a
**6*b**10*x**6 + 15*a**5*b**11*x**8 + 5*a**4*b**12*x**10) - 16*b**(29/2)*x
**10*sqrt(a/(b*x**2) + 1)/(5*a**7*b**9*x**4 + 15*a**6*b**10*x**6 + 15*a**5
*b**11*x**8 + 5*a**4*b**12*x**10)) + B*(-a**3*b**(9/2)*sqrt(a/(b*x**2) + 1
)/(3*a**5*b**4*x**2 + 6*a**4*b**5*x**4 + 3*a**3*b**6*x**6) + 3*a**2*b**(11
/2)*x**2*sqrt(a/(b*x**2) + 1)/(3*a**5*b**4*x**2 + 6*a**4*b**5*x**4 + 3*a**
3*b**6*x**6) + 12*a*b**(13/2)*x**4*sqrt(a/(b*x**2) + 1)/(3*a**5*b**4*x**2
+ 6*a**4*b**5*x**4 + 3*a**3*b**6*x**6) + 8*b**(15/2)*x**6*sqrt(a/(b*x**2)
+ 1)/(3*a**5*b**4*x**2 + 6*a**4*b**5*x**4 + 3*a**3*b**6*x**6)) + C*(-1/(a*
sqrt(b)*x**2*sqrt(a/(b*x**2) + 1)) - 2*sqrt(b)/(a**2*sqrt(a/(b*x**2) + 1))
) + D*x/(a**(3/2)*sqrt(1 + b*x**2/a))
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.37

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6\left(a+bx^2\right)^{3/2}}\,dx = \frac{Dx}{\sqrt{bx^2+a}a} - \frac{2\,Cbx}{\sqrt{bx^2+a}a^2} + \frac{8\,Bb^2x}{3\,\sqrt{bx^2+a}a^3} - \frac{16\,Ab^3x}{5\,\sqrt{bx^2+a}a^4} - \frac{C}{\sqrt{bx^2+a}ax} + \frac{4\,Bb}{3\,\sqrt{bx^2+a}a^2x} - \frac{8\,Ab^2}{5\,\sqrt{bx^2+a}a^3x} - \frac{B}{3\,\sqrt{bx^2+a}ax^3} + \frac{2\,Ab}{5\,\sqrt{bx^2+a}a^2x^3} - \frac{A}{5\,\sqrt{bx^2+a}ax^5}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^6/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
D*x/(sqrt(b*x^2 + a)*a) - 2*C*b*x/(sqrt(b*x^2 + a)*a^2) + 8/3*B*b^2*x/(sqr
t(b*x^2 + a)*a^3) - 16/5*A*b^3*x/(sqrt(b*x^2 + a)*a^4) - C/(sqrt(b*x^2 + a
)*a*x) + 4/3*B*b/(sqrt(b*x^2 + a)*a^2*x) - 8/5*A*b^2/(sqrt(b*x^2 + a)*a^3*
x) - 1/3*B/(sqrt(b*x^2 + a)*a*x^3) + 2/5*A*b/(sqrt(b*x^2 + a)*a^2*x^3) - 1
/5*A/(sqrt(b*x^2 + a)*a*x^5)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 427 vs. $2(120) = 240$.

Time = 0.15 (sec) , antiderivative size = 427, normalized size of antiderivative = 3.19

$$\int \frac{A + Bx^2 + Cx^4 + Dx^6}{x^6 (a + bx^2)^{3/2}}\,dx = \frac{(Da^3 - Ca^2b + Bab^2 - Ab^3)x}{\sqrt{bx^2 + a}a^4}$$
$$+ \frac{2\left(15\left(\sqrt{b}x - \sqrt{bx^2+a}\right)^8 Ca^2\sqrt{b} - 15\left(\sqrt{b}x - \sqrt{bx^2+a}\right)^8 Bab^{\frac{3}{2}} + 15\left(\sqrt{b}x - \sqrt{bx^2+a}\right)^8 Ab^{\frac{5}{2}} - 60\left(\sqrt{\right.}\right.}{}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^6/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
(D*a^3 - C*a^2*b + B*a*b^2 - A*b^3)*x/(sqrt(b*x^2 + a)*a^4) + 2/15*(15*(sq
rt(b)*x - sqrt(b*x^2 + a))^8*C*a^2*sqrt(b) - 15*(sqrt(b)*x - sqrt(b*x^2 +
a))^8*B*a*b^(3/2) + 15*(sqrt(b)*x - sqrt(b*x^2 + a))^8*A*b^(5/2) - 60*(sqr
t(b)*x - sqrt(b*x^2 + a))^6*C*a^3*sqrt(b) + 90*(sqrt(b)*x - sqrt(b*x^2 + a
))^6*B*a^2*b^(3/2) - 90*(sqrt(b)*x - sqrt(b*x^2 + a))^6*A*a*b^(5/2) + 90*(
sqrt(b)*x - sqrt(b*x^2 + a))^4*C*a^4*sqrt(b) - 160*(sqrt(b)*x - sqrt(b*x^2
 + a))^4*B*a^3*b^(3/2) + 240*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*a^2*b^(5/2)
 - 60*(sqrt(b)*x - sqrt(b*x^2 + a))^2*C*a^5*sqrt(b) + 110*(sqrt(b)*x - sqr
t(b*x^2 + a))^2*B*a^4*b^(3/2) - 150*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A*a^3*
b^(5/2) + 15*C*a^6*sqrt(b) - 25*B*a^5*b^(3/2) + 33*A*a^4*b^(5/2))/(((sqrt(
b)*x - sqrt(b*x^2 + a))^2 - a)^5*a^3)
```

## Mupad [B] (verification not implemented)

Time = 1.79 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.05

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6\left(a+bx^2\right)^{3/2}}\,dx=\frac{x\,D}{a\,\sqrt{b\,x^2+a}}-\frac{\sqrt{b\,x^2+a}\,\left(\frac{C}{a}+\frac{2\,C\,b\,x^2}{a^2}\right)}{b\,x^3+a\,x}+\frac{B\,\left(-a^2+4\,a\,b\,x^2+8\,b^2\,x^4\right)}{3\,a^3\,x^3\,\sqrt{b\,x^2+a}}-\frac{A\,\left(a^3-2\,a^2\,b\,x^2+8\,a\,b^2\,x^4+16\,b^3\,x^6\right)}{5\,a^4\,x^5\,\sqrt{b\,x^2+a}}$$

input
```
int((A + B*x^2 + C*x^4 + x^6*D)/(x^6*(a + b*x^2)^(3/2)),x)
```

output
```
(x*D)/(a*(a + b*x^2)^(1/2)) - ((a + b*x^2)^(1/2)*(C/a + (2*C*b*x^2)/a^2))/
(a*x + b*x^3) + (B*(8*b^2*x^4 - a^2 + 4*a*b*x^2))/(3*a^3*x^3*(a + b*x^2)^(
1/2)) - (A*(a^3 + 16*b^3*x^6 - 2*a^2*b*x^2 + 8*a*b^2*x^4))/(5*a^4*x^5*(a +
 b*x^2)^(1/2))
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.56

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6\left(a+bx^2\right)^{3/2}}\,dx=\frac{-6\sqrt{b\,x^2+a}\,a^3b+2\sqrt{b\,x^2+a}\,a^2b^2x^2-30\sqrt{b\,x^2+a}\,a^2bc\,x^4+30\sqrt{b\,x^2+}$$

input
```
int((D*x^6+C*x^4+B*x^2+A)/x^6/(b*x^2+a)^(3/2),x)
```

output
```
( - 6*sqrt(a + b*x**2)*a**3*b + 2*sqrt(a + b*x**2)*a**2*b**2*x**2 - 30*sqr
t(a + b*x**2)*a**2*b*c*x**4 + 30*sqrt(a + b*x**2)*a**2*b*d*x**6 - 8*sqrt(a
 + b*x**2)*a*b**3*x**4 - 60*sqrt(a + b*x**2)*a*b**2*c*x**6 - 16*sqrt(a + b
*x**2)*b**4*x**6 - 45*sqrt(b)*a**3*d*x**5 + 60*sqrt(b)*a**2*b*c*x**5 - 45*
sqrt(b)*a**2*b*d*x**7 + 16*sqrt(b)*a*b**3*x**5 + 60*sqrt(b)*a*b**2*c*x**7
+ 16*sqrt(b)*b**4*x**7)/(30*a**3*b*x**5*(a + b*x**2))
```

# 3.238 $\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8(a+bx^2)^{3/2}}\,dx$



## Optimal result

Integrand size = 32, antiderivative size = 183

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8\left(a+bx^2\right)^{3/2}}\,dx = \frac{b(Ab^3-a(b^2B-abC+a^2D))\,x}{a^5\sqrt{a+bx^2}}-\frac{A\sqrt{a+bx^2}}{7a^2x^7}+\frac{(13Ab-7aB)\sqrt{a+bx^2}}{35a^3x^5}-\frac{(87Ab^2-63abB+35a^2C)\sqrt{a+bx^2}}{105a^4x^3}+\frac{(279Ab^3-7a(33b^2B-25abC+15a^2D))\sqrt{a+bx^2}}{105a^5x}$$

output
```
b*(A*b^3-a*(B*b^2-C*a*b+D*a^2))*x/a^5/(b*x^2+a)^(1/2)-1/7*A*(b*x^2+a)^(1/2
)/a^2/x^7+1/35*(13*A*b-7*B*a)*(b*x^2+a)^(1/2)/a^3/x^5-1/105*(87*A*b^2-63*B
*a*b+35*C*a^2)*(b*x^2+a)^(1/2)/a^4/x^3+1/105*(279*A*b^3-7*a*(33*B*b^2-25*C
*a*b+15*D*a^2))*(b*x^2+a)^(1/2)/a^5/x
```

## Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.74

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8\left(a+bx^2\right)^{3/2}}\,dx=\frac{384Ab^4x^8+48ab^3x^6(4A-7Bx^2)+8a^2b^2x^4(-6A-21Bx^2+35Cx^4)+2}{105a}$$

input
```
Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(x^8*(a + b*x^2)^(3/2)),x]
```

output
```
(384*A*b^4*x^8 + 48*a*b^3*x^6*(4*A - 7*B*x^2) + 8*a^2*b^2*x^4*(-6*A - 21*B
*x^2 + 35*C*x^4) + 2*a^3*b*x^2*(12*A + 21*B*x^2 + 70*C*x^4 - 105*D*x^6) -
a^4*(15*A + 21*B*x^2 + 35*x^4*(C + 3*D*x^2)))/(105*a^5*x^7*Sqrt[a + b*x^2]
)
```

## Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2334, 2089, 1588, 359, 245, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8\left(a+bx^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{2334}$$

$$-\frac{\int \frac{8Ab-7a\left(Dx^4+Cx^2+B\right)}{x^6\left(bx^2+a\right)^{3/2}}dx}{7a}-\frac{A}{7ax^7\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{2089}$$

$$-\frac{\int \frac{-7aDx^4-7aCx^2+8Ab-7aB}{x^6\left(bx^2+a\right)^{3/2}}dx}{7a}-\frac{A}{7ax^7\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{1588}$$

$$-\frac{-\frac{\frac{\int \frac{35Dx^2a^2+35Ca^2-42bBa+48Ab^2}{x^4(bx^2+a)^{3/2}}dx}{5a}-\frac{8Ab-7aB}{5ax^5\sqrt{a+bx^2}}}{7a}-\frac{A}{7ax^7\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{ 359}$$

$$-\frac{-\frac{-\frac{(-105a^3D-28ab(6bB-5aC)+192Ab^3)\int \frac{1}{x^2(bx^2+a)^{3/2}}dx}{3a}-\frac{35a^2C-42abB+48Ab^2}{3ax^3\sqrt{a+bx^2}}}{5a}-\frac{8Ab-7aB}{5ax^5\sqrt{a+bx^2}}}{7a}-\frac{A}{7ax^7\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{ 245}$$

$$-\frac{-\frac{-\frac{(-105a^3D-28ab(6bB-5aC)+192Ab^3)\left(-\frac{2b\int \frac{1}{(bx^2+a)^{3/2}}dx}{a}-\frac{1}{ax\sqrt{a+bx^2}}\right)}{3a}-\frac{35a^2C-42abB+48Ab^2}{3ax^3\sqrt{a+bx^2}}}{5a}-\frac{8Ab-7aB}{5ax^5\sqrt{a+bx^2}}}{7a}-\frac{A}{7ax^7\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{ 208}$$

$$-\frac{-\frac{-\frac{35a^2C-42abB+48Ab^2}{3ax^3\sqrt{a+bx^2}}-\frac{\left(-\frac{2bx}{a^2\sqrt{a+bx^2}}-\frac{1}{ax\sqrt{a+bx^2}}\right)(-105a^3D-28ab(6bB-5aC)+192Ab^3)}{3a}}{5a}-\frac{8Ab-7aB}{5ax^5\sqrt{a+bx^2}}}{7a}-\frac{A}{7ax^7\sqrt{a+bx^2}}$$

input

```
Int[(A + B*x^2 + C*x^4 + D*x^6)/(x^8*(a + b*x^2)^(3/2)),x]
```

output

```
-1/7*A/(a*x^7*Sqrt[a + b*x^2]) - (-1/5*(8*A*b - 7*a*B)/(a*x^5*Sqrt[a + b*x
^2]) - (-1/3*(48*A*b^2 - 42*a*b*B + 35*a^2*C)/(a*x^3*Sqrt[a + b*x^2]) - ((
192*A*b^3 - 28*a*b*(6*b*B - 5*a*C) - 105*a^3*D)*(-(1/(a*x*Sqrt[a + b*x^2])
) - (2*b*x)/(a^2*Sqrt[a + b*x^2])))/(3*a))/(5*a))/(7*a)
```

### Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 1588
```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Simp[1/(d*f
^2*(m + 1))   Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x
) - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

rule 2089
```
Int[(u_)^(p_.)*((f_.)*(x_))^(m_.)*(z_)^(q_.), x_Symbol] :> Int[(f*x)^m*Expa
ndToSum[z, x]^q*ExpandToSum[u, x]^p, x] /; FreeQ[{f, m, p, q}, x] && Binomi
alQ[z, x] && TrinomialQ[u, x] &&  !(BinomialMatchQ[z, x] && TrinomialMatchQ
[u, x])
```

rule 2334
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coef
f[Pq, x, 0], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[A*
x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))   Int[
x^(m + 2)*(a + b*x^2)^p*(a*(m + 1)*Q - A*b*(m + 2*(p + 1) + 1)), x], x]] /;
 FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && IntegerQ[m/2] && ILtQ[(m + 1)/2 + p,
 0] && LtQ[m + Expon[Pq, x] + 2*p + 1, 0]
```

## Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.68

| method | result |
|---|---|
| pseudoelliptic | $\frac{(-105Dx^6-35Cx^4-21x^2B-15A)a^4+24(-\frac{35}{4}Dx^6+\frac{35}{6}Cx^4+\frac{7}{4}x^2B+A)x^2ba^3-48(-\frac{35}{6}Cx^4+\frac{7}{2}x^2B+A)x^4b^2a^2+192(}{105\sqrt{bx^2+a}\,x^7a^5}$ |
| gosper | $-\frac{-384Ax^8b^4+336Bx^8ab^3-280Ca^2b^2x^8+210Da^3bx^8-192Ax^6ab^3+168Bx^6a^2b^2-140Ca^3bx^6+105Da^4x^6+48Ax^4a}{105x^7\sqrt{bx^2+a}\,a^5}$ |
| trager | $-\frac{-384Ax^8b^4+336Bx^8ab^3-280Ca^2b^2x^8+210Da^3bx^8-192Ax^6ab^3+168Bx^6a^2b^2-140Ca^3bx^6+105Da^4x^6+48Ax^4a}{105x^7\sqrt{bx^2+a}\,a^5}$ |
| orering | $-\frac{-384Ax^8b^4+336Bx^8ab^3-280Ca^2b^2x^8+210Da^3bx^8-192Ax^6ab^3+168Bx^6a^2b^2-140Ca^3bx^6+105Da^4x^6+48Ax^4a}{105x^7\sqrt{bx^2+a}\,a^5}$ |
| default | $A\left(-\frac{1}{7ax^7\sqrt{bx^2+a}}-\frac{8b\left(-\frac{1}{5ax^5\sqrt{bx^2+a}}-\frac{6b\left(-\frac{1}{3ax^3\sqrt{bx^2+a}}-\frac{4b\left(-\frac{1}{ax\sqrt{bx^2+a}}-\frac{2bx}{a^2\sqrt{bx^2+a}}\right)}{3a}\right)}{5a}\right)}{7a}\right)+B\left(-\frac{}{5a}\right.$ |

input
```
int((D*x^6+C*x^4+B*x^2+A)/x^8/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/105*((-105*D*x^6-35*C*x^4-21*B*x^2-15*A)*a^4+24*(-35/4*D*x^6+35/6*C*x^4+
7/4*x^2*B+A)*x^2*b*a^3-48*(-35/6*C*x^4+7/2*x^2*B+A)*x^4*b^2*a^2+192*(-7/4*
x^2*B+A)*x^6*b^3*a+384*A*x^8*b^4)/(b*x^2+a)^(1/2)/x^7/a^5
```

## Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.83

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8\left(a+bx^2\right)^{3/2}}\,dx =$$

$$-\frac{(2\,(105\,Da^3b-140\,Ca^2b^2+168\,Bab^3-192\,Ab^4)x^8+(105\,Da^4-140\,Ca^3b+168\,Ba^2b^2-192\,Aab^3)x^6}{105\,(a^5bx^9+a^6x^7)}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^8/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
-1/105*(2*(105*D*a^3*b - 140*C*a^2*b^2 + 168*B*a*b^3 - 192*A*b^4)*x^8 + (1
05*D*a^4 - 140*C*a^3*b + 168*B*a^2*b^2 - 192*A*a*b^3)*x^6 + 15*A*a^4 + (35
*C*a^4 - 42*B*a^3*b + 48*A*a^2*b^2)*x^4 + 3*(7*B*a^4 - 8*A*a^3*b)*x^2)*sqr
t(b*x^2 + a)/(a^5*b*x^9 + a^6*x^7)
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1316 vs. $2(175) = 350$.

Time = 12.36 (sec) , antiderivative size = 1316, normalized size of antiderivative = 7.19

$$\int \frac{A + Bx^2 + Cx^4 + Dx^6}{x^8 (a + bx^2)^{3/2}} \, dx = \text{Too large to display}$$

input
```
integrate((D*x**6+C*x**4+B*x**2+A)/x**8/(b*x**2+a)**(3/2),x)
```

output
```
A*(-5*a**7*b**(33/2)*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16*x**6 + 140*a**8*b
**17*x**8 + 210*a**7*b**18*x**10 + 140*a**6*b**19*x**12 + 35*a**5*b**20*x*
*14) - 7*a**6*b**(35/2)*x**2*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16*x**6 + 14
0*a**8*b**17*x**8 + 210*a**7*b**18*x**10 + 140*a**6*b**19*x**12 + 35*a**5*
b**20*x**14) - 7*a**5*b**(37/2)*x**4*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16*x
**6 + 140*a**8*b**17*x**8 + 210*a**7*b**18*x**10 + 140*a**6*b**19*x**12 +
35*a**5*b**20*x**14) + 35*a**4*b**(39/2)*x**6*sqrt(a/(b*x**2) + 1)/(35*a**
9*b**16*x**6 + 140*a**8*b**17*x**8 + 210*a**7*b**18*x**10 + 140*a**6*b**19
*x**12 + 35*a**5*b**20*x**14) + 280*a**3*b**(41/2)*x**8*sqrt(a/(b*x**2) +
1)/(35*a**9*b**16*x**6 + 140*a**8*b**17*x**8 + 210*a**7*b**18*x**10 + 140*
a**6*b**19*x**12 + 35*a**5*b**20*x**14) + 560*a**2*b**(43/2)*x**10*sqrt(a/
(b*x**2) + 1)/(35*a**9*b**16*x**6 + 140*a**8*b**17*x**8 + 210*a**7*b**18*x
**10 + 140*a**6*b**19*x**12 + 35*a**5*b**20*x**14) + 448*a*b**(45/2)*x**12
*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16*x**6 + 140*a**8*b**17*x**8 + 210*a**7
*b**18*x**10 + 140*a**6*b**19*x**12 + 35*a**5*b**20*x**14) + 128*b**(47/2)
*x**14*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16*x**6 + 140*a**8*b**17*x**8 + 21
0*a**7*b**18*x**10 + 140*a**6*b**19*x**12 + 35*a**5*b**20*x**14)) + B*(-a*
*5*b**(19/2)*sqrt(a/(b*x**2) + 1)/(5*a**7*b**9*x**4 + 15*a**6*b**10*x**6 +
 15*a**5*b**11*x**8 + 5*a**4*b**12*x**10) - 5*a**3*b**(23/2)*x**4*sqrt(a/(
b*x**2) + 1)/(5*a**7*b**9*x**4 + 15*a**6*b**10*x**6 + 15*a**5*b**11*x**...
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.46

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8\left(a+bx^2\right)^{3/2}}\,dx=-\frac{2\,Dbx}{\sqrt{bx^2+a}a^2}+\frac{8\,Cb^2x}{3\,\sqrt{bx^2+a}a^3}-\frac{16\,Bb^3x}{5\,\sqrt{bx^2+a}a^4}+\frac{128\,Ab^4x}{35\,\sqrt{bx^2+a}a^5}-\frac{D}{\sqrt{bx^2+a}ax}+\frac{4\,Cb}{3\,\sqrt{bx^2+a}a^2x}-\frac{8\,Bb^2}{5\,\sqrt{bx^2+a}a^3x}+\frac{64\,Ab^3}{35\,\sqrt{bx^2+a}a^4x}-\frac{C}{3\,\sqrt{bx^2+a}ax^3}+\frac{2\,Bb}{5\,\sqrt{bx^2+a}a^2x^3}-\frac{16\,Ab^2}{35\,\sqrt{bx^2+a}a^3x^3}-\frac{B}{5\,\sqrt{bx^2+a}ax^5}+\frac{8\,Ab}{35\,\sqrt{bx^2+a}a^2x^5}-\frac{A}{7\,\sqrt{bx^2+a}ax^7}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^8/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
-2*D*b*x/(sqrt(b*x^2 + a)*a^2) + 8/3*C*b^2*x/(sqrt(b*x^2 + a)*a^3) - 16/5*
B*b^3*x/(sqrt(b*x^2 + a)*a^4) + 128/35*A*b^4*x/(sqrt(b*x^2 + a)*a^5) - D/(
sqrt(b*x^2 + a)*a*x) + 4/3*C*b/(sqrt(b*x^2 + a)*a^2*x) - 8/5*B*b^2/(sqrt(b
*x^2 + a)*a^3*x) + 64/35*A*b^3/(sqrt(b*x^2 + a)*a^4*x) - 1/3*C/(sqrt(b*x^2
 + a)*a*x^3) + 2/5*B*b/(sqrt(b*x^2 + a)*a^2*x^3) - 16/35*A*b^2/(sqrt(b*x^2
 + a)*a^3*x^3) - 1/5*B/(sqrt(b*x^2 + a)*a*x^5) + 8/35*A*b/(sqrt(b*x^2 + a)
*a^2*x^5) - 1/7*A/(sqrt(b*x^2 + a)*a*x^7)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 776 vs. 2(165) = 330.

Time = 0.15 (sec) , antiderivative size = 776, normalized size of antiderivative = 4.24

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8\left(a+bx^2\right)^{3/2}}\,dx=\text{Too large to display}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^8/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
-(D*a^3*b - C*a^2*b^2 + B*a*b^3 - A*b^4)*x/(sqrt(b*x^2 + a)*a^5) + 2/105*(
105*(sqrt(b)*x - sqrt(b*x^2 + a))^12*D*a^3*sqrt(b) - 105*(sqrt(b)*x - sqrt
(b*x^2 + a))^12*C*a^2*b^(3/2) + 105*(sqrt(b)*x - sqrt(b*x^2 + a))^12*B*a*b
^(5/2) - 105*(sqrt(b)*x - sqrt(b*x^2 + a))^12*A*b^(7/2) - 630*(sqrt(b)*x -
 sqrt(b*x^2 + a))^10*D*a^4*sqrt(b) + 840*(sqrt(b)*x - sqrt(b*x^2 + a))^10*
C*a^3*b^(3/2) - 840*(sqrt(b)*x - sqrt(b*x^2 + a))^10*B*a^2*b^(5/2) + 840*(
sqrt(b)*x - sqrt(b*x^2 + a))^10*A*a*b^(7/2) + 1575*(sqrt(b)*x - sqrt(b*x^2
 + a))^8*D*a^5*sqrt(b) - 2485*(sqrt(b)*x - sqrt(b*x^2 + a))^8*C*a^4*b^(3/2
) + 3045*(sqrt(b)*x - sqrt(b*x^2 + a))^8*B*a^3*b^(5/2) - 3045*(sqrt(b)*x -
 sqrt(b*x^2 + a))^8*A*a^2*b^(7/2) - 2100*(sqrt(b)*x - sqrt(b*x^2 + a))^6*D
*a^6*sqrt(b) + 3640*(sqrt(b)*x - sqrt(b*x^2 + a))^6*C*a^5*b^(3/2) - 5040*(
sqrt(b)*x - sqrt(b*x^2 + a))^6*B*a^4*b^(5/2) + 6720*(sqrt(b)*x - sqrt(b*x^
2 + a))^6*A*a^3*b^(7/2) + 1575*(sqrt(b)*x - sqrt(b*x^2 + a))^4*D*a^7*sqrt(
b) - 2835*(sqrt(b)*x - sqrt(b*x^2 + a))^4*C*a^6*b^(3/2) + 4011*(sqrt(b)*x
- sqrt(b*x^2 + a))^4*B*a^5*b^(5/2) - 5019*(sqrt(b)*x - sqrt(b*x^2 + a))^4*
A*a^4*b^(7/2) - 630*(sqrt(b)*x - sqrt(b*x^2 + a))^2*D*a^8*sqrt(b) + 1120*(
sqrt(b)*x - sqrt(b*x^2 + a))^2*C*a^7*b^(3/2) - 1512*(sqrt(b)*x - sqrt(b*x^
2 + a))^2*B*a^6*b^(5/2) + 1848*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A*a^5*b^(7/
2) + 105*D*a^9*sqrt(b) - 175*C*a^8*b^(3/2) + 231*B*a^7*b^(5/2) - 279*A*a^6
*b^(7/2))/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^7*a^4)
```

## Mupad [B] (verification not implemented)

Time = 2.10 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.21

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8 (a+bx^2)^{3/2}}\,dx = \frac{\frac{93\,A\,b^3}{35\,a^4}+\frac{128\,A\,b^4\,x^2}{35\,a^5}}{x\,\sqrt{b\,x^2+a}} - \frac{a^2\,D+2\,b^2\,x^4\,D+3\,a\,b\,x^2\,D}{a^2\,x\,{\left(b\,x^2+a\right)}^{3/2}} - \frac{A\,\sqrt{b\,x^2+a}}{7\,a^2\,x^7} + \frac{13\,A\,b\,\sqrt{b\,x^2+a}}{35\,a^3\,x^5} + \frac{C\,\left(-a^2+4\,a\,b\,x^2+8\,b^2\,x^4\right)}{3\,a^3\,x^3\,\sqrt{b\,x^2+a}} - \frac{29\,A\,b^2\,\sqrt{b\,x^2+a}}{35\,a^4\,x^3} - \frac{B\,\left(a^3-2\,a^2\,b\,x^2+8\,a\,b^2\,x^4+16\,b^3\,x^6\right)}{5\,a^4\,x^5\,\sqrt{b\,x^2+a}}$$

input
```
int((A + B*x^2 + C*x^4 + x^6*D)/(x^8*(a + b*x^2)^(3/2)),x)
```

output
```
((93*A*b^3)/(35*a^4) + (128*A*b^4*x^2)/(35*a^5))/(x*(a + b*x^2)^(1/2)) - (
a^2*D + 2*b^2*x^4*D + 3*a*b*x^2*D)/(a^2*x*(a + b*x^2)^(3/2)) - (A*(a + b*x
^2)^(1/2))/(7*a^2*x^7) + (13*A*b*(a + b*x^2)^(1/2))/(35*a^3*x^5) + (C*(8*b
^2*x^4 - a^2 + 4*a*b*x^2))/(3*a^3*x^3*(a + b*x^2)^(1/2)) - (29*A*b^2*(a +
b*x^2)^(1/2))/(35*a^4*x^3) - (B*(a^3 + 16*b^3*x^6 - 2*a^2*b*x^2 + 8*a*b^2*
x^4))/(5*a^4*x^5*(a + b*x^2)^(1/2))
```

## Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.40

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8 (a+bx^2)^{3/2}} \, dx = \frac{-15\sqrt{b\,x^2+a}\,a^4+3\sqrt{b\,x^2+a}\,a^3b\,x^2-35\sqrt{b\,x^2+a}\,a^3c\,x^4-105\sqrt{b\,x^2+}$$

input
```
int((D*x^6+C*x^4+B*x^2+A)/x^8/(b*x^2+a)^(3/2),x)
```

output
```
( - 15*sqrt(a + b*x**2)*a**4 + 3*sqrt(a + b*x**2)*a**3*b*x**2 - 35*sqrt(a
+ b*x**2)*a**3*c*x**4 - 105*sqrt(a + b*x**2)*a**3*d*x**6 - 6*sqrt(a + b*x*
*2)*a**2*b**2*x**4 + 140*sqrt(a + b*x**2)*a**2*b*c*x**6 - 210*sqrt(a + b*x
**2)*a**2*b*d*x**8 + 24*sqrt(a + b*x**2)*a*b**3*x**6 + 280*sqrt(a + b*x**2
)*a*b**2*c*x**8 + 48*sqrt(a + b*x**2)*b**4*x**8 + 210*sqrt(b)*a**3*d*x**7
- 280*sqrt(b)*a**2*b*c*x**7 + 210*sqrt(b)*a**2*b*d*x**9 - 48*sqrt(b)*a*b**
3*x**7 - 280*sqrt(b)*a*b**2*c*x**9 - 48*sqrt(b)*b**4*x**9)/(105*a**4*x**7*
(a + b*x**2))
```

# 3.239 $\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{10}(a+bx^2)^{3/2}}\,dx$



## Optimal result

Integrand size = 32, antiderivative size = 235

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{10}(a+bx^2)^{3/2}}\,dx = -\frac{b^2(Ab^3-a(b^2B-abC+a^2D))\,x}{a^6\sqrt{a+bx^2}} - \frac{A\sqrt{a+bx^2}}{9a^2x^9} + \frac{(17Ab-9aB)\sqrt{a+bx^2}}{63a^3x^7} - \frac{(55Ab^2-39abB+21a^2C)\sqrt{a+bx^2}}{105a^4x^5} + \frac{(325Ab^3-3a(87b^2B-63abC+35a^2D))\sqrt{a+bx^2}}{315a^5x^3} - \frac{b(965Ab^3-837ab^2B+693a^2bC-525a^3D)\sqrt{a+bx^2}}{315a^6x}$$

output

```
-b^2*(A*b^3-a*(B*b^2-C*a*b+D*a^2))*x/a^6/(b*x^2+a)^(1/2)-1/9*A*(b*x^2+a)^(
1/2)/a^2/x^9+1/63*(17*A*b-9*B*a)*(b*x^2+a)^(1/2)/a^3/x^7-1/105*(55*A*b^2-3
9*B*a*b+21*C*a^2)*(b*x^2+a)^(1/2)/a^4/x^5+1/315*(325*A*b^3-3*a*(87*B*b^2-6
3*C*a*b+35*D*a^2))*(b*x^2+a)^(1/2)/a^5/x^3-1/315*b*(965*A*b^3-837*B*a*b^2+
693*C*a^2*b-525*D*a^3)*(b*x^2+a)^(1/2)/a^6/x
```

## Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.71

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{10}\left(a+bx^2\right)^{3/2}}\,dx = \frac{-1280Ab^5x^{10}+128ab^4x^8(-5A+9Bx^2)+16a^2b^3x^6(10A+36Bx^2-63C$$

input
```
Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(x^10*(a + b*x^2)^(3/2)),x]
```

output
```
(-1280*A*b^5*x^10 + 128*a*b^4*x^8*(-5*A + 9*B*x^2) + 16*a^2*b^3*x^6*(10*A
+ 36*B*x^2 - 63*C*x^4) - 8*a^3*b^2*x^4*(10*A + 18*B*x^2 + 63*C*x^4 - 105*D
*x^6) - a^5*(35*A + 45*B*x^2 + 63*C*x^4 + 105*D*x^6) + 2*a^4*b*x^2*(25*A +
 36*B*x^2 + 63*C*x^4 + 210*D*x^6))/(315*a^6*x^9*Sqrt[a + b*x^2])
```

## Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.89, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {2334, 2089, 1588, 359, 245, 245, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{10}\left(a+bx^2\right)^{3/2}}dx$$

$$\Big\downarrow \; 2334$$

$$-\frac{\int \frac{10Ab-9a\left(Dx^4+Cx^2+B\right)}{x^8\left(bx^2+a\right)^{3/2}}dx}{9a}-\frac{A}{9ax^9\sqrt{a+bx^2}}$$

$$\Big\downarrow \; 2089$$

$$-\frac{\int \frac{-9aDx^4-9aCx^2+10Ab-9aB}{x^8\left(bx^2+a\right)^{3/2}}dx}{9a}-\frac{A}{9ax^9\sqrt{a+bx^2}}$$

$$\Big\downarrow \; 1588$$

$$-\frac{-\frac{\int \frac{63Dx^2a^2+63Ca^2-72bBa+80Ab^2}{x^6\left(bx^2+a\right)^{3/2}}dx}{7a}-\frac{10Ab-9aB}{7ax^7\sqrt{a+bx^2}}}{9a}-\frac{A}{9ax^9\sqrt{a+bx^2}}$$

$\Big\downarrow$ 359

$$-\frac{-\frac{-\frac{3\left(-105a^3D-18ab(8bB-7aC)+160Ab^3\right)\int \frac{1}{x^4\left(bx^2+a\right)^{3/2}}dx}{5a}-\frac{63a^2C-72abB+80Ab^2}{5ax^5\sqrt{a+bx^2}}}{7a}-\frac{10Ab-9aB}{7ax^7\sqrt{a+bx^2}}}{9a}-\frac{A}{9ax^9\sqrt{a+bx^2}}$$

$\Big\downarrow$ 245

$$-\frac{-\frac{-\frac{3\left(-105a^3D-18ab(8bB-7aC)+160Ab^3\right)\left(-\frac{4b\int \frac{1}{x^2\left(bx^2+a\right)^{3/2}}dx}{3a}-\frac{1}{3ax^3\sqrt{a+bx^2}}\right)}{5a}-\frac{63a^2C-72abB+80Ab^2}{5ax^5\sqrt{a+bx^2}}}{7a}-\frac{10Ab-9aB}{7ax^7\sqrt{a+bx^2}}}{9a}-\frac{A}{9ax^9\sqrt{a+bx^2}}$$

$\Big\downarrow$ 245

$$-\frac{-\frac{-\frac{3\left(-105a^3D-18ab(8bB-7aC)+160Ab^3\right)\left(-\frac{4b\left(-\frac{2b\int \frac{1}{\left(bx^2+a\right)^{3/2}}dx}{a}-\frac{1}{ax\sqrt{a+bx^2}}\right)}{3a}-\frac{1}{3ax^3\sqrt{a+bx^2}}\right)}{5a}-\frac{63a^2C-72abB+80Ab^2}{5ax^5\sqrt{a+bx^2}}}{7a}-\frac{10Ab-9aB}{7ax^7\sqrt{a+bx^2}}}{9a}-\frac{A}{9ax^9\sqrt{a+bx^2}}$$

$\Big\downarrow$ 208

$$-\frac{-\frac{-\frac{63a^2C-72abB+80Ab^2}{5ax^5\sqrt{a+bx^2}}-\frac{3\left(-\frac{4b\left(-\frac{2bx}{a^2\sqrt{a+bx^2}}-\frac{1}{ax\sqrt{a+bx^2}}\right)}{3a}-\frac{1}{3ax^3\sqrt{a+bx^2}}\right)\left(-105a^3D-18ab(8bB-7aC)+160Ab^3\right)}{5a}}{7a}-\frac{10Ab-9aB}{7ax^7\sqrt{a+bx^2}}}{9a}-\frac{A}{9ax^9\sqrt{a+bx^2}}$$

input
```
Int[(A + B*x^2 + C*x^4 + D*x^6)/(x^10*(a + b*x^2)^(3/2)),x]
```

output
```
-1/9*A/(a*x^9*Sqrt[a + b*x^2]) - (-1/7*(10*A*b - 9*a*B)/(a*x^7*Sqrt[a + b*
x^2]) - (-1/5*(80*A*b^2 - 72*a*b*B + 63*a^2*C)/(a*x^5*Sqrt[a + b*x^2]) - (
3*(160*A*b^3 - 18*a*b*(8*b*B - 7*a*C) - 105*a^3*D)*(-1/3*1/(a*x^3*Sqrt[a +
 b*x^2]) - (4*b*(-(1/(a*x*Sqrt[a + b*x^2])) - (2*b*x)/(a^2*Sqrt[a + b*x^2]
)))/(3*a)))/(5*a))/(7*a))/(9*a)
```

## Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 1588
```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Simp[1/(d*f
^2*(m + 1))   Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x
) - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

rule 2089
```
Int[(u_)^(p_.)*((f_.)*(x_))^(m_.)*(z_)^(q_.), x_Symbol] :> Int[(f*x)^m*Expa
ndToSum[z, x]^q*ExpandToSum[u, x]^p, x] /; FreeQ[{f, m, p, q}, x] && Binomi
alQ[z, x] && TrinomialQ[u, x] &&  !(BinomialMatchQ[z, x] && TrinomialMatchQ
[u, x])
```

rule 2334
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coef
f[Pq, x, 0], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[A*
x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))   Int[
x^(m + 2)*(a + b*x^2)^p*(a*(m + 1)*Q - A*b*(m + 2*(p + 1) + 1)), x], x]] /;
 FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && IntegerQ[m/2] && ILtQ[(m + 1)/2 + p,
 0] && LtQ[m + Expon[Pq, x] + 2*p + 1, 0]
```

## Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.66

| method | result |
|---|---|
| pseudoelliptic | $\frac{(-105Dx^6-63Cx^4-45x^2B-35A)a^5+50\left(\frac{42}{5}Dx^6+\frac{63}{25}Cx^4+\frac{36}{25}x^2B+A\right)x^2ba^4-80\left(-\frac{21}{2}Dx^6+\frac{63}{10}Cx^4+\frac{9}{5}x^2B+A\right)x^4b^2a}{315\sqrt{bx^2+a}\,x^9a^6}$ |
| gosper | $-\frac{1280Ab^5x^{10}-1152Bab^4x^{10}+1008Ca^2b^3x^{10}-840Da^3b^2x^{10}+640aAb^4x^8-576Ba^2b^3x^8+504Ca^3b^2x^8-420Da^4bx^8-}{315x^9}$ |
| trager | $-\frac{1280Ab^5x^{10}-1152Bab^4x^{10}+1008Ca^2b^3x^{10}-840Da^3b^2x^{10}+640aAb^4x^8-576Ba^2b^3x^8+504Ca^3b^2x^8-420Da^4bx^8-}{315x^9}$ |
| orering | $-\frac{1280Ab^5x^{10}-1152Bab^4x^{10}+1008Ca^2b^3x^{10}-840Da^3b^2x^{10}+640aAb^4x^8-576Ba^2b^3x^8+504Ca^3b^2x^8-420Da^4bx^8-}{315x^9}$ |
| default | $A\left(-\frac{1}{9a\,x^9\sqrt{bx^2+a}}-\frac{10b\left(-\frac{1}{7a\,x^7\sqrt{bx^2+a}}-\frac{8b\left(-\frac{1}{5a\,x^5\sqrt{bx^2+a}}-\frac{6b\left(-\frac{1}{3a\,x^3\sqrt{bx^2+a}}-\frac{4b\left(-\frac{1}{ax\sqrt{bx^2+a}}-\frac{2bx}{a^2\sqrt{bx^2+a}}\right)}{3a}\right)}{5a}\right)}{7a}\right)}{9a}\right.$ |

input

```
int((D*x^6+C*x^4+B*x^2+A)/x^10/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
1/315*((-105*D*x^6-63*C*x^4-45*B*x^2-35*A)*a^5+50*(42/5*D*x^6+63/25*C*x^4+
36/25*x^2*B+A)*x^2*b*a^4-80*(-21/2*D*x^6+63/10*C*x^4+9/5*x^2*B+A)*x^4*b^2*
a^3+160*(-63/10*C*x^4+18/5*x^2*B+A)*x^6*b^3*a^2-640*(-9/5*x^2*B+A)*x^8*b^4
*a-1280*A*b^5*x^10)/(b*x^2+a)^(1/2)/x^9/a^6
```

## Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.83

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{10}\left(a+bx^2\right)^{3/2}}\,dx = \frac{\left(8\left(105\,Da^3b^2-126\,Ca^2b^3+144\,Bab^4-160\,Ab^5\right)x^{10}+4\left(105\,Da^4b-126\right.}{}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^10/(b*x^2+a)^(3/2),x, algorithm="fricas"
)
```

output
```
1/315*(8*(105*D*a^3*b^2 - 126*C*a^2*b^3 + 144*B*a*b^4 - 160*A*b^5)*x^10 +
4*(105*D*a^4*b - 126*C*a^3*b^2 + 144*B*a^2*b^3 - 160*A*a*b^4)*x^8 - (105*D
*a^5 - 126*C*a^4*b + 144*B*a^3*b^2 - 160*A*a^2*b^3)*x^6 - 35*A*a^5 - (63*C
*a^5 - 72*B*a^4*b + 80*A*a^3*b^2)*x^4 - 5*(9*B*a^5 - 10*A*a^4*b)*x^2)*sqrt
(b*x^2 + a)/(a^6*b*x^11 + a^7*x^9)
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2134 vs. 2(228) = 456.

Time = 17.37 (sec) , antiderivative size = 2134, normalized size of antiderivative = 9.08

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{10}\left(a+bx^2\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((D*x**6+C*x**4+B*x**2+A)/x**10/(b*x**2+a)**(3/2),x)
```

output
```
A*(-7*a**9*b**(51/2)*sqrt(a/(b*x**2) + 1)/(63*a**11*b**25*x**8 + 315*a**10
*b**26*x**10 + 630*a**9*b**27*x**12 + 630*a**8*b**28*x**14 + 315*a**7*b**2
9*x**16 + 63*a**6*b**30*x**18) - 18*a**8*b**(53/2)*x**2*sqrt(a/(b*x**2) +
1)/(63*a**11*b**25*x**8 + 315*a**10*b**26*x**10 + 630*a**9*b**27*x**12 + 6
30*a**8*b**28*x**14 + 315*a**7*b**29*x**16 + 63*a**6*b**30*x**18) - 18*a**
7*b**(55/2)*x**4*sqrt(a/(b*x**2) + 1)/(63*a**11*b**25*x**8 + 315*a**10*b**
26*x**10 + 630*a**9*b**27*x**12 + 630*a**8*b**28*x**14 + 315*a**7*b**29*x*
*16 + 63*a**6*b**30*x**18) - 63*a**5*b**(59/2)*x**8*sqrt(a/(b*x**2) + 1)/(
63*a**11*b**25*x**8 + 315*a**10*b**26*x**10 + 630*a**9*b**27*x**12 + 630*a
**8*b**28*x**14 + 315*a**7*b**29*x**16 + 63*a**6*b**30*x**18) - 630*a**4*b
**(61/2)*x**10*sqrt(a/(b*x**2) + 1)/(63*a**11*b**25*x**8 + 315*a**10*b**26
*x**10 + 630*a**9*b**27*x**12 + 630*a**8*b**28*x**14 + 315*a**7*b**29*x**1
6 + 63*a**6*b**30*x**18) - 1680*a**3*b**(63/2)*x**12*sqrt(a/(b*x**2) + 1)/
(63*a**11*b**25*x**8 + 315*a**10*b**26*x**10 + 630*a**9*b**27*x**12 + 630*
a**8*b**28*x**14 + 315*a**7*b**29*x**16 + 63*a**6*b**30*x**18) - 2016*a**2
*b**(65/2)*x**14*sqrt(a/(b*x**2) + 1)/(63*a**11*b**25*x**8 + 315*a**10*b**
26*x**10 + 630*a**9*b**27*x**12 + 630*a**8*b**28*x**14 + 315*a**7*b**29*x*
*16 + 63*a**6*b**30*x**18) - 1152*a*b**(67/2)*x**16*sqrt(a/(b*x**2) + 1)/(
63*a**11*b**25*x**8 + 315*a**10*b**26*x**10 + 630*a**9*b**27*x**12 + 630*a
**8*b**28*x**14 + 315*a**7*b**29*x**16 + 63*a**6*b**30*x**18) - 256*b**...
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.49

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{10}\left(a+bx^2\right)^{3/2}}\,dx = \frac{8\,Db^2x}{3\,\sqrt{bx^2+a}a^3} - \frac{16\,Cb^3x}{5\,\sqrt{bx^2+a}a^4} + \frac{128\,Bb^4x}{35\,\sqrt{bx^2+a}a^5} - \frac{256\,Ab^5x}{63\,\sqrt{bx^2+a}a^6} + \frac{4\,Db}{3\,\sqrt{bx^2+a}a^2x} - \frac{8\,Cb^2}{5\,\sqrt{bx^2+a}a^3x} + \frac{64\,Bb^3}{35\,\sqrt{bx^2+a}a^4x} - \frac{128\,Ab^4}{63\,\sqrt{bx^2+a}a^5x} - \frac{D}{3\,\sqrt{bx^2+a}ax^3} + \frac{2\,Cb}{5\,\sqrt{bx^2+a}a^2x^3} - \frac{16\,Bb^2}{35\,\sqrt{bx^2+a}a^3x^3} + \frac{32\,Ab^3}{63\,\sqrt{bx^2+a}a^4x^3} - \frac{C}{5\,\sqrt{bx^2+a}ax^5} + \frac{8\,Bb}{35\,\sqrt{bx^2+a}a^2x^5} - \frac{16\,Ab^2}{63\,\sqrt{bx^2+a}a^3x^5} - \frac{B}{7\,\sqrt{bx^2+a}ax^7} + \frac{10\,Ab}{63\,\sqrt{bx^2+a}a^2x^7} - \frac{A}{9\,\sqrt{bx^2+a}ax^9}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^10/(b*x^2+a)^(3/2),x, algorithm="maxima"
)
```

output
```
8/3*D*b^2*x/(sqrt(b*x^2 + a)*a^3) - 16/5*C*b^3*x/(sqrt(b*x^2 + a)*a^4) + 1
28/35*B*b^4*x/(sqrt(b*x^2 + a)*a^5) - 256/63*A*b^5*x/(sqrt(b*x^2 + a)*a^6)
 + 4/3*D*b/(sqrt(b*x^2 + a)*a^2*x) - 8/5*C*b^2/(sqrt(b*x^2 + a)*a^3*x) + 6
4/35*B*b^3/(sqrt(b*x^2 + a)*a^4*x) - 128/63*A*b^4/(sqrt(b*x^2 + a)*a^5*x)
- 1/3*D/(sqrt(b*x^2 + a)*a*x^3) + 2/5*C*b/(sqrt(b*x^2 + a)*a^2*x^3) - 16/3
5*B*b^2/(sqrt(b*x^2 + a)*a^3*x^3) + 32/63*A*b^3/(sqrt(b*x^2 + a)*a^4*x^3)
- 1/5*C/(sqrt(b*x^2 + a)*a*x^5) + 8/35*B*b/(sqrt(b*x^2 + a)*a^2*x^5) - 16/
63*A*b^2/(sqrt(b*x^2 + a)*a^3*x^5) - 1/7*B/(sqrt(b*x^2 + a)*a*x^7) + 10/63
*A*b/(sqrt(b*x^2 + a)*a^2*x^7) - 1/9*A/(sqrt(b*x^2 + a)*a*x^9)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1001 vs. $2(213) = 426$.

Time = 0.16 (sec) , antiderivative size = 1001, normalized size of antiderivative = 4.26

$$\int \frac{A + Bx^2 + Cx^4 + Dx^6}{x^{10} (a + bx^2)^{3/2}} \, dx = \text{Too large to display}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^10/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
(D*a^3*b^2 - C*a^2*b^3 + B*a*b^4 - A*b^5)*x/(sqrt(b*x^2 + a)*a^6) - 2/315*
(315*(sqrt(b)*x - sqrt(b*x^2 + a))^16*D*a^3*b^(3/2) - 315*(sqrt(b)*x - sqr
t(b*x^2 + a))^16*C*a^2*b^(5/2) + 315*(sqrt(b)*x - sqrt(b*x^2 + a))^16*B*a*
b^(7/2) - 315*(sqrt(b)*x - sqrt(b*x^2 + a))^16*A*b^(9/2) - 3150*(sqrt(b)*x
 - sqrt(b*x^2 + a))^14*D*a^4*b^(3/2) + 3150*(sqrt(b)*x - sqrt(b*x^2 + a))^
14*C*a^3*b^(5/2) - 3150*(sqrt(b)*x - sqrt(b*x^2 + a))^14*B*a^2*b^(7/2) + 3
150*(sqrt(b)*x - sqrt(b*x^2 + a))^14*A*a*b^(9/2) + 12810*(sqrt(b)*x - sqrt
(b*x^2 + a))^12*D*a^5*b^(3/2) - 14490*(sqrt(b)*x - sqrt(b*x^2 + a))^12*C*a
^4*b^(5/2) + 14490*(sqrt(b)*x - sqrt(b*x^2 + a))^12*B*a^3*b^(7/2) - 14490*
(sqrt(b)*x - sqrt(b*x^2 + a))^12*A*a^2*b^(9/2) - 28350*(sqrt(b)*x - sqrt(b
*x^2 + a))^10*D*a^6*b^(3/2) + 35910*(sqrt(b)*x - sqrt(b*x^2 + a))^10*C*a^5
*b^(5/2) - 40950*(sqrt(b)*x - sqrt(b*x^2 + a))^10*B*a^4*b^(7/2) + 40950*(s
qrt(b)*x - sqrt(b*x^2 + a))^10*A*a^3*b^(9/2) + 37800*(sqrt(b)*x - sqrt(b*x
^2 + a))^8*D*a^7*b^(3/2) - 51408*(sqrt(b)*x - sqrt(b*x^2 + a))^8*C*a^6*b^(
5/2) + 64512*(sqrt(b)*x - sqrt(b*x^2 + a))^8*B*a^5*b^(7/2) - 80640*(sqrt(b
)*x - sqrt(b*x^2 + a))^8*A*a^4*b^(9/2) - 31290*(sqrt(b)*x - sqrt(b*x^2 + a
))^6*D*a^8*b^(3/2) + 43722*(sqrt(b)*x - sqrt(b*x^2 + a))^6*C*a^7*b^(5/2) -
 55818*(sqrt(b)*x - sqrt(b*x^2 + a))^6*B*a^6*b^(7/2) + 66570*(sqrt(b)*x -
sqrt(b*x^2 + a))^6*A*a^5*b^(9/2) + 15750*(sqrt(b)*x - sqrt(b*x^2 + a))^4*D
*a^9*b^(3/2) - 21798*(sqrt(b)*x - sqrt(b*x^2 + a))^4*C*a^8*b^(5/2) + 26...
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{A + Bx^2 + Cx^4 + Dx^6}{x^{10}\left(a + bx^2\right)^{3/2}}\,dx = \int \frac{A + B\,x^2 + C\,x^4 + x^6\,D}{x^{10}\,{\left(b\,x^2 + a\right)}^{3/2}}\,dx$$

input
```
int((A + B*x^2 + C*x^4 + x^6*D)/(x^10*(a + b*x^2)^(3/2)),x)
```

output
```
int((A + B*x^2 + C*x^4 + x^6*D)/(x^10*(a + b*x^2)^(3/2)), x)
```

## Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.36

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{10}\left(a+bx^2\right)^{3/2}}\,dx = \frac{-35\sqrt{b\,x^2+a}\,a^5+5\sqrt{b\,x^2+a}\,a^4b\,x^2-63\sqrt{b\,x^2+a}\,a^4c\,x^4-105\sqrt{b\,x^2+}}{}$$

input
```
int((D*x^6+C*x^4+B*x^2+A)/x^10/(b*x^2+a)^(3/2),x)
```

output
```
( - 35*sqrt(a + b*x**2)*a**5 + 5*sqrt(a + b*x**2)*a**4*b*x**2 - 63*sqrt(a
+ b*x**2)*a**4*c*x**4 - 105*sqrt(a + b*x**2)*a**4*d*x**6 - 8*sqrt(a + b*x*
*2)*a**3*b**2*x**4 + 126*sqrt(a + b*x**2)*a**3*b*c*x**6 + 420*sqrt(a + b*x
**2)*a**3*b*d*x**8 + 16*sqrt(a + b*x**2)*a**2*b**3*x**6 - 504*sqrt(a + b*x
**2)*a**2*b**2*c*x**8 + 840*sqrt(a + b*x**2)*a**2*b**2*d*x**10 - 64*sqrt(a
 + b*x**2)*a*b**4*x**8 - 1008*sqrt(a + b*x**2)*a*b**3*c*x**10 - 128*sqrt(a
 + b*x**2)*b**5*x**10 - 840*sqrt(b)*a**3*b*d*x**9 + 1008*sqrt(b)*a**2*b**2
*c*x**9 - 840*sqrt(b)*a**2*b**2*d*x**11 + 128*sqrt(b)*a*b**4*x**9 + 1008*s
qrt(b)*a*b**3*c*x**11 + 128*sqrt(b)*b**5*x**11)/(315*a**5*x**9*(a + b*x**2
))
```

# 3.240 $\int \frac{x^6(A+Bx^2+Cx^4+Bx^6)}{(a+bx^2)^{9/2}}\,dx$



## Optimal result

Integrand size = 32, antiderivative size = 289

$$\begin{aligned}\int \frac{x^6(A+Bx^2+Cx^4+Bx^6)}{(a+bx^2)^{9/2}}\,dx &= -\frac{a^2(Ab^3-a(a^2B+b^2B-abC))\,x}{7b^6\,(a+bx^2)^{7/2}} \\ &+ \frac{a(15Ab^3-a(36a^2B+22b^2B-29abC))\,x}{35b^6\,(a+bx^2)^{5/2}} \\ &- \frac{(45Ab^3-a(381a^2B+122b^2B-234abC))\,x}{105b^6\,(a+bx^2)^{3/2}} \\ &+ \frac{(15Ab^3-2a(669a^2B+88b^2B-291abC))\,x}{105ab^6\sqrt{a+bx^2}} - \frac{(19aB-4bC)x\sqrt{a+bx^2}}{8b^6} \\ &+ \frac{Bx^3\sqrt{a+bx^2}}{4b^5} + \frac{(99a^2B+8b^2B-36abC)\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{13/2}}\end{aligned}$$

output
```
-1/7*a^2*(A*b^3-a*(B*a^2+B*b^2-C*a*b))*x/b^6/(b*x^2+a)^(7/2)+1/35*a*(15*A*
b^3-a*(36*B*a^2+22*B*b^2-29*C*a*b))*x/b^6/(b*x^2+a)^(5/2)-1/105*(45*A*b^3-
a*(381*B*a^2+122*B*b^2-234*C*a*b))*x/b^6/(b*x^2+a)^(3/2)+1/105*(15*A*b^3-2
*a*(669*B*a^2+88*B*b^2-291*C*a*b))*x/a/b^6/(b*x^2+a)^(1/2)-1/8*(19*B*a-4*C
*b)*x*(b*x^2+a)^(1/2)/b^6+1/4*B*x^3*(b*x^2+a)^(1/2)/b^5+1/8*(99*B*a^2+8*B*
b^2-36*C*a*b)*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(13/2)
```

## Mathematica [A] (verified)

Time = 1.42 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.74

$$\int \frac{x^6(A+Bx^2+Cx^4+Bx^6)}{(a+bx^2)^{9/2}}\,dx = \frac{x(-10395a^6B+120Ab^6x^6+630a^5b(6C-55Bx^2)+2ab^5x^6(210Cx^2\ldots}{}$$
$$+\frac{(99a^2B+8b^2B-36abC)\,\text{arctanh}\left(\frac{\sqrt{b}x}{-\sqrt{a}+\sqrt{a+bx^2}}\right)}{4b^{13/2}}$$

input
```
Integrate[(x^6*(A + B*x^2 + C*x^4 + B*x^6))/(a + b*x^2)^(9/2),x]
```

output
```
(x*(-10395*a^6*B + 120*A*b^6*x^6 + 630*a^5*b*(6*C - 55*B*x^2) + 2*a*b^5*x^
6*(210*C*x^2 + B*(-704 + 105*x^4)) + a^2*b^4*x^4*(6336*C*x^2 - 7*B*(464 +
165*x^4)) + 42*a^4*b^2*(300*C*x^2 - B*(20 + 957*x^4)) - 8*a^3*b^3*x^2*(-18
27*C*x^2 + B*(350 + 2178*x^4))))/(840*a*b^6*(a + b*x^2)^(7/2)) + ((99*a^2*
B + 8*b^2*B - 36*a*b*C)*ArcTanh[(Sqrt[b]*x)/(-Sqrt[a] + Sqrt[a + b*x^2])])
/(4*b^(13/2))
```

## Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 245, normalized size of antiderivative = 0.85, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {2335, 9, 27, 1586, 9, 27, 363, 252, 252, 262, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6(A+Bx^6+Bx^2+Cx^4)}{(a+bx^2)^{9/2}}\,dx$$

$$\Big\downarrow \text{2335}$$

$$\frac{x^7\left(A-\frac{a(a^2B-abC+b^2B)}{b^3}\right)}{7a\,(a+bx^2)^{7/2}}-\frac{\int -\frac{7x^5\left(aBx^5-a\left(\frac{aB}{b}-C\right)x^3+a\left(\frac{Ba^2}{b^2}-\frac{Ca}{b}+B\right)x\right)}{(bx^2+a)^{7/2}}dx}{7ab}$$

$$\Big\downarrow \text{9}$$

$$\frac{x^7\left(A-\frac{a(a^2B-abC+b^2B)}{b^3}\right)}{7a\,(a+bx^2)^{7/2}}-\frac{\int -\frac{7x^6\left(aBx^4-a\left(\frac{aB}{b}-C\right)x^2+\frac{a\left(Ba^2-bCa+b^2B\right)}{b^2}\right)}{(bx^2+a)^{7/2}}dx}{7ab}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{x^6\left(aBx^4-a\left(\frac{aB}{b}-C\right)x^2+\frac{a\left(Ba^2-bCa+b^2B\right)}{b^2}\right)}{(bx^2+a)^{7/2}}dx}{ab}+\frac{x^7\left(A-\frac{a(a^2B-abC+b^2B)}{b^3}\right)}{7a\,(a+bx^2)^{7/2}}$$

$$\Big\downarrow \; 1586$$

$$\frac{\frac{x^7(3a^2B-2abC+b^2B)}{5b^2(a+bx^2)^{5/2}}-\frac{\int \frac{x^5\left(\frac{a\left(16Ba^2-9bCa+2b^2B\right)x}{b^2}-\frac{5a^2Bx^3}{b}\right)}{\left(bx^2+a\right)^{5/2}}dx}{5a}}{ab}+\frac{x^7\left(A-\frac{a(a^2B-abC+b^2B)}{b^3}\right)}{7a\,(a+bx^2)^{7/2}}$$

$$\Big\downarrow \; 9$$

$$\frac{\frac{x^7(3a^2B-2abC+b^2B)}{5b^2(a+bx^2)^{5/2}}-\frac{\int \frac{ax^6\left(16Ba^2-5bBx^2a-9bCa+2b^2B\right)}{b^2\left(bx^2+a\right)^{5/2}}dx}{5a}}{ab}+\frac{x^7\left(A-\frac{a(a^2B-abC+b^2B)}{b^3}\right)}{7a\,(a+bx^2)^{7/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{x^7(3a^2B-2abC+b^2B)}{5b^2(a+bx^2)^{5/2}}-\frac{\int \frac{x^6\left(16Ba^2-5bBx^2a-9bCa+2b^2B\right)}{\left(bx^2+a\right)^{5/2}}dx}{5b^2}}{ab}+\frac{x^7\left(A-\frac{a(a^2B-abC+b^2B)}{b^3}\right)}{7a\,(a+bx^2)^{7/2}}$$

$$\Big\downarrow \; 363$$

$$\frac{\frac{x^7(3a^2B-2abC+b^2B)}{5b^2(a+bx^2)^{5/2}}-\frac{\frac{1}{4}\left(99a^2B-36abC+8b^2B\right)\int \frac{x^6}{\left(bx^2+a\right)^{5/2}}dx-\frac{5aBx^7}{4\left(a+bx^2\right)^{3/2}}}{5b^2}}{ab}+\frac{x^7\left(A-\frac{a(a^2B-abC+b^2B)}{b^3}\right)}{7a\,(a+bx^2)^{7/2}}$$

$$\Big\downarrow \; 252$$

$$\frac{\frac{x^7(3a^2B-2abC+b^2B)}{5b^2(a+bx^2)^{5/2}}-\frac{\frac{1}{4}(99a^2B-36abC+8b^2B)\left(\frac{5\int\frac{x^4}{(bx^2+a)^{3/2}}dx}{3b}-\frac{x^5}{3b(a+bx^2)^{3/2}}\right)-\frac{5aBx^7}{4(a+bx^2)^{3/2}}}{5b^2}}{ab}+\frac{x^7\left(A-\frac{a(a^2B-abC+b^2B)}{b^3}\right)}{7a(a+bx^2)^{7/2}}$$

$$\Big\downarrow \; 252$$

$$\frac{\frac{x^7(3a^2B-2abC+b^2B)}{5b^2(a+bx^2)^{5/2}}-\frac{\frac{1}{4}(99a^2B-36abC+8b^2B)\left(\frac{5\left(\frac{3\int\frac{x^2}{\sqrt{bx^2+a}}dx}{b}-\frac{x^3}{b\sqrt{a+bx^2}}\right)}{3b}-\frac{x^5}{3b(a+bx^2)^{3/2}}\right)-\frac{5aBx^7}{4(a+bx^2)^{3/2}}}{5b^2}}{ab}+\frac{x^7\left(A-\frac{a(a^2B-abC+b^2B)}{b^3}\right)}{7a(a+bx^2)^{7/2}}$$

$$\Big\downarrow \; 262$$

$$\frac{\frac{x^7(3a^2B-2abC+b^2B)}{5b^2(a+bx^2)^{5/2}}-\frac{\frac{1}{4}(99a^2B-36abC+8b^2B)\left(\frac{5\left(\frac{3\left(\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\int\frac{1}{\sqrt{bx^2+a}}dx}{2b}\right)}{b}-\frac{x^3}{b\sqrt{a+bx^2}}\right)}{3b}-\frac{x^5}{3b(a+bx^2)^{3/2}}\right)-\frac{5aBx^7}{4(a+bx^2)^{3/2}}}{5b^2}}{ab}+\frac{x^7\left(A-\frac{a(a^2B-abC+b^2B)}{b^3}\right)}{7a(a+bx^2)^{7/2}}$$

$$\Big\downarrow \; 224$$

$$\frac{\frac{x^7(3a^2B-2abC+b^2B)}{5b^2(a+bx^2)^{5/2}}-\frac{\frac{1}{4}(99a^2B-36abC+8b^2B)\left(\frac{5\left(\frac{3\left(\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2b}\right)}{b}-\frac{x^3}{b\sqrt{a+bx^2}}\right)}{3b}-\frac{x^5}{3b(a+bx^2)^{3/2}}\right)-\frac{5aBx^7}{4(a+bx^2)^{3/2}}}{5b^2}}{ab}$$

$$\frac{x^7\left(A-\frac{a(a^2B-abC+b^2B)}{b^3}\right)}{7a(a+bx^2)^{7/2}}$$

$\downarrow$ 219

$$\frac{x^7\left(A-\frac{a(a^2B-abC+b^2B)}{b^3}\right)}{7a(a+bx^2)^{7/2}}+$$

$$\frac{\frac{x^7(3a^2B-2abC+b^2B)}{5b^2(a+bx^2)^{5/2}}-\frac{\frac{1}{4}\left(\frac{5\left(\frac{3\left(\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}\right)}{b}-\frac{x^3}{b\sqrt{a+bx^2}}\right)}{3b}-\frac{x^5}{3b(a+bx^2)^{3/2}}\right)(99a^2B-36abC+8b^2B)-\frac{5aBx^7}{4(a+bx^2)^{3/2}}}{5b^2}}{ab}$$

input
```
Int[(x^6*(A + B*x^2 + C*x^4 + B*x^6))/(a + b*x^2)^(9/2),x]
```

output
```
((A - (a*(a^2*B + b^2*B - a*b*C))/b^3)*x^7)/(7*a*(a + b*x^2)^(7/2)) + (((3
*a^2*B + b^2*B - 2*a*b*C)*x^7)/(5*b^2*(a + b*x^2)^(5/2)) - ((-5*a*B*x^7)/(
4*(a + b*x^2)^(3/2)) + ((99*a^2*B + 8*b^2*B - 36*a*b*C)*(-1/3*x^5/(b*(a +
b*x^2)^(3/2)) + (5*(-(x^3/(b*Sqrt[a + b*x^2])) + (3*((x*Sqrt[a + b*x^2])/(
2*b) - (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(3/2))))/b))/(3*b)))/
4)/(5*b^2))/(a*b)
```

### Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 1586
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, d + e*x^2, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^
p, d + e*x^2, x], x, 0]}, Simp[(-R)*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(2*d
*f*(q + 1))), x] + Simp[f/(2*d*(q + 1))   Int[(f*x)^(m - 1)*(d + e*x^2)^(q
+ 1)*ExpandToSum[2*d*(q + 1)*x*Qx + R*(m + 2*q + 3)*x, x], x], x]] /; FreeQ
[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[q, -1]
&& GtQ[m, 0]
```

rule 2335
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Simp[c/(2*a*b*(p + 1))   Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSu
m[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
 b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

## Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.70

| method | result |
|---|---|
| pseudoelliptic | $\frac{\frac{693a\left(a^2B+\frac{8}{99}Bb^2-\frac{4}{11}Cab\right)\left(bx^2+a\right)^{\frac{7}{2}}\operatorname{arctanh}\left(\frac{\sqrt{bx^2+a}}{x\sqrt{b}}\right)}{8}+\left(\frac{7ax^6\left(Bx^4+2Cx^2-\frac{704}{105}B\right)b^{\frac{11}{2}}}{4}-\frac{6699a^4\left(Bx^4-\frac{100}{319}Cx^2+\frac{20}{957}B\right)}{20}\right.}{7b^{\frac{13}{2}}(b}$ |
| default | $A\left(-\frac{x^5}{2b(bx^2+a)^{\frac{7}{2}}}+\frac{5a\left(-\frac{x^3}{4b\left(bx^2+a\right)^{\frac{7}{2}}}+\frac{3a\left(-\frac{x}{6b\left(bx^2+a\right)^{\frac{7}{2}}}+\frac{a\left(\frac{x}{7a\left(bx^2+a\right)^{\frac{7}{2}}}+\frac{\frac{6x}{35a\left(bx^2+a\right)^{\frac{5}{2}}}+\frac{6\left(\frac{4x}{15a\left(bx^2+a\right)^{\frac{3}{2}}}+\frac{}{15}\right.}{7a}}{a}\right)}{6b}\right)}{4b}\right)}{2b}\right.$ |

input
```
int(x^6*(B*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
1/7/b^(13/2)*(693/8*a*(a^2*B+8/99*B*b^2-4/11*C*a*b)*(b*x^2+a)^(7/2)*arctan
h((b*x^2+a)^(1/2)/x/b^(1/2))+(7/4*a*x^6*(B*x^4+2*C*x^2-704/105*B)*b^(11/2)
-6699/20*a^4*(B*x^4-100/319*C*x^2+20/957*B)*b^(5/2)-726/5*a^3*x^2*(B*x^4-2
03/242*C*x^2+175/1089*B)*b^(7/2)-77/8*(B*x^4-192/35*C*x^2+464/165*B)*a^2*x
^4*b^(9/2)-1155/4*(x^2*B-6/55*C)*a^5*b^(3/2)+A*b^(13/2)*x^6-693/8*B*b^(1/2
)*a^6)*x)/(b*x^2+a)^(7/2)/a
```

## Fricas [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 816, normalized size of antiderivative = 2.82

$$\int \frac{x^6(A+Bx^2+Cx^4+Bx^6)}{(a+bx^2)^{9/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^6*(B*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

output
```
[1/1680*(105*((99*B*a^3*b^4 - 36*C*a^2*b^5 + 8*B*a*b^6)*x^8 + 99*B*a^7 - 3
6*C*a^6*b + 8*B*a^5*b^2 + 4*(99*B*a^4*b^3 - 36*C*a^3*b^4 + 8*B*a^2*b^5)*x^
6 + 6*(99*B*a^5*b^2 - 36*C*a^4*b^3 + 8*B*a^3*b^4)*x^4 + 4*(99*B*a^6*b - 36
*C*a^5*b^2 + 8*B*a^4*b^3)*x^2)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sq
rt(b)*x - a) + 2*(210*B*a*b^6*x^11 - 105*(11*B*a^2*b^5 - 4*C*a*b^6)*x^9 -
8*(2178*B*a^3*b^4 - 792*C*a^2*b^5 + 176*B*a*b^6 - 15*A*b^7)*x^7 - 406*(99*
B*a^4*b^3 - 36*C*a^3*b^4 + 8*B*a^2*b^5)*x^5 - 350*(99*B*a^5*b^2 - 36*C*a^4
*b^3 + 8*B*a^3*b^4)*x^3 - 105*(99*B*a^6*b - 36*C*a^5*b^2 + 8*B*a^4*b^3)*x)
*sqrt(b*x^2 + a))/(a*b^11*x^8 + 4*a^2*b^10*x^6 + 6*a^3*b^9*x^4 + 4*a^4*b^8
*x^2 + a^5*b^7), -1/840*(105*((99*B*a^3*b^4 - 36*C*a^2*b^5 + 8*B*a*b^6)*x^
8 + 99*B*a^7 - 36*C*a^6*b + 8*B*a^5*b^2 + 4*(99*B*a^4*b^3 - 36*C*a^3*b^4 +
 8*B*a^2*b^5)*x^6 + 6*(99*B*a^5*b^2 - 36*C*a^4*b^3 + 8*B*a^3*b^4)*x^4 + 4*
(99*B*a^6*b - 36*C*a^5*b^2 + 8*B*a^4*b^3)*x^2)*sqrt(-b)*arctan(sqrt(-b)*x/
sqrt(b*x^2 + a)) - (210*B*a*b^6*x^11 - 105*(11*B*a^2*b^5 - 4*C*a*b^6)*x^9
- 8*(2178*B*a^3*b^4 - 792*C*a^2*b^5 + 176*B*a*b^6 - 15*A*b^7)*x^7 - 406*(9
9*B*a^4*b^3 - 36*C*a^3*b^4 + 8*B*a^2*b^5)*x^5 - 350*(99*B*a^5*b^2 - 36*C*a
^4*b^3 + 8*B*a^3*b^4)*x^3 - 105*(99*B*a^6*b - 36*C*a^5*b^2 + 8*B*a^4*b^3)*
x)*sqrt(b*x^2 + a))/(a*b^11*x^8 + 4*a^2*b^10*x^6 + 6*a^3*b^9*x^4 + 4*a^4*b
^8*x^2 + a^5*b^7)]
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9649 vs. $2(279) = 558$.

Time = 128.11 (sec) , antiderivative size = 9649, normalized size of antiderivative = 33.39

$$\int \frac{x^6(A+Bx^2+Cx^4+Bx^6)}{(a+bx^2)^{9/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x**6*(B*x**6+C*x**4+B*x**2+A)/(b*x**2+a)**(9/2),x)
```

output
```
A*x**7/(7*a**(9/2)*sqrt(1 + b*x**2/a) + 21*a**(7/2)*b*x**2*sqrt(1 + b*x**2
/a) + 21*a**(5/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 7*a**(3/2)*b**3*x**6*sqrt
(1 + b*x**2/a)) + B*(105*a**(205/2)*b**45*sqrt(1 + b*x**2/a)*asinh(sqrt(b)
*x/sqrt(a))/(105*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*
b**(101/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(
1 + b*x**2/a) + 2100*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*
a**(197/2)*b**(107/2)*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*
x**10*sqrt(1 + b*x**2/a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2
/a)) + 630*a**(203/2)*b**46*x**2*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a
))/(105*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(101/2
)*x**2*sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**
2/a) + 2100*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/2
)*b**(107/2)*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*sqr
t(1 + b*x**2/a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) + 15
75*a**(201/2)*b**47*x**4*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(105*
a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*s
qrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2
100*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(10
7/2)*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*
x**2/a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) + 2100*a*...
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 986 vs. $2(257) = 514$.

Time = 0.06 (sec) , antiderivative size = 986, normalized size of antiderivative = 3.41

$$\int \frac{x^6(A + Bx^2 + Cx^4 + Bx^6)}{(a + bx^2)^{9/2}}\, dx = \text{Too large to display}$$

input
```
integrate(x^6*(B*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

output
```
1/4*B*x^11/((b*x^2 + a)^(7/2)*b) - 11/8*B*a*x^9/((b*x^2 + a)^(7/2)*b^2) +
1/2*C*x^9/((b*x^2 + a)^(7/2)*b) - 1/35*(35*x^6/((b*x^2 + a)^(7/2)*b) + 70*
a*x^4/((b*x^2 + a)^(7/2)*b^2) + 56*a^2*x^2/((b*x^2 + a)^(7/2)*b^3) + 16*a^
3/((b*x^2 + a)^(7/2)*b^4))*B*x - 99/280*(35*x^6/((b*x^2 + a)^(7/2)*b) + 70
*a*x^4/((b*x^2 + a)^(7/2)*b^2) + 56*a^2*x^2/((b*x^2 + a)^(7/2)*b^3) + 16*a
^3/((b*x^2 + a)^(7/2)*b^4))*B*a^2*x/b^2 + 9/70*(35*x^6/((b*x^2 + a)^(7/2)*
b) + 70*a*x^4/((b*x^2 + a)^(7/2)*b^2) + 56*a^2*x^2/((b*x^2 + a)^(7/2)*b^3)
 + 16*a^3/((b*x^2 + a)^(7/2)*b^4))*C*a*x/b - 33/40*B*a^2*x*(15*x^4/((b*x^2
 + a)^(5/2)*b) + 20*a*x^2/((b*x^2 + a)^(5/2)*b^2) + 8*a^2/((b*x^2 + a)^(5/
2)*b^3))/b^3 + 3/10*C*a*x*(15*x^4/((b*x^2 + a)^(5/2)*b) + 20*a*x^2/((b*x^2
 + a)^(5/2)*b^2) + 8*a^2/((b*x^2 + a)^(5/2)*b^3))/b^2 - 1/15*B*x*(15*x^4/(
(b*x^2 + a)^(5/2)*b) + 20*a*x^2/((b*x^2 + a)^(5/2)*b^2) + 8*a^2/((b*x^2 +
a)^(5/2)*b^3))/b - 1/2*A*x^5/((b*x^2 + a)^(7/2)*b) - 33/8*B*a^2*x*(3*x^2/(
(b*x^2 + a)^(3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2))/b^4 + 3/2*C*a*x*(3*x^2
/((b*x^2 + a)^(3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2))/b^3 - 1/3*B*x*(3*x^2
/((b*x^2 + a)^(3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2))/b^2 - 99/8*B*a^3*x^3
/((b*x^2 + a)^(5/2)*b^5) + 9/2*C*a^2*x^3/((b*x^2 + a)^(5/2)*b^4) - B*a*x^3
/((b*x^2 + a)^(5/2)*b^3) - 5/8*A*a*x^3/((b*x^2 + a)^(7/2)*b^2) + 4587/280*
B*a^2*x/(sqrt(b*x^2 + a)*b^6) + 561/280*B*a^3*x/((b*x^2 + a)^(3/2)*b^6) -
2871/280*B*a^4*x/((b*x^2 + a)^(5/2)*b^6) - 417/70*C*a*x/(sqrt(b*x^2 + a...
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 265, normalized size of antiderivative = 0.92

$$\int \frac{x^6(A+Bx^2+Cx^4+Bx^6)}{(a+bx^2)^{9/2}}\,dx = \frac{\left(\left(\left(\left(105\left(\frac{2Bx^2}{b}-\frac{11Ba^4b^9-4Ca^3b^{10}}{a^3b^{11}}\right)x^2-\frac{8(2178Ba^5b^8-792Ca^4b^9+176Ba^3b^{10}}{a^3b^{11}}\right.\right.\right.\right.}{}$$

$$-\frac{(99Ba^2-36Cab+8Bb^2)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{8b^{\frac{13}{2}}}$$

input
```
integrate(x^6*(B*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="giac")
```

output
```
1/840*((((105*(2*B*x^2/b - (11*B*a^4*b^9 - 4*C*a^3*b^10)/(a^3*b^11))*x^2 -
 8*(2178*B*a^5*b^8 - 792*C*a^4*b^9 + 176*B*a^3*b^10 - 15*A*a^2*b^11)/(a^3*
b^11))*x^2 - 406*(99*B*a^6*b^7 - 36*C*a^5*b^8 + 8*B*a^4*b^9)/(a^3*b^11))*x
^2 - 350*(99*B*a^7*b^6 - 36*C*a^6*b^7 + 8*B*a^5*b^8)/(a^3*b^11))*x^2 - 105
*(99*B*a^8*b^5 - 36*C*a^7*b^6 + 8*B*a^6*b^7)/(a^3*b^11))*x/(b*x^2 + a)^(7/
2) - 1/8*(99*B*a^2 - 36*C*a*b + 8*B*b^2)*log(abs(-sqrt(b)*x + sqrt(b*x^2 +
 a)))/b^(13/2)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^6(A+Bx^2+Cx^4+Bx^6)}{(a+bx^2)^{9/2}}\,dx = \int \frac{x^6\,(B\,x^6+C\,x^4+B\,x^2+A)}{(b\,x^2+a)^{9/2}}\,dx$$

input
```
int((x^6*(A + B*x^2 + B*x^6 + C*x^4))/(a + b*x^2)^(9/2),x)
```

output
```
int((x^6*(A + B*x^2 + B*x^6 + C*x^4))/(a + b*x^2)^(9/2), x)
```

## Reduce [F]

$$\int \frac{x^6(A + Bx^2 + Cx^4 + Bx^6)}{(a + bx^2)^{9/2}}\,dx = \int \frac{x^6(B\,x^6 + C\,x^4 + B\,x^2 + A)}{(b\,x^2 + a)^{\frac{9}{2}}}dx$$

input
```
int(x^6*(B*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x)
```

output
```
int(x^6*(B*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x)
```

# 3.241 $\int \frac{x^4(A+Bx^2+Cx^4+Bx^6)}{(a+bx^2)^{9/2}}\,dx$



## Optimal result

Integrand size = 32, antiderivative size = 249

$$\begin{aligned}\int \frac{x^4(A+Bx^2+Cx^4+Bx^6)}{(a+bx^2)^{9/2}}\,dx &= \frac{a(Ab^3-a(a^2B+b^2B-abC))x}{7b^5(a+bx^2)^{7/2}} \\ &- \frac{(8Ab^3-a(29a^2B+15b^2B-22abC))x}{35b^5(a+bx^2)^{5/2}} \\ &+ \frac{(3Ab^3-a(234a^2B+45b^2B-122abC))x}{105ab^5(a+bx^2)^{3/2}} \\ &+ \frac{(6Ab^3+a(582a^2B+15b^2B-176abC))x}{105a^2b^5\sqrt{a+bx^2}} \\ &+ \frac{Bx\sqrt{a+bx^2}}{2b^5} - \frac{(9aB-2bC)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{11/2}}\end{aligned}$$

output
```
1/7*a*(A*b^3-a*(B*a^2+B*b^2-C*a*b))*x/b^5/(b*x^2+a)^(7/2)-1/35*(8*A*b^3-a*
(29*B*a^2+15*B*b^2-22*C*a*b))*x/b^5/(b*x^2+a)^(5/2)+1/105*(3*A*b^3-a*(234*
B*a^2+45*B*b^2-122*C*a*b))*x/a/b^5/(b*x^2+a)^(3/2)+1/105*(6*A*b^3+a*(582*B
*a^2+15*B*b^2-176*C*a*b))*x/a^2/b^5/(b*x^2+a)^(1/2)+1/2*B*x*(b*x^2+a)^(1/2
)/b^5-1/2*(9*B*a-2*C*b)*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(11/2)
```

## Mathematica [A] (verified)

Time = 1.45 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.71

$$\int \frac{x^4(A+Bx^2+Cx^4+Bx^6)}{(a+bx^2)^{9/2}}\,dx = \frac{x(945a^6B+12Ab^6x^6-210a^5b(C-15Bx^2)+6ab^5x^4(7A+5Bx^2)+\ldots}{2\ldots}$$

$$+\frac{(-9aB+2bC)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{-\sqrt{a}+\sqrt{a+bx^2}}\right)}{b^{11/2}}$$

input
```
Integrate[(x^4*(A + B*x^2 + C*x^4 + B*x^6))/(a + b*x^2)^(9/2),x]
```

output
```
(x*(945*a^6*B + 12*A*b^6*x^6 - 210*a^5*b*(C - 15*B*x^2) + 6*a*b^5*x^4*(7*A
 + 5*B*x^2) + a^2*b^4*x^6*(-352*C + 105*B*x^2) + 14*a^4*b^2*x^2*(-50*C + 2
61*B*x^2) + 4*a^3*b^3*x^4*(-203*C + 396*B*x^2)))/(210*a^2*b^5*(a + b*x^2)^
(7/2)) + ((-9*a*B + 2*b*C)*ArcTanh[(Sqrt[b]*x)/(-Sqrt[a] + Sqrt[a + b*x^2]
)])/b^(11/2)
```

## Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.93, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {2335, 9, 25, 1586, 9, 27, 360, 1471, 27, 299, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(A+Bx^6+Bx^2+Cx^4)}{(a+bx^2)^{9/2}}dx$$

$$\Big\downarrow \text{2335}$$

$$\frac{x^5\left(A-\frac{a(a^2B-abC+b^2B)}{b^3}\right)}{7a\,(a+bx^2)^{7/2}}-\frac{\int -\frac{x^3\left(7aBx^5-7a\left(\frac{aB}{b}-C\right)x^3+\left(2Ab+\frac{5a\left(Ba^2-bCa+b^2B\right)}{b^2}\right)x\right)}{(bx^2+a)^{7/2}}dx}{7ab}$$

$$\Big\downarrow \text{9}$$

$$\frac{x^5\left(A-\frac{a(a^2B-abC+b^2B)}{b^3}\right)}{7a\left(a+bx^2\right)^{7/2}}-\frac{\int -\frac{x^4\left(7aBx^4-7a\left(\frac{aB}{b}-C\right)x^2+2Ab+\frac{5a\left(Ba^2-bCa+b^2B\right)}{b^2}\right)}{\left(bx^2+a\right)^{7/2}}dx}{7ab}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{x^4\left(7aBx^4-7a\left(\frac{aB}{b}-C\right)x^2+2Ab+\frac{5a\left(Ba^2-bCa+b^2B\right)}{b^2}\right)}{\left(bx^2+a\right)^{7/2}}dx}{7ab}+\frac{x^5\left(A-\frac{a(a^2B-abC+b^2B)}{b^3}\right)}{7a\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \ 1586$$

$$\frac{\frac{x^5\left(\frac{a\left(19a^2B-12abC+5b^2B\right)}{b^2}+2Ab\right)}{5a(a+bx^2)^{5/2}}-\frac{\int \frac{35x^3\left(\frac{a^2(2aB-bC)x}{b^2}-\frac{a^2Bx^3}{b}\right)}{\left(bx^2+a\right)^{5/2}}dx}{5a}}{7ab}+\frac{x^5\left(A-\frac{a(a^2B-abC+b^2B)}{b^3}\right)}{7a\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \ 9$$

$$\frac{\frac{x^5\left(\frac{a\left(19a^2B-12abC+5b^2B\right)}{b^2}+2Ab\right)}{5a(a+bx^2)^{5/2}}-\frac{\int \frac{35a^2x^4\left(-bBx^2+2aB-bC\right)}{b^2\left(bx^2+a\right)^{5/2}}dx}{5a}}{7ab}+\frac{x^5\left(A-\frac{a(a^2B-abC+b^2B)}{b^3}\right)}{7a\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{x^5\left(\frac{a\left(19a^2B-12abC+5b^2B\right)}{b^2}+2Ab\right)}{5a(a+bx^2)^{5/2}}-\frac{7a\int \frac{x^4\left(-bBx^2+2aB-bC\right)}{\left(bx^2+a\right)^{5/2}}dx}{b^2}}{7ab}+\frac{x^5\left(A-\frac{a(a^2B-abC+b^2B)}{b^3}\right)}{7a\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \ 360$$

$$\frac{\frac{x^5\left(\frac{a\left(19a^2B-12abC+5b^2B\right)}{b^2}+2Ab\right)}{5a(a+bx^2)^{5/2}}-\frac{7a\left(\frac{ax(3aB-bC)}{3b^2\left(a+bx^2\right)^{3/2}}-\frac{\int \frac{3b^3Bx^4-3b^2(3aB-bC)x^2+ab(3aB-bC)}{\left(bx^2+a\right)^{3/2}}dx}{3b^3}\right)}{b^2}}{7ab}+\frac{x^5\left(A-\frac{a(a^2B-abC+b^2B)}{b^3}\right)}{7a\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \ 1471$$

$$\frac{\frac{x^5\left(\frac{a(19a^2B-12abC+5b^2B)}{b^2}+2Ab\right)}{5a(a+bx^2)^{5/2}}-\frac{7a\left(\frac{ax(3aB-bC)}{3b^2(a+bx^2)^{3/2}}-\frac{\frac{bx(15aB-4bC)}{\sqrt{a+bx^2}}-\frac{\int\frac{3ab(-bBx^2+4aB-bC)}{\sqrt{bx^2+a}}dx}{a}}{3b^3}\right)}{b^2}}{7ab}+\frac{x^5\left(A-\frac{a(a^2B-abC+b^2B)}{b^3}\right)}{7a(a+bx^2)^{7/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{x^5\left(\frac{a(19a^2B-12abC+5b^2B)}{b^2}+2Ab\right)}{5a(a+bx^2)^{5/2}}-\frac{7a\left(\frac{ax(3aB-bC)}{3b^2(a+bx^2)^{3/2}}-\frac{\frac{bx(15aB-4bC)}{\sqrt{a+bx^2}}-3b\int\frac{-bBx^2+4aB-bC}{\sqrt{bx^2+a}}dx}{3b^3}\right)}{b^2}}{7ab}+\frac{x^5\left(A-\frac{a(a^2B-abC+b^2B)}{b^3}\right)}{7a(a+bx^2)^{7/2}}$$

$$\Big\downarrow \; 299$$

$$\frac{\frac{x^5\left(\frac{a(19a^2B-12abC+5b^2B)}{b^2}+2Ab\right)}{5a(a+bx^2)^{5/2}}-\frac{7a\left(\frac{ax(3aB-bC)}{3b^2(a+bx^2)^{3/2}}-\frac{\frac{bx(15aB-4bC)}{\sqrt{a+bx^2}}-3b\left(\frac{1}{2}(9aB-2bC)\int\frac{1}{\sqrt{bx^2+a}}dx-\frac{1}{2}Bx\sqrt{a+bx^2}\right)}{3b^3}\right)}{b^2}}{7ab}+\frac{x^5\left(A-\frac{a(a^2B-abC+b^2B)}{b^3}\right)}{7a(a+bx^2)^{7/2}}$$

$$\Big\downarrow \; 224$$

$$\frac{\frac{x^5\left(\frac{a(19a^2B-12abC+5b^2B)}{b^2}+2Ab\right)}{5a(a+bx^2)^{5/2}}-\frac{7a\left(\frac{ax(3aB-bC)}{3b^2(a+bx^2)^{3/2}}-\frac{\frac{bx(15aB-4bC)}{\sqrt{a+bx^2}}-3b\left(\frac{1}{2}(9aB-2bC)\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}-\frac{1}{2}Bx\sqrt{a+bx^2}\right)}{3b^3}\right)}{b^2}}{7ab}+\frac{x^5\left(A-\frac{a(a^2B-abC+b^2B)}{b^3}\right)}{7a(a+bx^2)^{7/2}}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{x^5\left(\frac{a\left(19a^2B-12abC+5b^2B\right)}{b^2}+2Ab\right)}{5a(a+bx^2)^{5/2}}-\frac{7a\left(\frac{ax(3aB-bC)}{3b^2\left(a+bx^2\right)^{3/2}}-\frac{\frac{bx(15aB-4bC)}{\sqrt{a+bx^2}}-3b\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(9aB-2bC)}{2\sqrt{b}}-\frac{1}{2}Bx\sqrt{a+bx^2}\right)}{3b^3}\right)}{b^2}}{7ab}+\frac{x^5\left(A-\frac{a(a^2B-abC+b^2B)}{b^3}\right)}{7a\left(a+bx^2\right)^{7/2}}$$

input
```
Int[(x^4*(A + B*x^2 + C*x^4 + B*x^6))/(a + b*x^2)^(9/2),x]
```

output
```
((A - (a*(a^2*B + b^2*B - a*b*C))/b^3)*x^5)/(7*a*(a + b*x^2)^(7/2)) + (((2
*A*b + (a*(19*a^2*B + 5*b^2*B - 12*a*b*C))/b^2)*x^5)/(5*a*(a + b*x^2)^(5/2
)) - (7*a*((a*(3*a*B - b*C)*x)/(3*b^2*(a + b*x^2)^(3/2)) - ((b*(15*a*B - 4
*b*C)*x)/Sqrt[a + b*x^2] - 3*b*(-1/2*(B*x*Sqrt[a + b*x^2]) + ((9*a*B - 2*b
*C)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b])))/(3*b^3)))/b^2)/(7*
a*b)
```

## Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 1586
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, d + e*x^2, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^
p, d + e*x^2, x], x, 0]}, Simp[(-R)*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(2*d
*f*(q + 1))), x] + Simp[f/(2*d*(q + 1))   Int[(f*x)^(m - 1)*(d + e*x^2)^(q
+ 1)*ExpandToSum[2*d*(q + 1)*x*Qx + R*(m + 2*q + 3)*x, x], x], x]] /; FreeQ
[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[q, -1]
&& GtQ[m, 0]
```

rule 2335
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Simp[c/(2*a*b*(p + 1))   Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSu
m[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
 b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

## Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.69

| method | result |
|---|---|
| pseudoelliptic | $-\frac{\frac{9a^2\left(b\,x^2+a\right)^{\frac{7}{2}}\left(Ba-\frac{2Cb}{9}\right)\operatorname{arctanh}\left(\frac{\sqrt{b\,x^2+a}}{x\sqrt{b}}\right)}{2}+\frac{2\left(\frac{315B\sqrt{b}\,a^6}{4}+\left(\frac{35\left(15x^2B-C\right)a^5}{2}+\frac{609\left(x^2B-\frac{50C}{261}\right)x^2b\,a^4}{2}+132\left(x^2B-\frac{203C}{396}\right)\cdots\right.\right.}{35}}{b^{\frac{11}{2}}\left(b\,x^2+a\right)^{\frac{7}{2}}a^2}$ |
| default | $A\left(-\frac{x^3}{4b(b\,x^2+a)^{\frac{7}{2}}}+\frac{3a\left(-\frac{x}{6b\left(b\,x^2+a\right)^{\frac{7}{2}}}+\frac{a\left(\frac{x}{7a\left(b\,x^2+a\right)^{\frac{7}{2}}}+\frac{\frac{6x}{35a\left(b\,x^2+a\right)^{\frac{5}{2}}}+\frac{6\left(\frac{4x}{15a\left(b\,x^2+a\right)^{\frac{3}{2}}}+\frac{8x}{15a^2\sqrt{b\,x^2+a}}\right)}{7a}}{a}\right)}{6b}\right)}{4b}\right)\cdots$ |

input
```
int(x^4*(B*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
2/35/(b*x^2+a)^(7/2)/b^(11/2)*(-315/4*a^2*(b*x^2+a)^(7/2)*(B*a-2/9*C*b)*ar
ctanh((b*x^2+a)^(1/2)/x/b^(1/2))+(315/4*B*b^(1/2)*a^6+(35/2*(15*B*x^2-C)*a
^5+609/2*(x^2*B-50/261*C)*x^2*b*a^4+132*(x^2*B-203/396*C)*x^4*b^2*a^3+35/4
*(x^2*B-352/105*C)*x^6*b^3*a^2+7/2*(5/7*x^2*B+A)*x^4*b^4*a+b^5*A*x^6)*b^(3
/2))*x)/a^2
```

## Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 653, normalized size of antiderivative = 2.62

$$\int \frac{x^4(A+Bx^2+Cx^4+Bx^6)}{(a+bx^2)^{9/2}}\,dx = \left[-\frac{105\,((9\,Ba^3b^4-2\,Ca^2b^5)x^8+9\,Ba^7-2\,Ca^6b+4\,(9\,Ba^4b^3-2\,Ca\right.$$

input
```
integrate(x^4*(B*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

output
```
[-1/420*(105*((9*B*a^3*b^4 - 2*C*a^2*b^5)*x^8 + 9*B*a^7 - 2*C*a^6*b + 4*(9
*B*a^4*b^3 - 2*C*a^3*b^4)*x^6 + 6*(9*B*a^5*b^2 - 2*C*a^4*b^3)*x^4 + 4*(9*B
*a^6*b - 2*C*a^5*b^2)*x^2)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b
)*x - a) - 2*(105*B*a^2*b^5*x^9 + 2*(792*B*a^3*b^4 - 176*C*a^2*b^5 + 15*B*
a*b^6 + 6*A*b^7)*x^7 + 14*(261*B*a^4*b^3 - 58*C*a^3*b^4 + 3*A*a*b^6)*x^5 +
 350*(9*B*a^5*b^2 - 2*C*a^4*b^3)*x^3 + 105*(9*B*a^6*b - 2*C*a^5*b^2)*x)*sq
rt(b*x^2 + a))/(a^2*b^10*x^8 + 4*a^3*b^9*x^6 + 6*a^4*b^8*x^4 + 4*a^5*b^7*x
^2 + a^6*b^6), 1/210*(105*((9*B*a^3*b^4 - 2*C*a^2*b^5)*x^8 + 9*B*a^7 - 2*C
*a^6*b + 4*(9*B*a^4*b^3 - 2*C*a^3*b^4)*x^6 + 6*(9*B*a^5*b^2 - 2*C*a^4*b^3)
*x^4 + 4*(9*B*a^6*b - 2*C*a^5*b^2)*x^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*
x^2 + a)) + (105*B*a^2*b^5*x^9 + 2*(792*B*a^3*b^4 - 176*C*a^2*b^5 + 15*B*a
*b^6 + 6*A*b^7)*x^7 + 14*(261*B*a^4*b^3 - 58*C*a^3*b^4 + 3*A*a*b^6)*x^5 +
350*(9*B*a^5*b^2 - 2*C*a^4*b^3)*x^3 + 105*(9*B*a^6*b - 2*C*a^5*b^2)*x)*sqr
t(b*x^2 + a))/(a^2*b^10*x^8 + 4*a^3*b^9*x^6 + 6*a^4*b^8*x^4 + 4*a^5*b^7*x^
2 + a^6*b^6)]
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6467 vs. 2(238) = 476.

Time = 86.91 (sec) , antiderivative size = 6467, normalized size of antiderivative = 25.97

$$\int \frac{x^4(A+Bx^2+Cx^4+Bx^6)}{(a+bx^2)^{9/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x**4*(B*x**6+C*x**4+B*x**2+A)/(b*x**2+a)**(9/2),x)
```

output
```
A*(7*a*x**5/(35*a**(11/2)*sqrt(1 + b*x**2/a) + 105*a**(9/2)*b*x**2*sqrt(1
+ b*x**2/a) + 105*a**(7/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 35*a**(5/2)*b**3
*x**6*sqrt(1 + b*x**2/a)) + 2*b*x**7/(35*a**(11/2)*sqrt(1 + b*x**2/a) + 10
5*a**(9/2)*b*x**2*sqrt(1 + b*x**2/a) + 105*a**(7/2)*b**2*x**4*sqrt(1 + b*x
**2/a) + 35*a**(5/2)*b**3*x**6*sqrt(1 + b*x**2/a))) + B*x**7/(7*a**(9/2)*s
qrt(1 + b*x**2/a) + 21*a**(7/2)*b*x**2*sqrt(1 + b*x**2/a) + 21*a**(5/2)*b*
*2*x**4*sqrt(1 + b*x**2/a) + 7*a**(3/2)*b**3*x**6*sqrt(1 + b*x**2/a)) + B*
(-315*a**(311/2)*b**66*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(70*a**
(309/2)*b**(143/2)*sqrt(1 + b*x**2/a) + 420*a**(307/2)*b**(145/2)*x**2*sqr
t(1 + b*x**2/a) + 1050*a**(305/2)*b**(147/2)*x**4*sqrt(1 + b*x**2/a) + 140
0*a**(303/2)*b**(149/2)*x**6*sqrt(1 + b*x**2/a) + 1050*a**(301/2)*b**(151/
2)*x**8*sqrt(1 + b*x**2/a) + 420*a**(299/2)*b**(153/2)*x**10*sqrt(1 + b*x*
*2/a) + 70*a**(297/2)*b**(155/2)*x**12*sqrt(1 + b*x**2/a)) - 1890*a**(309/
2)*b**67*x**2*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(70*a**(309/2)*b
**(143/2)*sqrt(1 + b*x**2/a) + 420*a**(307/2)*b**(145/2)*x**2*sqrt(1 + b*x
**2/a) + 1050*a**(305/2)*b**(147/2)*x**4*sqrt(1 + b*x**2/a) + 1400*a**(303
/2)*b**(149/2)*x**6*sqrt(1 + b*x**2/a) + 1050*a**(301/2)*b**(151/2)*x**8*s
qrt(1 + b*x**2/a) + 420*a**(299/2)*b**(153/2)*x**10*sqrt(1 + b*x**2/a) + 7
0*a**(297/2)*b**(155/2)*x**12*sqrt(1 + b*x**2/a)) - 4725*a**(307/2)*b**68*
x**4*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(70*a**(309/2)*b**(143...
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 753 vs. $2(221) = 442$.

Time = 0.06 (sec) , antiderivative size = 753, normalized size of antiderivative = 3.02

$$\int \frac{x^4(A + Bx^2 + Cx^4 + Bx^6)}{(a + bx^2)^{9/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^4*(B*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

output
```
1/2*B*x^9/((b*x^2 + a)^(7/2)*b) - 1/35*(35*x^6/((b*x^2 + a)^(7/2)*b) + 70*
a*x^4/((b*x^2 + a)^(7/2)*b^2) + 56*a^2*x^2/((b*x^2 + a)^(7/2)*b^3) + 16*a^
3/((b*x^2 + a)^(7/2)*b^4))*C*x + 9/70*(35*x^6/((b*x^2 + a)^(7/2)*b) + 70*a
*x^4/((b*x^2 + a)^(7/2)*b^2) + 56*a^2*x^2/((b*x^2 + a)^(7/2)*b^3) + 16*a^3
/((b*x^2 + a)^(7/2)*b^4))*B*a*x/b + 3/10*B*a*x*(15*x^4/((b*x^2 + a)^(5/2)*
b) + 20*a*x^2/((b*x^2 + a)^(5/2)*b^2) + 8*a^2/((b*x^2 + a)^(5/2)*b^3))/b^2
 - 1/15*C*x*(15*x^4/((b*x^2 + a)^(5/2)*b) + 20*a*x^2/((b*x^2 + a)^(5/2)*b^
2) + 8*a^2/((b*x^2 + a)^(5/2)*b^3))/b - 1/2*B*x^5/((b*x^2 + a)^(7/2)*b) +
3/2*B*a*x*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2))/b^3
- 1/3*C*x*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2))/b^2
+ 9/2*B*a^2*x^3/((b*x^2 + a)^(5/2)*b^4) - C*a*x^3/((b*x^2 + a)^(5/2)*b^3)
- 5/8*B*a*x^3/((b*x^2 + a)^(7/2)*b^2) - 1/4*A*x^3/((b*x^2 + a)^(7/2)*b) -
417/70*B*a*x/(sqrt(b*x^2 + a)*b^5) - 51/70*B*a^2*x/((b*x^2 + a)^(3/2)*b^5)
 + 261/70*B*a^3*x/((b*x^2 + a)^(5/2)*b^5) + 139/105*C*x/(sqrt(b*x^2 + a)*b
^4) + 17/105*C*a*x/((b*x^2 + a)^(3/2)*b^4) - 29/35*C*a^2*x/((b*x^2 + a)^(5
/2)*b^4) + 1/14*B*x/((b*x^2 + a)^(3/2)*b^3) + 1/7*B*x/(sqrt(b*x^2 + a)*a*b
^3) + 3/56*B*a*x/((b*x^2 + a)^(5/2)*b^3) - 15/56*B*a^2*x/((b*x^2 + a)^(7/2
)*b^3) + 3/140*A*x/((b*x^2 + a)^(5/2)*b^2) + 2/35*A*x/(sqrt(b*x^2 + a)*a^2
*b^2) + 1/35*A*x/((b*x^2 + a)^(3/2)*a*b^2) - 3/28*A*a*x/((b*x^2 + a)^(7/2)
*b^2) - 9/2*B*a*arcsinh(b*x/sqrt(a*b))/b^(11/2) + C*arcsinh(b*x/sqrt(a*...
```

## Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.82

$$\int \frac{x^4(A+Bx^2+Cx^4+Bx^6)}{(a+bx^2)^{9/2}}\,dx = \frac{\left(\left(\left(\left(\frac{105\,Bx^2}{b}+\frac{2\,(792\,Ba^4b^7-176\,Ca^3b^8+15\,Ba^2b^9+6\,Aab^{10})}{a^3b^9}\right)x^2+\frac{14\,(261\,Ba^5b^6-58\cdots}{a^3\cdots}\cdots\right.\right.\right.}{210\,(bx^2+\cdots} + \frac{(9\,Ba-2\,Cb)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2\,b^{\frac{11}{2}}}$$

input
```
integrate(x^4*(B*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="giac")
```

output
```
1/210*((((105*B*x^2/b + 2*(792*B*a^4*b^7 - 176*C*a^3*b^8 + 15*B*a^2*b^9 +
6*A*a*b^10)/(a^3*b^9))*x^2 + 14*(261*B*a^5*b^6 - 58*C*a^4*b^7 + 3*A*a^2*b^
9)/(a^3*b^9))*x^2 + 350*(9*B*a^6*b^5 - 2*C*a^5*b^6)/(a^3*b^9))*x^2 + 105*(
9*B*a^7*b^4 - 2*C*a^6*b^5)/(a^3*b^9))*x/(b*x^2 + a)^(7/2) + 1/2*(9*B*a - 2
*C*b)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(11/2)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(A+Bx^2+Cx^4+Bx^6)}{(a+bx^2)^{9/2}}\,dx = \int \frac{x^4\,(B\,x^6+C\,x^4+B\,x^2+A)}{{(b\,x^2+a)}^{9/2}}\,dx$$

input
```
int((x^4*(A + B*x^2 + B*x^6 + C*x^4))/(a + b*x^2)^(9/2),x)
```

output
```
int((x^4*(A + B*x^2 + B*x^6 + C*x^4))/(a + b*x^2)^(9/2), x)
```

## Reduce [B] (verification not implemented)

Time = 8.85 (sec) , antiderivative size = 728, normalized size of antiderivative = 2.92

$$\int \frac{x^4(A+Bx^2+Cx^4+Bx^6)}{(a+bx^2)^{9/2}}\,dx = \text{Too large to display}$$

input
```
int(x^4*(B*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x)
```

output
```
(945*sqrt(a + b*x**2)*a**5*b*x + 3150*sqrt(a + b*x**2)*a**4*b**2*x**3 - 21
0*sqrt(a + b*x**2)*a**4*b*c*x + 3654*sqrt(a + b*x**2)*a**3*b**3*x**5 - 700
*sqrt(a + b*x**2)*a**3*b**2*c*x**3 + 1584*sqrt(a + b*x**2)*a**2*b**4*x**7
- 812*sqrt(a + b*x**2)*a**2*b**3*c*x**5 + 105*sqrt(a + b*x**2)*a*b**5*x**9
 + 42*sqrt(a + b*x**2)*a*b**5*x**5 - 352*sqrt(a + b*x**2)*a*b**4*c*x**7 +
42*sqrt(a + b*x**2)*b**6*x**7 - 945*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b
)*x)/sqrt(a))*a**6 - 3780*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(
a))*a**5*b*x**2 + 210*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*
a**5*c - 5670*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**4*b**
2*x**4 + 840*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**4*b*c*
x**2 - 3780*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**3*b**3*
x**6 + 1260*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**3*b**2*
c*x**4 - 945*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2*b**4
*x**8 + 840*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2*b**3*
c*x**6 + 210*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a*b**4*c*
x**8 - 639*sqrt(b)*a**6 - 2556*sqrt(b)*a**5*b*x**2 + 112*sqrt(b)*a**5*c -
3834*sqrt(b)*a**4*b**2*x**4 + 18*sqrt(b)*a**4*b**2 + 448*sqrt(b)*a**4*b*c*
x**2 - 2556*sqrt(b)*a**3*b**3*x**6 + 72*sqrt(b)*a**3*b**3*x**2 + 672*sqrt(
b)*a**3*b**2*c*x**4 - 639*sqrt(b)*a**2*b**4*x**8 + 108*sqrt(b)*a**2*b**4*x
**4 + 448*sqrt(b)*a**2*b**3*c*x**6 + 72*sqrt(b)*a*b**5*x**6 + 112*sqrt(...
```

# 3.242 $\int \frac{x^2(A+Bx^2+Cx^4+Bx^6)}{(a+bx^2)^{9/2}}\,dx$



## Optimal result

Integrand size = 32, antiderivative size = 219

$$\int \frac{x^2(A+Bx^2+Cx^4+Bx^6)}{(a+bx^2)^{9/2}}\,dx = -\frac{(Ab^3-a(a^2B+b^2B-abC))\,x}{7b^4(a+bx^2)^{7/2}} + \frac{(Ab^3-a(22a^2B+8b^2B-15abC))\,x}{35ab^4(a+bx^2)^{5/2}} + \frac{(4Ab^3+a(122a^2B+3b^2B-45abC))\,x}{105a^2b^4(a+bx^2)^{3/2}} + \frac{(8Ab^3-a(176a^2B-6b^2B-15abC))\,x}{105a^3b^4\sqrt{a+bx^2}} + \frac{B\,\mathrm{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{9/2}}$$

output

```
-1/7*(A*b^3-a*(B*a^2+B*b^2-C*a*b))*x/b^4/(b*x^2+a)^(7/2)+1/35*(A*b^3-a*(22
*B*a^2+8*B*b^2-15*C*a*b))*x/a/b^4/(b*x^2+a)^(5/2)+1/105*(4*A*b^3+a*(122*B*
a^2+3*B*b^2-45*C*a*b))*x/a^2/b^4/(b*x^2+a)^(3/2)+1/105*(8*A*b^3-a*(176*B*a
^2-6*B*b^2-15*C*a*b))*x/a^3/b^4/(b*x^2+a)^(1/2)+B*arctanh(b^(1/2)*x/(b*x^2
+a)^(1/2))/b^(9/2)
```

## Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.67

$$\int \frac{x^2(A+Bx^2+Cx^4+Bx^6)}{(a+bx^2)^{9/2}}\,dx = \frac{-105a^6Bx-350a^5bBx^3-406a^4b^2Bx^5+8Ab^6x^7-176a^3b^3Bx^7+2a}{105a^3b^4\,(a+bx^2)^{7}}$$
$$-\frac{B\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{b^{9/2}}$$

input
```
Integrate[(x^2*(A + B*x^2 + C*x^4 + B*x^6))/(a + b*x^2)^(9/2),x]
```

output
```
(-105*a^6*B*x - 350*a^5*b*B*x^3 - 406*a^4*b^2*B*x^5 + 8*A*b^6*x^7 - 176*a^
3*b^3*B*x^7 + 2*a*b^5*x^5*(14*A + 3*B*x^2) + a^2*b^4*x^3*(35*A + 21*B*x^2
+ 15*C*x^4))/(105*a^3*b^4*(a + b*x^2)^(7/2)) - (B*Log[-(Sqrt[b]*x) + Sqrt[
a + b*x^2]])/b^(9/2)
```

## Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.01, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.344$, Rules used = {2335, 9, 25, 1586, 9, 25, 27, 357, 252, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\left(A+Bx^6+Bx^2+Cx^4\right)}{\left(a+bx^2\right)^{9/2}}\,dx$$

$$\Big\downarrow \text{2335}$$

$$\frac{x^3\left(A-\frac{a\left(a^2B-abC+b^2B\right)}{b^3}\right)}{7a\left(a+bx^2\right)^{7/2}}-\frac{\int -\frac{x\left(7aBx^5-7a\left(\frac{aB}{b}-C\right)x^3+\left(4Ab+\frac{3a\left(Ba^2-bCa+b^2B\right)}{b^2}\right)x\right)}{\left(bx^2+a\right)^{7/2}}dx}{7ab}$$

$$\Big\downarrow \text{9}$$

$$\frac{x^3\left(A-\frac{a(a^2B-abC+b^2B)}{b^3}\right)}{7a\left(a+bx^2\right)^{7/2}}-\frac{\int -\frac{x^2\left(7aBx^4-7a\left(\frac{aB}{b}-C\right)x^2+4Ab+\frac{3a\left(Ba^2-bCa+b^2B\right)}{b^2}\right)}{(bx^2+a)^{7/2}}dx}{7ab}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{x^2\left(7aBx^4-7a\left(\frac{aB}{b}-C\right)x^2+4Ab+\frac{3a\left(Ba^2-bCa+b^2B\right)}{b^2}\right)}{(bx^2+a)^{7/2}}dx}{7ab}+\frac{x^3\left(A-\frac{a(a^2B-abC+b^2B)}{b^3}\right)}{7a\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \; 1586$$

$$\frac{\frac{x^3\left(\frac{a\left(17a^2B-10abC+3b^2B\right)}{b^2}+4Ab\right)}{5a(a+bx^2)^{5/2}}-\frac{\int -\frac{x\left(\frac{35a^2Bx^3}{b}+\left(8Ab-\frac{3a\left(12Ba^2-5bCa-2b^2B\right)}{b^2}\right)x\right)}{\left(bx^2+a\right)^{5/2}}dx}{5a}}{7ab}+\frac{x^3\left(A-\frac{a(a^2B-abC+b^2B)}{b^3}\right)}{7a\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \; 9$$

$$\frac{\frac{x^3\left(\frac{a\left(17a^2B-10abC+3b^2B\right)}{b^2}+4Ab\right)}{5a(a+bx^2)^{5/2}}-\frac{\int -\frac{x^2\left(35a^2Bx^2+b\left(8Ab-\frac{3a\left(12Ba^2-5bCa-2b^2B\right)}{b^2}\right)\right)}{b\left(bx^2+a\right)^{5/2}}dx}{5a}}{7ab}+\frac{x^3\left(A-\frac{a(a^2B-abC+b^2B)}{b^3}\right)}{7a\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{x^2\left(8Ab^2+35a^2Bx^2-3a\left(\frac{12Ba^2}{b}-5Ca-2bB\right)\right)}{b\left(bx^2+a\right)^{5/2}}dx}{5a}+\frac{x^3\left(\frac{a\left(17a^2B-10abC+3b^2B\right)}{b^2}+4Ab\right)}{5a(a+bx^2)^{5/2}}}{7ab}+\frac{x^3\left(A-\frac{a(a^2B-abC+b^2B)}{b^3}\right)}{7a\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{x^2\left(8Ab^2+35a^2Bx^2-3a\left(\frac{12Ba^2}{b}-5Ca-2bB\right)\right)}{\left(bx^2+a\right)^{5/2}}dx}{5ab}+\frac{x^3\left(\frac{a\left(17a^2B-10abC+3b^2B\right)}{b^2}+4Ab\right)}{5a(a+bx^2)^{5/2}}}{7ab}+\frac{x^3\left(A-\frac{a(a^2B-abC+b^2B)}{b^3}\right)}{7a\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \; 357$$

$$\frac{\frac{\frac{35a^2B\int \frac{x^2}{\left(bx^2+a\right)^{3/2}}dx}{b}+\frac{x^3\left(8Ab^3-a\left(71a^2B-15abC-6b^2B\right)\right)}{3ab\left(a+bx^2\right)^{3/2}}}{5ab}+\frac{x^3\left(\frac{a\left(17a^2B-10abC+3b^2B\right)}{b^2}+4Ab\right)}{5a(a+bx^2)^{5/2}}}{7ab}+\frac{x^3\left(A-\frac{a(a^2B-abC+b^2B)}{b^3}\right)}{7a\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \; 252$$

$$\frac{\frac{\frac{35a^2B\left(\frac{\int \frac{1}{\sqrt{bx^2+a}}dx}{b}-\frac{x}{b\sqrt{a+bx^2}}\right)}{b}+\frac{x^3\left(8Ab^3-a\left(71a^2B-15abC-6b^2B\right)\right)}{3ab\left(a+bx^2\right)^{3/2}}}{5ab}+\frac{x^3\left(\frac{a\left(17a^2B-10abC+3b^2B\right)}{b^2}+4Ab\right)}{5a(a+bx^2)^{5/2}}}{7ab}+\frac{x^3\left(A-\frac{a(a^2B-abC+b^2B)}{b^3}\right)}{7a\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \; 224$$

$$\frac{\frac{\frac{35a^2B\left(\frac{\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{b}-\frac{x}{b\sqrt{a+bx^2}}\right)}{b}+\frac{x^3\left(8Ab^3-a\left(71a^2B-15abC-6b^2B\right)\right)}{3ab\left(a+bx^2\right)^{3/2}}}{5ab}+\frac{x^3\left(\frac{a\left(17a^2B-10abC+3b^2B\right)}{b^2}+4Ab\right)}{5a(a+bx^2)^{5/2}}}{7ab}+\frac{x^3\left(A-\frac{a(a^2B-abC+b^2B)}{b^3}\right)}{7a\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{\frac{x^3\left(8Ab^3-a\left(71a^2B-15abC-6b^2B\right)\right)}{3ab\left(a+bx^2\right)^{3/2}}+\frac{35a^2B\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}-\frac{x}{b\sqrt{a+bx^2}}\right)}{b}}{5ab}+\frac{x^3\left(\frac{a\left(17a^2B-10abC+3b^2B\right)}{b^2}+4Ab\right)}{5a(a+bx^2)^{5/2}}}{7ab}+\frac{x^3\left(A-\frac{a\left(a^2B-abC+b^2B\right)}{b^3}\right)}{7a\left(a+bx^2\right)^{7/2}}$$

input
```
Int[(x^2*(A + B*x^2 + C*x^4 + B*x^6))/(a + b*x^2)^(9/2),x]
```

output
```
((A - (a*(a^2*B + b^2*B - a*b*C))/b^3)*x^3)/(7*a*(a + b*x^2)^(7/2)) + (((4
*A*b + (a*(17*a^2*B + 3*b^2*B - 10*a*b*C))/b^2)*x^3)/(5*a*(a + b*x^2)^(5/2
)) + (((8*A*b^3 - a*(71*a^2*B - 6*b^2*B - 15*a*b*C))*x^3)/(3*a*b*(a + b*x^
2)^(3/2)) + (35*a^2*B*(-(x/(b*Sqrt[a + b*x^2])) + ArcTanh[(Sqrt[b]*x)/Sqrt
[a + b*x^2]]/b^(3/2)))/b)/(5*a*b))/(7*a*b)
```

## Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 357
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_
Symbol] :> Simp[(b*c - a*d)*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*b*e*(m +
1))), x] + Simp[d/b   Int[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b*c - a*d, 0] && EqQ[m + 2*p + 3, 0] && LtQ[p, -1]
```

rule 1586
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, d + e*x^2, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^
p, d + e*x^2, x], x, 0]}, Simp[(-R)*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(2*d
*f*(q + 1))), x] + Simp[f/(2*d*(q + 1))   Int[(f*x)^(m - 1)*(d + e*x^2)^(q
+ 1)*ExpandToSum[2*d*(q + 1)*x*Qx + R*(m + 2*q + 3)*x, x], x], x]] /; FreeQ
[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[q, -1]
&& GtQ[m, 0]
```

rule 2335
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Simp[c/(2*a*b*(p + 1))   Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSu
m[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
 b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

## Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.67

| method | result |
|---|---|
| pseudoelliptic | $\frac{3\left(b\,x^{2}+a\right)^{\frac{7}{2}} B\,a^{3} \operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{x\sqrt{b}}\right)+x\left(x^{2} a^{2} \left(\frac{3}{7} C\,x^{4}+\frac{3}{5} x^{2} B+A\right) b^{\frac{9}{2}}+\frac{4\left(\frac{3 x^{2} B}{14}+A\right) a\,x^{4} b^{\frac{11}{2}}}{5}+\frac{8 A\,b^{\frac{13}{2}} x^{6}}{35}-10 a^{3}\left(\frac{88 b^{\frac{7}{2}} x^{6}}{175} \cdots\right.\right.}{3 b^{\frac{9}{2}} \left(b\,x^{2}+a\right)^{\frac{7}{2}} a^{3}}$ |
| default | $A\left(-\frac{x}{6b\left(b\,x^{2}+a\right)^{\frac{7}{2}}}+\frac{a\left(\frac{x}{7a\left(b\,x^{2}+a\right)^{\frac{7}{2}}}+\frac{\frac{6x}{35a\left(b\,x^{2}+a\right)^{\frac{5}{2}}}+\frac{6\left(\frac{4x}{15a\left(b\,x^{2}+a\right)^{\frac{3}{2}}}+\frac{8x}{15a^{2}\sqrt{b\,x^{2}+a}}\right)}{7a}}{a}\right)}{6b}\right)+B\left(-\frac{x^{3}}{4b\left(b\,x^{2}+a \cdots\right.}\right.$ |

input
```
int(x^2*(B*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/b^(9/2)/(b*x^2+a)^(7/2)*(3*(b*x^2+a)^(7/2)*B*a^3*arctanh((b*x^2+a)^(1/
2)/x/b^(1/2))+x*(x^2*a^2*(3/7*C*x^4+3/5*x^2*B+A)*b^(9/2)+4/5*(3/14*x^2*B+A
)*a*x^4*b^(11/2)+8/35*A*b^(13/2)*x^6-10*a^3*(88/175*b^(7/2)*x^6+29/25*a*b^
(5/2)*x^4+a^2*b^(3/2)*x^2+3/10*a^3*b^(1/2))*B))/a^3
```

## Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 491, normalized size of antiderivative = 2.24

$$\int \frac{x^2(A+Bx^2+Cx^4+Bx^6)}{(a+bx^2)^{9/2}}\,dx = \left[\frac{105\,(Ba^3b^4x^8+4\,Ba^4b^3x^6+6\,Ba^5b^2x^4+4\,Ba^6bx^2+Ba^7)\sqrt{b}\log\left(-\right.}{}\right.$$

$$-\frac{105\,(Ba^3b^4x^8+4\,Ba^4b^3x^6+6\,Ba^5b^2x^4+4\,Ba^6bx^2+Ba^7)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)+(105\,Ba^6bx+(176\,B}{105\,(a^3b^9x^8+4\,a^4b^8x^6}$$

input
```
integrate(x^2*(B*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

output
```
[1/210*(105*(B*a^3*b^4*x^8 + 4*B*a^4*b^3*x^6 + 6*B*a^5*b^2*x^4 + 4*B*a^6*b
*x^2 + B*a^7)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*
(105*B*a^6*b*x + (176*B*a^3*b^4 - 15*C*a^2*b^5 - 6*B*a*b^6 - 8*A*b^7)*x^7
+ 7*(58*B*a^4*b^3 - 3*B*a^2*b^5 - 4*A*a*b^6)*x^5 + 35*(10*B*a^5*b^2 - A*a^
2*b^5)*x^3)*sqrt(b*x^2 + a))/(a^3*b^9*x^8 + 4*a^4*b^8*x^6 + 6*a^5*b^7*x^4
+ 4*a^6*b^6*x^2 + a^7*b^5), -1/105*(105*(B*a^3*b^4*x^8 + 4*B*a^4*b^3*x^6 +
 6*B*a^5*b^2*x^4 + 4*B*a^6*b*x^2 + B*a^7)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(
b*x^2 + a)) + (105*B*a^6*b*x + (176*B*a^3*b^4 - 15*C*a^2*b^5 - 6*B*a*b^6 -
 8*A*b^7)*x^7 + 7*(58*B*a^4*b^3 - 3*B*a^2*b^5 - 4*A*a*b^6)*x^5 + 35*(10*B*
a^5*b^2 - A*a^2*b^5)*x^3)*sqrt(b*x^2 + a))/(a^3*b^9*x^8 + 4*a^4*b^8*x^6 +
6*a^5*b^7*x^4 + 4*a^6*b^6*x^2 + a^7*b^5)]
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3803 vs. $2(214) = 428$.

Time = 74.11 (sec) , antiderivative size = 3803, normalized size of antiderivative = 17.37

$$\int \frac{x^2(A + Bx^2 + Cx^4 + Bx^6)}{(a + bx^2)^{9/2}}\, dx = \text{Too large to display}$$

input
```
integrate(x**2*(B*x**6+C*x**4+B*x**2+A)/(b*x**2+a)**(9/2),x)
```

output
```
A*(35*a**5*x**3/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a**(17/2)*b*x**2*s
qrt(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 420*a**(1
3/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*sqrt(1 + b*x**
2/a)) + 63*a**4*b*x**5/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a**(17/2)*b
*x**2*sqrt(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 42
0*a**(13/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*sqrt(1
+ b*x**2/a)) + 36*a**3*b**2*x**7/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a
**(17/2)*b*x**2*sqrt(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x*
*2/a) + 420*a**(13/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x*
*8*sqrt(1 + b*x**2/a)) + 8*a**2*b**3*x**9/(105*a**(19/2)*sqrt(1 + b*x**2/a
) + 420*a**(17/2)*b*x**2*sqrt(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt
(1 + b*x**2/a) + 420*a**(13/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2
)*b**4*x**8*sqrt(1 + b*x**2/a))) + B*(7*a*x**5/(35*a**(11/2)*sqrt(1 + b*x*
*2/a) + 105*a**(9/2)*b*x**2*sqrt(1 + b*x**2/a) + 105*a**(7/2)*b**2*x**4*sq
rt(1 + b*x**2/a) + 35*a**(5/2)*b**3*x**6*sqrt(1 + b*x**2/a)) + 2*b*x**7/(3
5*a**(11/2)*sqrt(1 + b*x**2/a) + 105*a**(9/2)*b*x**2*sqrt(1 + b*x**2/a) +
105*a**(7/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 35*a**(5/2)*b**3*x**6*sqrt(1 +
 b*x**2/a))) + B*(105*a**(205/2)*b**45*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/
sqrt(a))/(105*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**
(101/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(...
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 533 vs. $2(197) = 394$.

Time = 0.06 (sec) , antiderivative size = 533, normalized size of antiderivative = 2.43

$$\int \frac{x^2(A+Bx^2+Cx^4+Bx^6)}{(a+bx^2)^{9/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(B*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

output
```
-1/35*(35*x^6/((b*x^2 + a)^(7/2)*b) + 70*a*x^4/((b*x^2 + a)^(7/2)*b^2) + 5
6*a^2*x^2/((b*x^2 + a)^(7/2)*b^3) + 16*a^3/((b*x^2 + a)^(7/2)*b^4))*B*x -
1/15*B*x*(15*x^4/((b*x^2 + a)^(5/2)*b) + 20*a*x^2/((b*x^2 + a)^(5/2)*b^2)
+ 8*a^2/((b*x^2 + a)^(5/2)*b^3))/b - 1/2*C*x^5/((b*x^2 + a)^(7/2)*b) - 1/3
*B*x*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2))/b^2 - B*a
*x^3/((b*x^2 + a)^(5/2)*b^3) - 5/8*C*a*x^3/((b*x^2 + a)^(7/2)*b^2) - 1/4*B
*x^3/((b*x^2 + a)^(7/2)*b) + 139/105*B*x/(sqrt(b*x^2 + a)*b^4) + 17/105*B*
a*x/((b*x^2 + a)^(3/2)*b^4) - 29/35*B*a^2*x/((b*x^2 + a)^(5/2)*b^4) + 1/14
*C*x/((b*x^2 + a)^(3/2)*b^3) + 1/7*C*x/(sqrt(b*x^2 + a)*a*b^3) + 3/56*C*a*
x/((b*x^2 + a)^(5/2)*b^3) - 15/56*C*a^2*x/((b*x^2 + a)^(7/2)*b^3) + 3/140*
B*x/((b*x^2 + a)^(5/2)*b^2) + 2/35*B*x/(sqrt(b*x^2 + a)*a^2*b^2) + 1/35*B*
x/((b*x^2 + a)^(3/2)*a*b^2) - 3/28*B*a*x/((b*x^2 + a)^(7/2)*b^2) - 1/7*A*x
/((b*x^2 + a)^(7/2)*b) + 8/105*A*x/(sqrt(b*x^2 + a)*a^3*b) + 4/105*A*x/((b
*x^2 + a)^(3/2)*a^2*b) + 1/35*A*x/((b*x^2 + a)^(5/2)*a*b) + B*arcsinh(b*x/
sqrt(a*b))/b^(9/2)
```

## Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.73

$$\int \frac{x^2(A+Bx^2+Cx^4+Bx^6)}{(a+bx^2)^{9/2}}\,dx =$$

$$-\frac{\left(\left(x^2\left(\frac{(176\,Ba^3b^6-15\,Ca^2b^7-6\,Bab^8-8\,Ab^9)x^2}{a^3b^7}+\frac{7\,(58\,Ba^4b^5-3\,Ba^2b^7-4\,Aab^8)}{a^3b^7}\right)+\frac{35\,(10\,Ba^5b^4-Aa^2b^7)}{a^3b^7}\right)x^2+\frac{105\,Ba^3}{b^4}\right)x}{105\,(bx^2+a)^{\frac{7}{2}}} - \frac{B\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{b^{\frac{9}{2}}}$$

input
```
integrate(x^2*(B*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="giac")
```

output
```
-1/105*((x^2*(((176*B*a^3*b^6 - 15*C*a^2*b^7 - 6*B*a*b^8 - 8*A*b^9)*x^2/(a^
3*b^7) + 7*(58*B*a^4*b^5 - 3*B*a^2*b^7 - 4*A*a*b^8)/(a^3*b^7)) + 35*(10*B*
a^5*b^4 - A*a^2*b^7)/(a^3*b^7))*x^2 + 105*B*a^3/b^4)*x/(b*x^2 + a)^(7/2) -
 B*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(9/2)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(A+Bx^2+Cx^4+Bx^6)}{(a+bx^2)^{9/2}}\,dx=\int \frac{x^2\,(B\,x^6+C\,x^4+B\,x^2+A)}{(b\,x^2+a)^{9/2}}\,dx$$

input
```
int((x^2*(A + B*x^2 + B*x^6 + C*x^4))/(a + b*x^2)^(9/2),x)
```

output
```
int((x^2*(A + B*x^2 + B*x^6 + C*x^4))/(a + b*x^2)^(9/2), x)
```

## Reduce [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 519, normalized size of antiderivative = 2.37

$$\int \frac{x^2(A+Bx^2+Cx^4+Bx^6)}{(a+bx^2)^{9/2}}\,dx=\frac{-105\sqrt{b\,x^2+a}\,a^5bx-350\sqrt{b\,x^2+a}\,a^4b^2x^3-406\sqrt{b\,x^2+a}\,a^3b^3x^5-}{}$$

input
```
int(x^2*(B*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x)
```

output
```
( - 105*sqrt(a + b*x**2)*a**5*b*x - 350*sqrt(a + b*x**2)*a**4*b**2*x**3 -
406*sqrt(a + b*x**2)*a**3*b**3*x**5 - 176*sqrt(a + b*x**2)*a**2*b**4*x**7
+ 35*sqrt(a + b*x**2)*a**2*b**4*x**3 + 49*sqrt(a + b*x**2)*a*b**5*x**5 + 1
5*sqrt(a + b*x**2)*a*b**4*c*x**7 + 14*sqrt(a + b*x**2)*b**6*x**7 + 105*sqr
t(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**6 + 420*sqrt(b)*log((s
qrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**5*b*x**2 + 630*sqrt(b)*log((sqrt(
a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**4*b**2*x**4 + 420*sqrt(b)*log((sqrt(a
 + b*x**2) + sqrt(b)*x)/sqrt(a))*a**3*b**3*x**6 + 105*sqrt(b)*log((sqrt(a
+ b*x**2) + sqrt(b)*x)/sqrt(a))*a**2*b**4*x**8 + 56*sqrt(b)*a**6 + 224*sqr
t(b)*a**5*b*x**2 + 15*sqrt(b)*a**5*c + 336*sqrt(b)*a**4*b**2*x**4 - 14*sqr
t(b)*a**4*b**2 + 60*sqrt(b)*a**4*b*c*x**2 + 224*sqrt(b)*a**3*b**3*x**6 - 5
6*sqrt(b)*a**3*b**3*x**2 + 90*sqrt(b)*a**3*b**2*c*x**4 + 56*sqrt(b)*a**2*b
**4*x**8 - 84*sqrt(b)*a**2*b**4*x**4 + 60*sqrt(b)*a**2*b**3*c*x**6 - 56*sq
rt(b)*a*b**5*x**6 + 15*sqrt(b)*a*b**4*c*x**8 - 14*sqrt(b)*b**6*x**8)/(105*
a**2*b**4*(a**4 + 4*a**3*b*x**2 + 6*a**2*b**2*x**4 + 4*a*b**3*x**6 + b**4*
x**8))
```

# 3.243 $\int \frac{A+Bx^2+Cx^4+Bx^6}{(a+bx^2)^{9/2}}\,dx$



## Optimal result

Integrand size = 29, antiderivative size = 191

$$\int \frac{A+Bx^2+Cx^4+Bx^6}{(a+bx^2)^{9/2}}\,dx = \frac{\left(\frac{A}{a}-\frac{a^2B+b^2B-abC}{b^3}\right)x}{7(a+bx^2)^{7/2}} + \frac{(6Ab^3+a(15a^2B+b^2B-8abC))\,x}{35a^2b^3(a+bx^2)^{5/2}} + \frac{(24Ab^3-a(45a^2B-4b^2B-3abC))\,x}{105a^3b^3(a+bx^2)^{3/2}} + \frac{(48Ab^3+a(15a^2B+8b^2B+6abC))\,x}{105a^4b^3\sqrt{a+bx^2}}$$

output
```
1/7*(A/a-(B*a^2+B*b^2-C*a*b)/b^3)*x/(b*x^2+a)^(7/2)+1/35*(6*A*b^3+a*(15*B*
a^2+B*b^2-8*C*a*b))*x/a^2/b^3/(b*x^2+a)^(5/2)+1/105*(24*A*b^3-a*(45*B*a^2-
4*B*b^2-3*C*a*b))*x/a^3/b^3/(b*x^2+a)^(3/2)+1/105*(48*A*b^3+a*(15*B*a^2+8*
B*b^2+6*C*a*b))*x/a^4/b^3/(b*x^2+a)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.52

$$\int \frac{A+Bx^2+Cx^4+Bx^6}{(a+bx^2)^{9/2}}\,dx = \frac{48Ab^3x^7+8ab^2x^5(21A+Bx^2)+2a^2bx^3(105A+14Bx^2+3Cx^4)+a^3(10}{105a^4\,(a+bx^2)^{7/2}}$$

input
```
Integrate[(A + B*x^2 + C*x^4 + B*x^6)/(a + b*x^2)^(9/2),x]
```

output
```
(48*A*b^3*x^7 + 8*a*b^2*x^5*(21*A + B*x^2) + 2*a^2*b*x^3*(105*A + 14*B*x^2
 + 3*C*x^4) + a^3*(105*A*x + 21*C*x^5 + 5*B*x^3*(7 + 3*x^4)))/(105*a^4*(a
+ b*x^2)^(7/2))
```

## Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.96, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {2344, 2089, 1586, 9, 25, 27, 362, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^6+Bx^2+Cx^4}{(a+bx^2)^{9/2}}dx$$

$$\Big\downarrow \; 2344$$

$$\frac{\int \frac{x^2(6Ab+a(Bx^4+Cx^2+B))}{(bx^2+a)^{9/2}}dx}{a}+\frac{Ax}{a\,(a+bx^2)^{7/2}}$$

$$\Big\downarrow \; 2089$$

$$\frac{\int \frac{x^2(aBx^4+aCx^2+6Ab+aB)}{(bx^2+a)^{9/2}}dx}{a}+\frac{Ax}{a\,(a+bx^2)^{7/2}}$$

$$\Big\downarrow \; 1586$$

$$\frac{\frac{x^3\left(\frac{a\left(a^2B-abC+b^2B\right)}{b^2}+6Ab\right)}{7a(a+bx^2)^{7/2}}-\frac{\int -\frac{x\left(\frac{7a^2Bx^3}{b}+\left(24Ab-\frac{a\left(3Ba^2-3bCa-4b^2B\right)}{b^2}\right)x\right)}{\left(bx^2+a\right)^{7/2}}dx}{7a}}{a}+\frac{Ax}{a\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \; 9$$

$$\frac{\frac{x^3\left(\frac{a\left(a^2B-abC+b^2B\right)}{b^2}+6Ab\right)}{7a(a+bx^2)^{7/2}}-\frac{\int -\frac{x^2\left(7a^2Bx^2+b\left(24Ab-\frac{a\left(3Ba^2-3bCa-4b^2B\right)}{b^2}\right)\right)}{b\left(bx^2+a\right)^{7/2}}dx}{7a}}{a}+\frac{Ax}{a\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{x^2\left(24Ab^2+7a^2Bx^2-a\left(\frac{3Ba^2}{b}-3Ca-4bB\right)\right)}{b\left(bx^2+a\right)^{7/2}}dx}{7a}+\frac{x^3\left(\frac{a\left(a^2B-abC+b^2B\right)}{b^2}+6Ab\right)}{7a(a+bx^2)^{7/2}}}{a}+\frac{Ax}{a\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{x^2\left(24Ab^2+7a^2Bx^2-a\left(\frac{3Ba^2}{b}-3Ca-4bB\right)\right)}{\left(bx^2+a\right)^{7/2}}dx}{7ab}+\frac{x^3\left(\frac{a\left(a^2B-abC+b^2B\right)}{b^2}+6Ab\right)}{7a(a+bx^2)^{7/2}}}{a}+\frac{Ax}{a\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \; 362$$

$$\frac{\frac{\frac{\left(a\left(15a^2B+6abC+8b^2B\right)+48Ab^3\right)\int \frac{x^2}{\left(bx^2+a\right)^{5/2}}dx}{5ab}+\frac{x^3\left(24Ab^3-a\left(10a^2B-3abC-4b^2B\right)\right)}{5ab\left(a+bx^2\right)^{5/2}}}{7ab}+\frac{x^3\left(\frac{a\left(a^2B-abC+b^2B\right)}{b^2}+6Ab\right)}{7a(a+bx^2)^{7/2}}}{a}+\frac{Ax}{a\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \; 242$$

$$\frac{\frac{x^3\left(\frac{a\left(a^2B-abC+b^2B\right)}{b^2}+6Ab\right)}{7a(a+bx^2)^{7/2}}+\frac{\frac{x^3\left(a\left(15a^2B+6abC+8b^2B\right)+48Ab^3\right)}{15a^2b\left(a+bx^2\right)^{3/2}}+\frac{x^3\left(24Ab^3-a\left(10a^2B-3abC-4b^2B\right)\right)}{5ab\left(a+bx^2\right)^{5/2}}}{7ab}}{a}+\frac{Ax}{a\left(a+bx^2\right)^{7/2}}$$

input
```
Int[(A + B*x^2 + C*x^4 + B*x^6)/(a + b*x^2)^(9/2),x]
```

output
```
(A*x)/(a*(a + b*x^2)^(7/2)) + (((6*A*b + (a*(a^2*B + b^2*B - a*b*C))/b^2)*
x^3)/(7*a*(a + b*x^2)^(7/2)) + (((24*A*b^3 - a*(10*a^2*B - 4*b^2*B - 3*a*b
*C))*x^3)/(5*a*b*(a + b*x^2)^(5/2)) + ((48*A*b^3 + a*(15*a^2*B + 8*b^2*B +
 6*a*b*C))*x^3)/(15*a^2*b*(a + b*x^2)^(3/2)))/(7*a*b))/a
```

### Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

rule 1586
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, d + e*x^2, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^
p, d + e*x^2, x], x, 0]}, Simp[(-R)*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(2*d
*f*(q + 1))), x] + Simp[f/(2*d*(q + 1))   Int[(f*x)^(m - 1)*(d + e*x^2)^(q
+ 1)*ExpandToSum[2*d*(q + 1)*x*Qx + R*(m + 2*q + 3)*x, x], x], x]] /; FreeQ
[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[q, -1]
&& GtQ[m, 0]
```

rule 2089
```
Int[(u_)^(p_.)*((f_.)*(x_))^(m_.)*(z_)^(q_.), x_Symbol] :> Int[(f*x)^m*Expa
ndToSum[z, x]^q*ExpandToSum[u, x]^p, x] /; FreeQ[{f, m, p, q}, x] && Binomi
alQ[z, x] && TrinomialQ[u, x] &&  !(BinomialMatchQ[z, x] && TrinomialMatchQ
[u, x])
```

rule 2344
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coeff[Pq, x, 0
], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[A*x*((a + b*
x^2)^(p + 1)/a), x] + Simp[1/a   Int[x^2*(a + b*x^2)^p*(a*Q - A*b*(2*p + 3)
), x], x]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && ILtQ[p + 1/2, 0] && LtQ
[Expon[Pq, x] + 2*p + 1, 0]
```

## Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.47

| method | result |
|---|---|
| pseudoelliptic | $\frac{\left(\left(\frac{1}{7}B x^6+\frac{1}{5}C x^4+\frac{1}{3}x^2B+A\right)a^3+2\left(\frac{1}{35}C x^4+\frac{2}{15}x^2B+A\right)x^2b a^2+\frac{8b^2\left(\frac{x^2B}{21}+A\right)x^4a}{5}+\frac{16A b^3x^6}{35}\right)x}{(b x^2+a)^{\frac{7}{2}}a^4}$ |
| gosper | $\frac{x\left(48A b^3x^6+15B a^3x^6+8Ba b^2x^6+6C a^2b x^6+168aA b^2x^4+28B a^2b x^4+21C a^3x^4+210a^2Ab x^2+35B a^3x^2+105a^3A\right)}{105(b x^2+a)^{\frac{7}{2}}a^4}$ |
| trager | $\frac{x\left(48A b^3x^6+15B a^3x^6+8Ba b^2x^6+6C a^2b x^6+168aA b^2x^4+28B a^2b x^4+21C a^3x^4+210a^2Ab x^2+35B a^3x^2+105a^3A\right)}{105(b x^2+a)^{\frac{7}{2}}a^4}$ |
| orering | $\frac{x\left(48A b^3x^6+15B a^3x^6+8Ba b^2x^6+6C a^2b x^6+168aA b^2x^4+28B a^2b x^4+21C a^3x^4+210a^2Ab x^2+35B a^3x^2+105a^3A\right)}{105(b x^2+a)^{\frac{7}{2}}a^4}$ |
| default | $A\left(\frac{x}{7a(b x^2+a)^{\frac{7}{2}}}+\frac{\frac{6x}{35a\left(b x^2+a\right)^{\frac{5}{2}}}+\frac{6\left(\frac{4x}{15a\left(b x^2+a\right)^{\frac{3}{2}}}+\frac{8x}{15a^2\sqrt{b x^2+a}}\right)}{7a}}{a}\right)+B\left(-\frac{x^5}{2b(b x^2+a)^{\frac{7}{2}}}+\frac{5a\left(-\frac{x^3}{4b\left(b x^2+\right.}\right.}{}\right.$ |

input
```
int((B*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
((1/7*B*x^6+1/5*C*x^4+1/3*x^2*B+A)*a^3+2*(1/35*C*x^4+2/15*x^2*B+A)*x^2*b*a
^2+8/5*b^2*(1/21*x^2*B+A)*x^4*a+16/35*A*b^3*x^6)/(b*x^2+a)^(7/2)*x/a^4
```

## Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.74

$$\int \frac{A+Bx^2+Cx^4+Bx^6}{(a+bx^2)^{9/2}}\,dx = \frac{((15\,Ba^3+6\,Ca^2b+8\,Bab^2+48\,Ab^3)x^7+7\,(3\,Ca^3+4\,Ba^2b+24\,Aab^2)x}{105\,(a^4b^4x^8+4\,a^5b^3x^6+6\,a^6b^2x^4+4\,a}$$

input
```
integrate((B*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

output
```
1/105*((15*B*a^3 + 6*C*a^2*b + 8*B*a*b^2 + 48*A*b^3)*x^7 + 7*(3*C*a^3 + 4*
B*a^2*b + 24*A*a*b^2)*x^5 + 105*A*a^3*x + 35*(B*a^3 + 6*A*a^2*b)*x^3)*sqrt
(b*x^2 + a)/(a^4*b^4*x^8 + 4*a^5*b^3*x^6 + 6*a^6*b^2*x^4 + 4*a^7*b*x^2 + a
^8)
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2088 vs. 2(187) = 374.

Time = 62.26 (sec) , antiderivative size = 2088, normalized size of antiderivative = 10.93

$$\int \frac{A+Bx^2+Cx^4+Bx^6}{(a+bx^2)^{9/2}}\,dx = \text{Too large to display}$$

input
```
integrate((B*x**6+C*x**4+B*x**2+A)/(b*x**2+a)**(9/2),x)
```

output
```
A*(35*a**14*x/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt
(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2
)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a
) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*
sqrt(1 + b*x**2/a)) + 175*a**13*b*x**3/(35*a**(37/2)*sqrt(1 + b*x**2/a) +
210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 +
 b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b*
*4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) +
 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a)) + 371*a**12*b**2*x**5/(35*a**
(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*
a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 +
b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**
5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a)) +
 429*a**11*b**3*x**7/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x*
*2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a
**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b
*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6
*x**12*sqrt(1 + b*x**2/a)) + 286*a**10*b**4*x**9/(35*a**(37/2)*sqrt(1 + b*
x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**
4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525...
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.75

$$\int \frac{A+Bx^2+Cx^4+Bx^6}{(a+bx^2)^{9/2}}\,dx = -\frac{Bx^5}{2\,(bx^2+a)^{\frac{7}{2}}b} - \frac{5\,Bax^3}{8\,(bx^2+a)^{\frac{7}{2}}b^2} - \frac{Cx^3}{4\,(bx^2+a)^{\frac{7}{2}}b} + \frac{16\,Ax}{35\,\sqrt{bx^2+a}a^4} + \frac{8\,Ax}{35\,(bx^2+a)^{\frac{3}{2}}a^3} + \frac{6\,Ax}{35\,(bx^2+a)^{\frac{5}{2}}a^2} + \frac{Ax}{7\,(bx^2+a)^{\frac{7}{2}}a} + \frac{Bx}{14\,(bx^2+a)^{\frac{3}{2}}b^3} + \frac{Bx}{7\,\sqrt{bx^2+a}ab^3} + \frac{3\,Bax}{56\,(bx^2+a)^{\frac{5}{2}}b^3} - \frac{15\,Ba^2x}{56\,(bx^2+a)^{\frac{7}{2}}b^3} + \frac{3\,Cx}{140\,(bx^2+a)^{\frac{5}{2}}b^2} + \frac{2\,Cx}{35\,\sqrt{bx^2+a}a^2b^2} + \frac{Cx}{35\,(bx^2+a)^{\frac{3}{2}}ab^2} - \frac{3\,Cax}{28\,(bx^2+a)^{\frac{7}{2}}b^2} - \frac{Bx}{7\,(bx^2+a)^{\frac{7}{2}}b} + \frac{8\,Bx}{105\,\sqrt{bx^2+a}a^3b} + \frac{4\,Bx}{105\,(bx^2+a)^{\frac{3}{2}}a^2b} + \frac{Bx}{35\,(bx^2+a)^{\frac{5}{2}}ab}$$

input
```
integrate((B*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

output
```
-1/2*B*x^5/((b*x^2 + a)^(7/2)*b) - 5/8*B*a*x^3/((b*x^2 + a)^(7/2)*b^2) - 1
/4*C*x^3/((b*x^2 + a)^(7/2)*b) + 16/35*A*x/(sqrt(b*x^2 + a)*a^4) + 8/35*A*
x/((b*x^2 + a)^(3/2)*a^3) + 6/35*A*x/((b*x^2 + a)^(5/2)*a^2) + 1/7*A*x/((b
*x^2 + a)^(7/2)*a) + 1/14*B*x/((b*x^2 + a)^(3/2)*b^3) + 1/7*B*x/(sqrt(b*x^
2 + a)*a*b^3) + 3/56*B*a*x/((b*x^2 + a)^(5/2)*b^3) - 15/56*B*a^2*x/((b*x^2
 + a)^(7/2)*b^3) + 3/140*C*x/((b*x^2 + a)^(5/2)*b^2) + 2/35*C*x/(sqrt(b*x^
2 + a)*a^2*b^2) + 1/35*C*x/((b*x^2 + a)^(3/2)*a*b^2) - 3/28*C*a*x/((b*x^2
+ a)^(7/2)*b^2) - 1/7*B*x/((b*x^2 + a)^(7/2)*b) + 8/105*B*x/(sqrt(b*x^2 +
a)*a^3*b) + 4/105*B*x/((b*x^2 + a)^(3/2)*a^2*b) + 1/35*B*x/((b*x^2 + a)^(5
/2)*a*b)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.69

$$\int \frac{A+Bx^2+Cx^4+Bx^6}{(a+bx^2)^{9/2}}\,dx = \frac{\left(\left(x^2\left(\frac{(15\,Ba^3b^3+6\,Ca^2b^4+8\,Bab^5+48\,Ab^6)x^2}{a^4b^3}+\frac{7\,(3\,Ca^3b^3+4\,Ba^2b^4+24\,Aab^5)}{a^4b^3}\right)+\frac{35\,(Ba^3b\ldots}{\ldots}\right.\right.}{105\,(bx^2+a)^{\frac{7}{2}}}$$

input
```
integrate((B*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="giac")
```

output
```
1/105*((x^2*((15*B*a^3*b^3 + 6*C*a^2*b^4 + 8*B*a*b^5 + 48*A*b^6)*x^2/(a^4*
b^3) + 7*(3*C*a^3*b^3 + 4*B*a^2*b^4 + 24*A*a*b^5)/(a^4*b^3)) + 35*(B*a^3*b
^3 + 6*A*a^2*b^4)/(a^4*b^3))*x^2 + 105*A/a)*x/(b*x^2 + a)^(7/2)
```

## Mupad [B] (verification not implemented)

Time = 1.48 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.16

$$\int \frac{A+Bx^2+Cx^4+Bx^6}{(a+bx^2)^{9/2}}\,dx = \frac{x\left(\frac{a\left(\frac{B}{5b^2}+\frac{Ba-Cb}{5ab^2}\right)}{b}+\frac{Ba^3-Ca^2b+Bab^2+6Ab^3}{35a^2b^3}\right)}{(bx^2+a)^{5/2}} + \frac{x\left(\frac{A}{7a}-\frac{a\left(\frac{B}{7a}+\frac{a\left(\frac{B}{7b}-\frac{C}{7a}\right)}{b}\right)}{b}\right)}{(bx^2+a)^{7/2}} - \frac{x\left(\frac{B}{3b^3}-\frac{-10Ba^3+3Ca^2b+4Bab^2+24Ab^3}{105a^3b^3}\right)}{(bx^2+a)^{3/2}} + \frac{x\,(15Ba^3+6Ca^2b+8Bab^2+48Ab^3)}{105a^4b^3\sqrt{bx^2+a}}$$

input
```
int((A + B*x^2 + B*x^6 + C*x^4)/(a + b*x^2)^(9/2),x)
```

output
```
(x*((a*(B/(5*b^2) + (B*a - C*b)/(5*a*b^2)))/b + (6*A*b^3 + B*a^3 + B*a*b^2
 - C*a^2*b)/(35*a^2*b^3)))/(a + b*x^2)^(5/2) + (x*(A/(7*a) - (a*(B/(7*a) +
 (a*(B/(7*b) - C/(7*a)))/b))/b))/(a + b*x^2)^(7/2) - (x*(B/(3*b^3) - (24*A
*b^3 - 10*B*a^3 + 4*B*a*b^2 + 3*C*a^2*b)/(105*a^3*b^3)))/(a + b*x^2)^(3/2)
 + (x*(48*A*b^3 + 15*B*a^3 + 8*B*a*b^2 + 6*C*a^2*b))/(105*a^4*b^3*(a + b*x
^2)^(1/2))
```

## Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.84

$$\int \frac{A+Bx^2+Cx^4+Bx^6}{(a+bx^2)^{9/2}}\,dx = \frac{105\sqrt{bx^2+a}\,a^3b^3x+15\sqrt{bx^2+a}\,a^2b^4x^7+245\sqrt{bx^2+a}\,a^2b^4x^3+21\sqrt{b\,x}}{}$$

input
```
int((B*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x)
```

output
```
(105*sqrt(a + b*x**2)*a**3*b**3*x + 15*sqrt(a + b*x**2)*a**2*b**4*x**7 + 2
45*sqrt(a + b*x**2)*a**2*b**4*x**3 + 21*sqrt(a + b*x**2)*a**2*b**3*c*x**5
+ 196*sqrt(a + b*x**2)*a*b**5*x**5 + 6*sqrt(a + b*x**2)*a*b**4*c*x**7 + 56
*sqrt(a + b*x**2)*b**6*x**7 + 15*sqrt(b)*a**6 + 60*sqrt(b)*a**5*b*x**2 - 6
*sqrt(b)*a**5*c + 90*sqrt(b)*a**4*b**2*x**4 - 56*sqrt(b)*a**4*b**2 - 24*sq
rt(b)*a**4*b*c*x**2 + 60*sqrt(b)*a**3*b**3*x**6 - 224*sqrt(b)*a**3*b**3*x*
*2 - 36*sqrt(b)*a**3*b**2*c*x**4 + 15*sqrt(b)*a**2*b**4*x**8 - 336*sqrt(b)
*a**2*b**4*x**4 - 24*sqrt(b)*a**2*b**3*c*x**6 - 224*sqrt(b)*a*b**5*x**6 -
6*sqrt(b)*a*b**4*c*x**8 - 56*sqrt(b)*b**6*x**8)/(105*a**3*b**3*(a**4 + 4*a
**3*b*x**2 + 6*a**2*b**2*x**4 + 4*a*b**3*x**6 + b**4*x**8))
```

# 3.244 $\int \frac{A+Bx^2+Cx^4+Bx^6}{x^2(a+bx^2)^{9/2}}\,dx$



## Optimal result

Integrand size = 32, antiderivative size = 217

$$\int \frac{A+Bx^2+Cx^4+Bx^6}{x^2(a+bx^2)^{9/2}}\,dx = -\frac{(Ab^3-a(a^2B+b^2B-abC))\,x}{7a^2b^2(a+bx^2)^{7/2}} - \frac{(13Ab^3+a(8a^2B-6b^2B-abC))\,x}{35a^3b^2(a+bx^2)^{5/2}} - \frac{A}{a^3x(a+bx^2)^{3/2}} - \frac{(192Ab^3-a(3a^2B+24b^2B+4abC))\,x}{105a^4b^2(a+bx^2)^{3/2}} - \frac{2(192Ab^3-a(3a^2B+24b^2B+4abC))\,x}{105a^5b^2\sqrt{a+bx^2}}$$

output
```
-1/7*(A*b^3-a*(B*a^2+B*b^2-C*a*b))*x/a^2/b^2/(b*x^2+a)^(7/2)-1/35*(13*A*b^
3+a*(8*B*a^2-6*B*b^2-C*a*b))*x/a^3/b^2/(b*x^2+a)^(5/2)-A/a^3/x/(b*x^2+a)^(
3/2)-1/105*(192*A*b^3-a*(3*B*a^2+24*B*b^2+4*C*a*b))*x/a^4/b^2/(b*x^2+a)^(3
/2)-2/105*(192*A*b^3-a*(3*B*a^2+24*B*b^2+4*C*a*b))*x/a^5/b^2/(b*x^2+a)^(1/
2)
```

## Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.60

$$\int \frac{A+Bx^2+Cx^4+Bx^6}{x^2\left(a+bx^2\right)^{9/2}}\,dx = \frac{-384Ab^4x^8+48ab^3x^6(-28A+Bx^2)+8a^2b^2x^4(-210A+21Bx^2+Cx^4)}{105a^5}$$

input
```
Integrate[(A + B*x^2 + C*x^4 + B*x^6)/(x^2*(a + b*x^2)^(9/2)),x]
```

output
```
(-384*A*b^4*x^8 + 48*a*b^3*x^6*(-28*A + B*x^2) + 8*a^2*b^2*x^4*(-210*A + 2
1*B*x^2 + C*x^4) - 7*a^4*(15*A - 5*C*x^4 - 3*B*x^2*(5 + x^4)) + 2*a^3*b*x^
2*(-420*A + 14*C*x^4 + 3*B*x^2*(35 + x^4)))/(105*a^5*x*(a + b*x^2)^(7/2))
```

## Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.86, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2334, 2087, 1469, 362, 245, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^6+Bx^2+Cx^4}{x^2\left(a+bx^2\right)^{9/2}}dx$$

$$\Big\downarrow \text{ 2334}$$

$$-\frac{\int \frac{8Ab-a(Bx^4+Cx^2+B)}{(bx^2+a)^{9/2}}dx}{a}-\frac{A}{ax\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \text{ 2087}$$

$$-\frac{\int \frac{-aBx^4-aCx^2+8Ab-aB}{(bx^2+a)^{9/2}}dx}{a}-\frac{A}{ax\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \text{ 1469}$$

$$-\frac{\frac{\int \frac{x^2\left(-Bx^2a^2-Ca^2+6b(8Ab-aB)\right)}{\left(bx^2+a\right)^{9/2}}dx}{a}+\frac{x(8Ab-aB)}{a(a+bx^2)^{7/2}}}{a}-\frac{A}{ax\left(a+bx^2\right)^{7/2}}$$

$\Big\downarrow$ 362

$$-\frac{\frac{\frac{\left(-3a^3B-4ab(aC+6bB)+192Ab^3\right)\int \frac{x^2}{\left(bx^2+a\right)^{7/2}}dx}{7ab}+\frac{x^3\left(a\left(a^2B-abC-6b^2B\right)+48Ab^3\right)}{7ab\left(a+bx^2\right)^{7/2}}}{a}+\frac{x(8Ab-aB)}{a(a+bx^2)^{7/2}}}{a}-\frac{A}{ax\left(a+bx^2\right)^{7/2}}$$

$\Big\downarrow$ 245

$$-\frac{\frac{\frac{\left(-3a^3B-4ab(aC+6bB)+192Ab^3\right)\left(\frac{2b\int \frac{x^4}{\left(bx^2+a\right)^{7/2}}dx}{3a}+\frac{x^3}{3a\left(a+bx^2\right)^{5/2}}\right)}{7ab}+\frac{x^3\left(a\left(a^2B-abC-6b^2B\right)+48Ab^3\right)}{7ab\left(a+bx^2\right)^{7/2}}}{a}+\frac{x(8Ab-aB)}{a(a+bx^2)^{7/2}}}{a}-\frac{A}{ax\left(a+bx^2\right)^{7/2}}$$

$\Big\downarrow$ 242

$$-\frac{\frac{\frac{x^3\left(a\left(a^2B-abC-6b^2B\right)+48Ab^3\right)}{7ab\left(a+bx^2\right)^{7/2}}+\frac{\left(\frac{2bx^5}{15a^2\left(a+bx^2\right)^{5/2}}+\frac{x^3}{3a\left(a+bx^2\right)^{5/2}}\right)\left(-3a^3B-4ab(aC+6bB)+192Ab^3\right)}{7ab}}{a}+\frac{x(8Ab-aB)}{a(a+bx^2)^{7/2}}}{a}-\frac{A}{ax\left(a+bx^2\right)^{7/2}}$$

input
```
Int[(A + B*x^2 + C*x^4 + B*x^6)/(x^2*(a + b*x^2)^(9/2)),x]
```

output
```
-(A/(a*x*(a + b*x^2)^(7/2))) - (((8*A*b - a*B)*x)/(a*(a + b*x^2)^(7/2)) +
(((48*A*b^3 + a*(a^2*B - 6*b^2*B - a*b*C))*x^3)/(7*a*b*(a + b*x^2)^(7/2))
+ ((192*A*b^3 - 3*a^3*B - 4*a*b*(6*b*B + a*C))*(x^3/(3*a*(a + b*x^2)^(5/2)
) + (2*b*x^5)/(15*a^2*(a + b*x^2)^(5/2))))/(7*a*b))/a)/a
```

### Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

rule 1469
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[a^p*x*((d + e*x^2)^(q + 1)/d), x] + Simp[1/d   Int[x^2*(d
 + e*x^2)^q*(d*PolynomialQuotient[(a + b*x^2 + c*x^4)^p - a^p, x^2, x] - e*
a^p*(2*q + 3)), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && ILtQ[q + 1/2, 0] && LtQ[4
*p + 2*q + 1, 0]
```

rule 2087
```
Int[(u_)^(q_.)*(v_)^(p_.), x_Symbol] :> Int[ExpandToSum[u, x]^q*ExpandToSum
[v, x]^p, x] /; FreeQ[{p, q}, x] && BinomialQ[u, x] && TrinomialQ[v, x] &&
 !(BinomialMatchQ[u, x] && TrinomialMatchQ[v, x])
```

rule 2334
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coef
f[Pq, x, 0], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[A*
x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))   Int[
x^(m + 2)*(a + b*x^2)^p*(a*(m + 1)*Q - A*b*(m + 2*(p + 1) + 1)), x], x]] /;
 FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && IntegerQ[m/2] && ILtQ[(m + 1)/2 + p,
 0] && LtQ[m + Expon[Pq, x] + 2*p + 1, 0]
```

## Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.57

| method | result |
|---|---|
| pseudoelliptic | $-\frac{\left(-\frac{1}{5} B\,x^{6}-\frac{1}{3} C\,x^{4}-x^{2} B+A\right) a^{4}+8\left(-\frac{1}{140} B\,x^{6}-\frac{1}{30} C\,x^{4}-\frac{1}{4} x^{2} B+A\right) x^{2} b\,a^{3}+16\left(-\frac{1}{210} C\,x^{4}-\frac{1}{10} x^{2} B+A\right) x^{4} b^{2} a^{2}+\frac{64\left(-\frac{x^{2}}{2}\cdots\right.}{\cdots}}{\left(b\,x^{2}+a\right)^{\frac{7}{2}} x\,a^{5}}$ |
| gosper | $-\frac{384 A\,x^{8} b^{4}-6 B\,a^{3} b\,x^{8}-48 B\,x^{8} a\,b^{3}-8 C\,a^{2} b^{2} x^{8}+1344 A\,x^{6} a\,b^{3}-21 B\,a^{4} x^{6}-168 B\,x^{6} a^{2} b^{2}-28 C\,a^{3} b\,x^{6}+1680 A\,x^{4} a^{2} b^{2}-\cdots}{105 x \left(b\,x^{2}+a\right)^{\frac{7}{2}} a^{5}}$ |
| trager | $-\frac{384 A\,x^{8} b^{4}-6 B\,a^{3} b\,x^{8}-48 B\,x^{8} a\,b^{3}-8 C\,a^{2} b^{2} x^{8}+1344 A\,x^{6} a\,b^{3}-21 B\,a^{4} x^{6}-168 B\,x^{6} a^{2} b^{2}-28 C\,a^{3} b\,x^{6}+1680 A\,x^{4} a^{2} b^{2}-\cdots}{105 x \left(b\,x^{2}+a\right)^{\frac{7}{2}} a^{5}}$ |
| orering | $-\frac{384 A\,x^{8} b^{4}-6 B\,a^{3} b\,x^{8}-48 B\,x^{8} a\,b^{3}-8 C\,a^{2} b^{2} x^{8}+1344 A\,x^{6} a\,b^{3}-21 B\,a^{4} x^{6}-168 B\,x^{6} a^{2} b^{2}-28 C\,a^{3} b\,x^{6}+1680 A\,x^{4} a^{2} b^{2}-\cdots}{105 x \left(b\,x^{2}+a\right)^{\frac{7}{2}} a^{5}}$ |
| risch | $-\frac{A\sqrt{b\,x^{2}+a}}{a^{5} x}-\frac{\sqrt{b\,x^{2}+a}\,x\left(279 A\,b^{4} x^{6}-6 B\,a^{3} b\,x^{6}-48 B a\,b^{3} x^{6}-8 C\,a^{2} b^{2} x^{6}+924 A a\,b^{3} x^{4}-21 B\,a^{4} x^{4}-168 B\,a^{2} b^{2} x^{4}-28 C\cdots\right.}{105 a^{5} \left(b^{4} x^{8}+4 a\,b^{3} x^{6}+6 a^{2} b^{2} x^{4}+4 a^{3} b\,x^{2}\cdots\right)}$ |
| default | $B\left(\frac{x}{7 a \left(b\,x^{2}+a\right)^{\frac{7}{2}}}+\frac{\frac{6 x}{35 a \left(b\,x^{2}+a\right)^{\frac{5}{2}}}+\frac{6\left(\frac{4 x}{15 a \left(b\,x^{2}+a\right)^{\frac{3}{2}}}+\frac{8 x}{15 a^{2} \sqrt{b\,x^{2}+a}}\right)}{7 a}}{a}\right)+A\left(-\frac{1}{a x \left(b\,x^{2}+a\right)^{\frac{7}{2}}}-\frac{8 b\left(\frac{x}{7 a \left(b\,x^{2}+a\right)\cdots}\cdots\right)}{\cdots}\cdots\right)$ |

input
```
int((B*x^6+C*x^4+B*x^2+A)/x^2/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-((-1/5*B*x^6-1/3*C*x^4-x^2*B+A)*a^4+8*(-1/140*B*x^6-1/30*C*x^4-1/4*x^2*B+
A)*x^2*b*a^3+16*(-1/210*C*x^4-1/10*x^2*B+A)*x^4*b^2*a^2+64/5*(-1/28*x^2*B+
A)*x^6*b^3*a+128/35*A*x^8*b^4)/(b*x^2+a)^(7/2)/x/a^5
```

## Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.84

$$\int \frac{A+Bx^2+Cx^4+Bx^6}{x^2\left(a+bx^2\right)^{9/2}}\,dx = \frac{(2\,(3\,Ba^3b+4\,Ca^2b^2+24\,Bab^3-192\,Ab^4)x^8+7\,(3\,Ba^4+4\,Ca^3b+24\,B}{105\,(a^5b^4x^9}$$

input
```
integrate((B*x^6+C*x^4+B*x^2+A)/x^2/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

output
```
1/105*(2*(3*B*a^3*b + 4*C*a^2*b^2 + 24*B*a*b^3 - 192*A*b^4)*x^8 + 7*(3*B*a
^4 + 4*C*a^3*b + 24*B*a^2*b^2 - 192*A*a*b^3)*x^6 - 105*A*a^4 + 35*(C*a^4 +
 6*B*a^3*b - 48*A*a^2*b^2)*x^4 + 105*(B*a^4 - 8*A*a^3*b)*x^2)*sqrt(b*x^2 +
 a)/(a^5*b^4*x^9 + 4*a^6*b^3*x^7 + 6*a^7*b^2*x^5 + 4*a^8*b*x^3 + a^9*x)
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2392 vs. 2(212) = 424.

Time = 115.87 (sec) , antiderivative size = 2392, normalized size of antiderivative = 11.02

$$\int \frac{A+Bx^2+Cx^4+Bx^6}{x^2\left(a+bx^2\right)^{9/2}}\,dx = \text{Too large to display}$$

input
```
integrate((B*x**6+C*x**4+B*x**2+A)/x**2/(b*x**2+a)**(9/2),x)
```

output
```
A*(-35*a**4*b**(33/2)*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b**17
*x**2 + 210*a**7*b**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8) -
280*a**3*b**(35/2)*x**2*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b**
17*x**2 + 210*a**7*b**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8)
- 560*a**2*b**(37/2)*x**4*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b
**17*x**2 + 210*a**7*b**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8
) - 448*a*b**(39/2)*x**6*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b*
*17*x**2 + 210*a**7*b**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8)
 - 128*b**(41/2)*x**8*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b**17
*x**2 + 210*a**7*b**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8)) +
 B*(7*a*x**5/(35*a**(11/2)*sqrt(1 + b*x**2/a) + 105*a**(9/2)*b*x**2*sqrt(1
 + b*x**2/a) + 105*a**(7/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 35*a**(5/2)*b**
3*x**6*sqrt(1 + b*x**2/a)) + 2*b*x**7/(35*a**(11/2)*sqrt(1 + b*x**2/a) + 1
05*a**(9/2)*b*x**2*sqrt(1 + b*x**2/a) + 105*a**(7/2)*b**2*x**4*sqrt(1 + b*
x**2/a) + 35*a**(5/2)*b**3*x**6*sqrt(1 + b*x**2/a))) + B*(35*a**14*x/(35*a
**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 52
5*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1
+ b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b
**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a))
 + 175*a**13*b*x**3/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*...
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.44

$$\int \frac{A+Bx^2+Cx^4+Bx^6}{x^2\left(a+bx^2\right)^{9/2}}\,dx = -\frac{Bx^3}{4\,(bx^2+a)^{\frac{7}{2}}b} + \frac{16\,Bx}{35\,\sqrt{bx^2+a}a^4} + \frac{8\,Bx}{35\,(bx^2+a)^{\frac{3}{2}}a^3} + \frac{6\,Bx}{35\,(bx^2+a)^{\frac{5}{2}}a^2} + \frac{Bx}{7\,(bx^2+a)^{\frac{7}{2}}a} + \frac{3\,Bx}{140\,(bx^2+a)^{\frac{5}{2}}b^2} + \frac{2\,Bx}{35\,\sqrt{bx^2+a}a^2b^2} + \frac{Bx}{35\,(bx^2+a)^{\frac{3}{2}}ab^2} - \frac{3\,Bax}{28\,(bx^2+a)^{\frac{7}{2}}b^2} - \frac{Cx}{7\,(bx^2+a)^{\frac{7}{2}}b} + \frac{8\,Cx}{105\,\sqrt{bx^2+a}a^3b} + \frac{4\,Cx}{105\,(bx^2+a)^{\frac{3}{2}}a^2b} + \frac{Cx}{35\,(bx^2+a)^{\frac{5}{2}}ab} - \frac{128\,Abx}{35\,\sqrt{bx^2+a}a^5} - \frac{64\,Abx}{35\,(bx^2+a)^{\frac{3}{2}}a^4} - \frac{48\,Abx}{35\,(bx^2+a)^{\frac{5}{2}}a^3} - \frac{8\,Abx}{7\,(bx^2+a)^{\frac{7}{2}}a^2} - \frac{A}{(bx^2+a)^{\frac{7}{2}}ax}$$

input
```
integrate((B*x^6+C*x^4+B*x^2+A)/x^2/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

output
```
-1/4*B*x^3/((b*x^2 + a)^(7/2)*b) + 16/35*B*x/(sqrt(b*x^2 + a)*a^4) + 8/35*
B*x/((b*x^2 + a)^(3/2)*a^3) + 6/35*B*x/((b*x^2 + a)^(5/2)*a^2) + 1/7*B*x/(
(b*x^2 + a)^(7/2)*a) + 3/140*B*x/((b*x^2 + a)^(5/2)*b^2) + 2/35*B*x/(sqrt(
b*x^2 + a)*a^2*b^2) + 1/35*B*x/((b*x^2 + a)^(3/2)*a*b^2) - 3/28*B*a*x/((b*
x^2 + a)^(7/2)*b^2) - 1/7*C*x/((b*x^2 + a)^(7/2)*b) + 8/105*C*x/(sqrt(b*x^
2 + a)*a^3*b) + 4/105*C*x/((b*x^2 + a)^(3/2)*a^2*b) + 1/35*C*x/((b*x^2 + a
)^(5/2)*a*b) - 128/35*A*b*x/(sqrt(b*x^2 + a)*a^5) - 64/35*A*b*x/((b*x^2 +
a)^(3/2)*a^4) - 48/35*A*b*x/((b*x^2 + a)^(5/2)*a^3) - 8/7*A*b*x/((b*x^2 +
a)^(7/2)*a^2) - A/((b*x^2 + a)^(7/2)*a*x)
```

## Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.97

$$\int \frac{A+Bx^2+Cx^4+Bx^6}{x^2\left(a+bx^2\right)^{9/2}}\,dx = \frac{\left(\left(x^2\left(\frac{\left(6\,Ba^{12}b^4+8\,Ca^{11}b^5+48\,Ba^{10}b^6-279\,Aa^9b^7\right)x^2}{a^{14}b^3}+\frac{7\,\left(3\,Ba^{13}b^3+4\,Ca^{12}b^4+24\,Ba^{11}b^5-13\cdots}{a^{14}b^3}\cdots\right.\right.\right.}{105\,\left(bx^2+a\cdots\right.}$$
$$+\frac{2\,A\sqrt{b}}{\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)a^4}$$

input
```
integrate((B*x^6+C*x^4+B*x^2+A)/x^2/(b*x^2+a)^(9/2),x, algorithm="giac")
```

output
```
1/105*((x^2*((6*B*a^12*b^4 + 8*C*a^11*b^5 + 48*B*a^10*b^6 - 279*A*a^9*b^7)
*x^2/(a^14*b^3) + 7*(3*B*a^13*b^3 + 4*C*a^12*b^4 + 24*B*a^11*b^5 - 132*A*a
^10*b^6)/(a^14*b^3)) + 35*(C*a^13*b^3 + 6*B*a^12*b^4 - 30*A*a^11*b^5)/(a^1
4*b^3))*x^2 + 105*(B*a^13*b^3 - 4*A*a^12*b^4)/(a^14*b^3))*x/(b*x^2 + a)^(7
/2) + 2*A*sqrt(b)/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)*a^4)
```

## Mupad [B] (verification not implemented)

Time = 1.62 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.12

$$\int \frac{A+Bx^2+Cx^4+Bx^6}{x^2\left(a+bx^2\right)^{9/2}}\,dx=\frac{x\left(\frac{3\,B\,a^3+4\,C\,a^2\,b+24\,B\,a\,b^2-52\,A\,b^3}{105\,a^4\,b^2}-\frac{A\,b}{3\,a^4}\right)}{\left(b\,x^2+a\right)^{3/2}}-\frac{x\left(\frac{a\left(\frac{B}{5\,a\,b}+\frac{B\,a^3-C\,a^2\,b-6\,B\,a\,b^2+6\,A\,b^3}{35\,a^4\,b}\right)}{b}+\frac{A\,b}{5\,a^3}\right)}{\left(b\,x^2+a\right)^{5/2}}+\frac{x\left(\frac{a\left(\frac{a\left(\frac{B}{7\,a}-\frac{C\,b}{7\,a^2}\right)}{b}+\frac{B\,b}{7\,a^2}\right)}{b}-\frac{A\,b}{7\,a^2}\right)}{\left(b\,x^2+a\right)^{7/2}}-\frac{\frac{A}{a^4}-x^2\left(\frac{6\,B\,a^3+8\,C\,a^2\,b+48\,B\,a\,b^2-174\,A\,b^3}{105\,a^5\,b^2}-\frac{2\,A\,b}{a^5}\right)}{x\,\sqrt{b\,x^2+a}}$$

input
```
int((A + B*x^2 + B*x^6 + C*x^4)/(x^2*(a + b*x^2)^(9/2)),x)
```

output
```
(x*((3*B*a^3 - 52*A*b^3 + 24*B*a*b^2 + 4*C*a^2*b)/(105*a^4*b^2) - (A*b)/(3
*a^4)))/(a + b*x^2)^(3/2) - (x*((a*(B/(5*a*b) + (6*A*b^3 + B*a^3 - 6*B*a*b
^2 - C*a^2*b)/(35*a^4*b)))/b + (A*b)/(5*a^3)))/(a + b*x^2)^(5/2) + (x*((a*
((a*(B/(7*a) - (C*b)/(7*a^2)))/b + (B*b)/(7*a^2)))/b - (A*b)/(7*a^2)))/(a
+ b*x^2)^(7/2) - (A/a^4 - x^2*((6*B*a^3 - 174*A*b^3 + 48*B*a*b^2 + 8*C*a^2
*b)/(105*a^5*b^2) - (2*A*b)/a^5))/(x*(a + b*x^2)^(1/2))
```

## Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 414, normalized size of antiderivative = 1.91

$$\int \frac{A+Bx^2+Cx^4+Bx^6}{x^2\left(a+bx^2\right)^{9/2}}\,dx=\frac{336\sqrt{b}\,b^6x^9-336\sqrt{b\,x^2+a}\,b^6x^8-6\sqrt{b}\,a^6x+21\sqrt{b\,x^2+a}\,a^3b^3x^6+6\sqrt{b\,x}}{}$$

input
```
int((B*x^6+C*x^4+B*x^2+A)/x^2/(b*x^2+a)^(9/2),x)
```

output
```
( - 105*sqrt(a + b*x**2)*a**4*b**2 + 21*sqrt(a + b*x**2)*a**3*b**3*x**6 -
735*sqrt(a + b*x**2)*a**3*b**3*x**2 + 35*sqrt(a + b*x**2)*a**3*b**2*c*x**4
 + 6*sqrt(a + b*x**2)*a**2*b**4*x**8 - 1470*sqrt(a + b*x**2)*a**2*b**4*x**
4 + 28*sqrt(a + b*x**2)*a**2*b**3*c*x**6 - 1176*sqrt(a + b*x**2)*a*b**5*x*
*6 + 8*sqrt(a + b*x**2)*a*b**4*c*x**8 - 336*sqrt(a + b*x**2)*b**6*x**8 - 6
*sqrt(b)*a**6*x - 24*sqrt(b)*a**5*b*x**3 - 8*sqrt(b)*a**5*c*x - 36*sqrt(b)
*a**4*b**2*x**5 + 336*sqrt(b)*a**4*b**2*x - 32*sqrt(b)*a**4*b*c*x**3 - 24*
sqrt(b)*a**3*b**3*x**7 + 1344*sqrt(b)*a**3*b**3*x**3 - 48*sqrt(b)*a**3*b**
2*c*x**5 - 6*sqrt(b)*a**2*b**4*x**9 + 2016*sqrt(b)*a**2*b**4*x**5 - 32*sqr
t(b)*a**2*b**3*c*x**7 + 1344*sqrt(b)*a*b**5*x**7 - 8*sqrt(b)*a*b**4*c*x**9
 + 336*sqrt(b)*b**6*x**9)/(105*a**4*b**2*x*(a**4 + 4*a**3*b*x**2 + 6*a**2*
b**2*x**4 + 4*a*b**3*x**6 + b**4*x**8))
```

# 3.245 $\int \frac{A+Bx^2+Cx^4+Bx^6}{x^4(a+bx^2)^{9/2}}\,dx$



## Optimal result

Integrand size = 32, antiderivative size = 246

$$\int \frac{A+Bx^2+Cx^4+Bx^6}{x^4(a+bx^2)^{9/2}}\,dx = \frac{(Ab^3-a(a^2B+b^2B-abC))\,x}{7a^3b\,(a+bx^2)^{7/2}} + \frac{(20Ab^3+a(a^2B-13b^2B+6abC))\,x}{35a^4b\,(a+bx^2)^{5/2}} + \frac{(185Ab^3+a(4a^2B-87b^2B+24abC))\,x}{105a^5b\,(a+bx^2)^{3/2}} + \frac{(790Ab^3+a(8a^2B-279b^2B+48abC))\,x}{105a^6b\sqrt{a+bx^2}} - \frac{A\sqrt{a+bx^2}}{3a^5x^3} + \frac{(14Ab-3aB)\sqrt{a+bx^2}}{3a^6x}$$

output

```
1/7*(A*b^3-a*(B*a^2+B*b^2-C*a*b))*x/a^3/b/(b*x^2+a)^(7/2)+1/35*(20*A*b^3+a
*(B*a^2-13*B*b^2+6*C*a*b))*x/a^4/b/(b*x^2+a)^(5/2)+1/105*(185*A*b^3+a*(4*B
*a^2-87*B*b^2+24*C*a*b))*x/a^5/b/(b*x^2+a)^(3/2)+1/105*(790*A*b^3+a*(8*B*a
^2-279*B*b^2+48*C*a*b))*x/a^6/b/(b*x^2+a)^(1/2)-1/3*A*(b*x^2+a)^(1/2)/a^5/
x^3+1/3*(14*A*b-3*B*a)*(b*x^2+a)^(1/2)/a^6/x
```

## Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.66

$$\int \frac{A+Bx^2+Cx^4+Bx^6}{x^4\left(a+bx^2\right)^{9/2}}\,dx = \frac{1280Ab^5x^{10}+128ab^4x^8(35A-3Bx^2)+16a^2b^3x^6(350A-84Bx^2+3Cx^4)}{}$$

input
```
Integrate[(A + B*x^2 + C*x^4 + B*x^6)/(x^4*(a + b*x^2)^(9/2)),x]
```

output
```
(1280*A*b^5*x^10 + 128*a*b^4*x^8*(35*A - 3*B*x^2) + 16*a^2*b^3*x^6*(350*A
 - 84*B*x^2 + 3*C*x^4) + 8*a^3*b^2*x^4*(350*A + 21*C*x^4 + B*x^2*(-210 + x^
4)) + 14*a^4*b*x^2*(25*A + 15*C*x^4 + 2*B*x^2*(-30 + x^4)) - 35*a^5*(A - 3
*C*x^4 - B*x^2*(-3 + x^4)))/(105*a^6*x^3*(a + b*x^2)^(7/2))
```

## Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.88, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {2334, 2089, 1588, 298, 209, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^6+Bx^2+Cx^4}{x^4\left(a+bx^2\right)^{9/2}}dx$$

$$\Big\downarrow \text{2334}$$

$$-\frac{\int \frac{10Ab-3a\left(Bx^4+Cx^2+B\right)}{x^2\left(bx^2+a\right)^{9/2}}dx}{3a}-\frac{A}{3ax^3\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \text{2089}$$

$$-\frac{\int \frac{-3aBx^4-3aCx^2+10Ab-3aB}{x^2\left(bx^2+a\right)^{9/2}}dx}{3a}-\frac{A}{3ax^3\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \text{1588}$$

$$-\frac{-\frac{\int \frac{80Ab^2+3a^2Bx^2-3a(8bB-aC)}{\left(bx^2+a\right)^{9/2}}dx}{a}-\frac{10Ab-3aB}{ax(a+bx^2)^{7/2}}}{3a}-\frac{A}{3ax^3\left(a+bx^2\right)^{7/2}}$$

$\downarrow$ 298

$$-\frac{-\frac{\frac{3\left(a\left(a^2B+6abC-48b^2B\right)+160Ab^3\right)\int \frac{1}{\left(bx^2+a\right)^{7/2}}dx}{7ab}+\frac{x\left(80Ab^3-3a\left(a^2B-abC+8b^2B\right)\right)}{7ab\left(a+bx^2\right)^{7/2}}}{a}-\frac{10Ab-3aB}{ax(a+bx^2)^{7/2}}}{3a}-\frac{A}{3ax^3\left(a+bx^2\right)^{7/2}}$$

$\downarrow$ 209

$$-\frac{-\frac{\frac{3\left(a\left(a^2B+6abC-48b^2B\right)+160Ab^3\right)\left(\frac{4\int \frac{1}{\left(bx^2+a\right)^{5/2}}dx}{5a}+\frac{x}{5a\left(a+bx^2\right)^{5/2}}\right)}{7ab}+\frac{x\left(80Ab^3-3a\left(a^2B-abC+8b^2B\right)\right)}{7ab\left(a+bx^2\right)^{7/2}}}{a}-\frac{10Ab-3aB}{ax(a+bx^2)^{7/2}}}{3a}-\frac{A}{3ax^3\left(a+bx^2\right)^{7/2}}$$

$\downarrow$ 209

$$-\frac{-\frac{\frac{3\left(a\left(a^2B+6abC-48b^2B\right)+160Ab^3\right)\left(\frac{4\left(\frac{2\int \frac{1}{\left(bx^2+a\right)^{3/2}}dx}{3a}+\frac{x}{3a\left(a+bx^2\right)^{3/2}}\right)}{5a}+\frac{x}{5a\left(a+bx^2\right)^{5/2}}\right)}{7ab}+\frac{x\left(80Ab^3-3a\left(a^2B-abC+8b^2B\right)\right)}{7ab\left(a+bx^2\right)^{7/2}}}{a}-\frac{10Ab-3aB}{ax(a+bx^2)^{7/2}}}{3a}-\frac{A}{3ax^3\left(a+bx^2\right)^{7/2}}$$

$\downarrow$ 208

$$-\frac{-\frac{\frac{x\left(80Ab^3-3a\left(a^2B-abC+8b^2B\right)\right)}{7ab\left(a+bx^2\right)^{7/2}}+\frac{3\left(\frac{4\left(\frac{2x}{3a^2\sqrt{a+bx^2}}+\frac{x}{3a\left(a+bx^2\right)^{3/2}}\right)}{5a}+\frac{x}{5a\left(a+bx^2\right)^{5/2}}\right)\left(a\left(a^2B+6abC-48b^2B\right)+160Ab^3\right)}{7ab}}{a}-\frac{10Ab-3aB}{ax(a+bx^2)^{7/2}}}{3a}-$$

$$\frac{A}{3ax^3\left(a+bx^2\right)^{7/2}}$$

input
```
Int[(A + B*x^2 + C*x^4 + B*x^6)/(x^4*(a + b*x^2)^(9/2)),x]
```

output
```
-1/3*A/(a*x^3*(a + b*x^2)^(7/2)) - (-((10*A*b - 3*a*B)/(a*x*(a + b*x^2)^(7
/2))) - (((80*A*b^3 - 3*a*(a^2*B + 8*b^2*B - a*b*C))*x)/(7*a*b*(a + b*x^2)
^(7/2)) + (3*(160*A*b^3 + a*(a^2*B - 48*b^2*B + 6*a*b*C))*(x/(5*a*(a + b*x
^2)^(5/2)) + (4*(x/(3*a*(a + b*x^2)^(3/2)) + (2*x)/(3*a^2*Sqrt[a + b*x^2])
))/(5*a)))/(7*a*b))/a)/(3*a)
```

## Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 1588
```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Simp[1/(d*f
^2*(m + 1))   Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x
) - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

rule 2089
```
Int[(u_)^(p_.)*((f_.)*(x_))^(m_.)*(z_)^(q_.), x_Symbol] :> Int[(f*x)^m*Expa
ndToSum[z, x]^q*ExpandToSum[u, x]^p, x] /; FreeQ[{f, m, p, q}, x] && Binomi
alQ[z, x] && TrinomialQ[u, x] &&  !(BinomialMatchQ[z, x] && TrinomialMatchQ
[u, x])
```

rule 2334
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coef
f[Pq, x, 0], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[A*
x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))   Int[
x^(m + 2)*(a + b*x^2)^p*(a*(m + 1)*Q - A*b*(m + 2*(p + 1) + 1)), x], x]] /;
 FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && IntegerQ[m/2] && ILtQ[(m + 1)/2 + p,
 0] && LtQ[m + Expon[Pq, x] + 2*p + 1, 0]
```

## Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.63

| method | result |
|---|---|
| pseudoelliptic | $-\frac{\left(-B\,x^{6}-3C\,x^{4}+3x^{2}B+A\right)a^{5}-10\left(\frac{2}{25}B\,x^{6}+\frac{3}{5}C\,x^{4}-\frac{12}{5}x^{2}B+A\right)x^{2}b\,a^{4}-80\left(\frac{1}{350}B\,x^{6}+\frac{3}{50}C\,x^{4}-\frac{3}{5}x^{2}B+A\right)x^{4}b^{2}a^{3}-160}{3\left(b\,x^{2}+a\right)^{\frac{7}{2}}x^{3}a^{6}}$ |
| gosper | $-\frac{-1280A\,b^{5}x^{10}-8B\,a^{3}b^{2}x^{10}+384Ba\,b^{4}x^{10}-48C\,a^{2}b^{3}x^{10}-4480aA\,b^{4}x^{8}-28B\,a^{4}b\,x^{8}+1344B\,a^{2}b^{3}x^{8}-168C\,a^{3}b^{2}x^{8}-56}{105a}$ |
| trager | $-\frac{-1280A\,b^{5}x^{10}-8B\,a^{3}b^{2}x^{10}+384Ba\,b^{4}x^{10}-48C\,a^{2}b^{3}x^{10}-4480aA\,b^{4}x^{8}-28B\,a^{4}b\,x^{8}+1344B\,a^{2}b^{3}x^{8}-168C\,a^{3}b^{2}x^{8}-56}{105a}$ |
| orering | $-\frac{-1280A\,b^{5}x^{10}-8B\,a^{3}b^{2}x^{10}+384Ba\,b^{4}x^{10}-48C\,a^{2}b^{3}x^{10}-4480aA\,b^{4}x^{8}-28B\,a^{4}b\,x^{8}+1344B\,a^{2}b^{3}x^{8}-168C\,a^{3}b^{2}x^{8}-56}{105a}$ |
| risch | $-\frac{\sqrt{b\,x^{2}+a}\,\left(-14Ab\,x^{2}+3Ba\,x^{2}+Aa\right)}{3a^{6}x^{3}}+\frac{\sqrt{b\,x^{2}+a}\,x\left(790b^{5}A\,x^{6}+8B\,a^{3}b^{2}x^{6}-279Ba\,b^{4}x^{6}+48C\,a^{2}b^{3}x^{6}+2555aA\,b^{4}x^{4}+2\right.}{}$ |
| default | $C\left(\frac{x}{7a\left(b\,x^{2}+a\right)^{\frac{7}{2}}}+\frac{\frac{6x}{35a\left(b\,x^{2}+a\right)^{\frac{5}{2}}}+\frac{6\left(\frac{4x}{15a\left(b\,x^{2}+a\right)^{\frac{3}{2}}}+\frac{8x}{15a^{2}\sqrt{b\,x^{2}+a}}\right)}{7a}}{a}\right)+A\left(-\frac{1}{3a\,x^{3}\left(b\,x^{2}+a\right)^{\frac{7}{2}}}-\frac{10b\left(-\frac{}{ax\left(b\right.}\right.}{}\right.$ |

input
```
int((B*x^6+C*x^4+B*x^2+A)/x^4/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*((-B*x^6-3*C*x^4+3*B*x^2+A)*a^5-10*(2/25*B*x^6+3/5*C*x^4-12/5*x^2*B+A
)*x^2*b*a^4-80*(1/350*B*x^6+3/50*C*x^4-3/5*x^2*B+A)*x^4*b^2*a^3-160*(3/350
*C*x^4-6/25*x^2*B+A)*x^6*b^3*a^2-128*(-3/35*x^2*B+A)*x^8*b^4*a-256/7*A*b^5
*x^10)/(b*x^2+a)^(7/2)/x^3/a^6
```

## Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.91

$$\int \frac{A+Bx^2+Cx^4+Bx^6}{x^4\left(a+bx^2\right)^{9/2}}\,dx = \frac{(8\,(Ba^3b^2+6\,Ca^2b^3-48\,Bab^4+160\,Ab^5)x^{10}+28\,(Ba^4b+6\,Ca^3b^2-48\,\ldots}{}$$

input
```
integrate((B*x^6+C*x^4+B*x^2+A)/x^4/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

output
```
1/105*(8*(B*a^3*b^2 + 6*C*a^2*b^3 - 48*B*a*b^4 + 160*A*b^5)*x^10 + 28*(B*a
^4*b + 6*C*a^3*b^2 - 48*B*a^2*b^3 + 160*A*a*b^4)*x^8 + 35*(B*a^5 + 6*C*a^4
*b - 48*B*a^3*b^2 + 160*A*a^2*b^3)*x^6 - 35*A*a^5 + 35*(3*C*a^5 - 24*B*a^4
*b + 80*A*a^3*b^2)*x^4 - 35*(3*B*a^5 - 10*A*a^4*b)*x^2)*sqrt(b*x^2 + a)/(a
^6*b^4*x^11 + 4*a^7*b^3*x^9 + 6*a^8*b^2*x^7 + 4*a^9*b*x^5 + a^10*x^3)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4+Bx^6}{x^4\left(a+bx^2\right)^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate((B*x**6+C*x**4+B*x**2+A)/x**4/(b*x**2+a)**(9/2),x)
```

output
```
Timed out
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.37

$$\int \frac{A + Bx^2 + Cx^4 + Bx^6}{x^4 (a + bx^2)^{9/2}} dx = \frac{16 Cx}{35 \sqrt{bx^2 + a}a^4} + \frac{8 Cx}{35 (bx^2 + a)^{\frac{3}{2}} a^3} + \frac{6 Cx}{35 (bx^2 + a)^{\frac{5}{2}} a^2} + \frac{Cx}{7 (bx^2 + a)^{\frac{7}{2}} a} - \frac{Bx}{7 (bx^2 + a)^{\frac{7}{2}} b} + \frac{8 Bx}{105 \sqrt{bx^2 + a}a^3 b} + \frac{4 Bx}{105 (bx^2 + a)^{\frac{3}{2}} a^2 b} + \frac{Bx}{35 (bx^2 + a)^{\frac{5}{2}} ab} - \frac{128 Bbx}{35 \sqrt{bx^2 + a}a^5} - \frac{64 Bbx}{35 (bx^2 + a)^{\frac{3}{2}} a^4} - \frac{48 Bbx}{35 (bx^2 + a)^{\frac{5}{2}} a^3} - \frac{8 Bbx}{7 (bx^2 + a)^{\frac{7}{2}} a^2} + \frac{256 Ab^2 x}{21 \sqrt{bx^2 + a}a^6} + \frac{128 Ab^2 x}{21 (bx^2 + a)^{\frac{3}{2}} a^5} + \frac{32 Ab^2 x}{7 (bx^2 + a)^{\frac{5}{2}} a^4} + \frac{80 Ab^2 x}{21 (bx^2 + a)^{\frac{7}{2}} a^3} - \frac{B}{(bx^2 + a)^{\frac{7}{2}} ax} + \frac{10 Ab}{3 (bx^2 + a)^{\frac{7}{2}} a^2 x} - \frac{A}{3 (bx^2 + a)^{\frac{7}{2}} ax^3}$$

input
```
integrate((B*x^6+C*x^4+B*x^2+A)/x^4/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

output
```
16/35*C*x/(sqrt(b*x^2 + a)*a^4) + 8/35*C*x/((b*x^2 + a)^(3/2)*a^3) + 6/35*
C*x/((b*x^2 + a)^(5/2)*a^2) + 1/7*C*x/((b*x^2 + a)^(7/2)*a) - 1/7*B*x/((b*
x^2 + a)^(7/2)*b) + 8/105*B*x/(sqrt(b*x^2 + a)*a^3*b) + 4/105*B*x/((b*x^2
+ a)^(3/2)*a^2*b) + 1/35*B*x/((b*x^2 + a)^(5/2)*a*b) - 128/35*B*b*x/(sqrt(
b*x^2 + a)*a^5) - 64/35*B*b*x/((b*x^2 + a)^(3/2)*a^4) - 48/35*B*b*x/((b*x^
2 + a)^(5/2)*a^3) - 8/7*B*b*x/((b*x^2 + a)^(7/2)*a^2) + 256/21*A*b^2*x/(sq
rt(b*x^2 + a)*a^6) + 128/21*A*b^2*x/((b*x^2 + a)^(3/2)*a^5) + 32/7*A*b^2*x
/((b*x^2 + a)^(5/2)*a^4) + 80/21*A*b^2*x/((b*x^2 + a)^(7/2)*a^3) - B/((b*x
^2 + a)^(7/2)*a*x) + 10/3*A*b/((b*x^2 + a)^(7/2)*a^2*x) - 1/3*A/((b*x^2 +
a)^(7/2)*a*x^3)
```

## Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.42

$$\int \frac{A+Bx^2+Cx^4+Bx^6}{x^4\left(a+bx^2\right)^{9/2}}\,dx = \frac{\left(\left(x^2\left(\frac{\left(8\,Ba^{15}b^5+48\,Ca^{14}b^6-279\,Ba^{13}b^7+790\,Aa^{12}b^8\right)x^2}{a^{18}b^3}+\frac{7\left(4\,Ba^{16}b^4+24\,Ca^{15}b^5-132\,Ba^{14}b\right.}{a^{18}b^3}\right.\right.\right.}{1}$$

$$+\frac{2\left(3\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4Ba\sqrt{b}-12\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4Ab^{\frac{3}{2}}-6\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2Ba^2\sqrt{b}+30\left(\sqrt{b}\right.\right.}{3\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^3a^5}$$

input
```
integrate((B*x^6+C*x^4+B*x^2+A)/x^4/(b*x^2+a)^(9/2),x, algorithm="giac")
```

output
```
1/105*((x^2*((8*B*a^15*b^5 + 48*C*a^14*b^6 - 279*B*a^13*b^7 + 790*A*a^12*b
^8)*x^2/(a^18*b^3) + 7*(4*B*a^16*b^4 + 24*C*a^15*b^5 - 132*B*a^14*b^6 + 36
5*A*a^13*b^7)/(a^18*b^3)) + 35*(B*a^17*b^3 + 6*C*a^16*b^4 - 30*B*a^15*b^5
+ 80*A*a^14*b^6)/(a^18*b^3))*x^2 + 105*(C*a^17*b^3 - 4*B*a^16*b^4 + 10*A*a
^15*b^5)/(a^18*b^3))*x/(b*x^2 + a)^(7/2) + 2/3*(3*(sqrt(b)*x - sqrt(b*x^2
+ a))^4*B*a*sqrt(b) - 12*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*b^(3/2) - 6*(sq
rt(b)*x - sqrt(b*x^2 + a))^2*B*a^2*sqrt(b) + 30*(sqrt(b)*x - sqrt(b*x^2 +
a))^2*A*a*b^(3/2) + 3*B*a^3*sqrt(b) - 14*A*a^2*b^(3/2))/(((sqrt(b)*x - sqr
t(b*x^2 + a))^2 - a)^3*a^5)
```

## Mupad [B] (verification not implemented)

Time = 1.57 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.20

$$\int \frac{A+Bx^2+Cx^4+Bx^6}{x^4(a+bx^2)^{9/2}}\,dx = \frac{\frac{4\,B\,a^3+24\,C\,a^2\,b-192\,B\,a\,b^2+640\,A\,b^3}{105\,a^5\,b^2}+\frac{2\,x^2\,\left(4\,B\,a^3+24\,C\,a^2\,b-192\,B\,a\,b^2+640\,A\,b^3\right)}{105\,a^6\,b}}{x\,\sqrt{b\,x^2+a}} + \frac{x\,\left(\frac{A\,b^2}{7\,a^3}-\frac{a\,\left(\frac{B\,b^2}{7\,a^3}-\frac{a\,\left(\frac{C\,b^2}{7\,a^3}-\frac{B\,b}{7\,a^2}\right)}{b}\right)}{b}\right)}{{\left(b\,x^2+a\right)}^{7/2}} - \frac{x^2\,\left(\frac{a\,\left(\frac{B\,a^3+6\,C\,a^2\,b-6\,B\,a\,b^2+6\,A\,b^3}{35\,a^4\,b}+\frac{8\,b\,\left(A\,b-B\,a\right)}{15\,a^4}\right)}{b}-\frac{A\,b-B\,a}{3\,a^3}+\frac{8\,A\,b}{15\,a^3}\right)+\frac{A}{3\,a^2}}{x^3\,{\left(b\,x^2+a\right)}^{5/2}} - \frac{B\,a^3+6\,C\,a^2\,b-48\,B\,a\,b^2+160\,A\,b^3}{105\,a^4\,b^2\,x\,{\left(b\,x^2+a\right)}^{3/2}}$$

input
```
int((A + B*x^2 + B*x^6 + C*x^4)/(x^4*(a + b*x^2)^(9/2)),x)
```

output
```
((640*A*b^3 + 4*B*a^3 - 192*B*a*b^2 + 24*C*a^2*b)/(105*a^5*b^2) + (2*x^2*(
640*A*b^3 + 4*B*a^3 - 192*B*a*b^2 + 24*C*a^2*b))/(105*a^6*b))/(x*(a + b*x^
2)^(1/2)) + (x*((A*b^2)/(7*a^3) - (a*((B*b^2)/(7*a^3) - (a*((C*b^2)/(7*a^3
) - (B*b)/(7*a^2)))/b))/b))/(a + b*x^2)^(7/2) - (x^2*((a*((6*A*b^3 + B*a^3
 - 6*B*a*b^2 + 6*C*a^2*b)/(35*a^4*b) + (8*b*(A*b - B*a))/(15*a^4)))/b - (A
*b - B*a)/(3*a^3) + (8*A*b)/(15*a^3)) + A/(3*a^2))/(x^3*(a + b*x^2)^(5/2))
 - (160*A*b^3 + B*a^3 - 48*B*a*b^2 + 6*C*a^2*b)/(105*a^4*b^2*x*(a + b*x^2)
^(3/2))
```

## Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 474, normalized size of antiderivative = 1.93

$$\int \frac{A+Bx^2+Cx^4+Bx^6}{x^4(a+bx^2)^{9/2}}\,dx = \frac{-35\sqrt{b\,x^2+a}\,a^5b-896\sqrt{b}\,b^6x^{11}+896\sqrt{b\,x^2+a}\,b^6x^{10}-8\sqrt{b}\,a^6x^3+35\sqrt{}}{}$$

input
```
int((B*x^6+C*x^4+B*x^2+A)/x^4/(b*x^2+a)^(9/2),x)
```

output

```
( - 35*sqrt(a + b*x**2)*a**5*b + 35*sqrt(a + b*x**2)*a**4*b**2*x**6 + 245*
sqrt(a + b*x**2)*a**4*b**2*x**2 + 105*sqrt(a + b*x**2)*a**4*b*c*x**4 + 28*
sqrt(a + b*x**2)*a**3*b**3*x**8 + 1960*sqrt(a + b*x**2)*a**3*b**3*x**4 + 2
10*sqrt(a + b*x**2)*a**3*b**2*c*x**6 + 8*sqrt(a + b*x**2)*a**2*b**4*x**10
+ 3920*sqrt(a + b*x**2)*a**2*b**4*x**6 + 168*sqrt(a + b*x**2)*a**2*b**3*c*
x**8 + 3136*sqrt(a + b*x**2)*a*b**5*x**8 + 48*sqrt(a + b*x**2)*a*b**4*c*x*
*10 + 896*sqrt(a + b*x**2)*b**6*x**10 - 8*sqrt(b)*a**6*x**3 - 32*sqrt(b)*a
**5*b*x**5 - 48*sqrt(b)*a**5*c*x**3 - 48*sqrt(b)*a**4*b**2*x**7 - 896*sqrt
(b)*a**4*b**2*x**3 - 192*sqrt(b)*a**4*b*c*x**5 - 32*sqrt(b)*a**3*b**3*x**9
 - 3584*sqrt(b)*a**3*b**3*x**5 - 288*sqrt(b)*a**3*b**2*c*x**7 - 8*sqrt(b)*
a**2*b**4*x**11 - 5376*sqrt(b)*a**2*b**4*x**7 - 192*sqrt(b)*a**2*b**3*c*x*
*9 - 3584*sqrt(b)*a*b**5*x**9 - 48*sqrt(b)*a*b**4*c*x**11 - 896*sqrt(b)*b*
*6*x**11)/(105*a**5*b*x**3*(a**4 + 4*a**3*b*x**2 + 6*a**2*b**2*x**4 + 4*a*
b**3*x**6 + b**4*x**8))
```

# 3.246 $\int \frac{A+Bx^2+Cx^4+Bx^6}{x^6(a+bx^2)^{9/2}}\,dx$



## Optimal result

Integrand size = 32, antiderivative size = 277

$$\int \frac{A+Bx^2+Cx^4+Bx^6}{x^6(a+bx^2)^{9/2}}\,dx = -\frac{(Ab^3-a(a^2B+b^2B-abC))x}{7a^4(a+bx^2)^{7/2}} - \frac{(27Ab^3-a(6a^2B+20b^2B-13abC))x}{35a^5(a+bx^2)^{5/2}} - \frac{(318Ab^3-a(24a^2B+185b^2B-87abC))x}{105a^6(a+bx^2)^{3/2}} - \frac{(1686Ab^3-a(48a^2B+790b^2B-279abC))x}{105a^7\sqrt{a+bx^2}} - \frac{A\sqrt{a+bx^2}}{5a^5x^5} + \frac{(24Ab-5aB)\sqrt{a+bx^2}}{15a^6x^3} - \frac{(198Ab^2-70abB+15a^2C)\sqrt{a+bx^2}}{15a^7x}$$

output
```
-1/7*(A*b^3-a*(B*a^2+B*b^2-C*a*b))*x/a^4/(b*x^2+a)^(7/2)-1/35*(27*A*b^3-a*
(6*B*a^2+20*B*b^2-13*C*a*b))*x/a^5/(b*x^2+a)^(5/2)-1/105*(318*A*b^3-a*(24*
B*a^2+185*B*b^2-87*C*a*b))*x/a^6/(b*x^2+a)^(3/2)-1/105*(1686*A*b^3-a*(48*B
*a^2+790*B*b^2-279*C*a*b))*x/a^7/(b*x^2+a)^(1/2)-1/5*A*(b*x^2+a)^(1/2)/a^5
/x^5+1/15*(24*A*b-5*B*a)*(b*x^2+a)^(1/2)/a^6/x^3-1/15*(198*A*b^2-70*B*a*b+
15*C*a^2)*(b*x^2+a)^(1/2)/a^7/x
```

## Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.73

$$\int \frac{A+Bx^2+Cx^4+Bx^6}{x^6\left(a+bx^2\right)^{9/2}}\,dx = \frac{-3072Ab^6x^{12}+256ab^5x^{10}(-42A+5Bx^2)-128a^2b^4x^8(105A-35Bx^2+\cdots}{}$$

input
```
Integrate[(A + B*x^2 + C*x^4 + B*x^6)/(x^6*(a + b*x^2)^(9/2)),x]
```

output
```
(-3072*A*b^6*x^12 + 256*a*b^5*x^10*(-42*A + 5*B*x^2) - 128*a^2*b^4*x^8*(10
5*A - 35*B*x^2 + 3*C*x^4) + 14*a^5*b*x^2*(6*A - 60*C*x^4 + 5*B*x^2*(5 + 3*
x^4)) + 56*a^4*b^2*x^4*(-15*A - 30*C*x^4 + B*x^2*(50 + 3*x^4)) + 16*a^3*b^
3*x^6*(-420*A - 84*C*x^4 + B*x^2*(350 + 3*x^4)) - 7*a^6*(3*A + 5*x^2*(B +
3*C*x^2 - 3*B*x^4)))/(105*a^7*x^5*(a + b*x^2)^(7/2))
```

## Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.84, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.281$, Rules used = {2334, 2089, 1588, 27, 359, 209, 209, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^6+Bx^2+Cx^4}{x^6\left(a+bx^2\right)^{9/2}}\,dx$$

$$\Big\downarrow \text{2334}$$

$$-\frac{\int \frac{12Ab-5a\left(Bx^4+Cx^2+B\right)}{x^4\left(bx^2+a\right)^{9/2}}dx}{5a}-\frac{A}{5ax^5\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \text{2089}$$

$$-\frac{\int \frac{-5aBx^4-5aCx^2+12Ab-5aB}{x^4\left(bx^2+a\right)^{9/2}}dx}{5a}-\frac{A}{5ax^5\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \text{1588}$$

$$-\frac{-\frac{\int \frac{5\left(24Ab^2+3a^2Bx^2-a(10bB-3aC)\right)}{x^2\left(bx^2+a\right)^{9/2}}dx}{3a}-\frac{12Ab-5aB}{3ax^3(a+bx^2)^{7/2}}}{5a}-\frac{A}{5ax^5\left(a+bx^2\right)^{7/2}}$$

$\Big\downarrow$ 27

$$-\frac{-\frac{5\int \frac{24Ab^2+3a^2Bx^2-a(10bB-3aC)}{x^2\left(bx^2+a\right)^{9/2}}dx}{3a}-\frac{12Ab-5aB}{3ax^3(a+bx^2)^{7/2}}}{5a}-\frac{A}{5ax^5\left(a+bx^2\right)^{7/2}}$$

$\Big\downarrow$ 359

$$-\frac{-\frac{5\left(-\frac{\left(192Ab^3-a\left(3a^2B-24abC+80b^2B\right)\right)\int \frac{1}{\left(bx^2+a\right)^{9/2}}dx}{a}-\frac{24Ab^2-a(10bB-3aC)}{ax\left(a+bx^2\right)^{7/2}}\right)}{3a}-\frac{12Ab-5aB}{3ax^3(a+bx^2)^{7/2}}}{5a}-\frac{A}{5ax^5\left(a+bx^2\right)^{7/2}}$$

$\Big\downarrow$ 209

$$-\frac{-\frac{5\left(-\frac{\left(192Ab^3-a\left(3a^2B-24abC+80b^2B\right)\right)\left(\frac{6\int \frac{1}{\left(bx^2+a\right)^{7/2}}dx}{7a}+\frac{x}{7a\left(a+bx^2\right)^{7/2}}\right)}{a}-\frac{24Ab^2-a(10bB-3aC)}{ax\left(a+bx^2\right)^{7/2}}\right)}{3a}-\frac{12Ab-5aB}{3ax^3(a+bx^2)^{7/2}}}{5a}-\frac{A}{5ax^5\left(a+bx^2\right)^{7/2}}$$

$\Big\downarrow$ 209

$$-\frac{-\frac{5\left(-\frac{\left(192Ab^3-a\left(3a^2B-24abC+80b^2B\right)\right)\left(\frac{6\left(\frac{4\int\frac{1}{\left(bx^2+a\right)^{5/2}}dx}{5a}+\frac{x}{5a\left(a+bx^2\right)^{5/2}}\right)}{7a}+\frac{x}{7a\left(a+bx^2\right)^{7/2}}\right)}{a}-\frac{24Ab^2-a(10bB-3aC)}{ax\left(a+bx^2\right)^{7/2}}\right)}{3a}-\frac{12Ab-5aB}{3ax^3(a+bx^2)}}{5a}$$

$$\frac{A}{5ax^5\left(a+bx^2\right)^{7/2}}$$

$\downarrow$ 209

$$-\frac{-\frac{5\left(-\frac{\left(192Ab^3-a\left(3a^2B-24abC+80b^2B\right)\right)\left(\frac{6\left(\frac{4\left(\frac{2\int\frac{1}{\left(bx^2+a\right)^{3/2}}dx}{3a}+\frac{x}{3a\left(a+bx^2\right)^{3/2}}\right)}{5a}+\frac{x}{5a\left(a+bx^2\right)^{5/2}}\right)}{7a}+\frac{x}{7a\left(a+bx^2\right)^{7/2}}\right)}{a}-\frac{24Ab^2-a(10bB-3aC}{ax\left(a+bx^2\right)^{7/2}}\right)}{3a}}{5a}$$

$$\frac{A}{5ax^5\left(a+bx^2\right)^{7/2}}$$

$\downarrow$ 208

$$-\frac{-\frac{5\left(-\frac{\left(\frac{6\left(\frac{4\left(\frac{2x}{3a^2\sqrt{a+bx^2}}+\frac{x}{3a\left(a+bx^2\right)^{3/2}}\right)}{5a}+\frac{x}{5a\left(a+bx^2\right)^{5/2}}\right)}{7a}+\frac{x}{7a\left(a+bx^2\right)^{7/2}}\right)\left(192Ab^3-a\left(3a^2B-24abC+80b^2B\right)\right)}{a}-\frac{24Ab^2-a(10bB-3aC)}{ax\left(a+bx^2\right)^{7/2}}\right)}{3a}-}{5a}$$

$$\frac{A}{5ax^5\left(a+bx^2\right)^{7/2}}$$

input
```
Int[(A + B*x^2 + C*x^4 + B*x^6)/(x^6*(a + b*x^2)^(9/2)),x]
```

output
```
-1/5*A/(a*x^5*(a + b*x^2)^(7/2)) - (-1/3*(12*A*b - 5*a*B)/(a*x^3*(a + b*x^
2)^(7/2)) - (5*(-((24*A*b^2 - a*(10*b*B - 3*a*C))/(a*x*(a + b*x^2)^(7/2)))
 - ((192*A*b^3 - a*(3*a^2*B + 80*b^2*B - 24*a*b*C))*(x/(7*a*(a + b*x^2)^(7
/2)) + (6*(x/(5*a*(a + b*x^2)^(5/2)) + (4*(x/(3*a*(a + b*x^2)^(3/2)) + (2*
x)/(3*a^2*Sqrt[a + b*x^2])))/(5*a)))/(7*a)))/a))/(3*a))/(5*a)
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 1588
```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Simp[1/(d*f
^2*(m + 1))   Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x
) - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

rule 2089
```
Int[(u_)^(p_.)*((f_.)*(x_))^(m_.)*(z_)^(q_.), x_Symbol] :> Int[(f*x)^m*Expa
ndToSum[z, x]^q*ExpandToSum[u, x]^p, x] /; FreeQ[{f, m, p, q}, x] && Binomi
alQ[z, x] && TrinomialQ[u, x] &&  !(BinomialMatchQ[z, x] && TrinomialMatchQ
[u, x])
```

rule 2334
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coef
f[Pq, x, 0], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[A*
x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))   Int[
x^(m + 2)*(a + b*x^2)^p*(a*(m + 1)*Q - A*b*(m + 2*(p + 1) + 1)), x], x]] /;
 FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && IntegerQ[m/2] && ILtQ[(m + 1)/2 + p,
 0] && LtQ[m + Expon[Pq, x] + 2*p + 1, 0]
```

## Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.67

| method | result |
|---|---|
| pseudoelliptic | $-\frac{(-5Bx^6+5Cx^4+\frac{5}{3}x^2B+A)a^6-4(\frac{5}{2}Bx^6-10Cx^4+\frac{25}{6}x^2B+A)x^2ba^5+40(-\frac{1}{5}Bx^6+2Cx^4-\frac{10}{3}x^2B+A)x^4b^2a^4+320(}{5(bx^2+a)^{\frac{7}{2}}}$ |
| gosper | $-\frac{3072Ab^6x^{12}-48Ba^3b^3x^{12}-1280Bab^5x^{12}+384Ca^2b^4x^{12}+10752Aab^5x^{10}-168Ba^4b^2x^{10}-4480Ba^2b^4x^{10}+1344Ca^3}{}$ |
| trager | $-\frac{3072Ab^6x^{12}-48Ba^3b^3x^{12}-1280Bab^5x^{12}+384Ca^2b^4x^{12}+10752Aab^5x^{10}-168Ba^4b^2x^{10}-4480Ba^2b^4x^{10}+1344Ca^3}{}$ |
| orering | $-\frac{3072Ab^6x^{12}-48Ba^3b^3x^{12}-1280Bab^5x^{12}+384Ca^2b^4x^{12}+10752Aab^5x^{10}-168Ba^4b^2x^{10}-4480Ba^2b^4x^{10}+1344Ca^3}{}$ |
| risch | $-\frac{\sqrt{bx^2+a}\,(198Ab^2x^4-70Babx^4+15Ca^2x^4-24aAbx^2+5Ba^2x^2+3a^2A)}{15a^7x^5}-\frac{\sqrt{bx^2+a}\,x(1686Ab^6x^6-48Ba^3b^3x^6-79}{}$ |
| default | $B\left(\frac{x}{7a(bx^2+a)^{\frac{7}{2}}}+\frac{\frac{6x}{35a(bx^2+a)^{\frac{5}{2}}}+\frac{6\left(\frac{4x}{15a(bx^2+a)^{\frac{3}{2}}}+\frac{8x}{15a^2\sqrt{bx^2+a}}\right)}{7a}}{a}\right)+A\left(-\frac{1}{5ax^5(bx^2+a)^{\frac{7}{2}}}-\frac{12b\left(-\frac{}{3ax^3}\right.}{}\right.$ |

input
```
int((B*x^6+C*x^4+B*x^2+A)/x^6/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5*((-5*B*x^6+5*C*x^4+5/3*x^2*B+A)*a^6-4*(5/2*B*x^6-10*C*x^4+25/6*x^2*B+
A)*x^2*b*a^5+40*(-1/5*B*x^6+2*C*x^4-10/3*x^2*B+A)*x^4*b^2*a^4+320*(-1/140*
B*x^6+1/5*C*x^4-5/6*x^2*B+A)*x^6*b^3*a^3+640*(1/35*C*x^4-1/3*x^2*B+A)*x^8*
b^4*a^2+512*x^10*b^5*(-5/42*x^2*B+A)*a+1024/7*A*b^6*x^12)/(b*x^2+a)^(7/2)/
x^5/a^7
```

## Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 270, normalized size of antiderivative = 0.97

$$\int \frac{A+Bx^2+Cx^4+Bx^6}{x^6\left(a+bx^2\right)^{9/2}}\,dx = \frac{(16\,(3\,Ba^3b^3-24\,Ca^2b^4+80\,Bab^5-192\,Ab^6)x^{12}+56\,(3\,Ba^4b^2-24\,Ca^3b$$

input
```
integrate((B*x^6+C*x^4+B*x^2+A)/x^6/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

output
```
1/105*(16*(3*B*a^3*b^3 - 24*C*a^2*b^4 + 80*B*a*b^5 - 192*A*b^6)*x^12 + 56*
(3*B*a^4*b^2 - 24*C*a^3*b^3 + 80*B*a^2*b^4 - 192*A*a*b^5)*x^10 + 70*(3*B*a
^5*b - 24*C*a^4*b^2 + 80*B*a^3*b^3 - 192*A*a^2*b^4)*x^8 - 21*A*a^6 + 35*(3
*B*a^6 - 24*C*a^5*b + 80*B*a^4*b^2 - 192*A*a^3*b^3)*x^6 - 35*(3*C*a^6 - 10
*B*a^5*b + 24*A*a^4*b^2)*x^4 - 7*(5*B*a^6 - 12*A*a^5*b)*x^2)*sqrt(b*x^2 +
a)/(a^7*b^4*x^13 + 4*a^8*b^3*x^11 + 6*a^9*b^2*x^9 + 4*a^10*b*x^7 + a^11*x^
5)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4+Bx^6}{x^6\left(a+bx^2\right)^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate((B*x**6+C*x**4+B*x**2+A)/x**6/(b*x**2+a)**(9/2),x)
```

output
```
Timed out
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.44

$$\int \frac{A+Bx^2+Cx^4+Bx^6}{x^6(a+bx^2)^{9/2}}\,dx = \frac{16\,Bx}{35\,\sqrt{bx^2+a}a^4} + \frac{8\,Bx}{35\,(bx^2+a)^{\frac{3}{2}}a^3} + \frac{6\,Bx}{35\,(bx^2+a)^{\frac{5}{2}}a^2} + \frac{Bx}{7\,(bx^2+a)^{\frac{7}{2}}a} - \frac{128\,Cbx}{35\,\sqrt{bx^2+a}a^5} - \frac{64\,Cbx}{35\,(bx^2+a)^{\frac{3}{2}}a^4} - \frac{48\,Cbx}{35\,(bx^2+a)^{\frac{5}{2}}a^3} - \frac{8\,Cbx}{7\,(bx^2+a)^{\frac{7}{2}}a^2} + \frac{256\,Bb^2x}{21\,\sqrt{bx^2+a}a^6} + \frac{128\,Bb^2x}{21\,(bx^2+a)^{\frac{3}{2}}a^5} + \frac{32\,Bb^2x}{7\,(bx^2+a)^{\frac{5}{2}}a^4} + \frac{80\,Bb^2x}{21\,(bx^2+a)^{\frac{7}{2}}a^3} - \frac{1024\,Ab^3x}{35\,\sqrt{bx^2+a}a^7} - \frac{512\,Ab^3x}{35\,(bx^2+a)^{\frac{3}{2}}a^6} - \frac{384\,Ab^3x}{35\,(bx^2+a)^{\frac{5}{2}}a^5} - \frac{64\,Ab^3x}{7\,(bx^2+a)^{\frac{7}{2}}a^4} - \frac{C}{(bx^2+a)^{\frac{7}{2}}ax} + \frac{10\,Bb}{3\,(bx^2+a)^{\frac{7}{2}}a^2x} - \frac{8\,Ab^2}{(bx^2+a)^{\frac{7}{2}}a^3x} - \frac{B}{3\,(bx^2+a)^{\frac{7}{2}}ax^3} + \frac{4\,Ab}{5\,(bx^2+a)^{\frac{7}{2}}a^2x^3} - \frac{A}{5\,(bx^2+a)^{\frac{7}{2}}ax^5}$$

input
```
integrate((B*x^6+C*x^4+B*x^2+A)/x^6/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

output
```
16/35*B*x/(sqrt(b*x^2 + a)*a^4) + 8/35*B*x/((b*x^2 + a)^(3/2)*a^3) + 6/35*
B*x/((b*x^2 + a)^(5/2)*a^2) + 1/7*B*x/((b*x^2 + a)^(7/2)*a) - 128/35*C*b*x
/(sqrt(b*x^2 + a)*a^5) - 64/35*C*b*x/((b*x^2 + a)^(3/2)*a^4) - 48/35*C*b*x
/((b*x^2 + a)^(5/2)*a^3) - 8/7*C*b*x/((b*x^2 + a)^(7/2)*a^2) + 256/21*B*b^
2*x/(sqrt(b*x^2 + a)*a^6) + 128/21*B*b^2*x/((b*x^2 + a)^(3/2)*a^5) + 32/7*
B*b^2*x/((b*x^2 + a)^(5/2)*a^4) + 80/21*B*b^2*x/((b*x^2 + a)^(7/2)*a^3) -
1024/35*A*b^3*x/(sqrt(b*x^2 + a)*a^7) - 512/35*A*b^3*x/((b*x^2 + a)^(3/2)*
a^6) - 384/35*A*b^3*x/((b*x^2 + a)^(5/2)*a^5) - 64/7*A*b^3*x/((b*x^2 + a)^
(7/2)*a^4) - C/((b*x^2 + a)^(7/2)*a*x) + 10/3*B*b/((b*x^2 + a)^(7/2)*a^2*x
) - 8*A*b^2/((b*x^2 + a)^(7/2)*a^3*x) - 1/3*B/((b*x^2 + a)^(7/2)*a*x^3) +
4/5*A*b/((b*x^2 + a)^(7/2)*a^2*x^3) - 1/5*A/((b*x^2 + a)^(7/2)*a*x^5)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 592 vs. $2(249) = 498$.

Time = 0.14 (sec) , antiderivative size = 592, normalized size of antiderivative = 2.14

$$\int \frac{A+Bx^2+Cx^4+Bx^6}{x^6\left(a+bx^2\right)^{9/2}}\,dx = \frac{\left(\left(x^2\left(\frac{(48\,Ba^{18}b^6-279\,Ca^{17}b^7+790\,Ba^{16}b^8-1686\,Aa^{15}b^9)x^2}{a^{22}b^3}+\frac{7\,(24\,Ba^{19}b^5-132\,Ca^{18}b^6+365\,\cdots}{a^{22}b^3}\right.\right.\right.}{} + \frac{2\left(15\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^8Ca^2\sqrt{b}-60\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^8Bab^{\frac{3}{2}}+150\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^8Ab^{\frac{5}{2}}-60\left(\cdots\right.\right.}{}$$

input
```
integrate((B*x^6+C*x^4+B*x^2+A)/x^6/(b*x^2+a)^(9/2),x, algorithm="giac")
```

output
```
1/105*((x^2*((48*B*a^18*b^6 - 279*C*a^17*b^7 + 790*B*a^16*b^8 - 1686*A*a^1
5*b^9)*x^2/(a^22*b^3) + 7*(24*B*a^19*b^5 - 132*C*a^18*b^6 + 365*B*a^17*b^7
 - 768*A*a^16*b^8)/(a^22*b^3)) + 35*(6*B*a^20*b^4 - 30*C*a^19*b^5 + 80*B*a
^18*b^6 - 165*A*a^17*b^7)/(a^22*b^3))*x^2 + 105*(B*a^21*b^3 - 4*C*a^20*b^4
 + 10*B*a^19*b^5 - 20*A*a^18*b^6)/(a^22*b^3))*x/(b*x^2 + a)^(7/2) + 2/15*(
15*(sqrt(b)*x - sqrt(b*x^2 + a))^8*C*a^2*sqrt(b) - 60*(sqrt(b)*x - sqrt(b*
x^2 + a))^8*B*a*b^(3/2) + 150*(sqrt(b)*x - sqrt(b*x^2 + a))^8*A*b^(5/2) -
60*(sqrt(b)*x - sqrt(b*x^2 + a))^6*C*a^3*sqrt(b) + 270*(sqrt(b)*x - sqrt(b
*x^2 + a))^6*B*a^2*b^(3/2) - 720*(sqrt(b)*x - sqrt(b*x^2 + a))^6*A*a*b^(5/
2) + 90*(sqrt(b)*x - sqrt(b*x^2 + a))^4*C*a^4*sqrt(b) - 430*(sqrt(b)*x - s
qrt(b*x^2 + a))^4*B*a^3*b^(3/2) + 1260*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*a
^2*b^(5/2) - 60*(sqrt(b)*x - sqrt(b*x^2 + a))^2*C*a^5*sqrt(b) + 290*(sqrt(
b)*x - sqrt(b*x^2 + a))^2*B*a^4*b^(3/2) - 840*(sqrt(b)*x - sqrt(b*x^2 + a)
)^2*A*a^3*b^(5/2) + 15*C*a^6*sqrt(b) - 70*B*a^5*b^(3/2) + 198*A*a^4*b^(5/2
))/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^5*a^6)
```

## Mupad [B] (verification not implemented)

Time = 1.68 (sec) , antiderivative size = 541, normalized size of antiderivative = 1.95

$$\int \frac{A+Bx^2+Cx^4+Bx^6}{x^6\left(a+bx^2\right)^{9/2}}\,dx = \frac{\frac{6\,B\,a^4-48\,C\,a^3\,b+160\,B\,a^2\,b^2-377\,A\,a\,b^3}{70\,a^6\,b}-\frac{a\,\left(\frac{6\,B\,a^4-48\,C\,a^3\,b+160\,B\,a^2\,b^2-377\,A\,a\,b^3}{42\,a^7}-\frac{4\,A\,b^3}{15\,a^6}\right)}{b}}{x\,{\left(b\,x^2+a\right)}^{3/2}}$$
$$-\frac{\frac{-24\,B\,a^3+192\,C\,a^2\,b-640\,B\,a\,b^2+1536\,A\,b^3}{105\,a^6\,b}+\frac{2\,x^2\,\left(-24\,B\,a^3+192\,C\,a^2\,b-640\,B\,a\,b^2+1536\,A\,b^3\right)}{105\,a^7}}{x\,\sqrt{b\,x^2+a}}$$
$$+\frac{x^2\,\left(\frac{189\,A\,a^2\,b^4-35\,B\,a^5\,b}{105\,a^6\,b^2}+\frac{a\,\left(\frac{13\,A\,b^3}{25\,a^5}-\frac{8\,\left(189\,A\,a^2\,b^4-35\,B\,a^5\,b\right)}{525\,a^7\,b}\right)}{b}-\frac{8\,\left(15\,B\,a^6-15\,C\,a^5\,b+50\,B\,a^4\,b^2-183\,A\,a^3\,b^3\right)}{525\,a^7\,b}\right)-\frac{15\,B\,a^6-15\,C}{}}{x^3\,{\left(b\,x^2+a\right)}^{5/2}}$$
$$-\frac{\frac{5\,B\,a^5-24\,A\,a^4\,b}{20\,a^6}-\frac{a\,\left(\frac{5\,C\,a^5-46\,A\,a^3\,b^2}{20\,a^6}+\frac{11\,b\,\left(5\,B\,a^5-24\,A\,a^4\,b\right)}{140\,a^7}-\frac{a\,\left(\frac{5\,B\,a^5-44\,A\,a^2\,b^3}{20\,a^6}+\frac{11\,b\,\left(5\,C\,a^5-46\,A\,a^3\,b^2\right)}{140\,a^7}-\frac{a\,\left(\frac{17\,A\,b^4}{35\,a^5}+\frac{11\,b\,\left(5\,B\,a^5-44\,A\,a\right.}{140\,a^7}\right.}{b}\right.}{b}\right)}{b}}{x^3\,{\left(b\,x^2+a\right)}^{7/2}}$$
$$-\frac{A\,\sqrt{b\,x^2+a}}{5\,a^5\,x^5}$$

input
```
int((A + B*x^2 + B*x^6 + C*x^4)/(x^6*(a + b*x^2)^(9/2)),x)
```

output
```
((6*B*a^4 + 160*B*a^2*b^2 - 377*A*a*b^3 - 48*C*a^3*b)/(70*a^6*b) - (a*((6*
B*a^4 + 160*B*a^2*b^2 - 377*A*a*b^3 - 48*C*a^3*b)/(42*a^7) - (4*A*b^3)/(15
*a^6)))/b)/(x*(a + b*x^2)^(3/2)) - ((1536*A*b^3 - 24*B*a^3 - 640*B*a*b^2 +
 192*C*a^2*b)/(105*a^6*b) + (2*x^2*(1536*A*b^3 - 24*B*a^3 - 640*B*a*b^2 +
192*C*a^2*b))/(105*a^7))/(x*(a + b*x^2)^(1/2)) + (x^2*((189*A*a^2*b^4 - 35
*B*a^5*b)/(105*a^6*b^2) + (a*((13*A*b^3)/(25*a^5) - (8*(189*A*a^2*b^4 - 35
*B*a^5*b))/(525*a^7*b)))/b - (8*(15*B*a^6 - 183*A*a^3*b^3 + 50*B*a^4*b^2 -
 15*C*a^5*b))/(525*a^7*b)) - (15*B*a^6 - 183*A*a^3*b^3 + 50*B*a^4*b^2 - 15
*C*a^5*b)/(105*a^6*b^2))/(x^3*(a + b*x^2)^(5/2)) - ((5*B*a^5 - 24*A*a^4*b)
/(20*a^6) - (a*((5*C*a^5 - 46*A*a^3*b^2)/(20*a^6) + (11*b*(5*B*a^5 - 24*A*
a^4*b))/(140*a^7) - (a*((5*B*a^5 - 44*A*a^2*b^3)/(20*a^6) + (11*b*(5*C*a^5
 - 46*A*a^3*b^2))/(140*a^7) - (a*((17*A*b^4)/(35*a^5) + (11*b*(5*B*a^5 - 4
4*A*a^2*b^3))/(140*a^7)))/b))/b))/b)/(x^3*(a + b*x^2)^(7/2)) - (A*(a + b*x
^2)^(1/2))/(5*a^5*x^5)
```

## Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 521, normalized size of antiderivative = 1.88

$$\int \frac{A+Bx^2+Cx^4+Bx^6}{x^6 (a+bx^2)^{9/2}}\,dx = \frac{-1792\sqrt{b\,x^2+a}\,b^6x^{12}+1792\sqrt{b}\,b^6x^{13}+49\sqrt{b\,x^2+a}\,a^5b\,x^2-105\sqrt{b\,x^2+}$$

input
```
int((B*x^6+C*x^4+B*x^2+A)/x^6/(b*x^2+a)^(9/2),x)
```

output
```
( - 21*sqrt(a + b*x**2)*a**6 + 105*sqrt(a + b*x**2)*a**5*b*x**6 + 49*sqrt(
a + b*x**2)*a**5*b*x**2 - 105*sqrt(a + b*x**2)*a**5*c*x**4 + 210*sqrt(a +
b*x**2)*a**4*b**2*x**8 - 490*sqrt(a + b*x**2)*a**4*b**2*x**4 - 840*sqrt(a
+ b*x**2)*a**4*b*c*x**6 + 168*sqrt(a + b*x**2)*a**3*b**3*x**10 - 3920*sqrt
(a + b*x**2)*a**3*b**3*x**6 - 1680*sqrt(a + b*x**2)*a**3*b**2*c*x**8 + 48*
sqrt(a + b*x**2)*a**2*b**4*x**12 - 7840*sqrt(a + b*x**2)*a**2*b**4*x**8 -
1344*sqrt(a + b*x**2)*a**2*b**3*c*x**10 - 6272*sqrt(a + b*x**2)*a*b**5*x**
10 - 384*sqrt(a + b*x**2)*a*b**4*c*x**12 - 1792*sqrt(a + b*x**2)*b**6*x**1
2 - 48*sqrt(b)*a**6*x**5 - 192*sqrt(b)*a**5*b*x**7 + 384*sqrt(b)*a**5*c*x*
*5 - 288*sqrt(b)*a**4*b**2*x**9 + 1792*sqrt(b)*a**4*b**2*x**5 + 1536*sqrt(
b)*a**4*b*c*x**7 - 192*sqrt(b)*a**3*b**3*x**11 + 7168*sqrt(b)*a**3*b**3*x*
*7 + 2304*sqrt(b)*a**3*b**2*c*x**9 - 48*sqrt(b)*a**2*b**4*x**13 + 10752*sq
rt(b)*a**2*b**4*x**9 + 1536*sqrt(b)*a**2*b**3*c*x**11 + 7168*sqrt(b)*a*b**
5*x**11 + 384*sqrt(b)*a*b**4*c*x**13 + 1792*sqrt(b)*b**6*x**13)/(105*a**6*
x**5*(a**4 + 4*a**3*b*x**2 + 6*a**2*b**2*x**4 + 4*a*b**3*x**6 + b**4*x**8)
)
```

# 3.247 $\int \frac{A+Bx^2+Cx^4+Bx^6}{x^8(a+bx^2)^{9/2}}\,dx$



## Optimal result

Integrand size = 32, antiderivative size = 330

$$\int \frac{A+Bx^2+Cx^4+Bx^6}{x^8(a+bx^2)^{9/2}}\,dx = \frac{b(Ab^3-a(a^2B+b^2B-abC))\,x}{7a^5(a+bx^2)^{7/2}} + \frac{b(34Ab^3-a(13a^2B+27b^2B-20abC))\,x}{35a^6(a+bx^2)^{5/2}} + \frac{b(486Ab^3-a(87a^2B+318b^2B-185abC))\,x}{105a^7(a+bx^2)^{3/2}} + \frac{b(3072Ab^3-a(279a^2B+1686b^2B-790abC))\,x}{105a^8\sqrt{a+bx^2}} - \frac{A\sqrt{a+bx^2}}{7a^5x^7} + \frac{(34Ab-7aB)\sqrt{a+bx^2}}{35a^6x^5} - \frac{(486Ab^2-168abB+35a^2C)\sqrt{a+bx^2}}{105a^7x^3} + \frac{(3072Ab^3-7a(15a^2B+198b^2B-70abC))\sqrt{a+bx^2}}{105a^8x}$$

output

```
1/7*b*(A*b^3-a*(B*a^2+B*b^2-C*a*b))*x/a^5/(b*x^2+a)^(7/2)+1/35*b*(34*A*b^3
-a*(13*B*a^2+27*B*b^2-20*C*a*b))*x/a^6/(b*x^2+a)^(5/2)+1/105*b*(486*A*b^3-
a*(87*B*a^2+318*B*b^2-185*C*a*b))*x/a^7/(b*x^2+a)^(3/2)+1/105*b*(3072*A*b^
3-a*(279*B*a^2+1686*B*b^2-790*C*a*b))*x/a^8/(b*x^2+a)^(1/2)-1/7*A*(b*x^2+a
)^(1/2)/a^5/x^7+1/35*(34*A*b-7*B*a)*(b*x^2+a)^(1/2)/a^6/x^5-1/105*(486*A*b
^2-168*B*a*b+35*C*a^2)*(b*x^2+a)^(1/2)/a^7/x^3+1/105*(3072*A*b^3-7*a*(15*B
*a^2+198*B*b^2-70*C*a*b))*(b*x^2+a)^(1/2)/a^8/x
```

## Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.70

$$\int \frac{A+Bx^2+Cx^4+Bx^6}{x^8\left(a+bx^2\right)^{9/2}}\,dx = \frac{6144Ab^7x^{14}-3072ab^6x^{12}(-7A+Bx^2)+256a^2b^5x^{10}(105A-42Bx^2+5C}{}$$

input
```
Integrate[(A + B*x^2 + C*x^4 + B*x^6)/(x^8*(a + b*x^2)^(9/2)),x]
```

output
```
(6144*A*b^7*x^14 - 3072*a*b^6*x^12*(-7*A + B*x^2) + 256*a^2*b^5*x^10*(105*
A - 42*B*x^2 + 5*C*x^4) + 14*a^6*b*x^2*(3*A + 6*B*x^2 + 25*C*x^4 - 60*B*x^
6) + 112*a^4*b^3*x^6*(15*A + 50*C*x^4 - 12*B*x^2*(5 + x^4)) + 128*a^3*b^4*
x^8*(105*A + 35*C*x^4 - 3*B*x^2*(35 + x^4)) - 56*a^5*b^2*x^4*(3*A - 50*C*x
^4 + 15*B*(x^2 + 2*x^6)) - a^7*(15*A + 35*C*x^4 + 21*B*(x^2 + 5*x^6)))/(10
5*a^8*x^7*(a + b*x^2)^(7/2))
```

## Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.79, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {2334, 27, 2089, 1588, 359, 245, 209, 209, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^6+Bx^2+Cx^4}{x^8\left(a+bx^2\right)^{9/2}}\,dx$$

$$\Big\downarrow \text{2334}$$

$$-\frac{\int \frac{7\left(2Ab-a\left(Bx^4+Cx^2+B\right)\right)}{x^6\left(bx^2+a\right)^{9/2}}dx}{7a}-\frac{A}{7ax^7\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{2Ab-a\left(Bx^4+Cx^2+B\right)}{x^6\left(bx^2+a\right)^{9/2}}dx}{a}-\frac{A}{7ax^7\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \ 2089$$

$$-\frac{\int \frac{-aBx^4-aCx^2+2Ab-aB}{x^6(bx^2+a)^{9/2}}dx}{a}-\frac{A}{7ax^7(a+bx^2)^{7/2}}$$

$$\Big\downarrow \ 1588$$

$$-\frac{-\frac{\int \frac{24Ab^2+5a^2Bx^2-a(12bB-5aC)}{x^4(bx^2+a)^{9/2}}dx}{5a}-\frac{2Ab-aB}{5ax^5(a+bx^2)^{7/2}}}{a}-\frac{A}{7ax^7(a+bx^2)^{7/2}}$$

$$\Big\downarrow \ 359$$

$$-\frac{-\frac{-\frac{5(48Ab^3-a(3a^2B-10abC+24b^2B))\int \frac{1}{x^2(bx^2+a)^{9/2}}dx}{3a}-\frac{24Ab^2-a(12bB-5aC)}{3ax^3(a+bx^2)^{7/2}}}{5a}-\frac{2Ab-aB}{5ax^5(a+bx^2)^{7/2}}}{a}-\frac{A}{7ax^7(a+bx^2)^{7/2}}$$

$$\Big\downarrow \ 245$$

$$-\frac{-\frac{-\frac{5(48Ab^3-a(3a^2B-10abC+24b^2B))\left(-\frac{8b\int \frac{1}{(bx^2+a)^{9/2}}dx}{a}-\frac{1}{ax(a+bx^2)^{7/2}}\right)}{3a}-\frac{24Ab^2-a(12bB-5aC)}{3ax^3(a+bx^2)^{7/2}}}{5a}-\frac{2Ab-aB}{5ax^5(a+bx^2)^{7/2}}}{a}-\frac{A}{7ax^7(a+bx^2)^{7/2}}$$

$$\Big\downarrow \ 209$$

$$-\frac{-\frac{-\frac{5(48Ab^3-a(3a^2B-10abC+24b^2B))\left(-\frac{8b\left(\frac{6\int \frac{1}{(bx^2+a)^{7/2}}dx}{7a}+\frac{x}{7a(a+bx^2)^{7/2}}\right)}{a}-\frac{1}{ax(a+bx^2)^{7/2}}\right)}{3a}-\frac{24Ab^2-a(12bB-5aC)}{3ax^3(a+bx^2)^{7/2}}}{5a}-\frac{2Ab-aB}{5ax^5(a+bx^2)^{7/2}}}{a}-\frac{A}{7ax^7(a+bx^2)^{7/2}}$$

$$\Big\downarrow \ 209$$

$$-\frac{-\frac{-\frac{5\left(48Ab^3-a\left(3a^2B-10abC+24b^2B\right)\right)\left(-\frac{8b\left(\frac{6\left(\frac{4\int\frac{1}{\left(bx^2+a\right)^{5/2}}dx}{5a}+\frac{x}{5a\left(a+bx^2\right)^{5/2}}\right)}{7a}+\frac{x}{7a\left(a+bx^2\right)^{7/2}}\right)}{a}-\frac{1}{ax\left(a+bx^2\right)^{7/2}}\right)}{3a}-\frac{24Ab^2-a(12bB-5aC)}{3ax^3\left(a+bx^2\right)^{7/2}}}{5a}}{a}$$

$$\frac{A}{7ax^7\left(a+bx^2\right)^{7/2}}$$

$\downarrow$ 209

$$-\frac{-\frac{-\frac{5\left(48Ab^3-a\left(3a^2B-10abC+24b^2B\right)\right)\left(-\frac{8b\left(\frac{6\left(\frac{4\left(\frac{2\int\frac{1}{\left(bx^2+a\right)^{3/2}}dx}{3a}+\frac{x}{3a\left(a+bx^2\right)^{3/2}}\right)}{5a}+\frac{x}{5a\left(a+bx^2\right)^{5/2}}\right)}{7a}+\frac{x}{7a\left(a+bx^2\right)^{7/2}}\right)}{a}-\frac{1}{ax\left(a+bx^2\right)^{7/2}}\right)}{3a}}{5a}}{a}$$

$$\frac{A}{7ax^7\left(a+bx^2\right)^{7/2}}$$

$\downarrow$ 208

$$-\frac{-\frac{5\left(-\frac{8b\left(\frac{6\left(\frac{4\left(\frac{2x}{3a^2\sqrt{a+bx^2}}+\frac{x}{3a\left(a+bx^2\right)^{3/2}}\right)}{5a}+\frac{x}{5a\left(a+bx^2\right)^{5/2}}\right)}{7a}+\frac{x}{7a\left(a+bx^2\right)^{7/2}}\right)}{a}-\frac{1}{ax\left(a+bx^2\right)^{7/2}}\right)\left(48Ab^3-a\left(3a^2B-10abC+24b^2B\right)\right)}{3a}-\frac{24Ab^2}{3ax}}{5a}}{a}$$

$$\frac{A}{7ax^7\left(a+bx^2\right)^{7/2}}$$

input
```
Int[(A + B*x^2 + C*x^4 + B*x^6)/(x^8*(a + b*x^2)^(9/2)),x]
```

output
```
-1/7*A/(a*x^7*(a + b*x^2)^(7/2)) - (-1/5*(2*A*b - a*B)/(a*x^5*(a + b*x^2)^
(7/2)) - (-1/3*(24*A*b^2 - a*(12*b*B - 5*a*C))/(a*x^3*(a + b*x^2)^(7/2)) -
 (5*(48*A*b^3 - a*(3*a^2*B + 24*b^2*B - 10*a*b*C))*(-(1/(a*x*(a + b*x^2)^(
7/2))) - (8*b*(x/(7*a*(a + b*x^2)^(7/2)) + (6*(x/(5*a*(a + b*x^2)^(5/2)) +
 (4*(x/(3*a*(a + b*x^2)^(3/2)) + (2*x)/(3*a^2*Sqrt[a + b*x^2])))/(5*a)))/(
7*a)))/a))/(3*a))/(5*a))/a
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 1588
```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Simp[1/(d*f
^2*(m + 1))   Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x
) - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

rule 2089
```
Int[(u_)^(p_.)*((f_.)*(x_))^(m_.)*(z_)^(q_.), x_Symbol] :> Int[(f*x)^m*Expa
ndToSum[z, x]^q*ExpandToSum[u, x]^p, x] /; FreeQ[{f, m, p, q}, x] && Binomi
alQ[z, x] && TrinomialQ[u, x] &&  !(BinomialMatchQ[z, x] && TrinomialMatchQ
[u, x])
```

rule 2334
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coef
f[Pq, x, 0], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[A*
x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))   Int[
x^(m + 2)*(a + b*x^2)^p*(a*(m + 1)*Q - A*b*(m + 2*(p + 1) + 1)), x], x]] /;
 FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && IntegerQ[m/2] && ILtQ[(m + 1)/2 + p,
 0] && LtQ[m + Expon[Pq, x] + 2*p + 1, 0]
```

## Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.66

| method | result |
|---|---|
| pseudoelliptic | $\frac{(-105B\,x^6-35C\,x^4-21x^2B-15A)a^7+42(-20B\,x^6+\frac{25}{3}C\,x^4+2x^2B+A)x^2b\,a^6-168(10B\,x^6-\frac{50}{3}C\,x^4+5x^2B+A)x^4b^2a^5}{}$ |
| gosper | $-\frac{-6144A\,b^7x^{14}+384B\,a^3b^4x^{14}+3072Ba\,b^6x^{14}-1280C\,a^2b^5x^{14}-21504Aa\,b^6x^{12}+1344B\,a^4b^3x^{12}+10752B\,a^2b^5x^{12}-44}{}$ |
| trager | $-\frac{-6144A\,b^7x^{14}+384B\,a^3b^4x^{14}+3072Ba\,b^6x^{14}-1280C\,a^2b^5x^{14}-21504Aa\,b^6x^{12}+1344B\,a^4b^3x^{12}+10752B\,a^2b^5x^{12}-44}{}$ |
| orering | $-\frac{-6144A\,b^7x^{14}+384B\,a^3b^4x^{14}+3072Ba\,b^6x^{14}-1280C\,a^2b^5x^{14}-21504Aa\,b^6x^{12}+1344B\,a^4b^3x^{12}+10752B\,a^2b^5x^{12}-44}{}$ |
| risch | $-\frac{\sqrt{b\,x^2+a}\,(-3072A\,b^3x^6+105B\,a^3x^6+1386Ba\,b^2x^6-490C\,a^2b\,x^6+486aA\,b^2x^4-168B\,a^2b\,x^4+35C\,a^3x^4-102a^2Ab\,x^2+}{105a^8x^7}$ |
| | $\left(2b\left(-\frac{1}{5a\,x^5\left(b\,x^2+a\right)^{\frac{7}{2}}}-\frac{12b\left(-\frac{1}{3a\,x^3\left(b\,x^2+a\right)^{\frac{7}{2}}}-\frac{10b\left(-\frac{1}{ax\left(b\,x^2+a\right)^{\frac{7}{2}}}-\frac{8b\left(\frac{x}{7a\left(b\,x^2+a\right)^{\frac{7}{2}}}+\frac{}{35a(}\right.}{}\right.}{}\right)}{5a}\right.\right.$ |

input
```
int((B*x^6+C*x^4+B*x^2+A)/x^8/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
1/105*((-105*B*x^6-35*C*x^4-21*B*x^2-15*A)*a^7+42*(-20*B*x^6+25/3*C*x^4+2*
x^2*B+A)*x^2*b*a^6-168*(10*B*x^6-50/3*C*x^4+5*x^2*B+A)*x^4*b^2*a^5+1680*(-
4/5*B*x^6+10/3*C*x^4-4*x^2*B+A)*x^6*b^3*a^4+13440*(-1/35*B*x^6+1/3*C*x^4-x
^2*B+A)*x^8*b^4*a^3+26880*(1/21*C*x^4-2/5*x^2*B+A)*x^10*b^5*a^2+21504*(-1/
7*x^2*B+A)*x^12*b^6*a+6144*A*b^7*x^14)/(b*x^2+a)^(7/2)/x^7/a^8
```

## Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 311, normalized size of antiderivative = 0.94

$$\int \frac{A + Bx^2 + Cx^4 + Bx^6}{x^8 (a + bx^2)^{9/2}} \, dx =$$
$$-\frac{(128\,(3\,Ba^3b^4 - 10\,Ca^2b^5 + 24\,Bab^6 - 48\,Ab^7)x^{14} + 448\,(3\,Ba^4b^3 - 10\,Ca^3b^4 + 24\,Ba^2b^5 - 48\,Aab^6)x^{12}}{}$$

input
```
integrate((B*x^6+C*x^4+B*x^2+A)/x^8/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

output
```
-1/105*(128*(3*B*a^3*b^4 - 10*C*a^2*b^5 + 24*B*a*b^6 - 48*A*b^7)*x^14 + 44
8*(3*B*a^4*b^3 - 10*C*a^3*b^4 + 24*B*a^2*b^5 - 48*A*a*b^6)*x^12 + 560*(3*B
*a^5*b^2 - 10*C*a^4*b^3 + 24*B*a^3*b^4 - 48*A*a^2*b^5)*x^10 + 280*(3*B*a^6
*b - 10*C*a^5*b^2 + 24*B*a^4*b^3 - 48*A*a^3*b^4)*x^8 + 15*A*a^7 + 35*(3*B*
a^7 - 10*C*a^6*b + 24*B*a^5*b^2 - 48*A*a^4*b^3)*x^6 + 7*(5*C*a^7 - 12*B*a^
6*b + 24*A*a^5*b^2)*x^4 + 21*(B*a^7 - 2*A*a^6*b)*x^2)*sqrt(b*x^2 + a)/(a^8
*b^4*x^15 + 4*a^9*b^3*x^13 + 6*a^10*b^2*x^11 + 4*a^11*b*x^9 + a^12*x^7)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{A + Bx^2 + Cx^4 + Bx^6}{x^8 (a + bx^2)^{9/2}} \, dx = \text{Timed out}$$

input
```
integrate((B*x**6+C*x**4+B*x**2+A)/x**8/(b*x**2+a)**(9/2),x)
```

output
```
Timed out
```

## Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 489, normalized size of antiderivative = 1.48

$$\int \frac{A+Bx^2+Cx^4+Bx^6}{x^8(a+bx^2)^{9/2}}\,dx = -\frac{128\,Bbx}{35\,\sqrt{bx^2+a}a^5} - \frac{64\,Bbx}{35\,(bx^2+a)^{\frac{3}{2}}a^4} - \frac{48\,Bbx}{35\,(bx^2+a)^{\frac{5}{2}}a^3} - \frac{8\,Bbx}{7\,(bx^2+a)^{\frac{7}{2}}a^2} + \frac{256\,Cb^2x}{21\,\sqrt{bx^2+a}a^6} + \frac{128\,Cb^2x}{21\,(bx^2+a)^{\frac{3}{2}}a^5} + \frac{32\,Cb^2x}{7\,(bx^2+a)^{\frac{5}{2}}a^4} + \frac{80\,Cb^2x}{21\,(bx^2+a)^{\frac{7}{2}}a^3} - \frac{1024\,Bb^3x}{35\,\sqrt{bx^2+a}a^7} - \frac{512\,Bb^3x}{35\,(bx^2+a)^{\frac{3}{2}}a^6} - \frac{384\,Bb^3x}{35\,(bx^2+a)^{\frac{5}{2}}a^5} - \frac{64\,Bb^3x}{7\,(bx^2+a)^{\frac{7}{2}}a^4} + \frac{2048\,Ab^4x}{35\,\sqrt{bx^2+a}a^8} + \frac{1024\,Ab^4x}{35\,(bx^2+a)^{\frac{3}{2}}a^7} + \frac{768\,Ab^4x}{35\,(bx^2+a)^{\frac{5}{2}}a^6} + \frac{128\,Ab^4x}{7\,(bx^2+a)^{\frac{7}{2}}a^5} - \frac{B}{(bx^2+a)^{\frac{7}{2}}ax} + \frac{10\,Cb}{3\,(bx^2+a)^{\frac{7}{2}}a^2x} - \frac{8\,Bb^2}{(bx^2+a)^{\frac{7}{2}}a^3x} + \frac{16\,Ab^3}{(bx^2+a)^{\frac{7}{2}}a^4x} - \frac{C}{3\,(bx^2+a)^{\frac{7}{2}}ax^3} + \frac{4\,Bb}{5\,(bx^2+a)^{\frac{7}{2}}a^2x^3} - \frac{8\,Ab^2}{5\,(bx^2+a)^{\frac{7}{2}}a^3x^3} - \frac{B}{5\,(bx^2+a)^{\frac{7}{2}}ax^5} + \frac{2\,Ab}{5\,(bx^2+a)^{\frac{7}{2}}a^2x^5} - \frac{A}{7\,(bx^2+a)^{\frac{7}{2}}ax^7}$$

input
```
integrate((B*x^6+C*x^4+B*x^2+A)/x^8/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

output
```
-128/35*B*b*x/(sqrt(b*x^2 + a)*a^5) - 64/35*B*b*x/((b*x^2 + a)^(3/2)*a^4)
- 48/35*B*b*x/((b*x^2 + a)^(5/2)*a^3) - 8/7*B*b*x/((b*x^2 + a)^(7/2)*a^2)
+ 256/21*C*b^2*x/(sqrt(b*x^2 + a)*a^6) + 128/21*C*b^2*x/((b*x^2 + a)^(3/2)
*a^5) + 32/7*C*b^2*x/((b*x^2 + a)^(5/2)*a^4) + 80/21*C*b^2*x/((b*x^2 + a)^
(7/2)*a^3) - 1024/35*B*b^3*x/(sqrt(b*x^2 + a)*a^7) - 512/35*B*b^3*x/((b*x^
2 + a)^(3/2)*a^6) - 384/35*B*b^3*x/((b*x^2 + a)^(5/2)*a^5) - 64/7*B*b^3*x/
((b*x^2 + a)^(7/2)*a^4) + 2048/35*A*b^4*x/(sqrt(b*x^2 + a)*a^8) + 1024/35*
A*b^4*x/((b*x^2 + a)^(3/2)*a^7) + 768/35*A*b^4*x/((b*x^2 + a)^(5/2)*a^6) +
 128/7*A*b^4*x/((b*x^2 + a)^(7/2)*a^5) - B/((b*x^2 + a)^(7/2)*a*x) + 10/3*
C*b/((b*x^2 + a)^(7/2)*a^2*x) - 8*B*b^2/((b*x^2 + a)^(7/2)*a^3*x) + 16*A*b
^3/((b*x^2 + a)^(7/2)*a^4*x) - 1/3*C/((b*x^2 + a)^(7/2)*a*x^3) + 4/5*B*b/(
(b*x^2 + a)^(7/2)*a^2*x^3) - 8/5*A*b^2/((b*x^2 + a)^(7/2)*a^3*x^3) - 1/5*B
/((b*x^2 + a)^(7/2)*a*x^5) + 2/5*A*b/((b*x^2 + a)^(7/2)*a^2*x^5) - 1/7*A/(
(b*x^2 + a)^(7/2)*a*x^7)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 938 vs. $2(298) = 596$.

Time = 0.15 (sec) , antiderivative size = 938, normalized size of antiderivative = 2.84

$$\int \frac{A + Bx^2 + Cx^4 + Bx^6}{x^8 (a + bx^2)^{9/2}} \, dx = \text{Too large to display}$$

input
```
integrate((B*x^6+C*x^4+B*x^2+A)/x^8/(b*x^2+a)^(9/2),x, algorithm="giac")
```

output

```
-1/105*((x^2*((279*B*a^21*b^7 - 790*C*a^20*b^8 + 1686*B*a^19*b^9 - 3072*A*
a^18*b^10)*x^2/(a^26*b^3) + 7*(132*B*a^22*b^6 - 365*C*a^21*b^7 + 768*B*a^2
0*b^8 - 1386*A*a^19*b^9)/(a^26*b^3)) + 35*(30*B*a^23*b^5 - 80*C*a^22*b^6 +
 165*B*a^21*b^7 - 294*A*a^20*b^8)/(a^26*b^3))*x^2 + 105*(4*B*a^24*b^4 - 10
*C*a^23*b^5 + 20*B*a^22*b^6 - 35*A*a^21*b^7)/(a^26*b^3))*x/(b*x^2 + a)^(7/
2) + 2/105*(105*(sqrt(b)*x - sqrt(b*x^2 + a))^12*B*a^3*sqrt(b) - 420*(sqrt
(b)*x - sqrt(b*x^2 + a))^12*C*a^2*b^(3/2) + 1050*(sqrt(b)*x - sqrt(b*x^2 +
 a))^12*B*a*b^(5/2) - 2100*(sqrt(b)*x - sqrt(b*x^2 + a))^12*A*b^(7/2) - 63
0*(sqrt(b)*x - sqrt(b*x^2 + a))^10*B*a^4*sqrt(b) + 2730*(sqrt(b)*x - sqrt(
b*x^2 + a))^10*C*a^3*b^(3/2) - 7140*(sqrt(b)*x - sqrt(b*x^2 + a))^10*B*a^2
*b^(5/2) + 14700*(sqrt(b)*x - sqrt(b*x^2 + a))^10*A*a*b^(7/2) + 1575*(sqrt
(b)*x - sqrt(b*x^2 + a))^8*B*a^5*sqrt(b) - 7210*(sqrt(b)*x - sqrt(b*x^2 +
a))^8*C*a^4*b^(3/2) + 19950*(sqrt(b)*x - sqrt(b*x^2 + a))^8*B*a^3*b^(5/2)
- 42840*(sqrt(b)*x - sqrt(b*x^2 + a))^8*A*a^2*b^(7/2) - 2100*(sqrt(b)*x -
sqrt(b*x^2 + a))^6*B*a^6*sqrt(b) + 9940*(sqrt(b)*x - sqrt(b*x^2 + a))^6*C*
a^5*b^(3/2) - 28560*(sqrt(b)*x - sqrt(b*x^2 + a))^6*B*a^4*b^(5/2) + 64680*
(sqrt(b)*x - sqrt(b*x^2 + a))^6*A*a^3*b^(7/2) + 1575*(sqrt(b)*x - sqrt(b*x
^2 + a))^4*B*a^7*sqrt(b) - 7560*(sqrt(b)*x - sqrt(b*x^2 + a))^4*C*a^6*b^(3
/2) + 21966*(sqrt(b)*x - sqrt(b*x^2 + a))^4*B*a^5*b^(5/2) - 49812*(sqrt(b)
*x - sqrt(b*x^2 + a))^4*A*a^4*b^(7/2) - 630*(sqrt(b)*x - sqrt(b*x^2 + a...
```

## Mupad [B] (verification not implemented)

Time = 2.49 (sec) , antiderivative size = 522, normalized size of antiderivative = 1.58

$$\int \frac{A+Bx^2+Cx^4+Bx^6}{x^8\,(a+bx^2)^{9/2}}\,dx = \frac{16\,B}{35\,a^3\,x\,(b\,x^2+a)^{3/2}} - \frac{64\,B}{35\,a^4\,x\,\sqrt{b\,x^2+a}} - \frac{A\,\sqrt{b\,x^2+a}}{7\,a^5\,x^7} + \frac{8\,B}{35\,a^2\,x\,(b\,x^2+a)^{5/2}} - \frac{B\,\sqrt{b\,x^2+a}}{5\,a^5\,x^5} - \frac{10\,C}{21\,a^2\,x^3\,(b\,x^2+a)^{5/2}} - \frac{B}{7\,b\,x^3\,(b\,x^2+a)^{7/2}} + \frac{C}{7\,a\,x^3\,(b\,x^2+a)^{7/2}} + \frac{34\,A\,b\,\sqrt{b\,x^2+a}}{35\,a^6\,x^5} + \frac{2048\,A\,b^4\,x}{35\,a^8\,\sqrt{b\,x^2+a}} + \frac{61\,B\,b}{35\,a^3\,x^3\,(b\,x^2+a)^{5/2}} - \frac{1024\,B\,b^3\,x}{35\,a^7\,\sqrt{b\,x^2+a}} - \frac{B\,b}{7\,a^2\,x^3\,(b\,x^2+a)^{7/2}} + \frac{128\,C\,b}{21\,a^5\,x\,\sqrt{b\,x^2+a}} - \frac{32\,C\,b}{21\,a^4\,x\,(b\,x^2+a)^{3/2}} - \frac{16\,C\,b}{21\,a^3\,x\,(b\,x^2+a)^{5/2}} + \frac{256\,C\,b^2\,x}{21\,a^6\,\sqrt{b\,x^2+a}} + \frac{1024\,A\,b^3}{35\,a^7\,x\,\sqrt{b\,x^2+a}} - \frac{58\,A\,b^3}{7\,a^6\,x\,(b\,x^2+a)^{3/2}} - \frac{167\,A\,b^2}{35\,a^4\,x^3\,(b\,x^2+a)^{5/2}} - \frac{191\,A\,b^3}{35\,a^5\,x\,(b\,x^2+a)^{5/2}} + \frac{A\,b^2}{7\,a^3\,x^3\,(b\,x^2+a)^{7/2}} + \frac{B}{7\,a\,b\,x^3\,(b\,x^2+a)^{5/2}} - \frac{512\,B\,b^2}{35\,a^6\,x\,\sqrt{b\,x^2+a}} + \frac{27\,B\,b^2}{7\,a^5\,x\,(b\,x^2+a)^{3/2}} + \frac{78\,B\,b^2}{35\,a^4\,x\,(b\,x^2+a)^{5/2}} - \frac{128\,B\,b\,x}{35\,a^5\,\sqrt{b\,x^2+a}}$$

input
```
int((A + B*x^2 + B*x^6 + C*x^4)/(x^8*(a + b*x^2)^(9/2)),x)
```

output
```
(16*B)/(35*a^3*x*(a + b*x^2)^(3/2)) - (64*B)/(35*a^4*x*(a + b*x^2)^(1/2))
- (A*(a + b*x^2)^(1/2))/(7*a^5*x^7) + (8*B)/(35*a^2*x*(a + b*x^2)^(5/2)) -
 (B*(a + b*x^2)^(1/2))/(5*a^5*x^5) - (10*C)/(21*a^2*x^3*(a + b*x^2)^(5/2))
 - B/(7*b*x^3*(a + b*x^2)^(7/2)) + C/(7*a*x^3*(a + b*x^2)^(7/2)) + (34*A*b
*(a + b*x^2)^(1/2))/(35*a^6*x^5) + (2048*A*b^4*x)/(35*a^8*(a + b*x^2)^(1/2
)) + (61*B*b)/(35*a^3*x^3*(a + b*x^2)^(5/2)) - (1024*B*b^3*x)/(35*a^7*(a +
 b*x^2)^(1/2)) - (B*b)/(7*a^2*x^3*(a + b*x^2)^(7/2)) + (128*C*b)/(21*a^5*x
*(a + b*x^2)^(1/2)) - (32*C*b)/(21*a^4*x*(a + b*x^2)^(3/2)) - (16*C*b)/(21
*a^3*x*(a + b*x^2)^(5/2)) + (256*C*b^2*x)/(21*a^6*(a + b*x^2)^(1/2)) + (10
24*A*b^3)/(35*a^7*x*(a + b*x^2)^(1/2)) - (58*A*b^3)/(7*a^6*x*(a + b*x^2)^(
3/2)) - (167*A*b^2)/(35*a^4*x^3*(a + b*x^2)^(5/2)) - (191*A*b^3)/(35*a^5*x
*(a + b*x^2)^(5/2)) + (A*b^2)/(7*a^3*x^3*(a + b*x^2)^(7/2)) + B/(7*a*b*x^3
*(a + b*x^2)^(5/2)) - (512*B*b^2)/(35*a^6*x*(a + b*x^2)^(1/2)) + (27*B*b^2
)/(7*a^5*x*(a + b*x^2)^(3/2)) + (78*B*b^2)/(35*a^4*x*(a + b*x^2)^(5/2)) -
(128*B*b*x)/(35*a^5*(a + b*x^2)^(1/2))
```

## Reduce [B] (verification not implemented)

Time = 5.34 (sec) , antiderivative size = 585, normalized size of antiderivative = 1.77

$$\int \frac{A+Bx^2+Cx^4+Bx^6}{x^8\left(a+bx^2\right)^{9/2}}\,dx = \frac{21\sqrt{b\,x^2+a}\,a^6 b\,x^2 - 35\sqrt{b\,x^2+a}\,a^6 c\,x^4 - 84\sqrt{b\,x^2+a}\,a^5 b^2 x^4 + 840\sqrt{b\,x^2}}{}$$

input
```
int((B*x^6+C*x^4+B*x^2+A)/x^8/(b*x^2+a)^(9/2),x)
```

output
```
( - 15*sqrt(a + b*x**2)*a**7 - 105*sqrt(a + b*x**2)*a**6*b*x**6 + 21*sqrt(
a + b*x**2)*a**6*b*x**2 - 35*sqrt(a + b*x**2)*a**6*c*x**4 - 840*sqrt(a + b
*x**2)*a**5*b**2*x**8 - 84*sqrt(a + b*x**2)*a**5*b**2*x**4 + 350*sqrt(a +
b*x**2)*a**5*b*c*x**6 - 1680*sqrt(a + b*x**2)*a**4*b**3*x**10 + 840*sqrt(a
 + b*x**2)*a**4*b**3*x**6 + 2800*sqrt(a + b*x**2)*a**4*b**2*c*x**8 - 1344*
sqrt(a + b*x**2)*a**3*b**4*x**12 + 6720*sqrt(a + b*x**2)*a**3*b**4*x**8 +
5600*sqrt(a + b*x**2)*a**3*b**3*c*x**10 - 384*sqrt(a + b*x**2)*a**2*b**5*x
**14 + 13440*sqrt(a + b*x**2)*a**2*b**5*x**10 + 4480*sqrt(a + b*x**2)*a**2
*b**4*c*x**12 + 10752*sqrt(a + b*x**2)*a*b**6*x**12 + 1280*sqrt(a + b*x**2
)*a*b**5*c*x**14 + 3072*sqrt(a + b*x**2)*b**7*x**14 + 384*sqrt(b)*a**6*b*x
**7 + 1536*sqrt(b)*a**5*b**2*x**9 - 1280*sqrt(b)*a**5*b*c*x**7 + 2304*sqrt
(b)*a**4*b**3*x**11 - 3072*sqrt(b)*a**4*b**3*x**7 - 5120*sqrt(b)*a**4*b**2
*c*x**9 + 1536*sqrt(b)*a**3*b**4*x**13 - 12288*sqrt(b)*a**3*b**4*x**9 - 76
80*sqrt(b)*a**3*b**3*c*x**11 + 384*sqrt(b)*a**2*b**5*x**15 - 18432*sqrt(b)
*a**2*b**5*x**11 - 5120*sqrt(b)*a**2*b**4*c*x**13 - 12288*sqrt(b)*a*b**6*x
**13 - 1280*sqrt(b)*a*b**5*c*x**15 - 3072*sqrt(b)*b**7*x**15)/(105*a**7*x*
*7*(a**4 + 4*a**3*b*x**2 + 6*a**2*b**2*x**4 + 4*a*b**3*x**6 + b**4*x**8))
```

# 3.248 $\int \frac{A+Bx^2+Cx^4+Bx^6}{x^{10}(a+bx^2)^{9/2}}\,dx$



## Optimal result

Integrand size = 32, antiderivative size = 388

$$\begin{aligned}
\int \frac{A+Bx^2+Cx^4+Bx^6}{x^{10}\left(a+bx^2\right)^{9/2}}\,dx &= -\frac{b^2(Ab^3-a(a^2B+b^2B-abC))\,x}{7a^6\left(a+bx^2\right)^{7/2}} \\
&\quad -\frac{b^2(41Ab^3-a(20a^2B+34b^2B-27abC))\,x}{35a^7\left(a+bx^2\right)^{5/2}} \\
&\quad -\frac{b^2(689Ab^3-a(185a^2B+486b^2B-318abC))\,x}{105a^8\left(a+bx^2\right)^{3/2}} \\
&\quad -\frac{b^2(5053Ab^3-2a(395a^2B+1536b^2B-843abC))\,x}{105a^9\sqrt{a+bx^2}}-\frac{A\sqrt{a+bx^2}}{9a^5x^9} \\
&\quad +\frac{(44Ab-9aB)\sqrt{a+bx^2}}{63a^6x^7}-\frac{(298Ab^2-102abB+21a^2C)\sqrt{a+bx^2}}{105a^7x^5} \\
&\quad +\frac{(3292Ab^3-3a(35a^2B+486b^2B-168abC))\sqrt{a+bx^2}}{315a^8x^3} \\
&\quad -\frac{b(17609Ab^3-6a(245a^2B+1536b^2B-693abC))\sqrt{a+bx^2}}{315a^9x}
\end{aligned}$$

output
```
-1/7*b^2*(A*b^3-a*(B*a^2+B*b^2-C*a*b))*x/a^6/(b*x^2+a)^(7/2)-1/35*b^2*(41*
A*b^3-a*(20*B*a^2+34*B*b^2-27*C*a*b))*x/a^7/(b*x^2+a)^(5/2)-1/105*b^2*(689
*A*b^3-a*(185*B*a^2+486*B*b^2-318*C*a*b))*x/a^8/(b*x^2+a)^(3/2)-1/105*b^2*
(5053*A*b^3-2*a*(395*B*a^2+1536*B*b^2-843*C*a*b))*x/a^9/(b*x^2+a)^(1/2)-1/
9*A*(b*x^2+a)^(1/2)/a^5/x^9+1/63*(44*A*b-9*B*a)*(b*x^2+a)^(1/2)/a^6/x^7-1/
105*(298*A*b^2-102*B*a*b+21*C*a^2)*(b*x^2+a)^(1/2)/a^7/x^5+1/315*(3292*A*b
^3-3*a*(35*B*a^2+486*B*b^2-168*C*a*b))*(b*x^2+a)^(1/2)/a^8/x^3-1/315*b*(17
609*A*b^3-6*a*(245*B*a^2+1536*B*b^2-693*C*a*b))*(b*x^2+a)^(1/2)/a^9/x
```

## Mathematica [A] (verified)

Time = 1.00 (sec) , antiderivative size = 269, normalized size of antiderivative = 0.69

$$\int \frac{A+Bx^2+Cx^4+Bx^6}{x^{10}\left(a+bx^2\right)^{9/2}}\,dx = \frac{-32768Ab^8x^{16}+2048ab^7x^{14}(-56A+9Bx^2)-1024a^2b^6x^{12}(140A-63Bx$$

input
```
Integrate[(A + B*x^2 + C*x^4 + B*x^6)/(x^10*(a + b*x^2)^(9/2)),x]
```

output
```
(-32768*A*b^8*x^16 + 2048*a*b^7*x^14*(-56*A + 9*B*x^2) - 1024*a^2*b^6*x^12
*(140*A - 63*B*x^2 + 9*C*x^4) - 56*a^6*b^2*x^4*(4*A + 9*B*x^2 + 45*C*x^4 -
 150*B*x^6) + 112*a^5*b^3*x^6*(8*A + 45*B*x^2 - 180*C*x^4 + 150*B*x^6) + 4
480*a^4*b^4*x^8*(-2*A - 9*C*x^4 + 3*B*x^2*(3 + x^4)) + 256*a^3*b^5*x^10*(-
280*A - 126*C*x^4 + 15*B*x^2*(21 + x^4)) - a^8*(35*A + 63*C*x^4 + 15*B*x^2
*(3 + 7*x^4)) + 2*a^7*b*x^2*(40*A + 21*(6*C*x^4 + B*x^2*(3 + 25*x^4))))/(3
15*a^9*x^9*(a + b*x^2)^(7/2))
```

## Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 289, normalized size of antiderivative = 0.74, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.344$, Rules used = {2334, 2089, 1588, 27, 359, 245, 245, 209, 209, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^6+Bx^2+Cx^4}{x^{10}\left(a+bx^2\right)^{9/2}}dx$$

$$\Big\downarrow \text{2334}$$

$$-\frac{\int \frac{16Ab-9a\left(Bx^4+Cx^2+B\right)}{x^8\left(bx^2+a\right)^{9/2}}dx}{9a}-\frac{A}{9ax^9\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \text{2089}$$

$$-\frac{\int \frac{-9aBx^4-9aCx^2+16Ab-9aB}{x^8\left(bx^2+a\right)^{9/2}}dx}{9a}-\frac{A}{9ax^9\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \text{1588}$$

$$-\frac{-\frac{\int \frac{7\left(32Ab^2+9a^2Bx^2-9a(2bB-aC)\right)}{x^6\left(bx^2+a\right)^{9/2}}dx}{7a}-\frac{16Ab-9aB}{7ax^7\left(a+bx^2\right)^{7/2}}}{9a}-\frac{A}{9ax^9\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{-\frac{\int \frac{32Ab^2+9a^2Bx^2-9a(2bB-aC)}{x^6\left(bx^2+a\right)^{9/2}}dx}{a}-\frac{16Ab-9aB}{7ax^7\left(a+bx^2\right)^{7/2}}}{9a}-\frac{A}{9ax^9\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \text{359}$$

$$-\frac{-\frac{-\frac{3\left(-15a^3B-36ab(2bB-aC)+128Ab^3\right)\int \frac{1}{x^4\left(bx^2+a\right)^{9/2}}dx}{5a}-\frac{32Ab^2-9a(2bB-aC)}{5ax^5\left(a+bx^2\right)^{7/2}}}{a}-\frac{16Ab-9aB}{7ax^7\left(a+bx^2\right)^{7/2}}}{9a}-\frac{A}{9ax^9\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \text{245}$$

$$-\frac{-\frac{-\frac{3\left(-15a^3B-36ab(2bB-aC)+128Ab^3\right)\left(-\frac{10b\int \frac{1}{x^2\left(bx^2+a\right)^{9/2}}dx}{3a}-\frac{1}{3ax^3\left(a+bx^2\right)^{7/2}}\right)}{5a}-\frac{32Ab^2-9a(2bB-aC)}{5ax^5\left(a+bx^2\right)^{7/2}}}{a}-\frac{16Ab-9aB}{7ax^7\left(a+bx^2\right)^{7/2}}}{9a}-\frac{A}{9ax^9\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \text{245}$$

$$-\frac{-\frac{-\frac{3\left(-15a^3B-36ab(2bB-aC)+128Ab^3\right)\left(-\frac{10b\left(-\frac{8b\int\frac{1}{\left(bx^2+a\right)^{9/2}}dx}{a}-\frac{1}{ax\left(a+bx^2\right)^{7/2}}\right)}{3a}-\frac{1}{3ax^3\left(a+bx^2\right)^{7/2}}\right)}{5a}-\frac{32Ab^2-9a(2bB-aC)}{5ax^5\left(a+bx^2\right)^{7/2}}}{a}-\frac{16Ab-9}{7ax^7(a+bx}}{9a}$$

$$\frac{A}{9ax^9\left(a+bx^2\right)^{7/2}}$$

$\downarrow$ 209

$$-\frac{-\frac{-\frac{3\left(-15a^3B-36ab(2bB-aC)+128Ab^3\right)\left(-\frac{10b\left(-\frac{8b\left(\frac{6\int\frac{1}{\left(bx^2+a\right)^{7/2}}dx}{7a}+\frac{x}{7a\left(a+bx^2\right)^{7/2}}\right)}{a}-\frac{1}{ax\left(a+bx^2\right)^{7/2}}\right)}{3a}-\frac{1}{3ax^3\left(a+bx^2\right)^{7/2}}\right)}{5a}-\frac{32Ab^2-9a(2bB}{5ax^5\left(a+bx^2\right.}}{a}}{9a}$$

$$\frac{A}{9ax^9\left(a+bx^2\right)^{7/2}}$$

$\downarrow$ 209

$$-\frac{-\frac{-\frac{3\left(-15a^3B-36ab(2bB-aC)+128Ab^3\right)\left(-\frac{10b\left(-\frac{8b\left(\frac{6\left(\frac{4\int\frac{1}{\left(bx^2+a\right)^{5/2}}dx}{5a}+\frac{x}{5a\left(a+bx^2\right)^{5/2}}\right)}{7a}+\frac{x}{7a\left(a+bx^2\right)^{7/2}}\right)}{a}-\frac{1}{ax\left(a+bx^2\right)^{7/2}}\right)}{3a}-\frac{1}{3ax^3\left(a+bx^2\right)}\right.}{5a}}{a}}{9a}$$

$$\frac{A}{9ax^9\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \; 209$$

$$-\frac{-\frac{-\frac{3\left(-15a^3B-36ab(2bB-aC)+128Ab^3\right)\left(-\frac{10b\left(-\frac{8b\left(\frac{6\left(\frac{4\left(\frac{2\int\frac{1}{\left(bx^2+a\right)^{3/2}}dx}{3a}+\frac{x}{3a\left(a+bx^2\right)^{3/2}}\right)}{5a}+\frac{x}{5a\left(a+bx^2\right)^{5/2}}\right)}{7a}+\frac{x}{7a\left(a+bx^2\right)^{7/2}}\right)}{a}-\frac{1}{ax\left(a+bx^2\right.}\cdots}{3a}\cdots}{5a}\cdots}{a}\cdots}{9a}$$

$$\frac{A}{9ax^9\left(a+bx^2\right)^{7/2}}$$

$\Big\downarrow$ 208

$$-\frac{-\frac{3\left(-\frac{10b\left(-\frac{8b\left(\frac{6\left(\frac{4\left(\frac{2x}{3a^2\sqrt{a+bx^2}}+\frac{x}{3a\left(a+bx^2\right)^{3/2}}\right)}{5a}+\frac{x}{5a\left(a+bx^2\right)^{5/2}}\right)}{7a}+\frac{x}{7a\left(a+bx^2\right)^{7/2}}\right)}{a}-\frac{1}{ax\left(a+bx^2\right)^{7/2}}\right)}{3a}-\frac{1}{3ax^3\left(a+bx^2\right)^{7/2}}\right)\left(-15a^3B-36a}{5a}}{a}}{9a}$$

$$\frac{A}{9ax^9\left(a+bx^2\right)^{7/2}}$$

input
```
Int[(A + B*x^2 + C*x^4 + B*x^6)/(x^10*(a + b*x^2)^(9/2)),x]
```

output
```
-1/9*A/(a*x^9*(a + b*x^2)^(7/2)) - (-1/7*(16*A*b - 9*a*B)/(a*x^7*(a + b*x^
2)^(7/2)) - (-1/5*(32*A*b^2 - 9*a*(2*b*B - a*C))/(a*x^5*(a + b*x^2)^(7/2))
 - (3*(128*A*b^3 - 15*a^3*B - 36*a*b*(2*b*B - a*C))*(-1/3*1/(a*x^3*(a + b*
x^2)^(7/2)) - (10*b*(-(1/(a*x*(a + b*x^2)^(7/2))) - (8*b*(x/(7*a*(a + b*x^
2)^(7/2)) + (6*(x/(5*a*(a + b*x^2)^(5/2)) + (4*(x/(3*a*(a + b*x^2)^(3/2))
+ (2*x)/(3*a^2*Sqrt[a + b*x^2])))/(5*a)))/(7*a)))/a))/(3*a)))/(5*a))/a)/(9
*a)
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 1588
```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Simp[1/(d*f
^2*(m + 1))   Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x
) - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

rule 2089
```
Int[(u_)^(p_.)*((f_.)*(x_))^(m_.)*(z_)^(q_.), x_Symbol] :> Int[(f*x)^m*Expa
ndToSum[z, x]^q*ExpandToSum[u, x]^p, x] /; FreeQ[{f, m, p, q}, x] && Binomi
alQ[z, x] && TrinomialQ[u, x] &&  !(BinomialMatchQ[z, x] && TrinomialMatchQ
[u, x])
```

rule 2334
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coef
f[Pq, x, 0], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[A*
x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))   Int[
x^(m + 2)*(a + b*x^2)^p*(a*(m + 1)*Q - A*b*(m + 2*(p + 1) + 1)), x], x]] /;
 FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && IntegerQ[m/2] && ILtQ[(m + 1)/2 + p,
 0] && LtQ[m + Expon[Pq, x] + 2*p + 1, 0]
```

## Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.64

| method | result |
|---|---|
| pseudoelliptic | $\frac{(-105B\,x^6-63C\,x^4-45x^2B-35A)a^8+80(\frac{105}{8}B\,x^6+\frac{63}{20}C\,x^4+\frac{63}{40}x^2B+A)x^2b\,a^7-224(-\frac{75}{2}B\,x^6+\frac{45}{4}C\,x^4+\frac{9}{4}x^2B+A)x^4}{}$ |
| gosper | $-\frac{32768A\,b^8x^{16}-3840B\,a^3b^5x^{16}-18432Ba\,b^7x^{16}+9216C\,a^2b^6x^{16}+114688Aa\,b^7x^{14}-13440B\,a^4b^4x^{14}-64512B\,a^2b^6x^{14}}{}$ |
| trager | $-\frac{32768A\,b^8x^{16}-3840B\,a^3b^5x^{16}-18432Ba\,b^7x^{16}+9216C\,a^2b^6x^{16}+114688Aa\,b^7x^{14}-13440B\,a^4b^4x^{14}-64512B\,a^2b^6x^{14}}{}$ |
| orering | $-\frac{32768A\,b^8x^{16}-3840B\,a^3b^5x^{16}-18432Ba\,b^7x^{16}+9216C\,a^2b^6x^{16}+114688Aa\,b^7x^{14}-13440B\,a^4b^4x^{14}-64512B\,a^2b^6x^{14}}{}$ |
| risch | $-\frac{\sqrt{b\,x^2+a}\,(17609A\,x^8b^4-1470B\,a^3b\,x^8-9216B\,x^8a\,b^3+4158C\,a^2b^2x^8-3292A\,x^6a\,b^3+105B\,a^4x^6+1458B\,x^6a^2b^2-504C}{315a^9x^9}$ |
| | $\left(\left(2b\left(-\frac{1}{5a\,x^5\left(b\,x^2+a\right)^{\frac{7}{2}}}-\frac{12b\left(-\frac{1}{3a\,x^3\left(b\,x^2+a\right)^{\frac{7}{2}}}-\frac{10b\left(-\frac{1}{ax\left(b\,x^2+a\right)^{\frac{7}{2}}}-\right.}{}\right)}{}\right.\right.\right.$ |

input
```
int((B*x^6+C*x^4+B*x^2+A)/x^10/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
1/315*((-105*B*x^6-63*C*x^4-45*B*x^2-35*A)*a^8+80*(105/8*B*x^6+63/20*C*x^4
+63/40*x^2*B+A)*x^2*b*a^7-224*(-75/2*B*x^6+45/4*C*x^4+9/4*x^2*B+A)*x^4*b^2
*a^6+896*(75/4*B*x^6-45/2*C*x^4+45/8*x^2*B+A)*x^6*b^3*a^5-8960*x^8*(-3/2*B
*x^6+9/2*C*x^4-9/2*x^2*B+A)*b^4*a^4-71680*(-3/56*B*x^6+9/20*C*x^4-9/8*x^2*
B+A)*x^10*b^5*a^3-143360*x^12*(9/140*C*x^4-9/20*x^2*B+A)*b^6*a^2-114688*x^
14*b^7*(-9/56*x^2*B+A)*a-32768*A*b^8*x^16)/(b*x^2+a)^(7/2)/x^9/a^9
```

## Fricas [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 354, normalized size of antiderivative = 0.91

$$\int \frac{A + Bx^2 + Cx^4 + Bx^6}{x^{10} (a + bx^2)^{9/2}} \, dx = \frac{(256\,(15\,Ba^3b^5 - 36\,Ca^2b^6 + 72\,Bab^7 - 128\,Ab^8)x^{16} + 896\,(15\,Ba^4b^4 - 36$$

input
```
integrate((B*x^6+C*x^4+B*x^2+A)/x^10/(b*x^2+a)^(9/2),x, algorithm="fricas"
)
```

output
```
1/315*(256*(15*B*a^3*b^5 - 36*C*a^2*b^6 + 72*B*a*b^7 - 128*A*b^8)*x^16 + 8
96*(15*B*a^4*b^4 - 36*C*a^3*b^5 + 72*B*a^2*b^6 - 128*A*a*b^7)*x^14 + 1120*
(15*B*a^5*b^3 - 36*C*a^4*b^4 + 72*B*a^3*b^5 - 128*A*a^2*b^6)*x^12 + 560*(1
5*B*a^6*b^2 - 36*C*a^5*b^3 + 72*B*a^4*b^4 - 128*A*a^3*b^5)*x^10 - 35*A*a^8
 + 70*(15*B*a^7*b - 36*C*a^6*b^2 + 72*B*a^5*b^3 - 128*A*a^4*b^4)*x^8 - 7*(
15*B*a^8 - 36*C*a^7*b + 72*B*a^6*b^2 - 128*A*a^5*b^3)*x^6 - 7*(9*C*a^8 - 1
8*B*a^7*b + 32*A*a^6*b^2)*x^4 - 5*(9*B*a^8 - 16*A*a^7*b)*x^2)*sqrt(b*x^2 +
 a)/(a^9*b^4*x^17 + 4*a^10*b^3*x^15 + 6*a^11*b^2*x^13 + 4*a^12*b*x^11 + a^
13*x^9)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4+Bx^6}{x^{10}\left(a+bx^2\right)^{9/2}}\,dx=\text{Timed out}$$

input
```
integrate((B*x**6+C*x**4+B*x**2+A)/x**10/(b*x**2+a)**(9/2),x)
```

output
```
Timed out
```

## Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 579, normalized size of antiderivative = 1.49

$$\int \frac{A+Bx^2+Cx^4+Bx^6}{x^{10}\left(a+bx^2\right)^{9/2}}\,dx=\text{Too large to display}$$

input
```
integrate((B*x^6+C*x^4+B*x^2+A)/x^10/(b*x^2+a)^(9/2),x, algorithm="maxima"
)
```

output
```
256/21*B*b^2*x/(sqrt(b*x^2 + a)*a^6) + 128/21*B*b^2*x/((b*x^2 + a)^(3/2)*a
^5) + 32/7*B*b^2*x/((b*x^2 + a)^(5/2)*a^4) + 80/21*B*b^2*x/((b*x^2 + a)^(7
/2)*a^3) - 1024/35*C*b^3*x/(sqrt(b*x^2 + a)*a^7) - 512/35*C*b^3*x/((b*x^2
+ a)^(3/2)*a^6) - 384/35*C*b^3*x/((b*x^2 + a)^(5/2)*a^5) - 64/7*C*b^3*x/((
b*x^2 + a)^(7/2)*a^4) + 2048/35*B*b^4*x/(sqrt(b*x^2 + a)*a^8) + 1024/35*B*
b^4*x/((b*x^2 + a)^(3/2)*a^7) + 768/35*B*b^4*x/((b*x^2 + a)^(5/2)*a^6) + 1
28/7*B*b^4*x/((b*x^2 + a)^(7/2)*a^5) - 32768/315*A*b^5*x/(sqrt(b*x^2 + a)*
a^9) - 16384/315*A*b^5*x/((b*x^2 + a)^(3/2)*a^8) - 4096/105*A*b^5*x/((b*x^
2 + a)^(5/2)*a^7) - 2048/63*A*b^5*x/((b*x^2 + a)^(7/2)*a^6) + 10/3*B*b/((b
*x^2 + a)^(7/2)*a^2*x) - 8*C*b^2/((b*x^2 + a)^(7/2)*a^3*x) + 16*B*b^3/((b*
x^2 + a)^(7/2)*a^4*x) - 256/9*A*b^4/((b*x^2 + a)^(7/2)*a^5*x) - 1/3*B/((b*
x^2 + a)^(7/2)*a*x^3) + 4/5*C*b/((b*x^2 + a)^(7/2)*a^2*x^3) - 8/5*B*b^2/((
b*x^2 + a)^(7/2)*a^3*x^3) + 128/45*A*b^3/((b*x^2 + a)^(7/2)*a^4*x^3) - 1/5
*C/((b*x^2 + a)^(7/2)*a*x^5) + 2/5*B*b/((b*x^2 + a)^(7/2)*a^2*x^5) - 32/45
*A*b^2/((b*x^2 + a)^(7/2)*a^3*x^5) - 1/7*B/((b*x^2 + a)^(7/2)*a*x^7) + 16/
63*A*b/((b*x^2 + a)^(7/2)*a^2*x^7) - 1/9*A/((b*x^2 + a)^(7/2)*a*x^9)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1162 vs. $2(352) = 704$.

Time = 0.16 (sec) , antiderivative size = 1162, normalized size of antiderivative = 2.99

$$\int \frac{A + Bx^2 + Cx^4 + Bx^6}{x^{10} (a + bx^2)^{9/2}} \, dx = \text{Too large to display}$$

input
```
integrate((B*x^6+C*x^4+B*x^2+A)/x^10/(b*x^2+a)^(9/2),x, algorithm="giac")
```

output
```
1/105*((x^2*((790*B*a^24*b^8 - 1686*C*a^23*b^9 + 3072*B*a^22*b^10 - 5053*A
*a^21*b^11)*x^2/(a^30*b^3) + 7*(365*B*a^25*b^7 - 768*C*a^24*b^8 + 1386*B*a
^23*b^9 - 2264*A*a^22*b^10)/(a^30*b^3)) + 35*(80*B*a^26*b^6 - 165*C*a^25*b
^7 + 294*B*a^24*b^8 - 476*A*a^23*b^9)/(a^30*b^3))*x^2 + 105*(10*B*a^27*b^5
 - 20*C*a^26*b^6 + 35*B*a^25*b^7 - 56*A*a^24*b^8)/(a^30*b^3))*x/(b*x^2 + a
)^(7/2) - 2/315*(1260*(sqrt(b)*x - sqrt(b*x^2 + a))^16*B*a^3*b^(3/2) - 315
0*(sqrt(b)*x - sqrt(b*x^2 + a))^16*C*a^2*b^(5/2) + 6300*(sqrt(b)*x - sqrt(
b*x^2 + a))^16*B*a*b^(7/2) - 11025*(sqrt(b)*x - sqrt(b*x^2 + a))^16*A*b^(9
/2) - 10710*(sqrt(b)*x - sqrt(b*x^2 + a))^14*B*a^4*b^(3/2) + 27720*(sqrt(b
)*x - sqrt(b*x^2 + a))^14*C*a^3*b^(5/2) - 56700*(sqrt(b)*x - sqrt(b*x^2 +
a))^14*B*a^2*b^(7/2) + 100800*(sqrt(b)*x - sqrt(b*x^2 + a))^14*A*a*b^(9/2)
 + 39270*(sqrt(b)*x - sqrt(b*x^2 + a))^12*B*a^5*b^(3/2) - 105840*(sqrt(b)*
x - sqrt(b*x^2 + a))^12*C*a^4*b^(5/2) + 223020*(sqrt(b)*x - sqrt(b*x^2 + a
))^12*B*a^3*b^(7/2) - 405300*(sqrt(b)*x - sqrt(b*x^2 + a))^12*A*a^2*b^(9/2
) - 81270*(sqrt(b)*x - sqrt(b*x^2 + a))^10*B*a^6*b^(3/2) + 226800*(sqrt(b)
*x - sqrt(b*x^2 + a))^10*C*a^5*b^(5/2) - 495180*(sqrt(b)*x - sqrt(b*x^2 +
a))^10*B*a^4*b^(7/2) + 927360*(sqrt(b)*x - sqrt(b*x^2 + a))^10*A*a^3*b^(9/
2) + 103950*(sqrt(b)*x - sqrt(b*x^2 + a))^8*B*a^7*b^(3/2) - 297108*(sqrt(b
)*x - sqrt(b*x^2 + a))^8*C*a^6*b^(5/2) + 666036*(sqrt(b)*x - sqrt(b*x^2 +
a))^8*B*a^5*b^(7/2) - 1291374*(sqrt(b)*x - sqrt(b*x^2 + a))^8*A*a^4*b^(...
```

## Mupad [B] (verification not implemented)

Time = 3.12 (sec) , antiderivative size = 594, normalized size of antiderivative = 1.53

$$\int \frac{A+Bx^2+Cx^4+Bx^6}{x^{10}\,(a+bx^2)^{9/2}}\,dx = \frac{B}{7\,a\,x^3\,\left(b\,x^2+a\right)^{7/2}}-\frac{10\,B}{21\,a^2\,x^3\,\left(b\,x^2+a\right)^{5/2}}-\frac{A\,\sqrt{b\,x^2+a}}{9\,a^5\,x^9}-\frac{B\,\sqrt{b\,x^2+a}}{7\,a^5\,x^7}-\frac{C\,\sqrt{b\,x^2+a}}{5\,a^5\,x^5}+\frac{44\,A\,b\,\sqrt{b\,x^2+a}}{63\,a^6\,x^7}-\frac{32768\,A\,b^5\,x}{315\,a^9\,\sqrt{b\,x^2+a}}+\frac{128\,B\,b}{21\,a^5\,x\,\sqrt{b\,x^2+a}}-\frac{32\,B\,b}{21\,a^4\,x\,\left(b\,x^2+a\right)^{3/2}}-\frac{16\,B\,b}{21\,a^3\,x\,\left(b\,x^2+a\right)^{5/2}}+\frac{256\,B\,b^2\,x}{21\,a^6\,\sqrt{b\,x^2+a}}+\frac{34\,B\,b\,\sqrt{b\,x^2+a}}{35\,a^6\,x^5}+\frac{2048\,B\,b^4\,x}{35\,a^8\,\sqrt{b\,x^2+a}}+\frac{61\,C\,b}{35\,a^3\,x^3\,\left(b\,x^2+a\right)^{5/2}}-\frac{1024\,C\,b^3\,x}{35\,a^7\,\sqrt{b\,x^2+a}}-\frac{C\,b}{7\,a^2\,x^3\,\left(b\,x^2+a\right)^{7/2}}-\frac{16384\,A\,b^4}{315\,a^8\,x\,\sqrt{b\,x^2+a}}-\frac{298\,A\,b^2\,\sqrt{b\,x^2+a}}{105\,a^7\,x^5}+\frac{998\,A\,b^4}{63\,a^7\,x\,\left(b\,x^2+a\right)^{3/2}}+\frac{3337\,A\,b^3}{315\,a^5\,x^3\,\left(b\,x^2+a\right)^{5/2}}+\frac{3616\,A\,b^4}{315\,a^6\,x\,\left(b\,x^2+a\right)^{5/2}}-\frac{A\,b^3}{7\,a^4\,x^3\,\left(b\,x^2+a\right)^{7/2}}+\frac{1024\,B\,b^3}{35\,a^7\,x\,\sqrt{b\,x^2+a}}-\frac{58\,B\,b^3}{7\,a^6\,x\,\left(b\,x^2+a\right)^{3/2}}-\frac{167\,B\,b^2}{35\,a^4\,x^3\,\left(b\,x^2+a\right)^{5/2}}-\frac{191\,B\,b^3}{35\,a^5\,x\,\left(b\,x^2+a\right)^{5/2}}+\frac{B\,b^2}{7\,a^3\,x^3\,\left(b\,x^2+a\right)^{7/2}}-\frac{512\,C\,b^2}{35\,a^6\,x\,\sqrt{b\,x^2+a}}+\frac{27\,C\,b^2}{7\,a^5\,x\,\left(b\,x^2+a\right)^{3/2}}+\frac{78\,C\,b^2}{35\,a^4\,x\,\left(b\,x^2+a\right)^{5/2}}$$

input

```
int((A + B*x^2 + B*x^6 + C*x^4)/(x^10*(a + b*x^2)^(9/2)),x)
```

output
```
B/(7*a*x^3*(a + b*x^2)^(7/2)) - (10*B)/(21*a^2*x^3*(a + b*x^2)^(5/2)) - (A
*(a + b*x^2)^(1/2))/(9*a^5*x^9) - (B*(a + b*x^2)^(1/2))/(7*a^5*x^7) - (C*(
a + b*x^2)^(1/2))/(5*a^5*x^5) + (44*A*b*(a + b*x^2)^(1/2))/(63*a^6*x^7) -
(32768*A*b^5*x)/(315*a^9*(a + b*x^2)^(1/2)) + (128*B*b)/(21*a^5*x*(a + b*x
^2)^(1/2)) - (32*B*b)/(21*a^4*x*(a + b*x^2)^(3/2)) - (16*B*b)/(21*a^3*x*(a
 + b*x^2)^(5/2)) + (256*B*b^2*x)/(21*a^6*(a + b*x^2)^(1/2)) + (34*B*b*(a +
 b*x^2)^(1/2))/(35*a^6*x^5) + (2048*B*b^4*x)/(35*a^8*(a + b*x^2)^(1/2)) +
(61*C*b)/(35*a^3*x^3*(a + b*x^2)^(5/2)) - (1024*C*b^3*x)/(35*a^7*(a + b*x^
2)^(1/2)) - (C*b)/(7*a^2*x^3*(a + b*x^2)^(7/2)) - (16384*A*b^4)/(315*a^8*x
*(a + b*x^2)^(1/2)) - (298*A*b^2*(a + b*x^2)^(1/2))/(105*a^7*x^5) + (998*A
*b^4)/(63*a^7*x*(a + b*x^2)^(3/2)) + (3337*A*b^3)/(315*a^5*x^3*(a + b*x^2)
^(5/2)) + (3616*A*b^4)/(315*a^6*x*(a + b*x^2)^(5/2)) - (A*b^3)/(7*a^4*x^3*
(a + b*x^2)^(7/2)) + (1024*B*b^3)/(35*a^7*x*(a + b*x^2)^(1/2)) - (58*B*b^3
)/(7*a^6*x*(a + b*x^2)^(3/2)) - (167*B*b^2)/(35*a^4*x^3*(a + b*x^2)^(5/2))
 - (191*B*b^3)/(35*a^5*x*(a + b*x^2)^(5/2)) + (B*b^2)/(7*a^3*x^3*(a + b*x^
2)^(7/2)) - (512*C*b^2)/(35*a^6*x*(a + b*x^2)^(1/2)) + (27*C*b^2)/(7*a^5*x
*(a + b*x^2)^(3/2)) + (78*C*b^2)/(35*a^4*x*(a + b*x^2)^(5/2))
```

## Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 647, normalized size of antiderivative = 1.67

$$\int \frac{A+Bx^2+Cx^4+Bx^6}{x^{10}\left(a+bx^2\right)^{9/2}}\,dx = \frac{-105\sqrt{b\,x^2+a}\,a^7b\,x^6+35\sqrt{b\,x^2+a}\,a^7b\,x^2-63\sqrt{b\,x^2+a}\,a^7c\,x^4+1050\sqrt{}}{}$$

input
```
int((B*x^6+C*x^4+B*x^2+A)/x^10/(b*x^2+a)^(9/2),x)
```

output
```
( - 35*sqrt(a + b*x**2)*a**8 - 105*sqrt(a + b*x**2)*a**7*b*x**6 + 35*sqrt(
a + b*x**2)*a**7*b*x**2 - 63*sqrt(a + b*x**2)*a**7*c*x**4 + 1050*sqrt(a +
b*x**2)*a**6*b**2*x**8 - 98*sqrt(a + b*x**2)*a**6*b**2*x**4 + 252*sqrt(a +
 b*x**2)*a**6*b*c*x**6 + 8400*sqrt(a + b*x**2)*a**5*b**3*x**10 + 392*sqrt(
a + b*x**2)*a**5*b**3*x**6 - 2520*sqrt(a + b*x**2)*a**5*b**2*c*x**8 + 1680
0*sqrt(a + b*x**2)*a**4*b**4*x**12 - 3920*sqrt(a + b*x**2)*a**4*b**4*x**8
- 20160*sqrt(a + b*x**2)*a**4*b**3*c*x**10 + 13440*sqrt(a + b*x**2)*a**3*b
**5*x**14 - 31360*sqrt(a + b*x**2)*a**3*b**5*x**10 - 40320*sqrt(a + b*x**2
)*a**3*b**4*c*x**12 + 3840*sqrt(a + b*x**2)*a**2*b**6*x**16 - 62720*sqrt(a
 + b*x**2)*a**2*b**6*x**12 - 32256*sqrt(a + b*x**2)*a**2*b**5*c*x**14 - 50
176*sqrt(a + b*x**2)*a*b**7*x**14 - 9216*sqrt(a + b*x**2)*a*b**6*c*x**16 -
 14336*sqrt(a + b*x**2)*b**8*x**16 - 3840*sqrt(b)*a**6*b**2*x**9 - 15360*s
qrt(b)*a**5*b**3*x**11 + 9216*sqrt(b)*a**5*b**2*c*x**9 - 23040*sqrt(b)*a**
4*b**4*x**13 + 14336*sqrt(b)*a**4*b**4*x**9 + 36864*sqrt(b)*a**4*b**3*c*x*
*11 - 15360*sqrt(b)*a**3*b**5*x**15 + 57344*sqrt(b)*a**3*b**5*x**11 + 5529
6*sqrt(b)*a**3*b**4*c*x**13 - 3840*sqrt(b)*a**2*b**6*x**17 + 86016*sqrt(b)
*a**2*b**6*x**13 + 36864*sqrt(b)*a**2*b**5*c*x**15 + 57344*sqrt(b)*a*b**7*
x**15 + 9216*sqrt(b)*a*b**6*c*x**17 + 14336*sqrt(b)*b**8*x**17)/(315*a**8*
x**9*(a**4 + 4*a**3*b*x**2 + 6*a**2*b**2*x**4 + 4*a*b**3*x**6 + b**4*x**8)
)
```

# 3.249 $\int \frac{Ax^5+Bx^7+Cx^9+Dx^{11}}{\sqrt{a+bx^2}}\,dx$



## Optimal result

Integrand size = 33, antiderivative size = 216

$$\int \frac{Ax^5+Bx^7+Cx^9+Dx^{11}}{\sqrt{a+bx^2}}\,dx = \frac{a^2(Ab^3-a(b^2B-abC+a^2D))\sqrt{a+bx^2}}{b^6} - \frac{a(2Ab^3-a(3b^2B-4abC+5a^2D))\left(a+bx^2\right)^{3/2}}{3b^6} + \frac{(Ab^3-a(3b^2B-6abC+10a^2D))\left(a+bx^2\right)^{5/2}}{5b^6} + \frac{(b^2B-4abC+10a^2D)\left(a+bx^2\right)^{7/2}}{7b^6} + \frac{(bC-5aD)\left(a+bx^2\right)^{9/2}}{9b^6} + \frac{D\left(a+bx^2\right)^{11/2}}{11b^6}$$

output
```
a^2*(A*b^3-a*(B*b^2-C*a*b+D*a^2))*(b*x^2+a)^(1/2)/b^6-1/3*a*(2*A*b^3-a*(3*
B*b^2-4*C*a*b+5*D*a^2))*(b*x^2+a)^(3/2)/b^6+1/5*(A*b^3-a*(3*B*b^2-6*C*a*b+
10*D*a^2))*(b*x^2+a)^(5/2)/b^6+1/7*(B*b^2-4*C*a*b+10*D*a^2)*(b*x^2+a)^(7/2
)/b^6+1/9*(C*b-5*D*a)*(b*x^2+a)^(9/2)/b^6+1/11*D*(b*x^2+a)^(11/2)/b^6
```

## Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.73

$$\int \frac{Ax^5+Bx^7+Cx^9+Dx^{11}}{\sqrt{a+bx^2}}\,dx$$
$$= \frac{\sqrt{a+bx^2}(-1280a^5D+128a^4b(11C+5Dx^2)-16a^3b^2(99B+44Cx^2+30Dx^4)+8a^2b^3(231A+99Bx^2}{}$$

input
```
Integrate[(A*x^5 + B*x^7 + C*x^9 + D*x^11)/Sqrt[a + b*x^2],x]
```

output
```
(Sqrt[a + b*x^2]*(-1280*a^5*D + 128*a^4*b*(11*C + 5*D*x^2) - 16*a^3*b^2*(9
9*B + 44*C*x^2 + 30*D*x^4) + 8*a^2*b^3*(231*A + 99*B*x^2 + 66*C*x^4 + 50*D
*x^6) - 2*a*b^4*x^2*(462*A + 297*B*x^2 + 220*C*x^4 + 175*D*x^6) + b^5*x^4*
(693*A + 5*(99*B*x^2 + 77*C*x^4 + 63*D*x^6))))/(3465*b^6)
```

## Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {2029, 2331, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{Ax^5+Bx^7+Cx^9+Dx^{11}}{\sqrt{a+bx^2}}\,dx$$
$$\Big\downarrow \text{2029}$$
$$\int \frac{x^5\left(A+Bx^2+Cx^4+Dx^6\right)}{\sqrt{a+bx^2}}dx$$
$$\Big\downarrow \text{2331}$$
$$\frac{1}{2}\int \frac{x^4\left(Dx^6+Cx^4+Bx^2+A\right)}{\sqrt{bx^2+a}}dx^2$$
$$\Big\downarrow \text{2123}$$

$$\frac{1}{2}\int \left(\frac{D(bx^2+a)^{9/2}}{b^5}+\frac{(bC-5aD)\,(bx^2+a)^{7/2}}{b^5}+\frac{(10Da^2-4bCa+b^2B)\,(bx^2+a)^{5/2}}{b^5}+\frac{(Ab^3-a(10Da^2-$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{2(a+bx^2)^{5/2}\,(Ab^3-a(10a^2D-6abC+3b^2B))}{5b^6}-\frac{2a(a+bx^2)^{3/2}\,(2Ab^3-a(5a^2D-4abC+3b^2B))}{3b^6}+\frac{2a$$

input
```
Int[(A*x^5 + B*x^7 + C*x^9 + D*x^11)/Sqrt[a + b*x^2],x]
```

output
```
((2*a^2*(A*b^3 - a*(b^2*B - a*b*C + a^2*D))*Sqrt[a + b*x^2])/b^6 - (2*a*(2
*A*b^3 - a*(3*b^2*B - 4*a*b*C + 5*a^2*D))*(a + b*x^2)^(3/2))/(3*b^6) + (2*
(A*b^3 - a*(3*b^2*B - 6*a*b*C + 10*a^2*D))*(a + b*x^2)^(5/2))/(5*b^6) + (2
*(b^2*B - 4*a*b*C + 10*a^2*D)*(a + b*x^2)^(7/2))/(7*b^6) + (2*(b*C - 5*a*D
)*(a + b*x^2)^(9/2))/(9*b^6) + (2*D*(a + b*x^2)^(11/2))/(11*b^6))/2
```

## Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2029
```
Int[(Fx_.)*((d_.)*(x_)^(q_.) + (a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.) + (c_.)*
(x_)^(t_.))^(p_.), x_Symbol] :> Int[x^(p*r)*(a + b*x^(s - r) + c*x^(t - r)
+ d*x^(q - r))^p*Fx, x] /; FreeQ[{a, b, c, d, r, s, t, q}, x] && IntegerQ[p
] && PosQ[s - r] && PosQ[t - r] && PosQ[q - r] &&  !(EqQ[p, 1] && EqQ[u, 1]
)
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

rule 2331
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/2   S
ubst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
 FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

## Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.66

| method | result |
|---|---|
| pseudoelliptic | $\frac{8\left(\frac{3\left(\frac{5}{11}Dx^6+\frac{5}{9}Cx^4+\frac{5}{7}x^2B+A\right)x^4b^5}{8}-\frac{\left(\frac{25}{66}Dx^6+\frac{10}{21}Cx^4+\frac{9}{14}x^2B+A\right)x^2ab^4}{2}+a^2\left(\frac{50}{231}Dx^6+\frac{2}{7}Cx^4+\frac{3}{7}x^2B+A\right)b^3-\frac{6\left(\frac{10}{33}Dx^4\right.}{}\right.}{15b^6}$ |
| gosper | $\sqrt{bx^2+a}\,(315Dx^{10}b^5+385Cb^5x^8-350Dab^4x^8+495Bb^5x^6-440Cab^4x^6+400Da^2b^3x^6+693Ab^5x^4-594Bab^4x^4+528C$ |
| trager | $\sqrt{bx^2+a}\,(315Dx^{10}b^5+385Cb^5x^8-350Dab^4x^8+495Bb^5x^6-440Cab^4x^6+400Da^2b^3x^6+693Ab^5x^4-594Bab^4x^4+528C$ |
| orering | $(315Dx^{10}b^5+385Cb^5x^8-350Dab^4x^8+495Bb^5x^6-440Cab^4x^6+400Da^2b^3x^6+693Ab^5x^4-594Bab^4x^4+528Ca^2b^3x^4-$ |
| default | $A\left(\frac{x^4\sqrt{bx^2+a}}{5b}-\frac{4a\left(\frac{x^2\sqrt{bx^2+a}}{3b}-\frac{2a\sqrt{bx^2+a}}{3b^2}\right)}{5b}\right)+B\left(\frac{x^6\sqrt{bx^2+a}}{7b}-\frac{6a\left(\frac{x^4\sqrt{bx^2+a}}{5b}-\frac{4a\left(\frac{x^2\sqrt{bx^2+a}}{3b}-\frac{2a\sqrt{bx^2}}{3b^2}\right.}{5b}\right.}{7b}\right.$ |

input
```
int((D*x^11+C*x^9+B*x^7+A*x^5)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
8/15*(3/8*(5/11*D*x^6+5/9*C*x^4+5/7*x^2*B+A)*x^4*b^5-1/2*(25/66*D*x^6+10/2
1*C*x^4+9/14*x^2*B+A)*x^2*a*b^4+a^2*(50/231*D*x^6+2/7*C*x^4+3/7*x^2*B+A)*b
^3-6/7*(10/33*D*x^4+4/9*C*x^2+B)*a^3*b^2+16/21*(5/11*D*x^2+C)*a^4*b-160/23
1*D*a^5)*(b*x^2+a)^(1/2)/b^6
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.82

$$\int \frac{Ax^5 + Bx^7 + Cx^9 + Dx^{11}}{\sqrt{a + bx^2}}\,dx$$
$$= \frac{(315\,Db^5x^{10} - 35\,(10\,Dab^4 - 11\,Cb^5)x^8 + 5\,(80\,Da^2b^3 - 88\,Cab^4 + 99\,Bb^5)x^6 - 1280\,Da^5 + 1408\,Ca^4b$$

input
```
integrate((D*x^11+C*x^9+B*x^7+A*x^5)/(b*x^2+a)^(1/2),x, algorithm="fricas"
)
```

output
```
1/3465*(315*D*b^5*x^10 - 35*(10*D*a*b^4 - 11*C*b^5)*x^8 + 5*(80*D*a^2*b^3
- 88*C*a*b^4 + 99*B*b^5)*x^6 - 1280*D*a^5 + 1408*C*a^4*b - 1584*B*a^3*b^2
+ 1848*A*a^2*b^3 - 3*(160*D*a^3*b^2 - 176*C*a^2*b^3 + 198*B*a*b^4 - 231*A*
b^5)*x^4 + 4*(160*D*a^4*b - 176*C*a^3*b^2 + 198*B*a^2*b^3 - 231*A*a*b^4)*x
^2)*sqrt(b*x^2 + a)/b^6
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 442 vs. 2(207) = 414.

Time = 0.45 (sec) , antiderivative size = 442, normalized size of antiderivative = 2.05

$$\int \frac{Ax^5 + Bx^7 + Cx^9 + Dx^{11}}{\sqrt{a + bx^2}}\,dx$$
$$= \begin{cases} \frac{8Aa^2\sqrt{a+bx^2}}{15b^3} - \frac{4Aax^2\sqrt{a+bx^2}}{15b^2} + \frac{Ax^4\sqrt{a+bx^2}}{5b} - \frac{16Ba^3\sqrt{a+bx^2}}{35b^4} + \frac{8Ba^2x^2\sqrt{a+bx^2}}{35b^3} - \frac{6Bax^4\sqrt{a+bx^2}}{35b^2} + \frac{Bx^6\sqrt{a+bx^2}}{7b} + \frac{128C \\ \frac{\frac{Ax^6}{6} + \frac{Bx^8}{8} + \frac{Cx^{10}}{10} + \frac{Dx^{12}}{12}}{\sqrt{a}} \end{cases}$$

input
```
integrate((D*x**11+C*x**9+B*x**7+A*x**5)/(b*x**2+a)**(1/2),x)
```

output
```
Piecewise((8*A*a**2*sqrt(a + b*x**2)/(15*b**3) - 4*A*a*x**2*sqrt(a + b*x**
2)/(15*b**2) + A*x**4*sqrt(a + b*x**2)/(5*b) - 16*B*a**3*sqrt(a + b*x**2)/
(35*b**4) + 8*B*a**2*x**2*sqrt(a + b*x**2)/(35*b**3) - 6*B*a*x**4*sqrt(a +
 b*x**2)/(35*b**2) + B*x**6*sqrt(a + b*x**2)/(7*b) + 128*C*a**4*sqrt(a + b
*x**2)/(315*b**5) - 64*C*a**3*x**2*sqrt(a + b*x**2)/(315*b**4) + 16*C*a**2
*x**4*sqrt(a + b*x**2)/(105*b**3) - 8*C*a*x**6*sqrt(a + b*x**2)/(63*b**2)
+ C*x**8*sqrt(a + b*x**2)/(9*b) - 256*D*a**5*sqrt(a + b*x**2)/(693*b**6) +
 128*D*a**4*x**2*sqrt(a + b*x**2)/(693*b**5) - 32*D*a**3*x**4*sqrt(a + b*x
**2)/(231*b**4) + 80*D*a**2*x**6*sqrt(a + b*x**2)/(693*b**3) - 10*D*a*x**8
*sqrt(a + b*x**2)/(99*b**2) + D*x**10*sqrt(a + b*x**2)/(11*b), Ne(b, 0)),
((A*x**6/6 + B*x**8/8 + C*x**10/10 + D*x**12/12)/sqrt(a), True))
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.61

$$\int \frac{Ax^5 + Bx^7 + Cx^9 + Dx^{11}}{\sqrt{a + bx^2}}\,dx = \frac{\sqrt{bx^2 + a}Dx^{10}}{11\,b} - \frac{10\,\sqrt{bx^2 + a}Dax^8}{99\,b^2} + \frac{\sqrt{bx^2 + a}Cx^8}{9\,b} + \frac{80\,\sqrt{bx^2 + a}Da^2x^6}{693\,b^3} - \frac{8\,\sqrt{bx^2 + a}Cax^6}{63\,b^2} + \frac{\sqrt{bx^2 + a}Bx^6}{7\,b} - \frac{32\,\sqrt{bx^2 + a}Da^3x^4}{231\,b^4} + \frac{16\,\sqrt{bx^2 + a}Ca^2x^4}{105\,b^3} - \frac{6\,\sqrt{bx^2 + a}Bax^4}{35\,b^2} + \frac{\sqrt{bx^2 + a}Ax^4}{5\,b} + \frac{128\,\sqrt{bx^2 + a}Da^4x^2}{693\,b^5} - \frac{64\,\sqrt{bx^2 + a}Ca^3x^2}{315\,b^4} + \frac{8\,\sqrt{bx^2 + a}Ba^2x^2}{35\,b^3} - \frac{4\,\sqrt{bx^2 + a}Aax^2}{15\,b^2} - \frac{256\,\sqrt{bx^2 + a}Da^5}{693\,b^6} + \frac{128\,\sqrt{bx^2 + a}Ca^4}{315\,b^5} - \frac{16\,\sqrt{bx^2 + a}Ba^3}{35\,b^4} + \frac{8\,\sqrt{bx^2 + a}Aa^2}{15\,b^3}$$

input
```
integrate((D*x^11+C*x^9+B*x^7+A*x^5)/(b*x^2+a)^(1/2),x, algorithm="maxima"
)
```

output
```
1/11*sqrt(b*x^2 + a)*D*x^10/b - 10/99*sqrt(b*x^2 + a)*D*a*x^8/b^2 + 1/9*sq
rt(b*x^2 + a)*C*x^8/b + 80/693*sqrt(b*x^2 + a)*D*a^2*x^6/b^3 - 8/63*sqrt(b
*x^2 + a)*C*a*x^6/b^2 + 1/7*sqrt(b*x^2 + a)*B*x^6/b - 32/231*sqrt(b*x^2 +
a)*D*a^3*x^4/b^4 + 16/105*sqrt(b*x^2 + a)*C*a^2*x^4/b^3 - 6/35*sqrt(b*x^2
+ a)*B*a*x^4/b^2 + 1/5*sqrt(b*x^2 + a)*A*x^4/b + 128/693*sqrt(b*x^2 + a)*D
*a^4*x^2/b^5 - 64/315*sqrt(b*x^2 + a)*C*a^3*x^2/b^4 + 8/35*sqrt(b*x^2 + a)
*B*a^2*x^2/b^3 - 4/15*sqrt(b*x^2 + a)*A*a*x^2/b^2 - 256/693*sqrt(b*x^2 + a
)*D*a^5/b^6 + 128/315*sqrt(b*x^2 + a)*C*a^4/b^5 - 16/35*sqrt(b*x^2 + a)*B*
a^3/b^4 + 8/15*sqrt(b*x^2 + a)*A*a^2/b^3
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.20

$$\int \frac{Ax^5 + Bx^7 + Cx^9 + Dx^{11}}{\sqrt{a + bx^2}}\,dx = -\frac{(Da^5 - Ca^4b + Ba^3b^2 - Aa^2b^3)\sqrt{bx^2 + a}}{b^6} + \frac{315\,(bx^2 + a)^{\frac{11}{2}}D - 1925\,(bx^2 + a)^{\frac{9}{2}}Da + 4950\,(bx^2 + a)^{\frac{7}{2}}Da^2 - 6930\,(bx^2 + a)^{\frac{5}{2}}Da^3 + 5775\,(bx^2 + a}{}$$

input
```
integrate((D*x^11+C*x^9+B*x^7+A*x^5)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
-(D*a^5 - C*a^4*b + B*a^3*b^2 - A*a^2*b^3)*sqrt(b*x^2 + a)/b^6 + 1/3465*(3
15*(b*x^2 + a)^(11/2)*D - 1925*(b*x^2 + a)^(9/2)*D*a + 4950*(b*x^2 + a)^(7
/2)*D*a^2 - 6930*(b*x^2 + a)^(5/2)*D*a^3 + 5775*(b*x^2 + a)^(3/2)*D*a^4 +
385*(b*x^2 + a)^(9/2)*C*b - 1980*(b*x^2 + a)^(7/2)*C*a*b + 4158*(b*x^2 + a
)^(5/2)*C*a^2*b - 4620*(b*x^2 + a)^(3/2)*C*a^3*b + 495*(b*x^2 + a)^(7/2)*B
*b^2 - 2079*(b*x^2 + a)^(5/2)*B*a*b^2 + 3465*(b*x^2 + a)^(3/2)*B*a^2*b^2 +
 693*(b*x^2 + a)^(5/2)*A*b^3 - 2310*(b*x^2 + a)^(3/2)*A*a*b^3)/b^6
```

## Mupad [B] (verification not implemented)

Time = 1.27 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.13

$$\int \frac{Ax^5 + Bx^7 + Cx^9 + Dx^{11}}{\sqrt{a+bx^2}}\,dx$$
$$= \sqrt{b\,x^2+a}\left(\frac{128\,C\,a^4}{315\,b^5} + \frac{C\,x^8}{9\,b} - \frac{8\,C\,a\,x^6}{63\,b^2} + \frac{16\,C\,a^2\,x^4}{105\,b^3} - \frac{64\,C\,a^3\,x^2}{315\,b^4}\right)$$
$$- \sqrt{b\,x^2+a}\left(\frac{16\,B\,a^3}{35\,b^4} - \frac{B\,x^6}{7\,b} + \frac{6\,B\,a\,x^4}{35\,b^2} - \frac{8\,B\,a^2\,x^2}{35\,b^3}\right)$$
$$+ \sqrt{b\,x^2+a}\left(\frac{8\,A\,a^2}{15\,b^3} + \frac{A\,x^4}{5\,b} - \frac{4\,A\,a\,x^2}{15\,b^2}\right)$$
$$+ \frac{\frac{(b\,x^2+a)^{11/2}\,D}{11} - \frac{5\,a\,(b\,x^2+a)^{9/2}\,D}{9} - a^5\,\sqrt{b\,x^2+a}\,D + \frac{5\,a^4\,(b\,x^2+a)^{3/2}\,D}{3} - 2\,a^3\,(b\,x^2+a)^{5/2}\,D + \frac{10\,a^2\,(b\,x^2+a)^{7/2}}{7}}{b^6}$$

input
```
int((A*x^5 + B*x^7 + C*x^9 + x^11*D)/(a + b*x^2)^(1/2),x)
```

output
```
(a + b*x^2)^(1/2)*((128*C*a^4)/(315*b^5) + (C*x^8)/(9*b) - (8*C*a*x^6)/(63
*b^2) + (16*C*a^2*x^4)/(105*b^3) - (64*C*a^3*x^2)/(315*b^4)) - (a + b*x^2)
^(1/2)*((16*B*a^3)/(35*b^4) - (B*x^6)/(7*b) + (6*B*a*x^4)/(35*b^2) - (8*B*
a^2*x^2)/(35*b^3)) + (a + b*x^2)^(1/2)*((8*A*a^2)/(15*b^3) + (A*x^4)/(5*b)
 - (4*A*a*x^2)/(15*b^2)) + (((a + b*x^2)^(11/2)*D)/11 - (5*a*(a + b*x^2)^(
9/2)*D)/9 - a^5*(a + b*x^2)^(1/2)*D + (5*a^4*(a + b*x^2)^(3/2)*D)/3 - 2*a^
3*(a + b*x^2)^(5/2)*D + (10*a^2*(a + b*x^2)^(7/2)*D)/7)/b^6
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.74

$$\int \frac{Ax^5 + Bx^7 + Cx^9 + Dx^{11}}{\sqrt{a+bx^2}}\,dx$$
$$= \frac{\sqrt{b\,x^2+a}\,(315b^5d\,x^{10} - 350a\,b^4d\,x^8 + 385b^5c\,x^8 + 400a^2b^3d\,x^6 - 440a\,b^4c\,x^6 + 495b^6x^6 - 480a^3b^2d\,x^4 +}{3465b}$$

input
```
int((D*x^11+C*x^9+B*x^7+A*x^5)/(b*x^2+a)^(1/2),x)
```

output

```
(sqrt(a + b*x**2)*( - 1280*a**5*d + 1408*a**4*b*c + 640*a**4*b*d*x**2 + 26
4*a**3*b**3 - 704*a**3*b**2*c*x**2 - 480*a**3*b**2*d*x**4 - 132*a**2*b**4*
x**2 + 528*a**2*b**3*c*x**4 + 400*a**2*b**3*d*x**6 + 99*a*b**5*x**4 - 440*
a*b**4*c*x**6 - 350*a*b**4*d*x**8 + 495*b**6*x**6 + 385*b**5*c*x**8 + 315*
b**5*d*x**10))/(3465*b**6)
```

# 3.250 $\int \frac{Ax^3+Bx^5+Cx^7+Dx^9}{\sqrt{a+bx^2}}\,dx$



## Optimal result

Integrand size = 33, antiderivative size = 168

$$\int \frac{Ax^3+Bx^5+Cx^7+Dx^9}{\sqrt{a+bx^2}}\,dx = -\frac{a(Ab^3-a(b^2B-abC+a^2D))\sqrt{a+bx^2}}{b^5} + \frac{(Ab^3-a(2b^2B-3abC+4a^2D))\,(a+bx^2)^{3/2}}{3b^5} + \frac{(b^2B-3abC+6a^2D)\,(a+bx^2)^{5/2}}{5b^5} + \frac{(bC-4aD)\,(a+bx^2)^{7/2}}{7b^5} + \frac{D(a+bx^2)^{9/2}}{9b^5}$$

output
```
-a*(A*b^3-a*(B*b^2-C*a*b+D*a^2))*(b*x^2+a)^(1/2)/b^5+1/3*(A*b^3-a*(2*B*b^2
-3*C*a*b+4*D*a^2))*(b*x^2+a)^(3/2)/b^5+1/5*(B*b^2-3*C*a*b+6*D*a^2)*(b*x^2+
a)^(5/2)/b^5+1/7*(C*b-4*D*a)*(b*x^2+a)^(7/2)/b^5+1/9*D*(b*x^2+a)^(9/2)/b^5
```

## Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.73

$$\int \frac{Ax^3+Bx^5+Cx^7+Dx^9}{\sqrt{a+bx^2}}\,dx$$
$$= \frac{\sqrt{a+bx^2}(128a^4D-16a^3b(9C+4Dx^2)+24a^2b^2(7B+3Cx^2+2Dx^4)-2ab^3(105A+42Bx^2+27Cx^4}{315b^5}$$

input
```
Integrate[(A*x^3 + B*x^5 + C*x^7 + D*x^9)/Sqrt[a + b*x^2],x]
```

output
```
(Sqrt[a + b*x^2]*(128*a^4*D - 16*a^3*b*(9*C + 4*D*x^2) + 24*a^2*b^2*(7*B +
 3*C*x^2 + 2*D*x^4) - 2*a*b^3*(105*A + 42*B*x^2 + 27*C*x^4 + 20*D*x^6) + b
^4*x^2*(105*A + 63*B*x^2 + 45*C*x^4 + 35*D*x^6)))/(315*b^5)
```

## Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {2029, 2331, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{Ax^3+Bx^5+Cx^7+Dx^9}{\sqrt{a+bx^2}}\,dx$$
$$\Big\downarrow \text{2029}$$
$$\int \frac{x^3\left(A+Bx^2+Cx^4+Dx^6\right)}{\sqrt{a+bx^2}}dx$$
$$\Big\downarrow \text{2331}$$
$$\frac{1}{2}\int \frac{x^2\left(Dx^6+Cx^4+Bx^2+A\right)}{\sqrt{bx^2+a}}dx^2$$
$$\Big\downarrow \text{2123}$$

$$\frac{1}{2}\int \left(\frac{D(bx^2+a)^{7/2}}{b^4}+\frac{(bC-4aD)\,(bx^2+a)^{5/2}}{b^4}+\frac{(6Da^2-3bCa+b^2B)\,(bx^2+a)^{3/2}}{b^4}+\frac{(Ab^3-a(4Da^2-3b\right.$$

$\downarrow$ 2009

$$\frac{1}{2}\left(\frac{2(a+bx^2)^{3/2}\,(Ab^3-a(4a^2D-3abC+2b^2B))}{3b^5}-\frac{2a\sqrt{a+bx^2}(Ab^3-a(a^2D-abC+b^2B))}{b^5}+\frac{2(a+bx^2)^5}{}\right.$$

input
```
Int[(A*x^3 + B*x^5 + C*x^7 + D*x^9)/Sqrt[a + b*x^2],x]
```

output
```
((-2*a*(A*b^3 - a*(b^2*B - a*b*C + a^2*D))*Sqrt[a + b*x^2])/b^5 + (2*(A*b^
3 - a*(2*b^2*B - 3*a*b*C + 4*a^2*D))*(a + b*x^2)^(3/2))/(3*b^5) + (2*(b^2*
B - 3*a*b*C + 6*a^2*D)*(a + b*x^2)^(5/2))/(5*b^5) + (2*(b*C - 4*a*D)*(a +
b*x^2)^(7/2))/(7*b^5) + (2*D*(a + b*x^2)^(9/2))/(9*b^5))/2
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2029
```
Int[(Fx_.)*((d_.)*(x_)^(q_.) + (a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.) + (c_.)*
(x_)^(t_.))^(p_.), x_Symbol] :> Int[x^(p*r)*(a + b*x^(s - r) + c*x^(t - r)
+ d*x^(q - r))^p*Fx, x] /; FreeQ[{a, b, c, d, r, s, t, q}, x] && IntegerQ[p
] && PosQ[s - r] && PosQ[t - r] && PosQ[q - r] &&  !(EqQ[p, 1] && EqQ[u, 1]
)
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

rule 2331
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/2   S
ubst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
 FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

## Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.66

| method | result |
|---|---|
| pseudoelliptic | $-\frac{2\left(-\frac{\left(\frac{1}{3}Dx^6+\frac{3}{7}Cx^4+\frac{3}{5}x^2B+A\right)x^2b^4}{2}+a\left(\frac{4}{21}Dx^6+\frac{9}{35}Cx^4+\frac{2}{5}x^2B+A\right)b^3-\frac{4\left(\frac{2}{7}Dx^4+\frac{3}{7}Cx^2+B\right)a^2b^2}{5}+\frac{24\left(\frac{4Dx^2}{9}+C\right)a^3b}{35}-\right.}{3b^5}$ |
| gosper | $-\frac{\sqrt{bx^2+a}\,(-35Dx^8b^4-45Cb^4x^6+40Dab^3x^6-63Bx^4b^4+54Cab^3x^4-48Da^2b^2x^4-105Ab^4x^2+84Bab^3x^2-72Ca^2b^2x^2}{315b^5}$ |
| trager | $-\frac{\sqrt{bx^2+a}\,(-35Dx^8b^4-45Cb^4x^6+40Dab^3x^6-63Bx^4b^4+54Cab^3x^4-48Da^2b^2x^4-105Ab^4x^2+84Bab^3x^2-72Ca^2b^2x^2}{315b^5}$ |
| orering | $-\frac{(-35Dx^8b^4-45Cb^4x^6+40Dab^3x^6-63Bx^4b^4+54Cab^3x^4-48Da^2b^2x^4-105Ab^4x^2+84Bab^3x^2-72Ca^2b^2x^2+64Da^3}{315b^5(Dx^6+Cx^4+x^2B+A)x^3}$ |
| default | $A\left(\frac{x^2\sqrt{bx^2+a}}{3b}-\frac{2a\sqrt{bx^2+a}}{3b^2}\right)+B\left(\frac{x^4\sqrt{bx^2+a}}{5b}-\frac{4a\left(\frac{x^2\sqrt{bx^2+a}}{3b}-\frac{2a\sqrt{bx^2+a}}{3b^2}\right)}{5b}\right)+C\left(\frac{x^6\sqrt{bx^2+a}}{7b}-\frac{6a}{}\right.$ |

input
```
int((D*x^9+C*x^7+B*x^5+A*x^3)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(-1/2*(1/3*D*x^6+3/7*C*x^4+3/5*x^2*B+A)*x^2*b^4+a*(4/21*D*x^6+9/35*C*
x^4+2/5*x^2*B+A)*b^3-4/5*(2/7*D*x^4+3/7*C*x^2+B)*a^2*b^2+24/35*(4/9*D*x^2+
C)*a^3*b-64/105*D*a^4)*(b*x^2+a)^(1/2)/b^5
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.80

$$\int \frac{Ax^3+Bx^5+Cx^7+Dx^9}{\sqrt{a+bx^2}}\,dx = \frac{(35\,Db^4x^8-5\,(8\,Dab^3-9\,Cb^4)x^6+128\,Da^4-144\,Ca^3b+168\,Ba^2b^2-210\,Aab^3+3\,(16\,Da^2b^2-18\,C}{315\,b^5}$$

input
```
integrate((D*x^9+C*x^7+B*x^5+A*x^3)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
1/315*(35*D*b^4*x^8 - 5*(8*D*a*b^3 - 9*C*b^4)*x^6 + 128*D*a^4 - 144*C*a^3*
b + 168*B*a^2*b^2 - 210*A*a*b^3 + 3*(16*D*a^2*b^2 - 18*C*a*b^3 + 21*B*b^4)
*x^4 - (64*D*a^3*b - 72*C*a^2*b^2 + 84*B*a*b^3 - 105*A*b^4)*x^2)*sqrt(b*x^
2 + a)/b^5
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 340 vs. $2(160) = 320$.

Time = 0.37 (sec) , antiderivative size = 340, normalized size of antiderivative = 2.02

$$\int \frac{Ax^3 + Bx^5 + Cx^7 + Dx^9}{\sqrt{a + bx^2}}\,dx$$
$$= \begin{cases} -\frac{2Aa\sqrt{a+bx^2}}{3b^2} + \frac{Ax^2\sqrt{a+bx^2}}{3b} + \frac{8Ba^2\sqrt{a+bx^2}}{15b^3} - \frac{4Bax^2\sqrt{a+bx^2}}{15b^2} + \frac{Bx^4\sqrt{a+bx^2}}{5b} - \frac{16Ca^3\sqrt{a+bx^2}}{35b^4} + \frac{8Ca^2x^2\sqrt{a+bx^2}}{35b^3} - \frac{6Cax}{} \\ \frac{\frac{Ax^4}{4} + \frac{Bx^6}{6} + \frac{Cx^8}{8} + \frac{Dx^{10}}{10}}{\sqrt{a}} \end{cases}$$

input
```
integrate((D*x**9+C*x**7+B*x**5+A*x**3)/(b*x**2+a)**(1/2),x)
```

output
```
Piecewise((-2*A*a*sqrt(a + b*x**2)/(3*b**2) + A*x**2*sqrt(a + b*x**2)/(3*b
) + 8*B*a**2*sqrt(a + b*x**2)/(15*b**3) - 4*B*a*x**2*sqrt(a + b*x**2)/(15*
b**2) + B*x**4*sqrt(a + b*x**2)/(5*b) - 16*C*a**3*sqrt(a + b*x**2)/(35*b**
4) + 8*C*a**2*x**2*sqrt(a + b*x**2)/(35*b**3) - 6*C*a*x**4*sqrt(a + b*x**2
)/(35*b**2) + C*x**6*sqrt(a + b*x**2)/(7*b) + 128*D*a**4*sqrt(a + b*x**2)/
(315*b**5) - 64*D*a**3*x**2*sqrt(a + b*x**2)/(315*b**4) + 16*D*a**2*x**4*s
qrt(a + b*x**2)/(105*b**3) - 8*D*a*x**6*sqrt(a + b*x**2)/(63*b**2) + D*x**
8*sqrt(a + b*x**2)/(9*b), Ne(b, 0)), ((A*x**4/4 + B*x**6/6 + C*x**8/8 + D*
x**10/10)/sqrt(a), True))
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.57

$$\int \frac{Ax^3+Bx^5+Cx^7+Dx^9}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{bx^2+a}Dx^8}{9\,b} - \frac{8\,\sqrt{bx^2+a}Dax^6}{63\,b^2} + \frac{\sqrt{bx^2+a}Cx^6}{7\,b} + \frac{16\,\sqrt{bx^2+a}Da^2x^4}{105\,b^3} - \frac{6\,\sqrt{bx^2+a}Cax^4}{35\,b^2} + \frac{\sqrt{bx^2+a}Bx^4}{5\,b} - \frac{64\,\sqrt{bx^2+a}Da^3x^2}{315\,b^4} + \frac{8\,\sqrt{bx^2+a}Ca^2x^2}{35\,b^3} - \frac{4\,\sqrt{bx^2+a}Bax^2}{15\,b^2} + \frac{\sqrt{bx^2+a}Ax^2}{3\,b} + \frac{128\,\sqrt{bx^2+a}Da^4}{315\,b^5} - \frac{16\,\sqrt{bx^2+a}Ca^3}{35\,b^4} + \frac{8\,\sqrt{bx^2+a}Ba^2}{15\,b^3} - \frac{2\,\sqrt{bx^2+a}Aa}{3\,b^2}$$

input
```
integrate((D*x^9+C*x^7+B*x^5+A*x^3)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/9*sqrt(b*x^2 + a)*D*x^8/b - 8/63*sqrt(b*x^2 + a)*D*a*x^6/b^2 + 1/7*sqrt(
b*x^2 + a)*C*x^6/b + 16/105*sqrt(b*x^2 + a)*D*a^2*x^4/b^3 - 6/35*sqrt(b*x^
2 + a)*C*a*x^4/b^2 + 1/5*sqrt(b*x^2 + a)*B*x^4/b - 64/315*sqrt(b*x^2 + a)*
D*a^3*x^2/b^4 + 8/35*sqrt(b*x^2 + a)*C*a^2*x^2/b^3 - 4/15*sqrt(b*x^2 + a)*
B*a*x^2/b^2 + 1/3*sqrt(b*x^2 + a)*A*x^2/b + 128/315*sqrt(b*x^2 + a)*D*a^4/
b^5 - 16/35*sqrt(b*x^2 + a)*C*a^3/b^4 + 8/15*sqrt(b*x^2 + a)*B*a^2/b^3 - 2
/3*sqrt(b*x^2 + a)*A*a/b^2
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.14

$$\int \frac{Ax^3+Bx^5+Cx^7+Dx^9}{\sqrt{a+bx^2}}\,dx = \frac{(Da^4-Ca^3b+Ba^2b^2-Aab^3)\sqrt{bx^2+a}}{b^5} + \frac{35\,(bx^2+a)^{\frac{9}{2}}D - 180\,(bx^2+a)^{\frac{7}{2}}Da + 378\,(bx^2+a)^{\frac{5}{2}}Da^2 - 420\,(bx^2+a)^{\frac{3}{2}}Da^3 + 45\,(bx^2+a)^{\frac{7}{2}}Cb - \cdots}{3\cdots}$$

input
```
integrate((D*x^9+C*x^7+B*x^5+A*x^3)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
(D*a^4 - C*a^3*b + B*a^2*b^2 - A*a*b^3)*sqrt(b*x^2 + a)/b^5 + 1/315*(35*(b
*x^2 + a)^(9/2)*D - 180*(b*x^2 + a)^(7/2)*D*a + 378*(b*x^2 + a)^(5/2)*D*a^
2 - 420*(b*x^2 + a)^(3/2)*D*a^3 + 45*(b*x^2 + a)^(7/2)*C*b - 189*(b*x^2 +
a)^(5/2)*C*a*b + 315*(b*x^2 + a)^(3/2)*C*a^2*b + 63*(b*x^2 + a)^(5/2)*B*b^
2 - 210*(b*x^2 + a)^(3/2)*B*a*b^2 + 105*(b*x^2 + a)^(3/2)*A*b^3)/b^5
```

## Mupad [B] (verification not implemented)

Time = 1.37 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.23

$$\int \frac{Ax^3+Bx^5+Cx^7+Dx^9}{\sqrt{a+bx^2}}\,dx = \sqrt{b\,x^2+a}\left(\frac{8\,B\,a^2}{15\,b^3}+\frac{B\,x^4}{5\,b}-\frac{4\,B\,a\,x^2}{15\,b^2}\right) - \sqrt{b\,x^2+a}\left(\frac{16\,C\,a^3}{35\,b^4}-\frac{C\,x^6}{7\,b}+\frac{6\,C\,a\,x^4}{35\,b^2}-\frac{8\,C\,a^2\,x^2}{35\,b^3}\right) + \frac{x^8\,\sqrt{b\,x^2+a}\,D}{9\,b} - \frac{A\,\sqrt{b\,x^2+a}\,(2\,a-b\,x^2)}{3\,b^2} - \frac{4\,a\,D\left(\frac{2\,(b\,x^2+a)^{7/2}}{7\,b^4}-\frac{6\,a\,(b\,x^2+a)^{5/2}}{5\,b^4}-\frac{2\,a^3\,\sqrt{b\,x^2+a}}{b^4}+\frac{2\,a^2\,(b\,x^2+a)^{3/2}}{b^4}\right)}{9\,b}$$

input
```
int((A*x^3 + B*x^5 + C*x^7 + x^9*D)/(a + b*x^2)^(1/2),x)
```

output
```
(a + b*x^2)^(1/2)*((8*B*a^2)/(15*b^3) + (B*x^4)/(5*b) - (4*B*a*x^2)/(15*b^
2)) - (a + b*x^2)^(1/2)*((16*C*a^3)/(35*b^4) - (C*x^6)/(7*b) + (6*C*a*x^4)
/(35*b^2) - (8*C*a^2*x^2)/(35*b^3)) + (x^8*(a + b*x^2)^(1/2)*D)/(9*b) - (A
*(a + b*x^2)^(1/2)*(2*a - b*x^2))/(3*b^2) - (4*a*D*((2*(a + b*x^2)^(7/2))/
(7*b^4) - (6*a*(a + b*x^2)^(5/2))/(5*b^4) - (2*a^3*(a + b*x^2)^(1/2))/b^4
+ (2*a^2*(a + b*x^2)^(3/2))/b^4))/(9*b)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.74

$$\int \frac{Ax^3 + Bx^5 + Cx^7 + Dx^9}{\sqrt{a + bx^2}}\,dx$$
$$= \frac{\sqrt{b\,x^2 + a}\,(35b^4d\,x^8 - 40a\,b^3d\,x^6 + 45b^4c\,x^6 + 48a^2b^2d\,x^4 - 54a\,b^3c\,x^4 + 63b^5x^4 - 64a^3bd\,x^2 + 72a^2b^2c\,x^2}{315b^5}$$

input
```
int((D*x^9+C*x^7+B*x^5+A*x^3)/(b*x^2+a)^(1/2),x)
```

output
```
(sqrt(a + b*x**2)*(128*a**4*d - 144*a**3*b*c - 64*a**3*b*d*x**2 - 42*a**2*
b**3 + 72*a**2*b**2*c*x**2 + 48*a**2*b**2*d*x**4 + 21*a*b**4*x**2 - 54*a*b
**3*c*x**4 - 40*a*b**3*d*x**6 + 63*b**5*x**4 + 45*b**4*c*x**6 + 35*b**4*d*
x**8))/(315*b**5)
```

# 3.251 $\int \frac{Ax+Bx^3+Cx^5+Dx^7}{\sqrt{a+bx^2}}\,dx$



## Optimal result

Integrand size = 31, antiderivative size = 121

$$\int \frac{Ax+Bx^3+Cx^5+Dx^7}{\sqrt{a+bx^2}}\,dx = \frac{(Ab^3-a(b^2B-abC+a^2D))\sqrt{a+bx^2}}{b^4} + \frac{(b^2B-2abC+3a^2D)(a+bx^2)^{3/2}}{3b^4} + \frac{(bC-3aD)(a+bx^2)^{5/2}}{5b^4} + \frac{D(a+bx^2)^{7/2}}{7b^4}$$

output
```
(A*b^3-a*(B*b^2-C*a*b+D*a^2))*(b*x^2+a)^(1/2)/b^4+1/3*(B*b^2-2*C*a*b+3*D*a
^2)*(b*x^2+a)^(3/2)/b^4+1/5*(C*b-3*D*a)*(b*x^2+a)^(5/2)/b^4+1/7*D*(b*x^2+a
)^(7/2)/b^4
```

## Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.76

$$\int \frac{Ax+Bx^3+Cx^5+Dx^7}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{a+bx^2}(105Ab^3-48a^3D+8a^2b(7C+3Dx^2)-2ab^2(35B+14Cx^2+9Dx^4)+b^3x^2(35B+21Cx^2+}{105b^4}$$

input
```
Integrate[(A*x + B*x^3 + C*x^5 + D*x^7)/Sqrt[a + b*x^2],x]
```

output
```
(Sqrt[a + b*x^2]*(105*A*b^3 - 48*a^3*D + 8*a^2*b*(7*C + 3*D*x^2) - 2*a*b^2
*(35*B + 14*C*x^2 + 9*D*x^4) + b^3*x^2*(35*B + 21*C*x^2 + 15*D*x^4)))/(105
*b^4)
```

## Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {2029, 2331, 2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{Ax+Bx^3+Cx^5+Dx^7}{\sqrt{a+bx^2}}dx$$

$$\Big\downarrow \text{2029}$$

$$\int \frac{x\left(A+Bx^2+Cx^4+Dx^6\right)}{\sqrt{a+bx^2}}dx$$

$$\Big\downarrow \text{2331}$$

$$\frac{1}{2}\int \frac{Dx^6+Cx^4+Bx^2+A}{\sqrt{bx^2+a}}dx^2$$

$$\Big\downarrow \text{2389}$$

$$\frac{1}{2}\int \left(\frac{D\left(bx^2+a\right)^{5/2}}{b^3}+\frac{(bC-3aD)\left(bx^2+a\right)^{3/2}}{b^3}+\frac{\left(3Da^2-2bCa+b^2B\right)\sqrt{bx^2+a}}{b^3}+\frac{Ab^3-a\left(Da^2-bCa+\right.}{b^3\sqrt{bx^2+a}}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{2\sqrt{a+bx^2}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{b^4}+\frac{2\left(a+bx^2\right)^{3/2}\left(3a^2D-2abC+b^2B\right)}{3b^4}+\frac{2\left(a+bx^2\right)^{5/2}(bC-3a}{5b^4}\right.$$

input
```
Int[(A*x + B*x^3 + C*x^5 + D*x^7)/Sqrt[a + b*x^2],x]
```

output
```
((2*(A*b^3 - a*(b^2*B - a*b*C + a^2*D))*Sqrt[a + b*x^2])/b^4 + (2*(b^2*B -
 2*a*b*C + 3*a^2*D)*(a + b*x^2)^(3/2))/(3*b^4) + (2*(b*C - 3*a*D)*(a + b*x
^2)^(5/2))/(5*b^4) + (2*D*(a + b*x^2)^(7/2))/(7*b^4))/2
```

**Defintions of rubi rules used**

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2029
```
Int[(Fx_.)*((d_.)*(x_)^(q_.) + (a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.) + (c_.)*
(x_)^(t_.))^(p_.), x_Symbol] :> Int[x^(p*r)*(a + b*x^(s - r) + c*x^(t - r)
+ d*x^(q - r))^p*Fx, x] /; FreeQ[{a, b, c, d, r, s, t, q}, x] && IntegerQ[p
] && PosQ[s - r] && PosQ[t - r] && PosQ[q - r] &&  !(EqQ[p, 1] && EqQ[u, 1]
)
```

rule 2331
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/2   S
ubst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
 FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

## Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.65

| method | result |
|---|---|
| pseudoelliptic | $\frac{\left(\left(\frac{1}{7}Dx^6+\frac{1}{5}C\,x^4+\frac{1}{3}x^2B+A\right)b^3-\frac{2\left(\frac{9}{35}Dx^4+\frac{2}{5}C\,x^2+B\right)a\,b^2}{3}+\frac{8\left(\frac{3Dx^2}{7}+C\right)a^2b}{15}-\frac{16a^3D}{35}\right)\sqrt{b\,x^2+a}}{b^4}$ |
| gosper | $\frac{\sqrt{b\,x^2+a}\,(15b^3Dx^6+21b^3C\,x^4-18Da\,b^2x^4+35b^3B\,x^2-28Ca\,b^2x^2+24Da^2b\,x^2+105b^3A-70a\,b^2B+56a^2bC-48a^3D)}{105b^4}$ |
| trager | $\frac{\sqrt{b\,x^2+a}\,(15b^3Dx^6+21b^3C\,x^4-18Da\,b^2x^4+35b^3B\,x^2-28Ca\,b^2x^2+24Da^2b\,x^2+105b^3A-70a\,b^2B+56a^2bC-48a^3D)}{105b^4}$ |
| orering | $\frac{(15b^3Dx^6+21b^3C\,x^4-18Da\,b^2x^4+35b^3B\,x^2-28Ca\,b^2x^2+24Da^2b\,x^2+105b^3A-70a\,b^2B+56a^2bC-48a^3D)\sqrt{b\,x^2+a}\,(Dx}{105b^4(Dx^6+C\,x^4+x^2B+A)x}$ |
| default | $B\left(\frac{x^2\sqrt{b\,x^2+a}}{3b}-\frac{2a\sqrt{b\,x^2+a}}{3b^2}\right)+C\left(\frac{x^4\sqrt{b\,x^2+a}}{5b}-\frac{4a\left(\frac{x^2\sqrt{b\,x^2+a}}{3b}-\frac{2a\sqrt{b\,x^2+a}}{3b^2}\right)}{5b}\right)+D\left(\frac{x^6\sqrt{b\,x^2+a}}{7b}-\frac{6a}{}\right.$ |

input
```
int((D*x^7+C*x^5+B*x^3+A*x)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((1/7*D*x^6+1/5*C*x^4+1/3*x^2*B+A)*b^3-2/3*(9/35*D*x^4+2/5*C*x^2+B)*a*b^2+
8/15*(3/7*D*x^2+C)*a^2*b-16/35*a^3*D)*(b*x^2+a)^(1/2)/b^4
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.78

$$\int \frac{Ax+Bx^3+Cx^5+Dx^7}{\sqrt{a+bx^2}}\,dx = \frac{(15\,Db^3x^6-3\,(6\,Dab^2-7\,Cb^3)x^4-48\,Da^3+56\,Ca^2b-70\,Bab^2+105\,Ab^3+(24\,Da^2b-28\,Cab^2+35}{105\,b^4}$$

input
```
integrate((D*x^7+C*x^5+B*x^3+A*x)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
1/105*(15*D*b^3*x^6 - 3*(6*D*a*b^2 - 7*C*b^3)*x^4 - 48*D*a^3 + 56*C*a^2*b
- 70*B*a*b^2 + 105*A*b^3 + (24*D*a^2*b - 28*C*a*b^2 + 35*B*b^3)*x^2)*sqrt(
b*x^2 + a)/b^4
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 238 vs. $2(110) = 220$.

Time = 0.31 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.97

$$\int \frac{Ax + Bx^3 + Cx^5 + Dx^7}{\sqrt{a + bx^2}}\,dx = \begin{cases} \frac{A\sqrt{a+bx^2}}{b} - \frac{2Ba\sqrt{a+bx^2}}{3b^2} + \frac{Bx^2\sqrt{a+bx^2}}{3b} + \frac{8Ca^2\sqrt{a+bx^2}}{15b^3} - \frac{4Cax^2\sqrt{a+bx^2}}{15b^2} + \frac{Cx^4\sqrt{a+bx^2}}{5b} - \frac{16Da^3\sqrt{a+bx^2}}{35b^4} + \frac{8Da^2x^2\sqrt{a+b}}{35b^3} \\ \frac{\frac{Ax^2}{2} + \frac{Bx^4}{4} + \frac{Cx^6}{6} + \frac{Dx^8}{8}}{\sqrt{a}} \end{cases}$$

input
```
integrate((D*x**7+C*x**5+B*x**3+A*x)/(b*x**2+a)**(1/2),x)
```

output
```
Piecewise((A*sqrt(a + b*x**2)/b - 2*B*a*sqrt(a + b*x**2)/(3*b**2) + B*x**2
*sqrt(a + b*x**2)/(3*b) + 8*C*a**2*sqrt(a + b*x**2)/(15*b**3) - 4*C*a*x**2
*sqrt(a + b*x**2)/(15*b**2) + C*x**4*sqrt(a + b*x**2)/(5*b) - 16*D*a**3*sq
rt(a + b*x**2)/(35*b**4) + 8*D*a**2*x**2*sqrt(a + b*x**2)/(35*b**3) - 6*D*
a*x**4*sqrt(a + b*x**2)/(35*b**2) + D*x**6*sqrt(a + b*x**2)/(7*b), Ne(b, 0
)), ((A*x**2/2 + B*x**4/4 + C*x**6/6 + D*x**8/8)/sqrt(a), True))
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.49

$$\begin{aligned}\int \frac{Ax + Bx^3 + Cx^5 + Dx^7}{\sqrt{a + bx^2}}\,dx = {} & \frac{\sqrt{bx^2 + a}Dx^6}{7\,b} - \frac{6\,\sqrt{bx^2 + a}Dax^4}{35\,b^2} + \frac{\sqrt{bx^2 + a}Cx^4}{5\,b} \\ & + \frac{8\,\sqrt{bx^2 + a}Da^2x^2}{35\,b^3} - \frac{4\,\sqrt{bx^2 + a}Cax^2}{15\,b^2} \\ & + \frac{\sqrt{bx^2 + a}Bx^2}{3\,b} - \frac{16\,\sqrt{bx^2 + a}Da^3}{35\,b^4} \\ & + \frac{8\,\sqrt{bx^2 + a}Ca^2}{15\,b^3} - \frac{2\,\sqrt{bx^2 + a}Ba}{3\,b^2} + \frac{\sqrt{bx^2 + a}A}{b}\end{aligned}$$

input
```
integrate((D*x^7+C*x^5+B*x^3+A*x)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/7*sqrt(b*x^2 + a)*D*x^6/b - 6/35*sqrt(b*x^2 + a)*D*a*x^4/b^2 + 1/5*sqrt(
b*x^2 + a)*C*x^4/b + 8/35*sqrt(b*x^2 + a)*D*a^2*x^2/b^3 - 4/15*sqrt(b*x^2
+ a)*C*a*x^2/b^2 + 1/3*sqrt(b*x^2 + a)*B*x^2/b - 16/35*sqrt(b*x^2 + a)*D*a
^3/b^4 + 8/15*sqrt(b*x^2 + a)*C*a^2/b^3 - 2/3*sqrt(b*x^2 + a)*B*a/b^2 + sq
rt(b*x^2 + a)*A/b
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.06

$$\int \frac{Ax + Bx^3 + Cx^5 + Dx^7}{\sqrt{a + bx^2}}\,dx = -\frac{(Da^3 - Ca^2b + Bab^2 - Ab^3)\sqrt{bx^2 + a}}{b^4} + \frac{15\,(bx^2 + a)^{\frac{7}{2}}D - 63\,(bx^2 + a)^{\frac{5}{2}}Da + 105\,(bx^2 + a)^{\frac{3}{2}}Da^2 + 21\,(bx^2 + a)^{\frac{5}{2}}Cb - 70\,(bx^2 + a)^{\frac{3}{2}}Cab + 35}{105\,b^4}$$

input
```
integrate((D*x^7+C*x^5+B*x^3+A*x)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
-(D*a^3 - C*a^2*b + B*a*b^2 - A*b^3)*sqrt(b*x^2 + a)/b^4 + 1/105*(15*(b*x^
2 + a)^(7/2)*D - 63*(b*x^2 + a)^(5/2)*D*a + 105*(b*x^2 + a)^(3/2)*D*a^2 +
21*(b*x^2 + a)^(5/2)*C*b - 70*(b*x^2 + a)^(3/2)*C*a*b + 35*(b*x^2 + a)^(3/
2)*B*b^2)/b^4
```

## Mupad [B] (verification not implemented)

Time = 1.98 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.09

$$\int \frac{Ax + Bx^3 + Cx^5 + Dx^7}{\sqrt{a + bx^2}}\,dx = \sqrt{b\,x^2 + a}\left(\frac{8\,C\,a^2}{15\,b^3} + \frac{C\,x^4}{5\,b} - \frac{4\,C\,a\,x^2}{15\,b^2}\right) + \frac{A\,\sqrt{b\,x^2 + a}}{b} - \frac{B\,\sqrt{b\,x^2 + a}\,(2\,a - b\,x^2)}{3\,b^2} - \frac{\sqrt{b\,x^2 + a}\,D\left(6\,a\,(b\,x^2 + a)^2 - 20\,a^2\,(b\,x^2 + a) + 30\,a^3 - 5\,b^3\,x^6\right)}{35\,b^4}$$

input
```
int((A*x + B*x^3 + C*x^5 + x^7*D)/(a + b*x^2)^(1/2),x)
```

output
```
(a + b*x^2)^(1/2)*((8*C*a^2)/(15*b^3) + (C*x^4)/(5*b) - (4*C*a*x^2)/(15*b^
2)) + (A*(a + b*x^2)^(1/2))/b - (B*(a + b*x^2)^(1/2)*(2*a - b*x^2))/(3*b^2
) - ((a + b*x^2)^(1/2)*D*(6*a*(a + b*x^2)^2 - 20*a^2*(a + b*x^2) + 30*a^3
- 5*b^3*x^6))/(35*b^4)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.74

$$\int \frac{Ax + Bx^3 + Cx^5 + Dx^7}{\sqrt{a + bx^2}}\,dx$$
$$= \frac{\sqrt{b\,x^2 + a}\,(15b^3d\,x^6 - 18a\,b^2d\,x^4 + 21b^3c\,x^4 + 24a^2bd\,x^2 - 28a\,b^2c\,x^2 + 35b^4x^2 - 48a^3d + 56a^2bc + 35a\,b}{105b^4}$$

input
```
int((D*x^7+C*x^5+B*x^3+A*x)/(b*x^2+a)^(1/2),x)
```

output
```
(sqrt(a + b*x**2)*( - 48*a**3*d + 56*a**2*b*c + 24*a**2*b*d*x**2 + 35*a*b*
*3 - 28*a*b**2*c*x**2 - 18*a*b**2*d*x**4 + 35*b**4*x**2 + 21*b**3*c*x**4 +
 15*b**3*d*x**6))/(105*b**4)
```

# 3.252 $\int (cx)^m \left(a+bx^2\right)^3 \left(A+Bx^2+Cx^4+Dx^6\right) dx$



## Optimal result

Integrand size = 32, antiderivative size = 204

$$\int (cx)^m \left(a+bx^2\right)^3 \left(A+Bx^2+Cx^4+Dx^6\right) dx = \frac{a^3A(cx)^{1+m}}{c(1+m)}+\frac{a^2(3Ab+aB)(cx)^{3+m}}{c^3(3+m)}+\frac{a(3Ab^2+a(3bB+aC))\,(cx)^{5+m}}{c^5(5+m)} + \frac{(Ab^3+a(3b^2B+3abC+a^2D))\,(cx)^{7+m}}{c^7(7+m)} + \frac{b(b^2B+3abC+3a^2D)\,(cx)^{9+m}}{c^9(9+m)}+\frac{b^2(bC+3aD)(cx)^{11+m}}{c^{11}(11+m)}+\frac{b^3D(cx)^{13+m}}{c^{13}(13+m)}$$

output
```
a^3*A*(c*x)^(1+m)/c/(1+m)+a^2*(3*A*b+B*a)*(c*x)^(3+m)/c^3/(3+m)+a*(3*A*b^2
+a*(3*B*b+C*a))*(c*x)^(5+m)/c^5/(5+m)+(A*b^3+a*(3*B*b^2+3*C*a*b+D*a^2))*(c
*x)^(7+m)/c^7/(7+m)+b*(B*b^2+3*C*a*b+3*D*a^2)*(c*x)^(9+m)/c^9/(9+m)+b^2*(C
*b+3*D*a)*(c*x)^(11+m)/c^11/(11+m)+b^3*D*(c*x)^(13+m)/c^13/(13+m)
```

## Mathematica [A] (verified)

Time = 1.30 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.78

$$\int (cx)^m \left(a+bx^2\right)^3 \left(A+Bx^2+Cx^4+Dx^6\right) \, dx = x(cx)^m \left(\frac{a^3A}{1+m}+\frac{a^2(3Ab+aB)x^2}{3+m}+\frac{a(3Ab^2+a(3bB+aC))\,x^4}{5+m}+\frac{(Ab^3+a(3b^2B+3abC+a^2D))\,x^6}{7+m}+\frac{b(b^2B+3abC+3a^2D)\,x^8}{9+m}+\frac{b^2(bC+3aD)x^{10}}{11+m}+\frac{b^3Dx^{12}}{13+m}\right)$$

input
```
Integrate[(c*x)^m*(a + b*x^2)^3*(A + B*x^2 + C*x^4 + D*x^6),x]
```

output
```
x*(c*x)^m*((a^3*A)/(1 + m) + (a^2*(3*A*b + a*B)*x^2)/(3 + m) + (a*(3*A*b^2
 + a*(3*b*B + a*C))*x^4)/(5 + m) + ((A*b^3 + a*(3*b^2*B + 3*a*b*C + a^2*D)
)*x^6)/(7 + m) + (b*(b^2*B + 3*a*b*C + 3*a^2*D)*x^8)/(9 + m) + (b^2*(b*C +
 3*a*D)*x^10)/(11 + m) + (b^3*D*x^12)/(13 + m))
```

## Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^3 (cx)^m \left(A+Bx^2+Cx^4+Dx^6\right) \, dx$$

$$\Big\downarrow \text{2333}$$

$$\int \left(a^3A(cx)^m+\frac{(cx)^{m+6}\left(a\left(a^2D+3abC+3b^2B\right)+Ab^3\right)}{c^6}+\frac{a^2(cx)^{m+2}(aB+3Ab)}{c^2}+\frac{b(cx)^{m+8}\left(3a^2D+3abC\right.}{c^8}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3A(cx)^{m+1}}{c(m+1)}+\frac{(cx)^{m+7}\left(a\left(a^2D+3abC+3b^2B\right)+Ab^3\right)}{c^7(m+7)}+\frac{a^2(cx)^{m+3}(aB+3Ab)}{c^3(m+3)}+\frac{b(cx)^{m+9}\left(3a^2D+3abC+b^2B\right)}{c^9(m+9)}+\frac{a(cx)^{m+5}\left(a(aC+3bB)+3Ab^2\right)}{c^5(m+5)}+\frac{b^2(cx)^{m+11}(3aD+bC)}{c^{11}(m+11)}+\frac{b^3D(cx)^{m+13}}{c^{13}(m+13)}$$

input
```
Int[(c*x)^m*(a + b*x^2)^3*(A + B*x^2 + C*x^4 + D*x^6),x]
```

output
```
(a^3*A*(c*x)^(1 + m))/(c*(1 + m)) + (a^2*(3*A*b + a*B)*(c*x)^(3 + m))/(c^3
*(3 + m)) + (a*(3*A*b^2 + a*(3*b*B + a*C))*(c*x)^(5 + m))/(c^5*(5 + m)) +
((A*b^3 + a*(3*b^2*B + 3*a*b*C + a^2*D))*(c*x)^(7 + m))/(c^7*(7 + m)) + (b
*(b^2*B + 3*a*b*C + 3*a^2*D)*(c*x)^(9 + m))/(c^9*(9 + m)) + (b^2*(b*C + 3*
a*D)*(c*x)^(11 + m))/(c^11*(11 + m)) + (b^3*D*(c*x)^(13 + m))/(c^13*(13 +
m))
```

**Defintions of rubi rules used**

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1334 vs. 2(204) = 408.

Time = 0.55 (sec) , antiderivative size = 1335, normalized size of antiderivative = 6.54

| method | result | size |
|---|---|---|
| gosper | Expression too large to display | 1335 |
| orering | Expression too large to display | 1335 |
| parallelrisch | Expression too large to display | 1896 |

input
```
int((c*x)^m*(b*x^2+a)^3*(D*x^6+C*x^4+B*x^2+A),x,method=_RETURNVERBOSE)
```

output
```
x*(D*b^3*m^6*x^12+36*D*b^3*m^5*x^12+C*b^3*m^6*x^10+3*D*a*b^2*m^6*x^10+505*
D*b^3*m^4*x^12+38*C*b^3*m^5*x^10+114*D*a*b^2*m^5*x^10+3480*D*b^3*m^3*x^12+
B*b^3*m^6*x^8+3*C*a*b^2*m^6*x^8+555*C*b^3*m^4*x^10+3*D*a^2*b*m^6*x^8+1665*
D*a*b^2*m^4*x^10+12139*D*b^3*m^2*x^12+40*B*b^3*m^5*x^8+120*C*a*b^2*m^5*x^8
+3940*C*b^3*m^3*x^10+120*D*a^2*b*m^5*x^8+11820*D*a*b^2*m^3*x^10+19524*D*b^
3*m*x^12+A*b^3*m^6*x^6+3*B*a*b^2*m^6*x^6+613*B*b^3*m^4*x^8+3*C*a^2*b*m^6*x
^6+1839*C*a*b^2*m^4*x^8+14039*C*b^3*m^2*x^10+D*a^3*m^6*x^6+1839*D*a^2*b*m^
4*x^8+42117*D*a*b^2*m^2*x^10+10395*D*b^3*x^12+42*A*b^3*m^5*x^6+126*B*a*b^2
*m^5*x^6+4528*B*b^3*m^3*x^8+126*C*a^2*b*m^5*x^6+13584*C*a*b^2*m^3*x^8+2290
2*C*b^3*m*x^10+42*D*a^3*m^5*x^6+13584*D*a^2*b*m^3*x^8+68706*D*a*b^2*m*x^10
+3*A*a*b^2*m^6*x^4+679*A*b^3*m^4*x^6+3*B*a^2*b*m^6*x^4+2037*B*a*b^2*m^4*x^
6+16627*B*b^3*m^2*x^8+C*a^3*m^6*x^4+2037*C*a^2*b*m^4*x^6+49881*C*a*b^2*m^2
*x^8+12285*C*b^3*x^10+679*D*a^3*m^4*x^6+49881*D*a^2*b*m^2*x^8+36855*D*a*b^
2*x^10+132*A*a*b^2*m^5*x^4+5292*A*b^3*m^3*x^6+132*B*a^2*b*m^5*x^4+15876*B*
a*b^2*m^3*x^6+27688*B*b^3*m*x^8+44*C*a^3*m^5*x^4+15876*C*a^2*b*m^3*x^6+830
64*C*a*b^2*m*x^8+5292*D*a^3*m^3*x^6+83064*D*a^2*b*m*x^8+3*A*a^2*b*m^6*x^2+
2259*A*a*b^2*m^4*x^4+20335*A*b^3*m^2*x^6+B*a^3*m^6*x^2+2259*B*a^2*b*m^4*x^
4+61005*B*a*b^2*m^2*x^6+15015*B*b^3*x^8+753*C*a^3*m^4*x^4+61005*C*a^2*b*m^
2*x^6+45045*C*a*b^2*x^8+20335*D*a^3*m^2*x^6+45045*D*a^2*b*x^8+138*A*a^2*b*
m^5*x^2+18840*A*a*b^2*m^3*x^4+34986*A*b^3*m*x^6+46*B*a^3*m^5*x^2+18840*...
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 962 vs. $2(204) = 408$.

Time = 0.09 (sec) , antiderivative size = 962, normalized size of antiderivative = 4.72

$$\int (cx)^m \left(a+bx^2\right)^3 \left(A+Bx^2+Cx^4+Dx^6\right) \, dx = \text{Too large to display}$$

input
```
integrate((c*x)^m*(b*x^2+a)^3*(D*x^6+C*x^4+B*x^2+A),x, algorithm="fricas")
```

output
```
((D*b^3*m^6 + 36*D*b^3*m^5 + 505*D*b^3*m^4 + 3480*D*b^3*m^3 + 12139*D*b^3*
m^2 + 19524*D*b^3*m + 10395*D*b^3)*x^13 + ((3*D*a*b^2 + C*b^3)*m^6 + 38*(3
*D*a*b^2 + C*b^3)*m^5 + 555*(3*D*a*b^2 + C*b^3)*m^4 + 36855*D*a*b^2 + 1228
5*C*b^3 + 3940*(3*D*a*b^2 + C*b^3)*m^3 + 14039*(3*D*a*b^2 + C*b^3)*m^2 + 2
2902*(3*D*a*b^2 + C*b^3)*m)*x^11 + ((3*D*a^2*b + 3*C*a*b^2 + B*b^3)*m^6 +
40*(3*D*a^2*b + 3*C*a*b^2 + B*b^3)*m^5 + 613*(3*D*a^2*b + 3*C*a*b^2 + B*b^
3)*m^4 + 45045*D*a^2*b + 45045*C*a*b^2 + 15015*B*b^3 + 4528*(3*D*a^2*b + 3
*C*a*b^2 + B*b^3)*m^3 + 16627*(3*D*a^2*b + 3*C*a*b^2 + B*b^3)*m^2 + 27688*
(3*D*a^2*b + 3*C*a*b^2 + B*b^3)*m)*x^9 + ((D*a^3 + 3*C*a^2*b + 3*B*a*b^2 +
 A*b^3)*m^6 + 42*(D*a^3 + 3*C*a^2*b + 3*B*a*b^2 + A*b^3)*m^5 + 679*(D*a^3
+ 3*C*a^2*b + 3*B*a*b^2 + A*b^3)*m^4 + 19305*D*a^3 + 57915*C*a^2*b + 57915
*B*a*b^2 + 19305*A*b^3 + 5292*(D*a^3 + 3*C*a^2*b + 3*B*a*b^2 + A*b^3)*m^3
+ 20335*(D*a^3 + 3*C*a^2*b + 3*B*a*b^2 + A*b^3)*m^2 + 34986*(D*a^3 + 3*C*a
^2*b + 3*B*a*b^2 + A*b^3)*m)*x^7 + ((C*a^3 + 3*B*a^2*b + 3*A*a*b^2)*m^6 +
44*(C*a^3 + 3*B*a^2*b + 3*A*a*b^2)*m^5 + 753*(C*a^3 + 3*B*a^2*b + 3*A*a*b^
2)*m^4 + 27027*C*a^3 + 81081*B*a^2*b + 81081*A*a*b^2 + 6280*(C*a^3 + 3*B*a
^2*b + 3*A*a*b^2)*m^3 + 25979*(C*a^3 + 3*B*a^2*b + 3*A*a*b^2)*m^2 + 47436*
(C*a^3 + 3*B*a^2*b + 3*A*a*b^2)*m)*x^5 + ((B*a^3 + 3*A*a^2*b)*m^6 + 46*(B*
a^3 + 3*A*a^2*b)*m^5 + 835*(B*a^3 + 3*A*a^2*b)*m^4 + 45045*B*a^3 + 135135*
A*a^2*b + 7540*(B*a^3 + 3*A*a^2*b)*m^3 + 34759*(B*a^3 + 3*A*a^2*b)*m^2 ...
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7201 vs. $2(190) = 380$.

Time = 1.09 (sec) , antiderivative size = 7201, normalized size of antiderivative = 35.30

$$\int (cx)^m \left(a+bx^2\right)^3 \left(A+Bx^2+Cx^4+Dx^6\right) \, dx = \text{Too large to display}$$

input
```
integrate((c*x)**m*(b*x**2+a)**3*(D*x**6+C*x**4+B*x**2+A),x)
```

output
```
Piecewise(((-A*a**3/(12*x**12) - 3*A*a**2*b/(10*x**10) - 3*A*a*b**2/(8*x**
8) - A*b**3/(6*x**6) - B*a**3/(10*x**10) - 3*B*a**2*b/(8*x**8) - B*a*b**2/
(2*x**6) - B*b**3/(4*x**4) - C*a**3/(8*x**8) - C*a**2*b/(2*x**6) - 3*C*a*b
**2/(4*x**4) - C*b**3/(2*x**2) - D*a**3/(6*x**6) - 3*D*a**2*b/(4*x**4) - 3
*D*a*b**2/(2*x**2) + D*b**3*log(x))/c**13, Eq(m, -13)), ((-A*a**3/(10*x**1
0) - 3*A*a**2*b/(8*x**8) - A*a*b**2/(2*x**6) - A*b**3/(4*x**4) - B*a**3/(8
*x**8) - B*a**2*b/(2*x**6) - 3*B*a*b**2/(4*x**4) - B*b**3/(2*x**2) - C*a**
3/(6*x**6) - 3*C*a**2*b/(4*x**4) - 3*C*a*b**2/(2*x**2) + C*b**3*log(x) - D
*a**3/(4*x**4) - 3*D*a**2*b/(2*x**2) + 3*D*a*b**2*log(x) + D*b**3*x**2/2)/
c**11, Eq(m, -11)), ((-A*a**3/(8*x**8) - A*a**2*b/(2*x**6) - 3*A*a*b**2/(4
*x**4) - A*b**3/(2*x**2) - B*a**3/(6*x**6) - 3*B*a**2*b/(4*x**4) - 3*B*a*b
**2/(2*x**2) + B*b**3*log(x) - C*a**3/(4*x**4) - 3*C*a**2*b/(2*x**2) + 3*C
*a*b**2*log(x) + C*b**3*x**2/2 - D*a**3/(2*x**2) + 3*D*a**2*b*log(x) + 3*D
*a*b**2*x**2/2 + D*b**3*x**4/4)/c**9, Eq(m, -9)), ((-A*a**3/(6*x**6) - 3*A
*a**2*b/(4*x**4) - 3*A*a*b**2/(2*x**2) + A*b**3*log(x) - B*a**3/(4*x**4) -
 3*B*a**2*b/(2*x**2) + 3*B*a*b**2*log(x) + B*b**3*x**2/2 - C*a**3/(2*x**2)
 + 3*C*a**2*b*log(x) + 3*C*a*b**2*x**2/2 + C*b**3*x**4/4 + D*a**3*log(x) +
 3*D*a**2*b*x**2/2 + 3*D*a*b**2*x**4/4 + D*b**3*x**6/6)/c**7, Eq(m, -7)),
((-A*a**3/(4*x**4) - 3*A*a**2*b/(2*x**2) + 3*A*a*b**2*log(x) + A*b**3*x**2
/2 - B*a**3/(2*x**2) + 3*B*a**2*b*log(x) + 3*B*a*b**2*x**2/2 + B*b**3*x...
```

## Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.58

$$\int (cx)^m \left(a+bx^2\right)^3 \left(A+Bx^2+Cx^4+Dx^6\right) dx = \frac{Db^3c^mx^{13}x^m}{m+13} + \frac{3\,Dab^2c^mx^{11}x^m}{m+11} + \frac{Cb^3c^mx^{11}x^m}{m+11} + \frac{3\,Da^2bc^mx^9x^m}{m+9} + \frac{3\,Cab^2c^mx^9x^m}{m+9} + \frac{Bb^3c^mx^9x^m}{m+9} + \frac{Da^3c^mx^7x^m}{m+7} + \frac{3\,Ca^2bc^mx^7x^m}{m+7} + \frac{3\,Bab^2c^mx^7x^m}{m+7} + \frac{Ab^3c^mx^7x^m}{m+7} + \frac{Ca^3c^mx^5x^m}{m+5} + \frac{3\,Ba^2bc^mx^5x^m}{m+5} + \frac{3\,Aab^2c^mx^5x^m}{m+5} + \frac{Ba^3c^mx^3x^m}{m+3} + \frac{3\,Aa^2bc^mx^3x^m}{m+3} + \frac{(cx)^{m+1}Aa^3}{c(m+1)}$$

input
```
integrate((c*x)^m*(b*x^2+a)^3*(D*x^6+C*x^4+B*x^2+A),x, algorithm="maxima")
```

output
```
D*b^3*c^m*x^13*x^m/(m + 13) + 3*D*a*b^2*c^m*x^11*x^m/(m + 11) + C*b^3*c^m*
x^11*x^m/(m + 11) + 3*D*a^2*b*c^m*x^9*x^m/(m + 9) + 3*C*a*b^2*c^m*x^9*x^m/
(m + 9) + B*b^3*c^m*x^9*x^m/(m + 9) + D*a^3*c^m*x^7*x^m/(m + 7) + 3*C*a^2*
b*c^m*x^7*x^m/(m + 7) + 3*B*a*b^2*c^m*x^7*x^m/(m + 7) + A*b^3*c^m*x^7*x^m/
(m + 7) + C*a^3*c^m*x^5*x^m/(m + 5) + 3*B*a^2*b*c^m*x^5*x^m/(m + 5) + 3*A*
a*b^2*c^m*x^5*x^m/(m + 5) + B*a^3*c^m*x^3*x^m/(m + 3) + 3*A*a^2*b*c^m*x^3*
x^m/(m + 3) + (c*x)^(m + 1)*A*a^3/(c*(m + 1))
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1895 vs. $2(204) = 408$.

Time = 0.15 (sec) , antiderivative size = 1895, normalized size of antiderivative = 9.29

$$\int (cx)^m \left(a+bx^2\right)^3 \left(A+Bx^2+Cx^4+Dx^6\right) \, dx = \text{Too large to display}$$

input
```
integrate((c*x)^m*(b*x^2+a)^3*(D*x^6+C*x^4+B*x^2+A),x, algorithm="giac")
```

output
```
((c*x)^m*D*b^3*m^6*x^13 + 36*(c*x)^m*D*b^3*m^5*x^13 + 3*(c*x)^m*D*a*b^2*m^
6*x^11 + (c*x)^m*C*b^3*m^6*x^11 + 505*(c*x)^m*D*b^3*m^4*x^13 + 114*(c*x)^m
*D*a*b^2*m^5*x^11 + 38*(c*x)^m*C*b^3*m^5*x^11 + 3480*(c*x)^m*D*b^3*m^3*x^1
3 + 3*(c*x)^m*D*a^2*b*m^6*x^9 + 3*(c*x)^m*C*a*b^2*m^6*x^9 + (c*x)^m*B*b^3*
m^6*x^9 + 1665*(c*x)^m*D*a*b^2*m^4*x^11 + 555*(c*x)^m*C*b^3*m^4*x^11 + 121
39*(c*x)^m*D*b^3*m^2*x^13 + 120*(c*x)^m*D*a^2*b*m^5*x^9 + 120*(c*x)^m*C*a*
b^2*m^5*x^9 + 40*(c*x)^m*B*b^3*m^5*x^9 + 11820*(c*x)^m*D*a*b^2*m^3*x^11 +
3940*(c*x)^m*C*b^3*m^3*x^11 + 19524*(c*x)^m*D*b^3*m*x^13 + (c*x)^m*D*a^3*m
^6*x^7 + 3*(c*x)^m*C*a^2*b*m^6*x^7 + 3*(c*x)^m*B*a*b^2*m^6*x^7 + (c*x)^m*A
*b^3*m^6*x^7 + 1839*(c*x)^m*D*a^2*b*m^4*x^9 + 1839*(c*x)^m*C*a*b^2*m^4*x^9
 + 613*(c*x)^m*B*b^3*m^4*x^9 + 42117*(c*x)^m*D*a*b^2*m^2*x^11 + 14039*(c*x
)^m*C*b^3*m^2*x^11 + 10395*(c*x)^m*D*b^3*x^13 + 42*(c*x)^m*D*a^3*m^5*x^7 +
 126*(c*x)^m*C*a^2*b*m^5*x^7 + 126*(c*x)^m*B*a*b^2*m^5*x^7 + 42*(c*x)^m*A*
b^3*m^5*x^7 + 13584*(c*x)^m*D*a^2*b*m^3*x^9 + 13584*(c*x)^m*C*a*b^2*m^3*x^
9 + 4528*(c*x)^m*B*b^3*m^3*x^9 + 68706*(c*x)^m*D*a*b^2*m*x^11 + 22902*(c*x
)^m*C*b^3*m*x^11 + (c*x)^m*C*a^3*m^6*x^5 + 3*(c*x)^m*B*a^2*b*m^6*x^5 + 3*(
c*x)^m*A*a*b^2*m^6*x^5 + 679*(c*x)^m*D*a^3*m^4*x^7 + 2037*(c*x)^m*C*a^2*b*
m^4*x^7 + 2037*(c*x)^m*B*a*b^2*m^4*x^7 + 679*(c*x)^m*A*b^3*m^4*x^7 + 49881
*(c*x)^m*D*a^2*b*m^2*x^9 + 49881*(c*x)^m*C*a*b^2*m^2*x^9 + 16627*(c*x)^m*B
*b^3*m^2*x^9 + 36855*(c*x)^m*D*a*b^2*x^11 + 12285*(c*x)^m*C*b^3*x^11 + ...
```

## Mupad [F(-1)]

Timed out.

$$\int (cx)^m \left(a+bx^2\right)^3 \left(A+Bx^2+Cx^4+Dx^6\right)\,dx = \int (c\,x)^m\,\left(b\,x^2+a\right)^3\,\left(A+B\,x^2+C\,x^4+x^6\,D\right)\,dx$$

input
```
int((c*x)^m*(a + b*x^2)^3*(A + B*x^2 + C*x^4 + x^6*D),x)
```

output
```
int((c*x)^m*(a + b*x^2)^3*(A + B*x^2 + C*x^4 + x^6*D), x)
```

## Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 1072, normalized size of antiderivative = 5.25

$$\int (cx)^m \left(a+bx^2\right)^3 \left(A+Bx^2+Cx^4+Dx^6\right) dx = \text{Too large to display}$$

input
```
int((c*x)^m*(b*x^2+a)^3*(D*x^6+C*x^4+B*x^2+A),x)
```

output
```
(x**m*c**m*x*(a**4*m**6 + 48*a**4*m**5 + 925*a**4*m**4 + 9120*a**4*m**3 +
48259*a**4*m**2 + 129072*a**4*m + 135135*a**4 + 4*a**3*b*m**6*x**2 + 184*a
**3*b*m**5*x**2 + 3340*a**3*b*m**4*x**2 + 30160*a**3*b*m**3*x**2 + 139036*
a**3*b*m**2*x**2 + 292216*a**3*b*m*x**2 + 180180*a**3*b*x**2 + a**3*c*m**6
*x**4 + 44*a**3*c*m**5*x**4 + 753*a**3*c*m**4*x**4 + 6280*a**3*c*m**3*x**4
 + 25979*a**3*c*m**2*x**4 + 47436*a**3*c*m*x**4 + 27027*a**3*c*x**4 + a**3
*d*m**6*x**6 + 42*a**3*d*m**5*x**6 + 679*a**3*d*m**4*x**6 + 5292*a**3*d*m*
*3*x**6 + 20335*a**3*d*m**2*x**6 + 34986*a**3*d*m*x**6 + 19305*a**3*d*x**6
 + 6*a**2*b**2*m**6*x**4 + 264*a**2*b**2*m**5*x**4 + 4518*a**2*b**2*m**4*x
**4 + 37680*a**2*b**2*m**3*x**4 + 155874*a**2*b**2*m**2*x**4 + 284616*a**2
*b**2*m*x**4 + 162162*a**2*b**2*x**4 + 3*a**2*b*c*m**6*x**6 + 126*a**2*b*c
*m**5*x**6 + 2037*a**2*b*c*m**4*x**6 + 15876*a**2*b*c*m**3*x**6 + 61005*a*
*2*b*c*m**2*x**6 + 104958*a**2*b*c*m*x**6 + 57915*a**2*b*c*x**6 + 3*a**2*b
*d*m**6*x**8 + 120*a**2*b*d*m**5*x**8 + 1839*a**2*b*d*m**4*x**8 + 13584*a*
*2*b*d*m**3*x**8 + 49881*a**2*b*d*m**2*x**8 + 83064*a**2*b*d*m*x**8 + 4504
5*a**2*b*d*x**8 + 4*a*b**3*m**6*x**6 + 168*a*b**3*m**5*x**6 + 2716*a*b**3*
m**4*x**6 + 21168*a*b**3*m**3*x**6 + 81340*a*b**3*m**2*x**6 + 139944*a*b**
3*m*x**6 + 77220*a*b**3*x**6 + 3*a*b**2*c*m**6*x**8 + 120*a*b**2*c*m**5*x*
*8 + 1839*a*b**2*c*m**4*x**8 + 13584*a*b**2*c*m**3*x**8 + 49881*a*b**2*c*m
**2*x**8 + 83064*a*b**2*c*m*x**8 + 45045*a*b**2*c*x**8 + 3*a*b**2*d*m**...
```

# 3.253 $\int (cx)^m \left(a+bx^2\right)^2 \left(A+Bx^2+Cx^4+Dx^6\right) \, dx$



## Optimal result

Integrand size = 32, antiderivative size = 155

$$\int (cx)^m \left(a+bx^2\right)^2 \left(A+Bx^2+Cx^4+Dx^6\right) \, dx = \frac{a^2A(cx)^{1+m}}{c(1+m)}+\frac{a(2Ab+aB)(cx)^{3+m}}{c^3(3+m)}+\frac{\left(Ab^2+a(2bB+aC)\right)(cx)^{5+m}}{c^5(5+m)}+\frac{\left(b^2B+2abC+a^2D\right)(cx)^{7+m}}{c^7(7+m)}+\frac{b(bC+2aD)(cx)^{9+m}}{c^9(9+m)}+\frac{b^2D(cx)^{11+m}}{c^{11}(11+m)}$$

output
```
a^2*A*(c*x)^(1+m)/c/(1+m)+a*(2*A*b+B*a)*(c*x)^(3+m)/c^3/(3+m)+(A*b^2+a*(2*
B*b+C*a))*(c*x)^(5+m)/c^5/(5+m)+(B*b^2+2*C*a*b+D*a^2)*(c*x)^(7+m)/c^7/(7+m
)+b*(C*b+2*D*a)*(c*x)^(9+m)/c^9/(9+m)+b^2*D*(c*x)^(11+m)/c^11/(11+m)
```

## Mathematica [A] (verified)

Time = 0.96 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.75

$$\int (cx)^m \left(a+bx^2\right)^2 \left(A+Bx^2+Cx^4+Dx^6\right) \, dx = x(cx)^m\left(\frac{a^2A}{1+m}+\frac{a(2Ab+aB)x^2}{3+m}+\frac{\left(Ab^2+a(2bB+aC)\right)x^4}{5+m}+\frac{\left(b^2B+2abC+a^2D\right)x^6}{7+m}+\frac{b(bC+2aD)x^8}{9+m}+\frac{b^2Dx^{10}}{11+m}\right)$$

input
```
Integrate[(c*x)^m*(a + b*x^2)^2*(A + B*x^2 + C*x^4 + D*x^6),x]
```

output
```
x*(c*x)^m*((a^2*A)/(1 + m) + (a*(2*A*b + a*B)*x^2)/(3 + m) + ((A*b^2 + a*(
2*b*B + a*C))*x^4)/(5 + m) + ((b^2*B + 2*a*b*C + a^2*D)*x^6)/(7 + m) + (b*
(b*C + 2*a*D)*x^8)/(9 + m) + (b^2*D*x^10)/(11 + m))
```

## Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^2 (cx)^m \left(A+Bx^2+Cx^4+Dx^6\right) \, dx$$

$$\Big\downarrow \text{2333}$$

$$\int \left(a^2A(cx)^m+\frac{(cx)^{m+6}\left(a^2D+2abC+b^2B\right)}{c^6}+\frac{(cx)^{m+4}\left(a(aC+2bB)+Ab^2\right)}{c^4}+\frac{a(cx)^{m+2}(aB+2Ab)}{c^2}+\frac{b(\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2A(cx)^{m+1}}{c(m+1)}+\frac{(cx)^{m+7}\left(a^2D+2abC+b^2B\right)}{c^7(m+7)}+\frac{(cx)^{m+5}\left(a(aC+2bB)+Ab^2\right)}{c^5(m+5)}+\frac{a(cx)^{m+3}(aB+2Ab)}{c^3(m+3)}+\frac{b(cx)^{m+9}(2aD+bC)}{c^9(m+9)}+\frac{b^2D(cx)^{m+11}}{c^{11}(m+11)}$$

input
```
Int[(c*x)^m*(a + b*x^2)^2*(A + B*x^2 + C*x^4 + D*x^6),x]
```

output
```
(a^2*A*(c*x)^(1 + m))/(c*(1 + m)) + (a*(2*A*b + a*B)*(c*x)^(3 + m))/(c^3*(
3 + m)) + ((A*b^2 + a*(2*b*B + a*C))*(c*x)^(5 + m))/(c^5*(5 + m)) + ((b^2*
B + 2*a*b*C + a^2*D)*(c*x)^(7 + m))/(c^7*(7 + m)) + (b*(b*C + 2*a*D)*(c*x)
^(9 + m))/(c^9*(9 + m)) + (b^2*D*(c*x)^(11 + m))/(c^11*(11 + m))
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 792 vs. $2(155) = 310$.

Time = 0.51 (sec) , antiderivative size = 793, normalized size of antiderivative = 5.12

| method | result |
|---|---|
| gosper | $\frac{x(Db^2m^5x^{10}+25Db^2m^4x^{10}+C\,b^2m^5x^8+2Dab\,m^5x^8+230Db^2m^3x^{10}+27C\,b^2m^4x^8+54Dab\,m^4x^8+950Db^2m^2x^{10}+B\,b^2m}{}$ |
| orering | $\frac{x(Db^2m^5x^{10}+25Db^2m^4x^{10}+C\,b^2m^5x^8+2Dab\,m^5x^8+230Db^2m^3x^{10}+27C\,b^2m^4x^8+54Dab\,m^4x^8+950Db^2m^2x^{10}+B\,b^2m}{}$ |
| parallelrisch | Expression too large to display |

input
```
int((c*x)^m*(b*x^2+a)^2*(D*x^6+C*x^4+B*x^2+A),x,method=_RETURNVERBOSE)
```

output
```
x*(D*b^2*m^5*x^10+25*D*b^2*m^4*x^10+C*b^2*m^5*x^8+2*D*a*b*m^5*x^8+230*D*b^
2*m^3*x^10+27*C*b^2*m^4*x^8+54*D*a*b*m^4*x^8+950*D*b^2*m^2*x^10+B*b^2*m^5*
x^6+2*C*a*b*m^5*x^6+262*C*b^2*m^3*x^8+D*a^2*m^5*x^6+524*D*a*b*m^3*x^8+1689
*D*b^2*m*x^10+29*B*b^2*m^4*x^6+58*C*a*b*m^4*x^6+1122*C*b^2*m^2*x^8+29*D*a^
2*m^4*x^6+2244*D*a*b*m^2*x^8+945*D*b^2*x^10+A*b^2*m^5*x^4+2*B*a*b*m^5*x^4+
302*B*b^2*m^3*x^6+C*a^2*m^5*x^4+604*C*a*b*m^3*x^6+2041*C*b^2*m*x^8+302*D*a
^2*m^3*x^6+4082*D*a*b*m*x^8+31*A*b^2*m^4*x^4+62*B*a*b*m^4*x^4+1366*B*b^2*m
^2*x^6+31*C*a^2*m^4*x^4+2732*C*a*b*m^2*x^6+1155*C*b^2*x^8+1366*D*a^2*m^2*x
^6+2310*D*a*b*x^8+2*A*a*b*m^5*x^2+350*A*b^2*m^3*x^4+B*a^2*m^5*x^2+700*B*a*
b*m^3*x^4+2577*B*b^2*m*x^6+350*C*a^2*m^3*x^4+5154*C*a*b*m*x^6+2577*D*a^2*m
*x^6+66*A*a*b*m^4*x^2+1730*A*b^2*m^2*x^4+33*B*a^2*m^4*x^2+3460*B*a*b*m^2*x
^4+1485*B*b^2*x^6+1730*C*a^2*m^2*x^4+2970*C*a*b*x^6+1485*D*a^2*x^6+A*a^2*m
^5+812*A*a*b*m^3*x^2+3489*A*b^2*m*x^4+406*B*a^2*m^3*x^2+6978*B*a*b*m*x^4+3
489*C*a^2*m*x^4+35*A*a^2*m^4+4524*A*a*b*m^2*x^2+2079*A*b^2*x^4+2262*B*a^2*
m^2*x^2+4158*B*a*b*x^4+2079*C*a^2*x^4+470*A*a^2*m^3+10706*A*a*b*m*x^2+5353
*B*a^2*m*x^2+3010*A*a^2*m^2+6930*A*a*b*x^2+3465*B*a^2*x^2+9129*A*a^2*m+103
95*A*a^2)*(c*x)^m/(11+m)/(9+m)/(7+m)/(5+m)/(3+m)/(1+m)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 575 vs. $2(155) = 310$.

Time = 0.09 (sec) , antiderivative size = 575, normalized size of antiderivative = 3.71

$$\int (cx)^m \left(a+bx^2\right)^2 \left(A+Bx^2+Cx^4+Dx^6\right)\,dx$$
$$= \frac{((Db^2m^5+25\,Db^2m^4+230\,Db^2m^3+950\,Db^2m^2+1689\,Db^2m+945\,Db^2)x^{11}+((2\,Dab+Cb^2)m^5+2}{}$$

input
```
integrate((c*x)^m*(b*x^2+a)^2*(D*x^6+C*x^4+B*x^2+A),x, algorithm="fricas")
```

output
```
((D*b^2*m^5 + 25*D*b^2*m^4 + 230*D*b^2*m^3 + 950*D*b^2*m^2 + 1689*D*b^2*m
+ 945*D*b^2)*x^11 + ((2*D*a*b + C*b^2)*m^5 + 27*(2*D*a*b + C*b^2)*m^4 + 26
2*(2*D*a*b + C*b^2)*m^3 + 2310*D*a*b + 1155*C*b^2 + 1122*(2*D*a*b + C*b^2)
*m^2 + 2041*(2*D*a*b + C*b^2)*m)*x^9 + ((D*a^2 + 2*C*a*b + B*b^2)*m^5 + 29
*(D*a^2 + 2*C*a*b + B*b^2)*m^4 + 302*(D*a^2 + 2*C*a*b + B*b^2)*m^3 + 1485*
D*a^2 + 2970*C*a*b + 1485*B*b^2 + 1366*(D*a^2 + 2*C*a*b + B*b^2)*m^2 + 257
7*(D*a^2 + 2*C*a*b + B*b^2)*m)*x^7 + ((C*a^2 + 2*B*a*b + A*b^2)*m^5 + 31*(
C*a^2 + 2*B*a*b + A*b^2)*m^4 + 350*(C*a^2 + 2*B*a*b + A*b^2)*m^3 + 2079*C*
a^2 + 4158*B*a*b + 2079*A*b^2 + 1730*(C*a^2 + 2*B*a*b + A*b^2)*m^2 + 3489*
(C*a^2 + 2*B*a*b + A*b^2)*m)*x^5 + ((B*a^2 + 2*A*a*b)*m^5 + 33*(B*a^2 + 2*
A*a*b)*m^4 + 406*(B*a^2 + 2*A*a*b)*m^3 + 3465*B*a^2 + 6930*A*a*b + 2262*(B
*a^2 + 2*A*a*b)*m^2 + 5353*(B*a^2 + 2*A*a*b)*m)*x^3 + (A*a^2*m^5 + 35*A*a^
2*m^4 + 470*A*a^2*m^3 + 3010*A*a^2*m^2 + 9129*A*a^2*m + 10395*A*a^2)*x)*(c
*x)^m/(m^6 + 36*m^5 + 505*m^4 + 3480*m^3 + 12139*m^2 + 19524*m + 10395)
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4068 vs. $2(141) = 282$.

Time = 0.87 (sec) , antiderivative size = 4068, normalized size of antiderivative = 26.25

$$\int (cx)^m \left(a+bx^2\right)^2 \left(A+Bx^2+Cx^4+Dx^6\right)\,dx = \text{Too large to display}$$

input
```
integrate((c*x)**m*(b*x**2+a)**2*(D*x**6+C*x**4+B*x**2+A),x)
```

output
```
Piecewise(((-A*a**2/(10*x**10) - A*a*b/(4*x**8) - A*b**2/(6*x**6) - B*a**2
/(8*x**8) - B*a*b/(3*x**6) - B*b**2/(4*x**4) - C*a**2/(6*x**6) - C*a*b/(2*
x**4) - C*b**2/(2*x**2) - D*a**2/(4*x**4) - D*a*b/x**2 + D*b**2*log(x))/c*
*11, Eq(m, -11)), ((-A*a**2/(8*x**8) - A*a*b/(3*x**6) - A*b**2/(4*x**4) -
B*a**2/(6*x**6) - B*a*b/(2*x**4) - B*b**2/(2*x**2) - C*a**2/(4*x**4) - C*a
*b/x**2 + C*b**2*log(x) - D*a**2/(2*x**2) + 2*D*a*b*log(x) + D*b**2*x**2/2
)/c**9, Eq(m, -9)), ((-A*a**2/(6*x**6) - A*a*b/(2*x**4) - A*b**2/(2*x**2)
- B*a**2/(4*x**4) - B*a*b/x**2 + B*b**2*log(x) - C*a**2/(2*x**2) + 2*C*a*b
*log(x) + C*b**2*x**2/2 + D*a**2*log(x) + D*a*b*x**2 + D*b**2*x**4/4)/c**7
, Eq(m, -7)), ((-A*a**2/(4*x**4) - A*a*b/x**2 + A*b**2*log(x) - B*a**2/(2*
x**2) + 2*B*a*b*log(x) + B*b**2*x**2/2 + C*a**2*log(x) + C*a*b*x**2 + C*b*
*2*x**4/4 + D*a**2*x**2/2 + D*a*b*x**4/2 + D*b**2*x**6/6)/c**5, Eq(m, -5))
, ((-A*a**2/(2*x**2) + 2*A*a*b*log(x) + A*b**2*x**2/2 + B*a**2*log(x) + B*
a*b*x**2 + B*b**2*x**4/4 + C*a**2*x**2/2 + C*a*b*x**4/2 + C*b**2*x**6/6 +
D*a**2*x**4/4 + D*a*b*x**6/3 + D*b**2*x**8/8)/c**3, Eq(m, -3)), ((A*a**2*l
og(x) + A*a*b*x**2 + A*b**2*x**4/4 + B*a**2*x**2/2 + B*a*b*x**4/2 + B*b**2
*x**6/6 + C*a**2*x**4/4 + C*a*b*x**6/3 + C*b**2*x**8/8 + D*a**2*x**6/6 + D
*a*b*x**8/4 + D*b**2*x**10/10)/c, Eq(m, -1)), (A*a**2*m**5*x*(c*x)**m/(m**
6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 35*A*
a**2*m**4*x*(c*x)**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**...
```

## Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.48

$$\begin{aligned}\int (cx)^m \left(a+bx^2\right)^2 \left(A+Bx^2+Cx^4+Dx^6\right)\,dx &= \frac{Db^2c^mx^{11}x^m}{m+11}+\frac{2\,Dabc^mx^9x^m}{m+9}+\frac{Cb^2c^mx^9x^m}{m+9}+\frac{Da^2c^mx^7x^m}{m+7}\\&+\frac{2\,Cabc^mx^7x^m}{m+7}+\frac{Bb^2c^mx^7x^m}{m+7}+\frac{Ca^2c^mx^5x^m}{m+5}+\frac{2\,Babc^mx^5x^m}{m+5}\\&+\frac{Ab^2c^mx^5x^m}{m+5}+\frac{Ba^2c^mx^3x^m}{m+3}+\frac{2\,Aabc^mx^3x^m}{m+3}+\frac{(cx)^{m+1}Aa^2}{c(m+1)}\end{aligned}$$

input
```
integrate((c*x)^m*(b*x^2+a)^2*(D*x^6+C*x^4+B*x^2+A),x, algorithm="maxima")
```

output
```
D*b^2*c^m*x^11*x^m/(m + 11) + 2*D*a*b*c^m*x^9*x^m/(m + 9) + C*b^2*c^m*x^9*
x^m/(m + 9) + D*a^2*c^m*x^7*x^m/(m + 7) + 2*C*a*b*c^m*x^7*x^m/(m + 7) + B*
b^2*c^m*x^7*x^m/(m + 7) + C*a^2*c^m*x^5*x^m/(m + 5) + 2*B*a*b*c^m*x^5*x^m/
(m + 5) + A*b^2*c^m*x^5*x^m/(m + 5) + B*a^2*c^m*x^3*x^m/(m + 3) + 2*A*a*b*
c^m*x^3*x^m/(m + 3) + (c*x)^(m + 1)*A*a^2/(c*(m + 1))
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1152 vs. $2(155) = 310$.

Time = 0.15 (sec) , antiderivative size = 1152, normalized size of antiderivative = 7.43

$$\int (cx)^m (a+bx^2)^2 (A+Bx^2+Cx^4+Dx^6)\, dx = \text{Too large to display}$$

input
```
integrate((c*x)^m*(b*x^2+a)^2*(D*x^6+C*x^4+B*x^2+A),x, algorithm="giac")
```

output
```
((c*x)^m*D*b^2*m^5*x^11 + 25*(c*x)^m*D*b^2*m^4*x^11 + 2*(c*x)^m*D*a*b*m^5*
x^9 + (c*x)^m*C*b^2*m^5*x^9 + 230*(c*x)^m*D*b^2*m^3*x^11 + 54*(c*x)^m*D*a*
b*m^4*x^9 + 27*(c*x)^m*C*b^2*m^4*x^9 + 950*(c*x)^m*D*b^2*m^2*x^11 + (c*x)^
m*D*a^2*m^5*x^7 + 2*(c*x)^m*C*a*b*m^5*x^7 + (c*x)^m*B*b^2*m^5*x^7 + 524*(c
*x)^m*D*a*b*m^3*x^9 + 262*(c*x)^m*C*b^2*m^3*x^9 + 1689*(c*x)^m*D*b^2*m*x^1
1 + 29*(c*x)^m*D*a^2*m^4*x^7 + 58*(c*x)^m*C*a*b*m^4*x^7 + 29*(c*x)^m*B*b^2
*m^4*x^7 + 2244*(c*x)^m*D*a*b*m^2*x^9 + 1122*(c*x)^m*C*b^2*m^2*x^9 + 945*(
c*x)^m*D*b^2*x^11 + (c*x)^m*C*a^2*m^5*x^5 + 2*(c*x)^m*B*a*b*m^5*x^5 + (c*x
)^m*A*b^2*m^5*x^5 + 302*(c*x)^m*D*a^2*m^3*x^7 + 604*(c*x)^m*C*a*b*m^3*x^7
 + 302*(c*x)^m*B*b^2*m^3*x^7 + 4082*(c*x)^m*D*a*b*m*x^9 + 2041*(c*x)^m*C*b^
2*m*x^9 + 31*(c*x)^m*C*a^2*m^4*x^5 + 62*(c*x)^m*B*a*b*m^4*x^5 + 31*(c*x)^m
*A*b^2*m^4*x^5 + 1366*(c*x)^m*D*a^2*m^2*x^7 + 2732*(c*x)^m*C*a*b*m^2*x^7 +
 1366*(c*x)^m*B*b^2*m^2*x^7 + 2310*(c*x)^m*D*a*b*x^9 + 1155*(c*x)^m*C*b^2*
x^9 + (c*x)^m*B*a^2*m^5*x^3 + 2*(c*x)^m*A*a*b*m^5*x^3 + 350*(c*x)^m*C*a^2*
m^3*x^5 + 700*(c*x)^m*B*a*b*m^3*x^5 + 350*(c*x)^m*A*b^2*m^3*x^5 + 2577*(c*
x)^m*D*a^2*m*x^7 + 5154*(c*x)^m*C*a*b*m*x^7 + 2577*(c*x)^m*B*b^2*m*x^7 + 3
3*(c*x)^m*B*a^2*m^4*x^3 + 66*(c*x)^m*A*a*b*m^4*x^3 + 1730*(c*x)^m*C*a^2*m^
2*x^5 + 3460*(c*x)^m*B*a*b*m^2*x^5 + 1730*(c*x)^m*A*b^2*m^2*x^5 + 1485*(c*
x)^m*D*a^2*x^7 + 2970*(c*x)^m*C*a*b*x^7 + 1485*(c*x)^m*B*b^2*x^7 + (c*x)^m
*A*a^2*m^5*x + 406*(c*x)^m*B*a^2*m^3*x^3 + 812*(c*x)^m*A*a*b*m^3*x^3 + ...
```

## Mupad [F(-1)]

Timed out.

$$\int (cx)^m (a+bx^2)^2 (A+Bx^2+Cx^4+Dx^6) \, dx = \int (c\,x)^m \,(b\,x^2+a)^2 \,(A+B\,x^2+C\,x^4+x^6\,D) \,dx$$

input
```
int((c*x)^m*(a + b*x^2)^2*(A + B*x^2 + C*x^4 + x^6*D),x)
```

output
```
int((c*x)^m*(a + b*x^2)^2*(A + B*x^2 + C*x^4 + x^6*D), x)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 661, normalized size of antiderivative = 4.26

$$\int (cx)^m (a+bx^2)^2 (A+Bx^2+Cx^4+Dx^6) \, dx = \frac{x^m c^m x (b^2 d\, m^5 x^{10} + 25 b^2 d\, m^4 x^{10} + 2abd\, m^5 x^8 + b^2 c\, m^5 x^8 + 230 b^2 d\, m^3 x^{10} + 54abd\, m^4 x^8 + 27 b^2 c\, m^4 x^8 +}{}$$

input
```
int((c*x)^m*(b*x^2+a)^2*(D*x^6+C*x^4+B*x^2+A),x)
```

output
```
(x**m*c**m*x*(a**3*m**5 + 35*a**3*m**4 + 470*a**3*m**3 + 3010*a**3*m**2 +
9129*a**3*m + 10395*a**3 + 3*a**2*b*m**5*x**2 + 99*a**2*b*m**4*x**2 + 1218
*a**2*b*m**3*x**2 + 6786*a**2*b*m**2*x**2 + 16059*a**2*b*m*x**2 + 10395*a*
*2*b*x**2 + a**2*c*m**5*x**4 + 31*a**2*c*m**4*x**4 + 350*a**2*c*m**3*x**4
+ 1730*a**2*c*m**2*x**4 + 3489*a**2*c*m*x**4 + 2079*a**2*c*x**4 + a**2*d*m
**5*x**6 + 29*a**2*d*m**4*x**6 + 302*a**2*d*m**3*x**6 + 1366*a**2*d*m**2*x
**6 + 2577*a**2*d*m*x**6 + 1485*a**2*d*x**6 + 3*a*b**2*m**5*x**4 + 93*a*b*
*2*m**4*x**4 + 1050*a*b**2*m**3*x**4 + 5190*a*b**2*m**2*x**4 + 10467*a*b**
2*m*x**4 + 6237*a*b**2*x**4 + 2*a*b*c*m**5*x**6 + 58*a*b*c*m**4*x**6 + 604
*a*b*c*m**3*x**6 + 2732*a*b*c*m**2*x**6 + 5154*a*b*c*m*x**6 + 2970*a*b*c*x
**6 + 2*a*b*d*m**5*x**8 + 54*a*b*d*m**4*x**8 + 524*a*b*d*m**3*x**8 + 2244*
a*b*d*m**2*x**8 + 4082*a*b*d*m*x**8 + 2310*a*b*d*x**8 + b**3*m**5*x**6 + 2
9*b**3*m**4*x**6 + 302*b**3*m**3*x**6 + 1366*b**3*m**2*x**6 + 2577*b**3*m*
x**6 + 1485*b**3*x**6 + b**2*c*m**5*x**8 + 27*b**2*c*m**4*x**8 + 262*b**2*
c*m**3*x**8 + 1122*b**2*c*m**2*x**8 + 2041*b**2*c*m*x**8 + 1155*b**2*c*x**
8 + b**2*d*m**5*x**10 + 25*b**2*d*m**4*x**10 + 230*b**2*d*m**3*x**10 + 950
*b**2*d*m**2*x**10 + 1689*b**2*d*m*x**10 + 945*b**2*d*x**10))/(m**6 + 36*m
**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395)
```

# 3.254 $\int (cx)^m (a+bx^2) (A+Bx^2+Cx^4+Dx^6) \, dx$



## Optimal result

Integrand size = 30, antiderivative size = 106

$$\int (cx)^m (a+bx^2) (A+Bx^2+Cx^4+Dx^6) \, dx = \frac{aA(cx)^{1+m}}{c(1+m)} + \frac{(Ab+aB)(cx)^{3+m}}{c^3(3+m)} + \frac{(bB+aC)(cx)^{5+m}}{c^5(5+m)} + \frac{(bC+aD)(cx)^{7+m}}{c^7(7+m)} + \frac{bD(cx)^{9+m}}{c^9(9+m)}$$

output
```
a*A*(c*x)^(1+m)/c/(1+m)+(A*b+B*a)*(c*x)^(3+m)/c^3/(3+m)+(B*b+C*a)*(c*x)^(5
+m)/c^5/(5+m)+(C*b+D*a)*(c*x)^(7+m)/c^7/(7+m)+b*D*(c*x)^(9+m)/c^9/(9+m)
```

## Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.71

$$\int (cx)^m (a+bx^2) (A+Bx^2+Cx^4+Dx^6) \, dx = x(cx)^m \left( \frac{aA}{1+m} + \frac{(Ab+aB)x^2}{3+m} + \frac{(bB+aC)x^4}{5+m} + \frac{(bC+aD)x^6}{7+m} + \frac{bDx^8}{9+m} \right)$$

input
```
Integrate[(c*x)^m*(a + b*x^2)*(A + B*x^2 + C*x^4 + D*x^6),x]
```

output
```
x*(c*x)^m*((a*A)/(1 + m) + ((A*b + a*B)*x^2)/(3 + m) + ((b*B + a*C)*x^4)/(
5 + m) + ((b*C + a*D)*x^6)/(7 + m) + (b*D*x^8)/(9 + m))
```

## Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^2)(cx)^m(A+Bx^2+Cx^4+Dx^6)\,dx$$

$$\Big\downarrow \text{2333}$$

$$\int \left(\frac{(cx)^{m+2}(aB+Ab)}{c^2}+aA(cx)^m+\frac{(cx)^{m+4}(aC+bB)}{c^4}+\frac{(cx)^{m+6}(aD+bC)}{c^6}+\frac{bD(cx)^{m+8}}{c^8}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{(cx)^{m+3}(aB+Ab)}{c^3(m+3)}+\frac{aA(cx)^{m+1}}{c(m+1)}+\frac{(cx)^{m+5}(aC+bB)}{c^5(m+5)}+\frac{(cx)^{m+7}(aD+bC)}{c^7(m+7)}+\frac{bD(cx)^{m+9}}{c^9(m+9)}$$

input
```
Int[(c*x)^m*(a + b*x^2)*(A + B*x^2 + C*x^4 + D*x^6),x]
```

output
```
(a*A*(c*x)^(1 + m))/(c*(1 + m)) + ((A*b + a*B)*(c*x)^(3 + m))/(c^3*(3 + m)
) + ((b*B + a*C)*(c*x)^(5 + m))/(c^5*(5 + m)) + ((b*C + a*D)*(c*x)^(7 + m)
)/(c^7*(7 + m)) + (b*D*(c*x)^(9 + m))/(c^9*(9 + m))
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 370 vs. $2(106) = 212$.

Time = 0.12 (sec) , antiderivative size = 371, normalized size of antiderivative = 3.50

| method | result |
|---|---|
| gosper | $x(D b m^4 x^8+16 D b m^3 x^8+C b m^4 x^6+D a m^4 x^6+86 D b m^2 x^8+18 C b m^3 x^6+18 D a m^3 x^6+176 D b m x^8+B b m^4 x^4+C a m^4 x^4+$ |
| orering | $x(D b m^4 x^8+16 D b m^3 x^8+C b m^4 x^6+D a m^4 x^6+86 D b m^2 x^8+18 C b m^3 x^6+18 D a m^3 x^6+176 D b m x^8+B b m^4 x^4+C a m^4 x^4+$ |
| parallelrisch | $105 D x^9 (cx)^m b+135 C x^7 (cx)^m b+135 D x^7 (cx)^m a+189 B x^5 (cx)^m b+189 C x^5 (cx)^m a+315 A x^3 (cx)^m b+315 B x^3 (cx)^m a+94$ |

input
```
int((c*x)^m*(b*x^2+a)*(D*x^6+C*x^4+B*x^2+A),x,method=_RETURNVERBOSE)
```

output
```
x*(D*b*m^4*x^8+16*D*b*m^3*x^8+C*b*m^4*x^6+D*a*m^4*x^6+86*D*b*m^2*x^8+18*C*
b*m^3*x^6+18*D*a*m^3*x^6+176*D*b*m*x^8+B*b*m^4*x^4+C*a*m^4*x^4+104*C*b*m^2
*x^6+104*D*a*m^2*x^6+105*D*b*x^8+20*B*b*m^3*x^4+20*C*a*m^3*x^4+222*C*b*m*x
^6+222*D*a*m*x^6+A*b*m^4*x^2+B*a*m^4*x^2+130*B*b*m^2*x^4+130*C*a*m^2*x^4+1
35*C*b*x^6+135*D*a*x^6+22*A*b*m^3*x^2+22*B*a*m^3*x^2+300*B*b*m*x^4+300*C*a
*m*x^4+A*a*m^4+164*A*b*m^2*x^2+164*B*a*m^2*x^2+189*B*b*x^4+189*C*a*x^4+24*
A*a*m^3+458*A*b*m*x^2+458*B*a*m*x^2+206*A*a*m^2+315*A*b*x^2+315*B*a*x^2+74
4*A*a*m+945*A*a)*(c*x)^m/(9+m)/(7+m)/(5+m)/(3+m)/(1+m)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 272 vs. 2(106) = 212.

Time = 0.08 (sec) , antiderivative size = 272, normalized size of antiderivative = 2.57

$$\int (cx)^m (a+bx^2) (A+Bx^2+Cx^4+Dx^6) \, dx = \frac{((Dbm^4+16\,Dbm^3+86\,Dbm^2+176\,Dbm+105\,Db)x^9+((Da+Cb)m^4+18\,(Da+Cb)m^3+104\,(D}{}$$

input
```
integrate((c*x)^m*(b*x^2+a)*(D*x^6+C*x^4+B*x^2+A),x, algorithm="fricas")
```

output
```
((D*b*m^4 + 16*D*b*m^3 + 86*D*b*m^2 + 176*D*b*m + 105*D*b)*x^9 + ((D*a + C
*b)*m^4 + 18*(D*a + C*b)*m^3 + 104*(D*a + C*b)*m^2 + 135*D*a + 135*C*b + 2
22*(D*a + C*b)*m)*x^7 + ((C*a + B*b)*m^4 + 20*(C*a + B*b)*m^3 + 130*(C*a +
 B*b)*m^2 + 189*C*a + 189*B*b + 300*(C*a + B*b)*m)*x^5 + ((B*a + A*b)*m^4
+ 22*(B*a + A*b)*m^3 + 164*(B*a + A*b)*m^2 + 315*B*a + 315*A*b + 458*(B*a
+ A*b)*m)*x^3 + (A*a*m^4 + 24*A*a*m^3 + 206*A*a*m^2 + 744*A*a*m + 945*A*a)
*x)*(c*x)^m/(m^5 + 25*m^4 + 230*m^3 + 950*m^2 + 1689*m + 945)
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1911 vs. 2(92) = 184.

Time = 0.61 (sec) , antiderivative size = 1911, normalized size of antiderivative = 18.03

$$\int (cx)^m (a+bx^2) (A+Bx^2+Cx^4+Dx^6) \, dx = \text{Too large to display}$$

input
```
integrate((c*x)**m*(b*x**2+a)*(D*x**6+C*x**4+B*x**2+A),x)
```

output
```
Piecewise(((-A*a/(8*x**8) - A*b/(6*x**6) - B*a/(6*x**6) - B*b/(4*x**4) - C
*a/(4*x**4) - C*b/(2*x**2) - D*a/(2*x**2) + D*b*log(x))/c**9, Eq(m, -9)),
((-A*a/(6*x**6) - A*b/(4*x**4) - B*a/(4*x**4) - B*b/(2*x**2) - C*a/(2*x**2
) + C*b*log(x) + D*a*log(x) + D*b*x**2/2)/c**7, Eq(m, -7)), ((-A*a/(4*x**4
) - A*b/(2*x**2) - B*a/(2*x**2) + B*b*log(x) + C*a*log(x) + C*b*x**2/2 + D
*a*x**2/2 + D*b*x**4/4)/c**5, Eq(m, -5)), ((-A*a/(2*x**2) + A*b*log(x) + B
*a*log(x) + B*b*x**2/2 + C*a*x**2/2 + C*b*x**4/4 + D*a*x**4/4 + D*b*x**6/6
)/c**3, Eq(m, -3)), ((A*a*log(x) + A*b*x**2/2 + B*a*x**2/2 + B*b*x**4/4 +
C*a*x**4/4 + C*b*x**6/6 + D*a*x**6/6 + D*b*x**8/8)/c, Eq(m, -1)), (A*a*m**
4*x*(c*x)**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 24*A*
a*m**3*x*(c*x)**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) +
206*A*a*m**2*x*(c*x)**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 9
45) + 744*A*a*m*x*(c*x)**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m
+ 945) + 945*A*a*x*(c*x)**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m
 + 945) + A*b*m**4*x**3*(c*x)**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1
689*m + 945) + 22*A*b*m**3*x**3*(c*x)**m/(m**5 + 25*m**4 + 230*m**3 + 950*
m**2 + 1689*m + 945) + 164*A*b*m**2*x**3*(c*x)**m/(m**5 + 25*m**4 + 230*m*
*3 + 950*m**2 + 1689*m + 945) + 458*A*b*m*x**3*(c*x)**m/(m**5 + 25*m**4 +
230*m**3 + 950*m**2 + 1689*m + 945) + 315*A*b*x**3*(c*x)**m/(m**5 + 25*m**
4 + 230*m**3 + 950*m**2 + 1689*m + 945) + B*a*m**4*x**3*(c*x)**m/(m**5 ...
```

## Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.30

$$\int (cx)^m \left(a+bx^2\right) \left(A+Bx^2+Cx^4+Dx^6\right) \, dx = \frac{Dbc^m x^9 x^m}{m+9} + \frac{Dac^m x^7 x^m}{m+7} + \frac{Cbc^m x^7 x^m}{m+7} + \frac{Cac^m x^5 x^m}{m+5} + \frac{Bbc^m x^5 x^m}{m+5} + \frac{Bac^m x^3 x^m}{m+3} + \frac{Abc^m x^3 x^m}{m+3} + \frac{(cx)^{m+1} Aa}{c(m+1)}$$

input
```
integrate((c*x)^m*(b*x^2+a)*(D*x^6+C*x^4+B*x^2+A),x, algorithm="maxima")
```

output
```
D*b*c^m*x^9*x^m/(m + 9) + D*a*c^m*x^7*x^m/(m + 7) + C*b*c^m*x^7*x^m/(m + 7
) + C*a*c^m*x^5*x^m/(m + 5) + B*b*c^m*x^5*x^m/(m + 5) + B*a*c^m*x^3*x^m/(m
 + 3) + A*b*c^m*x^3*x^m/(m + 3) + (c*x)^(m + 1)*A*a/(c*(m + 1))
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 569 vs. $2(106) = 212$.

Time = 0.13 (sec) , antiderivative size = 569, normalized size of antiderivative = 5.37

$$\int (cx)^m (a+bx^2) (A+Bx^2+Cx^4+Dx^6) \, dx = \frac{(cx)^m Dbm^4x^9 + 16 (cx)^m Dbm^3x^9 + (cx)^m Dam^4x^7 + (cx)^m Cbm^4x^7 + 86 (cx)^m Dbm^2x^9 + 18 (cx)^m D}{}$$

input
```
integrate((c*x)^m*(b*x^2+a)*(D*x^6+C*x^4+B*x^2+A),x, algorithm="giac")
```

output
```
((c*x)^m*D*b*m^4*x^9 + 16*(c*x)^m*D*b*m^3*x^9 + (c*x)^m*D*a*m^4*x^7 + (c*x
)^m*C*b*m^4*x^7 + 86*(c*x)^m*D*b*m^2*x^9 + 18*(c*x)^m*D*a*m^3*x^7 + 18*(c*
x)^m*C*b*m^3*x^7 + 176*(c*x)^m*D*b*m*x^9 + (c*x)^m*C*a*m^4*x^5 + (c*x)^m*B
*b*m^4*x^5 + 104*(c*x)^m*D*a*m^2*x^7 + 104*(c*x)^m*C*b*m^2*x^7 + 105*(c*x)
^m*D*b*x^9 + 20*(c*x)^m*C*a*m^3*x^5 + 20*(c*x)^m*B*b*m^3*x^5 + 222*(c*x)^m
*D*a*m*x^7 + 222*(c*x)^m*C*b*m*x^7 + (c*x)^m*B*a*m^4*x^3 + (c*x)^m*A*b*m^4
*x^3 + 130*(c*x)^m*C*a*m^2*x^5 + 130*(c*x)^m*B*b*m^2*x^5 + 135*(c*x)^m*D*a
*x^7 + 135*(c*x)^m*C*b*x^7 + 22*(c*x)^m*B*a*m^3*x^3 + 22*(c*x)^m*A*b*m^3*x
^3 + 300*(c*x)^m*C*a*m*x^5 + 300*(c*x)^m*B*b*m*x^5 + (c*x)^m*A*a*m^4*x + 1
64*(c*x)^m*B*a*m^2*x^3 + 164*(c*x)^m*A*b*m^2*x^3 + 189*(c*x)^m*C*a*x^5 + 1
89*(c*x)^m*B*b*x^5 + 24*(c*x)^m*A*a*m^3*x + 458*(c*x)^m*B*a*m*x^3 + 458*(c
*x)^m*A*b*m*x^3 + 206*(c*x)^m*A*a*m^2*x + 315*(c*x)^m*B*a*x^3 + 315*(c*x)^
m*A*b*x^3 + 744*(c*x)^m*A*a*m*x + 945*(c*x)^m*A*a*x)/(m^5 + 25*m^4 + 230*m
^3 + 950*m^2 + 1689*m + 945)
```

## Mupad [F(-1)]

Timed out.

$$\int (cx)^m (a+bx^2) (A+Bx^2+Cx^4+Dx^6) \, dx = \int (c\,x)^m\,\left(b\,x^2+a\right)\,\left(A+B\,x^2+C\,x^4+x^6\,D\right) \,dx$$

input
```
int((c*x)^m*(a + b*x^2)*(A + B*x^2 + C*x^4 + x^6*D),x)
```

output
```
int((c*x)^m*(a + b*x^2)*(A + B*x^2 + C*x^4 + x^6*D), x)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 338, normalized size of antiderivative = 3.19

$$\int (cx)^m (a+bx^2) (A+Bx^2+Cx^4+Dx^6) \, dx = \frac{x^m c^m x(bd\, m^4x^8 + 16bd\, m^3x^8 + ad\, m^4x^6 + bc\, m^4x^6 + 86bd\, m^2x^8 + 18ad\, m^3x^6 + 18bc\, m^3x^6 + 176bdm\, x^8}{}$$

input
```
int((c*x)^m*(b*x^2+a)*(D*x^6+C*x^4+B*x^2+A),x)
```

output
```
(x**m*c**m*x*(a**2*m**4 + 24*a**2*m**3 + 206*a**2*m**2 + 744*a**2*m + 945*
a**2 + 2*a*b*m**4*x**2 + 44*a*b*m**3*x**2 + 328*a*b*m**2*x**2 + 916*a*b*m*
x**2 + 630*a*b*x**2 + a*c*m**4*x**4 + 20*a*c*m**3*x**4 + 130*a*c*m**2*x**4
 + 300*a*c*m*x**4 + 189*a*c*x**4 + a*d*m**4*x**6 + 18*a*d*m**3*x**6 + 104*
a*d*m**2*x**6 + 222*a*d*m*x**6 + 135*a*d*x**6 + b**2*m**4*x**4 + 20*b**2*m
**3*x**4 + 130*b**2*m**2*x**4 + 300*b**2*m*x**4 + 189*b**2*x**4 + b*c*m**4
*x**6 + 18*b*c*m**3*x**6 + 104*b*c*m**2*x**6 + 222*b*c*m*x**6 + 135*b*c*x*
*6 + b*d*m**4*x**8 + 16*b*d*m**3*x**8 + 86*b*d*m**2*x**8 + 176*b*d*m*x**8
+ 105*b*d*x**8))/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945)
```

# 3.255 $\int (cx)^m (A + Bx^2 + Cx^4 + Dx^6) dx$



## Optimal result

Integrand size = 23, antiderivative size = 69

$$\int (cx)^m (A+Bx^2+Cx^4+Dx^6) \, dx = \frac{A(cx)^{1+m}}{c(1+m)} + \frac{B(cx)^{3+m}}{c^3(3+m)} + \frac{C(cx)^{5+m}}{c^5(5+m)} + \frac{D(cx)^{7+m}}{c^7(7+m)}$$

output
```
A*(c*x)^(1+m)/c/(1+m)+B*(c*x)^(3+m)/c^3/(3+m)+C*(c*x)^(5+m)/c^5/(5+m)+D*(c
*x)^(7+m)/c^7/(7+m)
```

## Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.65

$$\int (cx)^m (A+Bx^2+Cx^4+Dx^6) \, dx = x(cx)^m \left(\frac{A}{1+m} + \frac{Bx^2}{3+m} + \frac{Cx^4}{5+m} + \frac{Dx^6}{7+m}\right)$$

input
```
Integrate[(c*x)^m*(A + B*x^2 + C*x^4 + D*x^6),x]
```

output
```
x*(c*x)^m*(A/(1 + m) + (B*x^2)/(3 + m) + (C*x^4)/(5 + m) + (D*x^6)/(7 + m)
)
```

## Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^m \left(A+Bx^2+Cx^4+Dx^6\right) \, dx$$

$$\Big\downarrow \ 2010$$

$$\int \left(A(cx)^m+\frac{B(cx)^{m+2}}{c^2}+\frac{D(cx)^{m+6}}{c^6}+\frac{C(cx)^{m+4}}{c^4}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{A(cx)^{m+1}}{c(m+1)}+\frac{B(cx)^{m+3}}{c^3(m+3)}+\frac{D(cx)^{m+7}}{c^7(m+7)}+\frac{C(cx)^{m+5}}{c^5(m+5)}$$

input
```
Int[(c*x)^m*(A + B*x^2 + C*x^4 + D*x^6),x]
```

output
```
(A*(c*x)^(1 + m))/(c*(1 + m)) + (B*(c*x)^(3 + m))/(c^3*(3 + m)) + (C*(c*x)
^(5 + m))/(c^5*(5 + m)) + (D*(c*x)^(7 + m))/(c^7*(7 + m))
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

## Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.99

| method | result |
|---|---|
| norman | $\frac{Ax\,\mathrm{e}^{m\ln(cx)}}{1+m}+\frac{B\,x^{3}\mathrm{e}^{m\ln(cx)}}{3+m}+\frac{C\,x^{5}\mathrm{e}^{m\ln(cx)}}{5+m}+\frac{Dx^{7}\mathrm{e}^{m\ln(cx)}}{7+m}$ |
| gosper | $\frac{x\left(Dm^{3}x^{6}+9Dm^{2}x^{6}+C\,m^{3}x^{4}+23Dm\,x^{6}+11C\,m^{2}x^{4}+15Dx^{6}+B\,m^{3}x^{2}+31Cm\,x^{4}+13B\,m^{2}x^{2}+21C\,x^{4}+A\,m^{3}+47Bm\,x^{2}-\right.}{(7+m)(5+m)(3+m)(1+m)}$ |
| orering | $\frac{x\left(Dm^{3}x^{6}+9Dm^{2}x^{6}+C\,m^{3}x^{4}+23Dm\,x^{6}+11C\,m^{2}x^{4}+15Dx^{6}+B\,m^{3}x^{2}+31Cm\,x^{4}+13B\,m^{2}x^{2}+21C\,x^{4}+A\,m^{3}+47Bm\,x^{2}-\right.}{(7+m)(5+m)(3+m)(1+m)}$ |
| parallelrisch | $\frac{Dx^{7}(cx)^{m}m^{3}+9Dx^{7}(cx)^{m}m^{2}+C\,x^{5}(cx)^{m}m^{3}+23Dx^{7}(cx)^{m}m+11C\,x^{5}(cx)^{m}m^{2}+15Dx^{7}(cx)^{m}+B\,x^{3}(cx)^{m}m^{3}+31C\,x^{5}(c}{(7+m)(5}$ |

input
```
int((c*x)^m*(D*x^6+C*x^4+B*x^2+A),x,method=_RETURNVERBOSE)
```

output
```
A/(1+m)*x*exp(m*ln(c*x))+B/(3+m)*x^3*exp(m*ln(c*x))+C/(5+m)*x^5*exp(m*ln(c
*x))+D/(7+m)*x^7*exp(m*ln(c*x))
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.70

$$\int (cx)^m\left(A+Bx^2+Cx^4+Dx^6\right)\,dx$$
$$=\frac{((Dm^3+9\,Dm^2+23\,Dm+15\,D)x^7+(Cm^3+11\,Cm^2+31\,Cm+21\,C)x^5+(Bm^3+13\,Bm^2+47\,B}{m^4+16\,m^3+86\,m^2+176\,m+105}$$

input
```
integrate((c*x)^m*(D*x^6+C*x^4+B*x^2+A),x, algorithm="fricas")
```

output
```
((D*m^3 + 9*D*m^2 + 23*D*m + 15*D)*x^7 + (C*m^3 + 11*C*m^2 + 31*C*m + 21*C
)*x^5 + (B*m^3 + 13*B*m^2 + 47*B*m + 35*B)*x^3 + (A*m^3 + 15*A*m^2 + 71*A*
m + 105*A)*x)*(c*x)^m/(m^4 + 16*m^3 + 86*m^2 + 176*m + 105)
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 624 vs. $2(58) = 116$.

Time = 0.45 (sec) , antiderivative size = 624, normalized size of antiderivative = 9.04

$$\int (cx)^m \left(A + Bx^2 + Cx^4 + Dx^6\right) dx$$
$$= \begin{cases} \frac{-\frac{A}{6x^6} - \frac{B}{4x^4} - \frac{C}{2x^2} + D\log(x)}{c^7} \\ \frac{-\frac{A}{4x^4} - \frac{B}{2x^2} + C\log(x) + \frac{Dx^2}{2}}{c^5} \\ \frac{-\frac{A}{2x^2} + B\log(x) + \frac{Cx^2}{2} + \frac{Dx^4}{4}}{c^3} \\ \frac{A\log(x) + \frac{Bx^2}{2} + \frac{Cx^4}{4} + \frac{Dx^6}{6}}{c} \\ \frac{Am^3x(cx)^m}{m^4+16m^3+86m^2+176m+105} + \frac{15Am^2x(cx)^m}{m^4+16m^3+86m^2+176m+105} + \frac{71Amx(cx)^m}{m^4+16m^3+86m^2+176m+105} + \frac{105Ax(cx)^m}{m^4+16m^3+86m^2+176m+105} \end{cases}$$

input

```
integrate((c*x)**m*(D*x**6+C*x**4+B*x**2+A),x)
```

output

```
Piecewise((((-A/(6*x**6) - B/(4*x**4) - C/(2*x**2) + D*log(x))/c**7, Eq(m,
-7)), ((-A/(4*x**4) - B/(2*x**2) + C*log(x) + D*x**2/2)/c**5, Eq(m, -5)),
((-A/(2*x**2) + B*log(x) + C*x**2/2 + D*x**4/4)/c**3, Eq(m, -3)), ((A*log(
x) + B*x**2/2 + C*x**4/4 + D*x**6/6)/c, Eq(m, -1)), (A*m**3*x*(c*x)**m/(m*
*4 + 16*m**3 + 86*m**2 + 176*m + 105) + 15*A*m**2*x*(c*x)**m/(m**4 + 16*m*
*3 + 86*m**2 + 176*m + 105) + 71*A*m*x*(c*x)**m/(m**4 + 16*m**3 + 86*m**2
+ 176*m + 105) + 105*A*x*(c*x)**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105)
 + B*m**3*x**3*(c*x)**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 13*B*m*
*2*x**3*(c*x)**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 47*B*m*x**3*(c
*x)**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 35*B*x**3*(c*x)**m/(m**4
 + 16*m**3 + 86*m**2 + 176*m + 105) + C*m**3*x**5*(c*x)**m/(m**4 + 16*m**3
 + 86*m**2 + 176*m + 105) + 11*C*m**2*x**5*(c*x)**m/(m**4 + 16*m**3 + 86*m
**2 + 176*m + 105) + 31*C*m*x**5*(c*x)**m/(m**4 + 16*m**3 + 86*m**2 + 176*
m + 105) + 21*C*x**5*(c*x)**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + D
*m**3*x**7*(c*x)**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 9*D*m**2*x*
*7*(c*x)**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 23*D*m*x**7*(c*x)**
m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 15*D*x**7*(c*x)**m/(m**4 + 16
*m**3 + 86*m**2 + 176*m + 105), True))
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96

$$\int (cx)^m \left(A+Bx^2+Cx^4+Dx^6\right) dx = \frac{Dc^m x^7 x^m}{m+7} + \frac{Cc^m x^5 x^m}{m+5} + \frac{Bc^m x^3 x^m}{m+3} + \frac{(cx)^{m+1} A}{c(m+1)}$$

input
```
integrate((c*x)^m*(D*x^6+C*x^4+B*x^2+A),x, algorithm="maxima")
```

output
```
D*c^m*x^7*x^m/(m + 7) + C*c^m*x^5*x^m/(m + 5) + B*c^m*x^3*x^m/(m + 3) + (c
*x)^(m + 1)*A/(c*(m + 1))
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 214 vs. 2(69) = 138.

Time = 0.13 (sec) , antiderivative size = 214, normalized size of antiderivative = 3.10

$$\int (cx)^m \left(A+Bx^2+Cx^4+Dx^6\right) dx = \frac{(cx)^m Dm^3x^7 + 9(cx)^m Dm^2x^7 + (cx)^m Cm^3x^5 + 23(cx)^m Dmx^7 + 11(cx)^m Cm^2x^5 + 15(cx)^m Dx^7 +$$

input
```
integrate((c*x)^m*(D*x^6+C*x^4+B*x^2+A),x, algorithm="giac")
```

output
```
((c*x)^m*D*m^3*x^7 + 9*(c*x)^m*D*m^2*x^7 + (c*x)^m*C*m^3*x^5 + 23*(c*x)^m*
D*m*x^7 + 11*(c*x)^m*C*m^2*x^5 + 15*(c*x)^m*D*x^7 + (c*x)^m*B*m^3*x^3 + 31
*(c*x)^m*C*m*x^5 + 13*(c*x)^m*B*m^2*x^3 + 21*(c*x)^m*C*x^5 + (c*x)^m*A*m^3
*x + 47*(c*x)^m*B*m*x^3 + 15*(c*x)^m*A*m^2*x + 35*(c*x)^m*B*x^3 + 71*(c*x)
^m*A*m*x + 105*(c*x)^m*A*x)/(m^4 + 16*m^3 + 86*m^2 + 176*m + 105)
```

## Mupad [F(-1)]

Timed out.

$$\int (cx)^m \left(A+Bx^2+Cx^4+Dx^6\right) \, dx = \int (c\,x)^m \left(A+B\,x^2+C\,x^4+x^6\,D\right) \, dx$$

input
```
int((c*x)^m*(A + B*x^2 + C*x^4 + x^6*D),x)
```

output
```
int((c*x)^m*(A + B*x^2 + C*x^4 + x^6*D), x)
```

## Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.99

$$\int (cx)^m \left(A+Bx^2+Cx^4+Dx^6\right) \, dx = \frac{x^m c^m x(d\,m^3x^6+9d\,m^2x^6+c\,m^3x^4+23dm\,x^6+11c\,m^2x^4+15d\,x^6+b\,m^3x^2+31cm\,x^4+13b\,m^2x^2+}{m^4+16m^3+86m^2+176m+105}$$

input
```
int((c*x)^m*(D*x^6+C*x^4+B*x^2+A),x)
```

output
```
(x**m*c**m*x*(a*m**3 + 15*a*m**2 + 71*a*m + 105*a + b*m**3*x**2 + 13*b*m**
2*x**2 + 47*b*m*x**2 + 35*b*x**2 + c*m**3*x**4 + 11*c*m**2*x**4 + 31*c*m*x
**4 + 21*c*x**4 + d*m**3*x**6 + 9*d*m**2*x**6 + 23*d*m*x**6 + 15*d*x**6))/
(m**4 + 16*m**3 + 86*m**2 + 176*m + 105)
```

# 3.256 $\int \frac{(cx)^m(A+Bx^2+Cx^4+Dx^6)}{a+bx^2}\,dx$



## Optimal result

Integrand size = 32, antiderivative size = 155

$$\int \frac{(cx)^m\left(A+Bx^2+Cx^4+Dx^6\right)}{a+bx^2}\,dx = \frac{\left(b^2B-abC+a^2D\right)(cx)^{1+m}}{b^3c(1+m)}+\frac{(bC-aD)(cx)^{3+m}}{b^2c^3(3+m)}+\frac{D(cx)^{5+m}}{bc^5(5+m)} + \frac{\left(Ab^3-a\left(b^2B-abC+a^2D\right)\right)(cx)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{bx^2}{a}\right)}{ab^3c(1+m)}$$

output
```
(B*b^2-C*a*b+D*a^2)*(c*x)^(1+m)/b^3/c/(1+m)+(C*b-D*a)*(c*x)^(3+m)/b^2/c^3/
(3+m)+D*(c*x)^(5+m)/b/c^5/(5+m)+(A*b^3-a*(B*b^2-C*a*b+D*a^2))*(c*x)^(1+m)*
hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-b*x^2/a)/a/b^3/c/(1+m)
```

## Mathematica [A] (verified)

Time = 0.90 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.79

$$\int \frac{(cx)^m\left(A+Bx^2+Cx^4+Dx^6\right)}{a+bx^2}\,dx = \frac{x(cx)^m\left(\frac{b^2B-abC+a^2D}{1+m}+\frac{b(bC-aD)x^2}{3+m}+\frac{b^2Dx^4}{5+m}+\frac{\left(Ab^3-a\left(b^2B-abC+a^2D\right)\right)\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{bx^2}{a}\right)}{a(1+m)}\right)}{b^3}$$

input
```
Integrate[((c*x)^m*(A + B*x^2 + C*x^4 + D*x^6))/(a + b*x^2),x]
```

output
```
(x*(c*x)^m*((b^2*B - a*b*C + a^2*D)/(1 + m) + (b*(b*C - a*D)*x^2)/(3 + m)
+ (b^2*D*x^4)/(5 + m) + ((A*b^3 - a*(b^2*B - a*b*C + a^2*D))*Hypergeometri
c2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(a*(1 + m))))/b^3
```

## Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^m \left(A+Bx^2+Cx^4+Dx^6\right)}{a+bx^2} \, dx$$

$$\Big\downarrow \text{ 2333}$$

$$\int \left(\frac{(cx)^m \left(a^2D-abC+b^2B\right)}{b^3}+\frac{(cx)^m \left(a^3(-D)+a^2bC-ab^2B+Ab^3\right)}{b^3 \left(a+bx^2\right)}+\frac{(cx)^{m+2}(bC-aD)}{b^2c^2}+\frac{D(cx)^{m+4}}{bc^4}\right)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{(cx)^{m+1} \text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\frac{bx^2}{a}\right) \left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{ab^3c(m+1)}+\frac{(cx)^{m+1} \left(a^2D-abC+b^2B\right)}{b^3c(m+1)}+\frac{(cx)^{m+3}(bC-aD)}{b^2c^3(m+3)}+\frac{D(cx)^{m+5}}{bc^5(m+5)}$$

input
```
Int[((c*x)^m*(A + B*x^2 + C*x^4 + D*x^6))/(a + b*x^2),x]
```

output
```
((b^2*B - a*b*C + a^2*D)*(c*x)^(1 + m))/(b^3*c*(1 + m)) + ((b*C - a*D)*(c*
x)^(3 + m))/(b^2*c^3*(3 + m)) + (D*(c*x)^(5 + m))/(b*c^5*(5 + m)) + ((A*b^
3 - a*(b^2*B - a*b*C + a^2*D))*(c*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/
2, (3 + m)/2, -((b*x^2)/a)])/(a*b^3*c*(1 + m))
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

## Maple [F]

$$\int \frac{(cx)^m (Dx^6 + C x^4 + x^2 B + A)}{b x^2 + a} dx$$

input
```
int((c*x)^m*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a),x)
```

output
```
int((c*x)^m*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a),x)
```

## Fricas [F]

$$\int \frac{(cx)^m (A + Bx^2 + Cx^4 + Dx^6)}{a + bx^2} dx = \int \frac{(Dx^6 + Cx^4 + Bx^2 + A)(cx)^m}{bx^2 + a} dx$$

input
```
integrate((c*x)^m*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a),x, algorithm="fricas")
```

output
```
integral((D*x^6 + C*x^4 + B*x^2 + A)*(c*x)^m/(b*x^2 + a), x)
```

## Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.56 (sec) , antiderivative size = 405, normalized size of antiderivative = 2.61

$$\int \frac{(cx)^m (A + Bx^2 + Cx^4 + Dx^6)}{a + bx^2} \, dx = \frac{Ac^m m x^{m+1} \Phi\left(\frac{bx^2 e^{i\pi}}{a}, 1, \frac{m}{2} + \frac{1}{2}\right) \Gamma\left(\frac{m}{2} + \frac{1}{2}\right)}{4a\Gamma\left(\frac{m}{2} + \frac{3}{2}\right)} + \frac{Ac^m x^{m+1} \Phi\left(\frac{bx^2 e^{i\pi}}{a}, 1, \frac{m}{2} + \frac{1}{2}\right) \Gamma\left(\frac{m}{2} + \frac{1}{2}\right)}{4a\Gamma\left(\frac{m}{2} + \frac{3}{2}\right)} + \frac{Bc^m m x^{m+3} \Phi\left(\frac{bx^2 e^{i\pi}}{a}, 1, \frac{m}{2} + \frac{3}{2}\right) \Gamma\left(\frac{m}{2} + \frac{3}{2}\right)}{4a\Gamma\left(\frac{m}{2} + \frac{5}{2}\right)} + \frac{3Bc^m x^{m+3} \Phi\left(\frac{bx^2 e^{i\pi}}{a}, 1, \frac{m}{2} + \frac{3}{2}\right) \Gamma\left(\frac{m}{2} + \frac{3}{2}\right)}{4a\Gamma\left(\frac{m}{2} + \frac{5}{2}\right)} + \frac{Cc^m m x^{m+5} \Phi\left(\frac{bx^2 e^{i\pi}}{a}, 1, \frac{m}{2} + \frac{5}{2}\right) \Gamma\left(\frac{m}{2} + \frac{5}{2}\right)}{4a\Gamma\left(\frac{m}{2} + \frac{7}{2}\right)} + \frac{5Cc^m x^{m+5} \Phi\left(\frac{bx^2 e^{i\pi}}{a}, 1, \frac{m}{2} + \frac{5}{2}\right) \Gamma\left(\frac{m}{2} + \frac{5}{2}\right)}{4a\Gamma\left(\frac{m}{2} + \frac{7}{2}\right)} + \frac{Dc^m m x^{m+7} \Phi\left(\frac{bx^2 e^{i\pi}}{a}, 1, \frac{m}{2} + \frac{7}{2}\right) \Gamma\left(\frac{m}{2} + \frac{7}{2}\right)}{4a\Gamma\left(\frac{m}{2} + \frac{9}{2}\right)} + \frac{7Dc^m x^{m+7} \Phi\left(\frac{bx^2 e^{i\pi}}{a}, 1, \frac{m}{2} + \frac{7}{2}\right) \Gamma\left(\frac{m}{2} + \frac{7}{2}\right)}{4a\Gamma\left(\frac{m}{2} + \frac{9}{2}\right)}$$

input
```
integrate((c*x)**m*(D*x**6+C*x**4+B*x**2+A)/(b*x**2+a),x)
```

output
```
A*c**m*m*x**(m + 1)*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma
(m/2 + 1/2)/(4*a*gamma(m/2 + 3/2)) + A*c**m*x**(m + 1)*lerchphi(b*x**2*exp
_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(4*a*gamma(m/2 + 3/2)) + B*
c**m*m*x**(m + 3)*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 3/2)*gamma(m
/2 + 3/2)/(4*a*gamma(m/2 + 5/2)) + 3*B*c**m*x**(m + 3)*lerchphi(b*x**2*exp
_polar(I*pi)/a, 1, m/2 + 3/2)*gamma(m/2 + 3/2)/(4*a*gamma(m/2 + 5/2)) + C*
c**m*m*x**(m + 5)*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 5/2)*gamma(m
/2 + 5/2)/(4*a*gamma(m/2 + 7/2)) + 5*C*c**m*x**(m + 5)*lerchphi(b*x**2*exp
_polar(I*pi)/a, 1, m/2 + 5/2)*gamma(m/2 + 5/2)/(4*a*gamma(m/2 + 7/2)) + D*
c**m*m*x**(m + 7)*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 7/2)*gamma(m
/2 + 7/2)/(4*a*gamma(m/2 + 9/2)) + 7*D*c**m*x**(m + 7)*lerchphi(b*x**2*exp
_polar(I*pi)/a, 1, m/2 + 7/2)*gamma(m/2 + 7/2)/(4*a*gamma(m/2 + 9/2))
```

## Maxima [F]

$$\int \frac{(cx)^m (A + Bx^2 + Cx^4 + Dx^6)}{a + bx^2} \, dx = \int \frac{(Dx^6 + Cx^4 + Bx^2 + A)(cx)^m}{bx^2 + a} \, dx$$

input
```
integrate((c*x)^m*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a),x, algorithm="maxima")
```

output
```
integrate((D*x^6 + C*x^4 + B*x^2 + A)*(c*x)^m/(b*x^2 + a), x)
```

## Giac [F]

$$\int \frac{(cx)^m (A + Bx^2 + Cx^4 + Dx^6)}{a + bx^2} \, dx = \int \frac{(Dx^6 + Cx^4 + Bx^2 + A)(cx)^m}{bx^2 + a} \, dx$$

input
```
integrate((c*x)^m*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a),x, algorithm="giac")
```

output
```
integrate((D*x^6 + C*x^4 + B*x^2 + A)*(c*x)^m/(b*x^2 + a), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^m (A+Bx^2+Cx^4+Dx^6)}{a+bx^2}\,dx=\int \frac{(c\,x)^m\,\left(A+B\,x^2+C\,x^4+x^6\,D\right)}{b\,x^2+a}\,dx$$

input
```
int(((c*x)^m*(A + B*x^2 + C*x^4 + x^6*D))/(a + b*x^2),x)
```

output
```
int(((c*x)^m*(A + B*x^2 + C*x^4 + x^6*D))/(a + b*x^2), x)
```

## Reduce [F]

$$\int \frac{(cx)^m (A+Bx^2+Cx^4+Dx^6)}{a+bx^2}\,dx$$
$$=\frac{c^m\left(15x^m b^3x+3x^m b^2d\,x^5-9\left(\int \frac{x^m}{b\,x^2+a}dx\right)a^3d\,m^2-23\left(\int \frac{x^m}{b\,x^2+a}dx\right)a^3dm+15\left(\int \frac{x^m}{b\,x^2+a}dx\right)a^2bc-\left(\int \frac{x^m}{b\,x^2}\right.\right.}{}$$

input
```
int((c*x)^m*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a),x)
```

output
```
(c**m*(x**m*a**2*d*m**2*x + 8*x**m*a**2*d*m*x + 15*x**m*a**2*d*x - x**m*a*
b*c*m**2*x - 8*x**m*a*b*c*m*x - 15*x**m*a*b*c*x - x**m*a*b*d*m**2*x**3 - 6
*x**m*a*b*d*m*x**3 - 5*x**m*a*b*d*x**3 + x**m*b**3*m**2*x + 8*x**m*b**3*m*
x + 15*x**m*b**3*x + x**m*b**2*c*m**2*x**3 + 6*x**m*b**2*c*m*x**3 + 5*x**m
*b**2*c*x**3 + x**m*b**2*d*m**2*x**5 + 4*x**m*b**2*d*m*x**5 + 3*x**m*b**2*
d*x**5 - int(x**m/(a + b*x**2),x)*a**3*d*m**3 - 9*int(x**m/(a + b*x**2),x)
*a**3*d*m**2 - 23*int(x**m/(a + b*x**2),x)*a**3*d*m - 15*int(x**m/(a + b*x
**2),x)*a**3*d + int(x**m/(a + b*x**2),x)*a**2*b*c*m**3 + 9*int(x**m/(a +
b*x**2),x)*a**2*b*c*m**2 + 23*int(x**m/(a + b*x**2),x)*a**2*b*c*m + 15*int
(x**m/(a + b*x**2),x)*a**2*b*c))/(b**3*(m**3 + 9*m**2 + 23*m + 15))
```

# 3.257 $\int \frac{(cx)^m(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^2}\,dx$



## Optimal result

Integrand size = 32, antiderivative size = 186

$$\int \frac{(cx)^m\left(A+Bx^2+Cx^4+Dx^6\right)}{\left(a+bx^2\right)^2}\,dx = \frac{(bC-2aD)(cx)^{1+m}}{b^3c(1+m)}+\frac{D(cx)^{3+m}}{b^2c^3(3+m)}+\frac{\left(\frac{A}{a}-\frac{b^2B-abC+a^2D}{b^3}\right)(cx)^{1+m}}{2c\left(a+bx^2\right)} + \frac{\left(Ab^3(1-m)+a(b^2B(1+m)-abC(3+m)+a^2D(5+m))\right)(cx)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2}\ldots\right)}{2a^2b^3c(1+m)}$$

output
```
(C*b-2*D*a)*(c*x)^(1+m)/b^3/c/(1+m)+D*(c*x)^(3+m)/b^2/c^3/(3+m)+1/2*(A/a-(
B*b^2-C*a*b+D*a^2)/b^3)*(c*x)^(1+m)/c/(b*x^2+a)+1/2*(A*b^3*(1-m)+a*(b^2*B*
(1+m)-a*b*C*(3+m)+a^2*D*(5+m)))*(c*x)^(1+m)*hypergeom([1, 1/2+1/2*m],[3/2+
1/2*m],-b*x^2/a)/a^2/b^3/c/(1+m)
```

## Mathematica [A] (verified)

Time = 1.10 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.78

$$\int \frac{(cx)^m (A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^2}\,dx$$
$$= \frac{x(cx)^m\left(\frac{bC-2aD}{1+m}+\frac{bDx^2}{3+m}+\frac{(b^2B-2abC+3a^2D)\text{ Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{bx^2}{a}\right)}{a(1+m)}+\frac{(Ab^3-a(b^2B-abC+a^2D))\text{ Hypergeo}}{a^2(1+m}\right.}{b^3}$$

input
```
Integrate[((c*x)^m*(A + B*x^2 + C*x^4 + D*x^6))/(a + b*x^2)^2,x]
```

output
```
(x*(c*x)^m*((b*C - 2*a*D)/(1 + m) + (b*D*x^2)/(3 + m) + ((b^2*B - 2*a*b*C
+ 3*a^2*D)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(a*(1
 + m)) + ((A*b^3 - a*(b^2*B - a*b*C + a^2*D))*Hypergeometric2F1[2, (1 + m)
/2, (3 + m)/2, -((b*x^2)/a)])/(a^2*(1 + m))))/b^3
```

## Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2337, 25, 1584, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^m (A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^2}\,dx$$
$$\Big\downarrow \text{ 2337}$$
$$\frac{(cx)^{m+1}\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{2ac\,(a+bx^2)}-\frac{\int -\frac{(cx)^m\left(\frac{2aDx^4}{b}+\frac{2a(bC-aD)x^2}{b^2}+A(1-m)+\frac{a\left(Da^2-bCa+b^2B\right)(m+1)}{b^3}\right)}{bx^2+a}dx}{2a}$$
$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{(cx)^m\left(\frac{2aDx^4}{b}+\frac{2a(bC-aD)x^2}{b^2}+A-Am+\frac{a\left(Da^2-bCa+b^2B\right)(m+1)}{b^3}\right)}{bx^2+a}dx}{2a}+\frac{(cx)^{m+1}\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{2ac\,(a+bx^2)}$$

$$\Big\downarrow \text{ 1584}$$

$$\frac{\int \left(\frac{2a(bC-2aD)(cx)^m}{b^3}+\frac{(5Da^3+Dma^3-3bCa^2-bCma^2+b^2Ba+b^2Bma+Ab^3-Ab^3m)(cx)^m}{b^3(bx^2+a)}+\frac{2aD(cx)^{m+2}}{b^2c^2}\right)dx}{2a}+\frac{(cx)^{m+1}\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{2ac\,(a+bx^2)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{(cx)^{m+1}\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\frac{bx^2}{a}\right)\left(a\left(a^2D(m+5)-abC(m+3)+b^2B(m+1)\right)+Ab^3(1-m)\right)}{ab^3c(m+1)}+\frac{2a(cx)^{m+1}(bC-2aD)}{b^3c(m+1)}+\frac{2aD(c...}{b^2c^3(...}}{2a}+\frac{(cx)^{m+1}\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{2ac\,(a+bx^2)}$$

input
```
Int[((c*x)^m*(A + B*x^2 + C*x^4 + D*x^6))/(a + b*x^2)^2,x]
```

output
```
((A - (a*(b^2*B - a*b*C + a^2*D))/b^3)*(c*x)^(1 + m))/(2*a*c*(a + b*x^2))
+ ((2*a*(b*C - 2*a*D)*(c*x)^(1 + m))/(b^3*c*(1 + m)) + (2*a*D*(c*x)^(3 + m
))/(b^2*c^3*(3 + m)) + ((A*b^3*(1 - m) + a*(b^2*B*(1 + m) - a*b*C*(3 + m)
+ a^2*D*(5 + m)))*(c*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2,
 -((b*x^2)/a)])/(a*b^3*c*(1 + m)))/(2*a)
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1584
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2337
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(-(c*x)^(m + 1))*(f + g*x)*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1)))
, x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*ExpandToSum[2
*a*(p + 1)*Q + f*(m + 2*p + 3) + g*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a
, b, c, m}, x] && PolyQ[Pq, x] && LtQ[p, -1] &&  !GtQ[m, 0]
```

## Maple [F]

$$\int \frac{(cx)^m (D x^6 + C x^4 + x^2 B + A)}{(b x^2 + a)^2} dx$$

input
```
int((c*x)^m*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^2,x)
```

output
```
int((c*x)^m*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^2,x)
```

## Fricas [F]

$$\int \frac{(cx)^m (A + Bx^2 + Cx^4 + Dx^6)}{(a + bx^2)^2} dx = \int \frac{(Dx^6 + Cx^4 + Bx^2 + A)(cx)^m}{(bx^2 + a)^2} dx$$

input
```
integrate((c*x)^m*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
integral((D*x^6 + C*x^4 + B*x^2 + A)*(c*x)^m/(b^2*x^4 + 2*a*b*x^2 + a^2),
x)
```

## Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 38.21 (sec) , antiderivative size = 2062, normalized size of antiderivative = 11.09

$$\int \frac{(cx)^m (A + Bx^2 + Cx^4 + Dx^6)}{(a + bx^2)^2} \, dx = \text{Too large to display}$$

input
```
integrate((c*x)**m*(D*x**6+C*x**4+B*x**2+A)/(b*x**2+a)**2,x)
```

output
```
A*(-a*c**m*m**2*x**(m + 1)*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2
)*gamma(m/2 + 1/2)/(8*a**3*gamma(m/2 + 3/2) + 8*a**2*b*x**2*gamma(m/2 + 3/
2)) + 2*a*c**m*m*x**(m + 1)*gamma(m/2 + 1/2)/(8*a**3*gamma(m/2 + 3/2) + 8*
a**2*b*x**2*gamma(m/2 + 3/2)) + a*c**m*x**(m + 1)*lerchphi(b*x**2*exp_pola
r(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(8*a**3*gamma(m/2 + 3/2) + 8*a**
2*b*x**2*gamma(m/2 + 3/2)) + 2*a*c**m*x**(m + 1)*gamma(m/2 + 1/2)/(8*a**3*
gamma(m/2 + 3/2) + 8*a**2*b*x**2*gamma(m/2 + 3/2)) - b*c**m*m**2*x**2*x**(
m + 1)*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(
8*a**3*gamma(m/2 + 3/2) + 8*a**2*b*x**2*gamma(m/2 + 3/2)) + b*c**m*x**2*x*
*(m + 1)*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)
/(8*a**3*gamma(m/2 + 3/2) + 8*a**2*b*x**2*gamma(m/2 + 3/2))) + B*(-a*c**m*
m**2*x**(m + 3)*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 3/2)*gamma(m/2
 + 3/2)/(8*a**3*gamma(m/2 + 5/2) + 8*a**2*b*x**2*gamma(m/2 + 5/2)) - 4*a*c
**m*m*x**(m + 3)*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 3/2)*gamma(m/
2 + 3/2)/(8*a**3*gamma(m/2 + 5/2) + 8*a**2*b*x**2*gamma(m/2 + 5/2)) + 2*a*
c**m*m*x**(m + 3)*gamma(m/2 + 3/2)/(8*a**3*gamma(m/2 + 5/2) + 8*a**2*b*x**
2*gamma(m/2 + 5/2)) - 3*a*c**m*x**(m + 3)*lerchphi(b*x**2*exp_polar(I*pi)/
a, 1, m/2 + 3/2)*gamma(m/2 + 3/2)/(8*a**3*gamma(m/2 + 5/2) + 8*a**2*b*x**2
*gamma(m/2 + 5/2)) + 6*a*c**m*x**(m + 3)*gamma(m/2 + 3/2)/(8*a**3*gamma(m/
2 + 5/2) + 8*a**2*b*x**2*gamma(m/2 + 5/2)) - b*c**m*m**2*x**2*x**(m + 3...
```

## Maxima [F]

$$\int \frac{(cx)^m (A + Bx^2 + Cx^4 + Dx^6)}{(a + bx^2)^2} dx = \int \frac{(Dx^6 + Cx^4 + Bx^2 + A)(cx)^m}{(bx^2 + a)^2} dx$$

input
```
integrate((c*x)^m*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
integrate((D*x^6 + C*x^4 + B*x^2 + A)*(c*x)^m/(b*x^2 + a)^2, x)
```

## Giac [F]

$$\int \frac{(cx)^m (A + Bx^2 + Cx^4 + Dx^6)}{(a + bx^2)^2} dx = \int \frac{(Dx^6 + Cx^4 + Bx^2 + A)(cx)^m}{(bx^2 + a)^2} dx$$

input
```
integrate((c*x)^m*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
integrate((D*x^6 + C*x^4 + B*x^2 + A)*(c*x)^m/(b*x^2 + a)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^m (A + Bx^2 + Cx^4 + Dx^6)}{(a + bx^2)^2} dx = \int \frac{(c\,x)^m\,(A + B\,x^2 + C\,x^4 + x^6\,D)}{(b\,x^2 + a)^2} dx$$

input
```
int(((c*x)^m*(A + B*x^2 + C*x^4 + x^6*D))/(a + b*x^2)^2,x)
```

output
```
int(((c*x)^m*(A + B*x^2 + C*x^4 + x^6*D))/(a + b*x^2)^2, x)
```

## Reduce [F]

$$\int \frac{(cx)^m (A + Bx^2 + Cx^4 + Dx^6)}{(a + bx^2)^2} \, dx = \text{too large to display}$$

input
```
int((c*x)^m*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^2,x)
```

output
```
(c**m*(x**m*a**2*d*m**2*x + 8*x**m*a**2*d*m*x + 15*x**m*a**2*d*x - x**m*a*
b*c*m**2*x - 6*x**m*a*b*c*m*x - 9*x**m*a*b*c*x - x**m*a*b*d*m**2*x**3 - 4*
x**m*a*b*d*m*x**3 + 5*x**m*a*b*d*x**3 + x**m*b**3*m**2*x + 4*x**m*b**3*m*x
 + 3*x**m*b**3*x + x**m*b**2*c*m**2*x**3 + 2*x**m*b**2*c*m*x**3 - 3*x**m*b
**2*c*x**3 + x**m*b**2*d*m**2*x**5 - x**m*b**2*d*x**5 - int(x**m/(a**2*m -
 a**2 + 2*a*b*m*x**2 - 2*a*b*x**2 + b**2*m*x**4 - b**2*x**4),x)*a**4*d*m**
4 - 8*int(x**m/(a**2*m - a**2 + 2*a*b*m*x**2 - 2*a*b*x**2 + b**2*m*x**4 -
b**2*x**4),x)*a**4*d*m**3 - 14*int(x**m/(a**2*m - a**2 + 2*a*b*m*x**2 - 2*
a*b*x**2 + b**2*m*x**4 - b**2*x**4),x)*a**4*d*m**2 + 8*int(x**m/(a**2*m -
a**2 + 2*a*b*m*x**2 - 2*a*b*x**2 + b**2*m*x**4 - b**2*x**4),x)*a**4*d*m +
15*int(x**m/(a**2*m - a**2 + 2*a*b*m*x**2 - 2*a*b*x**2 + b**2*m*x**4 - b**
2*x**4),x)*a**4*d + int(x**m/(a**2*m - a**2 + 2*a*b*m*x**2 - 2*a*b*x**2 +
b**2*m*x**4 - b**2*x**4),x)*a**3*b*c*m**4 + 6*int(x**m/(a**2*m - a**2 + 2*
a*b*m*x**2 - 2*a*b*x**2 + b**2*m*x**4 - b**2*x**4),x)*a**3*b*c*m**3 + 8*in
t(x**m/(a**2*m - a**2 + 2*a*b*m*x**2 - 2*a*b*x**2 + b**2*m*x**4 - b**2*x**
4),x)*a**3*b*c*m**2 - 6*int(x**m/(a**2*m - a**2 + 2*a*b*m*x**2 - 2*a*b*x**
2 + b**2*m*x**4 - b**2*x**4),x)*a**3*b*c*m - 9*int(x**m/(a**2*m - a**2 + 2
*a*b*m*x**2 - 2*a*b*x**2 + b**2*m*x**4 - b**2*x**4),x)*a**3*b*c - int(x**m
/(a**2*m - a**2 + 2*a*b*m*x**2 - 2*a*b*x**2 + b**2*m*x**4 - b**2*x**4),x)*
a**3*b*d*m**4*x**2 - 8*int(x**m/(a**2*m - a**2 + 2*a*b*m*x**2 - 2*a*b*x...
```

# 3.258 $\int \frac{(cx)^m(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^3}\,dx$



## Optimal result

Integrand size = 32, antiderivative size = 242

$$\int \frac{(cx)^m\,(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^3}\,dx = \frac{D(cx)^{1+m}}{b^3c(1+m)} + \frac{\left(\frac{A}{a}-\frac{b^2B-abC+a^2D}{b^3}\right)(cx)^{1+m}}{4c\,(a+bx^2)^2}$$
$$+ \frac{\left(\frac{A(3-m)}{a}+\frac{b^2B(1+m)-abC(5+m)+a^2D(9+m)}{b^3}\right)(cx)^{1+m}}{8ac\,(a+bx^2)}$$
$$+ \frac{(Ab^3(3-4m+m^2)+a(abC(3+4m+m^2)-a^2D(15+8m+m^2)+b^2(B-Bm^2)))\,(cx)^{1+m}\,\text{Hyperg}}{8a^3b^3c(1+m)}$$

output

```
D*(c*x)^(1+m)/b^3/c/(1+m)+1/4*(A/a-(B*b^2-C*a*b+D*a^2)/b^3)*(c*x)^(1+m)/c/
(b*x^2+a)^2+1/8*(A*(3-m)/a+(b^2*B*(1+m)-a*b*C*(5+m)+a^2*D*(9+m))/b^3)*(c*x
)^(1+m)/a/c/(b*x^2+a)+1/8*(A*b^3*(m^2-4*m+3)+a*(a*b*C*(m^2+4*m+3)-a^2*D*(m
^2+8*m+15)+b^2*(-B*m^2+B)))*(c*x)^(1+m)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*
m],-b*x^2/a)/a^3/b^3/c/(1+m)
```

## Mathematica [A] (verified)

Time = 1.30 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.64

$$\int \frac{(cx)^m (A + Bx^2 + Cx^4 + Dx^6)}{(a + bx^2)^3} dx$$
$$= \frac{x(cx)^m \left(D + \frac{(bC-3aD)\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{bx^2}{a}\right)}{a} + \frac{(b^2B-2abC+3a^2D)\,\text{Hypergeometric2F1}\left(2,\frac{1+m}{2},\frac{3+m}{2},-\frac{bx^2}{a}\right)}{a^2} \right.}{b^3(1+m)}$$

input
```
Integrate[((c*x)^m*(A + B*x^2 + C*x^4 + D*x^6))/(a + b*x^2)^3,x]
```

output
```
(x*(c*x)^m*(D + ((b*C - 3*a*D)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2,
-((b*x^2)/a)])/a + ((b^2*B - 2*a*b*C + 3*a^2*D)*Hypergeometric2F1[2, (1 +
m)/2, (3 + m)/2, -((b*x^2)/a)])/a^2 + ((A*b^3 - a*(b^2*B - a*b*C + a^2*D))
*Hypergeometric2F1[3, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/a^3))/(b^3*(1 +
 m))
```

## Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.90, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2337, 25, 1674, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^m (A + Bx^2 + Cx^4 + Dx^6)}{(a + bx^2)^3} dx$$
$$\Big\downarrow \text{2337}$$
$$\frac{(cx)^{m+1} \left(A - \frac{a(a^2D-abC+b^2B)}{b^3}\right)}{4ac\,(a+bx^2)^2} - \frac{\int -\frac{(cx)^m \left(\frac{4aDx^4}{b} + \frac{4a(bC-aD)x^2}{b^2} + A(3-m) + \frac{a\left(Da^2-bCa+b^2B\right)(m+1)}{b^3}\right)}{(bx^2+a)^2}dx}{4a}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{(cx)^m\left(\frac{4aDx^4}{b}+\frac{4a(bC-aD)x^2}{b^2}+A(3-m)+\frac{a\left(Da^2-bCa+b^2B\right)(m+1)}{b^3}\right)}{(bx^2+a)^2}dx}{4a}+\frac{(cx)^{m+1}\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{4ac\left(a+bx^2\right)^2}$$

$$\Big\downarrow \ 1674$$

$$\frac{\int \left(\frac{4aD(cx)^m}{b^3}-\frac{4a(3aD-bC)(cx)^m}{b^3(bx^2+a)}+\frac{\left(A(3-m)b^3+a\left(D(m+9)a^2-bC(m+5)a+b^2B(m+1)\right)\right)(cx)^m}{b^3(bx^2+a)^2}\right)dx}{4a}+\frac{(cx)^{m+1}\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{4ac\left(a+bx^2\right)^2}$$

$$\Big\downarrow \ 2009$$

$$\frac{\frac{(cx)^{m+1}\,\text{Hypergeometric2F1}\left(2,\frac{m+1}{2},\frac{m+3}{2},-\frac{bx^2}{a}\right)\left(a\left(a^2D(m+9)-abC(m+5)+b^2B(m+1)\right)+Ab^3(3-m)\right)}{a^2b^3c(m+1)}+\frac{4(cx)^{m+1}(bC-3aD)\,\text{Hypergeome}}{b^3c(m-}}{4a}$$

$$\frac{(cx)^{m+1}\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{4ac\left(a+bx^2\right)^2}$$

input

```
Int[((c*x)^m*(A + B*x^2 + C*x^4 + D*x^6))/(a + b*x^2)^3,x]
```

output

```
((A - (a*(b^2*B - a*b*C + a^2*D))/b^3)*(c*x)^(1 + m))/(4*a*c*(a + b*x^2)^2
) + ((4*a*D*(c*x)^(1 + m))/(b^3*c*(1 + m)) + (4*(b*C - 3*a*D)*(c*x)^(1 + m
)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(b^3*c*(1 + m)
) + ((A*b^3*(3 - m) + a*(b^2*B*(1 + m) - a*b*C*(5 + m) + a^2*D*(9 + m)))*(
c*x)^(1 + m)*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(a^
2*b^3*c*(1 + m)))/(4*a)
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1674
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2337
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(-(c*x)^(m + 1))*(f + g*x)*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1)))
, x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*ExpandToSum[2
*a*(p + 1)*Q + f*(m + 2*p + 3) + g*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a
, b, c, m}, x] && PolyQ[Pq, x] && LtQ[p, -1] &&  !GtQ[m, 0]
```

## Maple [F]

$$\int \frac{(cx)^m (Dx^6 + C x^4 + x^2 B + A)}{(b x^2 + a)^3} dx$$

input
```
int((c*x)^m*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^3,x)
```

output
```
int((c*x)^m*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^3,x)
```

## Fricas [F]

$$\int \frac{(cx)^m (A + Bx^2 + Cx^4 + Dx^6)}{(a + bx^2)^3} \, dx = \int \frac{(Dx^6 + Cx^4 + Bx^2 + A)(cx)^m}{(bx^2 + a)^3} \, dx$$

input
```
integrate((c*x)^m*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
integral((D*x^6 + C*x^4 + B*x^2 + A)*(c*x)^m/(b^3*x^6 + 3*a*b^2*x^4 + 3*a^
2*b*x^2 + a^3), x)
```

## Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 107.43 (sec) , antiderivative size = 6484, normalized size of antiderivative = 26.79

$$\int \frac{(cx)^m (A + Bx^2 + Cx^4 + Dx^6)}{(a + bx^2)^3} \, dx = \text{Too large to display}$$

input
```
integrate((c*x)**m*(D*x**6+C*x**4+B*x**2+A)/(b*x**2+a)**3,x)
```

output
```
A*(a**2*c**m*m**3*x**(m + 1)*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1
/2)*gamma(m/2 + 1/2)/(32*a**5*gamma(m/2 + 3/2) + 64*a**4*b*x**2*gamma(m/2
+ 3/2) + 32*a**3*b**2*x**4*gamma(m/2 + 3/2)) - 3*a**2*c**m*m**2*x**(m + 1)
*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(32*a**
5*gamma(m/2 + 3/2) + 64*a**4*b*x**2*gamma(m/2 + 3/2) + 32*a**3*b**2*x**4*g
amma(m/2 + 3/2)) - 2*a**2*c**m*m**2*x**(m + 1)*gamma(m/2 + 1/2)/(32*a**5*g
amma(m/2 + 3/2) + 64*a**4*b*x**2*gamma(m/2 + 3/2) + 32*a**3*b**2*x**4*gamm
a(m/2 + 3/2)) - a**2*c**m*m*x**(m + 1)*lerchphi(b*x**2*exp_polar(I*pi)/a,
1, m/2 + 1/2)*gamma(m/2 + 1/2)/(32*a**5*gamma(m/2 + 3/2) + 64*a**4*b*x**2*
gamma(m/2 + 3/2) + 32*a**3*b**2*x**4*gamma(m/2 + 3/2)) + 8*a**2*c**m*m*x**
(m + 1)*gamma(m/2 + 1/2)/(32*a**5*gamma(m/2 + 3/2) + 64*a**4*b*x**2*gamma(
m/2 + 3/2) + 32*a**3*b**2*x**4*gamma(m/2 + 3/2)) + 3*a**2*c**m*x**(m + 1)*
lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(32*a**5
*gamma(m/2 + 3/2) + 64*a**4*b*x**2*gamma(m/2 + 3/2) + 32*a**3*b**2*x**4*ga
mma(m/2 + 3/2)) + 10*a**2*c**m*x**(m + 1)*gamma(m/2 + 1/2)/(32*a**5*gamma(
m/2 + 3/2) + 64*a**4*b*x**2*gamma(m/2 + 3/2) + 32*a**3*b**2*x**4*gamma(m/2
 + 3/2)) + 2*a*b*c**m*m**3*x**2*x**(m + 1)*lerchphi(b*x**2*exp_polar(I*pi)
/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(32*a**5*gamma(m/2 + 3/2) + 64*a**4*b*x
**2*gamma(m/2 + 3/2) + 32*a**3*b**2*x**4*gamma(m/2 + 3/2)) - 6*a*b*c**m*m*
*2*x**2*x**(m + 1)*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gam...
```

## Maxima [F]

$$\int \frac{(cx)^m (A + Bx^2 + Cx^4 + Dx^6)}{(a + bx^2)^3}\, dx = \int \frac{(Dx^6 + Cx^4 + Bx^2 + A)(cx)^m}{(bx^2 + a)^3}\, dx$$

input
```
integrate((c*x)^m*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
integrate((D*x^6 + C*x^4 + B*x^2 + A)*(c*x)^m/(b*x^2 + a)^3, x)
```

## Giac [F]

$$\int \frac{(cx)^m (A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^3} \, dx = \int \frac{(Dx^6+Cx^4+Bx^2+A)(cx)^m}{(bx^2+a)^3} \, dx$$

input
```
integrate((c*x)^m*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
integrate((D*x^6 + C*x^4 + B*x^2 + A)*(c*x)^m/(b*x^2 + a)^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^m (A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^3} \, dx = \int \frac{(c\,x)^m\,(A+B\,x^2+C\,x^4+x^6\,D)}{(b\,x^2+a)^3} \, dx$$

input
```
int(((c*x)^m*(A + B*x^2 + C*x^4 + x^6*D))/(a + b*x^2)^3,x)
```

output
```
int(((c*x)^m*(A + B*x^2 + C*x^4 + x^6*D))/(a + b*x^2)^3, x)
```

## Reduce [F]

$$\int \frac{(cx)^m (A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^3} \, dx = \text{too large to display}$$

input
```
int((c*x)^m*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^3,x)
```

output
```
(c**m*(x**m*a**2*d*m**2*x + 8*x**m*a**2*d*m*x + 15*x**m*a**2*d*x - x**m*a*
b*c*m**2*x - 4*x**m*a*b*c*m*x - 3*x**m*a*b*c*x - x**m*a*b*d*m**2*x**3 - 2*
x**m*a*b*d*m*x**3 + 15*x**m*a*b*d*x**3 + x**m*b**3*m**2*x - x**m*b**3*x +
x**m*b**2*c*m**2*x**3 - 2*x**m*b**2*c*m*x**3 - 3*x**m*b**2*c*x**3 + x**m*b
**2*d*m**2*x**5 - 4*x**m*b**2*d*m*x**5 + 3*x**m*b**2*d*x**5 - int(x**m/(a*
*3*m**2 - 4*a**3*m + 3*a**3 + 3*a**2*b*m**2*x**2 - 12*a**2*b*m*x**2 + 9*a*
*2*b*x**2 + 3*a*b**2*m**2*x**4 - 12*a*b**2*m*x**4 + 9*a*b**2*x**4 + b**3*m
**2*x**6 - 4*b**3*m*x**6 + 3*b**3*x**6),x)*a**5*d*m**5 - 5*int(x**m/(a**3*
m**2 - 4*a**3*m + 3*a**3 + 3*a**2*b*m**2*x**2 - 12*a**2*b*m*x**2 + 9*a**2*
b*x**2 + 3*a*b**2*m**2*x**4 - 12*a*b**2*m*x**4 + 9*a*b**2*x**4 + b**3*m**2
*x**6 - 4*b**3*m*x**6 + 3*b**3*x**6),x)*a**5*d*m**4 + 10*int(x**m/(a**3*m*
*2 - 4*a**3*m + 3*a**3 + 3*a**2*b*m**2*x**2 - 12*a**2*b*m*x**2 + 9*a**2*b*
x**2 + 3*a*b**2*m**2*x**4 - 12*a*b**2*m*x**4 + 9*a*b**2*x**4 + b**3*m**2*x
**6 - 4*b**3*m*x**6 + 3*b**3*x**6),x)*a**5*d*m**3 + 50*int(x**m/(a**3*m**2
 - 4*a**3*m + 3*a**3 + 3*a**2*b*m**2*x**2 - 12*a**2*b*m*x**2 + 9*a**2*b*x*
*2 + 3*a*b**2*m**2*x**4 - 12*a*b**2*m*x**4 + 9*a*b**2*x**4 + b**3*m**2*x**
6 - 4*b**3*m*x**6 + 3*b**3*x**6),x)*a**5*d*m**2 - 9*int(x**m/(a**3*m**2 -
4*a**3*m + 3*a**3 + 3*a**2*b*m**2*x**2 - 12*a**2*b*m*x**2 + 9*a**2*b*x**2
+ 3*a*b**2*m**2*x**4 - 12*a*b**2*m*x**4 + 9*a*b**2*x**4 + b**3*m**2*x**6 -
 4*b**3*m*x**6 + 3*b**3*x**6),x)*a**5*d*m - 45*int(x**m/(a**3*m**2 - 4*...
```

# 3.259 $\int (cx)^m \left(a+bx^2\right)^{3/2} \left(A+Bx^2+Cx^4+Dx^6\right) dx$



## Optimal result

Integrand size = 34, antiderivative size = 295

$$\int (cx)^m \left(a+bx^2\right)^{3/2} \left(A+Bx^2+Cx^4+Dx^6\right) dx = \frac{\left(a^2D(15+8m+m^2)-abC(30+13m+m^2)+b^2B(80+18m+m^2)\right)(cx)^{1+m}\left(a+bx^2\right)^{5/2}}{b^3c(6+m)(8+m)(10+m)} - \frac{(aD(5+m)-bC(10+m))(cx)^{3+m}\left(a+bx^2\right)^{5/2}}{b^2c^3(8+m)(10+m)} + \frac{D(cx)^{5+m}\left(a+bx^2\right)^{5/2}}{bc^5(10+m)} + \frac{a\left(\frac{A}{1+m}-\frac{a\left(a^2D(15+8m+m^2)-abC(30+13m+m^2)+b^2B(80+18m+m^2)\right)}{b^3(6+m)(8+m)(10+m)}\right)(cx)^{1+m}\sqrt{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\right.}{c\sqrt{1+\frac{bx^2}{a}}}$$

output
```
(a^2*D*(m^2+8*m+15)-a*b*C*(m^2+13*m+30)+b^2*B*(m^2+18*m+80))*(c*x)^(1+m)*(
b*x^2+a)^(5/2)/b^3/c/(6+m)/(8+m)/(10+m)-(a*D*(5+m)-b*C*(10+m))*(c*x)^(3+m)
*(b*x^2+a)^(5/2)/b^2/c^3/(8+m)/(10+m)+D*(c*x)^(5+m)*(b*x^2+a)^(5/2)/b/c^5/
(10+m)+a*(A/(1+m)-a*(a^2*D*(m^2+8*m+15)-a*b*C*(m^2+13*m+30)+b^2*B*(m^2+18*
m+80))/b^3/(6+m)/(8+m)/(10+m))*(c*x)^(1+m)*(b*x^2+a)^(1/2)*hypergeom([-3/2
, 1/2+1/2*m],[3/2+1/2*m],-b*x^2/a)/c/(1+b*x^2/a)^(1/2)
```

## Mathematica [A] (verified)

Time = 2.04 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.61

$$\int (cx)^m \left(a+bx^2\right)^{3/2} \left(A+Bx^2+Cx^4+Dx^6\right) dx = \frac{ax(cx)^m\sqrt{a+bx^2}\left(\frac{A\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1+m}{2},\frac{3+m}{2},-\frac{bx^2}{a}\right)}{1+m}+\frac{Bx^2\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{3+m}{2},\frac{5+m}{2},-\frac{bx^2}{a}\right)}{3+m}\right.}{\sqrt{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(c*x)^m*(a + b*x^2)^(3/2)*(A + B*x^2 + C*x^4 + D*x^6),x]
```

output
```
(a*x*(c*x)^m*Sqrt[a + b*x^2]*((A*Hypergeometric2F1[-3/2, (1 + m)/2, (3 + m
)/2, -((b*x^2)/a)])/(1 + m) + (B*x^2*Hypergeometric2F1[-3/2, (3 + m)/2, (5
 + m)/2, -((b*x^2)/a)])/(3 + m) + (C*x^4*Hypergeometric2F1[-3/2, (5 + m)/2
, (7 + m)/2, -((b*x^2)/a)])/(5 + m) + (D*x^6*Hypergeometric2F1[-3/2, (7 +
m)/2, (9 + m)/2, -((b*x^2)/a)])/(7 + m)))/Sqrt[1 + (b*x^2)/a]
```

## Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$, Rules used = {2340, 1590, 363, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^{3/2} (cx)^m \left(A+Bx^2+Cx^4+Dx^6\right) dx$$

$$\Big\downarrow \text{ 2340}$$

$$\frac{\int (cx)^m \left(bx^2+a\right)^{3/2} \left(-\left((aD(m+5)-bC(m+10))x^4\right)+bB(m+10)x^2+Ab(m+10)\right) dx}{b(m+10)} + \frac{D\left(a+bx^2\right)^{5/2} (cx)^{m+5}}{bc^5(m+10)}$$

$$\Big\downarrow \text{ 1590}$$

$$\frac{\frac{\int (cx)^m \left(bx^2+a\right)^{3/2} \left(A(m+8)(m+10)b^2+\left(D\left(m^2+8m+15\right)a^2-bC\left(m^2+13m+30\right)a+b^2B\left(m^2+18m+80\right)\right)x^2\right)dx}{b(m+8)} - \frac{\left(a+bx^2\right)^{5/2}(cx)^{m+3}(aD(m}{bc^3(m+8)}}{b(m+10)}$$

$$\frac{D\left(a+bx^2\right)^{5/2}(cx)^{m+5}}{bc^5(m+10)}$$

$\downarrow$ 363

$$\frac{\frac{\frac{\left(Ab^3(m+8)(m+10)-\frac{a(m+1)\left(a^2D\left(m^2+8m+15\right)-abC\left(m^2+13m+30\right)+b^2B\left(m^2+18m+80\right)\right)}{m+6}\right)\int (cx)^m \left(bx^2+a\right)^{3/2}dx}{b}+\frac{\left(a+bx^2\right)^{5/2}(cx)^{m+1}\left(a^2D\left(m^2+8m\right.\right.}{}}{b(m+8)}}{b(m+10)}$$

$$\frac{D\left(a+bx^2\right)^{5/2}(cx)^{m+5}}{bc^5(m+10)}$$

$\downarrow$ 279

$$\frac{\frac{\frac{a\sqrt{a+bx^2}\left(Ab^3(m+8)(m+10)-\frac{a(m+1)\left(a^2D\left(m^2+8m+15\right)-abC\left(m^2+13m+30\right)+b^2B\left(m^2+18m+80\right)\right)}{m+6}\right)\int (cx)^m \left(\frac{bx^2}{a}+1\right)^{3/2}dx}{b\sqrt{\frac{bx^2}{a}+1}}+\frac{\left(a+bx^2\right)^{5/2}(cx)^{m+1}\left(a\right.}{}}{b(m+8)}}{b(m+10)}$$

$$\frac{D\left(a+bx^2\right)^{5/2}(cx)^{m+5}}{bc^5(m+10)}$$

$\downarrow$ 278

$$\frac{\frac{\frac{a\sqrt{a+bx^2}(cx)^{m+1}\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{m+1}{2},\frac{m+3}{2},-\frac{bx^2}{a}\right)\left(Ab^3(m+8)(m+10)-\frac{a(m+1)\left(a^2D\left(m^2+8m+15\right)-abC\left(m^2+13m+30\right)+b^2B\left(m^2+18m+80\right)\right.}{m+6}\right.}{bc(m+1)\sqrt{\frac{bx^2}{a}+1}}}{b(m+8)}}{b(m+10}$$

$$\frac{D\left(a+bx^2\right)^{5/2}(cx)^{m+5}}{bc^5(m+10)}$$

input
```
Int[(c*x)^m*(a + b*x^2)^(3/2)*(A + B*x^2 + C*x^4 + D*x^6),x]
```

output
```
(D*(c*x)^(5 + m)*(a + b*x^2)^(5/2))/(b*c^5*(10 + m)) + (-(((a*D*(5 + m) -
b*C*(10 + m))*(c*x)^(3 + m)*(a + b*x^2)^(5/2))/(b*c^3*(8 + m))) + (((a^2*D
*(15 + 8*m + m^2) - a*b*C*(30 + 13*m + m^2) + b^2*B*(80 + 18*m + m^2))*(c*
x)^(1 + m)*(a + b*x^2)^(5/2))/(b*c*(6 + m)) + (a*(A*b^3*(8 + m)*(10 + m) -
 (a*(1 + m)*(a^2*D*(15 + 8*m + m^2) - a*b*C*(30 + 13*m + m^2) + b^2*B*(80
+ 18*m + m^2)))/(6 + m))*(c*x)^(1 + m)*Sqrt[a + b*x^2]*Hypergeometric2F1[-
3/2, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(b*c*(1 + m)*Sqrt[1 + (b*x^2)/a]
))/(b*(8 + m)))/(b*(10 + m))
```

### Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 1590
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[c^p*(f*x)^(m + 4*p - 1)*((d + e*x^2)^
(q + 1)/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1))), x] + Simp[1/(e*(m + 4*p + 2*q
 + 1))   Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a +
b*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x],
x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p,
0] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

rule 2340
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Simp[1/(b*(m
+ q + 2*p + 1))   Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)
*Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /;
GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ
[Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

## Maple [F]

$$\int (cx)^m \left(b\,x^2+a\right)^{\frac{3}{2}} \left(Dx^6+C\,x^4+x^2B+A\right) dx$$

input
```
int((c*x)^m*(b*x^2+a)^(3/2)*(D*x^6+C*x^4+B*x^2+A),x)
```

output
```
int((c*x)^m*(b*x^2+a)^(3/2)*(D*x^6+C*x^4+B*x^2+A),x)
```

## Fricas [F]

$$\int (cx)^m \left(a+bx^2\right)^{3/2} \left(A+Bx^2+Cx^4+Dx^6\right) dx = \int \left(Dx^6+Cx^4+Bx^2+A\right)\left(bx^2+a\right)^{\frac{3}{2}}(cx)^m\, dx$$

input
```
integrate((c*x)^m*(b*x^2+a)^(3/2)*(D*x^6+C*x^4+B*x^2+A),x, algorithm="fric
as")
```

output
```
integral((D*b*x^8 + (D*a + C*b)*x^6 + (C*a + B*b)*x^4 + (B*a + A*b)*x^2 +
A*a)*sqrt(b*x^2 + a)*(c*x)^m, x)
```

## Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 17.09 (sec) , antiderivative size = 495, normalized size of antiderivative = 1.68

$$\int (cx)^m \left(a+bx^2\right)^{3/2} \left(A+Bx^2+Cx^4+Dx^6\right) dx = \text{Too large to display}$$

input
```
integrate((c*x)**m*(b*x**2+a)**(3/2)*(D*x**6+C*x**4+B*x**2+A),x)
```

output
```
A*a**(3/2)*c**m*x**(m + 1)*gamma(m/2 + 1/2)*hyper((-1/2, m/2 + 1/2), (m/2
+ 3/2,), b*x**2*exp_polar(I*pi)/a)/(2*gamma(m/2 + 3/2)) + A*sqrt(a)*b*c**m
*x**(m + 3)*gamma(m/2 + 3/2)*hyper((-1/2, m/2 + 3/2), (m/2 + 5/2,), b*x**2
*exp_polar(I*pi)/a)/(2*gamma(m/2 + 5/2)) + B*a**(3/2)*c**m*x**(m + 3)*gamm
a(m/2 + 3/2)*hyper((-1/2, m/2 + 3/2), (m/2 + 5/2,), b*x**2*exp_polar(I*pi)
/a)/(2*gamma(m/2 + 5/2)) + B*sqrt(a)*b*c**m*x**(m + 5)*gamma(m/2 + 5/2)*hy
per((-1/2, m/2 + 5/2), (m/2 + 7/2,), b*x**2*exp_polar(I*pi)/a)/(2*gamma(m/
2 + 7/2)) + C*a**(3/2)*c**m*x**(m + 5)*gamma(m/2 + 5/2)*hyper((-1/2, m/2 +
 5/2), (m/2 + 7/2,), b*x**2*exp_polar(I*pi)/a)/(2*gamma(m/2 + 7/2)) + C*sq
rt(a)*b*c**m*x**(m + 7)*gamma(m/2 + 7/2)*hyper((-1/2, m/2 + 7/2), (m/2 + 9
/2,), b*x**2*exp_polar(I*pi)/a)/(2*gamma(m/2 + 9/2)) + D*a**(3/2)*c**m*x**
(m + 7)*gamma(m/2 + 7/2)*hyper((-1/2, m/2 + 7/2), (m/2 + 9/2,), b*x**2*exp
_polar(I*pi)/a)/(2*gamma(m/2 + 9/2)) + D*sqrt(a)*b*c**m*x**(m + 9)*gamma(m
/2 + 9/2)*hyper((-1/2, m/2 + 9/2), (m/2 + 11/2,), b*x**2*exp_polar(I*pi)/a
)/(2*gamma(m/2 + 11/2))
```

## Maxima [F]

$$\int (cx)^m \left(a+bx^2\right)^{3/2} \left(A+Bx^2+Cx^4+Dx^6\right) dx = \int \left(Dx^6+Cx^4+Bx^2+A\right)\left(bx^2+a\right)^{\frac{3}{2}}(cx)^m\, dx$$

input
```
integrate((c*x)^m*(b*x^2+a)^(3/2)*(D*x^6+C*x^4+B*x^2+A),x, algorithm="maxi
ma")
```

output
```
integrate((D*x^6 + C*x^4 + B*x^2 + A)*(b*x^2 + a)^(3/2)*(c*x)^m, x)
```

## Giac [F]

$$\int (cx)^m \left(a+bx^2\right)^{3/2} \left(A+Bx^2+Cx^4+Dx^6\right) \, dx = \int \left(Dx^6+Cx^4+Bx^2+A\right)\left(bx^2+a\right)^{\frac{3}{2}}(cx)^m \, dx$$

input
```
integrate((c*x)^m*(b*x^2+a)^(3/2)*(D*x^6+C*x^4+B*x^2+A),x, algorithm="giac
")
```

output
```
integrate((D*x^6 + C*x^4 + B*x^2 + A)*(b*x^2 + a)^(3/2)*(c*x)^m, x)
```

## Mupad [F(-1)]

Timed out.

$$\int (cx)^m \left(a+bx^2\right)^{3/2} \left(A+Bx^2+Cx^4+Dx^6\right) \, dx = \int (c\,x)^m \left(b\,x^2+a\right)^{3/2} \left(A+B\,x^2+C\,x^4+x^6\,D\right) \, dx$$

input
```
int((c*x)^m*(a + b*x^2)^(3/2)*(A + B*x^2 + C*x^4 + x^6*D),x)
```

output
```
int((c*x)^m*(a + b*x^2)^(3/2)*(A + B*x^2 + C*x^4 + x^6*D), x)
```

## Reduce [F]

$$\int (cx)^m (a + bx^2)^{3/2} (A + Bx^2 + Cx^4 + Dx^6)\, dx = c^m \left( \left( \int x^m \sqrt{b\,x^2 + a}\, x^8 dx \right) bd + \left( \int x^m \sqrt{b\,x^2 + a}\, x^6 dx \right) ad + \left( \int x^m \sqrt{b\,x^2 + a}\, x^6 dx \right) bc + \left( \int x^m \sqrt{b\,x^2 + a}\, x^4 dx \right) ac + \left( \int x^m \sqrt{b\,x^2 + a}\, x^4 dx \right) b^2 + 2\left( \int x^m \sqrt{b\,x^2 + a}\, x^2 dx \right) ab + \left( \int x^m \sqrt{b\,x^2 + a} dx \right) a^2 \right)$$

input
```
int((c*x)^m*(b*x^2+a)^(3/2)*(D*x^6+C*x^4+B*x^2+A),x)
```

output
```
c**m*(int(x**m*sqrt(a + b*x**2)*x**8,x)*b*d + int(x**m*sqrt(a + b*x**2)*x*
*6,x)*a*d + int(x**m*sqrt(a + b*x**2)*x**6,x)*b*c + int(x**m*sqrt(a + b*x*
*2)*x**4,x)*a*c + int(x**m*sqrt(a + b*x**2)*x**4,x)*b**2 + 2*int(x**m*sqrt
(a + b*x**2)*x**2,x)*a*b + int(x**m*sqrt(a + b*x**2),x)*a**2)
```

# 3.260 $\int (cx)^m \sqrt{a+bx^2}\left(A+Bx^2+Cx^4+Dx^6\right)\,dx$



## Optimal result

Integrand size = 34, antiderivative size = 294

$$\int (cx)^m \sqrt{a+bx^2}\left(A+Bx^2+Cx^4+Dx^6\right)\,dx$$
$$= \frac{\left(a^2D(15+8m+m^2)-abC(24+11m+m^2)+b^2B(48+14m+m^2)\right)(cx)^{1+m}\left(a+bx^2\right)^{3/2}}{b^3c(4+m)(6+m)(8+m)}$$
$$-\frac{(aD(5+m)-bC(8+m))(cx)^{3+m}\left(a+bx^2\right)^{3/2}}{b^2c^3(6+m)(8+m)}+\frac{D(cx)^{5+m}\left(a+bx^2\right)^{3/2}}{bc^5(8+m)}$$
$$+\frac{\left(\frac{A}{1+m}-\frac{a\left(a^2D(15+8m+m^2)-abC(24+11m+m^2)+b^2B(48+14m+m^2)\right)}{b^3(4+m)(6+m)(8+m)}\right)(cx)^{1+m}\sqrt{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{2}\right.}{c\sqrt{1+\frac{bx^2}{a}}}$$

output
```
(a^2*D*(m^2+8*m+15)-a*b*C*(m^2+11*m+24)+b^2*B*(m^2+14*m+48))*(c*x)^(1+m)*(
b*x^2+a)^(3/2)/b^3/c/(4+m)/(6+m)/(8+m)-(a*D*(5+m)-b*C*(8+m))*(c*x)^(3+m)*(
b*x^2+a)^(3/2)/b^2/c^3/(6+m)/(8+m)+D*(c*x)^(5+m)*(b*x^2+a)^(3/2)/b/c^5/(8+
m)+(A/(1+m)-a*(a^2*D*(m^2+8*m+15)-a*b*C*(m^2+11*m+24)+b^2*B*(m^2+14*m+48))
/b^3/(4+m)/(6+m)/(8+m))*(c*x)^(1+m)*(b*x^2+a)^(1/2)*hypergeom([-1/2, 1/2+1
/2*m],[3/2+1/2*m],-b*x^2/a)/c/(1+b*x^2/a)^(1/2)
```

## Mathematica [A] (verified)

Time = 1.32 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.61

$$\int (cx)^m\sqrt{a+bx^2}\left(A+Bx^2+Cx^4+Dx^6\right)\,dx$$
$$=\frac{x(cx)^m\sqrt{a+bx^2}\left(\frac{A\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},-\frac{bx^2}{a}\right)}{1+m}+\frac{Bx^2\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{3+m}{2},\frac{5+m}{2},-\frac{bx^2}{a}\right)}{3+m}+\frac{Cx^4\,\text{Hyperg}}{}\right.}{\sqrt{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(c*x)^m*Sqrt[a + b*x^2]*(A + B*x^2 + C*x^4 + D*x^6),x]
```

output
```
(x*(c*x)^m*Sqrt[a + b*x^2]*((A*Hypergeometric2F1[-1/2, (1 + m)/2, (3 + m)/
2, -((b*x^2)/a)])/(1 + m) + (B*x^2*Hypergeometric2F1[-1/2, (3 + m)/2, (5 +
 m)/2, -((b*x^2)/a)])/(3 + m) + (C*x^4*Hypergeometric2F1[-1/2, (5 + m)/2,
(7 + m)/2, -((b*x^2)/a)])/(5 + m) + (D*x^6*Hypergeometric2F1[-1/2, (7 + m)
/2, (9 + m)/2, -((b*x^2)/a)])/(7 + m)))/Sqrt[1 + (b*x^2)/a]
```

## Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.147, Rules used = {2340, 1590, 363, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx^2}(cx)^m\left(A+Bx^2+Cx^4+Dx^6\right)\,dx$$

$$\Big\downarrow \text{ 2340}$$

$$\frac{\int (cx)^m\sqrt{bx^2+a}\left(-\left((aD(m+5)-bC(m+8))x^4\right)+bB(m+8)x^2+Ab(m+8)\right)dx}{b(m+8)}+\frac{D\left(a+bx^2\right)^{3/2}(cx)^{m+5}}{bc^5(m+8)}$$

$$\Big\downarrow \text{ 1590}$$

$$\frac{\frac{\int (cx)^m\sqrt{bx^2+a}\left(A(m+6)(m+8)b^2+\left(D\left(m^2+8m+15\right)a^2-bC\left(m^2+11m+24\right)a+b^2B\left(m^2+14m+48\right)\right)x^2\right)dx}{b(m+6)}-\frac{(a+bx^2)^{3/2}(cx)^{m+3}(aD(m+5)}{bc^3(m+6)}}{b(m+8)}$$

$$\frac{D\left(a+bx^2\right)^{3/2}(cx)^{m+5}}{bc^5(m+8)}$$

$\downarrow$ 363

$$\frac{\frac{\frac{\left(Ab^3(m+6)(m+8)-\frac{a(m+1)\left(a^2D\left(m^2+8m+15\right)-abC\left(m^2+11m+24\right)+b^2B\left(m^2+14m+48\right)\right)}{m+4}\right)\int (cx)^m\sqrt{bx^2+a}dx}{b}+\frac{\left(a+bx^2\right)^{3/2}(cx)^{m+1}\left(a^2D\left(m^2+8m+15\right.\right.}{bc}}{b(m+6)}}{b(m+8)}$$

$$\frac{D\left(a+bx^2\right)^{3/2}(cx)^{m+5}}{bc^5(m+8)}$$

$\downarrow$ 279

$$\frac{\frac{\frac{\sqrt{a+bx^2}\left(Ab^3(m+6)(m+8)-\frac{a(m+1)\left(a^2D\left(m^2+8m+15\right)-abC\left(m^2+11m+24\right)+b^2B\left(m^2+14m+48\right)\right)}{m+4}\right)\int (cx)^m\sqrt{\frac{bx^2}{a}+1}dx}{b\sqrt{\frac{bx^2}{a}+1}}+\frac{\left(a+bx^2\right)^{3/2}(cx)^{m+1}\left(a^2D\left(m^2\right.\right.}{}}{b(m+6)}}{b(m+8)}$$

$$\frac{D\left(a+bx^2\right)^{3/2}(cx)^{m+5}}{bc^5(m+8)}$$

$\downarrow$ 278

$$\frac{\frac{\frac{\sqrt{a+bx^2}(cx)^{m+1}\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},-\frac{bx^2}{a}\right)\left(Ab^3(m+6)(m+8)-\frac{a(m+1)\left(a^2D\left(m^2+8m+15\right)-abC\left(m^2+11m+24\right)+b^2B\left(m^2+14m+48\right)\right)}{m+4}\right)}{bc(m+1)\sqrt{\frac{bx^2}{a}+1}}}{b(m+6)}}{b(m+8)}$$

$$\frac{D\left(a+bx^2\right)^{3/2}(cx)^{m+5}}{bc^5(m+8)}$$

input
```
Int[(c*x)^m*Sqrt[a + b*x^2]*(A + B*x^2 + C*x^4 + D*x^6),x]
```

output
```
(D*(c*x)^(5 + m)*(a + b*x^2)^(3/2))/(b*c^5*(8 + m)) + (-(((a*D*(5 + m) - b
*C*(8 + m))*(c*x)^(3 + m)*(a + b*x^2)^(3/2))/(b*c^3*(6 + m))) + (((a^2*D*(
15 + 8*m + m^2) - a*b*C*(24 + 11*m + m^2) + b^2*B*(48 + 14*m + m^2))*(c*x)
^(1 + m)*(a + b*x^2)^(3/2))/(b*c*(4 + m)) + ((A*b^3*(6 + m)*(8 + m) - (a*(
1 + m)*(a^2*D*(15 + 8*m + m^2) - a*b*C*(24 + 11*m + m^2) + b^2*B*(48 + 14*
m + m^2)))/(4 + m))*(c*x)^(1 + m)*Sqrt[a + b*x^2]*Hypergeometric2F1[-1/2,
(1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(b*c*(1 + m)*Sqrt[1 + (b*x^2)/a]))/(b
*(6 + m)))/(b*(8 + m))
```

## Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 1590
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[c^p*(f*x)^(m + 4*p - 1)*((d + e*x^2)^
(q + 1)/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1))), x] + Simp[1/(e*(m + 4*p + 2*q
 + 1))   Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a +
b*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x],
x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p,
0] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

rule 2340
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Simp[1/(b*(m
+ q + 2*p + 1))   Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)
*Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /;
GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ
[Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

## Maple [F]

$$\int (cx)^m \sqrt{b\,x^2+a}\,\left(Dx^6+C\,x^4+x^2B+A\right)dx$$

input
```
int((c*x)^m*(b*x^2+a)^(1/2)*(D*x^6+C*x^4+B*x^2+A),x)
```

output
```
int((c*x)^m*(b*x^2+a)^(1/2)*(D*x^6+C*x^4+B*x^2+A),x)
```

## Fricas [F]

$$\int (cx)^m\sqrt{a+bx^2}\left(A+Bx^2+Cx^4+Dx^6\right)\,dx$$
$$=\int \left(Dx^6+Cx^4+Bx^2+A\right)\sqrt{bx^2+a}(cx)^m\,dx$$

input
```
integrate((c*x)^m*(b*x^2+a)^(1/2)*(D*x^6+C*x^4+B*x^2+A),x, algorithm="fric
as")
```

output
```
integral((D*x^6 + C*x^4 + B*x^2 + A)*sqrt(b*x^2 + a)*(c*x)^m, x)
```

## Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.94 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.83

$$\int (cx)^m\sqrt{a+bx^2}\left(A+Bx^2+Cx^4+Dx^6\right)\,dx$$
$$=\frac{A\sqrt{a}c^m x^{m+1}\Gamma\left(\frac{m}{2}+\frac{1}{2}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{m}{2}+\frac{1}{2}\\ \frac{m}{2}+\frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}+\frac{B\sqrt{a}c^m x^{m+3}\Gamma\left(\frac{m}{2}+\frac{3}{2}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{m}{2}+\frac{3}{2}\\ \frac{m}{2}+\frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)}+\frac{C\sqrt{a}c^m x^{m+5}\Gamma\left(\frac{m}{2}+\frac{5}{2}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{m}{2}+\frac{5}{2}\\ \frac{m}{2}+\frac{7}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{m}{2}+\frac{7}{2}\right)}+\frac{D\sqrt{a}c^m x^{m+7}\Gamma\left(\frac{m}{2}+\frac{7}{2}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{m}{2}+\frac{7}{2}\\ \frac{m}{2}+\frac{9}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{m}{2}+\frac{9}{2}\right)}$$

input
```
integrate((c*x)**m*(b*x**2+a)**(1/2)*(D*x**6+C*x**4+B*x**2+A),x)
```

output
```
A*sqrt(a)*c**m*x**(m + 1)*gamma(m/2 + 1/2)*hyper((-1/2, m/2 + 1/2), (m/2 +
 3/2,), b*x**2*exp_polar(I*pi)/a)/(2*gamma(m/2 + 3/2)) + B*sqrt(a)*c**m*x*
*(m + 3)*gamma(m/2 + 3/2)*hyper((-1/2, m/2 + 3/2), (m/2 + 5/2,), b*x**2*ex
p_polar(I*pi)/a)/(2*gamma(m/2 + 5/2)) + C*sqrt(a)*c**m*x**(m + 5)*gamma(m/
2 + 5/2)*hyper((-1/2, m/2 + 5/2), (m/2 + 7/2,), b*x**2*exp_polar(I*pi)/a)/
(2*gamma(m/2 + 7/2)) + D*sqrt(a)*c**m*x**(m + 7)*gamma(m/2 + 7/2)*hyper((-
1/2, m/2 + 7/2), (m/2 + 9/2,), b*x**2*exp_polar(I*pi)/a)/(2*gamma(m/2 + 9/
2))
```

## Maxima [F]

$$\int (cx)^m\sqrt{a+bx^2}\left(A+Bx^2+Cx^4+Dx^6\right)\,dx = \int (Dx^6+Cx^4+Bx^2+A)\sqrt{bx^2+a}(cx)^m\,dx$$

input
```
integrate((c*x)^m*(b*x^2+a)^(1/2)*(D*x^6+C*x^4+B*x^2+A),x, algorithm="maxi
ma")
```

output
```
integrate((D*x^6 + C*x^4 + B*x^2 + A)*sqrt(b*x^2 + a)*(c*x)^m, x)
```

## Giac [F]

$$\int (cx)^m\sqrt{a+bx^2}\left(A+Bx^2+Cx^4+Dx^6\right)\,dx = \int (Dx^6+Cx^4+Bx^2+A)\sqrt{bx^2+a}(cx)^m\,dx$$

input
```
integrate((c*x)^m*(b*x^2+a)^(1/2)*(D*x^6+C*x^4+B*x^2+A),x, algorithm="giac
")
```

output
```
integrate((D*x^6 + C*x^4 + B*x^2 + A)*sqrt(b*x^2 + a)*(c*x)^m, x)
```

## Mupad [F(-1)]

Timed out.

$$\int (cx)^m\sqrt{a+bx^2}\left(A+Bx^2+Cx^4+Dx^6\right)\,dx = \int {(c\,x)}^m\,\sqrt{b\,x^2+a}\,\left(A+B\,x^2+C\,x^4+x^6\,D\right)\,dx$$

input
```
int((c*x)^m*(a + b*x^2)^(1/2)*(A + B*x^2 + C*x^4 + x^6*D),x)
```

output
```
int((c*x)^m*(a + b*x^2)^(1/2)*(A + B*x^2 + C*x^4 + x^6*D), x)
```

## Reduce [F]

$$\int (cx)^m\sqrt{a+bx^2}\left(A+Bx^2+Cx^4+Dx^6\right)\,dx = c^m\left(\left(\int x^m\sqrt{b\,x^2+a}\,x^6dx\right)d+\left(\int x^m\sqrt{b\,x^2+a}\,x^4dx\right)c+\left(\int x^m\sqrt{b\,x^2+a}\,x^2dx\right)b+\left(\int x^m\sqrt{b\,x^2+a}dx\right)a\right)$$

input
```
int((c*x)^m*(b*x^2+a)^(1/2)*(D*x^6+C*x^4+B*x^2+A),x)
```

output
```
c**m*(int(x**m*sqrt(a + b*x**2)*x**6,x)*d + int(x**m*sqrt(a + b*x**2)*x**4
,x)*c + int(x**m*sqrt(a + b*x**2)*x**2,x)*b + int(x**m*sqrt(a + b*x**2),x)
*a)
```

# 3.261 $\int \frac{(cx)^m(A+Bx^2+Cx^4+Dx^6)}{\sqrt{a+bx^2}}\,dx$



## Optimal result

Integrand size = 34, antiderivative size = 294

$$\int \frac{(cx)^m(A+Bx^2+Cx^4+Dx^6)}{\sqrt{a+bx^2}}\,dx = \frac{(a^2D(15+8m+m^2)-abC(18+9m+m^2)+b^2B(24+10m+m^2))\,(cx)^{1+m}\sqrt{a+bx^2}}{b^3c(2+m)(4+m)(6+m)} - \frac{(aD(5+m)-bC(6+m))(cx)^{3+m}\sqrt{a+bx^2}}{b^2c^3(4+m)(6+m)} + \frac{D(cx)^{5+m}\sqrt{a+bx^2}}{bc^5(6+m)} + \frac{\left(\frac{A}{1+m}-\frac{a(a^2D(15+8m+m^2)-abC(18+9m+m^2)+b^2B(24+10m+m^2))}{b^3(2+m)(4+m)(6+m)}\right)(cx)^{1+m}\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}\right.}{c\sqrt{a+bx^2}}$$

output

```
(a^2*D*(m^2+8*m+15)-a*b*C*(m^2+9*m+18)+b^2*B*(m^2+10*m+24))*(c*x)^(1+m)*(b
*x^2+a)^(1/2)/b^3/c/(2+m)/(4+m)/(6+m)-(a*D*(5+m)-b*C*(6+m))*(c*x)^(3+m)*(b
*x^2+a)^(1/2)/b^2/c^3/(4+m)/(6+m)+D*(c*x)^(5+m)*(b*x^2+a)^(1/2)/b/c^5/(6+m
)+(A/(1+m)-a*(a^2*D*(m^2+8*m+15)-a*b*C*(m^2+9*m+18)+b^2*B*(m^2+10*m+24))/b
^3/(2+m)/(4+m)/(6+m))*(c*x)^(1+m)*(1+b*x^2/a)^(1/2)*hypergeom([1/2, 1/2+1/
2*m],[3/2+1/2*m],-b*x^2/a)/c/(b*x^2+a)^(1/2)
```

## Mathematica [A] (verified)

Time = 2.54 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.61

$$\int \frac{(cx)^m (A + Bx^2 + Cx^4 + Dx^6)}{\sqrt{a + bx^2}} \, dx$$
$$= \frac{x(cx)^m \sqrt{1 + \frac{bx^2}{a}} \left( \frac{A \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, -\frac{bx^2}{a}\right)}{1+m} + \frac{Bx^2 \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{3+m}{2}, \frac{5+m}{2}, -\frac{bx^2}{a}\right)}{3+m} + \frac{Cx^4 \operatorname{Hypergeom}}{} \right.}{\sqrt{a + bx^2}}$$

input
```
Integrate[((c*x)^m*(A + B*x^2 + C*x^4 + D*x^6))/Sqrt[a + b*x^2],x]
```

output
```
(x*(c*x)^m*Sqrt[1 + (b*x^2)/a]*((A*Hypergeometric2F1[1/2, (1 + m)/2, (3 +
m)/2, -((b*x^2)/a)])/(1 + m) + (B*x^2*Hypergeometric2F1[1/2, (3 + m)/2, (5
 + m)/2, -((b*x^2)/a)])/(3 + m) + (C*x^4*Hypergeometric2F1[1/2, (5 + m)/2,
 (7 + m)/2, -((b*x^2)/a)])/(5 + m) + (D*x^6*Hypergeometric2F1[1/2, (7 + m)
/2, (9 + m)/2, -((b*x^2)/a)])/(7 + m)))/Sqrt[a + b*x^2]
```

## Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.147, Rules used = {2340, 1590, 363, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^m (A + Bx^2 + Cx^4 + Dx^6)}{\sqrt{a + bx^2}} \, dx$$

$$\Big\downarrow \text{2340}$$

$$\frac{\int \frac{(cx)^m \left(-((aD(m+5) - bC(m+6))x^4) + bB(m+6)x^2 + Ab(m+6)\right)}{\sqrt{bx^2+a}} dx}{b(m+6)} + \frac{D\sqrt{a + bx^2}(cx)^{m+5}}{bc^5(m+6)}$$

$$\Big\downarrow \text{1590}$$

$$\frac{\frac{\int \frac{(cx)^m\left(A(m+4)(m+6)b^2+\left(D\left(m^2+8m+15\right)a^2-bC\left(m^2+9m+18\right)a+b^2B\left(m^2+10m+24\right)\right)x^2\right)}{\sqrt{bx^2+a}}dx}{b(m+4)}-\frac{\sqrt{a+bx^2}(cx)^{m+3}(aD(m+5)-bC(m+6))}{bc^3(m+4)}}{b(m+6)}+\frac{D\sqrt{a+bx^2}(cx)^{m+5}}{bc^5(m+6)}$$

$\Big\downarrow$ 363

$$\frac{\frac{\frac{\left(Ab^3(m+4)(m+6)-\frac{a(m+1)\left(a^2D\left(m^2+8m+15\right)-abC\left(m^2+9m+18\right)+b^2B\left(m^2+10m+24\right)\right)}{m+2}\right)\int \frac{(cx)^m}{\sqrt{bx^2+a}}dx}{b}+\frac{\sqrt{a+bx^2}(cx)^{m+1}\left(a^2D\left(m^2+8m+15\right)-abC\left(m\right.\right.}{bc(m+2)}}{b(m+4)}}{b(m+6)}$$

$$\frac{D\sqrt{a+bx^2}(cx)^{m+5}}{bc^5(m+6)}$$

$\Big\downarrow$ 279

$$\frac{\frac{\frac{\sqrt{\frac{bx^2}{a}+1}\left(Ab^3(m+4)(m+6)-\frac{a(m+1)\left(a^2D\left(m^2+8m+15\right)-abC\left(m^2+9m+18\right)+b^2B\left(m^2+10m+24\right)\right)}{m+2}\right)\int \frac{(cx)^m}{\sqrt{\frac{bx^2}{a}+1}}dx}{b\sqrt{a+bx^2}}+\frac{\sqrt{a+bx^2}(cx)^{m+1}\left(a^2D\left(m^2+8m+15\right.\right.}{b}}{b(m+4)}}{b(m+6)}$$

$$\frac{D\sqrt{a+bx^2}(cx)^{m+5}}{bc^5(m+6)}$$

$\Big\downarrow$ 278

$$\frac{\frac{\frac{\sqrt{\frac{bx^2}{a}+1}(cx)^{m+1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},-\frac{bx^2}{a}\right)\left(Ab^3(m+4)(m+6)-\frac{a(m+1)\left(a^2D\left(m^2+8m+15\right)-abC\left(m^2+9m+18\right)+b^2B\left(m^2+10m+24\right)\right)}{m+2}\right)}{bc(m+1)\sqrt{a+bx^2}}+}{b(m+4)}}{b(m+6)}$$

$$\frac{D\sqrt{a+bx^2}(cx)^{m+5}}{bc^5(m+6)}$$

input
```
Int[((c*x)^m*(A + B*x^2 + C*x^4 + D*x^6))/Sqrt[a + b*x^2],x]
```

output
```
(D*(c*x)^(5 + m)*Sqrt[a + b*x^2])/(b*c^5*(6 + m)) + (-(((a*D*(5 + m) - b*C
*(6 + m))*(c*x)^(3 + m)*Sqrt[a + b*x^2])/(b*c^3*(4 + m))) + (((a^2*D*(15 +
 8*m + m^2) - a*b*C*(18 + 9*m + m^2) + b^2*B*(24 + 10*m + m^2))*(c*x)^(1 +
 m)*Sqrt[a + b*x^2])/(b*c*(2 + m)) + ((A*b^3*(4 + m)*(6 + m) - (a*(1 + m)*
(a^2*D*(15 + 8*m + m^2) - a*b*C*(18 + 9*m + m^2) + b^2*B*(24 + 10*m + m^2)
))/(2 + m))*(c*x)^(1 + m)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/2, (1 +
m)/2, (3 + m)/2, -((b*x^2)/a)])/(b*c*(1 + m)*Sqrt[a + b*x^2]))/(b*(4 + m))
)/(b*(6 + m))
```

## Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 1590
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[c^p*(f*x)^(m + 4*p - 1)*((d + e*x^2)^
(q + 1)/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1))), x] + Simp[1/(e*(m + 4*p + 2*q
 + 1))   Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a +
b*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x],
x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p,
0] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

rule 2340
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Simp[1/(b*(m
+ q + 2*p + 1))   Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)
*Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /;
GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ
[Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

## Maple [F]

$$\int \frac{(cx)^m (D x^6 + C x^4 + x^2 B + A)}{\sqrt{b x^2 + a}} dx$$

input
```
int((c*x)^m*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(1/2),x)
```

output
```
int((c*x)^m*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(1/2),x)
```

## Fricas [F]

$$\int \frac{(cx)^m (A + Bx^2 + Cx^4 + Dx^6)}{\sqrt{a + bx^2}} dx = \int \frac{(Dx^6 + Cx^4 + Bx^2 + A)(cx)^m}{\sqrt{bx^2 + a}} dx$$

input
```
integrate((c*x)^m*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="fric
as")
```

output
```
integral((D*x^6 + C*x^4 + B*x^2 + A)*(c*x)^m/sqrt(b*x^2 + a), x)
```

## Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.58 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.80

$$\int \frac{(cx)^m (A+Bx^2+Cx^4+Dx^6)}{\sqrt{a+bx^2}}\,dx = \frac{Ac^m x^{m+1}\Gamma\left(\frac{m}{2}+\frac{1}{2}\right) {}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{m}{2}+\frac{1}{2}\\ \frac{m}{2}+\frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)} + \frac{Bc^m x^{m+3}\Gamma\left(\frac{m}{2}+\frac{3}{2}\right) {}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{m}{2}+\frac{3}{2}\\ \frac{m}{2}+\frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)} + \frac{Cc^m x^{m+5}\Gamma\left(\frac{m}{2}+\frac{5}{2}\right) {}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{m}{2}+\frac{5}{2}\\ \frac{m}{2}+\frac{7}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}\Gamma\left(\frac{m}{2}+\frac{7}{2}\right)} + \frac{Dc^m x^{m+7}\Gamma\left(\frac{m}{2}+\frac{7}{2}\right) {}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{m}{2}+\frac{7}{2}\\ \frac{m}{2}+\frac{9}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}\Gamma\left(\frac{m}{2}+\frac{9}{2}\right)}$$

input
```
integrate((c*x)**m*(D*x**6+C*x**4+B*x**2+A)/(b*x**2+a)**(1/2),x)
```

output
```
A*c**m*x**(m + 1)*gamma(m/2 + 1/2)*hyper((1/2, m/2 + 1/2), (m/2 + 3/2,), b
*x**2*exp_polar(I*pi)/a)/(2*sqrt(a)*gamma(m/2 + 3/2)) + B*c**m*x**(m + 3)*
gamma(m/2 + 3/2)*hyper((1/2, m/2 + 3/2), (m/2 + 5/2,), b*x**2*exp_polar(I*
pi)/a)/(2*sqrt(a)*gamma(m/2 + 5/2)) + C*c**m*x**(m + 5)*gamma(m/2 + 5/2)*h
yper((1/2, m/2 + 5/2), (m/2 + 7/2,), b*x**2*exp_polar(I*pi)/a)/(2*sqrt(a)*
gamma(m/2 + 7/2)) + D*c**m*x**(m + 7)*gamma(m/2 + 7/2)*hyper((1/2, m/2 + 7
/2), (m/2 + 9/2,), b*x**2*exp_polar(I*pi)/a)/(2*sqrt(a)*gamma(m/2 + 9/2))
```

## Maxima [F]

$$\int \frac{(cx)^m (A+Bx^2+Cx^4+Dx^6)}{\sqrt{a+bx^2}}\,dx = \int \frac{(Dx^6+Cx^4+Bx^2+A)(cx)^m}{\sqrt{bx^2+a}}\,dx$$

input
```
integrate((c*x)^m*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate((D*x^6 + C*x^4 + B*x^2 + A)*(c*x)^m/sqrt(b*x^2 + a), x)
```

## Giac [F]

$$\int \frac{(cx)^m (A+Bx^2+Cx^4+Dx^6)}{\sqrt{a+bx^2}}\,dx = \int \frac{(Dx^6+Cx^4+Bx^2+A)(cx)^m}{\sqrt{bx^2+a}}\,dx$$

input
```
integrate((c*x)^m*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="giac
")
```

output
```
integrate((D*x^6 + C*x^4 + B*x^2 + A)*(c*x)^m/sqrt(b*x^2 + a), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^m (A+Bx^2+Cx^4+Dx^6)}{\sqrt{a+bx^2}}\,dx = \int \frac{(c\,x)^m\,(A+B\,x^2+C\,x^4+x^6\,D)}{\sqrt{b\,x^2+a}}\,dx$$

input
```
int(((c*x)^m*(A + B*x^2 + C*x^4 + x^6*D))/(a + b*x^2)^(1/2),x)
```

output
```
int(((c*x)^m*(A + B*x^2 + C*x^4 + x^6*D))/(a + b*x^2)^(1/2), x)
```

## Reduce [F]

$$\int \frac{(cx)^m (A + Bx^2 + Cx^4 + Dx^6)}{\sqrt{a + bx^2}} \, dx = c^m \left( \left( \int \frac{x^m}{\sqrt{b\,x^2 + a}} dx \right) a + \left( \int \frac{x^m x^6}{\sqrt{b\,x^2 + a}} dx \right) d + \left( \int \frac{x^m x^4}{\sqrt{b\,x^2 + a}} dx \right) c + \left( \int \frac{x^m x^2}{\sqrt{b\,x^2 + a}} dx \right) b \right)$$

input
```
int((c*x)^m*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(1/2),x)
```

output
```
c**m*(int(x**m/sqrt(a + b*x**2),x)*a + int((x**m*x**6)/sqrt(a + b*x**2),x)
*d + int((x**m*x**4)/sqrt(a + b*x**2),x)*c + int((x**m*x**2)/sqrt(a + b*x*
*2),x)*b)
```

# 3.262 $\int \frac{(cx)^m(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{3/2}}\,dx$



## Optimal result

Integrand size = 34, antiderivative size = 290

$$\int \frac{(cx)^m\left(A+Bx^2+Cx^4+Dx^6\right)}{(a+bx^2)^{3/2}}\,dx = \frac{\left(\frac{A}{a}-\frac{b^2B-abC+a^2D}{b^3}\right)(cx)^{1+m}}{c\sqrt{a+bx^2}}$$
$$+\frac{(bC(4+m)-aD(7+2m))(cx)^{1+m}\sqrt{a+bx^2}}{b^3c(2+m)(4+m)}+\frac{D(cx)^{3+m}\sqrt{a+bx^2}}{b^2c^3(4+m)}$$
$$-\frac{(Ab^3m(8+6m+m^2)-ab^2B(8+14m+7m^2+m^3)+a^2bC(12+19m+8m^2+m^3)-a^3D(15+23m}{ab^3c(1+m)(2+m)(4+m)\sqrt{a+}}$$

output
```
(A/a-(B*b^2-C*a*b+D*a^2)/b^3)*(c*x)^(1+m)/c/(b*x^2+a)^(1/2)+(b*C*(4+m)-a*D
*(7+2*m))*(c*x)^(1+m)*(b*x^2+a)^(1/2)/b^3/c/(2+m)/(4+m)+D*(c*x)^(3+m)*(b*x
^2+a)^(1/2)/b^2/c^3/(4+m)-(A*b^3*m*(m^2+6*m+8)-a*b^2*B*(m^3+7*m^2+14*m+8)+
a^2*b*C*(m^3+8*m^2+19*m+12)-a^3*D*(m^3+9*m^2+23*m+15))*(c*x)^(1+m)*(1+b*x^
2/a)^(1/2)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],-b*x^2/a)/a/b^3/c/(1+m)/
(2+m)/(4+m)/(b*x^2+a)^(1/2)
```

## Mathematica [A] (verified)

Time = 2.40 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.62

$$\int \frac{(cx)^m (A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{3/2}}\,dx = \frac{x(cx)^m\sqrt{1+\frac{bx^2}{a}}\left(\frac{A\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1+m}{2},\frac{3+m}{2},-\frac{bx^2}{a}\right)}{1+m}+\frac{Bx^2\,\text{Hypergeon}}{}\right.}{}$$

input
```
Integrate[((c*x)^m*(A + B*x^2 + C*x^4 + D*x^6))/(a + b*x^2)^(3/2),x]
```

output
```
(x*(c*x)^m*Sqrt[1 + (b*x^2)/a]*((A*Hypergeometric2F1[3/2, (1 + m)/2, (3 +
m)/2, -((b*x^2)/a)])/(1 + m) + (B*x^2*Hypergeometric2F1[3/2, (3 + m)/2, (5
 + m)/2, -((b*x^2)/a)])/(3 + m) + (C*x^4*Hypergeometric2F1[3/2, (5 + m)/2,
 (7 + m)/2, -((b*x^2)/a)])/(5 + m) + (D*x^6*Hypergeometric2F1[3/2, (7 + m)
/2, (9 + m)/2, -((b*x^2)/a)])/(7 + m)))/(a*Sqrt[a + b*x^2])
```

## Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 280, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2337, 1590, 27, 363, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^m (A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{3/2}}\,dx$$

$$\Big\downarrow \; 2337$$

$$\frac{(cx)^{m+1}\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{ac\sqrt{a+bx^2}}-\frac{\int \frac{(cx)^m\left(-\frac{aDx^4}{b}-\frac{a(bC-aD)x^2}{b^2}+\frac{-D(m+1)a^3+bC(m+1)a^2-b^2B(m+1)a+Ab^3m}{b^3}\right)}{\sqrt{bx^2+a}}dx}{a}$$

$$\Big\downarrow \; 1590$$

$$\frac{\frac{(cx)^{m+1}\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{ac\sqrt{a+bx^2}}-\frac{\frac{\int \frac{(cx)^m\left((m+4)\left(-D(m+1)a^3+bC(m+1)a^2-b^2B(m+1)a+Ab^3m\right)-ab(bC(m+4)-aD(2m+7))x^2\right)}{b^2\sqrt{bx^2+a}}dx}{b(m+4)}-\frac{aD\sqrt{a+bx^2}(cx)^{m+3}}{b^2c^3(m+4)}}{a}}{}$$

$\Big\downarrow$ 27

$$\frac{(cx)^{m+1}\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{ac\sqrt{a+bx^2}}-\frac{\frac{\int \frac{(cx)^m\left((m+4)\left(-D(m+1)a^3+bC(m+1)a^2-b^2B(m+1)a+Ab^3m\right)-ab(bC(m+4)-aD(2m+7))x^2\right)}{\sqrt{bx^2+a}}dx}{b^3(m+4)}-\frac{aD\sqrt{a+bx^2}(cx)^{m+3}}{b^2c^3(m+4)}}{a}$$

$\Big\downarrow$ 363

$$\frac{(cx)^{m+1}\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{ac\sqrt{a+bx^2}}-\frac{\frac{\left(\frac{a^2(m+1)(bC(m+4)-aD(2m+7))}{m+2}+(m+4)(a^3(-D)(m+1)+a^2bC(m+1)-ab^2B(m+1)+Ab^3m)\right)\int\frac{(cx)^m}{\sqrt{bx^2+a}}dx-\frac{a\sqrt{a+bx^2}(cx)^{m+1}(bC(m+4)-aD(2}{c(m+2)}}{b^3(m+4)}}{a}$$

$\Big\downarrow$ 279

$$\frac{(cx)^{m+1}\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{ac\sqrt{a+bx^2}}-\frac{\frac{\frac{\sqrt{\frac{bx^2}{a}+1}\left(\frac{a^2(m+1)(bC(m+4)-aD(2m+7))}{m+2}+(m+4)\left(a^3(-D)(m+1)+a^2bC(m+1)-ab^2B(m+1)+Ab^3m\right)\right)\int\frac{(cx)^m}{\sqrt{\frac{bx^2}{a}+1}}dx}{\sqrt{a+bx^2}}-\frac{a\sqrt{a+bx^2}(cx)^{m+1}(bC(m+4)-aD(}{c(m+2)}}{b^3(m+4)}}{a}$$

$\Big\downarrow$ 278

$$\frac{(cx)^{m+1}\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{ac\sqrt{a+bx^2}}-\frac{\frac{\frac{\sqrt{\frac{bx^2}{a}+1}(cx)^{m+1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},-\frac{bx^2}{a}\right)\left(\frac{a^2(m+1)(bC(m+4)-aD(2m+7))}{m+2}+(m+4)\left(a^3(-D)(m+1)+a^2bC(m+1)-ab^2B(m+1)+Ab^3m\right)\right)}{c(m+1)\sqrt{a+bx^2}}}{b^3(m+4)}}{a}$$

input
```
Int[((c*x)^m*(A + B*x^2 + C*x^4 + D*x^6))/(a + b*x^2)^(3/2),x]
```

output
```
((A - (a*(b^2*B - a*b*C + a^2*D))/b^3)*(c*x)^(1 + m))/(a*c*Sqrt[a + b*x^2]
) - (-((a*D*(c*x)^(3 + m)*Sqrt[a + b*x^2])/(b^2*c^3*(4 + m))) + (-((a*(b*C
*(4 + m) - a*D*(7 + 2*m))*(c*x)^(1 + m)*Sqrt[a + b*x^2])/(c*(2 + m))) + ((
(4 + m)*(A*b^3*m - a*b^2*B*(1 + m) + a^2*b*C*(1 + m) - a^3*D*(1 + m)) + (a
^2*(1 + m)*(b*C*(4 + m) - a*D*(7 + 2*m)))/(2 + m))*(c*x)^(1 + m)*Sqrt[1 +
(b*x^2)/a]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(c*
(1 + m)*Sqrt[a + b*x^2]))/(b^3*(4 + m)))/a
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 1590
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[c^p*(f*x)^(m + 4*p - 1)*((d + e*x^2)^
(q + 1)/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1))), x] + Simp[1/(e*(m + 4*p + 2*q
 + 1))   Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a +
b*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x],
x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p,
0] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

rule 2337
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(-(c*x)^(m + 1))*(f + g*x)*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1)))
, x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*ExpandToSum[2
*a*(p + 1)*Q + f*(m + 2*p + 3) + g*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a
, b, c, m}, x] && PolyQ[Pq, x] && LtQ[p, -1] &&  !GtQ[m, 0]
```

## Maple [F]

$$\int \frac{(cx)^m (Dx^6 + C\,x^4 + x^2 B + A)}{(b\,x^2 + a)^{\frac{3}{2}}}dx$$

input
```
int((c*x)^m*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(3/2),x)
```

output
```
int((c*x)^m*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(3/2),x)
```

## Fricas [F]

$$\int \frac{(cx)^m (A + Bx^2 + Cx^4 + Dx^6)}{(a + bx^2)^{3/2}}\,dx = \int \frac{(Dx^6 + Cx^4 + Bx^2 + A)(cx)^m}{(bx^2 + a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x)^m*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="fric
as")
```

output
```
integral((D*x^6 + C*x^4 + B*x^2 + A)*sqrt(b*x^2 + a)*(c*x)^m/(b^2*x^4 + 2*
a*b*x^2 + a^2), x)
```

## Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 23.21 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.81

$$\int \frac{(cx)^m (A + Bx^2 + Cx^4 + Dx^6)}{(a + bx^2)^{3/2}}\,dx = \frac{Ac^m x^{m+1}\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)\,{}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{m}{2}+\frac{1}{2}\\ \frac{m}{2}+\frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)} + \frac{Bc^m x^{m+3}\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)\,{}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{m}{2}+\frac{3}{2}\\ \frac{m}{2}+\frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)} + \frac{Cc^m x^{m+5}\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)\,{}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{m}{2}+\frac{5}{2}\\ \frac{m}{2}+\frac{7}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}\Gamma\left(\frac{m}{2}+\frac{7}{2}\right)} + \frac{Dc^m x^{m+7}\Gamma\left(\frac{m}{2}+\frac{7}{2}\right)\,{}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{m}{2}+\frac{7}{2}\\ \frac{m}{2}+\frac{9}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}\Gamma\left(\frac{m}{2}+\frac{9}{2}\right)}$$

input
```
integrate((c*x)**m*(D*x**6+C*x**4+B*x**2+A)/(b*x**2+a)**(3/2),x)
```

output
```
A*c**m*x**(m + 1)*gamma(m/2 + 1/2)*hyper((3/2, m/2 + 1/2), (m/2 + 3/2,), b
*x**2*exp_polar(I*pi)/a)/(2*a**(3/2)*gamma(m/2 + 3/2)) + B*c**m*x**(m + 3)
*gamma(m/2 + 3/2)*hyper((3/2, m/2 + 3/2), (m/2 + 5/2,), b*x**2*exp_polar(I
*pi)/a)/(2*a**(3/2)*gamma(m/2 + 5/2)) + C*c**m*x**(m + 5)*gamma(m/2 + 5/2)
*hyper((3/2, m/2 + 5/2), (m/2 + 7/2,), b*x**2*exp_polar(I*pi)/a)/(2*a**(3/
2)*gamma(m/2 + 7/2)) + D*c**m*x**(m + 7)*gamma(m/2 + 7/2)*hyper((3/2, m/2
+ 7/2), (m/2 + 9/2,), b*x**2*exp_polar(I*pi)/a)/(2*a**(3/2)*gamma(m/2 + 9/
2))
```

## Maxima [F]

$$\int \frac{(cx)^m (A + Bx^2 + Cx^4 + Dx^6)}{(a + bx^2)^{3/2}} \, dx = \int \frac{(Dx^6 + Cx^4 + Bx^2 + A)(cx)^m}{(bx^2 + a)^{\frac{3}{2}}} \, dx$$

input
```
integrate((c*x)^m*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="maxi
ma")
```

output
```
integrate((D*x^6 + C*x^4 + B*x^2 + A)*(c*x)^m/(b*x^2 + a)^(3/2), x)
```

## Giac [F]

$$\int \frac{(cx)^m (A + Bx^2 + Cx^4 + Dx^6)}{(a + bx^2)^{3/2}} \, dx = \int \frac{(Dx^6 + Cx^4 + Bx^2 + A)(cx)^m}{(bx^2 + a)^{\frac{3}{2}}} \, dx$$

input
```
integrate((c*x)^m*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="giac
")
```

output
```
integrate((D*x^6 + C*x^4 + B*x^2 + A)*(c*x)^m/(b*x^2 + a)^(3/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^m (A + Bx^2 + Cx^4 + Dx^6)}{(a + bx^2)^{3/2}} \, dx = \int \frac{(c\,x)^m\,(A + B\,x^2 + C\,x^4 + x^6\,D)}{(b\,x^2 + a)^{3/2}} \, dx$$

input
```
int(((c*x)^m*(A + B*x^2 + C*x^4 + x^6*D))/(a + b*x^2)^(3/2),x)
```

output
```
int(((c*x)^m*(A + B*x^2 + C*x^4 + x^6*D))/(a + b*x^2)^(3/2), x)
```

## Reduce [F]

$$\int \frac{(cx)^m (A + Bx^2 + Cx^4 + Dx^6)}{(a + bx^2)^{3/2}} dx = c^m \left( \left( \int \frac{x^m}{\sqrt{b x^2 + a}\, a + \sqrt{b x^2 + a}\, b x^2} dx \right) a + \left( \int \frac{x^m x^6}{\sqrt{b x^2 + a}\, a + \sqrt{b x^2 + a}\, b x^2} dx \right) d + \left( \int \frac{x^m x^4}{\sqrt{b x^2 + a}\, a + \sqrt{b x^2 + a}\, b x^2} dx \right) c + \left( \int \frac{x^m x^2}{\sqrt{b x^2 + a}\, a + \sqrt{b x^2 + a}\, b x^2} dx \right) b \right)$$

input
```
int((c*x)^m*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(3/2),x)
```

output
```
c**m*(int(x**m/(sqrt(a + b*x**2)*a + sqrt(a + b*x**2)*b*x**2),x)*a + int((
x**m*x**6)/(sqrt(a + b*x**2)*a + sqrt(a + b*x**2)*b*x**2),x)*d + int((x**m
*x**4)/(sqrt(a + b*x**2)*a + sqrt(a + b*x**2)*b*x**2),x)*c + int((x**m*x**
2)/(sqrt(a + b*x**2)*a + sqrt(a + b*x**2)*b*x**2),x)*b)
```

# 3.263 $\int \frac{(cx)^m(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{5/2}}\,dx$



## Optimal result

Integrand size = 34, antiderivative size = 301

$$\int \frac{(cx)^m\left(A+Bx^2+Cx^4+Dx^6\right)}{\left(a+bx^2\right)^{5/2}}\,dx=\frac{\left(\frac{A}{a}-\frac{b^2B-abC+a^2D}{b^3}\right)(cx)^{1+m}}{3c\left(a+bx^2\right)^{3/2}}$$

$$+\frac{\left(\frac{A(2-m)}{a}+\frac{b^2B(1+m)-abC(4+m)+a^2D(7+m)}{b^3}\right)(cx)^{1+m}}{3ac\sqrt{a+bx^2}}+\frac{D(cx)^{1+m}\sqrt{a+bx^2}}{b^3c(2+m)}$$

$$-\frac{(Ab^3m(4-m^2)+ab^2Bm(2+3m+m^2)-a^2bC(6+11m+6m^2+m^3)+a^3D(15+23m+9m^2+m^3)}{3a^2b^3c(1+m)(2+m)\sqrt{a+bx^2}}$$

output
```
1/3*(A/a-(B*b^2-C*a*b+D*a^2)/b^3)*(c*x)^(1+m)/c/(b*x^2+a)^(3/2)+1/3*(A*(2-
m)/a+(b^2*B*(1+m)-a*b*C*(4+m)+a^2*D*(7+m))/b^3)*(c*x)^(1+m)/a/c/(b*x^2+a)^
(1/2)+D*(c*x)^(1+m)*(b*x^2+a)^(1/2)/b^3/c/(2+m)-1/3*(A*b^3*m*(-m^2+4)+a*b^
2*B*m*(m^2+3*m+2)-a^2*b*C*(m^3+6*m^2+11*m+6)+a^3*D*(m^3+9*m^2+23*m+15))*(c
*x)^(1+m)*(1+b*x^2/a)^(1/2)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],-b*x^2/
a)/a^2/b^3/c/(1+m)/(2+m)/(b*x^2+a)^(1/2)
```

## Mathematica [A] (verified)

Time = 3.02 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.60

$$\int \frac{(cx)^m (A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{5/2}}\,dx = \frac{x(cx)^m\sqrt{1+\frac{bx^2}{a}}\left(\frac{A\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{1+m}{2},\frac{3+m}{2},-\frac{bx^2}{a}\right)}{1+m}+\frac{Bx^2\,\text{Hypergeon}}{}\right.}{}$$

input
```
Integrate[((c*x)^m*(A + B*x^2 + C*x^4 + D*x^6))/(a + b*x^2)^(5/2),x]
```

output
```
(x*(c*x)^m*Sqrt[1 + (b*x^2)/a]*((A*Hypergeometric2F1[5/2, (1 + m)/2, (3 +
m)/2, -((b*x^2)/a)])/(1 + m) + (B*x^2*Hypergeometric2F1[5/2, (3 + m)/2, (5
 + m)/2, -((b*x^2)/a)])/(3 + m) + (C*x^4*Hypergeometric2F1[5/2, (5 + m)/2,
 (7 + m)/2, -((b*x^2)/a)])/(5 + m) + (D*x^6*Hypergeometric2F1[5/2, (7 + m)
/2, (9 + m)/2, -((b*x^2)/a)])/(7 + m)))/(a^2*Sqrt[a + b*x^2])
```

## Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.206$, Rules used = {2337, 25, 1590, 27, 362, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^m (A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{2337}$$

$$\frac{(cx)^{m+1}\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{3ac(a+bx^2)^{3/2}}-\frac{\int -\frac{(cx)^m\left(\frac{3aDx^4}{b}+\frac{3a(bC-aD)x^2}{b^2}+A(2-m)+\frac{a\left(Da^2-bCa+b^2B\right)(m+1)}{b^3}\right)}{(bx^2+a)^{3/2}}dx}{3a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{(cx)^m\left(\frac{3aDx^4}{b}+\frac{3a(bC-aD)x^2}{b^2}+A(2-m)+\frac{a\left(Da^2-bCa+b^2B\right)(m+1)}{b^3}\right)}{(bx^2+a)^{3/2}}dx}{3a}+\frac{(cx)^{m+1}\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{3ac\,(a+bx^2)^{3/2}}$$

$\Big\downarrow$ 1590

$$\frac{\frac{\int \frac{(cx)^m\left(3ab(bC(m+2)-aD(2m+5))x^2+(m+2)\left(A(2-m)b^3+a\left(Da^2-bCa+b^2B\right)(m+1)\right)\right)}{b^2\left(bx^2+a\right)^{3/2}}dx}{b(m+2)}+\frac{3aD(cx)^{m+3}}{b^2c^3(m+2)\sqrt{a+bx^2}}}{3a}+\frac{(cx)^{m+1}\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{3ac\,(a+bx^2)^{3/2}}$$

$\Big\downarrow$ 27

$$\frac{\frac{\int \frac{(cx)^m\left(3ab(bC(m+2)-aD(2m+5))x^2+(m+2)\left(A(2-m)b^3+a\left(Da^2-bCa+b^2B\right)(m+1)\right)\right)}{\left(bx^2+a\right)^{3/2}}dx}{b^3(m+2)}+\frac{3aD(cx)^{m+3}}{b^2c^3(m+2)\sqrt{a+bx^2}}}{3a}+\frac{(cx)^{m+1}\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{3ac\,(a+bx^2)^{3/2}}$$

$\Big\downarrow$ 362

$$\frac{\frac{\frac{(cx)^{m+1}\left(a\left(a^2D\left(m^2+9m+17\right)-abC\left(m^2+6m+8\right)+b^2B\left(m^2+3m+2\right)\right)+Ab^3\left(4-m^2\right)\right)}{ac\sqrt{a+bx^2}}-\left(\frac{m(m+2)\left(a(m+1)\left(a^2D-abC+b^2B\right)+Ab^3(2-m)\right)}{a}-3a(m+1)(bC\right.}{b^3(m+2)}}{3a}$$

$$\frac{(cx)^{m+1}\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{3ac\,(a+bx^2)^{3/2}}$$

$\Big\downarrow$ 279

$$\frac{\frac{\frac{(cx)^{m+1}\left(a\left(a^2D\left(m^2+9m+17\right)-abC\left(m^2+6m+8\right)+b^2B\left(m^2+3m+2\right)\right)+Ab^3\left(4-m^2\right)\right)}{ac\sqrt{a+bx^2}}-\frac{\sqrt{\frac{bx^2}{a}+1}\left(\frac{m(m+2)\left(a(m+1)\left(a^2D-abC+b^2B\right)+Ab^3(2-m)\right)}{a}-3a(m\right.}{\sqrt{a+bx^2}}}{b^3(m+2)}}{3a}$$

$$\frac{(cx)^{m+1}\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{3ac\,(a+bx^2)^{3/2}}$$

$\Big\downarrow$ 278

$$\frac{\frac{\frac{(cx)^{m+1}\left(a\left(a^2D\left(m^2+9m+17\right)-abC\left(m^2+6m+8\right)+b^2B\left(m^2+3m+2\right)\right)+Ab^3\left(4-m^2\right)\right)}{ac\sqrt{a+bx^2}}-\frac{\sqrt{\frac{bx^2}{a}+1}(cx)^{m+1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},-\frac{bx^2}{a}\right)\left(\frac{m(m}{}\right.}{c}}{b^3(m+2)}}{3a}$$

$$\frac{(cx)^{m+1}\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{3ac\left(a+bx^2\right)^{3/2}}$$

input

```
Int[((c*x)^m*(A + B*x^2 + C*x^4 + D*x^6))/(a + b*x^2)^(5/2),x]
```

output

```
((A - (a*(b^2*B - a*b*C + a^2*D))/b^3)*(c*x)^(1 + m))/(3*a*c*(a + b*x^2)^(
3/2)) + ((3*a*D*(c*x)^(3 + m))/(b^2*c^3*(2 + m)*Sqrt[a + b*x^2]) + (((A*b^
3*(4 - m^2) + a*(b^2*B*(2 + 3*m + m^2) - a*b*C*(8 + 6*m + m^2) + a^2*D*(17
 + 9*m + m^2)))*(c*x)^(1 + m))/(a*c*Sqrt[a + b*x^2]) - (((m*(2 + m)*(A*b^3
*(2 - m) + a*(b^2*B - a*b*C + a^2*D)*(1 + m)))/a - 3*a*(1 + m)*(b*C*(2 + m
) - a*D*(5 + 2*m)))*(c*x)^(1 + m)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/
2, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(c*(1 + m)*Sqrt[a + b*x^2]))/(b^3*
(2 + m)))/(3*a)
```

### Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 278

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

rule 1590
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[c^p*(f*x)^(m + 4*p - 1)*((d + e*x^2)^
(q + 1)/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1))), x] + Simp[1/(e*(m + 4*p + 2*q
 + 1))   Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a +
b*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x],
x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p,
0] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

rule 2337
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(-(c*x)^(m + 1))*(f + g*x)*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1)))
, x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*ExpandToSum[2
*a*(p + 1)*Q + f*(m + 2*p + 3) + g*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a
, b, c, m}, x] && PolyQ[Pq, x] && LtQ[p, -1] &&  !GtQ[m, 0]
```

## Maple [F]

$$\int \frac{(cx)^m (Dx^6 + C x^4 + x^2 B + A)}{(b x^2 + a)^{\frac{5}{2}}} dx$$

input
```
int((c*x)^m*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(5/2),x)
```

output
```
int((c*x)^m*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(5/2),x)
```

## Fricas [F]

$$\int \frac{(cx)^m(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{5/2}}\,dx=\int \frac{(Dx^6+Cx^4+Bx^2+A)(cx)^m}{(bx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((c*x)^m*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="fric
as")
```

output
```
integral((D*x^6 + C*x^4 + B*x^2 + A)*sqrt(b*x^2 + a)*(c*x)^m/(b^3*x^6 + 3*
a*b^2*x^4 + 3*a^2*b*x^2 + a^3), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(cx)^m(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((c*x)**m*(D*x**6+C*x**4+B*x**2+A)/(b*x**2+a)**(5/2),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{(cx)^m(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{5/2}}\,dx=\int \frac{(Dx^6+Cx^4+Bx^2+A)(cx)^m}{(bx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((c*x)^m*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="maxi
ma")
```

output
```
integrate((D*x^6 + C*x^4 + B*x^2 + A)*(c*x)^m/(b*x^2 + a)^(5/2), x)
```

## Giac [F]

$$\int \frac{(cx)^m (A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{5/2}}\,dx=\int \frac{(Dx^6+Cx^4+Bx^2+A)(cx)^m}{(bx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((c*x)^m*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="giac
")
```

output
```
integrate((D*x^6 + C*x^4 + B*x^2 + A)*(c*x)^m/(b*x^2 + a)^(5/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^m (A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{5/2}}\,dx=\int \frac{(c\,x)^m\,(A+B\,x^2+C\,x^4+x^6\,D)}{(b\,x^2+a)^{5/2}}\,dx$$

input
```
int(((c*x)^m*(A + B*x^2 + C*x^4 + x^6*D))/(a + b*x^2)^(5/2),x)
```

output
```
int(((c*x)^m*(A + B*x^2 + C*x^4 + x^6*D))/(a + b*x^2)^(5/2), x)
```

## Reduce [F]

$$\int \frac{(cx)^m (A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{5/2}}\,dx=c^m\Bigg(\left(\int \frac{x^m}{\sqrt{b\,x^2+a}\,a^2+2\sqrt{b\,x^2+a}\,ab\,x^2+\sqrt{b\,x^2+a}\,b^2x^4}dx\right)a$$
$$+\left(\int \frac{x^m x^6}{\sqrt{b\,x^2+a}\,a^2+2\sqrt{b\,x^2+a}\,ab\,x^2+\sqrt{b\,x^2+a}\,b^2x^4}dx\right)d$$
$$+\left(\int \frac{x^m x^4}{\sqrt{b\,x^2+a}\,a^2+2\sqrt{b\,x^2+a}\,ab\,x^2+\sqrt{b\,x^2+a}\,b^2x^4}dx\right)c$$
$$+\left(\int \frac{x^m x^2}{\sqrt{b\,x^2+a}\,a^2+2\sqrt{b\,x^2+a}\,ab\,x^2+\sqrt{b\,x^2+a}\,b^2x^4}dx\right)b\Bigg)$$

input
```
int((c*x)^m*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(5/2),x)
```

output
```
c**m*(int(x**m/(sqrt(a + b*x**2)*a**2 + 2*sqrt(a + b*x**2)*a*b*x**2 + sqrt
(a + b*x**2)*b**2*x**4),x)*a + int((x**m*x**6)/(sqrt(a + b*x**2)*a**2 + 2*
sqrt(a + b*x**2)*a*b*x**2 + sqrt(a + b*x**2)*b**2*x**4),x)*d + int((x**m*x
**4)/(sqrt(a + b*x**2)*a**2 + 2*sqrt(a + b*x**2)*a*b*x**2 + sqrt(a + b*x**
2)*b**2*x**4),x)*c + int((x**m*x**2)/(sqrt(a + b*x**2)*a**2 + 2*sqrt(a + b
*x**2)*a*b*x**2 + sqrt(a + b*x**2)*b**2*x**4),x)*b)
```

# 3.264 $\int \frac{(cx)^m (A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{7/2}}\,dx$



## Optimal result

Integrand size = 34, antiderivative size = 278

$$\int \frac{(cx)^m (A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{7/2}}\,dx = \frac{\left(\frac{A}{a}-\frac{b^2B-abC+a^2D}{b^3}\right)(cx)^{1+m}}{5c(a+bx^2)^{5/2}} + \frac{\left(\frac{A(4-m)}{a}+\frac{b^2B(1+m)-abC(6+m)+a^2D(11+m)}{b^3}\right)(cx)^{1+m}}{15ac(a+bx^2)^{3/2}} + \frac{D(cx)^{1+m}}{b^3cm\sqrt{a+bx^2}} + \frac{\left(ab^2B(2-m)+a^2bC(3+m)+\frac{Ab^3(8-6m+m^2)}{1+m}-\frac{a^3D(15+8m+m^2)}{m}\right)(cx)^{1+m}\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}}{15a^3b^3c\sqrt{a+bx^2}}$$

output
```
1/5*(A/a-(B*b^2-C*a*b+D*a^2)/b^3)*(c*x)^(1+m)/c/(b*x^2+a)^(5/2)+1/15*(A*(4
-m)/a+(b^2*B*(1+m)-a*b*C*(6+m)+a^2*D*(11+m))/b^3)*(c*x)^(1+m)/a/c/(b*x^2+a
)^(3/2)+D*(c*x)^(1+m)/b^3/c/m/(b*x^2+a)^(1/2)+1/15*(a*b^2*B*(2-m)+a^2*b*C*
(3+m)+A*b^3*(m^2-6*m+8)/(1+m)-a^3*D*(m^2+8*m+15)/m)*(c*x)^(1+m)*(1+b*x^2/a
)^(1/2)*hypergeom([3/2, 1/2+1/2*m],[3/2+1/2*m],-b*x^2/a)/a^3/b^3/c/(b*x^2+
a)^(1/2)
```

## Mathematica [A] (verified)

Time = 2.99 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.65

$$\int \frac{(cx)^m (A + Bx^2 + Cx^4 + Dx^6)}{(a + bx^2)^{7/2}} dx = \frac{x(cx)^m \sqrt{1 + \frac{bx^2}{a}} \left( \frac{A \text{ Hypergeometric2F1}\left(\frac{7}{2}, \frac{1+m}{2}, \frac{3+m}{2}, -\frac{bx^2}{a}\right)}{1+m} + \frac{Bx^2 \text{ Hypergeon}}{} \right.}{}$$

input
```
Integrate[((c*x)^m*(A + B*x^2 + C*x^4 + D*x^6))/(a + b*x^2)^(7/2),x]
```

output
```
(x*(c*x)^m*Sqrt[1 + (b*x^2)/a]*((A*Hypergeometric2F1[7/2, (1 + m)/2, (3 +
m)/2, -((b*x^2)/a)])/(1 + m) + (B*x^2*Hypergeometric2F1[7/2, (3 + m)/2, (5
 + m)/2, -((b*x^2)/a)])/(3 + m) + (C*x^4*Hypergeometric2F1[7/2, (5 + m)/2,
 (7 + m)/2, -((b*x^2)/a)])/(5 + m) + (D*x^6*Hypergeometric2F1[7/2, (7 + m)
/2, (9 + m)/2, -((b*x^2)/a)])/(7 + m)))/(a^3*Sqrt[a + b*x^2])
```

## Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.14, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.206$, Rules used = {2337, 25, 1590, 27, 362, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^m (A + Bx^2 + Cx^4 + Dx^6)}{(a + bx^2)^{7/2}} dx$$

$$\Big\downarrow \text{ 2337}$$

$$\frac{(cx)^{m+1} \left(A - \frac{a(a^2D - abC + b^2B)}{b^3}\right)}{5ac(a + bx^2)^{5/2}} - \frac{\int -\frac{(cx)^m \left(\frac{5aDx^4}{b} + \frac{5a(bC - aD)x^2}{b^2} + A(4-m) + \frac{a(Da^2 - bCa + b^2B)(m+1)}{b^3}\right)}{(bx^2+a)^{5/2}}dx}{5a}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{(cx)^m\left(\frac{5aDx^4}{b}+\frac{5a(bC-aD)x^2}{b^2}+A(4-m)+\frac{a\left(Da^2-bCa+b^2B\right)(m+1)}{b^3}\right)}{(bx^2+a)^{5/2}}dx}{5a}+\frac{(cx)^{m+1}\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{5ac\left(a+bx^2\right)^{5/2}}$$

$\Big\downarrow$ 1590

$$\frac{\frac{\int \frac{(cx)^m\left(5a(bCm-aD(2m+3))x^2+bm\left(Ab(4-m)+\frac{a\left(Da^2-bCa+b^2B\right)(m+1)}{b^2}\right)\right)}{b\left(bx^2+a\right)^{5/2}}dx}{bm}+\frac{5aD(cx)^{m+3}}{b^2c^3m(a+bx^2)^{3/2}}}{5a}+\frac{(cx)^{m+1}\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{5ac\left(a+bx^2\right)^{5/2}}$$

$\Big\downarrow$ 27

$$\frac{\frac{\int \frac{(cx)^m\left(5a(bCm-aD(2m+3))x^2+bm\left(Ab(4-m)+\frac{a\left(Da^2-bCa+b^2B\right)(m+1)}{b^2}\right)\right)}{\left(bx^2+a\right)^{5/2}}dx}{b^2m}+\frac{5aD(cx)^{m+3}}{b^2c^3m(a+bx^2)^{3/2}}}{5a}+\frac{(cx)^{m+1}\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{5ac\left(a+bx^2\right)^{5/2}}$$

$\Big\downarrow$ 362

$$\frac{\frac{\frac{\left((2-m)m\left(a(m+1)\left(a^2D-abC+b^2B\right)+Ab^3(4-m)\right)+5a^2(m+1)(bCm-aD(2m+3))\right)\int \frac{(cx)^m}{\left(bx^2+a\right)^{3/2}}dx}{3ab}+\frac{(cx)^{m+1}\left(a^3D\left(m^2+11m+15\right)-a^2bCm(m+6)+ab^2Bm}{3abc\left(a+bx^2\right)^{3/2}}}{b^2m}}{5a}$$

$$\frac{(cx)^{m+1}\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{5ac\left(a+bx^2\right)^{5/2}}$$

$\Big\downarrow$ 279

$$\frac{\frac{\frac{\sqrt{\frac{bx^2}{a}+1}\left((2-m)m\left(a(m+1)\left(a^2D-abC+b^2B\right)+Ab^3(4-m)\right)+5a^2(m+1)(bCm-aD(2m+3))\right)\int \frac{(cx)^m}{\left(\frac{bx^2}{a}+1\right)^{3/2}}dx}{3a^2b\sqrt{a+bx^2}}+\frac{(cx)^{m+1}\left(a^3D\left(m^2+11m+15\right)-a^2bCm(m+}{3abc\left(a+bx^2\right)}}{b^2m}}{5a}$$

$$\frac{(cx)^{m+1}\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{5ac\left(a+bx^2\right)^{5/2}}$$

$\Big\downarrow$ 278

$$\frac{(cx)^{m+1}\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{5ac\left(a+bx^2\right)^{5/2}}+\frac{\frac{\frac{\sqrt{\frac{bx^2}{a}+1}(cx)^{m+1}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{m+1}{2},\frac{m+3}{2},-\frac{bx^2}{a}\right)\left((2-m)m\left(a(m+1)\left(a^2D-abC+b^2B\right)+Ab^3(4-m)\right)+5a^2(m+1)(bCm-aD(2m+3))\right)}{3a^2bc(m+1)\sqrt{a+bx^2}}+\frac{(cx)^{m+1}(\cdots}{\cdots}}{b^2m}}{5a}$$

input
```
Int[((c*x)^m*(A + B*x^2 + C*x^4 + D*x^6))/(a + b*x^2)^(7/2),x]
```

output
```
((A - (a*(b^2*B - a*b*C + a^2*D))/b^3)*(c*x)^(1 + m))/(5*a*c*(a + b*x^2)^(
5/2)) + ((5*a*D*(c*x)^(3 + m))/(b^2*c^3*m*(a + b*x^2)^(3/2)) + (((A*b^3*(4
 - m)*m + a*b^2*B*m*(1 + m) - a^2*b*C*m*(6 + m) + a^3*D*(15 + 11*m + m^2))
*(c*x)^(1 + m))/(3*a*b*c*(a + b*x^2)^(3/2)) + (((2 - m)*m*(A*b^3*(4 - m) +
 a*(b^2*B - a*b*C + a^2*D)*(1 + m)) + 5*a^2*(1 + m)*(b*C*m - a*D*(3 + 2*m)
))*(c*x)^(1 + m)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[3/2, (1 + m)/2, (3
+ m)/2, -((b*x^2)/a)])/(3*a^2*b*c*(1 + m)*Sqrt[a + b*x^2]))/(b^2*m))/(5*a)
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

rule 1590
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[c^p*(f*x)^(m + 4*p - 1)*((d + e*x^2)^
(q + 1)/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1))), x] + Simp[1/(e*(m + 4*p + 2*q
 + 1))   Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a +
b*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x],
x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p,
0] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

rule 2337
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(-(c*x)^(m + 1))*(f + g*x)*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1)))
, x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*ExpandToSum[2
*a*(p + 1)*Q + f*(m + 2*p + 3) + g*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a
, b, c, m}, x] && PolyQ[Pq, x] && LtQ[p, -1] &&  !GtQ[m, 0]
```

## Maple [F]

$$\int \frac{(cx)^m (Dx^6 + C\,x^4 + x^2 B + A)}{(b\,x^2 + a)^{\frac{7}{2}}}dx$$

input
```
int((c*x)^m*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(7/2),x)
```

output
```
int((c*x)^m*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(7/2),x)
```

## Fricas [F]

$$\int \frac{(cx)^m (A + Bx^2 + Cx^4 + Dx^6)}{(a + bx^2)^{7/2}}\, dx = \int \frac{(Dx^6 + Cx^4 + Bx^2 + A)(cx)^m}{(bx^2 + a)^{\frac{7}{2}}}\, dx$$

input
```
integrate((c*x)^m*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(7/2),x, algorithm="fric
as")
```

output
```
integral((D*x^6 + C*x^4 + B*x^2 + A)*sqrt(b*x^2 + a)*(c*x)^m/(b^4*x^8 + 4*
a*b^3*x^6 + 6*a^2*b^2*x^4 + 4*a^3*b*x^2 + a^4), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(cx)^m (A + Bx^2 + Cx^4 + Dx^6)}{(a + bx^2)^{7/2}}\, dx = \text{Timed out}$$

input
```
integrate((c*x)**m*(D*x**6+C*x**4+B*x**2+A)/(b*x**2+a)**(7/2),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{(cx)^m (A + Bx^2 + Cx^4 + Dx^6)}{(a + bx^2)^{7/2}}\, dx = \int \frac{(Dx^6 + Cx^4 + Bx^2 + A)(cx)^m}{(bx^2 + a)^{\frac{7}{2}}}\, dx$$

input
```
integrate((c*x)^m*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(7/2),x, algorithm="maxi
ma")
```

output
```
integrate((D*x^6 + C*x^4 + B*x^2 + A)*(c*x)^m/(b*x^2 + a)^(7/2), x)
```

## Giac [F]

$$\int \frac{(cx)^m (A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{7/2}}\,dx=\int \frac{(Dx^6+Cx^4+Bx^2+A)(cx)^m}{(bx^2+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate((c*x)^m*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(7/2),x, algorithm="giac
")
```

output
```
integrate((D*x^6 + C*x^4 + B*x^2 + A)*(c*x)^m/(b*x^2 + a)^(7/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^m (A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{7/2}}\,dx=\int \frac{(c\,x)^m\,(A+B\,x^2+C\,x^4+x^6\,D)}{(b\,x^2+a)^{7/2}}\,dx$$

input
```
int(((c*x)^m*(A + B*x^2 + C*x^4 + x^6*D))/(a + b*x^2)^(7/2),x)
```

output
```
int(((c*x)^m*(A + B*x^2 + C*x^4 + x^6*D))/(a + b*x^2)^(7/2), x)
```

## Reduce [F]

$$\int \frac{(cx)^m (A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{7/2}}\,dx=c^m\Bigg(\Bigg(\Bigg(\int \frac{x^m}{\sqrt{b\,x^2+a}\,a^3+3\sqrt{b\,x^2+a}\,a^2b\,x^2+3\sqrt{b\,x^2+a}\,a\,b^2x^4+\cdots}$$
$$+\left(\int \frac{x^m x^6}{\sqrt{b\,x^2+a}\,a^3+3\sqrt{b\,x^2+a}\,a^2b\,x^2+3\sqrt{b\,x^2+a}\,a\,b^2x^4+\sqrt{b\,x^2+a}\,b^3x^6}dx\right)d$$
$$+\left(\int \frac{x^m x^4}{\sqrt{b\,x^2+a}\,a^3+3\sqrt{b\,x^2+a}\,a^2b\,x^2+3\sqrt{b\,x^2+a}\,a\,b^2x^4+\sqrt{b\,x^2+a}\,b^3x^6}dx\right)c$$
$$+\left(\int \frac{x^m x^2}{\sqrt{b\,x^2+a}\,a^3+3\sqrt{b\,x^2+a}\,a^2b\,x^2+3\sqrt{b\,x^2+a}\,a\,b^2x^4+\sqrt{b\,x^2+a}\,b^3x^6}dx\right)b\Bigg)$$

input
```
int((c*x)^m*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(7/2),x)
```

output
```
c**m*(int(x**m/(sqrt(a + b*x**2)*a**3 + 3*sqrt(a + b*x**2)*a**2*b*x**2 + 3
*sqrt(a + b*x**2)*a*b**2*x**4 + sqrt(a + b*x**2)*b**3*x**6),x)*a + int((x*
*m*x**6)/(sqrt(a + b*x**2)*a**3 + 3*sqrt(a + b*x**2)*a**2*b*x**2 + 3*sqrt(
a + b*x**2)*a*b**2*x**4 + sqrt(a + b*x**2)*b**3*x**6),x)*d + int((x**m*x**
4)/(sqrt(a + b*x**2)*a**3 + 3*sqrt(a + b*x**2)*a**2*b*x**2 + 3*sqrt(a + b*
x**2)*a*b**2*x**4 + sqrt(a + b*x**2)*b**3*x**6),x)*c + int((x**m*x**2)/(sq
rt(a + b*x**2)*a**3 + 3*sqrt(a + b*x**2)*a**2*b*x**2 + 3*sqrt(a + b*x**2)*
a*b**2*x**4 + sqrt(a + b*x**2)*b**3*x**6),x)*b)
```

# 3.265 $\int (cx)^m \left(a+bx^2\right)^p \left(A+Bx^2+Cx^4+Dx^6\right) \, dx$



## Optimal result

Integrand size = 32, antiderivative size = 346

$$\int (cx)^m \left(a+bx^2\right)^p \left(A+Bx^2+Cx^4+Dx^6\right) \, dx$$
$$= \frac{\left(a^2D(15+8m+m^2)-abC(3+m)(7+m+2p)+b^2B(35+m^2+24p+4p^2+4m(3+p))\right)(cx)^{1+m}\,(}{b^3c(3+m+2p)(5+m+2p)(7+m+2p)}$$
$$-\frac{(aD(5+m)-bC(7+m+2p))(cx)^{3+m}\left(a+bx^2\right)^{1+p}}{b^2c^3(5+m+2p)(7+m+2p)}+\frac{D(cx)^{5+m}\left(a+bx^2\right)^{1+p}}{bc^5(7+m+2p)}$$
$$+\frac{\left(\frac{A}{1+m}-\frac{a(a^2D(15+8m+m^2)-abC(3+m)(7+m+2p)+b^2B(35+m^2+24p+4p^2+4m(3+p)))}{b^3(3+m+2p)(5+m+2p)(7+m+2p)}\right)(cx)^{1+m}\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)}{c}$$

output
```
(a^2*D*(m^2+8*m+15)-a*b*C*(3+m)*(7+m+2*p)+b^2*B*(35+m^2+24*p+4*p^2+4*m*(3+
p)))*(c*x)^(1+m)*(b*x^2+a)^(p+1)/b^3/c/(3+m+2*p)/(5+m+2*p)/(7+m+2*p)-(a*D*
(5+m)-b*C*(7+m+2*p))*(c*x)^(3+m)*(b*x^2+a)^(p+1)/b^2/c^3/(5+m+2*p)/(7+m+2*
p)+D*(c*x)^(5+m)*(b*x^2+a)^(p+1)/b/c^5/(7+m+2*p)+(A/(1+m)-a*(a^2*D*(m^2+8*
m+15)-a*b*C*(3+m)*(7+m+2*p)+b^2*B*(35+m^2+24*p+4*p^2+4*m*(3+p)))/b^3/(3+m+
2*p)/(5+m+2*p)/(7+m+2*p))*(c*x)^(1+m)*(b*x^2+a)^p*hypergeom([-p, 1/2+1/2*m
],[3/2+1/2*m],-b*x^2/a)/c/((1+b*x^2/a)^p)
```

## Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.51

$$\int (cx)^m \left(a+bx^2\right)^p \left(A+Bx^2+Cx^4+Dx^6\right) dx = x(cx)^m \left(a+bx^2\right)^p \left(1+\frac{bx^2}{a}\right)^{-p} \left(\frac{A\,\text{Hypergeometric2F1}\left(\frac{1+m}{2},-p,\frac{3+m}{2},-\frac{bx^2}{a}\right)}{1+m} + \frac{Bx^2\,\text{Hypergeometric2F1}\left(\frac{3+m}{2},-p,\frac{5+m}{2},-\frac{bx^2}{a}\right)}{3+m} + \frac{Cx^4\,\text{Hypergeometric2F1}\left(\frac{5+m}{2},-p,\frac{7+m}{2},-\frac{bx^2}{a}\right)}{5+m} + \frac{Dx^6\,\text{Hypergeometric2F1}\left(\frac{7+m}{2},-p,\frac{9+m}{2},-\frac{bx^2}{a}\right)}{7+m}\right)$$

input
```
Integrate[(c*x)^m*(a + b*x^2)^p*(A + B*x^2 + C*x^4 + D*x^6),x]
```

output
```
(x*(c*x)^m*(a + b*x^2)^p*((A*Hypergeometric2F1[(1 + m)/2, -p, (3 + m)/2, -
((b*x^2)/a)])/(1 + m) + (B*x^2*Hypergeometric2F1[(3 + m)/2, -p, (5 + m)/2,
 -((b*x^2)/a)])/(3 + m) + (C*x^4*Hypergeometric2F1[(5 + m)/2, -p, (7 + m)/
2, -((b*x^2)/a)])/(5 + m) + (D*x^6*Hypergeometric2F1[(7 + m)/2, -p, (9 + m
)/2, -((b*x^2)/a)])/(7 + m)))/(1 + (b*x^2)/a)^p
```

## Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {2340, 1590, 363, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^m \left(a+bx^2\right)^p \left(A+Bx^2+Cx^4+Dx^6\right) dx$$

$$\Big\downarrow \text{2340}$$

$$\frac{\int (cx)^m \left(bx^2+a\right)^p \left(-\left(\left(aD(m+5)-bC(m+2p+7)\right)x^4\right)+bB(m+2p+7)x^2+Ab(m+2p+7)\right)dx}{b(m+2p+7)}+\frac{D(cx)^{m+5}\left(a+bx^2\right)^{p+1}}{bc^5(m+2p+7)}$$

$$\Big\downarrow \text{1590}$$

$$\frac{\frac{\int (cx)^m \left(bx^2+a\right)^p \left(A(m+2p+5)(m+2p+7)b^2+\left(D\left(m^2+8m+15\right)a^2-bC(m+3)(m+2p+7)a+b^2B\left(m^2+4(p+3)m+4p^2+24p+35\right)\right)x^2\right)dx}{b(m+2p+5)}-\frac{(cx)\cdots}{\cdots}}{b(m+2p+7)}+\frac{D(cx)^{m+5}\left(a+bx^2\right)^{p+1}}{bc^5(m+2p+7)}$$

$$\Big\downarrow \text{363}$$

$$\frac{\frac{\frac{\left(Ab^3(m+2p+5)(m+2p+7)-\frac{a(m+1)\left(a^2D\left(m^2+8m+15\right)-abC(m+3)(m+2p+7)+b^2B\left(m^2+4m(p+3)+4p^2+24p+35\right)\right)}{m+2p+3}\right)\int (cx)^m\left(bx^2+a\right)^p dx}{b}+\frac{(cx)^{m+1}\left(a+b\cdots\right.}{\cdots}}{b(m+2p+5)}\cdots}{b(m+2p+\cdots}+\frac{D(cx)^{m+5}\left(a+bx^2\right)^{p+1}}{bc^5(m+2p+7)}$$

$$\Big\downarrow \text{279}$$

$$\frac{\frac{\frac{\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(Ab^3(m+2p+5)(m+2p+7)-\frac{a(m+1)\left(a^2D\left(m^2+8m+15\right)-abC(m+3)(m+2p+7)+b^2B\left(m^2+4m(p+3)+4p^2+24p+35\right)\right)}{m+2p+3}\right)\int (cx)^m\left(\frac{bx^2}{a}\cdots\right.}{b}\cdots}{b(m+2p+5)}\cdots}{b(\cdots}+\frac{D(cx)^{m+5}\left(a+bx^2\right)^{p+1}}{bc^5(m+2p+7)}$$

$$\Big\downarrow \text{278}$$

$$\frac{\frac{\frac{(cx)^{m+1}\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{m+1}{2},-p,\frac{m+3}{2},-\frac{bx^2}{a}\right)\left(Ab^3(m+2p+5)(m+2p+7)-\frac{a(m+1)\left(a^2D\left(m^2+8m+15\right)-abC(m+3)(m+2p+7)\cdots\right.}{m+2p+3}\cdots\right.}{bc(m+1)}\cdots}{b(m+2p+5)\cdots}\cdots}{\cdots}+\frac{D(cx)^{m+5}\left(a+bx^2\right)^{p+1}}{bc^5(m+2p+7)}$$

input
```
Int[(c*x)^m*(a + b*x^2)^p*(A + B*x^2 + C*x^4 + D*x^6),x]
```

output
```
(D*(c*x)^(5 + m)*(a + b*x^2)^(1 + p))/(b*c^5*(7 + m + 2*p)) + (-(((a*D*(5
+ m) - b*C*(7 + m + 2*p))*(c*x)^(3 + m)*(a + b*x^2)^(1 + p))/(b*c^3*(5 + m
 + 2*p))) + (((a^2*D*(15 + 8*m + m^2) - a*b*C*(3 + m)*(7 + m + 2*p) + b^2*
B*(35 + m^2 + 24*p + 4*p^2 + 4*m*(3 + p)))*(c*x)^(1 + m)*(a + b*x^2)^(1 +
p))/(b*c*(3 + m + 2*p)) + ((A*b^3*(5 + m + 2*p)*(7 + m + 2*p) - (a*(1 + m)
*(a^2*D*(15 + 8*m + m^2) - a*b*C*(3 + m)*(7 + m + 2*p) + b^2*B*(35 + m^2 +
 24*p + 4*p^2 + 4*m*(3 + p))))/(3 + m + 2*p))*(c*x)^(1 + m)*(a + b*x^2)^p*
Hypergeometric2F1[(1 + m)/2, -p, (3 + m)/2, -((b*x^2)/a)])/(b*c*(1 + m)*(1
 + (b*x^2)/a)^p))/(b*(5 + m + 2*p)))/(b*(7 + m + 2*p))
```

## Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 1590
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[c^p*(f*x)^(m + 4*p - 1)*((d + e*x^2)^
(q + 1)/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1))), x] + Simp[1/(e*(m + 4*p + 2*q
 + 1))   Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a +
b*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x],
x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p,
0] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

rule 2340
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Simp[1/(b*(m
+ q + 2*p + 1))   Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)
*Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /;
GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ
[Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

## Maple [F]

$$\int (cx)^m \left(b\,x^2+a\right)^p \left(Dx^6+C\,x^4+x^2B+A\right)dx$$

input
```
int((c*x)^m*(b*x^2+a)^p*(D*x^6+C*x^4+B*x^2+A),x)
```

output
```
int((c*x)^m*(b*x^2+a)^p*(D*x^6+C*x^4+B*x^2+A),x)
```

## Fricas [F]

$$\int (cx)^m \left(a+bx^2\right)^p \left(A+Bx^2+Cx^4+Dx^6\right)\,dx$$
$$=\int \left(Dx^6+Cx^4+Bx^2+A\right)\left(bx^2+a\right)^p (cx)^m\,dx$$

input
```
integrate((c*x)^m*(b*x^2+a)^p*(D*x^6+C*x^4+B*x^2+A),x, algorithm="fricas")
```

output
```
integral((D*x^6 + C*x^4 + B*x^2 + A)*(b*x^2 + a)^p*(c*x)^m, x)
```

## Sympy [F(-1)]

Timed out.

$$\int (cx)^m \left(a+bx^2\right)^p \left(A+Bx^2+Cx^4+Dx^6\right) dx = \text{Timed out}$$

input
```
integrate((c*x)**m*(b*x**2+a)**p*(D*x**6+C*x**4+B*x**2+A),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int (cx)^m \left(a+bx^2\right)^p \left(A+Bx^2+Cx^4+Dx^6\right) dx = \int (Dx^6+Cx^4+Bx^2+A)\left(bx^2+a\right)^p(cx)^m\, dx$$

input
```
integrate((c*x)^m*(b*x^2+a)^p*(D*x^6+C*x^4+B*x^2+A),x, algorithm="maxima")
```

output
```
integrate((D*x^6 + C*x^4 + B*x^2 + A)*(b*x^2 + a)^p*(c*x)^m, x)
```

## Giac [F]

$$\int (cx)^m \left(a+bx^2\right)^p \left(A+Bx^2+Cx^4+Dx^6\right) dx = \int (Dx^6+Cx^4+Bx^2+A)\left(bx^2+a\right)^p(cx)^m\, dx$$

input
```
integrate((c*x)^m*(b*x^2+a)^p*(D*x^6+C*x^4+B*x^2+A),x, algorithm="giac")
```

output
```
integrate((D*x^6 + C*x^4 + B*x^2 + A)*(b*x^2 + a)^p*(c*x)^m, x)
```

## Mupad [F(-1)]

Timed out.

$$\int (cx)^m \left(a+bx^2\right)^p \left(A+Bx^2+Cx^4+Dx^6\right)\,dx = \int (c\,x)^m\,\left(b\,x^2+a\right)^p\,\left(A+B\,x^2+C\,x^4+x^6\,D\right)\,dx$$

input
```
int((c*x)^m*(a + b*x^2)^p*(A + B*x^2 + C*x^4 + x^6*D),x)
```

output
```
int((c*x)^m*(a + b*x^2)^p*(A + B*x^2 + C*x^4 + x^6*D), x)
```

## Reduce [F]

$$\int (cx)^m \left(a+bx^2\right)^p \left(A+Bx^2+Cx^4+Dx^6\right)\,dx = \text{too large to display}$$

input
```
int((c*x)^m*(b*x^2+a)^p*(D*x^6+C*x^4+B*x^2+A),x)
```

output

```
(c**m*(2*x***m*(a + b*x**2)**p*a**3*d*m**2*p*x + 16*x**m*(a + b*x**2)**p*a*
*3*d*m*p*x + 30*x***m*(a + b*x**2)**p*a**3*d*p*x - 2*x***m*(a + b*x**2)**p*a
**2*b*c*m**2*p*x - 4*x***m*(a + b*x**2)**p*a**2*b*c*m*p**2*x - 20*x***m*(a +
 b*x**2)**p*a**2*b*c*m*p*x - 12*x***m*(a + b*x**2)**p*a**2*b*c*p**2*x - 42*
x***m*(a + b*x**2)**p*a**2*b*c*p*x - 2*x***m*(a + b*x**2)**p*a**2*b*d*m**2*p
*x**3 - 4*x***m*(a + b*x**2)**p*a**2*b*d*m*p**2*x**3 - 12*x***m*(a + b*x**2)
**p*a**2*b*d*m*p*x**3 - 20*x***m*(a + b*x**2)**p*a**2*b*d*p**2*x**3 - 10*x*
*m*(a + b*x**2)**p*a**2*b*d*p*x**3 + x***m*(a + b*x**2)**p*a*b**3*m**3*x +
8*x***m*(a + b*x**2)**p*a*b**3*m**2*p*x + 15*x***m*(a + b*x**2)**p*a*b**3*m*
*2*x + 20*x***m*(a + b*x**2)**p*a*b**3*m*p**2*x + 84*x***m*(a + b*x**2)**p*a
*b**3*m*p*x + 71*x***m*(a + b*x**2)**p*a*b**3*m*x + 16*x***m*(a + b*x**2)**p
*a*b**3*p**3*x + 108*x***m*(a + b*x**2)**p*a*b**3*p**2*x + 212*x***m*(a + b*
x**2)**p*a*b**3*p*x + 105*x***m*(a + b*x**2)**p*a*b**3*x + 2*x***m*(a + b*x*
*2)**p*a*b**2*c*m**2*p*x**3 + 8*x***m*(a + b*x**2)**p*a*b**2*c*m*p**2*x**3
+ 16*x***m*(a + b*x**2)**p*a*b**2*c*m*p*x**3 + 8*x***m*(a + b*x**2)**p*a*b**
2*c*p**3*x**3 + 32*x***m*(a + b*x**2)**p*a*b**2*c*p**2*x**3 + 14*x***m*(a +
b*x**2)**p*a*b**2*c*p*x**3 + 2*x***m*(a + b*x**2)**p*a*b**2*d*m**2*p*x**5 +
 8*x***m*(a + b*x**2)**p*a*b**2*d*m*p**2*x**5 + 8*x***m*(a + b*x**2)**p*a*b*
*2*d*m*p*x**5 + 8*x***m*(a + b*x**2)**p*a*b**2*d*p**3*x**5 + 16*x***m*(a + b
*x**2)**p*a*b**2*d*p**2*x**5 + 6*x***m*(a + b*x**2)**p*a*b**2*d*p*x**5 +...
```

# 3.266 $\int \frac{x^2(A+Bx^2+Cx^4+Dx^6+Fx^8)}{(a+bx^2)^{9/2}}\,dx$



## Optimal result

Integrand size = 37, antiderivative size = 285

$$\begin{aligned}\int \frac{x^2(A+Bx^2+Cx^4+Dx^6+Fx^8)}{(a+bx^2)^{9/2}}\,dx = &-\frac{(Ab^4-a(b^3B-ab^2C+a^2bD-a^3F))\,x}{7b^5\,(a+bx^2)^{7/2}}\\&+\frac{(Ab^4-a(8b^3B-15ab^2C+22a^2bD-29a^3F))\,x}{35ab^5\,(a+bx^2)^{5/2}}\\&+\frac{(4Ab^4+a(3b^3B-45ab^2C+122a^2bD-234a^3F))\,x}{105a^2b^5\,(a+bx^2)^{3/2}}\\&+\frac{(8Ab^4+a(6b^3B+15ab^2C-176a^2bD+582a^3F))\,x}{105a^3b^5\sqrt{a+bx^2}}\\&+\frac{Fx\sqrt{a+bx^2}}{2b^5}+\frac{(2bD-9aF)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{11/2}}\end{aligned}$$

output

```
-1/7*(A*b^4-a*(B*b^3-C*a*b^2+D*a^2*b-F*a^3))*x/b^5/(b*x^2+a)^(7/2)+1/35*(A
*b^4-a*(8*B*b^3-15*C*a*b^2+22*D*a^2*b-29*F*a^3))*x/a/b^5/(b*x^2+a)^(5/2)+1
/105*(4*A*b^4+a*(3*B*b^3-45*C*a*b^2+122*D*a^2*b-234*F*a^3))*x/a^2/b^5/(b*x
^2+a)^(3/2)+1/105*(8*A*b^4+a*(6*B*b^3+15*C*a*b^2-176*D*a^2*b+582*F*a^3))*x
/a^3/b^5/(b*x^2+a)^(1/2)+1/2*F*x*(b*x^2+a)^(1/2)/b^5+1/2*(2*D*b-9*F*a)*arc
tanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(11/2)
```

## Mathematica [A] (verified)

Time = 1.64 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.72

$$\int \frac{x^2(A+Bx^2+Cx^4+Dx^6+Fx^8)}{(a+bx^2)^{9/2}}\,dx = \frac{x(945a^7F+16Ab^7x^6+4ab^6x^4(14A+3Bx^2)-210a^6b(D-15}{}$$

$$+\frac{(2bD-9aF)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{-\sqrt{a}+\sqrt{a+bx^2}}\right)}{b^{11/2}}$$

input
```
Integrate[(x^2*(A + B*x^2 + C*x^4 + D*x^6 + F*x^8))/(a + b*x^2)^(9/2),x]
```

output
```
(x*(945*a^7*F + 16*A*b^7*x^6 + 4*a*b^6*x^4*(14*A + 3*B*x^2) - 210*a^6*b*(D
 - 15*F*x^2) + a^3*b^4*x^6*(-352*D + 105*F*x^2) + 14*a^5*b^2*x^2*(-50*D +
261*F*x^2) + 4*a^4*b^3*x^4*(-203*D + 396*F*x^2) + 2*a^2*b^5*x^2*(35*A + 21
*B*x^2 + 15*C*x^4)))/(210*a^3*b^5*(a + b*x^2)^(7/2)) + ((2*b*D - 9*a*F)*Ar
cTanh[(Sqrt[b]*x)/(-Sqrt[a] + Sqrt[a + b*x^2])])/b^(11/2)
```

## Rubi [A] (verified)

Time = 1.21 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.405$, Rules used = {2335, 9, 25, 2335, 9, 25, 1586, 9, 27, 360, 25, 27, 299, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\left(A+Bx^2+Cx^4+Dx^6+Fx^8\right)}{\left(a+bx^2\right)^{9/2}}dx$$

$$\Big\downarrow \text{ 2335}$$

$$\frac{x^3\left(\frac{A}{a}-\frac{a^3(-F)+a^2bD-ab^2C+b^3B}{b^4}\right)}{7\left(a+bx^2\right)^{7/2}}-\frac{\int -\frac{x\left(7aFx^7+7a\left(D-\frac{aF}{b}\right)x^5+\frac{7a\left(Fa^2-bDa+b^2C\right)x^3}{b^2}+\left(4Ab+\frac{3a\left(-Fa^3+bDa^2-b^2Ca+b^3B\right)}{b^3}\right)x\right)}{\left(bx^2+a\right)^{7/2}}dx}{7ab}$$

$$\Big\downarrow 9$$

$$\frac{x^3\left(\frac{A}{a}-\frac{a^3(-F)+a^2bD-ab^2C+b^3B}{b^4}\right)}{7\left(a+bx^2\right)^{7/2}}-\frac{\int -\frac{x^2\left(7aFx^6+7a\left(D-\frac{aF}{b}\right)x^4+7a\left(C-\frac{a(bD-aF)}{b^2}\right)x^2+4Ab+\frac{3a\left(-Fa^3+bDa^2-b^2Ca+b^3B\right)}{b^3}\right)}{\left(bx^2+a\right)^{7/2}}dx}{7ab}$$

$$\Big\downarrow 25$$

$$\frac{\int \frac{x^2\left(7aFx^6+7a\left(D-\frac{aF}{b}\right)x^4+7a\left(C-\frac{a(bD-aF)}{b^2}\right)x^2+4Ab+\frac{3a\left(-Fa^3+bDa^2-b^2Ca+b^3B\right)}{b^3}\right)}{\left(bx^2+a\right)^{7/2}}dx}{7ab}+\frac{x^3\left(\frac{A}{a}-\frac{a^3(-F)+a^2bD-ab^2C+b^3B}{b^4}\right)}{7\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow 2335$$

$$\frac{\frac{x^3\left(\frac{a\left(-24a^3F+17a^2bD-10ab^2C+3b^3B\right)}{b^3}+4Ab\right)}{5a\left(a+bx^2\right)^{5/2}}-\frac{\int -\frac{x\left(35a^2Fx^5+35a^2\left(D-\frac{2aF}{b}\right)x^3+\left(8Ab^2+3a\left(\frac{19Fa^3}{b^2}-\frac{12Da^2}{b}+5Ca+2bB\right)\right)x\right)}{\left(bx^2+a\right)^{5/2}}dx}{5ab}}{7ab}+\frac{x^3\left(\frac{A}{a}-\frac{a^3(-F)+a^2bD-ab^2C+b^3B}{b^4}\right)}{7\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow 9$$

$$\frac{\frac{x^3\left(\frac{a\left(-24a^3F+17a^2bD-10ab^2C+3b^3B\right)}{b^3}+4Ab\right)}{5a\left(a+bx^2\right)^{5/2}}-\frac{\int -\frac{x^2\left(35a^2Fx^4+35a^2\left(D-\frac{2aF}{b}\right)x^2+8Ab^2+3a\left(\frac{19Fa^3}{b^2}-\frac{12Da^2}{b}+5Ca+2bB\right)\right)}{\left(bx^2+a\right)^{5/2}}dx}{5ab}}{7ab}+\frac{x^3\left(\frac{A}{a}-\frac{a^3(-F)+a^2bD-ab^2C+b^3B}{b^4}\right)}{7\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow 25$$

$$\frac{\frac{\int \frac{x^2\left(35a^2Fx^4+35a^2\left(D-\frac{2aF}{b}\right)x^2+8Ab^2+3a\left(\frac{19Fa^3}{b^2}-\frac{12Da^2}{b}+5Ca+2bB\right)\right)}{\left(bx^2+a\right)^{5/2}}dx}{5ab}+\frac{x^3\left(\frac{a\left(-24a^3F+17a^2bD-10ab^2C+3b^3B\right)}{b^3}+4Ab\right)}{5a(a+bx^2)^{5/2}}}{7ab}+\frac{x^3\left(\frac{A}{a}-\frac{a^3(-F)+a^2bD-ab^2C+b^3B}{b^4}\right)}{7\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \text{ 1586}$$

$$\frac{\frac{\frac{x^3\left(a\left(\frac{162a^3F}{b^2}-\frac{71a^2D}{b}+15aC+6bB\right)+8Ab^2\right)}{3a\left(a+bx^2\right)^{3/2}}-\frac{\int -\frac{105x\left(\frac{Fx^3a^3}{b}+\frac{(bD-3aF)xa^3}{b^2}\right)}{\left(bx^2+a\right)^{3/2}}dx}{3a}}{5ab}+\frac{x^3\left(\frac{a\left(-24a^3F+17a^2bD-10ab^2C+3b^3B\right)}{b^3}+4Ab\right)}{5a(a+bx^2)^{5/2}}}{7ab}+\frac{x^3\left(\frac{A}{a}-\frac{a^3(-F)+a^2bD-ab^2C+b^3B}{b^4}\right)}{7\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \text{ 9}$$

$$\frac{\frac{\frac{x^3\left(a\left(\frac{162a^3F}{b^2}-\frac{71a^2D}{b}+15aC+6bB\right)+8Ab^2\right)}{3a\left(a+bx^2\right)^{3/2}}-\frac{\int -\frac{105a^3x^2\left(bFx^2+bD-3aF\right)}{b^2\left(bx^2+a\right)^{3/2}}dx}{3a}}{5ab}+\frac{x^3\left(\frac{a\left(-24a^3F+17a^2bD-10ab^2C+3b^3B\right)}{b^3}+4Ab\right)}{5a(a+bx^2)^{5/2}}}{7ab}+\frac{x^3\left(\frac{A}{a}-\frac{a^3(-F)+a^2bD-ab^2C+b^3B}{b^4}\right)}{7\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{\frac{35a^2\int \frac{x^2\left(bFx^2+bD-3aF\right)}{\left(bx^2+a\right)^{3/2}}dx}{b^2}+\frac{x^3\left(a\left(\frac{162a^3F}{b^2}-\frac{71a^2D}{b}+15aC+6bB\right)+8Ab^2\right)}{3a\left(a+bx^2\right)^{3/2}}}{5ab}+\frac{x^3\left(\frac{a\left(-24a^3F+17a^2bD-10ab^2C+3b^3B\right)}{b^3}+4Ab\right)}{5a(a+bx^2)^{5/2}}}{7ab}+\frac{x^3\left(\frac{A}{a}-\frac{a^3(-F)+a^2bD-ab^2C+b^3B}{b^4}\right)}{7\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \text{ 360}$$

$$\frac{\frac{\frac{35a^2\left(-\frac{\int -\frac{b\left(bFx^2+bD-4aF\right)}{\sqrt{bx^2+a}}dx}{b^2}-\frac{x(bD-4aF)}{b\sqrt{a+bx^2}}\right)}{b^2}+\frac{x^3\left(a\left(\frac{162a^3F}{b^2}-\frac{71a^2D}{b}+15aC+6bB\right)+8Ab^2\right)}{3a\left(a+bx^2\right)^{3/2}}}{5ab}+\frac{x^3\left(\frac{a\left(-24a^3F+17a^2bD-10ab^2C+3b^3B\right)}{b^3}+4Ab\right)}{5a(a+bx^2)^{5/2}}}{7ab}+\frac{x^3\left(\frac{A}{a}-\frac{a^3(-F)+a^2bD-ab^2C+b^3B}{b^4}\right)}{7\left(a+bx^2\right)^{7/2}}$$

$\big\downarrow$ 25

$$\frac{\frac{\frac{35a^2\left(\frac{\int \frac{b\left(bFx^2+bD-4aF\right)}{\sqrt{bx^2+a}}dx}{b^2}-\frac{x(bD-4aF)}{b\sqrt{a+bx^2}}\right)}{b^2}+\frac{x^3\left(a\left(\frac{162a^3F}{b^2}-\frac{71a^2D}{b}+15aC+6bB\right)+8Ab^2\right)}{3a\left(a+bx^2\right)^{3/2}}}{5ab}+\frac{x^3\left(\frac{a\left(-24a^3F+17a^2bD-10ab^2C+3b^3B\right)}{b^3}+4Ab\right)}{5a(a+bx^2)^{5/2}}}{7ab}+\frac{x^3\left(\frac{A}{a}-\frac{a^3(-F)+a^2bD-ab^2C+b^3B}{b^4}\right)}{7\left(a+bx^2\right)^{7/2}}$$

$\big\downarrow$ 27

$$\frac{\frac{\frac{35a^2\left(\frac{\int \frac{bFx^2+bD-4aF}{\sqrt{bx^2+a}}dx}{b}-\frac{x(bD-4aF)}{b\sqrt{a+bx^2}}\right)}{b^2}+\frac{x^3\left(a\left(\frac{162a^3F}{b^2}-\frac{71a^2D}{b}+15aC+6bB\right)+8Ab^2\right)}{3a\left(a+bx^2\right)^{3/2}}}{5ab}+\frac{x^3\left(\frac{a\left(-24a^3F+17a^2bD-10ab^2C+3b^3B\right)}{b^3}+4Ab\right)}{5a(a+bx^2)^{5/2}}}{7ab}+\frac{x^3\left(\frac{A}{a}-\frac{a^3(-F)+a^2bD-ab^2C+b^3B}{b^4}\right)}{7\left(a+bx^2\right)^{7/2}}$$

$\big\downarrow$ 299

$$\frac{\frac{\frac{35a^2\left(\frac{\frac{1}{2}(2bD-9aF)\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}Fx\sqrt{a+bx^2}}{b}-\frac{x(bD-4aF)}{b\sqrt{a+bx^2}}\right)}{b^2}+\frac{x^3\left(a\left(\frac{162a^3F}{b^2}-\frac{71a^2D}{b}+15aC+6bB\right)+8Ab^2\right)}{3a\left(a+bx^2\right)^{3/2}}}{5ab}+\frac{x^3\left(\frac{a\left(-24a^3F+17a^2bD-10ab^2C+3\right.}{b^3}\right.}{5a(a+bx^2)^{5/2}}}{7ab}+\frac{x^3\left(\frac{A}{a}-\frac{a^3(-F)+a^2bD-ab^2C+b^3B}{b^4}\right)}{7\left(a+bx^2\right)^{7/2}}$$

$\big\downarrow$ 224

$$\frac{\frac{\frac{35a^2\left(\frac{\frac{\frac{1}{2}(2bD-9aF)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}Fx\sqrt{a+bx^2}}{b}-\frac{x(bD-4aF)}{b\sqrt{a+bx^2}}}{b^2}\right)}{5ab}+\frac{x^3\left(a\left(\frac{162a^3F}{b^2}-\frac{71a^2D}{b}+15aC+6bB\right)+8Ab^2\right)}{3a\left(a+bx^2\right)^{3/2}}}{5ab}+\frac{x^3\left(\frac{a\left(-24a^3F+17a^2bD-1\right.}{b^3}\right.}{5a(a+bx}}{7ab}$$

$$\frac{x^3\left(\frac{A}{a}-\frac{a^3(-F)+a^2bD-ab^2C+b^3B}{b^4}\right)}{7\left(a+bx^2\right)^{7/2}}$$

$\downarrow$ 219

$$\frac{\frac{\frac{35a^2\left(\frac{\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bD-9aF)}{2\sqrt{b}}+\frac{1}{2}Fx\sqrt{a+bx^2}}{b}-\frac{x(bD-4aF)}{b\sqrt{a+bx^2}}\right)}{b^2}+\frac{x^3\left(a\left(\frac{162a^3F}{b^2}-\frac{71a^2D}{b}+15aC+6bB\right)+8Ab^2\right)}{3a\left(a+bx^2\right)^{3/2}}}{5ab}+\frac{x^3\left(\frac{a\left(-24a^3F+17a^2bD-10\right.}{b^3}\right.}{5a(a+bx^2}}{7ab}$$

$$\frac{x^3\left(\frac{A}{a}-\frac{a^3(-F)+a^2bD-ab^2C+b^3B}{b^4}\right)}{7\left(a+bx^2\right)^{7/2}}$$

input

```
Int[(x^2*(A + B*x^2 + C*x^4 + D*x^6 + F*x^8))/(a + b*x^2)^(9/2),x]
```

output

```
((A/a - (b^3*B - a*b^2*C + a^2*b*D - a^3*F)/b^4)*x^3)/(7*(a + b*x^2)^(7/2)
) + ((((4*A*b + (a*(3*b^3*B - 10*a*b^2*C + 17*a^2*b*D - 24*a^3*F))/b^3)*x^3
)/(5*a*(a + b*x^2)^(5/2)) + (((8*A*b^2 + a*(6*b*B + 15*a*C - (71*a^2*D)/b
+ (162*a^3*F)/b^2))*x^3)/(3*a*(a + b*x^2)^(3/2)) + (35*a^2*(-(((b*D - 4*a*
F)*x)/(b*Sqrt[a + b*x^2])) + ((F*x*Sqrt[a + b*x^2])/2 + ((2*b*D - 9*a*F)*A
rcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b]))/b))/b^2)/(5*a*b))/(7*a*b
)
```

### Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 1586
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, d + e*x^2, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^
p, d + e*x^2, x], x, 0]}, Simp[(-R)*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(2*d
*f*(q + 1))), x] + Simp[f/(2*d*(q + 1))   Int[(f*x)^(m - 1)*(d + e*x^2)^(q
+ 1)*ExpandToSum[2*d*(q + 1)*x*Qx + R*(m + 2*q + 3)*x, x], x], x]] /; FreeQ
[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[q, -1]
&& GtQ[m, 0]
```

rule 2335
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Simp[c/(2*a*b*(p + 1))   Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSu
m[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
 b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

## Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.68

| method | result |
|---|---|
| pseudoelliptic | $\frac{(b x^2+a)^{\frac{7}{2}}\left(Db-\frac{9Fa}{2}\right)a^3 \operatorname{arctanh}\left(\frac{\sqrt{b x^2+a}}{x\sqrt{b}}\right)+\frac{8\left(\frac{945\sqrt{b}\,F a^7}{16}+b^{\frac{3}{2}}\left(\frac{105\left(15F x^2-D\right)a^6}{8}-\frac{175\left(-\frac{261F x^2}{50}+D\right)x^2 b a^5}{4}-\frac{203\left(-3\right.}{}\right.\right.}{}}{(b x^2+a)^{\frac{7}{2}} b^{\frac{11}{2}} a}$ |
| default | $A\left(-\frac{x}{6b(b x^2+a)^{\frac{7}{2}}}+\frac{a\left(\frac{x}{7a\left(b x^2+a\right)^{\frac{7}{2}}}+\frac{\frac{6x}{35a\left(b x^2+a\right)^{\frac{5}{2}}}+\frac{6\left(\frac{4x}{15a\left(b x^2+a\right)^{\frac{3}{2}}}+\frac{8x}{15a^2\sqrt{b x^2+a}}\right)}{7a}}{a}\right)}{6b}\right)+B\left(-\frac{x^3}{4b(b x^2+a}\right.$ |

input
```
int(x^2*(F*x^8+D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x,method=_RETURNVERBOS
E)
```

output
```
8/105/(b*x^2+a)^(7/2)*(105/8*(b*x^2+a)^(7/2)*(D*b-9/2*F*a)*a^3*arctanh((b*
x^2+a)^(1/2)/x/b^(1/2))+(945/16*b^(1/2)*F*a^7+b^(3/2)*(105/8*(15*F*x^2-D)*
a^6-175/4*(-261/50*F*x^2+D)*x^2*b*a^5-203/4*(-396/203*F*x^2+D)*x^4*b^2*a^4
-22*(-105/352*F*x^2+D)*x^6*b^3*a^3+35/8*x^2*b^4*(3/7*C*x^4+3/5*x^2*B+A)*a^
2+7/2*(3/14*x^2*B+A)*x^4*b^5*a+A*b^6*x^6))*x)/b^(11/2)/a^3
```

## Fricas [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 705, normalized size of antiderivative = 2.47

$$\int \frac{x^2(A+Bx^2+Cx^4+Dx^6+Fx^8)}{(a+bx^2)^{9/2}}\,dx = \left[-\frac{105\,(9\,Fa^8-2\,Da^7b+(9\,Fa^4b^4-2\,Da^3b^5)x^8+4\,(9\,Fa^5b^3\right.$$

input
```
integrate(x^2*(F*x^8+D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="fr
icas")
```

output
```
[-1/420*(105*(9*F*a^8 - 2*D*a^7*b + (9*F*a^4*b^4 - 2*D*a^3*b^5)*x^8 + 4*(9
*F*a^5*b^3 - 2*D*a^4*b^4)*x^6 + 6*(9*F*a^6*b^2 - 2*D*a^5*b^3)*x^4 + 4*(9*F
*a^7*b - 2*D*a^6*b^2)*x^2)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b
)*x - a) - 2*(105*F*a^3*b^5*x^9 + 2*(792*F*a^4*b^4 - 176*D*a^3*b^5 + 15*C*
a^2*b^6 + 6*B*a*b^7 + 8*A*b^8)*x^7 + 14*(261*F*a^5*b^3 - 58*D*a^4*b^4 + 3*
B*a^2*b^6 + 4*A*a*b^7)*x^5 + 70*(45*F*a^6*b^2 - 10*D*a^5*b^3 + A*a^2*b^6)*
x^3 + 105*(9*F*a^7*b - 2*D*a^6*b^2)*x)*sqrt(b*x^2 + a))/(a^3*b^10*x^8 + 4*
a^4*b^9*x^6 + 6*a^5*b^8*x^4 + 4*a^6*b^7*x^2 + a^7*b^6), 1/210*(105*(9*F*a^
8 - 2*D*a^7*b + (9*F*a^4*b^4 - 2*D*a^3*b^5)*x^8 + 4*(9*F*a^5*b^3 - 2*D*a^4
*b^4)*x^6 + 6*(9*F*a^6*b^2 - 2*D*a^5*b^3)*x^4 + 4*(9*F*a^7*b - 2*D*a^6*b^2
)*x^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (105*F*a^3*b^5*x^9 +
2*(792*F*a^4*b^4 - 176*D*a^3*b^5 + 15*C*a^2*b^6 + 6*B*a*b^7 + 8*A*b^8)*x^7
 + 14*(261*F*a^5*b^3 - 58*D*a^4*b^4 + 3*B*a^2*b^6 + 4*A*a*b^7)*x^5 + 70*(4
5*F*a^6*b^2 - 10*D*a^5*b^3 + A*a^2*b^6)*x^3 + 105*(9*F*a^7*b - 2*D*a^6*b^2
)*x)*sqrt(b*x^2 + a))/(a^3*b^10*x^8 + 4*a^4*b^9*x^6 + 6*a^5*b^8*x^4 + 4*a^
6*b^7*x^2 + a^7*b^6)]
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6987 vs. $2(280) = 560$.

Time = 92.60 (sec) , antiderivative size = 6987, normalized size of antiderivative = 24.52

$$\int \frac{x^2(A+Bx^2+Cx^4+Dx^6+Fx^8)}{(a+bx^2)^{9/2}}\,dx = \text{Too large to display}$$

input

```
integrate(x**2*(F*x**8+D*x**6+C*x**4+B*x**2+A)/(b*x**2+a)**(9/2),x)
```

output

```
A*(35*a**5*x**3/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a**(17/2)*b*x**2*s
qrt(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 420*a**(1
3/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*sqrt(1 + b*x**
2/a)) + 63*a**4*b*x**5/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a**(17/2)*b
*x**2*sqrt(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 42
0*a**(13/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*sqrt(1
+ b*x**2/a)) + 36*a**3*b**2*x**7/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a
**(17/2)*b*x**2*sqrt(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x*
*2/a) + 420*a**(13/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x*
*8*sqrt(1 + b*x**2/a)) + 8*a**2*b**3*x**9/(105*a**(19/2)*sqrt(1 + b*x**2/a
) + 420*a**(17/2)*b*x**2*sqrt(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt
(1 + b*x**2/a) + 420*a**(13/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2
)*b**4*x**8*sqrt(1 + b*x**2/a))) + B*(7*a*x**5/(35*a**(11/2)*sqrt(1 + b*x*
*2/a) + 105*a**(9/2)*b*x**2*sqrt(1 + b*x**2/a) + 105*a**(7/2)*b**2*x**4*sq
rt(1 + b*x**2/a) + 35*a**(5/2)*b**3*x**6*sqrt(1 + b*x**2/a)) + 2*b*x**7/(3
5*a**(11/2)*sqrt(1 + b*x**2/a) + 105*a**(9/2)*b*x**2*sqrt(1 + b*x**2/a) +
105*a**(7/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 35*a**(5/2)*b**3*x**6*sqrt(1 +
 b*x**2/a))) + C*x**7/(7*a**(9/2)*sqrt(1 + b*x**2/a) + 21*a**(7/2)*b*x**2*
sqrt(1 + b*x**2/a) + 21*a**(5/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 7*a**(3/2)
*b**3*x**6*sqrt(1 + b*x**2/a)) + D*(105*a**(205/2)*b**45*sqrt(1 + b*x**...
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 826 vs. $2(256) = 512$.

Time = 0.06 (sec) , antiderivative size = 826, normalized size of antiderivative = 2.90

$$\int \frac{x^2(A+Bx^2+Cx^4+Dx^6+Fx^8)}{(a+bx^2)^{9/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(F*x^8+D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="ma
xima")
```

output
```
1/2*F*x^9/((b*x^2 + a)^(7/2)*b) - 1/35*(35*x^6/((b*x^2 + a)^(7/2)*b) + 70*
a*x^4/((b*x^2 + a)^(7/2)*b^2) + 56*a^2*x^2/((b*x^2 + a)^(7/2)*b^3) + 16*a^
3/((b*x^2 + a)^(7/2)*b^4))*D*x + 9/70*(35*x^6/((b*x^2 + a)^(7/2)*b) + 70*a
*x^4/((b*x^2 + a)^(7/2)*b^2) + 56*a^2*x^2/((b*x^2 + a)^(7/2)*b^3) + 16*a^3
/((b*x^2 + a)^(7/2)*b^4))*F*a*x/b + 3/10*F*a*x*(15*x^4/((b*x^2 + a)^(5/2)*
b) + 20*a*x^2/((b*x^2 + a)^(5/2)*b^2) + 8*a^2/((b*x^2 + a)^(5/2)*b^3))/b^2
 - 1/15*D*x*(15*x^4/((b*x^2 + a)^(5/2)*b) + 20*a*x^2/((b*x^2 + a)^(5/2)*b^
2) + 8*a^2/((b*x^2 + a)^(5/2)*b^3))/b - 1/2*C*x^5/((b*x^2 + a)^(7/2)*b) +
3/2*F*a*x*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2))/b^3
- 1/3*D*x*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2))/b^2
+ 9/2*F*a^2*x^3/((b*x^2 + a)^(5/2)*b^4) - D*a*x^3/((b*x^2 + a)^(5/2)*b^3)
- 5/8*C*a*x^3/((b*x^2 + a)^(7/2)*b^2) - 1/4*B*x^3/((b*x^2 + a)^(7/2)*b) -
417/70*F*a*x/(sqrt(b*x^2 + a)*b^5) - 51/70*F*a^2*x/((b*x^2 + a)^(3/2)*b^5)
 + 261/70*F*a^3*x/((b*x^2 + a)^(5/2)*b^5) + 139/105*D*x/(sqrt(b*x^2 + a)*b
^4) + 17/105*D*a*x/((b*x^2 + a)^(3/2)*b^4) - 29/35*D*a^2*x/((b*x^2 + a)^(5
/2)*b^4) + 1/14*C*x/((b*x^2 + a)^(3/2)*b^3) + 1/7*C*x/(sqrt(b*x^2 + a)*a*b
^3) + 3/56*C*a*x/((b*x^2 + a)^(5/2)*b^3) - 15/56*C*a^2*x/((b*x^2 + a)^(7/2
)*b^3) + 3/140*B*x/((b*x^2 + a)^(5/2)*b^2) + 2/35*B*x/(sqrt(b*x^2 + a)*a^2
*b^2) + 1/35*B*x/((b*x^2 + a)^(3/2)*a*b^2) - 3/28*B*a*x/((b*x^2 + a)^(7/2)
*b^2) - 1/7*A*x/((b*x^2 + a)^(7/2)*b) + 8/105*A*x/(sqrt(b*x^2 + a)*a^3*...
```

## Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.79

$$\int \frac{x^2(A+Bx^2+Cx^4+Dx^6+Fx^8)}{(a+bx^2)^{9/2}}\,dx = \frac{\left(\left(\left(\left(\frac{105\,Fx^2}{b}+\frac{2\,(792\,Fa^4b^7-176\,Da^3b^8+15\,Ca^2b^9+6\,Bab^{10}+8\,Ab^{11})}{a^3b^9}\right)x^2+\right.\right.\right.\right.}{}$$
$$+\frac{(9\,Fa-2\,Db)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2\,b^{\frac{11}{2}}}$$

input
```
integrate(x^2*(F*x^8+D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="gi
ac")
```

output
```
1/210*((((105*F*x^2/b + 2*(792*F*a^4*b^7 - 176*D*a^3*b^8 + 15*C*a^2*b^9 +
6*B*a*b^10 + 8*A*b^11)/(a^3*b^9))*x^2 + 14*(261*F*a^5*b^6 - 58*D*a^4*b^7 +
 3*B*a^2*b^9 + 4*A*a*b^10)/(a^3*b^9))*x^2 + 70*(45*F*a^6*b^5 - 10*D*a^5*b^
6 + A*a^2*b^9)/(a^3*b^9))*x^2 + 105*(9*F*a^7*b^4 - 2*D*a^6*b^5)/(a^3*b^9))
*x/(b*x^2 + a)^(7/2) + 1/2*(9*F*a - 2*D*b)*log(abs(-sqrt(b)*x + sqrt(b*x^2
 + a)))/b^(11/2)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(A+Bx^2+Cx^4+Dx^6+Fx^8)}{(a+bx^2)^{9/2}}\,dx = \int \frac{x^2\,(A+B\,x^2+C\,x^4+F\,x^8+x^6\,D)}{(b\,x^2+a)^{9/2}}\,dx$$

input
```
int((x^2*(A + B*x^2 + C*x^4 + F*x^8 + x^6*D))/(a + b*x^2)^(9/2),x)
```

output
```
int((x^2*(A + B*x^2 + C*x^4 + F*x^8 + x^6*D))/(a + b*x^2)^(9/2), x)
```

## Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 861, normalized size of antiderivative = 3.02

$$\int \frac{x^2(A+Bx^2+Cx^4+Dx^6+Fx^8)}{(a+bx^2)^{9/2}}\,dx = \text{Too large to display}$$

input
```
int(x^2*(F*x^8+D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x)
```

output
```
(945*sqrt(a + b*x**2)*a**6*b*f*x - 210*sqrt(a + b*x**2)*a**5*b**2*d*x + 31
50*sqrt(a + b*x**2)*a**5*b**2*f*x**3 - 700*sqrt(a + b*x**2)*a**4*b**3*d*x*
*3 + 3654*sqrt(a + b*x**2)*a**4*b**3*f*x**5 - 812*sqrt(a + b*x**2)*a**3*b*
*4*d*x**5 + 1584*sqrt(a + b*x**2)*a**3*b**4*f*x**7 + 70*sqrt(a + b*x**2)*a
**2*b**6*x**3 - 352*sqrt(a + b*x**2)*a**2*b**5*d*x**7 + 105*sqrt(a + b*x**
2)*a**2*b**5*f*x**9 + 98*sqrt(a + b*x**2)*a*b**7*x**5 + 30*sqrt(a + b*x**2
)*a*b**6*c*x**7 + 28*sqrt(a + b*x**2)*b**8*x**7 - 945*sqrt(b)*log((sqrt(a
+ b*x**2) + sqrt(b)*x)/sqrt(a))*a**7*f + 210*sqrt(b)*log((sqrt(a + b*x**2)
 + sqrt(b)*x)/sqrt(a))*a**6*b*d - 3780*sqrt(b)*log((sqrt(a + b*x**2) + sqr
t(b)*x)/sqrt(a))*a**6*b*f*x**2 + 840*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(
b)*x)/sqrt(a))*a**5*b**2*d*x**2 - 5670*sqrt(b)*log((sqrt(a + b*x**2) + sqr
t(b)*x)/sqrt(a))*a**5*b**2*f*x**4 + 1260*sqrt(b)*log((sqrt(a + b*x**2) + s
qrt(b)*x)/sqrt(a))*a**4*b**3*d*x**4 - 3780*sqrt(b)*log((sqrt(a + b*x**2) +
 sqrt(b)*x)/sqrt(a))*a**4*b**3*f*x**6 + 840*sqrt(b)*log((sqrt(a + b*x**2)
+ sqrt(b)*x)/sqrt(a))*a**3*b**4*d*x**6 - 945*sqrt(b)*log((sqrt(a + b*x**2)
 + sqrt(b)*x)/sqrt(a))*a**3*b**4*f*x**8 + 210*sqrt(b)*log((sqrt(a + b*x**2
) + sqrt(b)*x)/sqrt(a))*a**2*b**5*d*x**8 - 639*sqrt(b)*a**7*f + 112*sqrt(b
)*a**6*b*d - 2556*sqrt(b)*a**6*b*f*x**2 + 30*sqrt(b)*a**5*b**2*c + 448*sqr
t(b)*a**5*b**2*d*x**2 - 3834*sqrt(b)*a**5*b**2*f*x**4 - 28*sqrt(b)*a**4*b*
*4 + 120*sqrt(b)*a**4*b**3*c*x**2 + 672*sqrt(b)*a**4*b**3*d*x**4 - 2556...
```

# 3.267 $\int \frac{A+Bx^2+Cx^4+Dx^6+Fx^8}{(a+bx^2)^{9/2}}\,dx$



## Optimal result

Integrand size = 34, antiderivative size = 251

$$\int \frac{A+Bx^2+Cx^4+Dx^6+Fx^8}{(a+bx^2)^{9/2}}\,dx = \frac{\left(\frac{A}{a}-\frac{b^3B-ab^2C+a^2bD-a^3F}{b^4}\right)x}{7(a+bx^2)^{7/2}} + \frac{\left(\frac{6A}{a}+\frac{b^3B-8ab^2C+15a^2bD-22a^3F}{b^4}\right)x}{35a(a+bx^2)^{5/2}} + \frac{(24Ab^4+a(4b^3B+3ab^2C-45a^2bD+122a^3F))x}{105a^3b^4(a+bx^2)^{3/2}} + \frac{(48Ab^4+a(8b^3B+6ab^2C+15a^2bD-176a^3F))x}{105a^4b^4\sqrt{a+bx^2}} + \frac{F\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{9/2}}$$

output
```
1/7*(A/a-(B*b^3-C*a*b^2+D*a^2*b-F*a^3)/b^4)*x/(b*x^2+a)^(7/2)+1/35*(6*A/a+
(B*b^3-8*C*a*b^2+15*D*a^2*b-22*F*a^3)/b^4)*x/a/(b*x^2+a)^(5/2)+1/105*(24*A
*b^4+a*(4*B*b^3+3*C*a*b^2-45*D*a^2*b+122*F*a^3))*x/a^3/b^4/(b*x^2+a)^(3/2)
+1/105*(48*A*b^4+a*(8*B*b^3+6*C*a*b^2+15*D*a^2*b-176*F*a^3))*x/a^4/b^4/(b*
x^2+a)^(1/2)+F*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(9/2)
```

## Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.70

$$\int \frac{A+Bx^2+Cx^4+Dx^6+Fx^8}{(a+bx^2)^{9/2}}\,dx = \frac{x(-105a^7F-350a^6bFx^2-406a^5b^2Fx^4+48Ab^7x^6-176a^4b^3Fx^6}{} - \frac{F\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{b^{9/2}}$$

input
```
Integrate[(A + B*x^2 + C*x^4 + D*x^6 + F*x^8)/(a + b*x^2)^(9/2),x]
```

output
```
(x*(-105*a^7*F - 350*a^6*b*F*x^2 - 406*a^5*b^2*F*x^4 + 48*A*b^7*x^6 - 176*
a^4*b^3*F*x^6 + 8*a*b^6*x^4*(21*A + B*x^2) + 2*a^2*b^5*x^2*(105*A + 14*B*x
^2 + 3*C*x^4) + a^3*b^4*(105*A + 35*B*x^2 + 21*C*x^4 + 15*D*x^6)))/(105*a^
4*b^4*(a + b*x^2)^(7/2)) - (F*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/b^(9/2)
```

## Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.07, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {2345, 25, 2345, 25, 1471, 25, 27, 298, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4+Dx^6+Fx^8}{(a+bx^2)^{9/2}}\,dx$$

$$\Big\downarrow \text{ 2345}$$

$$\frac{x\left(\frac{A}{a}-\frac{a^3(-F)+a^2bD-ab^2C+b^3B}{b^4}\right)}{7\left(a+bx^2\right)^{7/2}}-\frac{\int -\frac{\frac{7aFx^6}{b}+\frac{7a(bD-aF)x^4}{b^2}+\frac{7a\left(Fa^2-bDa+b^2C\right)x^2}{b^3}+6A+\frac{a\left(-Fa^3+bDa^2-b^2Ca+b^3B\right)}{b^4}}{(bx^2+a)^{7/2}}dx}{7a}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{\frac{7aFx^6}{b}+\frac{7a(bD-aF)x^4}{b^2}+\frac{7a\left(Fa^2-bDa+b^2C\right)x^2}{b^3}+6A+\frac{a\left(-Fa^3+bDa^2-b^2Ca+b^3B\right)}{b^4}}{(bx^2+a)^{7/2}}dx}{7a}+\frac{x\left(\frac{A}{a}-\frac{a^3(-F)+a^2bD-ab^2C+b^3B}{b^4}\right)}{7\left(a+bx^2\right)^{7/2}}$$

$\Big\downarrow$ 2345

$$\frac{\frac{x\left(\frac{-22a^3F+15a^2bD-8ab^2C+b^3B}{b^4}+\frac{6A}{a}\right)}{5(a+bx^2)^{5/2}}-\frac{\int -\frac{\frac{35a^2Fx^4}{b^2}+\frac{35a^2(bD-2aF)x^2}{b^3}+24A+\frac{a\left(17Fa^3-10bDa^2+3b^2Ca+4b^3B\right)}{b^4}}{\left(bx^2+a\right)^{5/2}}dx}{5a}}{7a}+\frac{x\left(\frac{A}{a}-\frac{a^3(-F)+a^2bD-ab^2C+b^3B}{b^4}\right)}{7\left(a+bx^2\right)^{7/2}}$$

$\Big\downarrow$ 25

$$\frac{\frac{\int \frac{\frac{35a^2Fx^4}{b^2}+\frac{35a^2(bD-2aF)x^2}{b^3}+24A+\frac{a\left(17Fa^3-10bDa^2+3b^2Ca+4b^3B\right)}{b^4}}{\left(bx^2+a\right)^{5/2}}dx}{5a}+\frac{x\left(\frac{-22a^3F+15a^2bD-8ab^2C+b^3B}{b^4}+\frac{6A}{a}\right)}{5(a+bx^2)^{5/2}}}{7a}+\frac{x\left(\frac{A}{a}-\frac{a^3(-F)+a^2bD-ab^2C+b^3B}{b^4}\right)}{7\left(a+bx^2\right)^{7/2}}$$

$\Big\downarrow$ 1471

$$\frac{\frac{\frac{x\left(\frac{122a^3F-45a^2bD+3ab^2C+4b^3B}{b^4}+\frac{24A}{a}\right)}{3\left(a+bx^2\right)^{3/2}}-\frac{\int -\frac{-71Fa^4+105bFx^2a^3+15bDa^3+6b^2Ca^2+8b^3Ba+48Ab^4}{b^4\left(bx^2+a\right)^{3/2}}dx}{3a}}{5a}+\frac{x\left(\frac{-22a^3F+15a^2bD-8ab^2C+b^3B}{b^4}+\frac{6A}{a}\right)}{5(a+bx^2)^{5/2}}}{7a}+\frac{x\left(\frac{A}{a}-\frac{a^3(-F)+a^2bD-ab^2C+b^3B}{b^4}\right)}{7\left(a+bx^2\right)^{7/2}}$$

$\Big\downarrow$ 25

$$\frac{\frac{\frac{\int \frac{48Ab^4+105a^3Fx^2b+a\left(-71Fa^3+15bDa^2+6b^2Ca+8b^3B\right)}{b^4\left(bx^2+a\right)^{3/2}}dx}{3a}+\frac{x\left(\frac{122a^3F-45a^2bD+3ab^2C+4b^3B}{b^4}+\frac{24A}{a}\right)}{3\left(a+bx^2\right)^{3/2}}}{5a}+\frac{x\left(\frac{-22a^3F+15a^2bD-8ab^2C+b^3B}{b^4}+\frac{6A}{a}\right)}{5(a+bx^2)^{5/2}}}{7a}+\frac{x\left(\frac{A}{a}-\frac{a^3(-F)+a^2bD-ab^2C+b^3B}{b^4}\right)}{7\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{\int \frac{48Ab^4+105a^3Fx^2b+a\left(-71Fa^3+15bDa^2+6b^2Ca+8b^3B\right)}{\left(bx^2+a\right)^{3/2}}dx}{3ab^4}+\frac{x\left(\frac{122a^3F-45a^2bD+3ab^2C+4b^3B}{b^4}+\frac{24A}{a}\right)}{3\left(a+bx^2\right)^{3/2}}}{5a}+\frac{x\left(\frac{-22a^3F+15a^2bD-8ab^2C+b^3B}{b^4}+\frac{6A}{a}\right)}{5(a+bx^2)^{5/2}}}{7a}+\frac{x\left(\frac{A}{a}-\frac{a^3(-F)+a^2bD-ab^2C+b^3B}{b^4}\right)}{7\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \text{298}$$

$$\frac{\frac{\frac{105a^3F\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{x\left(-176a^4F+15a^3bD+6a^2b^2C+8ab^3B+48Ab^4\right)}{a\sqrt{a+bx^2}}}{3ab^4}+\frac{x\left(\frac{122a^3F-45a^2bD+3ab^2C+4b^3B}{b^4}+\frac{24A}{a}\right)}{3\left(a+bx^2\right)^{3/2}}}{5a}+\frac{x\left(\frac{-22a^3F+15a^2bD-8ab^2C+b^3B}{b^4}+\frac{6A}{a}\right)}{5(a+bx^2)^{5/2}}}{7a}+\frac{x\left(\frac{A}{a}-\frac{a^3(-F)+a^2bD-ab^2C+b^3B}{b^4}\right)}{7\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{\frac{105a^3F\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{x\left(-176a^4F+15a^3bD+6a^2b^2C+8ab^3B+48Ab^4\right)}{a\sqrt{a+bx^2}}}{3ab^4}+\frac{x\left(\frac{122a^3F-45a^2bD+3ab^2C+4b^3B}{b^4}+\frac{24A}{a}\right)}{3\left(a+bx^2\right)^{3/2}}}{5a}+\frac{x\left(\frac{-22a^3F+15a^2bD-8ab^2C+b^3B}{b^4}+\frac{6A}{a}\right)}{5(a+bx^2)^{5/2}}}{7a}+\frac{x\left(\frac{A}{a}-\frac{a^3(-F)+a^2bD-ab^2C+b^3B}{b^4}\right)}{7\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{x\left(\frac{A}{a}-\frac{a^3(-F)+a^2bD-ab^2C+b^3B}{b^4}\right)}{7\left(a+bx^2\right)^{7/2}}+\frac{\frac{x\left(\frac{-22a^3F+15a^2bD-8ab^2C+b^3B}{b^4}+\frac{6A}{a}\right)}{5(a+bx^2)^{5/2}}+\frac{\frac{x\left(\frac{122a^3F-45a^2bD+3ab^2C+4b^3B}{b^4}+\frac{24A}{a}\right)}{3\left(a+bx^2\right)^{3/2}}+\frac{\frac{105a^3F\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}+\frac{x\left(-176a^4F+15a^3bD+6a^2b^2C+8ab^3B+48Ab^4\right)}{a\sqrt{a+bx^2}}}{3ab^4}}{5a}}{7a}$$

input

```
Int[(A + B*x^2 + C*x^4 + D*x^6 + F*x^8)/(a + b*x^2)^(9/2),x]
```

output
```
((A/a - (b^3*B - a*b^2*C + a^2*b*D - a^3*F)/b^4)*x)/(7*(a + b*x^2)^(7/2))
+ ((((6*A)/a + (b^3*B - 8*a*b^2*C + 15*a^2*b*D - 22*a^3*F)/b^4)*x)/(5*(a +
 b*x^2)^(5/2)) + ((((24*A)/a + (4*b^3*B + 3*a*b^2*C - 45*a^2*b*D + 122*a^3
*F)/b^4)*x)/(3*(a + b*x^2)^(3/2)) + (((48*A*b^4 + 8*a*b^3*B + 6*a^2*b^2*C
+ 15*a^3*b*D - 176*a^4*F)*x)/(a*Sqrt[a + b*x^2]) + (105*a^3*F*ArcTanh[(Sqr
t[b]*x)/Sqrt[a + b*x^2]])/Sqrt[b])/(3*a*b^4))/(5*a))/(7*a)
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

## Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.71

| method | result |
|---|---|
| pseudoelliptic | $\frac{F\,a^4 \operatorname{arctanh}\left(\frac{\sqrt{b\,x^2+a}}{x\sqrt{b}}\right) b^4 (b\,x^2+a)^{\frac{7}{2}} + \frac{16 b^{\frac{9}{2}}\left(A\,b^7 x^6 + \frac{7\left(\frac{x^2 B}{21}+A\right) x^4 a\,b^6}{2} + \frac{35\left(\frac{1}{35} C\,x^4 + \frac{2}{15} x^2 B + A\right) x^2 a^2 b^5}{8} + \frac{35\left(\frac{1}{7} D x^6 + \frac{1}{5} C\,x\right.}{}\right.}{35}}{(b\,x^2+a)^{\frac{7}{2}} b^{\frac{17}{2}} a^4}$ |
| default | $A\left(\frac{x}{7a(b\,x^2+a)^{\frac{7}{2}}} + \frac{\frac{6x}{35a\left(b\,x^2+a\right)^{\frac{5}{2}}} + \frac{6\left(\frac{4x}{15a\left(b\,x^2+a\right)^{\frac{3}{2}}} + \frac{8x}{15a^2\sqrt{b\,x^2+a}}\right)}{7a}}{a}\right) + C\left(-\frac{x^3}{4b(b\,x^2+a)^{\frac{7}{2}}} + \frac{3a\left(-\frac{x}{6b\left(b\,x^2+\right.}\right.}{}\right.$ |

input
```
int((F*x^8+D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
(F*a^4*arctanh((b*x^2+a)^(1/2)/x/b^(1/2))*b^4*(b*x^2+a)^(7/2)+16/35*b^(9/2
)*(A*b^7*x^6+7/2*(1/21*x^2*B+A)*x^4*a*b^6+35/8*(1/35*C*x^4+2/15*x^2*B+A)*x
^2*a^2*b^5+35/16*(1/7*D*x^6+1/5*C*x^4+1/3*x^2*B+A)*a^3*b^4-11/3*F*a^4*b^3*
x^6-203/24*F*a^5*b^2*x^4-175/24*F*a^6*b*x^2-35/16*F*a^7)*x)/(b*x^2+a)^(7/2
)/b^(17/2)/a^4
```

## Fricas [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 567, normalized size of antiderivative = 2.26

$$\int \frac{A+Bx^2+Cx^4+Dx^6+Fx^8}{(a+bx^2)^{9/2}}\,dx = \left[\frac{105\,(Fa^4b^4x^8+4\,Fa^5b^3x^6+6\,Fa^6b^2x^4+4\,Fa^7bx^2+Fa^8)\sqrt{b}\log\right.$$

$$-\frac{105\,(Fa^4b^4x^8+4\,Fa^5b^3x^6+6\,Fa^6b^2x^4+4\,Fa^7bx^2+Fa^8)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)+((176\,Fa^4b^4-15\,Da}{}$$

input
```
integrate((F*x^8+D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="fricas
")
```

output
```
[1/210*(105*(F*a^4*b^4*x^8 + 4*F*a^5*b^3*x^6 + 6*F*a^6*b^2*x^4 + 4*F*a^7*b
*x^2 + F*a^8)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*
((176*F*a^4*b^4 - 15*D*a^3*b^5 - 6*C*a^2*b^6 - 8*B*a*b^7 - 48*A*b^8)*x^7 +
 7*(58*F*a^5*b^3 - 3*C*a^3*b^5 - 4*B*a^2*b^6 - 24*A*a*b^7)*x^5 + 35*(10*F*
a^6*b^2 - B*a^3*b^5 - 6*A*a^2*b^6)*x^3 + 105*(F*a^7*b - A*a^3*b^5)*x)*sqrt
(b*x^2 + a))/(a^4*b^9*x^8 + 4*a^5*b^8*x^6 + 6*a^6*b^7*x^4 + 4*a^7*b^6*x^2
+ a^8*b^5), -1/105*(105*(F*a^4*b^4*x^8 + 4*F*a^5*b^3*x^6 + 6*F*a^6*b^2*x^4
 + 4*F*a^7*b*x^2 + F*a^8)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + ((
176*F*a^4*b^4 - 15*D*a^3*b^5 - 6*C*a^2*b^6 - 8*B*a*b^7 - 48*A*b^8)*x^7 + 7
*(58*F*a^5*b^3 - 3*C*a^3*b^5 - 4*B*a^2*b^6 - 24*A*a*b^7)*x^5 + 35*(10*F*a^
6*b^2 - B*a^3*b^5 - 6*A*a^2*b^6)*x^3 + 105*(F*a^7*b - A*a^3*b^5)*x)*sqrt(b
*x^2 + a))/(a^4*b^9*x^8 + 4*a^5*b^8*x^6 + 6*a^6*b^7*x^4 + 4*a^7*b^6*x^2 +
a^8*b^5)]
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5071 vs. 2(246) = 492.

Time = 73.19 (sec) , antiderivative size = 5071, normalized size of antiderivative = 20.20

$$\int \frac{A+Bx^2+Cx^4+Dx^6+Fx^8}{(a+bx^2)^{9/2}}\,dx = \text{Too large to display}$$

input
```
integrate((F*x**8+D*x**6+C*x**4+B*x**2+A)/(b*x**2+a)**(9/2),x)
```

output
```
A*(35*a**14*x/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt
(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2
)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a
) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*
sqrt(1 + b*x**2/a)) + 175*a**13*b*x**3/(35*a**(37/2)*sqrt(1 + b*x**2/a) +
210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 +
 b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b*
*4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) +
 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a)) + 371*a**12*b**2*x**5/(35*a**
(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*
a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 +
b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**
5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a)) +
 429*a**11*b**3*x**7/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x*
*2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a
**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b
*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6
*x**12*sqrt(1 + b*x**2/a)) + 286*a**10*b**4*x**9/(35*a**(37/2)*sqrt(1 + b*
x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**
4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525...
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 597 vs. $2(231) = 462$.

Time = 0.06 (sec) , antiderivative size = 597, normalized size of antiderivative = 2.38

$$\int \frac{A + Bx^2 + Cx^4 + Dx^6 + Fx^8}{(a + bx^2)^{9/2}} \, dx = \text{Too large to display}$$

input
```
integrate((F*x^8+D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="maxima
")
```

output
```
-1/35*(35*x^6/((b*x^2 + a)^(7/2)*b) + 70*a*x^4/((b*x^2 + a)^(7/2)*b^2) + 5
6*a^2*x^2/((b*x^2 + a)^(7/2)*b^3) + 16*a^3/((b*x^2 + a)^(7/2)*b^4))*F*x -
1/15*F*x*(15*x^4/((b*x^2 + a)^(5/2)*b) + 20*a*x^2/((b*x^2 + a)^(5/2)*b^2)
+ 8*a^2/((b*x^2 + a)^(5/2)*b^3))/b - 1/2*D*x^5/((b*x^2 + a)^(7/2)*b) - 1/3
*F*x*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2))/b^2 - F*a
*x^3/((b*x^2 + a)^(5/2)*b^3) - 5/8*D*a*x^3/((b*x^2 + a)^(7/2)*b^2) - 1/4*C
*x^3/((b*x^2 + a)^(7/2)*b) + 16/35*A*x/(sqrt(b*x^2 + a)*a^4) + 8/35*A*x/((
b*x^2 + a)^(3/2)*a^3) + 6/35*A*x/((b*x^2 + a)^(5/2)*a^2) + 1/7*A*x/((b*x^2
 + a)^(7/2)*a) + 139/105*F*x/(sqrt(b*x^2 + a)*b^4) + 17/105*F*a*x/((b*x^2
+ a)^(3/2)*b^4) - 29/35*F*a^2*x/((b*x^2 + a)^(5/2)*b^4) + 1/14*D*x/((b*x^2
 + a)^(3/2)*b^3) + 1/7*D*x/(sqrt(b*x^2 + a)*a*b^3) + 3/56*D*a*x/((b*x^2 +
a)^(5/2)*b^3) - 15/56*D*a^2*x/((b*x^2 + a)^(7/2)*b^3) + 3/140*C*x/((b*x^2
+ a)^(5/2)*b^2) + 2/35*C*x/(sqrt(b*x^2 + a)*a^2*b^2) + 1/35*C*x/((b*x^2 +
a)^(3/2)*a*b^2) - 3/28*C*a*x/((b*x^2 + a)^(7/2)*b^2) - 1/7*B*x/((b*x^2 + a
)^(7/2)*b) + 8/105*B*x/(sqrt(b*x^2 + a)*a^3*b) + 4/105*B*x/((b*x^2 + a)^(3
/2)*a^2*b) + 1/35*B*x/((b*x^2 + a)^(5/2)*a*b) + F*arcsinh(b*x/sqrt(a*b))/b
^(9/2)
```

## Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.81

$$\int \frac{A + Bx^2 + Cx^4 + Dx^6 + Fx^8}{(a + bx^2)^{9/2}}\,dx =$$

$$-\frac{\left(\left(x^2\left(\frac{(176\,Fa^4b^6 - 15\,Da^3b^7 - 6\,Ca^2b^8 - 8\,Bab^9 - 48\,Ab^{10})x^2}{a^4b^7} + \frac{7\,(58\,Fa^5b^5 - 3\,Ca^3b^7 - 4\,Ba^2b^8 - 24\,Aab^9)}{a^4b^7}\right) + \frac{35\,(10\,Fa^6b^4 - Ba^3b^7 - 6\,A}{a^4b^7}}{105\,(bx^2 + a)^{\frac{7}{2}}}$$

$$-\frac{F\log\left(\left|-\sqrt{b}x + \sqrt{bx^2 + a}\right|\right)}{b^{\frac{9}{2}}}$$

input
```
integrate((F*x^8+D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="giac")
```

output
```
-1/105*((x^2*((176*F*a^4*b^6 - 15*D*a^3*b^7 - 6*C*a^2*b^8 - 8*B*a*b^9 - 48
*A*b^10)*x^2/(a^4*b^7) + 7*(58*F*a^5*b^5 - 3*C*a^3*b^7 - 4*B*a^2*b^8 - 24*
A*a*b^9)/(a^4*b^7)) + 35*(10*F*a^6*b^4 - B*a^3*b^7 - 6*A*a^2*b^8)/(a^4*b^7
))*x^2 + 105*(F*a^7*b^3 - A*a^3*b^7)/(a^4*b^7))*x/(b*x^2 + a)^(7/2) - F*lo
g(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(9/2)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{A + Bx^2 + Cx^4 + Dx^6 + Fx^8}{(a + bx^2)^{9/2}}\,dx = \int \frac{A + B\,x^2 + C\,x^4 + F\,x^8 + x^6\,D}{(b\,x^2 + a)^{9/2}}\,dx$$

input
```
int((A + B*x^2 + C*x^4 + F*x^8 + x^6*D)/(a + b*x^2)^(9/2),x)
```

output
```
int((A + B*x^2 + C*x^4 + F*x^8 + x^6*D)/(a + b*x^2)^(9/2), x)
```

## Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 660, normalized size of antiderivative = 2.63

$$\int \frac{A + Bx^2 + Cx^4 + Dx^6 + Fx^8}{(a + bx^2)^{9/2}} \, dx = \frac{15\sqrt{b}\, a^2 b^5 d\, x^8 - 6\sqrt{b}\, a\, b^6 c\, x^8 + 56\sqrt{b}\, a^3 b^4 f\, x^8 - 24\sqrt{b}\, a^2 b^5 c\, x^6 - 10}{}$$

input
```
int((F*x^8+D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x)
```

output
```
( - 105*sqrt(a + b*x**2)*a**6*b*f*x - 350*sqrt(a + b*x**2)*a**5*b**2*f*x**
3 - 406*sqrt(a + b*x**2)*a**4*b**3*f*x**5 + 105*sqrt(a + b*x**2)*a**3*b**5
*x - 176*sqrt(a + b*x**2)*a**3*b**4*f*x**7 + 245*sqrt(a + b*x**2)*a**2*b**
6*x**3 + 21*sqrt(a + b*x**2)*a**2*b**5*c*x**5 + 15*sqrt(a + b*x**2)*a**2*b
**5*d*x**7 + 196*sqrt(a + b*x**2)*a*b**7*x**5 + 6*sqrt(a + b*x**2)*a*b**6*
c*x**7 + 56*sqrt(a + b*x**2)*b**8*x**7 + 105*sqrt(b)*log((sqrt(a + b*x**2)
 + sqrt(b)*x)/sqrt(a))*a**7*f + 420*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b
)*x)/sqrt(a))*a**6*b*f*x**2 + 630*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*
x)/sqrt(a))*a**5*b**2*f*x**4 + 420*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)
*x)/sqrt(a))*a**4*b**3*f*x**6 + 105*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b
)*x)/sqrt(a))*a**3*b**4*f*x**8 + 56*sqrt(b)*a**7*f + 15*sqrt(b)*a**6*b*d +
 224*sqrt(b)*a**6*b*f*x**2 - 6*sqrt(b)*a**5*b**2*c + 60*sqrt(b)*a**5*b**2*
d*x**2 + 336*sqrt(b)*a**5*b**2*f*x**4 - 56*sqrt(b)*a**4*b**4 - 24*sqrt(b)*
a**4*b**3*c*x**2 + 90*sqrt(b)*a**4*b**3*d*x**4 + 224*sqrt(b)*a**4*b**3*f*x
**6 - 224*sqrt(b)*a**3*b**5*x**2 - 36*sqrt(b)*a**3*b**4*c*x**4 + 60*sqrt(b
)*a**3*b**4*d*x**6 + 56*sqrt(b)*a**3*b**4*f*x**8 - 336*sqrt(b)*a**2*b**6*x
**4 - 24*sqrt(b)*a**2*b**5*c*x**6 + 15*sqrt(b)*a**2*b**5*d*x**8 - 224*sqrt
(b)*a*b**7*x**6 - 6*sqrt(b)*a*b**6*c*x**8 - 56*sqrt(b)*b**8*x**8)/(105*a**
3*b**5*(a**4 + 4*a**3*b*x**2 + 6*a**2*b**2*x**4 + 4*a*b**3*x**6 + b**4*x**
8))
```

# 3.268 $\int \frac{A+Bx^2+Cx^4+Dx^6+Fx^8}{x^2(a+bx^2)^{9/2}}\,dx$



## Optimal result

Integrand size = 37, antiderivative size = 253

$$
\begin{aligned}
\int \frac{A+Bx^2+Cx^4+Dx^6+Fx^8}{x^2(a+bx^2)^{9/2}}\,dx = &-\frac{(Ab^4-a(b^3B-ab^2C+a^2bD-a^3F))\,x}{7a^2b^3(a+bx^2)^{7/2}} \\
&-\frac{(13Ab^4-a(6b^3B+ab^2C-8a^2bD+15a^3F))\,x}{35a^3b^3(a+bx^2)^{5/2}} \\
&-\frac{(87Ab^4-a(24b^3B+4ab^2C+3a^2bD-45a^3F))\,x}{105a^4b^3(a+bx^2)^{3/2}}-\frac{A}{a^4x\sqrt{a+bx^2}} \\
&-\frac{(384Ab^4-a(48b^3B+8ab^2C+6a^2bD+15a^3F))\,x}{105a^5b^3\sqrt{a+bx^2}}
\end{aligned}
$$

output
```
-1/7*(A*b^4-a*(B*b^3-C*a*b^2+D*a^2*b-F*a^3))*x/a^2/b^3/(b*x^2+a)^(7/2)-1/3
5*(13*A*b^4-a*(6*B*b^3+C*a*b^2-8*D*a^2*b+15*F*a^3))*x/a^3/b^3/(b*x^2+a)^(5
/2)-1/105*(87*A*b^4-a*(24*B*b^3+4*C*a*b^2+3*D*a^2*b-45*F*a^3))*x/a^4/b^3/(
b*x^2+a)^(3/2)-A/a^4/x/(b*x^2+a)^(1/2)-1/105*(384*A*b^4-a*(48*B*b^3+8*C*a*
b^2+6*D*a^2*b+15*F*a^3))*x/a^5/b^3/(b*x^2+a)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.55

$$\int \frac{A+Bx^2+Cx^4+Dx^6+Fx^8}{x^2\left(a+bx^2\right)^{9/2}}\,dx = \frac{-384Ab^4x^8+48ab^3x^6(-28A+Bx^2)+8a^2b^2x^4(-210A+21Bx^2 -}{}$$

input
```
Integrate[(A + B*x^2 + C*x^4 + D*x^6 + F*x^8)/(x^2*(a + b*x^2)^(9/2)),x]
```

output
```
(-384*A*b^4*x^8 + 48*a*b^3*x^6*(-28*A + B*x^2) + 8*a^2*b^2*x^4*(-210*A + 2
1*B*x^2 + C*x^4) + 2*a^3*b*x^2*(-420*A + 105*B*x^2 + 14*C*x^4 + 3*D*x^6) +
 a^4*(-105*A + 105*B*x^2 + 35*C*x^4 + 21*D*x^6 + 15*F*x^8))/(105*a^5*x*(a
+ b*x^2)^(7/2))
```

## Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.97, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.216$, Rules used = {2334, 2344, 1586, 9, 25, 27, 362, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4+Dx^6+Fx^8}{x^2\left(a+bx^2\right)^{9/2}}\,dx$$

$$\Big\downarrow \text{2334}$$

$$-\frac{\int \frac{8Ab-a\left(Fx^6+Dx^4+Cx^2+B\right)}{\left(bx^2+a\right)^{9/2}}dx}{a}-\frac{A}{ax\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \text{2344}$$

$$-\frac{\frac{\int \frac{x^2\left(-a^2Fx^4-a^2Dx^2+6b(8Ab-aB)-a^2C\right)}{\left(bx^2+a\right)^{9/2}}dx}{a}+\frac{x(8Ab-aB)}{a\left(a+bx^2\right)^{7/2}}}{a}-\frac{A}{ax\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \text{1586}$$

$$-\frac{\frac{\frac{x^3\left(48Ab^2-a\left(\frac{a^3F}{b^2}-\frac{a^2D}{b}+aC+6bB\right)\right)}{7a\left(a+bx^2\right)^{7/2}}-\frac{\int -\frac{x\left(\left(192Ab^2-a\left(-\frac{3Fa^3}{b^2}+\frac{3Da^2}{b}+4Ca+24bB\right)\right)x-\frac{7a^3Fx^3}{b}\right)}{\left(bx^2+a\right)^{7/2}}dx}{7a}}{a}+\frac{x(8Ab-aB)}{a\left(a+bx^2\right)^{7/2}}}{a}-\frac{A}{ax\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \; 9$$

$$-\frac{\frac{\frac{x^3\left(48Ab^2-a\left(\frac{a^3F}{b^2}-\frac{a^2D}{b}+aC+6bB\right)\right)}{7a\left(a+bx^2\right)^{7/2}}-\frac{\int -\frac{x^2\left(b\left(192Ab^2-a\left(-\frac{3Fa^3}{b^2}+\frac{3Da^2}{b}+4Ca+24bB\right)\right)-7a^3Fx^2\right)}{b\left(bx^2+a\right)^{7/2}}dx}{7a}}{a}+\frac{x(8Ab-aB)}{a\left(a+bx^2\right)^{7/2}}}{a}-\frac{A}{ax\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \; 25$$

$$-\frac{\frac{\frac{\int \frac{x^2\left(-7Fx^2a^3-\left(-\frac{3Fa^3}{b}+3Da^2+4bCa+24b^2B\right)a+192Ab^3\right)}{b\left(bx^2+a\right)^{7/2}}dx}{7a}+\frac{x^3\left(48Ab^2-a\left(\frac{a^3F}{b^2}-\frac{a^2D}{b}+aC+6bB\right)\right)}{7a\left(a+bx^2\right)^{7/2}}}{a}+\frac{x(8Ab-aB)}{a\left(a+bx^2\right)^{7/2}}}{a}-\frac{A}{ax\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{\frac{\int \frac{x^2\left(-7Fx^2a^3-\left(-\frac{3Fa^3}{b}+3Da^2+4bCa+24b^2B\right)a+192Ab^3\right)}{\left(bx^2+a\right)^{7/2}}dx}{7ab}+\frac{x^3\left(48Ab^2-a\left(\frac{a^3F}{b^2}-\frac{a^2D}{b}+aC+6bB\right)\right)}{7a\left(a+bx^2\right)^{7/2}}}{a}+\frac{x(8Ab-aB)}{a\left(a+bx^2\right)^{7/2}}}{a}-\frac{A}{ax\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \; 362$$

$$-\frac{\frac{\frac{\frac{\left(384Ab^4-a\left(15a^3F+6a^2bD+8ab^2C+48b^3B\right)\right)\int\frac{x^2}{(bx^2+a)^{5/2}}dx}{5ab}+\frac{x^3\left(192Ab^4-a\left(-10a^3F+3a^2bD+4ab^2C+24b^3B\right)\right)}{5ab\left(a+bx^2\right)^{5/2}}}{7ab}+\frac{x^3\left(48Ab^2-a\left(\frac{a^3F}{b^2}-\frac{a^2D}{b}+aC+6b\right.\right.}{7a\left(a+bx^2\right)^{7/2}}}{a}}{a}$$

$$\frac{A}{ax\left(a+bx^2\right)^{7/2}}$$

$\downarrow$ 242

$$-\frac{\frac{\frac{x^3\left(48Ab^2-a\left(\frac{a^3F}{b^2}-\frac{a^2D}{b}+aC+6bB\right)\right)}{7a\left(a+bx^2\right)^{7/2}}+\frac{\frac{x^3\left(384Ab^4-a\left(15a^3F+6a^2bD+8ab^2C+48b^3B\right)\right)}{15a^2b\left(a+bx^2\right)^{3/2}}+\frac{x^3\left(192Ab^4-a\left(-10a^3F+3a^2bD+4ab^2C+24b^3B\right)\right)}{5ab\left(a+bx^2\right)^{5/2}}}{7ab}}{a}+\frac{x(8A}{a(a+}}{a}$$

$$\frac{A}{ax\left(a+bx^2\right)^{7/2}}$$

input
```
Int[(A + B*x^2 + C*x^4 + D*x^6 + F*x^8)/(x^2*(a + b*x^2)^(9/2)),x]
```

output
```
-(A/(a*x*(a + b*x^2)^(7/2))) - ((((8*A*b - a*B)*x)/(a*(a + b*x^2)^(7/2)) +
(((48*A*b^2 - a*(6*b*B + a*C - (a^2*D)/b + (a^3*F)/b^2))*x^3)/(7*a*(a + b*
x^2)^(7/2)) + (((192*A*b^4 - a*(24*b^3*B + 4*a*b^2*C + 3*a^2*b*D - 10*a^3*
F))*x^3)/(5*a*b*(a + b*x^2)^(5/2)) + ((384*A*b^4 - a*(48*b^3*B + 8*a*b^2*C
 + 6*a^2*b*D + 15*a^3*F))*x^3)/(15*a^2*b*(a + b*x^2)^(3/2)))/(7*a*b))/a)/a
```

## Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

rule 1586
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, d + e*x^2, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^
p, d + e*x^2, x], x, 0]}, Simp[(-R)*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(2*d
*f*(q + 1))), x] + Simp[f/(2*d*(q + 1))   Int[(f*x)^(m - 1)*(d + e*x^2)^(q
+ 1)*ExpandToSum[2*d*(q + 1)*x*Qx + R*(m + 2*q + 3)*x, x], x], x]] /; FreeQ
[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[q, -1]
&& GtQ[m, 0]
```

rule 2334
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coef
f[Pq, x, 0], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[A*
x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))   Int[
x^(m + 2)*(a + b*x^2)^p*(a*(m + 1)*Q - A*b*(m + 2*(p + 1) + 1)), x], x]] /;
 FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && IntegerQ[m/2] && ILtQ[(m + 1)/2 + p,
 0] && LtQ[m + Expon[Pq, x] + 2*p + 1, 0]
```

rule 2344
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coeff[Pq, x, 0
], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[A*x*((a + b*
x^2)^(p + 1)/a), x] + Simp[1/a   Int[x^2*(a + b*x^2)^p*(a*Q - A*b*(2*p + 3)
), x], x]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && ILtQ[p + 1/2, 0] && LtQ
[Expon[Pq, x] + 2*p + 1, 0]
```

## Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.52

| method | result |
|---|---|
| pseudoelliptic | $\frac{(15F x^8+21Dx^6+35C x^4+105x^2B-105A)a^4-840\left(-\frac{1}{140}Dx^6-\frac{1}{30}C x^4-\frac{1}{4}x^2B+A\right)x^2 b a^3-1680\left(-\frac{1}{210}C x^4-\frac{1}{10}x^2B+A\right)\cdots}{105(b x^2+a)^{\frac{7}{2}} x a^5}$ |
| gosper | $-\frac{384A x^8 b^4-48B x^8 a b^3-8C a^2 b^2 x^8-6Da^3 b x^8-15F a^4 x^8+1344A x^6 a b^3-168B x^6 a^2 b^2-28C a^3 b x^6-21Da^4 x^6+1680\cdots}{105x(b x^2+a)^{\frac{7}{2}} a^5}$ |
| trager | $-\frac{384A x^8 b^4-48B x^8 a b^3-8C a^2 b^2 x^8-6Da^3 b x^8-15F a^4 x^8+1344A x^6 a b^3-168B x^6 a^2 b^2-28C a^3 b x^6-21Da^4 x^6+1680\cdots}{105x(b x^2+a)^{\frac{7}{2}} a^5}$ |
| orering | $-\frac{384A x^8 b^4-48B x^8 a b^3-8C a^2 b^2 x^8-6Da^3 b x^8-15F a^4 x^8+1344A x^6 a b^3-168B x^6 a^2 b^2-28C a^3 b x^6-21Da^4 x^6+1680\cdots}{105x(b x^2+a)^{\frac{7}{2}} a^5}$ |
| default | $B\left(\frac{x}{7a(b x^2+a)^{\frac{7}{2}}}+\frac{\frac{6x}{35a\left(b x^2+a\right)^{\frac{5}{2}}}+\frac{6\left(\frac{4x}{15a\left(b x^2+a\right)^{\frac{3}{2}}}+\frac{8x}{15a^2\sqrt{b x^2+a}}\right)}{7a}}{a}\right)+A\left(-\frac{1}{ax(b x^2+a)^{\frac{7}{2}}}-\frac{8b\left(\frac{x}{7a\left(b x^2+a\right)\cdots}\right.}{\cdots}\right.$ |

input
```
int((F*x^8+D*x^6+C*x^4+B*x^2+A)/x^2/(b*x^2+a)^(9/2),x,method=_RETURNVERBOS
E)
```

output
```
1/105*((15*F*x^8+21*D*x^6+35*C*x^4+105*B*x^2-105*A)*a^4-840*(-1/140*D*x^6-
1/30*C*x^4-1/4*x^2*B+A)*x^2*b*a^3-1680*(-1/210*C*x^4-1/10*x^2*B+A)*x^4*b^2
*a^2-1344*(-1/28*x^2*B+A)*x^6*b^3*a-384*A*x^8*b^4)/(b*x^2+a)^(7/2)/x/a^5
```

## Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.74

$$\int \frac{A+Bx^2+Cx^4+Dx^6+Fx^8}{x^2\left(a+bx^2\right)^{9/2}}\,dx = \frac{((15\,Fa^4+6\,Da^3b+8\,Ca^2b^2+48\,Bab^3-384\,Ab^4)x^8+7\,(3\,Da^4+}{1}$$

input
```
integrate((F*x^8+D*x^6+C*x^4+B*x^2+A)/x^2/(b*x^2+a)^(9/2),x, algorithm="fr
icas")
```

output
```
1/105*((15*F*a^4 + 6*D*a^3*b + 8*C*a^2*b^2 + 48*B*a*b^3 - 384*A*b^4)*x^8 +
 7*(3*D*a^4 + 4*C*a^3*b + 24*B*a^2*b^2 - 192*A*a*b^3)*x^6 - 105*A*a^4 + 35
*(C*a^4 + 6*B*a^3*b - 48*A*a^2*b^2)*x^4 + 105*(B*a^4 - 8*A*a^3*b)*x^2)*sqr
t(b*x^2 + a)/(a^5*b^4*x^9 + 4*a^6*b^3*x^7 + 6*a^7*b^2*x^5 + 4*a^8*b*x^3 +
a^9*x)
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2490 vs. 2(250) = 500.

Time = 106.67 (sec) , antiderivative size = 2490, normalized size of antiderivative = 9.84

$$\int \frac{A+Bx^2+Cx^4+Dx^6+Fx^8}{x^2\left(a+bx^2\right)^{9/2}}\,dx = \text{Too large to display}$$

input
```
integrate((F*x**8+D*x**6+C*x**4+B*x**2+A)/x**2/(b*x**2+a)**(9/2),x)
```

output
```
A*(-35*a**4*b**(33/2)*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b**17
*x**2 + 210*a**7*b**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8) -
280*a**3*b**(35/2)*x**2*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b**
17*x**2 + 210*a**7*b**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8)
- 560*a**2*b**(37/2)*x**4*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b
**17*x**2 + 210*a**7*b**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8
) - 448*a*b**(39/2)*x**6*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b*
*17*x**2 + 210*a**7*b**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8)
 - 128*b**(41/2)*x**8*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b**17
*x**2 + 210*a**7*b**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8)) +
 B*(35*a**14*x/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqr
t(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/
2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/
a) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12
*sqrt(1 + b*x**2/a)) + 175*a**13*b*x**3/(35*a**(37/2)*sqrt(1 + b*x**2/a) +
 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1
+ b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b
**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a)
+ 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a)) + 371*a**12*b**2*x**5/(35*a*
*(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + ...
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 421, normalized size of antiderivative = 1.66

$$\int \frac{A + Bx^2 + Cx^4 + Dx^6 + Fx^8}{x^2 (a + bx^2)^{9/2}} dx = -\frac{Fx^5}{2 (bx^2 + a)^{\frac{7}{2}} b} - \frac{5 Fax^3}{8 (bx^2 + a)^{\frac{7}{2}} b^2} - \frac{Dx^3}{4 (bx^2 + a)^{\frac{7}{2}} b} + \frac{16 Bx}{35 \sqrt{bx^2 + a} a^4} + \frac{8 Bx}{35 (bx^2 + a)^{\frac{3}{2}} a^3} + \frac{6 Bx}{35 (bx^2 + a)^{\frac{5}{2}} a^2} + \frac{Bx}{7 (bx^2 + a)^{\frac{7}{2}} a} + \frac{Fx}{14 (bx^2 + a)^{\frac{3}{2}} b^3} + \frac{Fx}{7 \sqrt{bx^2 + a} ab^3} + \frac{3 Fax}{56 (bx^2 + a)^{\frac{5}{2}} b^3} - \frac{15 Fa^2x}{56 (bx^2 + a)^{\frac{7}{2}} b^3} + \frac{3 Dx}{140 (bx^2 + a)^{\frac{5}{2}} b^2} + \frac{2 Dx}{35 \sqrt{bx^2 + a} a^2 b^2} + \frac{Dx}{35 (bx^2 + a)^{\frac{3}{2}} ab^2} - \frac{3 Dax}{28 (bx^2 + a)^{\frac{7}{2}} b^2} - \frac{Cx}{7 (bx^2 + a)^{\frac{7}{2}} b} + \frac{8 Cx}{105 \sqrt{bx^2 + a} a^3 b} + \frac{4 Cx}{105 (bx^2 + a)^{\frac{3}{2}} a^2 b} + \frac{Cx}{35 (bx^2 + a)^{\frac{5}{2}} ab} - \frac{128 Abx}{35 \sqrt{bx^2 + a} a^5} - \frac{64 Abx}{35 (bx^2 + a)^{\frac{3}{2}} a^4} - \frac{48 Abx}{35 (bx^2 + a)^{\frac{5}{2}} a^3} - \frac{8 Abx}{7 (bx^2 + a)^{\frac{7}{2}} a^2} - \frac{A}{(bx^2 + a)^{\frac{7}{2}} ax}$$

input
```
integrate((F*x^8+D*x^6+C*x^4+B*x^2+A)/x^2/(b*x^2+a)^(9/2),x, algorithm="ma
xima")
```

output
```
-1/2*F*x^5/((b*x^2 + a)^(7/2)*b) - 5/8*F*a*x^3/((b*x^2 + a)^(7/2)*b^2) - 1
/4*D*x^3/((b*x^2 + a)^(7/2)*b) + 16/35*B*x/(sqrt(b*x^2 + a)*a^4) + 8/35*B*
x/((b*x^2 + a)^(3/2)*a^3) + 6/35*B*x/((b*x^2 + a)^(5/2)*a^2) + 1/7*B*x/((b
*x^2 + a)^(7/2)*a) + 1/14*F*x/((b*x^2 + a)^(3/2)*b^3) + 1/7*F*x/(sqrt(b*x^
2 + a)*a*b^3) + 3/56*F*a*x/((b*x^2 + a)^(5/2)*b^3) - 15/56*F*a^2*x/((b*x^2
 + a)^(7/2)*b^3) + 3/140*D*x/((b*x^2 + a)^(5/2)*b^2) + 2/35*D*x/(sqrt(b*x^
2 + a)*a^2*b^2) + 1/35*D*x/((b*x^2 + a)^(3/2)*a*b^2) - 3/28*D*a*x/((b*x^2
+ a)^(7/2)*b^2) - 1/7*C*x/((b*x^2 + a)^(7/2)*b) + 8/105*C*x/(sqrt(b*x^2 +
a)*a^3*b) + 4/105*C*x/((b*x^2 + a)^(3/2)*a^2*b) + 1/35*C*x/((b*x^2 + a)^(5
/2)*a*b) - 128/35*A*b*x/(sqrt(b*x^2 + a)*a^5) - 64/35*A*b*x/((b*x^2 + a)^(
3/2)*a^4) - 48/35*A*b*x/((b*x^2 + a)^(5/2)*a^3) - 8/7*A*b*x/((b*x^2 + a)^(
7/2)*a^2) - A/((b*x^2 + a)^(7/2)*a*x)
```

## Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.87

$$\int \frac{A+Bx^2+Cx^4+Dx^6+Fx^8}{x^2(a+bx^2)^{9/2}}\,dx = \frac{\left(\left(x^2\left(\frac{(15\,Fa^{13}b^3+6\,Da^{12}b^4+8\,Ca^{11}b^5+48\,Ba^{10}b^6-279\,Aa^9b^7)x^2}{a^{14}b^3}+\frac{7\,(3\,Da^{13}b^3+4\,C\cdots}{\cdots}\right.\right.\right.\cdots}{\cdots} + \frac{2\,A\sqrt{b}}{\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)a^4}$$

input
```
integrate((F*x^8+D*x^6+C*x^4+B*x^2+A)/x^2/(b*x^2+a)^(9/2),x, algorithm="gi
ac")
```

output
```
1/105*((x^2*((15*F*a^13*b^3 + 6*D*a^12*b^4 + 8*C*a^11*b^5 + 48*B*a^10*b^6
- 279*A*a^9*b^7)*x^2/(a^14*b^3) + 7*(3*D*a^13*b^3 + 4*C*a^12*b^4 + 24*B*a^
11*b^5 - 132*A*a^10*b^6)/(a^14*b^3)) + 35*(C*a^13*b^3 + 6*B*a^12*b^4 - 30*
A*a^11*b^5)/(a^14*b^3))*x^2 + 105*(B*a^13*b^3 - 4*A*a^12*b^4)/(a^14*b^3))*
x/(b*x^2 + a)^(7/2) + 2*A*sqrt(b)/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)*a
^4)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4+Dx^6+Fx^8}{x^2(a+bx^2)^{9/2}}\,dx = \int \frac{A+B\,x^2+C\,x^4+F\,x^8+x^6\,D}{x^2\,(b\,x^2+a)^{9/2}}\,dx$$

input
```
int((A + B*x^2 + C*x^4 + F*x^8 + x^6*D)/(x^2*(a + b*x^2)^(9/2)),x)
```

output
```
int((A + B*x^2 + C*x^4 + F*x^8 + x^6*D)/(x^2*(a + b*x^2)^(9/2)), x)
```

## Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 512, normalized size of antiderivative = 2.02

$$\int \frac{A+Bx^2+Cx^4+Dx^6+Fx^8}{x^2\left(a+bx^2\right)^{9/2}}\,dx = \frac{-735\sqrt{b\,x^2+a}\,a^3b^5x^2-1470\sqrt{b\,x^2+a}\,a^2b^6x^4-1176\sqrt{b\,x^2+a}\,a\,b}{}$$

input
```
int((F*x^8+D*x^6+C*x^4+B*x^2+A)/x^2/(b*x^2+a)^(9/2),x)
```

output
```
( - 105*sqrt(a + b*x**2)*a**4*b**4 - 735*sqrt(a + b*x**2)*a**3*b**5*x**2 +
 35*sqrt(a + b*x**2)*a**3*b**4*c*x**4 + 21*sqrt(a + b*x**2)*a**3*b**4*d*x*
*6 + 15*sqrt(a + b*x**2)*a**3*b**4*f*x**8 - 1470*sqrt(a + b*x**2)*a**2*b**
6*x**4 + 28*sqrt(a + b*x**2)*a**2*b**5*c*x**6 + 6*sqrt(a + b*x**2)*a**2*b*
*5*d*x**8 - 1176*sqrt(a + b*x**2)*a*b**7*x**6 + 8*sqrt(a + b*x**2)*a*b**6*
c*x**8 - 336*sqrt(a + b*x**2)*b**8*x**8 + 20*sqrt(b)*a**7*f*x - 6*sqrt(b)*
a**6*b*d*x + 80*sqrt(b)*a**6*b*f*x**3 - 8*sqrt(b)*a**5*b**2*c*x - 24*sqrt(
b)*a**5*b**2*d*x**3 + 120*sqrt(b)*a**5*b**2*f*x**5 + 336*sqrt(b)*a**4*b**4
*x - 32*sqrt(b)*a**4*b**3*c*x**3 - 36*sqrt(b)*a**4*b**3*d*x**5 + 80*sqrt(b
)*a**4*b**3*f*x**7 + 1344*sqrt(b)*a**3*b**5*x**3 - 48*sqrt(b)*a**3*b**4*c*
x**5 - 24*sqrt(b)*a**3*b**4*d*x**7 + 20*sqrt(b)*a**3*b**4*f*x**9 + 2016*sq
rt(b)*a**2*b**6*x**5 - 32*sqrt(b)*a**2*b**5*c*x**7 - 6*sqrt(b)*a**2*b**5*d
*x**9 + 1344*sqrt(b)*a*b**7*x**7 - 8*sqrt(b)*a*b**6*c*x**9 + 336*sqrt(b)*b
**8*x**9)/(105*a**4*b**4*x*(a**4 + 4*a**3*b*x**2 + 6*a**2*b**2*x**4 + 4*a*
b**3*x**6 + b**4*x**8))
```

# 3.269 $\int \frac{A+Bx^2+Cx^4+Dx^6+Fx^8}{x^4(a+bx^2)^{9/2}}\,dx$



## Optimal result

Integrand size = 37, antiderivative size = 286

$$\begin{aligned}\int \frac{A+Bx^2+Cx^4+Dx^6+Fx^8}{x^4\left(a+bx^2\right)^{9/2}}\,dx &= \frac{\left(Ab^4-a(b^3B-ab^2C+a^2bD-a^3F)\right)x}{7a^3b^2\left(a+bx^2\right)^{7/2}} \\ &+ \frac{\left(20Ab^4-a(13b^3B-6ab^2C-a^2bD+8a^3F)\right)x}{35a^4b^2\left(a+bx^2\right)^{5/2}} \\ &+ \frac{\left(185Ab^4-a(87b^3B-24ab^2C-4a^2bD-3a^3F)\right)x}{105a^5b^2\left(a+bx^2\right)^{3/2}} \\ &+ \frac{\left(790Ab^4-a(279b^3B-48ab^2C-8a^2bD-6a^3F)\right)x}{105a^6b^2\sqrt{a+bx^2}} \\ &- \frac{A\sqrt{a+bx^2}}{3a^5x^3}+\frac{(14Ab-3aB)\sqrt{a+bx^2}}{3a^6x}\end{aligned}$$

output
```
1/7*(A*b^4-a*(B*b^3-C*a*b^2+D*a^2*b-F*a^3))*x/a^3/b^2/(b*x^2+a)^(7/2)+1/35
*(20*A*b^4-a*(13*B*b^3-6*C*a*b^2-D*a^2*b+8*F*a^3))*x/a^4/b^2/(b*x^2+a)^(5/
2)+1/105*(185*A*b^4-a*(87*B*b^3-24*C*a*b^2-4*D*a^2*b-3*F*a^3))*x/a^5/b^2/(
b*x^2+a)^(3/2)+1/105*(790*A*b^4-a*(279*B*b^3-48*C*a*b^2-8*D*a^2*b-6*F*a^3)
)*x/a^6/b^2/(b*x^2+a)^(1/2)-1/3*A*(b*x^2+a)^(1/2)/a^5/x^3+1/3*(14*A*b-3*B*
a)*(b*x^2+a)^(1/2)/a^6/x
```

## Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.63

$$\int \frac{A+Bx^2+Cx^4+Dx^6+Fx^8}{x^4\left(a+bx^2\right)^{9/2}}\,dx=\frac{1280Ab^5x^{10}+128ab^4x^8(35A-3Bx^2)+16a^2b^3x^6(350A-84Bx^2 \cdots}{}$$

input
```
Integrate[(A + B*x^2 + C*x^4 + D*x^6 + F*x^8)/(x^4*(a + b*x^2)^(9/2)),x]
```

output
```
(1280*A*b^5*x^10 + 128*a*b^4*x^8*(35*A - 3*B*x^2) + 16*a^2*b^3*x^6*(350*A
- 84*B*x^2 + 3*C*x^4) + 8*a^3*b^2*x^4*(350*A - 210*B*x^2 + 21*C*x^4 + D*x^
6) - 7*a^5*(5*A + 15*B*x^2 - 15*C*x^4 - 5*D*x^6 - 3*F*x^8) + 2*a^4*b*x^2*(
175*A - 420*B*x^2 + 105*C*x^4 + 14*D*x^6 + 3*F*x^8))/(105*a^6*x^3*(a + b*x
^2)^(7/2))
```

## Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 257, normalized size of antiderivative = 0.90, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}=0.243$, Rules used = {2334, 2334, 2087, 1469, 27, 2075, 362, 245, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4+Dx^6+Fx^8}{x^4\left(a+bx^2\right)^{9/2}}dx$$

$$\Big\downarrow \text{ 2334}$$

$$-\frac{\int \frac{10Ab-3a\left(Fx^6+Dx^4+Cx^2+B\right)}{x^2\left(bx^2+a\right)^{9/2}}dx}{3a}-\frac{A}{3ax^3\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \text{ 2334}$$

$$-\frac{-\frac{\int \frac{8b(10Ab-3aB)+3\left(a^2Fx^4+a^2Dx^2+a^2C\right)}{\left(bx^2+a\right)^{9/2}}dx}{a}-\frac{10Ab-3aB}{ax\left(a+bx^2\right)^{7/2}}}{3a}-\frac{A}{3ax^3\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \; 2087$$

$$-\frac{-\frac{\int \frac{3a^2Fx^4+3a^2Dx^2+80Ab^2-3a(8bB-aC)}{\left(bx^2+a\right)^{9/2}}dx}{a}-\frac{10Ab-3aB}{ax(a+bx^2)^{7/2}}}{3a}-\frac{A}{3ax^3\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \; 1469$$

$$-\frac{-\frac{\frac{\int \frac{3x^2\left(\left(Fx^2+D\right)a^3+2b\left(80Ab^2-3a(8bB-aC)\right)\right)}{\left(bx^2+a\right)^{9/2}}dx}{a}+\frac{x\left(80Ab^2-3a(8bB-aC)\right)}{a\left(a+bx^2\right)^{7/2}}}{a}-\frac{10Ab-3aB}{ax(a+bx^2)^{7/2}}}{3a}-\frac{A}{3ax^3\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \; 27$$

$$-\frac{-\frac{\frac{3\int \frac{x^2\left(\left(Fx^2+D\right)a^3+2b\left(80Ab^2-3a(8bB-aC)\right)\right)}{\left(bx^2+a\right)^{9/2}}dx}{a}+\frac{x\left(80Ab^2-3a(8bB-aC)\right)}{a\left(a+bx^2\right)^{7/2}}}{a}-\frac{10Ab-3aB}{ax(a+bx^2)^{7/2}}}{3a}-\frac{A}{3ax^3\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \; 2075$$

$$-\frac{-\frac{\frac{3\int \frac{x^2\left(Fx^2a^3-\left(-Da^2-6bCa+48b^2B\right)a+160Ab^3\right)}{\left(bx^2+a\right)^{9/2}}dx}{a}+\frac{x\left(80Ab^2-3a(8bB-aC)\right)}{a\left(a+bx^2\right)^{7/2}}}{a}-\frac{10Ab-3aB}{ax(a+bx^2)^{7/2}}}{3a}-\frac{A}{3ax^3\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \; 362$$

$$-\frac{-\frac{\frac{3\left(\frac{\left(640Ab^4-a\left(-3a^3F-4a^2bD-24ab^2C+192b^3B\right)\right)\int \frac{x^2}{\left(bx^2+a\right)^{7/2}}dx}{7ab}+\frac{x^3\left(160Ab^4-a\left(a^3F-a^2bD-6ab^2C+48b^3B\right)\right)}{7ab\left(a+bx^2\right)^{7/2}}\right)}{a}+\frac{x\left(80Ab^2-3a(8bB-aC)\right)}{a\left(a+bx^2\right)^{7/2}}}{a}-}{3a}-\frac{A}{3ax^3\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \; 245$$

$$-\frac{-\frac{\frac{3\left(\frac{\left(640Ab^4-a\left(-3a^3F-4a^2bD-24ab^2C+192b^3B\right)\right)\left(\frac{2b\int\frac{x^4}{\left(bx^2+a\right)^{7/2}}dx}{3a}+\frac{x^3}{3a\left(a+bx^2\right)^{5/2}}\right)}{7ab}+\frac{x^3\left(160Ab^4-a\left(a^3F-a^2bD-6ab^2C+48b^3B\right)\right)}{7ab\left(a+bx^2\right)^{7/2}}\right)}{a}+\frac{x(80A}{}}{a}}{3a}$$

$$\frac{A}{3ax^3\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \; 242$$

$$-\frac{-\frac{\frac{3\left(\frac{x^3\left(160Ab^4-a\left(a^3F-a^2bD-6ab^2C+48b^3B\right)\right)}{7ab\left(a+bx^2\right)^{7/2}}+\frac{\left(\frac{2bx^5}{15a^2\left(a+bx^2\right)^{5/2}}+\frac{x^3}{3a\left(a+bx^2\right)^{5/2}}\right)\left(640Ab^4-a\left(-3a^3F-4a^2bD-24ab^2C+192b^3B\right)\right)}{7ab}\right)}{a}+\frac{x(80Ab^2}{a(}}{a}}{3a}$$

$$\frac{A}{3ax^3\left(a+bx^2\right)^{7/2}}$$

input
```
Int[(A + B*x^2 + C*x^4 + D*x^6 + F*x^8)/(x^4*(a + b*x^2)^(9/2)),x]
```

output
```
-1/3*A/(a*x^3*(a + b*x^2)^(7/2)) - (-((10*A*b - 3*a*B)/(a*x*(a + b*x^2)^(7
/2))) - (((80*A*b^2 - 3*a*(8*b*B - a*C))*x)/(a*(a + b*x^2)^(7/2)) + (3*(((
160*A*b^4 - a*(48*b^3*B - 6*a*b^2*C - a^2*b*D + a^3*F))*x^3)/(7*a*b*(a + b
*x^2)^(7/2)) + ((640*A*b^4 - a*(192*b^3*B - 24*a*b^2*C - 4*a^2*b*D - 3*a^3
*F))*(x^3/(3*a*(a + b*x^2)^(5/2)) + (2*b*x^5)/(15*a^2*(a + b*x^2)^(5/2))))
/(7*a*b)))/a)/a)/(3*a)
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

rule 1469
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[a^p*x*((d + e*x^2)^(q + 1)/d), x] + Simp[1/d   Int[x^2*(d
 + e*x^2)^q*(d*PolynomialQuotient[(a + b*x^2 + c*x^4)^p - a^p, x^2, x] - e*
a^p*(2*q + 3)), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && ILtQ[q + 1/2, 0] && LtQ[4
*p + 2*q + 1, 0]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 2087
```
Int[(u_)^(q_.)*(v_)^(p_.), x_Symbol] :> Int[ExpandToSum[u, x]^q*ExpandToSum
[v, x]^p, x] /; FreeQ[{p, q}, x] && BinomialQ[u, x] && TrinomialQ[v, x] &&
 !(BinomialMatchQ[u, x] && TrinomialMatchQ[v, x])
```

rule 2334
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coef
f[Pq, x, 0], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[A*
x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))   Int[
x^(m + 2)*(a + b*x^2)^p*(a*(m + 1)*Q - A*b*(m + 2*(p + 1) + 1)), x], x]] /;
 FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && IntegerQ[m/2] && ILtQ[(m + 1)/2 + p,
 0] && LtQ[m + Expon[Pq, x] + 2*p + 1, 0]
```

## Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.58

| method | result |
|---|---|
| pseudoelliptic | $-\frac{\left(-\frac{3}{5}F\,x^8-Dx^6-3C\,x^4+3x^2B+A\right)a^5-10\left(\frac{3}{175}F\,x^8+\frac{2}{25}Dx^6+\frac{3}{5}C\,x^4-\frac{12}{5}x^2B+A\right)x^2b\,a^4-80\left(\frac{1}{350}Dx^6+\frac{3}{50}C\,x^4-\frac{3}{5}x^2B\right.}{3\left(b\,x^2+a\right)^{\frac{7}{2}}x^3a^6}$ |
| gosper | $-\frac{-1280A\,b^5x^{10}+384Ba\,b^4x^{10}-48C\,a^2b^3x^{10}-8Da^3b^2x^{10}-6F\,a^4b\,x^{10}-4480aA\,b^4x^8+1344B\,a^2b^3x^8-168C\,a^3b^2x^8-28}{}$ |
| trager | $-\frac{-1280A\,b^5x^{10}+384Ba\,b^4x^{10}-48C\,a^2b^3x^{10}-8Da^3b^2x^{10}-6F\,a^4b\,x^{10}-4480aA\,b^4x^8+1344B\,a^2b^3x^8-168C\,a^3b^2x^8-28}{}$ |
| orering | $-\frac{-1280A\,b^5x^{10}+384Ba\,b^4x^{10}-48C\,a^2b^3x^{10}-8Da^3b^2x^{10}-6F\,a^4b\,x^{10}-4480aA\,b^4x^8+1344B\,a^2b^3x^8-168C\,a^3b^2x^8-28}{}$ |
| default | $C\left(\frac{x}{7a\left(b\,x^2+a\right)^{\frac{7}{2}}}+\frac{\frac{6x}{35a\left(b\,x^2+a\right)^{\frac{5}{2}}}+\frac{6\left(\frac{4x}{15a\left(b\,x^2+a\right)^{\frac{3}{2}}}+\frac{8x}{15a^2\sqrt{b\,x^2+a}}\right)}{7a}}{a}\right)+A\left(-\frac{1}{3a\,x^3\left(b\,x^2+a\right)^{\frac{7}{2}}}-\frac{10b\left(-\frac{}{ax\left(b\right.}\right.}{}\right.$ |

input
```
int((F*x^8+D*x^6+C*x^4+B*x^2+A)/x^4/(b*x^2+a)^(9/2),x,method=_RETURNVERBOS
E)
```

output
```
-1/3/(b*x^2+a)^(7/2)*((-3/5*F*x^8-D*x^6-3*C*x^4+3*x^2*B+A)*a^5-10*(3/175*F
*x^8+2/25*D*x^6+3/5*C*x^4-12/5*x^2*B+A)*x^2*b*a^4-80*(1/350*D*x^6+3/50*C*x
^4-3/5*x^2*B+A)*x^4*b^2*a^3-160*(3/350*C*x^4-6/25*x^2*B+A)*x^6*b^3*a^2-128
*x^8*b^4*(-3/35*x^2*B+A)*a-256/7*A*b^5*x^10)/x^3/a^6
```

## Fricas [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.84

$$\int \frac{A+Bx^2+Cx^4+Dx^6+Fx^8}{x^4(a+bx^2)^{9/2}}\,dx = \frac{(2\,(3\,Fa^4b+4\,Da^3b^2+24\,Ca^2b^3-192\,Bab^4+640\,Ab^5)x^{10}+7\,(3\,F}{}$$

input
```
integrate((F*x^8+D*x^6+C*x^4+B*x^2+A)/x^4/(b*x^2+a)^(9/2),x, algorithm="fr
icas")
```

output
```
1/105*(2*(3*F*a^4*b + 4*D*a^3*b^2 + 24*C*a^2*b^3 - 192*B*a*b^4 + 640*A*b^5
)*x^10 + 7*(3*F*a^5 + 4*D*a^4*b + 24*C*a^3*b^2 - 192*B*a^2*b^3 + 640*A*a*b
^4)*x^8 + 35*(D*a^5 + 6*C*a^4*b - 48*B*a^3*b^2 + 160*A*a^2*b^3)*x^6 - 35*A
*a^5 + 35*(3*C*a^5 - 24*B*a^4*b + 80*A*a^3*b^2)*x^4 - 35*(3*B*a^5 - 10*A*a
^4*b)*x^2)*sqrt(b*x^2 + a)/(a^6*b^4*x^11 + 4*a^7*b^3*x^9 + 6*a^8*b^2*x^7 +
 4*a^9*b*x^5 + a^10*x^3)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4+Dx^6+Fx^8}{x^4(a+bx^2)^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate((F*x**8+D*x**6+C*x**4+B*x**2+A)/x**4/(b*x**2+a)**(9/2),x)
```

output
```
Timed out
```

## Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 426, normalized size of antiderivative = 1.49

$$\int \frac{A+Bx^2+Cx^4+Dx^6+Fx^8}{x^4(a+bx^2)^{9/2}}\,dx = -\frac{Fx^3}{4(bx^2+a)^{\frac{7}{2}}b} + \frac{16\,Cx}{35\sqrt{bx^2+a}a^4} + \frac{8\,Cx}{35(bx^2+a)^{\frac{3}{2}}a^3} + \frac{6\,Cx}{35(bx^2+a)^{\frac{5}{2}}a^2} + \frac{Cx}{7(bx^2+a)^{\frac{7}{2}}a} + \frac{3\,Fx}{140(bx^2+a)^{\frac{5}{2}}b^2} + \frac{2\,Fx}{35\sqrt{bx^2+a}a^2b^2} + \frac{Fx}{35(bx^2+a)^{\frac{3}{2}}ab^2} - \frac{3\,Fax}{28(bx^2+a)^{\frac{7}{2}}b^2} - \frac{Dx}{7(bx^2+a)^{\frac{7}{2}}b} + \frac{8\,Dx}{105\sqrt{bx^2+a}a^3b} + \frac{4\,Dx}{105(bx^2+a)^{\frac{3}{2}}a^2b} + \frac{Dx}{35(bx^2+a)^{\frac{5}{2}}ab} - \frac{128\,Bbx}{35\sqrt{bx^2+a}a^5} - \frac{64\,Bbx}{35(bx^2+a)^{\frac{3}{2}}a^4} - \frac{48\,Bbx}{35(bx^2+a)^{\frac{5}{2}}a^3} - \frac{8\,Bbx}{7(bx^2+a)^{\frac{7}{2}}a^2} + \frac{256\,Ab^2x}{21\sqrt{bx^2+a}a^6} + \frac{128\,Ab^2x}{21(bx^2+a)^{\frac{3}{2}}a^5} + \frac{32\,Ab^2x}{7(bx^2+a)^{\frac{5}{2}}a^4} + \frac{80\,Ab^2x}{21(bx^2+a)^{\frac{7}{2}}a^3} - \frac{B}{(bx^2+a)^{\frac{7}{2}}ax} + \frac{10\,Ab}{3(bx^2+a)^{\frac{7}{2}}a^2x} - \frac{A}{3(bx^2+a)^{\frac{7}{2}}ax^3}$$

input
```
integrate((F*x^8+D*x^6+C*x^4+B*x^2+A)/x^4/(b*x^2+a)^(9/2),x, algorithm="ma
xima")
```

output
```
-1/4*F*x^3/((b*x^2 + a)^(7/2)*b) + 16/35*C*x/(sqrt(b*x^2 + a)*a^4) + 8/35*
C*x/((b*x^2 + a)^(3/2)*a^3) + 6/35*C*x/((b*x^2 + a)^(5/2)*a^2) + 1/7*C*x/(
(b*x^2 + a)^(7/2)*a) + 3/140*F*x/((b*x^2 + a)^(5/2)*b^2) + 2/35*F*x/(sqrt(
b*x^2 + a)*a^2*b^2) + 1/35*F*x/((b*x^2 + a)^(3/2)*a*b^2) - 3/28*F*a*x/((b*
x^2 + a)^(7/2)*b^2) - 1/7*D*x/((b*x^2 + a)^(7/2)*b) + 8/105*D*x/(sqrt(b*x^
2 + a)*a^3*b) + 4/105*D*x/((b*x^2 + a)^(3/2)*a^2*b) + 1/35*D*x/((b*x^2 + a
)^(5/2)*a*b) - 128/35*B*b*x/(sqrt(b*x^2 + a)*a^5) - 64/35*B*b*x/((b*x^2 +
a)^(3/2)*a^4) - 48/35*B*b*x/((b*x^2 + a)^(5/2)*a^3) - 8/7*B*b*x/((b*x^2 +
a)^(7/2)*a^2) + 256/21*A*b^2*x/(sqrt(b*x^2 + a)*a^6) + 128/21*A*b^2*x/((b*
x^2 + a)^(3/2)*a^5) + 32/7*A*b^2*x/((b*x^2 + a)^(5/2)*a^4) + 80/21*A*b^2*x
/((b*x^2 + a)^(7/2)*a^3) - B/((b*x^2 + a)^(7/2)*a*x) + 10/3*A*b/((b*x^2 +
a)^(7/2)*a^2*x) - 1/3*A/((b*x^2 + a)^(7/2)*a*x^3)
```

## Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.28

$$\int \frac{A+Bx^2+Cx^4+Dx^6+Fx^8}{x^4(a+bx^2)^{9/2}}\,dx = \frac{\left(\left(x^2\left(\frac{(6Fa^{16}b^4+8Da^{15}b^5+48Ca^{14}b^6-279Ba^{13}b^7+790Aa^{12}b^8)x^2}{a^{18}b^3}+\frac{7(3Fa^{17}b^3+4\cdots}{\cdots}\right.\right.\right.}{\cdots}$$
$$+\frac{2\left(3\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4Ba\sqrt{b}-12\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4Ab^{\frac{3}{2}}-6\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2Ba^2\sqrt{b}+30\left(\sqrt{b}\cdots\right.\right.}{3\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^3a^5}$$

input
```
integrate((F*x^8+D*x^6+C*x^4+B*x^2+A)/x^4/(b*x^2+a)^(9/2),x, algorithm="gi
ac")
```

output
```
1/105*((x^2*((6*F*a^16*b^4 + 8*D*a^15*b^5 + 48*C*a^14*b^6 - 279*B*a^13*b^7
 + 790*A*a^12*b^8)*x^2/(a^18*b^3) + 7*(3*F*a^17*b^3 + 4*D*a^16*b^4 + 24*C*
a^15*b^5 - 132*B*a^14*b^6 + 365*A*a^13*b^7)/(a^18*b^3)) + 35*(D*a^17*b^3 +
 6*C*a^16*b^4 - 30*B*a^15*b^5 + 80*A*a^14*b^6)/(a^18*b^3))*x^2 + 105*(C*a^
17*b^3 - 4*B*a^16*b^4 + 10*A*a^15*b^5)/(a^18*b^3))*x/(b*x^2 + a)^(7/2) + 2
/3*(3*(sqrt(b)*x - sqrt(b*x^2 + a))^4*B*a*sqrt(b) - 12*(sqrt(b)*x - sqrt(b
*x^2 + a))^4*A*b^(3/2) - 6*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a^2*sqrt(b) +
 30*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A*a*b^(3/2) + 3*B*a^3*sqrt(b) - 14*A*a
^2*b^(3/2))/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^3*a^5)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4+Dx^6+Fx^8}{x^4(a+bx^2)^{9/2}}\,dx = \int \frac{A+B\,x^2+C\,x^4+F\,x^8+x^6\,D}{x^4\,(b\,x^2+a)^{9/2}}\,dx$$

input
```
int((A + B*x^2 + C*x^4 + F*x^8 + x^6*D)/(x^4*(a + b*x^2)^(9/2)),x)
```

output
```
int((A + B*x^2 + C*x^4 + F*x^8 + x^6*D)/(x^4*(a + b*x^2)^(9/2)), x)
```

## Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 599, normalized size of antiderivative = 2.09

$$\int \frac{A+Bx^2+Cx^4+Dx^6+Fx^8}{x^4\left(a+bx^2\right)^{9/2}}\,dx = \frac{105\sqrt{b\,x^2+a}\,a^4b^3c\,x^4+35\sqrt{b\,x^2+a}\,a^4b^3d\,x^6+21\sqrt{b\,x^2+a}\,a^4b^3f}{}$$

input
```
int((F*x^8+D*x^6+C*x^4+B*x^2+A)/x^4/(b*x^2+a)^(9/2),x)
```

output
```
( - 35*sqrt(a + b*x**2)*a**5*b**3 + 245*sqrt(a + b*x**2)*a**4*b**4*x**2 +
105*sqrt(a + b*x**2)*a**4*b**3*c*x**4 + 35*sqrt(a + b*x**2)*a**4*b**3*d*x*
*6 + 21*sqrt(a + b*x**2)*a**4*b**3*f*x**8 + 1960*sqrt(a + b*x**2)*a**3*b**
5*x**4 + 210*sqrt(a + b*x**2)*a**3*b**4*c*x**6 + 28*sqrt(a + b*x**2)*a**3*
b**4*d*x**8 + 6*sqrt(a + b*x**2)*a**3*b**4*f*x**10 + 3920*sqrt(a + b*x**2)
*a**2*b**6*x**6 + 168*sqrt(a + b*x**2)*a**2*b**5*c*x**8 + 8*sqrt(a + b*x**
2)*a**2*b**5*d*x**10 + 3136*sqrt(a + b*x**2)*a*b**7*x**8 + 48*sqrt(a + b*x
**2)*a*b**6*c*x**10 + 896*sqrt(a + b*x**2)*b**8*x**10 - 6*sqrt(b)*a**7*f*x
**3 - 8*sqrt(b)*a**6*b*d*x**3 - 24*sqrt(b)*a**6*b*f*x**5 - 48*sqrt(b)*a**5
*b**2*c*x**3 - 32*sqrt(b)*a**5*b**2*d*x**5 - 36*sqrt(b)*a**5*b**2*f*x**7 -
 896*sqrt(b)*a**4*b**4*x**3 - 192*sqrt(b)*a**4*b**3*c*x**5 - 48*sqrt(b)*a*
*4*b**3*d*x**7 - 24*sqrt(b)*a**4*b**3*f*x**9 - 3584*sqrt(b)*a**3*b**5*x**5
 - 288*sqrt(b)*a**3*b**4*c*x**7 - 32*sqrt(b)*a**3*b**4*d*x**9 - 6*sqrt(b)*
a**3*b**4*f*x**11 - 5376*sqrt(b)*a**2*b**6*x**7 - 192*sqrt(b)*a**2*b**5*c*
x**9 - 8*sqrt(b)*a**2*b**5*d*x**11 - 3584*sqrt(b)*a*b**7*x**9 - 48*sqrt(b)
*a*b**6*c*x**11 - 896*sqrt(b)*b**8*x**11)/(105*a**5*b**3*x**3*(a**4 + 4*a*
*3*b*x**2 + 6*a**2*b**2*x**4 + 4*a*b**3*x**6 + b**4*x**8))
```

# 3.270 $\int \frac{A+Bx^2+Cx^4+Dx^6+Fx^8}{x^6\left(a+bx^2\right)^{9/2}}\,dx$



## Optimal result

Integrand size = 37, antiderivative size = 324

$$\begin{aligned}\int \frac{A+Bx^2+Cx^4+Dx^6+Fx^8}{x^6\left(a+bx^2\right)^{9/2}}\,dx = &-\frac{\left(Ab^4-a(b^3B-ab^2C+a^2bD-a^3F)\right)x}{7a^4b\left(a+bx^2\right)^{7/2}}\\&-\frac{\left(27Ab^4-a(20b^3B-13ab^2C+6a^2bD+a^3F)\right)x}{35a^5b\left(a+bx^2\right)^{5/2}}\\&-\frac{\left(318Ab^4-a(185b^3B-87ab^2C+24a^2bD+4a^3F)\right)x}{105a^6b\left(a+bx^2\right)^{3/2}}\\&-\frac{\left(1686Ab^4-a(790b^3B-279ab^2C+48a^2bD+8a^3F)\right)x}{105a^7b\sqrt{a+bx^2}}-\frac{A\sqrt{a+bx^2}}{5a^5x^5}\\&+\frac{(24Ab-5aB)\sqrt{a+bx^2}}{15a^6x^3}-\frac{\left(198Ab^2-70abB+15a^2C\right)\sqrt{a+bx^2}}{15a^7x}\end{aligned}$$

output
```
-1/7*(A*b^4-a*(B*b^3-C*a*b^2+D*a^2*b-F*a^3))*x/a^4/b/(b*x^2+a)^(7/2)-1/35*
(27*A*b^4-a*(20*B*b^3-13*C*a*b^2+6*D*a^2*b+F*a^3))*x/a^5/b/(b*x^2+a)^(5/2)
-1/105*(318*A*b^4-a*(185*B*b^3-87*C*a*b^2+24*D*a^2*b+4*F*a^3))*x/a^6/b/(b*
x^2+a)^(3/2)-1/105*(1686*A*b^4-a*(790*B*b^3-279*C*a*b^2+48*D*a^2*b+8*F*a^3
))*x/a^7/b/(b*x^2+a)^(1/2)-1/5*A*(b*x^2+a)^(1/2)/a^5/x^5+1/15*(24*A*b-5*B*
a)*(b*x^2+a)^(1/2)/a^6/x^3-1/15*(198*A*b^2-70*B*a*b+15*C*a^2)*(b*x^2+a)^(1
/2)/a^7/x
```

## Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.68

$$\int \frac{A+Bx^2+Cx^4+Dx^6+Fx^8}{x^6 (a+bx^2)^{9/2}} dx = \frac{-3072Ab^6x^{12}+256ab^5x^{10}(-42A+5Bx^2)-128a^2b^4x^8(105A-35$$

input
```
Integrate[(A + B*x^2 + C*x^4 + D*x^6 + F*x^8)/(x^6*(a + b*x^2)^(9/2)),x]
```

output
```
(-3072*A*b^6*x^12 + 256*a*b^5*x^10*(-42*A + 5*B*x^2) - 128*a^2*b^4*x^8*(10
5*A - 35*B*x^2 + 3*C*x^4) + 16*a^3*b^3*x^6*(-420*A + 350*B*x^2 - 84*C*x^4
+ 3*D*x^6) + 8*a^4*b^2*x^4*(-105*A + 350*B*x^2 - 210*C*x^4 + 21*D*x^6 + F*
x^8) + 14*a^5*b*x^2*(6*A + 25*B*x^2 - 60*C*x^4 + 15*D*x^6 + 2*F*x^8) - 7*a
^6*(3*A + 5*x^2*(B + 3*C*x^2 - 3*D*x^4 - F*x^6)))/(105*a^7*x^5*(a + b*x^2)
^(7/2))
```

## Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 284, normalized size of antiderivative = 0.88, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.243$, Rules used = {2334, 2334, 27, 2089, 1588, 298, 209, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4+Dx^6+Fx^8}{x^6 (a+bx^2)^{9/2}} dx$$

$$\Big\downarrow \; 2334$$

$$-\frac{\int \frac{12Ab-5a\left(Fx^6+Dx^4+Cx^2+B\right)}{x^4\left(bx^2+a\right)^{9/2}}dx}{5a}-\frac{A}{5ax^5\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \; 2334$$

$$-\frac{-\frac{\int \frac{5\left(2b(12Ab-5aB)+3\left(a^2Fx^4+a^2Dx^2+a^2C\right)\right)}{x^2\left(bx^2+a\right)^{9/2}}dx}{3a}-\frac{12Ab-5aB}{3ax^3\left(a+bx^2\right)^{7/2}}}{5a}-\frac{A}{5ax^5\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \; 27$$

$$-\frac{-\frac{5\int \frac{2b(12Ab-5aB)+3\left(a^2Fx^4+a^2Dx^2+a^2C\right)}{x^2\left(bx^2+a\right)^{9/2}}dx}{3a}-\frac{12Ab-5aB}{3ax^3\left(a+bx^2\right)^{7/2}}}{5a}-\frac{A}{5ax^5\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \; 2089$$

$$-\frac{-\frac{5\int \frac{3a^2Fx^4+3a^2Dx^2+24Ab^2-a(10bB-3aC)}{x^2\left(bx^2+a\right)^{9/2}}dx}{3a}-\frac{12Ab-5aB}{3ax^3\left(a+bx^2\right)^{7/2}}}{5a}-\frac{A}{5ax^5\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \; 1588$$

$$-\frac{-\frac{5\left(-\frac{\int \frac{-3Fx^2a^3-\left(3Da^2-24bCa+80b^2B\right)a+192Ab^3}{\left(bx^2+a\right)^{9/2}}dx}{a}-\frac{24Ab^2-a(10bB-3aC)}{ax\left(a+bx^2\right)^{7/2}}\right)}{3a}-\frac{12Ab-5aB}{3ax^3\left(a+bx^2\right)^{7/2}}}{5a}-\frac{A}{5ax^5\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \; 298$$

$$-\frac{-\frac{5\left(-\frac{\frac{3\left(384Ab^4-a\left(a^3F+6a^2bD-48ab^2C+160b^3B\right)\right)\int \frac{1}{\left(bx^2+a\right)^{7/2}}dx}{7ab}+\frac{x\left(192Ab^4-a\left(-3a^3F+3a^2bD-24ab^2C+80b^3B\right)\right)}{7ab\left(a+bx^2\right)^{7/2}}}{a}-\frac{24Ab^2-a(10bB-3aC)}{ax\left(a+bx^2\right)^{7/2}}\right)}{3a}-}{5a}$$

$$\frac{A}{5ax^5\left(a+bx^2\right)^{7/2}}$$

$\downarrow$ 209

$$-\frac{-\frac{5\left(-\frac{\frac{3\left(384Ab^4-a\left(a^3F+6a^2bD-48ab^2C+160b^3B\right)\right)\left(\frac{4\int\frac{1}{\left(bx^2+a\right)^{5/2}}dx}{5a}+\frac{x}{5a\left(a+bx^2\right)^{5/2}}\right)}{7ab}+\frac{x\left(192Ab^4-a\left(-3a^3F+3a^2bD-24ab^2C+80b^3B\right)\right)}{7ab\left(a+bx^2\right)^{7/2}}}{a}-\frac{24A}{\cdots}\cdots\right.}{3a}}{5a}$$

$$\frac{A}{5ax^5\left(a+bx^2\right)^{7/2}}$$

$\downarrow$ 209

$$-\frac{-\frac{5\left(-\frac{\frac{3\left(384Ab^4-a\left(a^3F+6a^2bD-48ab^2C+160b^3B\right)\right)\left(\frac{4\left(\frac{2\int\frac{1}{\left(bx^2+a\right)^{3/2}}dx}{3a}+\frac{x}{3a\left(a+bx^2\right)^{3/2}}\right)}{5a}+\frac{x}{5a\left(a+bx^2\right)^{5/2}}\right)}{7ab}+\frac{x\left(192Ab^4-a\left(-3a^3F+3a^2bD-24\cdots\right.\right.}{7ab\left(a+bx^2\right)^{7/2}}}{a}\cdots\right.}{3a}}{5a}$$

$$\frac{A}{5ax^5\left(a+bx^2\right)^{7/2}}$$

$\downarrow$ 208

$$-\frac{-\frac{5\left(-\frac{\frac{x\left(192Ab^4-a\left(-3a^3F+3a^2bD-24ab^2C+80b^3B\right)\right)}{7ab\left(a+bx^2\right)^{7/2}}+\frac{3\left(\frac{4\left(\frac{2x}{3a^2\sqrt{a+bx^2}}+\frac{x}{3a\left(a+bx^2\right)^{3/2}}\right)}{5a}+\frac{x}{5a\left(a+bx^2\right)^{5/2}}\right)\left(384Ab^4-a\left(a^3F+6a^2bD-48ab^2C+1\right.\right.}{7ab}}{a}\right.}{3a}}{5a}$$

$$\frac{A}{5ax^5\left(a+bx^2\right)^{7/2}}$$

input
```
Int[(A + B*x^2 + C*x^4 + D*x^6 + F*x^8)/(x^6*(a + b*x^2)^(9/2)),x]
```

output
```
-1/5*A/(a*x^5*(a + b*x^2)^(7/2)) - (-1/3*(12*A*b - 5*a*B)/(a*x^3*(a + b*x^
2)^(7/2)) - (5*(-((24*A*b^2 - a*(10*b*B - 3*a*C))/(a*x*(a + b*x^2)^(7/2)))
 - (((192*A*b^4 - a*(80*b^3*B - 24*a*b^2*C + 3*a^2*b*D - 3*a^3*F))*x)/(7*a
*b*(a + b*x^2)^(7/2)) + (3*(384*A*b^4 - a*(160*b^3*B - 48*a*b^2*C + 6*a^2*
b*D + a^3*F))*(x/(5*a*(a + b*x^2)^(5/2)) + (4*(x/(3*a*(a + b*x^2)^(3/2)) +
 (2*x)/(3*a^2*Sqrt[a + b*x^2])))/(5*a)))/(7*a*b))/a))/(3*a))/(5*a)
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 1588
```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Simp[1/(d*f
^2*(m + 1))   Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x
) - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

rule 2089
```
Int[(u_)^(p_.)*((f_.)*(x_))^(m_.)*(z_)^(q_.), x_Symbol] :> Int[(f*x)^m*Expa
ndToSum[z, x]^q*ExpandToSum[u, x]^p, x] /; FreeQ[{f, m, p, q}, x] && Binomi
alQ[z, x] && TrinomialQ[u, x] &&  !(BinomialMatchQ[z, x] && TrinomialMatchQ
[u, x])
```

rule 2334
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coef
f[Pq, x, 0], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[A*
x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))   Int[
x^(m + 2)*(a + b*x^2)^p*(a*(m + 1)*Q - A*b*(m + 2*(p + 1) + 1)), x], x]] /;
 FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && IntegerQ[m/2] && ILtQ[(m + 1)/2 + p,
 0] && LtQ[m + Expon[Pq, x] + 2*p + 1, 0]
```

## Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.63

| method | result |
|---|---|
| pseudoelliptic | $-\frac{\left(A+\frac{5}{3}x^2B-\frac{5}{3}F\,x^8-5Dx^6+5C\,x^4\right)a^6-4x^2b\left(\frac{1}{3}F\,x^8+\frac{5}{2}Dx^6-10C\,x^4+\frac{25}{6}x^2B+A\right)a^5+40\left(-\frac{1}{105}F\,x^8-\frac{1}{5}Dx^6+2C\,x^4-\frac{1}{3}\right.}{}$ |
| gosper | $-\frac{3072A\,b^6x^{12}-1280Ba\,b^5x^{12}+384C\,a^2b^4x^{12}-48Da^3b^3x^{12}-8F\,a^4b^2x^{12}+10752Aa\,b^5x^{10}-4480B\,a^2b^4x^{10}+1344C\,a^3b^3}{}$ |
| trager | $-\frac{3072A\,b^6x^{12}-1280Ba\,b^5x^{12}+384C\,a^2b^4x^{12}-48Da^3b^3x^{12}-8F\,a^4b^2x^{12}+10752Aa\,b^5x^{10}-4480B\,a^2b^4x^{10}+1344C\,a^3b^3}{}$ |
| orering | $-\frac{3072A\,b^6x^{12}-1280Ba\,b^5x^{12}+384C\,a^2b^4x^{12}-48Da^3b^3x^{12}-8F\,a^4b^2x^{12}+10752Aa\,b^5x^{10}-4480B\,a^2b^4x^{10}+1344C\,a^3b^3}{}$ |
| default | $D\left(\frac{x}{7a(b\,x^2+a)^{\frac{7}{2}}}+\frac{\frac{6x}{35a\left(b\,x^2+a\right)^{\frac{5}{2}}}+\frac{6\left(\frac{4x}{15a\left(b\,x^2+a\right)^{\frac{3}{2}}}+\frac{8x}{15a^2\sqrt{b\,x^2+a}}\right)}{7a}}{a}\right)+A\left(-\frac{1}{5a\,x^5(b\,x^2+a)^{\frac{7}{2}}}-\frac{12b\left(-\frac{}{3a\,x^3}\right.}{}\right.$ |

input
```
int((F*x^8+D*x^6+C*x^4+B*x^2+A)/x^6/(b*x^2+a)^(9/2),x,method=_RETURNVERBOS
E)
```

output
```
-1/5/(b*x^2+a)^(7/2)*((A+5/3*x^2*B-5/3*F*x^8-5*D*x^6+5*C*x^4)*a^6-4*x^2*b*
(1/3*F*x^8+5/2*D*x^6-10*C*x^4+25/6*x^2*B+A)*a^5+40*(-1/105*F*x^8-1/5*D*x^6
+2*C*x^4-10/3*x^2*B+A)*x^4*b^2*a^4+320*(-1/140*D*x^6+1/5*C*x^4-5/6*x^2*B+A
)*x^6*b^3*a^3+640*x^8*b^4*(1/35*C*x^4-1/3*x^2*B+A)*a^2+512*(-5/42*x^2*B+A)
*x^10*b^5*a+1024/7*A*b^6*x^12)/x^5/a^7
```

## Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 289, normalized size of antiderivative = 0.89

$$\int \frac{A + Bx^2 + Cx^4 + Dx^6 + Fx^8}{x^6 (a + bx^2)^{9/2}} \, dx = \frac{(8 (Fa^4b^2 + 6 Da^3b^3 - 48 Ca^2b^4 + 160 Bab^5 - 384 Ab^6)x^{12} + 28 (F$$

input
```
integrate((F*x^8+D*x^6+C*x^4+B*x^2+A)/x^6/(b*x^2+a)^(9/2),x, algorithm="fr
icas")
```

output
```
1/105*(8*(F*a^4*b^2 + 6*D*a^3*b^3 - 48*C*a^2*b^4 + 160*B*a*b^5 - 384*A*b^6
)*x^12 + 28*(F*a^5*b + 6*D*a^4*b^2 - 48*C*a^3*b^3 + 160*B*a^2*b^4 - 384*A*
a*b^5)*x^10 + 35*(F*a^6 + 6*D*a^5*b - 48*C*a^4*b^2 + 160*B*a^3*b^3 - 384*A
*a^2*b^4)*x^8 - 21*A*a^6 + 35*(3*D*a^6 - 24*C*a^5*b + 80*B*a^4*b^2 - 192*A
*a^3*b^3)*x^6 - 35*(3*C*a^6 - 10*B*a^5*b + 24*A*a^4*b^2)*x^4 - 7*(5*B*a^6
- 12*A*a^5*b)*x^2)*sqrt(b*x^2 + a)/(a^7*b^4*x^13 + 4*a^8*b^3*x^11 + 6*a^9*
b^2*x^9 + 4*a^10*b*x^7 + a^11*x^5)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{A + Bx^2 + Cx^4 + Dx^6 + Fx^8}{x^6 (a + bx^2)^{9/2}} \, dx = \text{Timed out}$$

input
```
integrate((F*x**8+D*x**6+C*x**4+B*x**2+A)/x**6/(b*x**2+a)**(9/2),x)
```

output
```
Timed out
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 471, normalized size of antiderivative = 1.45

$$\int \frac{A+Bx^2+Cx^4+Dx^6+Fx^8}{x^6(a+bx^2)^{9/2}}\,dx = \frac{16\,Dx}{35\,\sqrt{bx^2+a}a^4}+\frac{8\,Dx}{35\,(bx^2+a)^{\frac{3}{2}}a^3}+\frac{6\,Dx}{35\,(bx^2+a)^{\frac{5}{2}}a^2}+\frac{Dx}{7\,(bx^2+a)^{\frac{7}{2}}a}-\frac{Fx}{7\,(bx^2+a)^{\frac{7}{2}}b}+\frac{8\,Fx}{105\,\sqrt{bx^2+a}a^3b}+\frac{4\,Fx}{105\,(bx^2+a)^{\frac{3}{2}}a^2b}+\frac{Fx}{35\,(bx^2+a)^{\frac{5}{2}}ab}-\frac{128\,Cbx}{35\,\sqrt{bx^2+a}a^5}-\frac{64\,Cbx}{35\,(bx^2+a)^{\frac{3}{2}}a^4}-\frac{48\,Cbx}{35\,(bx^2+a)^{\frac{5}{2}}a^3}-\frac{8\,Cbx}{7\,(bx^2+a)^{\frac{7}{2}}a^2}+\frac{256\,Bb^2x}{21\,\sqrt{bx^2+a}a^6}+\frac{128\,Bb^2x}{21\,(bx^2+a)^{\frac{3}{2}}a^5}+\frac{32\,Bb^2x}{7\,(bx^2+a)^{\frac{5}{2}}a^4}+\frac{80\,Bb^2x}{21\,(bx^2+a)^{\frac{7}{2}}a^3}-\frac{1024\,Ab^3x}{35\,\sqrt{bx^2+a}a^7}-\frac{512\,Ab^3x}{35\,(bx^2+a)^{\frac{3}{2}}a^6}-\frac{384\,Ab^3x}{35\,(bx^2+a)^{\frac{5}{2}}a^5}-\frac{64\,Ab^3x}{7\,(bx^2+a)^{\frac{7}{2}}a^4}-\frac{C}{(bx^2+a)^{\frac{7}{2}}ax}+\frac{10\,Bb}{3\,(bx^2+a)^{\frac{7}{2}}a^2x}-\frac{8\,Ab^2}{(bx^2+a)^{\frac{7}{2}}a^3x}-\frac{B}{3\,(bx^2+a)^{\frac{7}{2}}ax^3}+\frac{4\,Ab}{5\,(bx^2+a)^{\frac{7}{2}}a^2x^3}-\frac{A}{5\,(bx^2+a)^{\frac{7}{2}}ax^5}$$

input
```
integrate((F*x^8+D*x^6+C*x^4+B*x^2+A)/x^6/(b*x^2+a)^(9/2),x, algorithm="ma
xima")
```

output
```
16/35*D*x/(sqrt(b*x^2 + a)*a^4) + 8/35*D*x/((b*x^2 + a)^(3/2)*a^3) + 6/35*
D*x/((b*x^2 + a)^(5/2)*a^2) + 1/7*D*x/((b*x^2 + a)^(7/2)*a) - 1/7*F*x/((b*
x^2 + a)^(7/2)*b) + 8/105*F*x/(sqrt(b*x^2 + a)*a^3*b) + 4/105*F*x/((b*x^2
+ a)^(3/2)*a^2*b) + 1/35*F*x/((b*x^2 + a)^(5/2)*a*b) - 128/35*C*b*x/(sqrt(
b*x^2 + a)*a^5) - 64/35*C*b*x/((b*x^2 + a)^(3/2)*a^4) - 48/35*C*b*x/((b*x^
2 + a)^(5/2)*a^3) - 8/7*C*b*x/((b*x^2 + a)^(7/2)*a^2) + 256/21*B*b^2*x/(sq
rt(b*x^2 + a)*a^6) + 128/21*B*b^2*x/((b*x^2 + a)^(3/2)*a^5) + 32/7*B*b^2*x
/((b*x^2 + a)^(5/2)*a^4) + 80/21*B*b^2*x/((b*x^2 + a)^(7/2)*a^3) - 1024/35
*A*b^3*x/(sqrt(b*x^2 + a)*a^7) - 512/35*A*b^3*x/((b*x^2 + a)^(3/2)*a^6) -
384/35*A*b^3*x/((b*x^2 + a)^(5/2)*a^5) - 64/7*A*b^3*x/((b*x^2 + a)^(7/2)*a
^4) - C/((b*x^2 + a)^(7/2)*a*x) + 10/3*B*b/((b*x^2 + a)^(7/2)*a^2*x) - 8*A
*b^2/((b*x^2 + a)^(7/2)*a^3*x) - 1/3*B/((b*x^2 + a)^(7/2)*a*x^3) + 4/5*A*b
/((b*x^2 + a)^(7/2)*a^2*x^3) - 1/5*A/((b*x^2 + a)^(7/2)*a*x^5)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 618 vs. $2(295) = 590$.

Time = 0.15 (sec) , antiderivative size = 618, normalized size of antiderivative = 1.91

$$\int \frac{A + Bx^2 + Cx^4 + Dx^6 + Fx^8}{x^6 (a + bx^2)^{9/2}} \, dx = \frac{\left(\left(x^2 \left(\frac{(8\,Fa^{19}b^5 + 48\,Da^{18}b^6 - 279\,Ca^{17}b^7 + 790\,Ba^{16}b^8 - 1686\,Aa^{15}b^9)x^2}{a^{22}b^3} + \frac{7\,(4\,Fa^{20}b^4}{}\right.\right.\right.}{}$$
$$+ \frac{2\left(15\left(\sqrt{b}x - \sqrt{bx^2 + a}\right)^8 Ca^2\sqrt{b} - 60\left(\sqrt{b}x - \sqrt{bx^2 + a}\right)^8 Bab^{\frac{3}{2}} + 150\left(\sqrt{b}x - \sqrt{bx^2 + a}\right)^8 Ab^{\frac{5}{2}} - 60\left(\right.\right.}{}$$

input
```
integrate((F*x^8+D*x^6+C*x^4+B*x^2+A)/x^6/(b*x^2+a)^(9/2),x, algorithm="gi
ac")
```

output
```
1/105*((x^2*((8*F*a^19*b^5 + 48*D*a^18*b^6 - 279*C*a^17*b^7 + 790*B*a^16*b
^8 - 1686*A*a^15*b^9)*x^2/(a^22*b^3) + 7*(4*F*a^20*b^4 + 24*D*a^19*b^5 - 1
32*C*a^18*b^6 + 365*B*a^17*b^7 - 768*A*a^16*b^8)/(a^22*b^3)) + 35*(F*a^21*
b^3 + 6*D*a^20*b^4 - 30*C*a^19*b^5 + 80*B*a^18*b^6 - 165*A*a^17*b^7)/(a^22
*b^3))*x^2 + 105*(D*a^21*b^3 - 4*C*a^20*b^4 + 10*B*a^19*b^5 - 20*A*a^18*b^
6)/(a^22*b^3))*x/(b*x^2 + a)^(7/2) + 2/15*(15*(sqrt(b)*x - sqrt(b*x^2 + a)
)^8*C*a^2*sqrt(b) - 60*(sqrt(b)*x - sqrt(b*x^2 + a))^8*B*a*b^(3/2) + 150*(
sqrt(b)*x - sqrt(b*x^2 + a))^8*A*b^(5/2) - 60*(sqrt(b)*x - sqrt(b*x^2 + a)
)^6*C*a^3*sqrt(b) + 270*(sqrt(b)*x - sqrt(b*x^2 + a))^6*B*a^2*b^(3/2) - 72
0*(sqrt(b)*x - sqrt(b*x^2 + a))^6*A*a*b^(5/2) + 90*(sqrt(b)*x - sqrt(b*x^2
 + a))^4*C*a^4*sqrt(b) - 430*(sqrt(b)*x - sqrt(b*x^2 + a))^4*B*a^3*b^(3/2)
 + 1260*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*a^2*b^(5/2) - 60*(sqrt(b)*x - sq
rt(b*x^2 + a))^2*C*a^5*sqrt(b) + 290*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a^4
*b^(3/2) - 840*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A*a^3*b^(5/2) + 15*C*a^6*sq
rt(b) - 70*B*a^5*b^(3/2) + 198*A*a^4*b^(5/2))/(((sqrt(b)*x - sqrt(b*x^2 +
a))^2 - a)^5*a^6)
```

## Mupad [B] (verification not implemented)

Time = 2.47 (sec) , antiderivative size = 478, normalized size of antiderivative = 1.48

$$\int \frac{A+Bx^2+Cx^4+Dx^6+Fx^8}{x^6 (a+bx^2)^{9/2}}\,dx = \frac{\frac{61\,A\,b}{35\,a^3}+\frac{78\,A\,b^2\,x^2}{35\,a^4}}{x^3\,{\left(b\,x^2+a\right)}^{5/2}}+\frac{\frac{128\,B\,b}{21\,a^5}+\frac{256\,B\,b^2\,x^2}{21\,a^6}}{x\,\sqrt{b\,x^2+a}} + \frac{x\,D}{{\left(b\,x^2+a\right)}^{9/2}}-\frac{\frac{B}{3\,a^2}+\frac{19\,B\,b\,x^2}{21\,a^3}}{x^3\,{\left(b\,x^2+a\right)}^{5/2}}-\frac{\frac{C}{a^4}+\frac{128\,C\,b\,x^2}{35\,a^5}}{x\,\sqrt{b\,x^2+a}}-\frac{\frac{512\,A\,b^2}{35\,a^6}+\frac{1024\,A\,b^3\,x^2}{35\,a^7}}{x\,\sqrt{b\,x^2+a}} - \frac{F\,x}{7\,b\,{\left(b\,x^2+a\right)}^{7/2}}-\frac{A\,\sqrt{b\,x^2+a}}{5\,a^5\,x^5}+\frac{18\,b^2\,x^5\,D}{5\,a^2\,{\left(b\,x^2+a\right)}^{9/2}}+\frac{72\,b^3\,x^7\,D}{35\,a^3\,{\left(b\,x^2+a\right)}^{9/2}} + \frac{16\,b^4\,x^9\,D}{35\,a^4\,{\left(b\,x^2+a\right)}^{9/2}}-\frac{A\,b}{7\,a^2\,x^3\,{\left(b\,x^2+a\right)}^{7/2}}-\frac{32\,B\,b}{21\,a^4\,x\,{\left(b\,x^2+a\right)}^{3/2}} + \frac{B\,b^2\,x}{7\,a^3\,{\left(b\,x^2+a\right)}^{7/2}}+\frac{8\,F\,x}{105\,a^3\,b\,\sqrt{b\,x^2+a}}+\frac{4\,F\,x}{105\,a^2\,b\,{\left(b\,x^2+a\right)}^{3/2}} + \frac{F\,x}{35\,a\,b\,{\left(b\,x^2+a\right)}^{5/2}}+\frac{27\,A\,b^2}{7\,a^5\,x\,{\left(b\,x^2+a\right)}^{3/2}}+\frac{3\,b\,x^3\,D}{a\,{\left(b\,x^2+a\right)}^{9/2}} - \frac{29\,C\,b\,x}{35\,a^4\,{\left(b\,x^2+a\right)}^{3/2}}-\frac{13\,C\,b\,x}{35\,a^3\,{\left(b\,x^2+a\right)}^{5/2}}-\frac{C\,b\,x}{7\,a^2\,{\left(b\,x^2+a\right)}^{7/2}}$$

input
```
int((A + B*x^2 + C*x^4 + F*x^8 + x^6*D)/(x^6*(a + b*x^2)^(9/2)),x)
```

output
```
((61*A*b)/(35*a^3) + (78*A*b^2*x^2)/(35*a^4))/(x^3*(a + b*x^2)^(5/2)) + ((
128*B*b)/(21*a^5) + (256*B*b^2*x^2)/(21*a^6))/(x*(a + b*x^2)^(1/2)) + (x*D
)/(a + b*x^2)^(9/2) - (B/(3*a^2) + (19*B*b*x^2)/(21*a^3))/(x^3*(a + b*x^2)
^(5/2)) - (C/a^4 + (128*C*b*x^2)/(35*a^5))/(x*(a + b*x^2)^(1/2)) - ((512*A
*b^2)/(35*a^6) + (1024*A*b^3*x^2)/(35*a^7))/(x*(a + b*x^2)^(1/2)) - (F*x)/
(7*b*(a + b*x^2)^(7/2)) - (A*(a + b*x^2)^(1/2))/(5*a^5*x^5) + (18*b^2*x^5*
D)/(5*a^2*(a + b*x^2)^(9/2)) + (72*b^3*x^7*D)/(35*a^3*(a + b*x^2)^(9/2)) +
 (16*b^4*x^9*D)/(35*a^4*(a + b*x^2)^(9/2)) - (A*b)/(7*a^2*x^3*(a + b*x^2)^
(7/2)) - (32*B*b)/(21*a^4*x*(a + b*x^2)^(3/2)) + (B*b^2*x)/(7*a^3*(a + b*x
^2)^(7/2)) + (8*F*x)/(105*a^3*b*(a + b*x^2)^(1/2)) + (4*F*x)/(105*a^2*b*(a
 + b*x^2)^(3/2)) + (F*x)/(35*a*b*(a + b*x^2)^(5/2)) + (27*A*b^2)/(7*a^5*x*
(a + b*x^2)^(3/2)) + (3*b*x^3*D)/(a*(a + b*x^2)^(9/2)) - (29*C*b*x)/(35*a^
4*(a + b*x^2)^(3/2)) - (13*C*b*x)/(35*a^3*(a + b*x^2)^(5/2)) - (C*b*x)/(7*
a^2*(a + b*x^2)^(7/2))
```

## Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 678, normalized size of antiderivative = 2.09

$$\int \frac{A+Bx^2+Cx^4+Dx^6+Fx^8}{x^6\left(a+bx^2\right)^{9/2}}\,dx = \frac{-21\sqrt{b\,x^2+a}\,a^6b^2-1792\sqrt{b\,x^2+a}\,b^8x^{12}+1792\sqrt{b}\,b^8x^{13}-105\sqrt{b}$$

input
```
int((F*x^8+D*x^6+C*x^4+B*x^2+A)/x^6/(b*x^2+a)^(9/2),x)
```

output
```
( - 21*sqrt(a + b*x**2)*a**6*b**2 + 49*sqrt(a + b*x**2)*a**5*b**3*x**2 - 1
05*sqrt(a + b*x**2)*a**5*b**2*c*x**4 + 105*sqrt(a + b*x**2)*a**5*b**2*d*x*
*6 + 35*sqrt(a + b*x**2)*a**5*b**2*f*x**8 - 490*sqrt(a + b*x**2)*a**4*b**4
*x**4 - 840*sqrt(a + b*x**2)*a**4*b**3*c*x**6 + 210*sqrt(a + b*x**2)*a**4*
b**3*d*x**8 + 28*sqrt(a + b*x**2)*a**4*b**3*f*x**10 - 3920*sqrt(a + b*x**2
)*a**3*b**5*x**6 - 1680*sqrt(a + b*x**2)*a**3*b**4*c*x**8 + 168*sqrt(a + b
*x**2)*a**3*b**4*d*x**10 + 8*sqrt(a + b*x**2)*a**3*b**4*f*x**12 - 7840*sqr
t(a + b*x**2)*a**2*b**6*x**8 - 1344*sqrt(a + b*x**2)*a**2*b**5*c*x**10 + 4
8*sqrt(a + b*x**2)*a**2*b**5*d*x**12 - 6272*sqrt(a + b*x**2)*a*b**7*x**10
- 384*sqrt(a + b*x**2)*a*b**6*c*x**12 - 1792*sqrt(a + b*x**2)*b**8*x**12 -
 8*sqrt(b)*a**7*f*x**5 - 48*sqrt(b)*a**6*b*d*x**5 - 32*sqrt(b)*a**6*b*f*x*
*7 + 384*sqrt(b)*a**5*b**2*c*x**5 - 192*sqrt(b)*a**5*b**2*d*x**7 - 48*sqrt
(b)*a**5*b**2*f*x**9 + 1792*sqrt(b)*a**4*b**4*x**5 + 1536*sqrt(b)*a**4*b**
3*c*x**7 - 288*sqrt(b)*a**4*b**3*d*x**9 - 32*sqrt(b)*a**4*b**3*f*x**11 + 7
168*sqrt(b)*a**3*b**5*x**7 + 2304*sqrt(b)*a**3*b**4*c*x**9 - 192*sqrt(b)*a
**3*b**4*d*x**11 - 8*sqrt(b)*a**3*b**4*f*x**13 + 10752*sqrt(b)*a**2*b**6*x
**9 + 1536*sqrt(b)*a**2*b**5*c*x**11 - 48*sqrt(b)*a**2*b**5*d*x**13 + 7168
*sqrt(b)*a*b**7*x**11 + 384*sqrt(b)*a*b**6*c*x**13 + 1792*sqrt(b)*b**8*x**
13)/(105*a**6*b**2*x**5*(a**4 + 4*a**3*b*x**2 + 6*a**2*b**2*x**4 + 4*a*b**
3*x**6 + b**4*x**8))
```

# CHAPTER 4

## APPENDIX



## 4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

### Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

## Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                     return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                 convert(leaf_count_result,string),"$ vs. $2(",
                                 convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

## Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1]))  #max(3,ExpnType(op(1,expn)),Expn
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
else:
    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =""
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType(


#print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

## SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
   r"""
   Return the tree size of this expression.
   """
   #print("Enter tree_size, expr is ",expr)

   if expr not in SR:
      # deal with lists, tuples, vectors
      return 1 + sum(tree_size(a) for a in expr)
   expr = SR(expr)
   x, aa = expr.operator(), expr.operands()
   if x is None:
      return 1
   else:
      return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
   if expr.operator() == operator.pow:   #isinstance(expr,Pow):
      if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
         if debug: print ("expr is sqrt")
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   #debug=False
   m = func.name() in ['exp','log','ln',
         'sin','cos','tan','cot','sec','csc',
         'arcsin','arccos','arctan','arccot','arcsec','arccsc',
         'sinh','cosh','tanh','coth','sech','csch',
         'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
      'arctan2','floor','abs'
      ]
   if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral',
          'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
          'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
```

```
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

## 4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file